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A STUDY OF NINE GIRL'S LEARNING BEFORE, DURING AND AFTER THEIR INTRODUCTION TO SOME OF THE BASICS OF LOGO.

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by

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ABSTRACT

This dissertation describes an investigation into the learning done by a group of nine Standard Six girls at Immaculata High School in Wittebome, Cape Town before, during and after a five month course in Logo.

In an attempt to determine whether Logo is an effective environment for promoting real learning and conceptual development, as learning theories suggest it should be, the girls were examined in a number of ways. They were tested, along with a comparative group matched in terms of Intelligence Quotients and mathematics teachers, on algebra, ratio and proportion and measurement tests. These tests were based on those used by the CSMS (Hart 1981) project.

The Logo group worked through the algebra tests in individual interviews. They, and I, kept journals of the work done and of the feelings experienced during the course. A year after the course I interviewed the girls again asking them a variety of questions concerning their recall of the course, their feelings about it and learning in general. I examined their learning from three perspectives; as a product, as a process, and as a possibility in the light of the affective nature of the environment.

By comparison with the other group, the Logo girls did not show, on the written algebra and ratio and proportion tests, an improvement. Their performance on the measurement test shows a small comparative improvement. Their interviews provided valuable insight into their thought processes and mathematical problems. Their use of language indicates that they were more positive in their attitude to problem solving, and felt more responsible for their own learning after the LOGO course, a finding which is borne out in their journals and the later interview.

That they enjoyed the course, and were involved in their work with Logo there is no doubt. In the light of my observations I would contend that Logo is indeed a valuable learning environment, but that its immediate relevance from a cognitive perspective to school mathematics has yet to be established.

CONTENTS

1. Introduction

The seeds of this dissertation were sown in my frustration with my role as a high school mathematics teacher in South Africa. This frustration arose during years spent teaching children who were doing mathematics because they had to. They had to for a number of reasons amongst them parental pressure, career needs and school timetable constraints. They derived very little benefit, satisfaction or joy from the school subject we call mathematics. In many cases they hated it and considered it a waste of time, what they hated most was that it made them feel stupid. Yet they were not, in other spheres they were able to function as intelligent, rational people.

I can see now how some of the problems in learning mathematics arise. We have a set syllabus which has to be completed. Its successful completion is measured against the pupils' performance in a written examination. In order to complete this syllabus, especially with slower pupils, one often hurries through the concrete, language orientated, concept forming stage. This should have been an active and interactive time when children approached ideas from their particular perspectives and shared their insights. However it may be noisy, apparently without order and it does not produce a great deal of easily checked output and as such is untenable in a crowded classroom in a quiet school. We go on instead to the more easily organised symbolic manipulation stage. The class is quiet, apparently productive and learning. Some of the children, those who were ready for it, are indeed learning. The others are making what Erlwanger called arcane marks on paper (Skemp 1980)

As the teacher we take comfort from the thought that we are getting through the work and that we can consolidate the ideas next year. Next year however it is worse, time is in even shorter supply, and the pupils' layers of symbolic and algorithmic manipulation is teetering even more violently on its conceptless base of shifting rules and poorly remembered strategems. Their view of themselves as being 'bad at maths' is constantly reinforced by their inability to cope with increasingly sophisticated applications of ideas they have never understood.

This may seem an overstatement but when I consider how many people on hearing what I do, respond "Oh, I hated maths at school, I couldn't do it", I do not think it is. What puzzles me is that these people frequently use what I would consider mathematical ideas easily and fluently in practical and business situations.

My beliefs were formed in the classroom but they were focussed, reinforced and amplified by my reading of John Holt's books. These in turn led to my exploring a number of the ideas that are currently held on how children learn and understand and the manner in which concepts develop. It became apparent that while learning can be approached from a variety of perspectives there appear to be basic criteria for judging whether an environment is one which is likely to foster learning and development. That the classrooms in which I have taught in many ways did not satisfy these criteria reinforced my discontent.

This then was the ground into which the seed fell. It was Seymour Papert's "Mindstorms". In it he describes a computer language, a language for learning, called LOGO. He describes how LOGO puts the child in charge of the computer; how the children have the opportunity to explore ideas initially at a concrete level; how they can work at their own speed and developmental level; how they are involved in the creation and solution of problems; how they are encouraged to communicate with themselves, each other, and the computer and how the teacher becomes a learner and co-worker.

In contrast with the behaviour one generally sees in a mathematics classroom these would seem to satisfy many of the criteria mentioned above.

In the light of my discontent and these claims I had to take a closer look at LOGO. In this dissertation I describe why and how I did this.

2. Review of Research

2.1 Mathematics Learning

Mathematics learning is a particularly clear and concentrated example of the activity of human intelligence and is probably the best paradigm of cognitive education (Skemp 1980 and Freudenthal 1981). Mathematics is regarded as a necessary component of formal education and as a prerequisite to many fields of study. Teaching mathematics in schools is frequently fraught with problems and markedly unsuccessful. For all these reasons the teaching and learning of mathematics have received a great deal of attention from researchers and developmental psychologists, learning theorists and concerned educators.

In my reading I have become aware that two themes pervade their theories and ideas. The first is that what we learn depends upon what we already know. The second is that what we learn also depends on the way in which we encounter the field of knowledge. Both of these statements have obvious implications for us as mathematics teachers and I will now discuss these and the themes in detail.

2.1.1 What We Learn Depends Upon What We Already Know

John Holt (1964) in 'How Children Fail' made the statement "A piece of unreal learning has no hooks on it". By this he meant that if what we learn or what we are taught makes no connections with what we already know it does not become part of our knowledge. He formed this idea through observing children at school. He realised that the difference between intelligent children and the others was that the intelligent ones expected their answers to make sense, ie to be consistent with what they already knew. They felt, like Einstein, that God does not play dice with the universe. To the rest, the mathematics was meaningless and an answer, any answer, to a question relieved the pressure they were under to produce one and they would resort to any of a number of non-mathematical strategies to get one. The mathematics they had been taught had made no connections with their existing cognitive structures and was consequently extremely difficult for them to remember.

Richard Skemp (1980) has worked on developing theories on the structure and generation of knowledge, different modes of understanding and the functioning of intelligence. He visualised learning "as a change in an organism's director system towards a state of better function" and intelligence as a "kind of learning which results in the ability to achieve goal states in a wide variety of conditions, and by a wide variety of paths." (Skemp 1980, 3)

As a result of learning, cognitive maps develop which facilitate this goal-directed activity. When these cognitive maps are dense and well connected we have an understanding of the related concepts and can apply them intelligently in a variety of situations. Skemp maintained that this type of development is encouraged if there are strong links between the conceptual and the symbolic schemata and if input is structured in such a way as to favour initial resonance with the conceptual schemata.

Erich Wittmann (1981) visualises two paths along which mathematics exploration proceeds - the intuitive and the reflective. It seems that the creative behaviour of the intuitive mode is similar to the activities of Skemp's (1980) conceptual schemata. In reflecting (Wittmann 1981) and making explicit the formulation of general relationships we need the mathematical language of concepts and symbols of the symbolic schemata.

Piaget has emphasised that the origin of knowledge is in action, that knowledge is neither a copy of reality nor does it come to us ready made. We must construct it for ourselves and build upon that which we already know by a process of assimilation and accommodation. He viewed learning or the acquisition of knowledge as being subordinate to a child's level of development and this development as being generally impervious to the influences of deliberate intervention.

Wheeler (1975b) suggested that by substituting the goal of facilitating children's mathematical activity for the passing on of mathematical knowledge and by stressing the process and not the product of mathematics, we could humanise mathematics. He continued, "activity is personal whereas knowledge often seems impersonal, activity is dynamic whereas knowledge is frequently inert, activity implies involvement in one's own learning rather than passive acceptance of someone else's" (Wheeler 1975b). Bruner too has insisted that mathematics is a process and not a product.

Freudenthal (1981), Dienes (1966) and Bruner (1966) differ from Piaget in that while they view the learning of mathematics as developmental cycles Freudenthal's view is of the child repeating (with hindsight) the history of mankind's mathematics progressive schematising while Bruner and Dienes view each learning experience as going through a cycle of different ways of internalising a concept each of which is necessary if the subsequent one is to be meaningful.

Another protege of Piaget's is Margaret Donaldson. She has subsequently focussed her attention on the role of language in learning. (Donaldson 1978) Like Holt she believed that it is before we go to school that we learn best. He (Holt 1967) believed that children have a style of learning which fits their condition and which they use naturally and well until we train them out of it. Bruner (1966) too believed that we often teach children to think that they cannot do things which they previously could. Donaldson (1978) maintained that when children are allowed to use the skills which they bring with them and the language which relates to real life, meaningful situations and which is embedded in a matrix of human intentions which sustain and direct their thoughts and speech, then they can function intelligently in a wide range of situations. When they are forced to operate in a rootless, formal language in situations which are to them obscure they cannot function. This is obviously not only true of children.

Polya (1945) in discussing problem solving as a particular case of mathematical activity suggested that one should try to relate what one is trying to find to what one already knows. He also suggested that one take the problem and relate it to a similar one or simplify it. Both of these principles are founded firmly on the theories I have touched on.

Bruner (1966) stated that making children aware of generalisations and rules enables them to cope with cognitively more complex situations. On the basis of Kath Hart's (1983) story (Essay 1,8) it would seem that this is only true if the foundations of the work are sound and the children have reached a stage where a rule is needed. Scandura (1975) has a rule-orientated view of knowledge which presupposes that the learner has an innate capability (Skemp's Delta 2) which tells him how the various rules are to be applied (by Skemp's Delta 1). He too, claimed that children who are given higher rules make discoveries which they otherwise would not. He agreed however that what a child learns in a particular situation depends on what he already knows.

Caleb Gattegno (1972) has said "Only awareness is educable". In "The Common Sense of Teaching Mathematics" he has made the meaning of this statement more explicit. He said that instead of teaching mathematics we should be striving to make people into mathematicians. This idea is central to Seymour Papert's philosophy. Gattegno (1972) would have us make people aware of the powers which they already possess which they can use in the way mathematicians use them. Further we, as mathematics teachers, should become aware of our own functionings so that we can later educate these in our students. Papert (1980) spoke of the seeds of a mathematics culture which some children have sown in them by "math-speaking adults". Perhaps they are not so much seeds as heightened awarenesses; of patterns, possibilities, and generalisations of a numerical, analytical and structural nature and of the fact that mathematics is a language, a territory to be explored and a human activity conducted by human beings (Sturgess 1980), and not a pre-digested organised body of knowledge (Holt 1967) entirely contained within the covers of a book.

David Sturgess (1980) conducts his in-service training courses for teachers on this basis; he gives the teachers the chance to behave, with respect to themselves as teachers, as he would have them give children the opportunities to behave in the mathematics classroom.

Wheeler (1975b) has suggested that awareness has very nearly the same quality as insight which suggests that one is 'seeing into' things by means of one's inner sight. He and David Fielker (1981) are in agreement that the teacher's role is to facilitate the developing of awarenesses. They discuss ways in which children can be encouraged to focus their attention on suitable tasks and on their own actions in tackling them. They stress that it is essential that time be allowed for exploration, contemplation and communication and that awarenesses are related integrally to what individual children bring to the task.

Kent and Hedger (1982) have implemented these ideas. They share a belief in the power of thinking in images, Bruner's (1966) iconic stage, which is so often neglected. Piemonte (1982) has stressed the importance of visualisation in mathematics education and the relationship between mathematical and spatial ability. They (Kent and Hedger 1980) have encouraged, with great success, the children to visualise before they symbolise. It is also apparent in their writing that the language they use initially is the Donaldson's embedded language, relating what the children are exploring to the matrix of human behaviour in which they function efficiently. They believe that "all of their children are mathematically gifted and that it is our task and sometimes privelege to make them individually aware of it". (Kent and Hedger 1980, 150)

If we are going to base our mathematics teaching upon the idea that what you learn is determined in the first case by what you know we must take cognizance of what children do know and not what we feel they should know or what we think we have taught them. To this end an awareness of the methods they use, the difficulties they experience, the mistakes they make and the levels at which they operate in dealing with mathematical concepts is vital.

Holt (1966) has stressed that to children the central business of school is not learning but getting the daily tasks done with a minimum of effort and unpleasantness. He described how the children used a number of interesting work and thought evading strategies and were in fact discouraged from thinking in any other than rote fashion.

It is necessary to acknowledge that much of the mathematics teaching done in conventional chalk-and-talk classrooms is a waste of everyone's time. This has been borne out by the findings of the CSMS commission (Hart 1981) which by means of tests and interviews examined the mathematical knowledge of 10 000 British school children from 11 to 15 in the following fields; algebra, graphs, integers, matrices, vectors, rotations and reflections, decimals, fractions and measurement. They found that the children infrequently used the methods which they had been taught, they relied instead on their own 'child-methods'. The difference between these was articulated by one child, "I was trying to do it mathematically not logically". (Booth 1981, 35) They (Hart 1981) were able to identify in most areas four increasingly abstract conceptual levels at which the children functioned. In support of a Piagetian perspective of development they found a positive correlation between the levels in different topics at which a child coped adequately. They have described in detail the manner in which the concepts are understood at successive levels and have offered suggestions for teaching the topics based on their findings. They stress the importance of being aware of the methods which children use when faced with problems in which they have to think as opposed to problems which they can solve in a rote manner by applying a teacher-taught algorithm.

Lesley Booth (1981) examined these informal, naive, child-methods and gave concrete examples of them. The techniques she identified were largely based on primitive counting and/or combination based approaches eg repeated subtraction, doubling and halving. When children encountered a problem they could not solve using these techniques they tried, if motivated, to solve it in a human sense by relating it to what they knew. This they did not regard as mathematics; when doing mathematics one applied rules, one did not think logically in a common sense manner.

Booth (1981) discussed what must be one of the fundamental problems in mathaematics teaching. She said (Booth 1981, 40) "ways must be found of working from the child's own strategies but in such a manner as to ensure their replacement by a more mathematical approach. Perhaps this can be done by attempting to make the child aware of the inadequacy of his own methods (without making him feel stupid) and of the existence and superiority of the systemised approach; perhaps more indirect ways are required."

Vergnaud (1979) felt that the establishment of links between ordinary arithmetical situations and the relevant mathematical concepts was probably the most challenging question in mathematics education. It would seem to parallel the links between Wittmann's (1981) intuitive and reflective modes and the step from embedded to abstract language which Donaldson (1978) says is so problematic. Resek and Rupley (1980) in their work on mathophobic students found that performance on a computation test was the best predictor of those students who would successfully become concept, as opposed to rule, orientated.

The tasks through which arithmetrical concepts are made meaningful and useful to the child need to be understood if we are to propose better conditions for the acquisition of these concepts. Vergnaud (1979) has stressed the importance of assessing the relational calculus a child uses on a problem as a means of understanding what an operation means to him; and that the complexity of a child's handling of a problem is not directly dependent upon the difficulty of the task since the easiest way of tackling a difficult problem may be easier than the most sophisticated manner in which a simpler task is handled.

He felt that many school problems are solved intuitively by the children who then present them in a symbolic form to satisfy the teacher. He saw a major problem in mathematics as being the finding of problems which the children could not do in this manner.

Freudenthal (1981) put forward a similar viewpoint. He maintained that children acquire knowledge by insight and that it is vital that this insight remain with the child. To facilitate this he would encourage the learner to reflect on his learning processes, and to argue their intuitions and ultimately do their own formalising. He has said, like Papert (1972a), that it is necessary for the child to encounter first the real world, then mathematising and like Vergnaud (1981) saw one of the problems being the creation of suitable contexts for mathematising. Nick James (1982) said that in learning mathematics it is vital that the children proceed from activity to discussion in embedded language, and then only via Gattegno's negotiated meaning to a symbolic representation of the idea they have explored.

If we are to support children in these processes and to be aware of their understanding we need to do more than study their written words and signs.

Holt (1967) was sceptical about the amount one can understand of what goes on in other people's minds. He felt that we should be very modest and tentative about what we think we find out. Freudenthal (1981) felt that only by observing and understanding individuals can we hope to save the vast resources of human experience. Fielker (1981) too felt that as teachers we would do better to put our energy into listening to children and communicating with other teachers especially if we discuss our lessons which were failures. Streefland (1978) has also shown how much we can learn in a one-to-one situation in which the teacher is a listener who provides meaningful prompts. Clements (1980) found that it was futile to attempt to make valid statements about the characteristics of good and bad problem solvers. He (Clements 1980) said, "all that can be said is that an individual's error pattern can throw considerable light on why that individual makes mistakes in mathematical tasks." He discussed the value of error analysis procedures in providing gross diagnosis on the basis of which the teacher could then make more detailed probing. Af Ekenstam and Nilsson (1979) discussed a technique for generating one's own diagnostic tests by means of creating a sequence of progressively simpler problems deriving from one another.

Clements (1980) found in his interviews that it was nearly always possible to provide the child with some positive reinforcement and that it was enlightening to see how children, whom he had been teaching for some time, tackled the problems. The CSMS (Hart 1981) team found that while the interviews were extremely time consuming they provided a wealth of information about why the children made the mistakes they did and how their minds did and did not work. Sheila Hollander (1978) discussed Uhl's (1917) work in which he watched children at work and questioned them regarding the approaches they used in order to determine not only specific errors but some fundamental causes of these errors. She (Hollander 1978) suggested that a classroom teacher could employ a simple introspective-retrospective technique to encourage students to verbalise their thoughts while problem solving. She maintained that the written work tells the teacher what was done, the verablisation provides a guide as to the process by which the product was achieved. Hoyles (1982) has suggested a critical incident technique be used to get children to talk about their feelings.

If we as teachers knew more about what our children know we would be in a better position to provide mathematics instruction which resonates with, and not against, their prior knowledge and intuitions.

2.1.2 What We Learn Depends Upon the Manner in which We Encounter the Field of Knowledge

Holt (1964, 111) said "knowledge, learning and understanding are not linear. They are not little bits of facts lined up in rows or piled one on top of another. A field of knowledge is a matter of knowing how they (the items of knowledge) relate to, compare with and fit in with each other. Why do we talk and write about the world and our knowledge of it as if they were linear? Because it is the nature of talk. Words come out in single file, one at a time, there's no other way to talk or write. So, in order to talk about it... we make strings of talk... but our learning is not real until we convert the word strings into a model in our head... and what happens is that kids take these word strings and store them undigested in their minds so they can spit them back out on demand.... How can we make school a place where real learning goes on and not just word swallowing?"

In his second book, 'How Children Learn', (Holt 1967) he described how some children, on discovering some simple addition sums accidentally left on the board, extend and expand, through argument, concensus, curiosity and common sense, their addition capabilities - far more quickly and effectively than if they had been presented with a carefully structured, well explained lesson, As he (Holt 1967, 136) said, "I saw enough to make me feel that if arithmetic were treated as in fact it is, a territory to be explored, not a list of facts to be learned, children, or at least many children, would move into it faster than we would have dreamed possible."

Bruner (1966) and Donaldson (1978) suggest that school generally persuades children that they in fact do not know things they previously did. In extreme cases, which are unfortunately not rare, children become mathophobic. That they should become mathophobes rather than 'geographobes' would seem to be the result of mathematics' distanced nature and its position as a sine qua non of the educational curriculum. To some children it seems apparently that one either knows all the answers or one is stupid, there is no grey area between. A measure of the blame for this must surely rest on the shoulders of mathematics teachers who appear indeed to be omniscient. (Holt 1970) The techniques by which mathophobes can be helped are, by and large, the ones we should use when we introduce the concepts initially (Oxrieder and Ray 1982; Resek and Rupley 1980; Bell 1981).

How then can we present mathematics so that we encourage learning? How can we humanise mathematics education? Wheeler (1975b) focussed his attention on this in an address to the Association of Teachers of Mathematics.

He said that while he did not expect all children to be engrossed by mathematics he would however hope that every child could experience the power and excitement of mathematics for a few moments in his school career. He emphasised that humanising mathematics education was not to be confused with encouraging mathematics teachers to come on like warm accepting therapists. He wanted to eliminate the fear and anxiety often experienced by children doing mathematics, in order to develop the child's self-awareness, character and individuality.

He spoke of awareness, growth and mathematisation and his attempt to make explicit the nature of mathematisation synthesises and summarises much of what we need to be aware of if we are to teach in a manner which facilitates meaningful learning. He (Wheeler 1975b, 6) said,

> "I would include the following ingredients: the ability to perceive relationships, to idealise them into purely mental material, to operate on them mentally to produce other relationships. It is the capacity to internalise, or to virtualise, actions and perceptions so as to ask oneself the question 'What would happen if ...?', the ability to make transformations - from actions to perceptions, from perceptions to images, from images to concepts, as well as within each category - to alter frames of reference, to refocus on neglected attributes of a situation, to recast problems; the capacity to co-ordinate and contrast the real and the ideal and to synthesise the systems of perception, imagery, language and symbolism. When these functionings are applied to pure relationships, detached from specific examplars, the result will be mathematics."

In the following pages I will discuss a number of the aspects of this obviously complex process.

I have referred above to meaningful learning. What is meaningful as opposed to rote learning? It is the learning which results in the production of well connected schema within which the knowledge and concepts are securely linked and interconnected and on the basis of which we can behave in an intelligent manner in a variety of situations. What facilitates this type of learning? According to Skemp (1979) relational understanding does. He divides understanding into two main types, instrumental and relational.

He described 3 different ways of understanding. "Instrumental understanding is the ability to apply an appropriate remembered rule to the solution of a problem without knowing why the rule works. Relational understanding is the ability to deduce specific rules or procedures from more general mathematical relationships. Formal understanding is the ability to connect mathematics symbolism and notation with relevant mathematical ideas and to combine these ideas into chains of logical reasoning." (Skemp 1979, 45).

He differentiated between relational and formal in saying that when one has grasped an idea for oneself one has relational understanding. When you can explain it logically to someone else you have formal understanding.

He (Skemp 1982, 60) later differentiated formal into two categories, logical and symbolic. Logical understanding is needed to follow, for example, a formal geometric proof, while symbolic understanding is a mutual assimilation between a symbol system and an appropriate conceptual structure, dominated by the conceptual structure.

While it is easier in the short term to teach for instrumental understanding it does not provide bases for future work nor does it encourage the formation of a densely linked cognitive structure. To encourage the development of relational understanding it is essential to provide experiences from which concepts can be abstracted. In order to achieve what he calls symbolic understanding it is essential that the subject matter be attracted primarily to the conceptual schema and not to the symbolic. Since a good deal of the communication of mathematics at higher levels is purely symbolic it is vital that the conceptual structures be nurtured since they will otherwise cease to attract input. He insisted that we must sequence material in such a way that new material is presented which can be assimilated conceptually. To achieve this he suggested beginning mathematics in a physical manner, staying with informal spoken language as along as possible, resisting pressure to turn to written work so as to have something to show for our labours, and using the informal notation as a bridge to the highly condensed formal notation of established mathematics.

In this he concurred with, amongst others, Piaget and Nick James (1982) that the origin of knowledge is in action and with Donaldson (1978) on the importance of using language which is made meaningful by a matrix of human intentions.

In what ways can we sequence and structure material so as to provide a cycle of experience which supports learning? Piaget found that children's development went through a number of generally age-linked stages from sensori-motor to concrete to concrete-operational to symbolic. At three a child can physically order objects in a row, at seven he can think about ordering physical things and not until twelve can he order propositions mentally. On the basis of this one can provide the type of experience which is meaningful to a child at his stage.

Bruner (1966) analysed the processes of mathematics learning into three stages. In the initial enactive stage the child thinks in terms of actions and therefore if he cannot act out a solution he cannot solve the problem. In the next, iconic, stage the child manipulates images of objects. While images are easy to manipulate they do not facilitate the transformations in which mathematics abounds. Thus he believed that it is not until the child reaches the symbolic stage that he can handle most sophisticated mathematics. (Bruner 1966, 11) He has also said that mathematics should be viewed as a process and not as a product. While he viewed this as a general developmental pattern he also viewed each stage as being necessary in the formation of any concept. Thus even children who are doing formal mathematics need to, when they encounter a new idea, be able to meet it first in an enactive manner or, failing this, an iconic mode. Kent and Hedger (1980) base a good deal of their teaching upon this. Communication about one's visualisations encourages symbolic learning (Skemp 1982).

Wittmann (1981) visualised two paths along which mathematical exploration proceeds, intuitive and reflective. Intuitively objects are represented in a familiar, concrete-pictorial or symbolic way. This direct, immediate, often partly unconscious way of representation and information processing often leads to the discovery of general patterns of behaviour, creation of knowledge, checking of proofs and detecting of errors. Reflective thinking aims at the explicit formulation of general relationships; it is dependent on the mathematical language of concepts and symbols. On this path one reflects on and talks about one's activities. This abstraction, conceptual ordering and schematisation of sets of objects is essential to the growth of mathematics.

He (Wittmann 1981, 395) stated "The constructive branch of mathematics education faces the difficult problem of developing courses which base reflective thinking on intuitive activities and formal thinking on informal. This problem is difficult because it requires a sound feeling for changing levels of rigour. It is much easier to stick to the intuitive level - and to ignore advanced maths - or to rely on the formal systems of mathematicians - and to strive for the blessing of the watching Big Brother mathematician."

He made suggestions for encouraging both types of thought. He would have schools use a spiral approach in which children are exposed to and investigate a rich variety of examples and models and are then encouraged to reflect upon them gradually learning to analyse concepts and construct theorems and proofs.

Holt (1967) has emphasised the importance of allowing time for 'messing about' at this intuitive, experimental stage. Hawkins (in

Holt 1967) suggested that it is during this phase that the child builds up an apperceptive background against which a more analytical sort of knowledge could take form and make sense.

When we discuss the sequencing of the cognitive content of a course we speak of hierarchies of knowledge, this word can be used in a number of ways (Hart 1981b). It can describe the learning sequence or sequence of understanding which is essentially in the learner, or the teaching sequence which the teacher uses or the logic sequence which is in the topic.

These are not necessarily the same but can be considered interdependent. For successful learning in school, the three aspects must be closely matched or failure is the result. In each case, 'hierarchy' implies a string of skills/levels/stages/ concepts which are ordered from simple to complex.

That the sequence of material cannot simply be based on a Piagetian view should by now be obvious. Hart (1981b) quoted studies which showed that only a minority of adolescents seem capable of, or find it useful to use, formal operational thought.

While it is necessary for a teacher to consider carefully the sequencing of material this sequence is not always the one which students themselves would choose. Dr Hart (1981b) describes Robert Magers' experiment in which adult electronics students, through their questions, structure their own course. It highlighted the following points; the students initial interest was not in theory but in concrete examples and they brought to the course a wide range of background knowledge.

Hart suggested that the sequencing of material by an expert who knows 'the whole' is not necessarily the best way of matching the order of learning natural to a child. While Holt would agree with her there are those, Gagne (1968) and the Van Hieles (quoted in Hart 1981b) among them, who would not. The Van Hieles have sequenced a geometry course in a manner which matches the level of the child. They took cognisance of the fact that the learning process appears to be discontinuous with jumps revealing the presence of different levels of comprehension. They also view learning as being discontinuous in time. Their course has they said achieved in eight years what others achieve in eleven. The CSMS study has also shown evidence of these conceptual leaps. In providing a variety of experience, it is necessary to be aware that it is not only the cognitive content that needs to be varied but also the types of input. In this way one can provide footholds for many children into the same concept. I have taught children who, when encouraged to relate an idea to something in which they were interested, showed what seemed to me at the time incredible gains in insight. Approaching a new idea from a secure position and within one's own language is a potentially powerful position not a threatening one. In this regard it is also important that the teacher be involved as was David Kent in Linda's Story (1978) and Caleb Gattegno in the lesson John Holt (1967) described.

Children respond to a variety of stimuli. They may be visual, spatial, linguistic, kinetic and aesthetic. Any and all of these can be used to provide experience based learning.

It would seem that we need to encourage children to make jumps in comprehension without losing their footing on the supporting ideas and experiences. They need to be exposed to situations and problems which make the need for generalisations and mathematisation apparent while being supported in their search for them by spoken language and the knowledge that the advance is not irreversible.

When trying to solve a problem, possibly of their own creating, they need to be aware of the goal they are trying to achieve, the different ways in which it can be attained and the value and importance of the knowledge, skills and interests they bring to the situation.

If one can create a learning environment which begins to satisfy these criteria the chances of meaningful learning within it would seem to be good.

A man who has devoted a great deal of time to creating such an environment is Seymour Papert. In the next section I will discuss his ideas on children, learning, and computers and the way in which they have been manifested in the computer language called LOGO.

2.2 Seymour Papert and LOGO

2.2.1 Papert's Ideas on Children, Computers and Learning

He (Papert 1972b, 2) has said "I believe, with Dewey, Montesori and

Piaget that children learn by doing and by thinking about what they do. And so the fundamental ingredients of educational innovation must be better things to do and better ways to think about oneself doing these things. I believe that computation is by far the richest known source of these ingredients. We can give children unprecedented power to invent and carry out exciting projects by providing them with access to computers, with a suitably clear and intelligible programming language and with periferal devices capable of producing on-line real-time action."

He and his co-workers at MIT have developed a computer language, a language for learning, which they call LOGO. It is essentially a list processing language which has turtle-graphics. A turtle is a device which can be used for drawing, it may be mechanical or a representational triangle of light on the screen, which moves in response to keyboard instructions.

In LOGO they have aimed to put the child in charge of the computer, not vice versa, freeing him to explore in a potentially rich and varied environment. As Abelson (1982, 106) said "The best kind of LOGO activity is a synthesis of programming, mathematics, aesthetics and, above all, the opportunity to explore."

In the first part of this research review I discussed the importance of finding learning environments which encourage mathematising. It would seem that LOGO is one. That it is so is as a result of the congruence that I believe exists between Papert's views on learning and those held by many of the educators and researchers I have discussed and the manner in which these ideas have been implemented in the structure and nature of LOGO.

Papert has been influenced in his views on how children learn by, amongst others, Jean Piaget. He worked with the Swiss epistemologist for many years and took from him the model of children as builders of their own intellectual structures (Piaget's constructivism). A central problem in education is thus, to him, how to instruct while respecting this self-constructive character of the mind. (Lawler 1982)

He would have children learn mathematics in a Mathland as one learns French in France. (Papert 1980) In a Mathland children would get informal exposure to mathematics and the time and opportunity to learn to talk it. This he saw as the first step in how a computer presence can change not only the way we teach children mathematics but beyond that, and more fundamentally, the way in which our culture as a whole thinks about knowledge and learning. In a Mathland children would learn mathematics in what he terms a Piagetian manner, the same manner in which they learn so much of what they are not directly taught, the manner in which they learn best.

The learning that one does in this manner is real, the understanding relational and the language used is embedded.

He stresses throughout his writings the importance of communication, with oneself, with others and with the computer. He (Papert 1980, 96) said, appropos of the discussion about processes and procedures which cannot or should not be verbalised, that "My perspective is more flexible because it rejects the idea of the dichotomy verbalizable versus non-verbalizable.... An important component in the history of knowledge is the development of techniques that increase the potency of 'words and diagrams'. What is true historically is also true for the individual; an important part of becoming a good learner is learning how to push out the frontier of what we can express in words." Freudenthal (1981) has said that the history of mankind's progress in mathematics has been a learning process of progressive schematising and he feels that youngsters should repeat this history - though they should repeat the one that would have taken place if our ancestors had known what we do.

Papert is concerned with developing languages so that we can communicate, with ourselves and with others, about learning all sorts of things - eg riding a bicycle, juggling a ball, playing tennis, doing mathematics.

Papert (1972b) argued that it is essential for children to spend time on, and become involved in, projects which are extended in time, in order to develop the concepts and vocabulary skills needed for articulate discussion. He (Papert 1972b, 6) maintained that if mathematics classes were more like art classes "the duration of the process would be long enough for one to become involved, to try several ideas, to have the experience of putting something of oneself into the final result, to compare one's work with that of other children, to discuss, to criticise and to be criticised on some basis other than 'right or wrong'."

An environment like this is a more positively affective one than most mathematics classrooms, a factor which Papert believes to be vital in the learning process. He attributes Piaget's neglect of this component of learning to his having had a modest sense that little is known about the affective domain.

He is extremely critical of the conventional problems children are expected to churn out in silence in order to be rewarded with ticks and punished with crosses. To him the act of thinking is more important than the result which is produced. Many of the answers Piaget got to his questions were, in an adult sense, wrong, but to him they were evidence of the process of thought and as such valuable. To Papert (1980, viii) a computer is a "thing to think with". It is the Proteus of machines which through its universality and power to simulate can take on a thousand forms and appeal to a thousand tastes. In this way he sees each child as being able to relate to it in his own way which as we have seen is fundamental to learning.

Through exploration in LOGO children gain living experience of mathematising as an introduction to mathematics. He would have us select the content of courses on the basis of its concept-forming potential. He saw no value in introducing children to the process of mathematisation in an environment which is, as school mathematics frequently is, meaningless to them. He wanted them to come to understand what it means to mathematise via the mathematisation of familiar processes as for example using the turtle's body syntonicity to draw a circle.

Lawler (1982, 150) discussed this syntonicity, "Because LOGO is a vehicle for free exploration, knowledge built from LOGO is syntonic, appropriate to the person, and experienced as an authentic, intimate part of the self. Such is the power of an approach to learning that frees the individual to create within a social context that it makes our culture's most powerful ideas accessible."

Papert (1972b) wanted to teach children to be mathematicians, he did not want to teach them mathematics. When children work with LOGO they are learning by doing, and the knowledge becomes theirs. They are doing mathematics, not learning about it.

In LOGO he believed children can make genuine discoveries. They are also free to work on problems in a way which suits them best. He maintained that they are able to mobilise their multiple strengths in problem solving and creating.

"Turtle geometry serves as a carrier for the general ideas of heuristic strategy" (Papert 1980, 64). It is rich in situations in which simple, yet compelling, models of heuristic knowledge can be encountered and internalised by children.

When children 'play turtle' they are using a fundamental mathetic principle of relating what is new to that which you already know. A second principle is to take what is new and, by making sense of it, make it your own. Turtle geometry was designed to be something that children can make sense of and identify with. This facilitates learning, for as Carlson (1982, 42) said, "Anything is easy to learn if you can assimilate it to your collection of models." This is a view endorsed by all the theorists and educators discussed earlier.

It is essential that the teacher be aware of the different cognitive strategies and styles children are happy using. Solomon (1982) discussed three types of learners; the first is a planner who builds programs in a top-down manner following a coherent, formulated plan; the second is a macro-explorer who sets out without a specific goal intent on exploration of macroworlds and procedures; the third, often the most timid, is a micro-explorer who needs time to explore the environment on a microlevel before developing a design by trial and revision.

The effectiveness of LOGO as a learning environment is greatly influenced by the manner in which it is taught. To realise its potential it is vital that the teacher capitalise on the individual's predominant learning style while enriching it by suggesting other styles where appropriate. (Laridan 1984) The stage and manner at which the teacher intervenes is crucial in teaching LOGO. Moore (1983) has discussed the problem of intercepting children's frustrations without intercepting their discovery. Walt (1983) and Riordan (1982) offer a number of suggestions for helping a child overcome a problem without taking away the chance of his making a discovery. This problem is not peculiar to LOGO, it is just that it makes us more aware of children's mathematical behaviour in problem creating and solving. It seems to facilitate meaningful teaching by freeing the teacher from the drudgery of instruction and focussing him on the process as well as the product of mathematics.

Children who are made aware that the process of doing mathematics is important can no longer believe as John Holt's (1964) child did that the teacher and some children know all the answers while she did not. The idea of finding answers in an Answerland is very different from that of exploring a Mathland. Papert (1972b) is a firm believer in making children aware of how they think, exposing them to experiences which demonstrate that what he calls the Pop-Ed theories are wrong.

Children who are involved in experiences which provide for the growth of intuitions and concepts for dealing with thinking, learning and playing, discover for themselves how they think. Papert sees great value in thinking about thinking. "In most contemporary educational systems where children come into contact with computers the computer is programming the child. In the LOGO environment the relationship is reversed. The child is in control. The child programs the computer. And in teaching the computer how to think children embark on an exploration of how they themselves think. The experience can be heady. Thinking about thinking turns the child into an epistemologist, an experience not even shared by most adults." (Papert 1980, 19)

Reflecting upon the fruits of one's varied exploration and synthesising one's intuitions into generalisations is at the heart of mathematisation as defined by Wheeler (1975).

I feel that there is obvious agreement between Papert's ideas and those held by the theorists, researchers and educators I discussed earlier. I will now show how these ideas are manifested in LOGO's structure and nature.

2.2.2 The Nature of LOGO

LOGO's developers aimed at producing a language which developed thoughtful literacy and which grew by allowing children to program and encouraged the expression of ideas, mathematical and other. It is a user friendly language which aims at dispelling the illusion that learning about computing should be an activity of fiddling with an array of indexes and worrying whether x is a real number or an integer and focusses on programming as a source of ideas (Abelson 1982) and a way of thinking. Carlson (1982) has suggested that even seasoned programmers should learn LOGO for the new perspective it reveals about the art of thinking. For these reasons Charles Morgan (1982), editor of Byte, stated that LOGO was the language to choose for use in schools and for introducing children to computers.

The primary design specifications were that it be procedural, extendable, flexible and user-friendly.

Programs in LOGO are written as a number of linked procedures which, in keeping with Polya's (1945) problem solving principles, encourages the breaking down of problems into pieces which the programmer can cope with. Papert (1980) frequently speaks of the value of breaking powerful ideas down into mind-sized bites.

A program in LOGO is a hierarchy of procedures which call one another. (Carlson 1982) The procedures tend to be short but can be deeply nested. This makes it both simple and powerful, since simple procedures can be linked to achieve powerful ends. In order to write a program as a hierarchy of procedures it is vital to analyse the structure of the problem, relating it to what is known and where necessary to similar but simpler ideas. (Polya 1945) Using the technique of recursion, which is one of LOGO's powerful ideas, complex programs can be redefined in terms of simpler versions of themselves and solved. Papert has said that recursion is a fascinating idea to children, the procedure which calls itself as its last act can run literally for ever producing amazing effects quickly and easily. Many LOGO microworlds use this technique to facilitate the exploration of ideas and concepts.

Procedures are named by the programmer. The child teaches the computer new words defined in terms of LOGO primitives and existing procedures. It is thus a functional language in which the act of programming is in fact extending the language (McCauley 1982). The child is thus involved, active and creative and can use his own language in his programs. When a child draws, for example, a square he becomes aware of the meaning of squares in terms of how they are produced and used. A child's creations are his own, they relate to what he knows since that is how they were defined, he can re-use them either as building blocks in other programs or as a means of expressing an idea which was utilised. They become part of his vocabulary which is thus extended and enriched.

LOGO is, because of its procedural nature, comparatively easy to debug. That this is vital in a language which is to be used as a vehicle for creativity and development of ideas is obvious to anyone who has programmed and experienced the frustrations of seeing a good idea thwarted and sometimes destroyed by bugs.

