INVESTIGATION OF STABILITY FOR COMPOSITE OPERATIONAL AMPLIFIERS

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ABSTRACT

Evaluation of the stability of composite operational amplifiers is presented. Nyquist Diagram and Routh Criterion stability evaluation techniques are employed with different order models for the composite operational amplifier. The results here obtained are then compared with theoretical and experimental studies carried out by Campbell and Stephenson [1].

Although the approaches employed yield similar results, some are mathematically more complex than others and thus may not be suitable for rapid evaluation of the stability of composite operational amplifiers.

A modular approach to the evaluation of the stability of composite operational amplifiers for a particular network topology is followed and an analysis is done.

Finally, a brief summary of evaluation techniques is presented together with operational amplifier models where the experimental circuit results obtained by Campbell and Stephenson are compared.

DECLARATION

I declare that this dissertation is my own unaided work. It is being submitted for the degree of Master of Science of Engineering in the University of Cape Town. It has not been submitted previously for any degree or examination in any other University.

C. Schneider

C.Schneider

25th day of September ,1987.

ACKNOWLEDGEMENTS

To my supervisor Professor K.M.Reineck, many thanks for support and advice throughout the period during which this research was undertaken. Thanks are also due to Professor F.W.Stephenson for his advice and assistance while visiting the University of Cape Town as a visiting lecturer and Doctor C.R.W.Campbell for the useful discussions held.

Thanks to my wife, Nicolette, for her support and love, during the preparation of this manuscript.

Finally, thanks be to God, for creating the mysteries of this world and allowing mere mortals to unravel them.

PREFACE

In the study of a paper by Campbell and Stephenson [1] on the possibilities of employing composite operational amplifiers to extend the high frequency performance of conventional RC active filters it became evident that theoretical predictions of stability and experimental results did not agree.

Other publications concerned with composite operational amplifiers merely presented circuits where it was implied that these would work under most conditions. However when one set out to build such amplifiers and investigated their behaviour in frequency filters it emerged that severe stability problems beset such studies.

The work here presented was initiated by the fact that hitherto the problem of stability had not received the attention that it warranted. Experimental results obtained by Campbell and Stephenson made use of the Composite Two Operational Amplifier (C2OA) by Mikhael and Nessim [2]. It was for this reason that investigations presented here also made use of this amplifier.

Theoretical studies by Campbell and Stephenson showed significant deviations from the experimental results, something which obviously required further investigation. By using the Nyquist Diagram Stability analysis technique to determine the stability of the open loop system it became possible to investigate the effect of the higher order terms of operational amplifier models. In fact using Millman's [37] single, double and triple pole models of the operational amplifier the results obtained came close to those obtained by the experiments.

However, the procedure involves lengthy mathematical manipulations and it was therefore decided to apply a standard stability evaluation technique of sufficient accuracy. The relatively simple Routh Criterion Stability analysis technique was considered where stability could be very conveniently established.

TABLE OF ABBREVIATIONS

VSLI	Very Large Scale Integration
OA	Operational Amplifier
COA	Composite Operational Amplifier
C2OA	Composite Two Operational Amplifier
M1	C2OA-1 described by Mikhael and Nessim [2]
M2	C2OA-2 described by Mikhael and Nessim [2]
M3	C2OA-3 described by Mikhael and Nessim [2]
M4	C2OA-4 described by Mikhael and Nessim [2]
TF	Transfer Function
GF	Gain Function
SP	Single Pole Model of the Operational Amplifier
DP	Two Pole Model of the Operational Amplifier
TP	Three Pole Model of the Operational Amplifier
GFM2	Gain Function of the M2 COA
GFM2SP	Gain Function of the M2 COA including the Single Pole Model of the OA
GFM2DP	Gain Function of the M2 COA including the Two Pole Model of the OA
GFM2TP	Gain Function of the M2 COA including the Three Pole Model of the OA
SKB	Sallen and Key Bandpass Filter
CL	Closed Loop
OL	Open Loop

SKBCLTFM2SP Sallen and Key Bandpass Filter Closed Loop Transfer Function for the M2 COA with Single Pole Model of the OA included.

SKBCLTFM2DP Sallen and Key Bandpass Filter Closed Loop Transfer Function for the M2 COA including the Two Pole Model of the OA.

SKBCLTFM2TP Sallen and Key Bandpass Filter Closed Loop Transfer Function for the M2 COA using the Three Pole Model of the OA.

SKBOLTFM2SP Sallen and Key Bandpass Filter Open Loop Transfer Function for the M2 COA with Single Pole Model of the OA included.

SKBOLTFM2DP Sallen and Key Bandpass Filter Open Loop Transfer Function for the M2 COA including the Two Pole Model of the OA.

SKBOLTFM2TP Sallen and Key Bandpass Filter Open Loop Transfer Function for the M2 COA using the Three Pole Model of the OA.

M2SP M2 COA using the Single Pole Model of the OA.

M2DP M2 COA including the Two Pole Model of the OA.

M2TP M2 COA incorporating the Three Pole Model of the OA.

M4SP M4 COA using the Single Pole Model of the OA.

M4DP M4 COA including the Two Pole Model of the OA.

M4TP	M4 COA incorporating the Three Pole Model of the OA.
GFM3	Gain Function of the M3 COA
GFM3SP	Gain Function of the M3 COA including the Single Pole Model of the OA
GFM3DP	Gain Function of the M3 COA including the Two Pole Model of the OA
GFM3TP	Gain Function of the M3 COA including the Three Pole Model of the OA
SKBCLTFM3SP	Sallen and Key Bandpass Filter Closed Loop Transfer Function for the M3 COA with Single Pole Model of the OA included.
SKBCLTFM3DP	Sallen and Key Bandpass Filter Closed Loop Transfer Function for the M3 COA including the Two Pole Model of the OA.
SKBCLTFM3TP	Sallen and Key Bandpass Filter Closed Loop Transfer Function for the M3 COA using the Three Pole Model of the OA.
SKBOLTFM3SP	Sallen and Key Bandpass Filter Open Loop Transfer Function for the M3 COA with Single Pole Model of the OA included.
SKBOLTFM3DP	Sallen and Key Bandpass Filter Open Loop Transfer Function for the M3 COA including the Two Pole Model of the OA.

SKBOLTFM3TP	Sallen and Key Bandpass Filter Open Loop Transfer Function for the M3 COA using the Three Pole Model of the OA.
GFM4	Gain Function of the M4 COA
GFM4SP	Gain Function of the M4 COA including the Single Pole Model of the OA
GFM4DP	Gain Function of the M4 COA including the Two Pole Model of the OA
GFM4TP	Gain Function of the M4 COA including the Three Pole Model of the OA
SKBCLTFM4SP	Sallen and Key Bandpass Filter Closed Loop Transfer Function for the M4 COA with Single Pole Model of the OA included.
SKBCLTFM4DP	Sallen and Key Bandpass Filter Closed Loop Transfer Function for the M4 COA including the Two Pole Model of the OA.
SKBCLTFM4TP	Sallen and Key Bandpass Filter Closed Loop Transfer Function for the M4 COA using the Three Pole Model of the OA.
SKBOLTFM4SP	Sallen and Key Bandpass Filter Open Loop Transfer Function for the M4 COA with Single Pole Model of the OA included.
SKBOLTFM4DP	Sallen and Key Bandpass Filter Open Loop Transfer Function for the M4 COA including the Two Pole Model of the OA.

SKBOLTFM4TP Sallen and Key Bandpass Filter Open Loop Transfer Function for the M4 COA using the Three Pole Model of the OA.

CONTENTS

INTRODUCTION

With the steady technological progress in the electronics industry the cost, size, performance and power consumption limitations of the electronic vacuum tube was largely overcome by the introduction of the transistor.

As fabrication techniques advanced small, medium and large scale integration opened the door to extreme miniaturisation . Today we have reached the age of Very Large Scale Integration (VLSI) where it is possible to have a television set the size of a cigarette box.

In the past it has become evident in transistor design that better performance can be achieved by utilising more transistors. This is evident in the form of devices such as the Darlington pair, the push-pull amplifier and the operational amplifier (OA). (For an explanation of the abbreviations used please refer to the Table of Abbreviations)

OAs are used as the active element in the realisation of Active Filters. These active elements have frequency dependent gains, in the non-ideal case, resulting in limited high frequency operation. This limit in operating frequency can be related to the Gain Bandwidth Product of the active element. High frequency operation is limited by the passive components of the network. However this usually occurs at a much higher frequency than that at which the OA's limitations occur.

Research has revealed that extending the bandwidth can alleviate this frequency dependency. Techniques suggested by researchers include 'passive compensation', 'active compensation', 'active and/or passive compensation' and the use of more than one OA [1 - 36].

Passive compensation makes use of passive components and design distortion, while Active compensation uses additional operational amplifiers (OA's) to extend the bandwidth of the active elements. A more detailed explanation of Active and Passive compensation can be found in Chapter 1 page 32.

Extending the bandwidth by using more than one OA has given rise to the interesting contribution by Mikhael and Nessim [2]. Here the OA is replaced by two OAs, resulting in a three port device termed a Composite 2 Operational Amplifier (C2OA). Further papers by Mikhael and Nessim [2 - 4,7,33] dealt with applications for composite operational amplifiers (COA) and systematic approaches to generating the COA employing n OAs.

The paper by Campbell and Stephenson [1] investigates the possibilities of employing COAs to extend the high frequency performance of conventional RC active filters. The theoretical and experimental results obtained differed significantly and in some cases stable operation was difficult to achieve. Thus they were unable to duplicate the former published results [33].

The lack of information on evaluating the stability of the COA is currently a major obstacle to the successful use of this amplifier structure. From the work by Campbell and Stephenson it is indicated that research in this field must be done.

The objective of the study was to find a reasonably simple and reliable method of evaluating the stability of the COA in a particular network topology in order to obtain a stable extended bandwidth realisation.

Experimental results obtained by Campbell and Stephenson made use of the C2OA by Mikhael and Nessim [2] in a Sallen and Key Positive Feedback Bandpass network. For this reason investigations presented here also make use of this COA and network topology.

CHAPTER 1

Composite Operational Amplifiers

Various composite operational amplifiers (COA) have been presented in papers over the past few years. In this chapter a somewhat historical look is taken at the various topologies put forward.

In order to simplify the issue, the definition of a COA is taken to be any network having two or more operational amplifiers (OA) configured in such a way as to realise a device having the characteristics of an OA.

Or more simply put, a COA is a device, consisting of OAs, having an inverting-, a non-inverting-input and an output port and has the transfer characteristic of an OA.

Active compensation can be divided into two types of compensation, namely magnitude and phase compensation. Magnitude compensation realises an amplifier with a maximally flat magnitude response. To realise an amplifier with negligible phase error, phase compensation is used. These forms of compensation can be achieved by the appropriate choice of component values, as shown in a paper by Huertas and Rodriguez-Vazquez [24]. Figures 1 to 8 show various COAs discussed in their paper with the relevant component choices used to achieve either magnitude or phase compensation.

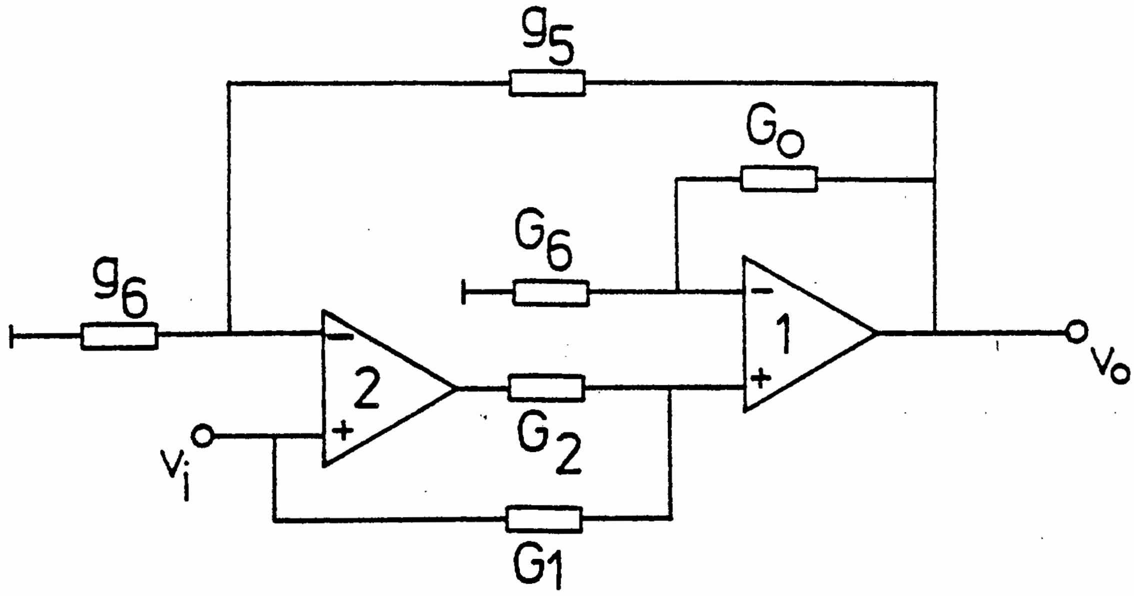


Figure 1

The above COA was discussed in a paper by Reddy [34].

Phase Compensation (for stability $\nu_o > 0$)

$g_6/g_5 = k_o - 1$

$G_2/G_1 = 1/(k_o\nu_o) - 1$

$G_6/G_o = 1/\nu_o - 1$

Magnitude Compensation ($b = GB_1/GB_2$)

$g_6/g_5 = k_o - 1$

$G_1 = 0$

$G_2 = \infty$

$G_6/G_o = \sqrt{(k_o b)/2} - 1$

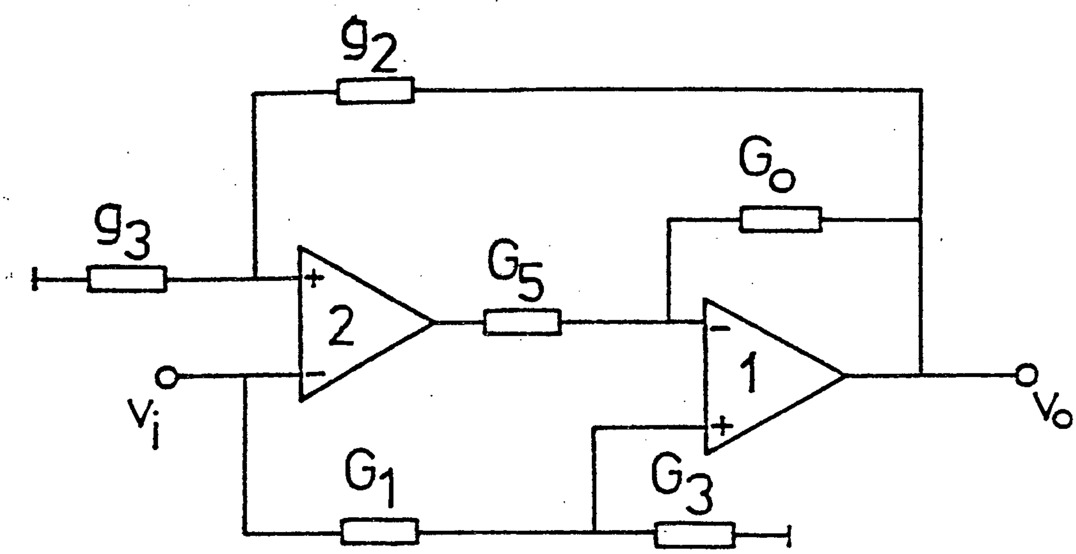


Figure 2

The above COA was shown in a paper by Reddy [34].

Phase Compensation (for stability $\nu_0 > 0$)

$g_3/g_2 = k_0 - 1$

$G_3/G_1 = 1/(k_0\nu_0) - 1$

$G_5/G_0 = 1/\nu_0 - 1$

Magnitude Compensation ($b = GB_1/GB_2$)

$g_3/g_2 = k_0 - 1$

$G_1 = \infty$

$G_3 = 0$

$G_5/G_0 = (-k_0 b/(1-\sqrt{(1+b^2+2k_0 b)})) - 1$

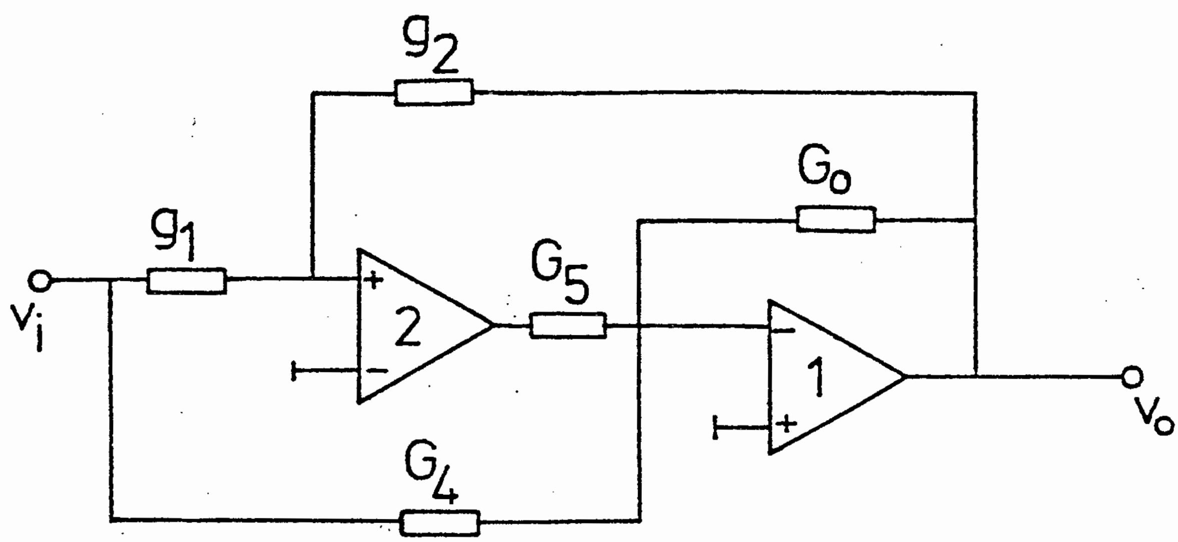


Figure 3

The above COA was presented in a paper by Soliman [35].

Phase Compensation (for stability $\nu_o > 0$)

$g_1/g_2 = G_4/G_o = k_o$

$G_5/G_o = (1 - \nu_o(k_o+1))/\nu_o$

Magnitude Compensation ($b = GB_1/GB_2$)

$g_1/g_2 = k_o$

$G_4 = 0$

$G_5/G_o = ((1+k_o)b/(\sqrt{(2b(1+k_o)+1} - 1)) - 1$

where k_o is an absolute value.

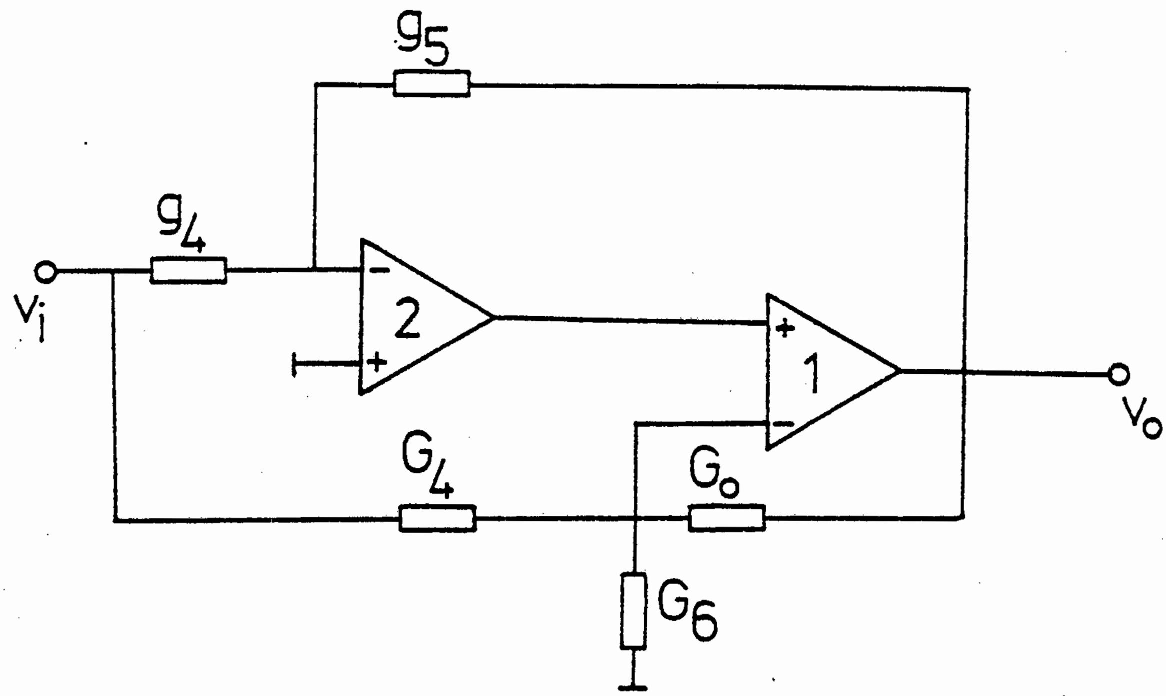


Figure 4

The above COA was derived in a paper by Reddy [34].

Phase Compensation (for stability $\nu_o > 0$)

$g_4/g_5 = G_4/G_o = k_o$

$G_6/G_o = 1/\nu_o - 1 - k_o$

Magnitude Compensation ($b = GB_1/GB_2$)

$g_4/g_5 = k_o$

$G_6 = 0$

$G_4/G_o = ((k^2_o - 1) / (\surd(k^2_o + 2k^2_o(k^2_o - 1)$

$/ (b(1 + k_o)) - 1)) - 1$

where k_o is the absolute value.

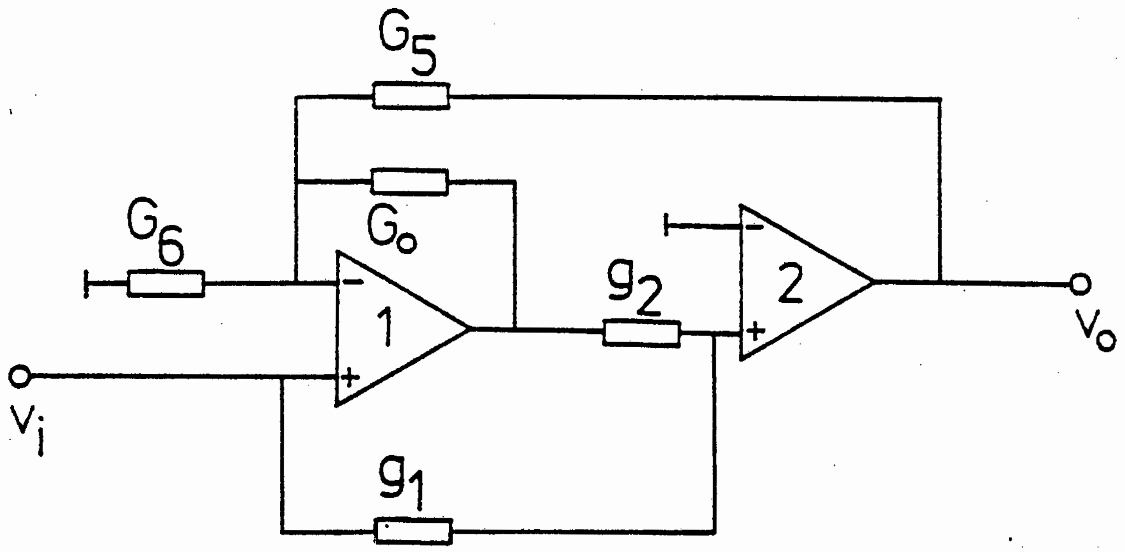


Figure 5

The above COA was referred to in a paper by Huertas and Rodriguez-Vazquez [24].

Phase Compensation ($b = GB_1/GB_2$ and for stability $\nu_0 > 0$)

$g_2/g_1 = 1/(\nu_0 k_0 b) - 1$

$G_5/G_0 = \nu_0 b/(1-\nu_0 k_0 b) + 1/(k_0 \nu_0)$

$G_6/G_0 = 1/\nu_0 - 1 - G_5/G_0$

Magnitude Compensation ($b = GB_1/GB_2$)

$g_1 = 0$

$g_2 = \infty$

$G_5/G_0 = \surd b/(\surd(2k_0))$

$G_6/G_0 = k_0 \surd b - \surd(k_0 2) - \surd b/(\surd(k_0 2))$

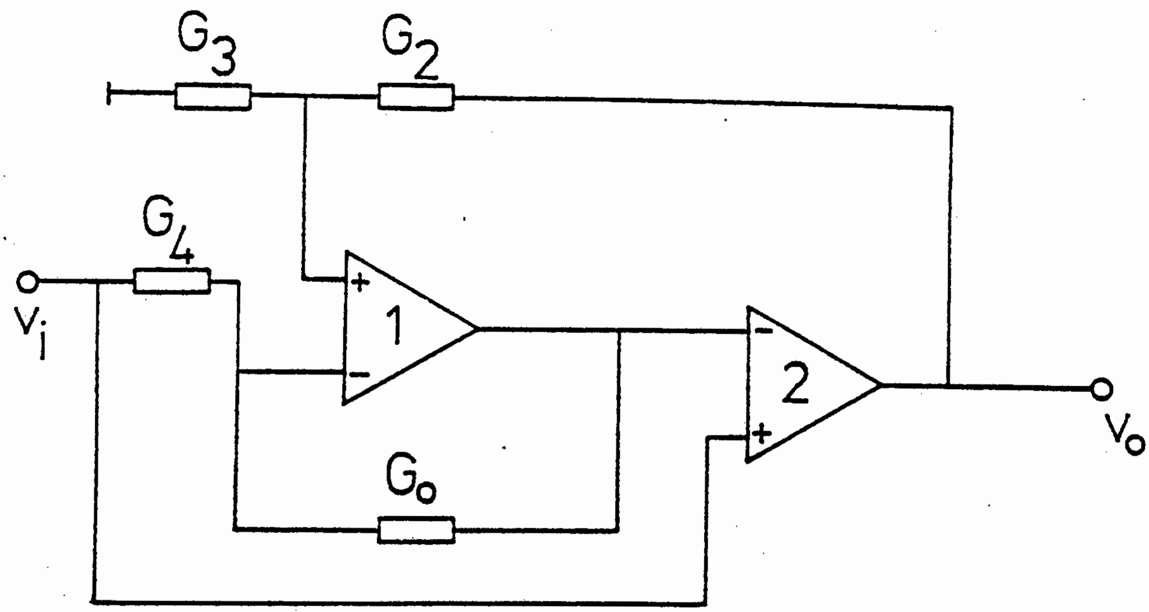


Figure 6

The above COA was presented in papers by Geiger [14] and Soliman [15].

Phase Compensation

$G_3/G_2 = k_0 - 1$

$G_4/G_0 = k_0b - 1$

Magnitude Compensation

$G_3/G_2 = k_0 - 1$

$G_4/G_0 = (bk_0/(\sqrt{(2k_0b+1)})) - 1$

where $b = GB_1/GB_2$.

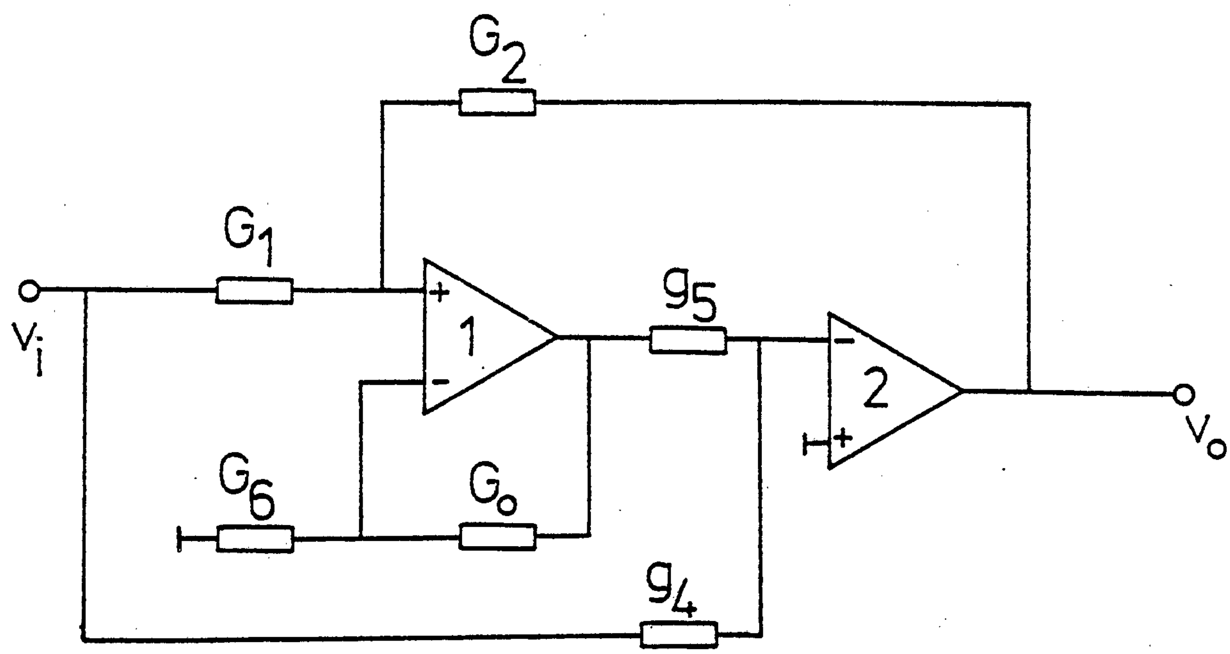


Figure 7

The above COA was discussed in a paper by Huertas and Rodriguez-Vazquez [24].

