THE COMPARISON AND EVALUATION OF DIFFERENT MATHEMATICAL MODELS FOR DEFORMATION ANALYSIS

by
Robert J Goullee, B.Sc. (Survey),
Cape Town

Submitted to the University of Cape Town in
fulfillment of the requirements for the
degree of Master of Science in Engineering,
April 1984

ABSTRACT

In the analysis of deformations using geodetic techniques, the errors in point positions due to observation errors must be distinguished from movements due to actual deformation. A number of models are available, which offer solutions to this problem.

In this study, four of such methods are described and compared:

1. Method using Invariant Functions.
2. Method using Direct Comparison of Co-ordinates.
3. Method using Direct Differences.
4. Method using Niemeier's Comparison of Co-ordinates.

The introduction of "false" deformations, caused by errors in translation, rotation and scale, is a very real problem which may be eliminated by processes such as the use of invariant functions (distances and angles) and the sound construction of constraint points. Niemeier's solution to this problem is the use of a free network adjustment which forces the new network into a best fit of the provisional co-ordinates, which generally would be the final co-ordinates of a previous epoch.

Although the model advocated for the first three methods above is the minimum constraint adjustment, the free network adjustment may also be used. Similarly, the minimum constraints technique may be employed for Niemeier's method, subject to some necessary modifications. The four methods have thus been compared using both adjustment techniques also.

The four methods using both adjustment techniques as well as some variations of methods 1. and 2. above are evaluated using a series of nine simulated test epochs, one reference and eight other, to which known deformations were applied.

From the results obtained from the various epochs, the methods are examined for reliability, accuracy and suitability.

ACKNOWLEDGEMENTS

I would like to thank the following people and bodies, without whom this study would not have been possible.

Dr. H. Rüther, University of Cape Town, supervisor, for his support and guidance, as well as for the use of two of his programmes, the free network adjustment and random error generator.

The firm Cabinet Albenque, Koeberg Nuclear Power Station, for time and facilities, enabling me to complete the thesis.

The Tobias van der Spuy Trust Bursary for their financial support.

Mr. J. Jackson, University of Cape Town, supervisor, for his guidance and support during the final stages of the thesis.

CONTENTS

1.	INTRODUCTION	Page
1.1	Observation of two epochs	1
1.2	Adjustment of the epochs	3
1.3	Comparison of the two epochs	3
1.4	Determination of the Deformations and magnitude of the displacements	5
2.	NETWORK ADJUSTMENT	6
2.1	Least squares adjustment of a network (two - dimensional)	6
2.2	Weights	9
2.3	Constraints	10
2.4	Error Analysis	10
2.4.1	Variance factor	11
2.4.2	Variance-covariance matrix	11
2.4.3	Absolute error ellipse	12
2.4.4	Relative error ellipses	13
2.4.5	A Posteriori variances of the observations	14
2.4.6	Confidence of variances	14
3.	DEFORMATION ANALYSIS	15
3.1	Method of Invariant Functions	15
3.1.1	Adjustment of the two epochs	17
3.1.2	Determination of suspected deformed points	17
3.1.3	Determination of deformed points and magnitude of deformations	23
3.1.4	Flow chart	28
3.1.5	Possible variation of the method	29
3.2	Direct Comparison of Co-ordinates	30
3.2.1	Reference Points	31
3.2.2	Scale Factor	31
3.2.3	Adjustment of two epochs	32
3.2.4	Helmert Transformation of second epoch onto the first	33
3.2.5	Determination of deformations and deformation magnitudes	34
3.2.6	Flow chart	36

Page

3.3 Method of Direct Differences 37
3.3.1 Adjustment of epochs 37
3.3.2 Weights 39
3.3.3 Determination of deformations and deformation magnitudes 40
3.3.3.1 Determination of distance from centre to curve of ellipse 41
3.3.4 Flow chart 43

3.4 Niemeier's Comparison of Co-ordinates 44
3.4.1 Adjustment of the epochs 44
3.4.2 Test on network to determine if deformations must be suspected 45
3.4.3 Partitioning of the network into reference and object points 46
3.4.4 Test on reference points 48
3.4.5 Identification of unstable point in reference points 49
3.4.6 Verification of deformed points and magnitude of deformations 50
3.4.7 Modifications to accommodate a minimum constraints adjustment 52
3.4.8 Flow chart 55

4. NETWORK USED FOR ANALYSIS 56

5. COMPARISON OF RESULTS 59

5.1 Epoch A 59
5.2 Epoch B 60
5.3 Epoch C 65
5.4 Epoch D 73
5.5 Epoch E 76
5.6 Epoch F 81
5.7 Epoch G 86
5.8 Epoch J 87
5.9 Epoch HR 92
5.10 Peculiarities in the methods 96
5.10.1 Absolute error ellipse parameter anomalies 96
5.10.1.1 Matrix configuration 100
5.10.1.2 Different Provisional Co-ordinates 100
5.10.1.3 Additional Observations 101

Page

5.10.1.4 Inversion of larger A^TPA matrix 101
5.10.2 Transformation of free network co-ordinates 102
5.10.3 Similarity of results from three of the methods 103

6. ANALYSIS OF RESULTS 105

6.1 Comparison of Methods 105
6.1.1 Mean of the squares of the differences 106
6.1.2 Position frequencies 107
6.1.3 Errors of Type I and Type II 110
6.1.3.1 Inclusion of method Ab 111
6.1.4 Execution time and programmes 112

6.2 Analysis of Methods 113
6.2.1 Epochs B to F 113
6.2.1.1 NF: Niemeier's comparison of co-ordinates 114
6.2.1.2 IM: Invariant functions (minimum constraints) 114
6.2.1.3 DF: Direct differences (free net.) 114
6.2.1.4 SF & Ab: Stable point transformation (free net) with minimum constraints adjustment holding stable points fixed 115
6.2.2 Epoch J 115
6.2.3 Epochs B to J pooled 116

6.3 Comparison of Adjustment techniques 116
6.4 The Deformation of constraint points 119
6.5 Combination of Methods 119
6.6 Network accuracy 119
6.7 Precautions in regard to Niemeier's method using minimum constraints 121

7. CONCLUSIONS 122

8. REFERENCES 125

ANNEXURE A : Theory of the Least squares adjustment
ANNEXURE B : The least squares solution of the Helmert transformation
ANNEXURE C : Programmes and sample Printouts.

CHAPTER 1

INTRODUCTION

As engineering projects become more complex and tolerances closer, there is an increasing demand for the monitoring of deformations in structures. Deformation monitoring fulfills the function of studying the, often, long term deformation of a deformable object or body. Examples of deformable bodies include dam walls, buildings, tectonic faults and survey networks used for precise engineering projects.

A variety of specialised equipment is available for the monitoring of deformations which include pendulums, extensometers and strain gauges. These instruments usually measure length changes and tilts within a deforming structure; this movement is known as relative deformation. In some cases, such as dam deformation, it is necessary to monitor movement relative to a fixed reference frame. This is known as absolute deformation. It requires that the reference points used to determine the deformation include points which are situated away from the object and which are therefore not subject to any deformation of the body. Relative deformation relies on reference points all of which are on the deformable object and thus any deformation of the body is related to these points which may also be subject to some displacement.

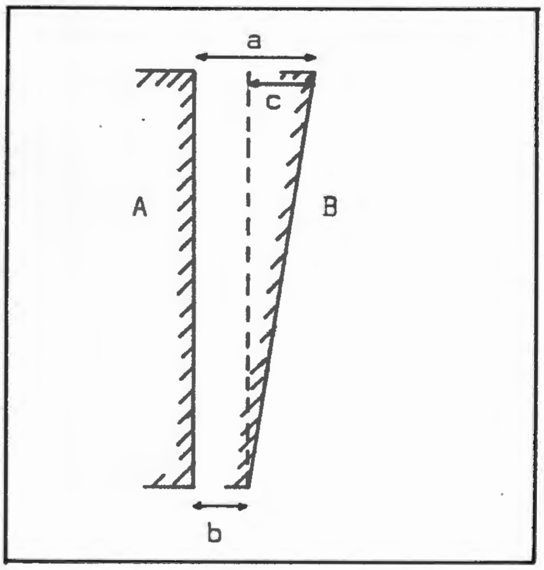


Figure 1.

The concept of absolute and relative deformation can be clearly seen from figure 1. The wall A has been deformed to position B in the time between two measurements. The absolute deformation of the wall is indicated by the displacements a and b. If, however, the displacement of the top of the wall is related to the bottom, then the displacement c would indicate this deformation. This displacement c is the relative deformation, which obviously may not always indicate the true displacement a.

Absolute deformation monitoring may employ photogrammetry or geodetic techniques for example.

The scope of this thesis encompasses the following requirements:

a) The comparison and evaluation of the four following approaches to deformation analysis for two dimensional networks using geodetic techniques.
   1. The Method of Invariant Functions - Chrzanowski 1981
   2. The Method of Direct Comparison of Co-ordinates - Ashkenazi and Dodson 1978.
   3. The Method of Direct Differences - Chrzanowski 1982
   4. Niemeier's Comparison of Co-ordinates - Niemeier 1976

b) The assessment of the approaches using free network as well as minimum constraints adjustments. This requirement would naturally include any theoretical modification needed to employ the different adjustment.

c) In the study of the four approaches to consider any variations which may be feasible. For example, in the method of direct comparison of co-ordinates, Ashkenazi (1981) mentioned the use of a Helmert transformation to minimise the effect of systematic errors on true deformation, i.e. that systematic errors could be incorrectly interpreted as deformation.

In the geodetic analysis of deformations, the procedures may be reduced to four not necessarily separated stages,viz

1. The observation of the two Epochs.
2. The adjustment of the two Epochs.
3. The comparison of the two Epochs.
4. The determination of the deformed points and the magnitude of the displacements.

These stages will be introduced briefly below.

1.1 <u>The observation of the two epochs</u>

Two sets of observations for a network are required for the analysis and each is assumed to take place over an instant of time referred to as an epoch. The relevant sets of measurements are differentiated by assigning numbers or letters to them, e.g. Epoch 1 and Epoch 2. Sufficient observations are required to allow the adjustment of each epoch to be performed separately. The observations usually combine either angles and distances or directions and distances, although networks relying on only one type of observation (eg. directions) are also used.
As is known, observations are made between stations which include beacon pillars and visual targets.

1.2 <u>The adjustment of the Epochs</u>

Of the four methods discussed, only the method of direct differences does not require the separate adjustment of the two Epochs, yielding two sets of "final" co-ordinates for the network.

Usually either a minimum constraint or a free network technique of adjustment is employed, but in this thesis both methods are used in each case.
As the adjustment using minimum constraints requires that certain co-ordinates be held fixed, assuming no movement in the points, it is preferable, and, in the case of direct co-ordinate comparison, essential to ensure that these fixed points are in fact stable (Ashkenazi and Dodson 1978). An initial hypothesis as to which stations are stable may be obtained by comparing the raw observations. The difference between corresponding observations from the two epochs are compared with a value equivalent to, say, twice the apriori standard deviation of the observation. If the majority of observation differences are less than this value then the point can be initially hypothesised as stable. It should be noted that although this scanning of the raw observations may give an indication of which points are doubtful, the full deformation procedure must still be executed to yield a true reflection of the deformations, if any.

1.3 The Comparison of the two epochs

Following the adjustment of the epochs, it is necessary to perform a comparison of the two sets of results. The first step of the comparison usually involves a global test on the network to determine whether the model is in fact subject to a deformation, while the second step would involve a test to determine which points in the network are doubtful. This stage will yield the points that have been deformed at the stipulated confidence level, but the following step is usually necessary to confirm these deformations, as stable points may be included as doubtful. This comparison may inter alia take the form of direct comparison of the adjusted co-ordinates, or a comparison of the adjusted distances and angles in the network.

1.4 The determination of the deformation and the magnitude of the displacement

This stage yields, finally, the points which are judged to have been deformed as well as the magnitude of their displacements. The determination of the deformation also involves a test. This test usually compares the displacement with the relative error ellipse or standard deviations of the displacements. If the test fails, the point is finally confirmed as having been deformed and its displacement given as the difference between its positions at the two epochs of measurement.

The four stages discussed above apply to all the methods of deformation analysis using geodetic monitoring. However, although in a method such as the invariant function procedure, the four are distinctly separated, a method such as the direct differences combines the last three stages into one operation. A method which combines the stages may appear to be preferable for economical reasons. However, other merits and/or disadvantages should be considered before such a method is finally chosen.

CHAPTER 2

NETWORK ADJUSTMENT

In geodetic networks, the observations for an individual epoch must be processed by a least squares adjustment before any meaningful results can be obtained. Two forms of constraining the net to the computing surface may be used:

1. Adjustment using Minimum Constraints
2. Adjustment using Inner Constraints (Free Network).

In the Minimum Constraints adjustment, a minimum number of co-ordinates are held fixed (therefore error free). These constraints serve the purpose of providing a reference for translations, rotations and scale factors.

In the free network adjustment, the net is "best fitted" onto a point field of provisional co-ordinates or co-ordinates established in a previous epoch, with the shape of the network remaining unaffected by the choice of these reference points. In this type of adjustment all co-ordinates are subject to change and point position accuracies are obtained for every point of the network.

These adjustments also yield the error properties of the network on which deformation analysis using geodetic methods depends.

2.1 The Least squares adjustment of a Network (two dimensional)

The least squares adjustment of a network provides a homogenous solution for a network, in that the effect of observational errors are distributed throughout the network.

For a two dimensional network, three types of observations can be made:

1. Directions
2. Angles
3. Distances

As the object of the adjustment is to provide "final" or most probable values for the co-ordinates, the above three types of observations are expressed as functions of the co-ordinates x and y.

A full description of the theory of least squares adjustment by the method of variation of co-ordinates is given in Annexure A. Its essential features are summarised below for purpose of establishing nomenclature. The observations are related to the components of x and y, and this to the unknowns(x_i) in an observation equation of the form:

$$L_i + v_i = f\,(x_1, x_2, x_3, - - - - x_n) \text{ weight } p_i \quad (1)$$

From (1) we may obtain the observation equations for direction, angle and distance, viz.

1) Direction. $$L_{ik} + v_{ik} = \tan^{-1} \frac{(y_k - y_i)}{(x_k - x_i)} - z_i \quad (2)$$

2) Angle. $$L_{jik} + v_{jik} = \tan^{-1} \frac{(y_j - y_i)}{(x_j - x_i)} - \tan^{-1} \frac{(y_k - y_i)}{(x_k - x_i)} \quad (3)$$

3) Distance. $$L_{ik} + v_{ik} = \sqrt{(x_k - x_i)^2 + (y_k - y_i)^2} \quad (4)$$

where :

L_i = observation

v_i = correction to the observation

y_i, x_i = final co-ordinates of the points involved in the observation

z_i = orientation correction at point i.

The least squares solution, as is well known, is only valid for linear functions and the equations (2), (3) and (4) must therefore first be linearised before being employed in the adjustment. This is achieved by the use of the linear terms of a Taylor's series expansion.

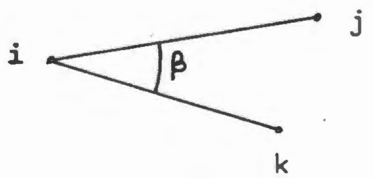
The above equations then become:

1) <u>Direction</u>

$$v_{ik} = a_{ik}\, dx_i + b_{ik}\, dy_i - a_{ik}\, dx_k - b_{ik}\, dy_k - dz_i - \ell_{ik} \qquad (5)$$

2) <u>Angle</u>

$$v_{jik} = (a_{ik} - a_{ij})\, dx_i + (b_{ik} - b_{ij})\, dy_i + a_{ij} dx_j + b_{ij} dy_j - a_{ik} dx_k - b_{ik} dy_k - \ell_{jik} \qquad (6)$$



3) <u>Distance</u>

$$\bar{v}_{ik} = \bar{a}_{ik}\, dx_i + \bar{b}_{ik}\, dy_i - \bar{a}_{ik}\, dx_k - \bar{b}_{ik}\, dy_k - \bar{\ell} \qquad (7)$$

where:

v_{ik}, v_{jik}, $\bar{v}_{ik}$	= corrections to observations
a_{ik}, b_{ik}, a_{ij}, b_{ij}, $\bar{a}_{ik}$, $\bar{b}_{ik}$	= co-efficients from the linearisation of the observation equations
dx, dy, dz	= correction to the unknowns
ℓ, $\bar{\ell}$	= free terms

The set of observation equations can be conveniently written as

$$\underline{v} = \underline{Ax} - \underline{\ell} \tag{8}$$

in matrix notation.

With the least squares principle of $\underline{v^TPv}$ = a minimum, the least squares solution for the adjustment becomes:

$$\underline{x} = (\underline{A}^T\underline{PA})^{-1}\ \underline{A}^T\underline{P\ell} = \underline{N}^{-1}\underline{A}^T\underline{P\ell} \tag{9}$$

2.2 Weights (Annex A, A.3)

In order to differentiate between different types of observations as well as observations of different quality, weights are applied to the observations. The weight p_i is given by

$$p_i = \frac{\sigma_o^2}{\sigma_i^2} \tag{10}$$

where:

σ_o^2 = variance factor

σ_o = standard deviation of unit weight

σ_i = a priori standard deviation of an observation L_i

As the variance factor is a function of the corrections to the observations, v, which are only determined after the adjustment, it becomes necessary to choose a value for the variance factor apriori (ie. before the adjustment). This can be chosen as being equal to 1 or may be set equal to the square of one of the observation standard deviations, σ_i, where σ_i would be chosen on the basis of a sound knowledge of a particular instrument, or from a series of test observations with the instrument.

2.3 Constraints (Annex A, A.4)

Constraints are applied to the adjustment in order to provide a reference for rotation, translation, compression or expansion in a network. In the minimum constraints method of adjustment, the constraints take the form of fixed (error free) co-ordinates. In a free network adjustment the reference is transferred to the centre of gravity of the system and the sum of the squares of the corrections to the unknown, $\underline{x}^T\underline{x}$ is a minimum, thereby removing the need to hold any particular co-ordinates fixed.

2.4 Error Analysis (Annex A, A.8)

The error analysis of a network, while normally important, becomes essential in the analysis of deformation. As is well known, observations are not error free, and, because of this, cause slight variations between network point co-ordinates, if these are determined more than once (viz. Epochs 1 and 2). The object of a deformation analysis is essentially that of differentiating between this slight variation (noise) and a real movement caused by deformation (signal). This concept can be clearly seen using the graphical representation of point accuracy, the absolute error ellipse (see Annexure A, A.8) which represents the area wherein variations, due to observation errors, may occur.

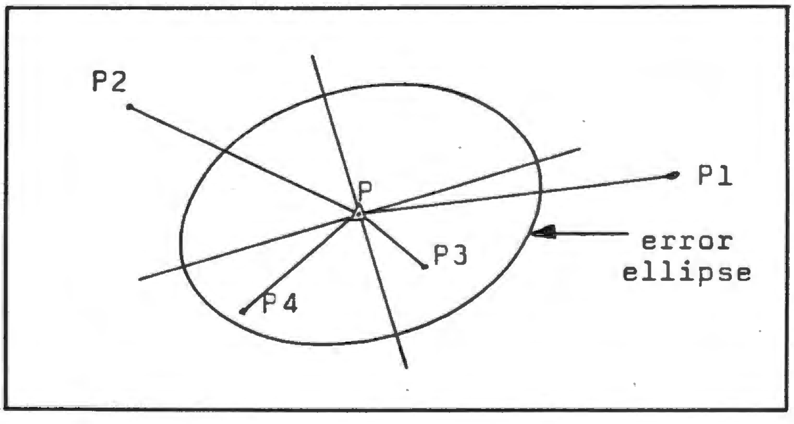


Figure 2.

In Figure 2, P represents a point fixed during one epoch and the ellipse represents the area within which the true value of the point may be expected to lie, at a particular confident level.

If P_1, P_2, P_3, P_4 are positions as determined in later epochs, then the apparent movements $P-P_1$ and $P-P_2$ probably represent signal or actual deformation, whereas $P-P_3$ and $P-P_4$ probably represent noise.

2.4.1 Variance Factor (Annex A, A.8.1)

The a posteriori value for the variance factor is found from

$$\sigma_o^2 = \frac{\underline{v}^T \underline{P} \underline{v}}{f} \qquad (11)$$

where:

$\underline{v}$ = vector of corrections to the observations
$\underline{P}$ = matrix of weights
f = degrees of freedom = n-u (min. constraints)or
= n-u+d (free network)
n = number of observations
u = number of unknowns
d = rank defect of the model

The aposteriori variance factor should be tested against the apriori value by means of the Chi-square test. The failure of this test indicates a significant difference between the aposteriori and apriori values and suggests inter alia, that the functional or stochastic model adopted for the adjustment may be incorrect, and the cause should be investigated before any further analysis is performed.

2.4.2 Variance - Covariance Matrix (V-C Matrix) (Annex A, A.8.2)

The V-C matrix is derived from:

$$\text{V-C matrix} = \sigma_o^2\ (\underline{A}^T\underline{P}\underline{A})^{-1} = \sigma_o^2\ \underline{Q} \qquad (12)$$

with the terms on the principal diagonal being the variances of the unknowns which may be written:

$$\sigma x^2_i = \sigma_o^2\ Qx_ix_i$$
$$\sigma y^2_i = \sigma_o^2\ Qy_iy_i$$
$$\sigma z^2_i = \sigma_o^2\ Qz_iz_i \qquad (13)$$

2.4.3 Absolute Error Ellipses (Annex A, A.8.3)

Absolute error ellipses are a graphical representation of the accuracy of a point position.

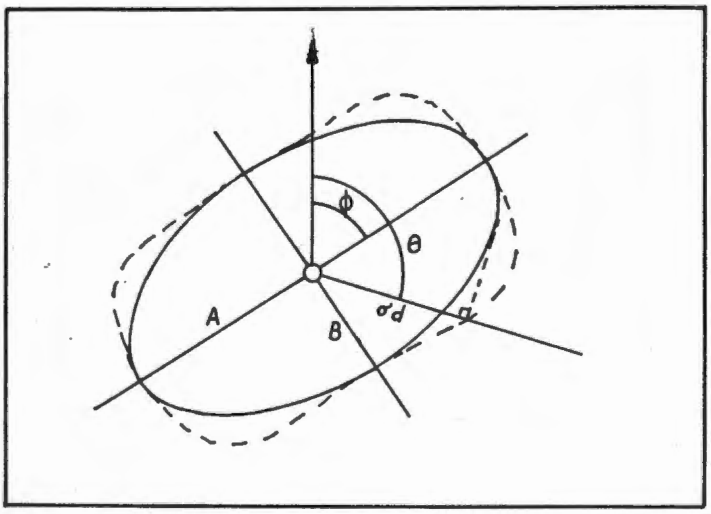


Figure 3

The ellipse parameters are obtained from:

Semi major axis $= A = \sigma_o \sqrt{D + \sqrt{E}}$ (14)

Semi minor axis $= B = \sigma_o \sqrt{D - \sqrt{E}}$ (15)

Rotation ϕ of A from $\tan 2\phi = \dfrac{2\ Qxy}{Qxx - Qyy}$ (16)

where:

$$D = \frac{Qyy + Qxx}{2}$$

$$E = \frac{(Qxx - Qyy)^2}{4} + Q^2xy$$

2.4.4 Relative Error Ellipses (Annex A, A.8.4)

Relative error ellipses are a measure of the accuracy between two points in a network. Although the formulae for the parameters are similar to those from the absolute error ellipses, they differ by the inclusion of elements of the Q matrix relating to both points involved.

The parameters for the relative ellipse are as follows:

$$\bar{A} = \sigma_o \sqrt{\bar{D} + \sqrt{\bar{E}}} \qquad (17)$$

$$\bar{B} = \sigma_o \sqrt{\bar{D} - \sqrt{\bar{E}}} \qquad (18)$$

$$\tan 2\bar{\phi} = \frac{2\,Q\bar{x}\bar{y}}{Q\bar{x}\bar{x} - Q\bar{y}\bar{y}} \qquad (19)$$

where:

$$Q\bar{x}\bar{x} = Qx_1x_1 - 2Qx_1x_2 + Qx_2x_2$$

$$Q\bar{y}\bar{y} = Qy_1y_1 - 2Qy_1y_2 + Qy_2y_2 \qquad (20)$$

$$Q\bar{x}\bar{y} = Qx_1y_1 - Qx_1y_2 - Qx_2y_1 + Qx_2y_2$$

$$\bar{D} = \frac{Q\bar{x}\bar{x} + Q\bar{y}\bar{y}}{2}$$

$$\bar{E} = \frac{(Q\bar{x}\bar{x} - Q\bar{y}\bar{y})^2}{4} + Q^2\bar{x}\bar{y}$$

where:

$\bar{A}$ = semi major axis

$\bar{B}$ = semi minor axis

$\bar{\phi}$ = orientation of semi major axis

2.4.5 A Posteriori Variance of the Observations
(Annexure A, A.8.5)

The variance of adjusted observations or a posteriori variance of the observations is obtained from:

$$\sigma_f^2 = \sigma_o^2 \, Q_{ff} \qquad (21)$$

where:

$$\underline{Q}_{ff} = \underline{f}^T \underline{\underline{Q}} \underline{f} \qquad \text{(scalar)} \qquad (22)$$

$$\underline{Q} = (\underline{A}^T \underline{P} \underline{A})^{-1}$$

$\underline{f}^T$ = vector of the differentials of the function with respect to the unknowns.

2.4.6 Confidence of Variables (Annexure A, A.8.6)

The standard deviation of a variable represents a confidence level of 68,3% for single variables and 39,4% for functions of two variables (e.g. error ellipses). Often it is desirable to modify the confidence level of a derived variable. This is achieved by scaling the standard deviation by a factor which may be found from statistical tables. These tables give factors which are dependent on confidence level required and on the number of degrees of freedom for a situation. For single variable standard deviations, eg σ_x, σ_y, a factor, c, is found directly from tables for the "Student's" t-distribution (Wells and Krakiwsky 1971), while in the case of bivariate standard deviations, eg error ellipses, a factor, d, may be derived from: $d = \sqrt{2.F(2,b;\alpha)}$

where $F(2,b;\alpha)$ = value found from tables for the Fisher Distribution

b = degrees of freedom

α = required probability e.g. 99%

(Heck, Kuntz and Meier-Hirmer 1977 & Chrzanowski 1977)

CHAPTER 3

DEFORMATION ANALYSIS

As mentioned earlier, the separation of noise (point shift due to observation errors incorrectly implying a deformation) from signal (actual movement) forms the basis of deformation analysis using geodetic methods.

Factors such as translation, rotation and scale also influence the final point positions. These factors however, can largely be eliminated (Ashkenazi and Dodson 1978) by the sound choice and construction of fixed points and the meticulous calibration of distance measuring equipment.

The various methods of deformation analysis offer differing approaches in overcoming the problem of separating noise from signal. These methods make use of either the adjusted geometry and precision or adjusted co-ordinates and precision in the determination of deformations. The four methods investigated are described hereunder.

3.1 METHOD OF INVARIANT FUNCTIONS (Chrzanowski 1981)

In a network where no deformations occur, certain functions remain invariant, irrespective of any translation or rotation applied to the map surface on which they are represented. These functions are distances between any two points and angles between any three points. This property of invariance is utilised for deformation analysis. Neither distances nor angles of a network derived in two epochs should differ significantly unless deformation is present in either one or both epochs.

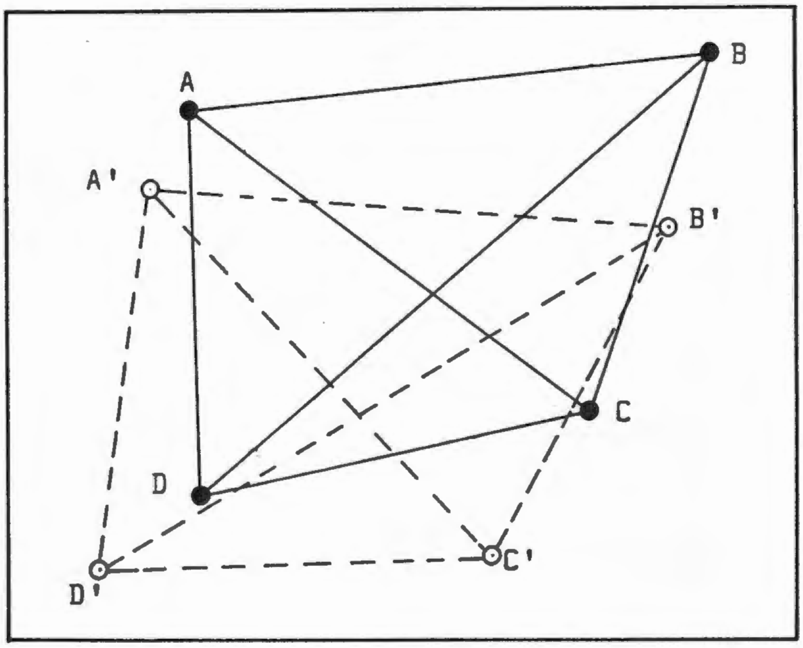


Figure 4

If one considers the network in figure 4, A,B,C & D represent the final positions of the points in the network for a set of observations at time 1, or epoch 1, and A',B',C' & D' the final positions of the same points for a second set of observations at time 2 or epoch 2. (Swing and translation exaggerated for clarity). Ignoring any differences due to observation errors, it is obvious that angles and distances between any of the points from epoch 1 will be equal to the corresponding angle or distance from the second epoch, irrespective of the fact that the final co-ordinates of the two epochs differ due to the swing and translation present, e.g. angle BAC = B'A'C' and distance DC = D'C' etc. It follows then, that if any of the points from the second epoch are deformed, the invariant functions involving these deformed points will no longer be equal between the two epochs.

The employment of this aspect. making due allowance for point position differences due to observation errors, forms the basis of this method.

The method may be reduced to three main stages:

1. Adjustment of the two epochs.
2. Determination of suspected deformed points.
3. Determination of deformed points and the magnitude of the deformations.

3.1.1 Adjustment of the Two Epochs

The adjustments of the two epochs are performed separately to yield two sets of final co-ordinates. Although Chrzanowski, Szostak - Chrzanowski and Tobin 1981 advocate the use of minimum constraints adjustment, the free network may also be employed for this method.

From each adjustment it is necessary to retain the following:

1) Value of $\underline{v}^T\underline{P}\underline{v}$
2) Number of degrees of freedom.
3) Final co-ordinates.
4) The $\underline{Q}$ matrix. $((\underline{A}^T\underline{P}\underline{A})^{-1})$

3.1.2 Determination of suspected deformed points

From the final co-ordinates of the two epochs all possible angles and distances are calculated, for each epoch, for the purpose of invariant function comparisons, and not only the angles and distances observed. As the following illustrates, calculating and comparing every possible angle in a network is time consuming and so only consecutive angles are considered.

As an example consider a system of four points, A,B,C & D. At one of the points, say A, the following angles are obtainable:

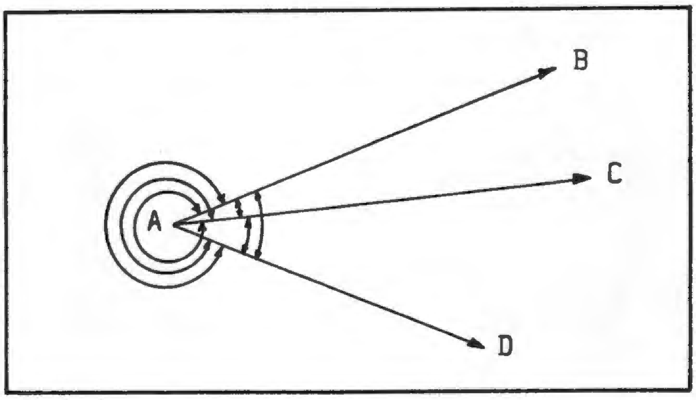


Figure 5

BAC, BAD, CAD, CAB, DAB, DAC.

Of these 6 angles, the last three are simply the explements of the first three and further BAD is the sum of the remaining two. To avoid having a distortion in an angle signalled more than once, only consecutive angles are used (e.g. BAC and CAD in Figure 5), and excluding exterior angles.

Obviously distances need only be determined in one direction as $d_{AB} = d_{BA}$

Thus, angle and distance differences between the two epochs are calculated from the final co-ordinates of the two epochs or:

$$\Delta dik = dik_2 - dik_1$$

$$\Delta \beta_{jik} = \beta_{jik2} - \beta_{jik1} \qquad (23)$$

where:

Δdik = difference between distances

$\Delta \beta_{jik}$ = difference between angles

$dik1, \beta_{jik1}$ = distance and angle from Epoch 1

d_{ik2}, β_{jik2} = distance and angle from Epoch 2

To separate signal noise from possible signal, these absolute differences are compared to the standard deviation of the adjusted distance or angle.

From Annexure A para. A.8.4, the variance of the adjusted observation σ_f^2 is given by:

$$\underline{\sigma_f^2 = \sigma_o^2 \, Q_{ff}} \tag{24}$$

where:

$$\sigma_o^2 = \frac{[Pvv]_1 + [Pvv]_2}{f_1 + f_2} \tag{25}$$

(a combined σ_o^2 to give a better estimate).

f_1 & f_2 = degrees of freedom for epoch 1 or 2 respectively

$$\underline{Q}_{ff} = \underline{f}^T \underline{Q} \underline{f} \tag{26}$$

where:

$$\underline{Q} = \underline{Q}_1 + \underline{Q}_2$$

(Chrzanowski, 1981)

$\underline{Q}_1$, $\underline{Q}_2$ = matrices $(A^T PA)^{-1}$ from epochs 1 & 2 respectively.

$\underline{f}$ is the vector of differentials of the function F (angle or distance) with respect to the unknowns of the adjustment,

so if $F = f(x_1, y_1, x_2, y_2, - - - -)$

then $$\underline{f}^T = \left(\frac{\partial F}{\partial x_1} \quad \frac{\partial F}{\partial y_1} \quad \frac{\partial F}{\partial x_2} \quad - - - - - \right) \tag{27}$$

These differentials are known from the linearisation of the observation equations. Only the elements corresponding to the points involved in the observation can be non-zero, the $\underline{f}$ vector can thus be reduced, and the $\underline{Q}$ matrix must then also be reduced to elements involving the points for the specific observation (Annex A, eqn. A-52).

From this the $\underline{f}$ vector for a distance and direction may be written as follows: (from Annex A, eqns. (A-6) & (A-7)).

<u>Distance</u> (between i & k)

$$\underline{f}^T = (a_{ik} \ b_{ik} \ -a_{ik} \ -b_{ik}) \quad (28)$$

<u>Angle</u> (between j&k).

$$\underline{f}^T = ((a_{ik} - a_{ij}) \ (b_{ik} - b_{ij}) \ a_{ij} \ b_{ij} \ - a_{ik} - b_{ik}) \quad (29)$$

The differences between distances and angles, Δd & $\Delta\beta$, are subjected to one-dimensional F - tests at 90%, 95% & 99% probability levels,

i.e.

$$\frac{|\Delta d| \ (\text{or} |\Delta\beta|)}{\sigma_f} < \sqrt{F\ (1,b,\alpha)} \quad (30)$$

where α = probability

b = degrees of freedom

In this test, values from the Fisher distribution table for probability levels of 90%, 95% & 99%, with infinite degrees of freedom, corresponding to values 1,65, 1,96 & 2,58 respectively were used. These values were chosen, as the degrees of freedom in this test was 106, and according to Spiegel 1961, samples with degrees of freedom exceeding 30 are considered to be "large samples", and for practical purposes may be equated to samples with infinite degrees of freedom. This becomes apparent when the distribution tables are examined and one notices that the values for large degrees of freedom differ insignificantly from the values for infinite degrees of freedom.

Any difference Δd or $\Delta\beta$ which fails any of the tests is listed in a table (example at Table 1) indicating the following:

a) The points involved in the failure (i.e. 2 for distance, 3 for angle). (Columns FROM, TO, TO, Table 1)

b) The standard deviation σ_f (Column SIGMA F, Table 1).

c) The difference Δd or $\Delta\beta$ (Column DIFF).

d) An indication showing at what probability levels the differences failed (Columns 90%, 95% & 99%).

If the failure is 95% or 99%, the lower probability level failures are also shown.

The two columns showing the difference Δd or $\Delta\beta$ and the standard deviations σ_f are included for interest only and are not required for the assessment of the table.

LIST OF ANGLES AND DISTANCES FAILING AT 90%, 95% AND 99% CONFIDENCE LEVELS

FROM	TO	TO	SIGMA F	DIFF	90%	95%	99%
P1	P2	P3	0.97	1.96	*		
P3	P17	CO2(P20)	1.42	-2.99	*		
P12	P4	P13	3.68	10.47	*	*	
P12	P13	P14	1.96	-5.59	*	*	
P13	P4	P12	4.06	10.67	*	*	
P14	P4	P12	2.00	4.68	*		
P14	P12	P13	2.43	-6.60	*	*	
P15	P4	P12	1.04	3.37	*	*	*
P15	P12	P13	0.76	-3.45	*	*	*
P16	P2	P3	1.05	-2.30	*		
P16	P12	P13	0.77	-1.91	*		
CO2(P20)	P12	P13	0.66	-2.98	*	*	*
P1	P12		1.20	4.78	*	*	*
P1	CO2(P20)		1.48	3.98	*	*	
P2	P12		0.81	4.59	*	*	*
P2	P16		0.86	2.52	*	*	
P2	CO2(P20)		1.25	4.73	*	*	*
P3	P12		1.08	6.40	*	*	*
P3	P14		1.03	7.78	*	*	
P3	CO2(P20)		1.81	5.09	*	*	
P4	P12		1.07	4.46	*	*	*
P12	P16		1.02	4.53	*	*	*
P17	P17		1.57	3.19	*		
P12	CO2(P20)		1.14	2.56	*		
P16	CO2(P20)		1.35	2.93	*		

P1	P2	P3	P4	P12	P13	P14	P15	P16	P17	CO2(P20)
3	5	6	5	16	7	4	2	5	2	7

Point Frequency.

P12, with the maximum frequency of 16, accounts for all but one of the 99% failures. The next frequency, P 13 and CD2 (P20), of 7 yields CD2 (P20), accounting for the last failure.

Table 1

On completion of the table, the frequency occurrence for each point in the table is determined from the columns FROM, TO, TO either manually or as a routine in the programme and set up in a separate table (see Table 1).

It follows logically that the points having the greatest frequency, in this example, (Table 1) P12, CD2 (P20) & P13, are the ones most likely to have been deformed.

Starting with the point having the highest frequency, P12 with 16, the failures in the 99% column are studied to determine if this point (P12) accounts for all these failures (ie. one of the points in the columns FROM, TO, TO is P12). If this point does account for all the failures, then this point is considered as the only suspect point and the next step is performed. If not, one or more doubtful points are present and the point with the next highest frequency is considered. (Here P13 and CD2 with a frequency of 7). This process is continued until all the 99% failures are accounted for.

In the example at Table 1, P12 accounts for all but one of the 99% failures. The remaining failure, the distance between P2 and CD2 (P20), has CD2 (P20) as the point, with the next highest frequency and thus the suspect points in this example would be P12 and CD2. Even though P13 has the same frequency as CD2 (P20) it is not involved in the last 99% failure and is thus ignored.

If however, point P2 and not P13 had the same frequency as CD2, then P2 would also be included as a doubtful point as it would be impossible to determine at this stage which of the two points was at fault.

It would appear that the 90% and 95% failure columns are superfluous, but as the frequency of 99% failures is usually low (in Table 1 only 9), which would result in a point frequency table of

P1	P2	P3	P4	P12	P13	P14	P15	P16	P17	P20
1	2	1	2	8	2	0	2	0	0	1

if only these failures were to be used.

Although P12 has still the largest frequency, the next highest points are P2, P4, P13 and P15 (2). Although in this case the first choice would still be P12, the second suspect point would now be P2. If this abridged table is compared with table 1, it is clear that the latter table would be far easier to analyse than the former. The 95% failure column is of minor importance and is only used if any clarification is required when difficulty is encountered from analysis of the 99% failure column, or if a confidence level of 95% is to be used for the testing of displacements (see sect. 3.1.3).

3.1.3 Determination of deformed points and the magnitude of the deformations

After finding which points are suspected of being deformed, this assumption has to be confirmed and the magnitude of deformations found.

This is achieved by a further adjustment in which the observations from the two epochs are combined. The points considered as being stable are maintained as a single point in the combined adjustment while the doubtful points are considered as two separate points representing their positions at the two epochs.

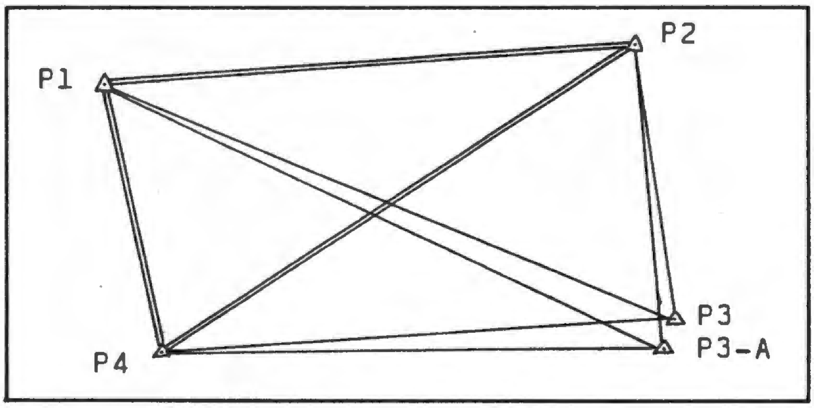


Figure 6

For example, the fully observed quadrilateral in Figure 6 has point P3 as doubtful. An extra point P3-A, representing the position of P3 in epoch 2, is added. The double lines between P1, P2 & P4 represent the double observations between these points (from epoch 1 & 2). While the single lines between P1, P2, P4 and P3 represent the observations to and from P3 taken at epoch 1 and those between P1, P2. P4 and P3-A, the observations to and from P3 at epoch 2. Thus this second adjustment would now contain five points; P1, P2, P3, P3-A and P4. It is therefore clear that the stable points will have two sets of observations, while the doubtful points only one. The observation equation from the two epochs would be:

1. Between two stable points

Direction (from point i)

$$v_{ik1} = a_{ik}dx_i + b_{ik}dy_i - a_{ik}dx_k - b_{ik}dy_k - dz_{i1} - \ell_{ik1} \qquad (31)$$

$$v_{ik2} = a_{ik}dx_i + b_{ik}dy_i - a_{ik}dx_k - b_{ik}dy_k - dz_{i2} - \ell_{ik2}$$

where:

v_{ik1} = correction to the observation epoch 1

v_{ik2} = correction to the observation epoch 2

dz_{i1} = orientation correction epoch 1

dz_{i2} = orientation correction epoch 2

ℓ_{ik1} = residual epoch 1

ℓ_{ik2} = residual epoch 2

a_{ik}, b_{ik} = co-efficients from linearisation of observation equation

dx, dy = corrections to the unknowns x & y.

Distance

$$\bar{v}_{ik1} = \bar{a}_{ik}dx_i + \bar{b}_{ik}dy_i - \bar{a}_{ik}dx_k - \bar{b}_{ik}dy_k - \bar{\ell}_{ik1}$$
$$\bar{v}_{ik2} = \bar{a}_{ik}dx_i + \bar{b}_{ik}dy_i - \bar{a}_{ik}dx_k - \bar{b}_{ik}dy_k - \bar{\ell}_{ik2} \qquad (32)$$

2. Between two doubtful points

Direction

$$v_{ik1} = a_{ik}dx_{i1} + b_{ik}dy_{i1} - a_{ik}dx_{k1} - b_{ik}dy_{k1} - dz_{i1} - \ell_{ik1} \qquad (33)$$

$$v_{ik2} = a_{ik}dx_{i2} + b_{ik}dy_{i2} - a_{ik}dx_{k2} - b_{ik}dy_{k2} - dz_{i2} - \ell_{ik2}$$

where:

$dx_{i1}, dy_{i1}, dx_{k1}, dy_{k1}$ = corrections to the unknowns for epoch 1.

$dx_{i2}, dy_{i2}, dx_{k2}, dy_{k2}$ = corrections to the unknowns for epoch 2.

