# *Teachers' Perceptions of Whole Number Acquisition and Associated Pedagogy in the Foundation Phase*

by

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*DECLARATION*

I the undersigned declare that this work has not been submitted in whole, or in part, for the award of any degree. It is my own work. Each significant contribution to, and quotation in, this dissertation from the work, or works, of other people has been attributed, and has been cited and referenced.

Signature: Signed by candidate    2 September 2004

# *ABSTRACT*


This dissertation is interested in the systematic analysis of teachers' comments regarding the ways in which young children learn number concepts. The motivation for this study is based on my experience with primary school teachers through the University of Cape Town's Schools Development Unit and my background in Foundation Phase mathematics learning and teaching.

The purpose of this investigation is to analyse the discourse that teachers use when describing their knowledge and understanding of number acquisition. The investigation used semi-structured interviews as a primary source of data. The analysis of teachers' comments focused on transcripts from interviews conducted with six Foundation Phase teachers from rural and peri-urban schools in the Cape West Coast Winelands region of South Africa.

Each interview consisted of constructed virtual vignettes and associated open-ended questions. The vignettes were designed and selected to elicit subjective responses from teachers regarding their understanding about learning and classroom practice pertaining to number. The questions were designed to act as prompts to encourage teachers to reflect on, and express their opinions about, certain aspects of learning and teaching number in different contexts.

In order to investigate teachers' comments, Steffe's theoretical framework for number development was used. This framework provided the reference for comparing teachers' comments and a language for interpretation. The analysis in this study presents the didactic discourse and highlights teachers' perceptions of how children learn number in two parts. The first part relates to cognitive development of number and the second part to general didactic themes.

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# 1 *INTRODUCTION*

## *1.1 Motivation for this Study*

The motivation for this study was generated by two interests. The first was through my experience as a staff member at the University of Cape Town's (UCT) Schools Development Unit (SDU). This work involved mathematics professional development with primary school teachers through courses and school based support programmes. The second motivation for my interest was stimulated by current curriculum developments in South Africa, at a time of political and policy reform.

This investigation sets out to explore how Foundation Phase teachers describe their knowledge and understanding of the ways in which children learn early number concepts. It hopes to present this in terms of accepted theories on learning and number acquisition. In my experience informal arithmetic learning and teaching methods are common in Foundation Phase classrooms and can be seen in classes when children are encouraged to use their own intuitive methods to solve problems. I have observed that many teachers find it difficult to identify different strategies used by learners and therefore cannot build upon and extend these. It is apparent that teachers generally accept the idea that allowing children to solve problems and to use their own methods in solving problems is sufficient; in that the maths will 'emerge' and that children will *discover* how to work with bigger numbers and more difficult problems.

Teachers use various methods to teach mathematics, which include the use of concrete apparatus, visual prompts and 'real situations' - as a means of enhancing and developing concepts and understandings - as well as references (curricula and text books) for supporting content knowledge and sequencing of concepts. What the current South African Mathematics curriculum does not elucidate, and what in my experience many teachers do not manage effectively, is the process of modelling concepts and operations which enable children to bridge the gap between informal, context-bound work and formal standardised operations. It became evident to me while working with primary school teachers on courses and in their classrooms that their discourse around mathematical content knowledge and their understanding of how children learn mathematics was not particularly extensive or detailed.

In order to establish what teachers think and their perceptions about learning mathematics I wanted to hear what they said about mathematics learning and teaching. This interest led to the systematic study of data generated from semi-structured interviews with six Foundation Phase teachers. The interviews were based on vignettes that captured hypothetical learner and classroom episodes, and highlighted specific number activities with the intention of eliciting teachers' personal responses to the following question:

***How do teachers understand the ways in which children learn number?***

The intention of this investigation was to explore the teachers' role in learning and teaching mathematics at the Foundation Phase and the nature of the current South African curriculum. The study specifically focused on Foundation Phase teachers in schools in the Cape West Coast Winelands region of South Africa. The main focus of this study was on teachers' perceptions of learners' understandings and knowledge of number development and how these are encouraged and developed. The content of the current South African curriculum document, the Revised National Curriculum Statements (RNCS), explicitly presents mathematics content for each grade. This study set out to establish whether professional discourse about mathematics content and pedagogy was evident in what teachers say.

The particular interest of this investigation was on Foundation Phase teachers' knowledge and perceptions of how children learn number. The motivation for the focus of this dissertation was that the content of the Number strand in the (RNCS) Mathematics Learning Area at the Foundation Phase forms the basis of all learning in mathematics and receives the greatest learning and teaching attention and time allocation.

*1.2 Organisation of Dissertation*

Chapter 1 describes the motivation and interest of this dissertation and presents the research focus and research question within the current South African Foundation Phase learning and teaching context.

Chapter 2 describes the recent developments in mathematics education in South Africa and reviews literature on international curriculum reform. Among other things, this chapter also contains a discussion on the teachers' role and beliefs about teaching mathematics and how this influences learning and understanding. Also included in this chapter is an overview of five reformed approaches and a description of the main features of these approaches and the connected programmes. This study is interested in these programmes because they describe how learners' mathematical understanding can develop and what can be expected in the learning process. Furthermore, they operate from a perspective that teachers' knowledge and understanding of children's mathematical thinking is a critical factor in supporting children's mathematics learning.

Chapter 3 provides the analytic framework which is used to analyse teachers' responses to the interviews. The chapter begins with a discussion of 'number' and presents a literature review that locates the discussion within a body of research specific to number development. This chapter presents the theories of von Glasersfeld and Steffe which describe cognitive development of number and the processes involved in this learning. Finally this chapter give an overview of the literature related to children's learning of the base ten (decimal) number system, various forms of representation, children's solution strategies and contextual and calculation problems.

Chapter 4 describes the sample and setting which locates this study in a particular context. The research design describes the method used to collect data. A detailed discussion about the design and selection of the interview instruments is presented which describes the process of constructing the vignettes and open ended questions. Of particular interest in this chapter is the motivation and rationale behind the selection of each vignette and related questions. The development of the model and the mode of analysis is presented, the limitations of the study and the validity and reliability of the instruments are discussed.

Chapter 5 focuses on the systematic analysis of the interview data according to cognitive stages of development and didactic issues related to teaching and learning number in the early grades of the foundation phase, as described in chapter 3. The analysis is intended to illustrate didactic discourse and highlight teachers' perceptions of how children learn number. The analysis of the data interprets teachers' responses to the vignettes in the interviews and is presented in two parts. The first part categorises teachers' comments in terms of what they say in relation to Steffe's theoretical perspective on the development of number. The second part categorises those comments which relate to didactic issues not included in Steffe's framework.

Chapter 6 concludes the dissertation and captures the main features of the investigation and related discussion. This chapter also presents limitations of this study and recommendations for possible future research related to the topic of investigation.

# 2 RECENT DEVELOPMENTS IN MATHEMATICS EDUCATION

## *2.1 Introduction*

The intention of this chapter is to describe the recent developments in mathematics education in South Africa and review literature on international curriculum reform. This chapter also discusses the role of the teacher in learning and teaching number, and how beliefs about teaching mathematics influence learning and understanding. Included in this chapter is an overview of five reformed approaches to teaching number and a description of the main features of these approaches and their connected programmes. They include Cognitive Guided Instruction, Problem Centred Primary Mathematics, Realistic Mathematics Education, Mathematics Recovery and Counting Me in Too.

This chapter contextualises this dissertation within the field of current mathematics curriculum reform, and provides examples of initiatives which have attempted to highlight professional knowledge and understanding in the area of early number development and teaching. It also provides background to the discussions in this study regarding appropriate mathematics content knowledge for teachers at the Foundation Phase. In addition to this it describes programmes which have been designed to bridge the gap between explicit curriculum content and to support the learning and teaching of this content in the classroom.

## *2.2 Recent Developments in Mathematics Education in South Africa*

The developments within the educational debates and curriculum innovations in South Africa have been many and varied over the past 10 years. In 1993 there was no single, core national curriculum and according to Chisholm (2001) there were 1 400 registered syllabi.

> *Whatever the interpretation of its role, there can be no doubt about the centrality of both the formal and hidden curriculum in shaping social and personal values. In the curriculum these values are embedded in its organization, and the particular selection of content and skills developed. How do these values find a place: How have they changed over time? (Chisholm, 2001)*

I would like to briefly outline the developments over the past few years which led to the creation of a common curriculum that attempted to address issues of inequality, principles of multiculturalism and notions of citizenship in South Africa. The desire for one, integrated system of education and training and a single national qualification framework helped shape the reform proposal (Young, 2001).

Political debates included critical discussions about education and 1990 saw a major turning point and significant participation by interested parties. There were several social movements and political 'actors' in the reform movement and they began to stake their curriculum positions vehemently (Jansen, 1999). The National Education Co-ordinating Committee (NECC) initiated the National Education Policy Investigation (NEPI) to develop educational 'policy options'. The Curriculum Group, a key research group of the NECC, developed an initial document on which the 1998 curriculum policy is based.

The private sector too had interests in curriculum developments. The Private Sector Education Council (PRISEC) and the Educational Policy and Systems Change Unit (EDUPOL), applied constant pressure for a more vocational and entrepreneurial focus to education rather than the traditional academic focus. The Non-governmental Organisation (NGO) sector also made significant contributions to the debate. Language based on 'competencies' emerged, specifically through the adult education curricula developed by the Independent Examinations Board (IEB) and the beginnings of an outcome-based curriculum gradually emerged.

On 24 March 1997 the South African Minister of Education (Bengu) launched Curriculum C2005, the South African version of Outcomes Based Education (OBE). It presented curriculum in a completely new way and represented a break from the heavy content-based curriculum of the past. It offered teachers a degree of freedom in control over the selection, organisation, pacing and timing of knowledge to be transmitted and received, and firmly located the notion that 'all learners can succeed' as central to education.

One feature of this curriculum framework was that the formal presentation of content was removed and teachers were expected to design their own curriculum. This particular aspect of C2005 'disabled' many teachers who already had difficulty selecting and teaching appropriate content. By emphasising broad competencies it attempted to define the needs of society through its outcomes, rather than subject content. The emphasis shifted from traditional subject-based knowledge and skills, to broader overarching outcomes.

There was much excitement and public debate around C2005; however this initial enthusiasm was soon replaced with concern about *what* to teach and *how* to teach. Concerns about (curriculum) content, sequencing of activities and conceptual development are expressed in these few words from a Grade 1 teacher:

*What is it; the maths that I must teach?*

Teachers began reacting to the pressure of raised expectations, complicated planning procedures and lack of resources. Implementation was not always carefully thought through, piloted or resourced and enormous stresses and strains were consequently placed on already over-burdened principals and teachers in widely divergent educational contexts (Report of the Review Committee, 2000). With little to guide teachers and 'outcomes that are often novel and capable of multiple interpretation' (Malcolm, 1999, chap. 4) they were expected to set standards for assessment and plan teaching programmes which would benefit the individual needs of their learners. Without guiding content on which to base planning and teaching, the decision about the scope and sequence of concepts was left to schools.

Resistance to C2005 continued amidst calls to abandon outcomes-based education and many classroom teachers introduced what they called 'back to basics' content. Criticism of the new curriculum increased and pressure to address the concerns led to the establishment of the Review Committee to comment amongst other things on the structure of the C2005 document.

The Review Committee (2000) recommended that terminology used in the curriculum should be clearly defined, accessible and that clarity should be a fundamental feature of all curriculum statements. The Revised National Curriculum Statement (RNCS) emerged as the most recent South African curriculum development and was implemented in the Foundation Phase in January 2004. The RNCS attempts to detail the knowledge and skills that should be taught and learnt, when these should be introduced and how they should be sequenced for progression of content within and across grades from Grade R to Grade 9.

### *2.2.1 Mathematics Content in the Revised National Curriculum Statement*

Although it is fair to argue that teachers are currently faced with considerable problems, including overcrowded classes, under-resourcing and extensive curriculum changes and policy demands, it would seem that many of the problems encountered in the area of mathematics learning and teaching are not new. In attempting to answer the questions of why this is so and what can be done to improve the situation, it is important to interrogate the curriculum and teacher knowledge and understanding of mathematics learning and pedagogy.

The RNCS Mathematics Learning Area begins with a general introduction, in the form of a Learning Area Statement, the intention of which it to give a definition and describe the purpose and unique features of the Learning Area, while giving a clear indication of the learning goals and expected minimum requirements of learners in each grade. The Learning Area Statement gives an overview of the Mathematics Learning Area and states the unique features and scope of the Learning Area in 5 Learning Outcomes (LOs): LO 1: Numbers, Operations and Relationships; LO 2: Patterns, Functions and Algebra; LO 3: Space and Shape; LO 4: Measurement; LO 5: Data Handling.

Learning Outcome 1: Numbers, Operations and Relationships (APPENDIX 1) states that 'the learner should be able to recognise, describe and represent numbers and their relationships, and to count, estimate, calculate and check with competence and confidence in solving problems' (Revised National Curriculum Statement, 2002). The focus of Learning Outcome 1 is on building learners' number sense and developing an understanding of what different kinds of numbers mean and how they relate to one another. The relative size of numbers and how they can be thought about in different ways and represented in various forms are important concepts, which need to be developed during this phase. Essential to the development of number sense is an understanding of the effect of operating with numbers, knowledge of basic number facts, the use of efficient and accurate methods for calculation and estimation and a range of strategies for estimating and checking results (Revised National Curriculum Statements, 2002).

*2.3 International Reformed Approaches to Early Number*

There can be no doubt that the first three years of school have a profound effect on the rest of the child's mathematical education (Wright, 1993). Wright adds that it is during this period that children experience success or failure, interest or boredom and challenges or frustration. The Foundation Phase has the greatest responsibility for the success of these Curriculum Statements as it is at this level that the foundations are laid for success in higher school mathematics and further education.

A good knowledge of how children form number concepts in their early years and the many different ways they use and understand number in different contexts is invaluable when applied in learning and teaching situations. If an awareness of the processes and stages involved in mathematical cognitive development informs instruction and takes into account children's informal intuitive knowledge, as well as the transition to formal abstract mathematics then 'teaching mathematics, will assuredly result in the learning of mathematics' (Neuman, 1987).

*2.3.1 Focus on Informal Knowledge*

Children come into Grade R (from around 5 years of age) with a wide range of numerical learning experiences and conceptual understandings and it is this knowledge that forms the basis for what comes later. Children's prior knowledge and informal problem solving abilities with regard to dealing with numbers and quantities in the pre-school period has to be taken into account in order to create the right perspective and frame of reference when designing learning programmes (Van den Heuvel-Panhuizen, 2001, pg 22).

Research over the past 30 years indicates that young children's informal mathematics is based on their active construction and is encouraged as well as constrained by social and cultural factors (Säljö in Becker and Selter, 1996). This informal mathematics includes abilities and knowledge acquired out of school as well as concepts developed at school, without actually being taught (Becker and Selter, 1996).

However, in many instances children are taught methods and procedures which do not take into account the mathematical experience, knowledge and understanding of concepts and skills which they bring to the school (Graeber, 1999).

*2.3.2 Developments in Conceptualisation of Mathematics as a School Subject*

Significant shifts and developments within the conceptualisation of mathematics as a domain, of mathematical competence as a goal for instruction and the ways in which this competence should be acquired through schooling are evident (Verschaffel and De Corte, 1996). Several countries have produced reform documents and reform movements, such as the Association of Teachers of Mathematics (1997) and writers such as Gattegno and Colburn. Current curriculum changes in South Africa reflect these international trends and developments (Galant, 1997).

The common philosophy of mathematics education presented in these documents represents a significant paradigm shift from teaching isolated mathematics content topics to learning the principles of mathematics with understanding. The general principles and characteristics underpinning this reform have significantly influenced curriculum innovations, in particular the whole number domain, which includes topics such as number concepts and number sense, the meaning of arithmetic operations, mastery of basic arithmetic facts, mental and written computation and word problems as application of numerical and arithmetical knowledge and skills (Verschaffel and De Corte, 1996).

Verschaffel and De Corte (1996) state that all (elementary mathematics curriculum) documents should include the development of number concepts and numeration skills and Van den Heuvel-Panhuizen's (1999 and 2001) definition of whole number arithmetic includes number knowledge, number sense, mental arithmetic, estimation and formal calculation. These world wide developments have led to significant changes in the content and practice of mathematics education and are reflected in the curriculum and policy documents of many countries. The reformed conceptualisation of mathematics learning and teaching has also led to extensive classroom based research.

*2.3.3 The Teachers' Role in Teaching Mathematics*

The rethinking of mathematics curriculum, pedagogy and epistemology of the past years has placed the responsibility for success of emerging curriculum and policies on the school and classroom teachers. These responsibilities include: an emphasis on group work, problem solving and reasoning; communication and discourse around mathematics topics (new elementary school content); integration within and across learning areas and an increase in the use of technology and manipulatives (Smith, Senger, 1999 and Clements, 1999).

Carpenter, Fennema and Franke (1996) found that teachers had a great deal of intuitive knowledge about children's mathematical thinking but that this was fragmented and generally did not contribute to the decisions they made. The studies they conducted found that when teachers understand the development of children's mathematical thinking this leads to fundamental changes in their beliefs and practices and that these changes are reflected in their learners' learning.

Becker and Selter (1996) present a perspective on elementary (foundation phase) mathematics education that stresses the active role of the learner, and identify important conditions for change that mathematics teachers need to address. They discuss examples of research projects, which focus on the learning and teaching process and instructional theories linked to mathematical reform and policy innovations.

These recent trends in mathematical education reflect the view that learning and teaching are inherently social and cultural activities (Vygotsky 1978, Gauvain and Cole, 1992, Cobb 1997, Gravemeijer, Cobb, Bowers and Whitenack, 2000) and that children actively construct their mathematical understandings while participating in evolving communal activities. The notion that knowledge is not passively received but actively built up and that cognition occurs gradually, involves the simultaneous modification and reinterpretation of past, present and projected actions and understandings (Davis, 1996) underpin attempts to transform these policies. The belief that individual interpretation of and abstractions from experience are shaped by the learning context and in particular social mediation have significantly influenced teacher training and educational policy.

It is in the social context of the classroom and the participation in activities that the condition for the possibility of mathematics learning exists (Von Glasersfeld, 1989 and Cobb, 1997). In view of this position the teachers' role needs to be seen as proactive in supporting the development of individual children's learning and the planning of sequences of activities that take the learning environment into account.

If individual learning is placed in the social classroom context the challenge for the teacher is to support the collective learning of children, as shared mathematical meaning emerges and the teacher and children negotiate interpretations and solutions to various problems. This approach to instruction can be contrasted with the traditional (transmission) view of direct instruction in which mathematics symbols and rules are seen as fixed and are explained and demonstrated so that children may memorise the facts (Gravemeijer, Cobb, Bowers and Whitenack, 2000).

### *2.3.4 Teachers' Beliefs about Teaching Mathematics*

The significance of teachers' beliefs, attitudes and conceptions as a factor in the process of learning and teaching is well documented in the field of mathematics education and has a strong influence on the development of teaching styles. Practicing teachers hold a wide range of different experiences, opinions, beliefs and conceptions of learning and teaching. Although the literature does not report specifically on teachers' perceptions and understanding of how children learn, a number of studies have found that teachers' beliefs about mathematics and mathematics teaching play a significant role in shaping their instructional practice and consequently influence their learners' attitudes, interests and achievements.

> *For many educated persons, mathematics is a discipline characterised by accurate results and infallible procedures, whose basic elements are arithmetic operations, algebraic procedures and geometric terms and theorems. For them knowing mathematics is equivalent to being skilful in performing procedures and being able to identify the basic concepts of the discipline*
> *(Thompson, 1992)*

Practicing teachers frame their beliefs in terms of their views on mathematics. With reference to the above quote, instruction should involve teaching and repeated practice of concepts and procedures and the emphasis should be on the manipulation of symbols rather than understanding. Thomson (1992) refers to the need to involve children in the process of doing mathematics and not just explaining its content. In order to understand the composition and structure of belief systems and conceptions, Thompson (1992) also suggests that it is necessary to identify teachers' underlying thoughts and decisions. Distinguishing between and defining concepts related to beliefs and knowledge is difficult. According to Thompson one feature which pertains to teachers' beliefs is that these can be held with varying degrees of conviction. Another feature is that they are not consensual, i.e. others might think differently.

Philippou and Christou (1995), Gellert (1999) and Thompson (1992) discuss beliefs in terms of conceptions that are based on subjective feelings and knowledge, which is objective and subject to validity and is associated with truth or certainty. Knowing changes over time as thinking and theories develop and what at one time was considered as knowledge may be judged later as a belief. Inversely, what was once held as a belief may become knowledge if it is supported by new theories. In education it is common for alternative theories to coexist, even when aspects of one theory contradict another which may help explain the difficulty of distinguishing between teachers' knowledge and beliefs (Thompson, 1992).

Nicol, Smith Senger and Ernest (1998) discuss the nature of teachers' beliefs and prior experiences with mathematics, and discuss the many preconceptions about learning and teaching mathematics and the practical understandings that inform content and pedagogy. Teachers' knowledge of mathematics forms part of their content or schemes, however it is not this knowledge alone that determines the differences between teachers' practice. The way teachers conceive the nature and meaning of mathematics and their mental models of learning and teaching influence the methods they use in the classroom (Thompson, 1992).

Ernest (1988) presents several views of mathematics. Firstly as a continually expanding field of human invention in which patterns are generated and then distilled into knowledge as part of a process of enquiry. Secondly, that mathematics is a static but unified body of knowledge, of interconnected structures and truths, bound together by filaments of logic and meaning which must be discovered, not created. Thirdly, that mathematics is an accumulation of facts, unrelated rules and skills which need to be learnt in order to achieve some external end.

According to Thompson (1992) it is possible to hold aspects of more than one view of mathematics and even to hold conflicting beliefs. These conflicting beliefs about mathematics learning and teaching are not necessarily held in isolated clusters but are integrated into conceptual systems. Skemp (1978) distinguishes between two types of mathematical knowledge: instrumental and relational. Instrumental knowledge is set out according to procedures specified for a particular task and corresponds with Ernest's (1988) third view of mathematics, whereas relational knowledge involves the formation of conceptual structures that enable the construction of several plans for performing a given task (Skemp, 1978).

> *It is difficult to conceive of teaching models without some underlying theory of how students learn mathematics, even if the theory is incomplete and implicit. Although it seems reasonable to expect a model of mathematics teaching to be somehow related to or derived form some model of mathematics learning, for most teachers it is unlikely that the two have been developed and articulated into a coherent theory of instructions. Rather, conceptions of learning and teaching tend to be eclectic collections of beliefs and views that appear to be more the result of their years of experience in the classroom than any type of formal or informal study.*
> *(Thompson, 1992)*

Thompson (1992) found that the majority of teachers possess mathematics knowledge that relates to mathematics of the school curriculum, for example arithmetic, algebra, and geometry. In addition to this Thompson (1992) noted that while individual teachers' conceptions of mathematics were consistent a diversity of conceptions existed across groups of teachers.

Gellert (1999) refers to different kinds of professional knowledge including content, pedagogical and curriculum knowledge. Mathematical content is the '*what*' to teach, pedagogical knowledge is the practical knowledge of representations, analogies, illustrations, examples and demonstrations, and curriculum knowledge pertains to materials, resources and media. Gellert (1999) and Carpenter et al (1988) indicate that the source of pre-teachers beliefs about mathematics learning and teaching stem from their own relationships and experiences with school and tertiary mathematics training and how this has informed their personal learning and teaching practices. Teachers' formative experiences in mathematics significantly vary and influence what they do in the classroom and this practice reflects what they think and feel about the subject (Philippou and Christou, 1995 and Cooney, 1999). Van den Heuvel-Panhuizen (1999) notes the discrepancy between teachers' ideas about teaching methods and what they report actually happens in their classrooms. Consistency between teachers' professed beliefs about the nature of mathematics and their instructional practice varies. Analysing teachers' conceptions should not be based only on their professed views but should also consider the classroom setting, practices and the relationship between the teacher's expressed views and their instructional practise (Thompson, 1992).

Zevenbergen (1997) reports that teachers' practices reflect behaviourist notions of learning and teaching and that the discourse that dominated their interviews was based on the Piagettian perspective that children's ability to learn mathematics is intrinsically related to an innate individual ability which has the capacity to alter or increase with maturity.

### *2.4. Five Reformed Programmes as a Reference for this Study*

Internationally there have been several projects and many groups of researchers who have worked in the field of learning and teaching early number. I would like to briefly discuss five large reformed programmes which focus on learning and teaching early number concepts.

The programmes include:

- Cognitive Guided Instruction
- Problem Centred Primary Mathematics Programme
- Realistic Mathematics Education
- Mathematics Recovery
- Count Me In Too

This study is interested in these programmes because each operates from a perspective that teachers' knowledge and understanding of children's mathematical thinking is a critical factor in supporting children's mathematics learning. The programmes focus on helping teachers understand the mathematics of specific content domains and children's mathematical thinking in those domains. These five programmes focus on teachers' knowledge and understanding of mathematics and children's thinking and are grounded in practice.

While none of the projects discussed in this chapter prescribe instructional programmes, each presents mathematical understanding and content in a progressive and structured way. A common feature of each programme is the emphasis on constructing relationships between various forms of representation. They also highlight different learner strategies and provide opportunities for children to connect new concepts and procedures to existing knowledge, and extend and apply this knowledge and understanding to new situations. Children are not presented with procedures to follow, however links between symbolic procedures involving basic number operations and properties of these operations are explicitly drawn.

### *2.4.1. Cognitive Guided Instruction*

Cognitively Guided Instruction (CGI) is a research-based professional development programme based at the University of Wisconsin-Madison, and was directed by Elizabeth Fennema and Thomas Carpenter. The research which informed the development of this programme found that although teachers have a great deal of intuitive knowledge of children's mathematical thinking, this knowledge was fragmented and did not play a significant role in decisions about instructional planning. CGI was designed to give teachers a coherent basis for making instructional decisions based on the explicit knowledge about the development of children's mathematical thinking (Carpenter, Fennema, Peterson, Chiang, Loef, 1988 and Carpenter, Fennema, Franke, 1996).

> *Two major assumptions underlie CGI. One is that instruction should develop understanding by stressing relationships between skills and problem solving with problem solving serving as the main organising focus of instruction. The second assumption is that instruction should build upon student's existing knowledge*
> *(Carpenter, Fennema, Peterson, Chiang, Loef, 1988)*

CGI highlights children's typical understanding and development of specific mathematical concept domains. The programme's main intention is to develop understanding of children's thinking as a basis for teachers to develop their knowledge more broadly. CGI provides a framework for teachers' knowledge of mathematics and curriculum and a context in which they can interpret and apply general pedagogic knowledge. The project engages teachers in learning about the development of children's mathematical thinking by building on their existing knowledge and focuses on how children intuitively solve different kinds of word problems by modelling the actions and relations described in them (Carpenter, Fennema, Peterson, Chiang, Loef, 1988; Fuson, 1992 and Carpenter, Fennema, Franke, 1996).

The research base which informs the model of children's thinking in CGI is drawn from the work of Carpenter et al. (1988), and is in line with the work of Fuson (1992), Greer (1992) and Verschaffel and De Corte (1996). Studies conducted found that by the end of kindergarten children could solve problems by modelling the action or relations described in the problem, which many teachers and curriculum developers believed too difficult for young children. The results supported the idea that young children can invent informal strategies to solve a variety of problems if they are given the opportunity to do so.

The studies conducted as part of the CGI professional development programme (Carpenter et al. 1988) showed that teachers' informal knowledge of children's thinking can be built upon and that they can identify between problem types and have some idea of the modelling and counting strategies used by children. These studies also showed that teachers' knowledge of their children's thinking was related to their learners' achievement.

The results of Carpenter et al's (1988) studies confirmed that developing an understanding of children's thinking provides a basis for change and that change occurs gradually while teachers apply their knowledge and instruction to their own teaching situations. The studies also demonstrate that much can be accomplished by teachers and children when children's thinking becomes the primary focus for instruction. This research captures the essence of recent mathematics reform recommendations and documents, and highlights how much children are capable of learning in supporting learning environments where instruction builds on what they already know – if children are presented with appropriate and meaningful tasks in order for them to actively construct their own knowledge.

In CGI teachers use an explicit framework for interpreting children's strategies for solving problems. They construct their own materials and practices based on their developing understanding and knowledge of children's thinking (Carpenter, Fennema. Peterson, Chiang, Loef, 1988). The analysis (Carpenter et al, 1988) of whole number concepts and operations is based on children's informal solutions to word problems which represent addition, subtraction, multiplication and division situations as well as the development of place value concepts. The progression of understanding more abstract symbolic procedures in this framework is characterised by progressive abstractions of children's attempts to model action and relations as represented in problems (Carpenter, Fennema, Peterson, Chiang, Loef, 1988). The critical analysis of basic word problems developed by Carpenter et al. (1988) is based on how children think about and solve a range of problems. The problems are classified based on the types of action of relations described in the problems.

### *2.4.2. Problem Centred Primary Mathematics Programme*

A large scale instructional research programme based at the University of Stellenbosch in South Africa is the Problem Centred Primary Mathematics Programme (PCM). This programme was initiated by Piet Human, Alwyn Olivier and Hanlie Murray in 1988 and began in eight Stellenbosch primary schools. By 1993 the programme had been implemented in more than a thousand main stream schools and fifty 'special schools' by five provincial Departments of Education. This wide implementation of the programme was not supported by Human, Olivier and Murray who maintained that a more organic development of the programme with lots of support for participating teachers was more beneficial to sustaining the programme (Murray, Human and Olivier 1989).

Although the project is based on similar assumptions as those discussed above in the CGI programmes, PCM has a slightly different approach to assisting teacher reflection on children's thinking and how this can be used to guide instruction. PCM's intention was to provide information on a long term school intervention with the purpose of identifying unforeseen gains in learners' conceptual development and understanding of the properties of numbers and operations and on the development of teacher's knowledge and understanding (Murray, Human and Olivier 1989).

PCM base their programme on the perspective that children construct their own mathematical knowledge regardless of how they are taught and set out to establish individual and social procedures to monitor and improve the nature and quality of children's constructions (Murray, Human and Olivier 1989). PCM regard problem solving as central to learning mathematical knowledge and skills and distinguish between learning to solve problems and learning through problem solving. They emphasise that well planned and designed number concept activities should include a range of well sequenced problems types and effective discussion and that this will encourage and promote building of patterns and relationships.

Murray, Human and Olivier (1989) suggest that classroom and peer interactions and discussions during the process of solving problems have a greater influence on young children's mathematical constructions than the facilitatory skills of the teacher, and therefore the mathematical knowledge and tasks presented should determine how children are grouped and what materials are made available to them.

The PCM's views on suitable problems and learning sequences were originally based on the work of Dutch researchers Pierre-Marie and Dina van Hiele and Treffers (cited in Murray, Human and Olivier, 1989). The principle of *progressive schematisation* (Murray, Human and Olivier, 1989) suggests that: firstly, concept formation develops across several levels. Secondly that it is best facilitated when children first work on tasks involving new concepts using their own intuitive, informal solution methods. Thirdly that as children's solution strategies become more numerically aware and advanced the teacher should gradually introduce necessary mathematical content and terminology and increasingly abstract representations and operations.

### *2.4.3. Realistic Mathematics Education*

Dutch research and development work in the field of mathematics education has been conducted over the past three decades and is based on the ideas of Freudenthal and the influence of the Wiskobas project (see Van den Heuvel-Panhuizen, 1999). This project was initiated by Wijdeveld and Goffree and Treffers was one of the leading researchers. The accumulation and revision of the ideas contained in this work are referred to as Realistic Mathematics Education (RME).

The core principles of RME are that mathematics should be developed from children's reality and experience and be relevant to their lives and contexts. Van den Heuvel-Panhuizen (1999) says the term *realistic* is not strictly connected to the notion of real world situation, but to the emphasis that RME puts on learning and teaching involving the use of problems that children can imagine or realise. She adds that RME is characterised by six principles based on a view of mathematics as a subject, how children learn mathematics and how mathematics should be taught.

Van den Heuvel-Panhuizen (1999), Becker and Selter (1996) note that these principles include:

- Mathematization is the idea that mathematics is best learnt when children actively participate in the educational process in which they develop various mathematical tools and insights. RME does not support the idea that structured curricula and content can be transferred directly to children;
- Teaching involves the use of problems that include word problems, fantasy stories and formal mathematics as these provide appropriate ideal contexts in which children can apply and use their mathematical understanding and tools - as long as they are real in children's minds;
- Children progress through various levels of understanding and learning is a long-term process. The ability to reflect on activities and thinking is the precondition for progressing from one level to another;
- Teaching involves the ability to create a learning environment in which the constructing process can emerge. It also involves guiding children from their informal context-bound solutions, through the creation of various short cuts and schematizations to formal mathematics where they acquire insights into underlying principles and broad relationships;
- Learning is embedded in a social-cultural context and therefore children should be encouraged to share their strategies and inventions. Reflection can be elicited by interaction in small groups or whole class discussions;

- Mathematics learning and teaching is not split into distinctive learning strands. Rich context problems (which can be solved on different levels of understanding) provide children with opportunities to apply a broad range of mathematical tools and understandings. This 'inter-twinement' principle provides coherency within and across the curriculum;

Van den Heuvel-Panhuizen (1999) reports that the Netherlands does not prescribe curriculum syllabi, textbooks and examinations for primary schools but that schools and teachers make decisions for themselves. In general they follow the same curriculum, and mathematical topics taught in primary schools do not differ. The question posed regarding this situation is 'what determines this curriculum?

The main determinants for curriculum in Dutch mathematics education in primary schools are textbooks and a document called 'Proeve'. This supports textbook writers by describing various domains within which mathematics is to be taught and the key goals suggested by the government. In 1993 the Dutch Ministry of Education produced a list of 23 'Key Goals' which Van den Heuvel-Panhuizen (1999) says did not mention some significant mathematical topics, for example problem solving, probability and logic. Nevertheless, the list confirmed and validated some important changes in the Dutch curriculum, such as paying more attention to mental arithmetic and estimation.

These 23 Key Goals were not sufficient to support improvements in classroom practice and discussion about whether to provide more detailed goals for each grade, or descriptions which could support teaching, were on going. In 1997 the government approached the Freudenthal Institute with the task of providing guidance on improving classroom practice in the early grades. This endeavour resulted in the TAL project. In 1998 the first description of a longitudinal learning-teaching trajectory (covering all primary school grades) on whole number arithmetic was published. This trajectory presents arithmetic in a broad sense and gives an overview of how number knowledge, number sense, mental arithmetic, estimation and algorithms are related to each other in a longitudinal and cross sectional way. It presents the 'stepping stones' that students will pass on their way to reaching the core goals at the end of primary school. The learning-teaching trajectory provides a narrative overview or map of how children's mathematical understanding can develop and what can be expected in the learning process. Central to the learning-teaching trajectory is the notion that the learning processes are not regarded as an hierarchical process of small steps, nor are they considered as a check list to determine children's progress according to pre-stated mathematics content (Van den Heuvel-Panhuizen, 1999).

Not only does the learning-teaching trajectory describe landmarks in children's learning that can be recognised in their number development, but it also portrays the key activities in teaching that lead to these landmarks. Another feature is its inherent coherence, based on the distinction of levels. The description of these levels makes it clear that what is learned at one stage is understood and performed on a higher level in a following stage. A recurring pattern of interlocking transitions to a higher level forms the connecting element in the trajectory (Van den Heuvel-Panhuizen, 2003a).

### *2.4.4. Mathematics Recovery*

The programme known as Mathematics Recovery was initially developed during 1992 -1995 in the north coast region of New South Wales in Australia. Development of the program was funded by a grant from the Australian Research Council and also by contributions from participating school systems. This initial development involved 20 teachers in 18 schools. Mathematics Recovery is a research-based programme and was developed as a systemic response to the problem of chronic failure in school mathematics (Wright, 1994b). Since 1995, the program has been implemented widely in the United States and England and also other countries. Wright (2000) reports that Mathematics Recovery draws on research into the learning and teaching of early mathematics and includes: the work of Steffe and his colleagues (e.g. Steffe, Cobb and Von Glasersfeld, 1988) whose focus was on conceptual change and also research by Wright (e.g. 1991 and 1994a). Wright (1994) argues that there are significant differences in the levels of children's mathematical knowledge when they begin school and that these differences tend to be maintained as they progress through school.

Participating teachers undertake a specialised, post-initial training, professional development course, which focuses on Mathematics Recovery theory and practice (Wright, 2000). The programme involves the identification of the lowest attainers in the first grade level (South African Grade R) and includes intensive, individualised teaching using a range of instructional procedures and materials (Wright, Martland and Stafford, 2000; Wright, Martland, Stanger and Stafford, 2002). Teaching is informed by on-going assessment of children's progress through careful observation and review. Initial assessment involves determining the child's level on each of five aspects of early number knowledge (Wright et al., 2000).

Assessment in the programme is concerned with determining the relative sophistication of: children's knowledge and strategies for adding and subtracting; their competence with number words; forward and backwards number sequences; and competence with numeral identification (Wright et al., 2000). Individual teaching frameworks are developed from assessment profiles of each child and an understanding of each child's strategies enables teachers to adapt and select appropriate activities and instructional settings, which are closely attuned to the child's current knowledge.

*2.4.5. Count Me In Too*

In the initial years the focus of Mathematics Recovery was on individualised teaching of low attainers (Wright, 2000), but over time teachers who participated in the programme applied the knowledge and techniques in their whole class teaching situations. In New South Wales in 1996, the theory and methods of Mathematics Recovery were adapted as the basis for a school-based, systemic initiative of the government school system – the New South Wales Department of Education and Training. This initiative, known as Count Me In Too (CMIT), focuses on assessment and teaching of mathematics in early years.