Phase Compensation ($b = GB_1/GB_2$ and for stability $\nu_0 > 0$)

$g_5/g_4 = 1/(\nu_0 k_0 b) - 1$

$G_1/G_2 = ((1 + k_0)(1 - k_0\nu_0 b) / (1 - \nu_0 k_0 b + \nu^2{}_0 k_0 b)) - 1$

$G_6/G_0 = 1/\nu_0 - 1$

Magnitude Compensation ($b = GB_1/GB_2$)

$g_4 = 0$

$g_5 = \infty$

$G_1/G_2 = k_0$

$G_6/G_0 = (\surd(b(1+k_0))/2) - 1$

where k_0 is the absolute value.

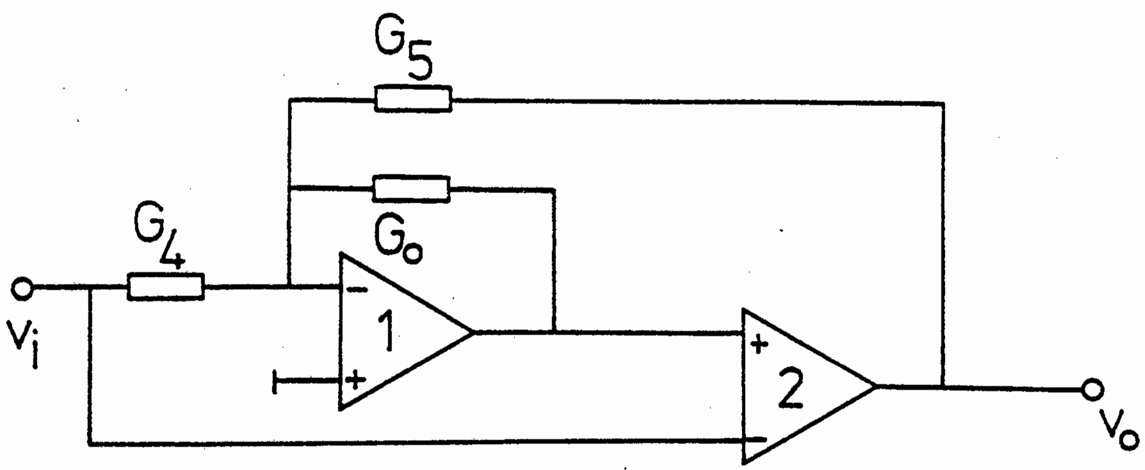


Figure 8

The above COA was presented in a paper by Huertas and Rodriguez-Vazquez [24].

Phase Compensation ($b = GB_1/GB_2$)

$G_4/G_o = k^2_o b/(1+k_o) - 1$

$G_5/G_o = bk_o/(1+k_o)$

Magnitude Compensation ($b = GB_1/GB_2$)

$G_4/G_o = G_5 / G_o + ((k_o b\sqrt{(1 + k_o)}) / (\sqrt{(1 + k_o + 2bk^2_o)})) - 1$

$G_5/G_o = k_o b/(\sqrt{((1+k_o)(1+k_o+2bk^2_o))})$

where k_o is the absolute value.

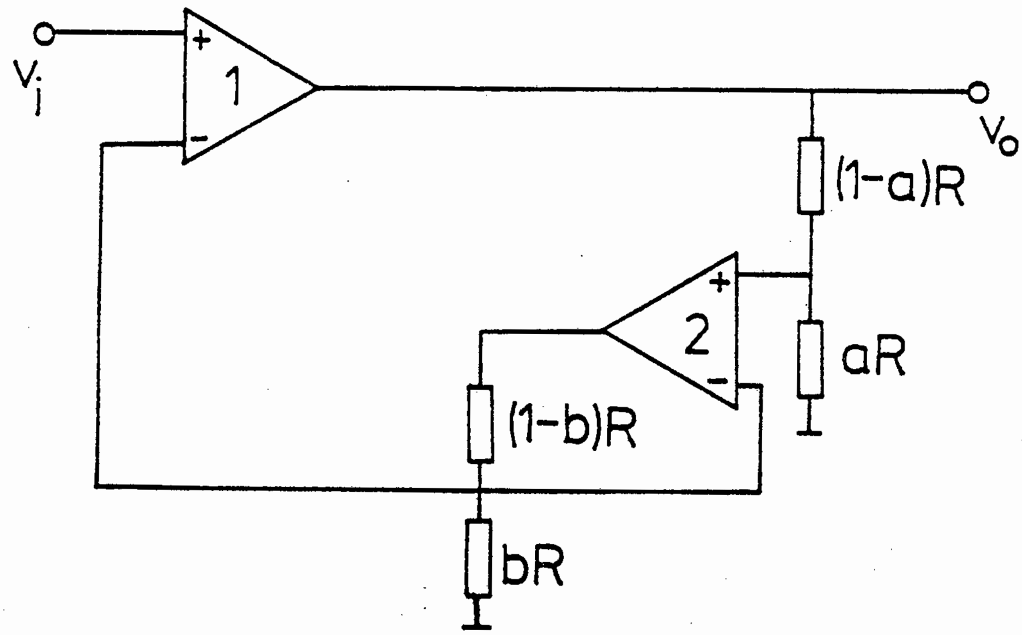


Figure 9

The above positive finite gain COA is due to Natarajan and Bhattacharyya [16].

The DC gain of this COA can be adjusted by varying 'a' and the first order coefficient matching can be achieved by varying 'b'. Where 'a' and 'b' are the resistor ratios as indicated in the above figure. Due to the possibility of small gain bandwidth differences in the two OAs it is useful to have the independent gain control adjustment.

A positive unity gain amplifier can be realized from this COA by taking the gain bandwidths of both OAs to be equal. Thus the resistor ratios are no longer required and the unity gain COA results having improved gain bandwidth. This is a realisation using only two OAs.

The tuning of the above circuit can be achieved by first adjusting 'a' to obtain the required gain and then 'b' to realise a zero phase difference between input and output.

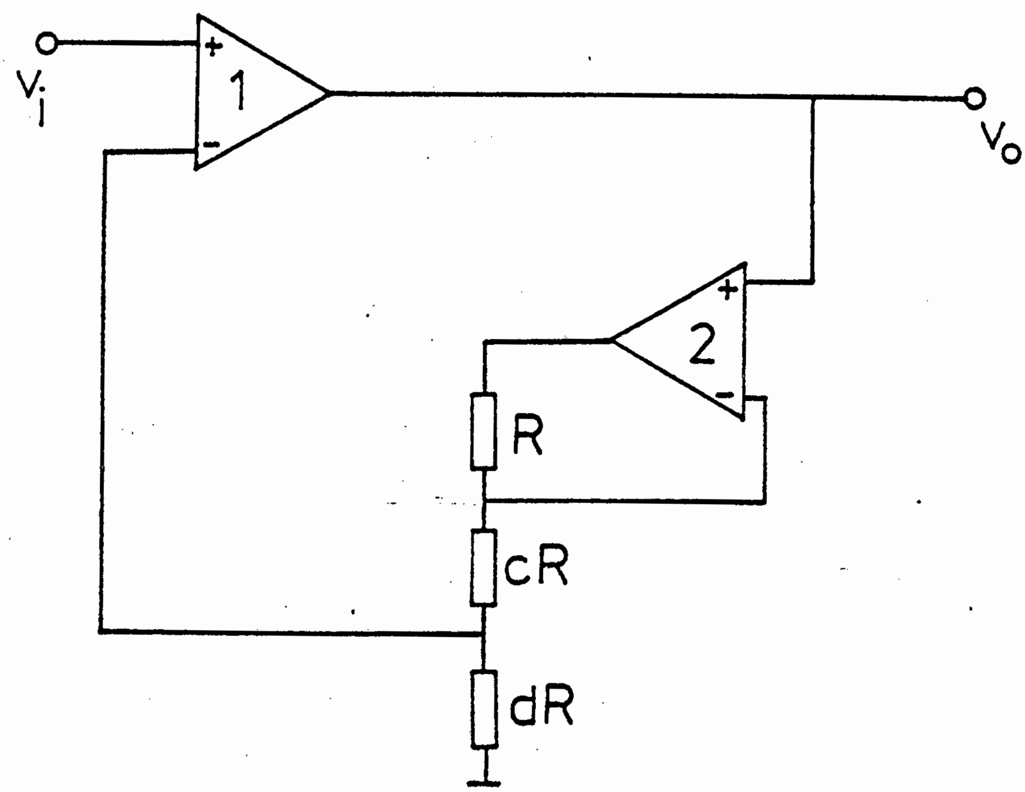


Figure 10

The above positive finite gain COA was mentioned in a paper by Natarajan and Bhattacharyya [16].

This COA is similar to that shown in the previous figure but lacks the independent DC gain adjustment. However this is only a three resistor realization which may be desirable in some instances where ease of post design adjustment of the actual response is not required.

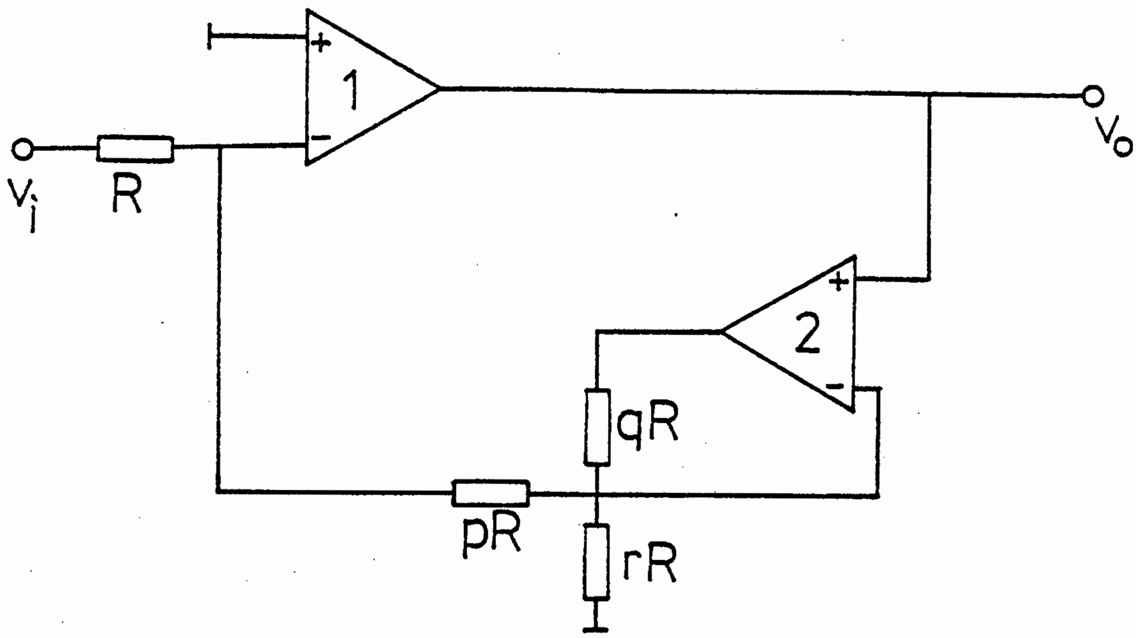


Figure 11

This is a negative finite gain COA reported by Natarajan and Bhattacharyya [16].

It is a four resistor realisation with independent DC gain control and the matching of the first order coefficients by adjustment of the resistor ratios. The resistor rR can be removed still allowing independent adjustment of gain and first order coefficient matching but the resistor qR will be of the order of K^2R. This is not a serious drawback for low values of K but could lead to unacceptably large component spreads for fabrication purposes.

The tuning of this circuit can be achieved by first adjusting 'a' to achieve the required gain and then 'b' to realise a zero phase difference between input and output.

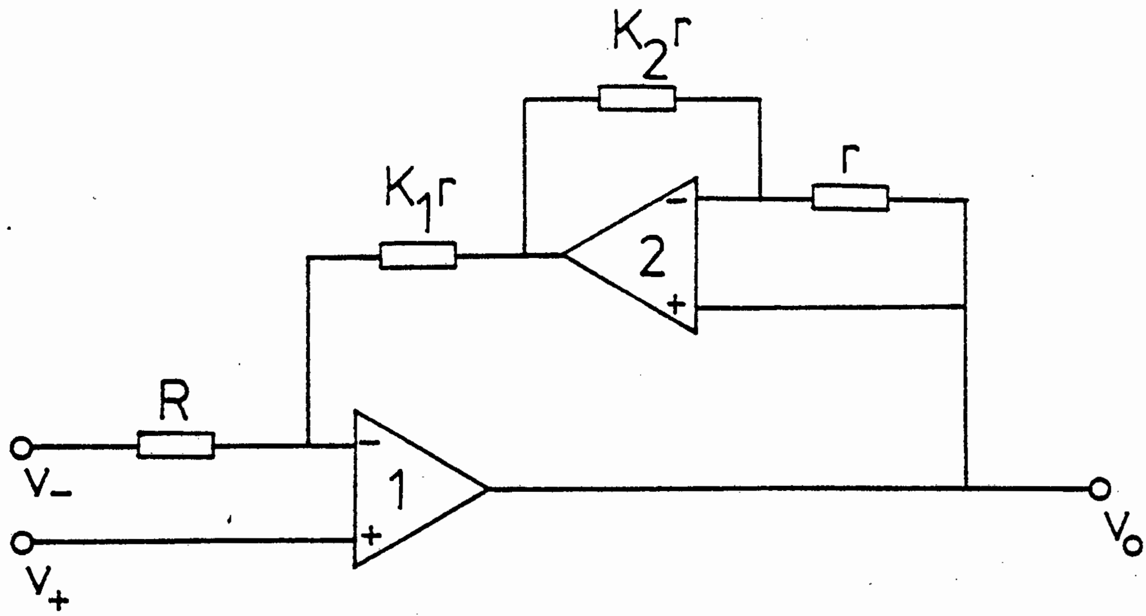


Figure 12

The above COA was referred to in a paper by Soliman and Ismail [32].

This three port device can be used in the inverting, non-inverting and differential input mode. Soliman and Ismail found that the magnitude error was minimal and the phase error such that it had to be compensated for. This is done by means of active compensation as can be seen from the figure above. The only limitation of this circuit is that the gain bandwidths of both OAs must be equal.

This equality of gain bandwidths is not easily achieved in practice thus seriously limiting the attractiveness of this particular arrangement where the magnitude error is no longer negligible and can not be compensated.

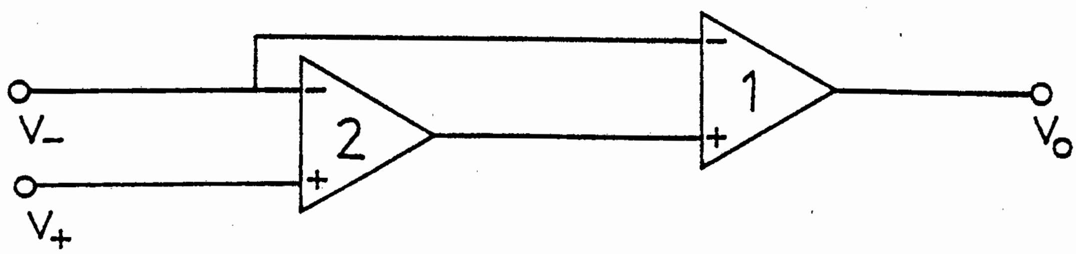


Figure 13

The above COA was discussed in a paper by Geiger and Budak [9].

In their paper Geiger and Budak show that when comparing this circuit with other zero-sensitivity bandpass filters it yields distinctly superior results. However the above COA does become unstable for $\omega_0/GB \approx 0{,}07$, $Q = 10$ and thus is only useful for sufficiently small values of ω_0/GB.

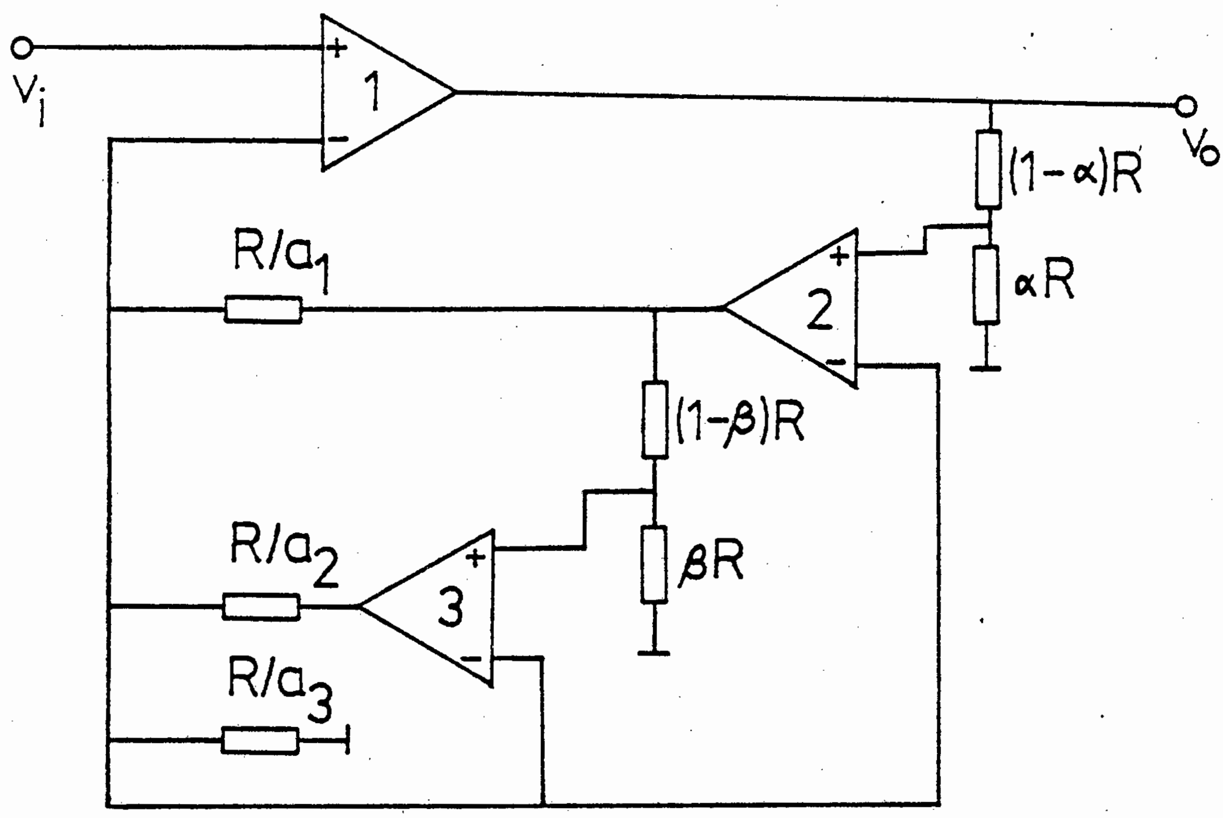


Figure 14

The above positive finite gain COA was presented in a paper by Natarajan and Bhattacharyya [18].

It is a COA actively compensated for the second order effects of the OAs finite gain bandwidth requiring two additional OAs and five resistors compared to a conventional realisation.

For a positive unity gain realisation, assuming that OAs are matched, only three OAs and two resistors are required as the overall gain $\mu_o = 1$ and $\alpha = 1/\mu_o$.

For OAs with unequal gain bandwidths suitable choices of β and a_2 or a_3 will realise the desired gain with a relatively low component spread.

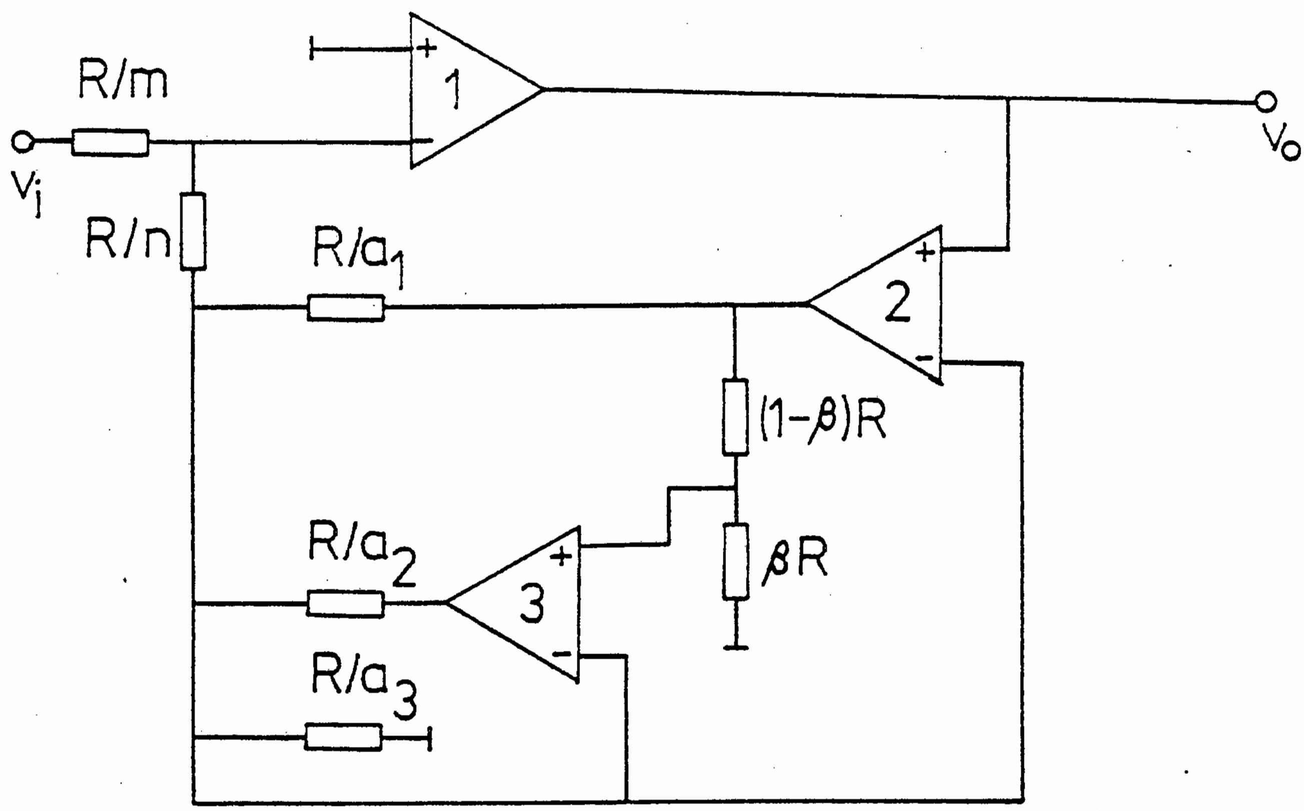


Figure 15

The above negative finite gain COA was reported on in a paper by Natarajan and Bhattacharyya [18].

Here the COA is actively compensated for the second order effects of the OAs finite gain bandwidth, again requiring two additional OAs and five resistors as opposed to the conventional realisation.

The COAs of Figures 16 through 19 were derived by Mikhael and Nessim [2].

The characteristics of these COAs are the same as that of an ordinary OA in that they can be used to realise positive finite gain and negative finite gain amplifiers. Both phase and magnitude compensation is achieved by the adjustment of the resistor ratio α. The authors claim that these COAs are stable and exhibit low sensitivity to variations in the resistor ratio α. The C2OA-2 of Figure 17 exhibits an infinite input impedance in both the inverting and non-inverting configurations.

The authors quoted experimental results in their paper but gave no indication of component values used and the value of α used to achieve these results. The authors also failed to indicate how the correct value of α was found and for which values of α the resultant network is unstable.

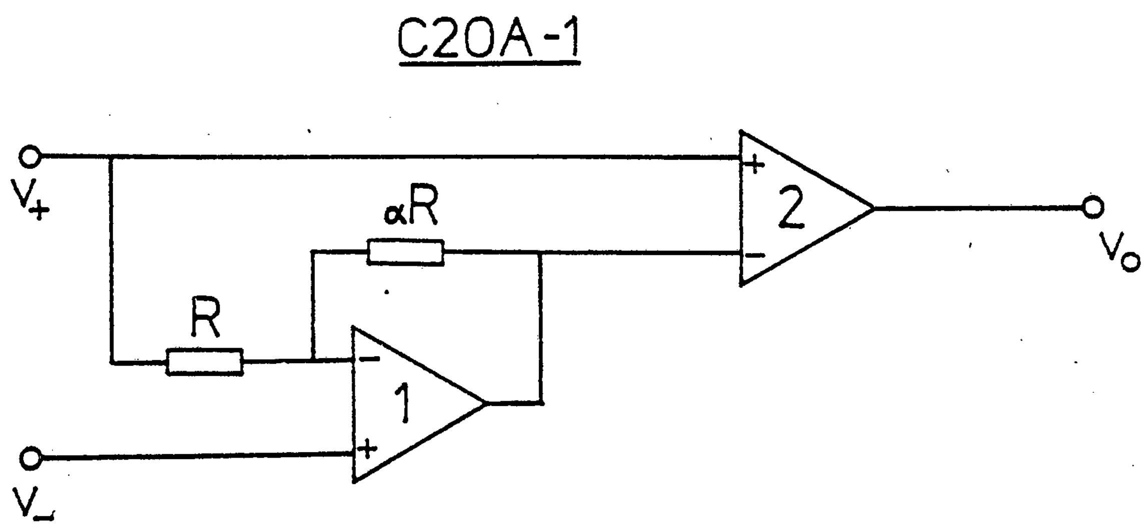


Figure 16

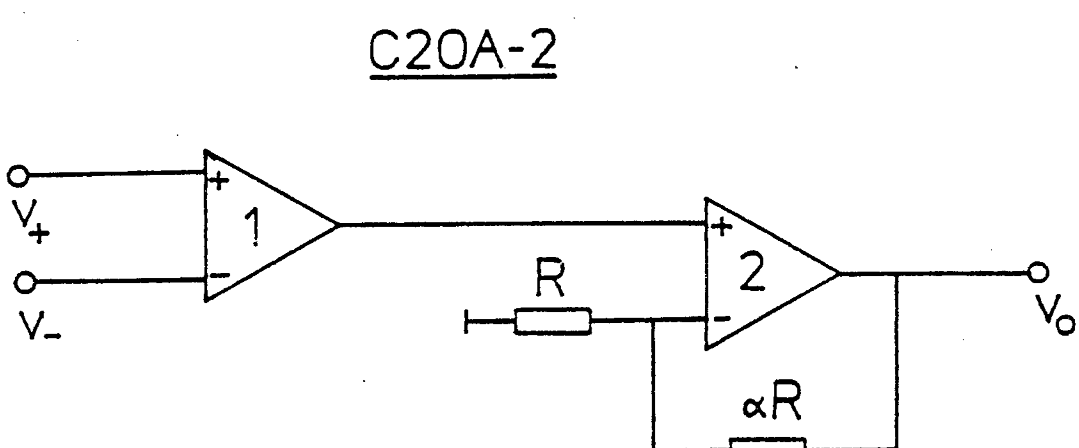


Figure 17

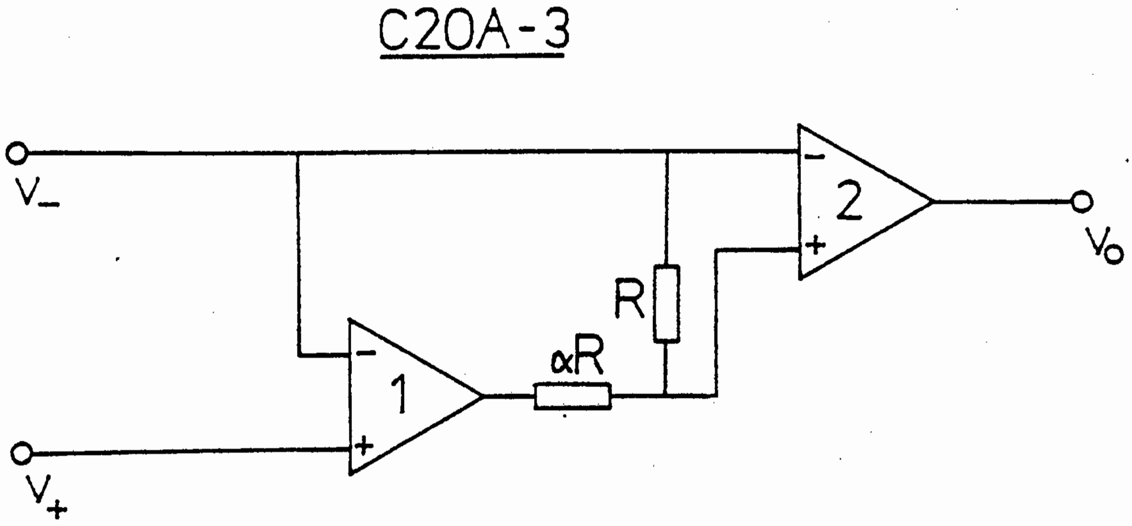


Figure 18

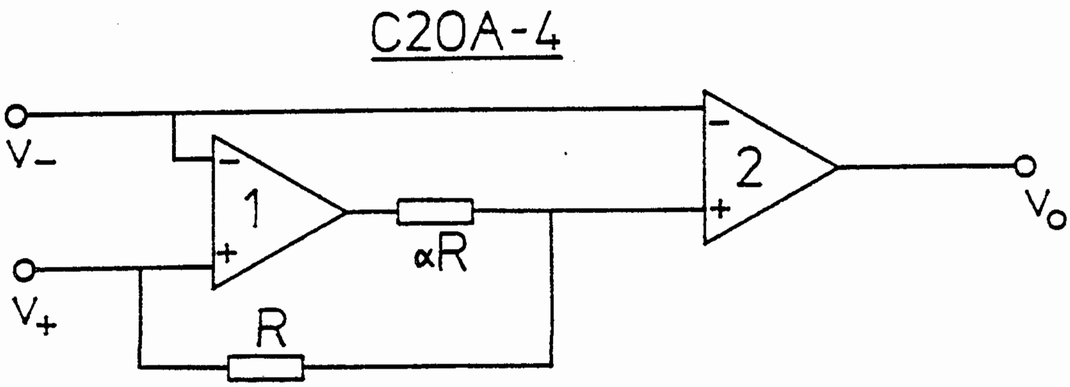


Figure 19

Figures 20 and 21 are COAs derived by Geiger and Budak [12].

