

Time diversification and holding periods on the Johannesburg Stock Exchange

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Abstract

This dissertation investigates the existence of time diversification on the Johannesburg Stock Exchange (JSE), with the goal of providing investor guidance toward their optimal investment horizon on the JSE. Focusing on the Random Walk and Mean Reversion Models, a variety of tests were employed to identify serial correlation within the JSE logarithmic total returns. By assessing the possibility of mean reversion or trending behavior in returns, this study aims to determine if short-term variance (as a risk measure) calculation intervals accurately describe the long-term risk on the JSE when scaled. Additionally, the skewness of the logarithmic and arithmetic return distributions on the JSE, as the return interval lengthens, was investigated.

The focus was on a composite JSE All Share Index (ALSI) resulting from the merger of the FTSE/JSE All Share Total Return Index (J203T), the JSE Actuaries Index (adjusted for dividends (AJ203)) and early JSE total return data (Firer & McLeod, 1999). The JSE All Bond Index (ALBI) was used in this study as an alternate asset class to JSE Equities. The dataset is comprised of 117 years (01/01/1900 to 31/12/2016) of ALSI and 18 years (31/12/1998 to 31/12/2016) of ALBI price and return series. The frequency of returns analyzed range from monthly to twenty-year total returns. The dataset was further analyzed, into a period before and after 1987 to observe the long and short-term serial correlation dynamics of the JSE, and to investigate how these change over time. This breakpoint (01/01/1987) was chosen due to the belief that structural change occurred on the JSE in 1987.

Data analysis included; descriptive statistics and tests for normality, the Augmented Dickey Fuller and Phillips-Perron tests for stationarity, the Autocorrelation Function tests for serial correlation, the Quandt-Andrews and Bai-Perron tests for structural breaks, the Variance Ratio Test, and the Runs Test. These parametric and nonparametric methods were performed on both the nominal and real total returns of the ALSI and ALBI.

This investigation uncovered significant short-term trending behavior in the ALSI returns, combined with evidence of medium-term mean reversion in this indexes returns. A lack of mean reversion and limited evidence of trending behavior in the ALBI returns were uncovered. ALSI returns have rejected the Random Walk Model over the short and medium-term, while ALBI returns have for the most part, failed to significantly reject the Random Walk Model.

The short-term trending behavior in ALSI returns was observed at the monthly, quarterly and semi-annual return frequencies. This behavior suggests that if variances (as risk measures) calculated over these shorter trending periods, are scaled to represent the risk of longer periods, they will underestimate the true period risk on the JSE. Furthermore, the implications of the mean reversion evidence in three yearly returns, suggest that if the variances (as risk measures) are calculated over these three-year periods, and were to be scaled to represent the variance of longer periods, then these longer periods would have their period risk overstated. This paper has documented the change in the logarithmic return distribution of the ALSI, that exhibited negative skewness as the return holding period lengthens. Paradoxically positive skewness is observed as the return holding period increased was observed for the arithmetic distribution of ALSI returns.

In the presence of autocorrelation in ALSI returns, portfolio and fund managers should employ the Lo and MacKinlay (1988) variance adjustment to unbiased their risk estimates - if they scale short or medium term variances. The existence of Mean Reversion at the three-year frequency in South African Equities, provides evidence to support Time Diversification. As a direct result of this, this study proposes that a five to six-year holding period is optimal to take advantage of these mean reverting returns.

Declaration

I, Akshay Kumar Panday (Student Number: PNDAKS001), hereby declare that the work contained herein is my original work (except where acknowledgements indicate otherwise). Neither the whole, nor part of this report are being or have been submitted for another degree at this, or any other university. The University of Cape Town is hereby empowered to reproduce this report, either wholly or partially, for research purposes in any manner whatsoever.

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Table of Contents

Abstract	ii
Declaration	iv
Acknowledgements	v
Table of Contents	vi
List of Appendices	viii
List of Tables	ix
List of Figures	xi
1. Introduction	1
2. Theory	4
2.1 Introduction	4
2.2 Auto-Regressive Series	4
2.3 Random Walk Models	5
2.3.1 Random Walk (without drift).....	5
2.3.2 Random Walk with positive drift.....	7
2.4 Mean Reversion Model	9
2.5 Autocorrelation Total Risk Adjustment	9
2.6 Summary and Conclusion	10
3. Prior Literature	11
3.1 Introduction	11
3.2 Prior Literature on the Fallacy of Time Diversification	11
3.3 Prior Literature on the existence of Time Diversification	14
3.4 Prior Literature on International studies on the distribution of returns	24
3.5 Prior Literature on South African JSE return distributions	26
3.6 Summary and conclusions	31
4. Data	33
4.1 Introduction	33
4.2 JSE All Share Index Data	34
4.2.1 Overview of JSE Index Series.....	34
4.2.2 J203T data	35
4.2.3 AJ203 data and total return index construction.....	35
4.2.4 Firer and McLeod data	36
4.2.5 Merging the Datasets.....	37
4.3 JSE All Bond Data	38
4.4 Nominal Return Calculations	39
4.5 South African Inflation Data	41
4.6 Real return calculation	42
4.7 Summary and Conclusion	42
5. Methodology	44
5.1 Introduction	44
5.2 Descriptive Statistics	44
5.2.1 Mean and variance	44
5.2.2 Skewness	45
5.2.3 Kurtosis	45
5.2.4 Jarque-Bera test.....	45

5.2.5 Return Distribution Histograms	46
5.3 Tests for Stationarity	46
5.3.1 Augmented Dickey Fuller Tests.....	46
5.3.2 Phillips-Perron Test.....	50
5.4 Autocorrelation Test	50
5.5 Unknown Structural Breakpoint Tests	51
5.5.1 Quandt-Andrews Single Unknown Breakpoint Test.....	52
5.5.2 Bai-Perron Multiple Unknown Breakpoint Test	53
5.6 Runs Test	54
5.7 Variance Ratio Tests	56
5.7.1 Single Interval Variance Ratio Test	56
5.7.2 Joint Interval Variance Ratio Test.....	57
5.8 Bradfield and Ardington (1997) Replication	58
5.9 Summary and Conclusion.....	58
6. Results	59
6.1 Introduction	59
6.2 Descriptive Statistics	59
6.2.1 Nominal returns.....	59
6.2.2 Real Returns	63
6.2.3 Return distribution Histograms	63
6.3 Tests for Stationarity.....	66
6.3.1 Augmented Dickey Fuller Tests	66
6.3.2 Phillips-Perron Test.....	67
6.4 Autocorrelation Test	68
6.4.1 Nominal Returns	68
6.4.2 Real Returns	70
6.5 Breakpoint Tests.....	71
6.5.1 Quandt-Andrews Breakpoint Test	71
6.5.2 Bai-Perron Breakpoint Test	72
6.5.3 Breakpoint Regressions.....	73
6.6 Runs Test.....	74
6.6.1 Nominal Returns	74
6.6.2 Real Returns	76
6.7 Variance Ratio Test.....	77
6.7.1 Entire Period	77
6.7.2 Pre-1987 Period.....	80
6.7.3 Post-1986 Period	81
6.7.4 ALBI Variance Ratio Tests.....	83
6.8 Bradfield and Ardington (1997) study replication	85
6.9 Summary and Conclusions	86
7. Conclusion	91
Reference List.....	93
Appendix.....	98

List of Appendices

A. Descriptive Statistics

A1. Nominal Returns

A2. Real Returns

B. Augmented Dickey-Fuller and Phillips-Perron tests

C. Autocorrelation test

C1. ALSI Nominal returns

C2. ALBI Nominal returns

C3. ALSI Real returns

C4. ALBI Real returns

D. Bai-Perron Unknown Structural Breakpoint test

E. Real Returns Runs Test

E1. ALSI

E2. ALBI

F. Variance Ratio Test

F1. ALSI Nominal returns

F2. ALBI Nominal returns

F3. Real returns

G. Bradfield and Ardington (1997) Replication

List of Tables

Table 3.1:	Showing the confidence intervals of terminal values of risky investments over various time horizons (Kritzman, 1994)
Table 3.2:	Showing the confidence intervals, shortfall risks and conditional means for various time horizons (Thorley, 1995)
Table 3.3:	Showing the Mean Reversion transitory component 1926-1985 (Lee, 1990)
Table 3.4:	Showing the serial correlations identified by Filer and McLeod (1999) for annual, three, five and 10 yearly returns over the period 1925 – 1998.
Table 3.5:	Showing the Runs test results for Bradfield and Ardington (1997)
Table 3.6:	Showing the Autocorrelation test results for Bradfield and Ardington (1997)
Table 3.7:	Showing the Variance Ratio test results for Bradfield and Ardington (1997)
Table 4.1	Data collection composition for JSE All Share Index Total return data
Table 6.1:	Descriptive statistics for nominal monthly total return (logarithmic and arithmetic) for the ALSI and ALBI over their maximum available period
Table 6.2:	Descriptive statistics for nominal monthly total (logarithmic and arithmetic) return data for the ALSI over the pre-1987 and post-1986 periods
Table 6.3:	Descriptive statistics for nominal annual total return data on the ALSI and ALBI over their maximum period
Table 6.4:	Descriptive statistics for nominal annual total (logarithmic and arithmetic) return data for the ALSI over the pre-1987 and post-1986 periods
Table 6.5:	Summary of Augmented Dickey-Fuller tests in Appendix B
Table 6.6:	Summary of Phillips-Perron tests in Appendix B
Table 6.7:	Autocorrelation test for ALSI logarithmic returns over various frequencies over the all three periods
Table 6.8:	Auto-Correlation Test for ALBI logarithmic total returns over various frequencies for the period 31/12/1998 to 31/12/2016
Table 6.9:	Autocorrelation Test for ALSI logarithmic real returns over the period 01/01/1987 to 31/12/2016 for all return frequencies
Table 6.10:	Autocorrelation Test for ALBI total real logarithmic returns over various frequencies for the period 31/12/1998 to 31/12/2016.
Table 6.11:	Quandt-Andrews Unknown Breakpoint test ALSI total nominal returns over the entire period.
Table 6.12:	Quandt-Andrews Unknown Breakpoint test ALBI total nominal returns over the entire period.
Table 6.13:	Autocorrelation test post results of Quandt-Andrews and Bai-Perron Breakpoint tests for monthly returns about the breakpoint 1972M09 for the ALSI

- Table 6.14: Autocorrelation test post-results of Quandt-Andrews and Bai-Perron Breakpoint tests for monthly returns about the breakpoint 2010m11 for the ALBI.
- Table 6.15: Runs Test for ALSI logarithmic total returns over all three periods
- Table 6.16: Runs Test for ALBI logarithmic total returns for the period 31/12/1998 to 31/12/2016
- Table 6.17: Overall Joint Variance Ratio tests for ALSI logarithmic total returns over the full period
- Table 6.18: Individual Variance Ratio Tests for ALSI logarithmic total returns over the full period
- Table 6.19: Overall Joint Variance Ratio Tests for ALSI logarithmic total returns over the pre-1987 period
- Table 6.20: Individual Joint Variance Ratio Tests for ALSI logarithmic total returns over the pre-1987 period
- Table 6.21: Overall Joint Variance Ratio Tests for ALSI logarithmic total returns over the post-1986 period
- Table 6.22: Individual Variance Ratio Tests for ALSI logarithmic total returns over the post-1986
- Table 6.23: Overall Joint Variance Ratio Tests for ALBI logarithmic total returns over the full period
- Table 6.24: Individual Variance Ratio Tests for ALBI logarithmic total returns over the full period

List of Figures

- Figure 3.1: Diagram showing the convergence of the return distribution as the horizon expands
Kritzman (1994)
- Figure 3.2: Showing the multi-period shortfall constraint graphically on the efficient frontier
(Leibowitz & Kogelman, 1991)
- Figure 3.3: Showing the convergence of the return distribution for developed markets (Anderson,
Malone & Marshall, 2012)
- Figure 3.4: Showing the convergence of the return distribution for emerging markets (Anderson,
Malone & Marshall, 2012)
- Figure 3.5: Showing the behavior of annual JSE All Share Index returns and the 91-Day Treasury
Bill Rate between 01/1985 and 01/1996 (Bradfield & Ardington, 1997)
- Figure 4.1: Showing the assumed 5% JSE All Share dividend yield amongst the historical JSE
dividend yields (Firer & McLeod, 1999).
- Figure 4.2: Showing JSE All Share Price Total Return Index over the period 31/01/1925 to
31/12/2016 using monthly price series data
- Figure 4.3: Showing JSE All Share Price Total Return Index over the period 01/01/1900 to
31/12/2016 using annual price series data
- Figure 4.4: Showing JSE All Bond Price Index over the period 31/12/1998 to 31/12/2016 using
monthly price series data
- Figure 4.5: Showing JSE All Bond Price Index over the period 31/12/1998 to 31/12/2016 using
annual price series data
- Figure 4.8: Showing Logarithmic JSE All Share Returns over the period 01/01/1900 to 31/12/2016
using annual return data
- Figure 4.9: Showing Arithmetic JSE All Share Returns over the period 01/01/1900 to 31/12/2016
using annual return data
- Figure 4.10: Showing Logarithmic JSE All Share Returns over the period 01/01/1900 to 31/12/2016
using monthly return data
- Figure 4.11: Showing Logarithmic JSE All Share Returns over the period 01/01/1900 to
31/12/2016 using annual return data
- Figure 4.12: Showing Logarithmic Inflation rate for South Africa over the period 31/12/1959 to
31/12/2016 using monthly return data
- Figure 4.13: Showing Logarithmic Inflation rate for South Africa over the period 01/01/1987 to
31/12/2016 using monthly return data
- Figure 4.14: Showing the Real return on the JSE All Share index over the period 01/01/1987 to
31/12/2016 using monthly return data

- Figure 4.15: Showing the Real and Nominal return on the JSE All Share index over the period 01/01/1987 to 31/12/2016 using monthly return data
- Figure 6.1: Variance Ratio Test for monthly ALSI total nominal returns over the entire period
- Figure 6.2: Variance Ratio Test for semi-annual ALSI total nominal returns over the entire period
- Figure 6.3: Variance Ratio Test for annual ALSI total nominal returns over the entire period
- Figure 6.4: Variance Ratio Test for five yearly ALSI total nominal returns over the entire period
- Figure 6.5: Variance Ratio Test for monthly ALSI total nominal returns over the pre-1987 period
- Figure 6.6: Variance Ratio Test for quarterly ALSI total nominal returns over the pre-1987 period
- Figure 6.7: Variance Ratio Test for semi-annual ALSI total nominal returns over the pre-1987 period
- Figure 6.8: Variance Ratio Test for annual ALSI total nominal returns over the pre-1987 period
- Figure 6.9: Variance Ratio Test for monthly ALSI total nominal returns over the post-1987 period
- Figure 6.10: Variance Ratio Test for quarterly ALSI total nominal returns over the post -1987 period
- Figure 6.11: Variance Ratio Test for semi-annual ALSI total nominal returns over the post -1987 period
- Figure 6.12: Variance Ratio Test for annual ALSI total nominal returns over the post -1987 period
- Figure 6.13: Variance Ratio Test for monthly ALBI total nominal returns over the post-1987 period
- Figure 6.14: Variance Ratio Test for quarterly ALBI total nominal returns over the post -1987 period
- Figure 6.15: Variance Ratio Test for semi-annual ALBI total nominal returns over the post -1987 period
- Figure 6.16: Variance Ratio Test for annual ALBI total nominal returns over the post -1987 period

1. Introduction

Throughout the 1970s, defined benefit pension plans were the most common in both South Africa and the rest of the world. These pension plans require that the investment risk of the fund be carried by the employer. However, since the 1990s, private pension funds in South Africa have migrated toward defined contribution plans. This places not only the investment risk in the hands of the employee, but also the investment decision (National Treasury, 2004). This investment decision requires that the investor make an informed long-term (asset allocation) decision regarding his/her future wealth.

This precarious asset allocation decision has accompanied investors since the emergence of more than one investment class. The asset allocation decision is vital, for both the long and short-term performance objectives of mutual funds, unit trusts, pension funds and general investors. The central tenet of this paper is to uncover whether diversifying risk across time, could make the asset allocation decision faced by investors more efficient. These time diversification benefits would give investors the best possible return exposure, while still attempting to reduce risk.

To emphasize the importance of the asset allocation decision further, South African asset class analysis since 1925, has shown that equities grossly outperform bonds, cash and inflation (Fier & McLeod, 1999). This analysis found that R1 invested in Johannesburg Stock Exchange (JSE) listed equities in 1925, would be worth R 12 951 by the completion of 1998, while R1 invested in JSE bonds at the same time would yield R 121 by the end of 1998. Although equity investments are inherently riskier, this paper aims to uncover whether an investor investing over the long-term, can diversify his/her equity risk through optimization of his/her holding period. This optimization would allow the investor to attain best equity exposure with a lower annualized risk.

Time diversification refers to the diversification benefit gained when investing over a longer time horizon. This benefit is grounded on the premise that risk decreases as ones' investment horizon expands. Time diversification advocates that below-average returns in one period will be more than offset by above-average returns in others. The investment decision that aims to take advantage of this, is one that allocates a larger proportion of its portfolio toward riskier

assets, over the most efficient time horizons. The advent of time diversification has been greeted by dismissive academic theory centered around utility theory, as well as paradoxically promoted by the widely-held practitioner belief, that risk reduction achieved through diversifying through time is not only possible - but exploitable.

It is imperative that we define risk early on, as some literature bias their analysis depending on their definition of risk. These vary between risk representing the variance of annualized returns or risk representing the variance of investment terminal values. For the purposes of our paper, risk can refer to either the variability in annual return or the variability in terminal wealth distributions. This study shall be specific when referring to each.

This research paper employs Auto-Regressive Models of order one (AR (1)) to determine the distribution of JSE returns. This investigation would yield, that if the returns on the JSE are non-random (i.e. Mean Revert or Trend), then the variance (risk) of returns would not scale proportionally with time - as is employed in practice. Thus, lead to under/overstatement of risk depending on the variance calculation interval. Time diversification benefits have been shown to exist in the presence of a mean-reverting underlying return series. Thus, if this study finds that JSE returns are random (as advocated under the EMH) and not mean-reverting, then we would expect to observe an apparent lack of time diversification. On the other hand, if returns are found to be mean-reverting, then this study aims to uncover the diversification of risk through time.

The remainder of this research paper is structured as follows: Chapter 2 will discuss the relevant academic theory relating to stochastic processes with emphasis on the Random Walk and Mean Reversion Models. Chapter 3 uncovers the debate of time diversification by presenting pertinent prior literature in favor and against, as well as prior literature on the distribution of returns both internationally and in South Africa. Chapter 4 expands on this report's data selection, collection and merging procedure for the real and nominal JSE All Share Index (ALSI) and the JSE All Bond Index (ALBI) price series. The methodology is outlined in Chapter 5, whose respective results are portrayed and analyzed in the Chapter 6. The Conclusion is presented in Chapter 7, that in addition to summarizing and concluding, will also present recommendations based on this academic report.

2. Theory

2.1 Introduction

Before delving into the idiosyncrasies of time diversification, this paper needs to assess the distribution of stock prices in a South African context. This analysis requires the consideration of the first order auto-regressive models (AR(1) models) to model stock prices. These time series models consist of the Random Walk Model (without drift), Random Walk Model with positive drift and the Mean Reversion Model. The term “auto” is used to indicate that current share prices are modelled as a function of their own past share price.

This Chapter comprises Section 2.2 that visits general theory of auto-regressive series. This is followed by Section 2.3 that sheds light on the Random Walk Model and Section 2.4 that outlines the Mean Reversion Model. Section 2.5 presents the autocorrelation risk adjustment that may need to be employed if serial correlation is found and Section 2.6 summarizes and concludes this Chapter.

2.2 Auto-Regressive Series

Any price series is auto-regressive of order one (AR(1)) if it can conform to the following stochastic AR(1) difference equation (Gujarati & Porter, 2003):

$$P_t = \alpha + \rho P_{t-1} + \varepsilon_t, \quad (1)$$

where P_t is the stock price at time t , P_{t-1} is the stock price at time $t - 1$ (previous period), α and ρ are constants and ε_t represents the independent and identically normally distributed zero-mean error term for time t ($\varepsilon_t \sim N(0, \sigma^2)$).

From the implicit difference Equation 1, it can be seen that P_t is a function of itself at a previous date (P_{t-1}). Calculating a general form for P_t , substituting for P_{t-1} , results in:

$$\begin{aligned} P_t &= \alpha + \rho(\alpha + \rho P_{t-2} + \varepsilon_{t-1}) + \varepsilon_t \\ P_t &= \alpha + \rho\alpha + \rho^2 P_{t-2} + \rho\varepsilon_{t-1} + \varepsilon_t. \end{aligned}$$

Substituting for P_{t-2} , results in:

$$\begin{aligned}
 P_t &= \alpha + \rho\alpha + \rho^2(\alpha + \rho P_{t-3} + \varepsilon_{t-2}) + \rho\varepsilon_{t-1} + \varepsilon_t \\
 P_t &= \alpha + \rho\alpha + \rho^2\alpha + \rho^3 P_{t-3} + \rho^2\varepsilon_{t-2} + \rho\varepsilon_{t-1} + \varepsilon_t \\
 P_t &= \alpha(1 + \rho + \rho^2) + \rho^3 P_{t-3} + \rho^2\varepsilon_{t-2} + \rho\varepsilon_{t-1} + \varepsilon_t.
 \end{aligned}$$

This general solution in summation form, yields the following result (Chikobvu and Knowledge, 2010):

$$P_t = \sum_{i=0}^{N-1} \alpha(\rho^i) + P_{t-N}\rho^N + \rho^i (\varepsilon_{t-i}). \quad (2)$$

From this simplification, the impact of changing/constraining the constants α and ρ is easily discernable. This leads to the following models; Random Walk (with no drift), Random Walk with positive drift and Mean Reversion.

2.3 Random Walk Models

Both Random Walk Models (with and without drift) are AR(1), with the constraint that $|\rho| = 1$ in Equation 1. However, the with and without drift models, differ due to their constraints on α in Equation 1. Section 2.3.1 outlines the Random Walk Model (without drift) and Section 2.3.2 describes the Random Walk Model (with drift).

2.3.1 Random Walk (without drift)

The AR(1) Random Walk without drift series specifies that $|\rho| = 1$ and $\alpha = 0$ in Equation 1. Therefore, if a series follows a Random Walk without drift Model, it takes the form (Gujarati & Porter, 2003):

$$P_t = P_{t-1} + \varepsilon_t. \quad (3)$$

This formula determines today's stock price as a function of the previous period's stock price - plus a random error. Substituting for P_{t-1} yields:

$$P_t = P_{t-2} + \varepsilon_{t-1} + \varepsilon_t .$$

Substituting for P_{t-2} results in:

$$P_t = P_{t-3} + \varepsilon_{t-2} + \varepsilon_{t-1} + \varepsilon_t .$$

Thus, a general form for the stock price at time t , under a Random Walk with no drift parameter is (Gujarati & Porter, 2003):

$$P_t = P_0 + \sum_{i=1}^t \varepsilon_t, \tag{4}$$

where P_0 is the stock price at time zero. The expected value of P_t results in:

$$E[P_t] = E[P_0] + E\left[\sum_{i=1}^t \varepsilon_t\right]$$

$$E[P_t] = E[P_0] + \sum_{i=1}^t E[\varepsilon_t]$$

$$E[P_t] = E[P_0] + \sum_{i=1}^t (0)$$

$$E[P_t] = E[P_0] + 0$$

$$E[P_t] = P_0.$$

From the above expectation, it can be concluded that the expected value of a Random Walk without drift model is stationary. Before drawing any series conclusions, the stationarity of its' variance needs to be analyzed:

$$Var[P_t] = Var[P_0] + Var\left[\sum_{i=1}^t \varepsilon_t\right]$$

$$\begin{aligned} \text{Var}[P_t] &= 0 + \sum_{i=1}^t \text{Var}[\varepsilon_t] \\ \text{Var}[P_t] &= 0 + t\sigma^2 \\ \text{Var}[P_t] &= t\sigma^2. \end{aligned}$$

The variance of the stock price series is dependent on the time horizon, and increases with time. Thus, this model of stock prices follows a non-stationary stochastic process. Considering the returns (first difference) of this non-stationary series leads to (Gujarati & Porter, 2003):

$$\begin{aligned} r_t &= P_t - P_{t-1} \\ r_t &= (P_{t-1} + \varepsilon_t) - P_{t-1} \\ r_t &= \varepsilon_t. \end{aligned}$$

The return series of a Random Walk with no drift stock price distribution is stationary and equal to the error variable ($\varepsilon_t \sim N(0, \sigma^2)$).

2.3.2 Random Walk with positive drift

This form of an AR(1) series has the same restriction on $|\rho| = 1$ as its' without drift counterpart, but the differentiating factor is the constraint $\alpha > 0$. This takes the form (Gujarati & Porter, 2003):

$$P_t = \alpha + P_{t-1} + \varepsilon_t, \quad (5)$$

where $\alpha > 0$ for the Random Walk Model with positive drift ($|\alpha| > 0$ for the general Random Walk Model). Substituting for P_{t-1} and P_{t-2} , the general form simplifies to (Gujarati & Porter, 2003):

$$P_t = t\alpha + P_0 + \sum_{i=1}^t \varepsilon_t. \quad (6)$$

Comparing this general form to that of the Random Walk with no drift, the observation that for a large time period (t), the positive drift co-efficient can have a considerable effect on future prices. Calculating its expectation takes the form:

$$E[P_t] = E[t\alpha] + E[P_0] + E\left[\sum_{i=1}^t \varepsilon_t\right]$$

$$E[P_t] = t\alpha + P_0 + 0$$

$$E[P_t] = t\alpha + P_0.$$

Thus, the expected stock price at time t is a function of P_0 , that increases with time (t) and drift component α . Hence, the expected stock price is non-stationary. The variance of the stock price at time t is:

$$Var[P_t] = Var[t\alpha] + Var[P_0] + Var\left[\sum_{i=1}^t \varepsilon_t\right]$$

$$Var[P_t] = 0 + 0 + Var\left[\sum_{i=1}^t \varepsilon_t\right]$$

$$Var[P_t] = \sum_{i=1}^t Var[\varepsilon_t]$$

$$Var[P_t] = t\sigma^2.$$

This variance is identical to the variance of the initial Random Walk Model (without drift), and this form of the Random Walk Model (with drift) is also a non-stationary stochastic process. The returns (first difference) of this series are (Gujarati & Porter, 2003):

$$r_t = P_t - P_{t-1}$$

$$r_t = (\alpha + P_{t-1} + \varepsilon_t) - P_{t-1}$$

$$r_t = \alpha + \varepsilon_t.$$

This result confirms that the return series of a Random Walk with positive drift is also a non-stationary stochastic process.

2.4 Mean Reversion Model

The final AR(1) model considered in this study is that of Mean Reversion. This Model constrains ρ in Equation 1 to be less than zero, such that $|\rho| < 1$, and takes the form (Gujarati & Porter, 2003):

$$P_t = \mu + \rho(P_{t-1} - \mu) + \epsilon_t \quad (7)$$

where μ is the mean of the long-term stock price series and $(P_{t-1} - \mu)$ is the prior period deviation in stock price from its long run mean. Intuitively, for a process to be mean reverting it must correct for unexpected deviations from the mean, and a $|\rho| < 1$ constraint is required as the model needs to pull back deviations from the mean. Attempting to understand this model in the context of financial markets, one would expect to observe a stock price rally to be followed by a stock price downwards mean reversion adjustment, and a stock price crash to be followed by a stock price upward mean reversion adjustment. Looking at Equation 7, observing a $|\rho| > 1$ implies that this series is not mean reverting, and is instead called an explosive or trending series, as deviations from the mean get amplified over every subsequent period. This would cause the share price series to either increase at an increasing rate, or approach its zero (its lower bound) at an increasing rate (Gujarati & Porter, 2003).

2.5 Autocorrelation Total Risk Adjustment

In the case of the existence of autocorrelation in share price returns, Lo and MacKinlay (1988) have identified that a total risk adjustment must be applied to the variances of returns to account for this autocorrelation. This total risk adjustment uses a ratio of the variance calculated over a longer period N to the variance over a single period, and adjusts this ratio using the autocorrelation coefficients. This adjustment is dependent on the order of the autocorrelation found, for this generalization, assume that there exists autocorrelation of the k th order. This adjustment and its general form is expressed as (Gujarati & Porter, 2003):

$$\frac{\hat{\sigma}_N^2}{N\hat{\sigma}_1^2} = 1 + \frac{2(N-1)}{N}(\widehat{\rho}_1) + \frac{2(N-2)}{N}(\widehat{\rho}_2) + \dots + \frac{2(N-(k-1))}{N}(\widehat{\rho}_{k-1}) + \frac{2(N-(k))}{N}(\widehat{\rho}_k)$$

$$\frac{\hat{\sigma}_N^2}{N\hat{\sigma}_1^2} = 1 + 2 \sum_{j=1}^k \frac{(N-j)}{N} (\hat{\rho}_j),$$

where $\hat{\sigma}_N^2$ represents the sample variance over the longer period N , $\hat{\sigma}_1^2$ represents the sample variance over the single period, N is the longer period, $\hat{\rho}_k$ represents autocorrelation coefficient at the k th lag and lastly, k represents the order of the autocorrelation.

To avoid risk over and under-estimation when scaling risks calculated over short and long period variances, this adjustment needs to be employed – in the case of serial correlation in returns.

2.6 Summary and Conclusion

This Chapter's exposition of pertinent academic theory relating to time diversification and stochastic processes, forms the grounding upon which this dissertation rests. This Chapter has presented an outline of AR(1) Models, with focus on the Random Walk Models and the Mean Reversion Model. Additionally, this paper presents the variance adjustment proposed by Lo and MacKinlay (1988), which will need to be employed in South Africa if serial correlation amongst returns is identified. These Models and phenomena reappear in later aspects of this study, in its investigation into the optimal JSE holding period based on the distribution of JSE returns and prices.

3. Prior Literature

3.1 Introduction

This Chapter reviews and critically compares prior literature on the topic of time diversification, and within a South African context. Section 3.2 presents the arguments of the academics that dismiss the existence of time diversification, while Section 3.3 assesses the arguments of propagators of time diversification. Section 3.4 discusses International research into the distribution of returns and South African literature on the distribution of returns on the JSE is presented in Section 3.5. Ultimately, Section 3.6 will summarize and conclude this Chapter.

3.2 Prior Literature on the Fallacy of Time Diversification

In one of the earlier time diversification studies by Kritzman (1994), he defined time diversification as a feature of returns that when observed over a long-time period, produce above-average returns in some periods that overcompensate investors regarding the below-average returns earned in other periods.

Samuelson (1963) claims that investment horizon has no effect on the asset allocation decision, with the following notable exceptions: if one's income/expected income is highly correlated with equity returns, in the case of mean-reverting price series and when one has decreasing relative risk aversion with increasing wealth. Samuelson (1963) uses the Bernoulli Law of Large Numbers to prove that any person who rejects a favorable bet when offered once, can never rationally accept a combination of the same bet, given the same utility function. This can be applied to the time diversification argument, where the author claims that an investor, will have the same decision on a risky investment irrespective of the time horizon (i.e. one period or multi-period investment period).

It has been established that there exists a mathematical truth, that under certain assumptions, prove that an investor who prefers a risk-free asset to a risky asset over a short time horizon, should also prefer the risk-free asset to a risky asset over a long-time horizon (Samuelson, 1969). In other words, that investors' asset allocation preferences are independent of the

investing time horizon. This premise is based on the following assumptions; the investors aversion to risk is constant irrespective of the investors change in wealth over the period, risky assets returns are random (zero autocorrelation) and the investors future wealth depends only on the investment terminal value.

The mathematical truth can be shown for both an inverse and log wealth utility functions. Using the example from Kritzman (1994:16) assuming a log wealth utility function and a starting wealth of \$100.00, with an opportunity to invest in a risky asset that has a 50% chance of yielding a 33.33% gain or a 50% chance of yielding a 25% loss. Demonstrating that the utility achieved from this investment, for a two-period investment horizon, gives the investor the same utility irrespective of investing time horizon and takes the form:

$$\begin{aligned} \text{Utility} &= \text{Log}(\text{Wealth}) \\ \text{Starting Utility} &= \text{Log}(100) = 4.60517, \end{aligned}$$

$$\text{Exected Wealth}_{\text{after 1 period}} = 0.5(100 \times 33.33\%) + 0.5(100 \times -25\%) = 104.17$$

$$\text{Utility}_{\text{after 1 period}} = \text{Log}(104.17) = 4.60517,$$

$$\text{Exected Wealth}_{\text{after 2 period}}$$

$$= 0.5[0.5(100 \times 33.33\% \times 33.33\%) + 0.5(100 \times 33.33\% \times -25\%)]$$

$$+ 0.5[0.5(100 \times -25\% \times -25\%) + 0.5(100 \times -25\% \times 33.33\%)] = 104.17$$

$$\text{Utility}_{\text{after 2 period}} = \text{Log}(104.17) = 4.60517.$$

As the utility of the investor is constant at 4.60517 for riskless investment of holding cash, the one period risky investment and the two-period risky investment, one can deduce that the investor receives no benefit from the time diversification concept - under these constraints.

The same can be shown for an investor with an inverse utility function.

Samuelson (1963) refuted the existence of time diversification, by showing that the terminal wealth of investors diverges as the time horizon grows. Samuelson (1969) initially referred to the time diversification conundrum as the fallacy of businessman risk. This is based on the notion that businessmen in their prime will take on investments with higher risks (and hence returns) than retirees/widows. His analysis refers to the following four points as the basis for

businessman risk; businessmen are affluent and have a higher propensity to accept risk. Businessmen have high future expected earnings, and being in the prime of their lives, they can still make up possible losses. Most importantly for the purposes of this paper, businessmen will have long investing horizons, and in accordance with the law of large numbers, their returns should average out.

Samuelson's (1969) frictionless isoelastic utility model concludes that investors have identical relative risk tolerances across all ages. This was used to invalidate the idea of businessman risk allowing for riskier investments during his/her youth. This inadvertently invalidated the existence of time diversification, and has since sparked the large body of literature on the topic.

Samuelson (1994) openly refutes the benefits of time diversification and bases his refusal on the following reasons. He claims that the law of large numbers does not support the notion of risk reducing to zero as the time horizon expands to infinity. Using log and square root utility functions, he again mentions that investors will have the same risk preference at all ages.

The argument sometimes used to convince younger investors into the long-term equity investment, is that when investors are young they have more time to recoup possible losses. Samuelson (1994) adds that a time diversification asset allocation strategy increases one's exposure to a possible market crashes. Due to the limited data time periods, Samuelson (1963) identifies that the long-term data observations that occur as overlapping periods are not independent. Furthermore, this results in a huge reduction in the number of independent long-term observations.

Time diversification is based on two tenets; the convergence of the distribution of annualized returns as the time horizon expands and that the shortfall probability of incurring a loss decreases as the time period expands. Many of the critics agree with these tenets, but conclude that in the event of losing money albeit with a small probability, the size of the loss increases with time. Also, using an expected utility theory approach, Olsen and Khaki (1998) express that the drop in the probability of losses are compensated by the extreme value of the possible loss as time increases.

The investing strategy based on time diversification, is one that allocates a larger proportion of its long-term investment towards a risky asset. This form of advice is often given to clients in the United States. Contenders of time diversification have consistently noted that because investors are faced with a lack of mean reversion in returns and a constant relative risk aversion, time diversification benefits cannot exist (Hansson & Persson, 2000).

Butler and Domian (1991) identify two problems with the estimation of the long-term distribution of equity returns. The lack of risk and return information over long-term holding periods and the time varying variability of returns and risk premia. Thus, we need to be very careful when inferring results from long-term historical data and need to support them with underlying theory and current market sentiment.

3.3 Prior Literature on the existence of Time Diversification

In a study by Kritzman (1994), one of his important discoveries supporting this notion, was that as the investing time horizon expands, the distribution of annualized returns converges towards its mean. This was succinctly expressed in Figure 3.1, presented in his study, that depicts the convergence of the 95% annualized return confidence interval as a function of investing time horizon. In his example, Kritzman (1994) assumes a lognormal distribution of returns where the expected return and standard deviation are assumed to be 10% and 15% respectively.

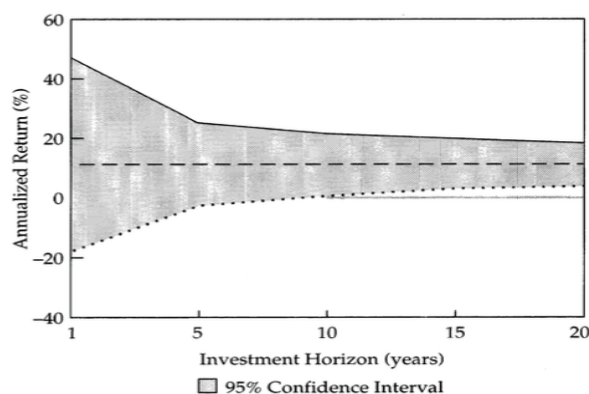


Figure 3.1: Showing the convergence of the return distribution as the horizon expands Kritzman (1994:14)

Samuelson (1963) refuted the existence of time diversification, by showing that the terminal wealth of investors diverges as the time horizon grows. The insights drawn from these results

allowed Kritzman (1994) to conclude that, while it is more probable to make money over a longer time horizon following the time diversification principle, if one were to lose money the scale of this loss would be greater.

To show the comparison between the terminal value of a risk-free asset over time, and a 95% confidence interval of a risky-asset over time, Kritzman (1994) summarized his findings in Table 3.1 below. These results are based on the assumptions of a risk-free rate of 3%, a Standard and Poor 500 (S&P 500) expected return of 10% with a 15% standard deviation in expected returns.

Table 3.1: Showing the confidence intervals of terminal values of risky investments over various time horizons (Kritzman, 1994:15)

The Table displays the S&P 500 Terminal Value 95% Confidence Interval Upper and Lower Bound as well as the guaranteed terminal value of a risk-free assets investment for varying time horizons. These varying time horizons span from one year to a maximum of twenty years.

Time Horizon	S&P 500 Terminal Value 95% Confidence Interval Lower Bound	S&P 500 Terminal Value 95% Confidence Interval Upper Bound	Risk-free Asset Terminal Wealth
1 Year	\$ 81 980	\$ 147 596	\$ 103 000
5 Year	\$ 83 456	\$ 310 792	\$ 115 927
10 Year	\$ 102 367	\$ 657 196	\$ 134 392
15 Year	\$ 133 776	\$ 1 304 376	\$ 155 797
20 Year	\$ 180 651	\$ 2 565 345	\$ 180 611

Table 3.1 displays the dispersion of terminal values between the upper and lower bounds of the risky asset (S&P 500) increases as the time increases. But more importantly, when considering the 95% confidence interval for the investment terminal value at the twenty-year investment horizon, the lower bound for the risky asset still exceeds the terminal value of the risk-free investment over the same horizon. This implies that if an investor were to find himself/herself at the bottom end of the twenty-year 95% confidence interval of the risky asset, he/she would still be better off than had he/she invested in the risk-free asset over the same period. This observation serves as a strong propagator of time diversification.

However, after further assessment of the lower bounds of the 99% and 99.99% terminal value confidence intervals, Kritzman (1994) found that these terminal values fall significantly below the riskless terminal value. These contradictory results cast doubt over the definitive existence of time diversification, which has spurred the extensive research into its validation and invalidation.

This argument does indeed seem convincing in its refutation of time diversification, but Kritzman (1994) puts forward the following reasons as to why an investor may still believe in the benefits of time diversification in the face of this mathematical truth.

The most discernable being that the returns of the risky investment may not be totally random, and instead follow a random walk with drift or a mean reverting process. As mentioned earlier, the extreme values of the confidence intervals that casted doubt on time diversification, could have been stimulated by specific events that could also affect the risk-free assets return. Another challenge is that investors are willing to take on more risk in the long run such that, if returns are lower than expected, a longer horizon could allow investors the opportunity to adjust their lifestyles' accordingly. As oppose to a shorter horizon that would leave investors worse off. Lastly, investors could have discontinuous utility functions, as oppose to those assumed in the mathematical proof above (Kritzman, 1994).

Given the inherent importance of time diversification to all types of investors, as well the broad divide in existing literature on the topic, Thorley (1995) attempts to answer the validity of time diversification by approaching this elusive concept from both a practitioner and academic viewpoint.

From the practitioner stand-point, the consensus on time diversification benefits is that it is fundamentally accepted. Practitioners attempt to take advantage of this belief by allocating a larger proportion of their long-term portfolios towards risky assets. The practitioner based methodology in determining the risk of such a strategy includes constructing tables that compare the confidence intervals of risky asset terminal values against the guaranteed risk-free asset terminal values, for varying time horizons. To take this further, another practitioner defined risk measure is that of shortfall risk (Thorley, 1995).

Shortfall risk is defined as the probability that the risky asset will underperform a pre-specified rate of return. In most cases this pre-specified rate of return is the risk-free rate, hence making shortfall risk the probability that the risky asset will underperform the risk-free asset (Stewart, Heisler & Piros, 2011).

Built on shortfall risk, is the notion of the mean of the risky asset terminal value, given that the risky asset underperforms. This conditional probability, gives practitioners an indication of how much they would stand to lose on average, should their asset underperform. A summary of these risk measures is shown below where the author assumes a risk-free return of 4% and a risky asset return of 12% with a standard deviation of 16% (Thorley, 1995):

Table 3.2: Showing the confidence intervals, shortfall risks and conditional means for various time horizons (Thorley, 1995:69)

The Table displays the guaranteed terminal value of a risk-free assets investment, the mean of the risky asset terminal value investment, the 10th and 90th risky asset terminal value percentile the probability of underperforming the risk free terminal value and the conditional terminal value on under performance of the risk-free investment. These varying time horizons span from one year to a maximum of forty years.

Time Horizon	Risk Free Value	Risky Mean	Risky 10th Percentile	Risky 90th Percentile	Under-performance Probability	Under-performance Risky Mean
1 Year	\$ 1 041	\$ 1 142	\$ 918	\$ 1 384	30.9%	\$ 942
5 Year	\$ 1 221	\$ 1 943	\$ 1 152	\$ 2 882	13.2%	\$ 1 032
10 Year	\$ 1 492	\$ 3 773	\$ 1 736	\$ 6 350	5.7%	\$ 1 222
20 Year	\$ 2 226	\$ 14 239	\$ 4 406	\$ 27 578	1.3%	\$ 1 776
40 Year	\$ 4 953	\$ 202 755	\$ 33 220	\$ 444 451	0.1%	\$ 3 875

Table 3.2 depicts the divergence of the risky asset terminal values as the horizon widens. More importantly for the propagators of time diversification, this divergence of terminal values still remains above the risk-free alternative for ten year and longer periods. Furthermore, the shortfall risk probability (the probability that the risky asset will underperform the risk-free) decreases significantly as the time horizon expands – supporting time diversification.

