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**MAKING CENTS OF YESTERDAY, TODAY AND TOMORROW:  
TRADING RULES FOR VOLATILITY ARBITRAGE**

A dissertation submitted to the  
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## Abstract

Institutional equity derivative activity typically focuses on both an underlying's expected future direction (or stability) and also identifying abnormally high or low volatility on the same underlying, or a correlated underlying. The objective of an equity derivative trade is classically to couple an option trade with a volatility wager that may be beneficial in terms of the profitability of the trade. Hence, one would aim to sell an option only in high volatility circumstances and purchase an option when volatility was cheap.

The rule that typically governs the decision as to what constitutes 'high' or 'low' volatility is comprised of a simple averaging of a rolling volatility window (at a certain term, say 3-months). The current (3-month) estimate of volatility is then computed. If the current estimate falls above or below, say a 1-standard deviation bound, a volatility opportunity is inferred, either a sell or a buy. This rule is in several senses statistically naive. The naive rule is biased by the error that autocorrelation introduces both to the average and standard deviation estimates. Furthermore the rule does not accommodate or emphasize recent changes in volatility more than it does volatility events in the distant past.

We propose several other statistical rules that are less immature – including combinations of resampling, exponentially weighted moving averages (EWMA) and GARCH volatility-updating models. Opportunities arising from these statistical rules are termed volatility arbitrages.

Historical and implied volatility for the Top-40 index and historical volatility for single stocks are considered. We then rigorously backtest the success of volatility arbitrage historically (using repeated out-of-sample testing) on both the Top-40 index as well as several large-capitalisation single stocks from several broad economic sectors.

The results indicate that, as expected, the market is generally efficient in terms of how it prices volatility. Captivatingly, the results also point towards several good opportunities still existing in both index and single stock volatility trades. The results further imply that better alternatives exist to the naive rule. The results suggest appropriate screening methods applicable to different underlyings in order to enhance the probability of success of a volatility trade.

## Motivation for Study

This study is intended to be a rigorous examination of a valid and attractive practical problem in financial mathematics in the real world rather than a theoretical contribution. It has foci in statistical modelling of derivative (Black-Scholes) mechanics. The impetus for this research was initially gleaned while working with an active South African derivatives brokerage house (Cadiz Holdings) as an intern.

It was immediately evident that there was a pressing need for an investigation into volatility trading rules to recommend trades (from the sell-side brokers) as well as to trigger trades (from the institutional investor's point of view). The rules that both parties typically used appeared too simple to be useful or profitable.

The question of when to trade volatility is highly relevant in this setting. In order to decide on the volatility aspect of an option trade, most investors utilise a basic rule of thumb along with some amount of instinct. The need to investigate the validity of such practices is obvious when one considers the financial risks involved.

The research methodology was designed after discussing relevant issues and practices with derivatives traders. The methodology was also presented to the majority of institutional and pension fund managers within the South African market and the interest and feedback obtained was positive and helped to further improve the study.

## Acknowledgments

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# 1. Introduction

The sole task of professional investors is to be able to identify undervalued or overvalued financial instruments, and buy and sell accordingly - thereby creating wealth. The extent to which their investment approach is innovative, their research process correct and their execution unblemished will determine their fund's performance, risks and their resulting reputation. The extent to which the market is efficient will detract from their ability to add value in these regards. There is a constant tussle to understand what market information is presently useful and what has already been assimilated.

Within the context of international institutional equity derivative play, two investment approaches are commonplace:

1. Long, short or neutral equity selection bets
2. Directional volatility wagers

These are also the two aspects of the market that equity investors can intuitively focus on and understand: market direction and volatility. The two approaches may be used in combination with one another, as well as independently.

The methodology for selecting a particular view on a stock varies from quantitative, fundamental or technical. For any view on an underlying, there will be a single or combination of derivative structures that can take advantage of this. Stock valuation is not the focus of the current research; rather, we turn our attention to volatility wagers.

An option's premium is directly related to the volatility at which it is priced (Hull 2000). Regardless of whether one's derivative structure is market neutral or not, one will aim to sell volatility high and purchase it low. Get this right and we maximize the profits from our volatility wager<sup>1</sup>. As we hope to identify undervalued or overvalued stocks through say, fundamental valuation, so too do we hope to identify aberrantly priced volatility, be it high or low. As there is no universally correct or accepted equity valuation toolkit there is thus risk associated with equity selection. Similarly, there is no universal rule for trading volatility.

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<sup>1</sup> Note: The typical institution will have both a directional stock outlook as well as a volatility view. The eventual P&L will be determined by both the success on the volatility aspect of the trade together with the success on the option trade. For example, if the spot market breaches the strike of a sold call – it is likely that the benefits attained from a correct volatility bet will be more than eroded by the opportunity costs of the option trade.

Interestingly, the rules that are typically utilized within South Africa are fairly rudimentary, despite the emphasis and advancements on volatility modelling internationally over the last two decades<sup>2</sup>.

In this study, we discuss what these common rules are and how we can modify them to better take into account the realities of the South African market. We suggest some alternatives and test them in the market on several different underlyings (Index and single stocks). The methods screen an underlying for a reasonable suggestion as to what modified rule is likely to work, and why.

The findings are fascinating, and point to previously under-utilized opportunities for profit-maximizing in derivative volatility play.

- We commence with an introductory section on volatility, where we discuss historical volatility and implied volatility as well as the volatility skew. We also present a note on the efficient market hypothesis in this section (section 2).
- Section 3 introduces volatility arbitrage trading rules and their application.
- Next, we examine their pros and cons and consider some alternative statistics for measuring volatility (section 4).
- Section 5 presents a synopsis of the methodology.
- The results are presented in section 6.
- Section 7 presents a comprehensive discussion of the results, and a conclusion.

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<sup>2</sup> For example, the ground breaking work of Engle (1982) and Bollerslev (1987).

## 2. Volatility

When trading options, significant consideration is given to:

- the price of the underlying asset,
- the volatility of the asset.

Market participants might consider taking a directional view on the underlying asset and complement this with a position in the option market based on their view. Typically professional risk-takers (banks) will take offsetting positions called delta neutral positions to hedge against directional changes in the price of underlying instruments.

Participants may also take a view on the volatility of the underlying asset. They will either be long volatility (i.e. net holders of options) or short volatility (i.e. net writers of options). A net long option position is a bet that volatility will increase in the future while a short option position is a bet that volatility will decrease in the future. Trading volatility is explained further in section 3.

There are two different measures of volatility, historical volatility (HV) and implied volatility (IV).

### 2.1 Historical Volatility

Volatility is a measure of dispersion in a probability density. Historical volatility refers to the volatility of the underlying asset measured over time and is measured using historical price data. The most common measure of dispersion is standard deviation. The simplest form of measuring historical volatility would be to take the standard deviation of asset returns over time.

For a series of daily closing prices  $\{s_0, s_1, \dots, s_n\}$ , the daily historical volatility  $\sigma_{daily}$  is defined as the annualized standard deviation of daily returns  $\{r_1, r_2, \dots, r_n\}$  where  $r_i = \frac{S_i - S_{i-1}}{S_{i-1}}$ .

Let  $\bar{r} = \frac{1}{n} \sum_{i=1}^n r_i$ , then

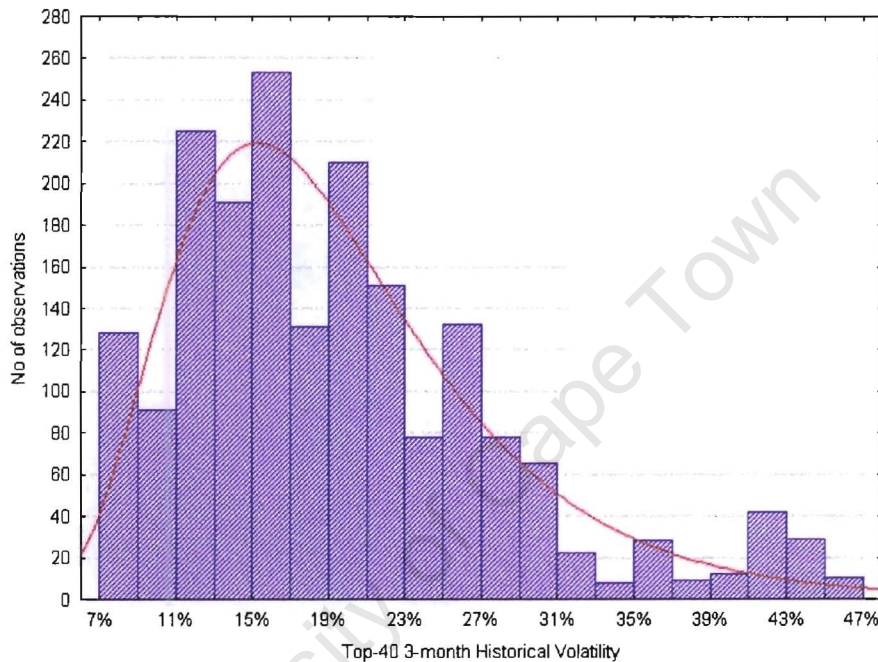
$$\sigma_{daily} = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (r_i - \bar{r})^2} \quad (1)$$

The rolling window approach (an equally weighted moving average model) is a frequently used method of estimating historical volatilities through time. It is common to assume that successive returns are relatively independent of each other and to express standard deviation of returns in annualised form

$$\sigma_{annual} = \sigma_{daily} \sqrt{k} \quad (2)$$

where  $\sigma_{daily}$  is a daily estimate of volatility and  $k$  is an annualising factor.

The annualising factor is simply a normalising constant and is usually set to the number of trading days in one year<sup>3</sup>. We use trading days rather than calendar days when calculating volatility because it has been observed that volatility is higher on days when the stock exchange is open than it is on days when the stock exchange is closed (French 1980).



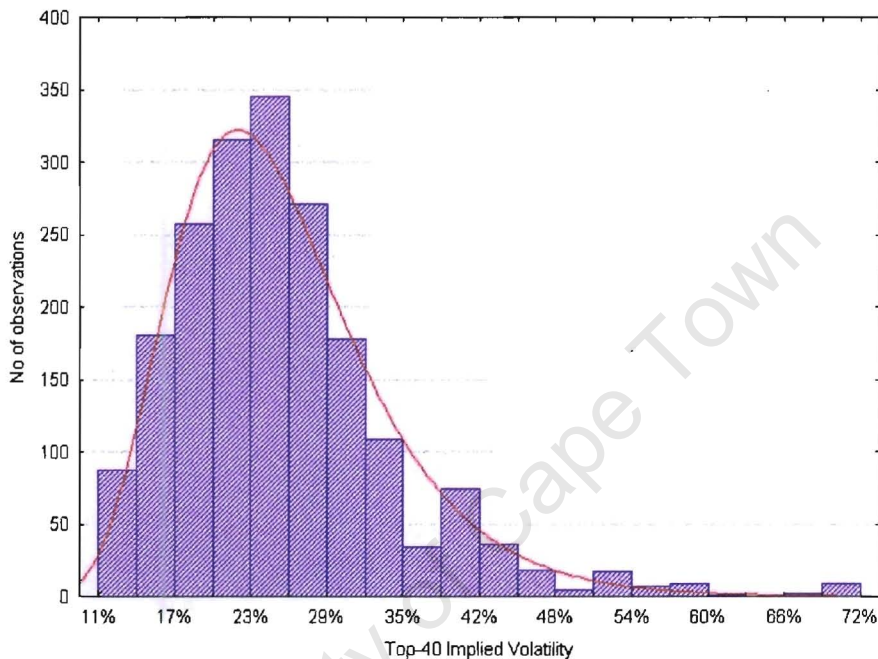
**Figure 1: Distribution of Top-40 3-month historical volatility (26 September 1995 – 29 April 2003)**

Historical volatility for the Top-40 as measured using the rolling standard deviation approach has a distribution that has a short left tail and a long, thin right tail. This is indicated in Figure 1.

<sup>3</sup> The number of trading days per year is usually set equal to 250 or 252 days. We adopt a convention of 240 trading days per year for computational convenience. Thus 1 month = 20 trading days, 3 months = 60 trading days and so on.

## 2.2 Implied Volatility

Implied volatility is the volatility forecast over the life of an option that equates an observed market price with the model price of an option. It is the volatility implied by an option price observed in the market and is the volatility that has been factored into an option pricing model at the time of pricing the option.



**Figure 2: Distribution of Top-40 3-month implied volatility (June 1995 – April 2003)**

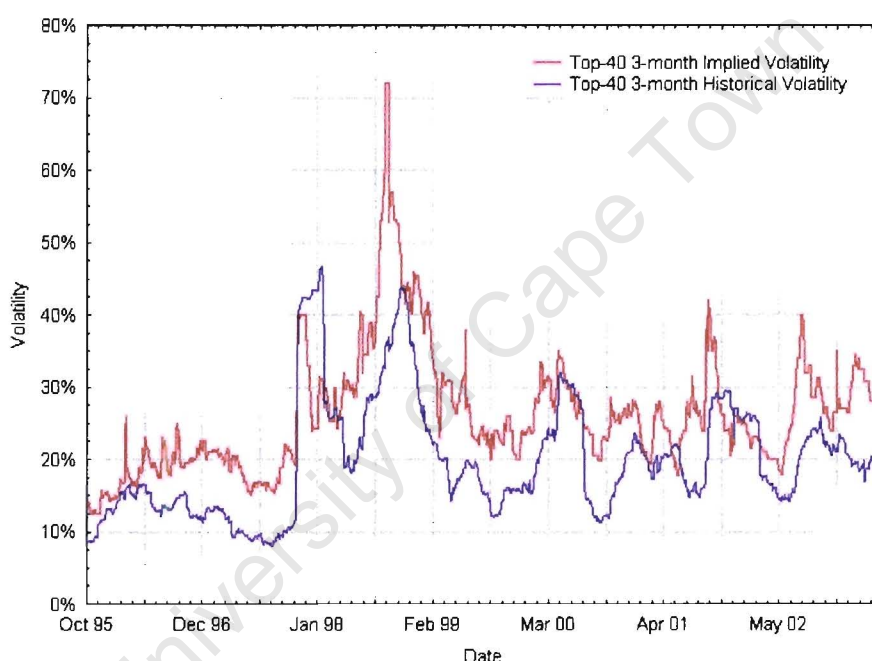
Implied volatility is sometimes thought of as being the market's forecast of the future return volatility of the underlying asset (Schwert 1990, Canina and Figlewski 1993). Implied volatility is the option market's view of expected price movement, reflected in the premium of an option. The market's view of volatility is governed by supply and demand. Some in the market may think that volatility is too cheap while others may be of the view that volatility is too expensive. The prevailing market volatility is a balance between the two views.

Implied volatilities are forward looking since they are based on the current prices which capture the market expectations of the future. Thus they should include some valuable information and market insight about future volatility.

One would expect that implied volatility should surpass historical volatility as a better forecast of future volatility. This idea is supported in several studies (Harvey and Whaley 1992, Christensen and Prabhala 1998). There is also contradictory evidence that implied volatility does not better forecast future volatility. Canina and Figlewski (1993) find that implied volatility has virtually no

correlation with future return volatility. They reject the hypothesis that implied volatility is an accurate forecast of future realised volatility. Lamoureux and Lastrapes (1993) also find that implied volatility is not an unbiased forecast of future volatility. They conclude that implied volatility is biased and inefficient and that historical volatility contains predictive information about future volatility beyond that contained in implied volatility.

It is often the case that implied volatilities cannot be obtained because there is no real market for options of the appropriate maturity. The lack of liquidity for options with appropriate maturities may cause inaccurate implied volatility quotes. This is also evident in the local index option market where far- and long-dated options (9- and 12- months respectively) are traded far less actively than near- and middle-dated options (3- and 6- months respectively).



**Figure 3: 3-month implied and historical volatility for the Top-40 (June 1995 – April 2003)**

We use implied volatility whenever reliable estimates for it are available. For index options, we make use of the SAFEX mark-to-market implied volatility<sup>4</sup>.

The 3-month implied volatility distribution for the Top-40 is shown in Figure 2. Implied volatility is distributed with a short left tail and a long, thin right tail. Both historical volatility and implied volatility exhibit negatively skewed distributions. We will make reference to this point in the discussions in section 7.

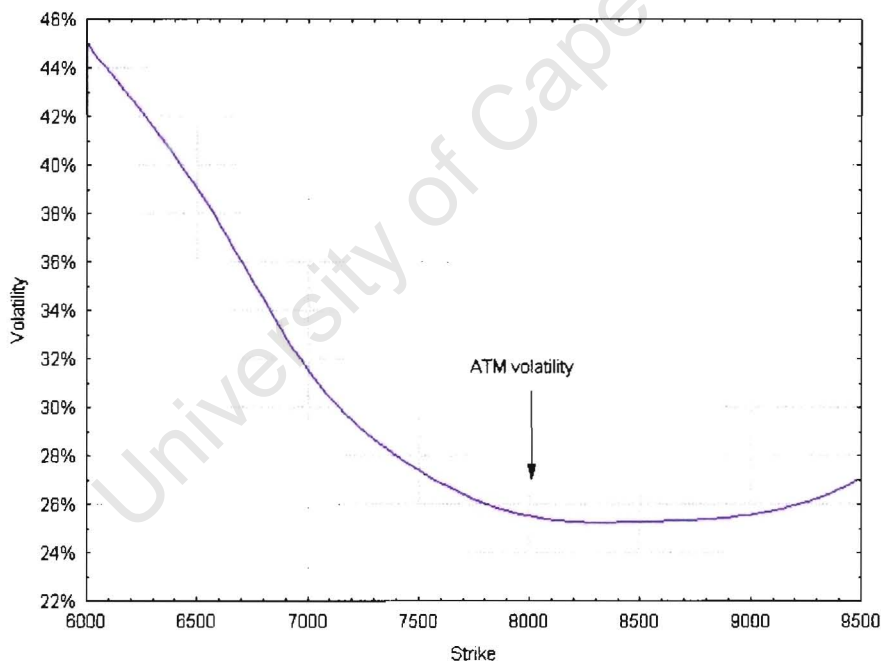
<sup>4</sup> See Appendix B for details of the SAFEX implied volatility methodology.

Implied volatility includes market sentiment as well as the market double (bid-ask spread) and impact costs. Thus implied volatility will usually be higher than historical volatility (see Figure 3).

## 2.3 The Volatility Skew

Since the crash of 1987, the market's implied Black-Scholes volatilities for index options have shown a pronounced negative relationship between implied volatilities and strike prices (Derman and Kani, 1994). That is to say, a plot of implied volatility as a function of strike price will generally indicate that volatility decreases as the strike price increases. Implied volatility as a function of strike price is often referred to as the volatility skew.

The reason for the existence of the skew is that the Black-Scholes model assumes that prices returns are normally distributed and have constant volatility. However, empirical research shows that many asset returns exhibit fatter tails in their distributions than are implied by a normal distribution and that conventional volatility of these distributions change over time.



**Figure 4: 3-month Top-40 implied volatility skew (June 2003)**

Volatility skews are a consequence of empirical violations of the normality assumption. This is because far OTM puts have a greater chance of becoming in-the-money than is assumed by the normality assumption of the Black-Scholes model. This means that the Black-Scholes model will underprice these options, so that the model price is less than the market price, and the only way the Black-Scholes model can reconcile this difference is to increase the implied volatility for OTM puts. The converse is true for OTM calls.

This also makes sense from a supply and demand perspective. OTM puts are more valuable because investors are scared of market crashes. There is a high demand for portfolio insurance offered by puts, and as such, traders will use higher volatilities to price OTM puts. For the purpose of illustration, we present the volatility skew for the Top-40 index in Figure 4.

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## 2.4 The Efficient Market Hypothesis

Fama (1965) defines an efficient market as one in which a large number of rational profit maximisers actively compete with each other. Each tries to predict the future market values of individual securities in an environment where important current information is almost freely available to all participants. In an efficient market, competition between the many participants will lead to a situation where at any point in time, the actual prices of individual securities already reflect the information that is based on events which have already occurred, or which the market expects to occur in the future.

If today's stock prices reflect all available information, then tomorrow's price movements must be unpredictable, since any information that might be used to forecast them will have already been incorporated by traders into today's prices.

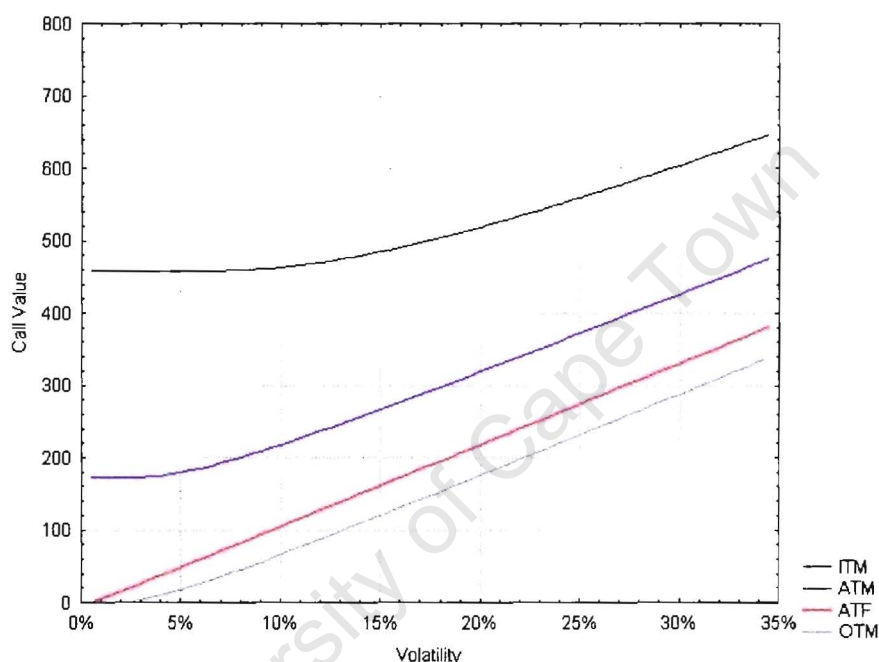
The efficient market hypothesis (EMH) can be subdivided into three categories:

1. The Weak form asserts that there is no information contained in historical stock prices which will allow an investor to earn abnormal returns based on that information. This hypothesis implies that technical analysis cannot be used to obtain abnormal returns. In other words, technical analysis is of no use.
2. The Semi-strong form of the EMH states that there is no publicly available information that will allow an investor to earn abnormal returns based on that information. Security prices react immediately to all new information. This hypothesis implies that fundamental analysis can't be used to obtain abnormal returns.
3. The Strong form of the EMH states that there is no public or private information that will allow an investor to earn abnormal returns based on that information. This hypothesis implies that even insider information can't be used to obtain abnormal returns.

The debate about efficient markets has resulted in numerous empirical studies attempting to determine whether specific markets are in fact efficient. Most empirical tests indicate that in general, markets are very close to being weak form efficient (Elton and Gruber, 1995).

### 3. Volatility arbitrage and trading rules for volatility arbitrage

It is commonly understood from the rudiments of option pricing that an option's premium is directly related to volatility, *ceteris paribus*. The relationship between an options' value and volatility is illustrated in Figure 5. We plot a call value as a function of volatility, for options struck in-the-money (ITM), at-the-money (ATM), at-the-forward<sup>5</sup> (ATF) and out-the-money (OTM). The same increasing relationship holds for put options.



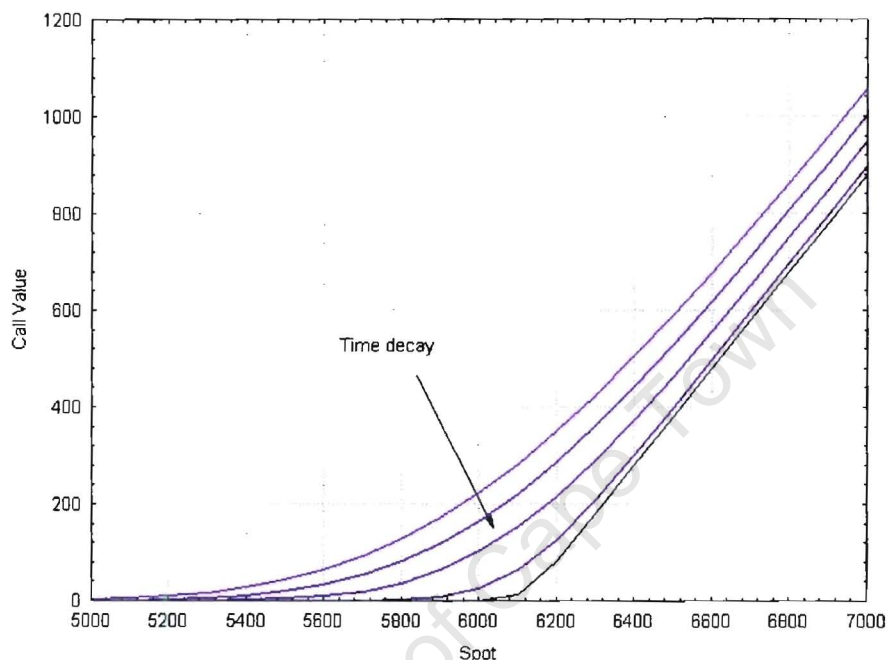
**Figure 5: Call value vs. volatility**

If volatility is mispriced, it is relatively easy to take advantage of this mispricing. A simple objective function exists in betting in the volatility market. If one believes that the market volatility is undervalued relative to where it should be (i.e. the market is cheap), then one wants to be on the long side of volatility. If one believes the market is overvalued (i.e. expensive), then one wants to be on the short side of volatility.

Purchasing volatility (through purchasing an option) at low volatility when volatility will rise within the term of the contract yields an accrued premium benefit to the option holder. The fair-value of the call option has increased, and this will manifest in the mark-to-market valuation of the position. Similarly, selling an option on the expectation of volatility falling will yield similar benefits.

<sup>5</sup> *At-the-forward* means that the option is struck at the forward price. This is explained in section 5.3.

The dynamics of calling the volatility direction correctly are not always appreciated. If we are long volatility, and the market volatility increases during the term of the trade, the net effect is to delay the degree to which time eats away at the option premium (Figures 6 and 7). Conversely, if we are short volatility and the market volatility decreases during the term of the trade, the net effect is to accelerate the rate at which the option decays into its final expiry profile.

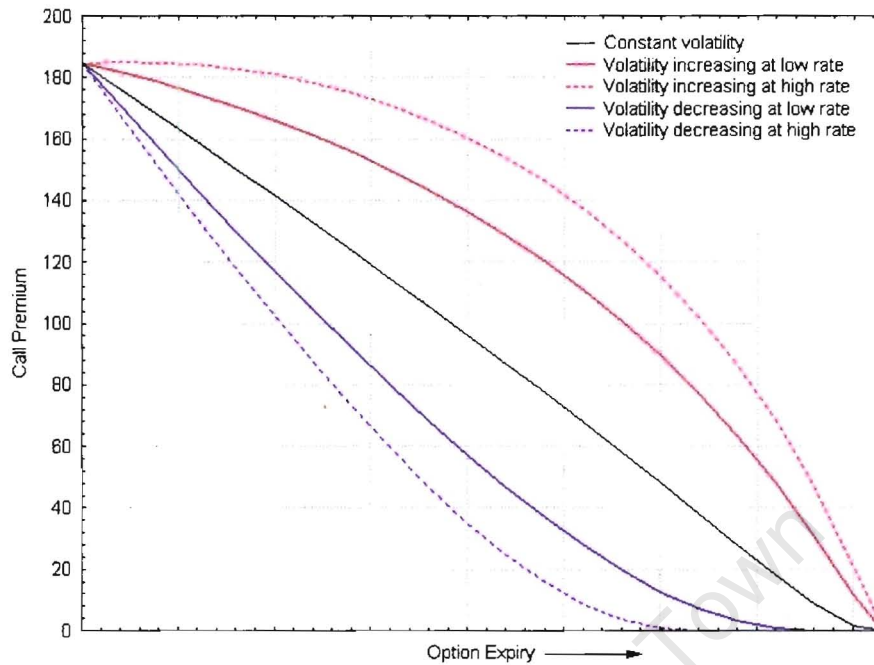


**Figure 6: Time decay of a purchased call vs. spot market**

The effects of volatility and time decay may best be understood by investigating the behaviour of an options Vega  $\mathcal{V}$  and Theta  $\Theta$ . Vega measures the impact of a change in volatility on an option's premium. It can be thought of as the rate of change of an option's value with respect to volatility

$$\mathcal{V} = \frac{\partial O}{\partial \sigma} \quad (3)$$

where  $O$  is the option's premium. As the volatility of the underlying increases both put and call values will increase (i.e. they have positive Vega). This is because puts and calls have no downside risk attached to them, and so as volatility increases, the likelihood that the options end in-the-money increases. Thus we expect both  $c$  and  $p$  to be an increasing function of volatility.