These COAs use three OAs and are designed to eliminate both the first- and second-order effects of the finite gain bandwidth of the OAs. The OAs for these circuits need not be matched.

The parasitic poles introduced by the OAs can lie in the right half of the s-plane for some values of β and θ. To ensure stability the values of β and θ must be chosen such that the parasitic poles remain in the left half of the s-plane.

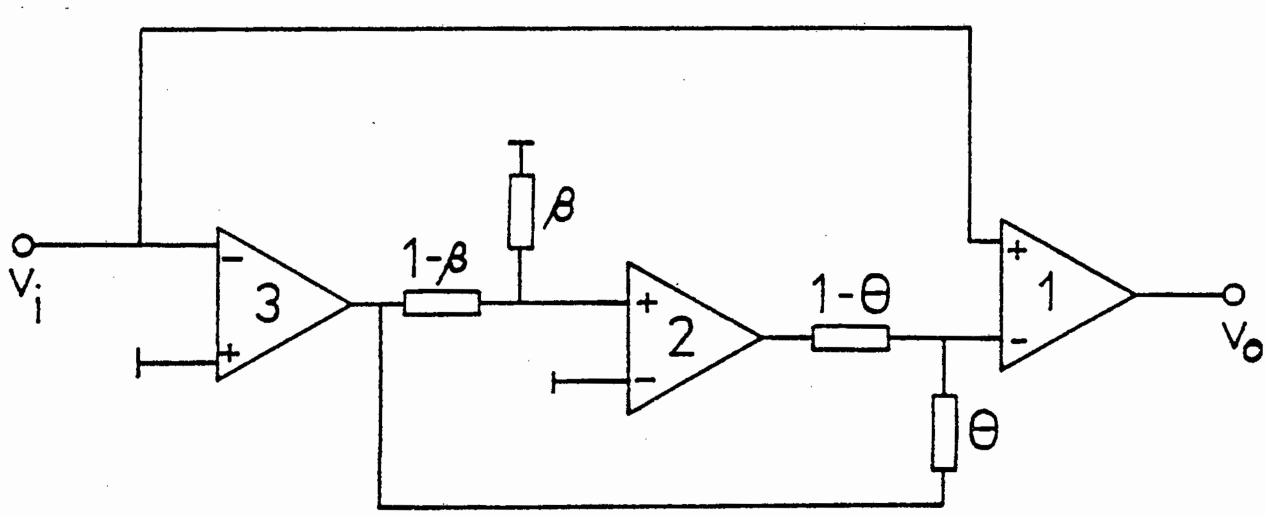


Figure 20

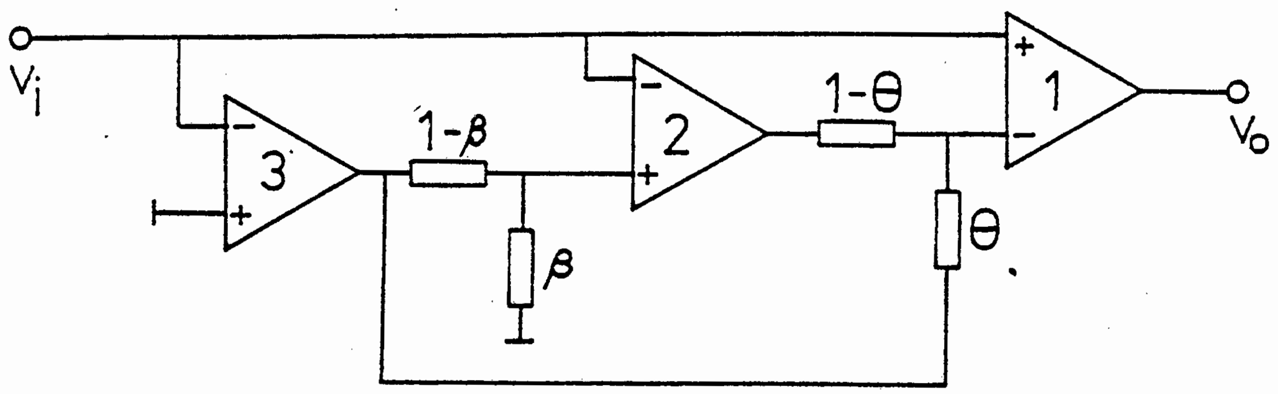


Figure 21

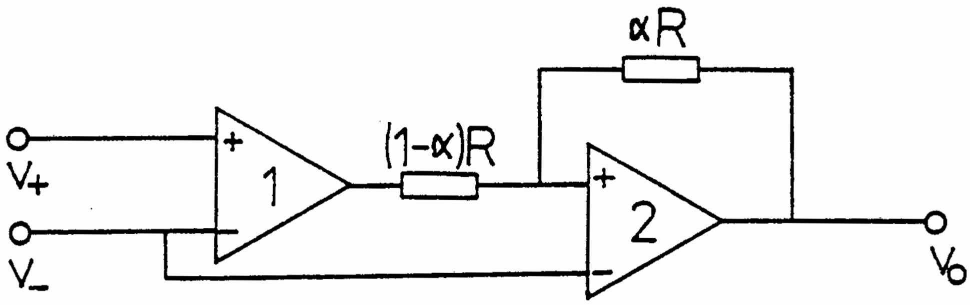


Figure 22

This COA was first shown in a paper by Natarajan [23] where positive feedback is used to reduce the second order effects of the OA's finite gain bandwidth. The amount of positive feedback must be carefully chosen to suit the near optimal solution. The choice of positive feedback is dependent on the topology into which the COA is being embedded.

The value of α is always less than 1 as is the amount of positive feedback.

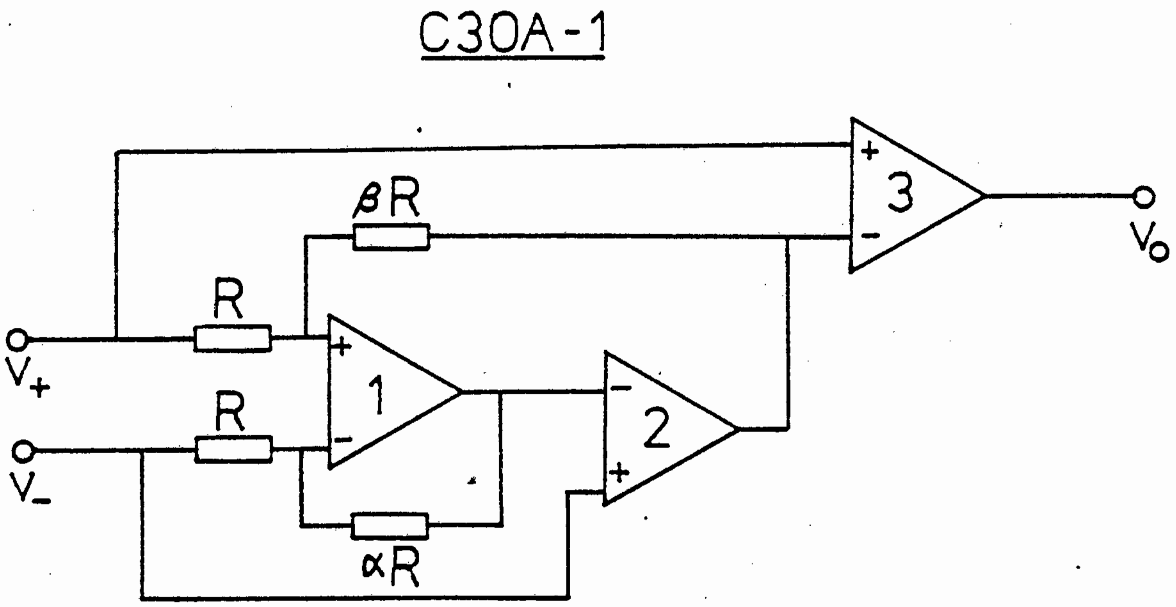


Figure 23

This COA was reported on by Mikhael and Nessim [4] and is obtained by taking the C2OA-1 [2], Figure 16, and replacing the first OA with a C2OA-1.

This COA is actively compensated for the first- and second-order effects of the OA's finite gain bandwidth.

The authors found this COA to be stable and to exhibit an extended bandwidth with low coefficient sensitivity and reasonable mismatch of the OAs allowed.

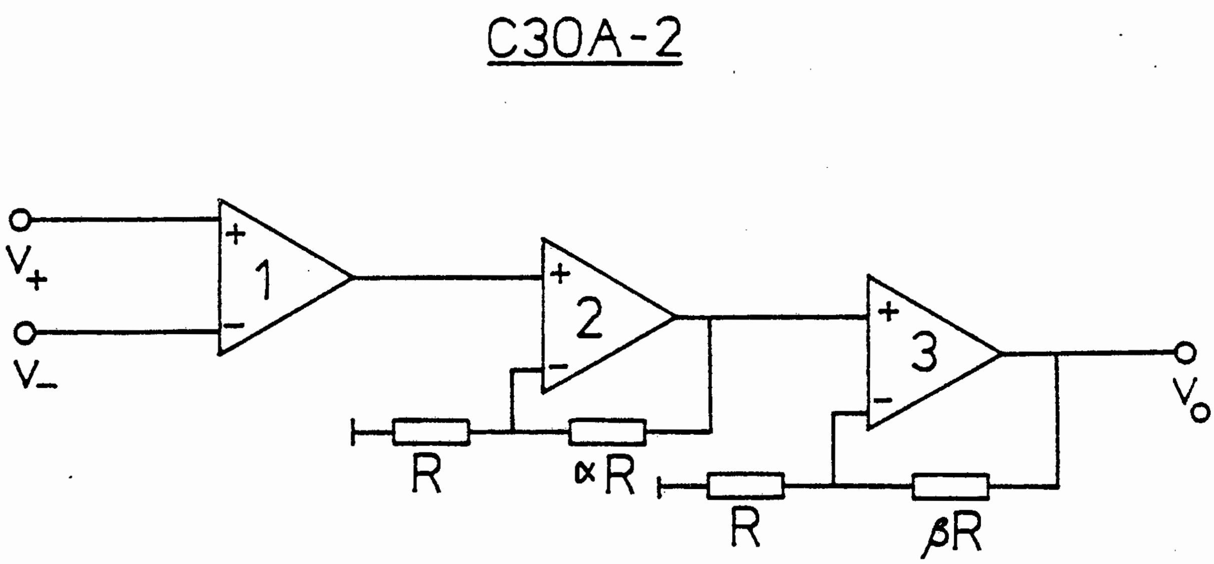


Figure 24

This is another COA generated by Mikhael and Nessim [4] obtained by taking the C2OA-2 [2], as shown in Figure 17, and replacing the first OA with a C2OA-2.

The authors found this COA to be stable with extended bandwidth exhibiting low coefficient sensitivity and allowing a certain amount of OA mismatch.

The above arrangement maintains the infinite input impedance characteristic associated with the C2OA-2 and actively compensates for the first- and second-order effects of the OA's finite gain bandwidth.

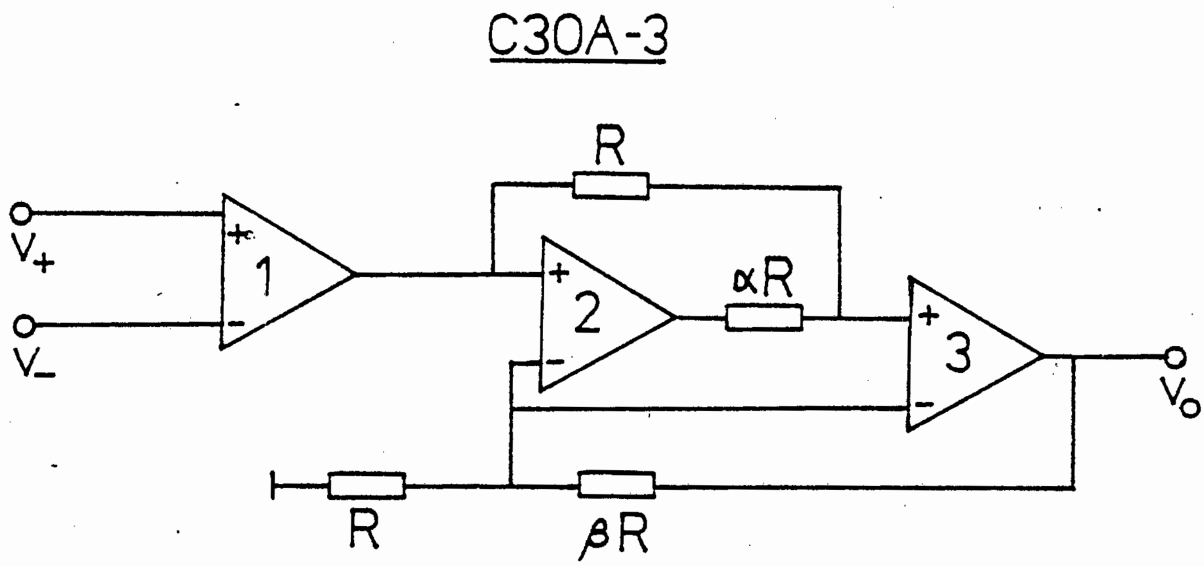


Figure 25

Mikhael and Nessim [4] also derived this COA by taking the C2OA-2 [2], depicted in Figure 17, and replacing the second OA with the C2OA-4 shown in Figure 19.

This circuit was found to be stable, to have extended bandwidth, low coefficient sensitivity, infinite input impedance, could allow a certain amount of mismatch between the OAs to exist and actively compensates for the first- and second-order effects of the OA's finite gain bandwidth.

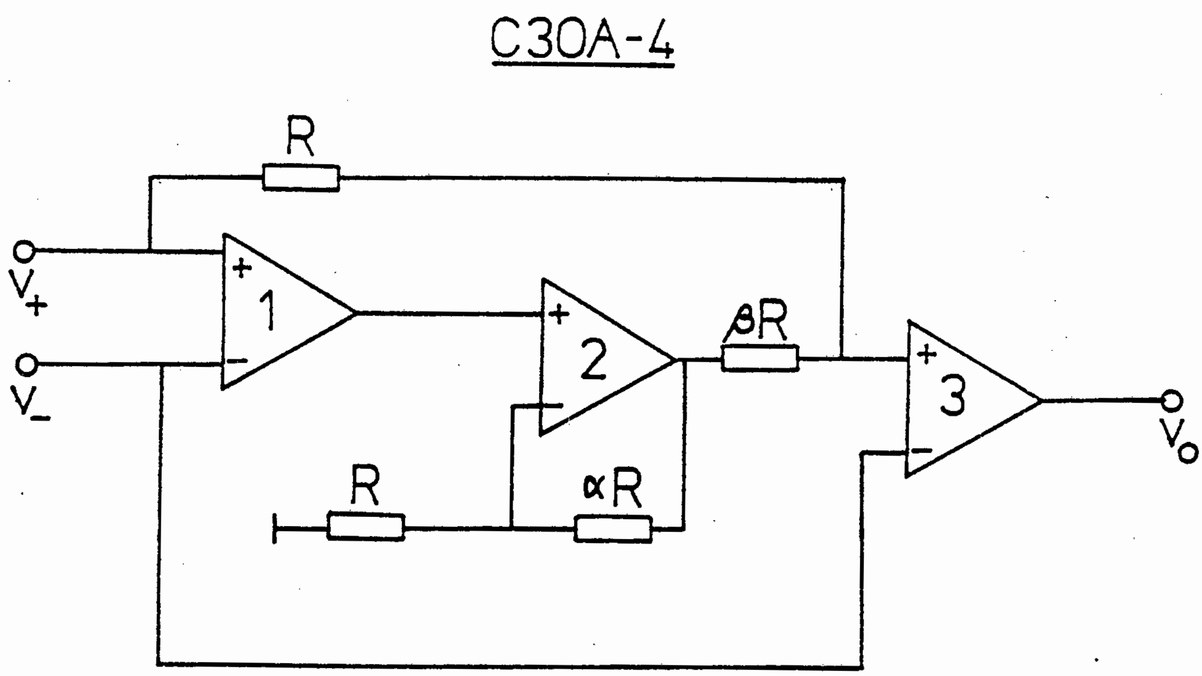


Figure 26

This COA was obtained by taking the C2OA-1 [2] and replacing the first OA with the C2OA-2, Figure 17, which actively compensates for the first- and second-order effects of the OA's finite gain bandwidth.

Mikhael and Nessim [4] found this COA to be stable with low coefficient sensitivity, extended bandwidth and allows for reasonable OA mismatch.

The COAs of Figures 27 through 30, by Mikhael and Nessim [4], were generated by replacing each single OA in the C2OA-1 and C2OA-2 circuits, Figures 16 and 17 respectively, with a C2OA-1 and/or C2OA-2 circuit.

The authors found these COAs to be stable, exhibiting extended bandwidth with low coefficient sensitivity and reasonable OA mismatch allowed.

These COAs are actively compensated for the first-, second- and third-order effects of the OA's finite gain bandwidth.

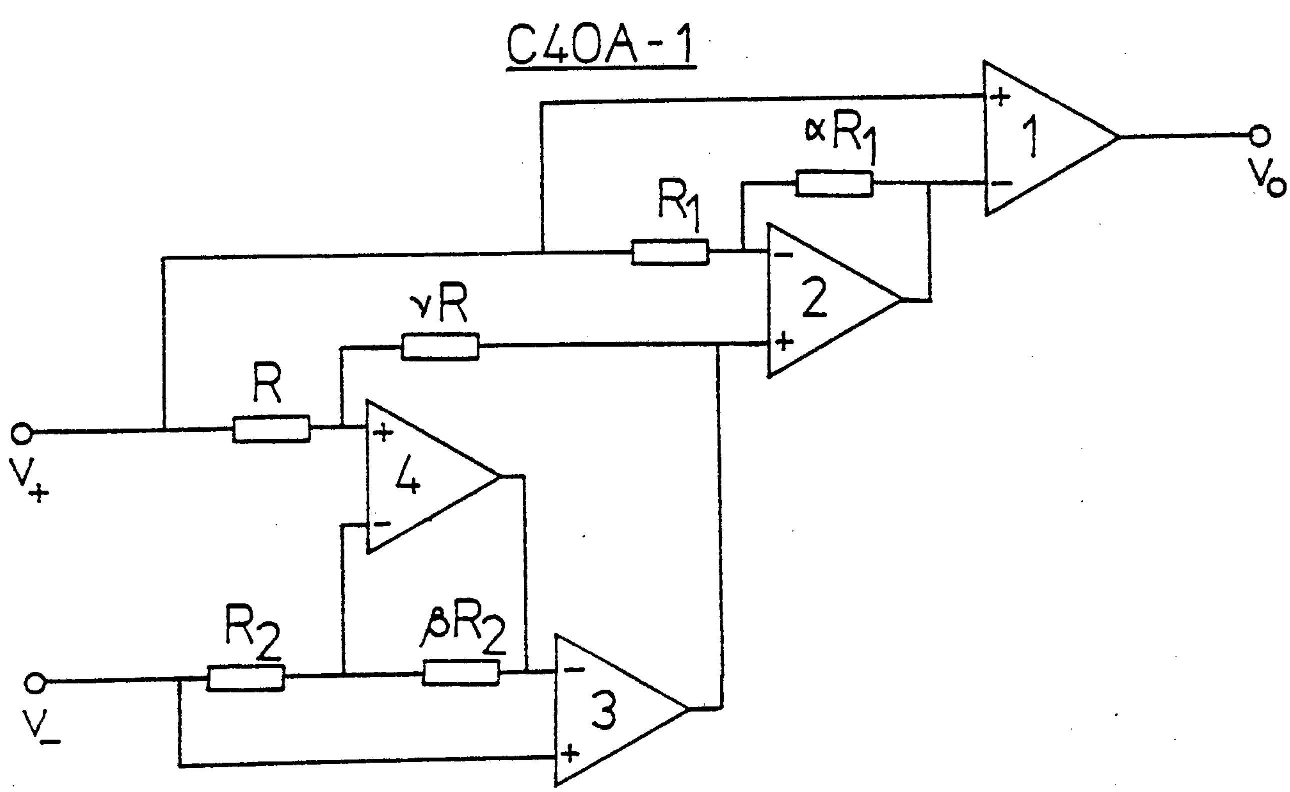


Figure 27

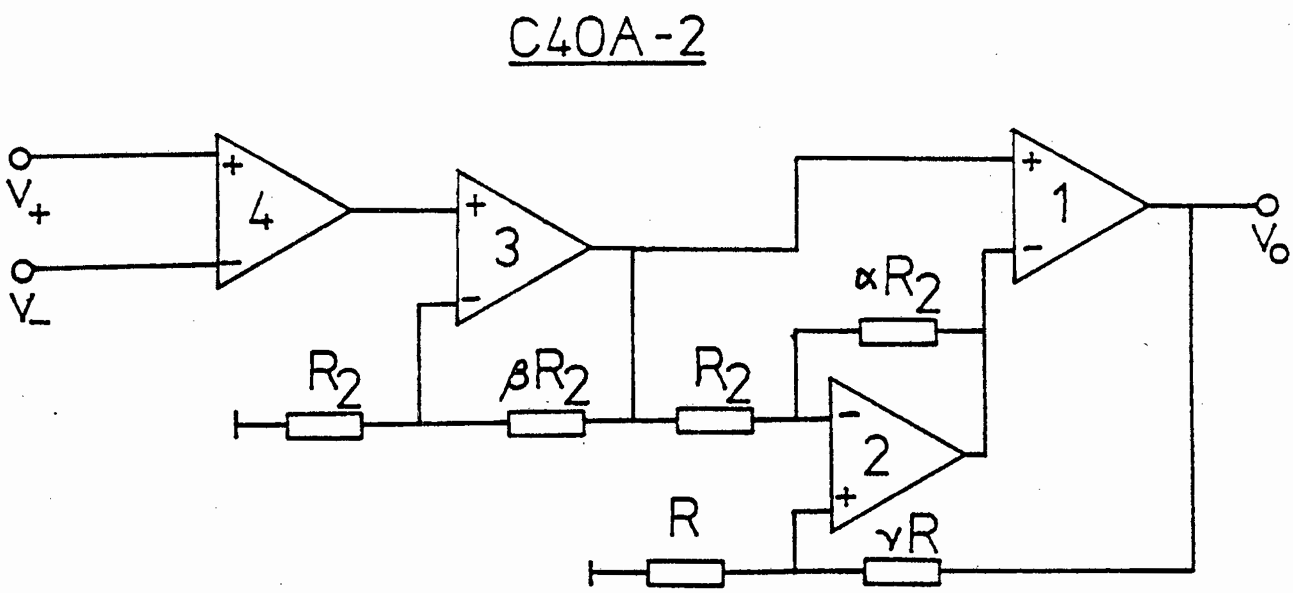


Figure 28

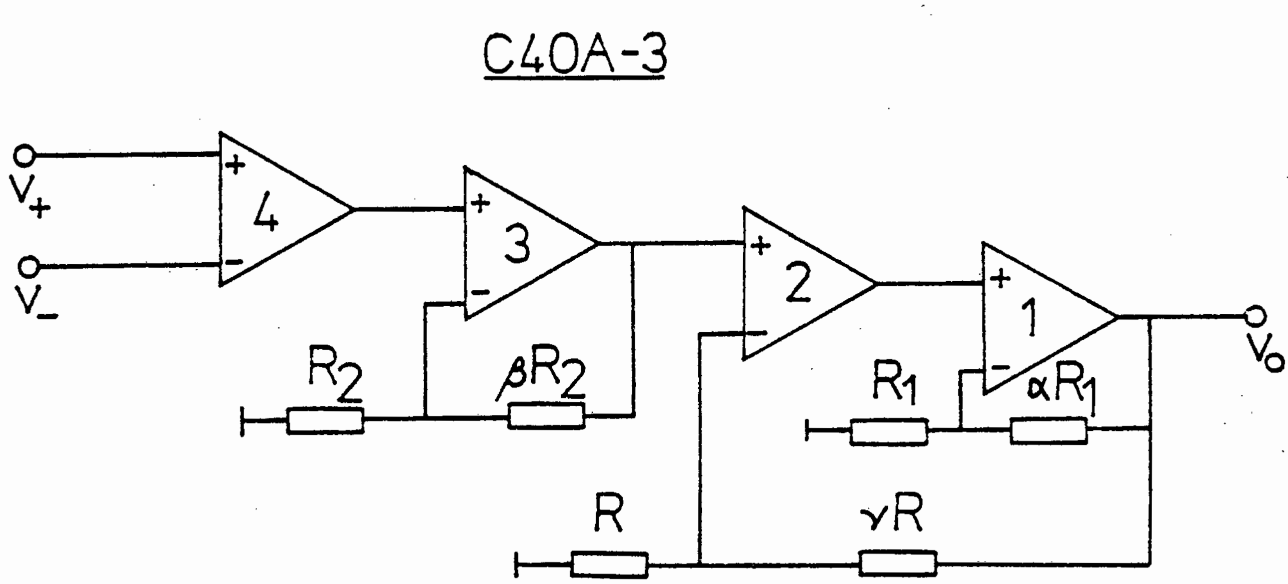


Figure 29

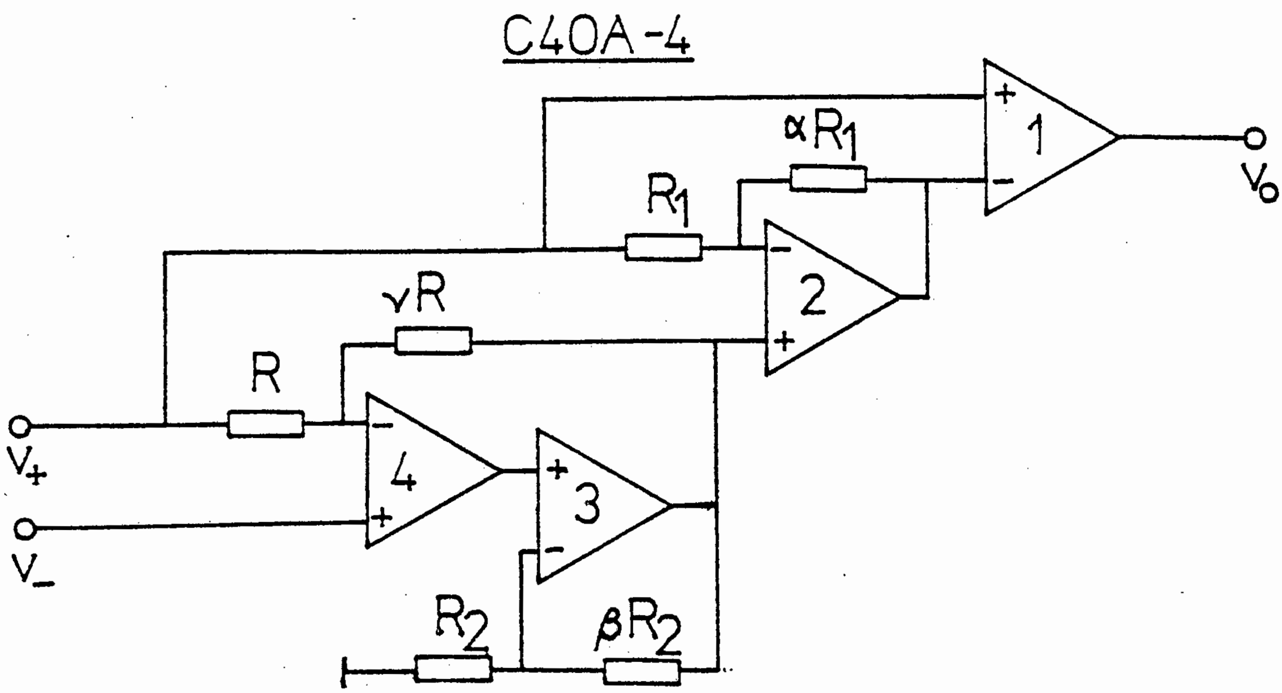


Figure 30

As can be seen from Figures 1 to 30 literature abounds with COAs of one type or another. Two types of compensation, namely active and passive compensation are widely applied to reduce the imperfections of the individual OA.