Distance

$$\begin{aligned} \bar{v}_{ik1} &= \bar{a}_{ik}dx_{i1} + \bar{b}_{ik}dy_{i1} - \bar{a}_{ik}dx_{k1} - \bar{b}_{ik}dy_{k1} - \bar{l}_{ik1} \\ \bar{v}_{ik2} &= \bar{a}_{ik}dx_{i2} + \bar{b}_{ik}dy_{i2} - \bar{a}_{ik}dx_{k2} - \bar{b}_{ik}dy_{k2} - \bar{l}_{ik2} \end{aligned} \qquad (34)$$

3. Between a stable and a doubtful point e.g. i stable

Direction

$$\begin{aligned} v_{ik1} &= a_{ik}dx_i + b_{ik}dy_i - a_{ik}dx_{k1} - b_{ik}dy_{k1} - \\ &\quad dz_{i1} - l_{ik1} \\ v_{ik2} &= a_{ik}dx_i + b_{ik}dy_i - a_{ik}dx_{k2} - b_{ik}dy_{k2} - \\ &\quad dz_{i2} - l_{ik2} \end{aligned} \qquad (35)$$

Distance

$$\begin{aligned} \bar{v}_{ik1} &= \bar{a}_{ik}dx_i + \bar{b}_{ik}dy_i - \bar{a}_{ik}dx_{k1} - \bar{b}_{ik}dy_{k1} - \bar{l}_{ik1} \\ \bar{v}_{ik2} &= \bar{a}_{ik}dx_i + \bar{b}_{ik}dy_i - \bar{a}_{ik}dx_{k2} - \bar{b}_{ik}dy_{k2} - \bar{l}_{ik2} \end{aligned} \qquad (36)$$

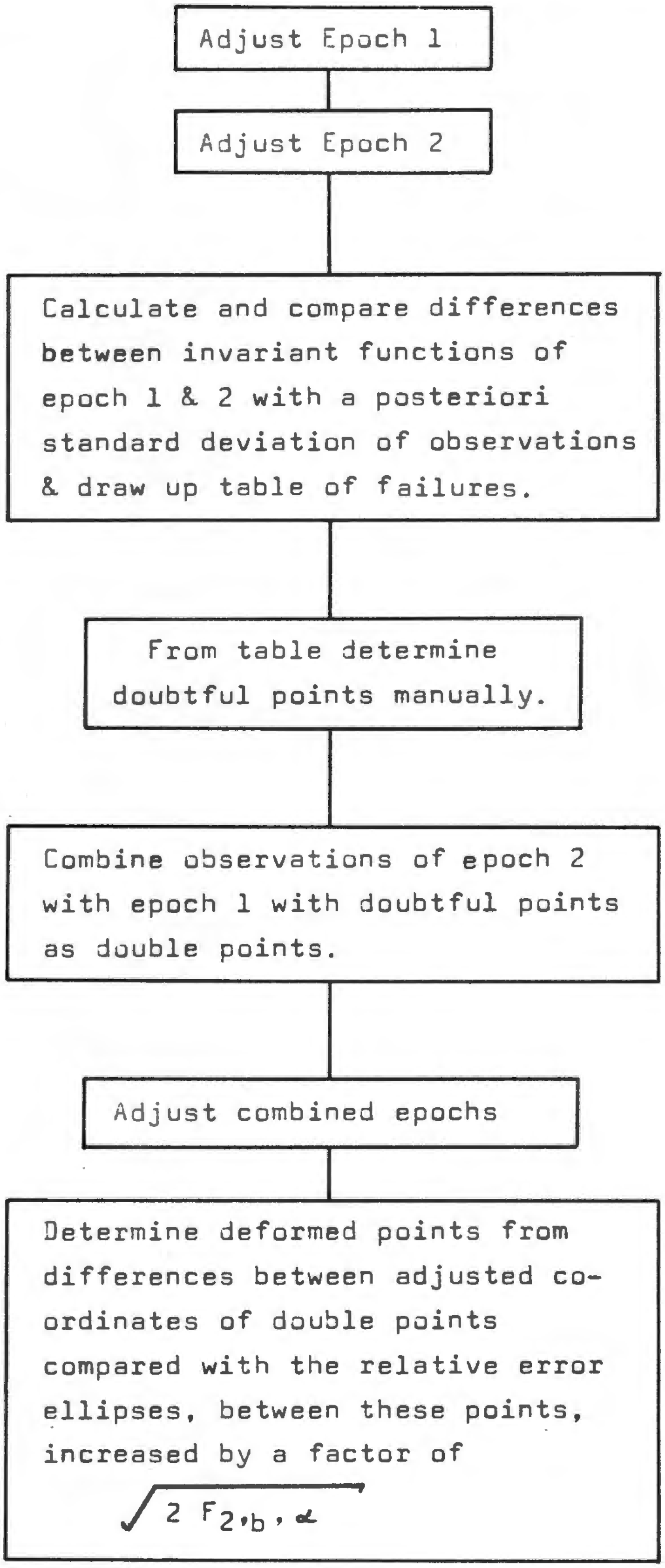
It should be noted that when using direction observations, the number of orientation unknowns increases when combining epochs, as at occupied stations there will be a different orientation correction for each epoch. Note also that in the equation involving doubtful points the co-efficients in the two equations ((33), (34), (35), & (36)) will be identical when the same provisional co-ordinate is used for both the points representing the doubtful point.

This combined adjustment results in one final value for the co-ordinates of the stable points and two sets of co-ordinates for the doubtful points. From these two sets of co-ordinates it is a simple matter to calculate the magnitude of the shift, either in polar co-ordinates or dx & dy differences.

It now becomes necessary to determine whether the shift calculated above is actually signal or just signal noise. This is achieved by drawing the relative error ellipse between the points in question, (from error analysis Annex A, Chap. A.8.4) scaled by a factor of $\sqrt{2.F\ (2,b,\alpha)}$ (b=n-u degrees of freedom, α= confidence interval = 95% or 99%) as described by Heck, Kuntz and Meier - Hirmer 1977. The shift, between the points representing the positions of a station for the two epochs, is superimposed on the relevant error ellipse. If the vector of the shift is greater than the ellipse, then the point is judged to have been deformed, while if it is within the ellipse, the point is taken to be stable.

Here, for checking for deformation, the procedure described by Ashkenazi and Dodson 1978 was adopted. This is described in full under the method of co-ordinate comparisons in section 3.2.4. The test involves determining the standard deviation of the shift, multiplying this by a factor of 2 or 2.5 to give a confidence level of 95% or 99% respectively, and then comparing the deformation magnitude to this value.

3.1.4 Flow chart for invariant function method



3.1.5 Possible variation in the method

A situation may occur where the surveyor has insufficient information to execute this method. This may arise where one has only one full set of data, for the second epoch, with only the final co-ordinates (without point accuracy estimates) of the first. In this instance, it is suggested that the invariant function comparisons may still be performed using only the available information.

For the comparison of the invariant functions, the one known value for σ_o would be employed while the value for $\underline{Q}$ (= $\underline{Q}_1 + \underline{Q}_2$ from Eqn 26) would now be replaced by $\underline{Q} = 2\,\underline{Q}_1$

The comparisons would then be performed as usual and the doubtful points adduced.

As the combined adjustment described in section 3.1.3 is no longer possible, owing to the unavailability of one epoch's observations, a second adjustment is suggested where the final co-ordinates from the existing adjustment would be used as provisional values for this new adjustment. The points judged to be stable by the invariant function comparisons would become constraint points and thus only the doubtful points would be adjusted.

The displacement vectors are found from the difference between the adjusted and provisional co-ordinates (i.e. values in the solution vector $\underline{x}$). The point position accuracy test used by Ashkenazi and Dodson 1978 (Section 3.2.4) is suggested as a means of testing for deformation.

3.2 DIRECT COMPARISON OF CO-ORDINATES (Ashkenazi and Dodson 1978)

As the name implies, the two epochs are adjusted separately and the final co-ordinates of the two epochs are directly compared.

3.2.1 Constraint Points

If one compares co-ordinates directly, it is obvious that, should there be any movement between epochs in the constraint parameters, then the network as a whole will be subject to some translation or rotation. This "movement" may then be falsely interpreted as deformation in other stable points of the network. To eliminate this type of situation, the points to be used as reference, should be placed such that they will be unaffected by any movement on the deformable body. This would be achieved by placing the points out of the area subject to deformation, yet still close enough so as to maintain a good network configuration. Further, these points would be anchored to bedrock and be of sound construction. With a minimum constraint adjustment this reference system would be reduced to two points, one to anchor the system and the second to provide a reference for orientation and scale if required.

3.2.2 Scale Factor

Another factor which may influence the correct determination of deformation is scale error. Scale errors may occur due to calibration errors in distance measuring equipment (especially E.D.M. equipment) and thus to eliminate this source of error, meticulous care must be taken when calibrating equipment. A scale factor in distances may be introduced as an unknown in the adjustment. but care must be exercised when using this, as actual deformations may be interpreted as a scale error, and thus suppress these deformations (Ashkenazi and Dodson 1981).

In Figure 7 an example of how this may occur is given.

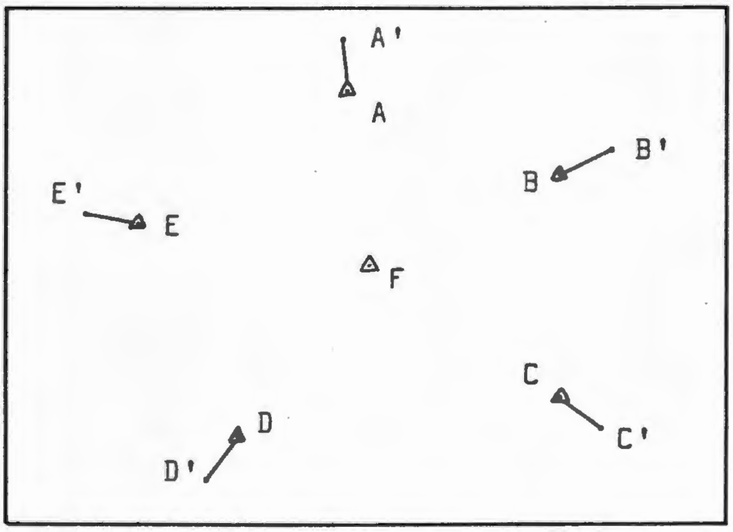


Figure 7

The points A - F indicate the original position of the points while A' - E' the deformed position of points A to E (points A to E possibly on a dam wall). From this it is clear that these deformations may be misinterpreted as a scale error.

3.2.3 Adjustment of the two Epochs

The adjustment advocated by Ashkenazi and Dodson 1978 is the minimum constraints method with special reference given to the factors discussed in 3.2.1 & 3.2.2 above. In the case of the second epoch adjustment, the final co-ordinates from the first epoch should be preferred as provisional co-ordinates.

In this case the corrections to the unknowns, dx and dy, are also the differences between co-ordinates of epochs 1 and 2.

3.2.4 Helmert Transformation of the second epoch onto the first

Although precau-tions may be taken with the construction and placing of constraint points (3.2.1), there still exists the possibility of movement in these points, which would affect the reliability of a model by introducing systematic errors such as rotation and origin shift. A Helmert transformation without scale factor may be used to "best fit" a network onto a given set of co-ordinates (usually those from a previous epoch), which would then show up any rotation or position error in the model. It must be noted however, (Ashkenazi, Dodson, Jones & Samson 1981) that care must be exercised when using a transformation, as any actual deformation may be interpreted as a systematic error.

The danger of scale error in EDM equipment is well known, and this error would normally be well determined prior to any measurement. For this reason, the consideration of scale error in this thesis is ignored and the unknown scale factor determined in a Helmert transformation is removed before any analysis is performed. The Helmert transformation is included in the thesis in the interest of completeness, even though the process would in general not be employed (Ashkenazi, Dodson, Jones and Samson 1981).

The equations for the Helmert Transformation are:

$$y = b_o + a_1 Y + b_1 X$$

$$x = a_o + a_1 X - b_1 Y \qquad (37)$$

(For the full derivation of the Helmert Transformation see Annexure B).

where:

x, y = co-ordinates of the New System (Epoch 1)

X, Y = co-ordinates of the Old System (Epoch 2)

a_0, b_0 = x & y translations respectively

a_1 = $m \cos\alpha$

b_1 = $m \sin\alpha$

m = scale factor

α = angle of rotation between the two systems.

To remove the scale factor one may either incorporate an additional condition in the transformation, viz

$$a_1^2 + b_1^2 = 1 \qquad (38)$$

(as $\sin^2\alpha + \cos^2\alpha = 1$ thus forcing $m = 1$)

or the scale factor may be removed by using the values for a_1 & b_1 in a simple trigonometrical equality viz

$$\frac{b_1}{a_1} = \frac{\not{m} \sin\alpha}{\not{m} \cos\alpha} = \tan\alpha \qquad (39)$$

To find the correct quadrant for the quotient in (39) may be treated as for a join calculation.

A simpler method is to assign the same signs to $\sin\alpha$ & $\cos\alpha$ as those for $m \sin\alpha$ & $m \cos\alpha$ respectively, as the actual value of α is of no interest.

In the case of deformation analysis any rotation will be either a small positive or small negative angle and the value for α as calculated directly from (39) may be used for calculating $\sin\alpha$ & $\cos\alpha$.

The values a_1 & b_1 in eqn. (37) are now replaced by Cosα & Sinα respectively and the transformation performed.

Two types of transformations were employed:

1) Considering all common points between the epochs (i.e. deformed and stable points were used to determine rotation and translations).

2) Transformation using only the points considered to be stable. The criterion for determining a stable point may be arbitrary. Here a factor of 2, modelled on Ashkenazi & Dodson 1978, is suggested as a multiple to be used on the standard deviations of the displacements. This would have the effect of only including points which have a large probability of being stable in the determination of the rotation and translation of the network. This selection process would only be performed after a previous all point transformation or after the initial adjustment of the second epoch if the translation and rotation errors appear to be absent. When testing points in the free network adjustments using a factor of 2, it was found that a larger proportion of points failed the test. As this would reduce the number of common points and possibly adversely affect the transformation, it was decided to increase the factor to 2,5.

3.2.5 Determination of deformations and deformation magnitudes

The differences between the co-ordinates of the two epochs must first be compared to the standard deviation of the displacement before they can be judged to be stable or deformed.
The differences dx and dy are found from:

$$dx = x_2 - x_1 \qquad (40)$$

$$dy = y_2 - y_1$$

or directly from the vector of unknowns $\underline{x}$ if the co-ordinates of the first epoch are used as provisionals for the second epoch.

The standard deviations for the differences σ_{dx} and σ_{dy} are found from:

$$\sigma_{dx} = \sqrt{\sigma_{x1}^2 + \sigma_{x2}^2} \approx \sqrt{2}\sigma_x \qquad (41)$$

$$\sigma_{dy} = \sqrt{\sigma_{y1}^2 + \sigma_{y2}^2} \approx \sqrt{2}\sigma_y$$

These values approximate $\sqrt{2}\,\sigma_x$ & $\sqrt{2}\,\sigma_y$ as the standard deviations from the two epochs would in general be similar,

i.e. $\sigma_{x1} \approx \sigma_{x2}$

$\sigma_{y1} \approx \sigma_{y2}$

the displacement, s, of a point is found from:

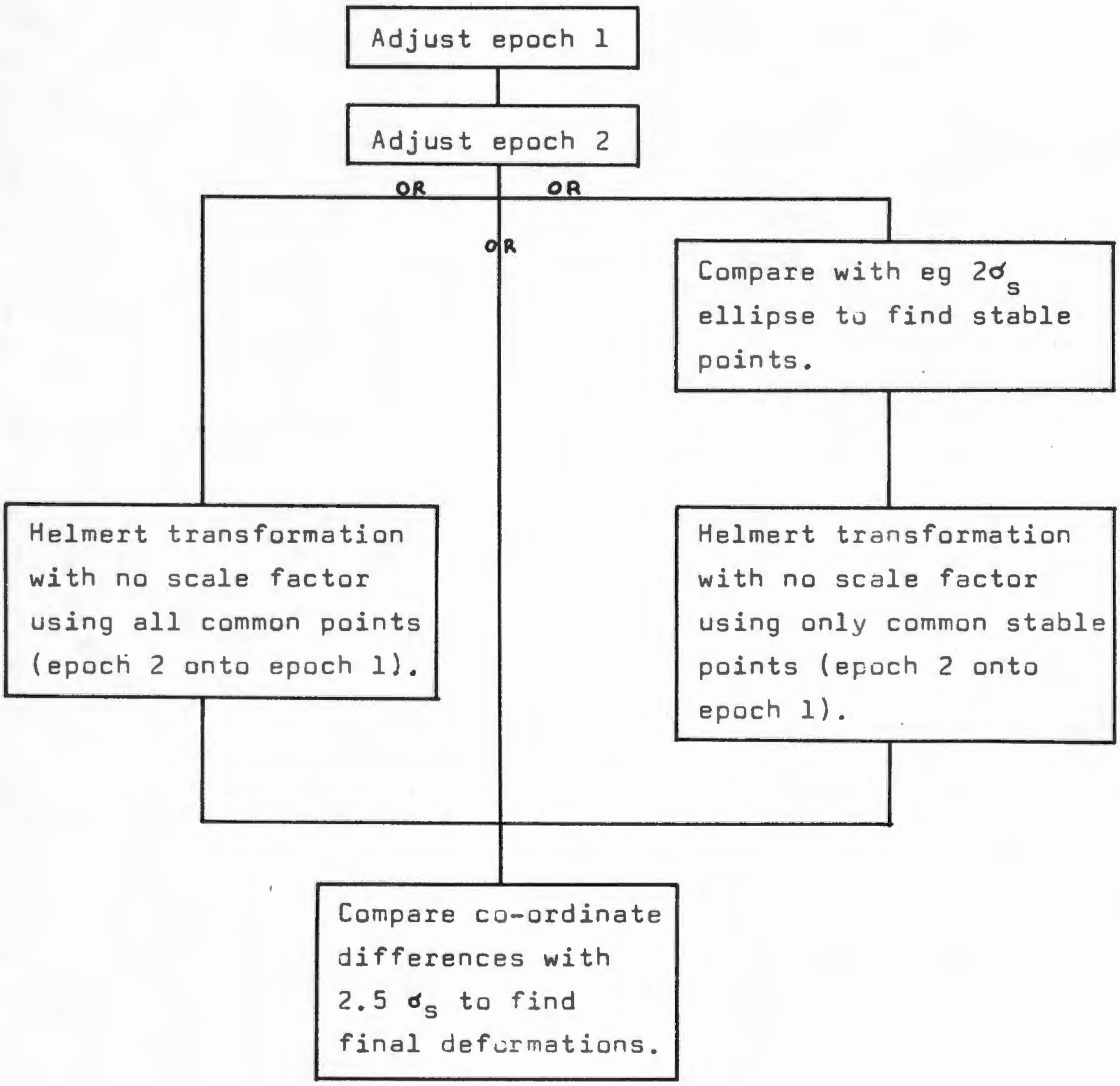
$$s = \sqrt{dx^2 + dy^2} \qquad (42)$$

and the standard deviation of the displacement σ_s is derived from:

$$\sigma^2_s = \frac{2}{s^2} (dx \;\; dy) \begin{vmatrix} \sigma^2_x & \sigma_{xy} \\ \sigma_{xy} & \sigma^2_y \end{vmatrix} \begin{vmatrix} dx \\ dy \end{vmatrix} \qquad (43)$$

When comparing the displacement with its standard deviation σ_s, the probability level is increased to 95% or 99% by multiplication with a factor of approximately 2 and 2.5 respectively.

3.2.6 Flow Chart for Direct Comparison of Co-ordinates



3.3 METHOD OF DIRECT DIFFERENCES (Chrzanowski 1982)

Using this method one solves directly for the co-ordinate differences between two epochs after subtracting one vector of observations from that taken at the other epoch.

3.3.1 Adjustment of Epochs

In this adjustment it is essential that the observation geometry be identical in both epochs, as the adjustment relies on the differences between the observations of Epoch 1 and Epoch 2.

Let us consider the observation equations for a direction:

$$v_{ik1} = a_{ik}dx_{i1} + b_{ik}dy_{i1} - a_{ik}dx_{k1} - b_{ik}dy_{k1} - dz_{i1} - \ell i_{k1} \qquad (45)$$

$$v_{ik2} = a_{ik}dx_{i2} + b_{ik}dy_{i2} - a_{ik}dx_{k2} - b_{ik}dy_{k2} - dz_{i2} - \ell i_{k2} \qquad (46)$$

where:

(45) = observation equation for epoch 1
(46) = observation equation for epoch 2
v_{ik} = correction to observation
a_{ik} b_{ik} = co-efficients from linearisation of observation equation
dx dy dz = corrections to the unknowns X, Y & Z.
ℓ_{ik} = free term = $t_i^k - t_{io}^k + z_{io}$
t_i^k = direction i to k
z_{io} = orientation correction at i (provisional)

If one now subtracts (45) from (46) one obtains:

$$v_{ik2} - v_{ik1} = a_{ik}(dx_{i2} - dx_{i1}) + b_{ik}(dy_{i2} - dy_{i1}) - a_{ik}(dx_{k2} - dx_{k1}) - b_{ik}(dy_{k2} - dy_{k1}) - (dz_{i2} - dz_{i1}) - (\ell_{ik2} - \ell_{ik1}) \quad (47)$$

or:

$$\Delta v_{ik} = a_{ik}\Delta x_i + b_{ik}\Delta y_i - a_{ik}\Delta x_k - b_{ik}\Delta y_k - \Delta z_i - \Delta \ell_{ik} \quad (48)$$

where: $\Delta \ell_{ik} = t^{k}_{i\,obs\,2} - t^{k}_{i\,obs\,1} + z_{io2} - z_{io1}$

Subscripts 1 & 2 indicate epochs 1 & 2.

If the two epochs are roughly pre-oriented making $z_{io} = 0$ in both cases

$$\Delta \ell_{ik} = t^{k}_{i\,obs2} - t^{k}_{i\,obs1} \quad (48a)$$

Since one is interested only in the co-ordinate differences, only a rough knowledge of the co-ordinates of the points is required to formulate the co-efficients a & b.

The observation equation for distance is similarly obtained, viz

$$\Delta \bar{v}_{ik} = \bar{a}_{ik}\,\Delta x_i + \bar{b}_{ik}\,\Delta y_i - \bar{a}_{ik}\,\Delta x_k - \bar{b}_{ik}\,\Delta y_k - \Delta \bar{\ell}_{ik} \quad (49)$$

where: $\Delta \bar{\ell}_{ik} = d_{ik\,obs\,2} - d_{ik\,obs\,1}$ (49a)

From the above, (48a) & (49a), it is clear that no provisional values for distances or directions are required for the formation of the free term , as is the case in the standard adjustment. Chrzanowski suggests that this fact may be put to use to speed up the observations of a network, as it would now not be necessary to observe directly to one single point on a pillar.

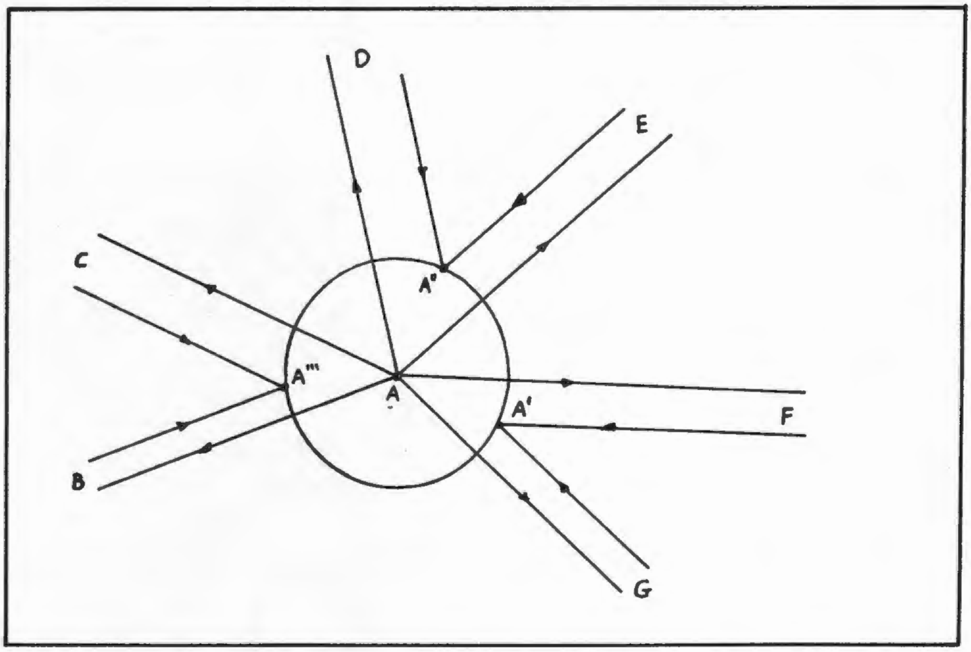


Figure 8

In figure 8, A represents the centre of a pillar and would in normal circumstances be the point observed to and from. However, in this method, as only observation differences are required, observations from distant stations B,C,D,E,F & G could be made to targets on the sides of the pillar (A''', A'' & A') while another observer is observing from the centre of the pillar A. Any movement at A due to deformation would naturally cause the identical deformation at A', A'' & A'''.

As the observation equation for directions if of the same form as in the standard adjustment, the elimination of the orientation unknowns may be performed as usual.

3.3.2 Weights

As the two sets of observations from the two epochs are subtracted to find the difference, the standard deviation of an observation must necessarily be modified to:

$$\sigma_{i\Delta} = \sqrt{\sigma_{i1}^{2} + \sigma_{i2}^{2}} \qquad (50)$$

where:

σ_{i1} = standard deviation for an observation from epoch 1.

σ_{i2} = standard deviation for an observation from epoch 2.

$\sigma_{i\Delta}$ = standard deviation of the observation difference .

If the standard deviations are identical for both epochs, $\sigma_{i\Delta}$ becomes:

$$\sigma_{i\Delta} = \sqrt{2}\ \sigma_{i} \qquad (51)$$

and the observation weight then becomes:

$$P_{i} = \frac{\sigma_{o}^{2}}{\sigma_{i\Delta}^{2}} \qquad (52)$$

In the case where $\sigma_{i1} = \sigma_{i2}$ and when σ_{o} is set equal to one of the observation standard deviations:

$$P_{i} = \frac{\sqrt{2}\ \bar{\sigma}_{o}^{2}}{\sqrt{2}\ \sigma_{i}^{2}} = \frac{\bar{\sigma}_{o}^{2}}{\sigma_{i}^{2}} \quad \text{where } \bar{\sigma}_{o} = \sigma_{i} \qquad (53)$$

It should be remembered that when using (53) that σ_{o} a priori is equal to $\sqrt{2}\ \sigma_{o}$ when σ_{o}^{2} is subjected to the χ^{2} test.

3.3.3 Determination of Deformations & Deformation Magnitudes

In this method, the standard deviations relate directly to the differences (from (50)) and the standard deviation of the difference vector may be immediately multiplied by a factor derived from $\sqrt{2\ F(2,b;\alpha)}$ (Chrzanowski 1977), (where $F(2,b;\alpha)$ is the value from the Fisher distribution tables, b, the degrees of freedom and α, the probability level) to give the required confidence level. Similarly, the absolute error ellipse may be used, multiplied by a factor 3,03 for a 99% confidence level.

The vector size of the ellipse in the direction of the shift may be determined directly as a routine in the adjustment thus immediately testing for deformation.

3.3.3.1 Determination of distance from centre to curve of the ellipse

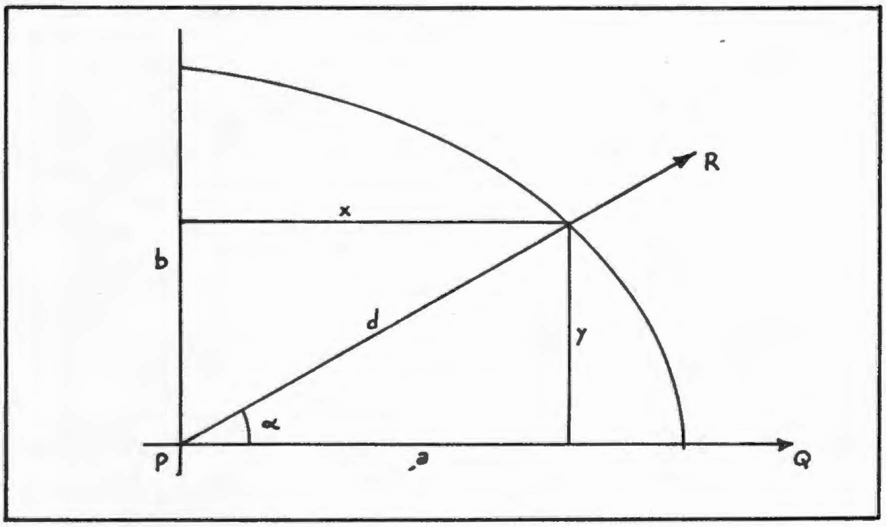


Figure 9

where:

PR = Direction of displacement.

PQ = Direction of Semi-major Axis.

a = Semi-major axis

b = Semi-minor axis.

d = Distance centre to curve.

α = Angle between PR & PQ.

The ellipse equation is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{54}$$

From Fig. 9:

$$\text{Tan}\,\alpha = \frac{y}{x} \tag{55}$$

and

$$d = \frac{x}{\text{Cos}\,\alpha} \tag{56}$$

multiplying (54) by $\frac{b^2}{x^2}$ and re-arranging, one gets:

$$\frac{b^2}{a^2} = \frac{b^2}{x^2} - \frac{y^2}{x^2} \tag{57}$$

or with (55):

$$\frac{b^2}{a^2} = \frac{b^2}{x^2} - \tan^2\alpha \tag{58}$$

$$\text{and } x = \sqrt{\frac{a^2b^2}{b^2 + a^2 \tan^2\alpha}} \tag{59}$$

from (56):

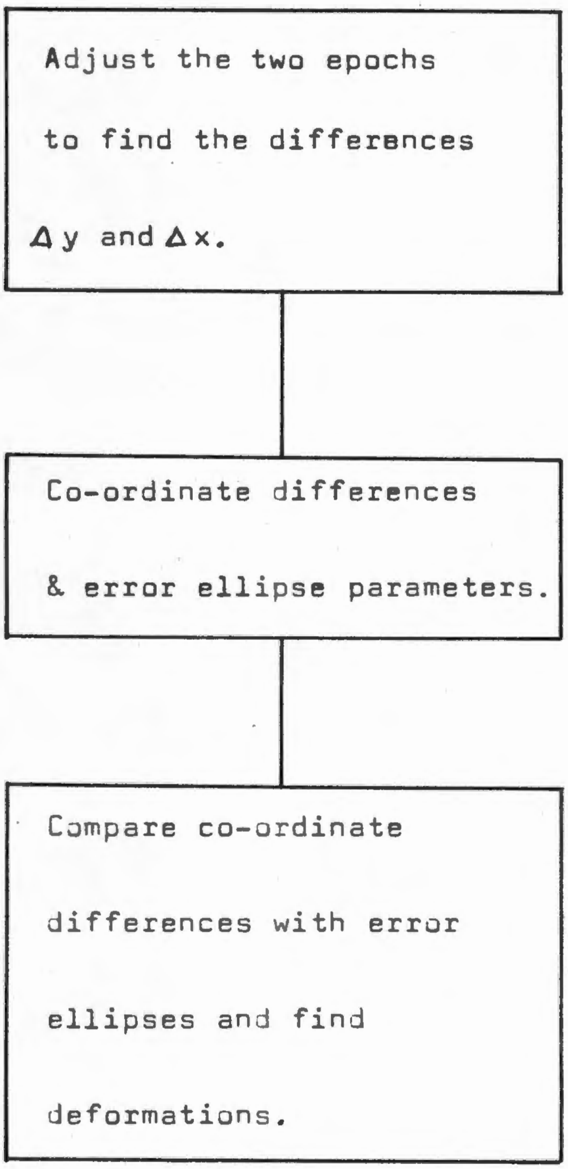
$$d = \frac{x}{\text{Cos}\,\alpha} \qquad \text{and} \qquad \tan\alpha = \frac{\text{Sin}\,\alpha}{\text{Cos}\,\alpha}$$

one obtains:

$$d = \sqrt{\frac{a^2b^2}{b^2\text{Cos}^2\alpha + a^2\text{Sin}^2\alpha}} \tag{60}$$

α may be found as the angle between the semi-major axis (PQ) and the direction of apparent point movement (PR). Once d has been found, its magnitude is compared with that of the shift to determine if any deformation has occurred.

3.3.4 <u>Flow Chart for Method of Direct Differences</u>



3.4 NIEMEIER'S COMPARISON OF CO-ORDINATES (Niemeier 1976)

This method involves modifying an initially assumed partitioning of points into "reference" (stable) and "object" (unstable) parts. Points are removed from the former list if it is found that this will significantly reduce the net strain. Final list sorting and determination of deformation magnitude can be carried out by methods already described for invariant functions.

Although Niemeier's method involves the use of free network adjustment, it was modified slightly (see section 3.4.7) to accomodate minimum constraints as well.

3.4.1 Adjustment of the epochs

The two epochs are first adjusted separately using a free network adjustment. From the adjustments the following information must be retained in each case:

1. Value for (pvv)
2. Number of degrees of freedom f $(n - u' + r)$
3. Number of unknown co-ordinates u.
4. Rank defect r
5. Final co-ordinate vector $\underline{x}$
6. $\underline{Q}$ matrix $((A^TPA + GG^T)^{-1} - GG^T) = Qxx$
7. $G\,G^T$ matrix or G matrix (matrix of normalised eigen vectors).

The observation of the two epochs may differ in number and type, but the point numbers must not be changed in the described procedure.

3.4.2 Test on the network to determine if deformations must be suspected

Combine the two $\bar{\sigma}_o$ to obtain a better estimate for $\bar{\sigma}$ from:

$$\bar{\sigma}^2 = \frac{v_o{}^T P_o v_o + v_1{}^T P_1 v_1}{f_o + f_1} \tag{61}$$

where:

v_o = vector of corrections epoch 0

v_1 = " " " " " 1

P_o = weight matrix epoch 0

P_1 = " " " 1

f_o = degrees of freedom epoch 0 = $n_o - u_o' + r_o$

f_1 = " " " " 1 = $n_1 - u_1' + r_1$

n_o, n_1 = number of obervations epoch 0 & 1 resp.

u'_o, u_1' = " " unknowns " " "

r_o, r_1 = " " rank defect " " "

find the vector d of differences between the 2 epochs.

$$\underline{\underline{d = x_1 - x_o}} \tag{62}$$

For the error propagation of d the two Q matrices are combined:

$$\underline{\underline{Q_{dd} = Q_{xxo} + Q_{xx1}}} \tag{63}$$

which is singular (Niemeier 1976).

To remove the singularity, the matrix $G\ G^T$ is added and the weight matrix for d, P_{dd}, is found from:

$$\underline{\underline{P_{dd} = (Q_{dd} + G\ G^T)^{-1} - G\ G^T}} \tag{64}$$

and the variance for the differences is found from:

$$\nu^2 = \frac{d^T P_{dd} d}{h} \tag{65}$$

where:

h = degrees of freedom for the differences = u - r

u - r = number of unknown co-ords - rank defect

The two variances are related to each other to determine if they belong to the same distribution by means of the Fisher Test:

$$\text{if} \quad \bar{F} = \frac{\nu^2}{\sigma^2} \tag{66}$$

$$\text{then} \quad P\left\{\bar{F} > F_{1-\alpha,h,f} \mid H_o\right\} = \alpha \tag{67}$$

where: h = degrees of freedom for differences

f = " " " $= f_o + f_1$

$\bar{F}$ = calculated value

F = value from Fisher tables

One stipulates the Null Hypothesis that the network is free from systematic errors (deformations), or that any difference in point positions between epochs is due only to random errors in the observations. The probability that $\bar{F}$ is larger than F, in spite of the fact that the Hypothesis is correct, is then α% (often 5%). So, if $\bar{F}>F$, then one assumes that there are deformations present.

3.4.3 Partitioning of the Network into Reference and object points

Obviously this and following steps are only performed if the test on the network indicates deformations within the network. To reduce the computation time and the chance of a large number of potentially unstable points adversely affecting the analysis, the network is partitioned into two groups:

1) Reference points - points which are initially assumed to be undisturbed but which may or may not be subject to deformations. These points would usually be removed from the deformable body.

2) Object points - points which are on a deformable body and are used for the actual deformation measurement of the body and would probably be subject to some deformation.

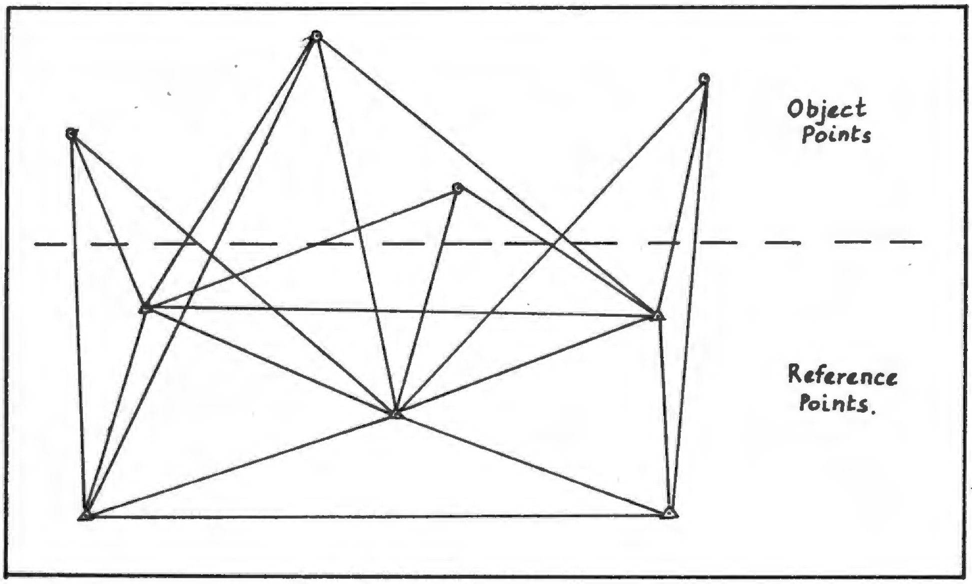


Figure 10

The weight matrix of the differences, P_{dd}, and the vector of differences, d, may be partitioned as follows:

$$P_{dd} = \begin{pmatrix} P_{ss} & P_{so} \\ P_{os} & P_{oo} \end{pmatrix}$$

$$d = \begin{pmatrix} d_s \\ d_o \end{pmatrix} \qquad (68)$$

where suffix "s" represents reference points and "o" the object points.

3.4.4 Test on Reference points

At this stage it becomes necessary to test the reference points to see if the assumption that they are in fact stable is justified and if not, which point is suspect.

For this test, one is required to determine the variance for d_s.

The quadratic form $d^T Pd$ then becomes:

$$d^T Pd = (d_s^T \; d_o^T) \begin{pmatrix} P_{ss} & P_{so} \\ P_{os} & P_{oo} \end{pmatrix} \begin{pmatrix} d_s \\ d_o \end{pmatrix} \tag{69}$$

$$d^T Pd = d_s^T P_{ss} d_s + 2d_s^T P_{so} d_o + d_o^T P_{oo} d_o \tag{70}$$

This quadratic form is used to determine $\underline{d}^T \underline{Pd}$ for the whole network. To find the corresponding forms for the reference and object points it becomes necessary to eliminate the mixed term in (70), using the following transformations:

$$\bar{d}_o = d_o + P_{oo}^{-1} P_{os} d_s \tag{71}$$

and

$$\bar{P}_{ss} = P_{ss} - P_{so} P_{oo}^{-1} P_{os} \tag{72}$$

Formula (70) becomes

$$d^T Pd = d_s^T \bar{P}_{ss} d_s + \bar{d}_o^T P_{oo} \bar{d}_o \tag{73}$$

Set $a = d_s^T \bar{P}_{ss} d_s$ (for reference points) (73a)

and $b = \bar{d}_o^T P_{oo} \bar{d}_o$ (for object points) (73b)

ie $d^T Pd = a + b$

To verify that (73) = (70) use (72) in (73a) to get

$$a = d_s^T (P_{ss} - P_{so} P_{oo}^{-1} P_{os}) d_s = d_s^T P_{ss} d_s - d_s^T P_{so} P_{oo}^{-1} P_{os} d_s$$

and (71) in (73b) one gets

$$b = \bar{d}_o^T P_{oo} \bar{d}_o = (d_o + P_{oo}^{-1} P_{os} d_s)^T \; P_{oo} (d_o + P_{oo}^{-1} P_{os} d_s)$$

$$= d_o^T P_{oo} d_o + d_s^T P_{os}^T (P_{oo}^{-1} P_{oo}) d_o + d_o^T (P_{oo} P_{oo}^{-1})$$

$$. P_{os} d_s + d_s^T P_{os}^T P_{oo}^{-1} (P_{oo} P_{oo}^{-1}) P_{os} d_s$$

with $(P_{oo} P_{oo}^{-1}) = (P_{oo}^{-1} P_{oo})$ = identity matrix

$$a + b \; = \; d_s^T P_{ss} d_s + d_s^T P_{os} \; d_o + d_o^T P_{os} \; d_s +$$

$$d_o^T P_{oo} d_o \qquad \text{(all are scalar)}$$

with $d_o^T P_{os} d_s = (d_o^T P_{os} d_s)^T = d_s^T P_{so} d_o$

and $P_{so} = P_{os}^T$ (P_{dd} symetrical)

$$\underline{d^T P d = a + b = d_s^T P_{ss} d_s + 2 d_s^T P_{so} d_o + d_o^T P_{oo} d_o \; =} \qquad (70)$$

with (73a)

$$\underline{\nu^2 = \frac{d_s^T \bar{P}_{ss} d_s}{h_s}} \qquad (74)$$

h_s = number of differences in d_s - rank defect r

applying the Fisher test, for the reference points, on ν^2

$$P \left\{ \frac{\nu^2}{\sigma^2} \quad > \quad F_{1-\alpha, h_s, f} \; \middle| \; Ho \right\} = \alpha \qquad (75)$$

so if $\frac{\nu^2}{\sigma^2} = \bar{F}_s > F$ then there are deformations in

the reference points with an error probability of α%. If not, then one proceeds to section 3.4.6 to test the object points for deformation.

3.4.5 Identification of unstable point in reference points

To identify an unstable point the P_{ss} matrix and ds vector are partitioned as for reference points and object points resulting in:

$$d_s = \begin{vmatrix} d_F \\ d_B \end{vmatrix}$$

where the vector d_B comprises the x & y difference for one point only.

$$P_s = \begin{pmatrix} P_{FF} & P_{FB} \\ P_{BF} & P_{BB} \end{pmatrix} \qquad (76)$$

Transform as before.

$$\bar{d}_B = d_B + P_{BB}^{-1} P_{BF} d_F$$
$$\bar{P}_{FF} = P_{FF} - P_{FB} P_{BB}^{-1} P_{BF} \qquad (77)$$

and $d_s^T P_{ss} d_s = d_F^T \bar{P}_{FF} d_F + \bar{d}_B^T P_{BB} \bar{d}_B$

The partitioning and transformation is performed for each point in turn (i.e. each point is set as the "object point" in turn) and

$$\nu_j^2 = \left(\frac{\bar{d}^T P_{BB} \bar{d}}{2}\right)_j \qquad j = 1 \text{ to } k/2 \qquad (78)$$

k = number of elements in the d_s vector. (dy's & dx's)

is found for each point.

The point yielding the highest ν^2 is regarded as unstable and transferred to the do vector from the ds vector.

The test for deformations in the reference points is now repeated on the remaining d_s (now k-2 elements) vector and further unstable points transferred to the do vector. It is necessary in each case to repartition the Pdd matrix also. This procedure is repeated until the equality

$$\nu_{Rest}^2 = \frac{d_s^T \bar{P}_{ss} d_s}{h_s - 2} \qquad (79)$$

satisfies the Fisher test or in other words, until no more deformations are believed to be present in the remaining reference points.

3.4.6 Verification of deformed points and magnitude of deformations

The verification of the deformed points may take two forms.