The children are encouraged to forgo the right-wrong dichotomy in favour of a right-fixable perpective. A program which does not do exactly what was planned for it is not wrong, it need not be scrapped. It needs fixing. The child identifies the errant procedures, isolates them, works on them and then tries again. He is responsible for his own learning, he has not abdicated and become dependent upon a teacher's red pen for the tick of approval. He may well be frustrated and irritated, but he is involved. 'Bugs in the works' can be the source of creative inspiration, exposing the child to new and unforeseen experiences.

LOGO has other attributes which make it a user friendly, flexible language. Its error messages are simple and informative. Its data is not typed. It makes explicit things which other languages ignore or make obscure. It is interactive both in the sense that it runs immediately and also in that procedures can be interactively defined. Its editing facilities both in and out of the edit mode are simple to use.

2.2.3 The Structure of LOGO

Carlson (1982) has described LOGO as having three faces. The first is 'turtle talk' - a set of graphic commands which serve as a splendid introduction to computing for a beginner, especially for younger children. Removing this turtle graphics mask reveals a general computer language on a par with Basic. An advanced user can remove the second mask to find a powerful lisp-like language which introduces techniques common to artificial intelligence programming.

Turtle geometry is a genuinely new mathematics based on turtle movements which emphasises transformations in local space rather than relationships to a fixed global referent (Goldenberg 1982). It is intrinsic and belongs to the family of differential geometries which have made possible much of modern physics. The turtle thus moves in the way we do, it moves in space by going forward and back, it changes its heading by turning left and right. It is easy for a child to identify with which they do when 'playing turtle' to facilitate problem solving by acting as if they were turtles. The ease of programming at this level and the manner in which it relates to what children already know makes working with the turtle extremely satisfying. The turtle can open mathematical doors in an experience based learning environment, it is an effective carrier of general mathematical ideas. The children can identify with the processes by which shapes are drawn, encounter theorems, and experiment with ideas in microworlds. A microworld is a well defined, but limited, learning environment in which interesting things happen and in which there are important ideas to be learned (Goldenberg 1982, 218). A feature of LOGO is the way in which children can learn the meaning of its primitives and techniques through using them in these microworlds rather than having to be formally taught.

In turtle geometry the ideas and concepts are not isolated examples, they are the language the children use to communicate with each other and the computer and through which they get the computer to obey their wishes.

Turtle geometry can facilitate the development of a number of mathematical concepts and ways of thinking. The child can play safely in an environment which provides body syntonic, visual experiences in a number of concepts usually taught in a purely symbolised fashion with no reference to concrete or iconic modes of thought.

Many microworlds and procedures use variables, one of the most powerful mathematical ideas. In many microworlds recursion is used to change the value/s of the variables used. This introduces iteration or variable-stepping which is an essential component of formal operational thought, an almost universal idea and one which is crucial to the process of scientific investigation. (Lawler 1982)

Procedures which accept inputs and which output are equivalent to mathematical functions. Its local perspective provides links with, metaphors for, and intuitive analogs of many of the processes involved in differential calculus and equations, limits and the infinitesimal.

When children draw shapes with a turtle they learn about angles, controlled repetition, heading and the Turtle Total Trip theorem, a far more generally applicable theorem than its Euclidean counterpart. They observe the conditions under which one gets rotations and reflections and the process by which these changes are achieved. Leron (1982) has found that a lot of group theory work goes on when children work in LOGO. They have created microworlds to encourage the children to become aware of this.

Dynaturtles, programmed to obey Newton's laws of matter, can be used to teach physics by experimentation and to provide a bridge from the complex real world to the stark symbolism of equations.

In biology turtles can be programmed for the senses of sight and smell and used in simulations.

List processing , the heart of LOGO, has all too often been ignored in the acclaim for turtle graphics (McCauley 1982) "Lists are", Abelson (1982) claimed, "a natural way to represent heirarchical structures, that is, structures composed of parts that are themselves composed of parts". A list is a sequence of data objects which may be either words or lists. Words are unbroken strings of characters which may be literal or numeric. When numerical LOGO is as efficient a computational language as Basic.

Lists can be operated on by a large number of primitives used as values for variables, passed to procedures as inputs and returned as outputs. One can combine operations on lists much as one combines operations on numbers in ordinary languages.

In LOGO children can create programs which generate random sentences, greetings, postcards and stories comparatively easily. This helps them become aware of the structure and function of grammar in an intensely personal manner (Papert 1972b). These programs can be interactive in the sense that they accept input from the keyboard. LOGO has the necessary screen editing facilities to be used as a word processor which can prepare children for more sophisticated systems. Children can rewrite and correct their creative writing with ease. This encourages them to re-read their creation with a critical eye since re-writing it is no longer a laborious task.

For children with physical handicaps LOGO can be used to write and in some cases to communicate. Procedures can be named by single letters which means that a child by pressing one key can initiate a communication.

Beyond simple list processing and turtle graphics LOGO has the potential to use procedures as trees and to make data lists into procedures. It can be used in manners similar to artificial intelligence programs. The turtle can be set free of the plane and allowed to explore spherical and cubical surfaces (Abelson and di Sessa 1981). The developers have plans to go still further utilising the freedoms of unlimited memory which now seems possible.

2.3 What Are the Criteria by which One Would Judge a Learning Environment?

It should be one which encourages meaningful learning and facilitates the development of concepts. How would it do this? It should provide a wealth of visual, spatial and verbal experiences. These experiences should be in the first instance concrete, concrete-pictorial and iconic, related to what the child already knows and capitalising on his strengths and existing awarenesses. It should encourage exploration of ideas in these modes and the development of language with which to link these experiences to subsequent symbolism and the awakening of awarenesses. It should foster a need to go beyond this intuitive stage to a more general, powerful symbolic one. In short it should provide a fruitful environment in which to mathematise and encourage the use of spoken language while encouraging the development of more formal language.

Does LOGO satisfy these criteria? To ask of one environment that it satisfy them all is unrealistic. We will never find a universal educational panacea. It does appear, from the perspective I have adopted this far, to be an environment worth introducing into the classroom, provided, as I have discussed in my fourth essay, we do so in a manner not at variance with its philosophy. We must introduce it as a field to be explored and not a body of knowledge to be absorbed. We must not evaluate the children's mastery of it by means of mark and comparison orientated tests.

2.4 Criticisms of LOGO and Computers in Education

There are other perspectives from which we can view the advent of computers in general and LOGO in particular, in education.

Sloan (1984, 539) raised critical questions based on the three-fold conviction that "the computer offers potential for human betterment and at the same time is fraught with great dangers to the human being; that neither the potential can be truly realised, nor the dangers avoided, without careful, far-reaching critical questions being asked about the computer in education; and that American educators in general have been almost totally remiss in their responsibility to raise and pursue these critical questions." Zajonc (1984) concurred in saying that we should submit each of our educational innovations to systematic theoretical and empirical scrutiny.

These critical questions begin with deciding where computers can be used in an appropriate and helpful manner in schools, what is appropriate in high school may be inappropriate or even harmful to young children. Cufforo (1984) substantiates the view that the environment young children need is not that offered by a VDU.
Sloan (1984) argued that the mode of thought and type of learning encouraged by programming takes no cognisance of the central part played in cognition by what John MacMurray (1935) called emotional rationality. He argued that it is only in the matrix of qualitative reality that all reason, including the logical and calculative, ultimately finds its ground and that qualitative knowing requires a rich, vital, emotional life. For this reason the healthy development of children requires an environment rich in sensory experience.
Sardello (1984) described children in an open field and watching a juggler, made oblivious, he claimed, to the richness of the scene, by having their attention focussed on the analysing of the creation of a square and the act of the juggler. He claimed that the computer has in fact the power to destroy education by transforming us into a culture of psychopaths, people who do everything effortlessly, freely without any sense of inhibition, restraint or suppression. He claimed that the poems children generate in LOGO look like poems, but they have no heart, just as a psychopath imitates behaviour without its going through the heart. He presented a frightening view of the possibilities of a bland acceptance of the computer culture and its values.

Yet one feels his is a one-sided argument. Papert (1980) said that true computer literacy is not just knowing how to make use of computers and computational ideas; it is knowing when it is appropriate to do so. He did not want to displace the teacher but to free him to do more meaningful teaching. To Simpson (1983) it made sense to use machines only to support teachers, not to replace or mimic them. The type of teaching Papert envisions could not be mimiced by a machine. He did not advocate depriving children of their childhood pursuits and sensory, emotional, social and physical stimulation and experiences.

His motivation was the enrichment of the environment with respect to ideas and concepts in which our culture is experientially poor. He did not advocate forcing all children to think in the same way but wanted to encourage them to be aware of the ways in which it is possible think and that it is possible to think about thinking itself. Davy (1984) maintained "despite Papert's (unsupported) claim that early thinking like a computer will promote awareness of other styles of thinking, the entire temper of his work is in the spirit of instrumental reason... Truth is what can be made to work, the means are the ends." He felt that this thought runs deep and strong through our technocratic society and that one challenges it at one's peril.

He emphasised that while computer systems can throw light on some aspects of human intelligence, we must not forget the others. In our culture the idolatry of the powerful ideas of mathematics and physics and our attitude that education is fundamentally a cognitive affair prevents us from asking questions from other perspectives. In illustration he (Davy 1984, 557) tells Weizenbaum's favourite drunk joke: The drunk is searching for lost keys in the pool of light beneath a streetlamp. A policeman asks him where he lost them. "Out there," says the drunk, gesturing vaguely into the darkness. "Then why are you looking here?" "Because the light's better," says the drunk.

Simpson (1983, 629) provided a perspective from which to view an educational innovation "Provided that we understand the limitations of each technology as well as its capabilities, and, more importantly, provided that we understand the people we are trying to educate and the kind of education we are trying to give them, we can use technology in ways that will really help. There is no technological panacea; there are only technological solutions to <u>some</u> educational problems. But if we start imagining that technology can bring about a quick and easy methodological revolution we should be heading straight for the ha-ha *."

So I have come from my discontent with the support of modern, educational theory and belief to a possible solution to my problem. It is, as we have seen, vital to examine any solution extremely carefully before embracing it. In the next section I will describe the structure of the investigation I made into LOGO as a learning environment.

* A sunken fence, invisible from a distance allowing a tantalising, unobstructed view of the verdant pastures on the other side, but forming and inpenetrable barrier.

3. The Investigation

There are powerful arguments to be made for LOGO as a language for learning. The arguments against it, and computers in general, as an educational medium counsel us to exercise caution in embracing innovations simply because they are new, exciting, popular and in step with the times.

In answer to the criticisms, I am sure that Papert would not have us to put away childish things and to rob our young children of the sensory, emotional and physical experiences which are so necessary for their development into thinking caring adults. Equally I am sure he would agree with Walter Hagen who insisted that it was essential to take time to smell the flowers along the way. There are however times when an environment like LOGO can focus our attention on the particularly mathematical, structured and analysable aspects of a situation. This selective focus may well give the impression that the environment is autistic and its purpose the development of instrumental reason.

If this were the only environment a child encountered he would indeed develop in the manner described by Sardello (1984) but it is not. Children do not live in classrooms and it is to the usual environment prevailing in high school mathematics classrooms that one should compare LOGO. Then it can be seen to be what Papert intended, an environment in which children can have experiences which bridge the concrete-formal jumps in understanding, experiences which link the intuitive, experimental concept forming stages of development to the formal symbolism of mathematics, experiences which facilitate the growth of concepts usually presented in a purely abstract and symbolic manner, in short experiences which encourage mathematisation and meaningful learning.

I have focussed my attention on the effectiveness of LOGO as a learning environment for children at the beginning of high school. There are many ways to examine and evaluate the impact and effectiveness of an idea or technological development. I have chosen to consider a number of facets of a small scale investiagation. In this dissertation I describe how I introduced nine standard six girls to LOGO over a period of five months and the manner in which I assessed its effectiveness as a learning environment.

3.1 The Students

The subjects of the investigation were nine girls from two standard six classes at Immaculata Convent in Wittbome, Cape. Those who were chosen lived close to the main railway line which meant that they could get home safely after staying late at school; this was the only criterion I used. They ranged in age from 13 years 11 months to 15 years and 6 months.

They worked in pairs and one threesome, based on existing relationships and the classes they were in. The groups were as follows: Liesle and Tracia, Ingrid and Anzonia, Mandy and Bonita; and Charlene, Lynn and Alitacia.

My Group	Comparison	Age	OTIS Score
Alitacia		13,11	102
	Bernadette	14,8	102
Anzonia		14,6	80
	Michelle	15,3	81
Bonita		14,5	95
	Rosali	13,11	93
Charlene		14,0	103
	Melanie	14,10	103
Ingrid		15,6	83
	Yvette	16,6	82
Liesle		14,3	81
	Felicity	15,0	84
Lynn		14,11	106
	Shireen	13,10	108
Mandy		13,11	97
	Ruby	14,0	97
Tracia		13,11	91
	Melaney	15,0	93

3.2 The Time

The girls started learning LOGO in August 1984 and, with breaks for holidays and examinations, continued until February 1985. For the first five weeks I worked with them for three hours a week, twice during school for half an hour at a time with half the group and once after school on a Monday for two hours with the whole group. Thereafter I was teaching elsewhere and could only see them on Monday afternoons.

They worked in their pairs for an hour a week after school and when I left also used the in-school time to work unsupervised.

They thus had a total of 35 hours of supervised time and at least 18 hours unsupervised. Those who were more interested worked more.

3.3 The Contents of the Course

The children encountered the following LOGO primitives:

Forward	Back	Left	Right
Hide Turtle	Show Turtle	Clear Screen	Home
Penup	Pendown	Print	
Repeat	To	End	
Edit facilities			
Save	Load	Catalog	Erase
Random	Make		
sum	difference	quotient	product
+	-	/	*
<	>	if	else
Fput	Lput	First	Last
Butfirst	Butlast		
Item	Memberp		

I have described in the diary in the Appendix the work done in each lesson. Briefly, we began with Turtle Graphics in the immediate mode. They then wrote graphics procedures some of which we saved and expanded on and used in other programs. They experimented in a number of graphical microworlds through which they met a large number of the primitives and programming techniques such as the use of recursion and conditionals. They did some arithmetic calculations and we ended with list processing programs which generated short, random sentences.

3.4 The Way of Teaching

I have discussed in the research review the importance of the manner of teaching LOGO. It is vital to teach it in a way which is consistent with its philosophy. I discussed in my fourth essay the problems of the structuring of a LOGO course. To an experienced teacher this dilemma is of less importance; there are a number of entry points to the language and many subsequent paths to follow. This is at once the solution and the problem; to match the child's needs to what is offered and possible while coping with the problems of working with a group of children. Riordan (1984) offers suggestions for creating a LOGO environment within a classroom which focusses on these problems.

It has been suggested that the effectiveness of the language lies primarily in the way in which it is taught (Wierzbicki 1984, Noss 1983). The secret of successful LOGO teaching appears to lie in knowing when to intervene and when not to. Informed intervention is essential to prevent an unproductive build up of frustration; yet if this intervention is premature and of the wrong type it can be harmful. If we intervene too soon the child has no chance to try for herself and possibly to succeed. Removing the possibility of a child making a personal discovery or reaching a solution unaided is more harmful than allowing frustrations to build up longer than necessary.

Frequently what children need are suggestions on techniques for solving problems (Riordan 1982 and Watt 1983) and time in which to implement them. As teachers we must be aware that children need to be responsible for their own learning, if we force them to abdicate this responsibility they are then dependent upon us which is not a healthy learning situation. Russell (1983) suggested that in training teachers to use computers the program developers should design learning experiences that encourage teachers to reflect on the learning process itself and that these reflections would help them understand how their students learn new material.

LOGO encourages teachers to become learners; of the language, of the learning process and of the ways in which different children respond to and cope with the same situation or problem.

We need to be aware of the different frames of references and awarenesses children bring to the learning situation and of their differing cognitive styles, strengths and weaknesses. Ideally we should capitalise on their strengths, while making them aware that the way in which they work best is not the only possible way(Laridan 1983).

To satisfy these criteria for being a successful LOGO teacher is extremely difficult. While I was aware, from my reading, of what I should and should not be doing I am not at all sure that I succeeded. My years of embedded notions about the role of the teacher influenced my behaviour; fortunately I was in the first stages of pregnancy for this helped me to take more of a back seat than usual. I found that the two most difficult things were to be aware of the child's individual style and to resist the impulse to hurry the children along. They too found the open ended exploration unnerving after all their years of being instructed in a very different way.

3.5 The Means of Testing

a) The CSMS project, Children's Understanding of Mathematics 11-16, attempted to take up where Piaget left off (Hart 1981). They tested and interviewed 10 000 British school children in the following areas; measurement, number operations, place value and decimals, fractions, positive and negative numbers, ratio and proportion, algebra, graphs, reflection and rotation and matrices. Their questions were chosen as far as possible to test understanding as compared with rote learning. They aimed to find out how children coped in these areas, all of which are taught in schools, when the questions are of a type that necessitates thinking as opposed to using a teacher-taught algorithm. The responses should therefore be indicative of the degree of development of the concepts involved.

Based on these tests and interviews in the various areas they have identified heirarchies of thought processes or levels of understanding. They discuss the types of problem and the sort of thought process needed to handle the problems at the different levels. In general a child who is capable of doing say level 3 questions will be able to do level 1 and 2 in that area, and further they found a high correlation between achievement in different areas. The levels go generally from simple to complex and concrete to abstract, enactive and iconic to symbolic, intuitive to reflective. Performance on the tests should thus give an indication of the level at which a child is functioning and if this level changes, the changes should show in the test results.

I decided to use these tests as yardsticks of conceptual development and to give them to the girls before and after the LOGO course. In order to have two tests on the same topic I divided the CSMS tests into what I felt were equivalent halves.

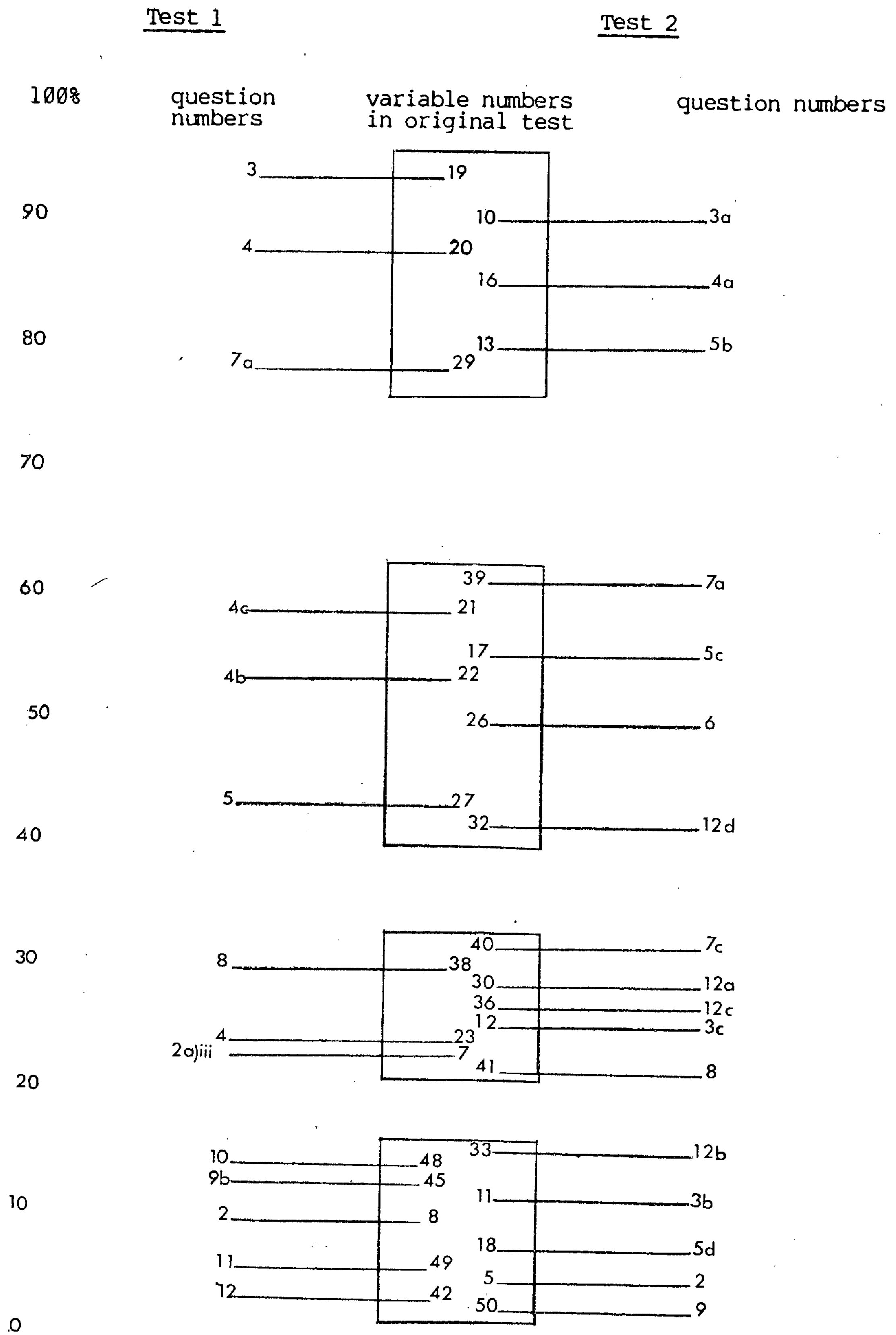
I did the split on the basis of the CSMS classification of questions according to the percentage of children who answered the questions correctly. There were times when questions were integrally linked and had to be kept together but as can be seen on the diagram the division of questions on this basis worked fairly easily. Where questions were unclassified they were assigned to a test either with other sections of the same question or in such a manner as to make both tests the same length.

I gave the whole of standard six the Test 1 in algebra, ratio and proportion, positive and negative numbers, measurement, graphs, decimals and fractions, but only administered Test 2 to the whole standard in algebra, ratio and proportion, and measurement since I felt these were the three areas most likely to have been affected. The LOGO group did Test 2 in all the areas.

The tests were originally used in Britain so I have used the levels of the age group 13+ since that is the equivalent school year.

I have included in the appendix the tests for the three areas analysed and an outline of the division of the questions in the other areas.

The diagram shows the division of the algebra into two halves.



b) Each of the LOGO girls worked through their algebra tests, soon after having written them, in an interview situation which I recorded and transcribed. In the interviews I encouraged them to think aloud wanting to gain an insight into their thought processes. I prompted them when I felt it appropriate, in general these prompts were of the "and then" variety. I tried when appropriate to keep quiet and allow them time to consider what they said or were about to say.

In general, I tried to use the techniques suggested by the CSMS interviewers (Hart 1981).

c) A year after completing the course I interviewed each of the nine girls asking them a variety of questions related to their recall of the content of the course, their feelings about the course, their feelings about school mathematics and their views on learning.

d) Each of the girls kept a journal in which they recorded not only what they did but also how they felt, while we were doing the course.

e) I kept a journal of their general progress, their behaviour, and their attitudes to work.

f) I interviewed Mrs Bank who is the teacher who took over the group when I left and who is now in charge of the computers at the school.

3.6 Plans for Analysis

On the basis of the research and the data available I have considered the environment's influence on learning from three perspectives:

a) we can treat **learning as a product** and measure whether, in relation to specific tasks which we believe are yardsticks, it has occurred.

I investigated this in three ways:

i) I compared the performances of the LOGO group and the comparison group on Test 1 and Test 2 of the algebra, ratio and proportion and measurement.

ii) I analysed the LOGO group's interviews in which they worked through Test 1 and Test 2 of the algebra.

iii) I analysed the LOGO group's answers to the relevant questions in the last interview.

b) we can try to assess whether the **process of learning** was occurring.

This is obviously more difficult to determine. I have used as indicators of its presence or absence the use of language and the articulation of ideas.

I analysed :

i) the difference between the individual girls' responses on the two algebra interviews

ii) the answers to relevant questions, and relevant answers to all questions, in the last interview.

c) we can assess whether, from an affective perspective, the **environment is one in which learning is likely to occur.**

I analysed and synthesised the ideas and feelings expressed and recorded in:

i) their journals

ii) the relevant questions in the last interview and relevant answers to any of the questions

iii) my journal

iv) the interview with Mrs Bank.

The three areas are interdependent, the product depends upon the child's motivation. What a child says depends upon what she knows but also, and more important mathematically, what she comes to know depends on what she says and how she feels about it.

I have created the divisions in order to be able to view the same situation in a number of ways. In an investigation of this size and duration one cannot hope to reach any conclusions about so complex a process as learning - I have tried to gain some insight into LOGO's effectiveness or otherwise and the way in which these particular girls think about algebra and learning.

4. Analysis 1: Learning as a Product

4.1 Introduction

I have approached this in the following three ways.

4.1.1 An Evaluation of Conceptual Development as Measured by Means of Tests and as Indicated in Interviews.

The research seems to indicate that concepts tend to develop in discrete steps and that LOGO is the type of environment which can help to facilitate this development. By mathematical concepts I mean the thought structures, processes and abstractions of experience which are necessary for the handling and understanding of ideas, questions and problems of a mathematical nature. Real learning is based on the development of these concepts through suitable experiences. The CSMS project (Hart 1981) found very little evidence of improved grasp of concepts in their study of children in successive school years which would seem to indicate that in school we are not teaching in a way which encourages real learning.

I am interested to see whether working with LOGO does in fact help children to develop their conceptual understanding of mathematics.

In order to determine whether the LOGO group's development is greater than the comparison group I have compared their performances on the algebra, ratio and proportion and measurement tests.

In order to assess the girls' development individually I have compared their answers to questions of similar facility from Test 1 and Test 2 at all four levels.

4.1.2 An Examination of the Mathematical Behaviour of the Group, and of Individuals in the Group, as Evidenced from Their Written Algebra Papers and the Subsequent Interviews.

I found that they exhibited a number of similar behaviour patterns. In some cases it is possible to discern a change in terms of frequency of occurrence of a behaviour after the LOGO course. I will discuss and give examples of all the behaviour I observed since I feel this gives an insight into the way in which they think.

4.1.3 A Test of Their Recall of the Content of the LOGO Course.

This is based on the last interview in which I asked a number of questions relating to the cognitive aspects of the course.

4.2.1.1 Transformation of the Test Results in Algebra, Ratio and Proportion and Measurement

Algebra

I constructed diagrams of the performances of each group of girls on specific questions of Test 1 and Test 2 of the algebra. I selected 12 questions from Test 1 and 13 from Test 2. Each of the questions fell definitely within a particular level as defined by Kuchemann (1981, 112).

As can be seen on the diagram each question is categorised in two ways:

a) From above with respect to the numbers of letters and/or numbers present within the question which relates to the "structural complexity" of the problem. (Collis 1975 and Halford 1978 discussed in Kuchemann 1981,103).

b) From below with respect to the least complex way in which the letter(s) needs to be perceived in order to do the question correctly. This is not exactly how Kuchemann (1981) used these categories; he examined the children's use of the letters in their answers.

Conclusions

It is visually apparent that all the children cope better on the second test, especially with the structurally more complex questions. There does not seem to be a difference in the improvement of the two groups.

Ratio and Proportion Test

Here I constructed charts of each girl's performance on questions which Kath Hart (1981, 99) found to be possible to classify with respect to her levels of understanding in this area.

Level	Description
1	No rate needed or rate given. Multiplication by 2, 3 or taking half.
2	Rate easy to find or answer can be obtained by taking an amount and then half as much again.
3	Rate must be found and is harder to find than above. Fraction operation also in this group.
4	Must recognise that ratio is needed, the questions are complex in either numbers needed or setting.

I compared each girl with her own performance on the preceding test to decide whether or not she had improved. I then rated the pairs

a) + if the girl from the LOGO group had done relatively better

b) = if they had performed in a similar manner

c) – if the other girl had done relatively better.

Conclusions

From the following table it can be seen that only three girls improved and that only one was from the LOGO group. It seems in fact that the second test was found to be more difficult.

Performance on Ratio and Proportion Tests.

	Test 1				Test 2					
Levels	1	2	3	4	1	2	3	4		
Alitacia	4	2	Ø	Ø	3	1	Ø	Ø	−	−
Bernadette	1	Ø	Ø	Ø	1	1	Ø	Ø	+	
Anzonia	1	1	Ø	Ø	Ø	1	Ø	Ø	−	=
Michelle	1	2	Ø	Ø	2	Ø	Ø	Ø	−	
Bonita	2	1	Ø	Ø	2	Ø	Ø	Ø	−	=
Rosalie	3	3	1	Ø	2	1	1	Ø	−	
Charlene	3	3	Ø	Ø	3	3	1	Ø	+	+
Melanie	1	1	Ø	Ø	3	Ø	Ø	Ø	=	
Ingrid	2	Ø	Ø	Ø	Ø	Ø	Ø	Ø	−	=
Yvette	4	Ø	Ø	Ø	1	Ø	Ø	Ø	−	
Liesle	4	3	Ø	Ø	3	Ø	Ø	Ø	−	=
Felicity	1	Ø	Ø	Ø	Ø	Ø	Ø	Ø	−	
Lynn	2	3	Ø	Ø	1	Ø	Ø	Ø	−	=
Shireen	2	3	Ø	Ø	2	Ø	Ø	Ø	−	
Mandy	3	1	1	Ø	2	Ø	Ø	Ø	−	=
Ruby	4	2	Ø	Ø	3	1	Ø	Ø	−	
Tracia	3	Ø	Ø	Ø	2	Ø	Ø	Ø	−	−
Melaney	1	Ø	Ø	Ø	3	Ø	Ø	Ø	=	
Possible	4	4	3	1	4	4	3	1		
Total no. of correct answers	42	25	20	Ø	33	8	2	Ø		
LOGO group	24	14	1	Ø	16	5	1	Ø		
Other group	18	11	1	Ø	17	3	1	Ø		

Measurement Test

I constructed charts in the same manner as for the ratio and proportion results. The levels were those given by Hart (1981, 20).

Level	Description of groups of items
1	Area found by counting squares. Value of cuboid when only one cube on each layer.
2	Volume can be found by counting cubes when not all of them are shown. Simple applications of the area formula.
3	Volumes of cuboids when dimensions but not cubes are shown. Formula for the area of a rectangle needed. Area of a triangle.
4	Application of area or valume formulae where the forumula has to be adapted eg half units or half formula are used.

Conclusions

The girls from the LOGO group have in only two cases performed comparatively poorly and in four cases have done better. With such a small group I am reluctant to make any general statement but it is worth noting that if one looks at the comparative scores on level 2 questions the LOGO group on Test 1 scored 8/22 correct answers while on Test 2 they score 18/24 and the two questions at level 3 which are answered correctly in Test 2 are from LOGO girls. This would suggest that their understanding of the concepts was comparatively deeper in the second test.

Performance on Measurement Tests.

	Test 1				Test 2					
	1	2	3	4	1	2	3	4		
Alitacia	2	2	1	Ø	2	3	Ø	Ø	=	+
Bernadette	3	1	Ø	Ø	1	Ø	Ø	Ø	–	
Anzonia	1	Ø	Ø	Ø	Ø	1	Ø	Ø	+	+
Michelle	1	Ø	Ø	Ø	1	Ø	Ø	Ø	=	
Bonita	Ø	1	Ø	Ø	1	3	Ø	Ø	+	+
Rosalie	3	4	2	Ø	1	2	Ø	Ø	–	
Charlene	4	2	1	Ø	2	3	1	Ø	–	=
Melanie	1	2	Ø	Ø	Ø	Ø	Ø	Ø	–	
Ingrid	1	1	Ø	Ø	Ø	1	Ø	Ø	–	=
Yvette	3	2	Ø	Ø	2	1	Ø	Ø	–	
Liesle	2	Ø	1	Ø	2	1	Ø	Ø	–	–
Felicity	Ø	1	Ø	Ø	1	1	Ø	Ø	+	
Lynn	1	1	Ø	Ø	Ø	1	Ø	Ø	–	–
Shireen	2	Ø	Ø	Ø	2	1	Ø	Ø	+	
Mandy	2	1	Ø	Ø	1	2	1	Ø	+	+
Ruby	2	3	1	Ø	2	2	Ø	Ø	–	
Tracia	3	Ø	Ø	Ø	3	1	Ø	Ø	+	+
Melaney	1	1	Ø	Ø	1	1	Ø	Ø	=	
Possible	3	4	2	2	3	4	2	3		
Total no. of correct answers	32	22	6	Ø	22	24	2	Ø		
LOGO group	16	8	3	Ø	11	16	2	Ø		
Other group	16	14	3	Ø	11	8	Ø	Ø		

4.2.2 Analysis of Interviews

They frequently performed better during the interview than they had on the written test. They did questions they had not done or they produced more general answers. This was true of both sets of interviews.

Q What can you say about c if $c + d = 10$ and $c < d$.
C Miss, c is less than ... Miss my answer is wrong. No. The answer is right.
Q Yes, c could be 1. And ...
C (Thinks). It could be 8 and 2, 7 and 3, 6 and 4, that's all.
Q OK. Is that it?
C Yes. c can be anything less than 5.

I could often see the comprehension dawning as they discussed the question with themselves, what had been a meaningless array of symbols seemed to take on meaning. Possibly what we see here is the input entering Skemp's (1982) conceptual schema and gaining meaning for the child as it is linked to the ideas she already has.

I do not feel that I prompted them, I think that an informed listener can help them to keep thinking instead of allowing them to switch off.

There was evidence, especially among the weaker pupils, of what I imagine Holt (1964) would call "number grabbing".

Once a number had been mentioned, the variable involved took on that value.

Test 1

Q If you add 4 onto $n + 5$?
B I said that n must be 4 Miss.
Q Why?
B Because they said 4 added onto n can be written as $n + 4$ and you should add 4 onto each of these so I thought it must be 4 so I said $n + 5$ is 9.

Charlene, Ingrid and Tracia said very similar things.
They also took the liberty of putting in their own values when it suited them. This is part of the syndrome Holt (1964) described when he said that any answer would do to relieve the tension and strain they are under to produce one.

B   I just added Miss. I just wrote this one down. I didn't know what to do Miss.
T   I just said make them all u.

There was less evidence of this type of behaviour on the second test.

In some cases they appeared to have their own system which they applied with reasonable consistency.

Anzonia for example believed that whenever a letter had no numerical coefficient it took the value 1.

Q   What is a + 4 if a is 2.
An   I added the 2 and said it was 5, the a is 1 and the four, that's 5.

Others needed reassurance from me that they were doing something mathematically reasonable.

C   No wait, r = s + t. Say r is 10 then s must and... No, I must work this out. So r is bigger than s and t, Miss, can you put in any number for them?
Q   Mmmmmmm

Their handling of variables was very inconsistent. As can be seen in the diagram of their performances on the algebra tests even on Test 2 where they handled questions with letters as objects or as specific unknowns with more skill they did not manage questions which required the idea of a letter used as a generalised number or as a variable.

They often seemed unaware that the letters stood in place of numbers and behaved like them. Some have no idea at all of how to use the variable.

Q   x ---> x + 3, 5 ---> 8, what will 4 map on to?
I   7
Q   and n?
I   (long pause) n?

Their inconsistency with respect to the way in which they treated the letters was interesting.

