Application of Process Data Reconciliation in Power Plants

Prepared by:

Muhluri Calvin Mathebula

MTHCAL001

Department of Mechanical Engineering

University of Cape Town

Supervisor:

Dr. Wim Fuls

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Abstract

The operation of power plants and chemical processes requires process measurements for optimal operations. Process measurements are essential for plant performance optimization, process monitoring and process control. It is vital to have reliable and accurate process data to achieve process optimization. However, process measurements are inevitably subject to measurement errors. These measurement errors are classified as random and gross errors.

Data reconciliation technique is an effective data treatment method that is used in chemical processes to enhance the quality of process data. The purpose of data reconciliation is to reduce random errors to achieve measurements which are as accurate and reliable as possible. Data reconciliation technique uses available process measurements to produce consistent and accurate estimates, so close to the true values that they satisfy model constraints. Further, data reconciliation technique depends on measurement redundancy to perform reconciliation and produce reliable estimates. In addition, data reconciliation can also provide estimates of unmeasured observable variables. Process data reconciliation is not complete without a gross error detection strategy that can effectively detect and eliminate gross errors in measurements.

Data reconciliation is applied to linear and nonlinear steady state processes with measured and partially measured variables. Heat exchanger and steam generator models with nonlinear mass and energy constraints are used. The reconciliation process is applied in a feed water flow measurements model to illustrate the applicability of data reconciliation.

Declaration

I, Muhluri Calvin Mathebula, hereby declare the work contained in this dissertation to be my own. All information which has been gained from various journal articles, text books or other sources has been referenced accordingly. I have not allowed, and will not allow, anyone to copy my work with the intention of passing it off as their own work or part thereof.

Muhluri Calvin Mathebula 2016-11-09

Name Date

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General symbols

$\dot{m}_{fw}$	Feedwater mass flow rate	kg/s
h_s	Steam enthalpy	kJ/kg
h_{fw}	Feedwater enthalpy	kJ/kg
u	Vector of unmeasured variables	
$\hat{u}$	Vector of unmeasured estimates	
x	Vector of unknown true values	
$\hat{x}$	Vector of reconciled values	
y	Vector of measured variables	
r	Residual	
cov	Covariance	
A, A_x, A_u	Incidence matrices	
S_X, V	Covariance matrix	
S_V	Measurement error covariance matrix	
W	Measurement weighting/ work done/ power	
Q	Energy	

Greek symbols

α	Level of significance
δ	Vector of gross errors
ε	Vector of random errors
χ^2	Chi-squared
γ	Gamma distribution / function
μ	Mean value
λ	Lagrange multipliers
Σ	Summation
σ	Standard deviation
σ^2	Variance

Acronyms and Abbreviations

DR	Data reconciliation
DVR	Data validation and reconciliation
GED	Gross error detection
RSA	Republic of South Africa
RTP	Reactor thermal Power
NPP	Nuclear power plant
SG	Steam generator
SGW	Steam generator water side
SGS	Steam generator steam side
HEX	Heat exchanger
HBD	Heat balance diagram
HR	Heat rate
PWR	Pressurized water reactor
KOU	Koeberg Operating Unit
UDE	User defined equations
UFM	Ultrasonic flow meter (measurement)
DCS	Distributed control systems
PIMS	Plant information management system
CPT	ChemPlant Technology s.r.o
GUI	Graphic user interface
DATACON	Reconciliation software package by SimSci-Esscor
BWR	Boiling water reactor
VALI	Reconciliation software package by BelSim
CCGT	Combined-cycle gas turbine
MUR	Measurement uncertainty recapture
MIMT	Modified iterative measurement test
QR	Matrix factorization technique
GT	Global test
GLR	Generalized likelihood ratio
NT	Nodal test
MT	Measurement test
PCA	Principal component analysis
PCT	Principal component test
RC	Reactor coolant

1. Introduction

Reliable and accurate measurements are essential for plant performance monitoring and process control. The power plant thermal efficiency calculation requires better control and reliable process measurement data which demand reliable instrumentation. In practice, instrumentations are used for on-line computation of performance monitoring, process control, heat rate, and cycle efficiency as well as reactor thermal power calculations. However, the instrumentations used are unreliable due to: failure, poor readability, drifts and process leaks, to name a few. Furthermore, measurement data are affected by measurement errors that are classified as random and gross errors [1].

Data validation and reconciliation (DVR) is a data treatment technique that utilizes process data and mathematical methods to automatically correct industrial process measurements. Data treatment focuses on delivering data as close as possible to the true process state variables that are presented by raw data. A DVR component which uses redundant process information to estimate process state is known as data reconciliation (DR) [2]. This technique is used for data treatment and gross error detection in process industries. The use of DR is increasing in chemical processing, nuclear power generation, conventional power generation and mining industries.

The primary focus of this research is, however, the application of DR in power generation plants. Power plant operations require accurate knowledge of process measurements for safe operation. Nuclear and conventional power plants are a potential threat to personnel and the environment when operated without safety measures in place. Thus process data treatment is one of the methods used for continuous monitoring of plant measurements and instrument failures.

Power generating plants require the capability to determine feedwater flow measurements with a high level of accuracy in order to calculate plant thermal efficiency and /or heat rate. In addition, nuclear power plant (NPP) operations require accurate feedwater flow measurements to calculate reactor thermal power [3].

In practice most power plants make use of flow nozzles, orifice plates and Venturi meters for feedwater flow measurements. However, with time, these flow meters are degraded by fouling, erosion and debris deposits. This in turn affects the respective discharge coefficients of the flow meters, resulting in inaccurate flow measurements of the feedwater, and also prevents accurate calculations of heat rate (HR) and reactor thermal power (RTP) [4], [5], [6].

Both heat rate and thermal power calculations depend largely on mass and energy balance which are a function of feedwater flow measurements. Therefore, feedwater flow measurement is an essential measurement variable in both nuclear and conventional power plants. Thus the accuracy of feedwater flow measurement is critical since it contributes to at least 80% of uncertainties in the estimation RTP in NPPs. The RTP is said to be directly proportional to feedwater flow as shown by the equation below:

$$RTP = \dot{m}_{fw}\left(h_s - h_{fw}\right) + \left(heat\,gain / loss\right) + heat\,removal \tag{1}$$

Where the feedwater mass flow rate is given by $\dot{m}_{fw}$, h_s and h_{fw} represent the enthalpies of steam and feedwater respectively [7].

The heat removal part of equation(1) represents the blowdown mass flow rate and its associated enthalpy from the steam generators (SG) of pressurized water reactors (PWR). The heat gain/ or loss represents the energy through the primary pumps and pipework.

The heat generated from the fission process in the primary loop of a nuclear plant is transferred to the feedwater in the secondary loop through the steam generator to produce steam. The steam is then used to drive the turbo-generator which generates electric power.

The primary loop refers to the nuclear reactor vessel, primary pumps, pressurizer, steam generators and their associated auxiliaries. The secondary loop refers to the high- and low-pressure turbines, the generator, condenser, feedwater preheaters and their associated auxiliaries. The tertiary loop being the ultimate heat sink required for component cooling. Figure 1 shows a schematic diagram of a PWR NPP with the three different loops.

Most power units in RSA are close to the end of their design life, for that reason ageing of components is expected. This contributes to the need to operate the power plants efficiently to continuously produce electricity. Furthermore, flow measuring devices in general are also affected by ageing, degradation and failures. Examples of such flow measurement devices commonly used in the power industry include flow nozzles, orifice plates and Venturi meters. The fouling and degradation of such devices result in flow rate values that are either higher or lower than expected [6].

The operations of NPPs require accurate, consistent and continual monitoring of the process state. This is achieved through performance monitoring and control of the critical plant components.

In power plant operations, numerous instrumentations have been installed for monitoring, control, safety and economic purposes. However, some of these instruments may not be maintained due to the cost of maintenance. The lack of maintenance therefore may results in instrument failure which in turn introduces measurement errors [1], [8].

Thermal power is an essential measurement for the safe operation of any NPP. Thermal power measurement is highly dependent in feedwater flow measurements and also the heat rate calculation in conventional power plants. As a result of inaccurate and unreliable feedwater measuring devices NPPs are forced to operate their reactors at power levels either below or higher than the licensed reactor power [6].

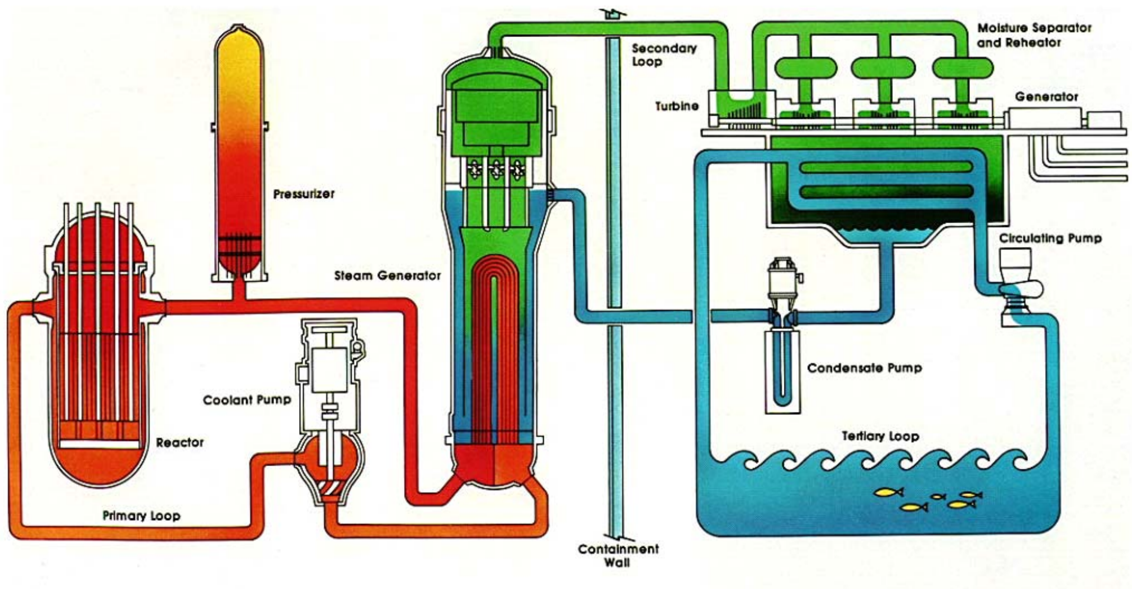

Figure 1: Simplified diagram of a PWR NPP *[9]*

The challenge for power plant operators is to obtain accurate and consistent feedwater flow measurements without errors. The estimation of the RTP in NPPs is usually calculated by means of heat balance analysis using the steam-water cycle flow measurements of the power plant. The uncertainty associated with this method cannot be determined with ease due to measurement errors.

A technique that is consistent, precise and has low uncertainty levels in the calculation of the RTP will ensure safe operation of power generation plants and other process plants. Acquiring a measuring technique with a reduced calorimetric uncertainty, high level of accuracy and significantly less susceptibility to feedwater fouling is essential in power plant operations.

A study of a data treatment technique to reduce the influence of errors in feedwater flow measurements and to reduce uncertainty in the calculations of both thermal power and heat rate is necessary. Data reconciliation (DR) is a technique that uses process plant models and incorporates existing measurement instrumentation to exploit the redundancy of measurements, providing accurate and reliable process data which approximates true values better than what is achieved by raw measurements [1].

In this research, the fundamental mathematical formulation of DR and its application in power plants were studied. Gross error detection (GED) and isolation form an integral part of the data treatment process applied in conjunction with the DR; this is further reviewed in the following chapters.

The ultrasonic flow measurement technology has been developed as a substitute to the conventional flow measuring techniques such as Venturi, flow nozzle and orifice. The feedwater measurement technique using ultrasonic flow meters (UFM) was developed for the recovery of electrical power loss in power plants due to overestimation of the feedwater flowrate. The overestimation of feedwater flowrate is attributed to the fouling of the conventional techniques of flow measurements. The ultrasonic technology uses two significant types of UFMs for flow measurement, the non-intrusive and intrusive ultrasonic meters. The clamp-on transit-time flow meters have the advantage of being non-intrusive.

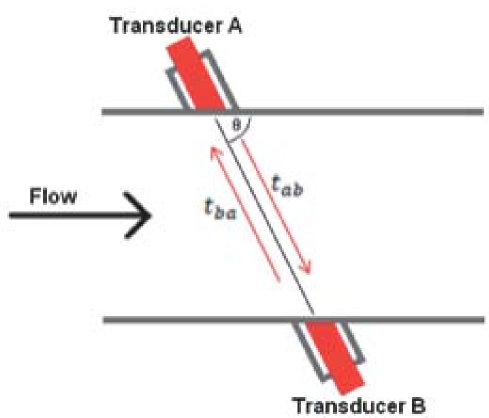

Figure 2: Transit-time ultrasonic flow meter *[*10*]*

The clamp-on meter is equipped with at least two transducers used to measure the transit-time of the measured fluid to calculate the volumetric flow. The ultrasonic flow meter is susceptible critical parameters associated with flow profile, propagation time difference, pipe dimensions, fluid characteristics, acoustical calibration factor and human error [11]. Thus, the study is focused feedwater flow measurement technique less vulnerable to flow characteristics and pipe dimensions.

1.1 Objectives

The nuclear power plant uses the secondary heat balance to estimate thermal power. This method uses raw feedwater measurements which have errors, resulting in high uncertainty of the RTP. It is therefore essential to enhance the quality of feedwater flow measurements to achieve measurements that are accurate and reliable. Data treatment techniques, such as DR for measurement improvements, are not applied in any of the power generating plants in South Africa. DR is thus an unknown technique in the South African power generation industry therefore there is a need to study the fundamentals of DR technique and its applicability in power plant operations.

In this study, process flow examples are used to demonstrate the application of DR in steady state process conditions. Examples are solved analytically using a mathematical solver then validated with DR software. In the quest to demonstrate the use of a DR problem in power plants, an application of real plant data was necessary to validate the use of this technique in feedwater flow measurements. Feedwater flow measurements data was extracted from the PWRNPP unit for the purpose of this study. The objective was to improve the quality and usability of process flow measurements in feedwater flow system.

1.2 Scope of research

The research focuses on the available literature on data reconciliation and the basis of data treatment and validation. The goal of literature review is to study how DR problem / the objective function is formulated using the generalized least squares technique of minimization. It evaluates the different approaches in solving linear and nonlinear systems using the DR problem and focuses on data validation techniques based on data reconciliation and gross error detection (GED) in order to enhance the quality of process data. Gross error detection strategies applicable in process data treatment are discussed.

The NPP feedwater flow measurement data are used to demonstrate the application of DR in power plants. Data from the plant's InSQL database was used to perform data reconciliation. A mathematical model simulating the process flow was constructed, incorporating the relationship between process variables and parameters physically observable. The constructed model enables the mathematical analysis of the physical process flow. The validation of the reconciliation technique is achieved through a mathematical model analysis of the process flow and comparing the analytical solutions to the solution obtained through a process data reconciliation software RECON.

1.3 Report outline

Chapter 2 describes the historical background and the fundamentals of process DR and its application. This chapter also covers the need for DR to enhance process measurements of industrial processes and highlights the purpose of DR in process industries. This chapter provides evidence of the importance of process variables classification and measurement uncertainty including the basis of the mathematical formulation of DR using the weighted least squares minimization problem, and the derivation of DR problem for steady state processes. The formulation includes linear and nonlinear cases and the application of the matrix decomposition technique to solve partially measured systems as well as the linearization of nonlinear systems using successive linearization by construction of the projection matrix.

Chapter 3 briefly discusses the statistical properties used in data analysis and their relationship with instruments uncertainty. It further provides an overview of statistical tests used for gross error detection in DR problem.

Chapter 4 opens with a discussion of some of the different DR software packages available commercially and their capabilities with relation to process measurement. The software used in this research is mentioned in this chapter. In addition, Chapter 4 demonstrates the application of process data reconciliation in process flows, and in different plant scenarios under steady state processes. The case studies involve linear and nonlinear cases with all variables measured and partially measured process flow. The analytical solutions in each case are compared with solutions from data reconciliation software for validation. Data reconciliation is applied in power plant feedwater measurements system using data from NPP to improve plant feedwater flow measurements.

A summary of the work described in this dissertation is presented in Chapter 5. Along with the summary of data reconciliation, gross error detection and possible future work are also presented.

2. Overview of process data reconciliation

2.1 Introduction

It is important for power plant operators and process engineers to have good understanding of the process state variables and parameters as in any process plant, like chemical plant, petrochemical plant, power generation plant, mineral processing plant or a refinery. Process performance monitoring of process plants requires balanced components and flow rates of the process streams [12]. Process data form the foundation on which the control and evaluation of process performance are based. Performance monitoring is described as a continuous evaluation of the production capability and efficiency of a process plant over a given period of time using online measured plant data [13].

In general, process plants contain large numbers of process variables and parameters that need to be measured and recorded. It is a common practice in process plants to measure mass flow rates, temperatures, pressures and concentrations of chemical components for process control and process performance evaluation. In steady state process condition, the measured data are expected to satisfy the process model mass and energy constraints. However, measurement data contain measurement errors, thus, the process model constraints are generally not satisfied [14].

The measurement errors are classified as random and systematic in nature. The existence of errors in measurements affects the consistency and the accuracy of process measurements, leading to a violation of the physical laws of conservation of mass and energy balance as well as poor decision-making [15]. To eliminate inaccuracies in process measurements and enhance plant performance, process plant operators have implemented estimation and data treatment techniques to continuously improve the reliability of process measurements. These estimation techniques are generally derived from the statistical concept of estimation theory which deals with estimation of values of parameters based on measured data that contain a random component [12].

Normally an estimation problem consists of a system and some measurements where the system describes the physical process, represented by mathematical equations. The redundancy and distribution of the measurements play an important role in the estimation problem. Measurements redundancy is often used as a safeguard when there are biases in the measurements or imperfections in the mathematical model of the process under consideration [12].

Process data reconciliation is a data treatment method used to enhance the accuracy of measurements by reducing the influence of random errors [13]. It is a subdivision of data validation and reconciliation (DVR) used in process plants for process variables enhancement. It permits the determination of estimated process parameters with a confidence level of 95%, considering mass and energy balances and assuming the Gaussian correction principle [16]. This principle refers to the correction made on measured variables with an aim to obtain estimated values [17]. DR technique uses mathematical statistics theory to adjust process data to satisfy mathematical model constraints, usually the conservation laws of mass and energy balance. The adjusted estimates will be more accurate and consistent with process model constraints than raw measurements. DR technique depends mostly on the redundancy of process measurements; hence to obtain reliable estimates process variables must contain a level of redundant information [18].

The main difference between data reconciliation and other methods is that data reconciliation explicitly uses process model constraints to obtain estimates of system parameters through the adjustment of system measurements so that the estimates will satisfy the model constraints. Thus DR problem is simply a method to enhance the accuracy of process data to a degree that they satisfy mass and energy balances of a system or process.

Reconciliation procedure is greatly affected by the presence of gross errors; therefore, to achieve effective DR procedure the measurements must be without gross errors or biases. It is therefore important to be able to identify and either correct or discard measurements with gross errors. Some statistical test methods for the detection of gross errors and identification have been researched in parallel with DR, as discussed below in Section 3.2 [15], [19]. As a result, data reconciliation and gross error detection are applied simultaneously during a reconciliation procedure [20].

The possibility of process control and optimization is achieved through knowledge of the current plant state. The process state variables are usually characterized by the process model representing the process flow or the physical plant. The process model generally comprises constraints equations of mass and energy balances. Romagnoli and Sanchez describe a mathematical model as a combination of two parts: the functional model and the stochastic model [12].

The functional model represents the physical model associated with process measurement data. The measurement data is collected in order to evaluate the parameter values of the functional model. The probabilistic property of measurement variables is represented by the stochastic model [12].

The commonly used data validation and reconciliation method in power plants and chemical processes is the mass and energy balances which form a substantial part of this study. In some cases the mathematical models may also include momentum balance which is usually applied in fluid flow problems in pipeline systems and the phase equilibrium. Madron et al [18] describe a mathematical model as a system that can truly describe the physical reality of a process. Valid mathematical models form the basis of data validation.

2.2 Historical background of process data reconciliation

The first industrial application of process data reconciliation was developed and introduced in 1961 by Kuehn and Davidson as a derivation of the analytical solution for linear systems with all measured variables [20]. Steady state process data reconciliation was mainly applicable in crude oil processing plants, and later applied in petrochemical, chemical, mineral processing and power generation plants, including nuclear power plants [21]. Mah et al further developed a graphical method to estimate unmeasured variables and the decomposition of the reconciliation problem. The concepts of redundancy and observability were also introduced. It was later demonstrated by Mah et al in a refinery process simulation that DR is capable of improving the accuracy of measured data, especially if there is enough redundancy in the measurements [17].

The matrix decomposition technique for the construction of a projection matrix was introduced by Crowe et al [22], for use in decomposing the reconciliation problem. This technique divides the reconciliation problem into two sub-problems, reducing it significantly. All the unmeasured variables are eliminated when multiplied with the projection matrix.

The approach uses a matrix decomposition method which is solved efficiently by QR factorization algorithm to construct the projection matrix. The projection method was later extended to cover bilinear systems and further developed for nonlinear functions by successive linearization method. In linear systems the projection matrix method solves the reconciliation problem without iteration [1], [23]. The steady state process data reconciliation was also developed for application in the mineral processing industry for material balance [15].

DR problem has been used in the power generation industry and applied in boiler heat balance analysis to improve the accuracy of feedwater flowrate, as well as in steam turbine heat balance analysis [24]. NPP operators have since taken advantage of the DR technique to estimate accurately feedwater flow measurements and the reduction in the uncertainty of the estimated reactor thermal power [25], [26]. Process data reconciliation is gradually being applied in most process industries and thus there is an increase in the development of commercial data reconciliation software packages, briefly discussed later, in Section2.5.

2.2.1 Purpose of process data reconciliation

Process information on industrial systems is obtained from the measurement of physical variables and parameters such as mass flowrates, temperatures, pressures, chemical concentration and so forth. Every measurement in process plant operations is subject to measurement errors, categorized as random and gross errors. These errors have undesirable consequences on process measurements required for process control, performance monitoring and determining other useful key performance indicators.

DR method is used to improve the quality of process measurements by minimizing the influence of random errors and also identifying and eliminating gross errors in measurements [21]. DR method is known as the methodology for improving the accuracy of process measurements so that they satisfy mass and energy balances of process flow. It is sometimes difficult to satisfy the process balance since not all variables can be measured from a process flow because of technical or, at times, financial constraints. It is thus considered necessary for DR method to adjust the measured variables and at the same time estimate unmeasured variables to satisfy balance constraints [14].

DR uses available process or plant information to estimate the true state process. This data processing technique is used for mass and energy balancing, process monitoring, on-line modelling and process control. Another valuable technique used in conjunction with DR is gross error detection and identification [27].

The benefits of applying DR in power plants to improve thermal analysis of the process are described below [28]:

- Determination of reliable thermal measurements,
- Estimation of the most probable unknown measurements in thermal processes,
- Assessment of the accuracy of the corrected measurements and estimated unknown measurements,
- Reduction of measurement uncertainties, and
- Gross error detection and elimination of corrupted measurements.

As previously mentioned, DR is known as the technique of improving the accuracy of process data to produce consistent measurement values which will satisfy mass and energy balances. It uses fundamental mathematical bases of a constrained weighted least squares optimization problem. Thus DR is a minimization problem which results in minimum values as close as possible to the true values of the process model. It was shown in literature that using DR in thermal processes has the advantage to reduce errors in measurements and to improve process measurements. As a result the quality of a measurement is indicated by its variation from the unknown true value.