1. The method of double point adjustment as described in the method of invariant functions.

2. A transformation of the object point differences.

This transformation takes the form of that mentioned in (71)

$$d = \begin{pmatrix} d_F \\ d_B \end{pmatrix} = \begin{pmatrix} d_s & - \text{ suspect reference points} \\ d_o & + \text{ suspect reference points} \end{pmatrix}$$

$$P_{dd} = \begin{pmatrix} P_{FF} & P_{FB} \\ P_{BF} & P_{BB} \end{pmatrix}$$

and the transformation of the object points d_B is

$$\bar{d}_B = d_B + P_{BB}^{-1} P_{BF} d_F \qquad (80)$$

where $\bar{d}_B$ can be interpreted as the shifts of the object points relative to the fixed reference points.

To test whether the object points have in fact been deformed, determine

$$\sigma_j = \bar{\sigma}\sqrt{\bar{Q}_{jj}} \qquad j = 1 \text{ to } \bar{k} \qquad (81)$$

where $\bar{Q}_{jj}$ = element in jth row and column of $\bar{Q}$ matrix

and $\bar{Q} = P_{BB}^{-1}$

form $q = \dfrac{\bar{d}_j}{\sigma_j} = \dfrac{\text{"signal"}}{\text{"noise"}}$ = signal noise ratio (82)

where $\bar{d}_j$ = element from vector $\bar{d}_b$

In the test according to Niemeier 1976,

1. If $q \geqslant 5$ then the point is deformed
2. If $q < 5$ then no deformation

It should be noted that a point is judged to be deformed if both or one of the components (dx or dy) is judged to be deformed.

Although Niemeier advocates a factor of 5 for the testing of q, this factor was reduced to 2,8 and 3,5 ($\sqrt{2}$*2 and $\sqrt{2}$ * 2,5 respectively as per section 3.2.5) giving confidence levels of 95% and 99% respectively as for the previous methods.

In this thesis the test Network comprises only reference points and therefore the vector of object points only comes into being once a reference point has been found to be unstable. Thus, in effect, this is just a special case of Niemeier's method.

3.4.7 Modifications to accomodate a minimum constraint adjustment

In the minimum constraints adjustment, constraints are applied directly to the network by holding certain co-ordinates fixed. This has the effect of reducing the size of the Q matrix by the number of constraints when compared with the corresponding Q matrix from the free network adjustment. The matrix of normalised eigen vectors is not part of the minimum constraints adjustment.

Similarly to the free network method:

$$\sigma_o^2 = \frac{v_o^T P_o v_o + v_1^T P_1 v_1}{f_o + f_1} \qquad (83)$$

$f = n - u'$ = number of observation - number of unknowns,

and

$$d = X_1 - X_o \qquad (84)$$

remain consistent.

However, in the formation of the weight matrix, P_{dd}, the formulae (63) and (64) change slightly to

$$P_{dd} = Q_{dd}^{-1} = (Qxx_0 + Qxx_1)^{-1} \tag{85}$$

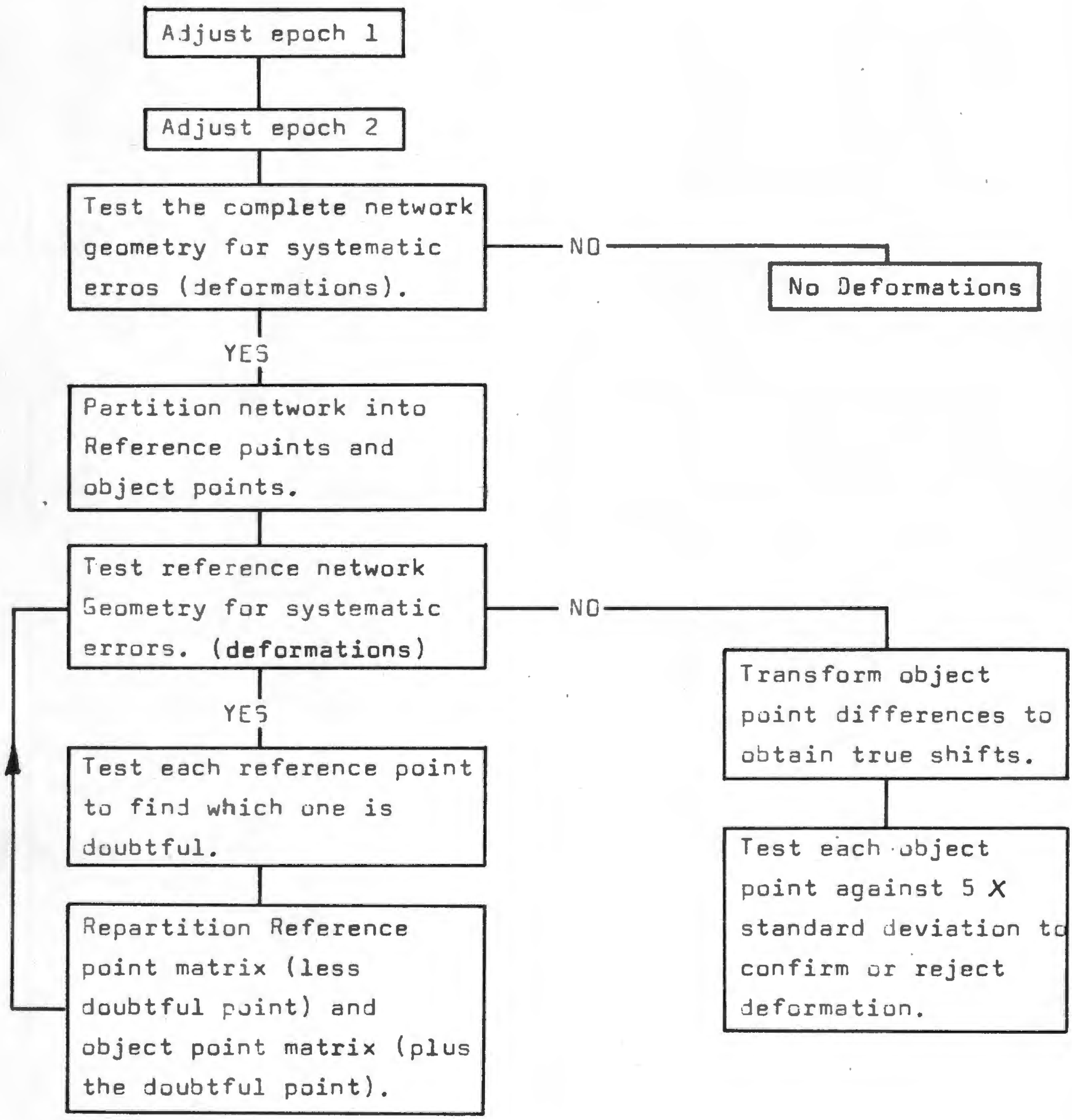
From this stage all the formulae remain the same as for the free network adjustment.

As is the case in a minimum constraints adjustment which includes distance observation (no scale factor) only three co-ordinates are held fixed, which constrain the network. it was decided to reduce the Q matrix and d vector by a further co-ordinate, which would be the second co-ordinate of the point used for the orientation constraint. This would have the effect of reducing the d vector to full points only, excluding the requirement for a separate routine to check the single co-ordinate, each time the reference points were checked for deformations. This assumption, that this single co-ordinate is not deformed, is logical as, should the point be subject to deformation the network would be able to rotate, depending on the degree of deformation, and thus possibly introduce false deformation elsewhere in the network. This problem will obviously not occur in a network comprising only angular observations or having distances with unknown scale factor. In such a case, two full points (4 co-ordinates) would be held fixed and the two full points would be eliminated from the Q matrix & d vector.

When forming v^2 and testing with the Fisher test, it should be noted that h will be equal to the number of elements in the d_s vector as the rank defect has been eliminated by the application of constraints.

A minor drawback to Niemeier's comparison of co-ordinates method is the fact that when executing the programme to determine deformation it is necessary to have a table of values for $F_{1-\alpha,h,f}$, for the comparison in the Fisher test, stored somewhere in the programme. Although this table could be stored in its entirety, it would be easier if only the necessary values for $F_{1-\alpha,h,f}$ are stored. If one examines the formulae involving F it is noted that the only variable for a particular network is h which begins at h = u-r = number of unknown co-ordinate - rank defect (eqn (53)), and thereafter reduces by 2 after each deformed point has been removed from the reference point matrix. Thus it is relatively simple to insert these values in a DATA statement in the programme. However, it must be noted that these values must be amended for each new network.

3.4.8 Flow chart for Niemeier's comparison of co-ordinates



CHAPTER 4

NETWORK USED FOR THE ANALYSIS

Before any deformation analysis can be performed, a minimum of two sets of observations for a network, corresponding to two separate epochs, is required. One, epoch 1, to be a reference epoch, with which the second epoch, which may have deformations present in some of the points, may be compared. The difficulty of obtaining reliable observations for a number of epochs makes it preferable to use simulated data, which can easily be generated on a computer. A further advantage is that one can apply purely random errors to the observations and eliminate the danger of arriving at false conclusions due to systematic errors which may be present in physically measured observations. A routine for generating normally distributed random errors, by H. Ruther, U.C.T., was employed. The standard deviation of the observations could be chosen prior to generation and the errors were added to the geometrically determined directions and distances. As the same error array is generated each time, a different, randomly selected section of the array was used to create the observations for each particular epoch. For the purpose of this thesis, the observation standard deviations were chosen to be ± 2" for directions and ± 1mm for distances.

For the network used in the tests it was felt that a "real life" design as used on an engineering project, and thus subject to limitations encountered in such a project (e.g. position and quantity of reference points and limits set on the number of observations), be adopted rather than a theoretical and therefore "perfect" network (i.e. a network resulting in ideal error ellipses). As the author was employed as a surveyor on the Koeberg Nuclear Power Station site, the network configuration (Fig. 11) used on this site was adopted as the test network. The observation geometry (see Fig. 11), apart from a number of extra distances, closely resembles that of the koeberg site.

NETWORK SHOWING OBSERVATION GEOMETRY

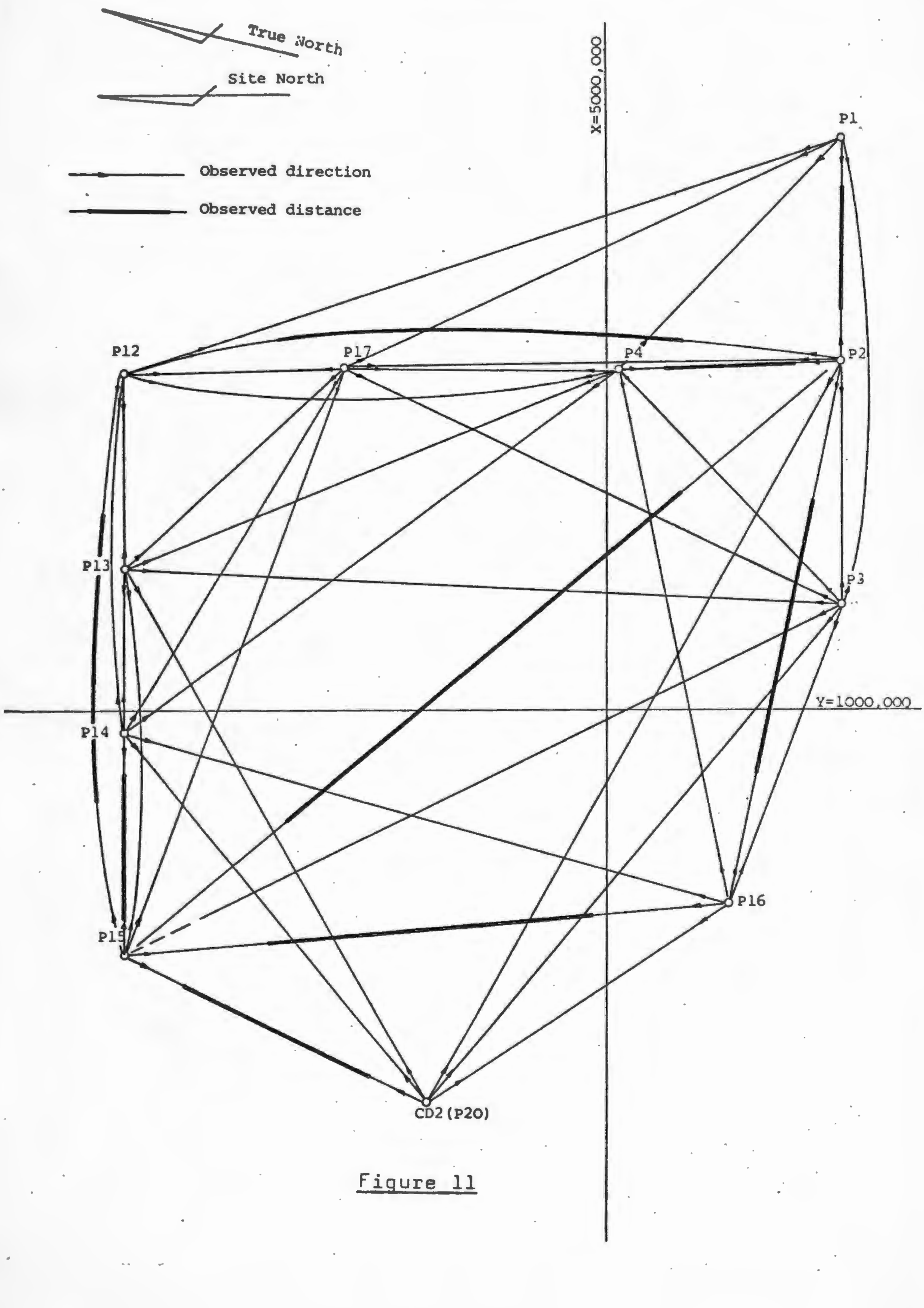


Figure 11

A consideration affecting the choice of epochs is whether one should create a large number of reference epochs, i.e. epochs with no deformations but varying in observations, resulting in a large number of solutions for this epoch, and similarly creating an equally large number of second epochs (same deformations in each case). This type of analysis would thoroughly investigate all types of solutions, but as this type of measurement would not be performed in practice, it was decided to adopt the more realistic approach of using one reference epoch and a number of second epochs (with differing deformations). It was felt that this would simulate actual situations with the difficulties and limitations encountered under these conditions.

The actual observations were then simulated from the network configuration, observation of geometry and observation standard deviations. Deformations were then applied to various points in the network and further epochs were generated (nine in all, including the reference epoch A, denoted A,B,C,D,E,F,G,J &HR).

CHAPTER 5

COMPARISON OF RESULTS

In the following comparisons, each epoch will be discussed separately, thus comparing the findings from the various methods epoch by epoch. For each of the methods, the adjustments have been executed using both the minimum constraints and free network techniques. The free network adjustment programme used was developed by Dr. H. Rüther of the University of Cape Town, who thereby holds its copyright.

In discussing the epochs and in sketches, use is made of abbreviations to distinguish the methods. The suffix M indicates a minimum constraint adjustment, while F indicates the free network technique.

1. IM, IF = Invariant Functions (Sect. 3.1).
2. Aa = Adjustment with stable points held fixed. Doubtful points from IM (Sect. 3.1.5).
3. Ab = As for Aa but doubtful points from SF.
4. CM, CF = Co-ordinate Comparison (Sect. 3.2).
5. CT = Co-ordinate Comparison with all point Transformation (Sect. 3.2.4).
6. SM, SF = Co-ordinate Comparison with stable point Transformation (Sect. 3.2.4).
7. DM, DF = Direct Differences (Sect. 3.3).
8. NM, NF = Niemeier's Co-ordinate Comparison (Sect. 3.4). ? 2.8 3.5

For the graphical representation of deformations, vectors are used to depict these shifts at a scale of 1:20. Unless otherwise indicated, Site North is towards the top of the page.

5.1 Epoch A

This is the reference epoch to which no deformations were applied.

All following epochs (B,C,D,E,F,G,J & HR) are compared to A. Thus epoch A would be an epoch 0 and the others all variations of an epoch 1.

5.2 Epoch B

Two points, P3 & P12 were deformed in this epoch (see figure 13, magnitudes table 2). The magnitude of the shift at P3 was relatively small and as a result only two methods (CF & DF) found this point to be definitely deformed (99% confidence level). For interest sake and to investigate the shifts indicated by a 95% confidence level, shifts falling between the 95% & 99% confidence levels are also included in the table, shown as magnitudes in brackets. With the 95% confidence level three further methods, i.e. CT, SM & SF also yielded P3 as disturbed.

From the invariant function comparison (table 3) it is clear that only P12 is doubtful, as it accounts for all the failures in the 99% column. From the 95% column three failures are not accounted for by P12. These three include points P2, P3 & CD2 (P20). In an attempt to highlight the shift at P3, these three points were included with P12 when the adjustment of method Aa was performed. However, only P12 showed a definite disturbance while P20 & P2 were found to be deformed with a 95% certainty. It would appear that, although type II errors (Hypothesis accepted even though it is incorrect - deformed points missed) may occur, it is reasonable to adopt the findings from the invariant function comparisons using the 99% failure column only. This would then result in P12 as being unstable for methods IM & Aa, and reduce the type 1 errors (Hypothesis rejected even though it is correct - stable points shown to be deformed).

Sketch indicating position of deformations for Epoch B

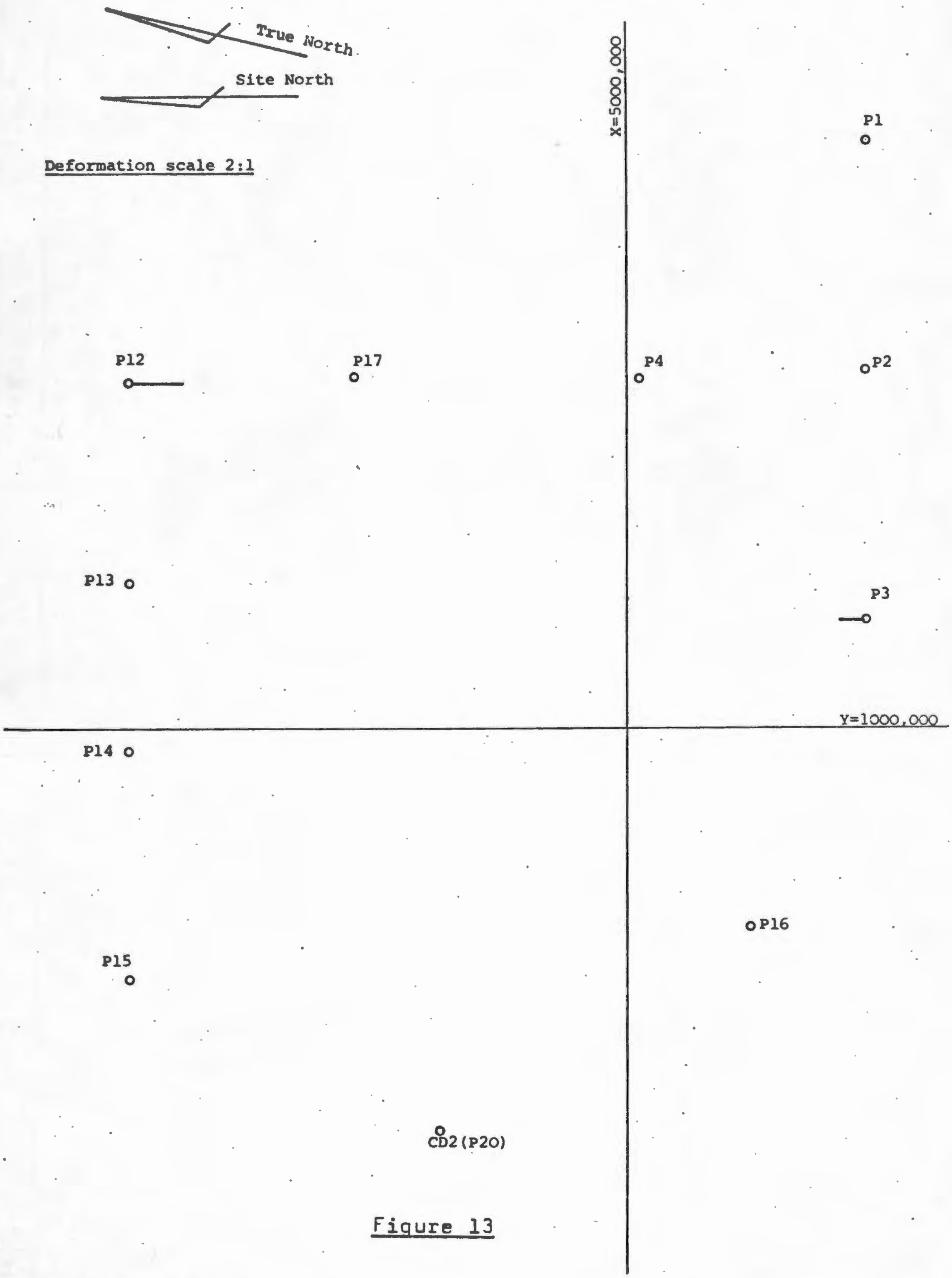


Figure 13

EPOCH B

Method	Shift P3 mm dy	dx	Shift P12 mm dy	dx	Shift P20 mm dy	dx	Shift P2 mm dy	dx	Shift mm dy	dx
Applied Shift	0	-2,5	0	+5,0	0	0	0	0		
Invariant Functions (Min Cons) IM			+0,2	+4,8						
Invariant Functions (Free Net) IF			+0,3	+4,9	-1,7	+2,8				
All fixed except doubtful points a			+0,1	+5,0	(-1,7	+2,6)	(+1,5	+0,3)		
A b			+0,2	+4,9	(-1,8	+2,7)				
Co-ord Comparison (Min Cons) CM			-0,9	+4,7	(-3,1	+3,0)				
Co-ord Comparison (Free Net) CF	+0,3	-2,8	0	+4,0	-2,0	+2,3	(+1,3	-0,6)		
Co-ord Comparison Min Cons + Transform. CT	(+0,4	-2,8)	+0,1	+4,0						
Direct Differences Min Cons DM			-1,0	+4,7						
Direct Differences Free Net DF	+0,4	-2,8	+0,1	+4,0						
Niemeier (Min Cons) NM			+0,2	+5,0						
Niemeier (Free Net) NF			-0,2	+4,8						
Co-ord Comparison (Min Cons) Partial Transformation SM	(-0,2	-2,4)	-0,2	+4,3						
Co-ord Comparison (Free) Partial Transformation SF	(+0,5	-2,4)	0	+4,5	(-2,0	+2,5)	(+1,4	-0,1)		

* Indicates shifts equal to the 99% or 95% confidence level test value

Values in brackets indicate shifts between 95% & 99% confidence levels.

Table 2

Invariant Function Comparisons

Epoch B.

LIST OF ANGLES AND DISTANCES FAILING AT 90%, 95% AND 99% CONFIDENCE LEVELS

FROM	TO	TO	SIGMA F	DIFF	90%	95%	99%
P12	P4	P13	5.21	10.47	*	*	
P12	P13	P14	2.77	-5.59	*	*	
P13	P4	P12	5.74	10.67	*		
P14	P4	P12	2.83	4.88	*		
P14	P12	P13	3.44	-6.60	*		
P15	P4	P12	1.47	3.37	*	*	
P15	P12	P13	1.08	-3.45	*	*	*✓
P16	P12	P13	1.09	-1.91	*		
CD2(P20)	P12	P13	0.94	-2.98	*	*	*✓
P1	P12		1.69	4.78	*	*	*✓
P1	CD2(P20)		2.09	3.98	*		
P2	P12		1.14	4.59	*	*	*✓
P2	P16		1.22	2.52	*	*	
P2	CD2(P20)		1.77	4.23	*	*	
P3	P12		1.52	6.40	*	*	*✓
P3	P14		1.45	2.78	*		
P3	CD2(P20)		2.56	5.09	*	*	
P4	P12		1.51	4.46	*	*	*✓
P12	P16		1.45	4.53	*	*	*✓

P1	P2	P3	P4	P12	P13	P14	P15	P16	P17
2	3	3	5	14	7	4	2	3	0

CD2(P20)
4

Table 3

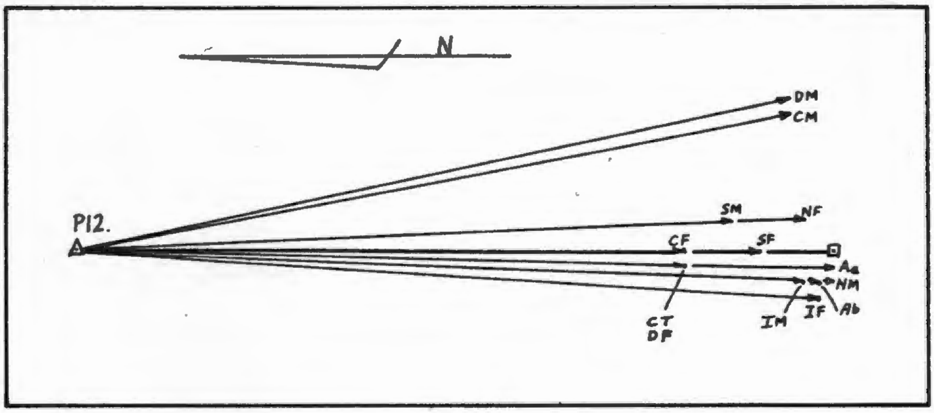


Figure 14

P12 was judged to be a definite deformation by all the methods. The graphical representation shown in figure 14 shows the good distribution of the derived deformations (vectors) with respect to the applied deformation ⊡, with methods Aa, Ab & NM falling very close to the true value; (the above deformations have been reduced to a common origin, △, to simplify the figure). In general, all the deformations fall within 1mm of the true position, which seems to confirm the logical assumption that larger shifts will give a better representation of the true deformation due to the ratio between position error and deformation magnitude being larger, and an error in point position will thus have less effect on the displacement vector than would a smaller one.

At the 99% confidence level, two type I errors were indicated, both by free network type methods. Methods IF & CF both resulted in P20 as being moved. At the 95% level method A, CM, CF & SF yielded type I errors for points P20 & P2 (CM and Ab only P20). In the case of Aa the two type I errors would not normally be encountered as the only doubtful point as indicated by the invariant function comparisons (IM) would be P12, and the other two points would not be included as doubtful.

5.3 Epoch C

The intention in this epoch was to create a situation where the points normally used as constraint points in the minimum constraint adjustment were suspect. Thus two points, P1 & P16 were deformed as well as a third point P15. It will be seen from Figure 15 that the deformations of these points would appear to introduce a rotation into the network, while this rotation would in itself not affect the invariant functions. From the initial adjustment, it was immediately apparent that resulting point positions were suspect, and a second adjustment was performed with points P3 & P13Y as the fixed parameters. This second adjustment was employed for further analysis.

The first faulty adjustment was used for the invariant function comparisons and confirmed the logical assumption that the origin and rotation do not affect the efficiency of the comparisons. From these comparisons (see table 4) the points P1, P15 & P16 were judged to be doubtful and introduced as double points in the combined adjustment. with P3 & P13Y as constraint points. The combined adjustment yielded only P16 as deformed at the 99% probability level with the remaining two points deformed when using a 95% probability level. A second combined adjustment was performed in an attempt to reduce the occurrence of type II errors. In this adjustment only the two constraint points were entered as single points, the remainder being defined as double points, (i.e. suspect). This adjustment, although time consuming did not result in a reduction of type II errors. In fact, all three of the deformed points were found to be stable even when using the 95% confidence level as the stability criterion. The use of the invariant function table to determine suspect points may therefore be confidently adopted.

Sketch indicating position of deformations for Epoch C

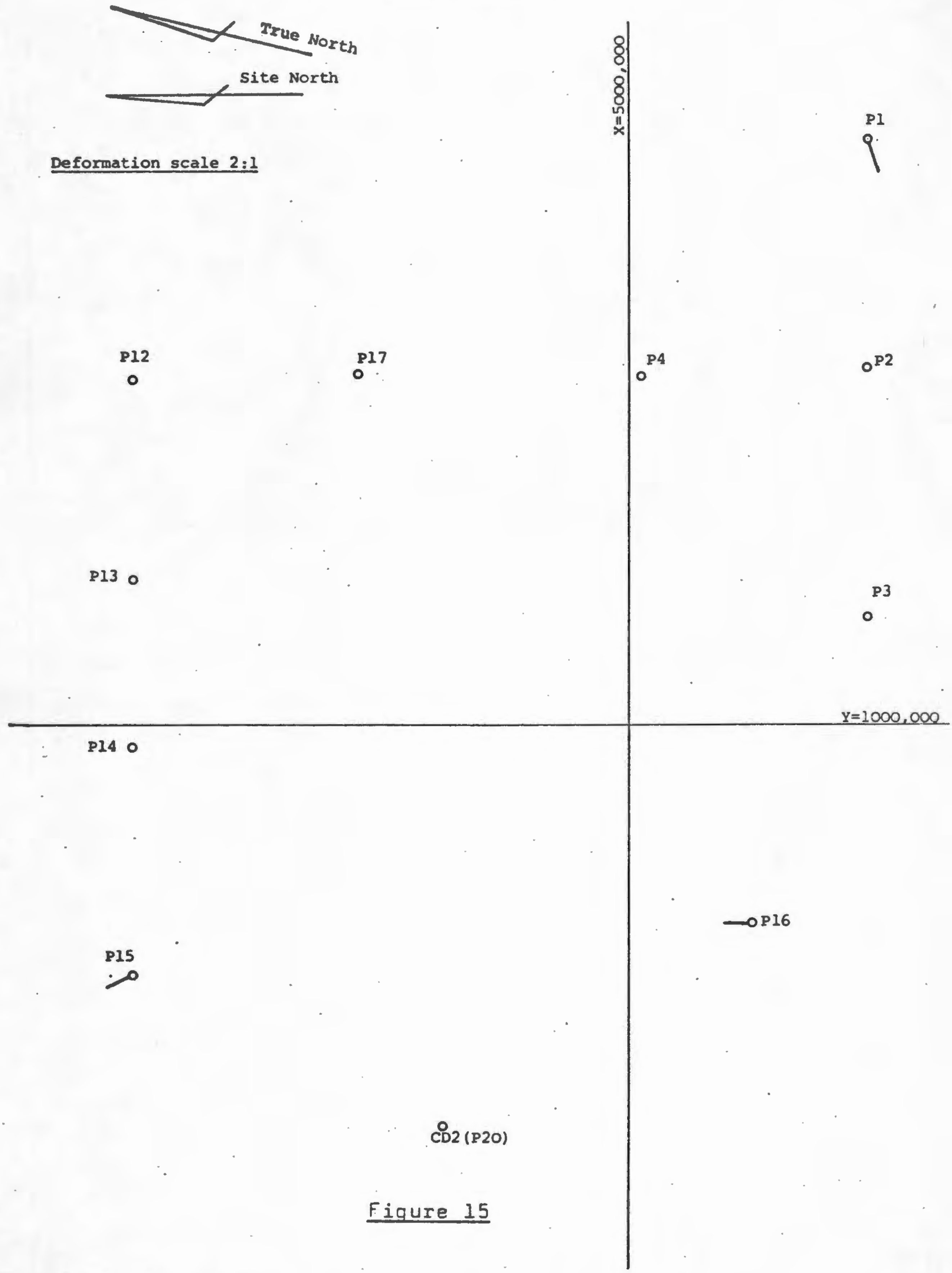


Figure 15

Invariant function Comparisons

Epoch C

LIST OF ANGLES AND DISTANCES FAILING AT 90%, 95% AND 99% CONFIDENCE LEVELS

FROM	TO	TO	SIGMA F	DIFF	90%	95%	99%
P1	P3	P4	1.30	4.95	*	*	*
P1	P4	P12	1.20	-2.86	*	*	
P1	P15	P16	0.60	1.08	*		
P1	P16	P17	1.00	1.99	*	*	
P1	P17	CD2(P20)	0.92	-4.11	*	*	*
P2	P15	P16	0.97	2.01	*	*	
P3	P14	P15	0.64	-1.08	*		
P3	P15	P16	1.36	4.82	*	*	*
P3	P16	P17	2.34	-4.56	*		
P4	P15	P16	1.66	3.12	*		
P12	P1	P2	0.68	-1.62	*	*	
P12	P14	P15	1.29	2.19	*		
P13	P1	P2	0.68	-1.15	*		
P14	P16	P17	2.24	-3.93	*		
P15	P1	P2	0.48	-1.12	*	*	
P16	P2	P3	1.33	2.71	*	*	
P16	P4	P12	0.97	-1.74	*		
P16	P14	P15	0.66	-2.54	*	*	*
P17	P1	P2	1.23	-2.57	*	*	
P17	P16	CD2(P20)	1.30	-3.25	*	*	
CD2(P20)	P1	P2	0.45	-0.93	*	*	
CD2(P20)	P14	P15	1.90	-3.66	*		
P1	P2		1.09	3.05	*	*	*
P1	P4		1.42	3.66	*	*	*
P1	P14		1.62	3.48	*	*	
P1	P16		1.53	3.31	*	*	
P1	CD2(P20)		1.88	4.45	*	*	
P2	P14		1.12	2.25	*	*	
P3	P15		1.43	-2.51	*		
P3	P17		2.10	-3.48	*		
P4	P15		1.20	-3.78	*	*	*
P12	P15		1.08	-2.06	*		
P13	P15		2.06	-3.44	*		
P13	P16		1.41	2.46	*		
P14	P15		1.11	-3.25	*	*	*
P14	P16		1.23	3.58	*	*	*
P15	P17		1.47	-2.48	*		
P15	CD2(P20)		1.10	-2.62	*	*	
P16	CD2(P20)		1.72	5.09	*	*	*

P1	P2	P3	P4	P12	P13	P14	P15	P16	P17	CD2(P20)
15	9	7	6	5	3	9	16	15	8	7

Table 4

EPOCH C

Method	Shift P1 mm dy	dx	Shift P15 mm dy	dx	Shift P16 mm dy	dx	Shift mm dy	dx	Shift mm dy	dx
Applied Shift	+3,0	+1,0	+1,0	-2,5	0	-2,5				
Invariant Functions (Min Cons) IM	(+3,9	+1,0)	(+1,9	-2,4)	-0,2	-3,8				
Invariant Functions (Free Net) IF	+6,2	+1,0	+2,3	-3,2	+2,0	-4,9	P2 +3,1	-0,6		
All fixed except doubtful points A a	+3,9	+1,0	+1,8	-2,4	-0,3	-3,8	P4			
A b	+3,2	+1,0	(+1,7	-1,3)	-0,7	-3,1	(-1,9	+0,4)		
Co-ord Comparison (Min Cons) CM					(-0,3	-3,2)	P17 (-1,0	-3,3)		
Co-ord Comparison (Free Net) CF	+3,4	+0,5	+2,1	-0,8	-0,2	-2,6	P4 -1,9 P14 (-1,1	+0,2 +1,3)	P20 -0,3	+3,3
Co-ord Comparison Min Cons + Transform. CT										
Direct Differences Min Cons DM a									P14 -1,6	+3,2
DM b							P4 -4,4	+0,7	P20 -2,0	+6,7
Direct Differences Free Net DF	+3,4	+0,5	+2,1	-0,8	-0,2	-2,6	P20 -0,3	+3,3		
Niemeier (Min Cons) NM	+6,2	+1,0	+2,3	-3,1	+2,0	-4,9	P2 (3,0	-0,6)		
Niemeier (Free Net) NF	(+3,4	+1,6)	+2,1	-2,4	-0,6	-3,9				
Co-ord Comparison (Min Cons) Partial Transformation SM	(+4,3	+0,2)			(+0,5	-3,8)				
Co-ord Comparison (Free) Partial Transformation SF	+3,5	+0,2	+2,5	-0,7	-0,1	-2,6	P4 -1,8 P14 (-0,7	+0,1 +1,3)	P20 0	+3,4

* Indicates shifts equal to the 99% or 95% confidence level test value

Values in brackets indicate shifts between 95% & 99% confidence levels.

Table 5

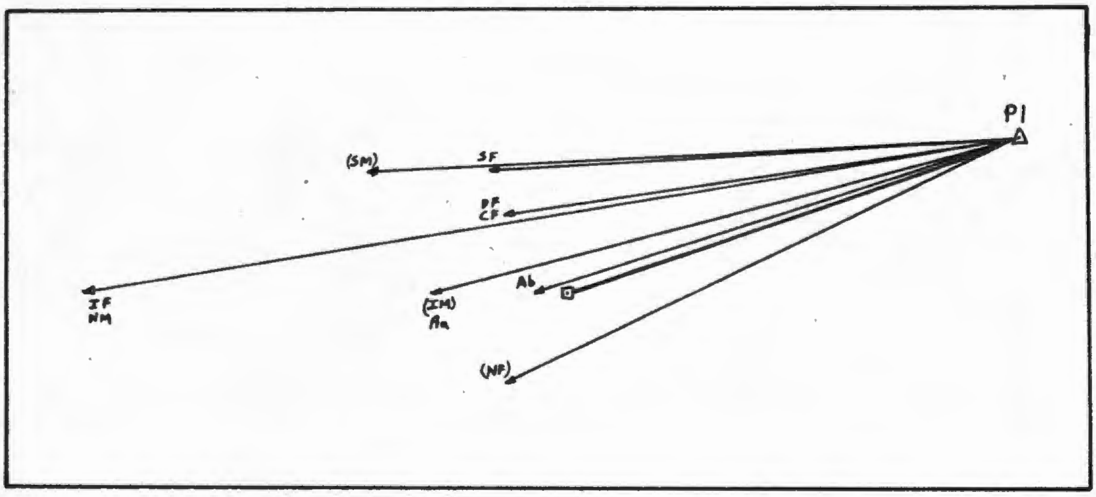


Figure 16

The distribution of deformations for P1 (Figure 16 & table 5) are reasonable, although not as good as that for P12 from epoch B. The deformation magnitude indicated by methods IF & NM are more than twice the applied deformation. Method Ab resulted in the best displacement vector with CF & DF also reasonably near.

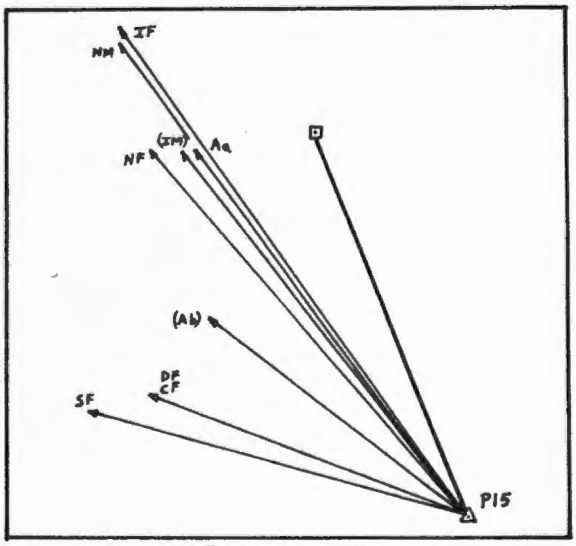


Figure 17

The displacements indicated for P15 in figure 17 show three groups, two of which are within 1,5mm of the applied shift.

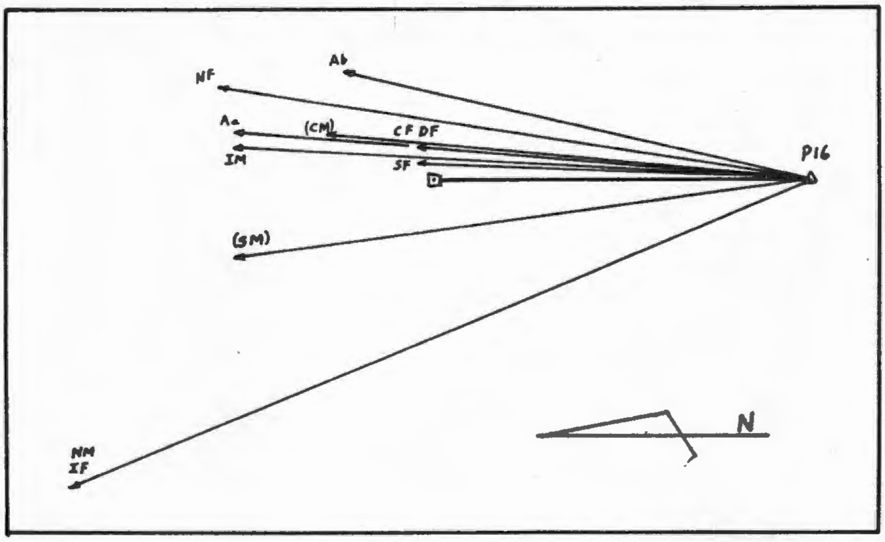


Figure 18

From Figure 18, it is clear that the direction distribution around P16 is good, except for methods IF & NM (again an exaggerated movement) with the magnitudes also giving reasonable findings.

In the above sketches all the disturbances have been reduced to a common origin. The actual positions as determined from the various adjustments are shown in Figure 19 below, where the large variation of actual point positions due to the adjustments can be clearly seen.

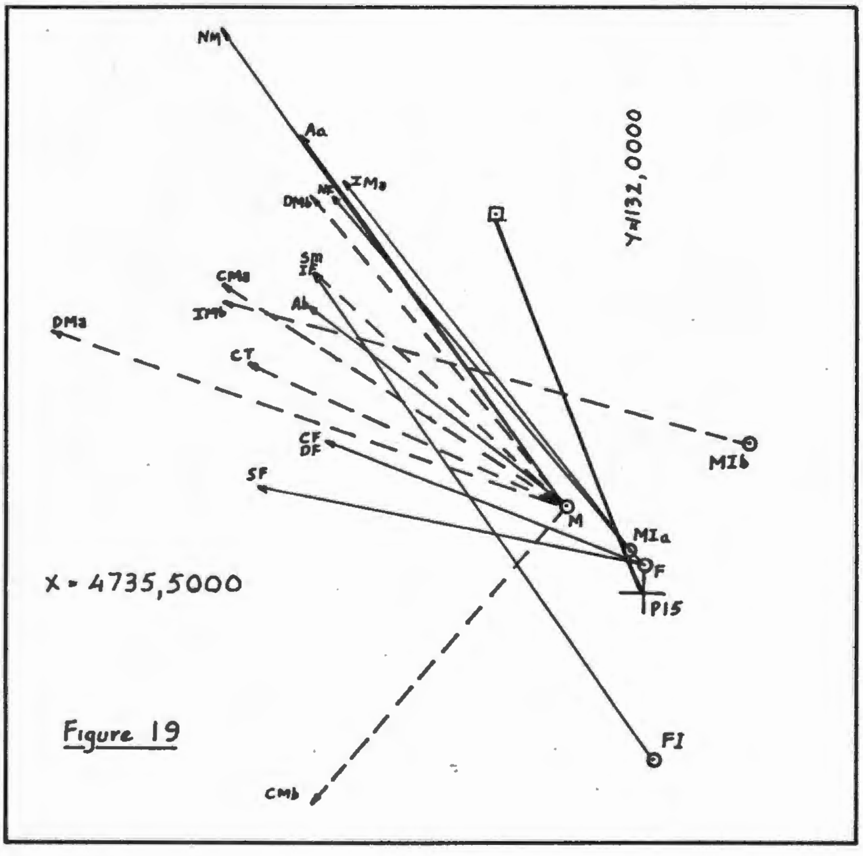


Figure 19

+ = true position of P15

⊡ = true position of deformation

M = epoch A Min. Constr.

F = epoch A Free Net.

MI = epoch A Min. Constr. combined A + C

FI = epoch A Free Net. combined A + C

——— = deformation

- - - = shift but no deformation

DMb = minimum constraints adjustments using P1 & P16x as fixed.

DMa = P3 & P13 fixed

IMa = invariant function normal procedure

IMb = invariant function only 2 points held as stable.

In the method of co-ordinate comparisons, CF (Free Net), although the three deformed points were indicated, two other points, P4, & P20 (table 5) were also shown to be unstable, i.e. errors of type I (the hypothesis is rejected although it is correct). A further point P14 was found to be unstable at 95% confidence level. In the case of the transformation of Epoch C onto stable points (Free Network) the identical points were found to be deformed with magnitudes of a similar order.
The reason for this confusion would be due to the distribution of the point positions from the two epochs' adjustments.