**Figure 7: Effect of changing volatility on call premium (all other parameters are fixed)**

We can then profit from changes in the volatility level of the underlying. A long volatility trade will profit from an increase in the level of volatility. It is important to note that any change in the underlying price will introduce a directional component to the trade as well.

Theta measures the impact of time on an option's premium. It is the rate of change of an option's value with respect to time

$$\Theta = \frac{\partial O}{\partial t} \quad (4)$$

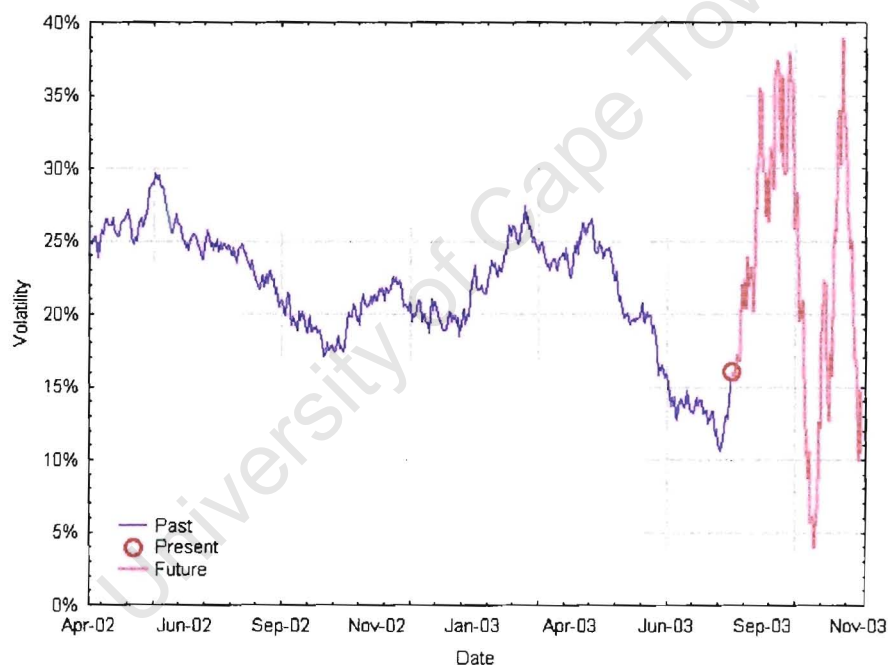
A purchased call will become less valuable as it approaches its expiry date (i.e.  $\Theta < 0$ ). This is because the more time there is until maturity, the greater the chance that the option will end in-the-money. That is, the further away the option is from its maturity, the greater the chance for variance to act thus resulting in a better chance that the option ends in-the-money. Another reason for purchased calls having negative theta is that the present value of the strike price will decrease the further away from the expiry date the option is, making the call less valuable as the maturity date draws closer.

Similar to the case for purchased calls, as time to maturity increases the put will become more valuable. However, as time to maturity increases, the present value of the payment for exercise declines, reducing the puts value. Thus the maximum payout needs to be balanced against the possibility of ending in-the-money.

In the analysis above, we are implicitly assuming that the spot market and the other peripheral variables to Black-Scholes (interest rates, dividends) have remained unchanged. This assertion is fair within the context of institutional volatility arbitrage strategies<sup>6</sup> where equity derivative decisions are typically based on both a market view and a valuation of volatility mispricing.

- How is volatility mispricing assessed?

Volatility mispricing is typically based on a historical assessment of where volatility has been trading, and where it is trading currently. If there is a large discrepancy between the two, volatility mispricing is inferred and opportunities for volatility arbitrage concluded. For the sake of understanding and brevity, we subdivide chronological time into three segments: 'past', 'present' and 'future' (Figure 8).



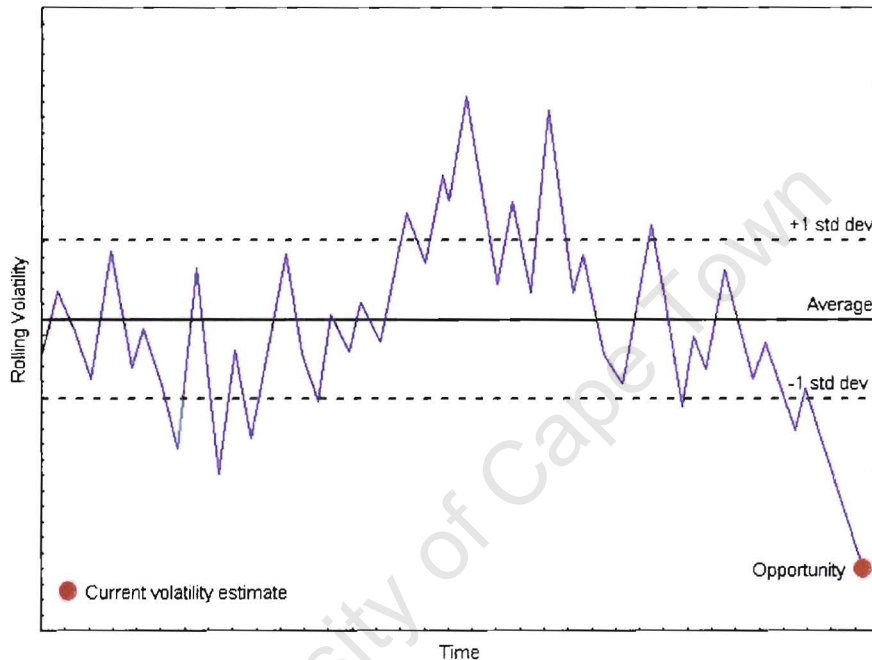
**Figure 8: A conceptual diagram of the relationship between financial time series from the past, the present and the future**

We are interested in understanding what present measure of the past volatility is the most appropriate predictor of the future volatility. Present estimates of past volatility are all statistical in nature, and differ dramatically in their reliability and usefulness (see section 4).

<sup>6</sup> We use the term *arbitrage* to refer to profit opportunities presented by assets that appear to be mispriced relative to publicly available information about fundamentals as well as relative to other assets. Such arbitrage is not necessarily always riskless.

- In order for volatility arbitrage strategies to be successful, we require that sometimes, the difference between the present statistical estimate of volatility and the past accurately reflects the realized future volatility.

If these instances do not predict future realized volatility, volatility play will most likely fail – and losses will be incurred. Typically, derivatives traders utilize a fairly simple rule in order to identify volatility trading opportunities. This rule is illustrated in Figure 9.



**Figure 9: A diagrammatical representation of a typical (naïve) volatility trading rule**

Assume we are interested in a 6-month trade (180-trading days). Characteristically, we would examine some length of a 180-day rolling historical volatility window, say one to three years, and then average the values over this window. One then investigates the level of variation about these values through using some statistic of dispersion, such as the standard deviation. Typically, standard deviation bounds of 1 are set about the average value (see Figure 9 above). Lastly, the present 180-day volatility is computed. If this figure falls within the standard deviation bound, it is concluded that no volatility opportunities exist. If, however, the present value falls well outside the bounds, some level of volatility opportunity is identified.

Implicit in this trading rule is the assumption that volatility reverts to a mean value. Mean reversion of volatility and implied volatility is well documented on indices and less so on single stocks. It has been argued that mean reversion makes better economic and practical sense than does the lognormal diffusion process invoked in option pricing models (Dueker 1997). Stein (1989) found strong evidence of mean-reversion on the S&P100 Index options for the period

1983 – 1987, and Merville and Piptea (1989) found similar evidence for the S&P 500 stock-index options for the period 1975 – 1985. Bates (1996) summarizes that most studies agree that implied volatility is mean-reverting. More recently, Fouque et al. (1998) found that implied volatility levels mean-revert more slowly than the tick-by-tick fluctuations on the S&P 500 Index. The rate of mean-reversion of volatility varies positively and non-linearly with the absolute level of volatility on the Chicago Board of Options VIX index (Dueker 1997). Polakow (2001) documented weak mean-reversion on implied volatility on the South African ALSI-40 exchange-traded option contracts.

The trading rule highlighted above is applied extensively locally to both implied and historical volatility (on the index) and historical volatility (on single stocks and the index) within the context of institutional equity derivatives activity.

- Interesting to point out is the fact that the typical trading rule highlighted above, while common both locally and abroad, is statistically unreliable for a number of reasons (see section 4).

Better alternatives to this rule may exist. The objective of this research is test the efficacy through which volatility trading rules forecast (or are reflective of) future realized volatility. In the ensuing section, we devise alternative rules that better accommodate statistical considerations. We then test these rules on both the Index and several single stocks in an attempt to discern what rules work on what underlyings and why.

## 4. Trading rules: Pros and cons

The trading rule referred to in the previous section (where standard deviation bounds were constructed about a rolling historical window average and then contrasted with a current volatility window) is common, but is also statistically naïve. The reason why it is unsophisticated is twofold:

- The rolling historical window model above assumes that asset returns are independent and identically distributed. The statistical measures of average and standard deviation are both biased due to the effect of overlapping, non-independent observations forced by the rolling window. The consequence is to bias the measure of 'average' in either direction (depending on the properties of the underlying distribution on which volatility is estimated – if this process is symmetrical and the observations are independent and identically distributed no bias will be incurred) and to reduce the estimate of variance.
- The measure weights all of history equally regardless of how representative the past volatility is of the present volatility regime; there is no time-varying volatility assumption. The volatility obtained in this way only provides an estimate of the unconditional volatility, which is assumed to be constant. However, returns are often autocorrelated and not independent. As time progresses, so the last observations in the distant past roll off, and the newest observations form part of the data window. There is no information bias towards weighting recent information more than older, possibly outdated information. If the underlying volatility regime is stationary (i.e. unchanging) then this concern is rendered trivial. If volatility is changing, and some mechanism of volatility updating is more likely to capture current estimates of variation, then this concern would need to be addressed.

It is therefore of interest to test whether a volatility trading rule that is based on volatility estimates that are statistically more rigorous perform better in assessing and identifying opportunities for volatility trading than does the simple naïve approach. We propose three refinements to the naïve statistical estimation procedure.<sup>7</sup> These are discussed briefly below.

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<sup>7</sup> *These approaches represent three fundamentally different statistical routes to circumventing the assumptions that concern us about the naïve rule. They are drawn from a spectrum of complexity, ranging from simple (EWMA) through to the more complex volatility updating models (GARCH). All are commonly understood and implemented.*

## 4.1 Bootstrap Resampling

The resampled approach is aimed at obviating the concerns of autocorrelation and non-independence by removing the degree that overlap between rolling observations detracts from the reliability on the estimates of volatility. Volatility moments are estimated from point samples in the historical volatility series.

Bootstrapping is a useful resampling technique that was initially proposed by Efron (1979) as a method for estimating standard errors of estimators, testing hypothesis or constructing confidence intervals for the parameters estimated by an estimator.

The ultimate goal in evaluating the standard error ( $SE$ ) of an estimator is to utilize this standard error for constructing confidence intervals. Unfortunately empirical formulas are not available for the standard error of many useful statistics. Bootstrap methods provide a means of readily obtaining good estimates of the standard errors in such cases. They allow us to avoid making restrictive parametric assumptions about unknown distributions while constructing confidence intervals. We use the original nonparametric bootstrap method of Efron (1979) to study the standard error of the estimator.

Suppose we wish to estimate the standard error of a statistic  $\theta = \theta(x_1, x_2, \dots, x_n)$  where the data  $x_1, x_2, \dots, x_n$  are a random sample from a population with an unknown distribution. We may estimate the parameter  $\theta$  by  $\hat{\theta}$  which is obtained by the following bootstrapping procedure:

1. Draw  $N$  bootstrap resamples from  $x_1, x_2, \dots, x_n$  by sampling randomly with replacement, we denote these resamples by  $x_{i,1}^*, x_{i,2}^*, \dots, x_{i,n}^*$  for  $i = 1 \dots N$ .
2. Evaluate the estimator  $\hat{\theta}_i^* = \hat{\theta}(x_{i,1}^*, x_{i,2}^*, \dots, x_{i,n}^*)$  for each resample.
3. Estimate the variance of the estimator  $\hat{\theta}$  by the empirical variance of the bootstrapped estimates  $\hat{\theta}_i^*$ .

The standard error  $SE$  is the square root of the estimated variance

$$SE = \left[ \frac{1}{N-1} \sum_{i=1}^N (\hat{\theta}_i^* - \theta') \right]^{\frac{1}{2}} \quad (5)$$

where  $\theta' = \frac{1}{N} \sum_{i=1}^N \hat{\theta}_i^*$

We may now calculate the  $(1-\alpha)100\%$  confidence interval for  $\hat{\theta}$  by taking the empirical  $\frac{\alpha}{2}$  and  $(1-\frac{\alpha}{2})$ -percentiles of the bootstrapped  $\hat{\theta}^{*(i)}$ . An alternative approach (the one we use) is to calculate  $\bar{\theta}^{*(i)} - t_{v,\alpha(2)}SE < \mu < \bar{\theta}^{*(i)} + t_{v,\alpha(2)}SE$ , where  $t_{v,\alpha}$  denotes the  $100\alpha^{th}$  percentile of the t-distribution with  $v = N - 1$  degrees of freedom.

The number of resamples  $N$  needs to be sufficiently large so that the random error due to the bootstrap process is small. We choose  $N=100$  because our samples are relatively small. As  $N \rightarrow \infty$ , the random error due to the bootstrap process will tend towards zero.

## 4.2 Time-Weighting Resampling

Time-weighted resampling removes the effects of autocorrelation from volatility estimates while weighting observations by time. Observations are weighted by a simple reciprocal function of time, giving greater weightings to more recent observations.

The resampling methodology is identical to that explained in section 4.1 above and the function that is used is presented in section 5.1.2.

## 4.3 Volatility updating – EWMA (Exponentially Weighted Moving Average)

Volatility updating is aimed at revising one's volatility forecast in terms of allowing more recent information to be given added influence to the estimated past volatility. Hence, observations are down-weighted the further from the present they occur in the rolling window sequence. EWMA is a simple mechanism that facilitates this desired weighting effect. The past volatility is thus brought closer to the present volatility, and hopefully, to future volatility.

We may model volatility by using the exponentially weighted moving average model (EWMA). The model weights historical observations exponentially, assigning heavier weights to recent return observations. The resulting volatility estimate will react immediately to unusually large returns. The effect of a large return then gradually diminishes over time with a strength governed by a decay factor  $\lambda \in [0;1]$ . The larger the value of  $\lambda$  the more weight is placed on past observations and so the smoother the series becomes.

The model may be written in recursive form as

$$\sigma_t^2 = \lambda \sigma_{t-1}^2 + (1 - \lambda) r_{t-1}^2 \quad (6)$$

Here,  $r_t$  represents the daily return at time  $t$  and  $\sigma_t^2$  is the volatility at time  $t$ .

Past observations are weighted by the decay factor so that an observation  $n$  days ago is multiplied by  $\lambda^n$ , which is very small for large  $n$ . Thus extreme events have less of an impact on volatility as they move further into the past. We use a decay factor of  $\lambda = 0.99$ .

#### **4.4 Volatility updating – GARCH (Generalized auto-regressive conditional heteroskedasticity)**

Both EWMA and time-weighted volatility updating are mechanisms that facilitate the more recent observations to impact the present volatility estimate more than older observations. However, both still afford some level of contribution to the oldest observations and a strong level of contribution to more recent observations. A more elaborate means of updating volatility than either simple time-weighting or EWMA is to unconstrain one's estimate of present volatility from the past observations. To this end, the GARCH model allows for current volatility to vary in accordance with the more recent volatility regime. The benefit of a GARCH-approach is that changing volatility will be reparameterized rapidly. In this way, we hope to encapsulate the nature of future volatility with more precision.

Many financial time series exhibit volatility clustering where periods of small returns and low volatility are interspersed with volatile periods of large returns. This phenomenon is termed autoregressive conditional heteroskedasticity. Volatility clustering implies a strong autocorrelation in squared returns at times. GARCH models impose more structure on the day-to-day volatility changes, with recent return shocks playing a driving role in determining conditional volatility.

We adopt a GARCH(1,1) specification because it is a parsimonious representation that seems to fit the data relatively well. The GARCH model (Bollerslev, 1986) postulates that the variance of returns follows a predictable process, driven by the latest squared innovation and by the previous conditional variance.

We use a simple constant conditional mean and a GARCH(1,1) conditional variance model to model the historical volatility.

The constant conditional mean model for the returns is

$$r_t = \mu + \varepsilon_t \quad (7)$$

Here,  $r_t$  represents the daily return at time  $t$ ;  $\mu$  is a constant term which relates to the mean of the return series. The disturbances or residuals,  $\{\varepsilon_t\}$ , are assumed to be random in time and normally distributed with mean zero and variance  $\sigma_\varepsilon^2$ .

The GARCH conditional variance model of orders  $p$  and  $q$ , GARCH( $p,q$ ) is given by the following recursive formula

$$\sigma_t^2 = \gamma + \sum_{i=1}^p \alpha_i \sigma_{t-i}^2 + \sum_{j=1}^q \beta_j \varepsilon_{t-j}^2 \quad (8)$$

This model says that the conditional variance at time  $t$  depends on the conditional variance in the previous periods and the squared residuals in the previous periods.

We restrict our attention to a GARCH(1,1) specification of this model that is given by

$$\sigma_t^2 = \gamma + \alpha \sigma_{t-1}^2 + \beta \varepsilon_{t-1}^2 \quad (9)$$

where  $\gamma > 0, \alpha > 0, \beta > 0$ . We note that  $\alpha + \beta < 1$  or else the volatility would expand without bound. This model assumes that the variance at time  $t$ , conditional on information available at time  $t-1$ , is a linear function of lagged conditional variance  $\sigma_{t-1}^2$  and of squared residuals  $\varepsilon_{t-1}^2$ .

The GARCH(1, 1) approach captures the persistence of a shock to the volatility estimate. This is captured by  $\alpha$  which measures the impact of the shock on current volatility and by  $\beta$  which measures the rate at which the effects of a shock dissipate.

By defining  $\varphi = \frac{\gamma}{1 - \alpha - \beta}$  we may rewrite the standard GARCH(1, 1) specification as

$$\sigma_t^2 = \varphi + \alpha(\sigma_{t-1}^2 - \varphi) + \beta(\varepsilon_{t-1}^2 - \varphi) \quad (10)$$

where  $\alpha$  is the persistence parameter,  $\beta$  is the response parameter and  $\varphi$  is the unconditional variance parameter. From (10) above, the mean reversion that the GARCH model may assume should be clear. If the conditional variance in the previous period is above the unconditional variance, then the next period's conditional variance increases in proportion to the persistence parameter  $\alpha$ . If the squared return residual in the previous period is above the unconditional variance, then the next period's return increases in proportion to the response parameter  $\beta$ .

## 5. Methodology

### 5.1 Statistical Trading Rules

As mentioned, we are interested in understanding what present statistical measure of the past most accurately reflects the realized future volatility.

- This research backtests different statistical rules for finding opportunities in estimating the 'past' volatility in a way that best agrees with the 'future' volatility – all conducted retrospectively.

In other words, we are hoping to maximize in- and out-of-sample agreement throughout our historical records. We focus on the dynamics of option volatility arbitrage. The movements of the underlying and their influence on option premiums are coincidental to this research.

We subdivide the empirical study into four sections, comprising four 'rules' (see Figure 10):

#### Rule 1: Past implied volatility vs. present implied volatility

- We consider only the Top-40 Index here<sup>8</sup>.
- We compare past implied volatility and present implied volatility.
- Past implied volatility is estimated by taking a 1-year<sup>9</sup> rolling average of past implied volatility and then determining a standard deviation bound about this average. Standard deviation bounds are set at 0.5, 1, and 2.
- Present implied volatility is the SAFEX mark-to-market volatility record on the day the option is traded.
- The term is set at 3-months and at 6-months.

The rule is as follows:

- We buy an Index option (and hence volatility) if the present implied volatility is less than the past implied volatility lower bound and sell an option if the present implied volatility is greater than the past implied volatility upper bound.
- Three further variations of this rule are investigated<sup>10</sup>.

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<sup>8</sup> *Reliable estimates of implied volatility are not available for single stock options as these are traded less frequently and are not marked-to-market if traded over-the-counter (OTC). This is discussed further in section 5.3.*

<sup>9</sup> *A 3-year window yields similar results.*

- Resampled averaging (section 5.1.1),
- Time-weighted resampled averaging (section 5.1.2),
- Exponentially weighted moving-average<sup>11</sup> (EWMA) (section 5.1.3).

Rule 2: Past historical volatility vs. present implied volatility

- We consider only the Top-40 Index here.
- We compare past historical volatility and present implied volatility.
- We calculate a rolling standard deviation of the index returns over the previous year and scale this to an annual volatility figure<sup>12</sup>. Past historical volatility is estimated by a rolling 1 year average of these historical volatilities. Standard deviation bounds are set at 0.5, 1, and 2.

The rule is as follows:

- We buy an option if the present implied volatility is less than the past historical volatility lower bound and sell an option if the present implied volatility is greater than the past historical volatility upper bound.
- Four further variations of this rule are investigated (details in Appendices A and B)
  - Resampled averaging (section 5.1.1),
  - Time-weighted resampled averaging (section 5.1.2),
  - Exponentially weighted moving-average (EWMA) (section 5.1.3),
  - Generalized auto-regressive conditional heteroskedasticity (GARCH) models (section 5.1.4).

Rule 3: Past historical volatility vs. present historical volatility (Index)

- We consider only the Top-40 Index here.
- We compare past historical volatility with present historical volatility. Past volatility is simply the rolling standard deviation of the index returns over the previous year (annualized). Past historical volatility is then estimated by a rolling 1 year average of these historical volatilities. We then construct standard deviation bounds (bounds = 0.5, 1 and 2) about this past volatility.

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<sup>10</sup> We do not implement GARCH on historical implied volatility as the relevance of modelling an implied volatility difference is obscure.

<sup>11</sup> We average implied volatility using an EWMA-type scheme.

<sup>12</sup> See discussion on annualising in section 2.1

- The 3 or 6-month rolling standard deviation of the index returns is used to estimate the present historical volatility.

The rule is as follows:

- We buy an option if the present historical volatility is less than the past historical volatility lower bound and sell an option if the present historical volatility is greater than the past historical volatility upper bound.
- Four further variations of this rule are investigated (details in Appendices A and B)
  - Resampled averaging (section 5.1.1),
  - Time-weighted resampled averaging (section 5.1.2),
  - Exponentially weighted moving-average (EWMA) (section 5.1.3),
  - Generalized auto-regressive conditional heteroskedasticity (GARCH) models (section 5.1.4).

Rule 4: Past historical volatility vs. present historical volatility (Single stocks)

- We selected six large capitalization stocks which are representative of three broad sectors; resource stocks (AGL, SOL), industrial stocks (RCH, SAB) and financial stocks (RMH, SBK).
- Terms are set at one of 3, 6, 9 or 12 months.
- We compare past historical volatility with present historical volatility. Past volatility is simply the rolling standard deviation of the index returns over the previous year (annualized). Past historical volatility is estimated by a rolling 1 year average of these historical volatilities. We then construct standard deviation bounds (bounds = 0.5, 1 and 2) about this past volatility.
- The rolling standard deviation (term based) of equity returns is used to estimate the present historical volatility.

The rule is as follows:

- We buy an option if the present historical volatility is less than the past historical volatility lower bound and sell an option if the present historical volatility is greater than the past historical volatility upper bound.
- Four further variations of this rule are investigated
  - Resampled averaging (section 5.1.1),
  - Time-weighted resampled averaging (section 5.1.2),
  - Exponentially weighted moving-average (EWMA) (section 5.1.3),
  - Generalized auto-regressive conditional heteroskedasticity (GARCH) models (section 5.1.4).

### 5.1.1 Resampling

We compare past volatility and present volatility according to each rule. In each rule, past volatility is measured as the equally weighted average volatility (either implied or historical) over a 1-year rolling window. In this variation, we have used a simple resampling method to estimate past volatility. We maintain the use of a 1-year rolling window and implement the following resampling procedure:

1. Randomly resample 5% of the points (say  $n$ ) in each 1-year rolling window  $t_1^{(i)}, t_2^{(i)}, \dots, t_n^{(i)}$ .
2. Average the volatilities at these points  $\bar{\sigma}^{(i)} = \frac{1}{n} \sum_{j=1}^n \sigma(t_j^{(i)})$ .
3. Repeat this procedure several times over, say for  $i = 1, 2, \dots, 30$ , the past volatility for each rolling window,  $\hat{\sigma}$ , is the average of the resampled averages,  $\hat{\sigma} = \frac{1}{30} \sum_{i=1}^{30} \bar{\sigma}^{(i)}$

The remainder of the rule is unchanged.

### 5.1.2 Time-weighted resampling

We further adapt the resampled variation by weighting our randomly resampled estimates by time. This is done in order to down-weight observations that are further out in the rolling window.

The time-weighted resampling procedure is similar to the resampling described above, except for a slight modification in the calculation of the average. We no longer calculate an equally weighted average of randomly resampled points. These are weighted instead by a function of time. The procedure is as follows:

1. Randomly resample 5% of the points (say  $n$ ) in each 1-year rolling window  $t_1^{(i)}, t_2^{(i)}, \dots, t_n^{(i)}$ .
2. Average (by a function of time) the volatilities at these points  $\bar{\sigma}^{(i)} = \sum_{j=1}^n \frac{1}{(k - t_j^{(i)})^{0.06}} \sigma(t_j^{(i)})$ .  $k$  is the number of days in one year. The constant 0.06 is chosen subjectively; the point is to ensure that the weights are not equal, but increasing with time. A linear function would also suffice.

3. Repeat this procedure several times over, say for  $i = 1, 2, \dots, 30$ , the past volatility for each rolling window,  $\hat{\sigma}$ , is the average of the resampled averages,  $\hat{\sigma} = \frac{1}{30} \sum_{i=1}^{30} \bar{\sigma}^{(i)}$

### 5.1.3 EWMA

EWMA is used in different contexts within the various rules.

Rule 1:

Just as in the time-weighted resampling approach, we obtain an average past volatility by down-weighting the observations that occur further out in the past. We use the exponentially weighted moving approach to do this. This method also implicitly includes volatility updating.

Rules 2 to 4:

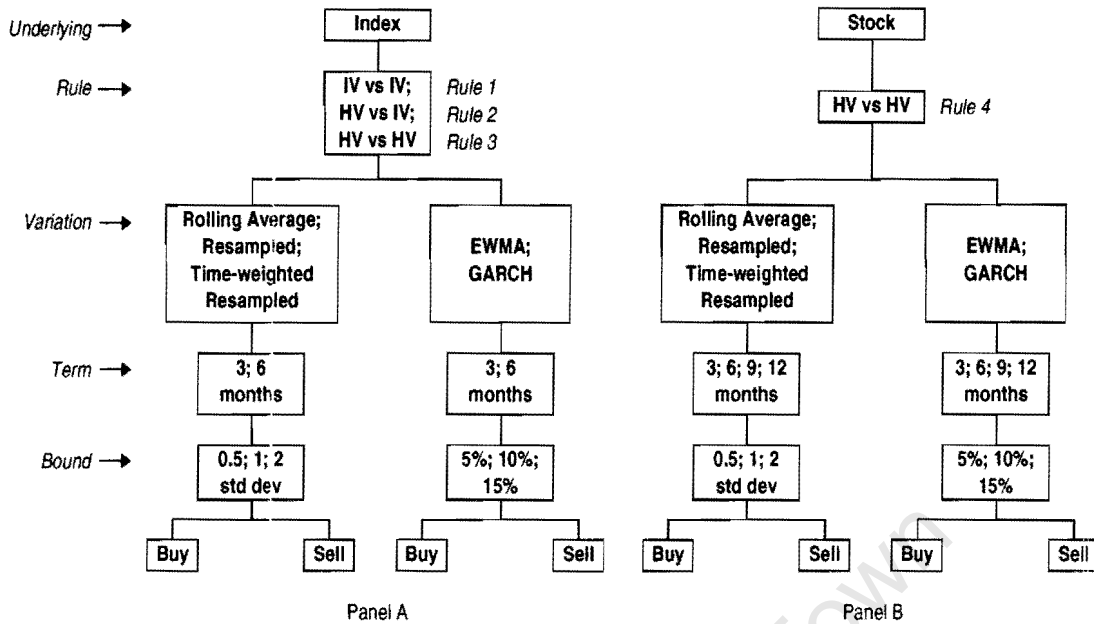
We use the EWMA methodology (described in section 4.3) to calculate the past volatility. Because volatility is being updated, we have to modify the rules slightly. We no longer calculate standard deviation bounds about the estimate of past volatility. Instead we trade if the present volatility is within  $\eta\%$  ( $\eta = 5, 10, 15$ ) of the past volatility.

### 5.1.4 GARCH

We use the GARCH methodology (described in section 4.4) to calculate the past volatility. As with the EWMA variation, we again modify the rules. We trade if the present volatility is within  $\eta\%$  ( $\eta = 5, 10, 15$ ) of the past volatility. The rules otherwise remain the same.

