

UNIVERSITY OF CAPE TOWN
DEPARTMENT OF STATISTICAL SCIENCES

**GAME-THEORETIC MODELS
FOR MERGERS AND ACQUISITIONS**

by

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A thesis prepared under the supervision of Professor
Theodor J Stewart in fulfilment of the requirements for the
degree of Doctor of Philosophy

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To Erica and Laura May

ACKNOWLEDGEMENTS

I am indebted to the following who have offered valuable assistance and support during the preparation of this thesis:

- 1) Professor Theodor Stewart, for his erudite and enthusiastic supervision throughout the entire duration of this thesis.
- 2) Professor Graham Barr and Dr David Bowie, for a continuous stream of helpful hints and advice about WordPerfect.
- 3) Dr Robert Bosch, who offered useful insights at crucial times.
- 4) My fellow colleagues in the Department of Statistical Sciences at The University of Cape Town, who showed constant interest in my work.
- 5) My wife Erica and my parents who encouraged me all the way through.

The financial assistance of the Centre for Science Development (HSRC, South Africa) towards this research is hereby acknowledged. Opinions expressed in this work, or conclusions arrived at, are those of the author and are not to be attributed to the Centre for Science Development.

ABSTRACT

This thesis examines the corporate merger process as a bargaining game, under the assumption that the two companies are essentially in conflict over the single issue of the price to be offered by the acquirer to the target.

The first part of the thesis deals with the construction and testing of analytical game-theoretic models to explain the proportion of the synergy gains accruing to the target company under different assumptions about the players' a priori knowledge. Assuming full certainty amongst the players about the pre- and post-merger values of the companies, the distribution of gains between target and acquiring companies that would be consistent with the Nash-Kalai axioms is determined in principle. The resulting model depends on the players' utility functions, and is parameterised by the relative bargaining strength of the players and their risk aversion coefficients. An operational version of the model is fitted to empirical data from a set of 24 recent mergers of companies quoted on the Johannesburg Stock Exchange. The model is shown to have good predictive power within this data set.

Under the more realistic assumption of shared uncertainty amongst the two players about the post-merger value of the combined company, a Nash-Kalai bargaining model incorporating this uncertainty is developed. This model is an improvement over those with complete certainty in that it offers improved model fit in terms of predicting the total amount paid by an acquirer, and is able to dichotomise this payment into a cash amount and a share transfer amount. The theoretical model produced some results of practical value. Firstly, a cash-only offer is never optimal. Conditions under which shares only should be tendered are identified. Secondly, the optimal offer amount depends on the form of payment and the level of perceived risk. In a share-only offer the amount is constant regardless of risk, whilst if cash is included an increase in risk will imply a decrease in the optimal amount of cash offered. The Nash-Kalai model incorporating shared uncertainty is empirically tested on the same data set used previously. This allows a comparison with earlier results and estimation of the extent of the uncertainty.

An extension of this model is proposed, incorporating an alternative form of the utility functions.

The second part of the thesis makes use of ideas from negotiation analysis to construct a dynamic model of the complex processes involved in negotiation. It offers prescriptive advice to one of the players on likely Pareto-optimal bargaining strategies, given a description of the strategy the other party is likely to employ. The model describes the negotiating environment and each player's negotiating strategy in terms of a few simple parameters.

The model is implemented via a Monte Carlo simulation procedure, which produces expected gains to each player and average transaction values for a wide range of each of the players' strategies. The resulting two-person game bimatrix is analysed to offer general insights into negotiated outcomes, and using conventional game-theoretic and Bayesian approaches to identify "optimal" strategies for each of the players. It is shown that for the purposes of identifying optimal negotiating strategies, the players' strategies (described by parameters which are continuous in nature) can be adequately approximated by a sparse grid of discrete strategies, providing that these discrete strategies are chosen so as to achieve an even spread across the set of continuous strategies. A sensitivity analysis on the contextual parameters shows that the optimal strategy pair is very robust to changes to the negotiating environment, and any such changes that have the players start negotiating from positions more removed from one another is more detrimental to the target.

A conceptual decision support system which uses the model and simulated results as key components is proposed and outlined.

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CHAPTER 1

INTRODUCTION

§1.1 MERGERS, ACQUISITIONS AND GAMES

Growth of a company by acquisition of another going concern (as opposed to internal expansion) is one strategic alternative that can be used by management to achieve its planned growth targets. A merger or acquisition can thus be considered as a special case of capital budgeting, where it is assumed that management's objective is to maximise the wealth of the shareholders who have placed their confidence in them by investing in the company. Mergers and acquisitions are not purely a phenomenon of the latter part of the twentieth century: amalgamations between businesses are as old as business itself. However the number of mergers that have taken place since the "frantic merger activity" (Rappaport (1979) p. 99) of the 1960's, and the corresponding total monetary value of the transactions indicate that merger activity is an integral part of modern industrial and commercial development. Indeed, in recent years an entire vocabulary has built up within the merger industry: today one hears of a threatened target company laying down a poison pill or a shark repellent to ward off a dawn raid, and companies attempting a bear-hug or a pac-man if a suitable white knight is unavailable!

Broadly speaking, a merger is simply the linking of two or more companies, either by consolidating the original companies into a single entity or by absorption of one (or more) by the other. In practice this can be achieved in a number of ways (Macgregor (1979) p.29, Jensen and Ruback (1983) p.6). A merger, in the strict sense of the word, refers to a combination in which all the assets and liabilities of the selling company are transferred to, and absorbed by, the buying company. The seller, in effect, disappears as a separate entity. Usually a merger is negotiated in an amicable manner between the two companies' managements, and if an agreement is reached it is put to the vote at a shareholder's meeting. In a hostile takeover attempt (i.e. one which does not have the

support of the selling company's management) the buying company can approach the seller's shareholders individually with a **tender offer**, and thereby bypass the seller's management completely. Providing that the buying company can accumulate enough of the selling company's voting stock it can call a shareholder's meeting and force through the merger. An alternative way of bypassing the selling company's management is to attempt to get the right to vote the selling shareholders' shares at an annual meeting of the selling company. This is called a **proxy contest**, and is often expensive and difficult to win. The buyer in a proxy contest generally has a large percentage holding in the selling company in the first place.

The acquiring company could pay for the purchase of the selling company (or its shares) by means of a cash payment, an exchange of common stock, preferred stock or debentures, or a combination of both cash and shares. The corporate merger is thus seen to be an extremely complex capital budgeting decision process: not only does a potential acquiring company's management team have to identify a potential target company and value it accurately, it must also consider issues such as potential synergies, the available financing options, a negotiating strategy, tax considerations, the timing of the offer and many other related issues. The finance literature abounds with work relevant to determining offers in a merger or acquisition situation. For instance, the question of how large the gains to the shareholders of the two companies are has been studied extensively by Mandelker (1974), Langetieg (1978), Dodd (1980), Asquith (1983), Bradley, Desai and Kim (1983), Jensen and Ruback (1983), Affleck-Graves, Flach and Jacobson (1988) and Van den Honert, Barr, Affleck-Graves and Smale (1988) amongst others, and the choice of a medium of exchange has been investigated by Hansen (1987), Fishman (1989), Carleton, Guilkey, Harris and Stewart (1983) and Travlos (1987). However, one area in the so-called market for corporate control that has not been adequately researched is that of the negotiating or bargaining strategies and processes that must of necessity occur between the competing management teams. Some attempts at quantitative modelling of general negotiation processes do exist (see Kersten (1985), Kersten and Szapiro (1986), Fraser and Hipel (1984), Jarke, Jelassi and Shakun (1988)). However these approaches tend to produce models and procedures which are formally elegant but inflexible and inaccessible to practitioners (Sycara (1990), Raiffa (1982)). Thus part of

this thesis will revolve around building and testing a model of the dynamic merger negotiation process which might prove useful in offering decision support to negotiating practitioners involved in mergers.

Since it is reasonable to assume that both companies are wealth maximisers, there will be an element of competition between the two "players" arising out of their conflicting objectives. They can thus be construed to be playing a "merger game" of strategy. The most significant contribution to the early study of game theory was Von Neumann and Morgenstern's (1947) *"The Theory of Games and Economic Behaviour"*, and this aroused such interest in the field that a complete literature now exists. Standard works include Luce and Raiffa (1957) and Owen (1982). An important class of game is the **bargaining game**, which was considered by Nash (1950), who proposed an axiomatic solution to the two-player case. A lot of game-theoretic literature on bargaining makes the assumption of complete information on the part of both players (see Harsanyi (1965) and Roth (1979, 1987)), but bargaining games with incomplete information have also been researched (Chatterjee and Samuelson (1983), Samuelson (1984) and Harsanyi (1968a,b,c)).

A special case of the two-player bargaining game, which in many respects closely resembles the merger bargaining situation, is the so-called **ultimatum game** (see Thaler (1988), Güth and Thietz (1990)). In this type of bargaining game Player 1 (called the Allocator) must divide some amount Ω (a monetary amount, or some other commodity measured on a continuous scale) between himself and Player 2 (the Recipient). If the offer x is accepted by the Recipient then the Allocator receives $\Omega - x$ and the Recipient receives x . If the offer is rejected then both players receive nothing. Rubinstein (1982) suggested that if both players act rationally then the Allocator need offer the Recipient only a small positive amount ϵ , and the Recipient should accept ϵ , since $\epsilon > 0$, the payoff due to him if he rejects the offer. The first laboratory experimental evidence on ultimatum games came from Güth, Schmittberger and Schwarze (1982), who showed that real subjects do not act in accordance with Rubinstein's model, with the mean offer by the Allocator being near 0.37Ω and almost all offers equal or close to zero being rejected by the Recipient. An explanation is that when the Recipient declines a small but positive

offer he is sending the signal that he would rather sacrifice ϵ than accept what he considers to be an unfair allocation of the stake. The actions on the part of the Allocator could be explained by the motive that he has a taste for fairness and/or is worried that the Recipient would be likely to decline an unfair offer of only ϵ . Experimental work on the ultimatum game described above, and on slight variations thereof, has been carried out by Kahneman, Knetsch and Thaler (1986) and Hoffman and Spitzer (1982, 1985), and all produced results similar to Güth et al. This game has been extended to two-stage bargaining (Binmore, Shaked and Sutton (1985), Güth and Tietz (1987)) and to multi-stage games (Neelin, Sonnenschein and Spiegel (1987), Ochs and Roth (1988)). In the context of the merger bargaining situation at hand, the acquirer assumes the role of the Allocator, and the target the Recipient. The amount Ω is the synergistic gain created by the unification of the two separate entities, and the offer amount x is the proportion of the synergy accruing to the target company's shareholders.

Analogous to the case of the ultimatum game in its general form, it is of interest to investigate how the overall synergistic gains arising out of a company merger (we will group together operational synergies and those arising from accounting/taxation advantages) are split between the shareholders of the two participating companies (Roy (1989) p. 594). A large portion of this thesis will be devoted to the construction and empirical testing of game-theoretic models to aid in this investigation. These models, if successful, will help to shed light on, and provide understanding of, the process of merger bargaining.

§1.2 PREVIOUS STUDIES OF MERGERS IN A GAME-THEORETIC CONTEXT

Little work has been published explicitly modelling a corporate merger or acquisition as a mathematical game. Milnor and Shapley (1978) developed the concept of an oceanic voting game in the early 1960's, in which certain fixed fractions of the voting strength are held by a few 'major' players, while the rest is scattered amongst a large number of individually insignificant and indistinguishable 'minor' players, collectively referred to as an 'ocean' to suggest the homogeneity of the group and its total lack of order or cohesion. As an example of such an oceanic game they studied a

corporation with common stock held by two large stockholders who own significant portions of the corporation, and numerous minor stockholders (aggregated into a group, the 'ocean') who individually own very small portions of the corporation. These minor stockholders hardly ever investigate policy matters, and most likely do not know each other, so that it is usually impractical for them to coordinate their actions. It was assumed that control of the corporation hinges entirely on a simple majority vote of the stock. They investigated the distribution of power of the two major stockholders as the distribution of stock among each of them and the ocean varies. They identified distributions of shares where each major stockholder has absolute control and no control over the policies of the corporation, and where they hold a balance of power, and where the ocean has a significant share of the power.

Powers' (1987) work was based on Milnor and Shapley's, but instead of being concerned with the distribution of power in terms of stockholdings, a model of exchange between cash and shares (voting power) was developed to study the feasibility and profitability of corporate takeovers. The threat-resistant Nash equilibrium strategy was found which would result in one major stockholder taking control of the company. Powers, however, did not consider uncertainty, and assumed that the transaction price was the highest price quoted by the seller. On the contrary, Roy (1989) argued that a merger or acquisition involves bargaining, i.e. that a buyer generally makes an offer and the seller merely accepts or rejects it. Furthermore, there is always an element of uncertainty in the valuation of the target company, since its value is merely the present value of its future cash-flows (Myers (1976)), which cannot be known in advance with certainty (Rappaport (1979)). Hence there may well be some disagreement on the price to be paid; the two parties will bargain, and may or may not agree on a price. Roy thus modelled the corporate takeover process as a bargaining game under uncertainty. He derived the optimal offer strategies for a buyer when there is uncertainty about the minimum price (which is dynamic and may change during the offer process) that the target shareholders would accept. The uncertainty about the target's minimum acceptable price was represented by some probability distribution. This was the first financial model which represented a merger as a bargaining game and which provided guidance to a firm as to the optimal bargaining strategies during a takeover.

§1.3 THE MERGER BARGAINING PROCESS

As Roy (1989) has argued, a merger or acquisition only takes place after some negotiating or bargaining between two or more parties, one of whom is the target (or more correctly, the group of target shareholders) and the others who are prospective acquirers. In the case of a friendly takeover attempt a prospective acquirer may bargain indirectly with the target shareholders, using the target firm's management as a negotiating agent. On the other hand, in a hostile tender offer the negotiations usually take place directly with the target shareholders, thus bypassing the target company's management who may not be acting in the best interests of the target shareholders, or who themselves might become a potential acquirer. The models we will develop throughout this thesis do not distinguish between direct and indirect bargaining: what is important is whether or not the offer is accepted. Whilst there may be hundreds or even thousands of individual target shareholders, the acquirer generally makes a single offer to all of them and does not negotiate prices with each of them individually. Thus we may treat the target shareholder group as a single bargaining entity.

There are several reasons why the parties bargain over an acceptable price at all. Firstly, as mentioned above there is uncertainty about the present value of the target's future cash-flows, which implies that the two parties may well arrive at different values for the target company. Secondly, at any stage in the bargaining process the target company forms an idea of its minimum acceptable price and the acquirer forms an idea of the maximum price it would be prepared to pay, given all available information, including the evaluation of any synergistic gains which may accrue from a successful merger. These prices are not likely to be disclosed to one another. Naturally, the target would like to settle on a price as much as possible over its minimum while the acquirer would prefer the transaction to occur at a price as close to the target's minimum as possible. The two parties may try to resolve this conflict of interests by means of bargaining.

A corporate merger can be thought of as a multi-stage, multi-party bargaining game, where the parties are the target and all prospective acquirers, and the stages are

each offer made by a prospective acquirer and the associated reaction from the target. A prospective acquirer makes a price offer, acting independently of the target. If the target accepts the offer, a transaction is concluded at the offer price. If the target rejects a prospective acquirer's offer at any stage, any of the prospective acquirers may make a new offer (the next stage). Since it is assumed that target management's prime objective is shareholder wealth-maximisation, it follows that rejection of an offer would only occur if the offer was less than a price which the target believes to be the minimum acceptable price given all available information and/or the target believes that a better price offer will be forthcoming from one of the prospective acquirers in due course. The offer process continues until either the target accepts an offer or the target rejects all offers and/or the acquirers all do not proceed with further offers. A prospective acquirer will, of course, withdraw from the offer process if there are no perceived economic benefits to be had from the merger. All offers are public in the sense that the parties involved have complete information about the offers.

§1.4 ORGANISATION OF THE THESIS

This thesis consists, in essence, of a set of mathematical models which describe the merger and acquisition conflict situation under various knowledge scenarios. In Part A we will propose and test several analytical models. To begin with, in Chapter 2 we put aside the complexities associated with the uncertainties that may exist in the real world and consider the completely deterministic case, in which both acquiring and target company has full and certain information about the financial worths of the two companies involved in the merger negotiations as separate entities, and as a combined unit. We will construct a model based on a generalisation of the axiomatic solution to the bargaining problem proposed by J.F. Nash (1950), which will explain how the synergistic gains from merger may be split between the acquiring and target companies. This level of model will be empirically tested on a data-base of recently-observed mergers on the Johannesburg Stock Exchange (JSE), on the assumption that observed market values can be used as surrogates for the (deterministically-known) financial "worth" of the individual companies.

This model is, of course, a gross simplification of reality, but does provide insights into key issues in the bargaining process. In particular though, the concept of uncertainty has been ignored entirely. Realistically stated, before negotiations actually commence it is extremely unlikely that the parties will have deterministic knowledge of the post-merger value of the combined entity; this amount will be estimated by both companies' experts and advisors subjectively and independently of each other, by evaluating aspects of the proposed combined company deemed important by the respective managements, which may of course differ from company to company. They would analyze various available company records in an attempt to predict future cash flows after merger which, after discounting, would yield a subjective estimate of the post-merger value of the merged company. In Chapter 3 we discuss the inclusion of uncertainty in the previously-developed generalised Nash model. In particular, the estimation of the uncertainty, or pre-merger perceived risk, present in the merger is one of the main aims of the modelling process. The presence of uncertainty in the model provides a further advantage: it facilitates a dichotomy of the payment by the acquirer into a cash portion and a share portion. Again the model will be tested empirically on the sample of recently-observed mergers on the JSE.

In Part B we will drop the assumption that the two players act in accordance with the precepts of game-theoretic rationality (a necessary assumption in Part A), and in so doing hopefully get closer to a model that more accurately represents the real-world bargaining situation. We will investigate how the merger participants formulate offers and responses to these offers, using ideas from negotiation analysis. Thus in Chapter 4 we will advance a parsimonious description of a "negotiation strategy" for each player, and build these into a dynamic, multi-stage model of the bargaining process, assuming symmetric uncertainty on the part of each player about the other's reservation price and strategy. The model may thus be useful in offering prescriptive advice to one party on what might prove to be a "good" strategy to employ, given some sort of description of the other player's negotiating strategy. Apart from the player's negotiating strategies, the model will also consider the effect of external conditions, such as the players' relative bargaining strengths.

The model we will develop in Chapter 4 will make use of parameters which are judgemental and subjective in nature, and so empirical data is not available at this level of modelling. In Chapter 5 we will thus implement a Monte Carlo simulation of the model which will describe typical long-term average outcomes of similar negotiations. In particular we will examine the probability of achieving successful merger agreement and the expected transaction value for various combinations of the two players' negotiating strategies. This will provide the players with expectations of the gains that they might expect to earn as a proportion of the overall synergy created from merger. This will allow a focus on what might prove to be "optimal" negotiating strategies for the two players.

In Chapter 6 we will investigate the effect of the bargaining environment (i.e. the external conditions that do not form part of the negotiators' strategies) on the optimal strategies and the associated expected payoffs to the players. We will specifically look at the robustness of the optimal strategies to changes in the players' perceptions of the magnitude of uncertainty and players' relative bargaining strengths.

We propose that the model to be developed in Chapter 4 and the simulation results of Chapters 5 and 6 could form the basis of a useful management decision support system regarding the choice of bargaining strategies to be employed by real-world merger negotiation practitioners. In Chapter 7 we will briefly examine the role of decision support systems in helping management involved in merger activity, and propose a conceptual system which will incorporate the model and a set of simulated outcomes to aid in determining optimal negotiating strategies.

The final chapter summarises the main findings and conclusions of the thesis and offers a few suggestions for extending this research.

PART A

ANALYTICAL MODELS AND EMPIRICAL RESULTS

CHAPTER 2

SIMPLE BARGAINING MODELS WITH FULL AND SHARED INFORMATION FOR MERGERS AND ACQUISITIONS

§2.1 INTRODUCTION

We consider the negotiations preceding a merger between two companies. We model the merger or acquisition as a conflict situation in which the conflicting parties may co-operate for mutual benefit. Thus some bargaining may ensue with both parties secure in the knowledge that if either company breaks off the negotiations they are left with what they started with; that is, the two separate entities. This specialised conflict situation is an example of the generalised bargaining problem which was solved in an axiomatic way by Nash (1950).

We assume in this chapter¹ that all relevant information relating to the financial worths of the two companies as separate entities pre-merger and as a single combined unit post-merger is known *with certainty* by both parties involved. We will employ a generalised version of Nash's solution to construct and empirically test a series of parsimonious descriptive models of the behaviour of acquirers and targets (represented by their respective management teams) during the negotiation phase using the above certain information. The aim of the models will be to give some explanation of how the synergistic gains to be had from the merger may be divided between the two parties. This will yield an understanding of a fair price that a prospective acquirer should be prepared to pay to a prospective target and as such may form the basis of a simple decision support system under certainty.

¹ Parts of this chapter have been published in *European Journal of Operational Research*, 59, 1992. The paper is entitled "A game-theoretic model for mergers and acquisitions" (see Van den Honert and Stewart (1992)).

In §2.2 we describe the assumptions of this chapter in detail and their implications, and introduce the notation to be used. The following section describes the Nash axioms and the generalised Nash solution to the bargaining problem, and discusses the application of this solution to the merger situation. As will be indicated in §2.3, the Nash solution requires that the gains to both parties be in utility terms. It is thus necessary to define utility functions for the acquiring and target companies so that the bargaining model may be tested empirically. Thus in §2.4 we propose two distinct families of utility functions: a simple linear family and a negative exponential family. We derive the bargaining models and discuss how the parameters may be estimated. We choose a sample of mergers from listed companies on the Johannesburg Stock Exchange, describe the collection of data, and present our empirical findings in §2.5. In the final section the two families of utility models are compared on the basis of their empirical results.

§2.2 NOTATION AND ASSUMPTIONS

We suppose that it is possible to associate precise financial worths with each company in the absence of a merger, which will be viewed conceptually as net present values. We call these PV_A and PV_B for the financial worths of the acquirer and target respectively. In §2.5.1 of this chapter we indicate how these may be assessed practically on the basis of current market values, i.e. by making use of the current stock price and the number of shares in issue. We suppose further that the corresponding worth of the merged company is also known with precision, which we shall call PV_{AB} . In this chapter we assume for simplicity that PV_A , PV_B and PV_{AB} are known to both parties with certainty. In the next chapter we will relax this assumption by assuming that the post-merger worth of the combined company is a random variables having some probability distribution. This certainty assumption about PV_A , PV_B and PV_{AB} implies that only net returns, and not any risk considerations, will be relevant to the analysis.

In this context we define the total net gain as

$$G = PV_{AB} - PV_A - PV_B.$$

On the assumption of rationality of net value maximisers, it is evident that no deal will be forthcoming if $G \leq 0$. Let Q be the net amount allocated to the target (that is, their shareholders). This may be any combination of cash and shares in the merged company, but under conditions of complete certainty these are effectively equivalent, and thus only Q is relevant to the present model. Invoking again the assumption of rational net value maximisers, it is clear that in any acceptable deal

- (i) $Q > PV_B$ (otherwise there is no benefit to B),
- (ii) $Q < PV_{AB} - PV_A$ (otherwise there is no benefit to A).

Thus PV_B is assumed to be the target company's **reservation price**; that is, the minimum price that the target would accept. Similarly, $PV_{AB} - PV_A$ is the acquiring company's reservation price, or the maximum price that the acquirer would be willing to offer. Since $G > 0$ by assumption, i.e. $PV_{AB} - PV_A > PV_B$, the **zone of agreement** for Q (the net amount allocated to the target) exists and is the interval from PV_B to $PV_{AB} - PV_A$. The acquirer's **net gain** or surplus value is

$$g_A = PV_{AB} - PV_A - Q$$

and the target's net gain or surplus value is

$$g_B = Q - PV_B.$$

At any stage of the bargaining game the target will be interested in setting Q as close to the maximum ($PV_{AB} - PV_A$) as possible, whilst the acquirer would be concerned with offering a value of Q as close to the minimum (PV_B) as possible. The above discussion is depicted in Figure 2.1.

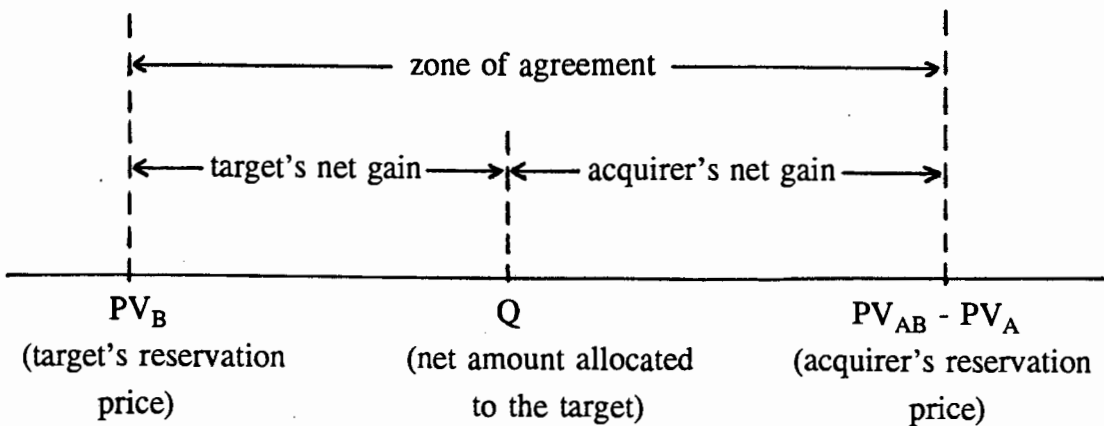


Figure 2.1. The geometry of the bargaining model with full and shared information

The sum of the net gains to both acquirer and target is simply G , which is independent of the intervening value Q . Thus providing the two parties come to a mutually acceptable agreement within the zone of agreement this bargaining game is constant-sum in net gains.

In practice, of course, situations do arise in which $Q \leq PV_B$ or $Q \geq PV_{AB} - PV_A$ i.e. the merger is irrational by our definition. In terms of our notation this implies that the agreed-upon value of Q lies outside the zone of agreement $[PV_B; PV_{AB} - PV_A]$. Many reasons may exist for this phenomenon. For example, the acquirer's management may perceive an increase in status or power in heading up a large conglomerate, and will actively pursue a growth policy through merger in order to achieve their objectives. They may well offer an "irrationally" large amount to ensure their goals. On the other hand, target management may well accept a "golden handshake" in return for accepting an offer favourable to the acquirer, and vacating their position. Furthermore, it is possible that certain company managements might be led to economically irrational behaviour simply through the thrill of achieving an apparent victory in bargaining over another, or the dread of being beaten. In any event the model in this chapter assumes that PV_{AB} is known with certainty a priori, and so an irrational merger should theoretically not arise. In practice PV_{AB} is only observable *after* the merger has occurred, and this explains why some "irrational" mergers do occur. We will not consider these "irrational" situations in our model.

We now define $Y = g_B / G$, the proportion of the net gain allocated to the target. In view of our rationality assumptions it follows that $0 \leq Y \leq 1$. Due to problems of scale it will be convenient to develop the model in terms of Y .

The aim of this model will be primarily to predict Y . By means of empirically fitting our model to a random set of mergers we will show that this model is a reasonable first representation of the observed situation and hence yields some understanding of the real-world bargaining process.

§2.3 A NASH-KALAI SOLUTION TO THE GAME

We model the negotiations between companies A and B as a co-operative two-person non-zero sum game. Nash (1950) considered the two-person bargaining problem in which the resulting feasible payoffs to the players form a closed bounded convex subset S of the plane (called the feasible region) with some status quo point (u^*, v^*) in S , which is the outcome if there is no agreement. Nash defined a **bargaining solution** (or arbitration scheme) as a function ϕ which maps the triple (u^*, v^*, S) into some unique point (\bar{u}, \bar{v}) in S , i.e.

$$\phi(u^*, v^*, S) = (\bar{u}, \bar{v}) .$$

Nash's axiomatic solution to the bargaining problem is in terms of payoffs to each player (in the sense that more of u is always preferred by one party and more of v is always preferred by the other party), relative to the status quo point (u^*, v^*) . These payoffs may directly be monetary returns, but need not be; we will call u and v "utilities". The only properties claimed for these utilities at this stage are those specified in Nash's axioms defined below (for example, these are not necessarily Von Neumann-Morgenstern utilities).

Nash defined the following axioms as being reasonable for some independent arbitrator to follow in arriving at a satisfactory bargaining solution ϕ :

N1 (Individual rationality)

$$\bar{u} > u^* , \bar{v} > v^* .$$

The arbitrated outcome must be at least as good as the status quo.

N2 (Feasibility)

$$(\bar{u}, \bar{v}) \in S .$$

The arbitrated solution must lie in the feasible region.

N3 (Pareto optimality)

$$\text{If } (u, v) \in S , u \geq \bar{u} \text{ and } v \geq \bar{v} , \text{ then } (u, v) = (\bar{u}, \bar{v}) .$$

The arbitrated value must not be bettered by any other feasible point.

N4 (Independence of irrelevant alternatives)

If $(\bar{u}, \bar{v}) \in T \subset S$ and $(\bar{u}, \bar{v}) = \phi(u^*, v^*, S)$, then $(\bar{u}, \bar{v}) = \phi(u^*, v^*, T)$.

If certain feasible outcomes are eliminated from a bargaining problem but the arbitrated solution (\bar{u}, \bar{v}) remains available, then this remains the arbitrated solution.

N5 (Invariance with respect to affine utility transformations)

Let S' be obtained from S by the transformation

$$u' = a_1 u + b_1 \quad (a_1 > 0)$$

$$v' = a_2 v + b_2 \quad (a_2 > 0).$$

Then if $\phi(u^*, v^*, S) = (\bar{u}, \bar{v})$ we require that

$$\phi(a_1 u^* + b_1, a_2 v^* + b_2, S') = (a_1 \bar{u} + b_1, a_2 \bar{v} + b_2).$$

The payoffs, or utilities, u and v are thus presumed to be measured on an interval preference scale. In other words, if every payoff is increased by the same fixed amount b , then preferences between outcomes remain unchanged. That is, the preferential value gained in moving from u' to u'' is the same as that gained in moving from $u'+b$ to $u''+b$. Financial gains do not in general satisfy this property however: for any single party the marginal utility gain of one further monetary unit is likely to decrease as the payoffs increase.

N6 (Symmetry)

If (u^*, v^*, S) is such that

$$(i) \quad u^* = v^*$$

$$(ii) \quad (u, v) \in S \Rightarrow (v, u) \in S$$

and $(iii) \quad \phi(u^*, v^*, S) = (\bar{u}, \bar{v})$,

then $\bar{u} = \bar{v}$.

If an abstract version of a bargaining game places the players in completely symmetric roles, the arbitrated outcome shall give them equal utility payoffs, where utility is measured in the units which made the game symmetric.

In the context of the merger bargaining problem, and assuming that the participating companies prefer more financial payoff to less, Nash's axioms may be interpreted as follows:

For an arbitrary merger the value of the two participating companies' shareholder's wealth after a merger is agreed upon must be at least as great as its value in the absence of a merger (i.e. they must not lose by participating in the merger) (N1) and must be realistically obtainable (N2). Furthermore it must not be possible to achieve any further simultaneous increase in shareholder wealth to both parties from *this* merger (N3). Assume that many potential deals exist which each could affect the synergy from the merger. If a number of these are withdrawn (i.e. the range of possible payoffs is reduced) but the current offer providing the greatest payoff to the shareholders is not withdrawn, then shareholder wealth will remain unchanged (N4). Payoff is in utility terms, and it is immaterial what units of utility payoff are used (N5). Finally, it is assumed that an acquirer and a target would bargain identically if their roles were reversed and they each had access to all the other's a priori information (N6).

In his pioneering work, Nash (1950) proved the following theorem:

Suppose there exist points $(u,v) \in S$ with $u > u^*$ and $v > v^*$, and that the maximum of

$$g(u,v) = (u - u^*) (v - v^*)$$

over this set is attained at (\bar{u}, \bar{v}) . Then the point (\bar{u}, \bar{v}) is *uniquely* determined, and the function $\phi(u^*, v^*, S) = (\bar{u}, \bar{v})$ is the unique function which satisfies axioms N1 to N6 above. The theorem is proved and extensively discussed in, for example, Luce and Raiffa (1957), Owen (1982) and Jones (1980).

In the problem at hand the feasible space S consists of union of $(0,0)$ (the status quo point, discussed below) and the utilities corresponding to monetary rewards satisfying $g_A + g_B = G$. The space S is non-convex, but can be made convex if we allow a randomised rule to apply; this implies the assumption that utilities for gambles are linear in probabilities (i.e. utilities satisfy the Von Neumann-Morgenstern expected utility hypothesis). The utilities u and v will be assumed to be derived as functions of the net gains to the acquirer A and to the target B , namely g_A and g_B , or equivalently of $1-Y$ and Y , where $Y = g_B/G$. We will call these utility functions $U_A(1-Y)$ and $U_B(Y)$.

For convenience we will define the origin for the utility functions such that $U_A(0) = U_B(0) = 0$, giving the status quo point (0,0). That is, if no agreement is reached, neither party gets anything of the overall net gain, and this situation has a utility value of zero to both parties. Under the assumption that each company is rational and prefers more financial net gain to less, all else being equal, $U_A(\cdot)$ and $U_B(\cdot)$ must be increasing in their arguments.

In the context of the merger bargaining game at hand, the Nash bargaining solution is defined by $g_A = (1-Y)G$ and $g_B = YG$, where Y maximises

$$U_A(1-Y) U_B(Y)$$

over $0 \leq Y \leq 1$.

A generalised version of the Nash solution is that due to Kalai (1976); he showed that by relaxing axiom N6 (symmetry) the **non-symmetric** Nash solution (or Nash-Kalai solution) would be to maximise

$$[U_A(1-Y)]^\gamma [U_B(Y)]^{1-\gamma}$$

where $0 < \gamma < 1$. Note that the symmetric Nash solution is equivalent to the non-symmetric case with $\gamma=0.5$. We term the parameter γ the **negotiating power** of company A; it is in some sense a measure of the relative power vested in the acquirer vis-a-vis the target at the bargaining table. Thus the model requires a knowledge of the functional form of the utility functions of the two companies involved in the merger, as well as the negotiating power pertaining to the pair.

§2.4 OPERATIONALISATION OF THE MODEL AND PARAMETER ESTIMATION

For a model such as the Nash-Kalai solution to be operationally successful, it must be possible to identify the utility functions $U_A(\cdot)$ and $U_B(\cdot)$. This is necessary both in fitting the model to empirical data as a test of its validity (as we do in §2.5), and in using the model as the basis of a decision support system to help the parties involved to predict outcomes of future mergers.

§2.4.1 A LINEAR UTILITY MODEL

As we have argued, $U_A(\cdot)$ and $U_B(\cdot)$ will be assumed to be increasing functions of their arguments. Initially we assume the *simplest* such function, that is where the utility value of net monetary gain is linear. We call these the family of **linear utility functions**, and thus suppose that both $U_A(\cdot)$ and $U_B(\cdot)$ have the general property that

$$U(x) = x$$

for all x , i.e. that

$$U_A(1-Y) = 1-Y$$

and

$$U_B(Y) = Y$$

where (0,0) is the status quo point.

The Nash-Kalai optimal solution is obtained from

$$\text{Maximise } F = (1-Y)^\gamma Y^{1-\gamma} \quad , \quad 0 < \gamma < 1 . \\ 0 \leq Y \leq 1$$

This is equivalent to solving

$$\text{Maximise } \Psi = \gamma \ln(1-Y) + (1-\gamma) \ln Y . \\ 0 \leq Y \leq 1$$

Since γ is assumed to lie inside the interval (0,1), the maximum of Ψ occurs away from the boundary, i.e. at $d\Psi / dY = 0$. We obtain

$$-\frac{\gamma}{1-Y} + \frac{1-\gamma}{Y} = 0$$

as the necessary condition for Y to be the Nash-Kalai solution. The solution of this is

$$Y = 1 - \gamma \tag{2.1}$$

which is a function only of the parameter γ . This is not yet sufficient to give a fully operational model, as it appears *prima facie* unlikely that γ will be a universal constant applicable to all mergers. It is likely that γ will be dependent at least on the *relative size* of the acquiring company to the target company.

For empirical testing of the model in §2.5 we have made the simplifying assumption that γ is a function only of the ratio of company sizes, PV_A / PV_B . It is tempting to use a linear functional relationship as an approximation, but this will not in

general obey the restriction that γ must lie in the unit interval. We thus propose functions of the form

$$\ln[\gamma / (1 - \gamma)] = \alpha_1 + \beta_1 (PV_A / PV_B) \quad (2.2a)$$

or

$$\ln[\gamma / (1 - \gamma)] = \alpha_2 + \beta_2 \ln(PV_A / PV_B). \quad (2.2b)$$

We now have a simple functional form of model which we may attempt to apply across a whole spectrum of mergers, and which would predict the final value of Y as a function of PV_A and PV_B . The general function depends on only two parameters (α_1 and β_1 , or alternatively α_2 and β_2) which in principle can be estimated from empirical data. This also enables us to evaluate the model fit.

To see how well our Nash-Kalai bargaining model fits actual observed values of Y , it is necessary to collect the information vector $\mathbf{x} = \begin{bmatrix} PV_A \\ PV_B \\ PV_{AB} \end{bmatrix}$ of ex-post assumed

known variables for each of a number of observed mergers. The estimation of \mathbf{x} from published data is described in §2.5.1. We will need to estimate a vector θ of unknown

parameters, where $\theta = \begin{bmatrix} \alpha_1 \\ \beta_1 \end{bmatrix}$ or $\theta = \begin{bmatrix} \alpha_2 \\ \beta_2 \end{bmatrix}$. For any θ , (2.2a) or (2.2b) allows us to

calculate γ for a particular merger, and thus also the predicted value of Y from (2.1). We call this predicted value $\hat{Y}(\mathbf{x}_i; \theta)$. Since Y and $\hat{Y}(\mathbf{x}_i; \theta)$ are standardised onto the unit interval, it is convenient to use the usual residual sum of squares to estimate an optimal value for θ , i.e. to estimate θ by minimising (over all possible values of θ)

$$\sum_{\text{all mergers}} (Y_i - \hat{Y}(\mathbf{x}_i; \theta))^2 \quad (2.3)$$

where Y_i is the actual proportion of the net merger gains accruing to the target in merger i , and $\hat{Y}(\mathbf{x}_i; \theta)$ is the predicted proportion of the net merger gains accruing to the target in merger i . Clearly (2.3) also provides a measure of the goodness-of-fit of the model.

§2.4.2 A FAMILY OF NEGATIVE EXPONENTIAL UTILITY MODELS

§2.4.2.1 THE GENERAL NEGATIVE EXPONENTIAL UTILITY MODEL

In many practical settings a linear utility function may not be representative of reality, and $U_A(\cdot)$ and $U_B(\cdot)$ might be expected to exhibit decreasing marginal returns to scale (i.e. to be concave) (see, for example, French (1988), p 172). The negative exponential function is one of the simplest such functional forms, and is in fact implied as the form of the utility function if certain behavioural properties are satisfied (cf. Keeney (1981)). In this section we suppose that both $U_A(\cdot)$ and $U_B(\cdot)$ have the general form

$$U(x) = ae^{-cx} + b \quad (a < 0).$$

Since we require that $U(0) = 0$, we have that $a = -b$. The scaling is irrelevant. We arbitrarily choose $a = -1$. Utility functions of the above form are well-known in decision-making *under uncertainty*, where the parameter c is called the **coefficient of risk aversion**. While not strictly true in the deterministic sense, we will continue to use this terminology for consistency. Thus we let $c=r_A$ be the coefficient of risk aversion for the acquirer and $c=r_B$ be the coefficient of risk aversion for the target. The coefficient of risk aversion is, in general defined as $-U''(x) / U'(x)$; for the negative exponential utility function above we have that $-U''(x) / U'(x) = c$ ($= r_A$ or r_B). Thus this form of the utility function implies *constant* risk aversion for all values of x for both acquirer and target. Then

$$U_A(1-Y) = 1 - e^{-r_A(1-Y)} \quad (r_A > 0)$$

and

$$U_B(Y) = 1 - e^{-r_B Y} \quad (r_B > 0),$$

and the Nash-Kalai optimal solution is found from

$$\text{Maximise } F' = [1 - e^{-r_A(1-Y)}]^\gamma [1 - e^{-r_B Y}]^{1-\gamma} \quad .$$

$$0 \leq Y \leq 1$$

This has the same solution as

$$\text{Maximise } \Psi' = \gamma \ln(1 - e^{-r_A(1-Y)}) + (1-\gamma) \ln(1 - e^{-r_B Y}) \quad .$$

$$0 \leq Y \leq 1$$

As in the case of the linear utility model, since γ lies inside the interval (0,1), the maximum of Ψ' occurs away from the boundary, i.e. at $d\Psi' / dY = 0$. This yields

$$\frac{-\gamma r_A e^{-r_A(1-Y)}}{1 - e^{-r_A(1-Y)}} + \frac{(1-\gamma) r_B e^{-r_B Y}}{1 - e^{-r_B Y}} = 0 \quad (2.4)$$

as the necessary condition for Y to be the Nash-Kalai optimum solution. Thus the model predicts Y as a function of only the parameters γ , r_A and r_B , and can be uniquely computed. From (2.4) we get

$$\gamma r_A e^{-r_A(1-Y)} [1 - e^{-r_B Y}] = (1-\gamma) r_B e^{-r_B Y} [1 - e^{-r_A(1-Y)}]$$

or

$$\gamma r_A e^{-r_A(1-Y)} - \gamma r_A e^{(r_A - r_B)Y - r_A} - (1-\gamma) r_B e^{-r_B Y} + (1-\gamma) r_B e^{(r_A - r_B)Y - r_A} = 0$$

which can be written as

$$A e^{r_A Y} - (B - D) e^{(r_A - r_B)Y} - C e^{-r_B Y} = 0$$

where

$$A = \gamma r_A e^{-r_A} > 0$$

$$C = (1-\gamma) r_B > 0$$

$$D = C e^{-r_A} > 0.$$

Now let $z = e^Y$. Then

$$A z^{r_A} + (D - B) z^{(r_A - r_B)} - C z^{-r_B} = 0.$$

Multiplying this by z^{r_B} yields

$$A z^{(r_A + r_B)} + (D - B) z^{r_A} - C = 0. \quad (2.5)$$

In principle, therefore, if we know the coefficients r_A , r_B and γ then the Nash-Kalai predicted point of agreement is given by $Y = \ln z$, where z is the solution (uniquely defined below) to (2.5). This is the basis of the negative exponential utility model for all empirical calculations.

In the special case in which $r_A = r_B$ (acquirer and target have equal risk aversion coefficients), (2.5) is exactly solved, since it reduces to

$$A z^{2r_A} + (D - B) z^{r_A} - C = 0$$

which is quadratic in z^{r_A} , and has solution

$$z^{r_A} = e^{r_A Y} = [(B - D) \pm \sqrt{(D - B)^2 + 4AC}] / 2A.$$

That is,

$$Y = \frac{1}{r_A} \ln \left(\frac{[(B - D) \pm \sqrt{(D - B)^2 + 4AC}]}{2A} \right) .$$

Since $A > 0$ and $C > 0$, it follows that for all values of A , B , C and D , the smaller root is always negative, and thus only the larger root yields a real-valued solution for Y . So in this case

$$Y = \frac{1}{r_A} \ln \left(\left[e^{-r_A} r_A (2\gamma - 1) + [e^{-2r_A} [r_A (1 - 2\gamma)]^2 + 4r_A^2 \gamma(1 - \gamma) e^{-r_A}]^{1/2}] / 2\gamma r_A e^{-r_A} \right) .$$

In the more general case where r_A cannot be assumed equal to r_B , a Newton-Raphson procedure may be employed to obtain the desired value of Y . As a starting point for the procedure we use the solution corresponding to the case where the sum of the risk aversion coefficients are set equal to 2 in (2.5), i.e. $x^2 = z^{r_A + r_B}$. Then $x = z^{(r_A + r_B)/2}$, and we start with the case $r_A = r_B$. Then

$$A x^2 + (D - B) x - C = 0 ,$$

which implies that

$$z = \exp \left[\left(\frac{2}{r_A + r_B} \right) \ln \left(\frac{[(B - D) \pm \sqrt{(D - B)^2 + 4AC}]}{2A} \right) \right] \quad (2.6)$$

is the required starting point. Providing that the initial values of the parameters are not too pathological, the procedure will converge to a point z^* within some suitable tolerance. We then calculate Y as $Y = \ln z^*$.

As in the case of the linear utility model, it appears prima facie that γ , r_A and r_B are likely to be dependent at least on the sizes of the organisations involved. We assume that r_A is a function of PV_A only, and r_B of PV_B only. Since r_A and r_B cannot be negative (to ensure an increasing concave function) we cannot use a linear function to approximate these relationships. We thus propose

$$\ln r_A = \alpha_A + \beta_A PV_A , \quad (2.7)$$

or equivalently $r_A = e^{\alpha_A} e^{\beta_A PV_A}$,

$$\ln r_B = \alpha_B + \beta_B PV_B , \quad (2.8)$$

or equivalently $r_B = e^{\alpha_B} e^{\beta_B PV_B}$.

It is interesting to note that (2.7) and (2.8) imply constant proportional rates of change of r_A and r_B with respect to company size, rather than the constant absolute rate of change implied by a linear relationship. We again assume that γ depends only on the ratio of company sizes, PV_A / PV_B , and in the same spirit as before we use

$$\ln [\gamma / (1-\gamma)] = \alpha^* + \beta^* (PV_A / PV_B), \quad (2.9)$$

that is,

$$\gamma = \frac{e^{\alpha^*} e^{\beta^* (PV_A / PV_B)}}{1 + e^{\alpha^*} e^{\beta^* (PV_A / PV_B)}}.$$

The general functional form of the model thus depends on six parameters which can, in principle, be estimated from empirical data.

We will need to estimate the 6-vector, θ , of unknown parameters for the negative

exponential model, where $\theta = \begin{bmatrix} \alpha_A \\ \alpha_B \\ \alpha^* \\ \beta_A \\ \beta_B \\ \beta^* \end{bmatrix}$. Once θ is determined the empirical values of r_A ,

r_B and γ for a particular merger can be found from (2.7), (2.8) and (2.9), as well as the predicted value of Y from (2.6) and the Newton-Raphson procedure described above. To estimate θ we need to minimise (over all values of θ)

$$\sum_{\text{all mergers}} (Y_i - \hat{Y}(x_i; \theta))^2 \quad (2.10)$$

where Y_i and $\hat{Y}(x_i; \theta)$ are the observed and predicted proportion respectively of the net gains accruing to the target in merger i .

§2.4.2.2 A HIERARCHY OF MODELS

The general negative exponential model described by (2.5) and (2.7) to (2.10) will be termed the **full negative exponential model (FNEM)** and has the six parameters found in the vector θ in §2.4.2.1. In an attempt to simplify this model and to remove partially the problem of any inter-relationships that might exist between the parameters, the negotiating power γ can be fixed at each of a sequence of values, leaving only a 4-

dimensional parameter vector $\theta = \begin{bmatrix} \alpha_A \\ \alpha_B \\ \beta_A \\ \beta_B \end{bmatrix}$ to be estimated. This would be expected to

result in a computationally more efficient and quicker solution to the problem, although the model fit might be expected to be poorer. Thus γ was varied in discrete steps over a range of values from 0.5 (equivalent to the Nash solution to the problem) to 0.9, and for each case an optimal solution was arrived at¹. This will be termed the **fixed γ model (F γ M)**.

The model F γ M can be further simplified in either of two different ways. Firstly, the risk aversion coefficients can be considered to be the *same* function of company size for both acquiring and target firms, whilst γ is held constant over the same range of values as in the model F γ M. Thus $\theta = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$, which is merely the model F γ M with $\alpha_A = \alpha_B = \alpha$ and $\beta_A = \beta_B = \beta$. We term this simplified model S1.

The second simplification amounts to the risk aversion coefficients being considered as different functions for acquirers and targets, but *not* dependent on company size. Again γ is held constant over the range of values as in the model F γ M. In this case $\theta = \begin{bmatrix} \alpha_A \\ \alpha_B \end{bmatrix}$, which is the model F γ M with $\beta_A = \beta_B = 0$. This simplified model is depicted in Figure 2.2, and the functional forms of the risk aversion coefficients and negotiating power for each model can be found in Table 2.1.

¹ The values actually used for γ were 0.50, 0.55, 0.60, 0.6667, 0.75, 0.793, 0.85 and 0.90. The value 0.793 was simply (average size of acquirer)/(average size of acquirer + average size of target), where these average sizes were calculated from the set of empirical data described in §2.5.1.

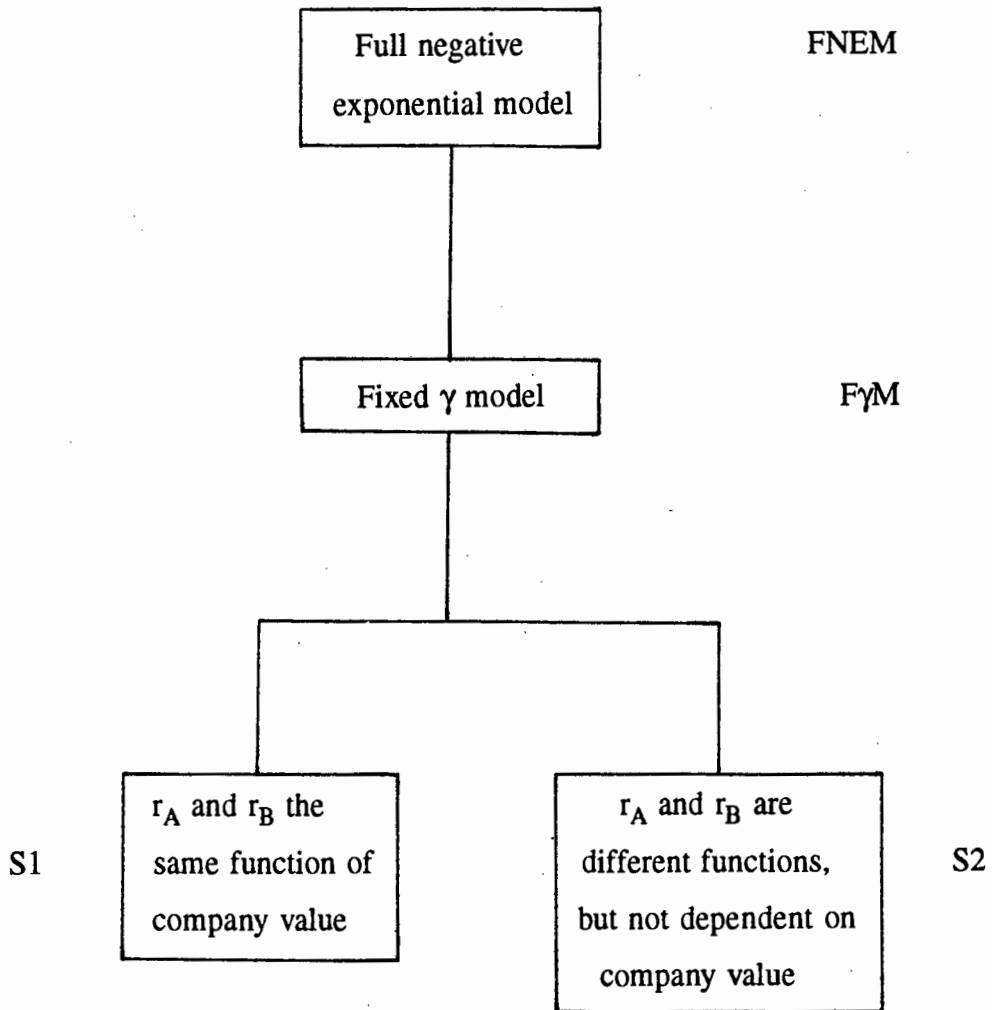


Figure 2.2. Model hierarchy for the negative exponential family of utility models

Table 2.1. Functional forms of the risk aversion coefficients
and negotiating power

Model	Risk aversion coefficients	Negotiating power	Length of θ
FNEM	$r_A = e^{\alpha_A} e^{\beta_A PV_A}$ $r_B = e^{\alpha_B} e^{\beta_B PV_B}$	$\gamma = \frac{e^t}{1 + e^t}$ <p>where $t = \alpha^* + \beta^* (PV_A / PV_B)$</p>	6
F γ M	$r_A = e^{\alpha_A} e^{\beta_A PV_A}$ $r_B = e^{\alpha_B} e^{\beta_B PV_B}$	Constant in [0.5;0.9]	4
S1	$r_A = e^{\alpha} e^{\beta PV_A}$ $r_B = e^{\alpha} e^{\beta PV_B}$	Constant in [0.5;0.9]	2
S2	$r_A = e^{\alpha_A}$ $r_B = e^{\alpha_B}$	Constant in [0.5;0.9]	2

§2.5 EMPIRICAL FITTING OF THE MODEL

§2.5.1 THE DATA

The input requirement for the model is the vector $x_i = \begin{bmatrix} PV_{A,i} \\ PV_{B,i} \\ PV_{AB,i} \end{bmatrix}$ for each

merger i considered. In §2.2 we defined PV_A and PV_B to be the financial worths of the acquirer and target respectively in the absence of a merger. To test the model empirically it is necessary to quantify these worths. Assuming an efficient market, we suppose that the pre-merger market values (MV) of A and B are surrogates for PV_A and PV_B respectively, that is $PV_A = MV_A = (\text{number of shares in issue for company A}) \times (\text{share price of company A})$ and $PV_B = MV_B = (\text{number of shares in issue for company B}) \times (\text{share price of company B})$. These may be converted to any monetary units for ease of calculation. Thus

$$PV_i = n_i p_i, \quad i = A \text{ or } B$$

where n_i is the pre-merger number of shares in issue for company i , and p_i is the pre-merger share price of company i .

As pointed out by, for example, Brealey and Myers (1981), it is important to bear in mind that if investors *expect* company A to acquire company B, the market value of A as defined above may be a poor measure of its value as a separate entity: the market value of company A will be its value in the absence of a merger *plus* some part of the benefits of the merger. That is, the market value of A will tend to overstate PV_A . On the other hand if investors perceive that no merger will occur the market value of A will indeed be its value as a separate entity. There may be similar aberrations when using the market value of company B to estimate its value as a separate entity. It is thus necessary to investigate when (relative to the merger announcement date) expectations of a successful merger start to become noticeable. The pre-merger share price of the company involved will then be measured at a point some time before this.

In all studies of these pre-announcement effects, the share price returns of the target companies are affected much more than those of the acquiring companies. In studies on US markets the pre-announcement effects have begun up to 6 or 7 months prior to the merger announcement. Examples of such empirical studies are those which were performed by Halpern (1973), Mandelker (1974), Langetieg (1978), Dodd (1980), Asquith (1983), Bradley, Desai and Kim (1983), Jensen and Ruback (1983) and Malatesta (1983), to name but a few. The above all performed event studies using the cumulative average abnormal return approach first suggested by Fama, Fisher, Jensen and Roll (1969). On the Johannesburg Stock Exchange (JSE) from which the data for this study were taken, studies have shown that this effect begins some three months prior to the announcement (Affleck-Graves, Flach and Jacobson (1988), Bhana (1987) and Van den Honert, Barr, Affleck-Graves and Smale (1988)). Thus the pre-merger share prices of acquirers and targets were calculated as the average of the high and low monthly prices for the three months before the merger announcement.

If a merger is financed by means of a share exchange, the acquirer swops k of the combined company's shares for each target share. Thus post-merger the new company AB will have $(n_A + kn_B)$ shares in issue, and PV_{AB} , the post-merger financial worth of the combined company, will be estimated by $(n_A + kn_B)P_{AB}$, where P_{AB} is the post-merger share price of company AB. P_{AB} was taken to be the average of the high and low prices of the resulting merged company in the month after the merger. The actual traded value of the target is $(kn_B P_A + zn_B)$, where the first term is the portion paid out as an exchange of shares and the second term is the portion paid out in cash. The term z is the cash payment per share.

The empirical data came from mergers between companies listed on the JSE and which took place between February 1972 and April 1987. For the purposes of this study mergers and acquisitions were only included if the following conditions held:

- (i) Full documentation regarding the exchange ratio was available (i.e. k and z above were available);
- (ii) The acquirer should not have undertaken any other merger in the six months prior to the current merger announcement;

- (iii) There should not have been any change to the issued ordinary shares of either participating company in the six months prior to the merger announcement;
- (iv) Only acquisitions of a single target at a time were considered;
- (v) Thinly and infrequently-traded shares were eliminated to ensure market efficiency. Thus it was required that at least 100 000 shares had to have been traded per annum on average over the five years prior to the merger and at least ten deals had to have taken place per month over the same period;
- (vi) The merger had to be "rational" in the sense that the net gains were positive, and the net amount allocated to the target was greater than PV_B but less than $PV_{AB} - PV_A$. Five mergers were eliminated from the sample through the enforcement of this condition, even though they satisfied conditions (i) to (v).

A set of 24 mergers remained, and the acquirer/target combinations as well as the values of PV_A , PV_B and PV_{AB} can be found in Appendix 2.A.

§2.5.2 EMPIRICAL RESULTS

§2.5.2.1 LINEAR UTILITY MODEL RESULTS

At convergence the linear utility model with γ related to PV_A / PV_B as in (2.2a), that is

$$\ln [\gamma / (1-\gamma)] = \alpha_1 + \beta_1 (PV_A / PV_B)$$

produced a solution vector of

$$\theta = \begin{bmatrix} \alpha_1 \\ \beta_1 \end{bmatrix} = \begin{bmatrix} 0.6514 \\ 0.0920 \end{bmatrix}$$

with residual sum of squares $\sum (Y_i - \hat{Y}(x_i; \theta))^2 = 0.7803$. We term this model **linear utility model 1 (LUM1)**.

Similarly, the solution vector for the linear utility model with γ related to PV_A/PV_B as in (2.2b), that is

$$\ln [\gamma / (1-\gamma)] = \alpha_2 + \beta_2 \ln(PV_A / PV_B)$$

converged to

$$\theta = \begin{bmatrix} \alpha_2 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} 0.5523 \\ 0.5007 \end{bmatrix}$$

with residual sum of squares 0.7127. We term this model **linear utility model 2 (LUM2)**. Table 2.2 shows the actual values of Y_i on the unit interval and the corresponding fitted values $\hat{Y}(x_i; \theta)$ and residuals $e_i = \hat{Y}(x_i; \theta) - Y_i$ for both the linear utility models considered here.

Since $\hat{Y}(x_i; \theta)$ is a *predicted* value of Y_i , an obvious measure of the quality of prediction is a least squares regression of Y_i on $\hat{Y}(x_i; \theta)$. Clearly a least squares regression of the form $Y = b_0 + b_1 \hat{Y}$ should have b_0 not different from 0, b_1 not different from 1 and a coefficient of determination r^2 as close as possible to 1. The OLS regression for model LUM1 for data in Table 2.2 yields

$$Y = 0.06136 + 0.7545 \hat{Y}$$

with correlation $r=0.4511$ and coefficient of determination $r^2=0.2035$. The predicted \hat{Y} thus explains 20.35% of the variation in the observed Y values. This indicates a clear linear trend with a significant correlation ($p = 0.0270$). Tests on the regression coefficients show that the intercept and slope are not significantly different from 0 and 1 respectively ($t_{b_0} = 0.8982$, $t_{b_1} = -0.7712$). The OLS regression for model LUM2 yields

$$Y = 0.007376 + 0.9683 \hat{Y}$$

with $r = 0.4960$ ($r^2 = 0.2460$). Thus the predicted \hat{Y} explains 24.60% of the variation in the observed Y values. There is a significant correlation ($p = 0.0138$) and again the intercept and slope are not statistically significantly different from 0 and 1 respectively ($t_{b_0} = 0.0928$, $t_{b_1} = -0.0879$). Table 2.3 provides a comparison of the models LUM1 and LUM2. Table 2.4 shows the actual monetary amount allocated to the target, Q , the value of Q estimated from \hat{Y} in our model, as well as the percentage error of the estimate for the better-fitting of the two models, LUM2. In this case it will be

Table 2.2. Values of actual Y_i , fitted Y_i , and residuals for linear utility models

Obs	Y_i	LUM1		LUM2	
		Fitted Y_i	Residual	Fitted Y_i	Residual
1	.7079	.3054	-.4024	.2973	-.4105
2	.0731	.3236	.2505	.3736	.3005
3	.0977	.0000	-.0977	.0515	-.0462
4	.0859	.1496	.0637	.1433	.0574
5	.2774	.0407	-.2366	.0991	-.1783
6	.2142	.0885	-.1258	.1185	-.0958
7	.2829	.3241	.0412	.3766	.0938
8	.0435	.3119	.2684	.3183	.2748
9	.4873	.3144	-.1729	.3277	-.1596
10	.0679	.0697	.0020	.1112	.0434
11	.2955	.0250	-.2705	.0912	-.2043
12	.3358	.3087	-.0270	.3075	-.0283
13	.0470	.0020	-.0449	.0688	.0219
14	.0483	.2460	.1977	.2031	.1548
15	.1004	.0440	-.0564	.1006	.0002
16	.0064	.0634	.0569	.1087	.1022
17	.5085	.2785	-.2300	.2415	-.2670
18	.1469	.2748	.1279	.2360	.0891
19	.1039	.2276	.1236	.1875	.0836
20	.0341	.2345	.2005	.1930	.1590
21	.0498	.1059	.0561	.1252	.0755
22	.0240	.1792	.1552	.1575	.1334
23	.6221	.3170	-.3051	.3388	-.2834
24	.0508	.0577	.0069	.1064	.0556

Table 2.3. A comparison of models LUM1 and LUM2

	LUM1	LUM2
θ	$\alpha = 0.6514$ $\beta = 0.0920$	$\alpha = 0.5523$ $\beta = 0.5007$
Σe^2	0.7803	0.7127
b_0	0.061355	0.007376
$se(b_0)$	0.068309	0.079503
$t(b_0)$	0.8982	0.0928
b_1	0.754518	0.968251
$se(b_1)$	0.318307	0.361359
$t(b_1)$	-0.7712	-0.0879
r	0.4511	0.4960
r^2	0.2035	0.2460
p	0.0270	0.0138

noted that the percentage errors are normally distributed ($\chi^2_1=0.502$). Figure 2.3 shows a scatterplot of Y against \hat{Y} for model LUM2 (the better-fitting of the two linear utility models) together with the fitted regression line and 95% and 99% confidence bands for the fitted regression line.

On the basis of all the above information LUM2 provides a somewhat better fit than does LUM1: the improvement in correlation is 9.95% and in percent of variation in the Y explained it is 20.88%. We note that 8 points have absolute residuals greater than 0.2 and 2 points have absolute residuals greater than 0.3 in LUM1, whilst the corresponding number of points in LUM2 are 6 and 2. If these ill-fitting points (those with absolute residual greater than 0.2) are omitted and the regression lines of Y against \hat{Y} recalculated for models LUM1 and LUM2, the functions become

Table 2.4. Actual traded values and estimates (in monetary terms)
for model LUM2

Merger	Q	Fitted Q	% error
1	333.775	303.390	-10.02
2	559.576	761.198	26.49
3	5.176	3.945	-31.20
4	67.573	82.249	17.84
5	17.187	10.774	-59.52
6	10.381	9.837	-1.06
7	280.017	283.696	1.30
8	278.122	294.166	5.45
9	263.088	239.461	-9.87
10	6.287	7.537	16.58
11	2.629	1.843	-42.65
12	92.478	91.609	-0.95
13	3.958	4.351	9.03
14	2.225	2.742	18.85
15	206.479	206.668	0.09
16	0.475	0.682	30.35
17	1.203	1.077	-11.70
18	3.339	3.799	12.11
19	3.276	3.653	10.32
20	1.872	3.744	50.00
21	1.643	3.700	55.59
22	24.121	40.602	40.59
23	135.365	122.857	-10.18
24	8.208	9.640	14.85

Average % error: 5.51%

Standard deviation of % error: 26.74%

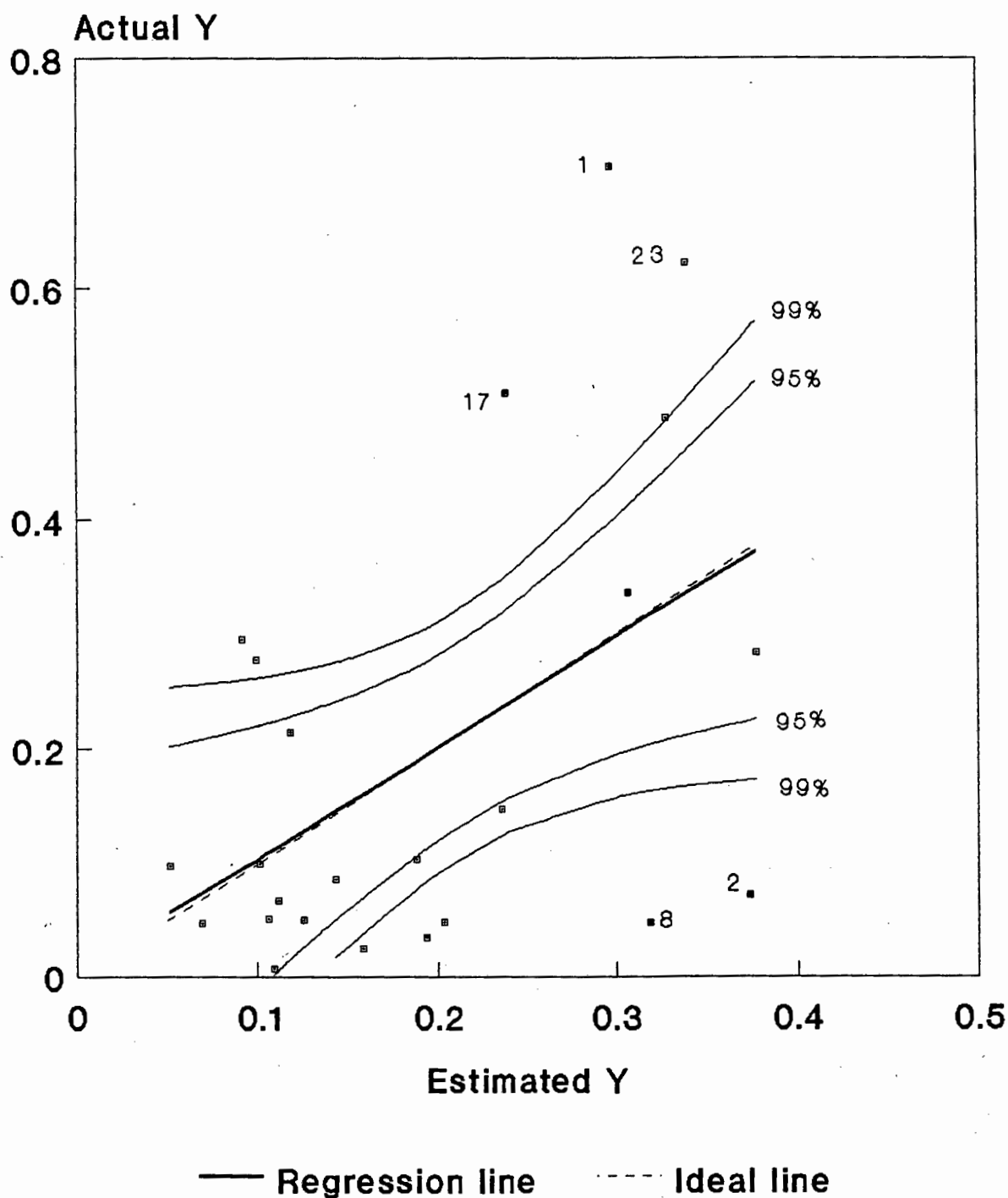


Figure 2.3. Scatterplot of actual Y against \hat{Y} for model LUM2 together with the fitted regression line and 95% and 99% confidence bands

$$\begin{aligned} \text{LUM1} \quad Y &= 0.01907 + 0.7510 \hat{Y} && \text{based on } n=16 \text{ points} \\ r &= 0.6530 \\ r^2 &= 0.4264, \quad p < 0.00005 \end{aligned}$$

and

$$\begin{aligned} \text{LUM2} \quad Y &= -0.02010 + 0.9339 \hat{Y} && \text{based on } n=18 \text{ points} \\ r &= 0.6448 \\ r^2 &= 0.4288, \quad p < 0.00005. \end{aligned}$$

Thus the relatively low predictive power (represented by r^2) of models LUM1 and LUM2 is caused by a fairly low proportion of outlying cases which are not well described by these models.

§2.5.2.2 NEGATIVE EXPONENTIAL UTILITY MODEL RESULTS

At convergence the full model (FNEM) produced a solution vector of

$$\theta = \begin{bmatrix} 2.5795 \\ 4.2792 \\ 1.1006 \\ -0.0070 \\ -0.0345 \\ 0.0151 \end{bmatrix}$$

with residual sum of squares of 0.6371. Table 2.5 shows the actual values of Y_i on the interval $[0;1]$, the corresponding values of $\hat{Y}(x_i; \theta)$ and the resulting residuals. The OLS regression of Y on \hat{Y} yields

$$Y = 0.01652 + 0.9259 \hat{Y}$$

with $r=0.5725$ and $r^2=0.3277$. The linear trend is very highly significant ($p = 0.0034$). Furthermore the intercept and slope are not significantly different from 0 and 1 respectively ($se(b_0) = 0.064041$, $t_{b_0} = 0.254$; $se(b_1) = 0.282735$, $t_{b_1} = -0.262$). Figure 2.4 shows a scatterplot of Y against \hat{Y} for model FNEM together with the fitted regression line and 95% and 99% confidence bands for the fitted regression line. A glance at Figure 2.4 shows that points 1, 2, 8 and 17 clearly do not fit the model well, all having absolute residuals of greater than 0.2. If these four points are omitted and the regression line recalculated on the remaining 20 points, the function becomes

$$Y = -0.009326 + 0.9762 \hat{Y}$$

Table 2.5. Actual values of Y_i , fitted values of Y_i and residuals for the full negative exponential utility model, FNEM

Merger	Actual Y_i	Fitted Y_i	Residual
1	.7079	.2690	-.4389
2	.0731	.2810	.2079
3	.0977	.0368	-.0609
4	.0859	.1109	.0250
5	.2774	.0775	-.1999
6	.2142	.0926	-.1216
7	.2829	.4164	.1335
8	.0435	.2962	.2527
9	.4873	.3892	-.0981
10	.0679	.1120	.0441
11	.2955	.1254	-.1701
12	.3358	.4167	.0809
13	.0470	.0647	.0177
14	.0483	.1608	.1125
15	.1004	.1709	.0705
16	.0064	.1512	.1448
17	.5085	.1624	-.3461
18	.1469	.1646	.0177
19	.1039	.1581	.0542
20	.0341	.1598	.1257
21	.0498	.1477	.0979
22	.0240	.1150	.0910
23	.6221	.4931	-.1290
24	.0508	.0884	.0376

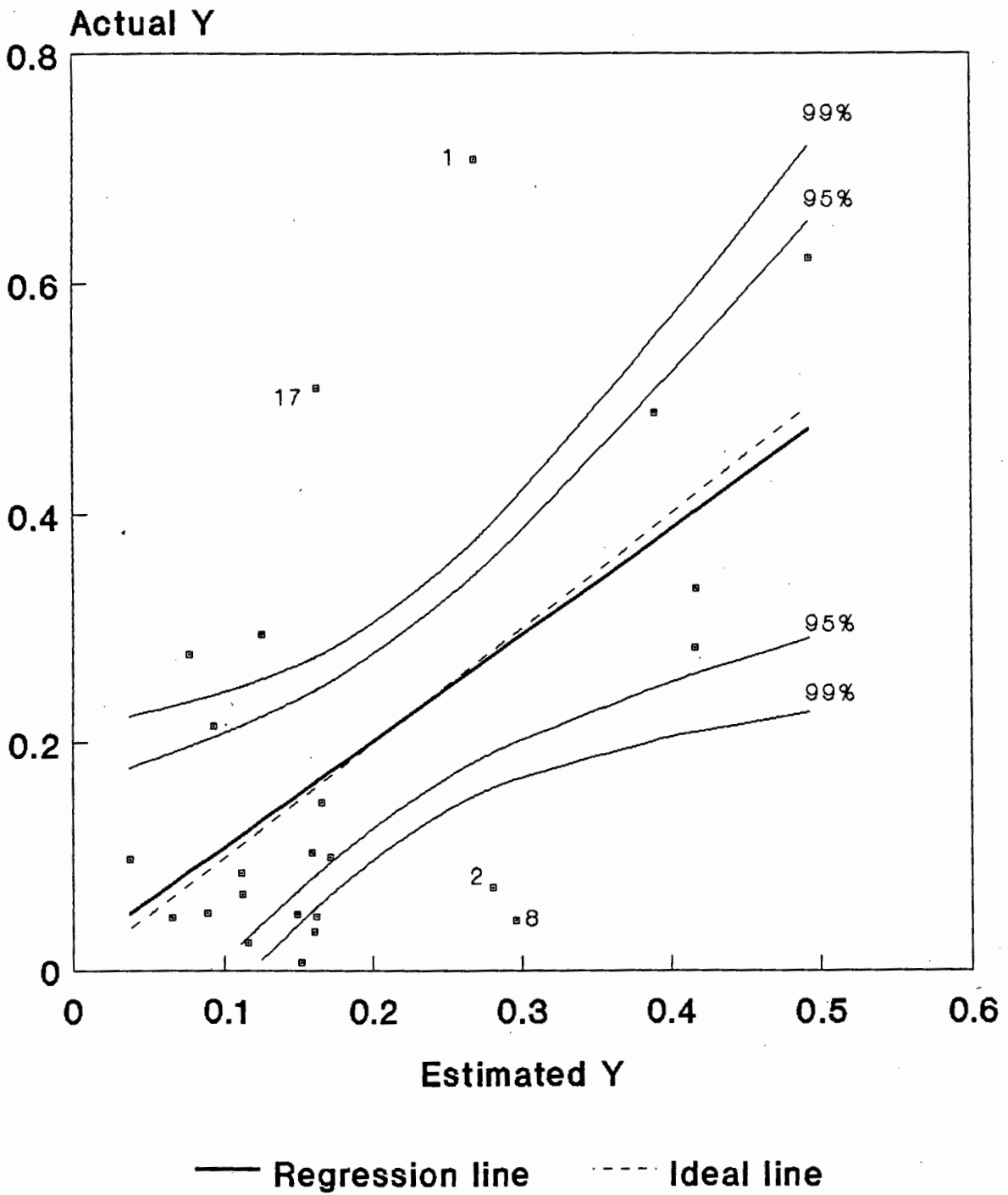


Figure 2.4. Scatterplot of Y against \hat{Y} for model FNEM together with the fitted regression line and 95% and 99% confidence bands

with $r = 0.7733$ and $r^2 = 0.5980$, i.e. a line even closer to the desired line, $Y = \text{estimated } Y$, and with a superior fit ($p < 0.00005$). Clearly the model FNEM has considerable predictive power for all but a small number of outlying cases.

Consider now the fixed γ model, $F\gamma M$. Table 2.6 shows the values of $\theta = \begin{bmatrix} \alpha_A \\ \alpha_B \\ \beta_A \\ \beta_B \end{bmatrix}$ for the fixed values of γ , as well as the goodness-of-fit statistics r , r^2 and

the residual sum of squares, SS_E . It will be noted that the correlation r remains virtually unchanged over a wide range of γ from 0.75 to 0.90, reaching a peak of 0.5695 when $\gamma=0.793$ and a low point in this range of 0.5597 when $\gamma=0.90$, a drop of only 1.75%. Furthermore over this range the regression of Y on \hat{Y} is significant at the 0.5% level. As γ tends towards 0.50, so the correlation decreases in significance until it is only significant at the 2% level for $\gamma \leq 0.55$. The symmetric Nash solution provides the least good fit of all the models considered here.

Comparing the model $F\gamma M$ with values of γ in the range [0.75 ; 0.90] to the model FNEM we see that the maximum loss in explanatory power occurs when $\gamma=0.90$. This loss amounts to only 1.91% (SS_E) and 2.29% (r). This is shown in Table 2.7.