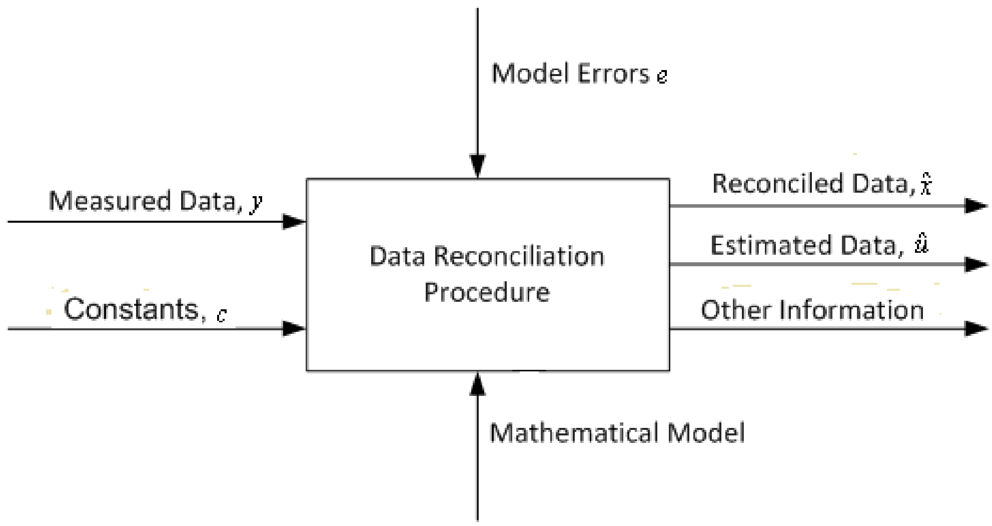

Figure 3: Data reconciliation procedure, modified *[21]*

Figure 3 shows the process of DR and its reliance on the mathematical model representing the physical process and other parameters. The mathematical model consists of the process model balance equations or model constraints equations. The constraints equations frequently used for DR applications in power plants analysis are:

- Mass (material) balance,
- Energy balance,
- Momentum balance, and
- Phase equilibrium.

The mass and energy balance (enthalpy balance) is the most used balance in power plants. Momentum balance is generally used in the modelling of pressure drops in pipeline systems for water and steam flows. Phase equilibrium is the relationship of both temperature and pressure at saturation, where the properties of water and steam are the same, defined as the wet steam conditions [27].

2.2.2 Process data reconciliation in other industries

Since the development of data reconciliation in the 1960s, more studies have been carried out to develop concepts and techniques for data reconciliation and gross error detection, focusing on the applications of data reconciliation in crude oil processing and chemical industries. Ishiyama et al applied DR in oil refinery distillation preheat trains to provide fouling rate parameters of heat exchangers for desalter inlet temperature control [29]. A simplified algorithm to reconcile energy balances was later proposed by Ijaz et al for large heat exchangers [30]. DR has been applied in a steady state process of a single-effect ammonia and water absorption chiller using a modified iterative measurement test (MIMT) method for gross error detection [31].

Jansky and Langenstein applied online data validation and reconciliation in nuclear plants to achieve the accuracy of various key performance parameters such as thermal reactor power and mean coolant temperature. These parameters are monitored to determine whether the NPP is operating within its safety margin and is able to operate at its full capacity [32].

In combine-cycle gas turbine plants (CCGT) data reconciliation has been used among other methods to monitor process efficiency and to obtain detailed information about the gas turbine [32]. Szega and Nowak applied DR for the optimization of redundant measurement locations in steam boilers [28]. The application of DR provides validated consistent and accurate data to determine the thermal power of SGs and reactors as well as thermal performance indicators for the monitoring of the water-steam cycle of NPPs [8], [27].

2.3 Classification of process variables

Classification of process variables plays an important role in DR procedure to reduce the size of the general estimation problem [22]. Variable classification generally reduces the dimension of the reconciliation problem, in organizing the analysis of process data and in structuring the estimation of measurement correction. It is also an important tool in monitoring systems design or refurbishment [20], [22]. The reduction of a set of constraints equations by eliminating unmeasured variables and non-redundant measurements allows for easy and quicker problem resolution. Process data reconciliation in a complex process plant comprises three significant parts:

- Classification of process variables,
- Gross error detection, and
- Co-variance estimation and variable rectification.

The concepts of observability and redundancy are closely related to how easily the variables can be solved and estimated. In steady state processes, variables are associated with mass and energy conservation laws. However, not all process variables are measured for economic reasons, plant accessibility or technical feasibility reasons, although some of these variables can be estimated through balance calculations from other measurements. We therefore can attain knowledge of whether a measured variable can be estimated even when its instruments have failed.

Observability and redundancy analysis is useful for introducing new measuring instruments. Observability and redundancy of variables are defined respectively by Narasimhan and Jordache [20] as;

- A variable is said to be observable if it can be estimated by using measurements and steady state constraints, and
- A measured variable is said to be redundant if it is observable even when its measurement is removed.

Process variables are classified according to how feasible it is to measure variables. Therefore, variables are divided into those measured and unmeasured. Measured variables are further classified into redundant and non-redundant types. A redundant measurement remains determinable even when the observation is deleted. Unmeasured variables are thus divided into those observable and unobservable. Unmeasured observable variables can be estimated from the measurements whilst unmeasured and unobservable variables cannot be determined. The estimation of unmeasured variables depends on process flowsheet structure and instruments placement [12], [20].

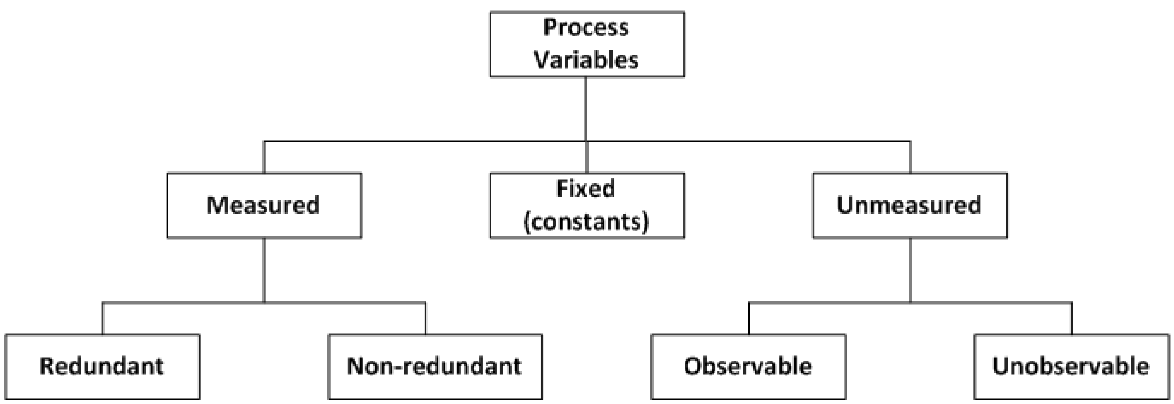

Figure 4: Process variable classification, modified *[26]*

In addition to measured variables, there are special variables, often obtained from precise measurement with negligible errors. Normally, they are not adjusted during reconciliation and are therefore known as fixed variables or constants.

The whole classification of process variables can be summarised in Figure 4. Redundant variables are adjusted during reconciliation whilst non-redundant cannot be adjusted.

Process variable classification by projection matrix was proposed by Crowe et al [22] for linear systems and later extended to bilinear systems. The extension of the projection matrix to cover bilinear systems was developed using nonlinear functions by successive linearization technique

[20], [23]. The method uses matrix decomposition to construct the projection matrix, decoupling measured variables from the process model (constraints) equations using the projection matrix to eliminate unmeasured variables. The projection matrix divides the general data reconciliation problem into two sub-problems, reducing the reconciliation problem [17], [22]. The unmeasured variables are eliminated when pre-multiplying with the projection matrix. The construction of the projection matrix is effectively solved by the QR factorization algorithm [20].

There is another important statistical property useful in gross error detection and data reconciliation; degree of redundancy. The degree of freedom is also a property used in chi-square distribution curves, given by:

$$v = m - n \tag{2}$$

Where the number of equations is denoted by m and the number of unmeasured variables is denoted by n [12]. A positive redundancy is essential for data validation and reconciliation. It is practically impossible to perform data validation and reconciliation when the redundancy is null or negative.

2.4 Measurement errors and uncertainty

In practice, all measurements are inevitably subjected to measurement errors, commonly classified as random, systematic and gross errors.

Random errors are characterized as unavoidable errors in measurements and are defined statistically by the Gaussian (normal) distribution with a mean $\mu = 0$ and variance-covariance matrix V. Random errors can possibly be characterized by the use of probability distributions [33].

Gross errors, on the other hand, are characterized by association with the system and they result from measurements bias, process flow leaks, process disturbances, instrumentation malfunction, instrumentation drift and, sometimes, human error. Through process data reconciliation, random errors can be significantly reduced whilst gross errors affect the validity of the reconciliation solution. As a result, gross error identification and elimination are necessary before performing a reconciliation procedure [34]. Gross error detection (GED) techniques have been developed in order to detect and eliminate gross errors in measurements, as discussed below in Section 3.2.

A measurement error is a measure of how accurate and precise is the measurement. The true value of a measurement is usually an unknown quantity, thus it is practically impossible to determine the accuracy of a measurement with ease. The quality of a measurement is defined by the knowledge of its uncertainty; however, a measurement is complete only when it is accompanied by the uncertainty in that estimate. Uncertainty can be estimated by understanding the error distribution and its statistics. Error distribution can be quantified by the standard deviation of the distribution, which is the square root of the distribution variance [35].

The measured quantity can be divided into three distinct components that fully describe a measurement: the estimated true value of state process x, the gross error δ and the random error ε. The measurement error model of the measurement y is thus defined by:

$$y = x + \delta + \varepsilon \tag{3}$$

These variables can be described as vectors, where $y(n \times 1)$ is the measurement vector, $x(n \times 1)$ is the estimated true value of measured variables, $\delta(n \times 1)$ the vector of gross errors, and $\varepsilon(n \times 1)$ the vector of random errors [20]. The measurement error can be described as the difference between the measured value and the estimated true value of a variable. The sum of the contributions from random errors and gross errors is given by:

$$e = \varepsilon + \delta \tag{4}$$

Since the true value of a measurement is always an unknown quantity, the term measurand is sometimes used to refer to the true value. A measurand is a quantity which is being measured. Therefore, a measurement uncertainty is said to be a parameter associated with the result of a measurement, which characterizes the dispersion of the values that are reasonably attributed to the measurand [33], [35]. The uncertainty in the result of a measurement is basically identical with the maximum error assumed in a measurement [21]. This is referred to as the tolerance in instrumentation and expressed as a percentage of the instrument range of measurement.

The measurement noise and fluctuations of process measurements are common in real plant applications and are assumed to be distributed around a mean value. The mean value μ, standard deviation σ and variance σ^2 are vital statistical properties useful in data analysis and data treatment. The mean value of a data set with y measurements and N observations is calculated as:

$$\mu_y = \frac{\sum_{i=1}^{N} y_i}{N} \tag{5}$$

The mean value is used to calculate the standard deviation of the measurement y given as:

$$\sigma_y = \sqrt{\frac{\sum_{i=1}^{N} \left(y_i - \mu_y\right)^2}{N-1}} \tag{6}$$

The standard deviation is given as the square root of the variance. The variance is defined as the expected square of the distance from the mean value and the standard deviation as the expected distance from the mean value [33].

$$\sigma_y^2 = \frac{\sum_{i=1}^{N}\left(y_i - \mu_y\right)^2}{N-1} \tag{7}$$

Another statistical measure that indicates how variables correlate to each other is the co-variance given as:

$$\operatorname{cov}\left(x, y\right) = \frac{\sum_{i=1}^{N}\left(x_i - \mu_x\right)\left(y_i - \mu_y\right)}{N-1} \tag{8}$$

In practice, process industries and power plants require the use of instrumentation for process monitoring and control. In power plants instruments generally measure mass flow rates, temperatures and pressures throughout the water, steam and flue gas systems [1]. However, these instruments typically have an inherent amount of uncertainty in their measurement.

Uncertainty is defined as the parameter associated with the result of a measurement that is characterized by the dispersion of the values that could be attributed to the measured value [36], [37]. Thus even the most precise instrument cannot give the exact true value of a measurement. The accuracy of an instrument is given by its uncertainty. The uncertainty of the measurement should be given with the actual measurement, for example, 31.64 ± 0.05 cm, 27±1^{o}C, etc. [21].

2.5 Tools and software

The need for accuracy, reliability and enhanced quality of process information in process plants requires the use of tools and techniques that are computer-based. Mathematical and other software packages are used to allow the analysis of industrial process data with ease and among them are several commercially available software packages used for DR applications in industrial process plants. These software packages utilize the available process data and are compatible with most plant information management systems (PIMS) and can be integrated into the distributed control systems (DCS).

Some DR software packages are presented here, highlighting their benefits to process measurements improvements as specified in their respective product brochures and developers' websites.

RECON software is considered a comprehensive interactive package for mass, energy and momentum balance for process /chemical plants. RECON is developed and marketed by ChemPlant Technology s.r.o, (CPT). RECON is useful for validation of process data and treatment of measured data for process optimization and control. As mentioned on the RECON website, it can be applied in complex systems for single- or multi-component balance of mass and energy balances and it is also able to perform momentum balancing for hydraulic calculations. RECON reconciles measured flow rates, temperatures, pressures, and concentrations and uses the reconciled values to calculate unmeasured variables. A DR problem on RECON graphic user interface (GUI) is defined by creating a process flowsheet and defining process variables. The process flowsheet is made up of nodes, mass and energy streams, and heat exchangers; the user can also add user-defined equations (UDE) in the balancing model. UDEs are used either to improve the reconciliation by adding redundant equations or to calculate other variables not defined by the model constraints equations.

Variables are specified in the GUI as:

- Measured (M), unmeasured (N) and fixed (F), as defined in variable classification,
- Values of the measured and fixed variables are defined in the flowsheet as well as initial guess values for unmeasured variables, and
- The maximum errors of measured variables are defined.

RECON is able to:

- reconcile redundant measured variables and calculate unmeasured variables,
- detect gross measurements errors, and
- perform a detailed variables classification.

In order to perform process state thermodynamic calculations, RECON contains three databases of physical properties:

- IAPWS IF 97 for water and steam properties,
- Parameters of BWR equations for density calculations, and
- Critical parameters of components for viscosity calculations of gaseous mixtures.

CPT claims RECON can provide an interactive solving of reconciliation problems, online industrial process monitoring and automatic historical data processing [21], [38].

SimSci-Esscor DATACON, developed by Schneider Electric, is a DR software package assumed to be able to turn real-time process data into consistent and reliable business information. DATACON uses statistics and mathematical techniques to reconcile flow, temperature and composition measurements to satisfy material and energy balances around a component in a process plant. It detects and locates gross errors in measurements and confirms measurement redundancy. It can offer full and automated links to the stored historic data and plant databases or directly to the DCS. As promoted by Schneider Electric, DATACON can be used in conventional and nuclear power generation for combustion optimization and increased efficiency through advanced process control and better control of turbomachinery and reactor vessels [39].

VALI is a data validation and reconciliation software package, developed by BelSim, and claims to offer a solution to enhance profitability of process industries by improving process performance. It can also be used for process monitoring and optimization and online component diagnosis, condition-based maintenance and instrument calibration. It is said to have been applied in NPP, combined-cycle gas turbine (CCGT) and fossil power plants, as well as in oil and gas, chemical and petrochemical industries. BelSim has identified the application of VALI in nuclear power generation for the measurement uncertainty recapture (MUR) and power uprate, increased plant safety, increased electrical output, detection for measuring devices, improved understanding of the heat cycle and condition-based maintenance [40].

3. Formulation of process data reconciliation problem

3.1 Mathematical formulation

In statistics, a measured variable is considered a random variable of a continuous distribution characterized by uncertainty around an estimated true value. The error in a measurement is a contribution of both random errors and gross errors.

The general formulation of process data reconciliation problem is described as the constrained weighted least squares optimization problem. It minimizes the difference between measurement variables y and reconciled variables $\hat{x}$ subject to the process model constraints [20]. Statistically, the reconciled variables are similar to the mean value. Vector and matrix notations are applied in the derivation of the data reconciliation problem because they provide a compact representation and allow the exploitation of powerful concepts from linear algebra and matrix theory [20].

The balance constraints represent the physical balance of the process model, which represents mass and energy balance in power plants. The data reconciliation procedure minimizes the generalized sum of squares of the adjustments constrained by:

$$f(x) = 0 \tag{9}$$

Where $f(x)$ is a vector of linear and / nonlinear functions of x which contains y with errors described in equation(3) and also contains unmeasured variables u and constants c in it. The DR problem is expressed mathematically as the least squares objective function:

$$\min \sum_{i=1}^{N} \left(\frac{y_i - \hat{x}_i}{\sigma_i} \right)^2 \tag{10}$$

Therefore, the reconciled values $\hat{x}$ will satisfy the condition of equation(9) and minimize the generalized sum of squares in a way that:

$$\gamma = r^T V^{-1} r \tag{11}$$

The gamma distribution γ is derived from the chi-square distribution χ^2 which describes a family of distribution curves [33] and will later be used as the hypothesis for gross error detection in Section 3.2 below. The co-variance matrix of the measurement errors is denoted by V and r is the vector of adjustments between the measured and the reconciled values, known as the residual:

$$r = y - \hat{x} \tag{12}$$

The weighted least squares regression method is commonly used to formulate data reconciliation problem because of its well-known optimal properties for unconstrained cases and for its ability to handle situations with varying data points [14], [41]. In a case where the standard deviation of the random errors is not constant, the weighted least squares with weights W are applied to yield the most precise estimates [41]. The formulation is based on the assumption that only random errors exist in process data, following a normal distribution with zero mean and known co-variance matrix.

Therefore the basic model of raw measurements vector can be represented by reducing equation(3) with only random errors ε available in measurements:

$$y = x + \varepsilon \tag{13}$$

The process model constraints equation for a linear system is given by:

$$Ax = 0 \tag{14}$$

Crowe et al describe the matrix A as the incidence or the functional matrix, representing the process structure or the process flow with the rows corresponding to the nodes and columns to the streams [22]. The incidence matrix is derived from the connectivity of the process flow sheet representing the physical process. In cases where complex model process equations are analysed, a mathematical approach to derive the incidence matrix can be used, applying the partial derivatives with respect to each variable of the constraint equations to determine the entries of matrix A. The entries or values in the incidence matrix are derived according to the flow stream and the nodes or junctions in a process flow [12]. Consider the incidence matrix of a process flow with j streams and i nodes in which the $(i,j)th$ entry is +1 if the stream j is an input to node i, -1 if the stream j is an output from node i, and 0 if there is no stream associated with node i [14].

The optimization of the objective function in equation(10) for a steady state process can be written in matrix notation given by:

$$\underset{x}{Min}\left[\left(y - \hat{x}\right)^T W \left(y - \hat{x}\right)\right] \tag{15}$$

The process model constraint equations are given in the form of equation(14). Where $W(n \times n)$ is a diagonal matrix with its elements representing the measurements weightings containing non-zero off-diagonal elements y is a vector of raw measurements for N variables and $\hat{x}$ is a vector of estimated or reconciled values for N variables. The analytical solution to equation(15) is obtained by applying the Lagrange multipliers [12].

The Lagrange multipliers are defined by λ and equation(15) becomes:

$$\mathrm{J}(x,\lambda) = (y-\hat{x})^T W (y-\hat{x}) + 2\lambda^T Ax \tag{16}$$

Where the $\lambda = [\lambda_i, \ldots, \lambda_n]^T$,

Taking partial derivatives of equation(16) with respect to $\hat{x}$ and λ equating them to zero gives the minimum:

$$\begin{aligned} \frac{\partial J}{\partial \hat{x}} &= -2W^{-1}(y-\hat{x}) - 2A^T\lambda = 0 \\ \frac{\partial J}{\partial \lambda} &= Ax = 0 \end{aligned} \tag{17}$$

Pre-multiplying each term of equation(17) by the weighting matrix W yields:

$$y - \hat{x} + WA^T\lambda = 0 \tag{18}$$

Pre-multiplying equation(18) by the incidence matrix A and applying equation(14) yields:

$$Ay + AWA^T\lambda = 0 \tag{19}$$

Rearrange equation(19) to get:

$$\lambda = -\left(AWA^T\right)^{-1} Ay \tag{20}$$

Substituting for λ in equation(18) and rearranging, making $\hat{x}$ the subject of the formula, gives the vector of the reconciled values:

$$\hat{x} = y - WA^T\left(AWA^T\right)^{-1} Ay \tag{21}$$

Therefore equation(21) gives the general analytical solution of the optimization problem of a linear steady state process model. The reconciled values provide balanced values of the constraints equation(14) satisfying conservation laws. The above solution is given only using the least-squares with weighted sum of the squares [12], [20], [22]. However, reconciliation is generally explained using statistical properties which helps to understand the technique and provides quantitative information about measurements improvement. The statistical properties also help in gross error identification and elimination, used in conjunction with data reconciliation

[20]. Also applying the assumption made earlier that only random errors exist in measured data. The co-variance matrix V described as the inverse of a positive-definite weighting matrix W contains information about the accuracy of the measurements and the correlations between them. The diagonal elements of the co-variance matrix V is the σ_i^2 variance of the measured variable i and the off-diagonal elements σ_{ij}^2 is the co-variance of the errors in variables i and j respectively. It is assumed that random measurements errors follow a Gaussian distribution with zero mean and co-variance matrix V and the probability density function $P_i(y_i)$ of the measurements y_i and expected estimates $\hat{x}$ of the reconciled data can be obtained statistically by the maximum likelihood equation [15]:

$$P_i = \frac{1}{\sigma_i\sqrt{2\pi}}\exp\left\{-\frac{1}{2}\left(\frac{y_i - \hat{x}_i}{\sigma_i}\right)^2\right\} \tag{22}$$

The quantity $\frac{1}{\sigma_i^2}$ is the weighting factor so that:

$$w_i = \frac{1}{\sigma_i^2} \tag{23}$$

Since the first factor of equation(22) is a constant, maximizing the probability function P_i is equivalent to minimizing the sum in the exponential. Therefore the gamma function is defined as the goodness-of-fit parameter, which minimizes the weighted sum of the squares of the deviations or the residual of the measured and reconciled values which is the same as equations(10) and (11) given by [42]:

$$\gamma = \Sigma\left(\frac{y_i - \hat{x}_i^2}{\sigma_i}\right)^2 \tag{24}$$

3.1.1 Linear steady state process data reconciliation

Linear steady state process data reconciliation problem is considered the simplest data treatment in process modeling. Linear data reconciliation is suitable if only overall mass flow balance is considered. It uses the same assumption that random measurements errors follow a Gaussian distribution with zero mean and covariance matrix V. The general DR mathematical formulation in Section 3.1 above gives the solution of linear steady state systems. For a linear system the constraints are given by $f(x) = Ax$ with A independent of x.

When replacing the variables weighting W by the inverse of the covariance matrix V^{-1} the objective function equation and the solution of the reconciled values are given below by equations(25) and (26) respectively;

$$J(x) = \left[(y-\hat{x})^T V^{-1} (y-\hat{x}) \right]$$
$$Subject\, to: \; f(x) = Ax = 0 \tag{25}$$

$$\hat{x} = y - VA^T (AVA^T)^{-1} Ay = \left[I - VA^T (AVA^T)^{-1} A \right] y = By \tag{26}$$

The reconciled estimates are therefore correlated to the measured values by a linear transformation. Their precision can be obtained from the measurement precision given the covariance matrix V and from the model equations from matrix A. The reconciled estimates are thus also normally distributed with an expected value and co-variance matrix respectively given by

$$E[\hat{x}] = BE(y) = B\hat{x} = \hat{x} \tag{27}$$

$$Cov[\hat{x}] = E\left\{ (By)(By)^T \right\} = BVB^T \tag{28}$$

The expected value in equation(27) forms a property of the maximum likelihood estimate for linear systems and the measure of the accuracy of the estimates given by equation(28) above. The above transformation is applicable only in cases of linear systems with all the process variables measured [12], [20], [22].