## 5.2 Outline of Study

The schemes for understanding the optimal statistical trading rules are highlighted below in Figure 10.



**Figure 10: A diagrammatical representation of the statistical trading rule methodologies for the Top-40 Index (Panel A) and Single Stocks (Panel B).**

## 5.3 Data

We consider both Top-40 index options and single stock options. Data on spot, dividend yield and interest rates was obtained from I-net Bridge. We use NCD (non-convertible debenture) rates as a proxy for 3, 6, 9 and 12-month risk-free rates. As described in section 2.1, historical volatility is computed as the annualized standard deviation of historical returns.

The volatility used for mark-to-market of index options is determined by the South African Futures Exchange (SAFEX) daily<sup>13</sup>. The index futures price  $F$  is given by<sup>14</sup>

$$F = Se^{(r-q)T} \quad (11)$$

<sup>13</sup> See Appendix B for details.

<sup>14</sup> (Hull 2000).

where  $S$  is the current index level,  $r$  is the continuously compounded risk-free rate,  $q$  is the dividend yield rate, and  $T$  is the time to maturity of the index future.

To avoid confusion we adopt the following convention:

- An index option that is struck at the spot index level is referred to as being *struck at-the-money* (ATM).
- An index option that is struck at the futures mark-to-market level (or within 100 index points thereof) is referred to as being *struck at-the-forward* (ATF).

The SAFEX mark-to-market implied volatilities are only applicable to options struck at the forward level<sup>15</sup>.

The term for the option trade is inferred. The volatility smile (skew) is circumvented by the fact that we are pricing options using at-the-money implied volatilities. The options are struck at-the-forward.

The Index and Single Stock analyses differ in three important respects:

- Implied Volatility is available historically for the Top-40 Index (formerly the ALSI-40 Index) as the SAFEX mark-to-market implied volatility but not on single stocks (for which we then need to infer a 'reasonable' market level for traded volatility). For 3-month Index contracts, we constructed a composite implied volatility index by rolling the implied volatility near (3-month) contract into the middle (6-month) contract a week prior to closeout to eliminate thin trading and closeout effects. For 6-month Index options, we use the mid-implied volatility.
- Implied volatility data on the near- and mid- contract are liquid and therefore reliable. Conversely, longer dated contracts are notoriously thinly-traded. Hence, we consider only 3 and 6-month trades on the Top-40 Index.
- For historical volatility estimation, we are unconstrained by 'implied' reporting. We therefore use short (3-month) through to long-term (12-month) trades on single stocks.

The Top-40 Index Data is gleaned for a period of just less than 8 years (30 June 1995 – 29 April 2003). The data for the single stocks is gleaned from a period of just over 9 years (31 March 1994 – 6 May 2003).

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<sup>15</sup> We use the forward price as a proxy for the futures mark-to-market level.

## 5.4 Evaluating Success

The choice of option pricing model is not important in this analysis as the methodology is independent of a pricing model. However, in order to gauge the success and establish profit and loss estimates for each of the trading rule variations, we will have to choose a pricing model.

To value the options, we use Merton's extension of the Black-Scholes formula (Merton 1973). The model is used to price European call or put options on a stock or an index, paying a known dividend yield  $q$ . Let  $c$  and  $p$  denote the price of a call and put option respectively. The formula states that:

$$c = Se^{-qt} N(d_1) - Xe^{-rt} N(d_2) \quad (12)$$

$$p = Xe^{-rt} N(-d_2) - Se^{-qt} N(-d_1) \quad (13)$$

where  $S$  is the spot level of the index,  $q$  is the continuous dividend yield and  $T$  is time (measured in years or days/365).  $N(\square)$  represents the cumulative normal distribution function and

$$d_1 = \frac{\ln(S/X) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}} \quad (14)$$

$$d_2 = \frac{\ln(S/X) + (r - q - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \quad (15)$$

- On each date we price two options,  $O_1$  and  $O_2$ . Both options are struck at the forward price<sup>16</sup>  $F = Se^{(r-q)T}$ , hence by put-call parity it is irrelevant whether the options are call or put options<sup>17</sup>.
- The volatility for option 1 ( $O_1$ ) is the present implied (rules 1 and 2) or historical (rules 3 and 4) volatility.
- The volatility for option 2 ( $O_2$ ) is the future historical volatility or realised volatility on the date that the option expires.
- We are thus assessing in retrospect what the realised volatility of an ATF (at-the-forward) option struck today would have been some term in the future.
- All other inputs for the two options are identical (spot, rate, dividend yield, strike, term).

<sup>16</sup> At-the-forward (ATF).

<sup>17</sup> For a proof of this, see Appendix A

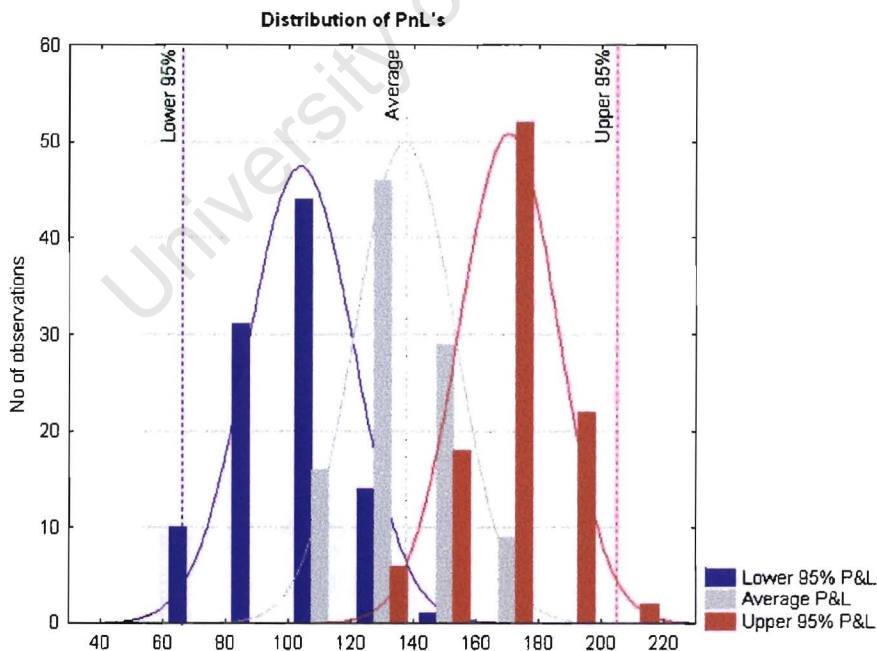
- This is not an analysis of intrinsic value at the term end. Rather, this is an analysis on the ability to identify volatility aberrations presently.

Because buying or selling volatility in the market requires trading at a premium or a discount we have to take a reasonable market double (the bid-ask spread) into account. Thus if the rule has indicated a buy, we add a premium to the volatility inputs. If the rule has indicated a sell, we will subtract a discount from the volatility inputs.

- We assume that the market double for options on the Top-40 Index varies between 0% and 0.25%. This is based on discussions with participants in the market. Similarly, we assume that the market double for single stocks is larger than for index options, and between 4% and 5%. The double may be higher than this for some single stock options. The extent of the double is selected at random for each instance of a buy or a sell signal between these upper and lower bounds. This is done to ensure that the effects of a varying double are factored into the results.
- If the trading rule has indicated that we should buy an option, we have to pay the option premium. The buy is thus a success if  $O_1 < O_2$  i.e.: if  $O_2 - O_1 > 0$ . If we sell an option, we receive the option premium. The sell is thus a success if  $O_1 > O_2$  i.e.: if  $O_1 - O_2 > 0$ .
- We assess the trading rules and their variations for all terms and bounds (as presented in Figure 10).
- For any rule, we have a historical volatility series and indications of when we ought to buy and when we ought to sell. Not surprisingly, many of these sell and buy signals are clumped together in time (when historical volatilities are high, they will tend to remain there for longer than a day). It is incorrect to report on the success of all such trades – as we again encounter the problem of autocorrelation biasing the results. In order to avoid this negative effect, we resample 5 % (say  $n$  observations) of the buy and sell trades randomly, and report only on these. The choice of 5% is not arbitrary. The average number of days that buy or sell sequences persisted was 21 days. In a sequence of 21 successive buy signals, we only need to look at one of these signals; so we resample approximately 1 day for every 21 days in the sample ( $\approx 5\%$ ).
- The mean P&L for the each resample is computed as well as the upper and the lower 95 % confidence bounds. The 95 % confidence bounds are computed as  $\text{mean} \pm \text{SE} * t\text{-stat}(\alpha=0.95, 2\text{-tailed}, d.f=n-1)$ . We repeat the resampling process 100 times. We have

effectively bootstrapped distributions for the mean profit and loss (P&L), the upper 95% P&L and the lower 95% P&L.

- We evaluate the success of buy and sell signals separately for each rule in two ways. First, we report on the average proportion of times that the trade has ended up in positive territory (with a positive P&L). We also report on the variation about this success proportion, this is obtained by resampling. We indicate a trading rule as successful if the percentage of successful trades is greater than or equal to 75%.
- Second, we examine the magnitude of the P&L of each trade. On each day, we calculate  $O_2 - O_1$  if there is a buy signal. If a sell signal was indicated we calculate  $O_1 - O_2$ .
- In order to get a true reflection of the average, extreme downside and extreme upside performance of each rule we report three P&L statistics: the lower 95% confidence (95% confidence bound =  $SE * t\text{-stat}(\alpha=0.95, 1\text{-tailed}, d.f.=100-1)$ ) bound of the lower 95% P&L distribution, the mean of the mean P&L distribution, and the upper 95% confidence bound of the upper 95% P&L distribution. These statistics are by definition 1-tailed. The results for this type of analysis are depicted in Figure 11 below.



**Figure 11: A diagrammatical representation of the lower, average and upper P&L performance of a hypothetical trading rule for a stock.**

Figure 11 would seem to indicate that the average success of the trading rule was normally distributed with an average P&L of, say, 136 cents. The downside risk is similarly distributed, but does not fall below 0 cents the vast majority of the time. On the upside, the profits range up to 220 cents.

- The results to each of the rules depicted in Figure 10 are presented in section 5.

## 5.5 Statistical Significance of Results

In the above section, we generate a number of samples in which the count for each sample represents the number of times that a buy or sell signal appeared. We examine buy signals and sell signals independently of each other. A successful trade is one that yields a positive P&L. If for example, we have a sample of 100 buy signals, and 30 of these buy signals yield positive P&L's, then the proportion of successful buy signals in the sample is 0.3. This proportion is an estimate of the probability of a buy signal being a success.

The underlying probability model for this type of problem is the binomial distribution. For each sample that is drawn, this distribution is characterized by the number of samples and the probability that a success occurs for each sample. This probability is called a proportion parameter.

For each rule variation and permutation thereof, we draw  $m$  resamples<sup>18</sup> (using the resampling procedure described in section 4.4 above). Each of these resamples yields a proportion parameter  $p_1, p_2, \dots, p_m$  that is the probability of success of the trade. Each  $p_i$  is obtained by dividing the number of successful buy (sell) signals in the  $i^{\text{th}}$  resample,  $n_{\text{success}}(i)$  by the total number of buy (sell) signals in the  $i^{\text{th}}$  resample,  $n_{\text{total}}(i)$  that were indicated by the rule

$$p_i = \frac{n_{\text{success}}(i)}{n_{\text{total}}(i)} \quad (16)$$

We then calculate the average proportion of successful trades,  $\bar{p}$  across all our samples (resamples) by taking the arithmetic average of the proportion parameters

$$\bar{p} = \frac{1}{m} \sum_{i=1}^m p_i \quad (17)$$

---

<sup>18</sup>  $m = 100$ , as indicated in section 4.4.

Each proportion  $p_i$  has an associated standard deviation  $\sigma(p_i)$ , given by<sup>19</sup>

$$\sigma(p_i) = \left[ \frac{1}{n_{total} - 1} \sum_{j=1}^{n_{total}} (s_j - p_i)^2 \right]^{\frac{1}{2}} \quad (18)$$

where  $s_j$  is either 1, if the  $j^{th}$  trade in the sample yields a positive P&L or 0 if it yields a negative P&L. The average standard deviation of the proportion of success is calculated as

$$\bar{\sigma}_p = \frac{1}{m} \sum_{i=1}^m \sigma(p_i) \quad (19)$$

The variance of the proportion of successful trades is given by  $\sigma_p^2 = \frac{1}{m-1} \sum_{i=1}^m (p_i - \bar{p})^2$  and the standard deviation is then given by  $\sigma_p$ .

Any permutation with an average proportion of success,  $\bar{p}$  of 0.75 or more is then considered a successful permutation<sup>20</sup>.

We need to show that the average proportion of success is statistically significant given the high variation about these average values (see result tables). This is done using a simple 1-sample hypothesis test at the alpha level of significance set at 0.95. All tests are one-tailed. We further utilise power analysis to assess the probability of a type II error (Appendix E).

Because the number of successful signals is quite small (recall that we first sample 5% of the buy/ sell signals – and these are not all going to yield positive P&L's) we want to ensure that the corresponding power is large enough for our results to be meaningful.

A meaningful result in this case is one for which average proportion of success,  $\bar{p}$  is statistically different from 0.5. If  $\bar{p}$  is not statistically different from 0.5 then the result can be likened to a series of coin tossing trials, where the probability of success (as indicated by either a head or a tail) is 0.5 if the coin is an unbiased one. In particular we are interested in determining whether  $\bar{p}$  is significantly greater than 0.5.

We will test the hypothesis  $H_0: \mu = 0.5$  versus  $H_A: \mu > 0.5$  with significance level  $\alpha = 0.05$  and assuming that  $\sigma^2$  is the same under both hypothesis and is unknown. To test the hypothesis we proceed as follows:

<sup>19</sup> A divisor of  $n_{total}$  (instead of  $n_{total} - 1$ ) will produce a biased estimator of the population variance.

<sup>20</sup> This is an arbitrary amount, it depends on how much of a bet one is willing to take.

- If  $\bar{p} > 0.5 + t_{m-1,0.95} \frac{\sigma_p}{\sqrt{m}}$  then  $H_0$  is rejected.
- If  $\bar{p} \leq 0.5 + t_{m-1,0.95} \frac{\sigma_p}{\sqrt{m}}$  then  $H_0$  is accepted.

The p-value for this test is given by the area to the right of  $(\bar{p} - 0.5) \frac{\sqrt{m}}{\sigma_p}$  under a  $t_{m-1}$  distribution. We note the following regarding the p-value:

- If  $0.01 \leq \text{p-value} < 0.05$ , then the results are significant.
- If  $0.001 \leq \text{p-value} < 0.01$ , then the results are highly significant.
- If  $\text{p-value} > 0.05$ , then the results are considered not statistically significant.

The power test for the hypothesis is given by

$$\Phi \left[ z_\alpha + (\bar{p} - 0.5) \sqrt{m} / \sigma_p \right] \quad (20)$$

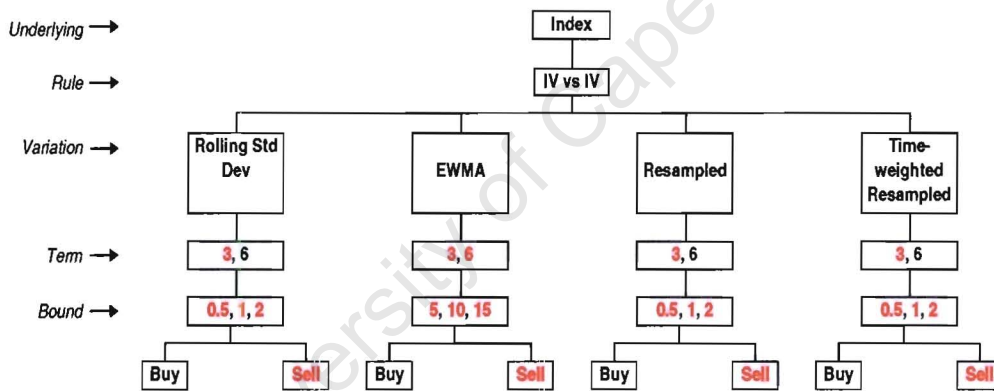
where  $z_\alpha$  is the  $100\% \alpha^{\text{th}}$  percentile of a standard normal distribution and  $\Phi(\cdot)$  is the cumulative distribution function of a standard normal distribution.

## 6. Results

The results of the analysis outlined in the previous sections are presented below. We indicate successful permutations of each rule in red. A successful trade is defined as where the average proportion of success is 75 % or greater. We require that the sampling effort be greater than 15 before a reliable inference can be made. We discuss each of the four rules (see Figure 10 for summary of what these rules are) in turn below.

### 6.1 Rule 1: Past implied volatility vs. present implied volatility

We base the decision on whether to trade or not by comparing past implied volatility and present implied volatility in Rule 1. Results are presented in both summarized graphical form (Figure 12) as well as in tabular form (Table 12).



**Figure 12: Permutations of Rule 1, with successes indicated in red**

Table 12 shows the permutations that yield successful volatility trades.

Variation	Term	Bound	Buy / Sell	Prop Success Ave	Prop Success Stdev	Sampling Effort	Power	Lower 95% bound of Lower 95% PnL distribution	Mean of Average PnL distribution	Upper 95% bound of Upper 95% PnL distribution
Rolling	3	0.5sd	Sell	0.772	0.417	32	0.979	38.64	84.05	128.74
Rolling	3	1sd	Sell	0.783	0.409	28	0.978	39.77	86.67	132.36
Rolling	3	2sd	Sell	0.757	0.426	24	0.905	27.87	79.14	128.70
EWMA	3	5%	Sell	0.808	0.393	34	0.998	34.37	78.87	122.30
EWMA	3	10%	Sell	0.824	0.375	24	0.995	34.84	84.26	132.40
EWMA	3	15%	Sell	0.821	0.367	18	0.981	27.69	89.61	149.04
EWMA	6	5%	Sell	0.793	0.402	32	0.993	57.48	114.55	170.56
EWMA	6	10%	Sell	0.787	0.399	21	0.950	25.72	99.37	170.55
Resampled	3	0.5sd	Sell	0.780	0.411	32	0.987	38.63	84.77	129.77
Resampled	3	1sd	Sell	0.774	0.414	27	0.964	31.92	82.20	131.22
Resampled	3	2sd	Sell	0.772	0.412	23	0.936	32.31	84.29	134.94
Time-weighted	3	0.5sd	Sell	0.787	0.409	32	0.990	42.00	87.05	131.13
Time-weighted	3	1sd	Sell	0.779	0.412	27	0.970	38.59	87.26	134.61
Time-weighted	3	2sd	Sell	0.755	0.427	23	0.888	26.51	79.46	130.56

**Table 12: Successful permutations of Rule 1**

The results from Rule 1 indicate that all methods, from the most primitive to the most advanced, yield reasonable levels of success along with moderate levels of variation of success. It is important to realize that the rule was only profitable when a sell signal was indicated. In other words, a volatility opportunity was correctly identified when present implied volatility was overpriced relative to measures of past implied volatility. Longer term (6-month) trades opportunities were only identified using the EWMA volatility updating technique. Variations of Rule 1 did not work for any 'buy' signals. A complete set of results for all permutations of Rule 1, successful or not, are indicated in Appendix C.

## 6.2 Rule 2: Past historical volatility vs. present implied volatility

We base the decision on whether to trade or not by comparing past historical volatility and present implied volatility in Rule 2. Results are presented in both summarized graphical form (Figure 13) as well as in tabular form (Table 13).

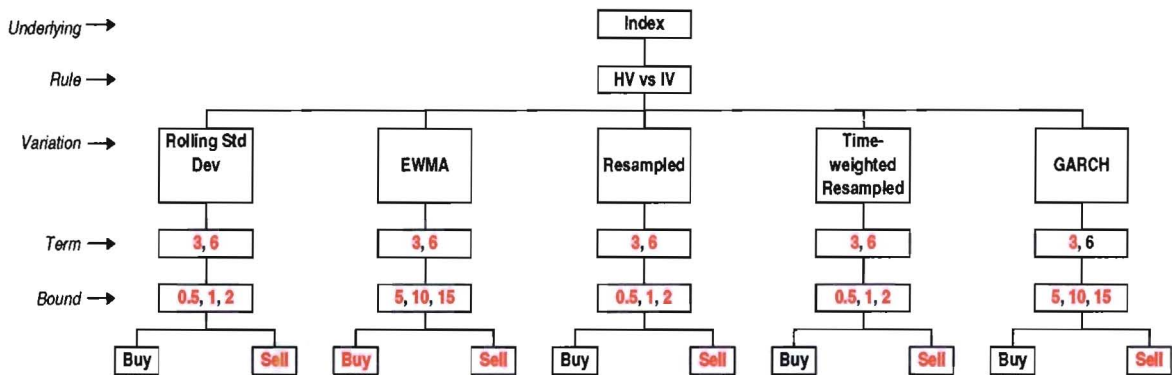


Figure 13: Permutations of Rule 2, with successes indicated in red

Table 13 shows the permutations that yield successful volatility trades.

Variation	Term	Bound	Buy / Sell	Prop Success Ave	Prop Success Stdev	Sampling Effort	Power	Lower 95% bound of Lower 95% PnL distribution	Mean of Average PnL distribution	Upper 95% bound of Upper 95% PnL distribution
Rolling	3	0.5sd	Sell	0.812	0.387	42	1.000	76.58	107.97	139.11
Rolling	3	1sd	Sell	0.814	0.386	41	1.000	77.76	109.78	141.60
Rolling	3	2sd	Sell	0.808	0.393	36	0.999	76.50	111.16	145.73
Rolling	6	0.5sd	Sell	0.866	0.337	45	1.000	99.73	134.68	169.63
Rolling	6	1sd	Sell	0.871	0.325	42	1.000	95.89	132.36	168.62
Rolling	6	2sd	Sell	0.880	0.316	34	1.000	97.14	137.37	177.42
EWMA	3	15%	Sell	0.827	0.376	57	1.000	48.49	81.27	113.13
EWMA	3	15%	Buy	0.940	0.239	100	1.000	5.83	7.98	10.13
Resampled	3	0.5sd	Sell	0.809	0.391	42	1.000	77.73	109.36	140.87
Resampled	3	1sd	Sell	0.816	0.385	41	1.000	77.81	109.13	140.35
Resampled	3	2sd	Sell	0.804	0.394	36	0.999	75.01	108.86	142.55
Resampled	6	0.5sd	Sell	0.876	0.324	45	1.000	100.05	134.68	169.29
Resampled	6	1sd	Sell	0.878	0.323	43	1.000	101.80	137.25	172.59
Resampled	6	2sd	Sell	0.882	0.312	34	1.000	98.07	138.56	178.80
Time-weighted	3	0.5sd	Sell	0.807	0.392	42	1.000	76.12	108.00	139.68
Time-weighted	3	1sd	Sell	0.822	0.382	41	1.000	78.85	109.95	140.98
Time-weighted	3	2sd	Sell	0.800	0.400	36	0.998	73.36	107.25	141.10
Time-weighted	6	0.5sd	Sell	0.878	0.324	45	1.000	98.06	132.64	167.08
Time-weighted	6	1sd	Sell	0.870	0.331	42	1.000	98.36	134.63	170.81
Time-weighted	6	2sd	Sell	0.874	0.328	34	1.000	94.88	135.54	176.01
GARCH	3	5%	Sell	0.805	0.394	76	1.000	47.71	74.17	100.03
GARCH	3	10%	Sell	0.812	0.387	71	1.000	47.16	74.22	100.65
GARCH	3	15%	Sell	0.810	0.390	66	1.000	45.15	73.55	101.54

Table 13: Successful permutations of Rule 2

It is evident that, once again, all variations appear to offer reasonably high levels of trading success on sell signals. There is one successful permutation on a buy signal (3-month EWMA). There is very little difference between the variation in terms of proportion success or variation in proportion success. It is worthwhile noting, however, that low P&L's were offered by the EWMA

approach (at low variation and high success) whereas the highest P&L's were offered by the resampled and time-weighted resampled approaches. A complete set of results for all permutations of Rule 2, successful or not, are indicated in Appendix C.

### 6.3 Rule 3: Past historical volatility vs. present historical volatility

In Rule 3 we base the decision on whether to trade or not by comparing past historical volatility and present historical volatility. Results are presented in both summarized graphical form (Figure 14) as well as in tabular form (Table 14).

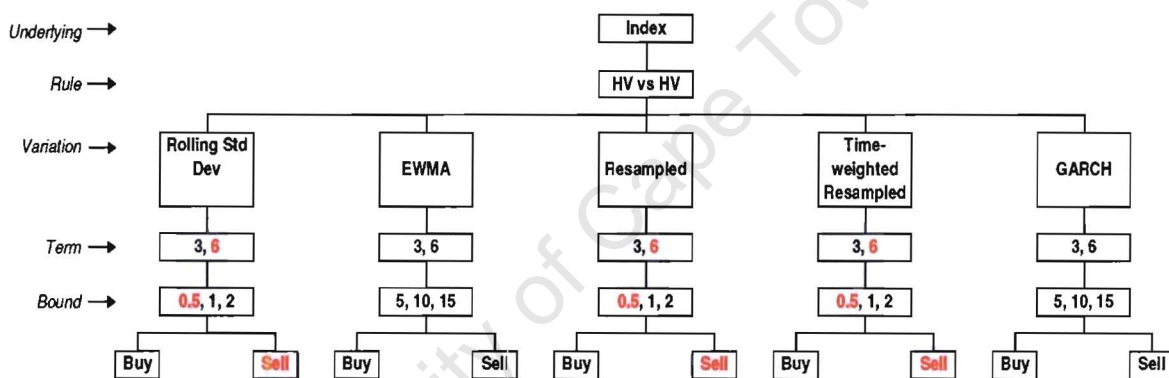


Figure 14: Permutations of Rule 3, with successes indicated in red

Table 14 shows the permutations that yield successful volatility trades.

Variation	Term	Bound	Buy / Sell	Prop Success Ave	Prop Success Stdev	Sampling Effort	Power	Lower 95% bound of Lower 95% PnL distribution	Mean of Average PnL distribution	Upper 95% bound of Upper 95% PnL distribution
Rolling	6	0.5sd	Sell	0.751	0.429	26	0.910	7.13	41.80	76.15
Resampled	6	0.5sd	Sell	0.742	0.436	26	0.882	5.12	40.14	75.00
Time-weighted	6	0.5sd	Sell	0.751	0.430	26	0.909	5.83	40.71	75.17

Table 14: Successful permutations of Rule 3

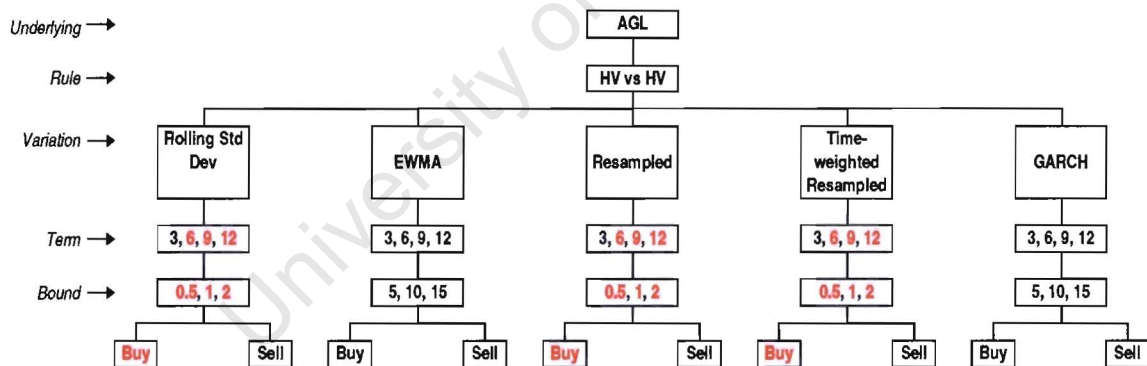
The success of Rule 3 is limited to three permutations only. Volatility updating mechanisms (EWMA and GARCH) do not add value to the successes of this rule. It is important to realize that even though this rule is not of practical importance as index options are priced and traded on

implied and not historical volatility, one still witnesses success on simple variations on a 6-month term. We include rule 3 as an introduction to the methodology adopted for the single stock case (Rule 4). A complete set of results for all permutations of Rule 3, successful or not, are indicated in Appendix C.

## 6.4 Rule 4: Past historical volatility vs. present historical volatility

We base the decision on whether to trade or not by comparing past historical volatility and present historical volatility in Rule 4 on several single stocks: Anglo American, Richemont, Rand Merchant Bank Holdings, South African Breweries, Standard Bank and Sasol. These are presented in turn below. A summary of the findings over all single stock trades using Rule 4 is presented in Table 21.

**Anglo American Corporation (AGL)** - Results are presented in both summarized graphical form (Figure 15) as well as in tabular form (Table 15).