From Tables 2.6 and 2.7 we note that the loss is minimised (for the values of γ considered here) when $\gamma=0.793$. At this value of γ three observations (numbered 1, 8 and 17) have absolute residuals greater than 0.2. Omitting these points we observe a correlation based on the remaining 21 points of 0.7519 ($r^2=0.5654$), i.e. an improvement of 32.03% in correlation and over 74% in the explanation of the variation of the Y values. Thus the model $F\gamma M$ could be considered an improvement on the full model in the sense that it offers an almost equivalent explanatory power whilst reducing the number of estimated parameters by two. From the above discussion it is clear that it is extremely difficult to estimate the value of γ accurately. Table 2.8 shows the values of Q (the actual monetary amount allocated to the target) for each merger in the sample, the values of Q estimated from the model, and the percentage error of the estimate for $\gamma =$

Table 2.6. Optimal values of θ for fixed values of γ using model $F\gamma M$ as well as goodness-of-fit statistics

γ	0.50 ¹	0.55	0.60	0.6667	0.75	0.793	0.85	0.90
α_A	2.116	2.481	2.808	2.807	2.756	2.653	2.372	2.296
α_B	4.111	4.345	4.646	4.602	4.457	4.364	4.141	4.068
β_A	-0.01275	-0.01244	-0.01211	-0.01106	-0.00823	-0.00654	-0.00405	-0.00271
β_B	-0.03274	-0.03504	-0.03329	-0.03348	-0.03554	-0.03391	-0.03373	-0.03383
r	0.4609 ¹	0.5147 ¹	0.5365 ²	0.5425 ²	0.5695 ³	0.5695 ³	0.5649 ³	0.5597 ³
r^2	0.2125	0.2649	0.2878	0.2943	0.3227	0.3243	0.3192	0.3122
SS_E	0.800386	0.727826	0.714044	0.677890	0.641658	0.639492	0.643000	0.649317

¹ corresponds to the symmetric Nash solution

¹ significant at the 2% level

² significant at the 1% level

³ significant at the 0.5% level

0.793. It may be noted that the percentage errors are normally distributed ($\chi^2_1=0.502$).

For the first simplification of the fixed γ model, S1, i.e. when the risk aversion

Table 2.7. Loss in explanatory power by model F γ M

Model		SS _E	% loss	r	% loss
FNEM		0.637120	-	0.5725	-
F γ M	$\gamma=0.75$	0.641658	0.71	0.5681	0.77
	0.793	0.639492	0.37	0.5695	0.53
	0.85	0.643800	1.05	0.5649	1.34
	0.90	0.649317	1.91	0.5597	2.29

coefficients are the same function of company size for both acquirers and targets, the resultant correlations never rise above 0.0558 (not significantly different from 0) for any value of γ between 0.50 and 0.90. The residual sum of squares ranges from 0.9419 when $\gamma=0.90$ to 3.1509 when $\gamma=0.50$. Clearly this model is thus an oversimplification of reality, despite saving on two further degrees of freedom, and indicates that the risk aversion coefficients r_A and r_B are indeed *different* functions of company size for targets and acquirers.

The second simplification of the fixed γ model, S2, i.e. when r_A and r_B are considered to be different functions, but not dependent on company size, yielded equally disappointing results with the correlations never rising above 0.0001 for any value of γ in the interval [0.5 ; 0.9]. The residual sum of squares is constant at 0.9449 for all values of γ considered. This model is clearly also an oversimplification of reality, and indicates that r_A and r_B are indeed functions of company size.

Table 2.8. Actual traded values and estimates (in monetary terms) for model FyM with $\gamma=0.793$

Merger	Q	Fitted Q	% error
1	333.775	299.075	-11.60
2	559.576	678.523	17.53
3	5.176	4.079	-26.66
4	67.573	72.355	6.61
5	17.187	10.133	-69.61
6	10.381	9.701	-7.01
7	280.017	284.912	1.72
8	278.122	291.398	4.56
9	263.088	247.025	-6.50
10	6.287	7.623	17.53
11	2.629	1.088	-32.24
12	92.478	94.782	2.43
13	3.958	4.468	11.41
14	2.225	2.589	14.06
15	206.479	278.446	25.85
16	0.475	0.768	38.18
17	1.203	1.039	-15.81
18	3.339	3.411	2.12
19	3.276	3.507	6.60
20	1.872	3.316	43.54
21	1.643	4.125	60.17
22	24.121	35.131	31.34
23	135.365	129.488	-4.54
24	8.208	9.256	11.32

Average % error: 5.04%

Standard deviation of % error: 26.62%

Thus the reduction of model F γ M to S1 and/or S2 yields essentially zero correlations. This implies the necessity of the F γ M model, and that the significant correlations in the F γ M model are not spurious.

§2.5.3 A COMPARISON OF EMPIRICAL RESULTS OF THE LINEAR AND NEGATIVE EXPONENTIAL FAMILIES OF MODELS

Table 2.9 summarises the empirical results of the goodness-of-fit statistics of the models considered in this chapter, that is when there is full and shared information

Table 2.9. Summary of empirical results of goodness-of-fit statistics of models (full and shared information)

Model	SS_E	r	r^2	number of estimated parameters
LUM1	0.7803	0.4511	0.2035	2
LUM2	0.7127	0.4960	0.2460	2
FNEM	0.6371	0.5725	0.3278	6
F γ M $\gamma=0.5$	0.8004	0.4609	0.2124	4
$\gamma=0.55$ to 0.90	0.6395 to 0.7278	0.5147 to 0.5695	0.2649 to 0.3243	5
S1 $\gamma=0.5$	3.1509	0.0118	0.0001	2
$\gamma=0.55$ to 0.90	0.9420 to 2.4888	0.0120 to 0.0558	0.0001 to 0.0031	3
S2 $\gamma=0.5$	0.9449	0.0000	0.0000	2
$\gamma=0.55$ to 0.90	0.9449	0.0000 to 0.0001	0.0000	3

amongst the parties concerned. Note that the table includes the number of parameters estimated in each model. In models F γ M, S1 and S2, the case $\gamma=0.5$ implies the symmetric Nash solution and hence requires one less parameter than for other γ values.

Since the two families of utility functions are not nested the usual model comparison procedures are not relevant. As a heuristic measure of goodness of fit we propose to use the root of the residual sum of squares, adjusted for the number of parameters estimated, as a means of comparing the models. Thus we use

$$R(SS_{adj}) = \sqrt{\frac{\sum (Y - \hat{Y})^2}{n - p}} \quad (2.11)$$

where n is the number of data points (in all cases $n=24$) and p is the number of parameters estimated. This measure is analagous to the usual measure used in multiple regression models. Table 2.10 displays the value of for $R(SS_{adj})$ for all the models considered and the number of parameters in the model, and a graphical presentation is

Table 2.10. $R(SS_{adj})$ values for models with full and shared information

Model		p	$RR(SS_{adj})$
LUM1		2	0.1883
LUM2		2	0.1800
FNEM		6	0.1881
F γ M	$\gamma=0.5$	4	0.2001
	$\gamma=0.55$ to 0.90	5	0.1835 to 0.1957
S1	$\gamma=0.5$	2	0.3784
	$\gamma=0.55$ to 0.90	3	0.2118 to 0.3443
S2	$\gamma=0.5$	2	0.2072
	$\gamma=0.55$ to 0.90	3	0.2072

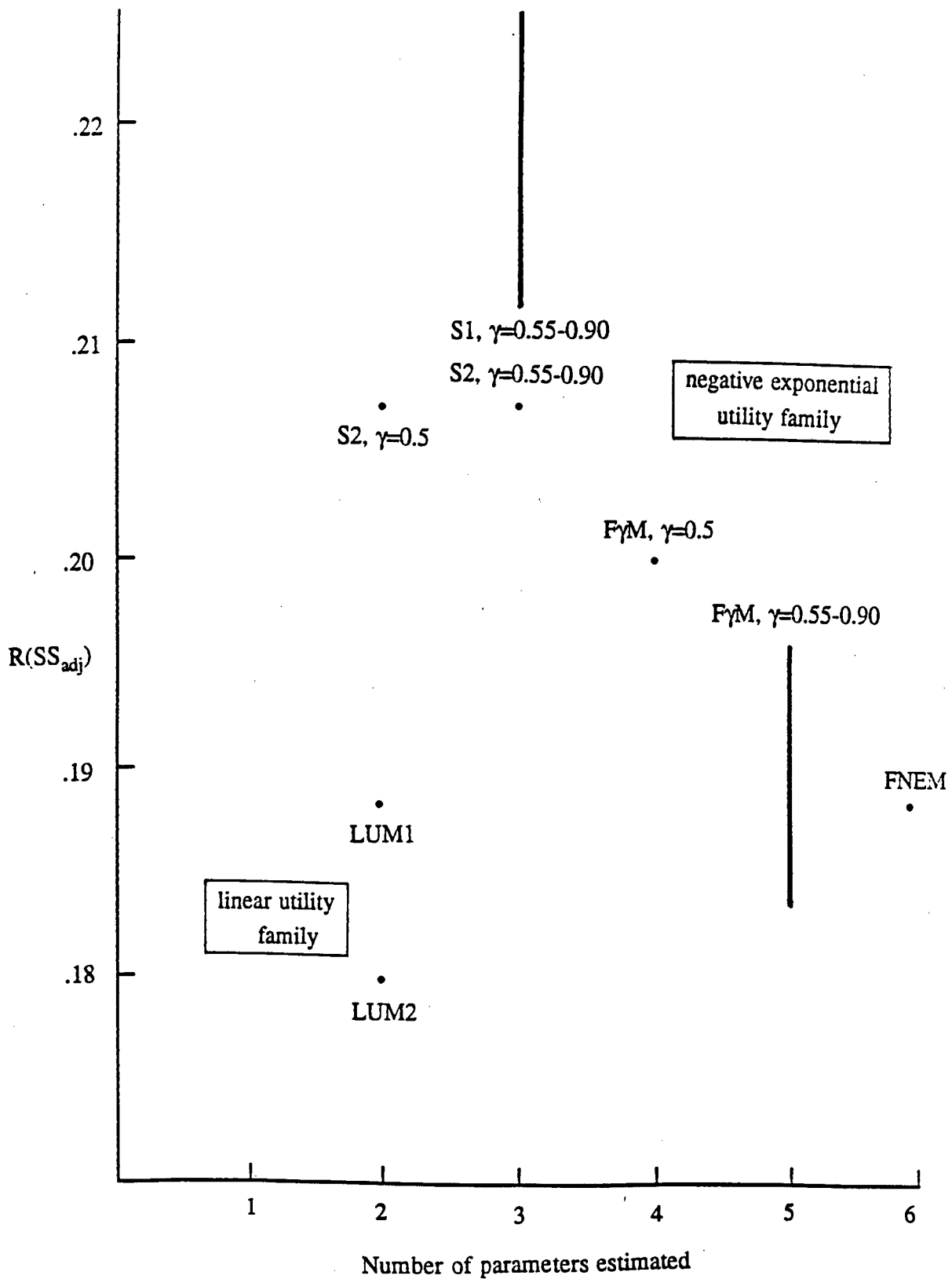


Figure 2.5. $R(SS_{adj})$ plotted against the number of parameters estimated for all models with full and shared information

given in Figure 2.5. Clearly the best model will provide the lowest $R(SS_{adj})$ value. Amongst the negative exponential family the model $F\gamma M$ with selected values of γ provides the best fit in terms of $R(SS_{adj})$. Amongst the linear family, model LUM2 is best and indeed this is the best model of all. Note that in Figure 2.5 the point representing model S1, $\gamma = 0.55$ has been completely omitted, and the range of points representing model S1, $\gamma = 0.55 - 0.90$ has been partially omitted due to the scaling chosen, but these are clearly the poorest models of all those considered.

We can thus conclude that on an *adjusted* basis and using (2.11) as a means of model comparison, the linear utility model LUM2 provides as much information (and possibly more) as any of the (more complex) negative exponential models.

§2.6 CONCLUSIONS

In this chapter we have constructed and analysed a number of relatively parsimonious models based upon, firstly, a linear utility function and, secondly, a negative exponential utility function, to explain the proportion of the synergy gains from merger that accrue to the target company. We showed that several provided a reasonably good description of the real-world decision-making process.

Amongst the linear utility family the best model comprised of only two parameters which were used in the estimation of the relative bargaining strength of the acquirer vis-a-vis the target, which was the only important measure used in explaining the variation in the proportion of the synergy gains accruing to the target.

Amongst the negative exponential utility family the best model comprised of four estimated parameters which were used in the estimation of the risk aversion coefficients of the acquiring and target companies. It was shown empirically that for this family, unlike the linear utility family, the fit of the model did *not* depend to any great extent on the negotiating power of the acquiring company for a wide range of such power. Simpler negative exponential models with fewer parameters did not provide any significant fit at all. Specifically, the risk aversion coefficients of the two companies involved were

- (i) different functions (although possibly of the same form), and
- (ii) dependent on company size.

This model is at best a simplification of reality, and many of the complications associated with a complex investment decision such as a corporate merger have been set aside. Thus any deviations between the actual value of the proportion of the synergy gains accruing to the target and our fitted model can be attributed to these simplifications in our assumptions. Furthermore, differing perceptions of the values of the input variables between the decision-makers and ourselves would contribute to the residual; it should be borne in mind that the empirical testing performed here used actual ex post data, some of which were unknown before the actual merger announcement. As regards the simplifying assumptions of the model, areas which could further contribute to our model deviating from reality are the form of the utility functions and the form of the relationship between company size and risk aversion coefficient, and between relative company size and negotiating power.

In the models presented in this chapter we assumed that the worth of the merged entity, and hence the synergy gains from merger, was known *with certainty*. In a real-world decision-making environment this value will have to be estimated a priori, and thus it is likely that acquirer and target (and any other interested party) will arrive at different values for the worth of the combined company. Decisions taken on the strength of these uncertain estimates will thus contain an element of risk. The current models do not consider the effects of uncertainty or risk in any way, and hence these models are unable to explain what proportion of the amount paid to the target was in the form of cash and what proportion shares. This question of uncertainty will be tackled in the next chapter.

APPENDIX 2.A. The merger sample

	Acquirer	Target	PV _A	PV _B	PV _{AB}	Actual
1	Anglo American	Rand Selections	520.314	281.385	875.711	333.775
2	General Mining	Union Corp	475.528	510.490	1657.074	559.576
3	Barlows	C J Fuchs	287.807	2.583	316.828	5.167
4	Transnata	Alfred McAlpine	538.388	45.613	839.659	67.573
5	C G Smith	Illovo	196.436	7.209	239.620	17.187
6	Volkscas	Bankovs	167.446	9.165	182.287	10.381
7	Welkom	FSI	244.143	268.917	552.300	280.017
8	UCI	Sentrust	418.470	275.580	752.423	278.122
9	Hulets	Tongaat	266.161	190.952	605.136	263.088
10	Amrel	ABC	91.179	4.331	124.335	6.287
11	Dorman	Bus Bodies	48.819	1.492	54.159	2.629
12	Clydesdale	Apex	138.040	82.164	250.921	92.478
13	Premier Mills	Propan	187.582	3.112	208.707	3.958
14	Globe	Ship & Engineer	10.506	2.064	15.903	2.225
15	Barlows	Plascon	3329.123	126.168	4255.488	206.479
16	Phil Morkel	Lewis	10.253	0.462	12.740	0.475
17	Brick & Clay	Eclipse	3.149	0.965	4.582	1.203
18	Boland	Voogdy	8.951	2.582	16.686	3.339
19	Kohler	Hayne & Gibson	17.410	2.807	24.730	3.276
20	Vavasseur	Criterion	8.496	1.471	21.743	1.872
21	Anglo Transvaal	Nail & Chain	20.288	1.260	29.242	1.643
22	Barlows	Rand Mines	199.991	21.151	344.643	24.121
23	West Rand Inv	OFS Inv	136.169	107.898	288.217	135.365
24	Standard Bank	UDC Holdings	160.631	6.900	193.276	8.208

CHAPTER 3

BARGAINING MODELS WITH SHARED UNCERTAINTY

§3.1 INTRODUCTION AND OBJECTIVES

If two companies enter into negotiations which may lead to merger, and one or both choose to employ the type of model developed in the previous chapter as a form of decision support, they will have to make an estimate of the post-merger value of the new company well before the merger occurs. Since not all the relevant information may be known at that time it is likely that the acquirer and target may well arrive at different estimates of this value. In this chapter we attempt to extend the realism of the model by incorporating and quantifying this uncertainty. For the time being we will suppose that while uncertainty exists, its extent is known to both parties with certainty. That is, we are dealing with a situation in which there is *shared uncertainty* amongst the two parties. Uncertainty may occur in at least two different forms: the uncertainty relating to an estimate of some future outcome (as above); and the uncertainty relating to the way in which each bargaining party assesses net present value in "utility" terms (quantified in terms of a company-specific risk aversion coefficient). Apart from adding realism to the model, another advantage of introducing a measure of uncertainty is that such a model will be able to discriminate between the amount paid to the target shareholders by means of cash and by means of a share transfer, thus adding an extra dimension of usefulness to the model as a decision support tool.

We again assume a single-stage two-party bargaining game between acquirer and target, and we develop the Nash-Kalai bargaining model in §3.2 for a simple form of utility function. In §3.3 we derive the optimal solution to the Nash-Kalai model. The important mathematical results are interpreted in a practical way for management in §3.4. In §3.5 we arbitrarily choose a single merger from our empirical data-set and show how in a particular case the various solution possibilities arise as the uncertainty associated

with the merger changes. We construct a simple global model in §3.6 to describe the behaviour of the bargaining parties, the aim being (i) to quantify the uncertainty involved in a merger, and (ii) to predict the split of cash and shares. We will test this model on a set of mergers from the Johannesburg Stock Exchange (JSE) and in so doing hope to show that by including an uncertainty term we obtain an improvement in model fit over the models of the previous chapter. Thus an understanding will be gained of how uncertainty fits into the complex bargaining process, and how this affects the form of payment. Finally, in §3.7 we look at an extension of this model, by examining how different utility functions might impact on the results.

§3.2 NOTATION, ASSUMPTIONS AND THE DEVELOPMENT OF A NASH-KALAI BARGAINING MODEL

As in the previous chapter we suppose that the pre-merger worths of the acquiring company (PV_A) and of the target company (PV_B) as separate entities are known to both parties. We suppose now that a priori there is uncertainty about the post-merger value of the merged entity. We model this uncertainty by assuming that the post-merger value of the combined company *viewed from the pre-merger perspective* (we call it $\tilde{\Pi}_{AB}$) is some random variable which has a probability distribution whose mean and variance are known to both parties. Thus μ_{AB} is the *expected* post-merger value of the combined company AB, and σ_{AB}^2 is the variance in the estimate of the post-merger value of the merged entity. We make no assumptions initially about the form of the probability distribution of $\tilde{\Pi}_{AB}$, and indeed there is no theoretical or empirical evidence to suggest that some particular functional form (for example, a normal distribution) should be chosen ahead of any other. Since a merger is a one-off event, the variance cannot be empirically measured. Thus σ_{AB}^2 is merely a *conceptual* variance, which gives an indication of how accurately the two parties are able to estimate the post-merger value of AB. All the above information is symmetric in the sense that both acquirer and target know the values of PV_A , PV_B and μ_{AB} with certainty, and have the same perception of the amount of uncertainty (σ_{AB}^2) involved. Empirical estimation of σ_{AB}^2 is the subject of a later section.

We noted in §2.2 that under conditions of complete certainty the form of payment by the acquirer to the target was irrelevant. That is, it was immaterial whether the payment was by means of cash, an exchange of shares, or a combination of both cash and shares. Since the present model contains an element of uncertainty we may be able to differentiate between the cash and share transfer portions of the payment. Thus we define β_S to be the portion of the value of the combined company allocated to the target (or more correctly, to the target shareholders) in the form of shares ($0 \leq \beta_S \leq 1$), and ω to be a cash side-payment ($\omega \geq 0$). The net amount allocated to the target shareholders, Q , is thus a random variable and can be written as $\beta_S \tilde{\Pi}_{AB} + \omega$. We will assume (as in the previous chapter) that the two parties are both rational, and we will restrict ourselves to the case where both parties are risk averse (we invoke this assumption later). Then the expected net value to A after the merger must exceed its value before merger, i.e.

$$\mu_{AB} - E(Q) = (1 - \beta_S) \mu_{AB} - \omega > PV_A$$

(otherwise there is no expected benefit to A). Similarly, the expected net value to B after the merger must exceed its value before merger, i.e. $E(Q) > PV_B$ (otherwise there is no expected benefit to B).

Employing the above notation the actual net gain to the acquirer is the random variable

$$g_A = (1 - \beta_S) \tilde{\Pi}_{AB} - PV_A - \omega .$$

This has expectation of

$$(1 - \beta_S) \mu_{AB} - PV_A - \omega$$

and variance of

$$(1 - \beta_S)^2 \sigma_{AB}^2 .$$

Similarly, the actual net gain to the target is the random variable

$$g_B = \beta_S \tilde{\Pi}_{AB} - PV_B + \omega ,$$

which has expectation of

$$\beta_S \mu_{AB} - PV_B + \omega$$

and variance of

$$\beta_S^2 \sigma_{AB}^2 .$$

The sum of the expectations of the actual net gains to the acquirer and the target is simply

$$\begin{aligned} & (1-\beta_S) \mu_{AB} - PV_A - \omega + \beta_S \mu_{AB} - PV_B + \omega \\ & = \mu_{AB} - PV_A - PV_B, \end{aligned}$$

which is merely the expected total net gain or synergy from the merger. To avoid problems of scale we will use the expected total net gain as a convenient standardisation. We will develop the model in terms of the **standardised net gain** to the acquirer and target companies. The standardised net gain to the acquirer is the net gain to the acquirer expressed as a fraction of the total expected synergy, and is defined as

$$\Gamma_A = g_A / (\mu_{AB} - PV_A - PV_B)$$

and the standardised net gain to the target is defined as

$$\Gamma_B = g_B / (\mu_{AB} - PV_A - PV_B).$$

The standardisation conveniently provides the result that

$$E(\Gamma_A) + E(\Gamma_B) = 1.$$

The expectation of Γ_A can be written as

$$E(\Gamma_A) = \frac{(1-\beta_S) \mu_{AB} - PV_A}{\mu_{AB} - PV_A - PV_B} - \frac{\omega}{\mu_{AB} - PV_A - PV_B}.$$

Recall from §1.3 that we are modelling an asymmetric bargaining relationship between an acquirer and a target; the acquirer makes an offer to the shareholders of the target, and the target merely reacts to this offer. Part of the offer may be in the form of a cash (as opposed to a share) deal; in any event the cash portion will be non-negative, i.e. a cash payment by the acquirer to the target and not vice versa. We have defined two policy variables to describe the offer: β_S , the portion of the value of the combined company allocated to the target, and ω , the cash side-payment. Now β_S is already standardised (and thus unitless), whilst ω is expressed in units of monetary value. We thus define $\lambda = \omega / (\mu_{AB} - PV_A - PV_B)$ as the standardised cash payment. Thus $\lambda \geq 0$. An upper limit on λ is not explicitly required as it is implied by the inequality $(1-\beta_S)\mu_{AB} - \omega > PV_A$. So

$$E(\Gamma_A) = \frac{(1-\beta_S) \mu_{AB} - PV_A}{\mu_{AB} - PV_A - PV_B} - \lambda,$$

which can be written as

$$E(\Gamma_A) = c_1 - c_2 \beta_S - \lambda \tag{3.1}$$

where $c_1 = (\mu_{AB} - PV_A) / (\mu_{AB} - PV_A - PV_B)$ and $c_2 = \mu_{AB} / (\mu_{AB} - PV_A - PV_B)$. Since A would only enter into the merger if the expectation of g_A exceeds zero, and since the total expected synergy gain $(\mu_{AB} - PV_A - PV_B) > 0$ (we assume that both parties are rational net value maximisers and would not enter into a deal that would result in an overall expected net loss), we have that $E(\Gamma_A) > 0$ is a necessary condition for a risk averse acquirer. The variance of Γ_A is

$$\text{Var}(\Gamma_A) = \frac{(1 - \beta_S)^2 \sigma_{AB}^2}{(\mu_{AB} - PV_A - PV_B)^2}.$$

We will find it convenient to develop the model in terms of the coefficient of variation of the post-merger value of the combined company, $CV_{AB} = \sigma_{AB} / \mu_{AB}$, which we call the **relative uncertainty** of the merger. Thus we can write

$$\begin{aligned} \text{Var}(\Gamma_A) &= \frac{(1 - \beta_S)^2 \mu_{AB}^2 CV_{AB}^2}{(\mu_{AB} - PV_A - PV_B)^2} \\ &= (1 - \beta_S)^2 c_2^2 CV_{AB}^2 \end{aligned} \quad (3.2)$$

using the notation developed above.

Similarly, the expected standardised net gain to the target is

$$\begin{aligned} E(\Gamma_B) &= \frac{\beta_S \mu_{AB} - PV_B}{\mu_{AB} - PV_A - PV_B} + \lambda \\ &= c_2 \beta_S - c_1 + 1 + \lambda \end{aligned} \quad (3.3)$$

where c_1 and c_2 are as previously defined. Γ_B has variance

$$\begin{aligned} \text{Var}(\Gamma_B) &= \frac{\beta_S^2 \mu_{AB}^2 CV_{AB}^2}{(\mu_{AB} - PV_A - PV_B)^2} \\ &= \beta_S^2 c_2^2 CV_{AB}^2. \end{aligned} \quad (3.4)$$

The aim of the models we build will primarily be to predict β_S and λ as a function of the known variables μ_{AB} , PV_A and PV_B and the uncertainty, and to show by means of empirical testing that this model provides an improvement on the models proposed in the previous chapter in at least two senses: firstly, that it offers a better fit (in terms of predicting the total amount paid by the acquirer) due to the presence of an extra explanatory parameter (the uncertainty, modelled as a variance), and secondly that it is able to model the mix of cash and shares in the payment.

We follow the spirit of the previous chapter and model the bargaining process as a co-operative two-person non-zero sum game, where the outcome of the game is the expected utility of the standardised net gains to A and B, that is $E[U_A(\Gamma_A)]$ and $E[U_B(\Gamma_B)]$. From a bargaining game point of view (in the sense of Nash (1950)) we define the origin of both of the utility functions such that $E[U_A(0)] = E[U_B(0)] = 0$ giving a status quo point $(\Gamma_A; \Gamma_B) = (0; 0)$, i.e. if the merger does not materialise, neither party has any utility gain. The expected utilities to both companies are monotone increasing in their standardised net gains. The Nash-Kalai solution to the problem is the point $(\lambda; \beta_S)$ which maximises the function

$$\Phi = \{E[U_A(\Gamma_A)]\}^\gamma \cdot \{E[U_B(\Gamma_B)]\}^{1-\gamma}, \quad 0 \leq \gamma \leq 1$$

in the range $\lambda \geq 0$, $0 \leq \beta_S \leq 1$. The parameter γ indicates the negotiating power of company A relative to company B.

A Taylor expansion of $U_A(\Gamma_A)$ around $E(\Gamma_A)$ can be written as

$$U_A(\Gamma_A) = U_A[E(\Gamma_A)] + \frac{\Gamma_A - E(\Gamma_A)}{1!} U_A'[E(\Gamma_A)] + \frac{(\Gamma_A - E(\Gamma_A))^2}{2!} U_A''[E(\Gamma_A)] + \dots$$

So

$$\begin{aligned} E[U_A(\Gamma_A)] &= U_A[E(\Gamma_A)] + \frac{E(\Gamma_A - E(\Gamma_A))}{1!} U_A'[E(\Gamma_A)] \\ &\quad + \frac{E(\Gamma_A - E(\Gamma_A))^2}{2!} U_A''[E(\Gamma_A)] + \dots \\ &= U_A[E(\Gamma_A)] + \frac{E(\Gamma_A - E(\Gamma_A))^2}{2} U_A''[E(\Gamma_A)] + \dots \end{aligned}$$

We assume concavity and monotonicity of $U_A(\Gamma_A)$. This implies that higher order derivatives cannot be very large; it is assumed that these derivatives are sufficiently small relative to the higher order moments of the distribution of Γ_A so that higher order terms can be neglected. Then

$$E[U_A(\Gamma_A)] \approx U_A[E(\Gamma_A)] - k_A^* \text{Var}(\Gamma_A) \quad \text{where } k_A^* = -\frac{U_A''[E(\Gamma_A)]}{2}$$

Concavity also implies the strict positivity of k_A^* , which is an indicator of the acquirer's attitude toward excessive deviations from $U_A[E(\Gamma_A)]$. A risk averse acquirer will have k_A^* large, whilst a risk-taker will have k_A^* close to 0. Similarly, it can be shown that

$$E[U_B(\Gamma_B)] \approx U_B[E(\Gamma_B)] - k_B^* \text{Var}(\Gamma_B)$$

where $k_B^* > 0$. This approximation in effect models uncertainty in such a way that the two sources of uncertainty, viz. the variance associated with the estimate of the post-merger value of the merged entity, and the acquirer's and the target's attitude towards risk, are additively independent. For purposes of empirical fitting we introduce the further approximation that for both parties $U[E(\Gamma)]$ can be approximated by some linear function of $E(\Gamma)$. With these approximations, maximisation of $E[U(\Gamma)]$ is equivalent to maximisation of $E(\Gamma) - k \text{Var}(\Gamma)$ for both parties, where k here is some positive constant times the k^* value introduced previously. From here onwards we will refer to k as the **risk aversion coefficient** of the party concerned. Then for the acquirer

$$\begin{aligned} E[U_A(\Gamma_A)] &= E(\Gamma_A) - k_A \text{Var}(\Gamma_A) \\ &= c_1 - c_2 \beta_S - \lambda - k_A (1-\beta_S)^2 c_2^2 CV_{AB}^2 . \end{aligned}$$

The two sources of uncertainty are represented by a single measure, ξ_A . Thus

$$E[U_A(\Gamma_A)] = c_1 - c_2 \beta_S - \lambda - (1-\beta_S)^2 c_2^2 \xi_A$$

where $\xi_A = k_A CV_{AB}^2$. Since $k_A > 0$ we have that $\xi_A > 0$. Similarly, for the target

$$\begin{aligned} E[U_B(\Gamma_B)] &= E(\Gamma_B) - k_B \text{Var}(\Gamma_B) \\ &= c_2 \beta_S - c_1 + 1 + \lambda - \beta_S^2 c_2^2 \xi_B \end{aligned}$$

where $\xi_B = k_B CV_{AB}^2 > 0$. ξ_A and ξ_B are the risk/uncertainty measures associated with the post-merger company AB as perceived from the acquirer's and target's point of view respectively. Thus these **additive mean/variance utility functions** have a *linear* component (the standardised net gain to the party concerned) and a *quadratic* component (the risk as perceived by the party concerned). The above remains an approximation which is of course only justifiable insofar as the underlying assumptions remain a reasonable approximation. For example, the approximation would no longer be appropriate if it led to non-monotonic behaviour over the range $0 < \beta_S < 1$.

The Nash-Kalai criterion based on the above is the point $(\lambda; \beta_S)$ which maximises the function

$$\Phi(\lambda; \beta_S) = \left[c_1 - c_2 \beta_S - \lambda - (1-\beta_S)^2 c_2^2 \xi_A \right]^\gamma \left[c_2 \beta_S - c_1 + 1 + \lambda - \beta_S^2 c_2^2 \xi_B \right]^{1-\gamma} \quad (3.5)$$

in the range $\lambda \geq 0$, $0 \leq \beta_S \leq 1$. This is equivalent to maximising $L = \ln \Phi(\lambda; \beta_S)$, i.e. maximising

$$L = \gamma \ln \left[c_1 - c_2 \beta_S - \lambda - (1-\beta_S)^2 c_2^2 \xi_A \right] + (1-\gamma) \ln \left[c_2 \beta_S - c_1 + 1 + \lambda - \beta_S^2 c_2^2 \xi_B \right] \quad (3.6)$$

This function describes a response surface for a given merger over the two-dimensional space defined by all $(\lambda; \beta_S)$ pairs, and is parameterised by the uncertainty terms ξ_A and ξ_B . The required non-negativity of $E[U_A(\Gamma_A)]$ and $E[U_B(\Gamma_B)]$ imply that

$$\lambda \leq - (c_2^2 \xi_A) \beta_S^2 + (2 c_2^2 \xi_A - c_2) \beta_S + c_1 - c_2^2 \xi_A \quad (3.7)$$

and

$$\lambda \geq (c_2^2 \xi_B) \beta_S^2 - c_2 \beta_S + c_1 - 1 \quad (3.8)$$

The feasible range for λ and β_S is thus an area bounded by the two quadratic equations implied by (3.7) and (3.8) in the range $\lambda \geq 0$ and $0 \leq \beta_S \leq 1$. The derivation of the roots of the two equations and the turning points can be found in Appendix 3.A. The feasible range depends only on the constants c_1 and c_2 , and the risk/uncertainty components ξ_A and ξ_B . A change in the perceived risk associated with the merger by both parties concerned will thus alter the shape and size of the feasible region, assuming that k_A and k_B do not change. As the perceived post-merger risk, σ_{AB} , increases, the feasible region will decrease in extent as the quadratics pass through one another, eventually becoming a single point on the β_S -axis (a share-only transaction), and then disappearing entirely from the region of all practical meaning (i.e. from the area $\lambda \geq 0$, $0 \leq \beta_S \leq 1$). This would imply that no rational merger would occur due to the large uncertainty involved.

§3.3 THE NASH-KALAI SOLUTION

The Nash-Kalai criterion in (3.5) can be written as

$$\text{Maximise}_{\lambda \geq 0, 0 \leq \beta_S \leq 1} \Phi(\lambda; \beta_S) = [Q_1(\beta_S) - \lambda]^\gamma [\lambda - Q_2(\beta_S)]^{1-\gamma} \quad (3.9)$$

where

$$Q_1(\beta_S) = c_1 - c_2 \beta_S - c_2^2 \xi_A (1-\beta_S)^2$$

and

$$Q_2(\beta_S) = c_1 - 1 - c_2 \beta_S + c_2^2 \xi_B \beta_S^2$$

We first investigate the *unconstrained* maximum of the Nash-Kalai criterion; we return later to investigate the effect of imposing the constraints on λ and β_S on this unconstrained solution. Necessary conditions for an unconstrained maximum are $\frac{\partial \Phi(\lambda; \beta_S)}{\partial \lambda} = 0$ and $\frac{\partial \Phi(\lambda; \beta_S)}{\partial \beta_S} = 0$ simultaneously, which give rise to the optimal solution point $(\lambda^*; \beta_S^*)$ where

$$\lambda^* = (1-\gamma) Q_1(\beta_S^*) + \gamma Q_2(\beta_S^*)$$

$$= c_1 - \gamma - c_2 \beta_S^* + c_2^2 CV_{AB}^2 \left[\gamma k_B (\beta_S^*)^2 - (1-\gamma) k_A (1-\beta_S^*)^2 \right] \quad (3.10)$$

$$\text{and } \beta_S^* = \frac{k_A}{k_A + k_B} \quad (3.11)$$

Derivation of these results is contained in Appendix 3.B.

Provided $\lambda^* \geq 0$, the solution given by (3.10) and (3.11) is feasible (since λ^* satisfies (3.7) and (3.8), and $0 \leq \beta_S^* \leq 1$ as both $k_A > 0$ and $k_B > 0$), and is the solution to the constrained problem. If $\lambda^* < 0$ (i.e. the unconstrained optimum is infeasible in the constrained problem), then we can set up the Kuhn-Tucker conditions on λ in the Nash-Kalai criterion to determine the optimal *constrained* value of λ . Since β_S^* was feasible in the constrained problem we do not consider the constraint on β_S at this stage. A derivation of the optimal constrained solution point $(\lambda^+; \beta_S^+)$ when $\lambda^* < 0$ can be found in Appendix 3.B, and is shown to be

$$\lambda^+ = 0$$

and β_S^+ the feasible root of the cubic equation

$$\gamma Q_2(\beta_S^+) Q_1'(\beta_S^+) + (1-\gamma) Q_1(\beta_S^+) Q_2'(\beta_S^+) = 0 \quad (3.12)$$

which maximises $\Phi(0; \beta_S)$ in (3.9). In the case where none of the roots proves feasible then no negotiated solution exists, and the status quo (i.e. no merger agreement) applies. The three possible solutions for β_S^+ arising from (3.12) have not been detailed here: their complicated and extremely lengthy structure mitigates against gaining further insights. We can however note that a resulting β_S^+ depends in a complex way not only on k_A and k_B , but also on the other parameters c_1 , c_2 , γ and the uncertainty CV_{AB}^2 . We have not been able to prove the general feasibility of any of the above three solutions for β_S^+ . However in principle in any specific case all real roots of (3.12) can be investigated for feasibility, and the resulting feasible ones examined to determine the optimal constrained solution, β_S^+ .

In summary, the optimal bargaining solution consists of two distinct cases:

- (a) that when the unconstrained $\lambda^* \geq 0$, in which case the unique solution is available in closed form by (3.10) and (3.11);

and

- (b) that when the unconstrained $\lambda^* < 0$, in which case the optimal solution has $\lambda^+ = 0$ and β_S^+ one of the feasible roots of a cubic equation, providing at least one exists (failing which the status quo applies).

§3.4 DISCUSSION AND PRACTICAL INTERPRETATION OF SOME MATHEMATICAL RESULTS

As already noted the optimal bargaining solution to the Nash-Kalai criterion consists of two distinct cases. In practice a merger can be financed by a share exchange and/or a cash payment; one aspect a rational acquirer must consider in formulating an offer is the level of the cash payment. This could be nothing or some positive amount: a negative cash payment (i.e. demanding cash back from the target shareholders) is not considered. Thus the solution given in (3.10) and (3.11) is important for practical application of the model and warrants further interpretation. Further work will be restricted to this case, with only brief observations made regarding the other solution case.

1. Cash alone is *never* the optimal form of payment according to this model: since the optimal $\beta_S = k_A / (k_A + k_B) > 0$, a share exchange will always form part of the optimal form of payment. On the basis of the assumptions of this model we note that the target shareholders are optimally required to bear at least a part of the risk (and also share in some of the gains) associated with the merger. This point is widely accepted in the Finance literature (see, for example, Brealey and Myers (1981)).

Note that there may, of course, be other external factors affecting the mix of cash and shares (for example, the acquirer may be cash-rich, and may be acquiring the target purely as a sensible means of redeploying this cash (Brealey and Myers (1981)), which we do not consider here.

2. The optimal amount offered (in cash and shares) for a specific target depends on the a priori perceived level of risk involved. The share exchange portion of the offer is independent of the uncertainty, CV_{AB} . However the cash portion of the offer *does* depend on CV_{AB} , and thus more uncertainty will imply a change in the amount of cash offered.

From (3.10) we observe that providing the risk aversion coefficient of the target is relatively large compared to that of the acquirer then an increase in uncertainty implies a decrease in the optimal cash offer quantity. This might be typical of a situation in which a large acquiring company in a healthy financial position attempts to acquire a small target with precarious finances. The acquirer demands "payment" for the extra uncertainty in the form of a lower (risk-free) cash portion in the offer. On the other hand, if the risk aversion coefficient of the target is small relative to that of the acquirer (typical of the case of the acquirer being smaller and/or less financially secure than the target) then an increase in uncertainty is accompanied by an increase in the cash portion of the offer. The acquirer here has to offer more in the form of (risk-free) cash to the target to tempt it to enter into the more risky deal.

In either of the above cases the uncertainty and risk aversion coefficients might be such that the optimal cash payment reduces to zero. Any further change in uncertainty in the same direction will result in a change in the share exchange portion of the payment whilst the cash portion of the offer remains zero.

Thus the decision as to whether to finance the merger by a share exchange alone or by a combination of cash and shares should depend (per the Nash-Kalai rationality assumption) entirely on the risk aversion coefficients of both target and acquirer, the relative bargaining strength of the players and the level of uncertainty present. The merger agreement does not merely involve the acceptance by the target of an amount offered by the acquirer: the form of payment is also central to the agreement, and so should be considered carefully by the acquiring company when making the offer. Management of a rational acquiring company might use the following steps in arriving at an acceptable bargaining solution:

- Step 1.** Determine the total amount T that the acquirer is willing to offer for the target company. This will be based in part on the overall uncertainty associated with the merger.
- Step 2.** Estimate the risk aversion coefficients of the acquirer (k_A) and of the target (k_B), and hence calculate $\bar{\beta}_S = k_A / (k_A + k_B)$.

Step 3. If $T \leq \bar{\beta}_S$, the acquirer should offer the full amount T in a share exchange. If $T > \bar{\beta}_S$, the acquirer should offer an amount of $\bar{\beta}_S$ in a share exchange and the balance $T - \bar{\beta}_S$ in cash.

§3.5 AN EMPIRICAL EXAMPLE

In this section we choose an empirical example from the Johannesburg Stock Exchange (JSE) to exhibit the results obtained in the previous sections. In particular, we examine the shape and extent of the feasible range of $(\lambda; \beta_S)$ points, identifying the position of the optimal point $(\lambda^*; \beta_S^*)$ for varying values of the risk/uncertainty parameters ξ_A and ξ_B .

In January 1982 Hulett's Corporation, a predominantly sugar-producing company with a listing in the Industrial sector of the JSE announced that it had concluded negotiations with Tongaat Foods to acquire all their listed stock, employing a share exchange of 117 Hulett's shares per 100 Tongaat shares. Tongaat was subsequently delisted. Six months¹ prior to the announcement Hulett's had 32.9 million shares outstanding, each trading at R8-09, for a total market capitalisation of R266.161 million, whilst Tongaat had 227.795 million shares outstanding, each trading at R6-87, for a total market capitalisation of R190.952 million. A month after the merger announcement the combined company had a market value of R605.136 million - we use this ex post observed value as an estimate of the a priori perception of the expected value of the merged entity. From the above we calculate

$$c_1 = (605.136 - 266.161) / (605.136 - 266.161 - 190.952) = 2.290$$

$$\text{and } c_2 = 605.136 / 148.023 = 4.088$$

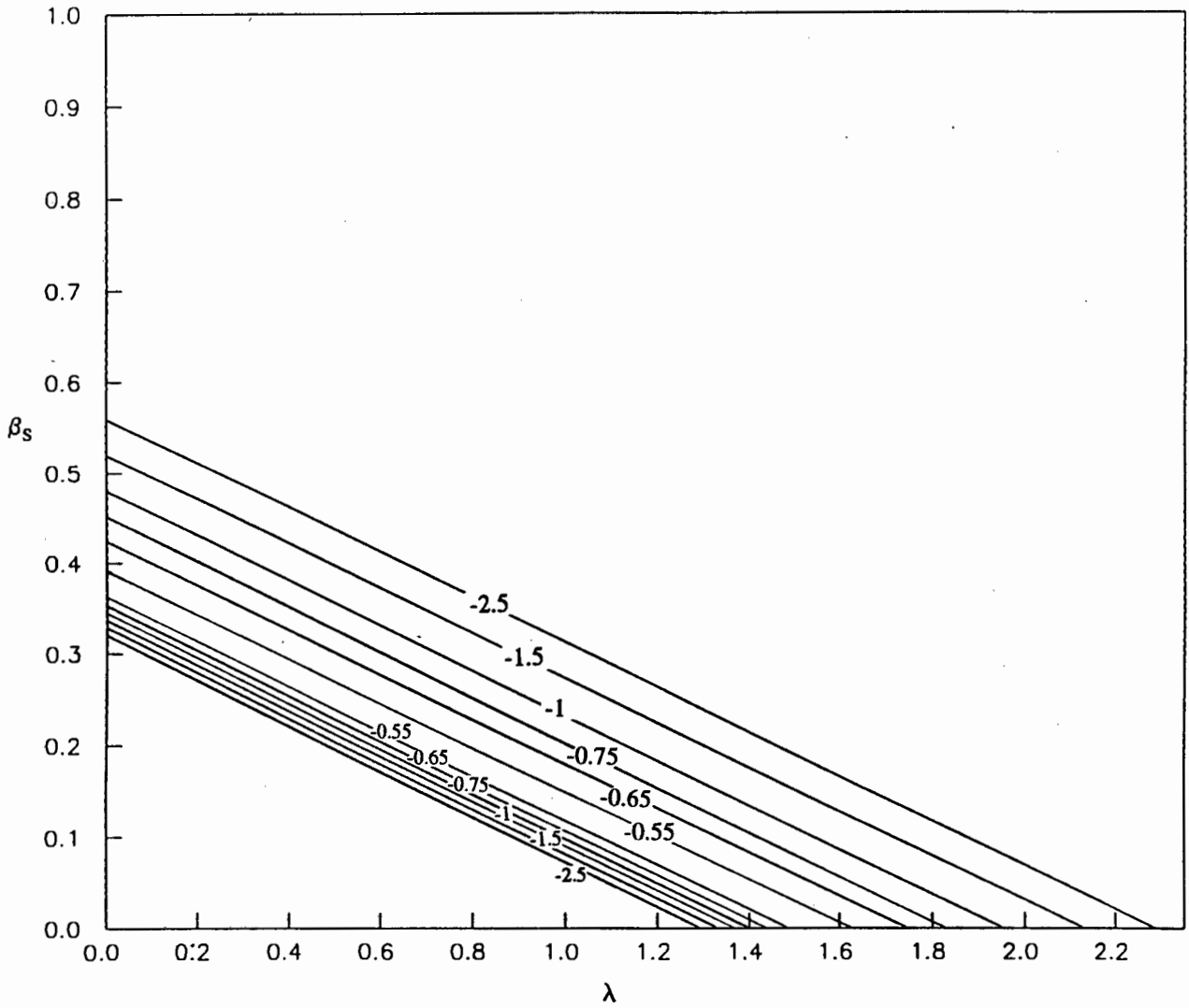
Figures 3.1 to 3.9 exhibit the extent of the feasible $(\lambda; \beta_S)$ space and the iso-Nash contours (lines of equal value of $L = \ln \Phi(\lambda; \beta_S)$ from (3.6)) within this feasible space for

¹ To avoid pre-announcement share-price effects (especially notable amongst target companies) a time period of six months prior to the merger announcement was chosen for the estimation of the pre-merger values of the two participants for the JSE (Affleck-Graves, Flach and Jacobson (1988), Bhana (1987), Van den Honert, Barr, Affleck-Graves and Smale (1988)). This period may well differ on other exchanges. See also §2.5.1. for a fuller discussion of the estimation of the financial worth of the participating companies.

the various values of ξ_A and ξ_B in the table below. We note that $\xi_A = k_A CV_{AB}^2$ and $\xi_B = k_B CV_{AB}^2$, and we call these the "risk parameters" of the acquirer and the target respectively; we do not attempt to differentiate between the risk aversion component and the a priori estimation uncertainty component. Figures 3.1 to 3.9 were created by evaluating the Nash-Kalai function L in (3.6) at each point in a 40x40 matrix of $(\lambda; \beta_S)$ points in the range $0 \leq \lambda \leq 1$, $0 \leq \beta_S \leq 1$, and then fitting contours through these values. The contour levels and the extent of the $(\lambda; \beta_S)$ space chosen for each plot may vary from

Figure	ξ_A	ξ_B
3.1	0.00	0.00
3.2	0.10	0.02
3.3	0.10	0.04
3.4	0.10	0.08
3.5	0.10	0.10
3.6	0.10	0.135
3.7	0.04	0.10
3.8	0.08	0.10
3.9	0.04	0.04

figure to figure to ensure ease of reading. Figure 3.1 is merely the risk-free case, and we observe that the feasible $(\lambda; \beta_S)$ region is a band of uniform width running from the interval (0.32; 0.56) on the β_S -axis in a south-easterly direction to the interval (1.29; 2.29) on the positive λ -axis, and is not defined for $(\lambda; \beta_S)$ values outside of this range. The plot shows the following iso-Nash contours: -2.5, -1.5, -1.0, -0.75, -0.65 and -0.55. All the iso-Nash contours are parallel to one another; the Nash-Kalai function reaches a maximum along a ridge situated in the feasible space at a value of just less than -0.50 and decreases in value towards both edges. The rate of decrease is much more rapid on the south-western edge than on the north-eastern edge. This model is unable to differentiate between cash and shares as a medium of payment and thus it is unable to locate a *single* optimal $(\lambda; \beta_S)$ point (the optimal locus consists of all $(\lambda; \beta_S)$ points along a line within the feasible space, i.e. all combinations of cash and shares which have equal utility value).



Iso-Nash contour levels:

-2.50

-1.50

-1.00

-0.75

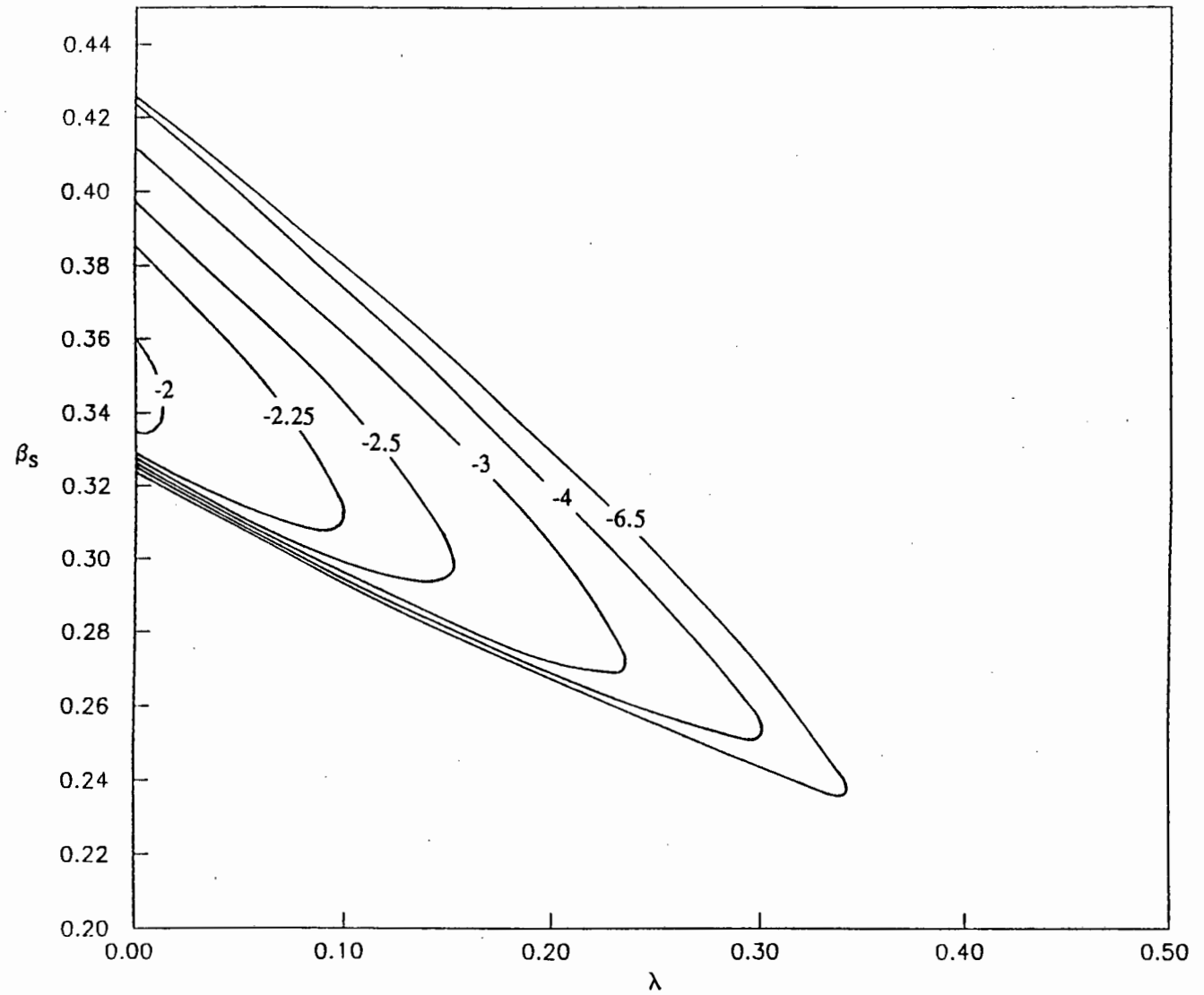
-0.65

-0.55

Figure 3.1. Feasible $(\lambda; \beta_S)$ space for the Hulett/Tonga merger

$$(\xi_A = \xi_B = 0)$$

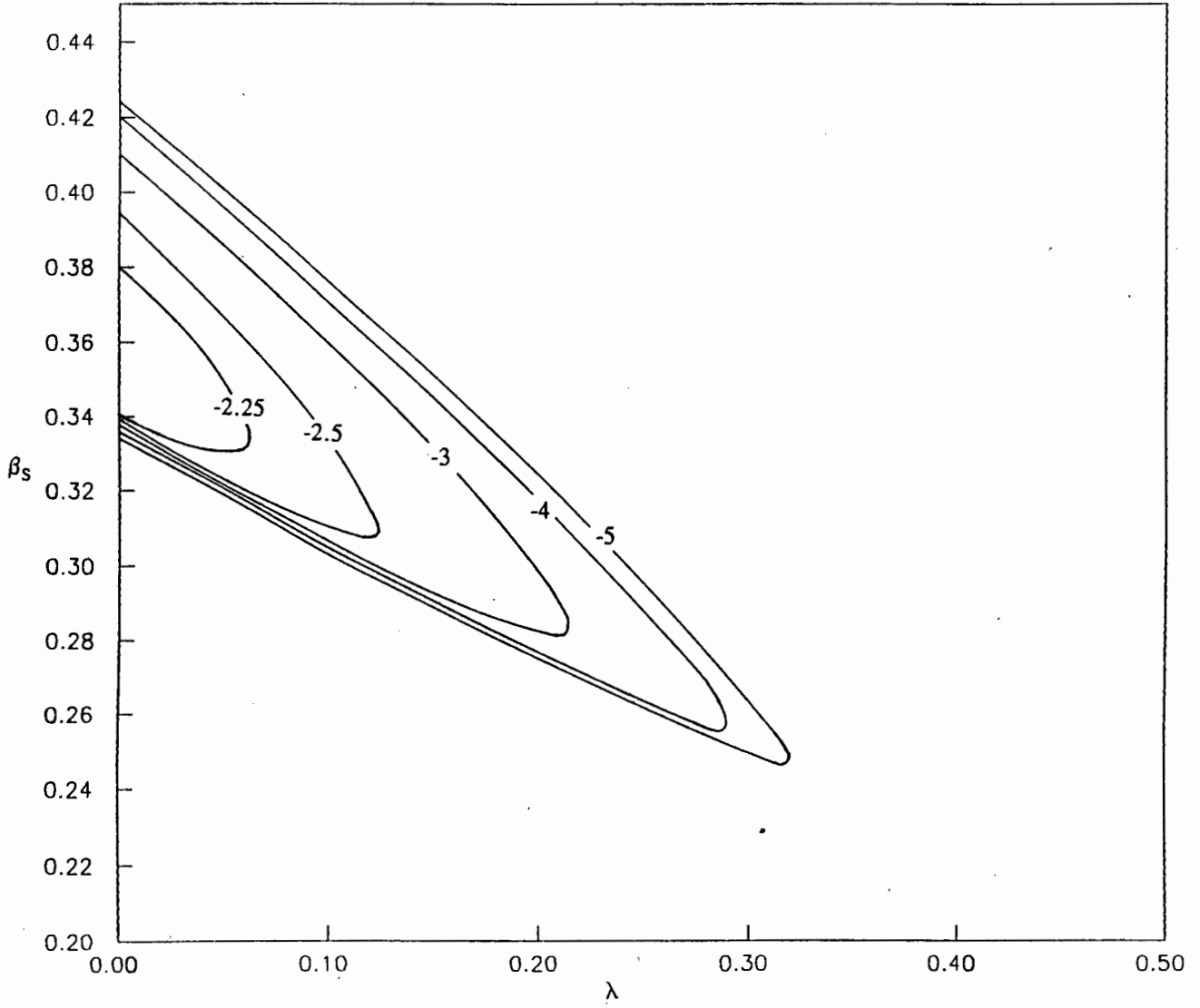
Figures 3.2 to 3.9 exhibit the effects of risk on the feasible space of the Nash-Kalai function. The sequence of Figures 3.2 to 3.6 show the effect of a change in ξ_B from 0.02 (low target risk parameter) through to 0.135 (high target risk parameter) whilst ξ_A is fixed at 0.1 (high acquirer's risk parameter). Whilst the actual contour levels indicated vary from plot to plot, it will be noticed that the upper boundary of the feasible space is entirely stationary (since the value of ξ_A is constant): the lowest contour level indicated passes through the point (0; 0.425) in all the plots. The lower boundary moves up towards the upper boundary as the target's risk parameter increases, hence reducing the size of the feasible space. Thus when $\xi_B=0.02$ the range of possible β_S values (when $\lambda=0$) is the interval (0.322; 0.425) and this reduces to the interval (0.41; 0.425) when $\xi_B=0.135$. At the same time the "point" of the feasible space is receding in a north-westerly direction: when $\xi_B=0.02$ the point of the feasible space is situated close to (0.34; 0.24), but when $\xi_B=0.135$ the point is at (0.19; 0.33). When ξ_B exceeds approximately 0.1483 the "lower boundary" is situated above the "upper boundary" and the feasible space disappears entirely. For risk levels larger than this the set of feasible $(\lambda; \beta_S)$ points is empty, indicating that there are no possible cash/share combinations which would satisfy both parties simultaneously, and thus there is no likelihood of a successful merger offer. In all the plots the contours increase in value towards the middle of the feasible space, forming a "shoulder"-like surface. The rate of increase is much slower on the north-eastern slope than on the south-western slope, where it is almost vertical in all cases. The Nash-Kalai function reaches a maximum on the β_S -axis (i.e. when the amount of cash offered is 0). Since the acquirer's risk parameter is large (i.e. the acquirer's perception of the post-merger risk is large and/or the acquirer has large risk aversion), a rational acquirer will not be prepared to enter into the merger unless the target shareholders share the merger risk fully, and so the optimal offer will be a share-only offer, regardless of the level of the target's risk parameter. We note that the smaller ξ_B is, the larger the value of the Nash-Kalai function at the optimum point.



Iso-Nash contour levels:

-6.50
 -4.00
 -3.00
 -2.50
 -2.25
 -2.00

Figure 3.2. Feasible $(\lambda; \beta_S)$ space for the Hulett/Tongaat merger
 $(\xi_A = 0.1; \xi_B = 0.02)$



Iso-Nash contour levels:

-5.00

-4.00

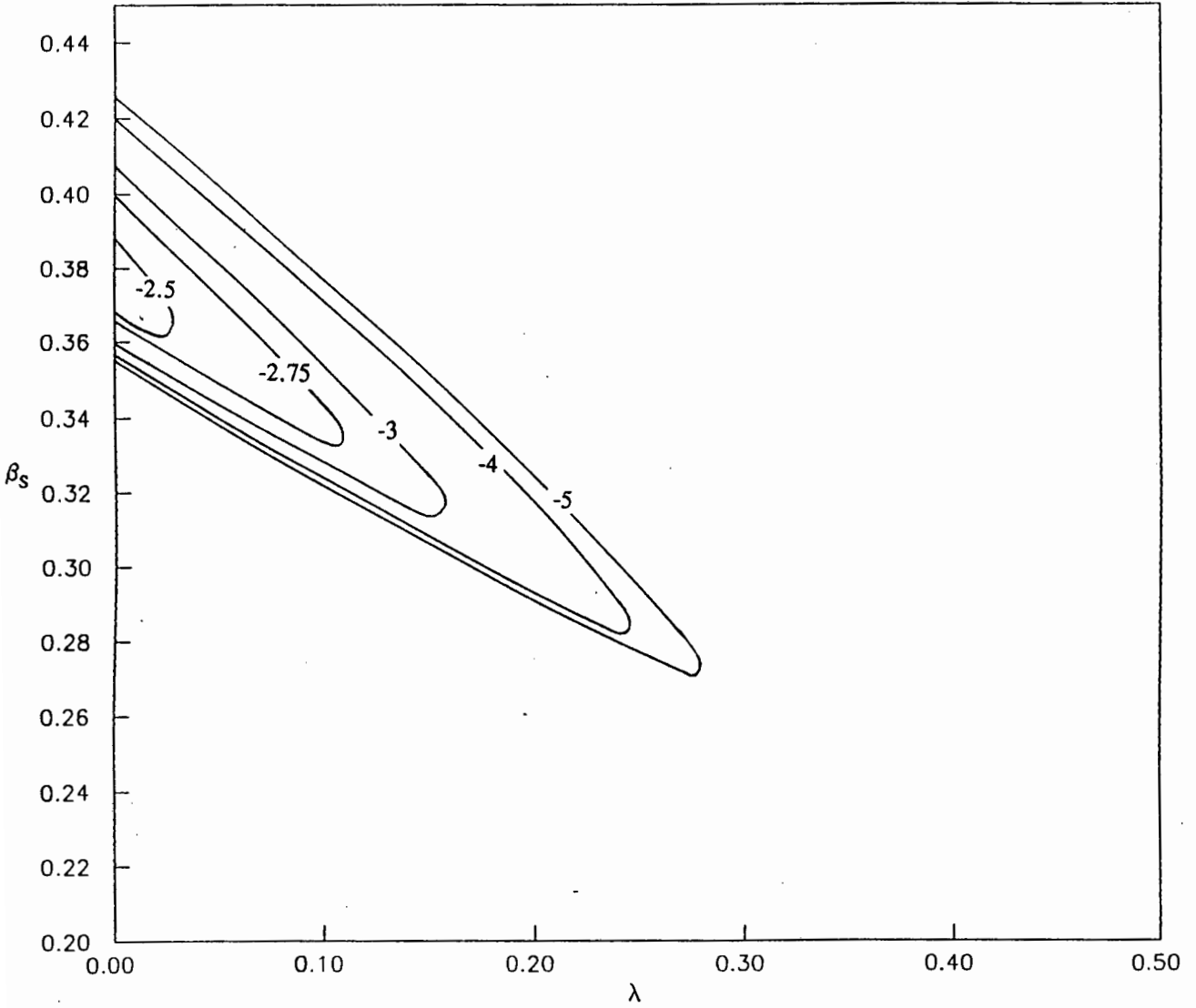
-3.00

-2.50

-2.25

Figure 3.3. Feasible $(\lambda; \beta_S)$ space for the Hulett/Tongaat merger

$(\xi_A = 0.1; \xi_B = 0.04)$



Iso-Nash contour levels:

-5.00

-4.00

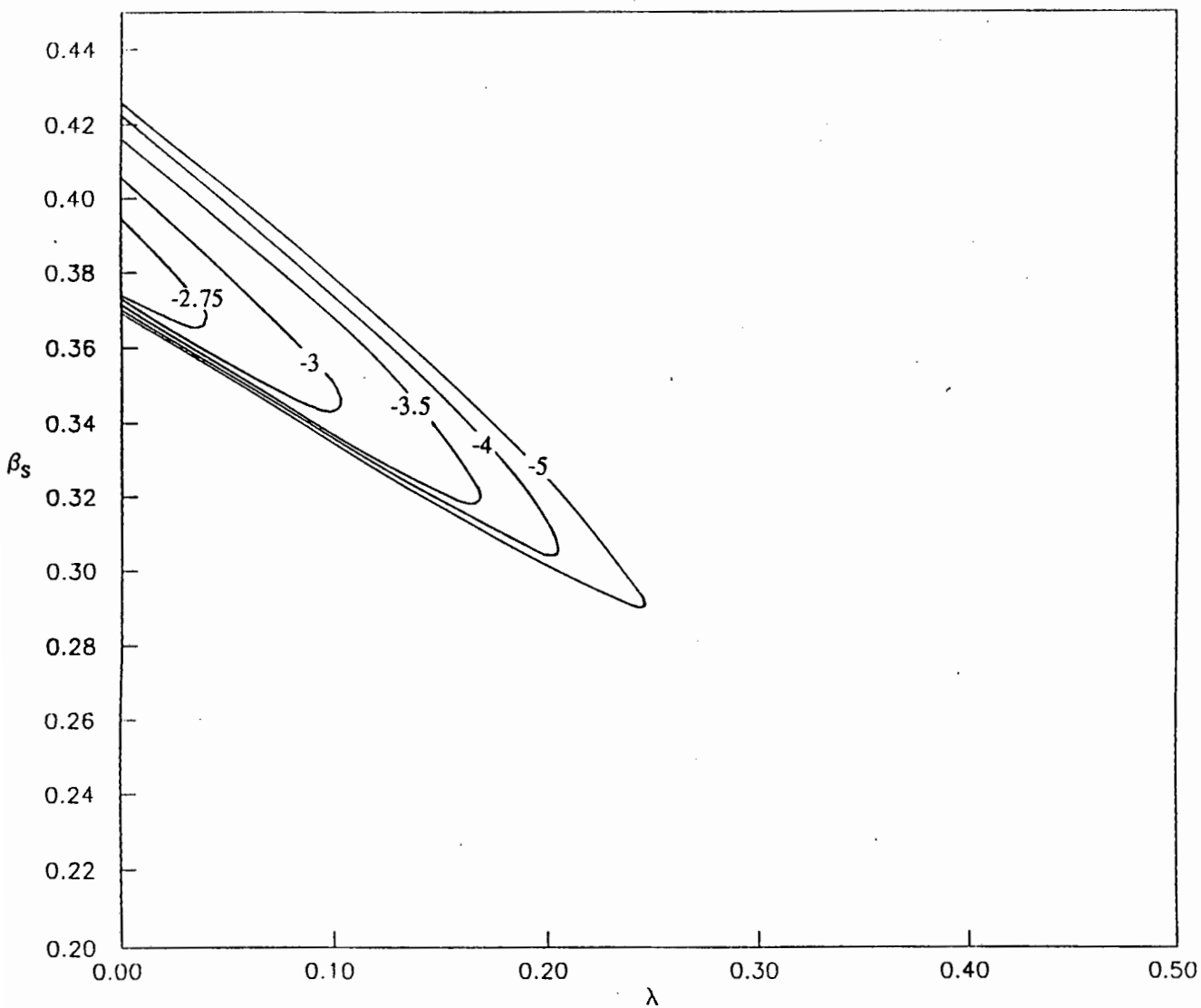
-3.00

-2.75

-2.50

Figure 3.4. Feasible $(\lambda; \beta_S)$ space for the Hulett/Tongaat merger

$(\xi_A = 0.1; \xi_B = 0.08)$



Iso-Nash contour levels:

-5.00

-4.00

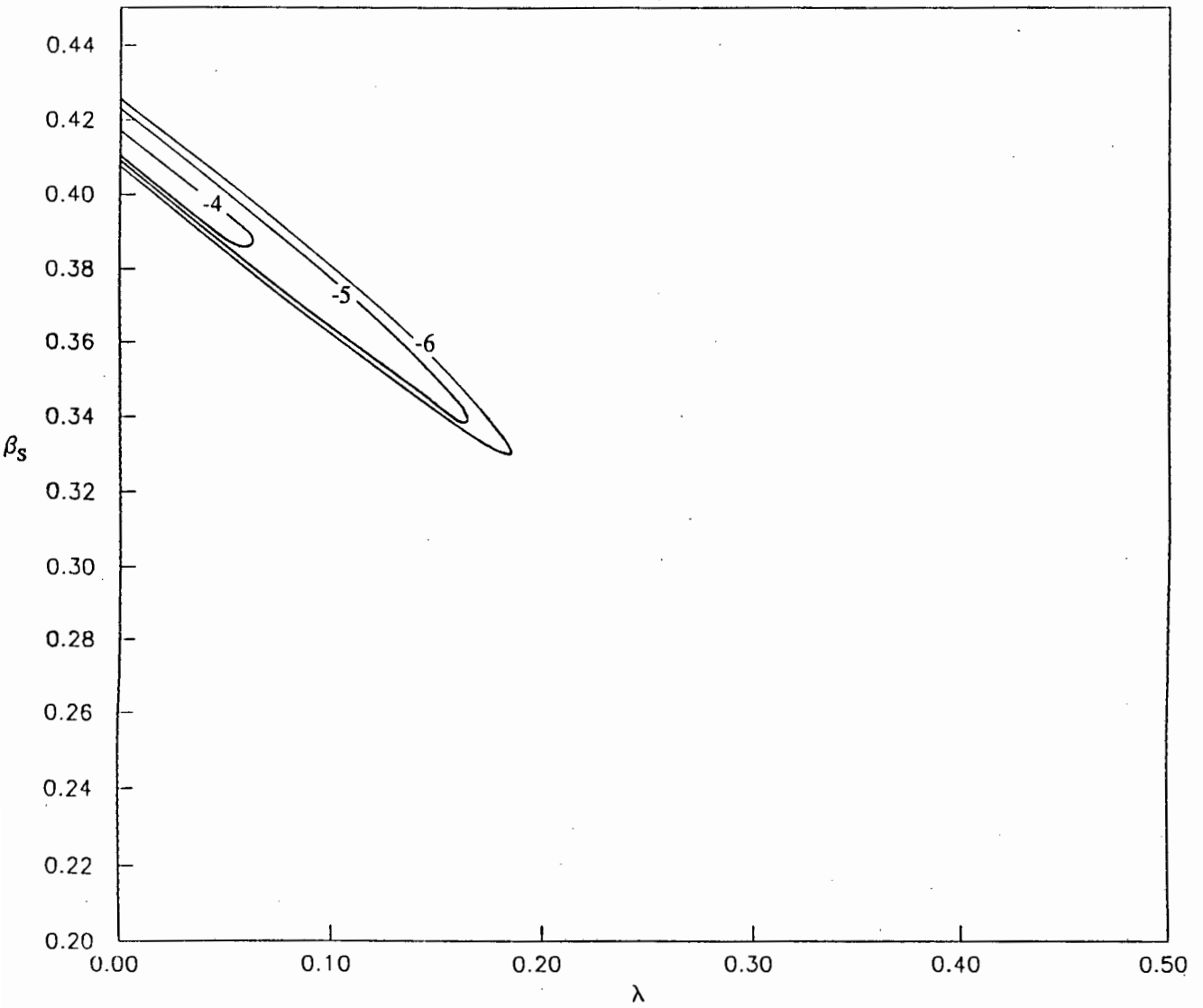
-3.50

-3.00

-2.75

Figure 3.5. Feasible ($\lambda; \beta_S$) space for the Hulett/Tongaat merger

($\xi_A = 0.1; \xi_B = 0.1$)



Iso-Nash contour levels:

-6.00

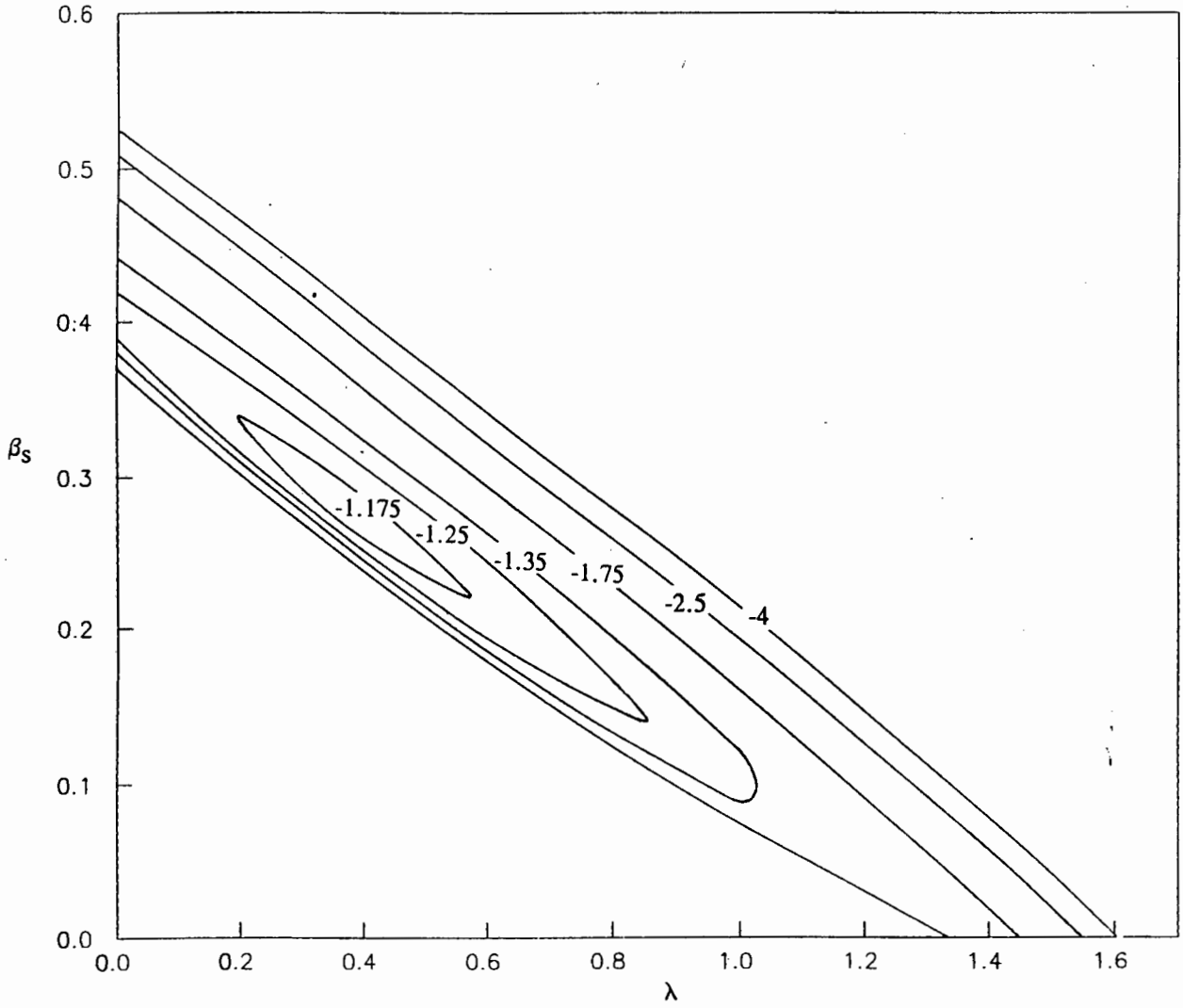
-5.00

-4.00

Figure 3.6. Feasible ($\lambda; \beta_S$) space for the Hulett/Tonga merger

($\xi_A = 0.1; \xi_B = 0.135$)

The sequence of Figures 3.7, 3.8 and 3.5 demonstrates the effect of a change in ξ_A from 0.04 (low acquirer's risk parameter) through to 0.1 (high acquirer's risk parameter) whilst ξ_B is fixed at 0.1 (high target risk parameter). For this sequence of diagrams the lower boundary of the feasible space remains stationary now since the value of ξ_B is constant: the lowest contour level indicated passes through the point (0; 0.37) in all cases. The upper boundary curve moves downwards as the acquirer's risk parameter increases, reducing the size of the feasible space. Thus when $\xi_A=0.04$ the range of possible β_S values (when $\lambda=0$) is (0.37; 0.525) and this reduces to (0.37; 0.425) when $\xi_A=0.1$. When ξ_A exceeds approximately 0.1160 the feasible space disappears entirely. At low values of ξ_A (Figure 3.7) the contours are almost parallel to one another and only curve gently inwards near the edges of the plot (i.e. near the axes). We note that a cash-only offer is feasible, since when $\beta_S=0$, values of λ in the interval (1.33; 1.61) are defined. The contours increase in value from both the north-east and south-west boundaries towards the middle of the plot (the rate of increase is slower on the north-eastern slope), reaching a maximum close to the point (0.4; 0.28), i.e. at some point where $\lambda \neq 0$. Thus an exchange of shares *together with* a cash payment form the optimal offer. Since the target's risk parameter is at such a high level (and thus the target perceives that the post-merger risk will be large and/or the target has a high level of risk aversion), the rational acquirer will be forced to include at least a portion of cash (which is, of course, risk-free) in the optimal offer to entice the target to accept. For higher values of ξ_A (Figures 3.8 and 3.5) a cash-only offer is no longer feasible; the feasible space returns to the familiar shoulder-like shape not intersecting the λ -axis. At these and higher levels of ξ_A the Nash-Kalai function is maximised at a point on the β_S -axis, implying that the optimal offer will be a share-only offer. This is due to the acquirer (who makes the offer anyway) having a large risk parameter, and thus demanding to be compensated for this risk by forcing it partly onto the target. The target's large risk parameter (which would seem to indicate the requirement for a portion of (risk-free) cash in the offer) appears to play a secondary role to that of the acquirer. This situation thus illustrates a further facet of the conflict: if both companies have large risk parameters, the acquirer will desire a share-only offer and the target will desire some form of cash payment as well as a share exchange. Finally, the smaller ξ_A is, the larger the value of the Nash-Kalai function at the optimum point.