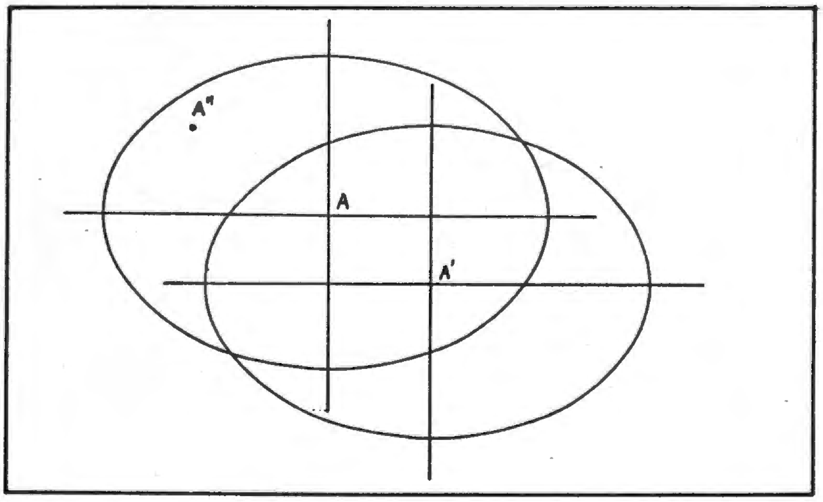


Figure 20

Consider Figure 20. Assume a known true position A, with A' the adjusted position from Epoch 0 and A" that from Epoch 1. It can be seen that, although A' & A" both fall within the 99% confidence ellipse area around point A, as may be expected, the distance between A' & A" may prove to be greater than the standard deviation of the displacement, particularly if the points A' & A" tend towards the outer limits of the absolute error ellipse.

Other errors of type I were present for the methods of invariant functions, direct differences (both free network) and direct differences, case b, using minimum constraints.

As can be seen from table 5, the method of direct difference, DMb, (fixed P1 & P16X), yielded 3 points as being unstable, P4, P14 & P20. These points were however not deformed and this indicates the necessity of ensuring the reliability of the fixed points when using minimum constraints adjustments. The adjusted differences from this model, although relatively large, would alone not indicate any fault with the constraint points and an initial scan of the raw observations could prevent this type of error from occuring, i.e. the creation of false deformations due to displacements in the constraint points introducing a swing and/or translation into the network. In the case of the direct differences method DMa, using P3 & P13Y as fixed, the method indicated that the whole metwork was stable. The use of method Ab (doubtful points from SF) showed all three points deformed at 95% confidence level as well as an extra point P4.

In relation to other epochs, this epoch showed a large number of type I errors particularly in methods CF & SF.

5.4 Epoch D

Three points. P12, P14 & P17, were deformed (Figure 21 & table 6). The displacement at P12 was detected by all the methods although in general the magnitude was exaggerated with large discrepancies from three of the methods, CM, DM & SM. In the case of the second point, P14, two methods showed type II errors at 99% confidence while this point was included at the 95% level.

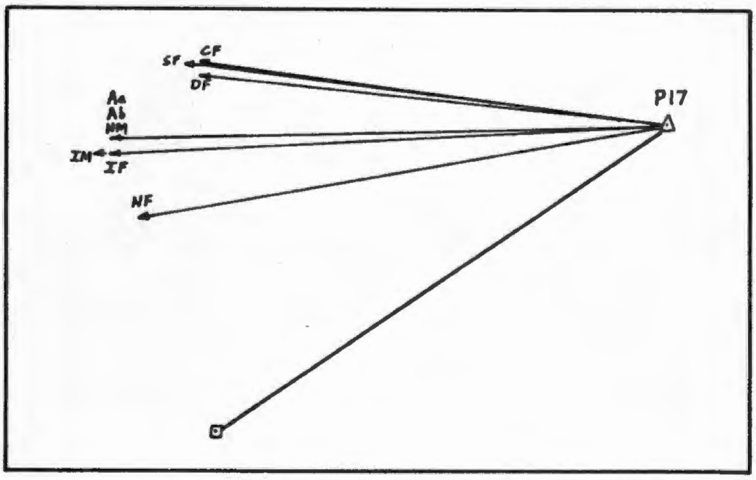


Figure 22

The distribution of the movements for P17 as derived from the various models (Figure 22) shows a mean deformation which is approximately due west, with all but one position being more than 2mm from the applied position. The method nearest the true shift was NF, being approximately 1,5mm away.

Three type I errors were disclosed at the 95% confidence level all for point P16 (see table 6).

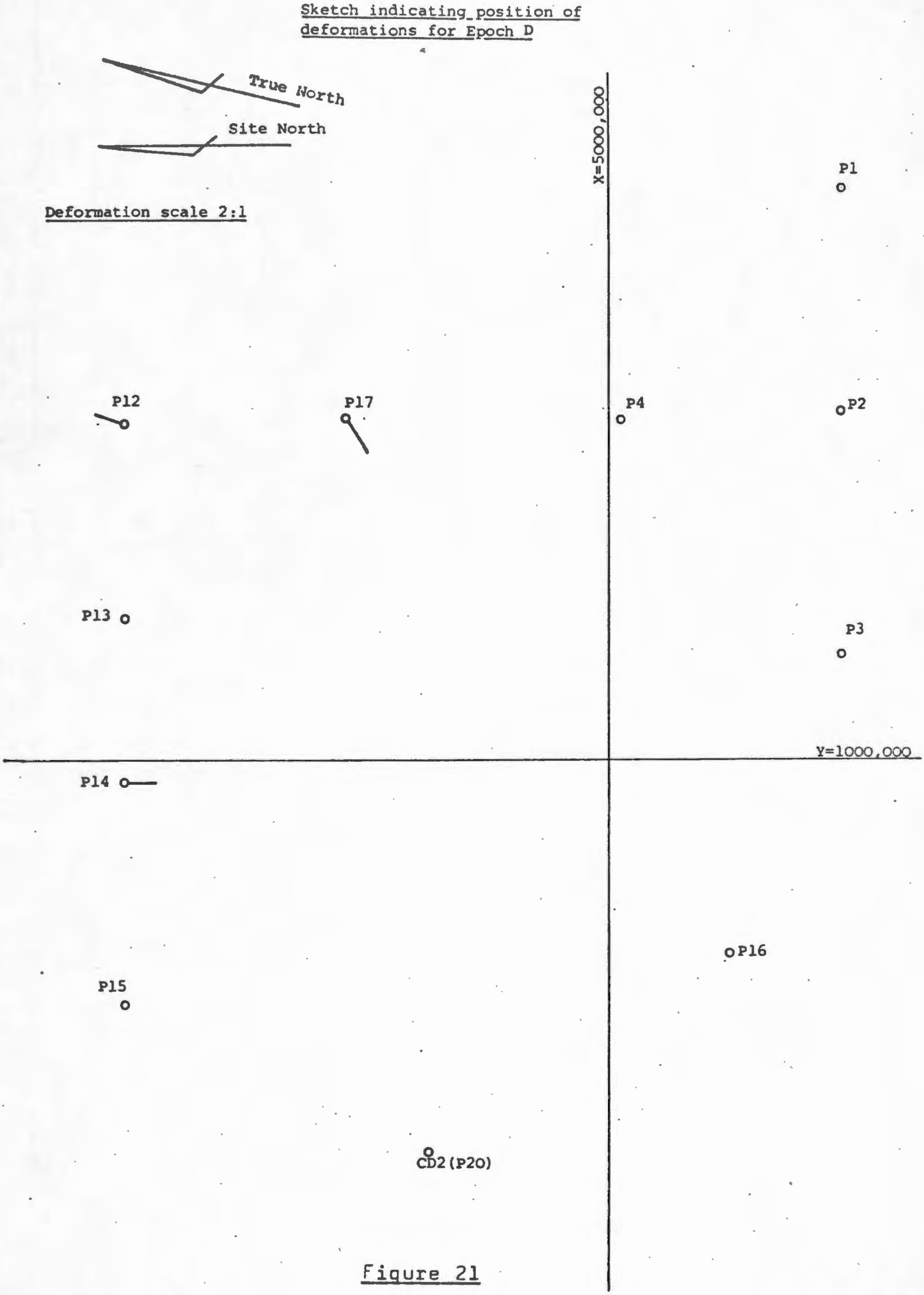

Sketch indicating position of deformations for Epoch D
True North
Site North
Deformation scale 2:1
X=5000,000
Y=1000,000
P1
P2
P3
P4
P12
P13
P14
P15
P16
P17
CD2 (P20)

Figure 21

EPOCH D

Method	Shift P12 mm dy	dx	Shift P14 mm dy	dx	Shift P17 mm dy	dx	Shift P16 mm dy	dx	Shift mm dy	dx
Applied Shift	-1,0	-3,0	0	+3,0	+3,0	+2,0				
Invariant Functions (Min Cons) IM	-0,9	-3,9	+0,5	+3,7	+3,8	+0,2				
Invariant Functions (Free Net) IF	-0,9	-3,9	+0,4	+3,7	+3,7	+0,2				
All fixed except doubtful points A a	-0,9	-3,9	+0,4	+3,7	+3,7	+0,1				
A b	-0,9	-4,0	+0,3	+3,7	+3,7	+0,1				
Co-ord Comparison (Min Cons) CM	-3,5	-4,4	(-2,4	+2,9)			(-3,2	0)		
Co-ord Comparison (Free Net) CF	-1,9	-4,3	-0,7	+3,3	+3,1	-0,4	(-1,9	+0,5)		
Co-ord Comparison Min Cons + Transform. CT	-1,9	-4,3	-0,7	+3,3						
Direct Differences Min Consr. DM	-3,6	-4,4	-2,4	+2,9						
Direct Differences Free Net DF	-1,9	-4,3	-0,7	+3,3	+3,1	-0,3				
Niemeier (Min Cons) NM	-0,9	-4,0	+0,3	+3,7	+3,7	+0,1				
Niemeier (Free Net) NF	-1,2	-3,8	+0,8	+3,8	+3,5	+0,6				
Co-ord Comparison (Min Cons) Partial Transformation SM	-3,3	-4,4	(-2,1	+2,9)						
Co-ord Comparison (Free) Partial Transformation SF	-1,7	-4,4	-0,6	+3,3	+3,2	-0,4	(-2,0	0,6)		

* Indicates shifts equal to the 99% or 95% confidence level test value

Values in brackets indicate shifts between 95% & 99% confidence levels.

Table 6

5.5 Epoch E

In the situation simulated in epoch C, the displacements applied to the constraint points were relatively large and could thus be reliably detected. In epoch E, a similar condition was simulated except that the deformations were relatively small but in opposite directions, falling within the detection threshold of the methods, but imposing a rotation on the network. Two further points were deformed also to see if this rotation would affect these displacements to the extent of either suppressing or magnifying them. The points displaced were P1, P16, P4 & P15 (see Figure 23 & table 9). The situation envisaged here would give the analyst the impression that the constraint points were stable. The minimum constraint adjustment yielded no abnormal results and the adjustment would in normal circumstances be adopted.

The comparison of invariant functions (minimum constraints) displayed a low failure frequency (table 7), with only one 99% failure.

LIST OF ANGLES AND DISTANCES FAILING AT 90%, 95% AND 99% CONFIDENCE LEVELS

FROM	TO	TO	SIGMA F	DIFF	90%	95%	99%
P1	P3	P4	1.46	3.91	*	*	*
P1	P4	P12	1.34	-3.32	*	*	
P12	P14	P15	1.44	2.56	*		
P15	P13	P14	2.06	3.61	*		
P1	P16		1.71	2.84	*		
P2	P12		1.16	-2.08	*		
P4	P15		1.35	-2.81	*	*	

P1	P2	P3	P4	P12	P13	P14	P15	P16	P17	CD2(P20)
3	1	1	3	3	1	2	3	1	0	0

table 7

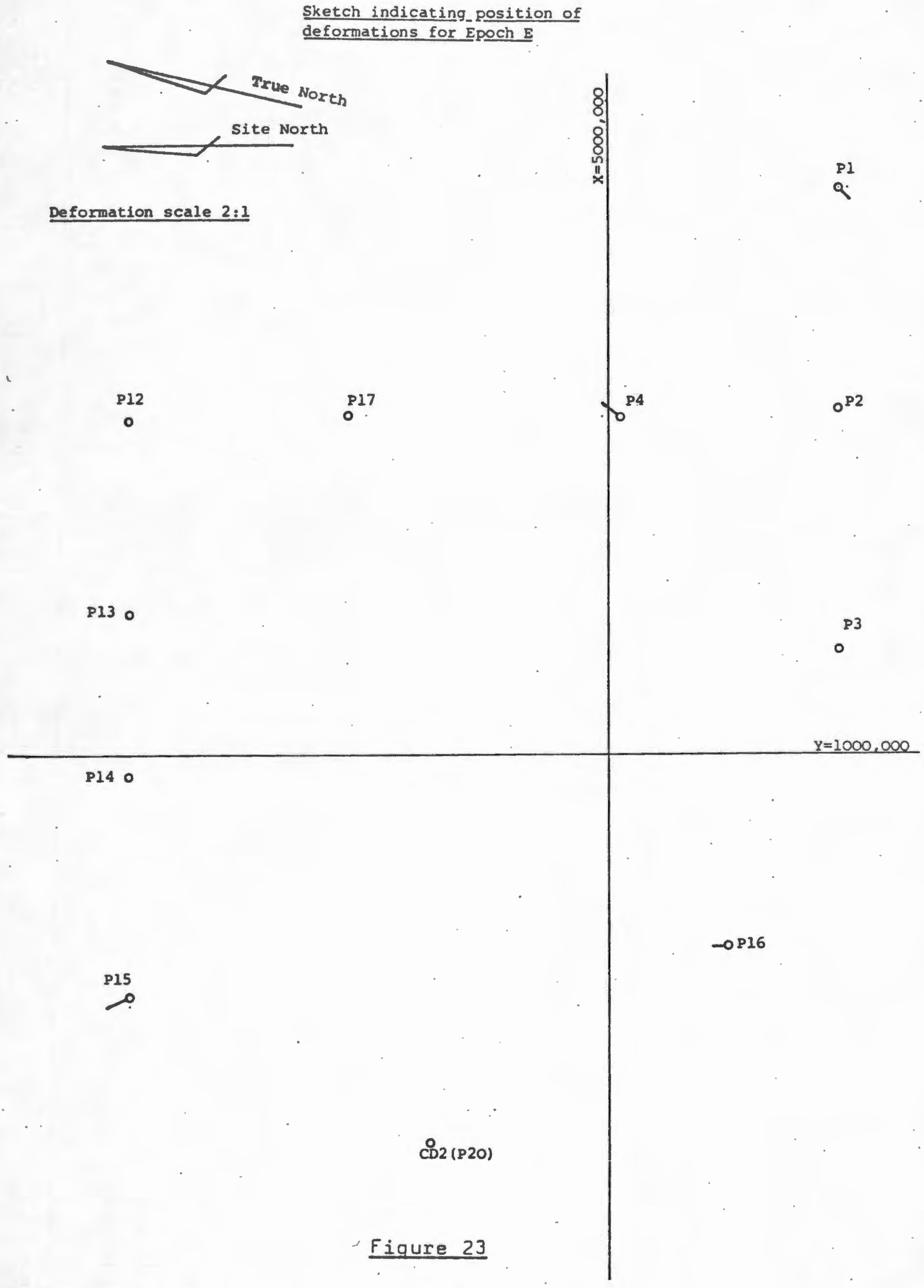

Sketch indicating position of
deformations for Epoch E
True North
Site North
X=5000,000
Deformation scale 2:1
P1
P12
P17
P4
P2
P13
P3
Y=1000,000
P14
P16
P15
CD2 (P20)

Figure 23

Of the four points having the highest occurrence frequency of 3, only P1 & P4 accounted for the one failure. Of the 95% failures P1 is involved in all three, while P4 only in two. The position displayed by the table 7 indicates two possibilities:

1. That the network is stable and the failures are due to point position distribution.

2. That the model has deformations present and a combined adjustment should be performed to verify or disprove this.

The second option was chosen, with P1 & P4 suspect, and the combined adjustment indicated a deformation-free network (see table 9).

LIST OF ANGLES AND DISTANCES FAILING AT 90%, 95% AND 99% CONFIDENCE LEVELS

FROM	TO	TO	SIGMA F	DIFF	90%	95%	99%
P1	P3	P4	1.72	3.91	*	*	
P1	P4	P12	1.45	-3.32	*	*	
P2	P3	P4	2.94	5.06	*		
P2	P4	P12	1.73	-4.52	*	*	*
P4	P14	P15	0.49	1.32	*		
P12	P1	P2	0.60	-1.09	*		
P12	P14	P15	1.02	2.56	*	*	
P13	P1	P2	0.56	-1.04	*		
P13	P14	P15	2.55	4.86	*		
P14	P13	P15	4.69	8.47	*		
P14	P15	P16	1.92	-4.56	*	*	
P15	P13	P14	1.28	3.61	*	*	*
P15	P14	P16	2.06	-4.64	*	*	
P15	P17	CD2(P20)	2.13	-3.57	*		
P1	P16		1.71	2.84	*		
P2	P12		1.16	-2.08	*		
P4	P15		1.35	-2.81	*	*	

P1	P2	P3	P4	P12	P13	P14	P15	P16	P17	CD2(P20)
5	5	2	6	5	4	7	9	3	1	1

<u>table 8</u>

The invariant function comparisons for the free network, (table 8), showed two 99% failures. The 95% column was again consulted, as for the minimum constraints, revealing P4 & P15 as doubtful.

Although P14 has a higher frequency than P4, all the failures at 95% involving P14 also involved P15. P4 was thus the obvious next choice. The combined adjustment rendered P4 as deformed with P15 showing a displacement at the 95% confidence level. In general P4 was adduced as being deformed while some doubt existed as to whether P15 was displaced, (table 9) with only three methods confirming shift at the 99% confidence level. The derived position for P4 was somewhat to the south south east of the true position (approx. - 0,7mm in Y & + 1,3mm in X), while the displacement for P15 was slightly shorter than the applied shift. An attempt was made to see if these displacements were due to the orientation swing introduced by the suspect points P1 & P16 or simply due to the observation sample.

In figure 24 the positions of P4 & P15, as they would appear if there were no observation error, were constructed, e.g. constructing the angle at P1' between P4', P1' & P16' (positions as at time of measurement), then transferring this angle to P1 P16 (positions as adopted by the adjustment). Similarly with angle P4' P16' P1' (at P16') and transferring this to P1 & P16. The intersection of these two rays would result in the position P4" if no errors existed in the observations. A third ray from P17 was included as a check. The position P15" was similarly constructed. The position P4''' & P15''' shows the approximate positions as found from the analysis. The absolute error ellipses at P15" & P4" have also been constructed to give an indication of the point accuracy. It can be seen that the derived positions P4''' & P15''' both fall outside these ellipses but would both fall within a 99% confidence level ellipse.

It appears from Figure 24 that there is more danger of a faulty analysis due to observation errors than there is to minor deformations in the fixed points.

EPOCH E

Method	Shift P1 mm dy	dx	Shift P4 mm dy	dx	Shift P15 mm dy	dx	Shift P16 mm dy	dx	Shift P12 mm dy	dx
Applied Shift	+1,0	+1,0	-1,5	-2,0	+1,0	-2,5	0	-1,3	0	0
Invariant Functions (Min Cons) IM										
Invariant Functions (Free Net) IF			-2,2	-0,6	(+0,3	-1,6)				
All fixed except doubtful points A a			-2,2	-0,6	+0,1	-2,1				
All fixed except doubtful points A b			-2,2	-0,7	+0,3	-2,1			(+0,1	-2,3)
Co-ord Comparison (Min Cons) CM			-3,8	-1,3						
Co-ord Comparison (Free Net) CF	(+2,1	+1,5)	-2,1	* -0,2	(+0,3	-1,6)			(-0,3	-1,8)
Co-ord Comparison Min Cons + Transform. CT										
Direct Differences Min Cons DM			-3,8	-1,2						
Direct Differences Free Net DF	(+2,1	+1,5)	-2,2	-0,2	(+0,3	-1,6)				
Niemeier (Min Cons) NM			-2,4	-1,0						
Niemeier (Free Net) NF			-2,5	-0,6						
Co-ord Comparison (Min Cons) Partial Transformation SM										
Co-ord Comparison (Free) Partial Transformation SF	(+2,4	+1,0)	(-1,9	-0,7)	+0,6	-2,2			0,0	-2,4

* Indicates shifts equal to the 99% or 95% confidence level test value

Values in brackets indicate shifts between 95% & 99% confidence levels.

Table 9

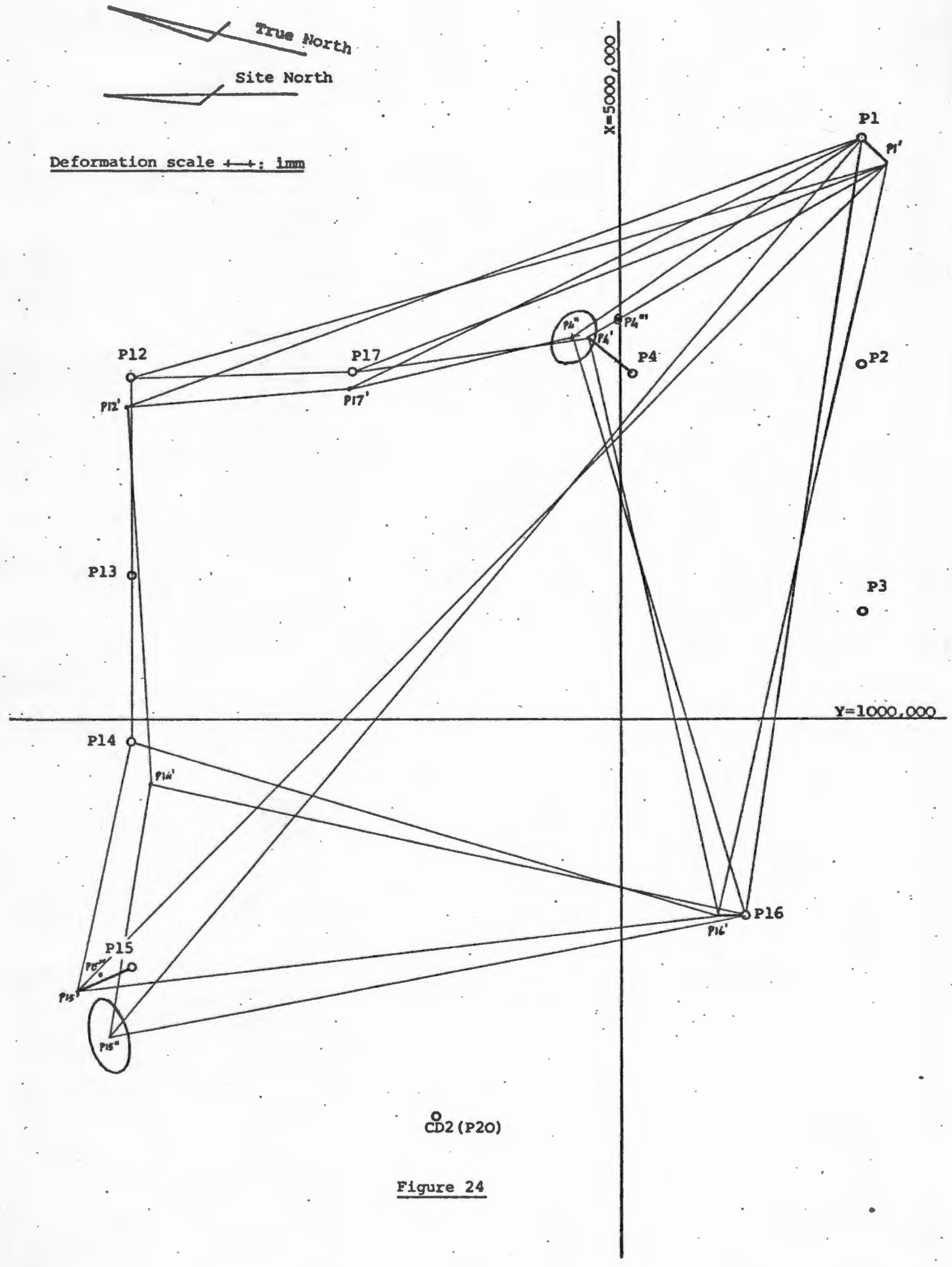

True North
Site North
Deformation scale +—+: 1mm
X=5000,000
Y=1000,000
P1
P1'
P2
P3
P4
P4'
P4"
P4'''
P12
P12'
P13
P14
P14'
P15
P15'
P15"
P16
P16'
P17
P17'
CD2 (P20)

Figure 24

5.6 Epoch F

In Figure 25 the displacements applied to points P3, P17 & CD2(P20) can be seen. In practical network design one would generally aim at a point accuracy of 1/3 the expected deformation. This value is derived from the factor of 3,03 (from table 3 annexure A) which if multiplied by the error ellipse parameters will result in a confidence level of 99% for the point position. This criterion would be applied to the error parameters of the weakest point thus ensuring that the accuracy of the other points would be in excess of the design requirements.

The three points deformed in this epoch were shifted by multiples of 3 (CD2), 3.5 (P3) & 4 (P17) times the absolute error ellipses for these points, the intention being to observe the effect of these types of shifts on the analysis. The invariant function comparisons both yielded P3 & P17 as doubtful (tables 8 & 9) with the combined adjustments producing results tabled under IMa & IFa in table 10. A second combined adjustment was performed in each case. For IM a third point CD2 was included because from the invariant function comparisons (table 8) this point had the next highest frequency and accounted for the one remaining failure at 95%. The author attempted this in an effort to see if the combined adjustment would show this point as deformed (which it was) contrary to the findings from the comparisons at 99%. This point was shown to be deformed at the 95% level only (table 10 IMb). In the case of IF two other points had higher frequencies than CD2 (table 9) i.e. P4 & P16 & these three points were thus included in the combined adjustment, with only CD2 (P20) being deformed along with the previous findings of P3 & P17 (table 10 IFb). Following the results from IMb, CD2 was included in method Aa, where it was judged to be displaced. Two other methods CF & SF yielded P20 as deformed at the 95% confidence level.

Sketch indicating position of deformations for Epoch F

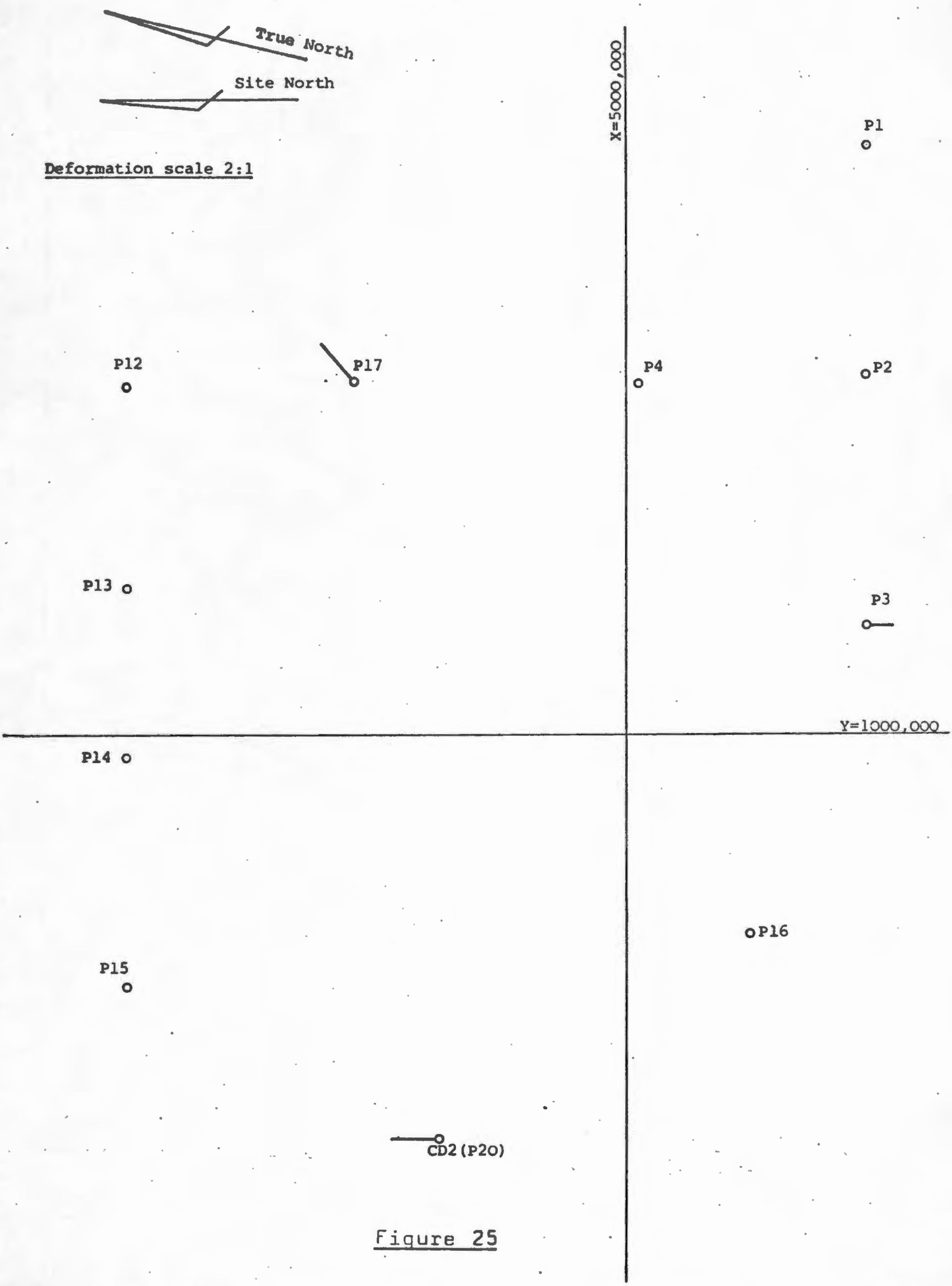


Figure 25

LIST OF ANGLES AND DISTANCES FAILING AT 90%, 95% AND 99% CONFIDENCE LEVELS

FROM	TO	TO	SIGMA F	DIFF	90%	95%	99%
P1	P2	P3	1.21	-2.42	*	*	
P1	P3	P4	1.28	2.73	*	*	
P1	P16	P17	0.98	3.70	*	*	*✓
P1	P17	CD2(P20)	0.90	-3.18	*	*	*✓
P2	P3	P4	4.57	8.25	*		
P2	P16	P17	1.87	3.86	*	*	
P2	P17	CD2(P20)	1.53	-3.31	*	*	
P3	P15	P16	1.33	2.60	*		
P13	P16	P17	3.06	-6.22	*	*	
P13	P17	CD2(P20)	3.92	7.79	*	*	
P14	P16	P17	2.20	-4.44	*	*	
P14	P17	CD2(P20)	3.17	6.75	*	*	
P15	P17	CD2(P20)	3.52	5.91	*		
P16	P2	P3	1.31	3.72	*	*	*✓
P17	P2	P3	1.58	-2.76	*		
P17	P14	P15	1.15	2.35	*	*	
P2	P15		0.88	1.55	*		
P3	P12		1.35	-4.27	*	*	*✓
P3	P13		1.25	-3.15	*	*	
P3	P14		1.29	-2.31	*		
P3	P17		2.06	-6.81	*	*	*✓
P3	CD2(P20)		2.26	-4.25	*		
P4	P12		1.34	-2.26	*		
P4	P17		2.08	-3.69	*		
P4	CD2(P20)		1.60	-3.36	*	*	
P16	P17		1.48	-4.77	*	*	*✓
P17	CD2(P20)		1.50	-5.73	*	*	*✓

P1	P2	P3	P4	P12	P13	P14	P15	P16	P17	CD2(P20)
4	7	11	5	2	3	4	4	7	15	8

table 8

LIST OF ANGLES AND DISTANCES FAILING AT 90%, 95% AND 99% CONFIDENCE LEVELS

FROM	TO	TO	SIGMA F	DIFF	90%	95%	99%
P1	P3	P4	1.51	2.73	*		
P1	P16	P17	1.04	3.70	*	*	*✓
P1	P17	CD2(P20)	0.87	-3.18	*	*	*✓
P2	P3	P4	2.58	8.25	*	*	*✓
P2	P16	P17	1.01	3.86	*	*	*✓
P2	P17	CD2(P20)	0.83	-3.31	*	*	*✓
P3	P1	P2	1.31	-2.20	*		
P3	P2	P4	2.07	4.52	*	*	
P3	P14	P15	0.57	1.02	*		
P3	P15	P16	1.53	2.60	*		
P4	P1	P2	1.83	3.32	*		
P4	P16	P17	1.80	3.12	*		
P4	P17	CD2(P20)	1.67	-2.96	*		
P12	P16	P17	2.11	-5.43	*	*	
P12	P17	CD2(P20)	2.25	6.23	*	*	*✓
P13	P16	P17	1.95	-6.22	*	*	*✓
P13	P17	CD2(P20)	2.13	7.79	*	*	*✓
P14	P16	P17	1.44	-4.44	*	*	*✓
P14	P17	CD2(P20)	1.72	6.75	*	*	*✓
P15	P16	P17	1.11	-2.82	*	*	
P15	P17	CD2(P20)	1.86	5.91	*	*	*✓
P16	P2	P3	1.33	3.72	*	*	*✓
P16	P3	P4	1.53	-2.67	*		
P17	P2	P3	1.57	-2.76	*		
P17	P3	P4	1.71	3.06	*		
P17	P4	P12	3.47	-9.49	*	*	*✓
P17	P14	P15	0.84	2.35	*	*	*✓
P17	P15	P16	0.69	2.58	*	*	*✓
P2	P15		0.88	1.55	*		
P3	P12		1.35	-4.27	*	*	*✓
P3	P13		1.25	-3.15	*	*	
P3	P14		1.29	-2.31	*		
P3	P17		2.06	-6.81	*	*	*✓
P3	CD2(P20)		2.26	-4.25	*		
P4	P12		1.34	-2.26	*		
P4	P17		2.08	-3.69	*		
P4	CD2(P20)		1.60	-3.36	*	*	
P16	P17		1.48	-4.77	*	*	*✓
P17	CD2(P20)		1.50	-5.73	*	*	*✓

P1	P2	P3	P4	P12	P13	P14	P15	P16	P17	CD2(P20)
5	9	15	12	5	3	5	7	12	23	10

table 9

EPOCH F

Method	Shift P3 mm dy	dx	Shift P17 mm dy	dx	Shift P20 mm dy	dx	Shift P4 mm dy	dx	Shift P12 mm dy	dx
Applied Shift	0	+2,5	-3,4	-2,8	0	-4,3	0	0		
Invariant Functions (Min Cons) IM a	+1,6	+3,0	-3,1	-4,0						
b	+1,8	+3,0	-3,1	-3,7	(+2,4	-2,2)				
Invariant Functions (Free Net) IF a	-0,1	+3,3	-3,7	-3,7						
b	+0,1	+3,3	-3,7	-3,4	+2,2	-2,2				
All fixed except doubtful points A a	+1,8	+3,0	-3,1	-2,7	+2,3	-2,2				
b	-0,3	+3,3	-4,2	-3,3			* -1,6	+1,3		
Co-ord Comparison (Min Cons) CM	-0,4	+3,2	-4,9	-2,2			(-1,7	+1,5)		
Co-ord Comparison (Free Net) CF	+0,2	+3,1	-3,8	-2,5	(+1,8	-1,5)				
Co-ord Comparison Min Cons + Transform. CT	+0,2	+3,1	-3,8	-2,5						
Direct Differences Min Cons DM	-0,3	+3,2	-4,9	-2,1						
Direct Differences Free Net DF	+0,2	+3,1	-3,8	-2,5						
Niemeier (Min Cons) NM	+1,5	+2,9	-3,1	-4,1						
Niemeier (Free Net) NF	+1,3	+2,9	-3,4	-3,3						
Co-ord Comparison (Min Cons) Partial Transformation SM	-0,1	+3,0	-4,4	-2,5						
Co-ord Comparison (Free) Partial Transformation SF	+0,1	+3,1	-4,2	-2,4	(+1,5	-1,8)	(-1,1	+1,3)	(-1,1	-1,0)

* Indicates shifts equal to the 99% or 95% confidence level test value
Values in brackets indicate shifts between 95% & 99% confidence levels.

Table 10

Only one definite type I error was present; P4 from Ab, with two methods showing type I errors at a 95% confidence level. These two were CM with P4 & SF with P4 & P12.

The distribution of the displacements for P3 was reasonably good, except for five methods which showed a discrepancy in the Y displacement (see table 10). P17 also showed a reasonable distribution around the true position. P20 however gave a very poor indication with all the derived deformations being more than 3mm from the true position.

5.7 Epoch G

This epoch, with no applied deformations, was used to test the methods for such an eventuality in a second epoch. It would be interesting to see whether the various methods would immediately reveal a lack of any displacements.

LIST OF ANGLES AND DISTANCES FAILING AT 90%, 95% AND 99% CONFIDENCE LEVELS

FROM	TO	TO	SIGMA F	DIFF	90%	95%	99%

P1	P2	P3	P4	P12	P13	P14	P15	P16	P17
0	0	0	0	0	0	0	0	0	0

CD2(P20)
0

table 11

As can be seen from table 11, the invariant function comparisons (identical for minimum constraint & free network adjustments) immediately indicated no deformations in the network. The methods relying on co-ordinate comparison & co-ordinate differences also showed no deformation. In Niemeier's method where the network is tested for deformation with the Fisher test using eqn. (67) section 3.4.2:

$$P\left\{\bar{F} \quad F_{1-\alpha,h,f} \,/\, Ho\right\} = \alpha \qquad \alpha = 0{,}05$$

The tests resulted in:

$$\left.\begin{array}{ll} \text{Minimum constraints} & \bar{F} = 0{,}45 \\ \text{Free Network} & \bar{F} = 0{,}45 \end{array}\right\} F_{,95,19,106} = 1{,}70$$

which also indicates no deformation in the network.

5.8 Epoch J

This case simulates a situation in which the majority of the points in the network are subject to deformation. Only P1, P16 & P12 were held stable while the remainder were deformed (see Figure 26).

In a situation where a large deformable body, e.g. a circular dam wall, is to be measured, it may be useful to establish the majority of the control network on such a body, maintaining a minimum number of points removed from such a body to anchor the system. It would be necessary that the anchor points be well established and free from movement, as if these points were also subject to movement, the whole system would be unreliable since all points would then be subject to deformation. In this example, points P1 & P16 are considered to be deformation free for some of the methods while P12, although in fact deformation free, was in general regarded as a normal point and thus may or may not be subject to deformation. Thus if a method indicated this point to be unstable it was assumed to be so. The shifts applied are in general roughly radially distributed, as, should they all be of the same order of magnitude and in the same direction, this would tend to indicate the three stable points as being deformed rather than the actual deformed points. In practice, this type of deformation would probably require a reference network which could first be independently checked for deformation, as, should a block translation occur, there would be no internal or observational evidence as to which block of points was subject to absolute movement.

In the method of invariant function comparisons (minimum constraints) the frequency of point failures was high, ranging from P14 (28) to 13 for P17 & P20. P16 (18) was the fourth highest after P14, P15 & P13 and as such would be included as a doubtful point if it accounted for 99% failures which were not a result of the previous

Sketch indicating position of deformations for Epoch J

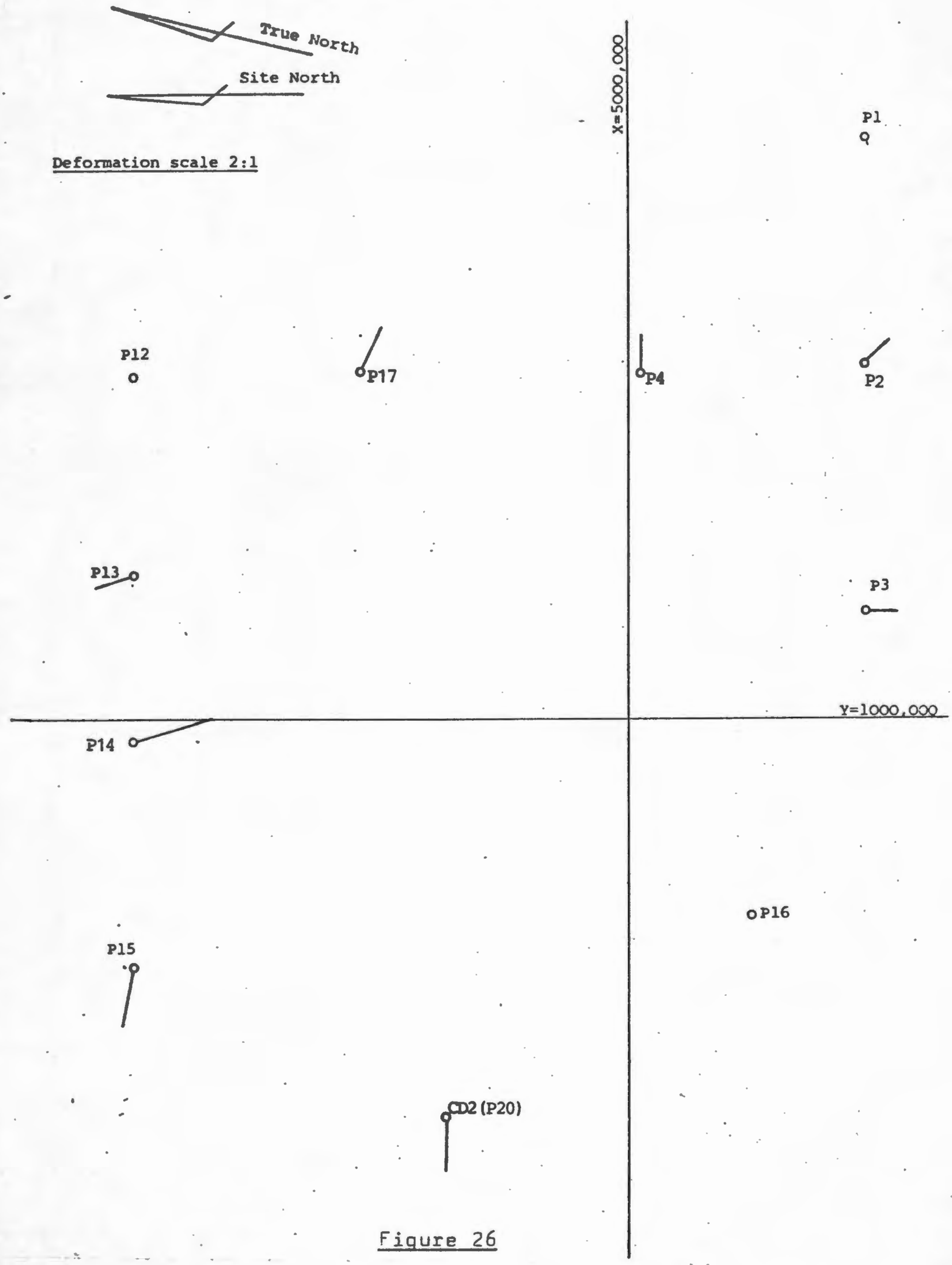


Figure 26

EPOCH J

Method	Shift P2 mm dy	dx	Shift P3 mm dy	dx	Shift P4 mm dy	dx	Shift P13 mm dy	dx	Shift P14 mm dy	dx
Applied Shift	-2,2	+2,2	0	+3,0	-3,5	0	+1,0	-3,5	-2,0	+7,0
Invariant Functions (Min Cons) IM	-1,5	+2,3	* +3,0	+3,3	-3,8	-0,2	+0,8	-4,7	-2,2	+7,0
Invariant Functions (Free Net) IF	-3,9	+2,4	(+0,2	+2,6)	-6,6	+0,2	-3,4	-4,7	-6,9	+6,7
All fixed except doubtful points A a	-1,5	+2,3	+3,0	+3,3	-3,8	-0,2	+0,8	-4,7	-2,2	+6,9
A b	-1,5	+2,9	+3,0	+3,7	-3,5	+0,2	+0,7	-3,4	-2,1	+7,9
Co-ord Comparison (Min Cons) CM	-3,1	+2,4	+1,2	+3,1	-5,3	0	-1,2	-4,6	-4,4	+6,9
Co-ord Comparison (Free Net) CF	-1,3	+2,0	+3,0	+2,6	-3,7	* -0,3	+0,4	-5,1	-2,9	+6,4
Co-ord Comparison Min Cons + Transform. CT	-1,3	+2,0	(+3,0	+2,6)	-3,7	-0,3	+0,4	-5,1	-2,9	+6,5
Direct Differences Min Cons DM	-3,1	+2,4	+1,3	+3,1	-5,4	+0,1	-1,2	-4,6	-4,4	+6,9
Direct Differences Free Net DF	-1,3	+2,0	+3,0	+2,6	-3,7	-0,3	+0,4	-5,1	-2,9	+6,4
Niemeier (Min Cons) NM a			+4,2	+3,3			+5,0	-4,1	+0,8	+7,9
NM b	-2,2	+2,8	+2,2	+3,5	-4,0	+0,3	+0,4	-3,3	-2,5	+8,2
Niemeier (Free Net) NF a							+8,8	-6,6	+3,6	+6,5
NF b	-1,5	+2,6	* +2,8	+3,4	-3,6	+0,5	+3,5	-3,4	-0,9	+8,3
Co-ord Comparison (Min Cons) Partial Transformation SM	-1,4	2,8	+3,0	+3,6	* -3,5	0,5	+1,0	-4,1	-2,2	+7,6
Co-ord Comparison (Free) Partial Transformation SF	-1,5	+2,3	+2,8	+3,1	-3,7	0	+0,7	-4,6	-2,6	+7,0

* Indicates shifts equal to the 99% or 95% confidence level test value

Values in brackets indicate shifts between 95% & 99% confidence levels.