Q   What is r if r = s + t and r + s + t = 30?
Al   r must be 15 and s + t must be 15

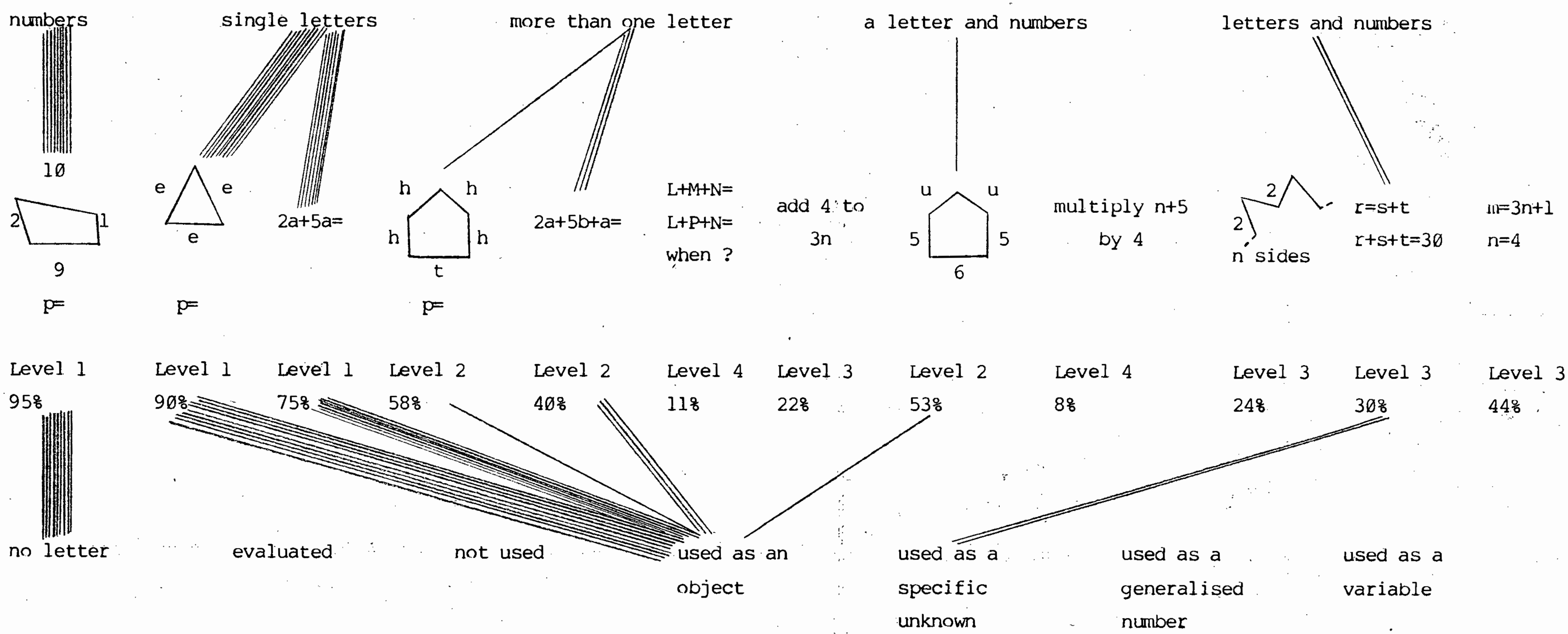
Test 1

In this Test

of

Algebra or Generalised Arithmetic

We Work with



LOGO group

Alitacia —

Anzonia —

Bonita —

Charlene —

Ingrid —

Liesle —

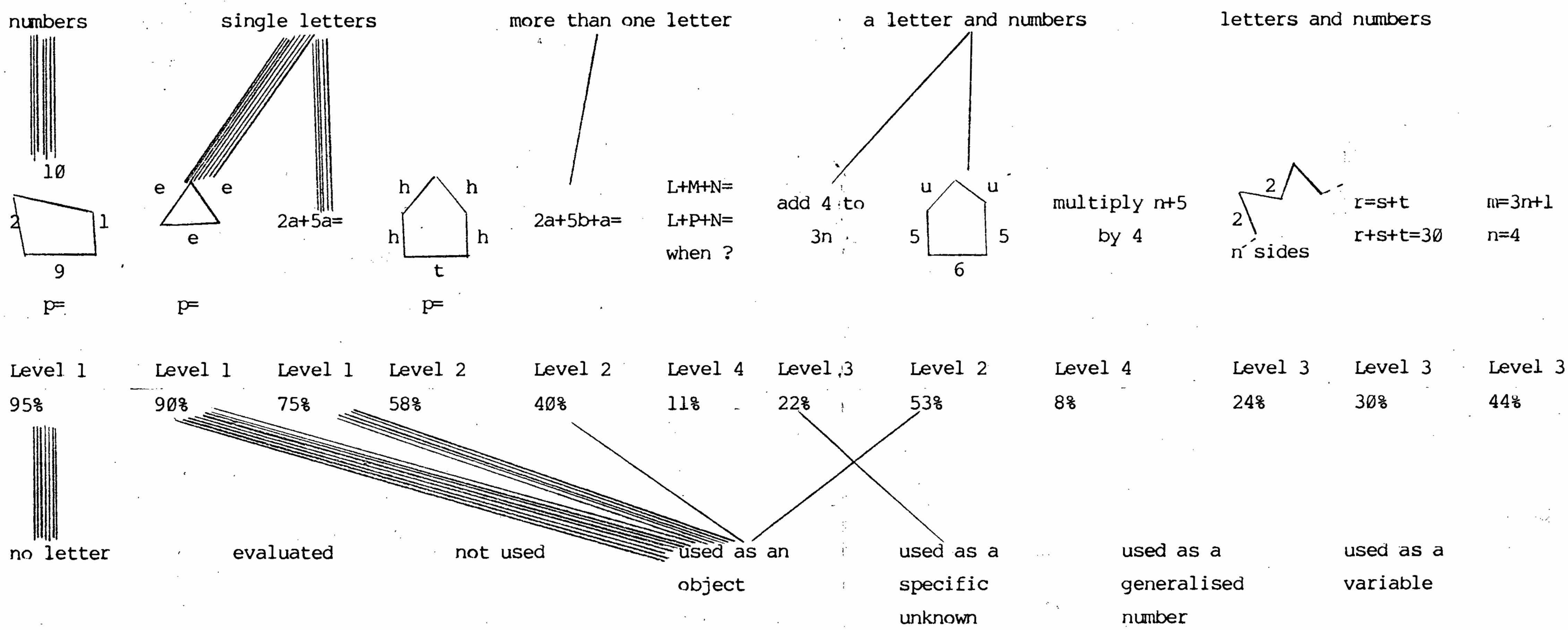
Lynn —

Mandy —

Tracia —

Test 1

In this Test
of
Algebra or Generalised Arithmetic
We Work with

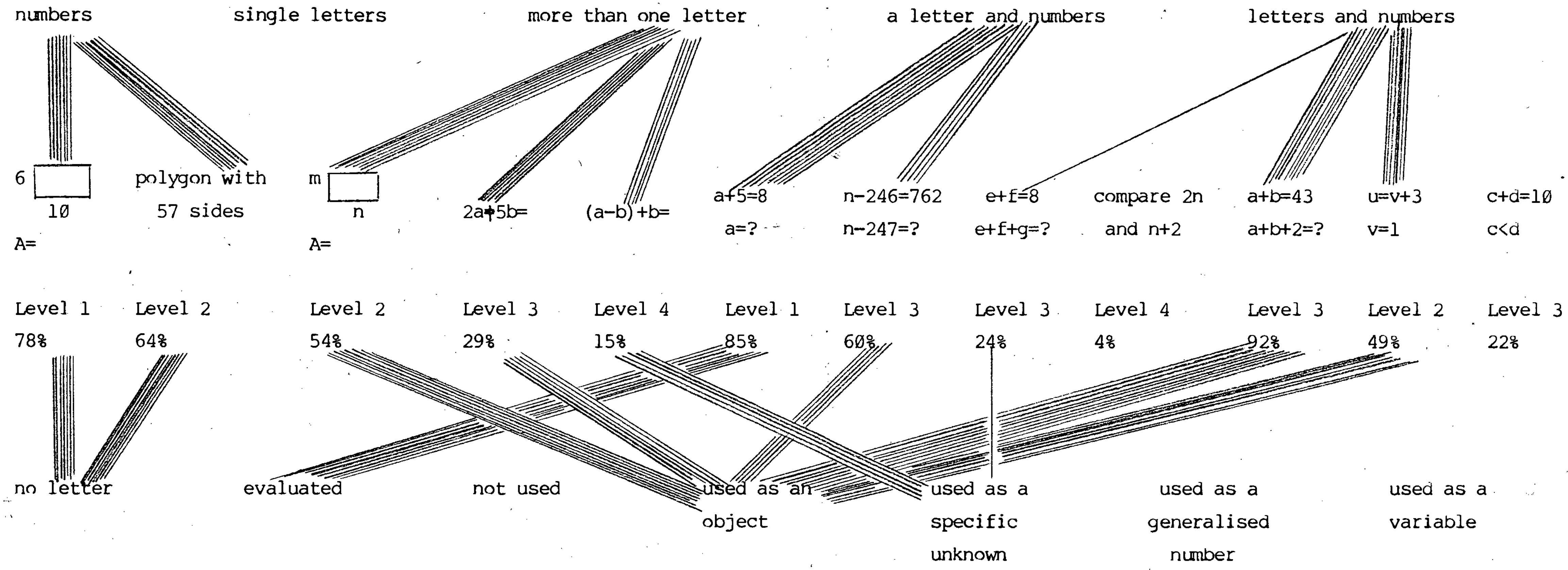


Comparison group

Bernadette —
Michelle —
Rosali —
Melanie —
Yvette —
Felicity —
Shireen —
Ruby —
Melaney —

Test 2

In this Test
of
Algebra or Generalised Arithmetic
We Work with

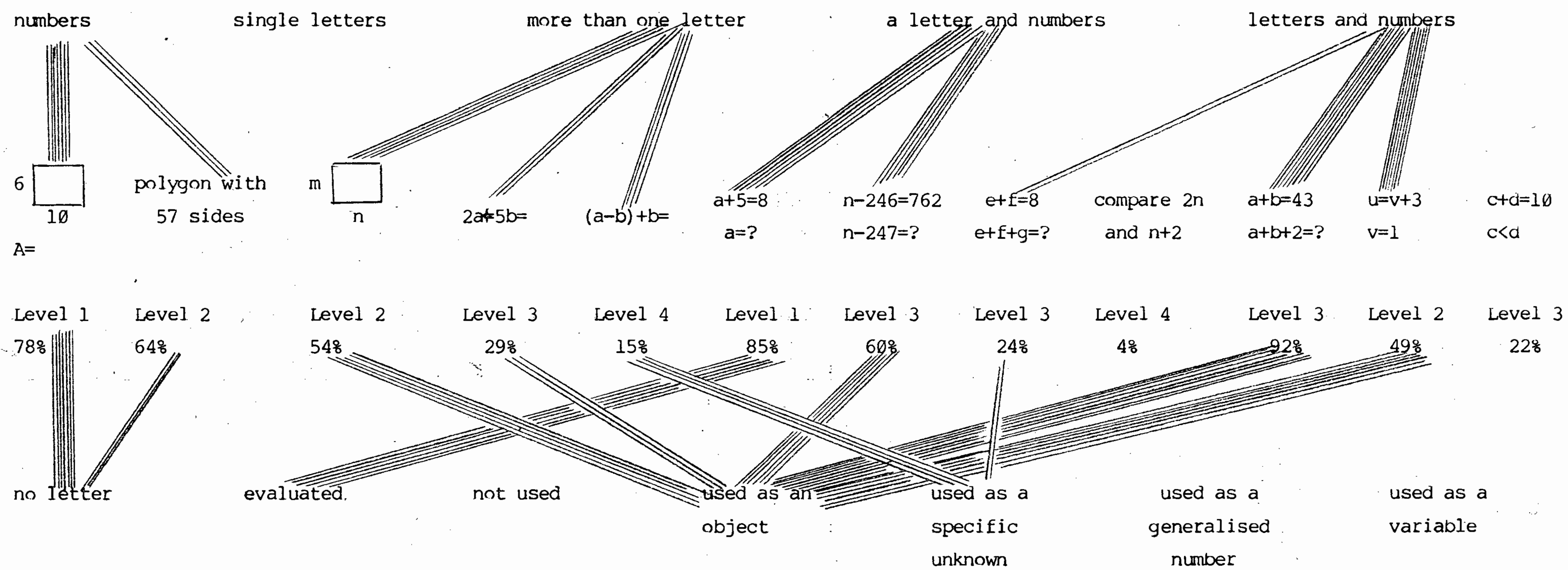


LOGO group

Alitacia —
Anzonia —
Bonita —
Charlene —
Ingrid —
Liesle —
Lynn —
Mandy —
Tracia —

Test 2

In this Test
of
Algebra or Generalised Arithmetic
We Work with



<u>Comparison group</u>

Bernadette —
Michelle —
Rosali —
Melanie —
Yvette —
Felicity —
Shireen —
Ruby —
Melaney —

Q Does A + B + C = A + C + B?
Al Yes, they're just letters in a different order.
Q If John has J marbles and Peter has P how many do they have together?
Al You find out how many J is in the alphabet and add it to P, that gives you 26.

This last strategy is extremely common both in Test 1 and Test 2, Mandy maintained that k - 3 = h and Charlene that n + 3 = q. They saw no flaw in the argument when I questioned them.

This behaviour seems to highlight the fact that they find the symbol manipulation meaningless and that assigning a cardinal value to the letters is perfectly acceptable, even obviously correct.

There was a marked contrast between the reasonable and logical argument which supported their solutions to the word questions and the illogical and inconsistent way in which they did the questions of a purely symbolic nature. In support of their answers they frequently mouthed statements like, "You can't add like terms" and "You must do brackets first".

Q 2a + 5a is?
B I said 2 + 5 is 7 and then there's 2 a's so it's $7a^2$.

In succeeding questions they sometimes apply different rules.

Q (a + b) + a
C That'll be 2ab
Q Will it?
C No, because of the brackets it'll be ab + a. Now 2a + 5b + a will be 3a + 5b and 3a - (b + a) will be 3a - ab because of the like terms.

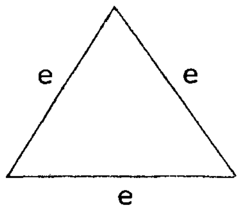
And yet Charlene, in the very next question, after some discussion, realises that r must be 15.

They were a little more consistent in Test 2 but this I feel can be ascribed to the amount of practice they had probably had in this type of question by then.

4.2.2.1 Comparison of the LOGO Group's Performance on Questions of Similar Facility at Each Level from Test 1 and Test 2.

Test 1 Level 1 90%

Q4.



p =

They all found this question easy. Some referred back to the example given.

Test 2 Level 1 87%

Q4.a) What can you say about a if a + 5 = 8?

This question they could all do. A number said, "The a stands for (is in place of) 3".

Test 1 Level 2 44%

Q5 What can you say about m
if m = 3n + 1
and n = 4

Only two of the girls could do this question, Mandy on her own and Charlene after some discussion.

C The m will be 3, n will be 1, Miss
Q Where does it say n is 1?
C Oh, m
Q What does it tell you?
C m is 4, oh, that's 4 so it's 12.....13, m is 13.

Liesle, Tracia and Anzonia made no attempt.

Q Do you know how much m is?
An No (flat and final)

Others did it incorrectly

Q How did you get that they were equal?
Al I think I said 3n + 1 is 4 and n is 4.

Ingrid and Lynn also thought n and m were both equal to 4.

Bonita thought that n was 1; she had equated 3n + 1 and 4 and solved the equation.

Test 2 Level 2 48%

Q6. What can you say about u

if u = v + 3

and v = 1

Seven of the girls managed this questions successfully. Only Anzonia and Liesle made no attempt to answer it. They were all more sure of their ideas. The following is typical of their responses.

Q How did you do this one?

C If u = v + 3 and v = 1 then u is 4 because if v is 1 then 1 + 3 is 4 Miss.

They have developed to a stage where the letter can be used as a specific unknown with ease and certainty. Whether this can be attributed to the LOGO course is doubtful since the comparison group show the same sort of improvement on the written test.

Q8 What can you say about r
if r = s + t
and r + s + t = 30?

Alitacia and Mandy did this question on their own. The discussion with them was not very fruitful since once you know r is 15 it seems very obvious. They said they had "just known" the answer.

Charlene and Lynn managed to work it out after talking about it.

Q What can you say about r?
Ly If r = s + t and r + s + t = 30, um ... (long pause) Should you divide 30 by 3 then you get 10. Then s + t would also be equal to 10 then r = 10, s = 10 and t = 10.
Q And does that make that true?
Ly r = s + t so maybe if you - I don't think it is, because s + t must be equal to r, so I think r will be equal to 20 and s and t will be equal to 5. Oh, sorry, not 20, yes r = 20... (long pause) r can also be like 15 then s and t can be two numbers which add up to 15 and r and s and t will be 30. (very flat - no "aha".)

Charlene took longer to explore the question, also substituting; in the end she was more convinced that r had be 15.

Ingrid had some idea that 15 was a significant number but did not pursue the idea. All that Bonita could offer was, "isn't r the other number to make up 30?" Between them they would probably have solved it.

Anzonia, Liesle and Tracia had nothing to offer.

Test 2 Level 3 22%

Q8 What can you say about c

if $c + d = 10$

and $c < d$

Only Anzonia and Liesle made no response. The remaining seven all gave c a specific value on their written paper but generalised the solution when discussing it. The ease with which they realised that c could vary within limits varied considerably.

Q How did you get c is 4 here?

Ly c could be, it could be 4 and it could be 2, or 3, or 1 or anything less than 5.

I think that we can see the beginnings of the idea of a letter being used as a generalised number. It is very clear that a concept in the formative stages needs discussion in order to be used.

Test 1 Level 4 4%

Q11 If this equation is true when $x = 6$

$(x + 1)^3 + x = 349$

What value of x makes this equation true

$(5x + 1)^3 + 5x = 349$?

Only Charlene was prepared to discuss this question. She could not however reach a solution.

Test 2 Level 4%

Q2 Which is the larger? 2n or n + 2? 4%

The only solution to this was Lynn's. It shows much more developed reasoning than her answer to the Level 3 question in Test 1.

Q Let's look at another one. You think that 2n is the larger.

Ly Ja, because that n + 2 will be a number plus 2 but 2 and n directly next to it means 2 times n.

Q OK. Can you think of any time when this will be the smaller one? When 2n will be smaller?

Ly When this one is smaller than that one?
(Some discussion leading on to...)

Ly (Pause) ... if n was 1 then 2n would be smaller than n + 2.

Q That's right.

Ly And if - mmm - no you can't do that - if it would be more than 2 smaller.

Q And when would it be bigger.

Ly If it was ... equal to 3 or any number from there on.

Q When would it be the same?

Ly Hmmm. If you say 2 + ... If n was now equal to 2 then 2 x 2 = 4 and 2 + 2 = 4 then they'd be the same.

Q Yes. Can you do this one then:
What can you say about b if b + 2 was equal to 2b. Can you tell me how much b must be?

Ly Umm. That will also be 2.

Q What's the difference between this question and that one?

Ly This one has a b and that an n.

4.2.3 A Third Investigation of Learning as a Product

Later I spoke to all the girls again. I asked a number of question relating to the cognitive aspects of the course.

1. What is a turtle?
All nine thought this a strange question to ask since they remembered it so well - eg "Oh yes - a triangle of light that draws what you tell it to".

2. Do you remember what procedural programming is?
Two said "no", four said "It's when you program the computer for a word", three gave an example of a procedure: "Like when we said 'TO SQUARE'".

3a) What would this draw?

FD 50 RT 120 FD 50 RT 120 FD 50

3b) Could you write that so that you needn't type so much?

3a) Two didn't know. Four said "A square - no - no - it can't be a a square - oh yes - a triangle." Three said without pause "A triangle."

Bonita was one of these three and yet we had the following discussion:

Q If you draw an equilateral triangle in class what are the angles?

B 90 degrees - oh no - not 90 degrees - um, um, I must work it out, 180 divided by 3, oh, it's 60 degrees.

Q So why have we got 120 degrees here instead of 60 degrees?

B Because say now you're facing this way and you want to turn to get to the next place you have to turn that much. [with appropriate gestures]

Her calculation of the angle seems purely instrumental while her understanding of the turtle's heading seems to have been internalised in a meaningful manner. This supports Papert's statements about the learning power of the turtle's body synonicity.

3b) Three made no attempt. Two could not sort out their ideas. Four used the Repeat command without hesitation. Eg

```
FD 50
REPEAT 2 [RT 120 FD 50]
```

4. What would this draw?

REPEAT 4 [FD 60 RT 90]

Eight said with little or no hesitation "A square". Tracia thought it was a rectangle.

5. What other commands do you remember?

They collectively recalled CS, HT, ST, LT, RT, FD, BK, REPEAT, PU, PD, PRINT, LOAD, DELETE, CONTROL KEYS for editing and RANDOM (Lynn).

6. How do you teach the computer words?

Only Anzonia could not respond meaningfully to this question. Most of them gave an example, "We said TO SQUARE and then we told it what to do and afterwards we only had to say SQUARE".

Four mentioned that it was possible to draw squares of different sizes if you had inputs.

Three (Lynn, Alitacia and Charlene) mentioned that one was then in the edit mode.

Six remembered teaching it to draw shapes; hexagons, houses with windows and doors, squares, circles and triangles and a "doily" (Bonita). When I asked Tracia how we drew the circle she said without hesitation "Ohh, RT 1 FD 1 REPEAT 360 times." Her knowledge really appears very spiky, with peaks of insight and hollows of ignorance.

Only Lynn remember the GOSSIP and CHAT programs.

8. Have you done any more LOGO?

Only Lynn had. She had been helping to teach it, competently according to Mrs Bank who is in charge of computer education at the school.

9. Have you done any other computer languages?

Four have done more work on a computer. They said, "We just wrote down notes and then we learnt the keys and wrote our name in coloured hearts."

10. What is a computer?

"A machine which records things and will only do what you tell it to do in a program. It stores information. You can use it to learn from and to find out how good you are at something."

11. If we have a procedure

```
TO POLY :SIDE :ANGLE
REPEAT 1Ø [FD :SIDE RT :ANGLE]
END
```

How do you use it?

They all, except Liesle, saw that it needed two inputs and remembered that if one did not give them, the message "not enough inputs" flashed on the screen. Their discussions showed that they clearly remembered using the program.

B We used it to draw shapes by putting in amounts.
Q How would you draw a hexagon?
B (with help) You would put in 6Ø for the angle and any side.

I We could draw squares if we made the angle 9Ø degrees.
Q Show me how.
I (drawing in the air) You'd go round 1Ø times.
Q What if the angle were 3Ø degrees?
I It'll turn less
Q And...
I (pause) and it won't close up

4.3 Conclusions and Observations

On the basis of the comparison between the LOGO girls' and the comparison group's performances on the written algebra tests it is not possible to say that LOGO was more effective in teaching the concepts involved in the time allowed. Nearly all the girls were, on Test 2 as compared with Test 1, better able to handle questions of a greater structural complexity but they could still generally only work with the letter used as an object. Only in a few cases did they manage to use a letter as a specific unknown. On the first test they were all successful only with Level 1 questions, a few girls coped with Level 2 questions of an iconic and rote-learned nature. On the second test they coped with Level 1 and Level 2 questions and a few girls were functioning at the third level. This improvement can probably be ascribed to a few months' additional algebraic experience in the classroom with questions such as these.

The level at which their algebra may have improved would need to be based on their existing concrete-pictorial or iconic understanding. This level is conceptually a long way from visualising a letter as a variable or even as a generalised number (Kuchemann 1981). As we have seen they did not use, of their own volition, the idea of a variable in programs, they only gave numbers as inputs to microworlds. One wonders if they eventually would have and what effect, if any, it would have had on their algebra.

It appears they all found the second test on the ratio and proportion more difficult. It would be valuable to compare the two halves experimentally in order to determine whether it was. The scarcity of questions classified with respect to levels and the children's generally poor performance makes comparisons extremely difficult.

The measurement results, while being far from conclusive, are interesting. It does seem as if the LOGO girls are, in the second test, functioning comparatively better with questions beyond Level 1. While this may be purely coincidental in view of the sample size and the manner of the test construction, this is the area in which the LOGO experience which the girls had would be most likely to have had a measurable effect.

In focussing my attention on so small a group of children I became aware that children who get the same mark on a test do not necessarily understand the same ideas. Nor do they do the same questions in the same manner. This has been discussed by Vergnaud (1979).

I feel that the algebra interviews, although extremely time consuming, were far more informative than the written tests. When we mark a written test a wrong answer does not often lead us to a deeper understanding of the child's though processes, in an interview situation it frequently does (Clements 1980). I can see from the response of the children to my prompts how much benefit they derived from an informed, uncritical sounding board.

It is unfortuante that I did not interview the comparison group. Consquently I can only make statements about the general mathematical behaviour of the LOGO group and indicate areas where their algebra seems to have improved or their understanding deepened. I gained a great deal of insight into the workings of their minds, the problems that they have with variables and symbols in general, and of the importance of language in facilitating development. The behaviour I observed strengthened my conviction that an environment like LOGO which provides mathematical experiences and encourages discussion is necessary for grasping and gaining control of many of the concepts underlying high school algebra.

Howe et al (1980) have shown that learning LOGO can conbtribute to the learning of some specific concepts in algebra. While I have not demonstrated this, the difference between the girls' inconsistent and unfounded algebraic notions and the clarity with which, a year later, they remembered the ideas and techniques we had used in LOGO argues that it is a situation which encourages real learning and that the problem lies in relating it to the content of the school syllabus ie in using the awarenesses that are generated in LOGO in other mathematical areas.

One of the main reasons for its value lies in its emphasising and encouraging communication, and in allowing children to use their own colloquial language to grasp abstractions (Schoen 1984, Donaldson 1978, Skemp 1982). Press (1975) has discussed the importance of helping children relate their common-sense English-language understanding of a system to an abstract mathematical description.

It was obvious from the interviews that the children gained insight into the questions through discussing them. That it is vital to discuss mathematical ideas was evidenced by the fact that the symbolic questions were apparently meaningless to the girls. They coped in a more logical and reasonable manner with those questions, even the more cognitively demanding, which were phrased in words or human terms.

Their inconsistency in handling variables related to the fact that letters are as Wagner (1983) said, like numbers and words, only different and that they do not know when they are which. She has discussed in detail the ideas used in variables which we as teachers need to articulate and be aware of.

I became aware through the interviews that we do not always function at the limit of our competence. At times they exhibited behaviour of different types in relation to the same, or what seems to us to be the same, concept,in consecutive questions.

It seems that while there are conceptual leaps of understanding, these leaps are more the sum of a number of insights, and that the development of these insights needs the support of informal discussion.

5. Analysis 2: Learning as a Process

5.1 Introduction

Howe et al (1980) showed that their pupils who did LOGO were subsequently able to argue more sensibly about mathematical issues and to articulate mathematical difficulties more clearly. Although school mathematics focusses more on the products and algorithms of mathematics than on the process and creative problem solving, I find it difficult to believe that becoming better at the latter is not of benefit to the former.

It has not been shown however to have a marked effect. Perhaps we are too impatient, wanting tangible results overnight while knowing that this is not the way that knowledge grows. Possibly the more project orientated subjects of the curriculum stand to benefit more directly from LOGO. I am also interested in whether LOGO does, as Papert claimed, encourage pupils to think about the process of learning itself, how they learn in particular, and whether it influences their approach to problem solving.

I have approached this chapter in two ways.

5.1.1 A Comparison on an Individual Basis of the Girl's Use of Language in the Two Algebra Interviews.

I did not have a group with which to compare the LOGO girls' performances on the algebra interviews so have compared them with themselves, before and after the LOGO course. I have considered their use of language and have looked for examples where, after the course, the girl:

-- expresses herself in a more articulate manner
-- is better able and more willing to discuss her thoughts
-- is clearer and more logical in her explanations
-- is coping better with the conflict situation in which she sees her written answer to be incorrect
-- has a more positive attitude to trying to understand the question, and is more prepared to justify her answers
-- is more prepared to offer suggestions and is looking within herself for answers instead of always asking whether she is correct.

If there are examples of this sort of change I believe that we can say that the child is learning in the sense that her cognitive structures have become more densely linked. More importantly I feel that these are the sort of changes that make future learning more likely to occur. One might say that these are indicators that the child is opening herself up to ideas, that her mind is a more fertile environment.

This type of data is obviously extremely difficult to quantify and is very open to differing interpretations. Given however the vital role that language is seen to play in learning, I feel that it is a fertile field for exploration.

5.1.2 Analysis of Questions Relating to the Process of Learning in the Last Interview.

I have also analysed their answers to the relevant questions in their last interview. These questions have to do with the process of learning and problem solving.

5.2.1 Individual Analyses

I have dealt with the girls who appear to have gained least from the course first.

Anzonia

Her maths was very weak and showed very little improvement. She became discouraged as the other progressed faster than she did and was at times bored with the course.

One of the most noticeable things about her was that she did not listen to herself, or perhaps she knows she sounds meaningless but to her all mathematics sounds like that so it does not concern her. John Holt (1966) maintained that the difference between the intelligent pupils and the others was that they demanded consistency of their answers.

She generally applied the rule that a number without a numerical coefficient has a value of 1.

Test 1 Test Item

Q What is a + 4 if a is 2

An I think I said a + 4 is 5 plus a is 6 minus 4 is 2, it should be 2.

She occasionally expresses herself well when the question is extremely simple

Test 2 Q3a 92% Level 1

Q How did you get 45?

An Here they said a + b is 43 and then a + b + 2 is 45 because 43 and 2 is 45.

Generally she is of interest for the manner in which she errs, often consistently. She said, for example, "2 goes into 7, 14 times and 5 goes into 3, 15 times" without it seeming to worry her at all.

Liesle

Liesle, like Anzonia, was extremely unsophisiticated algebraically. She could only function efficiently with specific numbers or with the letter as an object in questions of low structural complexity. Where there is an example given she is more relaxed and simply uses it as a pattern to be copied, the value of which is questionable. She does not appear to have thought at all in answering these questions.

Test 1 Q3 95% Level 1 and Q4a 90% Level 1

Q How did you get 22?

Li I added Miss, like they did.

Q And the 3e?

Li Added like the g's Miss.

She nearly always has an answer. It is often, like Anzonia's, based on a system of her own. Her system seemed to be, like many children I have taught, to write down the letters which had been mentioned in a shorter form than that in which they are given, which I suppose is often what we appear to do, and is not always wrong. Obviously she has no idea that these letters are place holders and stand in place of numbers.

Test 1 Q7a 75% Level 1 and Q7b

Q Can you write 2a + 5a more simply?

Li $7a^2$

Q (a + b) +a?

Li That's aba

Q Why?

Li It's shorter Miss.

I saw no noticeable change in her level of development or mode of expression, perhaps I was looking at too high a level.

Tracia

This is a strange child. The staff seemed uniformly convinced that she was extremely slow witted. They went out of their way to tell me that. I have a feeling that she might be a visual and audial learner who does not relate easily to written work.

There was a marked change in her attitude to understanding the questions in the two tests.

Test 1 Q4c 53% Level 2

Q How did you get your answer?

T I just said make them all u.

Q Why?

T (Puzzled silence) I don't know.

Q What's wrong with 5 and 5 and 6?

T (More puzzled silence.)

She seems more prepared, after the course, to make an effort to understand the questions and to justify her answers.

Test 2 Test item

Q What is a + 7 if a is 2?

T They said a is equal to 2, so I said 2 plus 7 gives 9.

She copes better, on both tests, when she hears the question read aloud.

Test 2 Q5 44% Level 2

Q You gave no answer to this one. (I read it to her.)
Does it mean anything to you? (I re-read it.)
So what will it be?

T 3 times 4 is 12 and one is 13.

Q Why can you do it now?

T Now I understand it.

In the first interview most of her answers are short and not very logical.Her answers on Test 2 were less terse and more assured.

Test 1 Q7d

Q   3a - (b + a)?

T   Miss I think I did it like the first two with that x.

Q   So ...

T   Miss, you can't add unlike terms so I just left it.

In the following question she seems to have the idea of a letter as a generalised number. Yet in Question 7 she reverts to the alphabet equivalent and says that k - 3 is h. It seems that while one is grasping an idea one's hold on it is fragile and if an older, more firmly embedded way is for some reason suggested, one tends to choose it.

Test 2 Q3c 24% Level 3

T   It can be anything because e and f and g can be any numbers.

Q   Yes.

T   So if I had to take g as 1 then it will be 9 and if I take it as 2 then it will be 10.

In the seventh week I have the following note in my journal:

"Even Tracia seems much more convinced of the correctness of her answers, and Charlene is volunteering more.

Charlene: LEFT-TRI might be REPEAT 3 [FD :60 LT :60]
Tracia: No, it wouldn't make any difference when you draw a triangle whether you turn left or right, it must be 120 degrees each time."

Ingrid

In the Test 1 interview, most of her answers are very short, most of her explanations are clearly reflections of instrumental meaning and rule orientated learning.

Test 1 Q7e

Q   a + 4 + a - 4, what can you do with that?

I   4 - 4 is 0.

Q   And the a's?

I a^2

Q   Is that how you write it?

I   2a

It seems that my questioning her made her think she was wrong so she said it in the other form which occurs in this type of question. Obviously neither a122 nor 2a means very much to her.

In the second interview she is a little more prepared to discuss her answers.

Test 2 Q

Q How did you decide which was biggest and which was smallest?

I It's, like you are on a number line and you get negative to positive. n - 7 is the smallest and n + 4 is the biggest.

This referring back to a concrete situation seems to me to be a positive step towards seeing mathematics as a reasonable and logical subject. In the first interview she kept asking me whether she was right.

She does not pursue her ideas in either interview. In the question where r = s + t she knew that 15 was significant but could go no further. These are the sort of links that Vergnaud (1979) says are so difficult to forge. Again and again in these interviews I found that when they had a chance to discuss their ideas they seemed to make this sort of progress but that when they were alone with a written question they did not.

She also tended to "number grab" in the question about adding 4 to n + 5.

Bonita

In her first interview she said "I just thought" and "I just wrote down" a great deal. She also frequently asked for my opinion. We see this in the following exerpt.

Test 1 Q4 58% Level 2

Q OK here

B Pause (her answer is 5 ht) I just thought up, I just wrote down ... I don't know what to do, Miss.

Q What do you think you should have done ... how do you get the perimeter?

B Pause. Can't you say ... length times, no length plus breadth, Miss.

Her response to a question of similar facility in the second test is very different. Here she expresses herself very clearly indeed.

Test 2 Q3b 60% Level 2

Q Now this one.
B They say n - 246 is 762, and they say n - 247, so that was 1 more, Miss, so surely the answer will be 1 less than the one that had so I got 761.

In the following Level 2 and Level 3 questions in Test 1 she is unsure of herself and her answers are extremely tentative.

Test 1 Q5 44% Level 2

Q And here. What can you say about m if m is 3n and 1 and n is 4. Can you work out anything about m?
B (Long pause) Mmmm ... no. You say 3n plus 1, is it one Miss, and n = 4 ... um. I can't work it out.

Test 1 Q8 3% Level 3

Q What can you say about r = s + t and r + s + t = 30; Can you tell me anything about r?
B (Long pause) Isn't r - um - the other number that makes up 30 because that's 30.
Q Yes. Can you tell me how much r is?
B Uhh (pause) no.

Her certainty in her use of language in the following comparable question in Test 2 contrasts sharply with that used above.

Test 2 Q6 49% Level 2

Q What can you say about u if u = v + 3 and v = 1.
B u must be 4 Miss.
Q How did you get it?
B Because Miss, if v = 1 and u = v + 3 and v + 3 is 4 and u equals that.

Test 2 Q8 22% Level 3

Q What can you say about c if c + d = 10 and c < d?
B Miss, I wrote that c can be equal to 4.
Q And can you say anything else, that's right.
B And d can be equal to 6.
Q Yes.
B And they add to 10 if c is 4.

Q Does c have to be 4?

B No, it can be any number - it can be 5.

Q Can it be 5?

B No it can be less than 5 Miss - a number less than 5.

Q So it could be?

B 2, 1, 3, 4.

Here we also see one of the many examples of the fact that when they get a chance to discuss the questions they are able to find a more general solution to the problem.

Alitacia

She is Tracia's twin sister. She is a quiet, hard working child, who remembers what she has been taught. Her understanding seems however to be, in many cases, purely instrumental.

Test 1 Q

Q When will A + B + C = A + C + B

Al It's always the same.

Q Why?

Al It's just a different order. that's the commutative law.

Q What do they stand for?

Al You can't add them.

Q Why?

Al They're not like terms.

Q What are they?

Al Letters.

In the second test the idea that a letter is a place holder seems to be developing.

Test 2 Q3c 24% Level 3

Q How did you get 12 for e + f +g?

Al I thought g could be 4, and so could e and f and that makes 12.

Q Must g be 4?

Al No, it actually can be anything.

Q And what about e and f?

Al They must be 8.

Q So ...

Al You could say g more than 8.

Q Does it have to be more?

A Yes.

In the first test there was also evidence that while she was doing the correct operation in her head eg adding the sides of a shape to get the perimeter, she did not translate this into the correct symbolic form which seemed to have no real meaning for her.

Test 1 Q4b 58% Level 2

Q How did you get 4ht?

Al You add the 4h's and the t.

Q And you get 4ht?

Al Yes?

In Test 2 she showed signs of an increased depth of understanding and better communication between the conceptual and the symbolic schemas.

Test 2 Q5c 54% Level 2

Q And for this one?

Al nm

Q What's that mean?

Al n times m

Q What if you wanted the perimeter?

Al Miss it would be 2n's and 2m's.

Q How would you write that?

Al (Pause) 2n + 2m.

In the first test she seemed scared to say anything wrong.

Test 1 Q4d 24% Level 3

Q What is the perimeter of this one?

Al (Pause) I don't know.

Q What do you think it ought to be?

Al I don't know.

She had probably never seen a shape like this and had no rule on which to base an answer.

In the second test she seemed more prepared to try and to step into the unknown. She did not seem as nervous of failing.

Test 2 Q11

Q If f = 3g + 1 what happens to f if g is increased by 2?

Al   It will get more Miss.

Q   Yes. Can you say anything else?

Al   It's like the last question. But that was b. I think it will get bigger by more than 2 times.

Q   Do you know by how much?

Al   Maybe 5 or 6 Miss.

Charlene

In Test 1 she tended to seek reassurance that her ideas were sound, or that what she was going to do was the right thing or at least permissible.

C   Miss, must we find out what x + 2 is equal to?

C   Miss, can you put any number in?

C   Miss, can I put in the amounts I think?

With this reassurance she tried questions of a high degree of difficulty eg Q11 from which all the others shied away. I think what helped her is that she always read the question aloud to herself while many of the others just looked at the paper.

Again and again I saw how she coped better when she talked about a problem.

In both tests she coped better when she discussed a question but whereas in Test 1 she needed to discuss the Level 2 questions in Test 2 she needed to discuss Level 3 and 4 while the Level 2 she found easy.

Mandy

Mandy's first interview was extremely quiet. In parts I could barely hear her. Many of her replies were monosyllabic, she was reluctant to discuss questions especially when she felt she had written down a wrong answer.

Test 1   Q7d

Q   3a - (b + a) ... Why can't you simplify it?

M   Miss ... the brackets can't ...

She had some strange idea about brackets which was still with her in Test 2.

Test 2   Q2   4%   Level 4

M   n + 2 is unlike terms so 2n is more.
Q   Always?
M   I think if you put it in brackets then you can multiply. I think its equal if it's in brackets.

I wonder if she means n(+2)?

In Test 2 she could cope with finding a mistake on the written paper, in general she was happier to discuss her thought processes.

Test 2   Q1

Q   Which is the longest?
M   I think n + 4 because n - 7 is less than n + 4.
Q   Why did you say n was the smallest?
M   n is to the power 1.
Q   Is n the smallest?
M   No, n - 7 is.

Lynn

Lynn benefitted in many ways the most from the course. It is very clear in the comparison of Test 1 and Test 2 that her confidence in her ability to solve a question increased.

Test 1   Q9b

Ly   It's never true. The m and p are different letters.
Q   Never?
Ly   I can't see how it can be.

Test Q   Q3c   24%   Level 3

Ly   Well, if e + f is 8 and then we have to add g then we'll get g more so I could just write g more than 8 which, oh yes, I could say 8 + g.

She could not cope with the following Level 2 question in Test 1. The comparable question in Test 2 she obviously found very easy. The difference in the certainty and consciousness with which she expresses herself is marked.

Test 1   Q5   44%   Level 2

Q   m = 2n + 1, n = 4. What can you say about m if m = 3n + 1 and n = 4?
Ly   Umm. They're equal.

Q What's equal?

Ly (Pause) Let's see m = 3n + 1. So that m should actually stand for any number there because it can be 3 multiplied by a number.

Q OK.

Ly It will be + 1.

Q So what number is it? What do they tell you?

Ly I think maybe it can be 3 x by 1 which will give you 4 so m and n are equal.

Test 2 Q6 49% Level 2

Q What can you say about u if u = v + 3 and v = 1?

Ly It's 4.

Q How did you get that 4?

Ly I subtracted 3 from the ... oh, I added the 3 and the 1 and it gave me 4.

I have already discussed her handling of the question in Test 2 concerning the relative size of 2n and n + 2. I cannot compare her handling of a comparable question in Test 2 since she did not attempt one.

5.2.2 Analysis of Data from the Last Interview

In their journals and their answers to the following question which I asked them in their last interview (February 1986) a number of themes recurred. They are concerned with matters other than mathematical that they felt they learned from doing the LOGO course. I have considered them in this section since I feel that they are to do with the processes of learning and problem solving.

Questions asked

1. What were the good points about the LOGO course?
2. What were the bad points about the LOGO course?
3. How did you feel when things didn't work out?
4. What did you do then?
5. How can you solve a problem? For example how did you draw a house with doors and windows?
6. What did you learn from the course?
7. Did it help you?
8. Why did the computer sometimes not do as you expected?
9. How do you learn?
10. a) Do you learn when you make mistakes?
    b) Do you learn when you get things right?
11. How do you feel when you learn something new?
12. Did you enjoy working with a partner?

They felt that the course had helped them understand their school mathematics better, especially their geometry. This ties in with a number of research findings. (Watt 1982) Most of them said this at some stage or another. From my journal observations regarding their progressively better handling of the turtle's heading I would say they are correct.

Sorting out the heading of the turtle when stringing procedures together highlighted the need to plan their programs. Lynn: "First we had to plan on paper what we wanted to do. It's important to use your brains and to think before you do something." We tried to break the problems down and to do them "step-by-step" using simple procedures as building blocks. They clearly remembered that this was how we had drawn a house with windows and doors! To quote Anzonia: "Mrs Paterson said to try to break the problem down and I think that's a good idea."