The problem of the finite gain bandwidth of the OA in a circuit can be overcome in the first instance by analysing the network using the single pole model and then pre-distorting the network components, this technique requires that the OA gain characteristics be accurately known. Another approach is to use post-design trimming of the components of the network, this technique is difficult and expensive especially with thin-film realisations. In the paper by Wilson [30] a passive compensation technique is outlined where one or two capacitors are connected to the OA inputs depending on the particular mode of operation.

For many applications the passive compensation technique is improved upon by an active compensation method where an additional OA is used to modify the imperfections of the first amplifier and in so doing the characteristics of the so generated COA are improved. This active compensation does however require an extra OA which will increase cost as well as power consumption.

It has been shown that COAs can contain two or more OAs. Taking the COAs of Figures 1 to 13, 16 to 19 and 22, comparison reveals that all of them use two OAs and a few resistors in a variety of topologies and all aim at improving the gain bandwidth product.

In Figures 20,21,23 to 26 three OAs with resistors are forming a COA while further enhancement is sought in COAs containing four OAs, Figures 27 to 30.

The various circuits presented are approximately in chronological order of publication. It can thus be seen how the COA has developed over the years.

In the following chapter a more detailed look is taken at the various COAs, their uses and specific characteristics.

CHAPTER 2

POSITIVE GAIN COMPOSITE OPERATIONAL AMPLIFIERS

As mentioned previously experimental results obtained by Campbell and Stephenson made use of the C20A-2 composite operational amplifier (COA) by Mikhael and Nessim [2] for a Sallen and Key positive feedback bandpass filter.

Negative gain and thus negative feedback networks are inherently stable and therefore only positive gain composite operational amplifiers (COA) require special attention.

The COA depicted in Figure 31 was presented in a paper by Reddy [34] and represents a magnitude compensated COA.

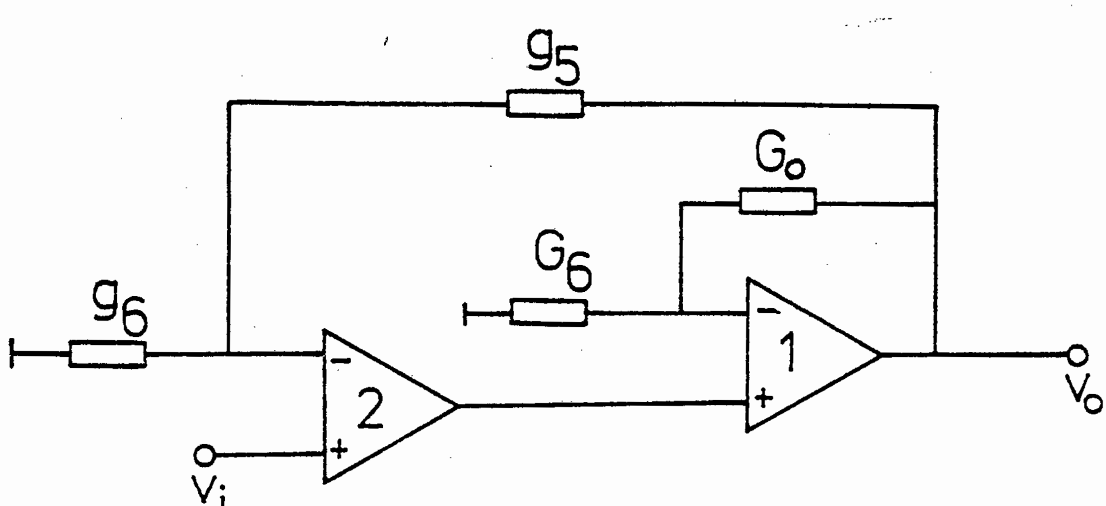


Figure 31

It has an infinite input impedance and as can be seen the impedance ratio g_6/g_5 sets the overall gain.

If now g_5 and g_6 are ignored a circuit as shown in Figure 32 emerges.

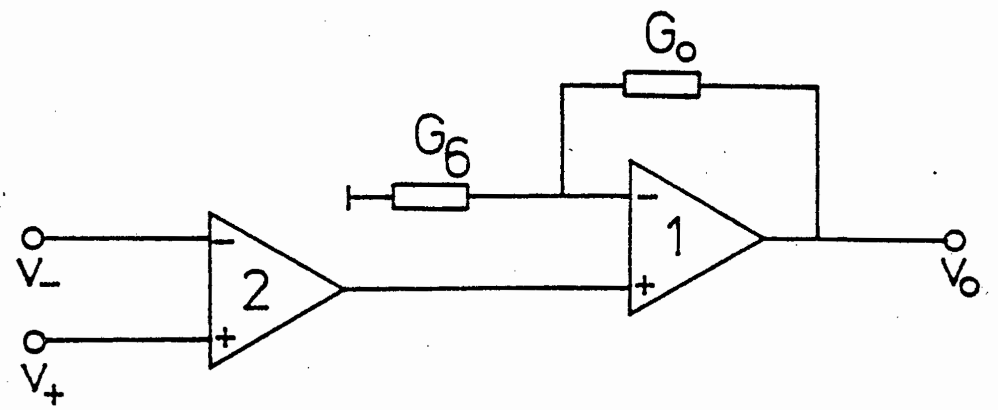


Figure 32

Comparing this COA with that of Figure 17 it can readily be seen that it is the C2OA-2 type COA also derived by Mikhael and Nessim [2].

The next COA of Figure 33, which is due to Huertas and Rodriguez-Vazquez [24], is magnitude compensated.

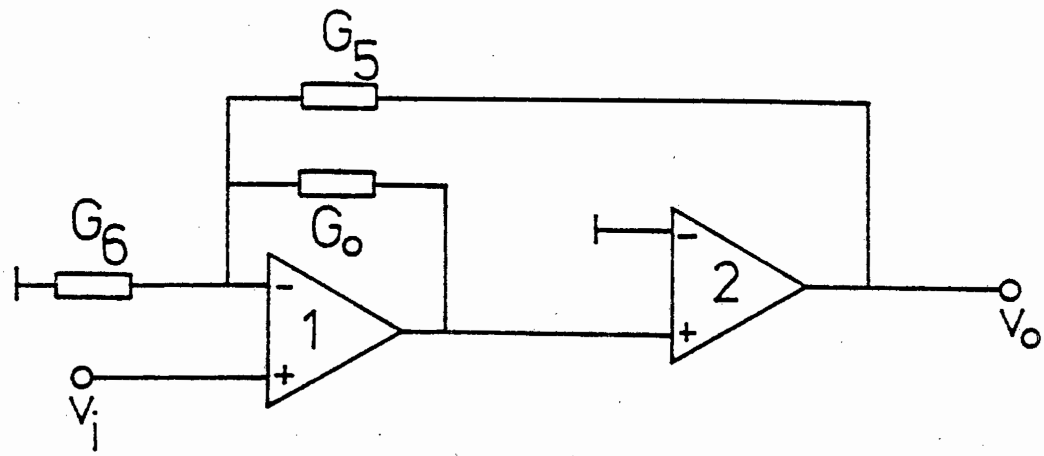


Figure 33

It exhibits infinite input impedance with maximum bandwidth, low phase error and was also studied in a paper by Geiger [14].

Another COA by Geiger [14], depicted in Figure 34 is again a magnitude compensated COA.

Geiger assumes that the active element can be modelled with a gain function of

$$A(s) = GB/s = 1/s_n$$

where s_n is the normalised complex frequency as used by Budak and Petrela [36].

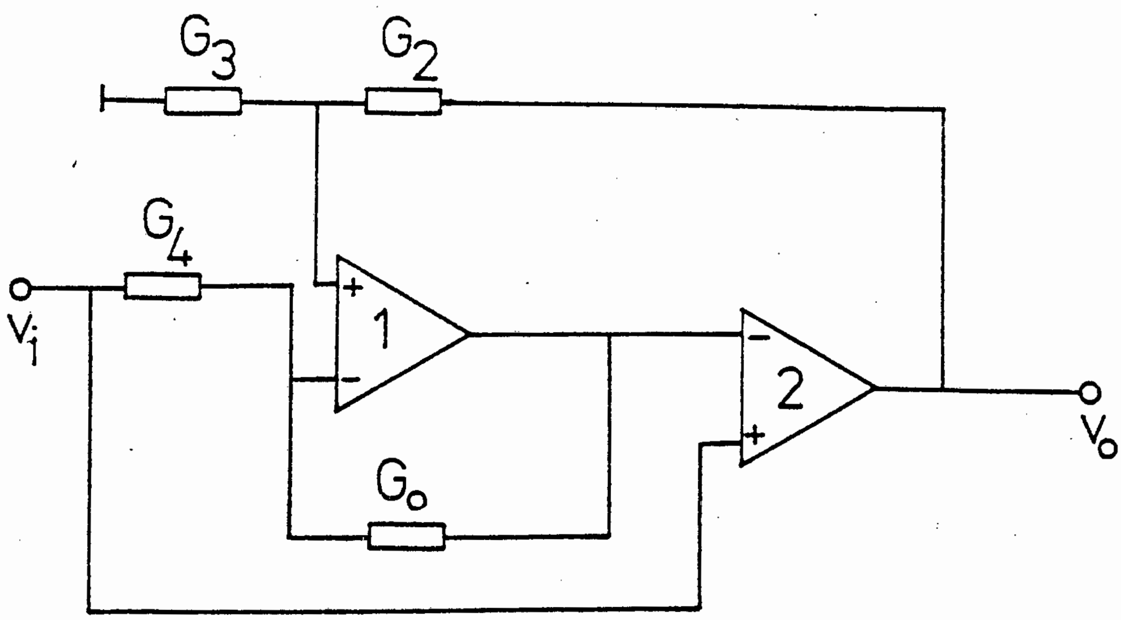


Figure 34

It will later be shown that this assumption does not sufficiently and fully characterise the OA.

Geiger uses G_3 and G_2 to set the overall gain and G_4 and G_0 to determine the desired bandwidth. If the gain setting resistor pair G_3 and G_2 is ignored the circuit reduces to that of Figure 35.

Now, comparing this COA with the one of Figure 16 it can readily be seen that this is equivalent to the C2OA-1 type derived by Mikhael and Nessim [2].

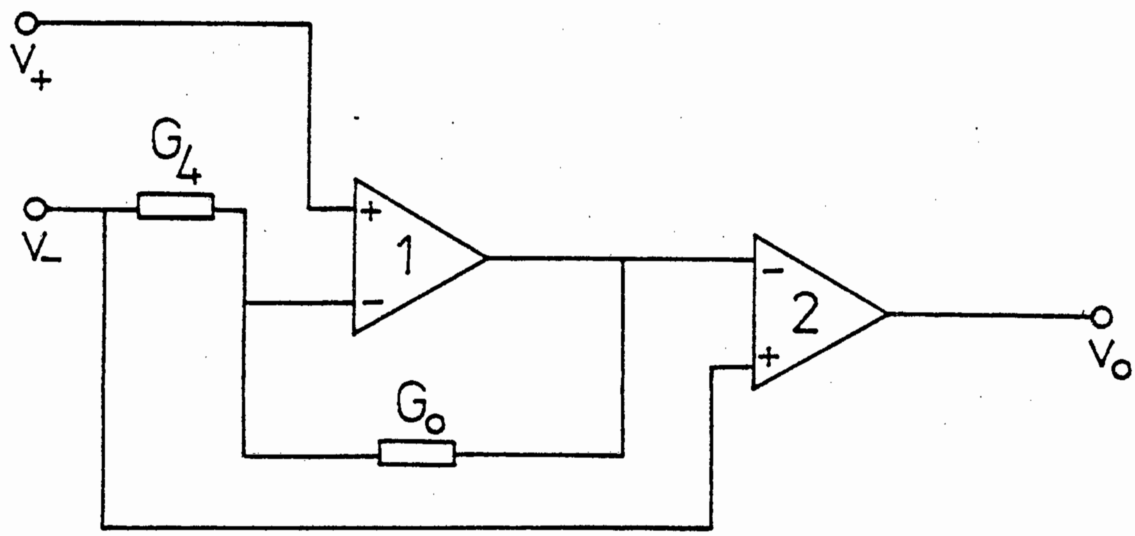


Figure 35

Soliman in his paper [15] makes use of the COA due to Geiger [14]. By a suitable choice of the resistors, he corrects for the phase error found to be an inherent disadvantage in Geiger's design when applied to a maximally flat magnitude realisation. In the paper by Soliman and Ismail [32] a novel active compensation method is proposed , realising a novel COA containing two resistive components, Figure 12. This circuit is basically equivalent to the COA given in Figure 35. A single pole model describing the individual operational amplifiers (OA) is used by the authors for the evaluation of the COA characteristic behaviour.

Natarajan and Bhattacharyya [16] in their paper present the COAs shown in Figures 9 and 10. Both are actively compensated and yield low phase errors for relatively low frequency operation. However, here too, the evaluation of the characteristics of the COA were derived using the single pole model.

A number of circuits where COAs comprising three OAs have been shown. Examples in particular are due to Natarajan and Bhattacharyya [18] Figures 14 and 15, Geiger and Budak [12] Figures 20 and 21 and Mikhael and Nessim [4] Figures 23 to 26.

All these circuits make use of active compensation to yield a COA made up of three OAs. Generally these COAs are fairly insensitive to first- and second-order effects of the finite gain bandwidth of the individual OAs.

Figures 27 to 30 are circuits by Mikhael and Nessim [4] where four OAs have been used to realise COAs relatively insensitive to first-, second- and third-order effects of the individual OAs finite gain bandwidth.

OAs, as well as COAs, find applications in active RC filters, instrumentation, oscillators and inductance simulation. A review of all available literature indicates that little research has been done into stability of COAs.

Campbell and Stephenson [1] reported on investigations into the possibilities of employing COAs to extend the high frequency performance of conventional RC active filters. However the studies were fairly inconclusive.

The authors found inconsistencies between theoretical predictions and experimental results. This was particularly evident with regard to stability and expected improvements in gain bandwidth products. The major problem was found to lie with the best choice of α, the gain bandwidth determining resistor ratio, or the degree of compensation. For some choices of α instability of the network resulted. The circuit used by Campbell and Stephenson [1] was a Sallen and Key positive gain bandpass structure. When a high Q was chosen the experimental value of α to achieve stability was markedly different from that of the theoretical prediction.

In the following an attempt will be made to identify the shortcomings which have beset investigations of stability and their previous prediction so far. The practical results obtained by Campbell and Stephenson [1] will therefore serve as a guideline.

CHAPTER 3

THE COMPOSITE OPERATIONAL AMPLIFIER

In order to evaluate the stability of a composite operational amplifier (COA) it is necessary to define the active elements which constitute the COA, namely the operational amplifiers (OA).

The Operational Amplifier

Considering an ideal OA the characteristics are summarised as follows :-

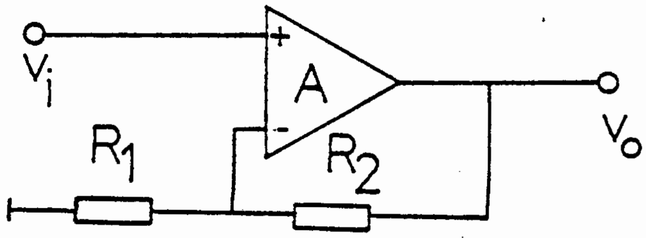
voltage gain : $A = \infty$;

input impedance : $R_{in} = \infty$;

output impedance : $R_{out} = 0$; and

bandwidth : $BW = \infty$.

This OA can be configured as a non-inverting device by the addition of passive components as shown below.



The ideal transfer function (TF) then becomes,

$$V_o/V_i = (R_1+R_2)/R_1 = K.$$

However, evaluation of the stability of an OA requires a closer look at the non-ideal characteristics of the OA. As a good approximation the input and output impedance may still be considered as ideal, however the gain bandwidth, ω_t, the voltage gain, A, are now finite and must be modelled taking into account first and higher terms.

A method of determining ω_t, the gain bandwidth is outlined in Appendix I. The value of ω_t as found by Campbell and Stephenson [1] has been used in this investigation. This allows for a fair comparison of theoretical and experimental results to be carried out.

As was shown above many previous studies have used the single pole model of the individual OAs in the COA.

Consider the single pole model transfer function of the OA as put forward by Millman [37],

$$A(s) = A_o\omega_1/(s+\omega_1) \qquad \ldots\{1\},$$

where $A_0\omega_1$, the gain bandwidth, is ω_t.

For the single pole model of the OA the gain function (GF) is,

$$A(s) = \omega_t/(s+\omega_1) \qquad \ldots\{2\}.$$

The two pole model (DP) gain function of the OA as proposed by Millman [37] is,

$$A(s) = \omega_t\omega_2/((s+\omega_1)(s+\omega_2)) \qquad \ldots\{3\},$$

where ω_t, the gain bandwidth, is $A_0\omega_1$.

Expanding this gain function yields,

$$A(s) = \omega_t(1-s/\omega_2)/s,$$

where it is assumed that $(1-s/\omega_2)$ tends to 1 as $\omega_2 >> \omega_t$ yielding the simplified form of the single pole model gain function of the OA,

$$A(s) = \omega_t/s \qquad \ldots\{4\}.$$

The three pole model (TP) of the OA due to Millman [37] is,

$$A(s) = A_0\omega_1\omega_2\omega_3/((s+\omega_1)(s+\omega_2)(s+\omega_3)), \qquad \ldots\{5\}$$

where $A_0\omega_1 = \omega_t$, the gain bandwidth.

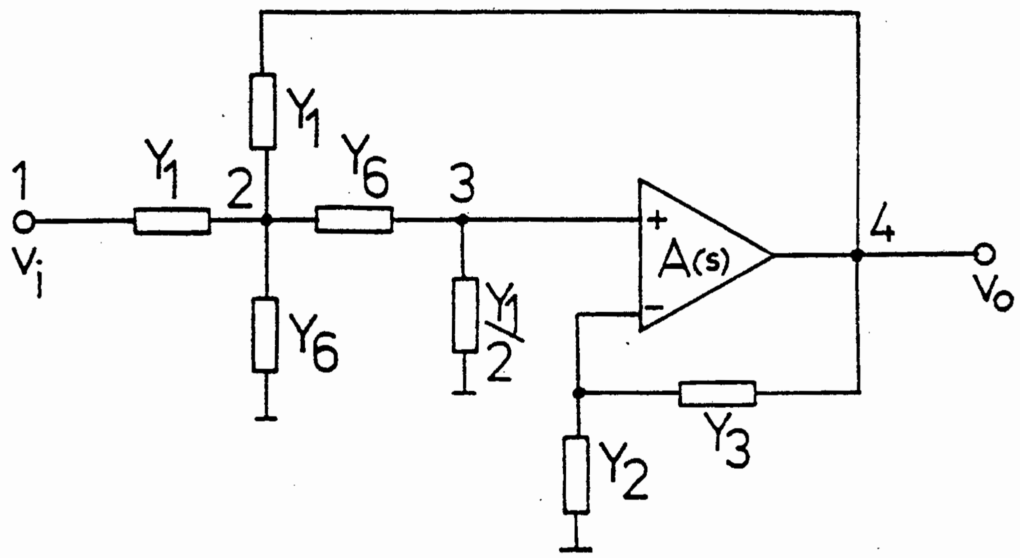
The gain function of the OA for the three pole model now is,

$$A(s) = \omega_t\omega_2\omega_3/((s+\omega_1)(s+\omega_2)(s+\omega_3)) \quad ...\{6\}.$$

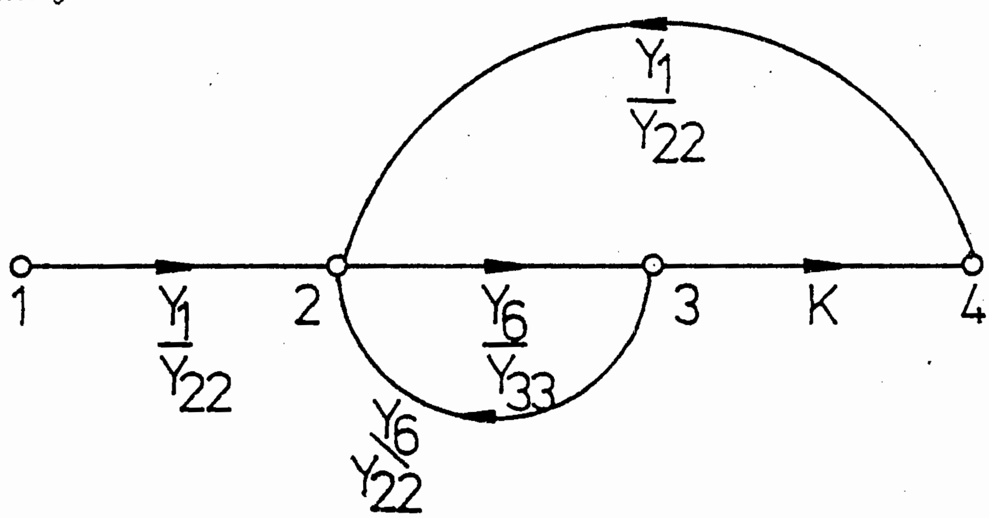
These various models of the OA will be evaluated in order to determine the stability of the COAs.

<u>The Sallen and Key Positive Gain Bandpass Network</u>

The network used to study the stability of the COA filter is derived from the Sallen and Key circuit shown.



Assuming the OA to be ideal and employing signal flow graph [42] analysis, the network transfer function for the closed loop configuration is obtained.



Thus with $A(s) = K = (Y_2+Y_3)/Y_3$ the network transfer function becomes,

$$V_o/V_i = (Y_1Y_6K / Y_{22}Y_{33}) / (1 - Y^2_6/(Y_{22}Y_{33}) + Y_1Y_6K/(Y_{22}Y_{33})),$$

where $Y_{22} = 2(Y_1+Y_6)$ and $Y_{33} = Y_1/2 + Y_6$.

Rationalisation yields,

$$V_o/V_i = Y_1Y_6 / (1/K(Y_1^2 + 3Y_1Y_6 + Y_6^2) - Y_1Y_6) \quad \ldots\{7\},$$

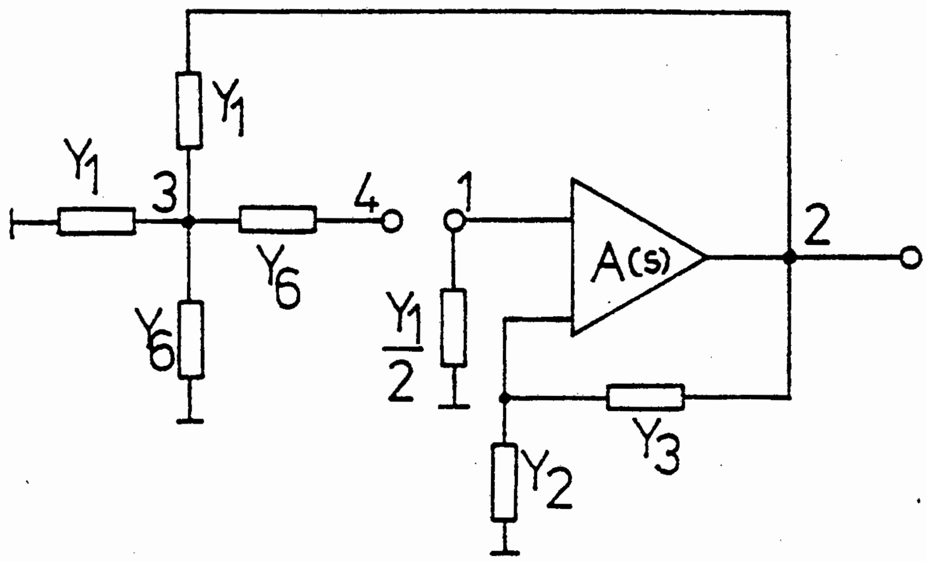
where Y_1 is a resistor and Y_6 a capacitor.

As is now evident the above equation can be used to realise a second order bandpass function,

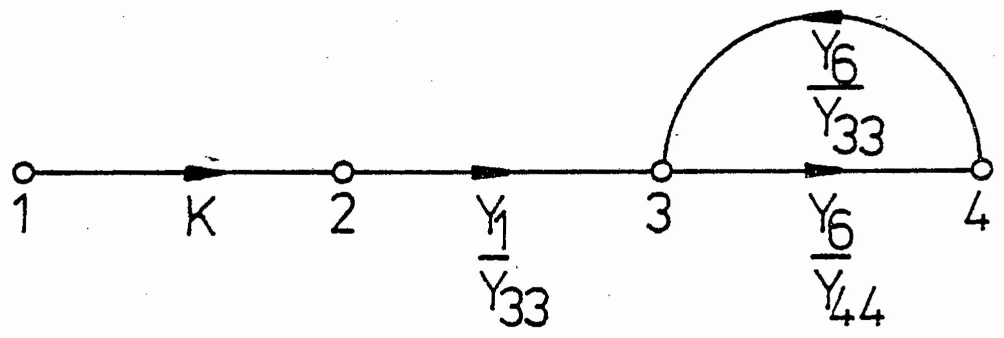
$$V_o/V_i = Hs/(s^2+s\omega_o/Q+\omega_o^2) \qquad \ldots\{8\},$$

where $H = K/(R_1C_1)$, $\omega_o = 1/(R_1C_1)$ and $Q = 1/(3-K)$.

The open loop transfer function may be derived from a configuration given below.



The corresponding signal flow graph is as shown.



As before, $A(s) = K = (Y_2+Y_3)/Y_3$, so that the network transfer function becomes,

$$V_o/V_i = (Y_1Y_6K/(Y_{33}Y_{44})) / (1 - Y_6^2 / (Y_{33}Y_{44})),$$

where $Y_{33} = 2(Y_1+Y_6)$ and $Y_{44} = Y_6$.

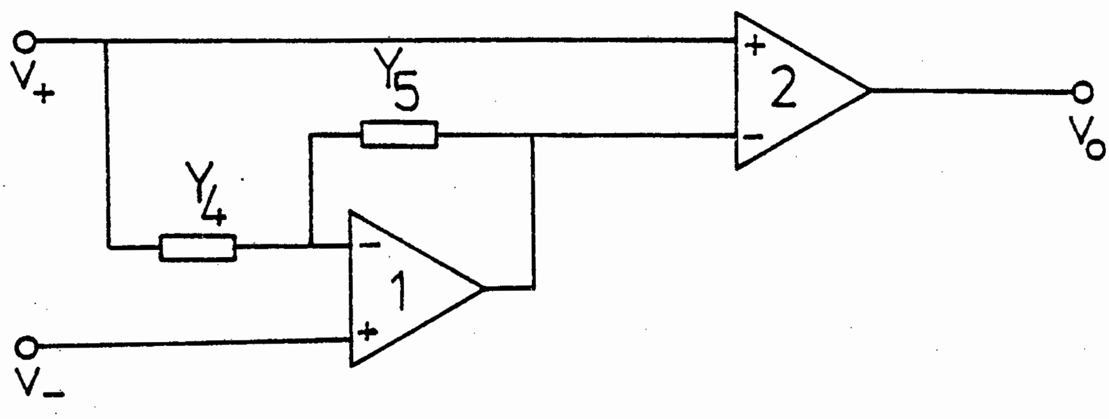
Rationalisation then yields,

$$V_o/V_i = Y_1/(1/K(2Y_1+Y_6)) \qquad \ldots\{9\}.$$

The Composite Operational Amplifier

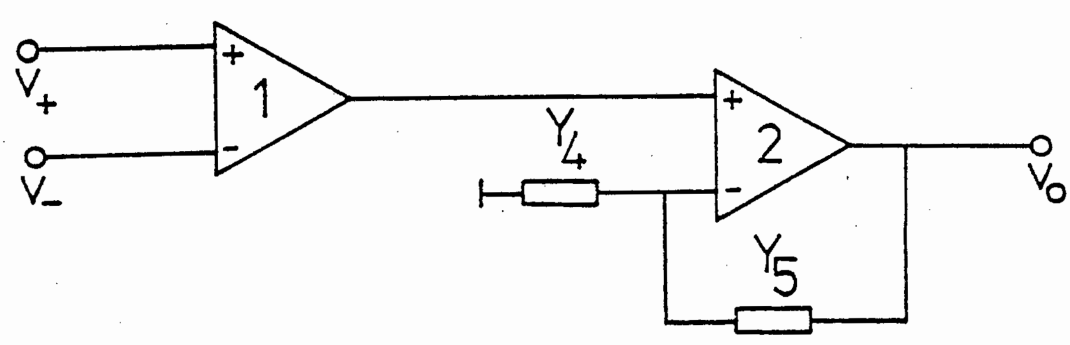
In the paper by Mikhael and Nessim [4] four COAs are discussed. These COAs are depicted below and identified as M1, M2, M3 and M4 referring to the same order which was chosen by Campbell and Stephenson [1].