Table 12 Continued overleaf

EPOCH J

Method	Shift P15 mm dy	dx	Shift P17 mm dy	dx	Shift P20 mm dy	dx	Shift P12 mm dy	dx	Shift P16 mm dy	dx
Applied Shift	+5,5	-1,0	-4,0	+2,0	+5,0	0	0	0	0	0
Invariant Functions (Min Cons) IM	+4,8	• -1,1	(-3,6	-0,7)	+5,5	+0,3				
Invariant Functions (Free Net) IF	(+0,2	-1,8	-7,2	-0,3			-4,8	-1,3	-4,5	-0,8
All fixed except doubtful points A a	+4,8	-1,1	(-3,6	-0,7)	+5,5	+0,3				
b	+5,0	-0,5	-3,1	+0,1	+5,5	+0,8				
Co-ord Comparison (Min Cons) CM			-5,4	-0,5						
Co-ord Comparison (Free Net) CF	+4,2	-1,7	-3,8	-0,8	+4,8	-0,5	-1,0	-1,8		
Co-ord Comparison Min Cons + Transform. CT	(+4,2	-1,7)	(-3,8	-0,8)	(+4,8	-0,5)				
Direct Differences Min Cons DM			-5,4	-0,4						
Direct Differences Free Net DF	+4,2	-1,7	-3,8	-0,8	+4,8	-0,5	(-1,0	-1,8)		
Niemeier (Min Cons) NM a	+7,7	+0,1			+7,0	+1,8				
b	+4,6	-0,2	-3,4	-0,1	(+4,5	+1,2)				
Niemeier (Free Net) NF a	+10,2	-0,2			+8,4	+2,0	+4,9	-5,2		
b	+5,7	0,0	-3,6	+3,1	+5,6	+1,7				
Co-ord Comparison (Min Cons) Partial Transformation SM	(+4,9	-0,4)			+5,1	+1,0				
Co-ord Comparison (Free) Partial Transformation SF	+4,5	-1,0	-3,6	-0,5	+4,9	+0,3	• (-0,7	-1,5)		

• Indicates shifts equal to the 99% or 95% confidence level test value
Values in brackets indicate shifts between 95% & 99% confidence levels.

Table 12 continued

three points. Due to the fact that points P2, P3, P4, P17 & P20 are known to be on the deformable body and also that P16, which is a constraint point, would normally be well established and thus not subject to deformation, it was decided to ignore the high frequency of this point, as indication of deformation, and assume it to be stable. This reasoning was applied to P1 also. With this assumption in mind the 99% failure column was investigated and it was found that all the remaining points should be considered doubtful. The results of the combined adjustment yielded results set out in table 12 under IM.

In the comparison of invariant functions (free network) the frequencies were higher than in the minimum constraint case. Again P16 had a high frequency and the adjustment being a free network type, it was decided to follow the usual procedure and assume P16 as a possible unstable point. In this case, P1 & P20 were kept stable for the combined adjustment (table 12 IF).

For the method where all stable points are held fixed, the two variations were continued as before. In Aa, points P1 & P16 were held fixed in accordance with the findings from the invariant function comparisons (IM), while for Ab, three points P1, P12 & P16 were constrained in accordance with the results from the stable point transformation SF at a 99% confidence level.

In the Niemeier method of analysis (for both free network & minimum constraints adjustments), the network was firstly assumed to be a reference network (i.e. no expected deformations) and as could be expected the results were meaningless with completely unrealistic deformations being yielded (table 12 NMa & NFa). Following the method as advocated by Niemeier 1976 and partitioning the network into reference (P1. P16 & P12) and object points (remainder), the results yield fairly good comparisons with actual applied shifts (table 12, NMb, NFb). As an exercise an extra deformed point was added to the reference points P1, P16 & P12 to see if the method would separate the correct point from the reference points. In both cases (minimum constraints and free network), this was successful and the results were identical to those as found in table 16 NMb & NFb.

For the stable point transformations, SM & SF table 12, the choice of stable points was P1, P16 & P12 for SM & P1 & P16 for SF. Errors of type II were due to points P15, P17 and P20. Type I errors for P12 were expected from the methods of invariant functions where this point was judged to be doubtful, IF. The method using Niemeier (NFa) also gave this type of error but as the results for this particular case are meaningless, this error is also meaningless. The one case where P16 is involved in this type of error is the invariant function method (IF).

In general, the various methods yielded a good analysis for the situation of majority point movement and it appears that for two of the methods (invariant functions & Niemeier) the assumption that the reference points are stable must be made, otherwise incorrect results may be adopted. In the case of Niemeier, the validity of this assumption is tested and this process should be followed in the invariant function case also to prevent type I and type II errors occuring in these points.

In general, themethods IMa, Ab, SF & SM tend to give the best distribution around the actual applied deformation. Only for points P3 & P17 are the distribution for these methods poor. One would expect the distribution of the various methods for a point with a large deformation (e.g. P12 from epoch 3) to be good, but in the case of P14 this is rather poor, except for the four methods mentioned above. This poor distribution may be characteristic of the majority point movement case.

5.9 Epoch HR

Until this stage, all applied deformations were known to the author. Although all results are viewed as objectively as possible, there remains the possibility of subjective bias in interpretation of the results. For this, epoch HR was created, in which the displacements were applied by an outside party, and were thus unknown to the author. The intention was to use this epoch as a test on the conclusions drawn from the analysis of the results from the previous epochs.

Although at this point no final conclusions as to the, as yet, unknown deformations have been made in this test network, the existing information indicates three points which have been deformed (table 13). Two of the points, P4 & P20, are shown by all the methods to have been displaced, whilst the third point, P15, is indicated as being stable by four of the methods at the 95% confidence level. Three methods, i.e. CF, DF & SF, have P12 as also being unstable with methods CF & DF also showing P13 as unstable. In the method of stable point transformation, SM, two sets of results appear. In case "a" P15 was considered as a stable point (shift less than 2 x the standard deviation) while, from the indication of this transformation showing the deformation at P15 to be greater than 2 x the standard deviation, a second transformation, which included P15 as a doubtful point, was calculated and the results from this second transformation, although again showing P15 as stable, yield results which compare more favourably with the other methods, than did the first set SMa.

EPOCH HR

Method	Shift P4 mm dy	dx	Shift P15 mm dy	dx	Shift P20 mm dy	dx	Shift P12 mm dy	dx	Shift P13 mm dy	dx
Applied Shift										
Invariant Functions (Min Cons) IM	-4,7	+3,2	(+4,0	0)	+4,7	+5,9				
Invariant Functions (Free Net) IF	-4,7	+3,3	+4,0	+0,1	+4,8	+5,9				
All fixed except doubtful points A a	-4,7	+3,2	+4,0	0	+4,5	+5,9				
b	-4,7	+3,1	+4,1	+0,1	+4,5	+6,3	(+0,1	-2,3)		
Co-ord Comparison (Min Cons) CM	-5,9	+3,0			+3,0	+5,7				
Co-ord Comparison (Free Net) CF	-4,7	+3,4	+2,7	-1,4	+3,8	+4,3	-1,9	-2,3	P13 +0,1 (P2 1,5	-2,3 -0,2)
Co-ord Comparison Min Cons + Transform. CT	-4,7	+3,4			+3,8	+4,3				
Direct Differences Min Cons DM	-5,9	+3,1			+3,0	+5,7				
Direct Differences Free Net DF	-4,7	+3,4	+2,7	-1,4	+3,8	+4,3	-1,9	-2,3	+0,1	-2,3
Niemeier (Min Cons) NM	-4,7	+3,3	+3,9	+0,1	+4,3	+6,0				
Niemeier (Free Net) NF	-4,9	+3,8	+4,3	+0,1	+4,4	+6,2				
Co-ord Comparison (Min Cons) Partial Transformation SM a	-5,2	+3,4			+3,6	+5,5				
b	-4,6	+3,2			+4,4	+6,4				
Co-ord Comparison (Free) Partial Transformation SF	-4,6	+3,3	+4,0	0	+4,3	+6,1	-0,6	-2,4		

* Indicates shifts equal to the 99% or 95% confidence level test value

Values in brackets indicate shifts between 95% & 99% confidence levels.

Table 13

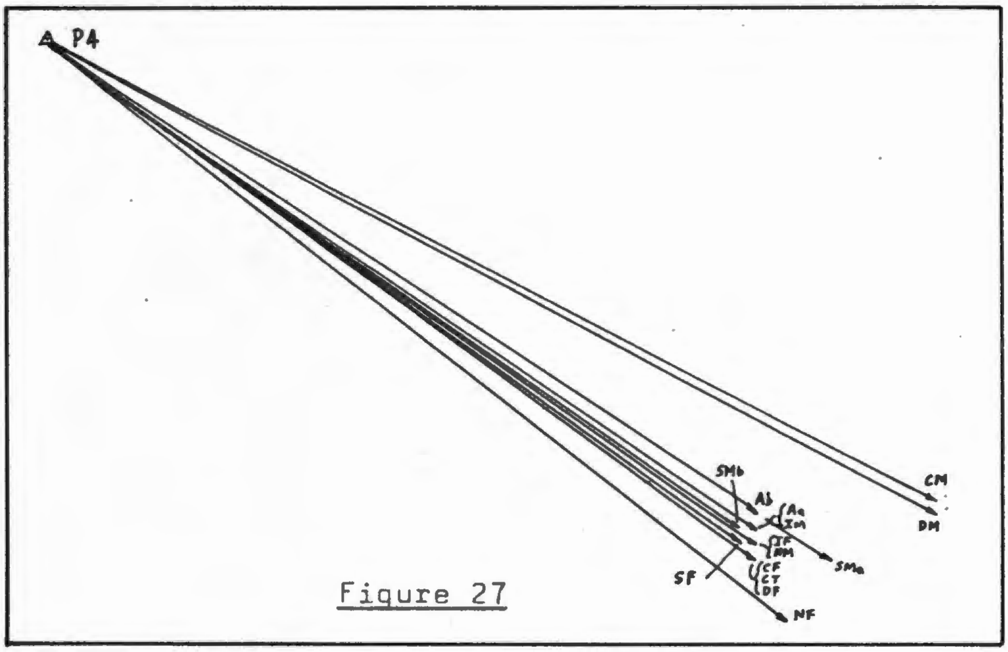


Figure 27

In Figure 27 the displacements for P4 are plotted. As is clear the distribution is very good except for methods CM & DM. This would indicate a deformation of the order of approximately dy = - 4,7 and dx = + 3,2 although the final result chosen will depend on the final conclusion.

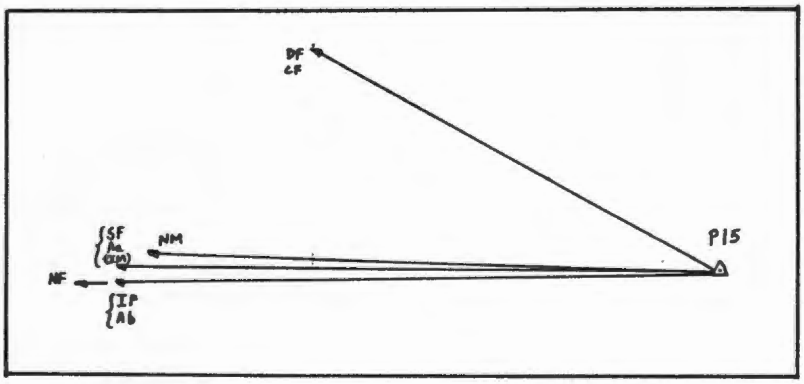


Figure 28

In the case of P15 (Figure 28), again there is a reasonably clear indication of the position of the deformation, i.e. approximately dy = + 4,0 and dx = 0 with methods DF & CF being away from the main group.

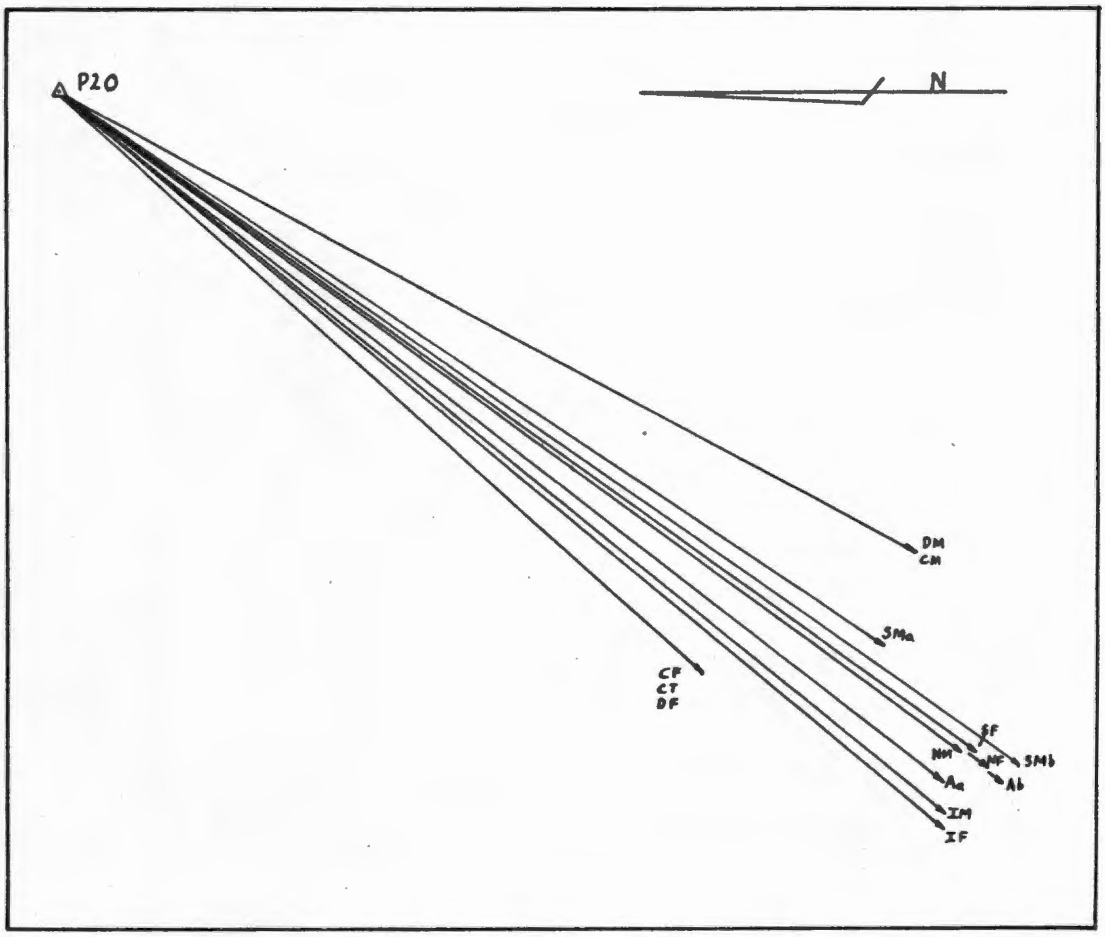


Figure 29

The distribution for P20 is not as clear as in the two previous points, although a reasonable indication of the deformation may be gauged from the Figure 29 which would be of the order of dy = + 4,5 and dx = + 6,0.

5.10 Peculiarities in the methods

In the course of the investigation, three features of the results were noticed which, while they had little bearing on the central purpose, were felt to be interesting enough to bear comment and investigation.

Firstly, it was found while investigating the method of invariant functions (minimum constraints) in combined adjustments, that some of the absolute error ellipse parameters of two points involved in double-point fixes, differed from each other. This phenomenon was unexpected as the observation geometry as well as observational accuracy for both epochs was identical.

Secondly, when carrying out co-ordinate comparisons of all-point transformations following a free network adjustment, it was found that no translation or swing was involved, indicating that the two epochs were already on the same system.

Thirdly, in the eight epochs where deformations have been applied, it was noted that in general three of the methods, direct co-ordinate comparison and direct difference using a free network adjustment and the all-point transformation of co-ordinates from the minimum constraints method, all yielded virtually identical deformations with in only three cases one of the components (dy or dx) differing by as much as 0,1mm.

5.10.1 Absolute Error Ellipse parameter anomalies

In the combined adjustment, for the invariant function method, doubtful points are entered as double points (one representing each epoch) and because there are two points, each of the points has its own error ellipse. The two points are labeled so as to distinguish the two epochs.

The point from the second epoch is labeled with a suffix - A on the original name, while the first epoch retains the original name, e.g. P2 for epoch 1, P2 - A for epoch 2 as in table 15. Due to the fact that the observation configuration onto each point is identical for each epoch, the observation accuracies are identical and the provisional co-ordinates for the two points are either identical or, if they differ, the differences being of the order of a few millimeters, one would expect the error ellipse parameters to be identical for the two points.

Contrary to expectations, differences were found, particularly in the ellipse orientation. This parameter is determined wholly by Q, $(A^TPA)^{-1}$ which in turn is a transformation of A^TPA. The cause of the discrepancy is therefore to be found in the formation of these matrices. Logically, one would expect the maximum difference in ϕ (ellipse orientation) to be in the region of 0^o and 90^o from Tan 2ϕ, (Annexure A, A.8.3) there being the greatest difference in ϕ for a small difference in x, when $Tan^{-1}x$ is of the magnitude of 0^o or 180^o. However, from table 14 there does not appear to be any correlation between ϕ and discrepancy.

In the investigation to determine the origin of these discrepancies, it was decided that four possible sources existed:

1. The Matrix (A^TPA) configuration was poor.
2. The use of different provisional co-ordinates for the doubtful points affected by the Q Matrix.
3. The double set of observations for the stable points was affecting the Q matrix.
4. The inversion of the larger and different A^TPA matrix caused the discrepancies.

Table of differences in absolute Ellipse parameters

	Epoch 0			Epoch 1					
Point	ø°	a	b	ø°	a	b	Δø°	Δa	Epoch
P12	106,4	1,3	0,8	106,2	1,3	0,8	0,2	0	B_1
P20	117,5	1,3	1,1	117,5	1,3	1,1	0	0	
P2	86,0	0,7	0,6	84,5	0,7	0,6	1,5	0	B_2
P3	90,8	1,7	0,7	90,7	1,7	0,7	0,1	0	
P12	106,7	1,2	0,8	106,5	1,2	0,8	0,2	0	
P1	49,0	1,5	1,0	49,0	1,5	1,0	0	0	C_1
P15	52,0	1,3	0,8	52,0	1,3	0,8	0	0	
P16	121,9	1,3	0,8	121,9	1,3	0,8	0	0	
P1	49,9	1,9	1,1	42,8	1,7	1,1	7,1	0,2	C_2
P2	69,2	1,6	0,7	68,0	1,4	0,7	1,2	0,2	
P4	41,8	1,3	0,9	36,9	1,1	0,9	4,9	0,2	
P12	109,0	1,5	0,8	108,0	1,5	0,8	1,0	0	
P14	81,1	1,6	0,8	81,1	1,6	0,8	0	0	
P15	53,9	1,7	0,9	55,6	1,6	0,9	1,7	0,1	
P16	121,1	1,7	0,9	122,1	1,5	0,9	1,0	0,2	
P17	158,1	1,8	0,9	156,6	1,8	0,9	1,5	0	
P20	173,0	1,8	1,3	177,0	1,8	1,3	4,0	0	
P12	98,5	1,1	0,6	98,2	1,1	0,6	0,3	0	D
P14	82,3	1,2	0,6	82,3	1,2	0,6	0	0	
P17	161,8	1,5	0,8	161,7	1,5	0,8	0,1	0	
P4	32,6	1,2	0,8	33,0	1,2	0,8	0,4	0	E_1
P1	49,1	1,7	1,1	49,1	1,7	1,1	0	0	E_2
P4	34,4	1,1	0,8	34,4	1,1	0,8	0	0	
P12	111,4	1,3	0,9	111,4	1,3	0,9	0	0	
P15	52,9	1,4	0,8	52,9	1,4	0,8	0	0	
P17	158,4	1,4	0,8	158,5	1,4	0,8	0,1	0	F_2

table 14

Investigation of Discrepancies in Ellipse Parameters

Point	Ellipse Param.	A	A (Shuf fled)	A (Prov. co-ords of C)	A + B	A + B -A	A + B (-A co-ords same)	A + B (-A co-ords same) -A	A + B Reduced	A + B Reduced -A
P1	amm bmm ø°									
P2	amm bmm ø°	0,8 0,6 93°2	0,8 0,6 93°2	0,8 0,6 93°2	0,7 0,6 86°0	0,7 0,6 84°5	0,7 0,6 86°0	0,7 0,6 84°5	0,6 0,5 81°8	0,7 0,6 79°4
P3	amm bmm ø°	1,8 0,7 92°0	1,8 0,7 92°0	1,8 0,7 92°0	1,7 0,7 90°8	1,7 0,7 90°7	1,7 0,7 90°8	1,7 0,7 90°7	1,5 0,6 91°0	1,7 0,7 90°4
P4	amm bmm ø°	1,1 0,8 122°8	1,1 0,8 122°8	1,1 0,8 122°8	0,8 0,6 121°8		0,8 0,6 121°8		0,7 0,6 120°0	
P12	"	1,6 0,9 105°9	1,6 0,9 105°9	1,6 0,9 105°9	1,2 0,8 106°7	1,2 0,8 106°5	1,2 0,8 106°7	1,2 0,8 106°5	1,2 0,8 103°6	1,2 0,8 104°9
P13	"	2,0 0,9 95°8	2,0 0,9 95°8	2,0 0,9 95°8	1,4 0,6 95°5		1,4 0,6 95°5		1,5 0,7 93°1	
P14	"	1,7 0,9 87,8	1,7 0,9 87,8	1,7 0,9 87,8	1,2 0,6 87,5		1,2 0,6 87,5		1,3 0,7 82,7	
P15	"	1,6 0,7 75°4	1,6 0,7 75°4	1,6 0,7 75°4	1,1 0,5 75°2		1,1 0,5 75°2		1,1 0,6 66°7	
P16	"	1,2 0,0 90°0	1,2 0,0 90°0	1,2 0,0 90°0	0,8 0 90°0		0,8 0 90°0		0,8 0 90°	
P17	"	1,8 1,1 151°1	1,8 1,1 151°1	1,8 1,1 151°1	1,3 0,8 151°0		1,3 0,8 151°0		1,4 0,9 163°9	
P20	"	1,5 1,3 92°2	1,5 1,3 92°2	1,5 1,3 92°2	1,0 0,9 90°8		1,0 0,9 90°8		1,2 1,1 143°9	

Table 15

5.10.1.1 Matrix Configuration

The concern here was that the original order of the unknowns in the A^TPA matrix resulted in a weak configuration, which, when the two epochs were combined, would cause discrepancies in the Q matrix and thus the ellipse parameters. If this was the case, then changing the order of the unknowns in the original matrix from Epoch A say, would result in different values for the ellipse parameters. Comparison between the original adjustment for A (table 15 column A) and the adjustment for A (table 15 column A (shuffled)) where the unknowns' order was shuffled showed identical parameters for the ellipses indicating that the matrix configuration was not at fault.

5.10.1.2 Different Provisional Co-ordinates

Initially, when combining the two epochs, two different values for the provisional co-ordinates of the doubtful points were used in the adjustment. There were the original provisional co-ordinates used for Epoch A and the provisional co-ordinates containing the applied error (from the co-ordinates used to generate the observations). As an exercise, the co-efficients a & b were determined using the above co-ordinates. When these are combined, with the factor ρ'' for direction and the large weight for the distances, the elements paa, pab & pbb show significant differences for the two sets of co-ordinate. Due to this finding, it was felt that this could be a reason for the discrepancies in the ellipse parameters. To investigate this point a second adjustment was performed where the provisional co-ordinates for the points of the network were those from epoch C and the observations from epoch A. As can be seen from the results (table 15 column A (Prov. co-ords. of C)), the ellipse parameters differ only in rightmost displayed values for ϕ, when the display format was widened. The effect of changing the provisional co-ordinates was therefore relevant but practically insignificant.

5.10.1.3 Additional Observations

It was thought that the extra observations, added by the second epoch, to the stable points could possibly be disturbing the $(A^TPA)^{-1}$ matrix in the combined adjustment, which could account for the anomalies in the ellipse parameters. A combined adjustment which eliminated all the second epoch observations between stable points was performed and although the ellipse parameters differed from the original combined adjustment as expected, the internal discrepancies of the ellipse parameters between the two points of the doubtful points still persisted; see table 15, column A + B reduced showing the cause to lie elsewhere.

5.10.1.4 Inversion of the Larger A^TPA Matrix

If the inversion of the larger A^TPA matrix were to cause discordance then this discordance should persist if slightly different provisional values for the co-ordinates are used. Epoch A & B were combined with points P2, P3 & P12 as doubtful. The provisional co-ordinates for the two points of each of these was identical (i.e. Y_o & X_o for P2 = Y_o & X_o for P2-A etc) as opposed to the original combined adjustment where the values differed slightly. (Results for original adjustment in table 15 column A + B). Also the co-efficients (paa), (pab) & (pbb) for the points P2 & P2-A would be identical and it would follow logically that the inverted matrix $(A^TPA)^{-1}$ should result in similar values for the two points in qxx, qxy and qyy which should then produce similar error ellipse parameters for the two points. The results from this combined adjustment, table 15 column A + B (co-ords. same) shows that the ellipse parameters are identical to those from the original combined adjustment. It would thus appear that the fault lies in an inherent instability of the larger matrix which may be exposed during inversion.

It was felt at first that the instability was a function of the size of the matrix, i.e. that the more double points present the greater would be the discrepancies, but with the addition of the combined adjustment from epoch J (only 2 points stable) it was found that this is not necessarily the case as in epoch J there were no discrepancies. The ellipse parameter differences between the combined adjustment and the single epoch adjustment are expected as the two Q matrices are different due to the different observation and network configuration (double points and double observations).

The ellipse parameter discrepancies appear to be a result of instability in the inversion of the larger A^TPA matrix, although the other factors mentioned could possibly contribute to some degree to this phenomenon.

5.10.2 Transformation of Free Network Co-ordinates

As mentioned in the description of the method of direct co-ordinate comparison, it was felt that better results may be obtained if the second epoch co-ordinates were transformed onto those of the first before any comparison was performed. In the case of a transformation, using all common points (stable and deformed), for the free network the two systems have already been transformed by the adjustment (Annexure A, A.7 Eqn A-33) and as such, the optimum fit is achieved without the necessity of a further transformation. This however obviously does not apply when transforming using only the stable points as common.

As a check on the programs and also as a practical test on the theory, results from a free network adjustment were compared to those using a Helmert transformation. Using the final co-ordinates of epoch A as provisionals for the free network adjustment, the adjustment of the second epoch resulted in the same values as those obtained by transforming the minimum constraints values of this second epoch onto the epoch A values (scale factor removed). This thus confirmed the anticipated theory as well as checking the programs.

5.10.3 Similarity of Results from three of the methods

Three methods, Direct comparison of co-ordinates, CF, (free network), Direct differences, DF, (free network) and Co-ordinate comparison (minimum constraints) with all point transformation, CT, yielded in general nearly identical deformations. If one considers the mechanics of the methods, it becomes clear that this result must be expected. In the case of method CT, as mentioned in section 5.10.2, this transformation results in the same values as the free network adjustment if the same provisional values are used. In the case of method DF, the co-ordinate differences must of necessity be the same as those from CF as the adjustment used for DF combines the two separate adjustments, used in the direct co-ordinate comparison method CF, into a simultaneous adjustment of the differences between observations. As these observation differences result in co-ordinate differences which, when the observation equations ((47) & (48)) are examined, are a function of the corrections to the unknowns for epoch 1 and epoch 2, or $\Delta x = dx_{i2} - dx_{i1}$, ($dx_{i1}$ & dx_{i2} = corrections to the unknowns epoch 1 & epoch 2 respectively), it is clear that the results from these two methods must be similar.

The explanation for the similarity between the results of methods DF & CF obviously also applies to these methods when a minimum constraint adjustment is used (i.e. DM & CM), where again similarities occurred.

CHAPTER 6

ANALYSIS OF RESULTS

6.1 Comparison of Methods

Examining the results as tabled in section 5, it is evident that no particular method stands out as yielding, consistently, the best information in regard to deformation magnitudes. Further, all the methods are subject to either type I or type II errors, (errors where stable points are found to be deformed or deformed points found to be stable respectively) and it thus becomes difficult to choose any one particular method as being the best. In an attempt to clarify the advantages and disadvantages of the various methods, tables were drawn up indicating various criteria, and the degree to which each method satisfied them.

These criteria are as follows:

1. Mean of the squares of the differences between applied and derived deformations for each method.

2. Frequency of a method indicating a shift closest to the applied shift.

3. Frequency of a method resulting in shifts of between 0-1mm, 1-2mm and >2mm from the applied shift.

4. Frequency of type I errors for each method.

5. Frequency of type II errors for each method.

6. Frequency of a method indicating a shift furthest from the applied shift.

7. The number of unstable points found (i.e. total unstable points - number of type II errors).

8. Approximate execution time of each method.

9. Number of different programmes required to perform a method.

6.1.1 Mean of the squares of the differences

As a means of determining the reliability of a method in giving good results, it was decided to calculate the mean amplitudes of the error vectors (applied to derived positions):

i.e.

$$e = \sqrt{(dx_a - dx_d)^2 + (dy_a - dy_d)^2} \qquad (86)$$

where:

dx_a, dy_a = x & y components of the applied deformation.

dx_d, dy_d = x & y components of the derived deformation.

the mean of the squares of the differences, m_e, would then be:

$$m_e = \sqrt{\frac{(ee)}{n}} \qquad (87)$$

where n = the number of deformed points found by a method.

The mean error vector amplitudes, m_e, are set out in table 16 below. The results have been separated into two groups, i.e. the values for 99% and 95% confidence levels. Also because of the difference in nature of epochs B to F & epoch J, the means for these two types of epoch as well as a pooled result are given as well.

Recall that in epochs B to F a small number of points were deformed (i.e. majority stable) while in epoch J, only three points were kept stable with the majority subject to some deformation.

The methods are ranked in order of derived M_e, indicating the method abbreviation with the value for M_e in each case.

M_e at 99% probability level													
Epochs B-J Pooled	CT 1,02	Ab 1,25	IM 1,30	SF 1,38	Aa 1,42	NF 1,45	SM 1,47	DF 1,51	CF 1,51	NM 1,70	CM 1,99	DM 2,05	IF 2,64
Epochs B F	CT 0,97	Ab 1,05	NF 1,11	IM 1,29	SF 1,29	DF 1,30	CF 1,31	Aa 1,48	SM 1,51	IF 1,77	NM 1,88	CM 1,91	DM 2,03
Epoch J	CT 1,11	IM 1,31	Aa 1,31	NM 1,42	SM 1,44	Ab 1,47	SF 1,48	NF 1,72	DF 1,75	CF 1,75	CM 2,06	DM 2,07	IF 3,82
M_e at 95% probability level													
Epochs B-J Pooled	Ab 1,26	IM 1,39	NF 1,43	SM 1,44	CT 1,45	SF 1,46	DF 1,49	Aa 1,51	CF 1,63	NM 1,66	CM 1,96	DM 2,05	IF 2,74
Epochs B-F	CT 0,91	Ab 1,09	NF 1,16	IM 1,21	DF 1,29	SF 1,45	Aa 1,48	SM 1,49	CF 1,55	IF 1,73	NM 1,84	CM 1,87	SM 2,03
Epoch J	SM 1,37	NM 1,40	Ab 1,47	SF 1,48	IM 1,56	Aa 1,56	NF 1,72	CF 1,75	DF 1,75	CT 1,75	CM 2,06	DM 2,07	IF 3,82

table 16

6.1.2 Position Frequencies

Although the means of the squares of the differences should indicate the method giving the best overall accuracy, it was felt that the inclusion of the following five criteria would serve as an indication as to how the value for M_e in each case was affected by varying sizes of discrepancies between applied and derived deformations.

i.e.

1. Frequency with which a method indicated a shift closest to the applied deformation.

2. Frequency with which a method indicated a shift falling between 0 & 1mm from the applied deformation.

3. Frequency as for 2., but falling between 1 & 2mm.

4. Frequency as for 2., but falling outside 2mm.

5. Frequency with which a method indicated a shift furthest from the applied deformation.

Position Frequency at 99% Probability Level

Criterion \ Method	IM	IF	AM	AF	CM	CF	CT	SM	SF	DM	DF	NM	NF
Epochs 3-F													
Nearest applied shift	0	0	3	2	0	2	1	1	3	0	1	0	3
0-1mm from shift	3	5	7	8	2	7	4	2	7	1	7	3	3
1-2mm from shift	4	3	3	1	1	2	1	1	1	2	2	3	5
> 2mm from shift	0	2	2	1	2	2	0	1	2	3	2	3	0
Furthest from applied shift	0	2	2	0	0	0	0	0	1.	5	2	1	0
Epoch J													
Nearest applied shift	2	0	1	3	1	0	0	0	2	0	0	1	1
0-1mm from shift	5	0	5	6	1	3	2	5	5	1	3	3	2
1-2mm from shift	1	1	1	0	2	3	2	0	1	2	3	2	4
> 2mm from shift	1	4	1	2	3	2	0	1	2	3	2	2	2
Furthest from shift	0	5	0	1	0	1	0	0	0	0	1	0	1
Position Frequency at 95% Probability Level													
Epochs 3-F													
Nearest applied shift	0	0	3	2	0	2	1	2	4	0	1	1	3
0-1mm from shift	5	5	7	8	3	7	5	3	8	1	7	3	4
1-2mm from shift	4	4	3	2	1	3	1	3	2	2	3	4	6
> 2mm from shift	0	2	2	1	3	3	0	2	3	3	2	3	0
Furthest from shift	0	3	2	0	1	1	0	0	2	5	3	1	0
Epoch J													
Nearest applied shift	2	1	1	3	1	0	0	0	2	0	0	1	1
0-1mm from shift	5	1	5	6	1	3	3	6	5	1	3	3	2
1-2mm from shift	1	1	1	0	2	3	3	0	1	2	3	3	4
>2mm from shift	2	5	2	2	3	2	2	1	2	3	2	2	2
Furthest from shift	0	6	0	1	0	1	0	0	0	0	1	0	1

table 17

At table 17, the frequencies as listed above have been shown again, separating the findings for 99% & 95% probability levels as well as epochs B-F and epoch J. The pooled frequencies are found by simply adding the two relevant values.

6.1.3 Errors of type I and type II

From the previous two sections a good indication of the accuracy of a particular method may be gauged, but it is obviously essential to have an indication of any shortcomings which may affect the reliability of a method. This is indicated by the tendency of a method to force false hypothesis decisions - that is, errors of types I and II. All methods resulted in such errors. Type II errors were found particularly when the deformation magnitude was close to the value specified for hypothesis rejection at the chosen confidence level. Type I errors were generally more frequent using free-net than in constrained adjustments, as shown in table 18 below.

Frequency of Type I & II Errors at 99% Probability Level													
Method	IM	IF	Aa	Ab	CM	CF	CT	SM	SF	DM	DF	NM	NF
Epochs B-F													
Type I	0	2	0	1	0	3	0	0	3	0	1	0	0
Type II	6	3	1	3	8	2	8	9	3	7	2	4	5
Epoch J													
Type I	0	2	0	0	0	1	0	0	0	0	1	0	0
Type II	1	3	1	0	2	0	4	2	0	2	0	1	0
Frequency of Type I & II Errors at 95% Probability Level													
Epochs B-F													
Type I	0	2	2	4	4	7	0	0	9	0	1	1	0
Type II	4	2	1	2	6	0	7	5	0	7	1	3	3
Epoch J													
Type I	0	2	0	0	0	1	0	0	1	0	0	0	0
Type II	0	1	0	0	2	0	0	1	0	2	0	0	0

table 18

6.1.3.1 Inclusion of Method Ab

The results for method SF show that the method has a fair degree of accuracy, a short execution time and needs two relatively simple programmes. However, as far as reliability is concerned, the method shows a large number of type I errors, particularly at the 95% probability level for epochs B-F (9).

In an attempt to reduce this large frequency, an A type adjustment was performed where all the points indicated as stable by SF were held fixed with the remainder to be adjusted. Although the type I errors were reduced to 4 (results under Ab) this was still regarded as excessive. The results at the 99% however, indicates the possibility of using this combination at that level only.

6.1.4 Execution Time and Programmes

Of somewhat lesser importance in choosing the best method to suit a situation, is the time involved in arriving at the deformations, and also the number of programmes required to derive these displacements. In the case of research, these may not be significant criteria, but in practice these two aspects may weigh heavily in favour of a less accurate or reliable method for deformation analysis. The times indicated (approximately) in table 19 apply to a Tektronix 4051 micro-computor with file manager and dot matrix printer. A related question is whether the process can be fully automated, or whether some degree of manual analysis is required.

Method	IM	IF	Aa	Ab	CM	CF	CT	SM	SF	DM	DF	NM	NF
Execution time Epochs B-F(min)	60	60	35	30	25	25	30	30	30	15	15	30	30
Execution time Epoch J (min)	80	80	35	30	25	25	30	30	30	15	15	30	30
Number of Programmes	3	3	2	3	1	1	2	2	2	1	1	2	2
Fully Automatic?	No	No	No	No	Yes	Yes	No	No	No	Yes	Yes	Yes	Yes

Table 19

6.2 Analysis of the methods

6.2.1 Epochs B-F

Examination of table 16 shows method CT as having the best value for m_e both at the 99% and 95% probability levels. However, although other factors such as time and programmes are favourable, the frequency of type II errors is large, table 18, making the method rather unreliable. The next methods, Ab, NF, IM, SF & DF have values for m_e within a reasonably small interval and show potential for use in deformation analysis. As mentioned earlier (6.1.3.1), methods Ab & SF would probably only be used at a 99% probability level due to the high rate of type I errors. In the determination of doubtful points for Ab, deformations at the 95% probability level from SF were also included. If the final choice of deformation depends on the point being shown as displaced at the 99% probability level for both methods, then the type I errors are entirely eliminated. However, with this argument the type II errors are increased from 3 to 4. The method NF, while being fully automatic, also has a low value for m_e as well as only 3 type II errors at a 95% probability level. The method may thus be employed at this level. Although no type I errors were encountered, this method relies on the free network adjustment and as the majority of this type of error occur in methods employing this adjustment technique, the danger of type I errors thus exists. Method IM with the next most favourable value may also be employed at the 95% probability level. The number of type II errors is 4 with a drawback to the method being the long execution time. Although method DF has a good all round reliability and accuracy, the method is limited to networks where the observation geometry is identical between epochs.

6.2.1.1 NF : Niemeier's Comparison of Co-ordinates (Free Network).

In favour of this method, apart from the accuracy and reliability, is the fact that only one special programme is required apart from the adjustment routine. The whole process is entirely automatic and deformations are directly available after the deformed points have been separated from the reference points.

6.2.1.2 IM : Invariant Functions (Minimum Constraints)

The advantages of this method are that errors in translation and orientation of the network do not affect the efficiency, and in the final determination of the deformation magnitudes, the two epochs are correlated in a combined adjustment.

The execution time is a disadvantage (+/- 60 minutes for a network of 11 points with 3 points deformed), and also the model is not fully automatic in that the choice of doubtful points is a manual process. Apart from the adjustment routine, two further programmes are required, one for the invariant function comparisons and a second for the combination of the observations of the two epochs and making provision for double points.

6.2.1.3 DF : Method of Direct Differences (Free Network)

This method is ideal in a situation where identical observation geometry is employed for each epoch, as observations may be made to targets painted on the sides of beacons, thus enabling more than one observing party to be employed at once. The adjustment is reasonably simple with only the single adjustment required to reach a conclusion in regard to any deformations.

The method is limited in that different observation geometries may not be used and extension or reduction of an existing network is not possible as part of the routine.

6.2.1.4 SF & Ab : Stable point transformation (free network) with minimum constraints adjustment holding stable points fixed

Alone, method SF would be unacceptable due to the large number of type I errors, while Ab is reliant on SF for the choice of stable points. Combined, the two methods result in a high accuracy as well as being reliable at a 99% probability level.

The method requires three separate programmes, free network adjustment, Helmert plain transformation (no scale factor) and a minimum constraints adjustment for its execution, but is reasonably time efficient. With the use of SF, the danger of type I errors exists.

6.2.2 Epoch J

At the 95% probability level, the "accuracies" of the first six methods are all within 0,2mm of each other (table 16). The best value for m_e comes from method SM, but at the same time has one type II error. The next, NM, seems fairly reliable and accurate (SM & NM are the minimum constraint versions of SF & NF respectively). In the case of Ab & SF, although no type I errors were found, there is a potential danger of this type of error occurring. The arguments as used for this combination in epochs B-F still hold, and if these methods were to be used only results at 99% probability should be considered. The following two methods, IM & Aa, yielded identical values for m_e. Due to the small interval of m_e between these methods, any of them could be employed depending on ease of use, time and user preference.

It must be noted that only one example of this type of deformation analysis (epoch J) was performed and due to this small sample, the indications could be suspect.

6.2.3 Epochs B-J pooled

The method Ab seems to yield a good all round reliability and accuracy if used as suggested for epochs B-F at 99% probability level. The next method, IM, offers a good solution without the apparent danger of type I errors encountered using Ab & SF. This method tends to be somewhat lengthy and method NF or SM could supply a quicker alternative. Although it appears from table 16 that in the pooled results these two methods have virtually identical values for m_e, SM seems to fall short in epochs B-F type analyses, while NF falls short in type J.

As a general method, the choice seems to be either Ab with SF or IM.

6.3 Comparison of Adjustment Techniques

Of the two adjustment techniques employed, minimum constraints and free network, it would appear from the results, depicted in tables 16 to 18, that the minimum constraints method provides the better reliability and accuracy.

Although three free network methods (SF, NF & DF) yielded reasonable accuracies, only the method advocated by Niemeier 1976 (NF) seems to combine both accuracy and reliability. Of the remaining two methods, SF (stable point transformation with co-ordinate comparison) appears to be very unreliable due to the large frequency of type I errors, while the method of direct differences, DF, although reasonably accurate, also tends to fall down on reliability to a minor degree with type I errors.

On investigation of the cause of the type I errors present in the free network method using epochs B to F (table 20), it was found that in general, although the final values from epoch A and the epoch 2 (B to F)both fell within the confines of the 3x error ellipse drawn around the actual point position, these positions tended towards opposite sides of the ellipses, thus creating a situation where the distance between these two positions was greater than the standard deviation of the displacement at 95% probability. To a lesser extent it was found that the adjusted position of a point actually fell outside the 99% probability error ellipse drawn around the actual position.

It would thus appear that the free network adjustment should be used with care for the analysis of deformations, particularly with the method SF.