CMIT was established to support teachers' developing a better understanding of children's mathematical strategies and how these progress from less sophisticated to more sophisticated strategies (Steward, Wright and Gould, 1998). A Learning Framework in Number was developed for the assessment and teaching of number. The framework also provided descriptions of children's strategies or knowledge, and tasks and guidelines for assessment (Wright 1998). CMIT has been implemented across all 40 administrative districts in the New South Wales government school system. The CMIT Learning Framework in Number was the major influence on the

Early Numeracy Research Project, which was established in 1999 by the Victorian Department of Education. This initiative aimed to improve numeracy learning in response to Australian community and political attention on measuring the outcomes of children's learning and an emphasis on the early years of schooling as a crucial period for providing a positive start to literacy and numeracy learning (Clarke and Sullivan, 2000).

The reform projects and their connected programmes for professional development discussed in this section each give an overview of the long term development of the learning process for number and an understanding of how the various aspects of numerosity develop. They also describe learner performance typical of the different stages of number development. My interest in these projects is from the perspective that teachers' knowledge and understanding of this development is a critical factor in supporting children's mathematics learning.

### *2.5. Summary*

This chapter presented an overview of recent mathematics curriculum reform and developments internationally and in South Africa. The discussion also focused on the role of the teacher and beliefs about teaching mathematics. Five systemic reform programmes were discussed, including Cognitive Guided Instruction, Problem Centred Primary Mathematics, Realistic Mathematics Education, Mathematics Recovery and Counting Me in Too. The main focus of each programme's approach to mathematics education was presented. The chapter was intended to provide background to the debate regarding appropriate mathematics content knowledge for teachers in the early grades of the primary school.

# 3 *LEARNING AND TEACHING EARLY NUMBER – A THEORETICAL FRAMEWORK*

### *3.1 Introduction*

As discussed in chapter 1, the research question I present focuses on how teachers understand the ways in which children learn number. Chapter 2 reviewed literature related to international mathematics curriculum reform and curriculum developments in South Africa. Chapter 2 also discussed the teachers' role in learning and teaching mathematics, and how beliefs about teaching mathematics influence classroom practice. Examples of empirical studies in the previous chapter focused on five reformed programmes in mathematics education. These programmes are of interest to this study as they highlight number development in the early years.

The intention of this chapter is to establish the body of knowledge, which gives a scientific explanation to the ways in which children learn, and in particular, the ways in which they acquire number concepts. It is well documented that when children enter school they have accumulated a wide range of number experiences and knowledge (Carpenter, Fennema and Franke, 1996; De Corte and Verschaffel, 1987; Ginsburg, 1977; Hughes, 1986; Ito-Hino, 1994; Olivier, Murray and Human, 1990; Resnick and Singer, 1993; Saxe, 1988; Schmidt and Weiser, 1982; Steffe et al: in Becker and Selter, 1996 and Van den Heuvel-Panhuizen, 1996).

This chapter begins with an overview of what this study accepts as a definition of number and a literature review that locates the discussion within a body of research specific to number development. The chapter also focuses on two theories of how children learn number and the processes involved in this learning. Von Glasersfeld's theory is of particular interest to this study since it deals directly with the ways in which children develop number concepts. Steffe's theory focuses on the development of counting and calculation. Steffe's framework will be used in the systematic analysis of teachers' knowledge and understanding pertaining to learning and teaching number at the early years as presented in the data collected in this investigation. Finally this chapter will briefly discuss features of the base ten (decimal) number system, various forms of representation, and give an overview of the literature related to children's solution strategies and contextual and calculation problems.

The Foundation Phase (Grade R to 3) Assessment Standards, which relate to number (APPENDIX 1), will guide the discussion in this chapter. These Assessment Standards are stated in the South African general Education and Training (GET) Mathematics Learning Area and set out in the Revised National Curriculum Statements (RNCS). The Assessment Standards prioritise knowledge and skills with a particular emphasis on: how numbers are represented, counting, calculations and (word) problem solving. I would like to consider the development of number concepts and skills in terms of these strands.

### *3.2 Understanding Number*

The term 'number' is used throughout this study to refer to topics related to the development of number concepts and skills. These involve the construction of number meanings, an understanding of the numeration system, developing number sense and understanding of the use of numbers in different contexts. This study focuses only on the development of number concepts, number relations and the number system within the domain of whole number and does not consider understanding and use of numbers beyond whole numbers i.e. rational numbers.

Learning number involves the development of *number concepts* and *number sense* and an understanding of numbers and numerical facts as they are encountered in everyday life. In addition to these skills and concepts, children should acquire a feeling for numbers, enjoy working with them and develop the ability to deal with them appropriately (Kamii, 2000 and Van den Heuvel-Panhuizen, 1999 and 2001). Critical to thinking about numbers is the idea and answer to 'How many?' questions. Any collection of things has 'numerosity', which is the number of things you get when you count a collection and refers to the 'how muchness' or magnitude of numbers. Learning number should also include competence at abstract number skills and relations (National Numeracy Strategy, 1999).

A young child is able to distinguish between small collections of objects (two, three or four objects) simply by looking at them (Von Glasersfeld, 1981). Butterworth (1999, p 107) proposes that children are 'born with special circuits in their brains for categorising the world in terms of numerosities, which he calls a 'number module'. He explains that young children categorise the world in terms of numerosities and they have to construct more advanced capacities onto this innate structure that enables them to recognise small numerosities.

Numerosities are ordered by the size of the collection that represents them, for example three sweets or the combined collection of two or more collections (addition). In other words every collection is the sum of other numerosities, for example 1+1=2, 2+1=3 (Butterworth, 1999). Piaget (1964) referred to this property as 'additive composition'. In order to understand numerosities, children need to understand the idea of an object as something that can be individualised. The objects can be concrete, tangible things, like toys, animals or flowers, they can be jumps, athletic teams at a sports day, they may be drum beats in a song or they may be even more abstract, like days of the week.

Children's basic number development appears to proceed slowly, especially when compared to the relative speed at which children learn language. The ability to recognise, describe and represent numbers and their relationships is one aspect of understanding number, which will be discussed in detail in section 3.7. Another is the awareness of the different ways and different contexts in which numbers are used in everyday life. English distinguishes between numbers that are used for numerosity, where numbers are used to signify the number of objects in a collection, and numbers that denote order, where they are used to identify an object's location in a sequence and each is either before or after any other number.

Dickson, Brown and Gibson (1984), Verschaffel and De Corte (1996) and Van den Heuvel-Panhuizen (2001) examine some of the different meanings for numbers. Numbers can also be used to label or name things, people or places, for example house numbers and motor car registration plates, and the ten symbols (0 to 9) allow for endless combinations. They are used to measure, for example 'how long, 'how much' or 'how heavy' and are cyclical, for example analogue clock time (which complicates the understanding of basic calculation, for example 9 + 5 = 2), days of the week and months.

Counting provides a conceptual tool that takes the child beyond what can be achieved simply by recognising numerosities, towards the idea of sequencing numerosities by size. The ability to recognise small collections of objects and recite numerals in order does not necessarily mean that children can count or understand what they are saying. Mastering the practical skills involved in order to count a collection of objects is a complex process and learners should engage in a range of counting activities before counting becomes meaningful (Dickson, Brown, Gibson, 1984 and Butterworth 1999).

The process of learning to count involves several complex understandings and abilities that include:

- realising that the first few number words are not one big, long word ('onetwothreefourfivesix') but actually six small, single words (one, two, three, four, five, six)
- assigning a number name or counting word to each object being counted (one-to-one correspondence)
- using each name only once
- counting all the objects
- assigning a number to indicate the position of an object in a (fixed) sequence of objects (known as the *ordinal* aspect of number)
- announcing the number of objects in the collection by using the last number counted (number name) to specify the size of a collection (known as the *cardinal* aspect of number)
- understanding that each number name represents a unique numerosity
- understanding that no matter which order you count the objects, the number remains the same (known as conservation).

> *To most adults, the knowledge and use of the first nine natural numbers (one, two, three...up to nine) appears to be a very simple and straightforward business. Yet an average child takes around five years, from the age of two to the age of seven, to learn to handle such numbers consistently and to apply them to a variety of everyday situations. And the period would become even longer if the use of number operations were included.*
> *(Dickson, Brown, Gibson, 1984)*

Butterworth (1999) explains that learning the sequence of counting words is one of the first ways in which a child connects their innate concept of numerosity with the practice of the culture they live in. He also advocates that counting practice builds a sense of numerosity into an entire numerosity system, which helps children distinguish among numerosities because different collections have different numerosities.

Explicit training in logical operations, including classification, ordering and conservation, has become less important in the early childhood mathematics curriculum. Greater emphasis is now placed on taking advantage of, and cultivating children's informal counting skills, which is now generally considered an essential aspect of the development of early number concepts. Appreciation of children's thinking provides an understanding of their emerging thinking and understanding (Verschaffel and De Corte, 1996, Hiebert and Carpenter 1992; Carpenter, Fennema and Frank 1996 and Van den Heuvel-Panhuizen, 1996).

### *3.3 Development of Number Structures*

Despite the influence that Piaget and Vygotsky's constructivist epistemologies (see APPENDIX 2) have had on mathematics education and teaching in general, neither specifically relate to the ways in which children learn number and number related concepts (Hughes, 1987; Donaldson, 1978). Both theories broadly explain children's development and learning which are not specifically related to learning number concepts. Although Piaget offers considerable insight into the growth of mathematical thinking, Hughes (1987) suggests that these ideas are related of Piaget's general notions of intellectual structures and that he sees mathematical concepts arising 'independently and spontaneously' (Hughes, 1987; 22). Piaget's interest was more about mathematical structure and relationships than function and his concern was in the processes children use in learning mathematics (Groen and Kieran, 1983).

Piaget believed that children's ability to reason emerges slowly and that each new concept builds on those previously developed. This includes the development of numerosity, which he suggested is connected to the ability to reason logically and abstractly and depends on the development of prior capacities and on interacting with the world and manipulating objects. One such ability is to recognise collections of different sizes and reason that, for example if collection A is bigger than B and B is bigger than C, then A is bigger than C.

Piaget proposed that without this reasoning capacity children will not be able to order numbers according to size. He also states that children's ability to conserve number (i.e. understand that regardless of the arrangement of objects in a collection or how the arrangement is changed the position of the objects does not affect the number) and one-to one correspondence are essential capacities in the development of numerosity (Piaget, 1964).

Piaget's view, according to Neuman (1987), is that children's development of number is connected to other logical concepts. They are not able to understand natural numbers before they understand the concepts of classification and seriation or, in mathematical terms, before they understand that the natural number is indissociably cardinal and ordinal. Von Glaserfeld (1981) states that Piaget's focus on number development was on the cognitive structures which determine the capacity for conservation of numerical collections (regardless of how they are arranged or represented), class inclusion (part-whole relationships) and order, and that he took the construction of a unit for granted. Dehaene (1997) believes that children are capable of mental representations of number from as early as birth and that their conceptual understanding of number develops with age and education.

Von Glasersfeld (1981) offers a theory that describes the development and acquisition of number concepts, which are essential in numerical operations; which are defined by Groen and Kieren (1983) as reversible transformations.

*3.4 Von Glasersfeld's Attentional Model for Learning Number*

The 'attentional model' outlined by Von Glasersfeld (1981) is a theoretical model for the development of concepts underlying numerical operations and outlines the conceptual construction that generates *units, pluralities and lots.*

> *First when we speak of 'things', 'wholes' 'units' and 'singulars', on the one hand and of 'plurals', 'pluralities', 'collections' and 'lots' on the other we refer to conceptual structures that are dependent on material supplied by sensory experience. Insofar as these concepts involve sensory-*

*motor signals, they do not belong to the realm of number. They enter that rarefied realm through the process of reflective abstraction, which extricates attentional patterns from instantiations in sensory-motor experiences and thus produces numerical concepts that are stripped of all sensory properties.*
*(Von Glasersfeld, 1981)*

Von Glasersfeld (1981) begins his explanation of the development of concepts underlying numerical operations by defining 'attention'. Attention in this discussion does not mean an extended period in time but rather a repeated sequence of 'focused' or 'unfocused' attentional pulses operating on visual, auditory or tactile signals. In the process of concept formation, we naturally distinguish and separate our perceptual experiences, which then become distinct things or 'items' in our cognitive organisation. These can be categorised and have a continuous existence in time. We perceive each object as a single or *unitary item*, and these can be distinguished from each other and the background in which they exist, and may be moved about or counted in the absence of sensory difference.

Although the creation of conceptual units is the result of operations that, in principle, are not dependent on sensory material, such material may be indispensable at the beginning of a child's development (Von Glasersfeld, 1981). Von Glasersfeld suggests that these unitising operations involve signals that are registered or focused and become a 'whole' or 'thing' or 'object'. An experiential object or *sensory-motor item* consists of more than one attentional pulse and this process in terminated when the signals are unfocused, as this establishes the equivalent of a 'blank space'.

*Second we may generalise and say that, conceptually, all whole numbers are indeed unitary wholes and are themselves composed of units; but we can now add, on the basis of our theoretical model, that whereas the component units are the non-numerical raw material of focused attentional pulses, the wholeness of the number concepts is made of different stuff – it derives from the structural pattern in which these pulses are arranged.*
*(Von Glasersfeld, 1981)*

When children begin naming (labelling) objects, each separate unitary item is a generalised construct of a unitary whole, which is linked to the linguistic form of the unitary singular. When individual sensory-motor items of a succession share at least one attribute that connects them, they are conceived as a *plurality.* A continuous string of unitary items can thus be thought of as a *collection* of items that are all individually different but share a common feature. This conceptual construct is related to what Piaget referred to as *classification.*

When a collection of items is conceived as a 'whole' with 'sameness' Von Glasersfeld calls this structure a *lot.* A lot is neither fully abstract nor is it a numerical construct as it is still dependent on sensory-motor material. In order for this construct to become abstracted from the sensory-motor level, or internalised as a higher order conceptual structure, what Piaget called 'reflective abstraction' is required. In other words, the child must focus attention on the results of operations on sensory-motor items and not on the items themselves. The attentional pattern that was constructed from the sensory-motor signals on the unitary items is now reprocessed and creates an *arithmetic unit,* which consists of abstract units that Von Glasersfeld calls unity of units or whole number and is the basis for the construction of number.

These arithmetic lots have numerosity but further operations are necessary to specify the numerosity of a particular arithmetic lot. The operations that are required in this process underlie the different forms of counting. The operations underlying the counting process require attributes and competencies that are not necessarily numerical. These include acquisition and application of the system of number names, the perception of small lots or patterns of items, called *subitising* and co-ordination that results in *one-to-one correspondence.*

According to this explanation each number is realised as an arithmetic lot regardless of the numerosity of the lot it incorporates. Children need to keep track of specific numerosities when they engage in arithmetic operations.

### *3.5 A Framework for Learning Number*

The following discussion will give an overview of the development of counting and distinguish between the stages or levels of numerical development. I will also present a framework for learning number, which is a research-based framework for the assessment and teaching of number in the early years of school. The framework describes different stages of arithmetical solution strategies. It is based on Steffe's

(2000) psychological model that describes four basic counting schemes, the notion of emergent counting as used by Wright (1998), and it incorporates Von Glasersfeld's (1981) attentional model.

Children progress through different levels of understanding in their development of counting, which include 'context-related' counting, 'object-connected' counting and 'formal ways of counting' (Van den Heuvel-Panhuizen, 1999, p15). The advancement through these levels begins from 'the ability to invent informal context-related solutions, to the creation of various levels of short cuts and schematizations, to the acquisition of insight into the underlying principles and the discernment of even broader relationships' (Van den Heuvel-Panhuizen, 1999, p 5). The ability to advance to another level is determined by the extent to which the child is able to reflect on activities and the nature of the interaction with others (Von Glasersfeld, 1989 and Van den Heuvel-Panhuizen, 1999). Knowledge and an understanding of these levels and how children develop through them will enable teachers to interpret strategies they use when solving problems (Carpenter, Fennema and Frank 1996 and Baroody, 1987).

### *3.5.1 Emergent Counting*

> *It might at first sight be thought that learning to count was merely a matter of reciting a string of words. However the art of counting involves a number of additional features, such as only pointing to one object at a time and keeping track of which objects have already been counted. One use of number is to specify the size of a collection of objects (known as the cardinal aspect of number). The process of counting i.e. the assigning of a number to a particular object which forms one of a sequence of objects, is known as the ordinal aspect of number. There is however a final step, that of knowing that the number the child 'finishes' on in counting a collection can be used to represent the size ('manyness', 'numerosity') of the whole collection. This is the link between the ordinal and cardinal aspects of number.*
> *(Dickson, Brown and Gibson, 1984)*

A child who is an emergent counter may have some number knowledge. For example, a child might be able to recite some of the sequence of number words and be able to identify some numerals. However, the results of the counting act does not signify the numerosity of the collection. A child at this stage is not able to count objects in a collection, either because they do not know the number words or because they are not able to co-ordinate the number words with objects (Bobis 1997, Wright 1998, Steward, Wright and Gould 1998; Gould & Wright, 2000). Schaeffer (in Dickson, Brown and Gibson, 1984) also describes four stages in the development of children's counting. His Stage One children are not able to count collections of five or more objects but can recognise small collections of objects (up to four) and attributes this to what Von Glasersfeld (1981) called subitising. In other words they are able to recognise a visual or auditory pattern in a small collection of sensory-motor items.

Schaeffer (in Dickson, Brown and Gibson, 1984) also notes that children are able to distinguish between larger and smaller collections of up to 5 and are able to recognise larger and smaller collections if they were displayed to show one-to-one correspondence. Piaget (1964) said seriation is intuitive in what he termed Stage One but that children at this stage had not established 'cardination' (numerosity). Neither are they able to use one-to-one correspondence and rely entirely on perceptual cues, for example spacing.

### *3.5.2 Steffe's Theory of the Development of Counting Scheme*

> *Counting as activity is a coordination of two productive activities: the projection of a number word sequence and the production of a sequence of countable items. A counting scheme includes counting as activity. But it also includes a situation of counting, a goal relative to the situation of counting and the results of counting.*
> *(Steffe, 2000)*

One of the most important mathematical structures that a child has to develop in the early years of life is that of a number sequence. The activity that leads to the construction of a number sequence is counting. Counting does not simply happen and as van Glasersfeld and Steffe explain, it is a dynamic process, which progresses through developmental stages, from counting perceptual units to counting abstract unit items. Steffe, in his paper for the Research Symposium at the 2000 National Council of Teachers of Mathematics (NCTM) Research Pre-session in Chicago, presented his position on the learning and teaching of number. He outlined four basic counting schemes as mathematics of children.

> *I traced the construction of number sequence through essentially four stages: the perceptual stage, the figurative stage, the stage of the initial number sequence, and the stage of the explicitly nested number sequence.*

*The tacitly nested number sequence is a more or less transitional stage between the preceding and succeeding number sequences.*
*(Steffe, 2000)*

Steffe (2000) describes these numerical counting schemes in terms of the progression from concrete motor counting, which involves counting external items, to abstract or internalised counting. Pre-numerical children are those who have not abstracted number (Wright, 1993) and are at either the perceptual or the figurative stage (Steffe, 2000).

*At every stage, I stress counting as a scheme rather than as an activity. As a scheme, counting is goal directed and purposeful. It serves in the construction of adding and subtracting schemes, child generated algorithms, the structure of a unit of units of units, multiplying and dividing schemes, and the establishment of units for measuring numbers. Children's number sequences are fundamental in their mathematical education, but they are not to be regarded as being constituted in the same way as the conventional concept of a number sequence in mathematics.*
*(Steffe, 2000)*

*3.5.2.1. Perceptual Stage of the Counting Scheme*

The first stage outlined by Steffe is the perceptual stage. A child who is at the perceptual stage of counting is only able to count *real things* that can be perceived (e.g see, hear or feel) by matching a number word in the sequence to each counted object or item. They are able to co-ordinate the act of pointing with their number word sequence and realise that this is necessary in order to find 'how many' objects they have. The items to be counted need to remain in their perceptual field and therefore the child is not able to count items that have been concealed in any way (Bobis, 1997; Wright, 1994 and 1998; Wright and Gould and Steward; 1998, Bobis, 1997; Gould and Wright, 2000 and Steffe, 2000).

*3.5.2.2. Figurative Stage of the Counting Scheme*

The second stage Steffe outlines is the figurative stage. A child at the figurative counting stage is able to count things that *stand in for* the objects being counted but counting is still a sensory-motor activity. A child who has reached this stage is able to count figural (picture, tally or numerals), motor (finger tapping, head nodding) or verbal items (Bobis, 1997; Wright 1994, 1998 and Steward, Wright and Gould, 1998

Bobis, 1997 and Gould and Wright, 2000, Steffe, 2000). This child is able to represent objects mentally even though they cannot be perceived directly. Objects being counted can be visualised even if they are hidden in some way and the child typically counts from one to do so. The counting process usually involves some sensory activity, like pointing or nodding while counting over a hidden number of objects. A child at this stage is able to use counting to solve contextual (word) problems in which some or all of the items to be counted are not visible. This child will count from one and count all the objects or items in a collection and rely on repeated re-counting of the whole collection (Dickson, Brown and Gibson, 1984).

Schaeffer's (in Dickson, Brown and Gibson, 1984) and Piaget's (1994) Stage Three child falls within this description of a figural counter in that by this stage they are able to conserve number and rely on one-to-one correspondence when counting and are not deceived by changes in perceptual cues. They add that a child at this stage is able to connect the ordinal aspect and cardinal aspect (understands that the last number counted represents the total size of the collection) of numbers. Carpenter, Fennema and Frank (1996) and Baroody (1987) discuss the 'counting all' strategy in their extensive work on problem solving and children's construction of knowledge.

*3.5.2.3. Abstract Stages*

The third and fourth stages in the development of the counting schemes according to Steffe are Abstract Stages and include the *Initial Number Sequence Stage,* the transitional *Tacitly-nested Number Sequence* and the *Explicitly-nested Number Sequence Stage.*

*3.5.2.3a. Initial Number Sequence Stage*

A child who 'counts all' can be encouraged to internalise (mentally re-represent) their counting acts by covering the first collection counted before adding more objects to be counted. The child then needs to visualise the covered objects and interiorise the number words and the results of counting. The child who has constructed the initial number sequence can generate an experience of counting hidden objects without actually counting and then extend counting beyond the imagined experience when counting the hidden objects (Steffe, 2000).

Children in this stage 'count on' from the larger or first number to solve additive or missing addend problems. Children at this stage have constructed numerical counting concepts or schemes and do not need to use 'count all' or count-by-one strategies which are entirely ordinal. Each number takes the role of a completed count and a child is able to count on or back from a given number (Wright 1994, 1998, Steward, Wright and Gould, 1998). Bobis (1997), Gould and Wright (2000), Carpenter, Fennema and Frank (1996) and Baroody (1987) include 'counting on' in their discussion on children's solution strategies. They are able to count on from the first or larger number to solve addition problems (Carpenter, Fennema and Frank, 1996) or might use 'counting-down-from (e.g. 17 – 3 as 16, 17, 18 answer 14)' (Wright, 1998, 1994, 1998; Steward, Wright and Gould, 1998 and Bobis, 1997).

Fuson (1992) reports that the 'counting on' strategy involves the ability to adapt from a cardinal number that represents the size of the collection, to an ordinal number that represents the last number counted for that particular collection.

Central to the whole 'counting on' strategy is the notion that a collection can be a collection in its own right and at the same time be part of a larger set (Dickson, Brown and Gibson, 1984). This Piaget refers to as 'class inclusion'. Steffe (2000) suggests that children at the perceptual and figurative stages should be encouraged to say the number words 'just before' and 'just after' a given number. Children can be encouraged to 'count forward' and 'backwards' from a given number, as this is important in the construction of addition and subtraction schemes.

Many children in the initial number sequence stage develop ways of keeping track of their 'counting on' actions which include using their fingers and pointing to represent counted items they cannot perceive directly. When children are able to represent the auditory records in their arithmetical units, Steffe refers to the image they produce as a verbal number sequence. He reports that children in the stage of the initial number sequence keep track of forward and backward counting using *numerical patterns* that involve spatial, auditory or rhythmic patterns of up to twenty items. Children at this stage are also able to count large collections in two's or five's (numerical composites). They do not however perceive each small collection as one countable item (composite unit item) and therefore would not be able to understand a question asking 'how many two's or five's' in the collection.

*3.5.2.3b. Tacitly-nested Number Sequence*

Steffe (2000) explains that the *Tacitly-nested Number Sequence* is a transitional stage between the *Initial Number Sequence Stage* and the *Explicitly-nested Number Sequence Stage* (discussed in 3.5.2.3c). When children have to count large amounts, like more than ten, they need to find a way of keeping track of their counting acts other than with their fingers. For example if a child has counted a collection of 13 objects and a collection of 16 objects in attempting to find out how many objects there are in both collections, he or she will begin with 13 and count on, raising a finger for each count. Having counted to 23 and raised ten fingers the child will have to lower his or her fingers and continue counting from 24 indicating one finger again. When the child completes the count he or she will have used a pattern of ten fingers and six more fingers. In this process of keeping track of counting on 16 more from 13, the child has to reprocess the abstract units in a sequence of counting act as countable unit items.

Steffe calls the tacitly nested sequence a *recursive counting scheme* and refers to this as 'double counting' because it is as if the child has two number sequences. The first the child uses as material to make countable items and the second the child uses as the counting operations used to count those countable items. For example a child may have developed an awareness of 16 as a composite whole and not just 16 single counted items. The collection of 16 items is taken as one thing, a *composite unit item*, when counted. In reprocessing the number sequence 1 to 16 the child has made an implicit or tacit nesting of the sequence within (for example) the number sequence 1 to 29. When keeping track of counting in this way the child experiences counting beyond the visible items and so does not count images of perceptual items but images of counting acts, and this is what Steffe refers to as double counting.

Wright (1998) describes an *Intermediate Number Sequence* stage in an adapted version of the Learning Framework in Number which falls within the definition of the tacitly nested number sequence and reports that it is at this stage that a child 'counts-down-to' when solving subtractive situations. For example, when a child is presented with the difference between 18 and 15, the child is able to 'count-down-from' 18 to 15 and keep track of the counting acts. In other words the child has an awareness of the number sequence from 1 to 15 as being part of the number sequence from 1 to 18 and is able to anticipate the result of counting down to 15. The child includes 15 in 18 but is yet to extract 15 from 18 without destroying the 18 (Steffe, 2000).

Both Wright (1998) and Steffe (2000) conclude that the child at this stage is able to choose the more efficient of the count-down-from and count-down-to strategies. All of the strategies above fall within the tacitly nested number sequence. All of the strategies take place during activity as the nested intervals are not yet explicit and still have to be produced (Olive, 2001).

Olive (2001) explains that children who have constructed the tacitly nested sequence are now able to answer the question 'How many two's or five's?' because they now 'see' the group of two or five as one thing or a composite unit and no longer simply as a numerical composite as described in the initial number sequence stage. Children at this stage are able to count in two's or five's for example and may be able to find how many groups of two or five there are in a specific collection. So if asked how many groups of two there are in 16 a child may count '1, 2, 3, 4, 5, 6, 7, 8'. The child could also count '2, 4, 6, 8, 10, 12, 14, 16' and keep track of how many times they counted two. Either way they will come to 8 groups of two as an answer.

Children at the tacitly nested number sequence stage are able to <u>begin</u> the production of a multiplicative structure whereas these constructs are not possible for children in the initial number sequence stage.

*3.5.2.3c. Explicitly-nested Number Sequence Stage*

The fourth stage Steffe (2000) describes in the development of the counting schemes is the *Explicitly-nested Number Sequence Stage*. In order for a child to have constructed the explicitly nested number sequence, two operations need to be established. Firstly the child has to engage with part-to-whole reasoning. For example, a child is able to 'disembed' or pull out the composite unit i.e. the elements of the number sequence 1 to 16 from the interval 1 to 29 without breaking down the whole sequence. Steffe says that when this operation emerges the child's number sequence is explicitly nested.

A second feature of the explicitly nested number sequence is the establishment of an abstract unit item, a single unit that can be repeated or iterated to complete the composite unit. For example, a child in the tacitly nested number sequence stage understands 'six' for example to be a composite unit containing a sequence of six unit items, the ordinal aspect of number which is represented as 1, 2, 3, 4, 5, 6. The *iterable unit* is a product of repeatedly applying the 'one more item' operation (Olive, 2001).

Steffe suggests that once a child has established the explicitly-nested number sequence they should be able to 'collapse' a number, such as sixteen, into its unitary items. They notice seven and the remainder of nine in sixteen, which is seven and also consider them as two component parts of sixteen, apart from sixteen, while leaving them in sixteen. The child is then able to produce three numbers, nine, seven and sixteen and realises the relationship among these three numbers in that seven (and nine) is contained in sixteen and can be compared to the whole unit, sixteen. The child also understands that if seven is taken away from sixteen the result will be nine remaining.

Children with a tacitly nested number sequence would generate the solution to this problem: 1 + 1 + 1 + 1 by repeatedly adding one: 1 + 1 is 2 + 1 is 3 + 1 is 4. A child with an explicitly-nested number sequence on the other hand would see the problem as four ones and know that four ones is the same as one four, hence their number concept of four is reversible (Olive 2001). Wright (1994) discusses the explicitly-nested number sequence in a study of the numerical development of 5 and 6 year olds. It is at this stage that he recognises that children have constructed the part-whole operation and have an awareness of addition and subtraction as inverse operations.

A real challenge for children at this stage is to construct what Steffe calls *composite units* as iterable in the same way as their unit of one is iterable. In order to answer the question 'If you count to 16 in two's how many two's (or three's or five's etc) would you count?' the child has to construct two (or three or five etc) as an iterable unit.

This multiplicative reasoning is what underlies the construction of multiplicative schemes (Olive, 2001) and Steffe suggests that this is within a child's 'zone of potential construction' at the stage of the explicitly nested number sequence. Olive asserts that children at this stage can use the composite unit items for counting, combining, comparing, segmenting and partitioning (breaking up) but are not able to find how many two's there are in 16 as this *commutative* solution requires further interiorising of the number sequence.

*3.6 Multi-digit Numbers*

Place value is derived from the concept of numerosity and is an important aspect of knowing and understanding our decimal number system and especially the multi-digit aspects of this system, which require the integration of three aspects of quantity. These are base names (e.g. 2 tens and 5 units), number names (e.g. twenty-five) and written numerals (e.g. 25) (Verschaffel and De Corte, 1996). Learning number names and their serial order and using the numbers to count quantities should include saying, writing and reading multi-digit numbers.

Fuson (1992) explains that understanding the 'marks' or written form of the number and the named-value word is difficult because spoken number words are explicitly named but the value of the written numerals are implicit in their positions. She says that for example, English-speaking children are confronted with the particular difficulty of understanding the differing features of the 'named-value number words' and the 'base ten positional written marks'. Children hear 'three hundred and forty-six' for example, and may wish to write the named value as 300406 but in fact have to write 346 which actually looks like 'three four six'. In English there are special names for numbers between 10 and 20 which do not reflect the rule of a simple additive combination, like 'ten-one, ten-two, ten-three etc' or 'tenty-one, tenty-two, tenty-three, etc'. There are also special names for the decades, 'ten, twenty, thirty, etc'. These irregularities make it difficult for children to learn the patterns and meanings of English number names (Fuson, 1992).

Carpenter, Fennema and Frank (1996) add that place value knowledge involves explicit knowledge of conventions of our base ten numeration system. Children cannot discover this on their own, for example that the value of a digit depends on its position in a sequence of digits. Although children may be able to count beyond ten and have implicit knowledge which they collected through learning to count, using money and from interaction with adults, this does not necessarily mean that a child understands or has any substantial knowledge of place value.

The fundamental concepts underlying an understanding of base ten numbers include: knowing that groupings or collections of ten (or 100 or 1000, etc) items can be counted; that we can talk about the number of tens (100 or 1000's, etc); and the number of individual units or ones. (Carpenter, Fennema and Frank 1996, Treffers and Buys 2001 and Van den Heuvel-Panhuizen, 2002). For example a collection of 27 counters can be thought about as 27 single counters, as 2 groups of ten counters and 7 single counters or 1 ten and 17 single counters, etc. In other words a child can count single counters (units) or group of counters into tens and units in order to find out the total number in a collection.

The idea of grouping by ten is found in skip counting activities and multiplication and division situations in which a child solves the problem by grouping objects in tens. An example of such a problem could be:

> Zadie has 27 crayons. She put them into boxes with 10 crayons in each box. How many boxes of crayons did Zadie fill?

Extending the concept of numerosity to include the idea of a collection with no members or an 'empty set' and that we should represent this nothing as a numeral (0) is difficult for children to comprehend. Although they are familiar with the idea of not having something, like sweets or a toy, the idea that no sweets, toys, birds, time or money are all the same and have no numerosity is very abstract and difficult to understand (Butterworth, 1999).

Children need perceptual support for the construction of multiunit items in the same way as they do for the construction of single unit items (Fuson 1992, Steffe, 2000). Fuson suggests that a range of materials be used to present collections of tens, hundreds and thousands so that children can construct multiunit quantity conceptual structures in order to understand and link them to written forms and multidigit words.

The materials can be used for supporting 'collected multiunits' and 'sequence multiunits'. Collected multiunits are based on collections or groupings of objects (eg counting rods). For example a ten-unit item is made conceptually by collecting ten single items, a hundred-unit item with one hundred single unit items and a thousand-unit item with one thousand single unit items. The multidigit number word is then conceptually made up of a certain number of these collected multiunit items (Fuson, 1992).

Fuson outlines the construction of sequence multiunits and explains that they are small groups within a number word sequence, which require the sequence skill of skip counting in tens (10, 20, 30 … 100) or (12, 22, 32 … 102) and by hundreds and thousands. This counting may initially be based on number word patterns, but for the child to construct sequence multiunits the counting must be associated with the quantity.

Having constructed multiunits using single unit item materials, children's understanding of multidigit addition and subtraction is facilitated by first using the physical objects and later mentally representing the unit items as collected tens being added to more collected tens (and hundreds etc). Fuson (1992) asserts that addition and subtraction of multidigit numbers thought of as a sequence of multinunit items are an extension of the unitary sequence counting procedures which involve counting on, counting up to and counting down, in which the counting is done using sequenced multiunit jumps of ten, hundred and thousand.

Fuson (1992) reports that many children demonstrate difficulties in carrying out the skills necessary for sequence counting on, even by tens let alone hundreds or thousands. The concern is that for children to construct these multiunit items they must first present the numbers by counting and reflecting on the collections of objects before attempting to present the numbers by counting their counting words.

Steffe (2000) expresses concern that children who have constructed the explicitly-nested number sequence might be expected to learn standard computational algorithms to add and subtract multi-digit numbers. Children at this stage are still constructing composite units as iterable units so in order for them to reorganise or break up numbers into groups of tens and ones means that they must have constructed ten as an iterable unit. Steffe also explains that for a composite unit to be considered iterable the child has to be aware of the composite unit before they operate.

Steffe (2000) maintains the importance of the composite unit of ten but cautions that children in the explicitly nested counting sequence stage are able to produce the number word sequence 'ten, twenty, thirty ... one hundred' before they have constructed the unit of ten as iterable. He notes that many children circumvent the construction of ten as an iterable unit simply by 'pseudo counting by ten' verbally the number word sequence and keeping track of the counting acts on their fingers.

Once ten has been constructed as an iterable unit children can take counting-by-ten as a given in the same way as they take counting-by-one as units of one. Once a child has established these constructs they would be able to solve a problem involving the combination of 27 counting rods and 34 counting rods by generating their own (invented) algorithm by counting-by-ten and –one (Steffe, 2000).

Van den Heuvel-Panhuizen (2002) emphasises the importance of supporting children's ability to operate with multi-digit numbers in such a way that their activity has quantitative significance and does not merely consist of manipulating symbols in meaningless algorithms. She acknowledges the importance of developing a strong conceptual understanding of place value to ground multi-digit algorithms. She also emphasises that children should be able to view 100 and 10 as composite units as well as individual units that can be counted, as well as composites of 10 and smaller units and that they are able to configure a collection in different ways. However, on the other hand the RME approach clearly rejects the narrow focus on algorithms in conjunction with a heavy emphasis on place value supported by Unifix cubes. Instead of this (within RME) written and mental calculations are integrated, numbers are kept as a whole and the number line forms an important basis for developing strategies to solve problems like 265–194.

*3.7 Representation*

Children represent their thinking and understanding of mathematical ideas in different ways. For example they may model problems by moving and rearranging objects or drawing pictures to represent a problem situation. They also express ideas and communicate their thinking through oral and written language, physical gestures and invented and conventional symbols, which form the basis for the use of symbols. Thompson (2001) reports that the dominant representations identified among low attaining children are associated with images, which range from pictorial representations of a hand with fingers to iconic representations of tally lines, number tracks or number lines. Higher attaining children on the other hand show evidence of an implicit appreciation of the information compressed into mathematical symbolism.

External representations are a record of children's thinking and by analysing and discussing their representations we can gain insight into their understanding and grasp of mathematical concepts (NCTM, 1989). Van den Heuvel-Panhuizen (1999 and 2003b) suggests that learning and teaching mathematics should involve tasks that enable children to create and elaborate symbolic *models of* their informal mathematical activities and that they should be guided towards *models for* increasingly abstract mathematically reasoning about different problem situations. She points out that the transition in modelling activity can involve drawings, diagrams and informal notation as well as conventional mathematical notation and that as children's understanding increases they would develop symbolic notations to communicate their mathematical thinking and reasoning.

> *The term representation is used for mental images that are supposed to reflect, or correspond to things that lie beyond our experiential interface. (Von Glasersfeld, 1997, p 2 of 9)*

Learning and teaching mathematics often involves the use of concrete educational materials (manipulatives) or visual models, which are designed to convey mathematical concepts and are provided with the intention to help children represent numerical size and 'develop and understand concepts, procedures and other aspects of mathematics' (Szendrei 1996, p. 427). While adults might be able to recognise the mathematical relationships that didactic apparatus represent based on their prior mathematical experiences and references, learners might only see the physical material being presented and therefore the meanings and interpretations of the materials may not be the same (Cobb, 1997; Gravemeijer, Cobb, Bowers and Whitenack, 2000; Van den Heuvel-Panhuizen, 2003c).

Clements (1999) offers a word of caution on the use of concrete manipulatives and their role in learning. He states that concrete materials or manipulatives do not necessarily convey the meaning of a mathematical idea or concept. He suggests that although manipulatives can be used in a rote manner or to convey meaning, unless children think about their actions with the material and teachers consider children's representations, the intended meaning may not be conveyed through the materials.