On the other hand, there exists contention amongst academic researchers, where the critics of time diversification base its irrelevancy on the lack of mean reverting stock prices and expected utility theory. Strong contenders of time diversification, rely on the investor preference models of expected utility theory to prove their views. Mean-Variance optimization leads to the underweighting of risky assets in long term portfolios, which is in direct contrast to the results shown above. Thorley (1995) claims that the use of Mean-Variance optimization effectively applies to portfolio theory under the constraint of a fixed time horizon, but fails to have a practical impact over varying time horizons.

Similarly, Thorley (1995) then analyses the constant relative risk aversion form expected utility theory for investor preferences, and these results contradict the existence of time diversification. The author concludes that while expected utility theory may be a very effective economic theory, it does not fully apply in the case of time diversification. Should a risky retirement investment return extremely bad returns, utility theory does not factor in that investors have other safety retirement nets such as social welfare, family wealth, other investments etc. Based on this, investors will not be as anxious of unlikely extreme negative returns as expected utility theory implies. The foundation of Thorley's (1995) paper is that the use of risk aversion models to refute time diversification, while seemingly convincing, is an inappropriate use of economic theory.

An alternate approach to the traditional time diversification analysis, was proposed by Merrill and Thorley (1996). They attempted to verify the benefit of time diversification by using option pricing theory. The idea of using the option pricing theory to critically analyze the validity of time diversification was first proposed by Bodie (1996). This is to assign a quantitative estimate to the benefit accompanying a time diversification strategy. This method proved insightful as the use of these derivatives precludes any use of economic theory, like expected utility theory and models of risk aversion that have raised concerns earlier (Samuelson, 1963; Kritzman, 1994 & Thorley, 1995).

Their derivative analysis used three derivative instruments as equity insurance for risky portfolios to guarantee the risk-free return if the portfolio underperforms. The three types of derivatives used were the simple put option that required an initial outlay, and two other derivatives that were self-funding, namely the Protected Equity Note and the Self-Funding Market Collar. Most notable amongst their results, was that the cost of one-year equity insurance (cost of option) was much higher than the annual cost of ten-year equity insurance (Merrill & Thorley, 1996). Implying that investing in risky portfolios over a longer time horizon is less risky on an annualized basis than investing over a shorter period.

Looking at time diversification through a behavioral finance lens, allows insight into the decision-making process of investors. Fisher and Statman (1999) used a behavioral framework to invalidate some assumptions used by Samuelson (1963) in his mathematical truth presented earlier. They questioned the assumption that an investors risk aversion is invariant to changes

in wealth, by using Kahneman and Tversky's (1979) prospect theory. They used the notion that investors, contrary to traditional finance assumptions, have asymmetrical utility functions with a strong aversion to losses. Investor's utility is dependent on their change in wealth - not initial wealth. Most importantly, investors exhibit different degrees of risk aversion depending on their change in wealth.

Although the mathematics of Samuelson (1963) is fault-less, his invalidation of time diversification is based on a spurious assumption. This allows us to overlook his mathematical truth in our consideration of time diversification.

In this study, the authors used a block bootstrap method to investigate whether the proportion of stocks to bonds for a mean-variance efficient portfolio does increase in keeping with the time horizon as postulated by time diversification (Hansson & Persson, 2000). They used a moving block bootstrapping to test time diversification in a mean-variance context. They found evidence that supports time diversification. They noted that the relative weights of stocks to bonds increased as the time horizon increased.

It can be shown that time diversification does not exist, under the premise that investors have constant relative risk aversion, asset prices are normally distributed and follow a random walk, and that future wealth is determined solely by asset returns. Strong and Taylor (2001) consider the trade-off between equity and bonds for both the United States (US) and United Kingdom (UK) markets for two centuries worth of data.

Using real returns, with the real return of bonds as the shortfall risk bound. They found that the shortfall risk of US (1837-1996) and UK (1807-1996) equity market, decreased monotonically as the investment horizon increases. The authors found more evidence for time diversification in the US market. To take this further they identified that the US market exhibited signs of time diversification consistently throughout time, but the UK market only exhibited time diversification under certain conditions. They attributed these results to the existence of mean-aversion in the US and UK fixed income market as well as due to the existence of cross-correlations amounts the asset returns in the UK (Strong & Taylor, 2001).

Leibowitz and Langetieg (1989) take a shortfall risk approach in determining the best asset allocation for given time horizons. Their approach was motivated by the asymmetry of returns that become more pronounced as the horizon lengthens, where the variance struggles to be a comprehensive measure of risk. They noted that investors showed more concern over downside portfolio performance. Thus, they focused on the downside risk of portfolio's underperforming a given return level, by using shortfall risk. Another benefit of shortfall risk is that it can be customized to meet the investors personal minimum return level. Comparing the returns of bonds and equities, Leibowitz and Langetieg (1989) computed a ratio of equity to bond portfolio terminal values after varying horizons.

They found that as the horizon expands, the stock to bond ratio increases significantly. From their findings, we can discern that the probability of stocks underperforming bonds decreases steadily as the time horizon increases. The opportunity for equities to outperform bonds also increases as the horizon expands. This exists for both the extreme positive values and the portfolio means. And supports increasing one's equity holdings over longer periods. However, there is an important downside to note, if stocks do underperform, this underperformance is considerably worse than bond underperformance. Overall the authors suggest increasing the weights of long horizon funds towards equities (Leibowitz & Langetieg, 1989). This emphasizes that there exists a time diversification benefit, that the annualized risk of a higher proportioned equity portfolio decreases as the horizon increases.

Leibowitz and Kogelman (1991) used the concept of shortfall probabilities to quantify that for a given shortfall probability level, an investor will increase his/her holdings in equities relative to bonds, as the period expands. They also found that the efficient frontier at longer time horizons is steeper than at lower time horizons. They attribute this to the fact that annualized volatility decreases as the time horizon increases. Their justification of this decreasing annual volatility over longer periods, is because of the square root of time rule (scaling volatility). This indirectly supports the time diversification argument, as the authors increased the weights of equities in longer holding periods, as the annual risk of these equities decreased.

Their method involved, superimposing a shortfall line that split the efficient frontier Cartesian plane into a portion that has a 90% or more probability of meeting the shortfall constraint and

a portion that has a 10% or less probability of failing to meet this minimum requirement (Leibowitz & Kogelman, 1991).

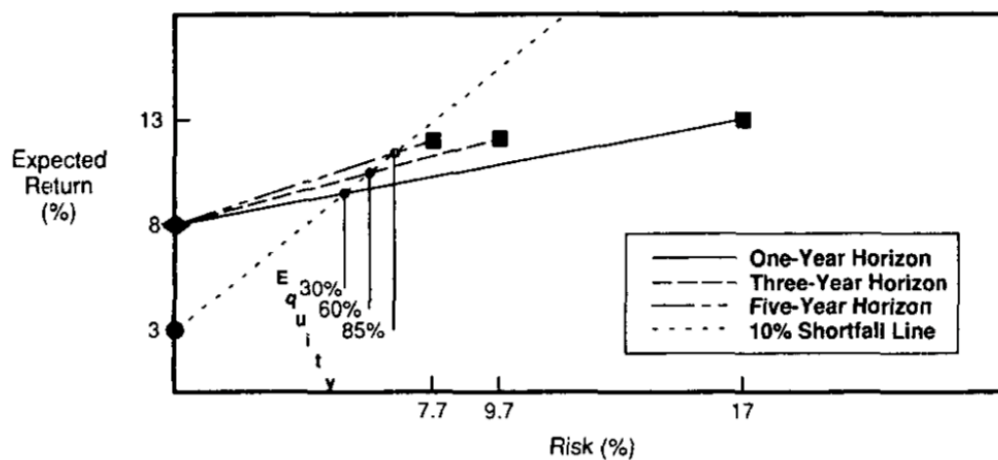


Figure 3.2: Showing the multi-period shortfall constraint graphically on the efficient frontier (Leibowitz & Kogelman, 1991)

In Figure 3.2, the 10% shortfall constraint (dotted line representing cash) is placed on the efficient frontier, that constrains our mean-variance efficient portfolios. The resulting asset allocations are shown for the different horizons, where the longest horizons correspond to the largest equity portfolio construction. But it is important to note, that at the 10% shortfall risk level, a time period of six years or greater results in a 100% equity allocation. They used a shortfall minimum return of 3% and shortfall probability level of 10% (i.e. that the probability that portfolio returns underperform the 3% minimum return is 10% or less). Subject to these constraints, their efficient frontier analysis prescribed a 30% equity and 70% bond portfolio over a one-year horizon. But more notably, when they conducted the same analysis over a five-year horizon their weighting in equities increased to 85% (Leibowitz & Kogelman, 1991).

The authors base the drop in annualized volatility from 17% to 7.7% on a Random Walk Model, due to the square root of time volatility scaling rule. According to the square root of time rule, the total volatility of the period will increase, but the annualized volatility will decrease. Over the longer horizon, they predict that one's risky asset appetite will be sensitive to the level of equity risk premium in the market (Leibowitz & Kogelman, 1991).

McEnally (1985) uses statistical theory in conjunction with the independent and identically distributed assumption of returns to conclude that an investor will be indifferent between investing over a long and short horizon as he/she is subject to the same set of risks every period.

Investors reduce their portfolio's risk through diversification of their portfolios by combining assets with less than perfect correlations. Lee (1990) attempts to determine whether investors can diversify their portfolios through optimization of their holding period as well. Lee (1990) goes further, and breaks down the diversification benefit into two aspects. He attributes the diversification benefit to the non-stationary asset return distributions instead of risk-pooling. Lee (1990) found that equity returns follow a mean-reverting pattern, and as a result have a predictable transitory component. This predictable transitory component, allows analysts to predict that periods of high equity returns will be followed by periods of low equity returns and vice-versa, both around the long-term mean.

To determine the maximum risk reduction benefit gained through time diversification, Lee (1990) found that this benefit does not increase with the horizon indefinitely, but rather crests at the 3-year horizon. He concludes that time diversification of investments in equities over bonds does benefit investors, but only in the absence of a Random Walk Return distribution.

Butler and Domain (1991) found that as equities are held for longer horizons, their average annual returns converge, while their terminal values diverge. The authors note the Black-Scholes option pricing model, assumes that the volatility of stock returns scale over different horizons according to the square root of time rule (Butler & Domian, 1991). McEnally (1985) uses this square root of time assumption to argue that the risk of an equity portfolio increases with time. Up until this point we have used the first definition of risk being the variability in returns, but here McEnally (1985) refers to the definition of risk such that risk represents the variability in terminal values.

However, Butler and Domian (1991) argue that a risky portfolio may outperform a less risky portfolio, if held for a long-time horizon with the diversification benefit of convergence of higher expected returns. Using an example with inflation-adjusted returns, they proved this using a combination of 39 twenty-five year periods. Their results indicated that their equity investment outperformed the treasury bond investment over every twenty-five-year period. To

stress the benefits of time diversification further, they found that the highest twenty-five-year bond terminal value was below the lowest twenty-five-year equity terminal value. Relatively, the highest twenty-five-year bond terminal value was just over an eighth of the value of the maximum equity terminal value.

Their twenty-five year periods overlap and hence are not independent. They used inflation adjusted returns, as the distribution of inflation adjusted returns is more stable over time. In addition, the removal of inflation from our returns allows us to compare the terminal value of investment returns according to today's purchasing power. With respect to shortfall risk, they found that there is a 5% chance over the entire period, that a twenty-year equity investment will underperform its treasury bond counterpart (Butler & Domian, 1991)

In a study by Gollier (2002), he found that liquidity constraints on investments inherently reduce the investors implicit investing horizon. After analyzing this further, it was found that the reduction in time horizon, causes an increase in one's risk aversion. Looking at the other side of the spectrum, richer people face less of a liquidity constraint than the poor, and are less risk averse as a result.

The only study (to the best of our knowledge), that encompassed time diversification directly in emerging markets is that of Anderson, Malone and Marshall (2012) where South Africa was included. The search strategy employed for time diversification studies involved a comprehensive search of Google Scholar, EBSCOHost and JSTOR. The authors took an American investor point of view, and thus their U.S. Dollar denominated returns encompass the currency fluctuations. Their holding periods ranged from one-year to forty years. Let us consider their distribution of annualized returns for both developed (Figure 3.3) and emerging (Figure 3.4) economies:

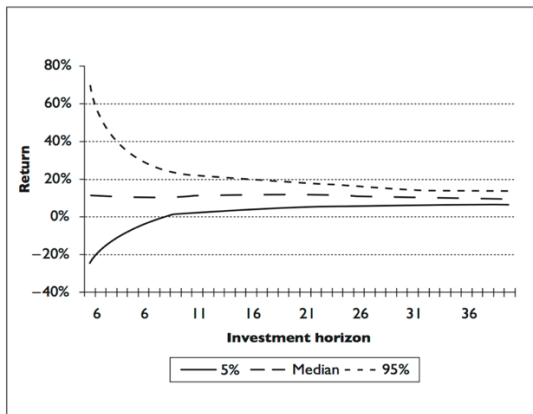


Figure 3.3: Showing the convergence of the return distribution for developed markets (Anderson, Malone & Marshall, 2012:135)

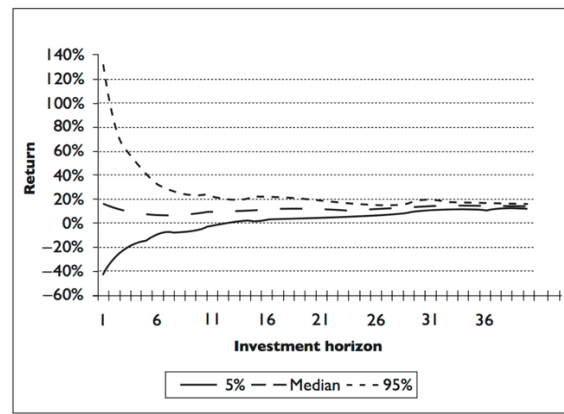


Figure 3.4: Showing the convergence of the return distribution for emerging markets (Anderson, Malone & Marshall, 2012:136)

It is observable that extreme positive and negative returns are very time horizon sensitive. At the one-year (smallest horizon considered in this study) time horizon, the 95% confidence interval of returns for a developed country ranged between -24% and 70%, while emerging economies yielded between -44% and 132%. However, as the time horizon expands, the distribution of annualized returns converges for both developed and emerging markets. Other downside metrics, like the Value at Risk, become positive after an eight-year holding period in developed countries and a thirteen-year holding period in emerging economies (Anderson, Malone & Marshall, 2012).

The smoothness of the develop market graph relative to the emerging could be signs of market inefficiencies, liquidity or concentration issues. Anderson, Malone and Marshall (2012) concluded that investors in both emerging and developed markets that have long time horizons, should increase their risky asset (equity) exposure to take advantage of the time diversification benefits.

3.4 Prior Literature on International studies on the distribution of returns

In Lo and MacKinlay's (1988) paper on stock price distributions, they tested the Random Walk Model of weekly stock prices over a twenty-three-year period. Their justifications for using weekly data are that although daily observations could have increased the number of observations, it would also induce unwanted biases (nontrading, bid-ask spread etc.). Using longer frequency data would have drastically reduced the number of observations and possibly smoothed out certain stock price effects. They felt that weekly data was the perfect

compromise, as it allows for a relatively large number of observations while minimizing the biases of smaller frequency data.

The authors rejected the Random Walk hypothesis using weekly data in their cross-sectional study. It is important to note, that the authors tested to see if the autocorrelation they found in returns was due to thin trading/nontrading. They found that their rejection of the Random Walk model was not due to thin trading of illiquid shares. Most importantly in their rejection, they did not indicate which stochastic process the distribution of stock prices more accurately conforms to, but rather that stock prices do not follow a Random Walk (Lo & MacKinlay, 1988).

Best, Hodges and Yoder (2015) analyzed US annual data since 1926. They found that in the absence of autocorrelation in returns, the percentage of bonds/equity in an optimal portfolio increases from 60%/40% at a one-year horizon to 95%/5% at a 25-year horizon. These results differ significantly when autocorrelation in returns is observed. In this case, they found that the optimal percentage of bonds/equities decreases from 60%/40% at a one-year horizon to 10%/90% for any horizon greater than 16 years. This further emphasizes the importance of our JSE stock price distribution (the degree of autocorrelation) on the existence of time diversification benefits.

In the study by Lee (1990), he found that stock prices follow Mean Reversion as their predictable transitory component (ρ in Equation 7) was statistically significant across all time horizons. Furthermore, it was notable that the explanatory power of the aforementioned component was maximized at the three-year horizon. This does indicate that the benefits of diversifying across time does not increase linearly to infinity. Furthermore, when calculating the standard errors in Table 3.3 below, Lee (1990) used the White correction for heteroscedasticity. Lee (1990) used logarithmic real returns of the Standard and Poor Composite index pre-1986. A range from one-year to ten year non-overlapping periods were used. His findings are summarized as:

$$\begin{aligned} [r_{t+N,N} - r_{t,N}] &= -\gamma[r_{t,N} - r_{t-N,N}] + e_{t+N} \\ r_{t+N,N} &= (1 - \gamma)r_{t,N} + \gamma r_{t-N,N} + e_{t+N} \end{aligned}$$

Horizon N Years	Sample Size	γ	R ²	Durbin- Watson
1	60	0.402*** (0.114)	0.16	2.26
2	30	0.586*** (0.163)	0.35	2.47
3	20	0.454** (0.179)	0.43	1.23
4	15	0.512** (0.182)	0.30	1.74
5	12	0.624** (0.205)	0.38	1.64
10	6	0.579* (0.271)	0.25	2.28

Table 3.3: Showing the Mean Reversion transitory component 1926-1985 (Lee, 1990:24)

3.5 Prior Literature on South African JSE return distributions

Cross-sectional mean reversion studies have been conducted on the JSE where Cubbin et al. (2006) followed the approach of De Bondt and Thaler (1985) using Price/Earnings ratio ranking methodology. From this analysis, they concluded that there existed strong evidence for mean reversion in share prices on the JSE between 1983-2005. However there exists a lack of time series studies of this nature conducted on the JSE.

An early study by Affleck-Graves and Money (1975) into the distribution of JSE returns, tested the Random Walk Model on the JSE, once again using cross-sectional data. For this study, the authors chose weekly data as well as fifty individual share returns to analyze. The period under analysis was from 30/04/1968 to 21/09/1973, using logarithmic returns. Initially they review the arguments in favor and against the Random Walk Model. This discussion is grounded on the ideology of fundamentalists who believe that equity markets are efficient and that there exists no consistently exploitable pattern in stock prices. This grounding is challenged by the technical school of thought, who believe that the distribution of information is imperfect.

After calculating the autocorrelation co-efficient for these fifty JSE listed shares, the authors found that most the autocorrelation coefficients were not significantly different from zero. But it is very important to note that this study only considered weekly data, with the maximum lag being twenty weeks in its cross-sectional analysis. Thus, these results do support the Random Walk Model, but this holds for short term (up to twenty week) returns.

Going back to the Firer and McLeod (1999) study, they found positive serial correlation in real equity returns at the one-year level, but for frequencies greater than one-year, they found significant negative serial correlation. As a result, this study needs to ensure that sufficient frequencies are used when analyzing the autocorrelation in stock prices and the behavior of returns at different measurement intervals. When analyzed over the period 1925 to 1998, the authors identified that equities performed the best when compared to bonds and cash. This additional return was accompanied by higher standard deviations. Most importantly, the authors found that when measured over longer return horizons, the annualized risk of equities decreased to within the risk levels of the other asset classes while maintaining their superior returns. This provides a basis for time diversification in historical South African markets, and this paper will endeavor to uncover if this phenomenon exists today.

Table 3.4 depicts the serial correlations identified by Firer and McLeod (1999) over the period 1925-1998 when assessing annual, three, five and ten yearly returns. It is very important to note that over this 74-year return period, the authors used non-overlapping data. The distinction between real and nominal bond and equity returns is significant, as the signs of the correlations change depending on the effect of inflation. This result prompted the use of both real and nominal returns for equities and bonds in our study. In the Table below, this study notes that the serial correlation for nominal equities at the five and ten-year level appear very high.

Table 3.4: Showing the serial correlations identified by Firer and McLeod (1999:23) for annual, three-year, five-year and ten yearly returns over the period 1925 – 1998.

The Table displays the serial correlations identified by Firer & McLeod (1999) for nominal and real equity, bonds and cash returns. Conducted over the period 1925 – 1998.

Serial Correlation	Nominal Equity	Nominal Bonds	Nominal Cash	Real Equity	Real Bonds	Real Cash
Annual	0.12	0.10	0.93	0.11	0.05	0.71
3 Years	0.89	-0.31	0.56	-0.41	0.41	0.50
5 Years	0.85	0.15	0.60	-0.07	0.37	0.55
10 Years	0.63	-0.45	0.82	-0.83	-0.30	-0.25

A study by Mangani (2007), investigated the distributional properties of JSE returns. The purpose of his study was to determine the relevance, of the application of traditional asset pricing models on the JSE. First, the author tested the stationarity of JSE logarithmic returns and prices, using the Dickey Fuller and Augmented Dickey Fuller tests. Additionally, the author employed the method of Perron that tested the stationarity in the presences of possible

structural breaks in the data. However, the author specified his structural break dates around socio-economic and political events in a South African context as oppose to using the unknown structural breakpoint tests used in this report. These ranged from the Soweto Uprising on June 16th 1976 to the Asian Crisis in late 1998.

These stationarity tests re-enforced that JSE prices are non-stationary, while logarithmic returns were found to contain a unit root (are stationary). The author went further to conclude that this formed prima facie evidence against the Random Walk hypothesis. This could mean that JSE returns are not in continuous stochastic equilibrium and could offer return predictability.

Mangani (2007) thereafter, used the normality and linearity tests to assess the properties of a normal strong Random Walk on the JSE. The normality tests comprised of the skewness statistic, level of kurtosis and the Jarque-Bera test for normality. The computed skewness and kurtosis statistics were compared to the normal distribution level of skewness of zero and kurtosis of three. The return series on the JSE was found to exhibit leptokurtosis, which again does not support the Random Walk hypothesis. This result is consistent with other local and international literature. In terms of skewness, it was found that the JSE All share index exhibits negative skewness. Their Jarque-Bera test, significantly rejected the null hypothesis of normality in JSE returns and prices. These results are consistent with result observed in other markets, but implies that normality is not an acceptable assumption in the analysis of JSE returns.

For the linearity tests, the author performed an Engel Test as well as a BDS independence test. Both were selected due to them being robust to non-linear data. The results of the Engel test, proved that the ALSI returns are non-linear. The BDS test confirmed that the JSE returns violate the independent and identically distributed (i.i.d) assumption, and together with the results of the Engel test, form evidence that the JSE returns do not follow a Random Walk stochastic process (Mangani, 2007).

The most pertinent South African study for the purposes of this paper is, *A Note on the riskiness of long term investments on the JSE* by Bradfield and Ardington (1997). This study on JSE returns, aimed to identify any predictable trends in returns and on the risk over long investment

horizons. In their paper, the authors used the sample variance as a proxy measure for risk. Under the Random Walk Model and Efficient Market Hypothesis, the series of returns are assumed to be random and to exhibit no serial correlation to their lagged values.

The purpose of the Bradfield and Ardington's (1997) paper is to determine whether JSE returns are non-random, and if so, to warn the dangers of looking at a risk measure like variance that has not been calculated at the same interval as the returns. This is due to non-random returns violating the square root of time rule, that states that the variance of an investment over n periods is n times the variance of one period. In the case of a non-random series, the variance of an investments returns will not scale up with time linearly (Bradfield & Ardington, 1997).

Bradfield employed ALSI total return data between 01/01/1980 to 01/01/1996. Preliminarily Bradfield and Ardington (1997) found that based on variances, that the JSE returns seem to become less risky over time, together with a perceived decrease in the market premium. Figure 3.6 shows these that the dispersion of returns seems to decrease over time, and more importantly, that the series of annual JSE returns seems to be mean reverting.

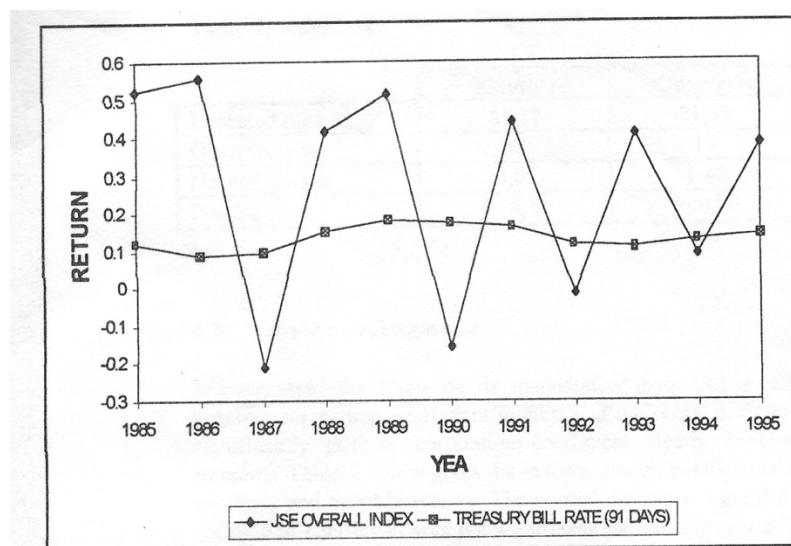


Figure 3.5: Showing the behavior of annual JSE All Share Index returns and the 91-Day Treasury Bill Rate between 01/1985 and 01/1996 (Bradfield & Ardington, 1997)

The first test employed by Bradfield and Ardington is the non-parametric Runs test (1997). This test tests whether the number of consecutive return runs (greater than or lower than the mean) are random. This is achieved by comparing the number of expected runs under a Random Walk Model against the number of observed runs under the JSE empirical data. If the

number of observed runs is below the number of expected runs under the Random Walk Model, it implies that the length of the runs is longer than expected under a Random Walk Model and thus the series is trending. If the number of expected runs under the Random Walk Model is below the number of empirically observed runs, it implies that the length of the runs is shorter than expected under a Random Walk Model and thus the series is mean reverting. This was performed for annual, quarterly and monthly data, where the authors found significant mean reverting behavior in annual returns. Table 3.5 summarizes their Runs test findings.

Table 3.5: Showing the Runs Test results for Bradfield and Ardington (1997)

The Table displays the results of the Runs test conducted by Bradfield and Ardington (1997) for monthly, quarterly and annual JSE All Share index returns over the period 01/1985 to 01/1996. This data is comprised of the JSE Actuaries Index total return.

	Monthly	Quarterly	Annual
Expected Runs	61.17	21.39	6.09
Observed Runs	61	17	9
Normal deviate	0.03	1.40	2.02
P-value	P>0.20	P>0.20	P<0.05*

*significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Thereafter, the authors performed an Autocorrelation Test (also referred to as the Autocorrelation Function), where they regressed the series on returns on its single-lagged version of itself. This was conducted for monthly, quarterly and annual returns. The results of this test are presented in Table 3.6. This test did find the autocorrelation coefficient for annual data to be weakly significant at the 10% probability level, but more importantly the authors observed positive autocorrelation coefficients for monthly and quarterly data. This implies a form of short term trending behavior. But the annual data had a negative autocorrelation coefficient, indicating mean reverting behavior in annual returns.

Table 3.6: Showing the Autocorrelation test results for Bradfield and Ardington (1997)

The Table displays the results of the Autocorrelation test conducted by Bradfield and Ardington (1997) for monthly, quarterly and annual JSE All Share index returns over the period 01/1985 to 01/1996. This data is comprised of the JSE Actuaries Index total return.

	Monthly	Quarterly	Annual
Auto Correlation Coefficient	0.098	0.134	-0.540
Normal Deviate	1.077	0.887	-1.709
P-value	P>0.20	P>0.20	P<0.10

*p-value<0.05, **p-value<0.01, ***p-value<0.001

Lastly, Bradfield and Ardington (1997) employed the Variance Ratio Test. This test, is used to determine whether the variance of returns scales up linearly with time or not. A Variance Ratio of one implies that the variance of short term returns scales up linearly to the variance of long term returns. If the variance ratio is below one it means that the shorter period returns mean revert, while a variance ratio of above one implies that the shorter period returns trend. These statistics were calculated for monthly, quarterly, annual, three and five-yearly. The results are shown in Table 3.7. The authors concluded that monthly and quarterly data trend within an annual period, while annual data exhibits mean reversion within the three and five-yearly return intervals.

Table 3.7: Showing the Variance Ratio Test results for Bradfield and Ardington (1997)

The Table displays the results of the Variance Ratio test conducted by Bradfield and Ardington (1997) for monthly, quarterly and annual JSE All Share index returns over the period 01/1985 to 01/1996. This data is comprised of the JSE Actuaries Index total return.

	Quarter/ Month	Year/ Quarter	Year/ Month	3 Year/ Year	5 Year/ Year
Variance Ratio	1.257	1.518	1.908**	0.287***	0.545*
Normal Deviate	1.898	1.772	2.664	-5.264	-2.285
P-value	P<0.10	P<0.10	P<0.01	P<0.0001	P<0.05

*p-value<0.05, **p-value<0.01, ***p-value<0.001

In their concluding remarks, the authors concede that there exists statistically significant mean reversion in annual returns with statistically significant trending behavior in monthly returns. These phenomena translate into the overstatement of longer period variances when scaling up annual variances, as well as an understatement in the annual variance when scaling monthly return variances.

3.6 Summary and conclusions

This Chapter has presented the arguments of the propagators of time diversification and their academic contenders. This Chapter has also highlighted the literature gap in emerging market literature on the time diversification topic. The inconclusiveness of this phenomenon was noted. However, the common ground between those in favor and those against is that there exist time diversification benefits if there exists mean reversion in returns (Samuelson, 1963: Kritzman, 1994).

This literature review provides the foundation of this study and identifies the need for a study of this nature to be conducted on the JSE. Internationally, rejection of the Random Walk Models has been noted as well as a three-year Mean Reversion benefits in the US. South African cross-sectional studies have differing results with Cubbin et al. (2006) identifying Mean Reversion in share prices and Affleck-Graves and Money (1975) finding a lack thereof. In terms of Time-Series studies, authors have identified autocorrelation, rejections of the Random Walk Model and Mean Reversion at the one-year level (Firer & McLeod, 1999; Mangani, 2007). The most important study conducted on the JSE was that of Bradfield and Ardington (1999) where the authors identified significant short-term trending behavior in monthly returns, followed by Mean Reverting behavior in annual returns over a 17-year period.

4. Data

4.1 Introduction

The purpose of this chapter is to familiarize the collection and construction methods of the datasets as well as to mention the datasets that are used for the analysis in later chapters. Due to the nature of time series and time diversification analysis, a very large set of time series data is required. Given South Africa's relatively small exchange history, this paper aims to maximize the possible data sample. This dataset is comprised of historic and current JSE ALSI price and total return data, JSE ALBI price and total return data, South African CPI Inflation data and lastly real (inflation adjusted) returns for the ALSI and ALBI datasets.

Each of the datasets were collected and returns were calculated individually for the following frequencies; monthly, quarterly, semi-annually, annually, two yearly, three yearly, five yearly, ten and twenty yearly. This was performed to ensure that tunnel vision was avoided in the Autocorrelation tests, as the serial correlation in South Africa is known to change sign at varying frequencies (Firer & McLeod, 1999).

This research paper employs total returns, because total returns by definition, include an income component. This mirrors more closely the returns that investors face, and avoids the issues of share prices dropping on their ex-dividend date, and biasing our results.

This dissertation has decided to partition each of the above datasets, (for all frequencies) into three samples. The first sample comprises all available data for that variable. The second sample comprises of all available data for that variable up to and including 31/12/1986. Ultimately, the last sample consists of all the available data for that variable from 01/01/1987.

Non-overlapping samples for each variable at each frequency were used, to avoid biased statistical test statistics and invalid p-values. The majority of the data analysis for this paper was conducted through Econometrics Views (E-Views) except for the merging of the data being conducted through Microsoft Excel.

The balance of the Chapter is organized as follows: Section 4.2 addresses data collection and construction of the ALSI data, as well as total returns calculations. Section 4.3 describes the collection and calculation of ALBI (used as an alternative investment to equities to later Chapters). Section 4.4 discusses the collection of historic South African Consumer Price Index (CPI) inflation data that is used in the Section 4.5. Section 4.5 uses the South African Inflation data to calculate a set of real (inflation-adjusted) total return. Finally, Section 4.6 summarizes and concludes.

4.2 JSE All Share Index Data

This dataset forms the most important for our series time analysis. As a result, we need JSE All Share price and total return data from as far back in time as possible.

ALSI data was collected over the period 01/01/1900 – 31/12/2016 for annual, two, three, five, ten and twenty yearly frequencies, and collected over the period 31/01/1925 – 31/12/2016 for monthly, quarterly and semi-annual frequencies. This was performed due to the lack of monthly, quarterly and semi-annual data between 01/01/1900 and 31/01/1925. A brief overview of the JSE index series is presented in Section 4.2.1. This dataset was collected from three sources. The most recent data collection using the FTSE/JSE All Share code J203T will be discussed in Section 4.2.2. Section 4.2.3 deals with the construction of a total return index for the JSE All Share code (under the JSE Actuaries Index Series) AJ203. The earliest set of data that was available for our analysis was that of Firer and McLeod (1999), this data set will be presented in Section 4.2.4. Lastly we will discuss the merging procedure in Section 4.2.5.

4.2.1 Overview of JSE Index Series

The JSE was founded in 1887, to aid in the finance of early mining operations in South Africa. Since then, the JSE has grown to the 19th largest exchange in the world by market capitalization and is currently the largest stock exchange in Africa based on market capitalization as at the 07/01/2017 (JSE, 2017).

Historically the JSE had used the JSE Actuaries Index Series. This Index Series was founded in 01/10/1978, but on the 24th of June 2002, in a joint venture between the JSE Limited and the Financial Times Stock Exchange (FTSE) Group, the JSE changed their index classification to

the FTSE/JSE Africa Index Series (JSE, 2017). This resulted in differing index and sector classifications, as well as a more internationally aligned set of indices that comply with the International Organization of securities Commissions (IOSCO) recommendations. Prior to the JSE Actuaries Index Series, the Rand Daily Mail Industrial Index (RDM100) and Bureau of Economics (BER) Indices were used (Firer & McLeod, 1999).

Limited JSE historical data combined with changes in the JSE indices, prompted the creation of this papers' merged JSE All Share Index dataset that encompasses the longest period of JSE All Share data available.

4.2.2 J203T data

The most current JSE All Share Index is the J203T (I-Net code) which forms part of the FTSE/JSE Index Series. This index represents the total return index of the FTSE/JSE All Share Index. This index also represents 99% of the free-float market capitalization on the JSE. The J203T dataset was collected from as far back as possible from 06/30/1995 to 12/31/2016, from the I-Net terminal at the University of Cape Town Libraries.

4.2.3 AJ203 data and total return index construction

The AJ203 was collected off the I-Net terminal at the University of Cape Town Libraries for the period 29/02/1960 to 31/12/2016. This index is not a total return index, and as a result, this paper was required to also collect the AJ203 Dividend Yield data over the period. This was performed to create a total return index. Utilizing the method of Collin and Firer (1999), this study accounted for dividends by using the annual dividend yield of the index to calculate the dividend that would have been received - given a few assumptions. These assumptions are that dividends are received mid-way through the month, and that these mid-monthly dividend receipts occur evenly throughout the year. The dividend paid adjustment employed by Firer and McLeod (1999) is:

$$Dividend\ Received_i = \frac{[DY_{i-1}Price\ Index_{i-1} + DY_iPrice\ Index_i]}{2400}, \quad (8)$$

where *Dividend Received*_{*i*} represents the dividend received for month *i*, *DY*_{*i-1*} represents the annual dividend yield at time *i - 1*, *DY*_{*i*} represents the dividend yield at time *i*, *Price Index*_{*i-1*} represents the value of the price index at time *i-1* and *Price Index*_{*i*} represents the value of the price index at time *i*.

To calculate a synthetic total return index for the AJ203 (hereafter referred to as AJ203T*), we need to make the following adjustment to our price index such that:

$$Total\ Return\ Index_i = Price\ Index_i + Dividend\ Received_i. \quad (9)$$

4.2.4 Firer and McLeod data

In a study by Firer and McLeod (1999), they compiled a dataset of JSE equity total returns pre-1960 based on a research paper by the Bureau for Economic Research (BER) published in 1948 in conjunction with the Rand Daily Mail Industrial Index (RDM100). In compilation of this dataset, the authors were faced with some missing data. They found only a price series for the RDM100 over the years 1946 and 1959, missing the dividend component.

To combat this lack of an income component, the authors assumed a constant dividend yield for that period of 5%. The authors tested this assumed dividend yield against alternatives 1% below, and found the change in total arithmetic returns to be immaterial. Figure 4.1 below is taken from Firer and McLeod (1999) study and depicts a graph of historical JSE All Share Index dividend yields. In addition, the Figure highlights the period over which the authors have made their assumption of a 5% dividend yield. We observe from this Figure, that the assumption of 5% appears reasonable in comparison to the pre-and post-dividend yields. The Firer & McLeod (1999) dataset was collected from Professor Paul van Rensburg from the University of Cape Town, who in turn, received the dataset directly from the authors.

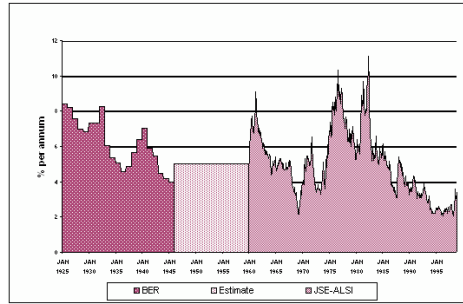


Figure 4.1: Showing the assumed 5% JSE All Share dividend yield amongst the historical JSE dividend yields (Firer & McLeod, 1999:10).

4.2.5 Merging the Datasets

Due to the ALSI data being collected from three different classifications, the accurate re-scaling at the transition points, to account for any index construction differences, was required. In doing so, this paper chose the J203T as the main classification metric, as it is current and will allow for a seamless addition to this study in the future. At the last overlapping date between the J203T and the AJ203T* (30/6/1995), the AJ203T* was re-scaled by multiplying the AJ203T* by a constant scaling factor, such that there is a seamless transition from the J203T to the AJ203T*. The Firer and McLeod (1999) total return index was recursively applied to the first total return index value of the AJ203T* on 02/29/1960 to produce a continuous total return index, using the scaling method mentioned above.

This unconventional method of data collection and merging is shown in Table 4.1. Table 4.1 shows the data collection composition for annual, two, three, five, ten and twenty yearly data, as well as the merging dates. Furthermore, Table 4.1 notes that the data collection period for monthly quarterly and semi-annual data began in 1925 as oppose to the annual frequency and above that was available from 1900.

Table 4.1: Data collection composition for JSE All Share Total Return Index data

The Table displays the monthly, quarterly, semi-annual, annual, two yearly, three yearly, five yearly, ten and twenty yearly JSE All Share Index Total Return Index composition. This composition is broken up into three data collection periods with different sources. This Table displays each of the three aforementioned data sources, the periods for which they were collection and their respective data tag. The entire data collection period spans 01/01/1900 to 31/12/2016. Apart from monthly, quarterly and semi-annual data being collected from 31/01/1925.

Period	Data tag	Data Source
01/01/1900 - 2/29/1960*	Firer and McLeod Data – BER and RDM100	Firer and McLeod Data
2/29/1960* – 30/6/1995*	AJ203T*– JSE Actuaries All Share Index	I-Net
30/6/1995* – 13/12/2016	J203T – FTSE/JSE All Share Africa Index Series	I-Net

*represents dataset merging dates

The above data merger, resulted in a JSE All Share Index data set that runs from 01/01/1900 to 31/12/2016 when using series data of a frequency higher than semi-annual, and a data set that runs from 31/01/1925 to 31/12/2016 when using series data of a frequency less than one-year. These merged datasets take the form of Figure 4.2 when using monthly price series (logged for the purpose of interpretation) and the form of Figure 4.3 when using annual price series (logged for the purpose of interpretation).

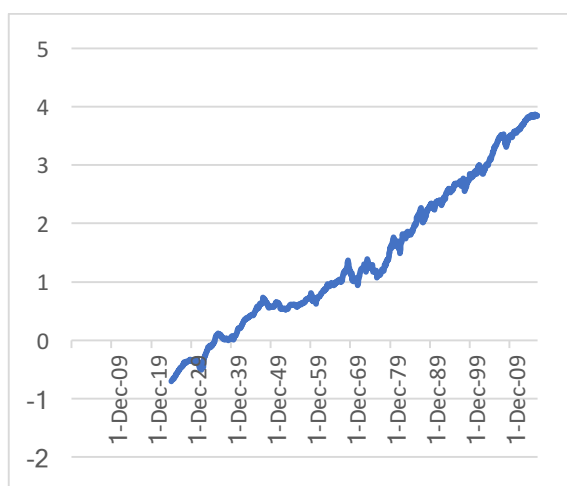


Figure 4.2: Showing Log JSE All Share Price Total Return Index over the period 31/01/1925 to 31/12/2016 using monthly price series data

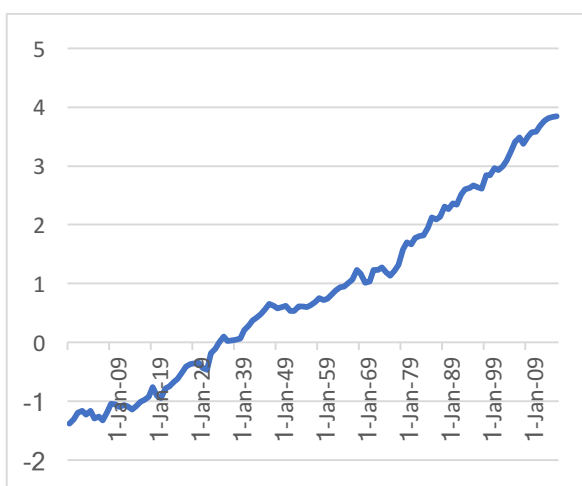


Figure 4.3: Showing JSE All Share Price Total Return Index over the period 01/01/1900 to 31/12/2016 using annual price series data

From both Figures above, the 1971 Latin Debt crisis and the 2008 Financial Crisis are easily discernable crashes that are apparent in both data frequencies. Figure 4.2 shows more variability in monthly returns, while Figure 4.3 seems to smooth out many of these effects - while keeping the general trends.

