

The University of Cape Town

FACULTY OF ENGINEERING AND THE BUILT ENVIRONMENT

Department of Civil Engineering



Dissertation for Master of Science in Structural Engineering and Materials

An investigation into the effects of the shifting centroid and shear centre in notched steel beams undergoing elastic lateral torsional buckling

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Abstract

Notched steel sections have been investigated since the late 1970s and early 1980s. Notched sections are interesting since, for a doubly symmetric member, part of the section is singly or monosymmetric – specifically the notched region – while the un-notched portion may be a standard section. Since the notched region is monosymmetric the centroid and shear centre in this region are at different vertical heights. The design for Lateral Torsional Buckling (LTB) in standard prismatic sections accounts for the monosymmetry through the monosymmetry factor which accounts for the additional twisting moment.

However, this is not used in the analysis of notched regions for beams that are laterally unrestrained. These regions are stiffened against local buckling failure using horizontal and vertical stiffeners. The horizontal stiffeners make the notched part appear as an H or I shaped section. Using downstands on the horizontal stiffener the centroid and shear centre in the notched region can be shifted relative to each other – and in some cases - to be at the same vertical height (which is the case for the un-notched sections). This study looks into how the addition of stiffeners in the notched region of a doubly symmetric beam affects the centroid and shear centre location for top notched steel sections undergoing elastic lateral torsional buckling, and how the shifting shear centre relates to the critical elastic moment.

A parametric study using a Finite Element Model (FEM) using the Finite Element Analysis (FEA) software Abaqus was used to conduct the study. This research tested 5 steel sections (203x133x25, 305x102x25, 533x210x82, 203x203x46 and 305x305x97) with four different spans (1m,3m and 5m,6m) and two loading cases (unit point load 600mm from notched end and a unit point load at mid span) to investigate the influence of the shifting centroid and shear centre. Initially, 13 stiffening techniques were tested on the 203x133x25 section to identify the five best performing techniques. These were improved, then tested on all five sections. Downstands were applied to the horizontal stiffener, their purpose being to shift the shear centre upwards and past the centroid. With each increase in downstand height the shear centre would approach the centroid and eventually move past it.

The critical elastic moment increased with the addition of downstands in almost all cases. The increase was independent of changes to the spans or loading configurations.

Changes in the trend were evident as changes in gradient or changes in curve type around the centroid shear centre coincidence point. This shows a link between the centroid, shear centre and the critical elastic moment.

Further research should be undertaken on the FHEN and FHHVNTS techniques by applying them to many steel sections to see if the results are consistent for a general case. Care should be taken to allow for downstands far beyond the centroid shear centre coincidence point to see if all sections tend to the same result. Future studies should focus on isolating the changes due to the shear centre centroid movement versus the change in overall geometric properties.

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List of Abbreviations

Abbreviation	Meaning
AISC	American Institute of Steel Construction
EPL	End Point Load
FD	Full Length Downstand
FEA	Finite Element Analysis
FEM	Finite Element Model
LRFD	Load and Resistance Factor Design
LTB	Lateral Torsional Buckling
MSL	Mid Span Load

1. Introduction

Structural engineers possess an understanding into the design of safe structures. The profession of structural engineering is a deep understanding into how key structural components behave under various loading conditions. The case of structural steel beams is no different. Structural steel beams are designed to be stable during their loading and life cycle because the instability of one member could cause the failure of the entire structure. Lateral Torsional Buckling (LTB) is a global instability failure characterised by the buckling of the compression flange and a twist about the shear centre. It can cause a beam to fail at a moment significantly lower than its yield or plastic moment. The beam must experience major axis bending (**Figure 1.1**) about its centroid which creates tension and compression in the member. If the compression flange were to be isolated, it would buckle like a column. Usually the web would prevent buckling about the minor axis but if the applied force is great enough - and no lateral support is provided along the compression flange - the flange and the web will buckle together about the minor axis (Ziemian, 2010). In a simply supported beam, the twist in the beam is due to the tension flange remaining stable thus restraining the bottom flange and web from buckling (Mudenda & Zingoni, 2018). This combination of buckling and twist results in LTB. There are two types of LTB: elastic (occurs before material yields) and inelastic (occurs after some material yields).

Extensive research has been completed by numerous researchers on LTB in singly, doubly and monosymmetric sections. Bleich (1952) cites Prandtl (1899) and Michell (1899) for the first theoretical models for LTB in long narrow rectangular sections. Timoshenko (1910) is credited with deriving the fundamental differential equation of torsion of transversely loaded I beams; and using the derived equation to investigate lateral buckling of these sections.

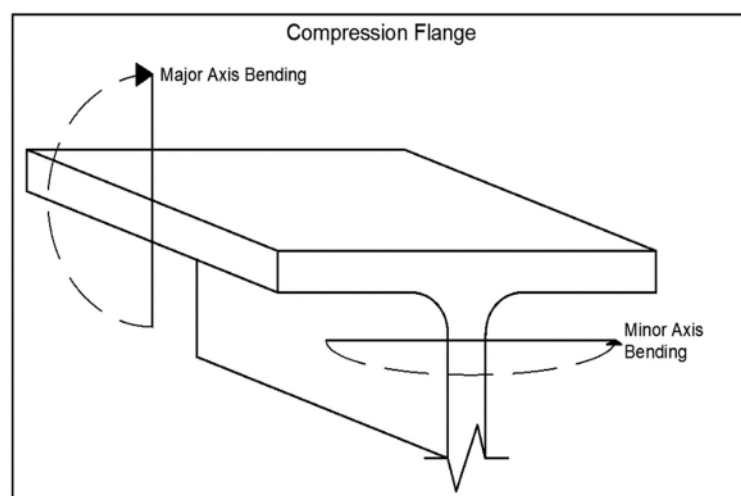


Figure 1.1: Isolated Compression Zones

Elastic and inelastic lateral torsional buckling of sections have been further studied by Vlasov (1940), Winter (1943), Hill (1954), Clark & Hill (1960) and Galambos (1963). Vincent (1969) summarised the early research into the resistance capacities of steel structural members which led to Load and Resistance Factor Design (LRFD).

Steel structures consist of several structural elements, beams and columns are just two examples. The connection between beams and columns can be straightforward, but the connection between two different beams such that the top flange of both is at the same level requires a special type of beam. This is because the beams must have flush web to web connections. This is difficult because one or both pairs of flanges of the two beams are in conflict. The practical solution is the removal of a portion of the conflicting flange from one of the beams to allow for the connection. The result is what is known as a notched or “coped” steel section (**Figure 1.2**). Notched sections are susceptible to instability failures because the removal of a portion of the flange reduces the stiffness of the member (Cheng & Yura, 1986). This, combined with stress concentrating at the notch corner, increases the instability of the member and the chance of local web buckling which reduces the LTB capacity of the member (Ibrahim et al., 2015).

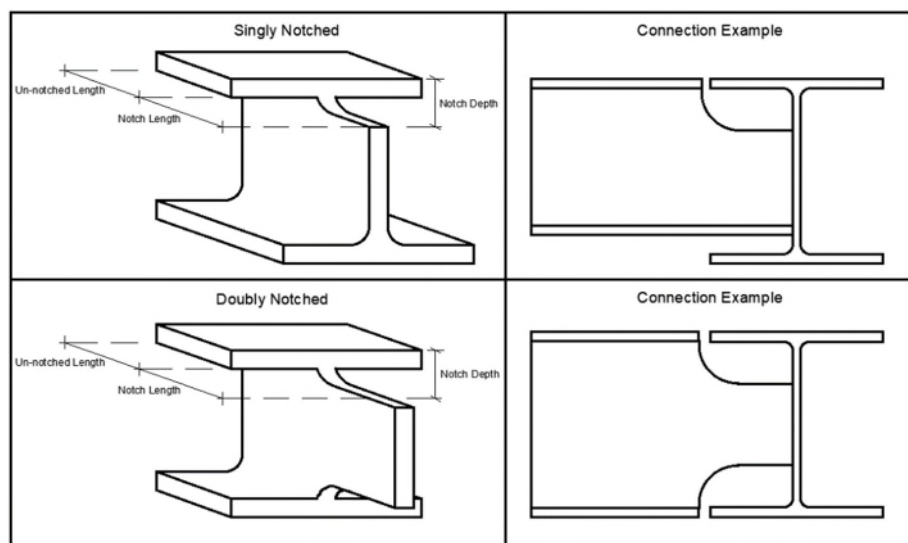


Figure 1.2: Notched Section

Research into the stability of notched or “coped” steel sections gathered interest in the late 1970s and early 1980s. The pioneers of this research, Cheng et al. (1984) and Cheng & Yura (1986), who conducted research into local buckling at the notched end, continued where Birkemoe & Gilmor (1978) began. Cheng & Yura (1986) presented Finite Element Analysis (FEA) simulations which generated data. They modified a classical plate buckling model to consider the effects of stress concentration, shear stress, the length and depth of the notch on local web buckling strength.

They proceeded to make design recommendations which were later adopted by the American Institute of Steel Construction (AISC) Manual of Steel Construction, Load & Resistance Factor Design (AISC-LRFD). Aalberg (2015b), Aalberg & Larsen (2001), Ibrahim et al. (2015), Lam et al. (2000), (2006), Yam et al. (2003), (2007), (2011), and Yam & Chung (2013) are a few of the authors who built upon the work of Cheng & Yura (1986). Yam et al. (2014) conducted a state-of-the-art review on the various failure modes of notched sections. Ibrahim et al. (2015) conducted research into strengthening techniques of notched sections and provided further insight into the influence of the notch length and depth.

For doubly symmetric members, notched sections are interesting because they comprise of both a monosymmetric (the notched region) and a doubly/singly symmetric section (the un-notched part). Therefore, the shear centre will have to shift to re-join the un-notched shear centre (**Figure 1.3**). Stiffeners must be used to control the notched region which is susceptible to local buckling failure, inelastic local web buckling, shear block failure and fatigue cracks (Yam et al., 2014). The use of horizontal stiffeners impacts the geometry of the section because it essentially adds the top or bottom flange back resulting in an I shape in the notched zone (**Figure 1.5**). This I shaped section is of a smaller depth than the un-notched region and will have a different centroid and shear centre location than the un-notched region of the beam. Within the notched region the centroid and shear centre may also not coincide, but this is dependent on the notched region height and the type of stiffeners used because a combination of the two could result in an I or H section which have the centroid and shear centre at the same vertical height.

The shear centre and LTB are closely linked. If an external force is not applied through the shear centre this results in a twist in the section. This twist can be corrected if the external load is applied with some eccentricity as it will counter the torsional moment created by the shear flow along the cross-sectional elements. The location of this eccentric point is known as the shear centre (Skatulla, 2017).

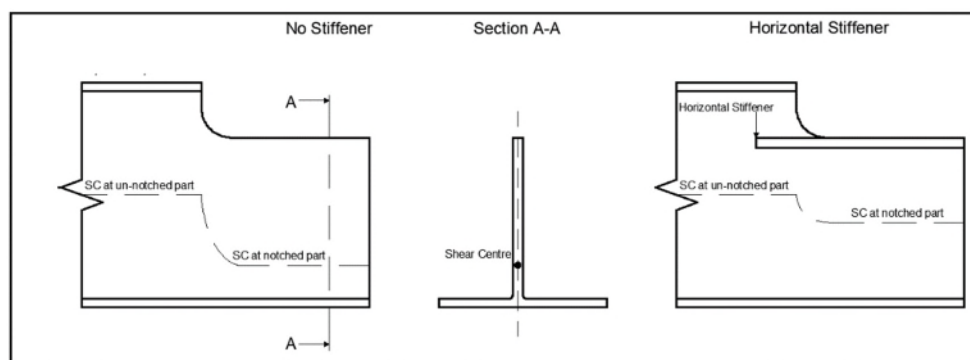


Figure 1.3: Shear Centre

A load applied below the shear centre stabilises the section ($M_{cr,1} > M_{cr}$) as shown in case a in **Figure 1.4**. A load applied at the shear centre will not produce a twist in the section (Case b **Figure 1.4**) whereas a load applied above the shear centre will destabilise the section ($M_{cr,2} < M_{cr}$) (case c in **Figure 1.4**) (da Silva et al., 2010).

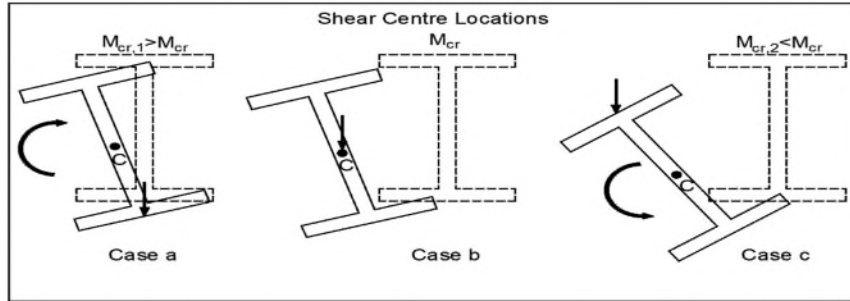


Figure 1.4: Effect of Location of Point Load (da Silva et al., 2010)

For H and I sections the centroid and shear centre coincide but in monosymmetric sections, which notched regions form part of, they do not. Galambos & Surovek (2008) state a factor is used to account for the additional twisting moment which Kitipornchai & Trahair (1980) simplified using numerical analysis. The factor is applied to monosymmetric sections, not notched portions of beams. There is uncertainty around the effects the location of the shear centre relative to the centroid in the notched region have on the stability of the beam and if shifting these properties to be at the same vertical height, as in a standard I and H section, will increase the section's overall stability and resistance to LTB. Mudenda & Zingoni (2018) investigated the influence of the shifting centroid and shear centre by using upstands on the top flange and found the introduction of the upstands increased the buckling capacity of the member to a point then decreased it. They propose this could be due to the shifting shear centre.

The objective of this study is to provide insight into the relationship between the shifting centroid and shear centre in a notched steel section and the critical elastic moment of the member. This study will investigate how the introduction of stiffeners affect the shear centre location in the notched region and if there is any link between the shifting shear centre relative to the centroid and the critical elastic moment of the section during LTB. This will be completed through a parametric study using the Finite Element Method (FEM). The key output of this research is a better understanding into how the geometry of a stiffener arrangement influences the shear centre, centroid location and how this affects the critical elastic moment. This understanding may prompt further research into alternative stiffening methods that could result in more efficient designs. This is key for the future in which sustainable and adaptable designs are required.

The resemblance between an I section and stiffened notched section is shown in **Figure 1.5**.

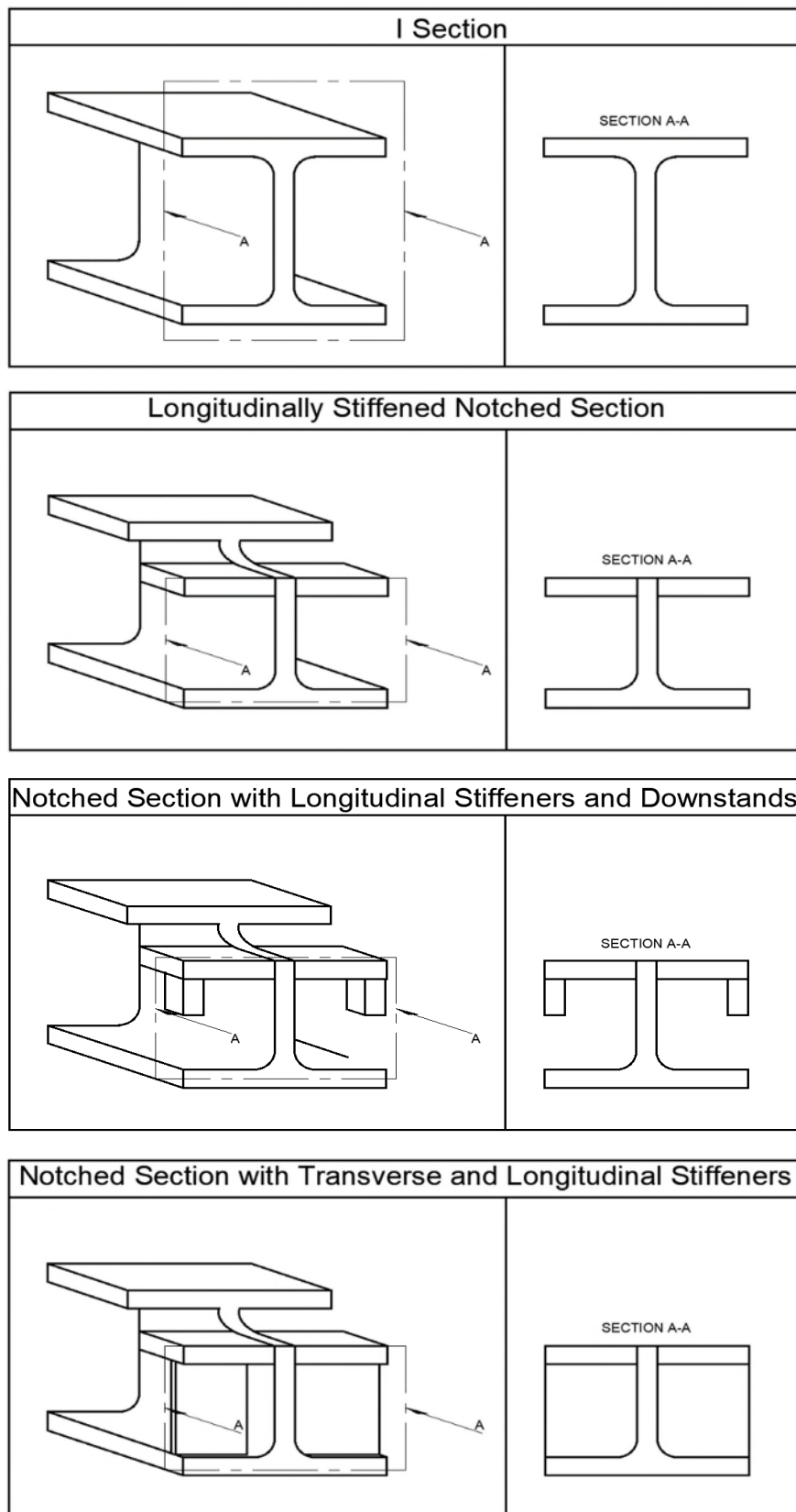


Figure 1.5: Resemblance Between a Standard I Section and Notched Section with Stiffeners

1.1 Methodology

The flow chart in **Figure 1.6** outlines the steps taken to conduct this research.

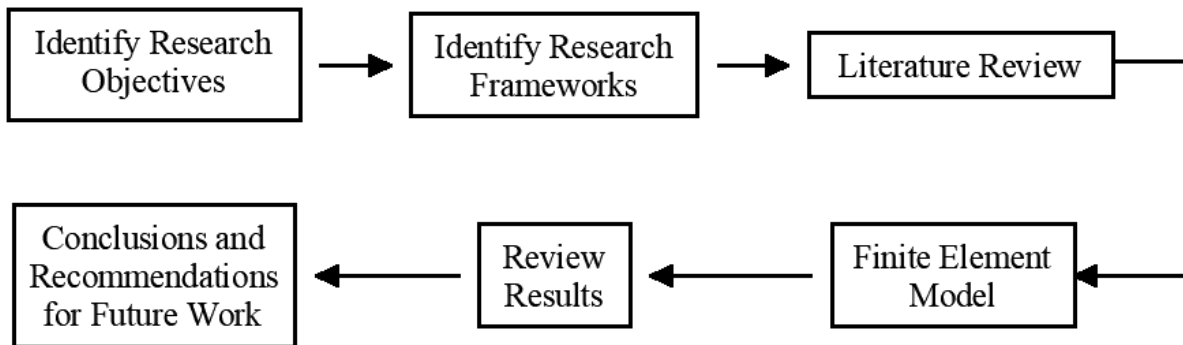


Figure 1.6: Flow Chart of Work

1.2 Research Objectives

The research question will be answered by the research objectives. The research question:

How does the addition of stiffeners in the notched region of a doubly symmetric beam affect the centroid and shear centre location for top notched steel sections undergoing elastic lateral torsional buckling, and how does the shifting shear centre relate to the critical elastic moment?

The research objectives are to:

- Investigate elastic LTB in notched beams.
- Investigate how downstands on longitudinal stiffeners on the notched portion of the beam shift the shear centre and if this movement affects the critical elastic moment of the beam.
- Use a FEM to test all proposed sections and observe trends.

1.3 Research Frameworks

The key ideas that must be investigated and how they link together are shown below in **Figure 1.7**.

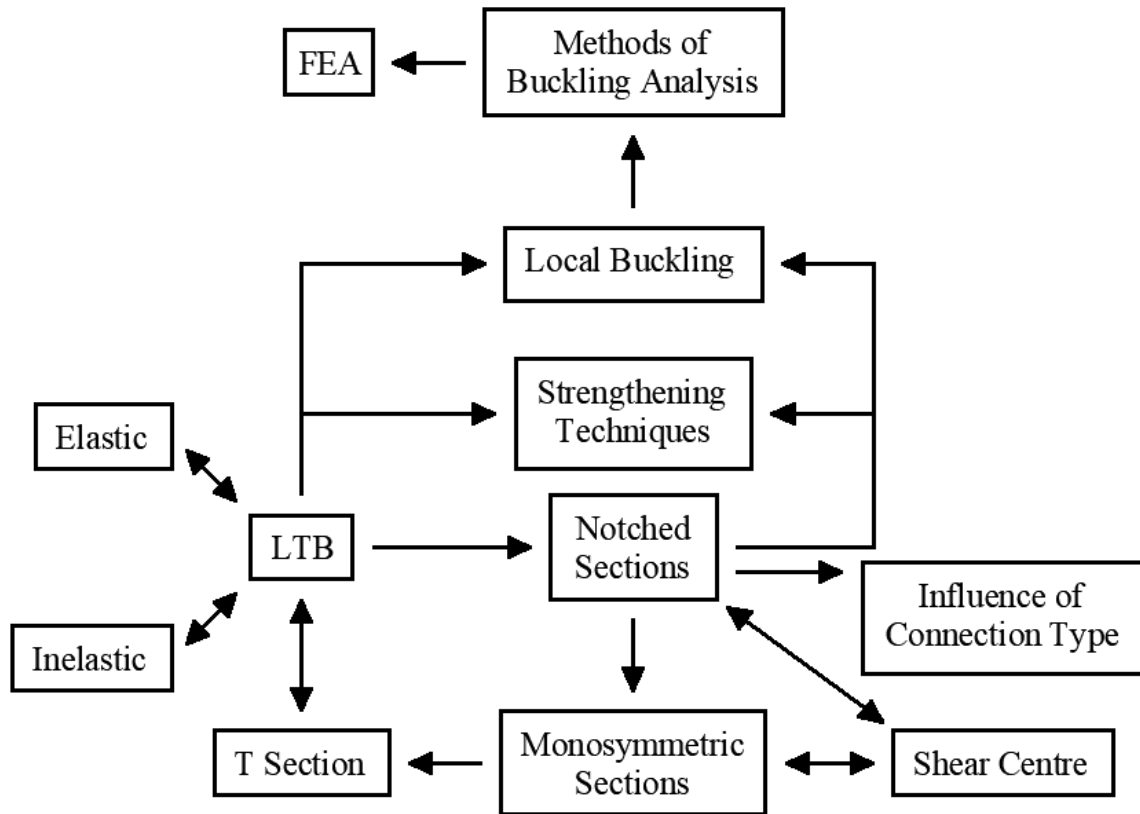


Figure 1.7: Research Framework

1.4 Parametric Study: Finite Element Model

A parametric study was conducted using the Finite Element Analysis (FEA) software Abaqus to generate the Finite Element Models (FEM). The results of the tests were used to identify trends and to understand if the shifting centroid and shear centre cause changes in the critical elastic moment. Two models were tested. Both models have the same notch and loading arrangements, but they have different spans. Model 1 has spans of 1m and 3m whereas Model 2 has 5m and 6m spans. Top notched steel sections were tested since they are more common than bottom notched sections. The single point load case is applied at an arbitrarily selected 600mm from only notched end. It was selected to test the effectiveness of the stiffening arrangements since this is close to the weakest part of the section (the notched region). It is most likely to fail by local web buckling because of the increase in stress in the notched region due to the missing restraint usually provided by the flange (Cheng & Yura, 1986). The midspan point load, applied to a doubly notched section, was selected to investigate the influence of the shifting centroid and shear centre on a different failure mode. **Figure 1.8** shows how the centroid and shear centre are shifted: using downstands and upstands of various heights. Failure by local web buckling was minimised as much as possible since LTB must be the principal failure mode in this investigation.

Section 3: Details of Beam and Stiffener Arrangement provides more details on the selection of the stiffener arrangements.

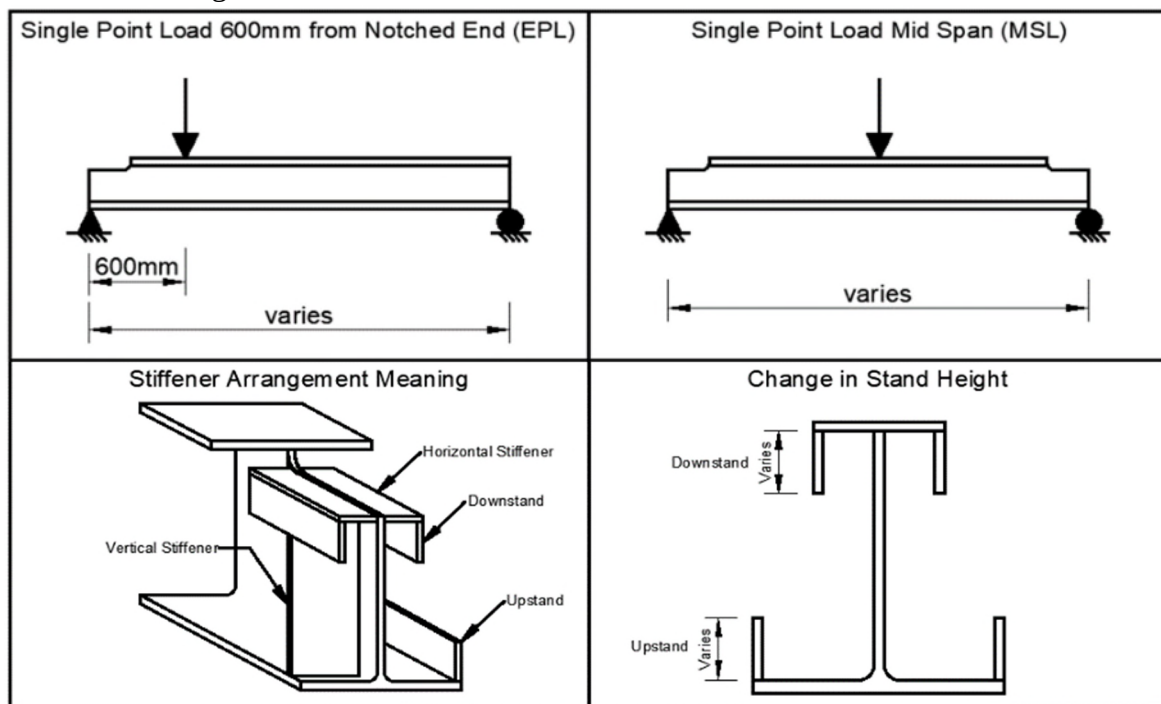


Figure 1.8: Loading, Downstand and Upstand Arrangements

The limits proposed by Ibrahim et al. (2015) were used to control local buckling. The notch depth to section depth ratio should be less than or equal to 0.25 (**Table 1.1**).

Table 1.1: Sections and Notch Depth to Section Depth Ratios

Section	Notch Depth	Section Depth	Ratio
203x133x25	50	203	0.246
305x102x25	70	305.1	0.229
533x210x82	130	528.3	0.246
203x203x46	50	203.2	0.246
305x305x97	70	307.8	0.227

Figure 1.9 shows the location of the notch depth, notch length, T section depth and section depth measurements.

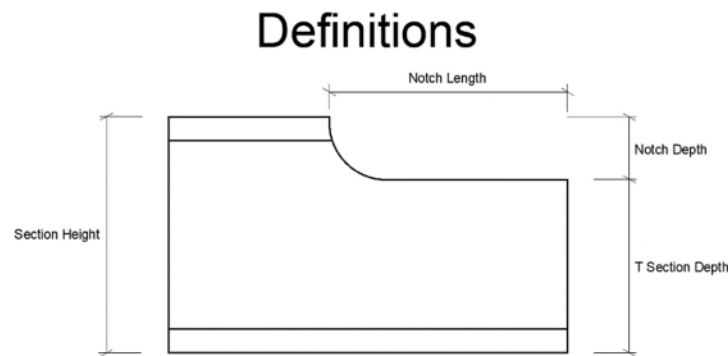


Figure 1.9: Explanation of Terms in Table 1.1

The notch length was selected to be in accordance with Ibrahim et al. (2015) who state the notch length to T section depth ratio should be less than or equal to 1 (**Table 1.2**).

Table 1.2: Notch Length to T Section Depth Ratios

Section	Notch Length	T Section Depth	Ratio
203x133x25	150	153	0.980
305x102x25	150	235.1	0.638
533x210x82	150	398.3	0.377
203x203x46	150	153.2	0.979
305x305x97	150	237.8	0.631

1.5 Workplan

The investigation started by identifying the research objectives. Thereafter, the research framework was established and investigated through the literature review. A parametric study followed in which the various loading, spans and stiffener arrangements were tested. The results were reviewed and following a discussion, conclusions were drawn. The workplan timeline can be found in *Appendix A: Workplan*.

1.6 Research Limitations

This research is limited to elastic lateral torsional buckling therefore only the effects in the elastic region will be considered and discussed. Further research should expand on this initial case by studying sections undergoing inelastic lateral torsional buckling.

Shifting the shear centre, by adding downstands or upstand, causes a change in the section properties and geometry. The proportion of the change in the critical elastic moment that can be attributed to the shear centre movement alone, and that which can be attributed to the change in section properties has not been investigated in this report. It is also acknowledged the shear centre does not move away from the line of load application but shifts vertically along the load application line.

1.7 Ethics

This dissertation aims to investigate how the addition of stiffeners affects the shear centre location for notched steel sections during LTB. The analysis was conducted using Abaqus FEA software. The aim of the research was to understand how the addition of stiffeners to notched sections influences the location of the shear centre, and if the shifting shear centre affects the stability of the section. This was achieved by stiffening the notched sections whilst tracking the location of the shear centre during LTB.

No human subjects or animals are used in this research. The ethical considerations will be the correct citation of all sources to prevent plagiarism. The sources must be cited and recorded according to the UCT Harvard referencing method.

2. Literature Review

2.1 Lateral Torsional Buckling

Lateral Torsional Buckling, as previously discussed, is an instability failure of a beam. The length of the unbraced compression flange of the beam defines the type of LTB as shown in **Figure 2.1** (Quimby, 2011). Beams are split into short, intermediate or long which depends on the sections attaining the full strength, elastic or inelastic strength (Canadian Institute of Steel Construction, 2005). The flexural strength is also limited by the section class. Class one and two sections develop the full plastic moment, Class three are restricted to the yield moment whereas Class four sections are limited by local buckling.

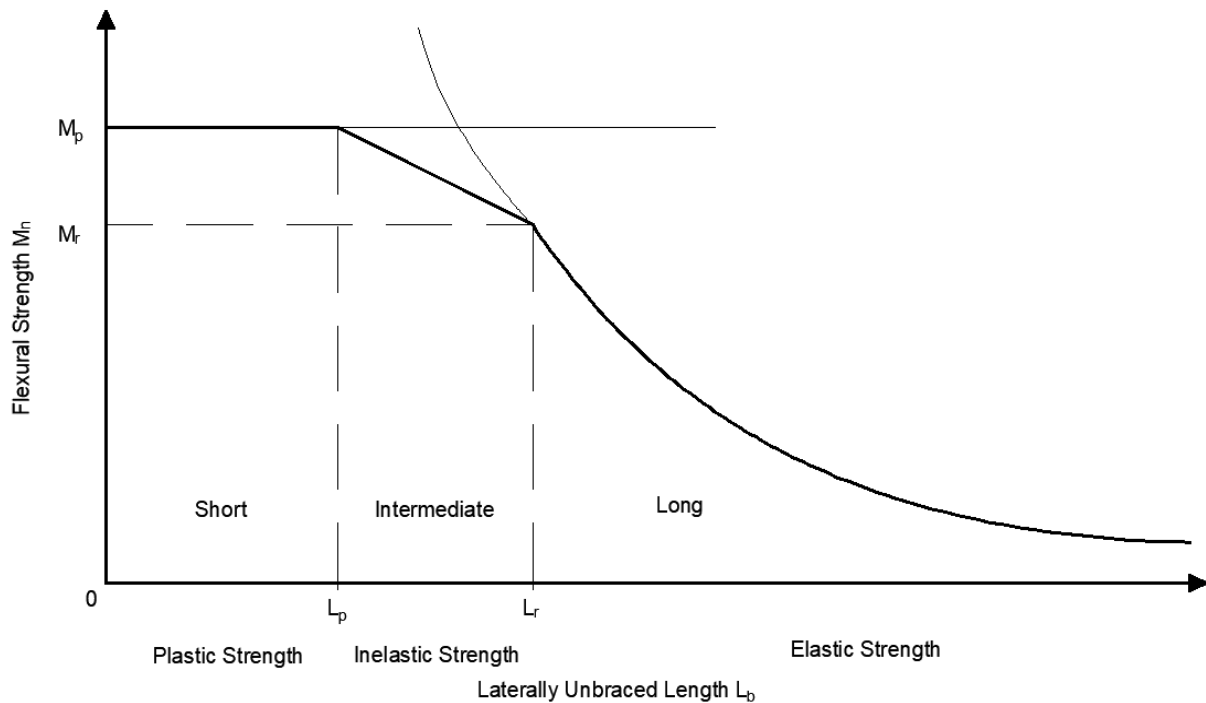


Figure 2.1: Lateral Torsional Buckling Strength and Length (Adapted from Quimby, 2011))

Figure 2.1 shows the limits of LTB regions: L_p (plastic limit) and L_r (inelastic limit). From **Figure 2.1**, if:

- $L_b \leq L_p$ – the strength of the section is governed by the plastic moment M_p .
- $L_p \leq L_b \leq L_r$ – then inelastic LTB occurs.
- $L_r < L_b$ – Elastic LTB occurs.

2.1.1 Elastic LTB

Notched (or coped) sections are normal I or H sections that have a portion of the top or bottom flange removed. This allows for web-to-web connections between members. The members are usually bolted together through a gusset plate welded to one beam. The critical moment calculation is found using elastic LTB. These equations were derived by Timoshenko and Gere (1961) and used a fork type support. Piotrowski et al. (2019) investigated an issue with these equations: in reality, the connection on site is not a perfect fork type support but an idealisation. Sophisticated support types exist which consider warping restraint and rotation about the major and minor axes which significantly influence the value of the critical elastic moment. Gizejowski (2001) (cited in Piotrowski et al. (2019)) found that the notched connection is one of these complex support types that lowers the critical resistance of the beam compared to a fork type support. Piotrowski et al. (2019) modified the equations for LTB to account for the unique behaviour and verified their results - which were found to be satisfactory from an engineering perspective - with a FEM.

2.1.2 Inelastic LTB

Galambos & Surovek (2008) state the analysis of inelastic LTB for beams are very complicated due to the complex geometry and the lateral, torsional deformations. They state there are various factors one must consider in inelastic LTB that are not considered in elastic LTB such as: initial imperfection, residual stress, tangent and reduced modulus. They recommend further reading on the subject by referring to Galambos (1968), Galambos (1998) and Trahair (1983), Trahair (1993).

Salvadori (1955) introduced a factor, C_b , the moment gradient factor, to account for loading that is not at the shear centre. Mohebbkhah (2011) investigated the effects the unbraced length and off-shear centre loading have on the moment gradient factor in the inelastic zone for I sections. This was to determine whether the moment gradient factor used in the AISC-LRFD Specification for structural steel buildings (AISC 360-05) and the Structural Stability Research Council Guidelines, which are based on elastic LTB, are appropriate. They found the factors are not accurate for centrally loaded or bottom flange point loaded I beams that buckle inelastically. The factors overestimate the actual moment capacity under moment gradient. Therefore, a simple equation was proposed for use in the inelastic zone.

2.2 Methods of Buckling Analysis

The notch influences the bending and shear stress in the notched region which is due to the discontinuity of the flange and web. This causes conventional bending and shear stress equations to yield results that are not true for the notched case. This prompted Cheng et al. (1984) and Cheng & Yura (1986) to research a modified classical plate buckling model to calculate the buckling capacity of the notched section. This was later modified by Yam et al. (2003) who presented a new calculation model based on a plate shear buckling model. Yoo & Lee (2011) explain the buckling of a plate has bending in two planes whereas in framed members bending only occurs in one plane. The ultimate load of a framed member is the critical load from buckling, whereas a plate element can carry more load after buckling (called post buckling strength). Local buckling is considered to be plate buckling; hence the terms are sometimes used interchangeably. Leonhard Euler proposed the first method to calculate the critical buckling load in 1774, the first accurate stability analysis of a column. Bryan (1891) proposed the first solution to a simply supported flat-plate stability problem. Further work on the subject was completed by Bleich (1952), Timoshenko & Gere (1961), Chajes (1974), Brush & Almroth (1975) and Allen & Bulson (1980).

Wang & Kitipornchai (1991) provided a great overview of the numerical techniques used for the elastic buckling load of structural members. Numerical techniques are used for cases in which the exact analytical solutions are tedious or impossible to obtain. Wang & Kitipornchai (1991) present a flow chart which shows the relationship between the methods used in buckling analysis (**Figure 2.2**). There are two approaches: static-kinematic and work-energy. The former utilises differential equations whereas the latter uses energy conditions via virtual work. *Section 2.2.1 Static-Kinematic* to *Section 2.2.2 Work-Energy* draws on the work of Wang & Kitipornchai (1991) to outline the major steps in each method.

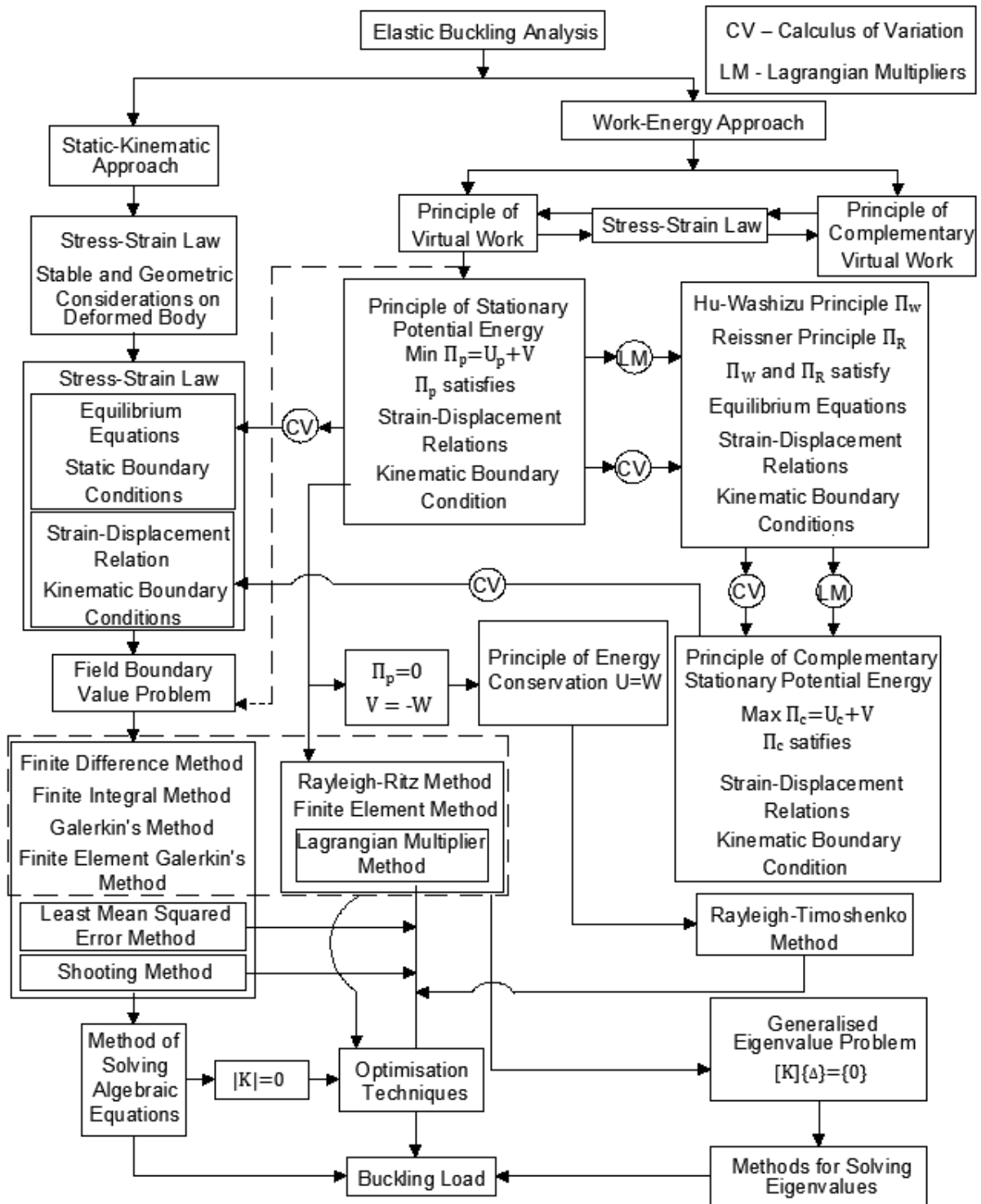


Figure 2.2: Relationships Between Methods of Buckling Analysis (Wang & Kitipornchai, 1991)

2.2.1 Static-Kinematic

2.2.1.1 Finite Difference Method

The Finite Difference Method converts differential equations into algebraic equations by replacing derivatives of the dependent variables using suitable finite differences at chosen nodal points. It can be used to solve partial differential equations; therefore also multi-dimensional structures (Kreyszig, 1983; Salvadori & Baron, 1961; Wang & Kitipornchai, 1991).

2.2.1.2 Finite Integral Method

The Finite Integral Method is the exact opposite of the Finite Difference Method because it transforms differential equations into integral equations in the highest derivative of the dependent variable. The integral equation is replaced with a finite set of algebraic equations using finite representations of integral operators. Using the same techniques as in the finite difference method, the algebraic equations are solved. The results of this method are more accurate than the Finite Difference Method as integration is used whereas differentiation is used in the Finite Difference Method (Brown & Trahair, 1968; Wang & Kitipornchai, 1991).

2.2.1.3 Least Mean Squared Error Method

Unlike the previous two methods, the Least Mean Squared Error Method approximates the solution and aims to minimise the error in the approximation (Duncan, 1937; Wang & Kitipornchai, 1991).

2.2.1.4 Galerkin Method

Like the Least Mean Squared Method, the Galerkin Method finds an approximate solution then minimises the error. Unlike the previous method, n coefficients are chosen such that n unique weighted means of the error, for the selected interval, shall be zero. The Finite Element Galerkin's Method is a special case in which piecewise polynomials are used instead of linear combinations of functions (Duncan, 1937; Wang & Kitipornchai, 1991).

2.2.1.5 Shooting Method

The Shooting Method does not require matrix manipulation or inversion to calculate the eigenfunction (the buckling mode and mode shape). Instead, the method converts the N^{th} order differential equations into first order differential equations which are then integrated using the Runge-Kutta algorithm. The results are very accurate as discretisation is not required, but the method is restricted to situations containing ordinary differential equations (Keller, 1968; Na, 1979; Wang & Kitipornchai, 1991).

2.2.2 Work-Energy

2.2.2.1 Rayleigh-Timoshenko

This method is based on the conservation of energy: the work done externally results in strain energy in the member (strain energy U = external work W). The buckling load is found by minimising the Rayleigh Quotient with respect to the displacement function y (Timoshenko & Gere, 1961; Wang & Kitipornchai, 1991).

$$\frac{\min \lambda}{y} = \frac{U}{W^*} \quad (2-1)$$

Where:

- λ – load parameter
- W^* – work done per unit load

2.2.2.2 Rayleigh-Ritz Method

Unlike the Rayleigh-Timoshenko method, the Rayleigh-Ritz method uses the principle of stationary potential energy. It is a more general principle of virtual work and states: if the virtual displacement of a body is zero, the total virtual work done by all internal or external forces will be in equilibrium. Consequently, neutral equilibrium results in a minimum state of total potential energy Π .

$$\partial \Pi = \partial(U + V) = 0 \quad (2-2a)$$

$$\frac{\partial \Pi}{\partial c_i} = 0 \quad i = 1, 2, 3, \dots, n \quad (2-2b)$$

Where:

- ∂ – change in quantity due to virtual displacement
- U – strain energy
- V – potential energy of the loads (the negative of value of the work done)

The displacement function is approximated using a linear combination of functions which must satisfy the kinematic boundary conditions. Through equation (2-2b) a system of linear algebraic homogenous equations is generated which are solved to find the buckling load (Ritz (1909) as cited in Wang & Kitipornchai (1991)).

2.2.2.3 Finite Element Method

The Finite Element Method involves sectioning a given domain into small elements connected at nodal points. By combining the governing equations of each element for the specified domain, an approximate solution is obtained. The strength of the Finite Element Method is it can calculate stress in complex shell and plate structures that would be complicated if classical plate and shell theories are used. Nonlinear analysis such as buckling can be very complicated because of large displacements. The stiffness matrix must also be redefined at each load increment because the structure has been displaced (Jawad, 2017).

The elemental stiffness matrix $[k_c]$ can be written from the total potential expression Π_e :

$$\Pi_e = (U_e + V_e) = \frac{1}{2} \{d\}^T [k_c] \{d\} \quad (2-3a)$$

Where $\{d\}$ is the element buckling nodal displacements vector. Using the compatibility relation:

$$\{d\} = [T] \{\Delta\} \quad (2-3b)$$

Where:

- $[T]$ – Transformation matrix
- $\{\Delta\}$ – Structure joint displacements vector

Therefore, equation (2-3a) can be written as:

$$\Pi_e = \frac{1}{2} \{\Delta\}^T [K_e] \{\Delta\} \quad (2-3c)$$

Where:

- $[K_e] = [T]^T [k_c] [T]$

Taking the sum for all the elements in the structure:

$$\Pi = \sum \Pi_e = \frac{1}{2} \{\Delta\}^T [K_T] \{\Delta\} \quad (2-3d)$$

Where:

- $[K_T] = \sum [K_e]$

The energy is conserved during buckling from a straight position to an adjacent out-of-plane equilibrium position. Consequently, the total energy change is zero (equation (2-3e)). Using an eigenvalue method on equation (2-3e) the buckling load can be calculated.

$$\Pi = \frac{1}{2} \{\Delta\}^T [K_T] \{\Delta\} = 0 \quad (2-3e)$$

The Finite Element Method is based off the Rayleigh-Ritz method. It is an improvement because it can cope well with rapidly changing displacement functions. For cases in which Ritz functions can approximate exact displacement functions closely; the resulting formulae are easier to work with and more accurate than those obtained using the Finite Element Method.

A special case of the Finite Element Method is the Finite Strip Method; named because its members are elongated. The displacement functions of the longer side can be approximated using trigonometric series instead of polynomials (Bathe & Wilson, 1976; Wang & Kitipornchai, 1991).

The Finite Element Method is used in FEA software. They can perform analyses of stability of members through eigenvalue extraction methods applied to the global system stiffness matrix. This approach is referred to as linearised eigenvalue buckling analysis (or buckling or eigenvalue buckling). This method is not computationally expensive because the eigenvalue buckling method takes one or two points on the equilibrium path, whereas the more general incremental analysis traces the entire non-linear equilibrium path.

Linearised eigenvalue buckling is used in stability research because it is very useful for including initial or seed imperfection for beam or frame analysis. This is a more detailed incremental nonlinear FEA (Ziemian, 2010).

2.2.2.4 Lagrangian Multiplier Method

A variant of the Rayleigh-Ritz method, the Lagrangian Multiplier Method requires the approximated function to be constrained such that the geometric boundary conditions are satisfied instead of satisfying the kinematic boundary conditions.

2.3 Notched Sections

Notched or “coped” steel sections have been used extensively in industry. They solve the problem of conflicting flanges at connections because - as previously discussed - a portion of the top, bottom or both flanges must be removed to enable web to web connection between the sections. The notch can be curved or at a right angle and results in the notched portion of the beam resembling a T section (**Figure 2.3**).

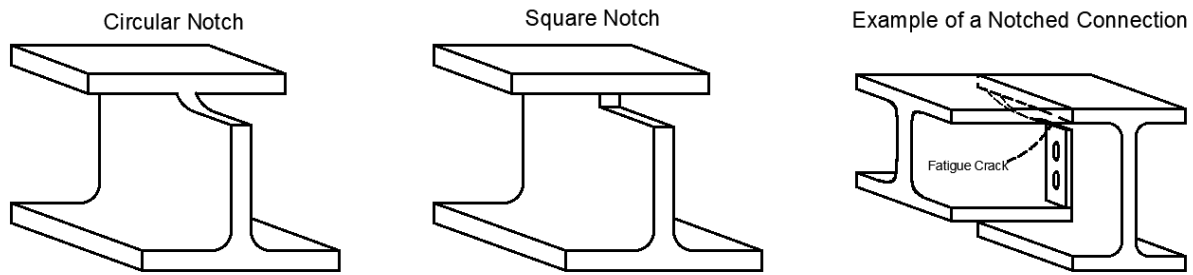


Figure 2.3: Types of Notched Sections

The discontinuity in the flanges for notched sections cause local failure at the notched ends to be the most likely failure mode. The removal of part of the section reduces the bending and torsional stiffness thus the upper notched edge can experience local web buckling and stress concentrating at the notch corner. This occurs if the connection at the notched end has insignificant rotational rigidity, consequently it can be taken as a pinned connection. Shear block failure is also possible because of the lack of restraint provided by the top flange. Tension at the notched connection is possible through a negative moment which is possible if the connection at the notch has a large rotational rigidity. This tension could cause fatigue cracks (Yam et al., 2014) as shown in **Figure 2.3**.

A stiffened notched section can fail at other locations of the beam. Yam et al. (2011) found in their experiments some sections failed due to flexural yielding of the full beam at the location of maximum bending after which local buckling occurred at the loading position. Ibrahim et al. (2015) found most test specimens failed by inelastic lateral torsional buckling with local buckling in the notched region.

Inelastic LTB can occur due to the geometric discontinuity which leads to a high stress concentration at the notch corner leading to local buckling after yielding. For bolted connections, shear block failure can occur in bolted top notched sections due to the lack of restraint provided by the top flange.

2.3.1 Local Web Buckling

Local web buckling occurs in notched sections due to the missing flange which normally provides some restraint. The discontinuity of the flange and web at the notch induces stress at the notch which can be quite significant depending on the type of notch. For example: square notches concentrate stress at the notch corner whereas circular notches distribute the stress evenly. This increase in stress in the notched zone could cause localised yielding followed by local buckling (inelastic local buckling). For thin webs, the notched zone may fail by elastic local buckling which could be at a load significantly lower than the yield stress (Cheng & Yura, 1986).

Cheng et al. (1984) and Cheng & Yura (1986) performed the first FEA and parametric studies on beam sections that have their top flange notched (top and bottom notched sections were researched in 1984). Through both their studies, they observed lateral movement was more significant at the top flange versus the bottom flange when local buckling occurred. An observation into the parametric study, completed by Cheng et al. (1984), shows a routine check of the bending and shear stress will always result in a conservative lower bound. Therefore, checking inelastic buckling is not necessary if flexural, shear yielding, and elastic buckling are checked. Cheng & Yura (1986) furthered their (1984) paper by testing sections that were deliberately designed to fail by elastic and inelastic buckling. They found elastic local web buckling occurred before yielding because an almost linear load-vertical deflection response until local web buckling was observed for the elastically designed sections. For the sections designed to fail by inelastic buckling, they observed a non-linear load-vertical deflection response, therefore yielding occurred before buckling. Strain gauges also showed high stress concentrations close to the notch.

Cheng et al. (1988) and Cheng & Yura (1988) proposed interaction equations that accounted for the effects of notch length, depth and their relationship with local web buckling of braced notched sections. Lam et al. (2000) noticed these equations may not account for local web buckling at the notch which occur in sections with a small notch length to web depth ratio. They also noticed the equations yield non-conservative results for symmetrical loading because the initial study only considered a mid-span point load. Therefore, they proceeded with an analytical study of notched sections to modify the interaction equations which are shown in equation (2-5a) and equation (2-5b). Note that cope is the same as notch.

Doubly notched sections are (2-5a) and one notched end are (2-5b):

$$\left(\frac{1}{M_{uncope}}\right) + \left(\frac{1}{\left(\frac{L_b}{2c}\right) M_{cope}}\right) = \left(\frac{1}{M_{cr}}\right) \quad (2-5a)$$

$$\left(\frac{1}{M_{uncope}}\right)^2 + \left(\frac{1}{\left(\frac{L_b}{c}\right) M_{cope}}\right)^2 = \left(\frac{1}{M_{cr}}\right)^2 \quad (2-5b)$$

Where:

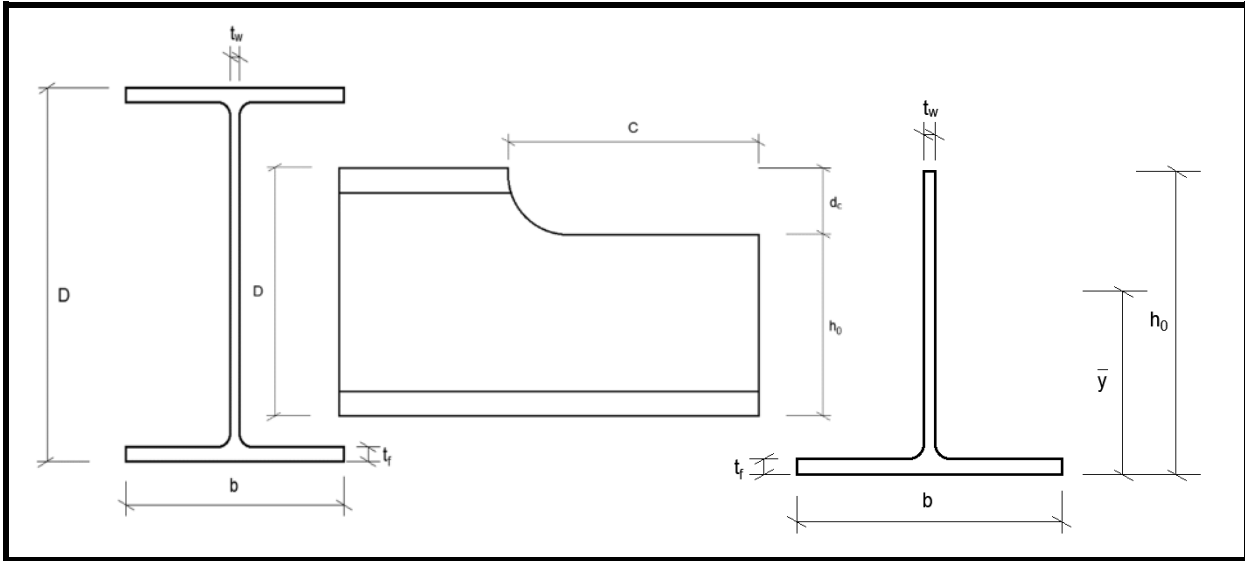
- L_b – unbraced length
- c – notch length
- M_{cr} – elastic lateral torsional buckling moment of the notched I section
- M_{uncope} – elastic lateral torsional buckling moment of the un-notched beam
- M_{cope} – elastic lateral torsional buckling moment of the T beam

See **Table 2.1** for explanation of M_{cope} .

Table 2.1: Explanation of Symbols

Equation 2.5c	
$M_{cope} = \left(\frac{\pi}{2c}\right) \sqrt{EI_y GJ} \left(\sqrt{1 + \left(\frac{\pi \gamma_m}{2}\right)^2} + \frac{\pi \gamma_m}{2} \right)$	
Symbol	Meaning
E	modulus of elasticity of steel
I_y	moment of inertia of an I section about the y-y axis
G	shear modulus of elasticity of steel
J	torsional constant of cross section
γ_m	monosymmetric parameter of tee-section lateral torsional buckling
Equation 2.5d	
$\gamma_m = \left(\frac{\beta_x}{2c}\right) \sqrt{\frac{EI_y}{GJ}}$	
Equation 2-5e	
$\beta_x = -\frac{1}{I_x} \left\{ \frac{t_w}{4} \left[(h_0 - \bar{y})^4 - (\bar{y} - t_f)^4 \right] - b t_f \left(\bar{y} - \frac{t_f}{2} \right) \left[\frac{b^2}{12} + \left(\bar{y} - \frac{t_f}{2} \right)^2 \right] \right\} - 2 \left(\bar{y} - \frac{t_f}{2} \right)$	
Symbol	Meaning
I_x	moment of inertia of I section about x-x axis
t_w	web thickness
h_0	depth of beam at coped or notched region = <i>section depth – notch depth</i>
$\bar{y}$	distance between neutral axis of the coped or notched section and the extreme fibre of the flange
t_f	thickness of the flange
b	flange width

Table 2.2: Explanation of Symbols (Continued)



Lam et al. (2000) found cases in which the local web buckling capacity may be smaller than the elastic LTB capacity if the notch depth to web depth (the T section depth) ratio is small. Through FEA, Lam et al. (2000) found for the cases where the notch length to T section depth ratio being less than 2.5: it would be more appropriate to use the local web buckling requirements in the interaction equations by replacing M_{cope} with M_{lwb} (local web buckling capacity of tee-sections) shown in equations (2-5f) and (2-5g). The only difference between equations (2-5a), (2-5b) and (2-5f) and (2-5g) is the replacement of M_{cope} with M_{lwb} .

$$\left(\frac{1}{M_{uncope}}\right) + \left(\frac{1}{\left(\frac{L_b}{2C}\right) M_{lwb}}\right) = \left(\frac{1}{M_{cr}}\right) \quad (2-5f)$$

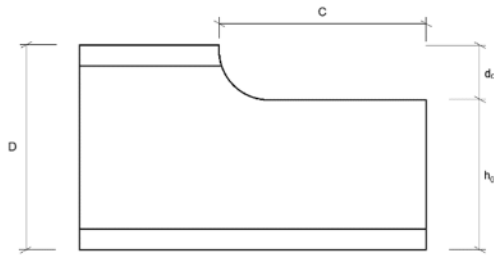
$$\left(\frac{1}{M_{uncope}}\right)^2 + \left(\frac{1}{\left(\frac{L_b}{C}\right) M_{lwb}}\right)^2 = \left(\frac{1}{M_{cr}}\right)^2 \quad (2-5g)$$

Where:

- $M_{lwb} = \frac{\sigma_{cr} I_{x-x}}{(h_0 - \bar{y})}$
- σ_{cr} – critical stress of the plate
- I_{x-x} – moment of inertia of the tee-section
- h_0 – web depth
- $\bar{y}$ – distance between extreme fibre of the flange and neutral axis

Lam et al. (2000) key findings regarding the conditions on when to use the equations are summarised in **Table 2.3**. They also found for short spans with small $\frac{c}{h_0}$ ratios: yielding should also be checked and compared with the elastic lateral torsional buckling capacity of notched I beams with the lower value being used.

Table 2.3: Summary of Conditions and Findings (Adapted from Lam et al. (2000))

Condition	Action
$\frac{c}{h_0} \leq 2.5$	For the unbraced single point load midspan and symmetric two point load use equation (2-5h): $\left(\frac{1}{M_{uncope}}\right) + \left(\frac{1}{\left(\frac{xL_b}{c}\right) M_{lwb}}\right) = \left(\frac{1}{M_{cr}}\right) \quad (2-5h)$ Where the loading is placed xL_b from the two end supports Braced case with one end notched equation (2-5g) is recommended
$\frac{c}{h_0} < 0.2 \left(\frac{d_c}{D} \leq 0.2\right)$	Unbraced Case
$\frac{c}{h_0} < 0.1 \left(\frac{d_c}{D} \leq 0.1\right)$	Braced Case
$\frac{c}{h_0} > 2.5$	Overall LTB should be checked against local web buckling and the lower value be taken as the local web buckling capacity of the notched portion
Short spans with small notch lengths	Yielding checked and compared with elastic LTB of notched I beam, and lower value used
Explanation of symbols	

Yam et al. (2003) presented a new calculation model based on a plate shear buckling model which would better represent the buckling behaviour of the notched region. They realised this when they noticed the buckling line in the web was often orientated at 45° , especially in sections which have a small notch length to section height ratio. This new model builds upon the work by Cheng et al. (1984).

Yam et al. (2014) state that under certain conditions (for example if the ratio of height between the flanges and the web thickness is low and the stress concentration at the notch is high) the notched beam can fail by inelastic local buckling which is the same result Cheng & Yura (1986) found. This was found for elastic lateral torsional buckling by Lam et al. (2000).

There is a relationship between the depth of the notch and the buckling capacity of the web: an increase in the depth of the notch will reduce the web buckling capacity (Yam et al., 2011). Ibrahim et al. (2015) also found this result but noted the notch length also influences the local buckling capacity. They suggested limits for notch depth and length that can control the buckling resistance. They state the notch length to T section depth should be less than or equal to 1 and the notch depth to section depth should be less than or equal to 0.25. They found increasing the notch depth for shorter notch lengths affects the lateral buckling capacity more than longer lengths which could be similar behaviour to slender columns. Both Yam et al. (2011) and Ibrahim et al. (2015) found the notch depth and length significantly influence the capacity of the notched section.

Aalberg (2015b) presented a paper in which an experimental and numerical parametric study on the capacity of notched beam ends was investigated, with a focus on local web buckling. Aalberg (2015b) proposed a new design formula for top notched beams supported at the bottom flange that determines the capacity of the section. This was done by reducing the capacity of an equivalent un-notched section based on the geometry of the notch. Aalberg (2015b) noticed similar behaviour to previous investigations by Yam et al. (2011) and Ibrahim et al. (2015). Namely the relationship between the notch depth and extent of the buckle: the shorter notch depths resulted in large buckle extensions whereas deeper notches resulted in more localised buckling. Aalberg (2015b) also found the relationship between the notch depth (or height) and notch length as other authors found.

Aalberg (2015b) included a comparison between the design models proposed by Cheng & Yura (1986) and Yam et al. (2003).

Both authors used regular web connections therefore a discrepancy existed between the boundary conditions of the model and the experiments conducted by Aalberg (2015b). The mismatch was explained to some extent through the FEA. Aalberg (2015b) found the shear model proposed by Yam et al. (2003) yielded the best buckling predictions. This agrees with the results of Yam et al. (2003) who state their model is an improvement on the work of Cheng & Yura (1986).

2.3.2 Common Strengthening Techniques

Aalberg & Larsen (2001) found that for the maximum practical notch dimensions, the capacity of the section would be reduced by half. This prompted renewed research into the arrangement and type of stiffeners for notched sections. Stiffeners are usually used for sections that have a tall web such as girders and take three common arrangements: transverse, longitudinal and combined transverse and longitudinal (**Figure 2.4**). The stiffeners brace the web against buckling that can occur at the supports or at the point of load application.

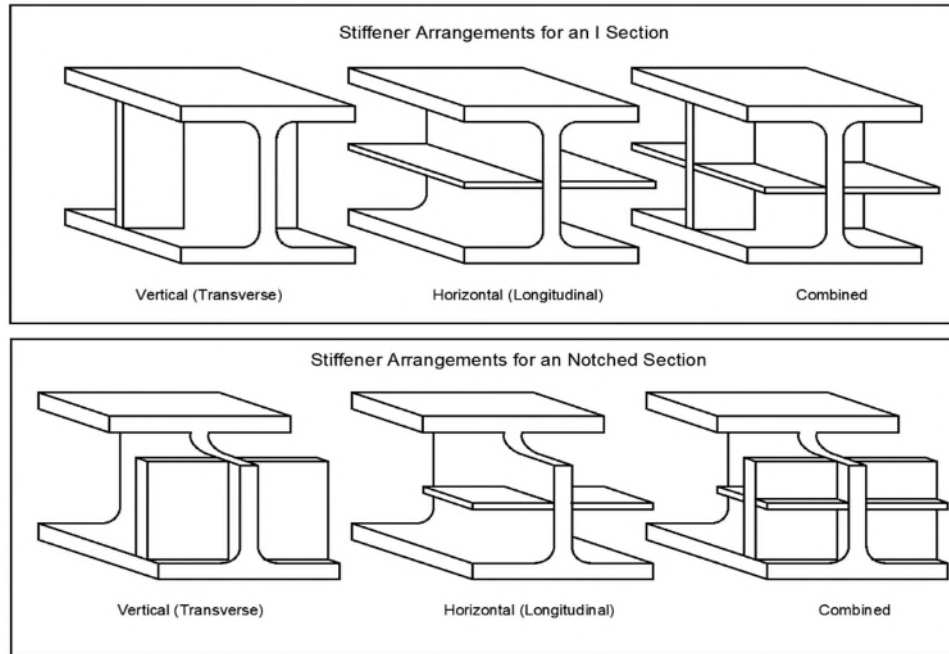


Figure 2.4: Stiffener Arrangements

Yam et al. (2011) conducted 10 experimental tests on reinforced notched I sections to investigate their strength and behaviour. They investigated numerous parameters: longitudinal and transverse stiffener lengths, individual and combined longitudinal, transverse stiffeners as well as the relationship between notch length and notch depth. Their work expands on previous research completed by Cheng et al. (1984), Lam et al. (2006) and Yam et al. (2007). As in previous research, the sections without stiffeners failed at the notch by local web buckling. By introducing longitudinal stiffeners, the section failed by flexural yielding at the location of maximum bending, then failure by web crippling at the notch between the stiffener and the top flange of the beam. Combined longitudinal and transverse stiffeners caused the section to fail in the same manner as longitudinal stiffeners only, but instead of web crippling at the notch, flange local buckling occurred near the load location because the combination of the stiffeners prevents lateral movement of the web. They also found an increase in the depth of the notch decreases the buckling capacity of the section. However, stiffeners can correct this reduction (**Figure 2.5** and **Figure 2.6**).

Yam & Chung (2013) expanded on Yam et al. (2011) through a numerical study using the finite element method. Their FEM was validated by comparing the results to the experimental study conducted by Yam et al. (2011) with good correlation (coefficient of variation was 0.058). Yam & Chung (2013) through a parametric study investigated the effects of longitudinal stiffener length, combined longitudinal and transverse stiffeners, notch length and depth. All the stiffened members did not experience flexural failure at the notch. However, they did experience shear yield failure and flexural failure near the location of loading.

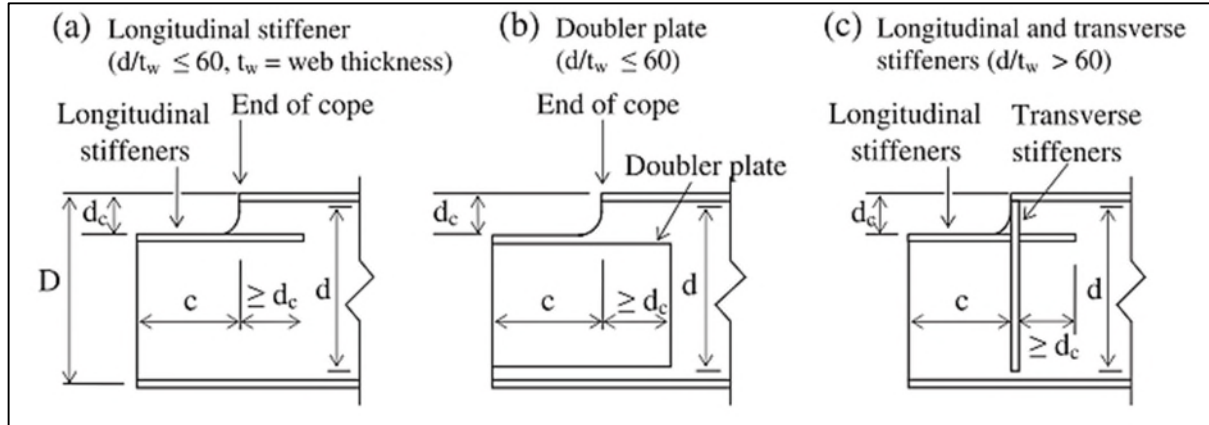


Figure 2.5: Reinforcement Details (Yam & Chung, 2013)

They found by increasing the notch depth, therefore increasing the gap (height of web) between the longitudinal stiffener and the top flange, causes web crippling to be critical. As in Yam et al. (2011), longitudinally reinforced beams usually failed by web crippling due to the movement of the longitudinal stiffeners (the longitudinal stiffener moves which causes the web of the notched portion to move with it thus resulting in web crippling). They found the ratio of the maximum observed moment to the plastic moment (M_{max}/M_p) decreases with an increasing notch depth (d_c) to beam depth (D) ratio regardless of the notch length (c) to D ratio and the type of stiffener used.

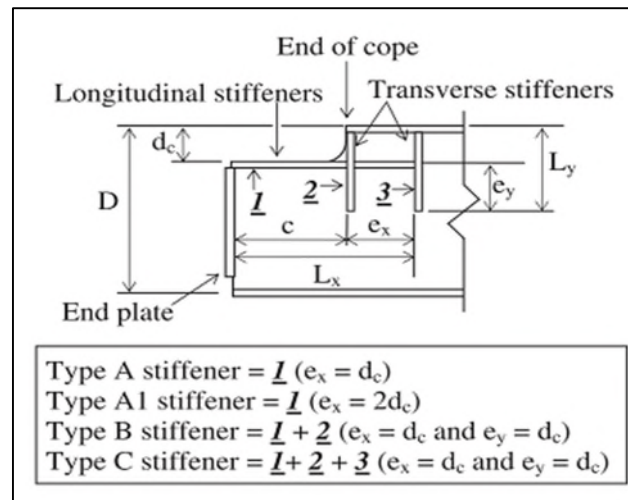


Figure 2.6: Types of Reinforcement (Yam & Chung, 2013)

The ratio of the maximum observed shear force to the shear yield capacity ($V_{co}/V_{co,y}$) and M_{max}/M_p in general decrease with an increasing c/D ratio. For sections with a web height to web thickness ratio (d/t_w) less than or equal to 52.7, using stiffener type A (**Figure 2.6**) will enable most of the tested sections to attain either their M_p or $V_{co,y}$ (depending on the notch details). Type A1 is more suitable for all the tested sections. In general, type A1 and C are the recommended stiffener types.

Yam & Chung (2013) found notch length affects the capacity but the depth has more of an effect. An increase in the notch length/section depth (c/D) ratio decrease the ratio of the maximum bending moment to the plastic moment of the section (M_{max}/M_p) and the ratio of the maximum shear force in the notched section to the shear yield capacity of the notched section ($V_{co}/V_{co,y}$). Consequently, when the beam fails due to web crippling from rigid body movement of the longitudinal stiffeners, the ultimate capacity of the section is reduced with an increase in the notch length. This is different to cases where the failure mode of the beam is due to flange local web buckling near the loading position since the effect of notch length on the capacity of the beam is insignificant.

Ibrahim et al. (2015) acknowledged the work completed by Cheng et al. (1984) and Yam & Chung (2013) but states their work did not consider inelastic LTB of stiffened notched I sections. Therefore, they proceeded to conduct an experimental study of notched I beams under two symmetrical point loads to investigate the influence of the notch on the type of failure. The general failure of the beams was by inelastic LTB with local buckling at the notch. They found the buckling starts as LTB followed by an inelastic buckling zone which forms at the notch corner.

They found each increase in applied load increased local buckling at the notch and LTB of the beam until the beam failed. Ibrahim et al. (2015) and Yam & Chung (2013) studied a square cut notch whereas Yam et al. (2011) studied a circular notch. A square notch concentrated stress at the corner of the notch which exceeded yield stress in all beams tested by Ibrahim et al. (2015). They found this stress concentration was the main reason for local web buckling.

From their experimental study, Ibrahim et al. (2015) identified three main buckling modes: local web buckling at the notched zone, web distortional buckling and lateral torsional buckling. Just as Yam & Chung (2013) found, Ibrahim et al. (2015) found an increase in notch length decreases the buckling capacity of the member.

In their parametric study, they tested the reinforcement strategy outlined by Cheng et al. (1984) and Yam et al. (2007) shown in **Figure 2.7**. The introduction of horizontal stiffeners prevented local buckling thus the beam failed by LTB. Increasing the horizontal stiffener length eliminates the web crippling and local buckling zone. Vertical stiffeners appeared to prevent lateral movement of the web, but it was more effective when used with horizontal stiffeners.

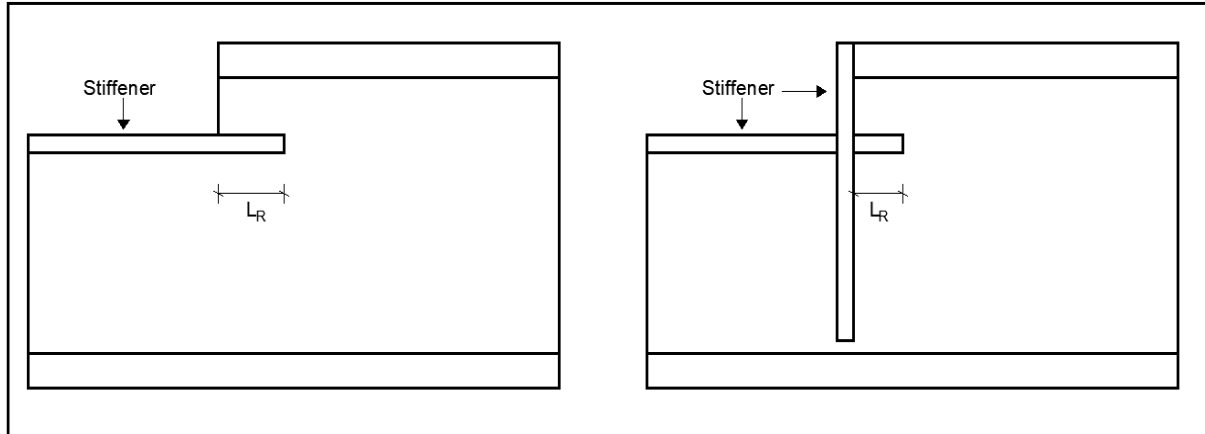


Figure 2.7: Reinforcement Strategy (Ibrahim et al., 2015)

2.3.3 Alternative Strengthening Technique: Corrugated Web

An alternative stiffening technique commonly used in girders is to use a corrugated web instead of a flat sheet web (**Figure 2.8**). The advantage of corrugated sheets is they reduce the total tonnage of the structure resulting in longer spans and less load on the foundation (Ibrahim et al., 2019). The main advantage is the increase in beam stability against LTB.

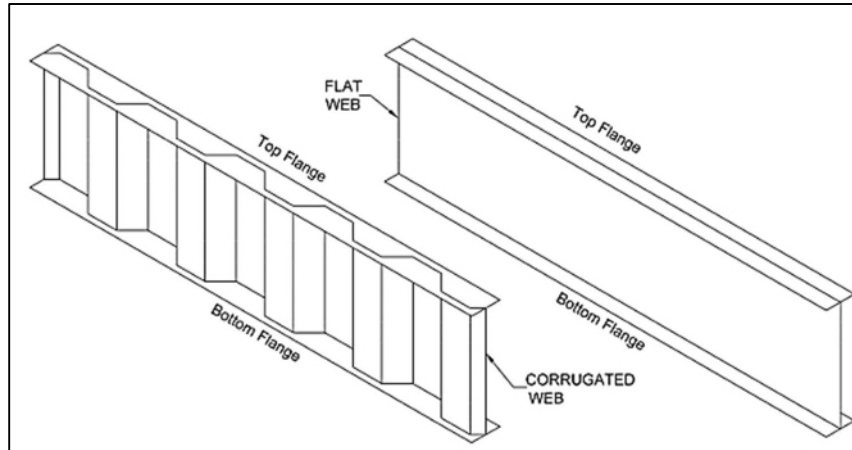


Figure 2.8: Corrugated I Section (Ibrahim et al., 2019)

Corrugated webs do not need transverse or longitudinal stiffeners which are used in flat webbed sections. They are expensive to fabricate and reduce the service fatigue life of the beam. Ibrahim et al. (2019) expanded on previous work completed by examining the buckling capacity of corrugated web beams with notches on either end using FEA. They also proposed stiffening techniques based off a parametric study. They found the LTB capacity is significantly reduced by the notch geometry (namely the depth and length just as for sections with flat webs). Other factors, some unique to corrugated webs, also influence the LTB capacity. The corrugation depth was found not to influence LTB capacity. In general the notched beams with corrugated webs increased its resistance to lateral torsional buckling by 15% when compared with notched flat web beams.

The location of the failure remains in the notched region. However, the web now fails by lateral elongation which causes the web to flatten. This is due to the flange not restraining the web in the notched region. Ibrahim et al. (2019) proposed three stiffening techniques (**Figure 2.9**), which all increase the beams resistance to LTB, to counter this failure. Using a flat web in the notched zone with a full depth web stiffener eliminates the lateral and vertical deformations but careful attention must be paid to the web configurations.

By testing full depth web stiffeners at one end only the authors were able to compare the change in capacity due to the introduction of the stiffeners. They found it increased the resistance to lateral and vertical deformations and was easy to install. The welded flange along the notch profile was found to be the best solution as it eliminated the movement deformations and effectively doubled the buckling capacity. Its drawback is the complexity of the manufacturing and installation of the beam.

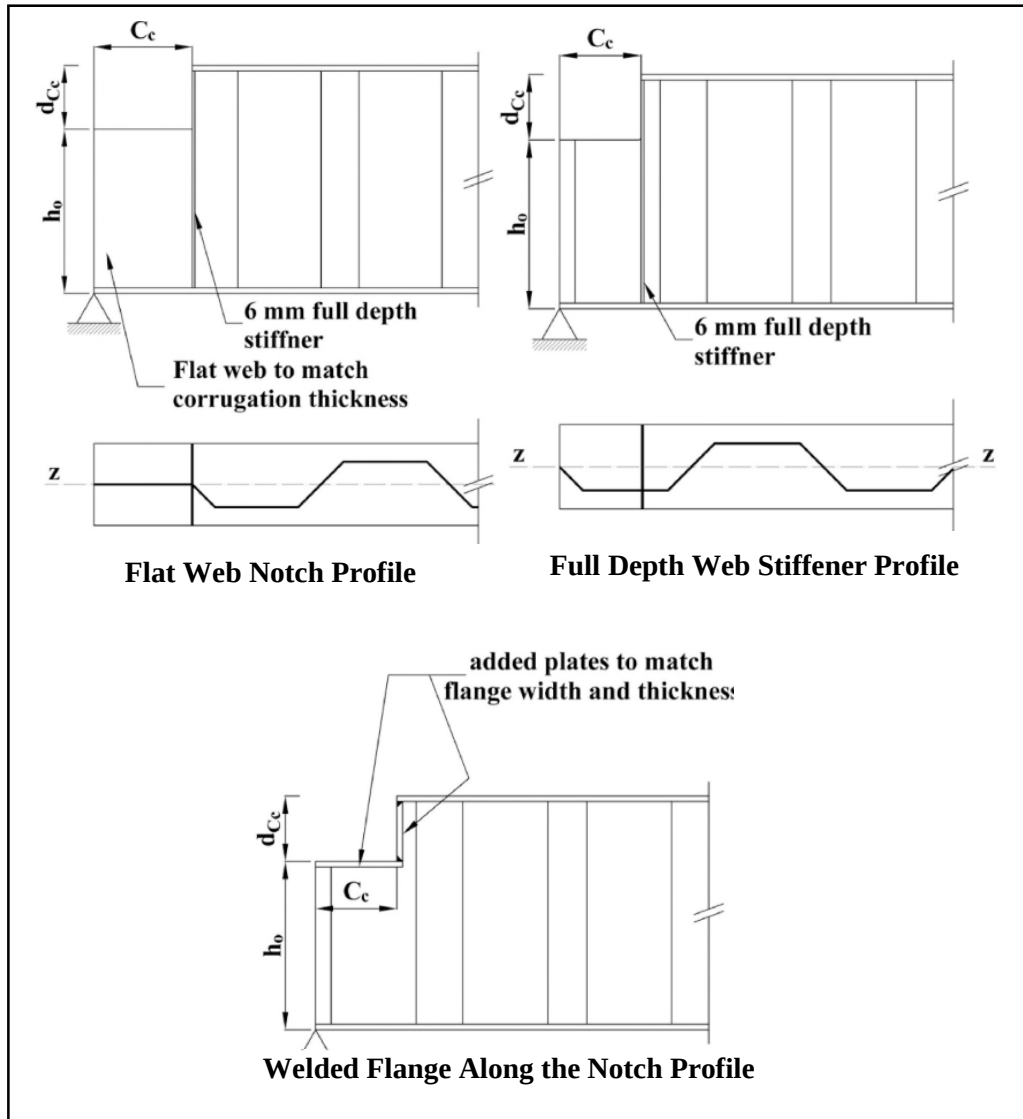


Figure 2.9: Corrugated Web Stiffening Techniques (Ibrahim et al., 2019)

2.3.4 Alternative Stiffening Technique: Bracing, Stepping

LTB can be prevented by laterally restraining the member or through torsional restraint as this will prevent the member from twisting and moving out-of-plane. An example of torsional bracing can be a rectangular plate that is secured at one end and welded or bolted to a stiffener which braces the web at the other as shown in **Figure 2.10**.

Mohammadi et al. (2016) investigated if using torsional stiffeners can increase the LTB capacity of a monosymmetric I section and derived an equation that includes the influence of the torsional bracing. Through a parametric study, they found web depth has a major influence on the dimensionless bracing stiffness requirement (the ratio of the bracing stiffness requirement to the torsional stiffness of the beam). They found by increasing the section height, the dimensionless bracing requirement increases thus increasing the need for torsional bracing.

Through Cheng (1993), it is known that notched sections have a reduced lateral buckling strength especially for long notch lengths or short spanning beams (because a greater percentage of the beam is notched).

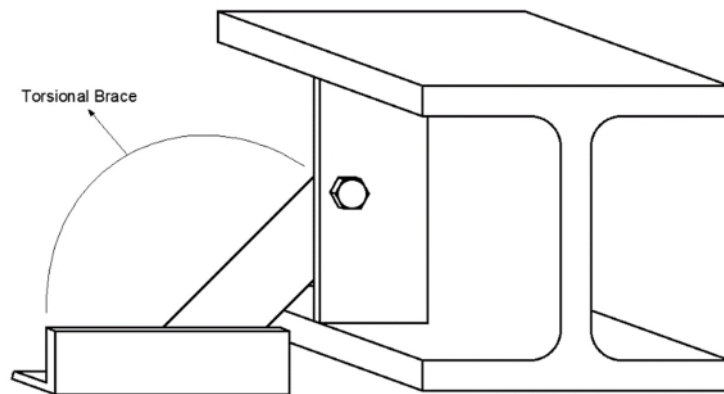


Figure 2.10: Torsional Bracing

Surla et al. (2015) investigated the inelastic LTB capacity of stepped I section beams under uniform bending. Using FEA they found stepped sections increased their resistance to LTB. Their results correlated well with existing equations so they could successfully provide a simpler expression to calculate the buckling strength of stepped sections that have non-compact flanges. Applying this stiffening technique at critical points is more economical than stiffening along the entire member's length. Webs are not usually stepped because increasing the web thickness has little effect on LTB and changing the web height would lead to discontinuous flanges.

Little research has yet been conducted into increasing the web thickness (“stepping” the web) of notched beams along the notch length shown in **Figure 2.11**. This part of the member is most susceptible to local buckling therefore making it less slender could decrease the risk of local buckling failure. This is what Yam et al. (2016) found: an increase in the web thickness and steel grade causes the buckling resistance to increase.

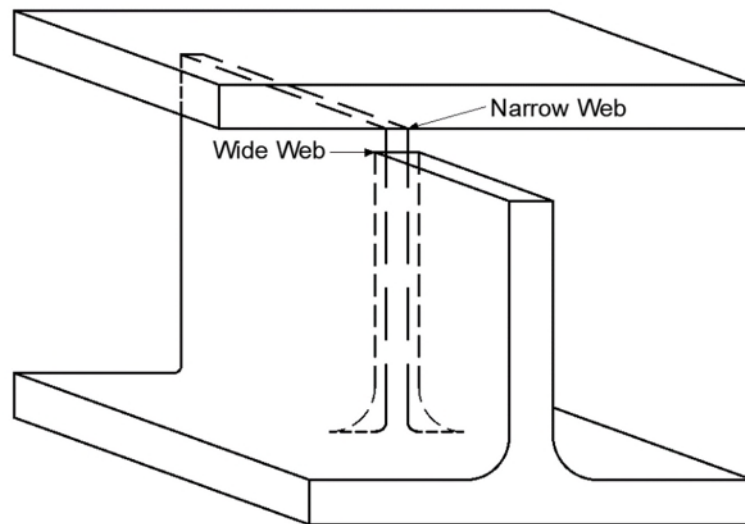


Figure 2.11: Notched Section with Increased Web Thickness

2.4 T Section Buckling

A notched section comprises of a singly or doubly symmetric section and a T section at the notched part. The monosymmetric portion resembles a T section (**Figure 2.12**). A comparison between the behaviour of a T section and notched section under LTB has not been fully explored by literature. In design offices, it is general practice to split a notched section into its two distinct parts when performing structural design checks such as shear failure or local buckling failure.

Bradford (1988) noticed the elastic distortional buckling moment is very similar to the elastic flexural torsional or lateral buckling. This was for the case of a T section loaded by end moments such that the free edge of the stem is in tension. Bradford (1988) also noticed if the free edge of the stem is put into compression from the loading, the lateral buckling interacts with local buckling which causes the critical moment to be less than the elastic lateral buckling moment. This reduction can be quite significant, especially if the beam is loaded with equal end moments or has a thin stem. Bradford (1988) suggested designers should be cautious of using lateral buckling solutions because the distortional buckling moment can often be much lower than the lateral or flexural torsional buckling moment. This is also the case for notched sections: distortional (or local) buckling is more critical than global LTB (as discussed in *Section 2.3 Notched Sections* and its sub sections).

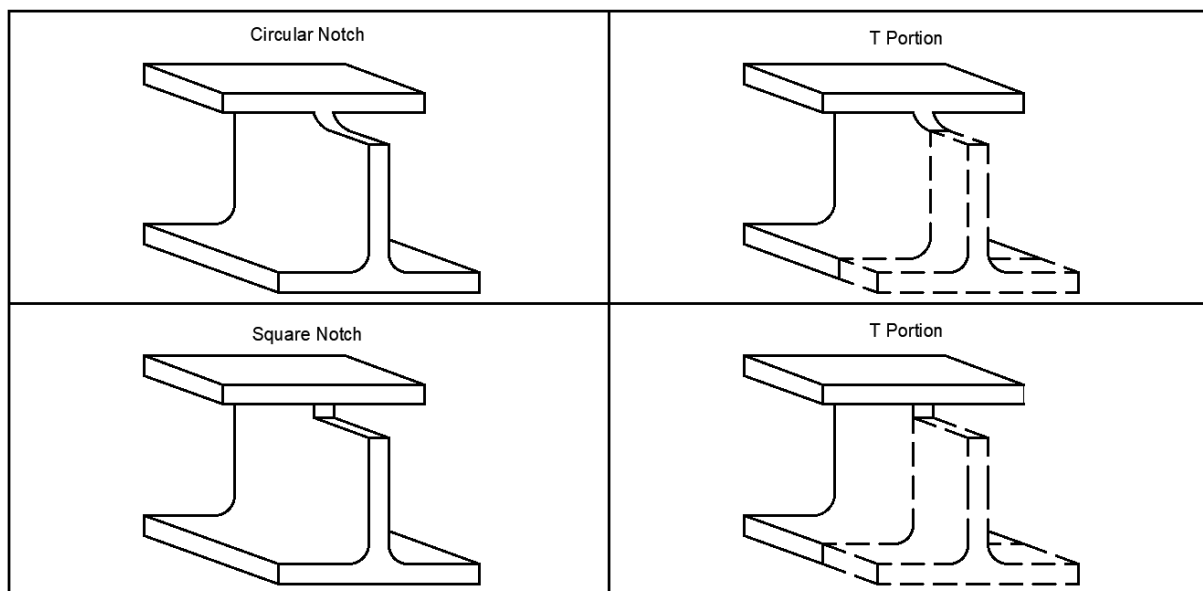


Figure 2.12: T Section Portion of a Notched Section

2.5 Influence of Connection Type

There are two types of connections commonly used to connect notched sections to other beams or columns: an end plate or using angles (cleats) shown in **Figure 2.13** (**Figure 2.14** shows the different restraint levels based on a beam to column connection). Yam et al. (2016) cites Cheng et al. (1984) as the pioneers in researching doubly notched steel connections. Yam et al. (2016) suggested by checking the flexural, shear yielding and elastic local buckling capacity the ultimate web buckling capacity of the notched section can be determined. Aalberg (2015a) conducted tests on aluminium doubly notched sections and found the design procedure proposed by Cheng et al. (1984) was limited as it did not account for the type of connection used. Aalberg (2015a) found this had an influence on the local web buckling capacity.

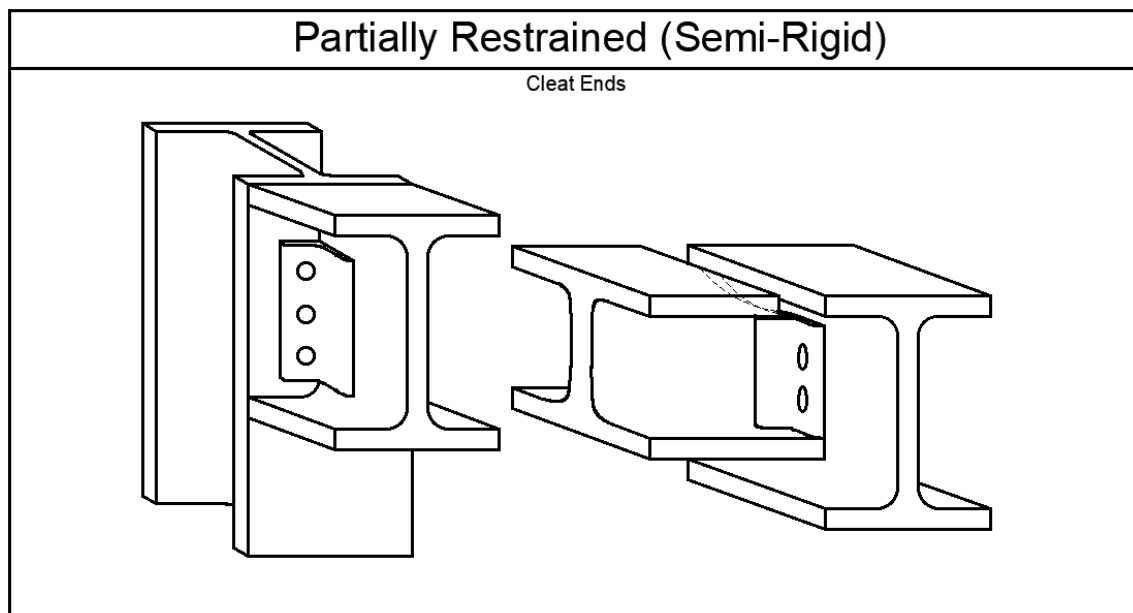


Figure 2.13: Cleated Connection (Mudenda, 2018)

Yam et al. (2016) expanded on Fang et al. published in (2017) experimental work to better understand the local web buckling response of these types of connections (end-plate, single and double cleats). The end plate does not provide as much support because it is only welded whereas the angle extends along the web and provides some restraint (**Figure 2.13**). A parametric study was also conducted in which various geometric and material variables were analysed. Yam et al. (2016) thought it was reasonable to think the end connection will influence the local buckling capacity of a notched section because the connection offers some rotational stiffness. Yam et al. (2014) found the rotational stiffness does indeed influence the local buckling capacity as discussed in *Section 2.3 Notched Sections*. The rotational stiffness, if sufficient, can allow for the presence of a negative moment which can reduce the “effective buckling length” thus reducing the chance for local web buckling.

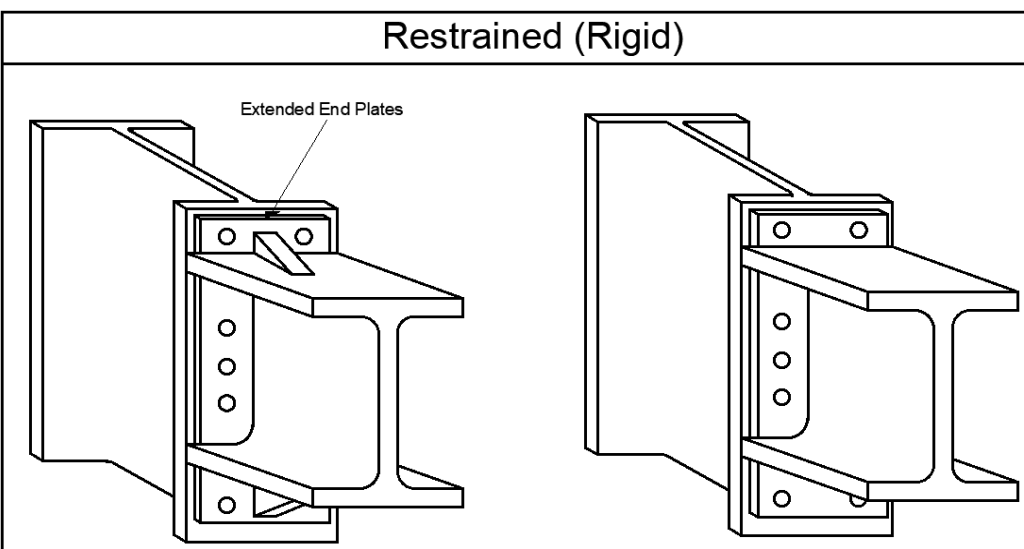
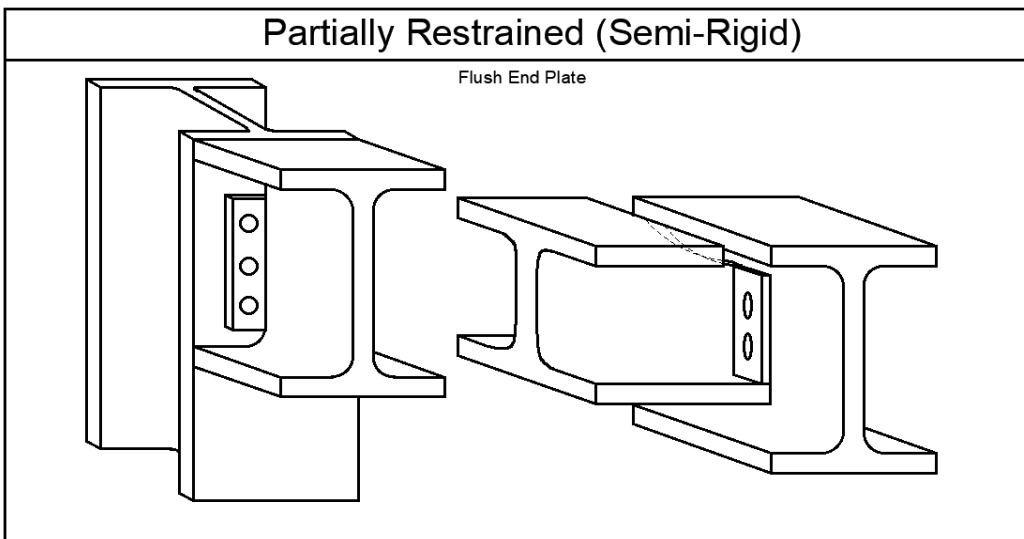
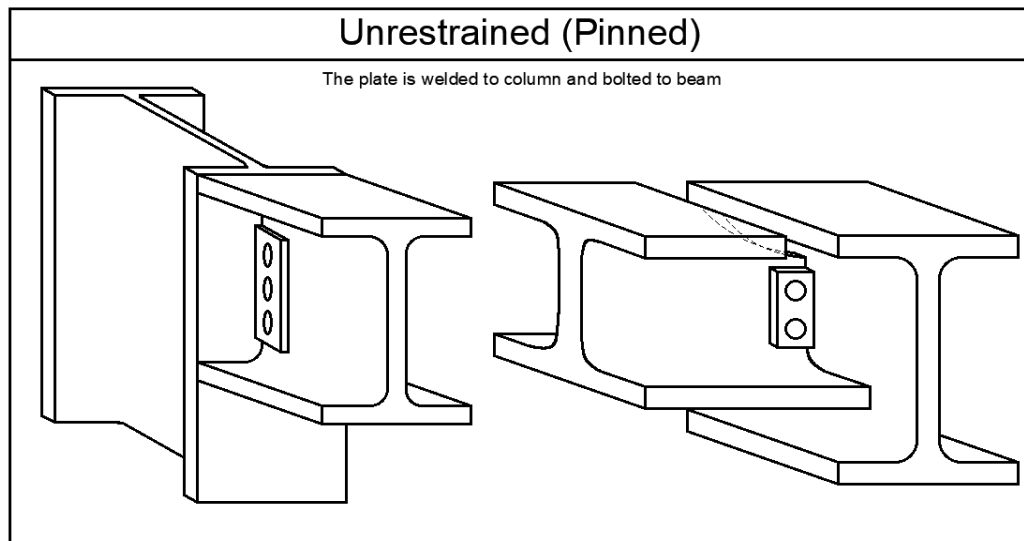


Figure 2.14: Degrees of Restraint (Mudenda, 2018)

Yam et al. (2016) confirmed their thought process: an increase in connection stiffness can increase the resistance to local web buckling in doubly notched sections. They found the main failure mode of their models was by elastic or inelastic local web buckling. They also found geometric and material parameters such as notch length, depth as well as web slenderness and steel grade have a clear impact on the buckling capacity.

With regards to the notch lengths, they found short lengths tended to fail by inelastic buckling accompanied by excessive shear yielding. Longer notch lengths could experience stable post buckling equilibriums after initial buckling. This was especially true for slender webbed sections. As a result, the ultimate reaction can be significantly lower than the elastic buckling case.

Yam et al. (2016) concludes by stating an increase in connection rotational stiffness is beneficial for local web buckling. The restraining effect developed by double angle cleat connections could improve the local web buckling capacity. However, single angle connections will counteract this gain through detrimental effects of out of plane eccentricity (**Figure 2.15**).

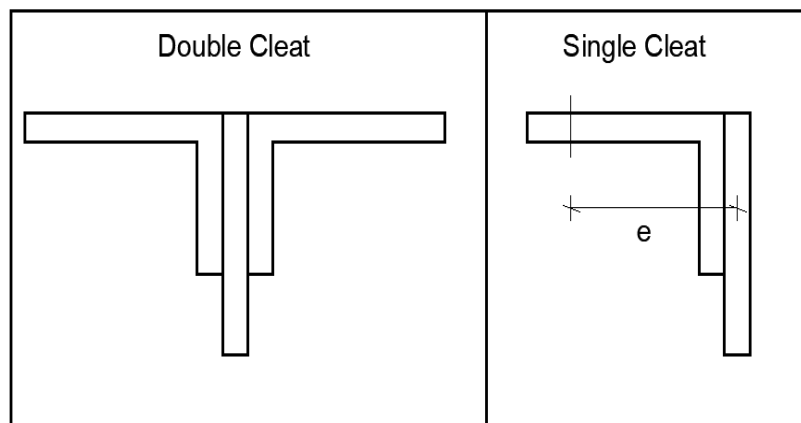


Figure 2.15: Out-of-Plane Eccentricity Yam et al. (2016)

2.6 Key Findings from Literature Review

The key findings from the Literature Review, which are most relevant to the modelling procedure and selection of the stiffening techniques, are listed below:

- LTB is an instability failure that is heavily determined by the unbraced length of the beam.
- FEA is one of many methods that can be used to determine the elastic buckling load of a beam.
- Notched sections can fail by local web buckling at the notch.
- It can be controlled through the:
 - notch depth to section depth ratio.
 - notch length to T section height.
 - Use of vertical and horizontal stiffeners.
- The type of connection, and its stiffness, affects the resistance to local web buckling.

These key findings will be used in the subsequent chapters which explain the selection of the techniques and the setting up of the FEM for the parametric study.

3. Details of Beam and Stiffener Arrangement

Using the Southern African Institute of Steel Construction Handbook (2016), five sections were selected for the study. Three are I sections and two are H sections as shown **Table 3.1** and **Table 3.2** as well as **Figure 3.2** and **Figure 3.3**. All sections are notched at the top flange. The top flange is notched because downstands on the longitudinal stiffener causes the shear centre to shift upwards. **Figure 3.2** and **Figure 3.3** show details of the un-notched part as well as an elevation view. The Prokon software module PROSEC was used to calculate the location of the centroid and shear centre for all the T-section parts of the beams.

As previously stated, the influence of local buckling was minimised using the limits proposed by Ibrahim et al. (2015) since the influence of the shifting centroid and shear centre on LTB was to be investigated. **Figure 3.1** shows the definitions of the terms used in **Table 3.1** and **Table 3.2**.

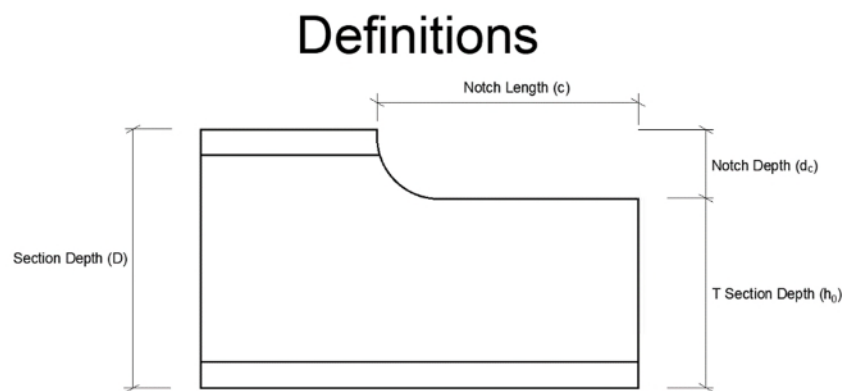


Figure 3.1: Explanation of Terms in Table 3.1

The notch depth (d_c) to section depth (D) ratio should be less than or equal to 0.25 which was achieved for all the tested sections as shown in **Table 3.2**.