A number of them felt that working through problems in LOGO had helped them develop patience and perseverance. Lynn said: "I was usually very impatient; if something was wrong I would not look over to see what I did; I would just ask someone else." I felt that they developed this attitude at least partly in response to the procedural structure of LOGO which facilitates debugging. This encouraged them to keep trying, to go home and to think about it, not to just give up.

This contrasts with their attitude to problems in "school" mathematics. In mathematics classes if they could not do a problem they asked either the teacher or another child to show them how to do it. They all said that was how they coped with wrong answers in class.

On the other hand, working with LOGO encouraged them to try different ways of doing problems. As Mandy said, "It was exciting to find different ways to do things, without using books." Charlene said, "It was not just writing down and working out like maths, the computer made you want to think about things for yourself because when they worked out it was so great." She continued, in response to my asking whether she had ever got excited, "Oh yes. Remember when I had to draw that house - I think it's the greatest thing - when we got that roof, me and Lynn working together, we got that triangular roof, I thought ohhhh!"

LOGO gave them what we all need to keep trying, positive reinforcement.

When they did ask someone for help it was one of their peers. As Ingrid said, "It's better when you ask your friend because she understands your problem and doesn't just say it all over again like teachers often do." In class too there is the problem of getting the teacher's attention. Ingrid said, "I sometimes think I'll go mad if I don't get an explanation." They also enjoyed working with a partner because, as Anzonia said, "We could pool our ideas when we got stuck."

It was very clear that in a mathematics class the pupils are dependent on the teacher to facilitate their development. In this environment they were free to interact as often as they pleased with the computer and each other and they found this very satisfying. A number of them felt they had grown more confident as a result of learning LOGO, because as they said, "We had a chance to try our own ideas and see if they worked."

They also felt that they had learned to cope with making mistakes in a more positive manner. Lynn: (early in the course) "I was very disappointed because in half an hour we did nothing except plenty of mistakes." And later in the course she said "It helps you to think out problems for yourself and find out what your mistakes are." And Alitacia: "I made a few mistakes but I learnt by my mistakes." She said this three times in her journal. They were almost unanimous that one learns when one makes mistakes. As Liesle said: "If you get something right then you know how to do it and you're not actually learning." This I think summarises much of what I have tried to say. Lynn too was articulate on this topic, "When you get it right you learn because it encourages you to go on but when you get it wrong you feel you have to get it right!"

Their attitude to learning new things was very positive. They found it exciting and stimulating and were not nervous about it. Lynn said, "The lines on the screen fell on top of each other. Maybe the problem will be solved when we learn more. Maybe we will find out why it did that." They said one of the good points about the course was that they were not bored and were learning new things. Alitacia, "I felt good and I felt I had learned something new."

When they did learn new things they could according to Charlene, "use the new ideas with the old problems because everything was connected". What an endorsement for a learning package!

5.3 Conclusions and Observations

The girls exhibit a number of behaviours which I felt would act as indicators that they were learning, and that they had a more positive attitude to making sense of the written questions with which they were presented. Since I have no group with which to compare them none of this is in no way conclusive, none the less I found the changes positive and in the light of the feelings they expressed in the last interview, hopeful.

They were generally open to discussion in the second interview and more prepared to justify their answers. They were less dependent upon my saying that they were correct or that they could proceed with what they were attempting.

There was more concise and reasoned argument to support their ideas and answers in the second interview than the first especially from the girls with higher Intelligence Quotients.

They were prepared to discuss their ideas and answers both with themselves and with me. It may be that our having worked together encouraged them to be more prepared to expose their thought processes, it could not, however, have accounted for the increased certainty with which they expressed themselves.

It was apparent that in situations where the questions were of a symbolic nature they frequently fixated on what they could recognise, eg the n and the 4 in question 4 of Test 1. In the same way as Bill Cosby, listening to his wife speaking Spanish and understanding nothing, says "I just listen for my name!"

Where there was an example given they frequently used it as a pattern, short circuiting any thought processes. The value of giving examples and then questions which exactly match them seems questionable.

The behaviour they exhibited in dealing with purely symbolic questions shows clearly that they are rote learned at best and are usually meaningless. Where the letters used are capitals there seems to be a strong tendency to view them either only as letters or as having the same value as their position in the alphabet.

Their behaviour does not demonstrate the effectiveness or otherwise of LOGO, it does however highlight the need for an alternative method of introducing children to algebra.

From their responses to the questions in the last interview it appears that their attitude to learning and problem solving was positively influenced, they perservered longer and were more confident. They felt less dependent upon a teacher for help in solving the problems. They coped better with mistakes and felt that they learned from them.

In the next chapter I will discuss how they felt about themselves and LOGO.

6. Analysis 3: The LOGO Learning Environment from an Affective Perspective

6.1 Introduction

O'Shea (1982, 247) said, "I have never been in an ordinary mathematics classroom which bustled with the activity of happy pupils in the way that classrooms equipped with LOGO computer equipment do." I have tried to assess whether, in addition to being happy, they were involved in their work and to compare how they felt doing LOGO with how they felt in a conventional mathematics classroom. If what we learn depends upon what we know then if we are involved in the learning process it indicates that we are putting ourselves into it which should facilitate learning. If in addition we are enjoying doing so then we are likely to continue.

I asked the children to record their feelings in their diaries. At the outset they considered this a strange thing to do in mathematics but we sorted out the problem by considering the journal as a diary. I emphasised that what they wrote would only be read by me and that I would not discuss it with other pupils and teachers in the school. There were times when I had to nag them to get it done but I felt that what they wrote was honest; the themes which recur in the journals were reflected in the interview and in Mrs Banks' views. In the interview a number of the questions are based on the ideas suggested by Hoyles (1982).

I have synthesised the data from the various sources under these themes.

6.2 Analysis and Synthesis of Data.

During the interview I asked the following questions.

1. How did you feel at first?
2. What advice would you give a little brother or sister who had a chance to do a Logo course?
3. Did you get bored?
   excited?
   frustrated?
   depressed?
   annoyed?

4. What do you remember best?
5. How did you feel when
   a) things didn't work
   b) things did work?
6. How do you feel in class when
   a) you can do the work?
   b) you can't do the work?
7. In class do you ever get bored?
   excited?
   depressed?
   annoyed?
   frustrated?
8. How do you feel when you learn something new?

One and all felt excited at first. Some of this excitement was due to having to travel up to the university for the first few weeks. They were sorry when we stopped going up there. A number of them said that they were surprised, and pleased, to have been chosen. They had assumed only mathematically able pupils who had done well on the written tests would be chosen. As Tracia said, "I did not think I'd ever get a chance like that." Charlene felt that it would give her a chance to prove to those who were "good at maths" that she could also do something.

Their initial attitude to the course was thus positive for a number of reasons; they felt they had been chosen to do something different and exciting, they were going to travel away from the school and they were doing something which was of relevance outside school. On the other hand they had to miss their music and gym lessons and had to stay late after school twice a week. After a while it would seem the pros and the cons would balance out and their feelings would reflect how they felt about the course itself.

Having completed the course they all said they would advise a younger brother or sister to learn LOGO. For example Alitacia said, "It was exciting and interesting because you learnt a lot of new things. He could use it later when he goes to work."

They all said that they had found the course exciting. In my journal and in theirs' one of the recurring themes is the excitement they felt and I saw on their faces when their programs worked out. They were very involved with their creations. As Mandy said, "After my squares worked on the computer I was so excited I just wanted to draw more and more but my time was finished." They found the rationing of time very frustrating.

The other side of the excitement coin was the frustration they said they felt when things did not work out as planned. Tracia: "You get so cross when you think you're on the right track and then all of a sudden it doesn't come out like you want it to be and you have to try again." Lynn: "When that was done we forgot to do Pendown. I was furious by all the mistakes we made." Liesle: "I am enjoying what I'm doing but getting very angry when I try something different ways and it still doesn't work out." This frustration was, I feel, an indication of their involvement with their work. They said that they did not get depressed. Some admitted that as afternoon wore on they got bored because they were tired and hungry!

At times they blamed the computer. Mandy: "At least I never got mad at the computer today!" Bonita: "I could not believe it; it was actually finding fault with directions!" and Alitacia: "I have realised how sensitive the computer is - if you make a simple mistake like leaving out a space it has to tell you."

At times they felt disappointed when they had problems. Lynn: "The first problem we came across was when the drawing did not turn out as expected. I felt down because I really felt it would not be a problem, but somewhere something went wrong."

They did not however refer to themselves as stupid in these situations. They said that they had made an error, or not planned enough or not written the program correctly. Their attitude was that they had done something which they could, if they tried, put right. Mrs Banks said that when they tried to show her programs that we had worked on they had a lot of trouble getting them to work and that this had frustrated them terribly. She unfortunately did not at that stage know enough to help them to see where they were going wrong. They could work happily on their own in the immediate mode but when it came to editing and running procedures they still needed some assistance. In a short while she was able to help them but it was obviously unfair of me to have left them in this situation.

In both the interviews and in their journals their pride in their achievements was obvious. I asked them how they felt when their plans succeeded. They all said that they felt happy and/or excited and very pleased with themselves. They said it gave them confidence to try more difficult things. The following are extracts from their journals:

Bonita: "When I did the measurements it worked out and I was very proud of myself. I felt like doing it over and over and watching how it moved." Ingrid: "Today we got a house and it was wonderful because it all worked. I am so happy. It is the first time we got something properly finished." And Charlene: "We have just finished doing a design which was right from the start and I am feeling happy. After Thursday I had just about had it with shapes!"

This pride must have outweighed the frustration since they all said they had enjoyed the course. Lynn's remark was typical of their responses, "I felt happy because I was doing things and learning new things. I felt good about myself and the LOGO."

The emotional involvement with their LOGO projects contrasted sharply with their attitude to their school mathematics. They said that they did not get excited in class, they were pleased if they got their work right, and were encouraged to keep working, especially if the teacher praised them; and disappointed if it was wrong because then they had to do it again. What they found frustrating in class was being interrupted by other children who were misbehaving or making a noise. They said they got bored when the teacher was repetititve and when they could not understand the work.

6.3 Conclusions and Observations

From this perspective, the claims made for LOGO are justified. The girls involved in their work, both their happiness and pride in their achievements and their disappointment and frustration with their problems are evidence of this.

7. Summary and Conclusions

7.1 Constraints and Problems

During the course I became aware of a number of constraints upon the experiment. The philosophy of the school and the structure of its timetable and lesson content were at variance with LOGO's philosophy. The children while educationally advantaged seemed in ways to be suppressed. They were used to receiving information in pre-digested chunks to fit into 35-minute periods. At first they found the afternoons long and the freedom to explore their own ideas difficult to cope with. I had to confront my embedded ideas about my role as teacher and intervene what was, to me, frighteningly seldom. They were initially very dependent on me for support and guidance. Fortunately in a way it was my introduction too and while this limited my awareness of the number of fruitful paths available it meant that I too was very definitely a learner. With hindsight I see that this was more of a problem than I realised as I was not sufficiently aware of their different cognitive styles.

We were short of time in the long term, a fact which tempted me to short cut their learning experiences by providing them with the quick ways. This time constraint was emphasised by the computer not being as accessible as I would have liked. We had to work in the library where while I was saying "talk", the librarian was saying "shush", and the children were never allowed in there alone.

I wonder if I did in fact teach in a manner consistent with LOGO's philosophy? I cannot say, I only know I tried to and was constantly aware of it.

7.2 Criticisms of this Work

In evaluating LOGO's worth and value as an educational environment we are in an extremely difficult position. As Breen (1983) has pointed out, Papert would prefer to have no relevance to school mathematics. But then who is going to use it? We live in a society that expects to do its learning in schools and we should, as teachers, offer the best opportunities for learning we can.

In order to convince others of the value of an environment one is forced to measure it by their yardsticks. Thus, to product orientated mathematics teachers who have a syllabus to teach and specific concepts to communicate, we must show that LOGO encourages the development of these ideas and that while it is initially time consuming, in the end the children will fare better in the traditional mathematics for having had the experience. Unfortunately this has yet to be proved.

The girls needed to spend longer, years and not months, working with the LOGO in order to have had the type of experiences which would possibly facilitate this conceptual development.

The evidence that the girls were in the process of learning, while interesting, proves nothing since I did not have a group with which to cocmpare them. It does however indicate that this is an area worth studying.

Only in the analysis of the environment from an affective perspective do I feel that it is safe to say that LOGO is as exciting an environment as is claimed. Here the problem is that the bulk of the research focusses on the cognitive realm.

7.3 Suggestions for Future Investigation

A number of types of studies which focus on different aspects of the situation are possible.

o Longer studies of a similar nature.

o Studies on children who meet algebra having already been exposed to LOGO at an age when their algebraic concepts were at a formative stage.

o Action-research in order to be more critical of one's teaching and of the girls' individual learning styles.

o Examination of pupils' attitudes to teachers who become involved in teaching them LOGO.

o Investigation into what the children learn about their own and other children's cognitive styles through working with LOGO.

o Investigations which focus on the language the children use while working with LOGO.

7.4 General Observations and Conclusions

In summary I feel that the experiment was a positive experience both for me and for the students. I have learned a great deal about learning, LOGO and the girls individually and as a group. They too felt that they had benefitted from learning LOGO.

It is possible, indeed probable, that I could have occupied them as profitably and enjoyably for the same amount of time in a non-computer based manner. They would however have been dependent on me and one of the benefits of LOGO is that it reduces the child's dependence upon its teacher, both for stimulation and correction. Further, the LOGO environment turned me into a co-learner the effect of which I feel we should not underestimate.

Their positive attitude may have stemmed initially from the pleasure that they felt in "being chosen" and the novelty of travelling away from school. Yet as I pointed out they had to stay late in the afternoon. They did not lose interest in the course as it progressed, on the contrary they were very sorry when it stopped.

They worked together well, both in their pairs and as a group. The environment was a positive one, they worked hard and were involved in their creations. The LOGO environment encourages pupils with a wide range of abilities to get involved in their work, not only the most talented. Their attitude to solving their problems was a positive one. A number of them said that they had become more perservering and patient during the course. They helped each other and shared ideas.

The school as a whole has benefitted from the experience, the computer which had sat on a shelf was no longer an item of mystery. A computer club was formed and the school has gone, computer-wise, from strength to strength. This is largely as a result of Mrs Banks' dedicated teaching but is also the result of their having received an entire computer laboratory.

I did not however prove that the LOGO made a difference to the girls' classroom mathematics in the three areas tested. It is probably unreasonable to have imagined in the time allowed that it would have.

While I have thus been unable to reach, with any certainty, what one could call a conclusion, I have however, based on the behaviour I observed in the interviews, found a new awareness of the manner in which children learn.

In order for them to go beyond the level at which they function happily and easily they appear to need a situation which stimulates them, inviting them to think beyond the present level. Having been stimulated they need the support of discussion and colloquial conversation to facilitate the perceiving of more general ideas than those with which they are already familiar. In addition they must remain aware of the base from which they operate and of the importance of not becoming cut off from their existing knowledge and experience. I believe that when, in connection with a particular concept or generalisation, a child has had a number of such experiences, the sum of these provides a new base level from which they can operate and progress in a similar manner. When this new level is the base level the child can synthesise the experiences which caused its development in generalisations and symbolism; which if rooted in these meaningful learning experiences are an expression of his understanding and not a substitute for it.

It is my contention that LOGO is the type of environment which, if supported by sensitive teaching, provides this type of environment.

APPENDIX 1

Diary of the Course.

Times of classes

Monday

6a first period (half-an-hour) } during gym (supervised in 3rd term
6b fourth period (half-an-hour) } 1984; thereafter not)
Both classes after school, 2:30 to 4:30 pm (supervised)

Tuesday

Liesli and Tracia after school (about one hour, unsupervised)

Wednesday

Ingrid and Anzonia after school (about one hour, unsupervised)

Thursday

6a third period (half-an-hour) } during singing (supervised in 3rd
6b fifth period (half-an-hour) } term 1984; thereafter not)

Friday

Mandy and Bonita after school (about one hour, unsupervised)
Charlene, Lynn and Alitacia during long break (unsupervised)

In detail

This diary is constructed from the journals which we all kept. I encouraged the girls to record not only factual matter but also their feelings about the experience. Mandy said that this was the most difficult part of the course. I have written it in the present and perfect in order to avoid hads and had hads.

24th - 31st July 1984

I chose nine girls from two standard six classes. At first we will be travelling up to the University of Cape Town on Monday afternoons.

Monday, 6th August

We nearly did not get started. The school misbehaved in assembly and was kept in. Fortunately I persuaded a prefect whom I teach to release my group in time to catch their lift! It is essential to be part of the informal fabric of the school in order to cope with the vagaries of its system. Otherwise as Burns said "The best laid plans ..."

This is our first encounter with Apple LOGO.

Alitacia: "after working with the computer I was excited. I drew a square using the words Mrs Paterson explained."

They worked in groups -

Liesle and Tracia - 6a

Lynn, Charlene and Alitacia - 6a

Ingrid and Anzonia - 6b

Bonita and Mandy - 6b

I introduced the following commands

	PRINT
ST ... Show Turtle	LT ... Left
FD ... Forward	RT ... Right
BK ... Backward	CS ... Clear Screen

They also asked how to hide the Turtle. They drew pictures, mostly of houses, wrote their names and greetings. One group worked out how to draw a circle.

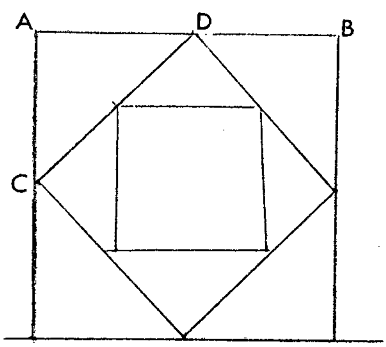
During the afternoon I showed them the Repeat command.

Thursday, 9th August

Our school Apple still needs upgrading to handle the LOGO so they planned Monday's activities.

Liesle and Tracia drew petals on paper. They had the idea that they could repeat the one petal but the change of the turtle's heading at the join puzzled them. Liesle tried to play Turtle.

I doubt whether their plan will work.
Lynn, Alitacia and Charlene wanted to draw



They made CD = 1/2 AB. I sowed the seed of the idea of procedural programming.

6b

Bonita and Mandy did the same. They worked well and seemed more at ease with angles than Ingrid and Anzonia for whom all did not go smoothly in their drawing of a tent.

Anzonia: "Yesterday we tried to make a tent but we could not get it right because could not do the number of angles. I think we did not try hard enough." They said they had learnt about triangles but seemed unable to bring any knowledge to bear on the situation.

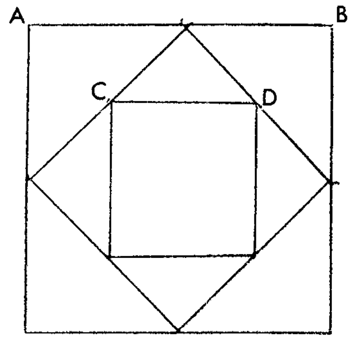
From my journal "The next major issue is to introduce procedures and teach them to debug them otherwise they will get very discouraged. I think I'll leave them one more week. I must try to keep my interference to a minimum."

Monday, 13th August

During school they all continued to plan.
After school I am very aware of the constraint of lack of time and my impulse to go too fast.

The girls are happier during the 'hands on' than the planning stage.

Introduced: PU ... Pen Up and PD ... Pen Down in response to their request for a way to move the turtle without having it draw over what they had already drawn.



Bonita and Mandy's was very neat and they saw that CD = 1/2 AB and used the fact.

Liesle and Tracia's was a mess but they said, "It's not bad for the first time".

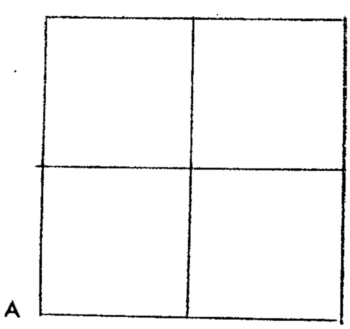
Charlene, Lynn and Alitacia are very perfectionistic and worked in small increments. They needed an introduction to PE (Pen Erase).

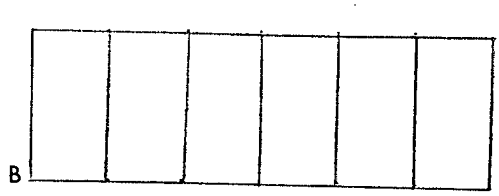
Anzonia and Ingrid took a long time over their tent and did not get very far.

From my journal: "It continues to be rewarding but it is difficult to be sufficiently aware of where each girl is and what sort of thinker she is. The exchange of ideas between groups is proving fruitful."

Thursday, 16th August

6a and 6b I discussed procedures land showed them how breaking down their drawings from Monday would have made things easier. They agreed. I showed them how to write a procedure to draw a square. We discussed the problem of the turtle's heading in using "To Square" to draw





These shapes are on Monday's agenda.

During school 6a and 6b planned for the afternoon.

Monday, 20th August

We no longer need to travel to UCT; they are quite sorry.

They all tried the above diagrams - most of them without using "To Square".

Liesle and Tracia also drew a very neat fence - picket by picket.

Lynn's group drew B with pride and aplomb in front of the headmistress!

Their input and arguments are generally more definite although they didn not use the idea of procedures unless prompted to do so.

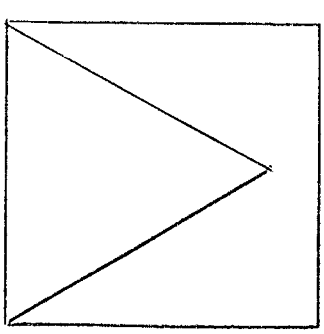
I would be bored by now by simply drawing, but they definitely are not. I have arranged for each group to have one afternoon a week to work on the computer after school, on their own in the library. They set themselves up to work in the immediate mode. From the school's perspective this is an enormous concession; till now the computer has been so carefully looked after that no-one used it!

Some of them - notably Lynn's group - are using their free time during the day to work on the computer.

Leaving them to their own devices has I am sure more positive then negative points.

Thursday, 23rd August

We worked at building a house from a square and a triangle. Drawing a square and triangle were easy by comparison with sorting out the turtle's heading. They find it difficult to see in their head what angle it would need to turn through so as not to get this



which is what

```
TO SQUARE
     REPEAT 4 [FD 50  RT 90]
```

immediately followed by

```
TO TRI
    REPEAT 3 [FD 50  RT 120]
```

draws.

We had to play turtle and plan on paper to sort it out. They would much rather have experimented on the actual computer - after all they had had pen, pencil and a body for ages!

Monday, 27th August

During school We very slowly and laboriously drew a circle.
After school I sat back and looked at the groups. Here are my thoughts:

"Liesle and Tracia: I don't know how much they are absorbing. The rest of the staff say Tracia is so slow (they go out of their way to tell me this) and yet sometimes her insight is good.

Lynn is the leader in her group. She has been trying procedures from the manual and is more independent.

Bonita and Mandy are 'bright eyed and bushy tailed' but they are still drawing in a very pedestrian manner.

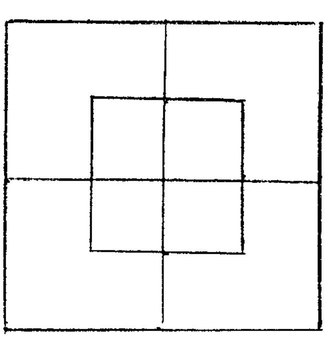
Ingrid and Anzonia are plodding happily along."

They tried to draw circles. This reinforced the usefulness of REPEAT.

Thursday, 30th August

Introduced the idea of a procedure with one input ie TO SQUARE :SIDE. I told them :SIDE is a promise to tell it how long the side is each time you say SQUARE.

6b used this happily to generate this very pleasing pattern. They were very proud of it.



With hindsight I realise that at this stage I should have shown them how to save and reload pictures and procedures on a disc and I should have made very sure that they could do it all alone. When I spoke to Mrs Bank she stressed the fact that their inability to do this frustrated and annoyed them.

Monday, 3rd September

am More experimentation with TO SQUARE :SIDE. They can use if when we are doing it but when they have to draw something with a square they revert to either drawing it or using SQUARE of a fixed size.

pm Presented the group, possibly too soon, with this mircroworld

```
TO SHAPE :SIDE   :ANGLE
REPEAT 10  [FD :SIDE    RT :ANGLE]
END
```

They played very happily with it but did not make remarks of a generalising nature unless forced into it, eg

Q: "What do you thinks happens when the side gets longer / the angle changes, etc."

They then derived a lot of satisfaction from checking their assertions. Those who were proved correct were very proud and freer with their opinions the next time around.

Thursday, 6th September

They continued to play with SHAPE which got altered by the 6a's to TO SHAPE :N :S   :A to cope with shapes with more than 10 sides.

Both groups generated regular polygons up to 20 sides.

Monday, 10th September

am I wanted them to explore the polygons on paper but they wanted to use the computer to do the drawings.

pm We continued to play with SHAPE; the general rule that the angle = 360/N became obvious.

They drew some aesthetically pleasing patterns using large N's and/or "angles that don't work out".

We could home in on the Turtle-Total-Trip theorem but I think I will wait for it to emerge naturally.

Thursday, 13th September

I gave them a new microworld.

```
TO MAZE :STEP   :ANGLE
     FD :STEP
     RT :ANGLE
MAZE  :STEP + 5  :ANGLE
END
```

which we played with as a group with different people calling out the inputs each time.

Monday, 17th September

6a: worked constructively with MAZE, under the leadership of Lynn and Charlene.

6b: worked in a more random manner.

pm In response to a request from Lynn's group I showed them how to halt MAZE with a conditional statement

```
IF :STEP > 150   [STOP].
```

Bonita and Mandy introduced the idea of using a negative quantity for :STEP. The other four enjoyed the drawings they created but did not do much innovation.

Thursday, 20th September

Start of September holidays.

In the fourth term I won't be teaching at Immaculata any more and will only be able to teach the girls on Monday afternoons. The girls will spend the other time working alone.

Monday, 8th October

It was very good to see the girls again. They set up happily. I wanted them to work with

```
TO SPINSQUARE :SIZE :ANGLE
IF :SIZE > 100 [STOP]
SQUARE :SIZE
RIGHT :ANGLE
SPINSQUARE :SIZE + 3  :ANGLE
END
TO SQUARE  :SIZE
     REPEAT 4  [FD :SIZE  RT :ANGLE]
```

It took us a long while to get it to work. Eventually by successive simplification we succeeded in debugging it. Their attitude to the problem was noticeably more positive. They wrote out the program to use in the week.

Monday, 15th October

We modified SPINSQUARE to SPINTRI and then drew, as a group, a beautiful spider's web by using SPINTRI 5 Ø six times and each time returning the turtle to the centre with the pen up. Their angles have definitely improved, they do not say so many obviously thoughtless things. The repetition fascinates them.

I tried to encourage them to calculate how many sides we would have before being stopped but they couldn't see how to work it out; they could however see, without trying it, that SPINSQUARE 1ØØ 3Ø wouldn't get us anywhere.

Monday, 22nd October

When I arrived they were clustered round the computer in an organised way using the MANYFLAGS procedure from the manual. Later I wanted to see how much of the procedure and variable ideas they could use for themselves.

I asked them to write procedures which

a) drew a square side S and which printed "sorry that's too big" if S> 1ØØ (Tracia)
b) drew triangles of side S and used this to generate a hexagon (Mandy)
c) drew rectangles with height H and breadth B and used this to draw a set of steps (Lynn)
d) drew hexagon of side S and used this to draw a beehive (Charlene)
e) drew a set of concentric circles (Bonita)
f) drew a flag and then a set of flags ((Liesle)
g) drew a set of concentric squares (Alitacia)
h) divided the screen up into squares (Ingrid)
i) drew a picket fence (Anzonia).

They tried very gamely but in general don't seem to have made the idea 'their own' yet. I wonder whether they wouldn't have fared better on questions that meant more to them. But, as others have observed, real, meaningful, personal problems of a mathematical nature are difficult to find.

At this stage there was a marked difference between two groups. The one group was Lynn, Charlene, Alitacia, Bonita and Mandy. The other Liesle, Tracia, Ingrid and Anzonia. When I became aware of their intelligence quotients it was apparent that the first group had an average IQ of 100, the second 84.

I showed them how to store and retrieve procedures. (See 30th August remark.) This is more complicated than it seems and I should have made more certain that they could do it.

We used the computer to do their mathematics homework:

```
TO AREACIRCLE  :R
PRINT  :R * :R *  22/7
```

and

```
TO AREATRI  :B  :H
PRINT  0.5 * :B * :H
```

While writing the second we had a quite heated discussion about whether it was the base or height which had to be halved. The group really is relaxed and open now.

I am very aware that while I am learning a lot myself I am not, because it is my first experience, guiding them as well as I might along their individual paths. In spite of this they are concentrating and working happily for two hours after school. LOGO must have something.

Monday, 29th October

Very much a continuation of last week. Lynn, Charlene, Alitacia and Bonita had made some headway with their problem. We worked as a group on the beehive and hexagons. We do a lot more group work now that I can only come once a week.

Monday, 5th November

They stored a lot of procedures from the manual using CIRCLER and CIRCLEL which our pack doesn't have. A very frustrating experience. They played with MANYFLAGS from the manual. It took them a while to see how the procedures depended on each other.

Monday, 12th November

Start of end of year examinations.

Monday, 28th January 1985

Liesle, Ingrid and Anzonia failed standard six and are busy elsewhere. Tracia is bored but all the rest are interested.

They drew a row of semi-detached houses and their reflections. The changing of the turtle's heading is no longer a problem.

Monday, 4th February

Liesle and Anzonia are absent again. We looked up how SENTENCE, FIRST, BUTFIRST, LAST, BUTLAST, ITEM and RANDOM, MAKE and SHOW work. We tried to write a program TO CHAT which generates simple three-word sentences of this form; noun verb adverb. We used their names for the noun. There was a problem with adverb which I stayed late to try and fix. Tracia stayed too and although I thought she was bored she was not slow to offer suggestions. In fact generally the girls are quicker to offer suggestions and seem more self-reliant, eg

Charlene: "Why don't we just draw one and move it on?"

Lynn: "OK - then we can see how to build the thing up."

The school has bought a mark processing and administration program package and three Commodores. A computer club has been started. All the standard sevens will do some LOGO this year during school in "Project Time".

Monday, 11th February

We worked on two word programs, TO CHAT (as above) and GOSSIP which writes four word sentences of the form "Lovely Lynn eats buns" which caused a lot of amusement. As I mentioned earlier the repetition fascinated them. This was the end of our LOGO course.

APPENDIX 2

Tests Used in Algebra, Ratio and Proportion, and Measurement.

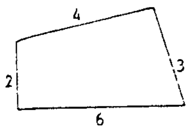
Algebra (Hart 1981c)

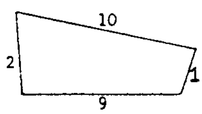
Level 1 (75% - 95% for age 13+)

Test 1

Q3.

The perimeter of this shape is equal to 6+3+4+2 which equals 15



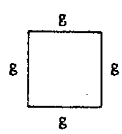


Work out the perimeter of this shape
p=.

(96%)

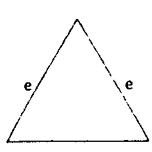
Q4.

This square has sides of length g, so for its perimeter, we can write p=4g.



What can we write for the perimeter of each of these shapes?

p=



(90%)

Q7.

a+3a can be written more simply as **4a.** Write these more simply where possible.

a) 2a+5a= (75%)

Test2

Q3a.

If a+b=43 then a+b+2=? (92%)

Q4a.

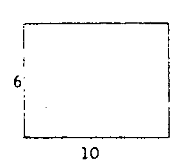
What can you say about a if a+5=8?

(87%)

Q5b.

What are the areas of these shapes?

A=



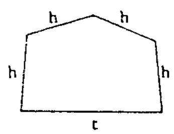
(78%)

Level 2 (40% - 63% for age 13+)

Test 1

Q4.

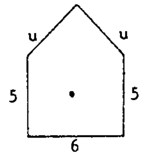
b)



p= (58%)

Q4.

c)



(53%)

Q5.

What can you say about m
if m=3n+1
and n=4? (44%)

Q7c.

See Q7 above.
2a+5b+a= (40%)

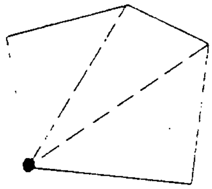
Test 2

Q7.

In a shape like this you can always work out the number of diagonals by takding three away from the number of sides.

So a shape with 57 sides has .... diagonals.

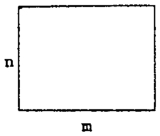
(64%)



Q5c.

What are the areas of these shapes?

c)



(55%)

Q6.

What can you say about u
if u=v+3
and v=1? (48%)

Level 3 (21% - 34% for Age 13+)

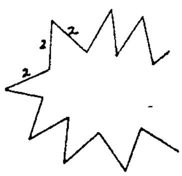
Test 1

Q8.

What can you say about r
if r=s+t
and r+s+t=30 (30%)

Q4. (see above)

a) Part of this figure is not drawn.
There are n sides altogether, all of length 2



p= (24%)

Q2.

4 added to n can be written as **n+4.**
Add 4 onto each of these.
a) iii) 3n (22%)

Test 2

Q7. (see above)

c) A shape with k sides has .... diagonals

Q12.

a+3a can be written more simply as **4a**
Write these more simply where possible

a) 2a+5b= (29%)

c) 3a-b+a= (27%)

Q3.

c) If e+f=8
then e+f+g= (24%)

Q8.

What can you say about c
if c+d=10
and c is less than 10 (22%)

Level 4 (1% - 15% of age 13+)

Test 1

Q1Ø.

Cakes cost c pence each and buns cost b pence each.
If I buy 4 cakes and 3 buns, what does
$4c+3b$ stand for? (14%)

Q9.

b) When are the following true - always, never or sometimes. Underline the correct answer
$L+M+N = L+P+N$
Always, never, sometimes, when .... (11%)

Q2.

n multiplied by 4 can be written as **4n.** Multiply each of these by 4.

b) $n+5$ (8%)

Q11.

If this equation is true when $x=6$

$(x+1)^3 +x = 349$

then, what value of x will make this equation true?

$(5x+1)^3 +5x = 349$ (4%)

Q12.

Blue pencils cost 5 pence each and red pencils cost 6 pence each. I buy some blue and some red pencils and altogethr it costs me 9Ø pence. If b is the number of blue pencils bought and r is the number of red pencils bought, what can you write down about b and r? (2%)

Test 2

Q12. (see above)

b) $(a-b)+b=$ (15%)

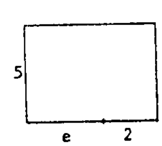
Q3.

b) If $n-246=762$ then
$n-247=$ (11%)

Q5.

What are the areas of these shapes?

d)



(7%)

Q5.

Which is the larger? 2n or n+2. Explain. (4%)

Q9.

Mary's basic wage is R2Ø per week. She is also paid R2 for each hour of overtime she works.
a) if h stands for the number of hours of overtime she worked and if w stands for her total wage, write down an equation connecting w and h (1%)

The rest of the questions were, according to the CSMS criteria (Hart, 1981a), not definitely of a specific level.

Test 1 **Test 2**

Trial questions (to which answers were given)

Q1. What number does a+4 stand for if a=2?

What number does 4a stand for if a=2?

Q2. Fill in the gaps (work down the page)

x -->3x x -->x+3 x -->7x x -->x+8
2 -->6 5 -->8 2 --> 3 -->
5 --> 4 -->
n -->

Questions

Q1. Fill in the gaps
x -->x+2 x -->4x
6 --> 3 -->
r -->

Q2. **4 added to n** can be written as **n+4.**
Add 4 onto each of these.
a) 8 b) n+s

n multiplied by 4 can be written as **4n.**
Multiply each of these by 4.
a) 8 b) 3n

Q6. If John has J marbles and Peter has P marbles, what could you write for the number of marbles they have altogether?

Q7. **a+3n** can be written more simply as **4a**
Write these more simply where possible.
b) (a+b)+a
d) 3a-(b+a)
e) a+4+a-4
f) (a+b)+(a-b)

Q9. When are the following true - always, never or sometimes?
Underline the correct answer.
A+B+C = C+A+B
Always, never, sometimes, when ....

Questions

Q1. Write down
a) the smallest
b) the largest, of these.
n+1; n+4; n-3; n; n-7.
a) ..... b) .....

Q4.
b) What can you say about b if b+2 is equal to 2b?

Q9.
b) What would Mary's wage be if she worked 4 hours overtime?

Q10. a=b+3. What happens to a if b is increased by 2?

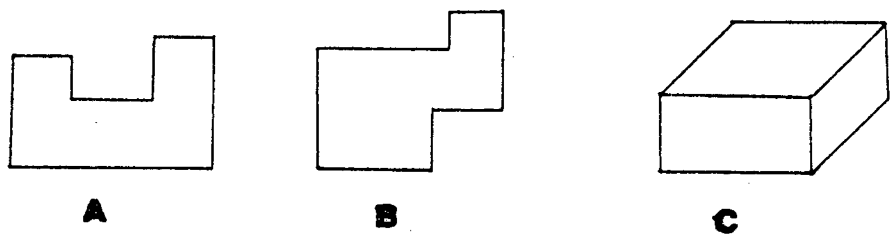
Q11. f=3g+1. What happens to f if g is increased by 2?

Test 1

Measurement

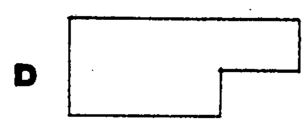
NAME: .... CLASS: ....

INTRODUCTION:

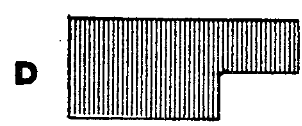


1. Trace your pencil round the edge of Figure A.
   We call the **distance** round the edge the **PERIMETER.**
2. Shade in the whole of Figure B.
   We call the **flat space** which it contains the **AREA.**
3. Figure C shows a box.
   We call the **amount of room inside** the box its **VOLUME.**

a) To find the **PERIMETER** of figure D you add the lengths of its six sides.



b) We have shown the **AREA** of figure D by shading.



c) What do we mean by the **VOLUME** of a sugar cube? Tick the correct answer:

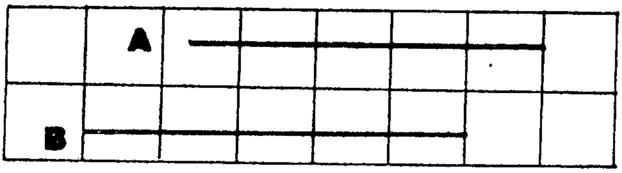
1. The shape of the cube ....
2. How big the outside faces are ......
3. The amount of sugar in the cube .....
4. How far it is all round the cube ....