4.3 JSE All Bond Data

The ALSI represents one significant portion of the JSE, but this report would not be complete without the consideration of an alternate asset class. Thus, bond data was also required. This prompted the use of either the JSE All Bond Index (ALBI) or the Government Bond Index (GOVI). The GOVI is investible, while a small portion of the ALBI is non-government bonds. Since the ALBI is a good representation of what a typical bond manager would hold against equities, we chose the ALBI for this study. Prior to 1998 there existed no central Bond data

collection index (Firer & McLeod, 1999). Due to the short nature of listed Bonds on the JSE, this paper requires ALBI data from as far back as possible.

The ALBI data was collected off the I-Net terminal at the University of Cape Town Libraries over the period 31/12/1998 to 31/12/2016. This data was collected at the aforementioned return frequencies. Figure 4.4 depicts the JSE All Bond Index price series using monthly data, while Figure 4.5 depicts the JSE All Bond Index price series using annual data.

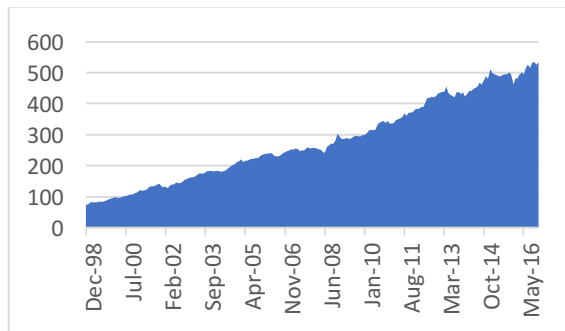


Figure 4.4: Showing JSE All Bond Price Index over the period 31/12/1998 to 31/12/2016 using monthly price series data

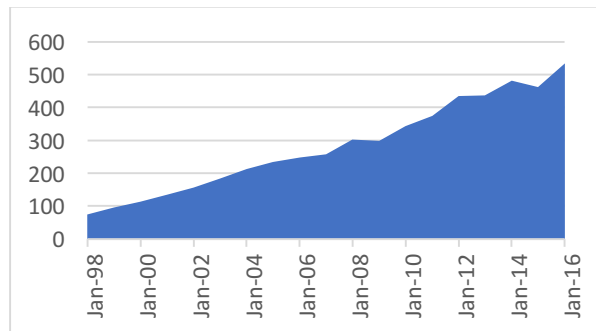


Figure 4.5: Showing JSE All Bond Price Index over the period 31/12/1998 to 31/12/2016 using annual price series data

From both Figures above, it is observed that the 2008 Financial Crisis did not have an extreme crash effect on the ALBI as was observed in Figures 4.2 and 4.3 for the ALSI. Less fluctuations in the price series distributions were observed for the ALBI than the ALSI, which stems from the ALBI being inherently less risky than the ALSI.

4.4 Nominal Return Calculations

Both arithmetic and logarithmic returns were calculated, to identify nuances in their return distributions. The calculation of the arithmetic total return for period i is as follows:

$$\text{Arithmetic Total Nominal Return}_i = \frac{\text{Total Return Index}_i - \text{Total Return Index}_{i-1}}{\text{Total Return Index}_{i-1}}, \quad (10)$$

where $\text{Total Arithmetic Return}_i$ is the total arithmetic return for period i , $\text{Total Return Index}_i$ represents the value of the total return index at the end of period i and $\text{Total Return Index}_{i-1}$ represents the value of the total return index at the end of the period $i-1$. The calculation of the log total return for period i is given as follows:

$$\text{Logarithmic Total Nominal Return}_i = \text{LN} \left(\frac{\text{Total Return Index}_i}{\text{Total Return Index}_{i-1}} \right), \quad (11)$$

where *Total Logarithmic Return_i* represents the logarithmic total return for period *i*.

Figure 4.6 depicts the nominal logarithmic return series over the period for annual returns, while Figure 4.7 depicts the nominal arithmetic return series for annual return over the entire period. The Figures below also plot the mean return over the period in orange.

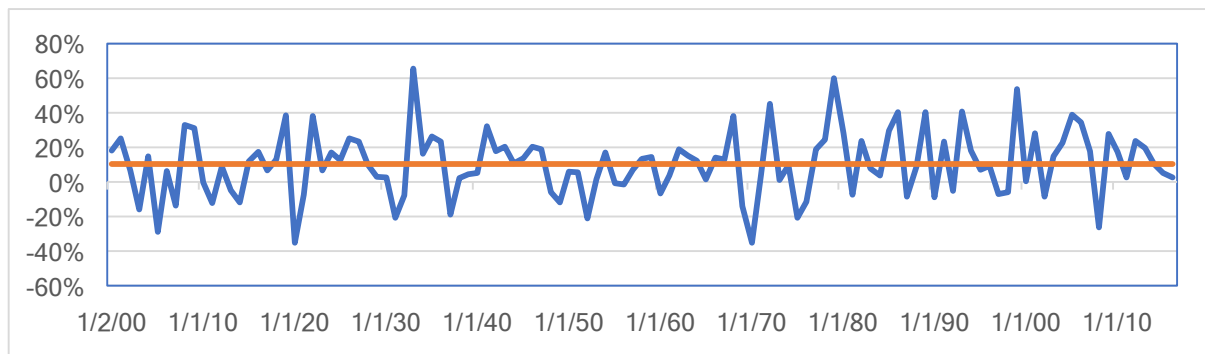


Figure 4.6: Showing Logarithmic JSE All Share Index Returns over the period 01/01/1900 to 31/12/2016 using annual return data

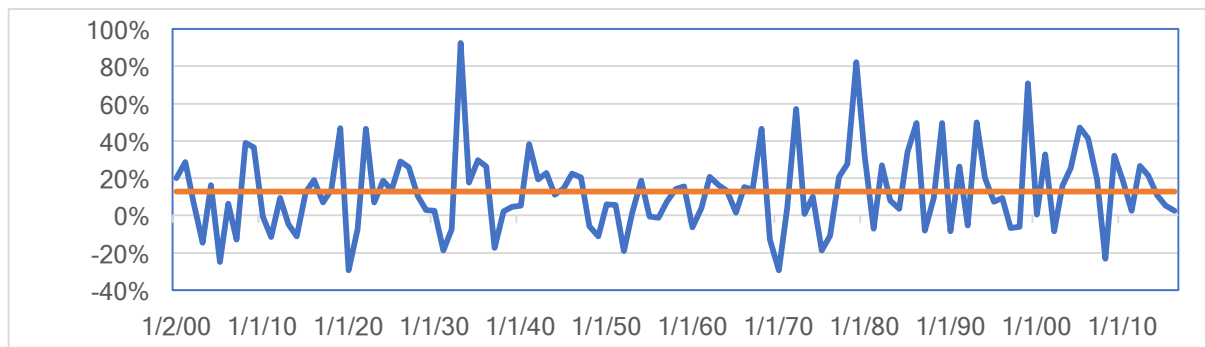


Figure 4.7: Showing Arithmetic JSE All Share Index Returns over the period 01/01/1900 to 31/12/2016 using annual return data

From Figures 4.6 and 4.7, it is observed that these Figures possibly exhibit mean reverting tendencies. This hypothesis will be statistically tested in Chapters 5 and 6 of this research paper. Focusing on the logarithmic returns, the logarithmic nominal ALBI returns are presented in Figure 4.8 for annual returns. These returns do not appear to follow a general trend over the entire period, but post-2008 crisis annual returns show signs of recurring behavior patterns.

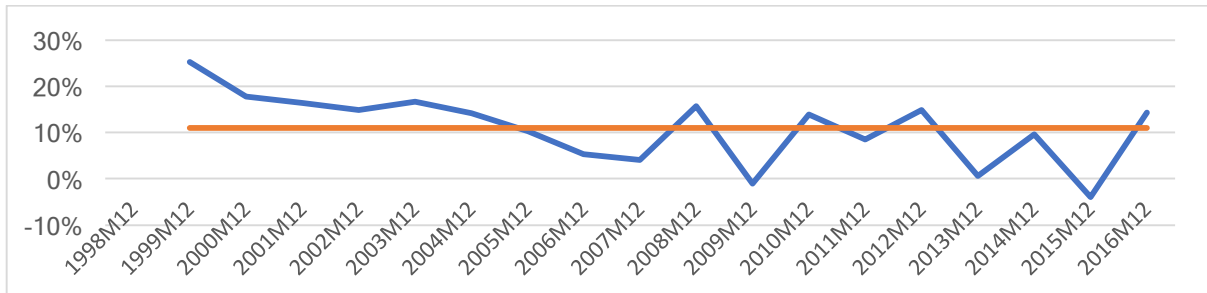


Figure 4.8: Showing Logarithmic JSE All Share Returns over the period 01/01/1900 to 31/12/2016 using annual return data

4.5 South African Inflation Data

This time diversification study analyzed both the real and nominal returns, as a result required Consumer Price Index (CPI) data connected from 31/12/1959. CPI is a measure used to quantify the rate of change of prices of consumer goods and services (International Monetary Fund, 2017). This data was collected at the aforementioned frequencies from the I-Net terminal using the ticker ECPI or Headline CPI where the index was specified such that the CPI value at December 2012 = 100. Thereafter the inflation rate was calculated using the following formula:

$$Inflation Rate_i = \ln\left(\frac{ECPI_i}{ECPI_{i-1}}\right), \quad (13)$$

where $Inflation Rate_i$ represents the inflation rate for period i , $ECPI_i$ represents the headline CPI value for period i and $ECPI_{i-1}$ represents the headline CPI value for period $i - 1$. This inflation rate series is depicted in Figure 4.9.

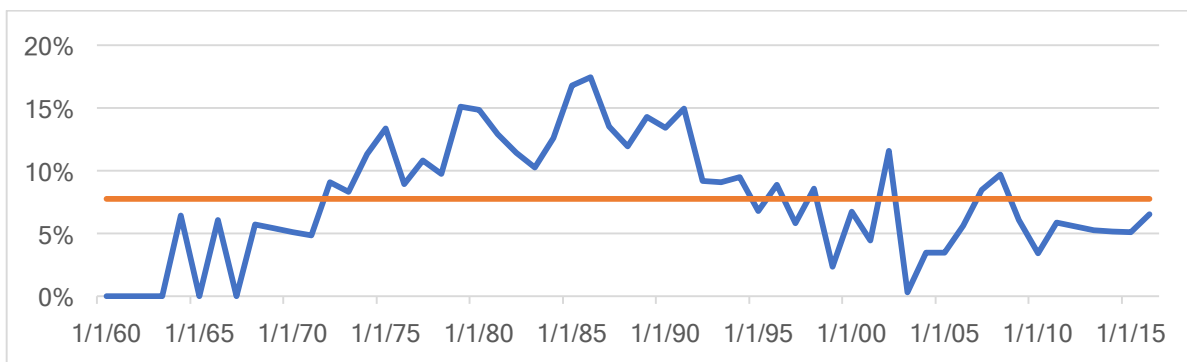


Figure 4.9: Showing the South African Inflation data over the period 31/12/1959 to 31/12/2016 using annual returns

As is evident from the above Figure 4.9, we observe that inflation rate data pre-1986 is very sporadic in its changes. After further investigation into the underlying CPI series data, we observe that these spikes seen in Figure 4.9 are due to the CPI value for those years only being updated once or twice every two-years. To avoid this possible collection error, we shall partition the inflation rate data in accordance with our sub-samples. In this case, this paper will only use inflation rate data in this paper's final sub-sample comprising of all data between 01/01/1987 to 31/12/2016. This partitioned inflation rate data is depicted in Figure 4.10 below, showing a more consistent and reliable dataset.

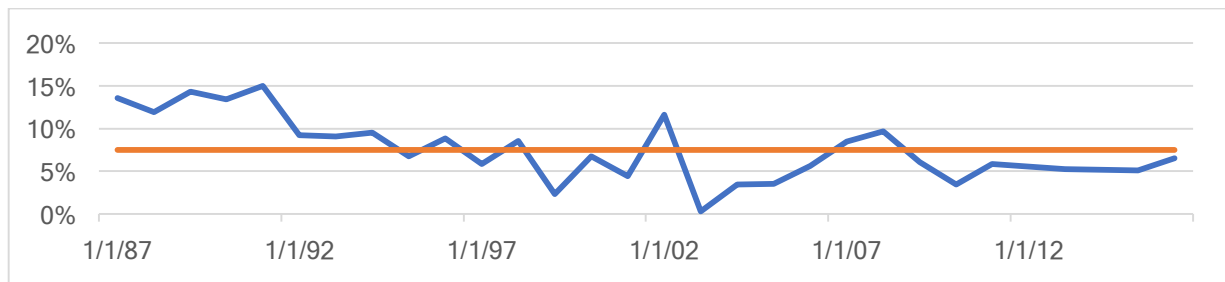


Figure 4.10: Showing the South African Inflation data over the period 01/01/1987 to 31/12/2016 using annual returns

4.6 Real return calculation

The real ALSI and real ALBI data, are calculated using the formula below:

$$Real\ Return_i = Logarithmic\ Total\ Nominal\ Return_i - Inflation\ Rate_i, \quad (13)$$

where $Real\ Return_i$ represents the inflation adjusted real return over period i . Due to the limitation place on the inflation rate in Section 4.5, we will only be considering real returns for our ultimate sub-sample between 01/01/1987 to 31/12/2016 for the ALSI. For the ALBI, we shall use real returns for its entire period of existence, between 31/12/1998 to 31/12/2016.

4.7 Summary and Conclusion

This Chapter has summarized this study's constituent datasets, their collection, calculation and merging procedures. Furthermore, this Chapter has depicted the price and return series used throughout this study. After visual inspection of the nominal ALSI monthly and annual datasets employed, we observe non-random behavior, that lead to the hypothesis of monthly returns

showing signs of trending non-random behavior while annual returns showed signs of mean reverting non-random behavior. This hypothesis was tested in Chapter 6 using the methods outlined in Chapter 5.

5. Methodology

5.1 Introduction

This chapter outlines the methods employed by this paper in its' investigation into time diversification on the JSE. This chapter is broken down into seven further Sections. Section 5.2 discusses the various methods of the descriptive statistics calculations and Section 5.3 addresses the tests for Stationarity. Section 5.4 discusses the Autocorrelation test, Section 5.5 outlines the methods employed in the use of the Structural Breakpoint tests. Section 5.6 discusses the Runs test, while the Variance Ratio test is presented in Section 5.7. Penultimately, this research paper presents the methodology employed in replication of the Bradfield and Ardington study on serial correlation (1997) in Section 5.8 followed by Section 5.9 that summarizes and concludes.

5.2 Descriptive Statistics

Descriptive statistics form part of the basis of any study, as it gives the authors a feel for the dataset they have chosen. For this study, the following descriptive statistics are particularly important, and are be discussed below. These include the mean, variance, skewness, kurtosis, Jarque Bera test statistic and the return distribution histograms.

5.2.1 Mean and variance

Initially this paper will analyze the means (first moment) of the JSE return data (both arithmetic and logarithmic). This uses the formula (Gujarati & Porter, 2003):

$$\bar{r}_j = \frac{\sum_i^n r_i}{n}, \quad (14)$$

where \bar{r} represents the sample mean return over period j , r_i represents the return for period i and n is the sample size. Secondly we will calculate the sample Variance (second moment) of this series using (Gujarati & Porter, 2003):

$$s_j^2 = \frac{\sum_i^n (r_i - \bar{r}_j)^2}{n - 1}. \quad (15)$$

5.2.2 Skewness

To measure the degree of symmetry (asymmetry) of logarithmic and arithmetic returns, the sample skewness (third moment) of returns was calculated. This statistic is normally symmetrical around zero and its calculation was based upon the following formula (Gujarati & Porter, 2003):

$$Skewness_j = \frac{1}{n} \left(\sum_i^n \left(\frac{(r_i - \bar{r}_j)^3}{s^3} \right) \right). \quad (16)$$

5.2.3 Kurtosis

The kurtosis of a distribution represents the shape of the distribution, and a normal distribution has a kurtosis of three. Thus, a kurtosis level of below three, is known as a platykurtic distribution, while a distribution with a kurtosis level of greater than three is known as leptokurtic. This statistic was calculated as follows (Gujarati & Porter, 2003):

$$Kurtosis_j = \frac{1}{n} \left(\sum_i^n \left(\frac{(r_i - \bar{r}_j)^4}{s^4} \right) \right). \quad (17)$$

5.2.4 Jarque-Bera test

The Jarque-Bera test is goodness of fit test that is used to test the normality of a series, based on its sample values for Skewness and Kurtosis. This statistic takes the form (Jarque & Bera, 1987):

$$Jarque - Bera Statistic_j = \frac{n}{6} \left(Skewness_j^2 + \frac{(Kurtosis_j - 3)^2}{4} \right). \quad (18)$$

This statistic is used to test the null hypothesis that the series is normally distributed. This statistic follows a Chi-squared distribution with 2 degrees of freedom ($Jarque - Bera Statistic_j \sim \chi_2^2$).

5.2.5 Return Distribution Histograms

The logarithmic and arithmetic return distributions had their histograms plotted as the return distributions increased in frequency from monthly to 50-year return periods.

5.3 Tests for Stationarity

The tests for stationarity are broken down into the Dickey Fuller tests (Standard and Augmented) that form Section 5.3.1 and the Phillips-Perron test for stationarity that comprises Section 5.3.2.

5.3.1 Augmented Dickey Fuller Tests

The first test that we used to test for stationarity of the ALSI and ALBI, was the Augmented Dickey Fuller Unit Root test.

We chose to perform the Augmented Dickey Fuller (ADF) test over the standard Dickey Fuller (DF) test as the standard Dickey Fuller test is only acceptable on the proviso that the underlying series being tested follows an AR(1) process (Eviews, 2017b). At this stage, we were unsure if our underlying series contained serial correlation of order one, and thus we chose to perform the augmented version of the test, which considers any serial correlation at higher order lags. If our underlying series were to have serial correlation at the second lag, for example, this distinction would violate the assumption of white noise disturbances that is assumed in the standard Dickey Fuller test. For the purposes of this study, these differences will be outlined as well as the explanation of the Augmented Dickey Fuller test in the specifications below.

5.3.1.1 Dickey Fuller Test Specification

The Dickey Fuller test is based on the three versions of the Random Walk. This includes the Random Walk, Random Walk with drift and the Random Walk with drift around a Stochastic Trend Model.

Due to this difference in modelling, the Dickey-Fuller test has three different specifications. The first deals with the Random Walk Model (without drift), and this first log difference test Equation takes the form (Gujarati & Porter, 2003):

$$\begin{aligned}
 r_t &= P_t - P_{t-1} & (19) \\
 r_t &= (\rho P_{t-1} + e_t) - P_{t-1} \\
 r_t &= (\rho - 1)P_{t-1} + e_t \\
 r_t &= \delta P_{t-1} + e_t, & (20)
 \end{aligned}$$

Where P_t represents the log price at time t , P_{t-1} represents the log price at time $t-1$ and $\delta = (\rho - 1)$ using ρ from Equation 1. In this form, δ is used in the null hypothesis that, for this version of the standard Dickey Fuller Test, is defined as $H_0: \delta = 0$. This null hypothesis implies that there exists a unit root and the series is non-stationary. This Null hypothesis is tested against the alternate hypothesis $H_1: \delta < 0$, such that the series is a stationary. Rejection of the Null, implies that the time series (P_t) is stationary with zero mean.

The second specification is the Random Walk with Drift Model. This first difference estimated Equation takes the form (Gujarati & Porter, 2003):

$$\begin{aligned}
 r_t &= P_t - P_{t-1} \\
 r_t &= (\alpha + \rho P_{t-1} + e_t) - P_{t-1} \\
 r_t &= \alpha + (\rho - 1)P_{t-1} + e_t \\
 r_t &= \alpha + \delta P_{t-1} + e_t, & (21)
 \end{aligned}$$

where α is the drift parameter in Equation 5. The Null hypothesis for this version of the standard Dickey Fuller Test is defined as $H_0: \delta = 0$, that implies that there exists a unit root and the series is non-stationary. This Null hypothesis is tested against the alternate hypothesis $H_1: \delta < 0$, such that the series is a stationary. Rejection of the Null, implies that the series (P_t) is stationary with non-zero mean.

The final specification of the Dickey Fuller Test is based on the Random Walk with drift around a Stochastic Trend Model. This model takes the form:

$$P_t = \alpha + P_{t-1} + \beta_1 t + e_t,$$

where P_t represents the price series at time t , P_{t-1} represents the price series at the previous period (first lag), α represents the drift term, where t represents the time or trend variable and β_1 is the co-efficient on the trend term t and lastly e_t represents the error term. The calculation of the return or first difference of this model yields (Gujarati & Porter, 2003):

$$\begin{aligned} r_t &= P_t - P_{t-1} \\ r_t &= (\alpha + \rho P_{t-1} + \beta_1 t + e_t) - P_{t-1} \\ r_t &= \alpha + (\rho - 1)P_{t-1} + \beta_1 t + e_t. \end{aligned} \quad (22)$$

This Equation 22 is the estimation Equation for the final version of the Dickey-Fuller Test. The Null hypothesis for this version of the standard Dickey Fuller Test is defined as $H_0: \delta = 0$, that implies that there exists a unit root and the series is non-stationary. This Null hypothesis is tested against the alternate hypothesis $H_1: \delta < 0$, such that the series is stationary. Rejection of the Null, implies that the time series (P_t) is stationary around the deterministic trend.

The Dickey-Fuller test is performed such that an ordinary least squares regression is estimated in Equation 20, 21 and 22. However, when testing the hypotheses specified above, the tau (τ) statistic must be calculated and the tau (τ) critical values must be employed (Dickey & Fuller, 1976). The tau (τ) statistic is calculated as follows:

$$\tau_{statistic} = \frac{\hat{\delta}}{Standard\ Error(\hat{\delta})}, \quad (23)$$

where $\tau_{statistic}$ is the tau test statistic for $\hat{\delta}$, $\hat{\delta}$ is the OLS estimated co-efficient of P_{t-1} and $Standard\ Error(\hat{\delta})$ represents the standard error of $\hat{\delta}$.

5.3.1.2 Augmented Dickey Fuller Test Specification

A key assumption throughout the Dickey-Fuller tests in the previous section (Equations 20, 21 and 22) was that the error terms (e_t) were not serially correlated and that the underlying series is AR (1). Because of this, Dickey and Fuller (1979) created an adjustment to their initial test,

that augments Equation 20, 21 and 22 with lagged versions of the dependent variable (r_t). This augmentation changes the Random Walk Model Equation 20 to:

$$r_t = \delta P_{t-1} + \sum_i^n \gamma_i r_{t-i} + e_t, \quad (24)$$

where r_{t-i} represents the first difference for period $t - 1$ (i.e. $r_{t-i} = P_{t-1} - P_{t-2}$) and γ_i represents the co-efficient of r_{t-i} . The Augmented Dickey Fuller Equations for the Random Walk without drift and the Random Walk with drift and trend changes Equations 21 and 22 to 25 and 26 respectively. These are shown below:

$$r_t = \alpha + \delta P_{t-1} + \sum_i^n \gamma_i r_{t-i} + e_t, \quad (25)$$

$$r_t = \alpha + \beta_1 t + \delta P_{t-1} + \sum_i^n \gamma_i r_{t-i} + e_t. \quad (26)$$

These estimation Equations thereafter follow the same steps as mentioned in 5.2.1, where this paper is required to run Ordinary Least Squares (OLS) regressions on the above Equations, compute the $\tau_{statistic}$ in Equation 23 and use the Tau (τ) critical values. The Null hypothesis for the Augmented Dickey Fuller Test remains unchanged from the standard Dickey Fuller test and is defined as $H_0: \delta = 0$, that implies that there exists a unit root and the series is non-stationary (Eviews, 2017b). This Null hypothesis is tested against the alternate hypothesis $H_1: \delta < 0$, that defines the series as stationary.

Misspecification in the model choice can significantly affect out results, as adding irrelevant regressors in our model will decrease the power of the test. Because of this, when running these tests, this paper performed all three specifications of the test. Together with these specifications, this paper not only performed the Augmented Dickey-Fuller test on the price series P_t , but also on its' first and second differences.

5.3.2 Phillips-Perron Test

Philips and Perron (1988) propose a non-parametric test for stationarity, that unlike the Augmented Dickey Fuller Test, does not include additional lagged differences when considering higher order serial correlation as well as heteroscedasticity in the error terms (e_t). This test makes non-parametric adjustments to the test statistics and is more robust to autocorrelation. This adjustment takes the form (Eviews, 2017b):

$$Z_\tau = \tau_{statistic} \delta \sqrt{\frac{\widehat{\sigma^2}}{\widehat{f}_0}} - \frac{T(\widehat{f}_0 - \widehat{\sigma^2}) (Standard Error(\widehat{\delta}))}{2 Standard Error (Test Regression) \sqrt{\widehat{f}_0}}, \quad (27)$$

where $\widehat{\sigma^2}$ represents a consistent estimator of the error variance, \widehat{f}_0 is an estimator of the residual spectrum at frequency zero, T is the number of observations and *Standard Error (Test Regression)* represents the standard error of the test regression.

Similar to the Augmented Dickey Fuller test process, this paper will perform the Phillips Perron test using all three exogenous variable test specifications. In other words, this paper will run OLS regressions on the Equations 24,25 and 26 (constant, none, constant and trend) and use the above test statistic. Similar to the above methodology, these regressions will be conducted on the logarithmic price series P_t , its' first difference and second difference. This adjustment does not change the asymptotic distribution of the underlying, hence this test uses the same tau (τ) critical values as before. Lastly this test requires specification regarding the estimation of \widehat{f}_0 . This paper selected the Kernel based sum of covariance's estimation method for \widehat{f}_0 , as it coherent with Eviews (Eviews, 2017b).

5.4 Autocorrelation Test

This test methodology was adapted from Bradfield and Ardington (1997), where the authors regressed a series of total ALSI returns between 1980 and 1996 on their prior period returns. This autocorrelation regression was performed for a one month lag when using monthly total return data, for a one quarter lag when using quarterly total return data and lastly for a one-year lag when using annual total returns. This method is also referred to as the Autocorrelation Function.

This paper will replicate this methodology, but also extrapolate this method further, to semi-annual, two yearly, three yearly, five yearly, ten yearly and twenty yearly lags using the respective frequency of total return data. In addition, Bradfield and Ardington (1997) regressed monthly data on the prior months returns (thus ensuring a non-overlapping dataset), but this paper will additionally regress monthly data on the prior month, quarter (three month), semi-annual (six month), annual (twelve month), two yearly (twenty-four month), three yearly (thirty-six month), five yearly (sixty month), ten yearly (one hundred and twenty month) and twenty-year (two hundred and forty month) period and perform the same test for other frequencies. Although this extrapolation will use an overlapping dataset, it shall only be presented in Appendix D for additional insights.

These regressions require an intercept co-efficient in their regression Equation, and this autocorrelation regression Equation would take the general form:

$$r_t = \alpha + \beta_{t-1}r_{t-1} + e_t,$$

where α represents the regression constant intercept term and β_{t-1} is the coefficient of the autocorrelation variable r_{t-1} . If this coefficient β_{t-1} were less than zero, it would imply that there exists negative serial correlation, while positive serial correlation would exist if the coefficient β_{t-1} were positive.

Gujarati and Porter (2003) proposes that to calculate the autocorrelation function at lag k, one should use the following formula (this is simply the estimation of the β_{t-1} in the regression equation above) :

$$\rho_k = \frac{\text{Covariance}(r_t, r_{t-k})}{\text{Variance}(r_{t-k})}. \quad (34)$$

5.5 Unknown Structural Breakpoint Tests

Thus far structural breaks in the data sample have not been considered. In a study by Smit and Wesso (1988) they found that South African financial models could experience structural

breaks due to the many socio-political and economic changes in these environments. To consider possible structural breakpoints, that are unknown to the authors, this paper shall employ two methods to take this into account. Firstly, this paper performed the Quandt-Andrews Single Unknown breakpoint test, which identifies and tests the single most significant structural break. Secondly this paper employed the Bai-Perron Multiple Unknown Breakpoint test, which identified and tested the significance of more than one structural breakpoint in our dataset.

5.5.1 Quandt-Andrews Single Unknown Breakpoint Test

The Quandt-Andrews Breakpoint test, tests the data sample for unknown structural breaks. This test relies on the Chow Breakpoint test, that requires a breakpoint to be specified and thereafter tests for a structural break at that pre-specified point. This Quandt-Andrews test performs multiple chow breakpoint tests with one at each observation. Thereafter these resulting F-statistics from the Chow test are combined into one test statistic for the Quandt-Andrews test is used to test the null hypothesis that there are no breakpoints in the sample.

A brief review of the Chow Breakpoint test is presented. The Chow test tests whether there exists any structural change in the regression Equation before and after a specified date. This is performed by comparing the sums of squared residuals over the full period to the separate regression sub-sample sums of squared residuals. The F-statistic based on a single breakpoint takes the form (Chow, 1960):

$$F_{Chow\ Breakpoint} = \frac{\frac{SSR_{Full\ Period} - (SSR_{Sub-Sample\ 1} + SSR_{Sub-Sample\ 2})}{m}}{\frac{SSR_{Sub-Sample\ 1} + SSR_{Sub-Sample\ 2}}{N - 2m}},$$

where $SSR_{Full\ Period}$ represents the sum of squared residuals for the entire period under analysis, $SSR_{Sub-Sample\ 1}$ represents the sum of squared residuals for the sub-sample prior to the breakpoint date, $SSR_{Sub-Sample\ 2}$ represents the squared residuals for the sub-sample after the breakpoint, m represents the number of parameters in the estimated Equations and N represents the total number of observations in the sample.

The Chow test also makes use of a log likelihood statistic and Wald statistic that both follow Chi square distributions (χ_m^2). An important restriction to note is that the number of observations in each sub sample must be less than the number of parameters to be estimated, but for the purposes of our study, this restriction does not affect us.

Returning to the Quandt-Andrews Test, this test utilizes three tests. Firstly, this test uses the maximum F-statistic that defined the highest observed F-statistic from the individual chow tests such that:

$$\text{Max } F_{\text{Quandt Andrews}} = \text{Maximum} (F_{\text{Chow Breakpoint}}).$$

Secondly, this test uses an $\text{Exp } F_{\text{Quandt Andrews}}$ statistic that takes the form (Andrews and Ploberger (1994));

$$\text{Exp } F_{\text{Quandt Andrews}} = \ln \left(\frac{1}{n_2 - n_1 + 1} \sum_{t=n_1}^{n_2} \exp \left(\frac{1}{2} m F_{\text{Chow Breakpoint}} \left(\frac{t}{n} \right) \right) \right),$$

where n_1 and n_2 represent the range of break dates, m represents the number of regressors, t and n . The final test statistic employed in this test is the $\text{Ave } F_{\text{Quandt Andrews}}$ statistic that takes the form (Andrews and Ploberger 1994):

$$\text{Ave } F_{\text{Quandt Andrews}} = \left(\frac{1}{n_2 - n_1 + 1} \sum_{t=n_1}^{n_2} \exp \left(m F_{\text{Chow Breakpoint}} \left(\frac{t}{n} \right) \right) \right).$$

When performing this test, it was ensured that the test specification included only the lagged variable as a breakpoint variable as well as the default 15% trimming of data.

5.5.2 Bai-Perron Multiple Unknown Breakpoint Test

Bai and Perron (1998), have extended on the work done by Quandt (1960) and Andrews (1993) by considering the possibility of multiple breakpoints that are unknown. This test considers the scenario where one is faced with n period with m possible breakpoints that produce $m+1$

breakpoint regimes. There after Bai and Perron (1998) specify that the regression Equation be split into a time-varying category and a time invariant category. For the purposes of this study, this autocorrelation analysis only considers one independent time varying variable being the JSE returns at different lags. This test has three different test specifications, namely the Global maximizer tests for the breakpoints, sequentially determined breakpoint tests and the Global information criteria tests (amalgamation of the two).

This paper performed the sequential breakpoint testing method, as this method tests breakpoints sequentially for differences. Based on these results, this test will determine the breakpoints and finally use sub-sample regressions to test the significance of the chosen breakpoint. This process was repeated until all possible breakpoints are tested. The Eviews statistical package uses a more robust F-Statistic for this test than the traditional Bai-Perron test statistic. This test statistic takes the form (Eviews, 2017a):

$$F(\beta) = \frac{1}{T} \left(\frac{T - (l + 1)q - p}{kq} \right),$$

where the critical values for this test form part of the Bai-Perron critical values (19980).

5.6 Runs Test

The Runs test is another non-parametric method used in this paper to determine the model that best describes the return series on the JSE. This test is also known as the Wald-Wolfowitz Runs Test. This test entails analyzing groups of consecutive net returns (relative to their median). Thereafter net returns are of the same sign are used to determine if these collections of positive and negative net returns are random or if they exhibit mean reverting (oscillating) or trending tendencies. A definition of a Run in this test, is a consecutive sequence of positive or negative net returns (relative to its median), while the length of a run is determined by the number of return observations within it. We chose to use the median in this study as the median of a distribution is more robust when explaining the distribution of non-parametric distributions (Bradley, 1968).

This test defines that the number of observed runs according to empirical results, be compared to the number of expected runs under the Random Walk hypothesis. If this paper observes more runs than expected, it implies that the length of each run is shorter (than expected under the random walk hypothesis). Hence, the series tends toward mean reversion. These shorter, faster sign changing and more frequent runs, lend themselves to negative serial correlation (Gujarati & Porter, 2003). On the other hand, if fewer runs than expected under the Random Walk hypothesis are empirically, it lends itself to positive serial correlation. This is due to the length of each run increasing as the number of runs decrease (Gujarati & Porter, 2003).

The null hypothesis states that all returns in our series are random. To test this, the number of expected runs in the return series were calculated and tested against the number of empirically observed runs. If the number of positive and negative runs was greater than ten each, then the expected number of return runs is approximately normally distributed with the expected number of runs and the sample variance of the number of runs taking the form (Bradley, 1968):

$$E(\text{Number of Runs}) = \bar{R} = \frac{2N_{Positive}N_{Negative}}{N_{Total}} + 1 \quad (28)$$

where $N_{Positive}$ is the number of positive return observations, $N_{Negative}$ is the number of negative return observations and N_{Total} represents the total number of observations. Thereafter this paper used the Z-Tables. Together with this, this paper employed the following test statistic to determine if the number of empirically observed runs of net returns relative to its median can be rejected (Bradley, 1968):

$$Z_{Runs} = \frac{R - \bar{R}}{S_{Number\ of\ Runs}}, \quad (29)$$

$$S_{Number\ of\ Runs}^2 = \frac{2N_{Positive}N_{Negative}(2N_{Positive}N_{Negative} - N_{Total})}{N_{Total}^2(N_{Total} - 1)}. \quad (30)$$

When specifying this test, this paper chose to discard all return observations that were tie with the median as oppose to randomizing them. Built on this test, is the notion of testing the observed and expected number of runs, to determine if the difference between them in statistically significant. This will allow the conclusion with a level of confidence, that if the

number of expected runs is statistically significantly higher than the number of observed runs, then it implies that the return series trends. On the other hand, if the number of expected runs is statistically significantly lower than the number of observed runs, then it implies that the return series exhibits negative serial correlation and mean reverts.

5.7 Variance Ratio Tests

To test whether our return series is Random, Mean Reverting or Trending, the Lo and MacKinlay (1988) Variance Ratio Test was performed. This test entailed, using the Square Root of Time Rule to verify that if data follows a Random Walk, the variance of returns calculated over different frequencies, scales up linearly with time such that:

$$\frac{\sigma_T^2}{\sigma_t^2 \left(\frac{T}{t}\right)} = 1, \quad (31)$$

where σ_T^2 represents the variance of returns over the longer period T, σ_t^2 represents the variance of returns over the shorter period t. This test can be broken down into a single interval test as well as a multiple interval joint tests. Section 5.5.1 and 5.5.2 deals with these tests respectively.

5.7.1 Single Interval Variance Ratio Test

In their 1988 paper, Lo and MacKinlay define two sets of null hypotheses regarding the error term e_t , as well as their respective test statistics. The first version of the Variance Ratio test has a null hypothesis that assumes that the e_t 's are i.i.d. The second version of this test has a null hypothesis that weakens the strict i.i.d. assumption, by allowing for heteroscedasticity and serial dependence. From Equation 31, the variance ratio of a series of returns takes the form:

$$\text{Variance Ratio } (T) = \frac{\sigma_T^2}{\sigma_t^2 \left(\frac{T}{t}\right)}. \quad (32)$$

If the Variance Ratio is equal to one, this implies that the volatility of returns scales up linearly with time and hence follows a Random Walk. If, however, the Variance Ratio is greater than unity, it implies that the shorter interval (t) of returns tend to trend within the longer interval

(T). If the Variance Ratio is less than one, it implies that within the longer interval of returns (T) the shorter interval of returns (t) tends to mean revert. The test statistic (Z-score) for this variance ratio can be calculated using (Lo & MacKinlay, 1988):

$$Z(T)_{Lo \text{ and } MacKinlay} = \frac{\text{Variance Ratio}(T) - 1}{\sqrt{s^2(T)}}, \quad (33)$$

where $s^2(T)$ is an estimator of the variance of returns over interval T . This estimator can be calculated by using the following formula under the i.i.d. null hypothesis:

$$s^2(T) = \frac{2(2T - 1)(T - 1)}{3Tn}.$$

Alternatively, this estimator $s^2(T)$ can be calculated using kernel estimators for the second null hypothesis that allows for heteroscedasticity and serial dependence. During this test specification, this paper analyzed the Heteroskedastic Random Walk Null hypothesis $H_0: \text{Variance Ratio}(T) = 1$. Furthermore, for the purposes of this study, we specified that our return data was log returns. The test probabilities were chosen to be asymptotically normal as outlined in Lo and MacKinlay (1998). To make our test more robust, this paper chose to use bias-corrected variance estimates, heteroskedastically robust test standard errors and a non-zero mean.

5.7.2 Joint Interval Variance Ratio Test

This supplemented result of the Variance Ratio Test, allows tested the hypothesis specified above (Heteroskedastic adjustment) for significance across all intervals. This Joint Test was designed by Chow and Denning (1993) and employs the Studentized Maximum Modulus distribution. This test statistic takes the form:

$$MV_1 = \sqrt{T} \max_{1 \leq i \leq m} |Z(T)_{Lo \text{ and } MacKinlay}|, \quad (33)$$

Where MV_1 represents the Chow-Denning Joint Variance Ratio Test statistic, m represents the number of individual sub-periods and $\max_{1 \leq i \leq m} |Z(T)_{Lo \text{ and } MacKinlay}|$ refers to the maximum Lo and MacKinlay Individual Variance Ratio test statistic in equation 33.

5.8 Bradfield and Ardington (1997) Replication

The methodology employed in the replication of the Bradfield and Ardington (1997), will follow that stipulated in their paper. This begins with the creation of a total return index for the ALSI, followed by Autocorrelation tests (as explained in Section 5.6.1) for monthly, quarterly and annual returns. Thereafter this study will perform the Runs test (outlined in Section 5.4) and Variance Ratio Test (described in Section 5.5) on monthly, quarterly and annual returns.

5.9 Summary and Conclusion

Chapter 5 has explained the methods and practices to be employed in the search for an answer to the Time Diversification debate. By assessing the stationarity of the ALSI and ALBI return and price series, this methodology aims to avoid the possibility of non-stationary returns being used in regression analysis. Furthermore, this Chapter has provided non-parametric methods of testing (Runs Test and Variance Ratio Test) as well as the parametric Autocorrelation tests.

6. Results

6.1 Introduction

In this investigation into the effects of time diversification on the JSE, Chapter 6 displays the results of the methods discussed in Chapter 5. This Chapter is broken down into nine Sections. Each Section deals with both real and nominal returns. Furthermore, each Section discusses its results over each of the pre-specified period partitions (01/01/1900 - 31/12/2016, 01/01/1900 - 31/12/1986 and 01/01/1987 - 31/12/2016), for both the ALSI and ALBI datasets. For clarification the pre-1987 period refers to the period (01/01/1900 – 31/12/1986) and the post-1986 period refers to the period (01/01/1987 – 31/12/2016).

The remainder of this Chapter is organized as follows; Section 6.2 presents the results of the descriptive statistical analysis, followed by Section 6.3 that assess the Stationarity of the return and price series. Section 6.4 deals with the results of the Autocorrelation test. Thereafter Breakpoint tests were performed to determine breakpoints in the Autocorrelation test and their results are presented in Section 6.5. The non-parametric Runs and Variance Ratio tests are presented in Sections 6.6 and 6.7 respectively. For added test validity, this study replicated the results of Bradfield and Ardington (1997) which are presented in Section 6.8. Finally, Section 6.9 will summarize and conclude this Chapter.

6.2 Descriptive Statistics

6.2.1 Nominal returns

To get a feel for the data, and identify time series trends, this paper initially employed descriptive statistical analysis. The descriptive statistics include the logarithmic and arithmetic returns; mean, standard deviation, skewness, kurtosis and the Jarque-Bera p-value. The results of the descriptive statistical tests run on monthly data for the ALSI and ALBI (arithmetic and logarithmic) returns are shown in Table 6.1 (over the entire period), and Table 6.2 displays the ALSI monthly descriptive statistics (over the pre-1987 and the post-1986 periods). The results of annual data for the ALSI and ALBI (arithmetic and logarithmic) returns are shown in Table

6.3 (over the entire period) and Table 6.4 displays the ALSI annual descriptive statistics (over the pre-1987 and the post-1986 periods).

Table 6.1: Descriptive statistics for nominal monthly total return (logarithmic and arithmetic) for the ALSI over their maximum available period

This Table displays the descriptive statistics calculated for the ALBI and ALSI (logarithmic and arithmetic) monthly returns over their entire period. The ALSI is calculated over the period 31/01/1925 to 31/12/2016, and the ALBI over the period 31/12/1998 to 31/12/2016. LR refers to logarithmic total return and AR refers to the arithmetic total return.