Thompson (2001) raises the question about the most appropriate representations to use when teaching. He reports that the language of teachers' representation and the procedure associated with it provide children with a metaphor for communicating their own methods of calculating, and that children's mental representations are influenced by the physical representations (verbal, pictorial, written or concrete) used by their teachers. Teachers need to encourage children to discuss their constructions with manipulatives in order to help them develop increasingly sophisticated and mathematical representations (Clements, 1999).

Hiebert and Carpenter (1992) discuss internal representations of knowledge and suggest that these are hierarchically structured. In order to communicate these representations it is required that they are initially represented externally through physical objects, spoken and written language and symbols. Mathematical ideas are internal and related or connected in different ways i.e. representing and operating on them cannot be observed. However, there is much discussion about whether the form of an internal representation matches exactly the external object, concept or event being represented, or whether the external representation reveals something about the way the child has represented the information internally (Hiebert and Carpenter 1992; Von Glasersfeld, 1997).

> *To know whether anything we derive from experience corresponds to or represents an aspect of an external world, we should have to be able to compare it to the real thing. But this we cannot do, because we can compare experiences only to more experiences. (Von Glasersfeld, 1997).*

This is not to suggest that children who use manipulatives such as counting rods for example to represent 24 + 37 will necessarily represent all two-digit number as mental images of counting rods. However Hiebert and Carpenter (1992) propose that children who interact with concrete materials will represent quantities differently than children who have only worked with the written form of the numbers. Connections *between* different forms of external representations are usually based on similarities or differences and connections can be built by examining these relationships. Connections *within* the same representation are usually generated by noticing consistent patterns (Hiebert and Carpenter, 1992), for example the connection between the written and spoken words for ones, tens and hundreds in our counting system.

Hiebert and Carpenter (1992) explain that understanding grows as networks of mental representations are progressively constructed and new information is connected to existing networks. New relationships are constructed between previously disconnected information. Growth occurs as changes take place in networks as well as when information is added to existing networks. Therefore children may construct relationships between internal networks which exist, and representations of concrete materials and their experience with them. Children's prior experience of operating with quantities and representing these quantities with objects like toys or sweets will prepare them for classroom activities which may involve counting rods, for example, to represent two-digit numbers.

Hiebert and Carpenter (1992) also caution that concrete materials do not essentially express the mathematical ideas and relationships we intend them to represent. They point out that using money, for example, to represent place value can be problematic as money has a socially assigned value and does not have physical characteristics that suggest the value of each coin or note. Children have already constructed internal networks or understandings of money depending on their associations and knowledge of the value through their interaction with each coin or note. Money plays a particular role in a child's cultural experience and is usually used in situations involving the purchase of items. The child's understanding of money may be associated with the way it is used out of the classroom and may not connected to the notion of place value that the teacher is trying to convey unless the ideas have been explicitly presented to the children (Hiebert and Carpenter, 1992).

Concrete manipulatives do, however allow the children and teacher to focus their attention and discussion on the same representation in order to share their interpretation and come to a common understanding of the items and relationship between the items. The use of contextually-related concrete manipulatives that capture certain features of quantitative relationships provide a useful connection for children between quantities and abstract, written symbols (Hiebert and Carpenter 1992).

Hiebert and Carpenter (1992) report that written symbols are important forms of representation and that children have to learn their particular meaning and how they connect with other forms of representations, like concrete manipulatives, pictures and spoken words, and within the same system of representation, i.e. our base ten number system.

> *In order for symbols to acquire meaning, learners must connect their mental representations of written symbols with their mental representations of concrete materials. The potential for these connections to create understanding is complicated by the fact that the concrete materials themselves are representation of mathematical relationships and quantities. Thus, the usefulness of concrete materials as referents for symbols depends both on their embodiments of mathematical relationships and on their connections to written symbols.*
> *(Hiebert and Carpenter, 1992)*

*3.8 Calculation*

There is substantial research evidence to suggest that counting should constitute the basis of the early year's number curriculum. This research indicates that even though some children know many number facts and have developed a range of sophisticated calculation strategies they combine these facts and strategies with counting techniques in order to derive unknown facts (Thompson, 2001). Thompson (2001) also reports that low attaining children carry out procedures in the mind in the same way as they would operate with tangible objects, whereas higher attaining children seem to focus on those abstractions that enable them to make choices.

Houlihan and Ginsburg (1981) found that children often use counting to add, and maintain that the strategies they use are partly based on what is taught in school and on their own invention. They say that children assimilate what is taught at school into what they already know and that the result is an 'invented strategy'. They also found that not all children use the same methods. Some children use methods taught to them by teachers while others use versions of these methods. Hughes (1987) noted in an experiment that although some children used conventional addition and subtraction symbols at school, none of them used this notation when asked to represent on paper the process of physically adding two bricks to a pile of three. Thompson (2001) says that the implication of the children's responses appear to be that they did not feel that these symbols were particularly relevant to the problem they had been asked to solve.

Connections and meaning between written symbols that represent collections of objects and the relationships between these collections and the actions on the symbols, for example numbers, should be established through interaction with concrete materials and in everyday situations which are shared and discussed (Hiebert and Carpenter, 1992). Operations and their related symbols, i.e. + - x and ÷, are understood as they are used in meaningful situations and connections are made with other representations (Greer 1992). Hiebert and Carpenter (1992) propose that only once the meaning of operations and actions and the results of these actions are understood can rules and procedures for operating with the symbols be introduced. They also suggest that symbols can be manipulated without reflection, but that manipulation without reflection is unlikely to stimulate construction of the relationships that lead to understanding (Hiebert and Carpenter, 1992 p. 73).

There is a growing body of research that suggests that teaching pencil and paper algorithms before basic part-whole thinking is established, impairs learners' number sense. Often children will attempt an algorithm that they have been taught, with very little understanding and often inaccurately. It has not been suggested that teaching algorithms should be delayed until appropriate part-whole understanding is well established with 'small' numbers before children are taught written procedures that are applied to larger numbers. The solution in figure 1 illustrates how a child attempts to solve the problem of 24 + 37 using a method taught by the teacher. The child first adds 2 + 4 to get 6 then adds 3 + 7 to get 10 and finally a total of 16.

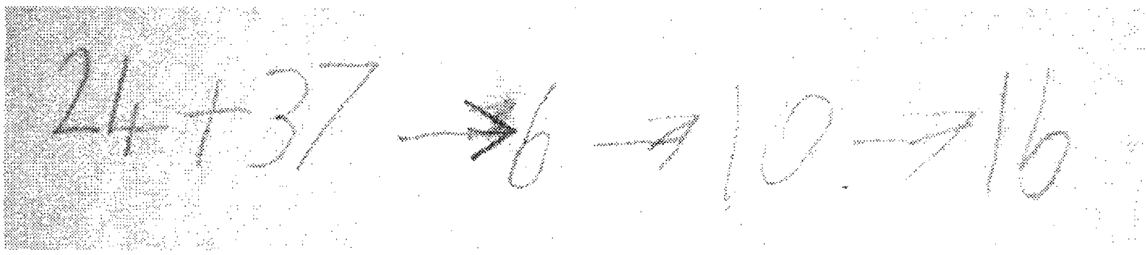



Figure 1: Grade 2 learner's attempt to solve 24 + 37

Mathematics education entails more than the acquisition of a variety of domain-specific concepts, relationships, rules, procedures and their application (Verschaffel and De Corte, 1996 and Butterworth, 1999). Olive (2001) says that children should not abandon their counting in favour of 'basic facts' but should be encouraged to build on their counting activities and knowledge in order to develop sophisticated counting strategies that will lead to understanding and modifications of their number sequences.

Thompson (2001) describes a taxonomy for mental strategies and for identifying the methods children use for the operations of addition with numbers to 20. The following strategies are generally accepted to represent increasing levels of sophistication:

- count all
- count on from first number
- count on from larger number
- use known number facts
- derive number facts.

The following classification system for strategies involving addition and subtraction of number from 20 to 100 includes:

- the partitioning or split method (47+36 as 40+30=70; 7+6=13; 70+1  83)
- the sequencing or jump method (83–47 as 83-40=43; 43-7=36)
- a variation of partitioning: mixed method (83-47 as 80-40=40; 40+3=43; 43-7=36)
- an extension of sequencing: compensation (47+36 as 50+36=86; 86-3=83)
- complementary addition: often used for solving difference problems (difference between 83 and 47 would be solved as: 47 to 50 (3); 50 to 80 (30); 80 to 83 (3), and the three steps 3, 30 and 3 would be added together to give 30)

Although a good knowledge and understanding of basic number facts, effective mental and written computation and estimation are essential, this knowledge should be integrated and acquired through meaningful and authentic contexts and should build on understanding arithmetic operations (Verschaffel and De Corte, 1996; Butterworth, 1999). Therefore learning and teaching basic facts should not focus on quick mastery and drilling of number computation but rather on children's own methods and the development of different strategies before the stage of recall.

Figure 2 shows how a child counts backwards in 5's from 60. As the mark above each 5 indicates the child then counted the number of times five was subtracted and notes that this occurred 12 times.

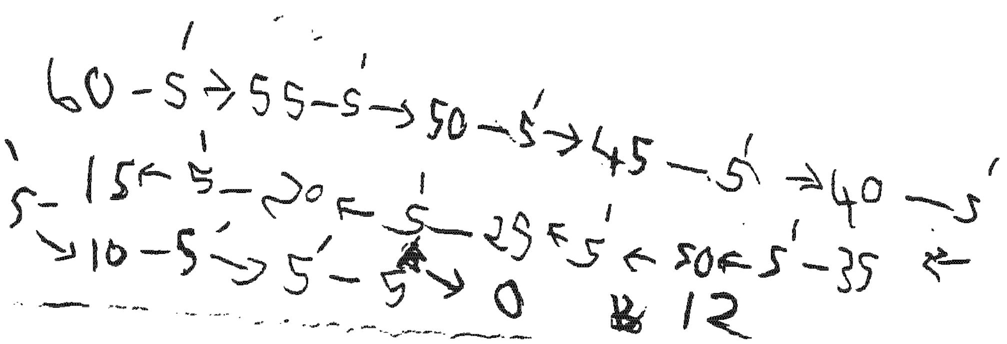


Figure 2: Grade 2 learner's solution using repeated subtraction

*3.9 Problem Solving*

Kilpatrick, Swafford and Findell (2001) note that an important part of our conception of mathematical proficiency involves the ability to formulate and solve problems coming form daily life or other domains, including mathematics itself. They note that problem solving provides and important contest win which children can learn about number and other mathematical topics. They also maintain that problem solving is important because it involves all strands of proficiency, thus increasing children's chances of integrating skills and concepts, and providing teachers with opportunities for assessing children's performance on all strands of mathematics proficiency.

Steffe (2000) suggests that asking children questions that encourage them to think about and regenerate parts of their solutions in order to consider the results of those solutions is important in the process of reflection. He emphasises that this process is essential in the construction and results of mental (internal) operations and representations that make up the mathematics of children. The process of reflecting on and reviewing solutions emphasises strong numerical connections among the numbers being operated on, and thereby children's basic number facts emerge as a product of their operating rather than as something they have had to memorise through drill and rote learning.

While refering to Freudenthal, Van den Heuvel-Panhuizen (1999) adds that mathematics is a human activity and that children should be given guided opportunities to 're-invent' mathematics in order for them to organise and solve problems which are close to and relevant to their own experiences. In other words, children should be presented with appropriate problems so that they can develop informal ways of finding the solution based on their own level of number development.

The didactical value of context-bound and object-bound counting and calculating, say Treffers (2001) and Van den Heuvel-Panhuizen (2001 and 1999), takes place in meaningful, problem related situations in which 'How many?' and comparison questions can be presented to children in an appropriate form. Gradually the focus becomes less on the context and more on the quantitative aspect of the situation or mathematical problem. Treffers (2001) suggests that the didactical benefit of gradually changing the nature of the questions we pose to children when solving problems, is that the original context-related situation remains available for those children as a model. This forms the basis as they begin to interpret the answer to a 'How many?' question, which leads to coordinated and resultative counting and calculating with concrete objects (Van den Heuvel-Panhuizen, 2001, 1999).

Carpenter, Fennema and Frank (1996), Baroody (1987), Treffers and Buys (2001) explain that without formal instruction of specific algorithms or procedures, children can intuitively solve problems by exploring and modelling the action and relations of the situations and that the operations take on real world meaning for children. For example figure 3 illustrates how a problem was modelled by a young child with drawings and numbers.

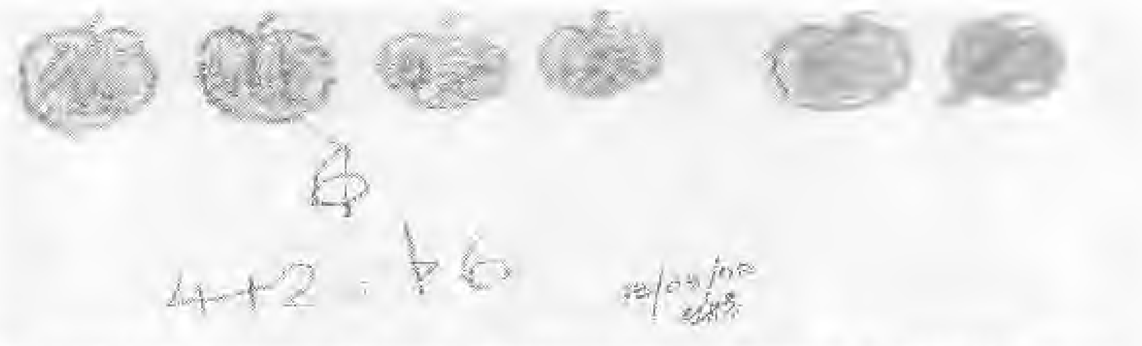


Figure 3: Grade 1 learner's solution to an addition problem with apples (Kühne and Ernstzen, 2003)

The problem illustrated in figure 4 was solved by breaking up numbers into groups of tens and ones and using tallies to represent the numbers being counted.

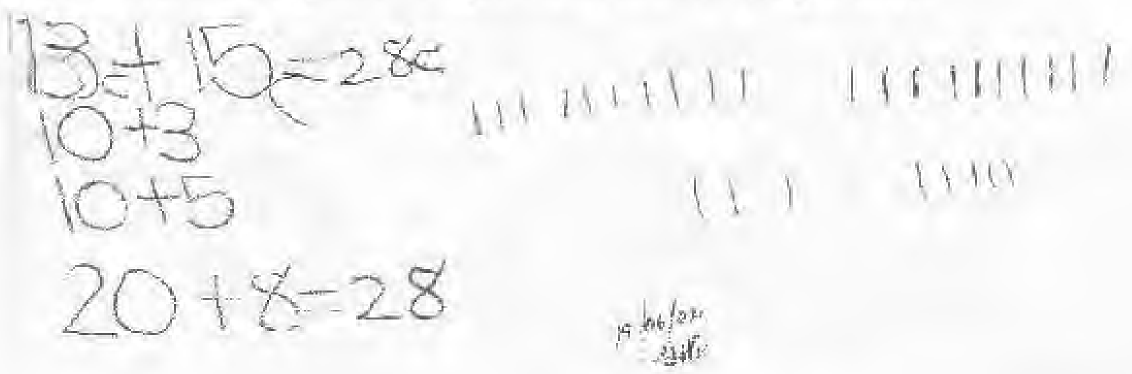



Figure 4: Grade 1 learner's solution to an addition problem using tallies (Kühne and Ernstzen, 2003)

Carpenter, Fennema and Frank (1996) and Baroody (1987) propose that children apply their knowledge of number facts to problems and use memorised facts to derive solutions for problems, for example using number combinations (2 + 6 = 8) or doubles (4 + 4 = 8). Murray, Olivier and Human (1998) however reject the notion of 'applying' mathematics to problem situations because of the implication that children would first have to begin with mathematical ideas and then find ways to use them. They argue that by starting with a problem, children will be left with a 'residue' of mathematics and that the mathematics is what is 'left over' after they have worked on problems.

As is the case with addition and subtraction facts, multiplication and division facts should develop from informal counting strategies. Greer (1992) suggests that by providing children with an increasing variety of different kinds of problem situations and allowing them to represent these in different ways their conceptual understanding of the number system and construction of the operations and links between situation and operations will be encouraged.

*3.9.1 Addition and Subtraction Situations*

Carpenter, Fennema, Franke (1996), Fuson (1992) and Greer (1992) describe the developmental progression of conceptual structures that children use to interpret and solve a range of problem types for addition, subtraction, multiplication and division. Carpenter, Fennema, Peterson, Chiang and Loef (1988) report that verbal problem types can be categorised into different classes according to their semantic characteristics. They distinguish whole number addition and subtraction according to four basic situations, which involve joining, separating, combining and comparing procedures.

These situations involve different actions or relations and by varying which quantity is unknown various problems may be generated. For example the following problems illustrate how the same context can be used to produce various problems with different structures.

> Themba has 6 sweets. Notozi gives him 9 more. How many sweets does Themba have altogether? (Result unknown)
>
> $6 + 9 = \square$

> Themba has 15 sweets. Notozi eats some. There are 9 left. How many sweets did Notozi eat? (Change unknown)
>
> $15 - \square = 9$

> Themba had some sweets. Notozi gives him 9 more. Now he has 15. How many sweets did Themba start with? (Start unknown)
>
> $\square + 9 = 15$

De Corte and Verschaffel (1987) report that children's strategies for addition and subtraction problems are strongly influenced by the semantic structure underlying the problem and on the sequence in which the known quantities are introduced in the problem text. They also note that children applied a great variety of addition and subtraction strategies, some of which were never explicitly taught. They criticise the traditional role word problems have played in learning and teaching early mathematics, where they were only assigned an application function whereby they were introduced to children only once they had learnt formal operations of addition and subtraction and successfully solved related number sentences.

Carpenter, Fennema, Franke (1996), De Corte and Verschaffel (1987), Carpenter, Fennema, Peterson, Chiang and Loef (1988) and Baroody (1987) reports that preschool children typically learn to use their internalised number sequence to determine the effects of 'adding one more' and that before they learn 'number facts' they are able to solve problems relying on various counting strategies.

Table 1, which I have taken from Carpenter, Fennema, Peterson, Chaing and Loef (1988) illustrates the categorisation of problems that can be used to assess progress in additive reasoning. Nunes (2001) maintains that research in the last decade has contributed to our understanding of several aspects of children's conceptual development. Investigations into children's understanding of additive reasoning indicate that their understanding of commutativity of addition develops relatively early but that it should not be taken for granted in the foundation phase (Nunes, 2001). Nunes adds that there is evidence that this understanding is related to the use of more efficient computation strategies and that it varies across problem types.

The 'join and separate' (change) problems involve active situations in which an event changes the value of a quantity. The 'combine and compare' problems relate to static situations consisting of two quantities that can be considered separately or in combination. The 'combine problems' involve part-whole relationships within a collection and the 'compare problems' involve the comparison of two separate collections.

The addition and subtraction problems discussed differ significantly in terms of their level of difficulty and the kinds of strategies children use to solve them (Verschaffel and De Corte, 1996). Verschaffel and De Corte (1996) propose that word problems be used to promote understanding of and give deeper meaning to the formal operations of addition and subtraction and in this way play a *concept acquisition* function.

Table 1: Classification of Addition and Subtraction Problem Situations

| Problem Type | Result Unknown | Change Unknown | Start unknown |
|---|---|---|---|
| Join | Notozi had 5 sweets. Themba gave her 8 more sweets. How many sweets does Notozi have altogether? | Notozi has some sweets. How many more sweets does she need to buy to have 13 sweets altogether? | Notozi had some sweets. Themba gave her 5 more sweets. Now she has 13 sweets. How many sweets did Notozi start with? |
| Separate | Notozi had 13 sweets. She gave 5 sweets to Themba. How many sweets does she have left? | Notozi had 13 sweets. She gave some to Themba. Now she has 5 sweets left. How many sweets did Notozi give to Themba? | Notozi had some marbles. She gave 5 to Themba. Now she has 8 sweets left. How many sweets did Notozi have to start with? |
| Combine | Notozi has 5 toffees and 8 chocolates. How many sweets does she have? | | Notozi has 13 sweets. Five are toffees and the rest of chocolates. How many chocolates does Notozi have? |
| Compare | Notozi has 13 sweets. Themba has 5 sweets. How many more sweets does Notoza have than Themba? | Themba has 5 sweets. Notozi has 8 more sweets than Themba. How many sweets does Notozi have? | Notozi has 13 sweets. She has 5 more sweets than Themba. How many sweets does Themba have? |

*3.9.2 Multiplication and Division Situations*

Dickson, Brown and Gibson (1984) report that understanding the meaning of multiplication and division operations is far more difficult than addition and subtraction. Children's understanding of commutativity of multiplication is a later development than commutativity of addition and is influenced by problem type, and understanding of distributivity is a later development (Nunes, 2001). Treffers and Buys (2001) suggest that the conceptual underpinning of multiplication and division problems and corresponding models is an important aspect of learning to multiply and divide because the problems and models reveal the underlying structure and offer insight into the operations and calculation.

Greer (1992) makes a distinction between different types of multiplication and division problems that are either commutative or non-commutative. Research on multiplication and division relates to analyses of the structure of the operations, and to children's performance in relation to different problem types, reports Anghileri (2001). Anghileri (2001) states that each type of multiplication and division problem may be related to a different contextual problem and this structure will influence the difficulty of the problem. Nunes (2001) reports that young children show some basic knowledge of multiplication and division and are able to use their understanding of multiplication to solve division questions much earlier than they are able to use division strategies to solve multiplication problems. Anghileri (2001) also found evidence that children as young as 5 years old have some understanding of equal grouping but have very little idea about the divisor/quotient relationship, even though they are able to share.

Table 2, which I have taken from Greer (1992) and Verschafffel and De Corte (1996) presents a summary of a classification of problems types for multiplication and division.

Table 2: Classification of Multiplication and Division Problem Situations

| | | |
|---|---|---|
| Asymmetric problem situations | | |
| Equal groups (Rate) | 3 children each have 4 sweets. How many sweets do they have altogether? | Multiplication problem |
| | 12 sweets are shared equally among 3 children. How many sweets does each child get? | Division by multiplier |
| | If you have 12 sweets, how many children can you give 4 sweets? | Division by multiplicand |
| Multiplicative comparison | Themba has 3 sweets. Notozi has 5 times more sweets than Themba. How many sweets does Notozi have? | Multiplication problem |
| | Themba has 15 sweets. Notozi has 5 times less sweets than Themba. How many sweets does Notozi have? | Division by multiplier |
| | Themba has 3 sweets. Notozi has 15 sweets. How many times as many apples does Notozi have? | Division by multiplicand |
| | | |
| Symmetric problem situations | | |
| Rectangular pattern (Array) | How many pieces are in a chocolate 3 pieces long and 5 pieces wide? | Multiplication problem |
| | A bar of chocolate has 15 pieces. The bar is 3 pieces long. How many pieces wide is the chocolate? | Division problem |
| Cartesian product | If there are 3 routes from A to B and 4 routes from B to C, how many different ways are there of going from A to C via B? | Multiplication problem |
| | If there are 12 different routes from A to C via B, and 3 routes from A to B, how many routes are there from B to C? | Division problem |

Brown (1978) reports that children recognise rate situations as repeated addition and usually represented these problems as collections of equal groups. Rate models can include groups of objects or continuous lengths. For example, figure 5, illustrates the solution to the following problem: *A hen lays 15 eggs. How many eggs can we put* *into 3 baskets?* The child represents the problem firstly by drawing 15 eggs, then by drawing three baskets with five eggs each and finally numerically in a number sentence.



Figure 5: Grade 1 learner's solution to a multiplication problem involving eggs (amaqanda is the Xhosa word for eggs). Kühne and Ernstzen (2003)

On the other hand, rectangular pattern (array) problems are more complex and therefore difficult for children to recognise as multiplication problems (Greer, 1992). Figure 6 is shows an example of how a child represents the following problem: *A farmer plants three rows of carrots. There are four carrots in each row. How many carrots does she plant altogether?* The child drew rows and columns of carrots, then indicated the number of carrots per row. The child then writes 3 x 4 but adds the numbers 3 + 4 and writes 7. This step is followed by writing 4 + 4 + 4, but in order to find the total the child uses tallies and counts in ones the number of carrots.

Figure 6: Grade 3 learner's solution to a multiplication problem involving carrots

Nunes (2001) states that research on children's understanding of multiplicative reasoning has had less impact on teaching than the research on additive reasoning. Nunes adds that classification of multiplicative reasoning problems do not have the same privileged treatment in primary school teaching and that problems of multiplicative reasoning are often ignored. Teachers encourage the common misconception that multiplication makes bigger and division makes smaller and that division is always division of the larger number by the smaller number (Nunes, 2001 and Anghileri, 2001). This misconception is connected to early ideas relating multiplication to 'repeated addition' and division to 'sharing or repeated subtraction' (Nunes, 2001; Anghileri, 2001). This interpretation is inadequate to deal with the variety of multiplicative situations (Hiebert and Carpenter, 1992; Greer 1992; Carpenter, Fennema and Franke, 1996) and has a long lasting effect and limit for later interpretations and for when integer numbers are introduced (Anghileri, 2001).

Anghileri, (2001) points out that the structure of a contextual problem, which involves equal groups, can influence which number is to be repeatedly added. The language used when presenting a contextual problem is an important factor as different phrases influence the strategy children will use to solve a problem. For example interpreting 8 x 5 as '8 times 5' or '8 lots of 5' could lead to a less efficient calculation than' 8 multiplied by 5' or '5 eights'. Anghileri (2001) states that children need help to interpret formal mathematical phrases and to identify meanings with appropriate solution strategies. She also reports that specific teaching is needed in order for children to develop the ability to use commutativity when solving multiplication problems. She adds that although division does not obey the commutative rule, young children will attempt to apply it when solving certain division problems, like $3 \div 6$.

It is accepted that children construct their (mathematical) knowledge and create their own mental (internal) representations while interacting with their world and build their own networks of these representations (Hiebert and Carpenter, 1992). Children's inventiveness does not always lead to productive mathematics, so if written symbols are not connected to understandings and networks of conceptual knowledge they may not be able to calculate accurately. However, if children's invented strategies are based on mental representations that are connected within a network and related to other existing knowledge, their understanding is more likely to connect with new representations.

Hiebert and Carpenter (1992) explain that the generation of understanding promotes remembering, as children represent information in ways that fit in with their existing networks of knowledge and structures. They impose meaning onto their representation while modifying these so that they can be more easily remembered.

Figure 7 illustrates several different ways a child keeps track of and represents his or her thinking while solving the following problem: *Four children can sit at one table. How many children can sit at 3 tables?*

Figure 7: Grade 3 learner's solution to a multiplication problem involving tables

*3.10 Summary*

Chapter 3 presents an overview of literature, which constitutes background research for this dissertation. The chapter presents a review of literature pertaining to children's learning of number and the development of number sense, in particular the ways in which number concepts and numerical facts are acquired. The focus in this review is on two theories of how children learn number and the processes involved in this learning. Von Glasersfeld and Steffe provide theoretical perspectives on the construction of number and offer a coherent theory of the development of scheme associated with learning number and related concepts and facts. Steffe's framework shaped the research design of this study and was used in the systematic analysis of the data collected in this investigation.

Von Glasersfeld and Steffe's theories focus on the cognitive processes involved in learning number. Other literature, which examines pedagogic processes involved in the learning and teaching of number, are also briefly presented in this chapter. These include an overview of the literature related to understanding the base-ten numeration system and the use of numbers in different contexts. Classification of addition, subtraction, multiplication and division problems and a review of literature on problem solving, number representation and children's solution strategies is also included in this chapter.

The main features of Piaget's and Vygotsky's work is contained in Appendix 2. These theoretical contributions to the field of cognitive development have influenced curriculum development and pedagogy in recent years and for this reason, I have included them in this dissertation.

The investigation reported in this dissertation focuses on the development of number concepts and number relations within the domain of whole number. In the chapters that follow, the key features of this discussion will be used to describe the research design used in this study; and to analyse the data collected in the interview transcripts.

# 4 RESEARCH DESIGN

## 4.1 Introduction

In the previous chapter I reviewed how learning and teaching of early number is understood in literature. I used Steffe's theoretical framework on the development of counting and calculation as this provided me with a model and language to describe teachers' discourse on early number acquisition and related pedagogy. The purpose of this chapter is to describe the process I followed in order to find out what teachers say about how children learn number. I describe the research design of the study, the sample and setting, instruments used for data collection, the data collected, the method used to analyse the data and an evaluation of the research process. Also in this section is a brief sketch of the pilot study which was used to determine the validity of the instruments.

## 4.2 Sample and Setting

This was an exploratory study which is qualitative in nature (Mouton, 2001) and set out to provide an in depth description of how Foundation Phase teachers understand the ways in which young children learn number. I selected 6 Foundation Phase teachers from 13 Cape West Coast Winelands schools who had all participated in a school-based professional development project called GetinSet between 1998 and 2001, and had volunteered to be part of a key teacher programme, the Professional Development Team (PDT). GetinSet was a partnership project which focused on the development of School Management, Mathematics, Science and Literacy in 85 schools in the Western Cape. Support was provided to teachers in the form of long and short non-accredited courses and school based workshops and classroom support by the University of Cape Town's Schools Development Unit (SDU), the University of the Western Cape's Teacher In-service Project (TIP) and the Primary Science Programme (PSP).

My decision to include 6 teachers from this project was based on the fact that they had all received considerable mathematics content and pedagogic input through the intervention's courses and classroom support at their schools. It was significant to this study that these teachers were part of the Professional Development Team as they offered content and pedagogic support to their colleagues at school. My interest in these key teachers was that they represented a sample of teachers with specialised mathematics knowledge and I was keen to explore how they understood the ways in which children learn number.

Since this study focuses on how teachers understand the ways in which children learn number, my sample selection took into account teachers who were currently teaching at the Foundation Phase level, or had done so in the past 5 years, and consequently have mediated the transition from 'informal' pre-school to 'formal' classroom teaching.

In order to collect data on how teachers understand the ways in which children learn number, I decided to conduct loosely-structured interviews based on short classroom episodes or vignettes. The structure and selection of the vignettes will be discussed below. My concern was with obtaining full and sincere responses to a relatively open-ended enquiry (Hitchcock and Hughes, 1989). Each teacher was interviewed once and the interviews were conducted at the teachers' convenience. This involved interviewing each teacher at her school, after the children had been dismissed.

### *4.3 Research Design*

As mentioned above, the instrument used to gather information about how teachers discuss and articulate their understanding of the ways in which children learn number was a semi-structured interview (APPENDIX 4) based on constructed virtual vignettes. The intention of the interview was to elicit information about teachers' knowledge and understanding of the developmental processes underpinning the learning of number, and the didactics they use when mediating this learning in the classroom. This investigation did not focus on learners and did not include any classroom observations.

The intention behind the development of these particular vignettes was that they should encourage fairly detailed responses from teachers to classroom situations and I hoped that these responses would reveal underlying thinking and perceptions related to learning and teaching early number. The design process and rationale for each is based on the developmental stages of Steffe's framework for number. The motivation for each vignette and related questions was that these would give teachers an opportunity to discuss particular features of children's' number development.

### *4.3.1 Method of Interview*

The interview method is similar to the Stimulated-Recall Interviews (Dunkin et al., 1998). A vignette represents a virtual lesson and participants were invited to respond to the situations presented in this form. Each vignette consisted of characteristics that relate to learning and teaching situations involving number activities. Each participant was presented with three vignettes focusing on different aspects of number concept development, in various contexts, and related questions. Vignettes describe short, self-contained, hypothetical episodes of classroom situations and provide images of mathematical teaching in practice, in such a way as to make the situation accessible to interpretation on various levels. The language used in the vignettes is authentic and presents the scenario from the teachers' perspective.

This method of interview provided a means of exploring subjective understandings and embedded attitudes and beliefs. It encouraged reflection and discussion at a practical and a theoretical level and gave participants the freedom to respond freely to familiar classroom situations. The situations offered to teachers were hypothetical and designed to reflect occurrences that teachers might recognise as part of their own teaching experience. Although vignettes provide a means to explore participants' perceptions, there is no guarantee that they will in fact offer their inner thoughts truthfully.

Steffe's (2000) theoretical framework[1] provides an explicit description of the development of the learning process and counting stages in the development of number. This framework details a hierarchy of necessary stages in the development of number and the learning process children follow, which can successfully be used to provide insights into, and description of, children's knowledge strategies and solutions to number problems.

[1] Steffe's (2000) theory is discussed in detail in Chapter 3 of this dissertation

The main components of Steffe's (2000) theoretical framework for learning number (presented in table 3 below) was used to guide the selection and design of the vignettes. These stages are not intended to be distinct from one another or to be interpreted as absolutely sequential but rather to be understood as overlapping and interrelated (Wright 1988). Furthermore this framework is not intended as an instructional sequence to be followed rigidly by teachers but as a tool for describing, assessing and documenting children's attainment levels.

### *4.3.2 Description of the Instrument*

The theoretical framework for learning early number (that is based on Steffe's theory) presented in table 3 was used to inform the design the vignettes (APPENDIX 4) used in the interviews with teachers.

Table 3: Theoretical Framework for Learning Early Number

| THEORETICAL FRAMEWORK FOR LEARNING EARLY NUMBER | |
|---|---|
| Stages | Expectations |
| Emergent Counting | - recognise patterns in small collections (subitise & sense of number)<br>- know difference between small collections<br>- recognise bigger collection<br>- understands different meanings/representations of numbers<br>- counting sequence as a verse (acoustic)<br>- symbolises using fingers<br>- does not know number words or cannot coordinate and assign number words with objects<br>- acoustic counting |
| Perceptual Stage | -counts what can be 'perceived'<br>- cannot determine total without contact<br>- process of counting & result not connected<br>- sequence number in correct order (ordinal aspect) |
| Figurative Stage | - sensory motor<br>- counts items that 'stand in for' objects<br>- represents physical numbers with fingers, lines, dots etc<br>- unitary strategies counts 'all'; re-counts in ones<br>- relies on one-to-one<br>- orders, compares and estimates<br>-last stated number indicates total quantity counted (resultative or cardinal aspect)<br>- connect ordinal and cardinal aspects<br>- equal grouping and counting without visible objects |
| Initial Number Sequence | - conceptual<br>- represents objects mentally<br>- counts concealed, imagined objects<br>- groups or chunks information<br>- counts 'on' from first or biggest number<br>- counts forward and backwards within a sequence |
| Tacitly-nested Number Sequence (Transitional stage) | - counting is facile (effortless)<br>- numerical composite – ten constructed as ten ones<br>- counting sequences become grouped or 'nested' within<br>numbers<br>- combines/joins and partitions/separates collections<br>- range of strategies<br>- automised basic addition and subtraction facts<br>- multiplication and division schemes – uses composite units in repeated addition and subtraction |
| Explicitly-nested Number Sequence Stage | - extracts from a sequence<br>- abstract composite unit – ten treated as single unit while recognising it contains ten ones<br>- iterable unit – ten is a unit which can be repeatedly constructed in place of ten individual ones<br>- abstract collectable unit (split method)<br>- structure multi-digit numbers<br>- contextualise multi-digit numbers<br>- coordinates composite units as an operation eg 2 threes<br>and 2 times 3 is 6 |

*4.3.3 Stages across Vignettes*

Table 4 indicates which vignettes were designed with the intended purpose of eliciting discussion across the various stages of number development. When I designed the vignettes I thought that the classroom situations would give participants opportunities to respond to particular aspects of learning and teaching number. In particular I wanted to encourage discussion which could be located within the conceptual framework for number development presented by Steffe.

I included characteristics of each stage described by Steffe, in the vignettes. As table 4 shows, two vignettes include elements of the Emergent Counting stage, while vignette 2, 3 and 4 contain aspects of the Perceptual Stage of number development. Vignettes 1, 2 and 4 cover the stages of the Initial Number Sequence Stage, Tacitly-nested Number Sequence and Explicitly-nested Number Sequence Stage. While examining the participants' transcripts I was interested to note how their comments related to and corresponded with the structure of this number framework.

Table 4: Design of Vignettes

| STAGES ACROSS VIGNETTES | | | | |
|---|---|---|---|---|
| Stages | V1 | V2 | V3 | V4 |
| Emergent Counting | | * | * | * |
| Perceptual Stage | | * | * | * |
| Figurative Stage | * | * | | * |
| Initial Number Sequence Stage | * | * | | * |
| Tacitly-nested Number Sequence | * | * | | * |
| Explicitly-nested Number sequence Stage | * | | | * |

V = vignette

*4.3.4 The Vignettes, Related Questions and Rationales*

**Interview 1: Vignette 1**

A class is investigating addition by using counting rods. The learners are working in pairs. The educator asks one learner in each pair to represent and write the number 34, and the other learner to represent and write the number 27. The learners are then asked to combine the rods and tell her how many there are altogether. She asks several pairs to explain how they found their answer. She then demonstrates a method for recording what the learners have done with the rods.

Please consider the following questions:

1. What do you think the educator is hoping to achieve from this activity?
2. What number concepts do you think are being developed?
3. How do you think learners will represent the numbers?
4. What method do you think the educator would have demonstrated?

*Rationale for Interview 1*

This vignette is not intended to illustrate good practice or to suggest that the approach being used is one that I would necessarily advocate. The situation describes a whole-class teacher-guided activity in which children are using structured apparatus to solve a problem. The strategies used by the educator include demonstration and discussion.

The scenario is designed to elicit discussion specifically about the learning and teaching of two-digit number representation and calculation. The questions were designed to encourage discussion about knowledge of number structure, basic calculation skills and strategies necessary to understand and successfully solve problems involving large numbers. Important aspects of the discussion could include comments on sequencing and recognising numbers up to 100, skip counting in tens and the need to memorise basic number facts to 10.