The investigation of Type I errors in epochs B to F

			A		Other		
Epoch	Method	Point	dy	dx	dy	dx	
B	IF	P20	+0,2	-2,1	-1,5	+0,7	
	CF	P20	+0,3	-1,9	-1,7	+0,4	
	SF	P20	+0,3	-1,9	-1,7	+0,7	
C	IF	P2	-2,4	+0,2	+0,7	-0,4	*
	CF	P4	+0,3	+0,1	-1,6	+0,3	*
		P14	+0,6	-0,6	-0,5	+0,7	
		P20	+P,3	-1,9	0	+1,4	
	SF	P4	+0,3	+0,1	-1,4	+0,1	*
		P14	+0,6	-0,6	-0,1	+0,7	
		P20	+0,3	-1,9	+0,3	+1,6	
D	SF	P16	-0,1	+0,4	-2,1	+1.0	*
E	SF	P12	+0,6	+0,5	+0,7	-2,0	*
F	SF	P4	+0,3	+0,1	-0,8	+1,3	
		P12	+0,2	+0,7	-0,9	-0,2	

dy & dx = differences from the actual position for a point.

* = case where the differences from one of the epochs fall outside the 3x ellipse drawn around the actual position.

table 20

6.4 The deformation of constraint points

It is evident from the findings for epoch C that it is essential to ensure that the constraint points are in fact stable when using the minimum constraint adjustment. In the invariant function method, this aspect is not critical as the nature of the process enables the user to confirm the stability of these points as well as determining doubtful points. However, in the other methods, deformations in these constraint points can lead to a false analysis by the introduction of "false deformations" due to the translation and rotation created by deformed constraint points.

6.5 Combination of Methods

An attempt at improving the reliability and accuracy of the analysis was made by combining the results from various combinations of two methods. Of the attempts, the only combination which achieved this object was that of Ab with SF (discussed in section 6.2.1.4). In the other cases, although a higher accuracy was attained, the reliability remained virtually the same. The increase in time of execution and extra programmes required did not seem to warrant this type of combination.

6.6 Network accuracy

When designing a network it is often the practice to aim for an accuracy which will be of the order of 1/3 the expected deformation or tolerance. This will give a confidence level of 99%, when applied to absolute error ellipses (see Annexure A, A.8.6). It seems logical that this would apply to the weakest point in the network which, depending on the strength of the network, would increase the accuracy of the other points.

(A strong network would have point accuracies which would be similar throughout the network). It is suggested that an accuracy of the order of $\frac{1}{4}$, or less, of the expected displacements be aimed for. This would have the effect of reducing the effect of observation error on the derived deformation.

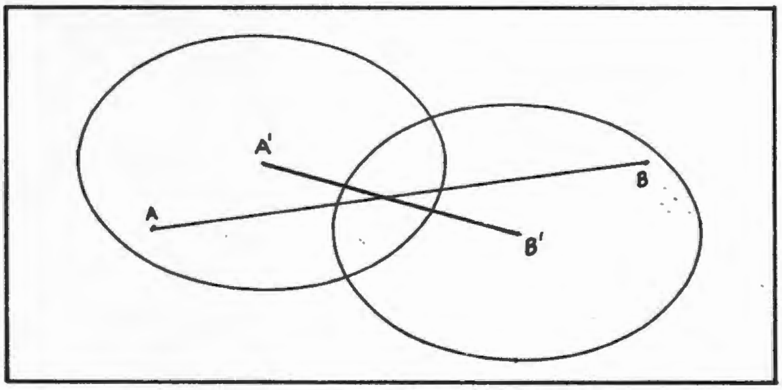


Figure 29

Consider figure 29. AB represents the true deformation of point A between epochs 1 & 2. It is possible for the separate adjustments of epoch 1 & 2 to yield point positions for A & B at A' & B' (such that A & B still fall within the respective 99% error ellipses). In this case, the deformation has been diminished but obviously the converse is also possible. It is clear that the greater the difference between deformation and point accuracy, the less the effect will be on the derived deformation. In other words, if the accuracy is say 1/10th of the deformation, then the amount of error to the displacement caused by errors in point position, will be less than say the case where the accuracy is only $\frac{1}{3}$ the deformation. Increasing the accuracy of the network will have the effect of reducing type II errors.

6.7 Precautions in regard to Niemeier's Method using minimum constraints

The use of Niemeier's method with a minimum constraints adjustment although not giving as accurate results as with the free network, still provides a reasonable alternative method. As the Q matrix is reduced by four columns and rows (corresponding to the minimum fixed points) care should be exercised when a case similar to epoch J is investigated. In the case of epoch J, three points were stable. Two of these were the fixed points leaving only one stable point in the reference network matrix. This obviously is a danger as should two or more other unstable points be included in the reference system, the possibility exists where the stable point may be found to be unstable and vice versa, resulting in false conclusions. It seems, however, from epoch J, that if the object points are well identified, the Niemeier transformation to separate reference and object points still operates, but one has no verification of the stability of the remaining reference point. From tests, it appears that the stability test on the reference points still operates on two points, thus to verify the remaining reference points, these should reduce to a minimum of two points. both of which must be stable. This implies that when using minimum constraints one requires a minimum of four stable points.

The method is, as with other co-ordinate comparison methods, sensitive to false deformations caused by shifts in the stable points, and if this type of error exists, a solution becomes impossible. This was the case with epoch C (P1 & P16X fixed) while epoch E yielded satisfactory results.

CHAPTER 7

CONCLUSIONS

In section 6, the analysis indicated four possible choices for use with networks with a small number of points unstable. Of these four methods, Ab, NF, IM & DF, only method IM relies entirely on a minimum constraint adjustment. Of the remaining three "free network dependent" methods, Ab seems to have a weakness as far as type I errors are concerned, while method DF, although very time efficient, is limited in its application, to networks where identical observation geometries are maintained between epochs. The author's choice of method would be Niemeier's comparison of co-ordinates, NF, while if the question of time was not of importance, the option would be the method of invariant functions, IM. As a general method, these two methods achieve similar reliabilities, but these differ when examining the accuracies for an epoch J type analysis (table 16). The choice as a general method would then be IM, as this supplies a reasonably consistent accuracy for all types of networks.

As a test on the choice of method, the deformations as derived by methods IM & NF for epoch HR, are compared with the applied displacements (table 21).

<u>Comparison of deformations as derived by methods IM & NF with applied displacement for epoch HR.</u>

Point deformed / Method	P4		P15		CD2(P20)	
	dy	dx	dy	dx	dy	dx
Applied shift	-4,0	+2,0	+5,0	0	+5,0	+6,0
Invariant Functions IM	-4,7	+3,2	+4,0	0	+4,7	+5,9
Niemeier NF	-4,9	+3,8	+4,3	+0,1	+4,4	+6,2

table 21

As mentioned previously, this epoch was created as a test on the final conclusions as to the preferred method. Until now, the applied displacements were unknown to the author. The deformations as indicated in table 21 were extracted from table 13, section 5.9. As can be seen from table 21, the derived deformations compare favourably with those applied, particularly in the case of CD2(P20). Although the comparisons for the other two points, P4 & P15, were not as good as was hoped, this order of displacement is repeated in all the other methods (table 13).

Displacement trends as seen in this case, seem to be caused by errors in the observations and not by any particular method. Trends such as this are encountered in other epochs also, notably in epoch D for P17 (table 6) and epoch E for P4 (table 9), and seems to confirm the suggestion of aiming for a higher accuracy than the minimum requirement for a network as discussed in section 6.6. As the network accuracy (at 99% confidence level) approaches the expected deformation magnitudes, there seems, from the networks analysed, a tendency towards a drop in reliability in the form of an increase in type II errors, and also because point position errors would have a larger effect on the derived deformations as shown in 6.6. This limitation should be borne in mind when designing a network for deformation measurements.

The methods discussed in this thesis, using geodetic models, offer a convenient and versatile solution for the measurement of deformations. The adjustment of small networks (up to +/- 25 points on the Tektronix 4051) may be achieved using desk top micro computers.

CHAPTER 8

REFERENCES

1. Ashkenazi, V. and Dodson, A.H. (1978), "Measurement of Deformations by Surveying Techniques : Compendium of Formulae", prepared for seminar at University of Nottingham, January 1978.

2. Ashkenazi, V., Dodson, A.H., Jones, D.E.B. and Samson, N. (1981), "Measurements of Deformations at the Queen Mother Reservoir, Datchet", Imperial College, London, November 1981.

3. Chrzanowski, A. (1977), "Design and Error Analysis of Surveying Projects : Selected Papers and Lecture Notes", Lecture Notes No. 47, Dept. of Surveying Engineering, University of New Brunswick, Fredericton N.B., 1977.

4. Chrzanowski, A. (1981), with contributions by members of the "Ad hoc" committee on the analysis of deformation measurements, "A Comparison of Different Approaches into the Analysis of Deformation Measurements", FIG XVI, Int. Congress, Montreux, Switzerland 1981.

5. Chrzanowski, A. (1982), "Measurement of Deformations by Geodetic Techniques", seminar at University of Cape Town, 1982.

6. Heck, B., Kuntz, E. and Meier-Hirmer, B. (1977), "Deformations-analyse Mittels Relativer Fehlerellipsen", AVN 1977, p. 78 - 87.

7. Krüger, J. (1979), "Numerische Behandlung von Datums und Konfigurations defekten", Geodätische Netze in Landes- und Ingenieursvermessungen 1979, p. 257 - 272.

8. Niemeier, W. (1976), "Grundprinzip und Rechenformeln einer Strengen Analyse Geodätische Deformationsvermess- ungen", VII Int. Kurs. für Ingenieursvermessungen hoher

Präzision Darmstadt 1976.

9. Rüther, H. (1982), "Relative Orientation with Limited Control in Close Range Photogrammetry", Ph. D Thesis, Dept. of Surveying, University of Cape Town 1982.

10. Spiegel, M.R. (1961), "Shaum's Outline Series : Theory and Problems of Statistics", McGraw-Hill Book Co., 1961.

11. Wells, D.E. and Krakiwsky, E.J. (1971), "The Method of Least Squares", Lecture Notes No. 18, Dept. of Surveying Engineering, University of New Brunswick, Fredericton N.B., May 1971, p. 68 - 85.

CONTENTS OF ANNEXURES

			Page
ANNEXURE A		THEORY OF THE LEAST SQUARES ADJUSTMENT	A-1
	A.1	The least squares adjustment of a two dimensional network	A-1
	A.2	Elimination of orientation unknowns	A-7
	A.3	Weights	A-9
	A.4	Constraints	A-11
	A.5	Solution vector	A-12
	A.6	Global check	A-13
	A.7	Free network adjustment	A-14
	A.8	Error analysis	A-18
	A.8.1	Variance factor	A-19
	A.8.2	Variance co-variance matrix	A-20
	A.8.3	Absolute error ellipses	A-20
	A.8.4	Relative error ellipses	A-22
	A.8.5	A Posteriori variances of the observations	A-23
	A.8.6	Confidence of variances	A-25
ANNEXURE B		THE LEAST SQUARES SOLUTION OF THE HELMERT TRANSFORMATIONS	B-1
ANNEXURE C		PROGRAMMES AND SAMPLE PRINTOUTS	
	C.1	Generation of observations	C-1
	C.2	Adjustment (minimum constraints)	C-6
	C.3	Invariant Function Comparisons	C-13
	C.4	Combination of Epochs	C-18
	C.5	Helmert Transformation without Scale Factor	C-23
	C.6	Adjustment of Differences	C-29
	C.7	Niemeier's Test for Deformation	C-37

ANNEXURE A

THEORY OF A NETWORK ADJUSTMENT

A.1 The Least Squares Adjustment of a two dimensional Network

The least squares adjustment of a network provides a homogenous solution, in that observational errors are distributed evenly throughout the network.

For a two dimensional network, three types of observations exist:

1. Directions
2. Angles
3. Distances

As the adjustment provides most probable values for the co-ordinates of the points in the network, the above three types of observations are related to the x and y co-ordinates of the points. This method of adjustment is also referred to as the "Variation of Co-ordinates" method. The observations, being related to the co-ordinates, are then also related to the unknowns (x_i) in observation equations of the form:

$$L_i + v_i = f\ (x_1, x_2, x_3 - - - - x_n) \qquad \text{(A-1)}$$

From (A-1) we may obtain the observation equations for direction, angle and distance.

1. Direction. $$L_{ik} + v_{ik} = \tan^{-1} \frac{(Y_k - Y_i)}{(x_k - x_i} - Z_i \qquad \text{(A-2)}$$

2. Angle. $$L_{jik} + v_{jik} = \tan^{-1} \frac{(Y_j - Y_i)}{(x_j - x_i)} - \frac{(y_k - y_i)}{(x_k - x_i)} \qquad \text{(A-3)}$$

3. Distance. $$L_{ik} + v_{ik} = (x_k - x_i)^2 + (y_k - y_i)^2 \qquad \text{(A-4)}$$

where: L_{ik} = observation

v_{ik} = correction to the observation

x_i, y_i = co-ordinates of the points involved in the observation

Z_i = orientation correction at point i.

The least squares solution, however, is only valid for linear functions and the equations, (A-1), (A-2) and (A-3), being non-linear, must first be linearised before being employed. This linearisation is achieved using the first term in the Taylor's series expansion for the functions. This is only valid if the provisional values for the co-ordinates are sufficiently close to the final result. If the the difference between provisional and final values is large, the linearisation technique is not valid and it becomes necessary to perform an iteration in the adjustment, where the final results are used as provisionals and if the differences are still large, further iterations are required until the differences are small.

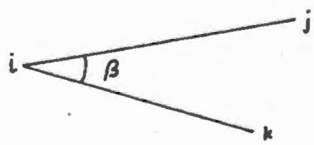
After linearisation, the above three observation equations become:

1. <u>Direction</u>

$$v_{ik} = a_{ik}dx_i + b_{ik}dy_i - a_{ik}dx_k - b_{ik}dy_k - dz_i - \ell_{ik} \qquad \text{(A-5)}$$

2. <u>Angle</u>

$$v_{jik} = (a_{ik} - a_{ij})dx_i + (b_{ik} - b_{ij})dy_i + a_{ij}dx_j + b_{ij}dy_j - a_{ik}dx_k - b_{ik}dy_k - \ell_{jik} \qquad \text{(A-6)}$$



3. Distance

$$\bar{v}_{ik} = \bar{a}_{ik}dx_i + \bar{b}_{ik}dy_i - \bar{a}_{ik}dx_k - \bar{b}_{ik}dy_k - \bar{\ell}_{ik} \qquad \text{(A-7)}$$

where:

$$a_{ik} = -a_{ki} = \frac{(y_{ko} - y_{io})}{d_{ik\,o}^{2}} \rho''$$

$$b_{ik} = -b_{ki} = \frac{-(x_{ko} - x_{io})}{d_{ik\,o}^{2}} \rho''$$

$$\bar{a}_{ik} = -\bar{a}_{ki} = \frac{-(x_{ko} - x_{io})}{d_{iko}}$$

$$\bar{b}_{ik} = -\bar{b}_{ik} = \frac{-(y_{ko} - y_{io})}{d_{iko}}$$

$$\ell_{ik} = t_{i\,obs}^{k} - t_{i\,o}^{k} + z_{io}$$

$$\ell_{jik} = \beta_{jik_{obs}} - \beta_{jiko}$$

$$\bar{\ell}_{ik} = d_{ik_{obs}} - d_{iko}$$

y_o, x_o = provisional co-ordinates for point

$t_{i\,o}^{k}$ = direction ik derived from provisional co-ordinates

β_{jiko} = angle jik derived from provisional co-ordinates

d_{iko} = distance ik derived from provisional co-ordinates

z_{io} = provisional orientation correction at i

$t_{i\,obs}^{k}$ = observed direction ik

$\beta_{jik_{obs}}$ = observed angle jik

$d_{ik_{obs}}$ = observed distance ik

ρ'' = 206264.8 (seconds in 1 radian)

v_{ik} = correction to observed direction ik.

v_{jik} = correction to observed angle jik

$\bar{v}_{ik}$ = correction to observed distance ik.

dx dy = unknown corrections to co-ordinates x_o, y_o

dz = unknown corrections to orientation correction z_o.

Although this type of observation is not used in this thesis, it is possible to introduce a scale factor unknown into the distances and the distance observation equation (A-7) becomes:

$$\bar{v}_{ik} = \bar{a}_{ik}dx_i+\bar{b}_{ik}dy_i-\bar{a}_{ik}dx_k-\bar{b}_{ik}dy_k-d_{ik}ds-\bar{\ell}_{ik} \qquad \text{(A-8)}$$

where:

ds = unknown correction to the scale factor s_o

$$\bar{\ell}_{ik} = \bar{d}_{ik_{obs}} - d_{ik_o}\, s_o$$

For each observation in a network, an observation equation (either (A-5), (A-6) or (A-7)) is formed. This set of observations can conveniently be combined in matrix notation, in the form:

$$\underline{v} = \underline{\underline{A}}\underline{x} - \underline{\ell} \qquad \text{(A-9)}$$

where:

$$\underline{v}^T = (v_1\ v_2\ v_3\ v_4\ v_5 - - - - - v_n)$$

$$\underline{x}^T = (dx_1\ dx_2\ dx_3\ dx_4 - - - - dx_{u-1}\ dx_u)$$

$$\underline{\ell}^T = (\ell_1\ \ell_2\ \ell_3\ \ell_4 - - - - - - - \ell_n)$$

$$\underline{\underline{A}} = \begin{pmatrix} a_1 & b_1 & 0 & 0 & \cdots & -a_1 & -b_1 & \cdots & 0 & 0 & - - - - \\ a_2 & b_2 & 0 & 0 & \cdots & \cdots & -a_2 & -b_2 & \cdots & 0 & - - - - \\ \ddots & & & & \ddots & & & \ddots & & & \\ 0 & 0 & \cdots & -a_n & -b_n & 0 & 0 & \cdots & a_n & b_n & \end{pmatrix}$$

n = number of observations

u = number of unknowns

dx = unknowns dx, dy and dz.

The set of observation equations in (A-9) can, as yet, not be solved and a further condition is required. This condition is given by the least squares principle;

that $[pvv]$ is a minimum

or $\underline{v}^T\underline{P}\underline{v}$ = a minimum in matrix notation. (A-10)

where $\underline{P}$ is the weight matrix, which is usually diagonal.

To minimise the function $\underline{v}^T\underline{P}\underline{v}$ one sets the differential of the function with respect to the unknowns $\underline{x} = 0$.

therefore set
$$\frac{d\,\underline{v}^T\underline{P}\underline{v}}{d\underline{x}} = 0 \qquad \text{(A-11)}$$

with (A-9) above

$$\begin{aligned}\underline{v}^T\underline{P}\underline{v} &= (\underline{A}\underline{x}-\underline{\ell})^T\underline{P}(\underline{A}\underline{x}-\underline{\ell})\\ &= (\underline{x}^T\underline{A}^T-\underline{\ell}^T)\underline{P}(\underline{A}\underline{x}-\underline{\ell})\\ &= \underline{x}^T\underline{A}^T\underline{P}\underline{A}\underline{x}-\underline{\ell}^T\underline{P}\underline{A}\underline{x}-\underline{x}^T\underline{A}^T\underline{P}\underline{\ell}+\underline{\ell}^T\underline{P}\,\underline{\ell}\end{aligned}$$

one finds the total differential

$$d\underline{x}^T\underline{A}^T\underline{P}\underline{A}\underline{x} + \underline{x}^T\underline{A}^T\underline{P}\underline{A}d\underline{x}-\underline{\ell}^T\underline{P}\underline{A}d\underline{x}-d\underline{x}^T\underline{A}^T\underline{P}\underline{\ell} = 0$$

with all four terms being scalars.
Also $\underline{A}^T\underline{P}\underline{A}$ is symmetrical and therefore unaffected by transposition

so $\underline{A}^T\underline{P}\underline{A} = (\underline{A}^T\underline{P}\underline{A})^T$

and as $\underline{P}$ is diagonal $\underline{P} = \underline{P}^T$

and one can write

$$dx^T \underline{A}^T \underline{PAx} + \underline{dx}^T (\underline{A}^T \underline{PA})^T \underline{x} - \underline{dx}^T \underline{A}^T \underline{P}^T \underline{\ell} - \underline{dx}^T \underline{A}^T \underline{P\ell} = 0$$

or $$2d\underline{x}^T \, (\underline{A}^T \underline{PAx} - \underline{A}^T \underline{P} \underline{\ell}) = 0 \qquad \text{(A-12)}$$

This equation is satisfied if either the term in brackets or $d\underline{x}^T$ is 0. $d\underline{x}^T = d\underline{x} = 0$ is the trivial case and is ignored.

This leaves the case where:

$$\underline{A}^T \underline{PAx} - \underline{A}^T \underline{P\ell} = 0 \qquad \text{(A-13)}$$

or $$\underline{A}^T \underline{PAx} = \underline{A}^T \underline{P\ell} \qquad \text{(A-14)}$$

pre-multiplying the two sides by $(A^T PA)^{-1}$, one obtains:

$$\underline{x} = (\underline{A}^T \underline{PA})^{-1} \, \underline{A}^T \underline{P\ell} = \underline{N}^{-1} \, \underline{A}^T \underline{P\ell} \qquad \text{(A-15)}$$

which represents the full solution of the least squares adjustment.

The corrections are given by:

$$\underline{v} = \underline{Ax} - \underline{\ell} \qquad \text{(A-16)}$$

The matrix $\underline{A}^T \underline{PA} = \underline{N}$ is known as the "normal equation matrix".

A.2 Elimination of orientation unknowns

In a network where direction observations are present, extra unknown corrections to the orientations arise. This obviously increases the size of the normal matrix system. When micro-computers are employed, it is desirable to reduce the use of the, already limited, memory space.

It has been proved that if a simulated observation equation is added to each station, then the orientation unknowns dz can be eliminated. The remaining unknowns dx_i and dy_i are unaffected by this procedure.

At a station P_1 the observation equations are:

$$
\begin{array}{l|l}
v_{11} = a_1dx_1+b_1dy_i-a_1dx_2-b_1dy_2+0 \cdots - dz_1-\ell_{11} & p_1 = 1 \\
v_{12} = a_2dx_1+b_2dy_i + 0-a_2dx_3-b_2dy_3+0 \cdots - dz_1-\ell_{12} & p_2 = 1 \\
\vdots \qquad \vdots \qquad \vdots & \\
v_{1n} = a_ndx_1+b_ndy_1 + 0 \cdots -a_ndx_k-b_ndy_k-dz_1-\ell_{1n} & p_n = 1
\end{array}
$$

The orientation unknowns dz_i are combined with the corrections v_i and all the observations equations are added to yield the simulated observation equation.

$$V_{11} = v_{11}+dz_1=a_1dx_1+b_1dy_1-a_1dx_2-b_1dy_2 + 0\cdots-\ell_{11} \quad p_1 = 1$$

$$V_{12} = v_{12}+dz_1=a_2dx_1+b_2dy_1 + \cdots -a_2dx_3-b_2dy_3 + 0-\ell_{12} \quad p_2 = 1$$

$$V_{1n} = v_{1n}+dz_1=a_ndx_1+b_2dy_1 + \cdots -a_ndx_k-b_2dy_k\cdots-\ell_{1n} \quad p_n = 1$$

$$V_1^* = v_n +ndz_1= [a]\,dx_1+ [b]\,dy_1-a_1dx_2-a_2dx_3-b_2dy_3\cdots-[\ell] \quad P_1^*=-\frac{1}{n}$$

(A-17)

In the sum equation one has $[v_n] = 0$ (A-18)

The sum equation is given the weight $-\frac{1}{n}$

It should be noted that because of (A-18) the observation weights for a station must be identical. If the weights are P_A and not 1 then the sum equation weight simply becomes $P_A^* = -\frac{P_A}{n}$

dz_a for a station is found from

$$dz_A = \frac{V_A^*}{n} \quad \text{(A-19)}$$

and $v_{Ai} = V_{Ai} - dz_A$ (A-20)

This operation is applied to all stations from which direction observations are taken.

This procedure has the effect of increasing the number of rows in the A matrix, but reducing the number of columns, and results in the matrix $\underline{A}^T\underline{PA}$ being reduced also.

Although the corrections V_{Ai} are quazi corrections, from $v_{Ai} = V_{Ai} + dz_A$, it can be shown that the sum of the squares of the corrections, V_i, of the new equation system with quazi weights and additional observation equations, is equal to the original $\underline{v}^T\underline{P}\underline{v}$

so that $\underline{V}^T\underline{P}^*\underline{V} = \underline{v}^T\underline{P}\underline{v}$ (A-21)

A.3 Weights

In an adjustment of a network where one has observations of different quality or mixed observations, i.e. directions (angles) and distances, it becomes necessary to reflect these differences in the adjustment, so that observations of better quality have more effect on the results of the adjustment than those of lesser quality. This is achieved by weighing the observations.

The weight p_i is given by:

$$p_i = \frac{\sigma_o^2}{\sigma_i^2} \qquad \text{(A-22)}$$

where:

σ_o = standard deviation of unit weight

σ_o^2 = variance factor

σ_i = standard deviation of an observation L_i

As σ_o^2 is dimensionless, the weight p_i has dimension

$$\frac{1}{(\text{units of observation})^2}$$

The variance factor σ_o^2 is usually chosen as being equal to 1 or equal to one of the observation variances σ_i^2 (often directions).

σ_i can be chosen on the basis of an educated guess derived from experience of the particular instrument used, or σ_i may be determined from a series of test observations using the specific instrument.

The validity of the adopted weight model may be tested by comparing the chosen, a priori, σ_o with the a posteriori σ_o obtained from the adjustment in a statistical test (e.g. χ^2 test).

σ_o^2 a posteriori is obtained from:

$$\sigma_o^2 = \frac{\underline{v}^T \underline{P} \underline{v}}{f} \qquad \text{(A-23)}$$

where:

f = degrees of freedom = n-u

n = number of observations

u = number of unknowns

In the formation of the $\underline{A}^T\underline{P}\underline{A}$ matrix, the weight has the effect of cancelling dimensions in the observation equation and in doing so, makes it possible to mix distance observations with direction observations.

Once calculated, the weights p_i are arranged in the principal diagonal of the $\underline{P}$ matrix in the same sequence as the observation equations in the $\underline{A}$ matrix.

$$\underline{P} = \begin{pmatrix} P_1 & 0 & 0 & \cdots & \cdots \\ \vdots & P_2 & & & \vdots \\ 0 & 0 & P_3 & & \vdots \\ 0 & 0 & & \ddots & \vdots \\ \vdots & \vdots & & & \\ \vdots & \vdots & \cdots & & P_n \end{pmatrix} \qquad \text{(A-24)}$$

The zero's elsewhere in the $\underline{P}$ matrix implies that the observations are non-correlated. Should these elements be non-zero, it implies that there is correlation between observations and these elements give a measure of the correlation between observations.

A.4 Constraints

If a network has no points held fixed, the normal equation system is singular, it has a rank defect and thus cannot be inverted. If one considers a network with no fixed points, it follows logically that the network is free to translate, rotate, compress or expand. It therefore becomes necessary to introduce some form of constraint to prevent this and thus to avoid a matrix with a rank defect. If one considers a triangulation network, it is clear that one requires one point to prevent translation and a second point to prevent rotation and provide a scale factor. Therefore four co-ordinates are held fixed and a triangulation network with no fixed points therefore has a rank defect of four.

In a trilateration system, with no scale factor, the required constraints are one point to prevent translation and a direction, in the form of 1 fixed co-ordinate of a second point to prevent swing. Thus a trilateration network with no scale factor would have a rank defect of three if no constraints were applied.

Rank defects in other types of networks follow logically and a table showing rank defect and thus the minimum constraints required may be formulated.

Type of Network	Rank Defect	Co-ordinates held fixed. (Constraints)
Triangulation	4	4 (2 points)
Trilateration (no scale factor)	3	3 (1 point 1 co-ordinate)
Combined Triangulation / Trilateration (no scale factor)	3	3 (1 point 1 co-ordinate)
Combined (with scale factor)	4	4 (2 points)
Trilateration (with scale factor)	4	4 (2 points)

Table A-1

A.5 Solution Vector

Once the observation equations have been formed and arranged in the $\underline{A}$ matrix, the normal equation system A^TPA and the vector $\underline{A}^T\underline{P}\underline{\ell}$ may be formed.

The solution is obtained from:

$$\underline{x} = (\underline{A}^T\underline{P}\underline{A})^{-1}\ \underline{A}^T\underline{P}\underline{\ell}$$

The vector $\underline{x}$ contains the unknown corrections dx, dy & dz which, once added to the provisional values of X, Y & Z, results in the final, or statistically, most probable, values for the co-ordinates and orientation corrections.

A.6 Global Check

To guard against errors in the formation of the mathematical model, incorrect signs for the free terms, ℓ, or corrections to the unknowns dx, dy & dz, it becomes necessary to incorporate some form of check in the adjustment. The simplest method is to compare the observations, derived from the non-linear form of the observation equations using final co-ordinates, with the adjusted observations derived from the observed observation L_i + the correction v_i generally of the form

$$L_i + v_i = F\ (X_1, Y_1, Z_1, X_2, Y_2, Z_2, X_3, Y_3 \ldots .)$$

where F is the non-linear function of the observation equation. The comparison should be identical giving due regard to the degree of accuracy of the computer employed (minor differences due to this may occur).

A.7 Free Network Adjustment (Kruger, J. 1979)

In the conventional constraints adjustment, a certain minimum number of constraints, in the form of fixed co-ordinates (see table A-1) must be applied, otherwise the normal equation system matrix has a rank defect d, is then singular and thus cannot be inverted.

When using minimum constraints in the adjustment, the shape of the network is unaffected, but should further constraints be added, the inherent errors of the additional constraint points could affect the shape of the network.

This type of adjustment has the disadvantage that no measure of accuracy can be obtained for the fixed co-ordinates, which are assumed to be without error. Although precautions may be taken to ensure the stability of the constraint points, (by fixing to bedrock, removing from the area subject to possible deformation), instances where this is not possible exist, (deformation measurements where all points may be subject to movement) and an alternative mathematical model is required to overcome this problem.

An adjustment, where all points are considered as having errors (Free network adjustment), offers a solution to this difficulty. Point positions and point accuracy, derived from the free network adjustment, differ from the minimum constraints adjustment, but scale, shape and a posteriori observation accuracies remain identical.

As mentioned above, the normal equation system matrix $\underline{A}^T\underline{P}\underline{A}$ is singular with a rank defect, d, if no constraints are applied to the network. This is overcome by methods which are referred to as the free network adjustment.

The basis of the technique is the addition of further fictitious or pseudo observations to the observation equation system. However, certain conditions must be applied to these observations, otherwise the system would either remain singular (1) below or the shape would be affected (2) below, or both. These conditions are:

1) The pseudo observations must be independent of each other and of the actual observations.
2) The pseudo observations vectors must be orthogonal (mathematical sense) to the actual observations.

It can be shown that the normalised eigen vectors of the existing $\underline{A}^T\underline{PA}$ matrix, corresponding to eigen values $\lambda = 0$, satisfy these two conditions, and may be added to the observation system. A non-singular matrix, leading to a unique solution results. A number (corresponding to the rank defect d) of pseudo observations in the form of eigen vectors (for $\lambda = 0$) must be added to the system. These are combined in a matrix G (order u by d, u = number of unknowns). The new matrix, which takes the place of the existing $\underline{A}^T\underline{PA}$ matrix, has the form:

$$\bar{\underline{N}} = (\underline{A}^T\underline{PA} + \underline{GG}^T) \tag{A-25}$$

with $$\bar{\underline{Q}} = \bar{\underline{N}}^{-1} = (\underline{A}^T\underline{PA} + \underline{GG}^T)^{-1} \tag{A-26}$$

To obtain the correct variance-covariance matrix, the matrix $\underline{GG}^T$ must first be subtracted from $\underline{Q}$

$$\underline{Q}_x = \bar{\underline{Q}} - \underline{GG}^T = (\underline{A}^T\underline{PA} + \underline{GG}^T)^{-1} - \underline{GG}^T \tag{A-27}$$

It can be proved that

$$\underline{Q}\underline{A}^T\underline{P}\,\underline{\ } = \underline{Q}_x\underline{A}^T\underline{P}\underline{\ell} \qquad \text{(A-28)}$$

so the solution vector can take the forms:

$$\underline{x} = \underline{Q}_x\underline{A}^T\underline{P}\underline{\ell} \qquad \text{(A-29)}$$

or

$$\underline{x} = \underline{Q}\underline{A}^T\underline{P}\underline{\ell} \qquad \text{(A-30)}$$

The free network adjustment not only leads to the least squares solution resulting in

$$\underline{v}^T\underline{v} = \text{minimum} \qquad \text{(A-31)}$$

but also to the case where the sum of the squares of the corrections to the unknowns is also a minimum.

$$\underline{x}^T\underline{x} = \text{minimum} \qquad \text{(A-32)}$$

This leads to the network being optimally fitted into the provisional co-ordinates, without changing its shape. This may be likened to a Helmert transformation having no scale factor. This transforms the network into the provisional points of the network. The derived normalised eigen vectors, corresponding to = 0, for the $\underline{N}$ matrix of a free network are:

$$G^T \doteq \begin{array}{c} \begin{array}{cccccccc} x_1 & y_1 & x_2 & y_2 & \cdots & x_m & y_m \end{array} \\ \left(\begin{array}{ccccccc} \frac{1}{\sqrt{m}} & 0 & \frac{1}{\sqrt{m}} & 0 & \cdots & \frac{1}{\sqrt{m}} & 0 \\ 0 & \frac{1}{\sqrt{m}} & 0 & \frac{1}{\sqrt{m}} & \cdots & 0 & \frac{1}{\sqrt{m}} \\ -cy_1' & cx_1' & -cy_2' & cx_2' & \cdots & -cy_m' & cx_m' \\ \hdashline cx_1' & cy_1' & cx_2' & cy_2' & \cdots & cx_m' & cy_m' \end{array}\right) \end{array} \begin{array}{l} \text{(translation in X)} \\ \text{(translation in Y)} \quad d=3 \text{ (rows 1–3)} \\ \text{(rotation)} \quad d = 4 \text{ (rows 1–4)} \\ \text{(scale factor)} \end{array} \qquad \text{(A-33)}$$

NB. <u>This solution requires the orientation unknowns dz to be eliminated</u>

where m = number of points in the network

y' & x' = co-ordinates reduced to the centre of gravity

$$y'_i = y_i - \bar{y} \qquad x'_i = x_i - \bar{x} \qquad \text{(A-34)}$$

$$\bar{y} = \frac{1}{m}\sum_{j=1}^{m} y_j \qquad \bar{x} = \frac{1}{m}\sum_{j=1}^{m} x_j \qquad \text{(A-35)}$$

To normalise the eigen vectors, the first two are multiplied by $1/n$ and the third and fourth by a constant, c, given by

$$c = \frac{1}{\sqrt{\sum_{j=1}^{m} (y'^2_j + x'^2_j)}} \qquad \text{(A-36)}$$

It can be shown that the four eigen vectors correspond to the elements required by a Helmert transformation i.e. translation rotation and scale factor. See matrix at (A-33).

When determining the a posteriori value for the variance σ_o^2 from:

$$\sigma_o^2 = \frac{\underline{v}^T \underline{\underline{P}} \underline{v}}{f}$$

The number of degrees of freedom must include the pseudo observations so

$$f = n - u + d$$

where: n = number of observations

u = number of unknowns

d = rank defect = number of pseudo observations.

A.8 Error Analysis

From the least squares adjustment, (both free network and minimum constraint), various measures of accuracy are available, from which the network quality may be assessed. These measures are:

1) A priori variances of the observations.
2) A posteriori variances of the observations.
3) A posteriori variances of the unknowns (dx dy & dz).
4) Error ellipses.

A knowledge of the a priori accuracies of the observations is essential for a correct weight model and thus adjustment.

A feature which is very useful. is that the whole error analysis of an adjustment may be performed without carrying out any observations in the field. This is due to the fact that 2), 3) and 4) above are functions of the network design and observation geometry only, which are both known beforehand. The only other requirement is that the a priori variances of the observations be known. This process is known as pre-analysis of a network. The advantage of this is that the optimum observation geometry and network design may be obtained, before the observations are actually performed; by testing various network models. It should be noted that if σ_0^2 a priori is incorrectly chosen, then the pre-analysis yields only relative measures of accuracy.

The following analysis applies to both minimum constraint and free network adjustments.

The only difference is that the free network provides accuracy measures for all the points in the network while the minimum constraints adjustment provides accuracies for all but the constraint points.

A.8.1 Variance Factor

The a priori value for the variance fact σ_0^2 should be tested against the a posteriori variance factor $\bar{\sigma}_0^2$.

where $$\bar{\sigma}_0^2 = \frac{v^T P v}{f} \qquad \text{(A-37)}$$

Ideally the two should agree, but if there is disagreement outside a statistical limit set by the χ^2 distribution,

where $$\frac{f\sigma_0^2}{\chi_1^2} \leq \bar{\sigma}_0^2 \leq \frac{f\sigma_0^2}{\chi_2^2} \qquad \text{(A-38)}$$

or using $\gamma_i = \sqrt{\frac{f}{\chi_i^2}}$ $\quad i = 1,2$

and $$\sigma_0\gamma_1 \leq \bar{\sigma}_0 \leq \sigma_0\gamma_2 \qquad \text{(A-39)}$$

where χ_i^2 & γ_i are obtained from tables.

factors such as poor observations, gross errors in observations and an incorrect weight model should be investigated, as these could cause this discrepancy. An incorrect mathematical model could also produce a discrepancy, but as this would be well established, the likelihood of this being the cause is minimal. It is essential that this test on σ_0 be confirmed before any further analysis is performed, as this test failure makes the adjustment meaningless. Although the aposteriori value $\bar{\sigma}_o^2$ is only an estimate derived from the adjustment, and the apriori value σ_o^2 should be preferred for analysis, the common practice of using the aposteriori estimate is followed here.

A.8.2 Variance - Covariance Matrix

The Variance - Covariance Matrix is derived from the inverse of the Normal equation matrix multiplied by the variance factor;

therefore: Variance - Covariance matrix =

$$\sigma_o^2(\underline{A}^T\underline{P}\underline{A})^{-1} = \sigma_o^2\underline{Q} \qquad \text{(A-40)}$$

The terms on the principal diagonal of the $\sigma_o^2\underline{Q}$ matrix are the variances of the unknowns (dx, dy). The variances are dependent on the points held fixed in a network and differ should different co-ordinates be held fixed. This problem does not occur in a Free Network. The variance of the unknowns can be directly extracted from the variance - covariance matrix, or calculated using the $\underline{Q}$ matrix from:

$$\sigma x_i^2 = \sigma_o^2 \, Qx_ix_i$$

$$\sigma y_i^2 = \sigma_o^2 \, Qy_iy_i \qquad \text{(A-41)}$$

$$\sigma z_i^2 = \sigma_o^2 \, Qz_iz_i$$

with point accuracy derived from:

$$\sigma p_i^2 = \sigma x_i^2 + \sigma y_i^2 \qquad \text{(A-42)}$$

A.8.3 Absolute Error Ellipses

Absolute error ellipses are graphical representations of the accuracy of a point position and ideally should be small circles of equal size throughout the network.

The parameters for the construction of the ellipses are:

A = Semi-major axis

B = Semi-minor axis

ϕ = Orientation of Semi-major axis

See Figure A-1.

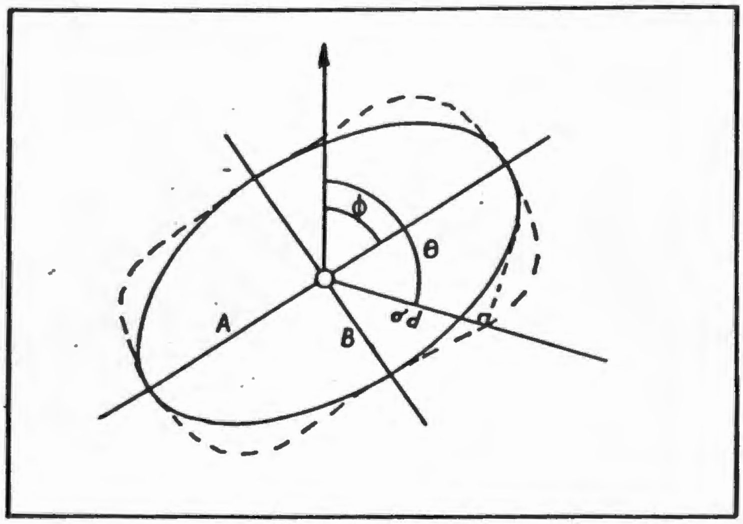


Figure A-1

Obtained from:

$$A^2 = \sigma_o^2 \, (D + \sqrt{E})$$

$$B^2 = \sigma_o^2 \, (D - \sqrt{E}) \qquad \text{(A-43)}$$

$$\operatorname{Tan} 2\phi = \frac{2Qxy}{Qxx - Qyy}$$

where:

$$D = \frac{Qyy + Qxx}{2}$$

$$E = \frac{(Qxx - Qyy)^2}{4} + Qxy^2 \qquad \text{(A-44)}$$

Linear variances, σ_d, for a point in a specific direction θ, may be obtained by drawing a line in the direction θ, from the centre of the ellipse. A line at right angles to this and tangential to the ellipse curve is constructed. The distance from the centre of the ellipse to this foot point is the linear variance σ_d. If all the footpoints for all directions are drawn, a new curve (dotted line in Figure A-1) is obtained which is known as the "pedal" or "error" curve, with its function being:

$$\sigma_\theta^{\ 2} = A^2 \mathrm{Cos}^2(\theta-\phi) + B^2 \mathrm{Sin}^2(\theta-\phi) \qquad \text{(A-45)}$$

A.8.4 Relative Error Ellipses

Relative error ellipses serve as a measure of accuracy between two points in a network. These ellipses are usually drawn between the two points involved. As two points are involved in the determination of a relative ellipse, all the elements pertinent to these points from the $\underline{Q}$ matrix must be employed.

The formulae for determining the parameters of the relative ellipse are similar to those for the absolute ellipse and are:

$$\bar{A}^2 = \sigma_o^{\ 2}\ (\bar{D}+\sqrt{\bar{E}})$$

$$\bar{B}^2 = \sigma_o^{\ 2}\ (\bar{D}-\sqrt{\bar{E}}) \qquad \text{(A-46)}$$

$$\tan 2\bar{\phi} = \frac{2Q\bar{x}\bar{y}}{Q\bar{x}\bar{x} - Q\bar{y}\bar{y}}$$

where: $\bar{A}$ = Semi-major axis

$\bar{B}$ = Semi-minor axis

$\bar{\phi}$ = Orientation of semi-major axis.

$$Q\bar{x}\bar{x} = Qx_1x_1 - 2Qx_1x_2 + Qx_2x_2$$

$$Q\bar{y}\bar{y} = Qy_1y_1 - 2Qy_1y_2 + Qy_2y_2 \qquad \text{(A-47)}$$

$$Q\bar{x}\bar{y} = Qx_1y_1 - Qx_2y_1 - Qx_1y_2 + Qx_2y_2$$

$$\bar{D} = \frac{Q\bar{x}\bar{x} + Q\bar{y}\bar{y}}{2}$$

$$\bar{E} = \frac{(Q\bar{x}\bar{x} - Q\bar{y}\bar{y})^2}{4} + Q^2\bar{x}\bar{y}$$

A.8.5 A Posteriori Variances of the Observations

These variances differ from the a priori variances, as the observations have been improved by the addition of the corrections **v**, and thus become a function of the unknowns of the adjustment as can be seen when inspecting the original observation equations.
For example:

$$d_{ik} = d_{ik_{obs}} + \bar{v}_{ik} = \sqrt{(y_{ko}+dy_k-(y_{io}+dy_i))^2+(x_{ko}+dx_k-(x_{io}+dx_i))^2}$$

The a posteriori variance of the observations or variance of the adjusted observations is given by:

$$\underline{\sigma_f^{\,2} = \sigma_o^{\,2}\, Q_{ff}} \qquad \text{(A-48)}$$

where $\underline{Q_{ff} = \underline{f}^T \underline{\underline{Q}}\, \underline{f}}$ (scalar) **(A-49)**

and $\underline{f}$ is the vector of differentials of the function in question (distance, direction or angle) with respect to the unknowns of the adjustment.

$$F = f\,(x_1, y_1, x_2, y_2, \text{------})$$

$$f^T = \left(\frac{\partial F}{\partial x_1} \; \frac{\partial F}{\partial y_1} \; \frac{\partial F}{\partial x_2} \; \text{------} \right) \qquad \text{(A-50)}$$

and $\underline{Q}$ is the inverse of matrix $\underline{A}^T\underline{P}\underline{A}$

If, for example, one considers a distance observation equation, it is clear that only two points are involved. Thus only four elements, corresponding to the differentials with respect to the four co-ordinates of the two points, can be non-zero. Therefore, the f vector can, in this case, be reduced to a vector of four elements. The other elements in the $\underline{f}$ vector are zero and from (A-49), it is clear that only the elements in the $\underline{Q}$ matrix corresponding to the four elements, in the $\underline{f}$ vector, will have an influence on the determination of Qff, and the $\underline{Q}$ matrix can be thus also reduced.