	ALSI LR	ALSI AR	ALBI LR	ALBI AR
Mean	0.9486%	1.0799%	0.9135%	0.9392%
Standard Deviation	5.0395%	5.0274%	2.0679%	2.0882%
Skewness	-0.6838	-0.2674	0.0109	0.1232
Kurtosis	7.4220	6.1121	4.6391	4.6391
Jarque-Bera P-value	0.0000***	0.0000***	0.000006***	0.000004***

* significant at $p < 0.05$; ** significant at $p < 0.01$; *** significant at $p < 0.001$

Table 6.2: Descriptive statistics for nominal monthly total (logarithmic and arithmetic) return data for the ALSI over the pre-1987 and post-1986 periods

This Table displays the descriptive statistics calculated for the ALBI and ALSI (logarithmic and arithmetic) monthly returns over their entire period. The pre-1986 period ranges from the 31/01/1925 to the 31/12/1986, while the post-1986 period ranges from the 31/12/1986 to the 31/12/2016. LR refers to logarithmic total return and AR refers to the arithmetic total return.

	ALSI LR pre-1987	ALSI AR pre-1986	ALSI LR post-1986	ALSI AR post-1986
Mean	0.8761%	0.9934%	1.0982%	1.2584%
Standard Deviation	4.7547%	4.7805%	5.5856%	5.4957%
Skewness	-0.2917	0.0541	-1.1930	-0.7250
Kurtosis	5.9535	5.8681	8.7574	6.2660
Jarque-Bera P-value	0.0000***	0.0000***	0.0000***	0.0000***

* significant at $p < 0.05$; ** significant at $p < 0.01$; *** significant at $p < 0.001$

Tables 6.1 and 6.2 refer: preliminarily monthly arithmetic returns for all statistics above, are observed to be both larger in magnitude than their logarithmic counterparts, as well as exhibit a more positively skewed distribution. Arithmetic returns, also display a slightly lower level of kurtosis, making the distribution of arithmetic returns less leptokurtic than logarithmic returns. The Jarque-Bera statistic for all return distributions are significant, that implies the rejection of normality entirely at a p-value of less than 0.001 for monthly returns.

Focusing on logarithmic returns, when comparing ALSI returns over time, this study has identified that the average monthly return on the ALSI has increased from 0.8761% in the pre-1987 period to 1.0982% in the post-1986 period. Together with this, the standard deviation of

logarithmic monthly returns has also increased from 4.7547% in the pre-1987 period to 5.5856% in the post-1986 period.

For arithmetic and logarithmic returns, the ALSI was found to offer a higher return combined with a higher level of risk, than the ALBI (as expected). All the ALSI logarithmic returns are negatively skewed, while the skewness of both logarithmic and arithmetic returns on the ALBI are slightly positive and close to zero. Over all the periods, the ALSI exhibits larger kurtosis values, while the ALBI is significantly less leptokurtic, and closer to appearing normal.

Turning the attention to the descriptive statistical analysis for annually calculated nominal returns depicted in Table 6.3 (over the entire period) and Table 6.4 (over the pre-1987 and the post-1986 periods). Preliminarily, the same trend as with monthly returns was found for annual data whereby the annual arithmetic returns for all statistics are both larger in magnitude than their logarithmic counterparts, as well as exhibit a more positively skewed distribution. However, the difference in the skewness of return distributions between arithmetic and logarithmic returns is lesser in magnitude for the ALBI when compared to the ALSI.

When comparing the annual data, we find that arithmetic returns, now display a higher level of kurtosis, making the distribution of arithmetic returns more leptokurtic than logarithmic returns. The Jarque-Bera statistics fail to reject the null hypothesis of normality - apart from ALSI arithmetic returns that is significantly non-normal.

Comparing the annual ALSI returns over time, has yielded that the average logarithmic annual return on the ALSI has increased from 9.3053% in the pre-1987 period to 13.1000% in the post-1986 period. In contrast to monthly returns, the standard deviation of annual returns was found to not increase over time, but instead remains approximately the same at 18.92917% in the pre-1987 period, to 18.1558% in the post-1986 period. This discrepancy in the behavior of the standard deviation of monthly and annual returns does somewhat provide doubt into the Random Walk Model.

Both arithmetic and logarithmic annual returns, have displayed that the ALSI offers a higher return combined with a higher level of risk, than the ALBI (as expected). Unlike monthly data, we find that the ALSI logarithmic returns are no longer negatively skewed. Over all periods the ALSI logarithmic returns exhibits larger kurtosis values than the ALBI. The kurtosis of the

ALSI and ALBI was observed being closer to the normal value of three for annual returns, than for monthly. This supports the notion that the returns on the JSE approach normality as the return interval increases.

Looking at the Jarque-Bera p-values over the annual return frequency indicates that normality is rejected for ALSI arithmetic returns over the entire period and the pre-1987 period. The logarithmic annual returns fail to reject normality, which supports the notion of returns following a log normal distribution, this is also one of the reasons why this paper focuses on logarithmic returns in the analysis presented in Sections 6.3 - 6.9. The ALBI also fails to reject normality at the annual level.

Table 6.3: Descriptive statistics for nominal annual total return data on the ALSI over their maximum period

This Table displays the descriptive statistics calculated for the ALBI and ALSI (logarithmic and arithmetic) monthly returns over their entire period. LR refers to logarithmic total return and AR refers to the arithmetic total return.

	ALSI LR	ALSI AR	ALBI LR	ALBI AR
Mean	10.2867%	12.8200%	10.7234%	11.4884%
Standard Deviation	18.7283%	22.0677%	5.6270%	6.4076%
Skewness	0.3168	1.0632	0.7711	0.8663
Kurtosis	3.5341	5.3755	2.7529	2.9141
Jarque-Bera P-value	0.1903	0.0000***	0.4006	0.3236

* significant at $p < 0.05$; ** significant at $p < 0.01$; *** significant at $p < 0.001$

Table 6.4: Descriptive Statistics for nominal annual total return data for the ALSI for data before 31/12/1986

This Table displays the descriptive statistics calculated for the ALBI and ALSI (logarithmic and arithmetic) monthly returns over their entire period. . The pre 1987 period ranges from the 31/01/1925 to the 31/12/1986, while the post-1986 period ranges from the 31/12/1986 to the 31/12/2016. LR refers to logarithmic total return and AR refers to the arithmetic total return.

	ALSI LR pre 1987	ALSI AR pre 1987	ALSI LR post 1986	ALSI AR post 1986
Mean	9.3053%	11.7668%	13.1000%	15.8392%
Standard Deviation	18.9291%	22.3381%	18.1558%	21.2510%
Skewness	0.3933	1.2421	0.1245	0.5471
Kurtosis	3.8608	6.2341	2.6063	2.9218
Jarque-Bera P-value	0.0875	0.0000***	0.8732	0.4713

* significant at $p < 0.05$; ** significant at $p < 0.01$; *** significant at $p < 0.001$

The Descriptive Statistics for quarterly, semi-annual, two, three, five, ten and twenty-year returns for all three period partitions yield similar results and are presented in Appendix A1. These results are in the same format as above and include both logarithmic and arithmetic returns.

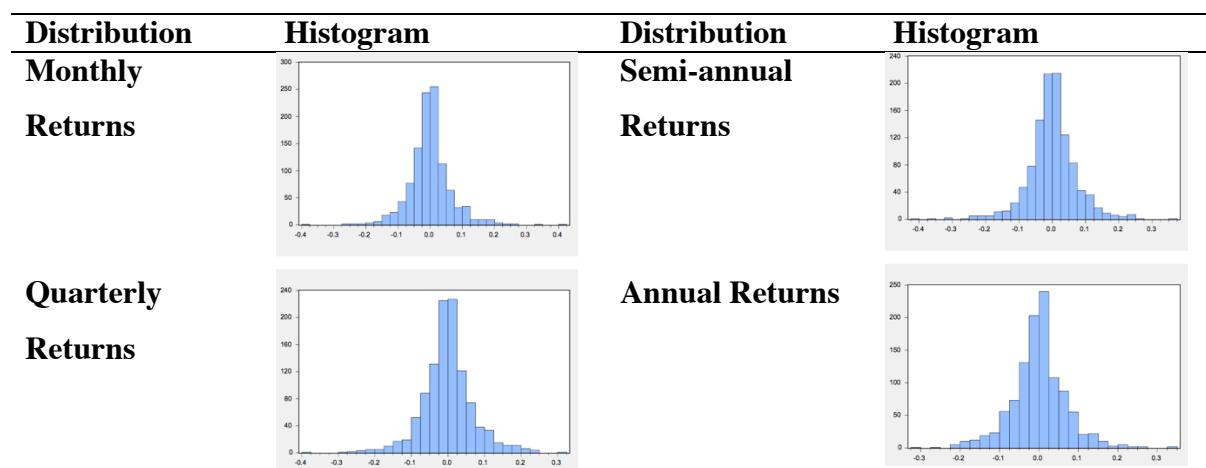
6.2.2 Real Returns

The descriptive statistics for real ALSI and ALBI data are available for the period 01/01/1987 – 31/12/2016 and 31/12/1998 – 31/12/2016 respectively. This is due to a lack of CPI data, and focuses on logarithmic returns. The results are appended in Appendix A2. Normality is rejected at the monthly level for the ALSI and ALBI, but at frequencies higher than monthly, no rejections of normality were observed.

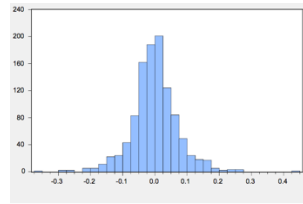
6.2.3 Return distribution Histograms

The Histograms depicted in the Figure 6.1 below, depict how the distribution of logarithmic returns on the ALSI change as the holding period of returns changes. These distributions are based on monthly returns and include the following frequency distributions; monthly, quarterly, semi-annual, annual, two-yearly, three-yearly, four-yearly, five-yearly, ten-yearly, 20-yearly, 30-yearly, 40-yearly, 50-yearly and 1000-month returns. For these distributions overlapping data samples were used to maximise the number of points in each histogram. It is abundantly clear from Figure 6.1 that logarithmic returns become more negatively skewed as their holding period increases. Furthermore, we observe a breakdown in the distribution at when looking at returns greater than 50-years.

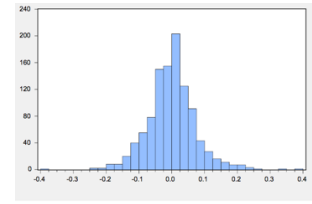
Figure 6.1: Showing the ALSI Logarithmic return distribution as the holding period of returns increases



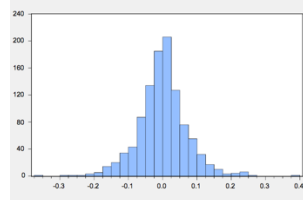
**Two-yearly
Returns**



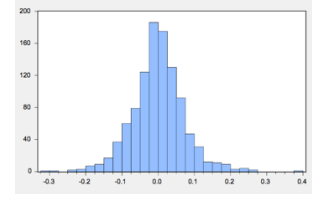
**Four-yearly
Returns**



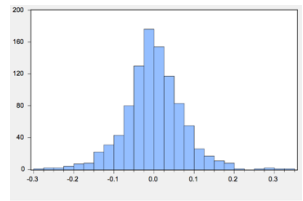
**Three-yearly
Returns**



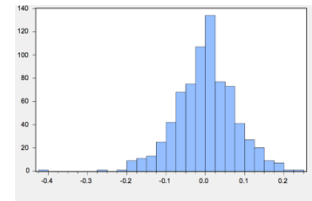
**Five-yearly
Returns**



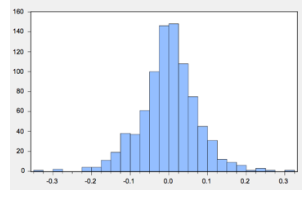
**Ten-yearly
Returns**



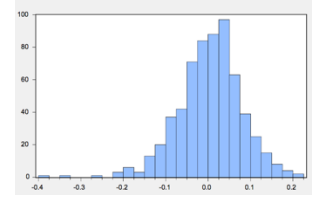
**30-yearly
Returns**



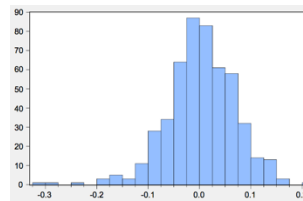
**20-yearly
Returns**



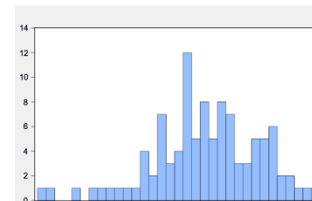
**40-yearly
Returns**



**50-yearly
Returns**

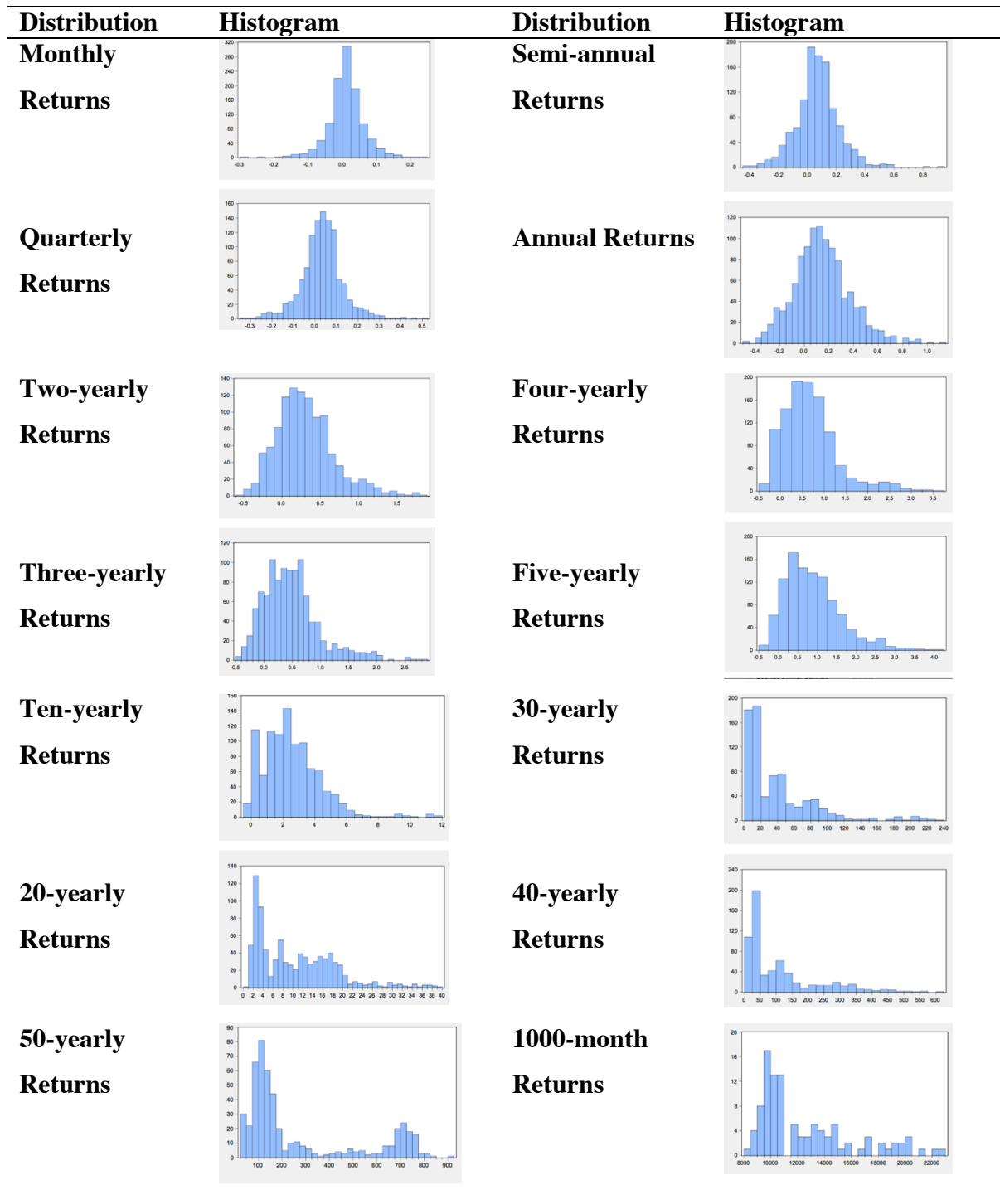


**1000-month
Returns**



The arithmetic returns distribution over time are presented in Figure 6.2. This Figure shows that the histograms of arithmetic returns exhibit contrasting results to that of the logarithmic returns in Figure 6.1. Instead of the negative skewness in returns observed as the holding period increases for logarithmic returns, arithmetic returns exhibit positive skewness as the return holding period increases.

Figure 6.2: Showing the ALSI Arithmetic return distribution as the holding period of returns increases



6.3 Tests for Stationarity

6.3.1 Augmented Dickey Fuller Tests

Table 6.5 summarizes the results of the Augmented Dickey Fuller (ADF) tests for real and nominal ALSI and ALBI returns. Detailed results that include the exogenous test factor specifications (constant, constant and trend and non), the t-statistic and the p-value for both indices, over all the return frequencies, for nominal and real returns, are appended in Appendix B.

From the results of the ADF tests, we find that both nominal and real ALSI logarithmic total returns for monthly, quarterly, semi-annually, annually, two-yearly and three-year frequencies are stationary. However, from the five-year return interval and longer, our results indicate that this return series is no longer stationary. The real return data has shown that the log-level price series is stationary as well.

The ALBI nominal returns are stationary for monthly, quarterly and semi-annual return intervals, while real ALBI returns provide stationarity until the annual frequency. Due to the non-stationary nature of longer return interval returns, this paper notes that regressions run on non-stationary series can result in spurious results (Van Rensburg, 2016).

Table 6.5: Summary of Augmented Dickey-Fuller tests in Appendix B

The Table summarizes the results of the Augmented Dickey Fuller tests for real and nominal data for the ALSI and the ALBI. These tests were performed over the period 01/01/1900 – 31/12/2016 (31/01/1925 for monthly, quarterly and semi-annual) for the nominal ALSI. Real ALSI tests were performed over the period 01/01/1987 – 31/12/2016, while real and nominal ALBI tests were conducted over the period 31/12/1998 – 31/12/2016.

Frequency	Nominal ALSI	Real ALSI	Nominal ALBI	Real ALBI
Month	First Log Difference is Stationary ***	Log-Level is Stationary ***	First Log Difference is Stationary ***	Log-Level is Stationary***
Quarter	First Log Difference is Stationary ***	Log-Level is Stationary ***	First Log Difference is Stationary ***	Log-Level is Stationary***
Semi-annual	First Log Difference is Stationary ***	Log-Level is Stationary ***	First Log Difference is Stationary ***	First Log Difference is Stationary*
Annual	First Log Difference is Stationary ***	Log-Level is Stationary ***	Second Log Difference is Stationary*	First Log Difference is Stationary*
2 Year	First Log Difference is Stationary***	Log-Level is Stationary *	Non-Stationary	Second Log Difference is Stationary *
3 Year	First Log Difference is Stationary ***	Log-Level is Stationary *	Non-Stationary	Non-Stationary
5 Year	Second Log Difference is Stationary*	Non-Stationary	Non-Stationary	Non-Stationary
10 Year	Second Log Difference is Stationary*	n/a	n/a	n/a
20 Year	Non-Stationary	n/a	n/a	n/a

* significant at $p < 0.05$; ** significant at $p < 0.01$; *** significant at $p < 0.001$

6.3.2 Phillips-Perron Test

Table 6.6 summarizes the results of the ALSI and ALBI non-parametric Phillips-Perron test. These results are appended in Appendix B. Although the t-statistics and p-values vary between the ADF and PP test (Shown in Appendix B), the net stationarity result of the nominal and real ALSI returns remains the same for both these tests for stationarity. This implies that the return series is stationary for frequencies up to five-year return intervals for both real and nominal ALSI returns.

The Phillips-Perron test on nominal ALBI returns presents the same stationarity conclusion as the ADF. This conclusion is that the return series on the ALBI is stationarity for frequencies up to six months. Real ALBI returns become non-stationary at the annual level, which makes inference for this series difficult due to its limited history and its non-stationarity at frequencies one-year and greater.

Table 6.6: Summary of Phillips-Perron tests in Appendix B

The Table summarizes the results of the Phillips Perron Tests for real and nominal data for the ALSI and the ALBI. These tests were performed over the period 01/01/1900 – 31/12/2016 (31/01/1925 for monthly, quarterly and semi-annual) for the nominal ALSI. Real ALSI tests were performed over the period 01/01/1987 – 31/12/2016, while real and nominal ALBI tests were conducted over the period 31/12/1998 – 31/12/2016.

Frequency	Nominal ALSI	Real ALSI	Nominal ALBI	Real ALBI
Month	First Log Difference is Stationary ***	Log-Level is Stationary ***	First Log Difference is Stationary ***	Log-Level is Stationary
Quarter	First Log Difference is Stationary ***	Log-Level is Stationary ***	First Log Difference is Stationary ***	Log-Level is Stationary
Semi-annual	First Log Difference is Stationary ***	Log-Level is Stationary ***	First Log Difference is Stationary ***	First Log Difference is Stationary
Annual	First Log Difference is Stationary ***	Log-Level is Stationary ***	Second Log Difference is Stationary ***	Second Log Difference is Stationary**
2 Year	First Log Difference is Stationary***	Log-Level is Stationary *	Non-Stationary	Non-Stationary
3 Year	First Log Difference is Stationary ***	Log-Level is Stationary *	Non-Stationary	Non-Stationary
5 Year	Second Log Difference is Stationary*	Non-Stationary	Non-Stationary	Non-Stationary
10 Year	Second Log Difference is Stationary*	n/a	n/a	n/a
20 Year	Non-Stationary	n/a	n/a	n/a

* significant at $p < 0.05$; ** significant at $p < 0.01$; *** significant at $p < 0.001$

6.4 Autocorrelation Test

6.4.1 Nominal Returns

The results of the Autocorrelation test when using non-overlapping ALSI nominal logarithmic total returns is shown in Table 6.7. These results represent the serial correlation coefficient when regressing non-overlapping data of the frequency specified, on its immediate prior value.

When assessing the entire sample without the partition in Table 6.7, statistically significant short-term (up to semi-annual) trending behavior in returns is observed due to the positive autocorrelation coefficients observed over this period. This trend being statistically significant short-term positive autocorrelation coefficients (monthly, quarterly, semi-annual), followed by (caution insignificant) negative serial correlations from two and three-year lags. This paper will cease from analyzing results from five-year return frequencies and above these regressions may be spurious due to non-stationary returns at those intervals. The overlapping data

regressions are presented in Appendix C1 and C2, but for the purposes of this analysis, non-overlapping samples will be focused on.

Observed from Table 6.7 the period prior to 1987 exhibited significant positive first order serial dependence in JSE returns for monthly, quarterly, semi-annual and annual returns. The positive autocorrelation coefficients are significant at the 0.1% significance level, and this positive serial dependence implies that short term returns (up to the one-year) are trending. For higher lags, the coefficients are no longer significant, but with caution, this paper would like to highlight that at the two and three-year interval, negative autocorrelation coefficients are noticed. These negative serial correlation coefficients, hint toward medium term mean reversion on the ALSI. This paper will cease from analyzing results at frequencies of five-year return frequencies and above. This is based on the possibility of these regressions being spurious due to non-stationary returns.

Observed in Table 6.7 during the post-1986 period, there existed significant positive serial dependence in JSE returns at quarterly lags. The positive autocorrelation coefficients are significant at the 1% p-value, and this positive serial dependence implies that short term returns (up to and including semi-annual) are trending. For higher lags, the coefficients are no longer significant, except for the negative serial correlation coefficient of -0.6968 with a p-value of 0.0255. This Table clearly depicts the short-term trending behavior (monthly, quarterly and semi-annually), as well as statistically significant mean reversion three-year lags. These negative serial correlation coefficients (one to three-year) hint toward medium term mean reversion on the JSE. This paper will cease from analyzing results from five-year return frequencies and above, as these regressions may be spurious due to non-stationary returns at those intervals.

Table 6.7: Autocorrelation test for ALSI logarithmic returns over various frequencies over the all three periods

The Table displays the autocorrelation coefficients and their respective p-value for each frequency of ALSI logarithmic total returns. This is shown below for month, quarter, semi-annual, annual, two-year, three-year, five-year, ten-year and twenty-year returns. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero. This was performed over the entire period 01/01/1900 (31/01/1925 for monthly, quarterly and semi-annual frequencies) to 31/12/2016, the pre-1987 period 01/01/1900 (31/01/1925 for monthly, quarterly and semi-annual frequencies) to 31/12/1986 and the post-1986 period 01/01/1987 to 31/12/2016.

Entire Period

Pre-1987

Post-1986

	Autocorrelation Coefficient	P-value	Autocorrelation Coefficient	P-value	Autocorrelation Coefficient	P-value
Month	0.1204***	0.0001	0.1956***	0.0001	0.0069	0.8960
Quarter	0.2826***	0.0000	0.3041***	0.0000	0.2396**	0.0084
Semi-Annual	0.2597***	0.0004	0.3346***	0.0002	0.1062	0.4083
Annual	0.0389	0.6777	0.1398	0.2044	-0.3018	0.0953
2 Year	-0.0583	0.0668	-0.0794	0.6244	-0.2613	0.3574
3 Year	-0.2152	0.0193	-0.2321	0.2426	-0.6968*	0.0255
5 Year	0.3235	0.1100	0.2728	0.2938	0.2610	0.6284
10 Year	0.1385	0.6743	-0.4449	0.4619	-0.4114	0.1886
20 Year	0.3662	0.6749	-2.007	0.6448	n/a	n/a

* significant at $p < 0.05$; ** significant at $p < 0.01$; *** significant at $p < 0.001$

The autocorrelation results of the nominal ALBI returns are presented in Table 6.8. From this Table, insignificantly slightly negative monthly co-efficient are observed, but more importantly the repeating significant short term trending behavior from the quarterly level is depicted. Interestingly, only positive autocorrelation coefficients for the ALBI at frequencies greater than quarterly are observed. This implies the lack of mean reversion in ALBI returns. However, the results of Section 6.3 concluded that the ALBI returns are non-stationary at frequencies of greater than one-year. Thus, the interpretation of results at frequencies greater than one-year were ignored.

Table 6.8: Auto-Correlation Test for ALBI logarithmic total returns over various frequencies for the period 31/12/1998 to 31/12/2016

The Table displays the autocorrelation coefficients and their respective p-value for each frequency of ALBI logarithmic total returns regressed on that respective frequencies previous periods return. These results were calculated using a non-overlapping sample of returns. This is shown below for month, quarter, semi-annual, annual, two-year, three-year and five-year returns. The p-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	Month	Quarter	Semi-annual	Annual	2 Year	3 Year	5 Year
Autocorrelation Coefficient	-0.0196	0.2073	0.1965	0.3321	0.7374	0.5695	0.2529
P-value	0.7750	0.0784	0.2527	0.2138	0.0413*	0.3192	n/a

* significant at $p < 0.05$; ** significant at $p < 0.01$; *** significant at $p < 0.001$

6.4.2 Real Returns

Table 6.9 displays the real results of the Autocorrelation tests for the ALSI real returns. When analyzing real returns for the Autocorrelation tests, significant short-term trending behavior in real ALSI returns were observed followed by statistically insignificant, medium-term negative autocorrelation coefficients. These coefficients hint towards mean reversion in the one, two and three-year return interval.

Table 6.9: Autocorrelation Test for ALSI logarithmic real returns over the period 01/01/1987 to 31/12/2016 for all return frequencies

The Table displays the autocorrelation coefficients and their respective p-value for each frequency of ALSI logarithmic total real returns regressed on that respective frequencies previous periods return. This test was performed on a non-overlapping sample over the period 01/01/1987 to 31/12/2016. This is shown below for month, quarter, semi-annual, annual, two-year, three-year, five-year and ten-year real returns. The p-value is specified by the Null hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	Month	Quarter	Semi-annual	Annual	2 Year	3 Year	5 Year	10 Year
Autocorrelation coefficient	0.0277	0.0675	0.1450	-0.2185	-0.1389	-0.5438	0.2238	-0.3344
P-value	0.6009	0.0045**	0.2768	0.2643	0.6388	0.1264	0.6869	0.296

* significant at $p < 0.05$; ** significant at $p < 0.01$; *** significant at $p < 0.001$

Table 6.10 displays the real results of the Autocorrelation test on the ALBI returns. No indication of mean reversion in the ALBI is presented, and a lack thereof is identified. However, significant short-term trending behavior, that is significant at the quarterly return interval was observed.

Table 6.10: Autocorrelation Test for ALBI total real logarithmic returns over various frequencies for the period 31/12/1998 to 31/12/2016.

The Table displays the autocorrelation coefficients and their respective p-value for each frequency of ALBI logarithmic total real returns regressed on that respective frequencies previous periods return. These results were calculated using a non-overlapping sample of returns. This is shown below for month, quarter, semi-annual, annual, two-year, and three-year returns. The p-value is specified by the Null hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	Monthly	Quarter	Semi-annual	Annual	2 Year	3 Year
Autocorrelation coefficient	0.0552	0.2424	0.2208	0.3094	0.3662	0.0310
P-value	0.4208	0.0387*	0.1967	0.2444	0.2855	0.9645

* significant at $p < 0.05$; ** significant at $p < 0.01$; *** significant at $p < 0.001$

6.5 Breakpoint Tests

6.5.1 Quandt-Andrews Breakpoint Test

This test for a single unknown structural break was performed using the Quandt-Andrews Breakpoint test, outlined in Chapter 5. The results of the test performed on the ALSI displayed in Table 6.11. Only the breakpoint 09/1927 was observed to be significant at the monthly return level for nominal ALSI return. On the inflation adjusted side, no significant structural breaks in real returns were shown.

Table 6.11: Quandt-Andrews Unknown Breakpoint test ALSI total nominal returns over the entire period.

This Table depicts the results of the Quandt-Andrews test performed on the ALSI returns over the period 01/01/1900 to 31/12/2016 (for monthly, quarterly and semi-annual frequencies 31/01/1925 to 31/12/2016).

	Month	Quarter	Semi-annual	Annual	2 Year	3 Year
Breakpoint Date	1927m9	1946Q3	1981S1	1921	1980	1981
Maximum LR F-Statistic	14.6619**	3.6202	5.3329	2.9154	4.2638	4.7071
p-value	0.0029	0.4361	0.2146	0.5717	0.3363	0.2798
Exp LR Fs-statistic	4.6301**	0.7975	0.6074	0.2611	0.6647	0.9188
p-value	0.0010	0.2678	0.3655	0.6969	0.3320	0.2220
Ave LR F-statistic	7.2079***	1.2561	1.0385	0.3980	0.9509	1.4919
p-value	0.0004	0.2516	0.3261	0.7370	0.3631	0.1923

* significant at $p < 0.05$; ** significant at $p < 0.01$; *** significant at $p < 0.001$

Table 6.12 describes the results of the Quandt-Andrews test performed on the ALBI. The ALBI finds a significant structural break on the 11/2010 for monthly data. The results of this test on real ALBI returns find the same structural break point at 11/2010. These breakpoints are significant at the 0.1% probability level.

Table 6.12: Quandt-Andrews Unknown Breakpoint test ALBI total nominal returns over the entire period.

This Table depicts the results of the Quandt-Andrews test performed on the ALBI returns over the period 31/12/1998 to 31/12/2016.

	Month	Quarter	Semi-annual	Annual	2-Year
Breakpoint Date	2010M11	2005Q2	2004S1	2006	2014
Maximum LR F-Statistic	12.9643**	3.3641	5.9259	5.9504	3.1367
p-value	0.0066	0.4821	0.1661	0.1643	0.5261
Exp LR Fs-statistic	4.6550**	0.8080	1.7401	1.6506	0.7411
p-value	0.0010	0.2634	0.0726	0.0813	0.2930
Ave LR F-statistic	7.8548***	1.4173	2.7393	2.6742	1.0899
p-value	0.0001	0.2091	0.0560	0.0594	0.3064

* significant at $p < 0.05$; ** significant at $p < 0.01$; *** significant at $p < 0.001$

6.5.2 Bai-Perron Breakpoint Test

This test was used to confirm the above breakpoints, as well as investigate the possibility of multiple unknown breakpoints in the sample. This test provides the same results as the Quandt-Andrews test for the ALBI real and nominal returns that are shown in. But for the ALSI, this Breakpoint test identifies a different structural breakpoint when using monthly data around the month 09/1972 instead of the 09/1927 breakpoint identified with nominal data. The Breakpoints at other frequencies are not significant. This breakpoint prompted a revised Autocorrelation regression for the dates in question. The Bai-Perron test results are presented in Appendix D.

6.5.3 Breakpoint Regressions

Due to the existence of structural breaks in our dataset, we decided to re-run the Autocorrelation test regressions in the Equations before the breakpoint, after the breakpoint and over the entire period. This was performed to calculate the change in the value of the autocorrelation coefficient around structural break. These results are depicted in Table 6.19 for the ALSI, and in Table 6.20 for the ALBI.

Table 6.13 refers, we notice that the sign of the autocorrelation coefficient displayed remains positive, however the autocorrelation coefficient statistical significance changes from significant in the pre-1972 and entire periods, to insignificant in the post 1972 period. This could induce spurious inference, however the period partition in 1987 does take this structural break into account.

Table 6.13: Autocorrelation test post results of Quandt-Andrews and Bai-Perron Breakpoint tests for monthly returns about the breakpoint 1972M09 for the ALSI

The Table displays the monthly results from Autocorrelation test performed on the ALSI (Logarithmic returns) about the breakpoint 1972m09. This regression does include an intercept term, and monthly returns were regressed on their immediate prior returns. These statistics include the autocorrelation co-efficient and the p-value that is specified by the Null hypothesis that the autocorrelation coefficient is not statistically different from zero.

Period	Entire Period (1925m1 2016m12)	Pre-Breakpoint (1925m1 1972m09)	Post Breakpoint (1972m09 2016m12)
Autocorrelation Co-efficient	0.120393 ***	0.310167 ***	0.041442
P-value	0.0001	0.0000	0.34009

* significant at $p < 0.05$; ** significant at $p < 0.01$; *** significant at $p < 0.001$

In Table 6.14 we observe a drastic change in the value of the autocorrelation coefficient as well as its sign. This breakpoint shows significantly different trends in ALBI returns, and this could be attributed to the after-math and change in regulation post the 2008 financial crisis. Due to the limited nature of our ALBI data, this paper decided against partitioning the eighteen-year period any further.

Table 6.14: Autocorrelation test post-results of Quandt-Andrews and Bai-Perron Breakpoint tests for monthly returns about the breakpoint 2010m11 for the ALBI.

The Table displays the monthly results from an Autocorrelation test performed on the ALSI (Logarithmic returns) about the breakpoint 2010m11. This regression does include an intercept term, and monthly returns were regressed on their immediate prior returns. These statistics include the autocorrelation co-efficient and the p-value that is specified by the null hypothesis that the autocorrelation coefficient is not statistically different from zero.

Period	Entire Period (1998m12 2016m12)	Pre-Breakpoint (1998m12 2010m11)	Post Breakpoint (2010m11 2016m12)
Autocorrelation Co-efficient	0.136269	-0.019587	-0.306805
P-value	0.1082	0.7750	0.0079

* significant at $p < 0.05$; ** significant at $p < 0.01$; *** significant at $p < 0.001$

6.6 Runs Test

6.6.1 Nominal Returns

The results of the nominal Runs test are depicted in Table 6.21 below. Looking at the entire period's results significant rejections of the Random Walk Model at monthly, quarterly, three-year and five-yearly return intervals are observed. Furthermore, these rejections of the Random Walk Model are based on a significant deviation in the expected number and observed number of runs. Delving deeper into the number of runs, it was observed that for monthly, quarterly, semi-annually and annual return intervals, the number of observed runs is less than the number of expected runs. This hints toward short-term trending behavior, and this behavior is significant at the monthly level. When analyzing the two, three and five-year return intervals the number of expected runs was observed to be less than the number of observed runs, hinting toward medium term mean reversion. This mean reversion is statistically significant at the three-year level.

Looking at the pre-1987 period, the Random Walk Model is rejected at monthly and semi-annual return intervals, with statistically significantly trending monthly returns – as shown in Table 6.15. Up till the one-year level, it is depicted that the number of observed runs are less than the number of expected runs, that once against hints toward short term trending behavior. significant mean reverting behavior was observed at the three-year level. In addition, it was observed at two, three and five-year return intervals, that the number of observed runs is larger than the number of expected runs, again reinforcing medium-term mean reversion.

Most importantly going forward, the assessment of the results of the most current period (01/01/1987 – 31/12/2016) is depicted in Table 6.15. Over this period, the Random Walk Model is rejected at the quarterly and three-year return intervals. These rejections are accompanied with significant trending behavior at the quarterly interval and significant mean reverting behavior at the three-year interval. However, when assessing the number of runs to their observed and expected values, this study notes the albeit insignificant, trending behavior in monthly, quarterly and semi-annual returns. As well as indications of Mean Reversion from the annual to three-yearly intervals.

For all three periods, long-term trending behavior (statistically insignificant) at return intervals of ten-years and above is observed. This test is non-parametric and is independent on the stationarity of the return series. The results are higher return frequencies that were shown to be non-stationary, clash with the results of the Autocorrelation test at the same frequencies. This further supports the notion of spurries regressions in non-stationary time series data.

Table 6.15: Runs Test for ALSI logarithmic total returns over all three periods

The Table displays the results of a Runs test on ALSI Logarithmic total returns. The Runs test p-value is specified by the Null hypothesis that the sequence of returns is random. This Runs test has the median selected as the threshold value. The p-value for trends and oscillations, test the Null hypothesis that returns follow a trending or oscillating behavior respectively.

Frequency	Period	Observed Number of Runs	Expected Number of Runs	P-value	P-value for Trends	P-value for Oscillations
Month	1925-2016	457	552.5	0.0000 ***	0.0000 ***	1
Month	1987-2016	168	181	0.17	0.7863	0.2137
Month	1925-1986	276	372.5	0.0000 ***	0.0000 ***	1
Quarter	1925-2016	164	184.5	0.0321*	0.2540	0.7460
Quarter	1987-2016	50	61	0.0437 *	0.0100*	0.9900
Quarter	1925-1986	115	124.5	0.2258	0.7602	0.2398
Semi-annual	1925-2016	82	92.497	0.1196	0.3845	0.6155
Semi-annual	1987-2016	30	31	0.7945	0.6608	0.3392
Semi-annual	1925-1986	51	62.496	0.0374*	0.2828	0.7172
Annual	1900-2016	58	59.5	0.7812	0.2089	0.7911
Annual	1987-2016	17	16	0.7102	0.2283	0.7717
Annual	1900-1986	41	44.494	0.4510	0.2466	0.7534
2 Year	1900-2016	32	30	0.5962	0.5835	0.4165
2 Year	1987-2016	10	8.497	0.4090	0.5862	0.4138
2 Year	1900-1986	21	22.488	0.6457	0.4510	0.5490
3 Year	1900-2016	28	20.487	0.0147*	0.9810	0.0190*
3 Year	1987-2016	10	6	0.0071**	0.9865	0.0135*
3 Year	1900-1986	19	15.48	0.1829	0.9138	0.0862*
5 Year	1900-2016	8	12.478	0.055	0.5	0.5

5 Year	1987-2016	3	4	0.3613	0.2199	0.7801
5 Year	1900-1986	11	9.471	0.4419	0.5	0.5
10 Year	1900-2016	6	6.455	0.7706	0.7830	0.2170
10 Year	1987-2016	2	2.333	0.4795	0.7659	0.2341
10 Year	1900-1986	4	5	0.4450	0.5	0.5
20 Year	1900-2016	4	5	0.4450	0.5	0.5

* significant at $p < 0.05$; ** significant at $p < 0.01$; *** significant at $p < 0.001$

As depicted in Table 6.16 the Runs test results on nominal ALBI returns, has no statistically significant results. Thus, this test fails to reject the null hypothesis that ALBI returns follow a Random Walk Model.

Table 6.16: Runs Test for ALBI logarithmic total returns for the period 31/12/1998 to 31/12/2016

The Table displays the results of a Runs test on ALBI logarithmic total returns. The Runs Test p-value is specified by the Null hypothesis that the sequence of returns is random. This Runs Test has the median selected as the threshold value. The p-value for trends and oscillations, test the Null hypothesis that returns follow a trending or oscillating behavior respectively. This test is conducted over the period 31/12/1998 to 31/12/2016.

Frequency	Observed Number of Runs	Expected Number of Runs	P-value	P-value for Trends	P-value for Oscillations
Month	113	109	0.5853	0.14	0.86
Quarter	36	37	0.8124	0.6471	0.3529
Semi-annual	22	19	0.3103	0.5538	0.4462
Annual	7	10	0.1449	0.5779	0.4221
2 Year	4	5.444	0.2964	0.6160	0.3840
3 Year	4	4	1.0000	0.2199	0.7801
5 Year	2	2.333	0.4795	0.0734	0.9266

* significant at $p < 0.05$; ** significant at $p < 0.01$; *** significant at $p < 0.001$

6.6.2 Real Returns

When assessing the Runs test for real ALSI returns, it indicated that the same results are observed as for its nominal counterpart, where the Random Walk Model is rejected at the semi-annual and three-yearly intervals. Furthermore, significant short-term trending behavior in returns (significant for quarterly returns) up to and including one-year was observed together with significant mean reverting behavior at three-years. These results are appended in Appendix E.

The real ALBI returns provide similar results to its nominal form, with no significant rejection of the Random Walk Model. However, significant mean reverting behavior at two-year returns

is observed as a result of the empirically observed number of runs of eight, being significantly higher than the expected number of runs of 5.44. These results are displayed in Appendix E.

6.7 Variance Ratio Test

6.7.1 Entire Period

Before analyzing the results of the Individual Variance Ratio tests, the Joint Variance Ratio test is presented first - that tests the overall null hypothesis that returns follow a Random Walk Model over all individual variance ratio test periods. These results are presented in Table 6.17. From this Table, significant rejections of the Random Walk Model at monthly and semi-annual return intervals, for the entire period is observed. The entire Variance Ratio test results are appended in Appendix F, while the tables that follow have summarized the important results.

Table 6.17: Overall Joint Variance Ratio tests for ALSI logarithmic total returns over the full period

The Table displays the Joint Variance Ratios tests of the ALSI logarithmic total return over the period 01/01/1900 to 31/12/2016 (31/01/1925 to 31/12/2016 for monthly, quarterly and semi-annual frequencies). The Variance ratio tests were conducted for monthly, quarterly, Semi-annually, annually, two-yearly, three-yearly, five-yearly and ten-yearly logarithmic total returns. In addition have reported the results of the Joint Test statistic, the degrees of freedom and the p-value of the test against the null hypothesis that the series of returns follow a Random Walk stochastic process.