**Figure 15: Permutations of Rule 4 on AGL, with successes indicated in red**

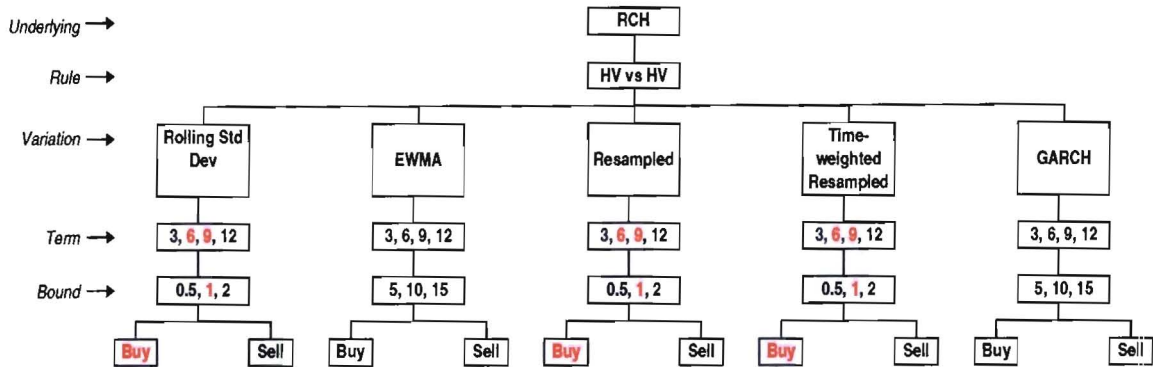
Table 15 shows the permutations that yield successful volatility trades on AGL.

Variation	Term	Bound	Buy / Sell	Prop Success Ave	Prop Success Stdev	Sampling Effort	Power	Lower 95% bound of Lower 95% PnL distribution	Mean of Average PnL distribution	Upper 95% bound of Upper 95% PnL distribution
Rolling	6	0.5sd	Buy	0.870	0.329	22	1.000	195.04	336.10	475.59
Rolling	6	1sd	Buy	0.899	0.277	20	1.000	219.17	363.46	505.33
Rolling	6	2sd	Buy	0.950	0.167	17	1.000	262.87	403.81	541.62
Rolling	9	0.5sd	Buy	0.954	0.176	22	1.000	292.40	430.67	567.60
Rolling	9	1sd	Buy	0.973	0.107	20	1.000	307.77	446.18	582.92
Rolling	9	2sd	Buy	1.000	0.000	17	1.000	360.03	494.94	627.84
Rolling	12	0.5sd	Buy	0.906	0.266	22	1.000	273.10	434.80	596.55
Rolling	12	1sd	Buy	0.936	0.202	20	1.000	280.00	446.58	611.93
Rolling	12	2sd	Buy	0.989	0.043	17	1.000	330.31	502.62	672.92
Resampled	6	0.5sd	Buy	0.862	0.326	22	1.000	187.61	331.32	473.14
Resampled	6	1sd	Buy	0.902	0.273	20	1.000	220.14	362.18	501.29
Resampled	6	2sd	Buy	0.949	0.169	17	1.000	255.19	395.96	533.97
Resampled	9	0.5sd	Buy	0.949	0.182	22	1.000	294.76	432.20	568.55
Resampled	9	1sd	Buy	0.981	0.079	20	1.000	313.18	447.18	580.46
Resampled	9	2sd	Buy	1.000	0.000	17	1.000	356.09	490.69	623.90
Resampled	12	0.5sd	Buy	0.913	0.248	22	1.000	271.04	432.80	594.73
Resampled	12	1sd	Buy	0.934	0.212	20	1.000	282.73	450.31	616.16
Resampled	12	2sd	Buy	0.984	0.065	17	1.000	315.66	488.83	659.84
Time-weighted	6	0.5sd	Buy	0.876	0.316	22	1.000	200.38	340.45	479.00
Time-weighted	6	1sd	Buy	0.902	0.269	20	1.000	222.85	365.31	505.15
Time-weighted	6	2sd	Buy	0.956	0.153	17	1.000	265.61	403.63	539.19
Time-weighted	9	0.5sd	Buy	0.944	0.193	22	1.000	275.17	415.96	555.82
Time-weighted	9	1sd	Buy	0.978	0.095	20	1.000	313.86	448.36	581.59
Time-weighted	9	2sd	Buy	1.000	0.000	17	1.000	362.56	497.81	630.78
Time-weighted	12	0.5sd	Buy	0.910	0.262	22	1.000	269.30	430.94	592.10
Time-weighted	12	1sd	Buy	0.930	0.213	20	1.000	283.92	449.31	613.45
Time-weighted	12	2sd	Buy	0.991	0.039	17	1.000	330.51	499.50	667.15

**Table 15: Successful permutations of Rule 4 applied to AGL**

Rule 4 yields successful buy trades for 6-month, 9-month and 12-month dated option contracts on AGL. The proportions of average success are particularly high and the variation about this average success quite low despite fairly low sampling effort. Also meaningful are the high absolute level of P&L's. Clearly, buy signals on AGL based on the abovementioned successful rules are underutilized as clear underpricing of volatility exists when present historical volatility is less than past historical volatility. Also worth noting is that volatility updating techniques do not yield successful trades. A complete set of results for all permutations of Rule 4 on AGL, successful or not, are indicated in Appendix C.

**Richemont (RCH)** - Results are presented in both summarized graphical form (Figure 16) as well as in tabular form (Table 16).



**Figure 16: Permutations of Rule 4 on RCH, with successes indicated in red**

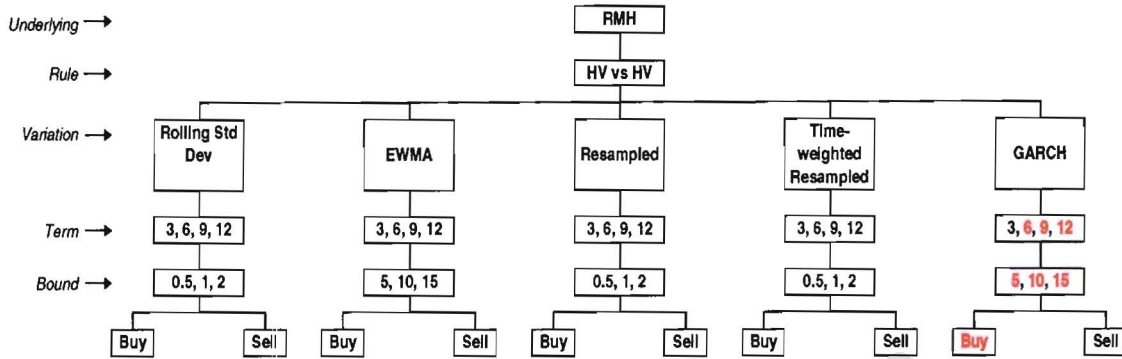
Table 16 shows the permutations that yield successful volatility trades on RCH.

Variation	Term	Bound	Buy / Sell	Prop Success Ave	Prop Success Stdev	Sampling Effort	Power	Lower 95% bound of Lower 95% PnL distribution	Mean of Average PnL distribution	Upper 95% bound of Upper 95% PnL distribution
Rolling	6	1sd	Buy	0.762	0.421	16	0.800	1.34	18.17	34.76
Rolling	9	1sd	Buy	0.772	0.415	16	0.835	2.96	19.69	36.45
Resampled	6	1sd	Buy	0.771	0.409	16	0.843	2.25	19.36	36.35
Resampled	9	1sd	Buy	0.783	0.401	16	0.880	3.09	20.20	37.35
Time-weighted	6	1sd	Buy	0.770	0.410	16	0.839	2.92	19.59	36.01
Time-weighted	9	1sd	Buy	0.773	0.410	16	0.845	2.67	19.25	35.88

**Table 16: Successful permutations of Rule 4 applied to RCH**

Rule 4 yields successful buy trades for 6-month and 9-month option contracts on RCH. It is worth noting that volatility updating techniques do not yield successful trades. A complete set of results for all permutations of Rule 4 on RCH, successful or not, are indicated in Appendix C.

**Rand Merchant Bank Holdings (RMH)** - Results are presented in both summarized graphical form (Figure 17) as well as in tabular form (Table 17).



**Figure 17: Permutations of Rule 4 on RMH, with successes indicated in red**

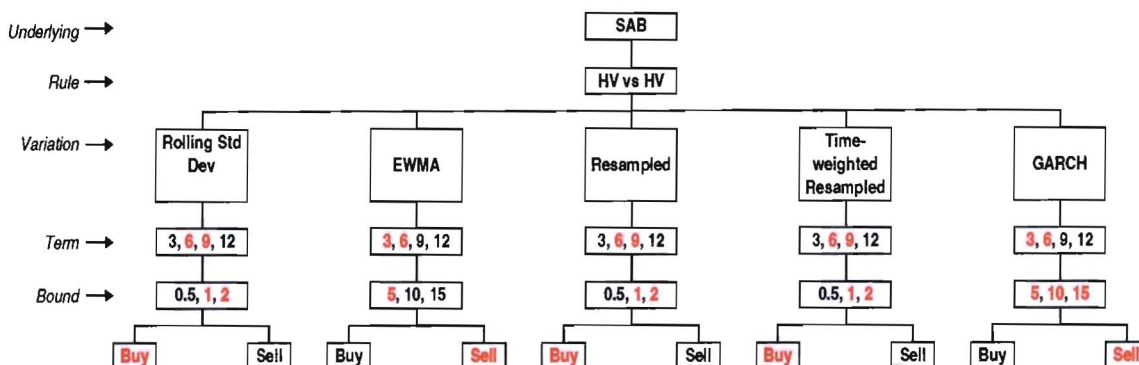
Table 17 shows the permutations that yield successful volatility trades on RMH.

Variation	Term	Bound	Buy / Sell	Prop Success Ave	Prop Success Stdev	Sampling Effort	Power	Lower 95% bound of 95% PnL distribution	Mean of Average PnL distribution	Upper 95% bound of 95% PnL distribution
Garch	6	0.05	Buy	0.758	0.426	41	0.987	4.80	19.11	33.57
Garch	6	0.1	Buy	0.775	0.414	35	0.989	4.43	19.02	34.02
Garch	6	0.15	Buy	0.782	0.410	29	0.980	3.63	19.12	35.08
Garch	9	0.05	Buy	0.766	0.423	40	0.990	9.67	30.54	51.75
Garch	9	0.1	Buy	0.757	0.428	35	0.972	7.33	29.42	52.06
Garch	9	0.15	Buy	0.759	0.422	29	0.951	5.18	30.20	56.04
Garch	12	5%	Buy	0.767	0.420	38	0.988	13.23	39.74	66.63
Garch	12	10%	Buy	0.765	0.421	34	0.978	11.70	40.14	68.99

**Table 17: Successful permutations of Rule 4 applied to RMH**

Rule 4 yields successful buy trades for 6 to 12-month option contracts on RMH. Only the GARCH technique yields successful trades. A complete set of results for all permutations of Rule 4 on RMH, successful or not, are indicated in Appendix C.

**South African Breweries (SAB)** - Results are presented in both summarized graphical form (Figure 18) as well as in tabular form (Table 18).



**Figure 18: Permutations of Rule 4 on SAB, with successes indicated in red**

Table 18 shows the permutations that yield successful volatility trades on SAB.

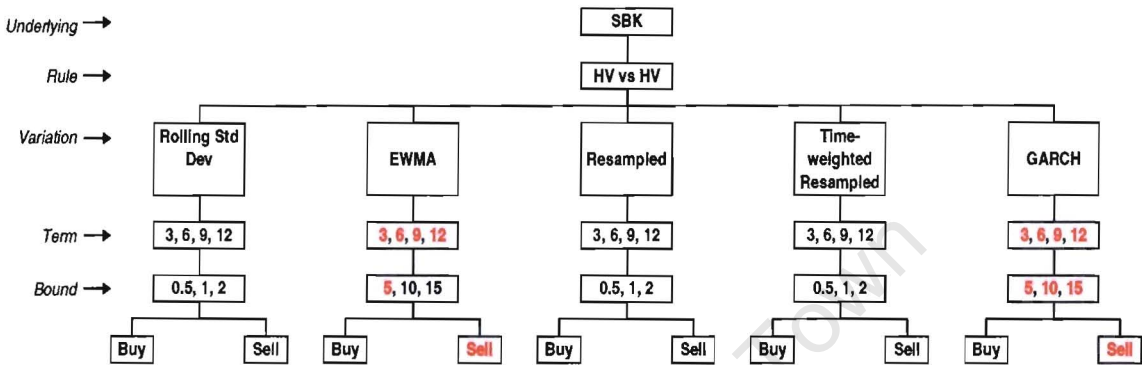
Variation	Term	Bound	Buy / Sell	Prop Success Ave	Prop Success Sdev	Sampling Effort	Power	Lower 95% bound of Lower 95% PnL distribution	Mean of Average PnL distribution	Upper 95% bound of Upper 95% PnL distribution
Rolling	6	1sd	Buy	0.814	0.384	21	0.982	47.77	133.11	219.50
Rolling	6	2sd	Buy	0.889	0.291	17	1.000	70.39	158.96	249.54
Rolling	9	1sd	Buy	0.814	0.385	21	0.982	44.98	145.90	249.60
Rolling	9	2sd	Buy	0.871	0.306	17	1.000	63.00	173.77	287.84
EWMA	3	0.05	Sell	0.873	0.325	30	1.000	34.76	69.07	102.08
EWMA	6	0.05	Sell	0.822	0.376	29	0.999	9.03	69.30	127.04
Resampled	6	1sd	Buy	0.808	0.389	21	0.976	47.95	131.64	216.62
Resampled	6	2sd	Buy	0.901	0.272	17	1.000	72.45	157.73	244.91
Resampled	9	1sd	Buy	0.787	0.404	21	0.946	37.11	136.06	238.70
Resampled	9	2sd	Buy	0.862	0.331	17	0.998	54.51	162.49	275.48
Time-weighted	6	1sd	Buy	0.807	0.389	21	0.975	40.25	125.79	212.59
Time-weighted	6	2sd	Buy	0.905	0.268	17	1.000	76.85	165.53	255.79
Time-weighted	9	1sd	Buy	0.816	0.380	21	0.985	50.49	152.34	257.46
Time-weighted	9	2sd	Buy	0.882	0.300	17	1.000	62.78	171.82	284.68
GARCH	3	0.05	Sell	0.763	0.425	47	0.995	14.00	45.48	76.18
GARCH	3	0.1	Sell	0.771	0.420	40	0.992	16.73	50.57	83.71
GARCH	3	0.15	Sell	0.807	0.390	32	0.998	30.02	64.79	98.77
GARCH	6	0.05	Sell	0.755	0.428	46	0.992	-4.16	45.11	92.46
GARCH	6	0.1	Sell	0.793	0.404	39	0.998	11.00	58.90	104.84
GARCH	6	0.15	Sell	0.837	0.366	32	1.000	33.15	77.90	121.02

**Table 18: Successful permutations of Rule 4 applied to SAB**

Selling SAB option contracts was successful only when indicated by one of the volatility updating techniques (EWMA, GARCH). Buy signals were only successful if they were indicated by one of

the non-volatility updating techniques. A complete set of results for all permutations of Rule 4 on SAB, successful or not, are indicated in Appendix C.

**Standard Bank (SBK)** - Results are presented in both summarized graphical form (Figure 19) as well as in tabular form (Table 19).



**Figure 19: Permutations of Rule 4 on SBK, with successes indicated in red**

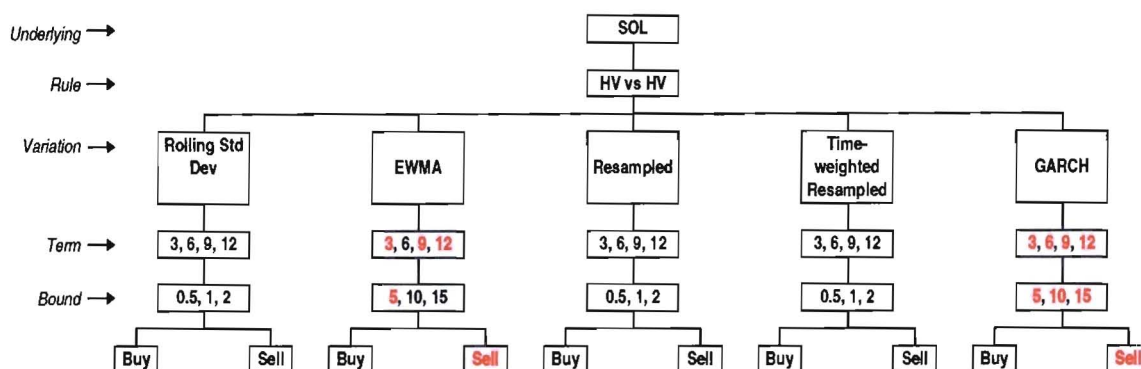
Table 19 shows the permutations that yield successful volatility trades on SBK.

Variation	Term	Bound	Buy / Sell	Prop Success Ave	Prop Success Stdev	Sampling Effort	Power	Lower 95% bound of Lower 95% PnL distribution	Mean of Average PnL distribution	Upper 95% bound of Upper 95% PnL distribution
EWMA	3	0.05	Sell	0.815	0.380	22	0.988	8.05	31.36	54.30
EWMA	6	0.05	Sell	0.854	0.335	22	1.000	16.55	45.89	74.57
EWMA	9	0.05	Sell	0.933	0.219	22	1.000	27.98	60.32	91.84
EWMA	12	0.05	Sell	0.900	0.284	22	1.000	21.28	61.10	99.38
GARCH	3	0.15	Sell	0.765	0.419	26	0.943	15.15	35.27	55.27
GARCH	6	0.1	Sell	0.751	0.432	30	0.938	15.19	39.74	63.84
GARCH	6	15%	Sell	0.818	0.377	24	0.993	24.05	50.23	75.91
GARCH	9	5%	Sell	0.796	0.398	35	0.997	4.41	36.76	67.86
GARCH	9	0.1	Sell	0.824	0.376	30	0.999	18.02	48.07	77.31
GARCH	9	0.15	Sell	0.870	0.324	24	1.000	30.12	60.47	89.90
GARCH	12	0.05	Sell	0.793	0.402	34	0.995	6.01	43.01	78.42
GARCH	12	0.1	Sell	0.828	0.375	29	0.999	20.73	57.17	92.09
GARCH	12	0.15	Sell	0.883	0.305	23	1.000	37.01	72.60	106.70

**Table 19: Successful permutations of Rule 4 applied to SBK**

Successful volatility trades were indicated by the EWMA and GARCH techniques only. These variations worked across all terms for sell signals only. A complete set of results for all permutations of Rule 4 on SBK, successful or not, are indicated in Appendix C.

**Sasol (SOL)** - Results are presented in both summarized graphical form (Figure 20) as well as in tabular form (Table 20).



**Figure 20: Permutations of Rule 4 on SOL, with successes indicated in red**

Table 20 shows the permutations that yield successful volatility trades on SOL.

Variation	Term	Bound	Buy / Sell	Prop Success Ave	Prop Success Stdev	Sampling Effort	Power	Lower 95% bound of Lower 95% PnL distribution	Mean of Average PnL distribution	Upper 95% bound of Upper 95% PnL distribution
EWMA	3	0.05	Sell	0.757	0.429	24	0.901	13.74	58.96	104.42
EWMA	9	0.05	Sell	0.818	0.381	23	0.991	54.70	115.70	178.06
EWMA	12	0.05	Sell	0.755	0.425	22	0.878	27.21	110.42	192.80
GARCH	3	0.1	Sell	0.777	0.416	36	0.991	6.19	43.24	79.90
GARCH	3	0.15	Sell	0.772	0.416	30	0.974	2.28	46.57	89.96
GARCH	6	0.15	Sell	0.764	0.421	29	0.959	16.11	69.49	122.57
GARCH	9	10%	Sell	0.784	0.408	34	0.992	13.89	69.52	124.64
GARCH	9	15%	Sell	0.802	0.396	28	0.992	19.73	80.69	141.07
GARCH	12	0.05	Sell	0.766	0.419	39	0.990	-2.86	59.52	120.84
GARCH	12	0.1	Sell	0.810	0.387	32	0.998	30.62	90.72	150.86
GARCH	12	0.15	Sell	0.834	0.369	27	0.999	39.74	104.47	168.92

**Table 20: Successful permutations of Rule 4 applied to SOL**

The only variations that work for SOL result from the use of the EWMA and GARCH. These are only successful for sell signals. A complete set of results for all permutations of Rule 4 on SOL, successful or not, are indicated in Appendix C.

We summarise the results in Table 21. We have also grouped the single stocks into broad economic sectors.

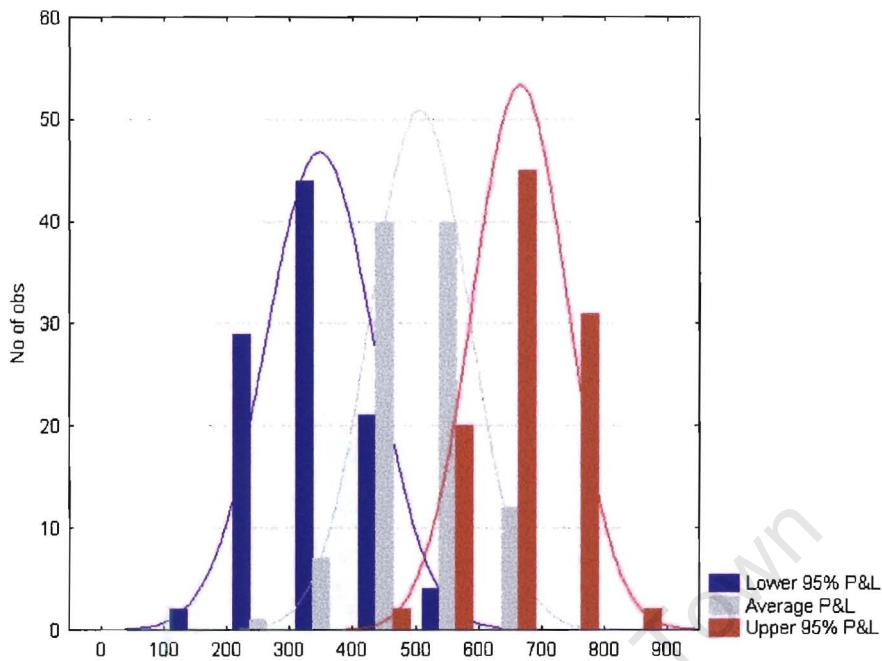
Underlying	Buy/Sell	Measure	Term
<b>Index</b>			
IV vs IV	Sell	All measures	3,6
HV vs IV	Buy	EWMA	3
	Sell	All measures	3,6
HV vs HV	Sell	All measures (excl. EWMA, GARCH)	6
<b>Resources</b>			
AGL	Buy	All measures (excl. EWMA, GARCH)	6,9,12
SOL	Sell	EWMA, GARCH	3,6,9,12
<b>Industrial</b>			
RCH	Buy	All measures (excl. EWMA, GARCH)	6,9
SAB	Buy	All measures (excl. EWMA, GARCH)	3,6,9
	Sell	EWMA, GARCH	3,6
<b>Financials</b>			
SBK	Sell	EWMA, GARCH	3,6,9,12
RMH	Buy	GARCH	6,9,12

*Table 21: Summary of successful permutations*

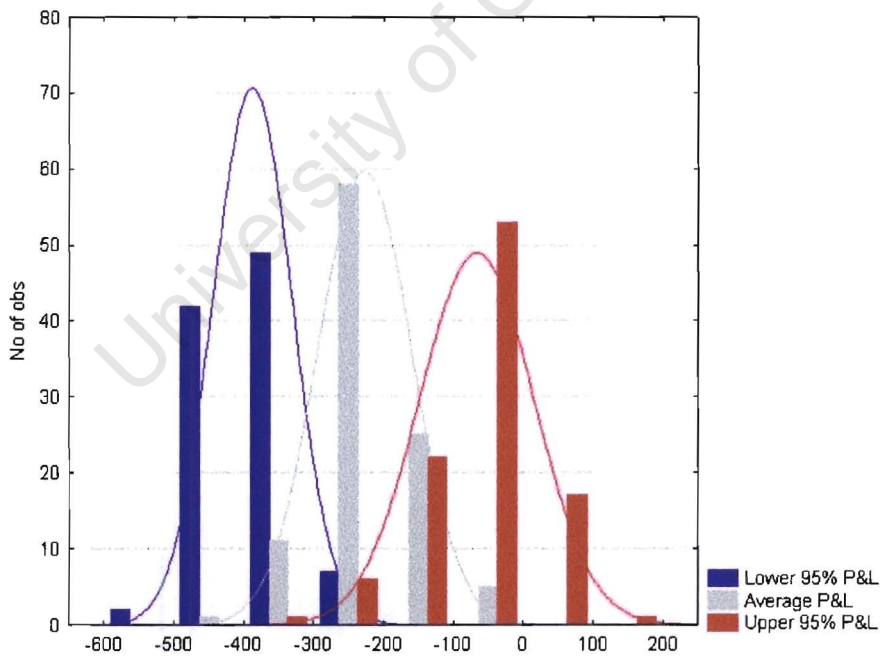
## 6.5 Some graphical representations of rule variations

The analysis described in section 5, and the results reported in section 6 make use of measures of both return and risk. In this subsection, we attempt to display a graphical aspect of trades and variations thereof that are successful or unsuccessful.

First, we examine the simple visual differences between distributions of P&L's on successful (Figure 21) and unsuccessful (Figure 22) trades. The rule, the underlying and the volatility updating regime that we use to demonstrate the relationship is not of central relevance here.



**Figure 21: Successful trade: P&L distributions for Rule 4 – AGL (using rolling standard deviation rule, term of 12-months, bound of 2 std dev, buying option contracts)**



**Figure 22: Unsuccessful trade: P&L distributions for Rule 4 – SAB (using resampling, term of 9-months, bound of 2 std dev, selling option contracts)**

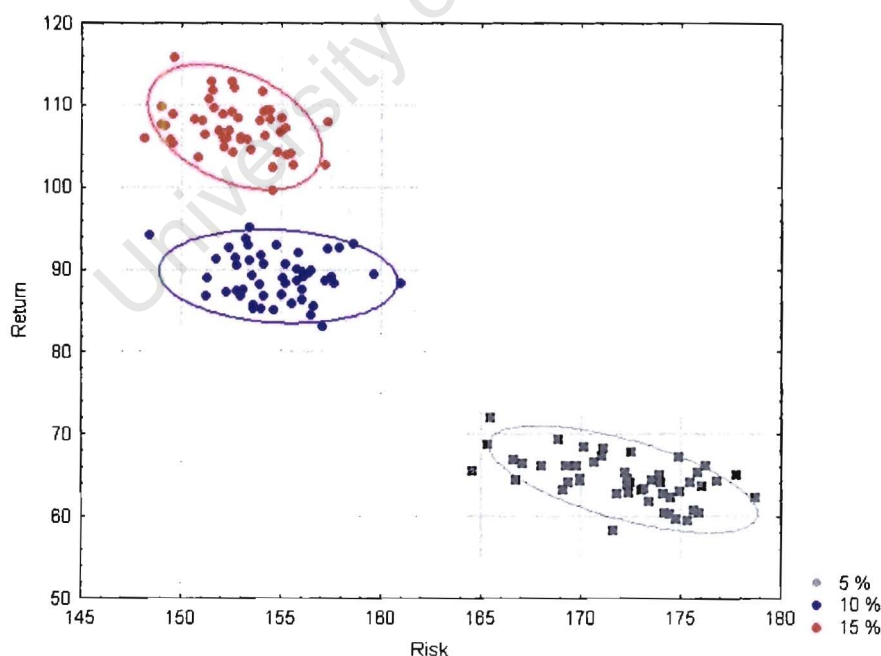
Clearly, the risks on a 12-month long volatility trade on Anglo American using the naïve (rolling standard deviation) rule is promising. In opposition are the risks we see on a 9-month short volatility trade on South African Breweries where the expected losses are considerable.

- It is important to interpret the proportion success in combination with an idea about the distribution of the expected P&L's for a comprehensive understanding to be realized.