However, in practice not all process variables are measured because of the physical constraints of the plant and economic limitations. The unmeasured variables in a process flow are sometimes required to complete the process model balance. One of the advantages of data reconciliation is the capability to estimation of unmeasured variables in the reconciliation process. The condition for estimating unmeasured variables is that they must be observable in accordance with variable classification discussed above. Crowe et al developed a method to eliminate unmeasured variables in the reconciliation problem using the projection matrix which forms two matrices [22]. There are many matrix decomposition methods available but the QR factorization algorithm was chosen because it is an efficient method of constructing the projection matrix.

In order to formulate the reconciliation problem for a partially measured linear system, let the variables be classified into two sets: vector x of measured variables and vector u of unmeasured variables. The model constraints are written in terms of the measured and unmeasured variables and are given by:

$$A_x x + A_u u = 0 \tag{29}$$

Where $x(n\times1)$ is a vector of the measured variables and $u(n\times1)$ is a vector of the unmeasured variables. $A_x(n\times n)$, and $A_u(n\times p)$ are the incidence matrices for measured and unmeasured sub-problems of the reconciliation problem of a partially measured system. These matrices are referred to as the Jacobian matrices obtained by taking partial derivatives of the constraint model equations with respect to the measured and unmeasured variables respectively. The incidence matrices can also be determined using numerical methods. The unmeasured variables u and matrix A_u are eliminated by pre-multiplying equation(29) with the projection matrix P yielding:

$$PA_u = 0 \tag{30}$$

Therefore the constraints equation(29) is reduced so that it has only the constraints of the measured variables as:

$$PA_x x = 0 \tag{31}$$

The projection matrix P is constructed by performing the QR factorization algorithm of the incident matrix A_u of unmeasured variables, which yields Q and R sub-matrices. The projection matrix is constructed from sub-matrix Q which is orthogonal to the matrix of the constraint equations corresponding to the measured and the unmeasured process variables. The reconciliation problem is reduced to a minimization problem of only redundant measured variables. Data reconciliation is performed first on the measured variables and utilise the reconciled values to estimate the solution to the unmeasured variables.

The reduced reconciliation problem subject to the reduced constraints equation(31) is given by:

$$\begin{aligned} &J(x,u) = \left[(y-\hat{x})^T V^{-1} (y-\hat{x}) \right] \\ &Subject\,to: \; PA_x x = 0 \end{aligned} \tag{32}$$

Therefore equation(32) takes the form of the linear constraint data reconciliation objective function equation. Substituting PA_x in equation(26) gives the solution of the reconciled values for a partially measured steady state linear system:

$$\hat{x} = y - V(PA_x)^T \left[(PA_x)V(PA_x)^T \right]^{-1} (PA_x)y \tag{33}$$

Consider the incident matrix A_u of the unmeasured streams of a steady state linear system. The projection matrix is constructed by decomposing A_u as derived below [20], [22]. The matrix A_u

contains linear independent columns and is decomposed into an orthogonal matrix $Q(m\times m)$ and upper triangular matrix $R(m\times p)$ such that $Q^TQ=I$

$$A_u = QR \tag{34}$$

The orthogonal matrix Q and the upper triangular matrix R are each sub-divided into two sub-matrices:

$$Q=[Q_1 \quad Q_2] \tag{35}$$

$$R = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} \tag{36}$$

Where Q_1 is an$(m\times m)$, Q_2 is an$(m\times m-n)$, R_1 is an $(n\times n)$, which is a non-singular upper triangular matrix and R_2 is an $(m-n\times n)$, which is a zero matrix.

Therefore equation(34) becomes:

$$A_u = [Q_1 \quad Q_2]\begin{bmatrix} R_1 \\ 0 \end{bmatrix} \tag{37}$$

Pre-multiplying both sides of equation(37) with the transpose matrix Q_2^T yields;

$$Q_2^T A_u = Q_2^T [Q_1 \quad Q_2]\begin{bmatrix} R_1 \\ 0 \end{bmatrix} \tag{38}$$

Recalling matrix Q is orthogonal, matrix Q_2 has the property to yield

$$Q_2^T [Q_1 \quad Q_2]\begin{bmatrix} R_1 \\ 0 \end{bmatrix} = [0 \quad I]\begin{bmatrix} R_1 \\ 0 \end{bmatrix} = 0 \tag{39}$$

From the above equation we have

$$Q_2^T A_u = 0 \tag{40}$$

Equation(40) has the matrix Q_2^T which satisfies the condition of equations(30), therefore the matrix Q_2^T is the required projection matrix,

$$P = Q_2^T \tag{41}$$

Substituting Q_2^T in equation(33) yields the generalized solution of the reconciled values of the measured variables using the projection matrix.

$$\hat{x} = y - V(Q_2^T A_x)^T \left[(Q_2^T A_x) \mathrm{V} (Q_2^T A_x)^T \right]^{-1} (Q_2^T A_x) y \tag{42}$$

The estimates of the unmeasured variables can be determined from the constraints equation(29) using the solution of the reconciled values. Rearranging equation(29) yields:

$$A_u \hat{u} = -A_x \hat{x} \tag{43}$$

In addition, rearranging the above equation then applying least squares techniques to give the generalized solution of the unmeasured variables given by:

$$\hat{u} = -\left(A_u^T A_u\right)^{-1} A_u^T \left(A_x \hat{x}\right) \tag{44}$$

Alternatively, the solution of the unmeasured variables can be computed from the QR factorization. Equation(29) can be written as:

$$A_x \hat{x} + QR\hat{u} = 0 \tag{45}$$

Pre-multiplying equation(45) by Q^T yields:

$$Q^T A_x \hat{x} + R\hat{u} = 0 \tag{46}$$

Rearrange to get

$$R\hat{u} = -Q^T A_x \hat{x} \tag{47}$$

Use equation(37) for R to get

$$\begin{bmatrix} R_1 \\ 0 \end{bmatrix} \hat{u} = -\begin{bmatrix} Q_1^T \\ Q_2^T \end{bmatrix} A_x \hat{x} \tag{48}$$

Rearranging equation(48) yields the generalized solution of the unmeasured variables:

$$\hat{u} = -R_1^{-1} Q_1^T A_x \hat{x} \tag{49}$$

Therefore the solution of the unmeasured and observable variables is given by equation(44) for partially measured steady state linear systems [12], [20].

3.1.2 Nonlinear steady state process data reconciliation

The constraint balances used to describe the operations of steady state power generation and chemical processing plants are nonlinear in nature. In power plants, the constraints are made up of mass and energy balances and may include thermodynamic relationships. In chemical processing plants the constraints consists of stream flow and stream composition measurements. These types of constraints are described as bilinear since they contain the stream flow and stream composition or the product of stream flows and temperature of energy flow. The data reconciliation for such cases is based on the solution of a nonlinear constrained optimization problem [12], [20].

The nonlinear data reconciliation problem is capable of reconciling the measurements of flow rates, temperatures, pressures and compositions of flow streams of a process to satisfy the process model constraints. In linear reconciliation, only equality constraints are used with boundaries. In nonlinear cases, boundaries are imposed on the constraints to avoid infeasible values of the estimates, such as negative values for flows or compositions. Imposing boundaries gives rise to inequality constraints in the reconciliation problem [20]. A simplified formulation of nonlinear data reconciliation will be considered below.

Similarly, as in the linear case it is assumed that random measurement errors follow a Gaussian distribution with zero mean and covariance matrix V given below. The nonlinear data reconciliation problem is formulated as the weighted least squares optimization problem as follows:

$$\underset{x,u}{Min}\left[\left(y-\hat{x}\right)^{T} V^{-1}\left(y-\hat{x}\right)\right] \tag{50}$$

$$\begin{aligned} subject\ f\left(x,u\right)&=0 \\ g\left(x,u\right)&\geq 0 \end{aligned} \tag{51}$$

In addition, x and u are vectors of measured and unmeasured variables respectively; $f\left(x,u\right)$ and $g\left(x,u\right)$ are the constraint functions [20].

The solution technique of nonlinear reconciliation problem with only equality constraints is considered in the study. For a nonlinear system $f\left(x\right)\neq Ax$ as was the case for a linear system, however, it is approximated with $f\left(x\right)\approx Ax+b$ using successive linearization to solve the reconciliation problem. Applying Langrage's optimization method, by adding the Langragian multipliers, minimizes the objective function to obtain the solution of the nonlinear reconciliation problem:

$$J\left(x,u,\lambda\right)=\left(y-\hat{x}\right)^{T}V^{-1}\left(y-\hat{x}\right)+2\lambda^{T}f\left(x,u\right) \tag{52}$$

The solution is obtained by setting the partial derivatives of equation(52) with respect to the variables for x, u and λ to zero and solving the resulting equations:

$$\begin{aligned}\frac{\partial J}{\partial x}&=-V^{-1}\left(y-\hat{x}\right)+A_{x}\lambda^{T}=0\\ \frac{\partial J}{\partial u}&=A_{u}\lambda^{T}=0\\ \frac{\partial J}{\partial \lambda}&=f\left(x,u\right)=0\end{aligned} \tag{53}$$

With A_x and A_u being the Jacobian matrices containing partial derivatives of the nonlinear functions of $f\left(x,u\right)$ with respect to x and u given by:

$$A_{x}=\frac{\partial f}{\partial x} \tag{54}$$

$$A_{u}=\frac{\partial f}{\partial u} \tag{55}$$

Solving nonlinear constraints will involve an iterative numerical procedure. An iterative approach for solving the normal equations(53) through (55) based on successive linearization is used. The first order Taylor expansion of the constraints equations is used to obtain a linear approximation, of which only the constant term and the first-order derivatives are retained [20]. The linearized system of the constraint equation becomes

$$f\left(x,u\right)=A_{x}x+A_{u}u-b \tag{56}$$

Only the equality constraint equation of equation(51) is considered for this transformation. The Jacobian matrices are derived through setting the partial derivatives of the constraints equations with respect to x and u given below. They are evaluated at $\hat{x}_i=x_i$ and $\hat{u}_i=u_i$ for the initial iteration.

$$A_x = \left.\frac{\partial f}{\partial \hat{x}}\right|_{\hat{x}_i,\hat{u}_i} = \begin{bmatrix} \frac{\partial f_1}{\partial \hat{x}_1} & \frac{\partial f_1}{\partial \hat{x}_2} & \cdots & \frac{\partial f_1}{\partial \hat{x}_M} \\ \frac{\partial f_2}{\partial \hat{x}_1} & \frac{\partial f_2}{\partial \hat{x}_2} & \cdots & \frac{\partial f_2}{\partial \hat{x}_M} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial f_C}{\partial \hat{x}_1} & \frac{\partial f_C}{\partial \hat{x}_2} & \cdots & \frac{\partial f_C}{\partial \hat{x}_M} \end{bmatrix} \tag{57}$$

$$A_u = \left.\frac{\partial f}{\partial \hat{u}}\right|_{\hat{x}_i,\hat{u}_i} = \begin{bmatrix} \frac{\partial f_1}{\partial \hat{u}_1} & \frac{\partial f_1}{\partial \hat{u}_2} & \cdots & \frac{\partial f_1}{\partial \hat{u}_N} \\ \frac{\partial f_2}{\partial \hat{u}_1} & \frac{\partial f_2}{\partial \hat{u}_2} & \cdots & \frac{\partial f_2}{\partial \hat{u}_N} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial f_C}{\partial \hat{u}_1} & \frac{\partial f_C}{\partial \hat{u}_2} & \cdots & \frac{\partial f_C}{\partial \hat{u}_N} \end{bmatrix} \tag{58}$$

Where M is the number of the measured variables, N is the number of the unmeasured variables, C is the number of constraint equations and i is the i^{th} iteration.

$$b = A_x x_i + A_u u_i - f\left(x_i, u_i\right) \tag{59}$$

The raw measurements are generally used as the initial estimates for the measured variables for the first iteration. Thus the general linearized nonlinear data reconciliation problem takes the form:

$$\begin{gathered} J\left(x,u\right) = \left[\left(y-\hat{x}\right)^T V^{-1}\left(y-\hat{x}\right)\right] \\ Subject\,to:\ A_x x + A_u u = b \end{gathered} \tag{60}$$

As in the linear case for partially measured systems, the projection matrix method is applied to reduce the reconciliation problem using the QR factorization algorithm to eliminate the unmeasured variables. The reduced reconciliation problem is given by:

$$\begin{gathered} J\left(x,u\right) = \left[\left(y-\hat{x}\right)^T V^{-1}\left(y-\hat{x}\right)\right] \\ Subject\,to:\ Q_2^T A_x x = Q_2^T b \end{gathered} \tag{61}$$

The solution of the reconciled estimates is given by:

$$\hat{x} = y - V(Q_2^T A_x)^T \left[(Q_2^T A_x)V(Q_2^T A_x)^T\right]^{-1}(Q_2^T A_x\,\mathrm{y} - Q_2^T b) \tag{62}$$

Then the solution of the unmeasured variables is given by:

$$\hat{u} = R_1^{-1}Q_1^T - R_1^{-1}Q_1^T A_x \hat{x} - R_1^{-1}R_2 u_{N-r} \tag{63}$$

From the QR factorization of the matrix A_u the last term of equation(63) falls out since R_2 is a zero matrix. Therefore the solution of the unmeasured variables is given by:

$$\hat{u} = R_1^{-1}Q_1^T - R_1^{-1}Q_1^T A_x \hat{x} \tag{64}$$

In addition, the validated results for measured variables x and unmeasured variables u are directly related to the covariance matrix V and process measurements. The derived data reconciliation methodology will be later applied in both linear and nonlinear steady state processes with measured and partially measured systems to validate the application of the technique.

3.2 Gross error detection (GED)

The existence gross errors in industrial measurements make it difficult for DR procedure to produce credible solutions consistent. [12], [20]. Gross errors are present in process measurements as result of instrument malfunction, miscalibration, poor sampling and process disturbances. These errors affect measurements in process plants.

Data reconciliation problem is traditionally used to adjust measured variables optimally to satisfy balance constraints. In addition, the adjustments of measured data should be used to detect the presence of gross errors to allow for appropriate action: this is known as gross error detection problem. The presence of gross errors in process data negatively impacts the solution of a reconciliation problem. It is therefore important for gross errors to be identified and either corrected or eliminated from the reconciliation problem.

Gross error detection strategies are derived from the statistical tests developed for gross error detection. The qualities of a good statistical test as described by Narasimhan and Jordache [20] must be the ability to:

- Detect the presence of gross errors,
- Identify the type and location of a gross error,
- Identify and locate multiple gross errors occurring simultaneously, and
- Estimate the magnitude of the gross error.

However, it must be noted that no single statistical test possesses all the above abilities hence the estimation of gross error magnitude is considered unnecessary [20].

The basis of statistical tests, hypothesis testing, is derived from the basic statistical principle for outlier detection. In the case of gross error detection, the null hypothesis H_0 is that there is no gross error, and the alternative hypothesis H_1 is that there is one gross error or more in the system. To test for the hypothesis the statistic value is compared with a criterion or critical value [12], [20]. Some of the basic statistical tests that have been proposed for gross error detection in process data are discussed below.

3.2.1 Global test (GT)

The global test uses the residual r of a linear process model in matrix A and the measured variable y . The residual represents the constraints violation. When there are no gross errors the residual follows a normal distribution and is computed as:

$$r = Ay \tag{65}$$

The global test is formed from residual r and the inverse of the covariance matrix V . The covariance is given by:

$$V = \operatorname{cov}(r) = AS_V A^T \tag{66}$$

The GT follows a chi-square distribution with degrees of freedom v equal to the rank of A :

$$\gamma = r^T V^{-1} r \tag{67}$$

The global test combines all the constraint residuals to complete the test statistic and is given by equation(67) which is equal to the minimum value of the objective function:

$$\gamma = \sum_{i=1}^{n} \left(\frac{x_i - \hat{x}_i}{\sigma_i} \right)^2 \tag{68}$$

Gross error is detected when the statistics test equation(67) is greater than or equal to the critical value given in equation(69):

$$\gamma \geq \chi^2_{1-\alpha,v} \tag{69}$$

The critical value of the gamma function γ is read off from the Chi-squared distribution using the degrees of freedom at a 95% confidence level. However, care should be taken in process models with large values and high standard deviations which have a tendency to conceal smaller values and cancel the possibility of detecting gross errors in them [20].

3.2.2 Nodal test (NT)

The nodal or constraint test is derived from the vector of residuals r for each constraint i. The test follows a normal distribution and can be evaluated by:

$$z_{r,i} = \frac{|r_i|}{\sqrt{V_{ii}}} \quad i = 1, 2 \ldots, m \tag{70}$$

Where r_i is a residual of constraint i and V_{ii} the corresponding co-variance value in the diagonal of V. The test can also be expressed in the form

$$z_r = \left[diag(V) \right]^{-1/2} r \tag{71}$$

Where $diag(V)$ defines the diagonal elements V_{ii} of matrix V. The test uses the $z_{r,i}$ statistics to detect gross error, which is detected when the statistic test exceeds the test criterion $Z_{1-\alpha/2}$ the critical value determined from a standard normal distribution table with the level of significance α

$$z_{r,i} \geq Z_{1-\alpha/2} \tag{72}$$

The nodal test treats each constraint residual separately and gives rise to m univariate tests [20].

3.2.3 Measurement test (MT)

The measurement test is based on the vector of measurement adjustments. The adjustments are computed only after reconciliation and the difference between measured variables y and the reconciled estimates $\hat{x}$ given by:

$$a = y - \hat{x} \tag{73}$$

The measurement adjustments can also be written as:

$$a = S_V A^T V^{-1} r \tag{74}$$

The measurement adjustment follows a multivariate normal distribution with zero mean and variance-covariance $\overline{W}$ given by

$$\bar{W} = \text{cov}(a) = S_V A^T V^{-1} S_V \tag{75}$$

The MT follows a standard normal distribution and is given by [20]:

$$z_{a,j} = \frac{|a_j|}{\sqrt{\bar{W}_{jj}}} \quad j = 1, 2 \ldots, n \tag{76}$$

3.2.4 General likelihood ratio (GLR)

The formulation of the general likelihood ratio test requires a process model, also referred to as a gross error model. GLR can detect gross errors and process leaks. The GLR test statistic is the ratio between the probability of the null hypothesis and the alternative hypothesis given by:

$$\gamma = \sup \frac{\Pr\{r/H_1\}}{\Pr\{r/H_0\}} \tag{77}$$

Where $\Pr\{r/H_0\}$, $\Pr\{r/H_1\}$ are the probabilities to obtain the residual vector r under the null and the alternative hypothesis respectively and the supremum (sup) in equation(77) is computed over all parameters in the hypothesis [1], [20], [43].

3.2.5 Principal component test (PCT)

The principal component test (PCT) is based on principal component analysis (PCA) which uses the entire co-variance matrix, not the diagonal terms of the matrix only. PCT is sensitive to subtle gross errors and can accurately identify variables in error. Gross errors from the principal components can be investigated rather than from the variables. The PCT is combined with NT and MT to form the principal component nodal test (PCNT) and principal component measurement test (PCMT) respectively. The PCNT for the constraint residual r is given by:

$$p_r = W_r^T r \tag{78}$$

Where the columns in W_r are eigenvectors of V and they satisfy

$$W_r = U_r \Lambda_r^{-1/2} \tag{79}$$

The matrix Λ_r is diagonal and consists of eigenvalues of V, $\lambda_{r,i}$ $i = 1 \ldots m$. It satisfies the variance matrix

$$\Lambda_r = U_r^T V U_r \tag{80}$$

The matrix U_r consists of orthonormal eigenvectors of V so that $U_r U_r^T = I$. Similarly, the PCMT for measurement adjustments is given by:

$$p_{ai} = \left(W_a^T a\right)_i \quad j = 1 \ldots n \tag{81}$$

Where the columns in W_a are eigenvectors of $\overline{W}$ and n is the number of retained principal components [12], [20], [44].

3.2.6 Closing remarks

Process data reconciliation procedure cannot be complete without a gross error detection strategy that will detect and identify gross errors in measurements. The confidence given to the credibility and the validity of data reconciliation solution require a good GED strategy, one which will detect, identify and locate gross errors in process data. The ability of a GED strategy to perform these roles is fundamentally important in data processing and validation. However, each of the strategies discussed above has limitations regarding the ability to detect and locate gross errors. Mei et al [45] have developed a combined NT-MT method with an aim to improve GED strategies. Another GED development by Bagajewicz et al [44] is the combination of the PCT with NT and MT to form PCNT and PCMT which are capable of detecting subtle gross errors and can precisely identify variables with errors

The GT as discussed can be applied to all measurements constraint residuals to detect gross error resulting in a multivariate test. While the NT treats each constraint at a time, resulting in a univariate test. For both the GT and NT gross error detection is usually performed before the reconciliation procedure as the first step in data processing. The MT on the other hand is different from the others since it is performed on the adjusted measurements after data reconciliation procedure. The MT results in a multivariate test similar to the NT. The GLR focuses on the entire process model and extends to detecting process flow leaks over and above gross errors.

4. Application of process data reconciliation

4.1 Linear steady state process data reconciliation models

As discussed earlier, DR problem was first formulated and applied in linear systems. The application of data reconciliation in linear systems is demonstrated by examples of two linear systems, one with all variables measured and the other with partially measured variables.

Consider a steady state process constrained by a set of linear process model equations. The measurements are assumed to contain random errors to a degree that the overall mass flow rates of the process cannot satisfy the mass balance of the systems. The partially measured system contains measured variables as well as unmeasured variables that will be estimated from the reconciled values, so that the model constraints equations are satisfied.

4.1.1 Process flow with all variables measured

The process flow shown in Figure 5 contains four junctions or nodes denoted by $N = [\mathrm{N}_1, \ldots, \mathrm{N}_4]$ and eight flow streams denoted by $X = [x_0, \ldots, x_7]$. The nodes represent various plant components in the process flow. All the process variables are measured. The process mass flow balances cannot be satisfied since the variables contain measurement errors. The linear steady-state DR problem derived in Section 3.1.1 is applied to solve the linear system derived from Figure 5. The unknown true values and the measured variables are denoted by y_i and x_i respectively, and i indicate the i^{th} stream. The measurement error model equation(13) relating the measured variables to the unknown true values is given by:

$$y_i = x_i + \varepsilon_i \quad i = 0 \ldots 7 \tag{82}$$

Where ε_i represents the random errors in the measurements.

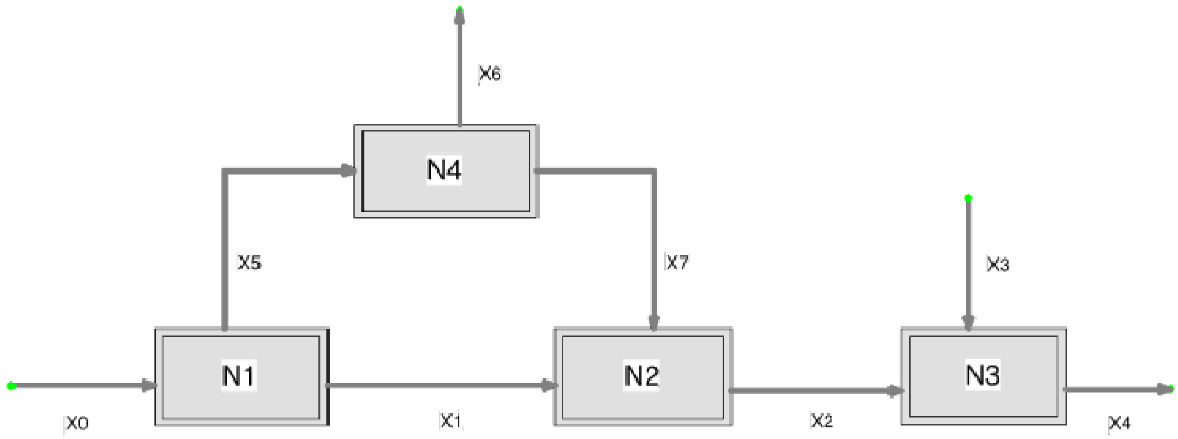

Figure 5: A simple process flowsheet with all flows measured *[21]*

The process flowsheet is produced using RECON software and generates four system equations around the nodes as shown in equation(83) below. The mathematical model given by the system equations represent the physical process of Figure 5.

$$\begin{aligned} x_0 - x_1 \qquad\qquad - x_5 \qquad\qquad \\ x_1 - x_2 \qquad\qquad\qquad + x_7 \\ x_2 + x_3 - x_4 \qquad\qquad \\ x_5 - x_6 - x_7 \end{aligned} = 0 \tag{83}$$

Equation(83) gives the constraint balanced equations of the process flowsheet shown in the figure above. The overall mass balance in matrix notation is given below in the form of equation(14)

$$f(x) = Ax = 0 \tag{84}$$

Where the incidence matrix A representing the flows associated with each node is given by:

$$A = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 \end{bmatrix} \tag{85}$$

The array representing the measured flows x is given by:

$$X = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} \quad (86)$$

The above example demonstrates how DR problem is applied in a linear steady state process flow with all measured variables. The solution of the DR problem is expected to produce estimates of the true values that are consistent and will satisfy the mass balance equations. Since all the streams are measured, the process is overdetermined; it has more measured variables than required to perform the reconciliation. The degree of redundancy of the system is calculated from the number of balanced equations less the unmeasured variables. In this process there are no unmeasured variables, therefore the degree of redundancy is calculated using equation(2) resulting in $v = 4$ since there are four balanced equations.