It was hoped that this scenario would lead to a discussion about the learning process involved in the development of multi-digit understanding and descriptions of strategies and solutions children might use to solve the problem. In particular, it was expected to elicit comment on 'grouping' while working with larger numbers and the special role played by ten in our number system. An important aspect of this discussion is understanding for example, that 27 involves knowing that 2 is to the left of the 7 and that 27 is either twenty-seven single one or two tens and seven separate ones (understanding the positional value of the digits or place value).

I also hoped that part of teachers' discussion would focus on the development of counting strategies, beginning with ten being treated as ten ones and the strategy children use to determine the ten at this stage which involves counting each single unit (numerical composite). I also hoped that teachers would discuss what progression from this level of understanding involves, in terms of treating ten as a single unit and still recognising that it consists of ten ones (abstract composite). Lastly I hoped that some discussion of the understanding that children should be able to treat ten as a unit, which can be repeatedly constructed in place of ten single counts or ones (iterable unit), before they are able to solve two digit addition and subtraction calculation problems successfully.

The learners in the scenario sketched above are expected to count rods by tens and units in order to find the total of the two numbers, and represent this. I hoped that teachers would describe the different models and representation that learners might use to solve this problem. Models that teachers could discuss for representing the structuring of two digit numbers could include: number lines; bundling groups of quantities, like the counting rods or drawings (tallies or dots); a 1 – 120 number square or individual expanding number cards; numerals and words.

An important element to this discussion pertains to strategies children use to count and combine counting rods and how their solutions depend on how advanced their counting skills are and the sophistication of their solution strategies. Much of this discussion depends on teachers' own instructional style and preferred method of teaching multi-digit concepts to children. The vignette was selected in the hope that this context would motivate teachers to talk about various components necessary for successful learning and teaching of place value. In addition, it involves discussion of the decimal number system as well as the understandings and knowledge needed to successfully calculate with large numbers and formal column algorithms.

**Interview 2: Vignette 2**

A group of learners sits around a Spaza shop food poster. Each learner has a pencil, crayons, paper and a bakkie of R1 and R5 plastic coins. The educator asks them to choose items from the poster that they would like to buy and to calculate what it would cost.

Deno chooses chips for R4 a chocolate for R3 and a coke for R5. The group discusses his choice and begins to solve the problem for themselves.

Deno: Draws the four packets of chips, three chocolates and five cokes. He counts them all and writes 12.

Freeda: Puts up 4 fingers. She then puts the tips of her fingers into her mouth and nods as she counts '5, 6, 7, 8, 9, 10, 11, 12'. '12!' she says

Zondi: Counts out R1 coins: 'R1, R2, R3, R4, R5, R6, R7, R8, R9, R10, R11, R12'

Lisa: Takes out R5 and R1 coins and counts 'R5, R10, R11, R12'

Ayanda: Looks at the teacher and says 'R12'.

Fran: Says '5 and 4 is 9 and 1 and 2 is 12'

The learners then explain what they have done and why they have presented the problem in their particular way.
(Adapted: Ans Veltman, Van den Heuvel-Panhuizen, 1999)

Please consider the following questions:

1. What do you think the educator is hoping to achieve from this activity?
2. Comment on each child's solution strategy.

*Rationale for Interview 2.*

This problem situation offers participants an opportunity to talk about the different solution strategies learners' use, depending on their understanding and knowledge of number. The solutions presented in this vignette offer a cross section of learners' understandings at different stages. They illustrate a range of strategies, which could be used to encourage learners to use more efficient strategies and to inform further planning and teaching. A cross section of strategies that informs the teachers about the longitudinal trajectory is one of the key characteristics of a learning-teaching trajectory (Van den Heuvel-Panhuizen, 2002).

This vignette attempts to highlight the learning process of individual learners in a whole class or group situation, and how they apply numbers in problem situations. The solutions reveal attainment at different stages of development and counting strategies commonly used by learners. Educators were encouraged to reflect on the problem and the different learners' responses and to describe them in terms of the progressive stages each one illustrates (outlined in table 3). This scenario was intended to elicit discussion on solutions in terms of different skills and levels of understanding from perceptual (concrete) stage to the abstract stage and describe how children symbolise and represent their thinking at each stage.

Teachers may discuss addition strategies as well as the use of money in the context. Each learner represents the solution to the problem according to the level at which they understand it. Deno, for example, operates at a Perceptual Stage. He illustrates the number of items he selects from the poster and counts each one. Freeda is operating within the Figurative Stage in that she is able to symbolise the items with raised fingers and a perceptual movement as she counts. Neither child responds to the problem involving money and appears to only recognise the numerical symbols.

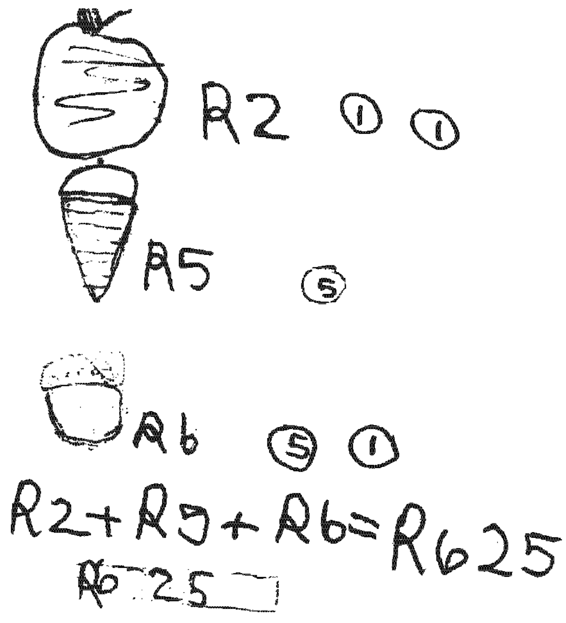
Zondi and Lisa physically count out coins to solve the problem and mentally re-represent the problem as they place the coins and count – Zondi in one's and Lisa in five's and one's. These strategies locate the two children in the stage of the Initial Number Sequence Stage. Ayanda and Fran have calculated the problem mentally, Fran by deriving number facts and breaking up numbers to make a ten, and Ayanda appears to have automated basic number facts and drawn on these to solve the problem.

This scenario also offers the possibility to extend the discussion about grouping collections and calculating with tens. This vignette has the potential to elicit responses regarding constructivist models of learning and teaching, in particular the use of contextualised problems, collaborative learning and the role of the teacher. This vignette lends itself to discussion about different kinds of problem contexts that generate different number possibilities and involve a range of mathematical tools and understandings. This discussion could include the critical role the teacher plays in guiding and determining the activities and outcomes of instruction.

Another point of discussion could focus on the recognition and inclusion of children's informal solutions and how they construct mathematical insights through their products and how the teacher can build on these to encourage more advanced insights and sophisticated solution strategies.

**Interview 3: Learner Solution**

**Look at this example of a copy of one Grade 3 learner's solution.**




Please consider the following questions:

1. What is your response to this learner's solution?
2. What prior knowledge and skills do you think the educator should have assessed before giving the learner a problem of this type?
3. What would you do to assist this learner to solve this type of problem accurately and efficiently in the future?

*Rationale for Interview 3*

The spaza shop activity described in vignette 2 was used with teachers as part of a professional development programme in one project school and offered to children in grades 1, 2 and 3. The solution in interview 3 was presented by a Grade 3 child and was selected for discussion on the basis of the range of strategies the learner appears to have selected in an effort to calculate the total amount spent on items. Although this is a simple addition problem the complexity of learning to calculate is demonstrated. This vignette was chosen for its potential in eliciting comments about learning and teaching basic operations and an understanding of number facts.

A critical point of this discussion is that the child's solution has two clearly distinct parts. The first is the representation of the problem and the second is the calculation and completion of the number sentence. The intention of this vignette is to motivate discussion about what appears to be a totally incorrect solution. In the first part of the solution the child has successfully represented the items selected from the spaza shop poster with corresponding prices. These have been accurately written and the coins drawn alongside. The indication is that this child has a good contextual understanding of the notation and corresponding representation of money and is able to break up coins to represent the equivalent value, for example R2 and the two one rand coins and R6 and the five and one rand coins. This child appears to assimilate symbolism of number in terms of what he or she already knows, and reflects an understanding of the different meanings of number (money) and ways of representing this at an emergent stage only.

Discussion on the second part of the solution could include the fact that the learner is not able to accurately add the amounts although does record the number sentence accurately. In terms of Emergent Counting, this learner has not demonstrated an understanding of the problem or the ability to calculate the total cost spent on the selected items. The ability to count and combine each amount and a good numerosity (cardinal value) are essential prerequisites to solving this problem. This child is not able to determine the value of the three amounts at a basic counting (perceptual) stage. The digits 6, 2 and 5 have been randomly placed following the equal sign even though they appear in the first and second representations as R2, R5 then R6.

The vignette is intended to elicit comments regarding these different aspects and representations and to encourage teachers to focus on the fact that even though this child is not able to complete the number sentence accurately they are able to break up the numbers when they are using (drawing) the coins and do have some concept of number.

**Interview 4**

Please consider the following questions:

1. Describe a learner in your class who you think has a good concept of number.

   * Why do you think this learner has a good concept of number?

2. Describe a learner who you think has a poor concept of number.

   * Why do you think this learner has a poor concept of number?

3. You are asked to support a student or colleague in the teaching of counting.

   * What would be the main points you would focus on?

   * What suggestions would you make on the learning and teaching of this topic?

*Rationale for Interview 4*

The questions in interview 4 have been selected in order to invite educators to talk about models for the learning and teaching of early number and their understanding of what constitutes this domain. Interview 4 and related questions allow teachers the opportunity to conclude the series of interviews with an overview of their understanding and to give descriptions of learners at different stages and examples of activities to support their comments. These questions are intended to elicit details about what teachers consider to be the essential content of instructional programmes designed to support the development of children's early number knowledge, and the particular features, range and diversity of children's strategies as they progress along a developmental path.

The last question regarding the suggestions for learning and teaching number is intended to draw on teachers' experience in practice. Didactic principles that inform the ways teachers organise and sequence learning and their instructional programmes will be described.

### *4.4 Data Collection*

Data was collected in the second term of 2003. Each participant was interviewed once. Five of the interviews lasted for approximately two hours and one interview lasted for one hour. Participants were first asked to complete the teacher questionnaire in order to establish accurate teacher profiles (APPENDIX 3). A series of questions were used during the interviews in an attempt to probe what teachers think about how children are learning in the different contexts. The questions were designed to act as prompts to encourage each respondent to express opinions about certain aspects of learning and teaching. These questions were open-ended and intended to elicit teachers' perceptions and deeply embedded beliefs about how children learn number.

#### *4.4.1 Teacher Profiles*

The six participants interviewed in this study were each asked to complete the teacher questionnaire. The teachers' questionnaire is an adapted version of the instrument used by Jaffer (2001). The following table summarises the biographical and professional data which was collected. The left hand column shows the question as stated on the questionnaire. The six right hand columns indicate participant responses to each of the items in the questionnaire.

Table 5: Teacher Profiles

| Teacher Profiles | | | | | | |
|---|---|---|---|---|---|---|
| Question | P1 | P2 | P3 | P4 | P5 | P6 |
| 1. Grade you are currently teaching | 5, 6, 7 | 2 | 1-6 | 3 | 2 | 1 |
| 2. How many learners are there in your class? | 40 | 35 | 42 | 42 | 36 | 70 |
| 3. Where is your school situated? | peri-urban | peri-urban | peri-urban | peri-urban | rural | peri-urban |
| 4. How many learners attend your school? | >500 | >500 | >500 | >500 | 200-300 | >500 |
| 5. Are you female or male? | female | female | female | female | female | female |
| 5. Which age group do you fall into? | 40-49 | 30-39 | 40-49 | 30-39 | 50-59 | 30-39 |
| 7. What is your medium of instruction? | Afrikaans | Afrikaans | Afrikaans | Afrikaans | Afrikaans | Xhosa |
| 8. By the end of this year, how many years will you have been teaching? | 30 | 11 | 21 | 19 | no answer | no answer |
| 9. At which of these grade levels have you taught in the past 5 years? | 3, 4, 5, 6, 7 | 1, 2 | 1-6 | 1, 3 | 1 | 1, 2, 3 |
| 10. How often do you have meetings with colleagues? | weekly | weekly | monthly | weekly | daily | Bi-monthly |
| 11. What is your highest level of formal education? | M+3 | M+4 or 5 | M+4 or 5 | M+3 | M+3 | M+3 |
| 12. What was the year in which you received this qualification? | 1990 (matric) | 1997 | 1992 | 1984 | 2003 | 1994 |
| 13. At which institution did you receive this qualification? | 1994 | Bellville Teachers Training College | Cape Teachers Training College | Hewat Teachers Training College | Bellville Teachers Training College | Good Hope Teachers Training College |
| 14. Have you been part of any Mathematics inset programmes or projects? | yes | yes | yes | yes | yes | yes |

P = participant

Afrikaans is the dominant language spoken in the West Coast Winelands area of the Western Cape. As table 5 shows four participants taught at Afrikaans medium primary schools. These schools were located in an industrial area outside Cape Town called Atlantis. One teacher taught at a small Afrikaans medium primary school situated on a wine farm outside Malmesbury, 75 kilometres from Cape Town. One teacher taught at a large Xhosa medium school situated in an informal settlement outside Malmesbury. The number of children in the Foundation Phase classes at these schools ranges from 40 to over 70. All the teachers involved in this study were women, each of them had taught at the primary level for more than 10 years, and each had experience teaching Grade 1.

As table 5 indicates, one participant indicated the year in which she graduated (1994) and not the institution at which she received her last professional qualification.

Participants were asked to indicate the highest formal qualification they had received. As table 5 shows three participants involved in this study had a matriculation certificate plus three additional years of tertiary training (M+3), while three of the participants had completed a matriculation certificate and had studied for a further four or five years (M+3 or M+4). Participant 1 indicated in table 5 that she had received her matric in 1990. The reason for this information is that she received her teacher training qualification prior to completing her senior school certificate (matric). One participant completed her studies in 1984, four participants completed their studies between 1990 and 1970 and one participant completed her studies in 2003. Table 5 indicates that two participants studied at Bellville Teachers Training College and one participant at Hewat Teachers Training College[2]. One participant studied at Cape Teachers Training College[3], and another at the Good Hope College of Education[4].

*4.4.2 Conducting the Interviews*

Participants were asked to respond to the vignettes, learner's solution and questions in the interviews. I did not prompt participants with additional questions or engage in any discussion about learning and teaching number prior to the interviews. I tried to

[2] These teacher training centres were under the previous South African segregated 'coloured' House of Representatives Education Department.

[3] This teacher training centre was under the previous South African segregated 'white' House of Assembly Education Department.

[4] This teacher training centre was under the previous South African segregated 'African' Department of Education and Training Department.

maintain the same interview conditions with each of the participants. This was difficult at times as I have a particularly good working relationship with these teachers, which includes reflecting on many aspects of teaching and learning. As I mentioned in section 4.2 none of the teachers who participated in this study spoke English as their first language. Participants 1 and 3 spoke Afrikaans and participant 6 spoke Xhosa but decided to conduct the interview in English. Participants 2, 4 and 5 responded entirely in Afrikaans (APPENDICES 11, 12 and 13). These transcripts were translated into English before analysis.

An audio recording captured the interview sessions and each participant's response was fully transcribed (APPENDICES 5 - 13). Participants were asked to read each vignettes and associated questions before the session began. I suggested that they make notes on the forms as this would help them remember their responses and provide necessary cues during the interview. Initially participants felt inhibited by the tape recording of the interviews. I explained how the tape recorder was switched on and off and asked participants to operate the machine themselves so that they could control the taping. I explained that we could not discuss the vignettes unless we were recording our conversation as I needed to capture everything we said. Three participants were initially inhibited but soon relaxed and conducted the interview without commenting on the tape recorder.

One participant was very anxious about the recording of the interview and I had to reassure her. Each time the recorder was switched on she giggled uncontrollably and switched it off again. Eventually she relaxed and the interview continued without further interruption. Another participant said very little while the recorder was switched on, and subsequently her interview is very limited. One participant insisted on switching the machine off and asking me what it was that I wanted her to say in response to particular questions. I assumed that she wanted to first gauge whether she was saying what I wanted her to say. I reassured her that there were no correct or incorrect responses and that I wanted to hear what different teachers had to say about their own teaching and learners. This participant accepted my explanation and the interview continued.

*4.5 Mode of Analysis*

The questionnaires and interviews in this study generated a set of both structured and unstructured data. The structured data is drawn from the teacher questionnaires and is shown in table 5 in section 4.4.1. The unstructured data is presented in the transcripts of the interviews. The analysis of the data proceeded in two ways. Initially I worked deductively with the transcripts, using the theoretical elements of Steffe's framework to describe and classify teachers' comments according to cognitive development of number. Thereafter I worked more inductively with the data in order to describe an emerging discourse based on teachers' ideas of didactics.

In the first phase of data analysis I read each interview transcript several times and noted significant comments in the margins. This process involved analysing each teachers' comments relating to each vignette. I then compared each interview and my notes and looked for common themes. Once I had completed this process I began the second phase of analysis which involved re-reading the transcripts and noting participant comments which related to cognitive development of number according to Steffe's theoretical framework. I also noted those themes which related to classroom management and teaching styles. I interpreted these comments according to various aspects of learning and teaching number, and classroom organisation. In this way I extracted comments which related directly to cognitive processes and number development, and comments which described more general pedagogic understandings of how children learn. In this phase I identified categories by coding comments which related to developmental sequences and factors which influence learning and teaching number.

*4.6 Validity and Reliability*

In order to check the validity of the instruments used to collect data, the vignettes were presented to two mathematics specialists at the University of Cape Town's Schools Development Unit (SDU) for comment. They were asked to read the four interviews and comment on the effectiveness of the classroom episodes, questions and prompts. This feedback was necessary in order to establish whether the instruments captured the complexity of learning and teaching in the situation and whether they offered a useful tool for engaging participants to share their thoughts and feelings. The suggestions and recommendations were used to adapt the first draft of the instruments. I then asked two colleagues at the SDU to participate in a pilot study. Both pilot participants had previously taught Grade 1 and 2 and one had taught Grade 6.

One pilot participant had 23 years teaching experience at the primary school level. She had been involved in mathematics key teacher programmes and had completed a Mathematics Further Diploma in Education (FDE) Certificate through the SDU and was registered for a Bachelor of Education (BEd) at the University of the Western Cape. She had been seconded by the SDU to provide support to teachers in the Overberg area (about 120 kilometres from Cape Town) through one of the SDU's school based projects, namely Management For Learning (MFL).

The other pilot participant had taught at the Foundation Phase level for 18 years and had a four year teacher training diploma. She had been involved in an environment teacher training programme for a year. She was contracted by the SDU to convene a Foundation Phase Mathematics course at the Western Cape Teachers Institute (CTI).

The interview was conducted with each pilot participant and their responses were audio taped and transcribed. They were also asked to comment on the teacher questionnaires. This process of validating the instruments made certain that the instruments did indeed provide a context for and would elicit discussion on the ways in which children learn number. Instructions and questions related to the vignettes that the specialists and pilot participants felt were not clear or did not achieve the intended results were either discarded or re-worded. I ensured that the vignettes and related questions were clear and dealt with underlying aspects of learning and teaching number that were contextualised in situations that Foundation Phase teachers would be familiar with. The opinions of these specialists and the pilot participants on the validity of the vignettes and questions ensured that the instrument would achieve the purpose of establishing how teachers understand the ways in which children learn number.

The reliability of the instruments was established during the process of analysis. The guidelines I used for analysing each set of data were precise. Initially each participants' comments relating to the vignettes and associated questions was analysed (see section 4.5) and the comments were compared across the vignettes. I was able to draw the same conclusions using the instruments for each of the participants' interviews. Based on the analysis I concluded that the instruments successfully illustrate the purpose for which they were constructed.

### *4.7 Summary*

The previous chapter provided the analytic framework which would be used for analysing teachers' responses to the interviews. This chapter frames the design of this research project. The sample and setting describe the background which locates this study in a particular context. The research design describes the method used to collect data. A detailed discussion of the interview instruments was presented for the purpose of highlighting the process of collecting the data. The discussion pertained to the vignettes and open ended questions which form the semi-structured interview presented to each participant. Of particular interest in this section was the motivation and rationale behind the vignettes and related questions. Also in this chapter is a brief discussion about the pilot test and validity and reliability of the instruments.

The following chapter in this thesis focuses on the results of the analysis of the data, which is presented in two parts. The first part presents participant comments related to the theoretical framework that was used for analysing the interview data and the second part presents participant comments on other didactic themes.

# 5 *RESULTS*

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*5.1 Introduction*

Chapter 5 presents the results of the analysis of the teacher interview data. The chapter presents the didactic discourse and highlights teachers' perceptions of how children learn number. Quotes from the English interview transcripts (APPENDIX 5 - 10) are incorporated in the discussion to illustrate the results.

In this study I analysed the data from the teacher interviews by looking for regularities and patterns in the ways teachers express their knowledge and understanding of children's learning of number. The findings of this investigation will be presented in two parts. Firstly, the results are given from the systematic analysis of which the focus was on teachers' discourse related to knowledge and perceptions of how children learn number according to Steffe's theoretical perspective on number development. Secondly, the results are given from the analysis in which I looked for didactic themes that emerged from the transcripts, which were not directly related to Steffe's framework, and which have been coded in terms of didactic categories.

The interviews conducted with Foundation Phase teachers were intended to examine their beliefs about the learning and teaching of early number. The design of the vignettes and related questions provided opportunities for teachers to discuss specific aspect of this development. Much of what participants expressed and described was stated explicitly and ideas about types of mathematical understandings and instructional strategies emerged through participants' statements. While reviewing the transcripts I noticed several common themes or threads emerging from the participants' interviews about the ways in which they think that children learn and should be taught number.

*5.2 Presentation of Results in Two Parts*

Having analysed the transcripts I chose to present the data in two parts. The first part relates to cognitive development and the second part relates to pedagogic themes. Table 6 reflects the participants' comments in terms of the stages outlined in Steffe's framework for learning number. These stages are indicated in the left hand column. *P* signifies the participant and the numbers in each participant column reflect the number of times a particular stage was referred to. Table 7 indicates the pedagogic categories that emerged from the analysis of the interview transcripts. Comments that fall into this category are not related directly to the theoretical framework, but reflect other didactic themes. The left hand columns in table 7 indicate the categories and subcategories I identified in the transcripts while the right hand columns indicate the participant and number of times comments were made regarding these didactic aspects of learning and teaching number.

*5.3 Part One: Cognitive Development*

Having coded the participant's transcripts according to the theoretical framework on number development I presented a brief overview of each stage and then discussed participants' responses in terms of the main features highlighted in the framework. These will be illustrated with quotes from the transcripts to support my conclusions. As mentioned above, Table 6 presents comments which relate directly to the stages of number concept development outlined by Steffe.

Table 6: Results Related to Cognitive Development

| COGNITIVE DEVELOPMENT | | | | | | | |
|---|---|---|---|---|---|---|---|
| Stages | P1 | P2 | P3 | P4 | P5 | P6 | Total |
| Emergent Counting | | | 1 | | | 1 | 2 |
| Perceptual Stage | 2 | | 1 | | | | 3 |
| Figurative Stage | 4 | | | | | 1 | 5 |
| Initial Number Sequence Stage | 3 | | 2 | 1 | 1 | 1 | 8 |
| Tacitly-nested Number Sequence | 4 | 5 | | | 1 | 7 | 17 |
| Explicitly-nested Number sequence Stage | 1 | | 3 | | | | 4 |
| Total | 14 | 5 | 7 | 1 | 2 | 10 | |

P = Participant

*5.3.1 Emergent Counting*

In this category I looked for comments that reflected an understanding of aspects of emergent number concepts and features of children's number development at this stage. Counting at this stage is not typically coordinated, i.e. number words and objects being counted are not matched. Children are however able to recite a sequence of number words but do not associate the act of counting with the numerosity of a collection. Vignette 2, 3 and 4 and related questions give participants an opportunity to discuss the features of emergent counting and number.

As table 6 indicates, two participants mentioned aspects of Emergent Counting. Although they did not elaborate on this topic their comments indicate an understanding that young children at this stage have very little knowledge of number and that counting constitutes acoustic rhymes which have been memorised. For example, children may be able to recite numbers, from 1 to 5 in sequence, but will not necessarily understand the distinct ordinal and cardinal qualities of those numbers. This participant's comments reflect her thoughts about children at the emergent level:

> *..can't write any number or show to a number or even count out. He can count without understanding, he's counting like the parrot but he doesn't understand what he is really doing. (P3:63– 65)*

The following quote captures the feature mentioned above related to emergent counting, i.e. that children's' counting sequence is typically repeated as a verse without an understanding of ordinal or positional value.

> *..when they are coming from Grade R they only come 1,2,3,4,5! (P6: 173)*

*5.3.2 Perceptual Stage*

Children's understanding of number is based on the continued development of mathematical structures that enable them to use different kinds of numbers and solve various number problems. One of the essential structures they develop in the early years of life is the construction of a number sequence. Interview 3 and interview 4 in particular encourage discussion about children's early strategies, representations and an understanding of the development of basic number facts and calculations. Two participants reflected this stage as mentioned in table 6. Two comments reflected an understanding that in order for children to have a sense of the value of number (cardinal aspect) they must be able to match a number word to each counted object or item and co-ordinate the act of pointing with the number word sequence in order to find 'how many' are in a collection. These participants realise that unless children are able to do this they will not be able to determine the total number of objects in a collection. In response to question 1 in interview 3, in which participants were asked to consider the learner's solution, this participant emphatically stated that the learner's work showed evidence that the child did not have any understanding of the value of the symbols.

> *If I look at this learner's solution I mean you can quite clearly see that this child has no understanding about the value of the R2 or R5 or R6. He writes down 2 and then R2 and two ones so all he knows is two 1's gives me 2. (P1:105-107)*

> *He will tell me, the poor one will tell me this is a 5 or this is a 10, but he won't know the value of the 5 or the 10. He won't be able to pack, he doesn't know how to pack out. (P1:195-197)*

This participant's comment reflects an awareness that learners at this stage are not able to combine two collections in order to find the total number in both collections. Furthermore the comment indicates that the items being counted must be within the child's perceptual field as he or she will not be able to count them if they are hidden or represented in abstract form.

> *..this learner writes down everything that he sees. He has no ability for adding, maybe he doesn't understand the sum or whatever he has to do. (P3:34-36)*

*5.3.3 Figurative Stage*

Two participants remarked on counting at this stage and recognised that children can count things that *stand in for* the objects being counted but still rely on sensory-motor activities, like pictures, fingers and verbal cues to support their mental representation of the counting activity. Vignette 1 and 2, which present situations in which counting activities and various strategies are illustrated, and the questions in interview 4, allow for discussion around this feature of number development.

One participant's comments indicate an awareness that children rely on sensory-motor activities and concrete objects in order to complete the counting act at this stage.

*The one with the poor concept of number must start from the beginning, he must every time have the picture from the beginning again and you have to tell him start in 3's from 3 to 18, then he'll say 3, then he'll count on his fingers to get to the 6 and maybe count again to get to the other one, but he won't off-hand be able to count (P1: 205-209)*

The same participant impressed the fact that counting should focus specifically on ordering and comparing natural numbers from 0 to 10. She emphasised that a deep understanding of the relationship between the numbers 1 to 10 and knowledge of each number as a single concept, is a pre-condition to learning the 4 basic operations (addition, subtraction, multiplication and division).

*If the child has a counting rods then most probably he'll just put everything together till he gets to the total he needs to be, make, to get to the 34 or he'll get to the 27. (P1:22-24)*

In addition to recognising that children at this stage represent numbers physically, one participant mentioned the central characteristic of the counting strategy employed by children at this stage, i.e. counting from one and counting all the items in a collection and repeatedly re-counting the whole collection in order to determine the total. This participant commented in response to interview 1:

*Counting...I think when you starting counting you must just start with the ...don't start with even numbers. Start with your natural numbers ne. For example they must start from 0 then 1 and then next to 1 so and so and so. The number before, you always start with the natural numbers when you are introducing counting. Focus is on the natural numbers. (P6:151-154)*

In response to question 2 in interview 4 this participant was discussing what she considered features of poor number concepts. This quote is included in this section because it highlights characteristics of counting in the tacitly-nested sequence stage.

*The one with the poor concept of number must start from the beginning, he must every time have the picture from the beginning again and you have to tell him start in 3's from 3 to 18, then he'll say 3, then he'll count on his fingers to get to the 6 and maybe count again to get to the other one, but he won't off-hand be able to count. (P1: 205-209)*

*5.3.4 Initial Number Sequence Stage*

Interviews 1, 2 and 4 were designed to evoke comments on number development typical of children at this stage. I hoped that participants would discuss development in terms of how children have abstracted their counting and internalised number words and the results of counting and appropriate solution strategies.

During the interview five participants commented on the use of the more efficient or advanced counting strategies children at this stage use, which involves counting on from the first or larger of two numbers in order to solve an additive problem. In response to question 1 in interview 1, in which participants were asked to consider what they thought the educator wanted to achieve, one participant began her discussion stating that the intention of this activity was to add tens and ones, and adds thoughts about the strategy the learner might use to solve the problem.

> *And the other case seems that they had to put the two totals together she wants to learn them the basics of counting on (P1: 9-11)*
>
> *But if they going to put it together they'll also just, if they clever enough, the one will take the 34 and they'll just count on, just add on the 27 to get to the new number. (P1: 24-26)*

Other comments which refer to the counting strategy discussed in the stage of the initial number sequence include simple statements such as *'..for the children to count on.' (P3: 24)*, *'Frieda she has the ability to count on' (P3:27)* and *'Frieda counted on, beginning with four' (P4: 42)* and one statement which implies that the participant feels that there is need for improvement and progression from this strategy: *'And Lisa is still counting on' (P5:34).*

One participant's comment reflects some recognition that the skill of counting back from a particular number is significant in the development of the ability to perform subtraction calculation and problems.

> *And um, maybe we can have some subtraction also here, if she needs to .....want to count back again. (P1: 17, 18)*

Another participant's comments show an understanding that a grasp of positive whole numbers must be established so that learners are able to build the knowledge and skills necessary for solving basic operations and problems in higher number ranges. She points out that a learner who does not have the conceptual understanding necessary to perform addition and subtraction operations is not able to recognise and understand mathematical symbols and basic operations: '*... this learner missed the signs of addition, the main concepts addition, subtraction, if a child knows that 4 signs and then again I can say the concept of numbers was...he didn't' get them.'(P6: 60-62)*

*5.3.5 Tacitly-nested Number Sequence*

In this section I was looking for evidence from participant comments that they recognised a range of strategies that involve procedures other than counting by ones, but also that they could describe more efficient strategies. One aspect of number development at this stage is that children begin to see a number as a composite whole, for example that 16 is 'sixteen' and not simply as sixteen single countable items. Another feature of this stage which I hoped to find in participants' comments is that children begin the production of a multiplicative structure. Interview 1, 2 and 4 provide the contexts from which the discussion could emerge.

The following comments capture the general understanding presented by participants in this interview that certain strategies used by children are more laborious and time consuming than others. The comments capture the notion that these skills are hierarchical and develop from situations in which children represent problems and physically count objects or items. The comments also indicate that some strategies are more desirable than other, i.e. immediate recall of basic operations or number facts and mental calculations (as opposed to counting based strategies). These are features of strategies that children in the tacitly-nested number sequence stage might use and some comments suggest that they are indications of good number concept and development.

Four participants in this study mention features of number development which relate to the Tacitly-nested Number Sequence. This participant refers to different strategies (counting physical objects on the one hand and using mental calculation on the other) and also notes that an understanding of the value of a number is important:

*The children have different strategies of how they get to, because what they going to buy costs say it's R12 but although they know it's 12 ....all of them doesn't know the value of the R12 because she's given them, they've got little bakkies here with R1 and R5 um, coins in, but because they don't know, because if they all, if only the one child ....he's Agatha...immediately know that she could cope with the numbers together, the 4 and the 3 and the 5 and say you get 12. (P1: 65 -71)*

One participant described learners' strategies in terms of efficiency and referred to these as 'long methods'. The comments indicate that this participant understands that strategies which necessitate drawing, using fingers or counters (coins) are not particularly efficient.

*Dino is taking the long method because he's counting the chips 4 packets of chips to come out to that 12 (P6: 44-46)*

*And then Freda is also using a long method because she is counting with her hands. Zondi is also using the long method. Lisa is using a little bit a short method because she didn't use a lot of coins to come up with that 12 (P6: 46-48)*

*Here the teacher wants to achieve... the child who don't have any problems in addition and here she wants to prove the child who is very slow in addition who is going to take a long road to addition and here she wants to see the child who is going to come first by addition he will not by making any steps or pictures but just add up mentally... whose going to use his mental memory and the child whose going to take a long time to come up with the answer (P6:36-41)*

The third quote above also indicates that this participant recognises more efficient strategies and recognises that these do not involve using physical objects mental operations. The following comments by two participants also reflect recognition that more efficient strategies are abstract in nature and involve knowledge of number facts and mental calculation.

*..so also number two there's only one child whose Ayanda......and uhhmmm it's Ayanda and the other one will be the last one, Fay, that knows a 5 and a 9 and you get to 12. So, that's it. (P1: 79-81)*

*Ayanda was very fast! Ayanda didn't use any concrete objects to come to that 12. Fran was also a little bit ...very...he was also fast because he just count 5 and 4 is 9 plus 1 is 10 and then 2 is 12. Ayanda was marvellous! ..the one that is going to take a loooong method and the one who is just counting and tell you, even when you doing some demonstration on the board just NO, they are calling me dadobawo (aunt), dadobawo that is 8 or that is 7 (P6: 49-54)*

The following comment shows an awareness that the child's solution in interview 3 illustrates the ability to break up numbers and represent them using coins, and that the child is not able to calculate the amounts or complete the number sentence.

> *And then when he comes to the R5 he only wrote.....5 because he couldn't even count on to say he had 1 and 1 and 1 or 2 plus a 2 plus a R1, if he had to break it up. You get to the R6 again, he saw he had a R5, so he knows 5 plus 1 gives me 6, that's why he could write 5 and 1 but when it comes to when he had to add it up and then write 2 plus 5 plus 6 gets him 625 he wrote down the 6 and he wrote down the 5 and wrote down the ..... (P1:107-113)*

This participant is describing the method she thinks the educator in vignette 1 could have demonstrated to the children. Her discussion places her comments into this category because although she is discussing tens and ones, the methods she talks about involve using counters to count out certain amounts, and breaking up numbers to represent them in different ways.

> *And also show them you can put all your, you can um, break up, maybe she should have showed them two or three ways how they could have get to the new total she wants, so that they could maybe they can break up the tens and the units and then they tell them okay then you count how many tens you have, how many units I have, put all your tens together, all your units together and get a new number. (P2: 38-43)*

> *I think she also showed them the easiest way to get to 34 to see what 34 looks like, he will first count out his thirty and say this is three tens, and than he will add his four units, and this will give him 34 altogether, because this is the easiest way to know what bigger numbers look like. (P2: 35-38)*

Children in the initial number sequence stage are able to count collections of tens (numerical composites) but their focus in on the individual things being counted. They do not recognise ten as a single unit of any kind (abstract composite). All participants made reference to models of representation and number structure which relate to place value. Three of these participants expressed an understanding that children at this stage are able to recognise ten as a unit consisting of ten ones and rely on representations of tens and groups of tens and ones when solving addition and subtraction problems involving tens and ones.

One participant's comments emphatically state that the reason why the child had not succeeded in solving the problem in interview 3 was that he or she did not have the necessary number concept and appreciation of cardinal value to understand the structure of numbers to enable operating on numbers, i.e. adding or subtracting.

> *My response is he didn't get that base of numbers to see how a number is built , how to dot his, plus...the 4 concepts that is addition, subtraction, multiplication and division. And again he didn't' get the number concepts right. (P6: 64-67)*

> *His main problem is addition here you see. If he had known addition he would have come up with something else here, not that. You only take that two rands, six rand...he started with the bigger number now he's doing the place value, starting with the biggest number to the smallest number. So if he have known his addition he would have no such a problem. (P6: 89-94)*

Two participants discussed representation and calculation of tens and ones. Although the comment made by the following participant refers to 'three tens' (a feature of the explicitly-nested number sequence stage), I have categorised this comment in the stage of the tacitly-nested number sequence because of the reference to the need to physically count concrete counters and related counting strategies.

> *He will count out his 34 counters, then he'll say three tens gives you thirty, then he'll add his four units, and this will give him 34 altogether, the number 34 and he will do the same with the number 27. (P2: 28-30)*

These two quotes illustrate the point that most comments about place value pertained to counting and grouping collections of objects into tens and ones.

> *..it seems as if the educator wants the child to understand tens and units a group together (P1:8, 9)*

> *I think, I feel, that he'll be able to count out the 34 and than he will probably explain to the teacher how he does this. (P2: 26-28)*

One participant's comments related to vignette 1 in which learners are representing 27 and 34 suggests that she interprets the activity as a skip counting task (counting in multiples of 2 *'even numbers'* and counting in multiples of 3 *'odd numbers'*)

*The students will first represent this number they will first put from 2 up to 34. Then they will start again from 3 up to 27. Will start from even numbers first and then odd numbers. (P6: 19-21*

This participant specifically mentions place value in her discussion of vignettes 1, 2 and 4 but does not elaborate on this topic. The comments suggests that she is refereeing to ways of representing multi-digit numbers

*What also comes in here is place value; it will fall under place value he must know what it looks like and how much the number is. (P2: 21-23)*

*5.3.6 Explicitly-nested Number Sequence Stage*

Interviews 1 and 4 were specifically selected in the hope of eliciting discussion around multi-digit representation and calculation. In the analysis of the transcripts I was looking in particular for comments which reflected an understanding of number structure and strategies necessary to solve problems involving large numbers. Two participants discussed this particular stage of development or specifically mentioned the aspects of multi-digit understanding characteristic of the explicitly nested number sequence. None of the participants interviewed mentioned sequencing of numbers up to 100, positional value of the digits, or skip counting in 10's (and 100).