Table 3.1: Sections and Notch Depth to Section Depth Ratios

Section	Notch Depth (d_c)	Section Depth (D)	Ratio (d_c/D)
203x133x25	50	203	0.246
305x102x25	70	305.1	0.229
533x210x82	130	528.3	0.246
203x203x46	50	203.2	0.246
305x305x97	70	307.8	0.227

The notch length was selected to be in accordance with Ibrahim et al. (2015) who state the notch length (c) to T section depth (h_0) ratio should be less than or equal to 1 (**Table 3.2**). The conditions also satisfy Lam et al. (2000) who proposed the notch length to T section depth ratio should be less than or equal to 2.5.

Table 3.2: Notch Length to T Section Depth Ratios

Section	Notch Length (c)	T Section Depth (h_0)	Ratio (c/h_0)
203x133x25	150	153	0.980
305x102x25	150	235.1	0.638
533x210x82	150	398.3	0.377
203x203x46	150	153.2	0.979
305x305x97	150	237.8	0.631

The limits from Ibrahim et al. (2015) for this study were:

- Notch depth/Section depth ≤ 0.25 .
- Notch length/T Section depth ≤ 1 .

These were selected to test sections with deeper notches because a deep notch pushes the shear centre far below the centroid and the section becomes very monosymmetric. This allows for more downstand cases since the shear centre has to travel a greater distance upwards. The reduction in the buckling capacity of notched beams can be more effectively controlled if the following limits are followed according to Ibrahim et al. (2015):

- Notch depth/Section depth ≤ 0.10 .
- Notch Length/T Section depth ≤ 1 .

Future studies should try these limits proposed by Ibrahim et al. (2015).

Section 3.2 Stiffening Techniques discusses the stiffening techniques investigated for each of the sections and how the sections were chosen. The techniques are the same across all sections but are implemented according to the unique geometric requirements of the section. For example: the number of downstands as well as their heights vary across each section.

3.1 Selected Sections

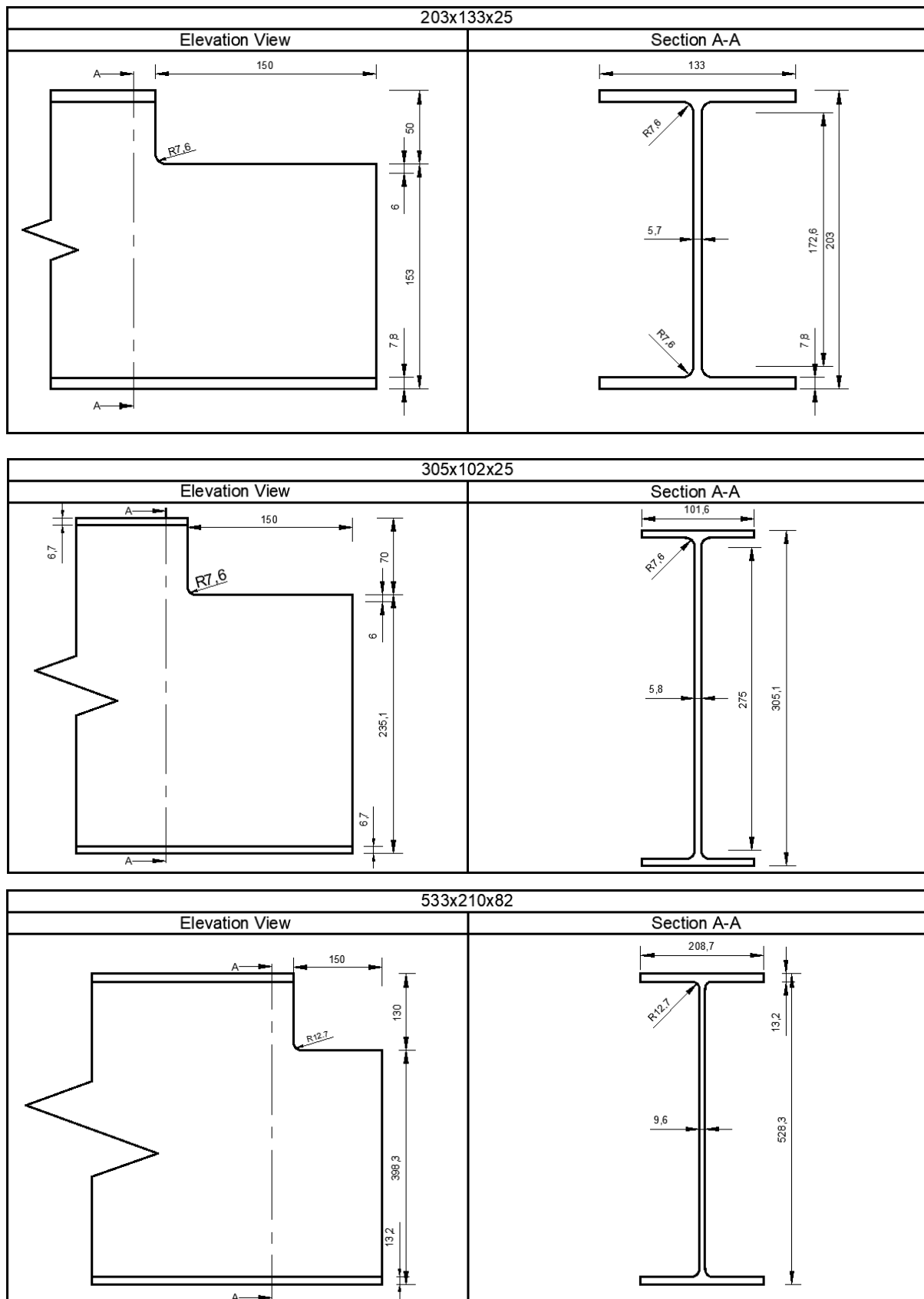


Figure 3.2: Selected I Sections

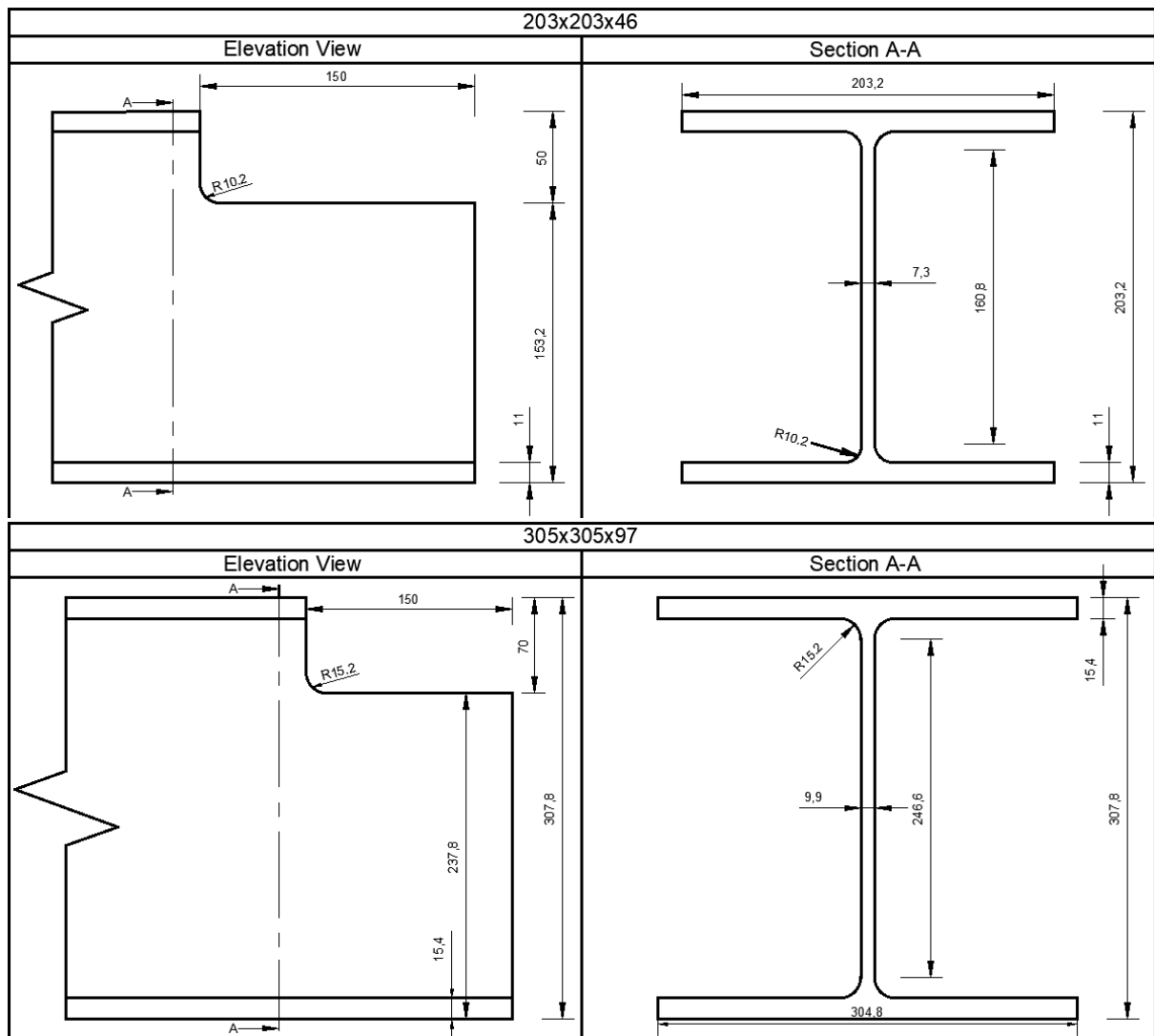


Figure 3.3: Selected H Sections

3.2 Stiffening Techniques

The procedure to choose the most suitable stiffening technique is summarised below in **Figure 3.4**. The five techniques that restored the notched 203x133x25 section closest to the un-notched 203x133x25 section capacity were carried forward to be applied to the other sections as discussed in *Section 3.2.3 Selected Stiffening Techniques*.

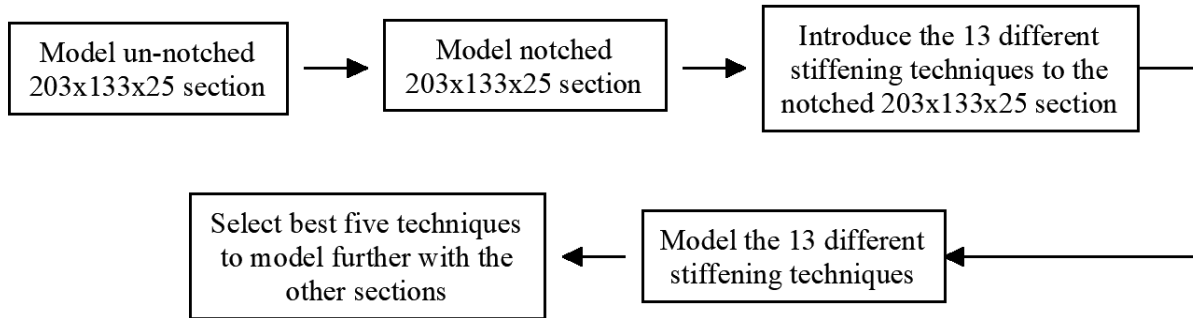


Figure 3.4: Procedure to Find Best Stiffening Techniques

3.2.1 Proposed Stiffening Techniques

This procedure was only applied to one section (203 x 133 x 25) for both loading and notched cases. The loading and support conditions are the same for all 13 stiffening techniques and the loading, spans and support cases are shown in **Figure 3.5**.

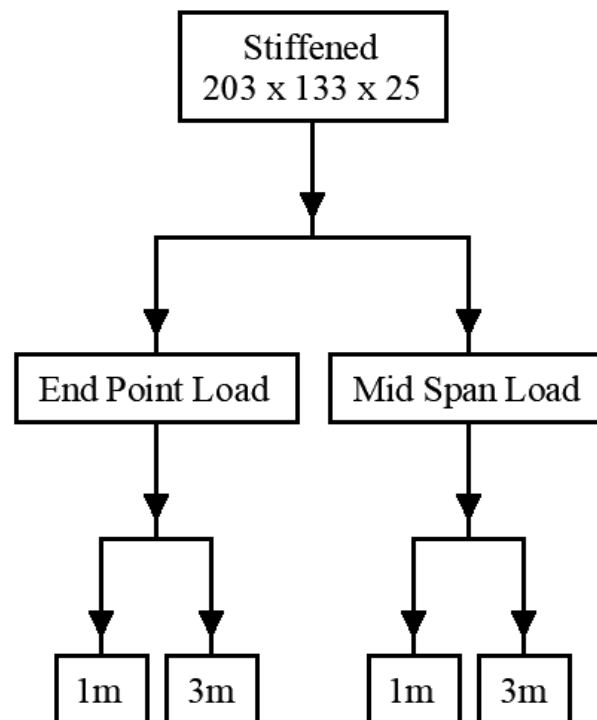


Figure 3.5: Modelling Procedure to Find the Best Stiffening Technique

Thirteen arrangements were selected and are shown in **Table 3.3** to **Table 3.5** with a small description of key points about the stiffener arrangement.

Table 3.3: Description of Stiffening Techniques

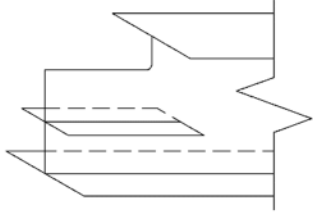
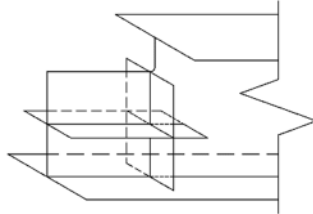
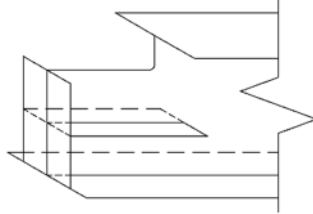
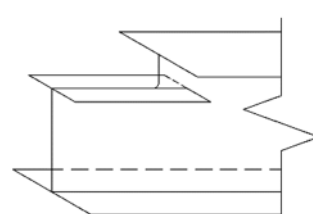
Stiffener Designation	Stiffener Name	Image
HAS	Horizontal (AS)	
HVASN	H and V at Notch (AS)	
HVASE	H and V at End (AS)	
FD	Full Length Downstand (FD) is 200mm long (same length as horizontal stiffener)	

Table 3.4: Description of Stiffening Techniques (Continued)

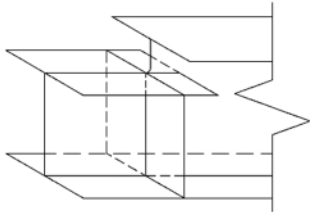
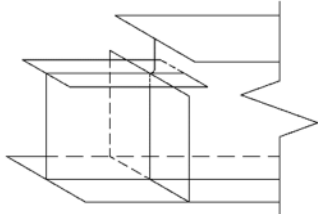
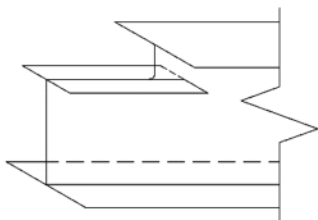
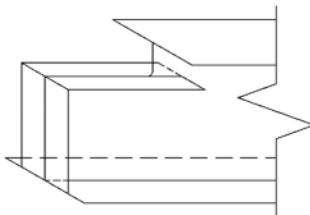
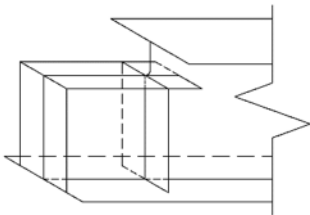
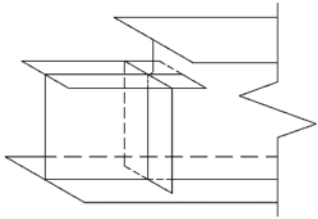
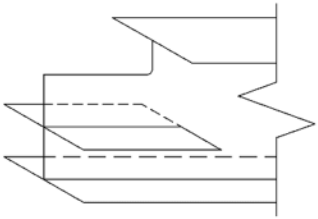
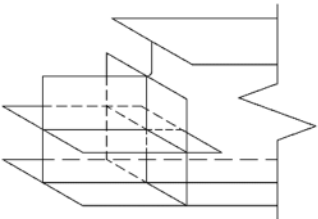
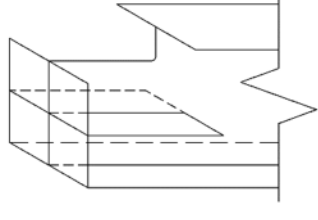
FHVS	Full Length H and V Stiffener (width of horizontal =vertical stiffener)	
FS	Full Length Stiffener (width of horizontal <vertical stiffener)	
HS	Horizontal Stiffener (also known as no vertical stiffener) Downstand is 150mm long (shorter than horizontal stiffener)	
HSE	Stiffener at End	
HSNE	Stiffener at End and Notch	

Table 3.5: Description of Stiffening Techniques (Continued)

HSN	Stiffener at Notch Downstand is 150mm in length (shorter than horizontal stiffener)	 A 3D perspective diagram showing a vertical stiffener (downstand) attached to a horizontal plate. The downstand is positioned at a notch in the plate. The downstand is shorter than the horizontal stiffener it is attached to.
HTS	Horizontal (TS)	 A 3D perspective diagram showing a horizontal stiffener (TS) attached to a vertical plate. The stiffener is positioned horizontally across the plate.
HVTS	H and V at Notch (TS)	 A 3D perspective diagram showing both a horizontal stiffener (TS) and a vertical stiffener (downstand) attached to a plate at a notch. The stiffeners are positioned to reinforce the notch area.
HVTSE	H and V at End (TS)	 A 3D perspective diagram showing both a horizontal stiffener (TS) and a vertical stiffener (downstand) attached to a plate at an end. The stiffeners are positioned to reinforce the end of the plate.

3.2.2 Comparison and Results

As previously mentioned, the thirteen proposed stiffening techniques were applied to the 203 x 233 x 25 section for the two loading cases: one side top notched with a point load 600mm from the notched end (“End Point Load” or EPL case) and doubly top notched with a point load applied at the mid span (“Mid Span Load” or MSL case) with both loading cases having spans of 1m and 3m as shown in **Figure 3.6**.

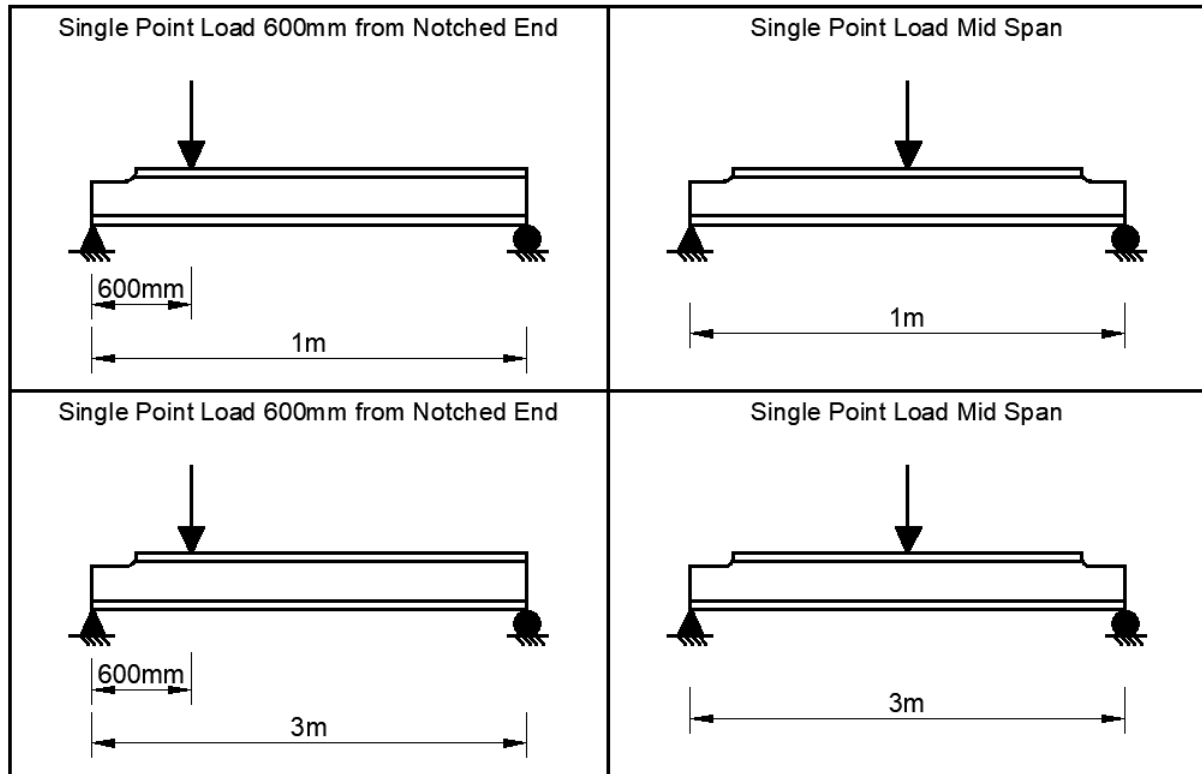


Figure 3.6: Loading Cases for Stiffening Comparison

The results for the 1m and 3m EPL case are shown in **Figure 3.7**. The graph shows the critical elastic moment for the un-notched section as the horizontal grey line. The vertical orange and blue columns represent the critical elastic moment for the notched section before the stiffening technique is applied and the critical elastic moment once the technique is applied. The five best stiffening techniques are: FHVS, FS, HSNE, HSN, HVTSE.

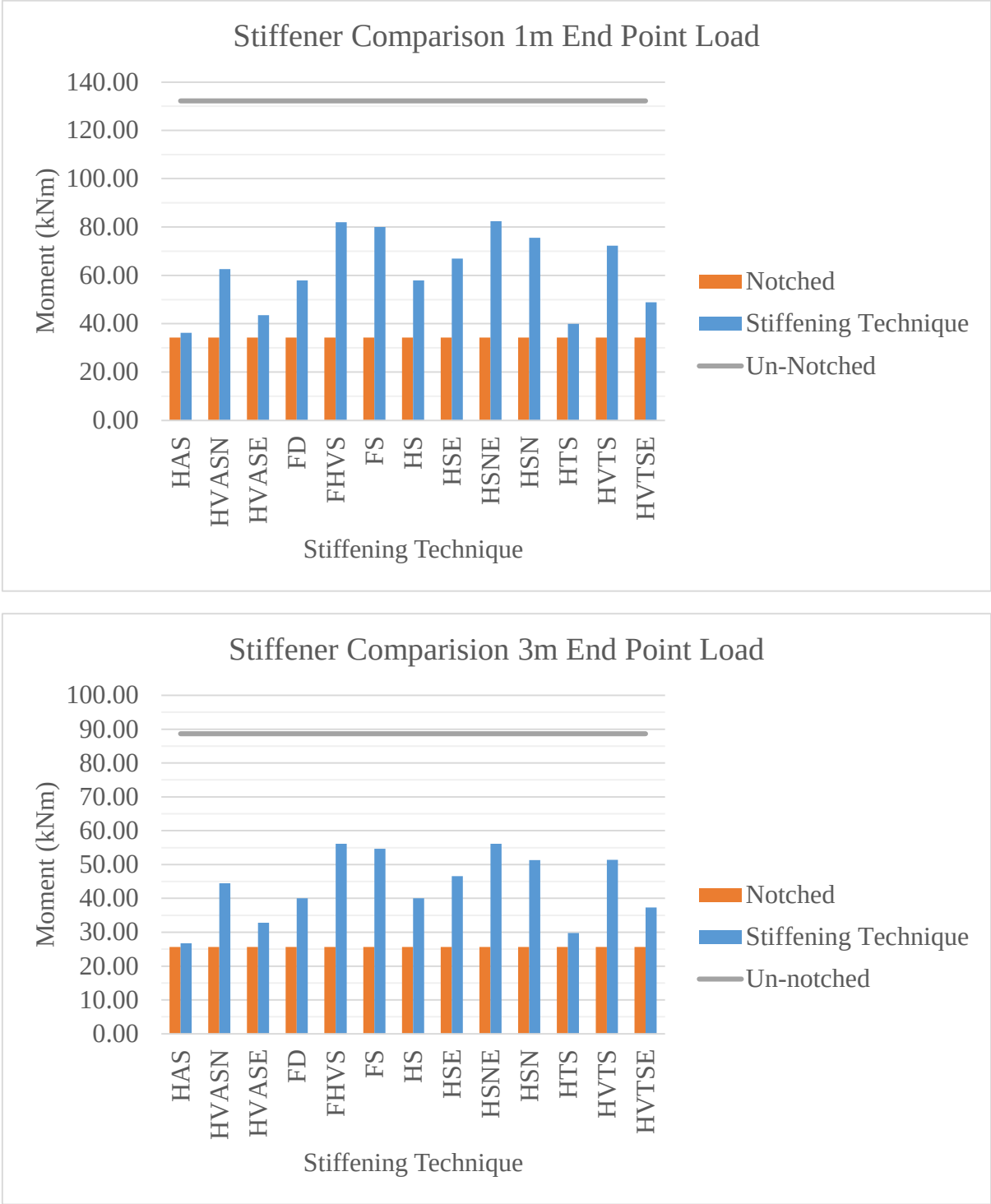


Figure 3.7: Results for 1m and 3m End Point Load Comparison

The results for the 1m and 3m MSL case are shown in **Figure 3.8**. The five best stiffening techniques are the same as the EPL case: FHVS, FS, HSNE, HSN, HVTS.

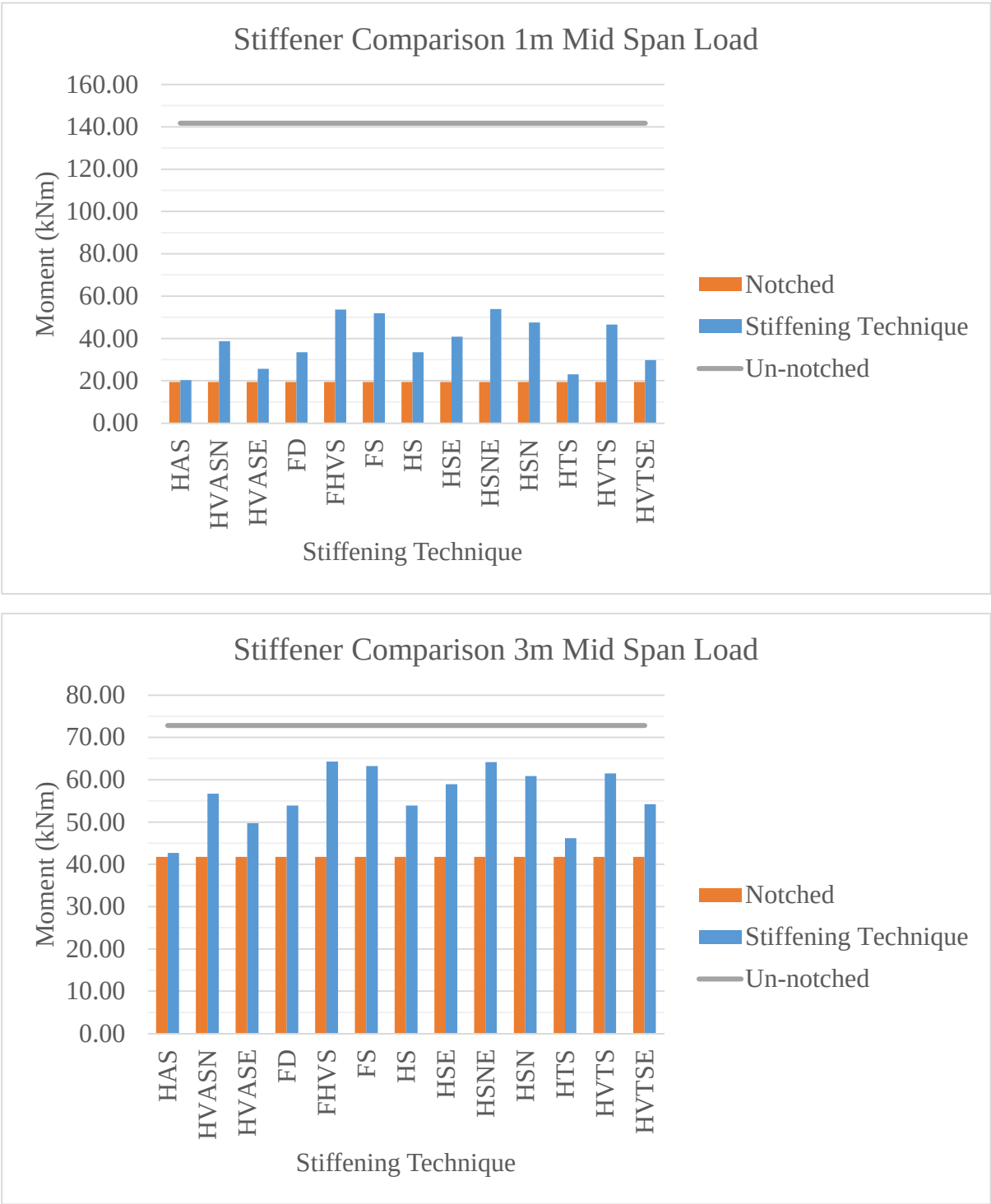


Figure 3.8: Results for 1m and 3m Mid Span Load Comparison

3.2.3 Selected Stiffening Techniques

The five best stiffening techniques were FHVS, FS, HSNE, HSN, HVTs. These techniques restored the notched sections closest to the un-notched section as shown in **Figures 3.7** and **Figure 3.8**. These techniques were further improved by extending the vertical stiffener to the top flange. **Figure 3.9** to **Figure 3.10** shows the new improved techniques and designations.

Selected Techniques

Improved Techniques

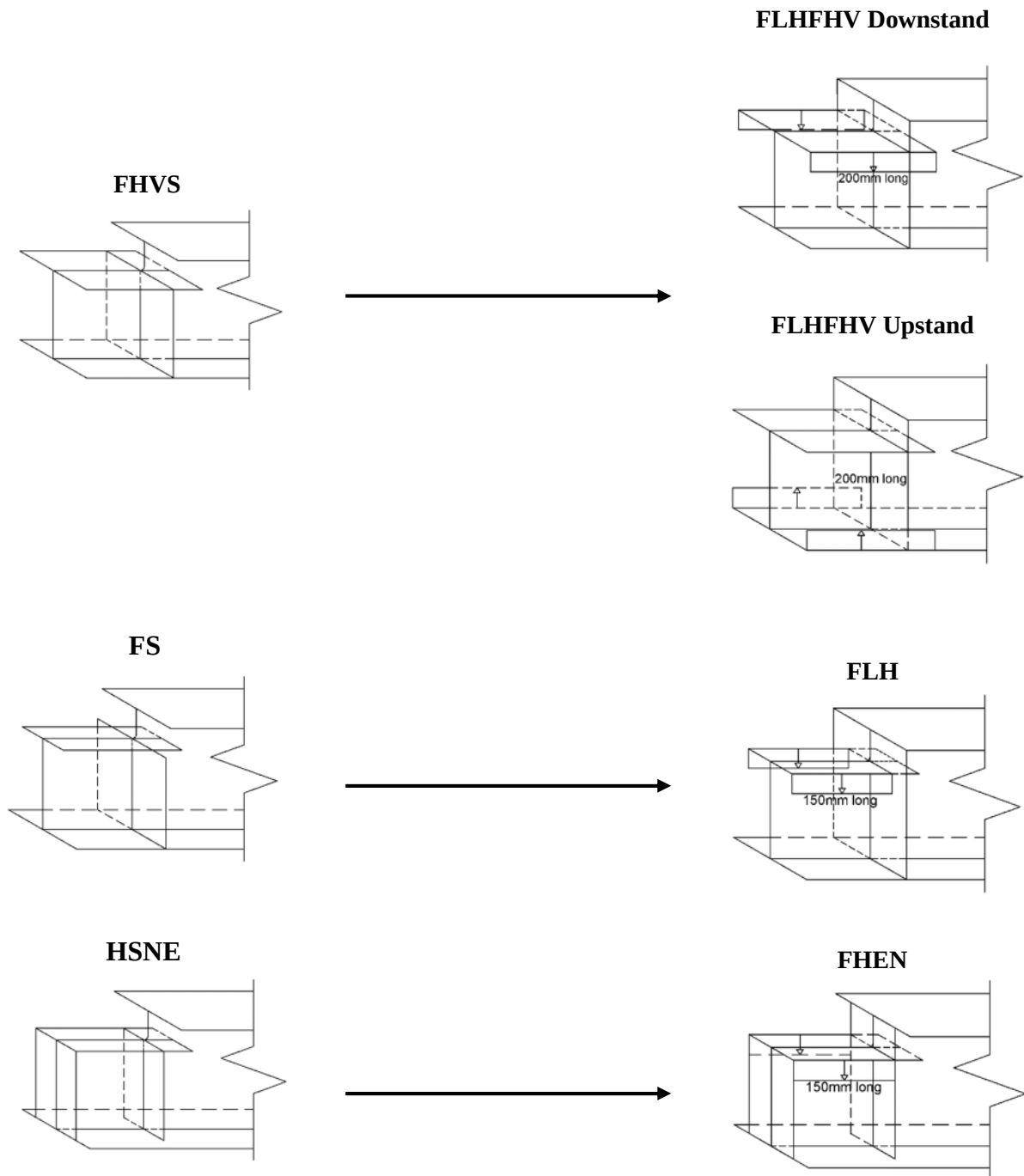


Figure 3.9: Selected and Improvements to Techniques

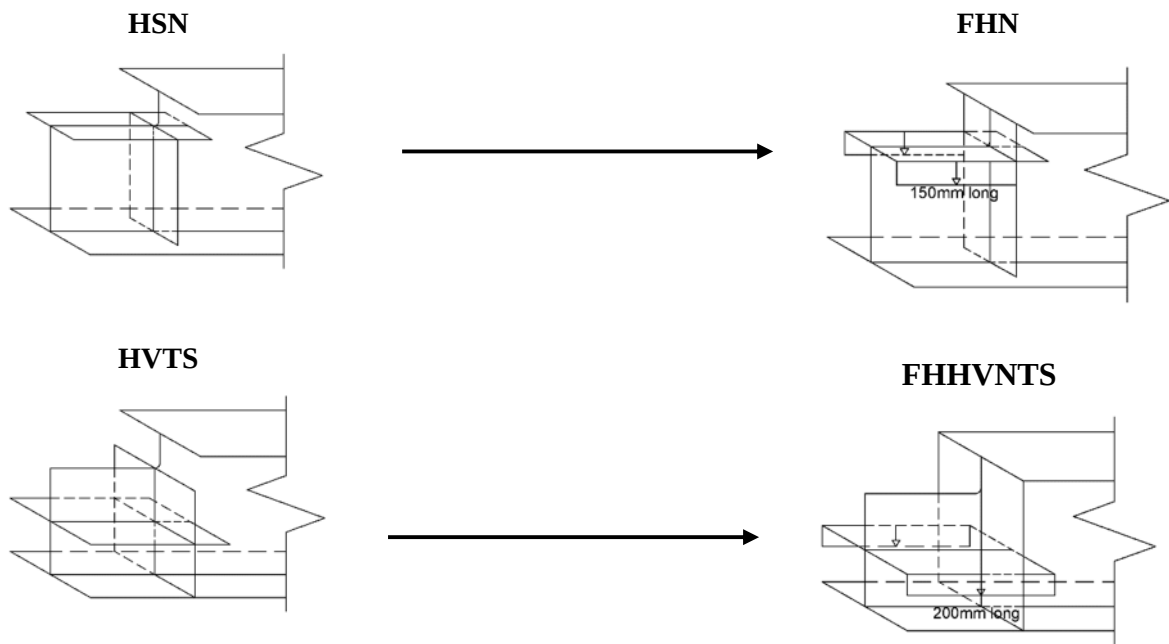


Figure 3.10: Selected and Improvements to Techniques (Continued)

The results of increasing the vertical height are shown in **Figure 3.11** to **Figure 3.14**. The aim is to try and restore the stiffened notched section critical moment capacity to be as close as possible to the un-notched capacity which was not close enough in the initial tests. If the capacity is sufficiently restored using the stiffening techniques: any further improvements are solely due to the introduction of the downstands or upstands. These improvements also reduce the chance of local web buckling occurring before lateral torsional buckling since notched beams are highly susceptible to local web buckling.

The graphs show the un-notched capacity of the section as a grey horizontal line. The orange column represents the notched capacity, and the yellow column is the initially proposed stiffening technique. The blue column represents the new technique which increased the height of the vertical stiffener to the top flange. This increase in height almost completely restores the critical elastic moment capacity for all stiffening techniques. Note the graphs use the old designation on the horizontal axis. **Figure 3.11** shows the results for the 1m EPL case.

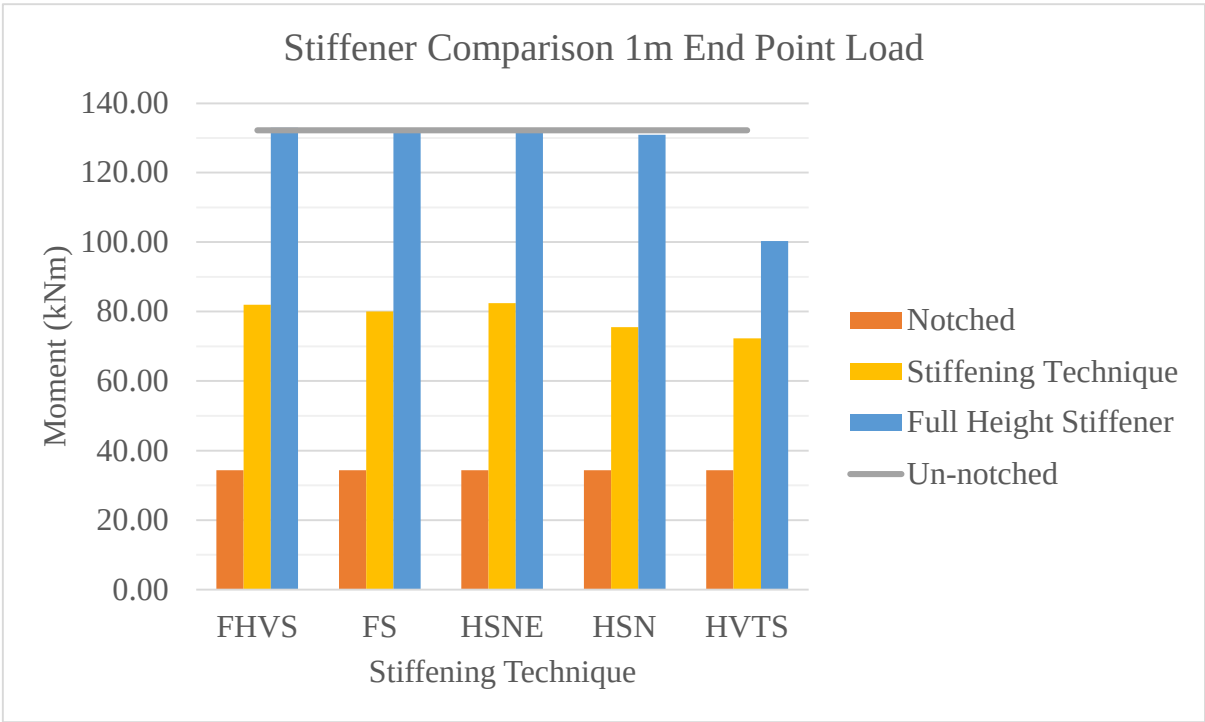


Figure 3.11: New Stiffening Technique 1m EPL Case

Similarly, **Figure 3.12** shows the improved moment capacity for the 3m EPL Case with similar results.

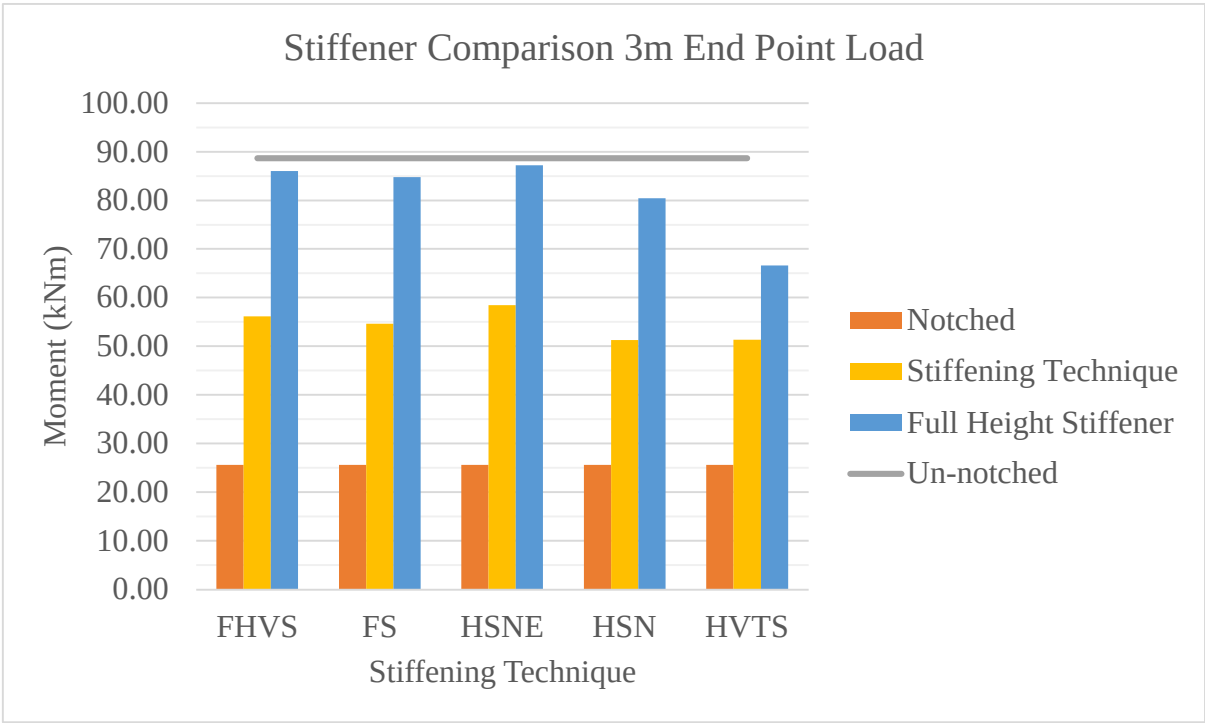


Figure 3.12: New Stiffening Technique 3m EPL Case

Figure 3.13 shows the results for the 1m MSL cases.

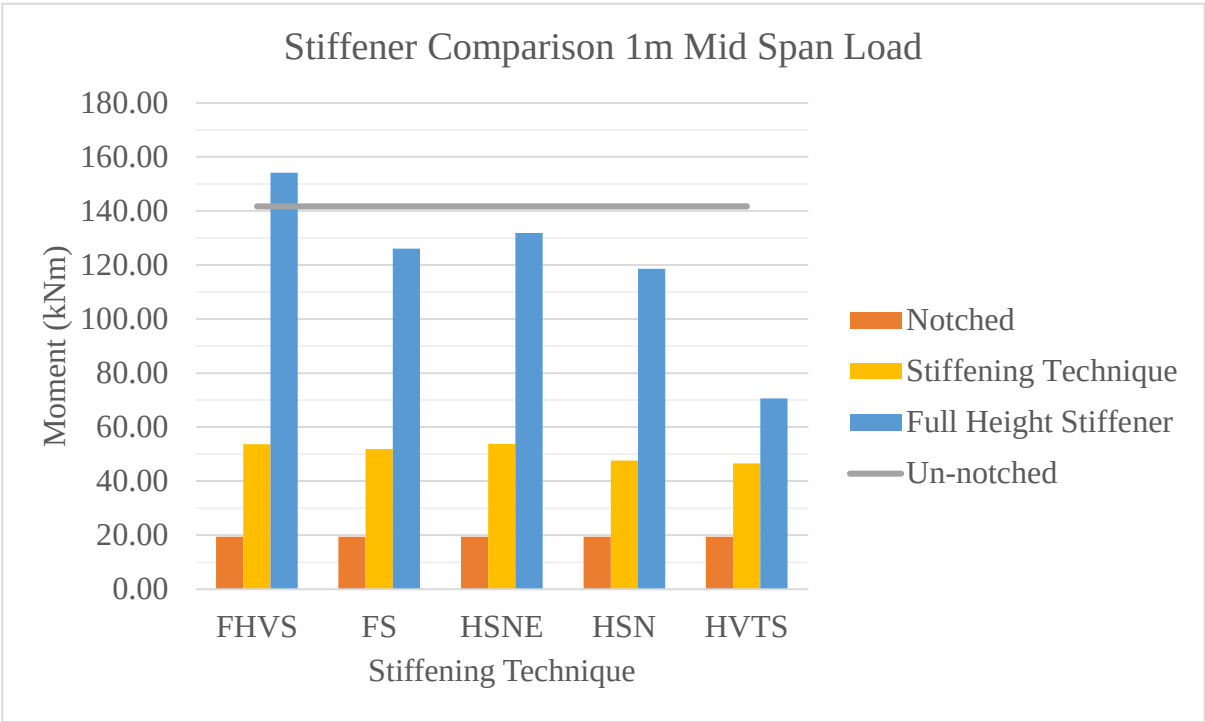


Figure 3.13: New Stiffening Technique 1m MSL Case

Figure 3.14 shows the results for the 3m MSL case.

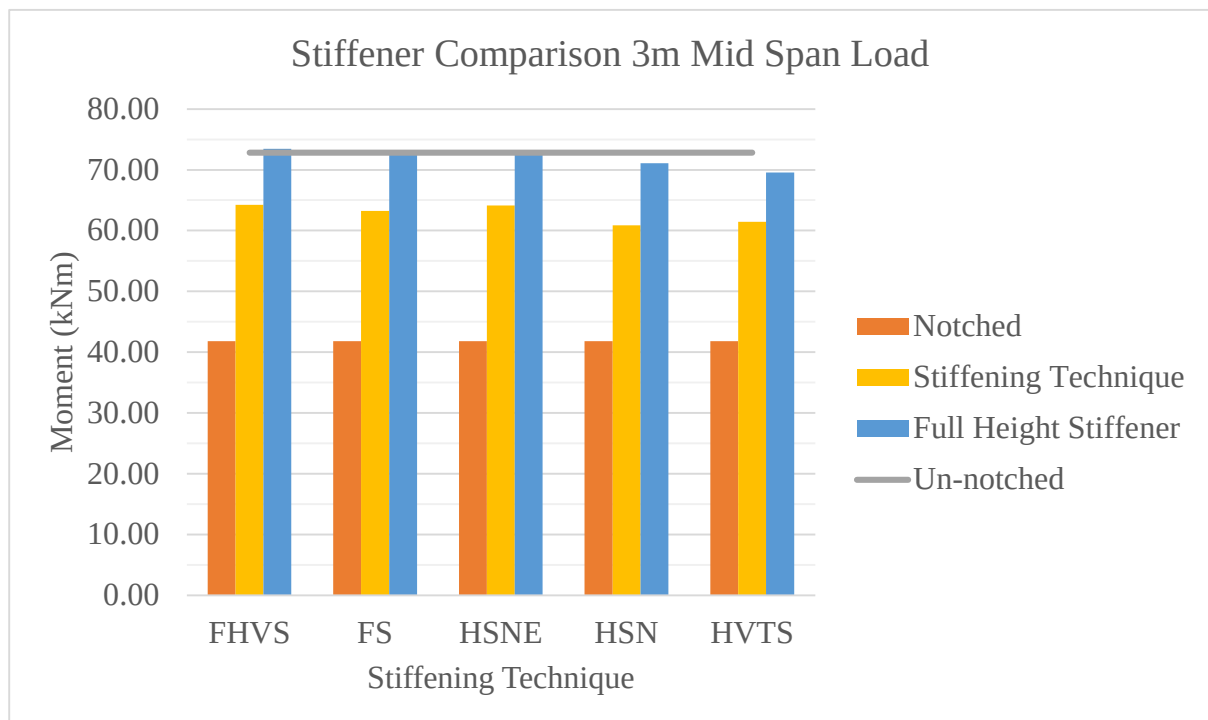
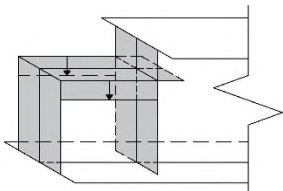
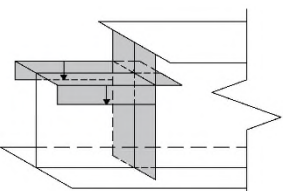
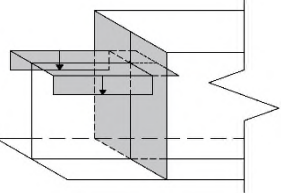
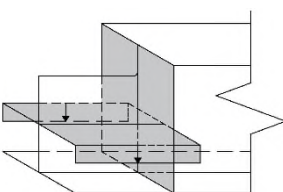
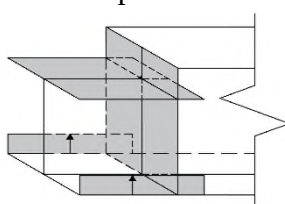
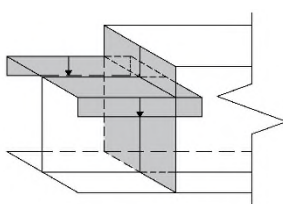


Figure 3.14: New Stiffening Technique 3m MSL Case

The techniques have been grouped according to **Table 3.6** and are shown in **Figure 3.15** to **Figure 3.16**.

Table 3.6: Description of Selected Stiffening Techniques

New Stiffening Techniques			
Description		Designation	Image
Stiffener Along Notch 150mm long downstand (arrows show direction of height increase) and 79.35mm wide horizontal stiffener for all cases Downstand is shorter than horizontal stiffener	Full Height Stiffener at End and Notch	FHEN	
	Full Height Stiffener at Notch	FHN	
	Full Length and Height Stiffener	FLH	
FHHVNTS 200mm long downstand (same length as horizontal stiffener)	Full Height Horizontal and Vertical Stiffener at Notch located at the centreline	FHHVNTS	
FLHFHV 200mm long downstand (same length as horizontal stiffener)	Full Length Horizontal and Full Height Vertical Stiffener	Upstand 	Downstand 

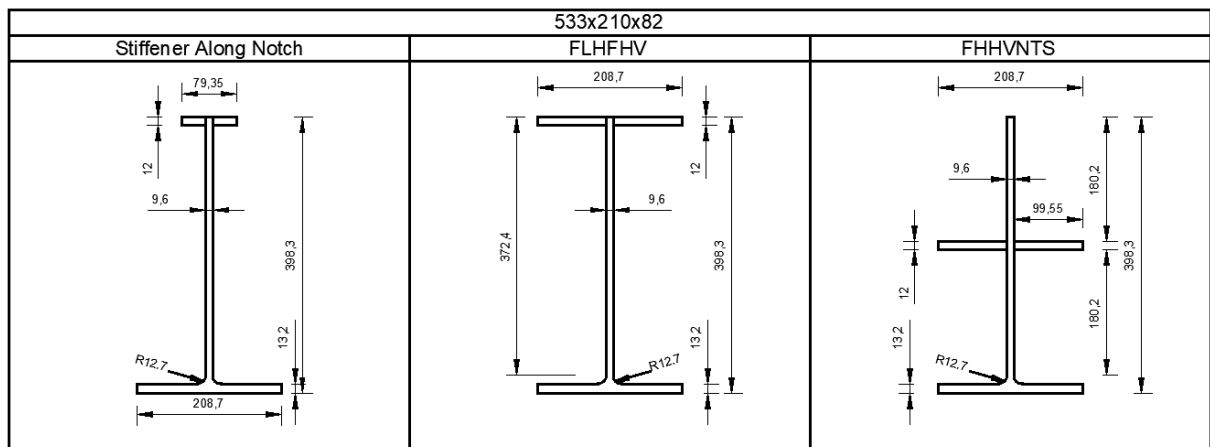
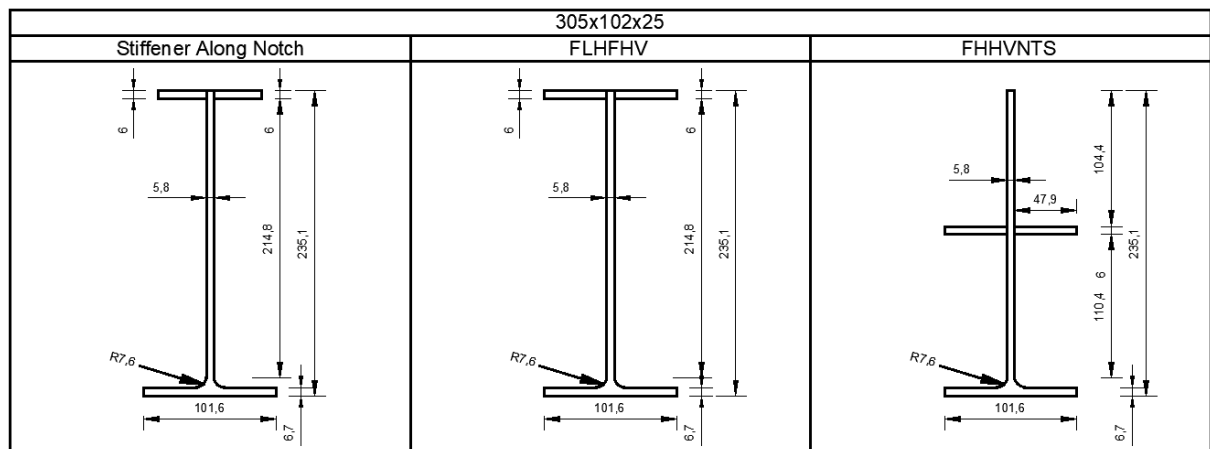
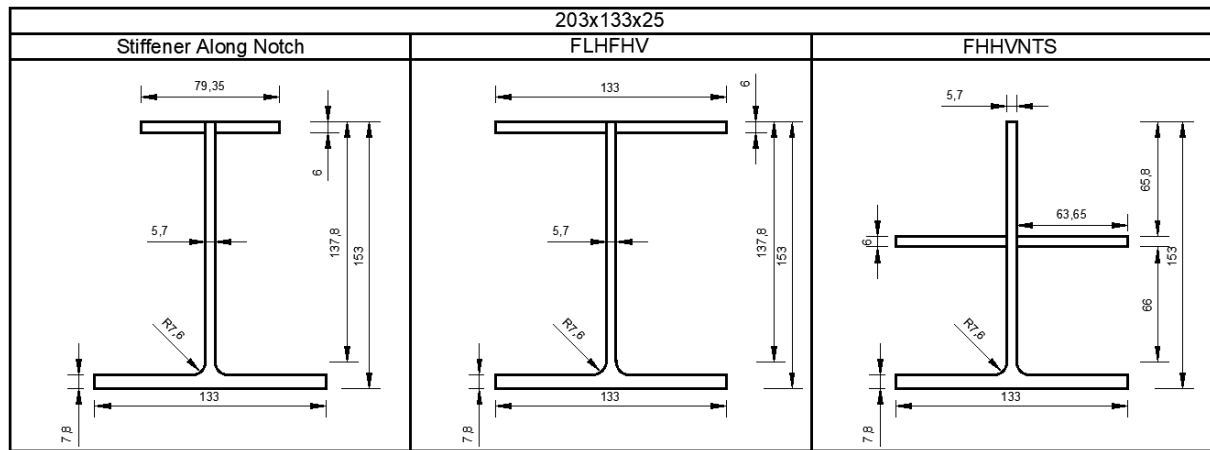


Figure 3.15: Stiffening Techniques for All Sections and Load Cases

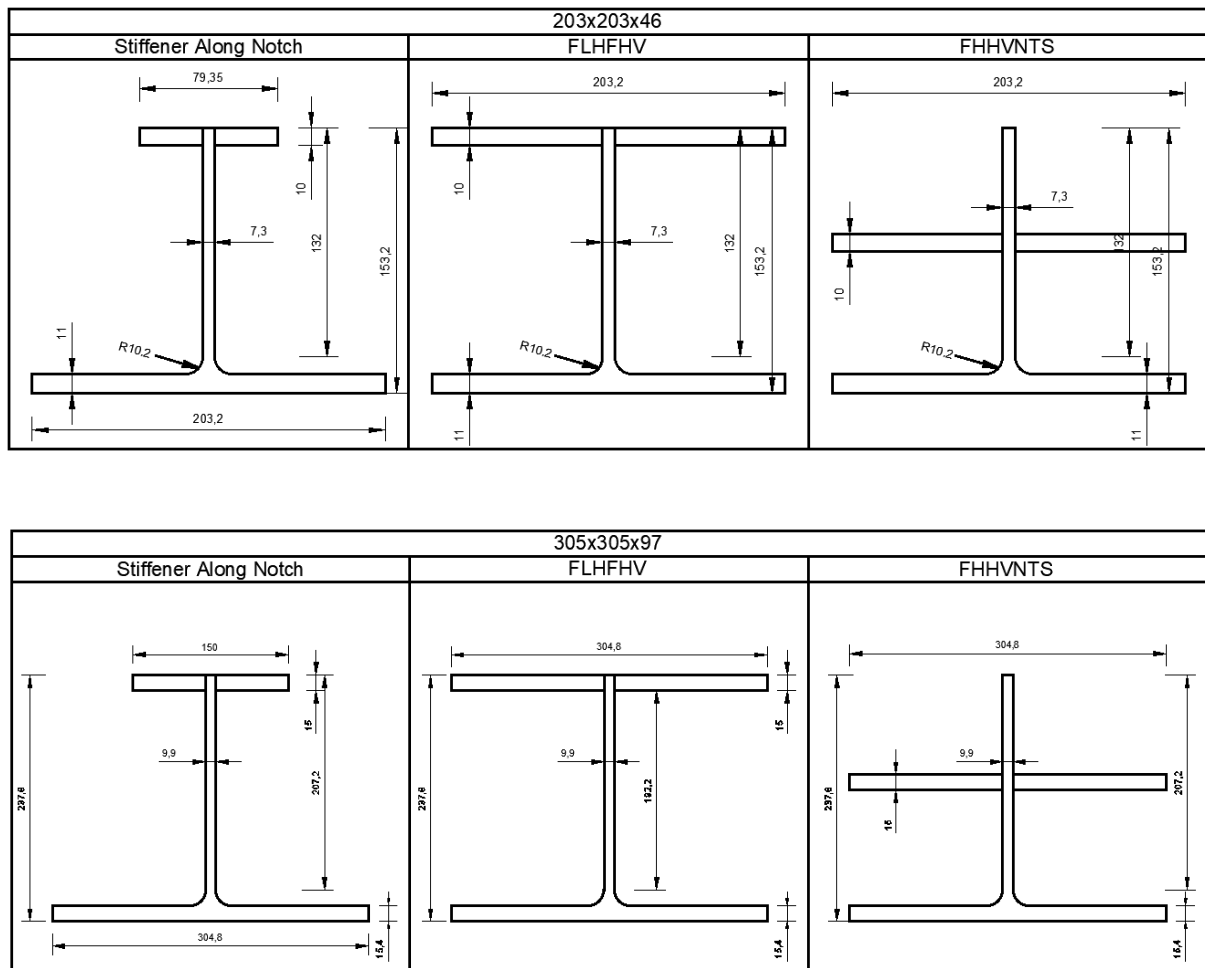


Figure 3.16: Stiffening Techniques for All Sections and Load Cases (Continued)

Figure 3.17 shows the modelling procedure with the selected stiffening techniques.

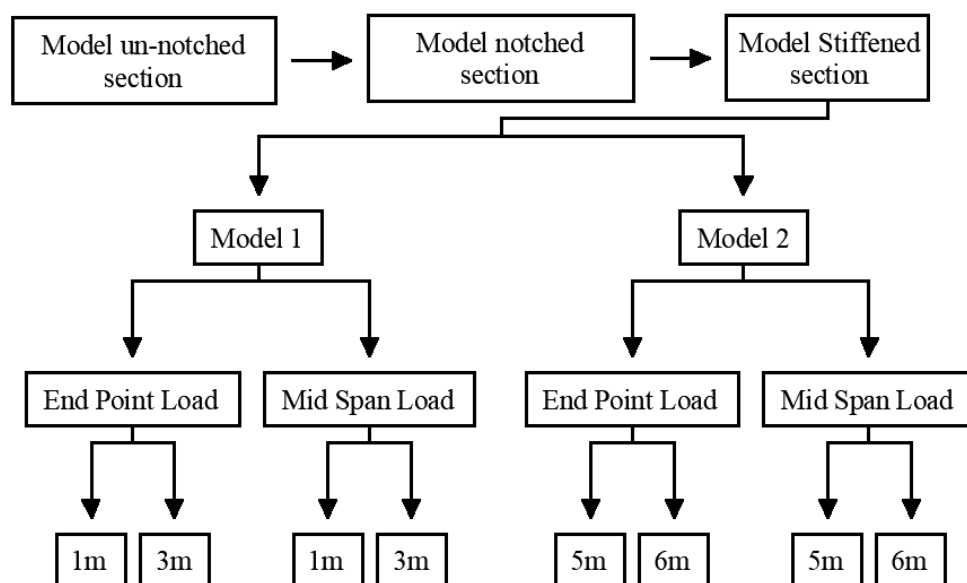


Figure 3.17: Modelling Process with Five Best Stiffening Techniques

3.2.4 Un-notched Capacity vs Stiffened Capacity

There were cases in which the stiffened section had a greater resistance to LTB than the un-notched section (FHVS in **Figure 3.13**). This is very interesting since the section should be weaker due to the notch but this shows the effectiveness of these stiffening techniques in combination with the recommendations made by Ibrahim et al. (2015).

The 533x210x82 section, for all five techniques, had a greater critical elastic moment using the stiffening techniques than the un-notched cases for Model 1 and 2 for the EPL case. The 305x305x97 section was similar but the FHHVNTS 3m and FHN 6m cases did not achieve an increase that would result in the stiffened section being at least 5kNm (regarded as a significant increase) greater than the un-notched case. The FLHFHV technique increased the critical elastic moment the most.

The MSL case has a different trend to the EPL case because the span appears to have an influence. For the 1m and 3m spans the 533x210x82, 203x203x46 and 305x305x97 sections had increases that were significant for all five techniques. The 5m and 6m spans followed a similar trend excluding the 533x210x82 section.

These trends are evident in the stiffener comparison graphs in *Section 6. Discussion of Results*.

4. Shifting Centroid and Shear Centre

The location of the centroid and shear centre in the notched region was tracked with each change in downstand (or upstand) height. The following sections discuss and show the effects on the centroid and shear centre location due to the addition of the downstands or upstands. The centroid and shear centre are measured from the bottom flange as indicated in **Figure 4.1**.

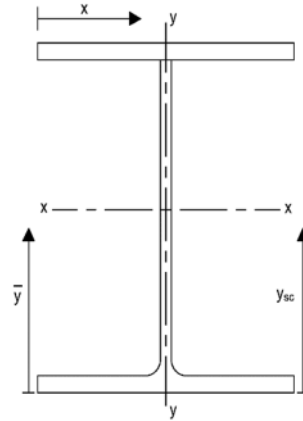


Figure 4.1: Sign Convention for Graphs

The three stiffening techniques: stiffener along notch (FHEN, FHN and FLH are grouped as “along notch”), FLHFHV and FHHVNTS required different numbers of downstand in each section because the shear centre was much lower in some techniques than others for the same section.

FHEN, FLH and FHN are grouped together because they required the same number of downstands since the same horizontal stiffener was used for each technique and was at the same height for each setup. The difference between these techniques is the number of vertical stiffeners (FHEN has two whereas FHN and FLH have one) and the width of the vertical stiffeners relative to the bottom flange as shown in **Figure 4.2**. The vertical stiffener is as wide as the bottom flange for the FLH technique but not for the FHEN and FHN techniques.

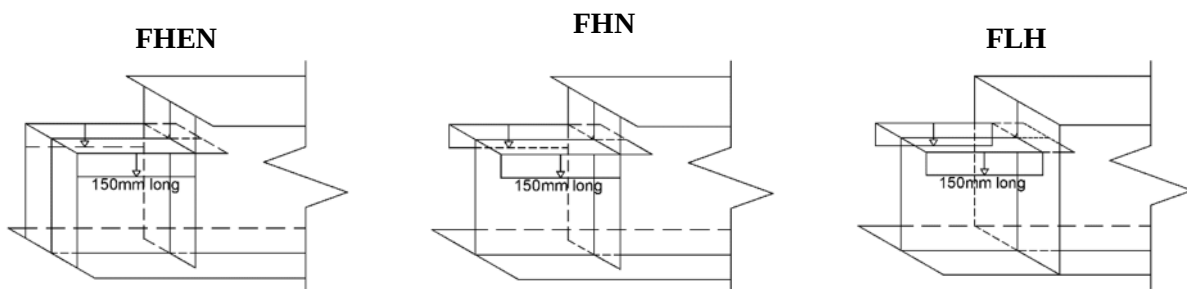


Figure 4.2: The "Along Notch" Group

Upstands were only used in the FLHFHV technique because of the proximity of the centroid and shear centre when the horizontal and vertical stiffeners were applied. That is: introducing downstands to shift the shear centre towards and past the centroid would require downstands of impractical heights since a height of 5mm would shift the shear centre above the centroid. Therefore, downstand heights of less than 5mm would have to be used which is impractical. Using downstands exclusively will only show the behaviour when the shear centre is above the centroid in the notched region. There would be insufficient data points to track the shear centre as it approaches and passes the centroid in the notched region if 5mm. Therefore, upstands were introduced to have more data points to track the shifting shear centre as it approaches and passes the centroid in the notched region.

The shear centre always moved towards and passed the centroid in the notched region, but it did not always pass the un-notched centroid due to height restrictions of some of the techniques (there was not enough space for the longer downstand heights necessary). This is because the point at which the shear centre in the notched region is at the same vertical height as the un-notched region was a point of interest.

4.1 203 x 133 x 25 Section

4.1.1 Along Notch Technique

Figure 4.3 shows the effects of introducing downstands for the FHEN, FHN and FLH techniques (which are grouped together as “along notch”). At 50mm downstand height, the centroid and shear centre in the notched region coincide. However, the centroid and shear centre of the un-notched region is much higher therefore the shear centre of the notched region only reaches it at the 90mm downstand.

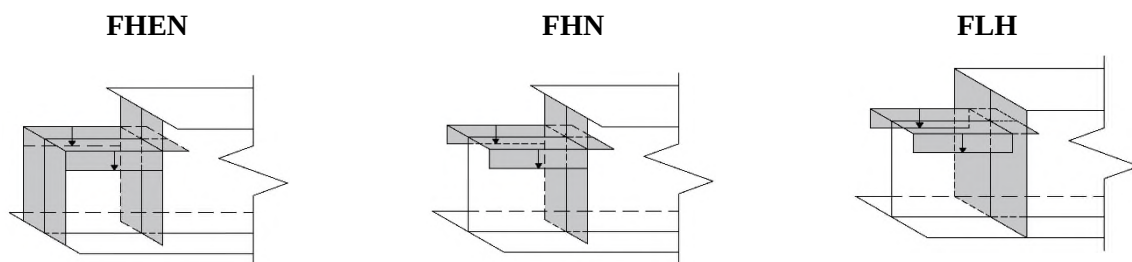
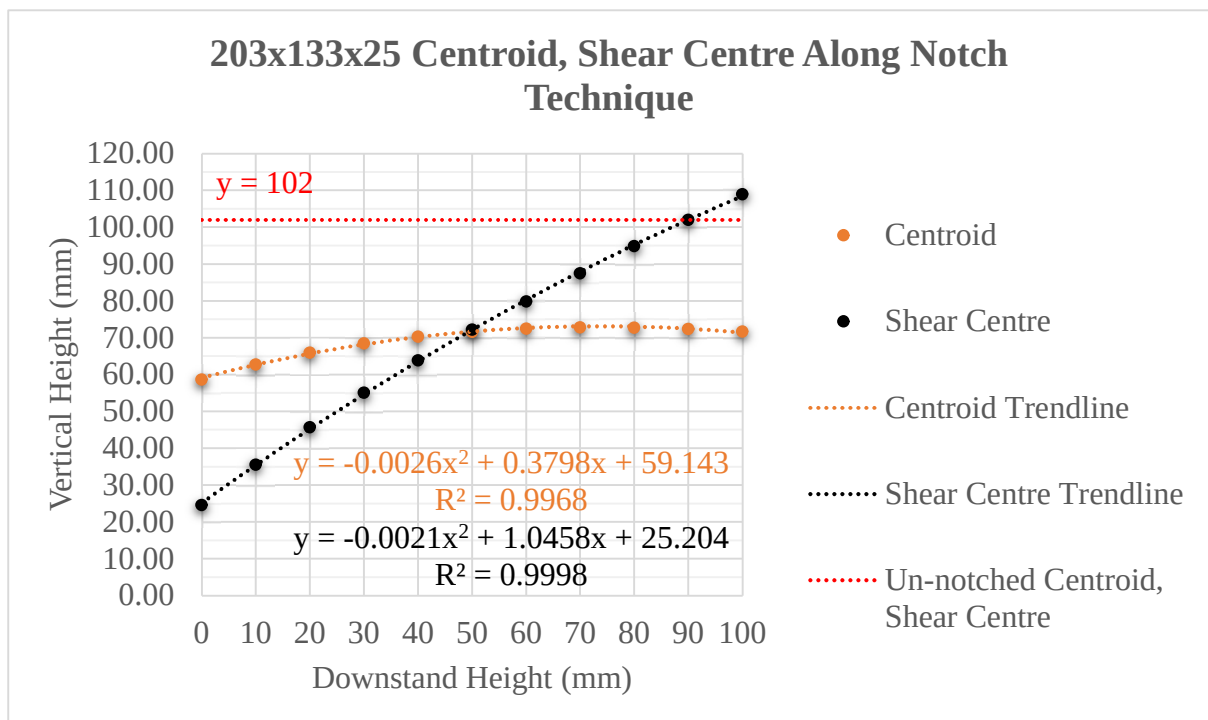


Figure 4.3: 203x133x25 Centroid, Shear Centre Along Notch Technique

4.1.2 FHHVNTS Technique

Figure 4.4 shows the results of the downstands on the FHHVNTS stiffening technique. The centroid and shear centre in the notched region meet at a downstand height of 17mm but the shear centre never reaches the centroid or shear centre of the un-notched region. This is due to a height restriction: since the horizontal stiffener lies at the midpoint of the notched part there is not enough space for the downstand height required to cause the shear centre of the notched region to meet, and pass, the centroid and shear centre of the un-notched region.

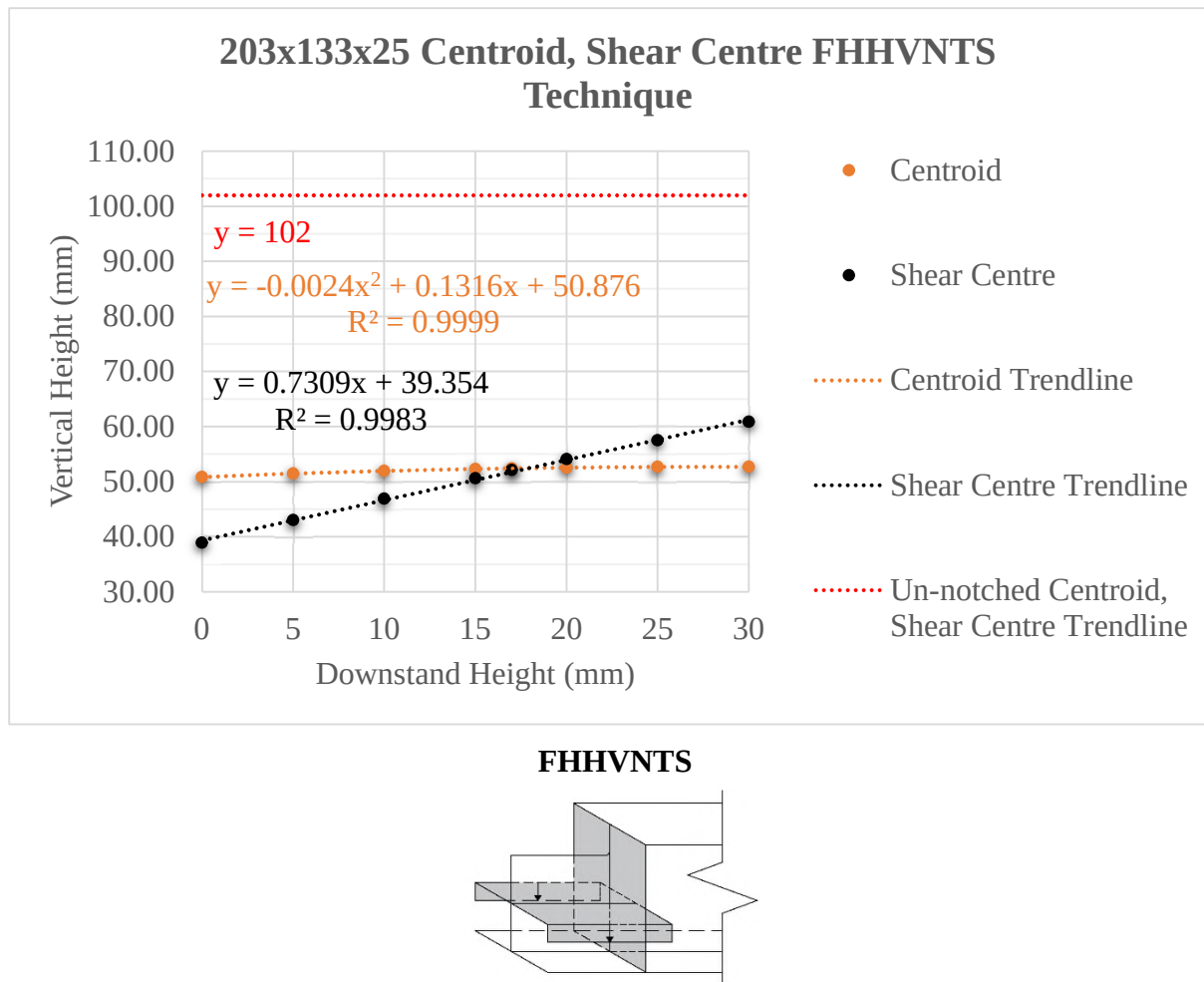


Figure 4.4: 203x133x25 Centroid, Shear Centre FHHVNTS Technique

4.1.3 FLHFHV Technique

Figure 4.5 shows the influence of the upstands and downstands on the centroid and shear centre for the FLHFHV stiffening technique. The centroid and shear centre coincide with no upstands or downstands therefore a combination of upstands and downstands was used. The upstands will shift the shear centre upward to have more data points to study the behaviour as the shear centre approaches the centroid. This is achieved by decreasing the upstand height until there are no upstands present because in this arrangement: the centroid and shear centre are at the same vertical height. Thereafter, downstands are added to shift the shear centre past the centroid. The shear centre of the un-notched region at a downstand height of 30mm.

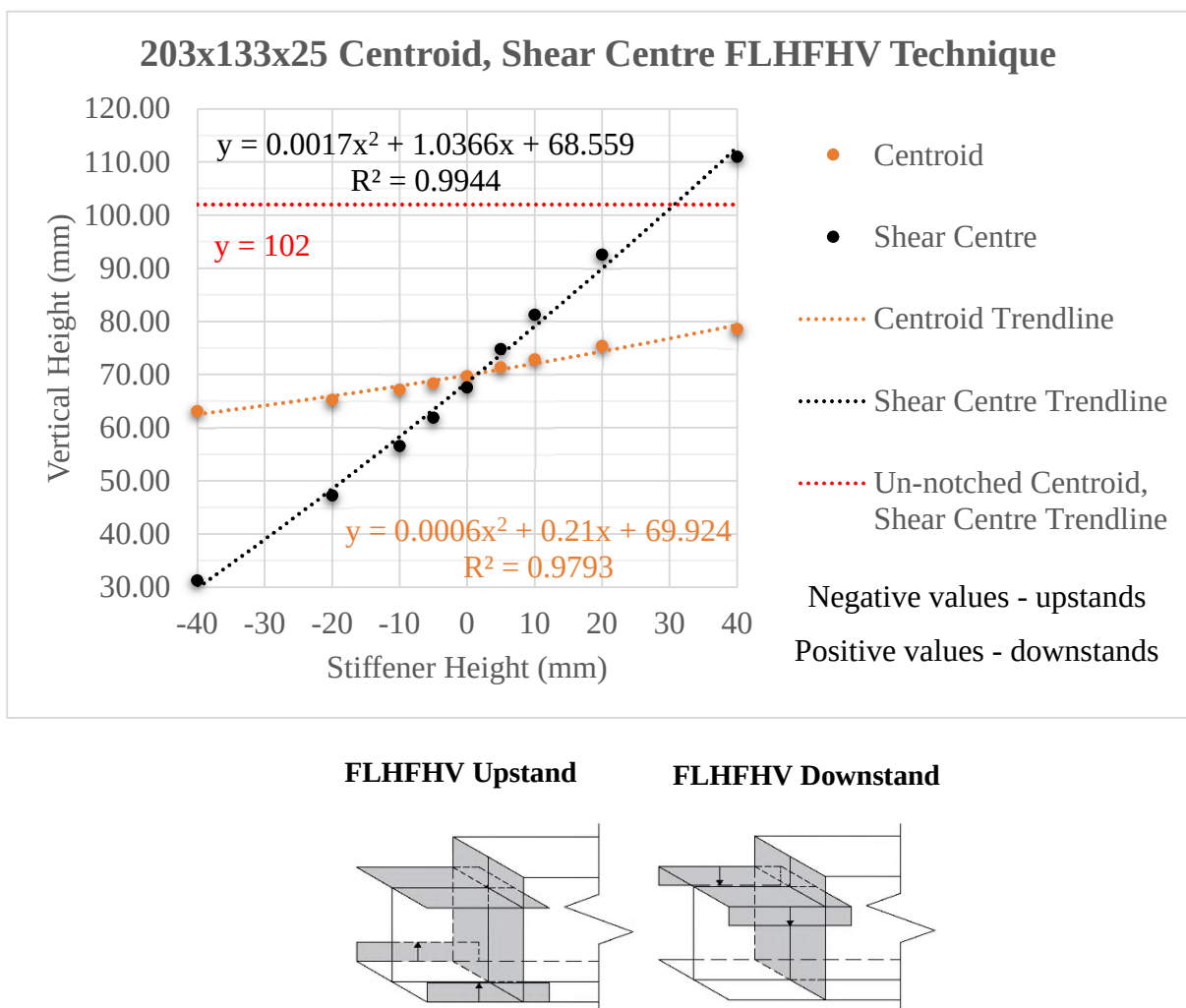


Figure 4.5: 203x133x25 Centroid, Shear Centre FLHFHV Technique

4.2 Remaining Sections

A similar process, as described in *Section 4.1 203 x 133 x 25 Section*, was followed for the remaining sections:

- 305x102x25 section
- 533x210x82 section
- 203x203x46 section
- 305x305x97 section

The graphs for each technique have been put in *Appendix B: Shifting Centroid and Shear Centre*.

5. Parametric Study: Finite Element Model

The variables investigated in the parametric study are related to the arrangement and geometry of the stiffeners, the location of the centroid and shear centre as well as the critical elastic moment. These variables are linked with the objectives of this investigation as well as the key research output.

5.1 Discussion of FEM

5.1.1 Aalberg (2015b)

Aalberg (2015b) investigated the reaction force capacity of top notched I sections experimentally and numerically to extend the experimental data. The length of the beam was 630mm and consisted of either solid or shell elements. The solid elements used four eight node elements through the plate thickness. The shell elements were type S4R with five integration points through the thickness of the shell and a mesh size of 7×7mm (web elements at the flange web interface were modelled with increased thickness). One material was used for the entire web to consider the nonlinear material response. Strain hardening was also considered. Nonlinear geometry was considered, and isotropic hardening and von Mises yield criterion were applied in the model.

The first elastic buckling mode was used for the imperfection with the same amplitude for all specimens. The imperfection had to be considered because buckling did not occur properly without imperfection present.

5.1.2 Yam et al. (2016)

Yam et al. (2016) investigated local web buckling mechanism and design of double notched beam connections using a finite element model. Yam et al. (2016) also used shell elements of type S4R. In general, the mesh was 10×10mm with a refined mesh of 5×5mm around the notch and connection. The nonlinear material property of steel was modelled using the isotropic hardening model with von Mises yield criterion (similar to Aalberg (2015b)).

The first step in the FEA involved an eigenvalue buckling analysis. This was followed by nonlinear load-deflection analysis. As in Aalberg (2015b), the first mode was used to assess model imperfection.

The second step used the first fundamental buckling mode as the initial imperfection shape (the amplitude of the imperfection can be scaled for sensitivity analysis of the imperfection). Four levels of imperfection were considered: $0.01t_w$, $0.1t_w$, $0.3t_w$ and $0.5t_w$ (t_w is the thickness of the web). The end plate connection, and the double cleat angle, was modelled as a tie interaction with weld failure not being considered. Lateral bracing was defined accordingly to match the actual test setup.

5.2 Model Description

The FEM considered the methods used by Aalberg (2015b) and Yam et al. (2016) outlined in *Section 5.1 Discussion of FEM* in an attempt to generate the most appropriate model. As with these two papers, the FEA software Abaqus was used to generate the model using the Abaqus/Standard model.

The properties of structural steel were used for the model: isotropic elastic material with a Young's modulus of 200GPa and Poisson's ration of 0.3. Elastic buckling analysis was used because the ideal behaviour of the beams was considered through an eigenvalue analysis. The eigenvalues of corresponding eigenmodes are requested using the linear perturbation buckling analysis. The first five eigenvalues were requested in the analysis.

The S4R shell element was selected because it is more computationally efficient than a solid element. The aspect ratio of the elements was aimed at being less than 2:1 as beyond this ratio the model will be too inaccurate. The stiffeners at the notched portion and at the point of load application would be welded to the section for a physical test. The failure of these welds were not considered in the FEM. A fork type support was used at the un-notched end (**Figure 5.1**). Due to the missing top flange at the notched end, a modified fork type support was used (**Figure 5.1**). The support arrangement was the same for all stiffener arrangements

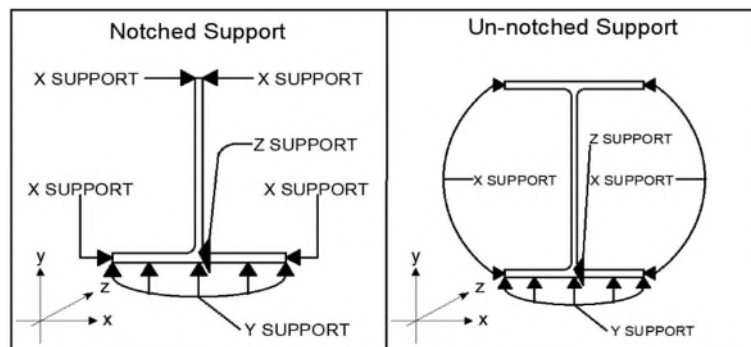


Figure 5.1: Support Restraint

5.3 Model Calibration

Yuan et al. (2013) was used to calibrate the model. The 3m case was used to calibrate the model and is shown in **Figure 5.2**. The FEM with shell elements and a global seed size of 10 resulted in a moment of 33.47kNm which is acceptable.

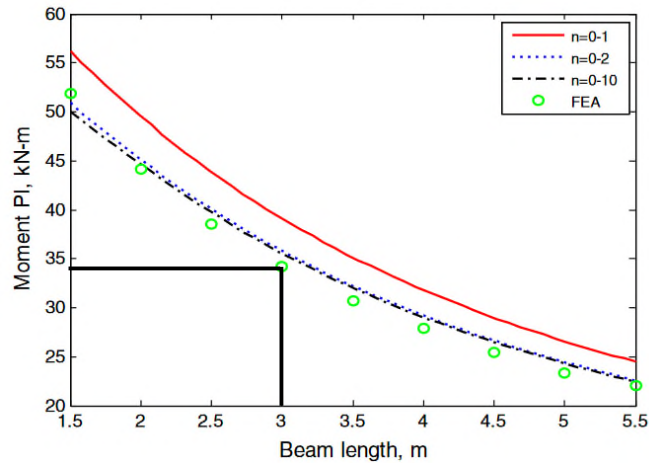


Figure 5.2: Calibration Graph (Yuan et al., 2013)

The calculation is shown below in **Table 5.1** where:

- P_{cr} – critical load (kN)
- L – span of the member (m)
- M_{cr} – critical moment (kNm)

Table 5.1: Calibration Accuracy

Source	P_{cr} (kN)	L (m)	$M_{cr} = P_{cr}L$ (kNm)
Yuan et al. (2013)	11,333	3	34
FEM	11,157	3	33.47

5.4 Convergence Study

A convergence study was conducted, using the first positive eigenvalue, to confirm if the meshing rules used in this report result in critical elastic moment values that are close to the actual value. Initially, the study used a global seeding size that resulted in a large mesh. The global seeding size was reduced which refined the mesh. The critical elastic moment at each global seeding value was recorded. This was completed until the critical elastic moment converged at the value stated in Yuan et al. (2013). The results of the study are shown in **Figure 5.3** and **Table C.1** in *Appendix C: Convergence Calculation*.

For the larger global seed sizes, it was noticed the mesh was inconsistent: that is some elements were distorted. This was not encountered in the smaller values. The global seed size that caused the critical elastic moment to be closest to Yuan et al. (2013), which is indicated by the black horizontal line at 34kNm, was in the range of 5 to 20 mesh sizes. Therefore, a mesh size of 10 was selected because it is the most computationally efficient for all the spans and downstand details.

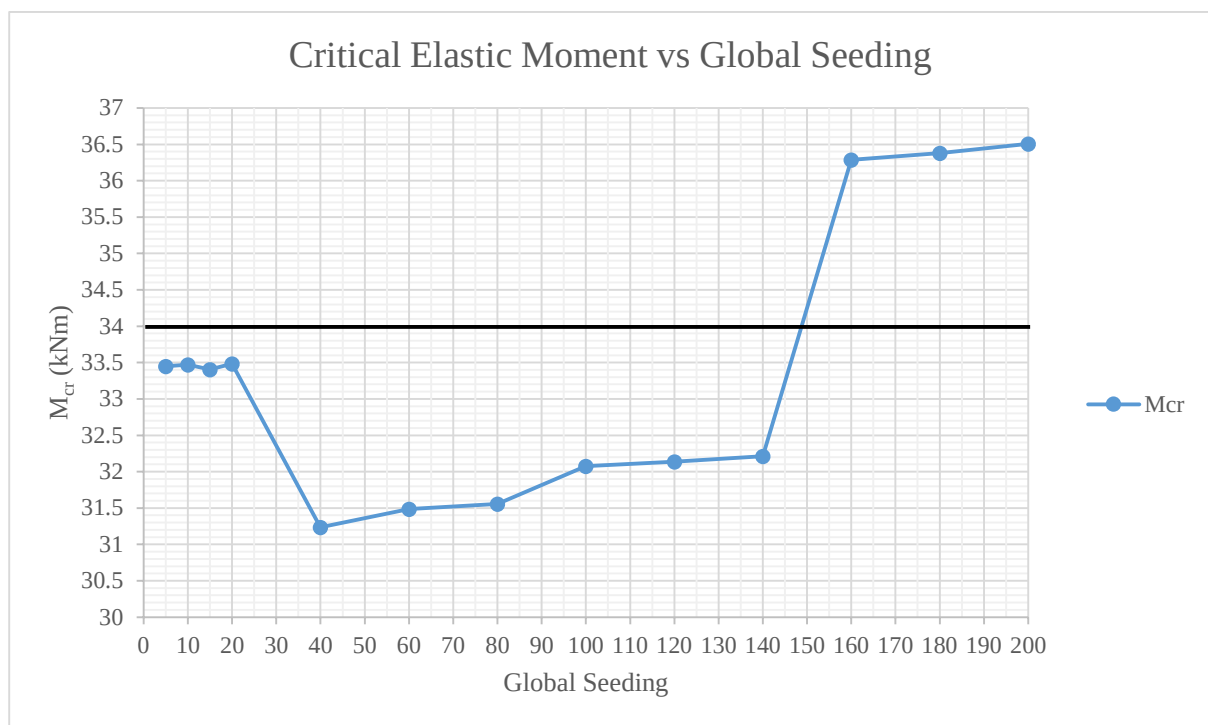


Figure 5.3: Convergence Curve

5.5 Modelling Process

The first FEM considers spans of 1m and 3m for all sections and the five stiffening techniques. The aim of the second model is to investigate the shear centre influence on longer spanning notched beams during LTB (5m and 6m were investigated). The procedure is the same for both models. The first step involved finding the critical elastic moment of standard un-notched section using the FEM for each loading and span case. Thereafter, the process is repeated but with the notches added to the section. This is to find the reduction in the moment capacity. Next, the stiffeners are added to mitigate the effects of the notches, and any failure associated with the notches, so any change in the critical elastic moment is due to the shifting shear centre. Thereafter the downstands are added the investigation into the shifting centroid and shear centre begins. A flow chart of the general process is shown in **Figure 5.4**.

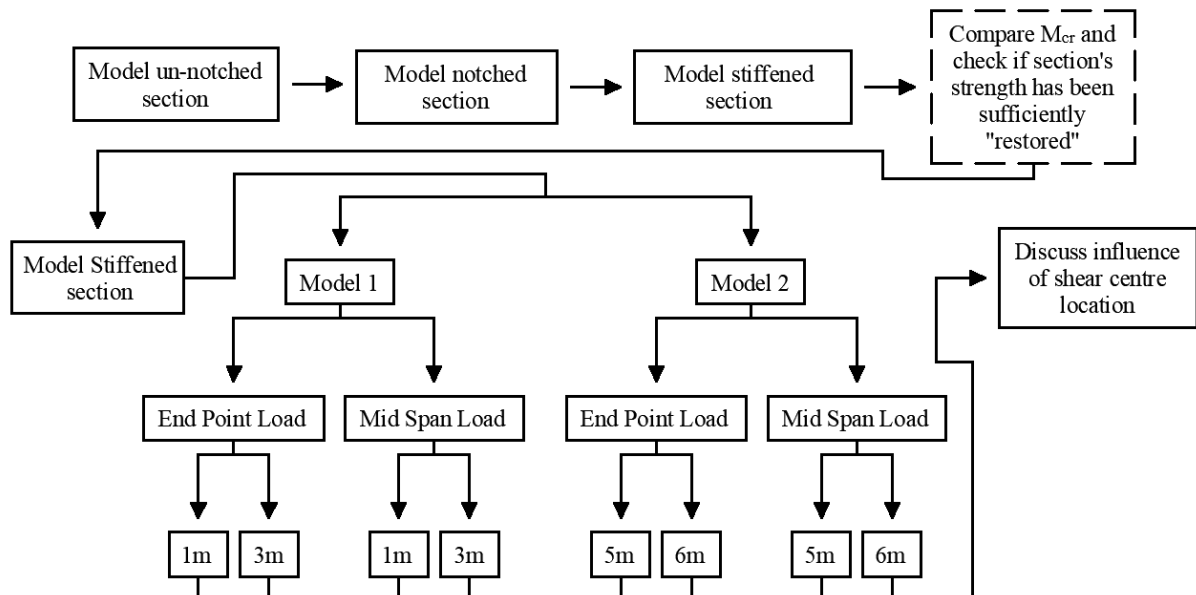
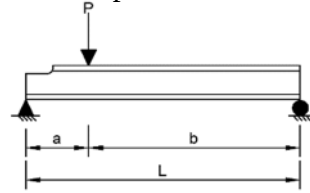


Figure 5.4: Flow Chart Modelling Procedure

5.6 Calculation of Critical Elastic Moment

The critical load was taken as the result of the eigenvalue buckling analysis. The critical elastic moment was then calculated according to equation (5-1) for the end point load case:

$$M = \frac{Pab}{L} \quad (5-1)$$


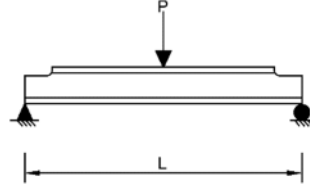
A sample calculation is shown below using the 203x133x25 section 1m span case with 0mm downstand height:

$$M = \frac{Pab}{L} \quad (5-1)$$

$$M = \frac{5.49 \times 10^5 \times 0.6 \times 0.4}{1}$$

$$M = 131760N = 131.76kNm$$

The critical elastic moment was calculated according to equation (5-2) for the mid span point load:

$$M = \frac{PL}{4} \quad (5-2)$$


A sample calculation is shown below using the 203x133x25 section 1m span case with 0mm downstand height:

$$M = \frac{PL}{4} \quad (5-2)$$

$$M = \frac{5.27 \times 10^5 \times 1}{4}$$

$$M = 131.75 \times 10^3Nm = 131.75kNm$$

6. Effects of Shifting the Centroid and Shear Centre

6.1 Influence of Downstand Length and Horizontal Stiffener

Another aspect that was investigated, before the vertical stiffener was fully extended, was the influence of the stiffener and downstand in the un-notched region because in some cases both these elements extend into the un-notched region. It is uncertain if the length of the downstand in the un-notched region influences the critical elastic moment. The stiffener and downstand will not be in the region for a significant length, say 50mm, but the influence of the stiffener must be investigated because in the FHHVNTS and FLHFHV techniques: the downstands and upstands extend into the un-notched region.

Two techniques were used to test the differences: the HS (horizontal stiffener with 150mm long downstand) and FD (full length downstand of 200mm) techniques are shown in **Figure 6.1**. Both techniques were tested on the 203x133x25 section with spans of 1 and 3m with the End Point Load (EPL) and Mid Span Load (MSL) cases.

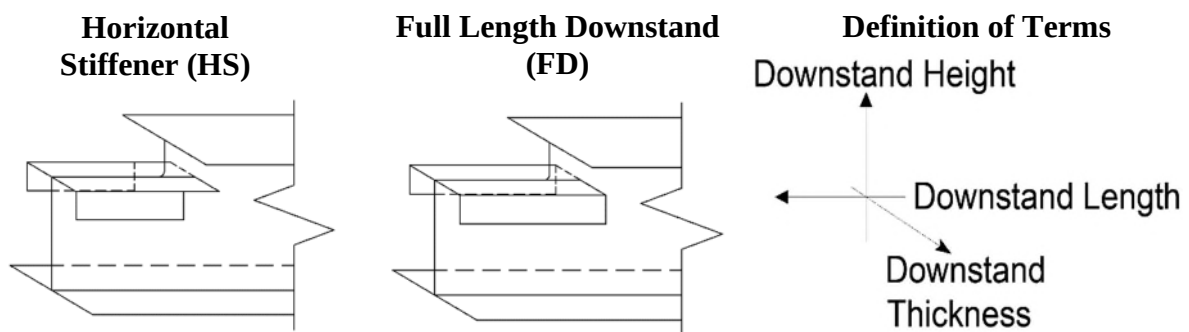


Figure 6.1: Tested Techniques

The standard 203x133x25 section has the centroid and shear centre at 101.6mm from the bottom flange. The HS and FD stiffening technique both use the same horizontal stiffener which is along the notch.

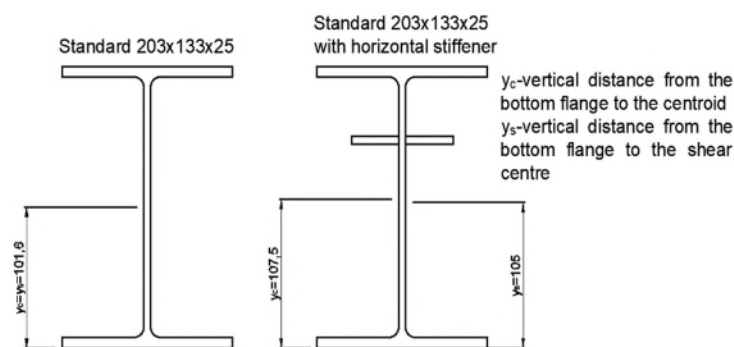


Figure 6.2: Change in Centroid and Shear Centre Location

The horizontal stiffener negligibly changes the location of the centroid and shear centre in the un-notched region from 101.6mm to 107.5mm and 105mm respectively as shown in **Figure 6.2**. This is a small shift which is why the effects are ignored for all tested sections.

Figure 6.3 shows the influence of the downstands on the standard 203x133x25 section using the FD technique (which has the downstand extending into the un-notched region). The centroid shifts upward 0.06mm until the 40mm downstand after which it shifts downwards 4.87mm which is negligible. The shear centre shifts upward 32mm.

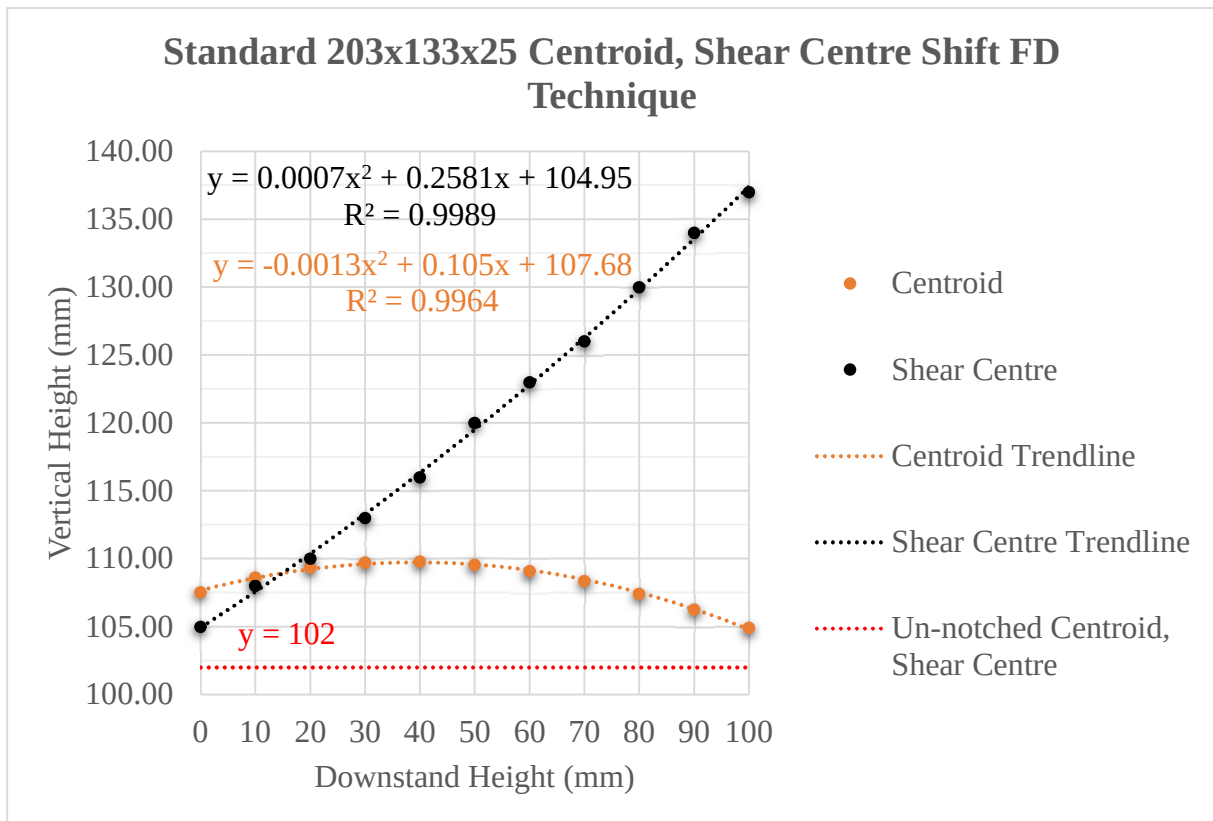
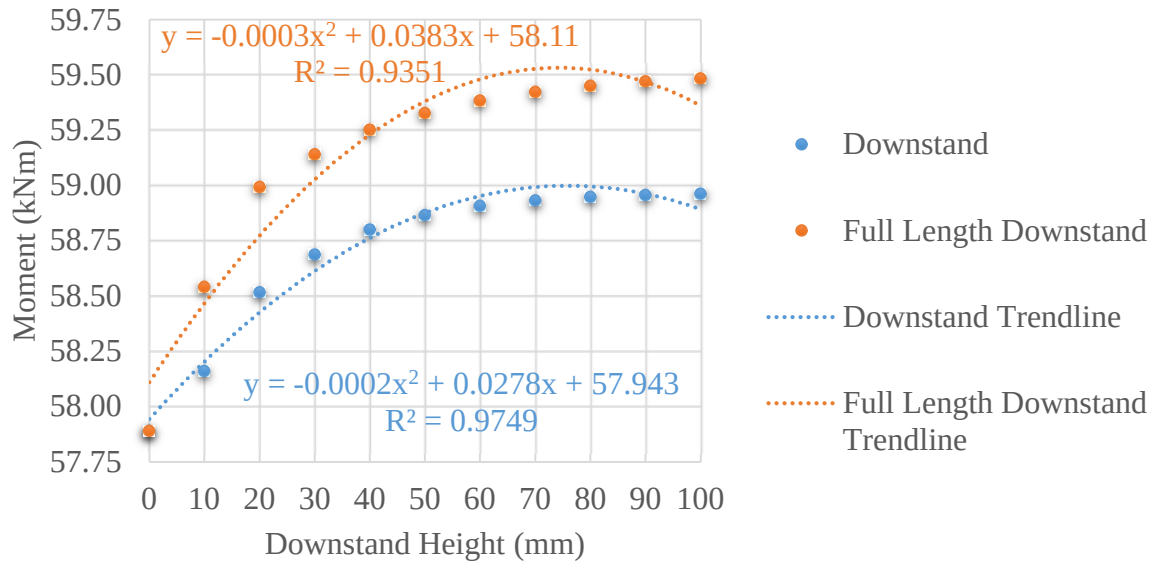


Figure 6.3: Shifting Centroid and Shear Centre in Un-Notched Region FD Technique

Figure 6.4 shows the influence of the downstands on the 1m and 3m EPL load case whereas **Figure 6.5** shows the 1m and 3m MSL load case. The trend is consistent across all span and load cases: the full length downstand increases the critical elastic moment capacity more but by an insignificant margin. The trend in the change in the critical elastic moment across each downstand height is also consistent. Consequently, there is no significant influence on the critical elastic moment if the downstand extends the full length of the horizontal stiffener or if it extends just the length of the notched region.