4.1.2 Input data

The input data table of the measured variables and their associated standard deviations, will also be referred to as maximum errors, are shown in Table 1 below; all the measured flow rates and standard deviations are measured in kg/s. The measurement values are used in the balanced equations and the constraints are not satisfied since the process flow measurements of the flowsheet contain errors.

Table 1: Process flow measurements with standard deviations

Stream	Measured Flow (kg/s)	Standard Deviation (kg/s)
X0	100.1	5.005
X1	41.1	2.055
X2	79	3.95
X3	30.6	1.53
X4	108.3	5.415
X5	56.8	2.84
X6	19.8	0.99
X7	38.8	1.94

A simple calculation of the mass balance using equation(83) is shown below, indicating the imbalances due to measurement errors;

$$f(x) = \begin{pmatrix} 2.2 \\ 0.9 \\ 1.3 \\ -1.8 \end{pmatrix}$$

The above calculation reveals the imbalances in every node. Process data reconciliation problem will be applied to correct the imbalances by adjusting the measured variables to satisfy the model constraint equations given by equation(83). The reconciliation problem uses the measurements redundancy to adjust the measured variables of the process flow. The mathematical model of the process flow is given by the constraints equations on which the reconciliation problem is performed. The analytical solution is performed using Mathcad, which is a general numerical tool for defining and solving analytical problems. The reconciled solution is validated using RECON software. The software is configured to detect the presence of imbalances as well as errors in the flow streams.

4.1.3 Gross error detection by GT

A GT strategy is applied on the measurement variables of vector X to test for gross errors. The process flow measurements contain random errors in accordance with the definition. However a gross error of -15kg/s is introduced in measurement x_3 as a result of the flow meter fouling or clogging.

The critical value of the above process flow as given by equation(69) is read off from the Chi-squared distribution table in Appendix A1 [33] with significance level of 95% and the associated degree of freedom of 4. Therefore for $\alpha = 0.05$ and $v = 4$ the critical value is $\chi^2_{0.05,4} = 9.488$.

The measurements covariance matrix is determined from equation(66) is given below;

$$V = \begin{pmatrix} 37.339 & -4.223 & 0 & -8.066 \\ -4.223 & 23.589 & -15.602 & -3.764 \\ 0 & -15.602 & 47.266 & 0 \\ -8.066 & -3.764 & 0 & 12.809 \end{pmatrix}$$

Figure 6: Covariance matrix of process flow with measured streams

Therefore the GT is calculated from equation(67) for measurement vectors X without GE and X_{GE} with gross error respectively,

$$\gamma_X := (A_X \cdot x)^T V^{-1} (A_X \cdot x) = 0.389$$

$$\gamma_{GE} := (A_X \cdot x_{GE})^T V^{-1} (A_X \cdot x_{GE}) = 11.921$$

According to the above test, the GT detects gross error in measurement vector X_{GE}, since the value of the test is greater than the critical value of the process flow. However due to the limitation of the GT, the test is unable to locate or identify which measurement or instrument contains a gross error and also give the magnitude of the gross error.

4.1.4 Reconciliation results

The analytical solution as calculated using equation(26) for a linear transformation of the measured variables and the RECON solution of the process flow are given in Table 2 below. The measurements adjustments between raw measurements and reconciled values are also shown in Table 2 for both analytical and RECON solutions.

Table 2: Reconciled flows and adjustments for both analytical and RECON

Stream	Measured Flow	Analytical	Analytical Adjustments	RECON	RECON Adjustments	Percentage Error
X0	100.1	98.946	-1.154	98.946	-1.154	1.17%
X1	41.1	41.026	-0.074	41.026	-0.074	0.18%
X2	79	79.237	0.237	79.237	0.237	0.30%
X3	30.6	30.486	-0.114	30.486	-0.114	0.37%
X4	108.3	109.723	1.423	109.723	1.423	1.30%
X5	56.8	57.92	1.12	57.92	1.12	1.93%
X6	19.8	19.709	-0.091	19.709	-0.091	0.46%
X7	38.8	38.211	-0.589	38.211	-0.589	1.54%

4.1.5 Discussion

For a linear steady state transformation the analytical and RECON solutions produce the same reconciled solutions. The reconciled solutions satisfy the mass balances of the process flow calculated from equation(83).

$$f(x') = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

The measurement adjustments from the analytical and RECON solutions are also the same, as indicated in Figure 7 below and also as shown in Table 2 above.

The accuracy of the reconciled values relies on the variance or the maximum errors of the measured variables. The variance of the measurements determines the adjustability of the measurements to give the reconciled values. The reconciled values are as close as possible to the true values of the process flow. The choice of the measurement variance determines how close reconciled values will be to the true process variables. The more precise the measurements values, the more accurate the reconciled values. In addition, values of the measurements adjustments will be smaller. The primary purpose of applying DR problem in process flows is to reduce or eliminate the influence of random errors found in process measurements which result in the violation of conservation laws of mass and energy balances.

The percentage error of the results is shown in Table 2 demonstrate the measurements improvement achieved through the use of data reconciliation on the measurement data.

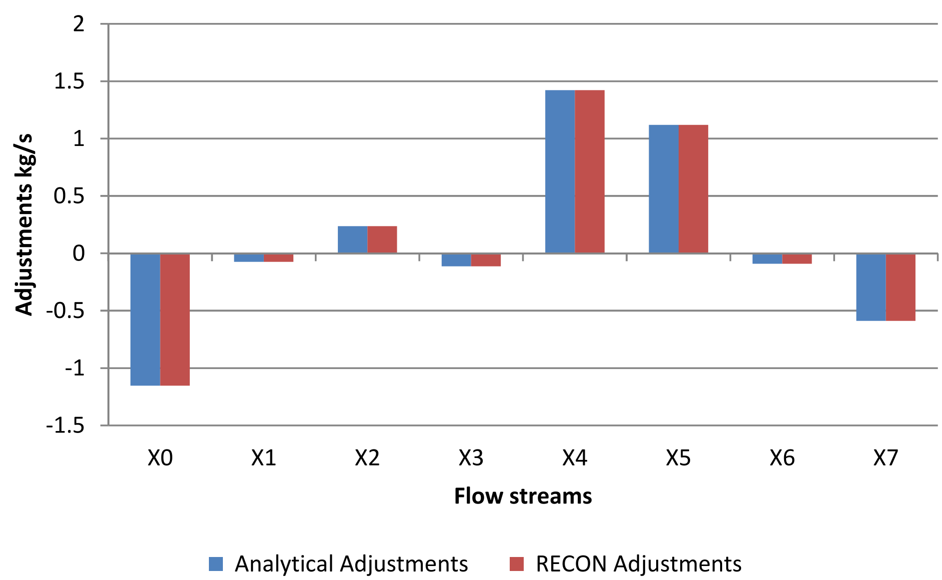

Figure 7: Adjustments of a linear system with all measured variables

Data reconciliation in linear steady state processes with measured variables forms the basis for the application of DR problem in partially measured linear steady state processes. The redundancy of the measurement has an impact on the reconciliation process, as demonstrated by a partially measured process flow (below).

4.1.6 Process flow with partially measured variables

Mah et al [17] and Crowe et al [22] developed a methodology for estimating unmeasured variables, using the reconciled values of the measured variables in the reconciliation process. The example below, adapted from Madron et al [21], is used to show the application of DR problem in a partially measured process flow. DR problem will estimate the values of the unmeasured variables to satisfy the model constraints balance. The process flowsheet with both measured and unmeasured variables is given in Figure 8 below; all the unmeasured flows are observable, as indicated by dashed lines.

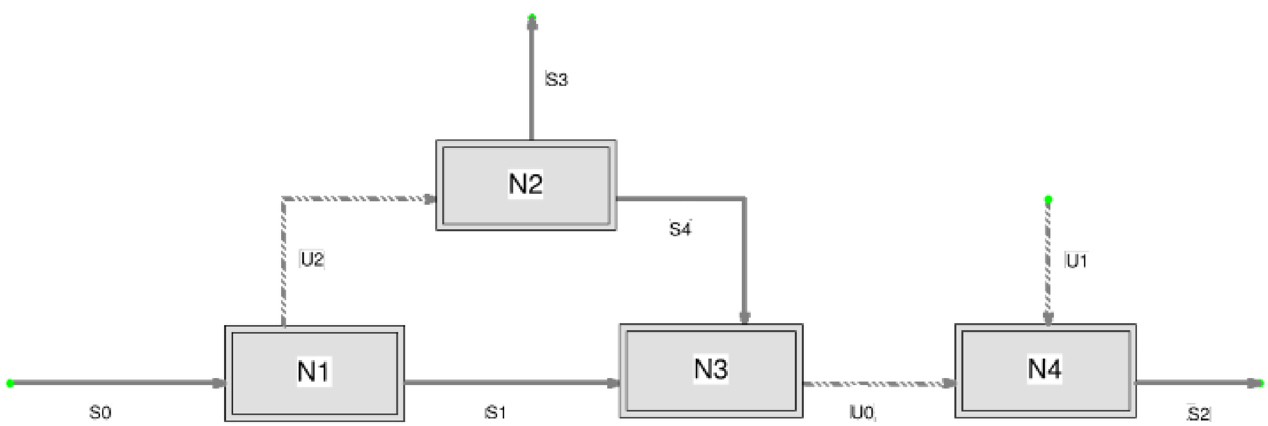

Figure 8: Process flowsheet with partially measured linear system *[21]*

The model constraint equations from the above process flowsheet can be written as shown below, with the unmeasured variables denoted by u, that is, let $s_5 = u_0$, $s_6 = u_1$ and $s_7 = u_2$ respectively. Therefore equations are given by:

$$\begin{array}{ccccccccc} s_0 & -s_1 & & & & & & -u_2 & \\ & & & -s_3 & -s_4 & & & +u_2 & \\ & s_1 & & & +s_4 & -u_0 & & & \\ & & -s_2 & & & +u_0 & +u_1 & & \end{array} = 0 \qquad (87)$$

The arrays of measured and unmeasured variables are given as:

$$S = \begin{pmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \\ s_4 \end{pmatrix} \quad \text{and} \quad U = \begin{pmatrix} u_0 \\ u_1 \\ u_2 \end{pmatrix}$$

4.1.7 Input data

The process input data of the measured flow rates and their maximum errors are given in Table 3 below; all measured in kg/s. The process flow contains unmeasured flows; consequently, the process model constraints will take the form of equation(29) with matrices A_s and A_u representing the incident matrices of the measured and unmeasured flows respectively. Therefore, the projection matrix technique is applied to eliminate the unmeasured variables from the DR problem. The reduced reconciliation problem given by equation(32) is then used to reconcile the values of the measured variables and the solution is used to estimate values of the unmeasured variables.

Table 3: Measured and unmeasured streams

Stream	Measured stream (kg/s)	Maximum error (kg/s)
S0	100.10	2.002
S1	41.10	0.206
S2	Unmeasured	-
S3	Unmeasured	-
S4	108.30	0.542
S5	19.80	0.099
S6	Unmeasured	-
S7	38.80	0.776

The Jacobian matrix of the process flow indicating both measured and unmeasured variables is given by:

$$A(s\,,\,u)\;=\begin{pmatrix}1 & -1 & 0 & 0 & 0 & 0 & 0 & -1\\ 0 & 0 & 0 & -1 & -1 & 0 & 0 & 1\\ 0 & 1 & 0 & 0 & 1 & -1 & 0 & 0\\ 0 & 0 & -1 & 0 & 0 & 1 & 1 & 0\end{pmatrix}$$

Figure 9: The incidence matrix of the partially measured process flow

The incidence matrices are shown Figure 10 below as determined using partial derivatives of the variables of the measured and unmeasured variables using equations(57) and (58) respectively:

$$A_s=\begin{pmatrix}1 & -1 & 0 & 0 & 0\\ 0 & 0 & 0 & -1 & -1\\ 0 & 1 & 0 & 0 & 1\\ 0 & 0 & -1 & 0 & 0\end{pmatrix}\quad\text{and}\quad A_u=\begin{pmatrix}0 & 0 & -1\\ 0 & 0 & 1\\ -1 & 0 & 0\\ 1 & 1 & 0\end{pmatrix}$$

Figure 10: Incidents matrices of measured and unmeasured variables

The incidence matrices are always a representation of the physical flow process. The QR factorization algorithm is applied to decompose the incidence matrix of the unmeasured variables A_u. The decomposed A_u matrix results in matrix $M=QR$ which consists of sub-matrices Q and R respectively.

$$M:=\begin{pmatrix}0 & 0 & -0.707 & -0.707 & -1.414 & -0.707 & 0\\ 0 & 0 & 0.707 & -0.707 & 0 & 0.707 & 0\\ 0.707 & 0.707 & 0 & 0 & 0 & 0 & 1.414\\ -0.707 & 0.707 & 0 & 0 & 0 & 0 & 0\end{pmatrix}$$

Figure 11: Matrix M from the decomposed A_u matrix

The sub-matrices are extracted from the decomposed matrix so that Q is an orthogonal matrix and R is an upper triangular matrix, as shown respectively in Figure 12 below. The required projection matrix is constructed from Q matrix, which is further sub-divided into two matrices in the form of equation(35); thus the transpose of matrix Q_2 is the required projection matrix. For partially measured systems the technique of first constructing the projection matrix to eliminate unmeasured variables is widely used in literature.

$$Q = \begin{pmatrix} 0 & 0 & -0.707 & -0.707 \\ 0 & 0 & 0.707 & -0.707 \\ 0.707 & 0.707 & 0 & 0 \\ -0.707 & 0.707 & 0 & 0 \end{pmatrix} \quad \text{and} \quad R = \begin{pmatrix} -1.414 & -0.707 & 0 \\ 0 & 0.707 & 0 \\ 0 & 0 & 1.414 \\ 0 & 0 & 0 \end{pmatrix}$$

Figure 12: Sub-matrices Q and R

The constructed projection matrix of the process flow computed from Q matrix is given by:

$$Q_2^T = \begin{pmatrix} -0.707 & -0.707 & 0 & 0 \end{pmatrix} \tag{88}$$

Therefore, the analytical reconciled values of the measured variables can be obtained using equation(42) and the estimates of the unmeasured variables can be obtained from either equations(44) or (49) as derived in Section 3.1.1.

4.1.8 Reconciliation results

The results of the analytical solution and the RECON solution for a partially measured system are given in Table 4 below, as well as the measurements adjustments made between the raw measurements and reconciled values of the measured variables. The solutions of the reconciliation and the estimated values of unmeasured variables satisfy the model mass balances for each node. The estimates of the unmeasured variables in both analyses are the same and validate the application of data reconciliation in partially measured systems. The measurement redundancy of the system was reduced because of the presence of unmeasured flows which has a slight effect on the adjustability of the measurements. However, the results are valid and acceptable.

Table 4: Reconciled values and estimates of unmeasured variables

Stream	Measured stream	Analytical	Analytical Adjustments	RECON	RECON Adjustments	Percentage Error
S0	100.10	99.756	-0.344	99.756	-0.344	0.34%
S1	41.10	41.104	0.004	41.104	0.004	0.01%
S2	Unmeasured	79.955	0.0	79.955	0.0	-
S3	Unmeasured	28.345	0.0	28.345	0.0	-
S4	108.30	108.300	0.0	108.300	0.0	0.00%
S5	19.80	19.801	0.001	19.801	0.001	0.01%
S6	Unmeasured	58.653	0.0	58.653	0.0	-
S7	38.80	38.852	0.052	38.852	0.052	0.13%

4.1.9 Discussion

Partially measured systems are a true representation of real plant systems since not all process flows are measured, due to physical constraints or for economic reasons. However, process plants require complete process measurements for process control and performance monitoring. As a result, a data processing system capable of estimating values of the unmeasured parameters is necessary. Process data reconciliation problem was successfully applied to a partially measured system as shown above. The measurement adjustments shown in Figure 13 below indicate a significant adjustment of two streams, S0 and S7, with the other measured streams having little or no adjustment as a result of the reduced level of redundancy of the measurements. The measurements redundancy affects the accuracy of the reconciliation results as shown in Figure 13. Some measurements could not be adjusted to improve the accuracy of the reconciled values. The higher the level of redundancy in a process flow the more accurate the reconciled estimates. The solutions of the reconciled and estimated values of the unmeasured variables satisfy the model constraints equations shown below:

$$A_s\hat{s}+A_u\hat{u}=0 \tag{89}$$

The reconciliation problem is required to adjust process measurements and estimate unmeasured variables in process flows to satisfy process model constraints. To achieve this, the reconciliation algorithm is performed in an iterative loop until the measurements converge to values that will satisfy the constraints equations. The model constraints could not be satisfied with the first analytical calculation as it is for linear system with all measured variables.

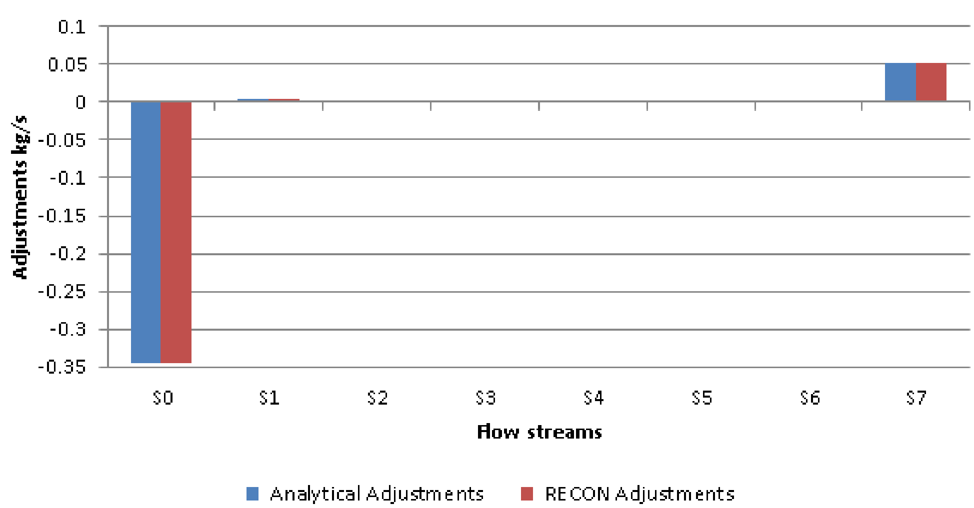

Figure 13: Measurement adjustments of a partially measured linear system

For the reconciliation to give estimates of unmeasured variables to satisfy the constraints, an iterative loop is applied to the DR algorithm to enable convergence of the measurements. The DR algorithm allows the repeat of the reconciliation problem calculation to achieve convergence. On the above process flow two (2) iterations were needed to achieve convergence. The DR algorithm is shown in Appendix A8. However, only the unmeasured variable estimates are affected. Similarly, the RECON software automatically runs two iterations to achieve convergence.

4.2 Nonlinear steady-state process data reconciliation models

Chemical processes as described by Narasimhan et al [20] contain flow streams with flow rates and compositions. As such the flow compositions are measured in the process. In a case when both the flow rate and composition measurements are reconciled the component balances are included in the constraints of the DR problem. Such constraints will contain component flow rate terms made up of the products of the flow rate and composition variables. These constraints are said to be bilinear, requiring a bilinear data reconciliation technique. Bilinear data reconciliation is used to refer to nonlinear problems with constraints resulting from a product of two variables.

Process plants are considered bilinear processes and are modelled as nonlinear systems. Mineral beneficiation is an example of a bilinear process where mineral compositions and flows are reconciled. In power generation plants, the reconciliation of flows and temperatures of energy flow subsystems are also bilinear problems if the specific enthalpy is only a function of temperature [20]. Therefore the mass and energy balance is bilinear; it consists of a product of two variables, mass flow rate and specific enthalpy, a function of temperature [21]. Bilinear problems are solved using bilinear or nonlinear data reconciliation [20].

In the following sections nonlinear data reconciliation is used to solve the bilinear systems. Two case studies of a general heat exchanger and steam generator are considered in the sections below to illustrate nonlinear DR problem in nonlinear steady state systems.

4.2.1 Heat exchanger with all variables measured

Madron et al [21] describe a general heat exchanger (HEX) as having the potential for more streams, unlike a simple heat exchanger that has only two streams and one node. A general HEX consists of two nodes, representing the hot stream and cold stream, connected by an energy stream representing the heat flow from the hot stream to the cold stream [21]. There are five flow streams, as shown in Figure 14, with two mass flow streams in each node and one heat flow connecting the two nodes. All the flow streams and temperatures are measured, except for the heat flow which is unmeasured; it is an observable variable which can be determined from the total mass flow rates and the enthalpies associated with the temperatures of the streams. The energy balance is calculated from the general equation of steady state flow systems [46] given by:

$$Q - W = \Sigma(mh)_{out} - \Sigma(mh)_{in} \tag{90}$$

Where the mass flowrate is given by m, the specific enthalpy h, energy supplied to the nodes Q and W is the work or power produced by the nodes. The energy or heat flow in the system is due to the temperature gradient between the hot and cold streams, but no work is produced in a heat exchanger, thus the general energy balance equation is reduced to:

$$Q = \Sigma(mh)_{out} - \Sigma(mh)_{in} \tag{91}$$

The process flow consists of three model constraint equations generated from the HEX model, two mass balance equations and an energy balance equation. The heat flow from the hot stream is determined using the enthalpy difference Δh across the streams. The enthalpy is calculated from the measured temperatures and the pressure of the system. The pressure in the HEX is assumed constant at atmospheric pressure $P_{atm} = 101.325 kPa$.