In response to question 1 in interview 2, in which participants were asked to consider what the educator in the vignette was hoping to achieve by presenting the learners with the spaza food poster problem, four participants remarked that the educator was wanting learners to demonstrate an understanding of addition across the ten by grouping the units and re-representing them as tens. The following quotes illustrate this point:

*..or do some um, plus sums to understand it goes from the one, to take the seven units and the four units you'll get eleven. So the child needs to know the eleven to come one ten and he'll have one unit left (P1: 11- 14)*

*..number concept, a new the units and to be taken over to the new ten must be taken over to the ten.(P3: 11-12)*

*adding with using the units to make a new ten (P3: 8)*

*making units into new tens (P3: 18)*

Neither of the participants quoted above indicated an awareness of progression from treating ten as ten ones and treating ten as a single unit. This characteristic is essential in development of place value understanding and calculation at this stage.

*5.4 Summary of Part One*

The discussion above relates to the first section of the analysis in terms of cognitive development across stages identified by Steffe in his theoretical framework. Table 6 reflects participants' comments in relation to each stage. The interviews were intended for teachers to reflect on their knowledge and understanding about learning and teaching early number. The vignettes and related questions provided opportunities for teachers to discuss specific aspect of number development. Participants expressed their thinking about teaching and learning number and I noted several common threads emerging from their comments about the ways in which they think that children learn and should be taught number.

The analysis reveals that although comments are made about counting and number development for each stage, the majority of comments pertained to the tacitly-nested number sequence. The initial number sequence stage was mentioned by most teachers. Quotes from the interview transcripts (APPENDIX 5 - 10) were extracted to support the interpretation and analysis of the interviews.

*5.5 Part Two: Didactic Themes*

This section of the study presents an analysis of the participants' comments which do not relate directly to Steffe's stages in the development of number. Comments which I interpreted as general pedagogic themes relate to any classroom learning and teaching and not specifically linked to number. Four dominant themes emerged from participants' comments. These include: the teachers' role, sequencing, perceptions of good and weak learners and the effect of environment on learning and teaching. Participants' comments were coded according to these themes and have been presented in table 7, in terms of four categories and five sub-categories. Extracts from participants' transcripts will be included to support the discussion.

The first category relates to the teacher's role and teaching styles which participants mentioned. This includes comments related to direct instruction or transmission modes of teaching, and includes responses that relate to indirect or constructivist modes of instruction. The second category relates to sequencing and pacing instruction and learning and teaching materials. This includes notions of concrete to abstract forms of representations, building on prior knowledge and experience, and presenting children with activities appropriate to their grade or age. The third category relates to participants' perceptions of good and weak learners. The fourth category relates to the perceived nature and effect of the child's environment on teaching, learning and personal disposition.

The headings in the left hand columns of table 7 reflect the main categories and sub-categories to be discussed, while the right hand columns reflect the participants and the number of times particular comments were made by each participant on a particular topic.

Table 7: Didactic Themes

| Didactic Themes | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Category | | P1 | P2 | P3 | P4 | P5 | P6 | Total |
| Role of the Teacher | Direct Instruction | 1 | 2 | | | 3 | 3 | 9 |
| | Indirect Instruction | 3 | 3 | | 2 | 1 | 1 | 10 |
| Sequencing Material | Concrete to Abstract | 1 | 1 | 1 | 4 | | | 7 |
| | ▫ Representation | 1 | | | 1 | 4 | 4 | 10 |
| | ▫ Range of manipulatives | 6 | 1 | 1 | 2 | 3 | 3 | 16 |
| | Prior-knowledge and experience | 3 | 2 | 2 | 3 | 3 | 3 | 16 |
| | Planning According to Age or Time | 1 | 1 | | | 3 | 3 | 8 |
| Perceptions of good and weak learners | | 12 | 3 | 2 | 3 | 1 | | 21 |
| Perceived Effect of Environment on Learning and Teaching | | | | 1 | 2 | 3 | | 6 |
| Total | | 28 | 13 | 7 | 17 | 21 | 17 | |

P = Participant

### *5.5.1 Role of the Teacher*

Mode of instruction in this study refers to all aspects of teaching, including the curriculum, the learning context, the educator and learners and the process of learning and teaching. The views expressed by participants about the nature of instruction include the role of practice, demonstration and guided support and responsibility for providing a learning environment that both facilitates and accommodates various learning needs. Five participants commented on teachers' role in the learning process.

*5.5.1.1 Direct Instruction*

This mode of instruction is based on the belief that learning is an individual effort and that learners should be explicitly taught to do certain tasks and that learning and teaching is usually guided by an accepted text, like a textbook, teachers' guide or syllabus which focuses on specific knowledge and skills. Generally the steps in a particular procedure are demonstrated and learners are expected to use standard methods to solve problems. Clear instructions are provided which involve the manipulation of symbols and widely used skills. Another feature of this style of instruction includes practice (repeated actions) as a means of creating new behavioural patterns for children to acquire the necessary reinforcement to establish and apply new skills.

Two participants commented specifically on the need to give clear, concise instructions and structure in order for children to proceed with and complete a task successfully.

In response to question 3 in vignette 1 in which participants were asked to say how they thought the learner in the vignette will represent the number, this participant said:

> *If the teacher wasn't clear how she wanted it, because in the question it just say she just asked them to tell how the s are then asked to combine the rods and tell her how many there are altogether. So, the instruction she has given just say put it altogether, tell me how many there are.*
> *(P1: 26-30)*

Question 4 in vignette 1 was selected because I hoped that discussion would relate to the teaching method participants thought the educator in the situation should use to demonstrate the concept for the learners. I also hoped that this question would generate a general discussion around pedagogy. This participant's comments suggest that the teachers direction should be clear:

> *..she demonstrated to them before she told them what to do, and how to get to their answer (P2: 33-35)*

*she should have demonstrated to them one way they could do it and then leave them with any other ways they want to do it, I mean the direction should be clear....what you want from them (P2: 46-49)*

One participant clearly states that showing or demonstrating a particular method to learners is essential in order for them to grasp a certain concept or replicate a particular procedure. For example, her comment relates to learning multiplication procedures and her comment indicates that she believes they are quicker and more efficient than using addition, including repeated addition, to calculate a problem.

*From this activity the educator wants to show the learners the short way of adding. She also wants to show them the shortest way of addition. (P6: 9, 10)*

One participant discusses the relation between counting and addition and multiplication and addition. She comments that by extending counting and addition children will be able to simplify and extend their knowledge by using repeated addition and multiplication. She sees an opportunity to demonstrate a more efficient method of adding, i.e. rather than using repeated addition to use multiplication.

*I think the educator would have demonstrated this... she will show them how fast is it...to do multiplication than addition the shortest way for addition is multiplication. The shortest way...of doing addition to multiplication. (P6: 24-26)*

*I will show them when we are doing addition it will take you a long time it will be a long method and then when you are doing multiplication it is the shortest way because you are not going to do all that steps. (P6: 27-30)*

*5.5.1.2 Indirect Instruction*

This mode of instruction is characterised by empiricist notions of guided practice, various perspectives of learning and teaching, a range of materials and explanations and multiple contexts. Opportunities are provided for learner interaction and desired performance is modelled as educators and learners work together on a range of appropriate tasks. The understanding is that learners will assume as much of the task as possible (which depends on the unique characteristics and diversity of each child), as soon as possible, as the educator gradually withdraws support as learners gain proficiency.

This learning environment is influenced by an understanding of learners' thinking and the processes involved in effective learning and teaching. Educators do not present procedures for learners to simply imitate but provide opportunities for learners to engage in contextualised reasoning and appreciate the variety of solutions that learners employ to solve problems. They accommodate different cognitive styles and present appropriate problems and questions accordingly, to elicit learners' thinking and provide activities that capture learners' interest and curiosity and inspire them to reflect on and discuss their thinking.

Most of the participants' comments in this study related to aspects of classroom learning and teaching and not specifically to mathematical development. This is best captured by one participant who stated that according to an outcomes-based education (OBE) theory, learners should be encouraged to solve problems using their own 'invented' strategies and that the educator should not teach a particular method.

> *They must use their own method to solve the problem. They must use their own methods to solve problems. We can't teach them a specific method and tell them they must do it that way. That is why OBE is so, this is part of OBE, each one must use his own method. (P2: 65-68)*

One participant commented that children learn for themselves and construct their own meaning and understanding when provided with opportunities to physically be involved in the learning experience.

> *A lot of practical work has been done with this learner and the learner has discovered many things himself because he has seen it, and if he sees and discovers for himself, this gives him an advantage, over a child that won't do it on his own. (P4: 88-91)*

Another participant's comments suggest that learning is best achieved by modelling a particular situation in which learners are guided through an experience by the educator who carries out a task while they observe and thereby build conceptual models of the process required to complete the task on their own. These comments indicate a strong position on the role of the teacher in the learning and teaching process.

> *I feel that the teacher should first have read the sum to the children, as there are many children who can't read and won't understand the sum*

*because they can't read it, she should first guide the child, guide the learner through the sum. She should first read the problem, the sum to the child and then give him an example...we often have problems with (word) problem solving in the classroom and with children we see cannot solve problems. (P2: 84-89)*

*This is why we need to first guide them towards the answer. For example, we don't give the same activity, we give another activity and then show them how to do it, and now if he does the activity you at least know that he understands how to do the sum. I feel the teacher should have first guided him to the sum and then given him an example of a similar sum, so that he sees how it should be done and then he could go back to his place and do it himself. (P2: 89-96)*

Two participants propose that instructional strategies should invite and support learner involvement and provide structures that enable collaborative learning, where learners work together as a means of coming to a shared understanding. The educator's role is to facilitate that process. The following statements from four participants regarding teaching strategies and classroom organisation suggest that instruction should encourage an active learning environment. Understanding and proficiency in such an environment would developed on the basis of experience and involvement which involves the use of physical objects, reasoning, solving problems and communicating mathematics with the educator and other learners.

*..she could have asked them to put the, she should have asked them to maybe have another way they can count all the tens and then count all the units, and then for the two of them could put their tens together and then their units together and then get their new number. (P1: 43-46)*

*..she is busy developing their skills in relation to their thinking abilities (P4: 12,13)*

Two participants mention the benefits of collaborative learning, which includes working in groups and in pairs.

*And then the last is group work. Facilitators will have to help learners and then learners will help each other (P5: 82, 83)*

*She also wants them to see when you are working in pairs how easy it is to come to the answer (P6: 10-12)*

Questions and discussion are highly effective methods of directing learners' attention to detail and particular learning material and ensuring that all learners participate and share their thinking. Two participants mention questioning and discussion in relation to facilitating learning and encouraging learners to use their own methods to solve a problem.

> *..the educator should have after they've counted out with the rods, the 34 or the 27, she then ask you how many tens you have, how many units you have. (P1: 36-38)*

> *Addition, how many tens there are, they count, she shows units, asks what happens, is the amount more than ten? (P5: 19, 20)*

One participant refers to problem solving and group discussions while reflecting on what the educator hoped to achieve in vignette 2.

> *And this is a problem solving, its number but it's also problem solving because the child, she gives the children after everybody has, one child has chosen what he wants to buy, they have a group discussion of how they going to calculate or to solve the problem. (P1: 60—63)*

*5.5.2 Sequencing*

Instruction can be planned so that activities are sequenced to accommodate learning at different difficulty levels. This enables learners to progress in their cumulative understanding of a particular concept or topic and make connections among and between ideas learned previously, and those they are currently learning and will learn in the future. This hierarchy of learning is organised in successive stages and learners can be located across these stages of development. An understanding of the developmental process enables educators to diagnose problems and plan activities appropriately according to these levels.

The reference to a hierarchy of relative difficulty of ideas and stages in pathways to understanding appears throughout the interviews in this study. Most participants' comments indicate an awareness of the idea of progression. This progression is expressed in terms of using manipulatives, building on prior-knowledge and experience and ways of organising learning according to time related aspects (including age or grade).

*5.5.2.1 Concrete to Abstract*

All participants commented on the activities presented in the interviews in terms of increasing complexity and diversity linked to various forms of representation, ranging from concrete to abstract, in their discussions. The majority of comments and the main focus of participants' discussions in the interviews suggest that they accept the notion of progression through various understandings as a structured process that develops from the use of concrete manipulatives to abstract symbolic forms of (mental) representation. All participants suggest that learners should be provided with learning experiences that facilitate the move from concrete to abstract forms of representation, and that they should be exposed to pictorial, concrete and symbolic representations of numbers in order to fully understand concepts and skills being taught. As table 7 indicates, 33 separate comments were made by participants in the interview which relate directly to this point.

One participant's comments indicate a strong position on the use of concrete manipulatives as a pre-requisite to understanding number in it's related abstract forms:

> *I suggest is that you should use concrete things to learn a child. He must use the real thing to count, we can't just count and we can't just do sums. Every time we do a sum when it comes to number you must give the child a problem, because if we just work from, if the child just see that number on the board or I just give them the number and he put out the sum, and there isn't a problem attached to it, he will never understand, he will never be able to make a, be a critical thinker when it comes to maths. Because if they understand number, they will be able to understand everything else in maths, because it starts with number. (P1: 245-253)*

Although participants in this study referred to learners' solution strategies, certain comments indicated that their understanding of this was in relation to progression from the use of concrete manipulatives to abstract symbolic forms of representation. For example, two participants referred to the use of drawing and physical objects (coins) in their discussions about the ways children solve problems:

> *Dino, he made an illustration by drawing a picture (P4: 41, 41)*

> *Some of them will just draw the whole amount in circles; some will maybe draw fruit, just as they see the amount. (P4: 22, 23)*

*He must be able to solve his problem on his own and I see here that Dino drew his to get his answer. He counted everything together and then got his answer. Zondi made use of the bakkie of R1's, counted all the R1's together, to get his answer.... (P2: 68-71)*

One participant's comment suggests that the notion of using practical manipulatives is essential in the early development of number. This participant for example insists that children need a lot of materials and that they should work with abstract number until they have a firm (concrete) grasp of numbers up to 5.

*Needs lots of apparatus. Will look to see where he must begin, never start at nought and then he must first, before teaching nought, he must know his numbers, up to five. (P5: 76-78)*

These two participants indicate that using manipulatives gives children the necessary support to begin working with different kinds of numbers.

*Frieda, she uses her fingers but needs much more practical work. Zondi, easier to work in Rands, to pack out, can help himself, knows his coins. (P5: 32, 33).*

*Maybe he didn't have enough practising in counting, didn't have practical exercises. I've left out here, I don't know what. He has not had enough practical experience with counting and numbers and with practical apparatus or whatever. (P3: 65-68)*

The following quotes from two participants extend the discussion on the use of concrete manipulatives by including that children should have fun while working practically on number tasks.

*More practical work should be done, with games in which he could buy and sell, so that he becomes aware of how to use money. (P5: 46, 47)*

*And I would also say this learner wasn't given enough opportunity to play with numbers, because a child that plays with number a lot, especially in Grade 1, is bound to be stronger as he moves into higher ability groups. (P4: 104-107)*

*I would begin with examples and when he begins with examples I would say it is most important to play with these, make it more, make it less, add on, take away and then, especially when working with numbers and counting work. (P4: 117-119)*

*5.5.2.1a Representation of Number*

The questions in interviews 3 and 4 encouraged discussion about children's early strategies, representation and understanding about the development of basic number facts and calculation. One participant's comment supports the suggestion that children represent numbers initially, physically or iconically, and then gradually in abstract forms. In response to question 2 in interview 2 in which this participant was considering children's solution strategies, she said:

> *Although most of...all of them had a 12 in mind, they don't really know the value of that number because, I mean the first one took out four packets, she drew four packets of chips although the chips cost say R4, so maybe there were he bought four packets of chips, I don't know and three chocolates, so one chocolate from the poster could have been R3 but he just chose to draw to get to the three, he knew he had to get to a 3, she doesn't understand the value of the R3 when it comes to break up. (P1:72-79)*

One participant indicates an awareness that children gradually develop more and more sophisticated ways of seeing and understanding numbers in this comment:

> *..she is also guiding them towards visualising numbers, so that they can write the numbers as well as understand it. (P4: 10-12)*

Another participant's comment indicates that she firmly believes that allowing children to physically represent numbers will enhance their learning (of the formation of the numeral):

> *I will suggest to a teacher when she is teaching counting numbers, teaching of counting. A child it is better when he do it on his self ne. By making some activities, body language ne, it is the best to do body language because if it is a 1, 1 is going down, the hands to show this is one, uyehla (straight down) and then if it is 2 uyenyaka uhle uhle (up and down and down), the shape, so the shape stays in his mind and also maybe when he's sitting there at the desk, OK 1 go down, 2 uyenyaka. (P6: 158-164)*

An important aspect of representing numbers at the foundation phase is the correct formation of numerals and writing number words. This participant mentions the process involved in teaching her learners to write numerals:

> *I'm doing Grade 1 ne, but I'm using methods and methods according to the problem of the children ne. I tell them 3 is facing to my left hand and*

*then when we are writing that we are talking about it. 3 uyenyaka uhle uhle (up down and down) that is a 3 ne. And always it is facing to my...to my left. (P6: 131-135)*

*..if you take him and ask him he will write that 3 like an E ne. And then you ask him, the E must look to the ..? He will tell you to the...to the left. But when it come to the...for writing in the book he will do the same mistake. Now you have to take him again and show him that 3 on the cardboard, that numbers that you give me, to show him, here is a 3. Now he take that 3 again and put it there on the table, on the wrong side ne, because some of them are making it like an m. Now you must put it there and ask him to come and place it on the chalkboard. (P6: 137-144)*

*..another thing is to do some pictures, this is 3 and number, this is 4 and so and so and so. On the chart will be, let's say it's the picture of 2 eggs ne and then you write the number underneath the 2 eggs. (P6: 168-170)*

*5.5.2.1b Range of Manipulatives*

I have separated comments in this category because in the interviews all participants discuss different apparatus and suggest that some manipulatives are more appropriate for developing number concepts and understanding initially than other. All participants in this investigation commented on the use of a range of manipulatives and apparatus and various forms of representations ranging from counters to numerals – in the form of expanding number cards and 1 – 120 number squares. Sixteen of the thirty-three comments relating to concrete and abstract sequencing of material refer to the use of a range of material which indicates that participants in this study consider this a significant feature of Foundation Phase mathematics learning and teaching.

This participant gives an overview of the kinds of apparatus she considers essential for learning and teaching number.

*..first the teacher or the student I will use, I'll use counters. The teacher has to focus on .....must be counters. You must have the real things with you. You must have your, the number blocks with you, like your 100's, and even maybe ... have loose one the 0 to 9...then the child understands ....the value of your units and then understand how you get the values if you add it it gets to the 10 and the units with, you have need to use, you have to .....but also need to have the numbers on cards, on individual cards and also on the number, the graph we use for the number. (P1: 222-229)*

Specific mention is made by all six participants in this study about the practical use of manipulatives in the learning environment. It would seem by these comments that participants believe that cognitive development is facilitated by providing a range of 'real' hands on activities and various representations, which assist learners to create conceptual understanding, for example counters (eggs) and expanding number cards. Of the six participants two comment directly on the need to use practical apparatus repeatedly during counting activities in order for the learners to physically count and represent numbers and operations.

Two participants' comment indicates an understanding that children need to be given opportunities to physically count objects and then draw and write the number.

> *I go back again to the picture...to the concrete, I see some of them are not getting the number concept right so I'm starting all together again now with them. Now I always again introduce that eggs, this is eggs and then if it is 3 eggs, if this is 3 eggs give me the number – write it in numbers ne, and then they are going to write 3. And then sometimes I am using the 3 and then I'm asking them to write in some eggs. (P6: 125-131)*

> *They have to pack them out and he must, you should have given him practical problems like you are going to buy say a bread and then ask him how much the bread costs, right so you have, how much money do you have. The child should have pack it out for you and say I have with me, say I have a R6, so you going to pay R4.20 for the bread, so pack out the R4.20 and then write down which coins you used to pay for the bread. And then what do you have left, how much do I have left. ....have R1.80 left and also pack out the coins you have left. (P1: 135-142)*

Two participants comment that manipulatives need to be appropriate and that there should be sufficient for each child to use.

> *..the learners must see and use materials and apparatus that they know, it must be life related. Use the apparatus within learners' experience, involving learners at all times. (P3: 78-80)*

> *..have enough learning apparatus for each child, for example when teaching this concept, I will give each child a number and as the class counts I'll say if anyone hears his number stand up, so that when there's a child in the class who doesn't recognise his number, his friends will remind him and he won't give that specific number that he for example holds against his chest, and his numbers can be swapped weekly, and this*

> *is also a method whereby counting can be taught and children won't easily forget these numbers. (P4: 123-129)*

Participants describe number competencies and infer that the meaning learners give to numerals and counting is modified from concrete to abstract symbolic forms in order to enable them to produce rapid, correct answers to calculation problems. They suggest that learners who have *not* made this transformation will need to rely on manipulatives to support their visualisation of the problem whereas more advanced learners will calculate mentally.

Three participants' comment suggests that counting charts and single counters are essential apparatus for early number development.

> *When children are starting at Grade 1 they always begin with eggs, this is not for the number standing, and then they first see the picture, and then the object and then the concrete number. He didn't get that. (P6: 76-78)*

> *..doesn't like to use his number square anymore, because his number concept is very good. (P5: 59-60)*

> *..learner must know what 34 and 27 looks like, especially if the teacher has the counting chart in front of him, and he also has his counting rods with him, then he must be able to see what 34 looks like on the counting chart and he must use the counting rods to count out the 34 and 27 in front of him and he must know how much 34 or 27 is. (P2: 15-19)*

Two participants emphatically state that expanding number cards are essential for place value learning and teaching.

> *..one of the most important apparatus are expanding number cards, because then the child can easily see if it's a one digit number or two-digit number. So, examples are most important. (P4: 119-122)*

> *Pack out, use of expanding number cards. (P5: 16)*

An understanding of money and its relative value as a teaching apparatus which can be used to represent number and used in contextualised number activities, like shopping, is expressed firmly by two participants. They suggest the usefulness of using money as a tool for teaching number concept and indicate that number skills can be easily understood by using coins of different denominations as they represent the cardinal value of number.

*When it comes to number, I just feel that children, especially when it comes, 'cos money is actually the easiest way or the most appropriate way to learn, to understand number, because it's got value and you working with that numbers. (P1: 84-86)*

*So the children need to let the children pack out that plastic money and he must really see it so that he really, or the child really knows the value of that coin, doesn't matter if it going to be one counter, he must know that counter is a 50c. (P1: 96-99)*

*So, if you like have that R1 the child should know before I can have a child to count out like Rands and R5 and R3, the child should know the value of that R1. He should know I have 100 tens will give me that R1 or that ten 10c pieces, or that 5c pieces are going to give him that R1, so he, because otherwise he's just going to see the 2 or the 1 and he doesn't, he won't know, I mean if he doesn't know the value he won't even be able to use that the money outside the classroom. (P1: 87-93)*

*If he go buy a bread, he must know if the have a R5, the bread is R4.50 I must get a 50c change because he knows the value of that, the bread is R4.50. So that R5 coin won't be one thing he have in his hand, he will know it is R5. And I have ten 50c pieces. (P1: 93-96)*

*I will have started from the numbers but not from the pictures but from the concrete numbers to show him if you add a R1 and also if you introduce the signs to him, addition signs. If you add one rand plus one rand equals to two rand I will start him from there. And then I also think that what he lack here is when there are no cents you have to put zero zero, you see. That's why it's coming up with twenty-five cents. He didn't get it, the rands and cents right. He just take that you see and place them there (the numerals 625), he didn't add. (P6: 82-89)*

*5.5.2.2 Building on Prior-knowledge and Experience*

The use of the term 'prior knowledge' in this study incorporates all of the learners' accumulated knowledge that is available at the time before a particular activity. This knowledge consists of all the facts, concepts, connections and structures which have been internalised as schema, and the methods and skills which are employed and applied during the activity. Prior knowledge may be explicit in that it is directly accessible and used directly in a situation, or it may be tacit, i.e. not directly accessible but able to be elicited through the activities and interactions with other learners and objects in the learning environment. This comment contains this notion very clearly:

*Fran began with what he knew in that he began with the five and counted the four together, and then added the units, so he starts with the known and moves towards the unknown, and he added.(P4: 45-47)*

One participant's comment indicates that learners need to see and internalise various representations as part of learning at different stages of their development in order to build on and connect their prior knowledge and skills to new mathematical ideas and contexts. She specifically refers to the use of money and the need for learners to have some knowledge of money and an understanding of the relative value of each coin before they are able to operate using the context of money.

*So before they do things, the children have exercises like this, they must first have a prior knowledge of the value of this money. (P1: 99-101)*

*As I just said previously is that the teacher should, before I can assess a child giving him a problem I must know the child understand in this instance it's money. He must know the value of that money, even if they have the plastic coins he must understand so the child should have been given the opportunity in the class beforehand to play around only say with the R2 and then he must have the different coins, the value coins like to have your R1, your 50c pieces, 20c coins and 10c coins and even 5c coins, so that he knows he will be able to count in two's to get to that how many 2c pieces, 5c pieces or 10c and then he should have had the opportunity to put whatever the different coins together, like having a 50c and 20c pieces and a R1 to get to the R2. (P1: 121-130)*

The assumption, that children learn about number bit by bit, in little chunks which have to be explained and modelled step by step, each step necessarily building on the previous one, emerged in the interviews and is evident in this participant's comment:

*..you must take it step by step, step by step, because when they are coming from Grade R they only come 1,2,3,4,5! (P6: 173)*

Two participants talk about establishing competence with 'easy' or 'small' numbers and simple counting strategies as a prerequisite to working with 'larger' numbers and more efficient computation.

*And then, I also feel that in future, if the teacher sees the child can't do the sum, that he has a problem, if he has such a poor concept of these numbers, she should give more easier activities, first before she can give him such an activity, she must work with the children even though they are in groups, maybe he is in the weak group, then I feel she should sit with* *them and give them easier activities, so that they can do it before they go on to more difficult ones. (P2: 101-107)*

*Then I feel that she should begin with smaller numbers and first establish a good number concept in the child before he can do more difficult sums. (P2: 107-109)*

*..the teacher has to first assess that the child could add with smaller numbers or with less amounts (P3: 41,42)*

*..how to add only two items until the learner understands before moving on to bigger numbers. (P3: 46, 47)*

One participant's comment shows an understanding of the links between operations and that children build on this knowledge.

*..when the time you have to teach the child to understand multiplication and division and subtraction, he will be able to do it, because the multiplication he will go adding on and work backwards and do subtraction and the division, to know it gets less and less (P1: 240-243)*

Two participants' comments suggest that single digit numbers and place value should be taught before double digit numbers can be introduced.

*I would never give a two-digit number if the grouping, if the child doesn't have a good understanding of tens and units, what are tens, what are units, what is more, what is less. If it's 34, for example, what is bigger or more, 3 or 4. Some will say the 4, and this actually isn't so, then you'll see that child doesn't have a good number concept. (P5: 93-98)*

*I was giving them...uh...single numbers ne, but now I'm using double because I found out they are very quick at counting ne, and their addition is very good, because if you write...I'm not even finished to write down on the chalkboard and they will come to me and show me our work. Very quickly. (P6: 105-109)*

The following comments from two participants express a position that if children do not have a sound understanding of cardinal value of numbers and knowledge of number facts, they should be re-taught these aspects of number.

*The value of numbers, I would say this child has no idea about solution strategies and this child has no idea about number combinations. In other words, even though he wrote down the numbers together, he knows what a plus sign is, he has no understanding of how to add the numbers together. So this learner must begin again with counting on, or counting everything, he hasn't mastered these basic skills. (P4: 60-65)*

*I would begin by going back to number combinations, for this learner, first so that he first masters the basics, his knowledge of numbers, go back to what was done in Grade 1 so that he will know when he packs out two, what a two looks like, what a five looks like, the place value versus the value of the number, because this is what this learner doesn't know. So, to back to a play-play method of packing out, showing the numbers in the class, I think these are the basics that I would use more to begin with the learner. (P4: 69-75)*

*As with values now, say for instance children find minus difficult. The problem is sometimes that they don't know their combinations, if he first knows his combinations, then he'll automatically know what 'friend' is missing. But, yes, that the child needs much more practical is definite. But I also found that, as they say in the new curriculum, the children must know their combinations, so the best would be if the child can say. (P5: 85-90)*

In response to the question relating to the learner's solution in interview 3 that asks what the educator would do to assist this learner to solve this type of problem, one participant's comment suggests that practicing the same type of problem will improve the learners' ability to solve the problem accurately and efficiently in the future as they will have had sufficient experience.

*Give the learners more of these type of problem sums to avoid confusion. (P5: 51)*

This participant's comment firmly expresses the notion that basic ideas should be revisited repeatedly in order to build on these, until learners fully understand the concept.

*I think it's because, when we were starting at the beginning of the year they were making, they were always asking to, why they are very good at this, it is because when I have teach them at the beginning of the year I say to them if you are adding you are, if you are doing addition you add, you are adding more – ukongeza, ne Xhosa ukongeza (add more). So all of the time they are...OK Dadobawo have said if it is add I have to add more, if it is addition I have to add more. That's why they are doing, and they don't forget it. And then when I'm asking the others some, maybe some others in the class. What I have said why do you add like this Mamie if you do this, if you are using addition you add more. They understand the concept of addition. (P6: 109-119)*

*5.5.2.3 Planning According to Age or Time*

Four participants' comments indicated that they had expectations of learners' performance based on grade or age-specific criteria. One participant asserts that a reason why learners may not acquire number concepts effectively is as a result of inadequate teaching. She suggests that the educator is responsible for planning the learning process by providing learners with relevant and appropriate experiences that will enable them to organise information in optimal ways and facilitate the development of competencies through successive stages of learning.

> *And also Cally I can blame his teacher for this, not the teacher where he is now, I think his Grade 1 teacher and his Grade 2 teachers have a big gap and a lack of showing. (P6: 73-75)*

One participant captured the notion of the central role of the educator's responsibility in the learning process as being essential to effective learning and teaching. This is captured in the comment below:

> *... the concept of numbers was...he didn't get them, the right concept of numbers from the foundation phase, from that level of Grade 1 where a child sees the picture, the number and the words. My response is he didn't get that base of numbers to see how a number is built. (P6: 62-65)*

Four participants' comments suggest that the teaching process should accommodate learners' progression at different grades gradually in order to facilitate the cumulative understanding and development of concepts. The following comments point to the notion that timing is important in the learning process and that learners construct knowledge at their own pace according to their ability and concept development at a particular time.

> *..if you for one week I focus on just counting, adding on, and I work with my 1's and the next week I start say with 2's and then with 3's until I get to 100's, and then count back again. And I have to start within any number range. If the child needs to say in Grade 1, needs to be able to know up to the value of 50, then the child needs to start anywhere, if I had to tell her count in 2's from 51 to 71 he have to start there. And important for counting is that you must from the beginning learn the child to count backwards, because if the child counts to the front and he counts backwards again. (P1: 233-240)*

*This is also why he had the wrong answer, because for a Grade 3 child, the sum is very easy and you can see that he can't do it because he isn't busy with big numbers, but small numbers under ten, and that's why he can't do it. He also has a poor concept of number. (P2: 95-99)*

*Say for instance in Grade 2 if you for example in Grade 2 you must do two-digit numbers. I would never give a sum where the child must carry over, because this would be too difficult. (P5: 92, 94)*

*I have a few, up to three learners, if you give them some work they are so fast. Sometimes I give them the difficult because they are, you see it is May now, from March. (P6: 103-105)*

*5.5.3 Teachers' Perceptions of Good and Weak Learners*

This category presents comments made by participants which related directly to their perceptions of the characteristics of 'good' and 'weak' learners. For example, that learners are able to recognise and count large numbers. This participant's comment captures this idea:

*A learner with a good concept of number is one that will identify number or any number within the context of, or maybe even further than, if say for instance he's in Grade 2, a Grade 2 learner and he just need to know numbers say till 100, he will be able to tell you any number say till 500, because he has a understanding of number. Because if he knows, if he can count from, he knows his number from 1 till 100, he will be able to identify any number, even if it's up to a 1000. Because if he knows the value of those numbers, then he will be able and this good learner will, is the one that will pack out your, the numbers correctly, he will know say what is a 100, what is tens, what is units. (P1: 149-157)*

Two participants' comments in response to question 1 in vignette 4 indicate that children with a good concept of number should be able to recognise and use different numbers in various contexts.

*A learner with a poor number concept usually makes many mistakes. He often transposes (reverses) numerals, can't count numbers out on a number chart. (P1: 96,97)*

*..he give his age, write his age, count it out, show it, can show any number and name it, can write numbers, maybe till 40. (P3:55, 56)*

Two participants' comments support the notion that problem solving is a context for learners to learn and demonstrate new concepts and for practicing new skills.

*I also see that it is problem solving. They must be able to solve their own problem, each one solved their problem in a different way, and they must also have good number concept to be able to do this. (P2: 48-51)*

*I think the educator is busy, what she actually wants is to instil the value of money in the learners. She wants them to be able to do a costing plan and at the same time she wants to develop their calculation strategies, and they must know how to calculate. (P4: 34-37)*

Another feature of learners with a good concept of number expressed by three participants is that they are able to perform basic operations and calculate efficiently.

*He has good number concept because he can add and subtract and maybe multiply till 25. (P3: 56-58)*

*I would say that a learner with a good number concept is a learner that can visualise numbers, he can recall numbers quickly mentally, he uses different solution methods, without illustrations, only using numbers. (P4: 83-85)*

*Lisa first made small groups of five and then added the Rands. Ayanda is very good, I think, ...her number concept is very good because she just visualised and did it with mentally. (P4:42-45)*

*I see that Ayanda gave her answer immediately. She added the numbers up immediately and gave her answer, this means she also has good number concept. She doesn't need to use concrete apparatus or use her fingers to solve the problem, where Dino drew his things. This is how a person sees that the different ways of the children in your classroom are not the same. (P2: 60-64)*

One participant notes that a learner with a good concept of number is able to estimate efficiently.

*He'll be able to estimate quite well also (P1: 170)*

A range of skills and knowledge is mentioned by one participant in her descriptions of learners she considers to have good number concepts, for example calculating with two-digit numbers, basic operations, time and fractions.

*He will be able to take away if he sees I have 11 units, he will immediately put the 1 with the tens and he will leave the 1 with the units. And the one* *who has a good concept of number is the one who will be able to add on quite well, be able to subtract quite well, even be able to multiply, to do all the basic concepts of multiplication, because he can add on and he will be able to subtract because he can take away, because he understands the value, he know what goes bigger and what goes smaller, if there's a number should be in the middle or to the left or to the right. (P1: 156-165)*

*He will be able to know that if I say for instance if you say what's nearer you have what is the half of 15, he'll be able to tell you whether 8 will be more to the half, because to have 15 is nearer to 16 and he won't say 7, or he'll tell you .. have to say 7 is you want the exact number...you must take 7 to......and you take the half of the 1 to have 7 ½ and another 7 ½, but the other one won't know that, that would be a good learner in the class..... (P1: 165-169)*

*..the good learner is the one that break up his number quite easily, and I mean that's also the one that will tell you even that you able to read his time well, because he knows, he would know what goes around to get the one, and he will be able to count that size of seconds in between, because he has the understanding of number and he will write his numbers correctly*
*(P1: 175-180)*

Participants' descriptions of children with weak number concepts in this study included lack of basic number knowledge and understanding, and not being able to solve contextualised problems. This participant comments that children need to know the value (cardinal) of numbers and coins in order to operate with them:

*..and they must know the value, they must know the value of money because many children don't know the value of money, if we work with money in the classroom with them, like here the answer is R12, they need to know what the value of R12 is, how to get the whole answer and then by doing this they must be able to add up the total cost of the sweets that they added together (P2: 51-56)*

One participant's comments reveal that she does not believe learners with weak number concepts are able to solve contextualised problems. She mentions that a weak learner does not have the cardinal aspect of number, and although they may respond with a correct solution to a calculation problem involving numbers only, they cannot solve the same problem in context.

*He will even give you the answer if you ask him 7 and 5, he will tell you it's 12 but if you give him a problem, he won't be able to get the solution* *to that problem but maybe he will just be able to write down let's say 7 plus 5 is 12, but the moment I'm going to put the 7 and the 5 in a problem, then he won't be able to give that solution. (P1: 197-201)*

*I can see that he has no number concept, or not a good concept of numbers, and this is a problem solving and I can see that the child can't solve the problem. He doesn't know how to solve the problem, and he doesn't know the value of money. He doesn't know what R2 is, how much R5 is and how much R6 is. (P1: 76-82)*

One participant adds that a learner with a poor concept of number does not have a sense of the magnitude of a number and therefore does not have the ability to estimate efficiently. Her comments show an understanding that this skill is important for 'having a sense of a number' and estimation.

*He is also the poor, the one who has poor concept of number is the one who can't estimate quickly. (P1: 201,202)*

Competencies in both reading and mathematics are mentioned by two participants who recognise the importance of reading and mathematical development and the impact a weakness or strength in reading can have on the acquisition of mathematical concepts and skills.

*And what could also be making the problem worse is that the child perhaps can't read. Maybe he can't read the sum. (P1: 82)*

*And he's also a poor reader as I said, because they won't, he doesn't know the value from left to right, and maybe what is in the middle if I tell him start counting in 3's from 71 to 100, he won't be able to do it. (P1: 203-205)*

*And naturally, writing of numerals plays a big role too. If the child reads incorrectly, he will also do his sum incorrectly. So, this is what you will pick up when you continue to work with a child. (P5: 98-101)*

*He will even be the good reader, because he knows his numbers goes from left to right, so he will be the good reader also, so your good readers will also be the ones that's able to be good in numbers and will know what's in the middle and what is left and what is right. (P1: 180-185)*

*5.5.4 Perceived Environment Factors Affecting Learning and Teaching*

This last category in the analysis of participants' transcripts serves to locate six comments from three participants, which pertain to the perceived affect of environmental factors, such as the organisation of the classroom and the home environment, on children and their learning. Five of these comments are included in this discussion as examples to illustrate the category. The first three comments selected to illustrate this point centre on learners who either work independently and who work as part of a collaborative group and assist others with their work. These comments from three participants indicate the value that teachers place on independent children.