From the above, if one considers, for example, a distance observation between points i & k, the $\underline{f}$ vector would be:

$$\underline{f}^T = \left(\frac{\partial F}{\partial x_i} \; \frac{\partial F}{\partial y_i} \quad 0 \quad 0 \cdots \frac{\partial F}{\partial x_k} \; \frac{\partial F}{\partial y_k} \quad 0 \quad 0 \cdots\cdots \right)$$

and reduces to four elements,

$$\underline{f}^T = \left(\frac{\partial F}{\partial x_i} \; \frac{\partial F}{\partial y_i} \; \frac{\partial F}{\partial x_k} \; \frac{\partial F}{\partial y_k} \right)$$

The values for $\frac{\partial F}{\partial x_i} \; \frac{\partial F}{\partial y_i} \; \frac{\partial F}{\partial x_k} \; \frac{\partial F}{\partial y_k}$ are known from the linearisation of the distance observation equation.

$$\frac{\partial F}{\partial x_i} = -\frac{\partial F}{\partial x_k} = \bar{a}_{ik} = -\bar{a}_{ki}$$

& $$\frac{\partial F}{\partial y_i} = -\frac{\partial F}{\partial y_k} = \bar{b}_{ik} = -\bar{b}_{ki}$$

and $\underline{f}$ becomes:

$$\underline{f}^T = (\bar{a}_{ik} \; \bar{b}_{ik} \; -\bar{a}_{ik} \; -\bar{b}_{ik}) \qquad \text{(A-51)}$$

and

$$Qff = (\bar{a}_{ik} \; \bar{b}_{ik} \; -\bar{a}_{ik} \; -\bar{b}_{ik}) \begin{pmatrix} Qx_ix_i & Qx_iy_i & Qx_ix_k & Qx_iy_k \\ Qy_ix_i & Qy_iy_i & Qy_ix_k & Qy_iy_k \\ Qx_kx_i & Qx_ky_i & Qx_kx_k & Qx_ky_k \\ Qy_kx_i & Qy_ky_i & Qy_kx_k & Qy_ky_k \end{pmatrix} \begin{pmatrix} \bar{a}_{ik} \\ \bar{b}_{ik} \\ -\bar{a}_{ik} \\ -\bar{b}_{ik} \end{pmatrix} \qquad \text{(A-52)}$$

↑ reduced Q matrix containing only those elements relating to points i & k.

A.8.6 <u>Confidence of Variances</u>

The standard deviation of a single variable (e.g. σx, σy, σ_f) represents a probability level of only 68,3%. This means that the probability of the value of a variable lying within the interval "true value + σ" and "true value $-\sigma$" is 68,3%. It sometimes becomes necessary to be more careful about quoted accuracies, and then higher confidence levels are required. To increase the confidence level of a standard deviation, σ must be multiplied by a factor c

$$\sigma_c = c.\sigma \qquad \text{(A-53)}$$

where σc is the higher confidence interval.

The value for c may be found from tables for the student t-distribution (Wells & Krakiwsky 1971). For example:

Confidence level %	68,3	80	90	95	98	99	99,9
c	1	1,28	1,64	1,96	2,33	2,58	3,29

<u>Table A-2</u>

Table A-2 is only a portion of the table and represents the values for c when one has an infinite number of degrees of freedom. Values for c for varying degrees of freedom at different confidence levels are available from these tables. If the aposteriori estimate of the standard deviation, $\hat{\sigma}$, is used, then it is necessary to extract a value for c dependent on the number of degrees of freedom in the model. However, according to Spiegel 1961, if the number of degrees of freedom exceeds 30, or the sample is "large", then for most practical purposes the value for c with infinite degrees of freedom may be used.

For standard deviations of two variables (error ellipses), the confidence level reduces to 39,4%. In other words, for an error ellipse, the true point lies within the area of the ellipse with a probability of 39,4%. To obtain a higher confidence level, the parameters of the error ellipse, i.e. semi-major and minor axes, must be multiplied by a factor d.

$$A_d = d.A$$

$$B_d = d.B \qquad \text{(A-54)}$$

The factor d may be derived from

$d = \sqrt{2.F(2,b;\alpha)}$ (Heck, Kuntz & Meier-Hirmer 1977 & Chrzanowski 1977)

where $F(2,b;\alpha)$ = value extracted from tables for the Fisher Distribution

b = degrees of freedom

α = required probability e.g. 95% 99% etc.

As an illustration, derived values for d with infinite degrees of freedom and varying probability are given in table A-3.

Confidence level %	39,4	80	90	95	98	99
d	1	1,79	2,15	2,45	2,80	3,03

Table A-3

In engineering surveying a confidence interval of 99% for ellipses and 99,9% for single variables is often adopted for tolerances. This means that a surveyor should aim for standard deviation of less than $\frac{1}{3}$ of the required tolerances.

ANNEXURE B

The Least Squares Solution of the Helmert Transformation (H. Rüther 1982)

The observation equations for the Helmert Transformation are:

$$\begin{aligned} x + v_x &= a_o + a_1 X - b_1 Y \\ y + v_y &= b_o + a_1 Y + b_1 X \end{aligned} \qquad \text{(B-1)}$$

where: Y & X are the co-ordinates from the old system
y & x are the co-ordinates from the new system
a_o b_o = translation unknowns in x & y respectively
a_1 b_1 = unknowns for rotation & scale (combined)

and the $\underline{A}$ matrix and $\underline{\ell}$ vectors are as follows:

$$\underline{A} = \begin{pmatrix} 1 & 0 & X_1 & -Y_1 \\ 0 & 1 & Y_1 & X_1 \\ 1 & 0 & X_2 & -Y_2 \\ 0 & 1 & Y_2 & X_2 \\ \vdots & & & \\ 1 & 0 & X_n & -Y_n \\ 0 & 1 & Y_n & X_n \end{pmatrix} \qquad \underline{\ell} = \begin{pmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ \vdots \\ x_n \\ y_n \end{pmatrix} \qquad \text{(B-2)}$$

and the $\underline{A}^T\underline{P}\underline{A}$ matrix and $\underline{A}^T\underline{P}\underline{\ell}$ matrix are (with n common points and all observations of equal weight, p = 1):

$$\underline{A}^T\underline{P}\underline{A} = \begin{vmatrix} n & 0 & [X] & [-Y] \\ 0 & n & [Y] & [X] \\ [X] & [Y] & [X^2+Y^2] & 0 \\ [-Y] & [X] & 0 & [Y^2+X^2] \end{vmatrix} \qquad \text{(B-3)}$$

$$([YX - XY] = 0)$$

$$\underline{A}^T \underline{P} \underline{\ell} = \begin{pmatrix} [x] \\ [y] \\ [xX + yY] \\ [yX - xY] \end{pmatrix} \qquad (B\text{-}4)$$

and the vector of unknowns $\underline{x}$

$$\underline{x} = \begin{pmatrix} a_0 \\ b_0 \\ a_1 \\ b_1 \end{pmatrix}$$

Reducing the two systems to their respective centres of gravity $\bar{Y}$ $\bar{X}$ $\bar{y}$ $\bar{x}$ simplifies the equation system.

The new co-ordinates reduced to their centres of gravity become:

$$Y'_i = Y_i - \bar{Y} \qquad X'_i = X_i - \bar{X}$$
$$y'_i = y_i - \bar{y} \qquad x'_i = x_i - \bar{x} \qquad (B\text{-}5)$$

where:

$$\bar{Y} = \frac{[Y]}{n} \qquad \bar{X} = \frac{[X]}{n}$$
$$\bar{y} = \frac{[y]}{n} \qquad \bar{x} = \frac{[x]}{n} \qquad (B\text{-}6)$$

from (B-5) & (B-6), it is clear that:

$$[Y'] = [X'] = [y'] = [x'] = 0$$

and the $\underline{A}^T \underline{P}\underline{A}$ matrix reduces to:

$$\begin{pmatrix} [Y'^2 + X'^2] & 0 \\ 0 & [Y'^2 + X'^2] \end{pmatrix} \qquad (B\text{-}7)$$

$\underline{A}^T\underline{P}\underline{\ell}$ to:

$$\begin{vmatrix} [x'X' + y'Y'] \\ [y'X' - x'Y'] \end{vmatrix} \qquad \text{(B-8)}$$

and results in the direct determination of unknowns a_1 & b_1 from:

$$a_1 = \frac{[x'X' + y'Y]}{[Y'^2 + X'^2]}$$

$$b_1 = \frac{[y'X' - x'Y']}{[Y'^2 + X'^2]} \qquad \text{(B-9)}$$

The translations a_o & b_o are found by applying the scale factor and rotation to the co-ordinates of the centre of gravity for the old system and subtracting there, from those of the new system, viz.

$$a_o = \bar{x} - a_1\bar{X} + b_1\bar{Y}$$

$$b_o = \bar{y} - a_1\bar{Y} - b_1\bar{X} \qquad \text{(B-10)}$$

Should error analysis be required, this is obtained from:

$$[vv] = [v_x^2] + [v_y^2]$$

$$vx_i = a_o + a_1X_i - b_1Y_i - x_i \qquad \text{(B-11)}$$

$$vy_i = b_o + a_1Y_i + b_1X_i - y_i$$

$$\sigma_o^2 = \frac{[vv]}{2n-4}$$

ANNEXURE C

PROGRAMMES AND SAMPLE PRINTOUTS

C.1 Generation of Observations

Requirements:

1. Co-ordinates of the network stored on data file in the following format:

   Number of points (N)

   List of co-ordinates with names:

   Y , X , "NAME".

2. A list of observations stored on a Programme file as DATA statements. The observations are divided into two blocks. Firstly, all direction observations and then distance observations. Following the order of co-ordinates in 1., the observations from each point in turn are listed (using point number) and terminated by a 0, e.g.

   ```
   100  DATA  2 , 3 , 6 , 0
   110  DATA  1 , 3 , 7 , 6 , 0
   ```

   etc., where line 100 indicates the observations (direction) from point 1, 110 from point 2 etc. A similar block is written for the distances. If no observation is required from a point, a zero only must be inserted.

3. The standard deviations for directions (seconds) and distances (mm).

4. A data file for the observations.

Sample printout on page C-5.

```
4 RUN 100
100 INIT
110 SET DEGREES
120 REM TO GENERATE OBSERVATIONS FOR A SET OF COORDS
130 PRINT "ENTER NAME OF FILE WITH COORDS"
140 INPUT C$
150 OPEN C$;1,"R",A$
160 READ #1:N1
170 DIM Y(N1),X(N1),A(N1,N1),B(N1,N1)
175 PRI @3: USI 176:"LIST OF COORDS SHOWING NUMBERS TO BE USED WHEN "
176 IMAGEFA"REFERING TO POINTS IN THIS PROGRAM"/L
180 FOR I=1 TO N1
190 READ #1:Y(I),X(I),P$
200 PRINT @3: USING 210:I,Y(I),X(I),P$
210 IMAGE4D2(10D.4D)5XFA
220 NEXT I
230 CLOSE 1
240 PRINT "ENTER MEAN SQUARE ERROR OF ANGLES (SECS)"
250 INPUT M1
260 PRINT "ENTER MEAN SQUARE ERROR OF DISTANCES (MM)"
270 INPUT M2
280 N2=0
290 A=0
300 B=0
305 PRINT "ENTER FILE WITH POINTS OBSERVED (PROGRAM FILE)"
306 INPUT C$
307 APPEND C$;3000
310 FOR I=1 TO N1
320 FOR J=1 TO N1
340 READ A(I,J)
350 IF A(I,J)=0 THEN 380
360 N2=N2+1
365 N3=0
370 NEXT J
380 NEXT I
390 FOR I=1 TO N1
400 FOR J=1 TO N1
420 READ B(I,J)
430 IF B(I,J)=0 THEN 460
440 N3=N3+1
450 NEXT J
460 NEXT I
470 PRINT "ENTER NAME OF FILE FOR OBSERVATIONS"
480 INPUT D$
490 OPEN D$;1,"F",A$
500 M=M1
510 N=N2+N3
515 GOSUB 970
516 PRINT @3: USING 517:"MSE- DIRECTION=",M1,"SEC","MSE= DISTANCE =";M2
517 IMAGE2(/L)FA5D2XFA/LFA5D2X,"MM"
521 PRINT @3: USING 522:"LIST OF OBSERVATIONS- DIRNS THEN DISTS"
522 IMAGE3(/L)FA2(/L)
523 PRINT @3: USING 524:"FROM","TO","CODE","DIRN(D.M.S)/DIST(M)"
524 IMAGE3XFA3XFA3XFA3XFA
525 REM   DIRECTION GENERATION
530 C=1
540 P=V
550 FOR I=1 TO N1
560 FOR J=1 TO N1
570 IF A(I,J)=0 THEN 690
580 L=A(I,J)
590 GOSUB 900
600 T=T1+X1(1,P)/3600
```

```
610 P=P+1
620 D=INT(T)
630 M3=INT((T-D)*60)
640 S=((T-D)*60-M3)*60
650 PRINT @3: USING 660:I,L,C,D,M3,S
660 IMAGE3(6D)2X2(6D)6D.D
670 WRITE #1:I,L,C,D,M3,S
680 NEXT J
690 NEXT I
695 REM   DISTANCE GENERATION
700 C=2
710 M3=0
720 S=0
730 P1=M2/M1
740 FOR I=1 TO N1
750 FOR J=1 TO N1
760 IF B(I,J)=0 THEN 850
770 L=B(I,J)
780 GOSUB 900
790 D=D1+X1(1,P)*P1/1000
800 P=P+1
810 PRINT @3: USING 820:I,L,C,D
820 IMAGE3(6D)9D.3D
830 WRITE #1:I,L,C,D,M3,S
840 NEXT J
850 NEXT I
860 C=0
870 WRITE #1:I,J,C,D,M3,S
880 CLOSE 1
885 PRINT @3: USING 886:"NUMBER OF OBS =",P-V
886 IMAGE/LFA5D
890 END
900 REM   SUB JOIN
910 Y2=Y(L)-Y(I)
920 X2=X(L)-X(I)
930 D1=SQR(Y2^2+X2^2)
940 T1=180-ATN(X2/(Y2+1.0E-50))-90*SGN(Y2+1.0E-50)
950 RETURN
970 REM                         SUB NORMAL DISTR. RANDOM VARIABLE GENERATOR
980 REM                                   CONSTANT-M.S.E.-NO OF RANDOM VALUES
984 V=1
985 PRINT "NEW STARTING POINT FOR RANDOM ERRORS Y/N"
986 INPUT E$
987 IF E$="N" THEN 990
988 PRINT "ENTER STARTING NUMBER"
989 INPUT V
990 K=6
1002 N9=N+10+V
1010 DIM X1(1,N9),Y1(14),Z1(K)
1020 Y1=0
1030 X1=0
1040 FOR I=1 TO N9
1050 B0=0
1060 FOR J=1 TO K
1070 REM                                                      SEED PI
1080 Z1(J)=RND(PI)
1090 B0=B0+Z1(J)
1100 NEXT J
1110 S0=(B0-3)*SQR(12/K)
1115 IF I<V THEN 1130
1120 X1(1,I)=(S0-(3*S0-S0^3)/(K*20))*M
1130 NEXT I
1140 B0=0
```

```
1150 FOR I=1 TO N9
1160 B0=B0+X1(1,I)
1170 NEXT I
1180 B0=B0/N9
1190 C0=B0*10/M
1200 D0=0
1210 FOR I=1 TO N9
1220 D0=D0+(X1(1,I)-B0)^2
1230 NEXT I
1240 D0=SQR(D0/(N9-1))
1250 PRINT @3: USING 1260:
1260 IMAGE 10X"GENERATED RANDOM ERRORS   VARIANCE"8X"M.S.E."10XS
1270 PRINT @3: USING 1280:D0^2,D0,M,B0
1280 IMA"DISTRIBUTION MEAN"/10X23"-"8D.1D,8D.1D" ("3D" )"13D.2D" ( 0 )"
1300 RETURN
1310 END
3000 DATA 2,3,4,0
3010 DATA 3,4,1,0
3020 DATA 4,1,2,0
3030 DATA 1,2,3,0
3040 DATA 2,3,4,0
3050 DATA 3,4,0
3060 DATA 4,0
3070 DATA 0
```

LIST OF COORDS SHOWING NUMBERS TO BE USED WHEN REFERING TO POINTS IN THIS PROGRAM

1	691.0000	5125.5000	P1
2	811.0000	5126.0000	P2
3	943.0000	5127.5025	P3
4	816.4000	5006.1000	P4
5	818.0000	4735.5500	P12
6	923.0000	4735.5100	P13
7	1012.0000	4735.5200	P14
8	1132.0000	4735.5000	P15
9	1105.2000	5084.4000	P16
10	814.5966	4855.6972	P17
11	1214.4000	4900.8957	CD2(P20)

GENERATED RANDOM ERRORS	VARIANCE	M.S.E.	DISTRIBUTION MEAN
-----------------------	2.0	1.4 (2)	0.01 (0)

MSE- DIRECTION= 2 SEC

MSE= DISTANCE = 1 MM

LIST OF OBSERVATIONS- DIRNS THEN DISTS

FROM	TO	CODE	DIRN(D.M.S)/DIST(M)		
1	2	1	89	45	41.0
1	3	1	89	32	38.3
1	4	1	133	35	45.8
1	5	1	161	57	41.2
1	10	1	155	23	12.4
2	1	1	269	45	40.2
2	3	1	89	20	49.3
2	4	1	177	25	17.6
2	5	1	178	58	21.8
2	9	1	98	2	55.2
2	10	1	179	14	17.3
2	11	1	119	9	43.8
3	1	1	269	32	40.3
3	2	1	269	20	53.1
3	4	1	226	12	6.5
3	6	1	182	55	16.4
3	8	1	154	15	33.7
3	9	1	104	52	55.1
3	10	1	205	17	10.8
3	11	1	129	51	36.9
4	1	1	313	35	46.0
4	2	1	357	25	18.3
4	3	1	46	12	3.2
4	5	1	179	39	39.2
4	6	1	158	29	49.9
4	7	1	144	8	13.4
4	9	1	74	49	49.6
4	10	1	180	41	14.8

C.2 Adjustment (Minimum Constraints)

Requirements:

1. Provisional co-ordinates in a data file with the first variable being N the number of points; then the list in the following format:

   Y , X , "NAME".

2. Observations on a data file in the following format:

   Point From, Point To, Code, Degrees (Distance), Minutes (0), Seconds (0).

   where:

   Point From & Point To = Point numbers corresponding to the Point order in 1.

   Code 1 = Directions

   2 = Distance

   0 = Last line of Data (i.e. dummy line).

3. Knowledge of a priori values for standard deviations, of unit weight, direction and distance units, none, seconds, mm respectively.

Sample printout on page C-12.

```
1 RUN 100
80 REM PROVISIONAL COORDS IN DATA FILE     1ST N THEN Y,X,"NAME"
81 REM
82 REM DATA FILE FOR OBSERVATIONS AS FOLLOWS
83 REM
84 REM FROM(POINT NO), TO(POINT NO), CODE, DEGREES(DISTANCE),
85 REM MINUTES(0), SECONDS(0)
86 REM
87 REM CODE=        1  DIRECTIONS
88 REM              2  DISTANCES
89 REM              0  LAST LINE OF DATA
90 REM
91 REM LAST LINE OF OBSERVATIONS TO BE A DUMMY WITH CODE = 0
92 REM
93 REM THIS PROGRAM IS TIME SAVING BUT SPACE COSTLY
100 INIT
110 PRINT @32,26:2
120 SET DEGREES
130 PRINT "ENTER NAME OF FILE WITH PROV COORDS"
140 INPUT C$
150 OPEN C$;2,"R",A$
160 READ #2:N1
170 DIM X(2*N1),N3(2*N1),N4(2*N1)
180 FOR I=1 TO N1
190 READ #2:X(2*I),X(2*I-1),D$
200 NEXT I
210 PRINT "ENTER NAME OF FILE WITH OBS"
220 INPUT C$
230 PRINT "ENTER NUMBER OF COORDS HELD FIXED"
240 INPUT N2
250 DIM N5(N2)
260 FOR I=1 TO N2
270 PRINT "ENTER NO OF POINT HELD FIXED + CODE 0=Y, 1=X"
280 INPUT N6,N
290 N5(I)=N6*2-N
300 NEXT I
310 FOR I=2 TO N2
320 FOR K=I TO 2 STEP -1
330 IF N5(K)>N5(K-1) THEN 380
340 B1=N5(K-1)
350 N5(K-1)=N5(K)
360 N5(K)=B1
370 NEXT K
380 NEXT I
390 N3=0
400 N4=0
410 N7=2*N1-N2
420 N=1
430 REM****** DETERMINATION OF POINTS IN A VECTOR  *******************
440 FOR I=1 TO 2*N1
450 IF N>N2 THEN 470
460 IF I=N5(N) THEN 500
470 N4(I)=I-N+1
480 N3(I-N+1)=I
490 GO TO 530
500 N4(I)=N7+N
510 N3(N7+N)=I
520 N=N+1
530 NEXT I
540 U=2*N1
550 DIM A2(4),A3(U),L(N7),X1(N7),X2(U),A4(4),B(U,U),M(U),A(N7,N7)
560 PRINT "ENTER VALUE FOR SIGMA 0"
570 INPUT P1
```

```
580 PRINT "ENTER VALUE FOR SIGMA T (SECS)"
590 INPUT P2
600 PRINT "ENTER VALUE FOR SIGMA D (MM)"
610 INPUT P3
620 OPEN C$; 1, "R", A$
630 B=0
640 Z1=0
650 Q=1
660 M=0
670 G=0
680 A3=0
690 L3=0
700 N8=0
710 REM ***************   ELIMINATION OF DZ'S   ***********************
720 READ #1: F, T, C, D, W, S
730 IF C=2 AND N8<=0 THEN 920
740 IF F=G AND C=1 THEN 920
750 G=F
760 IF N8=0 THEN 910
770 P=-1/N8
780 Z1=Z1+1
790 N8=0
800 FOR I=1 TO U
810 GO TO Q OF 820, 860
820 FOR J=I TO U
830 B(I, J)=B(I, J)+A3(I)*A3(J)*P
840 B(J, I)=B(I, J)
850 NEXT J
860 M(I)=M(I)+A3(I)*L3*P
870 NEXT I
880 A3=0
890 L3=0
900 REM ***************   FORMATION OF MATRICES ATPA AND ATPL  *********
910 IF C=0 THEN 1050
920 GOSUB C OF 2580, 2770
930 GO TO Q OF 960, 940
940 PRINT @32: USING 950: F, T, L2
950 IMAGE2(8D)8D 4D
960 FOR I=1 TO 4
970 GO TO Q OF 980, 1020
980 FOR J=I TO 4
990 B(A4(I), A4(J))=B(A4(I), A4(J))+A2(I)*A2(J)*P
1000 B(A4(J), A4(I))=B(A4(I), A4(J))
1010 NEXT J
1020 M(A4(I))=M(A4(I))+A2(I)*L2*P
1030 NEXT I
1040 GO TO 720
1050 REM********   REDUCTION OF MATRIX ATPA (IN*A)   ******************
1060 FOR I=1 TO N7
1070 GO TO Q OF 1080, 1120
1080 FOR J=I TO N7
1090 A(I, J)=B(N3(I), N3(J))
1100 A(J, I)=A(I, J)
1110 NEXT J
1120 L(I)=M(N3(I))
1130 NEXT I
1140 GO TO Q OF 1150, 1170
1150 A=INV(A)
1160 CLOSE 4
1170 X1=A MPY L
1180 REM********   EXPANSION OF Q MATRIX (IN B)   **********************
1190 B=0
1200 M=0
1210 FOR I=1 TO U
1220 IF N4(I)>N7 THEN 1290
1230 FOR J=I TO U
```

```
1240 IF N4(J)>N7 THEN 1270
1250 B(I,J)=A(N4(I),N4(J))
1260 B(J,I)=B(I,J)
1270 NEXT J
1280 M(I)=X1(N4(I))
1290 NEXT I
1300 GO TO 1420
1310 REM  *************   CHECK FOR ITERATION  ************************
1320 Q=1
1330 FOR I=1 TO N7
1340 IF ABS(X1(I))>5.0E-4 THEN 1370
1350 NEXT I
1360 GO TO 1420
1370 PRINT @32: USING 1380:"DIFF DX >0.5MM THEREFORE ITERATING"
1380 IMAGEP,FA/L"LIST OF FREE TERMS"2(/L)
1390 PRINT @32: USING 1400:"FROM","TO","L"
1400 IMAGE4XFA6XFA7XFA
1405 X=X+M
1410 Q=2
1420 CALL "REWIND",1
1430 GO TO Q OF 1440,660
1440 G=0
1450 DIM Z(Z1)
1460 Z=0
1470 Q=1
1480 N8=0
1490 REM *************  DETERMINATION OF DZ'S (IN Z)  *****************
1500 Z1=0
1510 READ #1:F,T,C,D,W,S
1520 IF C=2 AND N8<=0 THEN 1510
1530 IF F=G AND C=1 THEN 1660
1540 IF G=0 OR N8<=0 THEN 1600
1550 FOR I=1 TO U
1560 V1=V1+A3(I)*M(I)
1570 NEXT I
1580 Z1=Z1+1
1590 Z(Z1)=(V1-L3)/N8
1600 IF C=0 THEN 1680
1610 V1=0
1620 L3=0
1630 G=F
1640 N8=0
1650 A3=0
1660 GOSUB C OF 2580,2770
1670 GO TO 1510
1680 CALL "REWIND",1
1690 REM *****  DETERMINATION OF V'S , PVV, AND GLOBAL CHECK **********
1700 PRINT @3: USING 1710:"FROM","TO","DZ","V","OBS + CORRN","JOIN"
1710 IMAGEFA4XFA11XFA13XFA11XFA16XFA16X"DIFF"/L
1720 Q=3
1730 V2=0
1740 V3=0
1750 G=0
1760 Z1=0
1770 A3=0
1780 READ #1:F,T,C,D,W,S
1790 IF C=0 THEN 2050
1800 V3=V3+1
1810 GOSUB C OF 2580,2770
1820 V=M(A4(1))*A2(1)+M(A4(2))*A2(2)+M(A4(3))*A2(3)+M(A4(4))*A2(4)-L2
1822 Y0=X(2*T)+M(2*T)-X(2*F)-M(2*F)
1824 X0=X(2*T-1)+M(2*T-1)-X(2*F-1)-M(2*F-1)
1826 GOSUB 2950
1830 IF C=2 THEN 1990
1840 IF F=G THEN 1870
1850 Z1=Z1+1
```

```
1860 G=F
1870 D5=D+W/60+S/3600+V/3600
1880 V=V-Z(Z1)
1890 D1=INT(D5)
1900 T2=INT(T1)
1910 D2=INT((D5-D1)*60)
1920 T3=INT((T1-T2)*60)
1930 D3=((D5-D1)*60-D2)*60
1940 T4=((T1-T2)*60-T3)*60
1950 D4=(T1-D5)*3600
1960 PRINT @3: USING 1970: F, T, Z(Z1), V, D1, D2, D3, T2, T3, T4, D4, "SECS"
1970 IMAGE4D6D2(12D. D)2(13D4D4D. D)12D. 1D2XFA
1980 GO TO 2030
1990 D1=D+V
2000 D4=(S1-D1)*1000
2010 PRINT @3: USING 2020: F, T, V*1000, D1, S1, D4, "MM"
2020 IMAGE4D6D14X11D. 2D2(18D. 4D)10X2D. 2D2XFA
2030 V2=V2+V^2*P
2040 GO TO 1780
2050 V9=V3-N7-Z1
2060 V3=V2/V9
2070 REM *************   OUTPUT OF COORDS AND MSE'S   *****************
2080 PRINT @3: USING 2090: "SIGMA 0 APOSTERIORI =", SQR(V3), P1
2090 IMAGE2(/L)FA5D. 2D/L"SIGMA 0 APRIORI      ="5D. 2D/L
2100 PRINT @3: USING 2110: "DEGREES OF FREEDOM =", V9
2110 IMAGEFA5D
2115 X=X+M
2120 PRI @3: USI 2130: "NAME", "POINT", "YO", "XO", "Y", "MY(MM)", "X", "MX(MM)"
2130 IMAGEPXFA10XFA10XFA14XFA15XFA6XFA11XFA6XFA/L
2140 N=1
2150 DIM Z$(12*N1), Z2(N1)
2160 Z$=""
2170 CALL "REWIND", 2
2180 READ #2: N1
2190 FOR I=1 TO N1
2200 Q9=2*I
2210 Q1=SQR(V3*B(Q9, Q9))*1000
2220 Q9=Q9-1
2230 Q2=SQR(V3*B(Q9, Q9))*1000
2240 READ #2: Y, H, C$
2250 Z$=Z$&C$
2260 Z2(I)=LEN(C$)
2270 PRINT @3: USING 2280: C$, I, Y, H, X(Q9+1), Q1, X(Q9), Q2
2280 IMAGEX12A7D2(11D. 4D)2(11D. 4D6D. D)/L
2290 NEXT I
2300 REM *************   DETERMINATION OF ELLIPSE PARAMETERS **********
2310 PRINT @3: USING 2320: "ABSOLUTE ERROR ELLIPSE PARAMETERS"
2320 IMAGE PFA
2330 PRI @3: USI 2340: "POINT", "SEMI-MAJOR AXIS(MM)", "SEMI-MINOR AXIS(MM"
2340 IMAGE2(/L)FA10XFA6XFA")"6X"ORIENTATION"
2350 CALL "REWIND", 2
2360 READ #2: N1
2370 FOR I=1 TO N1
2380 D2=B(2*I, 2*I)
2390 D1=B(2*I-1, 2*I-1)
2400 D3=B(2*I-1, 2*I)
2410 D=(D2+D1)/2
2420 E1=D1-D2
2430 E=SQR(E1^2/4+D3^2)
2440 A1=SQR(V3*(D+E))*1000
2450 B1=SQR(V3*(D-E))*1000
2460 T1=(180-ATN(E1/(2*D3+1. 0E-90))-90*SGN(D3+1. 0E-90))/2
2470 READ #2: Y, H, C$
2480 PRINT @3: USING 2490: C$, A1, B1, T1
2490 IMAGE/L10A7X7D. D16X7D. D15X5D. D
2500 NEXT I
```

```
2510 CLOSE
2520 PRINT "ENTER FILE NAME FOR FINAL CO-ORDS"
2530 INPUT C$
2540 OPEN C$;3,"F",A$
2550 WRITE #3:N1,V2,V9,X,Z$,Z2,B
2560 CLOSE
2570 END
2580 REM *************    SUB DIRECTIONS    ****************************
2590 A2=0
2600 GOSUB 2880
2610 R0=180*3600/PI
2620 A2(1)=R0*Y0/S1^2
2630 A2(2)=-R0*X0/S1^2
2640 A2(3)=-A2(1)
2650 A2(4)=-A2(2)
2660 GO TO Q OF 2670,2670,2720
2670 A3(A4(1))=A3(A4(1))+A2(1)
2680 A3(A4(2))=A3(A4(2))+A2(2)
2690 A3(A4(3))=A3(A4(3))+A2(3)
2700 A3(A4(4))=A3(A4(4))+A2(4)
2710 N8=N8+1
2720 P=P1^2/P2^2
2730 L2=D+W/60+S/3600-T1
2740 L2=(L2+360*(L2<-90)-360*(L2>90))*3600
2750 L3=L3+L2
2760 RETURN
2770 REM *************    SUB DISTANCES    ****************************
2780 A2=0
2790 GOSUB 2880
2800 A2(1)=-X0/S1
2810 A2(2)=-Y0/S1
2820 A2(3)=-A2(1)
2830 A2(4)=-A2(2)
2840 P=P1^2/(P3/1000)^2
2850 N8=-1
2860 L2=D-S1
2870 RETURN
2880 REM *************    SUB JOIN         ****************************
2890 A4(1)=2*F-1
2900 A4(3)=2*T-1
2910 A4(2)=2*F
2920 A4(4)=2*T
2930 Y0=X(2*T)-X(2*F)
2940 X0=X(2*T-1)-X(2*F-1)
2950 T1=180-ATN(X0/(Y0+1.0E-50))-90*SGN(Y0+1.0E-50)
2960 S1=SQR(Y0^2+X0^2)
2970 RETURN
```

NAME	POINT	YO	XO	Y	MY(MM)	X	MX(MM)
P1	1	691.0000	5125.5000	691.0000	0.0	5125.5000	0.0
P2	2	810.9999	5125.9999	811.0002	0.7	5126.0000	0.5
P3	3	943.0002	5127.4998	942.9998	1.4	5127.5030	0.6
P4	4	816.4010	5006.0999	816.3993	0.8	5006.1014	0.7
P12	5	818.0006	4735.5505	817.9987	1.2	4735.5498	0.8
P13	6	923.0011	4735.5099	923.0009	1.6	4735.5100	0.7
P14	7	1012.0011	4735.5191	1012.0007	1.4	4735.5199	0.7
P15	8	1132.0005	4735.4994	1131.9985	1.2	4735.4996	0.7
P16	9	1105.2007	5084.4000	1105.2002	0.9	5084.4000	0.0
P17	10	814.6009	4855.7007	814.5960	1.0	4855.6985	1.3
CD2(P20)	11	1214.4009	4900.8976	1214.4017	1.2	4900.8956	1.0

ABSOLUTE ERROR ELLIPSE PARAMETERS

POINT	SEMI-MAJOR AXIS(MM)	SEMI-MINOR AXIS(MM)	ORIENTATION
P1	0.0	0.0	45.0
P2	0.7	0.5	93.2
P3	1.4	0.6	92.0
P4	0.9	0.6	122.8
P12	1.3	0.7	105.9
P13	1.6	0.7	95.8
P14	1.4	0.7	87.8
P15	1.3	0.6	75.4
P16	0.9	0.0	90.0
P17	1.4	0.9	151.1
CD2(P20)	1.2	1.0	92.2

C.3 Invariant Function Comparisons

Requirements:

1. Stored on data file from each epoch.

N = number of points

[FVV]

f = degrees of freedom

$\underline{x}$ = vector of final co-ordinates X(2*N)

ZØ = string variable of names of points ZØ(12*N)

Z2 = vector of the lengths of the names Z2(N)

$\underline{Q}$ = matrix $(A^TPA)^{-1}$ Q(2*N, 2*N)

The order of points in $\underline{x}$ must be identical for both epochs. The programme is designed for networks with identical observation geometry. To make the programme applicable to networks with differing geometries, the following changes are necessary:

Addition of lines 265 DIM Q2 (2*N2, 2*N2)

293 READ 2 : Q2

Change line 295 to 295 Q = Q + Q2.

Sample printouts in main body at page 63.

```
4 RUN 100
100 INIT
110 SET DEGREES
120 REM *********   COMPARISON OF INVARIANT FUNCTIONS  ****************
130 REM
140 REM SAVED FROM ADJUSTMENT N, PVV, DF, MAT X, Z$(NAMES), Z2(LEN NAME)
150 REM MAT Q
160 REM
170 PRINT "INPUT NAME OF FILE WITH DATA FOR EPOCH 1"
180 INPUT T$
190 OPEN T$;1,"R",A$
200 PRINT "ENTER NAME OF FILE WITH DATA FOR EPOCH 2"
210 INPUT T$
220 OPEN T$;2,"R",A$
230 READ #1:N1
240 READ #2:N2
250 DIM X1(2*N1),X2(2*N2),Q(2*N1,2*N1),F1(1,6),Q1(6,6),F2(6),A(1)
260 DIM G1(1,6),B$(12*N1),C$(12*N2),B(N1),C(N2),B1(N1),K(6),Z(N1)
270 READ #1:P1,D1,X1,B$,B
280 READ #2:P2,D2,X2,C$,C
290 READ #1:Q
295 Q=2*Q
300 CLOSE
310 Z=0
320 N=0
330 R1=1
340 B1=0
350 FOR I=1 TO N1
360 H$=SEG(B$,R1,B(I))
370 R2=1
380 FOR J=1 TO N2
390 G$=SEG(C$,R2,C(J))
400 IF H$=G$ THEN 440
410 R2=R2+C(J)
420 NEXT J
430 GO TO 450
440 B1(I)=J
450 R1=R1+B(I)
460 NEXT I
470 P=(P1+P2)/(D1+D2)
480 REM *****************     P= SIGMA 0 ^ 2    ************************
490 PRI @3: USI 500:"LIST OF ANGLES AND DISTANCES FAILING AT 90%, 95% "
500 IMAGEFA"AND 99% CONFIDENCE LEVELS"
510 PRI @3: USI 520:"FROM","TO","TO","SIGMA F","DIFF","90%","95%","99%"
520 IMAGE/L3(12A3X)3XFA6XFA7X3(FA6X)/L
530 REM*************  DETERMINATION OF ANGLES AND DIFFS  **************
540 R1=1
550 FOR I=1 TO N1
560 IF B1(I)=0 THEN 970
570 K1=2*B1(I)
580 K(1)=2*I-1
590 K(2)=2*I
600 G$=SEG(B$,R1,B(I))
610 R2=1
620 FOR J=1 TO N1-1
630 IF I=J OR B1(J)=0 THEN 950
640 H$=SEG(B$,R2,B(J))
650 U=1
660 K(3)=2*J-1
670 K(4)=2*J
680 R=3
690 GOSUB 1550
700 T1=T
```

```
710 L1=2*B1(J)
720 GOSUB 1760
730 T2=T1-T
740 L1=2*B1(J+1)
750 L6=J+1
760 R3=R2+B(J)
770 IF L6<>I AND B1(J+1)<>0 THEN 830
780 L6=J+2
790 IF L6>N1 THEN 970
800 R3=R3+B(J+1)
810 IF B1(L6)=0 THEN 950
820 L1=2*B1(L6)
830 J$=SEG(B$,R3,B(L6))
840 U=2
850 K(5)=2*L6-1
860 K(6)=2*L6
870 R=5
880 GOSUB 1570
890 T2=T2-T
900 GOSUB 1760
910 T2=(T2+T+180*(T2+T<-1)-180*(T2+T>1))*3600
920 T3=T2
930 V=6
940 GOSUB 1870
950 R2=R2+B(J)
960 NEXT J
970 R1=R1+B(I)
980 NEXT I
990 REM**********    DETERMINATION OF DISTANCE DIFFS    ***************
1000 J$=" "
1010 V=4
1020 R1=1
1030 FOR I=1 TO N1-1
1040 K=0
1050 IF B1(I)=0 THEN 1260
1060 K(1)=2*I-1
1070 K(2)=2*I
1080 K1=2*B1(I)
1090 G$=SEG(B$,R1,B(I))
1100 R2=R1+B(I)
1110 FOR J=I+1 TO N1
1120 IF B1(J)=0 THEN 1240
1130 H$=SEG(B$,R2,B(J))
1140 K(3)=2*J-1
1150 K(4)=2*J
1160 R=3
1170 GOSUB 1690
1180 T1=S
1190 L1=2*B1(J)
1200 GOSUB 1760
1210 T2=T1-S
1220 T3=T2*1000
1230 GOSUB 1870
1240 R2=R2+B(J)
1250 NEXT J
1260 R1=R1+B(I)
1270 NEXT I
1280 PRINT @3: USING 1290:
1290 IMAGE2(/L)3X
1300 R1=1
1310 J=1
1320 FOR I=1 TO N1
1330 IF I>10 THEN 1380
```

```
1340 M$=SEG(B$,R1,B(I))
1350 PRINT @3: USING 1360:M$
1360 IMAGE12A,S
1370 GO TO 1450
1380 PRINT @3:
1390 FOR J=I-10 TO I-1
1400 PRINT @3: USING 1410:Z(J)
1410 IMAGE4D8X,S
1420 NEXT J
1430 PRINT @3: USING 1290:
1440 GO TO 1340
1450 R1=R1+B(I)
1460 NEXT I
1470 PRINT @3:
1480 FOR I=J TO N1
1490 PRINT @3: USING 1410:Z(I)
1500 NEXT I
1510 PRINT @3:
1520 END
1530 REM **************    SUBROUTINES    *****************************
1540 REM
1550 REM **************   SUB DIRECTIONS   *****************************
1560 F2=0
1570 GOSUB 1810
1580 R0=180*3600/PI
1590 F2(1)=R0*Y/S^2-F2(1)
1600 F2(2)=-R0*X/S^2-F2(2)
1610 GO TO U OF 1620,1650
1620 F2(3)=-R0*Y/S^2
1630 F2(4)=R0*X/S^2
1640 RETURN
1650 F2(5)=R0*Y/S^2
1660 F2(6)=-R0*X/S^2
1670 RETURN
1680 REM**************   SUB  DISTANCES    ****************************
1690 F2=0
1700 GOSUB 1810
1710 F2(1)=-X/S
1720 F2(2)=-Y/S
1730 F2(3)=-F2(1)
1740 F2(4)=-F2(2)
1750 RETURN
1760 REM *************   SUB JOIN 2   *********************************
1770 Y=X2(L1)-X2(K1)
1780 X=X2(L1-1)-X2(K1-1)
1790 GO TO 1830
1800 REM***********    SUB  JOIN  1     ******************************
1810 Y=X1(K(R+1))-X1(K(2))
1820 X=X1(K(R))-X1(K(1))
1830 T=ATN(Y/(X+1.0E-50))
1840 S=SQR(Y^2+X^2)
1850 RETURN
1860 REM ********    SUB SIGMA F & COMPARISON    *********************
1870 Q1=0
1880 FOR O=1 TO V
1890 FOR M=O TO V
1900 Q1(O,M)=Q(K(O),K(M))
1910 Q1(M,O)=Q1(O,M)
1920 NEXT M
1930 NEXT O
1940 F1=TRN(F2)
1950 G1=F1 MPY Q1
1960 A=G1 MPY F2
```

```
1970 A1=SQR(P*A(1))
1980 A2=A1
1990 IF V=6 THEN 2010
2000 A2=A1*1000
2010 D$=" "
2020 E$=" "
2030 F$=" "
2040 T2=ABS(T2)
2050 IF 1.65*A1>T2 THEN 2170
2060 D$="*"
2070 Z(I)=Z(I)+1
2080 Z(J)=Z(J)+1
2090 IF V=4 THEN 2110
2100 Z(L6)=Z(L6)+1
2110 IF 1.96*A1>T2 THEN 2150
2120 E$="*"
2130 IF 2.58*A1>T2 THEN 2150
2140 F$="*"
2150 PRINT @3: USING 2160:G$,H$,J$,A2,T3,D$,E$,F$
2160 IMAGE3(12A3X)2(7D.2D)3(8XFA)
2170 RETURN
```

```
4 RUN 100
```

C.4 Combination of Epochs

Requirements:

1. Two data files with provisional co-ordinates with following format:

   N (Number of Points)

   Y , X , "NAME".