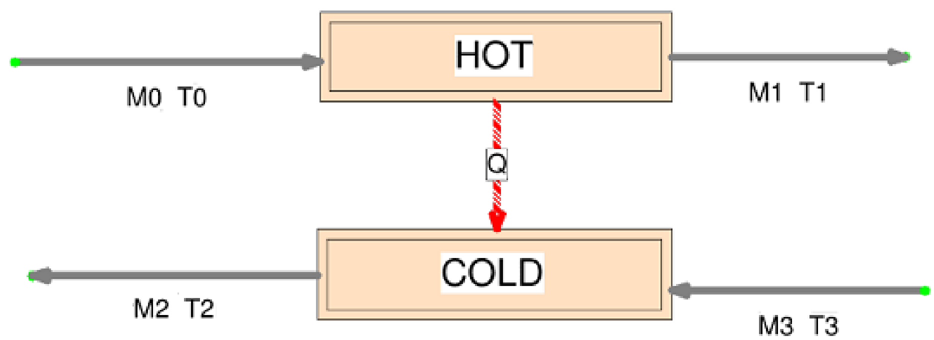

Figure 14: A nonlinear system of a HEX model with all variables measured

Since enthalpy is not a direct measured variable, it can be determined only from the measured temperature and pressure using the water and steam properties. The IAPWS-IF97 formulation for water and steam properties was used for the determination of the enthalpy of the fluid. The overall mass and energy balanced constraint equations for the HEX model is given by,

$$\begin{aligned} f_1 &= m_0 - m_1 \\ f_2 &= m_3 - m_2 \\ f_3 &= m_0(h_0 - h_1) - m_2(h_2 - h_3) \end{aligned} \tag{92}$$

The Jacobian matrix of the above system is shown below in terms of m and h;

$$A(m,h)=\begin{bmatrix} \frac{\partial f_1}{\partial m_0} & \frac{\partial f_1}{\partial m_1} & \frac{\partial f_1}{\partial m_2} & \frac{\partial f_1}{\partial m_3} & \frac{\partial f_1}{\partial h_0} & \frac{\partial f_1}{\partial h_1} & \frac{\partial f_1}{\partial h_2} & \frac{\partial f_1}{\partial h_3} \\ \frac{\partial f_2}{\partial m_0} & \frac{\partial f_2}{\partial m_1} & \frac{\partial f_2}{\partial m_2} & \frac{\partial f_2}{\partial m_3} & \frac{\partial f_2}{\partial h_0} & \frac{\partial f_2}{\partial h_1} & \frac{\partial f_2}{\partial h_2} & \frac{\partial f_2}{\partial h_3} \\ \frac{\partial f_3}{\partial m_0} & \frac{\partial f_3}{\partial m_1} & \frac{\partial f_3}{\partial m_2} & \frac{\partial f_3}{\partial m_3} & \frac{\partial f_3}{\partial h_0} & \frac{\partial f_3}{\partial h_1} & \frac{\partial f_3}{\partial h_2} & \frac{\partial f_3}{\partial h_3} \end{bmatrix} \tag{93}$$

Thus, the symbolic representation of the Jacobian matrix in equation(93) is given below as,

$$\mathrm{A}(\mathrm{m},\mathrm{h}) = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ h_0 - h_1 & 0 & h_3 - h_2 & 0 & m_0 & -m_0 & -m_2 & m_2 \end{pmatrix}$$

Figure 15: Symbolic Jacobian matrix of the HEX process model

The measured variables are represented by vector X , and the array of the measured variables is given below;

$$X = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} \tag{94}$$

Therefore the system of equations can be written in terms of variable x as;

$$\begin{matrix} x_0 & -x_1 & & & \\ & & -x_2 & +x_3 & =0 \\ x_0\left(x_4 - x_5\right) & & -x_2\left(x_6 - x_7\right) & & \end{matrix} \tag{95}$$

4.2.2 Input data

In power plants, the enthalpy of the steam plays a significant role in the energy balances however this thermodynamic property of steam cannot be measured directly. In order to determine enthalpy, other fluid properties are utilised [21]. Pressure and temperature of the streams indicated on the input

Table 5 below are used to calculate the enthalpy. The enthalpies associated with the flow streams are used as input data for the reconciliation procedure instead of the temperature for the analytical model. The enthalpy variation or measurement error is determined using the temperature variations. The enthalpy and its variation are calculated analytically by the following equations respectively:

$$h = h_{@T} \tag{96}$$

$$\Delta h = h_{@T+\Delta T} - h_{@T} \tag{97}$$

Note that the calculated enthalpies and enthalpy variations are used as input only for the analytical calculation. RECON calculates the enthalpy from the programmed water and steam properties IAPWS IF-97 using the thermodynamic state of the fluid [21].

The process flow is shown in matrix notation as:

$$f(x) = Ax = 0$$

The incidence matrix A evaluated using the input data in

Table ***5*** is shown in Figure 16 below;

Table 5: HEX process model input data

Stream	Measured	Maximum error
Flow rates (kg/s)		
m0	150.89	4.50
m1	153.15	3.00
m2	460.72	6.80
m3	450.31	5.70
Temperature (^{o}C)		
T0	90.90	1.50
T1	45.50	1.20
T2	44.90	1.80
T3	30.00	1.10
Enthalpy (kJ/kg)		
h0	380.777	6.31
h1	190.607	5.015
h2	188.099	7.522
h3	125.834	4.598

$$A_X = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 190.171 & 0 & -62.266 & 0 & 150.89 & -150.89 & -460.72 & 460.72 \end{pmatrix}$$

Figure 16: Incidence matrix of the HEX process model

As expected the process measurements given on the above table do not satisfy the constraint equations.

$$f(x) = \begin{pmatrix} -2.26 \\ -10.41 \\ 7.771 \end{pmatrix}$$

4.2.3 Gross error detection by GT

A GT strategy is applied on the measurement variables of vector X to test for gross errors. The process flow measurements contain random errors in accordance with definition. However a gross error of -15kg/s is introduced in measurement x_3 as a result of the flow meter fouling or clogging.

The critical value of the above process flow as given by equation(69) is read off from the Chi-squared distribution table in Appendix A1 [33] with significance level of 95% and the associated degree of freedom of 3. Therefore for $\alpha = 0.05$ and $v = 3$, the critical value is $\chi^2_{0.05,3} = 7.815$.

The measurement covariance matrix is determined from equation(66) is given below;

$$V = \begin{pmatrix} 29.244 & 0 & 3850.17 \\ 0 & 78.733 & 2879.433 \\ 3850.17 & 2879.433 & 18887659.519 \end{pmatrix}$$

Figure 17: Covariance matrix of process flow with measured streams

Therefore the GT calculated from equation(67) for measurement vectors X without GE and X_{GE} with gross error respectively,

$$\gamma_X := (A_X \cdot x)^T V^{-1} (A_X \cdot x) = 1.577$$

$$\gamma_{GE} := (A_X \cdot x_{GE})^T V^{-1} (A_X \cdot x_{GE}) = 8.46$$

According to the above test, the GT detects gross error in measurement vector X_{GE}, since the value of the test is greater than the critical value of the process flow. Also, due to the limitation of the GT, the test is unable to locate or identify which measurement or instrument contains a gross error and also is unable to give the magnitude of the gross error.

4.2.4 HEX model reconciliation results

The reconciled values and the measurement adjustments from the analytical and RECON solutions of the HEX model are presented in Appendix A2, with the analytical algorithm given in Appendix A9. The results are the same for both systems. The reconciled mass flowrates and enthalpies satisfy the mass and energy balance constraint equations.

$$A_X \cdot x' = \begin{pmatrix} 0 \\ -0 \\ -0 \end{pmatrix}$$

The heat flow between the hot and the cold streams calculated from the reconciled enthalpies of the flows using equation(91) is given below. The analytical and RECON heat flow calculations are the same, as shown by calculations;

$$Q_{hot} = Q_{cold} = m_0\left(h_0 - h_1\right) = 28.927MW\Big|_{ANALYTICAL} \quad (98)$$

$$Q_{hot} = Q_{cold} = m_0\left(h_0 - h_1\right) = 28.927MW\Big|_{RECON} \quad (99)$$

Equations(98) and (99) are the energy balance calculated from the analytical and RECON solutions respectively. The reconciled temperatures are then calculated using the reconciled enthalpies, applying the water and steam properties, also shown on the table in Appendix A2.

4.2.5 Discussion

Process data reconciliation was successfully applied in a HEX model with all measured variables. The mass and energy balance constraints are also satisfied. The measurements adjustments calculated between the measured variables and reconciled values are shown below. The adjustments on the flowrates are exactly the same in both analytical and RECON as shown in Figure 18. It should be noted that the considerable imbalance of the cold stream resulted in large adjustments between flow streams m_2 and m_3.

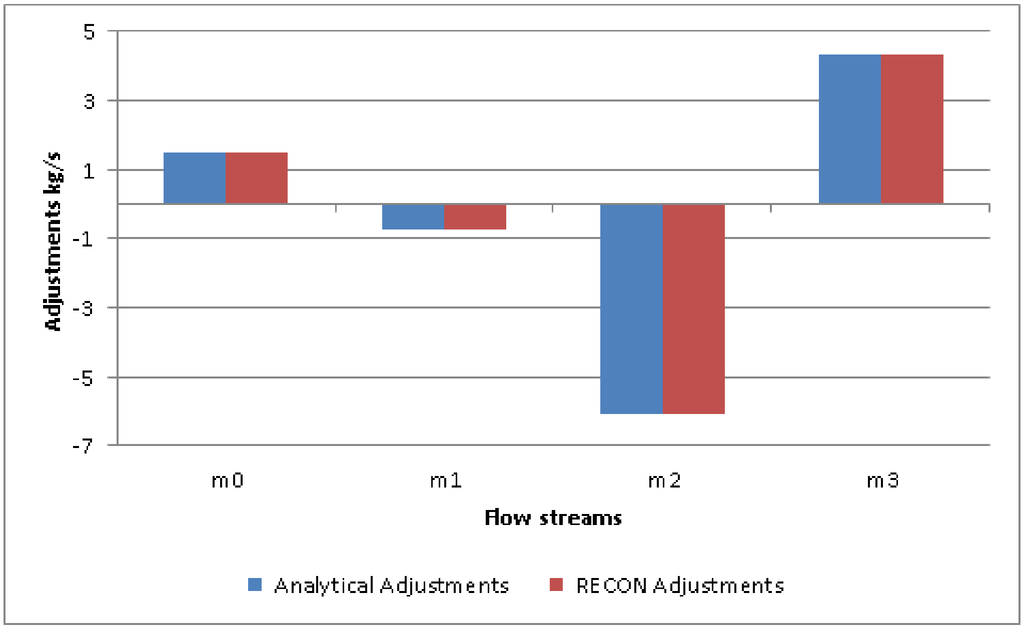

Figure 18: HEX model flow adjustments

The temperature adjustments indicate significant adjustments for both analytical and RECON as shown in Figure 19 and are both similar. The small differences in the reconciled temperatures have very little impact on either the enthalpies or the energy balance results. Therefore, the solutions from the analytical and RECON are acceptable and validate the application of process data reconciliation on nonlinear heat exchanger systems to improve process data.

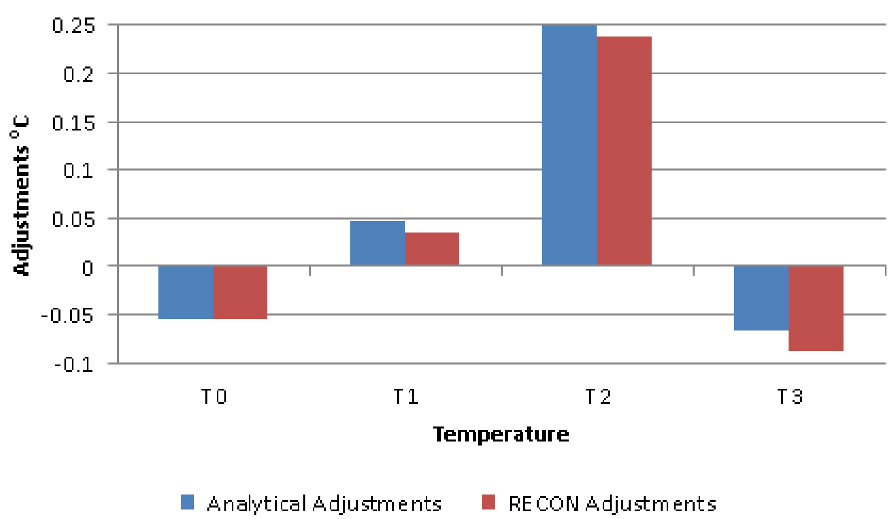

Figure 19: HEX model temperature adjustments

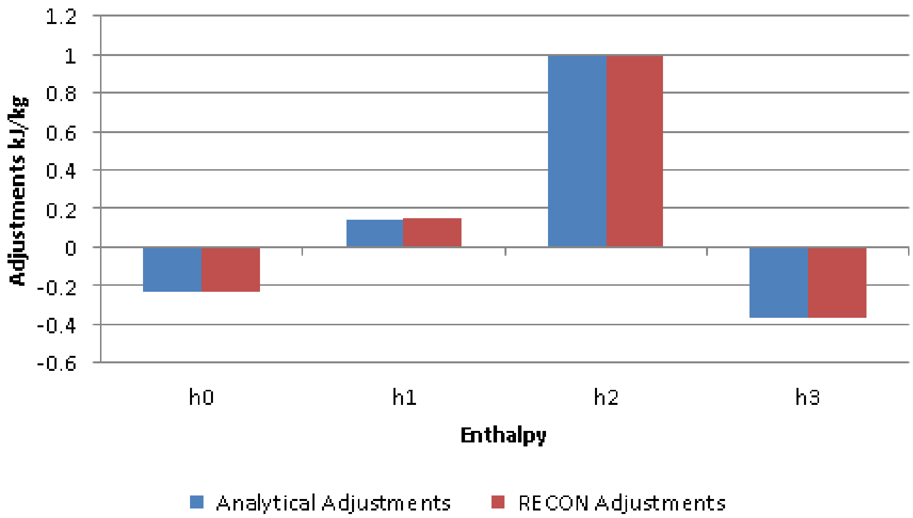

Figure 20: HEX model enthalpy adjustments

4.2.6 Steam generator flow with partially measured variables

A steam generator has been chosen to illustrate the application of data reconciliation for a partially measured nonlinear system and also to apply phase equilibrium on the water-steam in the steam generator. The SG model is shown in Figure 21 below with unmeasured flows represented by the dashed streams. The SG is assumed to be in equilibrium with the pressure expressed as a function of temperature, taking temperature as the measured variable. The pressure will be computed using the measured temperature T_{SG} of the steam generator. The feedwater and the outlet of the reactor coolant (RC) are the other unmeasured quantities. All the unmeasured variables in the SG flowsheet are observable. The heat flow between the two streams is determined from the reconciled enthalpy differences of the flow streams. The heat flow between the two nodes of the SG is equivalent to the RTP. The mass and energy balanced constraint equations are given below;

$$
\begin{aligned}
f_1 &= m_0 - m_1 - m_2 \\
f_2 &= m_3 - m_4 \\
f_1 &= (m_1 h_1 + m_2 h_2 - m_0 h_0) - m_3 (h_3 - h_4)
\end{aligned}
\tag{100}
$$

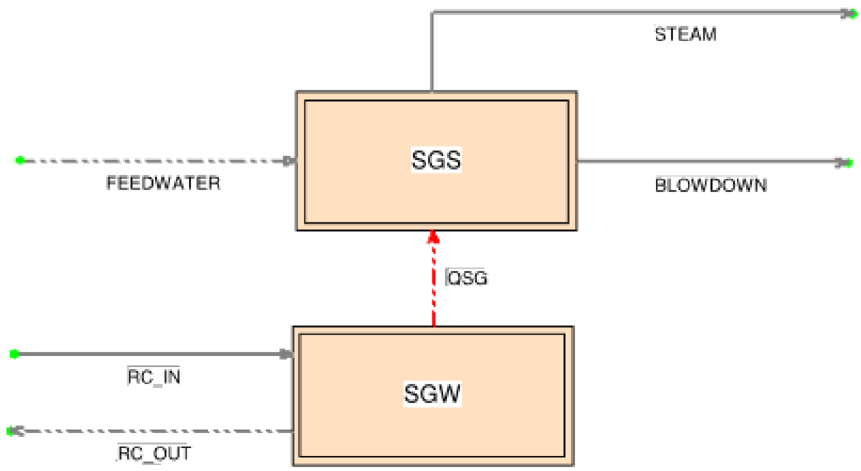

Figure 21: A nonlinear system of SG model with partially measured variables

The Jacobian matrix of the above system is shown below in terms of m and h respectively,

$$A(m,h)=\begin{bmatrix} \frac{\partial f_1}{\partial m_0} & \frac{\partial f_1}{\partial m_1} & \frac{\partial f_1}{\partial m_2} & \frac{\partial f_1}{\partial m_3} & \frac{\partial f_1}{\partial m_4} & \frac{\partial f_1}{\partial h_0} & \frac{\partial f_1}{\partial h_1} & \frac{\partial f_1}{\partial h_2} & \frac{\partial f_1}{\partial h_3} & \frac{\partial f_1}{\partial h_4} \\ \frac{\partial f_2}{\partial m_0} & \frac{\partial f_2}{\partial m_1} & \frac{\partial f_2}{\partial m_2} & \frac{\partial f_2}{\partial m_3} & \frac{\partial f_2}{\partial m_4} & \frac{\partial f_2}{\partial h_0} & \frac{\partial f_2}{\partial h_1} & \frac{\partial f_2}{\partial h_2} & \frac{\partial f_2}{\partial h_3} & \frac{\partial f_2}{\partial h_4} \\ \frac{\partial f_3}{\partial m_0} & \frac{\partial f_3}{\partial m_1} & \frac{\partial f_3}{\partial m_2} & \frac{\partial f_3}{\partial m_3} & \frac{\partial f_3}{\partial m_4} & \frac{\partial f_3}{\partial h_0} & \frac{\partial f_3}{\partial h_1} & \frac{\partial f_3}{\partial h_2} & \frac{\partial f_3}{\partial h_3} & \frac{\partial f_3}{\partial h_4} \end{bmatrix} \tag{101}$$

Thus, the symbolic representation of the Jacobian matrix of the SG model in equation(101) is given below as;

$$A(m,h) = \begin{pmatrix} 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ -h_0 & h_1 & h_2 & h_4 - h_3 & 0 & -m_0 & m_1 & m_2 & -m_3 & m_3 \end{pmatrix}$$

Figure 22: Symbolic Jacobian matrix of the SG model

The SG model's measured and unmeasured variables are represented by vectors X and U respectively as shown in the arrays below;

$$X = \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix} \quad \text{and} \quad U = \begin{pmatrix} u_0 \\ u_1 \end{pmatrix}$$

Therefore the system of equations can be written in terms of variables x and u as;

$$\begin{array}{llllll} u_0 & -x_0 & -x_1 & & & \\ & & & x_2 & & -u_1 = 0 \\ & \left(x_0 x_4 + x_1 x_5 - u_0 x_3\right) & & -x_2\left(x_6 - x_7\right) & & \end{array} \tag{102}$$

4.2.7 Input data

The table below contains the input data of the SG process model. The temperature measurements of the flow streams are used to calculate the enthalpy and enthalpy variations with respect to temperature using equations(96) and (97) respectively. The reconciliation problem is simplified by eliminating the unmeasured variables through the constructing the projection matrix. The process flow constraints equation is shown in matrix notation as in equation(56):

$$f\left(x,u\right) = A_x x + A_u u - b = 0 \tag{103}$$

The incidence matrices A_x and A_u representing the measured and unmeasured variables are given by Figure 23 and Figure 24 respectively:

$$A_X = \begin{pmatrix} -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1185.093 & 2789.69 & -202.71 & -1500 & 1510.01 & 10.95 & -13738.868 & 13738.868 \end{pmatrix}$$

Figure 23: Incidence matrix of the measured variables

$$A_u = \begin{pmatrix} 1 & 0 \\ 0 & -1 \\ -946.731 & 0 \end{pmatrix}$$

Figure 24: Incidence matrix of the unmeasured variables

Table 6: Input data of a partially measured SG model

Stream	Measured	Maximum error
Flow rates (kg/s)		
Feedwater	Unmeasured	-
Blowdown	10.95	0.055
Steam	1510.01	0.53
RC_in	13738.868	2.748
RC_out	Unmeasured	-
Temperature (^{o}C)		
Tfw	220.45	1.000
Tsg	270.00	1.000
T_in	316.50	1.000
T_out	279.30	1.000
Enthalpy (kJ/kg)		
hfw	946.731	4.589
hbd	1185.093	5.095
hst	2789.690	0.849
h_in	1431.809	6.014
h_out	1229.099	5.072

The decomposed A_u matrix results in matrix $M = QR$ which consists of sub-matrices Q and R respectively. The constraints equations of the SG are not balanced as shown below, thus nonlinear DR problem will be applied to give a solution of reconciled values and estimates of unmeasured variables.

$$f(x, u) = \begin{pmatrix} -20.96 \\ 738.868 \\ 20339.421 \end{pmatrix}$$

4.2.8 Results of a partially measured SG model

The reconciled results and the measurement adjustments of a partially measured nonlinear SG model are shown in Appendix A3, and the analytical algorithm in Appendix A10. The solutions of the reconciled values satisfy the constraints equations. The energy balances are calculated using the reconciled enthalpies and the flowrates from equation(91) and are given below for both the analytical and RECON solutions:

$$\left| A_x \cdot x' + A_u \cdot u' - b \right| = 0$$

The computed analytical energy balance is given by:

$$Q_{SGW} = Q_{SGS} = 2785.491MW\Big|_{ANALYTICAL} \tag{104}$$

In addition, the computed RECON energy balance for the SGW and SGS respectively are given below. Therefore the solutions produced by RECON and analytical are valid and acceptable since they have a very small difference between them:

$$Q_{SGW} = 2785.469MW\Big|_{RECON} \tag{105}$$

$$Q_{SGS} = 2785.492MW\Big|_{RECON} \tag{106}$$

4.2.9 Discussion

The reconciliation of a partially measured SG model using analytical method and RECON produced similar results for the reconciled and unmeasured estimated values. The reconciled values of flows and enthalpies together with the estimated values satisfy the mass and energy balance constraint equations. The results show a slight adjustment of the steam flowrate, with other flowrates not adjusted shown in Figure 25. However, the enthalpy and temperature adjustments indicate small adjustments of the reactor coolant temperatures and the enthalpies for the RECON solution. The analytical solution indicates small adjustments of the feedwater temperature and reactor coolant temperature. However the enthalpy variations are adjusted for the coolant to temperatures as shown in Figure 26.

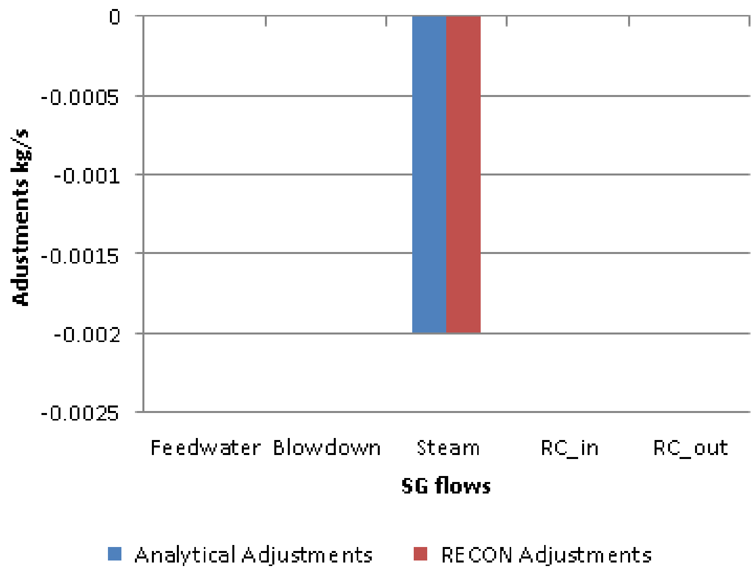

Figure 25: SG model flow adjustments

The computation of the SG pressure produces the same pressure using the analytical solution and the estimated RECON results. The SG pressure is determined in RECON by adding the equilibrium equation, referred to as user-defined equation (UDE) [21]. The number of constraint equations is thus increased, as well as the degree of redundancy of the reconciliation problem. As the degree of redundancy of process flow is increased, improved reconciled values are obtained.