> *Dino can't work on his own (P3: 27)*
>
> *I think of Jonathan. He is very self-assured. He is keen to learn, helps others, very observant, very obedient. (P5: 58, 58)*
>
> *I think the educator is busy teaching the children, so that they can add better. At the same time she is busy with co-operative learning, they work in groups so that they learn from each other. (P4: 8-10)*

This participant's comment reflects how she views a less confident learner:

> *This learner is very nervous, first looks at what others are doing and then he'll maybe try to do it himself. Not so self-assured and very reluctant. (P5: 66, 67)*

During their discussion on learner expertise with number and number concept acquisition, two participants commented on the influence and impact contexts other than the classroom have on children's performance and motivation. They suggest that problems at home which result in poor school attendance contribute to slow progress and that upsets at home cause emotional disturbances which result in poor concentration.

> *One of this learner's problems, because he has a poor number concept, is that he has maybe fallen behind, problems at home, absenteeism, he is possibly emotionally unstable. He isn't observant or we could also say there wasn't enough laying a thorough foundation, or too little practical work was done with him. (P4: 100-104)*
>
> *And then her background can play an important role in the fact that she is not concentrating on her numbers, and that can also be due to absenteeism. If she is absent often, then he or she can't progress well. (P5: 67-70)*

### *5.6 Summary of Part Two*

The second part of the analysis indicates that most participants' comments related to didactics and not specifically to number development. The results were presented according to four didactic categories which included the teachers' role, sequencing material, perceptions of good and weak learners and the effect of environment on learning and teaching. Table 7 shows that the majority of comments made by participants related to general pedagogic issues.

### *5.7 Summary of the Results*

The results of the teacher data analysis presented in this chapter indicates that the method of data collection was successful in that teachers commented on various aspects of learning and teaching number and reflected on the process of early number acquisition in the interviews. The results obtained from the interviews demonstrate that there is a degree of commonality between what teachers say about number development and their notions of effective classroom practice. There is however, considerable variation in participants' comments. To summarise the findings of this investigation the main points taken from the analysis of participants, responses in parts one and two, reflect an awareness of various aspects of how children learn number and how this should be taught.

In this chapter, the results of the analysis of the participants' responses to the vignettes and related questions were presented in two parts. In the first part, I presented the results of the analysis of the comments in terms of cognitive development across stages identified by Steffe. Table 6 reflects participants' comments in relation to each stage. In general comments made by participants regarding the development of number reflected an understanding that there are different stages in number concept development. My interpretation located participant comments within and across stages identified by Steffe. These included: Emergent counting, the Perceptual Stage, Figurative Stage, Initial Number Sequence Stage, Tacitly-nested Number Sequence and Explicitly-nested Number Sequence Stage. Participants did not, however make reference to specific stages or link their comments to these stages of development.

Most references to the early stages of number development – Emergent Counting, Perceptual stage and Figurative Stage – were contained in participant remarks in relation to weak number concepts or poor learner performance. Comments which related to the Initial Number Sequence Stage, Tacitly-nested Number Sequence and Explicitly-nested Number Sequence Stage were made in relation to performance and skills which participants considered most desirable. Participant comments did reveal an awareness that number development is progressive and that it occurs in particular ways.

In the second part of this chapter, table 7 shows that the majority of comments made by participants were related to general pedagogic issues. The analysis of the interview transcripts indicates that most participants' comments related to aspects of classroom learning and teaching and not specifically to mathematical development. I discussed these comments according to four general pedagogic themes which included the teachers' role, sequencing material, perceptions of good and weak learners and the effect of environment on learning and teaching.

Comments indicated that participants considered various ways of supporting number development, which included organising and sequencing material and learning in particular ways. Participants' comments also revealed an awareness that understanding and acquisition of skills are linked to various forms of representations and that this is a structured process that develops gradually from the use of concrete manipulatives to more abstract symbolic forms of (mental) representation.

All participants in this study suggested that learners should be provided with a range of learning experiences that facilitate the move from concrete to abstract forms of representation, and that these should include pictorial, concrete and symbolic forms. Most participants' comments indicated an awareness of the idea of progression. This progression was expressed in terms of the use of manipulatives, building on prior-knowledge and experience and ways of organising learning according to time related aspects (including age or grade). The analysis of comments in this study further revealed that participants recognise certain factors that impact on learners' performance. These factors include teaching styles and the role of the teacher in facilitating or supporting learning, and the effect of environments on children's learning.

# *6 CONCLUSION*

## *6.1 Introduction*

Chapter 6 concludes the dissertation and attempts to capture the main features of the investigation and related discussion. Furthermore, I discuss the achievements and limitations of this investigation. Finally, this chapter presents recommendations for supporting teachers in their teaching of number, using a research-based model, and overview of development within the domain of whole number.

## *6.2 Discussion*

### *6.2.1 General Remarks about the Study*

The research question which guided this dissertation is: *How do teachers understand the ways in which children learn number?* This dissertation set out to describe how teachers articulate their knowledge and understanding of the ways in which children learn early number concepts in the early grades of the primary school.

The data in this study was collected by means of stimulated-recall interviews based on vignettes and associated open-ended questions which were specifically designed and selected to elicit discussion from teachers regarding aspects of early number development. As far as I am aware vignettes have not been used expensively in interviews with teachers regarding opinions about mathematics content and didactics. The use of vignettes as a tool for encouraging discussion was successful in that they focused on different aspects of number concept development in situations that were familiar to participants' own teaching experiences. The unstructured nature of the interview and the open ended questions allowed participants the freedom to reflect on and discuss classroom situations, and provided in depth insights into participants' thinking and understanding of the ways in which children learn number.

The results of this investigation show that teachers understand children's number development in particular ways which they express within a framework of their own classroom experience. Teachers recognise that there are stages in number concept development and that children progress through these stages at different times; however their comments do not reveal that they know what these stages are. They also showed an understanding that number development should be supported and sequenced using particular methods and apparatus. Participants in this study also described teaching styles and the role of the teacher in facilitating learning in terms of general pedagogic notions and not specifically in relation to cognitive development.

It is evident from the analysis of participants' comments that they do not share a common discourse and understanding about number development. Many of the teachers' comments indicated a limited understanding of how children learn number and the complexity and diversity involved in this process. Participants did not describe an overview of how number develops or what can be expected in the learning process from a common understanding. Participants' comments also indicated an awareness that number development occurs in particular ways but did not distinguish between or elaborate on these stages of numerical development.

The analysis of comments in this dissertation reveal that teachers recognise that learning should be organised and sequencing in particular ways and that this is mainly in terms of notions of concrete to abstract representations. Discussions about sequencing number concepts progressively were not based on cognitive development but rather on generic teaching approaches.

### *6.2.2 Achievements of this Study*

My objective in this study was to describe the ways teachers understand how children learn number. I gained access to this information by means of interviews and analysed teachers' comments regarding learning and teaching early number in terms of Steffe's theoretical perspective on cognitive processes and other didactic processes.

Although Steffe's theory of number development has been applied to studies involving children and has informed instructional programmes, I am not aware of its application directly as a tool to interpret teachers' dialogue related to mathematics knowledge and didactics. I found this developmental theory useful as a framework for guiding the analysis of the data in this study.

Vignettes are not widely used in teacher interviews as a means of exploring subjective understandings and embedded attitudes and beliefs about mathematics content and didactics. The vignettes focused on different aspects of number concept development in various contexts and the open ended nature of the associated questions gave participants the freedom to reflect on and discuss familiar classroom situations at a practical and a theoretical level. This method of data collection provided in depth insights into how teachers understand the ways in which children learn number and allowed for a broad discussion of the topic.

This thesis contributes to numeracy education in the South African context by providing an extensive survey of literature on children's number development and empirical studies involving children's solutions. The study has given me the opportunity to map current accepted theories in the curriculum domain of whole number and consider a theoretical basis for instruction in the form of a framework or trajectory. This trajectory could support teachers in their implementation of the Revised National Curriculum Statement (RNCS) by providing a theoretical basis from which learning programmes and assessment tasks could be designed.

### *6.2.3 Limitations of the Study*

This study does have a number of limitations, which I would like to discuss in this section. Firstly, in order to establish further understandings of how teachers think about learning and teaching early number, I suggest that it would be necessary to include classroom observation as part of the investigation. This would provide insights into classroom practice and would give an opportunity to cross reference what teachers say and what they actually do when facilitating number activities.

This study was limited to six teachers. This decision was based on the nature of the data collection and the fact that all had worked directly with me as key teachers in a professional development programme. There had been no evaluation of teachers' mathematical knowledge and understanding prior to or on completion of the in-service intervention. There was no way of determining whether the intervention had improved or changed teachers' practice or knowledge base. I have made assumptions and generalised my findings based on the data collected. If such a model was to be repeated and a similar study carried out, I would suggest that a larger, more diverse cohort of teachers be identified so that the results can more easily be generalised.

None of the teachers who participated in this study spoke English as their mother tongue. The interviews were written and conducted in English. This situation was not ideal as I was obliged to check that the participants understood the instructions and questions prior to the interview. I invited teachers to speak Afrikaans as I understand the language and intended translating the full transcriptions. Three teachers however opted to speak English during the interview. This proved difficult for teachers when they were discussing children's mathematics as they did not have access to much of the English vocabulary. If a study of this nature were repeated it would be advisable to conduct the investigation in the language of the participants.

Participants' anxiety about the tape recording of their discussions resulted in several interview situations which were not ideal (as discussed in section 4.4.2). Unfortunately one participant was so anxious that her interview comments were limited to one word responses and short phrases and could not be included in the analysis (see APPENDIX 9).

### *6.3 Recommendations*

As discussed in chapter 4 I used Steffe's work as he specifically focused on cognitive development and 'number' structures or schemes of number development. His theory describes cognitive development of number across several stages and suggests that all learners' strategies will be determined by the cognitive development behind these.

Having analysed teachers' comments – including those related specifically to number development and other didactic themes – I end this thesis with a recommendation for a 'trajectory' which encompasses the cognitive and didactic continuum. The recommendation is for a framework that supports planning, teaching and assessment based on the cognitive theory as well as activities and a long term learning-teaching map. Steffe doesn't do this. Therefore, I think we need a trajectory that adds this necessary dimension and 'bridges' the RNCS and classroom practices.

In connection to the above, I would like to conclude this dissertation with some thoughts pertaining to further studies and developments. During the course of this study, and especially experiencing the use of the theoretical framework for learning early number several questions arose that need an answer:

- How can the domain of whole number which is contained in the Revised National Curriculum Statement Mathematics Learning Area best be described in order to promote teachers' knowledge and understanding of number development?

- How can teachers be assisted in the selection and sequencing of activities in order to accommodate conceptual development and progression during the transition from informal to formal abstract calculations?

- How can teacher training and professional development in-service programmes best address and support teachers in the implementation of Mathematics content at the Foundation Phase?

I refer back to Thompson (chapter 2, page 14) who suggests that a research-based overview of the learning path that children follow in their mathematical development in the early years would provide support for teachers of young children. I do not think that the RNCS Mathematical Learning Area Assessment Standards are sufficient to support improvements in classroom practice. Therefore, a more detailed description of the mathematics of the curriculum domain of whole number and children's mathematical thinking in that domain could support teaching and planning and assist with improving classroom practice in the Foundation Phase (Kilpatrick, Swafford and Findell, 2001).

I believe that a theoretical basis for instruction in the form of a framework or trajectory, which follows the recommendations and content of the RNCS and gives an overview of number development, could be a useful teaching, planning and assessment tool, which would give teachers a common understanding and language for understating how young children learn number. The framework could support teachers in their implementation of the RNCS by providing groundwork based on theory from which they could design programmes of instruction and improve their teaching methods. The framework could describe currently accepted principles of improving instruction and a basis for understanding how children learn number. Such a framework could extend the knowledge base of children's numeracy, as well as support pre and in-service training by providing teachers with a detailed overview and understanding of learning and teaching number across the Foundation Phase and into the Intermediate Phase.

The Dutch TAL Learning-Teaching Trajectory is an example of such a model. This trajectory gives an overview of children's learning and provides a basis for understanding how number develops in the early years and in the primary grades. In the course of writing this dissertation, the South African Netherlands Research Programme on Alternatives in Development (SANPAD) was approached to fund a join research project between the University of Cape Town, Utrecht University (Netherlands) and Cape Technikon. The project aims to develop and research the impact of a Learning Pathway for Number (LPN) and supporting professional development materials in 18 Foundation Phase classes at three rural primary schools in the Paarl region of the Western Cape. The proposal has three main priorities:

- The development of a learning pathway or trajectory for early number development in the primary grades

- An evaluation study that measures the impact of the Learning Pathway for Number (LPN) and related school based intervention in changing teachers' practices and improving learner performance in three project schools

- The development of an appropriate model of in-service professional development to implement and support the LPN

It is hoped that this initiative will benefit teacher training and in-service programmes and provide the support and content knowledge necessary to successfully teach whole number in the early grades of the primary school and improve children's number competence.

*6.4 Final Remark*

This dissertation has given me the chance to map theories of children's number development and consider a theoretical basis for instruction in the form of a framework or trajectory. I hope that this thesis is able to contribute to mathematics education in South Africa by providing an extensive survey of literature on young children's number development and empirical studies involving children's solutions.

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*APPENDIX 1* MATHEMATICS: Grade R – 4 Learning Outcomes 1 and 2 Assessment Standards

Learning Outcome 1: NUMBERS, OPERATIONS AND RELATIONSHIPS

The learner will be able to recognise, describe and represent numbers and their relationships, and to count, estimate, calculate and check with competence and confidence in solving problems.

| Grade R | Grade 1 | Grade 2 | Grade 3 | Grade 4 |
|---|---|---|---|---|
| Assessment Standards<br>We know this when the learner | Assessment Standards<br>We know this when the learner | Assessment Standards<br>We know this when the learner | Assessment Standards<br>We know this when the learner | Assessment Standards<br>We know this when the learner |
| R.1.1 Counts to at least 10 everyday objects reliably. | 1.1.1 Counts to at least 34 everyday objects reliably. | 2.1.1 Counts to at least 100 everyday objects reliably. | 3.1.1 | 4.1.1 |
| R.1.2 Says and uses number names in familiar contexts. | 1.1.2 | 2.1.2 | 3.1.2 | 4.1.2 |
| R.1.3 | 1.1.3 Counts forwards and backwards in:<br>• ones from any number between 0 and 100;<br>• tens from any multiple of 10 between 0 and 100. | 2.1.3 Counts forwards and backwards in:<br>• ones from any number between 0 and 200;<br>• tens from any multiple of 10 between 0 and 200;<br>• fives from any multiple of 5 between 0 and 200;<br>• twos from any multiple of 2 between 0 and 200. | 3.1.3 Counts forwards and backwards in:<br>• the intervals specified in Grade 2 with increased number ranges;<br>• twenties, twenty-fives, fifties and hundreds between 0 and at least 1 000. | Counts forwards and backwards in a variety of intervals (including 2s, 3s, 5s, 10s, 25s, 50s and 100s) between 0 and at least 10 000. |
| R.1.4 Knows the number names and symbols for 1 to 10. | 1.1.4 Knows and reads number symbols from 1 to at least 100 and writes number names from 1 to at least 34. | 2.1.4 Knows and reads number symbols from 1 to at least 200 and writes number names from 1 to at least 100. | 3.1.4 Knows number names from 1 to at least 10 in the mother tongue (if not the language of learning and teaching) and one other local language. | 4.1.4 Describes and illustrates various ways of counting in different cultures (including local) throughout history. |

| | | | | |
|---|---|---|---|---|
| R.1.5 | 1.1.5 | 2.1.5 | 3.1.5 Knows, reads and writes number symbols and names from 1 to at least 1 000. | 4.1.5 |
| R.1.6 Orders and compares collections of objects using the words 'more', 'less' and 'equal'. | 1.1.6 Orders, describes and compares whole numbers to at least 2-digit numbers. | 2.1.6 Orders, describes and compares the following numbers:<br><br>• whole numbers to at least 2-digit numbers;<br><br>• common fractions including halves and quarters. | 3.1.6 Orders, describes and compares the following numbers:<br><br>• whole numbers to at least 3-digit numbers;<br><br>• common fractions including halves, quarters and thirds. | 4.1.6 Recognises and represents the following numbers in order to describe and compare them:<br><br>• whole numbers to at least 4-digit numbers;<br><br>• common fractions with different denominators including halves, thirds, quarters, fifths, sixths, sevenths and eighths;<br><br>• common fractions in diagrammatic form;<br><br>• decimal fractions of the form 0,5; 1,5 and 2,5 and so on, in the context of measurement;<br><br>• odd and even numbers to at least 1 000;<br><br>• multiples of single-digit numbers to at least 100. |
| R.1.7 | 1.1.7 | 2.1.7 Recognises the place value of digits in whole numbers to at least 2-digit numbers. | 3.1.7 Recognises the place value of digits in whole numbers to at least 3-digit numbers. | 4.1.7 Recognises the place value of digits in whole numbers to at least 4-digit numbers. |
| R.1.8 | 1.1.8 | 2.1.8 | 3.1.8 | 4.1.8 Recognises and uses equivalent forms of the numbers listed above, including:<br><br>• common fractions with denominators that are multiples of each other;<br><br>• decimal fractions of the form 0,5; 1,5 and 2,5 and so on, in the context of measurement. |

| | | | | |
|---|---|---|---|---|
| R.1.9 | 1.1.9 Solves money problems involving totals and change in rands and cents. | 2.1.9 Solves money problems involving totals and change in rands and cents. | 3.1.9 Solves money problems involving totals and change in rands and cents, including converting between rands and cents. | 4.1.9 |
| R.1.10 | 1.1.10 | 2.1.10 | 3.1.10 | 4.1.10 Solves problems in context including contexts that may be used to build awareness of other Learning Areas, as well as human rights, social, economic and environmental issues such as:<br><br>• financial (including buying and selling, and simple budgets);<br><br>• measurements in Natural Sciences and Technology contexts. |
| R.1.11 Solves and explains solutions to practical problems that involve equal sharing and grouping with whole numbers of at least 10 and with solutions that include remainders. | 1.1.11 Solves and explains solutions to practical problems that involve equal sharing and grouping with whole numbers to at least 34 and with solutions that include remainders. | 2.1.11 Solves and explains solutions to practical problems that involve equal sharing and grouping and that lead to solutions that also include unitary fractions (e.g. ¼). | 3.1.11 Solves and explains solutions to practical problems that involve equal sharing and grouping and that lead to solutions that also include unitary and nonunitary fractions (e.g. ¼, ¾). | 4.1.11 |
| R.1.12 Solves verbally stated additions and subtraction problems with single-digit numbers and with solutions to at least 10. | 1.1.12 Can perform calculations, using appropriate symbols, to solve problems involving:<br><br>• addition and subtraction with whole numbers and solutions to at least 34;<br><br>• repeated addition with whole numbers and with solutions to at lease 34;<br><br>• estimation. | 2.1.12 Can perform calculations, using appropriate symbols, to solve problems involving:<br><br>• addition and subtraction of whole numbers with at least 2 digits;<br><br>• multiplication of whole 1-digit by 1-digit numbers with solutions to at least 50;<br><br>• estimation. | 3.1.12 Can perform calculations, using appropriate symbols, to solve problems involving:<br><br>• addition and subtraction of whole numbers with at least 3 digits;<br><br>• multiplication of at least whole 2-digit by 1-digit numbers;<br><br>• division of at least whole 2-digit by 1-digit numbers;<br><br>• estimation. | 4.1.12 Estimates and calculates by selecting and using operations appropriate to solving problems that involve:<br><br>• rounding off to the nearest 10, 100 or 1000;<br><br>• addition and subtraction of whole numbers with at least 4 digits;<br><br>• addition of common fractions in context;<br><br>• multiplication of at least whole 2-digit by 2-digit numbers. |

| | | | | |
|---|---|---|---|---|
| | | | | • division of at least whole 3-digit by 1-digit numbers;<br><br>• equal sharing with remainders; |
| R.1.13 | 1.1.13 Performs mental calculations involving addition and subtraction for numbers to at least 10. | 2.1.13 Performs mental calculations involving:<br><br>• addition and subtraction for numbers to at least 20;<br><br>• multiplication of whole numbers with solutions to at least 20. | 3.1.13 Performs mental calculations involving:<br><br>• addition and subtraction for numbers to at least 50;<br><br>• multiplication of whole numbers with solutions to at least 50. | 4.1.13 Performs mental calculations involving:<br><br>• addition and subtraction;<br><br>• multiplication of whole numbers to at least 10 x 10. |
| R.1.14 Uses the following techniques:<br><br>• building up and breaking down numbers to at least 10;<br><br>• doubling and halving to at least 10;<br><br>• using concrete apparatus (e.g. counters). | 1.1.14 Uses the following techniques:<br><br>• building up and breaking down numbers;<br><br>• doubling and halving;<br><br>• using concrete apparatus (e.g. counters);<br><br>• number-lines. | 2.1.14 Uses the following techniques:<br><br>• building up and breaking down numbers;<br><br>• doubling and halving;<br><br>• using concrete apparatus (e.g. counters);<br><br>• number-lines. | 3.1.14 Uses the following techniques:<br><br>• building up and breaking down numbers;<br><br>• doubling and halving;<br><br>• number-lines;<br><br>• rounding off in tens. | 4.1.14 Uses a range of techniques to perform written and mental calculations with whole numbers including:<br><br>• building up and breaking down numbers;<br><br>• rounding off and compensating;<br><br>• doubling and halving;<br><br>• using a number-line;<br><br>• using a calculator. |
| R.1.15 Explains own solutions to problems. | 1.1.15 Explains own solutions to problems. | 2.1.15 Explains own solutions to problems. | 3.1.15 Explains own solutions to problems. | 4.1.15 |
| R.1.16 | 1.1.16 Checks the solution given to problems by peers. | 2.1.16 Checks the solution given to problems by peers. | 3.1.16 Checks the solution given to problems by peers. | 4.1.16 Uses a range of strategies to check solutions and judges the reasonableness of solutions. |
| R.1.17 | 1.1.17 | 2.1.17 | 3.1.17 | 4.1.17 Recognises, describes and uses:<br><br>• the reciprocal relationship between multiplication and division (e.g. if 5 x 3 = 15 |

| | | | | |
|---|---|---|---|---|
| | | | | then 15 ÷ 3 = 5 and 15 ÷ 5 = 3)<br><br>• the equivalence of division and fractions (e.g. 1 ÷ 8 = ⅛)<br><br>• the commutative, associative and distributive properties with whole numbers (the expectation is that learners should be able to use the properties and not necessarily know the names) |
| Learning Outcome 2: PATTERNS, FUNCTIONS AND ALGEBRA<br><br>The learner will be able to recognise, describe and represent patterns and relationships, as well as to solve problems using algebraic language and skills. | | | | |
| Grade R | Grade 1 | Grade 2 | Grade 3 | Grade 4 |
| Assessment Standards<br><br>We know this when the learner | Assessment Standards<br><br>We know this when the learner | Assessment Standards<br><br>We know this when the learner | Assessment Standards<br><br>We know this when the learner | Assessment Standards<br><br>We know this when the learner |
| R.2.1 Copies and extends simple patterns using physical objects and drawings (e.g. using colours and shapes) | 1.2.1 Copies and extends simple patterns using physical objects and drawings (e.g. using colours and shapes) | 2.2.1 Copies and extends simple patterns using physical objects and drawings. | 3.2.1 Copies and extends simple patterns using physical objects and drawings. | 4.2.1 Investigates and extends numeric and geometric patterns looking for a relationship or rules, including patterns:<br><br>• represented in physical or diagrammatic form;<br><br>• not limited to sequences involving constant difference or ratio;<br><br>• found in natural and cultural contexts;<br><br>• of the learner's own creation |
| R.2.2 | 1.2.2 Copies and extends simple number sequences to at least 100 | 2.2.2 Copies and extends simple number sequences to at least 200 | 3.2.2 Copies and extends simple number sequences to at least 1 000. | 4.2.2 |
| R.2.3 Creates own patterns. | 1.2.3 Creates own patterns. | 2.2.3 Creates own patterns. | 3.2.3 Creates own patterns. | 4.2.3 |

| | | | | |
|---|---|---|---|---|
| R.2.4 | 1.2.4 Describes observed patterns | 2.2.4 Describes observed patterns. | 3.2.4 Describes observed patterns. | 4.2.4 Describes observed relationships or rules in own words |
| R.2.5 | 1.2.5 Identifies, describes and copies geometric patterns in natural and cultural artifacts of different cultures and times. | 2.2.5 Identifies, describes and copies geometric patterns in natural and cultural artifacts of different cultures and times. | 3.2.5 Identifies, describes and copies geometric patterns in natural and cultural artifacts of different cultures and times. | 4.2.5 |
| R.2.6 | 1.2.6 | 2.2.6 | 3.2.6 | 4.2.6 Determines output values for given input values using:<br>• verbal descriptions<br>• flow diagrams |
| R.2.7 | 1.2.7 | 2.2.7 | 3.2.7 | 4.2.7 Writes number sentences to describe a problem situation, including problems within contexts that may be used to build awareness of human rights, social, economic, cultural and environmental issues. |
| R.2.8 | 1.2.8 | 2.2.8 | 3.2.8 | 4.2.8 Solves or completes number sentences by inspection or by trial-and-improvement, checking the solutions by substitution (e.g. [illegible] ÷ 2 = 12). |
| R.2.9 | 1.2.9 | 2.2.9 | 3.2.9 | 4.2.9 Determines, through discussion and comparison, the equivalence of different descriptions of the same relationship or rule presented:<br>• verbally<br>• in flow diagrams<br>• by number sentences |

# *APPENDIX 2*

# *LEARNING THEORIES*

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*1 Introduction*

In Appendix 2 I would like to present two theories of how children learn and the processes of acquisition involved in this learning. This discussion was not included in chapter 3, as theories presented here do not specifically deal with number development. The first is Piaget's theory, which is situated within a cognitive constructivist framework and is interested in the successive restructuring of knowledge (Neuman, 1987). The second is Vygotsky's social constructivist developmental theory which offers an important insight into understanding the ways children think and learn. These two theories have been included as an appendix in this study as I would like to acknowledge the influence and contribution of their perspectives on current pedagogic and curriculum trends.

Jean Piaget and Lev Vygotsky are two of the most influential developmental psychologists of the twentieth century (Von Glasersfeld, 1989; Gauvain and Cole, 1992). Although they wrote at different times and for different forums, both Piaget and Vygotsky considered learning and development as important though distinct psychological processes and were concerned with the processes of knowledge construction.

*2. Piaget's Theory of Cognition*

Piaget's genetic epistemology distinguishes between development and learning. Development is described as a spontaneous process, tied to the whole process of embryogenesis, and involves not only the body but also the nervous system and mental functions. Learning is considered to be provoked, rather than spontaneous, and is in response to an external situation. Development is an essential process and each element of learning occurs as a function of total development (Gauvain and Cole, 1992).

### *2.1 Operations*

In order to fully understand the development of knowledge presented by Piaget we must understand what he meant by an operation. Knowledge is not a copy of reality that should simply be re-represented (Gauvain & Cole, 1992; von Glasersfeld, 1997). Consequently to know an object or event one has to act on it, to change it and to understand the transformation so as to eventually understand the way the object was constructed. It is an interiorised action, which modifies the object of knowledge (Piaget 1992 cited in Gauvain & Cole, 1992). For example an operation could involve the classification of objects according to similar features, ordering objects, counting or measuring. In other words an operation is a series of actions which change or modify an object.

Piaget explains an operation as a particular type of interiorised action, which is reversible, for example addition (joining) and subtraction (separating), which is always linked to other operations and therefore is always part of a total structure. For example a number does not exist in isolation but exists as part of a series of numbers that together make up a structure (Piaget in Gauvain & Cole, 1992).

### *2.2 Stages of Cognitive Development*

In this section I will elaborate on the formation, order and function of operational structures in terms four separate stages as described by Piaget (1964). The first stage outlined by Piaget is the *Sensori-motor stage.* This is a pre-verbal stage and lasts approximately 18 months. During this stage infants develop by exploring and discovering the relationships between their bodies and their environment. They rely on sensory perceptions and motor activities to learn about themselves and their environment. This practical knowledge that infants discover constitutes the substructure of later representational knowledge for example the construction of the schema of the permanent object. Infants realise that objects can be moved (concept of causality) and they discover object permanence.

Object permanence is the awareness that even though an object can not be seen it continues to exist. A young infant will appear to loose interest in an object if it is covered, apparently believing that it no longer exists. An older infant, realising the object still exists, will actively try to find it. A child at this stage will perform motor experiments and vary his or her movement on order to observe the different results. Consequently a series of structures are constructed, for example object permanence, sensory-motor space, temporal succession and elementary sensory-motor causality, and these sensory-motor structures constitute the source of the later operation of thought. This means that intelligence proceeds from action as a whole, in that it transforms objects and reality and these series of structures are indispensable for the structures of representational thought (Piaget 1964, Piaget & Inhelder 1969 and Gauvain & Cole, 1992).

The second stage outlined by Piaget (1994) is the *Pre-operational stage.* It is in this stage that the beginnings of language and words that children use to represent what they can not see – 'pre-operational representation – the beginnings of language, of symbolic function, and therefore of thought or representation' (Gauvain & Cole, 1992)

The first attempts at generalisations are evident as the child develops 'pre-concepts' which fall somewhere between mental images and true concepts (Spencer 1991; Pulaski, 1980 in Spencer). This stage lacks the stable, reversible character of operational thinking and is typified by the absence of conservation of quantity.

> *During the pre-operational phase the child focuses his attention on the separate states or configurations of an array rather than on the transformations by which one state is transformed into another and such mental images as exist move slowly from one static configuration to the next, with no reversibility or flexibility, rather like slides in a projector. (Spencer, 1991)*

The *Concrete Operational stage* is the third stage described by Piaget (1964). During this stage operations become evident and children begin to reason logically and organise their thoughts in more structured ways. However they operate on concrete objects and cannot reason abstractly and verbal reasoning is limited. During this stage children begin to conserve quantity and this requires knowledge of reversibility. Children are also able to order objects (seriation) according to a specific feature eg size.

The fourth and final stage in the development of cognition described by Piaget (1964) is the *Formal Operational stage* or hypothetic-deductive stage and is characterised by the ability to reason or reflect on hypothesis. A child is able to think abstractly and to understand the form or structure of a mathematical problem, plan, explain events, consider several possibilities and generalise. He or she constructs new operations and attains new structures that are complicated combinations of structures. This phase represents the peak of human cognitive development and functioning of the mature organism (Gauvain & Cole, 1992). Although it represents the final phase, not all adults necessarily reach the full potential of this stage (Spencer, 1991). Lemoyne and Favreau (1981) conclude that the processes used by children in solving arithmetic tasks will be called *operational* if children are able to demonstrate comprehension of the cardinal and ordinal character of numbers, the additive composition of numbers and the reversibility of numerical operations.

*2.3 Maturation, Experience and Social Transmission*

According to Spencer (1991) there are three dynamic factors which can disturb or upset an 'organism' once it has achieved equilibrium. These explain the development from one set of structures to another (Gauvain & Cole, 1992). These factors will receive a brief discussion as they are significant in understanding children's cognitive development.

Maturation is the physical and psychological growth that occurs in each individual. Although Piaget acknowledges that maturation plays an important role and is part of every transformation in a child's development, he believed that this was insufficient explanation on its own to explain development. Although the chronological ages of these stages vary a great deal, the ordering of these stages is constant (Piaget, 1964). Certain pedagogic interventions can accelerate and complete development, like teaching counting before children have a concept of number, but that these cannot change the order of constructions. Spencer (1991) says that according to Piaget's scheme theory maturation opens up future possibilities, but experience is necessary to actualize the child's full potential (Spencer, 1991).

Experience involves thinking and acting with real objects in an external environment. According to Piaget (1964) this experience is a basic factor in the development of cognitive structures but as a single factor is also inadequate in explaining development. One of the reasons he gives for this is that certain concepts that appear in the concrete operational stage cannot have been gained from experience. He bases this argument on a conservation experiment involving a plasticene ball, which a child has to mould into a sausage shape. The child is asked whether the sausage has the same amount of matter, whether it is the same weight and whether it has the same volume as the ball. Piaget reports consistent findings in this experiment and notes that the first conservation that children (around 8 years old) have is of substance. He argues that through perception you can feel the weight of the ball or a constant volume, but that no experience can show a child the same amount of substance. Piaget explains this as a logical necessity and an example of a progress in knowledge.

The second reason for stating that experience alone is inadequate in explaining development is based on the distinction between *physical and logical-mathematical knowledge* that a child constructs while interacting with his or her environment.

> *Logical relationships are constructed in the child's mind, such as higher, lower, faster. Mathematical relationships also are not implicit in objects but are constructed by the observing or counting child. Number and number related concepts are logico-mathematical types of knowledge, but number is not a quality or characteristic of an object, it is an invention of each child.*
> *(Spencer, 1991)*

Physical experience involves acting i.e. touching, rolling and throwing, and necessitates the active manipulation of concrete objects. In this way knowledge about the physical properties of objects is acquired. Logical-mathematical experience, on the other hand, is drawn from the effect of actions on objects, not from the objects themselves. Logical-mathematical knowledge is derived from acting upon objects and not from the objects themselves. (Spencer, 1991 and Gauvain & Cole, 1992)

Social transmission is a third factor that contributes to cognitive development in Piaget's theory (Gauvain & Cole, 1992) and involves linguistic or educational transmission (von Glasersfeld, 1989) between children, parents and teachers.

*To make the Piagetian definition of knowledge plausible, one must immediately take into account (which so many interpreters of Piaget seem to omit) that a human subject's experience always includes the social interaction with other cognizing subjects.*
*(Von Glasersfeld 1989)*

Gauvain & Cole (1992) state that this factor alone is insufficient, because a child can receive valuable information via language or education directed by an adult only if he is in a state where he can understand this information. In other words a structure, which enables the child to assimilate information, is necessary and only once the child has constructed this structure can we say that learning or understanding has occurred.

Spencer (1991) and Gauvain & Cole (1992) argue that the process of equilibrium co-ordinates and regulates the three factors of cognitive development (maturation, experience and social-transmission) and is the underlying factor that organises biological and intellectual intelligence. Piaget's (1964) theory of learning or scheme theory is founded on three principles. These include a recognised situation, an activity that has been associated with this situation and an expected result (Von Glasersfeld 1997). This theory is explained in terms of a regulatory process that is geared by *adaptation* to the environment and involves two processes, namely assimilation and accommodation.

*2.4 Assimilation and Accommodation*

Piaget defines *assimilation* as the integration of any sort of reality into a structure, and it is this assimilation which is fundamental in learning and didactic applications (Gauvain & Cole, 1992 and Piaget, 1964). Von Glasersfeld (1997) and Donaldson (1978) explain the process of assimilation as a process of categorising experience and perception in terms of what is already known. This is an ongoing process of experience where the child engages in an activity (an operation), accepts new encounters and deals with the environment by making it fit into already existing structures or schemes.

If a child encounters new experiences that are not consistent with existing schemes while investigating his or her environment this 'hitch' in the process of *assimilation* causes a disturbance or perturbation which results in a review that may lead to *accommodation.* In other words it may give rise to change in an existing structure or the formation of a new one (Von Glasersfeld 1997). The child must adjust to new and changing conditions and accept or *accommodate* the new information so that pre-existing patterns of behaviour are modified. Spencer (1991) refers to Piaget as stating that intellectual growth is propelled by disequilibrium and Von Glasersfeld (1997) points out that the fact that accommodation does *not* take place unless something unexpected happens, is important for any learning theory.

Donaldson (1978) points out that assimilation does not occur in pure form but is always balanced by some form of accommodation. She adds that although the two processes can be thought of separately they cannot be distinguished from one another in any act of adaptation. Adaptive behaviour according to Donaldson (1978) always contains aspects of each of these two functions and will be at its most effective when there is a balance between them.

*2.5 Equilibrium*

Equilibrium is the third element in Piaget's scheme theory and provides a mechanism for learning. Schemes are mental constructs that consist of three parts. The first is an assimilatory structure or recognition template through which a child recognises a certain situation, like the presence of a graspable object with a rounded shape at one end. The second is an action or operation associated with that situation, for example picking up and shaking the object. The third is the expectation of a certain result of the action or operation, which could be the reward of the noise make by shaking it (Von Glasersfeld, 1989 and Olive, 2001).

Von Glasersfeld (1989) points out that the scheme theory involves, on the part of the observer, certain presuppositions about cognising organisms. He suggests that the organism should posses at least the ability and the tendency to establish recurrences in the flow of experience. This involves remembering and retrieving (re-representing) experiences and the ability to make comparisons and judgements of similarity and difference. There is also the presupposition that the organism likes certain experiences better than others.

Assimilation is the fundamental relation involved in learning and development. Each stage of learning is governed by a specific set of criteria and these must be met and mastered at each stage in order to move on to the next stage and in this way it is assumed that learning is subordinate to development. Piaget argued that children form mental concepts and that these adapt during the course of their experiences and relate to each other to form a structure (Gauvain and Cole, 1992).

Equilibration or self-regulation is the principal factor which explains development from one set of structures to another. It is the most fundamental factor and occurs when maturation, experience and social interaction interact. Equilibrium is the dynamic, on going and self-regulating tendency to seek cognitive coherence and stability (Piaget cited in Gauvain and Cole, 1992).

According to Spencer (1991) it is the process of equilibration which co-ordinates and regulates development. Contradictory statements concerning a child's realm of knowledge will disturb his or her equilibrium which will result in that child searching for an answer which will reduce cognitive conflict. All development consists of contradictions, incompatibilities and conflicts and these must be overcome in order to achieve equilibrium or self-regulation.

*2.6 Reflective Abstraction*

Von Glasersfeld (1989, 121-140) comments that Piaget's theory of cognition involves a two-fold intrumentalism. The first is on the sensory-motor level and involves action schemes that are instrumental in assisting organisms (children) to achieve goals in their interaction with their experiential world. The second is on the level of reflective abstraction. Reflective abstraction is the focusing of attention on the results or products of prior attentional operation (Von Glasersfeld, 1981). Internal speech and reflective thought arise from the interactions between the child and person in his or her environment and these interactions provide the source of development of a child's voluntary behaviour (Gauvain and Cole, 1992). Von Glasersfeld (1989, 121-140) points out that on the level of reflective abstraction, operative schemes are instrumental in helping organisms achieve a coherent conceptual network that reflects the paths of acting as well as thinking, which at the organism's present point of experience have turned out to be feasible.