Critical Moment of Downstand and Full Length Downstand for 1m Span EPL



Critical Moment of Downstand and Full Length Downstand for 3m Span EPL

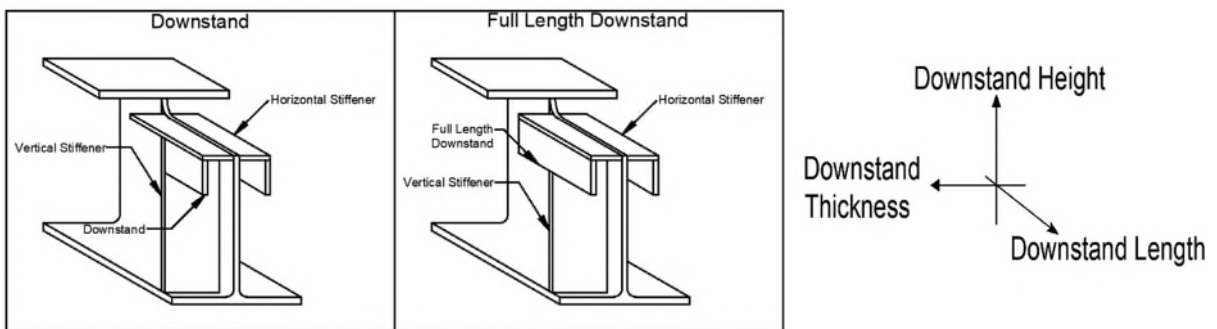
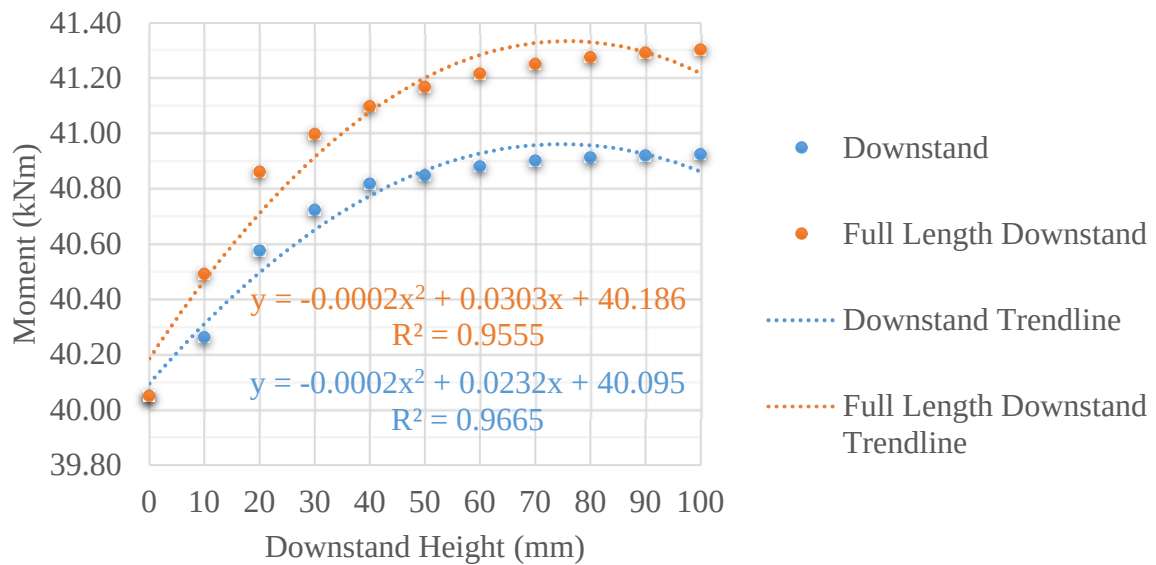
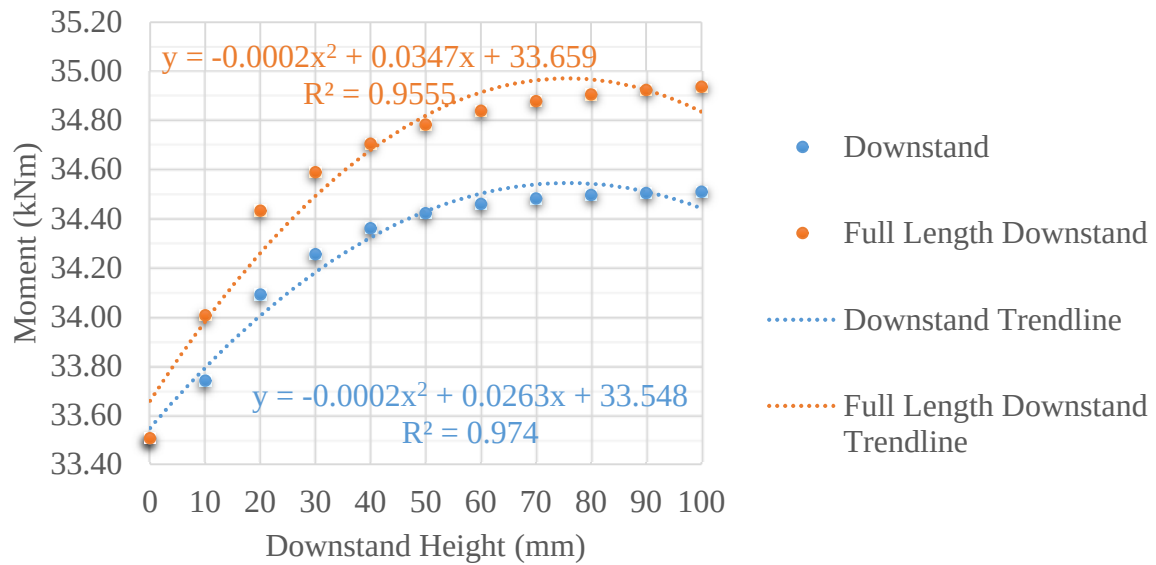


Figure 6.4: Downstand and Full Length Downstand 1m, 3m EPL Comparison

Critical Moment of Downstand and Full Length Downstand for 1m Span MSL



Critical Moment of Downstand and Full Length Downstand for 3m Span MSL

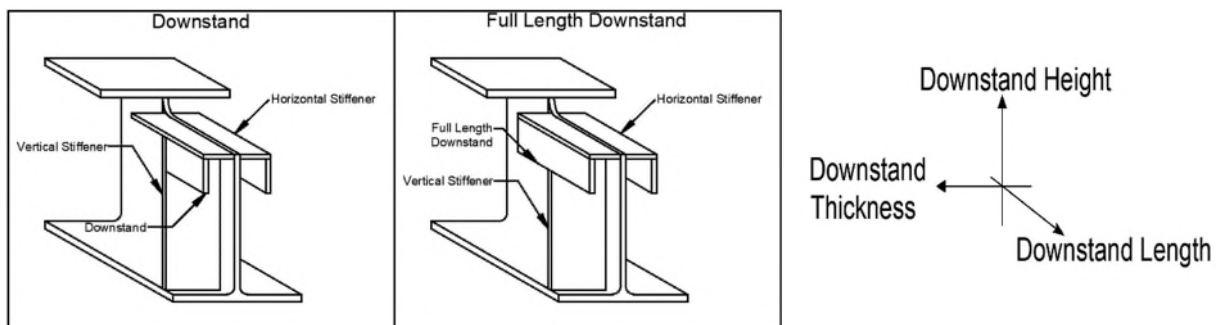
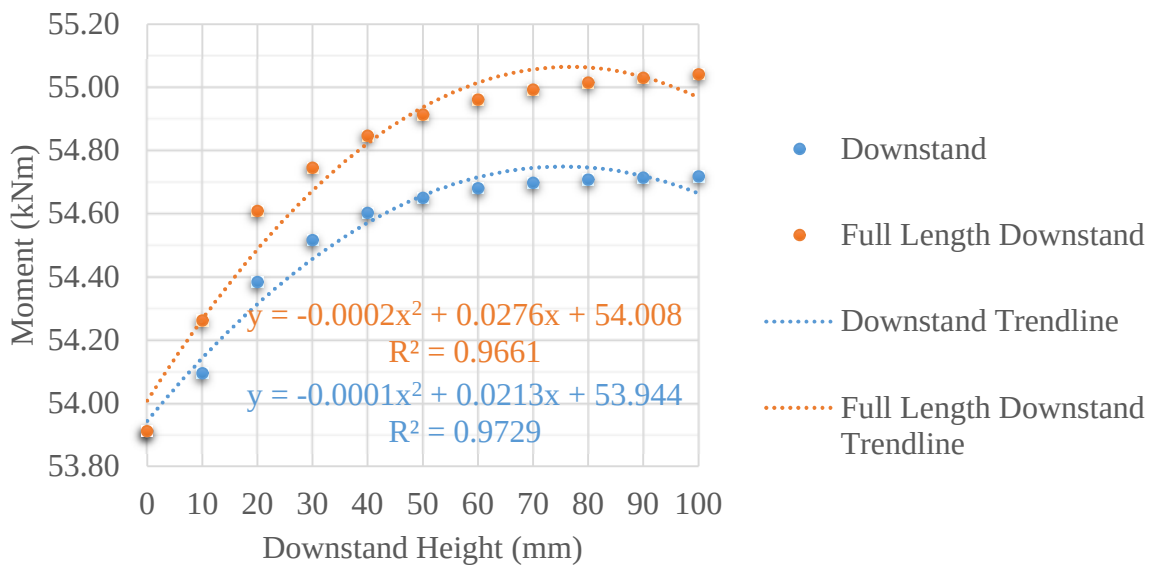


Figure 6.5: Downstand and Full Length Downstand 3m MSL Comparison

6.2 Influence of Vertical Stiffener Length Relative to Flange

The five selected stiffening techniques all utilise a vertical stiffener which fully extends to the top flange. However, the length relative to the bottom flange is different for some of the techniques. One of the initial tests, before the vertical stiffener was fully extended, was to investigate the influence of the length of the vertical stiffener relative to the width of the bottom flange. The FHEN and FHN techniques have a vertical stiffener that does not fully extend the width of the bottom flange whereas the remaining three techniques (FLH, FHHVNTS and FLHFHV) have lengths that are equal to the flange width. **Figure 6.6** shows the difference between the “stiffener” which does not extend the width of the flange and the “full length stiffener” which does.

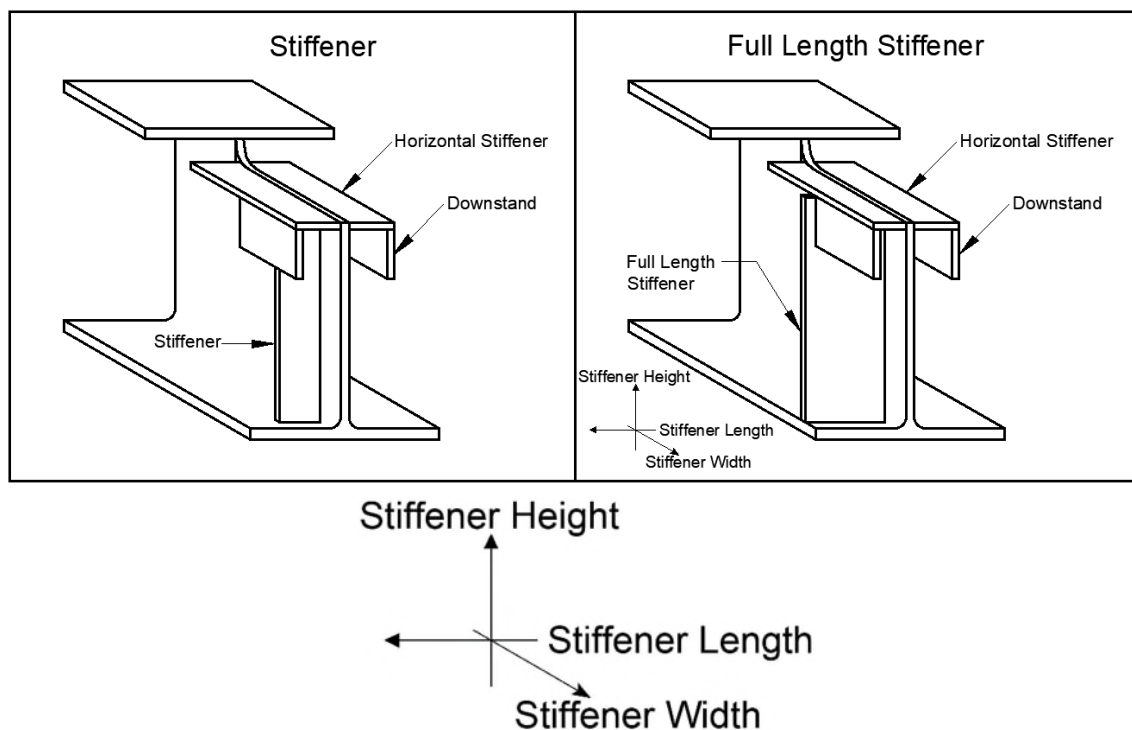
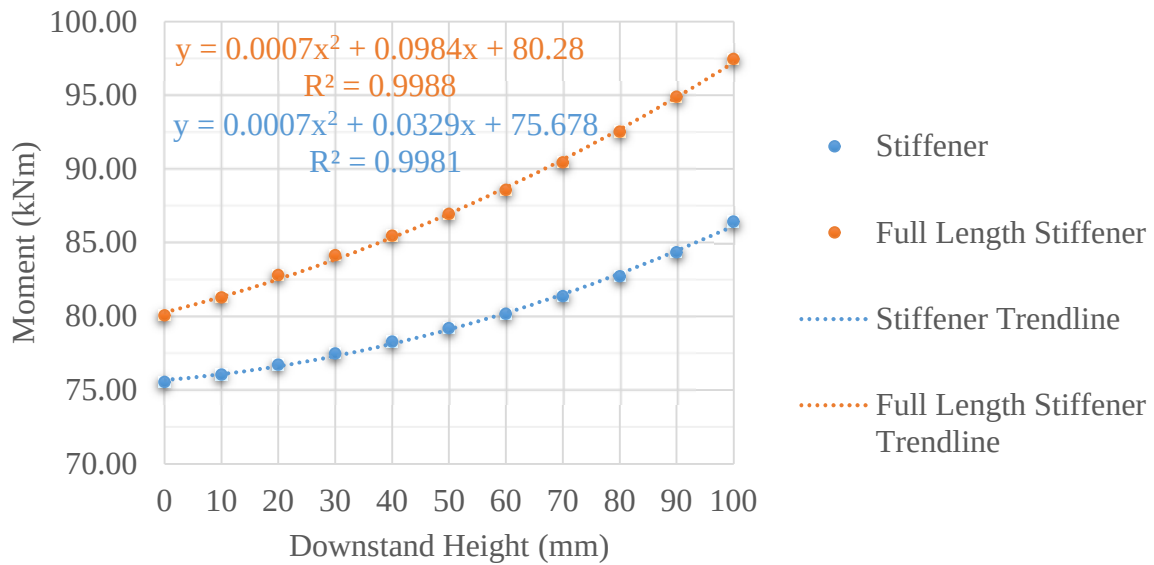


Figure 6.6: Vertical Stiffener Arrangement

The 203x133x25 section was used to investigate this influence. The EPL and MSL load cases were used with spans of 1m and 3m. **Figure 6.7** shows the 1m and 3m spans for the EPL case whereas **Figure 6.8** shows the 1m and 3m spans for the MSL load case. For both load and span cases, the trend is the same: as the downstand height increases, the critical elastic moment increases but the full length stiffener increases at a greater rate. Increasing the length of the vertical stiffener increases the critical elastic moment regardless of the load arrangement or span. This is also evident in the increase in the critical elastic moment to the notched sections: the techniques that have a full length vertical stiffener restore the section closest to the un-notched critical moment capacity.

The reason for this check is to understand if the different techniques have different outcomes due to the different stiffener lengths and if this will impact the investigation into the shifting centroid and shear centre. As discussed above, the trends are not skewed by the increase in the stiffener width, the critical elastic moment increases at a greater rate. Therefore, any changes to the critical elastic moment are due to the introduction of the downstands or upstands that shift the centroid and shear centre.

Critical Moment of Stiffener and Full Length Stiffener for 1m Span EPL



Critical Moment of Stiffener and Full Length Stiffener for 3m Span EPL

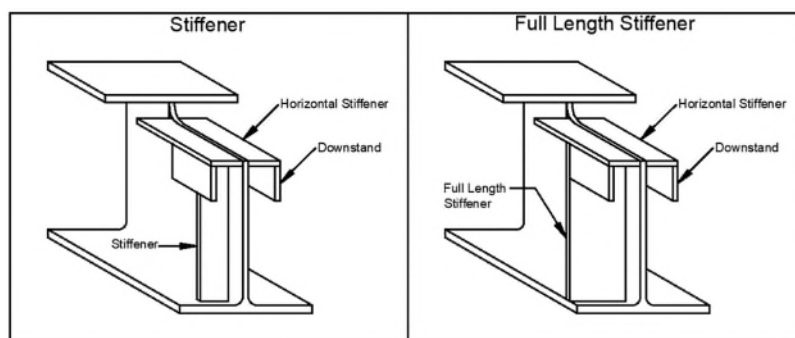
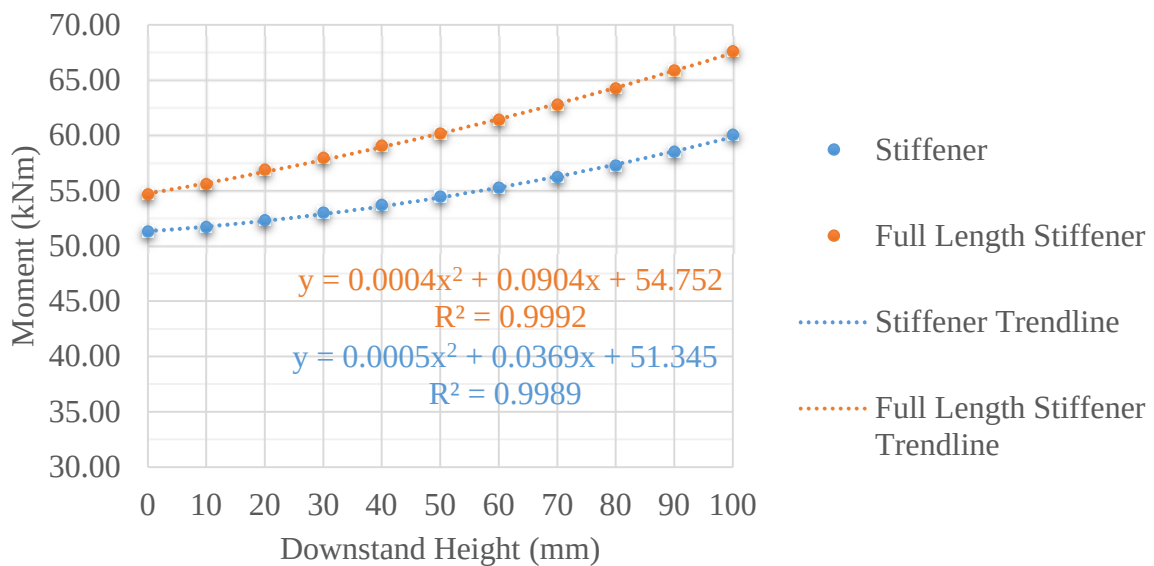
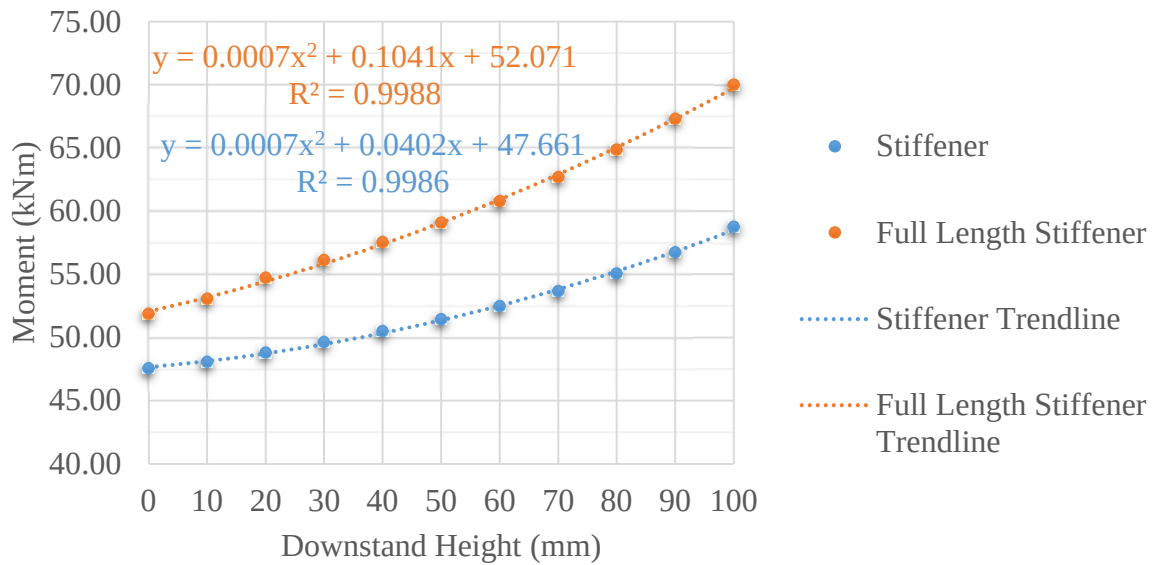


Figure 6.7: Stiffener and Full Length Stiffener 1m, 3m EPL Comparison

Critical Moment of Stiffener and Full Length Stiffener for 1m Span MSL



Critical Moment of Stiffener and Full Length Stiffener for 3m Span MSL

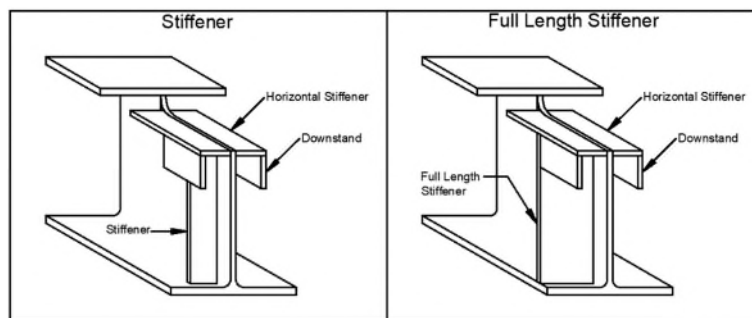
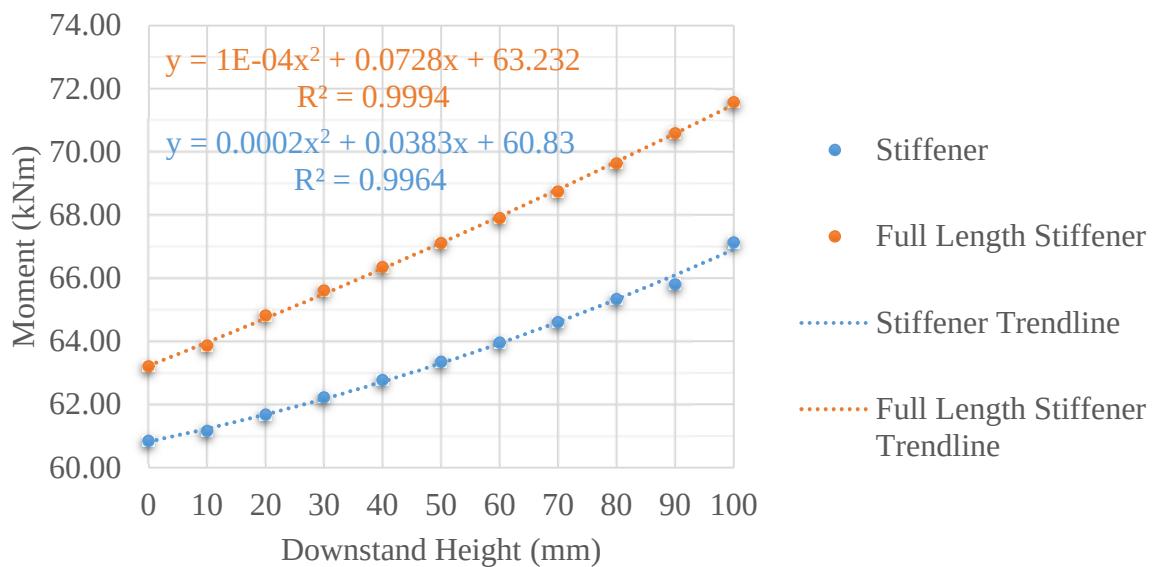


Figure 6.8: Stiffener and Full Length Stiffener 3m MSL Comparison

6.3 Order of Discussion

The discussion of the results for each beam is split into three parts. Part 1 and Part 2 discuss and show by how much the stiffening techniques, when applied to the notched section in Model 1 or Model 2, restores the critical elastic moment for the EPL and MSL cases. Part 1 discusses Model 1 which refers to spans of 1m and 3m whereas Part 2 discusses Model 2 which comprises of the 5m and 6m spans. A bar graph is used for this comparison with two bars for each technique: one for the notched section (orange) and one for when the stiffening technique is applied (blue). The horizontal grey line indicates the un-notched section's critical elastic moment.

Sub sections follow discussing the results of each technique with reference to a scatter plot for the two spans in the Model. The black dashed vertical line represents the downstand height at which the centroid and shear centre coincide in the notched region for that section and stiffening technique. The red dashed vertical line indicates the downstand height at which the shear centre of the notched region is at the same vertical height as the shear centre (and centroid) in the un-notched portion of the beam. This line will not always be shown as height restrictions in the notched region prevented the necessary downstand heights, which would cause the shear centre in the notched region to shift close to and past the centroid in the un-notched region, to be tested.

Certain techniques, which exhibit interesting behaviour, are discussed further in Part 3: the *Further Discussion* section. Trends across both models will also be discussed. Two graphs are used in these discussions: percentage change and average percentage change. The percentage change graph represents the incremental change with each increase in downstand height as a percentage of the total change. A sample calculation is completed in **Table D.1 Appendix D: Calculation of Percentage Change**. This graph is useful because it allows the general trend of the various spans and loading configurations for that stiffening technique when applied to the section to be discussed. The left axis plots Δ (%), the percentage change, whereas the centroid and shear centre are plotted on the right axis because these two data sets are of different units.

The average percentage change graph takes the average of the percentage value for all cases at each downstand height (see **Table D.2 in Appendix D: Calculation of Percentage Change** for calculation). This is to closely examine the general trend of the technique around the centroid and shear centre coincidence point. The data is split into two sets: “before” and “after” which represent the average behaviour before and after the downstand height that causes the centroid and shear centre to be at approximately the same vertical height.

This shows if there is any influence on the average trend due to the shifting centroid and shear centre in the notched region.

Outliers are defined as cases that follow a trend that is different to the general trend among the other spans, load cases or in the technique itself. Outliers have been excluded from the percentage change and average percentage change curves since they skew the average change calculations. Cases with a negligible increase or change are defined as cases that have a change in the critical elastic moment less than 0.5kNm. These have been excluded from the percentage change and average percentage change graphs and calculations because if a finer mesh were used in the FEM: the change is likely to tend to zero.

Appendix F: Buckled Shapes shows an image of the buckled shape for the 250mm and 350mm 5m EPL case for the FHEN technique when applied to the 533x210x82 section.

6.4 203 x 133 x 25

6.4.1 Model 1

As previously described, Model 1 involves the testing of the stiffened section for the 1m and 3m span cases with either the EPL (*Figure 6.9*) or MSL (*Figure 6.10*) load cases.

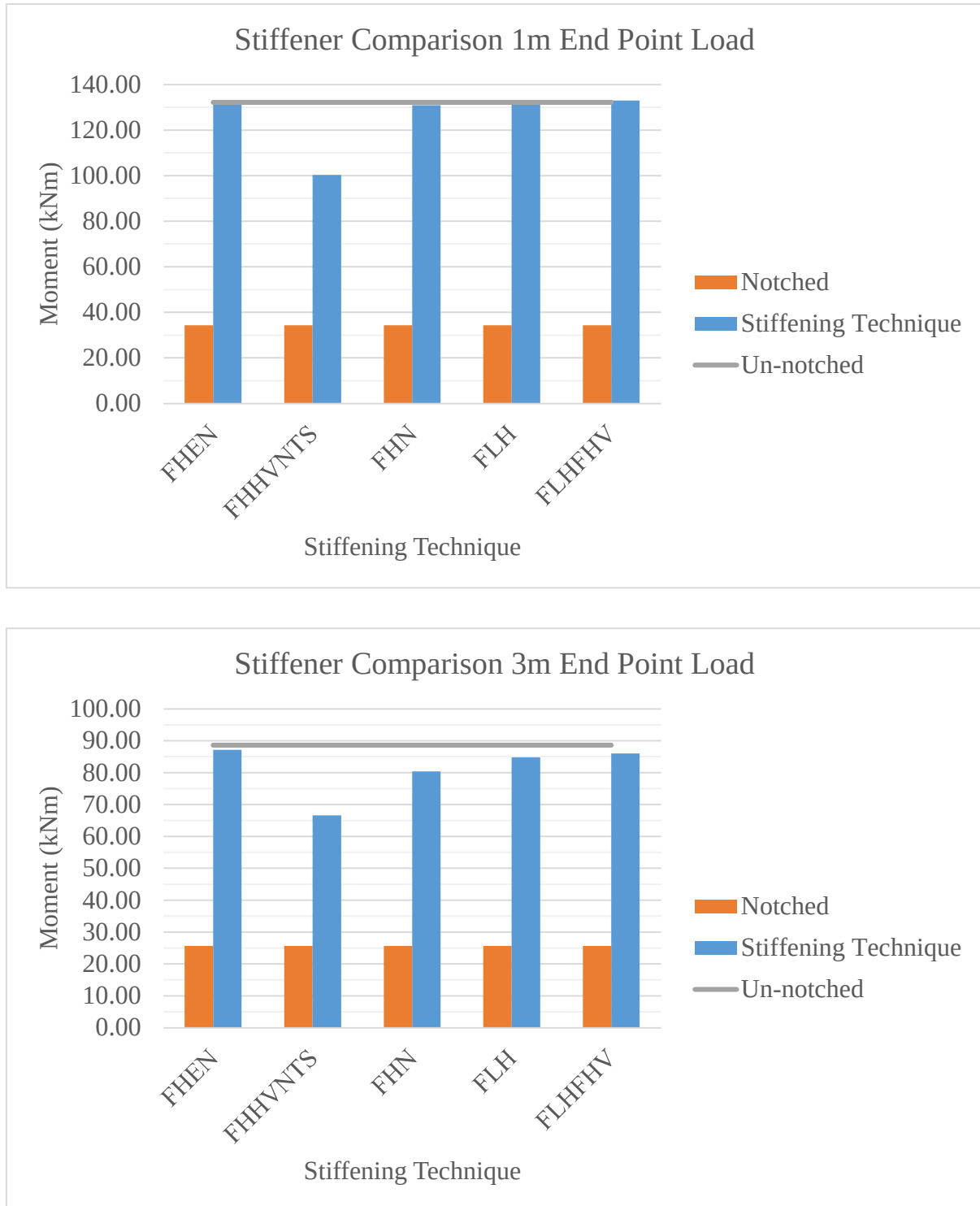


Figure 6.9: Stiffener Comparison 1m, 3m EPL 203x133x25

Figure 6.9 and **Figure 6.10** show the five stiffening techniques and by how much they restore the critical elastic moment to the notched section. It is important to restore the critical elastic moment to be as close as possible to the un-notched capacity because any increase will then be caused by the addition of upstands or downstands and the shifting centroid and shear centre.

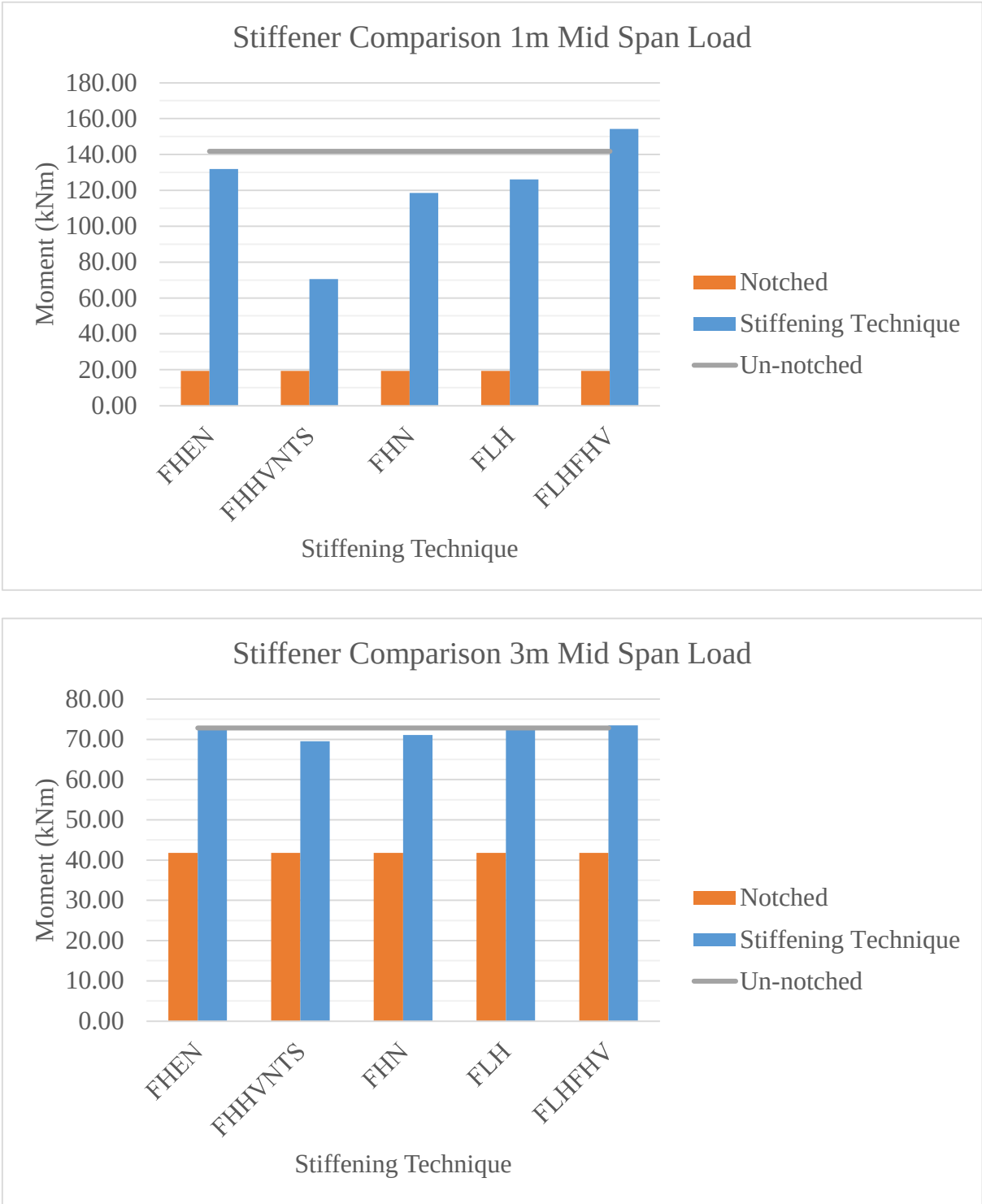


Figure 6.10: Stiffener Comparison 1m, 3m MSL 203x133x25

6.4.1.1 FHEN Stiffening Technique

Figure 6.11 shows the results of adding downstands to the horizontal stiffener and increasing the height of the downstands. The total change in the critical elastic moment for the 1m span is 2.26kNm whereas the 3m span had a total change of 26.61kNm which is more significant. Both spans increase their resistance to LTB. However, the longer span appears to have benefitted more from the introduction of the downstands.

It is clear the critical elastic moment increases as the shear centre approaches and passes the centroid in the notched region. The trend is similar in both cases: the curves start at a minimum value and increases in a slight parabolic manner. There appears to be a change in gradient at the 50mm downstand which is the downstand height that causes the centroid and shear centre of the notched region to be at the same vertical height. The change in gradient is different for each span: the 1m decreases its gradient whereas the 3m increases its gradient.

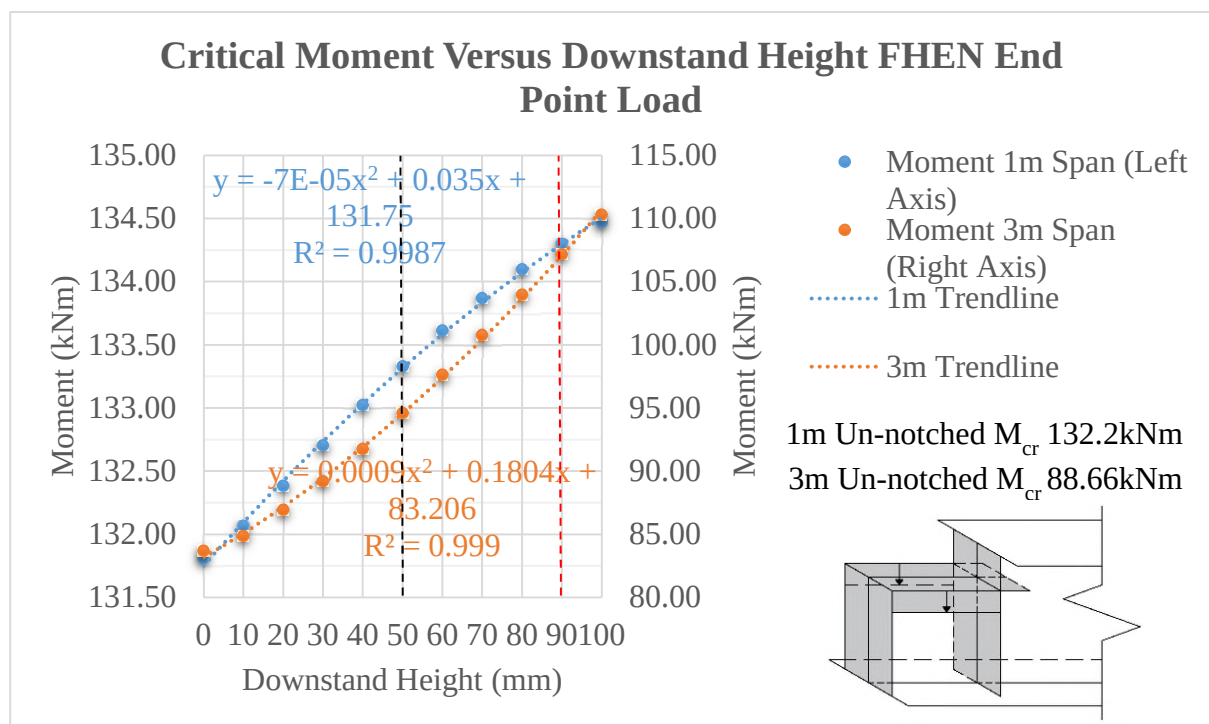


Figure 6.11: Critical Elastic Moment vs Downstand Height FHEN Technique EPL Case Model 1 203x133x25

Figure 6.12 shows the MSL case. The load applied at mid span appears more sensitive to the addition of the upstands for the shorter span. The 1m span increased its critical elastic moment by 21.37kNm whereas the 3m span 17.58kNm. This is different to the EPL case and must be due to the different location of the load.

Once again both spans increase their critical elastic moment with each increase in downstand height but there is a difference in the shape of the trends. The 1m span appears to approach an asymptote at 153kNm which is achieved at the 50mm downstand height: the point at which the centroid and shear centre coincide in the notched region. The 3m span behaves like the 3m EPL case in **Figure 6.11**: the critical elastic moment increases as the shear centre of the notched region approaches and passes the centroid of the notched region with a slight increase in gradient at the 50mm downstand height.

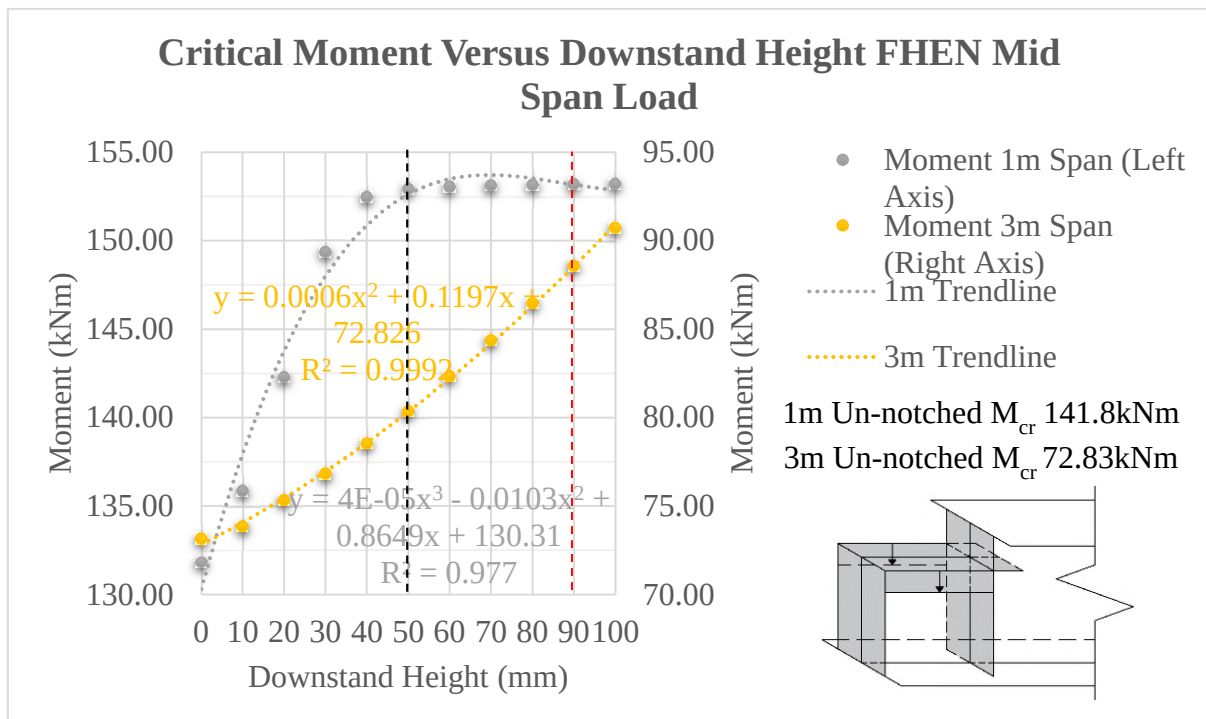


Figure 6.12: Critical Elastic Moment vs Downstand Height FHEN Technique MSL Case Model 1 203x133x25

6.4.1.2 FHHVNTS Technique

Figure 6.13 shows the results for the FHHVNTS EPL case. The total change in moment for the 1m span is 12.19kNm whereas the 3m span had a total change of 8.73kNm. Both spans increase their resistance to LTB however the shorter span appears to have benefitted more from the introduction of the downstands.

The trend is the same for both spans and is similar to the 3m spans in the EPL and MSL FHEN techniques (**Figure 6.11** and **Figure 6.12**). The critical elastic moment increases parabolically as the shear centre approaches and passes the centroid in the notched region (at 17mm) and the gradient increases with each increase in downstand height. As previously mentioned in *Section 4.1.2 FHHVNTS Technique* due to height restrictions in the notched region the required downstand height to shift the shear centre past the centroid in the un-notched region does not fit. It is unclear if there are any changes in the trend around the 17mm downstand height, but it appears the gradient is steeper after the 17mm downstand height.

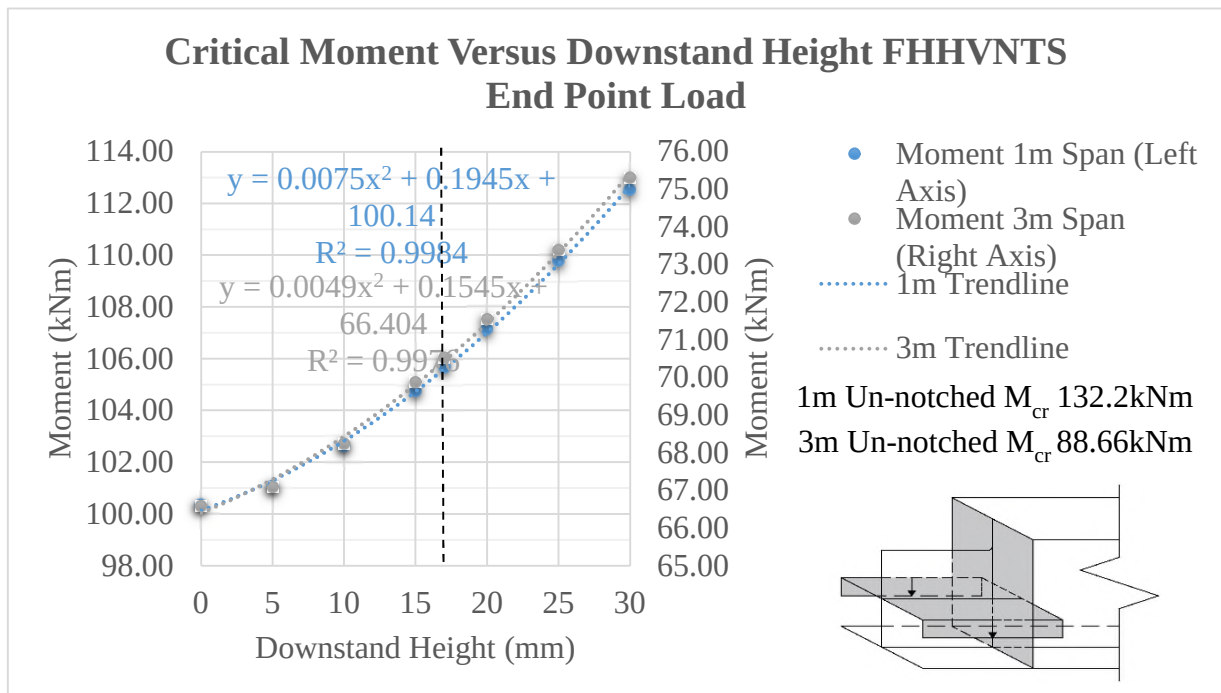


**Figure 6.13: Critical Elastic Moment vs Downstand Height FHHVNTS Technique EPL Case
Model 1 203x133x25**

Figure 6.14 shows the FHHVNTS MSL case. The 1m span increased its critical elastic moment by 15.42kNm whereas the 3m span 4.71kNm. This is consistent with the EPL case: the longer span did not increase its critical elastic moment as much as the shorter span but both spans increased their critical elastic moments.

There is a strong correlation between the results of the two spans: both start at minimum values and increase parabolically with each increase in downstand height. The gradient increases as the shear centre approaches and passes the centroid in the notched region but there are no obvious changes at the 17mm downstand height (just as in the EPL case in **Figure 6.13**). The trend is similar to the:

- 3m EPL FHEN (**Figure 6.11**)
- 3m MSL FHEN (**Figure 6.12**)
- 1m and 3m FHHVNTS (**Figure 6.13**)

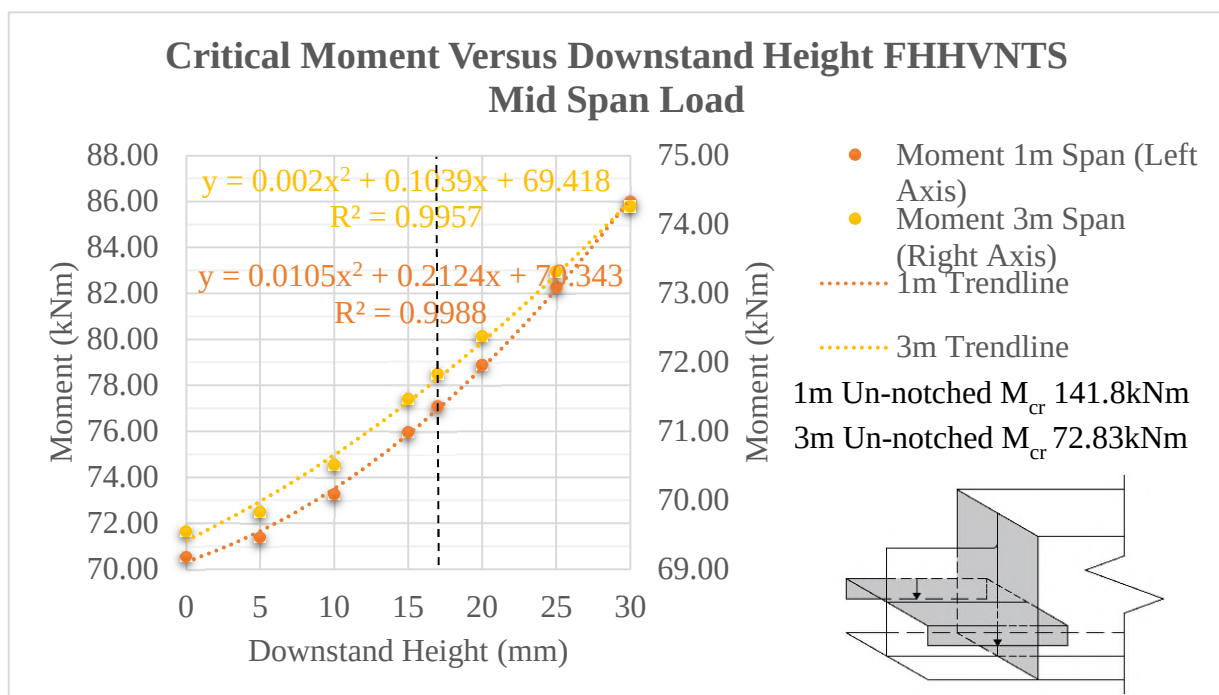


**Figure 6.14: Critical Elastic Moment vs Downstand Height FHHVNTS Technique MSL Case
Model 1 203x133x25**

6.4.1.3 FHN Technique

Figure 6.15 shows the results for the FHN EPL case. The total change in moment for the 1m span is 1.03kNm whereas the 3m span had a total change of 5.03kNm. Both spans increase their resistance to LTB however the 3m span appears to have benefitted more from the introduction of the downstands.

The trend in the increase in the critical elastic moment is the similar to the:

- 3m EPL and MSL cases for the FHEN technique (shown in **Figure 6.11** and **Figure 6.12**)
- FHHVNTS technique (**Figure 6.13** and **Figure 6.14**)

The critical elastic moment increases in a slightly parabolic trend that is concave up. As the shear centre approaches and passes the centroid in the notched region (which occurs at a downstand height of 50mm) it is clear the gradient increases with each increase in downstand height. This can be seen when comparing the gradient before and after the 50mm downstands: the gradient is significantly steeper after the 50mm downstand. Just as in the other cases listed above it appears the section is more sensitive to the changes in downstand height once the shear centre is above the centroid.

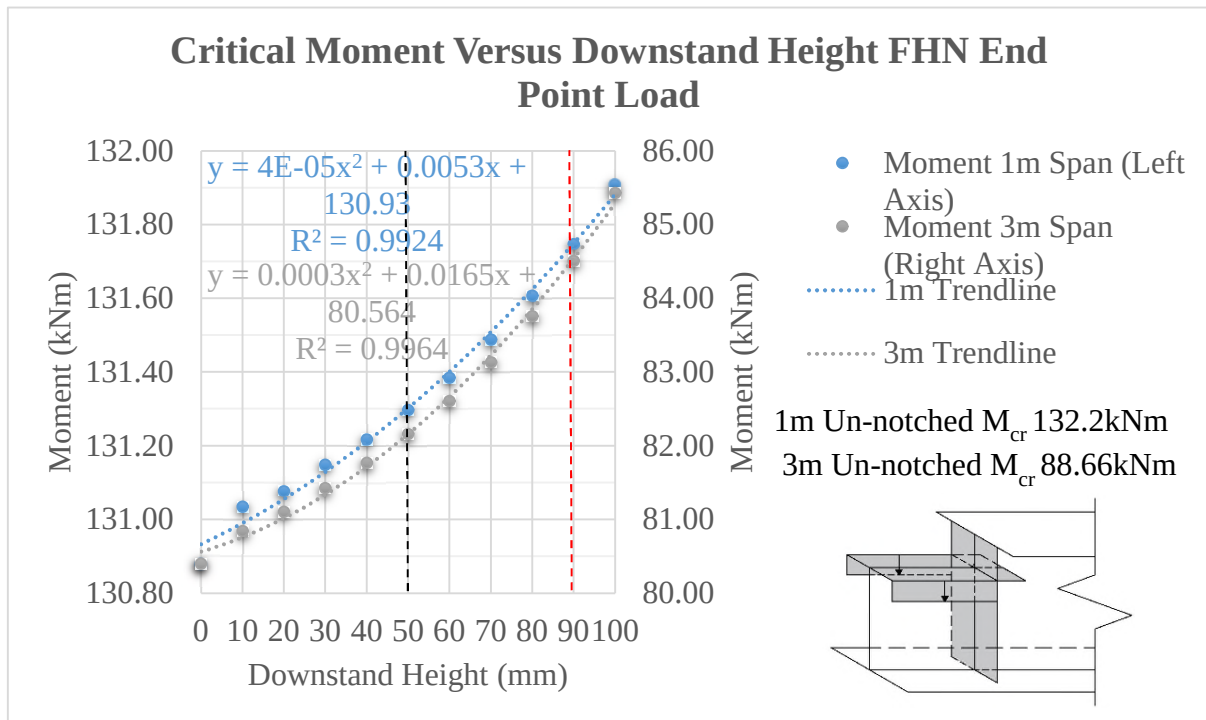


Figure 6.15: Critical Elastic Moment vs Downstand Height FHN Technique EPL Case Model 1 203x133x25

Figure 6.16 shows the results for the FHN MSL case. The total change in moment for the 1m span is 11.75kNm whereas the 3m span had a total change of 3.8kNm. Both spans increase their resistance to LTB however, just as for the FHEN and FHHVNTS techniques, the 1m span increases its critical elastic moment more than the 3m span.

The trend in the critical elastic moment is similar to the:

- 3m FHEN technique for both the EPL and MSL cases (**Figure 6.11** and **Figure 6.12**)
- FHHVNTS technique (**Figure 6.13** and **Figure 6.14**)
- FHN technique EPL case (**Figure 6.15**)

Albeit not as smooth as the above techniques, the trend that the critical elastic moment increases with each increase in downstand height in a slightly parabolic trend concave up remains. There is a distinct change in gradient at the 50mm downstand height for both spans which is the downstand height which the centroid and shear centre of the notched region coincide.

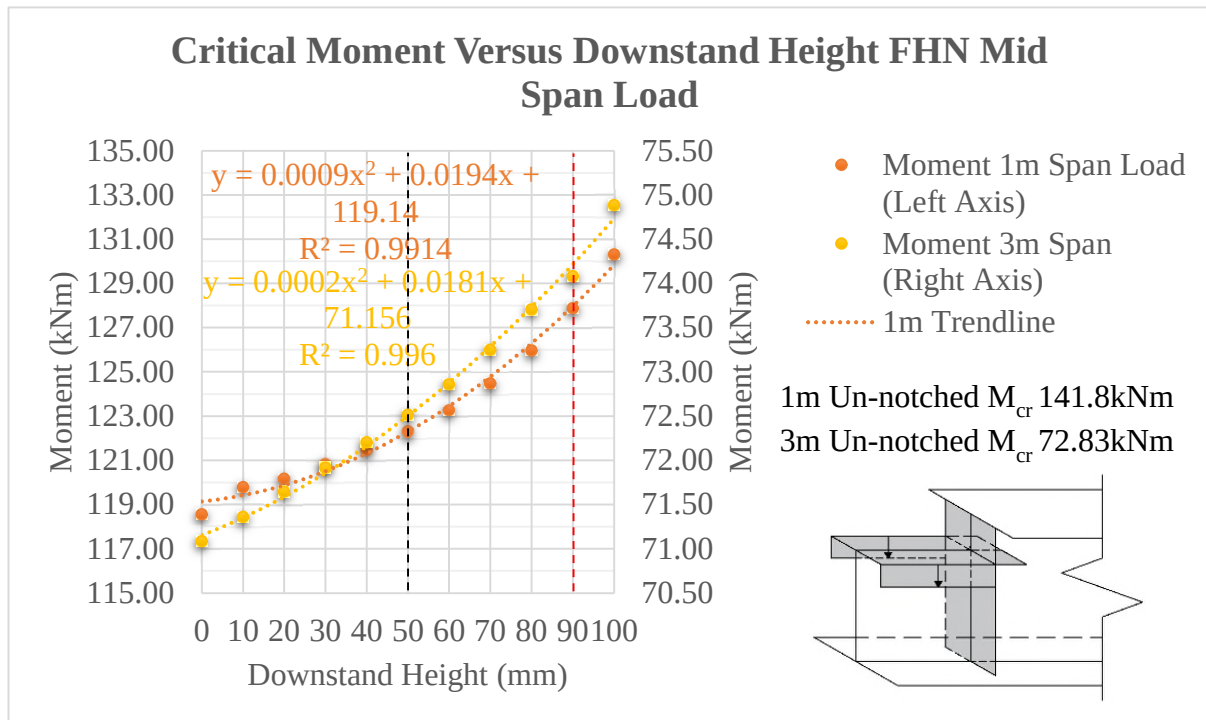


Figure 6.16: Critical Elastic Moment vs Downstand Height FHN Technique MSL Case Model 1
203x133x25

6.4.1.4 FLH Technique

Figure 6.17 shows the results for the FLH EPL case. The total change in moment for the 1m span is 0.64kNm whereas the 3m span had a total change of 6.62kNm. Both spans increase their resistance to LTB however the 3m span appears to have benefitted more from the introduction of the downstands and the change in the 1m span is almost negligible. The trend in both spans is the same as the:

- 3m FHEN technique for both the EPL and MSL cases (**Figure 6.11** and **Figure 6.12**)
- FHHVNTS technique (**Figure 6.13** and **Figure 6.14**)
- FHN technique (**Figure 6.15** and **Figure 6.16**)

Once again, the critical elastic moment increases in a concave up parabola starting at a minimum value. There is no obvious change in the trend around the 50mm downstand height but the gradient increases with each increase in downstand height.

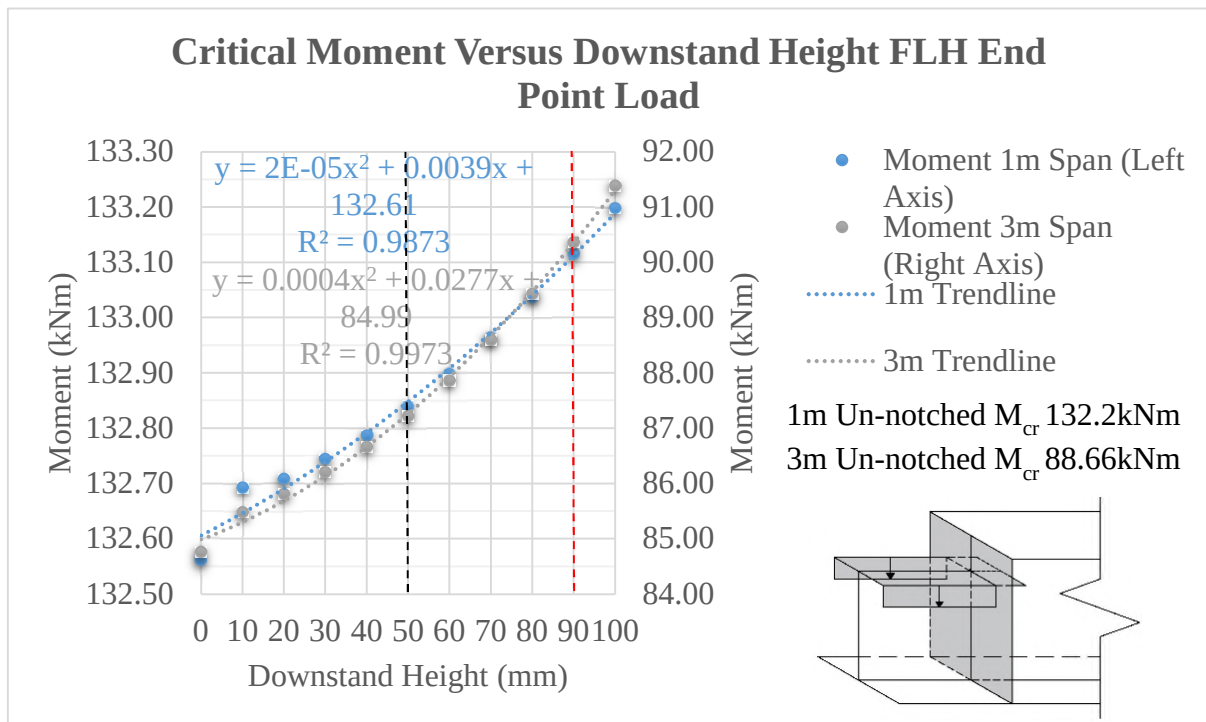


Figure 6.17: Critical Elastic Moment vs Downstand Height FLH Technique EPL Case Model 1
203x133x25

Figure 6.18 shows the results for the MSL case for the FLH stiffening technique. The 1m span had an increase of 5.57kNm whereas the 3m span increased by 5.47kNm. The 3m span case is very similar to the:

- 3m FHEN technique for both the EPL and MSL cases (**Figure 6.11** and **Figure 6.12**)
- FHHVNTS technique (**Figure 6.13** and **Figure 6.14**)
- FHN technique EPL case (**Figure 6.15**)
- FLH technique EPL case (**Figure 6.17**)

The 1m span appears to approach an asymptote at 154kNm which is achieved at the 50mm downstand height: the point at which the centroid and shear centre coincide in the notched region. The 1m span is the only other technique that exhibits similar behaviour to the 1m span FHEN mid span load shown in **Figure 6.12**. The 3m span increases parabolically concave up with each increase in downstand height.

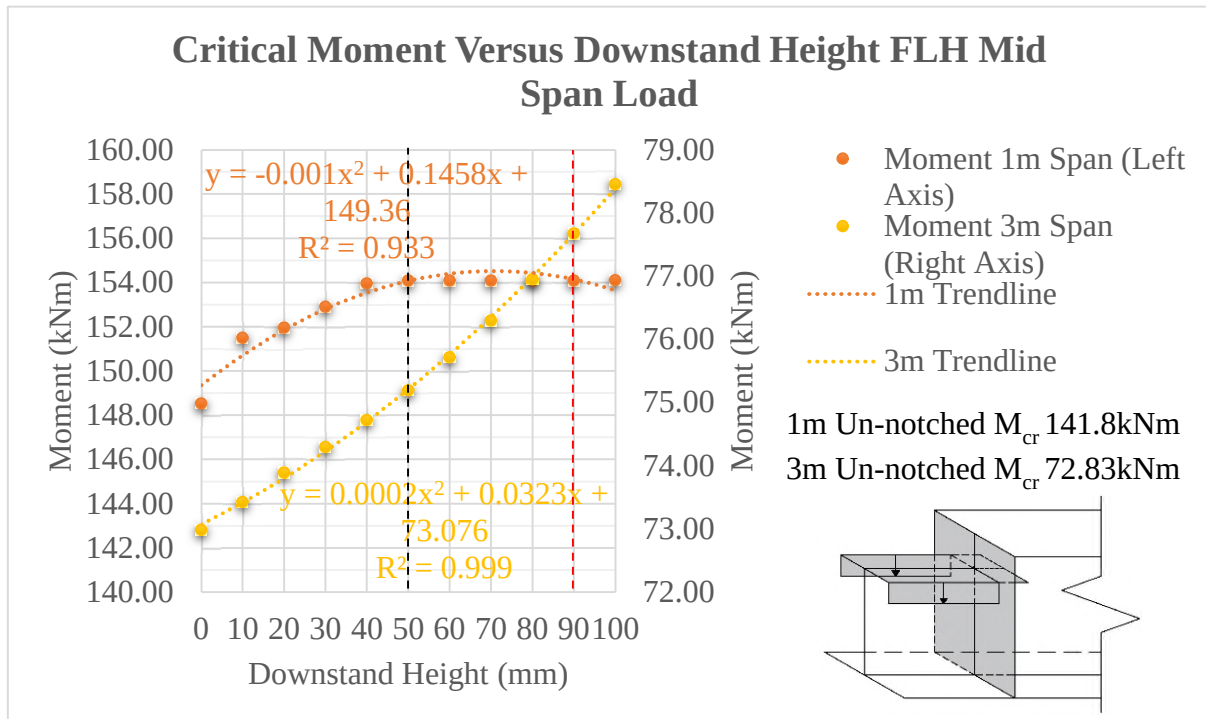


Figure 6.18: Critical Elastic Moment vs Downstand Height FLH Technique MSL Case Model 1
203x133x25

6.4.1.5 FLHFHV Technique

Figure 6.19 shows the results of the FLHFHV technique for the EPL case. The section decreased its critical elastic moment moving from the upstands to the no downstand/upstands at stand height of 0mm by 0.91Nm for the 1m span. The introduction of downstands then increased the critical elastic moment by 0.15kNm. The 3m span also decreased its critical elastic moment (by 6.47kNm) then increased it with the downstands (1.52kNm). The behaviour here is unlike any of the trends in the other techniques previously discussed. As the shear centre approaches the centroid in the notched section by reducing the height of the upstands, the critical elastic moment reduces and only increases, if by a negligible amount, once downstands are introduced. This stiffening technique could be considered an outlier to the other techniques.

As discussed in *Section 4.1.3 FLHFHV Technique* the centroid and shear centre coincide with no upstands or downstands therefore a combination of upstands and downstands was used.

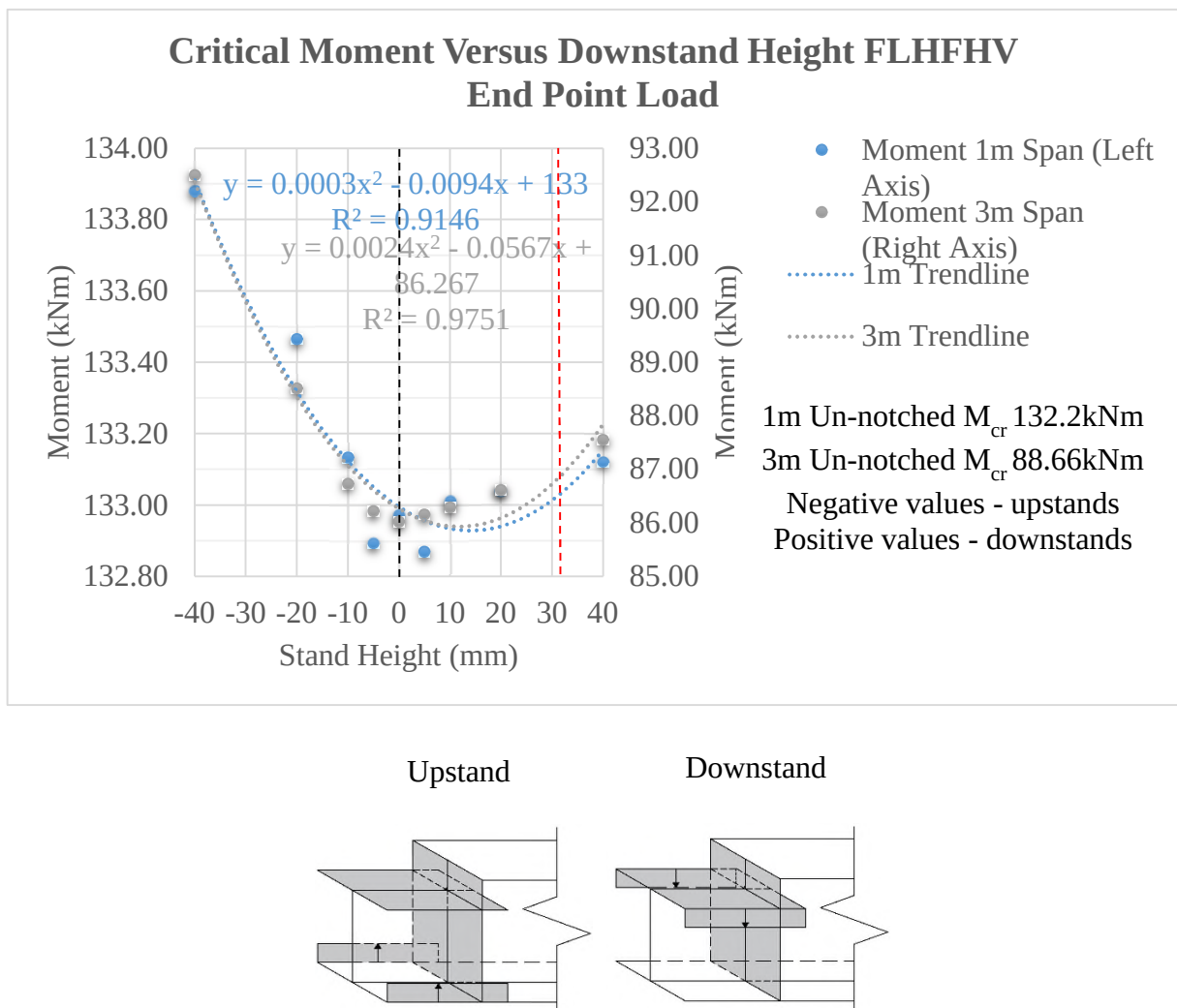


Figure 6.19: Critical Elastic Moment vs Stand Height FLHFHV Technique EPL Case Model 1 203x133x25

The trend in the FLHFHV MSL case shown in **Figure 6.20** follows the EPL case (**Figure 6.19**) but is also unique as it does not follow the trends shown in the previous stiffening techniques. There was also a reduction in the moment (0.95kNm) moving from the upstand to no upstand/downstand case then a further negligible reduction of 0.01kNm for the 1m span case. Similarly, the 3m span reduced its critical elastic moment by 4.63kNm then increased it by 1.36kNm.

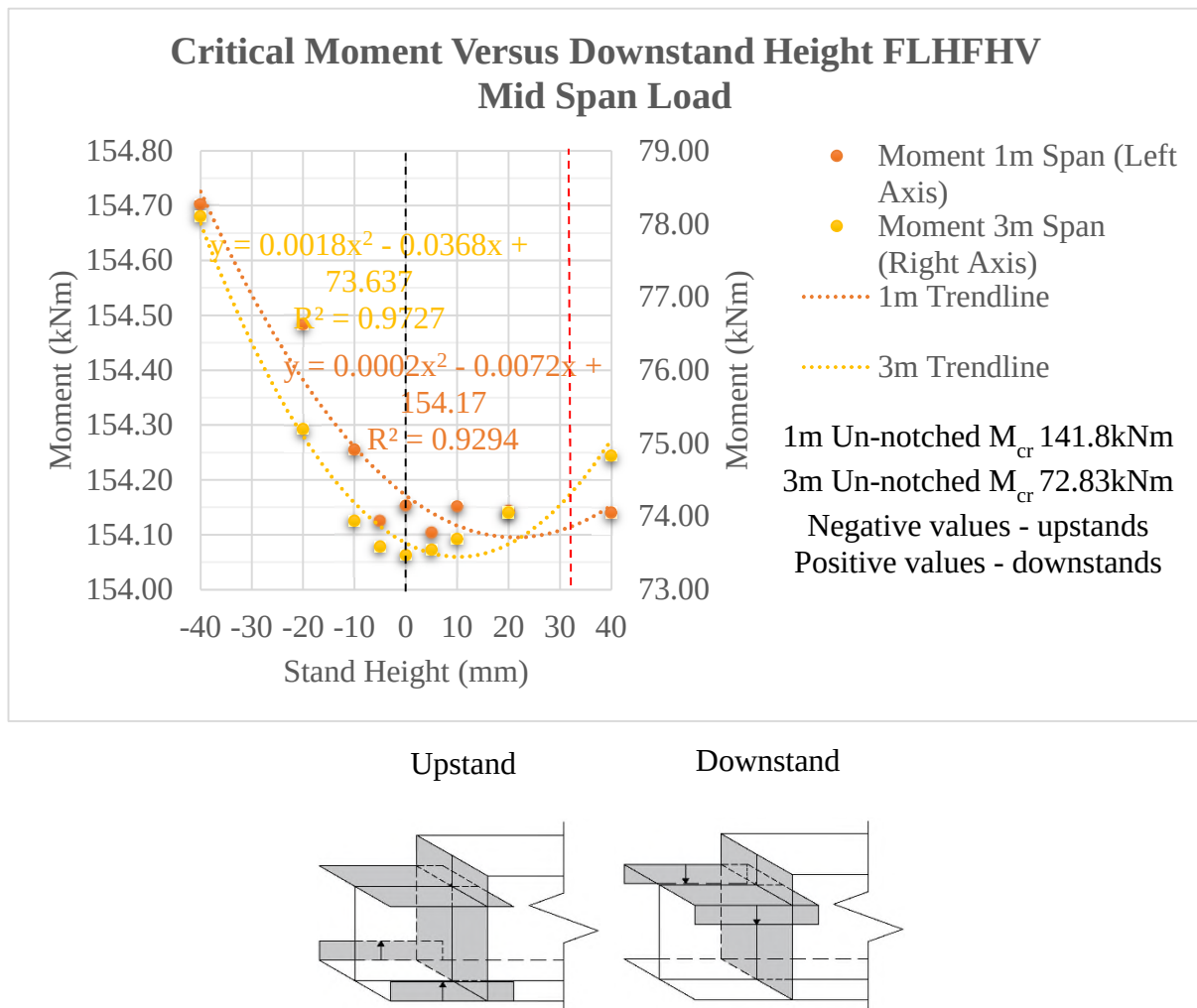


Figure 6.20: Critical Elastic Moment vs Stand Height FLHFHV Technique MSL Case Model 1 203x133x25

The FLHFHV technique is unusual because it uses upstands and downstands, as discussed in *Section 4.1.3 FLHFHV Technique*. This is due to the centroid and shear centre being almost at the same vertical height without any downstands in place. Therefore, it was necessary to shift the shear centre upwards to have data points when the shear centre is below the centroid. However, this causes a trend that is different to all the previous techniques if the entire trend is examined. This could be due to upstands being used since this is the only case that utilises them.

The case when the shear centre is above the centroid, when downstands are used, is similar to all the other trends. This is clear since with each increase in downstand height, there is an increase in the critical elastic moment as shown in **Figure 6.21**. Since the 1m spans have negligible increases, they have been excluded.

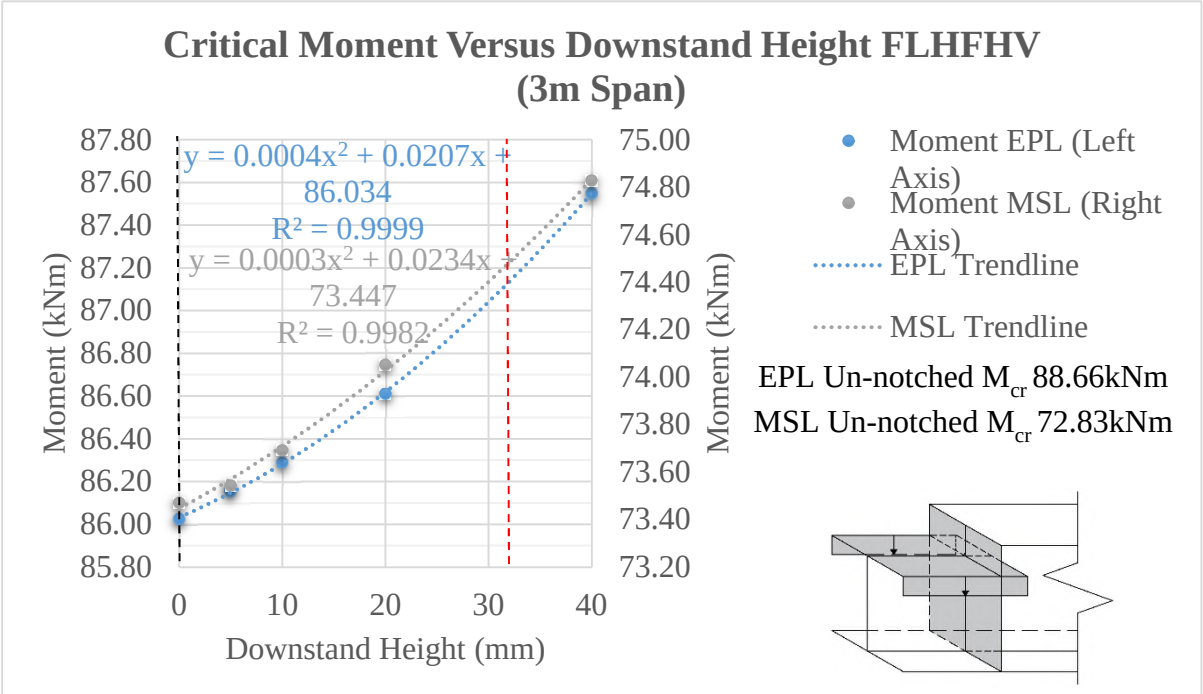


**Figure 6.21:Critical Elastic Moment vs Downstand Height FLHFHV Technique Model 1
203x133x25**

6.4.2 Model 2

Model 2 is the testing of the stiffened section for the 5m and 6m spans. **Figure 6.22** shows the five stiffening techniques and by how much they restore the critical elastic moment to the notched section for the EPL case.

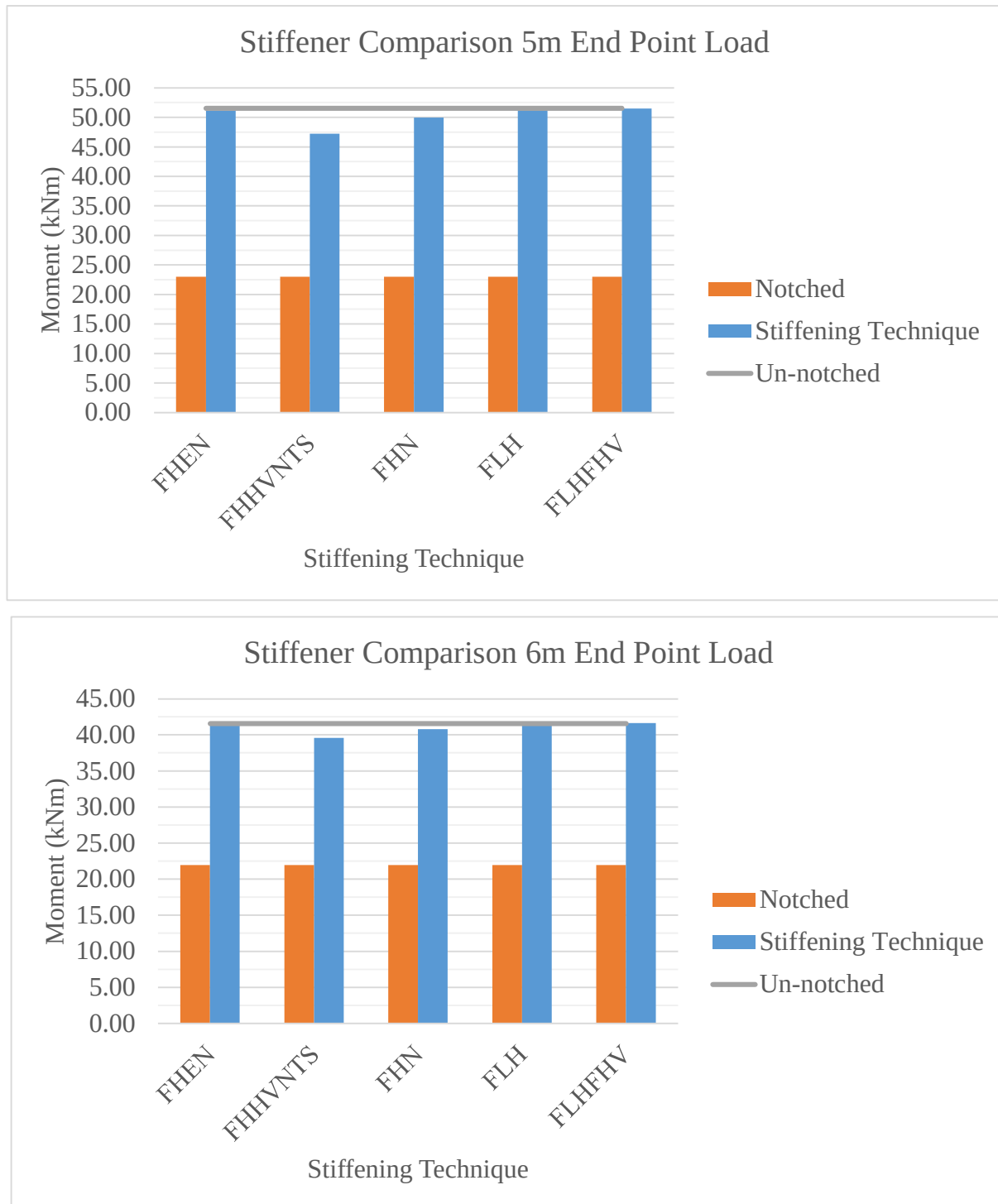


Figure 6.22: Stiffener Comparison 5m, 6m EPL 203x133x25

Figure 6.23 shows the 5m, 6m spans for the MSL case. **Figure 6.22** shows the five stiffening techniques and by how much they restore the critical elastic moment to the notched section for the MSL case.



Figure 6.23: Stiffener Comparison 5m, 6m MSL 203x133x25

6.4.2.1 FHEN Technique

Figure 6.24 shows the critical elastic moment versus downstand height for the FHEN EPL case for the 5m and 6m spans. The change in the critical elastic moment for the 5m and 6m spans are 8.38kNm and 4.74kNm, respectively (note the shorter span increased more).

The trend is similar in both spans: the first few downstand cases are slightly parabolic (a flat parabola) but increase linearly for the remainder of the downstand heights. It is unclear if the split in the trend occurs at the 50mm downstand height, but it certainly occurs close to it. There is no obvious change in the critical elastic moment as the shear centre approaches and passes the centroid in the notched region or the un-notched region. What is clear is the critical elastic moment increases with each increase in downstand height for both spans initially with an increasing gradient until approximately the 50mm downstand height but thereafter at a constant gradient.

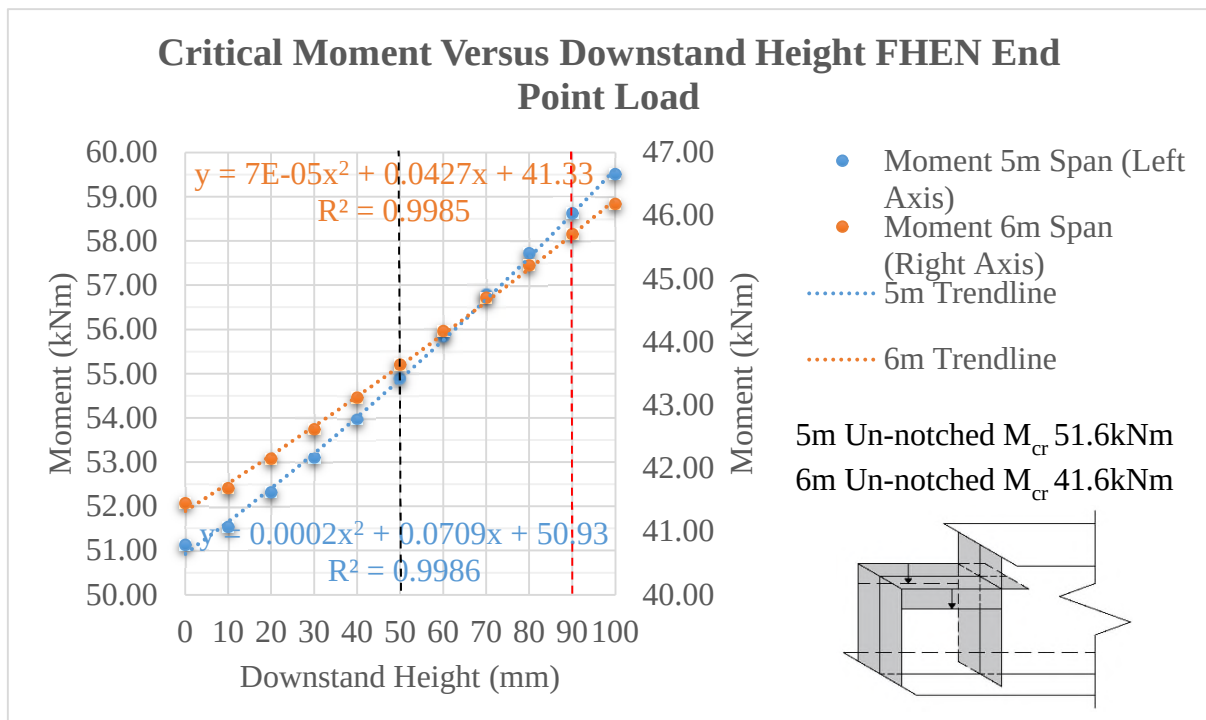


Figure 6.24: Critical Elastic Moment vs Downstand Height FHEN Technique EPL Case Model 2 203x133x25

Figure 6.25 shows the results for the MSL case. The 5m span increased its critical elastic moment by 5.13kNm and the 6m span by 3.19kNm which is similar the EPL case where the shorter span increased its critical elastic moment more.

The trend in the data is similar to the EPL case (**Figure 6.24**): the critical elastic moment increases with each increase in downstand height but there is no obvious change due to the shifting shear centre and centroid. The trend is strong between the two spans and is slightly parabolic across all the downstand heights. It increases at a constant gradient after the 50mm downstand height, but it could be from earlier heights as well.

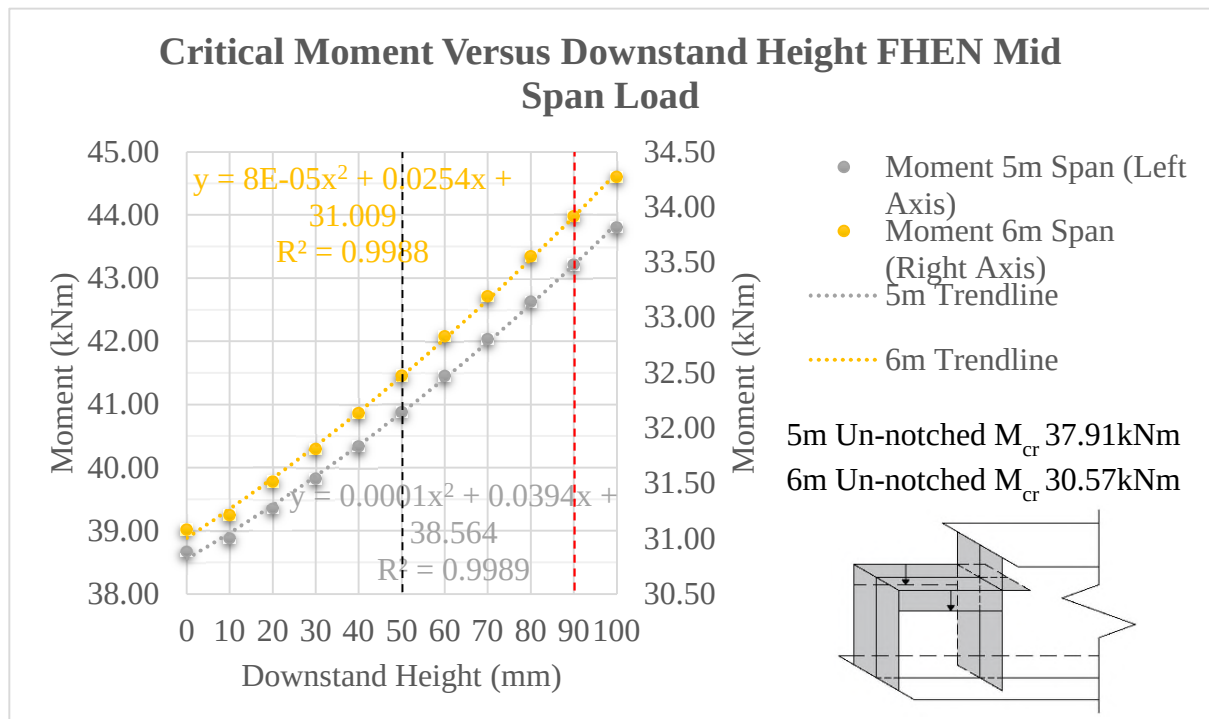


Figure 6.25: Critical Elastic Moment vs Downstand Height FHEN Technique MSL Case Model 2 203x133x25

6.4.2.2 FHHVNTS Technique

Figure 6.26 show the change in the critical elastic moment due to the introduction of downstands for the EPL FHHVNTS case. There was a small increase in both spans: 3.32kNm for the 5m span and 1.89kNm for the 6m span. Again, the shorter of the two spans increased its critical elastic moment more than the longer span.

Both spans have a similar trend: the critical elastic moment increases parabolically with each increase in downstand height as the shear centre approaches and passes the centroid in the notched region (the point of coincidence at 17mm downstand height). Unlike the FHEN case, the gradient of the curves seems to be increasing with each increase in downstand height. There is no obvious change in the general trend as the shear centre approaches and passes the centroid in the notched region, but it can be said the initial trend is parabolic (up until the 17mm downstand height) but afterwards it is linear.

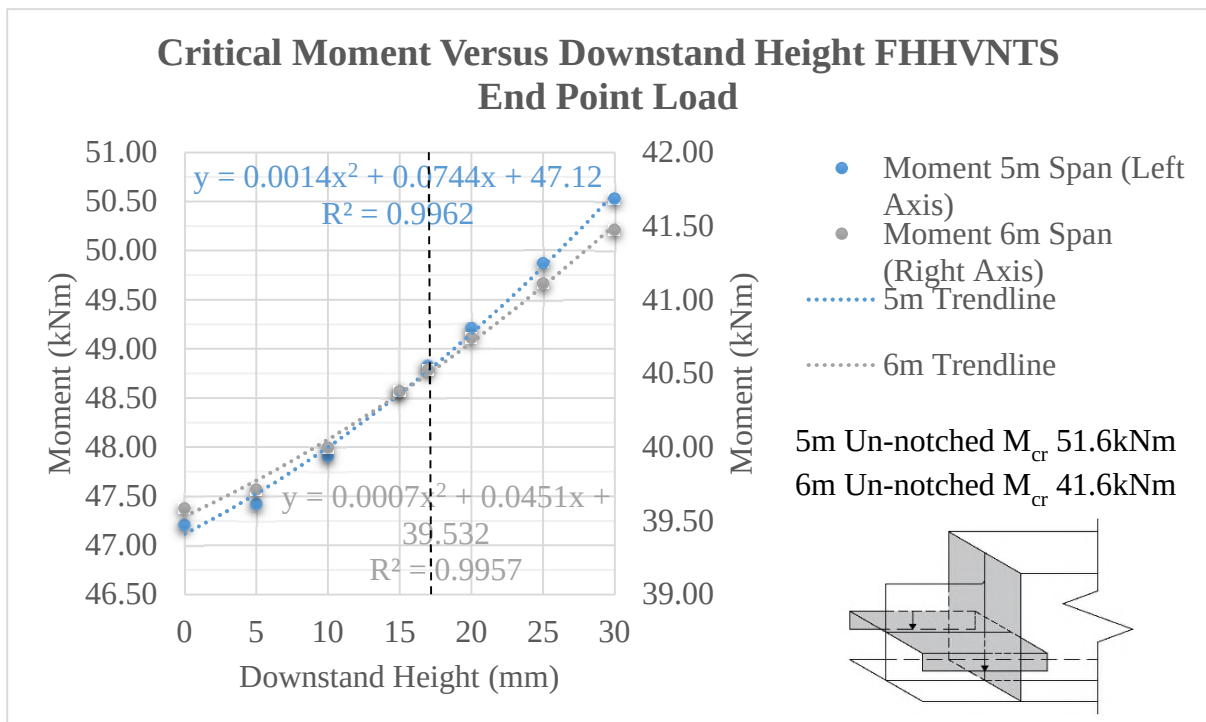


**Figure 6.26: Critical Elastic Moment vs Downstand Height FHHVNTS Technique EPL Case
Model 2 203x133x25**

Figure 6.27 shows the results for the MSL FHHVNTS technique. The increase in critical elastic moment is marginal: 1.13kNm and 0.69kNm for the 5m and 6m spans (note the shorter of the two spans increased more the longer span).

The trend is once again similar to the EPL case (**Figure 6.26**): the critical elastic moment started at a minimum value and increased parabolically with each increase in downstand height. Just as in the EPL case, there is no obvious change in behaviour as the shear centre approaches and passes the centroid in the notched region. The trend can be split: the initial trend is parabolic (from 0mm till 17mm downstand height) but afterwards it is linear.

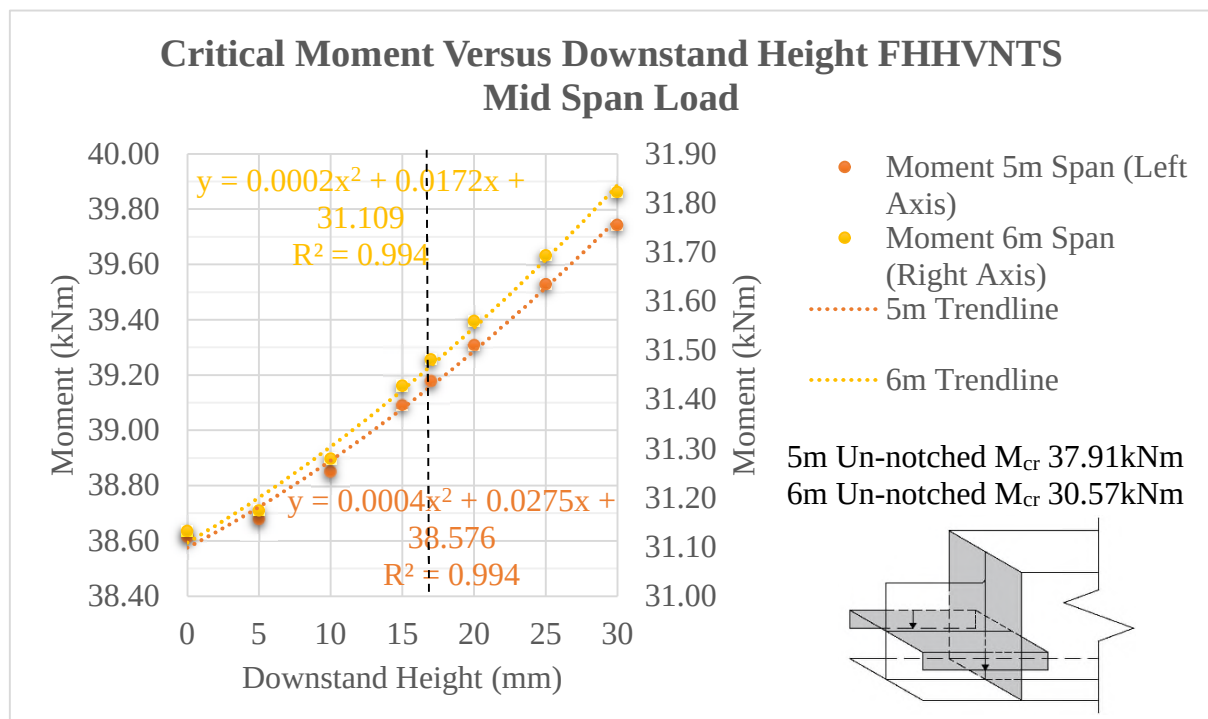


Figure 6.27: Critical Elastic Moment vs Downstand Height FHHVNTS Technique MSL Case Model 2 203x133x25

6.4.2.3 FHN Technique

The change in the critical elastic moment for the 5m and 6m spans for the EPL FHN technique is 2.03kNm and 1.24kNm respectively as shown in **Figure 6.28**. Note the shorter of the two spans increased more.

The trend is similar to the FHHVNTS technique: the curves start at a minimum value and the critical elastic moment increases with each increase in downstand height. The curves are similar for both spans: they are slightly parabolic but could be split into two straight lines because there is an almost constant gradient from 0mm to 50mm downstand and from 50mm to 100mm (with a gradient that is steeper than the 0mm to 50mm straight line) which is interesting since the centroid and shear centre coincidence point is at 50mm downstand height.

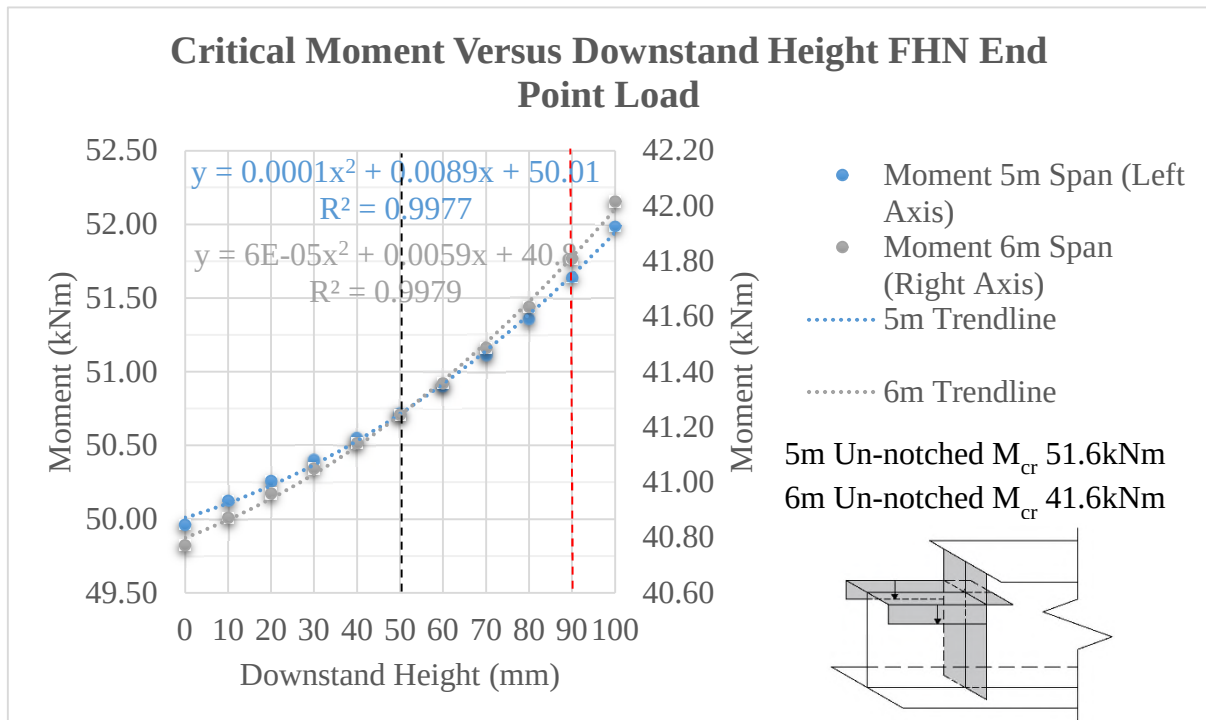


Figure 6.28: Critical Elastic Moment vs Downstand Height FHN Technique EPL Case Model 2
203x133x25

The results for the FHN MSL case are shown in **Figure 6.29**. There was a slight increase in the critical elastic moment of 1.25kNm and 0.80kNm for the 5m and 6m spans with the shorter of the two spans increasing more.

The trend is similar to the FHHVNTS technique and the EPL case (**Figure 6.28**). The two spans have a very similar trend: the curves start at a minimum value and increase slightly parabolically with each increase in downstand height. There is a small change in gradient, just as in the EPL case, at the 50mm downstand height. The data before can be approximated as a straight line and the data after into another straight line because the slight change in gradient appears at the 50mm downstand height. The gradient after the 50mm downstand height is slightly steeper than the gradient before (this is clear if lines are drawn on the graph). This is interesting because this is the height at which the centroid and shear centre in the notched region coincide.

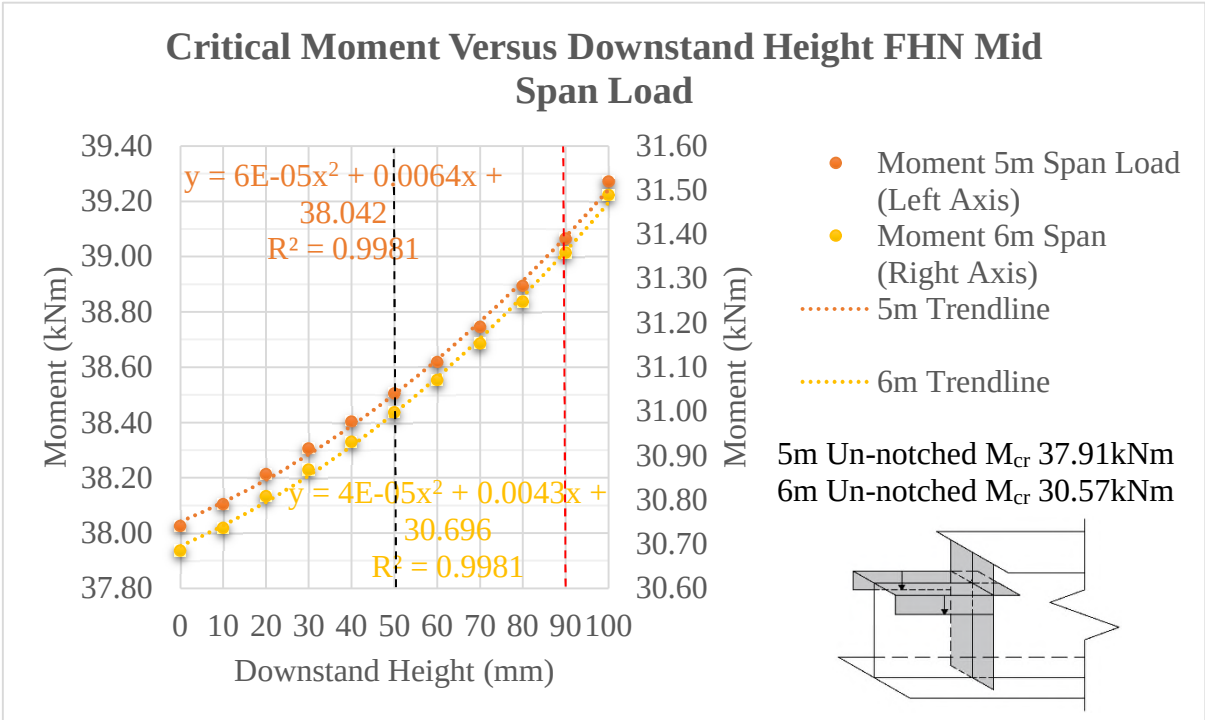


Figure 6.29: Critical Elastic Moment vs Downstand Height FHN Technique MSL Case Model 2
 203x133x25

6.4.2.4 FLH Technique

An increase of 2.68kNm and 1.64kNm is shown in **Figure 6.30** for the 5m and 6m span for the EPL case FLH technique. Once again, the shorter of the two spans increased more.