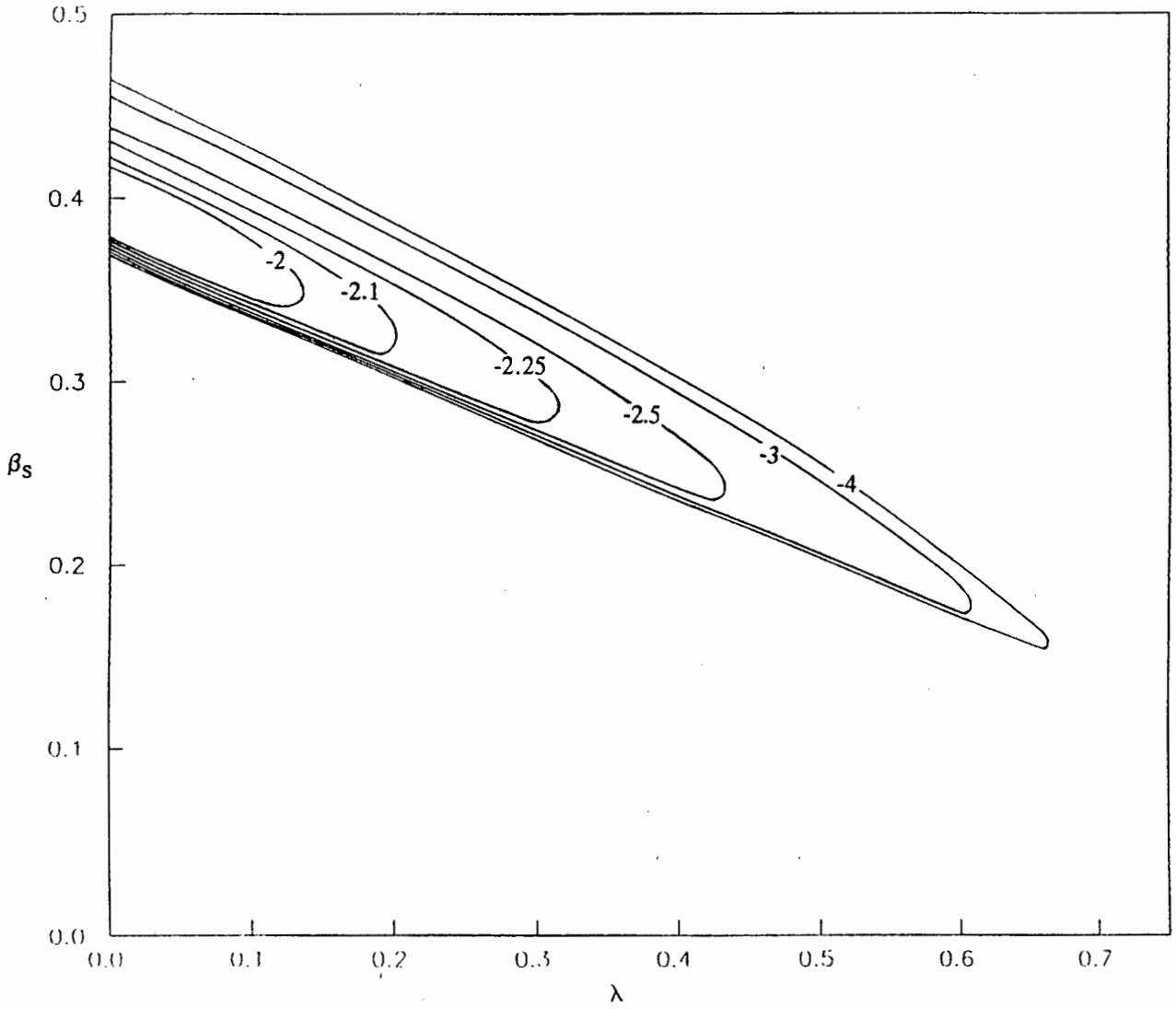


Iso-Nash contour levels:

-4.00
 -2.50
 -1.75
 -1.35
 -1.25
 -1.175

Figure 3.7. Feasible $(\lambda; \beta_S)$ space for the Huletts/Tongaat merger

$(\xi_A = 0.04; \xi_B = 0.1)$



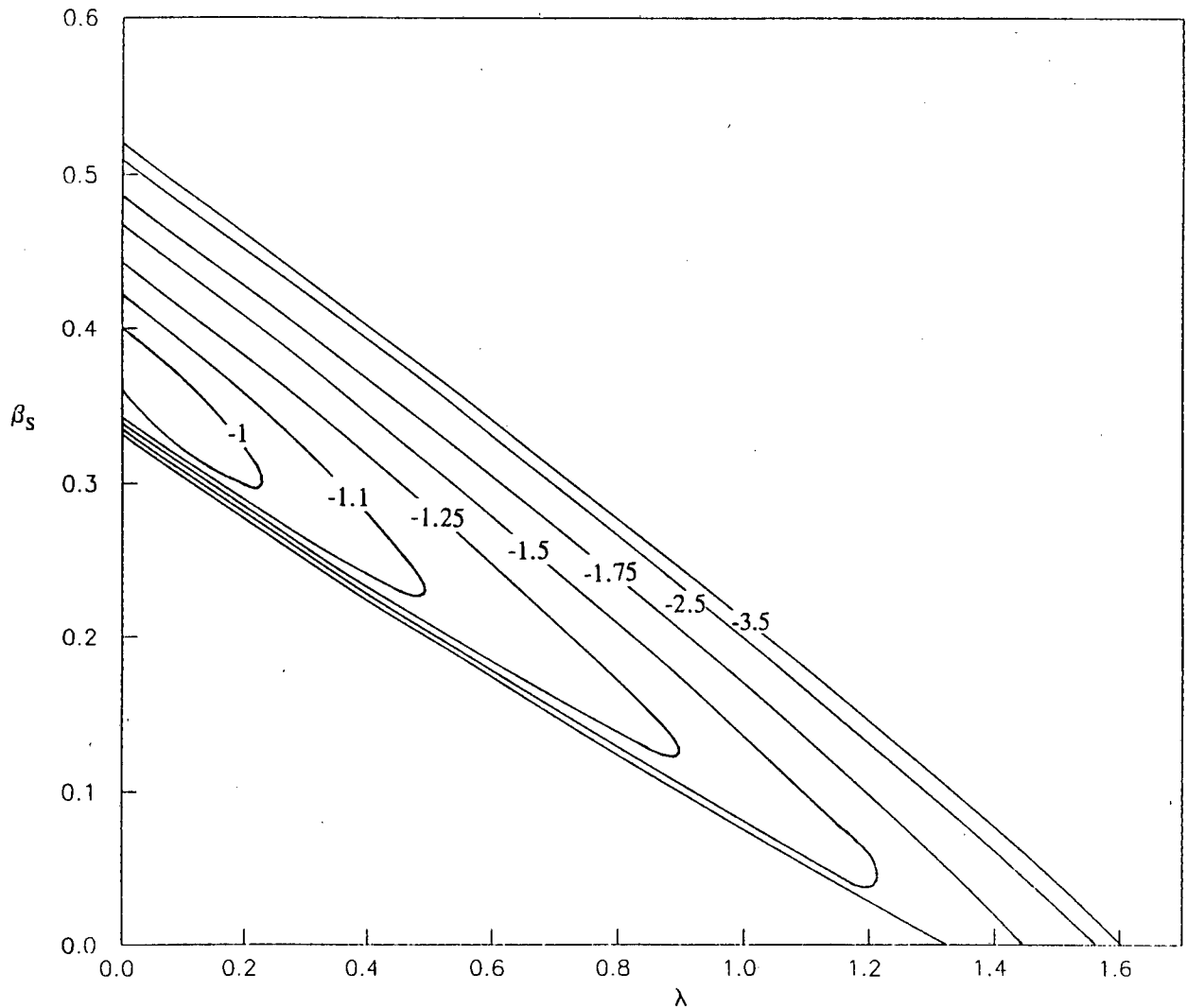
Iso-Nash contour levels:

-4.00
 -3.00
 -2.50
 -2.25
 -2.10
 -2.00

Figure 3.8. Feasible $(\lambda; \beta_S)$ space for the Huletts/Tongaat merger

$(\xi_A = 0.08; \xi_B = 0.1)$

The situation where both companies have small (but not zero) risk parameters is illustrated in Figure 3.9. Here both ξ_A and ξ_B are set to 0.04. The shape of the feasible space is very similar to that of Figure 3.7 in that it cuts both axes, and the outer contours are largely parallel to one another. The optimal point is on or very close to the



Iso-Nash contour levels:

-3.50

-2.50

-1.75

-1.50

-1.10

-1.00

Figure 3.9. Feasible $(\lambda; \beta_S)$ space for the Hulett/Tongaat merger

$(\xi_A = 0.04; \xi_B = 0.04)$

β_S -axis (thus implying a share exchange offer with possibly a very small cash side-payment). Since the target's risk parameter is small (and hence the target would not be unwilling to bear some degree of risk), the acquirer is able to capitalise by making what is largely a share-exchange offer, and hence ensuring that what risk there is will also fall on the shoulders of the target shareholders.

The table below summarises the above discussion on the optimal form of payment for this empirical example.

Figure	Risk parameters		Favoured outcome	
	acquirer	target	acquirer	target
3.8 / 3.5 / 3.6	large	large	shares	cash and shares
3.9	small	small	any form (shares)	any form
3.2 / 3.3	large	small	shares	any form
3.7	small	large	any form	cash and shares

The columns headed "favoured outcome" indicate the form of the optimal offer that would be favoured by each of the parties for their own level of risk parameter. For example, if the acquirer has a large risk parameter it would favour a share-only offer, to pass part of the risk on to the target. This is true irrespective of the size of the target's risk parameter. The modelled outcomes observed in Figures 3.2 to 3.9 are represented in bold in the table. It appears that the acquirer (the party that makes the offer) is dominant over the target when it comes to the optimal form of payment, the optimal offer always satisfying its favoured outcome. We see that if the acquirer has a small risk parameter (and hence would be prepared to tolerate any form of payment) its optimal offer comprises cash and shares if the target's risk parameter is high (to satisfy the target's desire to receive at least some cash) with a decreasing amount of cash as the target's risk parameter decreases. Eventually the cash payment will be zero (which, of course, will satisfy both parties). However, if the acquirer's risk parameter is large, a share-only offer is always the optimal offer, which satisfies a target with a small risk parameter (who is indifferent to the form of payment) but *does not* satisfy the target with a large risk parameter, which would prefer a cash payment.

§3.6 FITTING A GLOBAL MODEL WITH SHARED UNCERTAINTY

§3.6.1 CONSTRUCTION OF A GLOBAL MODEL WITH AN ADDITIVE MEAN / VARIANCE UTILITY FUNCTION

We now attempt to construct a general model which we will match to a set of empirical mergers, where the assumption is that there is some measure of uncertainty about the post-merger value of the merged entity. As in previous sections this uncertainty is modelled as the variance σ_{AB}^2 of the probability distribution of the post-merger value of the combined company (viewed from the pre-merger perspective), and is assumed to be the same for both the target and the acquirer. This variance cannot be measured empirically for any single merger since a merger is a one-off event, and thus its estimation will be of importance in this section. We furthermore aim to show that by including an uncertainty term in the model we achieve an improvement in fit relative to the models in Chapter 2, as well as being able to obtain estimates of the amount of cash and the monetary value of shares exchanged in the actual transaction. This model will thus provide a deeper understanding of the negotiation process as it may occur in practice, as well as offering decision support as to the form of payment and the optimum amount of cash and shares which should be offered.

The additive mean/variance utility function as used in this chapter (i.e. where the expected utility of the standardised net gain equals the expected standardised net gain less a risk penalty, for both parties) is simply the *risk-adjusted* analogue of the linear utility function used in Chapter 2 (in the case where $CV_{AB}^2 = 0$, the expected utility using the additive mean/variance utility function is identical to the utility using the linear utility function) and so any global models constructed using the additive mean/variance utility function can be directly compared to models LUM1 and LUM2.

In §2.4.1 of Chapter 2 we constructed an overall model (assuming no uncertainty) which employed a linear utility model and involved only the single parameter γ . We hypothesized that γ was some simple function of the ratio PV_A/PV_B (as in (2.2a) or (2.2b)) which comprised of only two parameters, and we estimated these parameters

directly by using a least squares objective function.¹ In the current chapter we have so far examined mergers *individually*: we constructed the Nash-Kalai model (see (3.5)), examined the feasible range of solution points for the cash and share exchange portions of the payment, and then identified the optimal (in the Nash-Kalai sense) cash/share combination within this range. This analysis gave us a clearer understanding of the conditions under which a potential acquirer might employ cash or shares as a medium of payment, and the amount he should be prepared to offer, given the assumptions of the model, and as such may be used as an a priori decision support tool. However we have thus far not determined whether this model provides a better fit (i.e. simulates a situation closer to that actually observed in reality) than do models LUM1 and LUM2. We now attempt to assemble a *global* model, based on the additive mean/variance utility functions for the target and the acquirer, which will allow us to determine whether the inclusion of an uncertainty term helps to provide a greater explanation of reality.

The Nash-Kalai model constructed in §3.2 and solved in §3.3 is not in a practically operational form. While we have already chosen a simple additive mean/variance utility model² we must still estimate the relative negotiating power γ and the risk/uncertainty parameters ξ_A and ξ_B of the acquirer and target, respectively, before the model can be used for practical decision support. We start by noting that *observed* values of Y , the proportion of the standardised net gains from merger ceded to the target, are of course available for each merger. We can thus calculate the values of γ for the risk-free case from (2.1), i.e. $\gamma = 1 - Y$ for each merger. Whilst uncertainty may now be present in the estimation of $\tilde{\Pi}_{AB}$, a function of the form

$$\ln [\gamma / (1 - \gamma)] = \alpha + \beta \ln (PV_A / PV_B) \quad (3.13)$$

may be fitted to the observed values of γ . It is thus assumed that the relative negotiating power γ is independent of any post-merger effects, including the risk involved, and depends purely on the relative sizes of the bargaining companies. The estimated

¹ We used a similar methodology in §2.4.2 but employed a negative exponential utility model involving three parameters γ , r_A and r_B which, we hypothesized, were simple functions of the known values PV_A , PV_B and PV_{AB} (as in (2.7) to (2.9)).

² We return later to consider the case of an alternative form for the utility function.

regression parameters α and β may then be used, together with PV_A , PV_B and the relationship in (3.13) to supply estimates of γ for any merger.

The optimal value of β_S was given in (3.11) as

$$\beta_S^* = \frac{k_A}{(k_A + k_B)} = \frac{\xi_A}{(\xi_A + \xi_B)}$$

and thus

$$\frac{\beta_S^*}{(1 - \beta_S^*)} = \frac{k_A}{k_B} = \frac{\xi_A}{\xi_B} .$$

We assume *prima facie* that the risk aversion coefficients are *inversely* related to the sizes of the respective companies. This assumption is based on the premise that all companies desire high rates of return, but dislike taking risk. Large companies are able to bear larger risks than small companies since they have a much larger asset-base. Thus the risk aversion coefficient of a large company would be expected to be small (close to 0) whilst the risk aversion coefficient of a small company would be expected to be large, and there appears to be no good reason why these risk aversions will be influenced by the role they play (i.e. whether they are the acquirer or the target). A simple model for the risk aversion coefficients might be

$$k_i = p_1 + p_2 / PV_i \quad \text{for } i = A \text{ or } B.$$

where $p_1 \geq 0$ and $p_2 > 0$. Then the ratio k_A/k_B can be written as

$$\frac{k_A}{k_B} = \frac{p_1 + p_2 / PV_A}{p_1 + p_2 / PV_B} \quad (3.14)$$

and thus the scaling of p_1 and p_2 is completely arbitrary. Let $p_2/p_1 = \tau$; we normalise (3.14) by

$$\frac{k_A}{k_B} = \frac{1 + \tau / PV_A}{1 + \tau / PV_B} . \quad (3.15)$$

Since *observed* values of β_S are available for all mergers, the observed $\beta_S/(1-\beta_S)$ can be calculated (i.e. values of the ratio k_A/k_B), and so in principle it is possible to estimate the

value of the single parameter τ which best fits (3.15), using the known values of PV_A and PV_B .¹

So far we have estimated only the value of the ratio of the risk aversion parameters, k_A/k_B , but have paid no attention to the second component of uncertainty, i.e. the variance of the estimate of the post-merger company size, σ_{AB}^2 . The estimation of σ_{AB}^2 (or equivalently, CV_{AB}^2) is important as together with μ_{AB} it allows probability statements to be made about the true post-merger value of the company (as a worst case Chebyshev's Inequality may be invoked; stronger statements can be made if the true distribution of $\tilde{\Pi}_{AB}$ is known), as well as allowing the relative riskiness of mergers to be estimated.

We make the simplifying assumption that the distribution of $\tilde{\Pi}_{AB}$ (about which we have so far made no assumptions) is such that CV_{AB} is non-decreasing with increasing μ_{AB} . That is, the a priori uncertainty in the perception of the post-merger value of the company increases at as least as great a rate as μ_{AB} does. This assumption arises from the observation that there may be little uncertainty in predicting the true post-merger value of a small combined company with limited operations and few assets, but as this true post-merger value increases, it becomes increasingly difficult to estimate, in part due to the difficulty in predicting the effect of the new business combination on the company's operations. Our assumption is that this uncertainty will increase faster than the expected mean μ_{AB} does. Two possible functions which might serve as approximations for the relationship between CV_{AB} and μ_{AB} are:

$$CV_{AB} = \sigma_{AB} / \mu_{AB} = \phi_1 + \phi_2 (\ln \mu_{AB})$$

i.e.
$$CV_{AB}^2 = (\phi_1 + \phi_2 (\ln \mu_{AB}))^2, \quad (3.16)$$

¹ A note on calculating the *observed* values of β_S and λ . By definition, $\lambda = \omega/G$ where $G = \mu_{AB} - PV_A - PV_B$ and $\omega = n_B z$, where n_B is the number of target shares outstanding, and z is the amount of cash paid for each target share. Now β_S is the proportion of the combined company paid to target shareholders as shares. The post-merger company value is $(n_A + kn_B) P_{AB}$ where n_A is the number of acquirer's shares outstanding, k is the number of acquirer's shares swapped for each target share and P_{AB} is the share price of the merged company. So $\beta_S = (\text{observed payment} - \omega) / ((n_A + kn_B) P_{AB})$.

$$\begin{aligned} \text{or} \quad & CV_{AB} = \phi_1' + \phi_2' \sqrt{\mu_{AB}} \\ \text{i.e.} \quad & CV_{AB}^2 = (\phi_1' + \phi_2' \sqrt{\mu_{AB}})^2. \end{aligned} \quad (3.17)$$

We will constrain the ϕ_i and ϕ_i' to non-negative values, which will ensure that CV_{AB} will not be zero merely by virtue of ϕ_1 and ϕ_2 having opposite signs. The model based on (3.13), (3.15) and (3.16) is termed **shared uncertainty model 1 (SU1)**, whilst the model based on (3.13), (3.15) and (3.17) is termed **shared uncertainty model 2 (SU2)**.

To see how well the Nash-Kalai models fit actual observed values of β_S and λ , it will be necessary to estimate the vector $\theta = \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}$ (for model SU1) or $\theta' = \begin{bmatrix} \phi_1' \\ \phi_2' \end{bmatrix}$ (for model SU2). For θ (or θ') known, (3.16) (or (3.17)) and the information vector $x = \begin{bmatrix} PV_A \\ PV_B \\ \mu_{AB} \end{bmatrix}$ of ex post assumed known variables allows us to calculate CV_{AB}^2 . For empirical testing purposes, for μ_{AB} we will use the ex post observed values of Π_{AB} , the post-merger value of the combined entity. Naturally, the quantity Π_{AB} is not known with certainty to the players before the merger and any difference between the observed Π_{AB} and the players' a priori perceptions of μ_{AB} will, of course, cause a residual between the model's results and reality. Knowledge of the estimated value of τ from (3.15) and x allows the determination of the ratio k_A/k_B , which will produce the model estimate of β_S . We call this estimate $\hat{\beta}_S(x_i)$, which is independent of the vector θ , i.e. $\hat{\beta}_S(x_i)$ is independent of the uncertainty term, CV_{AB}^2 . We now turn to the computation of a predicted value for λ . Recall that from (3.10) we had that

$$\hat{\lambda} = c_1 - \gamma - c_2 \beta_S + c_2^2 CV_{AB}^2 \left[\gamma k_B \beta_S^2 - (1-\gamma) k_A (1-\beta_S)^2 \right] \quad (3.18)$$

So far we have estimated only the ratio k_A/k_B : k_A and k_B *individually* have not been explicitly determined. However from (3.14) any function of the form

$$\frac{k_A}{k_B} = \frac{(1 + \tau/PV_A) c'}{(1 + \tau/PV_B) c'}$$

where c' is some constant, will provide an equally good fit. Thus we can assert that

$$k_i/c' = 1 + \tau/PV_i \quad , \quad i = A \text{ or } B \quad (3.19)$$

We rewrite (3.18) as

$$\hat{\lambda} = c_1 - \gamma - c_2 \beta_S + c_2^2 CV_{AB}^2 \left[\gamma \left(\frac{k_B}{c'} \right) \beta_S^2 - (1-\gamma) \left(\frac{k_A}{c'} \right) (1-\beta_S)^2 \right] c' \quad (3.20)$$

From (3.13) and \mathbf{x} we can derive an estimate of γ . From (3.16) or (3.17) we estimate the value of CV_{AB}^2 , and k_A/c' and k_B/c' are defined in (3.19). Using the estimate $\hat{\beta}_S(\mathbf{x}_i)$ of β_S we can compute a predicted value of λ , which we call $\hat{\lambda}(\mathbf{x}_i; \theta)$, as a function of the unknown constant c' . A suitable objective, then, is

$$\text{Minimise}_{\theta \geq 0, c' > 0} \left(\sum_{\text{all mergers}} (\lambda_i - \hat{\lambda}(\mathbf{x}_i; \theta))^2 \right). \quad (3.21)$$

where λ_i is the observed value of the cash payment expressed as a fraction of the expected total net gain for merger i . Once the optimal value of c' is obtained we will be able to calculate individual values of k_A and k_B from (3.19) for any pair of companies involved in merger.

By definition the observed $\lambda_i \geq 0$, i.e. any cash payment is *by the acquirer to the target*. In fitting the model, it may occur that the predicted value of $\hat{\lambda}(\mathbf{x}_i; \theta)$ is negative, especially if the observed λ_i is zero. In such cases, for purposes of empirical testing, we have simply truncated $\hat{\lambda}(\mathbf{x}_i; \theta)$ to zero, without attempting to derive more refined constrained estimates of the parameters.

Empirical testing of the models was carried out using the same set of 24 mergers on the Johannesburg Stock Exchange as was used in §2.5 to allow model comparisons. Appendix 3.C contains relevant information.

§3.6.2 MODEL RESULTS

The fitted regression parameters α and β based on (3.13), and where γ is the *observed* risk-free relative negotiating power, were found to be $\alpha = 1.0500$ and $\beta = 0.4376$, with $r=0.4252$ ($p=0.0384$), which indicates a statistically significant linear relationship between $\ln [\gamma / (1-\gamma)]$ and $\ln (PV_A / PV_B)$. Thus for given values of PV_A and PV_B , the observed value of γ can be estimated from

$$\gamma = \frac{2.8577 \exp [0.4376 \ln (PV_A / PV_B)]}{1 + 2.8577 \exp [0.4376 \ln (PV_A / PV_B)]}$$

An estimated value of τ was 2655.532 which produced a residual sum of squares of 0.2678 and a correlation of $r = 0.9412$ ($p < 0.0001$). Thus

$$\frac{k_A}{k_B} = \frac{1 + 2655.532 / PV_A}{1 + 2655.532 / PV_B}$$

and then β_S can be estimated from $\beta_S = 1 / (1 + (k_A / k_B)^{-1})$.

Within a suitable tolerance model SU1 produced a solution vector of $\theta = \begin{bmatrix} 0.069117 \\ 0.005987 \end{bmatrix}$ when c' was 0.008673, with a residual sum of squares for λ and β_S of 0.5262 and 0.0456 respectively. The solution vector for model SU2 was $\theta = \begin{bmatrix} 0.049285 \\ 0.002737 \end{bmatrix}$ when c' took on the value 0.009859, with residual sum of squares for λ and β_S of 0.4935 and 0.0456 respectively. Table 3.1 provides the resulting values of the relative bargaining strength parameter γ (which is independent of θ , and hence does not vary from model SU1 to SU2), the risk aversion coefficients k_A and k_B for each of the models, as well as CV_{AB}^2 and the fitted values of σ_{AB} (in monetary terms) for both models. The parameter γ varies between 0.7326 and 0.9574, covering approximately the same range as that over which the model F γ M (full and shared information) showed virtually no loss in explanatory power when γ was held constant (this range was shown to be [0.75 ; 0.90] in §2.5.2.2).

We observe that the estimated risk aversion in model SU1 drops very rapidly for a unit increase in company size for small companies (as company size increases from R5 million to R15 million there is a drop in risk aversion from 4.615 to 1.544 (66.5%)), but flattens out to drop far less rapidly for a similar increase in company size for large companies (an increase in company size from R100 million to R110 million results in a decrease in risk aversion from 0.239 to 0.218 (8.8%)). For this model the risk aversion of acquiring companies in the sample ranges from 0.0156 for the largest acquirer to 7.3226 for the smallest (and hence most risk-averse), and for target companies in the sample it ranges from 0.0538 for the largest target to 49.8603 for the smallest. For model SU2, k_A ranged from 0.0177 to 8.3239, and k_B ranged from 0.0611 to 56.6785.

Table 3.1. Values of γ , k_A , k_B , CV_{AB}^2 and σ_{AB} for models SU1 and SU2

Merger	γ	SU1					SU2				
		k_A	k_B	CV_{AB}^2	σ_{AB}	k_A	k_B	CV_{AB}^2	σ_{AB}		
1	.7890	.0529	.0905	.0120	96.047	.0602	.1029	.0170	114.087		
2	.7348	.0571	.0538	.0129	188.074	.0649	.0611	.0258	266.293		
3	.9574	.0887	8.9252	.0107	32.821	.1008	10.1457	.0096	31.050		
4	.8938	.0515	.5136	.0120	91.882	.0585	.5838	.0165	107.976		
5	.9239	.1259	3.2035	.0104	24.422	.1431	3.6416	.0084	21.962		
6	.9106	.1462	2.5216	.0101	18.280	.1662	2.8665	.0074	15.720		
7	.7326	.1030	.0943	.0114	59.052	.1171	.1072	.0129	62.745		
8	.7743	.0637	.0922	.0118	81.842	.0724	.1049	.0155	93.573		
9	.7677	.0952	.1293	.0115	65.032	.1082	.1470	.0136	70.567		
10	.9156	.2613	5.3265	.0096	12.184	.2970	6.0549	.0064	9.922		
11	.9293	.4804	15.4453	.0087	5.038	.5461	17.5574	.0048	3.760		
12	.7819	.1755	.2890	.0104	25.643	.1995	.3285	.0086	23.245		
13	.9450	.1315	7.4095	.0102	21.099	.1494	8.4227	.0079	18.539		
14	.8535	2.2009	11.1673	.0073	1.363	2.5019	12.6944	.0036	0.957		
15	.9229	.0156	.1912	.0142	507.017	.0177	.2174	.0519	969.531		
16	.9173	2.2550	49.8603	.0071	1.075	2.5633	56.6785	.0035	0.752		
17	.8274	7.3226	23.8754	.0061	0.358	8.3239	27.1403	.0030	0.253		
18	.8312	2.5817	8.9287	.0074	1.434	2.9348	10.1496	.0037	1.009		
19	.8640	1.3316	8.2137	.0078	2.184	1.5136	9.3369	.0040	1.555		
20	.8603	2.7195	15.6657	.0077	1.904	3.0914	17.8079	.0038	1.349		
21	.9060	1.1439	18.2876	.0080	2.612	1.3003	20.7883	.0041	1.874		
22	.8842	.1238	1.0976	.0108	35.876	.1408	1.2477	.0100	34.497		
23	.7598	.1778	.2221	.0106	29.694	.2021	.2525	.0092	27.597		
24	.9189	.1521	3.3466	.0101	19.450	.1728	3.8042	.0076	16.880		

The coefficient of variation, CV_{AB} , measures the **relative uncertainty** of the merger, and we assumed that this increases as μ_{AB} increases. The results in Table 3.1 show that the relative uncertainty of the merger in model SU1 shows a fairly rapid increase for a given increase in μ_{AB} for small companies (as μ_{AB} increases from R5 million to R15 million there is an increase in CV_{AB} from 0.0788 to 0.0853 (8.35%)), whilst the increase is much more gradual for a similar increase in company size for large companies (an increase in μ_{AB} from R100 million to R110 million results in an increase in CV_{AB} from 0.0967 to 0.0973 (0.59%)). The CV_{AB}^2 values in model SU1 range from a low of 0.0061 ($CV_{AB}=0.0781$) to a high of 0.0142 ($CV_{AB}=0.1192$) for the sample at hand. This range of CV_{AB} values is narrow due to the flattening effect of the logarithm used in (3.16) to estimate CV_{AB} . The corresponding σ_{AB} values range from R0.358 million to R507.017 million. Model SU2 employs a square root function to estimate CV_{AB} , and thus displays a much wider range in CV_{AB}^2 (and hence σ_{AB}) values than does model SU1. The CV_{AB}^2 values range from 0.0030 ($CV_{AB}=0.0547$) to 0.0519 ($CV_{AB}=0.2278$), and the corresponding σ_{AB} values range from R0.253 million to R969.531 million for the sample. This knowledge of μ_{AB} and σ_{AB} allows probability statements to be made about the true post-merger value of the merged company.

Table 3.2 shows the estimated values of $\hat{\lambda}(x_i; \theta)$ for both models SU1 and SU2, and the estimated values of $\hat{\beta}_S(x_i)$, which is model-independent. Also presented are these amounts converted into monetary terms, and the *observed* cash and share amounts for the 24 mergers in the sample. There is an extremely highly significant correlation between the estimated proportion of the value of the merged entity allocated to the target *as shares*, $\hat{\beta}_S(x_i)$, and the observed value of β_S ($r^2 = 0.9447$, $p < 0.0001$), indicating a model with excellent predictive power. Turning to the fraction of the synergistic gains paid as cash, $\hat{\lambda}(x_i; \theta)$, we observe that both models SU1 and SU2 indicate that cash should be used as a medium of payment in 10 mergers (in all of these cases a share transfer would also take place, as demonstrated in §3.3), whilst the observed data showed that in reality only 4 mergers made use of cash as a means of payment, and in two of these cases no share transfer took place. These 4 mergers (numbered 3, 5, 10 and 11) all correctly identified by the model for a cash payment, indicating that the model is able

Table 3.2. Model estimates of the cash and share fractions of the offer, these estimates expressed in monetary terms, and actual observed monetary amounts for models SU1 and SU2

Merger	SU1						SU2		
	Cash			Shares			Cash		
	$\hat{\lambda}(x_i; \theta)$	estimated	observed	$\beta_s(x_i)$	estimated	observed	$\hat{\lambda}(x_i; \theta)$	estimated	observed
1	0	0	0	.3690	323.140	333.775	0	0	0
2	0	0	0	.5150	853.320	559.576	0	0	0
3	.0179	0.474	2.050	.0098	3.118	3.117	.0179	.474	2.050
4	0	0	0	.0911	76.456	67.573	0	0	0
5	.0224	0.807	1.959	.0378	9.062	15.228	.0226	.813	1.959
6	0	0	0	.0548	9.993	10.381	0	0	0
7	0	0	0	.5220	288.312	280.017	0	0	0
8	0	0	0	.4085	307.322	278.076	0	0	0
9	0	0	0	.4241	256.633	263.088	0	0	0
10	.0313	0.902	6.287	.0468	5.814	0	.0317	.914	6.287
11	.0015	0.006	2.630	.0302	1.634	0	.0133	.051	2.630
12	0	0	0	.3779	94.814	92.477	0	0	0
13	.0191	0.345	0	.0174	3.638	3.959	.0199	.358	0
14	0	0	0	.1646	2.618	2.225	0	0	0
15	0	0	0	.0754	320.817	206.481	0	0	0
16	.0147	0.030	0	.0433	0.551	0.475	.0253	.051	0
17	.1408	0.066	0	.2347	1.075	1.203	.0519	.024	0
18	0	0	0	.2243	3.743	3.339	0	0	0
19	0	0	0	.1395	3.450	3.276	0	0	0
20	0	0	0	.1479	3.216	1.872	0	0	0
21	.0297	0.228	0	.0589	1.721	1.643	.0315	.242	0
22	.0040	0.493	0	.1014	34.942	24.121	.0040	.493	0
23	0	0	0	.4446	128.140	135.365	0	0	0
24	.0197	0.508	0	.0435	8.400	8.208	.0202	.520	0

to describe the bargaining process as it occurred in reality fairly well. Of the 6 remaining mergers identified by the model for a cash payment, the monetary amounts were small (between R0.006 million and R0.903 million for model SU1), and these all translate into a relatively small proportion of the total payment (between 1.38% and 11.76% for model SU1). In the 4 cases in which a cash payment was observed, a much larger amount of cash was actually paid than predicted by the model (at least twice as much in all cases). In absolute terms the amounts remain small, however. Whilst the reasons for this difference lie outside of the scope of the current modelling exercise, it is interesting to note that an often-quoted reason for merger is an excess of cash held by the acquirer: a cash purchase of some other company will reduce the acquirer's cash holdings and hence also their tax liability (Brealey and Myers (1981)). Such a motivation could well explain why the observed cash payments were much larger than those predicted by the model. The difference in the cash components predicted by models SU1 and SU2 is seen to be very small in all cases. Thus changing the functional form of the uncertainty component from (3.16) to (3.17) has not affected the estimate of the cash payment in any large way, and we conclude that the calculation of the uncertainty is fairly robust to changes in functional form, providing that the assumptions of the model (i.e. that CV_{AB} is some non-decreasing function of the mean, μ_{AB}), are not violated.

To be able to compare the fit of the models with shared uncertainty to those with full and shared information (Chapter 2) it is necessary to combine the estimated cash and share exchange payments into a single monetary value, the estimated total amount paid to the target, \hat{Q}' . This can be transformed into a variable \hat{Y}' which represents the proportion of the estimated net gain allocated to the target, i.e.

$$\hat{Y}' = \frac{\hat{Q}' - PV_B}{\mu_{AB} - PV_A - PV_B}$$

where $\hat{Q}' = \hat{\lambda}(x_i; \theta) (\mu_{AB} - PV_A - PV_B) + \hat{\beta}_S(x_i) \mu_{AB}$. This is directly comparable to the observed variable Y introduced in §2.2. Table 3.3 contains the values of the variables \hat{Y}' and Y for all 24 mergers, as well as the residual for each case for both models SU1 and SU2.

Table 3.3. Observed values of Y, estimated values of Y' and residuals for models SU1 and SU2

Merger	Y(obs)	SU1		SU2	
		Fitted Y	residual	Fitted Y	residual
1	.7079	.5642	.1437	.5642	.1437
2	.0731	.5109	-.4378	.5109	-.4378
3	.0977	.0382	.0595	.0382	.0595
4	.0859	.1206	-.0347	.1206	-.0347
5	.2774	.0740	.2034	.0741	.2033
6	.2142	.1459	.0683	.1459	.0683
7	.2829	.4943	-.2114	.4943	-.2114
8	.0435	.5449	-.5014	.5449	-.5014
9	.4873	.4437	.0436	.4437	.0436
10	.0679	.0828	-.0149	.0831	-.0152
11	.2955	.0383	.2572	.0502	.2453
12	.3358	.4118	-.0760	.4118	-.0760
13	.0470	.0483	-.0013	.0491	-.0021
14	.0483	.1663	-.1180	.1663	-.1180
15	.1004	.2433	-.1429	.2433	-.1429
16	.0064	.0587	-.0523	.0694	-.0630
17	.5085	.3768	.1317	.2879	.2206
18	.1469	.2252	-.0783	.2252	-.0783
19	.1039	.1424	-.0385	.1424	-.0385
20	.0341	.1482	-.1141	.1482	-.1141
21	.0498	.0896	-.0398	.0915	-.0417
22	.0240	.1157	-.0917	.1157	-.0917
23	.6221	.4585	.1636	.4585	.1636
24	.0508	.0780	-.0272	.0785	-.0277

A least squares regression of Y on \hat{Y}' for model SU1 is

$$Y = 0.0445 + 0.6483 \hat{Y}'$$

with correlation $r = 0.5917$ ($r^2 = 0.3502$, $p < 0.0025$), i.e. a very highly significant linear trend. Statistical tests on the regression coefficients show that the intercept is not significantly different from 0 ($t_{b_0} = 0.798$, $p > 0.2$) although the slope is significantly less than 1 ($t_{b_1} = -1.8677$, $p = 0.0376$). This may be expected, however: in fitting the model we truncated negative values of $\hat{\lambda}(x_i; \theta)$ to zero. This implies an overestimate of $\hat{\lambda}(x_i; \theta)$, and hence also of \hat{Y}' in these cases. This in turn implies that the fitted regression line has a slope somewhat less than 1. Figure 3.10 shows a scatterplot of Y against \hat{Y}' for this model, together with 95% and 99% confidence bands for the fitted regression line. Immediately noticeable from Figure 3.10 and Table 3.3 are the two outliers (points 2 and 8) which have residuals of as much as -0.4378 and -0.5014 respectively. This model has 5 cases with absolute residual greater than 0.2 (in 2 cases it is greater than 0.4), whilst model LUM1 has 8 cases with absolute residual greater than 0.2 (in 2 cases it is greater than 0.3) and model LUM2 has 6 cases (in 2 cases it is greater than 0.3). After removal of the two outlying points the revised OLS regression line (based on 22 observations) for model SU1 is

$$Y = 0.0010 + 1.0016 \hat{Y}'$$

with $r = 0.8153$ ($r^2 = 0.6648$, $p < 0.0001$). Clearly, hypothesis tests on the intercept and slope parameters show that they are not significantly different from 0 and 1 respectively ($t_{b_0} = 0.025$, $p > 0.2$; $t_{b_1} = 0.0101$, $p > 0.2$). Thus the smaller-than-expected slope parameter in model SU1 ($b_1 = 0.6483$) is largely attributable to the two outliers, and for 22 (out of 24) observations the fitted model SU1 explains over 66% of the variability of the observed Y , indicating that the model is clearly an extremely good representation of reality for all but a small number of mergers.

An OLS regression of Y on \hat{Y}' for model SU2 yields

$$Y = 0.0497 + 0.6330 \hat{Y}'$$

with correlation $r = 0.5687$ ($r^2 = 0.3224$). The regression parameters, correlation coefficient and outlying points of this model are very similar to those of model SU1, and we thus do not pursue the discussion further.

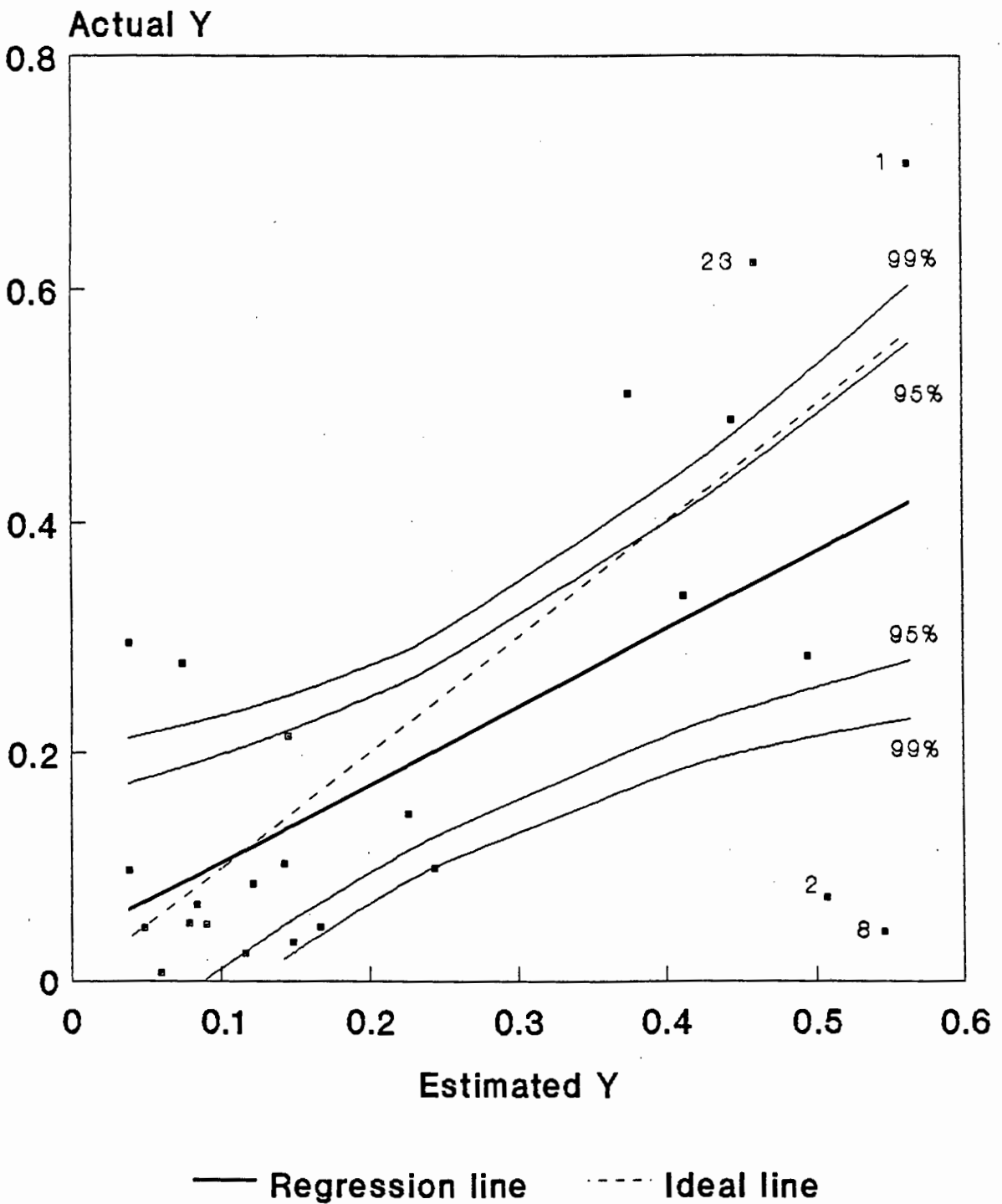


Figure 3.10. Scatterplot of actual Y against \hat{Y} for model SU1 together with the fitted regression line and 95% and 99% confidence bands

The current models with shared uncertainty are compared to several models with complete certainty from Chapter 2 in Table 3.4. We note that since SU1 and SU2 make use of an additive mean/variance utility function they should, strictly speaking, be compared only to models LUM1 and LUM2. Details of models FNEM and F γ M (with $\gamma = 0.793$) are included for interest. The analogue of models SU1 and SU2 with a negative exponential utility function is outlined in §3.7.

Table 3.4. A comparison of models with shared uncertainty (SU1 and SU2) with models with complete certainty (LUM1, LUM2, FNEM and F γ M)

Model	SU1	SU2	LUM1	LUM2	FNEM	F γ M
b_0	0.0445	0.0497	0.0614	0.0074	0.0165	0.0113
b_1	0.6483	0.6330	0.7545	0.9683	0.9259	0.9687
r	0.5917	0.5678	0.4511	0.4960	0.5725	0.5695
r^2	0.3502	0.3224	0.2035	0.2460	0.3277	0.3243
p	0.0024	0.0038	0.0270	0.0138	0.0034	0.0036

Model SU1 represents a slight improvement over model SU2, so model SU1 will be taken to be representative of the additive mean/variance utility model with shared uncertainty for future discussion. Model SU1 offers a superior fit to all the other models if correlation (or r^2) is taken as a measure of fit. Table 3.5 shows the percentage gain in explanatory power achieved by model SU1 vis-a-vis the four other models. Model SU1 has a vastly superior fit to either of the linear utility models with assumed certainty; this model explains 42.3% and 72.1% more of the variation in the observed Y values than do models LUM2 and LUM1 respectively. Model SU1 is also slightly better at predicting the observed phenomenon than models FNEM and F γ M (with $\gamma = 0.793$), which employ (more realistic) negative exponential utility functions, but lack the enhancements of the explanation of (i) the uncertainty involved, and (ii) the split of cash and shares in the merger. A model similar to SU1 (i.e. with shared uncertainty) but employing a more

realistic utility function may well improve the model fit still further (this model is outlined in §3.7).

Table 3.5. Gain in explanatory power by model SU1 relative to models with assumed certainty

Model	r	% gain by SU1	r ²	% gain by SU1
SU1	0.5917	-	0.3502	-
LUM1	0.4511	31.17	0.2035	72.09
LUM2	0.4960	19.29	0.2460	42.36
FNEM	0.5725	3.35	0.3277	6.87
F γ M($\gamma=0.793$)	0.5695	3.90	0.3243	7.99

§3.7 EXTENDING THE MODEL - A NEGATIVE EXPONENTIAL UTILITY MODEL

So far in this chapter we have restricted ourselves to the simplest increasing utility function containing uncertainty. In §2.4.2 we argued that in practical settings a linear utility function may not be representative of reality, and that the utility function may be expected to show decreasing marginal returns to scale (i.e. to be concave). We were able to show that by using negative exponential utility functions of the form

$$U_A(\cdot) = 1 - e^{-r_A(\cdot)} \quad , \quad r_A > 0$$

$$U_B(\cdot) = 1 - e^{-r_B(\cdot)} \quad , \quad r_B > 0$$

for the acquiring and target companies respectively in the Nash-Kalai model we achieved a correlation between the observed and fitted proportions of the synergy gains paid to the target which was significantly larger than the equivalent correlation using a linear utility

function. We now briefly examine the implications of using this more realistic utility model in the context of shared uncertainty amongst the players.

We let the utility to A for a standardised net gain of Γ_A be $1 - e^{-r_A \Gamma_A}$, which has expectation $1 - E(e^{-r_A \Gamma_A})$. Now $E(e^{-r_A \Gamma_A})$ is simply the moment generating function of the random variable Γ_A , which depends only on the probability distribution of $\tilde{\Pi}_{AB}$ (since PV_A , PV_B , r_A , γ and β_S are assumed constant). If the actual functional form $f(\Gamma_A)$ of Γ_A is known, then $1 - E(e^{-r_A \Gamma_A}) = 1 - \int_{-\infty}^{\infty} \exp(-r_A \Gamma_A) f(\Gamma_A) d\Gamma_A$. As a special case we assume that it is realistic that $\tilde{\Pi}_{AB}$ has an approximately normal distribution. Naturally if $\tilde{\Pi}_{AB}$ is approximated by a normal distribution the possibility arises that the value of the combined company might be negative. However it will be assumed here that σ_{AB}^2 is small enough so as to make $P(\tilde{\Pi}_{AB} < 0)$ negligible. Then

$$\begin{aligned} E[U_A(\Gamma_A)] &= 1 - E(e^{-r_A \Gamma_A}) \\ &= 1 - \exp[-r_A E(\Gamma_A) + r_A^2 \text{Var}(\Gamma_A)/2] \\ &= 1 - \exp[-r_A(c_1 - c_2 \beta_S - \lambda)] \cdot \exp[r_A^2 (1 - \beta_S)^2 c_2^2 CV_{AB}^2 / 2] \end{aligned}$$

from (3.1), (3.2) and the moment generating function of the normal distribution.

Similarly,

$$\begin{aligned} E[U_B(\Gamma_B)] &= 1 - E(e^{-r_B \Gamma_B}) \\ &= 1 - \exp[-r_B (c_2 \beta_S - c_1 + 1 + \lambda)] \exp[r_B^2 \beta_S^2 c_2^2 CV_{AB}^2 / 2] \end{aligned}$$

from (3.3), (3.4) and the m.g.f. of the normal distribution.

The Nash-Kalai solution based on the above is the point $(\lambda^*; \beta_S^*)$ which maximises the function

$$\begin{aligned} \Phi'(\lambda; \beta_S) &= \left[1 - \exp[-r_A (c_1 - c_2 \beta_S - \lambda)] \exp[r_A^2 (1 - \beta_S)^2 c_2^2 CV_{AB}^2 / 2] \right]^\gamma \\ &\quad \left[1 - \exp[-r_B (c_2 \beta_S - c_1 + 1 + \lambda)] \exp[r_B^2 \beta_S^2 c_2^2 CV_{AB}^2 / 2] \right]^{1-\gamma} \end{aligned}$$

in the range $\lambda \geq 0$, $0 \leq \beta_S \leq 1$. We have not succeeded in deriving explicit analytic expressions for the optimal values of λ^* and β_S^* from the above expression. Thus the model based on the negative exponential utility function lacks the analytical rigour of the model developed in §3.2 and §3.3. We will thus not pursue any empirical analysis using this model.

§3.8 CONCLUSIONS

In this chapter we have attempted to bring the concepts of uncertainty and risk aversion into the Nash-Kalai model. We assumed that the acquirer and target do not have certain knowledge of the post-merger value of the combined company, but rather estimate this value from some probability distribution, the mean and variance of which are symmetrically known to both companies. Naturally, the variance is merely a conceptual variance and it is thus not empirically measurable. Its estimation for each merger was one of the aims of the modelling process: a knowledge of the mean and variance of the distribution of merged company value allowed the decision-maker to make probability statements about the true post-merger value of the combined unit.

Due to the presence of uncertainty this model was able to differentiate between and quantify the cash payment and the share exchange portion of the merger offer amount. It was shown that cash alone would optimally never be used to finance a merger since then all the risk associated with the merger falls on the shoulders of the acquiring company's shareholders, and that the amount of cash offered depended on the level of uncertainty present. The share exchange portion was independent of uncertainty. Furthermore the optimal share exchange portion depended only on the relative sizes of the companies' risk aversion coefficients (and not on negotiating power); the cash portion in the optimal offer depended in a complex way on risk aversion, negotiating power, uncertainty and company sizes. A study of an empirical example offered further insights as to how the form of payment was affected by the two companies' risk perceptions.

Further empirical testing was carried out using an additive mean/variance utility function. This model provided a good description of the real-world decision-making process (the share exchange portion, in particular, was very accurately modelled) for all but a small number of observed mergers. Comparative empirical results show that the models of this chapter described the overall observed synergy gains to the target shareholders much better than do the models under assumed certainty, and this improvement can be directly attributed to the extra degree of freedom introduced in the form of the merger uncertainty. Due to the substantial improvement in explanatory power

by this model it is suggested that the incorporation of uncertainty is a necessary feature for any decision support tool of this kind.

In the last section of the chapter we have proposed an extension to this model by considering the (more realistic) negative exponential utility function to replace the additive mean/variance utility function. We were unable to derive a closed-form analytical solution for the proportion of the post-merger company transferred to the target shareholders and the amount of cash paid. The similarity of the theoretical model, however, leads to the conclusion that this model provides no additional insights into the bargaining process.

Whilst the models considered in this chapter are merely simplifications of reality and do not consider many of the complex financial intricacies of a corporate merger, they do help to shed light and offer further understanding of aspects of the bargaining process. Deviations from the observed real world situation will occur, largely because our perceptions of the values of the input variables (we used ex-post observed values in most cases) in all likelihood will differ from those perceived by the real-world decision-makers. Whilst promoting understanding of the process, our simplifying assumptions relating to the form of the negotiating power, the risk aversion coefficients and the uncertainty terms might also contribute to this residual.

APPENDIX 3.A. Derivation of the roots and turning points for the equations determining the feasible range for λ and β_S

In (3.7) we had

$$\lambda \leq -(c_2^2 \xi_A) \beta_S^2 + (2c_2^2 \xi_A - c_2) \beta_S + c_1 - c_2^2 \xi_A .$$

The axis of symmetry is given by

$$\beta_S = \frac{c_2 - 2c_2^2 \xi_A}{-2c_2^2 \xi_A} = 1 - \frac{1}{2c_2 \xi_A} ,$$

and the turning point is

$$\begin{aligned} \lambda &= -(c_2^2 \xi_A) \left(1 - \frac{1}{2c_2 \xi_A}\right)^2 + (2c_2^2 \xi_A - c_2) \left(1 - \frac{1}{2c_2 \xi_A}\right) + c_1 - c_2^2 \xi_A \\ &= (c_1 - c_2) + \frac{1}{4\xi_A} . \end{aligned}$$

The roots of the equation (at $\lambda = 0$) are

$$\left(1 - \frac{1}{2c_2 \xi_A}\right) \pm \left[\left(1 - \frac{1}{2c_2 \xi_A}\right)^2 + \left(\frac{c_1}{c_2^2 \xi_A} - 1\right) \right]^{1/2}$$

and the λ -intercept is when $\lambda = c_1 - c_2^2 \xi_A$.

Similarly, (3.8) had

$$\lambda \geq (c_2^2 \xi_B) \beta_S^2 - c_2 \beta_S + c_1 - 1 .$$

The axis of symmetry is

$$\beta_S = \frac{c_2}{2c_2^2 \xi_B} = \frac{1}{2c_2 \xi_B} ,$$

and the turning point is

$$\begin{aligned} \lambda &= (c_2^2 \xi_B) \left(\frac{1}{2c_2 \xi_B}\right)^2 - c_2 \left(\frac{1}{2c_2 \xi_B}\right) + c_1 - 1 \\ &= c_1 - 1 - \frac{1}{4\xi_B} . \end{aligned}$$

Since $c_2 > 0$ and $\xi_B > 0$ (by definition) the axis of symmetry of this function always lies in the positive β_S range. The roots of the equation (at $\lambda = 0$) are

$$\left(\frac{1}{2 c_2 \xi_B} \right) \pm \left[\left(\frac{1}{2 c_2 \xi_B} \right)^2 - \left(\frac{c_1 - 1}{c_2^2 \xi_B} \right) \right]^{1/2},$$

and the λ -intercept is when $\lambda = c_3$.

APPENDIX 3.B. Derivation of the Nash-Kalai solution for λ and β_S

The Nash-Kalai criterion in (3.5) can be written as

$$\text{Maximise } \Phi(\lambda; \beta_S) = [Q_1(\beta_S) - \lambda]^\gamma [\lambda - Q_2(\beta_S)]^{1-\gamma} \quad (3.B.1)$$

$\lambda \geq 0; 0 \leq \beta_S \leq 1$

where

$$Q_1(\beta_S) = c_1 - c_2 \beta_S - c_2^2 \xi_A (1 - \beta_S)^2$$

and

$$Q_2(\beta_S) = c_1 - 1 - c_2 \beta_S + c_2^2 \xi_B \beta_S^2$$

We first note that the Nash-Kalai criterion in (3.B.1) is undefined for $\lambda > Q_1(\beta_S)$ or for $\lambda < Q_2(\beta_S)$, since these correspond to cases in which at least one of the players is worse off than at the status quo. In fact we will assume here that there exist solutions which are strictly mutually advantageous, i.e. there exist solutions satisfying the condition

$$Q_2(\beta_S) < \lambda < Q_1(\beta_S) \quad (3.B.2)$$

The Nash-Kalai criterion is to be maximised subject to $\lambda \geq 0$ and $0 \leq \beta_S \leq 1$; if a non-zero maximum exists (i.e. one other than $\lambda=0, \beta_S=0$) then it will satisfy (3.B.2).

We start by investigating the *unconstrained* maximum of the Nash-Kalai criterion.

Necessary conditions for an unconstrained maximum of (3.B.1) are $\frac{\partial \Phi(\lambda; \beta_S)}{\partial \lambda} = 0$ and $\frac{\partial \Phi(\lambda; \beta_S)}{\partial \beta_S} = 0$. These partial derivatives yield the equations

$$\begin{aligned} \frac{\partial \Phi(\lambda; \beta_S)}{\partial \lambda} &= (1-\gamma) [Q_1(\beta_S) - \lambda]^\gamma [\lambda - Q_2(\beta_S)]^{-\gamma} \\ &\quad - \gamma [\lambda - Q_2(\beta_S)]^{1-\gamma} [Q_1(\beta_S) - \lambda]^{\gamma-1} = 0 \end{aligned}$$

and

$$\begin{aligned} \frac{\partial \Phi(\lambda; \beta_S)}{\partial \beta_S} &= \gamma [Q_1(\beta_S) - \lambda]^{\gamma-1} [\lambda - Q_2(\beta_S)]^{1-\gamma} Q_1'(\beta_S) \\ &\quad - (1-\gamma) [\lambda - Q_2(\beta_S)]^{-\gamma} [Q_1(\beta_S) - \lambda]^\gamma Q_2'(\beta_S) = 0 \end{aligned}$$

We note immediately that if either $\lambda = Q_1(\beta_S)$ or $\lambda = Q_2(\beta_S)$, the derivatives are undefined (the slope is infinite) while the Nash-Kalai criterion itself takes on the

minimum defined value of zero. Thus any point corresponding to $\lambda = Q_1(\beta_S)$ and/or $\lambda = Q_2(\beta_S)$ is neither a stationary point of the Nash-Kalai function nor a maximum (assuming that at least one solution satisfying (3.B.2) exists). Thus any stationary point satisfies the conditions

$$Q_1(\beta_S) - \lambda > 0$$

and
$$\lambda - Q_2(\beta_S) > 0.$$

We can thus factor out the term

$$\left[Q_1(\beta_S) - \lambda \right]^{\gamma-1} \left[\lambda - Q_2(\beta_S) \right]^{-\gamma}$$

in both partial derivatives to give the following necessary conditions for stationarity:

$$(1-\gamma) [Q_1(\beta_S) - \lambda] - \gamma [\lambda - Q_2(\beta_S)] = 0 \quad (3.B.3)$$

and

$$\gamma [\lambda - Q_2(\beta_S)] Q_1'(\beta_S) - (1-\gamma) [Q_1(\beta_S) - \lambda] Q_2'(\beta_S) = 0 \quad (3.B.4)$$

Now let $(\lambda^*; \beta_S^*)$ be the unconstrained maximum of the Nash-Kalai criterion. The first necessary condition (3.B.3) implies that

$$\lambda^* = (1-\gamma) Q_1(\beta_S^*) + \gamma Q_2(\beta_S^*) \quad (3.B.5)$$

$$= c_1 - \gamma - c_2 \beta_S^* + c_2^2 CV_{AB}^2 [\gamma k_B (\beta_S^*)^2 - (1-\gamma) k_A (1-\beta_S^*)^2] \quad (3.B.6)$$

Substituting this into the second necessary condition (3.B.4) yields

$$\gamma (1-\gamma) [Q_1(\beta_S^*) - Q_2(\beta_S^*)] [Q_1'(\beta_S^*) - Q_2'(\beta_S^*)] = 0 \quad (3.B.7)$$

Note that $Q_1(\beta_S^*) = Q_2(\beta_S^*)$ in (3.B.5) implies that $\lambda^* = Q_1(\beta_S^*) = Q_2(\beta_S^*)$, which we have already shown to be neither a stationary point nor a maximum of the Nash-Kalai function. The necessary condition for the unconstrained maximum from (3.B.7) is thus

$$Q_1'(\beta_S^*) - Q_2'(\beta_S^*) = 0$$

or equivalently

$$-c_2 + 2c_2^2 k_A CV_{AB}^2 (1-\beta_S^*) = -c_2 + 2c_2^2 k_B CV_{AB}^2 \beta_S^*$$

which has the unique solution

$$\beta_S^* = \frac{k_A}{k_A + k_B} \quad (3.B.8)$$

The stationary point $(\lambda^*; \beta_S^*)$ can be shown to be a maximum as follows. For any given β_S , $\lambda = (1-\gamma) Q_1(\beta_S) + \gamma Q_2(\beta_S)$ gives the maximum with respect to λ . With this expression for λ in terms of β_S the Nash-Kalai criterion itself reduces to

$$\begin{aligned}\Phi(\lambda; \beta_S) &= [\gamma (Q_1(\beta_S) - Q_2(\beta_S))]^\gamma [(1-\gamma) (Q_1(\beta_S) - Q_2(\beta_S))]^{1-\gamma} \\ &= \gamma^\gamma (1-\gamma)^{1-\gamma} [Q_1(\beta_S) - Q_2(\beta_S)]\end{aligned}$$

i.e. a concave quadratic in β_S ; the stationary point is thus a maximum.

Note that since k_A and k_B are strictly positive, the constraint on β_S , (i.e. that $0 \leq \beta_S \leq 1$), is automatically satisfied. However (3.B.6) could be negative, thus violating the constraint on λ . The unconstrained solution in (3.B.6) and (3.B.8) is thus only the solution to the constrained problem when $\lambda^* \geq 0$. If $\lambda^* < 0$ we determine the optimal *constrained* λ by setting up the Kuhn-Tucker conditions on λ . (Since β_S^* in (3.B.8) was feasible we do not at this stage consider the constraint on β_S). The Kuhn-Tucker objective is then to maximise an extended objective of the form

$$\theta = [Q_1(\beta_S) - \lambda]^\gamma [\lambda - Q_2(\beta_S)]^{1-\gamma} - M \lambda \quad (3.B.9)$$

under the condition that $-M\lambda=0$, where M is a Lagrange multiplier. Now let $(\lambda^+; \beta_S^+)$ be the constrained maximum of the Nash-Kalai criterion when $\lambda^* < 0$. The case $M=0$ in (3.B.9) produces the previous infeasible solution for λ . Thus in an optimal solution M can not equal 0, and by complementarity we have that $\lambda^+=0$. We thus need to maximise $\Phi(0; \beta_S)$ with respect to β_S subject to the condition $0 \leq \beta_S \leq 1$. The relevant necessary condition is

$$\gamma Q_2(\beta_S^+) Q_1'(\beta_S^+) + (1-\gamma) Q_1(\beta_S^+) Q_2'(\beta_S^+) = 0 \quad (3.B.10)$$

or else β_S^+ must be at one of the boundaries of β_S (i.e. $\beta_S=0$ or 1). Thus at most five possible solutions for β_S^+ emerge.

Under the assumption that the expected total net gain from the merger is positive, i.e. $\mu_{AB} - PV_A - PV_B > 0$, we observe that at $(\lambda=0, \beta_S=0)$ we have $\lambda - Q_2(\beta_S) = -(c_1-1) < 0$, and at $(\lambda=0, \beta_S=1)$ we have $Q_1(\beta_S) - \lambda = c_1 - c_2 < 0$, i.e. the Nash-Kalai criterion is not defined at these two points, i.e. both are strictly worse than the status quo for one of the players. Neither is thus an optimal solution to the problem. If a feasible solution better than the status quo exists, it must therefore be the feasible root of (3.B.10) giving the largest value of $\Phi(0; \beta_S)$.

APPENDIX 3.C. Observed values of β_S , λ and Y

Merger	k	z	β_S (obs)	λ (obs)
1	2.00		0.381147	0
2	0.80		0.337689	0
3	0.50	0.96	0.009838	0.077525
4	0.75		0.080477	0
5	0.50	0.375	0.063551	0.054454
6	0.125		0.056946	0
7	0.50		0.507001	0
8	0.72		0.369635	0
9	1.17		0.434758	0
10		1.80	0	0.218123
11		2.15	0	0.683411
12	3.30		0.368552	0
13	0.40		0.018967	0
14	0.90		0.139896	0
15	0.38		0.048521	0
16	0.75		0.037301	0
17	0.6		0.262637	0
18	0.34		0.200104	0
19	1.50		0.132471	0
20	0.1923		0.086105	0
21	0.83		0.056199	0
22	0.46		0.069987	0
23	1.00		0.469664	0
24	0.18		0.042468	0

$$\text{where } \beta_S (\text{obs}) = \frac{\text{observed payment} - n_B z}{(n_A + k n_B) P_{AB}}$$

$$\lambda (\text{obs}) = (n_B z) / (\mu_{AB} - PV_A - PV_B)$$

k = number of acquiring company's shares swapped per target share

z = number of rands cash paid per target share

PART B

NEGOTIATION-BASED MODELS AND DECISION-SUPPORT

CHAPTER 4

A MERGER BARGAINING MODEL WITH FULL AND SYMMETRIC
UNCERTAINTY

§4.1 INTRODUCTION AND OBJECTIVES

In the two previous chapters we assumed that certain information relating to the merger was known to the respective parties involved in the bargaining process. In Chapter 2 it was assumed that all relevant information relating to the financial worths of the two companies as separate entities pre-merger and as a single combined unit post-merger was known *with certainty* by both acquirer and target. Under this assumption we were able to construct and empirically test a series of descriptive models of the behaviour of an acquirer and a target during the negotiation phase, making use of Nash's solution to the generalised bargaining problem. In Chapter 3 we extended this model by assuming that the a priori estimate of the post-merger value of the newly-merged company as perceived by the target and the acquirer was not known with certainty, but instead was assumed to be a random variable whose distributional characteristics were known to both parties. Thus while uncertainty exists, its extent is known to both parties in probabilistic terms, i.e. we have a case of *shared uncertainty*. An advantage of this more general model of the bargaining behaviour of the two parties was its ability to discriminate between the cash portion and the share exchange portion of the offer amount, as well as offering a more realistic model of the merger bargaining process.

In this chapter we will assume that each party forms its own subjective perception of

- (1) its own reservation price¹, and
- (2) the other company's reservation price (together with some measure of uncertainty associated with this perception)

¹ Recall that the acquirer's reservation price is the maximum amount that it would be prepared to offer for the target, and the target's reservation price is the minimum amount that it would be prepared to accept.

independently of the other party, and that the two companies do not share this information with one another. Unlike the models of the previous chapters, the zone of agreement within which the two competing parties might bargain and ultimately achieve agreement is therefore not completely known to either side. Each has knowledge (with certainty) of its own reservation price, which may describe one end-point of the zone of agreement, but has an uncertain (at best probabilistic) estimate of the other's reservation price, which might be used to ascertain the other end-point. We are thus dealing with the canonical case of distributive bargaining, and we will use a formulation structure similar to that of Harsanyi (1965). The overall aim of this chapter will be to describe a parsimonious model representation of this decision problem, and we will later implement this model in an attempt to identify possible optimal negotiating "strategies" for the target and the acquiring companies. Whilst in reality a strategy is a complicated synthesis of a large number of variables, we represent a strategy here by a particular combination of *model* input parameter values. The model will thus be **normative** in the sense that it will prescribe general approaches to the negotiations for the players that in the long run will give rise to outcomes having "optimum" properties, and so might fulfil the role of a decision support system as it may be used by management of companies involved in merger in an attempt to isolate optimal (or near-optimal) strategies, or to investigate various possible bargaining strategies which might be under consideration.

Luce and Raiffa (1957) noted that a fundamental requirement of game theory is that

"each player is fully aware of the rules of the game and the utility functions of each of the players"

and that

"this is a serious idealization which only rarely is met in actual situations".

We will therefore now assume that each party does not necessarily regard the other party as acting in accordance with the precepts of game-theoretic rationality (as we did in the previous two chapters). Whilst a great deal of theory has been developed for such games of "incomplete information" (e.g. Harsanyi (1968a,b,c), Chatterjee and Samuelson (1983)), most of it rests on the assumption that the rules (axioms) of the game and the utility functions are common knowledge in the sense of Aumann (1976), i.e. that each

player knows this information, knows that the other knows it, that the other knows that he knows it, and so on. Another analytical approach which de-emphasises the game-theoretic solution concepts in a bargaining situation is **negotiation analysis**, which seeks to develop prescriptive theory and useful advice for negotiators and third parties given a (probabilistic) description of how others will behave. This fairly new approach is not an "alternative" to game-theory: Sebenius (1992) even offers that it might be called "non-equilibrium game-theory with bounded rationality and without common knowledge". We will employ some simple ideas from negotiation analysis (such as those proposed by Raiffa (1982)) to construct a parsimonious model of the merger negotiation process. Note that we are now concerning ourselves with the dynamics of the consensus-seeking process, rather than attempting to simply identify static trade values as in Chapters 2 and 3. The Nash Bargaining approach employed in these earlier chapters is thus no longer useful, and is replaced by a dynamic process model, following an approach similar to that of Balakrishnan and Eliashberg (1995), who conclude that this sort of approach to modelling the negotiation process can yield insights beyond those obtained by static outcome-oriented theories, by incorporating behavioural and economic aspects related to the negotiation.

In §4.2 we discuss some commonly-used and -studied forms of bargaining, and show how the corporate merger negotiation problem differs from these, thus highlighting the need for a model describing this situation. In §4.3 we present an overview of such a model representation. The chapter ends with some concluding remarks about the use of this type of model as a decision support tool.

§4.2 BARGAINING IN MERGERS AND ACQUISITIONS

There is a large variety of procedures in the literature whereby a prospective seller of a commodity and a group of prospective buyers can determine whether or not a deal will be clinched, and if so, who the buyer will be and what price the buyer will have to pay. For example, in a **sealed bid** procedure for selling a commodity, all bids by prospective buyers must be submitted by a certain date and remain concealed or confidential until they are all opened together. The seller's role is to choose a winning

bid from amongst those submitted. The winner may be the highest bidder who will have to pay his offer price, (i.e. the highest price bid), or may be chosen by some other decision rule (for example, in a Vickrey-type sealed bid auction, the highest bidder wins at the second-highest price offered (Vickrey (1961))). Thus the bids are not public, and in general each bidder only submits a single bid. In contrast, in an **open ascending auction**, monotonically increasing bids are submitted at random times by the prospective buyers in a random sequence and they are always public. A bidder may make more than one bid if he so desires. The seller plays virtually no role at all: in general the seller will merely accept the final (highest) bid providing it exceeds his previously-determined reservation price (which may not have been publicly announced). Thus in this kind of bidding procedure all the bargaining effectively takes place between the prospective buyers to determine who will ultimately put forward the single price offer to the seller. Rothkopf and Harstad (1994) discuss various other forms of auctions and offer a critical analysis of the models available to aid in competitive bidding decision-making in real transactions. In a **haggling** procedure both seller and prospective buyers are active in setting price offers: a sequence of public offers and counter-offers may converge to some mutually acceptable point, at which point a deal will be concluded. In general the seller will negotiate with a single prospective buyer until either a deal is reached or negotiations break off; the process may then restart with another prospective buyer.

The corporate merger bargaining process is unlike any of the procedures mentioned above. In Chapter 1 we argued that a merger or acquisition only takes place after some negotiating or bargaining between two or more parties, one of which is the target company and the others which are all prospective acquirers. Thus a merger can be seen as a multi-stage, multi-party bargaining game, where the parties are the target and all prospective acquirers, and the stages are each offer made by a prospective acquirer and the associated reaction from the target. In general, one of the prospective acquirers makes a price offer, acting independently of the target. If the target accepts the offer a transaction is concluded at the offer price. If the target rejects a prospective acquirer's offer at any stage, any of the prospective acquirers (including the one whose offer has most recently been rejected) may make a new offer (the next stage of the bargaining game). The offer process continues until either the target accepts an offer or the target

rejects all offers and/or the acquirers all do not proceed with further offers. Thus in some respects the merger bargaining procedure is similar to an open ascending auction in that all offers are public (allowing all interested parties access to complete information about the offer at each stage), they are therefore likely to be monotonically increasing, and each bidder is not restricted to a single offer. However the target plays a more active role in a corporate merger procedure than in a regular open ascending auction in that it undertakes activities designed to generate better offers (Roy (1989)), and is active during the bargaining process in accepting or rejecting the offer *at each stage*. At each stage the target company is faced with a dilemma similar to that in the well-known "secretary problem" (see, for example, Gilbert and Mosteller (1966) and Stewart (1981)): although there may be several other prospective acquirers who have shown an interest in acquiring the target and who may or may not have entered into the bargaining thus far, the target does not know with any degree of certainty whether or not there will be any further (larger) offers forthcoming if it rejects the current offer under consideration from one of the prospective acquirers.

§4.3 A MULTI-STAGE MERGER BARGAINING MODEL UNDER FULL AND SYMMETRIC UNCERTAINTY

Sycara (1990) states that

"Negotiation is an *ill-structured* and *complex* process, that to date has defied all attempts at analysis. The outcome of the process depends on such intangibles as the negotiators' skills and experience, uncertain and changing information, the parties' perceptions and idiosyncratic behaviours and on the exhaustive and systematic analysis of the problem There is no *typical* or *model* negotiator behaviour that can be codified and emulated."

She further suggests that the negotiation process exhibits several characteristics that give rise to various requirements for any model of the process. These are

- Parties to negotiation usually start having their goals far apart, and this distance is narrowed gradually in an *iterative* (rather than a one-shot) fashion.

- Each round of proposals effectively offers both negotiators *feedback* about the quality of its negotiation plan which they evaluate and use to modify their plan.
- Negotiations take place in a dynamically changing world. During the course of negotiations conditions in the world that affect the parties' behaviour and goals might *change*. A model must have a reactive component which displays the negotiators' response to these changes.
- Since final agreement is reached through narrowing the difference in the demands of the parties, a negotiation model must have a way of *evaluating* whether each new proposal indeed narrows these differences (this is trivial in the case of negotiations over a single issue, the level at which we will model the merger negotiations).

We now consider a multi-stage bargaining model for negotiations between a target company and one or more prospective acquiring companies. In the model to be constructed in this chapter we will abstract key features of the bargaining process, bearing in mind Sycara's modelling requirements. The aim here will be to simulate the strategic and psychological processes which go into an acquirer's selection of an offer amount at each stage, and a target's decision of whether or not to accept the offer for *any* specific acquirer/target combination. We are implicitly assuming that these processes have a generality which might be similar for all acquirer/target combinations. Thus the model we will develop will be in terms of a number of parameters, making the model of the bargaining process general to all mergers, and could thus lead to the selection of negotiating strategies for the parties which may be construed as being "good" in a variety of contexts. Furthermore, the way this abstraction is performed will effectively make the scenario of a single potential acquirer making multiple offers, and that of multiple potential acquirers each making one or more offers essentially equivalent as regards the bargaining mechanism, and thus we do not need to differentiate between them. Thus when we refer to "the acquiring company" it should be understood that we are in fact referring to the potential acquiring company in play *at that stage* of the bargaining process only; other potential acquirers may come into play at a later stage of the process. Figure 4.1 presents a simplified flow-diagram of the merger negotiation process in which multiple offers can occur, as described in Chapter 1 and §4.2. The left half of the figure

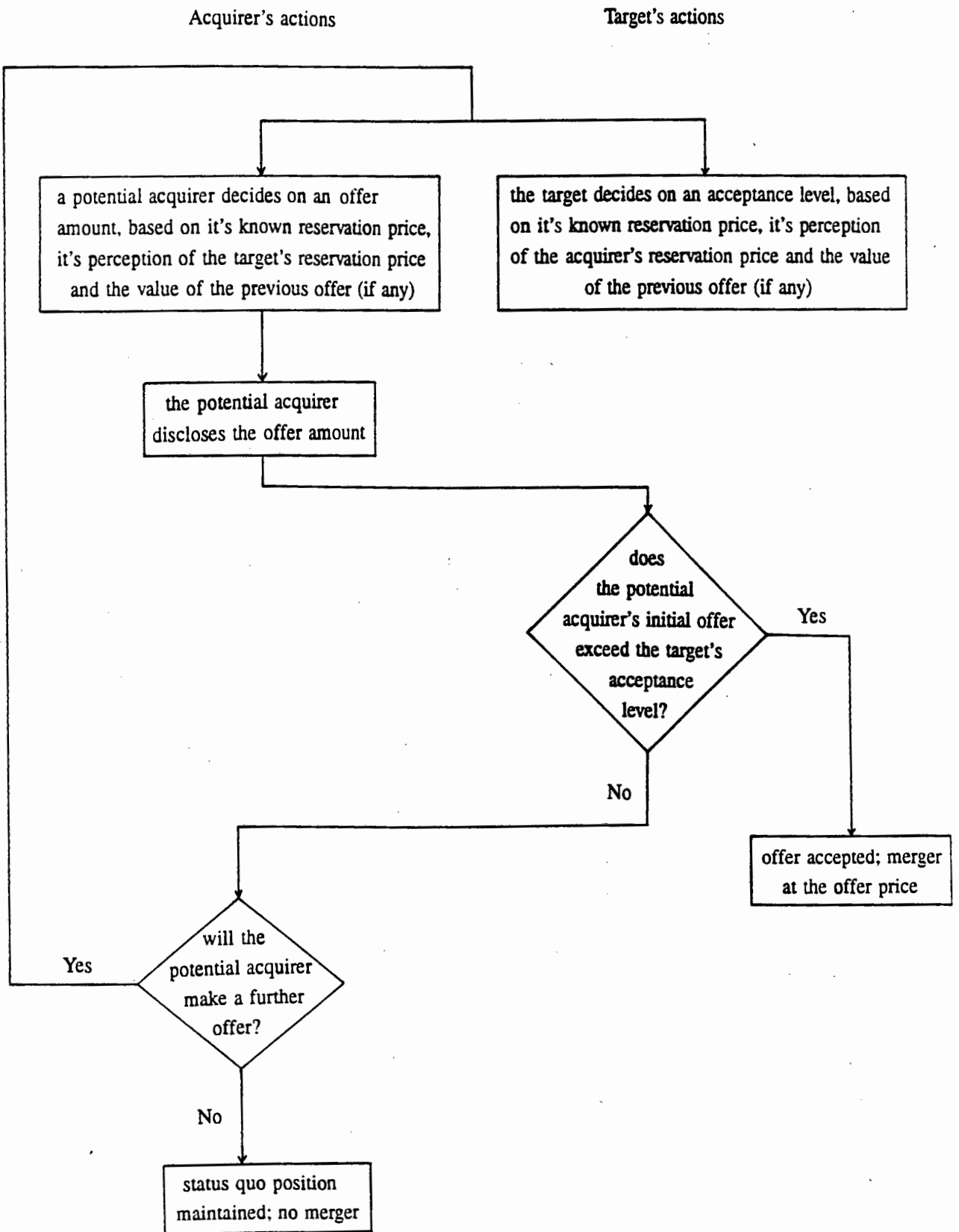


Figure 4.1. Simplified flow-diagram of the merger bargaining process

shows the acquirer's actions throughout this process, whilst the right half shows the target's actions. Several observations emerge from Figure 4.1:

- the **initial** offer and acceptance stage can be differentiated from the group of **subsequent** offer and acceptance stages, since there is less information available for the formulation of the initial offer (by the acquirer) and the initial acceptance level (the minimum acceptable offer to the target) due to the absence of any previous public offer;
- the acquirer publicly discloses its offer price at each stage, whilst the target does **not** in general disclose its acceptance level;
- the target appears to play a much more passive role in the process than the acquirer does: the target merely accepts or rejects each offer made by the acquirer;
- if an offer is accepted by the target, merger will take place at the offer price, no matter how much this is above the target's acceptance level for that round of bargaining;
- both the acquirer's offer **and** the target's acceptance level may be revised after each round of bargaining, in the light of any further available information;
- if all offers are rejected by the target, the status quo position is maintained for both companies, i.e. neither party gains or loses anything. (It might be argued here that costs were incurred by both parties in the negotiating process, in the form of time spent by experts and executives, consultation fees etc. Since these costs will vary considerably from one merger negotiating process to another, and should be small compared to the possible gains from merger, we will not consider them here.)