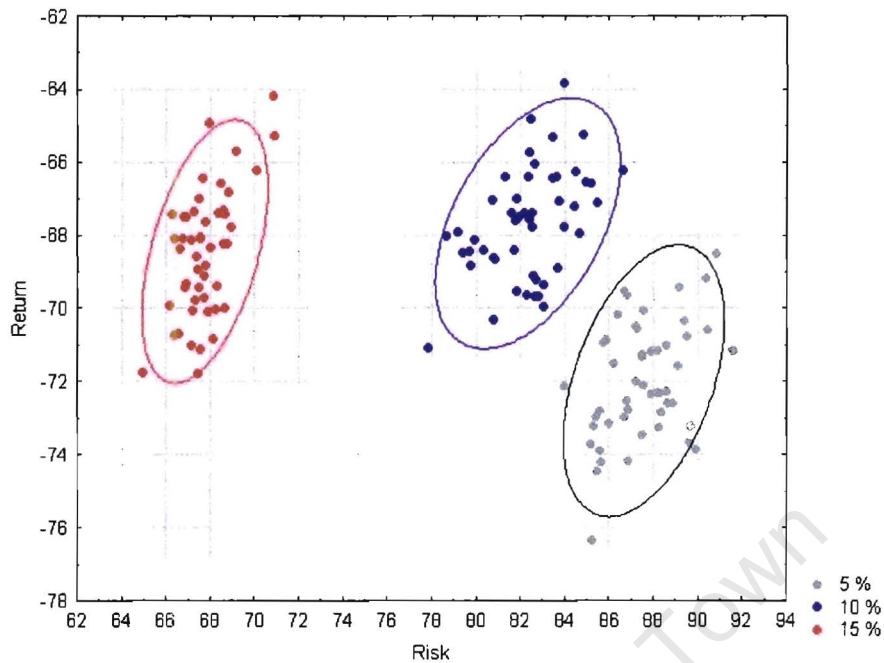
Next, we consider whether expectations of market efficiency and risk/return are borne out in the analysis.

- We are paying attention to whether the theoretical relationship between risk and return is maintained in unsuccessful trades as we would expect given an efficient market and Capital Asset Pricing Model (CAPM) (Sharpe 1964, Fama 1965) expectations. Conversely, we are also interested in assessing whether the converse holds in winning trades.

We first examine the relationship between varying percentage bounds (5 %, 10 % and 15%) on both successful (Figure 23) and unsuccessful (Figure 24) trades, here using GARCH and EWMA. The rule, the underlying and the volatility updating regime that we use to demonstrate the relationship is not of central relevance here.



**Figure 23: Successful trade: Varying percentage bounds in Rule 4 – SOL (using GARCH, term of 12-months, selling option contracts)**



**Figure 24: Unsuccessful trade: Varying percentage bounds in Rule 1 (EWMA, term of 3-months, buying option contracts)**

It is fascinating to note that our theoretical predictions hold true in the above two examples. A market where return is priced in proportion to increasing risk is a fundamental prediction of the CAPM model. Hence, we expect to see this relationship maintained in markets where volatility arbitrage (as defined here) is not feasible. This pattern is clear in Figure 24. In contrast, the inefficiency of the volatility markets is made noticeable by the negative relationship between risk and return seen in Figure 23.

- It is encouraging to see that simple risk/return predictions relate to market efficiency in an intuitive way.

## 7. Discussion and conclusion

South African institutional equity derivatives activity has focused and will continue to focus on identifiable and tradable aspects of the market that are thought to be knowable or predictable in some sense. First, the likely direction of an underlying stock or index is assessed - typically gleaned from earnings forecasts and fundamental accounting models and less often from technical or quantitative screening. Second, the volatility of the same underlying is appraised - characteristically from a fairly simple rule. This research has focused squarely on testing whether this commonly advocated volatility rule is robust, and whether better alternatives may exist.

We commenced with an introduction to efficient markets, defining arbitrage opportunities and the common rules utilized for the identification of such opportunities. We showed that the commonly used rule, while elegant in its simplicity, did bear several statistical imperfections. Several alternatives were proposed to better accommodate autocorrelation and volatility time-weighting and volatility updating. We performed an out-of-sample backtest on several underlying single stocks and the index, utilizing both implied volatility (where available) and historical volatility. Results were presented in terms of both proportion success and the distribution of the P&L's.

The results indicate five essential points.

- **First, the market is relatively efficient in terms of how volatility is priced.**

In general, most permutations of the trading rules that have been examined do not prove to be successful (see Appendix C). This suggests that the market is fairly efficient in terms of pricing volatility and that there is little room for profit-making based on volatility wagers.

- **Second, there are still good opportunities for profitable volatility arbitrage.**

A handful of successful permutations are statistically successful and prove to be lucrative with a concomitant controlled amount of risk. Based on discrepancies between past volatility and present volatility, we have shown (out of sample) that opportunities to profit from expensive or cheap volatility have existed in the past. The existence of opportunities allowing for profit taking by the use of a volatility trading rule violates the weak form of the efficient market hypothesis.

Interestingly, there are different volatility properties for different underlyings. The different statistical rules used in this study are resilient to different assumptions. Hence, different rules work better on different underlyings. This effect appears to be related to sectoral-type volatility qualities. We discuss this further below.

- **Third, the typical rule that is exploited to make judgments as to whether volatility is under- or over-priced is, in a real sense, naïve.**

Better alternatives to the naïve rolling volatility window approach certainly exist. Underutilized volatility updating techniques for measuring historical volatility, such as EWMA and GARCH, have achieved success in identifying and taking advantage of volatility arbitrage opportunities. The implications of this finding are motivating.

A conventional rolling historical window approach assumes two things: mean reversion of volatility and that the history of an underlying is bounded in terms of its variation, and any aberrant point measure of volatility that falls beyond these bounds represents an opportunity. Clearly, setting bounds of 1 standard deviation encapsulates only about 68 % of the variation of a stock. Approximately 32 % of the time, we expect the stock's volatility to be trading outside these ranges. Hence, even on an *a priori* basis, we can dismiss the effectiveness of this rule on grounds that the market has to be more efficient than we give it credit for using this rule. We see that as we increase the standard deviation bounds to 2 (95 %) the number of opportunities diminished, but the probability of identifying aberrant points of volatility increases (see Table 15; AGL 6-month buy).

The simple rule of resampling historical points from the rolling window renders our concern about autocorrelation of the rolling observations obsolete, and we see some success in this regard but less so than one would expect from simple statistical reflection.

Most appealing however, is the fact that our volatility time-weighting or updating schemes seem to offer opportunities where current opportunities would simply not previously be identified. The implication here is that present volatility is likely to be more related to future volatility rather than to past volatility. We have devised and tested statistical rules to update past volatility towards the present as a mechanism for trading. Clearly, these rules do identify volatility arbitrage opportunities on underlyings that no other rule can. The lesson: burying your head in history will often result in you not taking cognisance of what is presently going on, and what is presently going on is likely to be more closely related to what is just about to happen.

- **Fourth, index trades seem to sometimes offer opportunities for short volatility (sales) trades.**

Here, the results suggest that we cannot correctly identify when the Top-40 index demonstrates abnormally low volatility. We can, however, sometimes identify when volatility on the index is

overpriced. The volatility time-weighting and updating schemes seem to offer better general success on Index trades than do other statistical rules. In being able to identify such mispricings in volatility correctly, we have uncovered a level of inefficiency in the Index market.

It is sensible noting that the distribution of implied volatility is skewed, with a thick long tail to the right (high volatility). Hence, low relative volatility and opportunities to identify aberrantly-low volatility are in a true sense, constrained and truncated.

- **Fifth, and finally, no single rule seems to mirror opportunity for single stock volatility play.**

We have not found much evidence that points to a single rule which could or should be used for stocks in any general economic sector, apart from confirmation that volatility anomalies on two large-cap financial stocks seem to be well identified by GARCH and EWMA rules. It appears as though some form of time-weighted volatility (EWMA) or volatility updating (GARCH) seems particularly applicable to Index trades and financial stock trades. Different statistical rules seem to work better on Industrial and resource stocks. Each stock seems to exhibit a fairly unique volatility structure and thus different rules work for different underlying stocks.

Further research in the area of volatility trading could be aimed towards establishing a better understanding of the underlying characteristics of the index or stocks' volatility processes. An understanding of the volatility dynamics would explain why some statistical rules are better than others.

Examining volatility opportunities for single stocks should become easier as these options become more liquid and implied volatility estimates become more reliable.

It may also be interesting to examine the volatilities of stocks in particular economic sectors. Performing a principal component analysis might help to identify if there are sector level characteristics for volatility dynamics of single stocks. Intuitively one would expect to find relationships between the volatilities of stocks in the same economic sectors, the interesting question is to what extent sector volatility impacts on a single stocks volatility process.

We hope that this research will provide academics and practitioners with something of an initial stimulus for the application of rules to understanding and trading option volatility in the South African market.

## Bibliography

- Bates, D. 1986. Testing option pricing models. *Statistical Methods in Finance*, Edited by C. Rao and G. Maddala. North Holland, 567 - 611.
- Bollerslev, T. 1987. A conditionally heteroskedastic time series model for speculative prices and rates of returns. *Review of Economics and Statistics*, 69:542 - 547.
- Canina, L. and S. Figlewski. 1993. The informational content of implied volatility. *The Review of Financial Studies*, 6:3.
- Christensen, B. J. and N.R. Prabhala. The relation between implied and realized volatility. *Journal of Financial Economics* 50:125 - 150.
- Derman, E. and I. Kani. 1994. Riding on a smile. *RISK*, February:32 - 39.
- Dueker, M.J. 1997. Markov switching in GARCH processes and mean-reverting stock market volatility. *Journal of Business Economic Statistics*, 15:26 - 34.
- Elton, E.J. and M.J. Gruber. 1995. *Modern portfolio theory and investment analysis*. Wiley.
- Efron, B. 1979. Bootstrap methods: another look at the jackknife. *Ann. Statistics*, 7:1 - 26.
- Efron, B. and R. Tibshirani. 1986. Bootstrap methods for standard errors, confidence intervals, and other measures of statistical accuracy. *Statistical Science*, 1:45 - 96.
- Engle, R.F. 1982. Autoregressive conditional heteroskedasticity with estimates of the variance of U.K. inflation. *Econometrica*, 50:987 - 1008.
- Fama, E.F. 1965. Efficient capital markets: A review of theory and empirical work. *Journal of Finance*, 25:383 - 417.
- French, K.R. 1980. Stock returns and the week-end effect. *Journal of Financial Economics* 8:55 - 69.
- Fouque, J.P., G. Papanicolaou and K.R. Sgarbi. 1998. Asymptotics of a two-scale volatility model. In *Equations aux derives partielles et applications*, pages 517 - 525. Gauthier-Villars.
- Harvey, C.R. and R. Whaley. 1992. Market volatility prediction and the efficiency of the S&P 100 index option market. *Journal of Financial Economics*, 31:43 - 73.
- Hull, J.C. 2000. *Options, futures and other derivatives*, 4<sup>th</sup> edn. Prentice-Hall, London.
- Lamoureux, C.G. and W.D. Lastrapes. 1993. Forecasting stock return variances: Toward an understanding of stochastic implied volatilities. *Review of Financial Studies*, 6:293 - 326.
- Merton, R. C. 1973. The theory of rational option pricing. *Bell Journal of Economics and Management Science*, 4:141 - 183.
- Merville, L.J. and D.R. Piotea. 1989. Stock-price volatility, mean-reverting diffusion and noise. *Journal of Financial Economics*, 24:193 - 214.
- Polakow, D.A. September, 2001. Statistical properties of volatility and implied volatility on the South African futures exchange (SAFEX). (Unpublished report).

Rosner, B. 1990. Fundamentals of biostatistics. PWS-KENT, Boston.

Schwert, G.W. 1990. Stock volatility and the crash of '87. Review of Financial Studies, 3:77-102.

Sharpe, W.F. 1964. A simplified model for portfolio analysis. Management Science, 9:277-293.

Stein, J. 1989. Overreactions in the Options Market. Journal of Finance, XLIV:1011-1023.

Stoll, H.R. 1969. The relationship between put and call option prices. The Journal of Finance, XXIV.

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## Appendices

### A. Proof that put and call premiums are equal when struck at-the-forward

The put–call parity model was first developed by Stoll (1969), and then extended and modified by Merton (1973). Put–call parity does not rely for its validity on the restrictive Black–Scholes assumptions.

Put-call parity states that a long position in a put combined with a long position in the underlying is equivalent to a long call position plus a certain amount of cash

$$p + Se^{-qT} = c + Xe^{-rT} \quad (21)$$

Suppose we strike the options at-the-forward  $F = Se^{(r-q)T}$  so that the strike price is given by  $X = Se^{(r-q)T}$ . Substituting this strike price into (21) yields

$$p + Se^{-qT} = c + \left( Se^{(r-q)T} \right) e^{-rT} \quad (22)$$

which implies that  $p = c$ . So when striking options at-the-forward, it does not matter whether we price put options or call options, their price will be the same. Since put-call parity is independent of any particular pricing model, this relationship will always hold (otherwise there will be opportunities for arbitrage).

## **B. SAFEX mark-to-market implied volatility methodology**

The following procedure is used by SAFEX in arriving at the mark-to-market implied volatilities for index options.

1. The mark-to-market for futures and options is taken at any time between 17h00 and 17h05.
2. Options up to 100 points from the futures mark-to-market level will be considered at-the-money for the mark-to-market.
3. The options mark-to-market will be the *last* volatility displayed at the time of the mark-to-market unless it is better bid or better offered (in at least 100 contracts).
4. Where there are different *lasts* in acceptable strike prices, the average will be used.

Index options are considered to be at-the-money if their strikes are within 100 points of the futures mark-to-market level as determined by SAFEX. Thus the mark-to-market implied volatilities are for index options which are struck at the futures mark-to-market level and not at the index's spot level.

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## C. Results

### C1. Naïve

Underlying	Rule	Term	Std Dev Bound (σ)	Prop Success Ave(Buy)	Prop Success Stdev(Buy)	Sampling Effort (Buy)	p-value	Power	Lower 95% bound of Lower 95% PnL distribution (Buy)	Mean of Average PnL distribution (Buy)	Upper 95% bound of Upper 95% PnL distribution (Buy)	Prop Success Ave(Sell)	Prop Success Stdev(Sell)	Sampling Effort (Sells)	p-value	Power	Lower 95% bound of Lower 95% PnL distribution (Sell)	Mean of Average PnL distribution (Sell)	Upper 95% bound of Upper 95% PnL distribution (Sell)
Index	1	3	0.5	0.194	0.391	31	1.00	-	-100.66	-65.17	-27.93	0.772	0.417	32	0.00	0.98	38.64	84.05	128.74
Index	1	3	1.0	0.201	0.398	28	1.00	-	-101.38	-63.28	-23.37	0.783	0.409	28	0.00	0.98	39.77	86.67	132.36
Index	1	3	2.0	0.194	0.388	20	1.00	-	-100.13	-55.63	-8.39	0.757	0.426	24	0.00	0.90	27.87	79.14	128.70
Index	1	6	0.5	0.295	0.456	33	0.99	-	-104.98	-53.20	-0.15	0.667	0.472	29	0.03	0.60	26.29	92.42	157.59
Index	1	6	1.0	0.300	0.456	30	0.99	-	-101.39	-46.88	9.35	0.671	0.467	28	0.03	0.61	24.65	92.09	158.39
Index	1	6	2.0	0.333	0.470	24	0.95	-	-96.11	-32.39	33.24	0.666	0.471	20	0.07	0.47	-4.41	73.45	150.02
Index	2	3	0.5	0.246	0.433	13	0.97	-	-103.42	-58.85	-13.50	0.812	0.387	42	0.00	1.00	76.58	107.97	139.11
Index	2	3	1.0	0.253	0.425	11	0.96	-	-103.63	-55.37	-5.63	0.814	0.386	41	0.00	1.00	77.76	109.78	141.60
Index	2	3	2.0	0.238	0.395	8	0.95	-	-111.32	-56.08	1.82	0.808	0.393	36	0.00	1.00	76.50	111.16	145.73
Index	2	6	0.5	0.386	0.477	8	0.74	-	-150.93	-61.05	29.04	0.866	0.337	45	0.00	1.00	99.73	134.68	169.63
Index	2	6	1.0	0.351	0.479	7	0.78	-	-147.23	-56.25	34.78	0.871	0.325	42	0.00	1.00	95.89	132.36	168.62
Index	2	6	2.0	0.276	0.412	5	0.85	-	-168.18	-61.66	48.94	0.880	0.316	34	0.00	1.00	97.14	137.37	177.42
Index	3	3	0.5	0.468	0.499	23	0.62	-	-35.27	16.52	68.30	0.582	0.492	26	0.20	0.21	-16.82	19.95	58.76
Index	3	3	1.0	0.505	0.500	20	0.48	0.05	-22.85	31.41	86.17	0.473	0.501	18	0.59	-	-37.66	13.79	65.88
Index	3	3	2.0	0.464	0.498	14	0.60	-	-33.73	25.88	86.34	0.437	0.493	14	0.68	-	-52.37	12.82	79.24
Index	3	6	0.5	0.625	0.480	20	0.13	0.31	-18.92	50.94	118.67	0.751	0.429	26	0.00	0.91	7.13	41.80	76.15
Index	3	6	1.0	0.739	0.432	17	0.02	0.74	7.19	78.21	146.69	0.693	0.460	18	0.05	0.55	-13.55	33.91	81.32
Index	3	6	2.0	0.847	0.330	12	0.00	0.98	40.56	111.19	177.84	0.684	0.466	14	0.08	0.43	-29.82	28.23	86.42
AGL	4	3	0.5	0.608	0.489	25	0.14	0.30	4.71	121.36	239.63	0.436	0.496	27	0.75	-	-55.23	37.33	131.44
AGL	4	3	1.0	0.619	0.489	21	0.14	0.30	1.07	119.16	239.84	0.388	0.486	23	0.86	-	-60.99	40.09	144.81
AGL	4	3	2.0	0.689	0.464	17	0.06	0.51	35.68	162.73	294.16	0.327	0.470	19	0.94	-	-75.97	39.57	159.57
AGL	4	6	0.5	0.870	0.329	22	0.00	1.00	195.04	336.10	475.59	0.528	0.501	27	0.39	0.09	-23.31	63.79	152.21
AGL	4	6	1.0	0.899	0.277	20	0.00	1.00	219.17	363.46	505.33	0.470	0.500	23	0.61	-	-45.83	48.19	144.54
AGL	4	6	2.0	0.950	0.167	17	0.00	1.00	262.87	403.81	541.62	0.422	0.494	19	0.75	-	-73.18	29.47	136.29
AGL	4	9	0.5	0.954	0.176	22	0.00	1.00	292.40	430.67	567.60	0.686	0.464	26	0.03	0.66	-7.13	61.96	129.54
AGL	4	9	1.0	0.973	0.107	20	0.00	1.00	307.77	446.18	582.92	0.623	0.484	22	0.12	0.33	-33.94	42.46	118.07
AGL	4	9	2.0	1.000	0.000	17	0.00	1.00	360.03	494.94	627.84	0.583	0.490	19	0.24	0.18	-54.16	30.05	113.61
AGL	4	12	0.5	0.903	0.266	22	0.00	1.00	273.10	434.80	596.55	0.550	0.498	23	0.32	0.12	-23.97	78.18	179.74
AGL	4	12	1.0	0.936	0.202	20	0.00	1.00	280.00	446.58	611.93	0.507	0.501	20	0.48	0.06	-35.49	76.86	189.32
AGL	4	12	2.0	0.989	0.043	17	0.00	1.00	330.31	502.62	672.92	0.479	0.504	18	0.57	-	-54.82	64.00	183.42
RCH	4	3	0.5	0.503	0.501	21	0.49	0.05	-10.72	3.10	16.96	0.357	0.481	33	0.95	-	-16.63	-5.26	6.04
RCH	4	3	1.0	0.603	0.487	16	0.21	0.21	-5.59	9.16	23.93	0.270	0.439	27	0.99	-	-19.49	-7.77	4.00
RCH	4	3	2.0	0.756	0.414	11	0.03	0.66	0.61	16.34	31.97	0.328	0.462	20	0.94	-	-13.83	0.10	14.24
RCH	4	6	0.5	0.680	0.465	21	0.05	0.55	-4.53	12.58	29.64	0.401	0.489	32	0.87	-	-19.34	-7.61	3.86
RCH	4	6	1.0	0.762	0.421	16	0.01	0.80	1.34	18.17	34.76	0.340	0.471	27	0.95	-	-20.97	-9.07	2.58
RCH	4	6	2.0	0.945	0.162	11	0.00	1.00	12.94	25.57	38.05	0.395	0.490	20	0.83	-	-14.82	-2.56	9.67
RCH	4	9	0.5	0.695	0.458	21	0.03	0.62	0.00	16.70	33.53	0.324	0.467	31	0.98	-	-24.92	-13.49	-2.12
RCH	4	9	1.0	0.772	0.415	16	0.01	0.83	2.96	19.69	36.45	0.283	0.448	25	0.99	-	-24.99	-14.76	-4.65
RCH	4	9	2.0	0.913	0.216	11	0.00	1.00	8.51	24.79	41.10	0.326	0.468	20	0.94	-	-19.79	-9.84	-0.04
RCH	4	12	0.5	0.648	0.478	21	0.09	0.41	0.65	18.69	36.79	0.316	0.464	28	0.98	-	-28.18	-13.85	0.45
RCH	4	12	1.0	0.704	0.455	16	0.05	0.56	2.82	23.10	43.16	0.240	0.423	23	1.00	-	-29.20	-16.97	-4.88
RCH	4	12	2.0	0.764	0.415	11	0.03	0.68	6.00	27.17	48.16	0.256	0.429	20	0.99	-	-24.59	-13.33	-2.21

Notes: p-value < 0.05 indicated in blue italics; Power > 0.80 indicated in green italics; Negative P&L indicated in red.

Table C1: Results of naïve variation of rules.

Underlying	Rule	Term	Std Dev Bound ( $\phi$ )	Prop Success Ave(Buy)	Prop Success Stdev(Buy)	Sampling Effort (Buy)	P-value	Power	Lower 95% bound of Lower 95% PnL distribution (Buy)	Mean of Average PnL distribution (Buy)	Upper 95% bound of Upper 95% PnL distribution (Buy)	Prop Success Ave(Sell)	Prop Success Stdev(Sell)	Sampling Effort (Sells)	P-value	Power	Lower 95% bound of Lower 95% PnL distribution (Sell)	Mean of Average PnL distribution (Sell)	Upper 95% bound of Upper 95% PnL distribution (Sell)
RMH	4	3	0.5	0.404	0.493	20	0.80	-	-19.17	-7.46	4.17	0.394	0.488	37	0.90	-	-28.86	-12.82	3.04
RMH	4	3	1.0	0.418	0.490	14	0.73	-	-18.98	-5.37	8.32	0.367	0.482	35	0.94	-	-31.52	-15.73	-0.19
RMH	4	3	2.0	0.365	0.478	8	0.77	-	-20.93	-5.88	9.20	0.332	0.470	29	0.97	-	-38.37	-21.37	-4.80
RMH	4	6	0.5	0.354	0.476	19	0.90	-	-18.50	-5.92	6.44	0.311	0.462	35	0.99	-	-50.99	-26.75	-2.27
RMH	4	6	1.0	0.266	0.442	14	0.97	-	-19.93	-6.13	7.81	0.284	0.451	33	1.00	-	-58.67	-34.24	-9.57
RMH	4	6	2.0	0.003	0.007	8	1.00	-	-27.13	-15.70	-5.11	0.271	0.445	27	0.99	-	-65.80	-41.04	-16.16
RMH	4	9	0.5	0.348	0.476	19	0.91	-	-23.57	-8.44	6.57	0.322	0.467	32	0.98	-	-76.37	-41.81	-7.10
RMH	4	9	1.0	0.301	0.454	14	0.94	-	-27.52	-10.48	6.73	0.313	0.461	30	0.98	-	-84.17	-48.63	-12.67
RMH	4	9	2.0	0.091	0.216	8	1.00	-	-38.33	-23.59	-8.87	0.299	0.456	25	0.98	-	-98.36	-62.29	-25.76
RMH	4	12	0.5	0.341	0.473	19	0.92	-	-31.62	-13.53	4.22	0.339	0.474	29	0.96	-	-93.25	-49.20	-4.79
RMH	4	12	1.0	0.293	0.450	14	0.95	-	-32.05	-12.71	6.77	0.295	0.455	27	0.99	-	-109.85	-64.64	-18.68
RMH	4	12	2.0	0.066	0.160	8	1.00	-	-45.85	-26.52	-8.04	0.261	0.436	22	0.99	-	-121.45	-73.94	-25.38
SAB	4	3	0.5	0.543	0.501	28	0.33	0.12	-9.59	49.09	109.69	0.508	0.499	26	0.47	0.06	-96.16	-25.73	43.77
SAB	4	3	1.0	0.558	0.498	23	0.29	0.14	-8.89	58.85	130.71	0.486	0.501	21	0.55	-	-124.85	-44.78	34.03
SAB	4	3	2.0	0.578	0.496	19	0.25	0.17	-3.19	76.30	158.18	0.340	0.469	16	0.90	-	-176.42	-85.53	5.05
SAB	4	6	0.5	0.700	0.457	26	0.02	0.72	16.78	93.56	171.29	0.574	0.493	26	0.23	0.19	-151.48	-44.43	59.44
SAB	4	6	1.0	0.814	0.384	21	0.00	0.98	47.77	133.11	219.50	0.489	0.503	21	0.54	-	-202.58	-76.32	48.32
SAB	4	6	2.0	0.889	0.291	17	0.00	1.00	70.39	158.96	249.54	0.326	0.468	16	0.92	-	-303.29	-158.24	-14.40
SAB	4	9	0.5	0.716	0.451	26	0.01	0.79	16.73	103.24	192.88	0.552	0.496	24	0.31	0.13	-196.72	-61.85	70.64
SAB	4	9	1.0	0.814	0.385	21	0.00	0.98	44.98	145.90	249.60	0.427	0.495	19	0.74	-	-286.99	-128.21	30.32
SAB	4	9	2.0	0.871	0.306	17	0.00	1.00	63.00	173.77	287.84	0.258	0.428	15	0.98	-	-390.51	-218.31	-43.07
SAB	4	12	0.5	0.537	0.499	26	0.36	0.10	4.52	122.31	246.82	0.487	0.501	21	0.55	-	-257.16	-94.61	66.32
SAB	4	12	1.0	0.600	0.488	21	0.18	0.24	34.37	176.11	324.32	0.392	0.489	18	0.82	-	-326.34	-144.87	36.85
SAB	4	12	2.0	0.633	0.483	17	0.14	0.31	37.15	197.43	364.89	0.273	0.436	14	0.96	-	-422.13	-230.43	-34.50
SBK	4	3	0.5	0.435	0.495	31	0.77	-	-21.70	0.99	23.94	0.371	0.483	32	0.93	-	-40.76	-13.63	13.00
SBK	4	3	1.0	0.427	0.497	24	0.76	-	-26.37	0.59	27.89	0.331	0.471	27	0.96	-	-55.06	-24.97	4.50
SBK	4	3	2.0	0.603	0.490	11	0.25	0.17	-15.98	11.00	37.26	0.235	0.419	22	1.00	-	-71.50	-37.79	-4.41
SBK	4	6	0.5	0.478	0.499	31	0.60	-	-20.44	8.00	36.61	0.411	0.493	30	0.84	-	-83.93	-35.02	12.85
SBK	4	6	1.0	0.503	0.502	24	0.49	0.05	-15.99	16.24	48.80	0.420	0.496	26	0.79	-	-97.98	-46.83	3.61
SBK	4	6	2.0	0.618	0.479	11	0.22	0.20	-16.79	18.61	53.83	0.416	0.495	21	0.78	-	-125.92	-66.69	-8.57
SBK	4	9	0.5	0.451	0.497	31	0.71	-	-23.05	12.29	47.97	0.423	0.493	27	0.79	-	-121.95	-55.93	9.27
SBK	4	9	1.0	0.483	0.498	24	0.57	-	-21.02	21.11	63.60	0.433	0.497	23	0.74	-	-141.17	-69.28	2.25
SBK	4	9	2.0	0.469	0.503	11	0.58	-	-36.10	14.44	65.01	0.371	0.481	18	0.86	-	-192.67	-109.65	-26.62
SBK	4	12	0.5	0.472	0.499	31	0.62	-	-23.73	25.74	76.28	0.444	0.496	24	0.71	-	-144.95	-84.27	15.98
SBK	4	12	1.0	0.502	0.501	24	0.49	0.05	-21.73	36.32	95.60	0.448	0.497	20	0.68	-	-179.97	-99.76	0.47
SBK	4	12	2.0	0.464	0.505	11	0.59	-	-52.16	9.15	71.00	0.338	0.474	16	0.90	-	-226.01	-128.03	-28.98
SOL	4	3	0.5	0.369	0.483	27	0.91	-	-56.45	5.94	71.14	0.464	0.501	29	0.65	-	-77.62	-26.93	23.40
SOL	4	3	1.0	0.399	0.488	22	0.83	-	-51.11	14.74	84.52	0.417	0.492	27	0.81	-	-88.45	-34.70	19.32
SOL	4	3	2.0	0.428	0.489	12	0.69	-	-68.38	5.63	86.03	0.370	0.483	22	0.89	-	-109.22	-48.11	13.79
SOL	4	6	0.5	0.558	0.501	25	0.28	0.14	-9.87	88.39	189.01	0.461	0.499	29	0.66	-	-121.54	-50.02	20.64
SOL	4	6	1.0	0.595	0.488	22	0.18	0.23	1.32	103.25	207.79	0.387	0.487	26	0.88	-	-150.87	-77.29	-4.44
SOL	4	6	2.0	0.572	0.495	12	0.31	0.13	-26.49	113.53	257.82	0.304	0.457	22	0.97	-	-175.46	-95.50	-15.68
SOL	4	9	0.5	0.478	0.500	25	0.59	-	-7.52	114.47	239.48	0.427	0.494	26	0.77	-	-172.58	-76.27	18.83
SOL	4	9	1.0	0.493	0.503	22	0.53	-	5.68	135.48	268.62	0.355	0.478	23	0.92	-	-208.31	-110.93	-15.00
SOL	4	9	2.0	0.415	0.496	12	0.72	-	-38.93	153.74	351.45	0.300	0.458	21	0.97	-	-233.71	-134.74	-37.07
SOL	4	12	0.5	0.484	0.501	25	0.56	-	-11.84	145.28	306.43	0.536	0.499	23	0.37	0.10	-207.43	-79.23	46.93
SOL	4	12	1.0	0.499	0.504	22	0.50	-	-0.91	168.72	342.53	0.476	0.499	20	0.59	-	-258.53	-124.73	6.60
SOL	4	12	2.0	0.414	0.495	12	0.72	-	-59.81	190.39	445.72	0.411	0.488	18	0.78	-	-303.51	-166.02	-30.17

Notes: p-value < 0.05 indicated in blue italics; Power > 0.80 indicated in green italics; Negative P&L indicated in red.