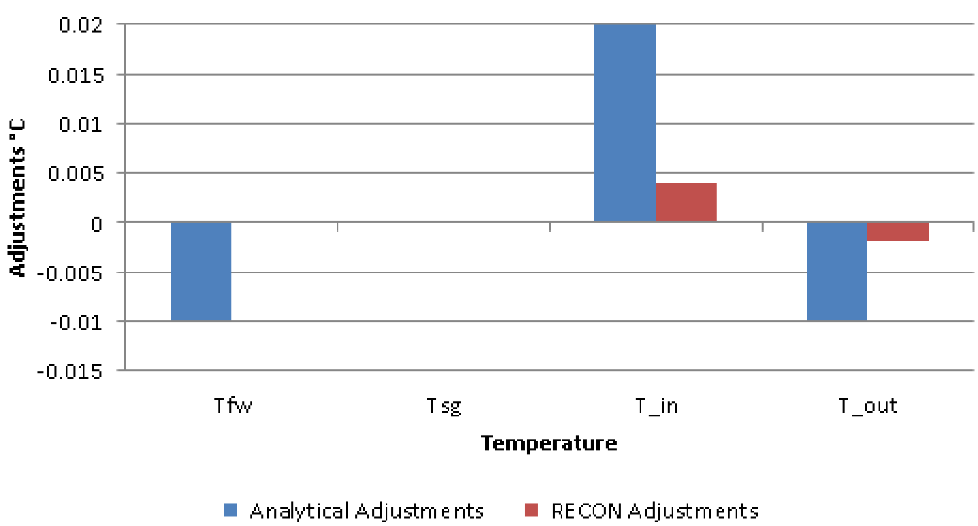

Figure 26: SG model temperature adjustments

Analytically, the reconciled enthalpy of the steam is used to compute the pressure in the SG. The SG pressure P_{SG} is computed analytically from the SG temperature T_{SG} and the reconciled enthalpy of the steam:

$$P_{SG} = P_{steam}\left(\mathrm{T}_{SG}\right) = 5502.839kPa \tag{107}$$

The reconciled SG pressure from the RECON solution is $P_{SG} = 5502.844kPa$. The results of the two systems are consistent. However the analytical and RECON reconciliation solutions produce the same results and validate DR as a useful tool in NPP to determine the reactor thermal power.

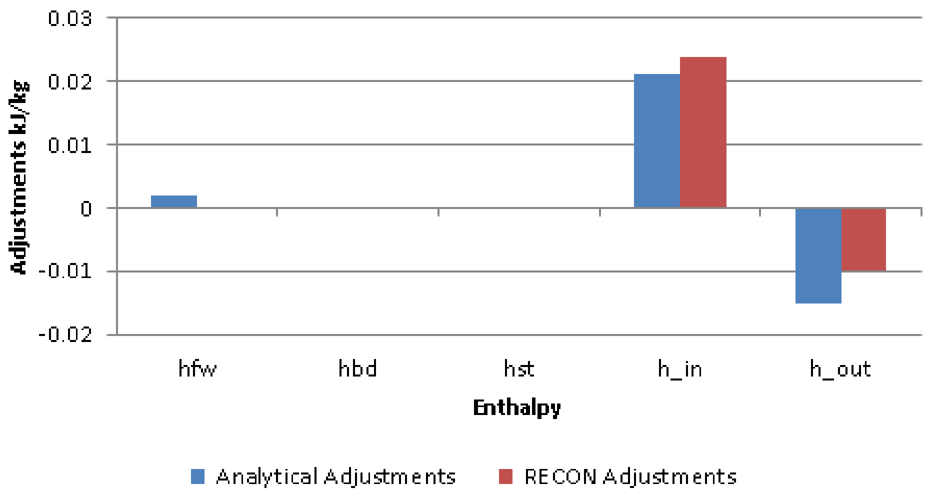

Figure 27: SG model enthalpy adjustments

4.3 Nuclear power plant feedwater flow measurements model

The study focuses on the development of a steady state feedwater flow measurements model of a PWR nuclear power plant. The power plant consists of two nuclear reactors with a rated reactor thermal power of $2775MWth$ [47] each, and three steam generators per reactor. In addition, the rated thermal power reactor circulation pumps contribute about $10MWth$ to the total thermal power of the reactor. The nuclear steam supply system (NSSS) has rated power of $2785MWth$ [48]. The reactors were commercially operated in 1984 and 1985 respectively.

In order to build the feedwater flow measurements model, plant data was collected from the distributed control systems of the power plant through an Industrial SQL (InSQL) server (database). The InSQL is an on-line system used for plant performance monitoring which collects real-time plant data from the units, taking into consideration only measurements from the feedwater flow systems. Statistical theory was used to analyse the plant data collected to determine the mean values and the standard deviations of the measurements. It was later realised that the data collected were insufficient because only a few measurements of the feedwater system are available in the InSQL server. It was found that other measurements of the feedwater and the condensate systems are locally measured on the plant, but not transmitted online.

As previously mentioned, performing data reconciliation requires sufficient level of measurement redundancy with observable unmeasured variables. The calculated mean values of the collected data are shown in Table *8* which contains only a few values of the feedwater flow measurements. Since only a few measurements were available the feedwater model was deemed unsolvable and it was impossible to perform data reconciliation. For the sake of running the case study, measurements taken from the heat balance diagrams (HBD) of the power plant were used to complete the model [47], [49].

The HBD of a thermal power plant is a schematic representation of the water-steam cycle. It contains information about the water-steam properties such as pressure, temperature, enthalpy and mass flowrate of the cycle. Although the information documented on these HBDs is used for determining the plant design specification, it provides sufficient information to complete the model and to perform the reconciliation. Information from both plant and HBDs was combined to complete the system model, the HBD information having been used to supplement the plant data.

The system process description in Table 7 defines the information as indicated on the simplified process flow diagram. The estimated mean value of the SG heat flow from the plant data is $916.10 MWth$ per SG, giving a total of $2748.3 MWth$ per unit. This value is less than the design NSSS-rated value of $2785 MWth$ per unit.

Table 7: System process flow description

System Process Flow Description			
F0	Condensate inlet 200RE preheaters	F13	Feedwater pump discharge
F1	200RE preheaters condensate outlet	F14	500RE preheaters feedwater outlet
F2	200RE preheaters steam inlet	F15	500RE preheaters steam inlet
F3	Normal drains from 200RE preheaters	F16	Normal drains from 500RE preheaters
F4	300RE preheaters condensate outlet	F17	Normal drains from 600RE preheaters
F5	300RE preheaters steam inlet	F18	Reheater drains
F6	Normal drains from 300RE preheaters	F19	Main steam valves drains
F7	400RE preheaters steam inlet	F20	600RE preheaters steam inlet
F8	Normal drains from 400RE preheaters	F21	Feedwater flow in steam generator
F9	400RE preheaters condensate outlet	F22	Blowdown flow
F10	Drains tank condensate outlet	F23	Steam flow
F11	Moisture separator drains	F24	Reactor vessel Hot water hot leg inlet flow
F12	Feedwater pump suction header	F25	Reactor vessel Hot water cold leg outlet flow

Table 8: NPP feedwater flow measurements

	Plant Data from InSQL						
Stream	Flow rate (kg/s)	Temperature (°C)	Pressure (kPa)	Stream	Flow rate (kg/s)	Temperature (°C)	Pressure (kPa)
F0	1078.26	-	4777.80	F13	-	-	5992.34
F1	-	92.99	-	F14	-	-	-
F2	-	95.73	-	F15	-	203.30	-
F3	-	-	-	F16	-	183.24	-
F4	-	134.43	-	F17	-	-	-
F5	-	137.98	329.61	F18	-	-	-
F6	-	-	-	F19	-	-	-
F7	-	184.87	-	F20	-	223.54	-
F8	155.27	185.11	-	F21	1508.10	220.45	5293.38
F9	-	180.94	-	F22	10.96	-	-
F10	-	-	-	F23	-	-	-
F11	-	-	-	F24	-	313.08	15408.38
F12	-	-	-	F25	-	279.02	-

The supplementary information from the HBDs is provided in Appendix A4. The HB diagrams were also unable to provide complete data as required for the feedwater flow system model. A reasonable combination of the feedwater measurements was achieved to complete the system model and generate sufficient redundancy to performance the reconciliation. The combined data are used as the input data for reconciliation for both analytical and RECON computations.

4.3.1 PWR nuclear power plant feedwater system description

The feedwater and condensate systems form part of the water-steam cycle of the secondary loop of the PWR nuclear plant. The combined feedwater and condensate system consists of the condensate system starting from the condenser outlet, including the low pressure feedwater heaters to the feedwater pumps, and the feedwater system starting from the discharge of the feedwater pumps through to the high pressure feedwater heaters up to the inlet of the SGs. The SGs form part of the primary loop, consisting of the pressure (reactor) vessel and its entire associated components. A simplified PWR process flow diagram is shown in Figure 28 for the purpose of the case study.

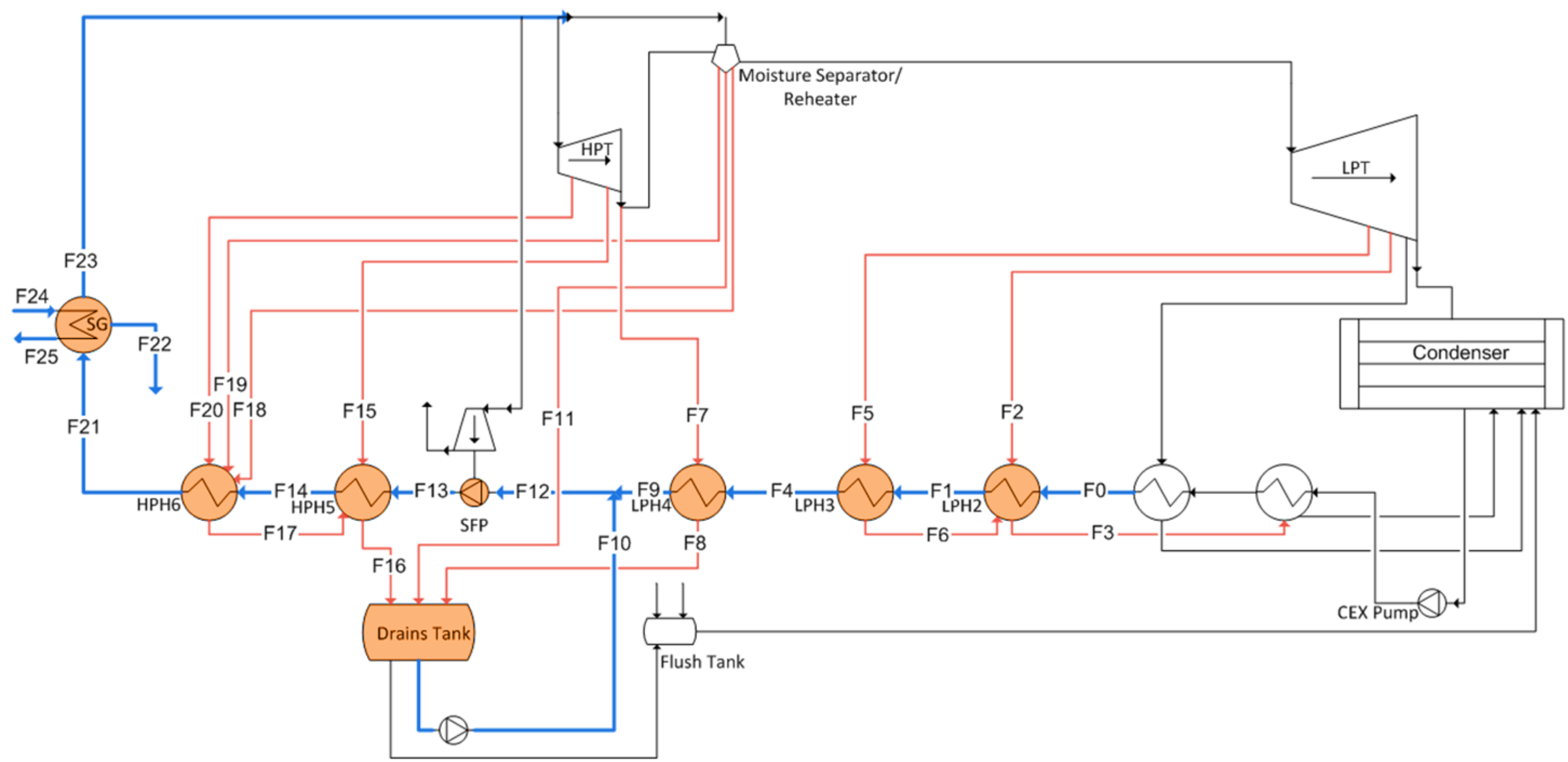

Figure 28: A simplified PWR nuclear power plant process flow diagram

4.3.2 Input data

The combined process measurements data in Table 9 provide a better and more realistic set of input data for the feedwater flow measurements system model. The process measurements could not be analysed using statistical methods to determine the standard deviations and the mean values because of lack of measurements. Therefore measurements are used as they appear on the HB diagrams. The maximum errors of the measured variables are assumed to be the weighting of the measurements. The maximum error is expressed as a percentage of the measured value at plant full capacity.

A typical orifice plate designed and manufactured in accordance with a standard for differential-pressure meters, ISO 5167, is expected to have a flowrate uncertainty that is approximately ±1% or less when operating at the maximum flowrate under ideal conditions [5], [50], [51]. The choice of the maximum error for the mass flowrates is based on the orifice plate error of ±1% with only a few measurements indicated in bold in Table 9 where the error is assumed to be either below 1% or above the maximum error.

The feedwater temperature measurements in a typical power plant have a measurement range of $0^{o}C$ - $300^{o}C$. For such a temperature range a thermocouple Type T is suitable. This thermocouple has two distinct temperature ranges and error limits. The thermocouple temperature range $-200^{o}C$ - $0^{o}C$ has a standard error limit of $\pm 1^{o}$ or ±1.5%, and that of $0^{o}C$ - $350^{o}C$ has a standard error limit of $\pm 1^{o}$ or ±0.75% [52], [53]. The standard error limit used for all temperature measurements set at $\pm 1^{o}C$; the enthalpy variations are computed based on the temperature variations.

The pressure vessel parameters are similar to those used for the SG problem. The system model is assumed to have three unmeasured and observable flows to be estimated through data reconciliation procedure. The analytical computation of the feedwater flow measurements model has a total of 21 model constraints equations, including nonlinear constraints for energy balance. The model has 44 measured flows and temperatures, 3 unmeasured flows and 9 pressure measurements. As in the previous HEX and SG models the enthalpies of the flows are used as input data for the reconciliation instead of the temperatures. The degree of redundancy of the model, calculated from equation(2) is 18. To simplify the analytical problem all the heat flows through the feedwater heaters are not calculated; only the RTP of the system is determined using the reconciled values. Note that for a complete model on RECON the energy flows through the SG and feedwater heaters are set as unmeasured parameters.

Table 9: Feedwater flow measurements system input data

Stream	Measured Flow (kg/s)	Maximum Error (kg/s)	Measured Temperature (^oC)	Pressure (kPa)
F0	1053.00	10.53	52.00	4004.05
F1	1052.76	10.528	92.99	-
F2	65.87	0.659	95.73	93.20
F3	Unmeasured	-	61.90	-
F4	1053.00	10.53	133.95	-
F5	78.09	0.781	137.98	329.61
F6	78.10	0.781	103.80	-
F7	121.90	1.219	184.87	1092.00
F8	122.00	1.22	184.90	-
F9	1053.00	**15.795**	180.94	-
F10	469.85	**3.289**	183.74	-
F11	115.20	1.152	-	-
F12	Unmeasured	-	183.10	-
F13	1522.00	**10.654**	183.60	5992.34
F14	1521.07	15.211	201.70	-
F15	58.00	0.58	203.30	1673.00
F16	232.61	2.326	188.45	-
F17	176.71	1.767	210.55	-
F18	56.65	0.567	260.40	4711.50
F19	55.98	0.56	-	-
F20	64.00	0.64	223.54	2583.95
F21	1521.54	**7.608**	220.45	-
F22	10.95	0.110	255.30	-
F23	1510.95	7.555	270.00	5502.00
F24	13738.868	**68.694**	316.50	15408.38
F25	Unmeasured	-	279.30	-

The analytical construction of the incidence matrices of the measured and unmeasured variables for the feedwater model consists of large and complex matrices. The incidence matrix A_x of the measured variables consists of 21 rows and 44 columns. The A_u matrix consists of 21 rows and 3 columns before decomposition and results in a M matrix with 21 rows and 24 columns after matrix decomposition as shown in Appendix A12. The construction of the projection matrix is more complicated than it is for simpler and smaller systems. The matrix $M = QR$ is large and a difficult matrix to handle manually without the aid of a mathematical tool such as Mathcad. The decomposed matrix is crucial in the computation of the reconciliation algorithm and estimates of the unmeasured variables.

The feedwater flow measurement process model constraint equations are given in Figure 29 below in terms of m and h:

$$f(m,h) := \begin{bmatrix} m_0 - m_1 \\ m_2 + m_6 - m_3 \\ m_2(h_2 - h_3) + m_6 \cdot (h_6 - h_3) - m_0 \cdot (h_1 - h_0) \\ m_1 - m_4 \\ m_5 - m_6 \\ m_5(h_5 - h_6) - m_4(h_4 - h_1) \\ m_4 - m_9 \\ m_7 - m_8 \\ m_7(h_7 - h_8) - m_4(h_9 - h_4) \\ m_9 + m_{10} - m_{12} \\ m_8 + m_{11} + m_{16} - m_{10} \\ m_{12} - m_{13} \\ m_{13} - m_{14} \\ m_{15} - m_{16} + m_{17} \\ m_{15}(h_{15} - h_{16}) + m_{17} \cdot (h_{17} - h_{16}) - m_{14}(h_{14} - h_{13}) \\ m_{14} - m_{21} \\ m_{18} + m_{19} + m_{20} - m_{17} \\ m_{20}(h_{20} - h_{17}) + m_{19}(h_{19} - h_{17}) + m_{18}(h_{18} - h_{17}) - m_{21}(h_{21} - h_{14}) \\ m_{21} - m_{22} - m_{23} \\ m_{24} - m_{25} \\ (m_{23} \cdot h_{23} + m_{22} \cdot h_{22} - m_{21} \cdot h_{21}) - m_{24}(h_{24} - h_{25}) \end{bmatrix}$$

Figure 29: PWR feedwater flow measurement process model constraints equations

4.3.3 Gross error detection by GT

Similarly, GT strategy is applied on the measurement vector X to test for gross error for the feedwater flow measurements. Due to the high measurement variables, the test is conducted without the introduction of a gross error. The critical value of the above process flow as given by equation(69) is read off from the Chi-squared distribution table in Appendix A1 [33] with significance level of 95% and the associated degree of freedom of 18. Therefore for $\alpha = 0.05$ and $v = 18$, the critical value is $\chi^2_{0.05,18} = 28.869$.

$$\gamma_x := \left(A_x \cdot x\right)^T V^{-1}\left(A_x \cdot x\right) = 250940.033$$

The large value of the test indicates the presence of a gross error in the measurement variables. This may be as a result of large measurement variables or that the variables contained gross errors since they are taken from various HBDs of the power plant as a result of limited real plant measurements in the plant's online DCS.

4.3.4 Reconciliation results

The reconciled and unmeasured values of the analytical and RECON solutions for the feedwater flow measurements model are shown in Appendix A5. Whereas Appendices A6 and A5 give both the temperature and enthalpy results for analytical and RECON respectively. The advantage with the RECON results, gives the reconciled pressure and all the heat flows through the feedwater heaters, which was not done with the analytical solution. In the analytical solution, the temperature values are reconciled the used for the computation of the enthalpy values. The unmeasured variables in the RECON computation were increased from 3 variables to 10, an increase caused by the heat flows through the SG, 5 feedwater heaters, extra temperature and pressure. This increases the constraint equations to 27 with the degree of redundancy reduced to 17.

The energy balances for the two analyses are computed from the reconciled data. The computed analytical energy balance is given by;

$$Q_{SGW} = Q_{SGS} = 2793.199MW \,\Big|_{ANALYTICAL} \qquad (108)$$

The computed RECON energy balance for the SGW and SGS respectively are given below:

$$Q_{SGW} = 2786.871MW \,\Big|_{RECON} \qquad (109)$$

$$Q_{SGS} = 2786.105MW \,\Big|_{RECON} \qquad (110)$$

The difference between the analytical and RECON energy balances is just over $7MW$. Although there are small differences in the results due to the data used and the way RECON is programmed to handle data reconciliation problem, the outcome is considered valid and acceptable.

4.3.5 Discussion

The application of process data reconciliation in feedwater flow measurements of a power plant was successfully demonstrated using PWR NPP flow measurements. Although the measurements used in the model were taken from two different HBDs combined with plant data. The analytical model required 2 iterations for the analytical model to get closer to realistic and acceptable estimates of the reconciled values. The RECON model was completed with 6 iterations to produce acceptable reconciled values. The results as indicated on Appendices A5 through to A5 are consistent but show small differences between the analytical and RECON solutions.

The measurements adjustments of the feedwater flow streams are similar. The temperature adjustment of the analytical solution indicates 3 large variations which can be attributed to the unpredictable data used in this case. However, the adjustments on the enthalpies indicate large variations as a result of the irregular measurements used for completing the feedwater flow model. A comparison of the two analyses is graphically represented on Figure 30 showing the feedwater flow adjustments, Figure 31 shows the temperature adjustments and Figure 32 showing the enthalpy adjustments.

Despite the imperfect and unreliable measurement data used in this case study, the goal of applying DR in power plant was achieved with very little errors in the reconciliation solutions. Data reconciliation was performed successfully in feedwater flow measurements as shown in the results given in Appendices A5 through to A5 using analytical method and RECON software. It is very important that reliable process measurements are used in data processing to achieve reliable and accurate outcomes. It can be noted from the above results that large adjustments were made even though very small measurement deviations were used. Inconsistent data will adversely impact the reconciliation solutions. In large systems, performing analytical computations like the feedwater flow measurements becomes more cumbersome as the system becomes large and complex. Thus the use of DR software is crucial in large process plants to ensure reliable outcomes of DR are obtained.

Insufficient process measurements become a hindrance when implementing process data reconciliation in process plants. Instrumentation placement also plays a critical role in process plants design to achieve a high number of measured variables and eliminate unobservable variables. However it must be noted that it is not practical to have more instruments in a process plant due to economical and plant physical constraints. In order for unmeasured variables to be estimated with ease they should be located so that their values can be estimated without the need to install another instrument.

It is imperative that process data have measurements redundancy which aid in performing reconciliation and the estimation of unmeasured variables. Therefore measurement weighting or the variance and redundancy of the measurements are vital in the performance of data reconciliation. When the redundancy of a process flow is reduced, the reconciliation procedure results in estimates that are less accurate and unreliable which impact the determination of reactor thermal power in NPP operation and also affect process performance monitoring and control.