*3 Vygotsky's Constructivist Epistemology*

Although Piaget considered children as active learners and as individual entities who construct knowledge and their own reality necessarily through interaction with their environment, he did not specifically focus his research on the socio-cultural aspects of learning and teaching. Vygotsky's constructivist epistemology on the other hand was more directly concerned with the social construction of knowledge and considered children in the light of social beings that learn from social interaction. Vygotsky was interested in language and the role of society in providing instruments to aid human development and how children gradually acquire the forms and tools of their culture and learn to use them appropriately.

Vygotsky (cited in Gauvain and Cole, 1992) states that maturation per se is a secondary factor in the development of the most complex, unique forms of human behaviour and that these behaviours are characterised by qualitative transformations of one form of behaviour into another. Spencer (1991) suggests that Vygotsky's objective was to explore how society provided instruments to aid the developing individual mind and how the child takes over the forms and tools of the culture and then uses them appropriately.

> *The most significant moment in the course of intellectual development, which gives birth to the purely human forms of practical and abstract intelligence, occurs when speech and practical activity converge.*
> *(Vygotsky, 1978)*

As soon as speech and the use of signs are incorporated into any action, the action becomes transformed and organised along entirely new lines. A child's speech and action in attaining a goal are part of one and the same complex psychological function (Vygotsky, 1978). Vygotsky suggests that if a young child were prohibited from speaking during the course of solving a problem, he or she would not be able to solve the problem. He states that by using hands, eyes (perception), speech and action together, internalisation of the visual field occurs. Vygotsky was particularly interested in the transforming power of language and the internalised linguistic system represented in thought (Spencer, 1991). This lead to his study of how children use hints and take advantage of other people helping them, to organise their thought processes until they are able to do so on their own.

As part of the process of solving a problem a child engages in a range of preparatory acts using instrumental or mediated (indirect), methods and in this way includes stimuli that are not within the immediate visual field. For example by using words to create a plan the child achieves a broader range of activity and applies not only available objects as tools but searches for and prepares such stimuli as can be useful in the solution of the task, and planning future actions (Vygotsky, 1978).

A child uses speech to plan how to solve a problem and then applies an action to complete the task. Direct manipulation is replaced by a complex psychological process through which inner motivation and intentions, postponed in time, stimulate their own development and realisation (Vygotsky, 1978).

When a task is too difficult for young children to solve, they will typically ask for assistance from an adult (interpersonal). As the child develops, this 'socialised speech' is *turned inward* and children are able to 'appeal to themselves' instead of to an adult (intrapersonal) for assistance. The history of the process of *internalising social speech* is part of the history of the socialisation of children's practical intellect (Vygotsky, 1978).

The initial stage of speech *accompanies* a young child's actions and is disorganised and unstructured during the process of solving a problem. This is followed by a stage in which speech *precedes* action as an aid in the process of solving a problem. At a later stage, speech guides and determines the course of action as a *planning function* at the beginning of an activity. It is in this way that children use language as a supporting tool when solving difficult tasks. This is part of the process of overcoming impulsive action in order to plan a solution before actually engaging in the problem.

Vygotsky did not believe that children invent or discover indirect operations as sudden insights ('aha' reactions); but that these appear as a result of a complex and gradual process after a series of *qualitative* transformations and that these transformations are linked as a series of stages which are part of one continuous process.

> *..the course of child development is characterized by a radical alteration in the very structure of behaviour; at each new stage the child changes not only her response but carries out the response in new ways, drawing on new "instruments" of behaviour and replacing one psychological function by another. Psychological operations that were achieved through direct forms of adaptation at early stages are later accomplished through indirect means. The growing complexity of children's behaviour is reflected in the changed means they use to fulfil new tasks and the corresponding reconstruction of their psychological processes.*
> *(Vygotsky, 1978)*

Vygotsky did not advocate that development and maturation are preconditions to learning, or that learning is development. Neither was he satisfied with the theoretical positions that suggested a combination of these first two notions. Two questions that Vygotsky attempted to answer were: 'What is the general relation between learning and development?' and 'What are the specific features of this relationship when children reach school age?'

It is accepted that children have a history of learning before they enter school and that they bring to Grade 1 a wealth of experiences, including quantity and early operations, for example adding, subtracting, sharing and being able to determine size and number of objects in small collections. Their learning and development has been completely interrelated since the day they were born. Learning language, learning a variety of skills and gathering information from imitating, asking questions and being instructed by adults. It is also generally understood that children's pre-school experiences are 'informal' and that their school experiences are more 'formal' and structured. In Vygotsky's view (cited in Gauvain and Cole, 1992) it is not only that formal school is structured and systematic, but that school learning introduces something fundamentally new into the child's development.

*3.1 The Process of Acquisition*

Vygotsky's concern was with the process of acquisition of meanings. He introduced the notion of a *Zone of Proximal Development* (ZPD) as an explanatory framework for formal and non-formal situations involving learning. He defined the ZPD as 'the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers (Vygotsky, 1978).

In order to understand the concept of the Zone of Proximal Development it is important to establish that Vygotsky did not believe that learning necessarily had to be set at a child's developmental level, nor that instruction should be determined by specific age or developmental levels. However what he did make clear is that we should not limit ourselves to 'determining developmental levels if we wish to discover the actual relations of the developmental process to learning capabilities' (Vygotsky, 1978; Gauvain and Cole, 1992).

He described two developmental levels: the first, the *actual developmental level,* which is the level that children perform at on certain tests and which is only indicative of mental abilities. These tests only succeed in assessing certain already completed developmental cycles and give us information based on how children solve these problems and at what level of difficulty. Vygotsky criticised assessment based on the actual level of development, claiming that what children can do with the assistance of another person (rather than on their own) might in fact be more indicative of their mental development (Spencer, 1991 and Gauvain and Cole, 1992).

The second and most significant level is the *Zone of Proximal Development.* According to Vygotsky (1978) if a child can successfully compete a task independently, that child has already developed the mental functions that make this possible. The zone of proximal development defines those functions that have not yet matured, when a child is unable to complete a task alone and requires assistance. The zone of proximal development is the range of a child's *potential* to solve a problem with assistance and guidance from an adult or more capable peer. Accepting that processes are in a state of formation, maturation and development, means that by raising our expectations and challenging children with activities that are beyond their actual developmental level, we increase the possibility of learning.

The significance of the Zone of Proximal Development is that it determines the lower and upper bounds of the zone within which instruction should be pitched (Wells, 1999). Gauvain and Cole (1992) propose that learning which is orientated towards developmental levels that have already been reached is ineffective and that good learning is that which is in advance of development.

*4. Summary*

The theories outlined in appendix 2 focus on children's learning in general. I have provided a brief sketch of Piaget and Vygotsky's constructivist theories as they are of particular interest to this study. The influence of the constructivist paradigm is evident in the methodology and discourse that I have encountered from teachers and has had a direct influence on the development of curriculum and policy in recent years.

*APPENDIX 3*

**TEACHER QUESTIONNAIRE**

1. Grade you are currently teaching: ........................

2. How many learners are there in your class? ................

3. Where is your school situated? *tick one box only*

   a). rural ☐

   b). per-urban ☐

   c). city ☐

4. How many learners attend your school? *tick one box only*

   a). under 200 ☐

   b). 200 – 300 ☐

   c). 300 – 400 ☐

   d). 400 – 500 ☐

   e). over 500 ☐

5. Are you female or male? *tick one box only*

   a). female ........................................................... ☐

   b). male ........................................................... ☐

6. Which age group do you fall into? *tick one box only*

   a). under 25 ........................................................... ☐

   b). 25 – 29 ........................................................... ☐

   c). 30 – 39 ........................................................... ☐

   d). 40 – 49 ........................................................... ☐

   e). 50 – 59 ........................................................... ☐

   f). 60 or older ........................................................... ☐

7. What is your medium of instruction? *tick one box only*

a). Afrikaans .......................................................... ☐

b). Xhosa .......................................................... ☐

c). English .......................................................... ☐

d). Other, specify .......................................................... ☐

8. By the end of this year, how many years will you have been teaching?

......................

9. At which of these grade levels have you taught in the past 5 years?

a). Pre-school ☐

b). Grade R ☐

c). Grade 1 ☐

d). Grade 2 ☐

e). Grade 3 ☐

f). Grade 4 ☐

g). Grade 5 ☐

h). Grade 6 ☐

i). Other, specify ☐

10. How often do you have meetings with colleagues at your school to reflect and plan teaching and learning?

a). never ☐

b). once or twice a year ☐

c). every second month ☐

d). once a month ☐

e). once a week ☐

f). 2 or 3 times a week ☐

g). almost every day ☐

11. What is the highest level of formal education you have completed? *tick one box only*

a). Std 8 ☐

b). Std 8 + 1 year teaching training ☐

c). Std 8 + 2 year teaching training ☐

d). Matric ☐

e). Std 8 + 1or 2 year teaching training + matric ☐

f). M + 1 year teaching training ☐

g). M + 2 year teaching training ☐

h). M + 3 year teaching training ☐

i). M + 4 or 5 year teaching training eg. HDE/FDE ☐

j). B. Degree, eg B.A., B.Sc + no teaching training ☐

k). B.Degree, eg B.A, B.Sc + teaching training ☐

l). Post graduate degree, eg B.Ed/B.Hons + no teaching training ☐

m). Post graduate degree, eg B.Ed/B.Hons + teaching training ☐

n). Masters + no teacher training ☐

o). Masters + teacher training ☐

12. What was the year in which you achieved your last formal academic/ teaching qualification?

..................

13. At which institution did you receive you last formal academic/ teaching qualification?

...........................................

14. Have you been part of any Mathematics professional development programmes or projects?

Yes ☐ No ☐

(Adapted S. Jaffer/2001)

## *APPENDIX 4*

### INTERVIEW 1

A class is investigating addition by using counting rods. The learners are working in pairs. The educator asks one leamer in each pair to represent and write the number 34, and the other leamer to represent and write the number 27. The leamers are then asked to combine the rods and tell her how many there are altogether. She asks several pairs to explain how they found their answer. She then demonstrates a method for recording what the leamers have done with the rods.

**Please consider the following questions:**

1. What do you think the educator is hoping to achieve from this activity?

2. What number concepts do you think are being developed?

3. How do you think learners will represent the numbers?

4. What method do you think the educator would have demonstrated?

**INTERVIEW 2**

A group of learners sits around a SPAZA food poster. Each learner has a pencil, crayons, paper and a bakkie of R1 and R5 plastic coins. The educator asks them to choose items from the poster that they would like to buy and to calculate what it would cost.

Deno chooses chips for R4 a chocolate for R3 and a coke for R5. The group discusses his choice and begins to solve the problem for themselves.

Deno: Draws the four packets of chips, three chocolates and five cokes. He counts them all and writes 12.

Freeda: Puts up 4 fingers. She then puts the tips of her fingers into her mouth and nods as she counts '5, 6, 7, 8, 9, 10, 11, 12'. '12!' she says

Zondi: Counts out R1 coins: 'R1, R2, R3, R4, R5, R6, R7, R8, R9, R10, R11, R12'

Lisa: Takes out R5 and R1 coins and counts 'R5, R10, R11, R12'

Ayanda: Looks at the teacher and says 'R12'.

Fran: Says '5 and 4 is 9 and 1 and 2 is 12'

The learners then explain what they have done and why they have presented the problem in their particular way.

(Adapted: Children Learn Mathematics, Freudenthal Institute, 2001)

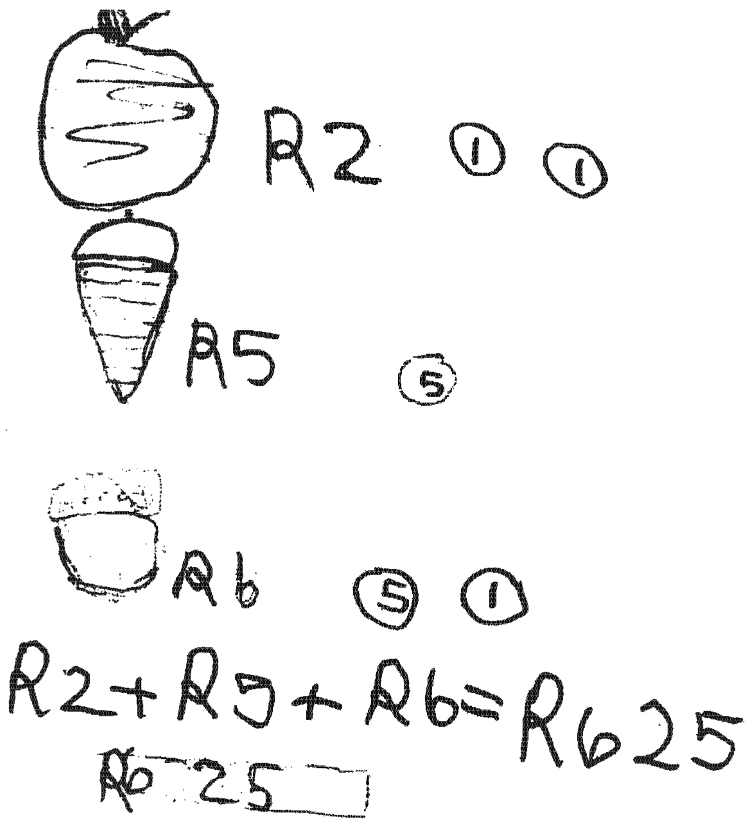
**Please consider the following questions:**

1. What do you think the educator is hoping to achieve from this activity?

2. Comment on each child's solution strategy.

## INTERVIEW 3

**Look at this example of a copy of one Grade 3 learner's solution.**




**Please consider the following questions:**

1. What is your response to this learner's solution?

2. What prior knowledge and skills do you think the educator should have assessed before giving the learner a problem of this type?

3. What would you do to assist this learner to solve this type of problem accurately and efficiently in the future?

**INTERVIEW 4**

**Please consider the following questions:**

1. Describe a learner in your class who you think has a good concept of number.

   - Why do you think this learner has a good concept of number?

2. Describe a learner who you think has a poor concept of number.

   - Why do you think this learner has a poor concept of number?

3. You are asked to support a student or colleague in the teaching of counting.

   - What would be the main points you would focus on?

   - What suggestions would you make on the teaching and learning of this topic?

*APPENDIX 5*

**Participant 1**

**Interview 1**
(Tape 1 Side 1: 290/Side 2: 000 – 232)

**1. What do you think the educator is hoping to achieve from this activity?**

Seeing that the educator wants to let the child pack out umm, tens and units has two learners doing it, it seems as if the educator wants the child to understand tens and units a group together. And the other case seems that they had to put the two totals together she wants to learn them the basics of counting on or do some um, plus sums to understand it goes from the one, to take the seven units and the four units you'll get eleven. So the child needs to know the eleven to come one ten and he'll have one unit left. And ....

**2. What number concepts do you think are being developed?**

As I said the concepts you do think are being developed will be tens and units and counting. And um, maybe we can have some subtraction also here, if she needs to .....want to count back again. You need to take the 27 away again and the 34 and put it together.

**3. How do you think learners will represent the numbers?**

And how the learners represented the numbers. If the child has a counting rods then most probably he'll just put everything together till he gets to the total he needs to be, make, to get to the 34 or he'll get to the 27. But if they going to put it together they'll also just, if they clever enough, the one will take the 34 and they'll just count on, just add on the 27 to get to the new number. If the teacher wasn't clear how she wanted it, because in the question it just say she just asked them to tell how the learners are then asked to combine the rods and tell her how many there are altogether. So, the instruction she has given just say put it altogether, tell me how many there are. So, if it's two, just children that can just count, they'll just put it together or they'll count from the beginning again and say lets count everything together.

**4. What method do you think the educator would have demonstrated?**

Right, what method do you think the educator should have demonstrated? I would have, I mean the educator should have after they've counted out with the rods, the 34 or the 27, she then ask you how many tens you have, how many units you have. And also show them you can put all your, you can um, break up, maybe she should have showed them two or three ways how they could have get to the new total she wants, so that they could maybe they can break up the tens and the units and then they tell them okay then you count how many tens you have, how many units I have, put all your tens together, all your units together and get a new number. Or she could have asked them to put the, she should have asked them to maybe have another way they can count all the tens and then count all the units, and then for the two of them could put their tens together and then their units together and then get their new number. But, she should have demonstrated to them one way they could do it and then leave them with any other ways they want to do it, I mean the direction should be clear....what you want from them.

**Interview 2**

**1. What do you think the educator is hoping to achieve from this activity?**

The educator ...to me is hoping that the child would understand again how to count, but also to put together different ways of exploring number here. Because he needs to, because they buying things, even if its from a poster, but they buying and then they have to calculate the cost of what they want to buy. And this is a problem solving, its number but it's also problem solving because the child, she gives the children after everybody has, one child has chosen what he wants to buy, they have a group discussion of how they going to calculate or to solve the problem. They need a solution how much he must, what's his name, Dino, must pay for the goods if he's going to buy....and

**2. Comment of each child's solution strategy.**

The children have different strategies of how they get to, because what they going to buy costs say it's R12 but although they know it's 12 ....all of them doesn't know the value of the R12 because she's given them, they've got little bakkies here with R1 and R5 um, coins in, but because they don't know, because if they all, if only the one child ....he's Agatha...immediately know that she could cope with the numbers together, the 4 and the 3 and the 5 and say you get 12. And then with the others, they on different levels when it gets to counting and to understanding the value of number. Although most of...all of them had a 12 in mind, they don't really know the value of that number because, I mean the first one took out four packets, she drew four packets of chips although the chips cost say R4, so maybe there were he bought four packets of chips, I don't know and three chocolates, so one chocolate from the poster could have been R3 but he just chose to draw to get to the three, he knew he had to get to a 3, she doesn't understand the value of the R3 when it comes to break up, so also number two there's only one child whose Ayanda......and uhhmmm it's Ayands and the other one will be the last one, Fay, that knows a 5 and a 9 and you get to 12. So, that's it. So the understanding.....

When it comes to number, I just feel that children, especially when it comes, 'cos money is actually the easiest way or the most appropriate way to learn, to understand number, because it's got value and you working with that numbers. So, if you like have that R1 the child should know before I can have a child to count out like Rands and R5 and R3, the child should know the value of that R1. He should know I have 100 tens will give me that R1 or that ten 10c pieces, or that 5c pieces are going to give him that R1, so he, because otherwise he's just going to see the 2 or the 1 and he doesn't, he won't know, I mean if he doesn't know the value he won't even be able to use that the money outside the classroom. If he go buy a bread, he must know if the have a R5, the bread is R4.50 I must get a 50c change because he knows the value of that, the bread is R4.50. So that R5 coin won't be one thing he have in his hand, he will know it is R5. And I have ten 50c pieces. So the children need to let the children pack out that plastic money and he must really see it so that he really, or the child really knows the value of that coin, doesn't matter if it going to be one counter, he must know that counter is a

50c. So before they do things, the children have exercises like this, they must first have a prior knowledge of the value of this money.

**Interview 3**

**1. What is your response to this learner's solution?**

If I look at this learner's solution I mean you can quite clearly see that this child has no understanding about the value of the R2 or R5 or R6. He writes down 2 and then R2 and two ones so all he knows is two 1's gives me 2. And then when he comes to the R5 he only wrote.....5 because he couldn't even count on to say he had 1 and 1 and 1 or 2 plus a 2 plus a R1, if he had to break it up. You get to the R6 again, he saw he had a R5, so he knows 5 plus 1 gives me 6, that's why he could write 5 and 1 but when it comes to when he had to add it up and then write 2 plus 5 plus 6 gets him 625 he wrote down the 6 and he wrote down the 5 and wrote down the .....

**2. What prior knowledge and skills do you think the educator should have assessed before giving the learner a problem of this type?**

So, to him it's money but he doesn't know or understand the value of that coin so the child has no or little understanding of numbers about the value of numbers and how to break up a number he doesn't know.

As I just said previously is that the teacher should, before I can assess a child giving him a problem I must know the child understand in this instance it's money. He must know the value of that money, even if they have the plastic coins he must understand so the child should have been given the opportunity in the class beforehand to play around only say with the R2 and then he must have the different coins, the value coins like to have your R1, your 50c pieces, 20c coins and 10c coins and even 5c coins, so that he knows he will be able to count in two's to get to that how many 2c pieces, 5c pieces or 10c and then he should have had the opportunity to put whatever the different coins together, like having a 50c and 20c pieces and a R1 to get to the R2.

**3. What would you do to assist this learner to solve this type of problem accurately and efficiently in the future?**

They have to pack them out and he must, you should have given him practical problems like you are going to buy say a bread and then ask him how much the bread costs, right so you have, how much money do you have. The child should have pack it out for you and say I have with me, say I have a R6, so you going to pay R4.20 for the bread, so pack out the R4.20 and then write down which coins you used to pay for the bread. And then what do you have left, how much do I have left. ....have R1.80 left and also pack out the coins you have left.

**Interview 4**

**1. Describe a learner in your class who you think has a good concept of number.**

A learner with a good concept of number is one that will identify number or any number within the context of, or maybe even further than, if say for instance he's in Grade 2, a Grade 2 learner and he just need to know numbers say till 100, he will be able to tell you any number say till 500, because he has a understanding of number. Because if he knows, if he can count from, he knows his number from 1 till 100, he will be able to identify any number, even if it's up to a 1000. Because if he knows the value of those numbers, then he will be able and this good learner will, is the one that will pack out your, the numbers correctly, he will know say what is a 100, what is tens, what is units. He will be able to take away if he sees I have 11 units, he will immediately put the 1 with the tens and he will leave the 1 with the units. And the one who has a good concept of number is the one who will be able to add on quite well, be able to subtract quite well, even be able to multiply, to do all the basic concepts of multiplication, because he can add on and he will be able to subtract because he can take away, because he understands the value, he know what goes bigger and what goes smaller, if there's a number should be in the middle or to the left or to the right. He will be able to know that if I say for instance if you say what's nearer you have what is the half of 15, he'll be able to tell you whether 8 will be more to the half, because to have 15 is nearer to 16 and he won't say 7, or he'll tell you .. have to say 7 is you want the exact number..you must take 7 to......and you take the half of the 1 to have 7 ½ and another 7 ½, but the other one won't know that, that would be a good learner in the class.... He'll be able to estimate quite well also.

- **Why do you think this learner has a good concept of number?**

I think I've answered that one now, the good concept of number he need to, I mean because of the good learner is the one that break up his number quite easily, and I mean that's also the one that will tell you even that you able to read his time well, because he knows, he would know what goes around to get the one, and he will be able to count that size of seconds in between, because he has the understanding of number and he will write his numbers correctly. He will even be the good, the one that, the learner in the class that has a good concept of numbers, of learners is also your good reader, because he knows his numbers goes from left to right, so he will be the good reader also, so your good readers will also be the ones that's able to be good in numbers and will know what's in the middle and what is left and what is right.

**2. Describe a learner who you think has a poor concept of number.**

- **Why do you think this learner has a poor concept of number?**

The learner with a poor concept of number is the learner that doesn't have any......maybe he's got a poor, his visual understanding is poor he's, his in Afrikaans we call it we say "latelariteit", it's like his left and his right is out, so and he'll quickly juggles the numbers, he like, he will write numbers upside down and he won't be able to, this learner is the one that can't break up, maybe he knows the number. He will tell me, the poor one will tell me this is a 5 or this is a 10, but he won't know the value of the 5 or the 10. He won't be able to pack, he doesn't know how to pack out. He will even give you the answer if you ask him 7 and 5, he will tell you it's 12 but if you give him a problem, he won't be able to get the solution to that problem but maybe he will just be able to write down let's say 7 plus 5 is 12, but the moment I'm going to put the 7 and the 5 in a problem, then he won't be able to give that solution. He is also the poor, the one who has poor concept of number is the one who can't estimate quickly. And he's also a poor reader as I said, because they won't, he doesn't know the value from left to right, and maybe what is in the middle if I tell him start counting in 3's from 71 to 100, he won't be able to do it. The one with the poor concept of number must start from the beginning, he must every time have the picture from the beginning again and you have to tell him start in 3's from 3 to 18, then he'll say 3, then he'll count on his fingers to get to the 6 and maybe count again to get to the other one, but he won't off-hand be able to count. And also to just to get to....the one with the poor concept of number is when you take your number block, they, and you take out numbers and you leave out numbers, they won't be able to put it in.

**3. You are asked to support a student or colleague in the teaching of counting.**

- **What would be the main points you would focus on?**

- **What suggestions would you make on the teaching and learning of this topic?**

If I should be asked, if I must now support a student or even a colleague...teaching of counting, is that first the teacher or the student I will use, I'll use counters. The teacher has to focus on .....must be counters. You must have the real things with you. You must have your, the number blocks with you, like your 100's, and even maybe ... have loose one the 0 to 9...then the child understands ....the value of your units and then understand how you get the values if you add it it gets to the 10 and the units with, you have need to use, you have to .....but also need to have the numbers on cards, on individual cards and also on the number, the graph we use for the number. And for counting, to me that is very important ....the student or the teacher needs to know the children have to count every day. They have, we don't want to, we know we not drilling the children anymore, but counting is most important, they need to start, if you for one week I focus on just counting, adding on, and I work with my 1's and the next week I start say with 2's and then with 3's until I get to 100's, and then count back again. And I have to start within any number range. If the child needs to say in Grade 1, needs to be able to know up to the value of 50, then the child needs to start anywhere, if I had to tell her count in 2's from 51 to 71 he have to start there. And important for counting is that you must from the beginning learn the child to count backwards, because if the child counts to the front and he counts backwards again, when the time you have to teach the child to understand multiplication and division and subtraction, he will be able to do it, because the multiplication he will go adding on and work backwards and do subtraction and the division, to know it gets less and less.

....whatever I suggest is that you should use concrete things to learn a child. He must use the real thing to count, we can't just count and we can't just do sums. Every time we do a sum when it comes to number you must give the child a problem, because if we just work from, if the child just see that number on the board or I just give them the number and he put out the sum, and there isn't a problem attached to it, he will never understand, he will never be able to make a, be a critical thinker when it comes to maths. Because if they understand number, they will be able to understand everything else in maths, because it starts with number.

*APPENDIX 6*

**Participant 2**

**Interview 1**
(Tape 1 Side 2: 233 – 285 / Tape 2 Side 1: 000 – 032)

**1. What do you think the educator is hoping to achieve from this activity?**

In answer to my first question, "What do you think the educator is hoping to achieve from this activity?", based on what I've just read, I think the teacher really wants the children to know how to count, and how to use the numbers she gave them to add and how to get their answers. That is my first answer.

**2. What number concepts do you think are being developed?**

...and for number two, "What number concepts do you think are being developed?", the child learner must know what 34 and 27 looks like, especially if the teacher has the counting chart in front of him, and he also has his counting rods with him, then he must be able to see what 34 looks like on the counting chart and he must use the counting rods to count out the 34 and 27 in front of him and he must know how much 34 or 27 is. This is number concept, how is it calculated and how does the child know, what 34 and 27 looks like, and he must be able to count. What also comes in here is place value, it will fall under place value he must know what it looks like and how much the number is. That is number two's answer, and number three.

**3. How do you think learners will represent the numbers?**

"How will the learner present the figures or the amounts?". I think, I feel, that he'll be able to count out the 34 and than he will probably explain to the teacher how he does this. He will count out his 34 counters, then he'll say three tens gives you thirty, then he'll add his four units, and this will give him 34 altogether, the number 34 and he will do the same with the number 27.

**4. What method do you think the educator would have demonstrated?**

And the method the teacher demonstrates, I think she did this, she demonstrated to them before she told them what to do, and how to get to their answer, I think she also showed them the easiest way to get to 34 to see what 34 looks like, he will first count out his thirty and say this is three tens, and than he will add his four units, and this will give him 34 altogether, because this is the easiest way to know what bigger numbers look like.

**Interview 2**

**1. What do you think the educator is hoping to achieve from this activity?**

As with answer to previous question, "What do you think the educator is hoping to achieve from this activity?", I think that the learner should know how to count and to add up, and to know how to, what the answer of this activity is. From what I can see, the activity that each one did, they used different strategies to get their answers. I also see that it is problem solving. They must be able to solve their own problem, each one solved their problem in a different way, and they must also have good number concept to be able to do this and they must know the value, they must know the value of money because many children don't know the value of money, if we work with money in the classroom with them, like here the answer is R12, they need to know what the value of R12 is, how to get the whole answer and then by doing this they must be able to add up the total cost of the sweets that they added together and what I also see here, each one used a different strategy to get their answer...that is basically all.

**2. Comment on each child's solution strategy.**

I see that Ayanda gave her answer immediately. She added the numbers up immediately and gave her answer, this means she also has good number concept. She doesn't need to use concrete apparatus or use her fingers to solve the problem, where Dino drew his things. This is how a person sees that the different ways of the children in your classroom are not the same. They must use their own method to solve the problem. They must use their won methods to solve problems. We can't teach them a specific method and tell them they must do it that way. That is why OBE is so, this is part of OBE, each one must use his own method. He must be able to solve his problem on his own and I see here that Dino drew his to get his answer. He counted everything together and then got his answer. Zondi made use of the bakkie of R1's, counted all the R1's together, to get his answer....

**Interview 3**

**1. What is your response to this learner's solution?**

Based on the activity I've seen, that the child did, I can see that he has no number concept, or not a good concept of numbers, and this is a problem solving and I can see that the child can't solve the problem. He doesn't know how to solve the problem, and he doesn't know the value of money. He doesn't know what R2 is, how much R5 is and how much R6 is. And what could also be making the problem worse is that the child perhaps can't read. Maybe he can't read the sum.

I feel that the teacher should first have read the sum to the children, as there are many children who can't read and won't understand the sum because they can't read it, she should first guide the child, guide the learner through the sum. She should first read the problem, the sum to the child and then give him an example...we often have problems with (word) problem solving in the classroom and with children we see cannot solve problems. This is why we need to first guide them towards the answer. For example, we don't give the same activity, we give another activity and then show them how to do it, and now if he does the activity you at least know that he understands how to do the sum. I feel the teacher should have first guided him to the sum and then given him an example of a similar sum, so that he sees how it should be done and then he could go back to his place and do it himself. This is also why he had the wrong answer, because for a Grade 3 child, the sum is very easy and you can see that he can't do it because he isn't busy with big numbers, but small numbers under ten, and that's why he can't do it. He also has a poor concept of number.

And then, I also feel that in future, if the teacher sees the child can't do the sum, that he has a problem, if he has such a poor concept of these numbers, she should give more easier activities, first before she can give him such an activity, she must work with the children even though they are in groups, maybe he is in the weak group, then I feel she should sit with them and give them easier activities, so that they can do it before they go on to more difficult ones. Then I feel that she should begin with smaller numbers and first establish a good number concept in the child before he can do more difficult sums.

**Interview 4**

1. **Describe a learner in your class who you think has a good concept of number.**

- **Why do you think this learner has a good concept of number?**

"Describe a leaner in your class with good number concept. Why do you think this child has good number concept?" I would say a learner in my class has good number concept if the learner can do all his basic operations. He can add, subtract, multiply and divide, and can also do problem solving. This is a child that has a good number concept. If I sit with him on the mat in the front of the classroom, and I maybe give him a problem to solve this child will give me the answer immediately, without counting on his fingers or using concrete apparatus, I feel this is a learner with good number concept, and also if I give them an activity at the table and I usually share counting charts out or give them sticks or something, then I know that I definitely don't need to give this learner any as he doesn't need to work with counting charts or sticks. This is how I can see that the learner has a good number concept.

2. **Describe a learner who you think has a poor concept of number.**

- **Why do you think this learner has a poor concept of number?**

And a learner with poor number concept, I notice immediately, I see that the learner can't count, before we do group work, we first do a class lesson in front on the mat and then I let them count and usually they can't count up to a certain amount, then I know the learner has a poor concept of number, and also if he does activities for me in the classroom or on the mat and I give him an easy sum, addition sum or subtraction sum, and the learner can't do it, then I know immediately that the learner has a poor concept of number. He can't count and he also doesn't know how to add numbers together or take numbers away.

**3. You are asked to support a student or colleague in the teaching of counting.**

- **What would be the main points you would focus on?**

- **What suggestions would you make on the teaching and learning of?**

The points that we need to concentrate on to encourage good number concepts in learners, that we need to count regularly, we must count every day in the classroom and also give the learner problems to solve every day, so that he can know how to solve the problem, how to count, because if the child has a poor number concept, than he cannot perform any of his basic operations. And what we can also do with children with a poor number concept, and we see can't count, we can work with them individually on the mat, or in a group, and that they do sums with the concrete apparatus because it's are difficult, because when you work with them on the mat, and you need to give them your attention, you must sit with them all the time, as soon as you move away from them and go to your desk and then come back, you see the child has done something incorrectly. So, you must keep saying to him, say you're doing a sum of 4 plus 2, now I must say first count 4 for me, he knows he has sticks with him but he doesn't know how to pack them out, how to arrive at his answer, now I need to guide him. Now I must first tell him to pack out 4 sticks and once he's counted out 4 sticks, how many must he add, now he adds another 2 sticks, now he must add all the sticks together to get his answer, now he will write his answer down. So, each time I must guide him to the answer. This is what the teacher must do in the classroom to help the child, to be able to see that the child progresses and has a good number concept, and at the end of the day, that he can develop.

# *APPENDIX 7*

**Participant 3**

**Interview 1**
(Tape 2 Side 1: 032 – 074)

1. **What do you think the educator is hoping to achieve from this activity?**

.....adding with using the units to make a new ten.

2. **What number concepts do you think are being developed?**

Number two, number concept, a new the units and to be taken over to the new ten must be taken over to the ten.

3. **How do you think learners will represent the numbers?**

Number three, 60 plus 11......

4. **What method do you think the educator would have demonstrated?**

Number four, making units into new tens.

**Interview 2**

1. **What do you think the educator is hoping to achieve from this activity?**

Number one, for the children to count on.

2. **Comment on each child's solution strategy.**

Dino can't work on his own, Frieda she has the ability to count on, Zondi number concept needs more practising, Lisa uses counting strategies. Ayanda, she has number concept as well as Fran.

**Interview 3**

1. **What is your response to this learner's solution?**

Number one, this learner writes down everything that he sees. He has no ability for adding, maybe he doesn't understand the sum or whatever he has to do.

**2. What prior knowledge and skills do you think the educator should have assessed before giving the learner a problem of this type?**

Number two, the teacher has to first assess that the child could add with smaller numbers or with less amounts.

**3. What would you do to assist this learner to solve this type of problem accurately and efficiently in the future?**

Number three, how to add only two items until the learner understands before moving on to bigger numbers.

**Interview 4**

**1. Describe a learner in your class who you think has a good concept of number.**

- **Why do you think this learner has a good concept of number?**

Number one, can he give his age, write his age, count it out, show it, can show any number and name it, can write numbers, maybe till 40. He has good number concept because he can add and subtract and maybe multiply till 25.

**2. Describe a learner who you think has a poor concept of number.**

- **Why do you think this learner has a poor concept of number?**

Number two, can't write any number or show to a number or even count out. He can count without understanding, he's counting like the parrot but he doesn't understand what he is really doing. Maybe he didn't have enough practising in counting, didn't have practical exercises. I've left out here, I don't know what. He has not had enough practical experience with counting and numbers and with practical apparatus or whatever.

3. **You are asked to support a student or colleague in the teaching of counting.**

- **What would be the main points you would focus on?**

- **What suggestions would you make on the teaching and learning of this topic?**

Number three, the learners must see and use materials and apparatus that they know, it must be life related. Use the apparatus within learners experience, involving learners at all times.

## *APPENDIX 8*

**Participant 4**

**Interview 1**
(Tape 2 Side 1: 074 – 165 )

1. **What do you think the educator is hoping to achieve from this activity?**

I think the educator is busy teaching the children, so that they can add better. At the same time she is busy with co-operative learning, they work in groups so that they learn from each other, and she is also guiding them towards visualising numbers, so that they can write the numbers as well as understand it, and then she is busy developing their skills in relation to their thinking abilities.

2. **What number concepts do you think are being developed?**

I think she is busy taking number concept up to 99 and at the same time I think she is also busy with crossing the 10, where they must make 10. She is busy laying the foundation of this understanding in the learners.

3. **How do you think learners will represent the numbers?**

I think some of the learners will pack out three tens and four units, two tens and seven units. Some of them will just draw the whole amount in circles; some will maybe draw fruit, just as they see the amount.

4. **What method do you think the educator would have demonstrated?**

I think they can first count the tens together, then the units and then everything together, because if they have a good understanding of tens and units it may be easier to work this way.

**Interview 2**

1. **What do you think the educator is hoping to achieve from this activity?**

I think the educator is busy, what she actually wants is to instil the value of money in the learners. She wants them to be able to do a costing plan and at the same time she wants to develop their calculation strategies, and they must know how to calculate.

**2. Comment on each child's solution strategy.**

What each child, how he saw it. Dino, he made an illustration by drawing a picture. Frieda counted on, beginning with four. Zondi counted everything. Lisa first made small groups of five and then added the Rands. Ayanda is very good, I think, ...her number concept is very good because she just visualised and did it with mentally. And Fran began with what he knew in that he began with the five and counted the four together, and then added the units, so he starts with the known and moves towards the unknown, and he added.

**Interview 3**

**1. What is your response to this learner's solution?**

I think the learner's knowledge of numbers is totally non-existent. The learner has not understanding of the value of numbers, or place value. And I think somehow the learner fell behind or missed out when calculation skills (numeracy) were being taught.

**2. What prior knowledge and skills do you think the educator should have assessed before giving the learner a problem of this type?**

The value of numbers, I would say this child has no idea about solution strategies and this child has no idea about number combinations. In other words, even though he wrote down the numbers together, he knows what a plus sign is, he has no understanding of how to add the numbers together. So this learner must begin again with counting on, or counting everything, he hasn't mastered these basic skills.