(3. is the correct answer)

1. The lines A, B, C, D, E, F are the dark lines on the squared paper below.

For each pair of lines, tick the answer you think is true.

a)



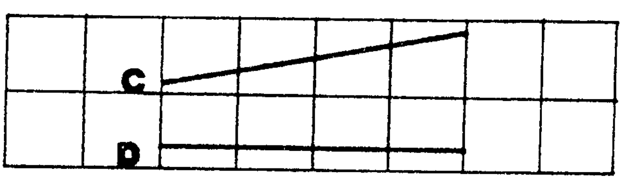
i) Line A is longer ......

ii) Line B is longer ......

iii) A and B are the same length ......

iv) You cannot tell ......

b)



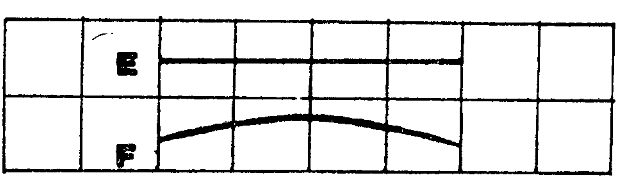
i) Line C is longer ......

ii) Line D is longer ......

iii) C and D are the same length ......

iv) You cannot tell ......

c)



i) Line E is longer ......

ii) Line F is longer ......

iii) E and F are the same length ......

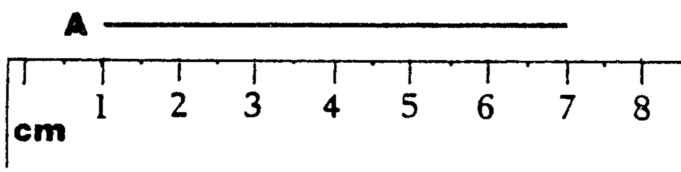
iv) You cannot tell ......

2. The marks on the line show <u>centimetres</u>
How long is the line?

Length of line: ......

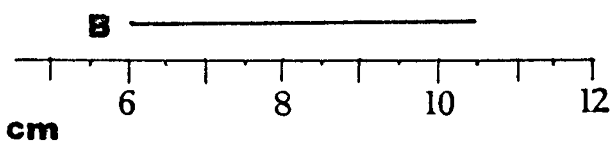
3. How long is each line in centimetres (cm):

a)



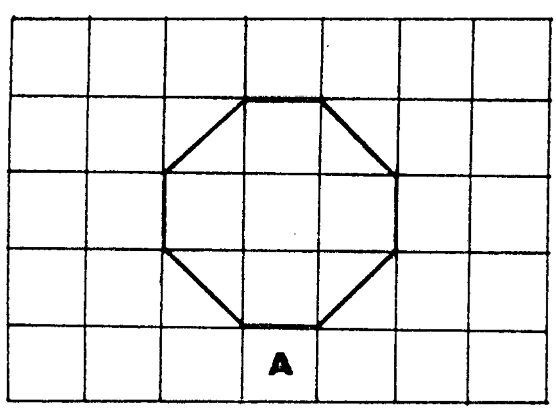
Length of A: ......

b)



Length of B: ......

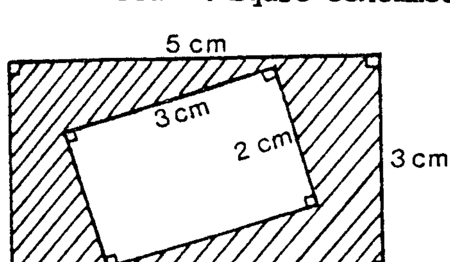
4. The 8-sided figure A is drawn below on **centimetre** paper.



Draw a ring round the correct answer:
The distance all round the edge of A is:

8cm More than 8cm less than 8cm you cannot tell

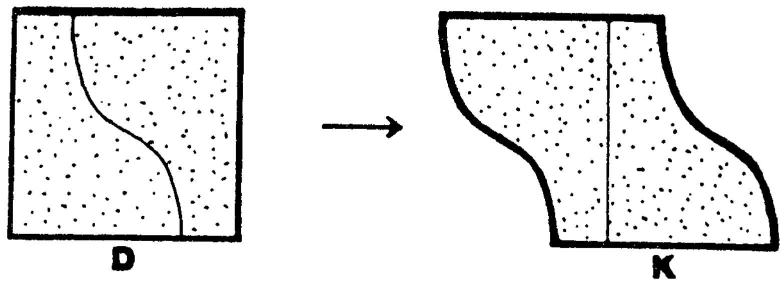
5. The area of the shaded figures measures 1 square centimetre. 
Find the area in **sqare centimetres** of each shape.



Shaded area = ...

(Level 3, 48%)

6. I cut a square D and arrange the pieces to make a new shape K like this:



Tick the answer you think is true in each question.

a) 1. D has the bigger AREA ......

2. K has the bigger AREA ......

3. D and K have equal AREA ......

4. You cannot tell if one AREA is bigger or not .....

Give a reason for your answer ....

:..

:..

b) 1. D has the bigger PERIMETER ......

2. K has the bigger PERIMETER ......

3. D and K have equal PERIMETERS ......

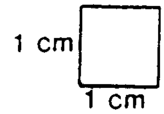
4. You cannot tell if one PERIMETER is bigger or not .....

Give a reason for your answer: ....

:..

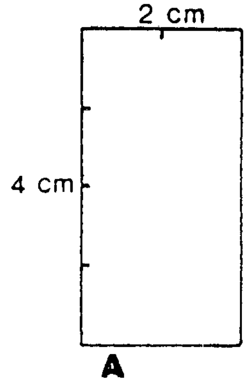
:..

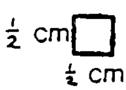
7.

a) How many tiles like this 

would be needed to cover shape A

Write your answer inside the shape.



b) If I used **smaller** tiles like this 

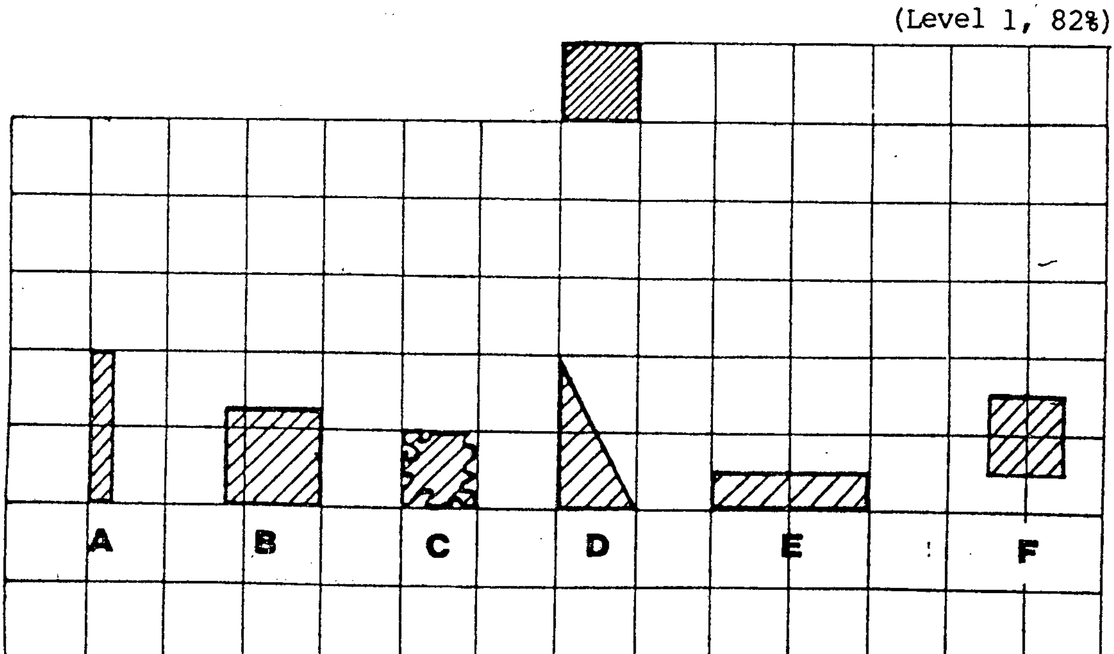
how many would be needed to cover each shape?

Shape A ......

8. The shaded area measures **1 square centimetre.**
Draw a ring round **each** shape which has area equal to 1 square centimetre.

(Level 1, 75%)

(Level 1, 82%)

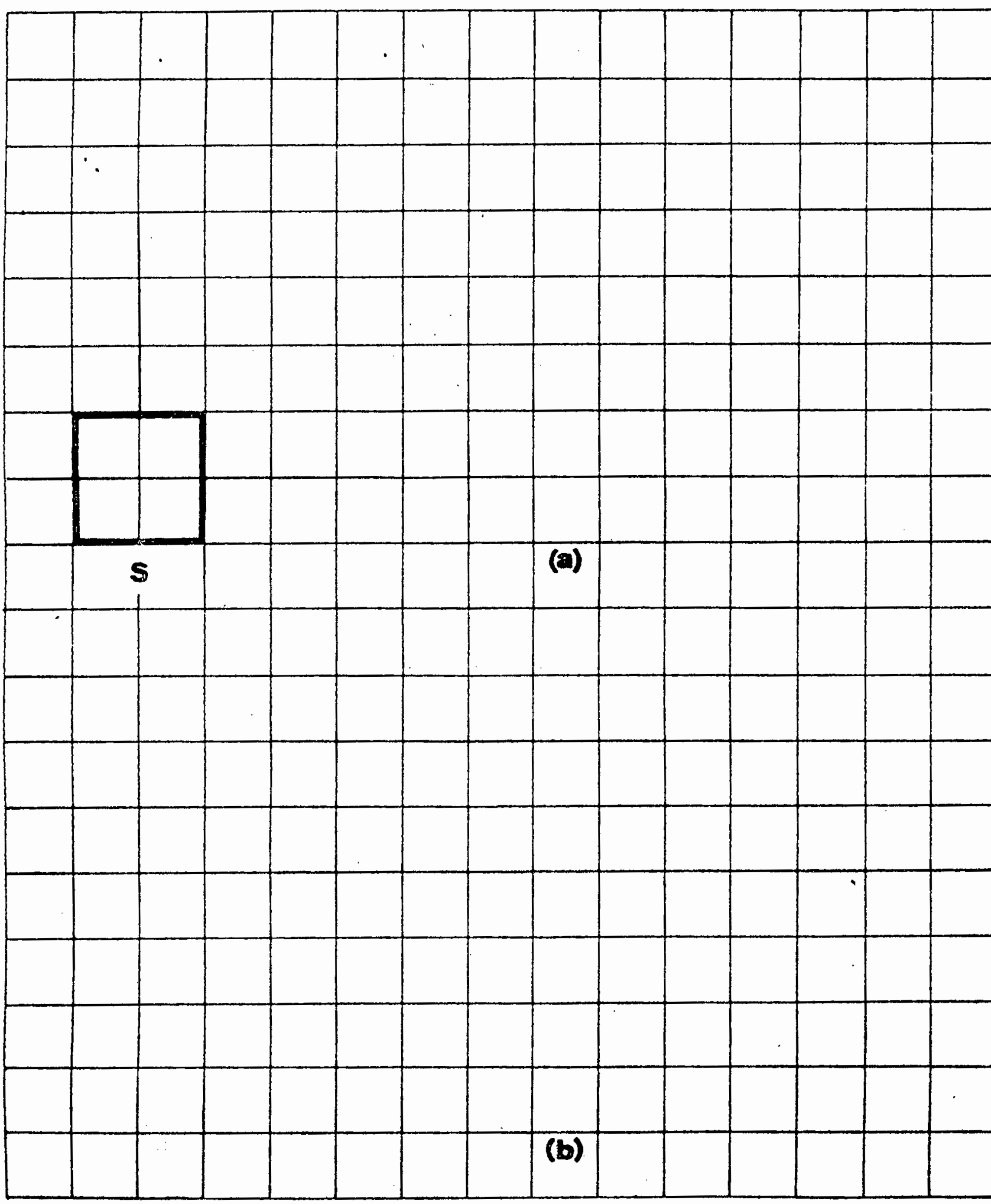


(Level 1, 83%)

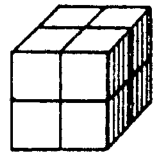
9. On the square paper below, draw:

a) A square whose **area** is **twice** (double) the area of square S. Put a cross on (a) if you think it is impossible.

b) A square whose **perimeter** is **twice** (double) the perimeter of square S. Put a cross on (b) if you think it is impossible.

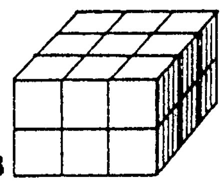


10. A Block 'A' is made by putting 8 small cubes like this  together.



A

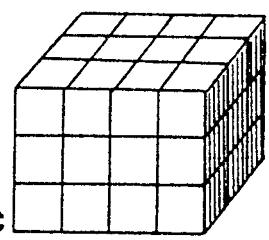
a) How many cubes make the block 'B' (there are no gaps inside)? ......



B

(Level 2, 58%)

b) Block 'C' is made by putting some small cubes together:

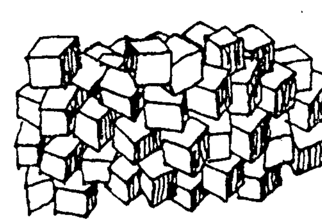


C

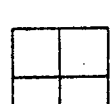
How many cubes make this block 'C' if there are no gaps inside? ......

(Level 2, 56%)

c) All the cubes from **block 'C'** are put in a pile:



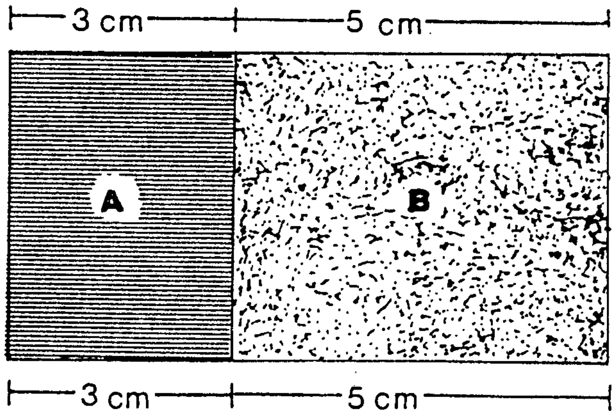
I am now going to use **all** these cubes from block 'C' to build a "sky-scraper" so that the bottom floor is 4 cubes



How many cubes high would this "sky-scraper" be from the the ground? ......

(Level 3, 43%0

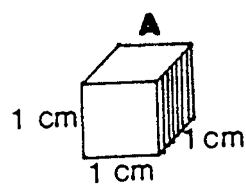
11. The area of the striped rectangle A is 12 square centimetres. What is the area of the dotted rectangle B?



Area of dotted rectangle B: ......

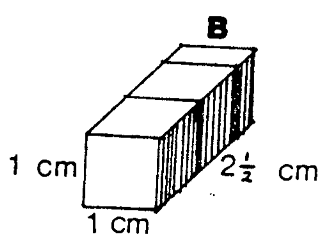
(Level 2, 61%)

12. The **amount of room** inside a block is called its VOLUME. The VOLUME of block A measures **1 cubic centimetre.**



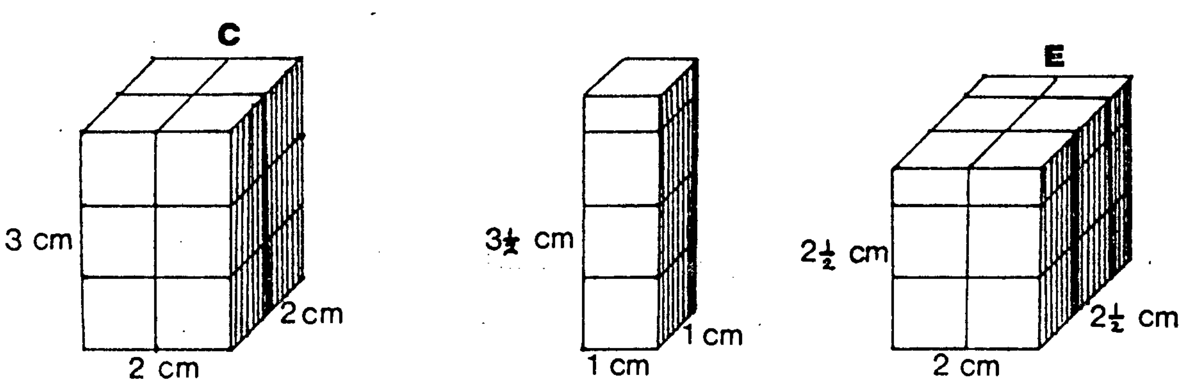
(Level 2, 56%)

The VOLUME of block B measures 2 1/2 cubic centimetres:



(Level 1, 70%)

Find the VOLUME in cubic centimetres of each block C, D, E.



Volume of C ...... Volume of D ...... Volume of E ......

(Level 4, 17%)

Test 2

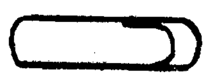
Measurement

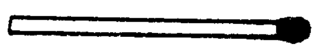
NAME: .... CLASS: ....

INTRODUCTION (as for Test 1)

2. I can measure how long an umbrella is by putting paper clips in a row next to the umbrella like this

and then counting when the line of clips is as long as the umbrella. I measure the same umbrella 3 times using

a) paper clips 

b) matches 

c) tacks 

The answers are 38 15 26

Which number goes with which object?

a) number of paper clips ......
b) number of match sticks ......
c) number of tacks ......

3. John measures how long paths A and B are, using a walking stick. Then he measures how long paths C and D are, using a metal rod. The answers are

Path A: 13 walking sticks

Path B: 14 1/2 walking sticks

Path C: 15 rods

Path D: 12 1/2 rods

Draw a ring round the answer you think is correct in each question.

a) Path B is longer than Path A

true / false / cannot tell

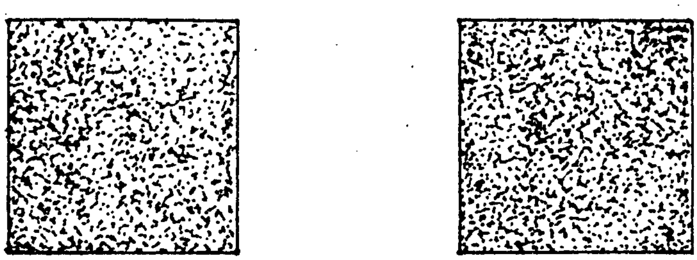
b) Path C is longer than Path B

true / false / cannot tell

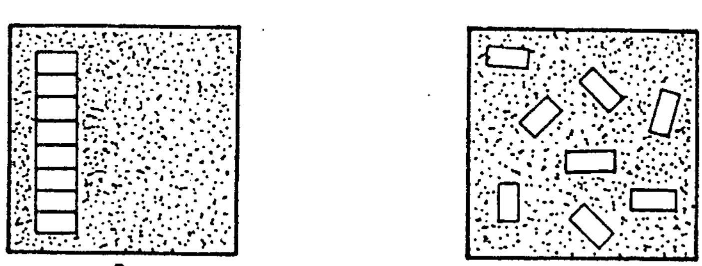
c) Path D is longer than Path C

true / false / cannot tell

4. This picture shows two tin squares which are the same size.



A machine makes 8 equal holes in each tin square:



A B

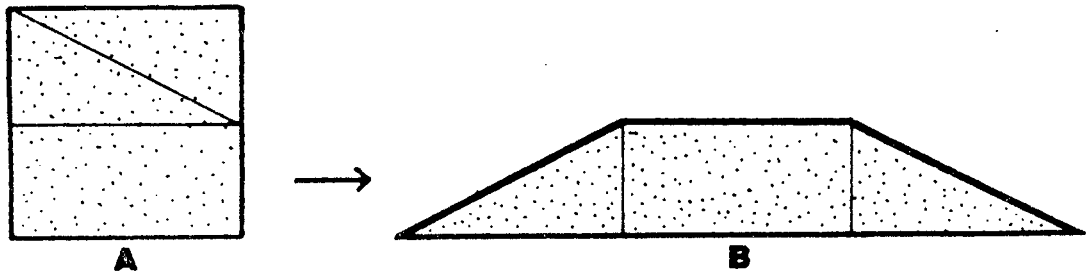
Tick the answer you think is true.

1. Sheet A now has more tin ......
2. Sheet B now has more tin ......
3. A and B have the same amount of tin ......
4. You cannot tell if one now has more tin or not ......

Give a reason for your answer: ....

:...

5. I cut a square A into 3 pieces and arrange the pieces without overlapping to make a new shape B like this:



Tick the answer you think is true in each question.

a) 1. A has the bigger AREA ......

2. B has the bigger AREA ......

3. A and B have equal AREA ......

4. You cannot tell if one area is bigger or not ......

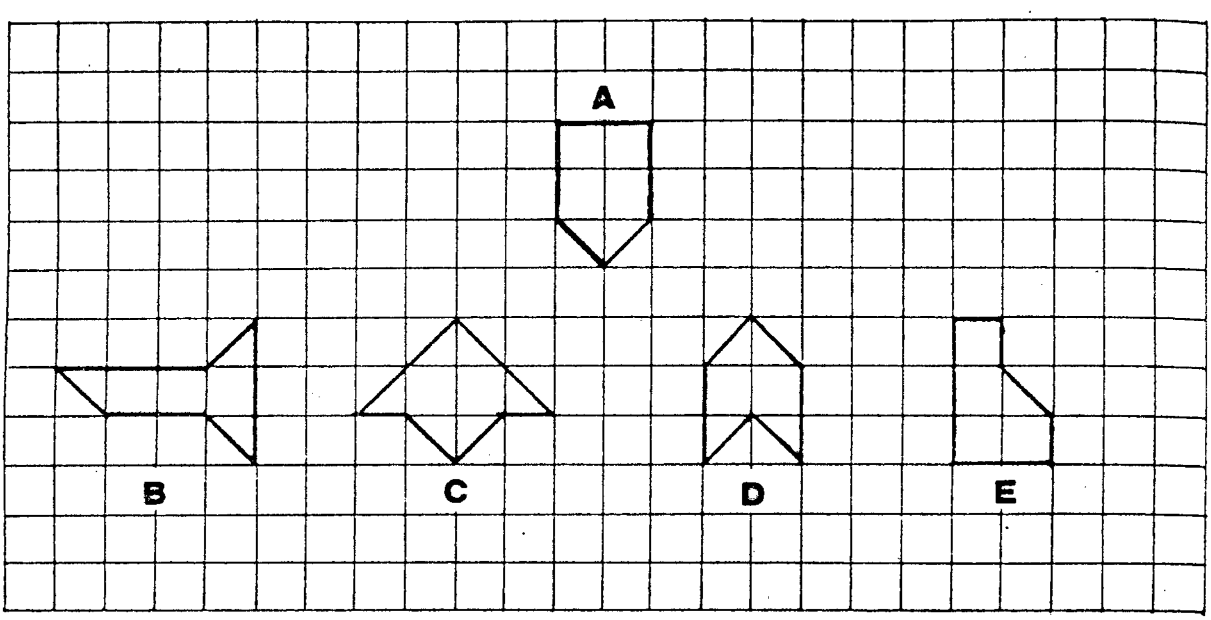
b) 1. A has the bigger PERIMETER ......

2. B has the bigger PERIMETER ......

3. A and B have equal PERIMETERS ......

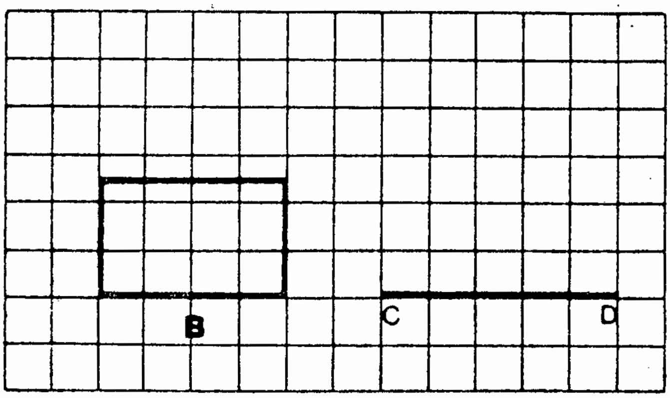
4. You cannot tell if one PERIMETER is bigger or not ......

6. Look at all the shapes A, B, C, D and E. Draw a ring around the shape which has the same area as A

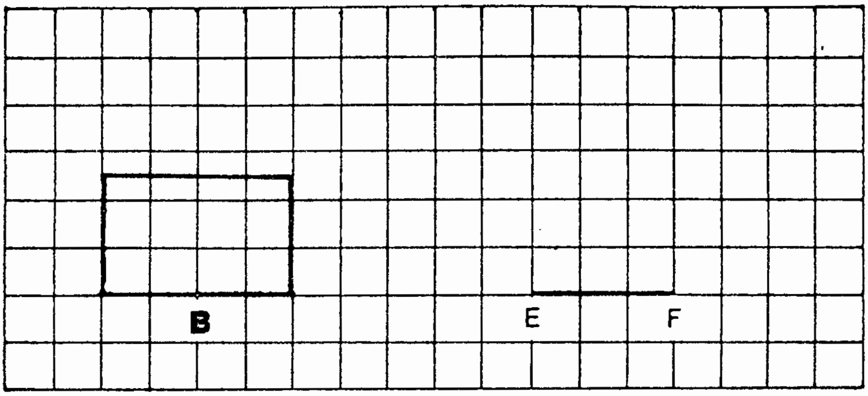


(Level 1, 75%)

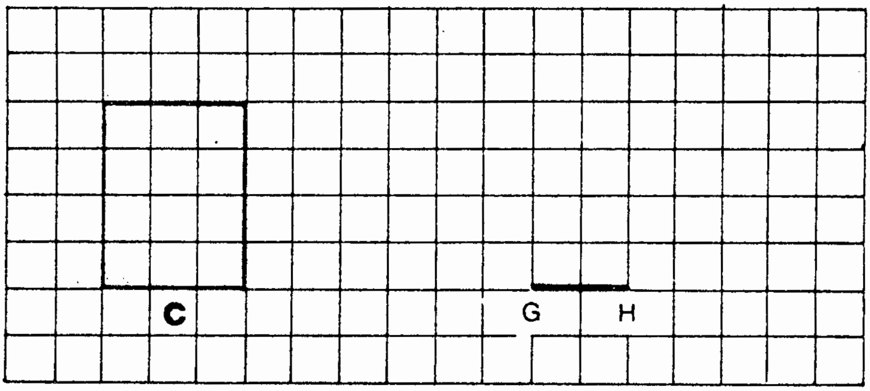
7. Using line CD as a base draw a RECTANGLE which as the **same area** as shape B. Put a large cross if you think it's impossible.



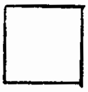
b) Using the line EF as base, draw a RECTANGLE which has the **same area** as shape B. Put a large cross if you think it is impossible.

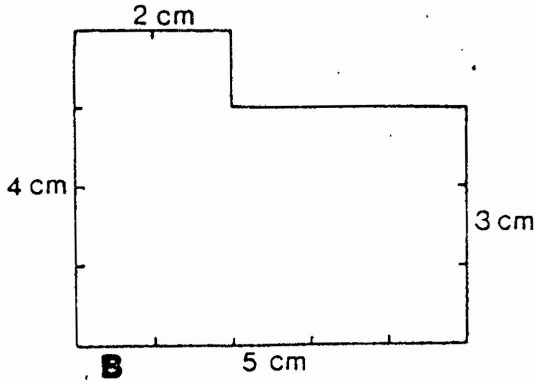


c) Using the line GH as base, draw a RECTANGLE which has the **same perimeter** as shape C. Put a large cross if you think it is impossible.



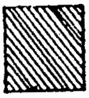
8.

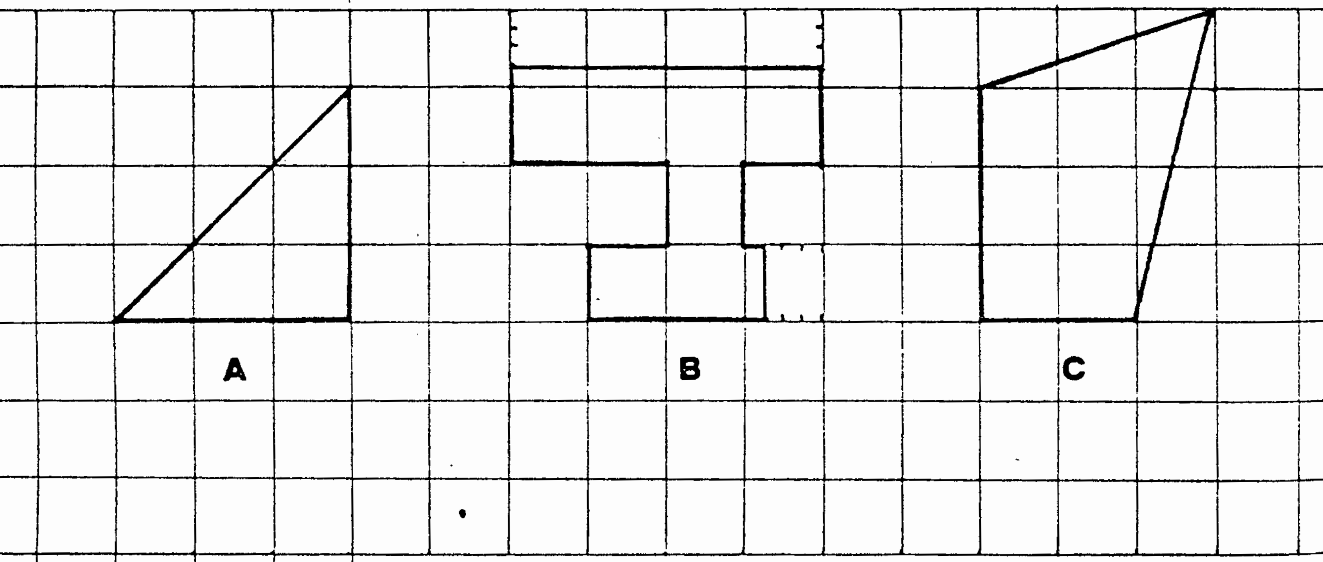
a) How many tiles like this  would be needed to cover this shape?

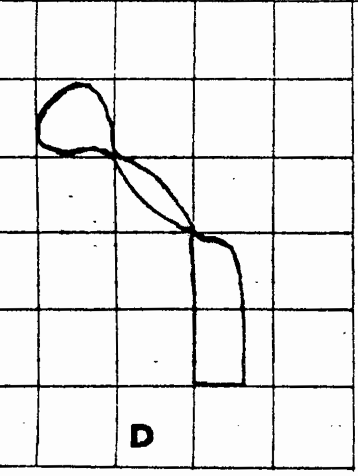


b) If I used smaller tiles like this  how many would be needed to cover the shape?

(Level 4, 20%)

9. The area of the shaded figure measures **1 square centimetre.** 
Find the area in square centimetres of each shape A, B, C.
Write down your best guess for the area of D.





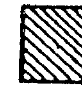
Area of A ......

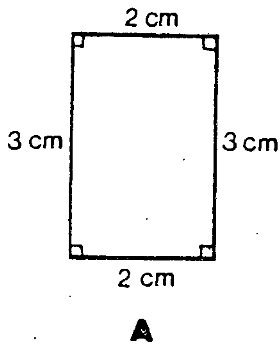
Area of B ......

Area of C ...... (Level 1, 85%)

(my best guess) Area of D ...... (Level 2, 58%)

10. The area of the shaded figure measures 1 square centimetre. Find the area in square centimetres of shape A:

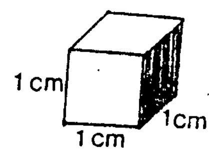


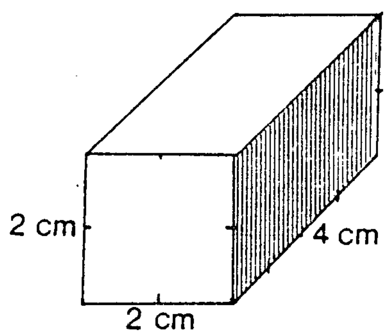


Area of A ......

(Level 1, 72%)

11. How many 1 centimetre cubes would fit into this box?

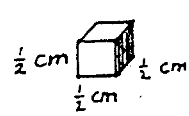




No. of 1 centimetre cubes ......

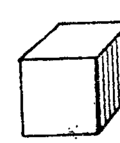
(Level 2, 56%)

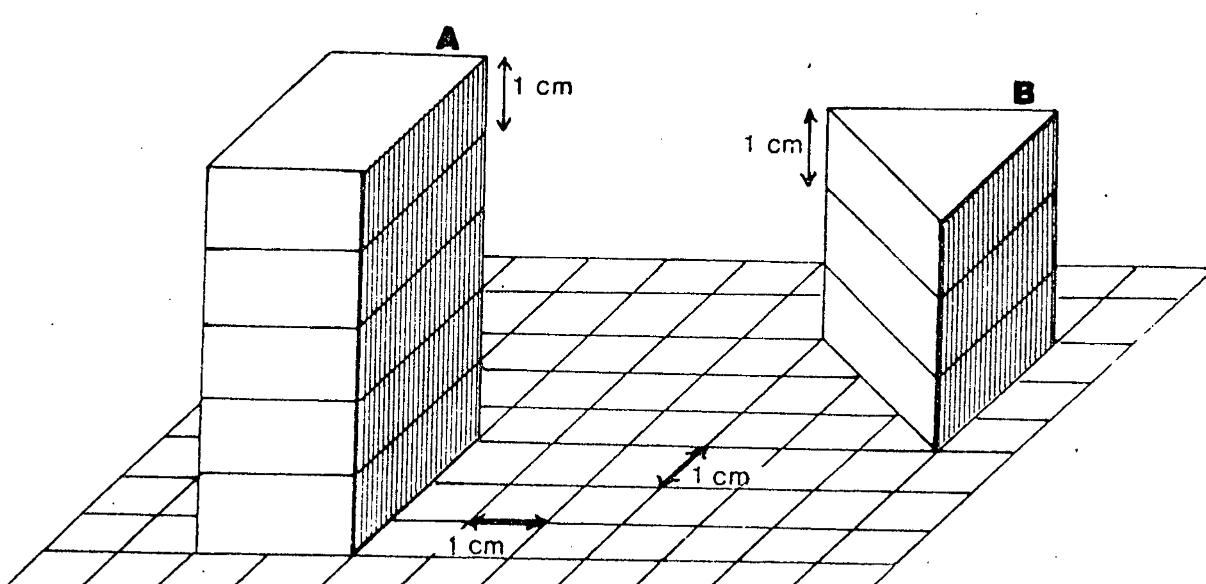
How many small cubes like this would fit in the same box?



No. of small cubes ......

12. The volume of the small block measures **1 cubic centimetre.** Find the volume in cubic centimetres of each block A and B.



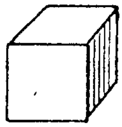


(Level 3, 45%)

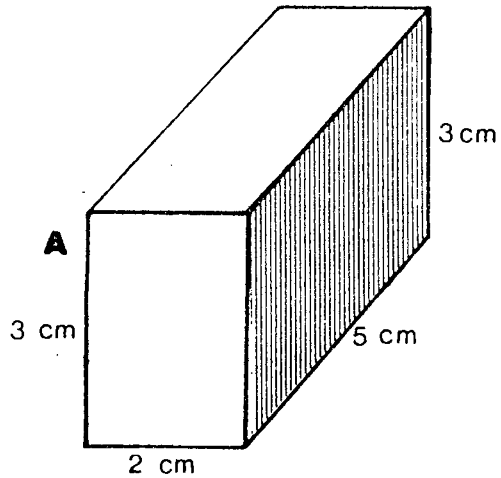
Volume of A .... Volume of B ......

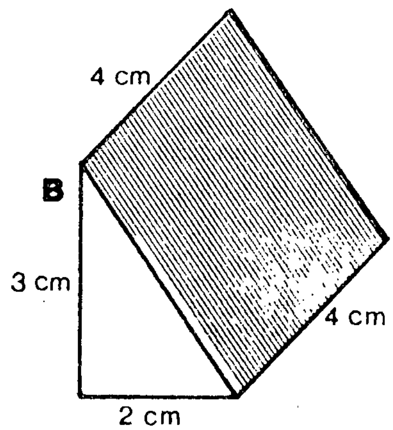
(Level 4, 16%)

13 The volume of ice in this ice cube measures **1 cubic centimetre.**



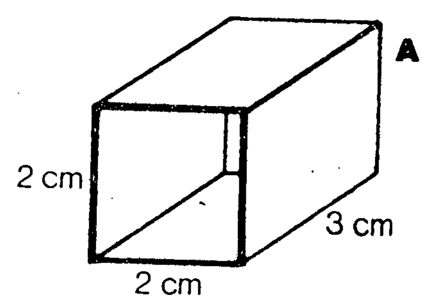
Find the volume in cubic centimetres of each block A and B.



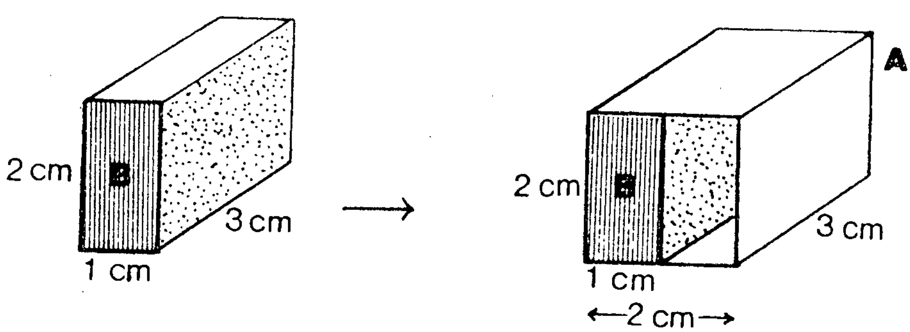


Volume of A .... Volume of B ...... (Level 4, 21%)

14. This question is about the volume of air-space **left** in a box when I put something in the box.
When box A is empty, the volume of air-space in box A measure **12 cubic centimetres.**



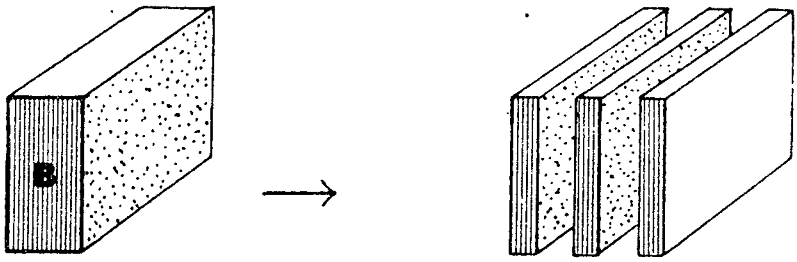
I put a block of plasticine B into the box.



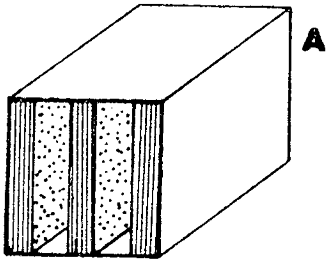
What do you think is the volume of airspace left in the box now that B is in the place: ......

(Level 2, 65%)

I take block B out of the box and cut it into 3 "slices".