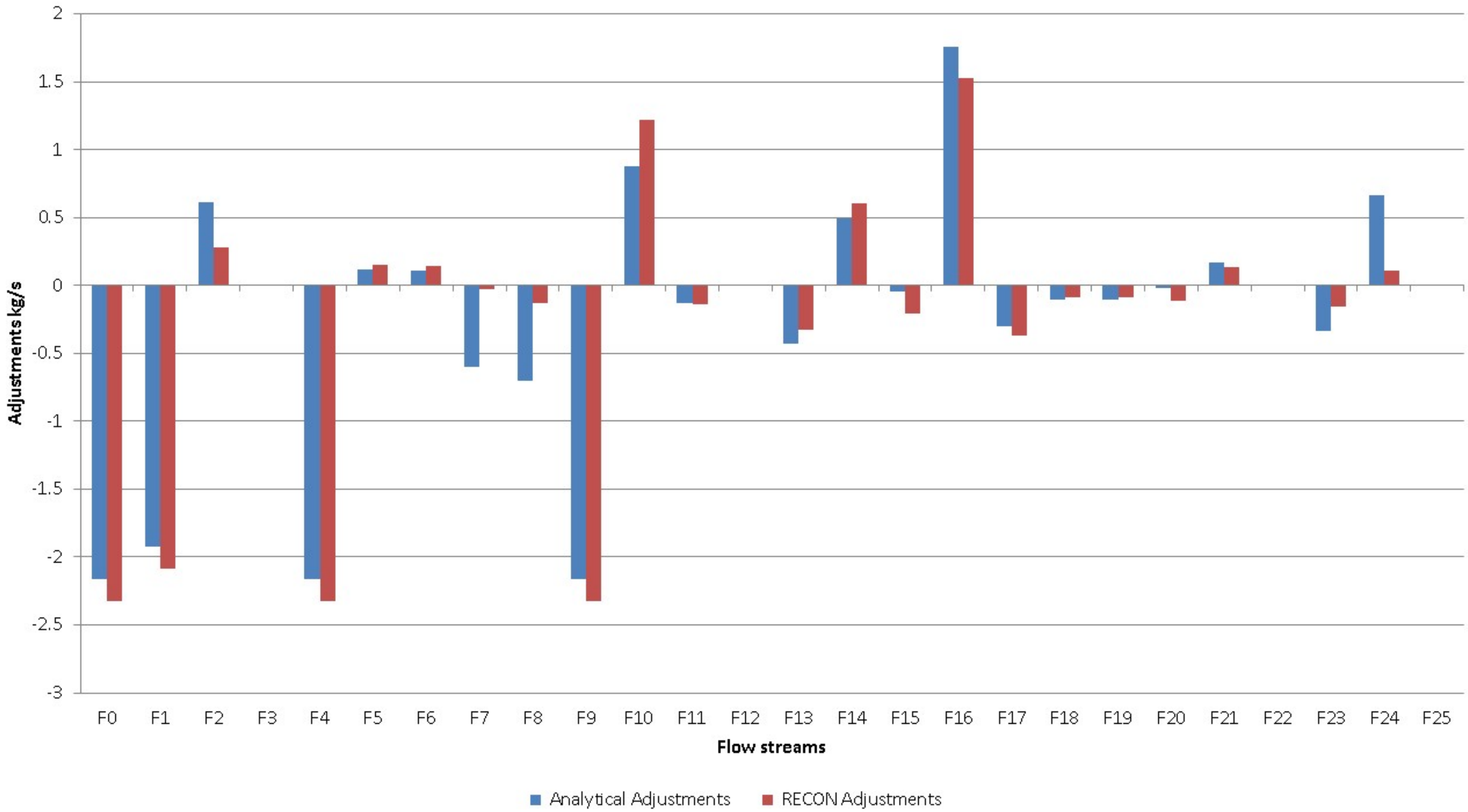

Figure 30: Adjustments of the feedwater flow measurements

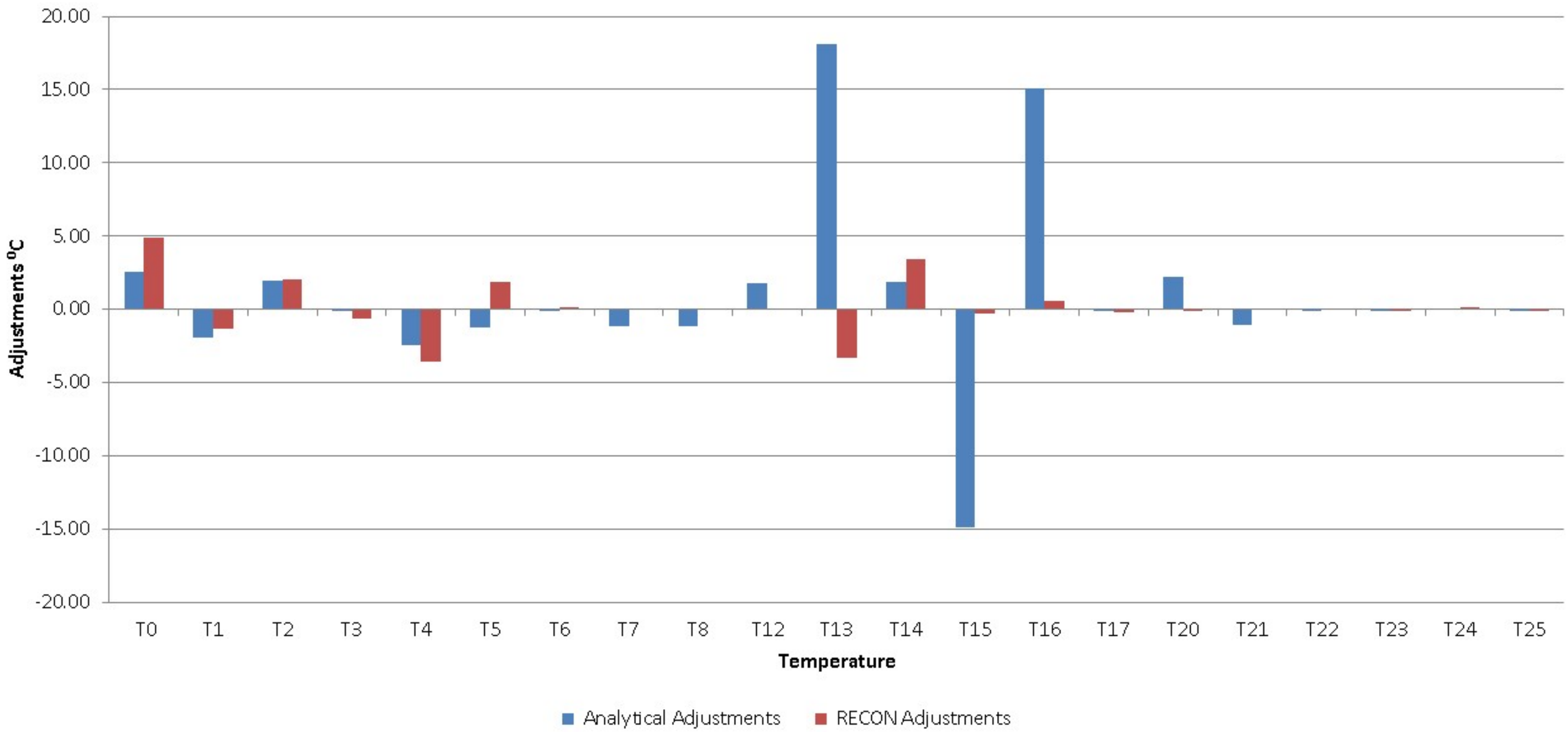

Figure 31: Adjustments of the feedwater flow temperatures

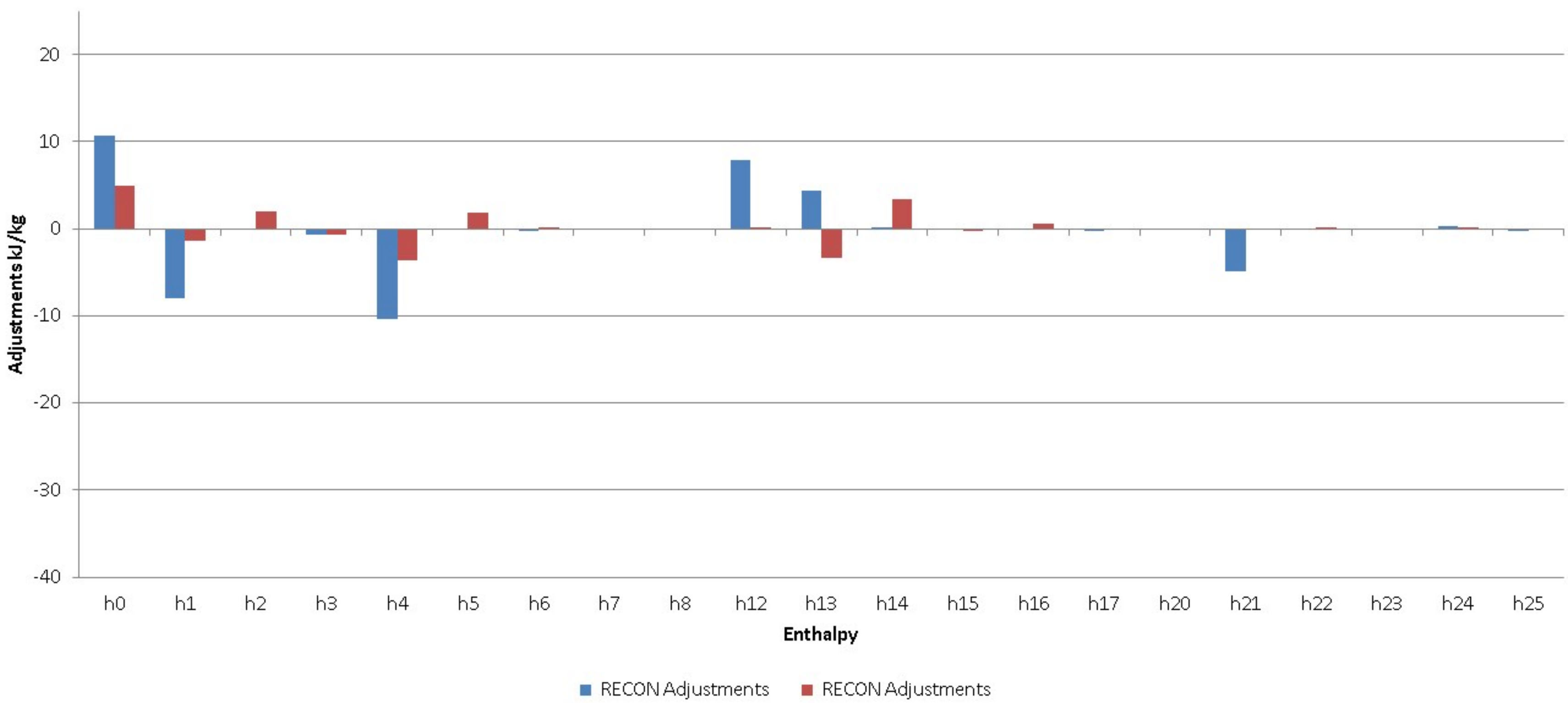

Figure 32: Adjustments of feedwater flow enthalpies

5. Conclusions and Recommendations

5.1 Conclusions

The measurement of mass flowrates, temperatures and pressures is crucial in power plant operations for process performance monitoring and control. The use of data reconciliation in power plant has a great potential to resolve the challenge presented by unreliable and inaccurate process measurements caused by faulty instrumentation and degraded plant components. The availability of accurate process measurements is capable to improving the plant thermal efficiency and also improve the overall power generation in power plant.

The application of data treatment strategies like data reconciliation has an added benefit in power plant by reducing errors in measurement data. However it's important to perform data filtration and pre-processing before performing reconciliation. This is achieved through gross error identification and elimination of measurements data using a suitable GED strategy. Thus it important to note that data reconciliation cannot be applied in isolation without a gross error detection strategy or a combination of strategies to detect and isolate gross errors. The existence of gross errors in process measurements affects the effective use of data reconciliation. Data reconciliation has the ability to enhance the quality of process measurements when coupled with an efficient GED strategy.

It is recommended for power plants to be fitted with adequate measuring devices to measure most and available process parameters for effective use of data reconciliation. This will ensure there is measurements redundancy which is important for data reconciliation. Since instrumentation retrofit programme can be an expensive project to implement. The emphasis for a cost effective placement of instrumentation should be done in the design and construction phase of a power plant. In the event that there are fewer instruments, variable classification can be introduced to identify variables that are observable and can be estimated through reconciliation process

The results obtained using analytical method and RECON software demonstrate that data reconciliation investigated in this study have a sufficient capability to accurately measure feedwater flowrate in power plant. The accurate measurements obtained from data reconciliation are expected to improve the quality of flow measurements for plant performance monitoring.

Process data reconciliation is considered a suitable technique that is capable to improve feedwater flow measurements in power generation plants, in both nuclear and fossil power plants. This can contribute to resolving problematic feedwater flow measurements experienced in power plant operations caused by conventional flow measurement methods.

5.2 Future work

This study focused on the basic fundamentals of process data reconciliation for steady state process and application of data reconciliation on feedwater flow measurements. It has inherent possibilities of leading to a number of further projects.

5.2.1 Model improvements

The feedwater flow measurements process model may be improved by the use of reliable plant data for better results. The collection of data can be extended to include local instrument measurements to improve measurement redundancy. The feedwater flow measurement model can be extended to include the remainder of the water-steam cycle, targeting the whole power plant. Another possibility is the application of data reconciliation to individual plant components such as heat exchangers, for the determination of the fouling factor.

5.2.2 Incorporation of fluid properties in the incidence matrix

In this study the energy balance equations have been simplified to reconciling mass flow and enthalpy. This implied that one could only reconcile either pressure or temperature when the fluid is not in the mixed state. A further enhancement to the calculation of the incidence matrix would be to incorporate the fluid property correlations into the energy balance equations. This will allow reconciliation of pressure and temperature as separate variables. Due to the complex nature of the fluid property correlations, it will not be possible to calculate the partial derivatives analytically. Numerical tools to calculate the Jacobian need to be used.

5.2.3 Data reconciliation in dynamic systems

Process plants are normally considered as dynamic systems, thus the study of data reconciliation may be extend to transient processes. However, the steady state data reconciliation problem forms the fundamental basis for dynamic / transient data reconciliation. As a first step, quasi-steady state DR can be considered.

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Appendix A. Tables and DR algorithms

A1. Chi-Squared distribution table

Chi-squared distribution with n degrees of freedom: a table of $\chi^2_{n,\alpha}$ in $P(D > \chi^2_{n,\alpha}) = \alpha$, for $\alpha = 0.005$ to 0.995, $n = 1$ to 30

n	α							
	0.995	0.99	0.975	0.95	0.05	0.025	0.01	0.005
1	0.0^4393	0.0^3157	0.0^3982	0.0^2393	3.841	5.024	6.635	7.879
2	0.0100	0.0201	0.0506	0.103	5.991	7.378	9.210	10.597
3	0.717	0.115	0.216	0.352	7.815	9.348	11.346	12.838
4	0.207	0.297	0.484	0.711	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	11.070	12.832	15.086	16.750
6	0.676	0.872	1.237	1.635	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	22.362	24.736	27.688	29.819
14	4.075	4.660	5.628	6.571	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	36.415	39.364	42.980	45.558
25	10.520	11.524	13.120	14.611	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	43.773	46.979	50.892	53.672

Taken from; Soong, T.T “Fundamentals of Probability and Statistics for Engineers” John Wiley & Sons, Ltd, New York, 2004.

A2. Reconciled measurements of the heat exchanger model

Stream	Measured	Analytical	Analytical Adjustments	RECON	RECON Adjustments	Percentage Error
Flow rates (kg/s)						
m0	150.89	152.410	1.520	152.409	1.519	1.00%
m1	153.15	152.410	-0.740	152.409	-0.741	0.49%
m2	460.72	454.651	-6.069	454.651	-6.069	1.33%
m3	450.31	454.651	4.341	454.651	4.341	0.95%
Temperature (^oC)						
T0	90.90	90.845	-0.055	90.846	-0.054	0.06%
T1	45.50	45.547	0.047	45.535	0.035	0.08%
T2	44.90	45.150	0.249	45.138	0.238	0.53%
T3	30.00	29.933	-0.067	29.913	-0.087	0.29%
Enthalpy (kJ/kg)						
h0	380.777	380.549	-0.228	380.550	-0.227	0.06%
h1	190.607	190.751	0.144	190.753	0.146	0.08%
h2	188.099	189.089	0.990	189.094	0.995	0.53%
h3	125.834	125.464	-0.370	125.470	-0.364	0.29%

A3. Reconciled measurements of the steam generator model

Stream	Measured	Analytical	Analytical Adjustments	RECON	RECON Adjustments
Flow rates (kg/s)					
Feedwater	Unmeasured	1520.958	-	1520.958	-
Blowdown	10.95	10.95	0.0	10.950	0.0
Steam	1510.01	1510.008	-0.002	1510.008	-0.002
RC_in	13738.868	13738.868	0.0	13738.868	0.0
RC_out	Unmeasured	13738.868	-	13738.868	-
Temperature (^oC)					
Tfw	220.45	220.44	-0.01	22.45	0.0
Tsg	270.00	270.00	0.0	270.00	0.0
T_in	316.50	316.52	0.02	316.504	0.004
T_out	279.30	279.29	-0.01	279.298	-0.002
Enthalpy (kJ/kg)					
hfw	946.731	946.733	0.002	946.731	0.0
hbd	1185.093	1185.093	0.0	1185.093	0.0
hst	2789.690	2789.690	0.0	2789.690	0.0
h_in	1431.809	1431.830	0.021	1431.833	0.024
h_out	1229.099	1229.084	-0.015	1229.089	-0.010

A4. Heat balance diagram data

	Secondary Steam Water Process Diagram			Heat Balance Diagram - Shaw Power International		
Stream	Flow rate (kg/s)	Temperature (^{o}C)	Pressure (kPa)	Flow rate (kg/s)	Temperature (^{o}C)	Pressure (kPa)
F0	1045.62		-	1053.00	52.00	4004.05
F1	1045.62	93.29	-	1053.00	94.65	-
F2	65.87	97.25	92.00	78.00	99.90	96.10
F3	145.00	66.47	-	153.00	61.90	70.25
F4	1045.62	133.81	-	1053.00	133.95	3838.75
F5	78.09	137.81	340.00	76.00	139.40	325.75
F6	78.09	103.25	-	76.00	103.80	325.65
F7	115.05	183.74	1092.00	120.00	188.15	1123.00
F8	115.05	183.74	-	122.00	184.90	1121.25
F9	1045.62	179.74	-	1053.00	181.75	3732.90
F10	469.85	183.74	-	469.00	94.65	4977.4
F11	122.19	-	-	115.00	-	-
F12	1515.48	181.14	-	1522.00	183.10	3730.40
F13	1515.48	181.68	-	1522.00	183.60	6232.80
F14	1515.48	199.52	-	1521.00	201.70	-
F15	55.90	203.52	1673.00	58.00	192.45	1749.60
F16	232.61	-	-	232.00	188.45	1756.40
F17	176.71	-	-	176.00	210.55	2581.95
F18	117.45	-	-	56.00	-	-
F19	3.60	-	-	56.00	-	-
F20	55.66	223.48	2479.00	64.00	213.35	2583.95
F21	1515.48	219.40	-	1521.00	223.00	5117.60
F22	2.50	-	-	0.00	255.30	5117.60
F23	1512.98	270.00	5502.00	1521.00	265.40	5117.60
F24	-	-	-	14935	316.50	32633.0
F25	-	-	-	14935	279.30	-

A5. Feedwater flow measurement reconciled flow rates

Stream	Measured Flow	Analytical	Analytical Adjustments	Percentage Error	RECON	RECON Adjustments	Percentage Error
F0	1053.00	1050.836	-2.164	0.21%	1050.674	-2.326	0.22%
F1	1052.76	1050.836	-1.924	0.18%	1050.674	-2.086	0.20%
F2	65.87	66.484	0.614	0.92%	66.146	0.276	0.42%
F3	Unmeasured	144.69	-	-	144.386	-	-
F4	1053	1050.836	-2.164	0.21%	1050.674	-2.326	0.22%
F5	78.09	78.205	0.115	0.15%	78.241	0.151	0.19%
F6	78.10	78.205	0.105	0.13%	78.241	0.141	0.18%
F7	121.90	121.301	-0.599	0.49%	121.872	-0.028	0.02%
F8	122.00	121.301	-0.699	0.58%	121.872	-0.128	0.11%
F9	1053	1050.836	-2.164	0.21%	1050.674	-2.326	0.22%
F10	469.85	470.731	0.881	0.19%	471.066	1.216	0.26%
F11	115.20	115.066	-0.134	0.12%	115.061	-0.139	0.12%
F12	Unmeasured	1521.567	-	-	1521.674	-	-
F13	1522.00	1521.567	-0.433	0.03%	1521.674	-0.326	0.02%
F14	1521.07	1521.567	0.497	0.03%	1521.674	0.604	0.04%
F15	58.00	57.958	-0.042	0.07%	57.794	-0.206	0.36%
F16	232.61	234.364	1.754	0.75%	234.133	1.523	0.65%
F17	176.71	176.406	-0.304	0.17%	176.339	-0.371	0.21%
F18	56.65	56.545	-0.105	0.19%	56.561	-0.089	0.16%
F19	55.98	55.877	-0.103	0.18%	55.893	-0.087	0.16%
F20	64.00	63.984	-0.016	0.03%	63.886	-0.114	0.18%
F21	1521.54	1521.707	0.167	0.01%	1521.674	0.134	0.01%
F22	10.95	10.95	0.0	0.00%	10.95	0.0	0.00%
F23	1510.95	1510.617	-0.333	0.02%	1510.790	-0.16	0.01%
F24	13738.868	13739.528	0.66	0.00%	13738.978	0.11	0.00%
F25	Unmeasured	13739.528	-	-	13738.978	-	-

A6. Analytical reconciled temperature and enthalpy

Measured Temperature (°C)		Reconciled Temperature (°C)	Analytical Adjustments	Enthalpy (kJ/kg)		Standard Deviation (kJ/kg)	Reconciled Enthalpy (kJ/kg)	Analytical Adjustments	Percentage Error
T0	52	54.58	2.58	h0	221.12	4.17	231.87	10.75	***4.64%***
T1	92.99	91.09	-1.90	h1	392.57	4.2	384.59	-7.98	2.07%
T2	95.73	97.65	1.92	h2	2671.84	0	2671.84	0	0.00%
T3	61.9	61.74	-0.16	h3	259.16	4.19	258.49	-0.67	0.26%
T4	133.95	131.52	-2.43	h4	565.74	4.26	555.33	-10.41	1.87%
T5	137.98	136.76	-1.22	h5	2729.21	0	2729.21	0	0.00%
T6	103.8	103.75	-0.05	h6	435.3	4.22	435.08	-0.22	0.05%
T7	184.87	183.75	-1.12	h7	2780.4	0	2780.4	0	0.00%
T8	184.9	183.75	-1.15	h8	779.76	0	779.76	0	0.00%
T12	183.1	184.84	1.74	h12	778.34	4.39	786.23	7.89	1.00%
T13	183.6	201.72	18.12	h13	781.53	4.39	785.95	4.42	0.56%
T14	201.7	203.53	1.83	h14	861.82	4.47	861.88	0.06	0.01%
T15	203.3	188.38	-14.93	h15	2794.11	0	2794.11	0	0.00%
T16	188.45	203.54	15.09	h16	800.66	4.43	800.47	-0.19	0.02%
T17	210.55	210.45	-0.10	h17	900.24	4.54	900.02	-0.22	0.02%
T20	223.54	225.72	2.18	h20	2802.39	0	2802.39	0	0.00%
T21	220.45	219.39	-1.06	h21	946.73	4.61	941.88	-4.85	0.51%
T22	255.3	255.28	-0.02	h22	1111.49	4.92	1111.49	0	0.00%
T23	270	270.00	0.00	h23	2789.7	0	2789.7	0	0.00%
T24	316.5	316.57	0.07	h24	1431.81	6.01	1432.15	0.34	0.02%
T25	279.3	279.25	-0.06	h25	1229.1	5.07	1228.86	-0.24	0.02%

A7. RECON reconciled temperature and enthalpy

Measured Temperature (^oC)		Reconciled Temperature (^oC)	RECON Adjustments	Enthalpy (kJ/kg)		Reconciled Enthalpy (kJ/kg)	RECON Adjustments	Percentage Error
T0	52	53.16	1.16	h0	221.12	225.98	4.86	2.15%
T1	92.99	92.66	-0.33	h1	392.57	391.21	-1.36	0.35%
T2	95.73	95.73	0.00	h2	2671.84	2673.88	2.04	0.08%
T3	61.9	61.75	-0.15	h3	259.16	258.52	-0.64	0.25%
T4	133.95	133.10	-0.85	h4	565.74	562.16	-3.58	0.64%
T5	137.98	137.98	0.00	h5	2729.21	2731.11	1.9	0.07%
T6	103.8	103.83	0.03	h6	435.3	435.42	0.12	0.03%
T7	184.87	184.87	0.00	h7	2780.4	2780.4	0	0.00%
T8	184.9	184.90	0.00	h8	779.76	779.76	0	0.00%
T12	183.1	183.10	0.00	h12	778.34	778.37	0.03	0.00%
T13	183.6	182.84	-0.76	h13	781.53	778.17	-3.36	0.43%
T14	201.7	202.48	0.78	h14	861.82	865.27	3.45	0.40%
T15	203.3	203.30	0.00	h15	2794.11	2793.84	-0.27	0.01%
T16	188.45	188.57	0.12	h16	800.66	801.18	0.52	0.06%
T17	210.55	210.46	-0.09	h17	900.24	900.05	-0.19	0.02%
T20	223.54	223.54	0.00	h20	2802.39	2802.38	-0.01	0.00%
T21	220.45	220.45	0.00	h21	946.73	946.73	0	0.00%
T22	255.3	255.30	0.00	h22	1111.49	1111.5	0.01	0.00%
T23	270	270.00	0.00	h23	2789.7	2789.67	-0.03	0.00%
T24	316.5	316.51	0.01	h24	1431.81	1431.91	0.1	0.01%
T25	279.3	279.29	-0.01	h25	1229.1	1229.07	-0.03	0.00%