Frequency	Test Statistic	Degrees of freedom	P-Value
Month	3.036879 *	1103	0.0236
Quarter	1.423506	367	0.7391
Semi-annual	3.047901 *	183	0.0160
Annual	0.963847	116	0.9136
2 Year	0.413804	58	0.9894
3 Year	1.195964	38	0.4097
5 Year	2.12	23	0.1302
10 Year	1.398427	11	0.2977

* significant at $p < 0.05$; ** significant at $p < 0.01$; *** significant at $p < 0.001$

With the Joint Variance Ratio tests analyzed, the single period Variance Ratio tests for the encompassing period including all data is presented. Variance Ratio tests have been performed at all return frequencies. Table 6.18 below presents the statistically significant Individual Variance Ratios for all return frequencies over the period.

Table 6.18: Individual Variance Ratio Tests for ALSI logarithmic total returns over the full period

The Table displays the Individual Variance Ratios Tests of the ALSI logarithmic total return over the period 01/01/1900 to 31/12/2016 (31/01/1925 to 31/12/2016 for monthly, quarterly and semi-annual frequencies). The Variance ratio tests were conducted for monthly, quarterly, Semi-annually, Annually, two-yearly, three-yearly, five-yearly and ten-yearly logarithmic total returns. In addition have reported the results of the Joint Test statistic, the degrees of freedom and the p-value of the test against the null hypothesis that the series of returns follow a Random Walk stochastic process.

Variance Ratio Test	Variance Ratio	P-value	Variance Ratio Test	Variance Ratio	P-value
Quarter/Month	1.1814**	0.0031	5 Year /Semi-annual	0.99	0.9909
Semi-annual/Month	1.2811**	0.0062	2 Year/Annual	1.05	0.5887
Annual/Month	1.3460*	0.0226	3 Year/Annual	0.99	0.9355
2 Year/Month	1.3546	0.0971	5 Year/Annual	0.80	0.3375
5 Year/Month	0.9625	0.9083	6 Year/3 Year	0.81	0.2317
3 Year/Quarter	0.2069	0.1546	10 Year/ 5 Year	1.35	0.0504
5 Year/Quarter	0.13935	0.1555	15 Year/ 5 Year	1.52*	0.0466
Year/Semi-annual	1.27**	0.0023	20 Year/ 5 Year	1.60	0.0651
2 Year /Semi-annual	1.36*	0.0311	30 Year/ 5 Year	1.89*	0.0343

* significant at $p < 0.05$; ** significant at $p < 0.01$; *** significant at $p < 0.001$

Monthly data reveals that monthly returns significantly trend within each quarter, semi-annual and annual period (weak significance at the two-year level) with Variance Ratios of greater than one. Furthermore, we observed a Variance Ratio of less than one at the five-year level. However, this result must be interpreted with caution as it is statistically insignificant.

Quarterly data provides no statistically significant results, but does again show statistically insignificant mean reverting behavior at the three and five-year intervals. Semi-annual data provides statistically significant trending behavior up to the two-year interval. Once again, insignificant Variance Ratio of less than one at the five-year level is observed when using semi-annual data.

Annual returns provide no statistically significant Variance Ratios. However, (with caution) a trending annual Variance Ratio of greater than one at the two and three-year level, and mean reverting annual Variance Ratios of less than one for three-years and greater are observed. Three-yearly data appears to mean revert with Variance Ratios below one that are statistically insignificant. Five-yearly Variance Ratios are statistically significantly greater than one and thus trend. Ten-yearly Variance Ratios trend although they are statistically insignificant.

Figures 6.3 – 6.6 below, presents a graphical representation of the result of the Variance Ratio Test. These Figures, show the Variance Ratio of the specified frequency over the entire

period. From these Figures, monthly, semi-annual and five-yearly returns trend in the short-term. More importantly, this paper identifies a pattern in the Variance Ratios, whereby the shapes of all the graphs seem to have a minimum point after their initial trending period. This pattern lends itself to the return series observing short-term trending followed by medium-term mean reversion (or lesser trending effect).

Figure 6.3: Variance Ratio Test for monthly ALSI nominal returns over the entire period

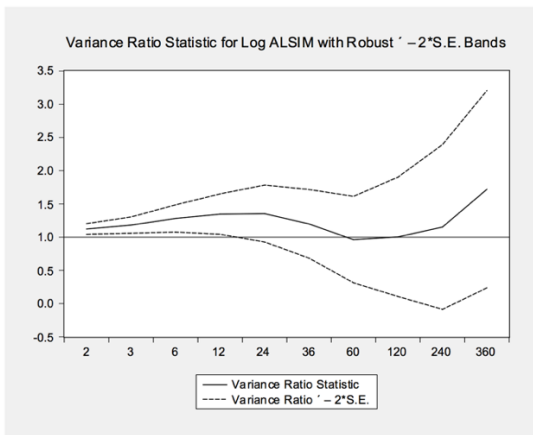


Figure 6.4: Variance Ratio Test for semi-annual ALSI nominal returns over the entire period

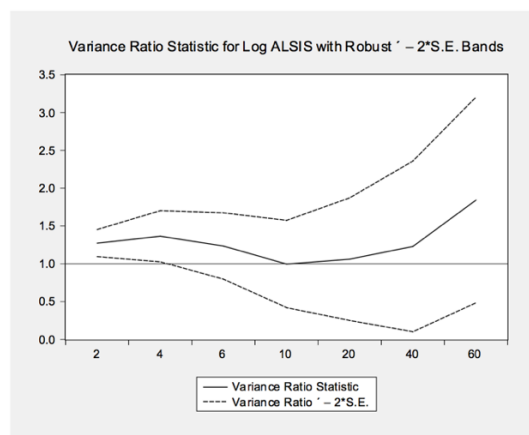


Figure 6.5: Variance Ratio Test for annual ALSI nominal returns over the entire period

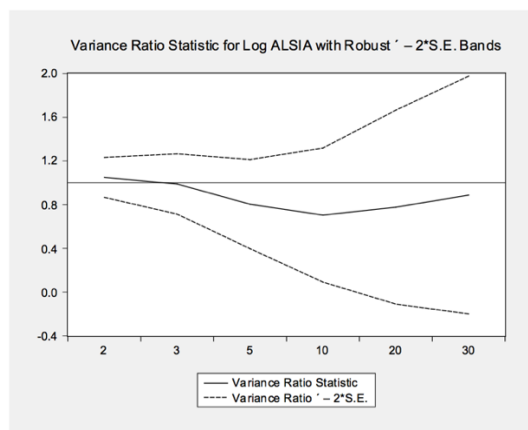
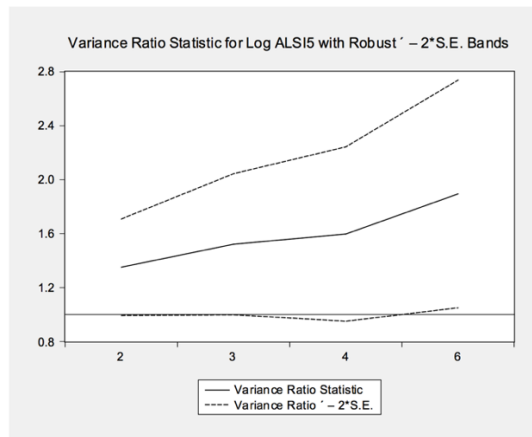


Figure 6.6: Variance Ratio Test for five-year ALSI nominal returns over the entire period



6.7.2 Pre-1987 Period

Table 6.19: Overall Joint Variance Ratio Tests for ALSI logarithmic total returns over the pre-1987 period

The Table displays the Joint Variance Ratios Tests of the ALSI logarithmic total return over the period 01/01/1900 to 31/12/2016 (31/01/1925 to 31/12/2016 for monthly, quarterly and semi-annual frequencies). The Variance ratio tests were conducted for monthly, quarterly, Semi-annually, annually, two-yearly, three-yearly, five-yearly and ten-yearly logarithmic total returns. In addition have reported the results of the Joint Test statistic, the degrees of freedom and the p-value of the test against the null hypothesis that the series of returns follow a Random Walk stochastic process.

Frequency	Test Statistic	Degrees of freedom	P-Value
Month	3.5064**	743	0.0045
Quarter	3.7652**	247	0.0013
Semi-annual	2.8131*	123	0.0338
Annual	1.2766	87	0.7412
2 Year	0.9566	43	0.8088
3 Year	1.4230	28	0.4895
5 Year	1.0132	17	0.7746
10 Year	2.1561	8	0.0612

* significant at $p < 0.05$; ** significant at $p < 0.01$; *** significant at $p < 0.001$

According to Table 6.19 for the period preceding 1987, rejections of the Random Walk Model for the Joint Variance Ratio tests are observed for monthly, quarterly and semi-annual returns. Table 6.20 however, deals with the Individual Variance Ratio Test where it is observed that short-term monthly returns up to two-years exhibit significant trending behavior, with periods greater than two-years exhibiting non-significant trending behavior. It is important to note that the Variance Ratio drops considerably at the five-year level. Quarterly and semi-annual returns provide the same phenomenon. Annual data however shows statistically insignificant trending in annual returns up to three-years, and mean reversion thereafter. Two and three-yearly data shows statistically insignificant mean reverting tendencies.

Table 6.20: Individual Joint Variance Ratio Tests for ALSI logarithmic total returns over the pre-1987 period

The Table displays the Individual Variance Ratios Tests of the ALSI logarithmic total return over the period 01/01/1900 to 31/12/1986 (31/01/1925 to 31/12/1986 for monthly, quarterly and semi-annual frequencies). The Variance Ratio tests were conducted for monthly, quarterly, Semi-annually, annually, two-yearly, three-yearly, five-yearly and ten-yearly logarithmic total returns. In addition have reported the results of the Joint Test statistic, the degrees of freedom and the p-value of the test against the null hypothesis that the series of returns follow a Random Walk stochastic process

Variance Ratio Test	Variance Ratio	p-value	Variance Ratio Test	Variance Ratio	p-value
Quarter/Month	1.2786***	0.0009	2 Year /Semi-annual	1.56*	0.0103
Semi-annual/Month	1.4045**	0.0027	5 Year /Semi-annual	1.12	0.7304
Annual/Month	1.5573**	0.0049	2 Year/Annual	1.14	0.2017

2 Year/Month	1.7248**	0.0094	3 Year/Annual	1.07	0.6637
5 Year/Month	1.2158	0.6131	5 Year/Annual	0.87	0.5887
Semi-annual/Quarter	1.3122***	0.0002	6 Year/3 Year	0.80	0.3003
Annual/Quarter	1.5848***	0.0002	10 Year/ 5 Year	1.22	0.3110
2 Year/Quarter	1.8024***	0.0006	20 Year/ 10 Year	0.58*	0.0311
5 Year/Quarter	1.2841	0.4498	30 Year/10 Year	0.47	0.0783
Year/Semi-annual	1.33**	0.0049			

* significant at $p < 0.05$; ** significant at $p < 0.01$; *** significant at $p < 0.001$

These Figures (6.7 – 6.8) again reinforce the short-term trending behavior of ALSI returns, but also exhibit the decrease in Variance Ratio that occurs after the short-term trending.

Figure 6.7: Variance Ratio Test for monthly ALSI nominal returns over the pre-1986 period

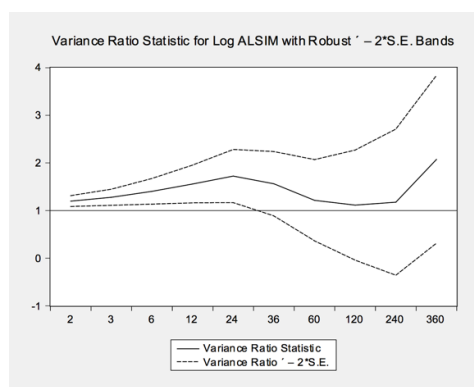
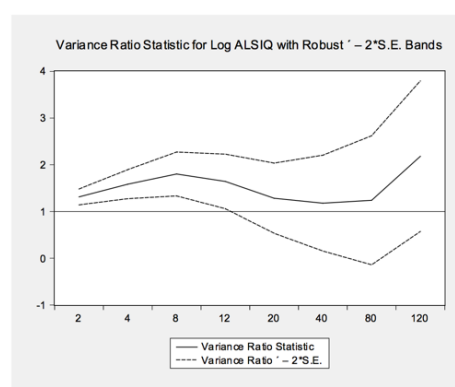


Figure 6.8: Variance Ratio Test for quarterly ALSI nominal returns over the pre-1986 period



6.7.3 Post-1986 Period

Table 6.21: Overall Joint Variance Ratio Tests for ALSI logarithmic total returns over the post-1986 period

The Table displays the Joint Variance Ratios Tests of the ALSI logarithmic total return over the period 01/01/1987 to 31/12/2016. The Variance ratio tests were conducted for monthly, quarterly, Semi-annually, Annually, two-yearly, three-yearly, five-yearly and ten-yearly logarithmic total returns. In addition have reported the results of the Joint Test statistic, the degrees of freedom and the p-value of the test against the null hypothesis that the series of returns follow a Random Walk stochastic process.

Frequency	Test Statistic	Degrees of freedom	P-Value
Monthly	1.0333	360	0.9604
Quarter	1.9273	120	0.3217
Semi-annual	1.2792	60	0.7395
Annual	1.6629	30	0.3974
2 Year	6.9514 ***	16	0.0000

* significant at $p < 0.05$; ** significant at $p < 0.01$; *** significant at $p < 0.001$

The Joint Variance Ratio tests presented in Table 6.21, show a rejection of the Random Walk Model at the two-year return interval.

Individual Variance Ratio tests as depicted in Table 6.22 for the final and most relevant period, no significant monthly results are observed, and our cautionary results support short term trending behavior up to the one-year level, combined with mean reversion thereafter. Quarterly results support short-term trending behavior (up till one-year), a Variance Ratio two-years of close to one, thereafter mean reverting Variance Ratios. Semi-annual data shows statistically insignificant mean reversion at intervals of two-years and greater. Annual Variance Ratios display weakly significant mean reverting behavior at the three-year level. Two-yearly Variance Ratios display statistically insignificant mean reverting tendencies. Three-yearly Variance Ratios exhibit statistically significant mean reversion in ALSI returns.

Table 6.22: Individual Variance Ratio Tests for ALSI logarithmic total returns over the post-1986

The Table displays the Individual Variance Ratios Tests of the ALSI logarithmic total return over the period 01/01/1987 to 31/12/2016. The Variance ratio tests were conducted for monthly, quarterly, Semi-annually, Annually, two-yearly, three-yearly, five-yearly and ten-yearly logarithmic total returns. In addition have reported the results of the Joint Test statistic, the degrees of freedom and the p-value of the test against the null hypothesis that the series of returns follow a Random Walk stochastic process.

Variance Ratio Test	Variance Ratio	p-value	Variance Ratio Test	Variance Ratio	p-value
Quarter/Month	1.0391	0.6565	Year/Semi-annual	1.16	0.2008
Semi-annual/Month	1.1041	0.5119	2 Year /Semi-annual	0.95	0.8443
Annual/Month	1.0426	0.8573	5 Year /Semi-annual	0.58	0.3032
2 Year/Month	0.8097	0.5618	2 Year/Annual	0.74	0.1193
5 Year/Month	0.4998	0.3015	3 Year/Annual	0.64	0.1502
Semi-annual/Quarter	1.2489	0.0539	5 Year/Annual	0.37***	0.0963
Annual/Quarter	1.27	0.2139	6 Year/3 Year	0.35*	0.0389
2 Year/Quarter	1.01	0.9842	10 Year/ 5 Year	1.42***	0.0000
3 Year/Quarter	0.81	0.5968	20 Year/ 10 Year	1.56*	0.0173
5 Year/Quarter	0.62	0.4026			

* significant at $p < 0.05$; ** significant at $p < 0.01$; *** significant at $p < 0.001$

Comparing Figures 6.9, 6.10, 6.11 and 6.12 to the Figures in the preceding period, it is observed over this post-1986 period, that the Variance Ratios of ALSI returns breach the unit axis of one earlier and remain below one for a greater period. This phenomenon supports mean reversion, as Variance Ratios of less than one indicate Mean Reversion.

Figure 6.9: Variance Ratio Test for monthly ALSI nominal returns over the post-1987 period

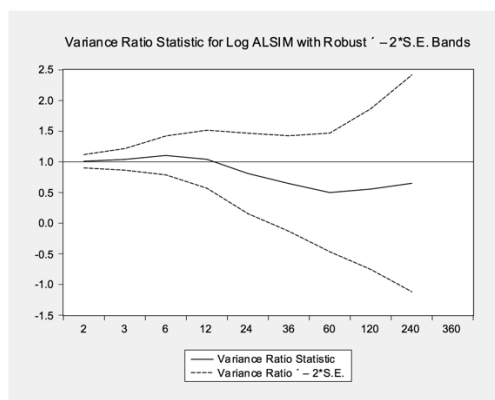


Figure 6.10: Variance Ratio Test for quarterly ALSI nominal returns over the post-1987 period

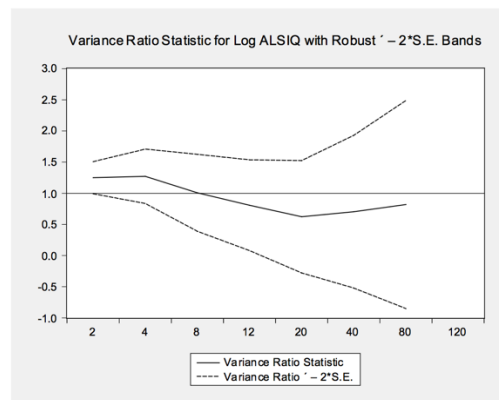


Figure 6.11: Variance Ratio Test for semi-annual ALSI nominal returns over the post-1987 period

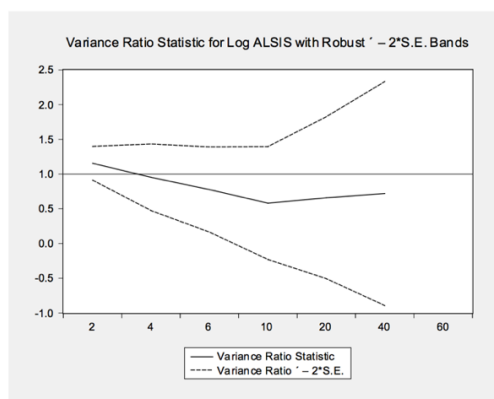
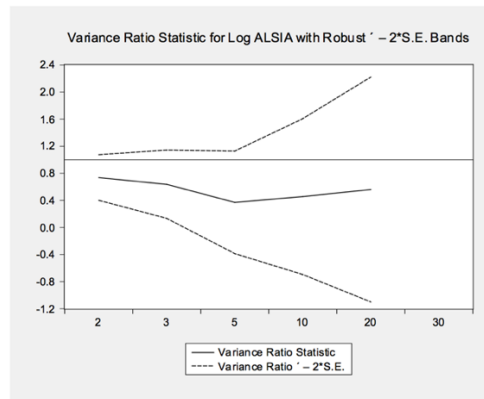


Figure 6.12: Variance Ratio Test for annual ALSI nominal returns over the post-1987 period



6.7.4 ALBI Variance Ratio Tests

The ALBI exhibits rejections of the Random Walk Model for the Joint Variance Ratio Tests at the annual, three and five-year frequencies over the entire period according to the results in Table 6.23

Table 6.23: Overall Joint Variance Ratio Tests for ALBI logarithmic total returns over the full period

The Table displays the Joint Variance Ratios Tests of the ALBI logarithmic total return over the period 31/12/1998 to 31/12/2016. The Variance ratio tests were conducted for monthly, quarterly, Semi-annually, Annually, two-yearly, three-yearly, five-yearly and ten-yearly logarithmic total returns. In addition have reported the results of the Joint Test statistic, the degrees of freedom and the p-value of the test against the null hypothesis that the series of returns follow a Random Walk stochastic process.

Frequency	Test Statistic	Degrees of freedom	P-Value
Month	1.0527	216	0.9684
Quarter	2.1305	72	0.1830

Semi-annual	2.1900	36	0.1349
Annual	2.6500 *	18	0.0315
2 Year	1.6946	9	0.1722
3 Year	3.2715 **	6	0.0021
5 Year	3.1300 ***	4	0.0035

* significant at $p < 0.05$; ** significant at $p < 0.01$; *** significant at $p < 0.001$

Delving deeper into the Individual Variance Ratio tests represented in Table 6.24, yield significant monthly results that indicate statistically significant Variance Ratios close to one, followed by statistically insignificant trending behavior at the three-year level. Quarterly Variance Ratios, provide significant trending behavior in returns up to the ten-year interval. Semi-annual, annual, three and five-year Variance Ratios indicate statistically significant trending behavior as well.

Table 6.24: Individual Variance Ratio Tests for ALBI logarithmic total returns over the full period

The Table displays the Individual Variance Ratios Tests of the ALBI logarithmic total return over the period 31/12/1998 to 31/12/2016. The Variance ratio tests were conducted for monthly, quarterly, Semi-annually, Annually, two-yearly, three-yearly, five-yearly and ten-yearly logarithmic total returns. In addition have reported the results of the Joint Test statistic, the degrees of freedom and the p-value of the test against the null hypothesis that the series of returns follow a Random Walk stochastic process.

Variance Ratio Test	Variance Ratio	p-value	Variance Ratio Test	Variance Ratio	p-value
Quarter/Month	0.98 **	0.0031	3 Year /Semi-annual	1.89 *	0.0343
Semi-annual/Month	0.99 **	0.0062	5 Year /Semi-annual	2.12 *	0.0416
Annual/Month	0.95 *	0.0226	10 Year/Semi-annual	2.63*	0.0286
2 Year/Month	0.97	0.0971	2 Year/Annual	1.40	0.0789
3 Year/Month	1.14	0.4382	3 Year/Annual	1.92 **	0.0080
Semi-annual/Quarter	1.22	0.0564	5 Year/Annual	2.24 *	0.0130
3 Year/Quarter	1.76	0.0800	10 Year/Annual	2.43 *	0.0399
5 Year/Quarter	1.98	0.0811	6 Year/ 3 Year	1.56 **	0.0011
10 Year/Quarter	2.63 *	0.0331	10 Year/5 Year	1.46 **	0.0056
Year/Semi-annual	1.25	0.1566	15 Year/5 Year	1.83 **	0.0017

* significant at $p < 0.05$; ** significant at $p < 0.01$; *** significant at $p < 0.001$

Figure 6.13: Variance Ratio Test for monthly ALBI nominal returns over the entire period

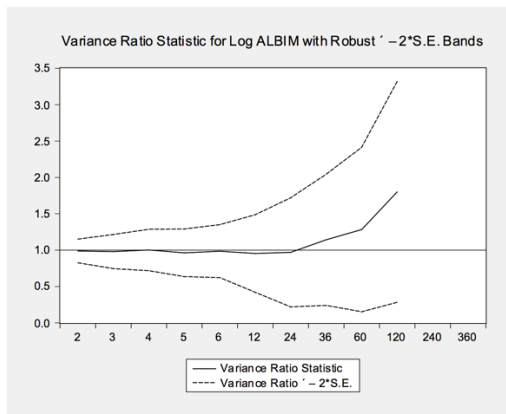


Figure 6.14: Variance Ratio Test for quarterly ALBI nominal returns over the entire period

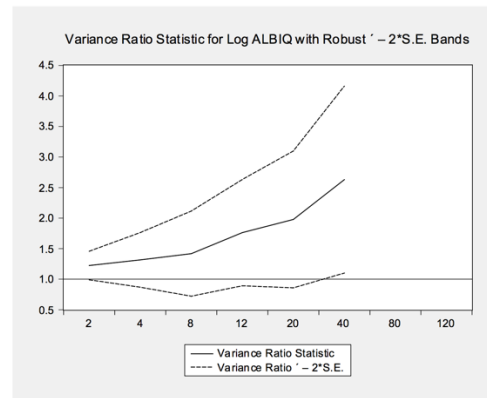


Figure 6.15: Variance Ratio Test for semi-annual ALBI nominal returns over the entire period

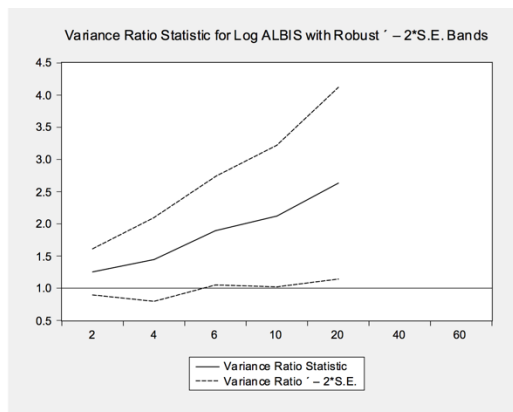
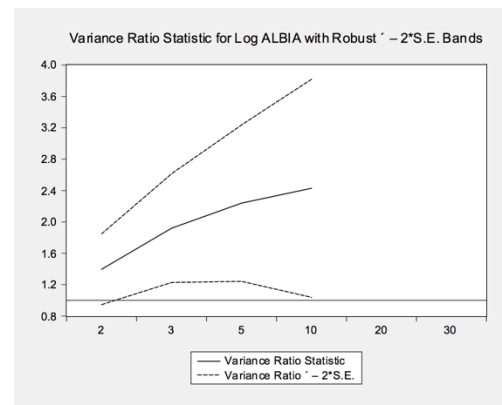


Figure 6.16: Variance Ratio Test for annual ALBI nominal returns over the entire period



The Variance Ratio plots depicted in Figures 6.12-6.16, depict the rejection of the Random Walk Model at monthly frequencies as the variance ratios are approximately one. This is followed by evidence of trending behavior in returns at higher frequencies.

6.8 Bradfield and Ardington (1997) study replication

The results of the Bradfield and Ardington study (1997) were presented earlier in Section 3.4. To provide more validity to our results, this study has replicated the Bradfield and Ardington (1997) paper. Bradfield and Ardington (1997) collected their JSE Actuaries Index data over the period 01/1980 to 01/1996 from the Department of Statistics at the University of Cape Town. Without a clear dataset to replicate, as well as no indication of the total return calculation method employed by Bradfield and Ardington (1997). This paper used the CI01 dataset collected from the I-Net terminal at the University of Cape Town, which acts as a proxy for

the data used by the authors. This replication employed non-overlapping monthly, quarterly and annual data.

The Autocorrelation Test, the Runs Test and the Variance Ratio Tests employed by Bradfield and Ardington (1997) were conducted in this study using the CI01 dataset and made use of the Filer and McLeod (1999) Total return index adjustment. Due to the difference in data collection and total return calculation, we expect slight deviations in results. These results provided similar results to the paper it was replicating, although we do have different test statistics and p-values over some frequencies. This could be due to differences in data collection and manipulation. The most notable difference in this study's replication of Bradfield and Ardington's (1997), is that in this study positive monthly serial correlation was observed, whilst it was previously found to be negative. These results are presented in Appendix G

6.9 Summary and Conclusions

Breaking the results down into the three periods for the ALSI, and the entire ALBI period will allow us to fully analyze these results and their changes over time. Before delving into the inter-period difference in return distributions, a brief review the results of the stationary tests is presented. Nominal ALSI returns were found to be stationary up to and including the three-year return frequency. This was found for both nominal and real ALSI returns. The ALBI returns were found to be stationary up to and including the semi-annual return frequency. The implications of these results, prohibit inference drawing conclusions from our parametric Autocorrelation tests when using the identified non-stationary return frequencies.

From this, the results period by period are assessed. Beginning with the ALSI over the entire period. Over this period, significant short-term trending behavior in returns was observed according to the Autocorrelation test up to and including the semi-annual frequency. Furthermore, statistically insignificant negative autocorrelation coefficients were observed at the two and three-year lag, which hinted toward medium-term mean reversion. This medium-term negative serial correlation was identified using stationary returns. Using the non-parametric Runs test, this analysis found evidence that rejects the Random Walk Model at monthly, quarterly, three-year and five-year return frequencies. Furthermore, statistically significant monthly trending behavior was found as well as significant three-yearly mean

reverting behavior. Building on this, indications of annual and five-year mean reversion due to the number of observed runs being greater than the number of expected runs (statistically insignificant) were observed.

The Joint Variance Ratio test provided significant rejection of the Random Walk Model at the semi-annual return interval. Furthermore, this test finds that monthly returns significantly trend within each quarter, semi-annual, and annual period - with weak significance at the two-year level (as these periods exhibit variance ratios of greater than one). Monthly returns insignificantly mean revert within five-year periods due to their Variance Ratio of less than one. Semi-annual returns provide a significant trending Variance Ratio of greater than one.

All the tests conducted over this entire period have identified short-term (monthly, quarterly and semi-annual) trending behavior. The Joint Variance Ratio test and the Runs test have rejected the Random Walk, in favor of this short-term trending behavior. The Autocorrelation and Variance Ratio tests found statistically insignificant Mean Reverting behavior at the annual, two, three and five-year return intervals. However, both these tests, display indications of mean reversion at these lags (negative coefficients in the Autocorrelation test and Variance Ratios of less one). The Runs test, has however, rejected the Random Walk Model at the three-year level and concluded significantly that returns Mean Revert at the three-year interval.

Over the ALSI pre-1987 period, this study found significant short-term trending behavior in monthly, quarterly and semi-annual returns from the Autocorrelation test. In addition, statistically insignificant but negative autocorrelation coefficients at the two and three-year lags were observed. The results of the Runs test further solidify the observed significant trending behavior in monthly returns, with rejections of the Random Walk Model at the monthly and semi-annual return frequencies. In keeping with the negative autocorrelation coefficients found in the Autocorrelation test, the Runs test also finds significant mean reversion at the three-year level with indications of mean reversion at two, three and five-year return frequencies (where the number of observed runs are below the number of expected runs).

The Variance Ratio tests significantly support the short-term trending nature of ALSI returns over this early period for monthly, quarterly and semi-annual return frequencies. This test does observe Variance Ratios of less than one, when assessing three-yearly returns.

Moving to the final and most important period for future extrapolation, the assessment of the post-1986 ALSI dataset. In the initial Autocorrelation test this study finds significant quarterly trending behavior together with insignificant but nonetheless negative autocorrelation coefficients at the annual, two and three-year return intervals. The non-parametric Runs test rejects the Random Walk model at the quarterly and three-year return frequency. Furthermore, these rejections are compounded with significant monthly trending behavior as well as significant mean reversion at the three-year interval.

The Variance Ratio Test over the final period provides weakly significant trending behavior in quarterly returns within semi-annual periods, as well as weakly significant mean reverting behavior in annual returns within five-year periods. Once again, these tests indicate that there exists statistically insignificant mean reverting behavior from the annual to five-year level. Most importantly, statistically significant behavior in three-year returns exhibiting mean reversion within each six-year interval was observed.

The short-term positive autocorrelation lends itself to short-term trending behavior in returns. The most important practitioner implication of this is that if short term risk (variances) were scaled up to an annual basis, this annual risk would understate the true risk over these short-term trending holding periods. This trending behavior invalidates a time diversification benefits over the short term. However, practitioners and investors may benefit from medium term mean reversion (negative autocorrelation) in returns. This lends medium-term returns to time diversification of risk. The most important practitioner implication of this is that if medium term risk (variances) were scaled down to an annual basis, this annual risk would overstate the true risk over these medium-term mean-reverting holding periods. Thus, one could benefit from time diversification when investing over these periods with true risk lower than what annualized risk may indicate. For the ALSI, time diversification benefits exist at the five to six year holding period.

The ALBI results are considerably different from that of the ALSI. Instead of short term trending behavior compounded with medium-term mean reversion as seen in the ALSI returns the ALBI exhibits weakly significant short-term trending behavior with significant two-year trending. However, the stationary tests proved that two-year returns are non-stationary and as

a result we cannot draw inferences from the Autocorrelation test regarding two-year trending behavior. Moving to the non-parametric Runs test, the ALBI yet again displays different characteristics to that of the ALSI. This nonparametric method has failed to reject the Random Walk model for all frequencies, but finds weakly significant trending behavior at the five-year return interval.

The Variance Ratio Test, has found statistically significant short-term Variance Ratios for monthly returns up to the semi-annual frequency that are extremely close to one – supporting the short-term Random Walk behavior of ALBI returns. This test also finds significant trending behavior in returns from the quarterly frequency and higher.

For the Autocorrelation and Runs tests, the real and nominal results provide the same outcome. When analyzing the distribution of returns, we find that as the return holding period increases we find negative skewness in logarithmic returns and positive skewness in arithmetic returns.

Considering the change in the return distribution over the three periods, when attempting to provide future investor guidance, this study focuses on the most recent post-1986 period. For the ALSI return analysis, the optimal holding period to benefit from Mean Reversion and Time Diversification, is a five to six-year holding period. This period maximizes the benefit of three-year mean reversion in ALSI returns.

The ALBI offers no Mean Reversion or Time Diversification benefit, but these traits support the ALBI being a good diversification metric to investors exposed to the ALSI. On the flipside, this study would like to emphasize the risks employing semi-annual holding periods that are exposed to the trending behavior of quarterly returns. The trending behavior in ALBI returns (positive autocorrelation), provides evidence against the existence of time diversification in bond returns.

The significant rejection of the Random Walk Models for ALSI returns over the quarterly and three-year return intervals, have implications for investment practitioners. These implications apply to practitioners if they scale risk (variance) measures calculated over these periods, to represent the risk (variance) of longer/shorter periods. If quarterly ALSI variances were calculated and scaled to represent the risk of annual returns, this annual risk measure would

understate the true annual risk. On the other hand, if three-year variances were scaled up to represent the risk of six-year period, this risk would overstate the true risk over the six-year periods.

This risk over and under-estimation, can be avoided by employing the Lo and MacKinlay (1988) adjustment presented in Section 2.5. This paper advises the use of this adjustment when using scaling monthly, quarterly or three-year variance calculation intervals.

7. Conclusion

This research paper set out to determine if the phenomenon of time diversification existed and/or exists on the South African Equity and Bond markets. In doing so, this paper tested for serial correlation in the ALSI and ALBI returns over varying periods. Built on these tests is the ideology of the skewness of returns and the validity of using risk measures calculated over shorter periods to estimate the long-term risk on the JSE (and vice-versa). To answer the above questions in a South African context, the tests employed included tests for Stationarity and Structural Breaks followed by parametric methods like the Autocorrelation Test. Based on the hypothesis of JSE returns not following the Random Walk Model, the non-parametric methods of the Variance Ratio Test and the Runs test were employed.

After accounting for structural breaks and identifying non-stationary return series, these tests in aggregate have all identified the same underlying trends for the ALSI as well as distinct but recurring ALBI return trends. Over the partitioned sub-periods, a small change in the serial correlation of ALSI returns was observed within the general trend of short-term trending behavior followed by medium-term mean reversion. Focusing on the most recent post-1986 period, the ALSI return series rejected the Random Walk Model at quarterly and three-year return intervals, together with significant short-term trending behavior in quarterly returns followed by mean reversion in three-year returns. Thus evidence of three-year mean reversion in returns, supports the existence of time diversification in JSE returns. This knowledge of mean reversion, will aid investors in their holding period decisions.

This result has significant practitioner implications, whereby the optimal holding period that maximizes the benefits of Mean Reversion and Time Diversification is a five to six-year ALSI holding period. Furthermore, this study would like to highlight the risk of employing a short-term investment strategy that leaves investors exposed to the short-term trending ALSI returns.

Due to the non-Random Walk nature of ALSI returns at these intervals, another implication of this study is that the scaling of variances (as risk measures) when measured over these periods, can lead to the under and overstatement of risk. If quarterly ALSI variances were calculated and scaled to represent the risk of annual returns, this annual risk measure would understate

the true annual risk. On the other hand, if three-year variances were scaled up to represent the risk of six-year period, this risk would overstate the true risk over the six-year period.

This risk over and under-estimation, can be avoided by employing the Lo and MacKinlay (1988) adjustment presented in Section 2.5. This paper advises the use of this adjustment when scaling monthly, quarterly or three-year variance calculation intervals.

The ALBI returns exhibited a lack of mean reversion with the failure to reject the Random Walk Model at all frequencies. The returns of this index provided no time diversification benefits, and this Index may be combined with the ALSI for diversification benefits based on their different distributions. The arithmetic return distribution of the ALSI exhibited positive skewness as the return interval increased, while the logarithmic return distribution of the ALSI exhibited negative skewness as the return interval increased.

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Appendix

A. Descriptive Statistics

A1. Nominal returns

Table A1.1: Descriptive Statistics for nominal quarterly total return data for the JSE All Share Index, JSE All Bond Index over their maximum period

The table displays the Mean, Standard Deviation, Skewness, Kurtosis and the Jarque-Bera p-value for the JSE All Share Index (Logarithmic and Arithmetic returns), the JSE All Bond Index (Logarithmic and Arithmetic Returns). These statistics were calculated over the entire period that its' respective dataset was available. The JSE All Share Index's descriptive statistics were calculated over the period 31/01/1925 to 31/12/2016. The JSE All Bond Index's descriptive statistics were calculated over the period 31/12/1998 to 31/12/2016.

	ALSI LR	ALSI AR	ALBI LR	ALBI AR
MEAN	0.028424	0.032177	0.009135	0.009392
SD	0.081188	0.082527	0.020679	0.020882
SKEWNESS	-0.499871	-0.054657	0.010859	0.123211
KURTOSIS	4.970753	4.651574	4.639052	4.639054
JB PR	0.000000***	0.000000***	0.000006***	0.000004***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table A1.2: Descriptive Statistics for nominal quarterly total return data for the JSE All Share Index for data before 31/12/1986

The table displays the Mean, Standard Deviation, Skewness, Kurtosis and the Jarque-Bera p-value for the JSE All Share Index (Logarithmic and Arithmetic returns). These statistics were calculated over the entire period that its' respective dataset was available before 31/12/1986. The JSE All Share Index's descriptive statistics were calculated over the period 31/01/1925 to 31/12/1986.

	ALSI LR	ALSI AR
MEAN	0.026201	0.029794
SD	0.079522	0.082478
SKEWNESS	0.072116	0.486121
KURTOSIS	4.475141	4.763062
JB PR	0.000012***	0.000000***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table A1.3: Descriptive Statistics for nominal quarterly total return data for the JSE All Share Index for data between 31/12/1986 and 31/12/2016

The table displays the Mean, Standard Deviation, Skewness, Kurtosis and the Jarque-Bera p-value for the JSE All Share Index (Logarithmic and Arithmetic returns). These statistics were calculated over the entire period 31/12/1986 to 31/12/2016.

	ALSI LR	ALSI AR
MEAN	0.033000	0.037082
SD	0.084669	0.082757
SKEWNESS	-1.501782	-1.164684
KURTOSIS	6.081889	4.833148
JB PR	0.000000***	0.000000***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table A1.4: Descriptive Statistics for nominal semi-annual total return data for the JSE All Share Index, JSE All Bond Index over their maximum period

The table displays the Mean, Standard Deviation, Skewness, Kurtosis and the Jarque-Bera p-value for the JSE All Share Index (Logarithmic and Arithmetic returns), the JSE All Bond Index (Logarithmic and Arithmetic Returns). These statistics were calculated over the entire period that its' respective dataset was available. The JSE All Share Index's descriptive statistics were calculated over the period 31/01/1925 to 31/12/2016. The JSE All Bond Index's descriptive statistics were calculated over the period 31/12/1998 to 31/12/2016.

	ALSI LR	ALSI AR	ALBI LR	ALBI AR
MEAN	0.05688	0.06587	0.05452	0.05688
SD	0.1189	0.1241	0.04080	0.04291
SKEWNESS	-0.4220	-0.03363	-0.2441	-1.1874
KURTOSIS	3.4163	3.1574	1.9925	1.98550
JB PR	0.03419*	0.8943	0.3906	0.4156

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table A1.5: Descriptive Statistics for nominal semi-annual total return data for the JSE All Share Index for data before 31/12/1986

The table displays the Mean, Standard Deviation, Skewness, Kurtosis and the Jarque-Bera p-value for the JSE All Share Index (Logarithmic and Arithmetic returns). These statistics were calculated over the entire period that its' respective dataset was available before 31/12/1986. The JSE All Share Index's descriptive statistics were calculated over the period 31/01/1925 to 31/12/1986.

	ALSI LR	ALSI AR
MEAN	0.05196	0.0607
SD	0.1191	0.1262
SKEWNESS	-0.06942	0.2972
KURTOSIS	3.0905	3.1465

JB PR	0.9320	0.3828
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* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table A1.6: Descriptive Statistics for nominal semi-annual total return data for the JSE All Share Index for data between 31/12/1986 and 31/12/2016

The table displays the Mean, Standard Deviation, Skewness, Kurtosis and the Jarque-Bera p-value for the JSE All Share Index (Logarithmic and Arithmetic returns). These statistics were calculated over the entire period 31/12/1986 to 31/12/2016.

	ALSI LR	ALSI AR
MEAN	0.06696	0.07637
SD	0.1188	0.1200
SKEWNESS	-1.1627	-0.8040
KURTOSIS	4.4843	3.5709
JB PR	0.000074***	0.0263

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table A1.7: Descriptive Statistics for nominal 2-yearly total return data for the JSE All Share Index, JSE All Bond Index over their maximum period

The table displays the Mean, Standard Deviation, Skewness, Kurtosis and the Jarque-Bera p-value for the JSE All Share Index (Logarithmic and Arithmetic returns), the JSE All Bond Index (Logarithmic and Arithmetic Returns). These statistics were calculated over the entire period that its' respective dataset was available. The JSE All Share Index's descriptive statistics were calculated over the period 01/01/1900 to 31/12/2016. The JSE All Bond Index's descriptive statistics were calculated over the period 31/12/1998 to 31/12/2016.

	ALSI LR	ALSI AR	ALBI LR	ALBI AR
MEAN	0.2026	0.2560	0.2066	
SD	0.2277	0.2861	0.09741	
SKEWNESS	-0.07349	0.5069	0.3753	
KURTOSIS	2.7825	2.9689	2.1460	
JB PR	0.9201	0.2885	0.7848	

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table A1.8: Descriptive Statistics for nominal 2-yearly total return data for the JSE All Share Index for data before 31/12/1986

The table displays the Mean, Standard Deviation, Skewness, Kurtosis and the Jarque-Bera p-value for the JSE All Share Index (Logarithmic and Arithmetic returns). These statistics were calculated over the entire period that its' respective dataset was available before 31/12/1986. The JSE All Share Index's descriptive statistics were calculated over the period 01/01/1900 to 31/12/1986.