The trend is very similar to the FHN and slightly similar to the FHHVNTS technique. The trend between the two spans is very strong as well. The general trend is slightly parabolic starting from a minimum value increasing concave up. However, the data can be split into two straight lines at the 50mm downstand since the gradient is almost constant before and after this point. This is similar to the FHN technique: the 50mm downstand - which is the downstand height at which the centroid and shear centre in the notched region coincide - splits the data into two almost linear trends with the trend after the 50mm downstand being steeper than the trend before.

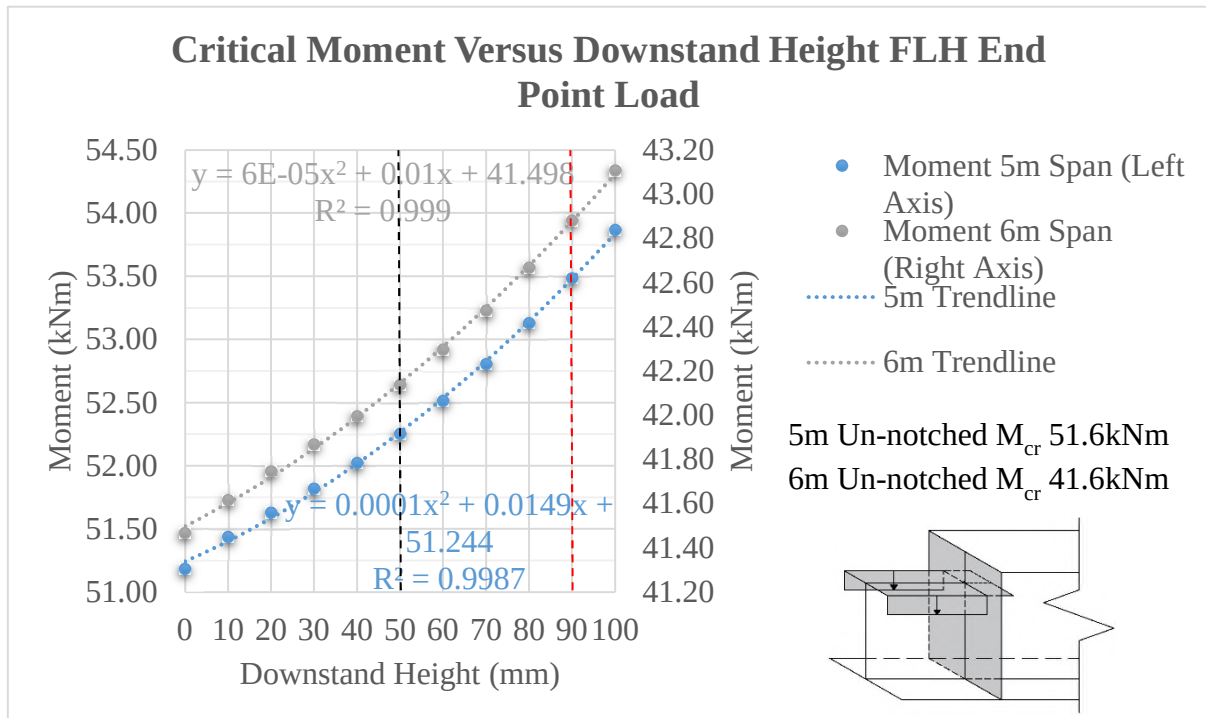


Figure 6.30: Critical Elastic Moment vs Downstand Height FLH Technique EPL Case Model 2 203x133x25

Figure 6.31 shows the results for the MSL case. Only a slight increase in critical elastic moment was achieved: 1.88kNm and 1.21kNm for the 5m and 6m spans respectively and the shorter of the two spans increased more.

The change in gradient at 50mm downstand height is not as distinct as in other techniques but is still present. Just as in the EPL case, the trend is strong between the two spans and is similar to the FHN technique and slightly similar to the FHHVNTS technique. The curves start at a minimum value and increase slightly parabolically concave up with each increase in downstand height. The gradient is steeper than the EPL case, but the data is still split at the 50mm downstand: the 50mm to 100mm trend is steeper than the initial 0mm to 50mm trend just as in the FHN technique.

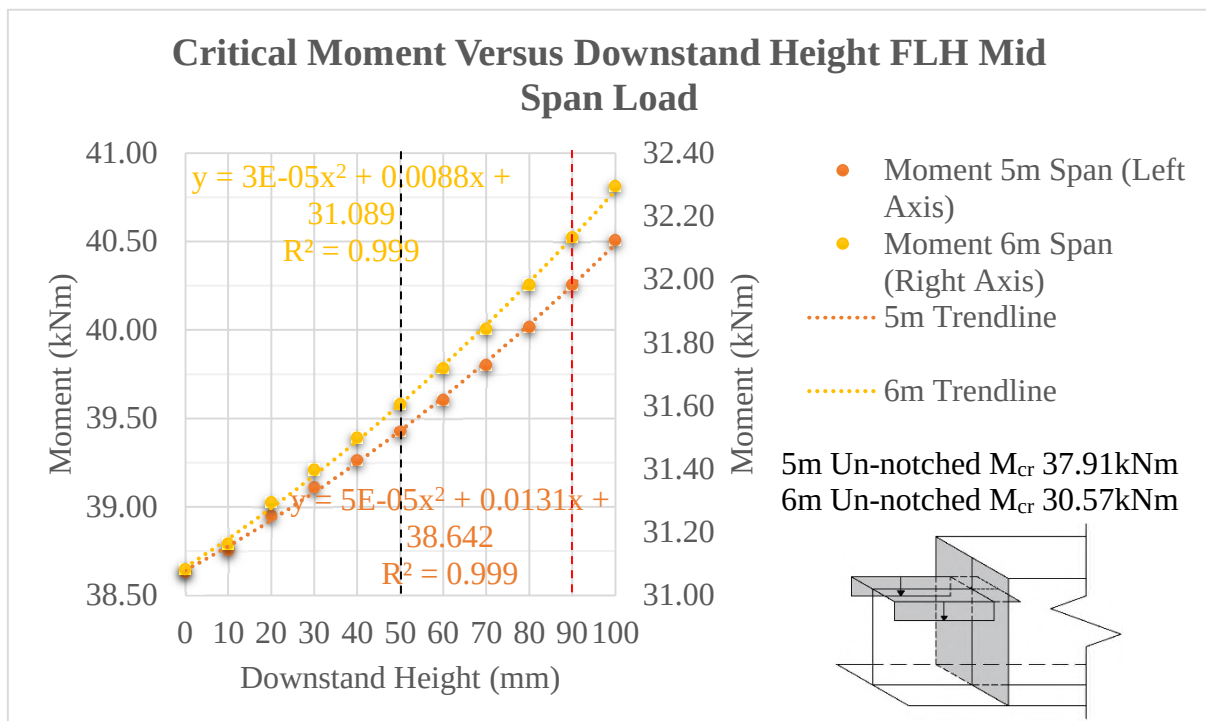


Figure 6.31: Critical Elastic Moment vs Downstand Height FLH Technique MSL Case Model 2
203x133x25

6.4.2.5 FLHFHV Technique

The 5m and 6m span EPL cases for the FLHFHV technique are shown in **Figure 6.32**. The trend is very similar to Model 1 (**Figure 6.19** and **Figure 6.20**): the change in the critical elastic moment is very small. The value decreases by 2.37kNm with the reduction in upstand height and increases with the introduction of downstands by 0.66kNm for the 5m span and reduces by 1.41kNm and recovers by 0.41kNm for the 6m span. The behaviour is unique to this technique, just as in Model 1, because the shear centre approaches the centroid in the notched section by reducing the height of the upstands, the critical elastic moment reduces and only increases, if by a negligible amount, once downstands are introduced. This stiffening technique could be considered an outlier to the other techniques.

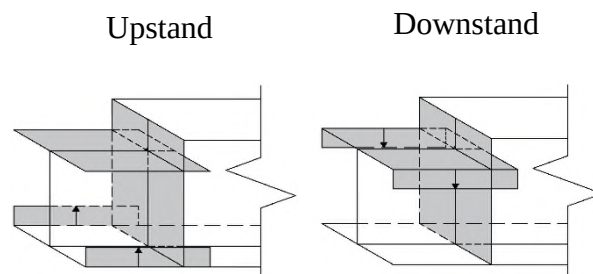
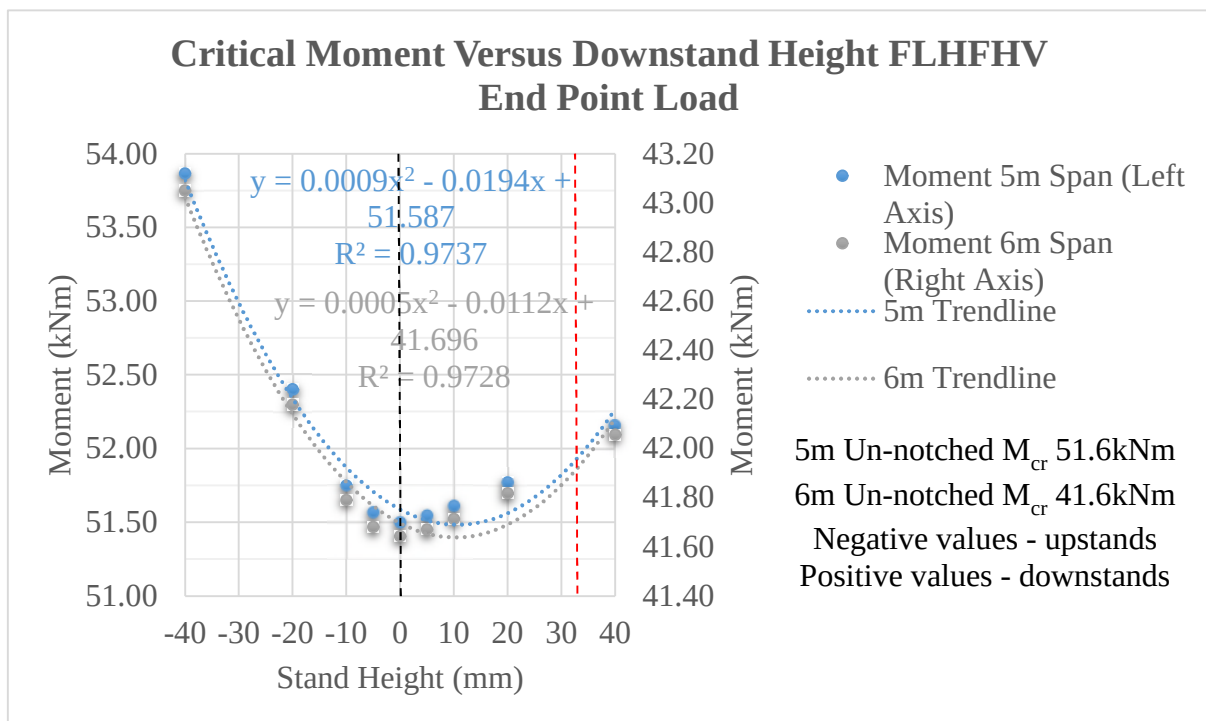


**Figure 6.32: Critical Elastic Moment vs Stand Height FLHFHV Technique EPL Case Model 2
203x133x25**

Figure 6.33 shows the results of the 5m and 6m spans for the FLHFHV technique. The unique behaviour is similar to **Figure 6.19**, **Figure 6.20** (Model 1) and **Figure 6.32** (EPL case for Model 2): the critical elastic moment decreases by 1.50kNm then recovers 0.51kNm for the 5m span whereas the 6m span reduces by 0.96kNm with the reduction in upstand height and recovers 0.33kNm.

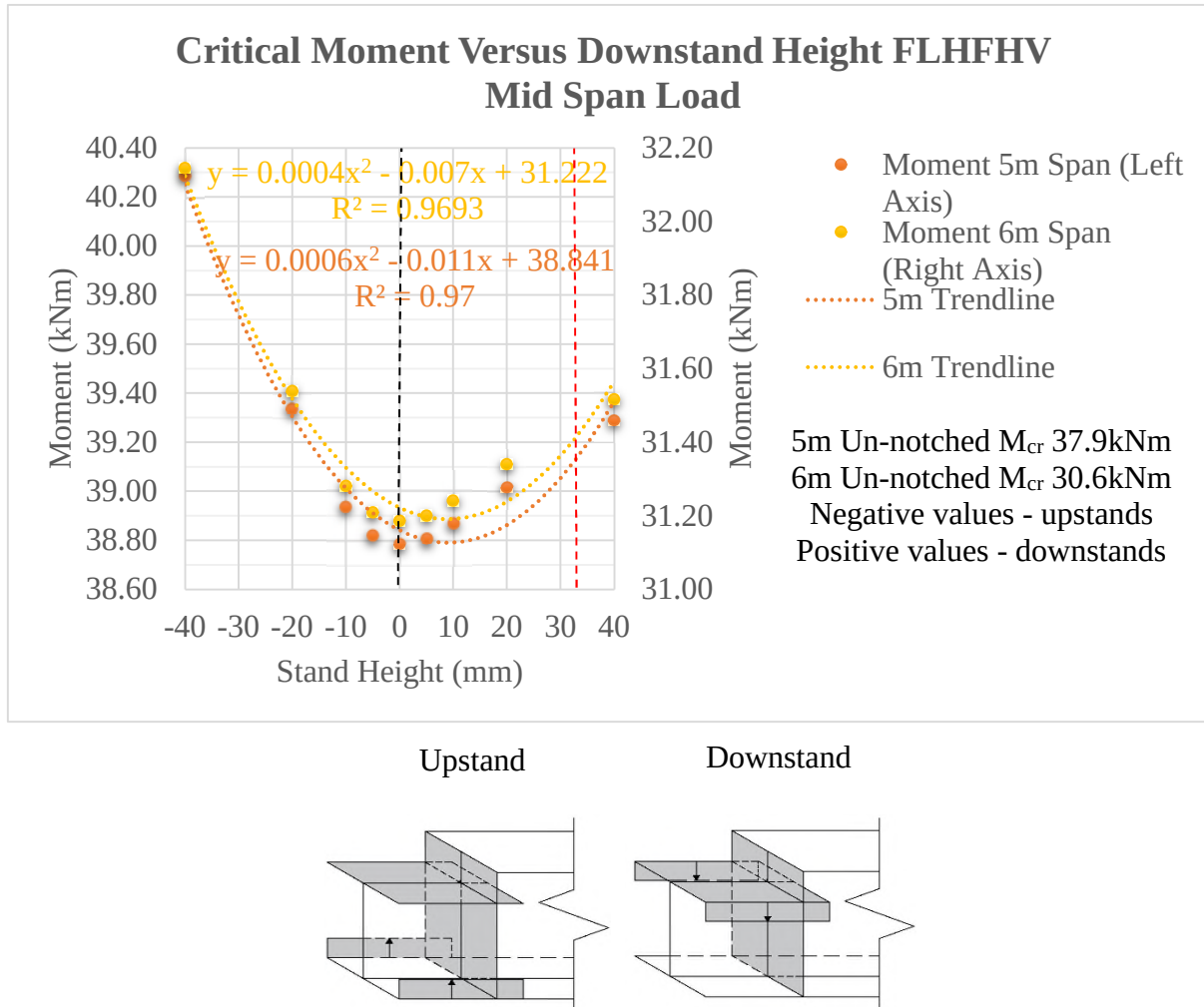


Figure 6.33: Critical Elastic Moment vs Stand Height FLHFHV Technique MSL Case Model 2 203x133x25

As discussed in Model 1: the case when the shear centre is above the centroid, when downstands are used, is similar to all the other trends. This is clear since with each increase in downstand height, there is an increase in the critical elastic moment as shown in **Figure 6.34**. The trend is similar to all techniques in Model 1 except the 1m MSL of the FHEN and FLH techniques. It is also similar to the FHHVNTS technique (Model 1 **Figure 6.13**, **Figure 6.14** and Model 2 **Figure 6.26**, **Figure 6.27**). The 6m EPL and MSL cases have been excluded since they do not meet the minimum required change of 0.5kNm.

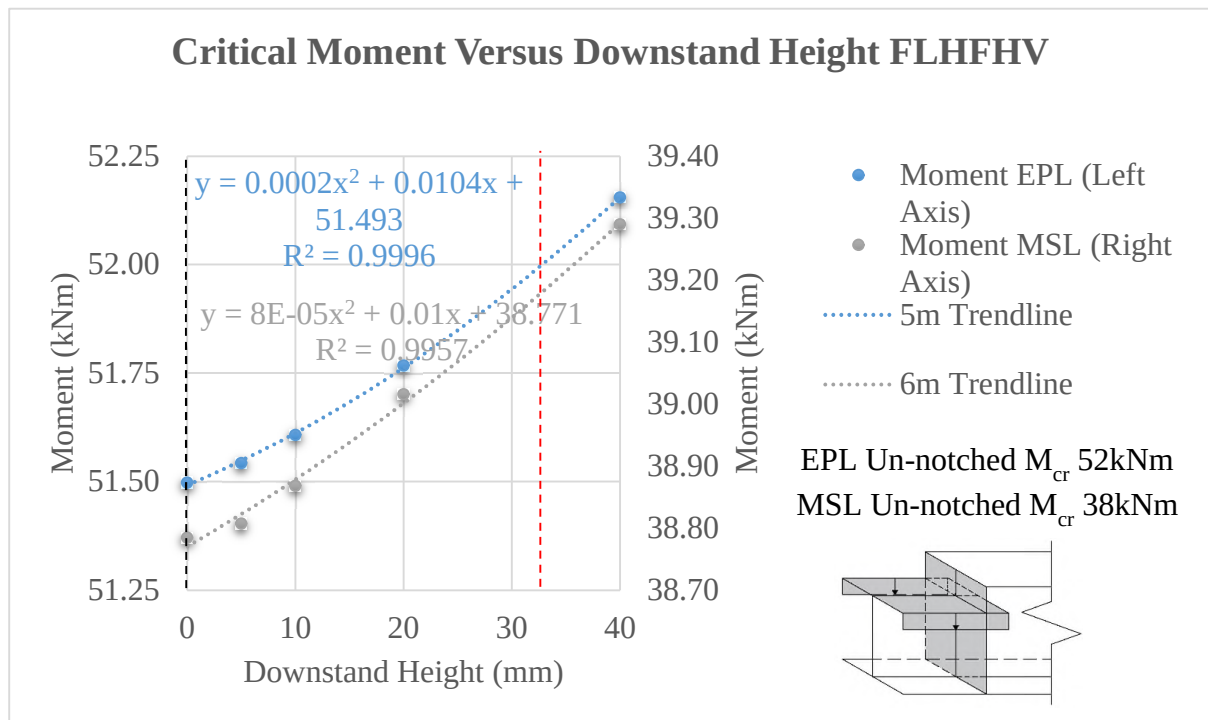


Figure 6.34: Critical Elastic Moment vs Downstand Height FLHFHV Technique Model 1
203x133x25

6.4.3 Further Discussion

An interesting trend was noticed in both models: in the 32 cases tested (four stiffening techniques each with two models of two load cases and two spans each) and excluding the FLHFHV technique 13/16 (four stiffening techniques with two models with 2 spans) load cases had the shorter spanning case increase its critical elastic moment more than the longer case. The FLHFHV technique was excluded because the entire behaviour of the shear centre as it approaches from below and eventually passed the centroid cannot be analysed in its entirety but has been discussed by analysing the isolated downstand behaviour. In Model 1, the 1m span increased its critical elastic moment more than the 3m span except for the FHEN, FHN and FLH EPL case where the 3m span increased more. In Model 2, the 5m span always increased its critical elastic moment more than the 6m span but it is worth noting the total increase for Model 2 was generally marginal.

In the FHEN technique, as previously discussed, the intersection of the centroid and shear centre occurs at a downstand height of 50mm. It is also clear that the 1m MSL case is an outlier and has been removed from the percentage change and average percentage change graph, calculation because it skews the trend. As discussed earlier it is difficult to see any changes in the trend around the 50mm downstand height. The only change is the trend could be initially parabolic and linear after the 50mm downstand because the gradient appears constant after this downstand height.

This is consistent with the percentage change curves in **Figure 6.35**: all the percentage change curves appear to generally increase in the same manner: only the 1m EPL increases at a slightly different rate. This is very interesting because it shows the trend is similar across all the other different loading and span configurations.

The average percentage change curve, which excludes the 1m MSL, is consistent with the percentage change curve which is further evidence of the consistency across the different load and span cases. It also shows there is a change at the 50mm downstand because there is a divergence in the trend. Using the trendline before the 50mm downstand height and extending it beyond this height: it is clear the initial trend has a changing gradient (parabolic trend) and the trend after the 50mm downstand height is linear.

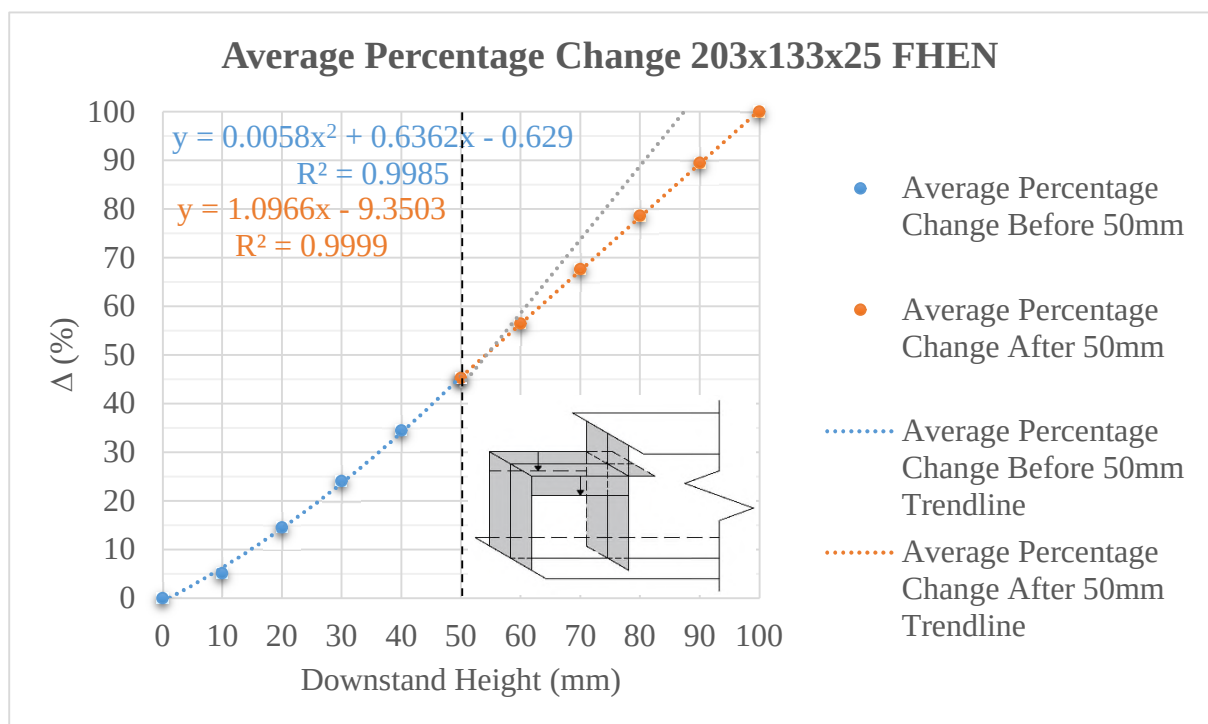
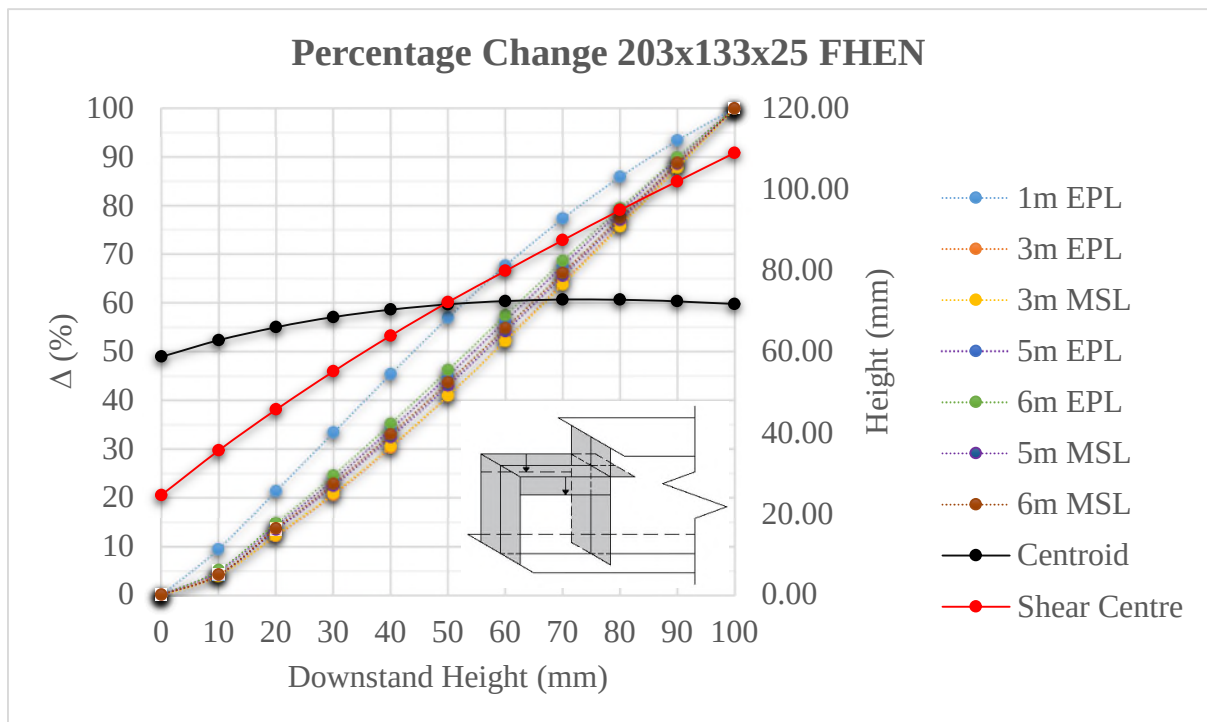


Figure 6.35: Percentage Change and Average Percentage Change Graph 203x133x25 FHEN Technique (Percentage change left axis, Centroid and Shear Centre right axis)

Figure 6.36 shows the percentage change and average percentage change graphs for the FHHVNTS stiffening technique. There is a strong trend in the data across all spans and loading configurations, even stronger than the FHEN technique, which was also evident in the critical elastic moment curves discussed in *Section 6.3.1.2 FHHVNTS Technique* and *Section 6.3.2.2 FHHVNTS Technique*. There are no outliers in this technique therefore all cases are included in the average percentage change calculation.

It appears there are two curves: an initial trend that is parabolic until the 17mm downstand height (which is the coincidence point of the centroid and shear centre) thereafter the trend is linear which is similar to the FHEN technique, but the parabolic curve is more pronounced in this case. The divergence is clear in the average percentage change graph as shown by extending the initial trendline across the remaining downstand heights. The divergence shows the initial downstand cases have a greater influence on the total change in the critical elastic moment. This is when the shear centre is below the centroid. The influence is less when the shear centre is above the centroid. This is clear because the gradient is increasing in the parabolic trend, but the gradient is constant in the linear trend.

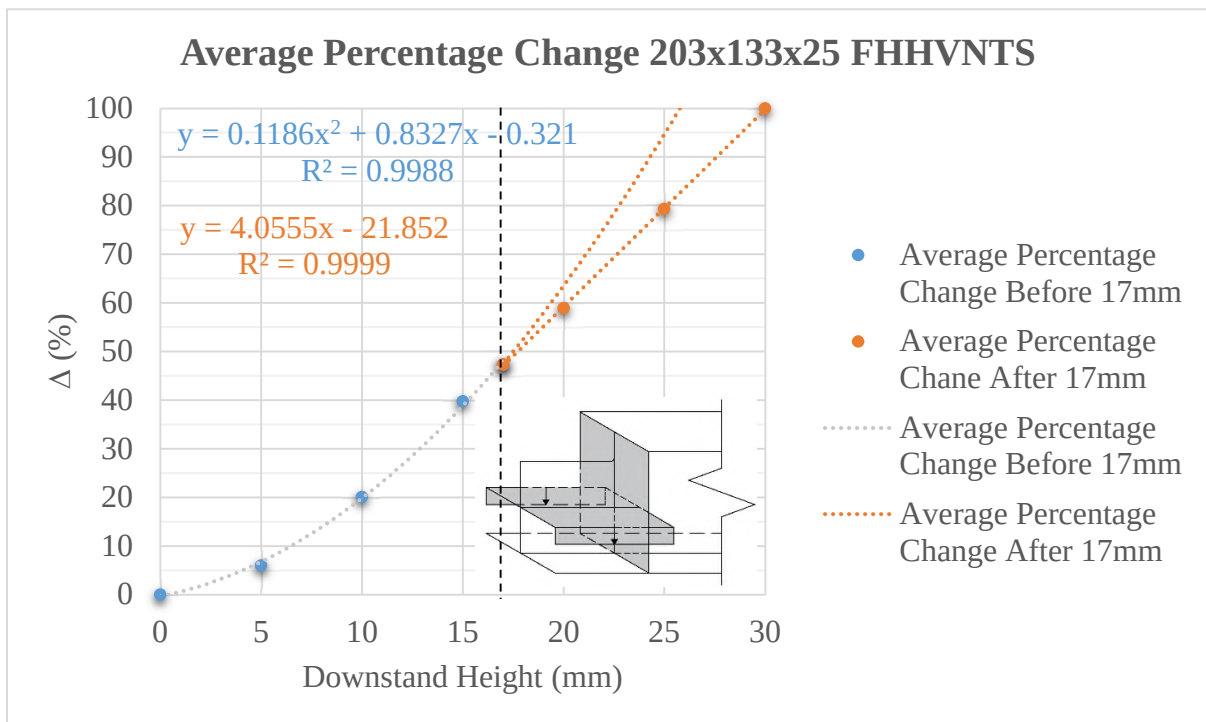
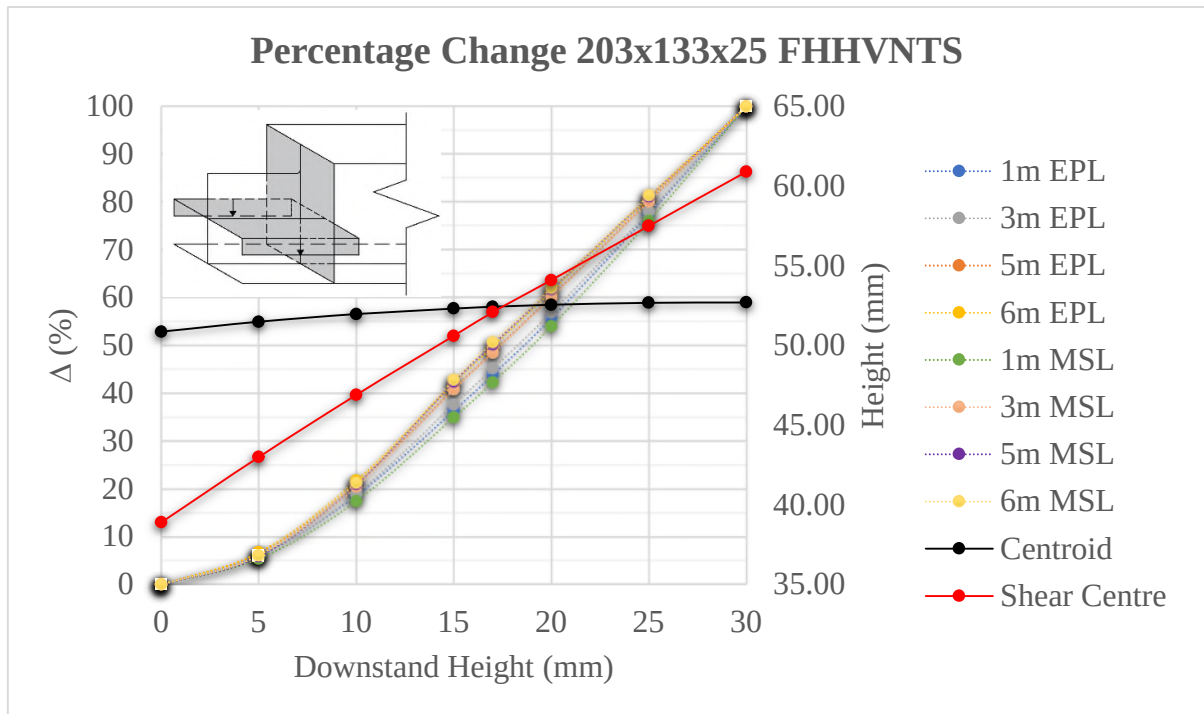


Figure 6.36: Percentage Change and Average Percentage Graph 203x133x25 FHHVNTS Technique (Percentage change left axis, Centroid and Shear Centre right axis)

Figure 6.37 shows the percentage change for the FHN technique. Once again, a strong trend exists because no outliers are evident. In the critical elastic moment discussion, the trend initially appeared to be split into two straight lines. However, it is better represented by a straight line followed by a parabola. The trend also looks similar to the FHHVNTS technique but on closer study there is a slight difference in the trend after the 50mm downstand height which is shown in the average percentage change graph.

The FHEN and FHHVNTS techniques have an initial parabolic trend followed by a linear trend after the coincidence point of the centroid and shear centre in the notched region. The FHN technique has a linear trend followed by a parabolic trend. Therefore, an increase in the downstand height affects the critical elastic moment more when the shear centre is above the centroid in the notched region when using the FHN technique which is different to the FHEN and FHHVNTS techniques.

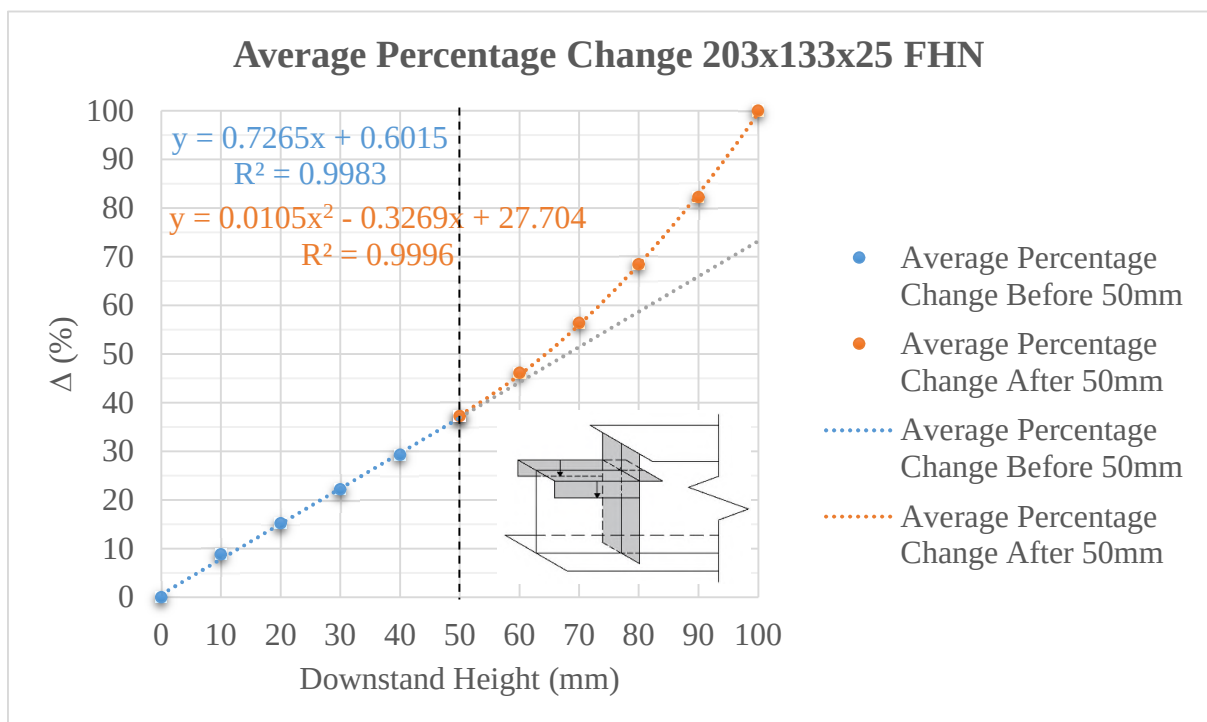
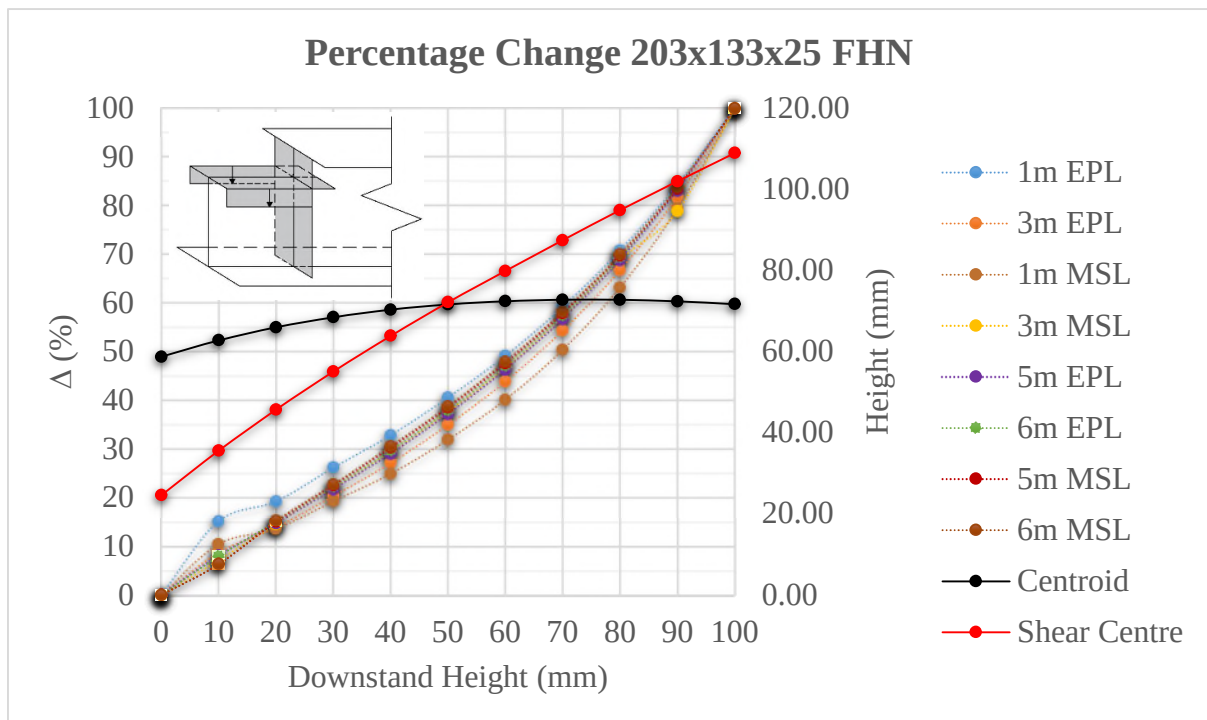


Figure 6.37: Percentage Change and Average Percentage Change Graph 203x133x25 FHN Technique (Percentage change left axis, Centroid and Shear Centre right axis)

Figure 6.38 shows the percentage change and average percentage change for the FLH technique. The 1m MSL is an outlier and was removed from the percentage change graph and the average percentage change curve, calculation because it skewed the average. Initially the trend appeared to be split into two straight lines and similar to the FHEN, FHHVNTS and FHN techniques. The trend can be split into two straight lines, when excluding the 1m MSL outlier, but the trend is not similar to the other two techniques. The FHEN and FHHVNTS technique are initially parabolic then linear whereas the FHN technique is linear then parabolic. The FLH is neither of these: it is uniquely two linear trends.

It is similar to both trends because the critical elastic moment increases as the shear centre approaches and passes the centroid in the notched region. It is also similar to the FHN technique because the gradient of the straight line is steeper after the coincidence point but different because the gradient is always increasing in the FHN technique after this point.

The downstand behaviour of the FLHFHV technique will be briefly discussed. The FLHFHV technique is unique in its behaviour and increased its critical elastic moment very marginally across all spans and loads whereas the other techniques had varying degrees of effectiveness. However, by isolating the downstand trend it is clear the trend is similar to all the other stiffening techniques. This is shown in **Figure 6.39**.

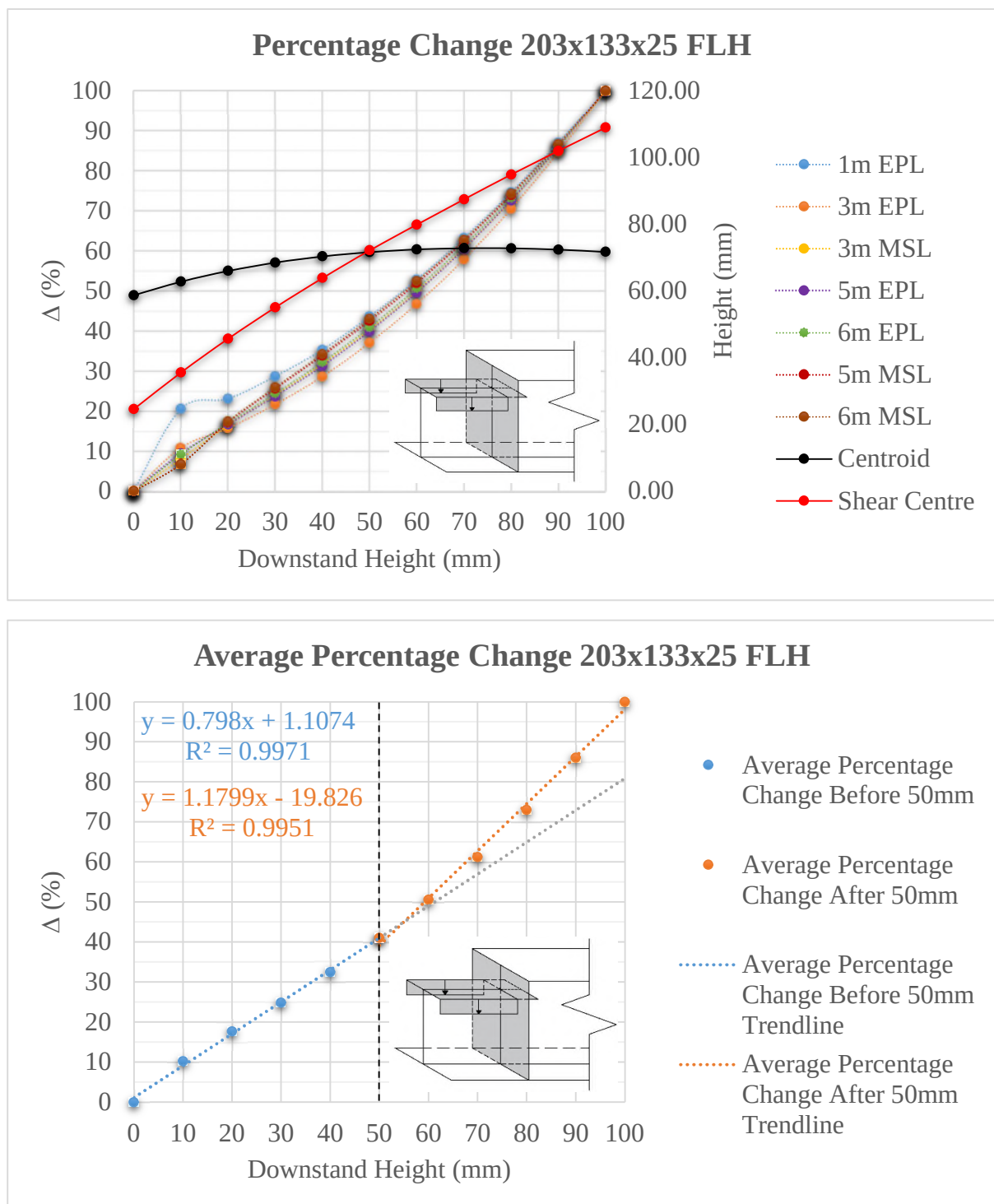


Figure 6.38: Percentage Change and Average Percentage Change Graph 203x133x25 FLH Technique (Percentage change left axis, Centroid and Shear Centre right axis)

Some key points to be taken from the discussion about the results for the 203x133x25 section:

- The shorter spanning members (1m and 5m) increase their critical elastic moment more than the longer members in the same model (3m and 6m) in almost all cases.
- There were only two outliers in all techniques (excluding the FLHFHV)
- There was remarkable similarity in the behaviour of the different loading and span cases within the techniques which shows span and loading arrangements do not influence the results.
- There is a change in the critical elastic moment as the shear centre approaches and passes the centroid. This is clearer in the percentage change and average percentage change curves.
- The change is evident as a change in gradient or change in curve type and occurs across all spans, loading configurations and stiffening techniques.
- Only the FHEN and FHHVNTS had similar changes: initially parabolic then linear after the coincidence of the centroid and shear centre.
- By isolating the downstand trend in the FLHFHV technique: it is clear there are similarities across all the techniques when the shear centre is above the centroid.

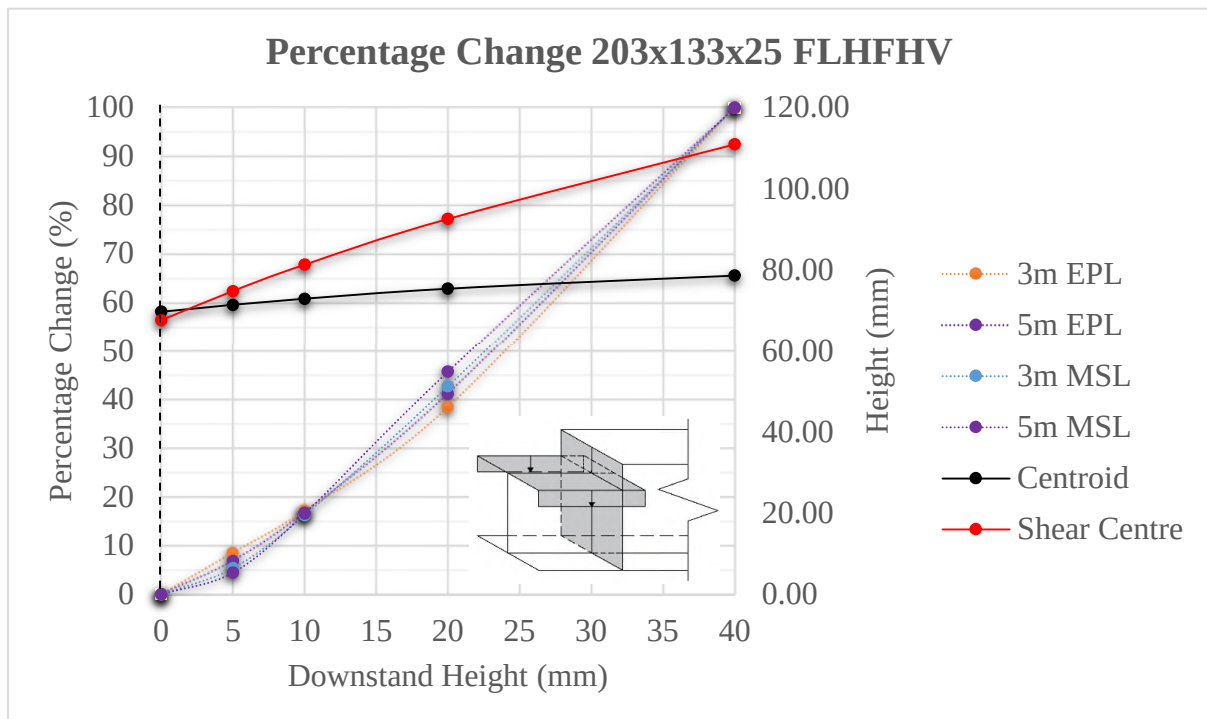


Figure 6.39: Percentage Change Graph 203x133x25 FLHFHV Technique (Percentage change left axis, Centroid and Shear Centre right axis)

6.5 305 x 102 x 25

6.5.1 Summary

Model 1 and Model 2 are presented in *Appendix E: Results of Model 1 and 2* since the discussion about Model 1 and Model 2 follows the same process as the 203x133x25 section. A summary of the trends is shown in **Figure 6.40** and techniques of interest will be further discussed in *6.5.2 Further Discussion*. Below is a list of techniques that do not meet the minimum change in critical elastic moment:

- 1m EPL FHEN
- 5m, 6m MSL FHHVNTS
- FHN and FLH techniques (excluding the 3m EPL and MSL cases)

The FLHFHV technique and isolated downstand FLHFHV are not included in the summary.

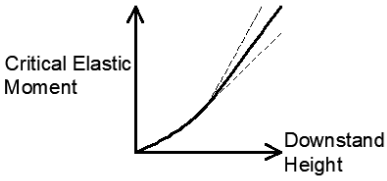
Trend Description	Trend Image	Techniques
M_{cr} increases more rapidly as shear centre approaches and passes the centroid in the notched region. Initially parabolic then after coincidence linear trend.		FHEN Model 1,2 FHHVNTS Model 1,2
The solid line is the average/approximated trend, the dashed lines are deviations from the average trend		

Figure 6.40: Summary of 305x102x25 Section

6.5.2 Further Discussion

Some interesting points to note from the data:

- Out of the 32 cases tested (excluding the FLHFHV technique because it is an outlier) 15 had negligible increases in the critical moment because the increase was less than 0.5kNm.
- The following cases were classified as having negligible increases:
 - o 1m EPL FHEN
 - o 5m and 6m MSL FHHVNTS
 - o Model 1 (except the 3m EPL, MSL) and Model 2 of the FHN
 - o Model 1 (except the 3m EPL, MSL) and Model 2 of the FLH
- The 3m MSL FHN and the 3m EPL and MSL FLH technique have changes above 0.5kNm but since they are the only cases that meet the requirement they have been excluded since they cannot be compared with other trends.
- Generally, the increase in the critical elastic moment were less than in the equivalent case for the 203x133x25 section.
- In 10 of the 16 load cases the shorter span increased its moment more which is consistent with the 203x133x25 section.

Figure 6.41 shows the percentage change and the average percentage change for the FHEN stiffening technique for the 305x102x25 section. The percentage change graph shows a clear trend in the percentage change across all loading and span configurations and this trend is similar (excluding the 1m MSL) to the 203x133x25 section trend shown in **Figure 6.35**. As the downstands increase in height, there is an increase in the critical elastic moment.

It appears there is a change around the 19mm downstand: the initial trend is parabolic but becomes linear after the 19mm downstand. This is similar to the 203x133x25 section shown in **Figure 6.35**. This change in trend is evident in the average percentage change graph because the initial trend, which is parabolic, diverges from the later trend, which is linear.

The data is split at the 15mm downstand height since it is before 19mm, the coincidence point, which was not tested. Clearly, the divergence in the trend occurs after the 19mm downstand height when the shear centre is above the centroid in the notched region.

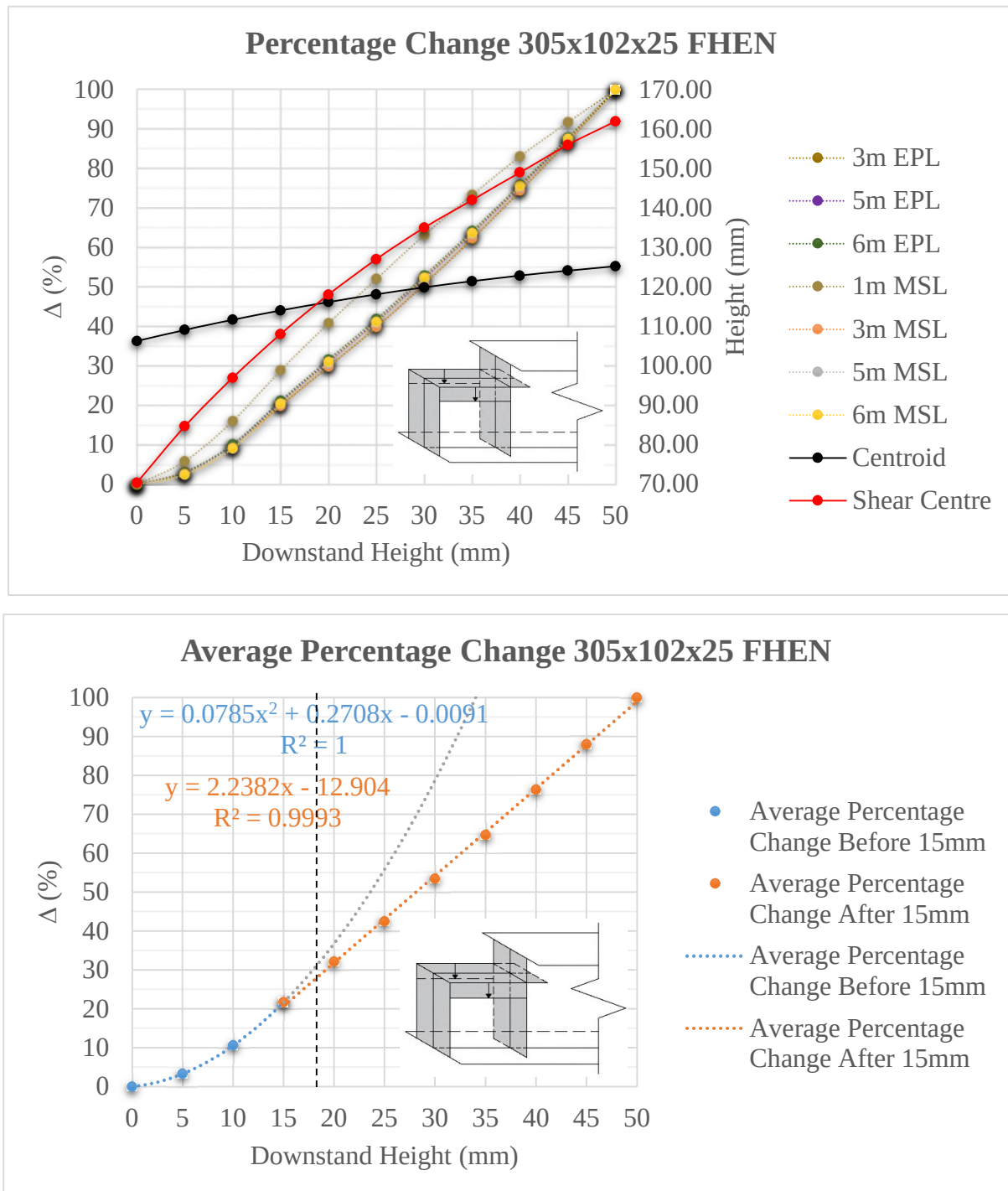


Figure 6.41: Percentage Change and Average Percentage Change Graph 305x102x25 FHEN Technique (Centroid, Shear Centre use right axis)

As previously mentioned, almost half of the data was excluded because the total change in the critical elastic moment was less than 0.5kNm which this report considers a negligible change. The FHHVNTS had two cases that were excluded for this reason. The cases that were excluded were the 5m, 6m MSL cases.

The critical elastic moment increases with each increase in downstand height. The increase starts off slightly parabolic then straightens out. In the FHEN case: the coincidence point of the centroid and shear centre in the notched region occurred within the first few downstands, the FHHVNTS occurred closer to the last few. This is why the trends appear similar because they are both initially parabolic, but the FHHVNTS is parabolic longer than the FHEN technique.

The trend is very strong across the various loading and span configurations. The trend is initially parabolic then linear after the 25mm downstand height: the intersection of the centroid and shear centre curves (**Figure 6.42**). This is similar to the 203x133x25 section in **Figure 6.36**, but the divergence is not as clear in the 305x102x25 section. Therefore, a change in the trend occurred at the coincidence point of the centroid and shear centre in the notched region but it is not significant.

There are similarities between the results of the 203x133x25 section and the 305x102x25 section across the stiffening techniques. The 203x133x25 section almost always increased its critical elastic moment more through the introduction of downstands than the 305x102x25 section. However, the trend in the increase is the same in both sections. That is: whether the total increase was 17kNm or 3kNm, the increase associated with the increase in downstand height was consistent across the techniques. This is clear in the percentage change graphs. Upon closer inspection, there are subtle differences in the average percentage change across all the span and loading configurations for each section, but the general trend is the same. This is evident in the average percentage change graphs: the trends may be similar but more prominent in either section.

Some key points to be taken from the discussion:

- The shorter spanning members (1m and 5m) increased their critical elastic moment more than the longer span (3m and 6m) just as in the 203x133x25.
- Almost half of the increases were negligible.
- Changes were evident around the centroid shear centre coincidence point in the form of a change in gradient or change in curve type.

Figure 6.42 shows the percentage change and average percentage change for the FHHVNTS technique.

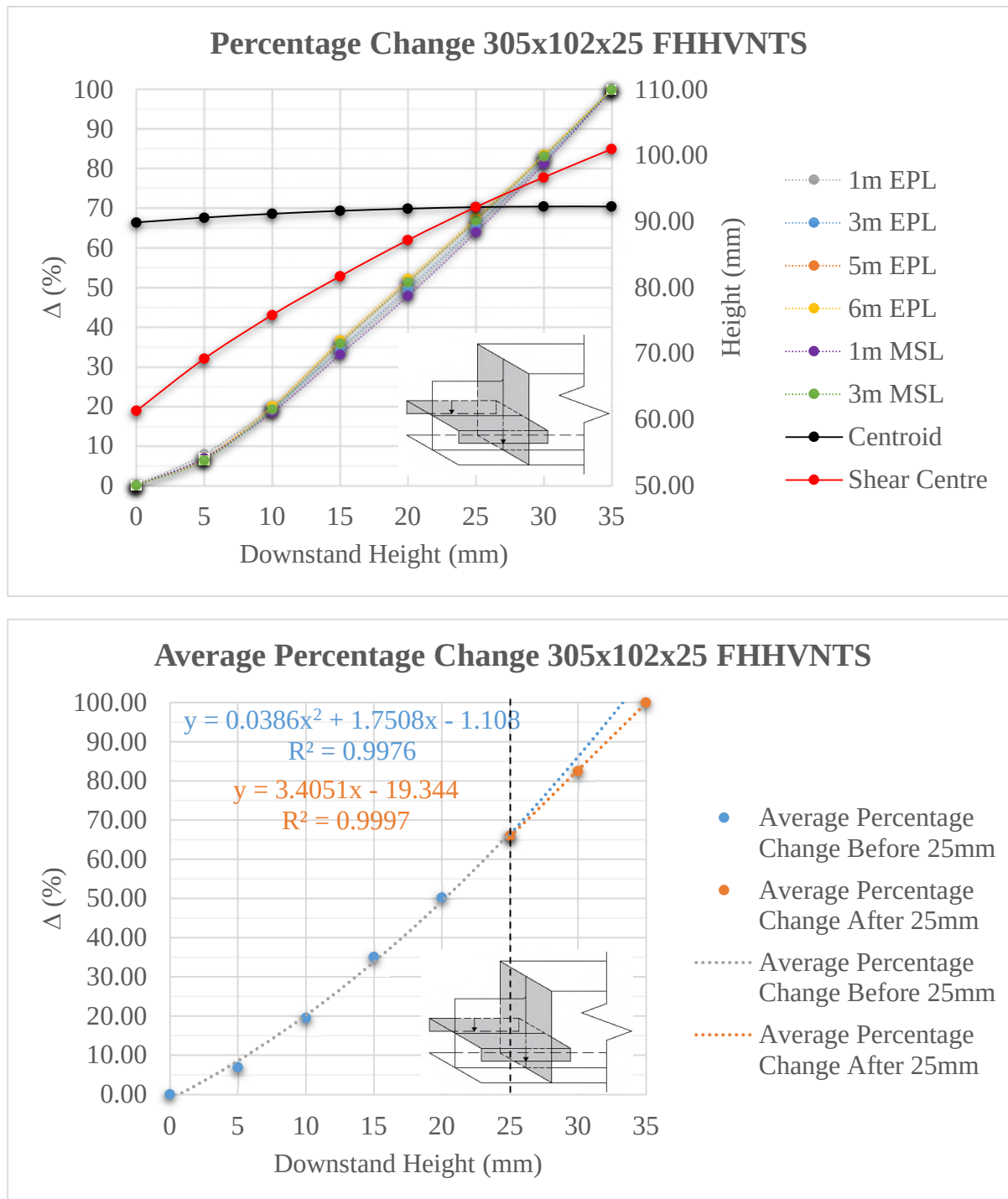


Figure 6.42: Percentage Change and Average Percentage Change Graph 305x102x25 FHHVNTS Technique (Centroid, Shear Centre use right axis)

6.6 533 x 210 x 82

6.6.1 Summary

Model 1 and Model 2 are presented in *Appendix E: Results of Model 1 and 2* since the discussion about Model 1 and Model 2 follows the same process as the previous sections. A summary of the trends is shown in **Figure 6.43** and techniques of interest will be further discussed in 6.6.2 *Further Discussion*. Below is a list of techniques that do not meet the minimum change in critical elastic moment:

- 1m EPL FLH

The FLHFHV technique and Isolated downstand FLHFHV are not included in the summary.

The 1m EPL and 3m MSL are outliers in the FHN technique, which is clear in **Figure 6.43** below, and were excluded from the average percentage change curves.

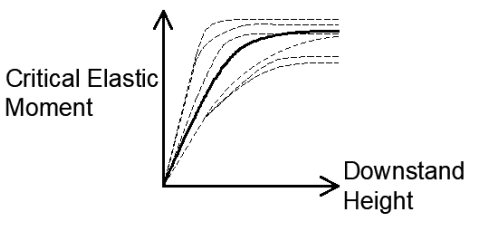
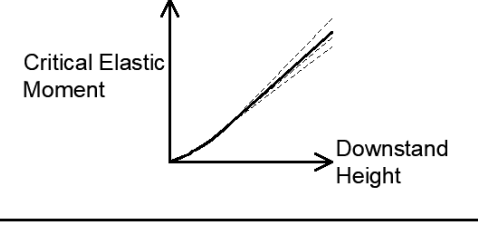
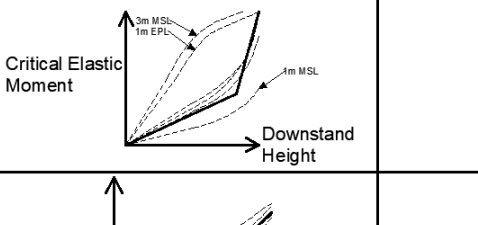
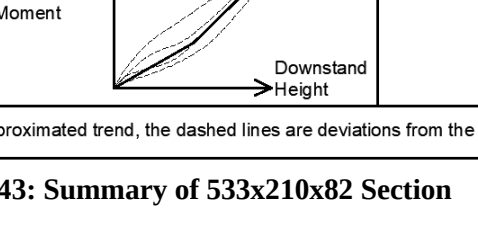
Trend Description	Trend Image	Techniques
M_{cr} increases less rapidly as shear centre approaches and passes the centroid in the notched region. The trend is initially parabolic then linear. The linear trend is almost horizontal as the curves appear to approach an asymptote.		FHN Model 1,2
M_{cr} increases more rapidly as shear centre approaches and passes the centroid in the notched region. Initially parabolic then after coincidence linear trend.		FHHVNTS Model 1,2
Trend Description	Trend Image	Techniques
M_{cr} exhibits different behavior which are dependent on the load or span. The trends can also be approximated as straight lines.		FHN Model 1,2
M_{cr} increased approximately in a linear trend with each increase in downstand height. However, close to the centroid shear centre coincidence, there is a change in gradient.		FLH Model 1,2
The solid line is the average/approximated trend, the dashed lines are deviations from the average trend		

Figure 6.43: Summary of 533x210x82 Section

6.6.2 Further Discussion

A trend was evident in the 203x133x25 and 305x102x25 sections: usually the shorter of the two spans tested in Model 1 and Model 2 increased their critical elastic moment more than the longer span. For the 533x210x82 section, 9/16 (four stiffening techniques with two models with 2 spans) load cases tested had the shorter of the two spans increase their critical elastic moment more which follows the trend seen in the previous two sections.

Other interesting points that were noticed:

- There was only one case that had a negligible increase in the critical elastic moment: the 1m span from the FLH technique in Model 1. This is in great contrast with the previous section, the 305x102x25, which had 18/32 cases with negligible increases (excluding the FLHFHV technique).
- There were two outliers: 1m EPL and 3m MSL FHN technique.
- In most cases, the total increase in the critical elastic moment for the 533x210x82 section were quite significant and much greater than the previous two tested sections.
- The 1m EPL and 3m MSL were excluded from the percentage change and average percentage change graphs for the FHN technique since they are outliers.

Figure 6.44 shows the percentage change and the average percentage change for the FHEN technique. The trend is different to the previous two sections: the 203x133x25 (shown in **Figure 6.35**) and the 302x105x25 (**Figure 6.41**). The trend in these two sections is slightly parabolic starting from a minimum value and increasing ever steeper with each increase in downstand height. The 533x210x82 section also increases its critical elastic moment but the percentage change approaches a maximum value. The trend is parabolic, but the trend seems to approach a turning point, maximum value or asymptote before the coincidence point of the centroid and shear centre in the notched region at a downstand height of 223mm.

The trend is not as strong as in the previous two sections, but it is not due to outliers. The 5m EPL, 3m MSL and 6m EPL are more “pronounced” or “extreme” trends of the other cases: the parabolic shape is more prominent. This distinction must be made because the outlier identified in the 203x133x25 (the 1m MSL) does not resemble the other cases at all.

Using the average percentage change graph, it appears the turning point is before the 223mm downstand, at the 200mm downstand. No downstand height was tested at 223mm or close enough to it to better see the behaviour around this point to directly check if the turning point occurs at this downstand height. It is clear that there is a change in the trend around the coincidence point of the centroid and shear centre.

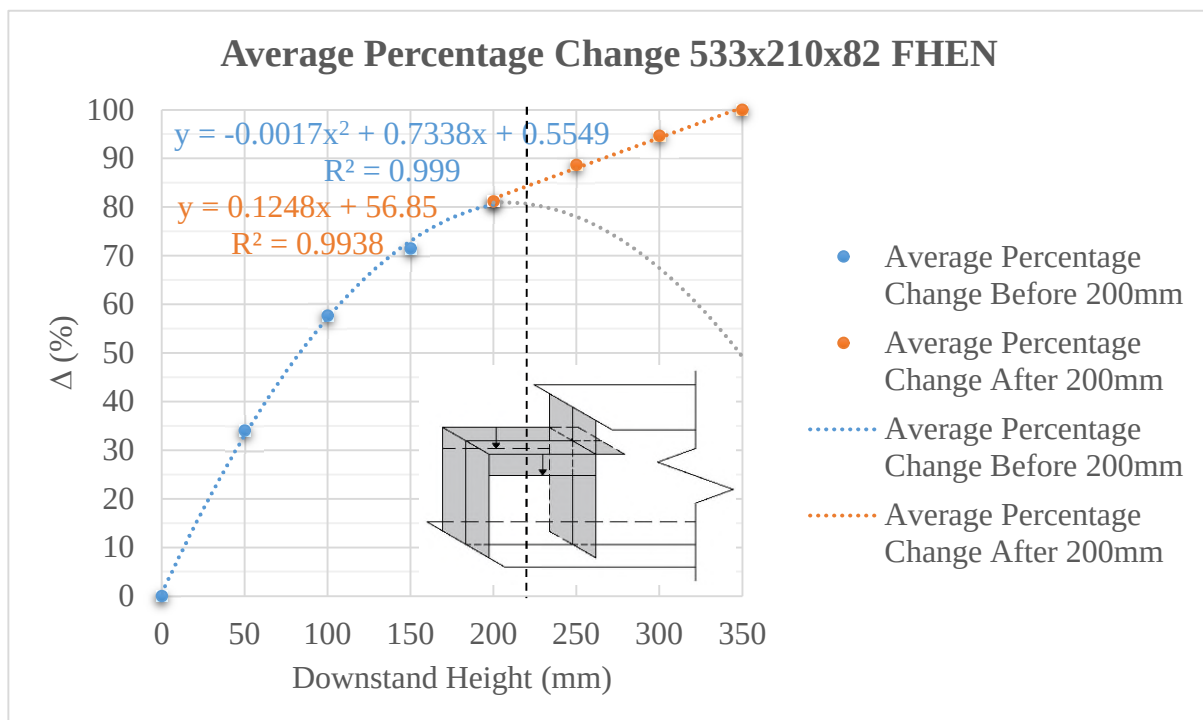
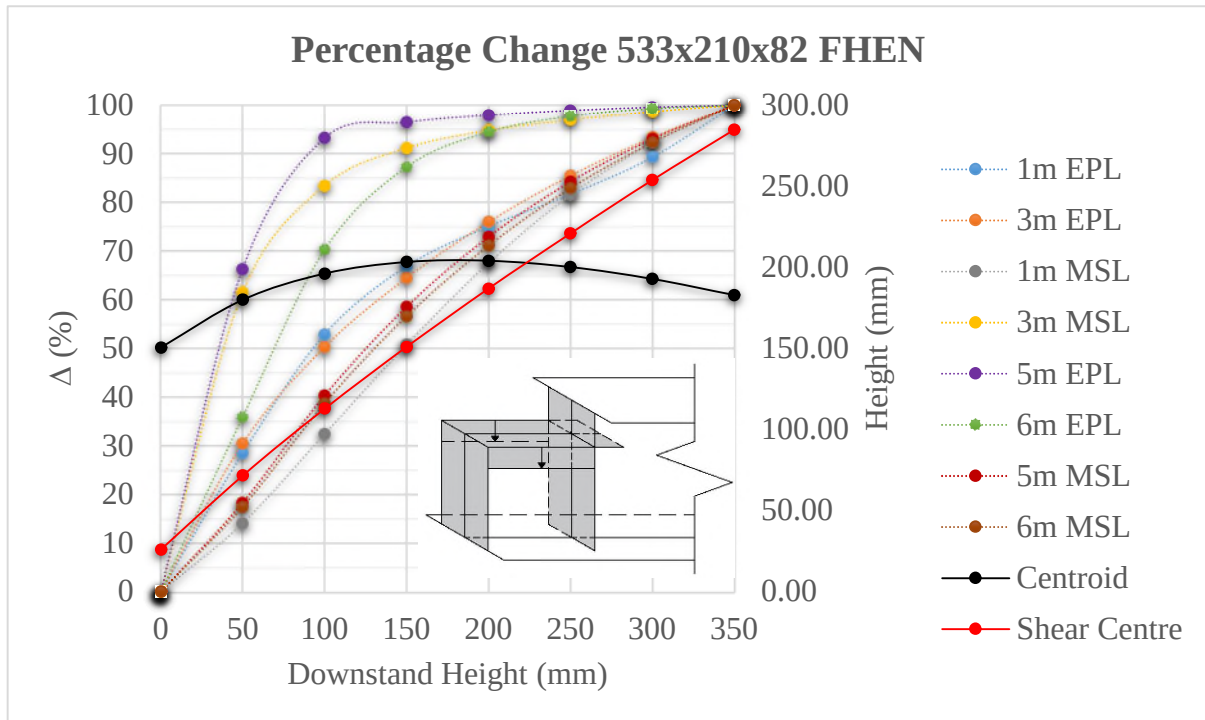


Figure 6.44: Percentage Change and Average Percentage Change Graph 533x210x82 FHEN Technique (Centroid, Shear Centre use right axis)

However, it is evident there are two distinct curves before and after the 230mm downstand: parabolic before and linear after. This is similar to the 305x102x25 section, but that trend is concave up whereas this trend is concave down. The 203x102x25 section had a very slight change in gradient between the two straight lines.

Figure 6.45 shows the percentage change and average percentage change graphs for the FHHVNTS technique. There is a strong trend evident in this technique just as in the 203x133x25 (**Figure 6.36**) and the 305x102x25 (**Figure 6.42**) sections. It is interesting the trend is the same despite the sections having different downstand heights and different number of downstand cases.

The percentage change curves resemble a parabola starting from a minimum value up till the coincidence of the centroid and shear centre at a downstand height of 40mm. Thereafter the curves are almost linear at a steep gradient. Using the average percentage change curve, it is clear there is a change in the curve type after the 40mm downstand height. This is evident through the divergence of the initial trend (before 40mm) and the final trend (after 40mm).

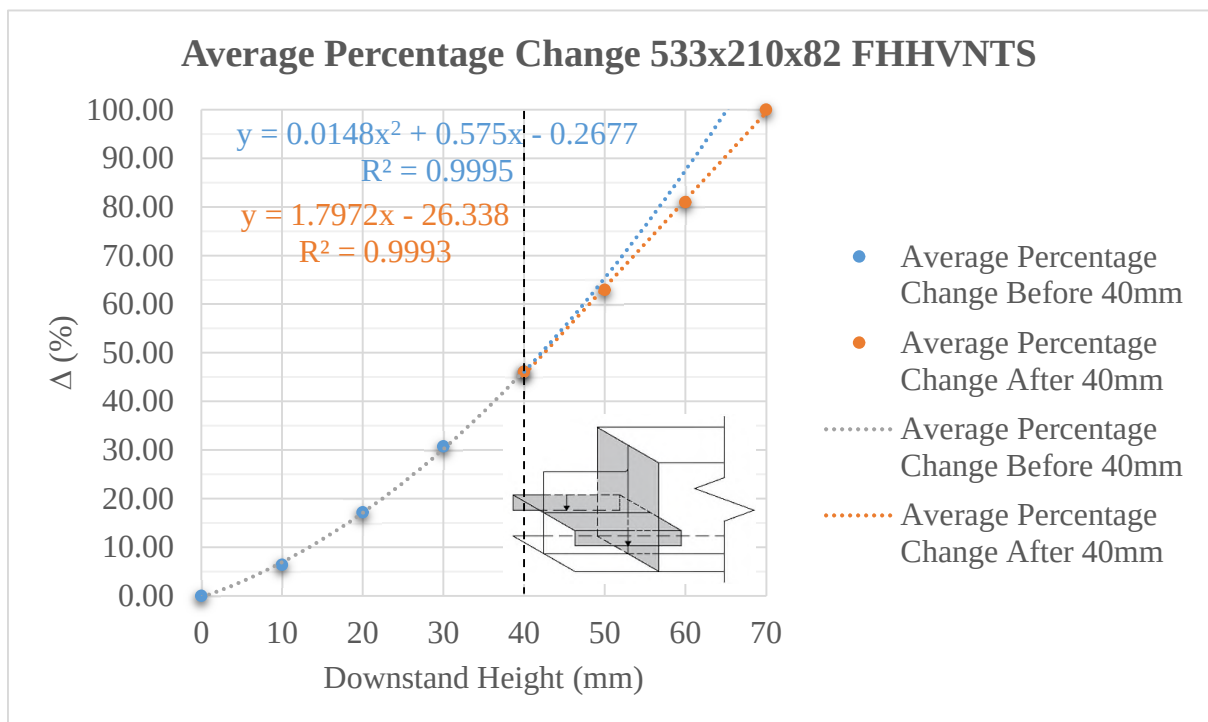
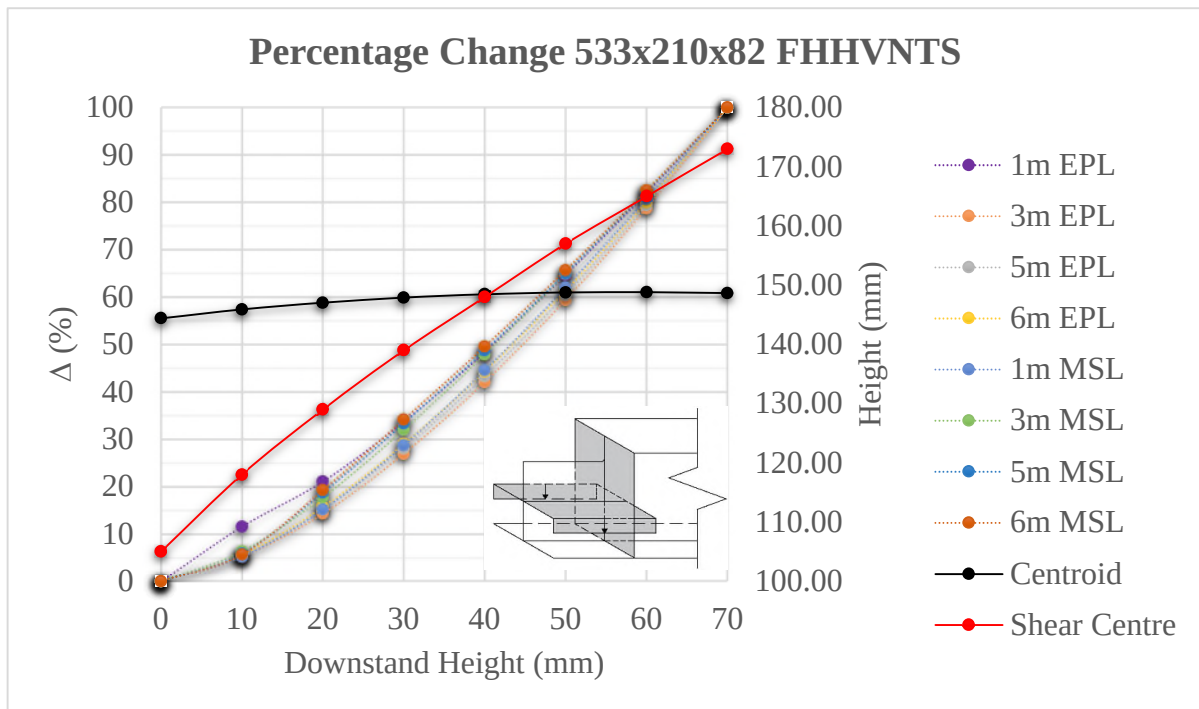


Figure 6.45: Percentage Change and Average Percentage Change Graph 533x210x82 FHHVNTS Technique (Centroid, Shear Centre use right axis)

Figure 6.46 shows the percentage change and average percentage change graphs for the FHN technique. The trend is similar - in most cases – to the 203x133x25 section (**Figure 6.37**). The 1m EPL and 3m MSL cases are outliers and have been excluded from the percentage change and average percentage change curves. The general trend is similar to the 203x133x25 section (**Figure 6.37**): the curve is slightly parabolic, starts at a minimum and is concave up.

There was some strange behaviour in the critical moment curves (**Figure E.31** and **Figure E.32**): the trends in the 1m and 3m spans swapped between load cases. The EPL case had the 1m approaching a maximum value whereas the MSL load the 1m span starts at a minimum value and is concave up. The same is true for the 3m span: the EPL case is concave up whereas the MSL 3m span tends to a maximum value. These trends are evident in the percentage change graph, but it is clear the 1m EPL and 3m MSL are the only two instances of this trend. Therefore, they are considered outliers and have been excluded from the average percentage change calculation.

There is a change in gradient as some of the curves approach and pass the 223mm downstand. The 1m MSL, 3m and 5m EPL decrease their gradient after the 223mm downstand whereas the other curves increase their gradient with each increase in downstand height. Using the average percentage change graph (which excludes the 1m EPL and 3m MSL from the calculation since these trends are outliers), it is clear there is a change in gradient of the curves after the 200mm downstand height. As with the FHEN technique, no heights around 223mm were tested. This means it is unclear if the change in gradient occurs at that height or before, but what can be drawn from the graph is the shifting centroid and shear centre has caused some change in behaviour when the shear centre approaches and passes the centroid in the notched region. It is unclear if this occurs at the coincidence point or before it.

The 305x102x25 section is not discussed for this technique since the percentage change for this technique was negligible.

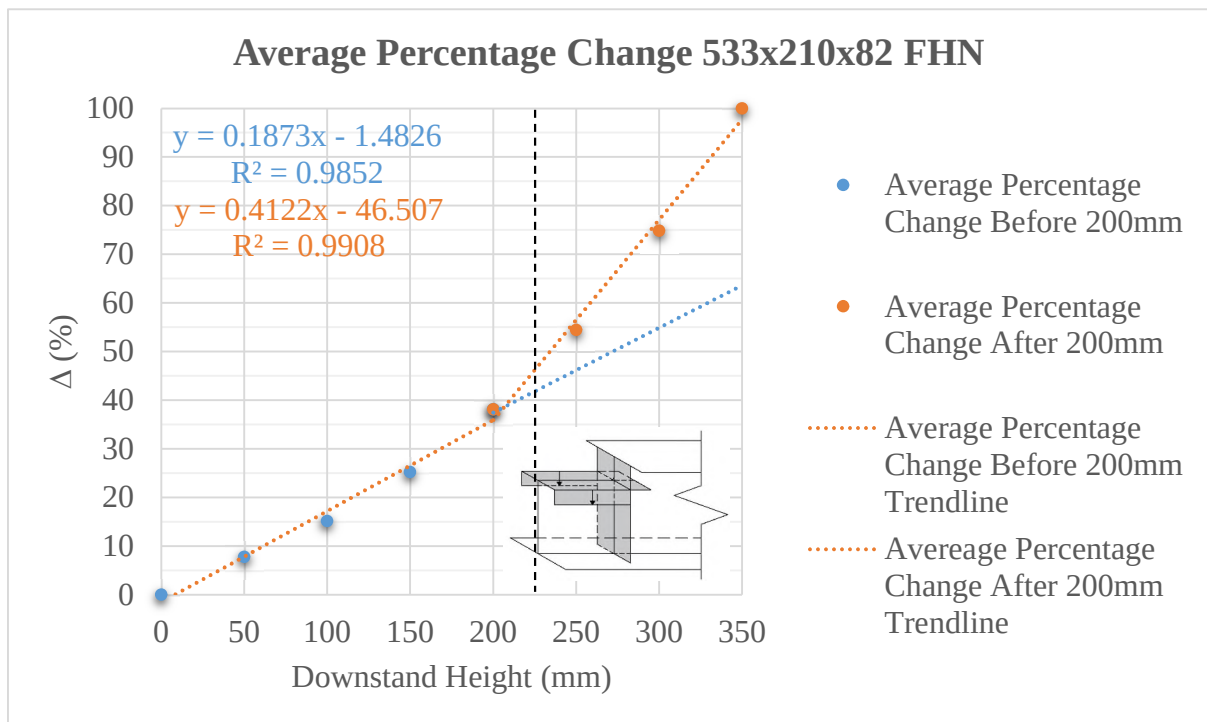
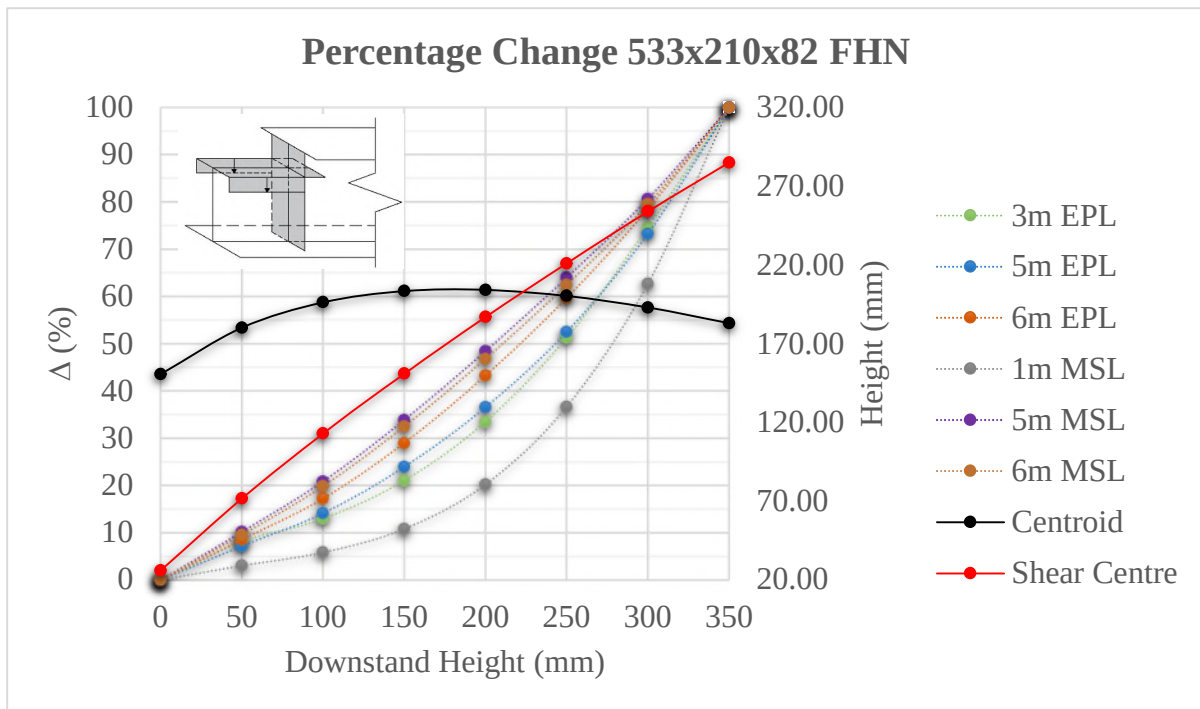


Figure 6.46: Percentage Change and Average Percentage Change Graph 533x210x82 FHN Technique (Centroid, Shear Centre use right axis)

Figure 6.47 shows the percentage change and average percentage change for the FLH stiffening technique. There is a common trend in the data but with some outliers and more pronounced cases like in the FHEN and FHN techniques. The outliers are due to the same behaviour evident in the FHN case with the swopping of the trends. The EPL case (**Figure E.33**) had the 1m approaching a maximum value whereas the MSL (**Figure E.34**) the 1m span starts at a minimum value and is concave up. The same is true for the 3m span: the EPL case is concave up whereas the MSL 3m span tends to a maximum value. These cases were still included in the average percentage change calculation because their trend does not skew the data. The 1m FLH EPL was excluded because the total change in the critical elastic moment was less than 0.5kNm which is less than the minimum required for this report.

There is no distinct change in the trends around the coincidence point of the centroid and shear centre at the 223mm downstand height. As with the FHEN and FHN techniques, no downstands were tested around or close enough to this downstand height. It is clear there is a change in gradient in the average percentage change curve as the trendlines diverge after the 200mm downstand. A clear gap exists between the curves at the 223mm downstand which could indicate the coincidence point could be the reason but, as previously mentioned, no downstands were tested close enough to this height to conclusively state this. What can be drawn from these graphs: as the shear centre approaches and passes the centroid in the notched region, there is an increase in the gradient of the percentage change and average percentage change curves. This shows the influence of these shifting properties. It remains unclear at which point the change occurs. That is: if it occurs exactly at the 223mm downstand height.

Some key points to be taken from the discussion:

- Just as in the 203x133x25 and 305x102x25 sections the shorter of the two spans (1m and 5m) increased their critical elastic moment more than the longer spans (3m and 6m).
- There was only one negligible increase, and two outliers were present.
- The trends were strong but, in some cases, slightly different: either more or less pronounced but not vastly different to the general trend.
- The total increase in critical elastic moment was significant.
- There was a change in the trend in the critical elastic moment around the centroid shear centre coincidence point which was seen as a change in gradient or curve type.

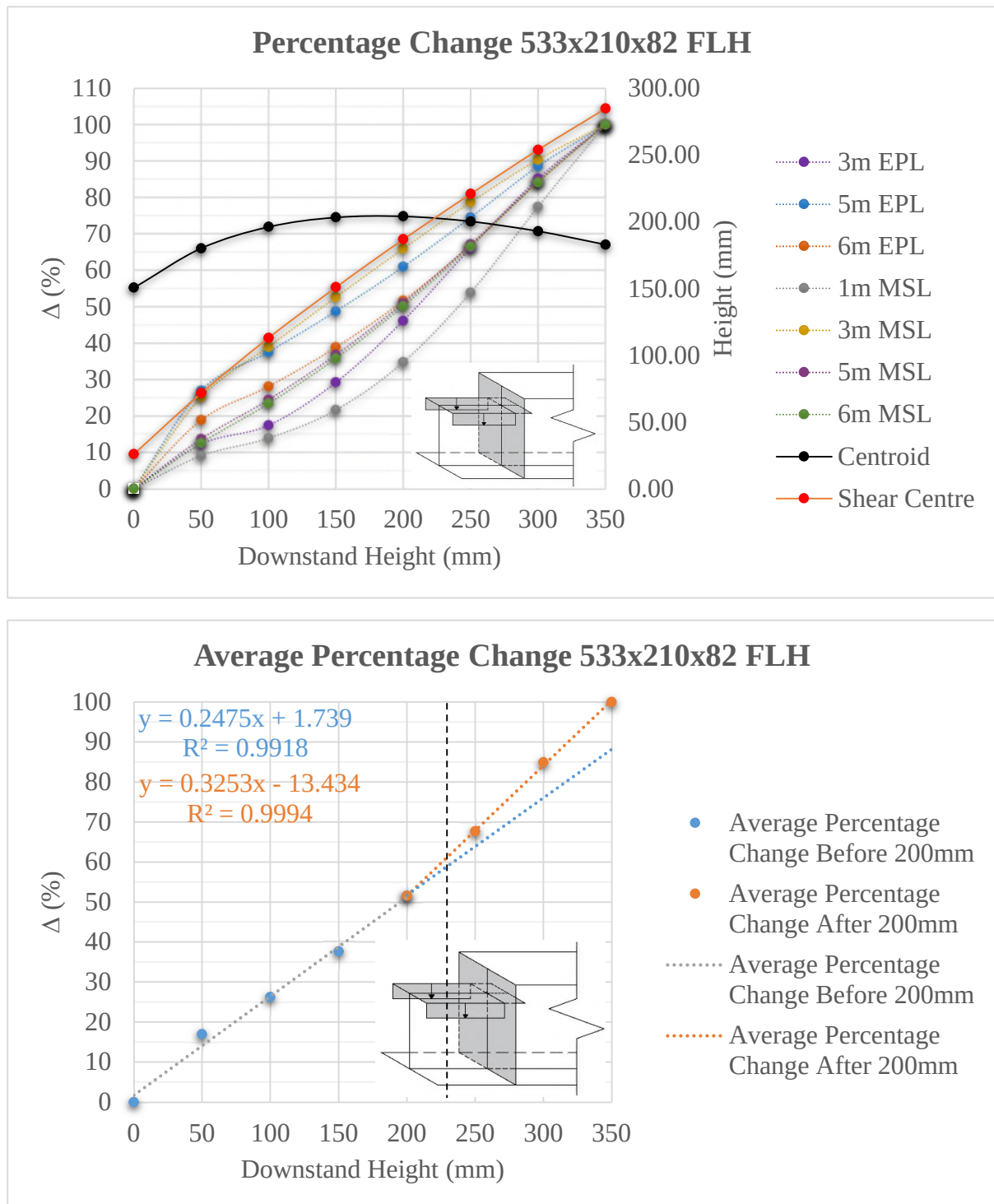


Figure 6.47: Percentage Change and Average Percentage Change Graph 533x210x82 FLH Technique (Centroid, Shear Centre use right axis)

6.7 203 x 203 x 46

6.7.1 Summary

Model 1 and Model 2 are presented in *Appendix E: Results of Model 1 and 2* since the discussion about Model 1 and Model 2 follows the same process as the previous sections. A summary of the trends is shown in **Figure 6.48** and techniques of interest will be further discussed in 6.7.2 *Further Discussion*. Below is a list of techniques that do not meet the minimum change in critical elastic moment:

- 1m MSL FHEN
- 1m MSL FLH (also an outlier)

The FLHFHV technique and Isolated downstand FLHFHV are not included in the summary.

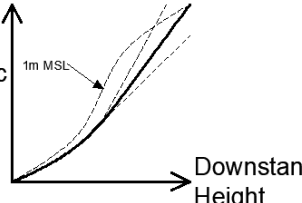
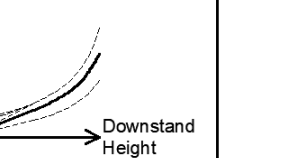
Trend Description	Trend Image	Techniques
M_{cr} increased linearly with each increase in downstand height. However, close to the centroid shear centre coincidence, there is a change in gradient.		FHEN Model 1,2 FLH Model 1,2
M_{cr} increases more rapidly as shear centre approaches and passes the centroid in the notched region. Initially parabolic then after coincidence linear trend. The 1m MSL is the only case with a slightly different trend.		FHHVNTS Model 1,2
Trend Description	Trend Image	Techniques
M_{cr} increases more rapidly as shear centre approaches and passes the centroid in the notched region. Initially parabolic then after coincidence linear trend.		FHN Model 1,2
The solid line is the average/approximated trend, the dashed lines are deviations from the average trend		

Figure 6.48: Summary of 203x203x46 Section

6.7.2 Further Discussion

The I sections (203x133x25, 305x102x25 and 533x210x82) have a trend in which the shorter of the two spans tested in the two Models increased their critical elastic moment more than the longer span. The 203x203x46 H section was no different: in 10/16 (four stiffening techniques with two models with 2 spans) of the load cases tested, the shorter spans increased more than the longer spans. As with the previous discussion sections, the FLHFHV case is not discussed.

Another point is there were two cases which had increases that were below the minimum of 0.5kNm. As previously discussed, this means those cases were ignored because their increase is considered negligible in this report and were therefore removed from the percentage change curves and the average percentage change calculation. These were the:

- 1m MSL FHEN
- 1m MSL FLH

Below are the two outliers that were excluded from the percentage change and average percentage change graphs, calculation:

- 1m MSL FHHVNTS
- 1m MSL FLH

Figure 6.49 shows the percentage change and average percentage change for the FHEN technique. There is a similar trend in the results across all spans and loading cases and no outliers exist. The trend is also similar to the 203x133x25 (**Figure 6.35**) and the 305x102x25 (**Figure 6.41**) FHEN technique. The trend starts at a minimum value and is parabolic for the first downstands then changes to a linear trend. There is no obvious change in the trend due to the shifting centroid and shear centre. This is very interesting due to the different shapes of these sections (I sections versus H section).

The 203x133x25 technique average percentage curve (**Figure 6.35**) is similar to the 203x203x46 average percentage change because both have linear trends before and after the coincidence point of the centroid and shear centre in the notched region (at 117mm downstand height). This is not the case for the 305x102x25 section which has a parabolic trend for the downstands before the coincidence point and linear thereafter. The 533x210x82 section has a completely different trend as the initial trend is parabolic but approaching a maximum value then a low gradient linear trend.

Figure 6.49 shows there is no clear divergence between the two curves in the average percentage change graph which suggests there is little to no influence on the critical elastic moment by the shifting centroid and shear centre.

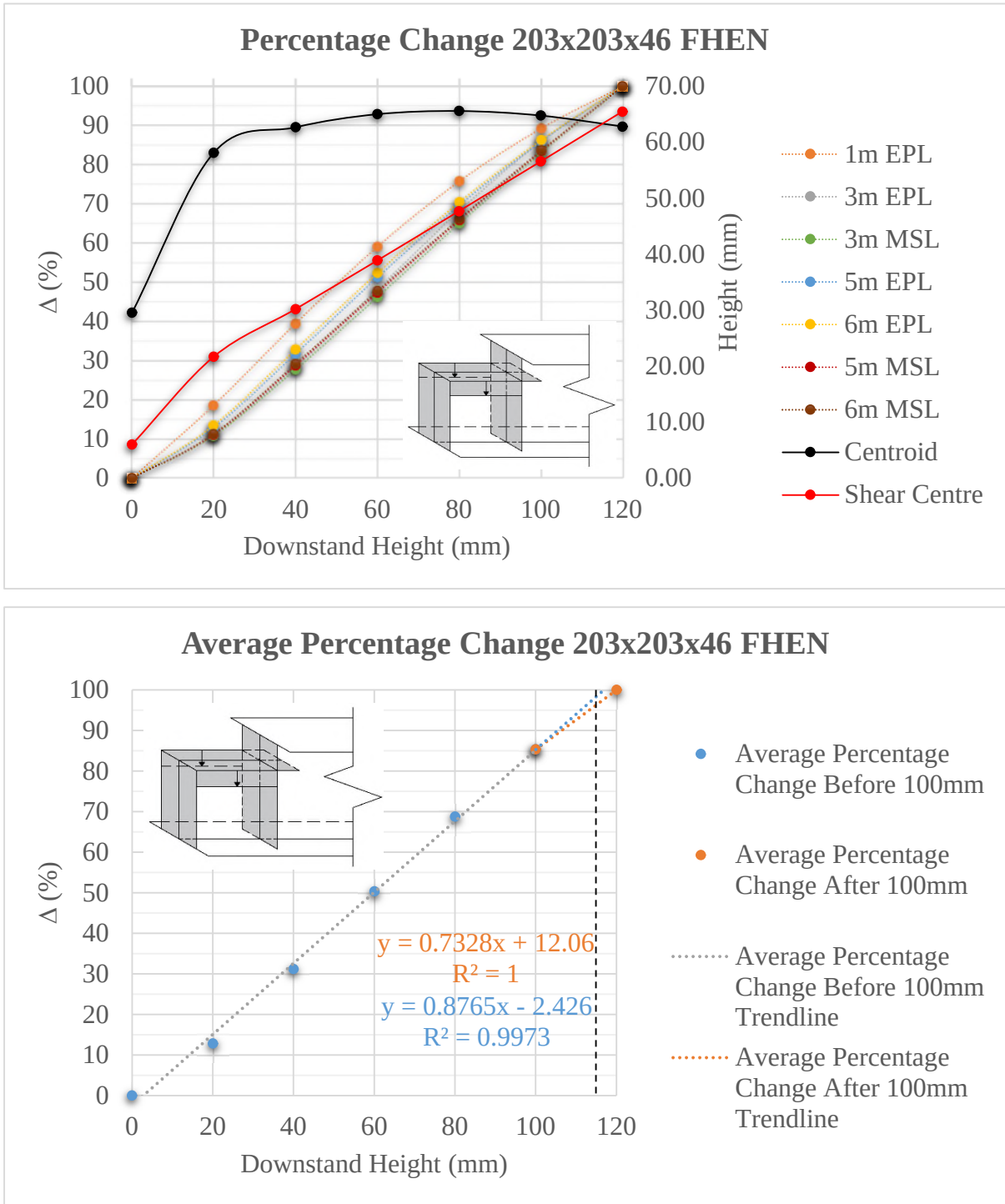


Figure 6.49: Percentage Change and Average Percentage Change Graph 203x203x46 FHEN Technique (Centroid, Shear Centre use right axis)

Figure 6.50 shows the percentage change and average percentage change graphs for the FHHVNTS technique. The trend is strong and similar to the FHEN technique (**Figure 6.49**) but with one outlier: the 1m MSL (which has been removed from the percentage change and average percentage change graph, calculation). It is quite interesting to see the trend is consistent with the other sections despite the completely different geometries of the sections.

There is no obvious change in the trend due to the shifting centroid and shear centre in the percentage change curves but closer inspection shows there is a change in the trendline. Before the 13mm downstand the trendline appears parabolic but after the trend looks linear. The average percentage change curves show this clear change close to the coincidence of the centroid and shear centre (here at the 13mm downstand). This change is clear because the initial trend diverges from the later trend. It is unclear if the change occurs exactly at the 13mm downstand because this height was not tested. This is why the 10mm downstand height is used as a reference for the before and after cases in the average percentage change curves.

The 203x133x25 section has a similar divergence to the 203x203x46 section but the 305x102x25 and 533x210x82 sections only slightly diverge. The details of the notch are almost identical between the 203x133x25 and 203x203x46 sections since they share the same height (un-notched height is 203mm and notch height is 153mm and 153.2mm respectively) but the number of downstands is different due to the different widths: 133mm and 203mm. This emphasises the behaviour in the trends: the FHHVNTS technique shows the influence of the shifting centroid and shear centre very clearly because in all sections, with different number of downstands and heights, around the coincidence of the centroid and shear centre there is a change in the gradient of the average percentage change. The change may be small, as in the 305x102x25 and 533x210x82 sections, or significant as shown through the divergence of the trends, as in the 203x133x25 and 203x203x46 sections, but the changes exist.