A8. DR algorithm for partially measured linear system

$$
\begin{pmatrix} s's' \\ u'u' \\ r \end{pmatrix} :=
\left|
\begin{array}{l}
s' \leftarrow s \\
s's' \leftarrow s' \\
Sx \leftarrow \mathrm{diag}\left(\sigma^{2}\right) \\
u' \leftarrow u \\
u'u' \leftarrow u' \\
tol \leftarrow 10^{-3} \\
i \leftarrow 1 \\
\text{while } \left|f(s', u')\right| > tol \wedge i \leq 2 \\
\quad \left|
\begin{array}{l}
A_s \leftarrow \mathrm{Jacob}(f(s', u'), s') \\
A_u \leftarrow \mathrm{Jacob}(f(s', u'), u') \\
M \leftarrow \mathrm{qr}\left(A_u\right) \\
Q_1 \leftarrow \mathrm{submatrix}(M, 0, 3, 0, 2) \\
Q_2 \leftarrow \mathrm{submatrix}(M, 0, 3, 3, 3) \\
R_1 \leftarrow \mathrm{submatrix}(M, 0, 2, 4, 6) \\
P \leftarrow Q_2^{T} \\
s_{new} \leftarrow s - Sx \cdot \left(Q_2^{T} \cdot A_s\right)^{T} \cdot \left[\left(Q_2^{T} \cdot A_s\right) \cdot Sx \cdot \left(Q_2^{T} \cdot A_s\right)^{T}\right]^{-1} \cdot \left(Q_2^{T} \cdot A_s\right) \cdot s \\
u'_{new} \leftarrow -\left(R_1^{-1} \cdot Q_1^{T} \cdot A_s \cdot s'\right) \\
r \leftarrow s_{new} - s' \\
s' \leftarrow s_{new} \\
s's' \leftarrow \mathrm{augment}\left(s's', s_{new}\right) \\
u'u' \leftarrow \mathrm{augment}\left(u'u', u'_{new}\right) \\
u' \leftarrow u'_{new} \\
i \leftarrow i + 1
\end{array}
\right. \\
\begin{pmatrix} s's' \\ u'u' \\ r \end{pmatrix}
\end{array}
\right.
$$

A9. DR algorithm for the heat exchanger model

$$A_x = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ h_1 - h_0 & 0 & h_3 - h_2 & 0 & -m_0 & m_0 & -m_2 & m_2 \end{pmatrix}$$

$$A_x = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 190.171 & 0 & -62.266 & 0 & 150.89 & -150.89 & -460.72 & 460.72 \end{pmatrix}$$

$$\begin{pmatrix} x'x' \\ r \end{pmatrix} :=$$

```
x' ← x
x'x' ← x'
Sx ← diag(sig^2)
Wx ← f(x')
tol ← 10^(-3)
i ← 1
while |f(x')| > tol ∧ i ≤ 1
    A_x ← Jacob(f(x') , x')
    x_new ← x − Sx·A_x^T ·(A_x·Sx·A_x^T)^(−1) ·A_x·x
    r ← x_new − x'
    x' ← x_new
    x'x' ← augment(x'x' , x_new)
    i ← i + 1
    Wx ← f(x')
(x'x'
 r)
```

A10. DR algorithm for the steam generator model

$$
\begin{pmatrix} x'x' \\ u'u' \\ r \end{pmatrix} :=
\left|
\begin{array}{l}
x' \leftarrow x \\
x'x' \leftarrow x' \\
Sx \leftarrow \mathrm{diag}\left(sig^{2}\right) \\
u' \leftarrow u \\
u'u' \leftarrow u' \\
tol \leftarrow 10^{-3} \\
i \leftarrow 1 \\
\text{while} \quad \left|f(x',u')\right| > tol \wedge i \le 3 \\
\quad \left|
\begin{array}{l}
A_x \leftarrow \mathrm{Jacob}(f(x',u'),x') \\
A_u \leftarrow \mathrm{Jacob}(f(x',u'),u') \\
b \leftarrow A_x \cdot x' + A_u \cdot u' - f(x',u') \\
M \leftarrow \mathrm{qr}\left(A_u\right) \\
Q_1 \leftarrow \mathrm{submatrix}(M,0,2,0,1) \\
Q_2 \leftarrow \mathrm{submatrix}(M,0,2,2,2) \\
R_1 \leftarrow \mathrm{submatrix}(M,0,1,3,4) \\
P \leftarrow {Q_2}^{T} \\
x_{new} \leftarrow x - Sx \cdot \left({Q_2}^{T} \cdot A_x\right)^{T} \cdot \left[\left({Q_2}^{T} \cdot A_x\right) \cdot Sx \cdot \left({Q_2}^{T} \cdot A_x\right)^{T}\right]^{-1} \cdot \left({Q_2}^{T} \cdot A_x \cdot x - {Q_2}^{T} \cdot b\right) \\
u'_{new} \leftarrow \left({R_1}^{-1} \cdot {Q_1}^{T} \cdot b\right) - \left({R_1}^{-1} \cdot {Q_1}^{T} \cdot A_x \cdot x'\right) \\
r \leftarrow x_{new} - x' \\
x' \leftarrow x_{new} \\
x'x' \leftarrow \mathrm{augment}\left(x'x', x_{new}\right) \\
u'u' \leftarrow \mathrm{augment}\left(u'u', u'_{new}\right) \\
u' \leftarrow u'_{new} \\
i \leftarrow i + 1
\end{array}
\right. \\
\begin{pmatrix} x'x' \\ u'u' \\ r \end{pmatrix}
\end{array}
\right.
$$

A11. PWR feedwater flow measurements systems equations

$$
F(x, u) = \begin{bmatrix}
x_0 - x_1 \\
x_2 + x_5 - u_0 \\
x_2(x_{25} - x_{26}) + x_5(x_{29} - x_{26}) - x_1(x_{24} - x_{23}) \\
x_1 - x_3 \\
x_4 - x_5 \\
x_4(x_{28} - x_{29}) - x_3(x_{27} - x_{24}) \\
x_3 - x_8 \\
x_6 - x_7 \\
x_6(x_{30} - x_{31}) - x_8(x_{32} - x_{27}) \\
x_8 + x_9 - u_1 \\
x_7 + x_{10} + x_{14} - x_9 \\
u_1 - x_{11} \\
x_{11} - x_{12} \\
x_{13} - x_{14} + x_{15} \\
x_{13}(x_{34} - x_{35}) + x_{15}(x_{36} - x_{35}) - x_{12}(x_{33} - x_{32}) \\
x_{12} - x_{19} \\
x_{16} + x_{17} + x_{18} - x_{15} \\
x_{16}(x_{37} - x_{36}) + x_{17}(x_{38} - x_{36}) + x_{18}(x_{39} - x_{36}) - x_{19}(x_{40} - x_{33}) \\
x_{19} - x_{20} - x_{21} \\
x_{22} - u_2 \\
(x_{21} \cdot x_{43} + x_{20} \cdot x_{41} - x_{19} \cdot x_{40}) - x_{22}(x_{43} - x_{44})
\end{bmatrix}
$$

A12. QR Matrix of unmeasured variables of the feedwater flow measurements

$M = qr(A_u) =$

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1.414	0
2	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1
3	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0.707	0	0	0	0	0	0	0	0	0	-0.707	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
11	0	-0.707	0	0	0	0	0	0	0	0	0	-0.707	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0
19	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0

$$M_{row} = last\left[(M)^{\langle 0 \rangle}\right] = 20$$

$$M_{column} = last\left[\left(M^T\right)^{\langle 0 \rangle}\right] = 23$$

A13. Analytical DR algorithm for the feedwater flow measurements

$$
\begin{pmatrix} x'x' \\ u'u' \\ r \end{pmatrix} :=
\left|
\begin{array}{l}
x' \leftarrow x \\
x'x' \leftarrow x' \\
u' \leftarrow u \\
u'u' \leftarrow u' \\
Sx \leftarrow \mathrm{diag}\left(sig^{2}\right) \\
tol \leftarrow 10^{-3} \\
i \leftarrow 1 \\
\text{while } \left|f(x', u')\right| > tol \wedge i \le 2 \\
\quad \left|
\begin{array}{l}
A_x \leftarrow \mathrm{Jacob}(f(x', u'), x') \\
A_u \leftarrow \mathrm{Jacob}(f(x', u'), u') \\
b \leftarrow A_x \cdot x' + A_u \cdot u' - f(x', u') \\
M \leftarrow \mathrm{qr}\left(A_u\right) \\
Q_1 \leftarrow \mathrm{submatrix}(M, 0, 20, 0, 2) \\
Q_2 \leftarrow \mathrm{submatrix}(M, 0, 20, 3, 20) \\
R_1 \leftarrow \mathrm{submatrix}(M, 0, 2, 21, 23) \\
P \leftarrow Q_2^{T} \\
x_{new} \leftarrow x - Sx \cdot \left(Q_2^{T} \cdot A_x\right)^{T} \cdot \left[\left(Q_2^{T} \cdot A_x\right) \cdot Sx \cdot \left(Q_2^{T} \cdot A_x\right)^{T}\right]^{-1} \cdot \left(Q_2^{T} \cdot A_x \cdot x - Q_2^{T} \cdot b\right) \\
u'_{new} \leftarrow \left(R_1^{-1} \cdot Q_1^{T} \cdot b\right) - \left(R_1^{-1} \cdot Q_1^{T} \cdot A_x \cdot x'\right) \\
r \leftarrow x_{new} - x' \\
x' \leftarrow x_{new} \\
x'x' \leftarrow \mathrm{augment}\left(x'x', x_{new}\right) \\
u'u' \leftarrow \mathrm{augment}\left(u'u', u'_{new}\right) \\
u' \leftarrow u'_{new} \\
i \leftarrow i + 1
\end{array}
\right. \\
\begin{pmatrix} x'x' \\ u'u' \\ r \end{pmatrix}
\end{array}
\right.
$$

Appendix B. RECON Solutions

B1. Linear system with partially measured flows

```
RECON 11.2.3-Acad  [University of Cape Town, Mechanical Engineering Department]
 Task: PARTIALLY MEASURED STREAMS (Single-component balance)
 I T E R A T I O N S
 Iter           Qeq            Qx              Qy            Qmin
 -----------------------------------------------------------------
 START     3.8896E+01
    1      4.3512E-15    8.6916E-02      3.3839E+01      1.3184E-01
    2      3.9721E-15    3.0879E-15      5.7053E-15      1.3184E-01
 Legend:
 Qeq   mean residual of equations
 Qx    mean increment of measured variables in iteration
 Qy    mean increment of non-measured variables in iteration
 Qmin  least-square function
 G L O B A L   D A T A
 Number of nodes                                        4
 Number of streams                                      8
 Number of components                                   1
 Number of measured variables                           5
 Number of adjusted variables                           4
 Number of non-measured variables                       3
 Number of observed variables                           3
 Number of non-observed variables                       0
 Number of free variables                               0
 Number of equations                                    4
 Number of independent equations                        4
 Number of user-defined equations                       0
 Degree of redundancy                                   1
 Mean residue of equations                              0
 Qmin                                          1.3184E-01
 Qcrit                                         3.8400E+00
 Status (Qmin/Qcrit)                             0.034333
```

S T R E A M S

Name	Type	Inp.value	Rec.value	Abs.error	
S0	MC	100.100	99.756	0.750	KG/S
S1	MC	41.100	41.104	0.205	KG/S
S2	MN	108.300	108.300	0.542	KG/S
S3	MC	19.800	19.801	0.099	KG/S
S4	MC	38.800	38.852	0.724	KG/S
U0	NO	1.000	79.955	0.745	KG/S
U1	NO	1.000	28.345	0.921	KG/S
U2	NO	1.000	58.653	0.729	KG/S

End of results

Calculations lasted 00:00:0.102

Report created 22.09.2015 02:33:36

B2. Nonlinear heat exchanger model

```
RECON 11.2.3-Acad  [University of Cape Town, Mechanical Engineering Department]
Task: ALL MEASURED VARIABLES HEX (Heat Exchanger)
Balance: [16.05.2014 13:00; 16.05.2014 14:00)
I T E R A T I O N S
Iter            Qeq            Qx              Qy            Qmin
-----------------------------------------------------------------
START     6.7706E+06
   1      1.1092E+03    9.5678E-01      1.8927E+07      6.0585E+00
   2      4.1742E-04    2.1107E-04      6.4419E+02      6.0598E+00
   3      6.2475E-09    1.1709E-10      7.3341E-04      6.0598E+00
Legend:
Qeq   mean residual of equations
Qx    mean increment of measured variables in iteration
Qy    mean increment of non-measured variables in iteration
Qmin  least-square function
G L O B A L   D A T A
Number of nodes                                         2
Number of heat nodes                                    2
Number of streams                                       5
Number of energy streams                                1
Number of components                                    1
Number of temperatures                                  4
Number of pressures                                     1
Number of measured variables                            8
Number of adjusted variables                            8
Number of non-measured variables                        1
Number of observed variables                            1
Number of non-observed variables                        0
Number of free variables                                0
Number of equations                                     4
Number of independent equations                         4
Number of user-defined equations                        0
Degree of redundancy                                    3

Mean residue of equations                      6.2475E-09
Qmin                                           6.0598E+00
Qcrit                                          7.8100E+00
Status (Qmin/Qcrit)                              0.775897
```

S T R E A M S

Name	Type	Inp.value	Rec.value	Abs.error	
M0	MC	150.890	152.409	2.480	KG/S
M1	MC	153.150	152.409	2.480	KG/S
M2	MC	460.720	454.651	4.359	KG/S
M3	MC	450.310	454.651	4.359	KG/S

E N E R G Y   S T R E A M S

Name	Type	Inp.value	Rec.value	Abs.error	
Q	NO	10.000	28.926	1.251	MW

T E M P E R A T U R E S

Name	Type	Inp.value	Rec.value	Abs.error	
T0	MC	90.900	90.846	1.461	C
T1	MC	45.500	45.535	1.180	C
T2	MC	44.900	45.138	1.058	C
T3	MC	30.000	29.913	0.956	C

P R E S S U R E S

Name	Type	Inp.value	Rec.value	Abs.error	
P	F	101.325	101.325		KPA

End of results
Calculations lasted 00:00:0.041
Report created 21.09.2015 23:35:08

B3. Nonlinear steam generator model

```
RECON 11.2.3-Acad  [University of Cape Town, Mechanical Engineering Department]
Task: PHASE EQUILIBRIUM (Balance of SG with phase equilibrium)
Balance: [19.09.2014 16:00; 19.09.2014 17:00)
I T E R A T I O N S
Iter            Qeq            Qx              Qy            Qmin
-----------------------------------------------------------------
START     9.3632E+08
   1      1.0046E+04    6.7677E-01      2.1373E+07      9.9821E-05
   2      3.1222E-01    5.1851E-02      4.5564E+03      8.2685E-05
   3      1.0014E-05    1.6117E-06      5.7800E+00      8.2685E-05
   4      3.1471E-06    5.1277E-11      9.2894E-06      8.2685E-05
Legend:
Qeq   mean residual of equations
Qx    mean increment of measured variables in iteration
Qy    mean increment of non-measured variables in iteration
Qmin  least-square function

G L O B A L   D A T A
Number of nodes                                     2
Number of heat nodes                                2
Number of streams                                   6
Number of energy streams                            1
Number of components                                1
Number of temperatures                              4
Number of pressures                                 3
Number of measured variables                        9
Number of adjusted variables                        9
Number of non-measured variables                    4
Number of observed variables                        4
Number of non-observed variables                    0
Number of free variables                            0
Number of equations (incl. UDE)                     5
Number of independent equations                     5
Number of user-defined equations (UDE)              1
Degree of redundancy                                1

Mean residue of equations                  3.1471E-06
Qmin                                       8.2685E-05
Qcrit                                      3.8400E+00
Status (Qmin/Qcrit)                          0.000022
```

S T R E A M S

Name	Type	Inp.value	Rec.value	Abs.error	
BLOWDOWN	MC	10.950	10.950	0.055	KG/S
FEEDWATER	NO	1500.000	1520.958	4.517	KG/S
RC_IN	MC	13738.868	13738.868	2.748	KG/S
RC_OUT	NO	10000.000	13738.868	2.748	KG/S
STEAM	MC	1510.010	1510.008	4.517	KG/S

E N E R G Y   S T R E A M S

Name	Type	Inp.value	Rec.value	Abs.error	
QSG	NO	2700.000	2785.490	10.889	MW

T E M P E R A T U R E S

Name	Type	Inp.value	Rec.value	Abs.error	
TFW	MC	220.450	220.450	0.998	C
TSG	MC	270.000	270.000	1.000	C
T_IN	MC	316.500	316.504	0.650	C
T_OUT	MC	279.300	279.298	0.767	C

P R E S S U R E S

Name	Type	Inp.value	Rec.value	Abs.error	
FW	MC	5992.340	5992.340	29.962	KPA
HW	MC	15408.380	15408.374	77.035	KPA
PSG	NO	5000.000	5502.844	86.073	KPA

W E T N E S S E S

Name	Type	Inp.value	Rec.value	Abs.error	
STEAM	F	0.00E+0	0.00E+0		%
WATER	F	100.000	100.000		%

End of results

Calculations lasted 00:00:0.070

Report created 23.09.2015 00:52:59

B4. A Simplified feedwater flow measurement model

```
RECON 11.2.3-Acad [University of Cape Town, Mechanical Engineering Department]
Task: PWR WATER STEAM 3 ( PWR Feedwater System)
Balance: [19.08.2014 22:00; 19.08.2014 23:00)
I T E R A T I O N S
Iter            Qeq            Qx              Qy            Qmin
-----------------------------------------------------------------
START     3.4972E+07
   1      3.7040E+05    1.6009E+03      1.3531E+07      1.8042E+01
   2      4.5966E+04    9.6882E+00      5.4497E+04      1.8080E+01
   3      7.5427E+02    2.5464E-04      8.7238E+03      1.8080E+01
   4      2.0503E-01    5.3001E-09      1.4794E+02      1.8080E+01
   5      3.7855E-07    4.9387E-09      4.0235E-02      1.8080E+01
   6      2.0973E-07    3.1704E-09      1.5008E-07      1.8080E+01
Legend:
Qeq   mean residual of equations
Qx    mean increment of measured variables in iteration
Qy    mean increment of non-measured variables in iteration
Qmin  least-square function

G L O B A L   D A T A
Number of nodes                                   15
Number of heat nodes                              12
Number of streams                                 32
Number of energy streams                           6
Number of components                               1
Number of temperatures                            23
Number of pressures                               10
Number of measured variables                      54
Number of adjusted variables                      43
Number of non-measured variables                  11
Number of observed variables                      11
Number of non-observed variables                   0
Number of free variables                           0
Number of equations                               27
Number of independent equations                   27
Number of user-defined equations                   0
Degree of redundancy                              16

Mean residue of equations                 2.0973E-07
Qmin                                      1.8080E+01
```

Qcrit	2.6300E+01
Status (Qmin/Qcrit)	0.687445

S T R E A M S

Name	Type	Inp.value	Rec.value	Abs.error	
F0	MC	1053.000	1050.674	3.606	KG/S
F1	MC	1052.760	1050.674	3.606	KG/S
F10	MC	469.850	471.066	1.497	KG/S
F11	MC	115.200	115.061	1.086	KG/S
F12	NO	1521.000	1521.740	3.551	KG/S
F13	MC	1522.000	1521.740	3.551	KG/S
F14	MC	1521.070	1521.740	3.551	KG/S
F15	MC	58.000	57.794	0.556	KG/S
F16	MC	232.610	234.133	0.923	KG/S
F17	MC	176.710	176.339	0.808	KG/S
F18	MC	56.650	56.561	0.533	KG/S
F19	MC	55.980	55.893	0.527	KG/S
F2	MC	65.870	66.146	0.634	KG/S
F20	MC	64.000	63.886	0.591	KG/S
F21	MC	1521.540	1521.740	3.551	KG/S
F22	MC	10.950	10.950	0.109	KG/S
F23	MC	1510.950	1510.790	3.552	KG/S
F24	MC	13738.868	13738.978	68.308	KG/S
F25	NO	13000.000	13738.978	68.308	KG/S
F3	NO	140.000	144.386	0.817	KG/S
F4	MC	1053.000	1050.674	3.606	KG/S
F5	MC	78.090	78.241	0.538	KG/S
F6	MC	78.100	78.241	0.538	KG/S
F7	MC	121.900	121.872	0.835	KG/S
F8	MC	122.000	121.872	0.835	KG/S
F9	MC	1053.000	1050.674	3.606	KG/S

E N E R G Y   S T R E A M S

Name	Type	Inp.value	Rec.value	Abs.error	
Q2	NO	120.000	173.606	1.770	MW
Q3	NO	120.000	179.617	1.506	MW
Q4	NO	120.000	243.509	2.515	MW
Q5	NO	120.000	132.549	1.724	MW
Q6	NO	120.000	123.948	8.512	MW
QSG	NO	2770.000	2786.912	21.243	MW

T E M P E R A T U R E S

Name	Type	Inp.value	Rec.value	Abs.error	
T0	MC	52.000	53.156	0.659	C
T1	MC	92.990	92.659	0.606	C
T12	MN	183.100	183.100	1.000	C
T13	MC	183.600	182.839	0.725	C
T14	MC	201.700	202.475	0.712	C
T15	MN	203.300	203.300	1.000	C
T16	MC	188.450	188.567	0.994	C
T17	MC	210.550	210.459	0.997	C
T18	MN	260.400	260.400	1.000	C
T2	MN	95.730	95.730	1.000	C
T20	MN	223.540	223.540	1.000	C
T21	MC	220.450	220.451	0.999	C
T22	MN	255.300	255.300	1.000	C
T23	MN	270.000	270.000	1.000	C
T24	MC	316.500	316.509	0.780	C
T25	MC	279.300	279.292	0.849	C
T3	MC	61.900	61.746	0.995	C
T4	MC	133.950	133.104	0.638	C
T5	MN	137.980	137.980	1.000	C
T6	MC	103.800	103.825	0.998	C
T7	MN	184.870	184.870	1.000	C
T8	MN	184.900	184.900	1.000	C
T9	NO	180.940	186.628	0.879	C

P R E S S U R E S

Name	Type	Inp.value	Rec.value	Abs.error	
P0	MC	4004.050	4048.472	1000.615	KPA
P13	MC	5992.336	5957.101	1497.665	KPA
P15	MC	1673.000	1656.655	417.979	KPA
P18	NO	4711.500	2113.733	686.218	KPA
P2	MC	93.195	97.615	23.133	KPA
P20	MC	2583.950	2581.100	645.982	KPA
P23	MC	5502.000	5505.047	1360.659	KPA
P24	MC	15408.380	15376.622	3244.842	KPA
P5	MC	329.606	343.538	81.502	KPA
P7	MN	1092.000	1092.000	273.000	KPA

W E T N E S S E S

Name	Type	Inp.value	Rec.value	Abs.error	
steam	F	0.00E+0	0.00E+0		%
water	F	100.000	100.000		%

End of results

Calculations lasted 00:00:0.096

Report created 02.05.2015 00:46:45