	ALSI LR	ALSI AR
MEAN	0.1810	0.233172
SD	0.1997	0.3016
SKEWNESS	0.0237	0.2904
KURTOSIS	2.6203	2.9488
JB PR	0.8771	0.2861

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table A1.9: Descriptive Statistics for nominal 2-yearly total return data for the JSE All Share Index for data between 31/12/1986 and 31/12/2016

The table displays the Mean, Standard Deviation, Skewness, Kurtosis and the Jarque-Bera p-value for the JSE All Share Index (Logarithmic and Arithmetic returns). These statistics were calculated over the entire period 31/12/1986 to 31/12/2016.

	ALSI LR	ALSI AR
MEAN	0.2786	0.3406
SD	0.1756	0.2379
SKEWNESS	0.1038	0.4470
KURTOSIS	2.3561	2.5632
JB PR	0.8585	0.7189

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table A1.10: Descriptive Statistics for nominal 3-yearly total return data for the JSE All Bond Index over their maximum period

The table displays the Mean, Standard Deviation, Skewness, Kurtosis and the Jarque-Bera p-value for the JSE All Share Index (Logarithmic and Arithmetic returns), the JSE All Bond Index (Logarithmic and Arithmetic Returns). These statistics were calculated over the entire period that its' respective dataset was available. The JSE All Share Index's descriptive statistics were calculated over the period 01/01/1900 to 31/12/2016. The JSE All Bond Index's descriptive statistics were calculated over the period 31/12/1998 to 31/12/2016.

	ALBI LR	ALBI AR
MEAN	0.3135	0.3739
SD	0.09946	0.1381
SKEWNESS	0.2369	0.3711
KURTOSIS	2.0201	2.1518
JB PR	0.8624	0.8532

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table A1.11: Descriptive Statistics for nominal 3-yearly total return data for the JSE All Share Index for data before 31/12/1986

The table displays the Mean, Standard Deviation, Skewness, Kurtosis and the Jarque-Bera p-value for the JSE All Share Index (Logarithmic and Arithmetic returns). These statistics were calculated over the entire period that its' respective dataset was available before 31/12/1986. The JSE All Share Index's descriptive statistics were calculated over the period 01/01/1900 to 31/12/1986.

	ALSI LR	ALSI AR
MEAN	0.2651	0.3618
SD	0.2871	0.4636
SKEWNESS	0.9908	2.0805
KURTOSIS	4.3257	8.2092
JB PR	0.0363	0.0000***

* significant at $p<0.05$; ** significant at $p<0.01$; *** significant at $p<0.001$

Table A1.12: Descriptive Statistics for nominal 3-yearly total return data for the JSE All Share Index for data between 31/12/1986 and 31/12/2016

The table displays the Mean, Standard Deviation, Skewness, Kurtosis and the Jarque-Bera p-value for the JSE All Share Index (Logarithmic and Arithmetic returns). These statistics were calculated over the entire period 31/12/1986 to 31/12/2016.

	ALSI LR	ALSI AR
MEAN	0.4216	0.5601
SD	0.2214	0.3741
SKEWNESS	0.7115	1.1933
KURTOSIS	3.0448	3.9369
JB PR	0.6555	0.2543

* significant at $p<0.05$; ** significant at $p<0.01$; *** significant at $p<0.001$

Table A1.13: Descriptive Statistics for nominal 5-yearly total return data for the JSE All Share Index, JSE All Bond Index over their maximum period

The table displays the Mean, Standard Deviation, Skewness, Kurtosis and the Jarque-Bera p-value for the JSE All Share Index (Logarithmic and Arithmetic returns), the JSE All Bond Index (Logarithmic and Arithmetic Returns). These statistics were calculated over the entire period that its' respective dataset was available. The JSE All Share Index's descriptive statistics were calculated over the period 01/01/1900 to 31/12/2016. The JSE All Bond Index's descriptive statistics were calculated over the period 31/12/1998 to 31/12/2016.

	ALSI LR	ALSI AR	ALBI LR	ALBI AR
MEAN	0.5059	0.7179	0.4604	0.6003
SD	0.2692	0.4795	0.1638	0.2511
SKEWNESS	0.2672	0.7450	-0.56668	-0.4413
KURTOSIS	2.3169	2.7621	1.8871	1.8183
JB PR	0.6974	0.3359	0.8103	0.8342

* significant at $p<0.05$; ** significant at $p<0.01$; *** significant at $p<0.001$

Table A1.14: Descriptive Statistics for nominal 5-yearly total return data for the JSE All Share Index for data before 31/12/1986

The table displays the Mean, Standard Deviation, Skewness, Kurtosis and the Jarque-Bera p-value for the JSE All Share Index (Logarithmic and Arithmetic returns). These statistics were calculated over the entire period that its' respective dataset was available before 31/12/1986. The JSE All Share Index's descriptive statistics were calculated over the period 01/01/1900 to 31/12/1986.

	ALSI LR	ALSI AR
MEAN	0.4552	0.6349
SD	0.2725	0.4758
SKEWNESS	0.5022	0.9417
KURTOSIS	2.4206	2.9651
JB PR	0.6212	0.2846

* significant at $p<0.05$; ** significant at $p<0.01$; *** significant at $p<0.001$

Table A1.15: Descriptive Statistics for nominal 5-yearly total return data for the JSE All Share Index for data between 31/12/1986 and 31/12/2016

The table displays the Mean, Standard Deviation, Skewness, Kurtosis and the Jarque-Bera p-value for the JSE All Share Index (Logarithmic and Arithmetic returns). These statistics were calculated over the entire period 31/12/1986 to 31/12/2016.

	ALSI LR	ALSI AR
MEAN	0.6772	1.0069
SD	0.2128	0.4298
SKEWNESS	0.02145	0.4622
KURTOSIS	2.5603	2.6071
JB PR	0.9719	0.8632

* significant at $p<0.05$; ** significant at $p<0.01$; *** significant at $p<0.001$

Table A1.16: Descriptive Statistics for nominal 10-yearly total return data for the JSE All Share Index, JSE All Bond Index over their maximum period

The table displays the Mean, Standard Deviation, Skewness, Kurtosis and the Jarque-Bera p-value for the JSE All Share Index (Logarithmic and Arithmetic returns), the JSE All Bond Index (Logarithmic and Arithmetic Returns). These statistics were calculated over the entire period that its' respective dataset was available. The JSE All Share Index's descriptive statistics were calculated over the period 01/01/1900 to 31/12/2016. The JSE All Bond Index's descriptive statistics were calculated over the period 31/12/1998 to 31/12/2016.

	ALSI LR	ALSI AR
MEAN	1.0365	2.0373
SD	0.4059	1.2318
SKEWNESS	0.6694	0.5509

KURTOSIS	1.8161	2.1980
JB PR	0.7223	0.6534

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table A1.17: Descriptive Statistics for nominal 10-yearly total return data for the JSE All Share Index for data before 31/12/1986

The table displays the Mean, Standard Deviation, Skewness, Kurtosis and the Jarque-Bera p-value for the JSE All Share Index (Logarithmic and Arithmetic returns). These statistics were calculated over the entire period that its' respective dataset was available before 31/12/1986. The JSE All Share Index's descriptive statistics were calculated over the period 01/01/1900 to 31/12/1986.

	ALSI LR	ALSI AR
MEAN	0.9239	1.7339
SD	0.4148	1.2728
SKEWNESS	0.6609	1.1814
KURTOSIS	2.3563	3.2619
JB PR	0.6975	0.3899

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table A1.18: Descriptive Statistics for nominal 10-yearly total return data for the JSE All Share Index for data between 31/12/1986 and 31/12/2016

The table displays the Mean, Standard Deviation, Skewness, Kurtosis and the Jarque-Bera p-value for the JSE All Share Index (Logarithmic and Arithmetic returns). These statistics were calculated over the entire period 31/12/1986 to 31/12/2016.

	ALSI LR	ALSI AR
MEAN	1.4206	3.2257
SD	0.2362	0.9770
SKEWNESS	-0.1450	0.0361
KURTOSIS	1.6118	1.5924
JB PR	0.8457	0.8474

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table A1.19: Descriptive Statistics for nominal 20-yearly total return data for the JSE All Share Index, JSE All Bond Index over their maximum period

The table displays the Mean, Standard Deviation, Skewness, Kurtosis and the Jarque-Bera p-value for the JSE All Share Index (Logarithmic and Arithmetic returns), the JSE All Bond Index (Logarithmic and Arithmetic Returns). These statistics were calculated over the entire period that its' respective dataset was available. The JSE All Share Index's descriptive statistics were calculated over the period 01/01/1900 to 31/12/2016. The JSE All Bond Index's descriptive statistics were calculated over the period 31/12/1998 to 31/12/2016.

	ALSI LR	ALSI AR
MEAN	2.1253	8.5195
SD	0.5618	5.3964
SKEWNESS	0.3122	0.4741
KURTOSIS	1.3198	1.4527
JB PR	0.7156	0.7096

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table A1.20: Descriptive Statistics for nominal 20-yearly total return data for the JSE All Share Index for data before 31/12/1986

The table displays the Mean, Standard Deviation, Skewness, Kurtosis and the Jarque-Bera p-value for the JSE All Share Index (Logarithmic and Arithmetic returns). These statistics were calculated over the entire period that its' respective dataset was available before 31/12/1986. The JSE All Share Index's descriptive statistics were calculated over the period 01/01/1900 to 31/12/1986.

	ALSI LR	ALSI AR
MEAN	2.0026	7.4786
SD	0.5660	5.6219
SKEWNESS	0.9359	1.0722
KURTOSIS	2.1271	2.2609
JB PR	0.7008	0.6513

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table A1.21: Descriptive Statistics for nominal 20-yearly total return data for the JSE All Share Index for data between 31/12/1986 and 31/12/2016

The table displays the Mean, Standard Deviation, Skewness, Kurtosis and the Jarque-Bera p-value for the JSE All Share Index (Logarithmic and Arithmetic returns). These statistics were calculated over the entire period 31/12/1986 to 31/12/2016.

	ALSI LR	ALSI AR
MEAN	2.7189	14.2443
SD	0.1454	2.2080
SKEWNESS	0	0
KURTOSIS	1	1
JB PR	0.8465	0.8465

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

A2. Real returns

Table A2.1: Descriptive Statistics for real monthly total return data for the JSE All Share Index, JSE All Bond Index over the period 01/01/1987 to 31/12/2016 (where available)

The table displays the Mean, Standard Deviation, Skewness, Kurtosis and the Jarque-Bera p-value for the JSE All Share Index (Logarithmic returns), the JSE All Bond Index (Logarithmic Returns) and CPI (Logarithmic and Arithmetic Returns). These statistics were calculated over the entire period that its' respective dataset was available. The JSE All Share Index's descriptive statistics were calculated over the period 01/01/1987 to 31/12/2016. The JSE All Bond Index's descriptive statistics were calculated over the period 31/12/1998 to 31/12/2016. CPI data was calculated over the period 01/01/1987 to 31/12/2016.

	R ALSI LR	R ALBI LR	CPI LR	CPI AR
MEAN	0.004556	0.004538	0.006255	0.006292
SD	0.056220	0.021523	0.005199	0.005232
SKEWNESS	-1.2168	-0.031024	0.482438	0.497093
KURTOSIS	8.936161	4.126394	3.305347	3.345288
JB PR	0.000000	0.0033**	0.000471	0.000247

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table A2.2: Descriptive Statistics for real annual total return data for the JSE All Share Index, JSE All Bond Index over the period 01/01/1987 to 31/12/2016 (where available)

The table displays the Mean, Standard Deviation, Skewness, Kurtosis and the Jarque-Bera p-value for the JSE All Share Index (Logarithmic returns), the JSE All Bond Index (Logarithmic Returns) and CPI (Logarithmic and Arithmetic Returns). These statistics were calculated over the entire period that its' respective dataset was available. The JSE All Share Index's descriptive statistics were calculated over the period 01/01/1987 to 31/12/2016. The JSE All Bond Index's descriptive statistics were calculated over the period 31/12/1998 to 31/12/2016. CPI data was calculated over the period 01/01/1987 to 31/12/2016.

	R ALSI LR	R ALBI LR	CPI LR	CPI AR
MEAN	0.06389	0.051219	0.07428	0.0805
SD	0.1920	0.065361	0.03334	0.3888
SKEWNESS	-0.05461	0.176601	0.6443	0.6894
KURTOSIS	2.7698	2.3917	2.5678	2.4690
JB PR	0.9615	0.830668	0.3276	0.2555

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table A2.3: Descriptive Statistics for real quarterly total return data for the JSE All Share Index, JSE All Bond Index over the period 01/01/1987 to 31/12/2016 (where available)

The table displays the Mean, Standard Deviation, Skewness, Kurtosis and the Jarque-Bera p-value for the JSE All Share Index (Logarithmic returns), the JSE All Bond Index (Logarithmic Returns) and CPI (Logarithmic and Arithmetic Returns). These statistics were calculated over the entire period that its' respective dataset was available. The JSE All Share Index's descriptive statistics were calculated over the period 01/01/1987 to 31/12/2016. The JSE All Bond Index's descriptive statistics were calculated over the period 31/12/1998 to 31/12/2016. CPI data was calculated over the period 01/01/1987 to 31/12/2016.

	R ALSI LR	R ALBI LR	CPI LR	CPI AR
MEAN	0.01387	0.013609	0.01871	0.01909
SD	0.08714	0.033190	0.01095	0.01123
SKEWNESS	-1.5130	-0.66949	0.2050	0.2148
KURTOSIS	6.2318	3.315003	2.5172	2.4642
JB PR	0.0000	0.058534	0.3700	0.3075

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table A2.4: Descriptive Statistics for real semi-annual total return data for the JSE All Share Index, JSE All Bond Index over the period 01/01/1987 to 31/12/2016 (where available)

The table displays the Mean, Standard Deviation, Skewness, Kurtosis and the Jarque-Bera p-value for the JSE All Share Index (Logarithmic returns), the JSE All Bond Index (Logarithmic Returns) and CPI (Logarithmic and Arithmetic Returns). These statistics were calculated over the entire period that its' respective dataset was available. The JSE All Share Index's descriptive statistics were calculated over the period 01/01/1987 to 31/12/2016. The JSE All Bond Index's descriptive statistics were calculated over the period 31/12/1998 to 31/12/2016. CPI data was calculated over the period 01/01/1987 to 31/12/2016.

	R ALSI LR	R ALBI LR	CPI LR	CPI AR
MEAN	0.02784	0.026895	0.03733	0.03890
SD	0.1237	0.046717	0.01888	0.0202
SKEWNESS	-1.1586	-0.500633	0.31607	0.3563
KURTOSIS	4.4213	2.39454	2.3205	2.2803
JB PR	0.000114	0.358235	0.3469	0.2774

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table A2.5: Descriptive Statistics for real 2-yearly total return data for the JSE All Share Index, JSE All Bond Index over the period 01/01/1987 to 31/12/2016 (where available)

The table displays the Mean, Standard Deviation, Skewness, Kurtosis and the Jarque-Bera p-value for the JSE All Share Index (Logarithmic returns), the JSE All Bond Index (Logarithmic Returns) and CPI (Logarithmic and Arithmetic Returns). These statistics were calculated over the entire period that its' respective dataset was available. The JSE All Share Index's descriptive statistics were calculated over the period 01/01/1987 to 31/12/2016. The JSE All Bond Index's descriptive statistics were calculated over the period 31/12/1998 to 31/12/2016. CPI data was calculated over the period 01/01/1987 to 31/12/2016.

	R ALSI LR	R ALBI LR	CPI LR	CPI AR
MEAN	0.1161	0.097436	0.1482	0.1759
SD	0.2018	0.106545	0.06698	0.09396

SKEWNESS	0.4984	0.193599	0.7449	0.8481
KURTOSIS	2.4773	2.057139	2.3648	2.5012
JB PR	0.6731	0.823003	0.4406	0.3528

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table A2.6: Descriptive Statistics for real 3-yearly total return data for the JSE All Share Index, JSE All Bond Index over the period 01/01/1987 to 31/12/2016 (where available)

The table displays the Mean, Standard Deviation, Skewness, Kurtosis and the Jarque-Bera p-value for the JSE All Share Index (Logarithmic returns), the JSE All Bond Index (Logarithmic Returns) and CPI (Logarithmic and Arithmetic Returns). These statistics were calculated over the entire period that its' respective dataset was available. The JSE All Share Index's descriptive statistics were calculated over the period 01/01/1987 to 31/12/2016. The JSE All Bond Index's descriptive statistics were calculated over the period 31/12/1998 to 31/12/2016. CPI data was calculated over the period 01/01/1987 to 31/12/2016.

	R ALSI LR	R ALBI LR	CPI LR	CPI AR
MEAN	0.1992	0.142952	0.2186	0.2788
SD	0.2783	0.114926	0.08917	0.1448
SKEWNESS	0.8163	-0.313966	1.0275	0.8998
KURTOSIS	2.9867	1.727599	3.1163	2.4094
JB PR	0.6067	0.777507	0.4519	0.4736

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table A2.7: Descriptive Statistics for real 5-yearly total return data for the JSE All Share Index, JSE All Bond Index and Inflation data over the period 01/01/1987 to 31/12/2016 (where available)

The table displays the Mean, Standard Deviation, Skewness, Kurtosis and the Jarque-Bera p-value for the JSE All Share Index (Logarithmic returns), the JSE All Bond Index (Logarithmic Returns) and CPI (Logarithmic and Arithmetic Returns). These statistics were calculated over the entire period that its' respective dataset was available. The JSE All Share Index's descriptive statistics were calculated over the period 01/01/1987 to 31/12/2016. The JSE All Bond Index's descriptive statistics were calculated over the period 31/12/1998 to 31/12/2016. CPI data was calculated over the period 01/01/1987 to 31/12/2016.

	R ALSI LR	R ALBI LR	CPI LR	CPI AR
MEAN	0.2955	0.19785	0.3537	0.5221
SD	0.2559	0.122046	0.1638	0.3157
SKEWNESS	0.9050	-0.203485	1.0086	0.6945
KURTOSIS	2.7084	1.376731	2.8351	1.8313
JB PR	0.6570	0.791848	0.5993	0.6184

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

B. Augmented Dickey-Fuller & Phillips-Perron tests

Table B.1: Augmented Dickey Fuller Tests for Stationarity of nominal monthly JSE All Share Logarithmic total returns for the period 31/01/1925 to 31/12/2016

The table presents the t statistic and respective Augmented Dickey Fuller test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 31/01/1925 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	0.428672	0.9841
Log Level	Constant & Linear Trend	-1.607755	0.7897
Log Level	None	4.271227	1.0000
1 st Log Difference	Constant	-29.39037	0.000***
1 st Log Difference	Constant & Linear Trend	-29.39298	0.000***
1 st Log Difference	None	-28.52050	0.000***
2 nd Log Difference	Constant	-18.09303	0.000***
2 nd Log Difference	Constant & Linear Trend	-18.08463	0.000***
2 nd Log Difference	None	-18.10128	0.000***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.2: Augmented Dickey Fuller Tests for Stationarity of nominal quarterly JSE All Share Logarithmic total returns for the period 31/01/1925 to 31/12/2016

The table presents the t statistic and respective Augmented Dickey Fuller test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 31/01/1925 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	0.354026	0.9807
Log Level	Constant & Linear Trend	-1.837643	0.6844
Log Level	None	3.572536	0.9999
1 st Log Difference	Constant	-14.26525	0.000***
1 st Log Difference	Constant & Linear Trend	-14.27167	0.000***
1 st Log Difference	None	-13.10913	0.000***
2 nd Log Difference	Constant	-10.91593	0.000***
2 nd Log Difference	Constant & Linear Trend	-10.90021	0.000***
2 nd Log Difference	None	-10.93033	0.000***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.3: Augmented Dickey Fuller Tests for Stationarity of nominal semi-annually JSE All Share Logarithmic total returns for the period 31/01/1925 to 31/12/2016

The table presents the t statistic and respective Augmented Dickey Fuller test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 31/01/1925 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	0.3747	0.9814
Log Level	Constant & Linear Trend	-1.8382	0.6821
Log Level	None	3.2874	0.9998
1 st Log Difference	Constant	-10.2819	0.0000***
1 st Log Difference	Constant & Linear Trend	-10.2923	0.0000***
1 st Log Difference	None	-8.8451	0.0000***
2 nd Log Difference	Constant	-10.2388	0.0000***
2 nd Log Difference	Constant & Linear Trend	-10.2093	0.0000***
2 nd Log Difference	None	-10.2671	0.0000***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.4: Augmented Dickey Fuller Tests for Stationarity of nominal annual JSE All Share Logarithmic total returns for the period 01/01/1900 to 31/12/2016

The table presents the t statistic and respective Augmented Dickey Fuller test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1900 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	1.1753	0.9979
Log Level	Constant & Linear Trend	-1.9793	0.6061
Log Level	None	3.6511	0.9999
1 st Log Difference	Constant	-10.2880	0.0000***
1 st Log Difference	Constant & Linear Trend	-10.5000	0.0000***
1 st Log Difference	None	-8.2964	0.0000***
2 nd Log Difference	Constant	-7.7016	0.0000***
2 nd Log Difference	Constant & Linear Trend	-7.671498	0.0000***
2 nd Log Difference	None	-7.7372	0.0000***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.5: Augmented Dickey Fuller Tests for Stationarity of nominal 2-yearly JSE All Share Logarithmic total returns for the period 01/01/1900 to 31/12/2016

The table presents the t statistic and respective Augmented Dickey Fuller test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1900 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	1.5147	0.9992
Log Level	Constant & Linear Trend	-1.9555	0.6127
Log Level	None	3.9077	1.0000
1 st Log Difference	Constant	-7.8353	0.0000***
1 st Log Difference	Constant & Linear Trend	-7.3201	0.0000***
1 st Log Difference	None	-0.7103	0.4044
2 nd Log Difference	Constant	-7.6941	0.0000***
2 nd Log Difference	Constant & Linear Trend	-7.6552	0.0000***
2 nd Log Difference	None	-7.7651	0.0000***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.6: Augmented Dickey Fuller Tests for Stationarity of nominal 3-year JSE All Share Logarithmic total returns for the period 01/01/1900 to 31/12/2016

The table presents the t statistic and respective Augmented Dickey Fuller test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1900 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	2.2932	0.9999
Log Level	Constant & Linear Trend	-1.8223	0.6740
Log Level	None	3.7620	0.9999
1 st Log Difference	Constant	-7.5012	0.0000***
1 st Log Difference	Constant & Linear Trend	-8.4222	0.0000***
1 st Log Difference	None	-0.7818	0.3702
2 nd Log Difference	Constant	-5.7213	0.0000***
2 nd Log Difference	Constant & Linear Trend	-8.5336	0.0000***
2 nd Log Difference	None	-5.7795	0.0000***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.7: Augmented Dickey Fuller Tests for Stationarity of nominal 5-yearly JSE All Share Logarithmic total returns for the period 01/01/1900 to 31/12/2016

The table presents the t statistic and respective Augmented Dickey Fuller test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1900 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	2.1558	0.9998
Log Level	Constant & Linear Trend	-1.6842	0.7253
Log Level	None	-0.0368	0.6596
1 st Log Difference	Constant	-3.4968	0.0181*
1 st Log Difference	Constant & Linear Trend	-3.5219	0.0616
1 st Log Difference	None	-1.1239	0.2289
2 nd Log Difference	Constant	-4.0901	0.0060**
2 nd Log Difference	Constant & Linear Trend	-4.0343	0.0257
2 nd Log Difference	None	-4.1433	0.0003***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.8: Augmented Dickey Fuller Tests for Stationarity of nominal 10-yearly JSE All Share Logarithmic total returns for the period 01/01/1900 to 31/12/2016

The table presents the t statistic and respective Augmented Dickey Fuller test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1900 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	1.8594	0.9989
Log Level	Constant & Linear Trend	-1.1699	0.8633
Log Level	None	-0.1721	0.5968
1 st Log Difference	Constant	-2.7745	0.0961
1 st Log Difference	Constant & Linear Trend	-3.1192	0.1556
1 st Log Difference	None	0.1243	0.6973
2 nd Log Difference	Constant	-4.6588	0.0073**
2 nd Log Difference	Constant & Linear Trend	-4.3413	0.0382*
2 nd Log Difference	None	-4.8732	0.0003***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.9: Augmented Dickey Fuller Tests for Stationarity of nominal 20-yearly JSE All Share Logarithmic total returns for the period 01/01/1900 to 31/12/2016

The table presents the t statistic and respective Augmented Dickey Fuller test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1900 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	1.6623	0.9944
Log Level	Constant & Linear Trend	-0.3478	0.9322
Log Level	None	1.2841	0.9209
1 st Log Difference	Constant	-0.8415	0.6863
1 st Log Difference	Constant & Linear Trend	-1.9015	0.4911
1 st Log Difference	None	0.2507	0.7086
2 nd Log Difference	Constant	-1.6747	0.3585
2 nd Log Difference	Constant & Linear Trend	N/A	N/A
2 nd Log Difference	None	-1.7088	0.0876

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.10: Phillips Perron Unit Root Tests for Stationarity of nominal monthly JSE All Share Logarithmic total returns for the period 31/01/1925 to 31/12/2016

The table presents the t statistic and respective Phillips-Perron Unit Root test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 31/01/1925 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	0.428672	0.9841
Log Level	Constant & Linear Trend	0.471637	0.9857
Log Level	None	-1.526405	0.8202
1 st Log Difference	Constant	4.282238	1.0000
1 st Log Difference	Constant & Linear Trend	-29.46536	0.0000***
1 st Log Difference	None	-29.46489	0.0000***
2 nd Log Difference	Constant	-28.95549	0.0000***
2 nd Log Difference	Constant & Linear Trend	-331.9112	0.0001***
2 nd Log Difference	None	-331.7142	0.0001***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.11: Phillips Perron Unit Root Tests for Stationarity of nominal quarterly JSE All Share Logarithmic total returns for the period 31/01/1925 to 31/12/2016

The table presents the t statistic and respective Phillips-Perron Unit Root test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 31/01/1925 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
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Log Level	Constant	0.435373	0.9842
Log Level	Constant & Linear Trend	-1.633339	0.7781
Log Level	None	3.993473	1.0000
1 st Log Difference	Constant	-14.21473	0.0000***
1 st Log Difference	Constant & Linear Trend	-14.21552	0.0000***
1 st Log Difference	None	-13.38755	0.0000***
2 nd Log Difference	Constant	-90.69320	0.0001***
2 nd Log Difference	Constant & Linear Trend	-90.39589	0.0001***
2 nd Log Difference	None	-90.86392	0.0001***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.12: Phillips Perron Unit Root Tests for Stationarity of nominal semi-annually JSE All Share Logarithmic total returns for the period 31/01/1925 to 31/12/2016

The table presents the t statistic and respective Phillips-Perron Unit Root test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 31/01/1925 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	0.7055	0.9920
Log Level	Constant & Linear Trend	-1.3508	0.8718
Log Level	None	4.3927	1.0000
1 st Log Difference	Constant	-9.9166	0.0000***
1 st Log Difference	Constant & Linear Trend	-9.9523	0.0000***
1 st Log Difference	None	-8.8085	0.0000***
2 nd Log Difference	Constant	-65.5882	0.0001***
2 nd Log Difference	Constant & Linear Trend	-65.4604	0.0001***
2 nd Log Difference	None	-65.8451	0.0001***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.13: Phillips Perron Unit Root Tests for Stationarity of nominal annual JSE All Share Logarithmic total returns for the period 01/01/1900 to 31/12/2016

The table presents the t statistic and respective Phillips-Perron Unit Root test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1900 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	3.8238	1.0000
Log Level	Constant & Linear Trend	-1.7753	0.7105
Log Level	None	3.2773	0.9997
1 st Log Difference	Constant	-10.3711	0.0000***
1 st Log Difference	Constant & Linear Trend	-13.7046	0.0000***
1 st Log Difference	None	-8.4260	0.0000***
2 nd Log Difference	Constant	-52.7335	0.0001***
2 nd Log Difference	Constant & Linear Trend	-52.4961	0.0001***
2 nd Log Difference	None	-53.4869	0.0000***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.14: Phillips Perron Unit Root Tests for Stationarity of nominal 2-yearly JSE All Share Logarithmic total returns for the period 01/01/1900 to 31/12/2016

The table presents the t statistic and respective Phillips-Perron Unit Root test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1900 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	2.2173	0.9999
Log Level	Constant & Linear Trend	-1.9072	0.6380
Log Level	None	2.3916	0.9955
1 st Log Difference	Constant	-7.8381	0.0000***
1 st Log Difference	Constant & Linear Trend	-8.5797	0.0000***
1 st Log Difference	None	-5.0388	0.0000***
2 nd Log Difference	Constant	-31.3532	0.0001***
2 nd Log Difference	Constant & Linear Trend	-31.9263	0.0001***
2 nd Log Difference	None	-31.2247	0.0000***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.15: Phillips Perron Unit Root Tests for Stationarity of nominal 3-yearly JSE All Share Logarithmic total returns for the period 01/01/1900 to 31/12/2016

The table presents the t statistic and respective Phillips-Perron Unit Root test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1900 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	2.2897	0.9999
Log Level	Constant & Linear Trend	-1.7036	0.7301
Log Level	None	1.9755	0.9870
1 st Log Difference	Constant	-7.3987	0.0000***
1 st Log Difference	Constant & Linear Trend	-8.4222	0.0000***

1 st Log Difference	None	-3.8214	0.0003***
2 nd Log Difference	Constant	-25.4606	0.0001***
2 nd Log Difference	Constant & Linear Trend	-25.4561	0.0000***
2 nd Log Difference	None	-23.0022	0.0000***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.16: Phillips Perron Unit Root Tests for Stationarity of nominal 5-yearly JSE All Share Logarithmic total returns for the period 01/01/1900 to 31/12/2016

The table presents the t statistic and respective Phillips-Perron Unit Root test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1900 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	2.0983	0.9998
Log Level	Constant & Linear Trend	-1.6842	0.7253
Log Level	None	1.7527	0.9771
1 st Log Difference	Constant	-3.4524	0.0199*
1 st Log Difference	Constant & Linear Trend	-3.3432	0.0855
1 st Log Difference	None	-0.9159	0.3086
2 nd Log Difference	Constant	-8.6255	0.0000***
2 nd Log Difference	Constant & Linear Trend	-9.7105	0.0000***
2 nd Log Difference	None	-8.5868	0.0000***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.17: Phillips Perron Unit Root Tests for Stationarity of nominal 10-yearly JSE All Share Logarithmic total returns for the period 01/01/1900 to 31/12/2016

The table presents the t statistic and respective Phillips-Perron Unit Root test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1900 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	3.0372	0.9999
Log Level	Constant & Linear Trend	-1.1699	0.8633
Log Level	None	1.1518	0.9235
1 st Log Difference	Constant	-2.7772	0.0957
1 st Log Difference	Constant & Linear Trend	-3.1190	0.1557
1 st Log Difference	None	-0.1483	0.6080
2 nd Log Difference	Constant	-7.3229	0.0003***
2 nd Log Difference	Constant & Linear Trend	-4.5780	0.0292*
2 nd Log Difference	None	-5.2035	0.0002***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.18: Phillips Perron Unit Root Tests for Stationarity of nominal 20-yearly JSE All Share Logarithmic total returns for the period 01/01/1900 to 31/12/2016

The table presents the t statistic and respective Phillips-Perron Unit Root test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1900 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	4.1680	0.9998
Log Level	Constant & Linear Trend	1.3404	0.9962
Log Level	None	0.6668	0.8228
1 st Log Difference	Constant	-0.8415	0.6863
1 st Log Difference	Constant & Linear Trend	-3.3268	0.2574
1 st Log Difference	None	0.6264	0.8004
2 nd Log Difference	Constant	-2.1795	0.2347
2 nd Log Difference	Constant & Linear Trend	N/A	N/A
2 nd Log Difference	None	-1.7088	0.0876

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.19: Augmented Dickey Fuller Tests for Stationarity of nominal monthly JSE All Bond Logarithmic total returns for the period 31/12/1998 to 31/12/2016

The table presents the t statistic and respective Augmented Dickey Fuller test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 31/12/1998 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-2.9533	0.0411
Log Level	Constant & Linear Trend	-2.7980	0.01997
Log Level	None	6.1259	1.0000
1 st Log Difference	Constant	-14.8997	0.0000***
1 st Log Difference	Constant & Linear Trend	-15.2847	0.0000***
1 st Log Difference	None	-12.6592	0.0000***
2 nd Log Difference	Constant	-13.5872	0.0000***
2 nd Log Difference	Constant & Linear Trend	-13.5510	0.0000***
2 nd Log Difference	None	-13.6206	0.0000***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.20: Augmented Dickey Fuller Tests for Stationarity of nominal quarterly JSE All Bond Logarithmic total returns for the period 31/12/1998 to 31/12/2016

The table presents the t statistic and respective Augmented Dickey Fuller test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 31/12/1998 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-3.5342	0.0097**
Log Level	Constant & Linear Trend	-2.5869	0.2874
Log Level	None	7.2757	1.0000
1 st Log Difference	Constant	-5.9506	0.0000***
1 st Log Difference	Constant & Linear Trend	-6.8897	0.0000***
1 st Log Difference	None	-1.6529	0.0926
2 nd Log Difference	Constant	-8.2049	0.0000***
2 nd Log Difference	Constant & Linear Trend	-8.1406	0.0000***
2 nd Log Difference	None	-8.1759	0.0000***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.21: Augmented Dickey Fuller Tests for Stationarity of nominal semi-annually JSE All Bond Logarithmic total returns for the period 31/12/1998 to 31/12/2016

The table presents the t statistic and respective Augmented Dickey Fuller test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 31/12/1998 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-0.0439	0.09602
Log Level	Constant & Linear Trend	-3.2374	0.6844
Log Level	None	4.1086	1.0000
1 st Log Difference	Constant	-6.8244	0.0000***
1 st Log Difference	Constant & Linear Trend	-6.7692	0.0000***
1 st Log Difference	None	-5.5135	0.0000***
2 nd Log Difference	Constant	-8.8775	0.0000***
2 nd Log Difference	Constant & Linear Trend	-8.7975	0.0000***
2 nd Log Difference	None	-8.9662	0.0000***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.22: Augmented Dickey Fuller Tests for Stationarity of nominal annual JSE All Bond Logarithmic total returns for the period 01/01/1900 to 31/12/2016

The table presents the t statistic and respective Augmented Dickey Fuller test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1900 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-3.9580	0.0082**
Log Level	Constant & Linear Trend	-1.5599	0.7681
Log Level	None	0.6468	0.8453
1 st Log Difference	Constant	-1.3434	0.5825
1 st Log Difference	Constant & Linear Trend	-2.1782	0.4686
1 st Log Difference	None	-1.3802	0.1492
2 nd Log Difference	Constant	-8.3266	0.0000***
2 nd Log Difference	Constant & Linear Trend	-8.0235	0.0001***
2 nd Log Difference	None	-8.3308	0.0000***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.23: Augmented Dickey Fuller Tests for Stationarity of nominal 2-yearly JSE All Bond Logarithmic total returns for the period 01/01/1900 to 31/12/2016

The table presents the t statistic and respective Augmented Dickey Fuller test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1900 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-5.7290	0.0020**
Log Level	Constant & Linear Trend	-2.0809	0.4872
Log Level	None	0.5113	0.8019
1 st Log Difference	Constant	-1.4517	0.5043
1 st Log Difference	Constant & Linear Trend	-2.2824	0.3943
1 st Log Difference	None	-2.4302	0.0227*
2 nd Log Difference	Constant	-2.2534	0.2085
2 nd Log Difference	Constant & Linear Trend	-2.8755	0.2523
2 nd Log Difference	None	-1.9060	0.0593

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.24: Augmented Dickey Fuller Tests for Stationarity of nominal 3-year JSE All Bond Logarithmic total returns for the period 01/01/1900 to 31/12/2016

The table presents the t statistic and respective Augmented Dickey Fuller test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1900 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-3.2846	0.0740
Log Level	Constant & Linear Trend	-0.6169	0.9134
Log Level	None	5.7035	0.9997
1 st Log Difference	Constant	-1.0569	0.6359
1 st Log Difference	Constant & Linear Trend	-2.9483	0.2608
1 st Log Difference	None	-0.828679	0.3083
2 nd Log Difference	Constant	-2.9681	0.1145
2 nd Log Difference	Constant & Linear Trend	-2.2205	0.4144
2 nd Log Difference	None	-2.5949	0.0237*

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.25: Augmented Dickey Fuller Tests for Stationarity of nominal 5-yearly JSE All Bond Logarithmic total returns for the period 01/01/1900 to 31/12/2016

The table presents the t statistic and respective Augmented Dickey Fuller test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1900 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-4.4068	0.0369*
Log Level	Constant & Linear Trend	4.0749	0.9996
Log Level	None	3.9942	0.9951
1 st Log Difference	Constant	3.1901	0.9968
1 st Log Difference	Constant & Linear Trend	-2.3145	0.0412*
1 st Log Difference	None	1.9390	0.9239
2 nd Log Difference	Constant	-7.6142	0.0054**
2 nd Log Difference	Constant & Linear Trend	10.9170	1.0000
2 nd Log Difference	None	3.9942	0.9951

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.26: Phillips Perron Unit Root Tests for Stationarity of nominal monthly JSE All Bond Logarithmic total returns for the period 31/12/1998 to 31/12/2016

The table presents the t statistic and respective Phillips Perron Unit Root test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 31/12/1998 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-3.7281	0.0043**
Log Level	Constant & Linear Trend	-2.7865	0.2039
Log Level	None	6.0499	1
1 st Log Difference	Constant	-14.90	0.0000***
1 st Log Difference	Constant & Linear Trend	-15.7190	0.0000***
1 st Log Difference	None	-13.2283	0.0000***
2 nd Log Difference	Constant	-82.4140	0.0001***
2 nd Log Difference	Constant & Linear Trend	-84.0730	0.0001***
2 nd Log Difference	None	-80.5509	0.0001***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.27: Phillips Perron Unit Root Tests for Stationarity of nominal quarterly JSE All Bond Logarithmic total returns for the period 31/12/1998 to 31/12/2016

The table presents the t statistic and respective Phillips Perron Unit Root test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 31/12/1998 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-4.5707	0.0004***
Log Level	Constant & Linear Trend	-2.8071	0.1997
Log Level	None	6.1125	1.0000
1 st Log Difference	Constant	-6.7695	0.0000***
1 st Log Difference	Constant & Linear Trend	-7.4824	0.0000***
1 st Log Difference	None	-4.3817	0.0000***
2 nd Log Difference	Constant	-49.6713	0.0001***
2 nd Log Difference	Constant & Linear Trend	-51.7576	0.0001***
2 nd Log Difference	None	-41.3175	0.0000***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.28: Phillips Perron Unit Root Tests for Stationarity of nominal semi-annually JSE All Bond Logarithmic total returns for the period 31/12/1998 to 31/12/2016

The table presents the t statistic and respective Phillips Perron Unit Root test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 31/12/1998 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
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Log Level	Constant	-5.2748	0.0001***
Log Level	Constant & Linear Trend	-3.8983	0.0224*
Log Level	None	5.1152	1.0000
1 st Log Difference	Constant	-4.7607	0.0005***
1 st Log Difference	Constant & Linear Trend	-5.7773	0.0002***
1 st Log Difference	None	-2.1829	0.0298*
2 nd Log Difference	Constant	-16.3935	0.0000***
2 nd Log Difference	Constant & Linear Trend	-15.9535	0.0000***
2 nd Log Difference	None	-15.5854	0.0000***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.29: Phillips Perron Unit Root Tests for Stationarity of annual nominal JSE All Bond Logarithmic total returns for the period 31/12/1998 to 31/12/2016

The table presents the t statistic and respective Phillips Perron Unit Root test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1900 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-3.9580	0.0082**
Log Level	Constant & Linear Trend	-1.5599	0.7681
Log Level	None	4.8726	1.0000
1 st Log Difference	Constant	-2.5944	0.1132
1 st Log Difference	Constant & Linear Trend	-4.4935	0.0125*
1 st Log Difference	None	-1.2135	0.1969
2 nd Log Difference	Constant	-9.5501	0.0000***
2 nd Log Difference	Constant & Linear Trend	-9.2158	0.0000***
2 nd Log Difference	None	-8.3308	0.0000***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.30: Phillips Perron Unit Root Tests for Stationarity of nominal 2 yearly JSE All Bond Logarithmic total returns for the period 31/12/1998 to 31/12/2016

The table presents the t statistic and respective Phillips Perron Unit Root test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1900 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-12.3128	0.0000***
Log Level	Constant & Linear Trend	-4.1055	0.0503
Log Level	None	4.0943	0.9993
1 st Log Difference	Constant	-1.6841	0.4021
1 st Log Difference	Constant & Linear Trend	-2.1655	0.4443
1 st Log Difference	None	-4.2693	0.0010**
2 nd Log Difference	Constant	-2.4896	0.1545
2 nd Log Difference	Constant & Linear Trend	-1.7854	0.6200
2 nd Log Difference	None	-1.8474	0.0655

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.31: Phillips Perron Unit Root Tests for Stationarity of nominal 3 yearly JSE All Bond Logarithmic total returns for the period 31/12/1998 to 31/12/2016

The table presents the t statistic and respective Phillips Perron Unit Root test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1900 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-6.3723	0.0035**
Log Level	Constant & Linear Trend	0.7022	0.9959
Log Level	None	4.8862	0.9992
1 st Log Difference	Constant	-0.7972	0.7297
1 st Log Difference	Constant & Linear Trend	-5.1152	0.0609
1 st Log Difference	None	-1.1421	0.1993
2 nd Log Difference	Constant	-4.7455	0.0290*
2 nd Log Difference	Constant & Linear Trend	-4.3221	0.1428
2 nd Log Difference	None	-2.5949	0.0237*

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.32: Phillips Perron Unit Root Tests for Stationarity of nominal 5 yearly JSE All Bond Logarithmic total returns for the period 31/12/1998 to 31/12/2016

The table presents the t statistic and respective Phillips Perron Unit Root test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1900 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-7.6142	0.0054**
Log Level	Constant & Linear Trend	10.9170	1.0000
Log Level	None	3.9942	0.9951
1 st Log Difference	Constant	7.0733	0.9998
1 st Log Difference	Constant & Linear Trend	n/a	0.5