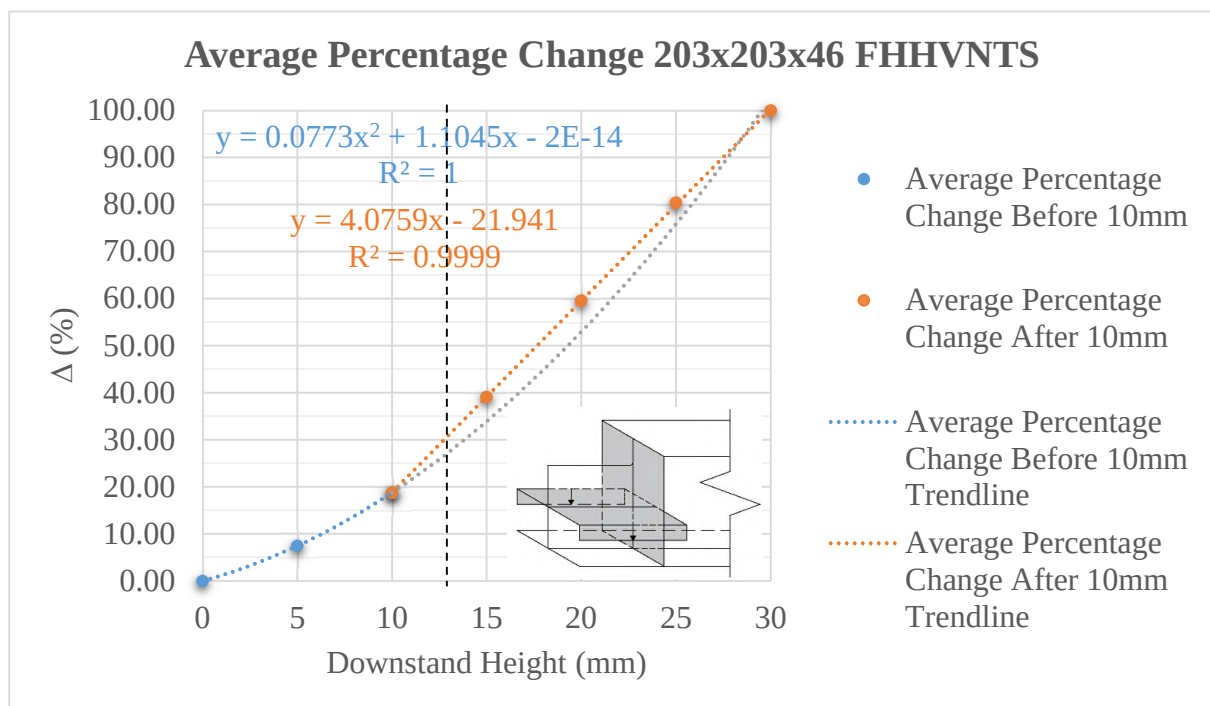
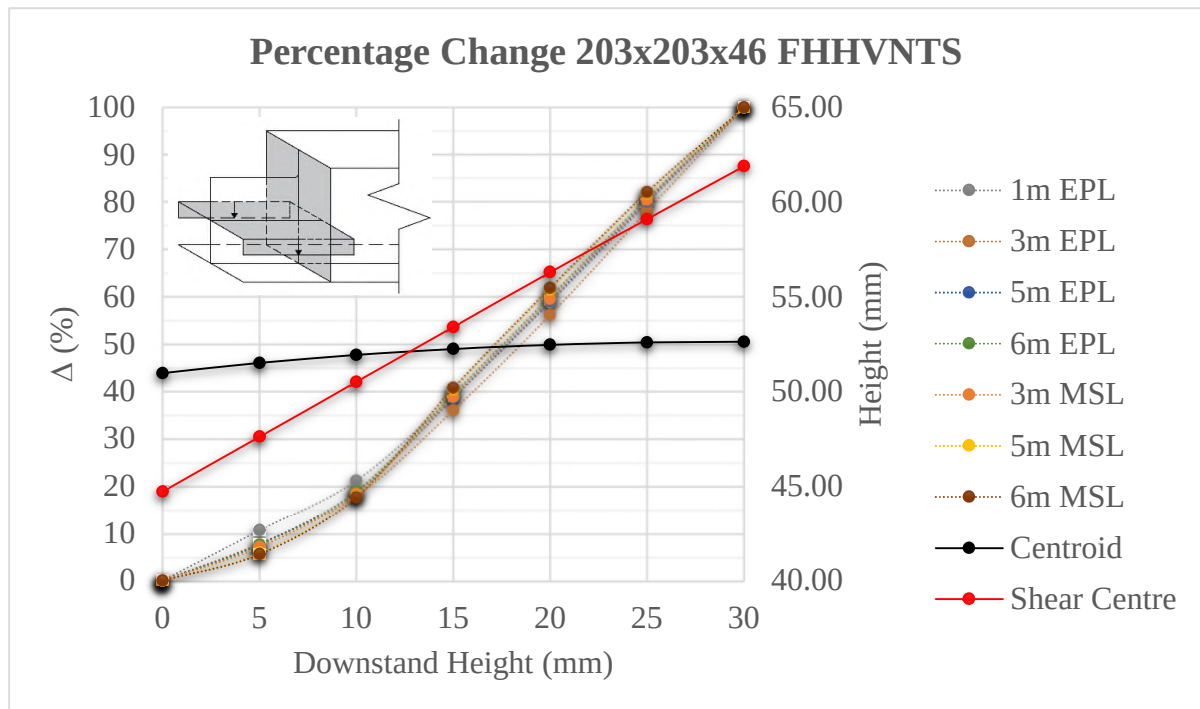


Figure 6.50: Percentage Change and Average Percentage Change Graph 203x203x46 FHHVNTS Technique (Centroid, Shear Centre use right axis)

Figure 6.51 shows the percentage change and average percentage change for the FHN technique. The trend in the 203x133x25 section (**Figure 6.37**) is similar: the trend is parabolic starting at a minimum value with no clear change in the trend around the centroid and shear centre. The trend is similar across all the span and loading configurations. Due to height restrictions in the notched region downstand heights beyond 120mm could not be tested so the trend beyond this height cannot be investigated. This means the behaviour after the coincidence point cannot be completely discussed.

Figure 6.52 shows the percentage change and average percentage change for the FLH technique. The change in the 1m MSL is negligible therefore it has been excluded from the graphs and the average moment calculation. The trend becomes more linear/smooth with the longer spans. There is no clear influence on the trend by the shifting centroid and shear centre in either graph which could be due to the lack of tests after the coincidence point due to the height restriction in the notched region (just as in the FHN technique).

Some key points that can be taken from the discussion:

- The shorter spans (1m and 5m) increased their critical elastic moment more than the longer spans (3m and 6m) in their models.
- There were only two negligible increases and one outlier.
- There was no obvious change in the trend of the trend in the critical elastic moment as the shear centre approached and passed the centroid in the notched region for the FHEN and FLH techniques.
- The FHN technique shows divergence but it is unclear if the divergence continues since one height was tested after the coincidence point.
- The FHHVNTS shows a clear divergence after the centroid shear centre coincidence point as there is a change in the curve type.

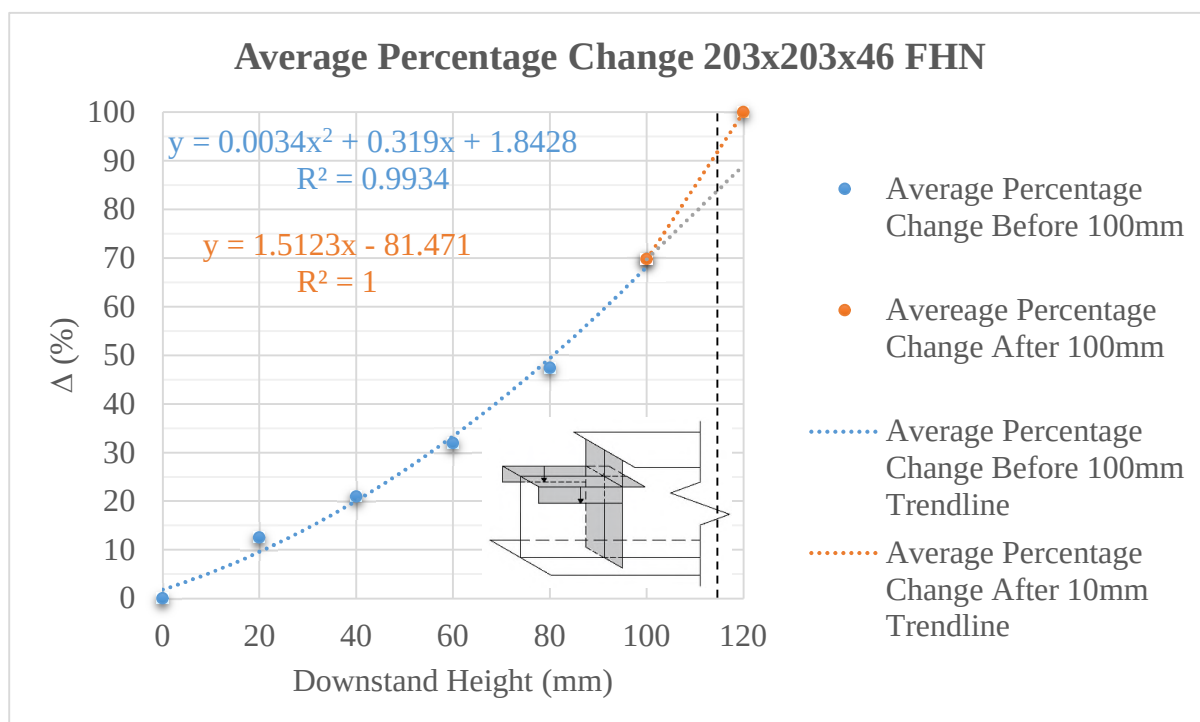
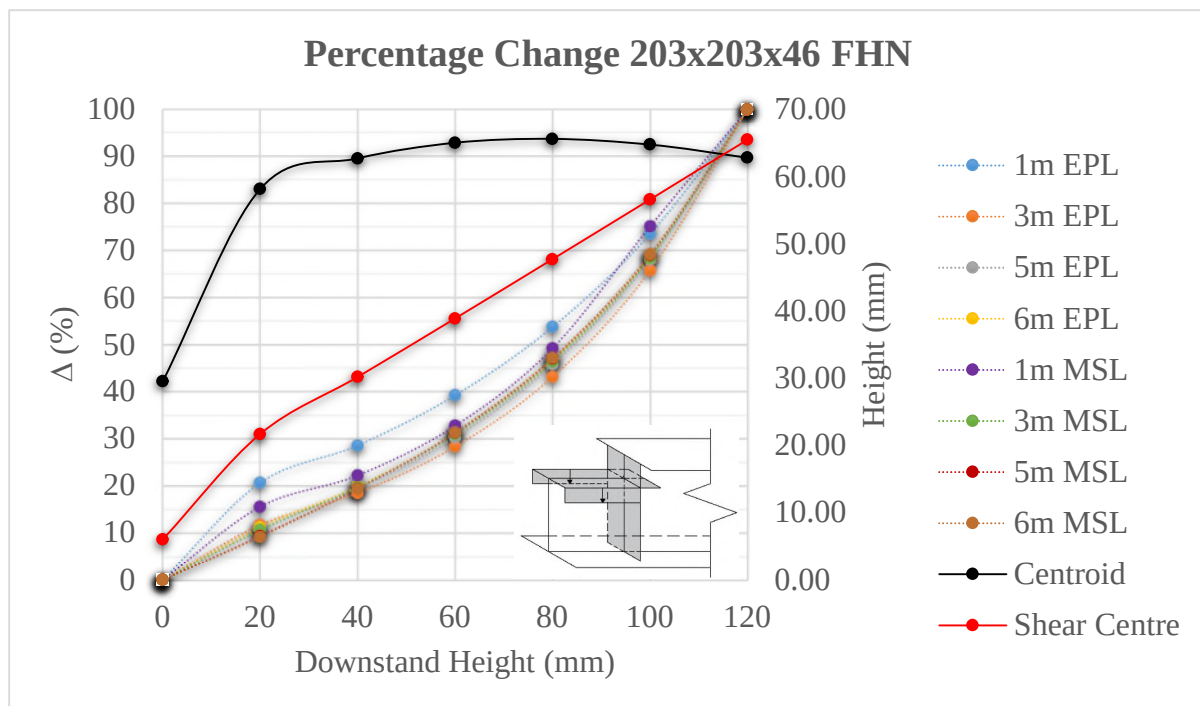


Figure 6.51: Percentage Change and Average Percentage Change Graph 203x203x46 FHN Technique (Centroid, Shear Centre use right axis)

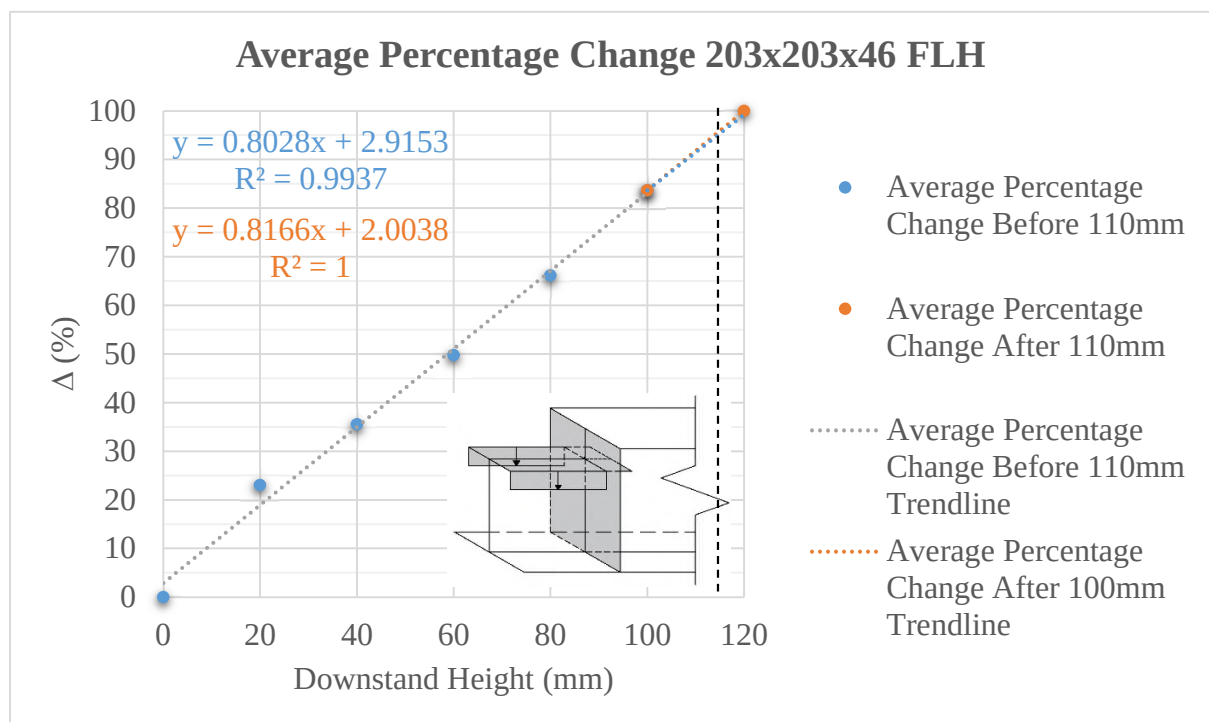
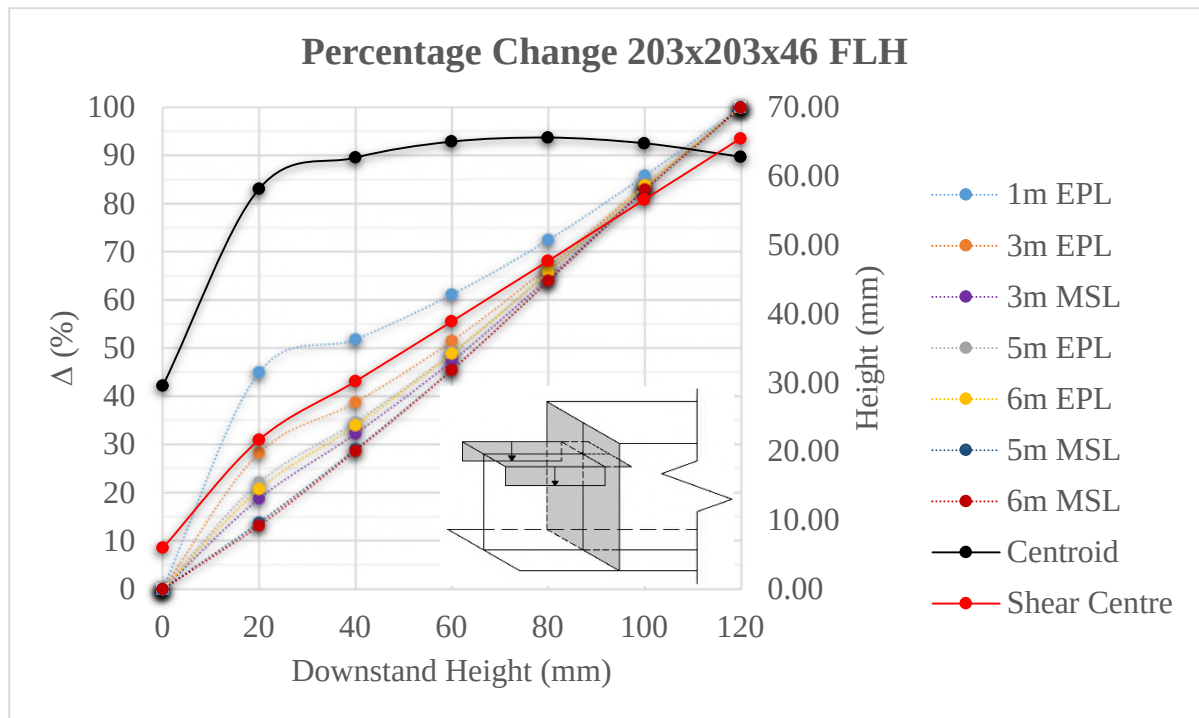


Figure 6.52: Percentage Change and Average Percentage Change Graph 203x203x46 FLH Technique (Centroid, Shear Centre use right axis)

6.8 305 x 305 x 97

6.8.1 Summary

Model 1 and Model 2 are presented in *Appendix E: Results of Model 1 and 2* since the discussion about Model 1 and Model 2 follows the same process as the previous sections. A summary of the trends is shown in **Figure 6.53** and techniques of interest will be further discussed in 6.8.2 *Further Discussion*. The 1m MSL, 3m and 5m EPL cases have been excluded because they are outliers that are difficult to show in the summary. Below is a list of techniques that do not meet the minimum change in critical elastic moment:

- 1m EPL and MSL FHHVNTS
- 3m EPL FLH

The FLHFHV technique and Isolated downstand FLHFHV are not included in the summary.

Trend Description	Trend Image	Techniques
M_{cr} increases less rapidly as shear centre approaches and passes the centroid in the notched region. The trend is initially parabolic then linear. The linear trend is almost horizontal as the curves appear to approach an asymptote.		FHEN Model 1,2
M_{cr} increases more rapidly as shear centre approaches and passes the centroid in the notched region. The curves are linear with a change to a steeper gradient at the coincidence point of the centroid and shear centre.		FHHVNTS Model 1,2
Trend Description	Trend Image	Techniques
M_{cr} increases more rapidly as shear centre approaches and passes the centroid in the notched region. Initially parabolic then after coincidence linear trend.		FHN Model 1,2
M_{cr} increased approximately in a linear trend with each increase in downstand height. However, close to the centroid shear centre coincidence, there is a change in gradient.		FLH Model 1,2
The solid line is the average/approximated trend, the dashed lines are deviations from the average trend		

Figure 6.53: Summary of 305x305x97 Section

6.8.2 Further Discussion

The previous sections had a trend in which the shorter span increased its critical elastic moment more than the longer span: the 1m and 5m span increased more than the 3m and 6m span. The 305x305x97 section was not as clear: the trend was split in half.

Some other key points:

- Three cases had increases in their critical elastic moment less than 0.5kNm: the 1m EPL, MSL for the FHHVNTS technique and the 3m EPL FLH case. These cases – as well as the 3m and 5m EPL, 1m MSL FLH case which are outliers - were removed from the percentage change and average percentage change graphs, calculation.
- In general, the 305x305x97 section increased its critical elastic moment the most, especially in Model 2, which is interesting because the deepest I section tested, the 533x210x82, increased in a similar manner.

Figure 6.54 shows the percentage change and average percentage change graphs for the FHEN technique. There is one outlier: the 1m MSL (which is excluded from the average percentage change calculation and graph) but all the other span and loading cases have a similar trend. The 1m MSL starts at a minimum value and increases parabolically with each increase in downstand height but concave up: the gradient increases with each increase in downstand height. This is different to the other cases which also start at a minimum value and increase parabolically but the gradient decreases with each increase in downstand height with some cases approaching a maximum value at the 100mm downstand and others approaching a turning point at 200mm downstand. The trend is very similar to the 533x210x82 section (**Figure 6.44**) which is very interesting because they are two very different types of sections, but they have one property in common: they are deep sections (the 305x305x97 is relatively deep for a H section). It appears the deeper sections, especially the longer spans in Model 2, increase their critical elastic moment the most when using the FHEN technique. It is also clear the manner in which they improve with each increase in downstand height is the same despite using a different number of downstands of different heights.

In both cases the maximum value, turning point or asymptote is approached as the shear centre approaches and passes the centroid in the notched region. It is clear the shifting centroid and shear centre have an impact on the results. The average percentage change curve shows the behaviour of the trend before the 100mm downstand and afterwards. No downstand height of 122mm was tested therefore the behaviour at the coincidence point of the centroid and shear centre is not completely known.

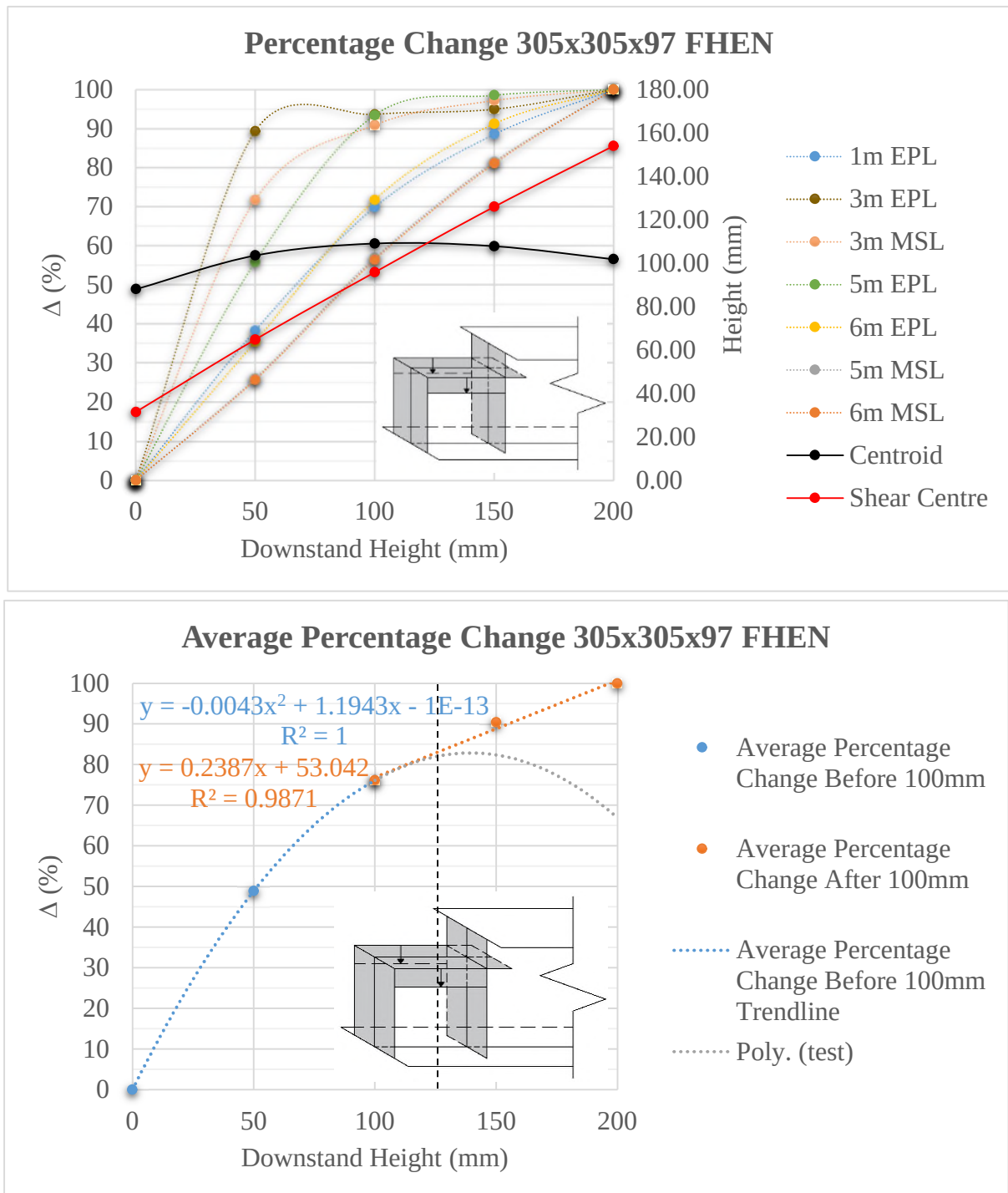


Figure 6.54: Percentage Change and Average Percentage Change Graph 305x305x97 FHEN Technique (Centroid, Shear Centre use right axis)

The average percentage change graph shows the change in the trend which occurs at the turning point of the parabola. Therefore, the initial trend when the shear centre is below the centroid and before the 100mm downstand, is parabolic but after the 100mm downstand the trend is linear which shows the location of the shear centre relative to the centroid influences the trend and therefore the critical elastic moment.

Figure 6.55 shows the percentage change and average percentage change curves for the FHHVNTS technique. As previously mentioned, the 1m EPL and 1m MSL cases were excluded because the total change in the critical elastic moment was less than the minimum set in this report of 0.5kNm. As with the previous tested sections (except the 203x203x46 section which had one outlier) there is a similar trend in the results. The trend starts at a minimum value then increases with each increase in downstand height concave up so the gradient increases with each increase in downstand height. There is no obvious change around the 18mm downstand height but before it the curve seems parabolic and after it seems linear. The 18mm downstand height – the downstand height needed for the centroid and shear centre in the notched region to be at the same vertical height - was not tested therefore the 15mm downstand was used as the reference height for the before and after series in the average percentage change graphs because it is the closest height before the coincidence of the centroid and shear centre.

It is clear there is a divergence in the trend after the 15mm downstand height. There is a change in gradient between the two series, but it cannot be conclusively stated it occurs at the 18mm downstand height since no downstand of that height was tested. It is clear close to the coincidence point of the centroid and shear centre in the notched region there are some changes in the trend.

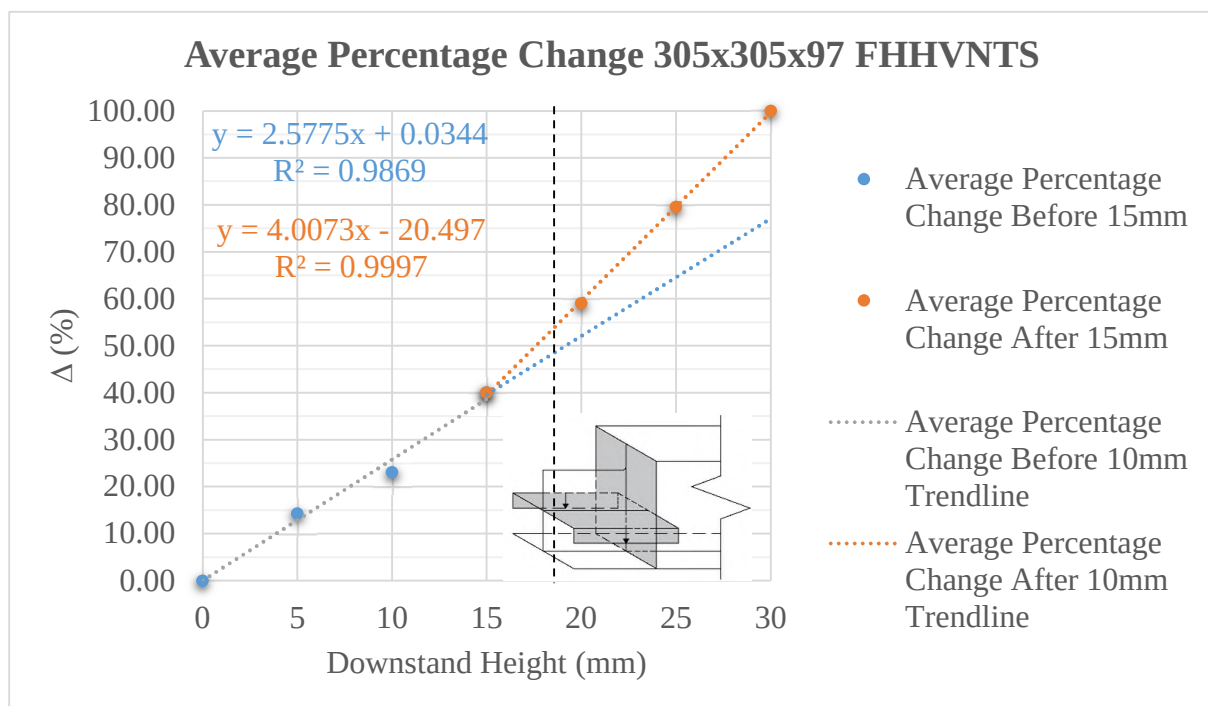
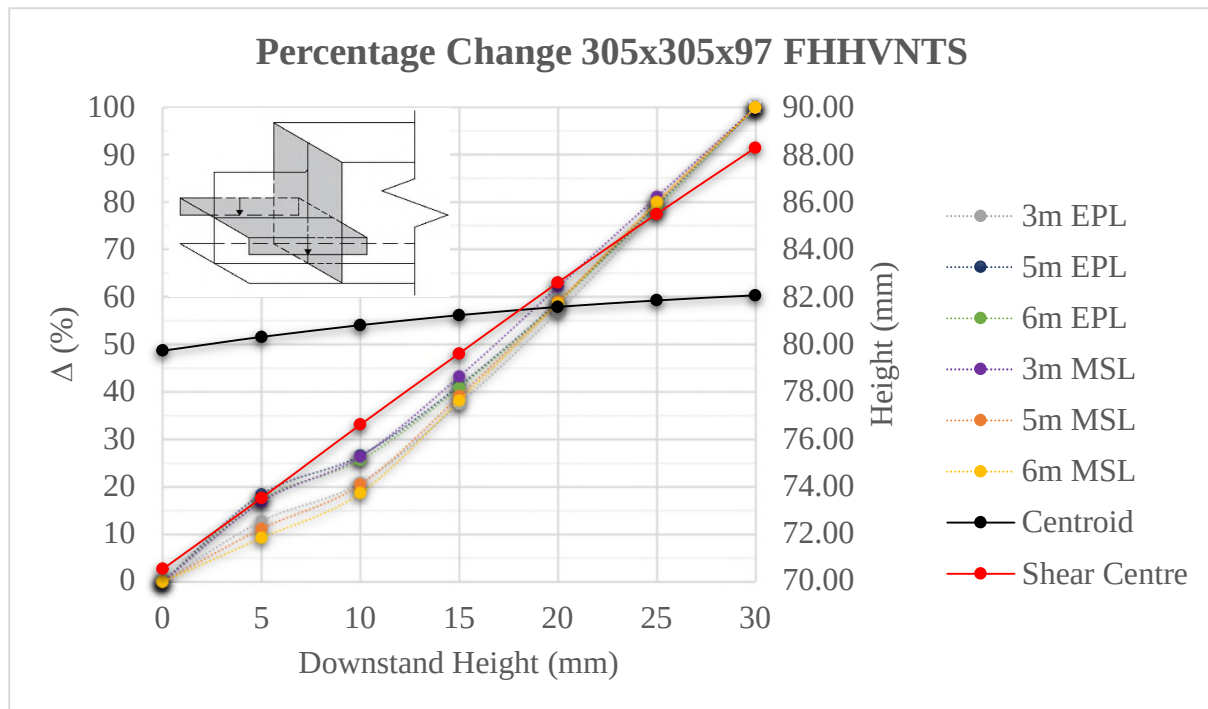


Figure 6.55: Percentage Change and Average Percentage Change Graph 305x305x97 FHHVNTS Technique (Centroid, Shear Centre use right axis)

Figure 6.56 shows the percentage change and average percentage change for the FHN technique. The trend is not as close as the 203x133x25 section (**Figure 6.37**) or the 203x203x46 (**Figure 6.51**) but closer than the 533x210x82 section (**Figure 6.46**). There are no outliers, it appears trend is more pronounced in some sections. The curves start at a minimum value then increase parabolically, some sections being slightly parabolic while others have the curve more pronounced, in a concave up trend. The gradient increases with each increase in downstand height. It is unclear if there is any change in the trend at the centroid shear centre coincidence point.

The centroid and shear centre are at the same vertical height at a downstand height of 122mm. No downstand of this height was tested therefore the closest downstand height before 122mm was selected, 100mm, as the reference height for the before and after series in the average percentage change graphs. It is clear from the average percentage change graph there is a change in the trend after the 100mm downstand since the trend is initially linear and becomes parabolic. It appears the curves start to diverge at the 122mm downstand, but it cannot be conclusively stated since this downstand height was not tested. It is clear there is a change in the trend around the coincidence point of the centroid and shear centre in the notched region.

Figure 6.57 shows the percentage change and average percentage change for the FLH technique. The three outliers are the 3m EPL (which also had a negligible increase), 5m EPL, the 1m MSL which are excluded as previously mentioned. The general trend, excluding the outliers, is different to the other sections: the curves start at a minimum value and increase parabolically but in a concave down trend. After which the trend is linear. It appears the trend is linear after the 100mm downstand height which is before the downstand height required to make the centroid and shear centre in the notched region to be at the same vertical height: 122mm.

Some key points from the discussion:

- Unlike all the other sections, the shorter of the spans tested (1m and 5m) increased just as many times as the longer of the spans (3m and 6m): the trend was split in half.
- The increase in the critical elastic moment was significant just as in the 533x210x82 section.
- There were three cases with negligible increases and with outliers in the data.

- There were clear changes in the trend around the centroid shear centre coincidence point shown as changes in gradient or curve type.

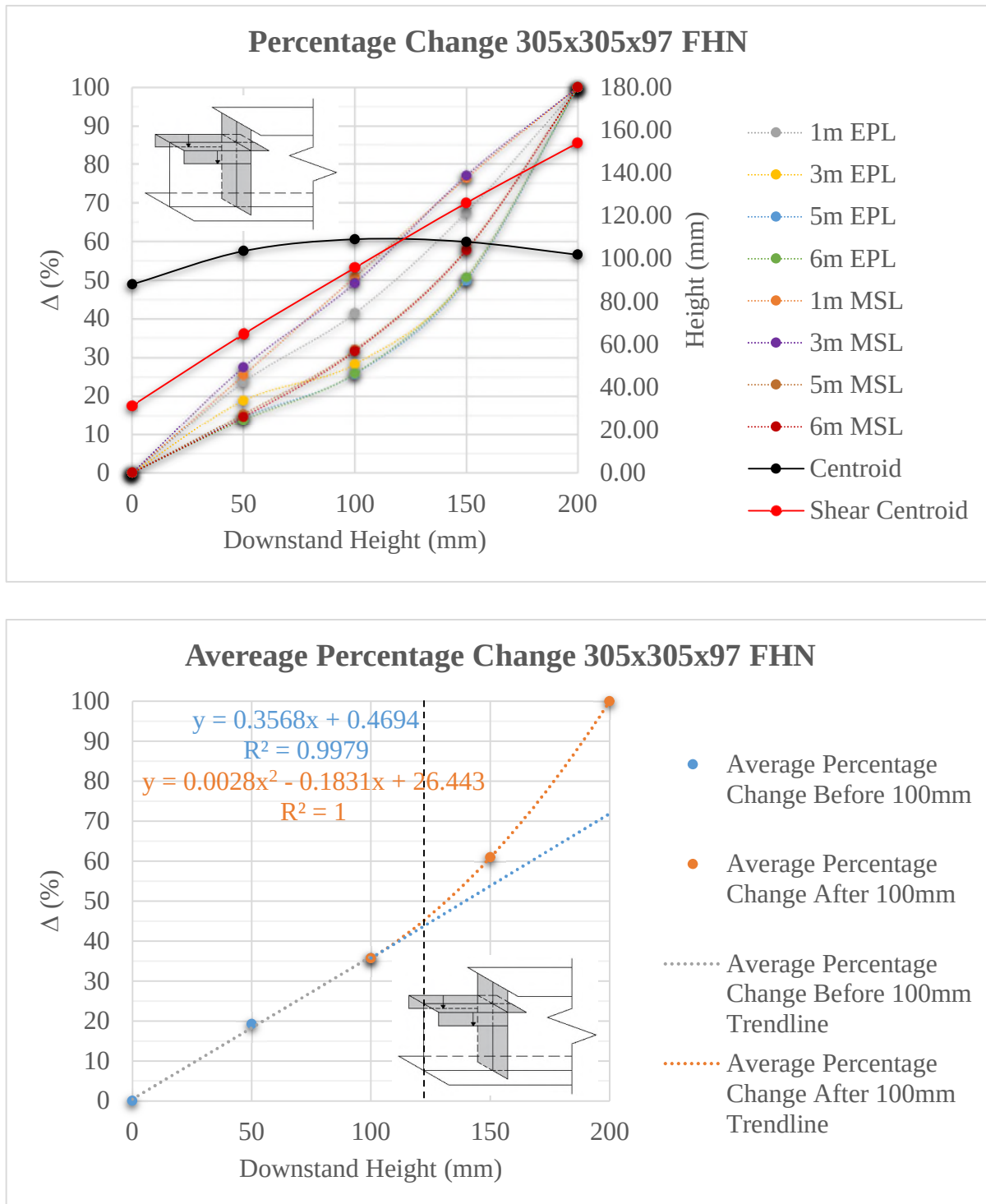


Figure 6.56: Percentage Change and Average Percentage Change Graph 305x305x97 FHN Technique (Centroid, Shear Centre use right axis)

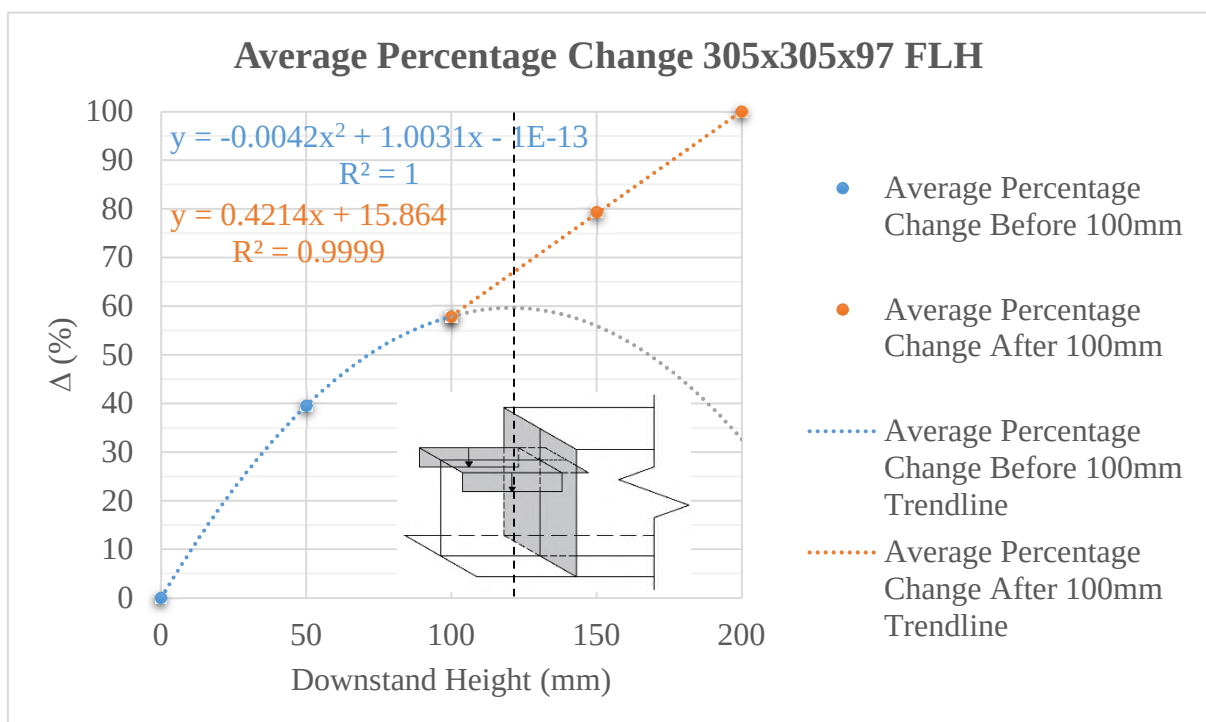
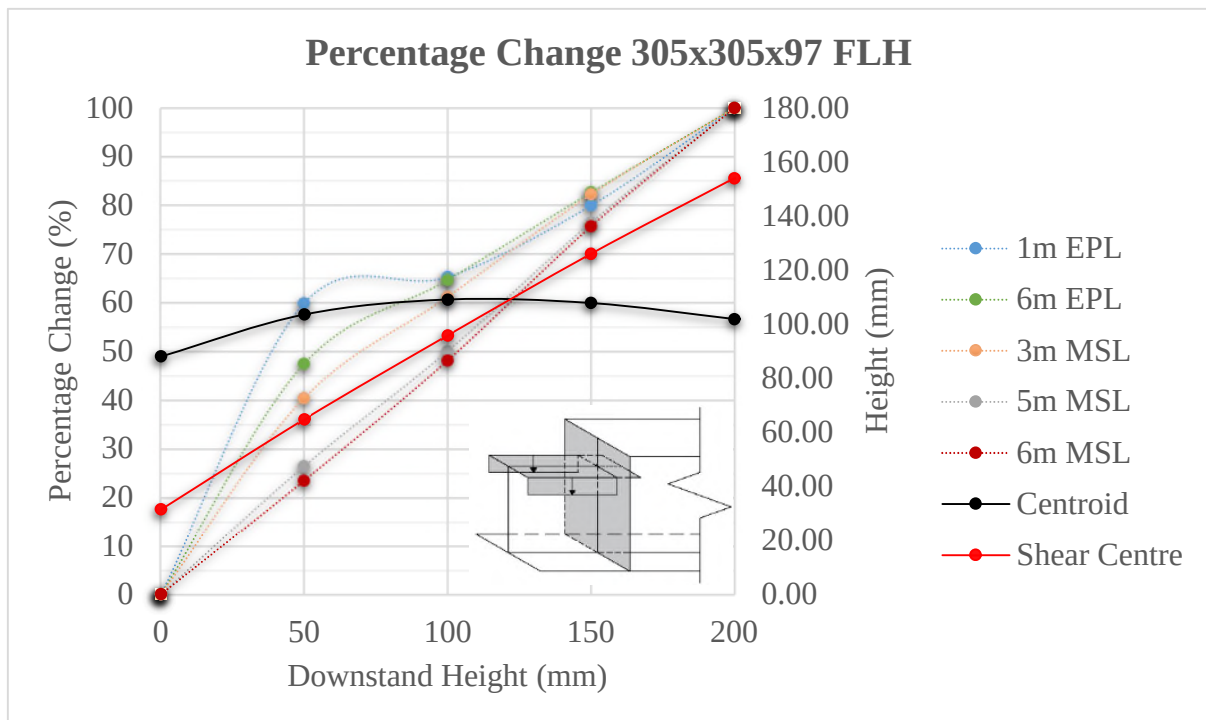


Figure 6.57: Percentage Change and Average Percentage Change Graph 305x305x97 FLH Technique (Centroid, Shear Centre use right axis)

6.9 Along Notch Technique

The “along notch” group (FHEN, FHN and FLH techniques) can be grouped within each section since these techniques use the same number of downstands at the same heights. This makes it possible to directly compare the average percentage change for these techniques when applied to these sections. This will show if the influence of the shifting centroid and shear centre is independent of the technique. **Figure 6.58** shows the average percentage change for the “along notch” technique when applied to the 203x133x25 section. The general trend of the techniques is similar: the average percentage change starts at a minimum value and increases parabolically. If the downstands before the 50mm downstand height are grouped together as a straight line and the downstands thereafter are grouped into another straight line it is clear there is a change in gradient. The gradient after the 50mm downstand height (the coincidence point of the centroid and shear centre) is steeper than the initial gradient. This trend exists for all the techniques and the change occurs at the 50mm downstand height for all techniques.

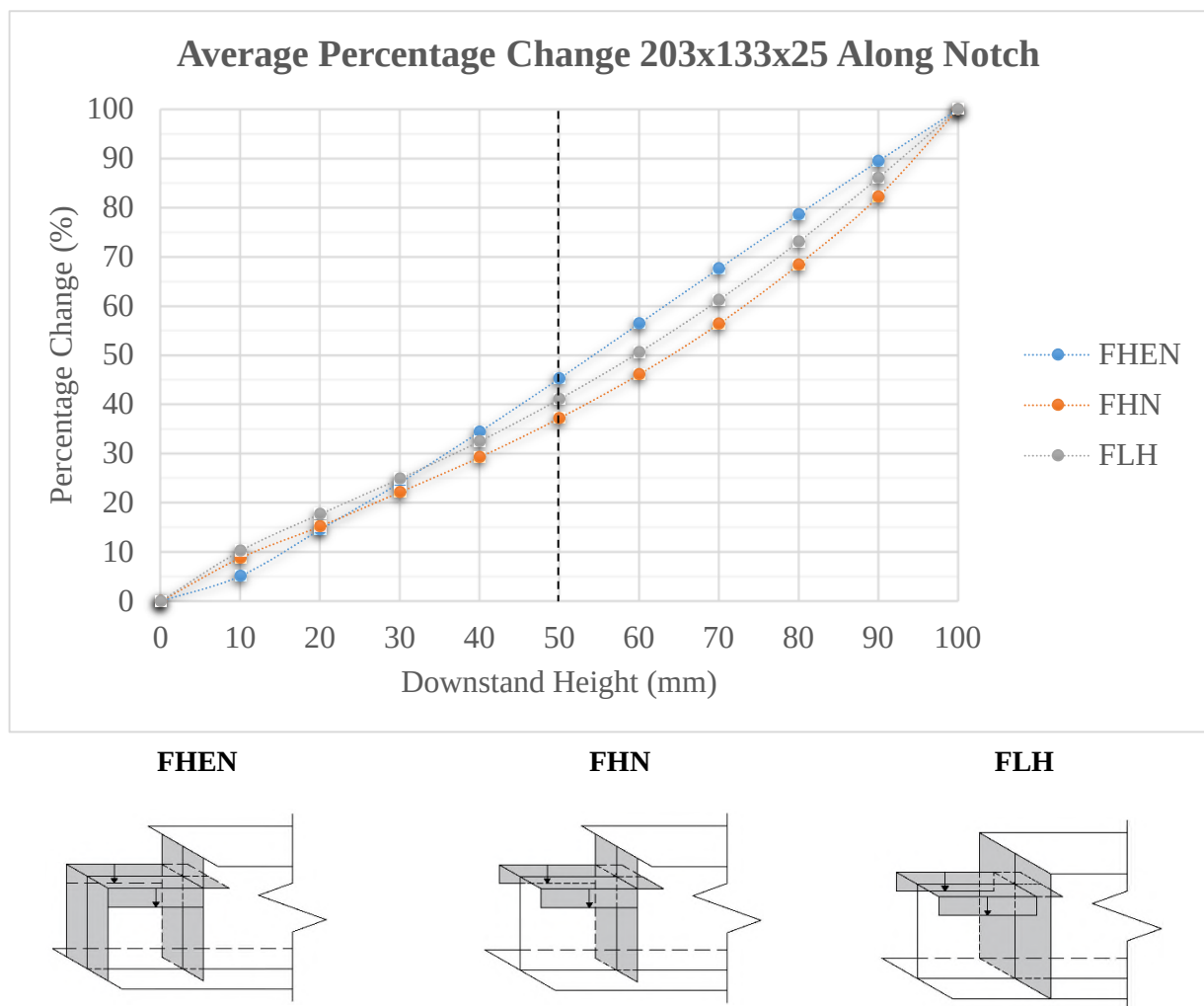


Figure 6.58: Average Percentage Change "Along Notch" Technique 203x133x25 Section

Figure 6.59 shows the average percentage change for the “along notch” technique when applied to the 305x102x25 section. The results that failed to reach the minimum change of 0.5kNm for the 305x102x25 section were excluded in the average calculation. Therefore, the FHN and FLH techniques are represented by the 3m MSL because this case is the only case of all the Models that meet the minimum requirement.

The three trends are similar to the 203x133x25 section: with each increase in downstand height the average percentage change increases. However, the three techniques all are slightly parabolic before the 19mm downstand height (the coincidence point of the centroid and shear centre) and almost perfectly linear after it. It is clear the influence of the shifting centroid and shear centre, although applied through different techniques, have similar effects.

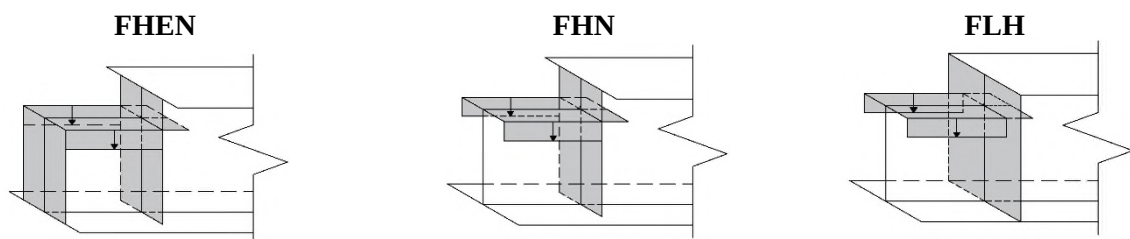
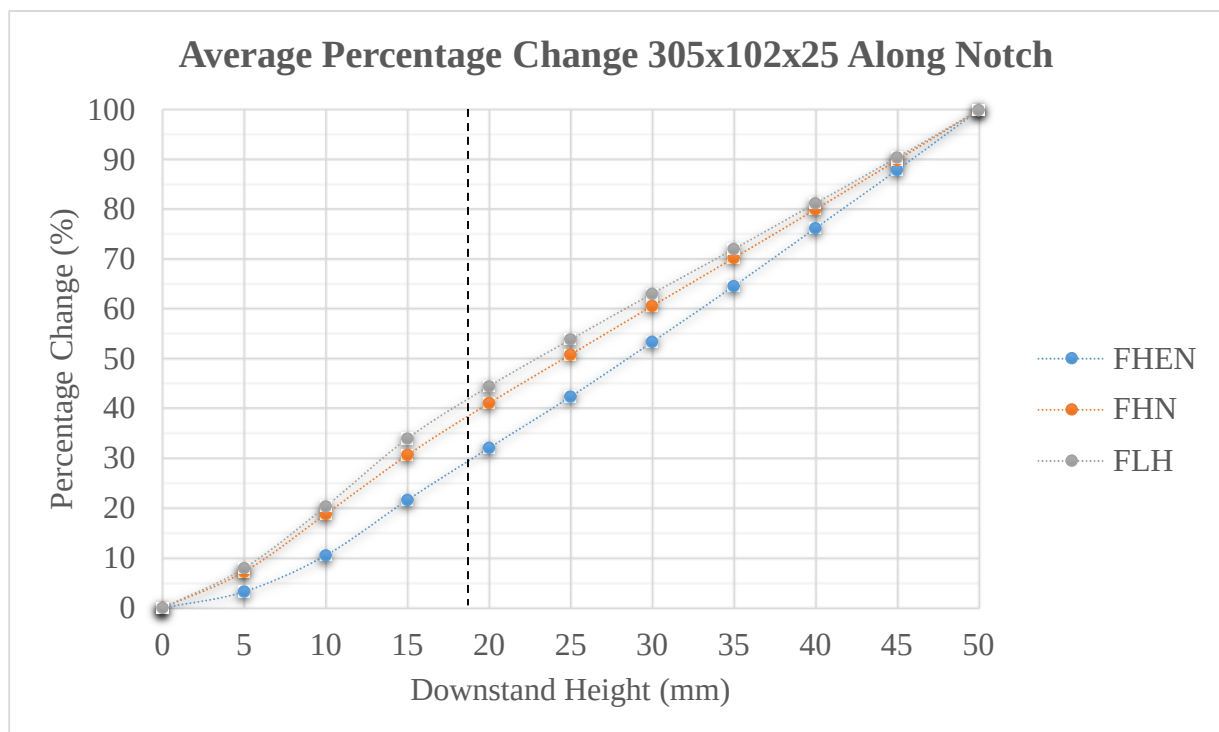


Figure 6.59: Average Percentage Change "Along Notch" Technique 305x102x25 Section

Figure 6.60 shows the average percentage change for the “along notch” technique when applied to the 533x210x82 section. The 1m EPL FLH technique case failed the minimum change of 0.5kNm. Therefore, it was excluded in the average calculation. Unlike the two previous sections, there is not a consistent trend across all the techniques. The FHEN technique appears to approach a maximum value as the downstands approach a height of 223mm (the coincidence point of the centroid and shear centre) whereas the FHN and FLH techniques start at a minimum and increase concave up in a trend similar to the previous two sections. Only one trend is different but all three have one thing in common: as the shear centre approaches and passes the centroid in the notched region the critical elastic moment increases. Also, a change exists after the coincidence of the centroid and shear centre in the notched region for all the techniques.

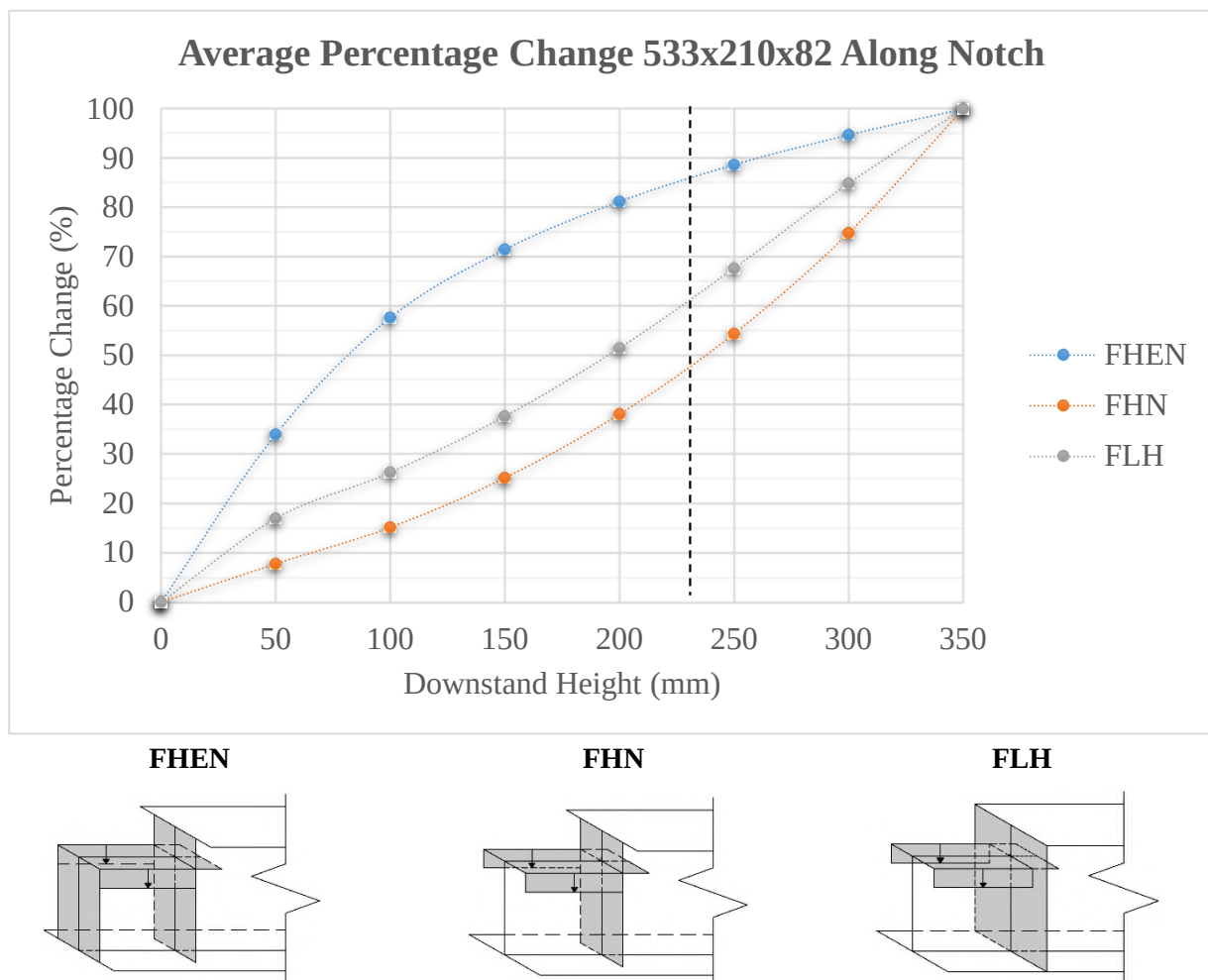


Figure 6.60: Average Percentage Change "Along Notch" Technique 533x210x82 Section

6.10 Non-Dimensional Discussion

The final aspect that was looked at was to discuss non-dimensional data. This is to look at the general case and see if the trends exist across all the different loads, spans and section types. The data was non-dimensionalised using the following equation:

$$\alpha = \gamma H$$

Where:

- α – downstand height
- γ – non-dimensional quantity
- H – section height (can also be written as $H = \text{notch depth} + \text{notched section depth}$)

Only the FHEN and FHHVNTS techniques were used because they had the least number of negligible changes. All cases with negligible increases (less than 0.5kNm) and outliers have been excluded. The equation can also be written in terms of notch depth and notched section height.

Figure 6.61 shows the FHEN non-dimensional curves and **Figure 6.62** the average percentage change for the dimensionless FHEN technique for each section. The trends in **Figure 6.61** are very interesting because they confirm the discussions earlier in the report and provide a possible explanation for the different trends. **Figure 6.62** shows the location of the coincidence point of the centroid and shear centre using dashed vertical lines (which are the same colour as the trend lines for the section they correspond to). For example, the red dashed vertical line at 0.062 corresponds to the 305x102x25 section because at 19mm downstand height, the centroid and shear centre coincide and $19/305=0.062$.

Figure 6.61 shows three distinct trends which were evident in the previous discussions. A linear trend, a parabola approaching a maximum value in three different ways (some approach it and remain at the maximum value, others are still approaching it with different gradients). **Figure 6.62** also shows these trends. Once the curves pass their respective vertical dashed lines, the coincidence of the centroid and shear centre, they approach a maximum value which acts as an asymptote (except the 305x102x25 section).

This is evident in the 305x102x25, 533x210x82 and 305x305x97. It would be more evident in the 203x203x46 section if height restrictions did not prevent further downstands being tested.

The shallower sections (203x102x25 and 305x102x25) are ever increasing, and the rate of increases is greater after the coincidence point. The 533x210x82 is the deepest I section tested and the 305x305x97 is relatively deep for a H section. Both these “deep” sections, in this context, behave similarly.

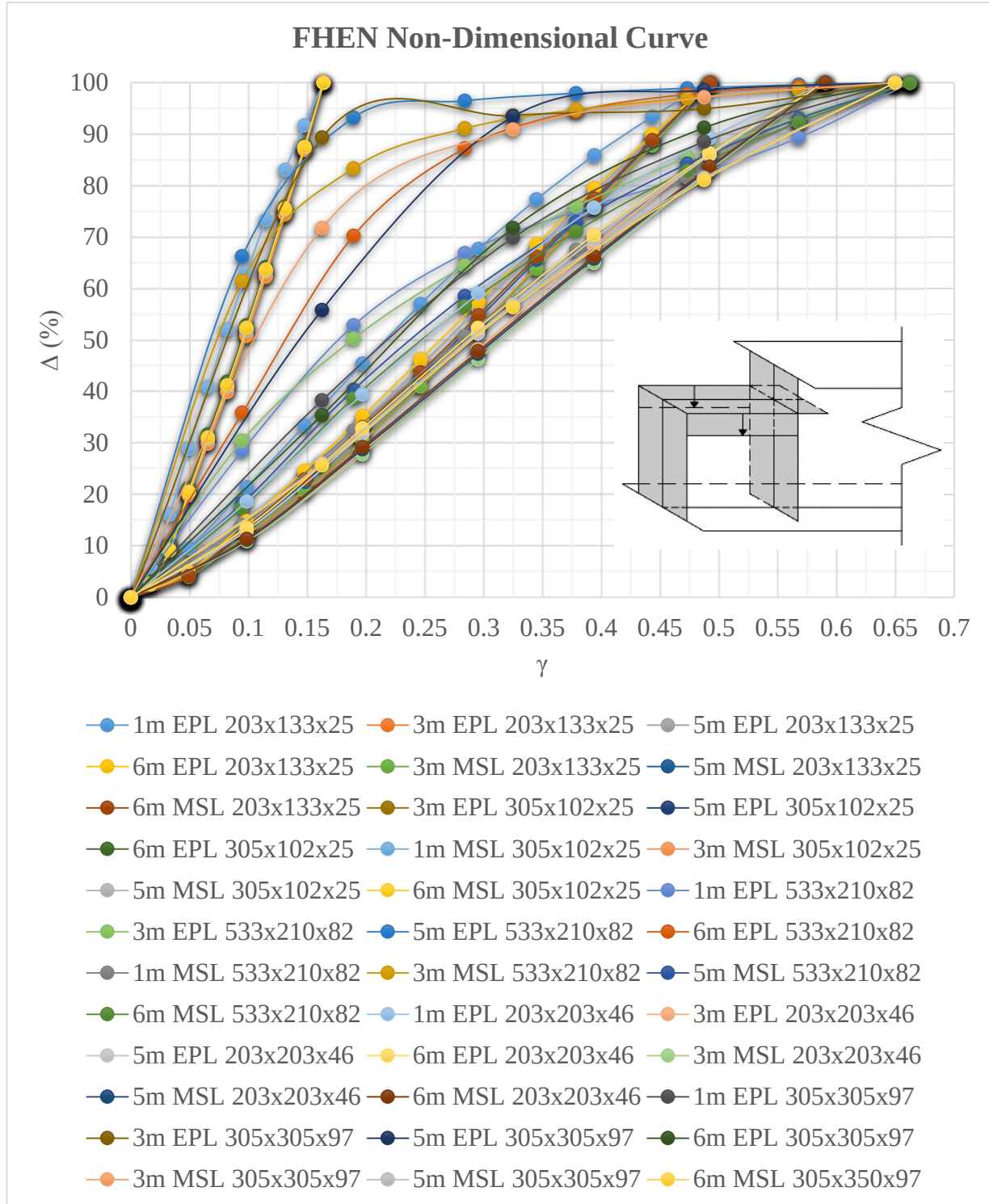


Figure 6.61: FHEN Non-Dimensional Curve

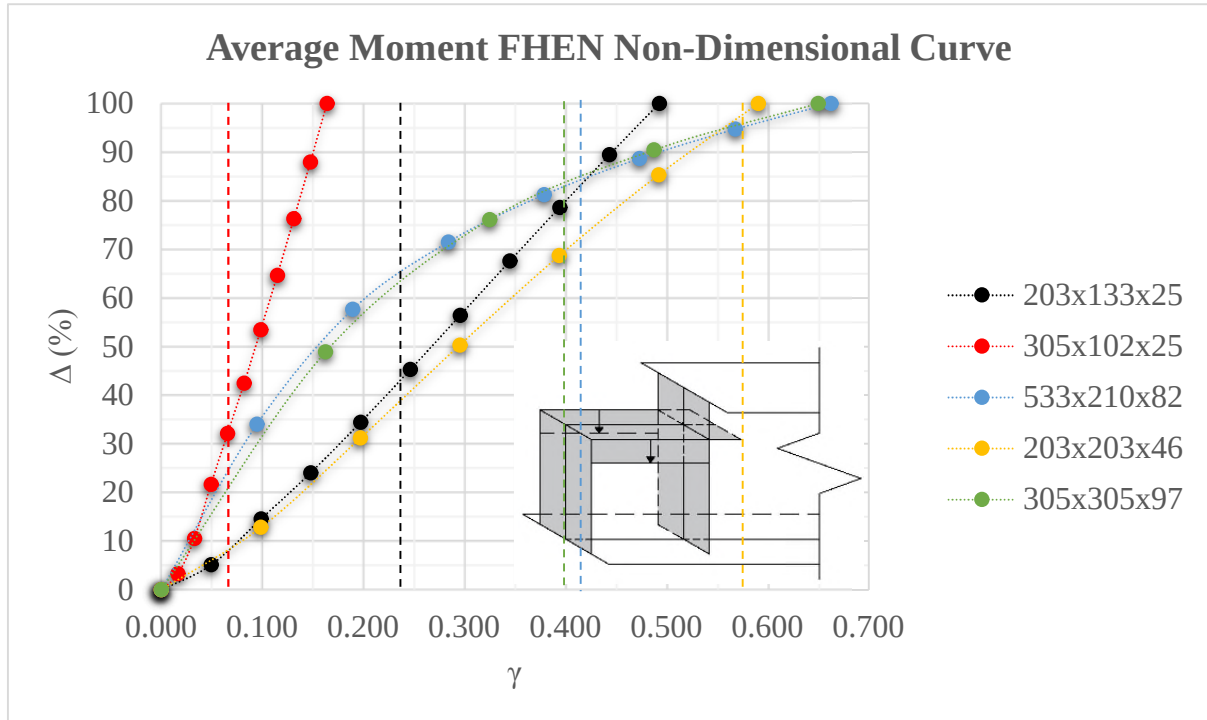


Figure 6.62: FHEN Non-Dimensional Curve (Average Moment)

Figure 6.63 shows the FHHVNTS non-dimensional curves and **Figure 6.64** the non-dimensional curves using the average moment. The trend in **Figure 6.63** is very strong, as seen in the previous discussions. There appears to be four distinct trends, but it is difficult to see what separates them. Using **Figure 6.64** it is clear the curves are separated by the section types, more specifically the section heights.

Moving left to right: the trends decrease in gradient with the relatively deep 305x305x97 H section having the steepest gradient followed by the 305x102x25 section. The 533x210x82 is steeper than the 203x133x25 and 203x203x46 sections which are very similar in behaviour. It is interesting to recall the notch depth to section height ratio for the sections in **Table 1.1**. The 305x305x97 (0.227) has the smallest ratio followed by the 305x102x25 (0.229) section. The 533x210x82, 203x203x46 and 203x133x25 sections have the same ratio of 0.246. This is very close to the limits proposed by Ibrahim et al. (2015) of 0.25 and these sections have the shallowest gradients. An explanation for the 533x210x82 section being steeper than the 203x203x46 and 203x133x25 sections is possibly because the 533x210x82 section is deeper. This appeared to be a factor in the section's response to the downstands as previously discussed and shown in **Figure 6.62**.

There are other ways to generate non-dimensional quantities using this data. The current definition can be expanded to write it in terms of notch depth and notched section depth for further study.

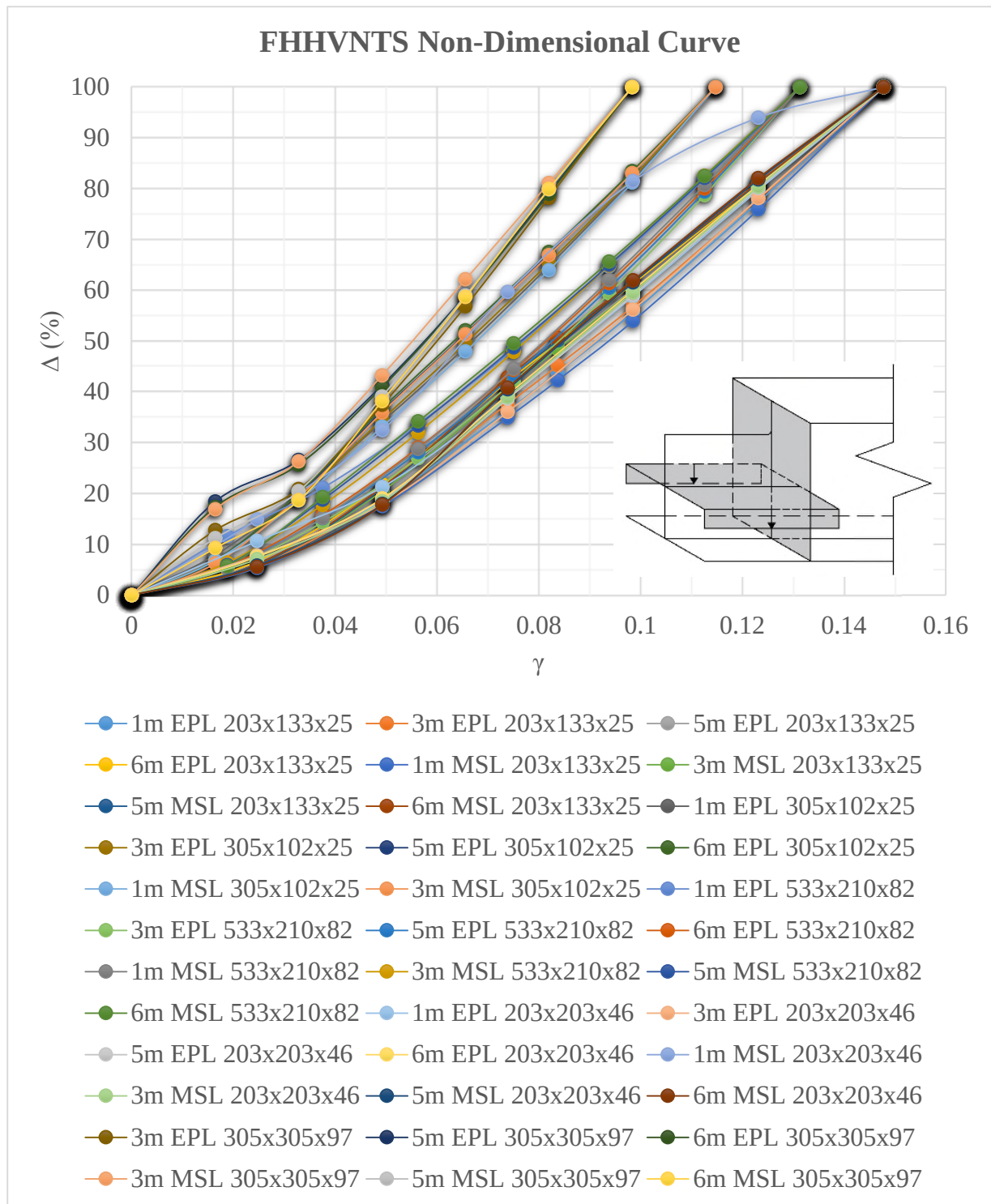


Figure 6.63: FHHVNTS Non-Dimensional Curve

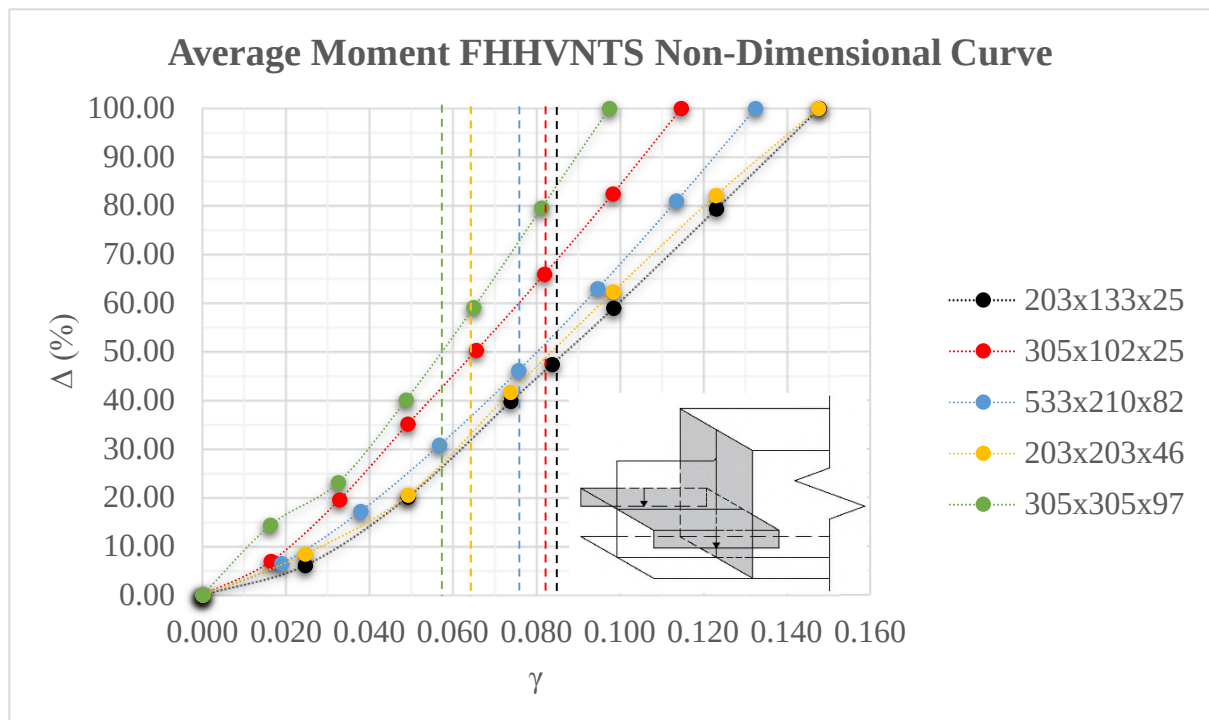


Figure 6.64: FHHVNTS Non-Dimensional Curve (Average Moment)

7. Conclusion

LTB is influenced by the span, support type, section geometry and connection type. In H and I sections, the centroid and shear centre are at the same vertical height (or approximately at the same vertical height). However, in monosymmetric sections this is not the case. Research has been conducted and accounts for this using the monosymmetry constant. This is not used when analysing notched regions of beams which are stiffened against local buckling failure. Using downstands or upstands on the stiffening techniques it is possible to shift the centroid and shear centre. In some cases they can coincide at the same vertical height just as in standard H and I sections. The influence of the shifting centroid and shear centre during elastic LTB in notched steel beams is not fully understood therefore this research attempted to fill the gap in knowledge.

Thirteen stiffening techniques were initially selected and the best performing 5 were taken for further testing; after being improved by extending the vertical stiffener to reach the top flange of the un-notched section. The tests were conducted on five different sections: 3 I and 2 H sections. There were two notch cases: single notch with a point load 600mm from the notched end and doubly notched with a point load mid span (both notches on the top flange) and two Models:

- Model 1: 1m and 3m span.
- Model 2: 5m and 6m span.

The FLHFHV section had results that were considered outliers and were excluded from this discussion. However, by isolating the downstand trend it is clear the trend is similar to the other sections. This was discussed for downstand trends that exceeded the minimum required change of 0.5kNm for this report. The 0.5kNm was selected since any change smaller than 0.5kNm would tend to zero if a finer mesh were used.

The remaining four of the stiffening techniques had similar behaviour and were further discussed. These were the:

- FHEN
- FHHVNTS
- FHN
- FLH

In all cases tested it was observed the shear centre shifted upwards towards and past the centroid in the notched region. This was achieved through the use of downstands. In almost all the cases tested, as the shear centre shifted upwards, the critical elastic moment increased with the addition of the downstands with the 533x210x82 and 305x305x97 sections generally benefitting the most. The aim of this research was to investigate if the position of the shear centre relative to the centroid in the notched region had any influence on the data. There were indications of changes in the trend after the centroid and shear centre shared the same vertical height for all the four techniques across all spans and loading configurations.

These changes were evident as changes in the gradient of the curves or changes in the curve type as shown in **Figure 7.1**. The trends for the FHEN technique appeared to be influenced by the depth of the section. The FHHVNTS technique had the most consistent behaviour and in almost all cases there was a change in the trendline once the shear centre was above the centroid in the notched region. Both these observations were evident in the non-dimensional discussion. The FHN and FLH techniques had many cases that had changes in the critical elastic moment that were less than the minimum threshold of 0.5kNm below which is considered insignificant in this report. Therefore, these were excluded from the non-dimensional discussion. There was also no apparent influence on the trends by the different spans and loading configurations.

This research study has met its research objectives because elastic LTB was investigated in various notched sections under different loading and span configurations. The influence of the shifting centroid and shear centre in the notched region was investigated and the results discussed. Therefore, the research question can be answered:

How does the addition of stiffeners in the notched region of a doubly symmetric beam affect the centroid and shear centre location for top notched steel sections undergoing elastic lateral torsional buckling, and how does the shifting shear centre relate to the critical elastic moment?

The addition of stiffeners increases the notched section's resistance to lateral torsional buckling for all sections tested in this study. This increase can be quite significant and can completely restore the "lost" critical elastic moment capacity to the un-notched value (and possibly surpass it). The shifting centroid and shear centre in the notched region does influence the critical elastic moment. Specifically, as the shear centre approaches and passes the centroid in the notched region, the trendline of the critical elastic moment changes by varying degrees for all the majority of the cases tested.

The change appears to be dependent on the stiffening technique, although occasionally there were similar trends evident in different techniques (**Figure 7.1** to **Figure 7.5**), and the section height. **Figure 7.1** to **Figure 7.5** below shows the general trend for the cases. The solid line is average and dashed lines are deviations from main trend.

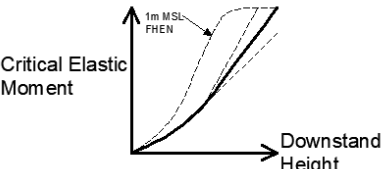
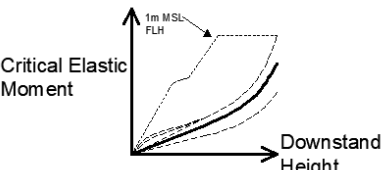
203x133x25 Summary Curves of the Trends for Each Technique		
Trend Description	Trend Image	Techniques
M_{cr} increases more rapidly as shear centre approaches and passes the centroid in the notched region. Initially parabolic then after coincidence linear trend.		FHEN Model 1,2 FHHVNTS Model 1,2
M_{cr} increases more rapidly as shear centre approaches and passes the centroid in the notched region. Initially parabolic then after coincidence linear trend.		FHN Model 1,2 FLH Model 1,2
The solid line is the average/approximated trend, the dashed lines are deviations from the average trend		

Figure 7.1: Summary of the Trends for Each Stiffening Technique for the 203x133x25 Section

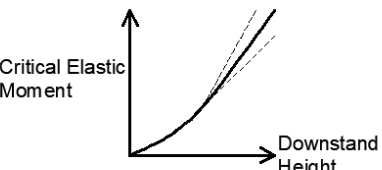
305x133x25 Summary Curves of the Trends for Each Technique		
Trend Description	Trend Image	Techniques
M_{cr} increases more rapidly as shear centre approaches and passes the centroid in the notched region. Initially parabolic then after coincidence linear trend.		FHEN Model 1,2 FHHVNTS Model 1,2
The solid line is the average/approximated trend, the dashed lines are deviations from the average trend		

Figure 7.2: Summary of the Trends for Each Stiffening Technique for the 305x133x25 Section

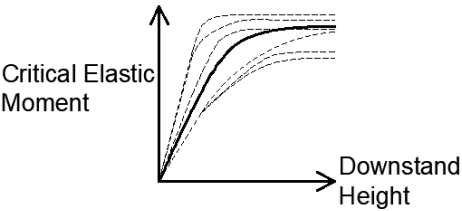
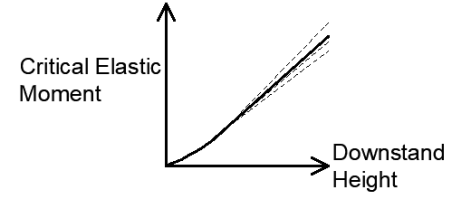
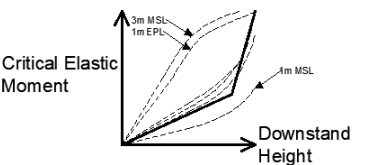
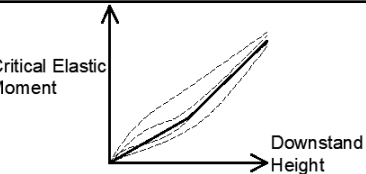
533x210x82 Summary Curves of the Trends for Each Technique		
Trend Description	Trend Image	Techniques
M_{cr} increases less rapidly as shear centre approaches and passes the centroid in the notched region. The trend is initially parabolic then linear. The linear trend is almost horizontal as the curves appear to approach an asymptote.		FHEN Model 1,2
M_{cr} increases more rapidly as shear centre approaches and passes the centroid in the notched region. Initially parabolic then after coincidence linear trend.		FHHVNTS Model 1,2
533x210x82 Summary Curves of the Trends for Each Technique		
Trend Description	Trend Image	Techniques
M_{cr} exhibits different behavior which are dependent on the load or span. The trends can also be approximated as straight lines.		FHN Model 1,2
M_{cr} increased approximately in a linear trend with each increase in downstand height. However, close to the centroid shear centre coincidence, there is a change in gradient.		FLH Model 1,2
The solid line is the average/approximated trend, the dashed lines are deviations from the average trend		

Figure 7.3: Summary of the Trends for Each Stiffening Technique for the 533x210x82 Section

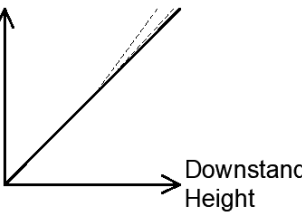
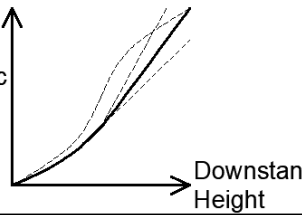
203x203x46 Summary Curves of the Trends for Each Technique		
Trend Description	Trend Image	Techniques
M_{cr} increased linearly with each increase in downstand height. However, close to the centroid shear centre coincidence, there is a change in gradient.	Critical Elastic Moment 	FHEN Model 1,2
M_{cr} increases more rapidly as shear centre approaches and passes the centroid in the notched region. Initially parabolic then after coincidence linear trend. The 1m MSL is the only case with a slightly different trend.	Critical Elastic Moment 	FHHVNTS Model 1,2
203x203x46 Summary Curves of the Trends for Each Technique		
Trend Description	Trend Image	Techniques
M_{cr} increases more rapidly as shear centre approaches and passes the centroid in the notched region. Initially parabolic then after coincidence linear trend.	Critical Elastic Moment 	FHN Model 1,2 FLH Model 1,2
The solid line is the average/approximated trend, the dashed lines are deviations from the average trend		

Figure 7.4: Summary of the Trends for Each Stiffening Technique for the 203x203x46 Section

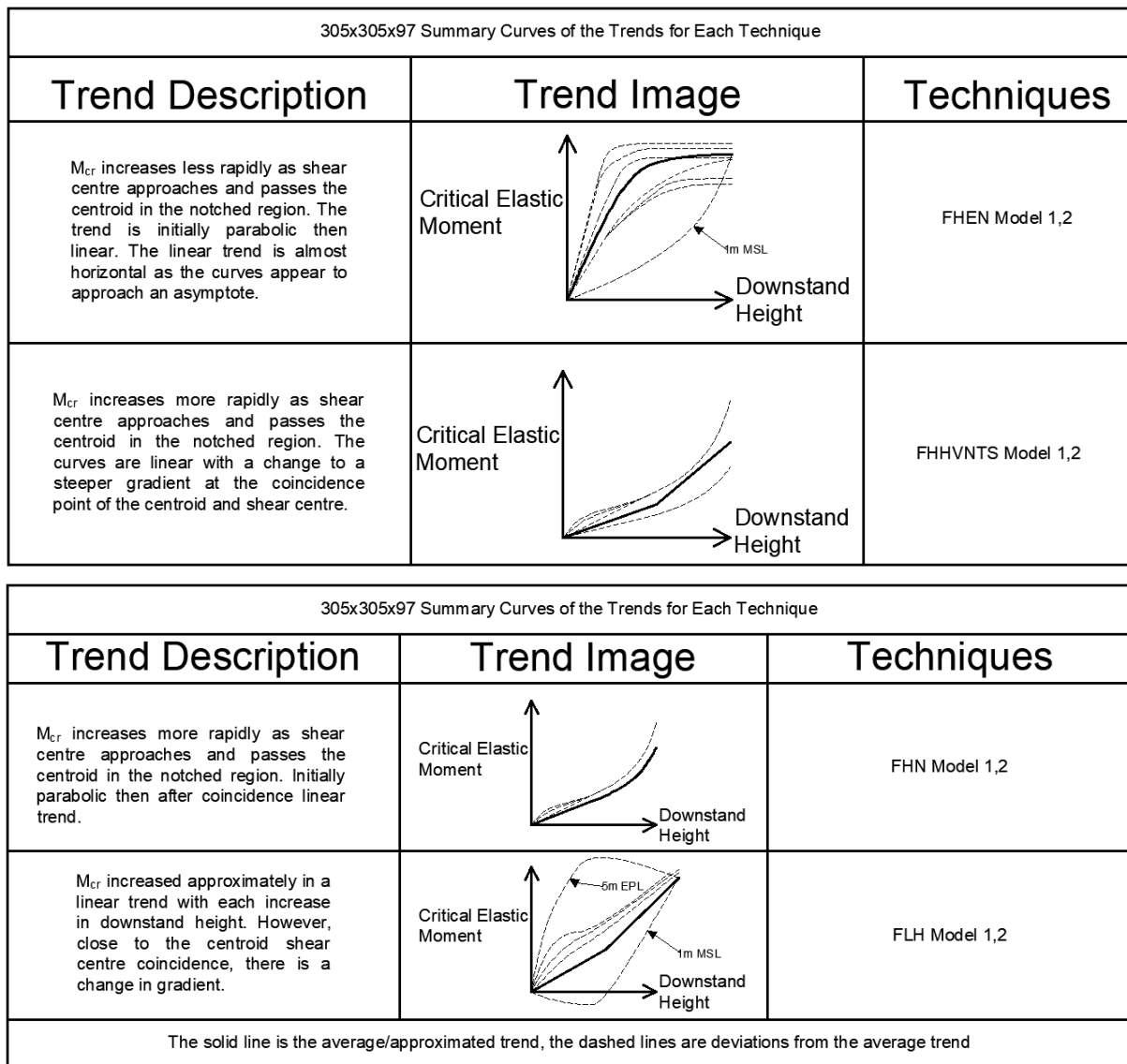


Figure 7.5: Summary of the Trends for Each Stiffening Technique for the 305x305x97 Section

8. Recommendations

Further research should be conducted on the FHEN and FHHVNTS techniques (since they had the fewest outliers and had changes that met the minimum requirement) by applying them to many steel sections. Care should also be taken to allow for further downstands beyond the coincidence point to see if all cases tend to the same result.

An equivalent bi-symmetric section should also be used in future investigations to verify the changes in the trend of the critical elastic moment are significantly due to the shifting shear centre and centroid in the notched region and not just due to the changes in the section properties.

9. References

- Aalberg, A. (2015a). Design of aluminium beam ends with flange copes. *Thin-Walled Structures*, 94(94), 593–602. <https://doi.org/10.1016/j.tws.2015.06.003>
- Aalberg, A. (2015b). Experimental and numerical parametric study on the capacity of coped beam ends. *Journal of Constructional Steel Research*, 113, 146–155. <https://doi.org/10.1016/j.jcsr.2015.06.004>
- Aalberg, A., & Larsen, P. (2001). Local Web Buckling of Coped Beams. *Proceedings of Nordic Steel Construction Conference NSCC*.
- Allen, H. G., & Bulson, P. S. (1980). *Background to Buckling*. McGraw-Hill.
- Bathe, K. J., & Wilson, E. L. (1976). *Numerical Methods in Finite Element Analysis*. Englewood Cliffs, N.J : Prentice-Hall.
- Birkemoe, P. C., & Gilmor, M. I. (1978). Behavior of Bearing Critical Double-Angle Beam Connections. *Engineering Journal*, 15(4), 109–115.
- Bleich, F. (1952). *Buckling Strength of Metal Structures*. McGraw-Hill Book Company, INC.
- Bradford, M. A. (1988). *Lateral - Distorsional Buckling of Tee - Section Beams*.
- Brown, P. T., & Trahair, N. S. (1968). Finite Integral Solution of Differential Equations. In C. E. Trans. (Ed.), *Institute Engineers* (2nd ed., pp. 193–196). Austrailia CE10.
- Brush, D. O., & Almroth, B. O. (1975). *Buckling of Bars, Plates, and Shells*. McGraw-Hill.
- Bryan, G. H. (1891). On the Stability of a Plane Plate Under Thrusts in Its Own Plane with Applications to the Buckling of the Sides of a Ship. *Proceedings London Mathematical Society*, 22, 54–67.
- Canadian Institute of Steel Construction. (2005). *CISC Commentary on CAN/CSA-S16-01 (S16S1-05)*. 2-37,2-38.
- Chajes, A. (1974). *Principles of Structural Stability Theory*. Englewood Cliffs, N.J : Prentice-Hall.
- Cheng, J. J. R. (1993). Design of steel beams with end copes. *Journal of Constructional Steel Research*, 25(1–2), 3–22. [https://doi.org/10.1016/0143-974X\(93\)90049-X](https://doi.org/10.1016/0143-974X(93)90049-X)
- Cheng, J. J. R., & Yura, J. A. (1986). Local Web Buckling of Coped Beams. *Journal of Structural Engineering*, 112(10), 2314–2331. [https://doi.org/10.1061/\(ASCE\)0733-](https://doi.org/10.1061/(ASCE)0733-)

9445(1986)112:10(2314)

- Cheng, J. J. R., & Yura, J. A. (1988). Lateral Buckling Tests on Coped Steel Beams. *Journal of Structural Engineering*, 114(1), 16–30. [https://doi.org/10.1061/\(ASCE\)0733-9445\(1988\)114:1\(16\)](https://doi.org/10.1061/(ASCE)0733-9445(1988)114:1(16))
- Cheng, J. J. R., Yura, J. A., & Johnson, C. P. (1984). *Design and behaviour of coped beams. Ferguson Structural Engineering Laboratory Report No. 84-1.*
- Cheng, J. J. R., Yura, J. A., & Johnson, C. P. (1988). Lateral Buckling of Coped Steel Beams. *Journal of Structural Engineering*, 114(1), 1–15. [https://doi.org/10.1061/\(ASCE\)0733-9445\(1988\)114:1\(1\)](https://doi.org/10.1061/(ASCE)0733-9445(1988)114:1(1))
- Clark, J. W., & Hill, H. N. (1960). Lateral Buckling of Beams. *Journal of the Structural Division*, 86, 175–196.
- da Silva, L. S., Simões, R., & Gervásio, H. (2010). Design of Members. In *Design of Steel Structures - Eurocode 3 Design of Steel Structures, Part 1-1 - General Rules and Rules for Buildings* (First, Issue May, pp. 205–208). European Convention for Constructional Steelwork : Ernst & Sohn Creation Date.
- Duncan, W. J. (1937). Galerkin's Method in Mechanics and Differential Equations. *Aeronautical Research Committee*, 1798, 484–516.
- Fang, C., Yam, M. C. H., Lam, A. C. C., Liu, Y., & Chung, K. F. (2017). Local web buckling of double-coped steel beam connections. *Journal of Constructional Steel Research*, 128, 166–178. <https://doi.org/10.1016/j.jcsr.2016.08.011>
- Galambos, T. V. (1963). Inelastic Lateral Buckling of Beams. *Journal of the Structural Division*, 89, 217–244.
- Galambos, T. V. (1968). *Structural members and frames*. Englewood Cliffs, N.J : Prentice-Hall.
- Galambos, T. V. (1998). Beams. In Structural Stability Research Council (Ed.), *Guide to Stability Design Criteria for Metal Structures* (pp. 192–217). John Wiley & Sons.
- Galambos, T. V., & Surovek, A. E. (2008). Lateral-Torsional Buckling. In *Structural Stability of Steel: Concepts and Applications for Structural Engineers* (pp. 236–289). John Wiley & Sons, Inc. <https://doi.org/10.1002/9780470261316.ch6>
- Gizejowski, M. (2001). Lateral buckling of steel beams with limited rotation ability at supports (In Polish). *Inżynieria I Bud*, 10, 589–594.

- Hill, H. N. (1954). Lateral Buckling of Channels and Z-Beams. *ASCE*, 119, 829–841.
- Ibrahim, S. A., Dessouki, A. K., & El-Sa’eed, S. A. (2015). Lateral buckling behavior and strengthening techniques of coped steel I-beams. *Journal of Constructional Steel Research*, 108, 11–22. <https://doi.org/10.1016/j.jcsr.2015.02.003>
- Ibrahim, S. A., Rizkalla, K. B., & Dessouki, A. K. (2019). Lateral torsional buckling and strengthening techniques of coped beams with corrugated webs. *Thin-Walled Structures*, 106533. <https://doi.org/10.1016/j.tws.2019.106533>
- Jawad, M. H. (2017). Finite Element Analysis. In *Stress in ASME Pressure Vessels, Boilers, and Nuclear Components* (pp. 283–308). John Wiley & Sons, Inc. <https://doi.org/10.1002/9781119259299.ch11>
- Keller, H. B. (1968). *Numerical Methods for Two-Point Boundary Value Problems*. Blaisdell.
- Kitipornchai, S., & Trahair, N. S. (1980). Buckling Properties of Mono-Symmetric I Beams. *ASCE J. Structural Division*, 106(5), 941–958.
- Kreyszig, E. (1983). *Advanced Engineering Mathematics* (5th ed.). John Wiley & Sons.
- Lam, A. C. C., Yam, M. C. H., & Fu, C. K. M. (2006). Experimental Investigation of the local web buckling strength of coped steel I-beam with and without stiffeners. *The 10th East Asia- Pacific Conference on Structural Engineering and Construction*, 559–564.
- Lam, A. C. C., Yam, M. C. H., Iu, V. P., & Cheng, J. J. R. (2000). Design for lateral torsional buckling of coped I-beams. *Journal of Constructional Steel Research*, 54(3), 423–443. [https://doi.org/10.1016/S0143-974X\(99\)00058-9](https://doi.org/10.1016/S0143-974X(99)00058-9)
- Michell, A. G. M. (1899). Elastic stability of long beams under transverse forces. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, 48(292), 298–309. <https://doi.org/10.1080/14786449908621336>
- Mohammadi, E., Hosseini, S. S., & Rohanimanesh, M. S. (2016). Elastic lateral-torsional buckling strength and torsional bracing stiffness requirement for monosymmetric I-beams. *Thin-Walled Structures*, 104, 116–125. <https://doi.org/10.1016/j.tws.2016.03.003>
- Mohebkah, A. (2011). Lateral buckling resistance of inelastic I-beams under off-shear center loading. *Thin-Walled Structures*, 49(3), 431–436. <https://doi.org/10.1016/j.tws.2010.11.007>
- Mudenda, K. 2018. Design of Steel Members Lateral Torsional Buckling (CIV3049S Lecture

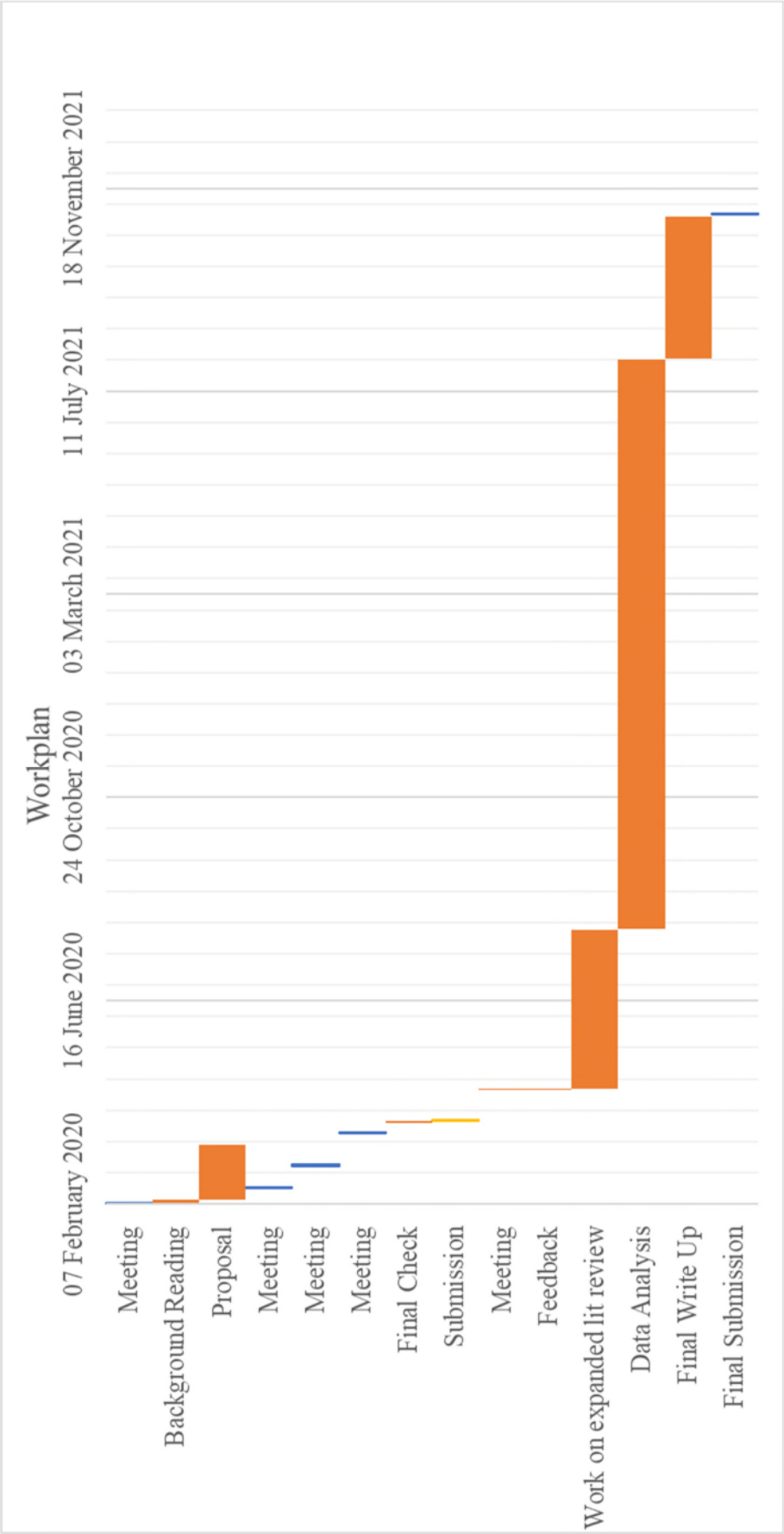
- Notes). Department of Civil Engineering, University of Cape Town.
- Mudenda, K., & Zingoni, A. (2018). Lateral-torsional buckling behavior of hot-rolled steel beams with flange upstands. *Journal of Constructional Steel Research*, 144, 53–64. <https://doi.org/10.1016/j.jcsr.2018.01.001>
- Na, Y. T. (1979). *Computational Methods in Engineering Boundary Value Problems*. Academic Press.
- Piotrowski, R., & Szychowski, A. (2019). Lateral torsional buckling of steel beams elastically restrained at the support nodes. *Applied Sciences (Switzerland)*, 9(9), 1–18. <https://doi.org/10.3390/app9091944>
- Prandtl, L. (1899). *Kipperscheinungen*. Polytechnical Institute.
- Quimby, T. B. (2011). *Lateral Torsional Buckling Limit State*. <https://www.bgstructuralengineering.com/BGSCM14/BGSCM008/Flexure/BGSCM0080204.htm>
- Ritz, W. (1909). Über Eine Neue Methode zur Lösung Gewisser Variationsprobleme der Mathematischen Physik. *Z. Reine Angew. Math.*, 135, 1–61.
- Salvadori, M. G. (1955). Lateral Buckling of I-Beams. *ASCE*, 120, 1165.
- Salvadori, M. G., & Baron, M. L. (1961). *Numerical Methods in Engineering* (2nd ed.). Englewood Cliffs, N.J : Prentice-Hall.
- Skatulla, S. 2017. Shear Centre Mechanics of Materials (CIV2011F Lecture Notes). Department of Civil Engineering, University of Cape Town.
- Southern African Institute of Steel Construction. (2016). *Southern African Steel Construction Handbook* (8th ed.). Southern African Institute of Steel Construction.
- Surla, A. S., Kang, S. Y., & Park, J. S. (2015). Inelastic buckling assessment of monosymmetric I-beams having stepped and non-compact flange sections. *Journal of Constructional Steel Research*, 114, 325–337. <https://doi.org/10.1016/j.jcsr.2015.08.019>
- Timoshenko, S. P. (1910). Einige Stabilitäts-probleme der Elastizitätstheorie. *Zeitschrift Fur Mathematik Und Physik*, 58, 337–385.
- Timoshenko, S. P., & Gere, J. M. (1961). *Theory of Elastic Stability*. Dover Publications Inc.
- Trahair, N. S. (1983). Inelastic lateral buckling of beams. In R. S. Narayanan (Ed.), *Beams and beam columns: stability and strength*. London : New York : Applied Science Publishers

- New York, NY, USA Sole distributors in the USA and Canada, Elsevier Science Pub. Co.
- Trahair, N. S. (1993). *Flexural, torsional, buckling of structures* (1st ed.). CRC Press.
- Vincent, G. S. (1969). Tentative Criteria for Load Factor Design of Steel Highway Bridges. *AISI Steel Research for Construction-Bul 15*.
- Vlasov, V. Z. (1940). *Tonkostennyye uprugie sterzhni (Thin-walled elastic beams)*.
- Wang, C. M., & Kitipornchai, S. (1991). Integrated View of Methods in Buckling Analysis. *International Journal of Applied Engineering*, 7(5), 381–387.
- Winter, G. (1943). Lateral Stability of Unsymmetrical I-Beams and Trusses in Bending. *ASCE*, 67, 1851–1864.
- Yam, M. C. H., & Chung, K. F. (2013). A numerical study of the strength and behaviour of reinforced coped beams. *Journal of Constructional Steel Research*, 80, 224–234. <https://doi.org/10.1016/j.jcsr.2012.07.032>
- Yam, M. C. H., Fang, C., & Lam, A. C. C. (2016). Local web buckling mechanism and practical design of double-coped beam connections. *Engineering Structures*, 125, 54–69. <https://doi.org/10.1016/j.engstruct.2016.06.049>
- Yam, M. C. H., Fang, C., Lam, A. C. C., & Cheng, J. J. R. (2014). Local failures of coped steel beams — A state-of-the-art review. *Journal of Constructional Steel Research*, 102, 217–232. <https://doi.org/10.1016/j.jcsr.2014.07.002>
- Yam, M. C. H., Lam, A. C. C., Iu, V. P., & Cheng, J. J. R. (2003). Local Web Buckling Strength of Coped Steel I Beams. *Journal of Structural Engineering*, 129(1), 3–11. [https://doi.org/10.1061/\(ASCE\)0733-9445\(2003\)129:1\(3\)](https://doi.org/10.1061/(ASCE)0733-9445(2003)129:1(3))
- Yam, M. C. H., Lam, A. C. C., Wei, F., & Chung, K. F. (2007). The local web buckling strength of stiffened coped steel I-beams. *International Journal Of Steel Structures*, 7(2), 129–138.
- Yam, M. C. H., Ma, H., Lam, A. C. C., & Chung, K. F. (2011). Experimental study of the strength and behaviour of reinforced coped beams. *Journal of Constructional Steel Research*, 67(11), 1749–1759. <https://doi.org/10.1016/j.jcsr.2011.04.015>
- Yoo, C. H., & Lee, S. C. (2011). Buckling of Plate Elements. In *Stability of Structures Principles and Applications* (pp. 373–440). Elsevier. <https://doi.org/10.1016/B978-0-12-385122-2.10008-9>
- Yuan, W., Kim, B., & Chen, C. (2013). Lateral–torsional buckling of steel web tapered tee-

section cantilevers. *Journal of Constructional Steel Research*, 87, 31–37.
<https://doi.org/10.1016/j.jcsr.2013.03.026>

Ziemian, R. D. (2010). Guide to Stability Design Criteria for Metal Structures. In R. D. Ziemian (Ed.), *Sixth Edition*. John Wiley & Sons, Inc. <https://doi.org/10.1002/9780470549087>

Appendix A: Workplan



Appendix B: Shifting Centroid and Shear Centre

B.1 305 x 102 x 25 Section

B.1.1 Along Notch Technique

Figure B.1 shows the centroid and shear centre of the notched region coincide at a downstand height of approximately 19mm. The shear centre of the notched region crosses the centroid and shear centre of the un-notched region at a downstand height of approximately 42mm.