C20A-1



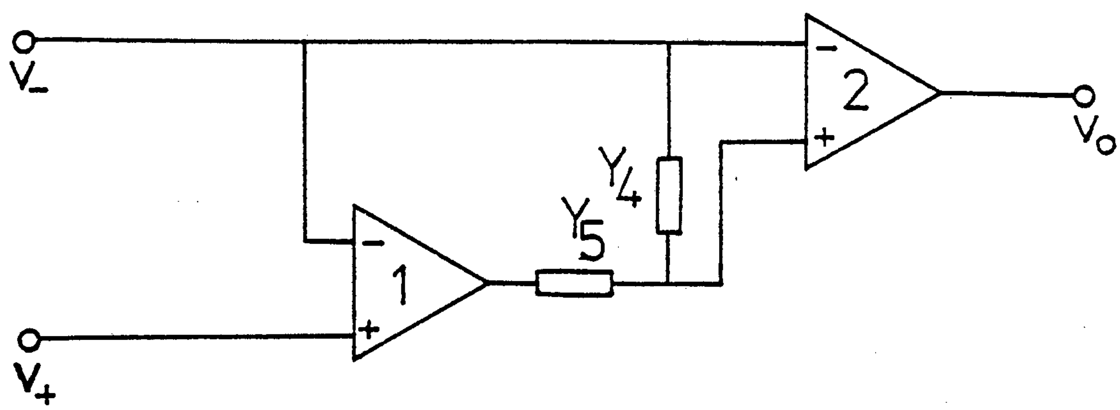
M1

C20A-2

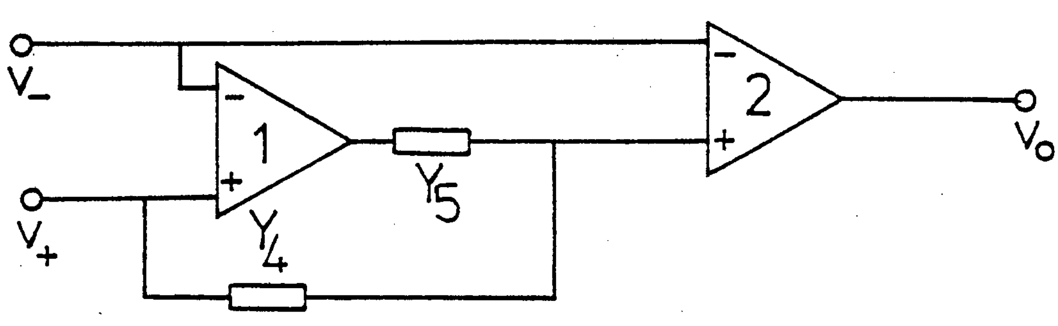


M2

C20A-3



M3



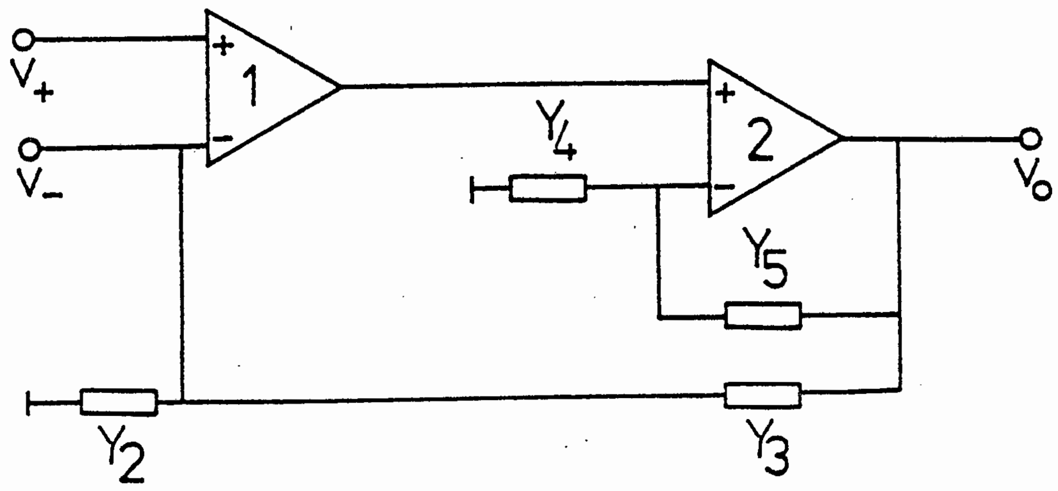
M4

In the above Figures $Y_5 = Y_4/\alpha$.

Campbell and Stephenson [1] studied the M2 and M4 COAs and found that especially the results for the M2 COA correlated poorly with theoretically predicted results. For this reason a closer look at the M2 COA case will be taken here.

The Composite Operational Amplifier Gain Function

Application of constrained nodal admittance matrix analysis [40,41] (Appendix II) allows for the evaluation of the gain function of the (C2OA-2) M2 COA as shown where the gain determining impedance pair Y_2 and Y_3 is included.



The constrained nodal admittance matrix is thus given by,

	1	2	3	4	5
1	0	0	0	0	0
2	0	Y_2+Y_3	$-Y_3$	0	0
3	0	$-Y_3$	$Y_3+Y_5+Y_{33}$	$A_2Y_{33}+Y_5$	$-A_2Y_{33}$
4	0	0	$-Y_5$	Y_4+Y_5	0
5	$-A_1Y_{55}$	A_1Y_{55}	0	0	Y_{55}

Using pivotal condensation techniques [40,41] the above constrained admittance matrix can be reduced.

	1	2	3	4
1	0	0	0	0
2	0	Y_2+Y_3	$-Y_3$	0
3	$-A_1A_2Y_{33}$	$A_1A_2Y_{33}-Y_3$	$Y_3+Y_5+Y_{33}$	$A_2Y_{33}-Y_5$
4	0	0	$-Y_5$	Y_4+Y_5

with the key terms of this matrix derived with certain assumptions made which are detailed as follows;

$$Y_{31} = -A_1A_2Y_{33}Y_{55}/Y_{55}$$

dividing both numerator and denominator by Y_{55} yields,

$$Y_{31} = -A_1A_2Y_{33}, \text{ and}$$

$$Y_{32} = -Y_3 + A_1A_2Y_{33}Y_{55} / Y_{55}.$$

Further division of both numerator and denominator of the second term by Y_{55} then yields,

$$Y_{32} = -Y_3 + A_1A_2Y_{33}.$$

A further reduction step leads to a 3x3 matrix from which

	1	2	3
1	0	0	0
2	0	Y_2+Y_3	$-Y_3$
3	$-A_1A_2Y_{33}$	$A_1A_2Y_{33}-Y_3$	$Y_3+Y_5+Y_{33}+Y_5(A_2Y_{33}-Y_5)/(Y_4+Y_5)$

the voltage transfer function emerges as,

$$V_0/V_I = -Y_{31}/Y_{33}.$$

Here Y_{31} and Y_{33} are the cofactors of the constrained nodal admittance matrix.

As only Y_{31} and Y_{33} terms are required and therefore a partial condensation of the [3x3] matrix will be sufficient, now

$$Y_{31} = -A_1A_2Y_{33}$$

and

$$Y_{33} = Y_3+Y_5+Y_{33}+Y_5(A_2Y_{33}-Y_5)/(Y_4+Y_5)$$
$$+ \; Y_3(A_1A_2Y_{33}-Y_3)/(Y_2+Y_3).$$

Dividing both Y_{31} and Y_{33} through by the Y_{33} term yields

$$Y_{31} = -A_1A_2$$

and

$$Y_{33} = 1 + A_2Y_5 / (Y_4 + Y_5) + A_1A_2Y_3 /$$
$$(Y_2 + Y_3).$$

The gain function is therefore

$$GFM2 = A_1A_2 / (1 + A_2Y_5 / (Y_4 + Y_5) +$$
$$A_1A_2Y_3 / (Y_2 + Y_3)).$$

Substituting for $Y_5/(Y_4+Y_5) = 1/(1+\alpha)$ and $Y_3/(Y_2+Y_3) = 1/K$ yields,

$$GFM2 = A_1A_2 / (1 + A_2/(1+\alpha) + A_1A_2/K) \quad ...\{10\}.$$

Dividing both numerator and denominator by A_1A_2 yields the gain function (GF) for the M2 COA,

$$GFM2 = 1 / (1/(A_1A_2) + 1/(A_1(1+\alpha)) + 1/K).$$

Assuming now $A_1 = A_2 = A$ which means that the two OAs show identical performance and possibly being in the same chip the transfer function reduces to

$$GFM2 = 1 / (1/A^2 + 1/(A(1+\alpha)) + 1/K) \qquad \ldots\{11\}.$$

The Single Pole Model

For the single pole model of the OA the amplitude function is chosen to be of the form

$$A(s) = \omega_t/s.$$

Substituting A(s) into GFM2 the gain function for the M2 COA is obtained. As can be seen the modified transfer function increases by an order of two, one for each amplifier, so that the single pole model is now superimposed on that of the second order network function.

$$GFM2SP = 1/((s/\omega_t)^2 + (s/(\omega_t(1+\alpha)) + 1/K) \quad ...\{12\}.$$

The Two Pole Model

For the two pole model a second order function is expected.

$$A(s) = \omega_t\omega_2/((s+\omega_1)(s+\omega_2)).$$

When including this model in the gain function for the M2 COA the transfer function for the COA alone is now a fourth order function.

$$GFM2DP = 1 / \{(s^4 + s^3 2(\omega_1+\omega_2) + \quad ...\{13\}$$
$$s^2(2\omega_1\omega_2+(\omega_1+\omega_2)^2) + s2\omega_1\omega_2(\omega_1+_2) +$$
$$\omega_1^2\omega_2^2)/(\omega_t^2\omega_2^2) + (s^2 + s(\omega_1+\omega_2) +$$
$$\omega_1\omega_2)/(\omega_t\omega_2(1+\alpha)) + 1/K\}.$$

The Three Pole Model

Now for the three pole model the gain function is of the third order.

$$A(s) = \omega_t\omega_2\omega_3/((s+\omega_1)(s+\omega_2)(s+\omega_3)).$$

Thus for the M2 COA the gain function yields a sixth order expression.

$$GFM2TP = 1 / \{(s^6 + s^5(\omega_1+\omega_2+\omega_3) + \qquad \ldots\{14\}$$
$$s^4(2(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) + (\omega_1+\omega_2+\omega_3)^2) +$$
$$s^3 2((\omega_1+\omega_2+\omega_3)(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) +$$
$$\omega_1\omega_2\omega_3) + s^2(2(\omega_1\omega_2\omega_3(\omega_1+\omega_2+\omega_3) +$$
$$(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)^2) +$$
$$s2\omega_1\omega_2\omega_3(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) +$$
$$\omega_1^2\omega_2^2\omega_3^2)/(\omega_t^2\omega_2^2\omega_3^2) + (s^3 +$$
$$s^2(\omega_1+\omega_2+\omega_3) + s(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) +$$
$$\omega_1\omega_2\omega_3) / ((1+\alpha)\omega_t\omega_2\omega_3) + 1/K\}.$$

The three gain function expressions are derived for COAs composed of two OAs may now be substituted into the second order transfer function for the Sallen and Key bandpass filter (SKB) to give the complete transfer function for the network.

When studying the stability of filter circuits using COAs it is useful to consider the application of different stability criteria. It is therefore necessary to have available both the closed loop (CL) and open loop (OL) transfer function expressions.

The Closed Loop Transfer Function

The Single Pole Model

Substituting GFM2SP {12} into the SKB closed loop transfer function {7},

$$V_o/V_i = Y_1Y_6 \;/\; (1/K(Y_1^2 + 3Y_1Y_6 + Y_6^2) - Y_1Y_6),$$

where GFM2SP = K , $Y_1 = 1/R_1$ and $Y_6 = sC_1$ of the SKB yields the SKB closed loop transfer function for M2.

$$SKBCLTFM2SP = s/\{s^4(R_1C_1/\omega_t^2) + s^3(3/\omega_t^2 + R_1C_1/(\omega_t(1+\alpha)) + s^2(1/(\omega_tR_1C_1) + 3/(\omega_t(1+\alpha)) + R_1C_1/K) + s(1/(\omega_t(1+\alpha)R_1C_1) + 3/K - 1) + 1/(KR_1C_1)\}. \quad ...\{15\}$$

The Two Pole Model

Similarly by substituting GFM2DP = K {13}, $Y_1 = 1/R_1$ and $Y_6 = sC_1$ into expression {7}, the SKB closed loop transfer function for M2 is obtained.

$$SKBCLTFM2DP \;=\; s/\{s^6(R_1C_1/(\omega_t^2\omega_2^2)) \qquad \ldots\{16\}$$

$$+ \; s^5(3/(\omega_t^2\omega_2^2) \; +$$

$$2(\omega_1+\omega_2)R_1C_1/(\omega_t^2\omega_2^2)) \; +$$

$$s^4(1/(\omega_t^2\omega_2^2R_1C_1) \; + \; 6(\omega_1+\omega_2)/(\omega_t^2\omega_2^2) \; +$$

$$(2\omega_1\omega_2+(\omega_1+\omega_2)^2)R_1C_1/(\omega_t^2\omega_2^2) \; +$$

$$R_1C_1/((1+\alpha)\omega_t\omega_2))) \; +$$

$$s^3(2(\omega_1+\omega_2)/(\omega_t^2\omega_2^2R_1C_1) \; + \; 3(2\omega_1\omega_2 \; +$$

$$(\omega_1+\omega_2)^2)/(\omega_t^2\omega_2^2) \; +$$

$$2\omega_1\omega_2(\omega_1+\omega_2)R_1C_1/(\omega_t^2\omega_2^2) \; +$$

$$3/((1+\alpha)\omega_t\omega_2) \; + \; (\omega_1+\omega_2)R_1C_1/((1+\alpha)\omega_t\omega_2))$$

$$+ \; s^2((2\omega_1\omega_2+(\omega_1+\omega_2)^2)/(\omega_t^2\omega_2^2R_1C_1) \; +$$

$$6\omega_1\omega_2(\omega_1+\omega_2)/(\omega_t^2\omega_2^2) \; +$$

$$\omega_1^2\omega_2^2R_1C_1/(\omega_t^2\omega_2^2) \; +$$

$$1/((1+\alpha)\omega_t\omega_2R_1C_1) \; + \; 3(\omega_1+\omega_2)/((1+\alpha)\omega_t\omega_2)$$

$$+ \; \omega_1\omega_2R_1C_1/((1+\alpha)\omega_t\omega_2) \; + \; R_1C_1/K) \; +$$

$$s(2\omega_1\omega_2(\omega_1+\omega_2)/(\omega_t^2\omega_2^2R_1C_1) \; +$$

$$3\omega_1^2\omega_2^2/(\omega_t^2\omega_2^2) \; +$$

$$(\omega_1+\omega_2)/((1+\alpha)\omega_t\omega_2R_1C_1) \; +$$

$$3\omega_1\omega_2/((1+\alpha)\omega_t\omega_2) \; + \; 3/K \; - \; 1) \; +$$

$$\omega_1^2\omega_2^2/(\omega_t^2\omega_2^2R_1C_1) \; +$$

$$\omega_1\omega_2/((1+\alpha)\omega_t\omega_2R_1C_1) \; + \; 1/(KR_1C_1)\}.$$

The Three Pole Model

Incorporating the model requires substituting of GFM2TP = K {14}, $Y_1 = 1/R_1$ and $Y_6 = sC_1$ into the SKB closed loop transfer function {7}. The expression is now an eighth order function which obviously becomes difficult to handle.

$$SKBCLTFM2TP = s/\{s^8(R_1C_1/(\omega_t^2\omega_2^2\omega_3^2)) \quad \ldots\{17\}$$
$$+ \; s^7(3/(\omega_t^2\omega_2^2\omega_3^2) \; +$$
$$2(\omega_1+\omega_2+\omega_3)R_1C_1/(\omega_t^2\omega_2^2\omega_3^2)) \; +$$
$$s^6(1/(\omega_t^2\omega_2^2\omega_3^2R_1C_1) \; +$$
$$6(\omega_1+\omega_2+\omega_3)/(\omega_t^2\omega_2^2\omega_3^2) \; +$$
$$(2(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) \; +$$
$$(\omega_1+\omega_2+\omega_3)^2)R_1C_1/(\omega_t^2\omega_2^2\omega_3^2)) \; +$$
$$s^5(2(\omega_1+\omega_2+\omega_3)/(\omega_t^2\omega_2^2\omega_3^2R_1C_1) \; +$$
$$3(2(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) \; +$$
$$(\omega_1+\omega_2+\omega_3)^2)/(\omega_t^2\omega_2^2\omega_3^2) \; +$$
$$2((\omega_1+\omega_2+\omega_3)(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) \; +$$
$$\omega_1\omega_2\omega_3)R_1C_1/(\omega_t^2\omega_2^2\omega_3^2) \; +$$
$$R_1C_1/((1+\alpha)\omega_t\omega_2\omega_3)) \; +$$
$$s^4((2(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) \; +$$
$$(\omega_1+\omega_2+\omega_3)^2)/(\omega_t^2\omega_2^2\omega_3^2R_1C_1) \; +$$
$$6((\omega_1+\omega_2+\omega_3)(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) \; +$$
$$\omega_1\omega_2\omega_3)/(\omega_t^2\omega_2^2\omega_3^2) \; +$$
$$(2\omega_1\omega_2\omega_3(\omega_1+\omega_2+\omega_3) \; +$$
$$(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)^2)R_1C_1/(\omega_t^2\omega_2^2\omega_3^2)$$

$$+ \ 3/((1+\alpha)\omega_t\omega_2\omega_3) \ +$$

$$(\omega_1+\omega_2+\omega_3)R_1C_1/((1+\alpha)\omega_t\omega_2\omega_3)) \ +$$

$$s^3(2((\omega_1+\omega_2+\omega_3)(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) \ +$$

$$\omega_1\omega_2\omega_3)/(\omega_t^2\omega_2^2\omega_3^2R_1C_1) \ +$$

$$(2\omega_1\omega_2\omega_3(\omega_1+\omega_2+\omega_3) \ +$$

$$(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)^2)3/(\omega_t^2\omega_2^2\omega_3^2) \ +$$

$$2\omega_1\omega_2\omega_3(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)R_1C_1$$

$$/(\omega_t^2\omega_2^2\omega_3^2) \ + \ 1/((1+\alpha)\omega_t\omega_2\omega_3R_1C_1) \ +$$

$$3(\omega_1+\omega_2+\omega_3)/((1+\alpha)\omega_t\omega_2\omega_3) \ +$$

$$(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)R_1C_1$$

$$/((1+\alpha)\omega_t\omega_2\omega_3)) \ + \ s^2((2\omega_1\omega_2\omega_3(\omega_1+\omega_2+\omega_3)$$

$$+ \ (\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)^2)$$

$$/(\omega_t^2\omega_2^2\omega_3^2R_1C_1) \ +$$

$$6\omega_1\omega_2\omega_3(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)$$

$$/(\omega_t^2\omega_2^2\omega_3^2) \ +$$

$$\omega_1^2\omega_2^2\omega_3^2R_1C_1/(\omega_t^2\omega_2^2\omega_3^2) \ +$$

$$(\omega_1+\omega_2+\omega_3)/((1+\alpha)\omega_t\omega_2\omega_3R_1C_1) \ +$$

$$3(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)/((1+\alpha)\omega_t\omega_2\omega_3) \ +$$

$$\omega_1\omega_2\omega_3R_1C_1/((1+\alpha)\omega_t\omega_2\omega_3) \ + \ R_1C_1/K) \ +$$

$$s(2\omega_1\omega_2\omega_3(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)$$

$$/(\omega_t^2\omega_2^2\omega_3^2R_1C_1) \ +$$

$$\omega_1^2\omega_2^2\omega_3^2/(\omega_t^2\omega_2^2\omega_3^2) \ +$$

$$(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)/((1+\alpha)\omega_t\omega_2\omega_3R_1C_1)$$

$$+ \ 3\omega_1\omega_2\omega_3/((1+\alpha)\omega_t\omega_2\omega_3) \ + \ 3/K \ - \ 1) \ +$$

$$\omega_1^2\omega_2^2\omega_3^2/(\omega_t^2\omega_2^2\omega_3^2R_1C_1) \ +$$

$$\omega_1\omega_2\omega_3/((1+\alpha)\omega_t\omega_2\omega_3R_1C_1) \ +$$

$$1/(KR_1C_1)\}.$$

The Open Loop Transfer Function

The Single Pole Model

Substituting the gain function GFM2SP {12} into the SKB open loop transfer function {9},

$$V_0/V_i = Y_1/(1/K(2Y_1+Y_6)),$$

where GFM2SP = K , $Y_1 = 1/R_1$ and $Y_6 = sC_1$ yields the SKB open loop transfer function for M2 which is a third order function.

$$SKBOLTFM2SP = 1/\{s^3(R_1C_1/(\omega_t^2)) + s^2(2/(\omega_t^2) + R_1C_1/((1+\alpha)\omega_t)) + s(2/((1+\alpha)\omega_t) + R_1C_1/K) + 2/K\}. \quad \ldots\{18\}$$

The Two Pole Model

Again substituting GFM2DP = K {13}, $Y_1 = 1/R_1$ and $Y_6 = sC_1$ into expression {9} yields the SKB open loop transfer function for M2, which is seen to be a fifth order function.

$$\mathrm{SKBOLTFM2TP} \;=\; 1/\{s^7(R_1C_1/(\omega_t^2\omega_2^2\omega_3^2)) \quad \ldots\{20\}$$

$$+ \;\; s^6(2/(\omega_t^2\omega_2^2\omega_3^2) \;\; +$$

$$2(\omega_1+\omega_2+\omega_3)R_1C_1/(\omega_t^2\omega_2^2\omega_3^2)) \;\; +$$

$$s^5(4(\omega_1+\omega_2+\omega_3)/(\omega_t^2\omega_2^2\omega_3^2) \;\; +$$

$$(2(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) \;\; +$$

$$(\omega_1+\omega_2+\omega_3)^2)R_1C_1/(\omega_t^2\omega_2^2\omega_3^2)) \;\; +$$

$$s^4(2(2(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) \;\; +$$

$$(\omega_1+\omega_2+\omega_3)^2)/(\omega_t^2\omega_2^2\omega_3^2) \;\; +$$

$$2((\omega_1+\omega_2+\omega_3)(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) \;\; +$$

$$\omega_1\omega_2\omega_3)R_1C_1/(\omega_t^2\omega_2^2\omega_3^2) \;\; +$$

$$R_1C_1/((1+\alpha)\omega_t\omega_2\omega_3)) \;\; +$$

$$s^3(4((\omega_1+\omega_2+\omega_3)(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) \;\; +$$

$$\omega_1\omega_2\omega_3)/(\omega_t^2\omega_2^2\omega_3^2) \;\; +$$

$$(2\omega_1\omega_2\omega_3(\omega_1+\omega_2+\omega_3) \;\; +$$

$$(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)^2)R_1C_1/(\omega_t^2\omega_2^2\omega_3^2)$$

$$+ \;\; 2/((1+\alpha)\omega_t\omega_2\omega_3) \;\; +$$

$$R_1C_1(\omega_1+\omega_2+\omega_3)/((1+\alpha)\omega_t\omega_2\omega_3)) \;\; +$$

$$s^2(2(2\omega_1\omega_2\omega_3(\omega_1+\omega_2+\omega_3) \;\; +$$

$$(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)^2)/(\omega_t^2\omega_2^2\omega_3^2) \;\; +$$

$$2\omega_1\omega_2\omega_3(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)R_1C_1$$

$$/(\omega_t^2\omega_2^2\omega_3^2) \;\; +$$

$$2(\omega_1+\omega_2+\omega_3)/((1+\alpha)\omega_t\omega_2\omega_3) \;\; +$$

$$(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)R_1C_1/((1+\alpha)\omega_t\omega_2\omega_3))$$

$$+ \;\; s(4\omega_1\omega_2\omega_3(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)$$

$$/(\omega_t^2\omega_2^2\omega_3^2) \;\; +$$

$$\omega_1^2\omega_2^2\omega_3^2R_1C_1/(\omega_t^2\omega_2^2\omega_3^2) \;\; +$$

$$2(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)/((1+\alpha)\omega_t\omega_2\omega_3) \;\; +$$

$\omega_1\omega_2\omega_3R_1C_1/((1+\alpha)\omega_t\omega_2\omega_3) + R_1C_1/K) +$

$2\omega_1^2\omega_2^2\omega_3^2/(\omega_t^2\omega_2^2\omega_3^2) +$

$2\omega_1\omega_2\omega_3/((1+\alpha)\omega_t\omega_2\omega_3) + 2/K\}.$

The above functions will later be employed to study the question of stability in COAs.

CHAPTER 4

COMPOSITE OPERATIONAL AMPLIFIER STABILITY INVESTIGATIONS

One of the classic methods, while being relatively simple, is the Routh Criterion (Appendix III). For its successful application certain conditions must be met [38].

Using this analysis it is possible to establish a point at which the system will become unstable. However this method does not give an indication of the margin of stability.

Routh Criterion Analysis

For the M2 composite operational amplifier (COA) used in a Sallen and Key bandpass network (SKB) the characteristic equation derived from SKBCLTFM2SP {15} is

$$s^4(R_1C_1/\omega_t^2) + s^3(3/\omega_t^2 + R_1C_1/(\omega_t(1+\alpha))) + s^2(1/(\omega_t R_1C_1) + 3/(\omega_t(1+\alpha)) + R_1C_1/K) + s(1/(\omega_t(1+\alpha)R_1C_1) + 3/K - 1) + 1/(KR_1C_1).$$

The Routh Criterion matrix here is

$$
\left|\begin{array}{lll}
a_4 & a_2 & a_0 \\
a_3 & a_1 & \\
b_1 & b_2 & \\
c_1 & & \\
d_1 & &
\end{array}\right.
$$

Where $a_4 = R_1C_1/\omega_t^2$

$a_3 = 3/\omega_t^2 + R_1C_1/(\omega_t(1+\alpha))$

$a_2 = 1/(\omega_t R_1C_1) + 3/(\omega_t(1+\alpha)) + R_1C_1/K$

$a_1 = 1/(\omega_t(1+\alpha)R_1C_1) + 3/K - 1$

$a_0 = 1/(KR_1C_1)$

$b_1 = (a_3a_2 - a_4a_1)/a_3$

$b_2 = a_0$

$c_1 = (b_1a_1 - a_3b_2)/b_1$

$d_1 = b_2$.

Using the component values shown in Table 1, Chapter 5, the above equations are evaluated for $\alpha = 1$. This computation results in a set of values for the coefficients of the Routh Criterion Matrix.

a_4 = 5,7E-19

a_3 = 2,5E-12

a_2 = 2,7E-3

a_1 = 2,2E-1

a_o = 9,5E3

b_1 = 2,7E-3

b_2 = 9,5E3

c_1 = 2,2E-1

d_1 = 9,5E3

Where the numbers are represented in scientific notation. Arranging these coefficients into the form shown below yields the Routh Criterion Matrix depicted in Table 2 of Chapter 5 for $\alpha = 1$.

5,7E-19	2,7E-3	9,5E3
2,5E-12	2,2E-1	
2,7E-3	9,5E3	
2,2E-1		
9,5E3		

Inspection of the coefficients a_1, a_2 and a_3 on page 66 reveals that the network is stable for $(\alpha+1)>0$ and for this the two conditions for stability are satisfied (see Appendix III). However it is necessary to exclude the range $-1<\alpha<0$ from consideration as this would yield a network containing negative components. Now α represents a resistor ratio which directly relates to the bandwidth improvement factor of the COA. As such improvement would mean that $\alpha \geqslant 1$ and consequently values of $0<\alpha \leqslant 1$ are undesireable. To demonstrate the principle involved a value of $\alpha=1$ has been chosen for the case study.