1 st Log Difference	None	-2.3145	0.0412*
2 nd Log Difference	Constant	n/a	0.5
2 nd Log Difference	Constant & Linear Trend	n/a	n/a
2 nd Log Difference	None	2.8999	0.9573

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.33: Augmented Dickey Fuller Tests for Stationarity of real monthly JSE All Bond Logarithmic total returns for the period 01/01/1987 to 31/12/2016

The table presents the t statistic and respective Augmented Dickey Fuller test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1987 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-13.81646	0.0000***
Log Level	Constant & Linear Trend	-13.81646	0.0000***
Log Level	None	-13.31202	0.0000***
1 st Log Difference	Constant	-13.06482	0.0000***
1 st Log Difference	Constant & Linear Trend	-13.03079	0.0000***
1 st Log Difference	None	-13.09652	0.0000***
2 nd Log Difference	Constant	-10.52077	0.0000***
2 nd Log Difference	Constant & Linear Trend		
2 nd Log Difference	None	-10.5483	0.0000***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.34: Augmented Dickey Fuller Tests for Stationarity of real quarterly JSE All Bond Logarithmic total returns for the period 01/01/1987 to 31/12/2016

The table presents the t statistic and respective Augmented Dickey Fuller test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1987 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-6.0440	0.0000***
Log Level	Constant & Linear Trend		
Log Level	None	-5.9832	0.0000***
1 st Log Difference	Constant	-7.7003	0.0000***
1 st Log Difference	Constant & Linear Trend	-7.6443	0.0000***
1 st Log Difference	None	-7.6919	0.0000***
2 nd Log Difference	Constant	-7.2885	0.0000***
2 nd Log Difference	None	-6.6588	0.0000***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.35: Augmented Dickey Fuller Tests for Stationarity of real semi-annually JSE All Bond Logarithmic total returns for the period 01/01/1987 to 31/12/2016

The table presents the t statistic and respective Augmented Dickey Fuller test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1987 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-0.3220	0.9119
Log Level	None	3.4864	0.9997
1 st Log Difference	Constant	-4.0213	0.0035
1 st Log Difference	Constant & Linear Trend	-3.9650	0.0189
1 st Log Difference	None	-0.8840	0.3264
2 nd Log Difference	Constant	-7.2658	0.0000***
2 nd Log Difference	Constant & Linear Trend	-7.1540	0.0000***
2 nd Log Difference	None	-7.3718	0.0000***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.36: Augmented Dickey Fuller Tests for Stationarity of real annual JSE All Bond Logarithmic total returns for the period 01/01/1987 to 31/12/2016

The table presents the t statistic and respective Augmented Dickey Fuller test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1987 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-0.1469	0.9303
Log Level	None	3.8143	0.9997
1 st Log Difference	Constant	-4.3870	0.0031
1 st Log Difference	Constant & Linear Trend	-4.1959	0.0190
1 st Log Difference	None	-1.0631	0.2493
2 nd Log Difference	Constant	-4.7603	0.0014
2 nd Log Difference	Constant & Linear Trend	-5.0263	0.0039
2 nd Log Difference	None	-4.7510	0.0001

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.37: Augmented Dickey Fuller Tests for Stationarity of real 2-yearly JSE All Bond Logarithmic total returns for the period 01/01/1987 to 31/12/2016

The table presents the t statistic and respective Augmented Dickey Fuller test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1987 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-2.029	0.2790
Log Level	None	-2.2748	0.0303
1 st Log Difference	Constant	-2.7896	0.1135
1 st Log Difference	Constant & Linear Trend	-3.522656	0.1399
1 st Log Difference	None	-3.1665	0.0068
2 nd Log Difference	Constant	-4.0482	0.0363
2 nd Log Difference	None	-4.7700	0.0014

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.38: Augmented Dickey Fuller Tests for Stationarity of real 3-year JSE All Bond Logarithmic total returns for the period 01/01/1987 to 31/12/2016

The table presents the t statistic and respective Augmented Dickey Fuller test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1987 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-1.5090	0.4504
Log Level	None	-1.1581	0.1945
1 st Log Difference	Constant	-2.8392	0.1280
1 st Log Difference	Constant & Linear Trend	-2.0570	0.4548
1 st Log Difference	None	-3.0392	0.0132
2 nd Log Difference	Constant	-2.0188	0.2684
2 nd Log Difference	None	-2.8664	0.0208

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.39: Augmented Dickey Fuller Tests for Stationarity of real 5-yearly JSE All Bond Logarithmic total returns for the period 01/01/1987 to 31/12/2016

The table presents the t statistic and respective Augmented Dickey Fuller test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1987 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-1.54	0.4380
Log Level	Constant & Linear Trend	-0.67	0.8878
Log Level	None	-0.71	0.3586
1 st Log Difference	Constant	-1.22	0.5466
1 st Log Difference	Constant & Linear Trend	-1.89	0.4923
1 st Log Difference	None	-1.55	0.1086
2 nd Log Difference	Constant	-1.86	0.3070
2 nd Log Difference	Constant & Linear Trend	n/a	n/a
2 nd Log Difference	None	-2.15	0.0512

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.40: Phillips Perron Unit Root Tests for Stationarity of real monthly JSE All Bond Logarithmic total returns for the period 01/01/1987 to 31/12/2016

The table presents the t statistic and respective Phillips Perron Unit Root test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1987 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-13.8294	0.000*
Log Level	Constant & Linear Trend	-14.1406	0.000*
Log Level	None	-13.31202	0.000*
1 st Log Difference	Constant	-80.1161	0.0001
1 st Log Difference	Constant & Linear Trend	-82.5433	0.0001
1 st Log Difference	None	-76.9510	0.0001
2 nd Log Difference	Constant	-151.9370	0.0001
2 nd Log Difference	Constant & Linear Trend	-151.7852	0.0001
2 nd Log Difference	None	-152.1319	0.0001

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.41: Phillips Perron Unit Root Tests for Stationarity of real quarterly JSE All Bond Logarithmic total returns for the period 01/01/1987 to 31/12/2016

The table presents the t statistic and respective Phillips Perron Unit Root test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1987 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-6.5049	0.000*
Log Level	Constant & Linear Trend	-6.9327	0.000*

Log Level	None	-5.9832	0.000*
1 st Log Difference	Constant	-39.0758	0.0001
1 st Log Difference	Constant & Linear Trend	-43.94557	0.0001
1 st Log Difference	None	-35.5004	0.0001
2 nd Log Difference	Constant	-57.2897	0.0001
2 nd Log Difference	Constant & Linear Trend	-57.2824	0.0001
2 nd Log Difference	None	-57.8686	0.0001

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.42: Phillips Perron Unit Root Tests for Stationarity of real semi-annually JSE All Bond Logarithmic total returns for the period 01/01/1987 to 31/12/2016

The table presents the t statistic and respective Phillips Perron Unit Root test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1987 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-0.2725	0.9196
Log Level	Constant & Linear Trend	-2.2391	0.4551
Log Level	None	11.87322	1.0000
1 st Log Difference	Constant	-3.7679	0.0069
1 st Log Difference	Constant & Linear Trend	-3.7004	0.0348
1 st Log Difference	None	-1.2549	0.1889
2 nd Log Difference	Constant	-16.9257	0.0000
2 nd Log Difference	Constant & Linear Trend	-16.5838	0.0000
2 nd Log Difference	None	17.2964	0.0000

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.43: Phillips Perron Unit Root Tests for Stationarity of real annual JSE All Bond Logarithmic total returns for the period 01/01/1987 to 31/12/2016

The table presents the t statistic and respective Phillips Perron Unit Root test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1987 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-0.1822	0.9256
Log Level	Constant & Linear Trend	-2.3480	0.3915
Log Level	None	10.8331	1.0000
1 st Log Difference	Constant	-3.2073	0.0355
1 st Log Difference	Constant & Linear Trend	-2.9166	0.1793
1 st Log Difference	None	-0.9410	0.2966
2 nd Log Difference	Constant	-5.2730	0.0004
2 nd Log Difference	Constant & Linear Trend	-5.5320	0.0015
2 nd Log Difference	None	-5.5709	0.0000

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.44: Phillips Perron Unit Root Tests for Stationarity of real 5-yearly JSE All Bond Logarithmic total returns for the period 01/01/1987 to 31/12/2016

The table presents the t statistic and respective Phillips Perron Unit Root test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1987 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-1.58	0.4207
Log Level	Constant & Linear Trend	0.76	0.9867
Log Level	None	-0.71	0.3586
1 st Log Difference	Constant	-1.22	0.5466
1 st Log Difference	Constant & Linear Trend	-3.07	0.2732
1 st Log Difference	None	-1.55	0.1086
2 nd Log Difference	Constant	-2.32	0.2085
2 nd Log Difference	Constant & Linear Trend	N/A	N/A
2 nd Log Difference	None	-2.15	0.0512

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.45: Augmented Dickey Fuller Tests for Stationarity of real monthly JSE All Share Logarithmic total returns for the period 01/01/1987 to 31/12/2016

The table presents the t statistic and respective Augmented Dickey Fuller test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1987 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-18.36	0.0000***
Log Level	Constant & Linear Trend	-18.37	0.0000***
Log Level	None	-18.27	0.0000***
1 st Log Difference	Constant	-11.81	0.0000***
1 st Log Difference	Constant & Linear Trend	-11.80	0.0000***
1 st Log Difference	None	-11.83	0.0000***
2 nd Log Difference	Constant	-11.71	0.0000***

2 nd Log Difference	Constant & Linear Trend	-11.68	0.0000***
2 nd Log Difference	None	-11.72	0.0000***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.46: Augmented Dickey Fuller Tests for Stationarity of real quarterly JSE All Share Logarithmic total returns for the period 01/01/1987 to 31/12/2016

The table presents the t statistic and respective Augmented Dickey Fuller test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1987 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-8.25	0.0000***
Log Level	Constant & Linear Trend	-8.27	0.0000***
Log Level	None	-8.14	0.0000***
1 st Log Difference	Constant	-9.93	0.0000***
1 st Log Difference	Constant & Linear Trend	-9.94	0.0000***
1 st Log Difference	None	-9.97	0.0000***
2 nd Log Difference	Constant	-8.40	0.0000***
2 nd Log Difference	Constant & Linear Trend	-8.34	0.0000***
2 nd Log Difference	None	-8.45	0.0000***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.47: Augmented Dickey Fuller Tests for Stationarity of real semi-annually JSE All Share Logarithmic total returns for the period 01/01/1987 to 31/12/2016

The table presents the t statistic and respective Augmented Dickey Fuller test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1987 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-6.48	0.0000***
Log Level	Constant & Linear Trend	-6.50	0.0000***
Log Level	None	-6.2428	0.0000***
1 st Log Difference	Constant	-8.7774	0.0000***
1 st Log Difference	Constant & Linear Trend	-8.7017	0.0000***
1 st Log Difference	None	-8.8645	0.0000***
2 nd Log Difference	Constant	-8.3513	0.0000***
2 nd Log Difference	Constant & Linear Trend	-8.2625	0.0000***
2 nd Log Difference	None	-8.4353	0.0000***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.48: Augmented Dickey Fuller Tests for Stationarity of real annual JSE All Share Logarithmic total returns for the period 01/01/1987 to 31/12/2016

The table presents the t statistic and respective Augmented Dickey Fuller test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1987 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-6.36	0.0000***
Log Level	Constant & Linear Trend	-6.27	0.0001***
Log Level	None	-5.65	0.0000***
1 st Log Difference	Constant	-5.55	0.0002***
1 st Log Difference	Constant & Linear Trend	-5.45	0.0011**
1 st Log Difference	None	-5.73	0.0000***
2 nd Log Difference	Constant	-5.84	0.0001***
2 nd Log Difference	Constant & Linear Trend	-5.65	0.0008***
2 nd Log Difference	None	-5.78	0.0000***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.49: Augmented Dickey Fuller Tests for Stationarity of real 2-yearly JSE All Share Logarithmic total returns for the period 01/01/1987 to 31/12/2016

The table presents the t statistic and respective Augmented Dickey Fuller test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1987 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-3.95	0.0111*
Log Level	Constant & Linear Trend	-3.88	0.0436*
Log Level	None	-0.68	0.4013
1 st Log Difference	Constant	-6.09	0.0005***
1 st Log Difference	Constant & Linear Trend	-6.25	0.0019**
1 st Log Difference	None	-6.39	0.0000***
2 nd Log Difference	Constant	-7.53	0.0001***
2 nd Log Difference	Constant & Linear Trend	-7.05	0.0010**
2 nd Log Difference	None	-7.89	0.0000***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.50: Augmented Dickey Fuller Tests for Stationarity of real 3-year JSE All Share Logarithmic total returns for the period 01/01/1987 to 31/12/2016

The table presents the t statistic and respective Augmented Dickey Fuller test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1987 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-5.03	0.0059**
Log Level	Constant & Linear Trend	-4.99	0.0235*
Log Level	None	-0.41	0.4979
1 st Log Difference	Constant	-8.51	0.0004***
1 st Log Difference	Constant & Linear Trend	-7.79	0.0038**
1 st Log Difference	None	-9.26	0.0001***
2 nd Log Difference	Constant	-10.71	0.0002***
2 nd Log Difference	Constant & Linear Trend	-9.1222	0.0035**
2 nd Log Difference	None	-11.9047	0.0001***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.51: Augmented Dickey Fuller Tests for Stationarity of real 5-yearly JSE All Share Logarithmic total returns for the period 01/01/1987 to 31/12/2016

The table presents the t statistic and respective Augmented Dickey Fuller test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1987 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-1.54	0.4380
Log Level	Constant & Linear Trend	-0.67	0.8878
Log Level	None	-0.71	0.3586
1 st Log Difference	Constant	-1.22	0.5466
1 st Log Difference	Constant & Linear Trend	-1.89	0.4923
1 st Log Difference	None	-1.55	0.1086
2 nd Log Difference	Constant	-1.86	0.3070
2 nd Log Difference	Constant & Linear Trend	n/a	n/a
2 nd Log Difference	None	-2.15	0.0512

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.52: Phillips Perron Unit Root Tests for Stationarity of real monthly JSE All Share Logarithmic total returns for the period 01/01/1987 to 31/12/2016

The table presents the t statistic and respective Phillips Perron Unit Root test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1987 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-18.41	0.0000***
Log Level	Constant & Linear Trend	-18.42	0.0000***
Log Level	None	-18.28	0.0000***
1 st Log Difference	Constant	-166.18	0.0001***
1 st Log Difference	Constant & Linear Trend	-166.21	0.0001***
1 st Log Difference	None	-166.29	0.0001***
2 nd Log Difference	Constant	-220.29	0.0001***
2 nd Log Difference	Constant & Linear Trend	-219.99	0.0001***
2 nd Log Difference	None	-220.65	0.0001***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.53: Phillips Perron Unit Root Tests for Stationarity of real quarterly JSE All Share Logarithmic total returns for the period 01/01/1987 to 31/12/2016

The table presents the t statistic and respective Phillips Perron Unit Root test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1987 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-8.12	0.0000***
Log Level	Constant & Linear Trend	-8.01	0.0000***
Log Level	None	-8.03	0.0000***
1 st Log Difference	Constant	-50.10	0.0000***
1 st Log Difference	Constant & Linear Trend	-47.84	0.0000***
1 st Log Difference	None	-50.86	0.0000***
2 nd Log Difference	Constant	-42.38	0.0001***
2 nd Log Difference	Constant & Linear Trend	-42.52	0.0001***
2 nd Log Difference	None	-42.58	0.0000***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.54: Phillips Perron Unit Root Tests for Stationarity of real semi-annually JSE All Share Logarithmic total returns for the period 01/01/1987 to 31/12/2016

The table presents the t statistic and respective Phillips Perron Unit Root test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1987 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-6.40	0.0000***
Log Level	Constant & Linear Trend	-6.43	0.0000***
Log Level	None	-6.18	0.0000***
1 st Log Difference	Constant	-18.52	0.0000***
1 st Log Difference	Constant & Linear Trend	-23.03	0.0001***
1 st Log Difference	None	-18.15	0.0000***
2 nd Log Difference	Constant	-43.92	0.0001***
2 nd Log Difference	Constant & Linear Trend	-47.10	0.0001***
2 nd Log Difference	None	-42.99	0.0000***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.55: Phillips Perron Unit Root Tests for Stationarity of real annual JSE All Share Logarithmic total returns for the period 01/01/1987 to 31/12/2016

The table presents the t statistic and respective Phillips Perron Unit Root test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1987 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-10.19	0.0000***
Log Level	Constant & Linear Trend	-11.54	0.0000***
Log Level	None	-5.64	0.0000***
1 st Log Difference	Constant	-21.56	0.0001***
1 st Log Difference	Constant & Linear Trend	-21.07	0.0000***
1 st Log Difference	None	-22.26	0.0000***
2 nd Log Difference	Constant	-41.66	0.0001***
2 nd Log Difference	Constant & Linear Trend	-44.13	0.0000***
2 nd Log Difference	None	-42.46	0.0000***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.56: Phillips Perron Unit Root Tests for Stationarity of real 2-yearly JSE All Share Logarithmic total returns for the period 01/01/1987 to 31/12/2016

The table presents the t statistic and respective Phillips Perron Unit Root test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1987 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-4.11	0.0083**
Log Level	Constant & Linear Trend	-4.64	0.0130*
Log Level	None	-2.98	0.0060**
1 st Log Difference	Constant	-8.04	0.0000***
1 st Log Difference	Constant & Linear Trend	-10.21	0.0000***
1 st Log Difference	None	-8.41	0.0000***
2 nd Log Difference	Constant	-12.36	0.0000***
2 nd Log Difference	Constant & Linear Trend	-11.85	0.0000***
2 nd Log Difference	None	-12.71	0.0001***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.57: Phillips Perron Unit Root Tests for Stationarity of real 3-yearly JSE All Share Logarithmic total returns for the period 01/01/1987 to 31/12/2016

The table presents the t statistic and respective Phillips Perron Unit Root test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1987 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
Log Level	Constant	-5.04	0.0059**
Log Level	Constant & Linear Trend	-4.97	0.0240*
Log Level	None	-2.60	0.0166*
1 st Log Difference	Constant	-15.60	0.0000***
1 st Log Difference	Constant & Linear Trend	-17.67	0.0001***
1 st Log Difference	None	-13.99	0.0001***
2 nd Log Difference	Constant	-22.09	0.0000***
2 nd Log Difference	Constant & Linear Trend	-28.59	0.0001***
2 nd Log Difference	None	-25.61	0.0001***

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table B.58: Phillips Perron Unit Root Tests for Stationarity of real 5-yearly JSE All Share Logarithmic total returns for the period 01/01/1987 to 31/12/2016

The table presents the t statistic and respective Phillips Perron Unit Root test p-values resulting from the various variable differencing and exogenous factor selection test specifications. The P-value is specified by the Null Hypothesis that the variable is non-stationary. These tests were performed over the period 01/01/1987 to 31/12/2016.

Difference	Exogenous	t-Statistic	P-Value
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Log Level	Constant	-1.58	0.4207
Log Level	Constant & Linear Trend	0.76	0.9867
Log Level	None	-0.71	0.3586
1 st Log Difference	Constant	-1.22	0.5466
1 st Log Difference	Constant & Linear Trend	-3.07	0.2732
1 st Log Difference	None	-1.55	0.1086
2 nd Log Difference	Constant	-2.32	0.2085
2 nd Log Difference	Constant & Linear Trend	N/A	N/A
2 nd Log Difference	None	-2.15	0.0512

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

C. Autocorrelation tests

C1. ALSI Nominal

Table C1.1: Auto-Correlation Test for JSE All Share monthly Total Logarithmic Returns for the period 31/01/1925 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Share Index Logarithmic total returns regressed on the returns of the previous month, quarter, semi-annual, annual, 2 year, 3 year, 5 year, 10 year, 20 year and 30 year period. These results were calculated using an overlapping sample of monthly returns. These returns and statistics were calculated over the period 31/01/1925 to 31/12/2016. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	Monthly	Quarterly	Semi-Annual	Annual	2 Year	3 Year	5 Year	10 Year	20 Year	30 Year
AC	0.120393	0.032075	-	0.046496	-	0.020591	0.011785	-	-	-
Co-eff			0.034485		0.000559			0.001932	0.009555	0.025241
P-value	0.0001***	0.2879	0.2536	0.1249	0.9854	0.5027	0.7052	0.9523	0.8013	0.5698

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C1.2: Auto-Correlation Test for JSE All Share monthly Total Logarithmic Returns for the period 31/01/1925 to 31/12/1986

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Share Index Logarithmic total returns regressed on the returns of the previous month, quarter, semi-annual, annual, 2 year, 3 year, 5 year, 10 year, 20 year and 30 year period. These results were calculated using an overlapping sample of monthly returns. These returns and statistics were calculated over the period 31/01/1925 to 31/12/1986. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	Monthly	Quarterly	Semi-Annual	Annual	2 Year	3 Year	5 Year	10 Year	20 Year	30 Year
AC	0.195586	0.010670	-	0.102952	0.025131	0.029833	0.071480	-	0.077871	-
Co-eff			0.060824					0.085897		0.011421
P-value	0.0001***	0.7721	0.1012	0.0058**	0.5082	0.4401	0.0882	0.0630	0.3002	0.9085

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C1.3: Auto-Correlation Test for JSE All Share monthly Total Logarithmic Returns for the period 01/01/1987 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Share Index Logarithmic total returns regressed on the returns of the previous month, quarter, semi-annual, annual, 2 year, 3 year, 5 year, 10 year, 20 year and 30 year period. These results were calculated using an overlapping sample of monthly returns. These returns and statistics were calculated over the period 01/01/1987 to 31/12/2016. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	Monthly	Quarterly	Semi-Annual	Annual	2 Year	3 Year	5 Year	10 Year	20 Year	30 Year
AC	0.006913	0.062923	0.001929	-	-	0.005958	-	0.061971	-	-
Co-eff				0.035620	0.038778		0.050295		0.039376	0.029452
P-value	0.8960	0.2335	0.9706	0.4949	0.4557	0.9077	0.2934	0.1863	0.3800	0.5450

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C1.4: Auto-Correlation Test for JSE All Share quarterly Total Logarithmic Returns for the period 31/01/1925 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Share Index Logarithmic total returns regressed on the returns of the previous quarter, semi-annual, annual, 2 year, 3 year, 5 year, 10 year, 20 year and 30 year period. These results were calculated using an overlapping sample of quarterly returns. These returns and statistics were calculated over the period 31/01/1925 to 31/12/2016. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	Quarterly	Semi-Annual	Annual	2 Year	3 Year	5 Year	10 Year	20 Year	30 Year
AC Co-eff	0.282554	-0.003660	0.037455	-0.053715	0.053744	0.005463	0.014008	-0.020764	-
P-value	0.0000***	0.9445	0.4772	0.3105	0.3129	0.9194	0.8003	0.7399	0.2752

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C1.5: Auto-Correlation Test for JSE All Share quarterly Total Logarithmic Returns for the period 31/01/1925 to 31/12/1986

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Share Index Logarithmic total returns regressed on the returns of the previous quarter, semi-annual, annual, 2 year, 3 year, 5 year, 10 year, 20 year and 30 year period. These results were calculated using an overlapping sample of quarterly returns. These returns and statistics were calculated over the period 31/01/1925 to 31/12/1986. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	Quarterly	Semi-Annual	Annual	2 Year	3 Year	5 Year	10 Year	20 Year	30 Year
AC Co-eff	0.304134	0.050488	0.111259	-0.041525	0.052918	0.102370	-0.091185	0.151828	-0.173120
P-value	0.0000***	0.4379	0.0833	0.5318	0.4324	0.1622	0.2513	0.1837	0.2193

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C1.6: Auto-Correlation Test for JSE All Share quarterly Total Logarithmic Returns for the period 01/01/1987 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Share Index Logarithmic total returns regressed on the returns of the previous month, quarter, semi-annual, annual, 2 year, 3 year, 5 year, 10 year, 20 year and 30 year period. These results were calculated using an overlapping sample of quarterly returns. These returns and statistics were calculated over the period 01/01/1987 to 31/12/2016. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	Quarterly	Semi-Annual	Annual	2 Year	3 Year	5 Year	10 Year	20 Year	30 Year
AC Co-eff	0.239629	-0.102497	-0.094586	-0.084014	0.046004	-	0.099017	-0.102246	-0.042024
P-value	0.0084**	0.2555	0.2942	0.3475	0.6035	0.112946 0.1620	0.2129	0.1664	0.6039

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C1.7: Auto-Correlation Test for JSE All Share semi-annual Total Logarithmic Returns for the period 31/01/1925 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Share Index Logarithmic total returns regressed on the returns of the previous semi-annual, annual, 2 year, 3 year, 5 year, 10 year, 20 year and 30 year period. These results were calculated using an overlapping sample of semi-annual returns. These returns and statistics were calculated over the period 31/01/1925 to 31/12/2016. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	Semi-Annual	Annual	2 Year	3 Year	5 Year	10 Year	20 Year	30 Year
AC Co-eff	0.259690	0.013356	-0.128469	-0.019207	0.011867	-0.010256	-0.053244	-0.158559
P-value	0.0004***	0.8583	0.0862	0.7993	0.8766	0.8948	0.5341	0.1002

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C1.8: Auto-Correlation Test for JSE All Share semi-annual Total Logarithmic Returns for the period 31/01/1925 to 31/12/1986

The table displays the Autocorrelation Coefficients and their respective p-value of semi-annual JSE All Share Index Logarithmic total returns regressed on the returns of the previous semi-annual, annual, 2 year, 3 year, 5 year, 10 year, 20 year and 30 year period. These results were calculated using an overlapping sample of semi-annual returns. These returns and statistics were calculated over the period 31/01/1925 to 31/12/1986. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	Semi-Annual	Annual	2 Year	3 Year	5 Year	10 Year	20 Year	30 Year
AC Co-eff	0.334567	0.131463	-0.171368	-0.024912	0.089473	-0.118124	0.105950	-0.260869
P-value	0.0002***	0.1576	0.0671	0.7936	0.3798	0.2786	0.4916	0.1287

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C1.9: Auto-Correlation Test for JSE All Share semi-annual Total Logarithmic Returns for the period 01/01/1987 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Share Index Logarithmic total returns regressed on the returns of the previous month, quarter, semi-annual, annual, 2 year, 3 year, 5 year, 10 year, 20 year and 30 year period. These results were calculated using an overlapping sample of semi-annual returns. These returns and statistics were calculated over the period 01/01/1987 to 31/12/2016. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	Semi-Annual	Annual	2 Year	3 Year	5 Year	10 Year	20 Year	30 Year
AC Co-eff	0.106208	-0.228239	-0.073711	-0.030294	-0.112490	0.080158	-0.140640	-0.106205
P-value	0.4083	0.0716	0.5659	0.8122	0.3375	0.4858	0.1629	0.3514

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C1.10: Auto-Correlation Test for JSE All Share annual Total Logarithmic Returns for the period 01/01/1900 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Share Index Logarithmic total returns regressed on the returns of the previous year, 2 year, 3 year, 5 year, 10 year, 20 year and 30 year period. These results were calculated using an overlapping sample of annual returns. These returns and statistics were calculated over the period 01/01/1900 to 31/12/2016. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	Annual	2 Year	3 Year	5 Year	10 Year	20 Year	30 Year
AC Co-eff	0.0389	-0.1311	-0.1123	-0.0359	0.1571	0.0077	-0.1206
P-value	0.6777	0.1619	0.2296	0.6997	0.0970	0.9393	0.2622

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C1.11: Auto-Correlation Test for JSE All Share annual Total Logarithmic Returns for the period 01/01/1900 to 31/12/1986

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Share Index Logarithmic total returns regressed on the returns of the previous year, 2 year, 3 year, 5 year, 10 year, 20 year and 30 year period. These results were calculated using an overlapping sample of annual returns. These returns and statistics were calculated over the period 01/01/1900 to 31/12/1986. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	Annual	2 Year	3 Year	5 Year	10 Year	20 Year	30 Year
AC Co-eff	0.1398	-0.1753	-0.1269	-0.0843	0.1083	-0.07941	-0.1030
P-value	0.2044	0.1151	0.2537	0.4444	0.3574	0.5449	0.4629

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C1.12: Auto-Correlation Test for JSE All Share annual Total Logarithmic Returns for the period 01/01/1987 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Share Index Logarithmic total returns regressed on the returns of the previous month, quarter, semi-annual, annual, 2 year, 3 year, 5 year, 10 year, 20 year and 30 year period. These results were calculated using an overlapping sample of annual returns. These returns and statistics were calculated over the period 01/01/1987 to 31/12/2016. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	Annual	2 Year	3 Year	5 Year	10 Year	20 Year	30 Year
AC Co-eff	-0.3018	-0.0727	-0.1288	0.0561	0.2358	0.1608	-0.1731
P-value	0.0953	0.6934	0.4811	0.7598	0.1837	0.4413	0.3233

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C1.13: Auto-Correlation Test for JSE All Share 2-yearly Total Logarithmic Returns for the period 01/01/1900 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Share Index Logarithmic total returns regressed on the returns of the previous 2 year, 3 year, 5 year, 10 year, 20 year and 30 year period. These results were calculated using an overlapping sample of 2-yearly returns. These returns and statistics were calculated over the period 01/01/1900 to 31/12/2016. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	2 Year	10 Year	20 Year	30 Year
AC Co-eff	-0.0583	0.1382	0.000038	-0.3293
P-value	0.6677	0.3091	0.9998	0.0192*

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C1.14: Auto-Correlation Test for JSE All Share 2 yearly Total Logarithmic Returns for the period 01/01/1900 to 31/12/1986

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Share Index Logarithmic total returns regressed on the returns of the previous 2 year, 3 year, 5 year, 10 year, 20 year and 30 year period. These results were calculated using an overlapping sample of 2-yearly returns. These returns and statistics were calculated over the period 01/01/1900 to 31/12/1986. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	2 Year	10 Year	20 Year	30 Year
AC Co-eff	-0.0794	0.2080	-0.1561	-0.2881
P-value	0.6244	0.2341	0.4435	0.1748

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C1.15: Auto-Correlation Test for JSE All Share 2-yearly Total Logarithmic Returns for the period 01/01/1987 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Share Index Logarithmic total returns regressed on the returns of the previous 2 year, 3 year, 5 year, 10 year, 20 year and 30 year period. These results were calculated using an overlapping sample of 2-yearly returns. These returns and statistics were calculated over the period 01/01/1987 to 31/12/2016. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	2 Year	10 Year	20 Year	30 Year
AC Co-eff	-0.2613	-0.4221	0.1803	-0.4878
P-value	0.3574	0.0514	0.3250	0.0011*

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C1.16: Auto-Correlation Test for JSE All Share 3-yearly Total Logarithmic Returns for the period 01/01/1900 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Share Index Logarithmic total returns regressed on the returns of the previous 3 year, 5 year, 10 year, 20 year and 30 year period. These results were calculated using an overlapping sample of 3-yearly returns. These returns and statistics were calculated over the period 01/01/1900 to 31/12/2016. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	3 Year	30 Year
AC Co-eff	-0.2152	-0.3211
P-value	0.1927	0.0880

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C1.17: Auto-Correlation Test for JSE All Share 3 yearly Total Logarithmic Returns for the period 01/01/1900 to 31/12/1986

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Share Index Logarithmic total returns regressed on the returns of the previous 3 year, 5 year, 10 year, 20 year and 30 year period. These results were calculated using an overlapping sample of 3-yearly returns. These returns and statistics were calculated over the period 01/01/1900 to 31/12/1986. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	3 Year	30 Year
AC Co-eff	-0.2321	-0.2452
P-value	0.2426	0.3579

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C1.18: Auto-Correlation Test for JSE All Share 3-yearly Total Logarithmic Returns for the period 01/01/1987 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Share Index Logarithmic total returns regressed on the returns of the previous 3 year, 5 year, 10 year, 20 year and 30 year period. These results were calculated using an overlapping sample of 3-yearly returns. These returns and statistics were calculated over the period 01/01/1987 to 31/12/2016. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	3 Year	30 Year
AC Co-eff	-0.6968	-0.6010
P-value	0.0255*	0.0096**

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C1.19: Auto-Correlation Test for JSE All Share 5-yearly Total Logarithmic Returns for the period 01/01/1900 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Share Index Logarithmic total returns regressed on the returns of the previous 5 year, 10 year, 20 year and 30 year period. These results were calculated using an overlapping sample of 5-yearly returns. These returns and statistics were calculated over the period 01/01/1900 to 31/12/2016. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	5 Year	10 Year	20 Year	30 Year
AC Co-eff	0.3235	0.002688	0.1661	-0.2475
P-value	0.1100	0.9898	0.4840	0.3376

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C1.20: Auto-Correlation Test for JSE All Share 5 yearly Total Logarithmic Returns for the period 01/01/1900 to 31/12/1986

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Share Index Logarithmic total returns regressed on the returns of the previous 5 year, 10 year, 20 year and 30 year period. These results were calculated using an overlapping sample of 5-yearly returns. These returns and statistics were calculated over the period 01/01/1900 to 31/12/1986. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	5 Year	10 Year	20 Year	30 Year
AC Co-eff	0.2728	-0.1606	-0.01323	-0.4274
P-value	0.2938	0.6285	0.9711	0.3004

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C1.21: Auto-Correlation Test for JSE All Share 5-yearly Total Logarithmic Returns for the period 01/01/1987 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Share Index Logarithmic total returns regressed on the returns of the previous 5 year, 10 year, 20 year and 30 year period. These results were calculated using an overlapping sample of 5-yearly returns. These returns and statistics were calculated over the period 01/01/1987 to 31/12/2016. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	5 Year	10 Year	20 Year	30 Year
AC Co-eff	0.2610	-0.6496	0.2386	-0.4073
P-value	0.6284	0.0604	0.4918	0.1649

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C1.22: Auto-Correlation Test for JSE All-Share 10-yearly Total Logarithmic Returns for the period 01/01/1900 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Share Index Logarithmic total returns regressed on the returns of the previous 10 year, 20 year and 30 year period. These results were calculated using an overlapping sample of 10-yearly returns. These returns and statistics were calculated over the period 01/01/1900 to 31/12/2016. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation coefficient is not significantly different from zero.

	10 Year	20 Year	30 Year
AC Co-eff	0.1358	0.1451	-0.1587
P-value	0.6743	0.6971	0.7078

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C1.23: Auto-Correlation Test for JSE All Share 10 yearly Total Logarithmic Returns for the period 01/01/1900 to 31/12/1986

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Share Index Logarithmic total returns regressed on the returns of the previous 10 year, 20 year and 30 year period. These results were calculated using an overlapping sample of 10-yearly returns. These returns and statistics were calculated over the period 01/01/1900 to 31/12/1986. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation coefficient is not significantly different from zero.

	10 Year	20 Year	30 Year
AC Co-eff	-0.4449	-0.2073	-0.4222
P-value	0.4619	0.7741	0.6028

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C1.24: Auto-Correlation Test for JSE All Share 10-yearly Total Logarithmic Returns for the period 01/01/1987 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Share Index Logarithmic total returns regressed on the returns of the previous 10 year, 20 year and 30 year period. These results were calculated using an overlapping sample of 10-yearly returns. These returns and statistics were calculated over the period 01/01/1987 to 31/12/2016. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation coefficient is not significantly different from zero.

	10 Year	20 Year	30 Year
AC Co-eff	-0.4114	0.0289	-0.4361
P-value	0.1886	0.9451	0.0282*

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C1.25: Auto-Correlation Test for JSE All Share 20-yearly Total Logarithmic Returns for the period 01/01/1900 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Share Index Logarithmic total returns regressed on the returns of the previous 20 year period. These results were calculated using a non-overlapping sample of 20-yearly returns. These returns and statistics were calculated over the period 01/01/1900 to 31/12/2016. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation coefficient is not significantly different from zero.

	20 Year
AC Co-eff	0.3662
P-value	0.6749

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C1.26: Auto-Correlation Test for JSE All-Share 20 yearly Total Logarithmic Returns for the period 01/01/1900 to 31/12/1986

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Share Index Logarithmic total returns regressed on the returns of the previous 20 year period. These results were calculated using a non-overlapping sample of 20-yearly returns. These returns and statistics were calculated over the period 01/01/1900 to 31/12/1986. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation coefficient is not significantly different from zero.

	20 Year
AC Co-eff	-2.007
P-value	0.6448

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C1.27: Auto-Correlation Test for JSE All Share Total Logarithmic Returns over various frequencies for the period 01/01/1900 (31/01/1925 for monthly, quarterly and semi-annual frequencies) to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value for each frequency of JSE All Share Index Logarithmic total returns regressed on that respective frequencies previous periods return. These results are performed using non-overlapping data of the respective frequency. This is shown below for month, quarter, semi-annual, annual, 2 year, 3 year, 5 year, 10 year and 20 year returns. The P-value is specified by the Null Hypothesis that the autocorrelation coefficient is not significantly different from zero.