Table C1 (continued): Results of naïve variation of rules.

## C2. Resampling

Underlying	Rule	Term	Std Dev Bound ( $\bar{\sigma}$ )	Prop Success Ave(Buy)	Prop Success Stdev(Buy)	Sampling Effort (Buy)	p-value	Power	Lower 95% bound of Lower 95% PnL distribution (Buy)	Mean of Average PnL distribution (Buy)	Upper 95% bound of Upper 95% PnL distribution (Buy)	Prop Success Ave(Sell)	Prop Success Stdev(Sell)	Sampling Effort (Sells)	p-value	Power	Lower 95% bound of Lower 95% PnL distribution (Sell)	Mean of Average PnL distribution (Sell)	Upper 95% bound of Upper 95% PnL distribution (Sell)
Index	1	3	0.5	0.206	0.402	31	1.00	-	-97.66	-62.23	-25.23	0.780	0.411	32	0.00	0.99	38.63	84.77	129.77
Index	1	3	1.0	0.199	0.398	28	1.00	-	-99.02	-62.70	-24.46	0.774	0.414	27	0.00	0.96	31.92	82.20	131.22
Index	1	3	2.0	0.207	0.393	20	1.00	-	-102.50	-57.04	-8.65	0.772	0.412	23	0.00	0.94	32.31	84.29	134.94
Index	1	6	0.5	0.302	0.460	33	0.99	-	-104.93	-51.23	3.59	0.670	0.469	29	0.03	0.62	26.79	94.20	160.62
Index	1	6	1.0	0.295	0.452	30	0.99	-	-104.49	-49.17	8.09	0.683	0.463	27	0.03	0.66	25.95	94.74	162.55
Index	1	6	2.0	0.311	0.460	24	0.97	-	-95.26	-32.55	32.01	0.646	0.478	20	0.09	0.39	-14.10	65.74	144.25
Index	2	3	0.5	0.255	0.431	13	0.97	-	-102.69	-58.26	-12.95	0.809	0.391	42	0.00	1.00	77.73	109.36	140.87
Index	2	3	1.0	0.253	0.418	11	0.96	-	-104.60	-55.24	-4.82	0.816	0.385	41	0.00	1.00	77.81	109.13	140.35
Index	2	3	2.0	0.255	0.407	8	0.93	-	-109.04	-51.05	9.05	0.804	0.394	36	0.00	1.00	75.01	108.86	142.55
Index	2	6	0.5	0.364	0.483	8	0.77	-	-152.00	-62.43	26.25	0.876	0.324	45	0.00	1.00	100.05	134.68	169.29
Index	2	6	1.0	0.334	0.470	7	0.81	-	-149.03	-60.43	29.16	0.878	0.323	43	0.00	1.00	101.80	137.25	172.59
Index	2	6	2.0	0.282	0.406	5	0.85	-	-161.92	-57.67	51.95	0.882	0.312	34	0.00	1.00	98.07	138.56	178.80
Index	3	3	0.5	0.468	0.498	23	0.62	-	-33.83	17.93	69.84	0.576	0.495	26	0.22	0.19	-16.35	20.83	58.14
Index	3	3	1.0	0.505	0.501	20	0.48	0.05	-25.97	29.09	84.21	0.468	0.499	17	0.60	-	-38.98	13.60	66.59
Index	3	3	2.0	0.484	0.503	14	0.55	-	-31.30	28.62	89.90	0.449	0.498	14	0.65	-	-52.63	13.32	80.37
Index	3	6	0.5	0.661	0.470	20	0.07	0.46	-8.01	58.94	124.38	0.742	0.436	26	0.00	0.88	5.12	40.14	75.00
Index	3	6	1.0	0.745	0.430	17	0.02	0.76	8.37	79.70	148.38	0.687	0.464	18	0.05	0.53	-13.79	33.11	80.12
Index	3	6	2.0	0.852	0.322	12	0.00	0.98	40.18	110.96	177.40	0.685	0.464	14	0.08	0.44	-28.46	29.91	88.47
AGL	4	3	0.5	0.609	0.484	25	0.14	0.30	4.72	120.75	238.75	0.440	0.493	27	0.73	-	-51.68	43.30	139.76
AGL	4	3	1.0	0.650	0.477	21	0.08	0.42	20.00	140.36	263.67	0.393	0.488	23	0.85	-	-55.22	48.12	154.46
AGL	4	3	2.0	0.678	0.465	17	0.07	0.47	27.67	149.30	275.70	0.315	0.461	19	0.95	-	-79.17	37.98	159.41
AGL	4	6	0.5	0.862	0.326	22	0.00	1.00	187.61	331.32	473.14	0.523	0.500	28	0.41	0.08	-20.20	64.77	151.67
AGL	4	6	1.0	0.902	0.273	20	0.00	1.00	220.14	362.18	501.29	0.475	0.500	23	0.59	-	-41.84	52.03	149.10
AGL	4	6	2.0	0.949	0.169	17	0.00	1.00	255.19	395.96	533.97	0.414	0.492	19	0.77	-	-76.50	26.76	134.54
AGL	4	9	0.5	0.949	0.182	22	0.00	1.00	294.76	432.20	568.55	0.675	0.467	26	0.03	0.61	-13.37	57.58	127.00
AGL	4	9	1.0	0.981	0.079	20	0.00	1.00	313.18	447.18	580.46	0.631	0.481	22	0.11	0.36	-29.88	45.68	120.38
AGL	4	9	2.0	1.000	0.000	17	0.00	1.00	356.09	490.69	623.90	0.582	0.494	19	0.24	0.18	-52.38	31.64	114.45
AGL	4	12	0.5	0.913	0.248	22	0.00	1.00	271.04	432.80	594.73	0.558	0.497	23	0.29	0.14	-21.82	80.15	181.29
AGL	4	12	1.0	0.934	0.212	20	0.00	1.00	282.73	450.31	616.16	0.505	0.497	19	0.48	0.05	-38.90	73.04	186.14
AGL	4	12	2.0	0.984	0.065	17	0.00	1.00	315.66	488.83	659.84	0.452	0.499	18	0.66	-	-66.48	52.40	172.47
RCH	4	3	0.5	0.492	0.501	21	0.53	-	-11.33	2.29	15.96	0.377	0.485	34	0.93	-	-16.06	-5.07	5.79
RCH	4	3	1.0	0.594	0.490	16	0.23	0.19	-5.75	8.89	23.60	0.293	0.454	27	0.99	-	-18.10	-6.09	6.04
RCH	4	3	2.0	0.768	0.417	11	0.03	0.69	1.24	17.17	33.07	0.316	0.463	20	0.95	-	-14.68	-0.73	13.24
RCH	4	6	0.5	0.642	0.479	21	0.09	0.39	-7.13	10.37	27.95	0.407	0.490	32	0.86	-	-20.11	-7.96	4.00
RCH	4	6	1.0	0.771	0.409	16	0.01	0.84	2.25	19.36	36.35	0.338	0.473	27	0.96	-	-20.63	-8.94	2.57
RCH	4	6	2.0	0.936	0.182	11	0.00	1.00	13.07	25.72	38.16	0.366	0.482	20	0.89	-	-15.53	-3.41	8.65
RCH	4	9	0.5	0.691	0.463	21	0.04	0.60	-0.92	15.12	31.40	0.328	0.467	31	0.98	-	-24.42	-13.37	-2.31
RCH	4	9	1.0	0.783	0.401	16	0.01	0.88	3.09	20.20	37.35	0.281	0.447	25	0.99	-	-25.79	-15.39	-5.08
RCH	4	9	2.0	0.914	0.229	11	0.00	1.00	7.76	23.80	40.00	0.322	0.470	20	0.95	-	-18.93	-9.41	0.01
RCH	4	12	0.5	0.645	0.479	21	0.09	0.40	0.32	18.13	36.00	0.315	0.465	28	0.98	-	-27.18	-13.22	0.78
RCH	4	12	1.0	0.706	0.453	16	0.04	0.57	4.77	24.87	44.81	0.253	0.430	23	0.99	-	-29.85	-16.91	-4.17
RCH	4	12	2.0	0.754	0.417	11	0.04	0.65	6.18	27.55	48.89	0.260	0.437	20	0.99	-	-25.39	-13.58	-2.07

Notes: p-value < 0.05 indicated in blue italics; Power > 0.80 indicated in green italics; Negative P&L indicated in red.

Table C2: Results of resampled variation of rules.

Underlying	Rule	Term	Std Dev Bound ( T )	Prop Success Ave(Buy)	Prop Success Stddev(Buy)	Sampling Effort (Buy)	p-value	Power	Lower 95% bound of Lower 95% PnL distribution (Buy)	Mean of Average PnL distribution (Buy)	Upper 95% bound of Upper 95% PnL distribution (Buy)	Prop Success Ave(Sell)	Prop Success Stddev(Sell)	Sampling Effort (Sells)	p-value	Power	Lower 95% bound of Lower 95% PnL distribution (Sell)	Mean of Average PnL distribution (Sell)	Upper 95% bound of Upper 95% PnL distribution (Sell)
RMH	4	3	0.5	0.420	0.495	20	0.76	-	-17.57	-8.05	5.41	0.401	0.490	37	0.89	-	-27.97	-12.08	3.74
RMH	4	3	1.0	0.404	0.493	14	0.76	-	-20.69	-8.81	7.11	0.367	0.483	35	0.94	-	-31.76	-18.13	-0.63
RMH	4	3	2.0	0.329	0.450	8	0.84	-	-22.54	-7.62	7.34	0.350	0.478	29	0.95	-	-38.30	-21.15	-4.36
RMH	4	6	0.5	0.359	0.476	19	0.89	-	-18.37	-5.81	6.59	0.315	0.465	35	0.99	-	-51.61	-27.13	-2.51
RMH	4	6	1.0	0.248	0.420	14	0.98	-	-21.58	-7.78	6.19	0.286	0.451	33	0.99	-	-56.84	-32.83	-8.60
RMH	4	6	2.0	0.005	0.014	8	1.00	-	-26.87	-15.57	-5.19	0.274	0.444	27	0.99	-	-65.35	-40.62	-15.70
RMH	4	9	0.5	0.344	0.475	19	0.92	-	-23.41	-8.40	6.43	0.329	0.470	33	0.98	-	-74.89	-40.48	-5.72
RMH	4	9	1.0	0.292	0.457	14	0.94	-	-27.83	-10.69	6.69	0.310	0.462	30	0.98	-	-84.77	-49.86	-14.66
RMH	4	9	2.0	0.108	0.233	8	1.00	-	-37.39	-22.63	-7.79	0.308	0.455	25	0.98	-	-97.42	-61.29	-24.69
RMH	4	12	0.5	0.368	0.482	19	0.88	-	-28.96	-11.43	5.92	0.340	0.473	29	0.96	-	-95.21	-50.27	-4.82
RMH	4	12	1.0	0.279	0.444	14	0.96	-	-33.87	-14.21	5.53	0.320	0.465	27	0.97	-	-104.69	-58.49	-11.84
RMH	4	12	2.0	0.073	0.178	8	1.00	-	-46.51	-28.86	-7.91	0.254	0.434	22	0.99	-	-120.77	-74.21	-26.60
SAB	4	3	0.5	0.528	0.500	28	0.39	0.09	-13.59	45.17	105.96	0.543	0.496	26	0.33	0.11	-84.55	-16.16	50.98
SAB	4	3	1.0	0.558	0.499	22	0.30	0.14	-7.73	63.84	137.78	0.483	0.502	21	0.56	-	-119.33	-40.88	36.71
SAB	4	3	2.0	0.588	0.493	19	0.22	0.19	-3.87	75.05	156.38	0.354	0.474	16	0.88	-	-180.43	-86.09	7.82
SAB	4	6	0.5	0.708	0.455	26	0.01	0.75	20.36	96.23	172.94	0.576	0.494	26	0.22	0.20	-155.08	-46.28	59.74
SAB	4	6	1.0	0.808	0.389	21	0.00	0.98	47.95	131.64	216.62	0.486	0.501	21	0.55	-	-203.90	-77.23	47.44
SAB	4	6	2.0	0.901	0.272	17	0.00	1.00	72.45	157.73	244.91	0.329	0.467	16	0.92	-	-289.90	-146.11	-4.23
SAB	4	9	0.5	0.722	0.447	26	0.01	0.81	21.58	106.55	194.54	0.546	0.500	24	0.33	0.12	-200.12	-64.29	69.63
SAB	4	9	1.0	0.787	0.404	21	0.00	0.95	37.11	136.06	238.70	0.435	0.494	19	0.71	-	-281.46	-121.92	36.94
SAB	4	9	2.0	0.862	0.331	17	0.00	1.00	54.51	162.49	275.48	0.265	0.429	15	0.97	-	-391.00	-218.13	-40.96
SAB	4	12	0.5	0.545	0.500	26	0.32	0.12	6.35	126.54	252.79	0.478	0.501	21	0.58	-	-263.89	-101.51	59.05
SAB	4	12	1.0	0.602	0.488	21	0.17	0.25	32.56	170.95	315.27	0.373	0.482	18	0.86	-	-329.46	-150.95	28.14
SAB	4	12	2.0	0.641	0.475	17	0.12	0.34	43.23	208.25	379.87	0.273	0.441	14	0.96	-	-415.92	-224.57	-29.01
SBK	4	3	0.5	0.432	0.495	31	0.77	-	-21.00	1.29	23.86	0.376	0.484	32	0.92	-	-41.16	-13.38	14.08
SBK	4	3	1.0	0.445	0.496	24	0.70	-	-23.88	2.52	29.19	0.337	0.473	27	0.96	-	-53.00	-23.39	5.73
SBK	4	3	2.0	0.601	0.488	12	0.24	0.18	-14.01	10.63	35.06	0.240	0.425	22	1.00	-	-68.56	-35.02	-2.19
SBK	4	6	0.5	0.476	0.500	31	0.60	-	-19.19	9.13	37.72	0.413	0.491	30	0.83	-	-88.05	-38.33	10.45
SBK	4	6	1.0	0.512	0.501	24	0.46	0.06	-15.44	16.94	49.51	0.413	0.493	26	0.81	-	-98.07	-46.76	3.59
SBK	4	6	2.0	0.588	0.488	11	0.29	0.14	-17.90	15.50	49.13	0.402	0.489	21	0.82	-	-129.55	-69.98	-11.82
SBK	4	9	0.5	0.465	0.497	31	0.65	-	-19.71	15.21	50.46	0.444	0.496	27	0.72	-	-115.35	-50.16	13.98
SBK	4	9	1.0	0.493	0.499	24	0.53	-	-19.05	22.10	64.02	0.432	0.492	23	0.74	-	-144.57	-71.64	0.92
SBK	4	9	2.0	0.454	0.502	11	0.62	-	-38.42	12.31	63.54	0.372	0.481	18	0.86	-	-189.59	-107.52	-25.28
SBK	4	12	0.5	0.472	0.501	31	0.62	-	-22.58	26.13	78.02	0.443	0.498	24	0.71	-	-148.08	-67.05	13.59
SBK	4	12	1.0	0.483	0.499	23	0.56	-	-27.90	30.81	90.99	0.429	0.497	20	0.74	-	-181.83	-92.30	-2.77
SBK	4	12	2.0	0.468	0.500	12	0.58	-	-47.62	11.72	71.55	0.326	0.467	16	0.92	-	-229.21	-131.33	-32.45
SOL	4	3	0.5	0.389	0.486	27	0.88	-	-51.30	11.99	78.69	0.471	0.501	29	0.62	-	-75.64	-25.89	23.86
SOL	4	3	1.0	0.407	0.491	22	0.81	-	-50.16	17.97	89.66	0.414	0.493	27	0.81	-	-91.40	-37.56	16.35
SOL	4	3	2.0	0.443	0.496	12	0.65	-	-70.27	13.09	102.15	0.346	0.475	22	0.93	-	-112.63	-52.97	7.64
SOL	4	6	0.5	0.551	0.497	25	0.31	0.13	-15.10	83.50	184.21	0.453	0.498	29	0.69	-	-121.43	-50.40	20.09
SOL	4	6	1.0	0.614	0.485	21	0.15	0.28	5.30	113.37	224.07	0.404	0.490	26	0.84	-	-146.51	-72.49	0.53
SOL	4	6	2.0	0.565	0.499	12	0.33	0.12	-26.60	110.29	253.41	0.292	0.453	22	0.98	-	-173.16	-95.05	-17.26
SOL	4	9	0.5	0.465	0.502	26	0.64	-	-6.03	115.01	239.38	0.413	0.492	26	0.81	-	-174.65	-79.80	13.95
SOL	4	9	1.0	0.499	0.502	21	0.50	-	8.90	144.40	282.59	0.347	0.477	23	0.93	-	-214.22	-115.88	-18.76
SOL	4	9	2.0	0.385	0.478	13	0.80	-	-38.99	141.55	329.19	0.285	0.449	21	0.98	-	-234.51	-136.24	-38.70
SOL	4	12	0.5	0.482	0.500	26	0.57	-	-10.47	144.35	303.87	0.535	0.499	23	0.37	0.09	-206.46	-79.78	44.48
SOL	4	12	1.0	0.505	0.502	22	0.48	0.05	4.35	176.36	353.49	0.467	0.500	20	0.61	-	-254.74	-121.95	9.21
SOL	4	12	2.0	0.423	0.489	12	0.70	-	-52.20	194.86	449.07	0.405	0.492	17	0.78	-	-308.82	-167.60	-28.39

Notes: p-value < 0.05 indicated in blue italics; Power > 0.80 indicated in green italics; Negative P&L indicated in red.

Table C2 (continued): Results of resampled variation of rules.

### C3. Time-weighted resampling

Underlying	Rule	Term	Std Dev Bound ( )	Prop Success Ave(Buy)	Prop Success Stdev(Buy)	Sampling Effort (Buy)	p-value	Power	Lower 95% bound of Lower 95% PnL distribution (Buy)	Mean of Average PnL distribution (Buy)	Upper 95% bound of Upper 95% PnL distribution (Buy)	Prop Success Ave(Sell)	Prop Success Stdev(Sell)	Sampling Effort (Sell)	p-value	Power	Lower 95% bound of Lower 95% PnL distribution (Sell)	Mean of Average PnL distribution (Sell)	Upper 95% bound of Upper 95% PnL distribution (Sell)
Index	1	3	0.5	0.187	0.384	31	1.00	-	-103.10	-68.32	-31.88	0.787	0.409	32	0.00	0.99	42.00	87.05	131.13
Index	1	3	1.0	0.202	0.394	28	1.00	-	-97.95	-60.42	-20.95	0.779	0.412	27	0.00	0.97	38.59	87.26	134.61
Index	1	3	2.0	0.203	0.396	20	1.00	-	-99.26	-54.75	-7.55	0.755	0.427	23	0.00	0.89	26.51	79.46	130.56
Index	1	6	0.5	0.295	0.457	33	0.99	-	-104.78	-51.10	3.75	0.680	0.467	29	0.02	0.67	31.50	96.79	161.36
Index	1	6	1.0	0.303	0.461	30	0.99	-	-98.94	-42.80	14.49	0.691	0.463	27	0.02	0.69	31.29	98.39	164.58
Index	1	6	2.0	0.327	0.469	24	0.96	-	-92.68	-28.10	38.19	0.660	0.476	20	0.08	0.44	-7.46	71.21	149.86
Index	2	3	0.5	0.238	0.408	13	0.98	-	-102.10	-57.59	-12.33	0.807	0.392	42	0.00	1.00	76.12	108.00	139.68
Index	2	3	1.0	0.254	0.430	11	0.96	-	-104.49	-55.78	-5.73	0.822	0.382	41	0.00	1.00	78.85	109.95	140.98
Index	2	3	2.0	0.274	0.423	8	0.91	-	-106.71	-49.51	9.47	0.800	0.400	36	0.00	1.00	73.36	107.25	141.10
Index	2	6	0.5	0.395	0.484	8	0.72	-	-138.52	-53.13	32.08	0.878	0.324	45	0.00	1.00	98.06	132.64	167.08
Index	2	6	1.0	0.359	0.479	7	0.77	-	-144.88	-53.48	38.55	0.870	0.331	42	0.00	1.00	98.38	134.63	170.81
Index	2	6	2.0	0.306	0.409	5	0.83	-	-158.05	-53.37	56.02	0.874	0.328	34	0.00	1.00	94.88	135.54	176.01
Index	3	3	0.5	0.470	0.500	23	0.61	-	-32.01	18.05	68.47	0.593	0.491	26	0.17	0.25	-13.53	22.72	59.31
Index	3	3	1.0	0.491	0.502	20	0.53	-	-26.87	28.31	83.37	0.479	0.499	18	0.57	-	-36.96	15.51	68.46
Index	3	3	2.0	0.459	0.499	14	0.62	-	-35.25	24.90	86.10	0.457	0.499	14	0.62	-	-51.15	15.22	82.99
Index	3	6	0.5	0.636	0.479	20	0.11	0.35	-18.39	50.93	118.70	0.751	0.430	26	0.00	0.91	5.83	40.71	75.17
Index	3	6	1.0	0.744	0.426	17	0.02	0.76	9.59	81.90	151.22	0.693	0.462	18	0.05	0.55	-15.18	33.29	81.69
Index	3	6	2.0	0.829	0.352	12	0.00	0.94	33.10	106.30	175.73	0.682	0.457	14	0.08	0.44	-28.15	30.73	89.00
AGL	4	3	0.5	0.600	0.489	25	0.16	0.27	-1.82	115.39	234.08	0.453	0.499	27	0.68	-	-45.56	50.33	148.29
AGL	4	3	1.0	0.643	0.478	21	0.09	0.39	12.91	132.49	254.40	0.390	0.488	23	0.85	-	-57.76	44.05	149.12
AGL	4	3	2.0	0.686	0.460	17	0.06	0.51	23.46	140.42	262.20	0.335	0.471	19	0.93	-	-70.76	43.53	163.52
AGL	4	6	0.5	0.876	0.316	22	0.00	1.00	200.38	340.45	479.00	0.513	0.499	27	0.45	0.07	-24.36	61.66	149.07
AGL	4	6	1.0	0.902	0.269	20	0.00	1.00	222.85	365.31	505.15	0.485	0.500	23	0.56	-	-41.17	51.43	146.58
AGL	4	6	2.0	0.956	0.153	17	0.00	1.00	265.61	403.63	539.19	0.426	0.494	19	0.74	-	-70.23	28.66	132.29
AGL	4	9	0.5	0.944	0.193	22	0.00	1.00	275.17	415.96	555.82	0.693	0.460	26	0.02	0.89	-10.40	60.50	130.02
AGL	4	9	1.0	0.978	0.095	20	0.00	1.00	313.86	448.36	581.59	0.617	0.485	22	0.13	0.30	-37.55	41.17	119.33
AGL	4	9	2.0	1.000	0.000	17	0.00	1.00	362.56	497.81	630.78	0.569	0.495	19	0.28	0.15	-61.55	23.29	107.01
AGL	4	12	0.5	0.910	0.262	22	0.00	1.00	269.30	430.94	592.10	0.568	0.495	23	0.26	0.16	-15.91	81.94	179.33
AGL	4	12	1.0	0.930	0.213	20	0.00	1.00	283.92	449.31	613.45	0.483	0.501	20	0.56	-	-44.97	66.63	178.58
AGL	4	12	2.0	0.991	0.039	17	0.00	1.00	330.51	499.50	667.15	0.448	0.498	18	0.67	-	-63.16	52.60	170.37
RCH	4	3	0.5	0.500	0.501	21	0.50	-	-10.40	3.17	16.81	0.353	0.478	33	0.96	-	-16.35	-5.25	5.74
RCH	4	3	1.0	0.613	0.480	16	0.18	0.24	-5.07	10.15	25.44	0.284	0.450	27	0.99	-	-18.12	-8.11	5.95
RCH	4	3	2.0	0.780	0.389	11	0.02	0.77	0.92	16.58	32.21	0.333	0.468	20	0.94	-	-13.69	0.47	14.79
RCH	4	6	0.5	0.650	0.475	21	0.08	0.42	-5.83	11.63	29.09	0.402	0.489	32	0.87	-	-19.30	-7.58	3.82
RCH	4	6	1.0	0.770	0.410	16	0.01	0.84	2.92	19.59	36.01	0.336	0.472	27	0.96	-	-21.09	-9.29	2.35
RCH	4	6	2.0	0.934	0.180	11	0.00	1.00	13.34	26.08	38.46	0.401	0.490	20	0.81	-	-15.15	-2.76	9.45
RCH	4	9	0.5	0.712	0.450	21	0.02	0.70	0.24	16.17	32.28	0.343	0.473	31	0.96	-	-22.98	-11.75	-0.53
RCH	4	9	1.0	0.773	0.410	16	0.01	0.84	2.67	19.25	35.88	0.280	0.447	25	0.99	-	-25.13	-15.01	-4.95
RCH	4	9	2.0	0.919	0.206	11	0.00	1.00	7.00	22.82	38.86	0.328	0.467	20	0.94	-	-18.91	-9.24	0.32
RCH	4	12	0.5	0.629	0.482	21	0.12	0.34	-0.31	17.55	35.56	0.320	0.466	28	0.97	-	-26.58	-12.55	1.46
RCH	4	12	1.0	0.717	0.442	16	0.03	0.62	4.29	24.17	43.86	0.255	0.432	23	0.99	-	-29.38	-16.70	-4.26
RCH	4	12	2.0	0.789	0.387	11	0.02	0.80	7.69	28.23	48.71	0.250	0.427	20	0.99	-	-25.00	-13.35	-1.93

Notes: p-value < 0.05 indicated in blue italics; Power > 0.80 indicated in green italics; Negative P&L indicated in red.

Table C3: Results of time-weighted resampled variation of rules.