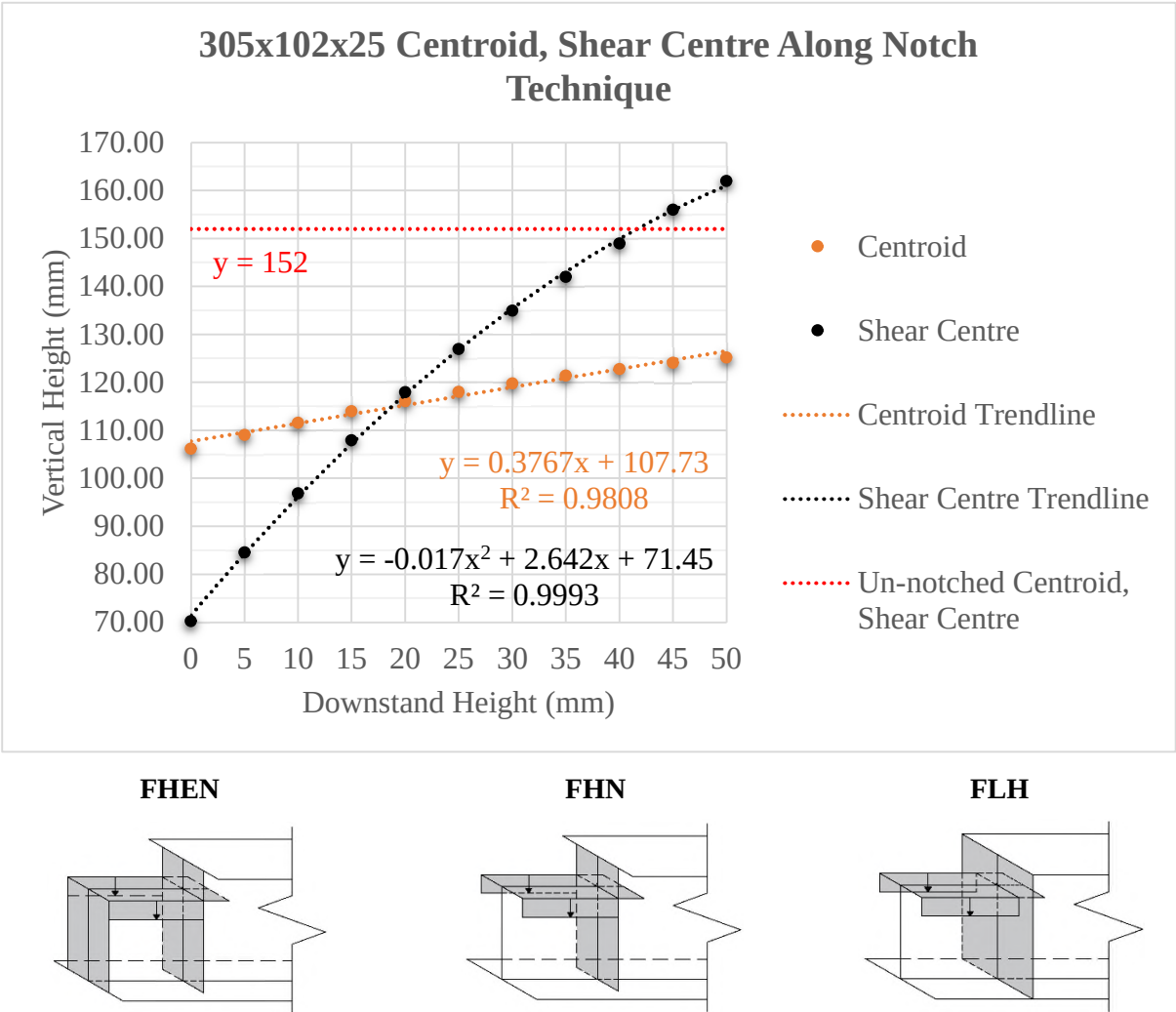
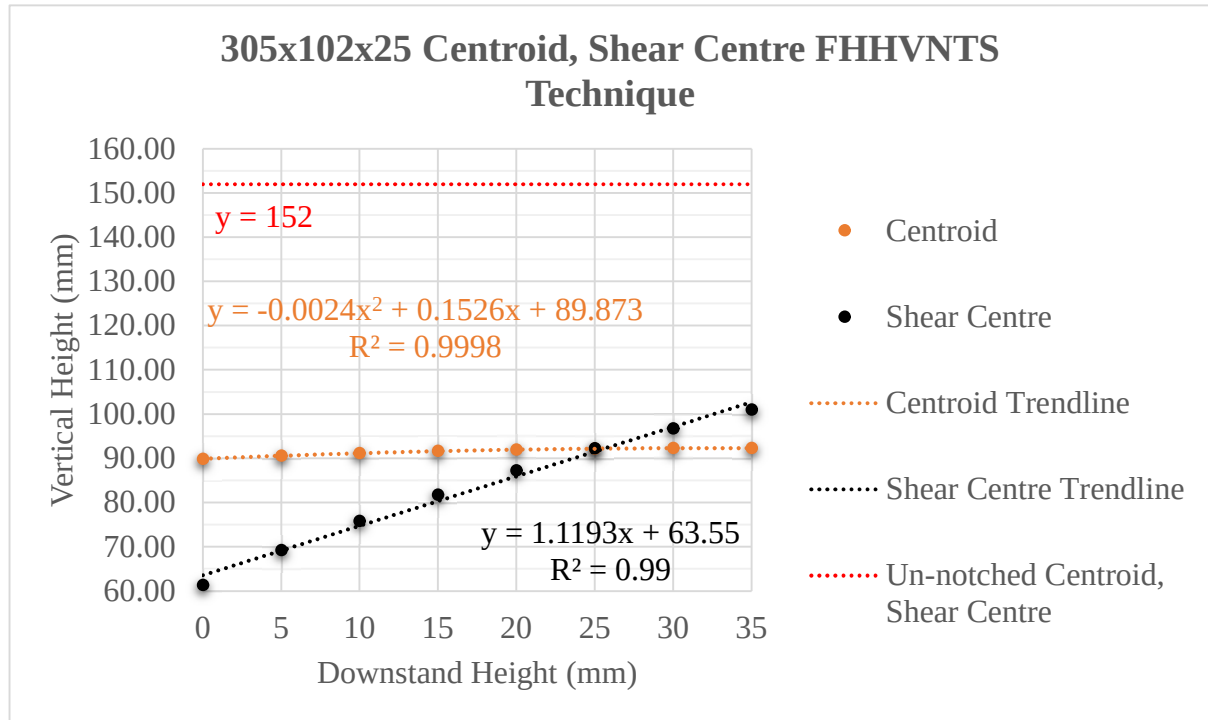


Figure B.1: 305x102x25 Centroid, Shear Centre Along Notch Technique

B.1.2 FHHVNTS Technique

Figure B.2 shows the shear centre passes the centroid of the notched region at a downstand height of 25mm, but it never crosses the centroid and shear centre of the un-notched region due to height restrictions in the notched region preventing the use of the necessary longer downstands.



FHHVNTS

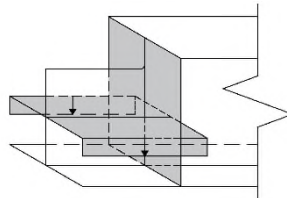


Figure B.2: 305x102x25 Centroid, Shear Centre FHHVNTS Technique

B.1.3 FLHFHV Technique

Figure B.3 shows the centroid and shear centre of the notched region are at the same vertical height with no downstands or upstands. The shear centre of the notched region intersects the centroid and shear centre of the un-notched region at a downstand height of 20mm.

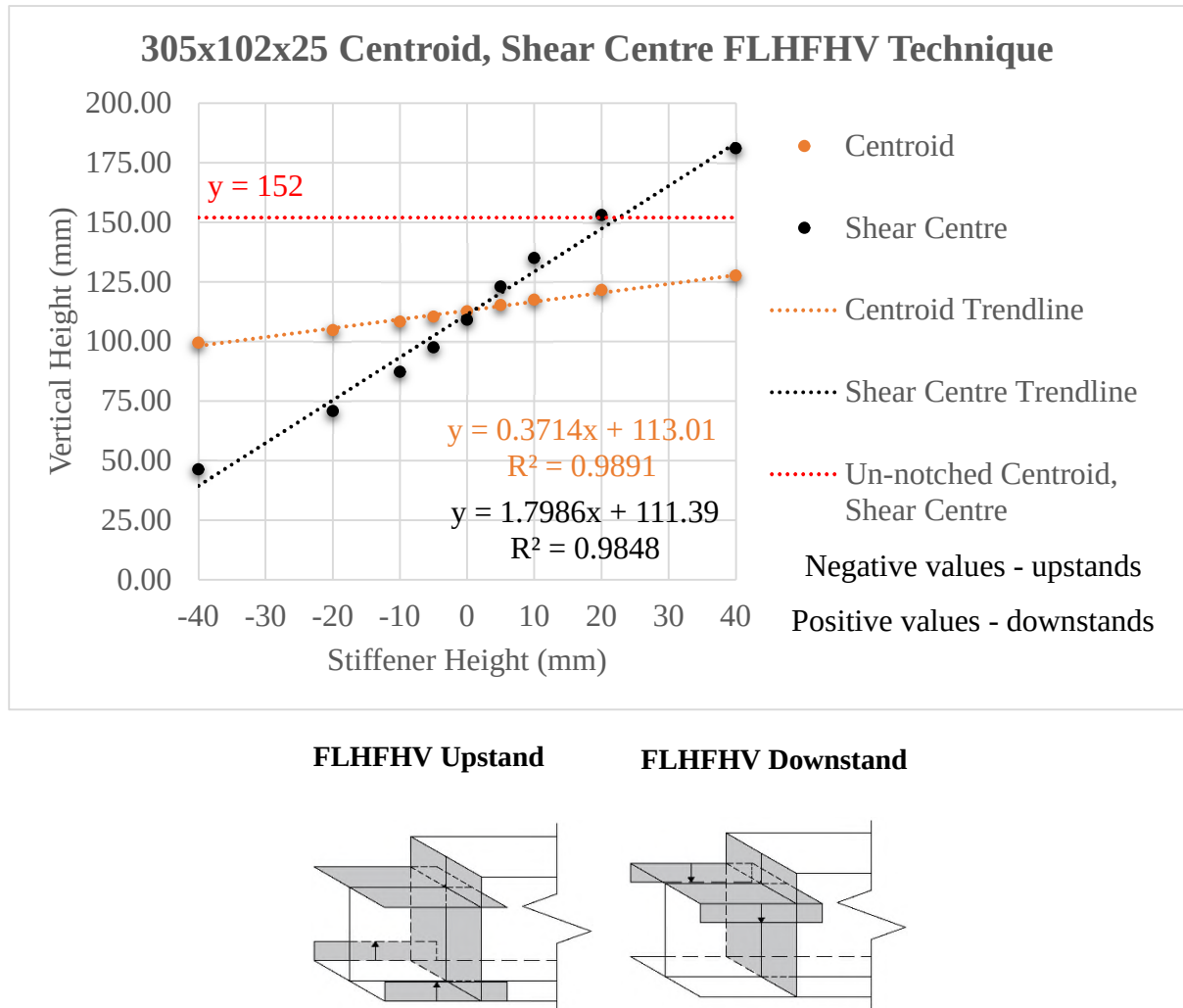


Figure B.3: 305x102x25 Centroid, Shear Centre FLHFHV Technique

B.2 533 x 210 x 82 Section

B.2.1 Along Notch Technique

Figure B.4 shows at a downstand height of 223mm the centroid and shear centre coincide. This downstand height was not tested. The shear centre of the notched region intersected the centroid and shear centre of the un-notched part at a downstand height of 310mm.

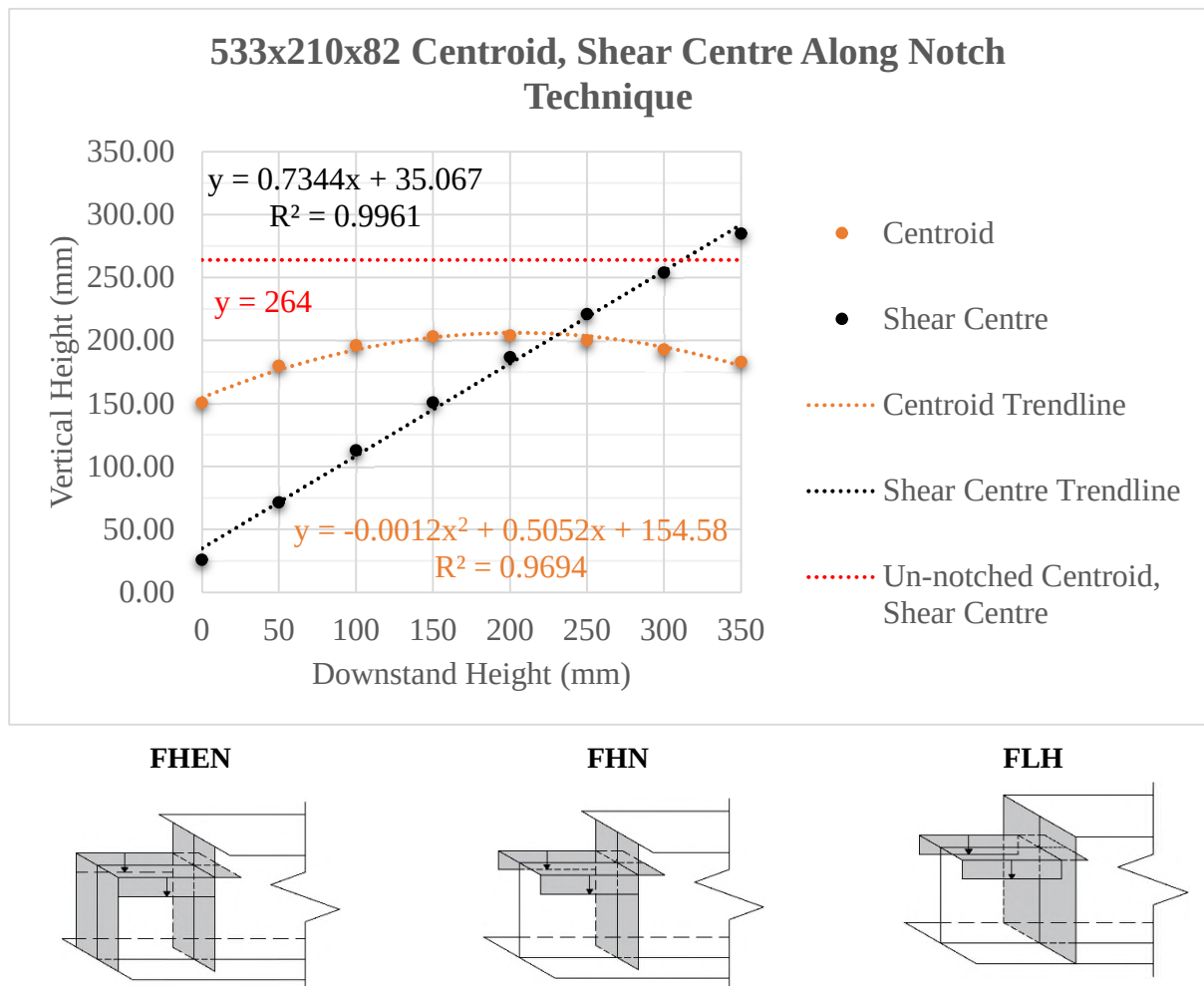
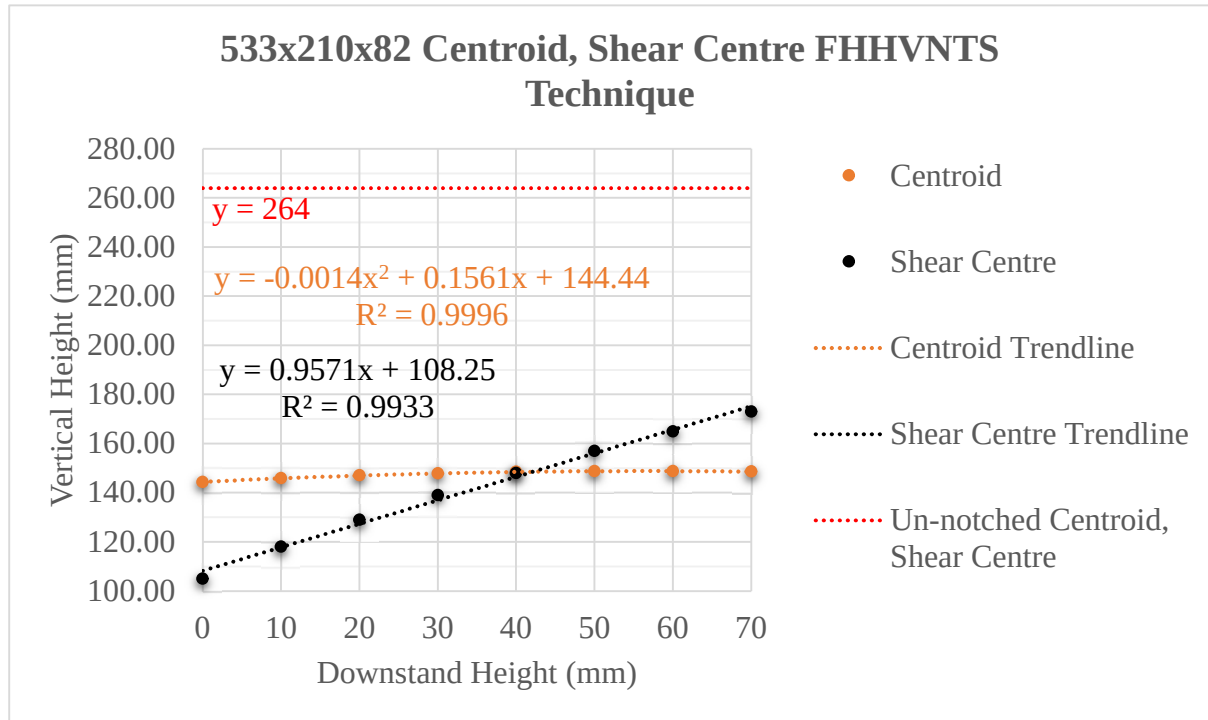


Figure B.4: 533x210x82 Centroid, Shear Centre Along Notch Technique

B.2.2 FHHVNTS Technique

Figure B.5 shows the influence of the downstands on the FHHVNTS stiffening technique. The shear centre passes the centroid of the notched region at a downstand height of 40mm but never passes the centroid, shear centre of the un-notched part. This is due to height limitations in the notched region preventing the use of the necessary longer downstand heights.



FHHVNTS

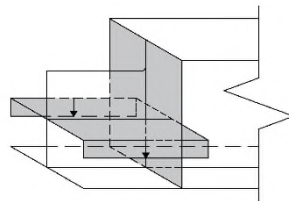


Figure B.5: 533x210x82 Centroid, Shear Centre FHHVNTS Technique

B.2.3 FLHFHV Technique

Figure B.6 shows the centroid and shear centre of the notched region intersecting with no upstands or downstands. The shear centre of the notched region coincides at the centroid and shear centre of the un-notched part at a downstand height of 40mm.

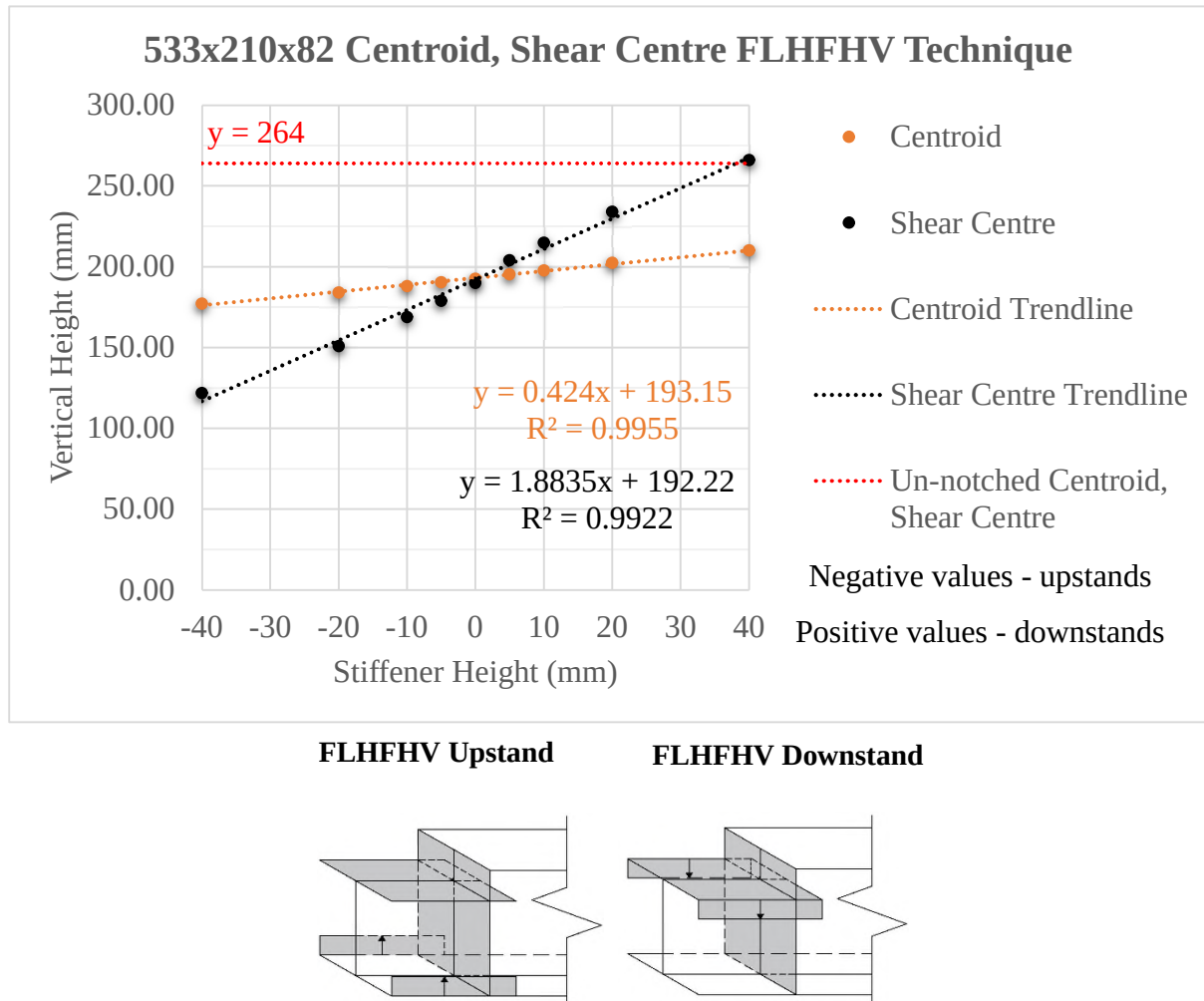


Figure B.6: 533x210x82 Centroid, Shear Centre FLHFHV Technique

B.3 203 x 203 x 46 Section

B.3.1 Along Notch Technique

Figure B.7 shows the shear centre and centroid coincide at a downstand height of approximately 117mm. Downstand heights beyond 120mm could not be tested due to height limitations in the notched region therefore the shear centre does not cross the centroid, shear centre of the un-notched region.

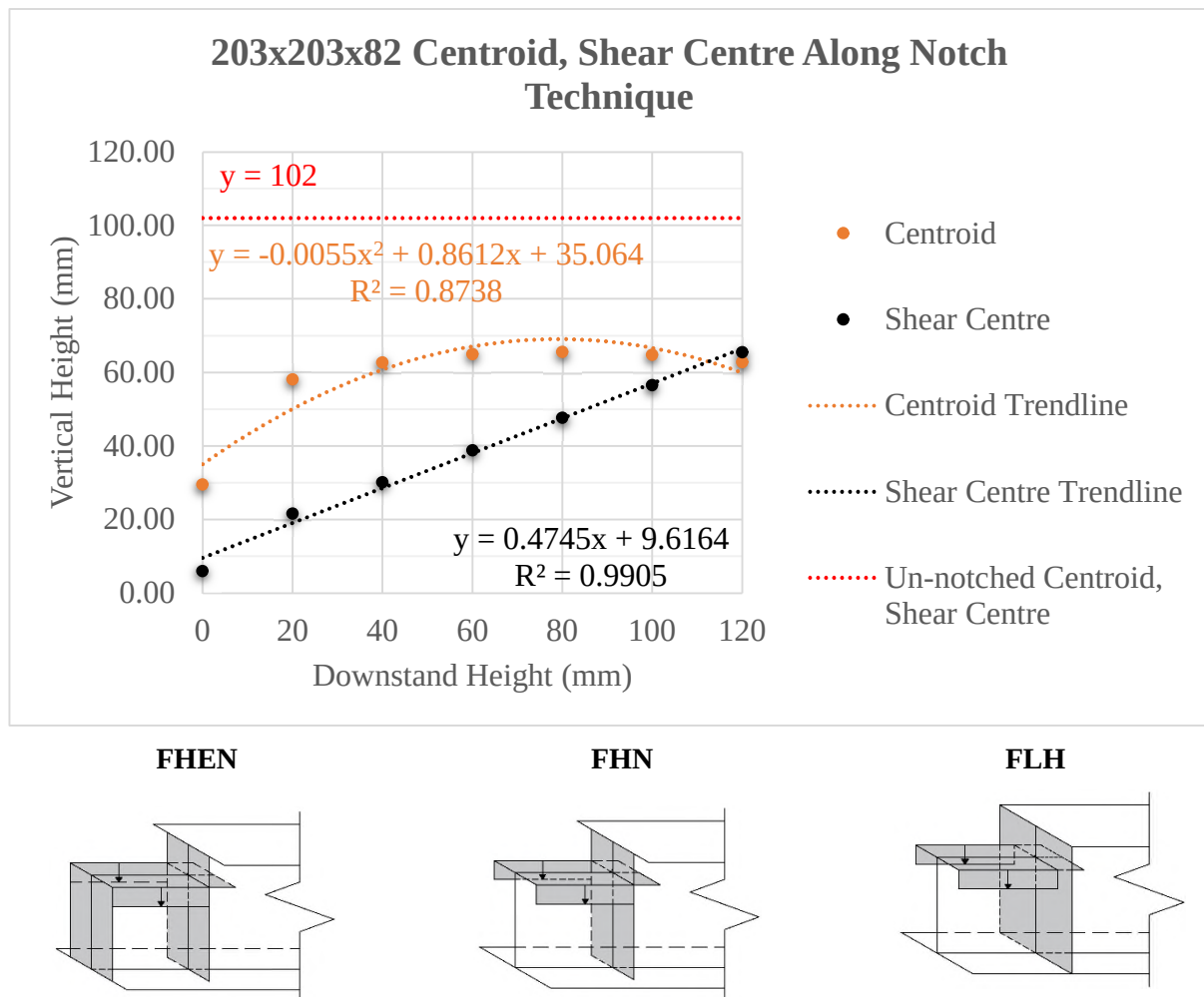
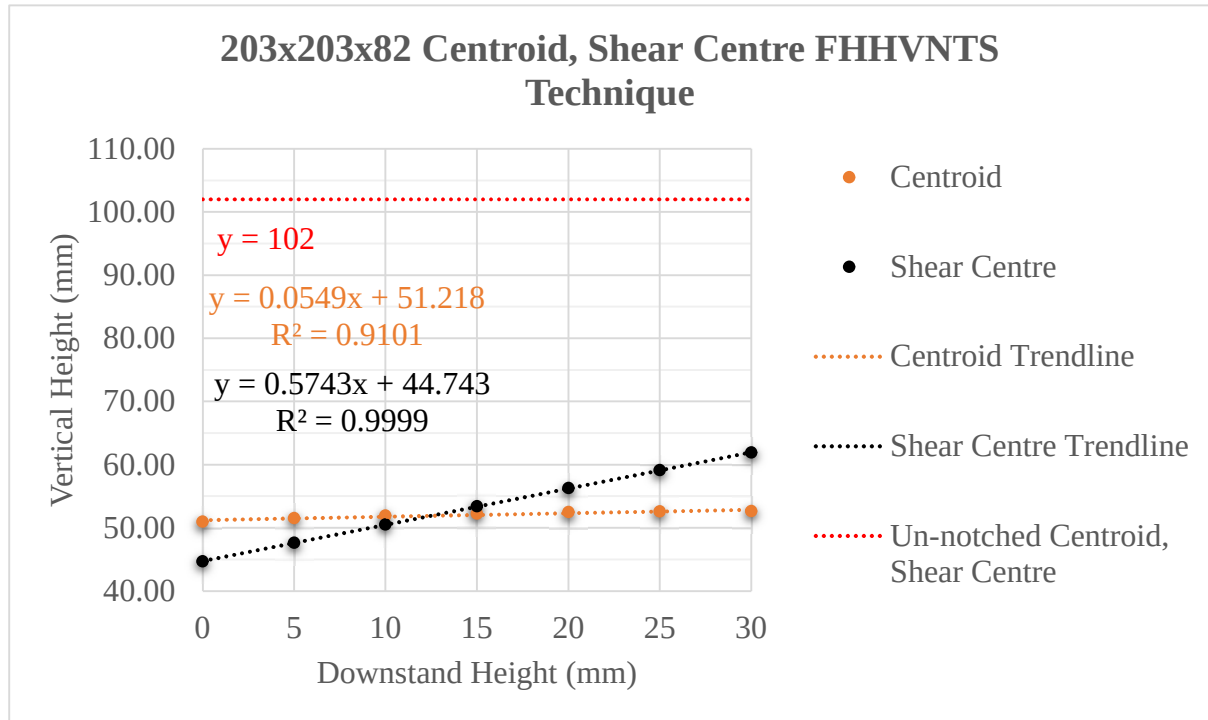


Figure B.7: 203x203x82 Centroid, Shear Centre Along Notch Technique

B.3.2 FHHVNTS Technique

Figure B.8 shows the centroid and shear centre at approximately 13mm downstand height coincide. Height restrictions in the notched region prevent the necessary longer downstands to cause the shear centre of the notched region to meet the centroid, shear centre of the un-notched part.



FHHVNTS

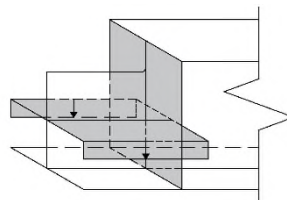


Figure B.8: 203x203x82 Centroid, Shear Centre FHHVNTS Technique

B.3.3 FLHFHV Technique

Figure B.9 shows the centroid and shear centre of the notched region coincide with no downstands or upstands present. The shear centre of the notched region passes the centroid, shear centre of the un-notched part at a downstand height of approximately 33mm.

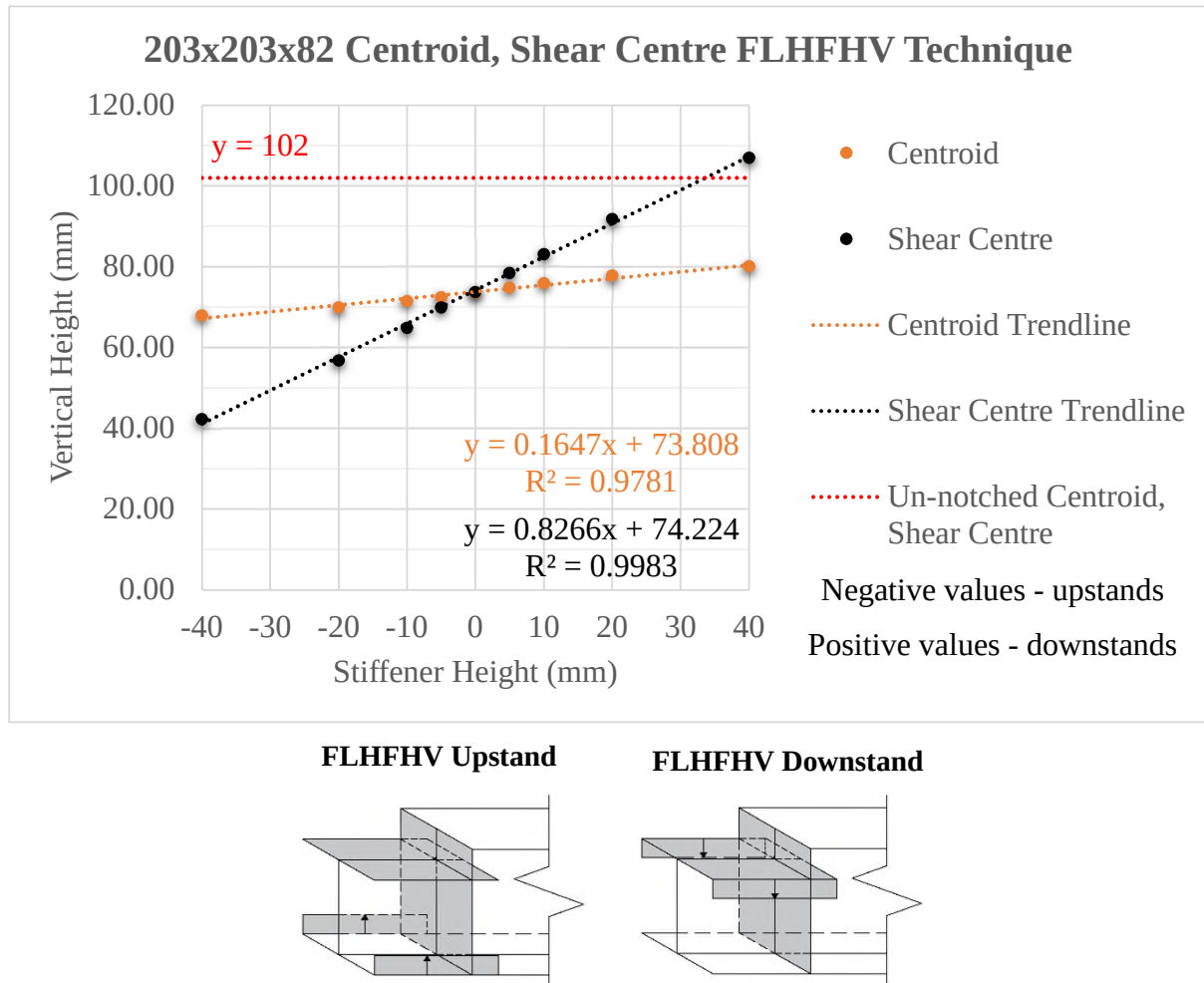


Figure B.9: 203x203x82 Centroid, Shear Centre FLHFHV Technique

B.4 305 x 305 x 97 Section

B.4.1 Along Notch Technique

Figure B.10 shows that at a downstand height of approximately 122mm the centroid and shear centre of the notched region are at the same vertical height. The shear centre of the notched region intersects the centroid, shear centre of the un-notched region at a downstand height of 200mm.

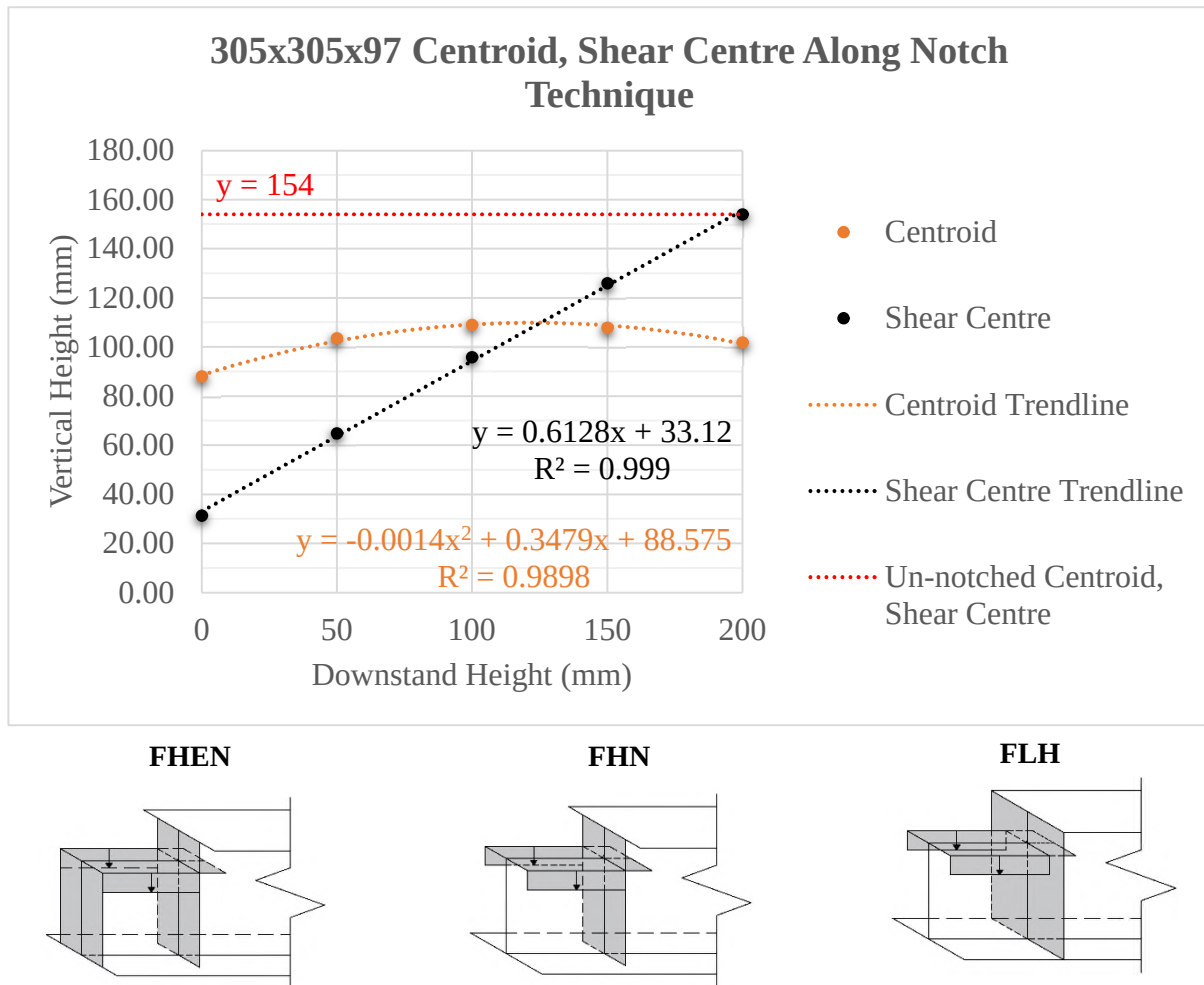


Figure B.10: 305x305x97 Centroid, Shear Centre Along Notch Technique

B.4.2 FHHVNTS Technique

Figure B.11 shows at a downstand height of 18mm the centroid and shear centre of the notched region coincide. Due to height limitations in the notched region, longer downstands that are necessary for the shear centre of the notched region to intersect the centroid, shear centre of the un-notched part cannot be used.

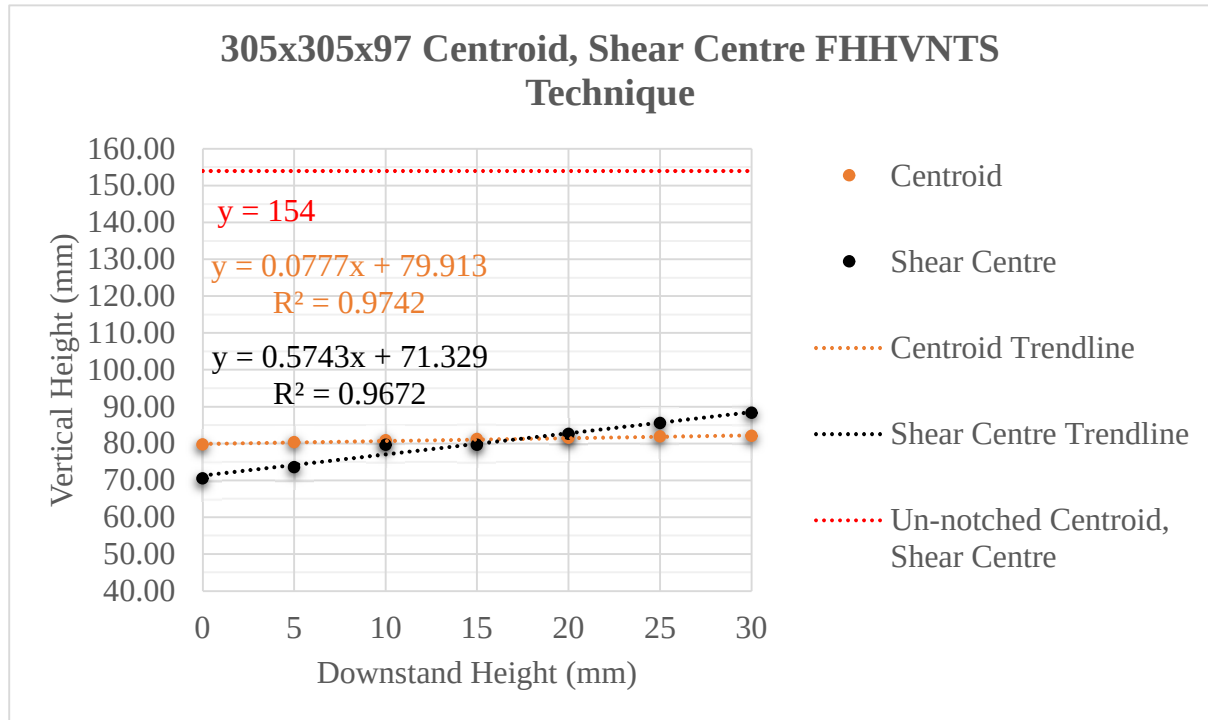


Figure B.11: 305x305x97 Centroid, Shear Centre FHHVNTS Technique

B.4.3 FLHFHV Technique

Figure B.12 shows the centroid and shear centre of the notched region coincide without any downstands or upstands. The shear centre of the notched region at a downstand height of approximately 30mm coincides with the centroid, shear centre of the un-notched part.

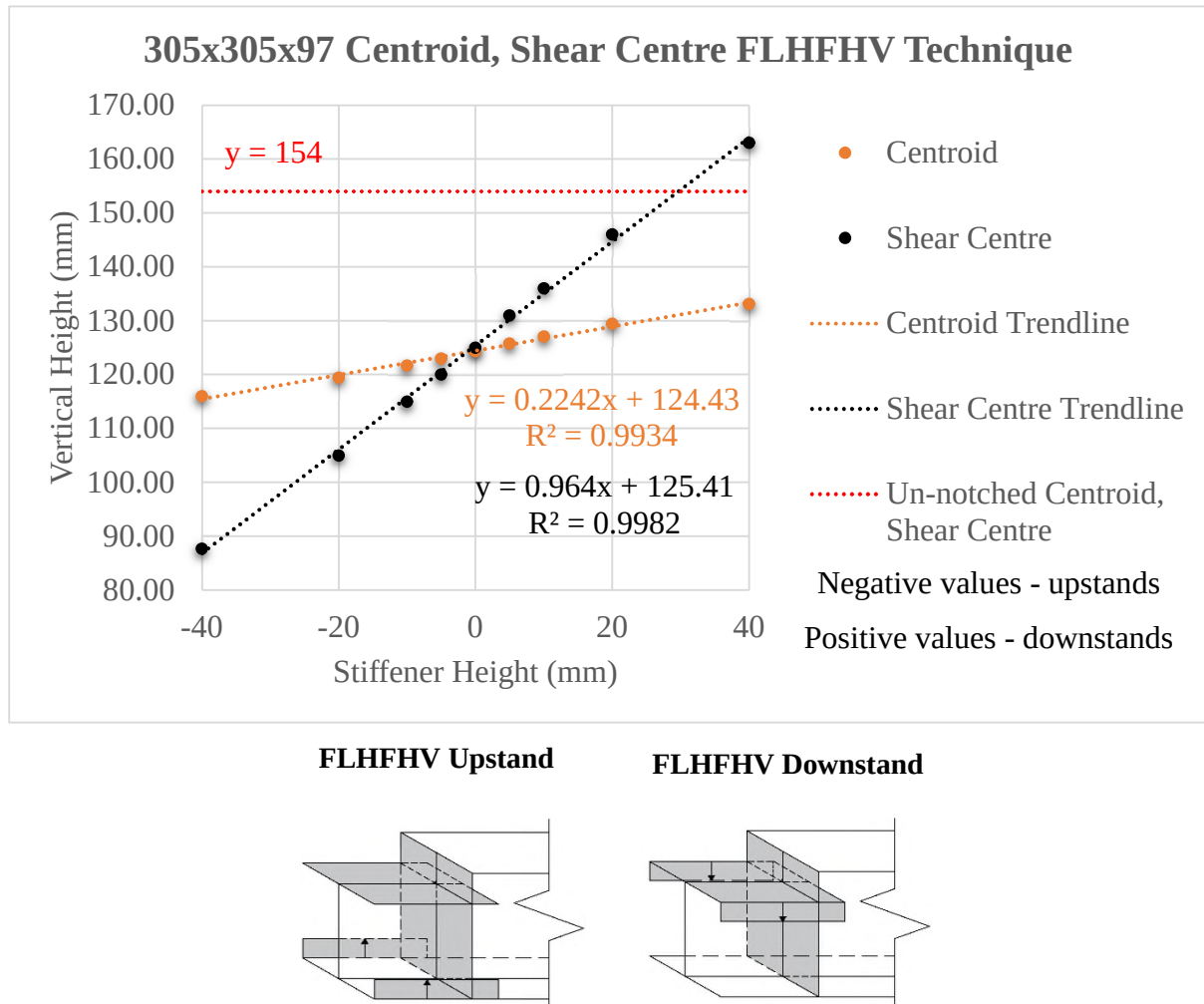


Figure B.12: 305x305x97 Centroid, Shear Centre FLHFHV Technique

Appendix C: Convergence Calculation

Table A.1 shows the data for **Figure 5.3**. A global seed size of 10 was selected and used in the modelling.

Source	Global Seed	P_{cr} (N)	P_{cr} (kN)	L (m)	$M_{cr} = P_{cr}L$ (kNm)
FEM	200	12169	12.169	3	36.507
FEM	180	12127	12.127	3	36.381
FEM	160	12096	12.096	3	36.288
FEM	140	10738	10.738	3	32.214
FEM	120	10713	10.713	3	32.139
FEM	100	10692	10.692	3	32.076
FEM	80	10518	10.518	3	31.554
FEM	60	10495	10.495	3	31.485
FEM	40	10412	10.412	3	31.236
FEM	20	11161	11.161	3	33.483
FEM	15	11135	11.135	3	33.405
FEM	10	11157	11.157	3	33.471
FEM	5	11150	11.15	3	33.45
Yuan <i>et al.</i> (2013)	N/A	11333	11.333	3	34

Table C.1: Divergence Calculation

Appendix D: Calculation of Percentage Change

The percentage change was calculated as follows:

$$\Delta = \% \text{ Change} = \frac{M_n - M_m}{M_f - M_i} \times 100$$

Where:

- M_n – moment at downstand height n
- M_m – moment at downstand height before downstand height n
- M_f – the final moment at the last downstand
- M_i – the initial moment where there is no downstand

Using the data from **Table D.1**, a sample calculation is shown below:

$$\Delta = \% \text{ Change for } 5\text{mm} = \frac{M_5 - M_0}{M_f - M_i} \times 100 = \frac{126.34 - 125.43}{140.67 - 125.43} \times 100$$

$$\Delta = \% \text{ Change for } 5\text{mm} = 5.99\%$$

1m FHHVNTS End Point Load					
Downstand (mm)	Force (N)	Force (kN)	Moment (kNm)	Δ (% Change)	Cumulative %
0	4.18E+05	418.10	125.43		0
5	4.21E+05	421.14	126.34	5.99	5.99
10	4.28E+05	427.58	128.27	12.67	18.66
15	4.37E+05	436.59	130.98	17.74	36.40
17	4.40E+05	440.39	132.12	7.49	43.88
20	4.46E+05	446.39	133.92	11.82	55.70
25	4.57E+05	457.43	137.23	21.73	77.43
30	4.69E+05	468.90	140.67	22.57	100.00
Length (m)	0.6				

Table D.1: 1m FHHVNTS EPL Percentage Change Calculation

The average percentage change (or average Δ) is calculated by taking the average Δ for each span tested for the technique applied to the section. Outliers and negligible data are excluded from this calculation, but these cases are stated in the discussion. A sample calculation is shown in **Table D.2** which uses the FHHVNTS technique applied to the 203x133x25 technique for the 10mm downstand height.

Span (m)	Load Case	Δ (%)
1	EPL	5.99
3		5.83
5		6.51
6		6.72
1	MSL	5.38
3		6.02
5		5.87
6		6.09
Average Δ		6.05

Table D.2: FHHVNTS 10mm Downstand Height Average Percentage Change Calculation

Appendix E: Results of Model 1 and 2

E.1 305 x 102 x 25 Section

E.1.1 Model 1

Figure E.1 shows the 1m, 3m spans for the EPL case.

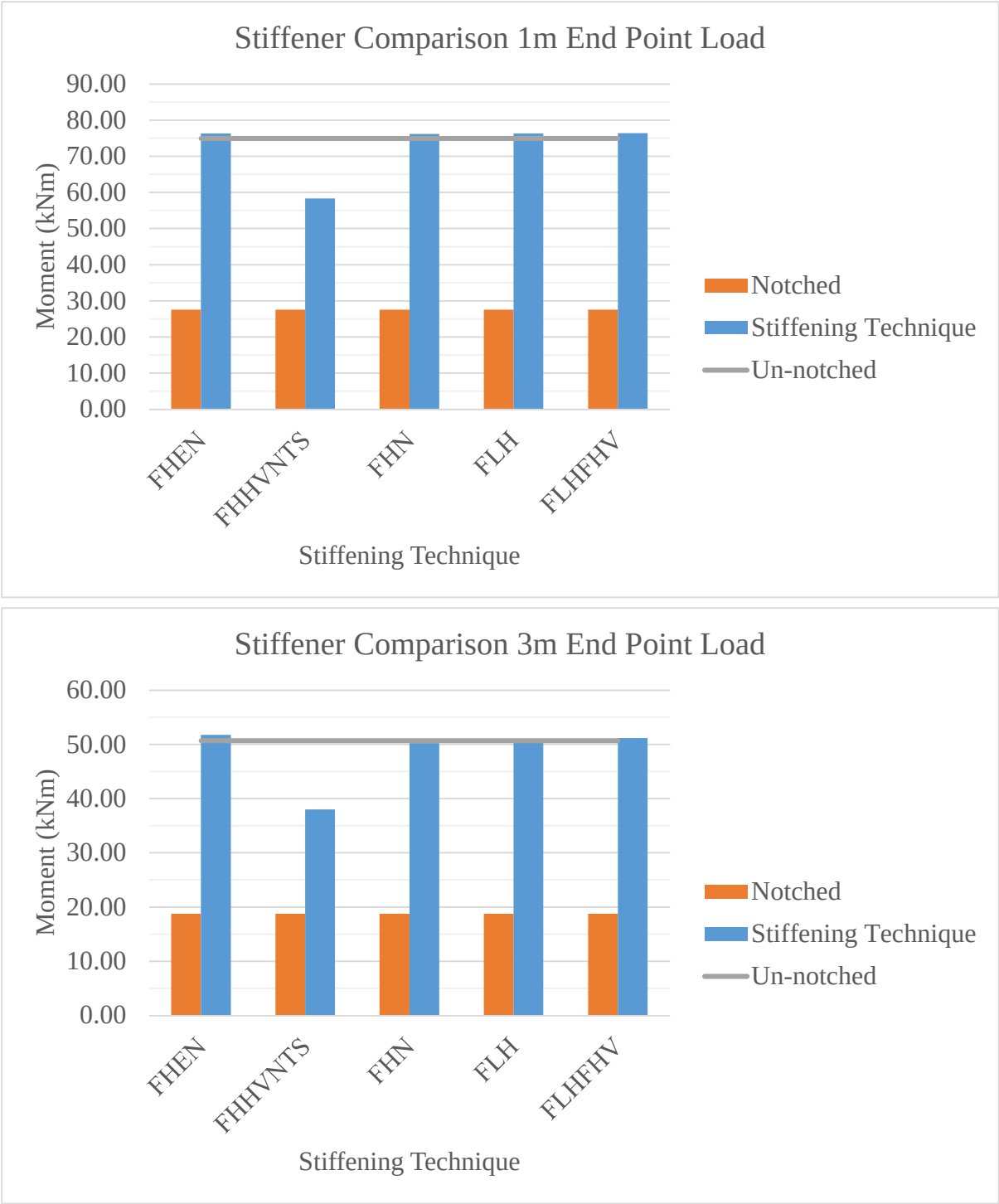


Figure E.1: Stiffener Comparison 1m, 3m EPL 305x102x25

Figure E.2 shows the 1m and 3m MSL. It also shows the five stiffening techniques and by how much they restored the critical elastic moment to the notched section.

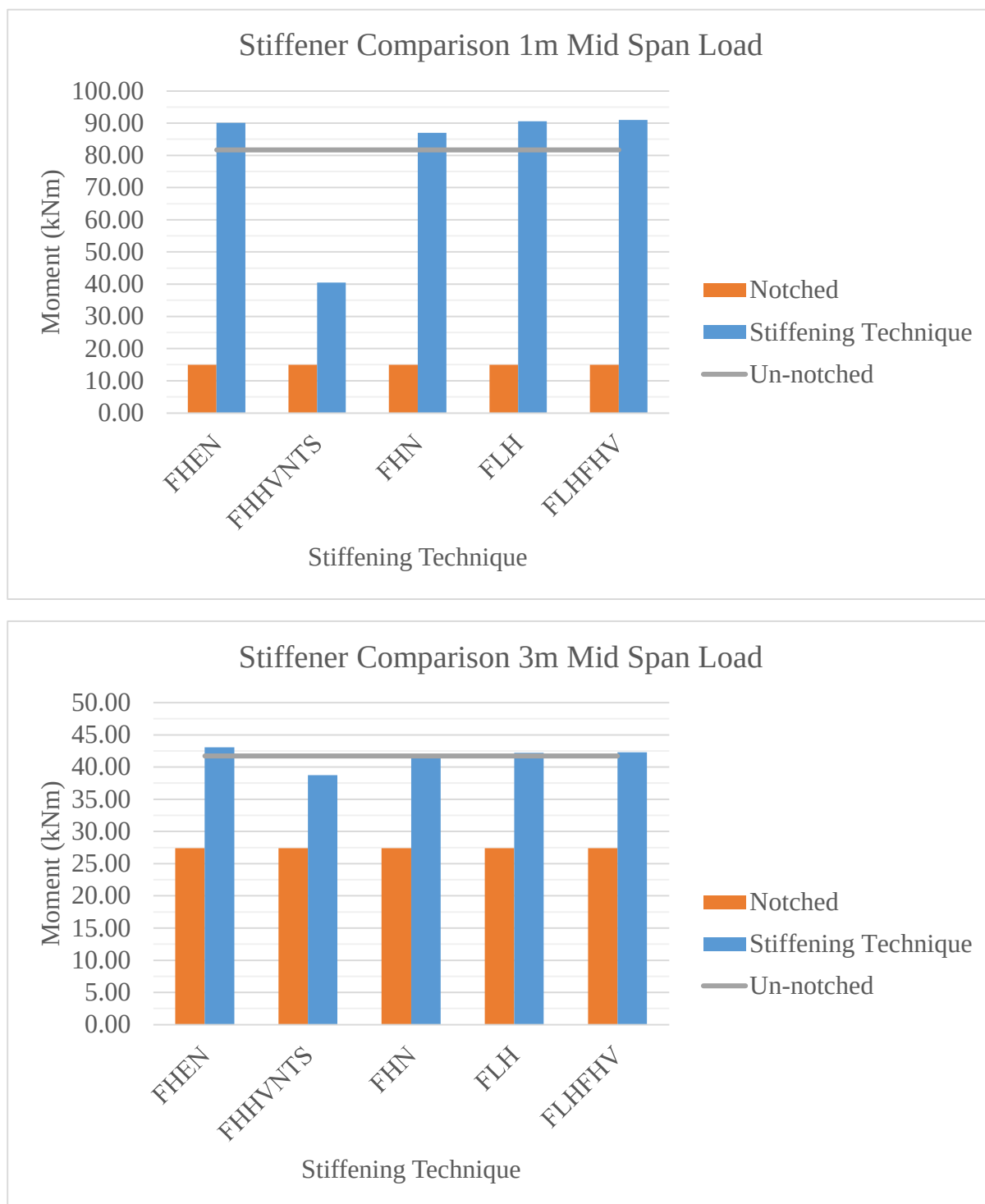


Figure E.2: Stiffener Comparison 1m, 3m MSL 305x102x25

E.1.1.1 FHEN Technique

Figure E.3 shows the critical elastic moment versus downstand height for the EPL FHEN technique. The 1m span increased its critical moment by a negligible 0.42kNm and the 3m span increased by 4.71kNm. This is similar to the 203x133x25 section: the 3m span increased its resistance more than the 1m span. The very marginal increase for the 1m span is unexpected since the 203x133x25 case increased by 3.32kNm.

Despite the small increase, there is a strong trend between the two spans. This trend is also similar to the 203x133x25 section:

- 3m span EPL FHEN (**Figure 6.11**) and 3m MSL (**Figure 6.12**)
- FHN technique (**Figure 6.15** and **Figure 6.16**)
- EPL FLH (**Figure 6.17**) and 3m MSL FLH (**Figure 6.18**)

This is very interesting considering they are completely different sections. The increase in the moment appears parabolic but only very slightly. The parabola seems to exist up till the 20mm downstand, close to the coincidence point of the centroid and shear centre in the notched region at 19mm, then a distinct straight line exists.

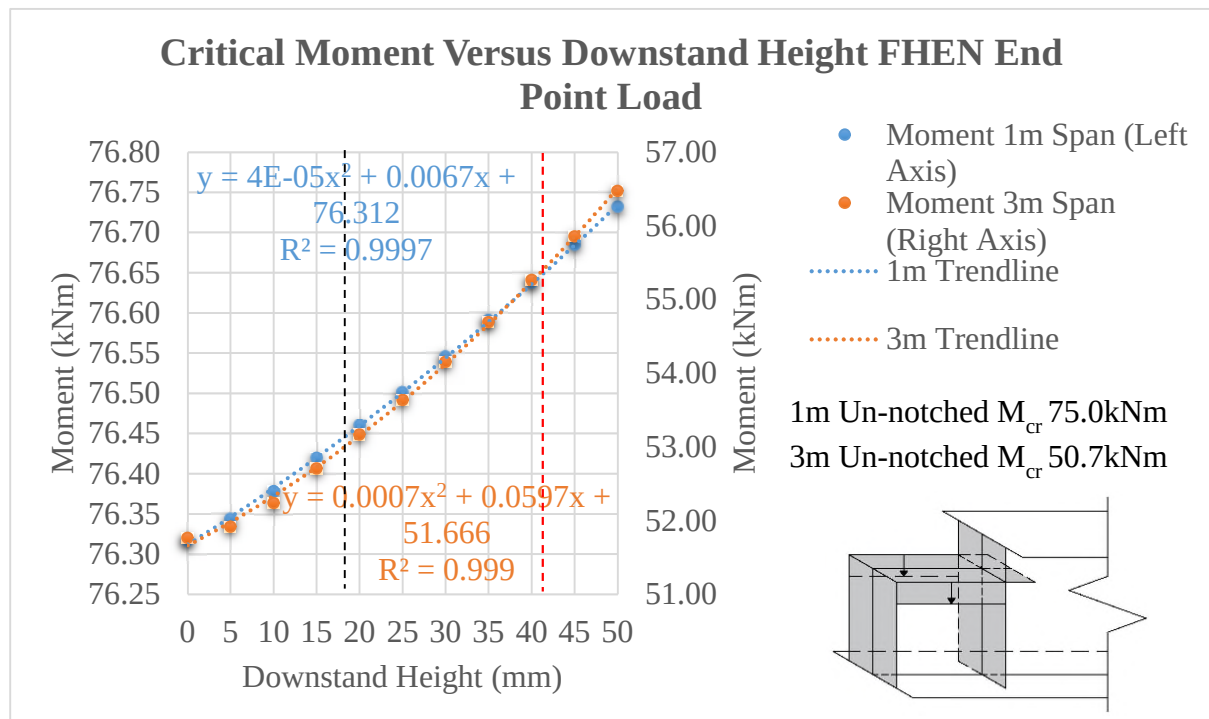


Figure E.3: Critical Moment Versus Downstand Height FHEN Technique EPL Model 1 305x102x25

Figure E.4 shows the critical elastic moment versus downstand height for the MSL FHEN technique. The 1m span increased its critical elastic moment by 2.02kNm and the 3m by 3.64kNm which is opposite to the EPL case: here the longer span increased more.

The trends for the 1m and 3m span appear similar to the EPL case in **Figure E.3** and the 3m span MSL FHEN 203x133x25 case (**Figure 6.12**). There are no outliers in the FHEN technique, unlike in the 203x133x25 section, and the trend is strong in the EPL and MSL cases. The only trend that is slightly different is the 1m MSL case, but this is not as much of an outlier as in the 203x133x25 case.

Just as in the EPL case in **Figure E.3**, the curves are initially parabolic until the 20mm downstand and linear thereafter. The parabolic nature is only for the first few downstand heights and the change in the curve type occurs close to the 20mm downstand.

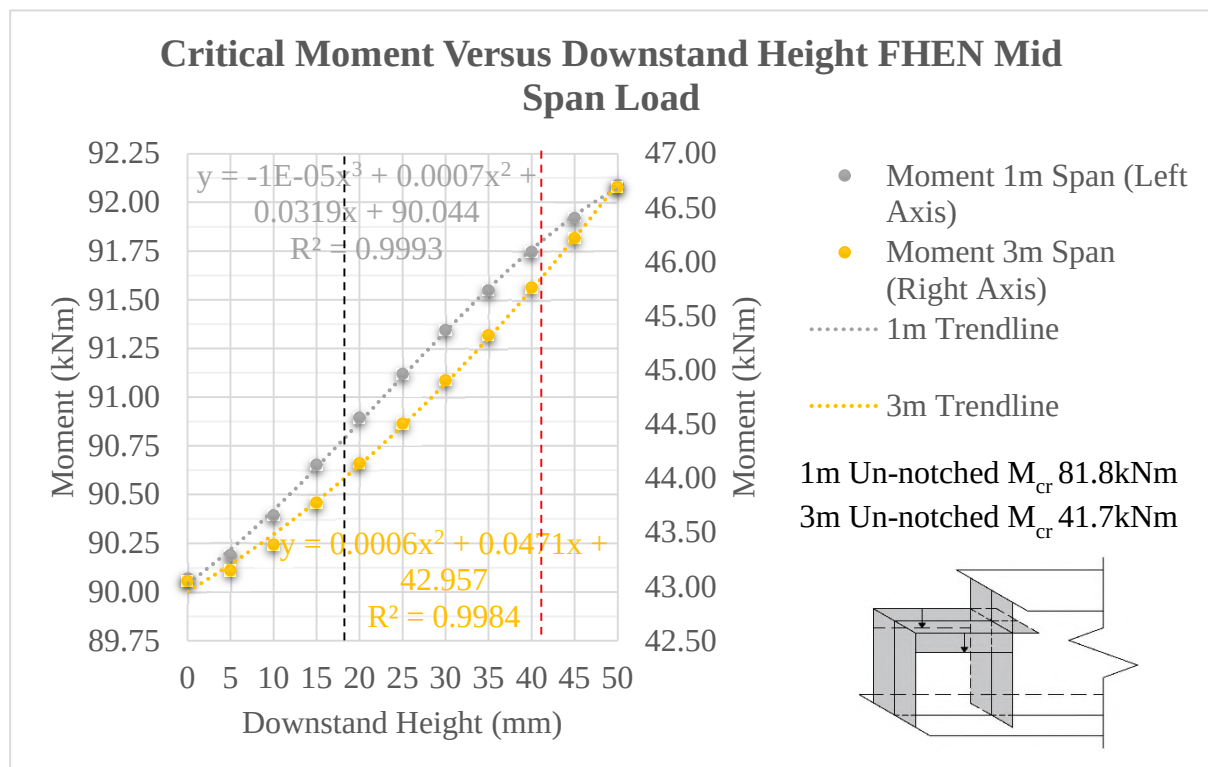


Figure E.4: Critical Moment Versus Downstand Height FHEN Technique MSL Model 1 305x102x25

E.1.1.2 FHHVNTS Technique

Figure E.5 shows the results for the FHHVNTS EPL case. The 1m span increased its critical elastic moment by 3.21kNm whereas the 3m span increased by 2.67kNm. There are similarities in the FHHVNTS EPL case between the 203x133x25 (**Figure 6.13**) and the 305x102x25 (**Figure E.5**) sections:

- The 1m span increased more than the 3m span.
- The trend in the increase is slightly parabolic but appears to resemble a straight line after the shear centre passes the centroid in the notched region (at 25mm downstand height for the 305x133x25 section).
- Due to height restrictions in the notched region the required downstand height to shift the shear centre past the centroid in the un-notched region does not fit.

Just as in the FHEN technique, these two very different sections exhibit similar behaviour despite completely different values in the critical elastic moment.

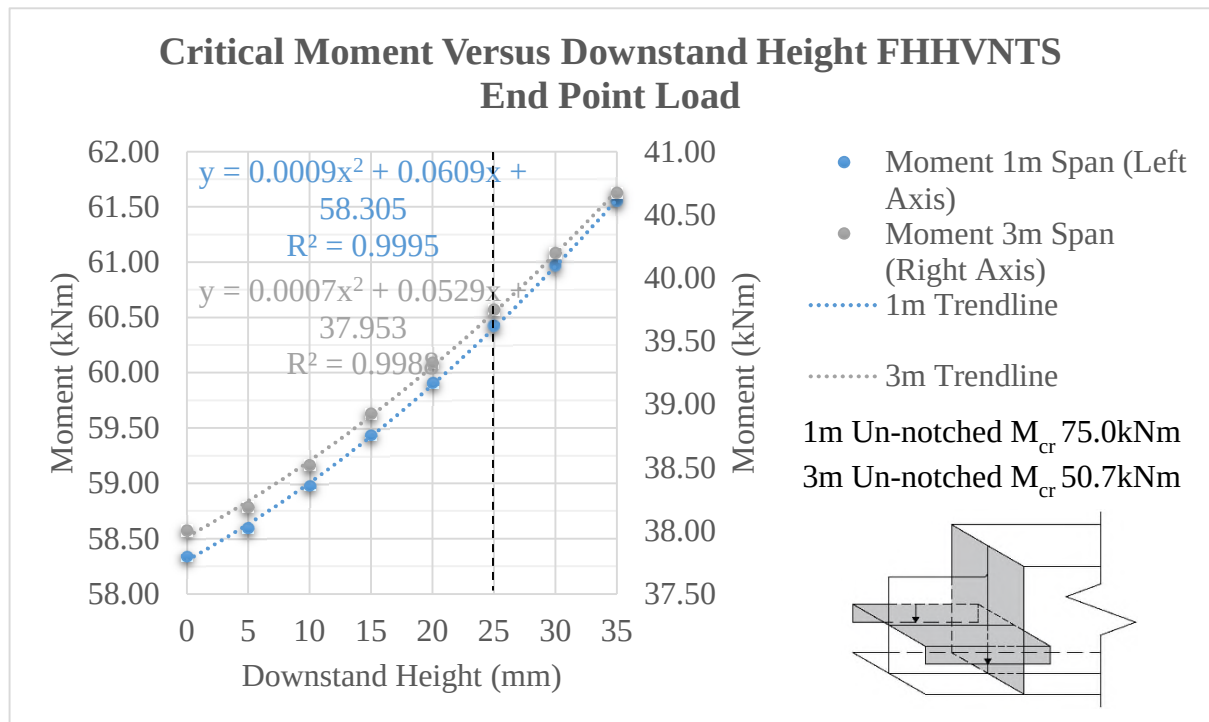


Figure E.5: Critical Moment Versus Downstand Height FHHVNTS Technique EPL Model 1 305x102x25

Figure E.6 shows the results for the FHHVNTS MSL case. The increase in the critical elastic moment for the 1m and 3m spans is 4.44kNm and 1.63kNm respectively. Just as in the EPL case (and also the FHHVNTS 203x133x25 EPL, MSL cases in **Figure 6.13** and **Figure 6.14**) the shorter span increased its critical elastic moment more.

Another similarity is the shape of the trend: almost parabolic up to the coincidence point of the centroid and shear centre in the notched region at 25mm downstand height. Thereafter, the trend can be approximated as a straight line. Due to height restrictions, more data points cannot be tested to check if the linear trend exist as in the 203x133x25 section.

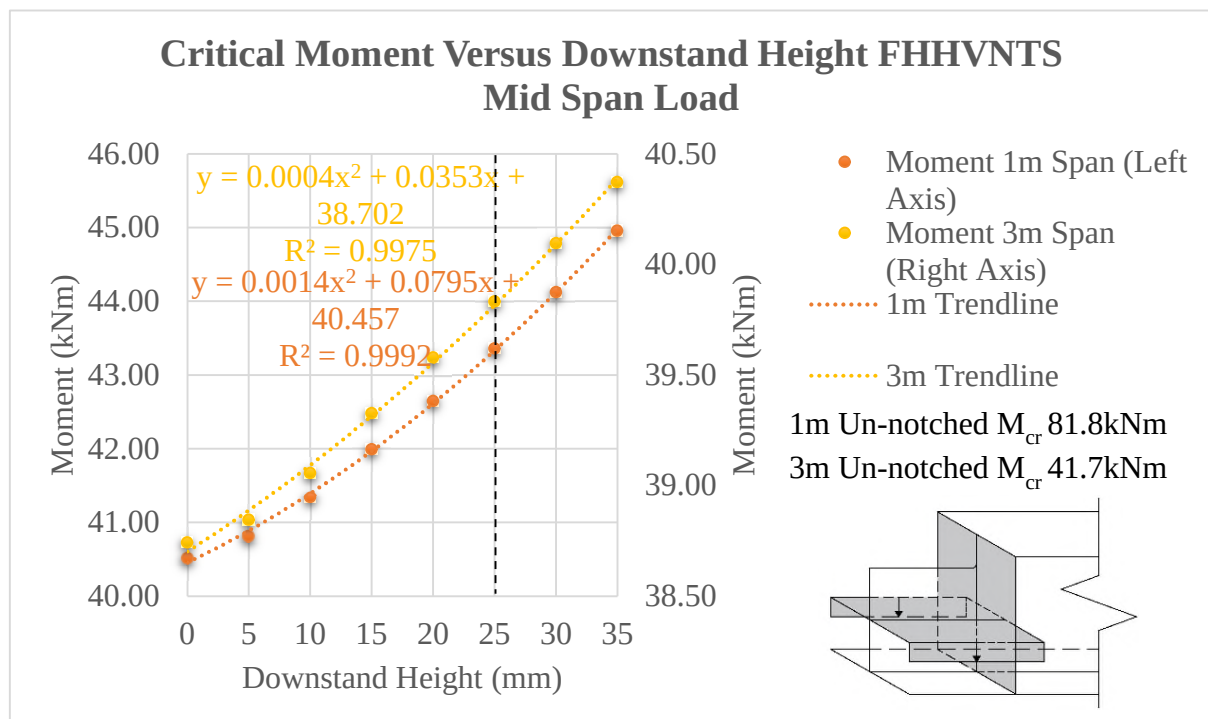


**Figure E.6: Critical Moment Versus Downstand Height FHHVNTS Technique MSL Model 1
305x102x25**

E.1.1.3 FHN Stiffening Technique

Figure E.7 shows the results for the EPL FHN technique. The increase in critical elastic moment is negligible for the 1m (0.08kNm). The 3m span increased by 0.60kNm which is slightly more than the 0.5kNm change which is the minimum acceptable value for change for this research.

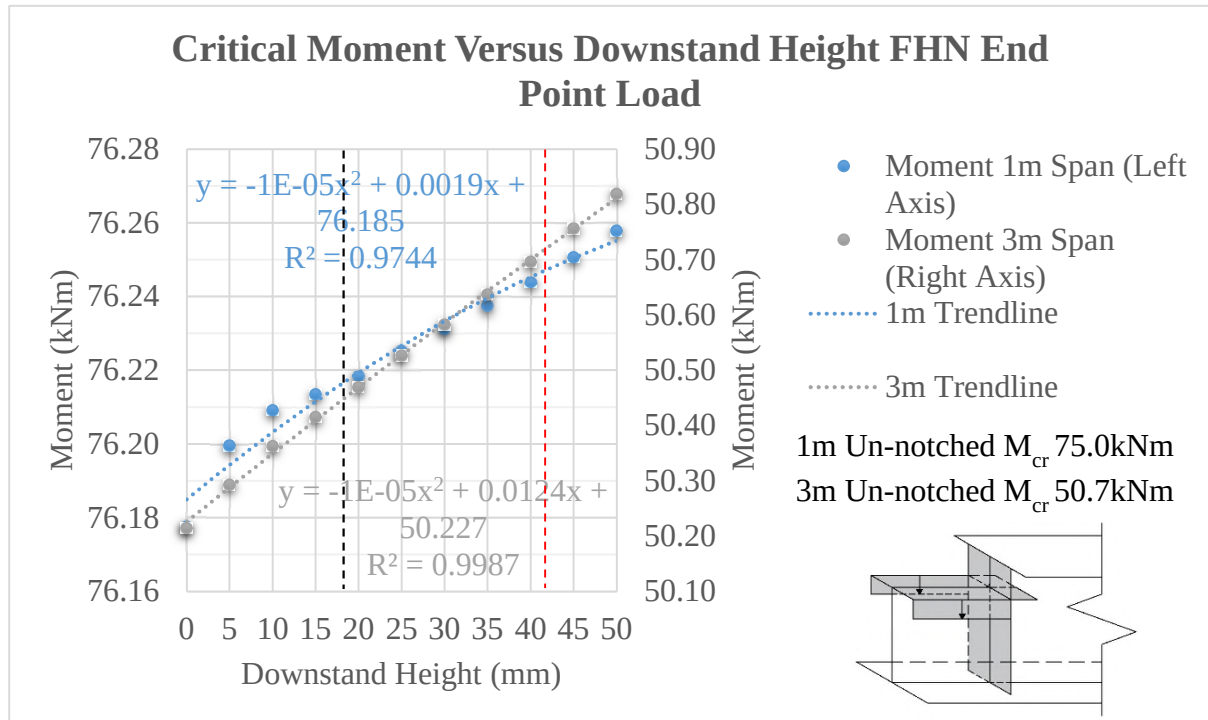


Figure E.7: Critical Moment Versus Downstand Height FHN Technique EPL Model 1 305x102x25

Figure E.8 shows the results for the MSL case for the FHN technique. There is also a marginal increase in the critical elastic moment of 0.45kNm for the 1m span load and 0.58kNm for the 3m span (which increased the most). The increase is around 0.5kNm which is the minimum value for change in this report.

The 3m span has a constant gradient and it is clear nothing changes around the 19mm downstand height. However, just as in all the other cases the critical elastic moment increased with each increase in downstand height.

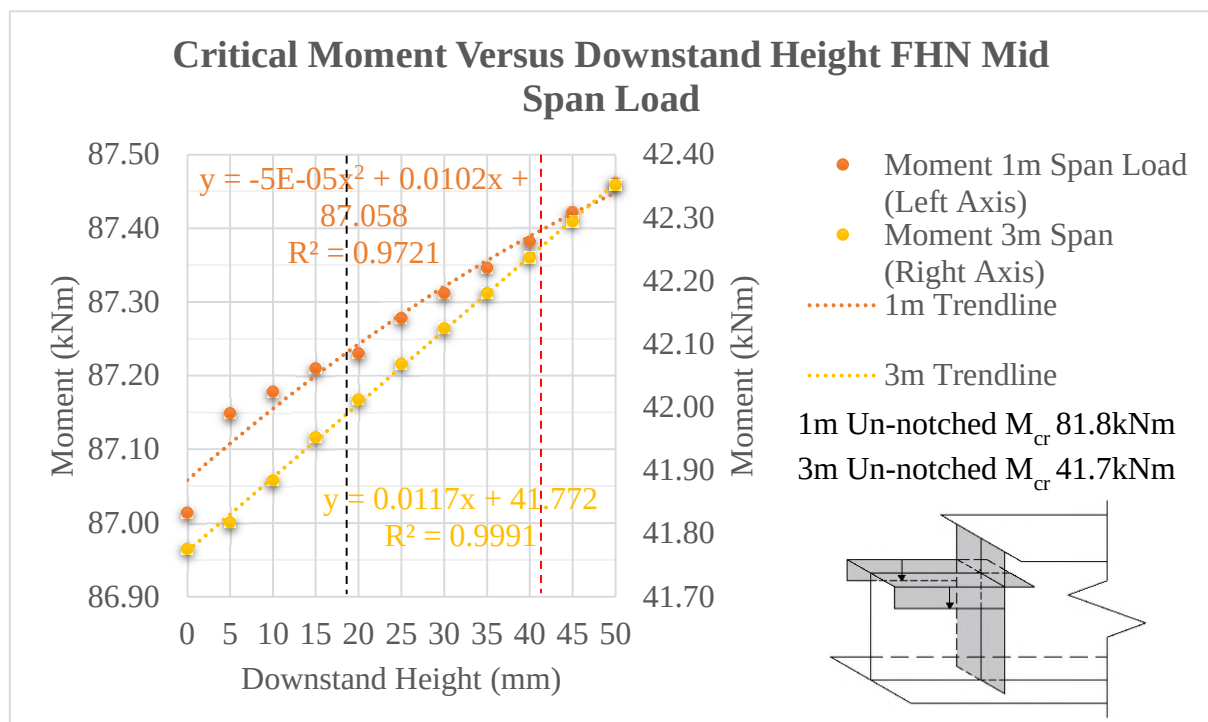


Figure E.8: Critical Moment Versus Downstand Height FHN Technique MSL Model 1 305x102x25

E.1.1.4 FLH Technique

Figure E.9 shows the results for the EPL FLH technique. The increase in critical elastic moment is negligible for both the 1m (0.07kNm) and 3m (0.68kNm) span since the increase is less than 0.5kNm which is the minimum acceptable value for change for this research.

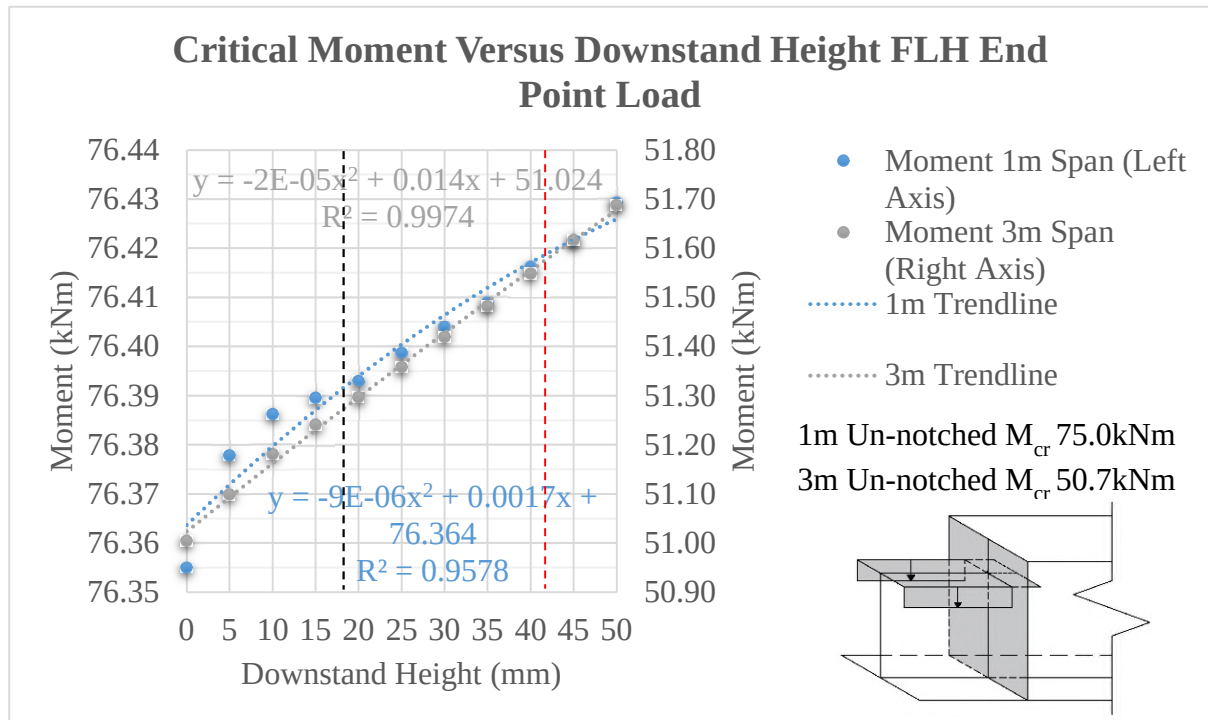


Figure E.9: Critical Moment Versus Downstand Height FLH Technique EPL Model 1 305x102x25

Figure E.10 shows the results for the MSL case for the FLH technique. There is also a marginal increase in the critical elastic moment of 0.30kNm for the 1m span load and 0.69kNm for the 3m span (which increased the most). The increase is around 0.5kNm for the 3m span which is the minimum value for change in this report.

The trend is similar to the 3m span in both EPL (**Figure 6.17**) and MSL (**Figure 6.18**) cases for the 203x133x25 section except the parabola seems to approach a minimum value. The gradient appears to decrease with each increase in downstand height. The curve could be a straight line as indicated by the very small coefficient of the x^2 term. The 1m FHEN MSL in **Figure E.4** is the only other case in Model 1 that is similar.

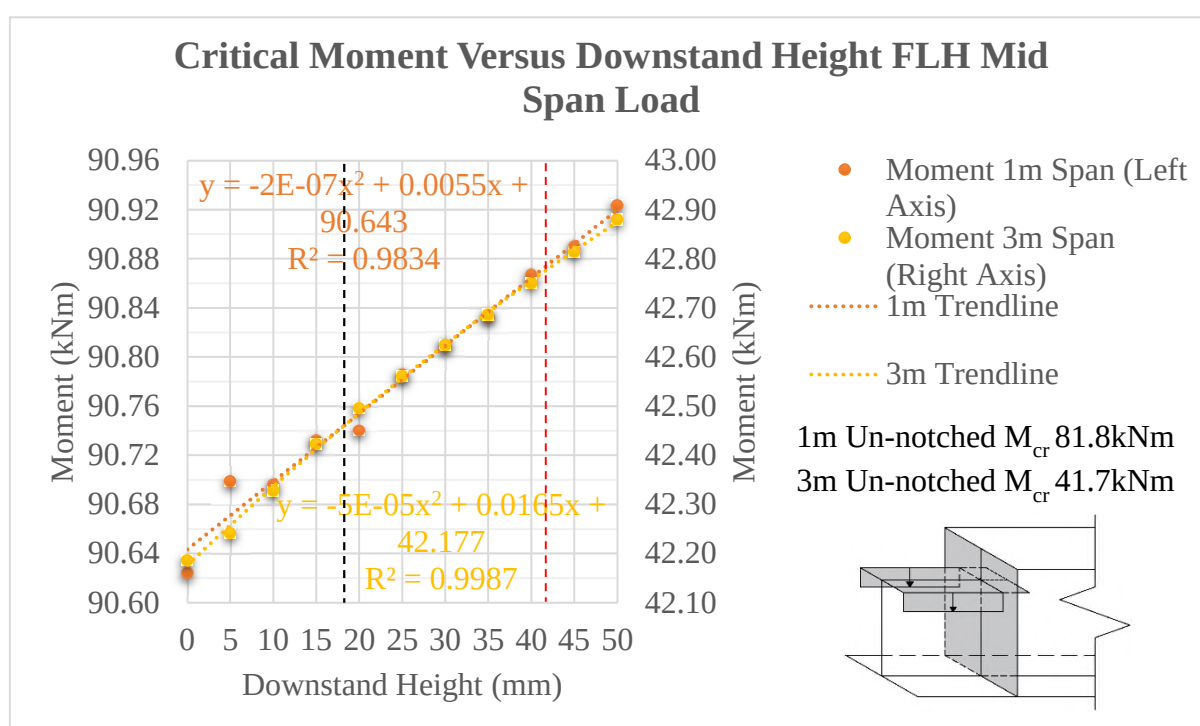


Figure E.10: Critical Moment Versus Downstand Height FLH Technique MSL Model 1 305x102x25

E.1.1.5 FLHFHV Technique

Figure E.11 shows the results of the FLHFHV technique for the EPL case. The section decreased its critical elastic moment moving from the upstands to the no downstand/upstands at stand height of 0mm by 0.33kNm for the 1m span. The introduction of downstands then increased the critical elastic moment by 0.06kNm. The changes are considered negligible since the increase is less than 0.5kNm which is the minimum acceptable value for change for this research.

The 3m span also decreased its critical elastic moment (by 2.49kNm) then increased it with the downstands (0.49kNm which is negligible). The behaviour here is unlike any of the trends in the other techniques previously discussed and is only similar to the FLHFHV technique for the 203x133x25 section. As the shear centre approaches the centroid in the notched section by reducing the height of the upstands, the critical elastic moment reduces and only increases, if by a negligible amount, once downstands are introduced. This stiffening technique could be considered an outlier to the other techniques.

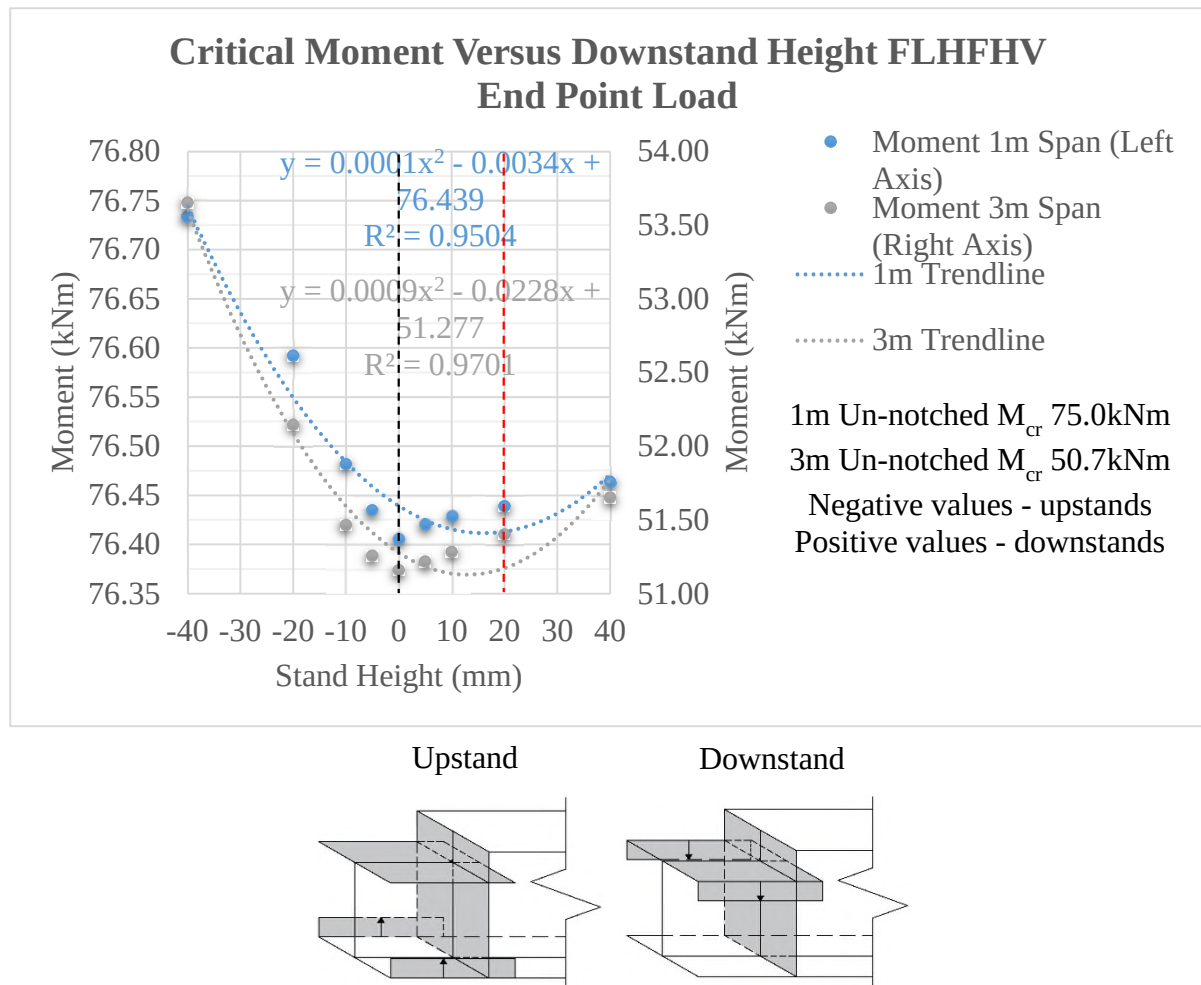


**Figure E.11: Critical Moment Versus Stand Height FLHFHV Technique EPL Model 1
305x102x25**

The trend in the FLHFHV MSL case shown in **Figure E.12** follows the EPL case (**Figure E.11**) but is also unique as it does not follow the trends shown in the previous stiffening techniques except for the FLHFHV technique applied to the 203x133x25 section. There was also a reduction in the moment (1.54kNm) moving from the upstand to no upstand/downstand case then a negligible increase of 0.25kNm for the 1m span case. Similarly, the 3m span reduced its critical elastic moment by 1.96kNm then increased it by 0.48kNm.

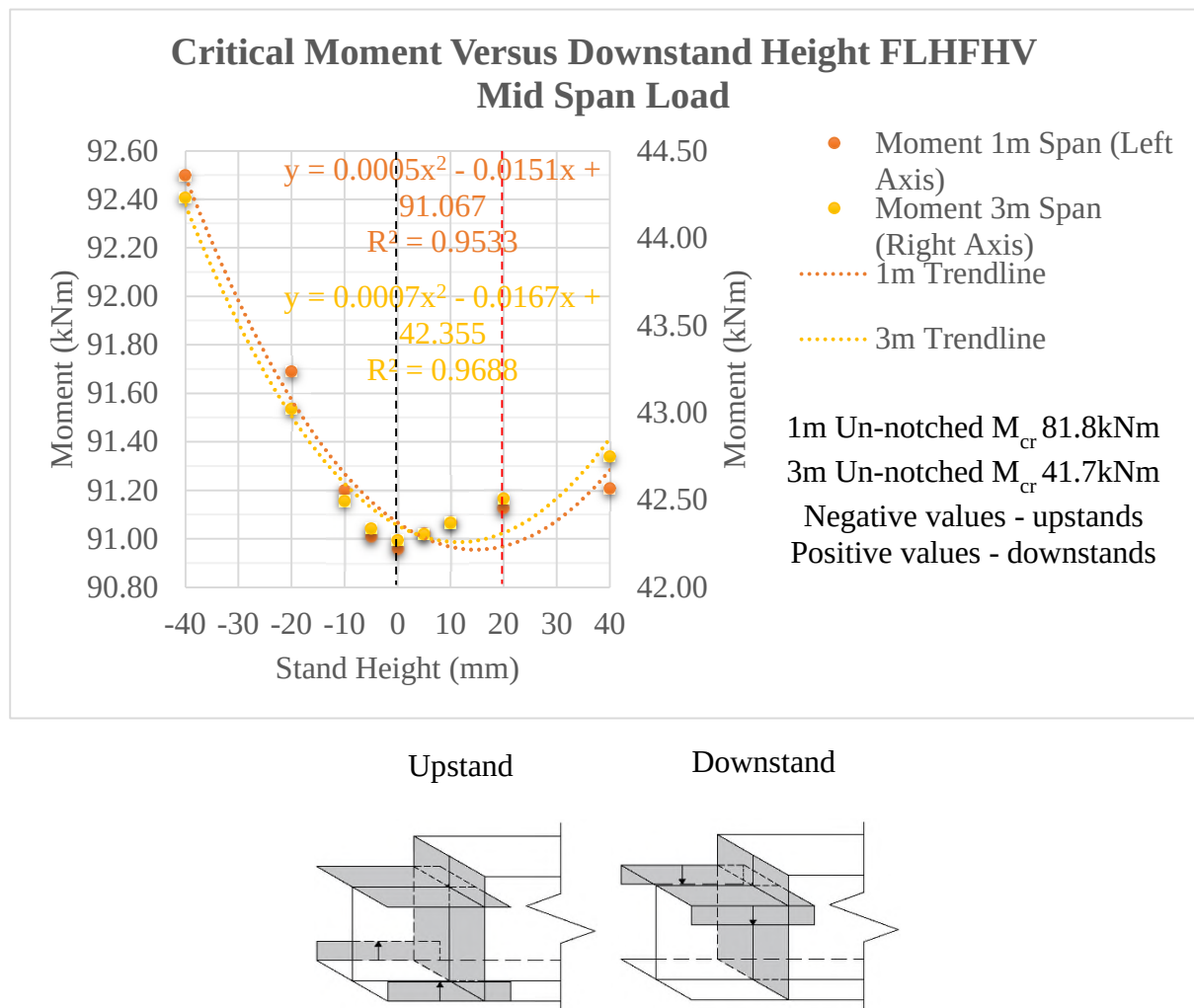


**Figure E.12: Critical Moment Versus Stand Height FLHFHV Technique MSL Model 1
305x102x25**

Just as in the 203x133x25 section, by isolating the downstand case it is clear the trend when the shear centre is above the centroid is similar to all the other sections. That is: there is an increase in the critical elastic moment as the shear centre moves further above the centroid. The graphs showing the isolated downstand are not shown since the total change in the critical elastic moment is less than 0.5kNm.

E.1.2 Model 2

Model 2 is the testing of the stiffened section for the 5m and 6m spans. The EPL case for the 5m and 6m spans are shown in **Figure E.13**. It also shows the five stiffening techniques and by how much they restore the critical elastic moment to the notched section.

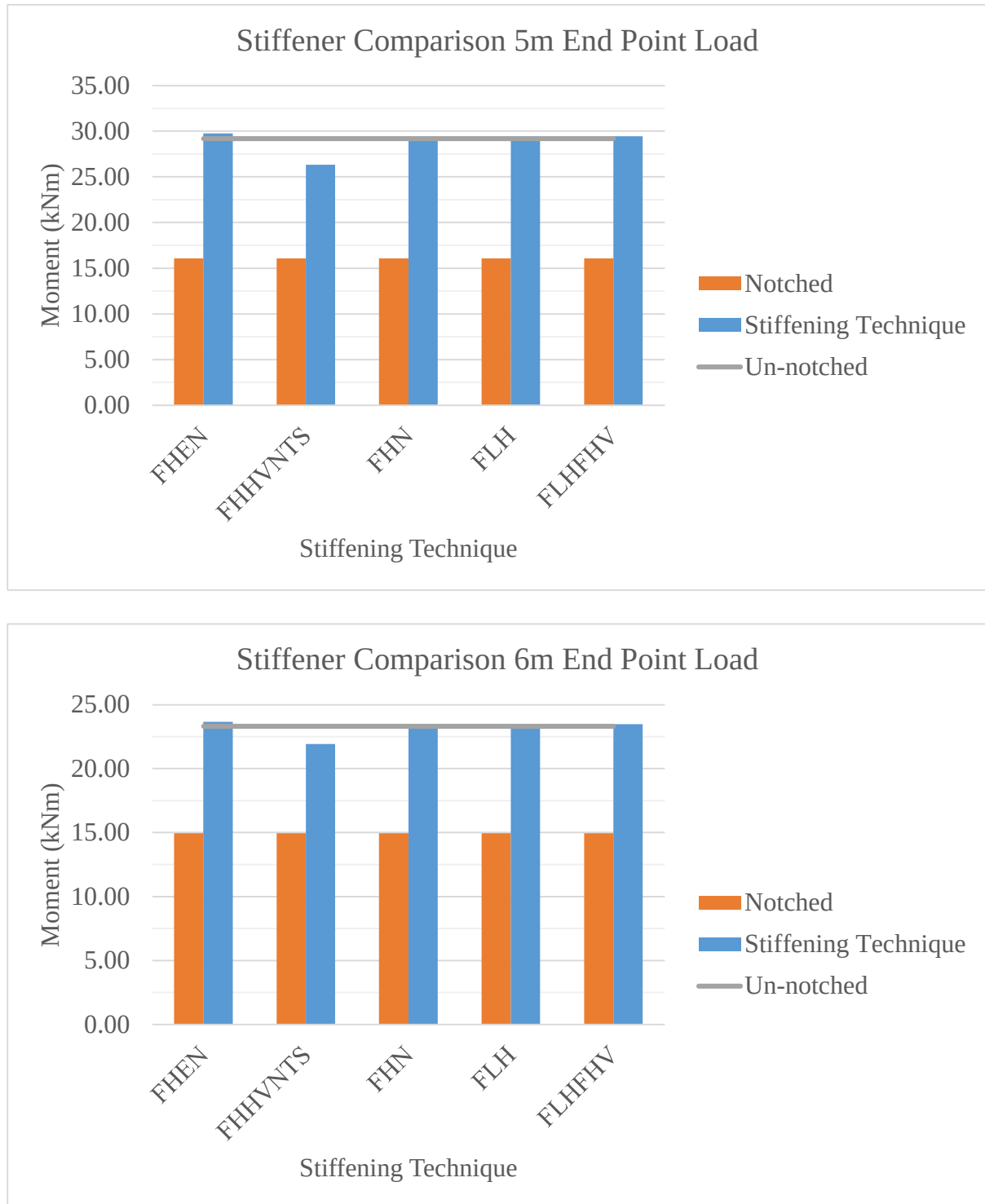


Figure E.13: Stiffener Comparison 5m, 6m EPL 305x102x25

Figure E.14 shows the 5m, 6m spans for the MSL case. It also shows the five stiffening techniques and by how much they restore the critical elastic moment to the notched section.

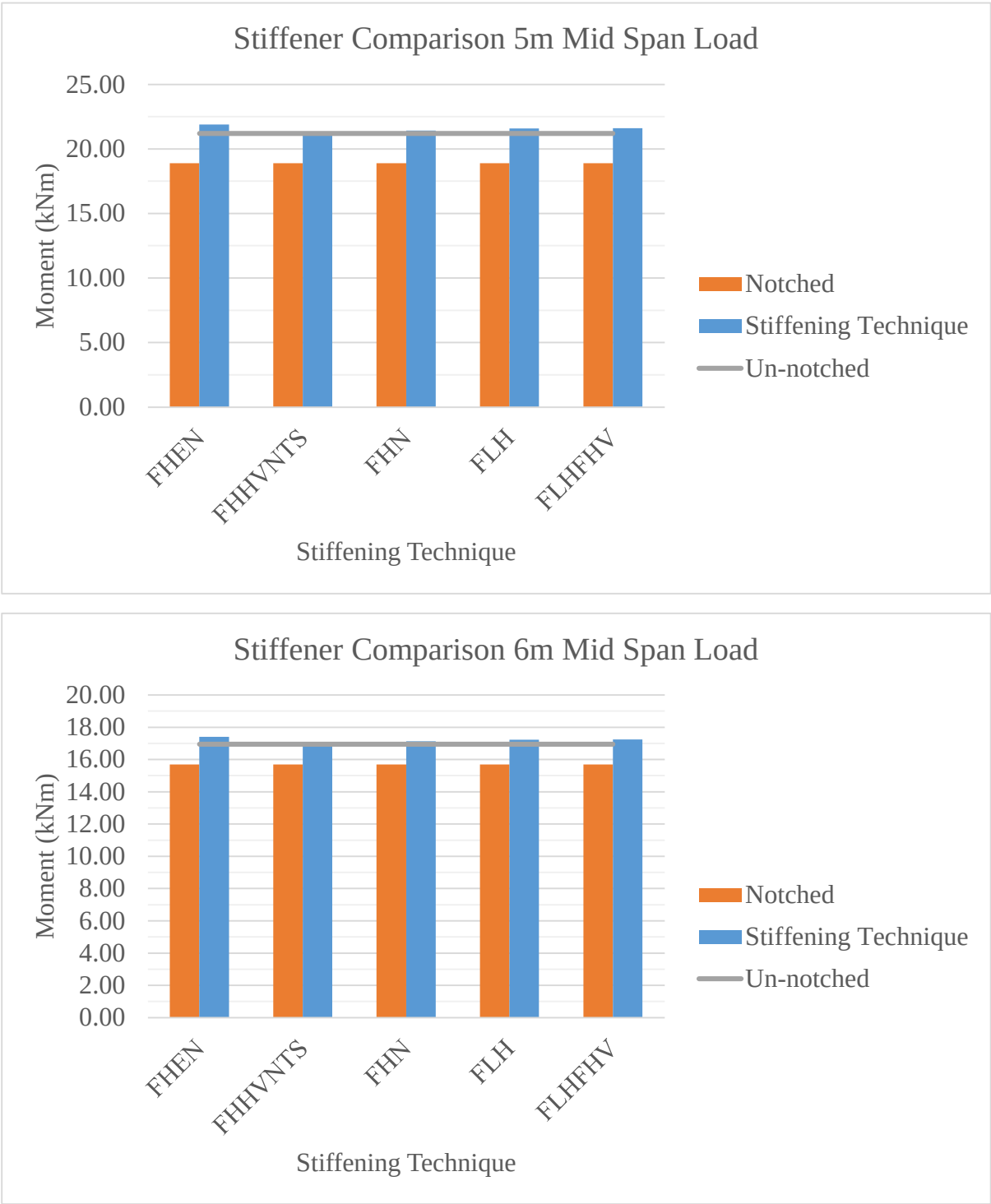


Figure E.14: Stiffener Comparison 5m, 6m MSL 305x102x25

E.1.2.1 FHEN Technique

Figure E.15 shows the results for the FHEN EPL case. The 5m span increased its critical elastic moment by 1.67kNm whereas the 6m span increased by an almost marginal 0.96kNm which is the same as the 203x133x25 Model 2 (**Figure 6.24**) in which the shorter of the two spans increased its critical elastic moment more.