In this section we will introduce a parsimonious representation of the merger bargaining process as depicted in Figure 4.1. We will explore how each party may use the knowledge (both certain and uncertain) at its disposal to formulate offers (by the acquirer) and responses to offers (by the target), and what other information might prove useful for this purpose. In practice, participating management teams may base their actions and reactions on different information (for example, the financial, structural and

operational characteristics of the two companies involved) after careful analysis by their own experts. Thus clearly the proposed model should be viewed only for what it is: a parsimonious mathematical **representation** of the real world, at a suitable level of detail, which serves the purpose of understanding and communicating the ideas contained in the negotiating process, and which allows inferences and conclusions to be drawn (Müller-Merbach (1985,1987)) about the process. It should furthermore be borne in mind that the individual pre-merger perceptions of the two companies' managements are unlikely ever to be made public, even after a successful merger or acquisition, and thus this model cannot be tested directly against real empirical evidence. In principle, however, we should still be able to make testable statements on the basis of the model, and to this end we will describe and implement a Monte Carlo simulation incorporating the ideas and assumptions of this model and propose that it might prove to be a useful form of decision support tool to assist the management teams of acquiring and target companies when making inferences and decisions involved in merger negotiations.

The **zone of agreement** for the negotiations associated with a particular merger is the region within which any negotiated agreement will lie. We assume that the acquiring company (which we term A) has some **true reservation price**, V_A , which represents the upper bound on the monetary amount it is prepared to offer for the target under consideration. The acquirer, of course, knows its own true reservation price. Now V_A is functionally dependent upon the acquirer's utility function, which is unknown to the target, resulting in the target having some uncertainty about the true value of V_A . In similar spirit, the target knows its own true reservation price, V_B , which is the lower bound on the monetary amount it would be prepared to accept from the acquirer and depends to some extent on its utility function. The acquirer, however, is uncertain of the true value of V_B .

Providing the actual value of V_A exceeds the actual value of V_B , a **true zone of agreement** exists, and is the interval from V_B to V_A . This true zone of agreement comprises of the set of all possible agreement points that, from the standpoint of each involved party, are better in value than no agreement at all. If the situation were to arise where $V_B > V_A$ there would be no true zone of agreement and no amount of bargaining

would lead the parties to a point acceptable to both. If the true zone of agreement existed, and if the values of V_B and V_A were made *public* (i.e. there was complete certainty about their values, as we assumed in Chapters 2 and 3), bargaining would take place within the interval $[V_B; V_A]$ and agreement would *always* be reached. However since the reservation prices are unknown to the other party, provided that for each party there is some non-zero probability on the other's reservation price such that $V_A > V_B$, then a **perceived zone of agreement** exists, and bargaining between the two parties may ensue, and possibly lead to a mutually acceptable agreement point. A particular negotiation between an acquirer and a target is thus in principle characterised by the parties' actual reservation prices V_A and V_B .

It will be convenient to develop the model of the bargaining process in terms of the *difference* in the two companies' reservation prices, $\Delta_{AB} = V_A - V_B$, i.e. in terms of the magnitude of the true zone of agreement. For any given acquirer/target combination the value of Δ_{AB} will not be precisely known to either party. We will assume that in any given negotiation the two parties have similar knowledge about the riskiness of the market in which they are operating and about the expectations of differences in reservation prices which will tend to occur in practice. We then model any specific Δ_{AB} by means of a probability distribution, which represents the common knowledge of both players about the propensity for any given value of Δ_{AB} to occur. Specifically, for the analysis in this chapter we will assume that Δ_{AB} is normally distributed with mean μ_Δ and variance $\sigma^2\mu_\Delta^2$, where the value of μ_Δ may be related to the reservation prices of the two companies. The choice of the normal distribution is arbitrary, but a priori it is reasonable to assume that the negotiators are equally likely to overestimate Δ_{AB} as they are to underestimate it. The general principles of the model will, however, continue to hold for any other continuous distribution. Thus $\Delta_{AB} = V_A - V_B \sim N(\mu_\Delta; \sigma^2\mu_\Delta^2)$ for modelling purposes. Since we are modelling the difference between two reservation prices, the actual values of V_A and V_B are irrelevant. Furthermore, since μ_Δ is specific to any particular acquirer/target combination, we can normalise the scale so that $\mu_\Delta = 1$, and then $\Delta_{AB} \sim N(1; \sigma^2)$.

At an individual company level we need to model the subjective uncertainty of each party regarding the other's reservation price. These uncertainties are modelled as subjective probability distributions to each party. Since $\Delta_{AB} = V_A - V_B$, the acquirer (party A) can be viewed as perceiving the value of V_B as the known value (to A) of V_A minus Δ_{AB} ; for a specific quantity for V_A (say v_A) the expectation of A's subjective probability distribution for V_B (conditional on the known value to A of V_A) is $v_A - E(\Delta_{AB})$. Similarly, from the point of view of the target (party B), the acquirer's reservation price V_A can be viewed as the known value (to B) of V_B plus Δ_{AB} , and B's subjective probability distribution for V_A (conditional on the known value to B of V_B) has expectation $v_B + E(\Delta_{AB})$, where v_B is a specific known value of V_B . In what went before we assumed that both parties had the same expectation for Δ_{AB} , which in terms of the normalised scale was 1. Thus in the rescaled model, A's expectation of V_B is $v_A - 1$, whilst B's expectation of V_A is $v_B + 1$. The variances of the parties' subjective conditional distributions on the other's reservation price will depend on what knowledge each is able to glean about the other. Thus these two variances (we term the acquirer's uncertainty about V_B as σ_A^2 , and the target's uncertainty about V_A as σ_B^2) may not be the same, and they may not equal the variance of the distribution representing the propensity of any given value of Δ_{AB} to occur. Furthermore, since the parties' individual subjective conditional distributions are derived from the distribution on Δ_{AB} , which was assumed normal, these two distributions are also normal. Thus from the acquirer's perspective,

$$V_B \sim N(v_A - 1; \sigma_A^2)$$

and from the target's perspective

$$V_A \sim N(v_B + 1; \sigma_B^2),$$

and the parties' bargaining strategies will be based in part upon their own subjective conditional distribution about the other's reservation price. Whilst it is possible that extreme measures (such as industrial espionage) might reveal information which would reduce σ_A^2 or σ_B^2 to below σ^2 , in the numerical simulation which follows in the next chapter we will restrict ourselves to the assumption that the players are entirely ethical, and that the parties' lack of full knowledge about the other's reservation price would contribute to an increase in the variance to $\sigma_A^2 \geq \sigma^2$ and $\sigma_B^2 \geq \sigma^2$.

Thus we explicitly recognise three separate distributions: two subjective conditional distributions which represent the parties' uncertainties about the other's reservation price, and a distribution which models the actual tendency for differences in the two reservation prices.

In our parsimonious representation of this bargaining scenario we will draw a clear distinction between the acquiring company's **initial offer** and its set of subsequent offers (in the event of the initial offer being rejected). We conjecture that the initial offer will primarily be based on the acquirer's view of the "true" fair price within its perceived zone of agreement (assuming that this exists). This fair price (we call it F_A) will lie between the acquirer's known reservation price v_A and the acquirer's view of the target's reservation price (i.e. the end-points of the zone of agreement as viewed by the acquirer); the position of F_A can be represented as a *proportion* of the distance between the perceived end points, the actual proportion being determined by some parameter which quantifies the *acquirer's perception* of its bargaining power relative to that of the target for the particular acquirer/target combination. We call this the **relative dominance parameter** of the acquirer, δ_A . The acquirer's perception of its relative dominance ranges between 0 and 1. The larger δ_A is, the greater is the acquirer's perception of its bargaining strength relative to that of the particular target. Since acquirers usually (but not always) have larger market values than the relevant target companies and often have greater access to resources, they might be inclined to perceive themselves to be dominant over the target in question. However the requirement that the target, too, must be satisfied to ensure a successful merger, and the presence in the market-place of other potential acquirers will ensure a downward pressure on an acquirer's perceived value of δ_A towards 0. Thus the relative dominance parameter describes the acquirer's perceived dominance at the negotiating table and may have little to do with any superiority enjoyed by the acquirer due to its physical or operational characteristics.

Since the acquirer's uncertainty in V_B is described by a probability distribution, it implies that the acquirer's uncertainty regarding a fair price can also be modelled by means of a probability distribution with mean $E(F_A)$ and variance $\text{Var}(F_A)$. We will

represent F_A as a linear combination of its own known reservation price and its perception of the target's reservation price. From the acquirer's perspective

$$\begin{aligned} F_A &= (1 - \delta_A) v_A + \delta_A V_B \\ &= v_A - \delta_A (v_A - V_B) \end{aligned}$$

where $V_B \sim N(v_A - 1; \sigma_A^2)$. This has expectation to the acquirer of

$$\begin{aligned} E(F_A) &= v_A - \delta_A (v_A - E(V_B)) \\ &= v_A - \delta_A \end{aligned}$$

and variance to the acquirer of

$$\text{Var}(F_A) = \delta_A^2 \sigma_A^2.$$

Since V_B is normally distributed from the acquirer's perspective, the fair price is distributed according to the normal distribution $N(v_A - \delta_A; \delta_A^2 \sigma_A^2)$. We will model the acquirer's policy for selecting an initial offer as being the choice of a particular quantile of the above distribution on fair price resulting from the perceived dominance and the uncertainty in V_B . We will term the quantile chosen by the acquirer to determine its initial offer strategy its **initial strategic concession** parameter, indicating that its choice will determine whether the acquirer will make a *hard-line* offer in the hope of acquiring the target at a "bargain" price (i.e. below its perception of an expected fair price) or a *generous* offer (above its perception of an expected fair price). Thus for the purposes of analysing strategies in a generic sense we will define a policy parameter β_{A1} such that the initial offer (which we term O_1) is the $100.\beta_{A1}^{\text{th}}$ percentile of the acquirer's distribution of fair price, i.e. O_1 is chosen so that

$$\Phi \left(\frac{O_1 - E(F_A)}{\sqrt{\text{Var}(F_A)}} \right) = \beta_{A1}$$

where Φ is the cumulative distribution function of the standard normal random variable. By the definition above, β_{A1} can range between 0 and 1; a value of $\beta_{A1} > 0.5$ implies an initial offer greater than $E(F_A)$, and a value of $\beta_{A1} < 0.5$ implies an initial offer less than $E(F_A)$.

Since from the acquirer's perspective the fair price is distributed according to $N(v_A - \delta_A; \delta_A^2 \sigma_A^2)$, the initial offer will be modelled as

$$O_1 = E(F_A) + \sqrt{\text{Var}(F_A)} \cdot \Phi^{-1}(\beta_{A1})$$

$$\begin{aligned}
&= v_A - \delta_A + \delta_A \sigma_A \Phi^{-1}(\beta_{A1}) \\
&= v_A - \delta_A (1 - \sigma_A \Phi^{-1}(\beta_{A1}))
\end{aligned} \tag{4.1}$$

provided that this amount is *less* than or equal to the acquirer's own known value of its reservation price, v_A . If O_1 exceeds v_A we will simply assume that $O_1 = v_A$. Thus the acquirer's initial offer is based on the two policy parameters δ_A (the acquirer's perception of its relative dominance) and β_{A1} (a parameter describing the acquirer's concessions made as part of its bargaining strategy), its own known reservation price v_A and the acquirer's uncertainty in its estimation of the target's reservation price. The construction of a typical initial offer is shown in Figure 4.2.

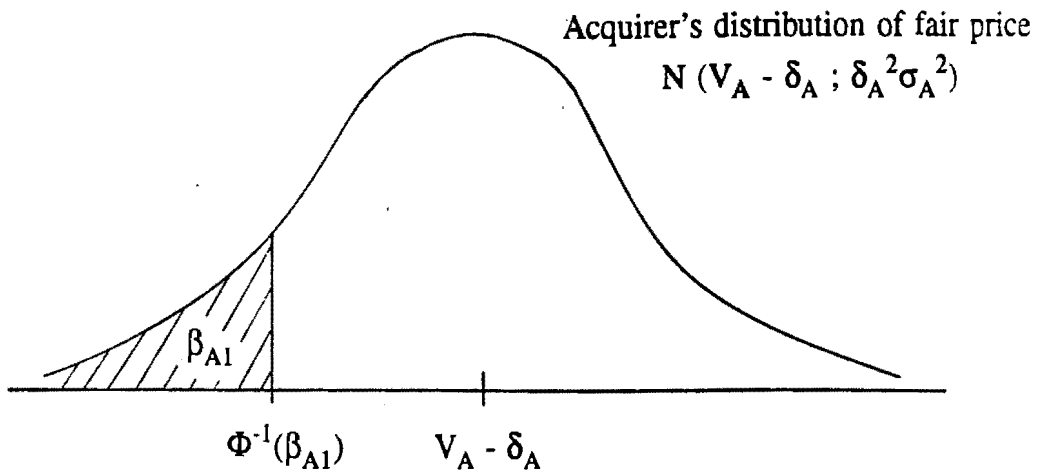


Figure 4.2. The construction of a typical initial offer when $\beta_{A1} < 0.5$

We will handle the target's choice of some acceptance level for the initial offer in a symmetrical way to the acquirer's choice of initial offer. We therefore assume that the target's decision to accept or reject the acquirer's initial offer will be based upon its own perception of the fair price within its perceived zone of agreement. In the same spirit as before the target's perception of a fair price (denoted by F_B) can be represented as a weighted average of its own known reservation price v_B and its expectation of V_A , the weights being determined by the relative dominance of the two companies *as viewed by the target*. The target's view of its relative dominance is represented by δ_B , with $0 \leq \delta_B \leq 1$. Despite the target's apparent inferior bargaining position in many cases due to its smaller size, the contribution of the target to the bargaining game is of paramount

importance, since if it does not accept the offer no agreement will be reached and neither party will partake of the merger gains. This knowledge, and the demand shown by other potential acquirers would help to ensure upward pressure on δ_B towards 1. From the above discussion it should be clear that in any given case it is unlikely that either δ_A or δ_B will be close to the end points of their respective ranges, 0 and 1. Now the probability distribution on V_A implies a probability distribution on fair price, F_B , which has mean $E(F_B)$ and variance $\text{Var}(F_B)$. The distribution on F_B from the target's standpoint will, of course, not necessarily be the same as that as seen from the acquirer's standpoint. From the target's perspective,

$$\begin{aligned} F_B &= \delta_B V_A + (1 - \delta_B) v_B \\ &= v_B + \delta_B (V_A - v_B) \end{aligned}$$

where $V_A \sim N(v_B + 1; \sigma_B^2)$. This has expectation to the target of

$$\begin{aligned} E(F_B) &= v_B + \delta_B (E(V_A) - v_B) \\ &= v_B + \delta_B \end{aligned}$$

and variance to the target of

$$\text{Var}(F_B) = \delta_B^2 \sigma_B^2.$$

Thus viewed by the target the fair price is distributed according to $N(v_B + \delta_B; \delta_B^2 \sigma_B^2)$. The target's policy for selecting an **initial acceptance level** (the level below which the initial offer will be rejected) will again be modelled in terms of a particular quantile (the target's **strategic concession**) of the resulting distribution on fair price, which can be interpreted in the same way as for the acquirer.

Depending on the strategy decided upon by the target, the target's initial acceptance level (termed A_1) may be above or below its expectation of a fair price, $E(F_B)$. To quantify A_1 we define a policy parameter β_B for the target such that A_1 is the $100.\beta_B^{\text{th}}$ percentile of the target's distribution of fair price, i.e. A_1 is chosen so that

$$\Phi \left(\frac{A_1 - E(F_B)}{\sqrt{\text{Var}(F_B)}} \right) = \beta_B.$$

We term β_B the target's **strategic concession parameter**. A target strategy of starting with a high initial acceptance level would be modelled by $\beta_B > 0.5$. Now since from the

target's point of view the fair price is distributed according to $N(v_B + \delta_B ; \delta_B^2 \sigma_B^2)$, the initial acceptance level will be

$$\begin{aligned} A_1 &= E(F_B) + \sqrt{\text{Var}(F_B)} \cdot \Phi^{-1}(\beta_B) \\ &= v_B + \delta_B + \delta_B \sigma_B \Phi^{-1}(\beta_B) \\ &= v_B + \delta_B (1 + \sigma_B \Phi^{-1}(\beta_B)) \end{aligned} \quad (4.2)$$

provided that A_1 is *greater* than or equal to the target's own known value of its reservation price, v_B . If not, we will assume $A_1 = v_B$. Thus the target's initial acceptance level is based, like the acquirer's formulation of an initial offer, upon a perception parameter (δ_B) and a strategy parameter (β_B), as well as its own known reservation price v_B and the target's uncertainty in the estimation of the acquirer's reservation price. The construction of a typical initial acceptance level is shown in Figure 4.3.

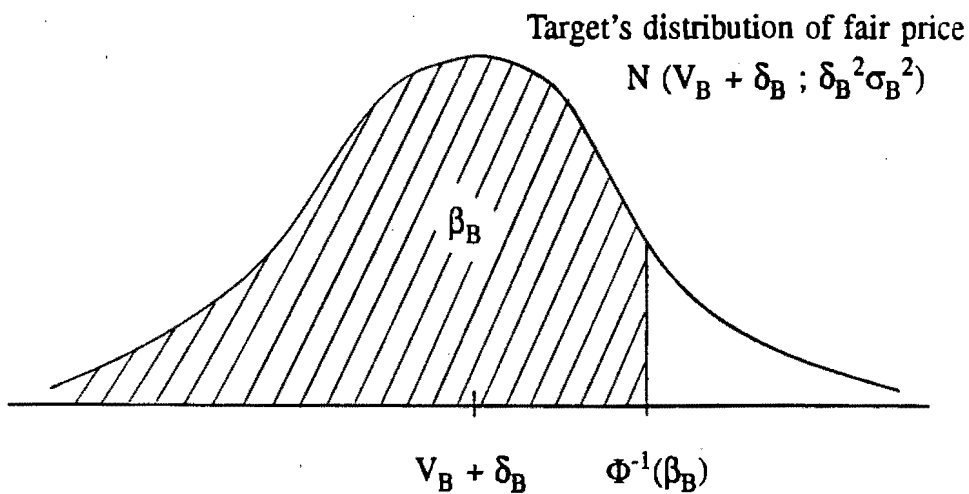


Figure 4.3. The construction of a typical initial acceptance level when $\beta_B > 0.5$

The acquirer's initial offer, O_1 , and the target's initial acceptance level, A_1 , are assumed to be arrived at by both parties concerned independently of one another. The acquirer discloses its initial offer to the target, either directly or through a negotiating agent. In general the target does not disclose its initial acceptance level. Agreement between the two parties will be reached if $O_1 > A_1$. If $O_1 \leq A_1$ the offer will be rejected. The target gains further information from this rejected offer: it learns that the acquirer's reservation price is at least as great as the initial offer value. The acquirer only learns that its offer is somewhat less than the target's initial acceptance level! This information

accruing from the initial bargaining stage may be incorporated into a subsequent (larger) offer by the acquirer (if it desires to make another offer), and into a revised acceptance level by the target (if another offer is forthcoming).

If the initial offer is rejected, the acquirer will decide whether or not to make one or more further offers. If it decides to do so, any further offer will be bounded below by the previous offer. We assume that the series of all possible **non-initial offers** by the acquirer is monotonically increasing, with the initial offer as the lower bound for the first such non-initial offer. Furthermore, the series is finite, and terminates in a final (largest) offer which has as upper bound the acquirer's true reservation price. That is,

$$O_1 \leq \text{all possible non-initial offers} \leq v_A.$$

This model thus allows for the situation in which the final (largest) possible offer tendered by the acquirer is *less* than the acquirer's a priori reservation price, v_A .

If a further offer (beyond the first) is forthcoming, the target company will necessarily have to accept or reject this offer. This decision may be based on a **revised acceptance level (RAL)**, which takes into account all public knowledge (i.e. the previous offer), and the target's private bargaining knowledge (i.e. its previous acceptance level), and indicates a concession that the target is prepared to make to meet the acquirer's offer. Thus the revised acceptance level at any stage is assumed to be always *less than or equal to* the acceptance level at the previous stage. If convergence is reached (i.e. a non-initial offer exceeds the target's (revised) acceptance level) at any stage, agreement is reached at the offer price. If the offer falls short of the acceptance level for that stage, the offer is rejected.

Thus in practice the negotiating "game" is *dynamic*. That is, each party will respond to the new information being made public at each stage of the process. We would like to model the *cumulative* effect of the acquirer's bargaining strategy, but naturally do not know a priori exactly when an acquirer might terminate its sequence of offers (assuming that none of them are accepted). Thus in our model we assume that a component of the acquirer's strategy would include its **degree of persistence**, the

implication of which would be the number of non-initial offers made. In reality the decision to make a further offer if the most recent one was rejected is a complex one, but we will parameterise this aspect of the acquirer's strategy by a single probability p that a rejected offer is not followed up. Thus if p is large there is little persistence on the part of the acquirer. We will further assume that the events (i.e. the making of offers) are independent of one another. Then the cumulative effect of the acquirer's bargaining strategy may be modelled by a *geometric* probability distribution on N , the number of non-initial offers made by the acquirer. That is, the probability that the acquirer makes exactly n non-initial offers is given by

$$P(N = n) = p(1 - p)^n \quad n = 0, 1, 2, 3, \dots$$

$$= 0 \quad \text{otherwise}$$

where p is the probability of termination at any stage. Our model will offer decision support by evaluating the strategies or policies which are defined by the resulting distribution on N (for a chosen p).

The number of non-initial offers n could be 0 (one offer only; initial offer = final offer), 1 (initial offer and a final offer only) etc. For modelling purposes, the acquirer's n non-initial offers will be generated on the interval $[O_1; O_L]$, where O_L is the value of the $(n+1)^{\text{th}}$ (i.e. final) offer. Clearly $O_L \geq O_1$ and $O_L \leq v_A$, and O_L can be generically characterised by some strategy parameter β_{AL} , which we call the acquirer's **final strategic concession parameter**. In similar spirit to the composition of the initial offer, and since the acquirer's perceived fair price is distributed as $N(v_A - \delta_A; \delta_A \sigma_A^2)$, the final (greatest) possible offer may be expressed as

$$O_L = v_A - \delta_A (1 - \sigma_A \Phi^{-1}(\beta_{AL})) \quad (4.3)$$

where Φ is the cumulative distribution function of the standard normal random variable. Since we assume that the offers are monotonically increasing we have that $\beta_{AL} \geq \beta_{A1}$; the equality only holds if all possible offers subsequent to the initial offer are *exactly* equal to the initial offer, which is unlikely in practice in corporate merger negotiations¹.

¹ The strategy of making a reasonable opening offer and holding firm is known as the Boulware strategy after Lemuel Boulware, former vice-president of the General Electric Company, who invariably used this strategy in wage negotiations.

The set of $n+1$ ordered offers represent the universe of possible offers that might be put forward by an acquirer in negotiations with a given target. Note that the model generates all offers which would be made conditional on no acceptance; the process of course terminates on acceptance, in which case one or more of the potential offers may go unused.

We now consider the modelling of the target's reaction to each non-initial offer $O_2, O_3, \dots, O_n \leq O_L$. The acquirer's concessions in the case of a rejected offer occurred in the form of a monotonically *increasing* series of offers. In our model representation we will assume that at each stage of the bargaining process beyond the first, the target's revised acceptance level (RAL) is positioned somewhere between its acceptance level at the previous stage and the acquirer's (now public) offer at the previous stage, i.e. the target yields decrementally from its previous position. The position of the RAL at the j^{th} offer stage within this interval may be expressed in the form

$$A_j = (1 - \gamma) A_{j-1} + \gamma O_{j-1} \quad (4.4)$$

where γ is defined as the target's **yield decrement**, $0 \leq \gamma \leq 1$, which characterizes the RAL and indicates the target's willingness to give up part of the merger profits to the acquirer in the bargaining process. Note that A_j must exceed the target's known reservation price, v_B . If not, we will assume that $A_j = v_B$. The closer γ is to 1, the more the target is prepared to concede at each stage. If $\gamma = 1$ then the target effectively reduces its acceptance level to the value of the acquirer's previous offer, i.e. $A_j = O_{j-1}$ (providing that this is greater than v_B). The implication of this is that provided a second offer O_2 is forthcoming, it will be at least as great as O_1 and will automatically be accepted. On the other hand, a value of $\gamma = 0$ implies that the target is not prepared to concede at all, i.e. $A_j = A_{j-1}$. The target's role is fairly passive: it merely accepts or rejects any offer from the acquirer. Thus the target's RAL continues as long as the acquirer's offer stream continues.

Given that the acquirer's offers are monotonically increasing, and that the target's acceptance levels are monotonically decreasing, the two will converge if the bargaining continues for long enough. The process of offers and responses continues until either an offer exceeds the target's acceptance level at that stage, and agreement is reached at the

offer price, or no further offers are forthcoming from the acquirer, i.e. the status quo position holds and both parties withdraw from the bargaining process.

The parameters of the bargaining model fall into two distinct categories:

(1) Contextual parameters

These describe the structure or environment within which the bargaining takes place, and are *perceptions* from the viewpoints of the two participating parties. This category includes the perceived uncertainties σ_A and σ_B , as well as the perceived dominances δ_A and δ_B . Furthermore, the players' fundamental propensity to identify the true difference in their reservation prices, σ , also falls into this category. Whilst each of these parameters are fixed a priori in any given case, their actual levels will play a prominent role in the outcome of the bargaining.

(2) Strategy parameters

The bargaining parties will face a choice from a range of values for each of their strategy parameters. Under individual or collective rationality the combination of values chosen for the strategy parameters will be those which the parties believe will offer the most acceptable agreement point. Thus the values of the strategy parameters are not fixed a priori, and their choice offers scope for optimisation (in some sense) of an agreement point.

The model we have defined characterizes the player's bargaining strategies in a generic parsimonious manner for the purposes of examining the general structure of "good" strategies. The acquirer's strategy can be divided into two distinct parts: one part which defines the initial offer, and a second part which defines the series of non-initial offers. The initial offer O_1 is parameterised by β_{A1} , whilst the non-initial offers O_2, O_3, \dots, O_L are parameterised by p and β_{AL} . We can thus represent the acquirer's **bargaining strategy** as

$$\bar{A}(\beta_{A1} : \beta_{AL}, p) .$$

The target's bargaining strategy can also be divided into two component parts: a part which defines the initial acceptance level (parameterised by β_B), and a part which defines

the revised acceptance level (parameterised by γ). We can write the target's bargaining strategy as

$$\bar{B}(\beta_B : \gamma) .$$

We assume that since both parties are in essence "competing" against one another (the acquirer to get an agreement on merger at as low a value as possible, and the target to get an agreement at as high a value as possible), they will attempt to keep their *true* bargaining strategies secret from one another. Even if the target does know the true value of \bar{A} it will not know the acquirer's reservation price and thus O_1 and O_L cannot be estimated with any certainty. Furthermore the target will not be able to determine at what stage in the offer process the acquirer will make its final offer. In similar spirit, even if the acquirer knows the true \bar{B} , it will not know the target's reservation price and thus A_1 remains uncertain to the acquirer, as do future values of A_i . Thus neither party will be able to predict the other's exact action or reaction at any stage in the negotiation process.

§4.4 CONCLUDING REMARKS

This negotiation model follows an entirely different approach to that presented in Chapters 2 and 3. The models in those chapters were based entirely on the utilities (or expected utilities) of the two opposing players, and were *symmetrically prescriptive* in the sense that they

"examine what ultra-smart, impeccably rational super-people *should* do in competitive situations. They are not interested in the way that erring folks like you and me actually behave, but in how we should behave if we were smarter, thought harder, were more consistent, were all knowing. Advice is given symmetrically to all parties about how to play the game" (Raiffa (1982)).

The resulting game-theoretic optimal solution is thus not a prediction of what actually happens in real-world bargains (the **positive** argument), but is merely a "fair" division of the merger synergy, reflecting the reasonable expectations of rational bargaining parties (the **normative** argument). These models are thus a description of the position

that the players *should* assume if expected utility gains alone were important, and says nothing of the multitude of other issues which should be considered in a complex transaction such as a corporate merger. Thus these fundamental analytical models, whilst offering a good degree of fit when compared to real empirical data, might be construed as being unrealistic and unbending representations of the real world.

The dynamic negotiation analysis model, on the other hand, focuses away from the game-theoretic idea of a static equilibrium solution, and is based on the players' perceptions of the zone of agreement and behavioural properties of the negotiating players. Several reasons exist for this approach. Firstly, whilst people might be construed to exhibit intelligent purposive behaviour in a negotiation situation, there are often important deviations from the "imaginary, idealised, super-rational people without psyches" (Bell, Raiffa and Tversky (1988)) needed by expected utility maximisers for a game-theoretic approach. Secondly, the negotiation's rules, structure and possible moves are often not common knowledge (in the sense of Aumann (1976)) to both parties. Whilst these reasons do not necessarily invalidate the game-theoretic models of the previous chapters, the model in the current chapter seeks to offer prescriptive advice to one of the players on likely Pareto-optimal bargaining strategies given a (probabilistic) description of how the other party will behave, which is in line with the general decision analytic approach. In the process we will assume that each party views the opposing party as intelligent and goal-seeking, but not necessarily fully rational in the game-theoretic sense (Sebenius (1992) p. 20).

The model satisfies all of Sycara's requirements for a "negotiation model". It makes no use of measurable data, such as the market values of the participating companies, and thus cannot be tested against real empirical data, as could the models of the previous chapters. Instead, the nature of the data employed in this model is entirely *judgemental, subjective and intangible*, and is thus unlikely to be documented in any company records or financial data-base. Whilst the negotiators may closely scrutinise publicly-available accounting- and market-related statistics to aid them in the bargaining process, the corresponding levels of the parameters of our model in any given instance may be arrived at in the negotiator's minds in a wholly unstructured way, and thus may

be very different for two apparently similar negotiating pairs. This is part of the reason why real-world negotiators often show deviations from the behaviour of the "ideal rational decision-maker".

In the next chapter we will describe and construct a Monte Carlo simulation to implement this model representation. We will report and discuss the simulation results for a wide range of values for the players' contextual and strategy parameters, and we will use these results to give general insights into possible solutions to the merger bargaining problem, and to help identify possible optimal negotiating strategies.

CHAPTER 5**A MONTE CARLO SIMULATION OF THE MULTI-STAGE MERGER
BARGAINING MODEL WITH FULL AND SYMMETRIC UNCERTAINTY****§5.1 INTRODUCTION**

Any rational bargaining party may be expected to attempt to negotiate a deal which is optimal for the company it represents, i.e. offers the maximum expected net gain to the company (Jones (1980) p. 78). It is thus important that the parties are sufficiently well-informed of which bargaining strategies will offer the greatest possible expected gain to each. This is especially true in merger negotiations, since the decision is a one-time decision for both parties, and will not be repeated for the same two parties under identical circumstances. Furthermore, the consequences of the decision will be far-reaching: millions of rands and thousands of investors (shareholders) may be involved.

A way of investigating merger strategies, and of using the model to offer decision support, is to construct a Monte Carlo simulation of the behaviour of each of the players under various scenarios (described by the contextual parameters) and for various discrete combinations of the bargaining strategies of both parties. Simulation involves the modelling of a process in such a way that the model mimics the response of the actual system to events that take place over time (Schriber (1987)), and can be used to experimentally evaluate various strategies for the operation of the process (Pegden, Shannon and Sadowski (1990)). A single replication of this simulation will effectively represent a typical observation of a negotiation between an acquirer and a target. Averaging over repeated replications of the simulation might yield information to the negotiators as to how successful they might expect merger negotiations to be for any particular bargaining strategy, and at what monetary amount they might expect the

merger transaction to occur (in the case of a successful merger) together with some idea of how far any particular transaction might vary from this amount. This sort of information would be useful to negotiators: both parties, being rational, would like to choose a strategy which is likely to maximise the probability of a successful merger, and each would like to ensure that the expected transaction value maximises their net gain. Brown (1951) p. 374 supports this sort of approach when he comments

"The iterative method ... can be loosely characterized by the fact that it rests on the traditional statistician's philosophy of basing future decisions on the relevant past history. Visualize two statisticians, perhaps ignorant of min-max theory, playing many plays of the same discrete ... game. One might naturally expect a statistician to keep track of the opponent's past plays and, in the absence of a more sophisticated calculation, perhaps to choose at each play the optimum pure strategy against the mixture represented by all the opponent's past plays."

Luce and Raiffa (1957) p. 84 comment further that

" ... if one wants to find the solution of a game which is to be played but once, he can set up two fictitious players, generate a fictitious iteration of games with the players behaving as naive statisticians, and observe the outcomes which generate a solution."

In this chapter we will construct and implement a Monte Carlo simulation of the multi-stage merger bargaining model developed in Chapter 4. In §5.2 we describe the construction of the simulation procedure and examine the ranges of values we will use for each of the input parameters. The results and some general observations from an initial simulation run are contained in §5.3. In this section we also attempt to identify strategies which could be construed as "optimal" to the players for this situation, using both conventional game-theoretic solution concepts as well as a softer, more subjective Bayesian decision-making approach. In §5.4 we compare the "optimal" solutions identified in the previous section, and we conclude the chapter with some possible implications of using this type of model for decision support.

§5.2 CONSTRUCTION AND IMPLEMENTATION OF A MONTE CARLO SIMULATION PROCEDURE

§5.2.1 THE MONTE CARLO SIMULATION PROCEDURE

A single replication of the simulation of the negotiating process will represent a typical negotiation between an acquirer and a target. But what is "typical"? Every prospective acquirer *expects* the value of the re-scaled difference between the two companies' reservation prices to be 1, and bases its initial offer (and, subsequently, its set of possible non-initial offers) partly on this expectation. The true difference $V_A - V_B$, however, is likely to differ from 1 ($V_A - V_B$ has variance σ^2), and this will vary from one acquirer/target combination to another. We are interested in long-run outcomes of the negotiating process, and the simulation procedure will therefore have to capture the effect of the differences between the estimated value of Δ_{AB} and the true value across all possible acquirer/target combinations. Thus the crucial element of the simulation will necessarily be a stochastic randomising term which is included to model the effect of the different initial and final possible offers for each acquirer on the negotiation process.

A single replication of the simulation procedure will effectively model the negotiations taking place within a single acquirer/target pair. Recall that the difference in re-scaled reservation price for an acquirer/target combination is assumed to be a random variable with probability distribution given by

$$\Delta_{AB} = V_A - V_B \sim N(1; \sigma^2).$$

From the point of view of the acquirer, V_A is known (with value v_A) and V_B is a random variable which has expectation $v_A - 1$. Similarly, from the target's point of view, V_B is known (with value v_B) and V_A is a random variable with expectation $v_B + 1$. The origin of the scale on which these variables are measured is irrelevant since we are dealing with the difference between two quantities. For simulation purposes, however, we will need an origin as reference point. To this end we will arbitrarily view the position from the target's perspective, and will standardize the target's reservation price to $v_B = 0$. Thus in the simulation we will fix one party's reservation price, and model the effect of the negotiating pair's uncertainty regarding the true difference in their reservation prices (and hence the parties' offers and acceptance levels) by means of the random variable Δ_{AB} .

The first stage of each replication of the simulation procedure deals with the *initial* offer and acceptance level. In the previous chapter we made the modelling assumption that the acquirer's initial offer be given by

$$O_1 = v_A - \delta_A (1 - \sigma_A \Phi^{-1}(\beta_{A1})) .$$

This (if accepted) will result in a net gain to the acquirer of

$$v_A - O_1 = \delta_A (1 - \sigma_A \Phi^{-1}(\beta_{A1})) .$$

Using the target's reservation price as origin, the initial offer is

$$O_1 = \eta_{AB}^i - \delta_A (1 - \sigma_A \Phi^{-1}(\beta_{A1}))$$

where η_{AB}^i is the value of Δ_{AB} in the i^{th} replication of the simulation. Since η_{AB}^i is chosen randomly, it is necessary to check that the i^{th} initial offer does not exceed the acquirer's own reservation price, since by individual rationality the acquirer would not be prepared to offer more than this. The acquirer's initial offer will be above its own reservation price if

$$O_1 = \eta_{AB}^i - \delta_A (1 - \sigma_A \Phi^{-1}(\beta_{A1})) > v_A = \eta_{AB}^i$$

or equivalently, if

$$1 - \sigma_A \Phi^{-1}(\beta_{A1}) < 0$$

$$\text{i.e.} \quad \sigma_A \Phi^{-1}(\beta_{A1}) > 1 .$$

In this case we will truncate the initial offer to η_{AB}^i .

We previously assumed that the target's initial acceptance level is given by

$$A_1 = v_B + \delta_B (1 + \sigma_B \Phi^{-1}(\beta_B))$$

which, expressed on the scale with origin at $v_B = 0$, is

$$\delta_B (1 + \sigma_B \Phi^{-1}(\beta_B)) .$$

In a precisely symmetric manner as before it is necessary to ensure that the target's acceptance level is above its reservation price. A_1 will fall below the target's reservation price if

$$A_1 = \delta_B (1 + \sigma_B \Phi^{-1}(\beta_B)) < v_B = 0$$

or equivalently, if

$$1 + \sigma_B \Phi^{-1}(\beta_B) < 0$$

$$\text{i.e.} \quad \sigma_B \Phi^{-1}(\beta_B) < -1 .$$

In this case we will truncate the initial acceptance level to 0. Note that in the simulation procedure the quantity A_1 remains constant for a given set of parameter values from one replication to another. The acquirer's initial offer will be accepted provided that $O_1 > A_1$, i.e. provided that

$$\eta_{AB}^i - \delta_A (1 - \sigma_A \Phi^{-1}(\beta_{A1})) > \delta_B (1 + \sigma_B \Phi^{-1}(\beta_B)) .$$

The target's gain from accepting the initial offer is measured relative to its reservation price $v_B = 0$. Thus the target's simulated gain in the i^{th} replication of the simulation will be simply

$$\eta_{AB}^i - \delta_A (1 - \sigma_A \Phi^{-1}(\beta_{A1})) - v_B = \eta_{AB}^i - \delta_A (1 - \sigma_A \Phi^{-1}(\beta_{A1}))$$

since $v_B = 0$. In the case of an accepted initial offer in replication i the simulation procedure records

- (1) that the offer was accepted; and
- (2) the offer value at which agreement was reached,

and proceeds with the $(i+1)^{\text{th}}$ replication of the simulation (i.e. the bargaining process of the $(i+1)^{\text{th}}$ acquirer/target combination using identical strategies).

In the event of a rejected initial offer in the i^{th} replication we need to simulate the acquirer's set of possible non-initial offers. In any replication all possible non-initial offers will lie between the initial offer, O_1 , and the simulated largest possible offer, O_L , where

$$O_L = \eta_{AB}^i - \delta_A (1 - \sigma_A \Phi^{-1}(\beta_{AL}))$$

which contains the same stochastic term η_{AB}^i as the initial offer, since a replication describes the behaviour of a single acquirer, whose known value of v_A remains constant throughout the bargaining process. Thus the interval $[O_1; O_L]$ is of constant length for any given set of parameter values for *all* replications; only the interval $[V_A; V_B]$ varies in length across the replications. Since $O_L > O_1$ (since $\beta_{AL} > \beta_{A1}$ by definition) it is necessary to check that O_L does not exceed the acquirer's own reservation price. If $O_L > v_A = \eta_{AB}^i$, then as before we will take this to imply that $O_L = \eta_{AB}^i$. The number of non-initial offers in the i^{th} replication of the simulation is a random variable N and is characterised by a probability (p , say) that the acquirer terminates the bargaining process at any stage after the initial offer. The parameter p is one of the acquirer's strategy parameters. We use the geometric distribution to model the exact number of non-initial offers in replication i , and thus

$$P(N = n_i) = p(1 - p)^{n_i} \quad n_i = 0, 1, 2, \dots$$

$$= 0 \quad \text{otherwise}$$

The case $n_i = 0$ implies that the offer stream terminates after the initial (rejected) offer, and the simulation proceeds with the $(i+1)^{\text{th}}$ replication. In all other cases the procedure chooses $(n_i - 1)$ non-initial offers uniformly from the interval $[O_1 ; O_L]$ (we assume that offers are equally likely to be positioned anywhere in this interval), orders them from smallest to largest, adds the n_i^{th} (largest possible) non-initial offer, O_L , and stores the resultant vector of length n_i . This n_i -vector constitutes the i^{th} acquirer's non-initial offer stream. The bargaining beyond the initial stage can now be simulated for replication i . The target's revised acceptance level for the j^{th} offer stage of the bargaining was given in the previous chapter as

$$A_j = (1 - \gamma) A_{j-1} + \gamma O_{j-1} \quad j = 2, \dots, n_i$$

provided that it is at least as large as $v_B = 0$, the lower bound on the target's acceptance level at any stage. The j^{th} offer is accepted if the stored $O_j > A_j$, in which case the simulation procedure records

(1) that agreement was reached; and

(2) the offer value O_j at which agreement was reached,

and then continues with replication $(i+1)$. If agreement is not reached, acceptance level A_{j+1} is generated and compared to the (stored) offer O_{j+1} , and the procedure continues in this way to test whether an offer is successful or not until either (1) agreement is eventually reached at some stage $j, j=2, \dots, n_i$, or (2) the set of non-initial offers is exhausted, in which case the procedure continues with replication $(i+1)$.

The simulation procedure may be repeated for a large number (say M) of replications of the process for given values of the contextual and strategy parameters. This approach effectively ensures that any concluding analysis will be based on the outcomes of a large number of independent acquirer/target negotiations, with the players using the same bargaining strategies each time, and will indicate how the players' uncertainty about the other's reservation price is transformed into useful decision-making measures for each acquirer/target bargaining strategy.

For each combination of contextual parameter values and player's bargaining strategies \bar{A} and \bar{B} (which together form a bargaining strategy pair $\langle \bar{A}; \bar{B} \rangle$) the

simulation procedure records the frequency of agreement, k , over all M replications. We can thus estimate the **probability of merger** occurring for the given combination of model parameters from

$$P(\text{merger occurring}) = \frac{\text{number of mergers occurring}}{\text{total number of replications}} = \frac{k}{M} .$$

Furthermore the simulation procedure records the offer value at which agreement was reached, otherwise called the transaction value, T_h , $h = 1, 2, \dots, k$ in all k cases in which agreement was indeed reached, and hence we may calculate the **average transaction value in successful mergers**, Ave, from

$$\text{average transaction value (successful mergers)} \quad (\text{Ave}) = \frac{\sum (\text{recorded traded values})}{(\text{number of successful trades})} = \frac{1}{k} \sum_{h=1}^k T_h .$$

The variance of the achieved transaction values can be calculated to indicate how far any arbitrary transaction value might lie from this average. The acquirer's **expected gain** from merger for any given set of values for the bargaining strategies \bar{A} and \bar{B} is denoted by $E(G_{\text{acq}})$ and is given by

$$E(G_{\text{acq}}) = P(\text{merger occurring}) \cdot \frac{1}{k} \sum_{h=1}^k (\eta_{AB}^h - T_h) \quad (5.1)$$

since $\frac{1}{k} \sum_{h=1}^k (\eta_{AB}^h - T_h)$ is the acquirer's average gain in successful mergers (i.e. measured relative to the acquirer's simulated reservation price). In the same way, the target's expected gain is denoted by $E(G_{\text{tar}})$ and is given by

$$E(G_{\text{tar}}) = P(\text{merger occurring}) \cdot \text{Ave} \quad (5.2)$$

since the average transaction value (Ave) is the target's average gain in successful mergers, i.e. an amount measured relative to the target company's reservation price (which was standardised to 0). Note that $E(G_{\text{acq}})$ and $E(G_{\text{tar}})$ always lie in the unit interval, and we will assume that these quantities represent the two parties' expected gains from arriving at a negotiated settlement using the particular combination of strategy parameters in question. If the simulation procedure is performed for a wide range of combinations of the parameter values, a **bimatrix game** representation of the negotiating process may be built up, where the two matrix entries in each cell are the expected gains to the target and the acquirer for any specific bargaining strategy pair $\langle \bar{A}; \bar{B} \rangle$ and for any combination of the contextual parameters. The game is two-person and non-constant (non-zero) sum. By identifying and analysing equilibrium points (if any exist) for the

bimatrix game (whose outcome in bimatrix form is available to both parties for a wide range of possible choices of bargaining strategies for the acquirer and the target) the merger participants may be able to establish strategies for themselves which are Pareto optimal, possibly under the assumption that the other party is doing the same. Thus to be a useful decision-tool, a simulation using this model will have to be run for a range of values for the acquirer's and target's bargaining strategies \bar{A} and \bar{B} which is wide enough to cover all reasonable bargaining strategies that may be used by the two sides, and would allow inferences to be made about the consequences of possible bargaining strategies.

The simulation procedure of the model as discussed above is diagrammatically described in the flow-diagram presented in Figure 5.1. The following symbols are used in the flow-chart representation of the simulation:

v_A	acquirer's known reservation price
v_B	target's known reservation price
M	maximum number of replications of the simulation
σ	uncertainty in the estimation of $V_A - V_B$ by both parties
σ_A	acquirer's uncertainty in the estimation of V_B
σ_B	target's uncertainty in the estimation of V_A
η_{AB}^i	a stochastic disturbance term chosen from the $N(1; \sigma^2)$ distribution and used to randomise the offers in the i^{th} replication
δ_A	acquirer's perceived relative dominance
β_{A1}	acquirer's initial strategic concession parameter
β_{AL}	acquirer's final strategic concession parameter
p	acquirer's degree of persistence (probability of terminating the offer stream at any stage)
δ_B	target's perceived relative dominance
β_B	target's strategic concession parameter
γ	target's yield decrement
$\Phi^{-1}(x)$	quantile of the $N(0;1)$ distribution pertaining to a cumulative probability of x
i	replication number
n	maximum number of acquirer's non-initial offers
j	current offer number ($j=1, \dots, n+1$)
k	number of successful mergers in i replications
O_1	acquirer's initial offer
O_L	acquirer's final (largest) possible offer
A_j	target's j^{th} acceptance level, $j=1, \dots, n+1$
T_k	agreed value of k^{th} transaction (only in the case of a successful agreement)
$E(G_{\text{acq}})$	expected gain as viewed by the acquirer
$E(G_{\text{tar}})$	expected gain as viewed by the target
Ave	average transaction value of successful mergers

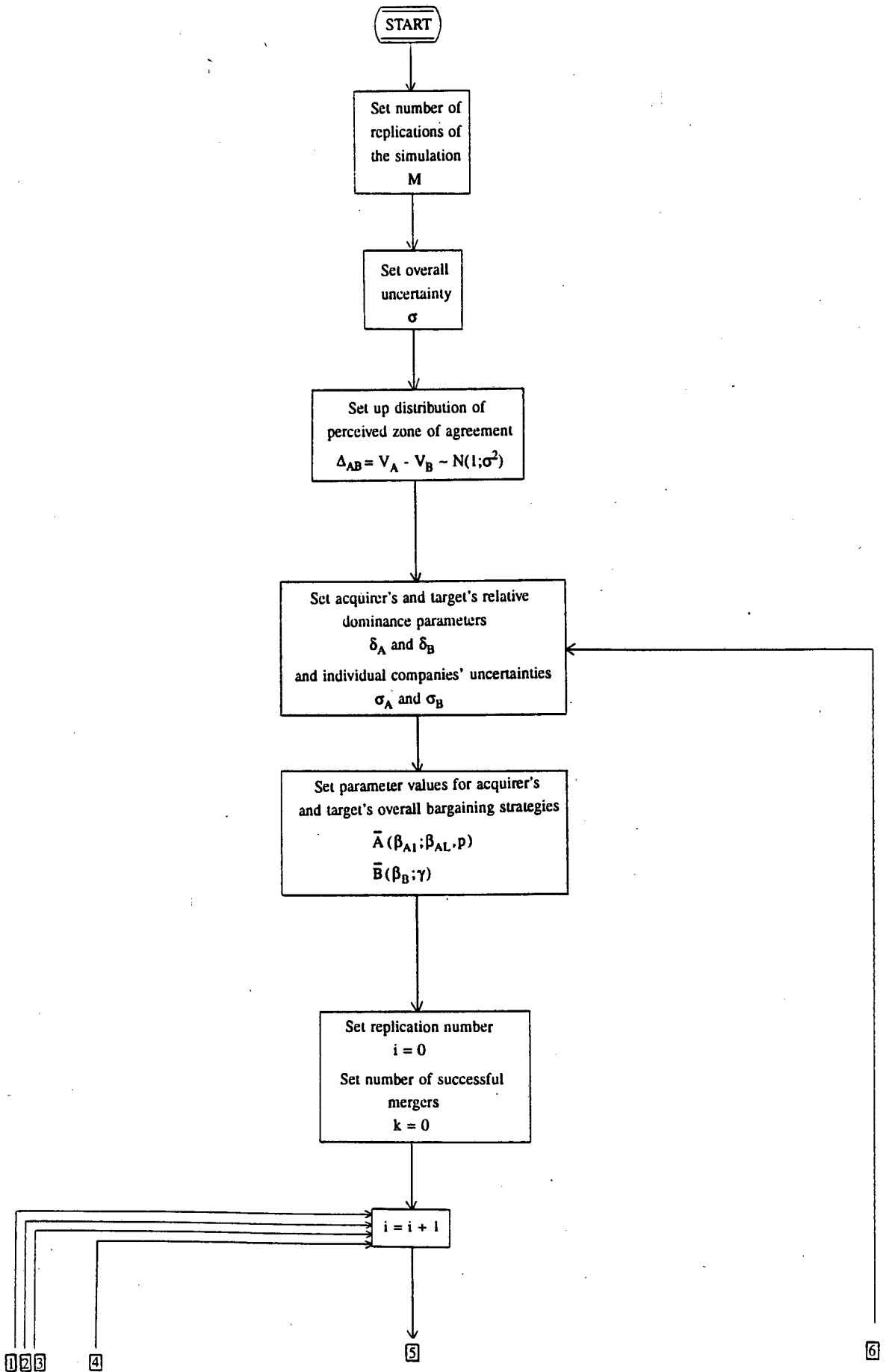


Figure 5.1. Flow-diagram of the simulation procedure

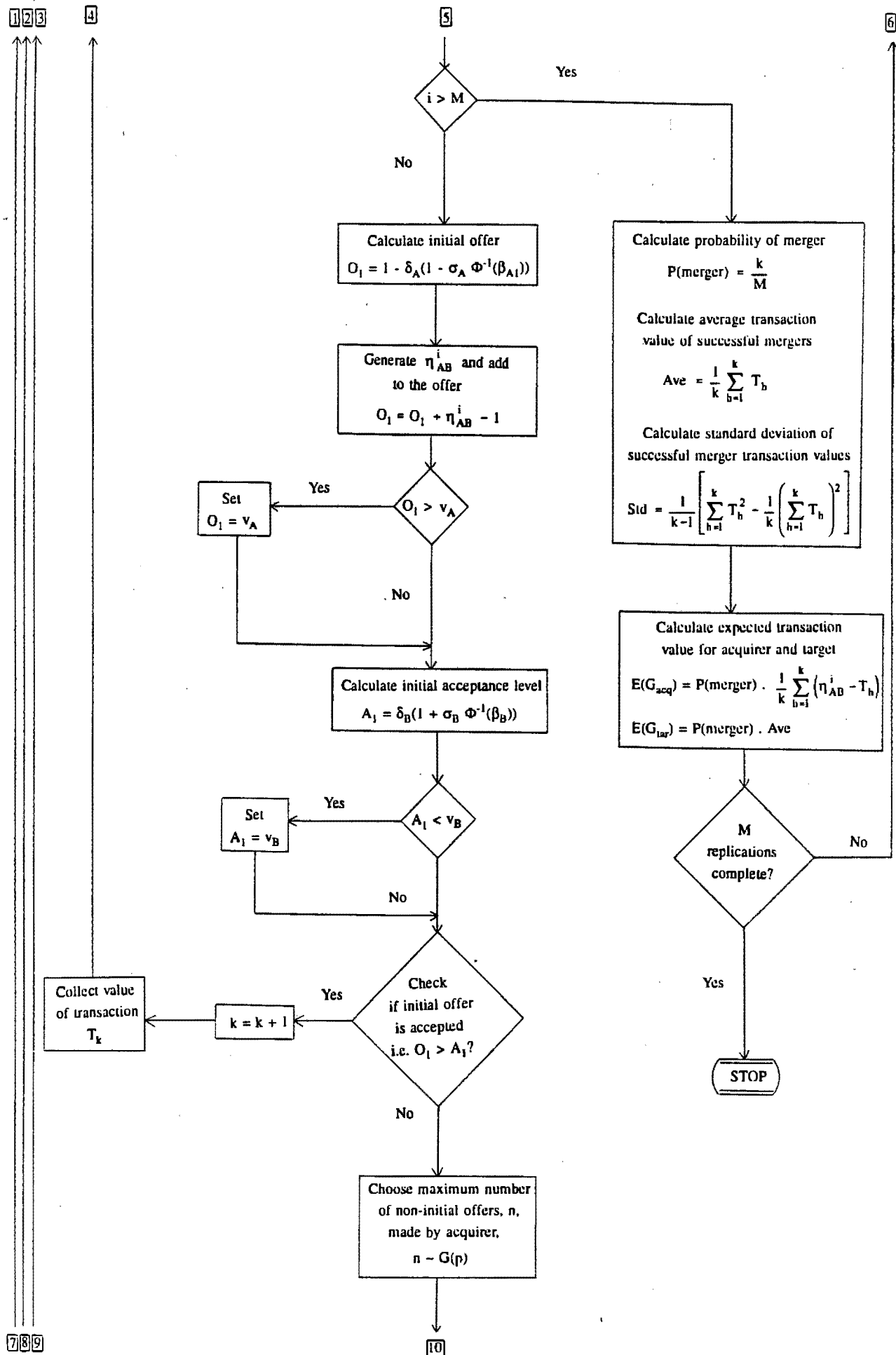


Figure 5.1. Flow-diagram of the simulation procedure (continued)

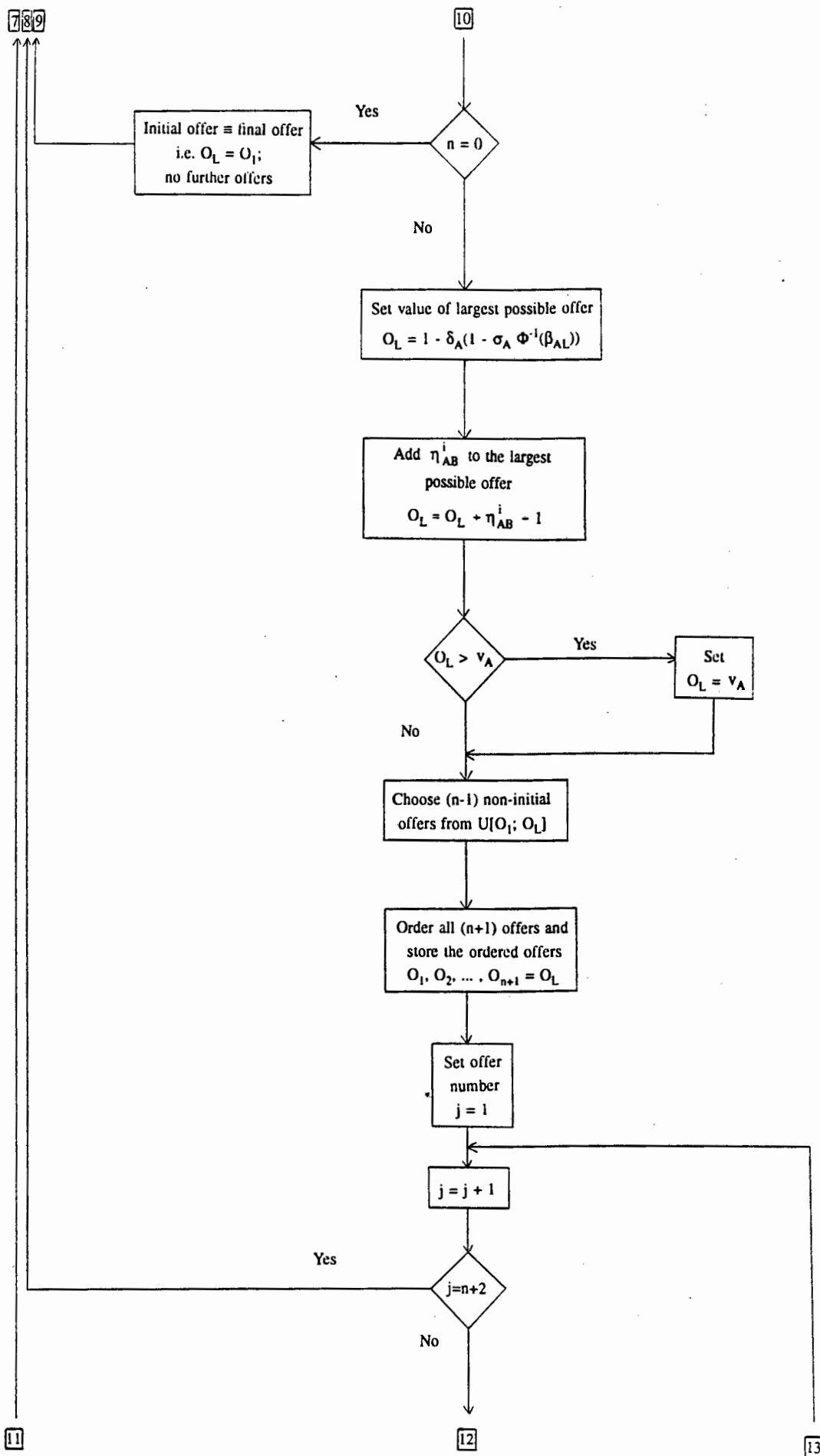


Figure 5.1. Flow-diagram of the simulation procedure (continued)

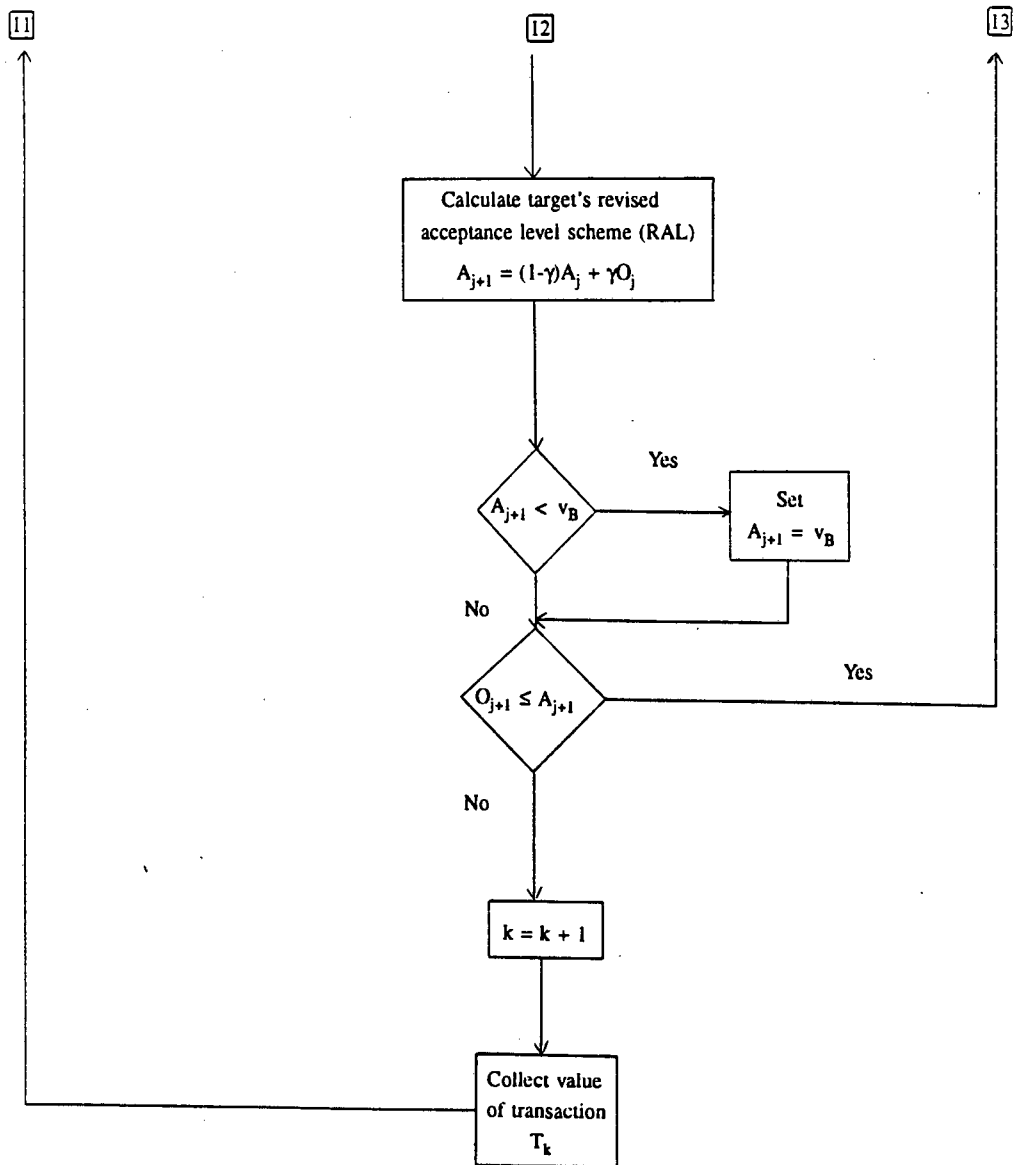


Figure 5.1. Flow-diagram of the simulation procedure (continued)

§5.2.2 INPUT DATA RANGES

To be a useful decision aid the simulation of the negotiating process must consider ranges of values for the strategy parameters contained in \bar{A} and \bar{B} and for the contextual parameters which are wide enough to cover all realistic bargaining scenarios that may arise, and all reasonable strategies that may be played by the two sides. In this section we describe the ranges we will consider for each model parameter in the implementation of the simulation.

We note first that all ten model parameters are *continuous* in nature; more specifically, the players' strategy sets are all real numbers in the closed interval from 0 to 1, i.e.

$$\bar{A}(\beta_{A1}; \beta_{AL}, p) = \{ (\beta_{A1}; \beta_{AL}, p) \mid 0 \leq \beta_{A1} \leq 1, 0 \leq \beta_{AL} \leq 1, 0 \leq p \leq 1 \} \subset \mathbb{R}^3$$

$$\bar{B}(\beta_B; \gamma) = \{ (\beta_B; \gamma) \mid 0 \leq \beta_B \leq 1, 0 \leq \gamma \leq 1 \} \subset \mathbb{R}^2$$

and thus in effect both players have strategy sets which are infinite in size. The expected gains to the acquirer and to the target should thus in actual fact be represented as a double response surface in five-dimensional unit space for any combination of the contextual parameters. Unfortunately no general characterisation of the optimal strategies for games with continuous payoffs over the unit square exists (let alone the unit interval in \mathbb{R}^5 !); we will resort to drawing conclusions about the merger game from the simulated negotiations. The expected gain to each party is a complex function of the bargaining strategy parameters of both of the parties. If the functional form of both of these multivariate relationships were known, the equilibrium points for the game (if any existed) could be identified analytically. Since the functional forms are not known we are restricted, without loss of generality, to numerical evaluation of the expected gains to the parties (by using our simulation procedure) over some five-dimensional finite grid consisting of discrete values of each of the parameters. The fineness of the grid depends largely on computer time; since ours is a pragmatic solution to a complex problem we will begin with a discrete, fairly coarse grid. We will term these discrete combinations of strategy parameter values for the two players their **discrete strategy scenarios**. Evaluating the expected gains to the two players over a finite grid can approximate a continuous mixture of the discrete strategy scenarios providing that we make the assumption that the response surfaces of the players' expected gains are continuous.

(1) **Strategy parameters**

The acquirer's strategic concession parameters β_{AI} and β_{AL} can range between 0 (a hard-line strategy for the offer(s)) and 1 (a generous strategy). We will examine the simulated merger outcomes when β_{AI} is set to 0.1, 0.3, 0.5, 0.7 and 0.9. We will use the same set of discrete values for β_{AL} as for β_{AI} , noting, however, that by definition the offer stream is monotonically increasing and so $\beta_{AL} \geq \beta_{AI}$. The target's strategic concession parameter β_B is defined in exactly the same way as β_{AI} , and thus we will compute the merger outcomes for the same set of discrete values for β_B .

The probability p that an acquirer terminates the bargaining at any stage after the initial offer could be any value in the unit interval. However, the smaller p is, the larger is the expected number of possible offers in the offer stream, as well as the probability of a large number of possible offers. For example, if $p = 0.1$ then the expected number of possible offers in the offer stream is 10, and the probability that there could be at least eight offers is

$$\begin{aligned} P(\text{at least 8 possible offers}) &= P(\text{at least 7 possible non-initial offers}) \\ &= 1 - \sum_{n=0}^6 0.1 (0.9)^n = 0.5206 \text{ ,} \end{aligned}$$

i.e. if a company pursuing a policy of growth through frequent merger activity always plays a strategy in which p is as low as 0.1, then approximately every second negotiation in which that acquirer is involved could last as long as eight rounds. Such strategies where the bargaining drags on over many offers (referred to by Roy (1989) as **minimal concession strategies**) are not generally observed in practice in the marketplace (Roy (1989) p. 597). We will thus restrict p to the values 0.3 (i.e. the expected number of offers is 3.33, and the probability of multiple offers in the offer stream is fairly large), 0.5, 0.7 and 0.9 (i.e. the expected number of offers is 1.11, and the probability of more than just 2 or 3 possible offers in the offer stream is extremely small).

The target's yield decrement, γ , ranges between 0 (no concessions are made at all) and 1 (the target capitulates completely). Since values near both extremes are possible in practice, we will consider γ at the discrete values 0.05, 0.35, 0.65 and 0.95.

Thus we will restrict ourselves to considering $\left(\sum_{N=1}^5 N \right) \cdot 4 = 60$ discrete acquirer's strategies and $(5)(4) = 20$ discrete target's strategies.

(2) Contextual parameters

The relative dominance parameters, δ_A and δ_B , range between 0 and 1. It was argued that an acquirer may be inclined to perceive itself dominant over the target due to its (usually) larger market value and greater access to resources. However the target must also be satisfied with the outcome of the merger, and the presence of other potential acquirers will ensure downward pressure on δ_A . By a symmetrical argument, there is upward pressure on δ_B . For simplicity we will initially consider both δ_A and δ_B set to 0.5, and later vary each of them down to 0.3 and up to 0.7.

On an individual company level we conjectured that it would be most likely that the acquirer and the target would each lack full knowledge in the estimation of the other's reservation price, and thus σ_A and σ_B would be *at least as great* as σ . Thus we will investigate the simulated merger outcomes when σ_A and σ_B are separately set equal to σ , 1.5σ , 2σ and 2.5σ when $\sigma = 0.1$.

In the simulation model the negotiating pair expects the difference in their reservation prices to be 1. However the actual value of this difference is unknown to the players, and the degree of uncertainty of this difference is likely to play a role in the outcome of a negotiation. We will initially set σ to 0.25, and later vary σ to as low as 0.1 (representing a small degree of uncertainty in the true magnitude of the zone of agreement) and as high as 0.5 (a large degree of uncertainty), keeping $\sigma_A = \sigma_B$ set to σ and 1.5σ .

Since the contextual parameters essentially describe the negotiating environment (the "playing field") as viewed by the two parties, they are effectively known a priori, and the players are likely to have at least fairly good estimates as to their values before any bargaining begins. The negotiators are thus likely to focus more of their attention on choosing a bargaining strategy which maximises their expected gain conditional on the given set of contextual parameters. Thus initially we will run the simulation and discuss

the results for the "median" case, i.e. the case where the contextual parameters are all set to values which are median in the set of values we have chosen to consider for those parameters.

§5.3 INITIAL SIMULATION RESULTS AND IDENTIFICATION OF POSSIBLE "OPTIMAL" STRATEGIES

§5.3.1 INITIAL SIMULATION RESULTS AND OBSERVATIONS

In this section we will report the results of the simulation for the case where the contextual parameters are set to the following values:

$$\sigma = \sigma_A = \sigma_B = 0.25$$

$$\delta_A = \delta_B = 0.5$$

Appendix 5.A displays the simulated expected gains to the target (above) and to the acquirer (below) for each of the (discrete) strategies under consideration for $M = 5000$ replications of the process.¹ The 20×60 bimatrix presented in Appendix 5.A extends over four pages, and as such is extremely cumbersome to assimilate. In a first attempt at making general observations about the expected gains to the two parties we will select a subsample of the acquirer's and the target's discrete strategies, and examine the resulting submatrix of expected gains to the parties. Later we will compare the relevant results from this submatrix to those gleaned from the entire 20×60 bimatrix in Appendix 5.A.

Assume that the set of the acquirer's discrete strategies is $A = \{a_1, a_2, \dots, a_{60}\}$ (in the same column order as in Appendix 5.A), and that the set of the target's discrete strategies is $B = \{b_1, b_2, \dots, b_{20}\}$ (in the same row order as in Appendix 5.A). We will choose arbitrarily to select 8 of the acquirer's strategies from A using random systematic sampling (i.e. one strategy is randomly chosen from each successive group of 8 strategies) to ensure a roughly even spread of acquirer's strategies across the columns in Appendix 5.A. Similarly, we choose to select 7 of the target strategies from B using 1-in-

¹ The generation of the 20×60 bimatrix in Appendix 5.A (with $M=5000$ replications) took over 6.5 hours of CPU time on a MicroVax 3100-90 mainframe computer. This justifies our use of a fairly coarse grid over which to evaluate the players' expected gains.

3 systematic sampling, which ensures an even spread of the target strategies across the rows in Appendix 5.A. The resulting 7×8 submatrix of expected gains is presented in Table 5.1, and is of a size which can be easily assimilated. Specifically, by using the above sampling plan we have restricted ourselves at this stage to the subset of acquirer's discrete strategies $A' = \{a_2, a_{13}, a_{24}, a_{30}, a_{33}, a_{42}, a_{51}, a_{59}\}$ and to the subset of target's discrete strategies $B' = \{b_1, b_4, b_7, b_{10}, b_{13}, b_{16}, b_{19}\}$. Note that the sampling procedure has succeeded in selecting strategies in such a way that each of the discrete values for each of the strategy parameters is represented in roughly equal proportions, ensuring a wide range of different negotiating strategies for the two parties.

For the subsets $A' \subset A$ and $B' \subset B$ of strategies the ranges of the expected gains to the target and to the acquirer (and the strategies which give rise to them) are laid out in Table 5.2. Immediately apparent is that the target's maximum possible expected gain is over 20% greater than that of the acquirer. Similarly, for this wide set of strategies the target's minimum possible expected gain is some 32% greater than that of the acquirer. Whilst this does not imply that the target's expected gain is greater than that of the acquirer for *all* strategies, it does indicate that over the long term possible opportunities may exist for target companies to earn a greater share of the expected synergistic gains from merger than acquiring companies. Further to this, the target's maximum possible expected gain is significantly greater than 0.5,¹ whilst that of the acquirer is only marginally greater than 0.5.

It will be noted that both parties' maximum possible expected gain occurs when the target invokes the strategy $\bar{B}(0.1; 0.95)$. This strategy is one which represents the target setting a very low initial acceptance level (that is, be prepared to accept a very low initial offer, i.e. one which is well below the target's perception of the expected fair price), and in any event rapidly capitulating and accepting a second offer (if it is forthcoming) provided that it is at least a little greater than the initial offer. Such a target strategy will certainly ensure a large probability of merger occurring, which is an important component of both companies' expected gain (see (5.1) and (5.2)). At this target strategy the acquirer achieves its maximum expected gain when it invokes the

¹ Remember that the perceived zone of agreement was scaled to $[0; 1]$, and thus a party employing a strategy with expected gain greater than 0.5 effectively expects to get more than half of the merger gains.

Table 5.1. Simulated expected gains to the target (above) and to the acquirer (below) for the subsets of strategies $A' \subset A$ and $B' \subset B$

	a_2	a_{13}	a_{24}	a_{30}	a_{33}	a_{42}	a_{51}	a_{59}
	$\bar{A}(1;3,3)$	$\bar{A}(7;7,3)$	$\bar{A}(3;9,5)$	$\bar{A}(9;9,5)$	$\bar{A}(1;5,7)$	$\bar{A}(5;9,7)$	$\bar{A}(3;3,9)$	$\bar{A}(7;9,9)$
b_1	0.3073	0.5230	0.4296	0.6234	0.3003	0.4753	0.3770	0.5235
	0.3920	0.3594	0.4168	0.3155	0.3704	0.3930	0.3704	0.3615
b_4	0.1796	0.3817	0.3343	0.5104	0.1748	0.3448	0.2140	0.3865
	0.1731	0.2243	0.2371	0.2307	0.1596	0.2290	0.1693	0.2269
b_7	0.3056	0.4801	0.4249	0.5900	0.2701	0.4404	0.3207	0.4824
	0.3819	0.3090	0.3701	0.2841	0.2995	0.3368	0.2856	0.3129
b_{10}	0.1944	0.2906	0.3288	0.4207	0.1414	0.2853	0.1476	0.3014
	0.1989	0.1581	0.2105	0.1805	0.1160	0.1723	0.1066	0.1637
b_{13}	0.3161	0.4245	0.4114	0.5625	0.2541	0.4132	0.2616	0.4534
	0.4139	0.2628	0.3422	0.2617	0.2740	0.3030	0.2201	0.2808
b_{16}	0.3431	0.5277	0.4381	0.6288	0.3152	0.4701	0.3723	0.5347
	0.5227	0.3811	0.4431	0.3162	0.4083	0.3983	0.3626	0.3689
b_{19}	0.3119	0.3801	0.3763	0.5098	0.2266	0.3737	0.2192	0.3920
	0.4465	0.2253	0.3069	0.2298	0.2459	0.2618	0.1732	0.2331

Table 5.2. Minimum and maximum expected gains for targets and acquirers over the subsets of strategies $A' \subset A$ and $B' \subset B$

	Minimum E(gain)	Maximum E(gain)
E(gain) to target	0.1414	0.6288
Corresponding E(gain) to acquirer	0.1160	0.3162
At strategy pair	$\bar{A}(0.1; 0.5, 0.7)$ $\bar{B}(0.9; 0.35)$	$\bar{A}(0.9; 0.9, 0.5)$ $\bar{B}(0.1; 0.95)$
E(gain) to acquirer	0.1066	0.5227
Corresponding E(gain) to target	0.1476	0.3431
At strategy pair	$\bar{A}(0.3; 0.3, 0.9)$ $\bar{B}(0.9; 0.35)$	$\bar{A}(0.1; 0.3, 0.3)$ $\bar{B}(0.1; 0.95)$

strategy $\bar{A}(0.1; 0.3, 0.3)$. This strategy indicates that the acquirer makes as low an initial offer as it deems reasonable, but would be prepared to negotiate upwards up to a point below its perceived expected fair price for the target. The acquirer would be prepared to negotiate fairly persistently within this range, i.e. there is a fairly high probability (probability = 0.7) that a rejected offer at any stage will be followed by a further (increased) offer. This combination of acquirer and target strategies (very low initial offer by the acquirer and very low initial acceptance level followed by virtual capitulation by the target) would clearly lead to a rapid acceptance of a low offer. Thus the probability of merger actually occurring is high with a low average transaction value, a combination clearly favouring the acquiring company. Table 5.3 shows the probability of agreement being reached (above), the average transaction value in successful mergers (middle) and the standard deviation of the transaction values in successful mergers (below) in the implemented simulation for the subsets of acquirer and target strategies A' and B' . (The complete 20×60 matrix of these three variables for the full strategy sets A and B can be found in Appendix 5.B). For this strategy combination we observe that $P(\text{merger occurring}) = 0.8178$ (high) with an average transaction value of 0.4196 (low -

Table 5.3. Probability of agreement (above), average transaction value in successful agreements (middle) and standard deviation of transaction values in successful agreements (below) for the subsets of strategies A' and B'

	a_2	a_{13}	a_{24}	a_{30}	a_{33}	a_{42}	a_{51}	a_{59}	
	$\bar{A}(1;3,3)$	$\bar{A}(7;7,3)$	$\bar{A}(3;9,5)$	$\bar{A}(9;9,5)$	$\bar{A}(1;5,7)$	$\bar{A}(5;9,7)$	$\bar{A}(3;3,9)$	$\bar{A}(7;9,9)$	
b_1	$\bar{B}(1;.05)$	0.6062 0.5069 0.1502	0.8164 0.6407 0.1807	0.7808 0.5502 0.1615	0.8980 0.6942 0.1909	0.5764 0.5210 0.1471	0.7944 0.5983 0.1764	0.6534 0.5769 0.1661	0.8226 0.6364 0.1814
b_4	$\bar{B}(7;.05)$	0.2728 0.6583 0.1053	0.5046 0.7564 0.1324	0.4832 0.6919 0.1093	0.6480 0.7876 0.1375	0.2578 0.6780 0.0996	0.4786 0.7204 0.1251	0.2980 0.7182 0.1186	0.5148 0.7508 0.1335
b_7	$\bar{B}(3;.35)$	0.6088 0.5019 0.1660	0.7016 0.6842 0.1615	0.7268 0.5846 0.1575	0.8096 0.7287 0.1688	0.4802 0.5625 0.1369	0.6962 0.6326 0.1614	0.5040 0.6363 0.1458	0.7122 0.6774 0.1655
b_{10}	$\bar{B}(9;.35)$	0.3288 0.5912 0.1870	0.3536 0.8218 0.1085	0.4760 0.6907 0.1423	0.5018 0.8384 0.1121	0.1996 0.7084 0.1113	0.3758 0.7591 0.1194	0.1868 0.7900 0.0997	0.3740 0.8059 0.1115
b_{13}	$\bar{B}(5;.65)$	0.6700 0.4717 0.1979	0.5948 0.7137 0.1470	0.7042 0.5842 0.1704	0.7374 0.7629 0.1552	0.4576 0.5553 0.1649	0.6448 0.6409 0.1624	0.3878 0.6745 0.1340	0.6418 0.7064 0.1516
b_{16}	$\bar{B}(1;.95)$	0.8178 0.4196 0.2060	0.8210 0.6428 0.1819	0.8322 0.5264 0.1895	0.9006 0.6982 0.1889	0.6480 0.4864 0.1733	0.8136 0.5778 0.1888	0.6396 0.5820 0.1654	0.8408 0.6359 0.1881
b_{19}	$\bar{B}(7;.95)$	0.7226 0.4316 0.2221	0.5092 0.7464 0.1375	0.6382 0.5897 0.1942	0.6482 0.7865 0.1374	0.4240 0.5345 0.2071	0.5660 0.6602 0.1813	0.3048 0.7192 0.1202	0.5376 0.7292 0.1573

the lowest in Table 5.3).¹ The target's maximum expected gain occurs if the acquirer invokes $\bar{A}(0.9; 0.9, 0.5)$ simultaneously with the target playing its abovementioned strategy. This strategy represents the acquirer making a very high initial offer (well above its perception of the expected fair price for the target), and then remaining firm (i.e. it may make a number of further offers *at exactly the same offer level*). Since the target's strategy is to accept even a low initial offer, the probability of the negotiations being successful will tend to be very high, and the average transaction value will be high, a combination thus favouring the target company. From Table 5.3 we observe that $P(\text{merger occurring}) = 0.9006$ here, with an average transaction value of 0.6982 (which is well above 0.5).