Underlying	Rule	Term	Std Dev Bound ( T )	Prop Success Ave(Buy)	Prop Success Sidev(Buy)	Sampling Effort (Buy)	p-value	Power	Lower 95% bound of Lower 95% PnL distribution (Buy)	Mean of Average PnL distribution (Buy)	Upper 95% bound of Upper 95% PnL distribution (Buy)	Prop Success Ave(Sell)	Prop Success Sidev(Sell)	Sampling Effort (Sells)	p-value	Power	Lower 95% bound of Lower 95% PnL distribution (Sell)	Mean of Average PnL distribution (Sell)	Upper 95% bound of Upper 95% PnL distribution (Sell)
RMH	4	3	0.5	0.423	0.498	20	0.75	-	-17.72	-6.26	5.07	0.385	0.489	37	0.92	-	-29.99	-13.82	2.15
RMH	4	3	1.0	0.413	0.490	14	0.74	-	-20.22	-6.42	7.44	0.361	0.477	35	0.95	-	-34.51	-18.24	-2.15
RMH	4	3	2.0	0.334	0.453	8	0.83	-	-22.23	-7.35	7.41	0.313	0.463	29	0.98	-	-40.88	-23.42	-6.34
RMH	4	6	0.5	0.326	0.468	19	0.94	-	-19.34	-6.77	5.61	0.312	0.461	35	0.99	-	-50.18	-25.94	-1.58
RMH	4	6	1.0	0.259	0.430	14	0.97	-	-20.54	-6.94	6.71	0.279	0.446	33	1.00	-	-57.06	-33.57	-9.86
RMH	4	6	2.0	0.011	0.032	8	1.00	-	-24.59	-14.30	-4.64	0.288	0.450	27	0.99	-	-62.68	-37.84	-12.82
RMH	4	9	0.5	0.367	0.479	19	0.88	-	-21.85	-6.87	7.88	0.331	0.471	33	0.98	-	-73.10	-38.69	-4.06
RMH	4	9	1.0	0.286	0.447	14	0.95	-	-28.34	-11.32	6.03	0.302	0.454	30	0.99	-	-84.99	-50.31	-14.94
RMH	4	9	2.0	0.095	0.223	8	1.00	-	-37.47	-22.68	-7.99	0.303	0.459	25	0.98	-	-97.74	-61.94	-25.53
RMH	4	12	0.5	0.375	0.482	19	0.86	-	-29.85	-11.31	6.74	0.335	0.471	29	0.96	-	-94.34	-50.45	-5.91
RMH	4	12	1.0	0.301	0.461	14	0.93	-	-33.81	-13.48	6.73	0.306	0.460	27	0.98	-	-105.44	-60.89	-15.69
RMH	4	12	2.0	0.079	0.183	8	1.00	-	-45.40	-25.98	-7.48	0.265	0.440	22	0.99	-	-118.60	-71.47	-23.45
SAB	4	3	0.5	0.530	0.500	27	0.38	0.09	-15.13	44.45	106.47	0.516	0.500	26	0.44	0.07	-93.98	-24.24	44.60
SAB	4	3	1.0	0.559	0.495	22	0.29	0.14	-9.50	60.80	133.62	0.485	0.502	21	0.55	-	-122.91	-43.75	34.09
SAB	4	3	2.0	0.598	0.487	19	0.20	0.22	-0.64	78.85	160.81	0.330	0.469	16	0.92	-	-180.80	-88.66	3.05
SAB	4	6	0.5	0.706	0.452	26	0.01	0.75	16.67	93.62	171.68	0.587	0.493	26	0.19	0.23	-148.34	-40.89	63.78
SAB	4	6	1.0	0.807	0.389	21	0.00	0.98	40.25	125.79	212.59	0.494	0.503	21	0.52	-	-195.88	-70.33	52.78
SAB	4	6	2.0	0.905	0.268	17	0.00	1.00	76.85	165.53	255.79	0.316	0.461	16	0.93	-	-295.27	-153.85	-13.88
SAB	4	9	0.5	0.715	0.447	26	0.01	0.79	23.43	114.35	207.19	0.532	0.502	24	0.38	0.09	-211.42	-78.42	56.84
SAB	4	9	1.0	0.816	0.380	21	0.00	0.99	50.49	152.34	257.46	0.410	0.490	19	0.78	-	-287.16	-131.09	26.04
SAB	4	9	2.0	0.882	0.300	17	0.00	1.00	62.78	171.82	284.68	0.253	0.426	14	0.98	-	-397.95	-222.13	-42.25
SAB	4	12	0.5	0.548	0.499	26	0.31	0.12	8.76	126.81	251.78	0.480	0.498	21	0.57	-	-265.32	-102.06	60.92
SAB	4	12	1.0	0.587	0.491	21	0.21	0.20	33.51	174.11	321.78	0.392	0.489	18	0.82	-	-321.85	-142.53	37.57
SAB	4	12	2.0	0.646	0.479	17	0.11	0.35	39.99	201.97	371.06	0.271	0.441	14	0.96	-	-414.34	-221.65	-24.73
SBK	4	3	0.5	0.435	0.494	31	0.76	-	-20.25	2.60	25.76	0.387	0.486	32	0.90	-	-40.39	-12.47	14.82
SBK	4	3	1.0	0.451	0.498	24	0.68	-	-24.62	2.72	30.47	0.351	0.476	28	0.95	-	-52.56	-22.83	6.14
SBK	4	3	2.0	0.572	0.497	12	0.31	0.13	-15.58	9.94	34.86	0.247	0.429	22	0.99	-	-70.03	-35.90	-2.45
SBK	4	6	0.5	0.475	0.500	31	0.61	-	-19.18	9.66	38.67	0.413	0.493	30	0.83	-	-81.47	-33.53	13.59
SBK	4	6	1.0	0.502	0.502	23	0.49	0.05	-15.97	16.64	49.34	0.401	0.488	26	0.85	-	-102.48	-50.35	0.54
SBK	4	6	2.0	0.573	0.501	11	0.32	0.12	-18.13	15.57	49.78	0.401	0.486	21	0.82	-	-131.39	-71.18	-12.15
SBK	4	9	0.5	0.451	0.498	31	0.71	-	-22.84	11.92	47.32	0.423	0.492	27	0.79	-	-119.81	-53.55	12.22
SBK	4	9	1.0	0.477	0.499	23	0.59	-	-21.54	21.15	64.43	0.435	0.497	23	0.73	-	-139.50	-68.09	2.39
SBK	4	9	2.0	0.467	0.502	11	0.58	-	-35.92	14.92	65.94	0.372	0.479	18	0.86	-	-191.58	-108.77	-25.58
SBK	4	12	0.5	0.482	0.502	31	0.58	-	-22.23	25.40	73.99	0.469	0.500	24	0.62	-	-136.63	-57.11	21.75
SBK	4	12	1.0	0.491	0.499	24	0.54	-	-24.83	32.89	91.71	0.436	0.496	20	0.72	-	-183.19	-92.77	-2.30
SBK	4	12	2.0	0.468	0.499	11	0.58	-	-51.73	10.12	72.55	0.341	0.476	16	0.90	-	-219.29	-122.30	-23.93
SOL	4	3	0.5	0.387	0.488	27	0.88	-	-51.10	13.64	80.63	0.457	0.497	29	0.68	-	-77.57	-27.58	22.17
SOL	4	3	1.0	0.417	0.496	22	0.78	-	-48.66	20.11	92.17	0.424	0.494	27	0.78	-	-86.10	-32.27	21.81
SOL	4	3	2.0	0.423	0.495	12	0.70	-	-69.50	2.97	81.28	0.356	0.481	22	0.91	-	-111.66	-51.61	9.26
SOL	4	6	0.5	0.563	0.498	26	0.26	0.16	-5.12	88.43	184.35	0.469	0.496	29	0.63	-	-117.43	-46.77	22.74
SOL	4	6	1.0	0.578	0.494	21	0.24	0.18	-3.62	102.90	212.55	0.407	0.491	26	0.83	-	-140.57	-68.18	3.64
SOL	4	6	2.0	0.561	0.502	13	0.33	0.11	-28.04	111.31	253.72	0.295	0.456	22	0.98	-	-178.18	-100.07	-21.88
SOL	4	9	0.5	0.484	0.502	25	0.56	-	-1.95	119.65	245.19	0.404	0.489	26	0.84	-	-178.38	-82.24	12.41
SOL	4	9	1.0	0.481	0.499	21	0.57	-	-2.13	130.43	267.25	0.359	0.479	23	0.91	-	-207.98	-110.75	-15.08
SOL	4	9	2.0	0.413	0.494	12	0.72	-	-38.83	156.89	356.19	0.289	0.451	20	0.97	-	-237.52	-136.83	-37.50
SOL	4	12	0.5	0.474	0.500	25	0.60	-	-15.36	142.98	306.50	0.537	0.499	23	0.36	0.10	-212.22	-83.27	43.63
SOL	4	12	1.0	0.497	0.499	21	0.51	-	-7.55	164.31	340.55	0.489	0.501	20	0.54	-	-245.09	-112.51	17.59
SOL	4	12	2.0	0.402	0.490	13	0.76	-	-55.17	183.16	427.88	0.421	0.492	17	0.74	-	-297.32	-156.31	-17.83

Notes: p-value < 0.05 indicated in blue italics; Power > 0.80 indicated in green italics; Negative P&L indicated in red.

**Table C3 (continued): Results of time-weighted resampled variation of rules.**

## C4. EWMA

Underlying	Rule	Term	Std Dev Bound ( )	Prop Success Ave(Buy)	Prop Success Stdev(Buy)	Sampling Effort (Buy)	p-value	Power	Lower 95% bound of Lower 95% PnL distribution (Buy)	Mean of Average PnL distribution (Buy)	Upper 95% bound of Upper 95% PnL distribution (Buy)	Prop Success Ave(Sell)	Prop Success Stdev(Sell)	Sampling Effort (Sell)	p-value	Power	Lower 95% bound of Lower 95% PnL distribution (Sell)	Mean of Average PnL distribution (Sell)	Upper 95% bound of Upper 95% PnL distribution (Sell)
Index	1	3	0.1	0.182	0.381	31	1.00	-	-104.86	-71.38	-36.93	0.808	0.393	34	0.00	1.00	34.37	78.87	122.30
Index	1	3	0.1	0.179	0.379	24	1.00	-	-105.13	-69.29	-32.14	0.824	0.375	24	0.00	1.00	34.84	84.26	132.40
Index	1	3	0.2	0.189	0.377	16	1.00	-	-105.76	-67.56	-28.13	0.821	0.367	18	0.00	0.98	27.69	89.61	149.04
Index	1	6	0.1	0.321	0.465	28	0.97	-	-106.79	-47.45	13.24	0.793	0.402	32	0.00	0.99	57.48	114.55	170.56
Index	1	6	0.1	0.305	0.456	20	0.96	-	-114.65	-44.27	28.51	0.787	0.399	21	0.00	0.95	25.72	99.37	170.55
Index	1	6	0.2	0.155	0.326	10	1.00	-	-182.48	-100.03	-13.74	0.789	0.398	14	0.01	0.86	14.71	108.05	199.63
Index	2	3	0.1	0.303	0.443	6	0.84	-	-120.10	-42.02	37.86	0.837	0.367	68	0.00	1.00	53.99	82.58	110.45
Index	2	3	0.1	0.510	0.396	2	0.49	0.05	-580.13	-12.67	536.84	0.834	0.371	64	0.00	1.00	52.89	83.01	112.27
Index	2	3	0.2	0.940	0.239	100	0.00	1.00	5.83	7.98	10.13	0.827	0.376	57	0.00	1.00	48.49	81.27	113.13
Index	2	6	0.1	0.562	0.466	5	0.39	0.09	-151.13	-13.27	117.38	0.761	0.426	68	0.00	1.00	53.25	92.37	131.09
Index	2	6	0.1	0.810	0.309	4	0.07	0.64	-32.54	35.39	107.73	0.776	0.415	65	0.00	1.00	55.00	95.58	135.51
Index	2	6	0.2	0.815	0.205	2	0.14	0.70	-252.34	25.33	307.39	0.769	0.419	59	0.00	1.00	50.36	94.30	137.33
Index	3	3	0.1	0.430	0.497	23	0.75	-	-45.27	0.39	47.01	0.721	0.447	34	0.00	0.89	-9.55	24.15	57.13
Index	3	3	0.1	0.474	0.503	13	0.57	-	-60.73	10.11	82.60	0.709	0.451	15	0.05	0.56	-3.39	50.80	104.48
Index	3	3	0.2	0.502	0.483	5	0.50	0.05	-103.38	-22.05	60.09	0.783	0.363	6	0.06	0.61	20.96	110.91	195.57
Index	3	6	0.1	0.578	0.494	21	0.24	0.18	-44.40	11.73	67.73	0.697	0.457	34	0.01	0.81	-44.33	13.68	70.09
Index	3	6	0.1	0.641	0.480	13	0.16	0.28	-58.57	14.32	87.35	0.713	0.454	15	0.05	0.57	-28.12	58.82	143.87
Index	3	6	0.2	0.546	0.466	5	0.42	0.08	-116.89	-24.97	63.26	0.835	0.306	6	0.02	0.85	64.05	164.45	256.95
AGL	4	3	0.1	0.540	0.498	26	0.34	0.11	-47.77	45.92	141.97	0.605	0.490	24	0.15	0.28	-66.70	-0.32	63.97
AGL	4	3	0.1	0.694	0.450	10	0.10	0.39	-78.10	46.01	169.69	0.591	0.483	10	0.28	0.15	-136.98	-35.61	59.14
AGL	4	3	0.2	0.940	0.112	4	0.00	1.00	7.05	95.46	184.71	0.470	0.339	2	0.54	-	-1637.34	-179.93	1276.32
AGL	4	6	0.1	0.513	0.501	26	0.45	0.06	-63.66	79.12	223.03	0.687	0.465	24	0.03	0.63	-41.99	19.65	77.63
AGL	4	6	0.1	0.716	0.434	10	0.08	0.47	-93.86	105.19	309.31	0.711	0.436	10	0.08	0.45	-43.87	13.77	68.11
AGL	4	6	0.2	1.000	0.000	4	0.00	1.00	100.03	144.48	191.61	0.535	0.318	2	0.45	0.07	-786.37	-28.95	723.43
AGL	4	9	0.1	0.548	0.494	26	0.31	0.13	-8.23	129.62	270.71	0.735	0.438	24	0.01	0.84	-64.26	18.97	97.59
AGL	4	9	0.1	0.591	0.488	10	0.28	0.15	-105.27	132.97	378.90	0.830	0.355	10	0.01	0.90	-59.17	35.78	123.58
AGL	4	9	0.2	1.000	0.000	4	0.00	1.00	167.86	229.12	288.54	1.000	0.000	2	0.00	1.00	-110.15	73.23	253.88
AGL	4	12	0.1	0.607	0.487	26	0.14	0.30	-7.15	131.88	275.02	0.508	0.499	21	0.47	0.06	-200.05	-78.13	42.27
AGL	4	12	0.1	0.714	0.437	10	0.08	0.46	-77.99	132.51	348.90	0.333	0.461	8	0.83	-	-309.11	-153.29	7.13
AGL	4	12	0.2	1.000	0.000	4	0.00	1.00	93.17	213.17	331.88	0.000	0.000	100	1.00	-	-317.93	-313.61	-309.29
RCH	4	3	0.1	0.592	0.490	33	0.15	0.28	-5.21	3.06	11.26	0.634	0.480	21	0.11	0.36	-10.49	-0.57	9.23
RCH	4	3	0.1	0.730	0.426	8	0.09	0.45	-13.48	0.80	13.87	0.825	0.348	11	0.01	0.93	-8.14	4.54	16.71
RCH	4	3	0.2	0.607	0.450	3	0.36	0.11	-8.23	2.32	13.09	0.997	0.006	3	0.00	1.00	-7.80	7.50	23.51
RCH	4	6	0.1	0.575	0.494	30	0.21	0.21	-5.01	5.98	17.00	0.637	0.482	21	0.10	0.37	-11.63	1.32	14.03
RCH	4	6	0.1	0.685	0.453	8	0.14	0.31	-13.90	0.51	13.29	0.839	0.337	11	0.00	0.96	-12.71	7.55	26.83
RCH	4	6	0.2	1.000	0.000	3	0.00	1.00	-1.71	6.34	14.51	0.977	0.040	3	0.00	1.00	-21.23	10.59	42.19
RCH	4	9	0.1	0.591	0.493	30	0.16	0.26	0.47	10.93	21.72	0.554	0.497	20	0.32	0.12	-15.20	-0.74	13.67
RCH	4	9	0.1	0.560	0.504	8	0.37	0.10	-10.03	3.40	16.39	0.674	0.467	11	0.12	0.34	-15.53	5.23	25.72
RCH	4	9	0.2	0.810	0.271	3	0.09	0.63	-5.83	11.47	28.04	0.537	0.404	3	0.44	0.07	-53.02	1.93	58.25
RCH	4	12	0.1	0.716	0.451	30	0.01	0.84	5.17	15.22	25.56	0.495	0.501	17	0.52	-	-23.13	-3.09	16.71
RCH	4	12	0.1	0.651	0.462	8	0.19	0.24	-1.61	9.56	20.70	0.487	0.487	9	0.53	-	-15.88	5.96	27.64
RCH	4	12	0.2	0.830	0.260	3	0.08	0.71	-8.04	17.67	40.15	0.293	0.352	3	0.79	-	-59.21	-5.79	50.74

Notes: p-value < 0.05 indicated in blue italics; Power > 0.80 indicated in green italics; Negative P&L indicated in red.

Table C4: Results of EWMA variation of rules.

Underlying	Rule	Term	Std Dev Bound (T)	Prop Success Ave(Buy)	Prop Success Stdev(Buy)	Sampling Effort (Buy)	P-value	Power	Lower 95% bound of Lower 95% PnL distribution (Buy)	Mean of Average PnL distribution (Buy)	Upper 95% bound of Upper 95% PnL distribution (Buy)	Prop Success Ave(Sell)	Prop Success Stdev(Sell)	Sampling Effort (Sells)	P-value	Power	Lower 95% bound of Lower 95% PnL distribution (Sell)	Mean of Average PnL distribution (Sell)	Upper 95% bound of Upper 95% PnL distribution (Sell)
RMH	4	3	0.1	0.667	0.468	34	<i>0.02</i>	0.67	0.19	11.29	22.82	0.625	0.481	17	0.15	0.28	3.57	20.12	36.78
RMH	4	3	0.1	0.598	0.490	12	0.25	0.17	<b>-5.53</b>	6.95	20.64	0.667	0.469	10	0.14	0.30	6.71	29.74	52.26
RMH	4	3	0.2	0.697	0.346	3	0.21	0.25	<b>-19.43</b>	6.22	33.97	0.942	0.130	5	<i>0.00</i>	<i>1.00</i>	27.64	48.98	68.73
RMH	4	6	0.1	0.729	0.442	34	<i>0.00</i>	<i>0.91</i>	9.63	28.28	47.29	0.498	0.494	16	0.51	-	3.14	27.39	51.90
RMH	4	6	0.1	0.780	0.406	12	<i>0.02</i>	0.77	1.59	35.50	70.13	0.625	0.486	10	0.22	0.20	7.47	39.82	71.67
RMH	4	6	0.2	0.907	0.139	3	<i>0.02</i>	<i>1.00</i>	<b>-53.31</b>	37.46	131.15	0.932	0.138	5	<i>0.00</i>	<i>1.00</i>	37.55	67.11	93.73
RMH	4	9	0.1	0.724	0.446	34	<i>0.00</i>	<i>0.90</i>	15.59	42.90	70.73	0.507	0.500	15	0.48	0.06	3.66	33.98	64.55
RMH	4	9	0.1	0.813	0.367	12	<i>0.01</i>	<i>0.90</i>	7.71	57.87	108.09	0.604	0.488	10	0.26	0.17	7.99	45.78	82.97
RMH	4	9	0.2	0.977	0.040	3	<i>0.00</i>	<i>1.00</i>	<b>-102.12</b>	66.44	243.86	0.930	0.143	5	<i>0.00</i>	<i>1.00</i>	38.19	76.88	112.28
RMH	4	12	0.1	0.662	0.473	33	<i>0.03</i>	0.62	13.74	46.81	80.37	0.491	0.500	15	0.53	-	5.61	40.76	76.10
RMH	4	12	0.1	0.828	0.353	12	<i>0.00</i>	<i>0.94</i>	12.68	73.04	133.20	0.616	0.485	10	0.23	0.19	10.86	53.65	95.59
RMH	4	12	0.2	1.000	0.000	3	<i>0.00</i>	<i>1.00</i>	<b>-124.24</b>	82.42	297.03	0.950	0.105	5	<i>0.00</i>	<i>1.00</i>	57.95	91.63	122.04
SAB	4	3	0.1	0.482	0.502	29	0.57	-	<b>-39.58</b>	15.77	72.41	0.873	0.325	30	<i>0.00</i>	<i>1.00</i>	34.76	69.07	102.08
SAB	4	3	0.1	0.683	0.448	12	0.09	0.41	<b>-38.24</b>	26.96	90.43	0.865	0.323	14	<i>0.00</i>	<i>1.00</i>	35.49	80.73	126.72
SAB	4	3	0.2	0.965	0.066	4	<i>0.00</i>	<i>1.00</i>	15.72	99.01	180.85	0.897	0.210	6	<i>0.00</i>	<i>1.00</i>	<b>-6.56</b>	99.55	202.59
SAB	4	6	0.1	0.551	0.499	29	0.29	0.14	<b>-26.49</b>	51.02	130.84	0.822	0.376	29	<i>0.00</i>	<i>1.00</i>	9.03	69.30	127.04
SAB	4	6	0.1	0.582	0.488	12	0.29	0.14	<b>-56.58</b>	56.20	168.35	0.852	0.327	14	<i>0.00</i>	<i>0.99</i>	<b>-7.29</b>	72.03	147.53
SAB	4	6	0.2	1.000	0.000	4	<i>0.00</i>	<i>1.00</i>	62.05	178.21	293.28	0.967	0.079	6	<i>0.00</i>	<i>1.00</i>	31.87	126.64	215.03
SAB	4	9	0.1	0.532	0.499	29	0.37	0.10	<b>-25.61</b>	76.69	181.29	0.617	0.486	26	0.12	0.34	<b>-45.58</b>	31.88	107.23
SAB	4	9	0.1	0.562	0.492	12	0.34	0.11	<b>-70.80</b>	117.52	309.25	0.611	0.487	14	0.20	0.22	<b>-69.12</b>	34.14	134.89
SAB	4	9	0.2	1.000	0.000	4	<i>0.00</i>	<i>1.00</i>	56.02	338.66	610.38	0.868	0.250	6	<i>0.01</i>	<i>0.97</i>	<b>-28.48</b>	108.42	238.12
SAB	4	12	0.1	0.442	0.496	29	0.73	-	<b>-49.70</b>	63.23	179.60	0.568	0.495	26	0.24	0.17	<b>-69.16</b>	<b>-4.40</b>	87.53
SAB	4	12	0.1	0.578	0.487	12	0.29	0.14	<b>-78.58</b>	147.59	375.96	0.647	0.475	14	0.13	0.31	<b>-100.61</b>	22.67	139.46
SAB	4	12	0.2	1.000	0.000	4	<i>0.00</i>	<i>1.00</i>	103.64	444.44	758.88	0.958	0.102	6	<i>0.00</i>	<i>1.00</i>	4.91	124.74	230.67
SBK	4	3	0.1	0.670	0.469	29	<i>0.03</i>	0.62	<b>-1.19</b>	27.66	57.33	0.815	0.380	22	<i>0.00</i>	<i>0.99</i>	8.05	31.36	54.30
SBK	4	3	0.1	0.751	0.413	9	0.05	0.57	<b>-1.78</b>	18.39	38.66	0.984	0.041	8	<i>0.00</i>	<i>1.00</i>	38.47	72.21	104.56
SBK	4	3	0.2	0.760	0.294	3	0.13	0.45	<b>-33.50</b>	17.78	68.72	0.974	0.058	5	<i>0.00</i>	<i>1.00</i>	42.98	87.08	127.94
SBK	4	6	0.1	0.626	0.485	29	0.09	0.40	7.93	51.11	95.46	0.854	0.335	22	<i>0.00</i>	<i>1.00</i>	16.55	45.89	74.57
SBK	4	6	0.1	0.562	0.493	9	0.36	0.10	<b>-12.65</b>	39.78	94.64	0.993	0.021	8	<i>0.00</i>	<i>1.00</i>	65.08	102.15	137.83
SBK	4	6	0.2	0.783	0.277	3	0.11	0.55	<b>-45.07</b>	53.46	147.77	0.982	0.040	5	<i>0.00</i>	<i>1.00</i>	80.15	121.09	158.92
SBK	4	9	0.1	0.599	0.492	29	0.14	0.29	13.85	69.97	127.00	0.933	0.219	22	<i>0.00</i>	<i>1.00</i>	27.98	60.32	91.84
SBK	4	9	0.1	0.538	0.499	9	0.41	0.08	<b>-10.50</b>	91.96	197.98	0.986	0.039	8	<i>0.00</i>	<i>1.00</i>	61.05	107.58	151.71
SBK	4	9	0.2	0.727	0.341	3	0.18	0.31	<b>-94.67</b>	122.99	337.13	0.966	0.076	5	<i>0.00</i>	<i>1.00</i>	77.32	129.93	177.92
SBK	4	12	0.1	0.541	0.500	28	0.33	0.11	8.38	72.60	137.91	0.900	0.284	22	<i>0.00</i>	<i>1.00</i>	21.28	61.10	99.38
SBK	4	12	0.1	0.491	0.497	9	0.52	-	<b>-23.24</b>	109.24	243.70	0.984	0.046	8	<i>0.00</i>	<i>1.00</i>	70.71	119.22	163.97
SBK	4	12	0.2	0.767	0.335	3	0.15	0.40	<b>-81.39</b>	201.39	466.10	0.974	0.058	5	<i>0.00</i>	<i>1.00</i>	99.00	146.09	187.59
SOL	4	3	0.1	0.463	0.499	26	0.65	-	<b>-26.70</b>	25.78	78.71	0.757	0.429	24	<i>0.00</i>	<i>0.90</i>	13.74	58.96	104.42
SOL	4	3	0.1	0.597	0.489	9	0.28	0.15	<b>-45.37</b>	56.43	155.96	0.954	0.123	8	<i>0.00</i>	<i>1.00</i>	35.19	123.37	209.85
SOL	4	3	0.2	1.000	0.000	3	<i>0.00</i>	<i>1.00</i>	41.20	154.34	261.65	0.985	0.021	2	<i>0.01</i>	<i>1.00</i>	<b>-621.66</b>	175.55	954.85
SOL	4	6	0.1	0.531	0.498	25	0.38	0.09	7.13	93.05	179.83	0.720	0.446	23	<i>0.01</i>	<i>0.77</i>	30.18	89.91	150.31
SOL	4	6	0.1	0.663	0.469	9	0.16	0.27	<b>-17.01</b>	154.75	322.39	0.958	0.115	8	<i>0.00</i>	<i>1.00</i>	53.68	185.83	313.79
SOL	4	6	0.2	1.000	0.000	3	<i>0.00</i>	<i>1.00</i>	161.74	288.53	412.45	0.985	0.021	2	<i>0.01</i>	<i>1.00</i>	<b>-571.78</b>	263.76	1073.91
SOL	4	9	0.1	0.548	0.498	25	0.32	0.12	34.94	144.30	254.16	0.818	0.381	23	<i>0.00</i>	<i>0.99</i>	54.70	115.70	178.06
SOL	4	9	0.1	0.673	0.459	9	0.15	0.30	37.31	262.22	478.15	0.989	0.032	8	<i>0.00</i>	<i>1.00</i>	84.43	209.86	327.43
SOL	4	9	0.2	1.000	0.000	3	<i>0.00</i>	<i>1.00</i>	363.60	474.34	587.41	0.985	0.021	2	<i>0.01</i>	<i>1.00</i>	<b>-550.44</b>	249.47	1026.06
SOL	4	12	0.1	0.556	0.498	25	0.29	0.14	38.58	168.71	300.88	0.755	0.425	22	<i>0.01</i>	<i>0.88</i>	27.21	110.42	192.80
SOL	4	12	0.1	0.694	0.454	9	0.12	0.36	63.80	330.73	590.29	0.863	0.289	8	<i>0.00</i>	<i>0.97</i>	34.11	218.12	388.39
SOL	4	12	0.2	1.000	0.000	3	<i>0.00</i>	<i>1.00</i>	341.30	624.27	902.13	0.950	0.071	2	<i>0.04</i>	<i>1.00</i>	<b>-757.62</b>	305.88	1331.99

Notes: p-value < 0.05 indicated in blue italics; Power > 0.80 indicated in green italics; Negative P&L indicated in red.