Then I put the 3 "slices" into Box A.



What do you think is the volume of airspace left in box A?

(Level 2, 58%)

Test 1

Ratio and Proportion

NAME: .... CLASS: ....

1. Onion soup recipe for 8 persons
   8 onions
   2 pints water
   4 chicken soup cubes
   2 dessert spoons butter
   1/2 pint cream

a) I am cooking onion soup for 4 people
   How many chicken soup cubes do I need? ...... (Level 1, 94%)

b) I am cooking onion soup for 6 people
   How much water do I need? ......
   How many chicken soup cubes do I need? ...... (Level 1, 84%)

2.

a) There are 3 eels A, B and C in a tank at the zoo.

15cm long A
10cm long B
5cm long C

The eels are fed sprats, the number depending on their length. If C is fed two sprats, how many sprats should A be fed to match? ......

If B eats 12 sprats, how many sprats should A be fed to match? ...... (Level 1, 76%)

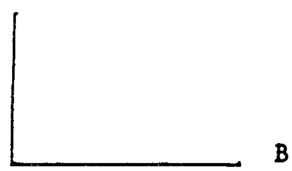
2b) Three other eels, X, Y and Z are fed with fishfingers, the length of the fishfinger depending on the length of the eel

25cm long Z
15cm long Y
10cm long X

If Z has a fishfinger 10cm long, how long should the fishfingers given to Y be? (Level 3, 28%)

3. In an office Mr Adams comes in to work 2 days a week.
Mr Brown comes in to work 4 days a week.
Mr Carter comes in 6 days a week.
The bill for lighting the office for these three men is 240p.
How much should each pay to be fair?
Mr Adams ...... Mr Brown ...... Mr Carter ......

(Level 2, 45%)

4. 

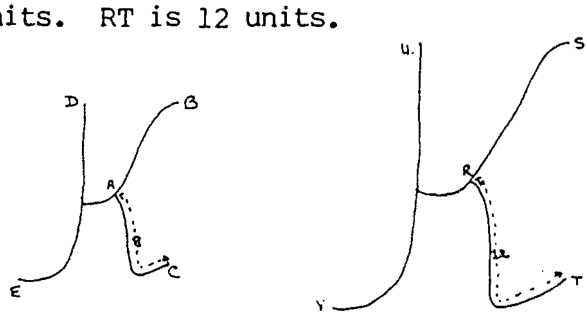
Work out how long the missing line
should be if this diagram ———————>
is to be the same shape but bigger
than the one above. .... cm

(Level 4, 12%)

5. In a particular metal alloy there are
1 part mercury to 5 parts copper
3 parts tin to 10 parts copper
8 parts zinc to 15 parts copper
You need how many parts mercury to how many parts tin?
...... parts mercury to ...... parts tin.

(Level 2, 32%)

6. These 2 letters are the same shape, one is larger than the other.
AC is 8 units. RT is 12 units.



The curve UV is 18 units. How long is the curve DE? ....

(Level 3, 21%)

7. % means **per cent** or **per 100**, so 3% is 3 out of every 100.

b) 6% of children in a school have free dinners. There are 250 children in the school.

How many children have free dinner? ......

(Level 2, 44%)

c) The newspaper says that 24 out of 800 Avenger cars have a faulty engine.

What percentage is this?

(Level 2, 38%)

d) The price of a coat is R20, in the sale it is reduced by 5%. How much does it now cost?

(Level 3, 25%)

Test 2

Ratio and Proportion

NAME: .... CLASS: ....

1. Onion soup receipe for 8 persons

   8 onions

   2 litres water

   4 chicken soup cubes

   2 dessert spoons butter

   1/2 litre cream

a) I am cooking soup for 4 people.
How much water do I need? ...... (Level 1, 94%)

b) I am cooking onion soup for 6 people.
How much cream do I need? ...... (Level 3, 24%)

2. There are 3 eels A, B and C in the tank at the zoo.

   15cm long A

   10cm long B

   5cm long C

   The eels are fed small fish, the number depending on their length.

   a) If C is fed 2 fish, how many fish should B be fed to match?....... (Level 1, 85%)

   b) If A gets 9 fish, how many fish should B get to match?....... (Level 2-3,50%)

3. Three other eels, X, Y and Z are fed with fishfingers, the length of the fishfinger depending on the length of the eel.

   25cm long Z

   15cm long Y

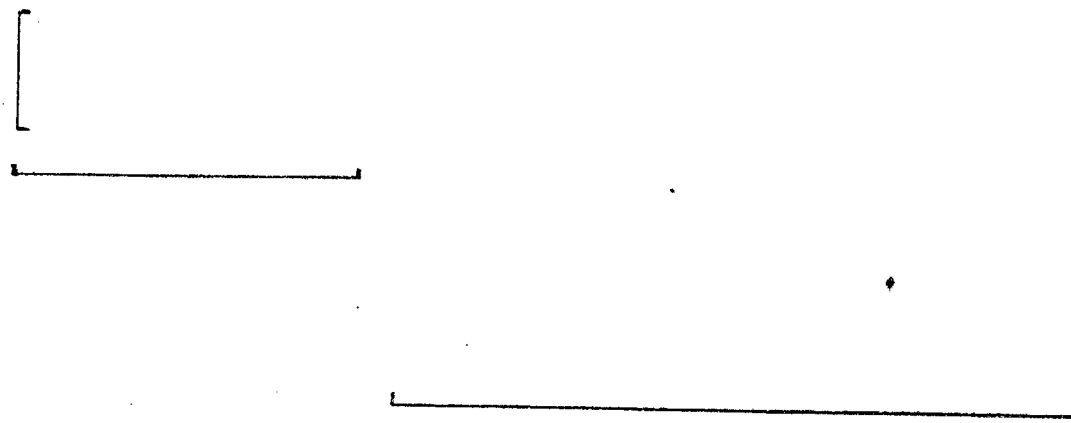
   10cm long X

   a) If X has a fishfinger 2cm long, how long a fishfinger should Z be given? ...... (Level 2, 47%)

b) If Y has a fishfinger 9cm long, how long a fishfinger should Z be given? ...... (Level 2-3,30%)

c) If Z has a fishfinger 10 cm long, how long should the fishfinger given to X be? (Level 3, 27%)

4. Finish drawing the diagram below so that it is the same shape but bigger than this diagram.



(Level 2, 42%)

5. In a particular metal alloy there are

1 part mercury to 5 parts copper

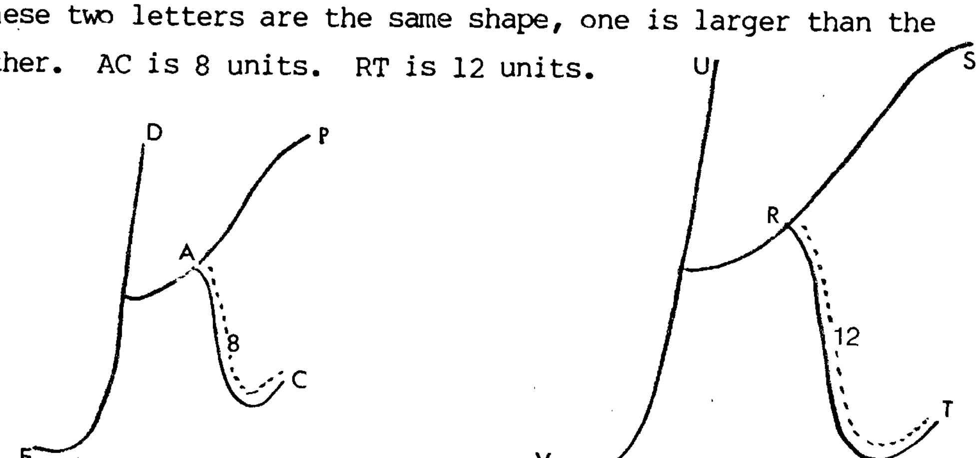
3 parts tin to 10 parts copper

8 parts zinc to 15 parts copper

You need how many parts zinc to how many parts tin?

...... parts zinc to ...... parts tin. (Level 4, 13%)

6. These two letters are the same shape, one is larger than the other. AC is 8 units. RT is 12 units.



The curve AB is 9 units. How long is the curve RS? ......

(Level 3, 20%)

7. % means per cent or out of 100.

a) 4 children out of the hundred on the school trip forgot to bring their lunch. What percentage is this? ...... (Level 1, 83%)

APPENDIX 3.

Division into Two Tests of Questions from the CSMS Papers. The questions from the CSMS papers were divided in two halves on the basis of the criteria described on page 34. The variable numbers are those used by Hart.(1981c) The variables are listed in order of decreasing facility.

Graphs.

	Test 1	**Test 2**
Level 1	10, 17, 14, 13	3, 2, 5
Level 2	11, 15, 29	9, 24, 4, 23
Level 3	30, 46, 48, 49, 50	36, 38, 31, 32, 39, 37

Decimals.

	Test 1	**Test 2**
Level 1	29, 36, 37, 41, 31, 63	6, 44, 25, 20, 3
Level 2	28, 42, 38, 39	4, 32, 26, 21
Level 3	64, 65, 72, 43, 2, 35, 61	15, 46, 16, 45, 23, 22, 34, 47, 49
Level 4	40, 67	50, 51

Fractions (words).

	Test 1	**Test 2**
Level 1	4, 1	8
Level 2	24, 35, 37, 25, 26	13, 62, 3, 10, 11
Level 3	6, 8, 7, 27, 28, 41	22, 23, 34, 30, 39
Level 4	40, 38	36, 31

Positive and Negative Numbers.

	Test 1	**Test 2**
Level 1	3Ø, 39, 24	35, 21, 29, 26, 23
Level 2	4Ø, 41, 27, 42, 49	22, 3, 19, 21, 36
Level 3	43, 44, 52, 5Ø, 51, 28, 56	2Ø, 16, 37, 14, 18, 34
Level 4	35	38, 57

APPENDIX 4

ESSAY 1 : An Examination of some Current Learning Theories and Research in Mathematics Education and their Implication for Mathematics Teaching.

ESSAY 2 : Can We Make Learning Mathematics a More Positive Experience? A Discussion of Three Approaches to this Question.

ESSAY 3 : "LOGO is a Language for Learning." A Discussion of Papert's Ideas on Learning and Computers and the Way in which LOGO Reflects them.

ESSAY 4 : The Introduction into the Classroom of LOGO as a Programming Language with Particular Reference to its Structure and its Potential for Encouraging Concept Formation in a Variety of Disciplines.

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16

UNIVERSITY OF CAPE TOWN

FACULTY OF EDUCATION

COURSE PAPERS

presented in partial fulfilment
of the requirements for the Degree of

MASTER OF EDUCATION

by

JUDITH EVELYN PATERSON

OCTOBER 1986

An Examination of Some Current Learning Theories and Research in Mathematics Education and their Implication for Mathematics Teaching.

JULY 1983

"My feelings on maths are : a very boring subject, as well as complicated. The only part of maths that I do enjoy is the practical part (I don't enjoy maths). I suppose if I did understand maths I would enjoy it more."

Sabine, Std 7a pupil, Cape Town High School

"Only a few children in school ever become good at learning in the way we try to make them learn. Most of them get humiliated, frightened and discouraged. They use their minds, not to learn, but to get out of doing the things we tell them to do to make them learn."

John Holt (1967, 7)

"The human mind after all, is a mystery and, in large part, will probably always be so. It takes even the most thoughtful, honest and introspective person many years to learn even a small part of what goes on in his own mind. How then can we be sure about what goes on in the mind of another? Yet many people talk as if we could measure and list the contents of another person's mind as easily, accurately and fully as the contents of a suitcase. This is not to say that we ought not to try to understand about other peoples' minds, and thoughts, but only that we must be very modest and tentative about what we think we have found out."

John Holt (1967, 9)

"Medical diagnosis in former times aimed at stating what was wrong as do the so-called diagnostic tests in education. True diagnosis tells you why something went wrong. The only way to know this is by observing the child's failure and trying to understand it. An expensive way? Would it be cheaper by computer? No, because in fact observing and understanding the individual child is not expensive. What is really expensive is wasting the vast resources of human experience."

Hans Freudenthal (1981, 134)

"Only awareness is educable."

Caleb Gattegno

"We get interested in what we get good at."

Jerome Bruner

CONTENTS

1. Introduction

In his paper on major problems in mathematics education, Freudenthal (1981) says, "In education all major problems are strongly interdependent ... and all major problems of mathematics education are problems of education as such ... the best paradigm of cognitive education is mathematics".

He poses a number of questions, the first of which is :
"Why can Jennifer not do arithmetic ?", and out of which grows the second :
"How should children learn ?, and further
"How do people learn ?".

He suggests that the way to answer would be : "By observing the learning processes, analysing and reporting learning processes within the total educational system. Learning to observe learning processes and therefore grasping :
how people learn,
how to teach learning, and
how to build a learning theory which should be based on evidence rather than preconceived ideas".

Many people doing research into learning generally, or mathematics learning specifically, have attempted to answer some of the above questions. In this section, I will look at their theories and ideas.

2.1 John Holt

To start with the ideas of John Holt is to go in at the deep end. His ideas are open-ended, thought provoking and do not lend themselves to being pinned down and categorised. As he says, the problem is that the words we use to discuss our ideas come out in single file, while the ideas are multi-dimensioned. But, as Macbeth said, in a slightly different context, "if 'twere done when 'tis done, 'twere well it were done quickly."

In "How Children Fail" (Holt 1964), we see the concerned observer-teacher realising that children in the class situation put their efforts and energies into work evasion. They employ a wide range of strategies to avoid learning, thinking and exposing their lack of knowledge and self-confidence.

He (Holt 1964,34) writes, "of all I saw and learned in the past half year, one thing stands out. What goes on in class is not what teachers think - certainly not what I always thought - you can't find out what a child does in class by only looking at him when he's called on. You have to watch him for long stretches of time without his knowing it - those who most needed to pay attention, usually paid the least. The kids who knew the answer - wanted to make sure you knew they knew." (Holt 1964, 34). He realised that the children and the teachers have totally different views of school. The teachers see themselves as guides on a fascinating journey in search of knowledge, the children as unwilling passengers, forced into being there. "For children the central business of school is not learning, whatever this vague word means, it is getting the daily tasks done, or at least out of the way, with a minimum of effort and unpleasantness."

The children are thus discouraged from thinking and are turned into 'producers of right answers.' "They see problems as a kind of announcement that, far off in some mysterious Answerland, there is an answer which they are supposed to go out and find .... or magic up with a wonderful formula." (Holt 1964, 95). To do this they use a number of interesting work- and thought-evading strategies:- "answer grabbing", "teacher pleasing", "numeral shoving", and "answer fishing" to name a few. And in school finding the right answer is the aim of the game. By contrast the problem-centred person will attack a problem by thinking about it and, if he uses a formula, it arises out of the problem. He will also get pleasure out of finding more than one way to tackle it. But this is not encouraged in most classrooms. For example, Betty, who says "2/4 + 3/5 is one or more. You need two more fifths to make one, and 2/4 is more than 2/5 so the answer must be more than one" would be considered a 'slow' pupil in most schools. (Holt 1964, 91). We are more used to Johnny who gives an answer quickly, whether it's right or not.

Children are terrified of failing. Holt (1964, 50) asked "'What do you think, what goes through your mind, when a teacher asks you a question and you don't know the answer? ' Instantly a paralysed silence fell on the classroom - Finally Ben, who is bolder than most, broke the tension, and also answered my question, by saying in a loud voice 'Gulp!' - I asked them why they felt Gulpish. They said they were afraid of failing, afraid of being kept back, afraid of being called stupid." How dreadful and how true. Yet some children when they start to succeed find that even more frightening for they worry that they won't be able to keep it up and will be expected to do more. It may well be that in being pushed harder, faster and further they lose the very thing they need to cope - freedom to slow down or to walk away from the tension for a while.

The difference he sees between intelligent children and others is that they check their answers against common sense and look at things from different angles; they feel like Einstein that "God does not play dice with the Universe." Other children do not expect answers to be in any way consistent. Once they've got an answer, any answer, the tension they were under is gone. The quicker you can get your work done the sooner you are relieved of having to worry. They have little sense of responsibility for their work. For example, "Monica wants the right answer, yes, but what she wants, first of all, is an answer, any old answer, and she will do almost anything to get some kind of answer. Once she gets it a large part of the pressure is off." (Holt 1964, 58).

In the same way kids may eventually stop trying to think things out for themselves as it is easier to just "take the teacher's word for it." This abdication of responsibility for one's own work must minimise real learning.

Holt describes a lesson Gattegno gave to a group of five retarded teenagers. Gattegno patiently involved them in a learning situation thus helping them to 'see' for themselves and to really learn. He "gave the children solid ground to stand and move on and he had to go way, way back to the beginning of learning and understanding to find it. He brought the conviction that under the right circumstances they could and would do first class thinking. There was not condescension or pity in his manner - for the duration of the lesson he and those children were no less than colleagues trying to work out a tough problem." (Holt 1964, 101).

In order for a person really to learn something it must be connected with reality in his mind, be part of, or appendable to, his existing schemata. (Skemp 1971). He must be able to use it and be able to make connections to it. "A piece of unreal learning has no hooks on it." (Holt 1964, 104). Through the book we see Holt becoming increasingly disenchanted with his own "good explanations." He says "knowledge, learning and understanding are not linear. They are not little bits of facts lined up in rows or piled one one top of another. A field of knowledge is a matter of knowing how they (the items of knowledge) relate to, compare with and fit in with each other. Why do we talk and write about the world and our knowledge of it as if they were linear? Because it is the nature of talk. Words come out in single file, one at a time, there's no other way to talk or write. So, in order to talk about it - we make strings of talk - but our learning is not real until we convert the word strings into a model in our head - and what happens is that kids take these word strings and store them undigested in their minds so they can spit them back out on demand - How can we make school a place where real learning goes on and not just word swallowing?" (Holt 1964, 111).

He tries to answer this question in his second book - "How Children Learn" where he describes children using their minds well, learning boldly and effectively (Holt 1967). Most of the observed children are pre-schoolers as he believes that it is then that we learn best. Because they use their minds in a special way children have a style of learning which fits their condition and which they use naturally and well until we - in teaching them to think - train them out of it. We make them give up their natural and powerful way of thinking in favour of a method which does not work well for them and that we rarely use ourselves. The result; children change from being excited learners into humiliated, frightened, discouraged failures.

He does not put forward a learning theory as such but his descriptions of situations in which learning does take place highlight the drawbacks of the conventional, compulsory, classroom-orientated, often silent, largely abstract school learning environment.

He describes how the children, on discovering some simple addition sums accidentally left on the board, extend and expand, through argument, concensus, curiosity and common sense, their addition capabilities - far more quickly and effectively than if they had been presented with a carefully structured, well explained lesson, As he says, "I saw enough to make me feel that if arithmetic were treated as in fact it is, a territory to be explored, not a list of facts to be learned, children, or at least many children, would move into it faster than we would have dreamed possible." (Holt 1967, 136).

Holt is a great believer in providing apparatus and then waiting, really waiting, till the child himself expresses an interest and even then "playing it very cool." In addition he feels that it is essential to allow time for "messing about." He quotes David Hawkins from an article "Messing about in Science" in which he says "there is a time, much greater in amount than commonly allowed which should be devoted to free and unguided exploratory work ... I call this phase 'Messing about'." (Holt 1967, 143). Professor Hawkins suggests that it is in this phase that the child builds an apperceptive background against which a more analytical sort of knowledge could take form and make sense. They found that during the time allowed the children were constructive and that a large number of the questions that would have been asked got spontaneously answered. Professor Hawkins again emphasises the folly in divorcing school learning (which many call the only learning) from life learning (at which so many children have already so obviously succeeded). (Holt 1967, 144).

Jerome Bruner has said that one thing that happens in school is that children are led to believe that they don't know, or can't do, something they knew, or could do, before they got to school. This idea is substantiated in a story John Holt tells in which children who could happily and easily fold paper fans before reading the instructions, were subsequently unable to. (Holt 1967, 165).

In conclusion Holt says "it is essential to realise that children learn independently, not in bunches, they learn out of interest and curiosity, not to please or appease the adults in power, and that they ought to be in control of their own learning." (Holt 1967, 172). He urges that the child be free to explore the culture in his own way so as to fill the gaps in his knowledge with something that makes sense.

2.2 Richard Skemp

Richard Skemp agrees with Freudenthal that mathematics is a "particularly clear and concentrated example of the activity of human intelligence." (Skemp 1980, 1). He has considered the structure and generation of what Holt calls "hookless knowledge" and has theories on intelligence, learning and understanding.

He conceptualises learning "as a change in an organisms director system towards a state of better functioning" and intelligence as a "kind of learning which results in the ability to achieve goal states in a wide variety of conditions, and by a wide variety of paths." (Skemp 198Ø, 3). He suggests a model of intelligence in which two director systems, delta-1 and delta-2, act to achieve learning by, optimally, establishing a cognitive map of paths from the initial states to the goals. These cognitive maps he sees as "mental models of certain features of the outside world, without which goal-directed activities cannot take place."

When, through intelligent learning, regularities are abstracted from our experiences, concepts develop and, are organised into conceptual structures or schema. These he views as being like "cognitive atlases" available to his second director system delta-two, and from which goal directed plans can be made for delta-one. If they are dense and well-connected a variety of reasonable options is open; if however the ideas are not interlinked delta-two will often have problems making connections. When we understand something it makes connections with existing schema, and we behave intelligently and remember the new facts easily. On the other hand, Holt's hookless knowledge and unrelated facts are difficult to remember or to use intelligently.

Skemp describes 3 different ways of understanding. "Instrumental understanding is the ability to apply an appropriate remembered rule to the solution of a problem without knowing why the rule works. Relational understanding is the ability to deduce specific rules or procedures from more general mathematical relationships. Formal understanding is the ability to connect mathematics symbolism and notation with relevant mathematical ideas and to combine these ideas into chains of logical reasoning." (Skemp 1979, 45).

He differentiates between relational and formal, in saying that when one has grasped an idea for oneself one has relational understanding. When you can explain it logically to someone else you have formal understanding.

Skemp (1982) later differentiates formal into two categories, logical and symbolic. Logical understanding is needed to follow, for example, a formal geometric proof. "Symbolic understanding is a mutual assimilation between a symbol system and an appropriate conceptual structure, dominated by the conceptual structure." (Skemp 82, 61).

We can see that different methods of teaching aim to achieve different sorts of understanding. Frequently, in encouraging the pupils to turn out pages of right answers by a uniform method we give them, (in Erlwanger's memorable words) a set of rules for making arcane marks on paper (Skemp 1980, 7), and only an instrumental understanding at best of the task in hand.

We need, instead, to try to provide the type of experiences which build concepts. At the highest level he suggests that input is attracted to either the symbol system or the conceptual structure. Further since communication is all symbolic it will go first to a symbol system. If, in the early years of mathematics the conceptual structures are neglected they will cease to attract input, and all learning will be symbolic and short-term. So he insists, "We must sequence material in such a way that new material is presented which can be assimilated conceptually" (Skemp 1982: 60).

To this end he suggests that in early years we begin with the physical embodiments of mathematics and that we stay with the spoken language longer, resisting the pressures to have 'something to show' in the form of pages of written work. Further we should use "transitional informal notation as bridges to the formal highly condensed notations of established mathematics." (Skemp 1982, 61). These ideas are echoed by Nick James (1982) of the Open University in his distance teaching course on "Developing Mathematical Thinking" where he insists that children be encouraged to follow the path

Do - Talk - Write in your own words - Write in generally negotiated symbolism - Write in universal symbolism

An experiment was described by Dr Hart at her Cape Town Workshop. In it nine children, who had completed the experimental and concrete section of learning about volumes of rectangular prisms, were presented with - or led to discover - the formula volume = length x breadth x height. The following day only those children who were on the verge of formalising the situation for themselves gained from having the formula given to them. The rest claimed never to have heard of it! (Hart 1983).

2.3 Seymour Papert

Like John Holt, Papert insists that child constructs - even if not ultimately true - are of more value in learning to think than true knowledge on a plate. (Papert 1972 a).

He has done a lot of work with children and computers at M.I.T. and believes "that children learn by doing and by thinking about what they do. And that the fundamental ingredient of educational innovation must be better things to do and better ways to think about oneself doing these things." (Papert, 1972a). He believes that it is important for children to develop their own theories and that by interacting with creative software they have the chance to do this. They also learn that there is a difference between not right and wrong; i.e. a program that is not perfect need not be scrapped, it can be debugged. In learning to debug programs children see the value of structured thought. He has a well formed philosophy of learning which I will discuss in detail in a later essay.

2.4 Hans Freudenthal

A third like-minded thinker is Hans Freudenthal. He believes that learning to observe mathematics learning processes is the easiest approach to the problem of learning to observe learning processes in general. To him the history of mankind's progress in maths has been a learning process of progressive schematising and he feels that youngsters should repeat this history - though they should repeat the one that would have taken place if our ancestors had known what we do.

In the beginning the child acquires knowledge by insight and to Freudenthal it is vital that this insight remains with the child. To facilitate this he would have the "learner reflect on his learning processes" (Freudenthal, 1981), and to argue their intuitions, reflecting on the obvious. Unfortunately teachers usually lack the patience to allow children to do their own formalising. The child must "encounter first the real world, then mathematatising" which leads to another of his problems, "How to create suitable contexts for mathematising."

2.5 Jean Piaget

For sheer volume of work and range of influence Jean Piaget led the field. For over forty years he studied children, in minute detail, from birth till their early teens. He, and his Geneva school, have studied the developmental stages through which children and certain types of concept formation pass and how the concepts are formed. While some of his conclusions and experimental situations have come in for criticism, his work has been indubitably seminal. Papert, Bruner and Donaldson all acknowledge his influence on their ideas.

He and his co-workers see the child as going through the following stages :

Ø-18 months	sensori-motor
1 -11 years	concrete operational
11+ years	formal operational

They see the three year old as being able to physically order objects in a row, the seven year old as capable of thinking about organising physical things and the twelve year old as being able to order propositions mentally. (Donaldson 1978, 138). During each period what was achieved at the preceeding level is restructured on a new plane. Thus he maintains that knowledge cannot come to us ready made, it is not a copy of reality, neither are we born with it. We must construct it for ourselves, slowly over the years.

The origins of thought, although at high levels, manifested in language and symbolic internalisation of knowledge, are in action. The amount we gain from our social environment depends on our ability to assimilate what is available. Social interchange, when meaningful, leads to development of thought, in particular to an awareness of different points of view. This ties in with Holt's view that it is the person who least needs to, who talks most in the classroom - the Teacher. (Holt 1976). The child who doesn't understand keeps quietest. Papert and Donaldson too emphasise the importance of language in the development of ideas.

In the same context Piaget regards the idea of decentration as being vital to the development of objective thought. Many of his early experiments - e.g. the child's apparent inability to see the mountain from another's viewpoint - have been criticised. People have pointed to the fact that the children being interviewed seem to spend most of their time looking at the interviewer as though the main aim was to guess what answer he wanted. In my own experience, children may frequently appear less perceptive and intelligent in experimental situations than when they are not on show.

It is extremely difficult to ask questions which are unambiguous, meaningful, right-for-age and which yield informative replies. Margaret Donaldson has a restructured version of the above experiment in which the child hides from a policeman and she says that this shows that the child can in fact decentre. (Donaldson 1978, 22). On the other hand he may well just know, from playing hide-and-seek, that usually if you cannot see, you cannot be seen.

To Piaget learning and development are not synonymous; learning he tends to equate with the acquisition of knowledge from an external source. This is not a generally held view. What most people would call meaningful learning he calls development. This takes place by active construction and self-regulation. In his view learning is subordinate to the subject's level of development.

2.6 Jerome Bruner

A one time co-worker of Piaget's is Jerome Bruner. His study of thought processes provided much of the force behind the Discovery Learning wave of thought. He analyses the processes of mathematics learning into three stages. In the initial enactive stage the child thinks in terms of actions and therefore if he cannot act out a solution he cannot solve the problem. In the next, iconic, stage the child manipulates images of objects. While images are easy to manipulate they do not facilitate the transformations in which mathematics abounds. Thus he believes that it is not until the child reaches the symbolic stage that he can handle most sophisticated mathematics. (Bruner 1966, 11).

While Bruner believes that each learning situation goes through a cycle of development from enactive to symbolic, Piaget regards the developmental sequences as essentially a sequence of stages which organically follow one another.

In the Chelsea project, Hart (1981a) and her co-workers found a high correlation between children's hierarchy levels in different areas - e.g. algebra and vectors. It appeared that children who were largely operating iconically were limited to the first two of the CSME's stages. The high correlations would seem to support a Piagetian idea of development. However, in an investigation into the sequence of acquision of concepts it was found by Kofsky (Hart 1981) that the pattern of success and failure in successively harder tasks was not consistent with a Piagetian prediction.

There seems, however, very little point in looking for conflict between theories when they are looking for the same thing - a better understanding of how children's minds work. Further, as Kath Hart says Bruner's early theory was based on Piaget. The distinction between the two appears to rest on Bruner's application of theory to classroom practice and his belief that by adaptation of the material to be taught the learning process can be accelerated.

2.7 Zoltan Dienes

A one time associate of Bruner's is Zoltan Dienes of Multibase block fame. His view too is that often the learning cycle is a microscopic copy of the developmental cycle. (Dienes 1966, 21 and 38). He postulated a number of different learning modes from the playing of games to abstraction, the underlying theme being the structure of mathematics. He has done a great deal of work creating situations in which, through creative and varied play, the children learn; John Holt writes with admiration of his multibase units, flats and longs. In these situations he provides a wide choice of paths to the solution. Sir Frederick Bartlett has found that subjects are more motivated in this situation. (Dienes 1965, 19).

2.8 Margaret Donaldson

Margaret Donaldson acknowledges two main influences on her thinking - Jean Piaget and Jerome Bruner. (Donaldson 1978). She is very concerned concerned with the change in children's attitudes to learning, life and school as they go up through the school - from excited, eager, curious, voracious learners at the outset to those at the other end who are often bored, alienated, illiterate, discontented and disillusioned. In contrast with Piaget she believes that "children are not nearly so limited in their ability to decentre, or to appreciate someone else's point of view ... as Piaget has for many years maintained." (Donaldson 1978, 3Ø). She gives examples of young children's spontaneous behaviour and conversations in which they are clearly capable of inferential reasoning while at the same time responding in contrived situations as if they are incapable of it.

She feels that their understanding and use of language may be the key to understanding this. She concludes that the heart of the matter is that children, by the time they come to school, are able to think and use language skilfully as long as, "they are dealing with 'real life', meaningful situations in which they have purposes and intentions and in which they can recognise and respond to similar purposes and intentions in others. These human intentions are the matrix in which the children's thinking is embedded. They sustain and direct their thoughts and speech, just as they sustain and direct the thought and speech of adults, even intellectually sophisticated adults, most of the time." (Donaldson 1978, 121). The child uses language but is largely unaware of its having a separate existence - it is a tool for coping with real, immediate problems, not problems which are presented in isolation, out of their useful matrix by some other person whose purposes are obscure.

Many concerned educators feel, she says, that the child should be offered opportunities to learn and a chance to gain a measure of control over his own thinking by being made aware of it and that in this way both his intellect and his consciousness can grow. She contends that while school is a wretched experience it is highly unlikely that it will foster this growth.

2.9 Erich Wittmann

In a similar vein of thought Erich Wittmann visualises two paths along which mathematical exploration proceeds. (Wittmann 1981).

The one is intuitive in which the objects are represented in a familiar, concrete pictorial or symbolic way. This is a direct, immediate, intuitive, often partly unconscious way of representation and information processing leading to the discovery of general patterns of behaviour. Very often it is intuitive thinking which leads to the creation of knowledge, the checking of proofs and the detecting of errors. (cf. Donaldson's embedded language).

Reflective thinking aims at the explicit formulation of general relationships; it is dependent on the mathematical language of concepts and symbols and the investigator does not simply work with objects and operations, he reflects on and talks about his activities. This abstraction, coneptual ordering and schematisation of sets of objects is essential to the growth of mathematics.

He suggests three types of activity to encourage the development of both types of thought :-

(a) The student must investigate and observe a rich variety of examples and models .... himself. He cannot learn intuitive experiences through verbal interaction - i.e. by chalk and talk.

(b) The student should be encouraged to reflect upon these discoveries, "to learn to test, demonstrate, improve and corroborate their intuitive ideas."

(c) The student should gradually learn to analyse concepts, to construct theorems and proofs.

Schools should do (a) and (b) above in a spiral approach as suggested by Bruner (Wittmann 1981, 395).

He (Wittmann 1981, 396) concludes "We need more knowledge about students' primary intuitions and the possibility of fostering intuition by operative activities. We also need further investigations on the development of reflective thinking. Of particular interest are longitudinal studies. The constructive branch of mathematics education faces the difficult problem of developing courses which base reflective thinking on intuitive activities and formal thinking on informal. This problem is difficult because it requires a sound feeling for changing levels of rigour. It is much easier to stick to the intuitive level - and to ignore advanced maths - or to rely on the formal systems of mathematicians - and to strive for the blessing of the watching Big Brother mathematicians."

2.10 Gerard Vergnaud

A study on the acquisition of arithmetical concepts by Gerard Vergnaud (1979) follows the direction of Wittmann's future problems and relates to Scandura's ideas on the learning process. He puts forward the idea that the problems children experience in the leap from 'lowly arithmetic to the noble mathematics' concepts has something to do with the prejudice against arithmetic. In Resek and Rupley's work on mathophobic students they found the best predictor of those students who could become concept oriented was a computation test. (Resek and Rupley 1980, 434).

As he says (Vergnaud, 1979), arithmetic, even in its elementary aspects, deals with very important mathematical concepts. To establish a link between ordinary arithmetical situations and the relevant mathematical concepts is probably the most challenging question in mathematics education. (Cf. facilitating Wittmann's intuitive - reflective jump and the step from embedded to abstract language).

He is interested in the child's developmental complexity and in how this relates to the mathematical complexity of different tasks. He suggests that as adult mathematicians we usually ignore the time - order - relationship and the dimension aspect of a situation. He (Vergnaud 1979, 266) says "it is hopeless to try to understand the aquisition of arithmetical concepts and to propose better conditions for the child to understand them if one does not make the effort to analyse the tasks through which these concepts are made meaningful and useful to the child."

Different children attempting the same task offer a variety of procedures - which are not necessarily equivalent from a cognitive point of view - be they successful or unsuccessful. He stresses that the problem of complexity is not directly dependent upon the difficulty of the task. The easiest way to tackle a difficult problem may be easier than a more sophisticated way of tackling a simpler one. You can learn more about the meaning of an operation to a child by studying how he deals with a problem and assessing the type of relational calculus he employs than by studying only his use of words and signs referring to the operation.

We, as teachers, need to find problems which the child cannot first solve, and then write down their symbolic representation to satisfy us. He finds that most arithmetical equations that children are supposed to use in primary school are not (as tested against this criterion) operations as they come after the solution has been found. They represent the operations the child made in solution rather than the relationships of the problem. For example, in $3x + 7 = 16$, you 'move' the 7 to the other side and change its sign.

He sees a need to differentiate between different situations even though they lead to the same equation, as that equation is frequently too abstract for the child. (Vergnaud 1979).

2.11 Lesley Booth

Lesley Booth gives us concrete examples of these "informal, naive, child-methods" used in secondary school mathematics. (Booth 1981). In the Concepts in Secondary Mathematics and Science study at Chelsea College they found that the majority of children used their own methods to solve the problems, not teacher-taught algorithms. Most children used an adding-on strategy, repeated addition and subtraction or doubling and halving to solve nearly all the problems they could solve. They used primitive counting and/or combination-based approaches. The brighter children managed well until the situations became too general or the arithmetic too complex. She suggests that to these children formal taught mathematics does not exist (and by an extension of the argument, might as well not have been taught). They attempt to solve a new problem in a human sense manner if they are motivated. Or they simply guess. It seemed as if two completely different types of mathematics were involved, one where the children use common sense, the other where they had to remember a rule. As the child said "I was trying to do it mathematically, not logically." (Booth 1981, 35). The difference between the two systems seems to relate to Margaret Donaldson's referenced and formal languages.
It appears that children often do not see things as we do. Vergnaud (1979) too reports that different axioms, for example commutativity and inversion of addition, which seem of equivalent cognitive complexity to an adult are of differing developmental complexities.

When children use their own methods they are in fact attempting feats of considerable difficulty as they work up from first principles; e.g. the child who attempted to divide 391 daffodils between 20 beds by successive subtraction and solved the remainder problem by putting them in a pot instead of simply saying "391 divided by 20". (Booth 1981, 37).

It has been shown by Bruner that by being made aware of a more efficient strategy children can be led to improve their apparent level of cognitive functioning. (Dienes 1965). The problem is to make the child see a real need for a more formal means of handling a situation, especially if he can manage nearly everything he is asked to do in his own way. The problems Vergnaud would have us find are the ones needed here. (Vergnaud 1979, 269).

In searching for Gattegno's negotiated meaning we must not however destroy the initial intuition by giving a general rule too soon. "Ways must be found of working from the child's own strategies but in such a manner as to ensure their replacement by a more mathematical approach. Perhaps this can be done by attempting to make the child aware of the inadequacy of his own methods (without making him feel stupid) and of the existence and superiority of the systemised approach; perhaps more indirect ways are required." (Booth 1981, 40).

In the Strategies and Errors in Secondary Mathematics project the Chelsea College team are hoping to translate the ideas they have gained in the C.S.M.S. study into effective classroom practice.

2.12 Dietmar Kuchemann

A co-worker of Lesley Booth's is Dietmar Kuchemann. He discusses the "mismatch between children's understanding of mathematics and the cognitive demand of much of what they are taught" in English secondary schools. (Kuchemann 1981, 301). He does not attempt to resolve contrasting views of how children's understanding develops but adopts as a framework for his discussion the view, held by Piaget and Inhelder amongst others, that "understanding develops in a sufficiently uniform way among different children and across different tasks for children usefully to be assigned to common stages of cognitive development but that intervention, for example through teaching, can have only a limited effect." (Kuchemann 1981, 301).

He examined the facilities of selected items and the correlation coefficients between the CSMS's different tests in an attempt to give horizontal and vertical comparisons. The vertical comparisons "should provide useful guide lines to teachers about the nature and order of magnitude of some of the factors which determine the difficulty of mathematical tasks. e.g. In algebra for 14 years olds

a) $a + 5 = 8$

$a = ?$ ... has a 92% facility

b) u = v + 3

v = 1

u = ? ... has a 62% facility

c) Add 4 to 3n .... has a 35% facility

d) Which is larger 2n or n + 2? ... has a 4% facility (Kuchemann 1981, 310).

He says "The purpose behind the horizontal comparisons is more ambitious but their value in practice is less certain." (Kuchemann 1981, 314). The tests can serve as predictors of a child's probable level in a different area of maths since the high correlations seem to indicate that a child is likely to be able to reach the same stage in different topics. He discusses in detail types of problems that fall into the 75%, 50% and 25% facility levels in each topic. It is eye opening to discover, for example, that three quarters of a class of fourteen year olds cannot divide decimals.