The characteristic equation for SKBCLTFM2DP {16} is,

$$s^6(R_1C_1/(\omega_t^2\omega_2^2)) + s^5(3/(\omega_t^2\omega_2^2) +$$
$$2(\omega_1+\omega_2)R_1C_1/(\omega_t^2\omega_2^2)) +$$
$$s^4(1/(\omega_t^2\omega_2^2R_1C_1) + 6(\omega_1+\omega_2)/(\omega_t^2\omega_2^2) +$$
$$(2\omega_1\omega_2+(\omega_1+\omega_2)^2)R_1C_1/(\omega_t^2\omega_2^2) +$$
$$R_1C_1/((1+\alpha)\omega_t\omega_2))) +$$
$$s^3(2(\omega_1+\omega_2)/(\omega_t^2\omega_2^2R_1C_1) + 3(2\omega_1\omega_2 +$$
$$(\omega_1+\omega_2)^2)/(\omega_t^2\omega_2^2) +$$
$$2\omega_1\omega_2(\omega_1+\omega_2)R_1C_1/(\omega_t^2\omega_2^2) +$$
$$3/((1+\alpha)\omega_t\omega_2) + (\omega_1+\omega_2)R_1C_1/((1+\alpha)\omega_t\omega_2))$$
$$+ s^2((2\omega_1\omega_2+(\omega_1+\omega_2)^2)/(\omega_t^2\omega_2^2R_1C_1) +$$
$$6\omega_1\omega_2(\omega_1+\omega_2)/(\omega_t^2\omega_2^2) +$$
$$\omega_1^2\omega_2^2R_1C_1/(\omega_t^2\omega_2^2) +$$
$$1/((1+\alpha)\omega_t\omega_2R_1C_1) + 3(\omega_1+\omega_2)/((1+\alpha)\omega_t\omega_2)$$
$$+ \omega_1\omega_2R_1C_1/((1+\alpha)\omega_t\omega_2) + R_1C_1/K) +$$
$$s(2\omega_1\omega_2(\omega_1+\omega_2)/(\omega_t^2\omega_2^2R_1C_1) +$$
$$3\omega_1^2\omega_2^2/(\omega_t^2\omega_2^2) +$$
$$(\omega_1+\omega_2)/((1+\alpha)\omega_t\omega_2R_1C_1) +$$
$$3\omega_1\omega_2/((1+\alpha)\omega_t\omega_2) + 3/K - 1) +$$
$$\omega_1^2\omega_2^2/(\omega_t^2\omega_2^2R_1C_1) +$$
$$\omega_1\omega_2/((1+\alpha)\omega_t\omega_2R_1C_1) + 1/(KR_1C_1).$$

The Routh Criterion matrix for the two pole model is

$$\begin{array}{|llll} a_6 & a_4 & a_2 & a_o \\ a_5 & a_3 & a_1 & \\ b_1 & b_2 & b_3 & \\ c_1 & c_2 & & \\ d_1 & d_2 & & \\ e_1 & & & \\ f_1 & & & \end{array}$$

Where $a_6 = R_1C_1/(\omega_t^2\omega_2^2)$

$a_5 = 3/(\omega_t^2\omega_2^2) + 2(\omega_1+\omega_2)R_1C_1/(\omega_t^2\omega_2^2)$

$a_4 = 1/(\omega_t^2\omega_2^2R_1C_1) + 6(\omega_1+\omega_2)/(\omega_t^2\omega_2^2)$

$+ (2\omega_1\omega_2+(\omega_1+\omega_2)^2)R_1C_1/(\omega_t^2\omega_2^2) +$

$R_1C_1/((1+\alpha)\omega_t\omega_2)$

$a_3 = 2(\omega_1+\omega_2)/(\omega_t^2\omega_2^2R_1C_1) + 3(2\omega_1\omega_2 +$

$(\omega_1+\omega_2)^2)/(\omega_t^2\omega_2^2) +$

$2\omega_1\omega_2(\omega_1+\omega_2)R_1C_1/(\omega_t^2\omega_2^2) +$

$3/((1+\alpha)\omega_t\omega_2) + (\omega_1+\omega_2)R_1C_1/((1+\alpha)\omega_t\omega_2)$

$$a_2 = (2\omega_1\omega_2+(\omega_1+\omega_2)^2)/(\omega_t{}^2\omega_2{}^2R_1C_1) +$$
$$6\omega_1\omega_2(\omega_1+\omega_2)/(\omega_t{}^2\omega_2{}^2) +$$
$$\omega_1{}^2\omega_2{}^2R_1C_1/(\omega_t{}^2\omega_2{}^2) +$$
$$1/((1+\alpha)\omega_t\omega_2R_1C_1) + 3(\omega_1+\omega_2)/((1+\alpha)\omega_t\omega_2)$$
$$+ \omega_1\omega_2R_1C_1/((1+\alpha)\omega_t\omega_2) + R_1C_1/K$$
$$a_1 = 2\omega_1\omega_2(\omega_1+\omega_2)/(\omega_t{}^2\omega_2{}^2R_1C_1) +$$
$$3\omega_1{}^2\omega_2{}^2/(\omega_t{}^2\omega_2{}^2) +$$
$$(\omega_1+\omega_2)/((1+\alpha)\omega_t\omega_2R_1C_1) +$$
$$3\omega_1\omega_2/((1+\alpha)\omega_t\omega_2) + 3/K - 1$$
$$a_0 = \omega_1{}^2\omega_2{}^2/(\omega_t{}^2\omega_2{}^2R_1C_1) +$$
$$\omega_1\omega_2/((1+\alpha)\omega_t\omega_2R_1C_1) + 1/(KR_1C_1)$$

$$b_1 = (a_5a_4-a_6a_3)/a_5$$
$$b_2 = (a_5a_2-a_6a_1)/a_5$$
$$b_3 = a_0$$
$$c_1 = (b_1a_3-a_5b_2)/b_1$$
$$c_2 = (b_1a_1-a_5b_3)/b_1$$
$$d_1 = (c_1b_2-b_1c_2)/c_1$$
$$d_2 = b_3$$
$$e_1 = (d_1c_2-d_2c_1)/d_1$$
$$f_1 = d_2.$$

From the inspection of the coefficients of the matrix it is evident that the system is not stable for all α.

The characteristic equation for SKCLTFM2TP {17} for the three pole model is

$s^8(R_1C_1/(\omega_t^2\omega_2^2\omega_3^2))$ +

$s^7(3/(\omega_t^2\omega_2^2\omega_3^2)$ +

$2(\omega_1+\omega_2+\omega_3)R_1C_1/(\omega_t^2\omega_2^2\omega_3^2))$ +

$s^6(1/(\omega_t^2\omega_2^2\omega_3^2R_1C_1)$ +

$6(\omega_1+\omega_2+\omega_3)/(\omega_t^2\omega_2^2\omega_3^2)$ +

$(2(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)$ +

$(\omega_1+\omega_2+\omega_3)^2)R_1C_1/(\omega_t^2\omega_2^2\omega_3^2))$ +

$s^5(2(\omega_1+\omega_2+\omega_3)/(\omega_t^2\omega_2^2\omega_3^2R_1C_1)$ +

$3(2(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)$ +

$(\omega_1+\omega_2+\omega_3)^2)/(\omega_t^2\omega_2^2\omega_3^2)$ +

$2((\omega_1+\omega_2+\omega_3)(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)$ +

$\omega_t\omega_2\omega_3)R_1C_1/(\omega_t^2\omega_2^2\omega_3^2)$ +

$R_1C_1/((1+\alpha)\omega_t\omega_2\omega_3))$ +

$s^4((2(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)$ +

$(\omega_1+\omega_2+\omega_3)^2)/(\omega_t^2\omega_2^2\omega_3^2R_1C_1)$ +

$6((\omega_1+\omega_2+\omega_3)(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)$ +

$\omega_1\omega_2\omega_3)/(\omega_t^2\omega_2^2\omega_3^2)$ +

$(2\omega_1\omega_2\omega_3(\omega_1+\omega_2+\omega_3)$ +

$(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)^2)R_1C_1/(\omega_t^2\omega_2^2\omega_3^2)$

$+\ 3/((1+\alpha)\omega_t\omega_2\omega_3)$ +

$(\omega_1+\omega_2+\omega_3)R_1C_1/((1+\alpha)\omega_t\omega_2\omega_3))$ +

$s^3(2((\omega_1+\omega_2+\omega_3)(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)$ +

$\omega_1\omega_2\omega_3)/(\omega_t^2\omega_2^2\omega_3^2R_1C_1)$ +

$(2\omega_1\omega_2\omega_3(\omega_1+\omega_2+\omega_3)$ +

$$(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)R_1C_1/((1+\alpha)\omega_t\omega_2\omega_3))$$

$$+ \quad 2\omega_1^2\omega_2^2\omega_3^2/(\omega_t^2\omega_2^2\omega_3^2) \quad +$$

$$2\omega_1\omega_2\omega_3/((1+\alpha)\omega_t\omega_2\omega_3) \quad + \quad 2/K) \quad +$$

$$J(-\omega^7(R_1C_1/(\omega_t^2\omega_2^2\omega_3^2)) \quad +$$

$$\omega^5(4(\omega_1+\omega_2+\omega_3)/(\omega_t^2\omega_2^2\omega_3^2) \quad +$$

$$(2(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) \quad +$$

$$(\omega_1+\omega_2+\omega_3)^2)R_1C_1/(\omega_t^2\omega_2^2\omega_3^2)) \quad -$$

$$\omega^3(4((\omega_1+\omega_2+\omega_3)(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) \quad +$$

$$\omega_1\omega_2\omega_3)/(\omega_t^2\omega_2^2\omega_3^2) \quad +$$

$$(2\omega_1\omega_2\omega_3(\omega_1+\omega_2+\omega_3) \quad +$$

$$(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)^2)R_1C_1/(\omega_t^2\omega_2^2\omega_3^2)$$

$$+ \quad 2/((1+\alpha)\omega_t\omega_2\omega_3)) \quad +$$

$$R_1C_1(\omega_1+\omega_2+\omega_3)/((1+\alpha)\omega_t\omega_2\omega_3)) \quad +$$

$$\omega(4\omega_1\omega_2\omega_3(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)$$

$$/(\omega_t^2\omega_2^2\omega_3^2) \quad +$$

$$\omega_1^2\omega_2^2\omega_3^2R_1C_1/(\omega_t^2\omega_2^2\omega_3^2) \quad +$$

$$2(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)/((1+\alpha)\omega_t\omega_2\omega_3) \quad +$$

$$\omega_1\omega_2\omega_3R_1C_1/((1+\alpha)\omega_t\omega_2\omega_3) \quad + \quad R_1C_1/K)\}.$$

Also here the Nyquist Diagram is obtained so that the stability behaviour can be determined.

All results from these stability evaluations are tabulated and discussed in the next chapter.

CHAPTER 5

STABILITY OF COMPOSITE OPERATIONAL AMPLIFIERS

In a publication by Mikhael and Nessim [4] composite operational amplifiers (COA) made up of three or four conventional operational amplifiers (OA) were presented. In a publication by Natarajan and Bhattacharyya [18] reference is made to Natarajan's Ph.D. thesis where a study of relative stability and expected bandwidth improvement was made which revealed that it was not worth going beyond the three OA COA.

Although techniques presented here are applicable to COAs containing three and four OAs it was found that the mathematical complexity did not warrant the analysis of such COAs.

To be able to compare stability results the component values derived by Campbell and Stephenson [1] are also used for the circuits under investigation in this dissertation. A summary is presented in Table 1.

Q	f_o KHz	R_2 KΩ	R_3 KΩ	C_1 nF	K
1,835	3,717	4,66	6,78	4,32	2,455
1,835	11,044	4,66	6,78	1,454	2,455
8,3	3,717	4,98	9,36	4,32	2,879
8,3	11,044	4,98	9,36	1,454	2,879

Table 1

where $K = (R_3/R_2 + 1)$, $Q = 1/(3 - K)$, $R_1 = 9{,}915$ kΩ , $R_4 = 10$ kΩ and $R_5 = \alpha 10$ kΩ.

It is important to note that the following assumptions have been made ;

(1) $\omega_2 = 2\omega_t$;

(2) $\omega_3 = 4\omega_t$; and

(3) $A_1 = A_2 = A$. (valid for equivalent and matched amplifiers)

These assumptions are based on preliminary experimental work carried out on COAs by the author. Operational amplifier data sheets do not usually provide second and higher pole frequencies and these must therefore be estimated or measured. Campbell and Stephenson [1] suggest ω_2 to be in the range $(2$ to $3)\omega_t$. It was found that for the approximations, chosen as above, good agreement with experimental results was obtained.

Assuming that the OAs are well matched the M1 and M2 COAs are in fact seen to be identical. For this reason the derivation and results associated with the M1 case have not been presented and it is understood that the results relating to the M2 apply equally to the M1 COA.

The various transfer functions for the M3 and M4 COAs are presented in Appendix IV and are derived in the same way as those of the M2 COA.

The tabulated results which follow were obtained using a spreadsheet program on an IBM-compatible PC with a numerical data processor installed. The spreadsheet's number crunching and graphics capabilities makes it possible to manipulate the various component values as well as α. This way it is possible to determine the boundary value for α for which instability occurs in the network.

The results shown in Table 2 are for the M2 COA, using a single pole OA model and applying the Routh Criterion analysis method shown in Chapter 4 page 65 - 67.

The value of α is varied between 1 and 1000000 for this computation. The upper limit has been taken as 1000000 due to the undesirable component spread, for larger values of α, when use is made of thin-film realisations.

Q = 1,835	f_o = 3,717 kHz	K = 2,455

α	Routh Criterion Matrix		
1	5,7E-19	2,7E-3	9,5E3
	2,5E-12	2,2E-1	
	2,7E-3	9,5E3	
	2,2E-1		
	9,5E3		
10	5,7E-19	2,7E-3	9,5E3
	4,9E-13	2,2E-1	
	2,7E-3	9,5E3	
	2,2E-1		
	9,5E3		
100	5,7E-19	2,7E-3	9,5E3
	8,9E-14	2,2E-1	
	2,7E-3	9,5E3	
	2,2E-1		
	9,5E3		
10000	5,7E-19	2,7E-3	9,5E3
	4,1E-14	2,2E-1	
	2,7E-3	9,5E3	
	2,2E-1		
	9,5E3		
100000	5,7E-19	2,7E-3	9,5E3
	4,0E-14	2,2E-1	
	2,7E-3	9,5E3	
	2,2E-1		
	9,5E3		
1000000	5,7E-19	2,7E-3	9,5E3
	4,0E-14	2,2E-1	
	2,7E-3	9,5E3	
	2,2E-1		
	9,5E3		

Table 2

As can be seen, for the chosen values of α, the network is stable.

The Routh Criterion results obtained for the M2 COA for a two pole model are given in Table 3.

Here the computations have been done for three values of α which bracket the marginally stable circuit.

Q = 1,835	f_o = 3,717 kHz	K = 2,455		
α	Routh Criterion Matrix			
1,7	1,9E-33	6,8E-19	1,7E-5	9,5E3
	6,6E-26	1,9E-12	2,2E-1	
	6,3E-19	1,7E-5	9,5E3	
	2,9E-14	2,2E-1		
	1,2E-5	9,5E3		
	2,2E-1			
	9,5E3			
1,72865	1,9E-33	6,8E-19	1,7E-5	9,5E3
	6,6E-26	1,9E-12	2,2E-1	
	6,3E-19	1,7E-5	9,5E3	
	8,0E-15	2,2E-1		
	4,6E-8	9,5E3		
	2,2E-1			
	9,5E3			
1,8	1,9E-33	6,8E-19	1,7E-5	9,5E3
	6,6E-26	1,9E-12	2,2E-1	
	6,3E-19	1,7E-5	9,5E3	
	-4,2E-14	2,2E-1		
	2,0E-5	9,5E3		
	2,2E-1			
	9,5E3			

<u>Table 3</u>

From Table 3 it can be seen that the transition from a stable to an unstable circuit occurs for a small change in α. Thus for a stable circuit the choice of α must be made such that a reasonable margin of error is allowed for.

$(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)^2)3/(\omega_t^2\omega_2^2\omega_3^2)\ +$

$2\omega_1\omega_2\omega_3(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)R_1C_1$

$/(\omega_t^2\omega_2^2\omega_3^2)\ +\ 1/((1+\alpha)\omega_t\omega_2\omega_3R_1C_1)\ +$

$3(\omega_1+\omega_2+\omega_3)/((1+\alpha)\omega_t\omega_2\omega_3)\ +$

$(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)R_1C_1$

$/((1+\alpha)\omega_t\omega_2\omega_3))\ +\ s^2((2\omega_1\omega_2\omega_3(\omega_1+\omega_2+\omega_3)$

$+\ (\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)^2)$

$/(\omega_t^2\omega_2^2\omega_3^2R_1C_1)\ +$

$6\omega_1\omega_2\omega_3(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)$

$/(\omega_t^2\omega_2^2\omega_3^2)\ +$

$\omega_1^2\omega_2^2\omega_3^2R_1C_1/(\omega_t^2\omega_2^2\omega_3^2)\ +$

$(\omega_1+\omega_2+\omega_3)/((1+\alpha)\omega_t\omega_2\omega_3R_1C_1)\ +$

$3(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)/((1+\alpha)\omega_t\omega_2\omega_3)\ +$

$\omega_1\omega_2\omega_3R_1C_1/((1+\alpha)\omega_t\omega_2\omega_3)\ +\ R_1C_1/K)\ +$

$s(2\omega_1\omega_2\omega_3(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)$

$/(\omega_t^2\omega_2^2\omega_3^2R_1C_1)\ +$

$\omega_1^2\omega_2^2\omega_3^2/(\omega_t^2\omega_2^2\omega_3^2)\ +$

$(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)/((1+\alpha)\omega_t\omega_2\omega_3R_1C_1)$

$+\ 3\omega_1\omega_2\omega_3/((1+\alpha)\omega_t\omega_2\omega_3)\ +\ 3/K\ -\ 1)\ +$

$\omega_1^2\omega_2^2\omega_3^2/(\omega_t^2\omega_2^2\omega_3^2R_1C_1)\ +$

$\omega_1\omega_2\omega_3/((1+\alpha)\omega_t\omega_2\omega_3R_1C_1)\ +\ 1/(KR_1C_1).$

The Routh Criterion matrix now becomes.

a_8	a_6	a_4	a_2	a_0
a_7	a_5	a_3	a_1	
b_1	b_2	b_3	b_4	
c_1	c_2	c_3		
d_1	d_2	d_3		
e_1	e_2			
f_1	f_2			
g_1				
h_1				

Where $a_8 \; = \; R_1C_1/(\omega_t^2\omega_2^2\omega_3^2)$

$a_7 \; = \; 3/(\omega_t^2\omega_2^2\omega_3^2) \; +$

$2(\omega_1+\omega_2+\omega_3)R_1C_1/(\omega_t^2\omega_2^2\omega_3^2)$

$a_6 \; = \; 1/(\omega_t^2\omega_2^2\omega_3^2R_1C_1) \; +$

$6(\omega_1+\omega_2+\omega_3)/(\omega_t^2\omega_2^2\omega_3^2) \; +$

$(2(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) \; +$

$(\omega_1+\omega_2+\omega_3)^2)R_1C_1/(\omega_t^2\omega_2^2\omega_3^2)$

$$a_5 \;=\; 2(\omega_1+\omega_2+\omega_3)/(\omega_t{}^2\omega_2{}^2\omega_3{}^2R_1C_1) \;+$$

$$3(2(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) \;+$$

$$(\omega_1+\omega_2+\omega_3)^2)/(\omega_t{}^2\omega_2{}^2\omega_3{}^2) \;+$$

$$2((\omega_1+\omega_2+\omega_3)(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) \;+$$

$$\omega_t\omega_2\omega_3)R_1C_1/(\omega_t{}^2\omega_2{}^2\omega_3{}^2) \;+$$

$$R_1C_1/((1+\alpha)\omega_t\omega_2\omega_3)$$

$$a_4 \;=\; (2(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) \;+$$

$$(\omega_1+\omega_2+\omega_3)^2)/(\omega_t{}^2\omega_2{}^2\omega_3{}^2R_1C_1) \;+$$

$$6((\omega_1+\omega_2+\omega_3)(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) \;+$$

$$\omega_1\omega_2\omega_3)/(\omega_t{}^2\omega_2{}^2\omega_3{}^2) \;+$$

$$(2\omega_1\omega_2\omega_3(\omega_1+\omega_2+\omega_3) \;+$$

$$(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)^2)R_1C_1/(\omega_t{}^2\omega_2{}^2\omega_3{}^2)$$

$$+\; 3/((1+\alpha)\omega_t\omega_2\omega_3) \;+$$

$$(\omega_1+\omega_2+\omega_3)R_1C_1/((1+\alpha)\omega_t\omega_2\omega_3)$$

$$a_3 \;=\; 2((\omega_1+\omega_2+\omega_3)(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) \;+$$

$$\omega_1\omega_2\omega_3)/(\omega_t{}^2\omega_2{}^2\omega_3{}^2R_1C_1) \;+$$

$$(2\omega_1\omega_2\omega_3(\omega_1+\omega_2+\omega_3) \;+$$

$$(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)^2)3/(\omega_t{}^2\omega_2{}^2\omega_3{}^2) \;+$$

$$2\omega_1\omega_2\omega_3(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)R_1C_1$$

$$/(\omega_t{}^2\omega_2{}^2\omega_3{}^2) \;+\; 1/((1+\alpha)\omega_t\omega_2\omega_3R_1C_1) \;+$$

$$3(\omega_1+\omega_2+\omega_3)/((1+\alpha)\omega_t\omega_2\omega_3) \;+$$

$$(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)R_1C_1 \;/((1+\alpha)\omega_t\omega_2\omega_3)$$

$a_2 = (2\omega_1\omega_2\omega_3(\omega_1+\omega_2+\omega_3) +$

$(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)^2)/(\omega_t^2\omega_2^2\omega_3^2R_1C_1)$

$+ 6\omega_1\omega_2\omega_3(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)$

$/(\omega_t^2\omega_2^2\omega_3^2) +$

$\omega_1^2\omega_2^2\omega_3^2R_1C_1/(\omega_t^2\omega_2^2\omega_3^2) +$

$(\omega_1+\omega_2+\omega_3)/((1+\alpha)\omega_t\omega_2\omega_3R_1C_1) +$

$3(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)/((1+\alpha)\omega_t\omega_2\omega_3) +$

$\omega_1\omega_2\omega_3R_1C_1/((1+\alpha)\omega_t\omega_2\omega_3) + R_1C_1/K$

$a_1 = 2\omega_1\omega_2\omega_3(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)$

$/(\omega_t^2\omega_2^2\omega_3^2R_1C_1) +$

$\omega_1^2\omega_2^2\omega_3^2/(\omega_t^2\omega_2^2\omega_3^2) +$

$(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)/((1+\alpha)\omega_t\omega_2\omega_3R_1C_1)$

$+ 3\omega_1\omega_2\omega_3/((1+\alpha)\omega_t\omega_2\omega_3) + 3/K - 1$

$a_0 = \omega_1^2\omega_2^2\omega_3^2/(\omega_t^2\omega_2^2\omega_3^2R_1C_1) +$

$\omega_1\omega_2\omega_3/((1+\alpha)\omega_t\omega_2\omega_3R_1C_1) + 1/(KR_1C_1)$

$b_1 = (a_7a_6-a_8a_5)/a_7$

$b_2 = (a_7a_4-a_8a_3)/a_7$

$b_3 = (a_7a_2-a_8a_1)/a_7$

$b_4 = a_0$

$c_1 = (b_1a_5-a_7b_2)/b_1$

$c_2 = (b_1a_3-a_7b_3)/b_1$

$c_3 = (b_1a_1-a_7b_4)/b_1$

$d_1 = (c_1b_2-b_1c_2)/c_1$

$d_2 = (c_1b_3-b_1c_3)/c_1$

$d_3 = b_4$

$e_1 = (d_1c_2 - c_1d_2)/d_1$

$e_2 = (d_1c_3 - c_1d_3)/d_1$

$f_1 = (e_1d_2 - d_1e_2)/e_1$

$f_2 = d_3$

$g_1 = (f_1e_2 - e_1f_2)/f_1$

$h_1 = f_2.$

Also here inspection of the coefficients of the matrix shows that the network is not stable for all α.

Nyquist Diagram Analysis

Another stability analysis technique which would be helpful here is that of the Nyquist Diagram (Appendix V). This provides a graphical representation of the open loop frequency response of a system as a polar- or s-plane-plot. Basically the Stability Criterion may be stated as follows [38]:

> 'a closed loop system will be stable if, when moving along the open loop frequency response, plotted on a Nyquist Diagram, in the direction of increasing frequency, the point (-1,j0) lies on the left of the locus.'

The degree of stability can be determined from the Nyquist Diagram in terms of gain and phase margins.

Using the open loop transfer function for the SKB with the M2 COA and choosing the single pole model (SP) we get the previously derived expression {18},

Substituting $s = j\omega$ and grouping terms yields,

$$1/\{(2/K - \omega^2(2/(\omega_t^2) + R_1C_1/((1+\alpha)\omega_t)) + j(\omega(2/((1+\alpha)\omega_t) + R_1C_1/K) - \omega^3(R_1C_1/(\omega_t^2)))\}. \quad \ldots\{21\}$$

Computing this expression and plotting the results in the s-plane will provide the Nyquist Diagram, from which the stability of the network can be established.

The open loop transfer function for the SKB using the M2 COA for a two pole model (DP), is expression {19}.

Substituting $s = j\omega$ and grouping terms yields,

$$1/\{(\omega^4(2/(\omega_t^2\omega_2^2) + 2(\omega_1+\omega_2)R_1C_1/(\omega_t^2\omega_2^2)) - \omega^2(2(2\omega_1\omega_2 + (\omega_1+\omega_2)^2)/(\omega_t^2\omega_2^2) + 2\omega_1\omega_2(\omega_1+\omega_2)R_1C_1/(\omega_t^2\omega_2^2) + 2/((1+\alpha)\omega_t\omega_2) + (\omega_1+\omega_2)R_1C_1/((1+\alpha)\omega_t\omega_2)) 2\omega_1^2\omega_2^2/(\omega_t^2\omega_2^2) + 2\omega_1\omega_2/((1+\alpha)\omega_t\omega_2) + 2/K) + j(\omega(4\omega_1\omega_2(\omega_1+\omega_2)/(\omega_t^2\omega_2^2) + \omega_1^2\omega_2^2R_1C_1/(\omega_t^2\omega_2^2) + \quad \ldots\{22\}$$

$2(\omega_1+\omega_2)/((1+\alpha)\omega_t\omega_2)$ +

$\omega_1\omega_2 R_1C_1/((1+\alpha)\omega_t\omega_2)$ + $R_1C_1/K)$ −

$\omega^3(4(\omega_1+\omega_2)/(\omega_t^2\omega_2^2)$ + $(2\omega_1\omega_2$ +

$(\omega_1+\omega_2)^2)R_1C_1/(\omega_t^2\omega_2^2)$ +

$R_1C_1/((1+\alpha)\omega_t\omega_2))$ +

$\omega^5(R_1C_1/(\omega_t^2\omega_2^2)))\}$.

The Nyquist Diagram is computed from this so that stability can be evaluated.

The open loop transfer function for the three pole model SKB with M2 COA was shown in expression {20}.

Substituting s = jω and grouping terms one gets,

$1/\{(-\omega^6(2/(\omega_t^2\omega_2^2\omega_3^2)$ + ...{23}

$2(\omega_1+\omega_2+\omega_3)R_1C_1/(\omega_t^2\omega_2^2\omega_3^2))$ +

$\omega^4(2(2(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)$ +

$(\omega_1+\omega_2+\omega_3)^2)/(\omega_t^2\omega_2^2\omega_3^2)$ +

$2((\omega_1+\omega_2+\omega_3)(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)$ +

$\omega_1\omega_2\omega_3)R_1C_1/(\omega_t^2\omega_2^2\omega_3^2)$ +

$R_1C_1/((1+\alpha)\omega_t\omega_2\omega_3))$ −

$\omega^2(2(2\omega_1\omega_2\omega_3(\omega_1+\omega_2+\omega_3)$ +

$(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)^2)/(\omega_t^2\omega_2^2\omega_3^2)$ +

$2\omega_1\omega_2\omega_3(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)R_1C_1$

$/(\omega_t^2\omega_2^2\omega_3^2)$ +

$2(\omega_1+\omega_2+\omega_3)/((1+\alpha)\omega_t\omega_2\omega_3)$ +

Again the results for the Routh Criterion as applied to the M2 COA for the three pole model are presented in Table 4.

As before the values of α are chosen to show the point of marginal stability.

Q = 1,835	f_o = 3,717 kHz	K = 2,455			
α	Routh Criterion Matrix				
0,8	1,6E-48	6,2E-33	8,2E-19	1,7E-5	9,5E3
	1,7E-40	1,0E-25	2,8E-12	2,2E-1	
	5,2E-33	7,9E-19	1,7E-5	9,5E3	
	7,9E-26	2,2E-12	2,2E-1		
	6,4E-19	1,7E-5	9,5E3		
	7,7E-14	2,2E-1			
	1,5E-5	9,5E3			
	2,2E-1				
	9,5E3				
0,84429	1,6E-48	6,2E-33	8,1E-19	1,7E-5	9,5E3
	1,7E-40	1,0E-25	2,7E-12	2,2E-1	
	5,2E-33	7,9E-19	1,7E-5	9,5E3	
	7,9E-26	2,2E-12	2,2E-1		
	6,4E-19	1,7E-5	9,5E3		
	8,1E-15	2,2E-1			
	3,1E-8	9,5E3			
	2,1E-1	9,5E3			
0,85	1,6E-48	6,2E-33	8,1E-19	1,7E-5	9,5E3
	1,7E-40	1,0E-25	2,7E-12	2,2E-1	
	5,2E-33	7,9E-19	1,7E-5	9,5E3	
	7,9E-26	2,2E-12	2,2E-1		
	6,4E-19	1,7E-5	9,5E3		
	-6,0E-16	2,2E-1			
	2,5E-4	9,5E3			
	2,2E-1				
	9,5E3				

Table 4

Thus it can be seen from inspection of Table 4 that a small change in α causes the circuit to exhibit instability. To avoid this instability a suitable choice of α must be made to ensure that the circuit remains stable. A comparison of the above results with those obtained by Campbell and Stephenson is made in Table 5, also included are results for the M2 COA for Q = 8,3.

M2 Composite Operational Amplifier					
Q = 1,835	Campbell and Stephenson [1]		Routh Criterion Analysis		
f_o kHz	Experimental α	Theoretical α	M2SP α	M2DP α	M2TP α
3,717	< 1,064	< 2,85	*	< 1,728	< 0,84
11,044	< 1,162	< 3,1	*	< 1,823	< 0,88
Q = 8,3 f_o kHz					
3,717	< 1,338	< 3,5	*	< 2,15	< 1,14
11,044	< 1,527	< 3,9	*	< 2,33	< 1,2

Table 5

Here * signifies that the network is stable for α between 1 and 1000000. Values of α above 1000000 lead to undesirably large component spreads for thin-film realisations.