**Table C4 (continued): Results of EWMA variation of rules.**

## C5. GARCH

Underlying	Rule	Term	Std Dev Bound (U)	Prop Success Ave(Buy)	Prop Success Stdev(Buy)	Sampling Effort (Buy)	p-value	Power	Lower 95% bound of Lower 95% PnL distribution (Buy)	Mean of Average PnL distribution (Buy)	Upper 95% bound of Upper 95% PnL distribution (Buy)	Prop Success Ave(Sell)	Prop Success Stdev(Sell)	Sampling Effort (Sells)	p-value	Power	Lower 95% bound of Lower 95% PnL distribution (Sell)	Mean of Average PnL distribution (Sell)	Upper 95% bound of Upper 95% PnL distribution (Sell)
Index	2	3	0.1	0.324	0.457	8	0.84	-	-154.36	-73.77	7.48	0.805	0.394	76	0.00	1.00	47.71	74.17	100.03
Index	2	3	0.1	0.342	0.440	5	0.77	-	-200.69	-79.68	41.69	0.812	0.387	71	0.00	1.00	47.16	74.22	100.65
Index	2	3	0.2	0.283	0.387	4	0.83	-	-253.28	-94.87	69.34	0.810	0.390	66	0.00	1.00	45.15	73.55	101.54
Index	2	6	0.1	0.142	0.293	9	1.00	-	-200.61	-114.86	-28.71	0.747	0.434	71	0.00	1.00	44.94	81.75	118.02
Index	2	6	0.1	0.145	0.280	6	0.99	-	-243.47	-123.94	-4.55	0.740	0.437	66	0.00	1.00	42.63	81.92	120.40
Index	2	6	0.2	0.098	0.164	4	0.99	-	-331.68	-158.74	22.04	0.738	0.439	62	0.00	1.00	37.53	78.59	118.94
Index	3	3	0.1	0.487	0.501	30	0.56	-	-4.77	48.74	104.42	0.672	0.469	43	0.01	0.78	11.79	41.36	70.61
Index	3	3	0.1	0.508	0.500	25	0.47	0.06	-6.36	52.93	114.43	0.677	0.466	38	0.01	0.76	15.46	46.87	78.13
Index	3	3	0.2	0.518	0.497	20	0.44	0.07	-12.95	54.00	123.61	0.679	0.465	33	0.02	0.72	18.61	51.85	84.62
Index	3	6	0.1	0.520	0.500	29	0.42	0.08	-17.45	45.31	110.19	0.646	0.476	42	0.03	0.63	-32.80	16.81	65.50
Index	3	6	0.1	0.545	0.500	24	0.33	0.11	-19.48	48.75	119.16	0.669	0.470	37	0.02	0.71	-20.83	30.45	80.89
Index	3	6	0.2	0.557	0.499	19	0.31	0.13	-31.19	46.52	126.84	0.683	0.463	32	0.02	0.72	-18.95	35.59	87.34
AGL	4	3	0.1	0.589	0.494	38	0.14	0.30	-14.39	61.29	138.62	0.717	0.450	43	0.00	0.93	6.41	65.07	123.74
AGL	4	3	0.1	0.600	0.491	32	0.13	0.31	-17.62	66.76	152.97	0.735	0.440	37	0.00	0.95	10.47	72.25	134.32
AGL	4	3	0.2	0.639	0.479	25	0.08	0.42	-20.80	77.69	178.87	0.730	0.441	32	0.00	0.90	-1.80	65.40	132.84
AGL	4	6	0.1	0.545	0.498	37	0.29	0.14	-31.22	84.85	202.34	0.644	0.478	41	0.03	0.61	-42.10	20.52	81.27
AGL	4	6	0.1	0.544	0.499	32	0.31	0.13	-38.32	88.23	215.76	0.660	0.473	35	0.03	0.64	-32.60	29.25	89.32
AGL	4	6	0.2	0.556	0.499	25	0.29	0.14	-56.45	86.20	230.69	0.667	0.469	30	0.03	0.62	-30.83	32.92	94.31
AGL	4	9	0.1	0.527	0.500	35	0.37	0.09	25.38	148.87	274.31	0.625	0.484	40	0.06	0.49	-65.60	5.20	73.36
AGL	4	9	0.1	0.496	0.498	30	0.52	-	5.17	138.57	275.66	0.663	0.472	35	0.02	0.66	-49.39	21.14	89.01
AGL	4	9	0.2	0.457	0.499	24	0.66	-	-29.42	124.82	283.13	0.700	0.457	30	0.01	0.77	-28.29	40.08	104.71
AGL	4	12	0.1	0.538	0.500	35	0.33	0.12	37.91	168.27	300.89	0.530	0.500	38	0.36	0.10	-107.97	-14.67	76.28
AGL	4	12	0.1	0.526	0.500	30	0.39	0.09	27.09	167.70	312.30	0.552	0.498	32	0.28	0.15	-98.20	0.05	95.39
AGL	4	12	0.2	0.501	0.499	24	0.50	0.05	-6.48	156.44	324.06	0.590	0.492	28	0.17	0.25	-70.68	23.39	114.91
RCH	4	3	0.1	0.646	0.479	39	0.03	0.60	-0.55	7.70	16.02	0.589	0.491	39	0.13	0.31	-5.52	2.78	10.94
RCH	4	3	0.1	0.658	0.473	32	0.03	0.60	-1.55	7.46	16.58	0.623	0.484	32	0.08	0.42	-3.98	4.83	13.54
RCH	4	3	0.2	0.670	0.472	26	0.04	0.58	-2.13	7.49	17.23	0.604	0.490	27	0.14	0.29	-3.63	5.20	14.01
RCH	4	6	0.1	0.681	0.466	38	0.01	0.77	0.50	10.50	20.52	0.546	0.498	38	0.28	0.14	-12.25	-2.06	7.90
RCH	4	6	0.1	0.681	0.466	31	0.02	0.70	-0.97	10.29	21.71	0.579	0.493	31	0.19	0.23	-10.85	0.06	10.69
RCH	4	6	0.2	0.691	0.460	25	0.02	0.67	-1.74	10.87	23.61	0.597	0.490	26	0.16	0.26	-9.47	2.06	13.29
RCH	4	9	0.1	0.694	0.459	36	0.01	0.81	4.39	14.63	24.96	0.479	0.500	38	0.60	-	-15.84	-5.18	5.36
RCH	4	9	0.1	0.710	0.453	30	0.01	0.82	4.14	15.48	26.90	0.515	0.502	31	0.44	0.07	-14.24	-2.50	9.04
RCH	4	9	0.2	0.686	0.462	24	0.03	0.63	0.22	12.40	24.61	0.517	0.502	26	0.43	0.07	-14.68	-1.82	10.82
RCH	4	12	0.1	0.739	0.438	35	0.00	0.94	7.78	19.05	30.33	0.411	0.491	36	0.86	-	-20.27	-7.85	4.82
RCH	4	12	0.1	0.718	0.448	29	0.01	0.84	6.03	19.12	32.31	0.451	0.500	30	0.70	-	-19.13	-5.20	8.63
RCH	4	12	0.2	0.714	0.452	24	0.01	0.75	3.79	18.36	32.94	0.484	0.499	25	0.56	-	-18.95	-3.43	11.88
RMH	4	3	0.1	0.640	0.479	41	0.03	0.59	0.58	9.79	19.33	0.557	0.499	40	0.24	0.18	-8.43	5.30	18.71
RMH	4	3	0.1	0.640	0.479	36	0.04	0.54	0.02	9.73	19.69	0.599	0.490	34	0.12	0.32	-5.82	8.87	23.19
RMH	4	3	0.2	0.620	0.485	29	0.10	0.38	-1.24	9.03	19.76	0.627	0.484	29	0.08	0.41	-4.32	12.52	28.89
RMH	4	6	0.1	0.758	0.426	41	0.00	0.99	4.80	19.11	33.57	0.574	0.495	38	0.18	0.23	-14.72	5.38	25.13
RMH	4	6	0.1	0.775	0.414	35	0.00	0.99	4.43	19.02	34.02	0.640	0.478	32	0.05	0.50	-10.09	11.87	33.38
RMH	4	6	0.2	0.782	0.410	29	0.00	0.98	3.63	19.12	35.08	0.658	0.474	27	0.05	0.53	-8.51	15.44	38.98
RMH	4	9	0.1	0.766	0.423	40	0.00	0.99	9.67	30.54	51.75	0.611	0.486	37	0.09	0.40	-15.36	9.00	33.05
RMH	4	9	0.1	0.757	0.428	35	0.00	0.97	7.33	29.42	52.06	0.638	0.482	31	0.06	0.48	-13.52	13.17	39.69
RMH	4	9	0.2	0.759	0.422	29	0.00	0.95	5.18	30.20	56.04	0.690	0.462	26	0.02	0.68	-8.09	20.98	49.46
RMH	4	12	0.1	0.767	0.420	38	0.00	0.99	13.23	39.74	66.63	0.554	0.497	36	0.26	0.16	-18.50	10.53	39.20
RMH	4	12	0.1	0.765	0.421	34	0.00	0.98	11.70	40.14	68.99	0.596	0.489	30	0.15	0.28	-12.13	18.75	49.31
RMH	4	12	0.2	0.749	0.431	28	0.00	0.92	8.49	39.92	72.00	0.663	0.471	25	0.05	0.53	-2.69	30.99	63.96

Notes: p-value < 0.05 indicated in blue italics; Power > 0.80 indicated in green italics; Negative P&L indicated in red.

Table C5: Results of GARCH variation of rules.

Underlying	Rule	Term	Std Dev Bound ( )	Prop Success Ave(Buy)	Prop Success Stdev(Buy)	Sampling Effort (Buy)	p-value	Power	Lower 95% bound of Lower 95% PnL distribution (Buy)	Mean of Average PnL distribution (Buy)	Upper 95% bound of Upper 95% PnL distribution (Buy)	Prop Success Ave(Sell)	Prop Success Stdev(Sell)	Sampling Effort (Sells)	p-value	Power	Lower 95% bound of Lower 95% PnL distribution (Sell)	Mean of Average PnL distribution (Sell)	Upper 95% bound of Upper 95% PnL distribution (Sell)
SAB	4	3	0.1	0.611	0.485	34	0.10	0.38	14.34	69.99	126.36	0.763	0.425	47	0.00	1.00	14.00	45.48	76.18
SAB	4	3	0.1	0.643	0.475	28	0.06	0.48	17.09	78.32	141.08	0.771	0.420	40	0.00	0.99	16.73	50.57	83.71
SAB	4	3	0.2	0.656	0.477	22	0.07	0.45	6.90	73.16	140.63	0.807	0.390	32	0.00	1.00	30.02	64.79	98.77
SAB	4	6	0.1	0.681	0.467	34	0.02	0.73	27.70	102.34	178.55	0.755	0.428	46	0.00	0.99	-4.16	45.11	92.46
SAB	4	6	0.1	0.693	0.463	28	0.02	0.71	29.08	109.65	192.35	0.793	0.404	39	0.00	1.00	11.00	58.90	104.84
SAB	4	6	0.2	0.689	0.460	22	0.03	0.61	8.23	91.68	177.38	0.837	0.366	32	0.00	1.00	33.15	77.90	121.02
SAB	4	9	0.1	0.646	0.476	34	0.04	0.55	34.19	127.92	222.89	0.635	0.482	43	0.04	0.58	-18.90	38.19	93.62
SAB	4	9	0.1	0.645	0.478	28	0.06	0.48	28.80	132.41	237.66	0.688	0.462	37	0.01	0.80	4.34	59.29	112.32
SAB	4	9	0.2	0.655	0.475	22	0.07	0.45	7.12	119.02	233.64	0.742	0.436	31	0.00	0.93	27.83	82.40	135.51
SAB	4	12	0.1	0.638	0.481	33	0.06	0.50	43.16	160.87	281.26	0.626	0.485	42	0.05	0.52	-28.90	33.10	93.32
SAB	4	12	0.1	0.627	0.484	27	0.09	0.39	29.64	161.27	296.22	0.655	0.475	36	0.03	0.62	-9.79	52.14	112.42
SAB	4	12	0.2	0.604	0.487	21	0.17	0.25	-8.12	140.29	294.46	0.706	0.456	30	0.01	0.80	17.08	77.61	136.68
SBK	4	3	0.1	0.539	0.499	38	0.32	0.12	-0.33	22.09	45.09	0.698	0.458	39	0.01	0.85	8.37	25.21	41.92
SBK	4	3	0.1	0.538	0.498	30	0.34	0.11	-0.78	24.96	51.73	0.710	0.455	32	0.01	0.83	10.69	29.14	47.56
SBK	4	3	0.2	0.555	0.499	23	0.30	0.13	-1.99	28.63	60.51	0.765	0.419	26	0.00	0.94	15.15	35.27	55.27
SBK	4	6	0.1	0.547	0.497	38	0.28	0.14	9.30	41.50	74.76	0.703	0.458	36	0.01	0.84	4.81	30.66	55.55
SBK	4	6	0.1	0.558	0.495	30	0.28	0.16	10.29	47.31	85.49	0.751	0.432	30	0.00	0.94	15.19	39.74	63.84
SBK	4	6	0.2	0.605	0.492	23	0.16	0.27	12.65	55.23	99.33	0.818	0.377	24	0.00	0.99	24.05	50.23	75.91
SBK	4	9	0.1	0.558	0.496	36	0.24	0.17	19.19	60.37	102.65	0.796	0.398	35	0.00	1.00	4.41	36.76	67.86
SBK	4	9	0.1	0.601	0.489	29	0.14	0.30	25.32	72.62	120.71	0.824	0.376	30	0.00	1.00	18.02	48.07	77.31
SBK	4	9	0.2	0.605	0.490	23	0.16	0.27	21.10	71.88	124.13	0.870	0.324	24	0.00	1.00	30.12	60.47	89.90
SBK	4	12	0.1	0.576	0.494	36	0.18	0.23	27.32	77.91	129.32	0.793	0.402	34	0.00	1.00	6.01	43.01	78.42
SBK	4	12	0.1	0.614	0.486	29	0.11	0.35	34.20	89.12	144.98	0.828	0.375	29	0.00	1.00	20.73	57.17	92.09
SBK	4	12	0.2	0.621	0.486	23	0.12	0.33	32.70	94.84	157.83	0.883	0.305	23	0.00	1.00	37.01	72.60	106.70
SOL	4	3	0.1	0.538	0.500	37	0.32	0.12	-3.23	50.51	106.35	0.736	0.438	43	0.00	0.97	-0.08	34.60	68.78
SOL	4	3	0.1	0.573	0.493	30	0.21	0.20	-5.92	53.76	115.48	0.777	0.416	36	0.00	0.99	6.19	43.24	79.90
SOL	4	3	0.2	0.618	0.485	23	0.13	0.32	-3.39	67.09	140.52	0.772	0.416	30	0.00	0.97	2.28	46.57	89.96
SOL	4	6	0.1	0.539	0.499	36	0.32	0.12	23.50	101.95	182.50	0.680	0.466	42	0.01	0.81	-7.12	39.08	84.74
SOL	4	6	0.1	0.539	0.497	29	0.34	0.11	18.67	109.14	201.12	0.732	0.442	35	0.00	0.93	10.03	58.97	107.77
SOL	4	6	0.2	0.510	0.500	23	0.46	0.06	0.33	99.25	200.62	0.764	0.421	29	0.00	0.96	16.11	69.49	122.57
SOL	4	9	0.1	0.624	0.484	35	0.07	0.45	67.41	164.39	263.49	0.736	0.441	41	0.00	0.96	-3.66	49.93	102.47
SOL	4	9	0.1	0.613	0.485	28	0.11	0.34	55.70	166.59	280.12	0.784	0.408	34	0.00	0.99	13.89	69.52	124.64
SOL	4	9	0.2	0.623	0.481	22	0.11	0.35	53.42	181.83	313.43	0.802	0.396	28	0.00	0.99	19.73	80.69	141.07
SOL	4	12	0.1	0.610	0.489	34	0.10	0.37	88.50	210.80	335.92	0.766	0.419	39	0.00	0.99	-2.86	59.52	120.84
SOL	4	12	0.1	0.621	0.484	28	0.10	0.37	97.11	242.60	391.42	0.810	0.387	32	0.00	1.00	30.62	90.72	150.86
SOL	4	12	0.2	0.638	0.482	21	0.10	0.37	79.77	247.47	419.39	0.834	0.369	27	0.00	1.00	39.74	104.47	168.92

Notes: p-value < 0.05 indicated in blue italics; Power > 0.80 indicated in green italics; Negative P&L indicated in red.

Table C5 (continued): Results of GARCH variation of rules.

## D. GARCH parameters

Parameter	Value	Standard Error	t-statistic
$\mu$	0.000645	0.000	2.702
$\gamma$	0.000003	0.000	4.912
$\alpha$	0.864680	0.008	102.893
$\beta$	0.130030	0.009	13.691

**Table D1: GARCH parameters for Top-40 Index.**

Parameter	Value	Standard Error	t-statistic
$\mu$	0.000526	0.000	1.451
$\gamma$	0.000005	0.000	4.619
$\alpha$	0.884470	0.007	121.122
$\beta$	0.115530	0.008	14.340

**Table D2: GARCH parameters for AGL.**

Parameter	Value	Standard Error	t-statistic
$\mu$	0.001051	0.000	3.099
$\gamma$	0.000010	0.000	6.767
$\alpha$	0.854970	0.008	109.580
$\beta$	0.133480	0.009	14.921

**Table D3: GARCH parameters for RCH.**

Parameter	Value	Standard Error	t-statistic
$\mu$	0.001126	0.000	2.855
$\gamma$	0.000010	0.000	11.502
$\alpha$	0.867300	0.007	128.951
$\beta$	0.122620	0.007	17.343

**Table D4: GARCH parameters for RMH.**

Parameter	Value	Standard Error	t-statistic
$\mu$	0.000246	0.000	0.659
$\gamma$	0.000012	0.000	8.689
$\alpha$	0.870780	0.008	106.428
$\beta$	0.100420	0.007	13.920

**Table D5: GARCH parameters for SAB.**

Parameter	Value	Standard Error	t-statistic
$\mu$	0.000963	0.000	2.339
$\gamma$	0.000026	0.000	12.712
$\alpha$	0.825910	0.012	67.839
$\beta$	0.125620	0.009	13.786

*Table D6: GARCH parameters for SBK.*

Parameter	Value	Standard Error	t-statistic
$\mu$	0.000976	0.000	2.322
$\gamma$	0.000020	0.000	6.451
$\alpha$	0.829860	0.011	75.919
$\beta$	0.147340	0.010	15.033

*Table D7: GARCH parameters for SOL.*

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## E. Statistical Appendix

Denote the null hypothesis by  $H_0$  and the alternative hypothesis by  $H_A$ . There are four possible outcomes of a hypothesis test of  $H_0$  versus  $H_A$ :

1. We fail to reject  $H_0$ , and  $H_0$  is in fact true.
2. We fail to reject  $H_0$ , and  $H_A$  is in fact true.
3. We reject  $H_0$ , and  $H_0$  is in fact true.
4. We reject  $H_0$ , and  $H_A$  is in fact true.

A type I error is defined as the probability of rejecting the null hypothesis given that  $H_0$  is true. The probability of a type I error is usually denoted by  $\alpha$  and is referred to as the significance level of a test.

A type II error is the probability of failing to reject the null hypothesis given that  $H_A$  is true. The probability of a type II error is usually denoted by  $\beta$ .

The power of a test is defined as  $1 - \beta = 1 - \text{probability of a type II error}$ .

The aim of hypothesis testing is to make  $\alpha$  and  $\beta$  as small as possible. However as  $\alpha$  increases,  $\beta$  will decrease and vice versa. This is because making  $\alpha$  as small as possible means that we will reject  $H_0$  less often, while making  $\beta$  as small as possible involves rejecting  $H_A$  less often.

The common approach is then to fix  $\alpha$  at some level (in our case  $\alpha = 0.05$ ) and to use a test that minimizes  $\beta$ , or equivalently, maximizes the power.

## F. Listing of Matlab code

All the analysis in this dissertation was performed in MATLAB<sup>21</sup>. The MATLAB code listed in this appendix and is available on the CD that accompanies this document. The code is also available for download on the following ftp site:

`ftp://ftp.gis.uct.ac.za/pub/polakow/DeAraujoCode.doc`

### F1. Main programs

The tables in Appendix C have been generated by the following scripts. The p-value and power columns in these tables are not outputted – the code which generates these (`ttest1.m` and `Power1.m`) require the MATLAB statistics toolbox. In order to ensure that the code will run (in case the user doesn't have the required toolbox) these columns are inserted manually.

**TableRolling.m** — Produces the table in Appendix C1.

**TableRS.m** — Produces the table in Appendix C2.

**TableRST.m** — Produces the table in Appendix C3.

**TableEWMA.m** — Produces the table in Appendix C4.

**TableGARCH.m** — Produces the table in Appendix C5.

To run the code, type the script name at the MATLAB command prompt. The code takes several hours to run.

### F2. Matlab scripts

**RunRule.m** — Run a rolling standard deviation (naive) variation.

**Rule1.m** — Naive variation of Rule 1 - IV vs. IV (Top40).

**Rule2.m** — Naive variation of Rule 2 - HV vs. IV (Top40).

**Rule3.m** — Naive variation of Rule 3 - HV vs. HV (Top40).

**Rule4.m** — Naive variation of Rule 4 - HV vs. HV (Single stocks).

**RunRuleRS.m** — Run a resampled variation.

**Rule1RS.m** — Resampled variation of Rule 1 - IV vs. IV (Top40).

**Rule2RS.m** — Resampled variation of Rule 2 - HV vs. IV (Top40).

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<sup>21</sup> <http://www.mathworks.com>

**Rule3RS.m** — Resampled variation of Rule 3 - HV vs. HV (Top40).

**Rule4RS.m** — Resampled variation of Rule 4 - HV vs. HV (Single stocks).

**RunRuleRST.m** — Run a time-weighted resampled variation.

**Rule1RST.m** — Time-weighted resampled variation of Rule 1 - IV vs. IV (Top40).

**Rule2RST.m** — Time-weighted resampled variation of Rule 2 - HV vs. IV (Top40).

**Rule3RST.m** — Time-weighted resampled variation of Rule 3 - HV vs. HV (Top40).

**Rule4RST.m** — Time-weighted resampled variation of Rule 4 - HV vs. HV (Single stocks).

**RunEWMA.m** — Run an EWMA rule.

**EWMARule1.m** — EWMA variation of Rule 1 - IV vs. IV (Top40).

**EWMARule2.m** — EWMA variation of Rule 2 - HV vs. IV (Top40).

**EWMARule3.m** — EWMA variation of Rule 3 - HV vs. HV (Top40).

**EWMARule4.m** — EWMA variation of Rule 4 - HV vs. HV (Single stocks).

**RunGARCH.m** — Run a GARCH variation.

**GARCHRule2.m** — GARCH variation of Rule 2 - HV vs. IV (Top40).

**GARCHRule3.m** — GARCH variation of Rule 3 - HV vs. HV (Top40).

**GARCHRule4.m** — GARCH variation of Rule 4 - HV vs. HV (Single stocks).

### F3. Matlab functions

**Bounds.m** — Determines standard deviation bounds.

**DiscreteRets.m** — Determines discrete return of price series.

**EWMAAverage.m** — Returns an EWMA average of a series.

**EWMAVol.m** — Returns EWMA volatility (annualised) of a series.

**GARCHInit.m** — Returns GARCH volatility (annualised) of a series (requires MATLAB GARCH toolbox).

**HistoricalVol.m** — Returns historical volatility (annualised) by rolling equally weighted standard deviation method.

**Index\_Call.m; Index\_d1.m; Index\_d2.m** — Black-Scholes-Merton formula

**Performance.m** — Returns a matrix whose columns are randomly resampled from an input vector.

**Power1.m** — Performs a one-sided power analysis (requires MATLAB Statistics toolbox).

**ResampledAverage.m** — Returns a resampled equally weighted average.

**ResampledAverageT.m** — Returns a resampled time-weighted average.

**RollingAverage.m** — Returns a rolling window average.

**RollingStdev.m** — Returns a rolling window standard deviation.  
**RunResample.m** — Randomly resample 5% of the elements of a vector.  
**Shift.m** — Shift each element in a vector forward.  
**TableEntry.m** — Returns rows of the tables in Appendix C.

## F4. Data Files

**Data4May03.mat** — Contains single stock data.

Dates — date vector; (2177, 1) vector.  
NC1YM — 1 year NCD rate vector; (2177, 1) vector.  
NC3MM — 3 month NCD rate vector; (2177, 1) vector.  
NC6MM — 6 month NCD rate vector; (2177, 1) vector.  
NC9MM — 9 month NCD rate vector; (2177, 1) vector.

For each of STOCK = { AGL, RCH, RMH, SAB, SBK, SOL } the following:

STOCK — price series for STOCK; (2177, 1) vector.  
STOCKdiv — dividend yield series for STOCK; (2177, 1) vector.  
GarchSTOCK — GARCH volatility for STOCK; (2177, 1) vector.

**DataMay03.mat** — Contains Top-40 (J200) Index data.

Dates — date vector; (1953, 1) vector.  
NC3MM — 3 month NCD rate vector; (1953, 1) vector.  
NC6MM — 6 month NCD rate vector; (1953, 1) vector.

J200 — price series for Top-40 Index; (1953, 1) vector.  
J200D — dividend yield series for Top-40 Index; (1953, 1) vector.  
GARCHJ200 — GARCH volatility for Top-40 Index; (1953, 1) vector.

MIV — middle (6-month) mark-to-market implied volatility for Top-40; (1953, 1) vector.  
NIV — near (3-month) mark-to-market implied volatility for Top-40; (1953, 1) vector.

## G. Glossary of Terms

**Arbitrage** — Profit opportunities presented by assets that appear to be mispriced relative to publicly available information about fundamentals as well as relative to other assets. These opportunities are usually low- or no-risk.

**Ask Price** — The price that a dealer is prepared to sell an asset for.

**At-the-money Option** — An option in which the strike price equals the price of the underlying asset.

**Backtesting** — Testing a model using historical data.

**Basis Point** — One hundredth of one percent.

**Bid-ask Spread** — The amount by which the ask price exceeds the bid price.

**Bid Price** — The price that a dealer is prepared to pay for an asset.

**Bootstrapping** — A resampling technique that was proposed by Efron (1979).

**Call** — An option to buy an asset at a certain price by a certain date.

**Capital Asset Pricing Model** — A model relating the expected return on an asset to its beta.

**Delta** — The rate of change of price of a derivative with the price of the underlying asset.

**Delta-neutral Portfolio** — A portfolio with a delta of zero so that there is no sensitivity to small changes in the price of the underlying asset.

**Derivative** — An instrument whose price depends on, or is derived from, the price of another asset.

**Dividend** — A cash payment made to the owner of a stock.

**Dividend Yield** — The dividend as a percentage of the stock price.

**Double** — See Bid-ask Spread.

**Efficient Market Hypothesis** — A hypothesis that asset prices reflect relevant information.

**European Option** — An option that can only be exercised at the end of its life.

**EWMA** — Exponentially weighted moving average. A model where exponential weighting is used to provide forecasts for a variable from historical data.

**Exercise Price** — The price at which the underlying asset may be bought or sold in an option contract.

**Expiration Date** — The end of the life of a contract.

**Exponential Weighting** — A weighting scheme where the weight given to an observation  $t$  time periods ago is  $\lambda$  times the weight given to an observation  $t-1$  time periods ago where  $\lambda < 1$ .

**Forward Contract** — A contract that obligates the holder to buy or sell an asset for a predetermined delivery price at a predetermined future time.

**Forward Price** — The delivery price in a forward contract that causes the contract to be worth zero.

**GARCH Model** — A model for forecasting volatility where the variance rate follows a mean-reverting process.

**Historic Volatility** — A volatility estimated from historical data.

**Impact Costs** — The result of a bid-ask spread and a dealer's price concession. This is also called market or price impact costs.

**Implied Volatility** — Volatility implied from an option price using an option pricing model.

**Index Futures** — A futures contract on a stock index or other index.

**Index Option** — An option contract on a stock index or other index.

**Initial Margin** — The cash required from a futures trader at the time of the trade.

**In-the-money Option** — A call option is in-the-money where the asset price is greater than the strike price. A put option is in-the-money where the asset price is less than the strike price.

**Intrinsic Value** — The difference between the exercise price of an option and the current cash value of the underlying security.

**Long Position** — A position involving the purchase of an asset.

**Maintenance Margin** -- When the balance in a trader's margin account falls below the maintenance margin level, the trader receives a margin call requiring the account to be topped up to the initial margin level.

**Margin** — The cash balance or security deposit required from a futures or options trader.

**Margin Call** — A request for extra margin when the balance in the margin account falls below the maintenance margin level.

**Market Double** — See Bid-ask Spread.

**Mark-to-market** — See Marking to Market.

**Marking to Market** — The practice of revaluing an instrument to reflect the current values of the relevant market variables.

**Maturity Date** — See Expiration Date.

**Nonstationary Model** -- A model where the volatility parameters are a function of time.

**Offer Price** — See Ask Price.

**Option** — The right to buy or sell a fixed amount of a given stock at a specified price within a limited period of time.

**Out-the-money Option** — A call option is out-the-money where the asset price is less than the strike price. A put option is out-the-money where the asset price is greater than the strike price.

**Payoff** — The cash realised by the holder of an option or other derivative at its expiration date.

**Premium** — The price of an option contract.

**Principal Components Analysis** — An analysis aimed at finding a small number of factors that describe most of the variation in a large number of correlated variables.

**Put** — An option to sell an asset at a certain price by a certain date.

**Put-call Parity** — The relationship between the price of a European call option and the price of a European put option with equal strike prices and maturity dates.

**SAFEX** — The South African Futures Exchange.

**Short Position** — A position assumed when traders sell shares that they do not own.

**Spot Price** — The price for immediate delivery.

**Stock Index** — An index monitoring the value of a portfolio of stocks.

**Stock Option** — An option on a stock.

**Strike Price** — See Exercise Price.

**Theta** — The rate of change of the price of an option or other derivative with the passage of time.

**Time Decay** — See Theta.

**Time Value** — The value of an option arising from the time left to maturity. This equals the option's price minus its intrinsic value.

**Underlying** — The security that one has the right to buy or sell.

**Variation Margin** — An extra margin required to bring the balance in a margin account up to the initial margin when there is a margin call.

**Vega** — The rate of change in the price of an option or other derivative with volatility.

**Volatility** — A measure of the uncertainty of the return realised on an asset.

**Volatility Skew** — A term used to describe the volatility smile when it is nonsymmetrical.

**Volatility Smile** — The variation of implied volatility with strike price.

**Writing an Option** — Selling an option.