Both parties achieve their minimum expected gains when the target invokes the strategy $\bar{B}(0.9; 0.35)$ (i.e. when the target will only accept an initial offer well above its expectation of a fair price, and will only gradually back down from this position). At this target strategy the acquirer will achieve its minimum expected gain if it implements the strategy $\bar{A}(0.3; 0.3, 0.9)$ (i.e. when the acquirer makes a low initial offer and may repeat the offer with little persistence, but not increase it). Clearly this combination of stubborn strategies is unlikely to result in a successful merger. We observe from Table 5.3 that $P(\text{merger occurring}) = 0.1864$ for this combination of strategies. The target's minimum expected gain occurs when the acquirer invokes $\bar{A}(0.1; 0.5, 0.7)$, i.e. when the acquirer makes a very low initial offer, but is prepared to negotiate (with little persistence) upwards to its perception of a fair price. Again $P(\text{merger occurring}) < 0.2$ here.

Several further general observations can be made from an examination of Tables 5.1 and 5.3.

¹ By definition of the expected gains to the players given in (5.1) and (5.2),
 $E(G_{\text{tar}}) = P(\text{merger occurring}) \times (\text{average transaction value})$
 $= 0.8178 \times 0.4196 = 0.3431$ here.

Similarly,

$$E(G_{\text{acq}}) = P(\text{merger occurring}) \times (1/k) \sum (\eta_{AB}^h - T_h)$$

$$= 0.8178 \times (1/k) \sum \eta_{AB}^h - 0.4196 = 0.5227 \text{ here.}$$

(1) The sum of the expected gains to the two negotiating parties always lies in the range

$$0 < E(\text{gain to acquirer}) + E(\text{gain to target}) < 1$$

For the range of parameter values under consideration here the two parties expect that their combined gains will have a minimum of 0.1429 and a maximum of 0.9450. In other words, the negotiating pair expect that any strategy played will have some possible joint gains "left on the table".

(2) The target's expected gain exceeds that of the acquirer for over 78% of the strategy pairs considered.¹ This offers further support to the conjecture that over the long term opportunities may exist for target companies to earn a greater share of the expected merger gains than acquiring companies.

Acquiring companies in general achieve greater expected gains than do target companies when their strategy is one of starting with an initial offer as far as possible below their perception of a fair price for the target, and are prepared to negotiate upwards to a point on or just below the perceived fair price, i.e. $\bar{A}(0.1; \sim 0.3, p)$. Target companies' expected gains tend to be greater than acquirer companies' expected gains when the target's strategy is one of starting with an initial acceptance level well above its perception of a fair price, followed by fairly rapid capitulation if it rejects the initial offer, i.e. $\bar{B}(\geq 0.7; \leq 0.35)$. It is worth noting here that so far we have concentrated on maximising the players' expected gains. But with human nature being what it is, we should ask ourselves whether or not expected gain is the only realistic criterion to consider. It is possible that some target companies may accept a somewhat lower expectation to get a higher conditional gain (conditional on successful agreement), or even consider a combination of maximising expected gains *and* conditional gains (i.e. a bi-criterion game). For the purposes of this study we will limit our analysis to the assumption that the players are expected value maximisers and base their strategy decisions solely on this. Thus an objective of merely attempting to earn greater expected gains than the other party would not be expected from a rational player in the merger

¹ The acquirer's expected gain exceeded the target's expected gain for 44 of the 56 strategy pairs under consideration here.

conflict: it is unlikely that the two parties are in total conflict, and their primary objective would more likely be to maximise their own *individual* expected gain. A player's secondary objective might take into account the other party's expected gain, but not at the expense of reducing its own expected gain by too much.

(3) In an attempt to understand the trends and tendencies in the expected gains to the players as their strategies vary, it is useful to examine the trends and tendencies of their two major component parts, i.e. the average transaction value in successful agreements (Ave) and the probability that merger agreement will be reached. The average transaction value in successful agreements ranges from 0.4196 to 0.8384, and lies above 0.5 for almost 93% of all strategy pairs considered. This result implies that if both sides were to choose their strategies randomly from those here, in the case of a successful merger the target more often than not would achieve a greater share of the expected merger gains than the acquirer would.¹ The probability that merger agreement will eventually be reached ranges from 0.1868 to 0.9006.

In general the more the target capitulates at each stage, the less the average transaction value in successful agreements tends to be, but the greater the chance of agreement being reached, all other things being equal. Also, the greater the probability that the acquirer terminates the offer stream at any stage, the greater the average transaction value in successful mergers, but the less the chances of successful agreement are, providing that all other strategy parameters are held constant. The rates at which these variables change, however, vary with the individual strategic concession parameters employed by both players. Furthermore, as the target's initial concession parameter increases, the average transaction value increases whilst the probability of successful

¹ This appears to be in line with the empirical findings of Malatesta (1983) who reported that target companies earned on average approximately 58% (i.e. more than half) of the gains in equity value in successful mergers, with a standard deviation, *s*, of 10%. Halpern (1973), however, found that targets gained on average only 46% of the total gain in equity value (*s* = 14%), whilst Chatterjee (1986) and Smale (1986) found that they gained 40% (*s* = 4%) and 39% (*s* = 2%) respectively. All of these studies show, however, that the target's proportion of equity gain is very variable. An interpretation of these empirical results *viz a vis* our simulated results is that the Malatesta study indicates that strategies tend to be chosen at *random* from the overall strategy sets we consider.

agreement decreases. (These trends are, however, more clearly observed by examining the matrix in Appendix 5.B).

From the above it will be realised that the trends in E(gain to target) and E(gain to acquirer) depend on the players' strategies in a complex manner, and generalisations about such trends can not easily be made.

(4) The standard deviation of the transaction value in the case of successful merger agreements is fairly large in all cases, ranging from a low of 0.0996 to a high of 0.2221, indicating that the actual transaction value achieved may differ markedly from one negotiation to another under identical conditions and strategies. This result is attributable to the structure of the model, which may be seen as representing the way that different individual negotiating parties deal with the various uncertainties during the course of negotiations.

A further observation that can be made from the full bimatrix of expected gains in Appendix 5.A (but not from the submatrix in Table 5.1) is that an acquirer strategy of making an opening offer and then remaining firm (i.e. $\beta_{A1} = \beta_{AL}$; all future offers are effectively at exactly the same level as the opening offer) is *always* inferior (in terms of the expected gains to both parties) to a strategy involving at least some concessions after the opening offer, no matter what strategy the target plays. In practice this so-called Boulware strategy of forcing the target to make all the concessions would more often than not antagonise the target (Raiffa (1982) p. 48); whilst we have not explicitly modelled this kind of target response, Appendix 5.B shows that for these acquirer strategies the probability of a successful agreement is always considerably lower than if some concessions are made.

How closely do the characteristics investigated above on the 7×8 submatrix of strategies compare to those measured on the full 20×60 bimatrix in Appendices 5.A and 5.B? Table 5.4 contains the relevant values for both matrices. A glance at Table 5.4 shows that for the characteristics considered here there is very little difference between

Table 5.4. A comparison of characteristics measured on both the full matrix of expected gains and on the 7×8 submatrix

Characteristic	7×8 submatrix (Tables 5.1 and 5.3)	Full 20×60 matrix (Appendices 5.A and 5.B)
target: min E(gain)	0.1414 $\bar{A}(0.1; 0.5, 0.7)$ $\bar{B}(0.9; 0.35)$	0.0741 $\bar{A}(0.1; 0.1, 0.5)$ $\bar{B}(0.9; 0.05)$
max E(gain)	0.6288 $\bar{A}(0.9; 0.9, 0.5)$ $\bar{B}(0.1; 0.95)$	0.6348 $\bar{A}(0.9; 0.9, 0.9)$ $\bar{B}(0.1; 0.95)$
acquirer: min E(gain)	0.1066 $\bar{A}(0.3; 0.3, 0.9)$ $\bar{B}(0.9; 0.35)$	0.0633 $\bar{A}(0.1; 0.1, 0.5)$ $\bar{B}(0.9; 0.05)$
max E(gain)	0.5227 $\bar{A}(0.1; 0.3, 0.3)$ $\bar{B}(0.1; 0.95)$	0.5235 $\bar{A}(0.1; 0.5, 0.3)$ $\bar{B}(0.1; 0.95)$
Σ E(gains): min	0.1429	0.1374
max	0.9450	0.9660
E(gain to target) > E(gain to acquirer)	$\frac{44}{56} = 78.6\%$ of pairs	$\frac{918}{1200} = 76.5\%$ of pairs
Ave: min	0.4196	0.4195
max	0.8384	0.8431
> 0.5	$\frac{52}{56} = 92.9\%$ of pairs	$\frac{1122}{1200} = 93.5\%$ of pairs
Prob: min	0.1868	0.0954
max	0.9006	0.9054
Standard deviation:		
min	0.0996	0.0800
max	0.2221	0.2359

the full matrix and its sampled submatrix, the only substantial differences occurring in the minimum possible expected gains to the two players and in the minimum probability of a merger (which are not altogether vital in strategy decision-making). It thus appears satisfactory to draw conclusions about various aspects of the merger bargaining process on the basis of a (carefully sampled) relatively sparse submatrix of the much larger bimatrix representation of expected gains. This adds further justification to our initial use of a fairly coarse grid over which to evaluate the expected gains, since the 20×60 bimatrix shown in Appendix 5.A is in itself merely a finite subset of the continuous set of double response surfaces in five-dimensional unit space, and we thus conjecture that, in the same way, it will offer a good approximation to this continuous set. For future use as a decision aid, careful sampling of a finite subset of acquirer and target strategies thus serves at least three purposes:

- (i) it affords a much more manageable matrix of expected gains by reducing the sheer size of the matrix, and
- (ii) it vastly reduces computational resources,

whilst still

- (iii) offering a good approximation to the general structure of the continuous merger bargaining problem.

§5.3.2 IDENTIFICATION OF POSSIBLE OPTIMAL STRATEGIES

We now turn to an examination of the bimatrix representation of this non-strictly competitive (i.e. non-zero sum) merger game in an attempt to identify strategies which might prove Pareto-optimal for each of the parties. By the "optimal" strategies we mean here the strategy (out of all of those considered) that each party would play to maximise its expected gain *in the full knowledge that the other party is doing the same*.

We will assume for the moment that both parties are able to generate the game matrix for the discrete strategies in Table 5.1. We call this matrix of expected gains G_0' , with

$$A' = \{a_2, a_{13}, a_{24}, a_{30}, a_{33}, a_{42}, a_{51}, a_{59}\}$$

the set of the acquirer's possible strategies, and

$$B' = \{b_1, b_4, b_7, b_{10}, b_{13}, b_{16}, b_{19}\}$$

the set of the target's possible strategies. In the previous section we surmised that we can approximate a continuous mixture of the strategies by a finite submatrix (together with a continuity assumption); we will thus assume that both parties know that they will restrict their attention to the discrete strategy sets A' and B' in G_0' . Now certain of each party's strategies may be **dominated** by others. The acquirer's dominated strategies in A' are those strategies $a_j \in A'$ for which there exists at least one other strategy $a_i \in A'$ such that $E(\text{gain to acquirer})$ is never worse under a_i than under a_j , no matter what strategy $b_k \in B'$ the target chooses to play, and is better for at least one target strategy; the target's dominated strategies $b_j \in B'$ are defined analogously. Thus in actual fact the acquirer need only consider the subset $A'_{(1)}$ of A' , where $A'_{(1)}$ consists of the set of its undominated strategies. The set $A'_{(1)} \subset A'$ is called a **minimal complete class** of strategies of A' relative to B' (Luce and Raiffa (1957), p. 108). Similarly, the target need only consider a minimal complete class of strategies $B'_{(1)} \subset B'$ relative to A' without any loss to itself. In general this will effect a reduction of the number of strategies under consideration. Thus if $(A', B': G_0')$ denotes the non-cooperative merger game with strategy sets A' and B' and expected gains to the parties given in G_0' , then the **associated reduced game** is defined to be $(A'_{(1)}, B'_{(1)}: G_1')$ where G_1' is the submatrix of G_0' formed by eliminating the rows and columns of G_0' not in $A'_{(1)}$ and $B'_{(1)}$. In other words, it is the same game except that the players confine themselves to their minimal complete classes. Close scrutiny of Table 5.1 shows that, based on the simulation, the acquirer would be prepared to eliminate 6 of the initial possible strategies in A' , and the target 5 of its strategies, leaving the minimal complete classes of strategies (relative to A' and B') as

$$A'_{(1)} = \{a_2, a_{24}\}$$

$$B'_{(1)} = \{b_1, b_{16}\}.$$

The associated reduced game has G_1' as shown in Table 5.5.

The conventional game-theoretic approach to selecting final strategies for the two parties from G_1' is to persist with the concept of eliminating dominated strategies. Thus starting from the reduced matrix G_1' , we note that since the players have nothing to lose by confining themselves to the strategy sets $A'_{(1)}$ and $B'_{(1)}$, we may be tempted to define

Table 5.5. The associated reduced game matrix, G_1'

		a_2 $\bar{A}(.1; .1, .3)$	a_{24} $\bar{A}(.3; .9, .5)$
b_1	$\bar{B}(.1; .05)$	0.3073	0.4296
		0.3920	0.4168
b_{16}	$\bar{B}(.1; .95)$	0.3431	0.4381
		0.5227	0.4431

the set $A'_{(2)}$ as the minimal complete class for $A'_{(1)}$ relative to $B'_{(1)}$, and the set $B'_{(2)}$ as the minimal complete class for $B'_{(1)}$ relative to $A'_{(1)}$, and to argue that the players should only consider a choice of strategy from $A'_{(2)}$ and $B'_{(2)}$. Since G_1' consists of only two possible strategies for each of the players in the present case, obviously each player can eliminate at most one further strategy here. It is necessary to note that from the target's point of view, choosing from $B'_{(2)}$ is, however, only advisable if it can be assumed that the acquirer will confine itself to $A'_{(1)}$. If the acquirer does *not* restrict its strategy choice to the set $A'_{(1)}$ (which appears irrational, but *is* plausible), then the target may actually suffer a disadvantage by restricting itself to $B'_{(2)}$. A symmetrical argument holds for the acquirer. On the assumption that both players are individually rational, calculating and forward-thinking, we will accept that both players will confine themselves to the game $(A'_{(2)}, B'_{(2)}; G_2')$. For the simulated game, the step from $B'_{(1)}$ to $B'_{(2)}$ eliminates one of the two target strategies (namely b_1); from $A'_{(1)}$ to $A'_{(2)}$ neither strategy is eliminated. The resulting minimal complete classes of strategies relative to $A'_{(1)}$ and $B'_{(1)}$ are thus

$$A'_{(2)} = \{a_2, a_{24}\}$$

$$B'_{(2)} = \{b_{16}\}$$

resulting in the associated reduced game matrix G_2' presented in Table 5.6.

Table 5.6. The associated reduced game matrix G_2'

		a_2 $\bar{A}(.1; .1, .3)$	a_{24} $\bar{A}(.3; .9, .5)$
b_{16}	$\bar{B}(.1; .95)$	0.3431	0.4381
		0.5227	0.4431

The target has effectively located its optimal strategy; the solution to the game depends only on the acquirer's choice between its two strategies in G_2' . Since it clearly would prefer strategy a_2 to a_{24} (since $0.5227 > 0.4431$) the "optimal solution" is the strategy pair $\langle a_2, b_{16} \rangle$. At this combination of strategies the acquirer can expect to gain considerably more of the expected synergy gains than the target; indeed this gain happens to be the acquirer's maximum possible expected gain. The target on the other hand expects to gain no more than 40% of the expected synergy gains, and this constitutes less than 55% of its maximum possible expected gain from the game.

The method of eliminating dominated strategies assumes **hyper-rationality** on the part of both players, i.e. both are able to identify and eliminate their *own* dominated strategies *and* those of their opponent, and continue through several rounds in this way until further reduction is no longer possible. If hyper-rationality of the players is questionable, an alternative means of identifying optimal strategies from G_1' is to follow a Bayesian decision-making approach to the analysis of games (such as that suggested by Kadane and Larkey (1982)). In this approach each player might assign some subjective probability distribution to the opponent's choice of strategies, and then choose the strategy which maximises the expectation of its expected gains. That is, the acquirer would choose its strategy j which maximises $\sum_{i=1}^I p_i E(G_{ij})$ where p_i ($i = 1, \dots, I$) is the acquirer's subjective probability of the target choosing strategy i , and $E(G_{ij})$ is the acquirer's expected gain from playing strategy j when the target plays strategy i . A symmetrical argument holds for the target's choice of strategy. The assignment of subjective probabilities to an opponent's strategy set represents the player's own judgement of the likelihood that each particular opponent strategy will be played, and will be based on the player's own beliefs, cognitive processes and interpretation of information (DeGroot (1975)). On this point, Young (1975) comments that

"... this problem (of assigning subjective probabilities) can be handled by introducing new assumptions (or empirical premises) about such things as the personality traits of the players. But such a course would carry the analyst far outside the basic structure of the theory of games, requiring a

fundamental revision of the basic perspective of game theory." ¹

In the absence of such cognitive information, the simplest subjective probability distribution that can be placed on the opponent's strategy set is a discrete uniform distribution, i.e. one in which each player assumes that the opposing player is equally likely to play each of its possible strategies. In this case the expectation to the acquirer (over all possible strategies that the target might play) of its expected gains for each of its possible strategies is

$$E(a_2) = (0.3920) (0.5) + (0.5227) (0.5) = 0.4574$$

and $E(a_{24}) = 0.4300$.

Similarly, the expectation to the target (over all possible acquirer strategies) of its expected gains for each of its two possible strategies is

$$E(b_1) = (0.3073) (0.5) + (0.4296) (0.5) = 0.3685$$

and $E(b_{16}) = 0.3906$.

(Since b_{16} so obviously dominates b_1 here, it is trivial to conclude that $E(b_{16}) > E(b_1)$).

Thus the acquirer would necessarily choose strategy a_2 , the target would choose strategy b_{16} , and the strategy pair $\langle a_2, b_{16} \rangle$ would be the "solution" to the game, which happens to be the same solution as identified by the more conventional game-theoretic approach. Note that players using the Bayesian approach to strategy choice in games act independently of their opponent, and thus do not necessarily concern themselves with their opponents' expectations or strategy choice.

We now return to the full matrix of expected gains (as in Appendix 5.A), which we will call G_0 , and investigate optimal solution concepts on the game $(A, B; G_0)$.² As before each party may eliminate its dominated strategies and arrive at a minimal complete class of its entire strategy set relative to the other party's entire strategy set. The associated reduced game for G_0 is $(A_{(1)}, B_{(1)}; G_1)$ and is given in Table 5.7.

¹ For a detailed overview of the role of cognitive processes in the assessment of subjective probability distributions, see Hogarth (1975).

² The prime (\prime) notation refers to the submatrix formed by sampling from the players' strategies. Thus $A' \subset A$, $B' \subset B$ and $G_0' \subset G_0$.

Table 5.7. The associated reduced game matrix, G_1

		a_2	a_3	a_4	a_5	a_8	a_9	a_{11}
		$\bar{A}(.1;.3;.3)$	$\bar{A}(.1;.5;.3)$	$\bar{A}(.1;.7;.3)$	$\bar{A}(.1;.9;.3)$	$\bar{A}(.3;.7;.3)$	$\bar{A}(.3;.9;.3)$	$\bar{A}(.5;.7;.3)$
b_1	$\bar{B}(.1;.05)$	0.3073	0.3446	0.3627	0.4006	0.4193	0.4520	0.4652
		0.3920	0.4363	0.4381	0.4523	0.4242	0.4464	0.3971
b_6	$\bar{B}(.1;.35)$	0.3390	0.3645	0.3753	0.4003	0.4406	0.4535	0.4780
		0.4637	0.4934	0.4889	0.4800	0.4643	0.4581	0.4191
b_7	$\bar{B}(.3;.35)$	0.3056	0.3403	0.3787	0.4047	0.4207	0.4484	0.4562
		0.3819	0.4151	0.4341	0.4152	0.4084	0.4108	0.3722
b_8	$\bar{B}(.5;.35)$	0.2787	0.3232	0.3653	0.4154	0.4015	0.4458	0.4138
		0.3281	0.3641	0.3842	0.3836	0.3650	0.3743	0.3358
b_{11}	$\bar{B}(.1;.65)$	0.3474	0.3543	0.3766	0.3881	0.4418	0.4515	0.4914
		0.5096	0.5084	0.5064	0.4880	0.4801	0.4699	0.4412
b_{16}	$\bar{B}(.1;.95)$	0.3431	0.3526	0.3586	0.3866	0.4355	0.4499	0.4881
		0.5227	0.5235	0.5068	0.5013	0.4812	0.4750	0.4505
b_{17}	$\bar{B}(.3;.95)$	0.3328	0.3475	0.3619	0.3924	0.4281	0.4442	0.4803
		0.4987	0.4808	0.4697	0.4558	0.4524	0.4355	0.4293

Proceeding along the lines of the Bayesian decision-making approach, the expectations to the two players (over all possible strategies of the opponent, all assumed equally likely) of their expected gains, for each possible strategy are as in Table 5.8. The best-choice strategy for the acquirer is a_4 , while strategies b_6 and b_{11} both offer the target the same maximum expectation of its expected gains. Thus, strictly speaking, either of the strategy pairs $\langle a_4, b_6 \rangle$ and $\langle a_4, b_{11} \rangle$ would be optimal for Bayesian game players. If, however, the target knew that the acquirer was also approaching the game as a Bayesian player (and thus would choose strategy a_4) it would observe that $\langle a_4, b_{11} \rangle$ **jointly dominates** $\langle a_4, b_6 \rangle$, i.e. the strategy $\langle a_4, b_{11} \rangle$ is preferable for *both* players (assuming that they are expected utility maximisers), and this is then the Bayesian optimal strategy pair for the merger game G_0 .

Examination of the game matrix G_1 reveals that it, too, has dominated target (row) and acquirer (column) strategies when measured relative to the strategy sets $A_{(1)}$ and $B_{(1)}$

Table 5.8. Players' expectations of their expected gains for the reduced game G_1

	Strategy	Expectation
Acquirer:	a_2	0.4424
	a_3	0.4602
	a_4	0.4612
	a_5	0.4537
	a_8	0.4394
	a_9	0.4386
	a_{11}	0.4065
Target:	b_1	0.3931
	b_6	0.4073
	b_7	0.3935
	b_8	0.3777
	b_{11}	0.4073
	b_{16}	0.4021
	b_{17}	0.3982

in G_1 respectively. Assuming that both players are individually rational, we will accept that each will confine its strategy choice to its minimal complete class relative to their opponent's (reduced) strategy set. This process of reducing the strategy sets for each player may continue, i.e. $A_{(n)}$ is the minimal complete class for $A_{(n-1)}$ relative to $B_{(n-1)}$ and $B_{(n)}$ is the minimal complete class for $B_{(n-1)}$ relative to $A_{(n-1)}$, and is a safe strategic move only as long as each player feels confident that the other is doing likewise. If both target and acquirer continue reducing their strategy spaces, the process will eventually terminate, in the sense that there is some integer N such that $A_{(N)} = A_{(N+1)}$ and $B_{(N)} = B_{(N+1)}$, i.e. no further reduction of the strategy sets is possible. When this occurs $A_{(N)}$ and $B_{(N)}$ are called **completely reduced strategy sets** and $(A_{(N)}, B_{(N)}; G_N)$ the **completely reduced game** associated with $(A, B; G_0)$ ¹. In the simulation of the negotiation process

¹ For early game theoretic discussions on the elimination of dominated strategies and thereby a reduction in the game matrix see, for example, Gale, Kuhn and Tucker (1950), Luce and Raiffa (1957) and Shapley (1964). Gilboa, Kalai and Zemel (1990) showed that in the case of finite games, providing that the dominances between the strategies are *strict*, the completely reduced game matrix is unique, regardless of the order of elimination of the dominated strategies. (A strictly dominated strategy for a player A is a strategy a_j for which there exists at least one other strategy a_i such that the payoff to A is always strictly greater under a_i than under a_j no matter what strategy the opposing

here we observe that there is no further reduction of strategies after $(A_{(3)}, B_{(3)}; G_3)$, where

$$A_{(3)} = \{a_2, a_3, a_4, a_5\}$$

$$B_{(3)} = \{b_1, b_6, b_7, b_8, b_{11}\}$$

i.e. $N = 3$, and G_3 (shown in Table 5.9) is the completely reduced game matrix.

Table 5.9. The completely reduced game matrix, G_3

		a_2	a_3	a_4	a_5
		$\bar{A}(.1;.3,.3)$	$\bar{A}(.1;.5,.3)$	$\bar{A}(.1;.7,.3)$	$\bar{A}(.1;.9,.3)$
b_1	$\bar{B}(.1;.05)$	0.3073	0.3446	0.3627	0.4006
		0.3920	0.4363	0.4381	0.4523
b_6	$\bar{B}(.1;.35)$	0.3390	0.3645	0.3753	0.4003
		0.4637	0.4934	0.4889	0.4800
b_7	$\bar{B}(.3;.35)$	0.3056	0.3403	0.3787	0.4047
		0.3819	0.4151	0.4341	0.4152
b_8	$\bar{B}(.5;.35)$	0.2787	0.3232	0.3653	0.4154
		0.3281	0.3641	0.3842	0.3836
b_{11}	$\bar{B}(.1;.65)$	0.3474	0.3543	0.3766	0.3881
		0.5096	0.5084	0.5064	0.4880

The acquirer's set $A_{(3)}$ corresponds to an initial offer as low as is reasonable to the acquirer (in any event well below the acquirer's perceived fair price for the target), but to be prepared to persistently negotiate upwards to varying final offer levels, the largest of which is as high as the acquirer would find reasonable. The target's set $B_{(3)}$ contains a variety of strategies, including to start with an initial acceptance level as low as is reasonable to the target and then give up ground at varying rates, or to start with an acceptance level which is closer to the target's perception of a fair price and give up ground fairly slowly.

player chooses to play). This condition holds for our game $(A, B; G_0)$. Furthermore, Kohlberg and Mertens (1982) have studied equilibrium solutions to the game that are invariant under successive elimination of dominated strategies.

The parties may now establish if one or more **equilibrium points** exist on the completely reduced game¹. An equilibrium point in this context is a pair of strategies $\langle a_i, b_j \rangle$, $a_i \in A_{(N)}$, $b_j \in B_{(N)}$ such that

- (1) E(gain to target) for acquirer strategy $a_i \in A_{(N)}$ is maximised over all given target strategies, and
- (2) E(gain to acquirer) for target strategy $b_j \in B_{(N)}$ is maximised over all given acquirer strategies.

For any strategy pair satisfying conditions (1) and (2) above, this acquirer strategy is better than any other acquirer strategy for the given target strategy, and analogously for the target strategy. The simulation of the negotiation process produces three such equilibrium points in the discrete strategies on G_3 , given in Table 5.10.

Table 5.10. Equilibrium points in the completely reduced game matrix, G_3

	Equilibrium points		
	1	2	3
Acquirer strategy	$a_2: \bar{A}(.1;.3,.3)$	$a_3: \bar{A}(.1;.5,.3)$	$a_4: \bar{A}(.1;.7,.3)$
Target strategy	$b_{11}: \bar{B}(.1;.65)$	$b_6: \bar{B}(.1;.35)$	$b_7: \bar{B}(.3;.35)$
E(gain to acquirer)	0.5096	0.4934	0.4341
E(gain to target)	0.3474	0.3645	0.3787

We note that the target's preference ordering increases with a move from equilibrium point 1 towards point 3 (a more preferred outcome has a higher expected gain), whilst the acquirer's preference ordering increases from point 3 towards point 1. Thus whilst the progression from $(A, B: G_0)$ to $(A_{(3)}, B_{(3)}: G_3)$ has rationally focused the players' strategy choice, it has not uniquely solved the problem. Any solution concept to the merger game should provide the two competing players with expectations of their

¹ Nash (1951) showed that any non-cooperative game with finite sets of discrete strategies (as we have here) has at least one mixed strategy equilibrium pair. For the present discussion we are restricting ourselves to an examination of the equilibrium points amongst pure (discrete) strategies.

gains from negotiating under the solution strategy pair, and an examination of matrix G_3 indicates more than one such solution.

Since both parties are able to generate the matrix G_3 and are acting rationally, any attempt to maximise their own expected gains should of necessity consider the other party's expected actions as well. Thus the players will at once realise that any strategy pair (a_i, b_j) in G_3 for which the expected gains are inferior for *both* parties compared to those of some other attainable strategy pair in G_3 are **jointly inadmissible**. In G_3 only the outcomes pertaining to acquirer strategy a_5 (for all target strategies) and target strategy b_{11} (for all acquirer strategies) are jointly admissible, and equilibrium point 1 is the only equilibrium point in this set of strategy pairs $\{<a_5, b_j>\} \cup \{<a_i, b_{11}>\}$. This appears to suggest that this point could be the solution to the game. For this strategy pair the sum of the expected gains is 0.8570, which is close to the maximum total expected gains (i.e. the players appear to be exhibiting a large degree of group rationality), and thus not a lot of the value which may possibly be created from merger will be left on the table. A way of choosing amongst multiple equilibrium points is to compare them to obvious status quo points. In this respect we could consider as status quo points the origin (0,0) (i.e. the negotiations are broken off entirely), and (0.3474, 0.3842) (i.e. the player's **security levels** in pure strategies, which is the expected gain which each player can guarantee to itself from playing the game). In both cases equilibrium point 1 proves superior to both of the others.

Equilibrium point 2, although jointly inadmissible, appears to offer a good alternative solution to point 1, in the sense that the relative loss in expected gains to the acquirer in moving from point 1 to point 2 (3.18%) is less than the relative gain in expected gains to the target (4.92%), i.e. the joint "cost" of the move is less than the joint "benefit". (The same can not be said of equilibrium point 3: the relative loss to the acquirer in moving from equilibrium point 1 (14.82%) is considerably greater than the relative gain to the target (9.01%)). Furthermore equilibrium point 2 offers larger maximum total expected gains (0.8579).

Whilst we have suggested that equilibrium point 1 could be the solution to the game, the existence of multiple equilibrium points might cause difficulties in the sense of creating a conflict of interest amongst rational negotiators. The rational acquirer would play strategy a_2 , and the rational target would play strategy b_7 , leading to an expected gain to the target of 0.3056 and to the acquirer of 0.3819, i.e. both expect to be very much worse off than at any of the three equilibrium points! Thus being strictly competitive in the presence of multiple equilibria appears to be a distinct disadvantage here¹. Noting that this is inevitable, one or both of the players may prefer to play the strategy that will lead to the other's preferred equilibrium point (under the assumption, of course, that the other remains a rational utility maximiser). Note that this does not imply pre-negotiation communication and agreement between the players: one or both merely deviate from their strictly competitive strategies to ensure a larger expected gain to themselves, given the knowledge of the game matrix G_3 and thus their opponent's preferred equilibrium strategy. Parties which alter their strategies in this way are thus in a sense acting in a **cooperative** manner. Thus if *both* play their cooperative strategies, the acquirer will play its best strategy against b_7 , i.e. a_4 , and the target will play its best strategy against a_2 , i.e. b_{11} . This joint choice leads to an expected gain to the target of 0.3766 (only 0.55% below the target's expected gain at its preferred equilibrium point $\langle a_4, b_7 \rangle$) and to the acquirer of 0.5064 (only 0.63% below the acquirer's expected gain at its preferred equilibrium point $\langle a_2, b_{11} \rangle$). Since both players give up very small proportions of what they could expect to achieve at their own preferred equilibrium points, the cooperative strategy pair $\langle a_4, b_{11} \rangle$ appears to offer a very satisfactory and safe compromise to both negotiating parties. This point is furthermore an element of the jointly admissible set.

At $\langle a_4, b_{11} \rangle$ either party can improve its own expected gain slightly by reverting back to its utility-maximising strategy (the "double-cross" strategy). If only one party does so the game reverts to one of the equilibrium points; if both do so the game reverts to $\langle a_2, b_7 \rangle$ which is extremely disagreeable to both. This threat alone would be inclined

¹ This situation is similar, but not identical, to the "Battle of the Sexes" described by Luce and Raiffa (1957), p. 90-94. Our jointly cooperative strategy produces payoffs that are much more favourable to both players than the traditional Battle of the Sexes does.

to ensure that both players play their cooperative strategies. Furthermore, each player notices that by unilaterally playing its own cooperative strategy it can improve its own expected gain, as well as that of the other player, quite considerably relative to the expected gains achievable with its strictly competitive strategy, and hence the strategy of both players playing their cooperative strategies appears a likely choice. Thus whilst game theory dictates that it would be folly to deviate from a single equilibrium strategy pair, the existence of multiple equilibrium strategies here produces a situation in which the strictly cooperative strategies are superior to the strictly competitive strategies.

The negotiators are furthermore frustrated by a **Prisoner's Dilemma**-type problem: examination of Tables 5.9 and 5.1 shows that each of the three equilibrium points is jointly dominated by at least one point occurring in a strictly dominated strategy of at least one of the players (i.e. a point in G_0 outside of G_3). To illustrate, we observe that equilibrium point 1 is jointly dominated by $\langle a_3, b_{16} \rangle$ and equilibrium points 2 and 3 are jointly dominated by $\langle a_5, b_{16} \rangle$, where b_{16} is a dominated target strategy. Furthermore equilibrium point 3 is also jointly dominated by a large number of the discrete strategy pairs, one of which is $\langle a_9, b_{16} \rangle$ which consists of strictly dominated strategies for *both* players. Since one or both of the negotiating parties will rationally not even consider the relevant strategy in each of the above cases, the rational-conforming players will miss the opportunity of taking advantage of these improved expected gains. Thus the ideal strategy choice remains somewhat elusive since there exist strategy pairs both inside and outside of G_3 offering greater expected gains to both players than do any of the equilibrium points¹, although no one strategy pair jointly dominates all the others. This indicates a temptation for the players to act "irrationally" and to move away from the equilibrium points; if the negotiations were held in a fully cooperative environment (i.e. in which pre-negotiation communication between the players was allowed) and supposing that the players agree that the payoff matrix G_0 remains invariant, it is possible (and indeed likely) that some non-equilibrium strategy pair would be jointly decided upon. This attraction to deviate from "optimality" (i.e. a solution given by an equilibrium point)

¹ We have noted that the joint cooperative strategy pair $\langle a_4, b_{11} \rangle$ in G_3 , whilst being "irrational" in terms of strict game theoretic mores, does jointly dominate equilibrium points 2 and 3, and hence warrants the attention of the negotiating parties.

might help to explain why several empirical studies (for example, those of Halpern (1973), Malatesta (1983), Chatterjee (1986) and Smale (1986)) have produced fairly large standard deviations in the estimation of the percentage split of the equity gains accruing to the target company.

In reality the equilibrium points and the cooperative strategy pair $\langle a_4, b_{11} \rangle$ offer very similar expected gains to the two players, although the strategies used by the players vary considerably from one point to another. In all these cases, however, the acquirer comes off best, both in absolute terms and relative to the parties' maximum expected gains. Table 5.11 shows the expected gains to the parties at all three equilibrium points and at the cooperative strategy pair.

Table 5.11. Comparison of selected strategies

	Strategy choice			
	$\langle a_2, b_{11} \rangle$	$\langle a_3, b_6 \rangle$	$\langle a_4, b_7 \rangle$	$\langle a_4, b_{11} \rangle$
E(gain to acquirer)	0.5096	0.4934	0.4341	0.5064
% of max E(gain to acquirer)	97.3	94.3	82.9	96.7
E(gain to target)	0.3478	0.3645	0.3787	0.3766
% of max E(gain to target)	54.8	57.4	59.7	59.3

Clearly the acquirer can expect to achieve a considerably greater share of the expected merger gains than can the target, and this amount is very close to its maximum expected gain at these strategy pairs, whilst the target can expect to gain little over half of its maximum expected gain at these strategy pairs. In absolute terms the target can expect to achieve around 40% - 45% of the expected combined merger gains, a figure which correlates well with the empirical observations of Halpern (1973), Chatterjee (1986) and Smale (1986). We conjecture that this was no chance event, but is instead directly attributable to the bargaining strategy employed by the negotiators involved. The implication is that the samples of acquiring and target companies selected in the above

empirical studies were, in the mean, all acting close to their "optimal" bargaining strategies. Halpern (1973) recognised this when he concluded

"...the (choice of) bargaining strategies will determine the split of the gains."

§5.4 A COMPARISON OF THE PARETO-OPTIMAL STRATEGIES ON THE FULL MATRIX G_0 AND THE SUBMATRIX G_0'

Table 5.12 consolidates all the optimal strategy pairs previously derived for both the Bayesian decision-making approach (BDM) and the more traditional elimination of dominated strategies (EDS) on the full matrix G_0 and its submatrix G_0' . From Table 5.12 we observe that no single strategy for either the acquirer or the target predominates. Amongst the eight strategy pairs there are three different acquirer strategies which might be considered as "optimal" (a_2, a_3, a_4). These collectively correspond to the scenario of an acquirer making a very low initial offer ($\beta_{A1} = 0.1$), and in the event of a rejection negotiating fairly persistently ($p = 0.3$) up to a final offer which varies from somewhat below the acquirer's perception of a fair price for the target up to a value somewhat above this fair price perception ($\beta_{AL} = 0.3, 0.5$ or 0.7). The four different target strategies which might be considered in a sense optimal (b_6, b_7, b_{11} and b_{16}) all correspond to the scenario of the target setting an initial acceptance level far below its perception of a fair price ($\beta_B = 0.1$ or 0.3), and then decreasing this demand towards the acquirer's price offer in various decrements ($\gamma = 0.35, 0.65$ or 0.95). The rational acquirer's strategy choice is thus limited to a choice of value for the final strategic concession parameter, β_{AL} , and its optimal bargaining strategy can be represented as

$$\bar{A}(0.1; \beta_{AL}, 0.3) \quad , \quad \beta_{AL} = 0.3, 0.5, 0.7 .$$

Clearly then, a good acquirer strategy is characterised by an extreme initial position, followed by a willingness to engage in vigorous upward negotiation. In similar spirit, the rational target's optimal bargaining strategy can be represented as

$$\bar{B}(0.1; \gamma) \quad , \quad \gamma = 0.35, 0.65, 0.95$$

and

$$\bar{B}(0.3; 0.35) .$$

Table 5.12. Possible optimal strategy pairs

Starting Matrix	Solutions on the completely reduced matrix (EDS) (E(gain to acquirer) ; E(gain to target))	Solutions using the BDM approach (E(gain to acquirer) ; E(gain to target))
G_0'	$\langle a_2, b_{16} \rangle$ (0.5227 ; 0.3431)	$\langle a_2, b_{16} \rangle$ (0.5227 ; 0.3431)
G_0	$\langle a_2, b_{11} \rangle$ (0.5096 ; 0.3474) (Equilibrium point 1)	$\langle a_4, b_{11} \rangle$ (0.5064 ; 0.3766)
	$\langle a_3, b_6 \rangle$ (0.4934 ; 0.3645) (Equilibrium point 2)	$\langle a_4, b_6 \rangle$ (0.4889 ; 0.3753)
	$\langle a_4, b_7 \rangle$ (0.4341 ; 0.3787) (Equilibrium point 3)	
	$\langle a_4, b_{11} \rangle$ (0.5064 ; 0.3766) (Cooperative solution)	

The submatrix of expected gains G_0' produced a single optimal strategy pair for both the BDM and EDS approaches to the analysis. In sharp contrast the solution to the full game matrix G_0 was somewhat more complex: the BDM approach yielded two solutions and EDS provided no less than three equilibrium points, as well as a possible cooperative solution (which was the same as one of the BDM solutions). We conclude from this observation that the finer the grid of strategies in the original matrix of expected gains, the richer the matrix is, and the more complex it is for decision-makers

and negotiators in terms of locating optimal bargaining strategies.¹ An analogy can be drawn with two sieves of differing mesh-widths: the sieve with coarser mesh (cf. G_0') allows all except a small number of large particles to pass through, whilst that with the finer mesh (cf. G_0) captures a large number of particles, both large and not so large. In effect the five-dimensional double response surface of expected gains has a large number of local maxima, and the finer the strategy-grid, the more of these local maxima are "caught" or identified as "optimal". We extrapolate this to argue that the set of expected gains defined on the five-dimensional product space of *continuous* strategies (of which G_0 is a finite subset) would have a much larger number of equilibrium points (and other points which might be construed as "optimal"), which of necessity adds more confusion and fuzziness to the decision of strategy choice. Clearly then from a decision-support point of view, to prove useful, a model such as the one used in this chapter should comprise of finite strategy sets for each of the negotiating players which are "small enough" to produce a manageable number of possible solutions, yet large enough to give a fairly wide coverage to all parameters and hence contain the vital information as regards the general structure of the merger bargaining problem. It appears that a matrix with dimensions approximately the same as G_0' might not be too far off the mark.

Before proceeding further we should ask ourselves whether the optimal solutions identified on G_0 and on G_0' are, from a negotiator's point of view, discernably different from one another at all. In the first place, the model parameters are merely abstractions of reality, useful for modelling purposes, and as such have little or no physical meaning to the negotiators. The parameters as they stand are certainly undocumented and intangible, to the extent that it is unlikely that a negotiator would be able to positively differentiate between two fairly closely-spaced values of each of the parameters. For example, a negotiating party asked to allocate a value to its initial strategic concession parameter, β_{A1} , would almost certainly be able to respond that it feels that this parameter has, say, a "low" value, but would have little grasp or "feeling" for the difference between $\beta_{A1} = 0.1$ and $\beta_{A1} = 0.2$. For the purposes of selecting strategies, it is

¹ This increasing complexity of the matrix as the strategy-grid is made finer is not restricted to the quantity of optimal strategy pairs. Other game-theoretic phenomena, such as Prisoner's Dilemma and a cooperative bargaining solution were evident in G_0 but not in G_0' , and they, too, further serve to add confusion to the strategy choice decision.

unreasonable to differentiate between players' expected gains that differ by very small amounts when there exists such a large degree of fuzziness in the estimation of parameter values in the first place. From Table 5.12 we note that for the optimal points considered here, the maximum difference in expected gain for the target is only 0.0356 and for the acquirer it is 0.0338 if equilibrium point 3 is ignored (if not, it is 0.0886). Further to this we have pointed out that the "optimal" strategies for each player are all fairly similar, in the sense that only one or two of the parameters vary within a fairly narrow range (i.e. they are all physically fairly close to one another in matrix G_0). In the light of the above we advocate that, for the purposes of selecting an "optimal" strategy from amongst those proposed here, there is little to choose between the strategies at hand. We contend that all offer sound guidance to negotiating parties.

Throughout the above discussion we should not lose sight of the fact that G_0' is merely a subset of G_0 . The strategies $A' \subset A$ and $B' \subset B$ were selected using statistical sampling methods to achieve an even spread of selected strategies across the rows and columns of G_0 , and it was for this reason that the general characteristics of G_0' were very similar to those of G_0 (see Table 5.4). Moreover, the statistical sampling of strategies ensured that the "optimal" strategies for the two parties were similar on G_0 and G_0' , and offered similar expected gains to each player. If the sampling had *not* ensured an even spread of A' and B' across A and B respectively, it is probable (and indeed likely) that the general characteristics and optimal solution(s) would not have resembled those of G_0 as closely as those of G_0' did. The implication of this could be far-reaching, as we illustrate by means of an example. Assume that Player A chooses to base its analysis and strategy-choice on the game matrix $G_{0(A)}^*$ of expected gains, with discrete strategy sets $A_{(A)}^* \subset A$ and $B_{(A)}^* \subset B$, on the assumption that Player B is doing exactly the same. Player B, ignorant of Player A's choice of $G_{0(A)}^*$, may choose some differently-selected game submatrix $G_{0(B)}^*$ of G_0 as the basis for strategy choice, with $A_{(B)}^* \subset A$ and $B_{(B)}^* \subset B$, and (incorrectly) assume that Player A will also employ $G_{0(B)}^*$ in determining its choice of strategy. Thus the two parties will, in effect, be playing different games, i.e. the assumption of players' "complete information" is clearly violated in this case. This is furthermore not a problem of decision-making under uncertainty either, in the sense of the participants being aware that their information is incomplete. The situation in which the two players have different perceptions of the game is known

as a **two-player hypergame**, which is characterised by each player's perceptions of (1) its own and its opponent's finite strategy sets, and (2) its own and its opponent's utilities defined over the product space of players' strategies (in the merger game the utilities are merely the expected gains to the two players). Hypergames are discussed by Bennett (1977 and 1980), and have been used extensively in the literature in an attempt to explain why certain military confrontations and other conflict situations (such as business and environmental conflicts) have produced surprising and unexpected outcomes (to one party, at least).¹ In the illustration at hand, Player A might, on the basis of game $(A_{(A)}^*, B_{(A)}^* : G_{0(A)}^*)$, arrive at an optimal strategy pair $\langle a_i, b_j \rangle$. This is A's perception of what the two players should do. Likewise, Player B may locate its perceived optimal strategy pair $\langle a_p, b_q \rangle$ on the basis of the game $(A_{(B)}^*, B_{(B)}^* : G_{0(B)}^*)$, where $i \neq p$ and $j \neq q$. Thus A would go ahead and play a_i , and B would play b_q . The expected gains to the players at $\langle a_i, b_q \rangle$, however, may leave both players far worse off than if they had settled on either $\langle a_i, b_j \rangle$ or $\langle a_p, b_q \rangle$! A numerical example of this using G_0 as starting point can be found in Appendix 5.C. On the other hand, if one player can anticipate that the other has different perceptions of the game being played, and can guess which strategies the opponent is basing his analysis on, the player might well be able to turn this situation to its strategic advantage.

§5.5 CONCLUSIONS AND IMPLICATIONS FOR DECISION SUPPORT

The model described in Chapter 4 and implemented here yielded several interesting findings which might have implications for decision-support.

Firstly, the use of a fairly coarse grid of strategy parameter values in the matrix of players' expected gains appears an adequate approximation of the game with players' continuous strategy sets. As such it is suitable as an aid to merger negotiators who require an understanding of the general structure of the expected gains to the players for various strategies they might choose to play. This also drastically limits computational

¹ For military applications see Bennett and Dando (1979), Said and Hartley (1982), Shupe, Wright, Hipel and Fraser (1980), Takahashi, Fraser and Hipel (1984), Hipel, Wang and Fraser (1988), Fraser, Wang and Hipel (1990) and Geldenhuys (1991). For business applications see Giesen and Bennett (1979), Bennett, Huxham and Dando (1981) and Stokes and Hipel (1983).

intensity, and approximates the main features of the continuous game by affording a matrix of more manageable dimensions. Assimilation by decision-makers will be simplified and human judgement will be clarified. Furthermore, an expected gains matrix with a coarse strategy-grid also appears to be suitable for choosing Pareto-optimal strategies, since it identifies only a very small number of "optimal" strategy pairs (possibly just one), which are not discernably different from those identified using a finer strategy-grid. This would greatly simplify a decision-maker's task of choosing a negotiating strategy. Care should, however, be taken to ensure that the players' strategy sets are chosen in such a way that an even spread of strategies across all those possible is achieved.

In Chapter 6 we will continue the simulations, focusing our investigations now on the effect of changes in the contextual parameters.

APPENDIX 5.A. Simulated expected gains to the target (above) and to the acquirer (below) for the full 20 × 60 matrix of strategies

Acquirer's strategy parameters

Target's strategy parameters		Acquirer's strategy parameters															P β _{AI} β _{AL}	
		0.1					0.3				0.5			0.7		0.9		
		0.1		0.3		0.5		0.7		0.9		0.5		0.7		0.9		
		0.1	0.3	0.5	0.7	0.9	0.3	0.5	0.7	0.9	0.5	0.7	0.9	0.7	0.9	0.9		
γ	β _B	0.05	0.1	0.2623	0.3073	0.3446	0.3627	0.4006	0.3748	0.3978	0.4193	0.4520	0.4473	0.4652	0.5000	0.5230	0.5447	0.6211
				0.3222	0.3920	0.4363	0.4361	0.4523	0.3692	0.4043	0.4242	0.4464	0.3729	0.3971	0.4181	0.3594	0.3794	0.3146
		0.3	0.1	0.2114	0.2619	0.3098	0.3417	0.3988	0.3144	0.3547	0.3937	0.4354	0.3928	0.4261	0.4732	0.4825	0.5208	0.5971
				0.2304	0.2950	0.3404	0.3564	0.3792	0.2823	0.3255	0.3574	0.3782	0.3009	0.3341	0.3631	0.3109	0.3409	0.2875
		0.5	0.1	0.1655	0.2242	0.2682	0.3205	0.3789	0.2655	0.3101	0.3538	0.4180	0.3496	0.3835	0.4521	0.4308	0.4814	0.5558
	0.1684			0.2319	0.2850	0.3030	0.3165	0.2275	0.2672	0.3001	0.3306	0.2537	0.2851	0.3241	0.2857	0.3006	0.2593	
	0.7	0.1	0.1264	0.1796	0.2336	0.2816	0.3607	0.2055	0.2592	0.3198	0.3893	0.2964	0.3287	0.4114	0.3817	0.4453	0.5066	
			0.1203	0.1731	0.2142	0.2397	0.2656	0.1622	0.2083	0.2492	0.2790	0.2025	0.2269	0.2735	0.2243	0.2620	0.2297	
	0.9	0.1	0.0802	0.1238	0.1674	0.2153	0.3208	0.1450	0.1779	0.2322	0.3315	0.2172	0.2614	0.3362	0.2832	0.3732	0.4249	
			0.0684	0.1053	0.1341	0.1574	0.2000	0.1052	0.1299	0.1623	0.2048	0.1368	0.1669	0.1991	0.1543	0.2019	0.1817	
	0.35	0.1	0.2666	0.3380	0.3845	0.3753	0.4003	0.3793	0.4197	0.4406	0.4535	0.4573	0.4780	0.5045	0.5291	0.5568	0.6283	
			0.3257	0.4637	0.4934	0.4889	0.4800	0.3709	0.4424	0.4643	0.4561	0.3767	0.4181	0.4350	0.3632	0.3969	0.3150	
		0.3	0.1	0.2134	0.3056	0.3403	0.3787	0.4047	0.3118	0.3812	0.4207	0.4464	0.3999	0.4562	0.4932	0.4801	0.5338	0.5952
				0.2343	0.3619	0.4151	0.4341	0.4152	0.2768	0.3753	0.4084	0.4108	0.3070	0.3722	0.3945	0.3090	0.3610	0.2886
		0.5	0.1	0.1663	0.2787	0.3232	0.3653	0.4154	0.2633	0.3578	0.4015	0.4468	0.3501	0.4318	0.4806	0.4410	0.5249	0.5807
	0.1728			0.3281	0.3641	0.3642	0.3836	0.2213	0.3294	0.3650	0.3743	0.2531	0.3358	0.3597	0.2899	0.3371	0.2926	
	0.7	0.1	0.1281	0.2399	0.3114	0.3550	0.4087	0.2169	0.3194	0.3714	0.4428	0.2954	0.3920	0.4591	0.3773	0.4929	0.5092	
			0.1195	0.2646	0.3245	0.3446	0.3427	0.1718	0.2775	0.3175	0.3397	0.2019	0.2891	0.3228	0.2224	0.3038	0.2280	
	0.9	0.1	0.0771	0.1944	0.2662	0.3228	0.4000	0.1403	0.2519	0.3282	0.4200	0.2042	0.3285	0.4299	0.2908	0.4441	0.4118	
			0.0658	0.1699	0.2546	0.2825	0.2923	0.1014	0.2040	0.2533	0.2851	0.1294	0.2249	0.2740	0.1561	0.2531	0.1773	
0.65	0.1	0.2688	0.3474	0.3843	0.3766	0.3881	0.3685	0.4324	0.4418	0.4515	0.4480	0.4914	0.5017	0.5235	0.5577	0.6260		
		0.3325	0.5096	0.5084	0.5064	0.4680	0.3643	0.4771	0.4801	0.4669	0.3659	0.4412	0.4423	0.3600	0.4048	0.3161		
	0.3	0.1	0.2200	0.3298	0.3566	0.3643	0.3987	0.3143	0.4013	0.4260	0.4499	0.4045	0.4722	0.4918	0.4774	0.5457	0.5919	
			0.2411	0.4533	0.4653	0.4525	0.4407	0.2945	0.4259	0.4368	0.4274	0.3092	0.4061	0.4103	0.3077	0.3769	0.2967	
	0.5	0.1	0.1872	0.3181	0.3510	0.3634	0.3919	0.2692	0.3668	0.4194	0.4444	0.3453	0.4515	0.5015	0.4245	0.5405	0.5605	
0.1697			0.4139	0.4365	0.4224	0.3968	0.2254	0.3946	0.4073	0.3956	0.2468	0.3785	0.3901	0.2828	0.3612	0.2514		
0.7	0.1	0.1347	0.2588	0.3363	0.3655	0.3984	0.2141	0.3629	0.4048	0.4487	0.2900	0.4359	0.4806	0.3750	0.5249	0.5135		
		0.1277	0.3621	0.4017	0.4017	0.3719	0.1690	0.3598	0.3795	0.3691	0.1974	0.3497	0.3581	0.2199	0.3366	0.2309		
0.9	0.1	0.0809	0.2694	0.3316	0.3608	0.4009	0.1425	0.3353	0.3972	0.4355	0.2081	0.4005	0.4591	0.2879	0.4698	0.4264		
		0.0683	0.3334	0.3701	0.3650	0.3375	0.1023	0.3121	0.3451	0.3335	0.1301	0.3058	0.3185	0.1570	0.2994	0.1637		
0.95	0.1	0.2730	0.3431	0.3526	0.3586	0.3666	0.3733	0.4245	0.4358	0.4489	0.4483	0.4681	0.4952	0.5277	0.5582	0.6312		
		0.3352	0.5227	0.5235	0.5068	0.5013	0.3676	0.4863	0.4812	0.4750	0.3745	0.4605	0.4434	0.3611	0.4078	0.3175		
	0.3	0.1	0.2074	0.3328	0.3475	0.3616	0.3924	0.3209	0.4179	0.4281	0.4442	0.3926	0.4803	0.4915	0.4750	0.5418	0.5792	
			0.2295	0.4987	0.4908	0.4697	0.4556	0.2906	0.4600	0.4529	0.4355	0.3009	0.4293	0.4163	0.3066	0.3663	0.2826	
	0.5	0.1	0.1684	0.3163	0.3381	0.3505	0.3865	0.2646	0.3949	0.4153	0.4383	0.3482	0.4651	0.4816	0.4280	0.5409	0.5576	
0.1741			0.4607	0.4573	0.4368	0.4175	0.2253	0.4319	0.4256	0.4120	0.2550	0.4066	0.3933	0.2945	0.3723	0.2802		
0.7	0.1	0.1405	0.3119	0.3380	0.3504	0.3832	0.2154	0.3903	0.4153	0.4272	0.2988	0.4529	0.4723	0.3801	0.5254	0.5070		
		0.1376	0.4465	0.4411	0.4207	0.3972	0.1711	0.4207	0.4090	0.3821	0.2041	0.3909	0.3695	0.2253	0.3531	0.2296		
0.9	0.1	0.0916	0.2989	0.3195	0.3474	0.3611	0.1471	0.3735	0.3932	0.4201	0.2142	0.4336	0.4623	0.2895	0.5077	0.4180		
		0.0851	0.4265	0.4144	0.3934	0.3685	0.1090	0.3992	0.3634	0.3546	0.1370	0.3693	0.3453	0.1568	0.3301	0.1806		

Acquirer's strategy parameters

γ β_B		0.5															
		0.1					0.3					0.5			0.7		0.9
		0.1	0.3	0.5	0.7	0.9	0.3	0.5	0.7	0.9	0.5	0.7	0.9	0.7	0.9	0.9	
Target's strategy parameters	0.05	0.1	0.2695	0.3009	0.3259	0.3373	0.3674	0.3759	0.3676	0.4053	0.4298	0.4437	0.4527	0.4666	0.5250	0.5406	0.6234
			0.3301	0.3782	0.4002	0.4035	0.4100	0.3684	0.3921	0.4073	0.4168	0.3690	0.3631	0.4040	0.3601	0.3746	0.3155
		0.3	0.2041	0.2488	0.2841	0.3132	0.3493	0.3228	0.3442	0.3632	0.4017	0.4009	0.4208	0.4429	0.4825	0.4997	0.5958
			0.2246	0.2764	0.3099	0.3249	0.3239	0.2869	0.3132	0.3285	0.3441	0.3080	0.3273	0.3355	0.3125	0.3238	0.2668
		0.5	0.1717	0.2160	0.2455	0.2749	0.3247	0.2683	0.3002	0.3366	0.3743	0.3498	0.3744	0.4182	0.4372	0.4841	0.5465
		0.1741	0.2211	0.2435	0.2540	0.2948	0.2251	0.2559	0.2626	0.2932	0.2531	0.2734	0.2978	0.2683	0.2871	0.2566	
	0.7	0.1251	0.1673	0.2018	0.2455	0.2975	0.2180	0.2479	0.2789	0.3343	0.2894	0.3299	0.3819	0.3801	0.4232	0.5104	
		0.1175	0.1574	0.1622	0.2077	0.2130	0.1718	0.1992	0.2168	0.2371	0.1970	0.2264	0.2503	0.2208	0.2476	0.2307	
	0.9	0.0741	0.1153	0.1394	0.1765	0.2433	0.1393	0.1782	0.2026	0.2756	0.2095	0.2366	0.2945	0.2654	0.3476	0.4326	
		0.0633	0.0981	0.1116	0.1286	0.1457	0.1010	0.1285	0.1410	0.1712	0.1322	0.1515	0.1733	0.1559	0.1670	0.1880	
	0.35	0.1	0.2692	0.3177	0.3279	0.3594	0.3722	0.3781	0.4061	0.4207	0.4380	0.4459	0.4721	0.4898	0.5241	0.5476	0.6318
			0.3312	0.4119	0.4232	0.4451	0.4279	0.3705	0.4165	0.4317	0.4291	0.3693	0.4018	0.4150	0.3563	0.3633	0.3180
		0.3	0.2094	0.2735	0.3028	0.3318	0.3633	0.3148	0.3535	0.3818	0.4249	0.3821	0.4372	0.4667	0.4718	0.5205	0.5800
			0.2287	0.3164	0.3473	0.3568	0.3526	0.2831	0.3307	0.3526	0.3701	0.3028	0.3451	0.3593	0.3071	0.3436	0.2841
		0.5	0.1689	0.2333	0.2741	0.3121	0.3539	0.2667	0.3174	0.3568	0.4019	0.3463	0.3982	0.4472	0.4267	0.4959	0.5547
		0.1728	0.2521	0.2663	0.3057	0.3027	0.2243	0.2793	0.3061	0.3233	0.2533	0.2952	0.3237	0.2649	0.3109	0.2609	
	0.7	0.1332	0.2024	0.2375	0.2923	0.3344	0.2224	0.2793	0.3334	0.3710	0.2851	0.3428	0.4206	0.3783	0.4575	0.5143	
		0.1253	0.2027	0.2297	0.2603	0.2970	0.1781	0.2285	0.2684	0.2718	0.1952	0.2433	0.2829	0.2216	0.2716	0.2334	
	0.9	0.0790	0.1399	0.1910	0.2534	0.3098	0.1474	0.2061	0.2492	0.3288	0.2108	0.2748	0.3538	0.2802	0.3630	0.4207	
		0.0680	0.1255	0.1643	0.1977	0.2042	0.1069	0.1537	0.1809	0.2105	0.1326	0.1771	0.2141	0.1536	0.2096	0.1805	
0.65	0.1	0.2656	0.3242	0.3371	0.3506	0.3671	0.3786	0.4119	0.4232	0.4338	0.4447	0.4756	0.4892	0.5200	0.5444	0.6294	
		0.3247	0.4480	0.4531	0.4520	0.4286	0.3699	0.4309	0.4446	0.4346	0.3703	0.4159	0.4174	0.3593	0.3691	0.3156	
	0.3	0.2126	0.2971	0.3227	0.3354	0.3649	0.3144	0.3622	0.3927	0.4133	0.3942	0.4419	0.4720	0.4783	0.5236	0.5966	
		0.2315	0.3752	0.3923	0.3808	0.3685	0.2805	0.3769	0.3827	0.3756	0.3028	0.3663	0.3743	0.3067	0.3532	0.2876	
	0.5	0.1879	0.2666	0.2983	0.3259	0.3488	0.2742	0.3408	0.3775	0.4114	0.3475	0.4196	0.4657	0.4338	0.5089	0.5625	
	0.1696	0.3230	0.3409	0.3432	0.3197	0.2284	0.3204	0.3464	0.3422	0.2511	0.3266	0.3487	0.2658	0.3298	0.2617		
0.7	0.1285	0.2539	0.2871	0.3091	0.3468	0.2134	0.3234	0.3605	0.3912	0.2936	0.3823	0.4370	0.3846	0.4914	0.5124		
	0.1210	0.2696	0.3069	0.3064	0.2914	0.1665	0.2670	0.3105	0.3004	0.2009	0.2970	0.3077	0.2241	0.3026	0.2307		
0.9	0.0797	0.2090	0.2592	0.2946	0.3296	0.1422	0.2620	0.3208	0.3634	0.2077	0.3297	0.3939	0.2821	0.4283	0.4215		
	0.0675	0.2243	0.2570	0.2629	0.2433	0.1027	0.2197	0.2572	0.2497	0.1317	0.2339	0.2536	0.1543	0.2466	0.1796		
0.95	0.1	0.2674	0.3226	0.3350	0.3397	0.3566	0.3666	0.4183	0.4215	0.4381	0.4444	0.4837	0.4914	0.5195	0.5473	0.6288	
		0.3304	0.4611	0.4602	0.4452	0.4278	0.3651	0.4522	0.4496	0.4431	0.3697	0.4313	0.4231	0.3575	0.3918	0.3182	
	0.3	0.2032	0.2993	0.3160	0.3307	0.3520	0.3146	0.3852	0.3934	0.4166	0.3924	0.4649	0.4658	0.4727	0.5334	0.5642	
		0.2220	0.4110	0.3986	0.3925	0.3698	0.2816	0.4021	0.3932	0.3855	0.2998	0.3971	0.3746	0.3043	0.3628	0.2805	
	0.5	0.1735	0.2674	0.2977	0.3145	0.3516	0.2649	0.3700	0.3765	0.4069	0.3377	0.4399	0.4539	0.4336	0.5117	0.5590	
	0.1766	0.3621	0.3844	0.3509	0.3385	0.2228	0.3772	0.3615	0.3488	0.2475	0.3544	0.3499	0.2658	0.3381	0.2617		
0.7	0.1324	0.2726	0.2885	0.3044	0.3260	0.2091	0.3442	0.3705	0.3763	0.2960	0.4129	0.4289	0.3876	0.4676	0.5098		
	0.1260	0.3595	0.3359	0.3235	0.2944	0.1646	0.3415	0.3413	0.3069	0.2029	0.3349	0.3154	0.2268	0.3118	0.2298		
0.9	0.0827	0.2387	0.2600	0.2850	0.3143	0.1405	0.3082	0.3228	0.3662	0.2077	0.3666	0.3915	0.2824	0.4488	0.4306		
	0.0716	0.3135	0.3035	0.2879	0.2578	0.1018	0.3075	0.2875	0.2755	0.1312	0.2938	0.2703	0.1547	0.2746	0.1844		

P
 β_{A1}
 β_{AL}

Acquirer's strategy parameters

γ β_B		0.7															P β_{A1} β_{A2}
		0.1					0.3				0.5			0.7		0.9	
		0.1	0.3	0.5	0.7	0.9	0.3	0.5	0.7	0.9	0.5	0.7	0.9	0.7	0.9	0.9	
		0.1	0.3	0.5	0.7	0.9	0.3	0.5	0.7	0.9	0.5	0.7	0.9	0.7	0.9	0.9	
Target's strategy parameters	0.05	0.1	0.2716 0.3309	0.2807 0.3508	0.3003 0.3704	0.3143 0.3744	0.3290 0.3725	0.3759 0.3691	0.3829 0.3784	0.3998 0.3955	0.4127 0.3962	0.4443 0.3678	0.4809 0.3858	0.4753 0.3930	0.5208 0.3579	0.5344 0.3678	0.8275 0.3154
		0.3	0.2102 0.2314	0.2334 0.2582	0.2471 0.2693	0.2695 0.2806	0.2896 0.2785	0.3207 0.2674	0.3315 0.2988	0.3521 0.3189	0.3696 0.3170	0.3948 0.3023	0.4144 0.3199	0.4293 0.3273	0.4848 0.3141	0.4910 0.3189	0.5937 0.2857
		0.5	0.1678 0.1710	0.1845 0.1989	0.2074 0.2069	0.2378 0.2232	0.2607 0.2259	0.2518 0.2120	0.2849 0.2433	0.3110 0.2815	0.3206 0.2537	0.3428 0.2482	0.3637 0.2868	0.3950 0.2801	0.4282 0.2834	0.4556 0.2812	0.5870 0.2661
		0.7	0.1282 0.1205	0.1464 0.1395	0.1748 0.1596	0.1983 0.1698	0.2314 0.1739	0.2185 0.1717	0.2300 0.1815	0.2550 0.1988	0.2918 0.2118	0.2855 0.1988	0.3124 0.2153	0.3448 0.2290	0.3781 0.2194	0.4168 0.2426	0.5118 0.2302
		0.9	0.0787 0.0649	0.0985 0.0840	0.1180 0.0958	0.1351 0.1010	0.1783 0.1143	0.1471 0.1060	0.1885 0.1214	0.1818 0.1287	0.2168 0.1390	0.2088 0.1325	0.2303 0.1457	0.2855 0.1598	0.2818 0.1534	0.3152 0.1708	0.4178 0.1791
	0.35	0.1	0.2700 0.3300	0.2931 0.3707	0.3163 0.3949	0.3185 0.3639	0.3315 0.3706	0.3737 0.3650	0.3954 0.3948	0.3997 0.3981	0.4117 0.4024	0.4538 0.3739	0.4541 0.3838	0.4731 0.3960	0.5285 0.3613	0.5368 0.3713	0.6296 0.3189
		0.3	0.2085 0.2316	0.2504 0.2814	0.2701 0.2995	0.2842 0.3014	0.2998 0.2933	0.3180 0.2841	0.3372 0.3103	0.3620 0.3299	0.3733 0.3243	0.3943 0.3032	0.4130 0.3238	0.4404 0.3368	0.4778 0.3068	0.4928 0.3214	0.5892 0.2844
		0.5	0.1726 0.1787	0.2094 0.2198	0.2338 0.2385	0.2484 0.2392	0.2625 0.2447	0.2648 0.2225	0.2848 0.2423	0.3151 0.2891	0.3470 0.2778	0.3523 0.2558	0.3752 0.2763	0.3975 0.2849	0.4411 0.2718	0.4857 0.2888	0.5612 0.2632
		0.7	0.1254 0.1195	0.1594 0.1549	0.1879 0.1747	0.2383 0.2081	0.2499 0.1910	0.2288 0.1797	0.2455 0.1978	0.2725 0.2142	0.3148 0.2293	0.2919 0.1995	0.3250 0.2252	0.3809 0.2399	0.3813 0.2241	0.4124 0.2429	0.5058 0.2280
		0.9	0.0781 0.0688	0.1137 0.0982	0.1414 0.1160	0.1728 0.1313	0.2235 0.1440	0.1485 0.1084	0.1729 0.1257	0.2048 0.1454	0.2521 0.1598	0.2099 0.1333	0.2383 0.1511	0.2853 0.1723	0.2921 0.1599	0.3341 0.1814	0.4163 0.1792
	0.65	0.1	0.2703 0.3352	0.3031 0.3973	0.3113 0.3983	0.3188 0.3975	0.3264 0.3781	0.3770 0.3707	0.3989 0.4073	0.4081 0.4129	0.4101 0.4055	0.4554 0.3778	0.4896 0.3995	0.4782 0.4015	0.5213 0.3583	0.5400 0.3771	0.8278 0.3149
		0.3	0.2186 0.2396	0.2646 0.3113	0.2850 0.3297	0.2931 0.3210	0.3098 0.3086	0.3153 0.2832	0.3588 0.3351	0.3697 0.3459	0.3836 0.3378	0.3980 0.3042	0.4310 0.3433	0.4427 0.3454	0.4853 0.3148	0.5081 0.3342	0.5858 0.2848
		0.5	0.1699 0.1738	0.2294 0.2523	0.2541 0.2740	0.2656 0.2687	0.2987 0.2580	0.2818 0.2194	0.3213 0.2836	0.3377 0.2933	0.3484 0.2842	0.3527 0.2553	0.3952 0.2984	0.4132 0.3030	0.4310 0.2892	0.4785 0.3010	0.5644 0.2928
		0.7	0.1258 0.1180	0.1878 0.1857	0.2208 0.2223	0.2394 0.2250	0.2650 0.2122	0.2204 0.1731	0.2818 0.2193	0.3075 0.2528	0.3242 0.2448	0.3012 0.2055	0.3473 0.2473	0.3797 0.2596	0.3807 0.2232	0.4378 0.2823	0.5135 0.2311
		0.9	0.0779 0.0663	0.1528 0.1474	0.1770 0.1837	0.2188 0.1817	0.2377 0.1672	0.1478 0.1063	0.1990 0.1541	0.2528 0.1908	0.2798 0.1873	0.2049 0.1289	0.2754 0.1822	0.3282 0.2053	0.2883 0.1578	0.3899 0.2082	0.4231 0.1811
	0.95	0.1	0.2718 0.3339	0.3052 0.4104	0.3152 0.4083	0.3258 0.4048	0.3317 0.3900	0.3737 0.3656	0.4082 0.4211	0.4088 0.4193	0.4128 0.4078	0.4531 0.3738	0.4858 0.4030	0.4701 0.3983	0.5214 0.3574	0.5396 0.3788	0.8229 0.3138
		0.3	0.2089 0.2282	0.2715 0.3423	0.2837 0.3345	0.2856 0.3188	0.3085 0.3147	0.3252 0.2902	0.3622 0.3552	0.3735 0.3546	0.3818 0.3380	0.3899 0.3010	0.4270 0.3515	0.4402 0.3458	0.4841 0.3132	0.5023 0.3368	0.5941 0.2875
		0.5	0.1643 0.1878	0.2435 0.2864	0.2517 0.2833	0.2803 0.2712	0.2828 0.2590	0.2745 0.2294	0.3312 0.3148	0.3435 0.3081	0.3512 0.2929	0.3457 0.2523	0.4031 0.3188	0.4154 0.3075	0.4248 0.2818	0.4781 0.3057	0.5543 0.2600
		0.7	0.1325 0.1239	0.2179 0.2800	0.2266 0.2459	0.2428 0.2375	0.2602 0.2183	0.2187 0.1712	0.2984 0.2788	0.3081 0.2843	0.3199 0.2485	0.2987 0.2051	0.3817 0.2780	0.3737 0.2818	0.3708 0.2179	0.4427 0.2736	0.5158 0.2322
		0.9	0.0743 0.0634	0.1795 0.2141	0.1918 0.2005	0.2180 0.1925	0.2353 0.1731	0.1512 0.1088	0.2508 0.2288	0.2855 0.2182	0.2827 0.1999	0.2121 0.1338	0.2836 0.2197	0.3364 0.2180	0.2987 0.1830	0.3832 0.2224	0.4215 0.1820

Acquirer's strategy parameters

Target's strategy parameters		Acquirer's strategy parameters															P β_{AI} β_{AL}	
		0.9																
		0.1					0.3					0.5			0.7			0.9
		0.1	0.3	0.5	0.7	0.9	0.3	0.5	0.7	0.9	0.5	0.7	0.9	0.7	0.9	0.9		
Y	β_B																	
		0.05	0.1	0.2648	0.2609	0.2842	0.2839	0.2911	0.3770	0.3811	0.3888	0.3858	0.4502	0.4456	0.4604	0.5253	0.5235	0.6288
0.3252	0.3425			0.3474	0.3459	0.3466	0.3704	0.3789	0.3632	0.3765	0.3721	0.3695	0.3798	0.3595	0.3615	0.3153		
0.3	0.2149		0.2204	0.2238	0.2311	0.2428	0.3124	0.3210	0.3218	0.3360	0.3940	0.3953	0.4063	0.4579	0.4639	0.5968		
	0.2345		0.2425	0.2436	0.2482	0.2511	0.2798	0.2879	0.2870	0.2958	0.3023	0.3044	0.3130	0.2960	0.3146	0.2888		
0.5	0.1747		0.1710	0.1940	0.1927	0.1991	0.2887	0.2742	0.2804	0.2908	0.3447	0.3487	0.3646	0.4428	0.4388	0.5530		
	0.1784		0.1747	0.1849	0.1905	0.1872	0.2240	0.2333	0.2353	0.2387	0.2508	0.2515	0.2628	0.2707	0.2710	0.2596		
0.7	0.1289		0.1384	0.1371	0.1504	0.1650	0.2140	0.2292	0.2254	0.2367	0.2880	0.3065	0.3108	0.3761	0.3665	0.5079		
	0.1212		0.1312	0.1273	0.1358	0.1392	0.1693	0.1810	0.1770	0.1798	0.1978	0.2061	0.2102	0.2207	0.2289	0.2279		
0.9	0.0799		0.0849	0.0886	0.0978	0.1100	0.1348	0.1463	0.1605	0.1709	0.2108	0.2217	0.2244	0.2840	0.3083	0.4232		
	0.0680		0.0725	0.0745	0.0783	0.0803	0.0974	0.1075	0.1182	0.1168	0.1339	0.1403	0.1389	0.1611	0.1679	0.1814		
0.35	0.1		0.2738	0.2754	0.2791	0.2834	0.2993	0.3763	0.3746	0.3808	0.3874	0.4450	0.4571	0.4547	0.5283	0.5389	0.6250	
			0.3353	0.3413	0.3451	0.3453	0.3522	0.3666	0.3696	0.3794	0.3801	0.3983	0.3803	0.3779	0.3614	0.3703	0.3146	
	0.3		0.2134	0.2279	0.2290	0.2374	0.2487	0.3207	0.3293	0.3310	0.3344	0.3944	0.4059	0.4109	0.4860	0.4824	0.5950	
			0.2315	0.2517	0.2511	0.2548	0.2583	0.2856	0.2937	0.2943	0.2998	0.3009	0.3134	0.3183	0.2997	0.3129	0.2871	
	0.5		0.1643	0.1781	0.1901	0.1981	0.2108	0.2878	0.2756	0.2941	0.2984	0.3497	0.3517	0.3618	0.4319	0.4384	0.5478	
			0.1678	0.1821	0.1903	0.1958	0.1973	0.2247	0.2339	0.2483	0.2428	0.2531	0.2549	0.2609	0.2680	0.2700	0.2569	
	0.7		0.1315	0.1447	0.1518	0.1585	0.1758	0.2177	0.2238	0.2346	0.2540	0.2971	0.3040	0.3147	0.3747	0.3914	0.5105	
			0.1243	0.1371	0.1407	0.1438	0.1478	0.1724	0.1772	0.1838	0.1940	0.2036	0.2105	0.2131	0.2195	0.2299	0.2303	
	0.9		0.0750	0.0930	0.0973	0.1019	0.1298	0.1478	0.1479	0.1641	0.1818	0.2111	0.2210	0.2216	0.2952	0.3014	0.4200	
			0.0643	0.0796	0.0812	0.0818	0.0928	0.1068	0.1069	0.1189	0.1228	0.1338	0.1397	0.1369	0.1618	0.1637	0.1801	
	0.85	0.1	0.2718	0.2803	0.2858	0.2872	0.2941	0.3757	0.3895	0.3907	0.3872	0.4507	0.4509	0.4644	0.5298	0.5221	0.6254	
			0.3336	0.3499	0.3538	0.3487	0.3523	0.3666	0.3624	0.3656	0.3805	0.3714	0.3813	0.3846	0.3614	0.3635	0.3140	
		0.3	0.2113	0.2325	0.2325	0.2398	0.2440	0.3148	0.3386	0.3337	0.3350	0.3875	0.4082	0.4114	0.4781	0.4882	0.5913	
			0.2205	0.2596	0.2601	0.2611	0.2524	0.2849	0.3051	0.3024	0.2989	0.2975	0.3161	0.3142	0.3073	0.3155	0.2860	
		0.5	0.1688	0.1817	0.1958	0.2068	0.2200	0.2816	0.2791	0.2983	0.3000	0.3441	0.3583	0.3764	0.4367	0.4534	0.5824	
			0.1703	0.1901	0.2032	0.2103	0.2075	0.2201	0.2383	0.2535	0.2461	0.2490	0.2640	0.2735	0.2694	0.2808	0.2832	
		0.7	0.1332	0.1480	0.1597	0.1624	0.1697	0.2109	0.2388	0.2475	0.2598	0.2941	0.3030	0.3283	0.3823	0.3971	0.5078	
			0.1251	0.1425	0.1552	0.1514	0.1498	0.1680	0.1915	0.1982	0.1993	0.2009	0.2103	0.2242	0.2228	0.2344	0.2293	
		0.9	0.0837	0.0993	0.1118	0.1239	0.1278	0.1445	0.1628	0.1775	0.1943	0.2188	0.2291	0.2438	0.2924	0.3042	0.4178	
			0.0718	0.0874	0.0973	0.1027	0.0938	0.1048	0.1192	0.1282	0.1331	0.1385	0.1458	0.1525	0.1608	0.1689	0.1791	
0.95		0.1	0.2693	0.2778	0.2824	0.2815	0.3010	0.3723	0.3768	0.3843	0.3892	0.4458	0.4582	0.4625	0.5197	0.5347	0.6348	
			0.3298	0.3529	0.3518	0.3439	0.3619	0.3628	0.3791	0.3822	0.3775	0.3742	0.3839	0.3619	0.3573	0.3688	0.3179	
		0.3	0.2084	0.2348	0.2378	0.2311	0.2438	0.3220	0.3250	0.3385	0.3413	0.4030	0.3993	0.4085	0.4795	0.4928	0.6003	
			0.2279	0.2704	0.2836	0.2528	0.2571	0.2873	0.3001	0.3076	0.3046	0.3068	0.3138	0.3150	0.3094	0.3202	0.2890	
		0.5	0.1893	0.1908	0.1959	0.2015	0.2045	0.2687	0.2943	0.2928	0.3028	0.3487	0.3688	0.3707	0.4387	0.4389	0.5591	
			0.1736	0.2079	0.2072	0.2022	0.1937	0.2247	0.2570	0.2512	0.2517	0.2501	0.2746	0.2878	0.2889	0.2732	0.2616	
		0.7	0.1283	0.1834	0.1838	0.1707	0.1750	0.2192	0.2445	0.2433	0.2570	0.2944	0.3182	0.3188	0.3779	0.3920	0.5167	
			0.1220	0.1731	0.1827	0.1820	0.1813	0.1732	0.2089	0.1978	0.1973	0.2013	0.2259	0.2183	0.2223	0.2331	0.2304	
		0.9	0.0774	0.1126	0.1242	0.1194	0.1273	0.1390	0.1835	0.1872	0.1903	0.2152	0.2453	0.2415	0.2903	0.3147	0.4278	
			0.0684	0.1137	0.1159	0.1024	0.0942	0.1005	0.1472	0.1431	0.1312	0.1387	0.1653	0.1535	0.1598	0.1764	0.1848	