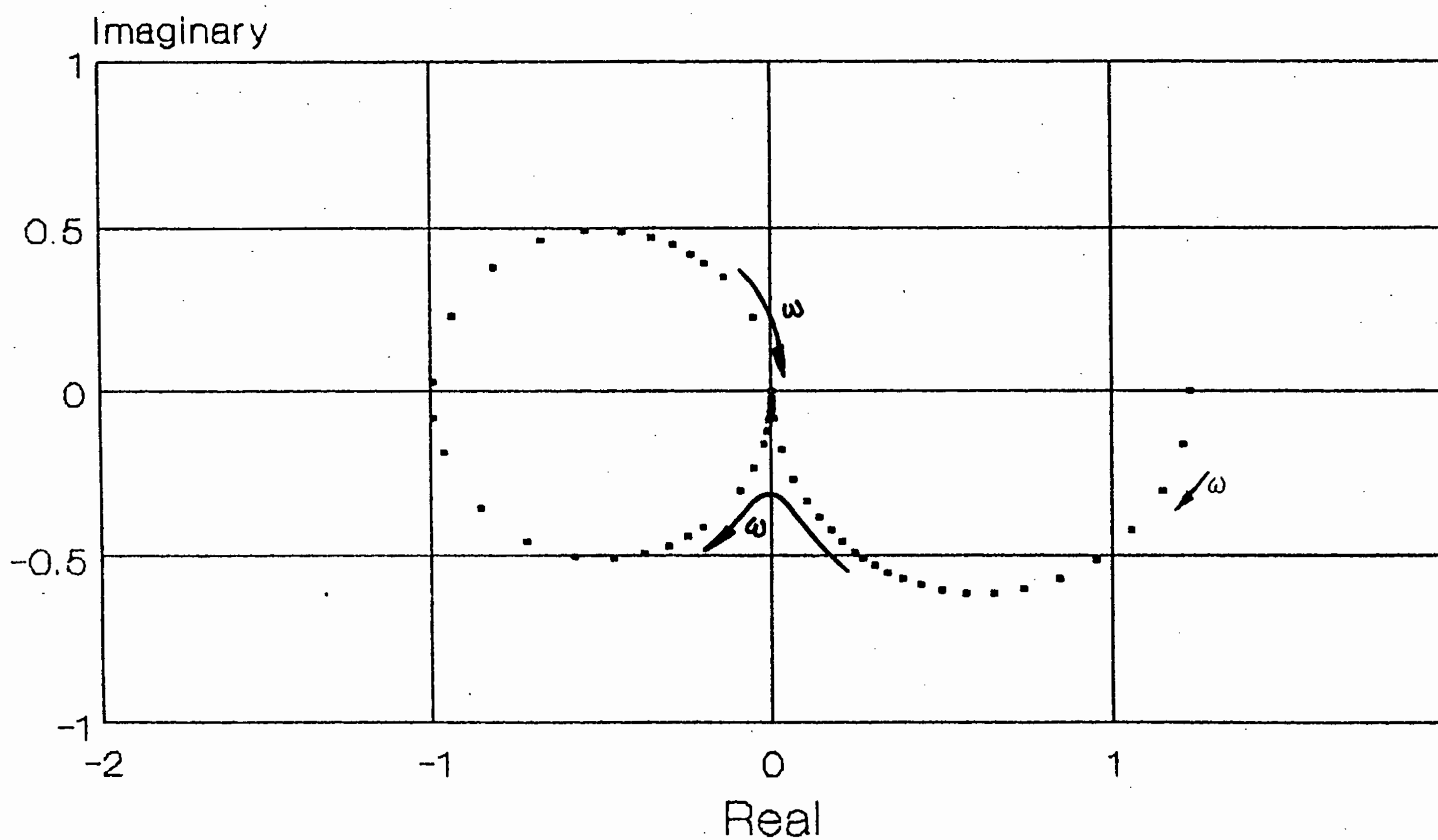
Comparison of the theoretical predictions and experimental values presented in Table 5 indicates that the single pole model does not satisfactorily model the OAs used. However, for the two pole model results are fairly close to those found experimentally. Nevertheless as can be seen further improvement is possible by making use of the three pole model.

While it is evident that the three pole model provides good results when compared with experimental findings, one gets no direct indication of the degree of stability for the particular value of α. To get this type of information it was considered convenient to make use of the Nyquist Diagram analysis technique.

For a particular choice of α, the gain and phase margin can be found graphically from the Nyquist Diagram, and this then allows for the degree of stability to be evaluated.

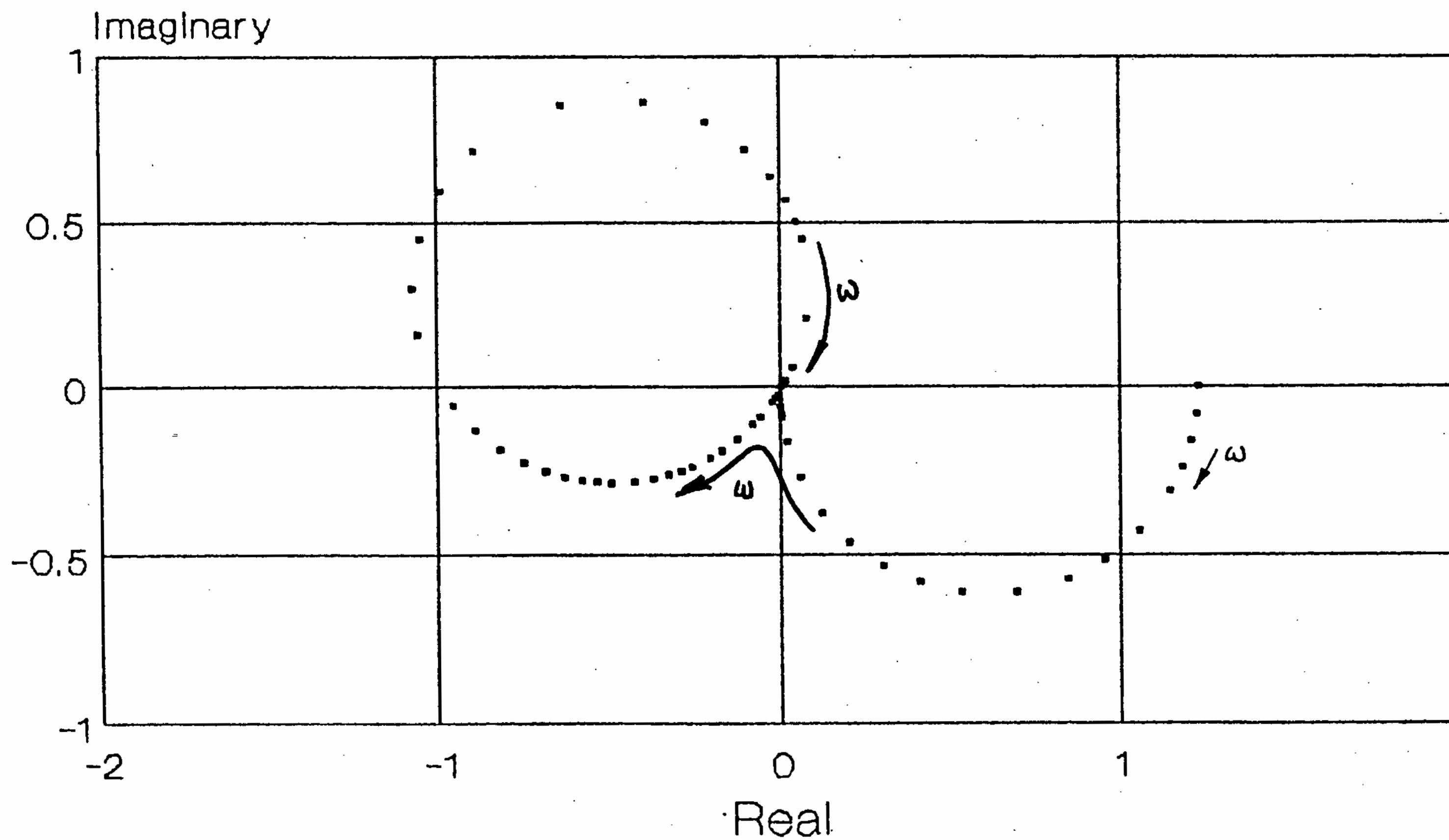
On pages 88,89 and 90 the Nyquist Diagrams for the one, two and three pole models respectively for the Sallen and Key bandpass network under consideration are presented. In this case the M2 COA circuit has been used. These Nyquist Diagrams show the frequency response locus with the direction of increasing ω as marked. Unfortunately however, the computer generated locus does not clearly indicate the characteristic spiralling of the locus in the vicinity of the s-plane origin. This lack of detail must be attributed to the order and composition of the denominator polynomial. In any event the usefulness of these diagrams is realised when considering the vicinity of the (-1,j0) point which allows for a visual determination of stability of the network for a particular value of α (see Appendix V).

Nyquist Diagram 1



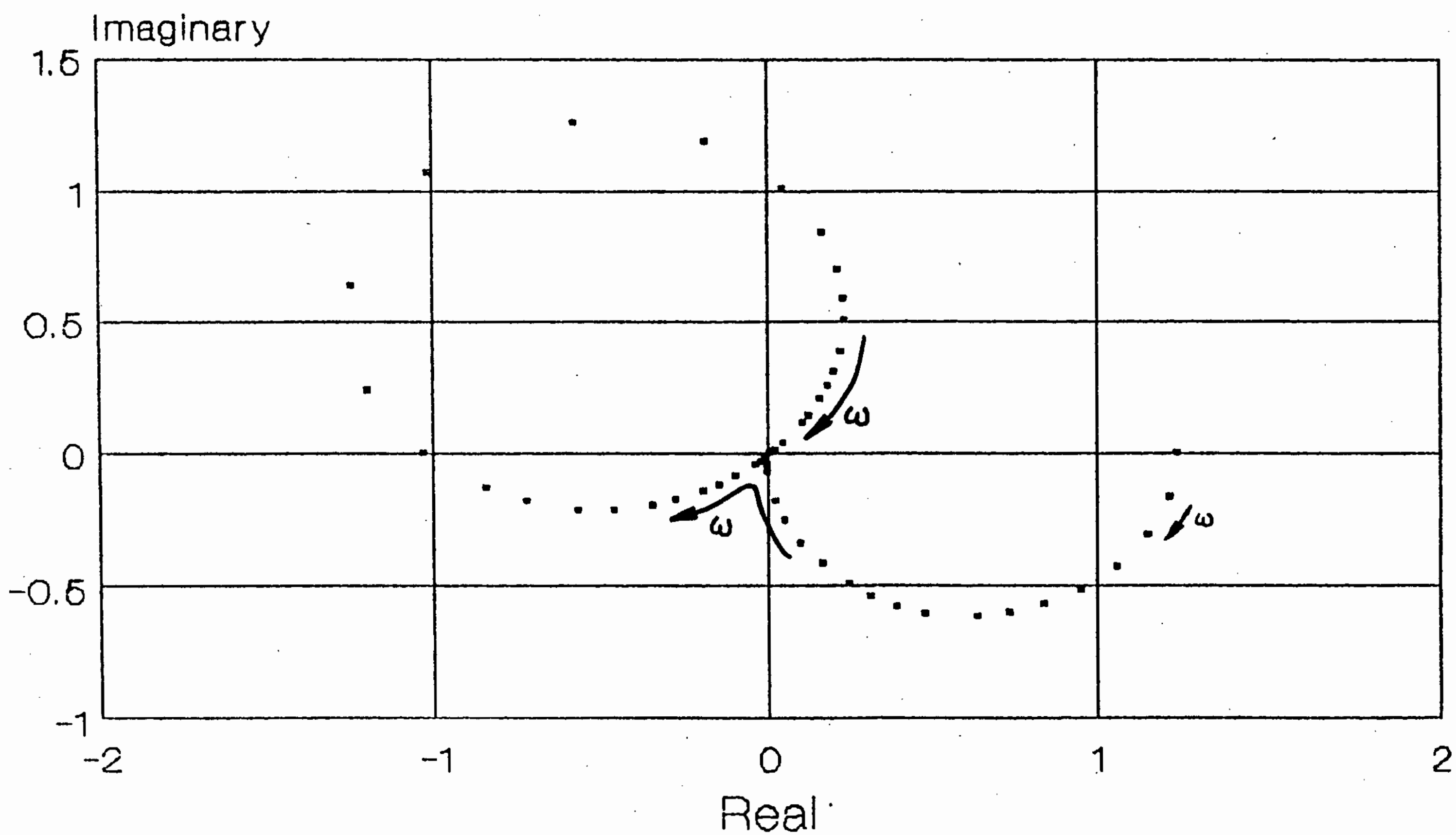
M2 Composite Operational Amplifier
Single Pole Model
$\alpha = 150$

Nyqusit Diagram 2



M2 Composite Operational Amplifier
Double Pole Model
$\alpha = 1.64$

Nyquist Diagram 3



M2 Composite Operational Amplifier
Triple Pole Model
$\alpha = 0{,}8$

These Nyquist Diagrams depict the open loop frequency response of the circuit, making use of the single, two and three pole models, for values of α at which the circuit is marginally stable. Increasing the gain or phase margin or α ,in the case of the M2 COA, will result in instability. Thus for a chosen α ensuring stability the Nyquist Diagram will yield the gain and phase margins which give a measure of the relative stability of the circuit.

A comparison of the Nyquist Diagram results with the practical results by Campbell and Stephenson is given in Table 6. Again the M2 case for Q = 8,3 is also shown.

M2 Composite Operational Amplifier					
Q = 1,835	Campbell and Stephenson [1]		Nyquist Diagram Analysis		
f_o kHz	Experimental α	Theoretical α	M2SP α	M2DP α	M2TP α
3,717	< 1,064	< 2,85	< 150	< 1,64	< 0,80
11,044	< 1,162	< 3,1	< 50	< 1,55	< 0,78
Q = 8,3 f_o kHz					
3,717	< 1,338	< 3,5	< 130	< 2,04	< 1,08
11,044	< 1,527	< 3,9	< 42	< 1,92	< 1,03

Table 6

It can readily be seen that the single pole model does not yield meaningful results while the two pole model come closer to those found experimentally. Also here it is striking that the three pole model correlates best with experimental results. It would appear that the improvement in the gain bandwidth product for the COAs is directly linked to the accuracy with which the individual OAs are modelled.

The Routh Criterion results for the M4 COA three pole model are presented in Table 7, along with the values from Campbell and Stephenson's paper [1]. Only the three pole model was used as this models the OA best. From these results it can be seen that the network will be stable for α greater than the lower limit predicted. This is more desirable than the M2 case as a greater bandwidth can be achieved. However the finite input impedance of the M4 COA is a disadvantage.

M4 Composite Operational Amplifier					
Q = 1,835	Campbell and Stephenson [1]		Routh Criterion Analysis		
f_o kHz	Experimental α	Theoretical α	M4SP α	M4DP α	M4TP α
3,717	> 0,67	> 0,65	-	-	> 0,37
11,044	> 0,60	> 0,65	-	-	> 0,33
Q = 8,3 f_o kHz					
3,717	> 0,68	> 0,65	-	-	> 0,37
11,044	> 0,49	> 0,65	-	-	> 0,32

Table 7

The Nyquist Diagram results for the M4 COA for the three pole model are shown in Table 8, and for comparison the results from Campbell and Stephenson's paper [1], are included.

M4 Composite Operational Amplifier					
Q = 1,835	Campbell and Stephenson [1]		Nyquist Diagram Analysis		
f_o kHz	Experimental α	Theoretical α	M4SP α	M4DP α	M4TP α
3,717	> 0,67	> 0,65	-	-	> 0,55
11,044	> 0,60	> 0,65	-	-	> 0,60
Q = 8,3 f_o kHz					
3,717	> 0,68	> 0,65	-	-	> 0,58
11,044	> 0,49	> 0,65	-	-	> 0,63

Table 8

The three pole model results are again very close to those found experimentally. This is the case for both Table 7 and 8.

When comparing the results shown in Tables 5 and 6 as well as those of Tables 7 and 8 it becomes evident that the α values for the two stability analysis techniques of Routh and Nyquist differ.

The Routh analysis technique (Appendix III) considers the coefficients of the denominator polynomial of the closed loop transfer function and the value of α for which the network becomes unstable is determined. The values given in Tables 5 and 7 are thus found by manually entering the value of α into the spreadsheet program on an IBM-compatible PC.

Visual inspection of the resultant coefficients of the Routh Criterion Matrix then yield the value of α for stability.

In the case of the Nyquist Diagram analysis method (Appendix V) the open loop transfer function of the network is used to predict the closed loop transfer function response. The frequency response locus is then obtained from the spreadsheet program with the values of α of Tables 6 and 8 determined from inspection of the Nyquist Diagram.

It must be thus appreciated that the α values are only approximate. Nevertheless it can be seen that reasonable agreement in α exists between the two stability techniques.

Results from all the cases studied clearly indicate that the three pole model gives the best correlated results. Thus for the Sallen and Key bandpass network using the M3 COA and taking the three pole model, stability is achieved for a choice of α > 18200000. This in turn requires that for R_4 = 10 kΩ , R_5 = 182 MΩ , a component spread which is far too large. However, inspection of the M3 transfer functions in Appendix IV reveals that resistor R_4 plays a significant role in the denominator. It is therefore possible to decrease the value of R_4 to 100 Ω, in which case for stability α > 3,6 and R_5 = 360 Ω. When considering thin film realisations this latter component spread would certainly be more desirable than the component spread required if R_4 = 10 kΩ.

As has been shown the composite amplifiers M1 - 4 may all be employed in circuits and filters where extended gain bandwidths are desirable. This does however require the careful selection of the scaling factor α as this is directly linked to the stability of the circuit.

CHAPTER 6

CONCLUSION

Information gained from the evaluation of the results discussed in the previous chapters indicates that composite operational amplifiers (COA) can be gainfully employed in cases where extended gain bandwidth is required. This is particularly so with active filters of higher order. However, as has been shown the careful selection of the scaling factor α is critical in producing a stable network.

The basic aim of this study was to find a theoretical approach which would allow the clear identification of stable circuits. Much depends on the choice of the operational amplifier (OA) model and in this only the three pole model gave consistent results which correlated well with experimental findings. While the mathematically simpler single and two pole models are fine for many other applications they are however not found accurate enough in cases where gains of individual OAs affect the coefficients of rather complicated transfer functions of active filters.

In the study of second order filter networks the three pole model has been seen to provide sufficient accuracy to effectively determine the stability of such networks when COAs are used. Preliminary results of these findings were reported, in a paper [39] at an International Conference, last year.

For the experimental work low cost OAs where used. It is considered that for this reason no good agreement between theory and practice could be gained other than using a three pole model. However, if high quality OAs had been used for the COA better correlation would be obtained with a single or two pole model. All this indicates that the shortcomings of low cost devices are in fact grossly accentuated when this becomes a multiplicative effect in a COA.

The two stability analysis methods used, namely, Routh Criterion and Nyquist Diagram are mathematically simpler than the Root Migration technique which gives a graphical representation of the stability margin [39]. Nevertheless the Nyquist Diagram is considered superior as both gain and phase margins for the network can be determined easily and it provides a more accurate indication of the likely behaviour of the composite operational amplifier network.

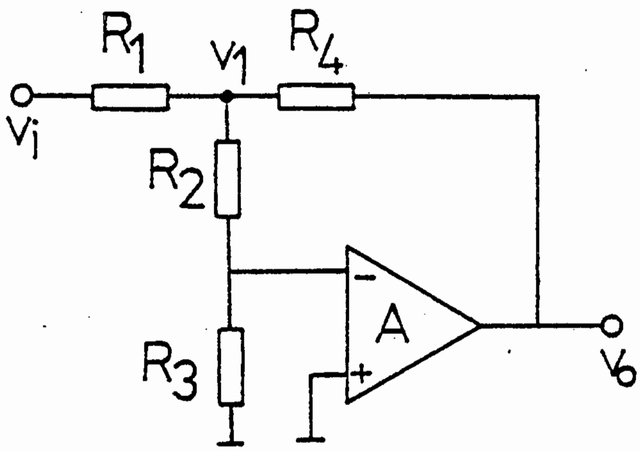
In order to carry out computer studies on the composite operational amplifier the value of ω_1 , ω_2 and ω_3 must be known. With the predictions made where $\omega_1 = \omega_t / A_0$, $\omega_2 = 2\omega_t$ and $\omega_3 = 4\omega_t$, good results were obtained. It is found that the value of ω_3 in particular plays an important role in determining the best value of the scaling factor α. Future investigations may have to address the question of accurately measuring the value of ω_3 for a particular OA so as to improve on the accuracy of the stability results.

APPENDIX I

A Method of Determining ω_t

To find the value of ω_t for an operational amplifier the following method may be employed.

Assemble the circuit as shown below.



where $R_1 = R_2 = R_4 = 47$ kΩ and $R_3 = 47$ Ω.

Using constrained nodal admittance matrix techniques on the above network yields

$$A(f) = -(1 + R_2/R_3)V_{out}/V_{in}.$$

Adjusting the frequency of V_{in} in such away as to realise $V_1 = V_{out}$ yields the value of ω_t.

Thus it can be seen that by definition ω_t is the frequency at which the operational amplifier exhibits unity gain.

APPENDIX II

The Constrained Nodal Admittance Matrix

The nodal admittance matrix [Y] is formed by inspection of the network topology using the following definitions[40,41];

(i) Y_{ii} is the sum of all admittances attached to node i;

and

(ii) Y_{ij} is the negative sum of all admittances connected directly between nodes i and j.

In a passive network, $Y_{ij} = Y_{ji}$, where $i \neq j$ and the off diagonal terms are symmetrical about the major diagonal. A constrained nodal admittance matrix is formed as indicated above, however, the inclusion of active elements causes the nodal admittance matrix to be constrained. The active elements destroy the symmetry of the off diagonal terms about the major diagonal.

Consider the M2 composite operational amplifier, as dealt with in Chapter 3 page 51. The constrained nodal admittance matrix was formed as follows; (where i = row and j = column)

Y_{11} : self admittance of node 1 : 0

Y_{12} : mutual admittance between nodes 1 and 2 : 0

Y_{13} : mutual admittance between nodes 1 and 3 : 0

Y_{14} : mutual admittance between nodes 1 and 4 : 0

Y_{15} : mutual admittance between nodes 1 and 5 : 0

Y_{21} : mutual admittance between nodes 2 and 1 : 0

Y_{22} : self admittance of node 2 : Y_2+Y_3

Y_{23} : mutual admittance between nodes 2 and 3 : $-Y_3$

Y_{24} : mutual admittance between nodes 2 and 4 : 0

Y_{25} : mutual admittance between nodes 2 and 5 : 0

Y_{31} : mutual admittance between nodes 3 and 1 : 0

Y_{32} : mutual admittance between nodes 3 and 2 : $-Y_3$

Y_{33} : self admittance of node 3 : $Y_3+Y_5+Y_{33}$

Y_{34} : mutual admittance between nodes 3 and 4 : $A_2Y_{33}+Y_5$

Y_{35} : mutual admittance between nodes 3 and 5 : $-A_2Y_{33}$

Y_{41} : mutual admittance between nodes 4 and 1 : 0

Y_{42} : mutual admittance between nodes 4 and 2 : 0

Y_{43} : mutual admittance between nodes 4 and 3 : $-Y_5$

Y_{44} : self admittance of node 4 : Y_4+Y_5

Y_{45} : mutual admittance between nodes 4 and 5 : 0

Y_{51} : mutual admittance between nodes 5 and 1 : $-A_1Y_{55}$

Y_{52} : mutual admittance between nodes 5 and 2 : A_1Y_{55}

Y_{53} : mutual admittance between nodes 5 and 3 : 0

Y_{54} : mutual admittance between nodes 5 and 4 : 0

Y_{55} : self admittance of node 5 : Y_{55}

where A_1 and A_2 are the gains of the two operational amplifiers and Y_{55} and Y_{33} are the respective output admittances.

As an example of how the active elements are determined consider the matrix entries Y_{35} and Y_{53}.

In the case of Y_{35} the forward path is from node 5, the matrix column number, to node 3, the matrix row number, through the inverting input of the operational amplifier thus yielding a constrained nodal admittance matrix entry of $-(-A_2Y_{33})$, where the '-' outside the bracket occurs because it is a non major diagonal term and the bracket is the product of the operational amplifier gain and the output admittance of the operational amplifier. In the case of Y_{53} the forward path is between node 3 to node 5, which represents a path from the output to the inverting input of the operational amplifier this clearly being an open path and hence has a value of zero.

This technique is applied to other nodes and associated active devices to yield the constrained nodal admittance matrix as given for the M2 composite operational amplifier on page 51.

To find the transfer function from this constrained nodal admittance matrix use is made of pivotal condensation. This technique is used to systematically reduce an [nxn] matrix about a desired node.

If any node i in an [nxn] matrix is to be suppressed, terms in the resulting [n-1xn-1] matrix are found by applying

$$Y'_{rc} = Y_{rc} - (Y_{ri}Y_{ic})/Y_{ii}$$

where Y'_{rc} is the modified version of Y_{rc} and Y_{ii} is the suppressed node.

This technique is applied successively until a [2x2] matrix is obtained.

The voltage transfer function , V_o/V_i , is formed from

$$V_o/V_i = Y_{io}/Y_{ii}$$

where Y_{io} and Y_{ii} are cofactors of the nodal admittance matrix and 'i' refers to the input node and 'o' refers to the output node.

APPENDIX III

The Routh Criterion

The Routh Criterion [38] is a method of determining whether a system is stable or unstable by examining the behaviour of the coefficients of the characteristic equation of a system. The characteristic equation has the form

$$a_n s^n + a_{n-1} s^{n-1} + \ldots + a_0 = 0$$

The system will be stable if

(i) all coefficients of s from n to 0 are present and are positive;

and

(ii) all terms in the first column of the array have the same sign.

The array is formed as follows :

$$
\begin{array}{|llll}
\hline
a_n & a_{n-2} & a_{n-4} & \cdots \\
a_{n-1} & a_{n-3} & a_{n-5} & \cdots \\
b_1 & b_2 & b_3 & \cdots \\
c_1 & c_2 & \cdots & \\
d_1 & d_2 & \cdots & \\
e_1 & \cdots & & \\
\cdot & & & \\
\cdot & & & \\
\cdot & & &
\end{array}
$$

where a_n, a_{n-1}, ..., are the coefficients of the characteristic equation and b_1, b_2, ..., c_1, c_2, ..., etc, are derived by the application of the formulae shown below.

$$b_1 = (a_{n-1}a_{n-2}-a_na_{n-3})/a_{n-1}$$

$$b_2 = (a_{n-1}a_{n-4}-a_na_{n-5})/a_{n-1}$$

$$c_1 = (b_1a_{n-3}-a_{n-1}b_2)/b_1$$

$$c_2 = (b_1a_{n-5}-a_{n-1}b_3)/b_1$$

$$d_1 = (c_1b_2-b_1c_2)/c_1$$

$$d_2 = (c_1b_3-b_1c_3)/c_1$$

etc.

This technique is used to evaluate the M2 COA on page 65.

APPENDIX IV

This appendix serves to list the various functions associated with the M3 and M4 composite operational amplifiers (COA) and are used, in a similar manner to those in the body of this dissertation, to obtain results for the M3 and M4 COA networks.

The abbreviations used in this appendix are similar to those used in the body of this dissertation and are summarised below.