The trend is strong between the spans and is also similar to both loading cases for Model 2 of the FHEN technique for the 203x133x25 section (**Figure 6.24** and **Figure 6.25**). This is very interesting because this shows consistency across different sections, spans, and loading cases. The critical elastic moment increases with each increase in downstand height. The first few downstand cases are parabolic but become linear around the 20mm downstand height which is close to the coincidence point of the centroid and shear centre at 19mm downstand height.

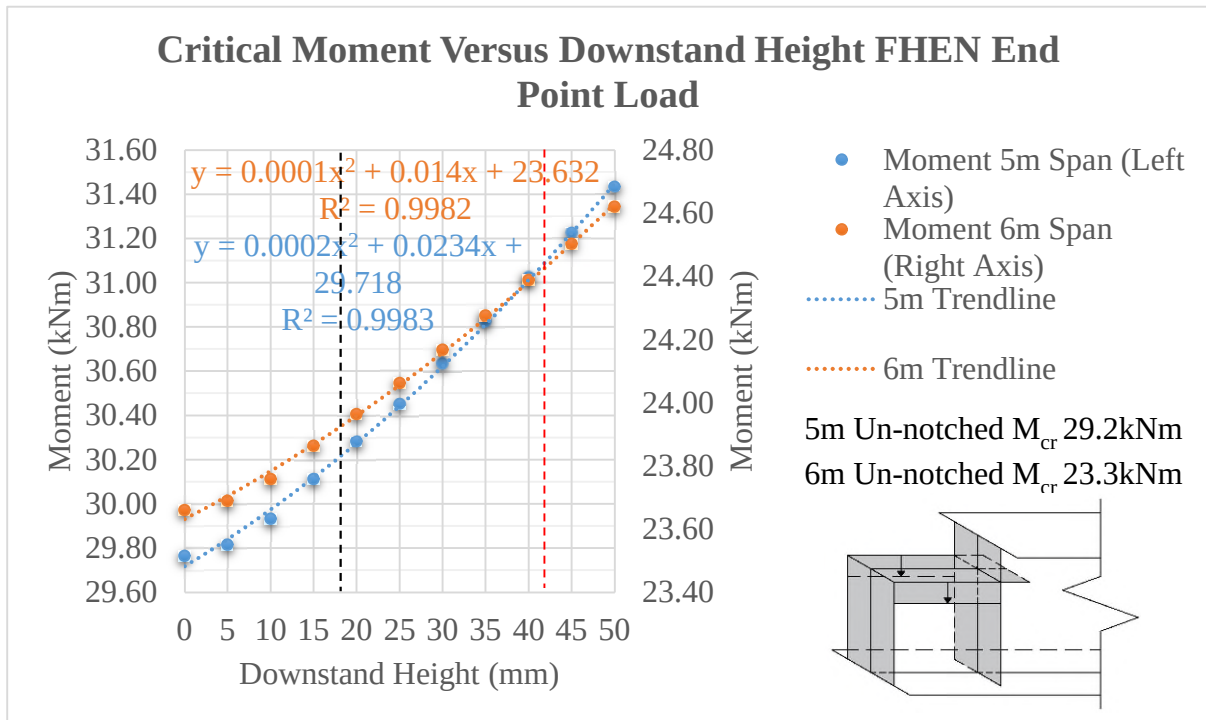


Figure E.15: Critical Moment Versus Downstand Height FHEN Technique EPL Model 2 305x102x25

Figure E.16 shows the critical elastic moment for the FHEN MSL case versus downstand height. The 5m span slightly increased its critical elastic moment by 1.21kNm and the 6m span by 0.77kNm. The shorter spanning member increased its critical elastic moment more than the longer span, just as in the EPL case.

The trend is strong between the two spans: both start at minimum values and increase parabolically. Closer inspection shows the trend may be split into two trendlines: initially the curves are parabolical then a straight line. It appears the change occurs around the 20mm downstand height which is close to the centroid shear centre coincidence point the downstand height of 19mm. This is similar to the MSL case in Model 2 for the 203x133x25 section (**Figure 6.25**). An interesting point is the 203x133x25 section increased its critical elastic more than the 305x102x25 section for all techniques.

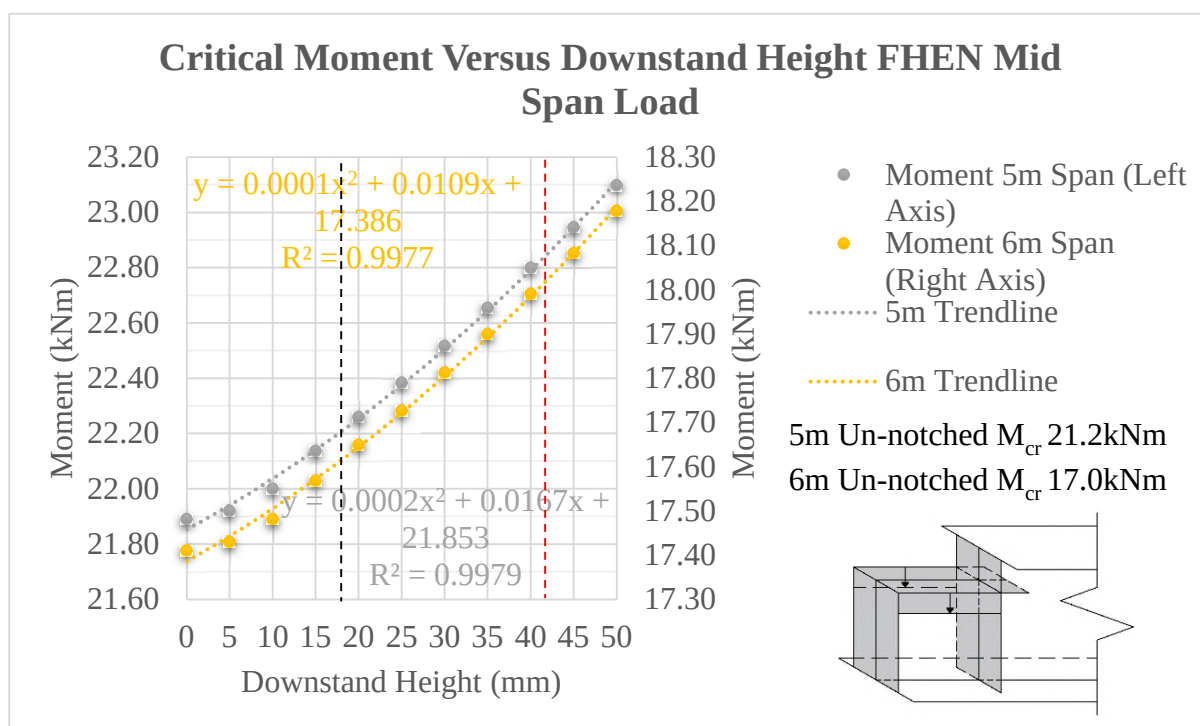


Figure E.16: Critical Moment Versus Downstand Height FHEN Technique MSL Model 2 305x102x25

E.1.2.2 FHHVNTS Technique

Figure E.17 shows the results for the FHHVNTS EPL case. The 5m and 6m spans increased their critical elastic moments by 1.09kNm and 0.63kNm, respectively.

The trend is strong between the two spans and is similar to the 203x133x35 EPL case (**Figure 6.26**). The critical elastic moment increases as the shear centroid approaches and passes the centroid in the notched region. The curve appears parabolic until the coincidence point, at 25mm downstand height, after which the curve is at a constant gradient.

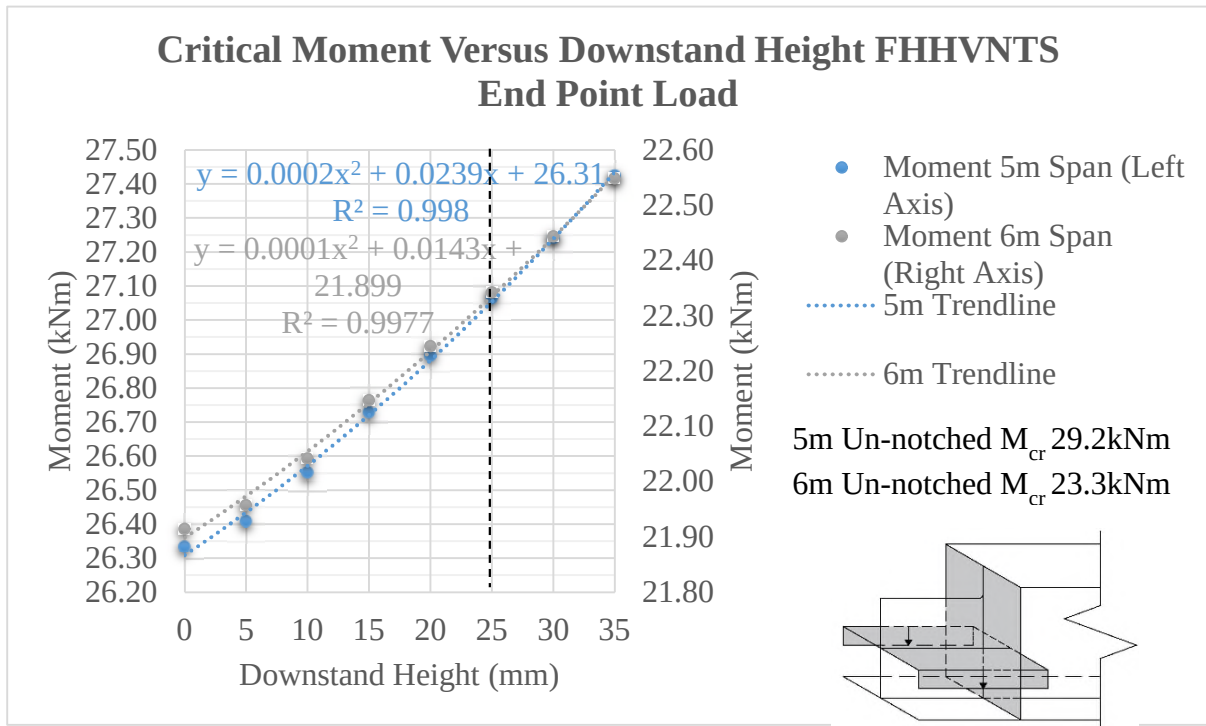


**Figure E.17: Critical Moment Versus Downstand Height FHHVNTS Technique EPL Model 2
305x102x25**

Figure E.18 shows the results for the FHHVNTS MSL case. The 5m and 6m spans increased their critical elastic moment by 0.44kNm and 0.27kNm respectively. These increases are less than the minimum of 0.5kNm set in this report therefore the increase is considered negligible.

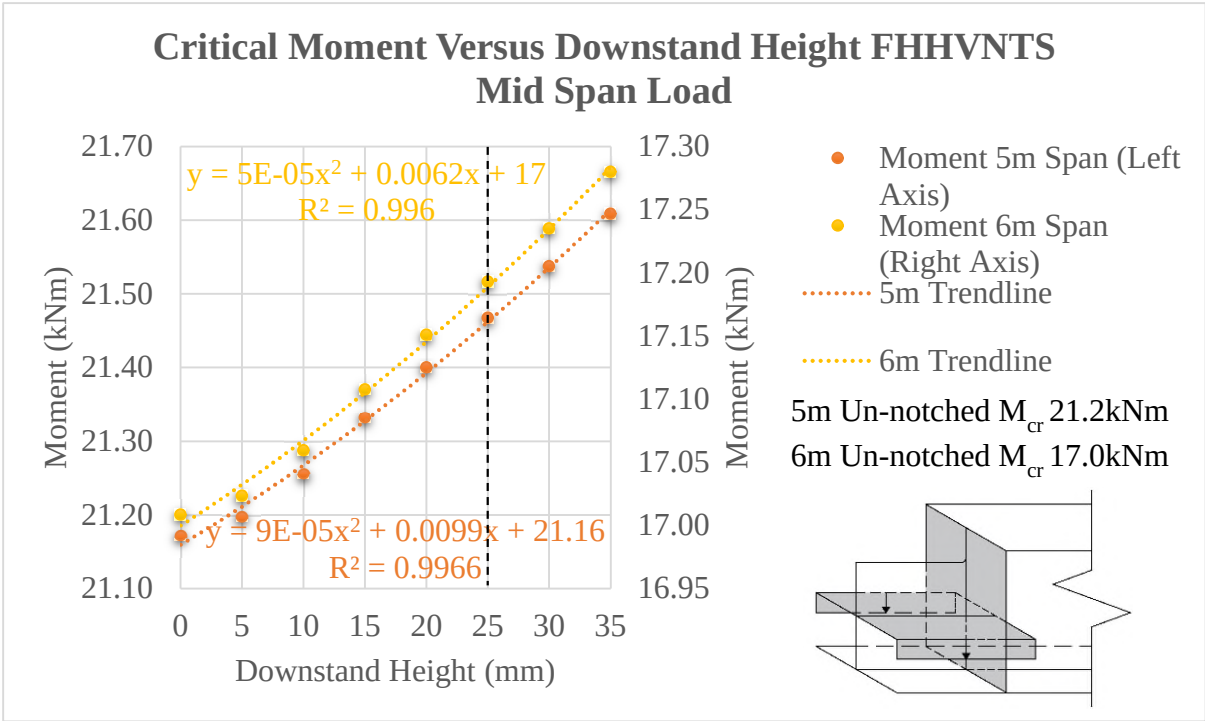


**Figure E.18: Critical Moment Versus Downstand Height FHHVNTS Technique MSL Model 2
305x102x25**

E.1.2.3 FHN Technique

Figure E.19 shows the results for the FHN EPL technique. Since the increase for the 5m (0.26kNm) and 6m (0.16kNm) are less than the minimum of 0.5kNm set in this report therefore the increase is considered negligible.

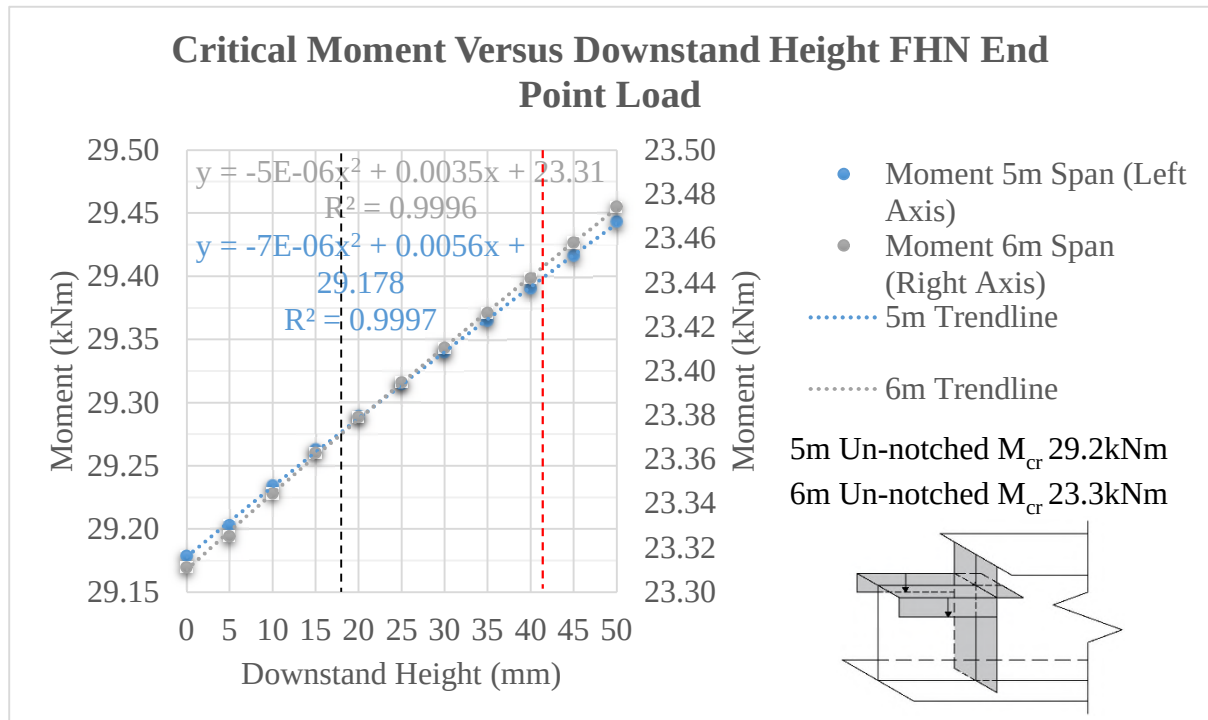


Figure E.19: Critical Moment Versus Downstand Height FHN Technique EPL Model 2 305x102x25

Figure E.20 shows the results for the FHN MSL technique. Since the increase for the 5m (0.21kNm) and 6m (0.14kNm) are less than the minimum of 0.5kNm set in this report therefore the increase is considered negligible.

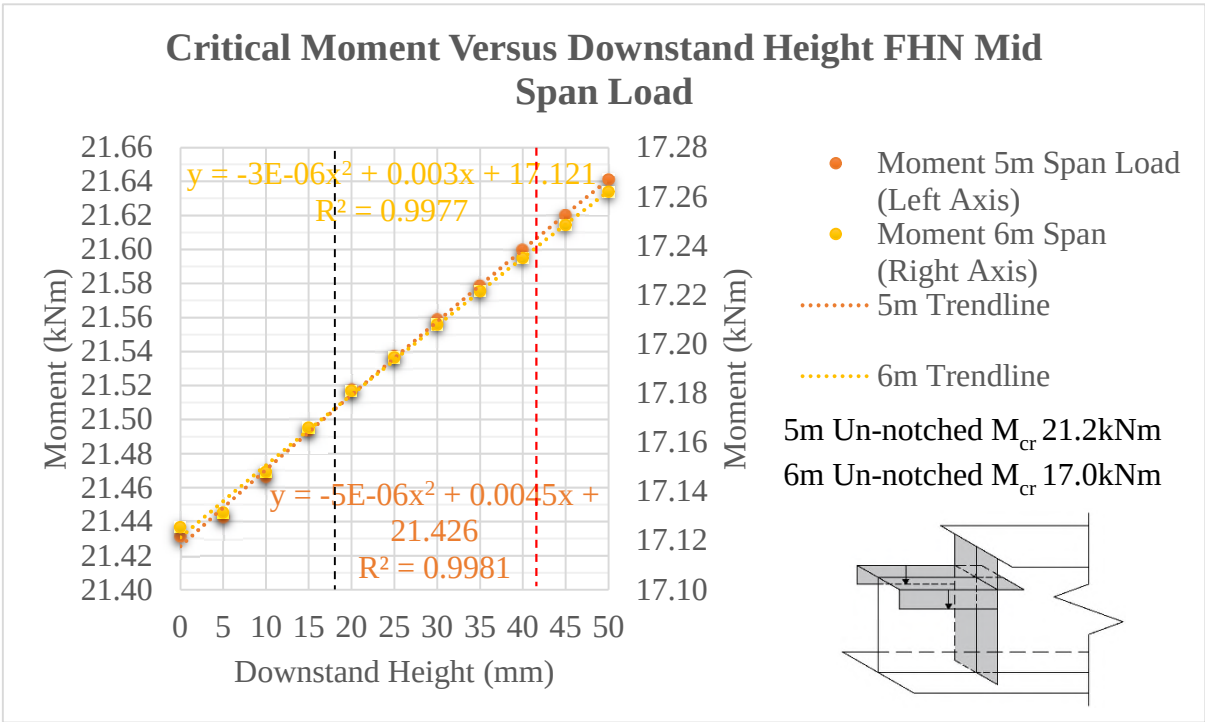


Figure E.20: Critical Moment Versus Downstand Height FHN Technique MSL Model 2 305x102x25

E.1.2.4 FLH Technique

Figure E.21 shows the results for the FLH EPL technique. Since the increase for the 5m (0.31kNm) and 6m (0.19kNm) are less than the minimum of 0.5kNm set in this report therefore the increase is considered negligible.

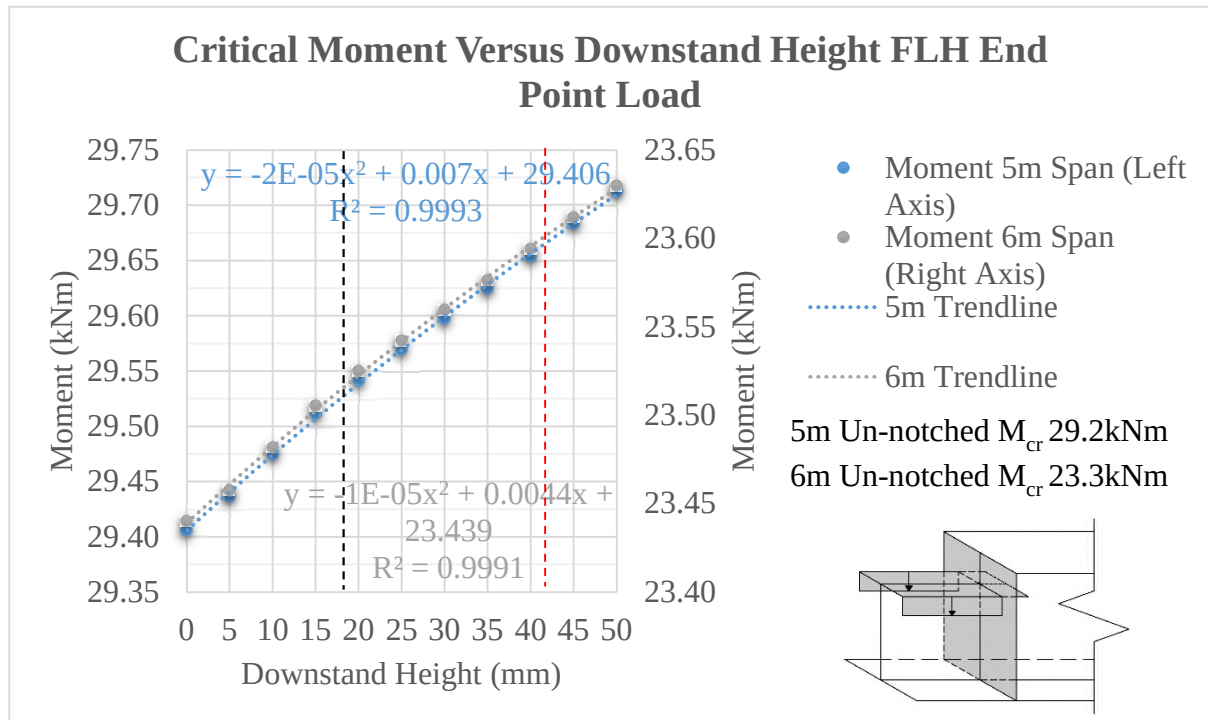


Figure E.21: Critical Moment Versus Downstand Height FLH Technique EPL Model 2 305x102x25

Figure E.22 shows the results for the FLH MSL technique. Since the increase for the 5m (0.26kNm) and 6m (0.17kNm) are less than the minimum of 0.5kNm set in this report therefore the increase is considered negligible.

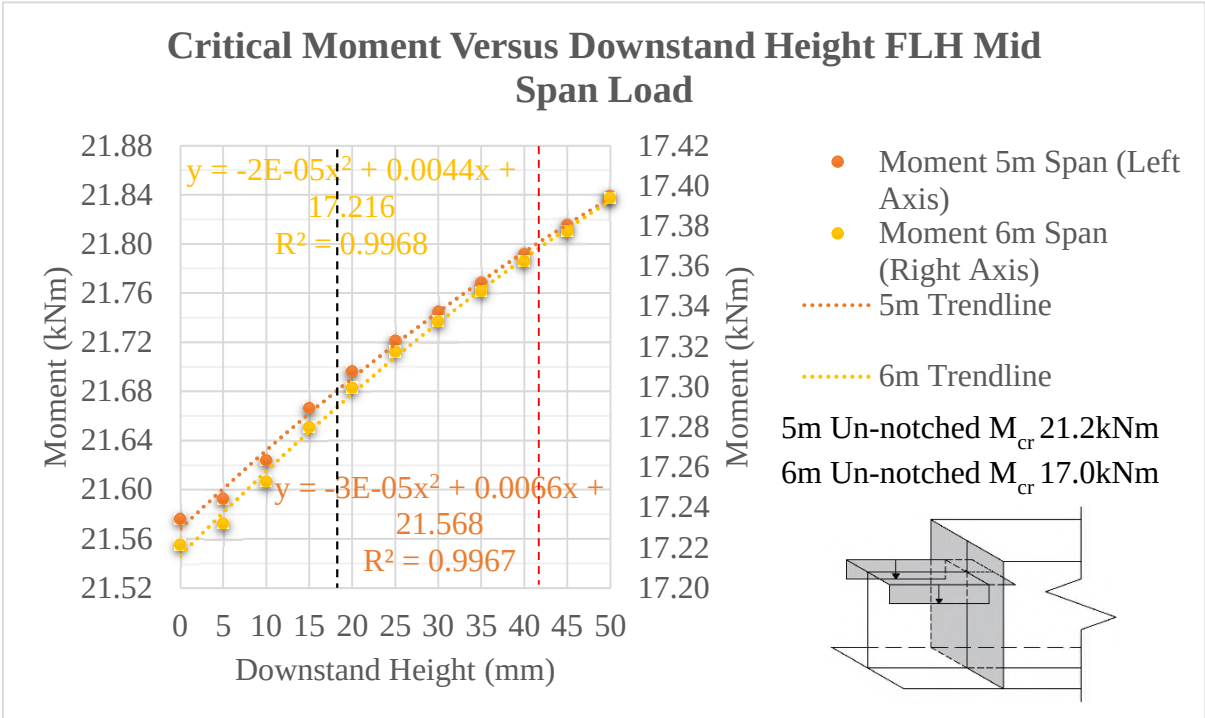


Figure E.22: Critical Moment Versus Downstand Height FLH Technique MSL Model 2 305x102x25

E.1.2.5 FLHFHV Technique

Figure E.23 shows the results of the FLHFHV technique for the EPL case. The section decreased its critical elastic moment moving from the upstands to the no downstand/upstands at stand height of 0mm by 0.92kNm for the 5m span. The introduction of downstands then increased the critical elastic moment by a negligible 0.21kNm. The 6m span also decreased its critical elastic moment (by 0.54kNm) then increased it with the downstands (0.13kNm).

The trends in the FLHFHV technique are unique to this technique. The trend is evident in both models for the 203x133x25 section and the 305x102x25 section.

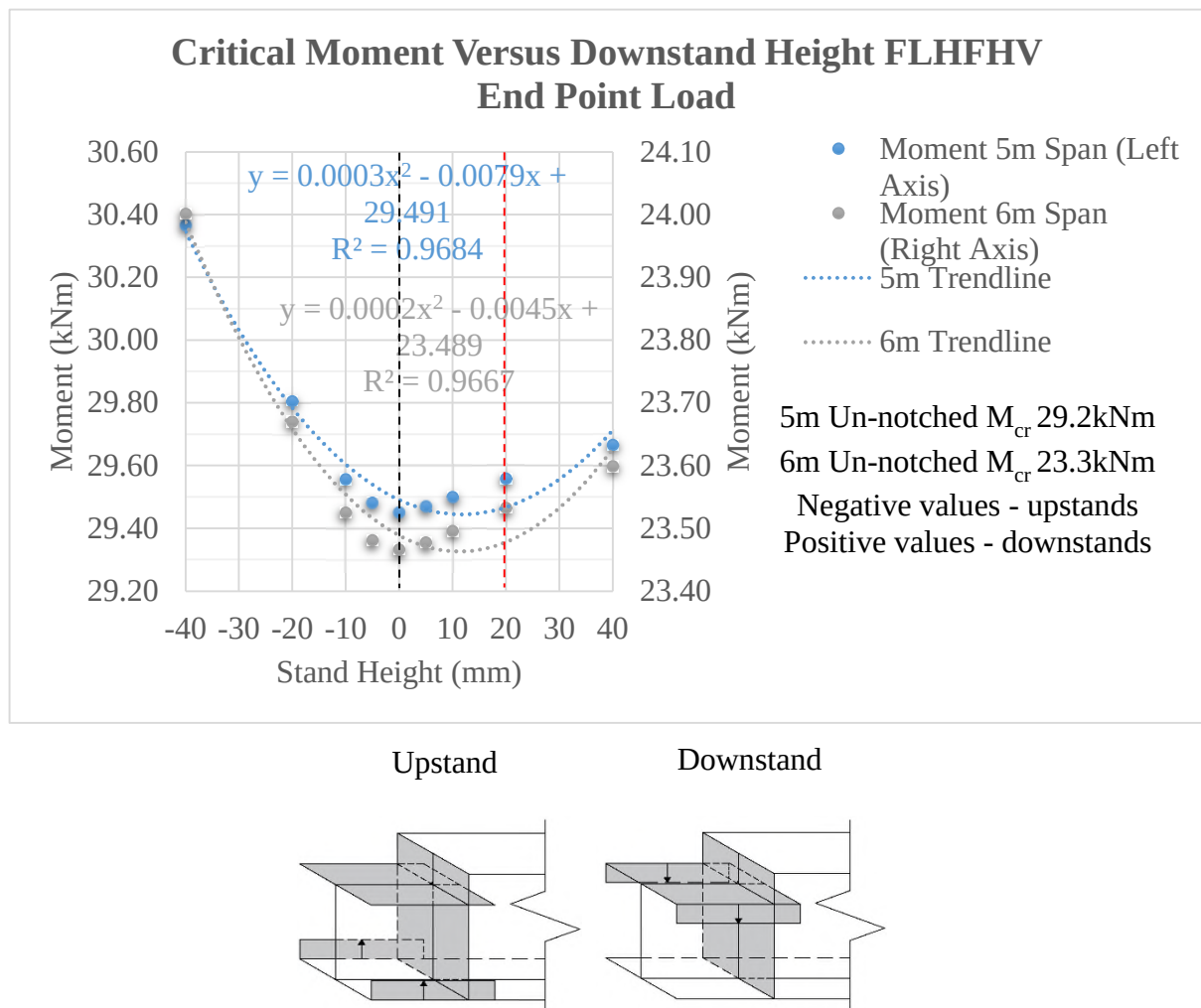


**Figure E.23: Critical Moment Versus Stand Height FLHFHV Technique EPL Model 2
305x102x25**

Figure E.24 shows the results of the FLHFHV technique for the MSL case. The section decreased its critical elastic moment moving from the upstands to the no downstand/upstands at stand height of 0mm by 0.68kNm for the 5m span. The introduction of downstands then increased the critical elastic moment by a negligible 0.18kNm. The 6m span also decreased its critical elastic moment (by 0.44kNm) then increased it with the downstands (0.11kNm). These changes are considered negligible as they are less than the minimum change of 0.5kNm for this report. Again, just as in the EPL case (**Figure E.23**), Model 1 and in both models in the 203x133x25 section, the trends are unique to the FLHFHV technique.

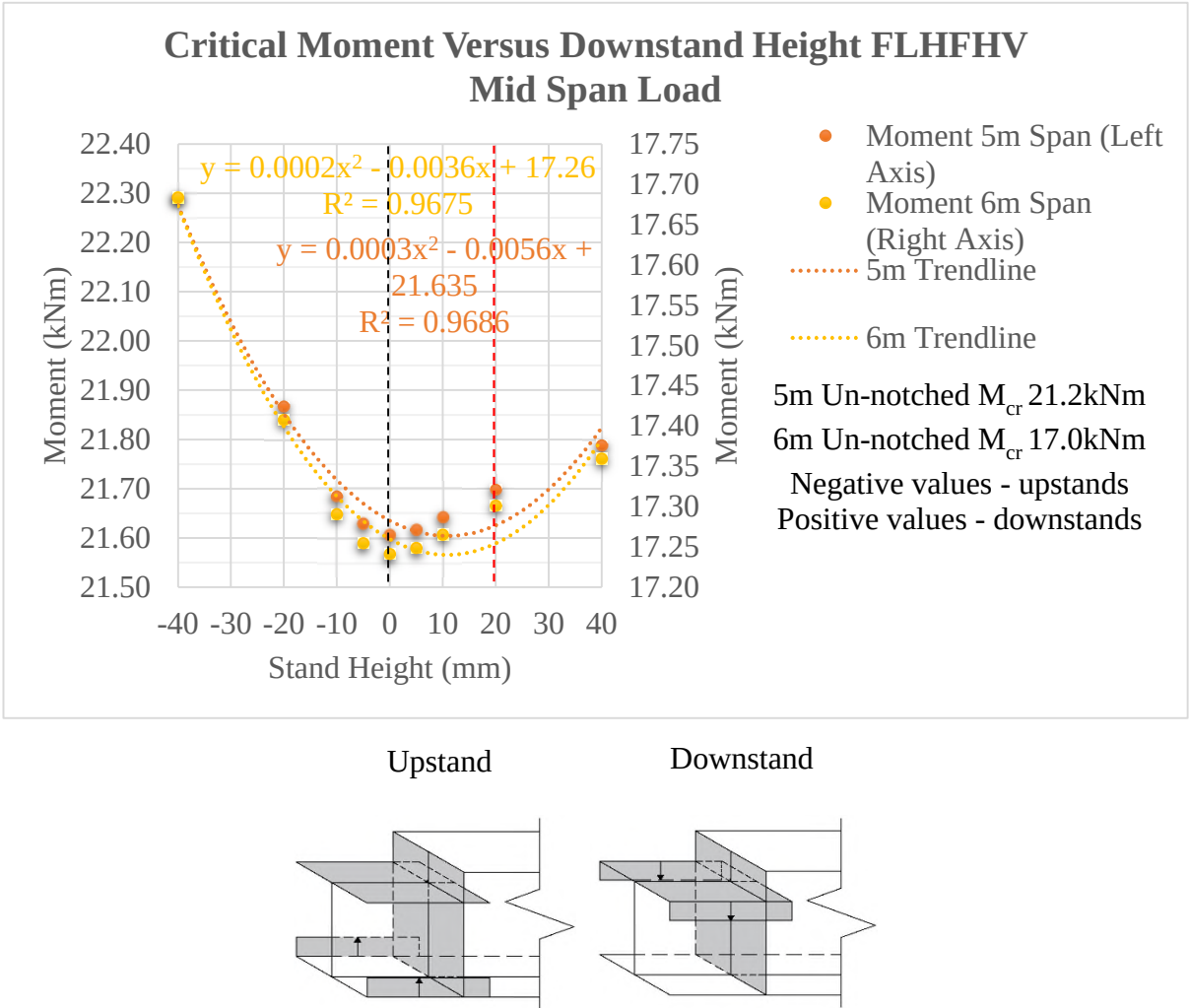


Figure E.24: Critical Moment Versus Stand Height FLHFHV Technique MSL Model 2
305x102x25

Model 2 is similar to Model 1. There is an increase in the critical elastic moment as the shear centre moves further above the centroid. The graphs showing the isolated downstand are not shown since the total change in the critical elastic moment is less than 0.5kNm.

E.2 533 x 210 x 82 Section

E.2.1 Model 1

Figure E.25 shows the 1m span and the 3m span for the EPL case. It also shows the five stiffening techniques and by how much they restore the critical elastic moment to the notched section.

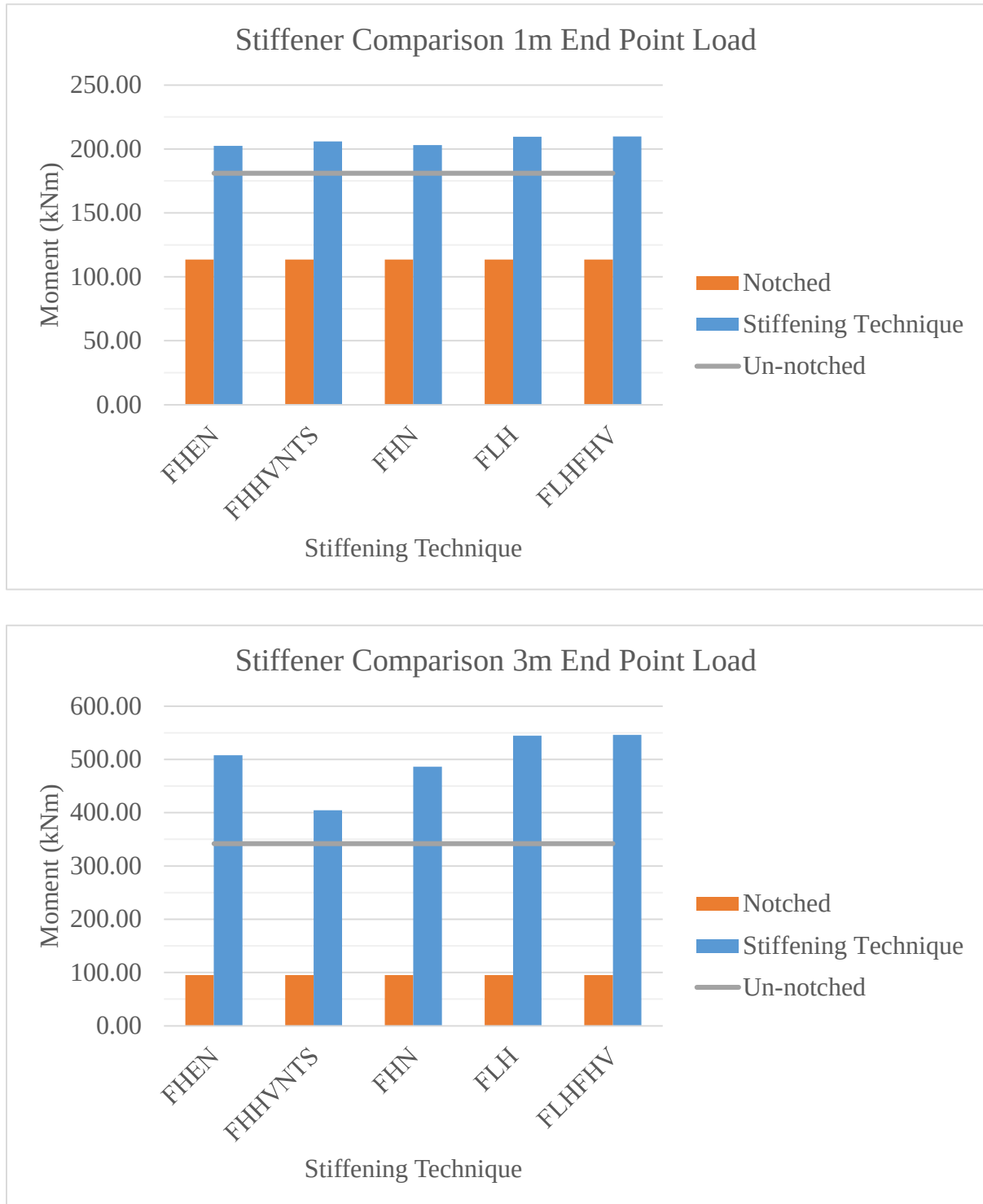


Figure E.25: Stiffener Comparison 1m, 3m EPL 533x210x82

Figure E.26 shows the 1m span and the 3m span for the MSL case. It also shows the five stiffening techniques and by how much they restore the critical elastic moment to the notched section.

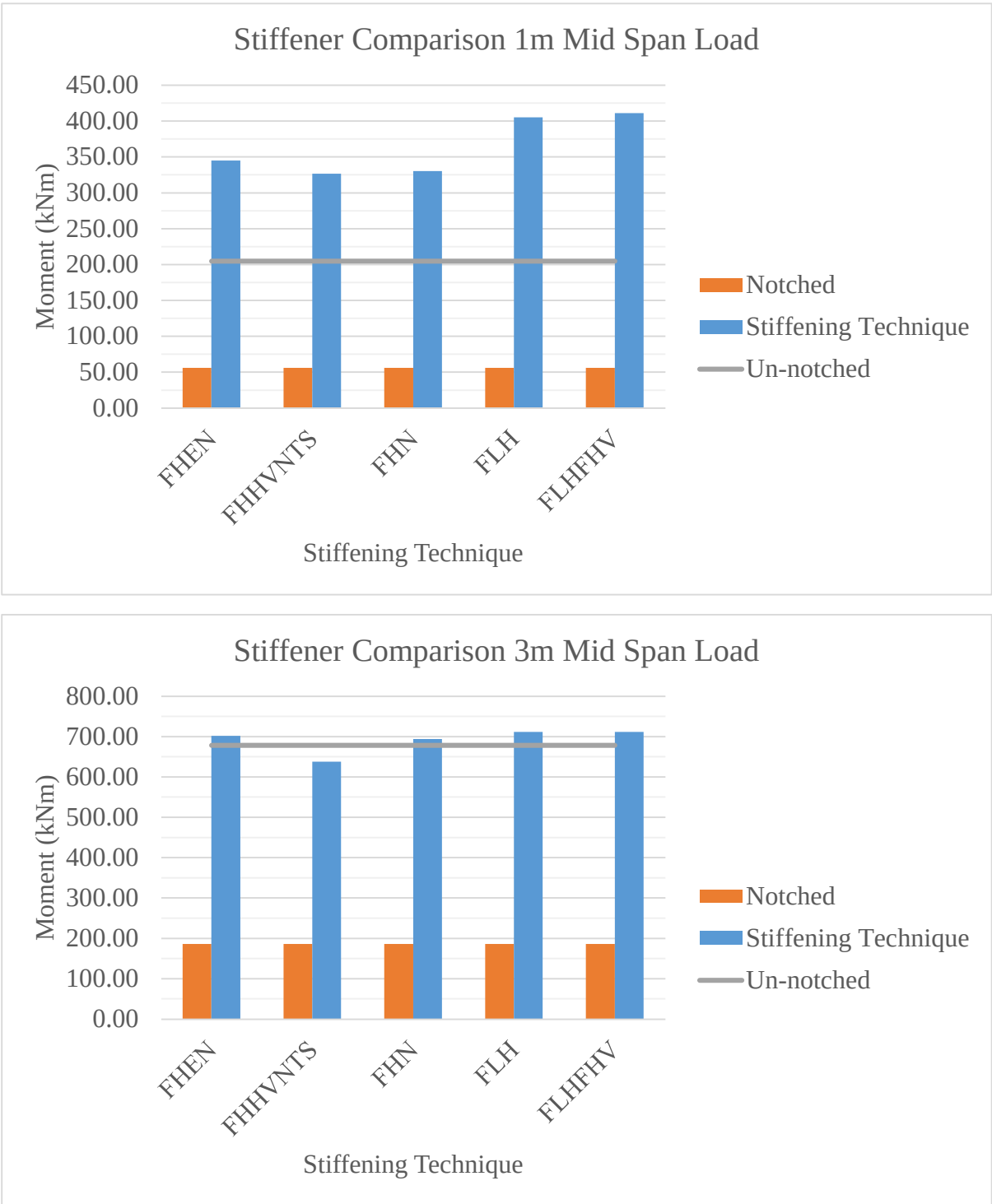


Figure E.26: Stiffener Comparison 1m, 3m MSL 533x210x82

E.2.1.1 FHEN Technique

Figure E.27 shows the critical elastic moment versus downstand height for the 1m and 3m spans for the FHEN EPL case. The 1m span increased its critical elastic moment by 2.21kNm and the 3m span increased by 19.41kNm. Once again, the longer span increased its critical elastic moment more than the shorter span just as in the 203x133x25 and 305x102x25 FHEN EPL cases. This is the first time this trend is evident in both spans. Previously, the trend was evident in the:

- FHEN EPL 203x133x25 case 1m span (**Figure 6.11**)
- FLH MSL 203x133x25 case 1m span (**Figure 6.18**)
- Model 1 FHN 305x102x25 (**Figure E.7** and **Figure E.8**)

However, they were considered outliers because all other cases did not follow the trend whereas in this case the trend is also present in the MSL case (**Figure E.28**). As the shear centre approaches the centroid in the notched region and passes it (at 223mm downstand height), the gradient of the critical elastic moment curve decreases. It is almost as if there is a turning point in the parabolic curve after the coincidence point or the curve approaches an asymptote. It is clear from the graph the curves are initially parabolic then linear after the 223mm downstand.

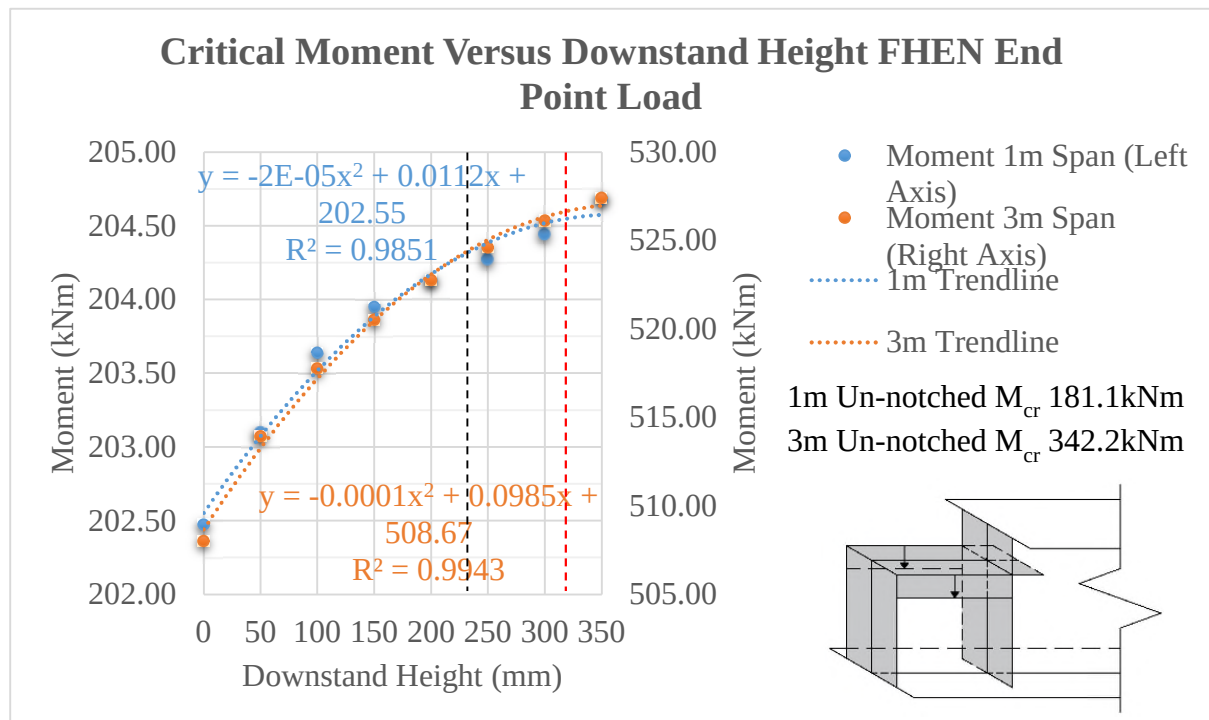


Figure E.27: Critical Moment Versus Downstand Height FHEN Technique EPL Model 1 533x210x82

Figure E.28 shows the critical elastic moment versus downstand height for the MSL FHEN technique. The 1m span increased its critical elastic moment by 51.08kNm and the 3m span by 9.60kNm. Note the shorter span increased its critical elastic moment more. There are some similarities between the:

- 1m span and the 3m span which is similar to the 1m span MSL for the 203x133x25 section (**Figure 6.12**).
- 1m MSL FHEN technique for the 305x102x25 section (**Figure E.4**).

The trends mentioned above were considered outliers for their sections because they were not similar to any other trends across all the techniques. Comparing the EPL (**Figure E.27**) to the MSL (**Figure E.28**): it appears in both cases the critical elastic moment approaches a maximum value or asymptote after the coincidence point at 223mm downstand height.

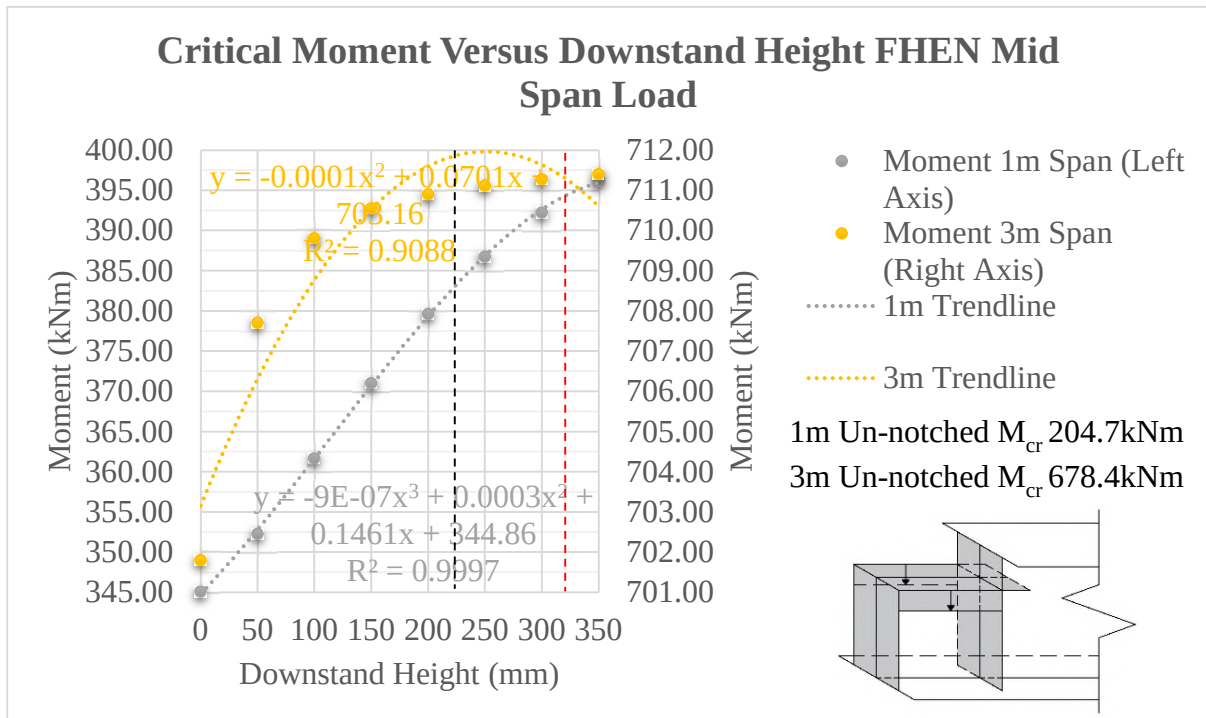


Figure E.28: Critical Moment Versus Downstand Height FHEN Technique MSL Model 1 533x210x82

E.2.1.2 FHHVNTS Technique

Figure E.29 shows the results for the FHHVNTS EPL case. The increase in the critical elastic moment for the 1m and 3m spans are 0.54kNm and 44.78kNm respectively. Unlike the previous tested sections, the longer of the two spans increased its critical elastic moment more for the EPL FHHVNTS technique.

The trend is similar to the 203x133x25 and 305x102x25 EPL and MSL FHHVNTS technique, 3m EPL and MSL case for the FHEN technique. There is an increase in the critical elastic moment with each increase in downstand height and the gradient of the trendline is steeper after the coincidence point of the centroid and shear centre (at 40mm downstand height). The trend also appears parabolic then linear after the 40mm downstand height which indicates the shifting centroid and shear centre influence the critical elastic moment more before the coincidence point for this load case and technique.

It is very interesting to note the three entirely different steel sections have a similar trend, with regards to their increase in critical elastic moment, when applying this stiffening technique. As with the previous sections due to height restrictions in the notched region, the required downstand height to shift the shear centre past the centroid in the un-notched region does not fit.

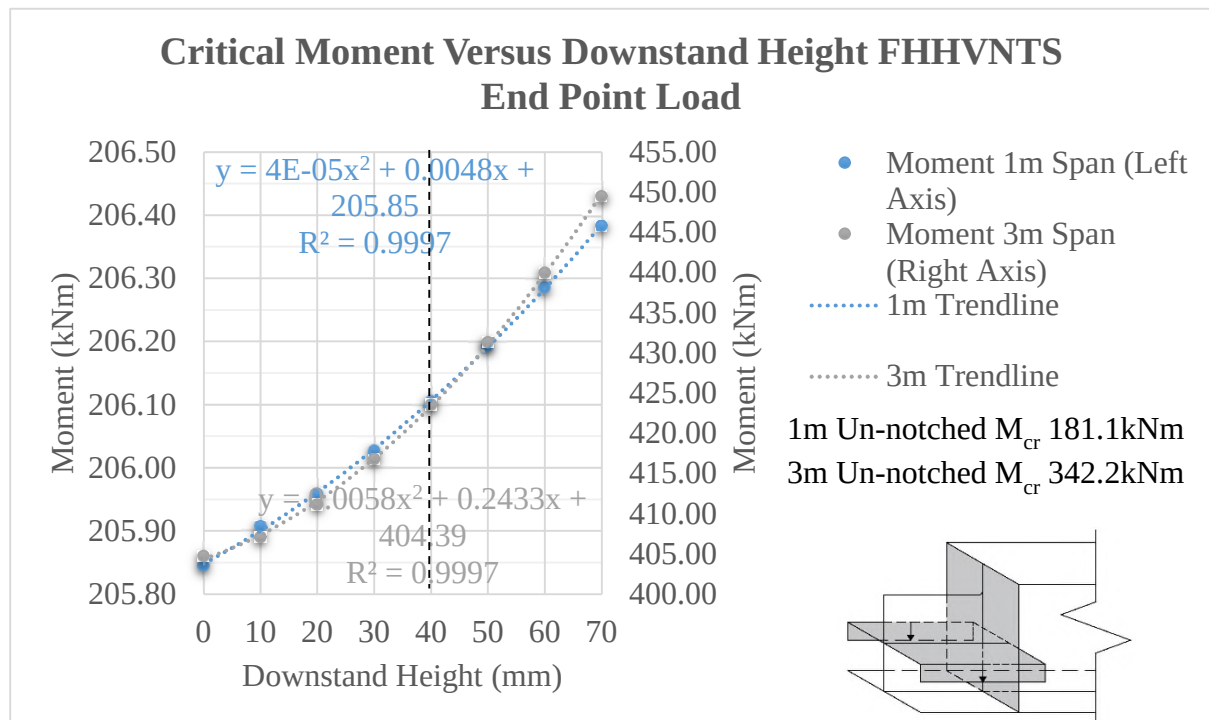


Figure E.29: Critical Moment Versus Downstand Height FHHVNTS Technique EPL Model 1 533x210x82

Figure E.30 shows the results for the FHHVNTS MSL case. The increase in the critical elastic moment for the 1m and 3m spans is 35.38kNm and 44.69kNm respectively. Just as in the EPL case (and unlike the FHHVNTS 203x133x25 and 305x102x25 sections) the longer span increased its critical elastic moment more.

The trend is strong between the two spans and is also similar to the EPL case. The trend is initially parabolic up to the coincidence point of the centroid and shear centre in the notched region (at 40mm downstand height) thereafter it is linear.

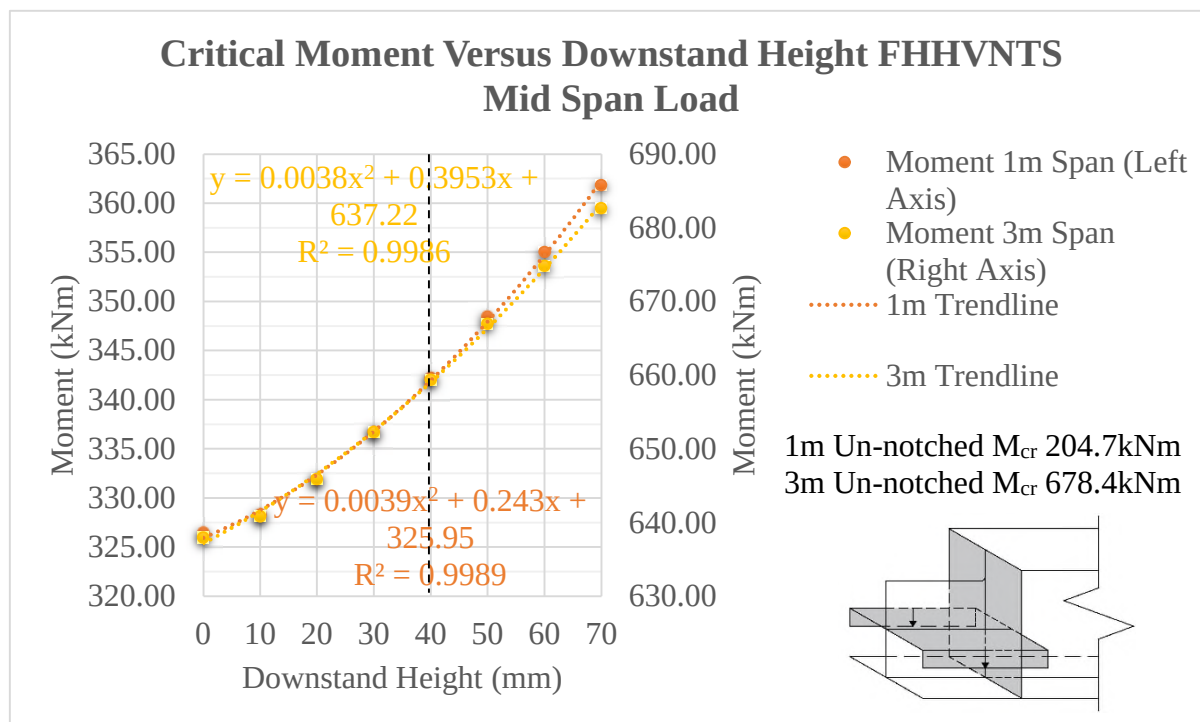


**Figure E.30: Critical Moment Versus Downstand Height FHHVNTS Technique MSL Model 1
533x210x82**

E.2.1.3 FHN Stiffening Technique

Figure E.31 shows the critical elastic moment for the EPL FHN technique. The increase for the 1m and 3m spans are 2.33kNm and 27.79kNm. Just as in the 203x133x25 section, the longer span increased its critical elastic moment more.

There is no similarity between the two sections trends whereas both spans in the 203x133x25 section had a strong trend (**Figure 6.15**). They both increase parabolically but the 1m span approaches a maximum value/asymptote just as in the FHEN technique whereas the 3m span is similar to the FHHVNTS technique because the gradient is ever increasing. Therefore, the two trends behave differently as the shear centre approaches and passes the centroid in the notched region (the coincidence point is at 223mm downstand height). The 1m span appears to approach a maximum value or a turning point/asymptote whereas the 3m span starts at a minimum value and becomes steeper with each increase in downstand height.

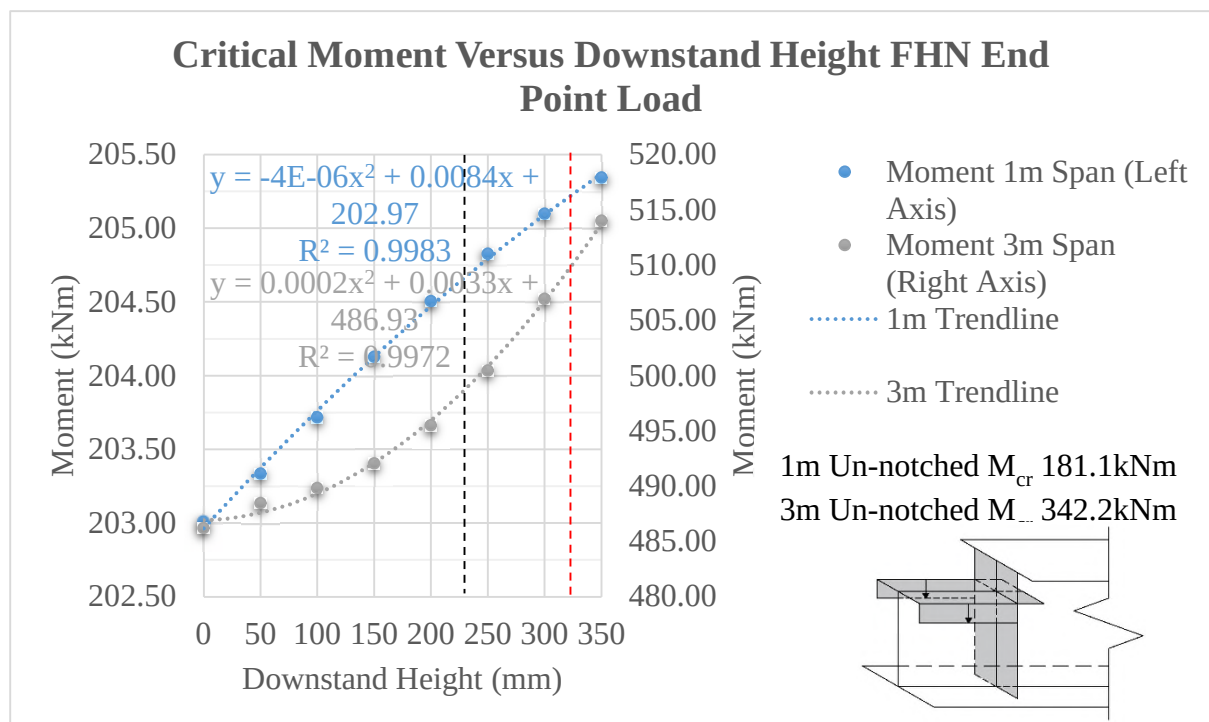


Figure E.31: Critical Moment Versus Downstand Height FHN Technique EPL Model 1 533x210x82

Figure E.32 shows the change in the critical elastic moment for the FHN MSL case. The increase in the critical elastic moment for the 1m and 3m spans are 24.17kNm and 15.52kNm respectively. The shorter span increased its critical elastic moment more just like the 203x133x25 section.

This case is unique because the trends evident in the EPL case have swapped around: the 1m span in the EPL case is the 3m MSL trend and the 3m EPL case is the 1m MSL trend.

The trend in the 305x102x25 section for the FHN MSL case (**Figure E.8**) is similar to the 3m span in **Figure E.32** because both appear to approach a maximum value after the coincidence point of the centroid and shear centre of the notched region (at 223mm downstand height). This is also evident in the FHEN technique for the 533x210x82 section. The 1m span is similar to the FHHVNTS technique as both increase their gradient with each increase in downstand height.

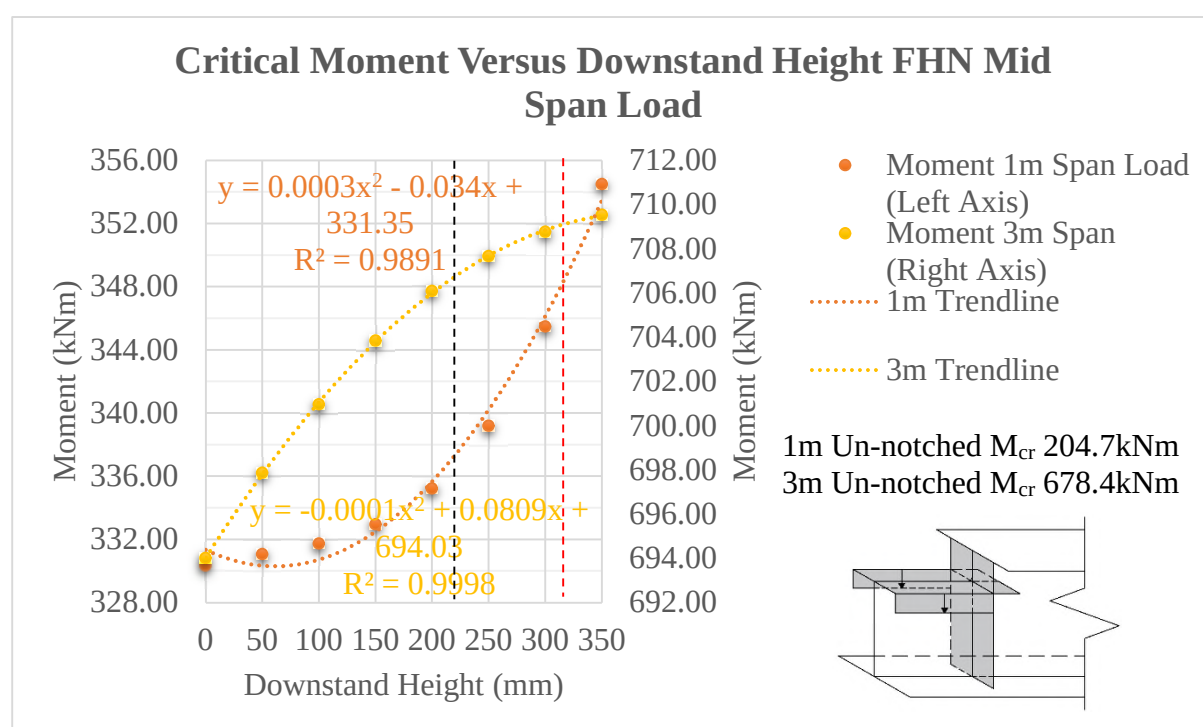


Figure E.32: Critical Moment Versus Downstand Height FHN Technique MSL Model 1 533x210x82

E.2.1.4 FLH Technique

Figure E.33 shows the results for the FLH EPL case. The increase for the 1m span was a negligible 0.25kNm and the 3m span increased marginally by 0.94kNm. The 3m span is similar to the trends in the:

- FLH EPL (**Figure 6.17**) 203x133x25 section.
- 3m FHN EPL (**Figure E.31**) 533x210x82 section
- 1m FHN MSL (**Figure E.32**) 533x210x82 section

As the shear centre approaches and passes the centroid of the notched region, the critical elastic moment increases with the gradient of the curve becoming steeper with each increase. The coincidence point is at a downstand height of 223mm.

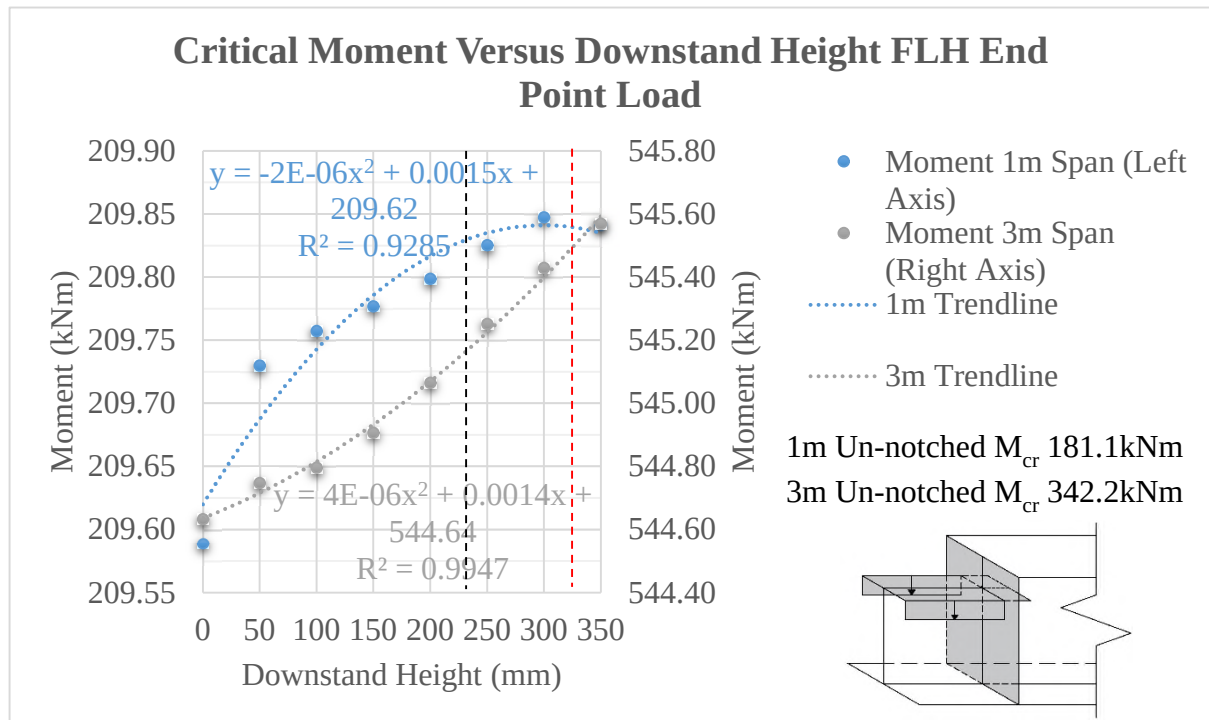


Figure E.33: Critical Moment Versus Downstand Height FLH Technique EPL Model 1 533x210x82

Figure E.34 shows the results for the MSL case for the FLH stiffening technique. The 1m span had an increase of 7.51kNm whereas the 3m span increased by 1.47kNm (the shorter span increased more). The trend is similar to the EPL case but the behaviour in the spans have switched around just as in the FHN technique.

The 3m span has a similar trend to the FLH MSL for the 305x102x25 section: the trendline appears to approach a maximum value or turning point/asymptote after the coincidence of the centroid and shear centre in the notched region. The 1m span is similar to both spans in the EPL FLH (**Figure 6.17**) and the 3m span in the FLH MSL (**Figure 6.18**) case for the 203x133x25 section. The 1m span is also similar to the 3m span in the EPL FLH case (**Figure E.33**) because both start at a minimum value and increase their critical elastic moment with each increase in upstand height. As the shear centre approaches and passes the shear centre in the notched region the gradient becomes ever steeper.

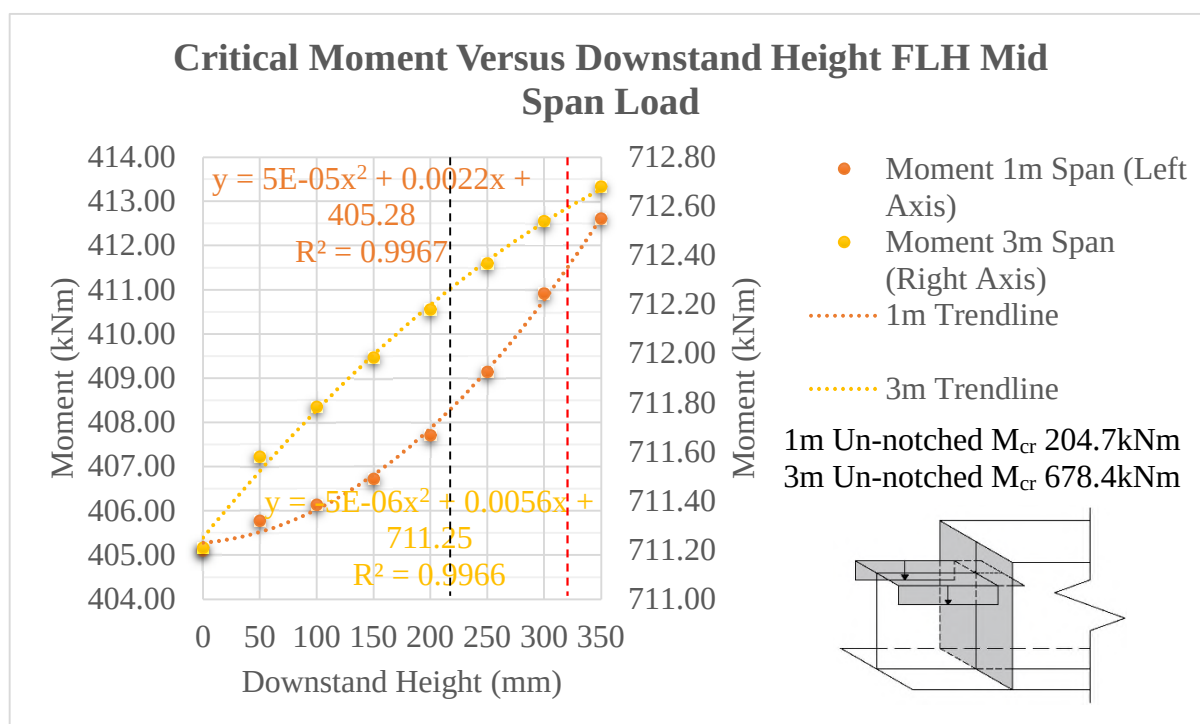


Figure E.34: Critical Moment Versus Downstand Height FLH Technique MSL Model 1 533x210x82

E.2.1.5 FLHFHV Technique

Figure E.35 shows the results of the FLHFHV technique for the EPL case. The section decreased its critical elastic moment moving from the upstands to the no downstand/upstands at stand height of 0mm by 0.81kNm for the 1m span. The introduction of downstands then increased the critical elastic moment by a negligible 0.06kNm.

The 3m span also decreased its critical elastic moment (by 0.71kNm) then increased with the downstands (0.09kNm which is negligible). The behaviour here is unlike any of the trends in the other techniques previously discussed and is only similar to the FLHFHV technique for the 203x133x25 and 305x102x25 sections. As the shear centre approaches the centroid in the notched section by reducing the height of the upstands, the critical elastic moment reduces and only increases, if by a negligible amount, once downstands are introduced. This stiffening technique could be considered an outlier to the other techniques.

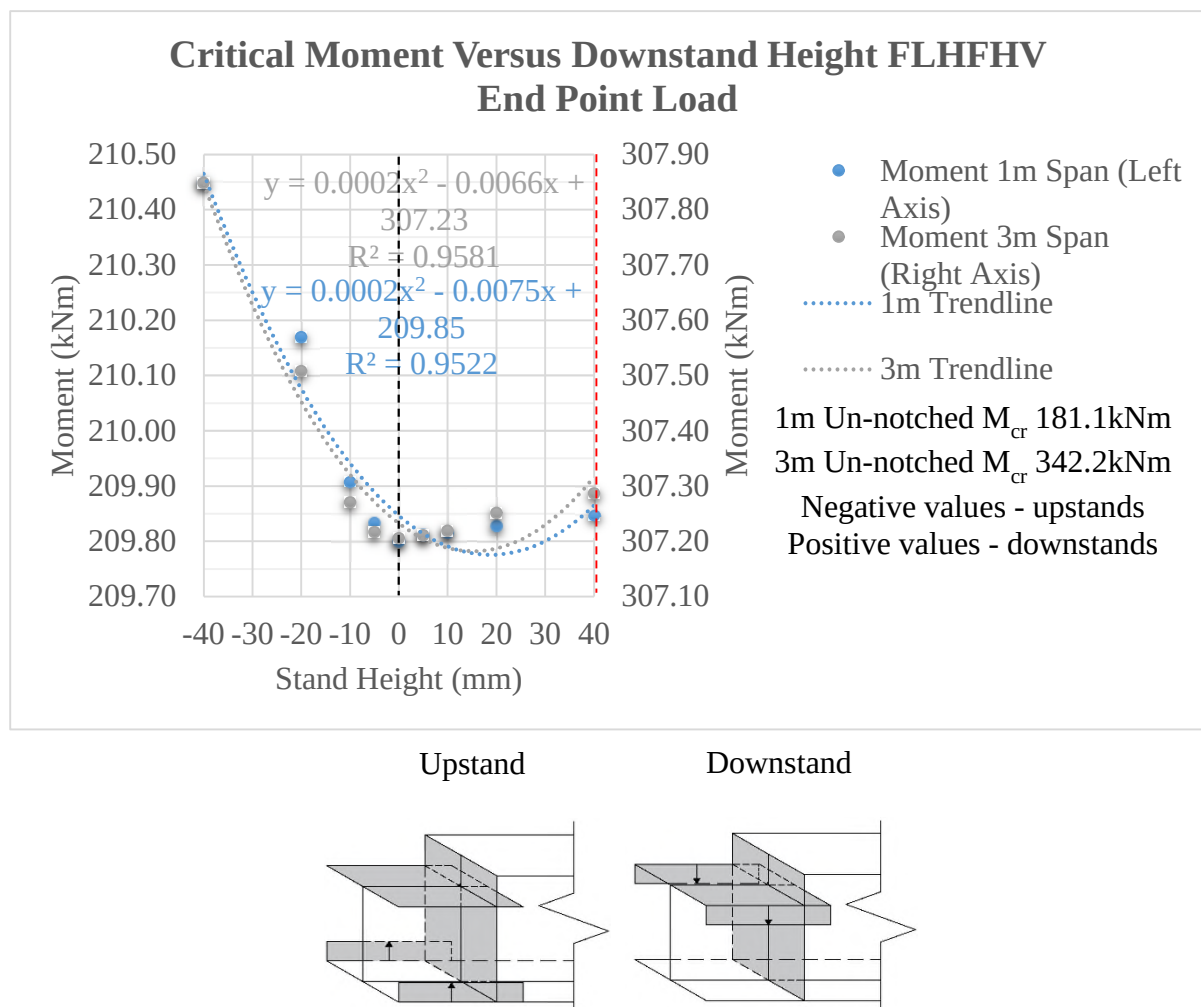


**Figure E.35: Critical Moment Versus Stand Height FLHFHV Technique EPL Model 1
533x210x82**

The trend in the FLHFHV MSL case shown in **Figure E.36** follows the EPL case (**Figure E.35**) but is also unique as it does not follow the trends shown in the previous stiffening techniques except for the FLHFHV technique applied to the 203x133x25 and 305x102x25 sections. There was also a reduction in the moment (2.83kNm) moving from the upstand to no upstand/downstand case then an increase of 0.61kNm for the 1m span case. Similarly, the 3m span reduced its critical elastic moment by 0.83kNm then increased it by a negligible 0.13kNm.

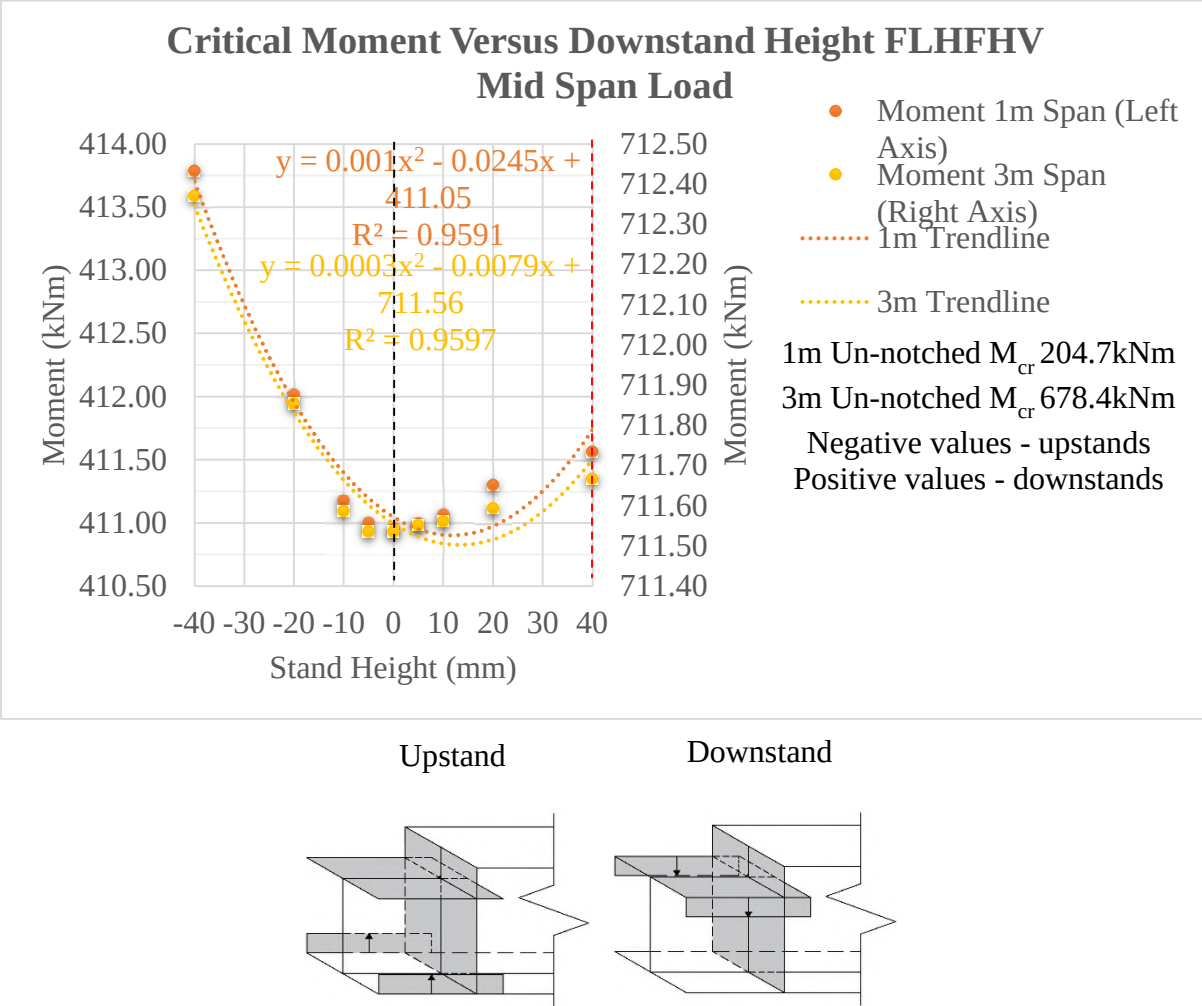


Figure E.36: Critical Moment Versus Stand Height FLHFHV Technique MSL Model 1 533x210x82

Just as in the 203x133x25 section, by isolating the downstand case it is clear the trend when the shear centre is above the centroid is similar to all the other sections. That is: there is an increase in the critical elastic moment as the shear centre moves further above the centroid. The graphs showing the isolated downstand are not shown since the total change in the critical elastic moment is less than 0.5kNm for the EPL case and only the 3m MSL has a significant increase.

E.2.2 Model 2

Figure E.37 shows the 5m span and the 6m span for the EPL case. It also shows the five stiffening techniques and by how much they restore the critical elastic moment to the notched section.

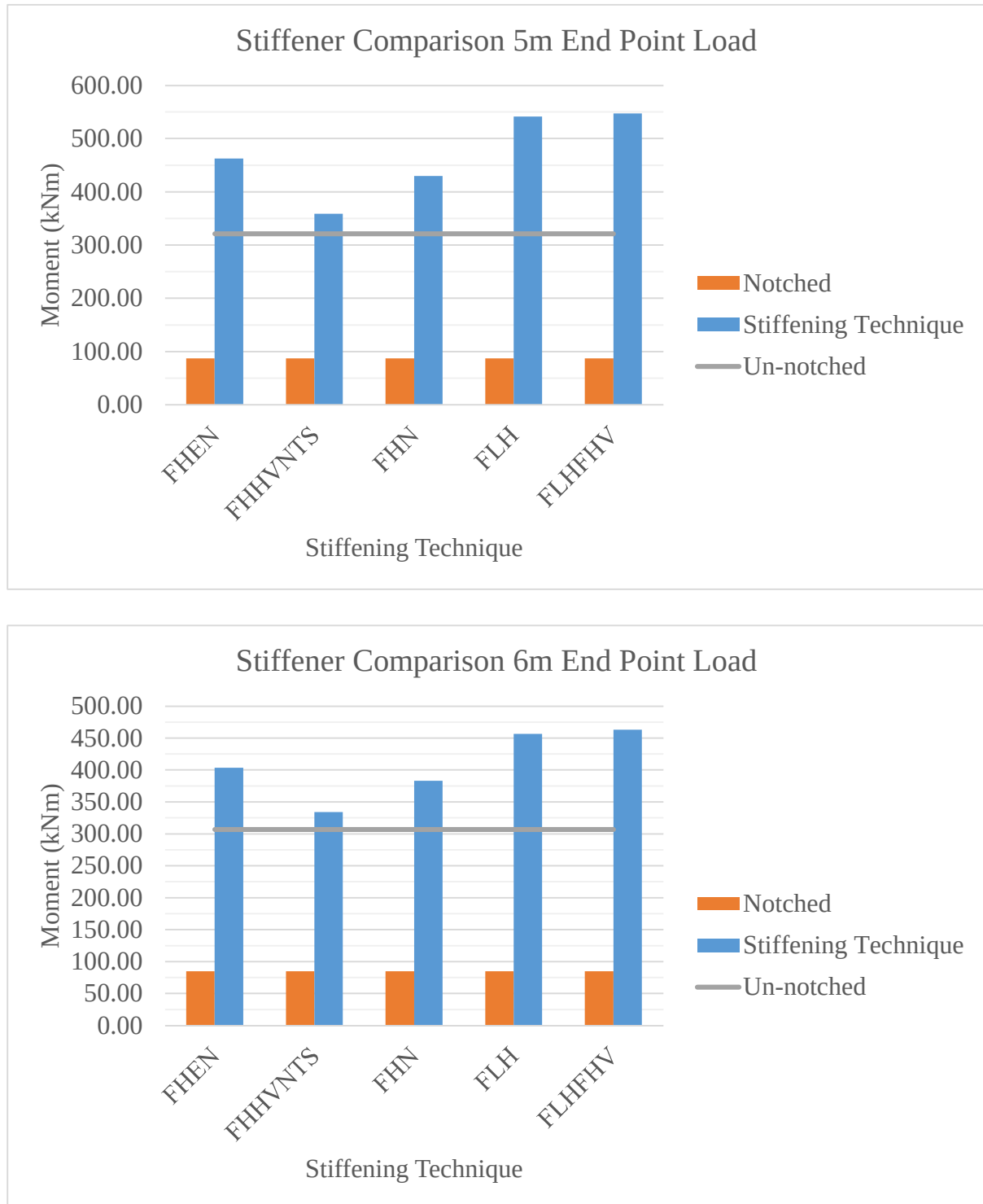


Figure E.37: Stiffener Comparison 5m, 6m EPL 533x210x82

Figure E.38 shows the 5m span and the 6m span for the MSL case. It also shows the five stiffening techniques and by how much they restore the critical elastic moment to the notched section.

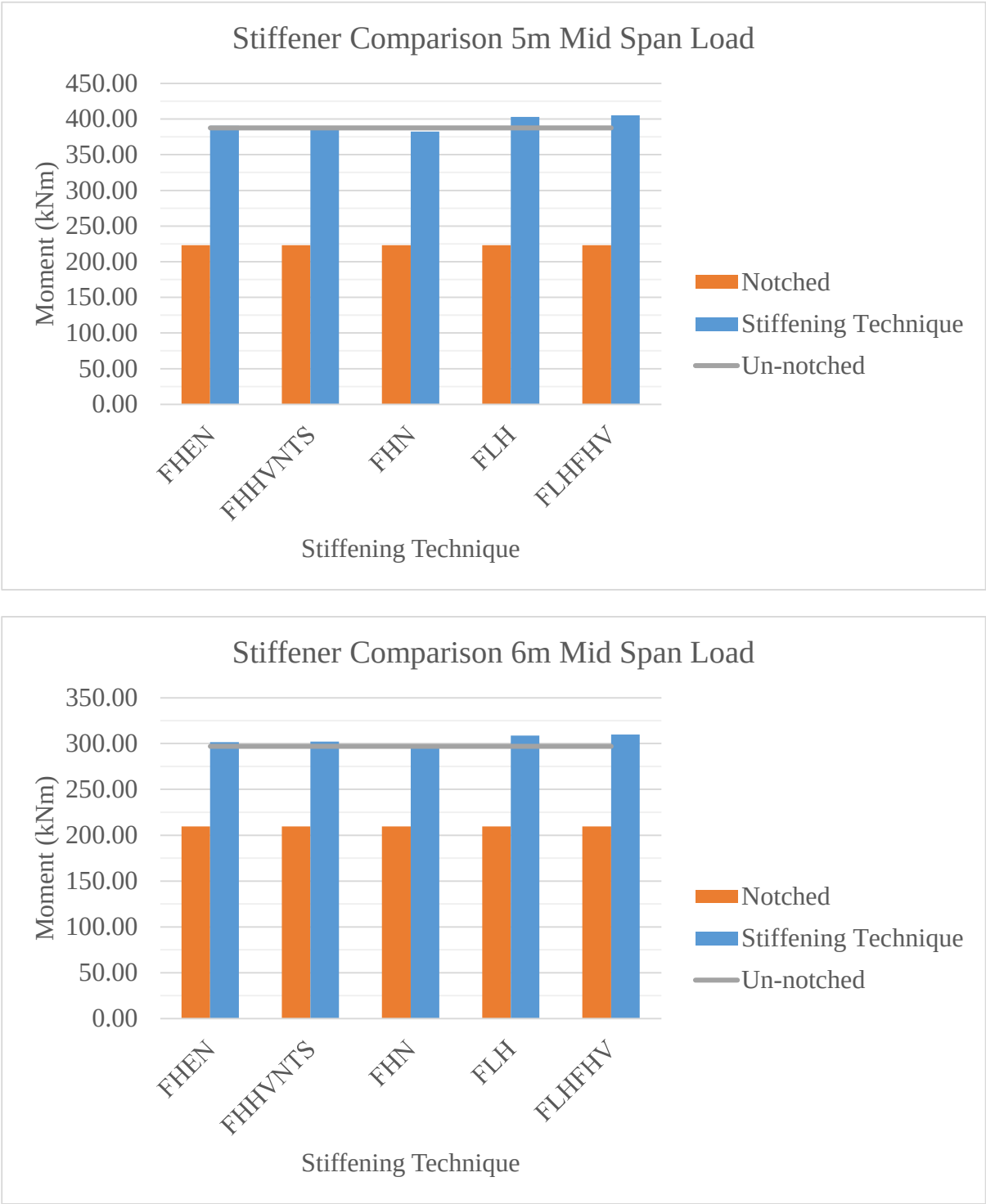


Figure E.38: Stiffener Comparison 5m, 6m MSL 533x210x82

E.2.2.1 FHEN Technique

Figure E.39 shows the critical elastic moment versus downstand height for the FHEN EPL case. The change in moment for the 5m and 6m spans are 76.11kNm and 97.11kNm. As with most of the FHEN cases: the longer span increased its critical elastic moment more. The critical elastic moment increases with each increase in downstand height but appears to approach a maximum value of around 305kNm (for the 5m span) and 277kNm (for the 6m span). The previous two sections (203x133x25 and 305x102x25) do not have this trend.

The trendline is best represented by a parabola but appears to be split into two parts: parabolic before the 223mm downstand and a straight line after it because the curves are approaching an asymptote. This is interesting because the 223mm downstand is the height at which the shear centre and centroid coincide.

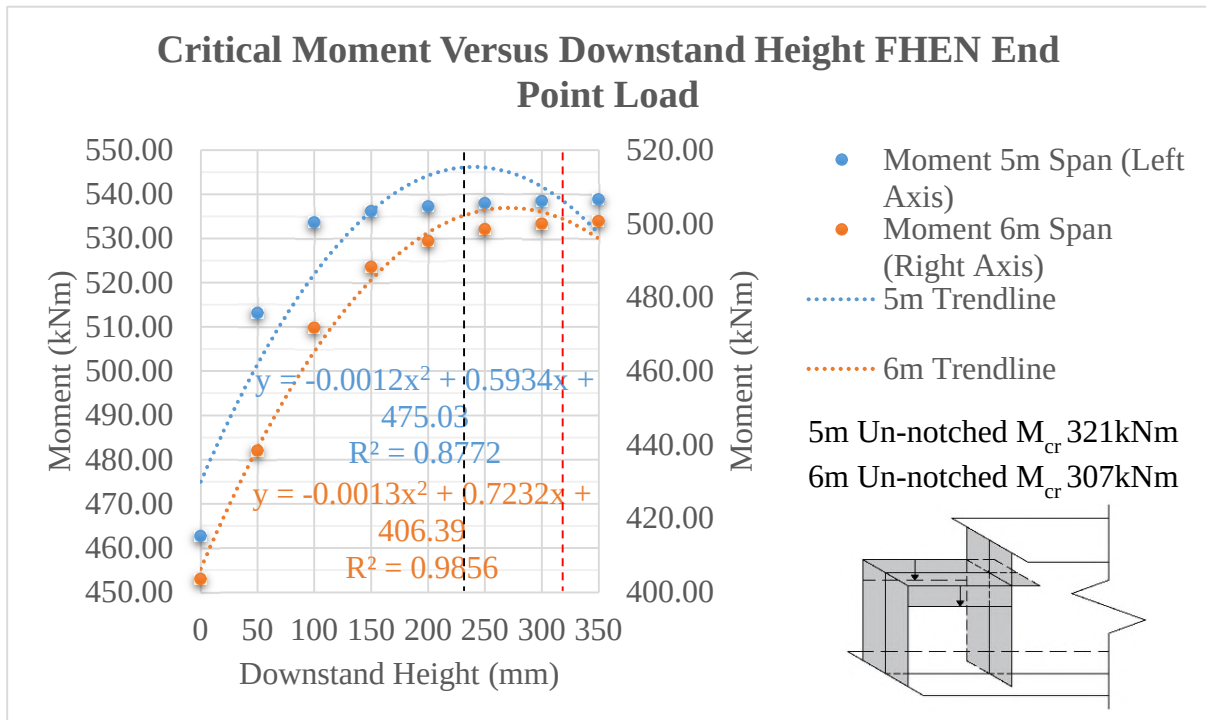


Figure E.39: Critical Moment Versus Downstand Height FHEN Technique EPL Model 2 533x210x82

Figure E.40 shows the results for the FHEN MSL case. There was an increase in the critical elastic moment of 76.31kNm and 49.45kNm for the 5m and 6m spans respectively. Just as in the EPL case, the longer span increased its critical elastic moment more.

The results are similar to the EPL case in **Figure E.39**. However, the results are different to the trends when the FHEN technique is applied to the 203x133x25 (**Figure 6.25**) and 305x102x25 (**Figure E.16**) sections: their trend is slightly parabolic starting from a minimum value whereas the trend for the MSL case for Model 2 appears to approach a maximum value (or asymptote) after the 223mm downstand. This is similar to the:

- 1m span MSL for the 203x133x25 section (**Figure 6.12**).
- 1m span for the MSL FHEN technique for the 305x102x25 section (**Figure E.4**).

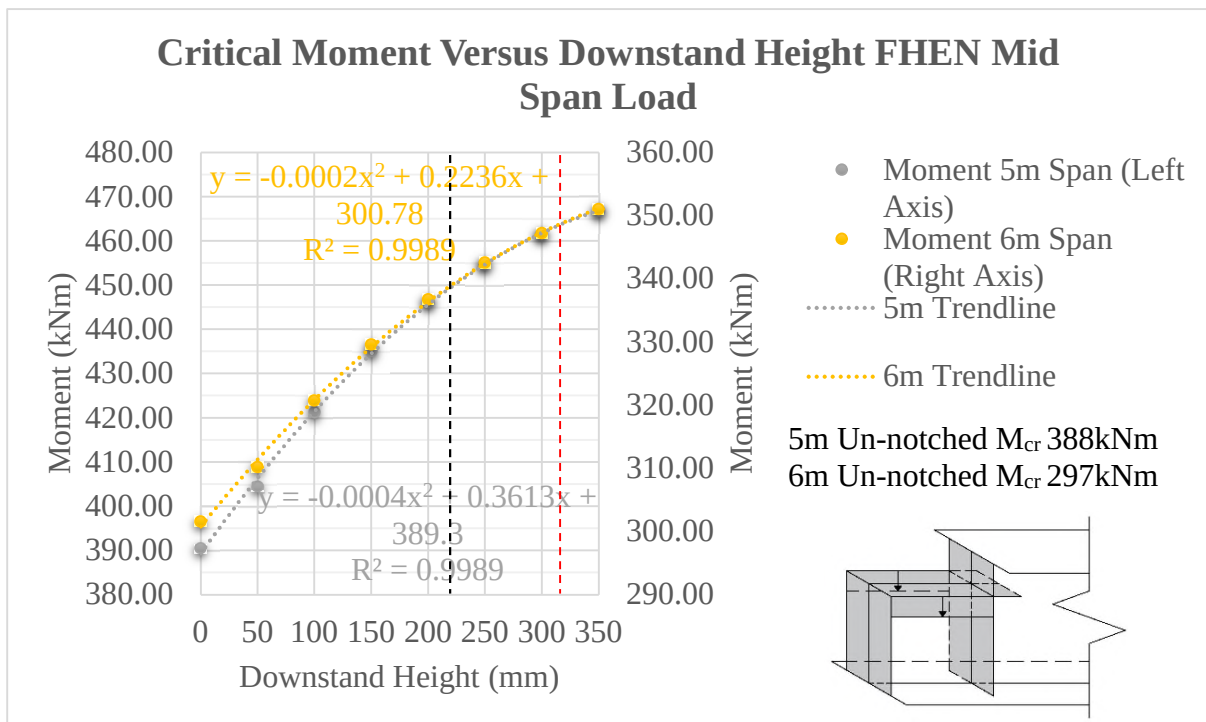


Figure E.40: Critical Moment Versus Downstand Height FHEN Technique MSL Model 2 533x210x82

E.2.2.2 FHHVNTS Technique

Figure E.41 shows the results for the FHHVNTS EPL case. The 5m span increased its critical elastic moment by 36.94kNm and the 6m span by 30.38kNm which continues the trend the shorter span increases its critical elastic moment more for the FHHVNTS technique.

The trend is strong between the two spans and the results are consistent with the 203x133x25 (**Figure 6.26**) and 305x102x25 (**Figure E.17**) sections. The increase in the critical elastic moment is initially parabolic with the gradient increasing with each increase in downstand height up till the 40mm downstand height (the coincidence point of the centroid and shear centre). After this downstand height the gradient appears constant. This is interesting because across three entirely different sections with completely different loading and span configurations the trend is the same.

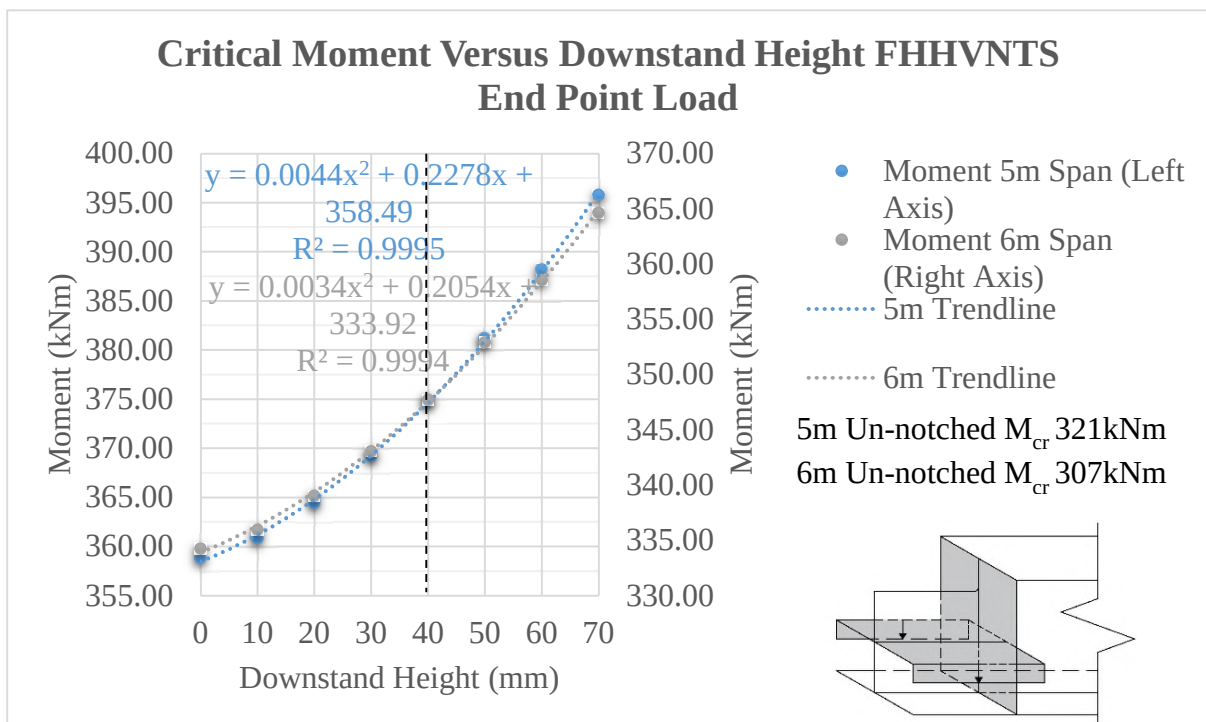


**Figure E.41: Critical Moment Versus Downstand Height FHHVNTS Technique EPL Model 2
533x210x82**

Figure E.42 shows the results for the MSL case. There was an increase of 11.66kNm for the 5m span and 6.93kNm for the 6m span. Once again, the shorter span increased its critical elastic moment more than the longer span.

The trend is again initially parabolic with the gradient of the trendline increasing with each increase in downstand height up till the 40mm downstand height (the coincidence point of the centroid and shear centre of the notched region.). After the 40mm downstand height the gradient is almost constant. This is similar to the EPL case (**Figure E.41**) and the 203x133x25 (**Figure 6.27**) and the 305x102x25 (**Figure E.18**) sections.