Acquirer's strategy parameters

γ		β_B		Acquirer's strategy parameters															P	β_{A1}	β_{AL}																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																										
				0.5					0.3					0.5								0.7					0.9																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																				
				0.1			0.3		0.5		0.7		0.9		0.3			0.5				0.7		0.9		0.5			0.7		0.9																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
Target's strategy parameters	0.05	0.1	0.1	0.4998	0.5826	0.6336	0.6568	0.7112	0.6498	0.6978	0.7366	0.7608	0.7346	0.7666	0.8270	0.8170	0.8580	0.8980	0.3	0.1	0.5392	0.5165	0.5144	0.5120	0.5167	0.5784	0.5557	0.5468	0.5502	0.6040	0.5905	0.5884	0.6428	0.6301	0.6942	0.3	0.3	0.1465	0.1506	0.1487	0.1426	0.1361	0.1635	0.1649	0.1627	0.1615	0.1710	0.1778	0.1777	0.1627	0.1667	0.1909	0.5	0.1	0.3402	0.4266	0.4998	0.5480	0.5960	0.5088	0.5576	0.6022	0.6668	0.6122	0.6566	0.7008	0.7108	0.7464	0.8128	0.5	0.3	0.5999	0.5805	0.5685	0.5715	0.5861	0.6344	0.6173	0.6032	0.6025	0.6548	0.6406	0.6322	0.6788	0.6695	0.7330	0.7	0.1	0.1293	0.1295	0.1205	0.1225	0.1212	0.1452	0.1464	0.1412	0.1391	0.1505	0.1574	0.1551	0.1584	0.1647	0.1709	0.7	0.3	0.2634	0.3446	0.3966	0.4430	0.5116	0.3968	0.4568	0.5228	0.5842	0.5022	0.5462	0.6268	0.6074	0.6648	0.7298	0.9	0.1	0.6520	0.6269	0.6191	0.6204	0.6347	0.6761	0.6572	0.6438	0.6407	0.6964	0.6829	0.6840	0.7197	0.6961	0.7515	0.9	0.3	0.1214	0.1196	0.1136	0.1070	0.1097	0.1335	0.1321	0.1278	0.1203	0.1366	0.1454	0.1395	0.1463	0.1488	0.1553	0.9	0.5	0.1774	0.2468	0.3014	0.3666	0.4362	0.3018	0.3538	0.4062	0.4632	0.3908	0.4598	0.5388	0.4968	0.5752	0.6480	0.9	0.5	0.7054	0.6781	0.6667	0.6696	0.6621	0.7225	0.7010	0.6966	0.6919	0.7328	0.7176	0.7068	0.7620	0.7358	0.7876	0.9	0.7	0.1139	0.1099	0.0994	0.0942	0.0988	0.1193	0.1190	0.1129	0.1093	0.1240	0.1282	0.1224	0.1315	0.1333	0.1375	0.9	0.9	0.0954	0.1538	0.1868	0.2362	0.3164	0.1774	0.2270	0.2662	0.3610	0.2590	0.3044	0.3788	0.3494	0.4368	0.5130	0.9	0.9	0.7771	0.7467	0.7461	0.7474	0.7642	0.7855	0.7716	0.7609	0.7640	0.8060	0.7778	0.7839	0.8169	0.7962	0.8431	0.9	0.9	0.0979	0.0879	0.0811	0.0800	0.0918	0.0970	0.1027	0.0962	0.0924	0.1068	0.1040	0.1021	0.1071	0.1110	0.1140	0.9	0.9	0.5012	0.6398	0.6800	0.7378	0.7504	0.6532	0.7428	0.7884	0.8114	0.7352	0.8064	0.8574	0.8150	0.8830	0.9030	0.9	0.9	0.5371	0.4966	0.4822	0.4673	0.4960	0.5768	0.5469	0.5336	0.5398	0.6064	0.5854	0.5712	0.6431	0.6201	0.6994	0.9	0.9	0.1456	0.1608	0.1571	0.1554	0.1517	0.1647	0.1737	0.1746	0.1720	0.1715	0.1661	0.1654	0.1612	0.1938	0.1888	0.9	0.9	0.3460	0.4972	0.5730	0.6228	0.6588	0.4968	0.5936	0.6556	0.7266	0.6024	0.6964	0.7578	0.6970	0.8004	0.8096	0.9	0.9	0.6052	0.5501	0.5265	0.5326	0.5514	0.6311	0.5956	0.5820	0.5846	0.6510	0.6278	0.6180	0.6770	0.6503	0.7287	0.9	0.9	0.1311	0.1465	0.1487	0.1457	0.1477	0.1439	0.1564	0.1592	0.1575	0.1511	0.1655	0.1671	0.1600	0.1723	0.1688	0.9	0.9	0.2616	0.4004	0.4808	0.5484	0.6090	0.3950	0.5034	0.5614	0.6612	0.5030	0.5976	0.6992	0.6010	0.7262	0.7358	0.9	0.9	0.6458	0.5826	0.5702	0.5691	0.5811	0.6752	0.6305	0.6171	0.6078	0.6885	0.6628	0.6398	0.7133	0.6829	0.7539	0.9	0.9	0.1163	0.1395	0.1403	0.1435	0.1463	0.1291	0.1428	0.1511	0.1496	0.1384	0.1522	0.1550	0.1433	0.1597	0.1545	0.9	0.9	0.1892	0.3248	0.3946	0.4658	0.5390	0.3096	0.4132	0.5092	0.5774	0.3858	0.4950	0.6222	0.5008	0.6406	0.6554	0.9	0.9	0.7039	0.6232	0.6019	0.6016	0.6205	0.7163	0.6760	0.6546	0.6425	0.7391	0.6926	0.6761	0.7553	0.7142	0.7848	0.9	0.9	0.1090	0.1369	0.1380	0.1430	0.1477	0.1190	0.1340	0.1441	0.1441	0.1266	0.1364	0.1441	0.1304	0.1479	0.1378	0.9	0.9	0.1024	0.2034	0.2906	0.3664	0.4640	0.1652	0.2796	0.3544	0.4760	0.2816	0.3620	0.4682	0.3440	0.5010	0.5018	0.9	0.9	0.7715	0.6880	0.6571	0.6557	0.6677	0.7958	0.7370	0.7033	0.6907	0.8051	0.7590	0.7246	0.8145	0.7645	0.8384	0.9	0.9	0.0930	0.1327	0.1446	0.1502	0.1576	0.1009	0.1313	0.1353	0.1423	0.1030	0.1246	0.1349	0.1092	0.1305	0.1121	0.9	0.9	0.4916	0.7034	0.7324	0.7506	0.7434	0.6524	0.7720	0.8160	0.8206	0.7356	0.8368	0.8648	0.8160	0.9000	0.9004	0.9	0.9	0.5407	0.4610	0.4603	0.4672	0.4938	0.5804	0.5335	0.5186	0.5267	0.6046	0.5671	0.5657	0.6357	0.6048	0.6991	0.9	0.9	0.1456	0.1856	0.1795	0.1747	0.1689	0.1671	0.1925	0.1879	0.1782	0.1731	0.1939	0.1953	0.1806	0.2022	0.1682	0.9	0.9	0.3504	0.5994	0.6542	0.6642	0.6838	0.4948	0.6808	0.7212	0.7426	0.6020	0.7450	0.7954	0.6958	0.8284	0.8208	0.9	0.9	0.6073	0.4965	0.4932	0.5050	0.5338	0.6355	0.5615	0.5445	0.5566	0.6548	0.5932	0.5935	0.6845	0.6321	0.7293	0.9	0.9	0.1334	0.1788	0.1794	0.1767	0.1660	0.1466	0.1850	0.1814	0.1722	0.1519	0.1636	0.1609	0.1585	0.1686	0.1676	0.9	0.9	0.2568	0.5210	0.5836	0.6256	0.6260	0.4022	0.5844	0.6642	0.7042	0.4978	0.6726	0.7558	0.6028	0.7768	0.7374	0.9	0.9	0.6538	0.5160	0.5111	0.5209	0.5541	0.6617	0.5829	0.5664	0.5842	0.6984	0.6239	0.6162	0.7199	0.6534	0.7629	0.9	0.9	0.1207	0.1765	0.1834	0.1618	0.1770	0.1351	0.1767	0.1763	0.1704	0.1395	0.1777	0.1766	0.1467	0.1776	0.1552	0.9	0.9	0.1630	0.4754	0.5404	0.5762	0.6038	0.2966	0.5268	0.6094	0.6434	0.3980	0.5924	0.6860	0.5068	0.7250	0.6492	0.9	0.9	0.7020	0.5341	0.5313	0.5364	0.5779	0.7195	0.6140	0.5916	0.6060	0.7378	0.6453	0.6371	0.7589	0.6778	0.7892	0.9	0.9	0.1062	0.1801	0.1658	0.1919	0.1654	0.1163	0.1793	0.1663	0.1772	0.1270	0.1601	0.1745	0.1262	0.1732	0.1361	0.9	0.9	0.1016	0.3742	0.4672	0.5210	0.5502	0.1798	0.4098	0.5182	0.5738	0.2588	0.4890	0.5982	0.3454	0.6062	0.5004	0.9	0.9	0.7829	0.5584	0.5526	0.5654	0.5990	0.7908	0.6393	0.6191	0.6332	0.8024	0.6743	0.6585	0.8167	0.7066	0.8424	0.9	0.9	0.0963	0.1992	0.1936	0.1943	0.2046	0.0994	0.2046	0.1852	0.1914	0.1651	0.1043	0.1635	0.1815	0.1097	0.1734	0.9	0.9	0.5002	0.7222	0.7400	0.7416	0.7340	0.6446	0.8106	0.8234	0.8322	0.7362	0.8700	0.8710	0.8114	0.9032	0.9006	0.9	0.9	0.5346	0.4470	0.4527	0.4561	0.4859	0.5667	0.5161	0.5119	0.5264	0.6036	0.5559	0.5642	0.6403	0.6059	0.6982	0.9	0.9	0.1472	0.1971	0.1845	0.1766	0.1706	0.1632	0.2020	0.1946	0.1895	0.1733	0.2067	0.1989	0.1793	0.2042	0.1889	0.9	0.9	0.3362	0.6562	0.6590	0.6762	0.6720	0.4968	0.7308	0.7356	0.7518	0.5952	0.8076	0.7906	0.6892	0.8476	0.7960	0.9	0.9	0.6043	0.4961	0.4795	0.4691	0.5239	0.6332	0.5271	0.5348	0.5571	0.6592	0.5755	0.5692	0.6659	0.6293	0.7340	0.9	0.9	0.1296	0.1997	0.1969	0.1894	0.1792	0.1461	0.2029	0.1983	0.1830	0.1529	0.2053	0.1913	0.1636	0.2002	0.1705	0.9	0.9	0.2672	0.6168	0.6170	0.6220	0.6382	0.3926	0.6914	0.6894	0.7046	0.4920	0.7476	0.7518	0.6010	0.7988	0.7386	0.9	0.9	0.6495	0.4659	0.4824	0.5057	0.5510	0.6747	0.5352	0.5490	0.5774	0.6863	0.5882	0.6037	0.7215	0.6406	0.7569	0.9	0.9	0.1204	0.2103	0.2015	0.1992	0.1918	0.1314	0.2124	0.1999	0.1865	0.1346	0.2085	0.1936	0.1476	0.1966	0.1536	0.9	0.9	0.1698	0.5860	0.5790	0.5868	0.5850	0.2896	0.6308	0.6602	0.6362	0.4016	0.6940	0.6968	0.5124	0.7450	0.6482	0.9	0.9	0.6977	0.4635	0.4983	0.5170	0.5572	0.7222	0.5457	0.5612	0.5697	0.7371	0.5949	0.6156	0.7565	0.6545	0.7665	0.9	0.9	0.1126	0.2220	0.2159	0.2076	0.2089	0.1206	0.2168	0.2099	0.1942	0.1254	0.2161	0.1978	0.1310	0.2001	0.1374	0.9	0.9	0.1060	0.5208	0.5398	0.5460	0.5476	0.1762	0.5780	0.5762	0.6052	0.2576	0.6198	0.6232	0.3454	0.6756	0.5136	0.9	0.9	0.7655	0.4583	0.4616	0.5220	0.5739	0.7867	0.5333	0.5602	0.6084	0.6056	0.5915	0.6282	0.8177	0.6640	0.8383	0.9	0.9	0.0968	0.2286	0.2264	0.2287	0.2265	0.0995	0.2353	0.2231	0.2152	0.1034	0.2265	0.2120	0.1094	0.2096	0.1113	0.9	0.9

Acquirer's strategy parameters

Target's strategy parameters

γ	β_B	0.7																		
		0.1					0.3					0.5					0.7		0.9	
		0.1	0.3	0.5	0.7	0.9	0.3	0.5	0.7	0.9	0.5	0.7	0.9	0.7	0.9	0.9				
0.05	0.1	0.5008	0.5370	0.5784	0.5960	0.6208	0.6514	0.6674	0.7104	0.7262	0.7322	0.7698	0.7944	0.8140	0.8414	0.8992	P β_{AI} β_{AL}			
		0.5423	0.5227	0.5210	0.5255	0.5300	0.5771	0.5738	0.5628	0.5668	0.6068	0.5987	0.5983	0.6395	0.6352	0.6978				
		0.1483	0.1497	0.1471	0.1459	0.1414	0.1634	0.1701	0.1641	0.1613	0.1709	0.1755	0.1764	0.1767	0.1838	0.1887				
	0.3	0.3500	0.3974	0.4264	0.4606	0.4854	0.5088	0.5302	0.5716	0.5976	0.6000	0.6394	0.6668	0.7128	0.7316	0.8108				
		0.6006	0.5873	0.5796	0.5852	0.5967	0.6330	0.6252	0.6158	0.6185	0.6561	0.6481	0.6421	0.6803	0.8711	0.7323				
		0.1287	0.1319	0.1230	0.1234	0.1249	0.1461	0.1478	0.1444	0.1409	0.1547	0.1562	0.1523	0.1603	0.1844	0.1697				
	0.5	0.2582	0.3078	0.3286	0.3742	0.4046	0.3740	0.4320	0.4792	0.4672	0.4830	0.5328	0.5816	0.5982	0.6448	0.7516				
		0.6501	0.6318	0.6312	0.6350	0.6439	0.6733	0.6594	0.6546	0.6580	0.6948	0.6828	0.6792	0.7163	0.7066	0.7542				
		0.1206	0.1151	0.1138	0.1100	0.1103	0.1295	0.1320	0.1285	0.1249	0.1396	0.1407	0.1365	0.1458	0.1471	0.1525				
	0.7	0.1822	0.2186	0.2578	0.2858	0.3306	0.3022	0.3228	0.3632	0.4150	0.3992	0.4312	0.4788	0.4950	0.5570	0.6502				
		0.7034	0.6760	0.6780	0.6873	0.6999	0.7183	0.7128	0.7020	0.7027	0.7336	0.7245	0.7204	0.7598	0.7462	0.7672				
		0.1119	0.1013	0.0996	0.0994	0.1049	0.1161	0.1167	0.1150	0.1104	0.1243	0.1248	0.1251	0.1340	0.1325	0.1368				
0.9	0.0978	0.1308	0.1566	0.1782	0.2306	0.1858	0.2162	0.2354	0.2804	0.2610	0.2908	0.3402	0.3438	0.3928	0.4966					
	0.7840	0.7541	0.7537	0.7580	0.7733	0.7917	0.7792	0.7721	0.7730	0.7991	0.7919	0.7808	0.8197	0.8028	0.8414					
	0.1013	0.0907	0.0884	0.0835	0.0956	0.1002	0.1000	0.0973	0.0931	0.1019	0.1053	0.0958	0.1088	0.1097	0.1128					
0.35	0.1	0.4996	0.5708	0.6208	0.6218	0.6378	0.6438	0.7024	0.7192	0.7484	0.7428	0.7576	0.8062	0.8204	0.8502	0.9044				
		0.5405	0.5134	0.5096	0.5082	0.5200	0.5804	0.5629	0.5558	0.5515	0.6108	0.5914	0.5854	0.6442	0.6311	0.6982				
		0.1472	0.1467	0.1541	0.1532	0.1470	0.1831	0.1672	0.1712	0.1662	0.1780	0.1797	0.1775	0.1801	0.1882	0.1859				
	0.3	0.3508	0.4378	0.4802	0.5020	0.5246	0.5012	0.5532	0.6042	0.6206	0.6024	0.6496	0.6962	0.6964	0.7422	0.8080				
		0.5948	0.5723	0.5625	0.5662	0.5711	0.6348	0.6098	0.5992	0.6016	0.6548	0.6337	0.6328	0.6858	0.6637	0.7292				
		0.1247	0.1398	0.1369	0.1363	0.1371	0.1469	0.1478	0.1499	0.1502	0.1532	0.1548	0.1514	0.1600	0.1664	0.1668				
	0.5	0.2674	0.3424	0.3886	0.4132	0.4604	0.3912	0.4328	0.4910	0.5432	0.5078	0.5550	0.6062	0.6146	0.6674	0.7462				
		0.6484	0.6116	0.6018	0.6012	0.6137	0.6770	0.6581	0.6417	0.6389	0.6940	0.6761	0.6668	0.7175	0.6978	0.7521				
		0.1197	0.1281	0.1290	0.1295	0.1437	0.1331	0.1378	0.1419	0.1360	0.1366	0.1461	0.1475	0.1467	0.1552	0.1521				
	0.7	0.1804	0.2438	0.2910	0.3086	0.3846	0.3162	0.3540	0.3992	0.4656	0.3942	0.4530	0.5146	0.5054	0.5656	0.6420				
		0.6949	0.6543	0.6487	0.6414	0.6499	0.7228	0.6936	0.6827	0.6760	0.7408	0.7174	0.7014	0.7544	0.7291	0.7878				
		0.1096	0.1109	0.1215	0.1299	0.1327	0.1207	0.1227	0.1311	0.1263	0.1285	0.1315	0.1329	0.1314	0.1388	0.1373				
0.9	0.1010	0.1556	0.1996	0.2474	0.3160	0.1868	0.2252	0.2760	0.3444	0.2820	0.3042	0.3758	0.3588	0.4224	0.4978					
	0.7737	0.7305	0.7084	0.6985	0.7072	0.7841	0.7677	0.7413	0.7320	0.8013	0.7833	0.7591	0.8142	0.7909	0.8363					
	0.0896	0.1039	0.1113	0.1232	0.1355	0.0984	0.1057	0.1159	0.1242	0.1037	0.1108	0.1194	0.1097	0.1180	0.1116					
0.65	0.1	0.5078	0.6160	0.6354	0.6470	0.6380	0.6540	0.7282	0.7504	0.7498	0.7510	0.8020	0.8206	0.8140	0.8688	0.8966				
		0.5325	0.4921	0.4899	0.4943	0.5116	0.5784	0.5493	0.5411	0.5470	0.6084	0.5858	0.5827	0.6404	0.6215	0.7000				
		0.1405	0.1678	0.1693	0.1593	0.1540	0.1645	0.1769	0.1805	0.1665	0.1723	0.1847	0.1685	0.1822	0.1918	0.1889				
	0.3	0.3828	0.4880	0.5380	0.5460	0.5510	0.4994	0.6010	0.6388	0.6516	0.6032	0.6934	0.7222	0.7138	0.7744	0.8084				
		0.5970	0.5422	0.5297	0.5368	0.5623	0.6314	0.5966	0.5806	0.5687	0.6598	0.6215	0.6129	0.6798	0.6562	0.7246				
		0.1263	0.1646	0.1662	0.1663	0.1545	0.1442	0.1680	0.1685	0.1641	0.1538	0.1728	0.1757	0.1823	0.1784	0.1667				
	0.5	0.2630	0.4014	0.4578	0.4744	0.4898	0.3982	0.5112	0.5484	0.5646	0.5056	0.6042	0.6448	0.6018	0.7028	0.7398				
		0.6481	0.5715	0.5553	0.5599	0.5854	0.6774	0.6285	0.6158	0.6135	0.6975	0.6541	0.6409	0.7183	0.6781	0.7629				
		0.1170	0.1625	0.1649	0.1678	0.1637	0.1351	0.1594	0.1632	0.1631	0.1391	0.1661	0.1624	0.1448	0.1679	0.1537				
	0.7	0.1782	0.3150	0.3824	0.4130	0.4308	0.3046	0.3974	0.4836	0.5030	0.4066	0.5016	0.5642	0.5038	0.6184	0.6512				
		0.7049	0.5955	0.5774	0.5768	0.6152	0.7235	0.6568	0.6357	0.6444	0.7406	0.6922	0.6730	0.7556	0.7099	0.7866				
		0.1120	0.1677	0.1706	0.1734	0.1615	0.1156	0.1507	0.1638	0.1639	0.1245	0.1595	0.1651	0.1309	0.1601	0.1367				
0.9	0.1000	0.2414	0.2922	0.3564	0.3690	0.1860	0.2614	0.3742	0.4140	0.2534	0.3730	0.4658	0.3514	0.4940	0.5038					
	0.7787	0.6332	0.6057	0.6078	0.6443	0.7935	0.7072	0.6756	0.6753	0.8068	0.7264	0.7004	0.8203	0.7489	0.8402					
	0.0956	0.1781	0.1798	0.1894	0.1958	0.1021	0.1639	0.1717	0.1725	0.1053	0.1497	0.1636	0.1094	0.1510	0.1131					
0.95	0.1	0.5050	0.6374	0.6480	0.6558	0.6506	0.6448	0.7534	0.7802	0.7540	0.7434	0.8102	0.8138	0.8104	0.8708	0.8972				
		0.5377	0.4788	0.4864	0.4970	0.5099	0.5796	0.5392	0.5377	0.5471	0.6094	0.5749	0.5776	0.6433	0.6198	0.6943				
		0.1487	0.1829	0.1733	0.1657	0.1619	0.1658	0.1963	0.1830	0.1769	0.1731	0.1975	0.1888	0.1831	0.1992	0.1865				
	0.3	0.3450	0.5408	0.5442	0.5374	0.5574	0.5116	0.6410	0.6538	0.6480	0.5974	0.7124	0.7234	0.7090	0.7816	0.8150				
		0.6054	0.5021	0.5214	0.5316	0.5535	0.6355	0.5650	0.5714	0.5868	0.6527	0.5994	0.6085	0.6829	0.6425	0.7289				
		0.1322	0.1934	0.1816	0.1754	0.1619	0.1485	0.1928	0.1832	0.1690	0.1521	0.1641	0.1606	0.1599	0.1678	0.1698				
	0.5	0.2538	0.4748	0.4742	0.4744	0.4874	0.4042	0.5722	0.5784	0.5744	0.5002	0.6504	0.6494	0.5916	0.7182	0.7360				
		0.6473	0.5128	0.5308	0.5487	0.5802	0.6781	0.5789	0.5960	0.6114	0.6911	0.6197	0.6397	0.7181	0.6675	0.7532				
		0.1177	0.2090	0.1887	0.1861	0.1776	0.1307	0.2016	0.1621	0.1727	0.1362	0.1653	0.1764	0.1470	0.1630	0.1533				
	0.7	0.1870	0.4236	0.4240	0.4282	0.4308	0.3008	0.5098	0.5068	0.5084	0.4068	0.5722	0.5680	0.4914	0.6476	0.6544				
		0.7085	0.5144	0.5345	0.5569	0.6040	0.7205	0.5853	0.6039	0.6203	0.7388	0.6321	0.6802	0.7542	0.6836	0.7878				
		0.1127	0.2223	0.2071	0.1991	0.1864	0.1207	0.2110	0.1967	0.1844	0.1240	0.2029	0.1613	0.1330	0.1644	0.1376				
0.9	0.0954	0.3558	0.3590	0.3710	0.3802	0.1912	0.4282	0.4302	0.4340	0.2828	0.4598	0.4938	0.3632	0.5370	0.5040					
	0.7787	0.6048	0.5337	0.5823	0.6190	0.7907	0.5879	0.6172	0.6513	0.8078	0.6369	0.6816	0.8189	0.7135	0.8362					
	0.0962	0.2359	0.2271	0.2182	0.2132	0.1008	0.2322	0.2144	0.2019	0.1045	0.2236	0.1948	0.1094	0.1940	0.1125					

Acquirer's strategy parameters

γ β_B		Acquirer's strategy parameters															P β_{A1} β_{AL}				
		0.9					0.7					0.5									
		0.1			0.3		0.5		0.7			0.9		0.1				0.3		0.5	
		0.1	0.3	0.5	0.7	0.9	0.3	0.5	0.7	0.9	0.5	0.7	0.9	0.7	0.9	0.9		0.7	0.9	0.9	
0.05	0.1	0.4922	0.5204	0.5316	0.5338	0.5446	0.6534	0.6666	0.8460	0.8744	0.7396	0.7360	0.7606	0.8178	0.8226	0.6970					
		0.5390	0.5397	0.5347	0.5319	0.5345	0.5789	0.5716	0.5679	0.5720	0.8067	0.8055	0.8053	0.8424	0.8364	0.6868					
		0.1461	0.1461	0.1488	0.1439	0.1441	0.1661	0.1632	0.1652	0.1627	0.1717	0.1743	0.1721	0.1606	0.1614	0.1907					
	0.3	0.3546	0.3692	0.3746	0.3876	0.4030	0.4932	0.5092	0.5108	0.5352	0.5998	0.6058	0.6294	0.6714	0.7162	0.8220					
		0.6059	0.5970	0.5974	0.5963	0.6026	0.6335	0.6304	0.6299	0.6315	0.6572	0.6526	0.6504	0.6619	0.6756	0.7289					
		0.1360	0.1291	0.1266	0.1298	0.1267	0.1428	0.1475	0.1420	0.1435	0.1541	0.1535	0.1535	0.1568	0.1616	0.1670					
	0.5	0.2698	0.2662	0.3012	0.2998	0.3070	0.3950	0.4116	0.4196	0.4332	0.4674	0.5006	0.5286	0.6136	0.6152	0.7344					
		0.6473	0.6422	0.6440	0.6429	0.6484	0.6751	0.6661	0.6683	0.6716	0.6930	0.6925	0.6901	0.7214	0.7130	0.7542					
		0.1146	0.1167	0.1179	0.1179	0.1142	0.1300	0.1299	0.1326	0.1285	0.1366	0.1411	0.1369	0.1460	0.1482	0.1545					
	0.7	0.1632	0.2004	0.1978	0.2156	0.2350	0.2980	0.3200	0.3164	0.3304	0.3912	0.4152	0.4252	0.4976	0.5146	0.6422					
		0.6926	0.6906	0.6930	0.6976	0.7022	0.7162	0.7163	0.7124	0.7165	0.7362	0.7363	0.7306	0.7556	0.7506	0.7906					
		0.1049	0.1100	0.1067	0.1064	0.1081	0.1186	0.1182	0.1153	0.1162	0.1261	0.1251	0.1236	0.1313	0.1335	0.1389					
0.9	0.1026	0.1108	0.1164	0.1274	0.1408	0.1706	0.1896	0.2056	0.2170	0.2626	0.2782	0.2816	0.3594	0.3804	0.5040						
	0.7783	0.7656	0.7610	0.7662	0.7615	0.7902	0.7874	0.7800	0.7673	0.8021	0.7969	0.7963	0.8179	0.8131	0.8397						
	0.0894	0.0935	0.0872	0.0865	0.0961	0.1027	0.1020	0.0961	0.0972	0.1060	0.1026	0.1034	0.1082	0.1091	0.1132						
0.35	0.1	0.5074	0.5198	0.5300	0.5370	0.5566	0.6466	0.6532	0.8756	0.6626	0.7326	0.7560	0.8202	0.8424	0.9960						
		0.5393	0.5298	0.5265	0.5277	0.5356	0.5619	0.5735	0.5632	0.5673	0.6072	0.6031	0.5999	0.6416	0.6397	0.6976					
		0.1472	0.1517	0.1526	0.1466	0.1482	0.1643	0.1679	0.1604	0.1656	0.1736	0.1722	0.1769	0.1625	0.1644	0.1694					
	0.3	0.3502	0.3642	0.3692	0.4016	0.4166	0.5040	0.5196	0.5330	0.5462	0.5974	0.6236	0.6372	0.6612	0.7122	0.8132					
		0.6094	0.5931	0.5864	0.5912	0.5921	0.6363	0.6279	0.6211	0.6123	0.6603	0.6507	0.6448	0.6541	0.6774	0.7317					
		0.1335	0.1316	0.1349	0.1341	0.1312	0.1456	0.1446	0.1445	0.1405	0.1550	0.1546	0.1530	0.1595	0.1655	0.1696					
	0.5	0.2536	0.2768	0.2966	0.3126	0.3294	0.3956	0.4146	0.4464	0.4474	0.5024	0.5070	0.5260	0.6024	0.6146	0.7276					
		0.6472	0.6369	0.6405	0.6334	0.6396	0.6761	0.6646	0.6603	0.6625	0.6941	0.6937	0.6853	0.7169	0.7101	0.7326					
		0.1155	0.1203	0.1274	0.1225	0.1253	0.1311	0.1314	0.1317	0.1355	0.1406	0.1406	0.1426	0.1481	0.1484	0.1541					
	0.7	0.1676	0.2096	0.2214	0.2330	0.2576	0.3036	0.3136	0.3304	0.3566	0.4024	0.4192	0.4326	0.4964	0.5244	0.6454					
		0.7004	0.6896	0.6848	0.6803	0.6819	0.7169	0.7135	0.7096	0.7064	0.7362	0.7252	0.7275	0.7549	0.7463	0.7909					
		0.1063	0.1150	0.1147	0.1142	0.1163	0.1172	0.1223	0.1217	0.1233	0.1269	0.1247	0.1275	0.1303	0.1349	0.1396					
0.9	0.0970	0.1224	0.1296	0.1376	0.1732	0.1866	0.1966	0.2106	0.2344	0.2624	0.2776	0.2814	0.3622	0.3740	0.4966						
	0.7733	0.7596	0.7496	0.7396	0.7425	0.7900	0.7836	0.7793	0.7754	0.8047	0.7957	0.7874	0.8151	0.8059	0.8420						
	0.0910	0.0910	0.1002	0.0992	0.1180	0.0997	0.1025	0.1036	0.1040	0.1040	0.1069	0.1079	0.1082	0.1115	0.1106						
0.65	0.1	0.5048	0.5346	0.5466	0.5446	0.5546	0.6476	0.6766	0.6864	0.6630	0.7364	0.7610	0.7716	0.8202	0.8264	0.8946					
		0.5390	0.5241	0.5227	0.5273	0.5302	0.5601	0.5757	0.5675	0.5669	0.6104	0.6025	0.6017	0.6456	0.6303	0.6991					
		0.1461	0.1577	0.1621	0.1579	0.1502	0.1646	0.1713	0.1702	0.1637	0.1768	0.1739	0.1766	0.1822	0.1854	0.1906					
	0.3	0.3464	0.3690	0.4094	0.4164	0.4116	0.5026	0.5412	0.5454	0.5454	0.5904	0.6304	0.6356	0.6964	0.7204	0.8116					
		0.6065	0.5626	0.5679	0.5753	0.5826	0.6260	0.6267	0.6119	0.6142	0.6563	0.6475	0.6471	0.6836	0.6749	0.7266					
		0.1356	0.1402	0.1541	0.1493	0.1475	0.1413	0.1536	0.1539	0.1506	0.1539	0.1560	0.1560	0.1636	0.1623	0.1655	0.1695				
	0.5	0.2574	0.2634	0.3216	0.3422	0.3516	0.3876	0.4232	0.4566	0.4614	0.4942	0.5276	0.5592	0.6094	0.6416	0.7412					
		0.6552	0.6193	0.6067	0.6130	0.6255	0.6745	0.6596	0.6504	0.6502	0.6963	0.6791	0.6731	0.7166	0.7064	0.7567					
		0.1226	0.1404	0.1453	0.1484	0.1446	0.1340	0.1365	0.1461	0.1456	0.1364	0.1446	0.1456	0.1481	0.1516	0.1549					
	0.7	0.1690	0.2214	0.2504	0.2554	0.2564	0.2916	0.3412	0.3594	0.3760	0.3976	0.4292	0.4624	0.5034	0.5356	0.6414					
		0.7047	0.6595	0.6376	0.6357	0.6320	0.7229	0.6999	0.6866	0.6910	0.7396	0.7210	0.7100	0.7595	0.7412	0.7917					
		0.1096	0.1327	0.1501	0.1566	0.1511	0.1191	0.1262	0.1436	0.1442	0.1264	0.1352	0.1425	0.1321	0.1431	0.1384					
0.9	0.1076	0.1366	0.1630	0.1630	0.1624	0.1834	0.2120	0.2370	0.2656	0.2720	0.2920	0.3196	0.3576	0.3826	0.4976						
	0.7763	0.7271	0.6844	0.6766	0.6996	0.7677	0.7669	0.7491	0.7310	0.8032	0.7846	0.7627	0.8172	0.7950	0.8392						
	0.0964	0.1250	0.1605	0.1641	0.1763	0.1908	0.1145	0.1366	0.1496	0.1055	0.1162	0.1359	0.1096	0.1265	0.1129						
0.95	0.1	0.4992	0.5404	0.5440	0.5356	0.5704	0.6396	0.6736	0.6816	0.6606	0.7450	0.7668	0.7874	0.8126	0.8406	0.9036					
		0.5395	0.5136	0.5191	0.5253	0.5276	0.5620	0.5590	0.5636	0.5716	0.5961	0.5936	0.6027	0.6395	0.6359	0.7025					
		0.1475	0.1673	0.1620	0.1512	0.1484	0.1654	0.1771	0.1713	0.1666	0.1716	0.1606	0.1792	0.1776	0.1661	0.1664					
	0.3	0.3446	0.4176	0.4144	0.4056	0.4192	0.5066	0.5342	0.5530	0.5566	0.6094	0.6266	0.6394	0.7014	0.7344	0.8234					
		0.6043	0.5619	0.5732	0.5699	0.5653	0.6354	0.6094	0.6122	0.6130	0.6614	0.6352	0.6399	0.6837	0.6706	0.7290					
		0.1362	0.1669	0.1613	0.1545	0.1410	0.1455	0.1694	0.1621	0.1516	0.1545	0.1696	0.1627	0.1625	0.1683	0.1666					
	0.5	0.2626	0.3236	0.3306	0.3266	0.3256	0.3954	0.4596	0.4552	0.4664	0.4960	0.5516	0.5474	0.6090	0.6272	0.7376					
		0.6441	0.5661	0.5921	0.6127	0.6277	0.6796	0.6404	0.6431	0.6469	0.6990	0.6650	0.6772	0.7204	0.6966	0.7580					
		0.1156	0.1732	0.1676	0.1530	0.1474	0.1329	0.1713	0.1592	0.1466	0.1406	0.1654	0.1600	0.1465	0.1613	0.1575					
	0.7	0.1644	0.2734	0.2652	0.2716	0.2662	0.3046	0.3764	0.3626	0.3750	0.3976	0.4546	0.4466	0.5032	0.5376	0.6496					
		0.6960	0.5977	0.6171	0.6261	0.6575	0.7192	0.6495	0.6706	0.6652	0.7400	0.6956	0.7044	0.7511	0.7292	0.7954					
		0.1066	0.1650	0.1895	0.1800	0.1502	0.1202	0.1856	0.1677	0.1504	0.1285	0.1673	0.1535	0.1267	0.1573	0.1366					
0.9	0.1002	0.1632	0.1950	0.1806	0.1654	0.1756	0.2670	0.2712	0.2646	0.2666	0.3360	0.3212	0.3572	0.4124	0.5066						
	0.7721	0.6145	0.6370	0.6606	0.6664	0.7905	0.6972	0.6904	0.7166	0.8011	0.7299	0.7472	0.8126	0.7630	0.8406						
	0.0923	0.2279	0.2204	0.1963	0.1616	0.1027	0.2035	0.1667	0.1742	0.1063	0.1639	0.1656	0.1074	0.1645	0.1145						

APPENDIX 5.C. A numerical example of a hypergame situation in merger negotiations

Assume that Player A (the acquirer) bases its decision regarding a choice of strategy on the submatrix G_0' of expected gains as defined in §5.3.1. Thus A's perception of the strategies available to itself and its opponent (Player B, the target) are

$$A' = \{a_2, a_{13}, a_{24}, a_{30}, a_{33}, a_{42}, a_{51}, a_{59}\}$$

and $B' = \{b_1, b_4, b_7, b_{10}, b_{13}, b_{16}, b_{19}\}.$

This matrix yielded a unique solution point $\langle a_2, b_{16} \rangle$ with $E(\text{gain to acquirer}) = 0.5227$ and $E(\text{gain to target}) = 0.3431.$

Assume also that Player B's perception of the situation is somewhat different: B perceives that the available strategies to the two players are

$$A^* = \{a_1, a_6, a_9, a_{13}\} \quad \text{to the acquirer,}$$

and $B^* = \{b_5, b_9, b_{15}, b_{18}\} \quad \text{to the target,}$

which yields a matrix G_0^* of expected gains as follows:

		a_1	a_6	a_9	a_{13}
		$\bar{A}(.1;.1,.3)$	$\bar{A}(.3;.3,.3)$	$\bar{A}(.3;.9,.3)$	$\bar{A}(.7;.7,.3)$
b_5	$\bar{B}(.9;.05)$	0.0802	0.1450	0.3315	0.2832
		0.0684	0.1052	0.2048	0.1543
b_9	$\bar{B}(.7;.35)$	0.1261	0.2169	0.4428	0.3773
		0.1195	0.1716	0.3397	0.2224
b_{15}	$\bar{B}(.9;.65)$	0.0809	0.1425	0.4355	0.2879
		0.0683	0.1023	0.3335	0.1570
b_{18}	$\bar{B}(.5;.95)$	0.1684	0.2646	0.4383	0.4280
		0.1741	0.2253	0.4120	0.2645

After elimination of dominated strategies relative to the other's complete strategy set, the associated reduced game G_1^* is

		a_9
		$\bar{A}(.3; .9, .3)$
b_9	$\bar{B}(.7; .35)$	0.4428 0.3397
b_{18}	$\bar{B}(.5; .95)$	0.4383 0.4120

Player B's choice is b_9 (since $0.4428 > 0.4383$), and the optimal strategy pair is $\langle a_9, b_9 \rangle$ with $E(\text{gain to acquirer}) = 0.3397$ and $E(\text{gain to target}) = 0.4428$.

The acquirer's view of the game gives itself an expected gain of 0.5227, considerably more than that afforded it by the target (0.3397) in its perception of the game. Similarly, the target sees itself as expecting a gain of 0.4428, a lot more than that expected in the acquirer's view of the game (0.3431).

Thus the acquirer will play strategy a_2 (anticipating the target to play b_{16}), and the target will play b_9 (in the expectation of the acquirer playing a_9). The actual outcome of $\langle a_2, b_9 \rangle$ yields

$$E(\text{gain to acquirer}) = 0.2648 < 0.3397 = E(G_{A(B)}) < 0.5227 = E(G_{A(A)})$$

$$\text{and } E(\text{gain to target}) = 0.2399 < 0.3431 = E(G_{B(A)}) < 0.4428 = E(G_{B(B)}),$$

where $E(G_{I(J)})$ is the expected gain for Player I as perceived by Player J.

Both A and B thus end up with expected gains far below what either player anticipated, since their initial perceptions of the game were different.

CHAPTER 6

FURTHER SIMULATION RESULTS: THE EFFECT OF THE CONTEXTUAL MODEL PARAMETERS

§6.1 INTRODUCTION

In Chapter 5 a Monte Carlo simulation procedure was constructed to execute the multi-stage merger bargaining model with full and symmetric uncertainty. This procedure was implemented for a particular set of contextual model parameter values, and the effect of varying both players' strategy parameters was investigated. Furthermore we were able to identify strategies which were Pareto-optimal to the players using a process of successive elimination of dominated strategies as well as applying a Bayesian approach.

This initial simulation did not take any account of the effect of changes to the contextual parameters. These parameters describe the environment within which the negotiations take place as perceived by the two players, and include the players' perceived uncertainties about the others' reservation price, the players' joint propensity to identify the difference in their reservation prices, and their perceived relative dominance at the negotiating table. In the next section of this chapter we will examine the effect of a change in the value of each of the contextual parameters separately, specifically focusing on (i) how the players' optimal bargaining strategies change, and (ii) how the players' expected gains at the optimal bargaining strategies change. The aim here is thus to perform a sensitivity analysis with respect to the parameters in question. The chapter concludes with some comments on the realism and applicability of the model for decision support, and some general observations which might prove useful from a decision support point of view.

§6.2 THE EFFECT OF A VARIATION IN THE VALUES OF THE MODEL'S CONTEXTUAL PARAMETERS

§6.2.1 VARIATION IN THE TARGET'S UNCERTAINTY, σ_B

In the development of the model in Chapter 4 we assumed that the acquirer's reservation price, V_A , might not be known with absolute certainty to the target. This uncertainty in V_A as perceived by the target was modelled in terms of a normal distribution with variance σ_B^2 . In this section we will examine the effect on the players' optimal bargaining strategies, and their expected gains at these optimal bargaining strategies, of changes in σ_B . It is important to note at this stage that σ_B is *not* a strategy parameter: it merely represents the degree to which the target's management believe that they are able to identify the true value of V_A , and only alters if further information concerning the acquirer (and its attitude towards the target) becomes available.

The contextual parameter σ_B only appears in setting the target's initial acceptance level, A_1 . However, since acceptance level A_j , $j \geq 2$, is based partially on acceptance level A_{j-1} , all acceptance levels A_j are dependent on σ_B . This parameter does not affect the acquirer's offer stream in any way. The model has

$$A_1 = v_B + \delta_B (1 + \sigma_B \Phi^{-1}(\beta_B))$$

where v_B is value of the target's known reservation price,

δ_B is the target's perceived relative dominance, $0 \leq \delta_B \leq 1$,

and β_B is the target's strategic concession parameter, $0 \leq \beta_B \leq 1$.

For simulation purposes we arbitrarily normalised the target's reservation price to 0, and thus

$$A_1 = \delta_B (1 + \sigma_B \Phi^{-1}(\beta_B)) .$$

The target's strategic concession parameter β_B determines whether the target will set its initial acceptance level high (i.e. above its own expectation of a fair price; $\beta_B > 0.5$) or low (i.e. below its own expectation of a fair price; $\beta_B < 0.5$), and σ_B serves only to act as a multiplicative "magnifying" factor of this strategy. Thus for any value of σ_B (≥ 0), if $\beta_B > 0.5$ (and so $\Phi^{-1}(\beta_B) > 0$) we have that $A_1 > \delta_B$ (the target's expectation of a fair

price), and if $\beta_B < 0.5$ (i.e. $\Phi^{-1}(\beta_B) < 0$) we have that $A_1 < \delta_B$. The effect of an increase in σ_B is merely to increase the amount that A_1 deviates from δ_B (above or below).

The simulation was rerun with both players' perceived dominances δ_A and δ_B set equal to 0.5, the acquirer's uncertainty in the estimation of V_B , σ_A , set to 0.1, and the players' propensity to estimate the difference in their reservation prices, σ , set to 0.1. The value of σ_B was varied from 0.1 to 0.25 in steps of 0.05.¹ For ease of assimilation the 7×8 submatrix G_0' formed by choosing subsets $A' \subset A$ and $B' \subset B$ of acquirer and target strategies (as in Chapter 5) will again be used as a starting matrix in analysing the effect of the change in σ_B . The matrices G_0' of players' expected gains for these values of σ_B can be found in Appendix 6.A. The players' optimal bargaining strategies on G_0' , and their resulting expected gains based on the simulations are displayed in Table 6.1.

We observe that, barring one exception, the optimal bargaining strategy is always $\langle a_2, b_{16} \rangle$ for all levels of $\sigma_B \geq \sigma = 0.1$ and for both approaches to identifying the optimal strategy. (The one exception is the BDM approach for $\sigma_B = 0.1$, where the optimal acquirer strategy is a_{24}). This very robust optimal strategy $\langle a_2, b_{16} \rangle$ implies that the acquirer should initially take a hard-line approach, with an initial offer well below its expectation of a fair price, but be prepared to negotiate persistently, if necessary, up to a final offer level still well below its expectation of a fair price. When the target is uncertain of the true value of V_A it should in general take a soft-line approach, starting with a very low initial acceptance level and rapidly yielding towards the acquirer's initial offer. The players' expected gains (under the EDS approach) as σ_B varies are graphically displayed in Figure 6.1.

¹ Since the two players do not share their reservation prices with each other, there is inherent uncertainty to both players as to the true value of the difference in their reservation prices, $V_A - V_B$. A lack of full knowledge about the other would have the effect of contributing to the uncertainty each has in estimating the other's reservation price. Thus for all practical purposes $\sigma_A \geq \sigma$ and $\sigma_B \geq \sigma$. The model could, of course, produce simulated results for the case $\sigma_A < \sigma$ or $\sigma_B < \sigma$, but since these would only arise out of extreme measures being taken by one or both players, and since we can not anticipate the players' behaviour for these levels of player's uncertainties, we will omit them entirely from this study.

Table 6.1. Optimal bargaining strategies and resulting expected gains as σ_B varies
 ($\delta_A = \delta_B = 0.5$; $\sigma = \sigma_A = 0.1$)

σ_B	Elimination of dominated strategies (EDS)		Bayesian decision-making approach (BDM)	
	Optimal strategy	E(gain to acquirer) E(gain to target)	Optimal strategy	E(gain to acquirer) E(gain to target)
0.10	$\langle a_2, b_{16} \rangle$	0.4716 0.3903	$\langle a_{24}, b_{16} \rangle$	0.4276 0.4256
0.15	$\langle a_2, b_{16} \rangle$	0.5007 0.4081	$\langle a_2, b_{16} \rangle$	0.5007 0.4081
0.20	$\langle a_2, b_{16} \rangle$	0.5158 0.4140	$\langle a_2, b_{16} \rangle$	0.5158 0.4140
0.25	$\langle a_2, b_{16} \rangle$	0.5346 0.4237	$\langle a_2, b_{16} \rangle$	0.5346 0.4237

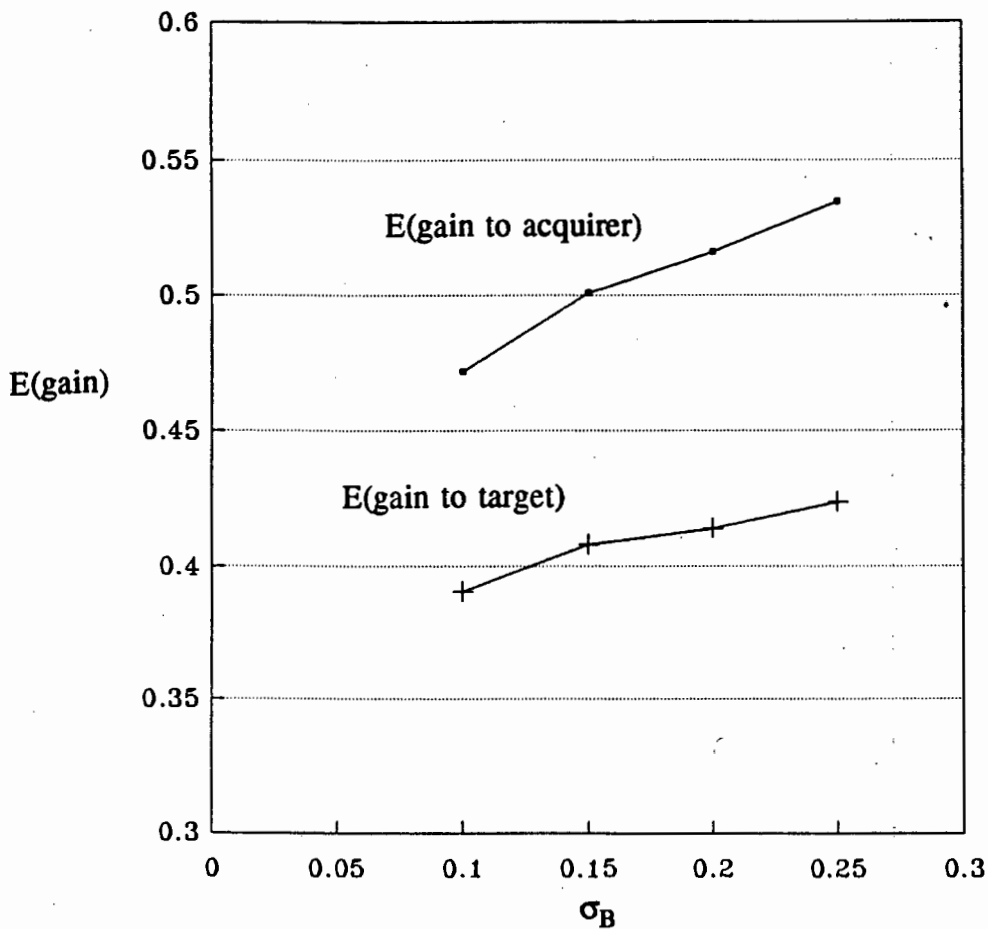


Figure 6.1. Players' expected gains as σ_B varies (under EDS)

Both players expect their gains to increase as the target's uncertainty regarding V_A increases, with the acquirer's gains increasing at a greater rate on average (the average slope of the expected gain to the acquirer is 0.4200, whilst the average slope of the target's expected gain is 0.2227). At first glance the concept of increasing uncertainty being rewarded with increasing expected gains might appear counter-intuitive. However the optimal target strategy here (b_{16}) has $\beta_B = 0.1$ and so $\Phi^{-1}(\beta_B) < 0$, implying $A_1 < \delta_B$. Thus an increase in σ_B will lead to an initial acceptance level A_1 which is further below δ_B , and thus closer to O_1 and O_L , the range within all offers will occur. This will necessarily increase the probability of merger agreement being reached, but at a lower transaction value if agreement is indeed reached. The probability of merger agreement and the average transaction value (under EDS) in the case of successful mergers for varying σ_B are shown in Figure 6.2.

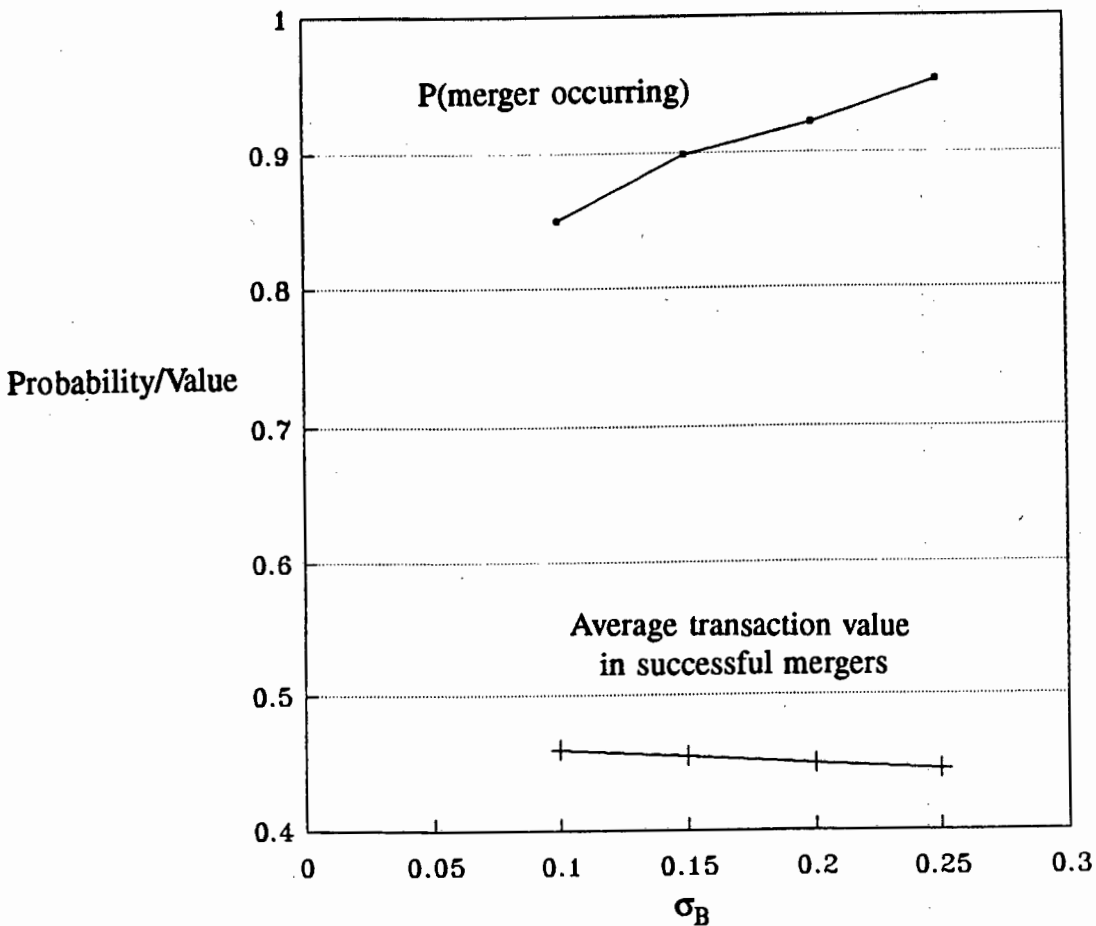


Figure 6.2. Probability of merger agreement and average transaction value in successful mergers as σ_B varies (under EDS)

In §5.2.1 we defined the expected gains to the two players in terms of the probability of merger agreement, $P(\text{merger occurring})$, and the average transaction value in successful merger agreements, Ave . On the basis of these definitions we would expect $E(\text{gain to the acquirer})$ to increase in any event, and $E(\text{gain to the target})$ to increase if $P(\text{merger occurring})$ increases faster than Ave (which it does, as is evident from the slopes of the respective lines in Figure 6.2). Thus the observed counter-intuitive effect of an increase in σ_B is due wholly to the optimal target strategy having $\beta_B < 0.5$; if the optimal target strategy had had $\beta_B > 0.5$ (i.e. the target choosing a more hard-line initial position relative to its expectation of a fair price, possibly a more intuitive response to increased uncertainty), the model predicts that both parties would have expected their gains to decrease as σ_B increased.

The important point in the above discussion is that A_1 decreases and lies closer to $[O_1; O_L]$, and this leads to increased expected gains to both players. Even if there is *no* change in σ_B the target could achieve the same result by simply *reducing its strategy parameter* β_B . Mathematically, β_B could be reduced to its lower limit (i.e. $\beta_B = 0$), leaving the simulated $A_1 = -\infty$ (which might not be an optimal strategy anyway). In practice, however, human nature on the part of the target would almost certainly play a role here: the target would balk at setting an obviously ridiculously low acceptance level. For this reason we have deemed 0.1 to be the lowest "reasonable" level for β_B in the simulation.

In terms of offering decision support, we thus note that the optimal bargaining strategy $\langle a_2, b_{16} \rangle$ appears to be relatively robust to variations in the target's uncertainty regarding the true value of the acquirer's reservation price, V_A . Furthermore, the nature of the target's strategy b_{16} is such that, providing that both parties stick to their optimal strategies, the probability of successful merger will increase rapidly as σ_B increases, whilst the average transaction value in successful agreements decreases only very slowly.

§6.2.2 VARIATION IN THE ACQUIRER'S UNCERTAINTY, σ_A

Recall that the acquirer's uncertainty regarding the target's true reservation price V_B was modelled as the standard deviation σ_A of some normal distribution on V_B . In this section we will investigate the effect of changes in this uncertainty about V_B on the

optimal strategies for the players and on the gains they would expect to receive at these optimal strategies.

The parameter σ_A appears in the model in setting the acquirer's initial offer O_1 , and the final possible offer O_L . Thus if σ_A was to change, so would the offer interval $[O_1; O_L]$ and thus all potential offers in this interval. Furthermore, since the target's acceptance level at each stage beyond the initial stage depends in part on the offer at the previous stage, each non-initial acceptance level A_2, A_3, \dots would also be affected by a change to σ_A . Note that the target's initial acceptance level A_1 is not affected by a change to σ_A . In the model the initial offer was represented as

$$O_1 = v_A - \delta_A (1 - \sigma_A \Phi^{-1}(\beta_{A1}))$$

and the final possible offer as

$$O_L = v_A - \delta_A (1 - \sigma_A \Phi^{-1}(\beta_{AL}))$$

where v_A is the known value of the acquirer's reservation price,

δ_A is the acquirer's perceived relative dominance, $0 \leq \delta_A \leq 1$,

β_{A1} is the acquirer's initial strategic concession parameter, $0 \leq \beta_{A1} \leq 1$,

and β_{AL} is the acquirer's final strategic concession parameter, $0 \leq \beta_{AL} \leq 1$, $\beta_{AL} > \beta_{A1}$.

For simulation purposes we arbitrarily viewed the negotiating position from the target's perspective; V_A was varied according to the distribution $\Delta_{AB} \sim N(1; \sigma^2)$, and we thus had

$$O_1 = \eta_{AB} - \delta_A (1 - \sigma_A \Phi^{-1}(\beta_{A1}))$$

and

$$O_L = \eta_{AB} - \delta_A (1 - \sigma_A \Phi^{-1}(\beta_{AL}))$$

where η_{AB} is a specific simulated outcome from Δ_{AB} . As in the case of the target's strategy, the acquirer's initial and final strategic concession parameters determine the charity of its offers; a value greater than 0.5 implies a generous offer (i.e. above the acquirer's expectation of a fair price) and a value less than 0.5 implies a hard-line one (below its expectation of a fair price). The uncertainty σ_A ($\geq \sigma$) again serves as a multiplicative factor of this strategy. Thus in the presence of acquirer uncertainty about V_B , if $\beta_{A1} > 0.5$ (and thus $\Phi^{-1}(\beta_{A1}) > 0$) we have that $O_1 > \eta_{AB} - \delta_A$, and if $\beta_{A1} < 0.5$ (i.e. $\Phi^{-1}(\beta_{A1}) < 0$) the initial offer is $O_1 < \eta_{AB} - \delta_A$. An increase in the value of σ_A results in a further increase of the deviation (up or down) of O_1 away from the acquirer's

expectation of a fair price, $\eta_{AB} - \delta_A$. An identical argument holds for determining the relative position of O_L from the value of β_{AL} .

To examine the effect of a change in the value of σ_A , the simulation was rerun with the relative dominances $\delta_A = \delta_B = 0.5$, the target's uncertainty about V_A set to 0.1, and σ set to 0.1. The value of σ_A was varied from 0.1 to 0.25 in discrete steps of 0.05, and the resulting submatrix G_0' was used as a starting matrix for determining optimal strategies. The submatrices G_0' for these values of σ_A can be found in Appendix 6.A. The optimal bargaining strategies on G_0' and their resulting expected gains to the players based on the simulation runs are displayed in Table 6.2.

Table 6.2. Optimal bargaining strategies and resulting expected gains as σ_A varies ($\delta_A = \delta_B = 0.5$; $\sigma = \sigma_B = 0.1$)

σ_A	Elimination of dominated strategies (EDS)		Bayesian decision-making approach (BDM)	
	Optimal strategy	E(gain to acquirer) E(gain to target)	Optimal strategy	E(gain to acquirer) E(gain to target)
0.10	$\langle a_2, b_{16} \rangle$	0.4716 0.3903	$\langle a_{24}, b_{16} \rangle$	0.4276 0.4256
0.15	$\langle a_2, b_{16} \rangle$	0.4694 0.3530	$\langle a_{24}, b_{16} \rangle$	0.4118 0.4060
0.20	$\langle a_2, b_{16} \rangle$	0.4665 0.3214	$\langle a_{59}, b_{16} \rangle$	0.3970 0.5081
	$\langle a_{59}, b_1 \rangle$	0.3979 0.5096		
0.25	$\langle a_2, b_{16} \rangle$	0.4668 0.2959	$\langle a_{59}, b_{16} \rangle$	0.3938 0.5280
	$\langle a_{59}, b_1 \rangle$	0.3962 0.5312		

Since the classical game-theoretic method of finding an optimal solution point and strategies by successive elimination of player's dominated strategies differs fundamentally

from the Bayesian approach to optima location, we might expect that the optimal strategies (and hence players' expected gains) determined by the two methods may differ from one another at different values of certain contextual parameter values (notably σ_A here). For this reason we will examine the effect of a change in σ_A for each of the two approaches (EDS and BDM) separately.

As σ_A increases from 0.1 the players' optimal strategies under EDS exhibit two distinct phases. Firstly, as the value of σ_A increases to moderate levels ($0.10 \leq \sigma_A \leq 0.15$) the acquirer's optimal strategy is a_2 , which indicates a hard-line positioning of all the acquirer's offers, but with a willingness to negotiate persistently within this fairly narrow range. The target's strategy b_{16} is to start with a low acceptance level but give up ground very rapidly as the negotiations proceed. Secondly, as σ_A increases to high levels ($\sigma_A \geq 0.2$) the players are faced with a choice of strategy, since there are two equilibrium points now. One possibility is for the players to remain with the strategy pair $\langle a_2, b_{16} \rangle$ (as at more moderate levels of acquirer uncertainty), and the other is to switch to $\langle a_{59}, b_1 \rangle$ (in which the acquirer starts with a very generous offer but with very little penchant to negotiate further, whilst the target still starts with a low acceptance level but is disinclined to yield much). The player's expected gains for varying σ_A for the EDS approach are depicted in Figure 6.3.

Whilst the optimal strategy pair remains the same (at $\langle a_2, b_{16} \rangle$) and σ_A increases above 0.1, the offer interval $[O_1; O_L]$ shifts downwards, lying completely below the acquirer's expectation of a fair price, $\eta_{AB} - \delta_A$ (since $\Phi^{-1}(\beta_{A1}) < 0$ and $\Phi^{-1}(\beta_{AL}) < 0$ at acquirer strategy a_2). Since the acquirer's degree of persistence remains constant, the probability of merger occurring can be expected to decrease, as will the average transaction value if agreement is reached, since the target will have to yield more in order to "reach" the acquirer's offer interval. Figure 6.4 shows the simulated probability of merger and the average transaction value in the case of successful agreement. The expected gain to the target for this strategy will thus decrease sharply as σ_A increases, whilst the acquirer's expected gain, made up of the product of a decreasing term (P(merger occurring)) and an increasing term ($\frac{1}{k} \sum_{h=1}^k \eta_{AB}^h$ - Average transaction value, where the term $\frac{1}{k} \sum_{h=1}^k \eta_{AB}^h$ effectively remains constant since it is independent of σ_A),

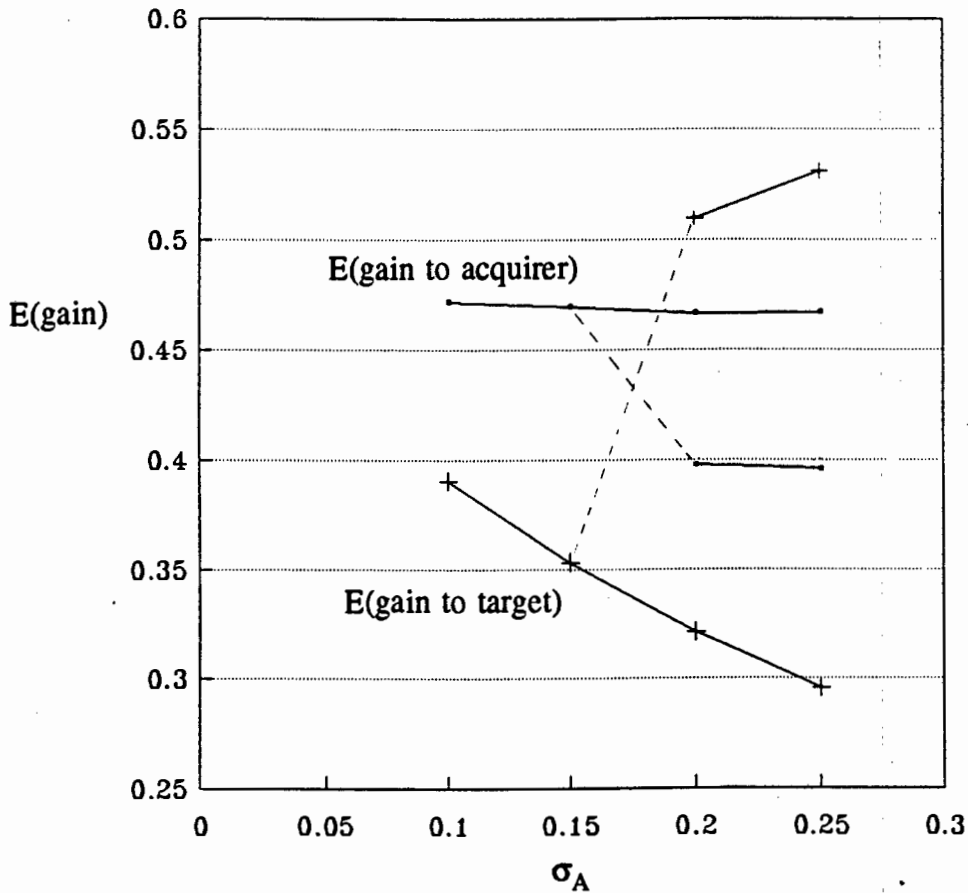


Figure 6.3. Player's expected gains as σ_A varies (EDS approach)

will not be expected to vary a lot. Finally, as σ_A reaches high levels, the strategy pair $\langle a_{59}, b_1 \rangle$ becomes a co-optimal equilibrium point. This strategy has $\Phi^{-1}(\beta_{A1}) > 0$, and thus the offer interval will be situated well above the acquirer's expectation of a fair price. Thus all offers are generous (even though the acquirer's propensity for making further offers is low), which, combined with the target's optimal strategy of a low initial acceptance level, will lead to a very high probability of success at a high transaction value. This combination clearly favours the target company, a fact borne out by Figure 6.3. Again we note that the EDS optimal strategy is stable for moderate levels of acquirer uncertainty. However when this uncertainty gets large the model is unable to identify a single optimal point (multiple equilibrium points exist), which renders it inadequate for meaningful decision support when σ_A alone reaches these levels of uncertainty.

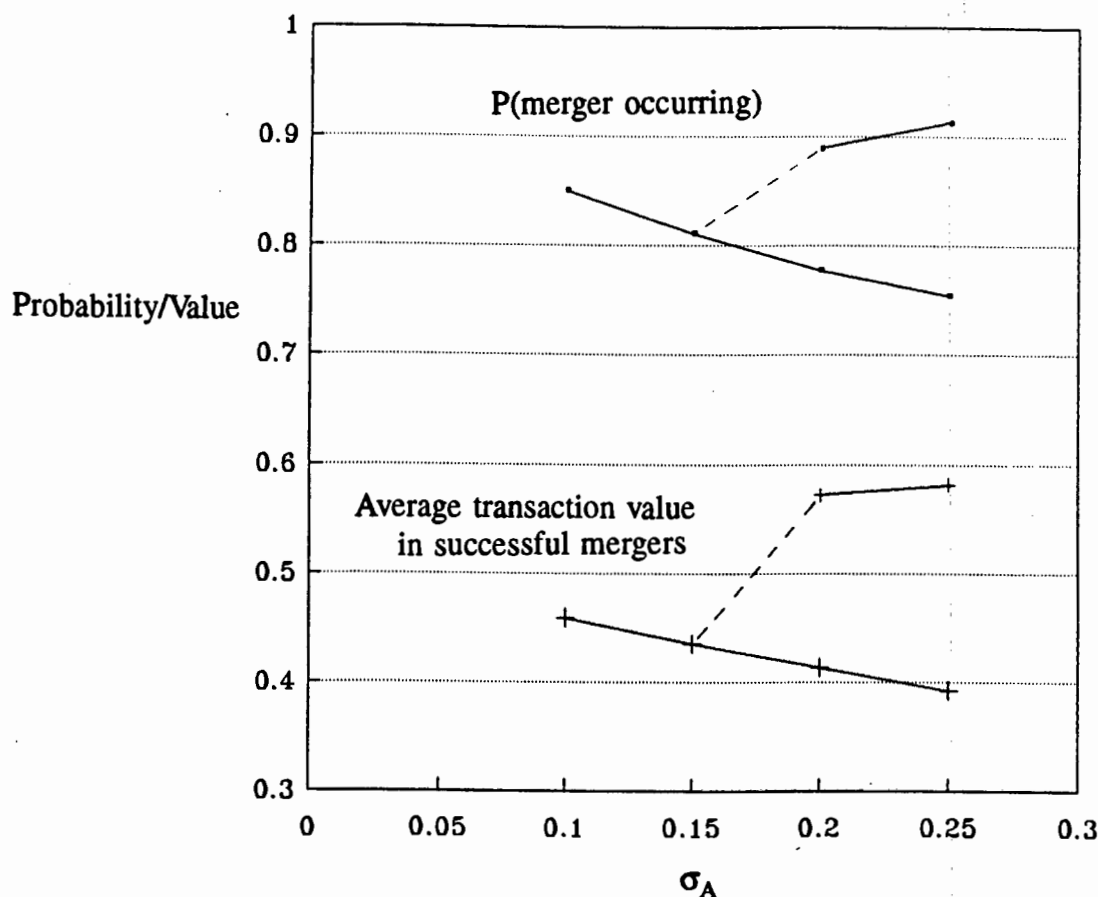


Figure 6.4. Probability of merger agreement and average transaction value in successful mergers as σ_A varies

Turning to the BDM approach we notice that an increase in σ_A has no effect on the target's optimal strategy, b_{16} . The acquirer's optimal strategy, however, does vary with σ_A . For moderate levels of σ_A ($0.1 \leq \sigma_A \leq 0.15$) the optimal acquirer strategy is a_{24} , switching to a_{59} for high levels of acquirer uncertainty about V_B , $\sigma_A \geq 0.2$. Two points are worth noting. Firstly, the strategy $\langle a_{59}, b_{16} \rangle$ is the cooperative strategy (in the sense of §5.3.2) arising out of the dual equilibrium points $\langle a_2, b_{16} \rangle$ and $\langle a_{59}, b_1 \rangle$. Secondly, a_{24} represents moderately persistent negotiating by the acquirer within a wide offer interval straddling the acquirer's expectation of a fair price, whilst a_{59} represents a very high initial offer but with a very low inclination to negotiate. Thus the BDM approach tends to indicate that increased acquirer uncertainty should lead to an increased initial offer position, but with less inclination to enter into protracted negotiations.

Thus whilst the two approaches to identifying optimal player strategies appear to yield slightly different outcomes, they both have highlighted the fact that as soon as the acquirer's uncertainty about V_B exceeds a certain value, an instability sets in, making strategy choice extremely difficult. The Bayesian approach might prove more useful as a tool for decision support in this instance since it offers a single optimal strategy pair which is in a sense a compromise between the two offered by the more conventional game-theoretic techniques.

§6.2.3 VARIATION IN THE LEVEL OF σ , THE PLAYERS' FUNDAMENTAL PROPENSITY TO DETERMINE THE TRUE DIFFERENCE IN THEIR RESERVATION PRICES

We now turn to investigating the effect on the players' optimal strategies and expected gains of a change in the level of σ , the inherent propensity that both players have to determine the true difference in the companies' reservation prices. We have argued that for the purposes of this study it is reasonable to assume that the individual players' uncertainties about the others' reservation price (σ_A and σ_B) are both at least as great as σ . We will start by observing the effect of a change in σ when $\sigma_A = \sigma_B = \sigma$, and later examine the situation when $\sigma_A = \sigma_B = 1.5\sigma$. Thus a change in σ will necessarily be accompanied by a change in σ_A and σ_B as well.

We have already examined the effect of a change in σ_A and in σ_B individually in §6.2.2 and §6.2.1 respectively. We showed that for the optimal strategy pair an increase in σ_B led to a decrease in the target's initial acceptance level, A_1 , and an increase in σ_A led to a downward shift in the entire offer interval $[O_1; O_L]$. The combined effect of these changes is that the entire negotiating process would simply take place at a somewhat lower level. This would have little effect on the probability of agreement being reached, but would naturally reduce the average transaction value in the case of successful agreements, facts borne out by the trends evident in Figures 6.2 and 6.4.

In the model we defined $\Delta_{AB} \sim N(1; \sigma^2)$, and in the simulation implementation of the model we had

$$O_1 = \eta_{AB} - \delta_A (1 - \sigma_A \Phi^{-1}(\beta_{A1}))$$

and

$$O_L = \eta_{AB} - \delta_A (1 - \sigma_A \Phi^{-1}(\beta_{AL}))$$

where η_{AB} is a specific simulated outcome from Δ_{AB} . The effect of an increase in σ would be to make the simulated position of the offer interval more variable, providing all the other parameters are held constant.

The simulation procedure was rerun with $\sigma = \sigma_A = \sigma_B$ varying from 0.1 to 0.5 in discrete steps of 0.1. The results (using submatrix G_0' as starting matrix for the analysis) are presented in Table 6.3. The submatrices G_0' for these cases can be found in Appendix 6.A.