GF Gain Function

M3 C2OA-3 described by Mikhael and Nessim [2]

M4 C2OA-4 described by Mikhael and Nessim [2]

SP Single Pole Model of the operational amplifier (OA)

DP Two Pole Model of the OA

TP Three Pole Model of the OA

SKB Sallen and Key Bandpass Network

CLTF Closed Loop Transfer Function

OLTF Open Loop Transfer Function

The M3 Composite Operational Amplifier

$$GFM3 = 1/\{(1+\alpha)/(A_1A_2) + Y_4/(A_2(Y_2+Y_3)) + 1/K\}$$

$$GFM3SP = 1/\{s^2(1+\alpha)/\omega_t^2 + s\beta/\omega_t + 1/K\}$$

where $\beta = Y_4/(Y_2+Y_3)$

$$GFM3DP = 1/\{(1+\alpha)(s^4 + s^3 2(\omega_1+\omega_2) + s^2(2\omega_1\omega_2 + (\omega_1+\omega_2)^2) + s2\omega_1\omega_2(\omega_1+\omega_2) + \omega_1^2\omega_2^2)/(\omega_t^2\omega_2^2) + \beta(s^2 + s(\omega_1+\omega_2) + \omega_1\omega_2)/(\omega_t\omega_2) + 1/K\}$$

$$GFM3TP = 1/\{(1+\alpha)(s^6 + s^5 2(\omega_1+\omega_2+\omega_3) + s^4(2(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) + (\omega_1+\omega_2+\omega_3)^2) + s^3 2((\omega_1+\omega_2+\omega_3)(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) + \omega_1\omega_2\omega_3) + s^2(2\omega_1\omega_2\omega_3(\omega_1+\omega_2+\omega_3) + (\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)^2) + s2\omega_1\omega_2\omega_3(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) + \omega_1^2\omega_2^2\omega_3^2)/(\omega_t^2\omega_2^2\omega_3^2) + \beta(s^3 + s^2(\omega_1+\omega_2+\omega_3) + s(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) + \omega_1\omega_2\omega_3)/(\omega_t\omega_2\omega_3) + 1/K\}$$

$$SKBCLTFM3SP = s/\{s^4((1+\alpha)R_1C_1/\omega_t^2) + s^3(3(1+\alpha)/\omega_t^2 + \beta R_1C_1/\omega_t) + s^2((1+\alpha)/(\omega_t^2R_1C_1) + 3\beta/\omega_t^2 + R_1C_1/K) + s(\beta/(\omega_tR_1C_1) + 3/K - 1) + 1/KR_1C_1\}$$

$SKBCLTFM3DP \;=\; s/\{s^6((1+\alpha)R_1C_1/(\omega_t^2\omega_2^2)) \;+$

$s^5(3(1+\alpha)/(\omega_t^2\omega_2^2) \;+$

$2(\omega_1+\omega_2)(1+\alpha)R_1C_1/(\omega_t^2\omega_2^2)) \;+$

$s^4((1+\alpha)/(\omega_t^2\omega_2^2R_1C_1) \;+$

$6(\omega_1+\omega_2)(1+\alpha)/(\omega_t^2\omega_2^2) \;+\; (2\omega_1\omega_2 \;+$

$(\omega_1+\omega_2)^2)(1+\alpha)R_1C_1/(\omega_t^2\omega_2^2) \;+$

$\beta R_1C_1/(\omega_t\omega_2)) \;+$

$s^3(2(\omega_1+\omega_2)(1+\alpha)/(\omega_t^2\omega_2^2R_1C_1) \;+\; (2\omega_1\omega_2$

$+\; (\omega_1+\omega_2)^2)(1+\alpha)3/(\omega_t^2\omega_2^2) \;+$

$2\omega_1\omega_2(\omega_1+\omega_2)R_1C_1(1+\alpha)/(\omega_t^2\omega_2^2) \;+$

$3\beta/(\omega_t\omega_2) \;+\; \beta(\omega_1+\omega_2)R_1C_1/(\omega_t\omega_2)) \;+$

$s^2((2\omega_1\omega_2 \;+\; (\omega_1+\omega_2)^2)(1+\alpha)/(\omega_t^2\omega_2^2R_1C_1) \;+$

$6\omega_1\omega_2(\omega_1+\omega_2)(1+\alpha)/(\omega_t^2\omega_2^2) \;+$

$\omega_1^2\omega_2^2R_1C_1(1+\alpha)/(\omega_t^2\omega_2^2) \;+$

$\beta/(\omega_t\omega_2R_1C_1) \;+\; 3\beta(\omega_1+\omega_2)/(\omega_t\omega_2) \;+$

$\beta\omega_1\omega_2R_1C_1/(\omega_t\omega_2) \;+\; R_1C_1/K) \;+$

$s(2\omega_1\omega_2(\omega_1+\omega_2)(1+\alpha)/(\omega_t^2\omega_2^2R_1C_1) \;+$

$3\omega_1^2\omega_2^2(1+\alpha)/(\omega_t^2\omega_2^2) \;+$

$\beta(\omega_1+\omega_2)/(\omega_t\omega_2R_1C_1) \;+\; 3\beta\omega_1\omega_2/(\omega_t\omega_2) \;+$

$3/K \;-\; 1) \;+\; \omega_1^2\omega_2^2(1+\alpha)/(1^2{}_t\omega_2^2R_1C_1) \;+$

$\beta\omega_1\omega_2/(\omega_t\omega_2R_1C_1) \;+\; 1/(KR_1C_1)\}$

$SKBCLTFM3TP \;=\; s/\{s^{8}(R_1C_1(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2)) \;+$

$s^{7}(3(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2) \;+$

$2(\omega_1+\omega_2+\omega_3)R_1C_1(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2)) \;+$

$s^{6}((1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2R_1C_1) \;+$

$6(\omega_1+\omega_2+\omega_3)(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2) \;+$

$(2(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) \;+$

$(\omega_1+\omega_2+\omega_3)^2)R_1C_1(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2)) \;+$

$s^{5}(2(\omega_1+\omega_2+\omega_3)(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2R_1C_1) \;+$

$(2(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) \;+$

$(\omega_1+\omega_2+\omega_3)^2)3(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2) \;+$

$2((\omega_1+\omega_2+\omega_3)(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) \;+$

$\omega_1\omega_2\omega_3)R_1C_1(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2) \;+$

$\beta R_1C_1/(\omega_t\omega_2\omega_3)) \;+$

$s^{4}((2(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) \;+$

$(\omega_1+\omega_2+\omega_3)^2)(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2R_1C_1) \;+$

$2((\omega_1+\omega_2+\omega_3)(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) \;+$

$\omega_1\omega_2\omega_3)3(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2) \;+$

$(2\omega_1\omega_2\omega_3(\omega_1+\omega_2+\omega_3) \;+$

$(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)^2)R_1C_1(1+\alpha)/$

$(\omega_t^2\omega_2^2\omega_3^2) \;+\; 3\beta/(\omega_t\omega_2\omega_3) \;+$

$(\omega_1+\omega_2+\omega_3)R_1C_1\beta/(\omega_t\omega_2\omega_3)) \;+$

$s^{3}(2((\omega_1+\omega_2+\omega_3)(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) \;+$

$\omega_1\omega_2\omega_3)(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2R_1C_1) \;+$

$(2\omega_1\omega_2\omega_3(\omega_1+\omega_2+\omega_3) \;+$

$(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)^2)3(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2) \;+$

$2\omega_1\omega_2\omega_3(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)R_1C_1(1+\alpha)/$

$(\omega_t^2\omega_2^2\omega_3^2) \;+\; \beta/(\omega_t\omega_2\omega_3R_1C_1) \;+$

$3\beta(\omega_1+\omega_2+\omega_3)/(\omega_t\omega_2\omega_3)\ +$

$(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)R_1C_1\beta/(\omega_t\omega_2\omega_3))\ +$

$s^2((2\omega_1\omega_2\omega_3(\omega_1+\omega_2+\omega_3)\ +$

$(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)^2)(1+\alpha)/$

$(\omega_t^2\omega_2^2\omega_3^2R_1C_1)\ +$

$6\omega_1\omega_2\omega_3(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)(1+\alpha)/$

$(\omega_t^2\omega_2^2\omega_3^2)\ +$

$\omega_1^2\omega_2^2\omega_3^2R_1C_1(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2)\ +$

$\beta(\omega_1+\omega_2+\omega_3)/(\omega_t\omega_2\omega_3R_1C_1)\ +$

$3\beta(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)/(\omega_t^2\omega_2^2\omega_3^2)\ +$

$\beta\omega_1\omega_2\omega_3R_1C_1/(\omega_t\omega_2\omega_3)\ +\ R_1C_1/K)\ +$

$s(2\omega_1\omega_2\omega_3(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)(1+\alpha)$

$/(\omega_t^2\omega_2^2\omega_3^2R_1C_1)\ +$

$\omega_1^2\omega_2^2\omega_3^2(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2)\ +$

$\beta(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)/(\omega_t\omega_2\omega_3R_1C_1)\ +$

$3\beta\omega_1\omega_2\omega_3/(\omega_t\omega_2\omega_3)\ +\ 3/K\ -\ 1)\ +$

$\omega_1^2\omega_2^2\omega_3^2(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2R_1C_1)\ +$

$\beta\omega_1\omega_2\omega_3/(\omega_t\omega_2\omega_3R_1C_1)\ +\ 1/(KR_1C_1)\}$

$SKBOLTFM3SP\ =\ 1/\{s^3(R_1C_1(1+\alpha)/(\omega_t^2))\ +$

$s^2(2(1+\alpha)/(\omega_t^2)\ +\ \beta R_1C_1/(\omega_t))\ +\ s(2\beta/(\omega_t)\ +$

$R_1C_1/K)\ +\ 2/K\}$

$\mathrm{SKBOLTFM3DP} \;=\; 1/\{s^5(R_1C_1(1+\alpha)/(\omega_t^2\omega_2^2)) \;+$

$s^4(2(1+\alpha)/(\omega_t^2\omega_2^2) \;+$

$2(\omega_1+\omega_2)R_1C_1(1+\alpha)/(\omega_t^2\omega_2^2)) \;+$

$s^3(4(\omega_1+\omega_2)(1+\alpha)/(\omega_t^2\omega_2^2) \;+\; (2\omega_1\omega_2 \;+$

$(\omega_1+\omega_2)^2)R_1C_1(1+\alpha)/(\omega_t^2\omega_2^2) \;+$

$2\beta R_1C_1/(\omega_t\omega_2)) \;+\; s^2(2(2\omega_1\omega_2 \;+$

$(\omega_1+\omega_2)^2)(1+\alpha)/(\omega_t^2\omega_2^2) \;+$

$2\omega_1\omega_2(\omega_1+\omega_2)R_1C_1(1+\alpha)/(\omega_t^2\omega_2^2) \;+$

$2\beta/(\omega_t\omega_2) \;+\; \beta(\omega_1+\omega_2)R_1C_1/(\omega_t\omega_2)) \;+$

$s(4\omega_1\omega_2(\omega_1+\omega_2)(1+\alpha)/(\omega_t^2\omega_2^2) \;+$

$\omega_1^2\omega_2^2R_1C_1(1+\alpha)/(\omega_t^2\omega_2^2) \;+$

$2\beta(\omega_1+\omega_2)/(\omega_t\omega_2) \;+\; \beta\omega_1\omega_2R_1C_1/(\omega_t\omega_2) \;+$

$R_1C_1/K) \;+2\omega_1^2\omega_2^2(1+\alpha)/(\omega_t^2\omega_2^2) \;+$

$2\beta\omega_1\omega_2/(\omega_t\omega_2) \;+\; 2/K\}$

$\mathrm{SKBOLTFM3TP} \;=\; 1/\{s^7(R_1C_1(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2)) \;+$

$s^6(2(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2) \;+$

$2(\omega_1+\omega_2+\omega_3)R_1C_1(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2)) \;+$

$s^5(4(\omega_1+\omega_2+\omega_3)(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2) \;+$

$(2(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) \;+$

$(\omega_1+\omega_2+\omega_3)^2)R_1C_1(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2)) \;+$

$s^4((2(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) \;+$

$(\omega_1+\omega_2+\omega_3)^2)2(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2) \;+$

$2((\omega_1+\omega_2+\omega_3)(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) \;+$

$\omega_1\omega_2\omega_3)R_1C_1(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2) \;+$

$\beta R_1C_1/(\omega_t\omega_2\omega_3)) \;+$

$s^3(4((\omega_1+\omega_2+\omega_3)(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) \;+$

$\omega_1\omega_2\omega_3)(1+\alpha)/(\omega_t{}^2\omega_2{}^2\omega_3{}^2)$ +

$(2\omega_1\omega_2\omega_3(\omega_1+\omega_2+\omega_3)$ +

$(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)^2)R_1C_1(1+\alpha)/$

$(\omega_t{}^2\omega_2{}^2\omega_3{}^2)$ + $2\beta/(\omega_t\omega_2\omega_3)$ +

$\beta R_1C_1(\omega_1+\omega_2+\omega_3)/(\omega_t\omega_2\omega_3))$ +

$s^2((2\omega_1\omega_2\omega_3(\omega_1+\omega_2+\omega_3)$ +

$(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)^2)2(1+\alpha)/(\omega_t{}^2\omega_2{}^2\omega_3{}^2)$ +

$2\omega_1\omega_2\omega_3(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)R_1C_1(1+\alpha)/$

$(\omega_t{}^2\omega_2{}^2\omega_3{}^2)$ + $2\beta(\omega_1+\omega_2+\omega_3)/(\omega_t\omega_2\omega_3)$ +

$\beta(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)R_1C_1/(\omega_t\omega_2\omega_3))$ +

$s(4\omega_1\omega_2\omega_3(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)(1+\alpha)$

$/(\omega_t{}^2\omega_2{}^2\omega_3{}^2)$ +

$\omega_1{}^2\omega_2{}^2\omega_3{}^2(1+\alpha)R_1C_1/(\omega_t{}^2\omega_2{}^2\omega_3{}^2)$ +

$2\beta(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)/(\omega_t\omega_2\omega_3)$ +

$\beta\omega_1\omega_2\omega_3R_1C_1/(\omega_t\omega_2\omega_3)$ + $R_1C_1/K)$ +

$2\omega_1{}^2\omega_2{}^2\omega_3{}^2(1+\alpha)/(\omega_t{}^2\omega_2{}^2\omega_3{}^2)$ +

$2\beta\omega_1\omega_2\omega_3/(\omega_t\omega_2\omega_3)$ + $2/K\}$

The M4 Composite Operational Amplifier

$$GFM4 = 1/\{(1+\alpha)/(A_1A_2) + (1+\alpha)/(A_1K) + 1/K\}$$

$$GFM4SP = 1/\{s^2(1+\alpha)/\omega_t^2 + s(1+\alpha)/(K\omega_t) + 1/K\}$$

$$GFM4DP = 1/\{(1+\alpha)(s^4 + s^3 2(\omega_1+\omega_2) + s^2(2\omega_1\omega_2 + (\omega_1+\omega_2)^2) + s2\omega_1\omega_2(\omega_1+\omega_2) + \omega_1^2\omega_2^2)/(\omega_t^2\omega_2^2) + (1+\alpha)(s^2 + s(\omega_1+\omega_2) + \omega_1\omega_2)/(K\omega_t\omega_2) + 1/K\}$$

$$GFM4TP = 1/\{(1+\alpha)(s^6 + s^5 2(\omega_1+\omega_2+\omega_3) + s^4(2(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) + (\omega_1+\omega_2+\omega_3)^2) + s^3 2((\omega_1+\omega_2+\omega_3)(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) + \omega_1\omega_2\omega_3) + s^2(2\omega_1\omega_2\omega_3(\omega_1+\omega_2+\omega_3) + (\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)^2) + s2\omega_1\omega_2\omega_3(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) + \omega_1^2\omega_2^2\omega_3^2)/(\omega_t^2\omega_2^2\omega_3^2) + (1+\alpha)(s^3 + s^2(\omega_1+\omega_2+\omega_3) + s(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) + \omega_1\omega_2\omega_3)/(K\omega_t\omega_2\omega_3) + 1/K\}$$

$$SKBCLTFM4SP = s/\{s^4((1+\alpha)R_1C_1/\omega_t^2) + s^3(3(1+\alpha)/\omega_t^2 + (1+\alpha)R_1C_1/(K\omega_t)) + s^2((1+\alpha)/(\omega_t^2R_1C_1) + 3(1+\alpha)/(K\omega_t) + R_1C_1/K) + s((1+\alpha)/(K\omega_tR_1C_1) + 3/K - 1) + 1/KR_1C_1\}$$

$SKBCLTFM4DP = s/\{s^6((1+\alpha)R_1C_1/(\omega_t^2\omega_2^2)) +$

$s^5(3(1+\alpha)/(\omega_t^2\omega_2^2) +$

$2(\omega_1+\omega_2)(1+\alpha)R_1C_1/(\omega_t^2\omega_2^2)) +$

$s^4((1+\alpha)/(\omega_t^2\omega_2^2R_1C_1) +$

$6(\omega_1+\omega_2)(1+\alpha)/(\omega_t^2\omega_2^2) + (2\omega_1\omega_2 +$

$(\omega_1+\omega_2)^2)(1+\alpha)R_1C_1/(\omega_t^2\omega_2^2) +$

$(1+\alpha)R_1C_1/(K\omega_t\omega_2)) +$

$s^3(2(\omega_1+\omega_2)(1+\alpha)/(\omega_t^2\omega_2^2R_1C_1) + (2\omega_1\omega_2$

$+ (\omega_1+\omega_2)^2)(1+\alpha)3/(\omega_t^2\omega_2^2) +$

$2\omega_1\omega_2(\omega_1+\omega_2)R_1C_1(1+\alpha)/(\omega_t^2\omega_2^2) +$

$3(1+\alpha)/(K\omega_t\omega_2) +$

$(1+\alpha)(\omega_1+\omega_2)R_1C_1/(K\omega_t\omega_2)) + s^2((2\omega_1\omega_2 +$

$(\omega_1+\omega_2)^2)(1+\alpha)/(\omega_t^2\omega_2^2R_1C_1) +$

$6\omega_1\omega_2(\omega_1+\omega_2)(1+\alpha)/(\omega_t^2\omega_2^2) +$

$\omega_1^2\omega_2^2R_1C_1(1+\alpha)/(\omega_t^2\omega_2^2) +$

$(1+\alpha)/(K\omega_t\omega_2R_1C_1) + 3(1+\alpha)(\omega_1+\omega_2)/(K\omega_t\omega_2)$

$+ (1+\alpha)\omega_1\omega_2R_1C_1/(K\omega_t\omega_2) + R_1C_1/K) +$

$s(2\omega_1\omega_2(\omega_1+\omega_2)(1+\alpha)/(\omega_t^2\omega_2^2R_1C_1) +$

$3\omega_1^2\omega_2^2(1+\alpha)/(\omega_t^2\omega_2^2) +$

$(1+\alpha)(\omega_1+\omega_2)/(K\omega_t\omega_2R_1C_1) +$

$3(1+\alpha)\omega_1\omega_2/(K\omega_t\omega_2) + 3/K - 1) +$

$\omega_1^2\omega_2^2(1+\alpha)/(\omega_t^2\omega_2^2R_1C_1) +$

$(1+\alpha)\omega_1\omega_2/(K\omega_t\omega_2R_1C_1) + 1/(KR_1C_1)\}$

$SKBCLTFM4TP = s/\{s^8(R_1C_1(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2) +$

$s^7(3(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2) +$

$2(\omega_1+\omega_2+\omega_3)R_1C_1(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2)) +$

$$s^6((1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2R_1C_1) +$$
$$6(\omega_1+\omega_2+\omega_3)(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2) +$$
$$(2(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) +$$
$$(\omega_1+\omega_2+\omega_3)^2)R_1C_1(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2)) +$$
$$s^5(2(\omega_1+\omega_2+\omega_3)(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2R_1C_1) +$$
$$(2(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) +$$
$$(\omega_1+\omega_2+\omega_3)^2)3(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2) +$$
$$2((\omega_1+\omega_2+\omega_3)(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) +$$
$$\omega_1\omega_2\omega_3)R_1C_1(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2) +$$
$$(1+\alpha)R_1C_1/(K\omega_t\omega_2\omega_3)) +$$
$$s^4((2(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) +$$
$$(\omega_1+\omega_2+\omega_3)^2)(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2R_1C_1) +$$
$$6((\omega_1+\omega_2+\omega_3)(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) +$$
$$\omega_1\omega_2\omega_3)(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2) +$$
$$(2\omega_1\omega_2\omega_3(\omega_1+\omega_2+\omega_3) +$$
$$(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)^2)R_1C_1(1+\alpha)/$$
$$(\omega_t^2\omega_2^2\omega_3^2) + 3(1+\alpha)/(K\omega_t\omega_2\omega_3) +$$
$$(\omega_1+\omega_2+\omega_3)R_1C_1(1+\alpha)/(K\omega_t\omega_2\omega_3)) +$$
$$s^3(2((\omega_1+\omega_2+\omega_3)(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3) +$$
$$\omega_1\omega_2\omega_3)(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2R_1C_1) +$$
$$(2\omega_1\omega_2\omega_3(\omega_1+\omega_2+\omega_3) +$$
$$(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)^2)3(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2) +$$
$$2\omega_1\omega_2\omega_3(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)R_1C_1(1+\alpha)/$$
$$(\omega_t^2\omega_2^2\omega_3^2) + (1+\alpha)/(K\omega_t\omega_2\omega_3R_1C_1) +$$
$$3(1+\alpha)(\omega_1+\omega_2+\omega_3)/(K\omega_t\omega_2\omega_3). +$$
$$(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)R_1C_1(1+\alpha)/(K\omega_t\omega_2\omega_3))$$

$+ \quad s^2((2\omega_1\omega_2\omega_3(\omega_1+\omega_2+\omega_3) \quad +$

$(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)^2)(1+\alpha)/$

$(\omega_t^2\omega_2^2\omega_3^2R_1C_1) \quad +$

$6\omega_1\omega_2\omega_3(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)(1+\alpha)/$

$(\omega_t^2\omega_2^2\omega_3^2) \quad +$

$\omega_1^2\omega_2^2\omega_3^2R_1C_1(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2) \quad +$

$(1+\alpha)(\omega_1+\omega_2+\omega_3)/(K\omega_t\omega_2\omega_3R_1C_1) \quad +$

$3(1+\alpha)(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)/(K\omega_t\omega_2\omega_3) \quad +$

$(1+\alpha)\omega_1\omega_2\omega_3R_1C_1/(K\omega_t\omega_2\omega_3) \quad + \quad R_1C_1/K) \quad +$

$s(2\omega_1\omega_2\omega_3(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)(1+\alpha)$

$/(\omega_t^2\omega_2^2\omega_3^2R_1C_1) \quad +$

$\omega_1^2\omega_2^2\omega_3^2(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2) \quad +$

$(1+\alpha)(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)/$

$(K\omega_t\omega_2\omega_3R_1C_1) \quad +$

$3(1+\alpha)\omega_1\omega_2\omega_3/(K\omega_t\omega_2\omega_3) \quad + \quad 3/K \quad - \quad 1) \quad +$

$\omega_1^2\omega_2^2\omega_3^2(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2R_1C_1) \quad +$

$(1+\alpha)\omega_1\omega_2\omega_3/(K\omega_t\omega_2\omega_3R_1C_1) \quad +$

$1/(KR_1C_1)\}$

$SKBOLTFM4SP \quad = \quad 1/\{s^3(R_1C_1(1+\alpha)/(\omega_t^2)) \quad +$

$s^2(2(1+\alpha)/(\omega_t^2) \quad + \quad (1+\alpha)R_1C_1/(K\omega_t)) \quad +$

$s(2(1+\alpha)/(K\omega_t) \quad + \quad R_1C_1/K) \quad + \quad 2/K\}$

$SKBOLTFM4DP = 1/\{s^5(R_1C_1(1+\alpha)/(\omega_t^2\omega_2^2))$ +

$s^4(2(1+\alpha)/(\omega_t^2\omega_2^2)$ +

$2(\omega_1+\omega_2)R_1C_1(1+\alpha)/(\omega_t^2\omega_2^2))$ +

$s^3(4(\omega_1+\omega_2)(1+\alpha)/(\omega_t^2\omega_2^2)$ + $(2\omega_1\omega_2$ +

$(\omega_1+\omega_2)^2)R_1C_1(1+\alpha)/(\omega_t^2\omega_2^2)$ +

$2(1+\alpha)R_1C_1/(K\omega_t\omega_2))$ + $s^2(2(2\omega_1\omega_2$ +

$(\omega_1+\omega_2)^2)(1+\alpha)/(\omega_t^2\omega_2^2)$ +

$2\omega_1\omega_2(\omega_1+\omega_2)R_1C_1(1+\alpha)/(\omega_t^2\omega_2^2)$ +

$2(1+\alpha)/(K\omega_t\omega_2)$ +

$(1+\alpha)(\omega_1+\omega_2)R_1C_1/(K\omega_t\omega_2))$ +

$s(4\omega_1\omega_2(\omega_1+\omega_2)(1+\alpha)/(\omega_t^2\omega_2^2)$ +

$\omega_1^2\omega_2^2R_1C_1(1+\alpha)/(\omega_t^2\omega_2^2)$ +

$2(1+\alpha)(\omega_1+\omega_2)/(K\omega_t\omega_2)$ +

$(1+\alpha)\omega_1\omega_2R_1C_1/(K\omega_t\omega_2)$ + $R_1C_1/K)$

$+2\omega_1^2\omega_2^2(1+\alpha)/(\omega_t^2\omega_2^2)$ +

$2(1+\alpha)\omega_1\omega_2/(K\omega_t\omega_2)$ + $2/K\}$

$SKBOLTFM4TP = 1/\{s^7(R_1C_1(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2))$ +

$s^6(2(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2)$ +

$2(\omega_1+\omega_2+\omega_3)R_1C_1(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2))$ +

$s^5(4(\omega_1+\omega_2+\omega_3)(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2)$ +

$(2(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)$ +

$(\omega_1+\omega_2+\omega_3)^2)R_1C_1(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2))$ +

$s^4((2(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)$ +

$(\omega_1+\omega_2+\omega_3)^2)2(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2)$ +

$2((\omega_1+\omega_2+\omega_3)(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)$ +

$\omega_1\omega_2\omega_3)R_1C_1(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2)$ +

$(1+\alpha)R_1C_1/(K\omega_t\omega_2\omega_3))$ +

$s^3(4((\omega_1+\omega_2+\omega_3)(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)$ +

$\omega_1\omega_2\omega_3)(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2)$ +

$(2\omega_1\omega_2\omega_3(\omega_1+\omega_2+\omega_3)$ +

$(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)^2)R_1C_1(1+\alpha)/$

$(\omega_t^2\omega_2^2\omega_3^2)$ + $2(1+\alpha)/(K\omega_t\omega_2\omega_3)$ +

$(1+\alpha)R_1C_1(\omega_1+\omega_2+\omega_3)/(K\omega_t\omega_2\omega_3))$ +

$s^2((2\omega_1\omega_2\omega_3(\omega_1+\omega_2+\omega_3)$ +

$(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)^2)2(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2)$ +

$2\omega_1\omega_2\omega_3(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)R_1C_1(1+\alpha)/$

$(\omega_t^2\omega_2^2\omega_3^2)$ +

$2(1+\alpha)(\omega_1+\omega_2+\omega_3)/(K\omega_t\omega_2\omega_3)$ +

$(1+\alpha)(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)R_1C_1/(K\omega_t\omega_2\omega_3))$

+ $s(4\omega_1\omega_2\omega_3(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)(1+\alpha)/$

$(\omega_t^2\omega_2^2\omega_3^2)$ +

$\omega_1^2\omega_2^2\omega_3^2(1+\alpha)R_1C_1/(\omega_t^2\omega_2^2\omega_3^2)$ +

$2(1+\alpha)(\omega_1\omega_2+\omega_1\omega_3+\omega_2\omega_3)/(K\omega_t\omega_2\omega_3)$ +

$(1+\alpha)\omega_1\omega_2\omega_3R_1C_1/(K\omega_t\omega_2\omega_3)$ + $R_1C_1/K)$ +

$2\omega_1^2\omega_2^2\omega_3^2(1+\alpha)/(\omega_t^2\omega_2^2\omega_3^2)$ +

$2(1+\alpha)\omega_1\omega_2\omega_3/(K\omega_t\omega_2\omega_3)$ + $2/K\}$

APPENDIX V

The Nyquist Diagram

The Nyquist Diagram [38] is a graphical method of determining the stability of a system under closed loop conditions based on the systems open loop frequency response.

Basically the Stability Criterion may be stated as follows :

> 'a closed loop system will be stable if, when moving along the open loop frequency response, plotted on a Nyquist Diagram, in the direction of increasing frequency, the point (-1,j0) lies on the left of the locus.'

The relative stability of a system can be determined from the Nyquist Diagram in terms of gain and phase margins.

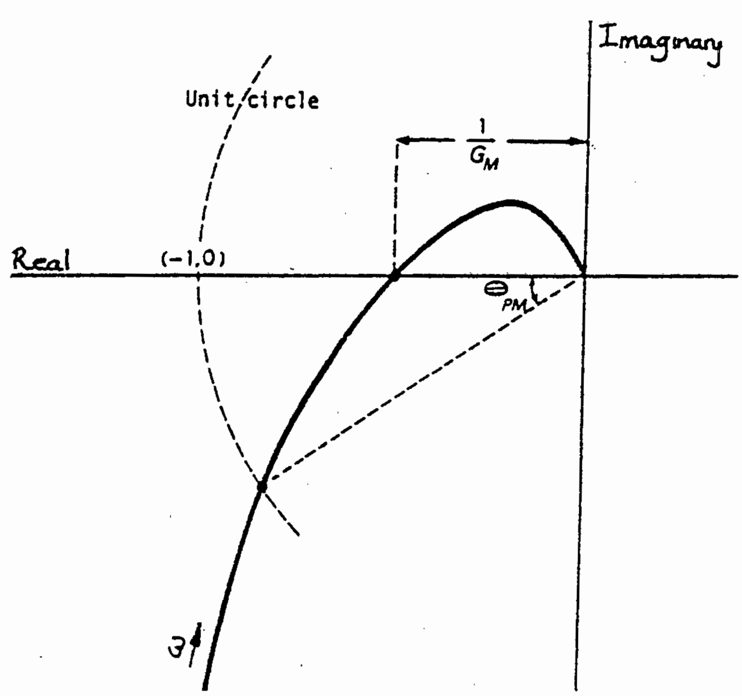
The gain margin is defined as the reciprocal of the magnitude of the open loop transfer function when the phase shift is 180 degrees. Thus the gain margin represents the factor by which the gain must be increased to cause the system to be unstable.

The other measure of relative stability is that of phase margin. This is defined as 180 degrees minus the phase angle of the open loop transfer function at the frequency where unity gain is achieved.

From the above definitions it can be seen that for a gain margin less than one or a negative phase margin the system is unstable.

The sketch below depicts a stable system, where $1/G_M$ is the gain margin and θ_{PM} the phase margin. Further, following the locus of the open loop frequency response in the direction of increasing frequency the point (-1,j0) is seen to lie to the left of the locus, thus indicating a stable system.

This technique of stability evaluation is used for the M2 COA on page 77.



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