2. The two data files with observations from the two epochs.

3. Data file for combined co-ordinates.

4. Data file for combined observations.

5. Names of Doubtful points.

Sample printout at page C-22.

```
4 RUN 100
100 INIT
110 REM    COMBINING EPOCHS
120 PRINT "ENTER NAME OF FILE WITH COORDS FOR EPOCH 1"
130 INPUT C$
140 PRINT "ENTER NAME OF FILE WITH COORDS FOR EPOCH 2"
150 INPUT D$
160 PRINT "ENTER NAME OF FILE FOR COMBINED COORDS"
170 INPUT E$
180 PRINT @3:"FILE WITH COMBINED COORDS IS ";E$
190 PRINT @3:
200 OPEN C$;1,"R",A$
210 OPEN D$;2,"R",A$
220 OPEN E$;3,"F",A$
230 READ #1:N1
240 READ #2:N2
250 PRINT "ENTER NUMBER OF POINTS UNSTABLE"
260 INPUT N5
270 DIM X1(2,N1),R(N2),X2(2,N2),P(N5),F$(N1*10),G$(N2*10),P$(N5*10)
280 DIM Q(N2),S1(N2),K$(20),P1(N5),F1(N1),G1(N2),W(3,50)
290 F$=""
300 P$=""
310 G$=""
320 H$=""
330 FOR I=1 TO N5
340 PRINT "ENTER NAME OF UNSTABLE POINT ",I
350 INPUT K$
360 P1(I)=LEN(K$)
370 P$=P$&K$
380 NEXT I
390 FOR I=1 TO N1
400 READ #1:X1(1,I),X1(2,I),K$
410 F1(I)=LEN(K$)
420 F$=F$&K$
430 NEXT I
440 FOR I=1 TO N2
450 READ #2:X2(1,I),X2(2,I),L$
460 G1(I)=LEN(L$)
470 G$=G$&L$
480 NEXT I
490 R1=1
500 S$=""
510 N4=0
520 Q=0
530 FOR I=1 TO N2
540 R2=1
550 M$=SEG(G$,R1,G1(I))
560 FOR J=1 TO N1
570 N$=SEG(F$,R2,F1(J))
580 IF M$=N$ THEN 670
590 R2=R2+F1(J)
600 NEXT J
610 S$=S$&M$
620 N4=N4+1
630 S1(N4)=LEN(M$)
640 R(N4)=I
650 Q(I)=N1+N4
660 GO TO 680
670 Q(I)=J
680 R1=R1+G1(I)
690 NEXT I
700 IF N4=0 THEN 750
710 DIM C(N4)
```

```
720 FOR K=1 TO N4
730 C(K)=R(K)
740 NEXT K
750 P2=1
770 J$="-A"
780 FOR I=1 TO N5
790 M$=SEG(P$,P2,P1(I))
795 R2=1
800 FOR K=1 TO N2
810 N$=SEG(G$,R2,G1(K))
820 IF M$=N$ THEN 860
830 R2=R2+G1(K)
840 NEXT K
850 GO TO 920
860 H$=H$&M$
870 H$=H$&J$
880 P2=P2+P1(I)
890 P1(I)=P1(I)+2
900 P(I)=K
910 Q(K)=N1+N4+I
920 NEXT I
930 N=N1+N4+N5
940 WRITE #3:N
950 R1=1
960 FOR I=1 TO N1
970 L$=SEG(F$,R1,F1(I))
980 WRITE #3:X1(1,I),X1(2,I),L$
990 R1=R1+F1(I)
1000 PRINT @3: USING 1010:X1(1,I),X1(2,I),L$
1010 IMAGE2(9D.3D)6XFA
1020 NEXT I
1030 IF N4=0 THEN 1110
1040 R1=1
1050 FOR I=1 TO N4
1060 L$=SEG(S$,R1,S1(I))
1070 WRITE #3:X2(1,C(I)),X2(2,C(I)),L$
1080 R1=R1+S1(I)
1090 PRINT @3: USING 1010:X2(1,C(I)),X2(2,C(I)),L$
1100 NEXT I
1110 R1=1
1120 FOR I=1 TO N5
1130 L$=SEG(H$,R1,P1(I))
1140 WRITE #3:X2(1,P(I)),X2(2,P(I)),L$
1150 R1=R1+P1(I)
1160 PRINT @3: USING 1010:X2(1,P(I)),X2(2,P(I)),L$
1170 NEXT I
1180 CLOSE
1190 PRINT "ENTER NAME OF FILE WITH OBSERVATIONS - EPOCH 1"
1200 INPUT C$
1210 PRINT "ENTER NAME OF FILE WITH OBSERVATIONS - EPOCH 2"
1220 INPUT D$
1230 PRINT "ENTER NAME OF FILE FOR COMBINED OBSERVATIONS"
1240 INPUT E$
1250 PRINT @3:
1260 PRINT @3:"FILE WITH COMBINED OBSERVATIONS IS ";E$
1270 PRINT @3:
1280 OPEN C$;1,"R",A$
1290 OPEN D$;2,"R",A$
1300 OPEN E$;3,"F",A$
1303 V=1
1306 W=0
1310 READ #1:F,T,C1,D,M,S
1320 IF C1=0 THEN 1360
```

```
1321 IF C1<>2 THEN 1330
1322 W(1,V)=F
1323 W(2,V)=T
1324 W(3,V)=D
1325 V=V+1
1326 GO TO 1310
1330 WRITE #3:F,T,C1,D,M,S
1340 PRINT @3: USING 1400:F,T,C1,D,M,S
1350 GO TO 1310
1360 READ #2:F,T,C1,D,M,S
1370 IF C1=0 THEN 1411
1380 WRITE #3:Q(F),Q(T),C1,D,M,S
1390 PRINT @3: USING 1400:Q(F),Q(T),C1,D,M,S
1400 IMAGE3(9D)9D.3D2(7D)
1410 GO TO 1360
1411 C2=2
1412 M=0
1413 S=0
1414 FOR I=1 TO 50
1415 IF W(1,I)=0 THEN 1420
1416 WRITE #3:W(1,I),W(2,I),C2,W(3,I),M,S
1417 PRINT @3: USING 1400:W(1,I),W(2,I),C2,W(3,I),M,S
1418 NEXT I
1420 WRITE #3:F,T,C1,D,M,S
1430 CLOSE
1440 END
```

FILE WITH COMBINED COORDS IS HTHR

690.999	5125.500	P1
810.999	5126.000	P2
942.999	5127.500	P3
816.400	5006.100	P4
818.000	4735.551	P12
923.001	4735.510	P13
1012.001	4735.519	P14
1132.000	4735.500	P15
1105.200	5084.400	P16
814.600	4855.701	P17
1214.400	4900.898	CD2(P20)
942.999	5127.500	P3-A
816.400	5006.100	P4-A
814.600	4855.701	P17-A
1105.200	5084.400	P16-A
1214.400	4900.898	CD2(P20)-A

FILE WITH COMBINED OBSERVATIONS IS HFOUR

1	2	1	89.000	45	40
1	3	1	89.000	32	44
1	4	1	133.000	35	48
1	5	1	161.000	57	34
1	10	1	155.000	23	13
2	1	1	269.000	45	42
2	3	1	89.000	20	54
2	4	1	177.000	25	17
2	5	1	178.000	58	23
2	9	1	98.000	2	54
2	10	1	179.000	14	13
2	11	1	119.000	9	43
3	1	1	269.000	32	43
3	2	1	269.000	20	56
3	4	1	226.000	12	4
3	6	1	182.000	55	15
3	8	1	154.000	15	34
3	9	1	104.000	52	50
3	10	1	205.000	17	14
3	11	1	129.000	51	34
4	1	1	313.000	35	44
4	2	1	357.000	25	11
4	3	1	46.000	12	3
4	5	1	179.000	39	42
4	6	1	158.000	29	50
4	7	1	144.000	8	15
4	9	1	74.000	49	48
4	10	1	180.000	41	6
5	1	1	341.000	57	35
5	2	1	358.000	58	21
5	4	1	359.000	39	42
5	6	1	90.000	1	20
5	7	1	90.000	0	37
5	8	1	90.000	0	36
5	10	1	358.000	22	48
6	4	1	338.000	29	51
6	5	1	270.000	1	18
6	7	1	89.000	59	36
6	8	1	90.000	0	12
6	10	1	317.000	57	6
6	11	1	60.000	25	20
7	4	1	324.000	8	13

etc.

C.5 Helmert Transformation without Scale Factor

Requirements:

1. Data File with co-ordinates of epoch 1 (New System) in format:

   N (Number of Points)

   Y , X , "NAME".

2. Data file with co-ordinates of epoch 2 (Old System) format as for 1.

3. Number and order of co-ordinates of 1. & 2. need not be equal.

4. Any common points from 1. which are not to be used for the transformation (point numbers in order of co-ordinates for 1.).

Sample printout at page C-28.

C-24

```
FE4 RUN 100
100 INIT
110 PRI @3: USI 115:"HELMERT PLAIN TRANSFORMATION WITHOUT SCALE FACTOR"
115 IMAGEFA/49"-"2(/L)
120 PRINT "ENTER NAME FILE DATA EPOCH 1 (NEW)"
130 INPUT C$
140 OPEN C$;1,"R",A$
150 PRINT "ENTER NAME FILE DATA EPOCH 2 (OLD)"
160 INPUT C$
170 OPEN C$;2,"R",A$
180 READ #1:N,C,E
190 READ #2:M,C,E
200 U=2*N
210 V=2*M
220 DIM X1(U),X2(V),S$(12*N),T$(12*M),S(N),T(M),Y1(2,N),Y2(N,2),P(2,N)
225 DIM K(2,2),D(2,2)
230 READ #1:X1,S$,S
240 READ #2:X2,T$,T
242 CLOSE
244 P=1
246 R1=1
248 FOR I=1 TO N
250 W$=SEG(S$,R1,S(I))
252 PRINT @32: USING 253:I,W$
253 IMAGE4D4XFA
254 R1=R1+S(I)
256 NEXT I
258 PRINT "INPUT NO OF POINTS TO BE ELIMINATED"
260 INPUT R2
262 FOR J=1 TO R2
264 PRINT "POINT NO TO BE ELIMINATED"
266 INPUT I
268 P(1,I)=0
270 P(2,I)=0
272 NEXT J
274 K=0
280 L=0
290 R1=1
300 FOR I=1 TO N
305 IF P(1,I)=0 THEN 460
310 W$=SEG(S$,R1,S(I))
320 R2=1
330 FOR J=1 TO M
340 Z$=SEG(T$,R2,T(J))
350 IF W$=Z$ THEN 390
360 R2=R2+T(J)
370 NEXT J
380 GO TO 470
390 P(1,I)=I
400 P(2,I)=J
410 K(1,1)=K(1,1)+X1(2*I-1)
420 K(1,2)=K(1,2)+X1(2*I)
430 K(2,1)=K(2,1)+X2(2*J-1)
440 K(2,2)=K(2,2)+X2(2*J)
450 L=L+1
460 R1=R1+S(I)
470 NEXT I
480 L=1/L
490 K=L*K
500 C=0
510 Y1=0
520 Y2=0
530 FOR I=1 TO N
```

```
540 IF P(1,I)=0 THEN 600
550 Y1(1,I)=X1(2*P(1,I)-1)-K(1,1)
560 Y1(2,I)=X1(2*P(1,I))-K(1,2)
570 Y2(I,1)=X2(2*P(2,I)-1)-K(2,1)
580 Y2(I,2)=X2(2*P(2,I))-K(2,2)
590 C=C+Y2(I,1)^2+Y2(I,2)^2
600 NEXT I
610 D=Y1 MPY Y2
620 A1=(D(2,2)+D(1,1))/C
630 B1=(D(2,1)-D(1,2))/C
640 A2=ATN(B1/A1)
650 A1=COS(A2)
660 B1=SIN(A2)
670 A0=-A1*K(2,1)+B1*K(2,2)+K(1,1)
680 B0=-A1*K(2,2)-B1*K(2,1)+K(1,2)
690 R1=1
700 PRINT @3: USING 710:A0,B0,A1,B1
710 IMAGE"A0="6D.5D4X"B0="6D.5D4X"A1="2D.8D4X"B1="2D.8D2(/L)
720 PRINT @3: USING 730:"NAME","OLD","NEW","CAL NEW","VY","VX"
730 IMAGE10A  14X3A27X3A25X7A17X2A9X2A/L
740 V=0
750 FOR I=1 TO N
760 W$=SEG(S$,R1,S(I))
770 R1=R1+S(I)
780 IF P(1,I)=0 THEN 900
790 Y3=X1(2*P(1,I))
800 X3=X1(2*P(1,I)-1)
810 Y4=X2(2*P(2,I))
820 X4=X2(2*P(2,I)-1)
830 Y=B0+A1*Y4+B1*X4
840 X=A0+A1*X4-B1*Y4
850 V1=Y-Y3
860 V2=X-X3
870 PRINT @3: USING 880:W$,Y4,X4,Y3,X3,Y,X,V1,V2
880 IMAGE10A3(8 D.4D 8D.4D4X)2(5D.5D)
890 V=V+V1^2+V2^2
900 NEXT I
910 L=1/L
920 V3=SQR(V)/(2*L-4)
930 PRINT @3: USING 940:"PVV=",V,"SIGMA0=",V3
940 IMAGE2(/L)8A4D.5DL/8A4D.5D/L
950 IF M=L THEN 1180
960 PRI @3: USI 970:"TRANSFORMATION OF REMAINING POINTS ONTO NEW SYSTEM"
970 IMAGE3(/L)FA2(/L)"NAME"25X"OLD"27X"NEW"16X,"DY",9X,"DX"/L
980 DIM Q(M-L),Q$((M-L)*12),T1(M),Q1(M-L)
990 T1=0
1000 FOR J=1 TO M
1010 FOR I=1 TO N
1020 IF P(2,I)=J THEN 1050
1030 NEXT I
1040 T1(J)=1
1050 NEXT J
1060 R1=1
1070 FOR I=1 TO M
1080 W$=SEG(T$,R1,T(I))
1090 R1=R1+T(I)
1100 IF T1(I)=0 THEN 1170
1110 Y3=X2(2*I)
1120 X3=X2(2*I-1)
1130 Y=B0+A1*Y3+B1*X3
1140 X=A0+A1*X3-B1*Y3
1150 PRINT @3: USING 1160:W$,Y3,X3,Y,X,Y-X1(2*I),X-X1(2*I-1)
1160 IMAGE12A2(  10D.4D10D.4D)2(6D.4D)
```

```
FE4 RUN 100
100 INIT
110 PRI @3: USI 115:"HELMERT-PLAIN TRANSFORMATION WITHOUT SCALE FACTOR"
115 IMAGEFA/49"-"2(/L)
120 PRINT "ENTER NAME FILE DATA EPOCH 1 (NEW)"
130 INPUT C$
140 OPEN C$;1,"R",A$
150 PRINT "ENTER NAME FILE DATA EPOCH 2 (OLD)"
160 INPUT C$
170 OPEN C$;2,"R",A$
180 READ #1:N,C,E
190 READ #2:M,C,E
200 U=2*N
210 V=2*M
220 DIM X1(U),X2(V),S$(12*N),T$(12*M),S(N),T(M),Y1(2,N),Y2(N,2),P(2,N)
225 DIM K(2,2),D(2,2)
230 READ #1:X1,S$,S
240 READ #2:X2,T$,T
242 CLOSE
244 P=1
246 R1=1
248 FOR I=1 TO N
250 W$=SEG(S$,R1,S(I))
252 PRINT @32: USING 253:I,W$
253 IMAGE4D4XFA
254 R1=R1+S(I)
256 NEXT I
258 PRINT "INPUT NO OF POINTS TO BE ELIMINATED"
260 INPUT R2
262 FOR J=1 TO R2
264 PRINT "POINT NO TO BE ELIMINATED"
266 INPUT I
268 P(1,I)=0
270 P(2,I)=0
272 NEXT J
274 K=0
280 L=0
290 R1=1
300 FOR I=1 TO N
305 IF P(1,I)=0 THEN 460
310 W$=SEG(S$,R1,S(I))
320 R2=1
330 FOR J=1 TO M
340 Z$=SEG(T$,R2,T(J))
350 IF W$=Z$ THEN 390
360 R2=R2+T(J)
370 NEXT J
380 GO TO 470
390 P(1,I)=I
400 P(2,I)=J
410 K(1,1)=K(1,1)+X1(2*I-1)
420 K(1,2)=K(1,2)+X1(2*I)
430 K(2,1)=K(2,1)+X2(2*J-1)
440 K(2,2)=K(2,2)+X2(2*J)
450 L=L+1
460 R1=R1+S(I)
470 NEXT I
480 L=1/L
490 K=L*K
500 C=0
510 Y1=0
520 Y2=0
530 FOR I=1 TO N
```

```
540 IF P(1,I)=0 THEN 600
550 Y1(1,I)=X1(2*P(1,I)-1)-K(1,1)
560 Y1(2,I)=X1(2*P(1,I))-K(1,2)
570 Y2(I,1)=X2(2*P(2,I)-1)-K(2,1)
580 Y2(I,2)=X2(2*P(2,I))-K(2,2)
590 C=C+Y2(I,1)^2+Y2(I,2)^2
600 NEXT I
610 D=Y1 MPY Y2
620 A1=(D(2,2)+D(1,1))/C
630 B1=(D(2,1)-D(1,2))/C
640 A2=ATN(B1/A1)
650 A1=COS(A2)
660 B1=SIN(A2)
670 A0=-A1*K(2,1)+B1*K(2,2)+K(1,1)
680 B0=-A1*K(2,2)-B1*K(2,1)+K(1,2)
690 R1=1
700 PRINT @3: USING 710:A0,B0,A1,B1
710 IMAGE"A0="6D.5D4X"B0="6D.5D4X"A1="2D.8D4X"B1="2D.8D2(/L)
720 PRINT @3: USING 730:"NAME","OLD","NEW","CAL NEW","VY","VX"
730 IMAGE10A  14X3A27X3A25X7A17X2A9X2A/L
740 V=0
750 FOR I=1 TO N
760 W$=SEG(S$,R1,S(I))
770 R1=R1+S(I)
780 IF P(1,I)=0 THEN 900
790 Y3=X1(2*P(1,I))
800 X3=X1(2*P(1,I)-1)
810 Y4=X2(2*P(2,I))
820 X4=X2(2*P(2,I)-1)
830 Y=B0+A1*Y4+B1*X4
840 X=A0+A1*X4-B1*Y4
850 V1=Y-Y3
860 V2=X-X3
870 PRINT @3: USING 880:W$,Y4,X4.Y3-X3,Y,X,V1,V2
880 IMAGE10A3(8 D.4D 8D.4D4X)2(5D.5D)
890 V=V+V1^2+V2^2
900 NEXT I
910 L=1/L
920 V3=SQR(V)/(2*L-4)
930 PRINT @3: USING 940:"PVV=",V."SIGMAD=",V3
940 IMAGE2(/L)8A4D.5DL/8A4D.5D/L
950 IF M=L THEN 1180
960 PRI @3: USI 970:"TRANSFORMATION OF REMAINING POINTS ONTO NEW SYSTEM"
970 IMAGE3(/L)FA2(/L)"NAME"25X"OLD"27X"NEW"16X,"DY",9X."DX"/L
980 DIM Q(M-L),Q$((M-L)*12),T1(M),Q1(M-L)
990 T1=0
1000 FOR J=1 TO M
1010 FOR I=1 TO N
1020 IF P(2,I)=J THEN 1050
1030 NEXT I
1040 T1(J)=1
1050 NEXT J
1060 R1=1
1070 FOR I=1 TO M
1080 W$=SEG(T$,R1,T(I))
1090 R1=R1+T(I)
1100 IF T1(I)=0 THEN 1170
1110 Y3=X2(2*I)
1120 X3=X2(2*I-1)
1130 Y=B0+A1*Y3+B1*X3
1140 X=A0+A1*X3-B1*Y3
1150 PRINT @3: USING 1160:W$,Y3,X3,Y,X.Y-X1(2*I).X-X1(2*I-1)
1160 IMAGE12A2(  10D.4D10D.4D)2(6D.4D)
1170 NEXT I
1180 END.
```

HELMERT PLAIN TRANSFORMATION WITHOUT SCALE FACTOR

A0= -0.00119 B0= 0.00552 A1= 1.00000000 B1=-0.00000103

NAME	OLD		NEW		CAL NEW		VY	VX
P1	691.0000	5125.5000	691.0000	5125.5000	691.0002	5125.4995	0.00024	-0.00048
P2	811.0002	5126.0000	810.9999	5125.9999	811.0005	5125.9997	0.00059	-0.00023
P13	923.0009	4735.5100	923.0011	4735.5099	923.0016	4735.5097	0.00048	-0.00018
P14	1012.0007	4735.5199	1012.0011	4735.5191	1012.0013	4735.5198	0.00021	0.00070
P15	1131.9985	4735.4996	1132.0005	4735.4994	1131.9992	4735.4996	-0.00128	0.00024
P16	1105.2002	5084.4000	1105.2007	5084.4000	1105.2004	5084.3999	-0.00024	-0.00005

PVV= 0.00000

SIGMAO= 0.00023

TRANSFORMATION OF REMAINING POINTS ONTO NEW SYSTEM

NAME	OLD		NEW		DY	DX
P3	942.9998	5127.5030	943.0001	5127.5028	-0.0001	0.0030
P4	816.3993	5006.1014	816.3997	5006.1010	-0.0013	0.0012
P12	817.9987	4735.5498	817.9993	4735.5494	-0.0013	-0.0011
P17	814.5960	4855.6985	814.5965	4855.6982	-0.0044	-0.0025
CD2(P20)	1214.4017	4900.8956	1214.4022	4900.8956	0.0013	-0.0020

C.6 Adjustment of Differences

Requirements:

1. Provisional co-ordinates as for programme C.2 (page C-6).

2. Observations for the two epochs (two sets identical) as for programme C.2 (page C-6).

3. A priori values for standard deviations of the observations for 1 epoch, i.e. σ_i and not $\sqrt{2}\,\sigma_i$.

Sample printout at pages C-35 & C-36.

```
80 REM PROVISIONAL COORDS IN DATA FILE    1ST N THEN Y,X,"NAME"
81 REM
82 REM DATA FILE FOR OBSERVATIONS AS FOLLOWS
83 REM
84 REM FROM(POINT NO), TO(POINT NO), CODE, DEGREES(DISTANCE),
85 REM MINUTES(0), SECONDS(0)
86 REM
87 REM CODE=        1  DIRECTIONS
88 REM              2  DISTANCES
89 REM              0  LAST LINE OF DATA
90 REM
91 REM LAST LINE OF OBSERVATIONS TO BE A DUMMY WITH CODE = 0
92 REM
93 REM THIS PROGRAM IS TIME SAVING BUT SPACE COSTLY
100 INIT
110 PRINT @32,26:2
120 SET DEGREES
130 PRINT "ENTER NAME OF FILE WITH PROV COORDS"
140 INPUT C$
150 OPEN C$;2,"R",A$
160 READ #2:N1
170 DIM X(2*N1),N3(2*N1),N4(2*N1)
180 FOR I=1 TO N1
190 READ #2:X(2*I),X(2*I-1),D$
200 NEXT I
210 PRINT "ENTER NAME OF FILE WITH OBS EPOCH1"
220 INPUT C$
225 PRINT "ENTER NAME OF FILE WITH OBS EPOCH2"
226 INPUT J$
230 PRINT "ENTER NUMBER OF COORDS HELD FIXED"
240 INPUT N2
250 DIM N5(N2)
260 FOR I=1 TO N2
270 PRINT "ENTER NO OF POINT HELD FIXED + CODE 0=Y, 1=X"
280 INPUT N6,N
290 N5(I)=N6*2-N
300 NEXT I
310 FOR I=2 TO N2
320 FOR K=I TO 2 STEP -1
330 IF N5(K)>N5(K-1) THEN 380
340 B1=N5(K-1)
350 N5(K-1)=N5(K)
360 N5(K)=B1
370 NEXT K
380 NEXT I
390 N3=0
400 N4=0
410 N7=2*N1-N2
420 N=1
430 REM****** DETERMINATION OF POINTS IN A VECTOR  *******************
440 FOR I=1 TO 2*N1
450 IF N>N2 THEN 470
460 IF I=N5(N) THEN 500
470 N4(I)=I-N+1
480 N3(I-N+1)=I
490 GO TO 530
500 N4(I)=N7+N
510 N3(N7+N)=I
520 N=N+1
530 NEXT I
540 U=2*N1
550 DIM A2(4),A3(U),L(N7),X1(N7),A4(4),B(U,U),M(U),A(N7,N7)
580 PRINT "ENTER VALUE FOR SIGMA T (SECS) 1 EPOCH ONLY"
```

```
590 INPUT P2
600 PRINT "ENTER VALUE FOR SIGMA D (MM) 1 EPOCH ONLY"
610 INPUT P3
620 OPEN C$;1,"R",A$
625 OPEN J$;3,"R",A$
630 B=0
640 Z1=0
650 Q=1
660 M=0
670 G=0
680 A3=0
690 L3=0
700 N8=0
710 REM ***************   ELIMINATION OF DZ'S   ***********************
720 READ #1:F,T,C,D,W,S
722 READ #3:F9,T9,C9,D9,W9,S9
724 IF F9=F AND C9=C AND T9=T THEN 736
726 PRINT "OBSERVATIONS DO NOT CORRESPOND!!!!!!!!!!!!!!!!!!!!!!!!!!!"
728 CLOSE
730 STOP
736 IF C=2 AND N8<=0 THEN 920
740 IF F=G AND C=1 THEN 920
750 G=F
760 IF N8=0 THEN 910
770 P=-1/N8
780 Z1=Z1+1
790 N8=0
800 FOR I=1 TO U
810 GO TO Q OF 820,860
820 FOR J=I TO U
830 B(I,J)=B(I,J)+A3(I)*A3(J)*P
840 B(J,I)=B(I,J)
850 NEXT J
860 M(I)=M(I)+A3(I)*L3*P
870 NEXT I
880 A3=0
890 L3=0
900 REM ***************   FORMATION OF MATRICES ATPA AND ATPL   *********
910 IF C=0 THEN 1050
920 GOSUB C OF 2580.2770
930 GO TO Q OF 960,940
940 PRINT @32: USING 950:F,T,L2
950 IMAGE2(8D)8D.4D
960 FOR I=1 TO 4
970 GO TO Q OF 980,1020
980 FOR J=I TO 4
990 B(A4(I),A4(J))=B(A4(I),A4(J))+A2(I)*A2(J)*P
1000 B(A4(J),A4(I))=B(A4(I),A4(J))
1010 NEXT J
1020 M(A4(I))=M(A4(I))+A2(I)*L2*P
1030 NEXT I
1040 GO TO 720
1050 REM********   REDUCTION OF MATRIX ATPA (IN*A)   ******************
1060 FOR I=1 TO N7
1070 GO TO Q OF 1080,1120
1080 FOR J=I TO N7
1090 A(I,J)=B(N3(I),N3(J))
1100 A(J,I)=A(I,J)
1110 NEXT J
1120 L(I)=M(N3(I))
1130 NEXT I
1140 GO TO Q OF 1150,1170
1150 A=INV(A)
```

```
1170 X1=A MPY L
1180 REM********   EXPANSION OF Q MATRIX (IN B)   **********************
1190 B=0
1200 M=0
1210 FOR I=1 TO U
1220 IF N4(I)>N7 THEN 1290
1230 FOR J=I TO U
1240 IF N4(J)>N7 THEN 1270
1250 B(I,J)=A(N4(I),N4(J))
1260 B(J,I)=B(I,J)
1270 NEXT J
1280 M(I)=X1(N4(I))
1290 NEXT I
1420 CALL "REWIND",1
1430 CALL "REWIND",3
1440 G=0
1450 DIM Z(Z1)
1460 Z=0
1470 Q=1
1480 N8=0
1490 REM *************  DETERMINATION OF DZ'S (IN Z)  *****************
1500 Z1=0
1510 READ #1:F,T,C,D,W,S
1515 READ #3:F9,T9,C9,D9,W9,S9
1520 IF C=2 AND N8<=0 THEN 1510
1530 IF F=G AND C=1 THEN 1660
1540 IF G=0 OR N8<=0 THEN 1600
1550 FOR I=1 TO U
1560 V1=V1+A3(I)*M(I)
1570 NEXT I
1580 Z1=Z1+1
1590 Z(Z1)=(V1-L3)/N8
1600 IF C=0 THEN 1680
1610 V1=0
1620 L3=0
1630 G=F
1640 N8=0
1650 A3=0
1660 GOSUB C OF 2580,2770
1670 GO TO 1510
1680 CALL "REWIND",1
1685 CALL "REWIND",3
1690 REM *****  DETERMINATION OF V'S , PVV, AND GLOBAL CHECK **********
1700 PRINT @3: USING 1710:"FROM","TO","DZ","DV","OBS + CORRN","JOIN"
1710 IMAGEFA4XFA11XFA12XFA7 XFA 7XFA11X"DIFF"/L
1720 Q=3
1730 V2=0
1740 V3=0
1750 G=0
1760 Z1=0
1770 A3=0
1780 READ #1:F,T,C,D,W,S
1785 READ #3:F9,T9,C9,D9,W9,S9
1790 IF C=0 THEN 2050
1800 V3=V3+1
1804 GOSUB C OF 2580,2770
1808 T2=T1
1810 S2=S1
1812 Y0=Y0-M(2*F)+M(2*T)
1814 X0=X0-M(2*F-1)+M(2*T-1)
1816 GOSUB 2950
1820 V=M(A4(1))*A2(1)+M(A4(2))*A2(2)+M(A4(3))*A2(3)+M(A4(4))*A2(4)-L2
1830 IF C=2 THEN 1990
```

```
1840 IF F=G THEN 1870
1850 Z1=Z1+1
1860 G=F
1870 D5=L2+V
1880 V=V-Z(Z1)
1890 T2=(T1-T2)*3600
1950 D4=T2-D5
1960 PRINT @3: USING 1970:F,T,Z(Z1),V,D5,T2,D4,"SECS"
1970 IMAGE4D6D5(12D.D)2XFA
1980 GO TO 2030
1990 D1=(D9-D+V)*1000
1995 S1=(S1-S2)*1000
2000 D4=S1-D1
2010 PRINT @3: USING 2020:F,T,V*1000,D1,S1,D4,"MM"
2020 IMAGE4D6D15X4(11D.2D)2XFA
2030 V2=V2+V^2*P
2040 GO TO 1780
2050 V9=V3-N7-Z1
2060 V3=V2/V9
2070 REM *************   OUTPUT OF COORDS AND MSE'S   ****************
2080 PRINT @3: USING 2090:"SIGMA 0 APOSTERIORI =",SQR(V3),P2*2^0.5
2090 IMAGE2(/L)FA5D.2D/L"SIGMA 0 APRIORI      ="5D.2D/L
2100 PRINT @3: USING 2110:"DEGREES OF FREEDOM =",V9
2110 IMAGEFA5D
2120 PRINT @3: USING 2130:"NAME","POINT","YO","XO","DY","DX","MY(MM)"
2130 IMAGEFXFA10XFA10XFA14XFA13XFA10XFA13XFA6X"MX(MM)"/L
2140 N=1
2150 DIM Z$(12*N1),Z2(N1)
2160 Z$=""
2170 CALL "REWIND",2
2180 READ #2:N1
2190 FOR I=1 TO N1
2200 Q9=2*I
2210 Q1=SQR(V3*B(Q9,Q9))*1000
2220 Q9=Q9-1
2230 Q2=SQR(V3*B(Q9,Q9))*1000
2240 READ #2:Y,H,C$
2250 Z$=Z$&C$
2260 Z2(I)=LEN(C$)
2270 PRINT @3: USING 2280:C$,I,Y,H,M(Q9+1)*1000,M(Q9)*1000,Q1,Q2
2280 IMAGEX12A7D2(11D.4D)2(10D.D)4X2(10D.D)/L
2290 NEXT I
2300 REM *************   DETERMINATION OF ELLIPSE PARAMETERS **********
2310 PRINT @3: USING 2320:"ABSOLUTE ERROR ELLIPSE PARAMETERS"
2320 IMAGE PFA
2330 PRINT @3: USING 2340:"POINT","A(MM)","B(MM)","ORIENT","3*ELL"
2340 IMAGE2(/L)FA11XFA5XFA3XFA5XFA3X"SHIFT(MM)"2X"ORIENT"
2350 CALL "REWIND",2
2360 READ #2:N1
2370 FOR I=1 TO N1
2380 D2=B(2*I,2*I)
2390 D1=B(2*I-1,2*I-1)
2400 D3=B(2*I-1,2*I)
2410 D=(D2+D1)/2
2420 E1=D1-D2
2430 E=SQR(E1^2/4+D3^2)
2440 A1=SQR(V3*(D+E))*1000
2450 B1=SQR(V3*(D-E))*1000
2460 T1=(180-ATN(E1/(2*D3+1.0E-90))-90*SGN(D3+1.0E-90))/2
2462 T2=180-ATN(M(2*I-1)/(M(2*I)+1.0E-90))-90*SGN(M(2*I)+1.0E-90)
2463 K1=B1*3
2464 T3=T1-T2
2466 K=SQR(M(2*I-1)^2+M(2*I)^2)*1000
```

```
2467 IF COS(T3)=0 THEN 2470
2468 K1=SQR(A1^2*B1^2/(B1^2+A1^2*TAN(T3)^2+1.0E-90))/COS(T3)*3
2470 READ #2:Y,H,C$
2480 PRINT @3: USING 2490:C$,A1,B1,T1,ABS(K1),K,T2
2490 IMAGE/L12A5D.D5(8D.D)
2500 NEXT I
2510 CLOSE
2570 END
2580 REM *************   SUB DIRECTIONS   *****************************
2590 A2=0
2600 GOSUB 2880
2610 R0=180*3600/PI
2620 A2(1)=R0*Y0/S1^2
2630 A2(2)=-R0*X0/S1^2
2640 A2(3)=-A2(1)
2650 A2(4)=-A2(2)
2660 GO TO Q OF 2670,2670,2720
2670 A3(A4(1))=A3(A4(1))+A2(1)
2680 A3(A4(2))=A3(A4(2))+A2(2)
2690 A3(A4(3))=A3(A4(3))+A2(3)
2700 A3(A4(4))=A3(A4(4))+A2(4)
2710 N8=N8+1
2720 P=1
2730 L2=(D9+W9/60+S9/3600-D-W/60-S/3600)*3600
2750 L3=L3+L2
2760 RETURN
2770 REM *************    SUB DISTANCES   *****************************
2780 A2=0
2790 GOSUB 2880
2800 A2(1)=-X0/S1
2810 A2(2)=-Y0/S1
2820 A2(3)=-A2(1)
2830 A2(4)=-A2(2)
2840 P=P2^2/(P3/1000)^2
2850 N8=-1
2860 L2=D9-D
2870 RETURN
2880 REM *************    SUB JOIN        *****************************
2890 A4(1)=2*F-1
2900 A4(3)=2*T-1
2910 A4(2)=2*F
2920 A4(4)=2*T
2930 Y0=X(2*T)-X(2*F)
2940 X0=X(2*T-1)-X(2*F-1)
2950 T1=ATN(Y0/(X0+1.0E-50))
2960 S1=SQR(Y0^2+X0^2)
2970 RETURN
```

NAME	POINT	YO	XO	DY	DX	MY(MM)	MX(MM)
P1	1	691.0000	5125.5000	0.0	0.0	0.0	0.0
P2	2	810.9999	5125.9999	0.3	0.1	1.0	0.7
P3	3	943.0002	5127.4998	-0.3	3.2	2.1	0.9
P4	4	816.4010	5006.0999	-1.7	1.5	1.2	1.1
P12	5	818.0006	4735.5505	-2.0	-0.7	1.8	1.1
P13	6	923.0011	4735.5099	-0.2	0.1	2.3	1.1
P14	7	1012.0011	4735.5191	-0.4	0.9	2.0	1.0
P15	8	1132.0005	4735.4994	-1.9	0.3	1.8	1.0
P16	9	1105.2007	5084.4000	-0.5	0.0	1.4	0.0
P17	10	814.6009	4855.7007	-4.9	-2.1	1.5	1.9
CD2(P20)	11	1214.4009	4900.8976	0.8	-2.0	1.7	1.6

ABSOLUTE ERROR ELLIPSE PARAMETERS

POINT	A(MM)	B(MM)	ORIENT	3*ELL	SHIFT(MM)	ORIENT
P1	0.0	0.0	45.0	0.0	0.0	90.0
P2	1.0	0.7	93.2	2.8	0.4	69.7
P3	2.1	0.9	92.0	2.6	3.2	354.3
P4	1.3	1.0	122.8	3.8	2.3	311.6
P12	1.9	1.1	105.9	4.3	2.1	249.1
P13	2.4	1.0	95.8	6.3	0.2	289.9
P14	2.0	1.0	87.8	3.3	1.0	333.2
P15	1.9	0.9	75.4	4.5	1.9	277.8
P16	1.4	0.0	90.0	4.2	0.5	270.0
P17	2.1	1.3	151.1	4.0	5.4	246.4
CD2(P20)	1.7	1.6	92.2	4.7	2.2	157.3

C.7 Niemeier's Test for Deformation

Requirements:

1. Two data files with information saved from the two adjustments (free network).

   N = number of points

   [PVV]

   f = degrees of freedom

   $\underline{x}$ = vector of final co-ordinates, X(2*N)

   ZØ = string of names, each name 10 characters long (name + blanks if not 10) ZØ(10*N)

   $\underline{Q}$ = matrix $(\underline{A}^T \underline{P}\underline{A} + GG^T)^{-1} - GG^T$, Q(2*N, 2*N)

2. Data file, name "TEN" with G matrix G(3, 2*N).

Sample printout at page C-41.

```
4 RUN 100
100 INIT
110 SET DEGREES
120 PRINT "INPUT FILE WITH DATA EPOCH 1"
130 INPUT D$
140 PRINT "INPUT FILE WITH DATA EPOCH 2"
150 INPUT E$
160 OPEN D$;1,"R",A$
170 OPEN E$;2,"R",B$
180 OPEN "TEN";3,"R",C$
190 READ #1:N1,P1,F1
200 READ #2:N2,P2,F2
210 K=N1*2
220 DIM X1(K),X2(K),Z$(N1*12),G(3,K),Q(K,K),G1(K,3),M(K,K),W(K),V(1,K)
230 DIM D1(1,2),T(1),D(2),C(2,2),D2(2),T1(N1),H(1,K),D3(2),D4(1,2)
240 FOR I=1 TO N1
250 T1(I)=I
260 NEXT I
270 READ #1:X1,Z$,Q
280 READ #2:X2
290 READ #3:G
300 CLOSE
305 REM********** FORMING Pdd AND d ************************************
310 G1=TRN(G)
320 M=G1 MPY G
330 Q=2*Q
340 Q=Q+M
350 Q=INV(Q)
360 M=Q-M
370 Q=M
380 W=X2-X1
390 DELETE X1,G;G1
400 G=0
410 X2=1000*W
420 W=X2
425 A1=2
430 P=(P1+P2)/(F1+F2)
435 REM********** TEST FOR DEFORMATION IN REF POINTS *******************
440 V=TRN(W)
450 H=V MPY Q
460 T=H MPY W
470 R=T(1)/(K-3)
480 F=R/P
490 READ A
495 PRINT A,F
500 IF F<A THEN 1430
520 G=G+2
524 K=K-2
529 REM********** DETERMINATION OF POINT DEFORMED IN REF MATRIX ********
530 GO TO A1 OF 540,550
540 DELETE X,B
550 DIM X(K),B(2,K)
560 U=0
570 FOR I=1 TO N1
580 L=1
590 FOR J=1 TO K+2
600 IF J=2*I OR J=2*I-1 THEN 650
610 B(1,L)=Q(I*2-1,J)
620 B(2,L)=Q(2*I,J)
630 X(L)=W(J)
640 L=L+1
650 NEXT J
660 C(1,1)=Q(I*2-1,2*I-1)
```

```
670 C(1,2)=Q(I*2-1,2*I)
680 C(2,1)=Q(I*2,2*I-1)
690 C(2,2)=Q(I*2,2*I)
700 D(1)=W(I*2-1)
710 D(2)=W(I*2)
720 C=INV(C)
730 D2=B MPY X
740 D3=C MPY D2
750 D=D+D3
760 D1=TRN(D)
770 C=INV(C)
780 D4=D1 MPY C
790 T=D4 MPY D
800 IF T(1)<U THEN 830
810 U=T(1)
820 U1=T1(I)
830 NEXT I
835 REM********* REFORMING OF Pss, Pso, Pos, Poo, ds, do *************
840 L=1
850 FOR I=1 TO N1
860 IF T1(I)=U1 THEN 880
870 T1(L)=T1(I)
875 L=L+1
880 NEXT I
890 T1(N1)=U1
900 N1=N1-1
910 GO TO A1 OF 920,930
920 DELETE B1,B2,C1,C2,E,Q1,B3
930 DELETE Q,W,V,H
940 DIM Q(K,K),W(K),B1(K,G),B2(G,K),C1(G,G),C2(G,G),E(G),V(1,K),Q1(K,K)
945 DIM H(1,K),B3(K,G)
950 FOR I1=1 TO N1
960 L=2*I1
980 S=2*T1(I1)
1000 FOR J=1 TO N1
1010 L1=2*J
1020 S1=2*T1(J)
1030 Q(L-1,L1-1)=M(S-1,S1-1)
1040 Q(L-1,L1)=M(S-1,S1)
1050 Q(L,L1)=M(S,S1)
1060 Q(L,L1-1)=M(S,S1-1)
1070 NEXT J
1080 W(L)=X2(S)
1090 W(L-1)=X2(S-1)
1100 NEXT I1
1110 FOR I=1 TO G/2
1120 L=2*I
1130 S=2*T1(N1+I)
1140 FOR J=1 TO N1
1150 L1=2*J
1160 S1=2*T1(J)
1170 B1(L1-1,L-1)=M(S-1,S1-1)
1180 B1(L1,L)=M(S,S1)
1183 B1(L1-1,L)=M(S-1,S1)
1186 B1(L1,L-1)=M(S,S1-1)
1190 B2(L-1,L1-1)=M(S1-1,S-1)
1200 B2(L,L1)=M(S1,S)
1203 B2(L,L1-1)=M(S1,S-1)
1206 B2(L-1,L1)=M(S1-1,S)
1210 NEXT J
1220 E(L)=X2(S)
1230 E(L-1)=X2(S-1)
1240 NEXT I
```

```
1250 FOR I=1 TO G/2
1260 L=2*I
1270 S=2*T1(N1+I)
1280 FOR J=1 TO G/2
1290 L1=2*J
1300 S1=2*T1(N1+J)
1310 C1(L-1,L1-1)=M(S-1,S1-1)
1320 C1(L-1,L1)=M(S-1,S1)
1330 C1(L,L1)=M(S,S1)
1340 C1(L,L1-1)=M(S,S1-1)
1350 NEXT J
1360 NEXT I
1365 REM******** TRANSFORMATION OF Pss ********************************
1370 C1=INV(C1)
1380 B3=B1 MPY C1
1390 Q1=B3 MPY B2
1400 Q=Q-Q1
1410 A1=1
1420 GO TO 440
1425 REM*** FINAL TRANSFORMATION OF do AND CHECK ON DEFORMED POINTS ***
1430 IF A1<>2 THEN 1460
1440 PRINT "NO DEFORMATION IN THIS NETWORK"
1450 END
1460 PRINT @3: USING 1470:"POINT","DY","DX","SHIFT","STATUS","DIRN"
1470 IMAGEFA12XFA8XFA14XFA9XFA6X"qy"7X"qx"/L38XFA5X"DIST(MM)"/L
1480 DIM Y(G),Y1(G)
1490 Y=B2 MPY W
1500 Y1=C1 MPY Y
1510 Y=E+Y1
1520 S2=0
1530 B$="DEFORMED"
1540 C$="STABLE"
1550 FOR I=1 TO G/2
1560 L=2*I
1565 R=T1(N1+I)*10-9
1570 S=SQR(C1(L-1,L-1)*P)
1580 S1=SQR(C1(L,L)*P)
1590 S=Y(L-1)/S
1600 S1=Y(L)/S1
1610 D$=C$
1620 IF ABS(S)<3 AND ABS(S1)<3 THEN 1640
1630 D$=B$
1640 A$=SEG(Z$,R,10)
1650 D6=180-ATN(Y(L-1)/(Y(L)+1.0E-90))-90*SGN(Y(L)+1.0E-90)
1660 D7=SQR(Y(L-1)^2+Y(L)^2)
1670 PRINT @3: USING 1680:A$,Y(L),Y(L-1),D6,D7,D$,S1,S
1680 IMAGE10A2(8D.D)10D.D8D.D5X8A2(6D.2D)
1690 NEXT I
1700 END
1710 DATA 1.7,1.73,1.77,1.82,1.89,1.97,2.1,2.31,2.7,3.94
```

28 JUN 1984

POINT	DY	DX	SHIFT		STATUS	qy	qx
			DIRN	DIST(MM)			
P3	1.3	2.9	23.7	3.2	DEFORMED	0.67	3.72
P17	-3.4	-3.3	225.7	4.8	DEFORMED	-4.67	-2.03

C-41.