Table 6.3. Optimal bargaining strategies and resulting expected gains
as $\sigma = \sigma_A = \sigma_B$ varies ($\delta_A = \delta_B = 0.5$)

σ	Elimination of dominated strategies (EDS)		Bayesian decision-making approach (BDM)	
	Optimal strategy	E(gain to acquirer) E(gain to target)	Optimal strategy	E(gain to acquirer) E(gain to target)
0.1	$\langle a_2, b_{16} \rangle$	0.4716 0.3903	$\langle a_{24}, b_{16} \rangle$	0.4276 0.4256
0.2	$\langle a_2, b_{16} \rangle$	0.5128 0.3569	$\langle a_2, b_{16} \rangle$	0.5128 0.3569
0.25*	$\langle a_2, b_{16} \rangle$	0.5227 0.3431	$\langle a_2, b_{16} \rangle$	0.5227 0.3431
0.3	$\langle a_2, b_{16} \rangle$	0.5290 0.3321	$\langle a_2, b_{16} \rangle$	0.5290 0.3321
0.4	$\langle a_2, b_{16} \rangle$	0.5376 0.3165	$\langle a_2, b_{16} \rangle$	0.5376 0.3165
0.5	$\langle a_2, b_{16} \rangle$	0.5519 0.3058	$\langle a_2, b_1 \rangle$	0.4908 0.3015

* This case corresponds to the initial simulation examined in some detail in §5.3.1

The EDS approach indicates that the optimal strategy pair, $\langle a_2, b_{16} \rangle$, is very stable under changes in σ , σ_A and σ_B within the ranges considered here. The BDM approach appears to again suffer from an "edge effect": at low levels of uncertainty the acquirer's strategy is at variance with that proposed by EDS and at high levels of uncertainty the target's optimal strategy differs. The expected gains to the players under EDS are presented in Figure 6.5.

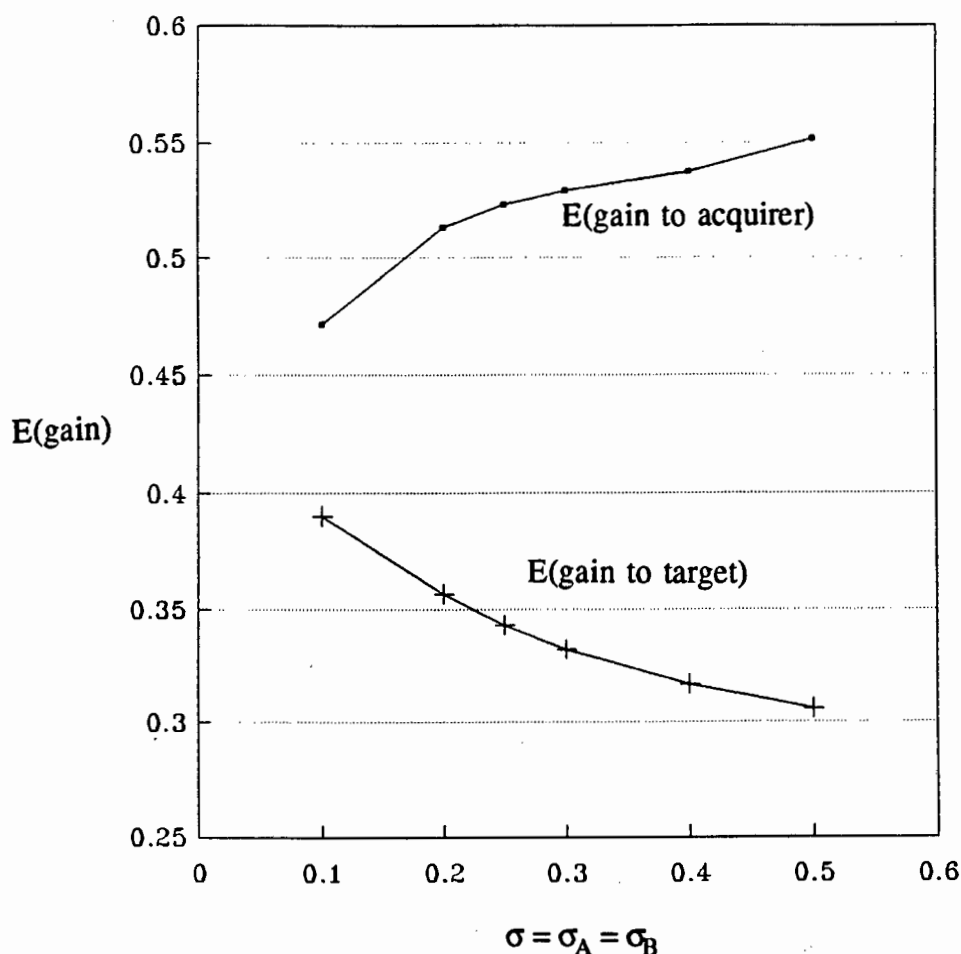


Figure 6.5. Players' expected gains under EDS for varying $\sigma = \sigma_A = \sigma_B$

We notice from Figure 6.5 that the acquirer's expected gain increases as the players' uncertainties increase (but at an ever-decreasing rate), whilst the target expects its gain to decrease. For the optimal strategy pair $\langle a_2, b_{16} \rangle$ we have shown that the combined effect of increasing σ , σ_A and σ_B is that the bargaining will take place at lower

levels, and the (simulated) position of the offer interval is more variable. In other words, increasing σ would increase the probability that $[O_U; O_L]$ is further from A_1 , thus reducing the chances of a successful bargaining agreement in these cases. This increase in the common propensity of the two players to determine the true magnitude of the zone of agreement, $V_A - V_B$, would similarly tend to increase the probability that the offer interval was closer to A_1 as well; this would tend to have little effect on the probability of agreement, since in these cases agreement would have been reached for smaller σ , but it would increase the average transaction value in successful cases. Thus combining the effect of an increase in σ_A and σ_B with that of an increase in σ indicates that we would expect a decrease in the probability of agreement without a material change in the average value at which agreement is reached. The components of the players' expected gains based on the simulated results are graphed in Figure 6.6.

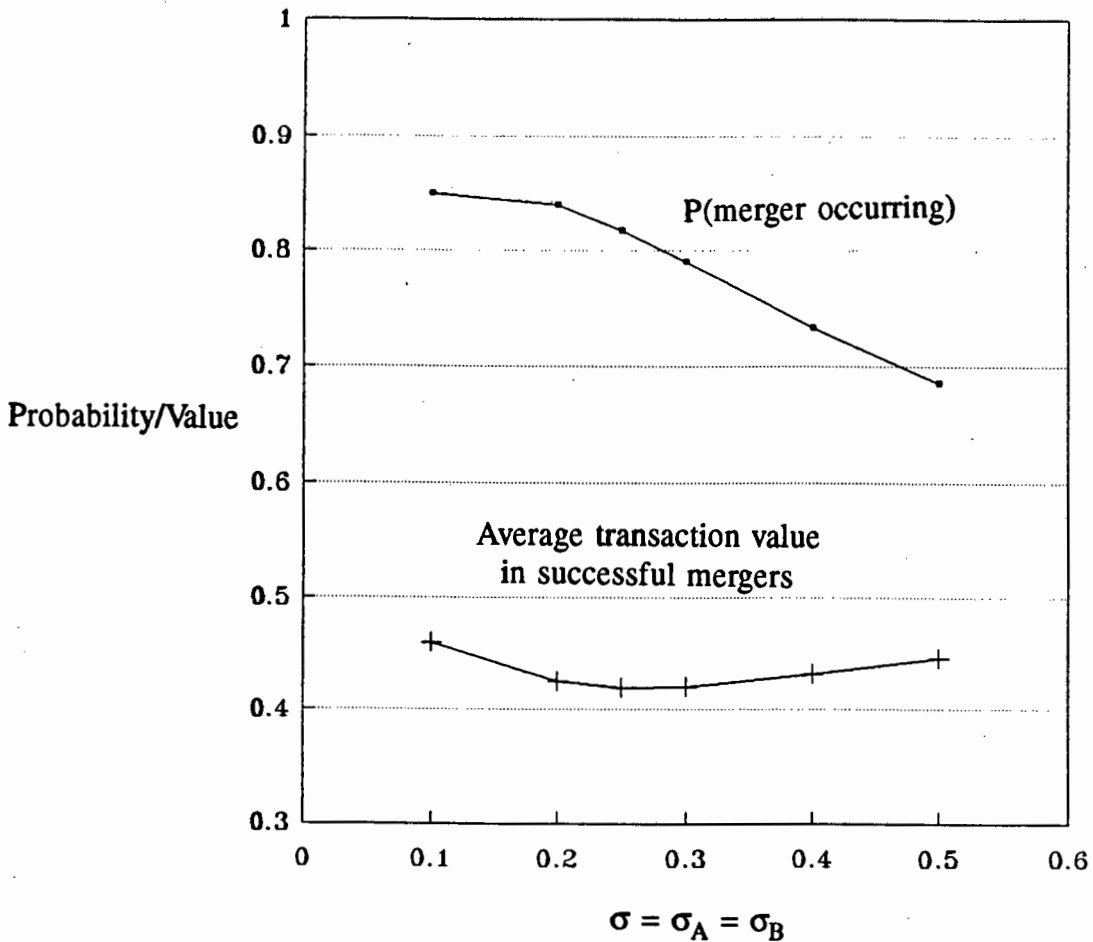


Figure 6.6. Probability of merger and average transaction value in successful mergers as $\sigma = \sigma_A = \sigma_B$ varies

The expected gain to the target comprises of the product of the above two terms. Since $P(\text{merger occurring})$ is decreasing whilst the average transaction value in successful mergers remains approximately constant, the expected gain to the target will decrease as σ , σ_A and σ_B increase. Turning now to the behaviour of the acquirer's expected gain as σ changes, recall that the acquirer's expected gain is

$$P(\text{merger occurring}) \cdot \frac{1}{k} \sum_{h=1}^k (\eta_{AB}^h - T_h)$$

where T_h is the simulated transaction value in the h^{th} successful merger, and k is the total number of successful agreements in M simulations, $k \leq M$. This can be written as

$$P(\text{merger occurring}) \cdot \frac{1}{k} \sum_{h=1}^k \eta_{AB}^h - E(\text{gain to target}) . \quad (6.1)$$

Now by definition $\Delta_{AB} \sim N(1; \sigma^2)$. Low values of η_{AB} where η_{AB} is a specific outcome from Δ_{AB} (say < 1) would tend to not lead to agreement, since this situation implies that the entire offer interval has been shifted downwards whilst A_1 remains constant. As σ increases, then, more and more cases would fall into this class. Thus the average simulated value of the η_{AB} *conditional on successful merger agreement* would tend to be somewhat *greater* than 1, and this would increase rapidly as σ increases. The first term in (6.1) thus consists of a decreasing term, $P(\text{merger})$, and an increasing term, $\frac{1}{k} \sum_{h=1}^k \eta_{AB}^h$, which results in a term which is approximately constant. Since we have already shown that $E(\text{gain to target})$ decreases as σ increases, the acquirer's expected gain increases with increasing σ . Thus greater levels of uncertainty appear to offer an opportunity to the acquirer, but to be detrimental to the target's aspirations of gains. In a real-world bargaining situation this means that as both players' uncertainties increase it becomes more likely that a very low offer will be accepted, an event which will, of course, strongly advantage the acquirer and disadvantage the target.

We turn now to the situation where $\sigma_A = \sigma_B > \sigma$. In particular, we examine the simulated results for the case $\sigma_A = \sigma_B = 1.5\sigma$. The results are displayed in Table 6.4. The optimal acquirer strategy is observed to remain unchanged at a_2 for all levels of σ and for both approaches to optima identification. The target's optimal strategy, however, is fairly unstable: at lowish levels of σ the optimal strategy is the familiar b_{16} , but switches

Table 6.4. Optimal bargaining strategies and resulting expected gains
as $\sigma_A = \sigma_B = 1.5\sigma$ varies ($\delta_A = \delta_B = 0.5$)

σ	Elimination of dominated strategies (EDS)		Bayesian decision-making approach (BDM)	
	Optimal strategy	E(gain to acquirer) E(gain to target)	Optimal strategy	E(gain to acquirer) E(gain to target)
0.1	$\langle a_2, b_{16} \rangle$	0.4991 0.3686	$\langle a_2, b_{16} \rangle$	0.4991 0.3686
0.2	$\langle a_2, b_{16} \rangle$	0.5627 0.3114	$\langle a_2, b_{16} \rangle$	0.5627 0.3114
0.3	$\langle a_2, b_{16} \rangle$	0.5848 0.2689	$\langle a_2, b_1 \rangle$	0.4988 0.2658
0.4	$\langle a_2, b_{13} \rangle$	0.4596 0.2518	$\langle a_2, b_1 \rangle$	0.5562 0.2427
0.5	$\langle a_2, b_{13} \rangle$	0.4694 0.2428	$\langle a_2, b_7 \rangle$	0.4903 0.2385

under both EDS and BDM to other strategies as σ increases. The Bayes optimal strategy is again more sensitive to the change in parameter values, switching at lower values of σ . This would tend to indicate that this approach is not sufficiently stable for use in a decision support role, and that the more traditional game-theoretic technique of eliminating player's dominated strategies should rather be pursued. Turning to the players' expected gains, where comparison to the case $\sigma = \sigma_A = \sigma_B$ is possible (i.e. where the optimal strategies are identical), the effect of increasing σ_A and σ_B to 1.5σ is to increase the acquirer's gains and to decrease the target's gains still further.

§6.2.4 VARIATION IN THE TARGET'S PERCEIVED RELATIVE DOMINANCE, δ_B

The target's perceived dominance, δ_B , appears in the model in determining the target's initial acceptance level, $A_1 = v_B + \delta_B (1 + \sigma_B \Phi^{-1}(\beta_B))$, and does not affect the

acquirer's offer stream. Providing that the target's optimal strategy and level of uncertainty about V_A are such that $\sigma_B \Phi^{-1}(\beta_B) > -1$ (the values for σ_B and β_B used so far in this study satisfy this criterion), the effect of an increase in δ_B is to shift the target's initial acceptance level upwards.¹

The simulation procedure was run with $\sigma = \sigma_A = \sigma_B = 0.25$ and the acquirer's relative dominance constant at 0.5 whilst δ was varied from 0.3 to 0.7 in steps of 0.1. The case $\delta_B = 0.5$ is simply the initial simulation discussed at length in §5.3. The optimal bargaining strategies on G_0' and the expected gains are shown in Table 6.5.

Table 6.5. Optimal bargaining strategies and resulting expected gains as δ_B varies ($\sigma = \sigma_A = \sigma_B = 0.25$; $\delta_A = 0.5$)

δ_B	Elimination of dominated strategies (EDS)		Bayesian decision-making approach (BDM)	
	Optimal strategy	E(gain to acquirer) E(gain to target)	Optimal strategy	E(gain to acquirer) E(gain to target)
0.3	$\langle a_2, b_{16} \rangle$	0.5742 0.3504	$\langle a_2, b_{16} \rangle$	0.5742 0.3504
0.4	$\langle a_2, b_{16} \rangle$	0.5557 0.3493	$\langle a_2, b_{16} \rangle$	0.5557 0.3493
0.5	$\langle a_2, b_{16} \rangle$	0.5227 0.3431	$\langle a_2, b_{16} \rangle$	0.5227 0.3431
0.6	$\langle a_2, b_{16} \rangle$	0.4944 0.3336	$\langle a_2, b_{16} \rangle$	0.4944 0.3336
0.7	$\langle a_2, b_{16} \rangle$	0.4733 0.3247	$\langle a_2, b_{16} \rangle$	0.4733 0.3247

¹ Recall that in the derivation of the model the target's expectation of a fair price offer was merely δ_B . Thus the more dominant a target believes itself to be, the more it expects to be able to achieve from the negotiations.

For the range of δ_B values considered here we note that EDS and BDM both offer a single optimal bargaining strategy: the well-frequented strategy $\langle a_2, b_{16} \rangle$. The expected gains to the players are displayed in Figure 6.7.

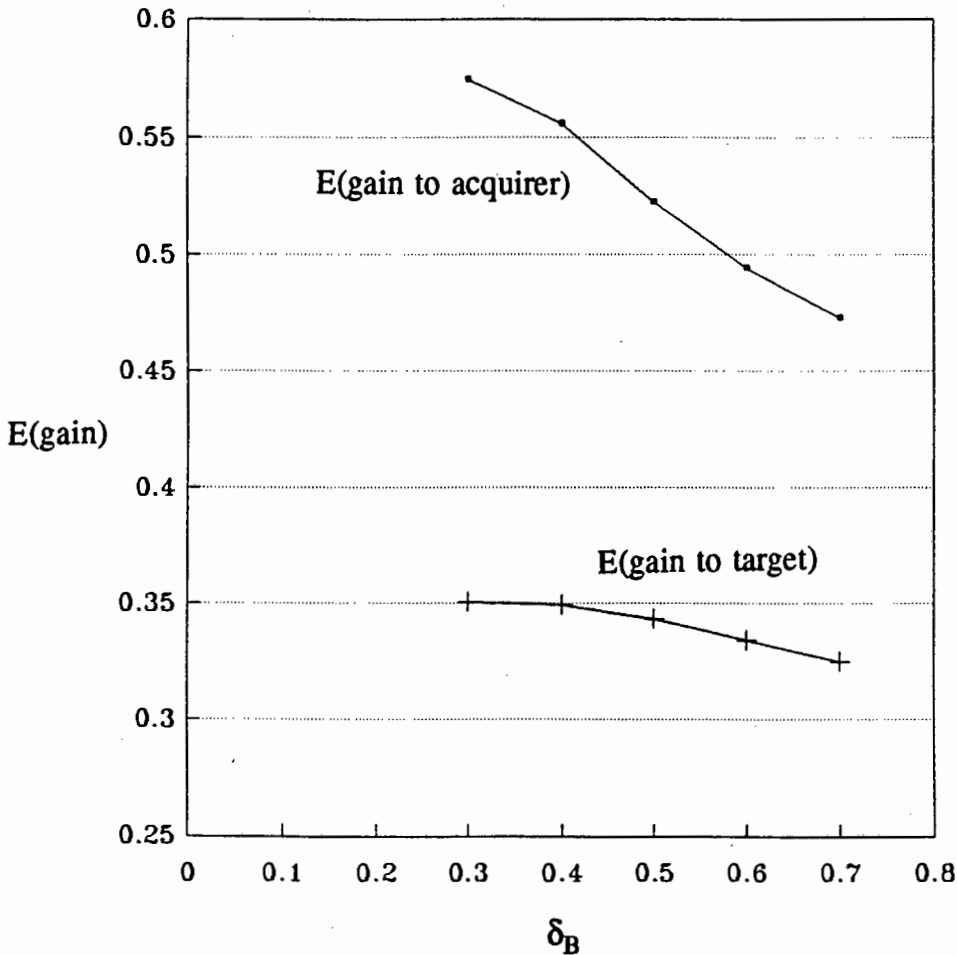


Figure 6.7. Players' expected gains for varying δ_B

Both players' expected gains decrease as δ_B (which in the model is also the target's expectation of a fair price) increases. The acquirer's expected gain, however, decreases at a far greater average rate than does the target's expected gain (the average rate of decrease for the acquirer is 0.2523, whilst for the target it is 0.0643). In practical terms, the more dominant the target perceives itself to be, the less the acquirer could expect to gain from the negotiations (providing the players stick to the optimal strategy pair). Now the target's initial acceptance level increases with increasing δ_B at the target's

optimal strategy, b_{16} (providing, of course, that σ_B remains at reasonable levels; if $\sigma_B > [-\Phi^{-1}(\beta_B)]^{-1} = (1.28)^{-1} = 0.7813$ then A_1 will, in fact, *decrease* with increasing δ_B , but we will not consider such large levels of uncertainty as being realistic), and the acquirer's offer interval is not affected. Obviously, then, the chances of reaching agreement become less. Since a modelling assumption was that the target revised its acceptance level downwards based partly on the acquirer's most recent offer, the target would tend to yield more as δ_B increases, leading to a transaction value which was, on average, only slightly higher than for lower values of δ_B . The probability of merger agreement and the average transaction value in successful mergers are shown in Figure 6.8.

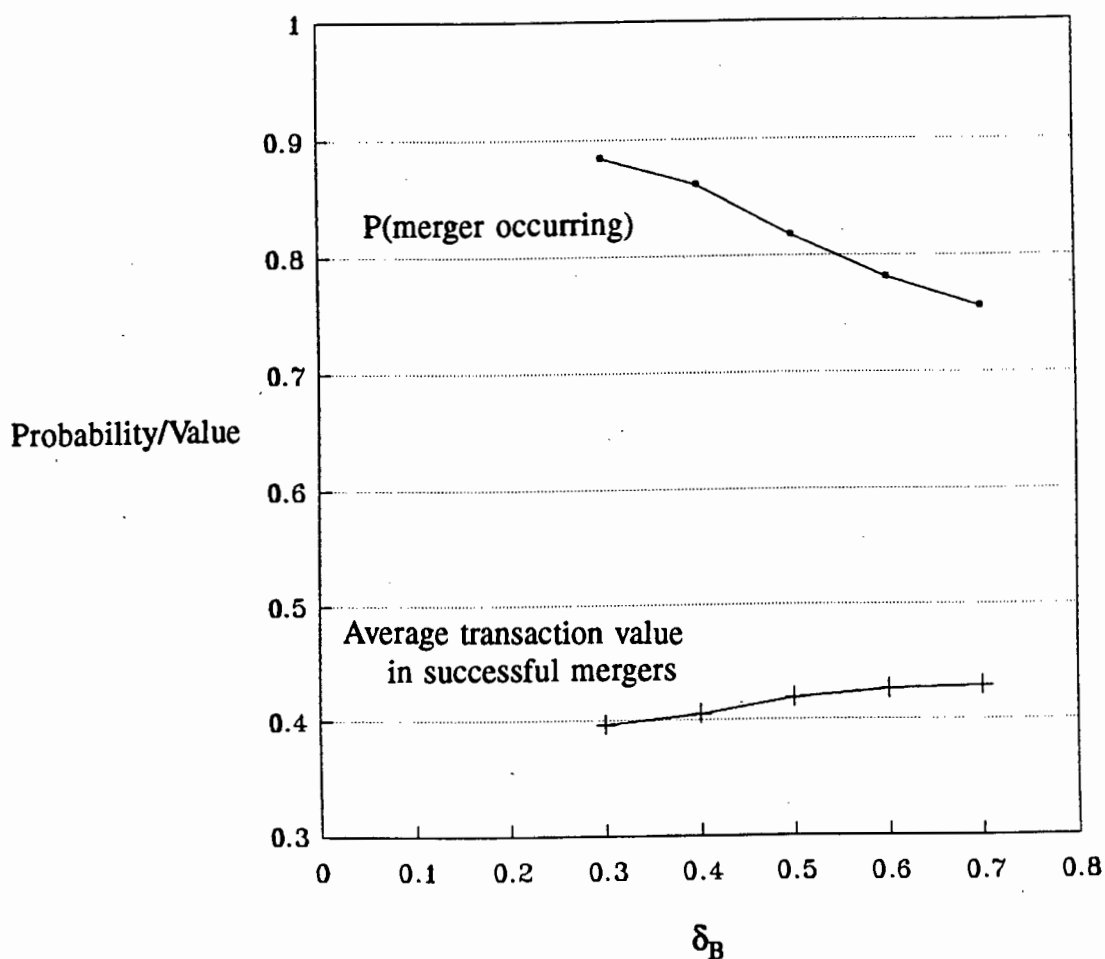


Figure 6.8. Probability of merger and average transaction value in successful mergers as δ_B varies

The expected gain to the target comprises of the product of the two terms graphed in Figure 6.8. Since $P(\text{merger occurring})$ decreases faster than the average transaction value increases, the target's expected gain will in actual fact decrease with increasing perceived negotiating power. This is due in the main to the target relenting quicker than the acquirer. The acquirer's expected gain is based on the product of two decreasing terms, implying a rapid decrease as δ_B increases.

From the above discussion, some useful observations can be made. Firstly, it is clear that the model provides a very stable optimal bargaining strategy as δ_B varies. Furthermore, the target is active in revising its acceptance levels downwards towards the acquirer's previous offer. Thus any change in the contextual parameter values that leads to the player's initial bargaining positions being further apart will be more detrimental to the target's expected gain than to the acquirer.

§6.2.5 VARIATION IN THE ACQUIRER'S PERCEIVED RELATIVE DOMINANCE, δ_A

The acquirer's perceived relative dominance, δ_A , appears in the model in setting up the acquirer's offer interval. In the simulation the offer interval was represented as

$$[O_1 = \eta_{AB} - \delta_A (1 - \sigma_A \Phi^{-1}(\beta_{A1})) ; O_L = \eta_{AB} - \delta_A (1 - \sigma_A \Phi^{-1}(\beta_{AL}))] .$$

Now providing that σ_A and the acquirer's strategy parameter β_{A1} are not too pathological (so that $\sigma_A \Phi^{-1}(\beta_{A1})$ remains less than 1), an increase in δ_A will lead to a downward shift in the offer interval providing that the players continue with their same strategies.

We examine the simulated results for $\sigma = \sigma_A = \sigma_B = 0.25$ and $\delta_B = 0.5$ while δ_A is varied from 0.3 to 0.7 in steps of 0.1. The results are shown in Table 6.6. We notice that the strategy pair $\langle a_2, b_{16} \rangle$ appears to be optimal for all values of δ_A except for large values: in this case the EDS approach suggests two possible solution (equilibrium) points (one of which is $\langle a_2, b_{16} \rangle$) and the Bayes optimal approach suggests that the acquirer should switch to strategy a_{24} .

The expected gains to the players are displayed in Figure 6.9. The acquirer's expected gain for the strategy pair $\langle a_2, b_{16} \rangle$ increases with increasing δ_A , flattening off for $\delta_A \geq 0.5$. The target's expected gain, on the other hand, decreases rapidly as δ_A increases. Thus if the acquirer's perception of its negotiating strength increases it will

Table 6.6. Optimal bargaining strategies and resulting expected gains as δ_A varies ($\sigma = \sigma_A = \sigma_B = 0.25$; $\delta_B = 0.5$)

δ_A	Elimination of dominated strategies (EDS)		Bayesian decision-making approach (BDM)	
	Optimal strategy	E(gain to acquirer) E(gain to target)	Optimal strategy	E(gain to acquirer) E(gain to target)
0.3	$\langle a_2, b_{16} \rangle$	0.3789 0.5881	$\langle a_2, b_{16} \rangle$	0.3789 0.5881
0.4	$\langle a_2, b_{16} \rangle$	0.4645 0.4600	$\langle a_2, b_{16} \rangle$	0.4645 0.4600
0.5	$\langle a_2, b_{16} \rangle$	0.5227 0.3431	$\langle a_2, b_{16} \rangle$	0.5227 0.3431
0.6	$\langle a_2, b_{16} \rangle$	0.5373 0.2321	$\langle a_2, b_{16} \rangle$	0.5373 0.2321
0.7	$\langle a_2, b_{16} \rangle$ $\langle a_{24}, b_7 \rangle$	0.5270 0.1532 0.3349 0.2687	$\langle a_{24}, b_{16} \rangle$	0.4224 0.2595

expect to gain more from any agreement, whilst the target will expect to lose. This is explained by referencing $P(\text{merger occurring})$ and the average transaction value in successful mergers, shown in Figure 6.10.

As mentioned earlier, an increase in δ_A will lead to a downward shift in the offer interval without affecting the position of A_1 , thus leaving the parties further apart at the initial bargaining stage. This clearly reduces the chances of agreement, and, since the target is yielding towards the acquirer's most recent offer, will reduce the transaction value on average if success is achieved. This leaves the target worse off and the acquirer better off as δ_A increases.

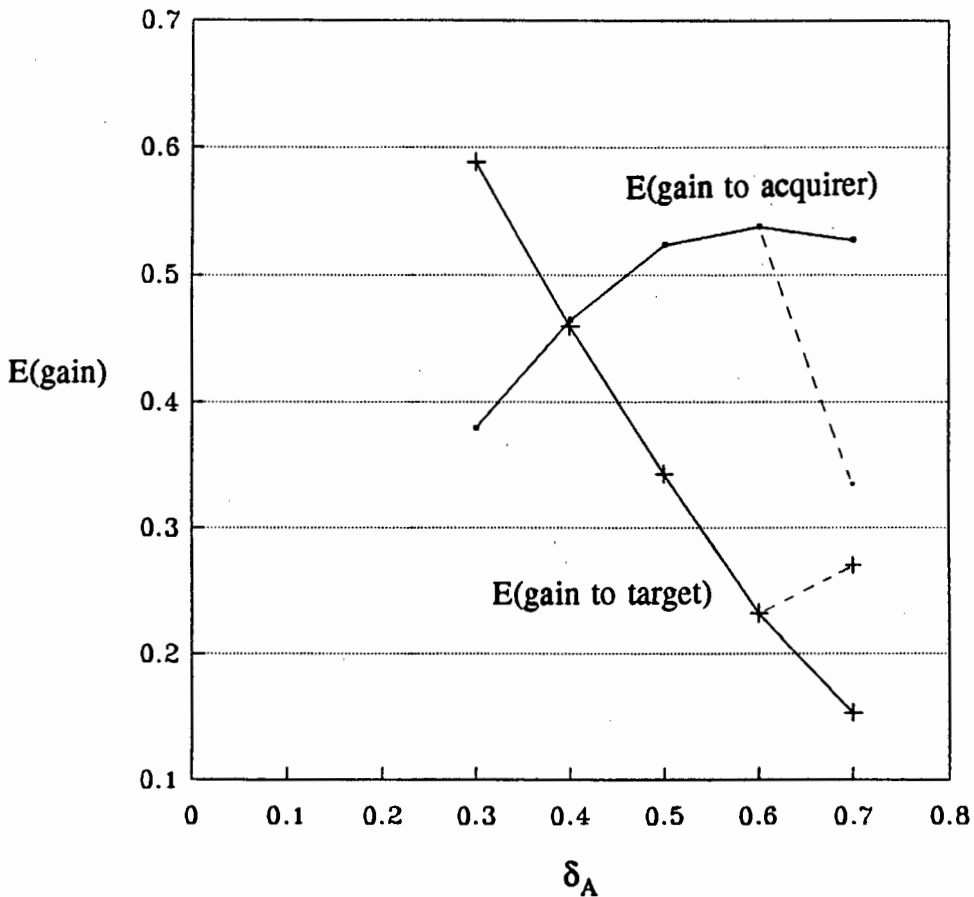


Figure 6.9. Players' expected gains for varying δ_A (under EDS)

Again the target's active role in responding to the acquirer's most recent offer leads to the target being worse off, as comparison to the case when δ_B was varied over the same range clearly shows. This is summarised in Table 6.7.

Table 6.7. Comparison of changes to δ_A and δ_B over the interval [0.3; 0.7]

	Average slope of E(gain to acquirer)	Average slope of E(gain to target)
δ_A	+ 0.3703	- 1.0873
δ_B	- 0.2523	- 0.0643

It can be seen that as the acquirer's perceived dominance increases over the range the acquirer expects to gain whilst the target expects to suffer massive losses. As the target's perceived dominance increases over the same range the acquirer expects to lose somewhat, but the target also expects to suffer a small loss. This apparent imbalance in reaction to a change in a player's perceived dominance is a consequence of the lack of symmetry in the bargaining mechanism.

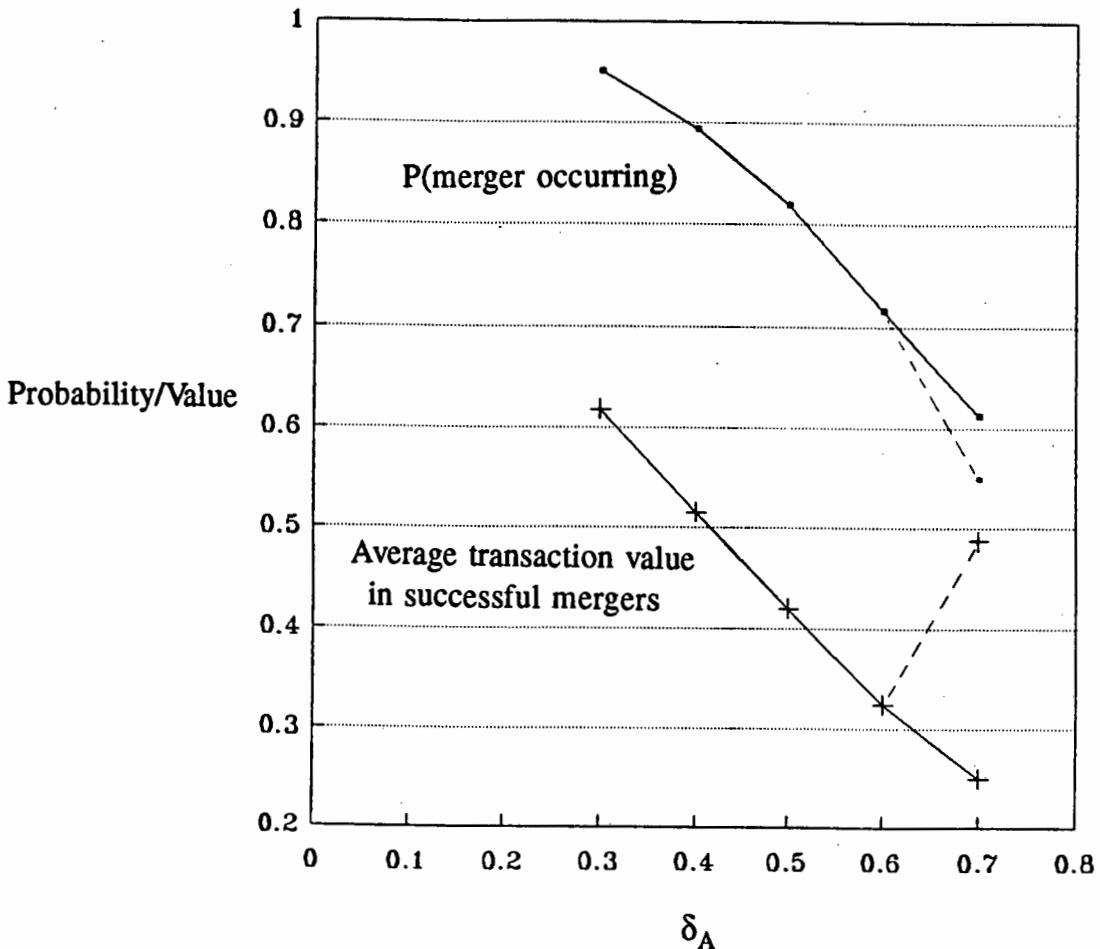


Figure 6.10. Probability of merger and average transaction value in successful mergers as δ_A varies (under EDS)

§6.3 SOME OVERALL CONCLUSIONS

The parameters examined in this chapter are merely abstractions of reality, and are thus neither documented nor empirically measurable - they are idealised representations of what are in fact intangible perceptions arrived at by the decision makers in an unstructured way. Their conceptual nature makes it virtually impossible to compare the effect of a change to a parameter's value in the model to changes induced in a real-world negotiating situation due to modifications to the bargaining environment. At best, in §6.2 we "translated" the effect of a change in each of the contextual model parameters into a change to the modelled negotiating environment, and in each case the simulated change induced in the probability of merger agreement and the average transaction value in successful mergers was entirely that which would have been anticipated had an equivalent alteration to the real-world negotiating situation in fact occurred. As such we can conclude that our model provides an adequate representation of the merger bargaining process insofar as environmental changes are concerned.

Several observations can be made from the simulations in §6.2 which might provide better understanding of the negotiation process and hence help in offering constructive decision support. Firstly, a striking feature is that the optimal bargaining strategy $\langle a_2, b_{16} \rangle$ is extremely robust to changes in the values of the contextual parameters, no matter which approach is applied to determine optimality and over fairly wide ranges for each of these parameters. The only exceptions to this tend to occur when the relevant parameters assume pathologically large or small values; then either a co-optimal point occurs or one of the players' strategies change. The analysis was, of course, performed on a fairly coarse grid of player's strategies (the submatrix G_0' , which consisted of only a subset of 7 target and 8 acquirer discrete strategies). If some finer grid of strategy pairs was used as the basis for analysis it is possible that slight deviations away from a_2 and/or b_{16} to strategies which represent fairly similar negotiating behaviour might be observed. However since the players' strategy parameters are merely abstractions of reality it would be impossible to differentiate between, for example, the target strategy represented by $(\beta_B = 0.1; \gamma = 0.95)$ and that represented by $(\beta_B = 0.15; \gamma = 0.9)$: when translated into practical negotiating actions there would be little to choose

between the two. Thus $\langle a_2, b_{16} \rangle$ will be considered as a **generic optimal strategy pair**, representing the acquirer being prepared to negotiate with great persistence within a narrow range situated well below its expectation of a fair price, and the target starting with a low initial acceptance level but being prepared to capitulate rapidly towards the acquirer's (low) initial offer.

Any change in perception by the acquirer of its contextual parameter values which leads to a downward shift in the offer interval (i.e. when σ_A or δ_A increase) will have the players start negotiating from positions more extreme from one another, and this is observed to leave the target severely worse off and the acquirer at worst no worse off in terms of expected gains. Similarly a change in perception by the target of its contextual parameter values leading to an upward shift in its initial acceptance level (i.e. when σ_B decreases or δ_B increases) is seen to leave the acquirer realising significantly smaller expected gains and the target only slightly reduced gains. In general then, the target appears to suffer more from changes which lead to an initial negotiating position which pitches the players further apart. The reason for this is that the model has the acquirer devising offers within the offer interval completely independently of the target's acceptance levels, whilst the target yields in its acceptance levels at a rate which depends on the value of the acquirer's offers. Thus extreme initial positions combined with a fairly narrow offer interval and a persistent acquirer (the case in acquirer strategy a_2 , where $\beta_{A1} = 0.1$, $\beta_{AL} = 0.3$ and $p = 0.3$) will lead to the target giving up considerably more than the acquirer does. The simulated probability of agreement changes much more rapidly than does the average transaction value in these cases, producing the observed results.

The flowchart of the merger bargaining process depicted in Figure 4.1 and which was modelled in §4.3 is an idealised representation of the merger negotiation process, and takes no account of anomalous individual behaviour. As an example we can cite the scenario just mentioned, where under certain conditions the model has the target giving up more than the acquirer. If this situation were to occur in practice it is possible that a target could act in one of a number of ways not catered for by the model. For example, the target could provide information, directly or indirectly, to the acquirer to indicate that

the acquirer's offers are all far below its acceptance levels, in the hope that this would drive the offers upwards at a greater rate. Alternatively, noticing that all the offers are occurring in a narrow range far below levels acceptable to itself, the target could simply break off negotiations. Thus instead of achieving agreement at lower levels, the probability of agreement is decreased. This anomalous behaviour (especially by the target) is a function of the individual concerned, and so is extraordinarily difficult to model. Whilst the above actions could occur (and if they do they would affect the negotiated outcomes) they are beyond the scope of this study. A much more refined version of our model, focusing on this issue, might provide scope for future research.

APPENDIX 6.A. Simulated submatrices G_0' used in investigating the effect of a change to contextual parameter values

$$\delta_A = \delta_B = 0.5; \sigma = \sigma_A = 0.1; \sigma_B = 0.1$$

		a_2	a_{13}	a_{24}	a_{30}	a_{33}	a_{42}	a_{51}	a_{59}
		$\bar{A}(1;3,3)$	$\bar{A}(7;7,3)$	$\bar{A}(3;9,5)$	$\bar{A}(9;9,5)$	$\bar{A}(1;5,7)$	$\bar{A}(5;9,7)$	$\bar{A}(3;3,9)$	$\bar{A}(7;9,9)$
b_1	$\bar{B}(1;.05)$	0.3049	0.4558	0.4065	0.5228	0.2932	0.4295	0.3473	0.4579
		0.3385	0.3870	0.4006	0.3915	0.3209	0.3944	0.3437	0.3896
b_4	$\bar{B}(7;.05)$	0.1538	0.3060	0.2792	0.4027	0.1473	0.2823	0.1754	0.3109
		0.1510	0.2392	0.2393	0.2825	0.1411	0.2343	0.1567	0.2434
b_7	$\bar{B}(3;.35)$	0.3058	0.4040	0.3884	0.4824	0.2522	0.3859	0.2798	0.4086
		0.3368	0.3326	0.3657	0.3530	0.2637	0.3427	0.2651	0.3369
b_{10}	$\bar{B}(9;.35)$	0.1771	0.2240	0.2747	0.3228	0.1165	0.2277	0.1151	0.2341
		0.1794	0.1676	0.2266	0.2188	0.1062	0.1807	0.0983	0.1763
b_{13}	$\bar{B}(5;.65)$	0.3314	0.3510	0.3765	0.4506	0.2393	0.3598	0.2214	0.3756
		0.3745	0.2831	0.3485	0.3215	0.2473	0.3139	0.2040	0.3031
b_{16}	$\bar{B}(1;.95)$	0.3903	0.4677	0.4256	0.5257	0.3219	0.4333	0.3414	0.4682
		0.4716	0.3999	0.4276	0.3927	0.3605	0.4035	0.3365	0.3980
b_{19}	$\bar{B}(7;.95)$	0.3499	0.3310	0.3427	0.4021	0.2195	0.3204	0.1795	0.3197
		0.4144	0.2671	0.3148	0.2826	0.2287	0.2740	0.1603	0.2535

$$\delta_A = \delta_B = 0.5; \sigma = \sigma_A = 0.1; \sigma_B = 0.15$$

		a_2	a_{13}	a_{24}	a_{30}	a_{33}	a_{42}	a_{51}	a_{59}
		$\bar{A}(1;3,3)$	$\bar{A}(7;7,3)$	$\bar{A}(3;9,5)$	$\bar{A}(9;9,5)$	$\bar{A}(1;5,7)$	$\bar{A}(5;9,7)$	$\bar{A}(3;3,9)$	$\bar{A}(7;9,9)$
b_1	$\bar{B}(1;.05)$	0.3544	0.4826	0.4344	0.5430	0.3388	0.4554	0.3892	0.4867
		0.4092	0.4166	0.4440	0.4117	0.3865	0.4308	0.3981	0.4226
b_4	$\bar{B}(7;.05)$	0.1325	0.2719	0.2608	0.3717	0.1279	0.2574	0.1529	0.2957
		0.1274	0.2097	0.2194	0.2575	0.1198	0.2103	0.1338	0.2282
b_7	$\bar{B}(3;.35)$	0.3237	0.4227	0.4002	0.5010	0.2713	0.4040	0.2969	0.4325
		0.3611	0.3522	0.3833	0.3689	0.2885	0.3632	0.2856	0.3600
b_{10}	$\bar{B}(9;.35)$	0.1401	0.1625	0.2417	0.2471	0.0873	0.1787	0.0653	0.1717
		0.1386	0.1178	0.1896	0.1619	0.0758	0.1352	0.0531	0.1242
b_{13}	$\bar{B}(5;.65)$	0.3341	0.3573	0.3648	0.4463	0.2292	0.3570	0.2185	0.3764
		0.3776	0.2865	0.3372	0.3198	0.2372	0.3102	0.2011	0.3038
b_{16}	$\bar{B}(1;.95)$	0.4081	0.4898	0.4412	0.5461	0.3567	0.4610	0.3879	0.4915
		0.5007	0.4262	0.4553	0.4135	0.4131	0.4378	0.3967	0.4255
b_{19}	$\bar{B}(7;.95)$	0.3448	0.3025	0.3431	0.3727	0.2105	0.3032	0.1525	0.3111
		0.4066	0.2427	0.3110	0.2576	0.2169	0.2569	0.1337	0.2440

$$\delta_A = \delta_B = 0.5; \sigma = \sigma_A = 0.1; \sigma_B = 0.2$$

		a_2	a_{13}	a_{24}	a_{30}	a_{33}	a_{42}	a_{51}	a_{59}
		$\bar{A}(1;3,3)$	$\bar{A}(7;7,3)$	$\bar{A}(3;9,5)$	$\bar{A}(9;9,5)$	$\bar{A}(1;5,7)$	$\bar{A}(5;9,7)$	$\bar{A}(3;3,9)$	$\bar{A}(7;9,9)$
b_1	$\bar{B}(1;.05)$	0.3874 0.4637	0.5039 0.4430	0.4519 0.4755	0.5527 0.4232	0.3769 0.4444	0.4720 0.4582	0.4241 0.4440	0.5091 0.4480
b_4	$\bar{B}(7;.05)$	0.1130 0.1066	0.2384 0.1816	0.2329 0.1905	0.3506 0.2394	0.1096 0.1005	0.2322 0.1868	0.1353 0.1166	0.2498 0.1901
b_7	$\bar{B}(3;.35)$	0.3439 0.3874	0.4416 0.3702	0.4049 0.3951	0.5164 0.3832	0.2919 0.3162	0.4156 0.3804	0.3240 0.3174	0.4472 0.3773
b_{10}	$\bar{B}(9;.35)$	0.1084 0.1061	0.1046 0.0732	0.1876 0.1414	0.1781 0.1128	0.0505 0.0418	0.1333 0.0968	0.0393 0.0307	0.1195 0.0829
b_{13}	$\bar{B}(5;.65)$	0.3372 0.3820	0.3515 0.2837	0.3725 0.3456	0.4484 0.3230	0.2391 0.2472	0.3559 0.3110	0.2236 0.2064	0.3749 0.3021
b_{16}	$\bar{B}(1;.95)$	0.4140 0.5158	0.5072 0.4483	0.4560 0.4830	0.5547 0.4240	0.3804 0.4543	0.4788 0.4623	0.4208 0.4404	0.5081 0.4475
b_{19}	$\bar{B}(7;.95)$	0.3428 0.4030	0.2946 0.2339	0.3360 0.2993	0.3532 0.2423	0.1971 0.2022	0.2923 0.2455	0.1333 0.1151	0.2716 0.2098

$$\delta_A = \delta_B = 0.5; \sigma = \sigma_A = 0.1; \sigma_B = 0.25$$

		a_2	a_{13}	a_{24}	a_{30}	a_{33}	a_{42}	a_{51}	a_{59}
		$\bar{A}(1;3,3)$	$\bar{A}(7;7,3)$	$\bar{A}(3;9,5)$	$\bar{A}(9;9,5)$	$\bar{A}(1;5,7)$	$\bar{A}(5;9,7)$	$\bar{A}(3;3,9)$	$\bar{A}(7;9,9)$
b_1	$\bar{B}(1;.05)$	0.4083 0.5026	0.5180 0.4592	0.4629 0.4976	0.5613 0.4313	0.4019 0.4888	0.4883 0.4775	0.4499 0.4823	0.5149 0.4583
b_4	$\bar{B}(7;.05)$	0.0955 0.0883	0.2208 0.1650	0.2133 0.1712	0.3243 0.2196	0.0891 0.0798	0.2062 0.1625	0.1126 0.0956	0.2275 0.1706
b_7	$\bar{B}(3;.35)$	0.3541 0.4048	0.4580 0.3881	0.4188 0.4138	0.5236 0.3927	0.3074 0.3373	0.4364 0.4021	0.3433 0.3411	0.4665 0.3966
b_{10}	$\bar{B}(9;.35)$	0.0813 0.0798	0.0658 0.0442	0.1582 0.1148	0.1183 0.0724	0.0342 0.0271	0.0908 0.0627	0.0211 0.0159	0.0704 0.0471
b_{13}	$\bar{B}(5;.65)$	0.3374 0.3820	0.3586 0.2885	0.3739 0.3468	0.4464 0.3205	0.2354 0.2437	0.3641 0.3166	0.2274 0.2096	0.3733 0.3022
b_{16}	$\bar{B}(1;.95)$	0.4237 0.5346	0.5181 0.4620	0.4622 0.4973	0.5625 0.4316	0.4105 0.4984	0.4910 0.4813	0.4447 0.4750	0.5179 0.4623
b_{19}	$\bar{B}(7;.95)$	0.3397 0.3988	0.2597 0.2047	0.3172 0.2805	0.3173 0.2143	0.1932 0.1981	0.2814 0.2328	0.1095 0.0928	0.2577 0.1971

$$\delta_A = \delta_B = 0.5; \sigma = \sigma_B = 0.1; \sigma_A = 0.15$$

		a_2	a_{13}	a_{24}	a_{30}	a_{33}	a_{42}	a_{51}	a_{59}
		$\bar{A}(1;3,3)$	$\bar{A}(7;7,3)$	$\bar{A}(3;9,5)$	$\bar{A}(9;9,5)$	$\bar{A}(1;5,7)$	$\bar{A}(5;9,7)$	$\bar{A}(3;3,9)$	$\bar{A}(7;9,9)$
b_1	$\bar{B}(1;.05)$	0.2634	0.4853	0.4015	0.5742	0.2393	0.4283	0.3186	0.4832
		0.3142	0.3932	0.3962	0.3814	0.2771	0.3920	0.3249	0.3935
b_4	$\bar{B}(7;.05)$	0.1262	0.3421	0.3077	0.4781	0.1205	0.3098	0.1482	0.3449
		0.1298	0.2576	0.2529	0.3036	0.1174	0.2523	0.1363	0.2602
b_7	$\bar{B}(3;.35)$	0.2789	0.4290	0.3800	0.5490	0.2133	0.3972	0.2480	0.4433
		0.3318	0.3399	0.3539	0.3597	0.2322	0.3485	0.2435	0.3513
b_{10}	$\bar{B}(9;.35)$	0.1794	0.2543	0.3087	0.4130	0.1100	0.2618	0.0906	0.2761
		0.1961	0.1846	0.2412	0.2546	0.1032	0.2015	0.0792	0.2007
b_{13}	$\bar{B}(5;.65)$	0.3136	0.3934	0.3591	0.5218	0.2030	0.3647	0.1976	0.3981
		0.3861	0.3045	0.3239	0.3367	0.2186	0.3119	0.1879	0.3083
b_{16}	$\bar{B}(1;.95)$	0.3530	0.4934	0.4060	0.5762	0.2678	0.4368	0.3109	0.4862
		0.4694	0.4039	0.4118	0.3828	0.3188	0.4016	0.3187	0.3970
b_{19}	$\bar{B}(7;.95)$	0.3216	0.3615	0.3421	0.4784	0.1884	0.3274	0.1495	0.3596
		0.4146	0.2815	0.3036	0.3032	0.2028	0.2722	0.1368	0.2726

$$\delta_A = \delta_B = 0.5; \sigma = \sigma_B = 0.1; \sigma_A = 0.2$$

		a_2	a_{13}	a_{24}	a_{30}	a_{33}	a_{42}	a_{51}	a_{59}
		$\bar{A}(1;3,3)$	$\bar{A}(7;7,3)$	$\bar{A}(3;9,5)$	$\bar{A}(9;9,5)$	$\bar{A}(1;5,7)$	$\bar{A}(5;9,7)$	$\bar{A}(3;3,9)$	$\bar{A}(7;9,9)$
b_1	$\bar{B}(1;.05)$	0.2280	0.5018	0.3947	0.6180	0.2069	0.4347	0.2885	0.5096
		0.2861	0.3908	0.3860	0.3626	0.2451	0.3933	0.3053	0.3979
b_4	$\bar{B}(7;.05)$	0.1042	0.3626	0.3163	0.5558	0.0933	0.3251	0.1295	0.3827
		0.1104	0.2644	0.2459	0.3149	0.0900	0.2567	0.1224	0.2774
b_7	$\bar{B}(3;.35)$	0.2601	0.4581	0.3811	0.6032	0.1800	0.3973	0.2131	0.4680
		0.3303	0.3488	0.3453	0.3497	0.1996	0.3431	0.2153	0.3557
b_{10}	$\bar{B}(9;.35)$	0.1783	0.2852	0.3245	0.4936	0.1130	0.2797	0.0771	0.3167
		0.2094	0.2006	0.2361	0.2738	0.1055	0.2059	0.0694	0.2197
b_{13}	$\bar{B}(5;.65)$	0.3021	0.4249	0.3523	0.5791	0.1740	0.3753	0.1727	0.4303
		0.4024	0.3170	0.3058	0.3329	0.1916	0.3123	0.1695	0.3201
b_{16}	$\bar{B}(1;.95)$	0.3214	0.5123	0.3940	0.6127	0.2279	0.4347	0.2860	0.5081
		0.4665	0.4019	0.3962	0.3599	0.2807	0.3961	0.3037	0.3970
b_{19}	$\bar{B}(7;.95)$	0.3013	0.3931	0.3380	0.5564	0.1702	0.3344	0.1239	0.3996
		0.4229	0.2931	0.2892	0.3145	0.1886	0.2681	0.1172	0.2900

$$\delta_A = \delta_B = 0.5; \sigma = \sigma_B = 0.1; \sigma_A = 0.25$$

		a_2	a_{13}	a_{24}	a_{30}	a_{33}	a_{42}	a_{51}	a_{59}
		$\bar{A}(1;3,3)$	$\bar{A}(7;7,3)$	$\bar{A}(3;9,5)$	$\bar{A}(9;9,5)$	$\bar{A}(1;5,7)$	$\bar{A}(5;9,7)$	$\bar{A}(3;3,9)$	$\bar{A}(7;9,9)$
b_1	$\bar{B}(1;.05)$	0.1997	0.5321	0.3900	0.6548	0.1674	0.4392	0.2562	0.5312
		0.2605	0.3940	0.3751	0.3359	0.1962	0.3952	0.2805	0.3962
b_4	$\bar{B}(7;.05)$	0.0888	0.4105	0.3407	0.6157	0.0860	0.3329	0.1092	0.4223
		0.0960	0.2870	0.2435	0.3097	0.0804	0.2545	0.1059	0.2939
b_7	$\bar{B}(3;.35)$	0.2545	0.4894	0.3724	0.6483	0.1649	0.4029	0.1867	0.4952
		0.3433	0.3581	0.3251	0.3302	0.1823	0.3413	0.1947	0.3613
b_{10}	$\bar{B}(9;.35)$	0.1737	0.3197	0.3330	0.5707	0.1197	0.2948	0.0590	0.3560
		0.2166	0.2165	0.2252	0.2804	0.1134	0.2067	0.0547	0.2370
b_{13}	$\bar{B}(5;.65)$	0.2826	0.4512	0.3512	0.6325	0.1609	0.3736	0.1456	0.4709
		0.4017	0.3227	0.2896	0.3199	0.1796	0.3046	0.1463	0.3366
b_{16}	$\bar{B}(1;.95)$	0.2959	0.5236	0.3832	0.6531	0.1899	0.4383	0.2573	0.5280
		0.4668	0.3918	0.3791	0.3341	0.2376	0.3929	0.2805	0.3938
b_{19}	$\bar{B}(7;.95)$	0.2832	0.4112	0.3332	0.6233	0.1524	0.3308	0.1062	0.4142
		0.4315	0.2880	0.2641	0.3125	0.1716	0.2567	0.1033	0.2888

$$\delta_A = \delta_B = 0.5; \sigma = \sigma_A = \sigma_B = 0.2$$

		a_2	a_{13}	a_{24}	a_{30}	a_{33}	a_{42}	a_{51}	a_{59}
		$\bar{A}(1;3,3)$	$\bar{A}(7;7,3)$	$\bar{A}(3;9,5)$	$\bar{A}(9;9,5)$	$\bar{A}(1;5,7)$	$\bar{A}(5;9,7)$	$\bar{A}(3;3,9)$	$\bar{A}(7;9,9)$
b_1	$\bar{B}(1;.05)$	0.3066	0.5026	0.4224	0.5941	0.2981	0.4612	0.3677	0.5036
		0.3740	0.3666	0.4109	0.3366	0.3537	0.3923	0.3608	0.3689
b_4	$\bar{B}(7;.05)$	0.1712	0.3586	0.3165	0.4789	0.1657	0.3250	0.2016	0.3635
		0.1656	0.2271	0.2373	0.2436	0.1534	0.2297	0.1647	0.2302
b_7	$\bar{B}(3;.35)$	0.3057	0.4565	0.4132	0.5580	0.2643	0.4233	0.3075	0.4600
		0.3676	0.3151	0.3681	0.3032	0.2874	0.3377	0.2783	0.3187
b_{10}	$\bar{B}(9;.35)$	0.1886	0.2703	0.3113	0.3923	0.1332	0.2672	0.1374	0.2807
		0.1928	0.1594	0.2154	0.1890	0.1126	0.1740	0.1033	0.1662
b_{13}	$\bar{B}(5;.65)$	0.3206	0.4025	0.4001	0.5297	0.2492	0.3965	0.2487	0.4295
		0.4046	0.2684	0.3442	0.2772	0.2651	0.3057	0.2142	0.2862
b_{16}	$\bar{B}(1;.95)$	0.3569	0.5128	0.4340	0.5988	0.3172	0.4586	0.3626	0.5146
		0.5128	0.3786	0.4380	0.3373	0.3932	0.3994	0.3534	0.3765
b_{19}	$\bar{B}(7;.95)$	0.3223	0.3785	0.3654	0.4776	0.2240	0.3569	0.2065	0.3696
		0.4427	0.2521	0.3095	0.2437	0.2413	0.2650	0.1684	0.2381

$$\delta_A = \delta_B = 0.5; \sigma = \sigma_A = \sigma_B = 0.3$$

		a_2	a_{13}	a_{24}	a_{30}	a_{33}	a_{42}	a_{51}	a_{59}
		$\bar{A}(1;3,3)$	$\bar{A}(7;7,3)$	$\bar{A}(3;9,5)$	$\bar{A}(9;9,5)$	$\bar{A}(1;.5,7)$	$\bar{A}(5;9,7)$	$\bar{A}(3;3,9)$	$\bar{A}(7;.9,9)$
b_1	$\bar{B}(1;.05)$	0.3074	0.5401	0.4356	0.6467	0.3020	0.4869	0.3849	0.5400
		0.4104	0.3555	0.4240	0.3003	0.3874	0.3961	0.3812	0.3575
b_4	$\bar{B}(7;.05)$	0.1875	0.4013	0.3509	0.5357	0.1833	0.3621	0.2253	0.4060
		0.1812	0.2250	0.2382	0.2240	0.1664	0.2307	0.1751	0.2271
b_7	$\bar{B}(3;.35)$	0.3052	0.5003	0.4352	0.6155	0.2754	0.4553	0.3324	0.5014
		0.3965	0.3062	0.3734	0.2715	0.3120	0.3381	0.2943	0.3105
b_{10}	$\bar{B}(9;.35)$	0.2001	0.3072	0.3453	0.4428	0.1492	0.3013	0.1565	0.3186
		0.2043	0.1605	0.2065	0.1782	0.1198	0.1726	0.1111	0.1648
b_{13}	$\bar{B}(5;.65)$	0.3126	0.4450	0.4215	0.5885	0.2588	0.4280	0.2731	0.4738
		0.4214	0.2625	0.3418	0.2531	0.2823	0.3023	0.2274	0.2789
b_{16}	$\bar{B}(1;.95)$	0.3321	0.5492	0.4413	0.6528	0.3132	0.4795	0.3806	0.5516
		0.5290	0.3647	0.4490	0.3011	0.4223	0.3989	0.3734	0.3644
b_{19}	$\bar{B}(7;.95)$	0.3043	0.4186	0.3871	0.5356	0.2293	0.3885	0.2306	0.4110
		0.4460	0.2450	0.3045	0.2222	0.2489	0.2605	0.1793	0.2316

$$\delta_A = \delta_B = 0.5; \sigma = \sigma_A = \sigma_B = 0.4$$

		a_2	a_{13}	a_{24}	a_{30}	a_{33}	a_{42}	a_{51}	a_{59}
		$\bar{A}(1;3,3)$	$\bar{A}(7;7,3)$	$\bar{A}(3;9,5)$	$\bar{A}(9;9,5)$	$\bar{A}(1;.5,7)$	$\bar{A}(5;9,7)$	$\bar{A}(3;3,9)$	$\bar{A}(7;.9,9)$
b_1	$\bar{B}(1;.05)$	0.3057	0.5655	0.4437	0.6781	0.3036	0.5037	0.3965	0.5644
		0.4494	0.3566	0.4421	0.2853	0.4236	0.4088	0.4072	0.3580
b_4	$\bar{B}(7;.05)$	0.2015	0.4312	0.3799	0.5719	0.1988	0.3900	0.2435	0.4361
		0.1992	0.2357	0.2445	0.2251	0.1816	0.2409	0.1910	0.2366
b_7	$\bar{B}(3;.35)$	0.3038	0.5301	0.4518	0.6512	0.2843	0.4781	0.3513	0.5301
		0.4238	0.3114	0.3838	0.2616	0.3386	0.3476	0.3163	0.3151
b_{10}	$\bar{B}(9;.35)$	0.2106	0.3299	0.3726	0.4728	0.1629	0.3270	0.1703	0.3431
		0.2155	0.1759	0.2046	0.1879	0.1291	0.1796	0.1243	0.1768
b_{13}	$\bar{B}(5;.65)$	0.3080	0.4751	0.4389	0.6251	0.2667	0.4517	0.2920	0.5054
		0.4346	0.2693	0.3429	0.2513	0.3011	0.3071	0.2460	0.2842
b_{16}	$\bar{B}(1;.95)$	0.3165	0.5725	0.4459	0.6856	0.3096	0.4925	0.3929	0.5758
		0.5376	0.3633	0.4624	0.2861	0.4512	0.4078	0.3991	0.3652
b_{19}	$\bar{B}(7;.95)$	0.2958	0.4460	0.4065	0.5716	0.2352	0.4126	0.2492	0.4394
		0.4421	0.2518	0.3022	0.2228	0.2570	0.2637	0.1957	0.2378

$$\delta_A = \delta_B = 0.5; \sigma = \sigma_A = \sigma_B = 0.5$$

		a_2	a_{13}	a_{24}	a_{30}	a_{33}	a_{42}	a_{51}	a_{59}
		$\bar{A}(1;3,3)$	$\bar{A}(7;7,3)$	$\bar{A}(3;9,5)$	$\bar{A}(9;9,5)$	$\bar{A}(1;5,7)$	$\bar{A}(5;9,7)$	$\bar{A}(3;3,9)$	$\bar{A}(7;9,9)$
b_1	$\bar{B}(1;.05)$	0.3015 0.4908	0.5811 0.3674	0.4471 0.4650	0.6964 0.2834	0.3026 0.4623	0.5136 0.4285	0.4031 0.4382	0.5789 0.3685
b_4	$\bar{B}(7;.05)$	0.2131 0.2195	0.4516 0.2557	0.4035 0.2562	0.5952 0.2391	0.2114 0.1997	0.4113 0.2576	0.2569 0.2117	0.4570 0.2551
b_7	$\bar{B}(3;.35)$	0.3017 0.4502	0.5503 0.3263	0.4637 0.3992	0.6737 0.2649	0.2909 0.3673	0.4936 0.3645	0.3656 0.3430	0.5492 0.3292
b_{10}	$\bar{B}(9;.35)$	0.2198 0.2292	0.3429 0.2008	0.3911 0.2116	0.4893 0.2112	0.1730 0.1421	0.3441 0.1951	0.1789 0.1427	0.3584 0.1979
b_{13}	$\bar{B}(5;.65)$	0.3056 0.4476	0.4953 0.2846	0.4535 0.3487	0.6485 0.2625	0.2726 0.3189	0.4695 0.3178	0.3065 0.2691	0.5276 0.2990
b_{16}	$\bar{B}(1;.95)$	0.3058 0.5519	0.5861 0.3704	0.4484 0.4812	0.7048 0.2847	0.3057 0.4816	0.4998 0.4237	0.4005 0.4296	0.5896 0.3768
b_{19}	$\bar{B}(7;.95)$	0.2925 0.4429	0.4488 0.2518	0.4224 0.3056	0.5943 0.2367	0.2409 0.2673	0.4302 0.2746	0.2631 0.2170	0.4588 0.2540

$$\delta_A = \delta_B = 0.5; \sigma = 0.1; \sigma_A = \sigma_B = 0.15$$

		a_2	a_{13}	a_{24}	a_{30}	a_{33}	a_{42}	a_{51}	a_{59}
		$\bar{A}(1;3,3)$	$\bar{A}(7;7,3)$	$\bar{A}(3;9,5)$	$\bar{A}(9;9,5)$	$\bar{A}(1;5,7)$	$\bar{A}(5;9,7)$	$\bar{A}(3;3,9)$	$\bar{A}(7;9,9)$
b_1	$\bar{B}(1;.05)$	0.3093 0.3867	0.5084 0.4223	0.4215 0.4378	0.5869 0.3935	0.2926 0.3560	0.4624 0.4342	0.3600 0.3816	0.5113 0.4231
b_4	$\bar{B}(7;.05)$	0.1100 0.1108	0.3088 0.2309	0.2844 0.2276	0.4552 0.2860	0.1004 0.0961	0.2837 0.2258	0.1270 0.1151	0.3198 0.2382
b_7	$\bar{B}(3;.35)$	0.2945 0.3558	0.4499 0.3596	0.3956 0.3755	0.5629 0.3701	0.2308 0.2580	0.4064 0.3620	0.2651 0.2642	0.4574 0.3655
b_{10}	$\bar{B}(9;.35)$	0.1493 0.1597	0.1902 0.1324	0.2778 0.2057	0.3421 0.2045	0.0804 0.0716	0.2140 0.1554	0.0591 0.0496	0.2009 0.1402
b_{13}	$\bar{B}(5;.65)$	0.3166 0.3898	0.3895 0.3026	0.3570 0.3231	0.5204 0.3363	0.2082 0.2239	0.3676 0.3125	0.2011 0.1906	0.4051 0.3149
b_{16}	$\bar{B}(1;.95)$	0.3686 0.4991	0.5116 0.4272	0.4241 0.4475	0.5862 0.3928	0.3020 0.3754	0.4614 0.4354	0.3669 0.3898	0.5089 0.4230
b_{19}	$\bar{B}(7;.95)$	0.3180 0.4082	0.3452 0.2651	0.3366 0.2935	0.4500 0.2826	0.1815 0.1949	0.3157 0.2581	0.1266 0.1143	0.3345 0.2503

$$\delta_A = \delta_B = 0.5; \sigma = 0.2; \sigma_A = \sigma_B = 0.3$$

		a_2	a_{13}	a_{24}	a_{30}	a_{33}	a_{42}	a_{51}	a_{59}
		$\bar{A}(1;3,3)$	$\bar{A}(7;7,3)$	$\bar{A}(3;9,5)$	$\bar{A}(9;9,5)$	$\bar{A}(1;5,7)$	$\bar{A}(5;9,7)$	$\bar{A}(3;3,9)$	$\bar{A}(7;9,9)$
b_1	$\bar{B}(1;.05)$	0.2877	0.5575	0.4231	0.6810	0.2793	0.4867	0.3659	0.5620
		0.4426	0.3879	0.4561	0.3057	0.4061	0.4308	0.4093	0.3882
b_4	$\bar{B}(7;.05)$	0.1227	0.3662	0.3280	0.5515	0.1141	0.3304	0.1471	0.3788
		0.1243	0.2124	0.2147	0.2230	0.1054	0.2154	0.1236	0.2179
b_7	$\bar{B}(3;.35)$	0.2773	0.5087	0.4163	0.6625	0.2330	0.4417	0.2850	0.5161
		0.3989	0.3304	0.3762	0.2874	0.2874	0.3534	0.2835	0.3347
b_{10}	$\bar{B}(9;.35)$	0.1543	0.2348	0.3238	0.4255	0.0926	0.2564	0.0722	0.2467
		0.1748	0.1231	0.1797	0.1614	0.0750	0.1402	0.0532	0.1279
b_{13}	$\bar{B}(5;.65)$	0.2882	0.4489	0.3828	0.6198	0.2103	0.4087	0.2253	0.4656
		0.4297	0.2777	0.3154	0.2613	0.2414	0.2995	0.2044	0.2873
b_{16}	$\bar{B}(1;.95)$	0.3114	0.5581	0.4209	0.6809	0.2797	0.4828	0.3721	0.5571
		0.5627	0.3902	0.4677	0.3048	0.4248	0.4319	0.4181	0.3880
b_{19}	$\bar{B}(7;.95)$	0.2778	0.3988	0.3652	0.5451	0.1800	0.3575	0.1472	0.3928
		0.4483	0.2405	0.2791	0.2206	0.2057	0.2429	0.1226	0.2273

$$\delta_A = \delta_B = 0.5; \sigma = 0.3; \sigma_A = \sigma_B = 0.45$$

		a_2	a_{13}	a_{24}	a_{30}	a_{33}	a_{42}	a_{51}	a_{59}
		$\bar{A}(1;3,3)$	$\bar{A}(7;7,3)$	$\bar{A}(3;9,5)$	$\bar{A}(9;9,5)$	$\bar{A}(1;5,7)$	$\bar{A}(5;9,7)$	$\bar{A}(3;3,9)$	$\bar{A}(7;9,9)$
b_1	$\bar{B}(1;.05)$	0.2658	0.5954	0.4229	0.7416	0.2656	0.5060	0.3699	0.6002
		0.4988	0.3647	0.4763	0.2515	0.4563	0.4324	0.4385	0.3660
b_4	$\bar{B}(7;.05)$	0.1352	0.4130	0.3694	0.6170	0.1275	0.3719	0.1655	0.4256
		0.1381	0.2044	0.2040	0.1909	0.1151	0.2102	0.1337	0.2098
b_7	$\bar{B}(3;.35)$	0.2617	0.5556	0.4349	0.7290	0.2353	0.4717	0.3029	0.5635
		0.4335	0.3130	0.3791	0.2377	0.3164	0.3500	0.3048	0.3154
b_{10}	$\bar{B}(9;.35)$	0.1603	0.2660	0.3631	0.4768	0.1041	0.2929	0.0831	0.2813
		0.1844	0.1274	0.1605	0.1505	0.0789	0.1309	0.0590	0.1268
b_{13}	$\bar{B}(5;.65)$	0.2659	0.4974	0.4070	0.6877	0.2123	0.4441	0.2476	0.5147
		0.4483	0.2647	0.3087	0.2177	0.2568	0.2921	0.2201	0.2709
b_{16}	$\bar{B}(1;.95)$	0.2689	0.5949	0.4183	0.7435	0.2602	0.4997	0.3757	0.5940
		0.5848	0.3655	0.4869	0.2489	0.4682	0.4328	0.4482	0.3642
b_{19}	$\bar{B}(7;.95)$	0.2505	0.4415	0.3943	0.6099	0.1808	0.3946	0.1657	0.4391
		0.4532	0.2293	0.2641	0.1888	0.2107	0.2322	0.1331	0.2162

$$\delta_A = \delta_B = 0.5; \sigma = 0.4; \sigma_A = \sigma_B = 0.6$$

		a_2	a_{13}	a_{24}	a_{30}	a_{33}	a_{42}	a_{51}	a_{59}
		$\bar{A}(1;3,3)$	$\bar{A}(7;7,3)$	$\bar{A}(3;9,5)$	$\bar{A}(9;9,5)$	$\bar{A}(1;5,7)$	$\bar{A}(5;9,7)$	$\bar{A}(3;3,9)$	$\bar{A}(7;9,9)$
b_1	$\bar{B}(1;05)$	0.2427 0.5562	0.6174 0.3575	0.4188 0.5003	0.7738 0.2257	0.2509 0.5078	0.5168 0.4426	0.3697 0.4727	0.6226 0.3595
b_4	$\bar{B}(7;05)$	0.1466 0.1529	0.4442 0.2122	0.4007 0.2032	0.6540 0.1873	0.1396 0.1262	0.4047 0.2137	0.1801 0.1475	0.4577 0.2165
b_7	$\bar{B}(3;35)$	0.2490 0.4623	0.5869 0.3112	0.4496 0.3861	0.7666 0.2170	0.2368 0.3463	0.4931 0.3554	0.3166 0.3303	0.5949 0.3120
b_{10}	$\bar{B}(9;35)$	0.1667 0.1936	0.2818 0.1470	0.3878 0.1558	0.4993 0.1683	0.1131 0.0852	0.3152 0.1358	0.0895 0.0693	0.3000 0.1416
b_{13}	$\bar{B}(5;65)$	0.2518 0.4596	0.5310 0.2668	0.4287 0.3055	0.7270 0.2027	0.2152 0.2681	0.4712 0.2931	0.2654 0.2404	0.5486 0.2699
b_{16}	$\bar{B}(1;95)$	0.2414 0.6021	0.6174 0.3556	0.4141 0.5085	0.7779 0.2212	0.2433 0.5087	0.5095 0.4411	0.3755 0.4819	0.6163 0.3552
b_{19}	$\bar{B}(7;95)$	0.2363 0.4487	0.4720 0.2346	0.4168 0.2561	0.6459 0.1858	0.1842 0.2126	0.4233 0.2302	0.1798 0.1478	0.4700 0.2205

$$\delta_A = \delta_B = 0.5; \sigma = 0.5; \sigma_A = \sigma_B = 0.75$$

		a_2	a_{13}	a_{24}	a_{30}	a_{33}	a_{42}	a_{51}	a_{59}
		$\bar{A}(1;3,3)$	$\bar{A}(7;7,3)$	$\bar{A}(3;9,5)$	$\bar{A}(9;9,5)$	$\bar{A}(1;5,7)$	$\bar{A}(5;9,7)$	$\bar{A}(3;3,9)$	$\bar{A}(7;9,9)$
b_1	$\bar{B}(1;05)$	0.2186 0.6103	0.6287 0.3609	0.4108 0.5282	0.7910 0.2148	0.2348 0.5595	0.5198 0.4605	0.3651 0.5110	0.6352 0.3629
b_4	$\bar{B}(7;05)$	0.1566 0.1692	0.4638 0.2315	0.4232 0.2114	0.6745 0.2001	0.1490 0.1390	0.4272 0.2274	0.1907 0.1655	0.4790 0.2339
b_7	$\bar{B}(3;35)$	0.2385 0.4903	0.6066 0.3210	0.4606 0.3974	0.7884 0.2120	0.2373 0.3760	0.5072 0.3682	0.3265 0.3595	0.6148 0.3201
b_{10}	$\bar{B}(9;35)$	0.1716 0.2057	0.2871 0.1771	0.4021 0.1618	0.5060 0.2020	0.1189 0.0948	0.3260 0.1521	0.0919 0.0836	0.3062 0.1689
b_{13}	$\bar{B}(5;65)$	0.2428 0.4694	0.5537 0.2797	0.4447 0.3093	0.7506 0.2034	0.2193 0.2811	0.4918 0.3014	0.2788 0.2650	0.5720 0.2792
b_{16}	$\bar{B}(1;95)$	0.2218 0.6188	0.6294 0.3567	0.4085 0.5329	0.7959 0.2099	0.2279 0.5506	0.5124 0.4572	0.3717 0.5192	0.6278 0.3574
b_{19}	$\bar{B}(7;95)$	0.2300 0.4501	0.4712 0.2384	0.4324 0.2573	0.6668 0.1979	0.1875 0.2174	0.4430 0.2379	0.1900 0.1665	0.4901 0.2362

$$\delta_A = 0.5; \delta_B = 0.3; \sigma = \sigma_A = \sigma_B = 0.25$$

		a_2	a_{13}	a_{24}	a_{30}	a_{33}	a_{42}	a_{51}	a_{59}
		$\bar{A}(.1;.3,.3)$	$\bar{A}(.7;.7,.3)$	$\bar{A}(.3;.9,.5)$	$\bar{A}(.9;.9,.5)$	$\bar{A}(.1;.5,.7)$	$\bar{A}(.5;.9,.7)$	$\bar{A}(.3;.3,.9)$	$\bar{A}(.7;.9,.9)$
b_1	$\bar{B}(.1;.05)$	0.3445	0.5614	0.4440	0.6493	0.3454	0.4984	0.4287	0.5577
		0.5175	0.4098	0.4931	0.3400	0.5006	0.4508	0.4677	0.4085
b_4	$\bar{B}(.7;.05)$	0.3141	0.5233	0.4297	0.6282	0.2988	0.4690	0.3842	0.5289
		0.4014	0.3604	0.4149	0.3149	0.3694	0.3910	0.3748	0.3654
b_7	$\bar{B}(.3;.35)$	0.3482	0.5455	0.4446	0.6441	0.3359	0.4918	0.4021	0.5435
		0.5170	0.3902	0.4745	0.3304	0.4614	0.4364	0.4267	0.3933
b_{10}	$\bar{B}(.9;.35)$	0.3196	0.5013	0.4288	0.6085	0.2831	0.4503	0.3434	0.5073
		0.4149	0.3294	0.3932	0.2983	0.3306	0.3580	0.3204	0.3368
b_{13}	$\bar{B}(.5;.65)$	0.3437	0.5374	0.4388	0.6397	0.3244	0.4822	0.3928	0.5388
		0.5208	0.3775	0.4554	0.3255	0.4381	0.4161	0.3979	0.3791
b_{16}	$\bar{B}(.1;.95)$	0.3504	0.5543	0.4445	0.6420	0.3441	0.4968	0.4135	0.5532
		0.5742	0.4068	0.5023	0.3383	0.5101	0.4553	0.4628	0.4086
b_{19}	$\bar{B}(.7;.95)$	0.3400	0.5248	0.4291	0.6249	0.3107	0.4672	0.3760	0.5275
		0.5220	0.3616	0.4324	0.3147	0.4044	0.3940	0.3684	0.3656