**3. What would you do to assist this learner to solve this type of problem accurately and efficiently in the future?**

To help this learner, I would begin by going back to number combinations, for this learner, first so that he first masters the basics, his knowledge of numbers, go back to what was done in Grade 1 so that he will know when he packs out two, what a two looks like, what a five looks like, the place value versus the value of the number, because this is what this learner doesn't know. So, to back to a play-play method of packing out, showing the numbers in the class, I think these are the basics that I would use more to begin with the learner.

**Interview 4**

**1. Describe a learner in your class who you think has a good concept of number.**

- **Why do you think this learner has a good concept of number?**

I would say that a learner with a good number concept is a learner that can visualise numbers, he can recall numbers quickly mentally, he uses different solution methods, without illustrations, only using numbers, and a leaner like this won't count everything. He will do most of his work by adding. And I'll also say why such a learner can do this, the basic skills are well developed, he is confident with his number chart and he is confident with numbers. A lot of practical work has been done with this learner and the learner has discovered many things himself because he has seen it, and if he sees and discovers for himself, this gives him an advantage, over a child that won't do it on his own.

**2. Describe a learner who you think has a poor concept of number.**

- **Why do you think this learner has a poor concept of number?**

A learner with a poor number concept usually makes many mistakes. He often transposes (reverses) numerals, can't count numbers out on a number chart. His solutions are done by counting everything out. He is confused, he can't differentiate. Such a learner is often a child that isn't observant in the class, because he battles with his numbers. One of this learners' problems, because he has a poor number concept, is that he has maybe fallen behind, problems at home, absenteeism, he is possibly emotionally unstable. He isn't observant or we could also say there wasn't enough laying a thorough foundation, or too little practical work was done with him. And I would also say this learner wasn't given enough opportunity to play with numbers, because a child that plays with number a lot, especially in Grade 1, is bound to be stronger as he moves into higher ability groups.

**3. You are asked to support a student or colleague in the teaching of counting.**

- **What would be the main points you would focus on?**

- **What suggestions would you make on the teaching and learning of this topic?**

I would begin with examples and when he begins with examples I would say it is most important to play with these, make it more, make it less, add on, take away and then, especially when working with numbers and counting work, one of the most important apparatus are expanding number cards, because then the child can easily see if it's a one digit number or two-digit number. So, examples are most important. And when such a child I'd say in Question 3, the second question under Question 3, have enough learning apparatus for each child, for example when teaching this concept, I will give each child a number and as the class counts I'll say if anyone hears his number stand up, so that when there's a child in the class who doesn't recognise his number, his friends will remind him and he won't give that specific number that he for example holds against his chest, and his numbers can be swopped weekly, and this is also a method whereby counting can be taught and children won't easily forget these numbers.

**Participant 5**

**Interview 1**
(Tape 2 Side 1: 167 - 238)

**1. What do you think the educator is hoping to achieve from this activity?**

What strategies did the learner uses? Not mainly to see what he used, what the learner's understanding is ... of tens and units, or if the learner can count, or break up numbers.

**2. What number concepts do you think are being developed?**

Addition.

**3. How do you think learners will represent the numbers?**

Pack out, use of expanding number cards.

**4. What method do you think the educator would have demonstrated?**

Addition, how many tens there are, they count, she shows units, asks what happens, is the amount more than ten?

**Interview 2**

**1. What do you think the educator is hoping to achieve from this activity?**

When a learner sees a Rand (R symbol) , he becomes confused, because maybe he thinks it is another number that he must add, and then to work with money.

**2. Comment on each child's solution strategy.**

Count everything, ...one. Easy to count, Frieda, she uses her fingers but needs much more practical work. Zondi, easier to work in Rands, to pack out, can help himself, knows his coins. And Lisa is still counting on. She ....two. Ayanda, very confident, good number concepts as well as of money. And Fran, knows his combinations and enjoys what he does.

**Interview 3**

**1. What is your response to this learner's solution?**

Has no number concept.

**2. What prior knowledge and skills do you think the educator should have assessed before giving the learner a problem of this type?**

More practical work should be done, with games in which he could buy and sell, so that he becomes aware of how to use money.

**3. What would you do to assist this learner to solve this type of problem accurately and efficiently in the future?**

Give the learners more of these type of problem sums to avoid confusion.

**Interview 4**

**1. Describe a learner in your class who you think has a good concept of number.**

- **Why do you think this learner has a good concept of number?**

I think of Johnathan. He is very self-assured. He is keen to learn, helps others, very observant, very obedient and doesn't like to use his number square (counting chart) anymore, because his number concept is very good.

**2. Describe a learner who you think has a poor concept of number.**

- **Why do you think this learner has a poor concept of number?**

This learner is very nervous, first looks at what others are doing and then he'll maybe try to do it himself. Not so self-assured and very reluctant. And then her background can play an important role in the fact that she is not concentrating on her numbers, and that can also be due to absenteeism. If she is absent often, the he or she can't progress well.

**3. You are asked to support a student or colleague in the teaching of counting.**

- **What would be the main points you would focus on?**

Needs lots of apparatus. Will look to see where he must begin, never start at nought and then he must first, before teaching nought, he must know his numbers, up to five.

- **What suggestions would you make on the teaching and learning of this topic?**

And then the last is group work. Facilitators will have to help learners and then learners will help each other.

As with values now, say for instance children find minus difficult. The problem is sometimes that they don't know their combinations, if he first knows his combinations, then he'll automatically know what 'friend' is missing. But, yes, that the child needs much more practical is definite. But I also found that, as they say in the new curriculum, the children must know their combinations, so the best would be if the child can say.

Say for instance in Grade 2 if you for example in Grade 2 you must do two-digit numbers. I would never give a sum where the child must carry over, because this would be too difficult. I would never give a two-digit number if the grouping, if the child doesn't have a good understanding of tens and units, what are tens, what are units, what is more, what is less. If it's 34, for example, what is bigger or more, 3 or 4. Some will say the 4, and this actually isn't so, then you'll see that child doesn't have a good number concept. And naturally, writing of numerals plays a big role too. If the child reads incorrectly, he will also do his sum incorrectly. So, this is what you will pick up when you continue to work with a child.

## *APPENDIX 10*

**Participant 6**

**Interview 1**
(Tape 2 Side 1: 239 - 541)

**Interview 1**

**1. What do you think the educator is hoping to achieve from this activity?**

From this activity the educator wants to show the learners the short way of adding. She also wants to show them the shortest way of addition. She also wants them to see when you are working in pairs how easy it is to come to the answer. That is what she wants to achieve from this activity.

**2. What number concepts do you think are being developed?**

The number concept, which is being developed here, is even numbers and odd numbers.

**3. How do you think learners will represent the numbers?**

The students will first represent this number they will first put from 2 up to 34. Then they will start again from 3 up to 27. Will start from even numbers first and then the odd numbers.

**4. What method do you think the educator would have demonstrated?**

I think the educator would have demonstrated this...she will show them how fast is it...to do multiplication than addition the shortest way for addition is multiplication. The shortest way...of doing addition to multiplication...the shortest way will be she will...I will show them when we are doing addition it will take you a long time it will be a long method and then when you are doing multiplication it is the shortest way because you are not going to do all that steps.

**Interview 2**

**1. What do you think the educator is hoping to achieve from this activity?**

Here the teacher wants to achieve... the child who don't have any problems in addition and here she wants to prove the child who is very slow in addition who is going to take a long road to addition and here she wants to see the child who is going to come first by addition he will not by making any steps or pictures but just add up mentally... whose going to use his mental memory and the child whose going to take a long time to come up with the answer.

**2. Comment on each child's solution strategy.**

And here to.. I'm going to describe the children now. Dino is taking the long method because he's counting the chips 4 packets of chips to come out to that 12. And then Freda is also using a long method because she is counting with her hands. Zondi is also using the long method. Lisa is using a little bit a short method because she didn't use a lot of coins to come up with that 12. Ayanda was very fast! Ayanda didn't use any concrete objects to come to that 12. Fran was also a little bit ...very...he was also fast because he just count 5 and 4 is 9 plus 1 is 10 and then 2 is 12. Ayanda was marvellous! ..the one that is going to take a loooong method and the one who is just counting and tell you, even when you doing some demonstration on the board just NO, they are calling me dadobawo (aunt), dadobawo that is 8 or that is 7

**Interview 3**

**1. What is your response to this learner's solution?**

Yes Cally, to this learner missed the signs of addition, the main concepts addition, subtraction, if a child knows that 4 signs and then again I can say, the concept of numbers was...he didn't get them, the right concept of numbers from the foundation phase, from that level of Grade 1 where a child sees the picture, the number and the words. My response is he didn't get that base of numbers to see how a number is built, how to do this, plus...the 4 concepts that is addition, subtraction, multiplication and division. And again he didn't get the number concept right

**2. What prior knowledge and skills do you think the educator should have assessed before giving the learner a problem of this type?**

I think the skills the learners should have is to know addition and also to know the number concept…natural numbers, even numbers and odd numbers…and this child didn't get to that level. And also Cally I can blame his teacher for this, not the teacher where he is now, I think his Grade 1 teacher and his Grade 2 teachers have a big gap and a lack of showing...wait I want to explain this. When children are starting at Grade 1 they always begin with eggs, this is not for the number standing, and then they first see the picture, and then the object and then the concrete number. He didn't get that.

**3. What would you do to assist this learner to solve this type of problem accurately and efficiently in the future?**

This child can, for instance now if he was in my class, the Grade 3 class I will have started from the numbers but not from the pictures but from the concrete numbers to show him if you add a R1 and also if you introduce the signs to him, addition signs. If you add one rand plus one rand equals to two rand I will start him from there. And then I also think that what he lack here is when there are no cents you have to put zero zero, you see. That's why it's coming up with twenty-five cents. He didn't get it, the rands and cents right. He just take that you see and place them there (the numerals 625), he didn't add. His main problem is addition here you see. If he had known addition he would have come up with something else here, not that. You only take that two rands, six rand...he started with the bigger number now he's doing the place value, starting with the biggest number to the smallest number. So if he have known his addition he would have no such a problem.

**Interview 4**

**1. Describe a learner in your class who you think has a good concept of number.**

- **Why do you think this learner has a good concept of number?**

OK number one Cally…I have a few, up to three learners, if you give them some work they are so fast. Sometimes I give them the difficult because they are, you see it is May now, from March I was giving them...uh...single numbers ne, but now I'm using double because I found out they are very quick at counting ne, and their addition is very good, because if you write...I'm not even finished to write down on the chalkboard and they will come to me and show me our work. Very quickly. I think it's because, when we were starting at the beginning of the year they were making, they were always asking to, why they are very good at this, it is because when I have teach them at the beginning of the year I say to them if you are adding you are, if you are doing addition you add, you are adding more – ukongeza, ne Xhosa ukongeza (add more). So all of the time they are…OK Dadobawo have said if it is add I have to add more, if it is addition I have to add more. That's why they are doing, and they don't forget it. And then when I'm asking the others some, maybe some others in the class. What I have said why do you add like this Mamie if you do this, if you are using addition you add more. They understand the concept of addition.

**2. Describe a learner who you think has a poor concept of number.**

- **Why do you think this learner has a poor concept of number?**

The poor concept of number is the learner who…the learner…to my class I've got some of them and now I go back again to the picture…to the concrete, I see some of them are not getting the number concept right so I'm starting all together again now with them. Now I always again introduce that eggs, this is eggs and then if it is 3 eggs, if this is 3 eggs give me the number – write it in numbers ne, and then they are going to write 3. And then sometimes I am using the 3 and then I'm asking them to write in some eggs. Sometimes they write it also wrong I don't know is it…now Cally I don't know, I'm doing Grade 1 ne, but I'm using methods and methods according to the problem of the children ne. I tell them 3 is facing to my left hand and then when we are writing that we are talking about it. 3 uyenyaka uhle uhle (up down and down) that is a 3 ne. And always it is facing to my…to my left.

And now this poor concept, if you take him and ask him he will write that 3 like an E ne. And then you ask him, the E must look to the ..? He will tell you to the…to the left. But when it come to the...for writing in the book he will do the same mistake. Now you have to take him again and show him that 3 on the cardboard, that numbers that you give me, to show him, here is a 3. Now he take that 3 again and put it there on the table, on the wrong side ne, because some of them are making it like an m. Now you must put it there and ask him to come and place it on the chalkboard.

**3. You are asked to support a student or colleague in the teaching of counting.**

- **What would be the main points you would focus on?**

Counting...I think when you are starting counting you must just start with the...don't start with even numbers. Start with your natural numbers ne. For example they must start from 0 then 1 and then next to 1 so and so and so. The number before, you always start with the natural numbers when you are introducing counting. Focus is on the natural numbers.

- **What suggestions would you make on the teaching and learning of this topic?**

I will suggest to a teacher when she is teaching counting numbers, teaching of counting. A child it is better when he do it on his self ne. By making some activities, body language ne, it is the best to do body language because if it is a 1, 1 is going down, the hands to show this is one, uyehla (straight down) and then if it is 2 uyenyaka uhle uhle (up and down and down), the shape, so the shape stays in his mind and also maybe when he's sitting there at the desk, OK 1 go down, 2 uyenyaka, and then you must always ask what is the number after 1, what is the number before 1? He will knows it is zero and then is 1 and then after 1 is 2.

And then another thing is to do some pictures, this is 3 and number, this is 4 and so and so and so. On the chart will be, let's say it's the picture of 2 eggs ne and then you write the number underneath the 2 eggs.

And another thing when you are teaching them counting, you mustn't rush them, you must take it step by step, step by step, because when they are coming from Grade R they only come 1,2,3,4,5! Now they have to learn this is a 1 and 1 is standing on it's own and if it is a zero, zero is that round shape.

## *APPENDIX 11*

**Participant 2**

**Interview 1**
(Tape 1 Side 2: 233 – 285 / Tape 2 Side 1: 000 – 032)

1. **What do you think the educator is hoping to achieve from this activity?**

Op my eerste vraag van "What do you think the educator is hoping to achieve from this activity?", volgens wat ek nou gelees het, sien ek dat die Juffrou will graag hê die kinders moet weet hoe om te tel en hoe om die syfers wat sy aan hulle voorgestel het, hoe moet dit kan optel and hoe om by hulle antwoorde uit te kom. Dit is my eerst antwoord

2. **What number concepts do you think are being developed?**

...en by nommer twee, "What number concepts do you think are being developed?", die kind leerder moet kan weet hoe lyk 34 end 27 as al die Juffrou die telkaart voor hom hê, en hy het mos ook sy counting rods wat hy by hom het, dan hy moet kan sien hoe lyk 34 op die telkaart en dan moet hy die counting rods wat hy gebruik moet hy die 34 end die 27 kan uit tel voor hom en hy moet kan weet hoe veel is 34 of 27. Dit is die getal begrip, number concept, hoe word dit ontwikkel en hoe weet die kind dit is, hoe lyk 34 en hoe lyk 27 en hy moet kan tel. Wat ook hier inkom is die plekwaarde, dit sal onder plekwaarde hy moet kan weet hoe lyk die en hy moet kan sê hoeveel is dit getal. Dit is nommer twee se antwoord, en nommer drie.

3. **How do you think learners will represent the numbers?**

"Hoe sal die leerder die syfers of die getalle kan voorstel?". Wat ek dink hy sal maak wat, hoe ek nou voel, hy sal sy 34 kan uit tel en dan sal hy nou seker verduidelik aan die Juffrou hoe hy kan doen. Hy sal sy 34 tellers sal uit tel, dan sal hy sê drie tiene gee vir jou dertig, dan sit hy sy vier ene by, dan dit gee dit altesaam vir hom 34, die getal 34 en ook so gaan hy ook maak met die getal 27.

**4. What method do you think the educator would have demonstrated?**

En die metode wat die Juffrou aangeleer het dink ek ook sy het dit so gedoen, sy het dit aan hulle demonstreer voordat sy vir hulle gesê het wat hulle moet doen, en om by hulle antwoord uit te kom, dink ek sy het ook vir hulle om die maklikste manier hoe om 34 kry om te sien hoe lyk 34, sal hy uiteens in drie, sy dertig sal hy eers uit tel en sal hy sê dis drie tiene, en dan hy die vier ene bysit, dan dit gee dit vir hom altesaam 34, want dit is die maklikste manier om grooter getalle, te weet hoe lyk grooter getalle.

**Interview 2**

**1. What do you think the educator is hoping to achieve from this activity?**

Volgens die onderhoud interview van nommer twee, "What do you think the educator is hoping to achieve from this activity?" I think that the learners should know how to count and to add up, and to know how to, what the answer of this activity is. En volgens wat ek kan sien, die aktiwiteit wat elkeen gedoen het sien ek dat hulle het verskillende strategië gebruik om by hulle antwoorde uit te kom. Ek kan ook sien dat dit 'n probleem oplossing is. Hulle moet hulle eie probleem moet hulle kan oplos, elkeen het sy probleem op 'n verskillende manier opgelos, en hulle moet ook 'n goeie getal begrip het om dit te kan doen en hulle moet ook weet wat die waarde, hulle moet weet wat die waarde van geld is want baie kinders weet nie wat is die waarde van geld nie as ons nou geld in the klas doen met hulle, soos hier wat hulle die antwoord wat R12 is, moet hulle kan weet wat is die waarde van 'n R12, hoe om by die hele antwoord uit te kom en dan deur dit te doen moet hulle die koste bereken van hulle lekkergoed wat hulle mos nou bymekaar getel het en wat ek ook hier kan sien, elkeen het mos 'n verskillende strategie gebruik om by sy antwoord uit te kom...dis maar basies al.

**2. Comment on each child's solution strategy.**

Ayanda sien ek dat sy het ontmiddelik haar antwoord gegee. Sy het die getalle ontmiddelik bymekaar getel en gesê wat haar antwoord is, dit wil sê dat sy het ook 'n goeie getal begrip. Sy hoef nie gebruik te maak van konkrete aparaat of van haar vingers om by 'n antwoord uit te kom nie, waar Dino hy het sy goed geteken. Dit is maar hoe 'n mens maar verskillende maniere van elke kind in jou klas is nie dieselfde nie. Hulle moet op hulle manier by hulle antwoord uitkom. Hulle moet weet wat hulle kan doen hoe om by hulle antwoord uit te kom. Ons kan nie regtig 'n metode vir hulle aanleer om vir hulle te sê hulle moet so doen nie. Dit is hoekom OBE so, dit is nou deel van OBE, elkeen moet by sy eie antwoord uitkom. Hy moet self kan sy probleem kan oplos en hier sien ek Dino het syne geteken om by sy antwoord uit te kom. Hy het alles bymekaar getel en toe het hy be sy antwoord uitgekom. Zondi het weer gebruik gemaak van die bakkie van R1e, al die R1e bymekaar getel om by sy antwoord uit te kom......

**Interview 3**

**1. What is your response to this learner's solution?**

Volgens die aktiwiteit wat ek nog gesien het, wat die kind gedoen het, kan ek sien dat hy het geen getal begrip nie, of nie 'n goeie getal begrip nie, en dit is 'n probleem oplossing en ek kan sien die kind kan nie die probleem oplos nie. Hy weet nie hoe om die probleem op te los nie, en dat hy ook geen waarde van geld het nie. Hy weet nie wat is die waarde van geld nie. Hy weet nie wat is 'n R2, hoeveel is 'n R5 en hoeveel is 'n R6 nie. En wat ook kan die probleem vererger is dit ook miskien dat die kind nie kan lees nie. Miskien kan dit wees dat hy nie die som kan lees nie. Wat ek ook voel, die Juffrou moet nou eers die probleem som eers aan die kind aan hulle gelees het, want daar is baie kinders wat nou nie kan lees nie, dat dis hoekom hy nie die som kan verstaan nie, want hy kan nie die som lees nie, voel ek dat sy eerste die kind moet lei, die leerder moet ly na die som toe. Sy moet eers die probleem, die som, aan die kind voorlees en dan moet sy 'n voorbeeld aan hom gee...ons het geweldige probleme met probleem oplossing self in die klas in met die kinders kan ons sien hulle het nie, hulle kan nie probleem oplossings doen nie. Dis hoekom ons maar eerste vir hulle lei na die antwoord toe. Byvoorbeeld ons gee nie dieselfde aktiwiteit, ons gee vir hom 'n ander een en dan wys ons vir hulle hoe dit gedoen moet word, en as hy nou 'n aktiwiteit moet doen, dan weet u teminste het hy 'n begrip hoe om die som te doen. Wat ek voel, die Juffrou moet vir hom eers gelei het na die som toe en vir hom soortgelyke som gegee het, sodat hy kan sien hoe word dit gedoen en dan kan hy dit self gaan doen by sy plek. Dis hoekom hy sy antwoord ook verkeerd het, want vir 'n Graad 3 kind is dit maar, die sommetjie is baie maklik en hy kan sien hy kan dit nie doen nie want dit is nie groot getalle waarmee hy besig is nie, dis maar klein getalle onder tien, en dis hoekom hy dit nie kan doen nie. Hy het ook 'n swak getal begrip.

En dan voel ek ook in die toekoms, as die Juffrou sien die kind kan nie som doen nie, hy het 'n probleem, voel ek as hy so 'n swak getal begrip het van hierdie syfers, voel ek dat sy moet meer makliker aktiwiteite eers vir hom gee voordat sy vir hom so 'n aktiwiteit kan gee, moet sy met die kind werk al hulle is in groepe gedeel, miskien sal hy onder die swak groepie, dan voel ek sy moet by hulle sit en vir hulle aktiwiteite gee wat makliker is vir hulle, om dit te kan doen voordat sy na die moeiliker kan gaan. Dan voel ek sy moet maar eers met die kleiner getalletjies begin en eers die getal begrip kan vaslê 'n goeie getal begrip kan vaslê by die kind voordat hy na die moeiliker somme kan gaan.

**Interview 4**

1. **Describe a learner in your class who you think has a good concept of number.**

   - **Why do you think this learner has a good concept of number?**

"Beskryf 'n leerder in jou klas met 'n goeie getal begrip. Hoekom dink jy die kind het 'n goeie getal begrip?". As ek 'n leerder in my klas het met 'n goeie getal begrip sal ek sê dis 'n leerder wat al sy basiese bewerkings kan doen. Hy kan optel, aftrek, vermeerdervuldig en deling, en ook probleem oplossings kan hy doen. Dit is 'n kind wat nou 'n goeie getal begrip het. As ek nou voor met hulle sit by my op die mat in die klas, en ek gee miskien 'n probleem oplossing en ontmiddelik sal die kind vir my die antwoord kan gee, sonder om op sy vingers te tel of van konkrete aparaat gebruik, maar voel ek dit is 'n leerder met 'n goeie getal begrip, en ook as ek vir hulle aktiwiteite gee by die tafel en gewoonlik deel ek telkaarte uit of ek gee vir hulle ook stokkies of so, dan weet ek presies ek hoef nie vir daai leerder te gee nie want hy het nie nodig om met die telkaart of met die stokkies te werk nie. Dis hoekom ek kan sien dis 'n leerder wat 'n goeie getal begrip het.

**2. Describe a learner who you think has a poor concept of number.**

- **Why do you think this learner has a poor concept of number?**

En 'n leerder met 'n swak getal begrip, om ontmiddelik op te let kan ek sien die leerder kan nie tel nie, voordat ons groepwerk doen, doen ons eers 'n klas les voor op die mat en dan laat ek vir hulle tel en gewoonlik vir die leerders om te tel en hulle kan nie tel nie tot by 'n seker getal nie, dan weet ek die leerder het 'n swak getal begrip, en ook as hy aktiwiteite vir my doen in die klas of voor op die mat en ek gee maar 'n maklike sommetjie, optel som of aftrek som, en die leerder kan dit nie doen nie, dan weet ek ontmiddelik dat die leerder 'n swak getal begrip het. Hy kan nie tel nie en hy weet ook nie hoe om die getalle bymekaar te tel nie of dit af te trek nie.

**3. You are asked to support a student or colleague in the teaching of counting.**

- **What would be the main points you would focus on?**
- **What suggestions would you make on the teaching and learning of?**

Die punte waarop ons moet fokus om 'n goeie getal begrip uit die leerders aan te kweek, dat ons gereeld moet tel, elke dag in die klas moet ons tel en ook dat ons elke dag vir die leerder 'n probleem oplossing kan gee, dat hy kan weet hoe om die probleem to doen, om te kan tel, want as die kind 'n swak getal begrip het, dan kan hy in al sy basiese bewerkings kan hy niks doen nie. En wat ons ook kan doen met kinders met 'n swak getal begrip, en ons sien hy kan nie tel nie, moet ons maar op die mat met hulle individueel werk, of in 'n groep in, en dat hulle maar die sommetjies kan doen met die konkrete aparaat waarmee hulle besig van is moeilik, want sodra jy met hulle werk op die mat en jy moet so jou aandag aan hulle gee, dat jy heeltyd met hulle moet sit, dan sodra jy weg beweeg van hulle en jy gaan oor na jou tafel toe en kom terug, dan sien jy die kind het verkeerd gedoen. So, jy moet heeltyd vir hom sê as hy nou 'n sommetjie doen van sê nou 4 plus 2, nou moet ek sê tel vir my nou eers die 4, hy weet hy sit met die stokkies by hom maar hy weet nie how om die uit te pak, hoe om by sy antwoord te kom, nou moet ek hom lei. Nou moet ek nou eers vir hom sê pak eers vir my die 4 stokkies uit en as nou 4 stokkies uitgetel het, hoeveel moet jy nou bysit, nou sit hy nog weer 2 by, nou moet hy al die stokkies bymekaar tel om by sy antwoord uit te kom, nou sal hy sy antwoord neerskryf. So, elke tyd moet ek hom lei na die antwoord toe. Dis maar wat die onderwyser maar in die klas moet doen om vir die kind te help, om te kan sien dat die kind vorder en dat hy 'n goeie getal begrip het en die einde van die dag sal hy kan ontwikkel.

## *APPENDIX 12*

**Participant 4**

**Interview 1**
(Tape 2 Side 1: 074 – 165)

1. **What do you think the educator is hoping to achieve from this activity?**

Ek dink die opvoederer is besig om die kinders aan te leer sodat hulle beter kan aantel. Sy's om terselfdetyd besig met ko-operatiewe leer juis omdat hulle in groepe werk dat hulle van mekaar kan leer en sy's ook besig om vir hulle te lei dat hulle getalle kan visualiseer, dat hulle die getal kan skryf maar terselfdetyd dit verstaan, en dan is sy ook besig om hulle vaardigheid te ontwikkel ten opsigte van hulle denkvermoë.

2. **What number concepts do you think are being developed?**

Ek dink sy's besig om die getal begrip nou tot 99 is sy besig daarmee en terselfetyd dink ek sy's ook besig om die oorgaan van 'n 10, waar hulle die 10 moet volmaak. Sy's besig met daardie begrip by die leerders vas te lê.

3. **How do you think learners will represent the numbers?**

Ek dink somige van die leerders sal uitpak drie tiene en vier ene, twee tiene en sewe ene. Somige van hulle sal net die hele getal in kringe uit trek, somige sal miskien vrugte teken, net soos hulle die getal dan sien.

4. **What method do you think the educator would have demonstrated?**

Ek dink hulle kan eers die tiene bymekaar tel, dan die ene en dan alles saamtel, want as hulle goeie begrip het van tiene en ene sal hulle miskien dit die maklikste vind om dit so te kan werk.

**Interview 2**

1. **What do you think the educator is hoping to achieve from this activity?**

Ek dink die opvoederer is besig, wat sy eintlik wil hê is, sy wil eintlik waarde van geld by die leerders vaslê. Sy wil hê hulle moet 'n kosteberaming moet hulle kan doen en terselfdetyd wil sy ook hê hulle moet hulle rekenstrategië moet kan ontwikkel, en hulle moet ook kan bereken.

**2. Comment on each child's solution strategy.**

Wat elke kind hoe hy dit gesien het. Dino, hy het 'n illustrasie gemaak deur middel van prent teken. Frieda het aantel gedoen deur by vier te begin. Zondi het alles getel. Lisa het eers groepe van vyf gemaak en toe die Rande bygesit. Ayanda, ek dink sy is baie goed ten..., haar getalbegrip is baie goed want sy het dit sommer gevisualiseer en dit deur midde van hoofreken gedoen. En Fran het begin met die bekende deur dat hy die vyf en die vier bymekaar getel het en daama toe die ene bygetel, so hy het begin by die bekende en beweeg na die onbekende, en hy het aantel gedoen.

**Interview 3**

**1. What is your response to this learner's solution?**

Ek dink die leerder se behoud van getal is heeltemal uiterverband. Die leerder het geen begrip van getalwaarde, sowel as plekwaarde nie. En ek dink iewers het die leerder 'n baie groot agterstand of hy het uitgemis terwyl daar besig was met syferkunde.

**2. What prior knowledge and skills do you think the educator should have assessed before giving the learner a problem of this type?**

Die waarde van getalle sou ek sê hierdie kind het geen idee van rekenstrategië nie en hierdie kind het geen idee van getalkombinasies nie. Met ander woorde, hy het die getalle wel bymekaar geskryf, hy weet van 'n plus teken, maar hy het geen begrip van hoe om die getalle bymekaar te tel nie. So hierdie leerder moet weer, sy begrip van aantel of alles tel, hy het nog nie daardie basiese begrip, het hy nog nie af onder die knie nie.

**3. What would you do to assist this learner to solve this type of problem accurately and efficiently in the future?**

Om hierdie leerder te help sou ek begin terug gaan na getal kombinasies toe, om vir hierdie leerder eers, dat hy eers die basies sal onder die blad kry, behoud van getalle, gaan maar weer terug wat in Graad 1 gedoen word sodat hy kan weet as soos hy twee uitpak hoe lyk 'n twee, hoe lyk 'n vyf, die plekwaarde teen die waarde die waarde van die getalle, want dit is wat hierdie leerder nie ken nie. So, om weer terug te gaan met 'n speel-speel metode van pak uit, wys die getalle in die klas uit, ek dink dit is die basiese ding wat ek meer sal begin om hierdie leerder, want hy het die .....heeltemal ..... Dit is die basiese waarmee ek sal begin.

**Interview 4**

1. **Describe a learner in your class who you think has a good concept of number.**

- **Why do you think this learner has a good concept of number?**

Ek sal sê dat 'n leerder met 'n goeie getalbegrip is 'n leerder wat, hy kan getalle visualiseer, hy kan getalle vinnig herroep, hy doen verskeie reken metodes, sonder illustrasie, slegs gebruik maak van getalle en so 'n leerder sal nie maklik alles tel nie. Hy sal die meeste van sy werking sal hy doen deur aantel. En ek sal ook sê hoekom so 'n leerder dit kan doen, die basiese vaardighede is goed vasgelê, hy is vertroud met sy telkaart en hy is vertroud met getalle. Baie praktiese werk is met so 'n leerder gedoen en hierdie leerder het baie dinge self ervaar omdat hy dit gesien het, en as hy sien en self ervaar, dan gee dit vir hom 'n beter voorsprong, voor die kind wat die nie self wil doen nie.

2. **Describe a learner who you think has a poor concept of number.**

- **Why do you think this learner has a poor concept of number?**

'n Leerder met 'n swak getalbegrip se werk is dikwels vol foute. Hy draai dikwels getalle om, kan nie getalle uit.... op 'n telkaart nie. Sy werkings word gedoen deur...deur alles getel. Hy is verwaard, hy kan nie onderskei nie. En so 'n leerder is gewoontlik 'n kind wat nie oplettend in die klas is nie, omdat hy sukkel met sy getalle. Een van die foute van hierdie leerder, omdat hy 'n baie swak getalbegrip het, daar is miskien hy het 'n groot agterstand, probleme tuis, afwesigheid, hy is miskien emosioneel ontstabiel. Hy is onoplettend of ons kan ook sê geen deeglike vaslegging was gedoen nie, of daar was te min praktiese werk met die leerder gedoen. En ek sal ook sê so 'n leerder het miskien ook te min kans gehad om te speel met getalle, want 'n kind wat baie gespeel het met getalle veral in Graad 1 is genuig om sterker voor te kom as hy hoër opbeweeg in die volgende groepe in.

**3. You are asked to support a student or colleague in the teaching of counting.**

- **What would be the main points you would focus on?**
- **What suggestions would you make on the teaching and learning of this topic?**

Ek sal begin met werk met voorwerpe en waneer hy begin nou werk met voorwerpe sal ek sê die belangrikste is om te speel daarmee, maak dit meer, maak die minder, voeg by, haal weg en dan veral waneer jy met getalle en met telwerk doen is een van die belangrikste aparate wat jy het is jou spreikaarte, want dan sal die kind baie maklik sien of dit 'n eensyfer getal is en as dit 'n tweesyfer getal is. So voorwerpe is die mees belangrikste ding daar. En wanneer so 'n kind ek sou sê by Question Three, the second question by Question Three, genoeg samehulpmiddels vir elke kind 'n nommer wanneer ek byvoorbeeld hierdie onderwerp sal aanleer, gee vir elke kind 'n nommer en soos daar getel word in die klas, sal ek sê as jy hierdie nommer hoor staan op, so wanneer daar iemand in die klas is wat nie sy nommer herken nie, sal sy maats hom herinner en daardie spesifieke nommer wat hy byvoorbeeld voor sy bors het, sal hy nie gegee het nie en so kan nommers dan weekliks kan hy nommers omgeruil word, en dit is ook 'n metode hoe telwerk dan aangeleer word en kinders sal nie maklik daardie getalle vergeet nie.

*APPENDIX 13*

**Participant 5**

**Interview 1**
(Tape 2 Side 1: 167 - 238)

1. **What do you think the educator is hoping to achieve from this activity?**

Watter strategië leerder gebruik het, nie sodanig die antwoord belê nie maar te kyk wat hy gebruik het, of leerder groep....van tiene en ene verstaan, of leerder optelling kan doen, of leerder getalle kan opbreek.

2. **What number concepts do you think are being developed?**

Optelling.

3. **How do you think learners will represent the numbers?**

Uitpak, gebruik van spreikaarte.

4. **What method do you think the educator would have demonstrated?**

Optelling, hoeveel tiene daar is, hulle tel, sy wys ene, vra wat gebeur, die getal meer as tien is.

**Interview 2**

1. **What do you think the educator is hoping to achieve from this activity?**

Wanneer 'n leerder 'n Rand teken sien, dan is hy soms verwaard, omdat hy dalk dink dis nog 'n getal wat hy moet bytel en dan om met geld te werk.

2. **Comment on each child's solution strategy.**

Alles tel, ...een. Maklik om te tel, Frieda, sy gebruik haar vingers maar het baie meer praktiese werk nodig. Zondi, makliker om in Rande te werk, om uit te pak, kan homself help, ken sy geldstukke. En Lisa is nog by aantel. Sy ....twee. Ayanda, baie selfvertroue, goeie begrip van getal sowel as geld. En Fran, ken sy kombinasies en geniet wat hy doen.

## Interview 3

**1. What is your response to this learner's solution?**
Ken geen getal begrip.

**2. What prior knowledge and skills do you think the educator should have assessed before giving the learner a problem of this type?**
Meer prakties gedoen het, deur speletjies waardeur hy kan koop en verkoop sodat hy bewiskund word van hoe om geld to gebruik.

**3. What would you do to assist this learner to solve this type of problem accurately and efficiently in the future?**
Gee meer van hierdie tiepe probleem somme aan leerders om verwaring uit te skakel.

## Interview 4

**1. Describe a learner in your class who you think has a good concept of number.**

- **Why do you think this learner has a good concept of number?**

Dink ek aan Johnathan. Hy is baie selfversekerd. Hy is ywerig om te leer, help graag ander, baie oplettend, baie gehoorsaam en hou nie daarvan om nou meer sy telkaart te gebruik nie, omdat sy getal begrip baie goed is.

**2. Describe a learner who you think has a poor concept of number.**

- **Why do you think this learner has a poor concept of number?**

Hierdie leerder is baie bang, kyk eers wat ander doen en dan sal sy miskien probeer om die selfde te doen. Nie so selfversekerd nie en baie hywerig. En dan haar agtergrond kan 'n belangrike rol speel in die feit dat sy nie so goed konsentreerd in haar getalle nie en dan kan dit ook wees weens afwesigheid. As sy baie afwesig is dan sa hy of sy dan nie kan vordering toon nie.

**3. You are asked to support a student or colleague in the teaching of counting.**

- **What would be the main points you would focus on?**

Baie aparaat nodig. Sal kyk waar hy moet begin, begin nooit by nil en dan moet hy eers, voordat hy nil wil aanleer, moet sy die behou van getal ken, tot by vyf.

- **What suggestions would you make on the teaching and learning of this topic?**

En dan die laaste is groepewerk. Fasiliteerders sal sy moet help en dan leerders sal dan so mekaar kan help.

Soos by waardes nou, sê nou byvoorbeeld kinders vind die minus baie mooilik. Die probleem is soms dat omdat hulle nie hulle kombinasies ken nie, as hy eers sy kombinasies ken, dan gaan hy outomaties watter maatjie is weg. Maar, ja, en dan so dis (seker maar rasing?) dat die kind baie meer praktiese, baie meer praktiese doen, is baie vasleggend. Maar ek het dit ook gevind as, soos hulle sê in die nuwe kurikulum, die kinders moet hulle kombinasies ken, so die beste sal ook wees as die kind nou kan sê.

Sê nou soos in Graad 2 as jy byvoorbeeld, in Graad 2 moet hulle twee-syfer getalle doen. Ek sal nooit een 'n som gee waar die kind moet oordra nie, omdat dit te mooilik gaan wees. Ek sal nooit 'n twee-syfer getal laat doen nie as die groepering, as die kind nie die begrip het van groepering van tien en ene, wat is tiene, wat is ene, wat is meer, wat is minder. As dit 34 is byvoorbeeld, wat is groter or meer, die 3 of die 4. Somige gaan sê die 4, en dan is dit werklik net nie so nie, dan gaan jy sommer sien daai kind het nie 'n begrip van getalle nie. En natuurlik ook die skryf van die getal speel 'n groot rol. As die kind nou verkeerd lees, gaan hy ook sy som verkeerd doen. So, dit is al nou goed wat 'n mens sou optel as jy nou aangaan met die kind.