2.13 Kath Hart

In her paper "Hierarchies in Mathematics Education" Dr Hart (1981b) discusses the work of a number of people in this field. "The word hierarchy when applied to how and in what order children learn mathematics, is used in a number of ways. It can be used to describe :-

a) a learning sequence or sequence of understanding, which is essentially in the learner

b) a teaching sequence which the teacher uses

and

c) a logic sequence which is in the topic.

These are not necessarily the same but can be considered interdependent. For successful learning in school, the three aspects must be closely matched or failure is the result. In each case 'hierarchy' implies a string of skills/levels/stages/concepts which are ordered from simple to complex. (Hart 1981, 203).

It is upon Piaget's idea of stages of cognitive development that much sequencing of mathematics experience has been based. Kath Hart (1981) quotes validation studies which show that only a minority of adolescents seem capable of, or find it useful to use, formal operational thought.

More recently, in the same field, the Van Hieles work in geometry has concentrated on matching the level of teaching to the child - "thus a teacher who is talking at a level higher than that of the pupil has no meaningful form of communication with him." (Hart 1981b, 207). They see the learning process as discontinuous with jumps revealing the presence of levels. They also see it as discontinuous in time. Their geometry levels are:-

a) child recognises shapes as distinct figures

b) he sees relationships within and between figures

c) he establishes properties of figures and can logically order them

d) he can see the significance of deductive geometric theory

e) he can abstract and deal with geometry without concrete representations.

One wonders how these stages fit into or compare with the stages 1 to 4 constructed in the C.S.M.S. project. In Russia children taught in conformity with this theory achieved in eight years what others achieved in eleven. And in our New Maths syllabus formal theorems are to be introduced in Std. 6!

She (Hart 1981) describes Robert Mager's experiment in which adult electronics students, through their questions, structure their own course. The experiment highlights the following points:-

a) when a question was unexpected it was difficult for the instructor to keep up with the material

b) all the students had different background knowledge which, even though not in the field of study, affected how they learned electronics

c) the initial interest was not in theory but in concrete examples.

Hart (1981b, 211) suggests that "sequencing of material by an expert who knows 'the whole' is not necessarily the best way of matching the order of learning natural to a child."

Another view of hierarchies is that used in the C.S.M.S. project where the children's performance on test items was used to group the items within a particular topic into homogeneous groups with a high level of association. Children were deemed to have succeeded at a level if they scored more than two thirds of the marks at all the levels up till then. The children appeared to go through four stages in all the topics tested and the correlation between levels attained by a particular child on different topics was high. (Hart 1981, 215).

She (Hart 1981, 216) concludes "What the teacher needs is well documented evidence that a particular teaching sequence is preferable to another and some means by which he/she can ascertain whether a child is ready (the criterion being either cognitive level or pre-requisite knowledge) for that sequence."

2.14 Joseph Scandura

This area has been explored by Joseph Scandura who puts forward an idea of how maths learning takes place in which, in contrast with many others, the teacher can play an important role. (Scandura 1975, 385). He has a rule-orientated view of knowledge which he illustrates by means of a trading game. He shows that in order to handle a problem the subject must not only know the components of a solution - he must also be able to put the components together appropriately. This integration is acocmplished by applying higher order rules to lower order ones (as in Dienes' generalisations?) and in addition the learner must have an innate capability which tells him how and when the various rules are to be used in tackling problems.

He suggests an innate mechanism which is basic to his structural theory - it explains how known rules interact and rests on three hypotheses :-

a) Simple Performance Hypothesis; Given a goal and the availability of one or more rules, each of which generates the desired response, the subject will use one of them.

b) Control Shift Hypothesis: If the subject does not have a rule immediately available for achieving his goal, control automatically shifts to the higher goal of deriving such a rule.

c) Learning and Reversion Hypothesis: Once a higher goal is satisfied the newly derived rule is added to the available knowledge (i.e. is learned) and control reverts back to the original goal.

He (Scandura 1975, 383) maintains that while this simple mechanism does not explain how all learning takes place it does account for a "whole lot."

Gagne (1968) has similar ideas. He assumes that any task can be analysed into a set of component tasks which should be identified if an optimal instruction design is to be obtained. Other researchers have found that "learning and retention of an hierarchical task are facilitated by mastering each successive component of the hierarchy before continuing in the instructional program." (Hart 1981, 216) It is however neither easy nor necessarily effective to dissect a topic into component skills. (Hart 1981, 214)

To return to Scandura's theories. He examines how a child finds the underlying rule in 1 - 3; 2 - 6; 3 - 9; 8 - ? He suggests that a higher order generalisation rule generates the discovery. Further, he shows that children who were given higher rules made discoveries which the control group did not.

In an example of two children, A and B, learning to use trigonometrical identities he explains how B, whose behaviour is more flexible, has a greater ability to retrieve and regenerate information and is therefore able to handle more than the routine tasks that A can handle with the same basic information.

In conclusion he says "We have found that what a child learns in a given situation depends on what he already knows. This in itself is not new ... but we have also learned that the concept of a rule which operates on other rules (which are functions defined on functions) is a critical idea that is missing in most existing theories of learning." (Scandura 1975, 383) Based on these ideas he sets out steps whereby a teacher can identify the rules and higher order rules which are necessary to cope with the tasks given - thus facilitating better teaching and, he maintains, transfer of learning.

While it may account for "a whole lot" in limited situations this view must come in for criticism from a number of sides. It is so rigidly focussed on the cognitive aspects of learning, ignores the role of intuition and appears to me to be be at variance with the finding of Brooks with respect to the role of searching for rules in a learning experience. (Dreyfus 1984, 598)

3. Conclusion

We, as teachers, have to get away from asking "What's the answer?" and "What's your score?" which encourage Holt's evasion strategies and "word swallowing" instead of real learning.

Through observing situations in which real learning does occur valid learning theories are developing. It seems commonly agreed that people learn for themselves, at different rates, by verbalising, not in silence, through different stages, and, it seems, in discrete jumps. We go from the concrete to the abstract; from an enactive to a symbolic stage; from embedded to asbstract language; from intuition to reflection and abstraction; from naive child methods to formal mathematical methods.

It seems that where we fail to take cognisance of the above "jumps", we often leave the bulk of the children floundering in a meaningless sea of symbols. Frequently our mathematics courses begin with a definition instead of with examples which would allow the child to formalise and internalise his own definition. Our school-set problems can generally be solved intuitively or by child methods and it is only to satisfy the teacher that the child puts it down in the learned pattern - he sees no need for the formal symbolic system, his way is often easier and, since it is logical, why relinquish it?

We must not destroy or deny their intuition but by providing meaningful examples lead them to see the need for and beauty of a general symbolic system. That is, of course, if they ever did basic arithmetic in any way other than by teacher taught over-drilled algorithms. For if the arithmetical concepts are absent we will spend forever constantly rebuilding a foundationless structure.

It appears vital for children to have time to "mess about" in a fruitful environment and then to be encouraged to reflect upon their intuitive findings - and to have the language with which to do it. The environment in which they find themselves is vital - but this is not to say that it needs to be full of things - it can just be a free, thinking environment in which people talk and interact. If the child feels more responsible for his learning, his learning will be less rote, rule-orientated and more schematic and concept-orientated.

It is vital that we acknowledge that in our system many children are unhappy and learn very little of what we are trying to teach them. I will explain in my next essay some of the ways we can begin to counteract this.

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Can We Make Learning Mathematics a More Positive Experience?
A Discussion of Three Approaches to this Question.

SEPTEMBER 1983

CONTENTS

1. Introduction

In the light of the learning theories discussed in my first essay I feel justified, as a high school mathematics teacher in South Africa, in putting my head in my hands and despairing. I have long felt that we, the pupils and teachers, were putting in a great deal of time and effort for very little positive result.

Now I can see that there is very little theoretical justification for the existing syllabuses or for the policy of lock-step classes who pass on to the next year after possibly not even attaining a pass mark of 33% the previous year. By the time they reach standard ten there must be hordes of pupils to whom mathematics is a meaningless, often terrifying, subject. Many of the matriculants I have taught resembled nothing so much as cows in quicksand flailing around and grasping at straws rules. Yet so many careers have mathematics as a prerequisite that many children have no option but to battle on.

It appears that our conventional chalk-and-talk content-orientated mathematics teaching is frequently almost useless. Further, that a teacher galloping through a syllabus in front of a large group of children can really only say with certainty that he has done the work.

This is all very depressing.

If one accepts that it is necessary to learn mathematics there are a number of approaches to teaching it which have a common basic belief; namely that it is the child and not the content which is central to a meaningful learning situation. I will consider three of them.

Ideally we should set out to **humanise mathematics education** as David Wheeler and others envisage. Failing this it is still possible even within one rigidly controlled system for us to **listen more effectively** to the pupils and to **provide them with more accurate diagnoses of their errors.** For these children and people who are already matho-phobes we can look at ways of **helping them overcome their fear.**

2. Humanising Mathematics Education

In 1975 David Wheeler (1975b) addressed the Association of Teachers of Mathematics Easter Conference on this topic. He said that having originally chosen a topic in which he felt sure he would find a place from which to speak when the time came he felt that he had chosen not too wisely, but too well. He suggested that a topic as powerful and evocative as this one should be the subject of a lecture at each of the ten succeeding conferences in order that the ATM could contribute to the extremely valuable task of humanising mathematics.

He (Wheeler 1975b,4) said "I don't expect, and I don't even want, every child to find maths an engrossing study - but I would hope that every child could experience for a few moments in his school career the power and excitement of maths," and further that "humanising maths education is not to be confused with encouraging maths teachers to come on like warm accepting therapists."

He wanted to humanise mathematics education in order to eliminate the fear and anxiety often experienced by children doing mathematics, in order to develop the child's self awareness, character and individuality. How does he suggest we do it? Primarily, "by the substitution of the goal of facilitating children's mathematical activity for the goal of passing on mathematical knowledge; by stressing the process not the product, by striving to encourage the children to the art of mathematising." He continued, "activity is personal whereas knowledge often seems impersonal; activity is dynamic whereas knowledge is frequently inert; activity implies involvement in one's own learning rather than passive acceptance of someone else's." (Wheeler 1975b, 5).

The concepts of awareness, growth and mathematisation are at the heart of his essay. He says, "In a crude attempt to make explicit the nature of mathematisation I would include the following ingredients: the ability to perceive relationships, to idealise them into purely mental material, to operate on them mentally to produce other relationships. It is the capacity to internalise, or to virtualise, actions and perceptions so as to ask oneself the question 'What would happen if ...?', the ability to make transformations - from actions to perceptions, from perceptions to images, from images to concepts, as well as within each category - to alter frames of reference, to refocus on neglected attributes of a situation, to recast problems; the capacity to co-ordinate and contrast the real and the ideal and to synthesise the systems of perception, imagery, language and symbolism. When these functionings are applied to pure relationships, detached from specific examplars, the result will be mathematics." (Wheeler 1975b, The games and activities suggested in Gattegno's "The Common Sense of Mathematics", Stern's equipment, Cuisenaire rods, Dienes' multi-base bricks and games, and Papert's turtles are designed with this in mind. But this equipment tends to sit on shelves and gather dust unless teachers are encouraged and reminded of its value. Also we have seen that the children must be given time to "mess about" with the apparatus if it is to be of any real and lasting value. The teacher needs to have faith that they are not just "wasting time". We need support from those "who know more about maths" as the fear of leaving out something which may be important later causes us to do everything we are told to do, probably without doing any of it well. Teachers should be encouraged to be discovery learners.

David Sturgess (1980) suggested at the ATM conference that when teachers go on courses they are looking for the "best" way to teach mathematics or even "how to teach fractions" and to be entertained and that they do not discuss the vital issues of teacher-pupil relationships - eg how the teacher feels about the child, the forbidden areas of their own fears about what they are doing, their feelings about the kids they teach, their feelings about themselves doing mathematics. He asked how many teachers use mathematics, often regarded as a "cold" impersonal subject, as a barrier between themselves and the kids.

Ken Dovey, in a series of lectures given at the University of Cape Town explored this idea from a socio-political perspective. He asked us to consider why we had become mathematics teachers. Was it because we shied away from involvement in moral, ethical and political issues and hid behind our "value-free" subject? That mathematics is value-free can be fiercely debated, that it should be, even more fiercely. To many people mathematics is in fact an oppressive subject in the sense that it perpetuates the status quo in an oppressive society, and that it is presented as a fixed body of knowledge to be assimilated instead of as a language, a territory to be explored in a variety of ways (Holt 1967, 136) and an activity conducted by human beings. (Sturgess 1980, 24).

He (Sturgess, 1980) suggested a means of building a meaningful autonomous in-service activity in which the teachers' past experience is shared and used; teachers get "space in which to grow" and "permission to do what they believe in" and time to examine their beliefs while being actively involved in tasks and having their own awareness educated and awakened. In short this training mirrors what he felt we as teachers should be offering our pupils.

What is this awareness of which we hear so much? What does Gattegno mean when he says, "Only awareness is educable"?

He has made his ideas more explicit as we see in this extract from the Common Sense of Teaching Mathematics:

"The most important lesson teachers can learn is that rather than teach mathematics we should strive to make people into mathematicians. That is to say, we should make people aware of certain powers they already possess which they can use in the same way that they are used by mathematicians. The professional mathematicians specialize in working on new challenges and publish their awarenesses; to a student, the challenge he is working on may seem just as new, though what he discovers may be already known. Since teachers should be more concerned with the dynamics of the mind as it functions in mathematical pursuits than in the novelty of the expression of an awareness, their preparations as mathematicians will be to become aware of particular functionings in themselves which they may later find to be educable in their students."

Sturgess summarised this in saying that it is the task of the teacher to help children feel what it is like to act as mathematicians. His choice of content is that which he believes will help teachers to develop this awareness since in order to reach others one must first become aware of one's own awareness.

These ideas are echoed by Papert in his paper "Teaching Children to be Mathematicians Versus Teaching about Mathematics".

In my experience the problem lies in making people aware that many of the things which they do competently are in fact mathematical.
As a sequitur they can be helped to realise that they are already, albeit unknowingly, doing mathematics and mathematising, and that their possible incompetence in the realm of school mathematics is a reflection upon its presentation and content and not upon their mental processes. At a less general level I would argue that an ability to do exacting patchwork, Fair Isle, or Aran style knitting shows a more highly developed and internalised feeling for symmetry, reflections and rotations and co-ordinate geometry than is evidenced by attaining a pass mark on a standard ten paper on these topics. Not many people

however would agree, the knitters often know that they are "useless at maths" and the mathematicians know that "anyone can knit and sew".

Wheeler (1975b,9), in a paper on humanising mathematics education suggests that "awareness is very nearly the same quality as insight, that subtle English word that hints at the possibility of 'seeing into' by means of one's inner sight." He suggests that in mathematics awarenesses are sometimes made explicit in defini-tions. He shows how the study of finding the area of an irregular shape makes us aware of:

a) the limitations of rule-of-thumb methods
b) lower and upper bounds
c) refinement of measuring devices
d) the meanings of area

He (Wheeler 1975b, 8) says "Awarenesses are of all types ... they may act as backgrounds - e.g. the context of geometry is images - or they may be more technical. Awarenesses often arrive unsought, in the middle of some other concern ... but we can seek them ... and by attempting to answer questions relevant to the topic ones understanding of a topic grows."

And when the awareness arrives - "Aha, says the child 'I've got it'." At the 1981 ATM conference David Fielker entitled his lecture "Communicating Mathematics is also a Human Activity." He said, "Perhaps the art of teaching maths is the art of asking the right questions ... and to enable and encourage the children to ask the questions."

This is in agreement with what Wheeler (1975a,25) said , "The teacher who approaches his job scientifically will start at the tasks to be done and will consider how the attention of the children can be focussed on them. He will consciously withdraw as much of himself as possible so that he will not be an interference to the activity he wants to promote ... everything he does directs the children to the problem and to their own actions in tackling it." He offered specific guidelines for the teacher who aims to achieve this.

Fielker (1981) focusses our attention on the communicating of mathematics. He stresses that mathematics, by its very nature, is concise and that we need time to think in between and during steps. He echoes John Holt in saying "Most teachers talk too much" and quotes a friend's daughter; "You know, daddy, she's the sort of teacher who explains so much you can't understand it." (Fielker 1981, 5). He would rather we put our energy as teachers into listening to the children and in communicating with other teachers - even and especially if the lessons we discuss are failures.

In "Linda's Story" David Kent (1978) gives a wonderful illustration of what humanising mathematics education is about. In a few months in pubs and houses and on the cliffs he, unconsciously on his part, made her realise that it was necessary to find "something to put my energy into". That she did not want it to be mathematics is irrelevant.

At Belper High School he and Keith Hedger (Kent and Hedger 1982, 137) have implemented their belief that "it is our responsibility to offer pupils situations in which they are able to express themselves to the extent of believing that they can make some contribution to mathematics, that our subject is not one they need regard with fear, but on the contrary it can make a positive contribution to their lives." Their satisfaction has been to observe the pupils "growing tall" within and through the subject.

By working together they expose the children to more than the usual ideas utilising the children's ability to visualise more than they can symbolise. "We share a belief that the ability to think in images, to construct and transform mind pictures is a fundamental thought process which many children lose. It is a power of the mind we feel may be lying dormant within so many kids." (Kent and Hodger 1980, 152)

Further they believe, influenced by Mary Boole, whose writings covered much that is to do with mathematics education, Caleb Gattegno (whose ideas they say, and I agree, are often difficult to grasp, and are almost impossible to summarise) and J. L. Nicolet (user of animated geometrical drawings and film in teaching geometry) that "all of our children are mathematically gifted and that it is our task and sometimes privilege to make them individually aware of it." (Kent and Hedger 1980, 150). They seem to have implemented this positive belief and their, or rather their pupils', results and achievements are staggering, exciting and encouraging.

But we could not do that at our school -

the classes are too big/slow/varied/bored,
and we have not the time/space/facilities/know-how/staff
and in any case we have to stick to the syllabus;
imagine what the inspectors/headmaster/head of department/parents
would say/do/feel.

3. Listening to Children and Diagnosing their Errors

But no-one is going to stop you listening to the children. Neither will you be forcibly prevented from trying to diagnose effectively their errors or trying to understand why they make them.

Much of the research in the area of investigating children's problem solving techniques, errors and thought processes has been via written tests. Yet as early as 1917 Uhl (In Hollander 1978) observed children as they worked and questioned each regarding approaches used. In this way he was able to determine not only specific errors but some of their fundamental causes. His study was hailed as a significant contribution which should become the basis of an effective plan of diagnosis.

In her conclusion 60 years later Sheila Hollander says "In a far less formal manner than that of the researcher, the classroom teacher would do well to employ a simple introspective-retrospective technique which requests that a student verbalise his thought while involved in a problem solving task. The written product provides the teacher with information on what was done, the verbalisation guides the teacher as to how the product was achieved" (Hollander, 1978, 333).

In the CSMS (Hart 1981) project they found that the interviews while being time consuming provided a wealth of information about why the children made the mistakes they did, and how their minds worked and didn't work. They tested 10 000 children from ages 11 to 15 on the following topics - algebra, reflection and rotation, ratio, decimals, integers, fractions, graphs, vectors and measurement - and in pre and post interviews explored the exposed areas of weakness in individual children.

Based upon this study they have produced at the end of each chapter soundly based practical guidelines for teachers. Eg "It was clear that many children still needed visual models of tenths, hundredths and so on ... the use of ... even just squared paper divided, or actually cut, into units, rows of ten, squares of a hundred and so on would obviously help." (Hart 1981a, 64) And on the subject of graphs, "... there appears to be a large gap between the relatively simple reading of information on a graph and the appreciation of an algebraic relationship." (Hart 1981a, 135) As described in Lesley Booth's paper they found that many children use very similar, untaught, child-methods. Dr Hart said "If all we have done is make teachers look out for the adding-on technique it has been worth it" (Hart 1981a, 100).

They also suggest that a child's performance on one of their tests can serve as a predictor for his level on another section. We are made aware of the different types of questions which have a similar facility level and of the types of questions which are, sometimes surprisingly, very difficult for all but a few school children. Dr Hart felt that English children no longer hated mathematics, possibly because of Miss Biggs and the Nuffield project "I do and I understand" series of books. (Hart 1983)

It may be that our white South African children would perform better on the computations since they have often had relentless drilling in applying algorithms. But would they have the initiative to develop their own child methods? Would we find a correlation between the absence of child methods and a dislike for mathematics?

Klaus Hasemann found that in his study on German school children their thinking was largely of the type Skemp calls "instrumental" not relational. Interestingly they did better on computational items than their problem counterparts - the opposite finding to that of the CSMS team. He quotes Hart (1981, 91) "they did not use the taught algorithms when they had available their own successful (even if naive) methods." While he (Hasemann 1981, 80) says "Our pupils behave rather in the opposite way. If they have an algorithm available which they have already practised, then they use it." Further "In purely computation exercises (the use of rules in clear situations) even the hauptschuler are quite successful; in comparison with the English and American pupils they are very successful."

A similar study by Af Ekenstam "On Children's Quantitative Understanding of Number" on Swedish children from 14 to 16 reveals many specific weaknesses that children have in handling decimals and fractions.

The value of the studies, even if they differ in minor ways, is that they give us increased insight into the workings of children's minds, and specific information about what our pupils probably can and cannot do.

Dr Hart suggested that we do not like gaps in our knowledge so we fill the gaps with matter that makes sense - even if only to us. The more confident the learner and his experiences the less likely he is to fill the gap with nonsense - as is often the case in school mathematics.

John Holt (1967) believed that we cannot find what the gaps are in another's mental model because:

a) the child is unaware of his own lack of understanding
b) he lacks the skill to verbalise his confusion
c) we have not the time because talking takes so long.

If researchers like the CSMS team, Hoyles, Clements, Af Ekenstam and Freudenthal agreed with him there would be no hope of helping children in our school system. Fortunately they do not, although they do all agree that diagnosis is difficult and time consuming.

In the CSMS interviews they taped and transcribed the interviews in which the children did the test items "out loud". They tried not to help the children at all and to wait patiently while the children thought. Their feedback is nearly all in the cognitive realm.

Another framework for interviewing children working on problems is the error-classifications devised by Newman. He developed a "Criterion for Error Causes" in the following steps - reading - comprehension - transformation - process skills - encoding. This can be used with a wide range of mathematical problems in order to help both the teacher and pupil find where errors are being made.

This hierarchy has been extended and generalised by Casey (1978) so as to be useful on many-step verbal problems. Both have error categories outside of their hierarchy - carelessness and motivation (Newman) and unknown and known blocks (Casey).

Clements (1980) found that it was futile to attempt to make valid general statements about the characteristics of good and bad problem solvers. "All that can be said is that an individual's error pattern can throw considerable light on why that individual makes mistakes on mathematical tasks." (Clements 198Ø, 15) He illustrated how the error analysis procedures can provide useful information for teachers by providing "gross diagnosis" from which "more detailed probing" can develop.

An additional advantage of an interview situation is that it is nearly always possible to tell the child that he had been suc- cessful at some particular stage thus providing positive re-inforcement. He (Clements 198Ø, 18) said that "Sometimes it was a pleasant surprise to both teacher and child to discover how much was successfully achieved. Even though he had been teaching these children for several months at the the time of the interview, he found it very enlightening to see and hear how each child tackled the problem."

A technique to explore children's attitude to mathematics is "critical incident" interview. In semi-structured "interviews" children were asked to tell stories about times when they felt particularly good or bad when learning (Hoyles 1982). This was found to be more productive than questions like "What makes you work hard?" They developed a technique for analysing the stories which enabled them to be compared, but for the classroom teacher it would often suffice to use the technique simply to get the child to talk about his feelings.

Streefland (1978) and Freudenthal (1981) show very clearly how much can be learnt in a one-to-one situation if one takes the trouble really to listen and provide meaningful prompts.

While the tests used by the CSMS team cover a great deal of work it would be of great value to be able to develop one's own diagnostic tests for a particular section. A method evolved by Adolf Af Ekenstam and Margita Nilsson is applicable.

"Starting with a complicated equation, e.g. the equation

$$(3x - 2)/2 = x/3$$

a sequence consisting of 5 - 15 problems was constructed. Each new problem followed the preceding one by taking away one or two details.

$$3(3x - 2) = 2x$$

$$9x - 6 = 2x$$

$$7x - 6 = 0$$

$$7x = 6$$

is an example of a sequence belonging to the equation above" (Ekenstam and Nilsson 1979, 41)

Their study aimed at diagnosing the retention of basic skills in some topics in algebra and geometry and revealing difficult steps in the learning processes in these topics. The topics are equations, simplifying algebraic expressions, changing the subject of a formula, use of formulas to find areas of circles and triangles. In each topic they make observations upon those things which appear difficult to the children, and on those things which don't appear to make any difference to the degree of difficulty of the problem.

It would seem that we have come a fair way since 1859 when the following two questions formed the basis for grading schools:

"What is the cost of five dozen eggs at five for twopence?"
(p = Ø,18)

"What do you mean by the state of life into which it shall please God to call you?" (p = Ø,Ø8)
(Howson 1978, 217)

4. Recognising and Combatting 'Mathophobia'

Many people in our and other school systems develop Mathophobia - which has been defined as an irrational and impeditive dread of mathematics. (Resek and Rupley 198Ø)

Most people view mathematics as an elitist subject understandable only to those few with mysterious insight who understand it. In this sense mathematics is certainly an oppressive subject. They feel that the teachers (and a few pupils) understand it in its entirety. As John Holt (197Ø, 144) says his student "in her confusion, ignorance and bafflement believed, because we as educators had led her to believe it, that we knew everything, knew where it fitted and how the paths related to each other." He wrote back to her that the difference was that he was not afraid of uncertainty, he further felt that uncertainty was necessary in order to cope with a changing world. She and many others had been led to believe that if it's not right it's wrong, and that if you get things wrong you are stupid.

Many people, talented in their own fields, report feeling physically ill when confronted with mathematics in its school form. They consequently avoid meeting it in other courses thus severely limiting their academic development. Nuria Cortada de Kohan (1969) reports that students in psychology are traditionally frightened by the requirement to learn statistics.

Oxrieder and Ray (1982) gave signs of mathematics anxiety as blocking out, tension, panic, paranoia, tuning out, guilt, physical reaction and avoidance; especially in stressful situations like tests.

Why do people develop this fear? Because they believe they cannot do mathematics. To them mathematics is school mathematics and is frequently unintelligible for a number of reasons I have touched on in these essays. To be forced to subject oneself to feeling inadequate once a day for the years from six to fourteen is an easy way to induce a phobia.

Fortunately, it appears that mathophobes can be helped.

In "Your Number's Up" Oxrieder and Ray (1982) help the mathophobe to realise why he is anxious, that he is not alone, that mathematics is not limited to school mathematics and that through better self management and a gradual introduction to problem solving skills he can cope with what he previously considered beyond him.

In a similar course, "Maths Without Fear", at San Francisco State University, Resek and Rupley (1980) helped mathophobic students try and prepare themselves for their next mathematics encounter. They concentrate on changing the students from being rule-oriented into being concept-oriented. The rule-oriented student finds mathematics very difficult to remember as it is just a collection of rules. The concept oriented student remembers by association and can employ the inter-related ideas and the growth of concepts.

To facilitate this change they use a number of techniques - guessing, use of concrete objects and visualisations, playing, especially in small groups, careful matching of problem to student level (challenge without the terror) and they try to support their students emotionally.

Alan Bell et al (1981) used similar techniques - the pupils are encouraged to rephrase the problem draw a diagram, use simpler numbers, try x and y, use calculators, work in pairs. (Bell 1981)

It appears that, once acknowledged, mathophobia can be alleviated using some or all of the above ideas in a classroom situation or in smaller groups.
How much better if it had never reared its ugly and debilitating head.

5. Concluding Remarks

We can, it seems, make mathematics a more personal, meaningful, happier subject. It is however not an easy task. John Holt (1967, 173) exhorts us to have faith in the fact that man is a thinking and learning animal; that if we brought the world to the learner and left him free, with help, guidance and respect, he would do the rest - without our needing to keep picking his mind to check his progress and with no wheedling, bullying, cajoling and bribing.

Jerome Bruner tells us "knowledge is a process not a product" and Caleb Gattegno's "Only awareness is educable" seems to sum up a multitude of ideas. We know that when we really need to know something we learn rapidly and permanently - we don't need drilling and testing. What then is the point of the teacher "getting through the syllabus" if most of the children learn only 40% of what is taught after endless boring drill and tests? And we also know that generalisations and insights are only of value if you have, or were about to have, formed them yourself. So we as teachers must stop showing people how to do the "easy" steps quickly so that they can get on with what we believe to be more important work. If they do not build their own knowledge from the bottom up they can go nowhere - fast or slowly. When necessary we must take the time and trouble to listen to children as they verbalise their mathematical efforts - instead of constantly examining them in stressful situations.

Then we can concentrate on what they do know and build from there. But it takes a lot of courage to admit that you standard eight class are still doing standard six algebra. But what is the point of the teacher doing standard eight algebra while a miserable, bored class never progresses past standard five? In the end what counts, surely, is what the child has learnt, not what the teacher has taught.

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"LOGO is a Language for Learning."

A Discussion of Papert's Ideas on Learning and Computers and the Way in which LOGO Reflects them.

MARCH 1984

CONTENTS

1. Introduction

We are living in what Barnes and Hill (1983) have called the years of transition from the industrial to the information age. They say it is an age of instant communication but limited understanding and that it is vital that children become comfortable with the tools and processes that have an impact on their lives. This age has brought with it a marvellous new tool, the microcomputer.

As Papert (1982) says, his discussion of a computer culture and its impact on thinking presupposes a massive penetration of powerful computers into people's lives. There is no doubt that this has happened. What can happen in the future is a technical question but what will happen is, he maintains, a political question, depending on social choices. How the children of the 1980s and 1990s will be influenced by computers depends on the people who are attracted to the world of computers, the talents they bring and the tastes and ideologies they impose on the computer culture.

In this essay I will discuss his views on learning and the computer language which he and his associates have developed.

2. "The Great Debate"

There has been what Carter (1981) calls a Great Debate among educators about how microcomputers could and should be used. There are three basic options, all of which have their proponents.

Firstly they can be used for computer aided instruction. This means using computers to provide drill and practice in the current curriculum activities. Papert (1972b, 2) is scathing about this use: "The phrase 'technology and education' usually means inventing new gadgets to teach the same old stuff in a thinly disguised version of the same old way. Moreover if the gadgets are computers the same old teaching becomes incredibly more expensive and biased towards its dullest parts, namely the kind of rote learning in which measurable results can be obtained by treating the children like pigeons in a Skinner box."

A lot of time and effort is being put into the development and marketing of this type of education software. It seems to me that people are trying to make money out of the flaws in the existing school system. They offer to patch up the gaps which must inevitably occur where most learning is rote learning. If we are to use computers profitably, in an educational sense, we must use them to encourage real, relational learning.

The second of these options is to use computers to promote computer literacy. Carter (1981, 48) said "Others argue that computers should be seen as a new social and technological phenomenon, and children should learn about them." If the children learn about computers by using them, in a variety of ways, they will certainly be better equipped to handle today's and tomorrow's world. "Hands-on" experience is vital, otherwise it is like learning to drive without ever getting in a car.

A third group view the computer as a potential "intellectual amplifier". (Schwartz quoted in Carter 1981, 48) They want children to learn to program and use computers. Unfortunately there are very few programming languages which are both easy to learn and powerful to use. So again we have the problem that children either have to spend a great deal of time learning to program, which changes programming from a means into an end, or the programs they can write are so trivial as to be boring.

LOGO may well be a potential peacemaker in this debate. It has even been suggested that 1982 may go down in history as "The year of the Turtle" (Olds 1982, 60).

3. What is LOGO?

LOGO is a Lisp-like language, ie it is a list processing language. It was developed by people who were concerned with artificial intelligence; a branch of computer science which has developed with cognitive psychology into the cognitive sciences - sciences concerned with natural language understanding, visual perception and knowledge acquisition. They, Seymour Papert and his associates at MIT, were computer scientists, mathematicians and educators. They were concerned with promoting Piagetian curriculum-free learning. They wanted to put the child in control of the computer, not vice versa, and to set him free in a new world in which he can develop his thinking skills. Abelson (1982, 106) says "The best kind of LOGO activity is a synthesis of programming, mathematics, aesthetics and, above all, the opportunity to explore." He sees LOGO as a computer based learning environment.

The primary design specifications for LOGO were that it be interactive, procedural and extensible. These are the characteristics which make it "a language for learning".

It aims at developing thoughtful literacy which grows by allowing children to program and to become fluent in expressing mathematical, and other, ideas.

As Abelson (1982, 112) says "If we can dispel the illusion that learning about computers should be an activity of fiddling with array indices and worrying about whether x is an integer or a real number, we can begin to focus on programming as a source of ideas."

To Papert, the computer is a "thing to think with" as he used gears in his childhood. "The computer is the Proteus of machines. Its essence is its universality, its power to simulate. Because it can take on a thousand forms and can serve a thousand functions, it can appeal to a thousand tastes." (Papert, 1982, viii)

Until recently the memory limitations of microcomputers meant that computer languages had to be compiler and not user orientated. Now we can have languages which think first about people and which are, in consequence, far easier to use. LOGO is a prime example of such a language. It is essentially a well structured procedural language which lets young people start with what they know best - their own everyday language.

It is extremely difficult to describe LOGO both comprehensively and concisely - it has the potential to be different things to different people. Carlson (1982) has described it as having three faces. The first is "turtle talk" - a set of graphics commands that serves as a splendid introduction to computing for any beginner, and especially for young children. When one removes the turtle graphics mask a general computer language on a par with Basic is revealed. The advanced user can remove the second mask and find a powerful lisp-like language which introduces techniques common to artificial intelligence programming.

LOGO is an easy language to learn and use; it makes it possible for people other than that small group of people who were at ease with the traditional mathematics curriculum to generate and solve interesting problems. Problems which no longer have to be limited to the traditional type which so few people find interesting, e.g. write a program to find the roots of a quadratic equation. Children are able to behave like mathematicians. As Carlson (1982, 41) says "LOGO is more than a computer language, it is an educational movement."

"It is not an introduction to something else, it is not specifically for computer science majors, it is not a tool for teaching the same mathematics curriculum that people are already teaching - it is a door into the territory of the computer as an object for intellectual exploration." (Harvey 1982, 188)

LOGO is a synthesis of Papert's beliefs about how children learn and his interest in computers as a learning tool. In the next section I will discuss two people who have exerted major influences on his thinking; Jean Piaget and George Polya.

4. People who have Influenced Papert's Learning Philosophy

4.1 Jean Piaget

Papert worked for many years with the Swiss psychologist and epistemologist, Jean Piaget. From him he took the model of children as builders of their own intellectual structures (Piaget's constructivism).

In "Mindstorms" Papert (1980) wrote "Children seem to be innately perfect learners, acquiring long before they go to school, a vast quantity of knowledge by a process I call 'Piagetian Learning' or 'learning without being taught'. For example children learn to speak, learn the intuitive geometry needed to get around in space and learn enough of logic and rhetoric to get around parents, all this without being 'taught'." In creating LOGO he focussed on the "child as builder" and set out to give the child things with which to build.

In this provision of experience he differed from Piaget since he feels that those logical structures which develop later in the Piagetian heirarchy often do so because the culture offers a paucity of experience in them - eg in combinatorial thinking. He felt that computers could offer a wider variety of personal and concrete experience and so facilitate the development of children's logical structures.

He also took to heart Piaget's statement that "If we desire to form individuals capable of inventive thought and of helping the society of tomorrow to achieve progress, then it is clear that an education which is an active discovery of reality is superior to one that consists merely in providing the young with ready-made wills to will with and ready-made truths to know with." (Piaget, 1971, quoted in Lawler 1982, 138) Thus a central problem of education is how to instruct while respecting the self-constructive character of the mind. (Lawler 1982) To Papert, LOGO and computer are a way out of this dilemma.

Piaget found that children will nearly always have an answer to a question - eg four-year-olds offer a variety of reasons for "What makes the wind blow" - nearly all of them wrong from an adult perspective. He was interested in the logic behind the wrong answers and Papert, like him, believes that it is more important for children to have ideas than for those ideas to be 'right'. It is the habit of thinking about things that it is important to cultivate.

Thus when a computer program does not work, it is not wrong, it simply needs 'debugging'. Carlson (1982) said that children learn in working with LOGO that debugging is a natural part of creation. The absolute right/wrong of ticks and crosses next to homework is gone. Frequently they find that a "bug in the works" leads to new and creative ideas.

It follows then that the question we should ask about the program is not whether it is right or wrong but whether it is fixable. If this way of looking at intellectual products were generalised to how the larger culture thinks about knowledge and its acquisition we might all be less intimidated by our fear of 'being wrong'.

In summary, evidence of the influence of Piagetian thinking (if not each of Piaget's specific notions about thinking) pervades the LOGO culture. The interest in debugging, the metaphor of objects, and the assimilation of computer technical ideas into familiar contexts (eg anthropomorphisation and playing turtle) all reflect that influence. (Goldenberg, 1982)

4.2 George Polya

A number of Polya's (1945) problem solving techniques are built into LOGO. Programs are written as a number of linked procedures. Each procedure is short and usually does only one thing. This encourages the programmer to break his problem down into bits that he can cope with. Papert frequently speaks of breaking powerful ideas down into mindsized bites. Children are encouraged, if they cannot solve the given problem, to think of something similar but simpler. These techniques are used by Polya and are also recommended by Bishop et al (1978) and Oxrieder and Ray (1982) in their work in combatting mathophobia.

Solving problems by doing something similar and/or simpler is another Polya problem solving principle. Playing Turtle gives a concrete example of how one can do this - we have an almost inexhaustible source of similar situations when we do turtle geometry since we can draw on our own behaviour and on how our own bodies move. "Turtle geometry becomes a bridge to Polya. The child who has worked extensively with Turtles becomes deeply convinced of the value of looking for something like it because the advice has so often paid off." (Papert 1980, 64)

5. Learning and LOGO - Papert's Ideas

Papert wants to use technology not in the form of machines that will process the child but as something the child himself will learn to manipulate, to extend, to apply to projects, thereby gaining greater and more articulate mastery of the world, a sense of the power of applied knowledge and a self-confidently realistic image of himself as an intellectual agent. "Stated more simply" he says "I believe, with Dewey, Montessori and Piaget that children learn by doing and by thinking about what they do. And so the fundamental ingredients of educational innovation must be better things to do and better ways to think about oneself doing these things. I believe that computation is by far the richest known source of these ingredients. We can give children unprecedented power to invent and carry out exciting projects by providing them with access to computers, with a suitably clear and intelligible programming language and with periferal devices capable of producing on-line real-time action." (Papert 1972b, 2)

In this section I will discuss a number of aspects of Papert's view of learning.