$$\delta_A = 0.5; \delta_B = 0.4; \sigma = \sigma_A = \sigma_B = 0.25$$

		a_2	a_{13}	a_{24}	a_{30}	a_{33}	a_{42}	a_{51}	a_{59}
		$\bar{A}(.1;.3,.3)$	$\bar{A}(.7;.7,.3)$	$\bar{A}(.3;.9,.5)$	$\bar{A}(.9;.9,.5)$	$\bar{A}(.1;.5,.7)$	$\bar{A}(.5;.9,.7)$	$\bar{A}(.3;.3,.9)$	$\bar{A}(.7;.9,.9)$
b_1	$\bar{B}(.1;.05)$	0.3320	0.5422	0.4451	0.6413	0.3208	0.4807	0.4080	0.5492
		0.4651	0.3854	0.4614	0.3295	0.4319	0.4177	0.4268	0.3907
b_4	$\bar{B}(.7;.05)$	0.2534	0.4631	0.3974	0.5924	0.2376	0.4239	0.3060	0.4650
		0.2794	0.2949	0.3305	0.2832	0.2514	0.3188	0.2691	0.2975
b_7	$\bar{B}(.3;.35)$	0.3333	0.5282	0.4278	0.6200	0.3086	0.4774	0.3707	0.5251
		0.4556	0.3601	0.4155	0.3127	0.3809	0.3932	0.3609	0.3588
b_{10}	$\bar{B}(.9;.35)$	0.2628	0.4064	0.3884	0.5369	0.2108	0.3883	0.2487	0.4207
		0.2998	0.2451	0.3015	0.2469	0.2061	0.2696	0.2021	0.2542
b_{13}	$\bar{B}(.5;.65)$	0.3372	0.4940	0.4267	0.6053	0.2869	0.4532	0.3391	0.5029
		0.4729	0.3274	0.3987	0.2978	0.3465	0.3615	0.3158	0.3352
b_{16}	$\bar{B}(.1;.95)$	0.3493	0.5422	0.4335	0.6369	0.3288	0.4896	0.4031	0.5389
		0.5557	0.3862	0.4667	0.3306	0.4566	0.4313	0.4227	0.3890
b_{19}	$\bar{B}(.7;.95)$	0.3230	0.4691	0.4222	0.5911	0.2676	0.4328	0.3054	0.4772
		0.4798	0.2990	0.3822	0.2838	0.3126	0.3357	0.2676	0.3082

$$\delta_A = 0.5; \delta_B = 0.6; \sigma = \sigma_A = \sigma_B = 0.25$$

		a_2	a_{13}	a_{24}	a_{30}	a_{33}	a_{42}	a_{51}	a_{59}
		$\bar{A}(1;3,3)$	$\bar{A}(7;7,3)$	$\bar{A}(3;9,5)$	$\bar{A}(9;9,5)$	$\bar{A}(1;5,7)$	$\bar{A}(5;9,7)$	$\bar{A}(3;3,9)$	$\bar{A}(7;9,9)$
b_1	$\bar{B}(1;.05)$	0.2768 0.3248	0.4958 0.3242	0.4162 0.3687	0.6112 0.2997	0.2651 0.2987	0.4395 0.3437	0.3286 0.3016	0.4991 0.3288
b_4	$\bar{B}(7;.05)$	0.1134 0.0948	0.2724 0.1470	0.2589 0.1546	0.4001 0.1698	0.1084 0.0855	0.2435 0.1444	0.1416 0.1003	0.2768 0.1488
b_7	$\bar{B}(3;.35)$	0.2651 0.3021	0.4126 0.2501	0.3934 0.3050	0.5453 0.2517	0.2233 0.2204	0.3971 0.2767	0.2483 0.2031	0.4317 0.2612
b_{10}	$\bar{B}(9;.35)$	0.1353 0.1302	0.1579 0.0800	0.2611 0.1397	0.2735 0.1113	0.0822 0.0576	0.1842 0.0961	0.0660 0.0430	0.1615 0.0809
b_{13}	$\bar{B}(5;.65)$	0.2848 0.3598	0.3447 0.1962	0.3848 0.2850	0.4828 0.2132	0.2095 0.2037	0.3579 0.2370	0.1902 0.1462	0.3644 0.2090
b_{16}	$\bar{B}(1;.95)$	0.3336 0.4944	0.4969 0.3261	0.4215 0.3995	0.6067 0.2955	0.2992 0.3624	0.4520 0.3627	0.3400 0.3126	0.5038 0.3333
b_{19}	$\bar{B}(7;.95)$	0.3000 0.4263	0.2783 0.1504	0.3514 0.2610	0.4003 0.1695	0.1863 0.1958	0.3156 0.2034	0.1361 0.0970	0.3068 0.1705

$$\delta_A = 0.5; \delta_B = 0.7; \sigma = \sigma_A = \sigma_B = 0.25$$

		a_2	a_{13}	a_{24}	a_{30}	a_{33}	a_{42}	a_{51}	a_{59}
		$\bar{A}(1;3,3)$	$\bar{A}(7;7,3)$	$\bar{A}(3;9,5)$	$\bar{A}(9;9,5)$	$\bar{A}(1;5,7)$	$\bar{A}(5;9,7)$	$\bar{A}(3;3,9)$	$\bar{A}(7;9,9)$
b_1	$\bar{B}(1;.05)$	0.2366 0.2533	0.4479 0.2804	0.3871 0.3126	0.5699 0.2698	0.2352 0.2410	0.4065 0.2980	0.2832 0.2430	0.4623 0.2892
b_4	$\bar{B}(7;.05)$	0.0573 0.0424	0.1582 0.0799	0.1819 0.0920	0.2876 0.1169	0.0523 0.0363	0.1581 0.0840	0.0664 0.0434	0.1834 0.0922
b_7	$\bar{B}(3;.35)$	0.2233 0.2358	0.3388 0.1914	0.3490 0.2407	0.4746 0.2072	0.1715 0.1507	0.3289 0.2089	0.1805 0.1360	0.3528 0.1996
b_{10}	$\bar{B}(9;.35)$	0.0887 0.0828	0.0682 0.0336	0.1932 0.0924	0.1413 0.0589	0.0374 0.0236	0.1061 0.0492	0.0229 0.0142	0.0820 0.0393
b_{13}	$\bar{B}(5;.65)$	0.2561 0.3103	0.2419 0.1286	0.3574 0.2370	0.3829 0.1613	0.1706 0.1521	0.2938 0.1759	0.1262 0.0874	0.2731 0.1453
b_{16}	$\bar{B}(1;.95)$	0.3247 0.4733	0.4476 0.2819	0.4005 0.3552	0.5777 0.2727	0.2600 0.3010	0.4231 0.3190	0.2872 0.2467	0.4587 0.2917
b_{19}	$\bar{B}(7;.95)$	0.2960 0.4185	0.1722 0.0881	0.3232 0.2244	0.2754 0.1119	0.1671 0.1751	0.2670 0.1587	0.0665 0.0429	0.2109 0.1107

$$\delta_A = 0.3; \delta_B = 0.5; \sigma = \sigma_A = \sigma_B = 0.25$$

		a_2	a_{13}	a_{24}	a_{30}	a_{33}	a_{42}	a_{51}	a_{59}
		$\bar{A}(1;3,3)$	$\bar{A}(7;7,3)$	$\bar{A}(3;9,5)$	$\bar{A}(9;9,5)$	$\bar{A}(1;.5,7)$	$\bar{A}(5;9,7)$	$\bar{A}(3;3,9)$	$\bar{A}(7;9,9)$
b_1	$\bar{B}(1;.05)$	0.5825 0.3586	0.7119 0.2682	0.6349 0.3197	0.7584 0.2253	0.5745 0.3509	0.6757 0.2965	0.6321 0.3159	0.7041 0.2663
b_4	$\bar{B}(7;.05)$	0.4651 0.2488	0.6254 0.2193	0.5706 0.2510	0.6887 0.1958	0.4610 0.2446	0.5864 0.2338	0.5102 0.2295	0.6206 0.2188
b_7	$\bar{B}(3;.35)$	0.5719 0.3396	0.6864 0.2508	0.6313 0.3051	0.7475 0.2207	0.5528 0.3203	0.6519 0.2758	0.5970 0.2886	0.6910 0.2537
b_{10}	$\bar{B}(9;.35)$	0.4466 0.2318	0.5422 0.1836	0.5317 0.2165	0.6155 0.1717	0.3897 0.1920	0.5285 0.1980	0.4309 0.1849	0.5522 0.1879
b_{13}	$\bar{B}(5;.65)$	0.5671 0.3365	0.6613 0.2367	0.6136 0.2885	0.7257 0.2103	0.5262 0.2960	0.6316 0.2604	0.5606 0.2608	0.6588 0.2375
b_{16}	$\bar{B}(1;.95)$	0.5881 0.3789	0.7069 0.2673	0.6459 0.3290	0.7550 0.2261	0.5839 0.3603	0.6712 0.2956	0.6286 0.3155	0.7089 0.2665
b_{19}	$\bar{B}(7;.95)$	0.5600 0.3369	0.6336 0.2254	0.5990 0.2735	0.6910 0.1971	0.4873 0.2673	0.6126 0.2448	0.5051 0.2281	0.6277 0.2208

$$\delta_A = 0.4; \delta_B = 0.5; \sigma = \sigma_A = \sigma_B = 0.25$$

		a_2	a_{13}	a_{24}	a_{30}	a_{33}	a_{42}	a_{51}	a_{59}
		$\bar{A}(1;3,3)$	$\bar{A}(7;7,3)$	$\bar{A}(3;9,5)$	$\bar{A}(9;9,5)$	$\bar{A}(1;.5,7)$	$\bar{A}(5;9,7)$	$\bar{A}(3;3,9)$	$\bar{A}(7;9,9)$
b_1	$\bar{B}(1;.05)$	0.4404 0.4081	0.6210 0.3214	0.5305 0.3852	0.6978 0.2740	0.4386 0.3932	0.5744 0.3527	0.5132 0.3700	0.6232 0.3244
b_4	$\bar{B}(7;.05)$	0.3101 0.2295	0.5011 0.2325	0.4415 0.2572	0.6028 0.2184	0.3035 0.2174	0.4681 0.2471	0.3462 0.2126	0.5098 0.2357
b_7	$\bar{B}(3;.35)$	0.4325 0.3814	0.5970 0.2954	0.5164 0.3470	0.6678 0.2557	0.4006 0.3294	0.5519 0.3230	0.4541 0.3058	0.5897 0.2935
b_{10}	$\bar{B}(9;.35)$	0.3128 0.2295	0.4124 0.1796	0.4389 0.2317	0.5268 0.1850	0.2438 0.1600	0.3985 0.1943	0.2684 0.1532	0.4301 0.1872
b_{13}	$\bar{B}(5;.65)$	0.4305 0.3877	0.5506 0.2630	0.5061 0.3275	0.6342 0.2353	0.3720 0.2981	0.5196 0.2918	0.4153 0.2663	0.5618 0.2706
b_{16}	$\bar{B}(1;.95)$	0.4600 0.4645	0.6240 0.3248	0.5283 0.3915	0.6981 0.2751	0.4354 0.4036	0.5782 0.3585	0.5019 0.3629	0.6183 0.3215
b_{19}	$\bar{B}(7;.95)$	0.4381 0.4057	0.5197 0.2469	0.4860 0.3033	0.6110 0.2220	0.3536 0.2749	0.4911 0.2675	0.3518 0.2145	0.5191 0.2415

$$\delta_A = 0.6; \delta_B = 0.5; \sigma = \sigma_A = \sigma_B = 0.25$$

		a_2	a_{13}	a_{24}	a_{30}	a_{33}	a_{42}	a_{51}	a_{59}
		$\bar{A}(.1;.3,.3)$	$\bar{A}(.7;.7,.3)$	$\bar{A}(.3;.9,.5)$	$\bar{A}(.9;.9,.5)$	$\bar{A}(.1;.5,.7)$	$\bar{A}(.5;.9,.7)$	$\bar{A}(.3;.3,.9)$	$\bar{A}(.7;.9,.9)$
b_1	$\bar{B}(.1;.05)$	0.1945 0.3231	0.4201 0.3692	0.3436 0.4078	0.5521 0.3482	0.1922 0.3011	0.3667 0.3890	0.2485 0.3174	0.4245 0.3747
b_4	$\bar{B}(.7;.05)$	0.0962 0.1131	0.2645 0.1906	0.2595 0.2098	0.4166 0.2271	0.0895 0.0972	0.2389 0.1903	0.1189 0.1164	0.2760 0.1981
b_7	$\bar{B}(.3;.35)$	0.2031 0.3447	0.3672 0.2969	0.3315 0.3516	0.5095 0.3042	0.1642 0.2281	0.3391 0.3232	0.1894 0.2150	0.3763 0.3088
b_{10}	$\bar{B}(.9;.35)$	0.1202 0.1735	0.1842 0.1219	0.2713 0.2051	0.3209 0.1647	0.0763 0.0762	0.2050 0.1463	0.0629 0.0552	0.2030 0.1336
b_{13}	$\bar{B}(.5;.65)$	0.2220 0.4245	0.3284 0.2506	0.3239 0.3312	0.4656 0.2665	0.1601 0.2312	0.3181 0.2900	0.1566 0.1647	0.3430 0.2645
b_{16}	$\bar{B}(.1;.95)$	0.2321 0.5373	0.4288 0.3892	0.3437 0.4477	0.5554 0.3518	0.2032 0.3584	0.3727 0.4091	0.2571 0.3278	0.4248 0.3790
b_{19}	$\bar{B}(.7;.95)$	0.2152 0.4745	0.2854 0.2239	0.2991 0.3118	0.4143 0.2274	0.1478 0.2240	0.2898 0.2567	0.1161 0.1134	0.2931 0.2157

$$\delta_A = 0.7; \delta_B = 0.5; \sigma = \sigma_A = \sigma_B = 0.25$$

		a_2	a_{13}	a_{24}	a_{30}	a_{33}	a_{42}	a_{51}	a_{59}
		$\bar{A}(.1;.3,.3)$	$\bar{A}(.7;.7,.3)$	$\bar{A}(.3;.9,.5)$	$\bar{A}(.9;.9,.5)$	$\bar{A}(.1;.5,.7)$	$\bar{A}(.5;.9,.7)$	$\bar{A}(.3;.3,.9)$	$\bar{A}(.7;.9,.9)$
b_1	$\bar{B}(.1;.05)$	0.1130 0.2356	0.3243 0.3568	0.2626 0.3626	0.4741 0.3673	0.1057 0.1967	0.2748 0.3585	0.1449 0.2285	0.3279 0.3595
b_4	$\bar{B}(.7;.05)$	0.0420 0.0575	0.1731 0.1496	0.1789 0.1565	0.3206 0.2081	0.0487 0.0597	0.1617 0.1477	0.0512 0.0594	0.1830 0.1571
b_7	$\bar{B}(.3;.35)$	0.1246 0.2913	0.2653 0.2616	0.2687 0.3349	0.4315 0.3095	0.0955 0.1649	0.2494 0.2849	0.1098 0.1506	0.2785 0.2742
b_{10}	$\bar{B}(.9;.35)$	0.0754 0.1481	0.1058 0.0829	0.2294 0.2030	0.2440 0.1464	0.0441 0.0525	0.1427 0.1164	0.0287 0.0299	0.1227 0.0950
b_{13}	$\bar{B}(.5;.65)$	0.1444 0.4040	0.2371 0.2186	0.2588 0.3278	0.3738 0.2569	0.1077 0.2120	0.2335 0.2613	0.0752 0.0953	0.2484 0.2329
b_{16}	$\bar{B}(.1;.95)$	0.1532 0.5270	0.3287 0.3762	0.2595 0.4224	0.4787 0.3704	0.1251 0.2944	0.2823 0.3887	0.1526 0.2444	0.3369 0.3732
b_{19}	$\bar{B}(.7;.95)$	0.1444 0.4784	0.1961 0.1953	0.2424 0.3157	0.3159 0.2064	0.0927 0.2019	0.2064 0.2234	0.0529 0.0613	0.1962 0.1778

CHAPTER 7

THE USE OF THE NEGOTIATION-BASED MODEL AND ITS MONTE CARLO SIMULATION IN DECISION SUPPORT

§7.1 INTRODUCTION

The model created in Chapter 4 was based upon the broad principles of negotiation analysis (Sebenius (1992)), and contains ideas and notions which are not dissimilar to some of those found in Raiffa (1982). In broad outline the model assumes that the competing parties would independently arrive at an estimate of an expected fair price for the target company. Providing that these estimates are situated within the players' zone of agreement, negotiation might take place. The model allows for a continuum of negotiating strategies for both players, and in the previous chapters these have been analysed to identify what negotiating behaviour might be construed as a "good" strategy for each player under various contextual scenarios.

The aim of the model is to focus on the dynamic nature of the bargaining or negotiating which might occur prior to agreement being reached regarding the merging of two companies. The extent of the dynamism of offers and responses to these offers is reflected in the values of the various strategy parameters. We have attempted to provide a model based on the way negotiators think in practice, making frequent use of "soft" and even qualitative parameters rather than restricting ourselves to hard, measurable parameters, such as accounting information. To be sure, the contextual and strategy parameters used in the model are just abstractions of reality comprising of a complicated combination of cognitive variables and thus will tend to be judgemental and subjective to any user of the model. Analysis of the model results will offer insights into the effects of important components which should be considered when formulating a merger bargaining strategy.

The potential usefulness of this model to merger and acquisition practitioners

would be best demonstrated by the design of a **decision support system (DSS)** incorporating the negotiating model as a key component. A range of software to aid in practical merger and acquisition decision-making is currently available but none to our knowledge focuses on the bargaining aspects of the process. In this chapter, we will outline the essential components of a DSS based on our model, avoiding the duplication of many features already provided by other more conventional merger and acquisition software, and thereby emphasising our different modelling approach. The construction and implementation of such a DSS is a major project in its own right, and so we have made no attempt at this in this thesis. This chapter will instead be restricted to an overview of a conceptual system based on the negotiating model. Detailed software design would best be left to analysts and/or software engineers actually involved in the programming function.

In the next section we introduce and define what is meant by a decision support system, and we will examine the applicability of DSSs in the area of mergers and acquisitions. We will briefly outline several systems which are currently available, highlighting gaps which our model might fill. In §7.3 and Appendix 7.A we will present the architecture for a conceptual DSS based on our model. The final section in the chapter contains a short discussion on the practical usefulness and possible impacts of the model/system, including perceived shortcomings and areas that could be improved upon.

§7.2 DECISION SUPPORT SYSTEMS AND MERGERS AND ACQUISITIONS

The decision to follow a strategy of growth by merger is an important one for any company. This will necessarily be followed by further decisions relating to the profile of potential targets, target valuation, bid timing, negotiation and post-merger integration, amongst others. Thus the decision processes involved in the lead-up to successful completion of a merger deal are many, interrelated in a complex way, and often have to be made under severe time pressure. The value of good decision-making in these circumstances is obvious; any system or means of making use of modern technology which might largely automate aspects of the decision processes (i.e. a DSS) should be welcomed by the decision-maker.

§7.2.1 DECISION SUPPORT SYSTEMS

Despite there being no established universal definition of a DSS (Keen (1987)), one widely accepted description is

"an interactive computer based system, which helps decision makers utilize data and models to solve unstructured problems" (Sprague (1980)).

A working description, provided by Turban (1993) says that

"A DSS is an interactive, flexible and adaptable computer based information system, especially developed for supporting the solution of a particular management problem for improved decision making. It utilizes data, it provides easy user interface, and it allows for the decision maker's own insights."

All characterisations of DSSs suggest that the computer system consists of a **knowledge base** (the system's database), a **model base** (the problem-processing system) and a **dialogue generation and management system** (the human user interface system) which all interact with each other and with the user/decision maker, who will have some feel for the rather fuzzy, unstructured and complex problem at hand.

The concept of a DSS has been around since the early 1970s, and literally thousands of DSSs have been built, implemented and recorded in the literature. Application areas are diverse: some areas of work include financial management, human resource management, marketing, transportation, production scheduling and natural resources management. A fairly comprehensive list of recent published material on DSS applications is contained in Eom and Lee (1990).

§7.2.2 COMPUTER TOOLS FOR MERGERS AND ACQUISITIONS

Finance theory has produced a number of models which might prove helpful during merger and acquisition deals (Myers (1976)), and several computer-based tools based on these models exist (Rock (1987)). Since up-to-date quantitative financial data

regarding various aspects of a company (eg. stock price, price/earnings ratios etc) are readily available from on-line databases (for example, Compustat and Value Line in the US, Datashare in South Africa), the above financial models can easily be implemented to generate numerical estimates of certain financial parameters such as forecasted cash flows, balance sheets etc.

Several software packages are commercially available which provide the type of facility mentioned above. Two programs (named *Value Planner* and *Merger Planner*) marketed by the Alcor Group Inc of Stokie, Illinois provide one such example. The *Value Planner* allows a user the opportunity to generate historical and forecasted financial statements, income statements, balance sheets, cash flow statements and other accounting ratios for a company. The *Merger Planner* builds on these abilities by allowing the user to analyse mergers and acquisitions between two or more companies, and divestures of a conglomerate company. A user can "combine" an acquirer and a target company into a single entity by specifying the deal structure, tax arrangements and accounting treatment.

Another example, marketed by Disclosure, Inc of Bethesda, Maryland, attempts to determine the financial viability of a possible merger through the review of income statement and balance sheets, examination of stock ownership (to judge the feasibility of a successful deal), and can even search amongst candidate targets to determine the most compatible. The output of these programs and financial models, together with subjective analyses of various qualitative factors (eg. an industry analyst's opinion of the future of a target company) can then be used to value the target company.

Bonissone and Dutta (1989) have suggested that this purely financial approach has certain limitations. For example, this approach is unable to take advantage of current state-of-the-art intelligent information retrieval methods which can process natural language information provided in business literature such as the Wall Street Journal and the Dow Jones News Service (in the US). This information, qualitative in nature, is usually in the form of editorial analyses, industry reports etc, and can play an important role in decisions regarding a merger deal. Financial models furthermore do not take

cognisance of precedents (within a particular industry sector, say) which might affect the structuring of various aspects of a particular merger deal. The ability of a decision support system to employ case-based reasoning would help here. Expert knowledge, expressed in the form of rules (as typified by conventional expert systems) is also lacking in most financial tools. Bonissone and Dutta have used conventional financial models, fused with a reasoning and simulation environment (to capture qualitative information), to overcome the above limitations in their MARS system ("Mergers and Acquisitions Reasoning System").

All the above software systems focus specifically on the valuation of the target company through quantitative and/or qualitative means. What the prospective acquirer or target company's management do with this information once negotiations actually begin is not touched on. We thus propose the development of a conceptual decision support system whose main aim is to answer this particular question. The emphasis will be on *what* negotiating strategy (given in terms of the qualitative/quantitative strategy parameters introduced in Chapter 4) a user (who will assume the role of one of the two main players: acquirer or target) should employ, given certain contextual conditions. Additional information supplied will relate to what share of the total merger gains each of the players can expect to achieve, i.e. how well each can expect to do. This proposed system thus picks up where existing computer-based tools leave off: it is assumed that the user has already developed an estimate of the value of the target company and some measure of the degree of precision of this estimate. This could have been arrived at by using one of the systems mentioned previously, or by some subjective method. Indeed it is envisaged that this conceptual system, once developed, could be attached as a back-end to an existing computer package similar to one of those mentioned above, thus offering enhanced decision support capabilities to management of acquiring and target companies involved in merger negotiations.

§7.3 CONCEPTUAL DESIGN FOR A DECISION SUPPORT SYSTEM BASED ON THE NEGOTIATION MODEL

In this section we propose the architecture of a conceptual DSS incorporating the negotiating model as a key component. We have not created an operational DSS here,

since our proposal requires the compilation of an external database consisting of a large number (possibly as many as 625) of matrices similar in dimension to that presented in Appendix 5.A (i.e. 20×60), each of which took almost 6.5 hours of CPU time on a MicroVax 3100-90 mainframe computer to generate. The compilation of the database and construction of the DSS is a major study in itself and beyond the scope of this thesis. Since the DSS will remain conceptual at this stage we will merely describe it in broad terms, and outline the operation of each proposed module of the system in an appendix to this chapter. We leave such details as input and output screen designs, retrieval methods etc to software engineers and analysts who might be involved in the operational construction of such a system, and who would base their decisions regarding these details on actual user requirements and programming language/environment capabilities.

§7.3.1 THE DATABASE

The conceptual DSS will extract information regarding the optimal strategy and expected gains at that strategy from an external database. The design and contents of this database warrant a short discussion.

The database should consist of the values of the two players' expected gains ($E(G_{acq})$ and $E(G_{tar})$) and the average transaction value for each of a number of both players' negotiating strategies (say 20 target strategies and 60 acquirer strategies, as in the matrices in Appendices 5.A and 5.B), for each of several values of the four contextual parameters σ_A , σ_B , δ_A and δ_B . For example, the chosen values of σ_A and σ_B could be 0.05, 0.10, 0.15, 0.20 and 0.25, whilst the chosen values for δ_A and δ_B could be 0.1, 0.3, 0.5, 0.7 and 0.9. The database might thus consist of $5^4 = 625$ matrices each of dimension 20×60 , where each matrix element consists of 3 items of numeric information. Each matrix would require approximately $20 \times 60 \times 6$ bytes = 7.2 Kb of fixed memory space; in the above configuration the total memory requirement would be approximately 625×7.2 Kb = 4.5 Mb. This could reside on the hard drive of the user's PC.

Whilst storage requirements are fairly moderate, the time and computing requirements to generate this quantity of simulated data is enormous however; for this

reason a somewhat sparser grid of contextual parameter values might make the task more practical. For the present study, time and computer availability do not allow us to compile such a database.

§7.3.2 A MODULAR DESCRIPTION OF THE DSS

A merger and acquisition DSS to help in the decision of how to proceed with negotiations could be used by either of the two main players: the acquirer and the target. The conceptual DSS therefore runs in either of two modes: Acquirer Support Mode or Target Support Mode. Note that another player of interest who is outside the structure of the actual merger deal, but who has a keen interest in the entire bargaining process is the arbitrageur, who attempts to make arbitrage profits by following and trying to accurately predict the course of a merger deal. Such an arbitrageur may use the DSS in either mode to determine optimal bargaining behaviour for both players, and may make arbitrage profits by acting on players' deviations from their optimal bargaining strategies.

We have chosen to display the conceptual DSS in modular form since this is good programming practice and enhances clarity and understanding. We will proceed with an overview of the DSS in the Acquirer Support Mode (ASM) configuration and later outline the DSS in Target Support Mode (TSM) configuration.

§7.3.2.1 ACQUIRER SUPPORT MODE (ASM) CONFIGURATION

The DSS operating in ASM allows a user to input his/her own reservation price, estimates of the contextual parameter values and perceptions about the target company in monetary terms, and outputs an optimal negotiating strategy for the acquirer in monetary and qualitative terms. Other relevant merger information relating to this optimal strategy is also provided, such as an outline of the target's optimal negotiating strategy. Between the input and output stages the DSS

- (a) rescales the input data into a form which is compatible with our model (and hence the database);
- (b) accesses the relevant information from the database, interpolating if necessary; and finally

- (c) transforms this and other calculated information for output back into units compatible with the input data.

These steps effectively comprise the model base, or problem processing system. Further to this the user is able to perform various sensitivity analyses around his/her optimal (acquirer) strategy. Specifically, if it is suspected before negotiations commence (or observed, if negotiations have already commenced) that the target will pursue a particular strategy different from its optimal one, a user in ASM may identify the acquirer strategy which is Pareto-optimal against this target strategy. Similar, a user in ASM may examine the effect of a deviation from the optimal acquirer strategy. In principle the DSS follows the same model base steps during the sensitivity analyses as during the running of the main program i.e. rescales input data, retrieves from the database, and transforms back into the original units.

The user will only see one or more input screens (which will guide his/her input) and one or more output screens (which will explain the necessary course of action). These are the dialogue generation and management system of the DSS and alone carry the responsibility of ensuring that the somewhat fuzzy input and output parameters are satisfactorily interpreted through the user/machine interface. The use of one or more interactive graphical user interfaces to achieve this (making use of tools such as graphical Likert scales) should be thoroughly explored. The architecture of the conceptual DSS in ASM is presented in Figure 7.1.

It will be noted from Figure 7.1 that each module has been given a short descriptive name and a code numbering. The letter "A" in the code indicates that the module forms part of the ASM configuration. Each module is more fully described in Appendix 7.A under the following headings:

- * *Function:* A concise statement of role of the module.
- * *Operational input:* The parameters that are actually used within the module.¹
- * *Operation:* A fuller description of how the operational input and other information is used to achieve the overall function of the module.

¹ These should be differentiated from parameters that are merely carried through for use in a later module. Such parameters have been totally omitted from this description to enhance clarity.

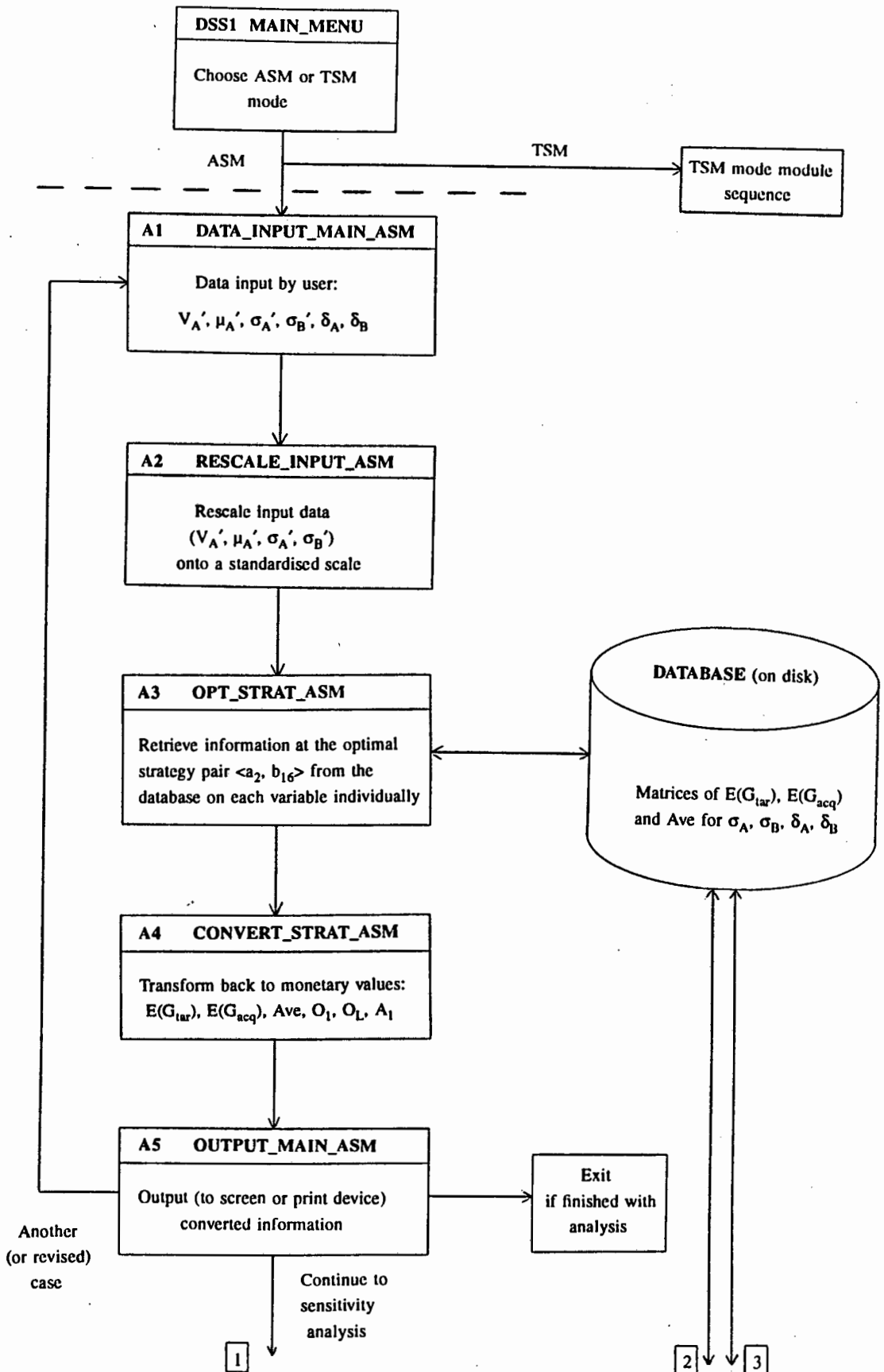


Figure 7.1. Architecture of the conceptual DSS in Acquirer Support Mode (ASM)

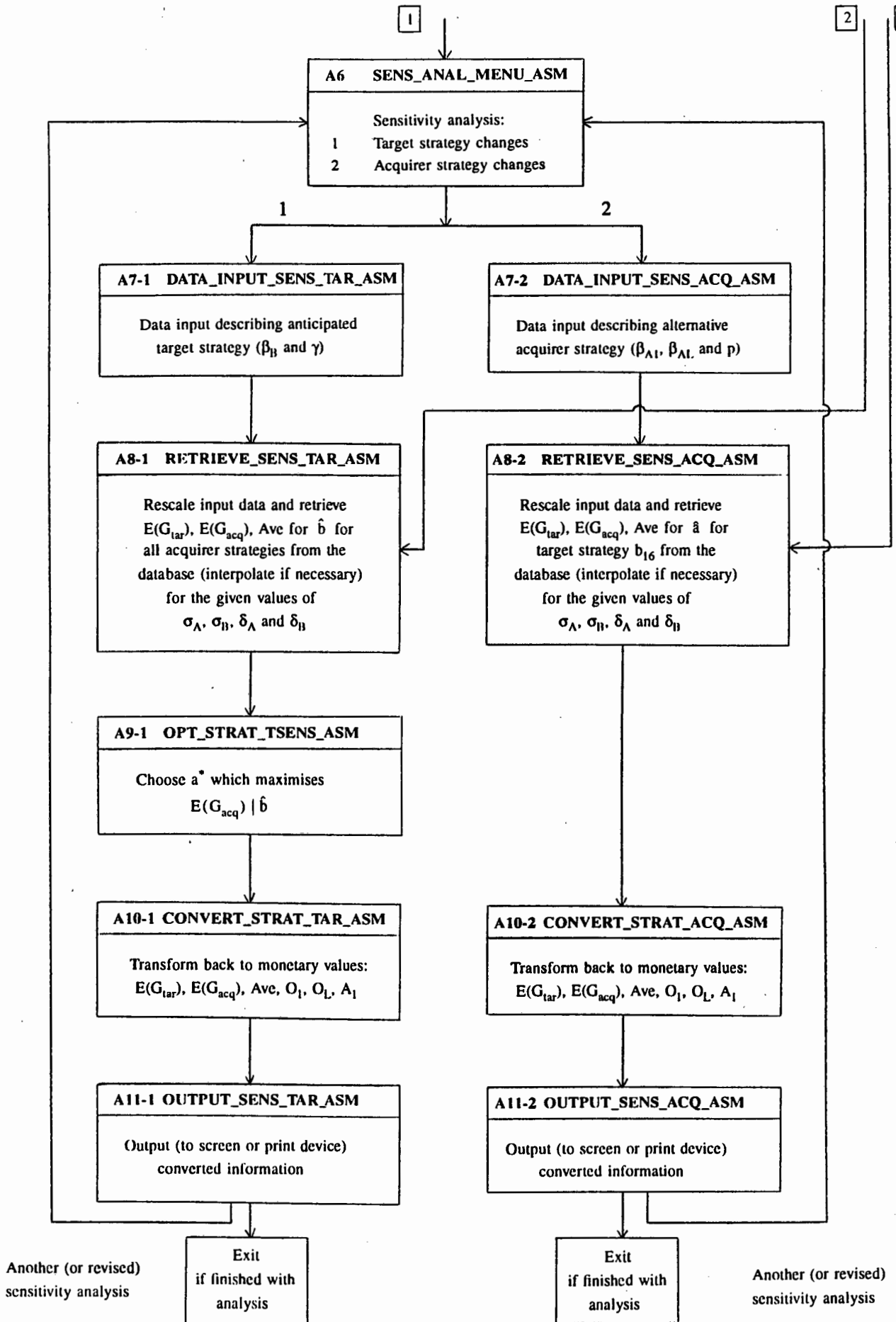


Figure 7.1. Architecture of the conceptual DSS in Acquirer Support Mode (ASM)
(continued)

Note that both the ASM module sequence and the TSM module sequence in the conceptual DSS are preceded by a module named MAIN_MENU and coded DSS1. This comprises a menu which passes control to either ASM or TSM, depending on the role chosen by the user/player. As indicated by the code, this module does not strictly form part of either the ASM or TSM configurations.

§7.3.2.2 TARGET SUPPORT MODE (TSM) CONFIGURATION

In this mode the user is required to input his/her own reservation price, perceptions about the target company's reservation price and estimates of the contextual parameter values. The output of the DSS in TSM is in the form of an optimal target negotiating strategy in monetary and qualitative terms, together with other relevant information relating to the optimal strategy, including an outline of the acquirers optimal negotiating strategy. The user is then invited to perform sensitivity analyses around the optimal strategies supplied here. A user may investigate the effect of a particular acquirer strategy (different from the optimal one), and is able to identify the target strategy which is Pareto-optimal against this acquirer strategy. Similarly, a user may examine the effect of a deviation from the optimal target strategy. The layout of the DSS in TSM is presented in Figure 7.2. Each module in TSM is recognisable by the letter "T" in the module code. It will be noted that TSM is the mirror image of ASM; for this reason we have felt it unnecessary to describe any of the modules in detail in an appendix.

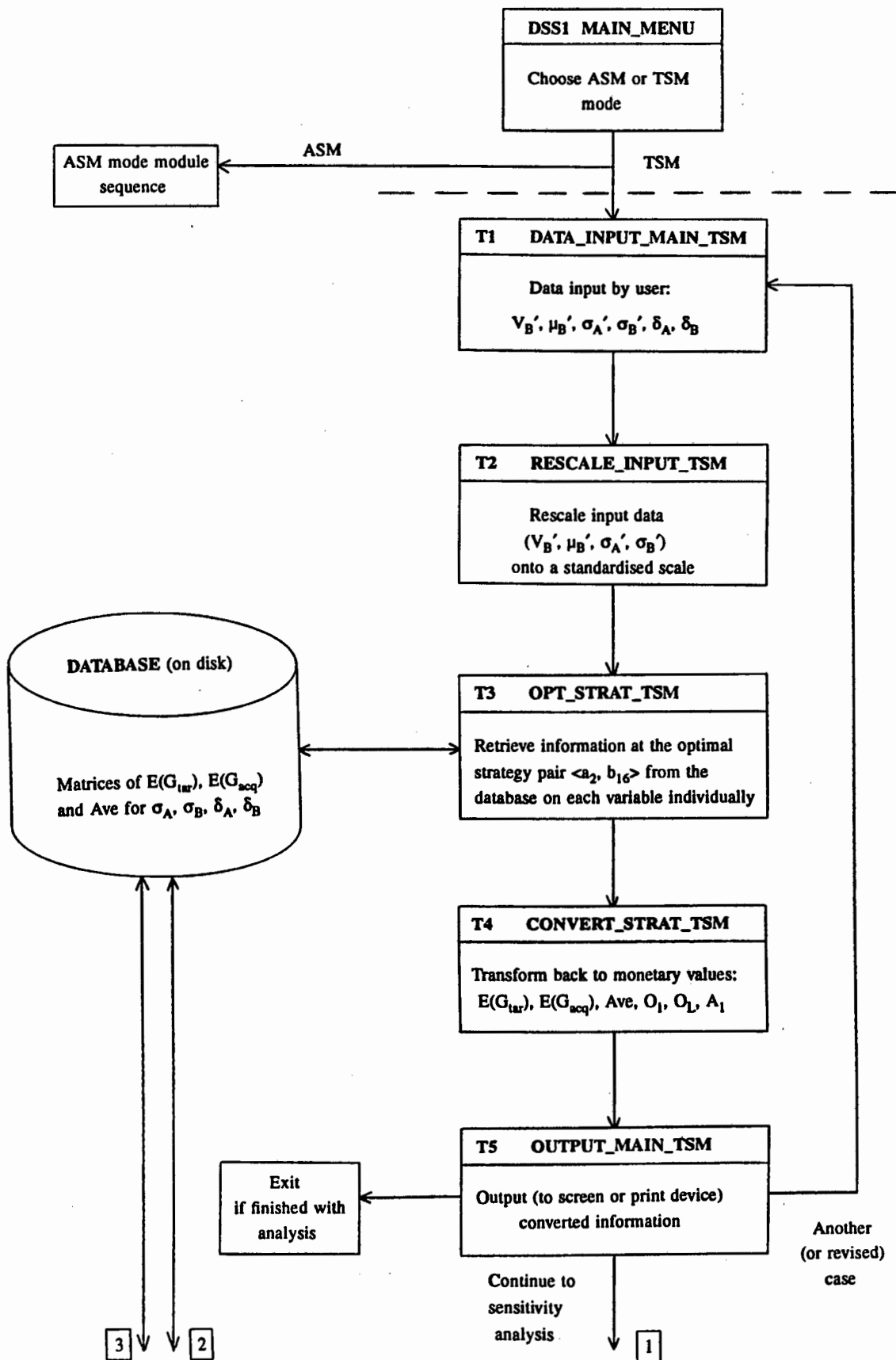


Figure 7.2. Architecture of the conceptual DSS in Target Support Mode (TSM)

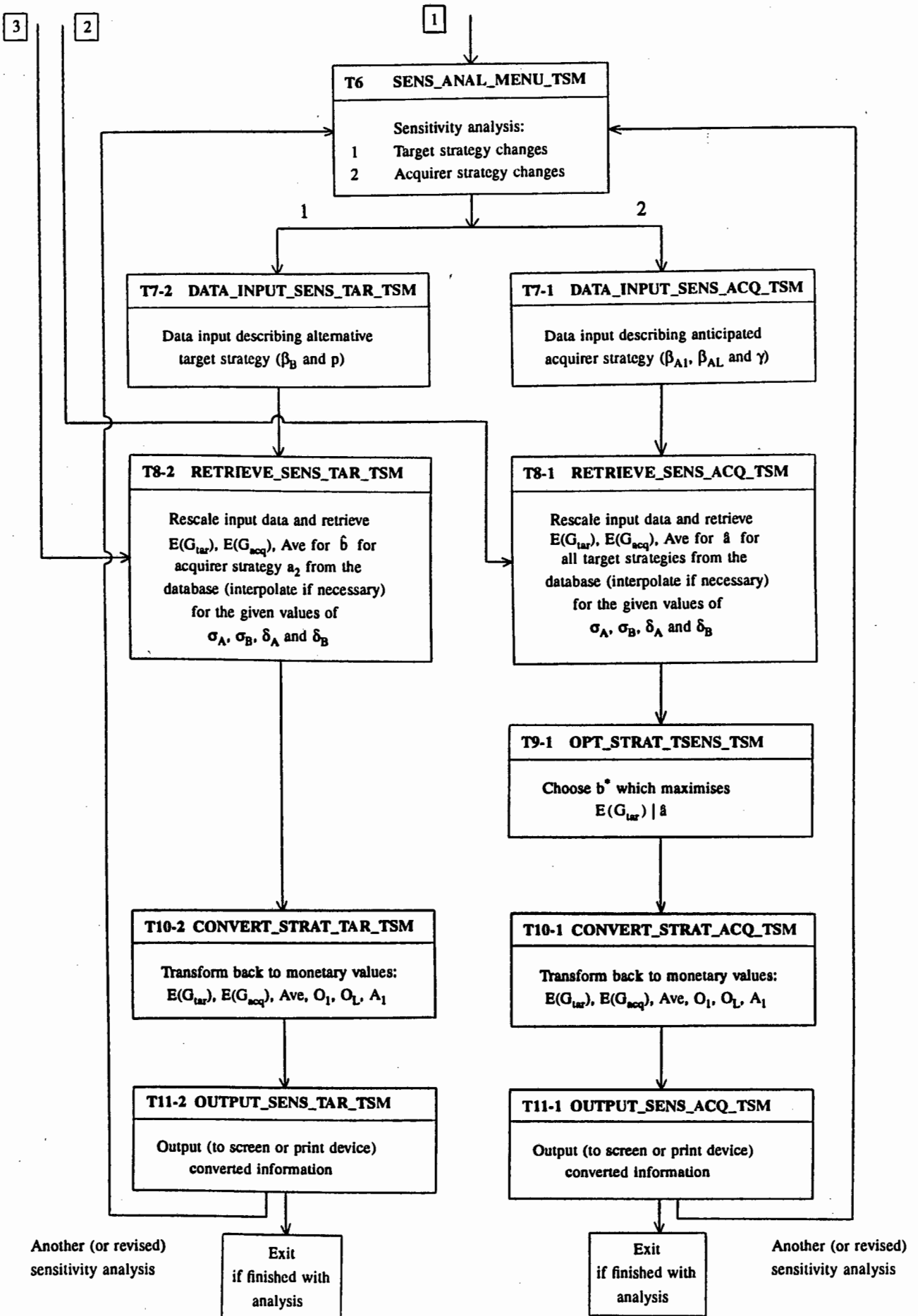


Figure 7.2. Architecture of the conceptual DSS in Target Support Mode (TSM) (continued)

§7.4 THE VALUE OF THE CONCEPTUAL DSS

The conceptual DSS outlined in this chapter (and more fully described in Appendix 7.A) is relatively simple in design and operation, yet offers an additional facet of decision support to currently existing computer based systems, i.e. information relating to *how* to negotiate. The model introduced in Chapter 4 described each of the players' negotiating strategies in terms of several aspects relating to the approach or tactics used by that player during the negotiating stage of the merger deal, and the conceptual DSS allows a user to investigate the effect of changes to one or more of these tactical aspects.

We have made no attempt to duplicate or improve the performance of any existing DSS - our aim here has been to emphasise the difference in our modelling approach. Indeed, an operational version of this conceptual DSS would be well suited to be used in conjunction with existing software, or even as an "add-on" module.

Since we do not have an operational DSS here, we cannot conclude that the DSS as proposed in this chapter is absolutely suitable for all users. As with all operations research studies, we should review an operational version of the DSS and possibly improve various aspects of the system, for example:

- the database design (are the functions of expected gains and average transaction values smooth enough to allow a sparser grid of contextual parameter values?)
- interactive input screens (are the graphical user interfaces as described sufficient to elicit an accurate value for the "fuzzy" parameters?)
- output information (could this be presented in a clearer, more usable way?)

Furthermore, the proposed use of the operational DSS should be considered. For example, if it is to be run in conjunction with some other existing DSS, input/output data requirements might have to be amended to ensure compatibility of the two systems.

To conclude, we have proposed a conceptual DSS in this chapter based on the model of an earlier chapter. Once an operational version has been constructed,

considerable work may have to be done to ensure its viability as a useful decision support tool. However, we believe that in concept it offers management support on one of the most crucial aspects of any merger deal - the tactics of negotiation.

APPENDIX 7.A. Description of the proposed modules comprising the conceptual DSS in Acquirer Support Mode (ASM)

A1 DATA_INPUT_MAIN_ASM

Function: A user-interface module to allow the user to input his/her perceptions of the contextual parameters and players' reservation prices.

Operational Input: None

Operation: This interactive user interface module prompts the user to enter his/her perceptions of the players' uncertainties (σ_A' and σ_B'), their negotiating strengths (δ_A and δ_B), the user's (in ASM) own reservation price (V_A') and the user's perception of the target's reservation price (μ_A'). The prime representation here indicates that the data is in *monetary terms* and will need to be rescaled later (see module A2) to be compatible with the model.

The data is captured when the user responds to either

- (1) a relevant question (with care being taken in the design stage so that technical language (such as the use of words "reservation price") be avoided); or
- (2) a graphical user interface.

Since V_A' is known to the user, its value may be elicited by means of a simple question, such as "*What is the maximum amount you would be prepared to offer for the target company?*"

The acquirer's uncertainty in its estimation of the target's reservation price is a somewhat fuzzy concept to a non-technical user, and is best elicited from the user by making use of an *interactive graphical user interface*. We propose that a one-dimensional axis be presented on the user's screen, with calibrations running from 0 up to at least V_A' (probably somewhat larger, say $1.3V_A'$). The cursor is mouse-controlled and moves up and down along this axis. The user is asked to identify a range within which he/she is 95%

sure that the target's true reservation price lies (again, technical language is to be avoided), and the upper and lower bounds are delimited by clicks of the mouse. The selected range is highlighted. Allowance is made within the module for the user to alter these bounds until he/she is satisfied with his/her choice. This effectively identifies a 95% confidence interval around μ_A' , which covers approximately $4\sigma_A'$. Then

$$\sigma_A' = \frac{\text{upper bound} - \text{lower bound}}{4}$$

$$\mu_A' = \frac{\text{upper bound} + \text{lower bound}}{2}$$

The user is required to estimate the target's uncertainty in its estimation of the acquirer's reservation price (σ_B'). This is likely to be an even more fuzzy concept for the user, and a similar graphical interface to the one outlined above is proposed. The acquirer's reservation price V_A' should always be positioned at the middle of any range chosen by the user, and thus the user's freedom to determine this range should extend to only one side of V_A' . The value of σ_B' is found identically to σ_A' .

The acquirer's perceptions of its own negotiating strength (δ_A) lies in the unit interval. It is proposed that a graphical Likert scale be used here. Thus the user will be exposed to a range of statements, ranging from

*I am very much weaker and feel that I will
be completely dominated during the negotiations* (if $\delta_A = 0.1$)

through

We are of about equal negotiating strength (if $\delta_A = 0.5$)

to

*I feel that I will be able to completely
dominate during negotiations* (if $\delta_A = 0.9$)

which are positioned alongside the graphical axis at the relevant value of δ_A . The cursor can be moved up and down the axis using a mouse, which is clicked when the user has made his/her choice, thereby selecting $\delta_A \in [0;1]$. The sum of the players' negotiating strengths should theoretically

equal 1. Thus the acquirer's perception of the target's negotiating strength is $\delta_B = 1 - \delta_A$. However we can allow deviations from this ideal behaviour by asking the user if he/she thinks that the target will overestimate, realistically estimate or underestimate its own negotiating strength. The parameter δ_B is adjusted accordingly by a multiplicative factor depending on the answer to the question. This raw input data is rescaled (if necessary) in module A2.

A2 RESCALE_INPUT_ASM

Function: To rescale the raw input data in monetary terms onto a standardised scale.

Operational Input: V_A' , μ_A' , σ_A' , σ_B' (from module A1)

Operation: The raw input data relating to the user's perception of the players' reservation prices in monetary terms are converted onto a standardised scale through the transformation

$$x_{\text{rescaled}} = \frac{x_{\text{raw}} - \mu_A'}{V_A' - \mu_A'}$$

This results in a rescaled acquirer's reservation price of $V_A = 1$ and a rescaled acquirer's perception of the target's reservation price of $\mu_A = 0$. The uncertainties σ_A' and σ_B' are rescaled through the transformation

$$\sigma_{i;\text{rescaled}} = \frac{\sigma_{i;\text{raw}}'}{V_A' - \mu_A'} \quad \text{where } i = A \text{ or } B.$$

Note that the players' perceived negotiating powers δ_A and δ_B , although being an integral part of the input, are not affected by this module.

On completion control is passed to module A3 which determines optimal strategies.

A3 OPT_STRAT_ASM

Function: To retrieve relevant information at the optimal strategy from the database, and interpolate on each variable individually, if necessary.

Operational Input: σ_A, σ_B (from module A2)
 δ_A, δ_B (from module A1)

Operation: One of the main conclusions of Chapter 6 was that the strategies comprising the optimal strategy pair $\langle a_2, b_{16} \rangle$ were extremely robust to changes in the values of the contextual parameters. This module thus retrieves from the database the relevant statistics ($E(G_{tar})$, $E(G_{acq})$ and Ave) for the given values of σ_A , σ_B , δ_A and δ_B at strategy pair $\langle a_2, b_{16} \rangle$. If the user-defined value of one or more of the parameters is not resident in the database, the module retrieves data for that parameter closest on either side of the user-defined value, and interpolation occurs on each variable individually. It is envisaged that a simple linear interpolation will be entirely adequate, although more complex (and computer-intensive) interpolation methods such as kernel methods, splining and kriging could be investigated (see Hastie and Tibshirani (1990) and Thisted (1988) for a full overview and discussion of such methods). This module also notes the values of β_{A1} , β_{AL} , p , β_B and γ at $\langle a_2, b_{16} \rangle$ for later use (modules A4 and A5).

A4 CONVERT_STRAT_ASM

Function: To convert the solution at the optimal strategy on the standardised scale into monetary terms.

Operational Input: $\sigma_A, \sigma_B, \delta_A, \delta_B, \mu_A', V_A'$ (from module A2)
 $E(G_{tar}), E(G_{acq}), Ave, \beta_{A1}, \beta_{AL}, \beta_B$ (from module A3)

Operation: The solution at the optimal strategy pair $\langle a_2, b_{16} \rangle$ is now converted back into monetary terms in preparation for output to the user. An inverse transformation to that used to standardise monetary amounts in module A2

is used here, i.e.

$$x_{\text{raw}} = \mu_A' + (V_A' - \mu_A') \cdot x_{\text{rescaled}}$$

Thus the average transaction value is

$$\text{Ave}' = \mu_A' + (V_A' - \mu_A') \cdot \text{Ave}$$

The players' expected gains $E(G_{\text{tar}})'$ and $E(G_{\text{acq}})'$ are not measured relative to μ_A' ; they are

$$E(G_{\text{tar}})' = (V_A' - \mu_A') \cdot E(G_{\text{tar}})$$

and
$$E(G_{\text{acq}})' = (V_A' - \mu_A') \cdot E(G_{\text{acq}})$$

The probability of merger agreement being reached is found from the relationship $E(G_{\text{tar}}) = P(\text{merger occurring}) \cdot \text{Ave}$ (see (5.2) in Chapter 5).

Thus

$$P(\text{merger occurring}) = \frac{E(G_{\text{tar}})}{\text{Ave}}$$

The optimal negotiating strategy will be presented to the user as an initial offer amount, a final offer amount and some indication of the persistence that the acquirer should maintain. The initial offer amount (in monetary terms) is thus

$$\begin{aligned} O_1' &= \mu_A' + (V_A' - \mu_A') \cdot O_1 \\ &= \mu_A' + (V_A' - \mu_A') \cdot [1 - \delta_A(1 - \sigma_A \cdot \Phi^{-1}(\beta_{A1}|a_2))] \end{aligned}$$

where $\beta_{A1}|a_2$ is the value of the acquirer's initial strategic concession parameter conditional on its strategy a_2 . Similarly, the final offer amount in monetary terms is calculated as

$$\begin{aligned} O_L' &= \mu_A' + (V_A' - \mu_A') \cdot O_L \\ &= \mu_A' + (V_A' - \mu_A') \cdot [1 - \delta_A(1 - \sigma_A \cdot \Phi^{-1}(\beta_{AL}|a_2))]. \end{aligned}$$

The target's co-optimal strategy will also be presented; it will be given in terms of an initial acceptance level and a yield decrement. The initial acceptance level in monetary terms is

$$\begin{aligned} A_1' &= \mu_A' + (V_A' - \mu_A') \cdot A_1 \\ &= \mu_A' + (V_A' - \mu_A') \cdot [\delta_B(1 + \sigma_B \cdot \Phi^{-1}(\beta_B|b_{16}))] \end{aligned}$$

where $\beta_B|b_{16}$ is the value of the target's strategic concession parameter conditional on its strategy b_{16} .

On completion control is passed to module A5 for output to the user.

A5 OUTPUT_MAIN_ASM

Function: To output to the screen or print device information relevant to the decision-maker regarding the players' optimal strategies.

Operational Input: p, γ (from module A3)
 $E(G_{tar})', E(G_{acq})', Ave', P(\text{merger occurring}), O_1', O_L', A_1'$
 (from module A4)

Operation: The players' optimal strategies and related information about the expected negotiated outcome (assuming both players play these optimal strategies) are presented to the user. The optimal acquirer strategy consists of

- the initial offer (in monetary terms), O_1'
- the final offer (in monetary terms), O_L'
- the degree of persistence, p , that the acquirer should maintain in his/her offers in pursuit of a negotiated outcome.

Since the acquirer's optimal strategy is the real essence of the DSS in ASM, it should be presented in as user-friendly and understandable a manner as possible. An example of a concise yet comprehensible exposition is as follows:

The optimal strategy to employ for this scenario is to begin with an initial offer of no lower than $R [O_1']$. Be prepared to negotiate upwards to a final offer no greater than $R [O_L']$.

The degree of persistence, p , is presented as a qualitative indicator variable on a Likert scale, rather than as a number on the unit interval, which would be difficult for a user to interpret, especially since he/she would be likely to be unfamiliar with the model. Thus the acquirer's degree of persistence could be termed

very reluctant to make further offers, if $p = 0.9$

hesitant to make further offers, if $p = 0.7$

ambivalent to making further offers, if $p = 0.5$

persistent in making further offers, if $p = 0.3$

extremely persistent in making further offers, if $p = 0.1$

The optimal target strategy is also presented to the user. This strategy consists of

- the initial acceptance level, A_1'
- the target's yield decrement, γ , which indicates how rapidly the target should decrease its acceptance level towards the acquirer's previous offer.

Again, as the user is likely to be unfamiliar with the terms and definitions used in the model, the information should be presented in layman's terms, for example:

Under this scenario the target should optimally not accept an offer less than $R [A_1']$.

The target's yield decrement, too, should be presented in qualitative terms on a Likert scale. Thus the target's reaction to non-initial offers could be

- maintain* the acceptance level, if $\gamma = 0.05$
- slight reduction* in acceptance level, if $\gamma = 0.35$
- large reduction* in acceptance level, if $\gamma = 0.65$
- capitulation* in acceptance level, if $\gamma = 0.95$

Other information relevant to the merger negotiations (assuming both players employ their optimal strategies, as outlined above) can now be presented. This includes

- the probability that agreement will be reached [$P(\text{merger occurring})$]
- the average transaction value if agreement is reached [Ave]
- the expected gains to the two players [$E(G_{tar})'$ and $E(G_{acq})'$].

Once the output has been presented the user has the option to either

- exit the system; or
- make changes to his/her data input (i.e. return to module A1); or
- continue with a sensitivity analysis relating to the optimal strategies (module A6).

A6 SENS_ANAL_MENU_ASM

Function: To pass control to one of two sensitivity analysis module sequences.

Operational Input: None

Operation: At this point the user is allowed to ask "what if...?" type questions. The conceptual system allows two questions here:

- (1) What if the target plays a strategy different from its optimal one as described in module A5?
- (2) What if I (the acquirer) play a strategy different from the optimal one described in module A5?

The module A6 prompts the user to make a choice of sensitivity analysis, and merely directs control to the sequence of modules which then answers the relevant question.

A7-1 DATA_INPUT_SENS_TAR_ASM

Function: To capture a user's perception of a target strategy.

Operational Input: None

Operation: A target strategy is characterised by the target's degree of concession, β_B (which determines its initial acceptance level), and its yield decrement, γ . In this sensitivity analysis we allow investigation of target strategies other than that described in module A5.

The user is prompted to enter a target's likely initial acceptance level and a likely yield decrement. The yield decrement γ is captured by using a graphical Likert scale as was the case for δ_A in module A1. Descriptive statements defining discrete points on this scale would be similar to those describing γ used in module A5.

A8-1 RETRIEVE_SENS_TAR_ASM

Function: To rescale the target strategy data onto a standardised scale, and to retrieve (from the database) and interpolate the relevant negotiating information for all acquirer strategies for the given target strategy.

Operational Input: $\sigma_A, \sigma_B, V_{A'}, \mu_{A'}$ (from module A2)
 δ_A, δ_B (from module A1)
 A_1', γ (from module A7-1)

Operation: Initially the given target initial acceptance level A_1' is converted to the standardised scale by the transformation

$$A_1 = \frac{A_1' - \mu_{A'}}{V_{A'} - \mu_{A'}}$$

This in turn gives rise to a value of β_B , since

$$\Phi^{-1}(\beta_B) = \frac{A_1 - \delta_B}{\delta_B \sigma_B}$$

The module then retrieves from the database the relevant statistics $E(G_{tar})$, $E(a_{acq})$ and Ave for the given values of the contextual parameters $\sigma_A, \sigma_B, \delta_A$ and δ_B for all strategy pairs $\langle a_i, \hat{b} \rangle$, where \hat{b} is the target strategy implied by the target strategy parameters β_B and γ . If the user-defined value of one or more of the parameters is not resident in the database, the module retrieves data for that parameter closest on either side of the user-defined value, and interpolates on each variable individually. The module also notes the values of $\beta_{A1}, \beta_{AL}, p, \beta_B$ and γ at $\langle a_i, \hat{b} \rangle$, for all acquirer strategies a_i .

A9-1 OPT_STRAT_TSENS_ASM

Function: To identify an optimal acquirer strategy for a given target strategy.

Operational Input: $E(G_{acq}) | \hat{b}, a_i$ $i = 1, 2, \dots, n$
 where $n =$ number of distinct acquirer strategies stored in the database.

Operation: The acquirer's optimal strategy conditional on the given target strategy \hat{b} is the one which maximises the acquirer's expected gain given \hat{b} . This module identifies the value of i , and hence $a^* = a_i$ which achieves this.
 Then

$$E(G_{acq}) | \hat{b}, a^* = \max_i [E(G_{acq}) | \hat{b}, a_i]$$

A10-1 CONVERT_STRAT_TAR_ASM

Function: To convert the solution at the optimal acquirer strategy for the given target strategy on the standardised scale into monetary terms.

Operational Input: $E(G_{acq}) | \hat{b}, a^*$ $E(G_{tar}) | \hat{b}, a^*$
 $Ave | \hat{b}, a^*$
 $\beta_{A1} | \hat{b}, a^*$ $\beta_{AL} | \hat{b}, a^*$ $\beta_B | \hat{b}, a^*$
 (all from module A8-1)
 $V_A', \mu_A', \sigma_A, \sigma_A, \delta_A, \delta_A$ (form module A2)

Operation: The operation of this module is identical to module A4, except that the optimal strategy pair $\langle a_2, b_{16} \rangle$ is replaced by $\langle a^*, \hat{b} \rangle$.

A11-1 OUTPUT_SENS_TAR_ASM

Function: To output to the screen or print device information relevant to the decision-maker (in ASM) regarding negotiating strategies and outcomes.

Operational Input: $E(G_{acq})' | \hat{b}, a^*$ $E(G_{tar})' | \hat{b}, a^*$
 $Ave' | \hat{b}, a^*$ $P(\text{merger occurring})' | \hat{b}, a^*$
 $O_1' | \hat{b}, a^*$ $O_L' | \hat{b}, a^*$ $A_1' | \hat{b}, a^*$
 (all from module A10-1)
 $p | \hat{b}, a^*$ $\gamma | \hat{b}, a^*$ (from module A8-1)

Operation: The operation of this module is identical to module A5. Once the output has been presented the user has the option to either

- exit the system; or
- try another sensitivity analysis (i.e. return to module A6).

A7-2 DATA_INPUT_SENS_ACQ_ASM

Function: To capture a user's acquirer strategy.

Operational Input: None

Operation: An acquirer strategy is characterised by the acquirer's initial and final degrees of concession (β_{A1} and β_{AL} respectively) which determine the initial and final offers, and its degree of persistence, p . In this sensitivity analysis we allow investigation of acquirer strategies other than that described in module A5.

The user is prompted to enter his/her initial and final offers. It is proposed that the initial and final offers at the optimal point $\langle a_2, b_{16} \rangle$ are presented on the screen for comparison whilst the decision is being made. The degree of persistence p is captured by using a graphical Likert scale as was the case for δ_A in module A1. Descriptive statements defining discrete points on this scale would be similar to those describing p used in module A5.

A8-2 RETRIEVE_SENS_ACQ_ASM

Function: To rescale the acquirer strategy data onto a standardised scale, and to retrieve (from the database) and interpolate the relevant negotiating information for the optimal target strategy (b_{16}).

Operational Input: $\sigma_A, \sigma_B, V_A', \mu_A'$ (from module A2)
 δ_A, δ_B (from module A1)
 O_1', O_L', p (from module A7-2)

Operation: The given initial offer O_1' is converted to the standardised scale by the transformation

$$O_1 = \frac{O_1' - \mu_A'}{V_A' - \mu_A'}$$

This gives rise to a value of β_{A1} , since

$$\Phi^{-1}(\beta_{A1}) = \frac{O_1 + \delta_A - 1}{\delta_A \sigma_A}$$

Likewise, $\Phi^{-1}(\beta_{AL}) = \frac{O_L + \delta_A - 1}{\delta_A \sigma_A}$ where $O_L = \frac{O_L' - \mu_A'}{V_A' - \mu_A'}$

This module retrieves from the database the relevant statistics $E(G_{tar})$, $E(acq)$ and Ave for the given values of the contextual parameters $\sigma_A, \sigma_B, \delta_A$ and δ_B for strategy pair $\langle \hat{a}, b_{16} \rangle$, where \hat{a} is the target strategy implied by the acquirer strategy parameters β_{A1}, β_{AL} and p . If the user-defined value of one or more of the parameters is not resident in the database, the module retrieves data for that parameter closest on either side of the user-defined value, and interpolates on each variable individually. The module also notes the values of $\beta_{A1}, \beta_{AL}, p, \beta_B$ and γ at $\langle \hat{a}, b_{16} \rangle$.

A10-2 CONVERT_STRAT_ACQ_ASM

Function: To convert the solution at the given acquirer strategy on the standardised scale into monetary terms.

Operational Input: $E(G_{acq}) | \hat{a}, b_{16}$ $E(G_{tar}) | \hat{a}, b_{16}$
 $Ave | \hat{a}, b_{16}$
 $\beta_{A1} | \hat{a}, b_{16}$ $\beta_{AL} | \hat{a}, b_{16}$ $\beta_B | \hat{a}, b_{16}$
 (all from module A8-2)
 $V_A', \mu_A', \sigma_A, \sigma_A, \delta_A, \delta_A$ (form module A2)

Operation: The operation of this module is identical to module A4, except that the optimal strategy pair $\langle a_2, b_{16} \rangle$ is replaced by $\langle \hat{a}, b_{16} \rangle$.

A11-2 OUTPUT_SENS_ACQ_ASM

Function: To output to the screen or print device information relevant to the decision-maker (in ASM) regarding negotiating strategies and outcomes.

Operational Input: $E(G_{acq})' | \hat{a}, b_{16}$ $E(G_{tar})' | \hat{a}, b_{16}$
 $Ave' | \hat{a}, b_{16}$ $P(\text{merger occurring})' | \hat{a}, b_{16}$
 $O_1' | \hat{a}, b_{16}$ $O_L' | \hat{a}, b_{16}$ $A_1' | \hat{a}, b_{16}$
 (all from module A10-2)
 $p | \hat{a}, b_{16}$ $\gamma | \hat{a}, b_{16}$ (from module A8-2)

Operation: The operation of this module is identical to module A5. Once the output has been presented the user has the option to either

- exit the system; or
- try another sensitivity analysis (i.e. return to module A6).

CHAPTER 8**SUMMARY, MAIN CONCLUSIONS AND RECOMMENDATIONS FOR
FURTHER RESEARCH****§8.1 SUMMARY AND MAIN CONCLUSIONS**

This study has examined mergers and acquisitions from the viewpoint of game-theoretic modelling. We have assumed that the players in a merger deal are essentially in conflict over the single issue of price; any such deal is thus characterised by some negotiating or bargaining between the management teams of the two companies concerned. Our aim has been twofold: to construct (and test where possible) bargaining models which might lead to greater understanding of the behaviour of the players involved in merger "games", and to develop a dynamic model of the negotiations in an attempt to offer useful decision support for merger players. These two aims were developed in Part A (Chapters 2 and 3) and Part B (Chapters 4 through to 7) respectively.

In Chapter 2 we began the study of analytic models for mergers and acquisitions by assuming full and shared information amongst the two players about the pre- and post-merger values of the companies, and modelled the merger bargaining game as a co-operative two person non-zero sum game, where the outcome was the net gains to the two players. A Nash-Kalai bargaining model, which depended on the players' utility functions, was implemented to explain the proportion of the synergy gains from merger that accrue to the target company.

Two separate families of utility functions for the model were proposed and tested: a linear utility family and a negative exponential utility family. It was shown that in the simple linear utility family the only determinant in explaining the variation in the

proportion of the synergy gains accruing to the target was the relative bargaining strength of the acquirer vis-a-vis the target, and this was successfully estimated by two different models, each comprising of only 2 parameters. Amongst the negative exponential utility family the best model comprised of four estimated parameters which were used in the estimation of the risk aversion coefficients of the two companies. Unlike in the case of the linear utility family the model fit here did not depend to any great extent on the relative bargaining strength of the two companies. Simplifications of this model employing fewer model parameters failed to provide significant model fit. Specifically, the risk aversion coefficients of the two companies involved were found to be different functions for the companies, and dependent on company size. In general, the best of the negative exponential utility models produced a far better empirical model fit (in terms of the coefficient of determination) than did the linear utility models, but employed many more parameters. On a parameter-adjusted basis, however, there was not much to choose between the two models.

Whilst the modelling approach of Chapter 2 succeeded in offering significant empirical fits, and hence provided a degree of understanding about the bargaining behaviour of merger participants, the oppressive (and unrealistic) assumption of complete certainty of information amongst both players needed to be addressed. In Chapter 3, therefore, we assumed that the two parties had complete certainty and agreement about the pre-merger values of the two companies, but shared uncertainty about the post-merger value of the combined entity (i.e. the extent of the uncertainty was known to both players with certainty, in the form of some probability distribution). A Nash-Kalai bargaining model incorporating the uncertainty was developed. The advantages offered by this model over those of the previous chapter were several. Firstly, the degree of empirical model fit (in terms of predicting the total amount paid by the acquirer) was significantly greater than before due to the presence of an extra explanatory parameter (the uncertainty), and secondly by introducing a measure of uncertainty, the model was able to discriminate between the amount that should be paid to the target shareholders by means of cash and by means of a share transfer for any given level of uncertainty. The theoretical model produced some results which have practical application. Firstly, a cash-only offer is never optimal: the optimal offer will consist of an exchange of shares and possibly a cash side

payment. This ensures that the target bears at least a portion of the risk involved in the merger. Conditions were identified under which shares only would be offered as a means of payment, and a combination of cash and shares. Secondly, the optimal amount offered by an acquirer depends on the form of payment (the cash/share split) and the level of perceived risk. In a pure share exchange the offer amount remains constant regardless of the risk. In general, if cash is included in the payment, an increase in risk implies a decrease in the optimal amount of cash offered. Examination of a specific empirical example indicated that the acquirer (who makes the offer) appears to be dominant over the target when it comes to choosing the optimal form of payment. The form of financing the optimal offer always appears to satisfy the acquirer's desired form of payment regardless of the levels of the players' risk parameters.

The uncertainty about the post-merger value of the combined entity is a *conceptual* variance, and cannot be empirically measured for any single merger. A Nash-Kalai model was thus constructed, utilising a family of additive mean/variance utility functions (the risk-adjusted analogue of the linear utility function used previously), which allowed empirical estimation of this uncertainty. Due to the presence of the uncertainty, the model was also able to provide estimates of the optimal cash and share portions of any offer. The model solution showed that the proportion of the post-merger company offered to the target by means of an exchange of shares was independent of the level of uncertainty in the post-merger value of the combined company; it depended only on the relative sizes of the risk aversion coefficients of the two companies. The amount of cash offered, however, was more complex: it was a function of the risk aversion coefficients, negotiating power, the uncertainty in the post-merger company value and the sizes of the companies involved. Model fits showed significant improvement over comparable simpler risk-free models, indicating that the incorporation of uncertainty is a necessary feature of any model attempting to provide understanding of this complex bargaining process.

An extension of this model was proposed, in which a risk-adjusted analogue of the negative exponential utility function was employed. Apart from possible improvement in model fit it did not offer any greater insights into the bargaining process.

The analytical models considered in Part A omitted a multitude of the complexities involved with a real-world merger or acquisitions. They did however, by virtue of their fit, help to shed understanding on aspects of the bargaining process.

In Part B a model based on the ideas of negotiation analysis was constructed which focused away from the usual game-theoretic idea of an equilibrium solution, and as a result offered a more positive solution to the merger bargaining problem (i.e. closer to what might really happen in real-world bargaining). This model is a first attempt at analysing the ill-structured and complex processes involved in negotiation, and was constructed to capture the dynamic, multi-stage nature of the negotiation process. It offered prescriptive advice to one of the players on likely Pareto-optimal bargaining strategies, given a description of how the other party might behave. The parsimonious model made use of several parameters, some contextual (describing the negotiating "environment", i.e. perceived bargaining strengths and risks) and some defining the players' strategies (describing a course of action to be followed by the players). The nature of the input data employed by this model is judgemental and subjective, and as such may be totally different for apparently similar negotiating pairs.

The model was implemented by means of a Monte Carlo simulation procedure which would tend to describe typical long-run outcomes of similar negotiations. The implementation produced a matrix of expected gains to both players and average transaction values for a wide range of each of the players' strategies. Several observations were evident from examination of the resulting game matrix. Firstly, each of the players can achieve a wide range of expected gains, depending on which combination of strategies the players implement. Significant opportunities appear to exist for the target company to claim a larger share of the merger synergy gains than the acquiring company. Secondly, the combined expected gains accruing to the two companies never reached the maximum synergy available, i.e. the players can expect that a Pareto-inefficient agreement will be reached. Thirdly, an acquirer offer of making an initial bid and holding firm is always inferior to one in which at least some increase in the offer is shown in later bargaining rounds.

The choice of an "optimal" negotiating strategy was examined using two differing approaches. Firstly, the conventional game-theoretic approach of eliminating players' dominated strategies was implemented, but this method makes the rather severe assumption that both players are hyper-rational. An alternative means of identifying "optimal" strategies is to follow a softer Bayesian decision-making approach. It was shown in the simulated merger game that in terms of expected gains there was little to be gained from using one method over the other. The "optimal" strategies identified showed that the acquirer should start with a low initial offer, but be prepared to negotiate vigorously up to a final possible offer still below the acquirer's perception of a fair price. The target, on the other hand, should be prepared to accept an offer far below its expectation of a fair price, and in any event rapidly decrease its acceptance level towards the initial offer in the next bargaining round.

The players' strategies were described in the model by parameters which were continuous in nature but were evaluated at only a discrete number of values in the Monte Carlo simulation. The question of how coarse the grid of discrete strategy parameter values should be to adequately identify features contained in the game with players' continuous strategy sets was raised, and it was found that a grid consisting of 7 or 8 discrete strategies for each player was sufficient, in the sense that a matrix of this dimension satisfactorily identified "optimal" negotiating strategies whilst avoiding issues which clouded the decision-making process, such as multiple equilibrium points and Prisoners' Dilemma problems which were evident in game matrices of larger dimensions. Furthermore, a matrix of this compact size could easily be assimilated by the human (user's) mind and drastically limited computational requirements. Care had to be taken to ensure that the subset of strategies used for this purpose were chosen in such a way that an even spread of strategies across all those available was achieved.

The simulations were repeated for a wide range of contextual conditions (players' perceived risks and negotiating powers), with specific attention being paid to how the optimal bargaining strategies changed, and how the players' expected gains at the optimal bargaining strategies changed. This effectively constituted a sensitivity analysis with respect to the contextual parameters. It was shown that the optimal bargaining strategy

pair arrived at in the initial simulation was extremely robust to contextual changes, a point useful for later decision support. Furthermore, changes to the contextual parameter values which led to the players starting negotiations from positions more extreme from one another appeared to have a greater negative effect (in terms of expected gains) on the target than on the acquirer. This was mainly due to the model having the target yielding towards the acquirer's offers, whilst the acquirer's offers were made independently of the target's reaction.

The model and its simulated results could provide useful decision support to merger and acquisition practitioners regarding strategies to be employed during negotiations. The final chapter of this study therefore proposes a conceptual decision support system that incorporates the model and a set of simulated outcomes as the model base and knowledge base respectively. Although not constructed, this DSS in concept was able to respond to a user's queries regarding an optimal strategy for either player, and expected outcomes from deviations from this optimal strategy.

§8.2 SUGGESTIONS FOR FURTHER RESEARCH IN THIS AREA

This study could be extended in several directions, some of which we will outline in this section.

The analytical models of Chapter 2 utilised a linear utility function and a family of negative exponential utility functions. Many other (more complex) functional forms could serve as utility functions. If the objective is to optimise model fit some other functional form might prove superior to those considered here. Apart from providing a "good" descriptive model, this research might shed light on the form of utilities actually employed by players in bargaining situations. The same comment applies to the models with shared uncertainty in Chapter 3.

We made several simplifying assumptions about the functional forms of the relationships linking the model "components" (negotiating power, risk aversions etc) to assumed known data (pre- and post-merger values or estimates of company sizes). These

were purposefully kept simple for clarity's sake. More complex forms here might also improve model fit and estimates of the cash portion and share portion of the actual offer amount.

The negotiation-based model of Chapter 4 was a first attempt at modelling the dynamic nature of the negotiation process, and as such showed promise to serve as a key component in a DSS. This model might provide a starting point for a more refined model which might take better cognisance of actual behavioural properties of human negotiators. Our model, for example, has a target responding to each acquirer offer by reducing its acceptance level. In practice the target might not react so passively; it might pass on to the acquirer (directly or indirectly) that the offers are far too low, or simply break off negotiations. Since a DSS should supply management information for real-world use, these options should also be considered. Furthermore, the model described the concept of a bargaining strategy in terms of just two or three fuzzy (but reasonably understandable) parameters. Practical research at the cognitive level might offer information as to whether this formulation of a strategy is sufficient for practical interpretation by a human user. Our model could be adapted accordingly.

Since the aim of the model was to promote understanding and supply decision support, the conceptual DSS (based on the existing model or some refined version) should be constructed and tested on a panel of human managers who have been or are currently involved in merger negotiation. This will provide feedback about further user requirements for a commercial version of this DSS.

Of course this model has considered a specific example of a negotiation scenario. Negotiation situations abound in many walks of life (e.g. political, business etc) and our model could be easily adapted to offer support to any such negotiation. An important adaptation here would be to extend the model to consider negotiations in which the conflict extends over multiple issues.

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