	Monthly	Quarterly	Semi-Annual	Annual	2 Year	3 Year	5 Year	10 Year	20 Year
AC Co-eff	0.120393	0.282554	0.259690	0.038929	-0.058303	-	0.323522	0.135818	0.366163
P-value	0.0001***	0.0000***	0.0004***	0.6777	0.6677	0.215150	0.1927	0.1100	0.6743

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C1.28: Auto-Correlation Test for JSE All Share Total Logarithmic Returns over various frequencies for the period 01/01/1900 (31/01/1925 for monthly, quarterly and semi-annual frequencies) to 31/12/1986

The table displays the Autocorrelation Coefficients and their respective p-value for each frequency of JSE All Share Index Logarithmic total returns regressed on that respective frequencies previous periods return. These results are performed using non-overlapping data of the respective frequency. This is shown below for month, quarter, semi-annual, annual, 2 year, 3 year, 5 year, 10 year and 20 year returns. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	Monthly	Quarterly	Semi-Annual	Annual	2 Year	3 Year	5 Year	10 Year	20 Year
AC Co-eff	0.195586	0.304134	0.334567	0.1398	-0.0794	-0.2321	0.2728	-0.4449	-2.007
P-value	0.0001***	0.0000***	0.0002***	0.2044	0.6244	0.2426	0.2938	0.4619	0.6448

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C1.29: Auto-Correlation Test for JSE All Share Total Logarithmic Returns over various frequencies for the period 01/01/1987 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value for each frequency of JSE All Share Index Logarithmic total returns regressed on that respective frequencies previous periods return. These results are performed using non-overlapping data of the respective frequency. This is shown below for month, quarter, semi-annual, annual, 2 year, 3 year, 5 year and 10 year returns. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	Monthly	Quarterly	Semi-Annual	Annual	2 Year	3 Year	5 Year	10 Year
AC Co-eff	0.006913	0.239629	0.106208	-0.3018	-0.2613	-0.6968	0.2610	-0.4114
P-value	0.8960	0.0084**	0.4083	0.0953	0.3574	0.0255*	0.6284	0.1886

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

C2. Nominal ALBI

Table C2.1: Auto-Correlation Test for JSE All Bond monthly Total Logarithmic Returns for the period 31/12/1998 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Bond Index Logarithmic total returns regressed on the returns of the previous month, quarter, semi-annual, annual, 2 year, 3 year, 5 year, 10 year, 20 year and 30 year period. These returns and statistics were calculated over the period 31/12/1998 to 31/12/2016. These results were calculated using an overlapping sample of monthly returns. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	Monthly	Quarterly	Semi-Annual	Annual	2 Year	3 Year	5 Year	10 Year
AC Co-eff	-0.0196	0.0315	-0.0444	0.0778	0.0920	0.1123	-0.0693	-0.0618
P-value	0.7750	0.6410	0.5156	0.2651	0.2278	0.1445	0.4051	0.5767

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C2.2: Auto-Correlation Test for JSE All Bond quarterly Total Logarithmic Returns for the period 31/12/1998 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Bond Index Logarithmic total returns regressed on the returns of the previous quarter, semi-annual, annual, 2 year, 3 year, 5 year, 10 year, 20 year and 30 year period. These returns and statistics were calculated over the period 31/12/1998 to 31/12/2016. These results were calculated using an overlapping sample of quarterly returns. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	Quarterly	Semi-Annual	Annual	2 Year	3 Year	5 Year	10 Year
AC Co-eff	0.2073	-0.082588	-0.009222	0.281722	0.003139	-0.122704	0.030250
P-value	0.0784	0.4907	0.9389	0.0227	0.9708	0.3143	0.8263

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C2.3: Auto-Correlation Test for JSE All Bond semi-annual Total Logarithmic Returns for the period 31/12/1998 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Bond Index Logarithmic total returns regressed on the returns of the previous semi-annual, annual, 2 year, 3 year, 5 year, 10 year, 20 year and 30 year period. These returns and statistics were calculated using an overlapping sample of semi-annual returns. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	Semi-Annual	Annual	2 Year	3 Year	5 Year	10 Year
AC Co-eff	0.1965	-0.069667	0.463741	-0.1114	-0.226912	0.084253
P-value	0.2527	0.6942	0.0077	0.3655	0.1717	0.6924

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C2.4: Auto-Correlation Test for JSE All Bond annual Total Logarithmic Returns for the period 31/12/1998 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Bond Index Logarithmic total returns regressed on the returns of the previous year, 2 year, 3 year, 5 year, 10 year, 20 year and 30 year period. These returns and statistics were calculated over the period 31/12/1998 to 31/12/2016. These results were calculated using an overlapping sample of annual returns. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	Annual	2 Year	3 Year	5 Year	10 Year
AC Co-eff	0.3321	0.501026	-0.2918	0.018108	0.120554
P-value	0.2138	0.5966	0.1244	0.9220	0.6091

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C2.5: Auto-Correlation Test for JSE All Bond 2-yearly Total Logarithmic Returns for the period 31/12/1998 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Bond Index Logarithmic total returns regressed on the returns of the previous 2 year, 3 year, 5 year, 10 year, 20 year and 30 year period. These results were calculated using an overlapping sample of 2-yearly returns. These returns and statistics were calculated over the period 31/12/1998 to 31/12/2016. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	2 Year	10 Year
AC Co-eff	0.7374	0.694246
P-value	0.0413	0.1145

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C2.6: Auto-Correlation Test for JSE All Bond 3-yearly Total Logarithmic Returns for the period 31/12/1998 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Bond Index Logarithmic total returns regressed on the returns of the previous 3 year, 5 year, 10 year, 20 year and 30 year period. These results were calculated using a non-overlapping sample of 3-yearly returns. These returns and statistics were calculated over the period 31/12/1998 to 31/12/2016. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	3 Year
AC Co-eff	0.5695
P-value	0.3192

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C2.7: Auto-Correlation Test for JSE All Bond 5-yearly Total Logarithmic Returns for the period 31/12/1998 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Bond Index Logarithmic total returns regressed on the returns of the previous 5 year, 10 year, 20 year and 30 year period. These results were calculated using an overlapping sample of 5-yearly returns. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	5 Year	10 Year
AC Co-eff	0.2529	2.691019
P-value	n/a	n/a

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

C3. Real ALSI

Table C3.1: Auto-Correlation Test for JSE All Share Total Logarithmic Monthly Real returns over the period 01/01/1987 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Share Index Logarithmic total real returns regressed on the real returns of the previous month, quarter, semi-annual, annual, 2 year, 3 year, 5 year, 10 year and 20 year period. This test was performed on an overlapping monthly return sample over the period 01/01/1987 to 31/12/2016. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	Monthly	Quarterly	Semi-Annual	Annual	2 Year	3 Year	5 Year	10 Year	20 Year
AC Co-eff	0.02773	0.06716	0.01041	-0.03640	-0.02835	-	-0.0305	0.0950	-0.07936
P-value	0.6009	0.2053	0.8444	0.4730	0.5801	0.001759	0.9722	0.5601	0.1068

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C3.2: Auto-Correlation Test for quarterly JSE All Share Total Logarithmic Real returns over the period 01/01/1987 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Share Index Logarithmic total real returns regressed on the real returns of the previous quarter, semi-annual, annual, 2 year, 3 year, 5 year, 10 year, and 20 year period. This test was performed on an overlapping quarterly return sample over the period 01/01/1987 to 31/12/2016. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	Quarterly	Semi-Annual	Annual	2 Year	3 Year	5 Year	10 Year	20 Year
AC Co-eff	0.06747	0.01525	-0.0224	-0.1459	0.003139	-0.0614	0.1842	-0.0897
P-value	0.0045	0.8863	0.8285	0.1593	0.9708	0.5383	0.0744	0.4354

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C3.3: Auto-Correlation Test for Semi-Annual JSE All Share Total Logarithmic Real returns over the period 01/01/1987 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Share Index Logarithmic total real returns regressed on the real returns of the previous semi-annual, annual, 2 year, 3 year, 5 year, 10 year and 20 year period. This test was performed on an overlapping semi-annual return sample over the period 01/01/1987 to 31/12/2016. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	Semi-Annual	Annual	2 Year	3 Year	5 Year	10 Year	20 Year
AC Co-eff	0.1450	-0.0508	-0.1007	-0.1114	-0.2106	-0.0304	0.2168
P-value	0.2768	0.7642	0.5503	0.3655	0.2031	0.8506	0.1969

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C3.4: Auto-Correlation Test for Annual JSE All Share Total Logarithmic Real returns over the period 01/01/1987 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Share Index Logarithmic total real returns regressed on the real returns of the previous annual, 2 year, 3 year, 5 year, 10 year and 20 year period. This test was performed on an overlapping annual return sample over the period 01/01/1987 to 31/12/2016. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	Annual	2 Year	3 Year	5 Year	10 Year	20 Year
AC Co-eff	-0.2185	-0.1236	-0.2918	-0.0571	0.2670	0.1524
P-value	0.2643	0.6044	0.1244	0.8059	0.2527	0.6979

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C3.5: Auto-Correlation Test for 2 Yearly JSE All Share Total Logarithmic Real returns over the period 01/01/1987 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Share Index Logarithmic total real returns regressed on the real returns of the previous 2 year and 10 year period. This test was performed on an overlapping 2-yearly return sample over the period 01/01/1987 to 31/12/2016. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	2 Year	10 Year
AC Co-eff	-0.1389	-0.2755
P-value	0.6388	0.4646

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C3.6: Auto-Correlation Test for 3 Yearly JSE All Share Total Logarithmic Real returns over the period 01/01/1987 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Share Index Logarithmic total real returns regressed on the real returns of the previous 3 year period. This test was performed on a non-overlapping 3-yearly return sample over the period 01/01/1987 to 31/12/2016. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	3 Year
AC Co-eff	-0.5438
P-value	0.1264

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C3.7: Auto-Correlation Test for 5 Yearly JSE All Share Total Logarithmic Real returns over the period 01/01/1987 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Share Index Logarithmic total real returns regressed on the real returns of the previous semi-annual, annual, 2 year, 3 year, 5 year, 10 year, 20 year and 30 year period. This test was performed on an overlapping 5-yearly return sample over the period 01/01/1987 to 31/12/2016. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	5 Year	10 Year
AC Co-eff	0.2238	-0.5368
P-value	0.6869	0.3216

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C3.8: Auto-Correlation Test for 10 Yearly JSE All Share Total Logarithmic Real returns over the period 01/01/1987 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of 10 Yearly JSE All Share Index Logarithmic total real returns regressed on the real returns of the previous 10 year period. This test was performed on a non-overlapping sample over the period 01/01/1987 to 31/12/2016. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	10 Year
AC Co-eff	-0.3344
P-value	0.296

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C3.9: Auto-Correlation Test for JSE All Share Total Logarithmic Real returns over the period 01/01/1987 to 31/12/2016 for all return frequencies

The table displays the Autocorrelation Coefficients and their respective p-value for each frequency of JSE All Share Index Logarithmic total real returns regressed on that respective frequencies previous periods return. This test was performed on a non-overlapping sample over the period 01/01/1987 to 31/12/2016. This is shown below for month, quarter, semi-annual, annual, 2 year, 3 year, 5 year, 10 year, 20 year and 30 year real returns. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	Monthly	Quarterly	Semi-Annual	Annual	2 Year	3 Year	5 Year	10 Year
AC Co-eff	0.02773	0.06747	0.1450	-0.2185	-0.1389	-0.5438	0.2238	-0.3344
P-value	0.6009	0.0045	0.2768	0.2643	0.6388	0.1264	0.6869	0.296

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

C4. Real ALBI

Table C4.1: Auto-Correlation Test for JSE All Bond monthly Total real Logarithmic Returns for the period 31/12/1998 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Bond Index Logarithmic total real returns regressed on the returns of the previous month, quarter, semi-annual, annual, 2 year, 3 year, 5 year, 10 year, 20 year and 30 year period. These returns and statistics were calculated over the period 31/12/1998 to 31/12/2016. These results were calculated using an overlapping sample of monthly returns. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	Monthly	Quarterly	Semi-Annual	Annual	2 Year	3 Year	5 Year	10 Year
AC Co-eff	0.0552	0.0080	-0.0695	0.0873	0.1250	0.0967	-0.0455	-0.0440
P-value	0.4208	0.9052	0.3075	0.2089	0.0976	0.2044	0.5761	0.6745

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C4.2: Auto-Correlation Test for JSE All Bond quarterly Total real Logarithmic Returns for the period 31/12/1998 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Bond Index Logarithmic total real returns regressed on the returns of the previous quarter, semi-annual, annual, 2 year, 3 year, 5 year, 10 year, 20 year and 30 year period. These returns and statistics were calculated over the period 31/12/1998 to 31/12/2016. These results were calculated using an overlapping sample of quarterly returns. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	Quarterly	3 Year
AC Co-eff	0.2424	-0.065346
P-value	0.0387	0.6019

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C4.3: Auto-Correlation Test for JSE All Bond semi-annual Total real Logarithmic Returns for the period 31/12/1998 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Bond Index Logarithmic total real returns regressed on the returns of the previous semi-annual, annual, 2 year, 3 year, 5 year, 10 year, 20 year and 30 year period. These results were calculated using an overlapping sample of semi-annual returns. These returns and statistics were calculated over the period 31/12/1998 to 31/12/2016. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	Semi-Annual	3 Year
AC Co-eff	0.2208	-0.1500
P-value	0.1967	0.3873

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C4.4: Auto-Correlation Test for JSE All Bond annual Total real Logarithmic Returns for the period 31/12/1998 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Bond Index Logarithmic total real returns regressed on the returns of the previous year, 2 year, 3 year, 5 year, 10 year, 20 year and 30 year period. These returns and statistics were calculated over the period 31/12/1998 to 31/12/2016. These results were calculated using an overlapping sample of annual returns. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	Annual	3 Year
AC Co-eff	0.3094	0.0176
P-value	0.2444	0.9416

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C4.5: Auto-Correlation Test for JSE All Bond 2-yearly Total real Logarithmic Returns for the period 31/12/1998 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Bond Index Logarithmic total real returns regressed on the returns of the previous 2 year, 3 year, 5 year, 10 year, 20 year and 30 year period. These results were calculated using an overlapping sample of 2-yearly returns. These returns and statistics were calculated over the period 31/12/1998 to 31/12/2016. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	2 Year
AC Co-eff	0.3662
P-value	0.2855

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C4.6: Auto-Correlation Test for JSE All Bond 3-yearly Total real Logarithmic Returns for the period 31/12/1998 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value of monthly JSE All Bond Index Logarithmic total real returns regressed on the returns of the previous 3 year, 5 year, 10 year, 20 year and 30 year period. These results were calculated using a non-overlapping sample of 3-yearly returns. These returns and statistics were calculated over the period 31/12/1998 to 31/12/2016. These auto-correlation tests included an intercept term in the regressions performed. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	3 Year
AC Co-eff	0.0310

P-value	0.9645
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* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table C4.7: Auto-Correlation Test for JSE All Bond Total real Logarithmic Returns over various frequencies for the period 31/12/1998 to 31/12/2016

The table displays the Autocorrelation Coefficients and their respective p-value for each frequency of JSE All Bond Index Logarithmic total real returns regressed on that respective frequencies previous periods return. These results were calculated using a non-overlapping sample of returns. This is shown below for month, quarter, semi-annual, annual, 2 year, 3 year, 5 year, 10 year, 20 year and 30 year returns. The P-value is specified by the Null Hypothesis that the autocorrelation co-efficient is not significantly different from zero.

	Monthly	Quarterly	Semi-Annual	Annual	2 Year	3 Year
AC Co-eff	0.0552	0.2424	0.2208	0.3094	0.3662	0.0310
P-value	0.4208	0.0387	0.1967	0.2444	0.2855	0.9645

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

D. Bai-Perron Unknown Structural Breakpoint tests

Table D.1: Bai-Perron Unknown breakpoint test ALBI over the period 31/12/1998 – 31/12/2016

This Table presents the results of the Bai-Perron multiple unknown breakpoint test performed on the ALBI nominal returns over the period 31/12/1998 to 31/12/2016.

	Month	Quarter	Semi-annual	Annual
Breakpoint Date	2010m11	No BP	2002S1	2002
F-Statistic	12.9643		9.7458	11.1247
Critical Value	8.58			
F-Statistic	<0.05			

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table D.2: Bai-Perron Unknown breakpoint test on the ALSI over the entire period

This Table depicts the results of the Bai-Perron unknown multiple structural breakpoint test performed on the ALSI returns over the period 01/01/1900 to 31/12/2016 (for monthly, quarterly and semi-annual frequencies 31/01/1925 to 31/12/2016).

	Month	Quarter	Semi-annual	Annual	2-Year	3-Year
Breakpoint Date	1972m09	No BP	No BP	No BP	No BP	No BP
F-Statistic	14.6619*					
F-Statistic Scaled	14.6619*					
Critical Value	8.58					
p-value	<0.05					

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

E. Real returns Runs test

E1. ALSI

Table E1: Runs Test over various return frequencies for Real JSE All Share Total Logarithmic Returns for the period 01/01/1987 to 31/12/2016

The table displays the results of a Runs Test on various frequencies of the real JSE All Share Index Logarithmic total return over the period 01/01/1987 to 31/12/2016. The Runs Test P-value is specified by the Null Hypothesis that the sequence of returns is random. This Runs Test has the Median selected as the threshold value. The P-value for trends and oscillations, test the Null hypothesis that returns follow a trending or oscillating behaviour respectively.

Frequency	Observed Number of Runs	Expected Number of Runs	P-value	P-value for Trends	P-value for Oscillations
Monthly	169	180.5	0.2242	0.7743	0.2257
Quarterly	56	60.496	0.4078	0.0005**	0.9995
Semi-Annually	23	30.492	0.0490*	0.5	0.5
Annually	15	15.483	0.8549	0.3246	0.6754
2 Yearly	9	8.467	0.7740	0.3316	0.6684
3 Yearly	9	5.44	0.0102*	0.9805	0.0195
5 Yearly	3	4	0.3613	0.0267*	0.9733
10 Yearly	2	2.333	0.1573	0.7659	0.2341

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

E2. Albi

Table E2: Runs Test for JSE All Bond Total Logarithmic Real returns for the period 31/12/1998 to 31/12/2016

The table displays the results of a Runs Test on JSE All Bond Index Logarithmic total real returns. The Runs Test P-value is specified by the Null Hypothesis that the sequence of real returns is random. This Runs Test has the Median selected as the threshold value. The P-value for trends and oscillations, test the Null hypothesis that real returns follow a trending or oscillating behaviour respectively. This test is conducted over the period 31/12/1998 to 31/12/2016.

Frequency	Observed Number of Runs	Expected Number of Runs	P-value	P-value for Trends	P-value for Oscillations
Monthly	99	109	0.1729	0.0801	0.9199
Quarterly	39	37.493	0.7224	0.5743	0.4257
Semi-Annual	17	19.486	0.4067	0.8568	0.1432
Annually	11	10	0.6270	0.5779	0.4221
2 Yearly	8	5.44	0.0647	0.9805	0.0195*
3 Yearly	6	4	0.0679	0.9389	0.0611
5 Yearly	4	3	0.2207	0.8575	0.1425

* significant at $p < 0.05$; ** significant at $p < 0.01$; *** significant at $p < 0.001$

F. Variance Ratio test

F1. Nominal ALSI Variance Ratio test

Table F1.1: Variance Ratio Tests for JSE All Share nominal Monthly total returns over the period 01/01/1925 to 31/12/2016

The table displays the Variance Ratios of the JSE All Share Index total logarithmic return over the period 01/01/1925 to 31/12/2016. The Variance ratios were conducted for monthly total logarithmic returns and calculated against quarterly, Semi-annually, Annually, 2 yearly, 3 Yearly, 5 Yearly, 10 Yearly, 20 Yearly and 30 Yearly logarithmic total returns. In addition we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

	Quarter/ Month	Semi Annual/ Month	Year/ Month	2 Year/ Month	3 Year/ Month	5 year/ Month	10 Year/ Month	20 Year/ Month	30 Year/ Month
Variance Ratio	1.181362	1.281059	1.346058	1.354698	1.200005	0.962518	1.004062	1.152852	1.720271
P-value	0.0031	0.0062	0.0226	0.0971	0.4382	0.9083	0.9928	0.8053	0.3310

* significant at $p < 0.05$; ** significant at $p < 0.01$; *** significant at $p < 0.001$

Table F1.2: Variance Ratio Tests for JSE All Share nominal Monthly total returns over the period 01/01/1925 to 31/12/1986

The table displays the Variance Ratios of the JSE All Share Index total logarithmic return over the period 01/01/1925 to 31/12/1986. The Variance ratios were conducted for monthly total logarithmic returns and calculated against quarterly, Semi-annually, Annually, 2 yearly, 3 Yearly, 5 Yearly, 10 Yearly, 20 Yearly and 30 Yearly logarithmic total returns. In addition we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	Quarter/ Month	Semi Annual/ Month	Year/ Month	2 Year/ Month	3 Year/ Month	5 year/ Month	10 Year/ Month	20 Year/ Month	30 Year/ Month
Variance Ratio	1.278557	1.404496	1.557351	1.724807	1.562253	1.215784	1.114020	1.174777	2.075394
P-value	0.0009	0.0027	0.0049	0.0094	0.0964	0.6131	0.8438	0.8199	0.2214

* significant at $p < 0.05$; ** significant at $p < 0.01$; *** significant at $p < 0.001$

Table F1.3: Variance Ratio Tests for JSE All Share nominal Monthly total returns over the period 01/01/1987 to 31/12/2016

The table displays the Variance Ratios of the JSE All Share Index total logarithmic return over the period 01/01/1987 to 31/12/2016. The Variance ratios were conducted for monthly total logarithmic returns and calculated against quarterly, Semi-annually, Annually, 2 yearly, 3 Yearly, 5 Yearly, 10 Yearly, 20 Yearly and 30 Yearly logarithmic total returns. In addition we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	Quarter/ Month	Semi Annual/ Month	Year/ Month	2 Year/ Month	3 Year/ Month	5 year/ Month	10 Year/ Month	20 Year/ Month
Variance Ratio	1.039114	1.104156	1.042552	0.809704	0.645764	0.499839	0.555260	0.649079
P-value	0.6565	0.5119	0.8573	0.5618	0.3630	0.3015	0.4974	0.6917

* significant at $p < 0.05$; ** significant at $p < 0.01$; *** significant at $p < 0.001$

Table F1.4: Variance Ratio Tests for Quarterly data over the full period

The table displays the Variance Ratios of the JSE All Share Index total logarithmic return over the period 01/01/1925 to 31/12/2016. The Variance ratios were conducted for quarterly total logarithmic returns and calculated against Semi-annually, Annually, 2 yearly, 3 Yearly, 5 Yearly, 10 Yearly, 20 Yearly and 30 Yearly logarithmic total returns. In addition we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	Semi Annual/ Quarter	Year/ Quarter	2 Year/ Quarter	3 Year/ Quarter	5 year/ Quarter	10 Year/ Quarter	20 Year/ Quarter	30 Year/ Quarter
Variance Ratio	0.991431	0.559974	0.297478	0.206903	0.139352	0.104340	0.128332	0.188246
P-value	0.3161	0.1783	0.1577	0.1546	0.1555	0.1636	0.1877	0.2243

* significant at $p < 0.05$; ** significant at $p < 0.01$; *** significant at $p < 0.001$

Table F1.5: Variance Ratio Tests for Quarterly data over the period 1986

The table displays the Variance Ratios of the JSE All Share Index total logarithmic return over the period 01/01/1925 to 31/12/2016. The Variance ratios were conducted for quarterly total logarithmic returns and calculated against Semi-annually, Annually, 2 yearly, 3 Yearly, 5 Yearly, 10 Yearly, 20 Yearly and 30 Yearly logarithmic total returns. In addition we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	Semi Annual/ Quarter	Year/ Quarter	2 Year/ Quarter	3 Year/ Quarter	5 year/ Quarter	10 Year/ Quarter	20 Year/ Quarter	30 Year/ Quarter
Variance Ratio	1.312154	1.584804	1.802376	1.642595	1.284118	1.178349	1.238228	2.183079
P-value	0.0002	0.0002	0.0006	0.0277	0.4498	0.7283	0.7301	0.1417

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table F1.6: Variance Ratio Tests for Quarterly data over the period 1987

The table displays the Variance Ratios of the JSE All Share Index total logarithmic return over the period 01/01/1925 to 31/12/2016. The Variance ratios were conducted for quarterly total logarithmic returns and calculated against Semi-annually, Annually, 2 yearly, 3 Yearly, 5 Yearly, 10 Yearly, 20 Yearly and 30 Yearly logarithmic total returns. In addition we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	Semi Annual/ Quarter	Year/ Quarter	2 Year/ Quarter	3 Year/ Quarter	5 year/ Quarter	10 Year/ Quarter	20 Year/ Quarter
Variance Ratio	1.248926	1.27	1.01	0.81	0.62	0.70	0.82
P-value	0.0539	0.2139	0.9842	0.5968	0.4026	0.6282	0.8280

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table F1.7: Variance Ratio Tests for Semi-Annual data over the full period

The table displays the Variance Ratios of the JSE All Share Index total logarithmic return over the period 01/01/1925 to 31/12/2016. The Variance ratios were conducted for semi-annual total logarithmic returns and calculated against Annually, 2 yearly, 3 Yearly, 5 Yearly, 10 Yearly, 20 Yearly and 30 Yearly logarithmic total returns. In addition we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	Year/ Semi annual	2 Year/ Semi annual	3 Year/ Semi annual	5 year/ Semi annual	10 Year/ Semi annual	20 Year/ Semi annual	30 Year/ Semi annual
Variance Ratio	1.27	1.36	1.23	0.99	1.06	1.23	1.84
P-value	0.0023	0.0311	0.2844	0.9909	0.8796	0.6849	0.2143

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table F1.8: Variance Ratio Tests for Semi-Annual data over the 1986

The table displays the Variance Ratios of the JSE All Share Index total logarithmic return over the period 01/01/1925 to 31/12/2016. The Variance ratios were conducted for semi-annual total logarithmic returns and calculated against Annually, 2 yearly, 3 Yearly, 5 Yearly, 10 Yearly, 20 Yearly and 30 Yearly logarithmic total returns. In addition we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	Year/ Semi annual	2 Year/ Semi annual	3 Year/ Semi annual	5 year/ Semi annual	10 Year/ Semi annual	20 Year/ Semi annual	30 Year/ Semi annual
Variance Ratio	1.33	1.56	1.44	1.12	1.03	1.06	1.85
P-value	0.0049	0.0103	0.1200	0.7304	0.9506	0.9263	0.2719

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table F1.9: Variance Ratio Tests for Semi-Annual data over the 1987

The table displays the Variance Ratios of the JSE All Share Index total logarithmic return over the period 01/01/1925 to 31/12/2016. The Variance ratios were conducted for semi-annual total logarithmic returns and calculated against Annually, 2 yearly, 3 Yearly, 5 Yearly, 10 Yearly, 20 Yearly and 30 Yearly logarithmic total returns. In addition we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	Year/ Semi annual	2 Year/ Semi annual	3 Year/ Semi annual	5 year/ Semi annual	10 Year/ Semi annual	20 Year/ Semi annual
Variance Ratio	1.16	0.95	0.78	0.58	0.66	0.72
P-value	0.2008	0.8443	0.4664	0.3032	0.5585	0.7279

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table F1.10: Variance Ratio Tests for Annual data over the full period

The table displays the Variance Ratios of the JSE All Share Index total logarithmic return over the period 01/01/1925 to 31/12/2016. The Variance ratios were conducted for annual total logarithmic returns and calculated against 2 yearly, 3 Yearly, 5 Yearly, 10 Yearly, 20 Yearly and 30 Yearly logarithmic total returns. In addition we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	2 Year/ Year	3 Year/ Year	5 year/ Year	10 Year/ Year	20 Year/ Year	30 Year/ Year
Variance Ratio	1.05	0.99	0.80	0.70	0.78	0.89
P-value	0.5887	0.9355	0.3375	0.3351	0.6160	0.8356

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table F1.11: Variance Ratio Tests for Annual data over the period 1986

The table displays the Variance Ratios of the JSE All Share Index total logarithmic return over the period 01/01/1925 to 31/12/2016. The Variance ratios were conducted for annual total logarithmic returns and calculated against 2 yearly, 3 Yearly, 5 Yearly, 10 Yearly, 20 Yearly and 30 Yearly logarithmic total returns. In addition we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	2 Year/ Year	3 Year/ Year	5 year/ Year	10 Year/ Year	20 Year/ Year	30 Year/ Year
Variance Ratio	1.14	1.07	0.87	0.59	0.48	0.38
P-value	0.2017	0.6637	0.5887	0.2457	0.3011	0.3065

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table F1.12: Variance Ratio Tests for Annual data over the period 1987

The table displays the Variance Ratios of the JSE All Share Index total logarithmic return over the period 01/01/1925 to 31/12/2016. The Variance ratios were conducted for annual total logarithmic returns and calculated against 2 yearly, 3 Yearly, 5 Yearly, 10 Yearly, 20 Yearly and 30 Yearly logarithmic total returns. In addition we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	2 Year/ Year	3 Year/ Year	5 year/ Year	10 Year/ Year	20 Year/ Year
Variance Ratio	0.74	0.64	0.37	0.46	0.56
P-value	0.1193	0.1502	0.0963	0.3447	0.5972

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table X.X: Variance Ratio Tests for 2 Yearly data over the full period

The table displays the Variance Ratios of the JSE All Share Index total logarithmic return over the period 01/01/1925 to 31/12/2016. The Variance ratios were conducted for 2 yearly total logarithmic returns and calculated against 10 Yearly, 20 Yearly and 30 Yearly logarithmic total returns. In addition we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	4 Year/ 2 Year	10 Year/ 2 Year	20 year/ 2 Year	30 Year/ 2 Year
Variance Ratio	0.97	0.87	1.02	1.18
P-value	0.8523	0.6790	0.9603	0.7390

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table F1.13: Variance Ratio Tests for 2 Yearly data over the full period 1986

The table displays the Variance Ratios of the JSE All Share Index total logarithmic return over the period 01/01/1925 to 31/12/2016. The Variance ratios were conducted for 2 yearly total logarithmic returns and calculated against 10 Yearly, 20 Yearly and 30 Yearly logarithmic total returns. In addition we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	4 Year/ 2 Year	10 Year/ 2 Year	20 year/ 2 Year	30 Year/ 2 Year
Variance Ratio	0.948144	0.69	0.53	0.41
P-value	0.7714	0.3795	0.3574	0.3388

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table F1.14: Variance Ratio Tests for 2 Yearly data over the full period 1987

The table displays the Variance Ratios of the JSE All Share Index total logarithmic return over the period 01/01/1925 to 31/12/2016. The Variance ratios were conducted for 2 yearly total logarithmic returns and calculated against 10 Yearly, 20 Yearly and 30 Yearly logarithmic total returns. In addition we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	4 Year/ 2 Year	10 Year/ 2 Year	20 year/ 2 Year	30 Year/ 2 Year
Variance Ratio	0.77	0.86	0.49	7.44
P-value	0.4348	0.8007	0.5193	0.7390

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table F1.15: Variance Ratio Tests for 3 Yearly data over the full period

The table displays the Variance Ratios of the JSE All Share Index total logarithmic return over the period 01/01/1925 to 31/12/2016. The Variance ratios were conducted for 3 Yearly total logarithmic returns and calculated against 6 yearly and 30 Yearly logarithmic total returns. In addition we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	6 Year/ 3 Year	30 Year/ 3 Year
Variance Ratio	0.81	1.16
P-value	0.2317	0.7407

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table F1.16: Variance Ratio Tests for 3 Yearly data over the full period 1986

The table displays the Variance Ratios of the JSE All Share Index total logarithmic return over the period 01/01/1925 to 31/12/2016. The Variance ratios were conducted for 3 Yearly total logarithmic returns and calculated against 6 yearly and 30 Yearly logarithmic total returns. In addition we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	6 Year/ 3 Year	30 Year/ 3 Year
Variance Ratio	0.80	0.41
P-value	0.3003	0.2732

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table F1.17: Variance Ratio Tests for 3 Yearly data over the full period 1986

The table displays the Variance Ratios of the JSE All Share Index total logarithmic return over the period 01/01/1925 to 31/12/2016. The Variance ratios were conducted for 3 Yearly total logarithmic returns and calculated against 6 yearly and 30 Yearly logarithmic total returns. In addition we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	6 Year/ 3 Year
Variance Ratio	0.35
P-value	0.0389

* significant at $p<0.05$; ** significant at $p<0.01$; *** significant at $p<0.001$

Table F1.18: Variance Ratio Tests for 5 Yearly data over the full period

The table displays the Variance Ratios of the JSE All Share Index total logarithmic return over the period 01/01/1925 to 31/12/2016. The Variance ratios were conducted for 5 Yearly total logarithmic returns and calculated against 10 yearly, 15 yearly, 20 yearly and 30 Yearly logarithmic total returns. In addition we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	10 Year/ 5 Year	15 Year/ 5 Year	20 year/ 5 Year	30 Year/ 5 Year
Variance Ratio	1.35	1.52	1.60	1.89
P-value	0.0504	0.0466	0.0651	0.0343

* significant at $p<0.05$; ** significant at $p<0.01$; *** significant at $p<0.001$

Table F1.19: Variance Ratio Tests for 5 Yearly data over the full period 1986

The table displays the Variance Ratios of the JSE All Share Index total logarithmic return over the period 01/01/1925 to 31/12/2016. The Variance ratios were conducted for 5 Yearly total logarithmic returns and calculated against 10 yearly, 15 yearly, 20 yearly and 30 Yearly logarithmic total returns. In addition we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	10 Year/ 5 Year	15 Year/ 5 Year	20 year/ 5 Year	30 Year/ 5 Year
Variance Ratio	1.22	0.99	0.82	0.65
P-value	0.3110	0.9746	0.6149	0.4598

* significant at $p<0.05$; ** significant at $p<0.01$; *** significant at $p<0.001$

Table F1.20: Variance Ratio Tests for 5 Yearly data over the full period 1986

The table displays the Variance Ratios of the JSE All Share Index total logarithmic return over the period 01/01/1925 to 31/12/2016. The Variance ratios were conducted for 5 Yearly total logarithmic returns and calculated against 10 yearly, 15 yearly, 20 yearly and 30 Yearly logarithmic total returns. In addition we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	10 Year/ 5 Year	15 Year/ 5 Year	20 year/ 5 Year	30 Year/ 5 Year
Variance Ratio	1.42	0.79	0.66	4.58
P-value	0.0000	0.4986	0.4636	0.0000

* significant at $p<0.05$; ** significant at $p<0.01$; *** significant at $p<0.001$

Table F1.21: Variance Ratio Tests for 10 Yearly data over the full period

The table displays the Variance Ratios of the JSE All Share Index total logarithmic return over the period 01/01/1925 to 31/12/2016. The Variance ratios were conducted for 10 Yearly total logarithmic returns and calculated against 20 yearly and 30 Yearly logarithmic total returns. In addition we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	20 Year/ 10 Year	30 Year/ 10 Year
Variance Ratio	1.262309	1.511086
P-value	0.2661	0.1620

* significant at $p<0.05$; ** significant at $p<0.01$; *** significant at $p<0.001$

Table F1.22: Variance Ratio Tests for 10 Yearly data over the full period 1986

The table displays the Variance Ratios of the JSE All Share Index total logarithmic return over the period 01/01/1925 to 31/12/2016. The Variance ratios were conducted for 10 Yearly total logarithmic returns and calculated against 20 yearly and 30 Yearly logarithmic total returns. In addition we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	20 Year/ 10 Year	30 Year/ 10 Year
Variance Ratio	0.58	0.47
P-value	0.0311	0.0783

* significant at $p<0.05$; ** significant at $p<0.01$; *** significant at $p<0.001$

Table F1.23: Variance Ratio Tests for 10 Yearly data over the full period 1987

The table displays the Variance Ratios of the JSE All Share Index total logarithmic return over the period 01/01/1925 to 31/12/2016. The Variance ratios were conducted for 10 Yearly total logarithmic returns and calculated against 20 yearly and 30 Yearly logarithmic total returns. In addition we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	20 Year/ 10 Year	30 Year/ 10 Year
Variance Ratio	1.56	8.29
P-value	0.0173	0.0000

* significant at $p<0.05$; ** significant at $p<0.01$; *** significant at $p<0.001$

F2. Nominal ALBI Variance Ratio test

Table F2.1: Variance Ratio Tests for JSE All Bond nominal Monthly total returns over the period 31/12/1998 to 31/12/2016

The table displays the Variance Ratios of the JSE All Bond Index total logarithmic return over the period 31/12/1998 to 31/12/2016. The Variance ratios were conducted for monthly total logarithmic returns and calculated against quarterly, Semi-annually, Annually, 2 yearly, 3 Yearly, 5 Yearly, 10 Yearly, 20 Yearly and 30 Yearly logarithmic total returns. In addition we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

	Quarter/ Month	Semi Annual/ Month	Year/ Month	2 Year/ Month	3 Year/ Month	5 year/ Month	10 Year/ Month
Variance Ratio	0.98	0.99	0.95	0.97	1.14	1.28	1.8
P-value	0.0031	0.0062	0.0226	0.0971	0.4382	0.9083	0.9928

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table F2.2: Variance Ratio Tests for Quarterly data over the full period

The table displays the Variance Ratios of the JSE All Bond Index total logarithmic return over the period 31/12/1998 to 31/12/2016. The Variance ratios were conducted for quarterly total logarithmic returns and calculated against Semi-annually, Annually, 2 yearly, 3 Yearly, 5 Yearly, 10 Yearly, 20 Yearly and 30 Yearly logarithmic total returns. In addition we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	Semi Annual/ Quarter	Year/ Quarter	2 Year/ Quarter	3 Year/ Quarter	5 year/ Quarter	10 Year/ Quarter
Variance Ratio	1.22	1.31	1.42	1.76	1.98	2.63
P-value	0.0564	0.1595	0.2304	0.0800	0.0811	0.0331

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table F2.3: Variance Ratio Tests for Semi-Annual data over the full period

The table displays the Variance Ratios of the JSE All Bond Index total logarithmic return over the period 31/12/1998 to 31/12/2016. The Variance ratios were conducted for semi-annual total logarithmic returns and calculated against Annually, 2 yearly, 3 Yearly, 5 Yearly, 10 Yearly, 20 Yearly and 30 Yearly logarithmic total returns. In addition we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	Year/ Semi annual	2 Year/ Semi annual	3 Year/ Semi annual	5 year/ Semi annual	10 Year/ Semi annual
Variance Ratio	1.25	1.45	1.89	2.12	2.63
P-value	0.1566	0.1713	0.0343	0.0416	0.0286

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table F2.4: Variance Ratio Tests for Annual data over the full period

The table displays the Variance Ratios of the JSE All Bond Index total logarithmic return over the period 31/12/1998 to 31/12/2016. The Variance ratios were conducted for annual total logarithmic returns and calculated against 2 yearly, 3 Yearly, 5 Yearly, 10 Yearly, 20 Yearly and 30 Yearly logarithmic total returns. In addition we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	2 Year/ Year	3 Year/ Year	5 year/ Year	10 Year/ Year
Variance Ratio	1.40	1.92	2.24	2.43
P-value	0.0789	0.0080	0.0130	0.0399

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table F2.5: Variance Ratio Tests for 2 Yearly data over the full period

The table displays the Variance Ratios of the JSE All Bond Index total logarithmic return over the period 31/12/1998 to 31/12/2016. The Variance ratios were conducted for 2 yearly total logarithmic returns and calculated against 10 Yearly, 20 Yearly and 30 Yearly logarithmic total returns. In addition we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	4 Year/ 2 Year	10 Year/ 2 Year
Variance Ratio	1.47	1.72
P-value	0.0902	0.1646

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table F2.6: Variance Ratio Tests for 3 Yearly data over the full period

The table displays the Variance Ratios of the JSE All Bond Index total logarithmic return over the period 31/12/1998 to 31/12/2016. The Variance ratios were conducted for 3 Yearly total logarithmic returns and calculated against 6 yearly and 30 Yearly logarithmic total returns. In addition we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	6 Year/ 3 Year	15 Year/ 3 Year
Variance Ratio	1.56	0.89
P-value	0.0011	0.8513

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table F2.7: Variance Ratio Tests for 5 Yearly data over the full period

The table displays the Variance Ratios of the JSE All Bond Index total logarithmic return over the period 31/12/1998 to 31/12/2016. The Variance ratios were conducted for 5 Yearly total logarithmic returns and calculated against 10 yearly, 15 yearly, 20 yearly and 30 Yearly logarithmic total returns. In addition we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	10 Year/ 5 Year	15 Year/ 5 Year
Variance Ratio	1.46	1.83
P-value	0.0056	0.0017

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

F3. Real Returns

Table F3.1: Variance Ratio Tests for JSE All Share Monthly total real returns over the period 01/01/1987 to 31/12/2016

The table displays the Variance Ratios of the JSE All Share Index total logarithmic real returns over the period 01/01/1987 to 31/12/2016. The Variance ratios were conducted for monthly total logarithmic real returns and calculated against quarterly, Semi-annually, Annually, 2 yearly, 3 Yearly, 5 Yearly, 10 Yearly and 20 logarithmic total real returns. In addition, we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	Quarter/ Month	Semi Annual/ Month	Year/ Month	2 Year/ Month	3 Year/ Month	5 year/ Month	10 Year/ Month	20 Year/ Month
Variance Ratio	0.3226	0.1721	0.0888	0.0463	0.03181	0.02201	0.01282	0.01258
P-value	0.0000	0.0000	0.0005	0.0053	0.0158	0.0484	0.1394	0.2709

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table F3.2: Variance Ratio test for JSE All Share quarterly total real returns over the period 01/01/1987 to 31/12/2016

The table displays the Variance Ratios of the JSE All Share Index total logarithmic real returns over the period 01/01/1987 to 31/12/2016. The Variance ratios were conducted for quarterly total logarithmic real returns and calculated against Semi-annually, Annually, 2 yearly, 3 Yearly, 5 Yearly, 10 Yearly and 20 logarithmic total real returns. In addition, we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	Semi Annual/ Quarter	Year/ Quarter	2 Year/ Quarter	3 Year/ Quarter	5 year/ Quarter	10 Year/ Quarter	20 Year/ Quarter
Variance Ratio	0.7409	0.3433	0.1766	0.1194	0.08426	0.04609	0.06822
P-value	0.0315	0.0036	0.0117	0.0225	0.0549	0.1357	0.2809

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table F3.3: Variance Ratio Tests for Semi-Annual real returns over the period 01/01/1987 to 31/12/2016

The table displays the Variance Ratios of the JSE All Share Index total logarithmic real returns over the period 01/01/1987 to 31/12/2016. The Variance ratios were conducted for semi-annual total logarithmic real returns and calculated against Annually, 2 yearly, 3 Yearly, 5 Yearly, 10 Yearly and 20 Yearly logarithmic total real returns. In addition, we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	Year/ Semi annual	2 Year/ Semi annual	3 Year/ Semi annual	5 year/ Semi annual	10 Year/ Semi annual	20 Year/ Semi annual
Variance Ratio	0.6432	0.3029	0.2315	0.1393	0.07311	0.009462
P-value	0.0330	0.0133	0.0249	0.0472	0.1291	0.2860

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table F3.4: Variance Ratio Tests for Annual real returns over the period 01/01/1987 to 31/12/2016

The table displays the Variance Ratios of the JSE All Share Index total logarithmic real returns over the period 01/01/1987 to 31/12/2016. The Variance ratios were conducted for annual total logarithmic real returns and calculated against 2 yearly, 3 Yearly, 5 Yearly, 10 Yearly and 20 Yearly logarithmic total real returns. In addition, we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	2 Year/ Year	3 Year/ Year	5 year/ Year	10 Year/ Year	20 Year/ Year
Variance Ratio	0.4286	0.3918	0.1776	0.0960	0.08807
P-value	0.0115	0.0581	0.0606	0.1365	0.2865

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table F3.5: Variance Ratio Tests for 2 Yearly real returns over the period 01/01/1987 to 31/12/2016

The table displays the Variance Ratios of the JSE All Share Index total logarithmic real returns over the period 01/01/1987 to 31/12/2016. The Variance ratios were conducted for 2 yearly total logarithmic real returns and calculated against 10 Yearly, 20 Yearly and 30 Yearly logarithmic total real returns. In addition, we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	4 Year/ 2 Year	10 Year/ 2 Year	20 year/ 2 Year
Variance Ratio	0.6388	0.3545	0.1394
P-value	0.2080	0.3033	0.3222

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table F3.7: Variance Ratio Tests for 5 Yearly real returns over the period 01/01/1987 to 31/12/2016

The table displays the Variance Ratios of the JSE All Share Index total logarithmic real returns over the period 01/01/1987 to 31/12/2016. The Variance ratios were conducted for 5 Yearly total logarithmic real returns and calculated against 10 yearly, 15 yearly, 20 yearly and 30 Yearly logarithmic total real returns. In addition, we have reported the p-value of these variance ratios against the null hypothesis that the variance ratio is not significantly different from 1.

All	10 Year/ 5 Year	20 year/ 5 Year
Variance Ratio	1.6570	0.3152
P-value	0.1709	0.3796

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table F3.8: Overall Joint Test Variance Ratio Tests for JSE All Share total logarithmic real returns over the period 01/01/1987 to 31/12/2016

The table displays the Joint Variance Ratios Tests of the JSE All Share Index total logarithmic real returns over the period 01/01/1987 to 31/12/2016. The Variance ratio tests were conducted for monthly, quarterly, Semi-annually, Annually, 2 yearly, 3 Yearly, 5 Yearly, 10 Yearly, 20 Yearly and 30 Yearly logarithmic total real returns. In addition, have reported the results of the Joint Test statistic, the degrees of freedom and the p-value of the test against the null hypothesis that the series of real returns follow a random walk stochastic process.

Frequency (Max Period)	Test Statistic	Degrees of freedom	P-Value
Monthly	5.056531	358	0.0000
Quarter	2.907553	118	0.0252
Semi-Annual	2.474348	58	0.0898
Annual	2.528265	28	0.0560
2 Year	1.259104	14	0.5032
5 year	1.369380	5	0.3126

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

G. Bradfield and Ardington (1997) replication

Table X.X: Bradfield (1997) Table 1 Runs Test Replication

The table displays the Monthly, Quarterly and Annual Results from a Runs test performed on the JSE All Share Index (Logarithmic returns) over the period 01/01/1980 to 01/01/1996. These statistics include the Expected number of runs away from the threshold value (in this case the mean), the observed number of runs in the return series and the p-value that is specified by the null hypothesis that the sequence of returns is random.

	Monthly	Quarterly	Annually
Expected Number of Runs	97	32.492	9
Observed Number of Runs	85	37	12
P-value	0.0825	0.2520	0.1205

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table X.X: Bradfield (1997) Table 2 Autocorrelation Test Replication (Intercept)

The table displays the Monthly, Quarterly and Annual Results from an Auto Correlation Test performed on the JSE All Share Index (Logarithmic returns) over the period 01/01/1980 to 01/01/1996. This regression does include an intercept term. These statistics include the autocorrelation co-efficient and the p-value that is specified by the null hypothesis that the autocorrelation coefficient is not statistically different from zero.

	Monthly	Quarterly	Annually
Autocorrelation Co-efficient	-0.146072	0.255557	-0.235511
P-value	0.0418*	0.0359*	0.2806

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001

Table X.X: Bradfield (1997) Table 3 Variance Test Replication

The table displays the Monthly, Quarterly and Annual Results from a Variance Ratio Test performed on the JSE All Share Index (Logarithmic returns) over the period 01/01/1980 to 01/01/1996. These statistics include the variance ratio and the p-value that is specified by the null hypothesis that the variance ratio is not statistically different from one.

	Quarter/ Month	Year/ Quarter	Year/ Month	3 Year/ Year	5 Year/ Year
Variance Ratio	1.2540	1.3326	1.0963	0.807570	0.741167
P-value	0.0416	0.0721	0.7742	0.5946	0.6262

* significant at p<0.05; ** significant at p<0.01; *** significant at p<0.001