5.1 Thinking about Thinking and Thinking about Learning

Papert has much to say about the manner in which children come to believe - or are made to believe - that they are "poor at" mathematics or "good at" art. He feels that many people's problems could be cured if alternative paths to mathematics could be used - experience based, spatially orientated as opposed to symbolically orientated, routes. He attributes many of the problems to the dissociated manner in which subjects like mathematics and grammar are taught and cites examples where, when the knowledge became meaningful and had purpose, the children no longer had problems. (Papert 1980, 49)

He (Papert 1972b) maintains there is no value in telling people that their ideas of how they learn - what he calls Pop-Ed theories like Blank-mind, Getting-it and Faculty theories - are wrong; people's intellectual growth must be rooted in experience. Children who are involved in experiences which provide for the growth of intuitions and concepts for dealing with thinking, learning and playing, discover for themselves how they think. Two such activi-ties are playing games of strategy on a computer or writing CAI programs. It has long been acknowledged that the best way to learn something is to teach it and writing a program which teaches is even more instructive. In the development of such programs children delve deep into the thought processes and concepts involved and in so doing learn how they themselves think.

Papert sees great value in thinking about thinking. "In most contemporary educational systems where children come into contact with computers the computer is programming the child. In the LOGO environment the relationship is reversed. The child is in control. The child programs the computer. And in teaching the computer how to think children embark on an exploration of how they themselves think. The experience can be heady. Thinking about thinking turns the child into an epistemologist, an experience not even shared by most adults." (Papert 1980, 19) He claims that the computer is unique in that it provides us with the means of addressing what Piaget and many others see as obstacles to be overcome in the passage from child to adult thinking. It allows us, he maintains, to shift the boundary separating concrete and formal so that knowledge which was previously only accessible through formal processes can now be approached concretely. Self-referential thinking, ie thinking about thinking itself, is a type of thought which clearly falls into this category.

5.2 Mathematising

Papert (1972a, 200) focusses our attention on the fact that while, for example, points and lines are primitive concepts in geometry they are not the only primitives. There are also epistemological primitives, such as the nature of a mathematical system itself, which we tend to ignore. He (Papert 1972a, 260) says "For most children at school the problem is not that they do not understand particular mathematical structures or concepts. Rather, they do not understand what kind of thing a mathematical structure is; they do not see the point of the whole enterprise. Asking them to learn it is like asking them to learn poetry in a completely foreign language." Under these conditions emphasising the process of mathematisation is surely a futile exercise.

He describes how, through exploration with LOGO, children have living experiences of mathematising as an introduction to mathematics. Thus the choice of content material, especially in the early years, should, he feels, be made primarily as a function of its suitability for developing concepts. When a child gets a turtle to draw a circle by observing how he himself makes one he is learning to mathematise and analyse.

He continues, "When mathematizing familiar processes is a fluent, natural and enjoyable activity, then is the time to talk about mathematizing mathematical structures, as in a good course of modern algebra." (Papert 1972a, 263)

5.3 Papert's Idea of a Mathland

Papert (1980) believes that most people who grow up loving mathematics do so because they "acquired the seeds of a maths culture" from "math-speaking" adults. These "math-speakers" argue logically, enjoy puns, paradoxes and puzzles; they don't necessary wander around solving equations. And this is what schools fail to provide - informal exposure to the language of mathematics and time and opportunity to learn to talk it. Papert would have them learn mathematics in "Mathland". He (Papert 1980, 39) argues that his Mathland concept "is the first step in a larger argument about how the computer presence could change not only the way we teach children mathematics, but, much more fundamentally, the way in which our culture as a whole thinks about knowledge and learning."

Early in "Mindstorms" Papert (1980, 6) says "Two fundamental ideas run through this book. The first is that it is possible to design computers so that learning to communicate with them can be a natural process, more like learning French by living in France than like trying to learn it through the unnatural process of American foreign-language instruction in classrooms. Second, learning to communicate with a computer may change the way other learning takes place. The computer can be a mathematics-speaking and alphabetic-speaking entity. We are learning how to make computers with which children love to communicate. When this communication occurs children learn mathematics as a living language."

5.4 The Importance of Communication

In response to the implications of the quote:

> The centipede was happy quite
> Until the toad in fun
> Said, Pray which leg comes after which?
> This wrought her mind to such a pitch
> She lay distracted in a ditch
> Considering how to run.
>
> Anonymous

he (Papert 1980, 96) says "My perspective is more flexible because it rejects the idea of the dichotomy verbalizable versus non-verbalizable.... An important component in the history of knowledge is the development of techniques that increase the potency of 'words and diagrams'. What is true historically is also true for the individual; an important part of becoming a good learner is learning how to push out the frontier of what we can express in words." Consequently he is concerned with developing languages so that we can communicate, with ourselves and with others, about learning all sorts of things - eg riding a bicycle, juggling a ball, playing tennis, doing mathematics.

This idea is implemented by Gallway (quoted in Papert 1980, 97) in his book "Inner Tennis" in which he attempts to make the player aware that he can choose between two types of thought - holistic and analytic - and that the player can and must choose the appropriate mode for the moment.

Descartre's analytic geometry is a striking example of how something which seemed too amorphous for systematic thought is now easily described. (Papert 1980, 97) Unfortunately most children are not ready psychologically when they are introduced to it and consequently its power and simplicity are not appreciated. In turtle geometry where the descriptions are local instead of global, children observe their own movements in making, for example, a circle, and program the turtle to do likewise. In this way they, like Descartre, get the reward which is "to describe analytically something that until then was known in a global, perceptual-kinesthetic way" (Papert 1980, 99). And, unlike the centipede, they are just as able to walk in circles afterwards as before.

5.5 The Need to Spend Time on and Become Involved in Projects

In addition to communicating with the computer, articulate discussion - with oneself or with others - facilitates learning. Endless similar sums done in silence do not. In order to develop skill in using the vocabulary children need time. Papert argues that they do in fact "acquire the ability and motivation to work on projects that extend in time over several days, or even weeks." (Papert 1972b, 6)

During the planning, choosing of strategies, finding solutions, debugging, extending of a project children have time to develop vocabulary for articulate discussion. They become involved. It is difficult to see how children can develop the degree of personal involvement essential to meaningful leaning while working in a typical problem-orientated environment.

If mathematics classes were more like art classes "the duration of the process would be long enough for one to become involved, to try several ideas, to have the experience of putting something of oneself into the final result, to compare one's work with that of other children, to discuss, to criticise and to be criticised on some basis other than 'right or wrong'." (Papert 1972b, 6)

In order to make the computer do more, children will use the appropriate terminology and concepts. In this way they become sophisticated in the art of setting up models and developing formal systems. In addition the computer culture provides a number of useful and descriptive terms, eg loop and debug.

Formal mathematics and grammar, usually pointless and meaningless to children, when needed, are worth understanding, or developing for oneself in order to, for example, generate meaningful sentences.

As well as being fun to play and work with, LOGO is aesthetically appealing. If offers an alternative to the current formal, purely logical, depersonalised mathematics which we teach in schools, and in which we ask children to forget their natural experience of mathematics and learn a new set of rules. Children now have a chance to experience personal mathematics. The "extralogical face" of mathematics with its beauty and pleasure can be generally experienced, the aesthetic awareness no longer limited to Poincarre's chosen few (Papert 1980, 190).

5.6 Children Work Best when They are Free to Choose How to Solve their Problems

When children work freely they are able to do things the way that suits them personally best. Thus they are able to use a love for language in understanding mathematics and a feeling for logic can be used to understand grammar. This begins to bridge the science - humanities gap and to break down the rigid subject compartmentalisation which now exists in schools and in people's minds. Children are able to mobilise their multiple strengths in problem solving and problem creating.

5.7 LOGO Develops Problem Solving Skills and Incorporates Sound Mathetic Principles

"Turtle geometry serves as a carrier for the general ideas of heuristic strategy." (Papert 1980, 64) It is rich in situations in which simple, yet compelling, models of heuristic knowledge can be encountered and internalised by children. Thus teachers can pay attention to the process as well as the product of mathematics.

One of Polya's (1945) basic strategies is: To solve a problem look for something like it that you do understand, ie use the mathetic principle relating what is new to that which you already know. In turtle geometry this can be rephrased as: Play Turtle. Do it yourself. Turtle geometry's ego and body syntonicity make it a far better introductory domain for learning heuristic thinking than simple arithmetic. "Mathetics is to learning as heuristics is to problem solving. Principles of mathetics are ideas that illuminate and facilitate the process of learning." (Papert 1980, 120)

A second important mathetic principle is: take what is new and make it your own. In general try to make sense of what you want to learn. Turtle geometry was specifically designed to be something that children can make sense of and identify with.

Children are encouraged to "play turtle", ie to relate the turtle's geometric movements to their own body movements. This encourages syntonic learning - ie learning which is related to the children's sense and knowledge about their own bodies. Carlson's (1982, 42) paraphrase "Anything is easy to learn if you can assimilate it to your collection of models" underlines the fact that, in the light of nearly all modern learning theories, what you learn relates to what you know, not to what other people decide you should learn.

Drawing a circle because we want to and expressing pride and excitement are in addition ego syntonic - coherent with our senses of ourselves as people. Turtle geometry also has cultural syntocity - many of its ideas relate easily to the world in which we live.

Turtle geometry is learnable because it is syntonic. And it is an aid to learning other things because it encourages the conscious deliberate use of problem solving and mathetic strategies.

Another of Polya's strategies is the breaking down of a problem into parts which can then be solved separately. The procedural nature of LOGO encourages children to break down problems. They are encouraged to "think like computers" when that is the style of thinking that suits the problem. They are made aware that different cognitive styles can be brought to bear on different problems or on different stages of the same problem. Papert (1980, 105) illustrates how this principle can be applied to describing the act of juggling.

A problem which is made of a number of sub-procedures is easy to understand and, perhaps even more important, to debug. Children learn that they do not have to be right first time. They realise that they can eradicate errors and create something that works. They also learn that errors, or bugs, can lead to some of the most exciting discoveries. Mistakes, which can be fixed, encourage children to examine their thinking, to understand what they did wrong, and then to fix it. They are learning to think about thinking and to think about learning.

One of the most powerful things children can learn from playing constructively with LOGO is to be unafraid of making errors.

5.8 Children Learn by Doing and Discovering

When children work with LOGO they are doing things, they are not learning about things. (Papert 1972, 249) When you learn by doing the knowledge becomes yours. This is one of Papert's chief concerns - that children be put in a position where they can do mathematics, as real mathematicians do, rather than learning about it. Genuine discovery, as opposed to discovering what the teacher wants you to discover, is very exciting. That many children do not realise that play with turtles is mathematics, is irrelevant. They are building up experience; time enough later to discover that what was fun was also mathematics.

Lawler (1982, 150) said "Because LOGO is a vehicle for free exploration, knowledge built from LOGO is syntonic, appropriate to the person, and experienced as an authentic, intimate part of the self. Such is the power of an approach to learning that frees the individual to create within a social context that it makes our culture's most powerful ideas accessible."

Children can discover things by creating their own programs or by exploring microworlds. A microworld is a well defined, but limited, learning environment in which interesting things happen and in which there are important ideas to be learned. It is a concept borrowed from artificial-intelligence research. (Goldenberg 1982, 218)

Papert (1972a) describes children exploring the POLY microworld

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TO POLY  :STEP  :ANGLE
FORWARD  :STEP
LEFT  :ANGLE
POLY  :STEP  :ANGLE
END
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They explore the behaviour of the procedure with different inputs. There is inevitable challenge, and competition, in producing beautiful or spectacular, or just different, effects. When children change the procedure itself they can make truly spontaneious discoveries. While not all children will initiate such changes the less adventurous will often follow the lead of those who do.

Nearly all children can make original discoveries, however minor. "The possibilities are endless. These are small discoveries. But perhaps one is already closer to mathematics in doing this than in learning new formal manipulations, transforming bases, intersecting sets and drifting through misty lessons on the difference between fractions, rationals and equivalence classes of pairs of integers. Perhaps learning to make small discoveries puts one more surely on a path to making big ones than does faultlessly learning any number of sound algebraic concepts." (Papert 1972a, 258)

5.9 The Importance of the Affective Domain in Learning

Papert believes that the affective is an important factor in learning. Children need to relate in a personal manner to the learning situation. Papert stresses the fact that he fell in love with "the gears of his childhood". (Papert 1980, viii) That children relate to turtles in this way is clearly seen in the examples quoted in Goldenberg (1982, 230) and Wier's (1981, 84) work where autistic children record their first words talking to the turtle. In my own experience local school children said "We loved the turtle, he was such a friendly creature. We wish we could each have one to take home." Piaget neglected the affective component and concentrated almost entirely on the cognitive. Papert (1980, vii) attributes this to his having "a modest sense that little is known about it more than from an arrogant sense of its irrelevance," since he believes that

Piaget agrees with him that a positive affective component is vital to learning.

6. Concluding Remarks

When LOGO and the turtle were invented their creators wanted them to be carriers of important, powerful ideas.

Powerful ideas should be simple, general, useful and syntonic. Ideas are powerful if they relate and unify knowledge gained in diverse experiences. Ideas gain power if they can be reduced to a concrete model which can serve as a metaphor for interpreting subsequent problems. Essentially powerful ideas relate to the individual's previous knowledge. Thus individuals make connections between the structures of different ideas at levels which suit them. This internalisation is the basis of an idea's power for the individual.

These powerful ideas are the result of real, relational learning. They are the fruit of well formed cognitive structures. They are what learning theorists feel we should strive to develop when we learn. From a theoretical viewpoint it appears that LOGO is likely to facilitate their development. If it does as Papert claims, we would be foolish not to introduce it into our classrooms.

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The Introduction into the Classroom of LOGO
as a Programming Language
with Particular Reference to its Structure and its Potential
for Encouraging Concept Formation in a Variety of Disciplines.

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CONTENTS

1. Introduction

In August 1982 Byte devoted their entire issue to LOGO. In the editorial Morgan explained why: "We chose LOGO this year for many reasons, but the most important is that the fist computer language you learn has a lifelong effect on how you think, computerwise. Thus the computer language we choose for use in schools is vital. I believe, for reasons detailed in various articles in this issue that LOGO is a much better language to use for introducing children to computers than, say, Basic."

In many ways a programming language is like a natural human language in that it favours certain metaphors, images and ways of thinking. The language used strongly colours the subsequent use made of the computer. It would seem to follow that educators interested in using computers and sensitive to cultural influences would pay particular attention to the choice of language. Further some types of problems are easier to solve in one language than in another and Carlson (1982) maintains that even seasoned programmers should learn LOGO for the new perspective it reveals about the art of thinking.

If a primary aim of education is to encourage children to think then LOGO should be introduced into the classroom. It should not however be treated as a universal panacea but as a learning environment which can be used to enrich and not replace children's physical and social experiences. Many classrooms are experientially barren environs where all knowledge flows from the teacher. This is one of the problems of introducing LOGO since its philosophy insists that the teacher become a co-learner, a position which a number of us find unnerving. It is thus essential that teachers be well trained as LOGO users in order that they do not introduce it in a manner at variance with its philosophy since it then loses a great deal of its potential.

In this essay I will discuss the manner in which the structure and nature of LOGO reflect its philosophy and the way in which these facilitate the development of various concepts and modes of thinking. I will also describe the creation of a LOGO environment within a classroom.

2. The Structure of LOGO

2.1 The First Face of LOGO - Turtle Geometry (Carlson 1982)

An important aspect of LOGO is turtle geometry. Turtle geometry is a "genuinely new mathematics based on turtle movements. It emphasises transformations in local space rather than relationships to a fixed global referent." (Goldenberg 1982, 214) "Local" means something which has meaning only in the specific context in which it is being used; "global" means something pertaining to the entire environment being considered. The turtle has a strictly local perspective on its turtle moves; it only needs to know how to move with respect to itself. (Harvey 1982) In LOGO most variables are local and have values only within the procedure in which they are defined. This makes for more orderly and debuggable programs.

To return to the turtle.

It may be a computer-controlled robot that has independently controlled position and heading, alternatively it may be a graphic representation, usually a pointy isosceles triangle, of a physical robot. The turtle is controlled using the graphics commands, or Turtle Talk which are now availble in a number of languages but which originated in LOGO. Some languages have Sprites which can be controlled as the turtle can, and which can be different shapes and colours. They make it possible to create exciting animations fairly easily and without having to worry about shape tables, co-ordinates and the like. Dynaturtles have been developed which have position, heading and velocity and are programmed to obey Newton's laws. They can be extensively used in physics courses.

Unlike a Euclidean point the turtle is easy to identify with. Like a person it has position and heading. The commands FORWARD and BACKWARD move it in a straight line in the direction of its heading and RIGHT and LEFT rotate it. SQUARE illustrates what can be achieved with only two commands. CIRCLE is a good deal more sophisticated. (All these microworlds and programs are in the Appendix to this essay). In order to write it children often "play turtle" ie they walk a arc and then tell the computer to do likewise. This body syntonicity is frequently used in the creation and debugging of programs. Even the powerful POLYSPI uses very few commands and provides a wealth of geometrical experiences.

Children playing in these microworlds often discover for themselves the Turtle Total Trip theorem, viz if a turtle takes a trip around the boundary of any area and ends up in the state in which it started then the sum of all turns will be a multiple of 360 degrees. (Papert 1980) He argues that this theorem is more powerful, general and intelligible than its Euclidean counterpart ie "the sum of the exterior angles of a polygon is 360 degrees." Children learn to use it, not just to memorise it rote fashion.

The turtle opens mathematical doors and is an effective carrier of general mathematical ideas. When children draw squares and stars they are learning about angles, controlled repetition, variables and state change operators. They are exposed in this way to the idea of ideas being tools with which to think - a very powerful idea. Drawing a turtle circle is an introduction to the meaning, as opposed to the formalism and symbolic manipulation of calculus. Many turtle programs are intuitive analogs of differential equations; a concept one finds in almost every example of traditional applied mathematics. This is the type of knowledge Papert (1980) feels should be considered for inclusion in the intellectual equipment of the educated citizen of the future.

Differential calculus describes growth by what is happening, locally, at the growing tip. The turtle does not concern itself with what is happening elsewhere. Turtle geometry is thus intrinsic and belongs to the family of differential geometries which have made possible much of modern physics.

Goldenberg (1982, 218) explained, "This local view of movement in space is not only easier to use for simple mathematical ideas, but lends itself quite beautifully to extensions into very fancy mathematics: calculus and limits come immediately to mind."

A fundamental concept in calculus is that of the infinitesimal. Lemmons (1982, 337) quotes an example of a 10-year-old child who, when his mathematics teacher described a circle as having no sides, remarked that it could be thought of as having an infinity of very small sides. He had internalised the concept of the infinitesimal. In the CSMS study, Hart (1981, 123) found that only 6,4% (19,6%) of 13 (15) year-old children showed evidence of understanding this concept. Leron (1982) and his colleagues at Haifa University found that a lot of group theory work goes on when children are working with the LOGO language. They used the basic object of the turtle group generated by the turtle operations FORWARD, BACK, RIGHT and LEFT with all possible inputs to make explicit the group theory concepts and structures that are implicit in LOGO.

They have also observed other group theory concepts in their children's work.

Group Theory Concept	LOGO Counterpart
products and decompositions	paths traced by the turtle
inverses	opposite operation
the order of a group element	the number of repetitions to close a path
conjugacy	transparent operations - procedures
subgroups	limiting oneself to a special subset of allowable inputs
tree groups	this is where the LOGO procedures live
homomorphisism	the relations between LOGO procedures and their products

To encourage the children to become aware of the concepts they use a special-purpose set of primitives. For example FIND.ORDER takes a list of turtle commands as inputs, repeats the list until the path closes and outputs the number of repetitions. In this way they direct the explorations into specific subject-matter areas without sacrificing the spirit of LOGO's spontaneous and meaningful learning.

In Turtle Geometry (Abelson and di Sessa, 1981) the authors show how the turtle can be used to model patterns in mathematical biology by programming the turtle for senses of "sight" and "smell". They also free the turtle from the plane and set it free to explore cubical and spherical surfaces. This book is for the more advanced and sophisticated mathematicians than would usually be found in schools. Exceptional children who are bored with school mathematics could use it as a guide to extension and enrichment.

In turtle geometry the ideas and concepts are not isolated examples. They, and the language, are usable and necessary if one is to get the computer to obey one's wishes. They have "math power" and children using them learn to talk mathematics and to mathematise the tasks in hand. When mathematising the familiar has become a fluent, natural and enjoyable activity they will be better equipped to mathematise mathematical structures of a more formal sort (Papert 1972a).

2.2 The Second Face of LOGO - a List Processing Language

List processing is the heart of LOGO. So much has been made of turtle graphics (which can be adapted to any language) that the computational beauty of list processing has been lost in the shuffle. (McCauley 1982, 59)

A list is a sequence of data objects eg ((PETER PAN) WENDY JOHN) has three items, the first of which is itself a list of two items. So we can have lists whose items are lists. As Abelson (1982, 102) explained "Lists, therefore, are a natural way to represent hierarchical structures, that is, structures composed of parts that are themselves composed of parts."

LOGO has a number of operations for manipulating lists as can be seen in GOSSIP. Writing a simple sentence generating program such as this helps children become aware of the structure and function of grammar in an intensely personal manner (Papert 1972b).

These lists can be manipulated as "first class data objects" ie they can be used as values for variables, passed as inputs to procedures and returned as outputs. You can write procedures to manipulate lists. Thus you can combine operations on lists much as you combine operations on numbers in ordinary languages. (Abelson 1982, 104)

A combination of the primitive RANDOM and ITEM can be used to generate random sentences and stories or questions and answers. Abelson (1982, 104) describes a program which generates random length postcards with random phrases and which every so often asks for a new phrase to add to its repertoire.

List processing enables children to use words in interesting ways while programming, making it possible for them to compose, edit and manipulate their own writing.

Papert (1980, 30) says "For most children re-writing a text is so laborious that the first draft is the final copy and the skill of re-reading with a critical eye is never acquired. This changes dramatically when children have access to computers capable of manipulating text. The first draft is composed at the keyboard. Corrections are made easily. The current copy is always neat and tidy. I have seen a child move from total rejection of writing to an intense involvement (accompanied by rapid improvement of quality) within a few weeks of beginning to write with a computer. Even more dramatic changes are seen when the child has physical handicaps that make writing by hand more than usually difficult or even impossible."

LOGO has many screen editing features that can prepare children for a more sophisticated system. Writing can be formatted using the SETCURSOR commands.

Inputs are accepted from the keyboard using the READCHARACTER (for single key responses) and READLIST (for anything longer) primitives. Interactive programs and games are relatively easy to write. Checking whether a keyboard response is a member of a list is done with the Apple LOGO MEMBERP primitive.

2.3 The Third Face of LOGO

At a more advanced level LOGO uses the procedures as trees. Carlson (1982, 43) describes this, "The data are trees in the form of lists, the primitives DEFINE and RUN make a data list into a procedure." In his book Abelson (quoted in Book Review by McCauley 1982, 59) deals with property list manipulatic, treatment of programs as data and artificial intelligence programming. McCauley maintained that this chapter gave good evidence to support the claim that LOGO has no ceiling.

3. The Nature of LOGO

3.1 It is Procedural

A program in LOGO is not, as in Basic, written as one long program. It is a hierarchy of procedures which call one another (Carlson 1982) This contributes to making it both simple and powerful. Each procedure has a name, given to it by the user, or defined for his use. The procedures are usually aptly named, tend to be short and may be deeply nested. LOGO has many procedures in memory at once. Breaking tasks down into sub-tasks and problems into "mindsized bites" is one of mathetics' basic principles which is implemented here.

Procedures are defined using the TO .... primitive, eg TO SQUARE (1). The computer is taught new words by the programmer. This is a fundamental difference between LOGO and many other languages. LOGO is a functional language, one in which the act of programming is in fact extending the language (McCawley 1982). Weir (1981, 78) explains "The meaning of a square is captured by the procedure which draws a square. When one thinks of a square in this way, when you are involved in the procedural thinking, you can have an understanding of the meaning of the square in terms of how you produce it an how you might want to use it."

Those procedures, which are built into LOGO, are called primitives. Many of them can be used to teach mathematical concepts. Higginson (1982, 328) in discussing this said "Negative numbers and co-ordinates, for instance, get gobbled up in a two-minutes discussion of SET x. The unforced way in which powerful ideas emerge from the turtle geometry microworld is in stark contrast to the struggles of traditional teaching."

LOGO's procedural nature makes debugging of programs comparatively easy. As anyone who has tried to program will realise this is very important. In long, unstructured programs fixing one bug all too often leads to the creation of at least one more; while when a program is written as a number of short procedures it is relatively easy to focus on the spot at which the bug creeps in. Errors are removed not by trial and error but by trial and revision. Debugging in LOGO fosters good problem solving skills.

LOGO is a good language for developing programs because of its interactive nature and procedural style. However, the program once developed would run much faster in a language which uses a compiler and not the interpreter LOGO needs and which barely fits into the 64K-byte Apple II memory.

To quote Harvey (1982, 93) "Do you want to write a video-game program? - it might well be worth while to develop it in LOGO, playing around with different ideas for your game in an environment which permits quick, easy modification of your program. The advantage of LOGO can be described partly in purely technical terms like "interactive". Another way of looking at it, however, is that LOGO encourages the playfulness you need to design the best possible game. If all you want to do is make an exact copy of Asteroids, the benefits of LOGO are less important."

Procedures can have inputs and outputs. The output of a procedure is a message sent back to the procedure that called it, which is expecting an input. An input is thought of as a message that the two inputs, one for the side length, the other for the angle through which the turtle will turn having completed the square.

Lawler (1982, 138) quotes di Sessa as saying that one of the most powerful ideas accessible through LOGO is embodied in procedures which output because they are equivalent to mathematical functions. Algebra is about mathematical functions.

One of the most powerful mathematical ideas ever invented is that of the variable. Many of LOGO's most fascinating and instructive micro-worlds depend on the use of one or more variables, as input to procedures. POLYSPI for example has three variables and produces a remarkable variety of polyspiral designs. As Lawler (1982, 138) says "Variable-stepping is an essential component of formal operational thought. The idea is a powerful one because it is almost universally useful; and is crucial to the process of scientific investigation."

POLYSPI is also an example of a recursive procedure, ie one which calls itself as its last action, thus, unless including a conditional statement, setting up a never ending process. Papert (1980, 71) has found that "of all the ideas I have introduced to children, recursion stands out as the one idea that is particularly able to evoke an excited response. I think this is partly because the idea of going on for ever touches on every child's fantasy and partly because recursion itself has roots in popular culture. For example, there is the recursion riddle: if you have two wishes what is the second?" Harvey (1982, 168) said "It is hard to explain in a simple way why recursion is important. The idea behind recursion, though, has profound mathematical importance. By allowing a complicated program to be described in terms of simpler versions of itself, recursion allows very large problems to be stated in a very compact form."

Thus calculating and solving the Towers of Hanoi problem can be reduced to solving progressively simpler and simpler problems and then using the outputs of the simplest solution as inputs to the next level.

3.2 LOGO is a User-Friendly Language

When you make a mistake the error messages are informative unlike those in languages geared for specialist programmers.

For example, instead of

SYNTAX ERROR

it will say

*** DOESN'T LIKE HELLO AS INPUT**

which tells you exactly what you did wrong. (Harvey 1982, 180)

LOGO is orientated toward children and program developers, not engineers and the commercial world. This is reflected in its handling of data.

Data-processing programs which manipulate data stored on disks are not feasible in LOGO. In LOGO what is elsewhere called data processing is often thought of as object manipulation. This arises out of the naturalness with which it passes data messages back and forth among procedures and the ubiquity of anthropomorphic images for these procedures. LOGO has two kinds of data objects, words and lists. (Goldenberg 1982, 220)

LOGO's word is written as "x and is like Basic's string constant. It is an unbroken string of characters, which can even contain spaces and control characters where necessary. These words can be concatenated into longer words, dissected into parts and used as elements of lists. Numbers, single characters and character strings are all treated as words. (Goldenberg 1982, 220) If the words are numeric LOGO is as efficient a computational language as Basic. It has primitives for all the fundamental arithmetical and trigonometrical and logical operations and functions.

:x is a 'thing', so that words have 'things' or values associated with them. The 'thing' can be a word or a list.

eg MAKE "TED "ANGRY

means that :TED = ANGRY

you can

MAKE "ANGRY "REDFACED

and begin to build a tree with 'things' as nodes. (Carlson 1982, 43)

'Things' do not have to be of a particular data type, they can be character strings, integers, real numbers, sentences. While it can be argued that declaring data types as, for example, in Fortran, Pascal and Basic makes for more disciplined programming, this is not the aim of LOGO. LOGO encourages creative programming and thinking.

There are also many ways in which LOGO makes explicit things that many languages leave hidden or obscure. In place of the confusing x = x + 1 which goes against one's algebraic intuition LOGO uses MAKE "x :x + 1 where "x is the variable name and :x is the old value of x (called dots x). x on its own would be a procedure name.

Another user-friendly aspect of LOGO is its facility for interactive definition of procedures. The procedure can be edited by using special control characters to move the cursor around the screen to change individual characters or insert lines. (Harvey 1982, 182)

3.3 LOGO is Extendable and Flexible

Children can be given pre-written programs to run and they can then modify and extend them. Teachers can simplify programming tasks by pre-defining essential but currently irrelevant procedures. For example, a biology simulation can have distance defined to give the distance between two interacting turtles while in mathematics the calculation of this distance is left to the children. Handicapped children can program using a minimum of keystrokes if procedures are given single letter names. Children can learn to program by being given transparent and modifiable environments with which to experiment. These are called microworlds. A microworld is a well defined, but limited, learning environment in which interesting things happen and in which there are important ideas to be learned. (Goldenberg 1982)

SPINSQUARE is a microworld in which children can experiment with different sizes and angles. It is easily modified so that the angle too is changed creating a whole new microworld. It takes very little programming skill to adapt this simple program to, for example, spin a polygon where the shape being spun would also be varied.

4. Introducing LOGO into the Classroom

In my third essay I discussed the philosophy of LOGO which appears in many ways to be congruent with current learning theories. In the first half of this essay I have described how this philosophy was manifest and what it has to offer us as teachers. However, we can not simply put a LOGO disc into a computer, sit a child before it and expect it to do all that is claimed for it. We need a plan of action for introducing it into the classroom. This is the subject of the latter half of this essay.

4.1 The Structured-Unstructured Dilemma

Howe and Ross (1981) describe in detail how the manufacturers of Meccano designed their series of graded projects. They (Howe and Ross 1981, 90) state "We can see that the implicit teaching strategy was to match the projects to a child's level of expertise in assembly, beginning with a small set of components and a few simple tasks and gradually widening the scope to keep pace with his growing confidence and experience. This approach provides the close guidance a novice needs on being introduced to new skills, new concepts and new ideas. Yet it allows him to experiment and, if able, to be creative by introducing structural variations and implementing his own designs."

They believe in a structured approach to LOGO instruction. They have developed worksheets which introduce a topic, provide specimen procedures and set exercises for using the procedures. A similar approach has been used by Laridan in training teachers at the University of the Witwatersrand (Laridan 1983b).

As Riordan (1982, 49) explained, "In the beginning you will need to demonstrate a number of things to the whole class. The question of scope and sequence is problematic. LOGO inventors fear that if a scope and sequence is published it will invite the belief that students should be accountable for learning programming concepts. This will inevitably lead to evaluation of student learning. This will lead to LOGO being a joyless, unnatural learning activity. LOGO was not meant to be taught, and the kind of learning that occurs was not meant to be evaluated like other school learning."

So we sit impaled on the horns of a dilemma - wanting to share LOGO since it appears so valuable a learning environment and yet being scared of ruining it by being over-prescriptive. Hopefully, if we remember Riordan's warning and steer well clear of conventional evaluation we may be able to lure teachers into using LOGO by presenting possible sequences while not being in direct conflict with Papert's beliefs. In other words we will have our cake and eat it too.

4.2 Creating a LOGO Environment

Tim Riordan (1982, 46) describes how he would create a LOGO environment in a situation where the class spends all, or most, of its time with one teacher. In our high schools LOGO would probably have to introduced preferably with students being allowed to miss parts of the regular curriculum or after school in computer clubs. A possibility is a room in a media centre which could be set up, with a LOGO teacher, in the manner he describes. As Upitis (1982, 31) suggests, such a centre could be set up in conjunction with other mathematics learning centres. Children of different ages would be freed simultaneously which opens up possibilities for the formation of inter-standard interest groups. Only one LOGO teacher would be needed.

In a self-contained classroom with one computer he (Riordan 1982, 46) sees a LOGO environment as "the entire context, made possible and managed by the teacher, in which students work with LOGO. It is more than a computer learning station. It includes psychological as well as physical space - how students feel, how students and adults interact." The children are allotted time on the computer which stands in a front corner, for three twenty-minute periods a week, alone or in pairs. For the rest of the time they are part of the main group.

While they are at the computer they can work on their own projects or on suggestions from a notice board. Alternatively they can use a program stored either on disk or in a storage file. If they have a problem they go, not to the teacher, but to one of the two "LOGO experts" for the week. This reduces the number of interruptions to the group and at the same time promotes a democratic learning environment.

Children are encouraged to teach one another and to borrow and lend programs. This makes them aware of differences in cognitive style. Bandelier (1982, 39) used a system of successive pairings to get the children to teach each other. Careful and varying choice of pairs can take a lot of the teaching load off the teacher.

It is vital that the children discuss what they are doing - with their peers, themselves and the teacher.

While the children are at the computer the teacher finds time to watch them for a few minutes. His intervention is generally limited to questions like, "Can you see a pattern?"; "Do you have a theory about what caused that?" In addition he should be observing the manner in which they approach problems. Moore (1983, 28) emphasises that it is the student not the screen which should be under scrutiny. Breen (1984) has suggested that the teacher should never touch the keyboard.

Once a day the teacher will have a time for the whole class to focus on the computer. He may then put a new idea to the class or he may get a pupil to demonstrate and discuss a procedure. Discussion focusses on queries from other classmates, what plans and/or problems the child has with his procedures and what the class could suggest in the way of changes.

As well as watching children on the computer the teacher sees children individually about once a week to ask them, "What are you doing with LOGO this week?" When he has heard their plans and/or that they are simply "messing around" with a procedure he usually continues "Have you had any unexpected results? Do you have a theory about what caused this? What did you try in attempting to solve the problem?" He (Riordan 1982, 48) is "using questions to help students focus upon their own theorising, and is promoting learning by having students theorise, test and revise, instead of always having them learn by direct instruction."

If the student cannot solve his problem the teacher has a number of options open

* either he or another child can help the pupil.
* he can find a similar simpler problem for the student.
* he can introduce a new programming concept or LOGO primitive which makes the problem easier to manage.
* he can pinpoint the problem area very precisely for the pupil.
* he can encourage the pupil to think of himself as a turtle, ie to play turtle, and to walk, tape or draw the procedure in question.
* he can introduce the child to a relevant microworld.

It is necessary to make the children realise that much of the most valuable time is spent away from the computer.

They need help in planning complicated projects. Dan Watt (1983, 40) gives the following suggestions:

a) Draw you idea first on paper.
b) Divide the drawing into parts.
c) Give each part a name.
d) Draw a picture showing how the parts fit together and number them in the order you want to put them together.
e) Simplify the parts that look too difficult.
f) Start writing procedures.

It is vital to be aware that different children have different cognitive styles and strengths and that they must be allowed to capitalise on them.

Cynthia Solomon (1982, 198) describes three types of learners. The first learning style she calls the planner who builds programs from the top level down or from the bottom level up but always from a coherent formulated plan. They would probably be happy with Watt's (1983, 41) top-down programming approach which may be too abstract for many other children.

A second learning style is the macro-explorer; they like to mess about with sub-procedures or building blocks to arrive at a product, rather than starting out with a specific goal. They are intent on exploring the effect of a particular building block and the result therefore is open-ended.

Finally some learners have to explore their environment on a micro-level before they can establish patterns of planning or directed exploration. These micro-explorers are often the most timid learners. Both explorer types need time to develop their designs by trial and revision.

Children may use all three styles and different styles are more productive in different situations. They can be encouraged to try different strategies at different times and to appreciate that there are a variety of ways of solving problems.

As Watt (1983, 40) summarises "In fact in teaching LOGO to many beginners, I have found that it is best for them to find their own ways to do things. For the helper the most important thing is to have patience, to understand how each particular person thinks about what he or she is doing and to offer help with that. It can be as counter-productive to force everyone into the same mode of learning as it would be to make everyone do the same project. In time, students come to 'own the process' as well as the product. This ownership cannot be forced. The trick of teaching is to learn to make a suggestion and then to be content with a learner who rejects it."

Laridan (1984, 38) emphasises that "The teacher needs to capitalise on each individual's most predominant style yet should also enrich it by helping the individual to use other styles as appropriate."

Moore (1983, 28) explores the problem of intercepting pupils' frustration and discouragement without intercepting their discovery. It would seem better to err on the side of allowing the child to be a bit discouraged rather than run the risk of doing his thinking for him and taking away his possible "Aha!" Children very often need time, not intervention.

4.3 LOGO Commands

The commands fall into a number of categories. Within each category there are more and less sophisticated commands. It would seem logical to introduce the more simple ideas from a number of categories before thoroughly exploring a particular one, since they support one another and this gives different children a chance to find an entry point which suits them into the language.

I have listed what I feel to be eleven main categories in a possible sequence of introduction based on Riordan's (1982) suggested approach. A complete list of the commands available will be in the LOGO manual. The list on the following page is obviously neither prescriptive nor exhaustive, merely suggestive.

A

Turtle Graphics

1. Moving the turtle around the screen
2. The Repeat Command REPEAT
3. Drawing techniques eg SHOW TURTLE
4. Colour drawing eg SET PEN
5. Turtle Heading eg TOWARDS
6. Plotting co-ordinates eg SET X
7. Screen adjustment eg WRAP

B

Saving of Programs and Pictures

eg SAVE

C

Editing in and out of the Editor

eg CONTROL N

D

Procedures

1) eg TO

2) Recursion

3) Taming Recursion using Conditionals and Controlling flow of procedures

eg IF a<b [STOP]

4) Using inputs and outputs to train procedures

E	F	G	H	I
Variables	Randomising	Doing Arithmetic and printing Arithmetic	Communication with outside world via keyboard	List Processing
eg MAKE	eg RANDOM	eg PRINT a+b	eg READLIST	eg BUTFIRST

Many of these concepts and commands can and should be introduced via the microworld concept. In the appendix is a list of a few which have been referred to in the text and others which illustrate particular ideas.

5. Concluding Remarks

Once children have started to program the order of introduction is largely dictated by their needs and interests. If we are sensitive to these and aware of their strengths and cognitive styles we can allow them to learn in the way that suits them best, freely, with a minimum of intervention and a total absence of mark-orientated evaluation. We have to believe that learning can occur without testing otherwise we cannot teach LOGO in the way in which Papert visualised.

These four essays have followed the path of my search for a possible means of making mathematics teaching and learning a more meaningful pastime. In LOGO I feel I have found one answer, I hope one of many. In my dissertation I am going to examine it in the light of the theories I have discussed and the claims made for it.

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