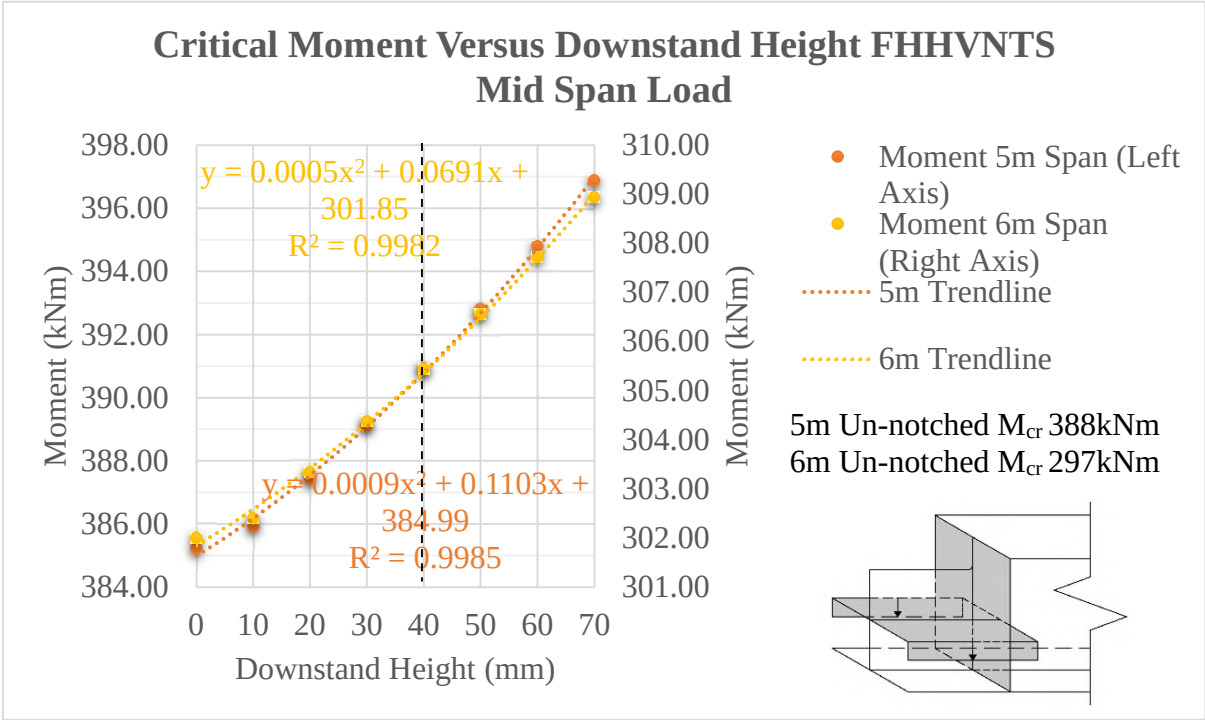


Figure E.42: Critical Moment Versus Downstand Height FHHVNTS Technique MSL Model 2 533x210x82

E.2.2.3 FHN Technique

Figure E.43 shows the results for the FHN EPL case. There were increases of 75.65kNm and 53.62kNm for the 5m and 6m spans respectively and the trend of the shorter span increasing its critical elastic moment more than the longer span continues.

The slightly parabolic trendline is similar to the 203x133x25 case (**Figure 6.28**) and the FHHVNTS technique. It is clear there is a change in gradient after the 223mm downstand height as the shear centre approaches and passes the centroid in the notched region. The trendline is initially parabolic till the 223mm downstand thereafter it is linear.

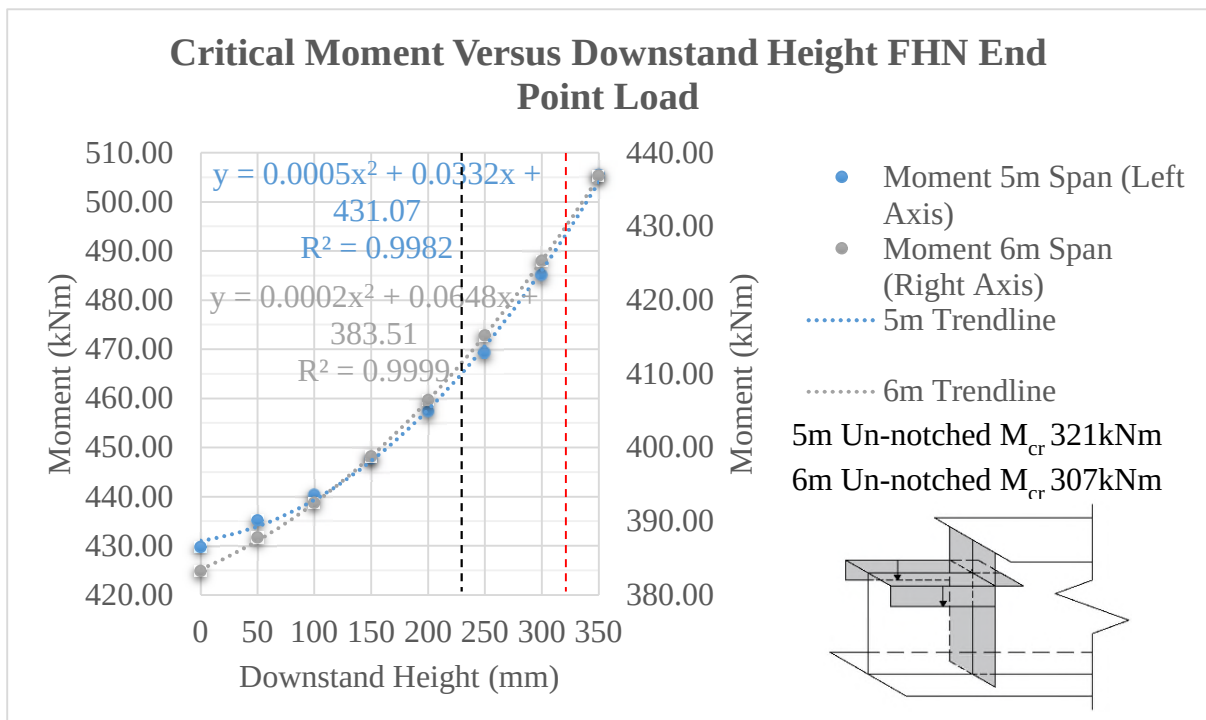


Figure E.43: Critical Moment Versus Downstand Height FHN Technique EPL Model 2 533x210x82

Figure E.44 shows the change in the critical elastic moment for the FHN MSL case. There was an increase of 22.43kNm and 14.22kNm for the 5m and 6m spans respectively showing again the shorter span increased its critical elastic moment more than the longer span.

The trend is similar to the EPL and the FHHVNTS technique. The trend is initially parabolic till the coincidence of the centroid and shear centre in the notched region at a downstand height of 223mm. Thereafter the trend appears linear.

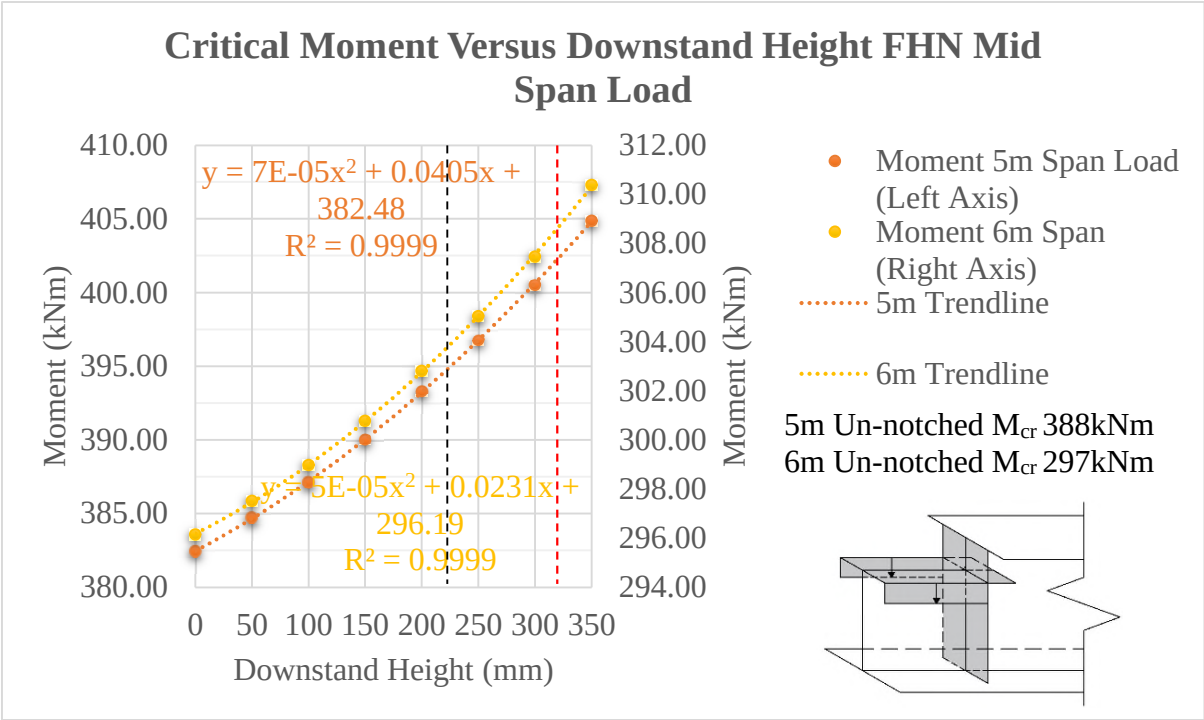


Figure E.44: Critical Moment Versus Downstand Height FHN Technique MSL Model 2 533x210x82

E.2.2.4 FLH Technique

Figure E.45 shows the results for the FLH EPL case. There was an increase of 16.73kNm and 26.89kNm for the 5m and 6m spans respectively. The longer span increased its critical elastic more than the shorter span.

The trend is strong between the two spans. The gradient of the curve decreases with each increase in upstand height, so it appears the curve is approaching a maximum value, turning point or asymptote. This occurs as the shear centre approaches and passes the centroid in the notched region.

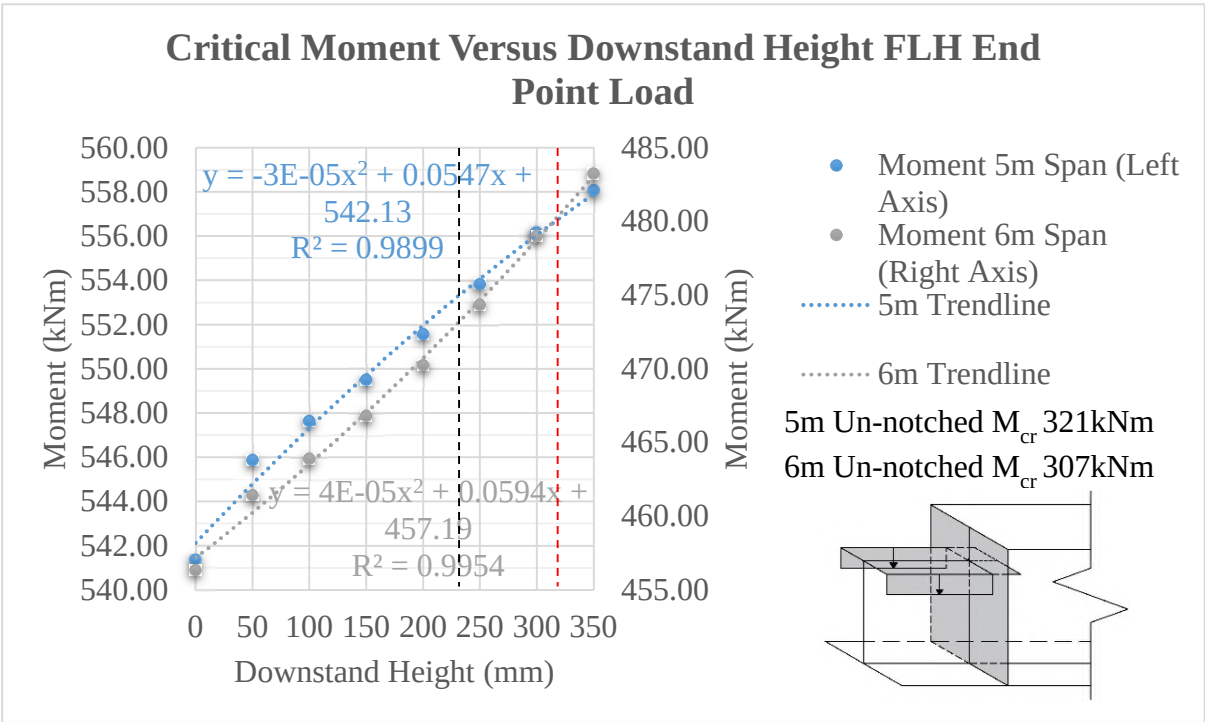


Figure E.45: Critical Moment Versus Downstand Height FLH Technique EPL Model 2 533x210x82

Figure E.46 shows the results for the FLH MSL case. There was an increase of 22.14kNm and 15.52kNm for the 5m and 6m spans. Note the shorter span increased its critical elastic moment more.

The trend is similar to the EPL case in that there is an increase in the critical elastic moment with each increase in downstand height, but the gradient increases slightly after the 223mm downstand height.

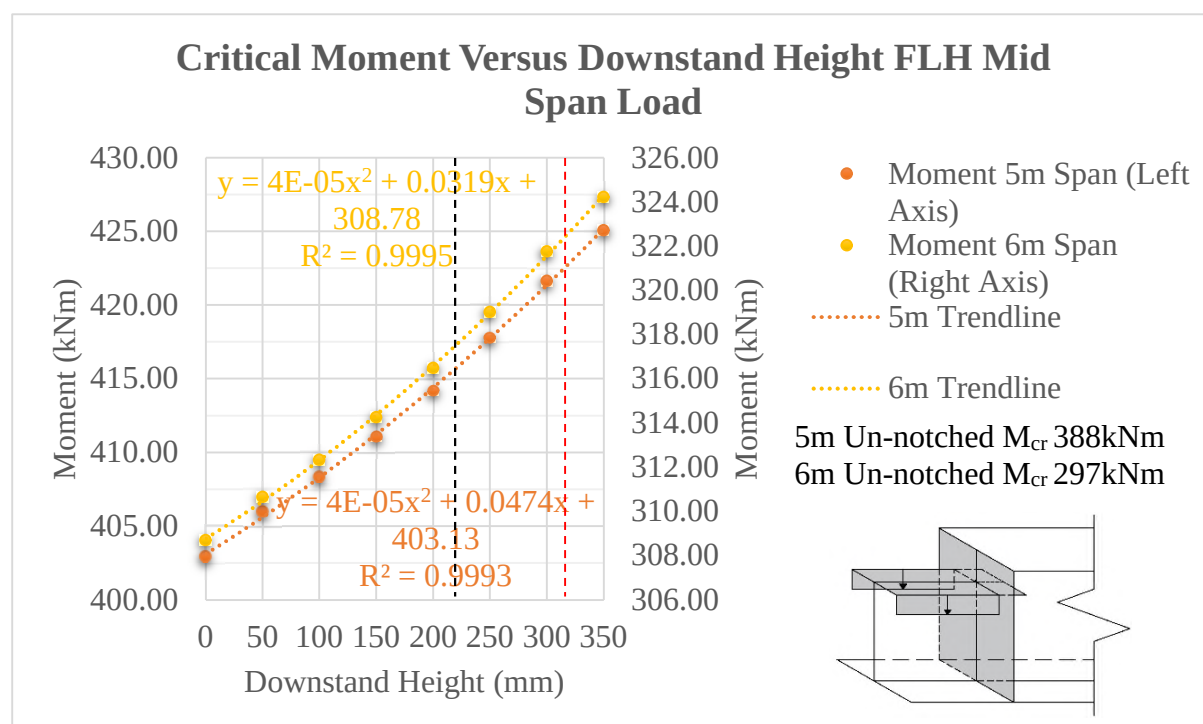


Figure E.46: Critical Moment Versus Downstand Height FLH Technique MSL Model 2
533x210x82

E.2.2.5 FLHFHV Technique

Figure E.47 shows the results of the FLHFHV technique for the EPL case. The section decreased its critical elastic moment moving from the upstands to the no downstand/upstands at stand height of 0mm by 8.73kNm for the 5m span. The introduction of downstands then increased the critical elastic moment by 1.16kNm. The 6m span also decreased its critical elastic moment (by 10.82kNm) then increased it with the downstands (1.60kNm). The trend is evident in both models for the 203x133x25 section and the 305x102x25 section, but they are unique to the FLHFHV technique.

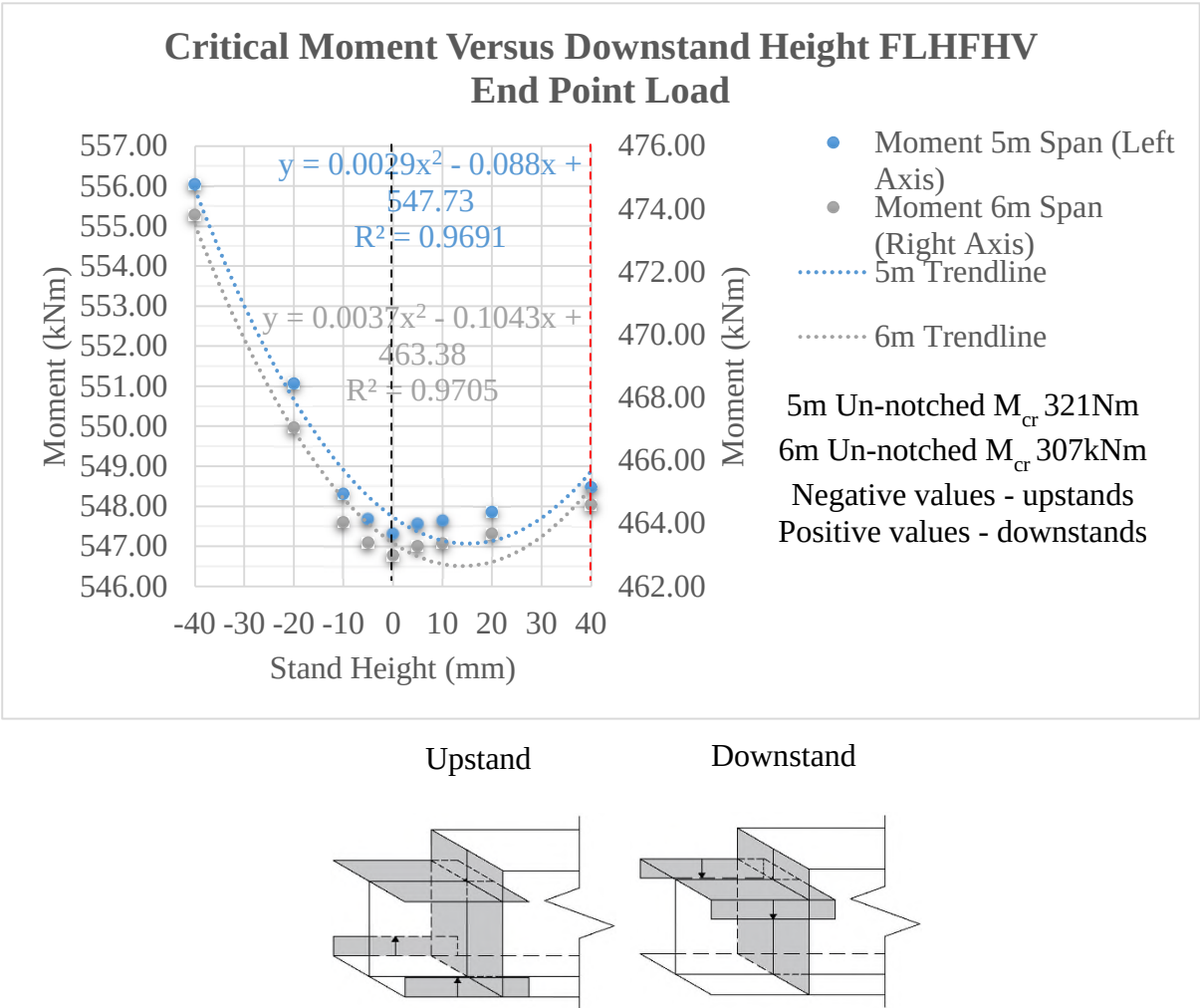


Figure E.47: Critical Moment Versus Stand Height FLHFHV Technique EPL Model 2
533x210x82

Figure E.48 shows the results of the FLHFHV technique for the MSL case. The section decreased its critical elastic moment moving from the upstands to the no downstand/upstands at stand height of 0mm by 7.97kNm for the 5m span. The introduction of downstands then increased the critical elastic moment by 1.37kNm. The 6m span also decreased its critical elastic moment (by 5.58kNm) then increased it with the downstands (0.98kNm). Again, just as in the EPL case (**Figure E.47**), Model 1 and in both models in the 203x133x25, 302x133x25 sections, the trends are unique to the FLHFHV technique.

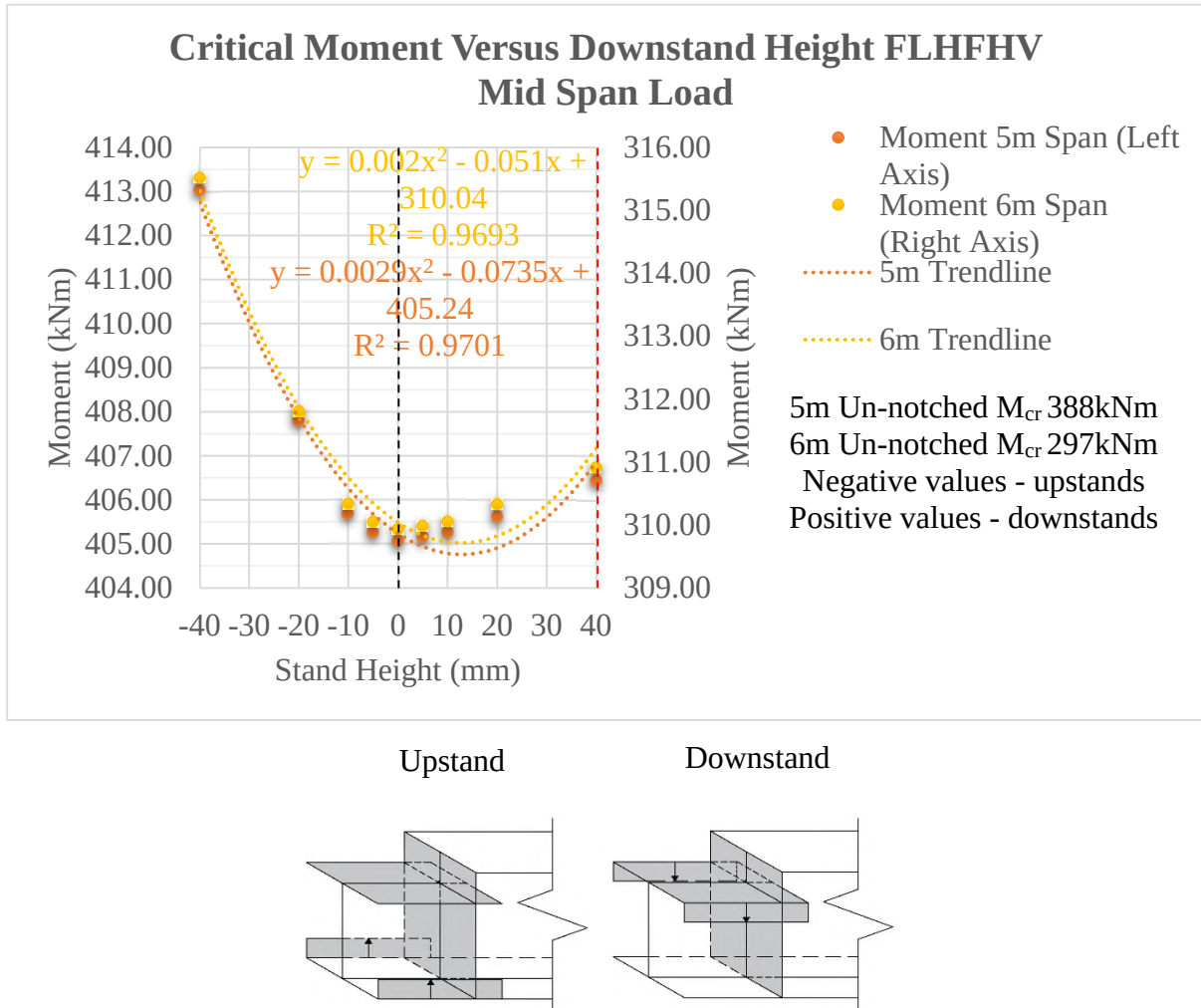


**Figure E.48: Critical Moment Versus Stand Height FLHFHV Technique MSL Model 2
533x210x82**

Just as in the previous sections: by isolating the downstand case of the FLHFHV technique it is clear the trend is similar to the other techniques. This is clear since with each increase in downstand height, there is an increase in the critical elastic moment as shown in **Figure E.49**.

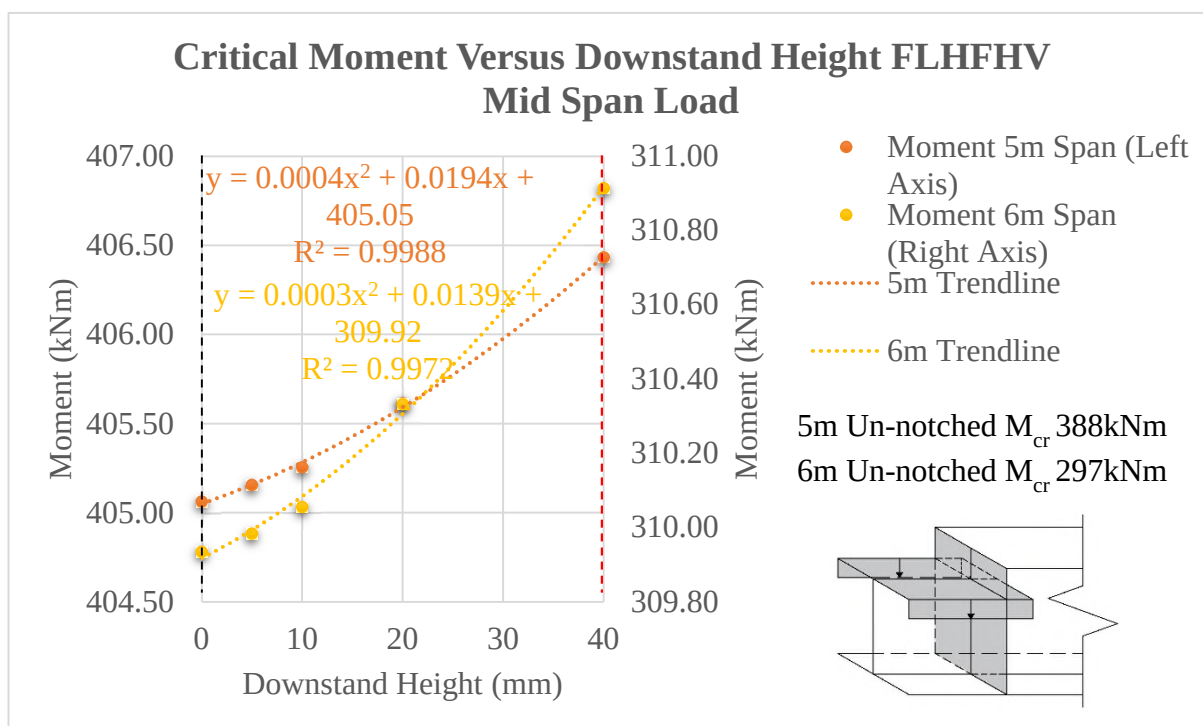
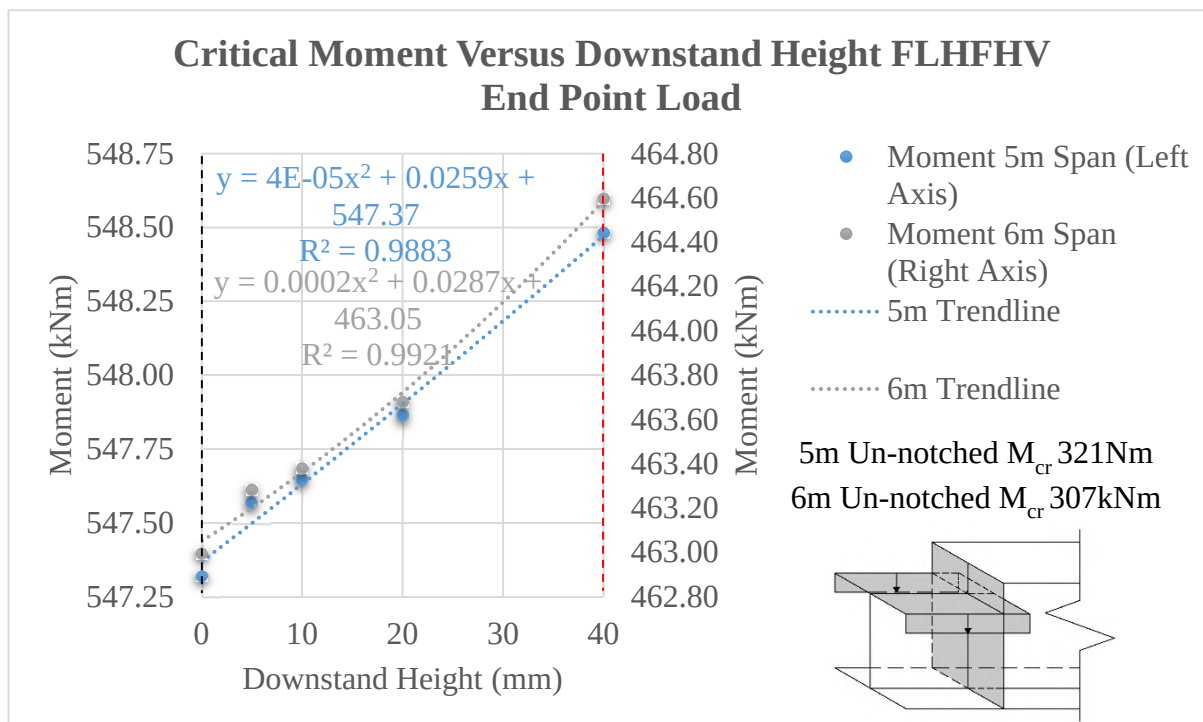


**Figure E.49: Critical Moment Versus Downstand Height FLHFHV Technique Model 2
533x210x82**

E.3 203 x 203 x 46 Section

E.3.1 Model 1

Figure E.50 shows the 1m span and the 3m span for the EPL case. It also shows the five stiffening techniques and by how much they restore the critical elastic moment to the notched section.

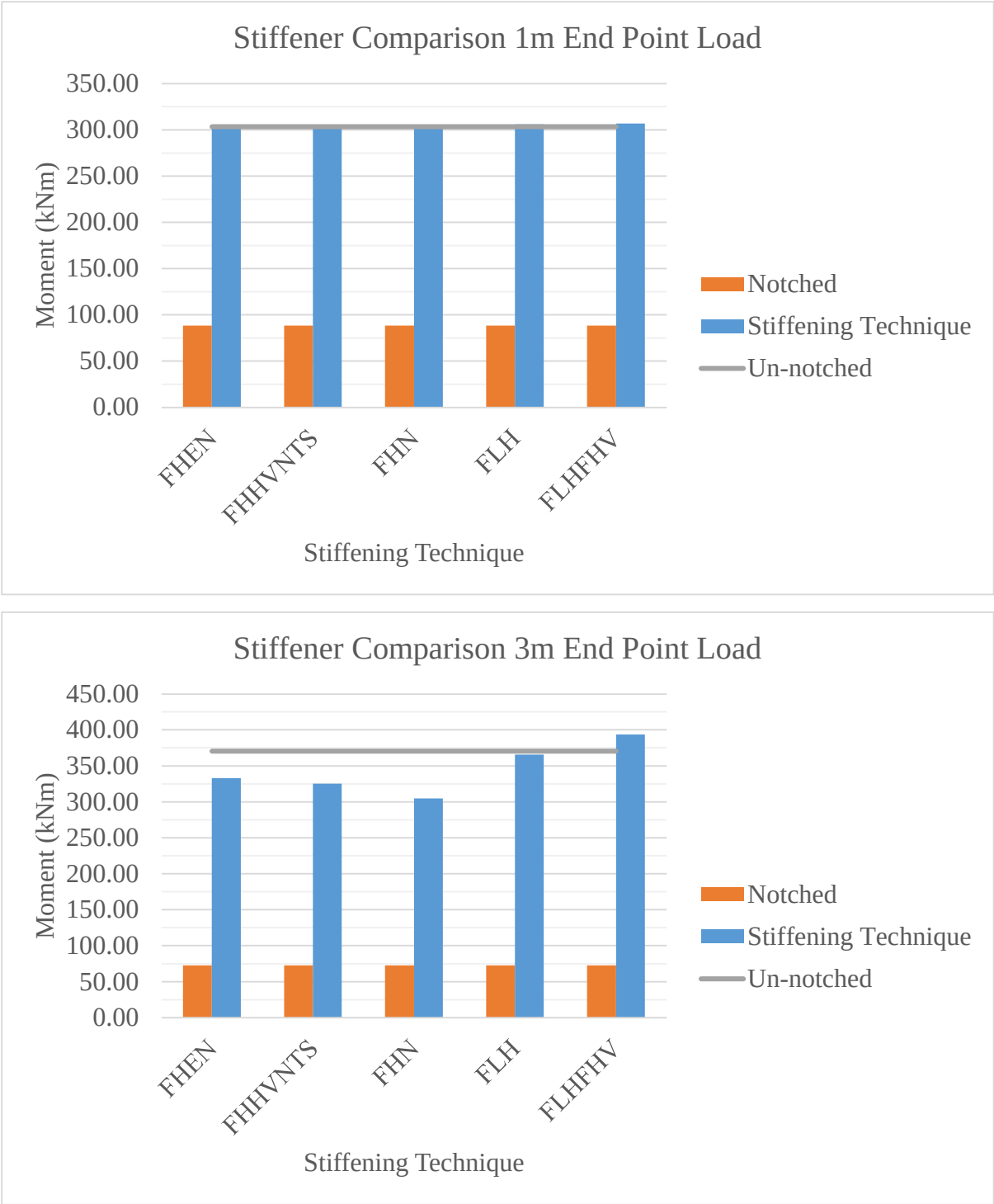


Figure E.50: Stiffener Comparison 1m, 3m EPL 203x203x46

Figure E.51 shows the 1m span and the 3m span for the EPL case. It also shows the five stiffening techniques and by how much they restore the critical elastic moment to the notched section.

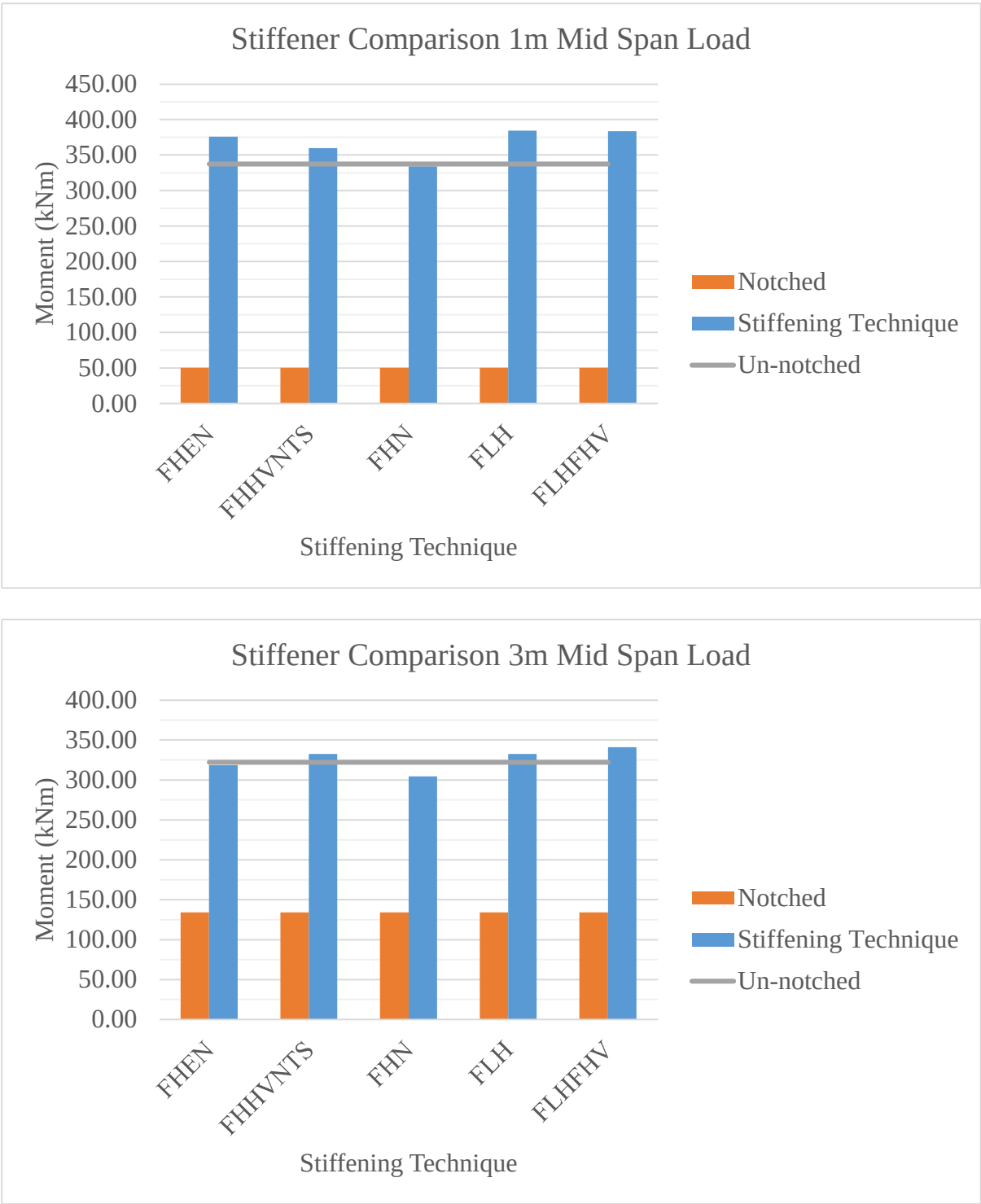


Figure E.51: Stiffener Comparison 1m, 3m MSL 203x203x46

E.3.1.1 FHEN Technique

Figure E.52 shows the critical elastic moment versus downstand height for the FHEN EPL case. The 1m span increased its critical elastic moment by 2.94kNm whereas the 3m increased by 162.59kNm. Once again, the longer span increased more.

The trends are similar to the 533x210x82 section (**Figure E.27**) because there is a slight parabolic curve that decreases in gradient as the shear centre approaches and passes the centroid in the notched region at the 117mm downstand height. However, the change in gradient is much more apparent in the 533x210x82 section whereas the data here seems to resemble the first few data points of that trend. This could be due to the limited number of downstands due to height restrictions in the notched region.

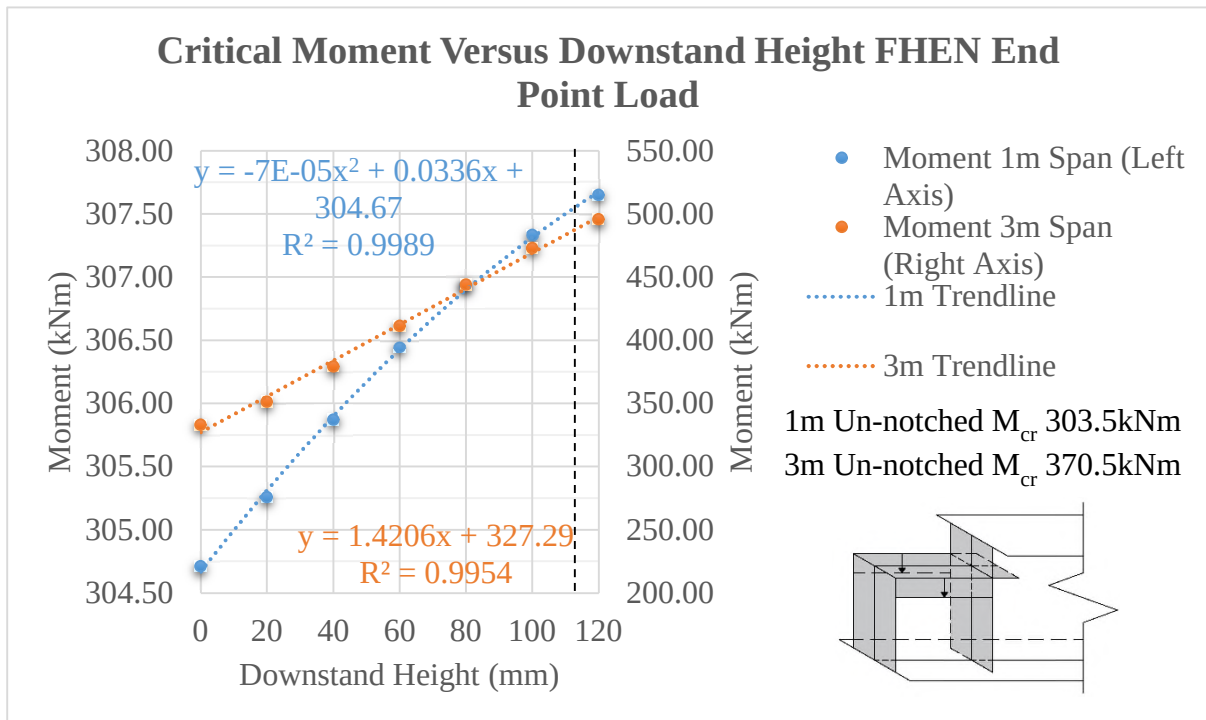


Figure E.52: Critical Moment Versus Downstand Height FHEN Technique EPL Model 1 203x203x46

Figure E.53 shows the results for the MSL case. There was a negligible change of 0.24kNm for the 1m span and 93.07kNm for the 3m (which is different to the usual trend of the longer span not improving as much as the shorter span).

Focussing on the 3m span: it is difficult to see any changes around the 117mm downstand height. It is unclear if the curve resembles a parabola starting at a minimum value, like the 3m span of the 203x133x25 (**Figure 6.12**) and 305x102x25 (**Figure E.4**) sections or approaches a maximum value like in the 533x210x82 section (**Figure E.28**). However, it can be said as the shear centre approaches and passes the centroid in the notched region the critical elastic moment increases with each increase in downstand height.

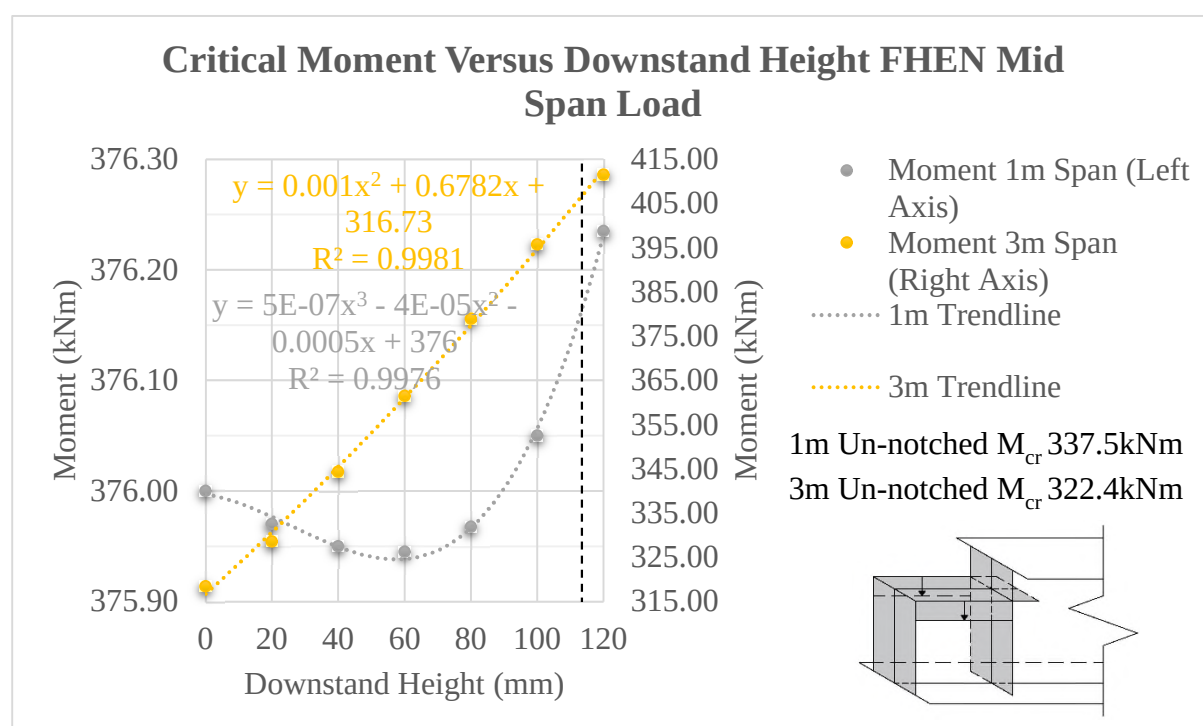


Figure E.53: Critical Moment Versus Downstand Height FHEN Technique MSL Model 1 203x203x46

E.3.1.2 FHHVNTS Technique

Figure E.54 shows the results for the EPL FHHVNTS technique. The 1m and 3m spans increased by 1.38kNm and 28.63kNm respectively. This is different to the previous trend in the FHHVNTS technique because usually the shorter of the two spans increases their critical elastic moment more.

The FHHVNTS technique is very consistent across all spans and loading arrangements: the trend starts at a minimum value then increases parabolically with each increase in downstand height. It appears the trendline changes from being initially parabolic to linear at the centroid shear centre coincidence point (at 13mm downstand height).

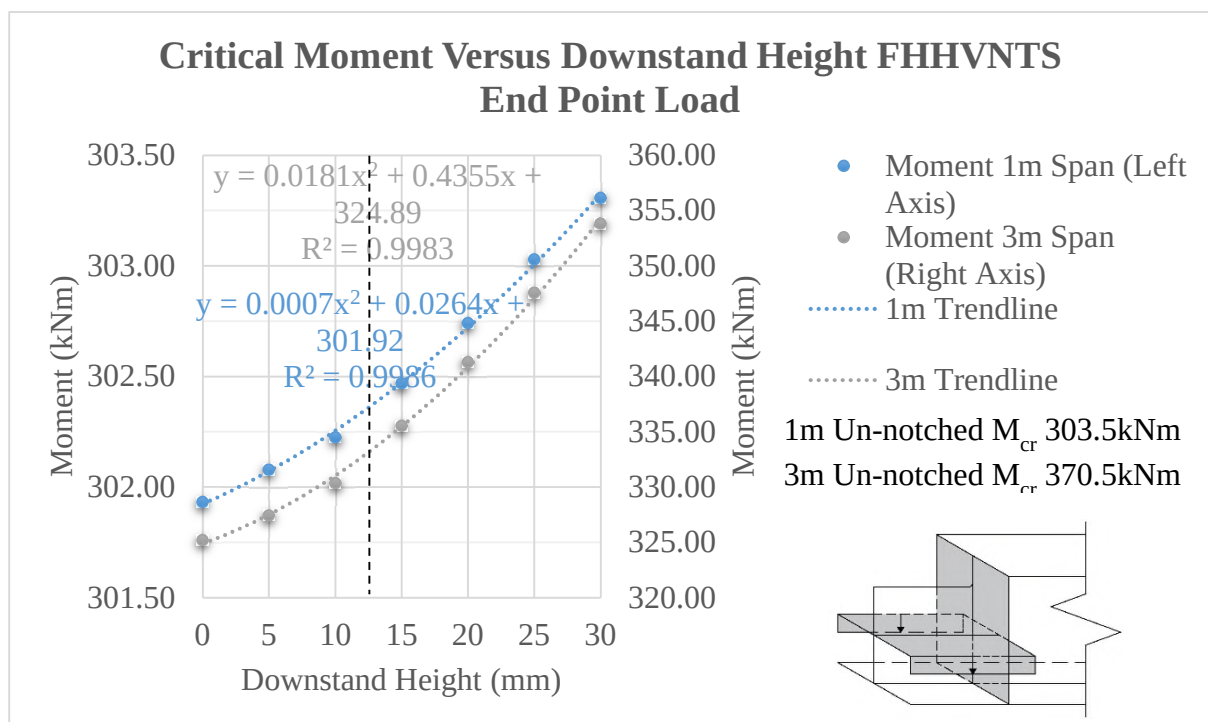


Figure E.54: Critical Moment Versus Downstand Height FHHVNTS Technique EPL Model 1
203x203x46

Figure E.55 shows the critical elastic moment versus downstand height for the MSL FHHVNTS technique. There was an increase of 23.08kNm and 15.42kNm for the 1m and 3m spans which is similar to the previous FHHVNTS techniques and other cases. The 1m span has a trend that is similar to the FHEN technique applied to the 533x210x82 section whereas the 3m is similar to all the other FHHVNTS techniques. This is the first time there has been a different trend between the spans in the FHHVNTS technique. It is also very interesting because the 3m trend is similar across completely different sections: the 533x210x82 is a very deep I section whereas the 203x203x46 is a shallower H section.

The only changes around the 13mm downstand height appear to be changes in gradient of both curves (a very slight change for the 3m span). The change in gradient could be a change in curve type as in the EPL case (**Figure E.54**).

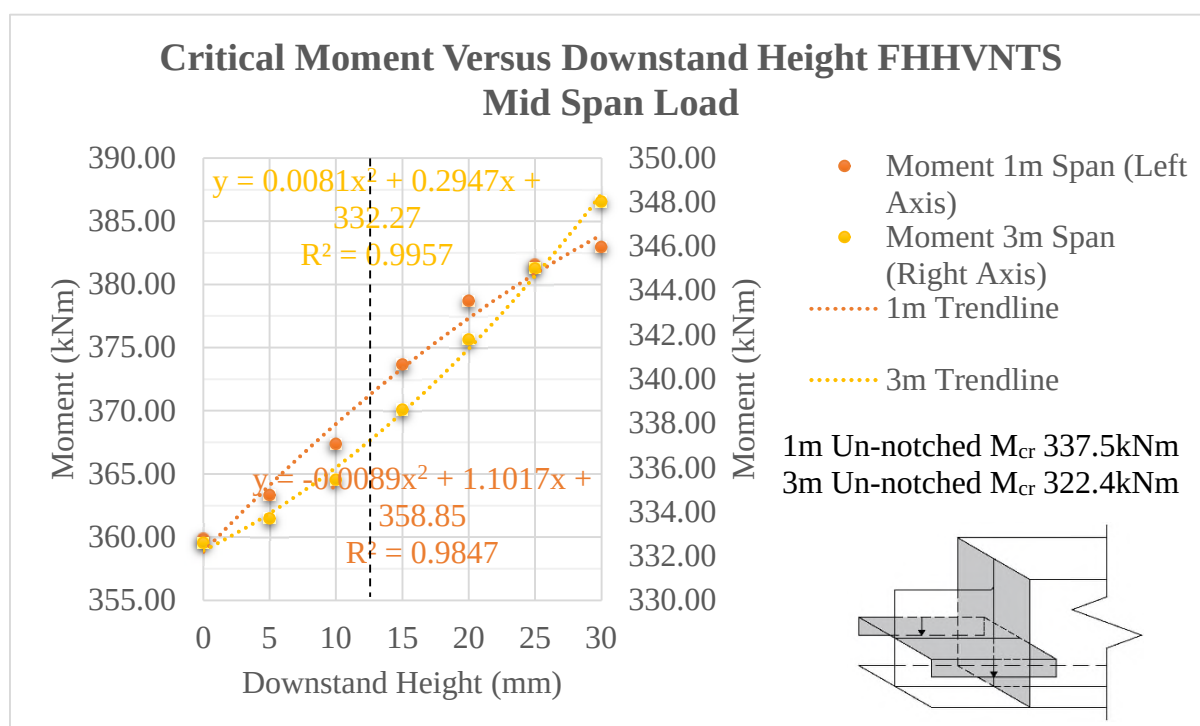


Figure E.55: Critical Moment Versus Downstand Height FHHVNTS Technique MSL Model 1 203x203x46

E.3.1.3 FHN Stiffening Technique

Figure E.56 shows the results for the EPL FHN technique. There was an increase of 1.24kNm for the 1m and 35.46kNm for the 3m span which is different to the other FHN cases where the shorter span would increase more than the longer span.

The trend is similar to the 203x133x25 section (**Figure 6.15**), the 3m span in the 533x210x82 section (**Figure E.31**) and the FHHVNTS technique: the trend starts at a minimum value then increases parabolically with each increase in downstand height. It is unclear if there are any changes at the 117mm downstand height.

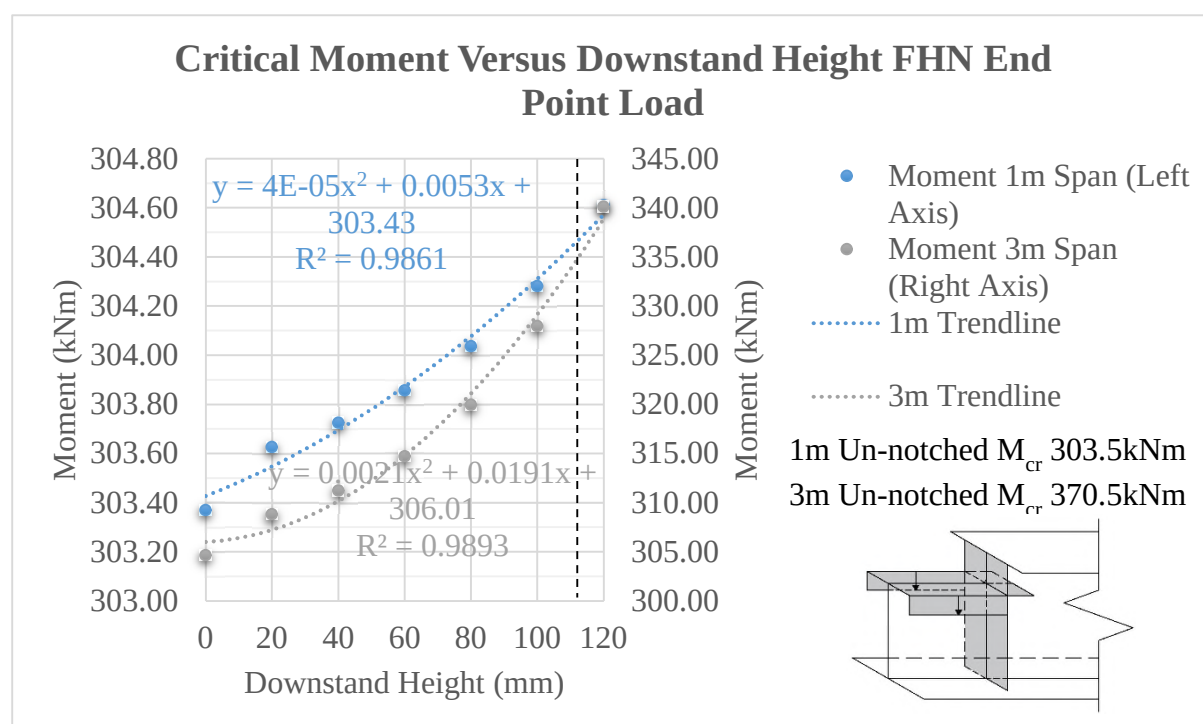


Figure E.56: Critical Moment Versus Downstand Height FHN Technique EPL Model 1 203x203x46

Figure E.57 shows the results for the FHN MSL case. There were increases of 42.94kNm and 24.46kNm for the 1m and 3m spans which is the opposite to the EPL case: the shorter span increased more here.

Once again, the trend is similar to the 203x133x25 section (**Figure 6.15** and **Figure 6.16**), the 1m span in the 533x210x82 section (**Figure E.32**) and the FHHVNTS technique. However, there is no obvious change in the trend when the centroid and shear centre in the notched region coincide at downstand height of 117mm.

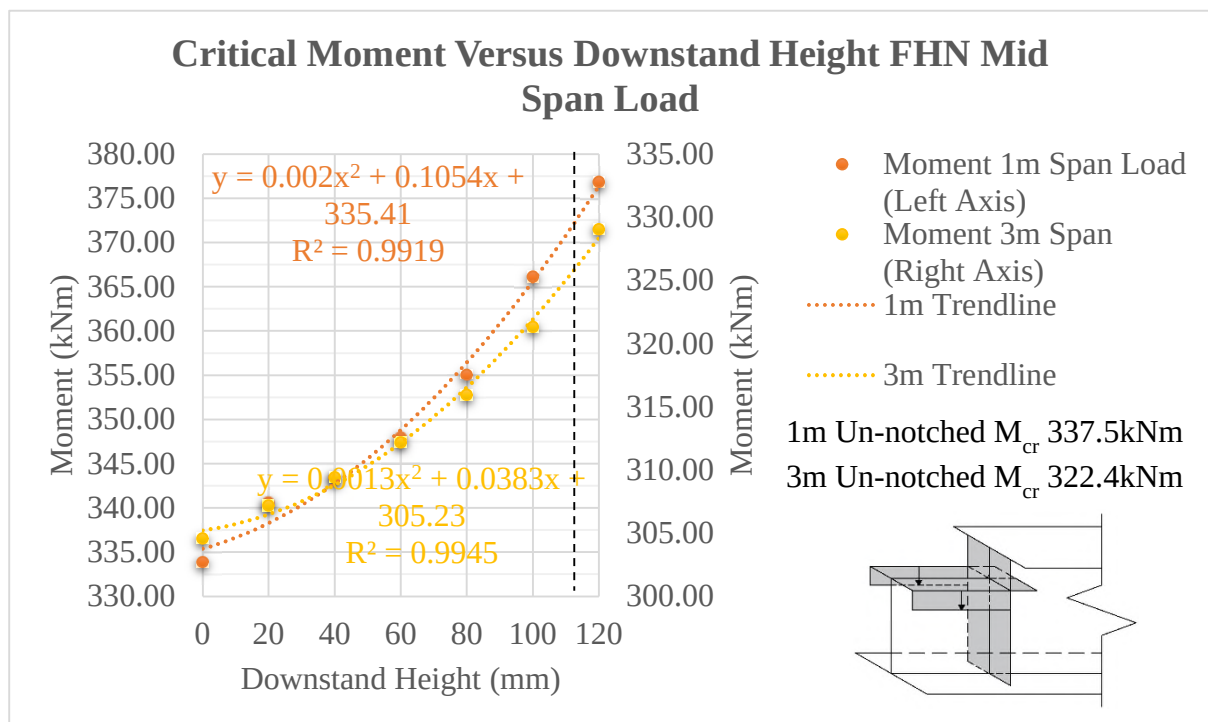


Figure E.57: Critical Moment Versus Downstand Height FHN Technique MSL Model 1 203x203x46

E.3.1.4 FLH Technique

Figure E.58 shows the critical elastic moment versus downstand height for the EPL FLH case. There was a slight increase of 0.59kNm for the 1m span and 39.95kNm for the 3m span (note the longer span increased more).

The trend is similar to the 1m span EPL of the FHHVNTS technique (**Figure E.55**). There is a reduction in gradient as the shear centre approaches and passes the centroid in the notched region at the 117mm downstand height. The trend starts at a minimum value and increases with each increase in downstand height like the FHEN technique in for the 203x203x46 section. This is also similar to the 1m span of the EPL FLH case for the 533x210x82 section (**Figure E.33**).

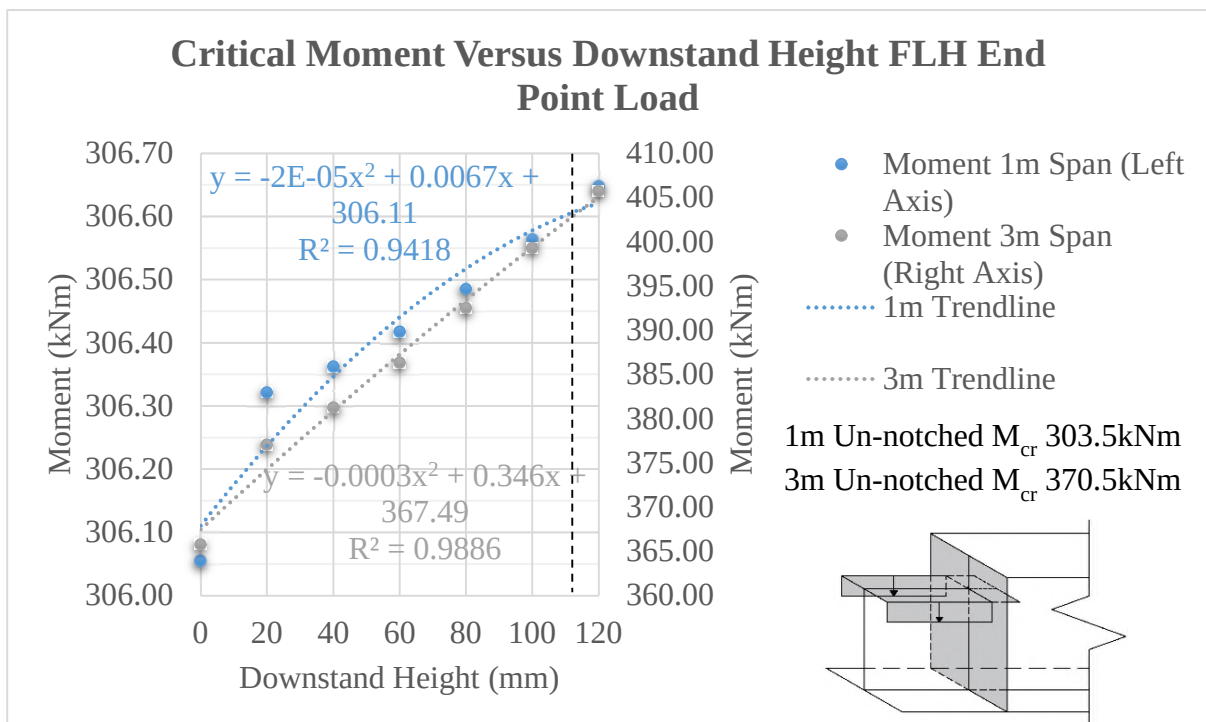


Figure E.58: Critical Moment Versus Downstand Height FLH Technique EPL Model 1 203x203x46

Figure E.59 shows the results for the MSL FLH case. There was a negligible increase of 0.11kNm for the 1m span and 29.85kNm for the 3m span.

There is no clear change in behaviour as the downstands approach 117mm in height (the coincidence point of the centroid and shear centre in the notched region) but it is clear the critical elastic moment increases with each increase in downstand height. The 3m span is similar to the 3m MSL FLH case for the 533x210x82 section (**Figure E.34**).

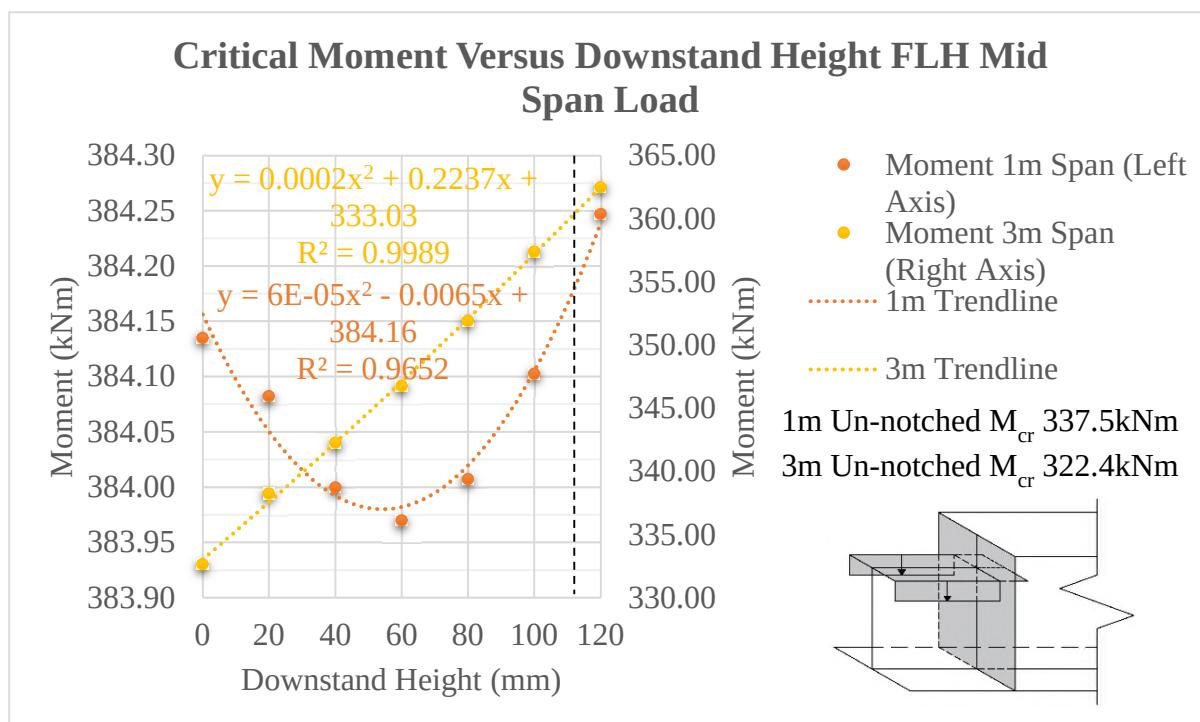


Figure E.59: Critical Moment Versus Downstand Height FLH Technique MSL Model 1 203x203x46

E.3.1.5 FLHFHV Technique

Figure E.60 shows the results of the FLHFHV technique for the EPL case. The section decreased its critical elastic moment moving from the upstands to the no downstand/upstands at stand height of 0mm by 0.93kNm for the 1m span. The introduction of downstands then increased the critical elastic moment by 0.08kNm.

The 3m span also decreased its critical elastic moment (by 37.42kNm) then increased it with the downstands (7.00kNm). The behaviour here is unlike any of the trends in the other techniques previously discussed and is only similar to the FLHFHV technique for the 203x133x25, 305x102x25 and 533x210x82 sections. As the shear centre approaches the centroid in the notched section by reducing the height of the upstands, the critical elastic moment reduces and only increases, if by a negligible amount, once downstands are introduced. This stiffening technique could be considered an outlier to the other techniques.

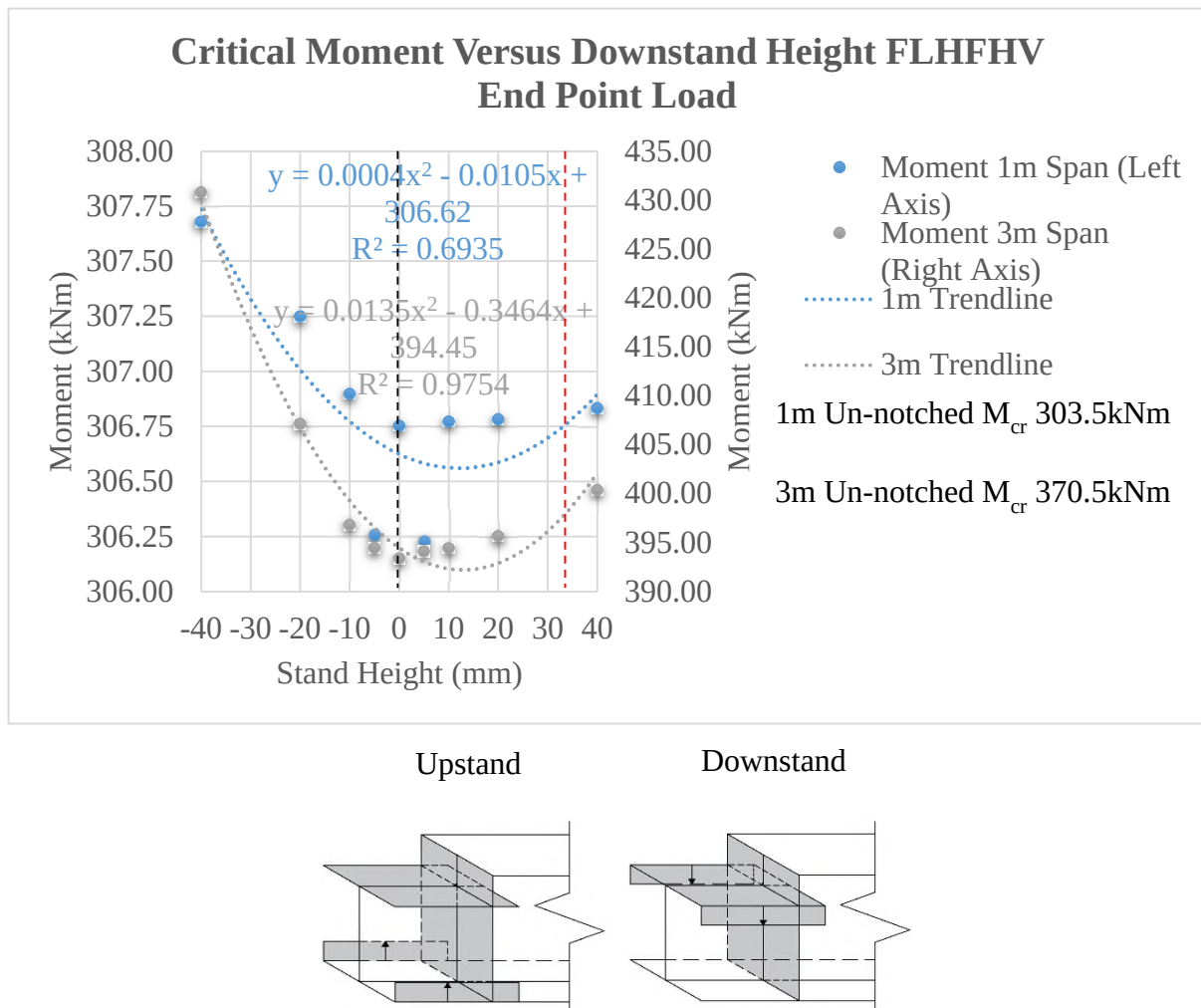


**Figure E.60: Critical Moment Versus Stand Height FLHFHV Technique EPL Model 1
203x203x46**

The trend in the FLHFHV MSL case shown in **Figure E.61** follows the EPL case (**Figure E.60**) but is also unique as it does not follow the trends shown in the previous stiffening techniques except for the FLHFHV technique applied to the previous sections. There was also a reduction in the moment (2.01kNm) moving from the upstand to no upstand/downstand case then followed by a further negligible reduction of 0.14kNm for the 1m span case. Similarly, the 3m span reduced its critical elastic moment by 28.28kNm then increased it by a 6.42kNm.

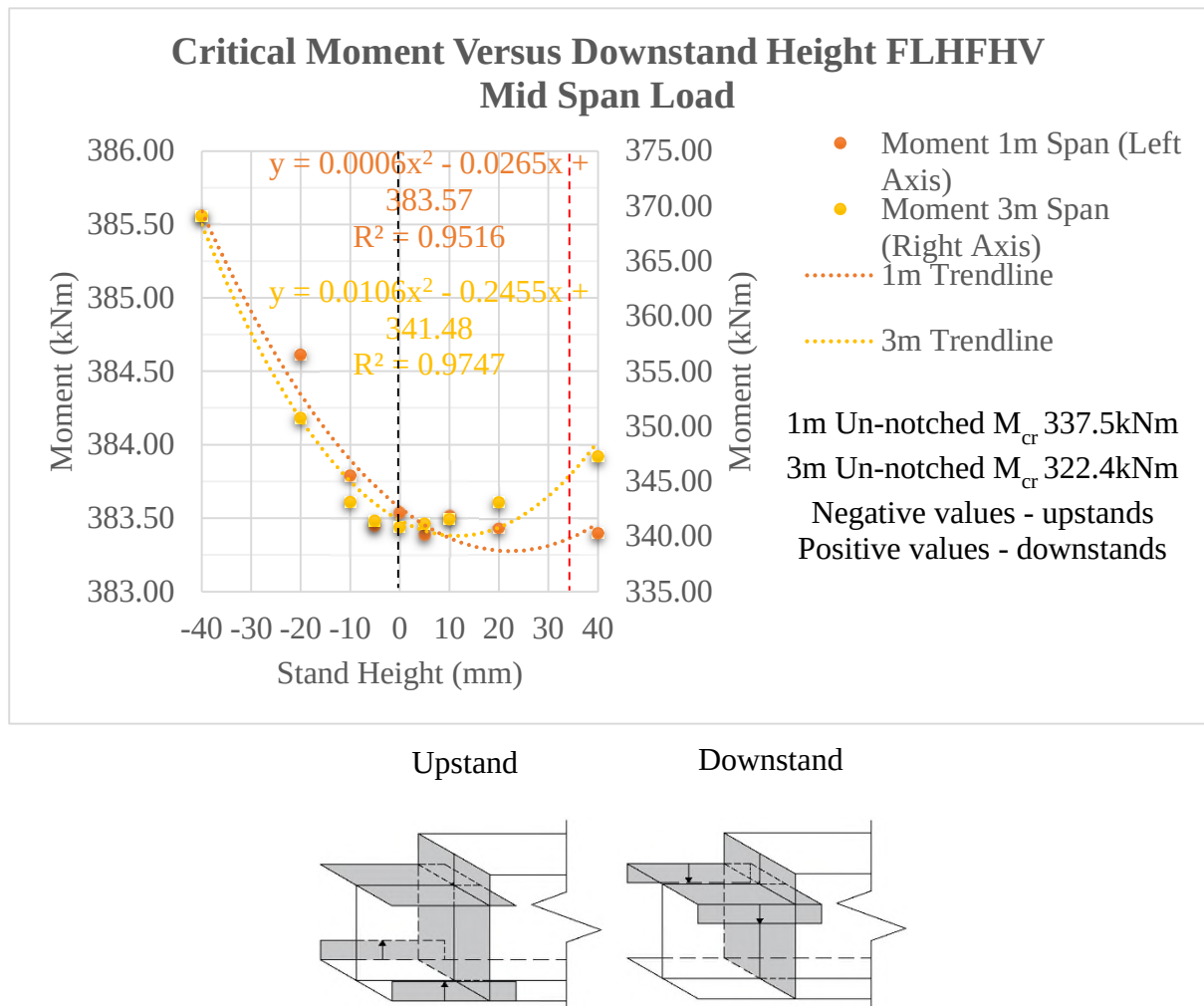


**Figure E.61: Critical Moment Versus Stand Height FLHFHV Technique MSL Model 1
203x203x46**

The isolated downstand case, as discussed in the previous sections and shown in **Figure E.62**, show similarities with the other stiffening techniques. The 1m EPL and MSL were excluded because they did not achieve changes in the critical elastic moment greater than 0.5kNm.

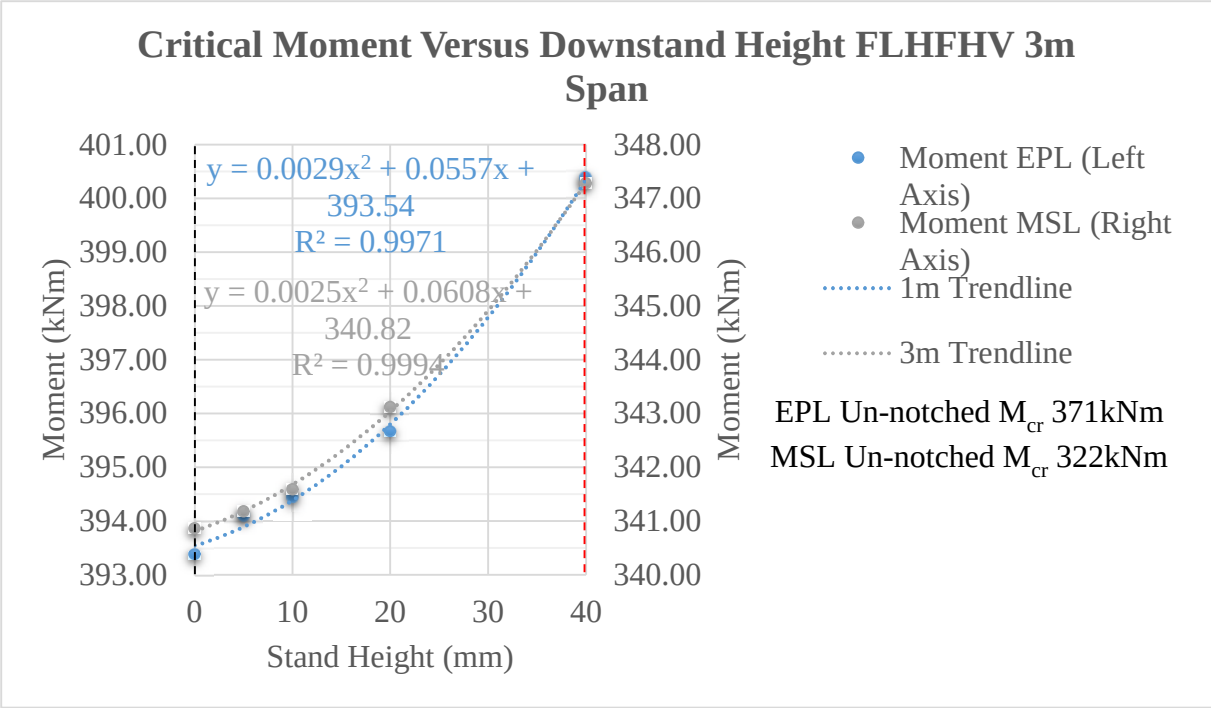


Figure E.62: Critical Moment Versus Downstand Height FLHFHV Technique Model
1 203x203x46

E.3.2 Model 2

Figure E.63 shows the 5m span and the 6m span for the EPL case. It also shows the five stiffening techniques and by how much they restore the critical elastic moment to the notched section.



Figure E.63: Stiffener Comparison 5m, 6m EPL 203x203x46

Figure E.64 shows the 5m span and the 6m span for the MSL case. It also shows the five stiffening techniques and by how much they restore the critical elastic moment to the notched section.

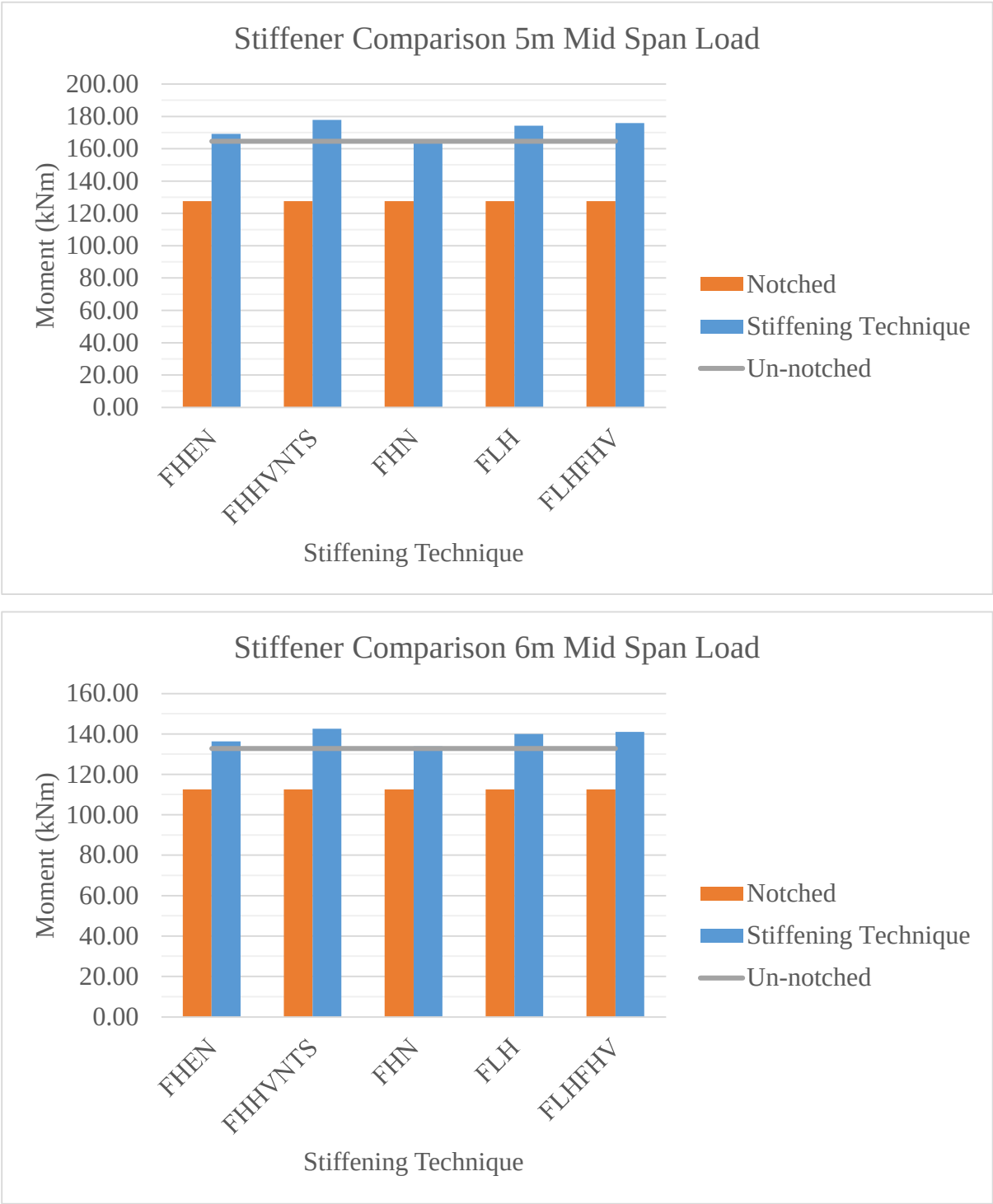


Figure E.64: Stiffener Comparison 5m, 6m MSL 203x203x46

E.3.2.1 FHEN Technique

Figure E.65 shows the results for the EPL FHEN case. There was an increase of 53.98kNm for the 5m span and 32.25kNm for the 6m span. Just as in the 203x133x25 and 305x102x25 sections, the shorter of the two spans increased more.

With each increase in downstand height, the critical elastic moment also increases in a linear trend with no clear change at the 117mm downstand height: the coincidence point of the centroid and shear centre of the notched region.

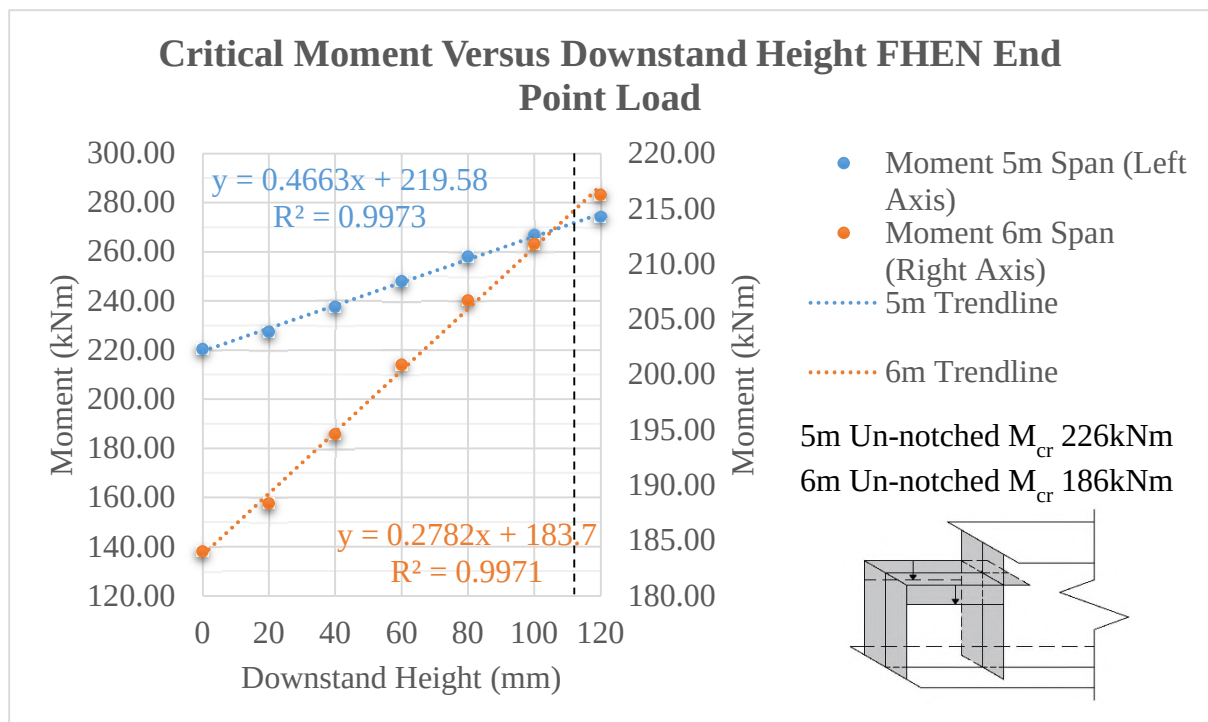


Figure E.65: Critical Moment Versus Downstand Height FHEN Technique EPL Model 2 203x203x46

Figure E.66 shows the results for the MSL FHEN technique. The 5m and 6m spans increased their critical elastic moment by 25.44kNm and 16.22kNm respectively. As in the EPL case, the shorter span increased its critical elastic moment more.

The critical elastic moment increased with each increase in downstand height in a linear trend, for both spans similar to the 203x133x25 section (**Figure 6.25**). There is no obvious change in the trend at the 117mm downstand height.

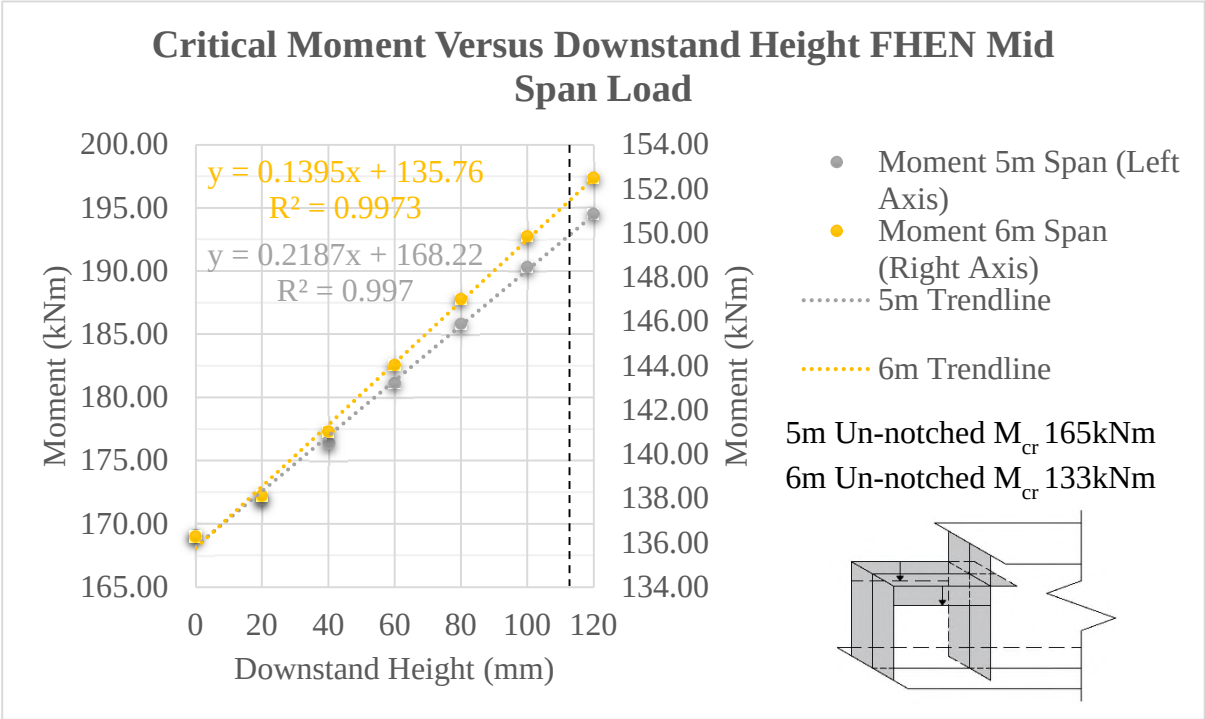


Figure E.66: Critical Moment Versus Downstand Height FHEN Technique MSL Model 2 203x203x46

E.3.2.2 FHHVNTS Technique

Figure E.67 shows the results for the FHHVNTS EPL case. There was an increase of 10.79kNm and 6.41kNm for the 5m and 6m spans. Once again, the shorter of the two spans increased its critical elastic moment more than the longer span.

The trend is consistent with the other FHHVNTS cases: the trend starts at a minimum value and increases parabolically with each increase in downstand height. There is no obvious change in the trend as the downstands approach a height of 13mm: the height required to shift the shear centre to the same vertical height as the centroid. However, it appears the trendline before the 13mm downstand height is parabolic and after it is linear. There is a clear trend, just as in almost all other the FHHVNTS cases, of the critical elastic moment increases with each increase in downstand height.

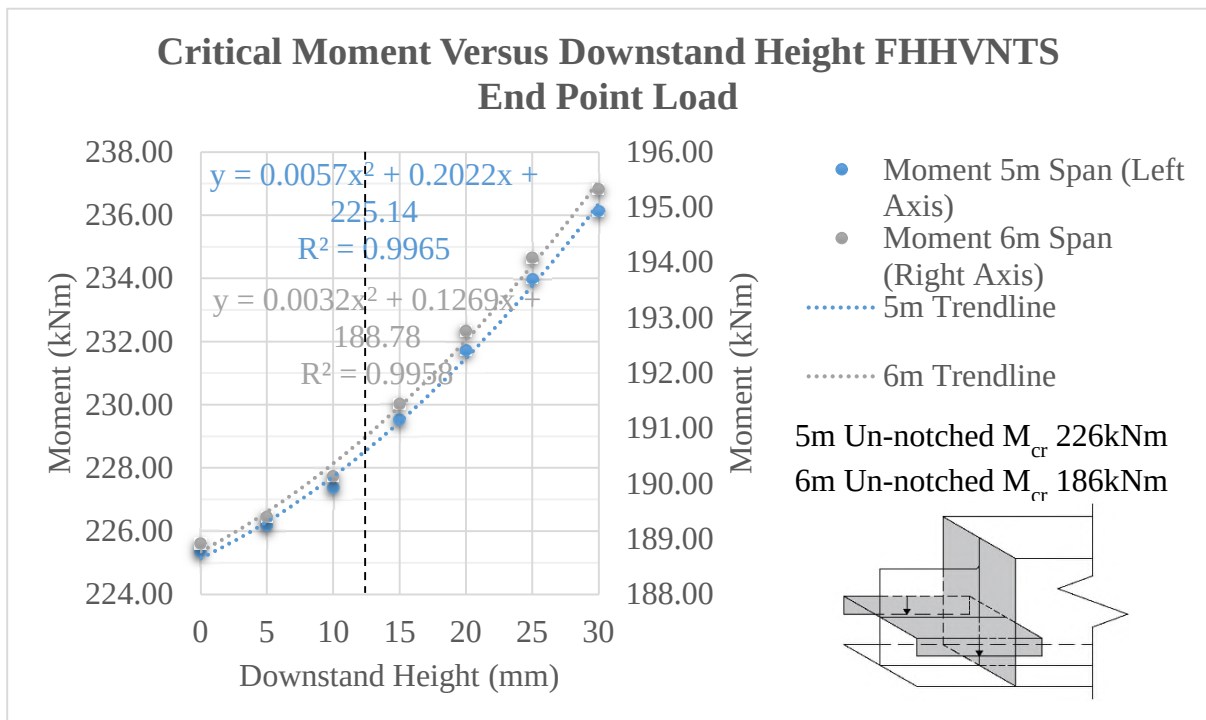


Figure E.67: Critical Moment Versus Downstand Height FHHVNTS Technique EPL Model 2
203x203x46

Figure E.68 shows the results for the MSL case. The 5m and 6m span had increases of 3.73kNm and 2.32kNm respectively with the shorter of the two spans increasing more.

The trend is similar to the EPL case and there was no obvious change in the trend around the 13mm downstand height. However, the change in curve type is more pronounced here. Before the coincidence point of the centroid and shear centre in the notched region (at 13mm) the trendline is parabolic but after, when the shear centre is above the centroid, the trendline is linear.

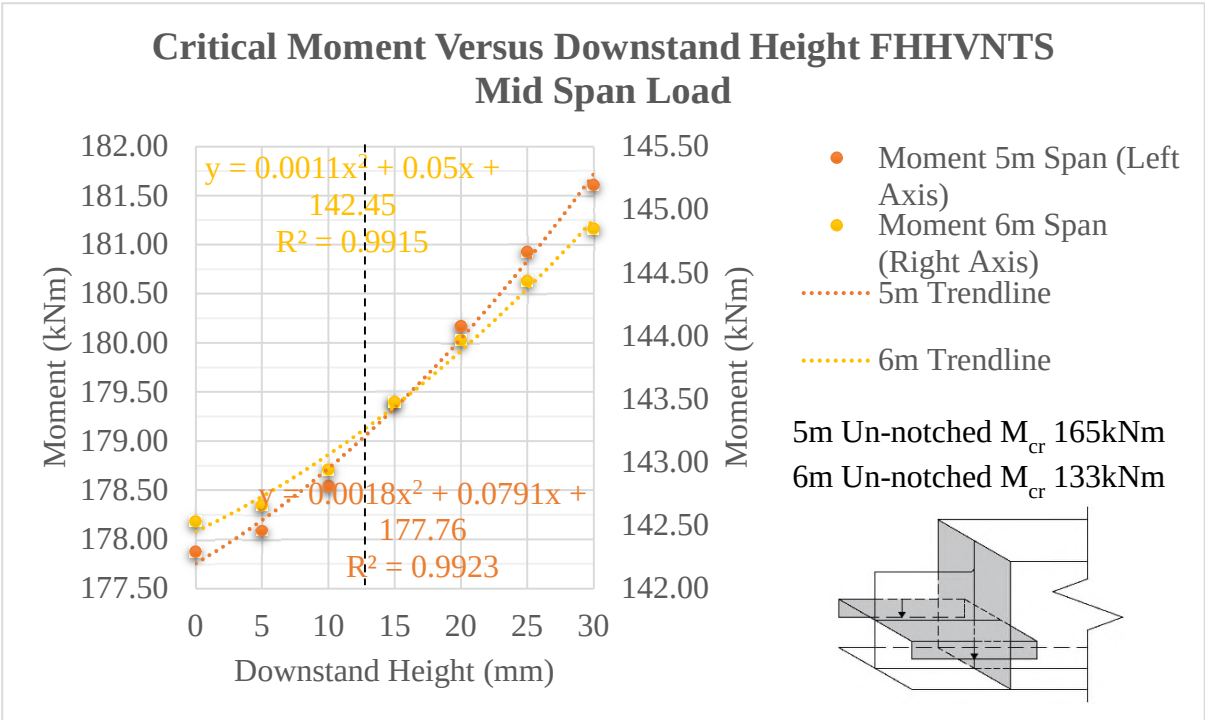


Figure E.68: Critical Moment Versus Downstand Height FHHVNTS Technique MSL Model 2 203x203x46

E.3.2.3 FHN Technique

Figure E.69 shows the results for the EPL FHN case. There was an increase of 16.17kNm and 10.54kNm for the 5m and 6m spans (notice the shorter span increased more just as in the other FHN cases).

The trend is similar to the 203x133x25 (**Figure 6.28**), 533x210x82 (**Figure E.43**) sections: the trend starts at a minimum value then increases parabolically with each increase in downstand height with no obvious change around the 117mm downstand height.

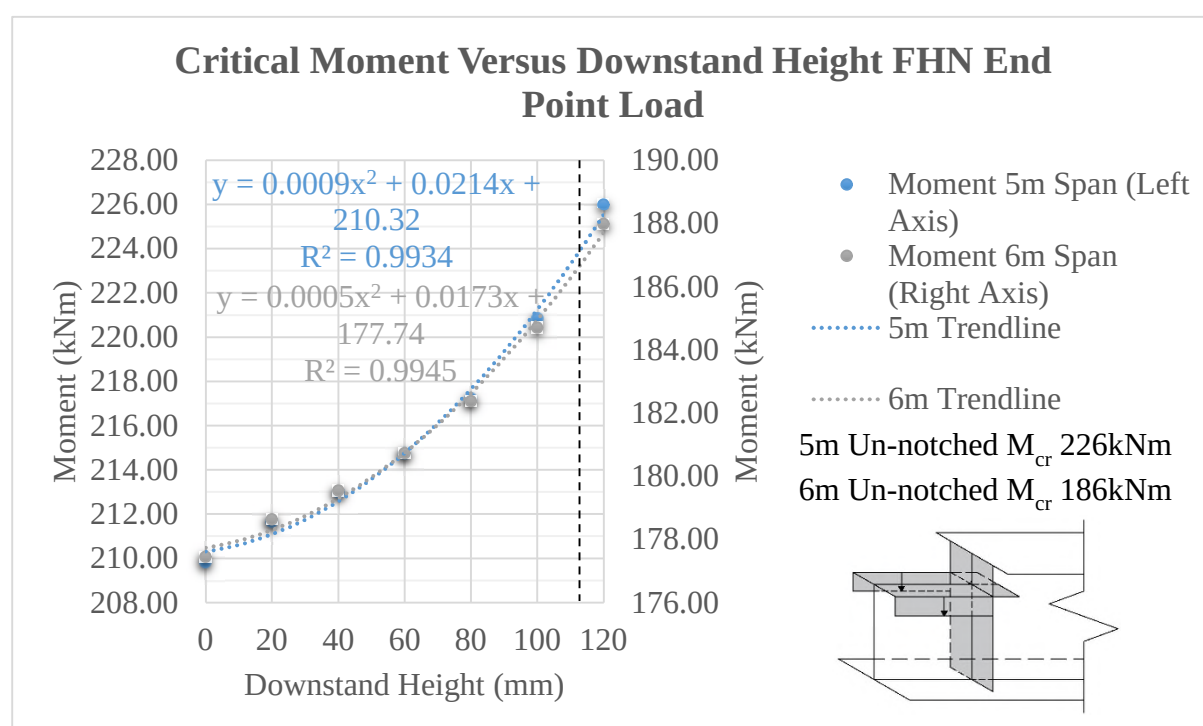


Figure E.69: Critical Moment Versus Downstand Height FHN Technique EPL Model 2 203x203x46

Figure E.70 shows the results for the MSL FHN case. The 5m and 6m spans increased their critical elastic moment by 7.37kNm and 4.84kNm respectively and again the shorter span increased more than the longer span.

There are no obvious changes in the trend as the shear centre approaches and passes the centroid at the 117mm downstand height, but the trend is similar to the EPL case (**Figure E.69**), the 203x133x25 (**Figure 6.29**) and the 533x210x82 section (**Figure E.44**). With each increase in downstand height there is an increase in the critical elastic moment.

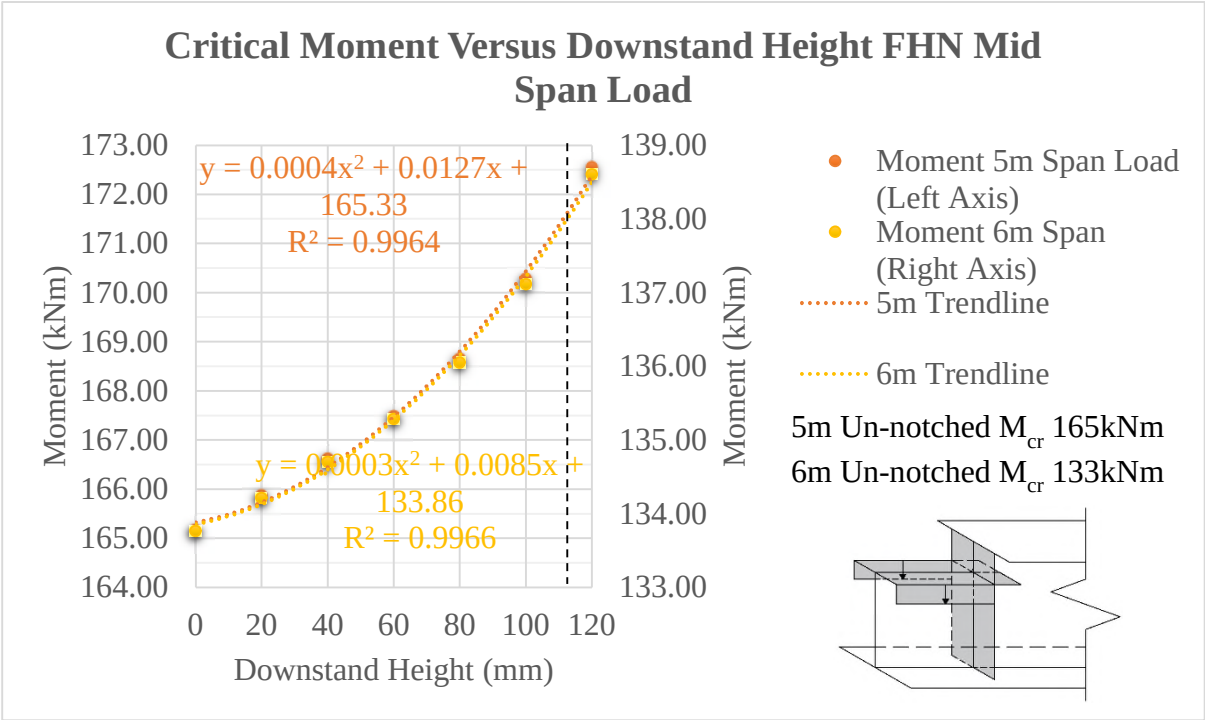


Figure E.70: Critical Moment Versus Downstand Height FHN Technique MSL Model 2 203x203x46

E.3.2.4 FLH Technique

Figure E.71 shows the critical elastic moment versus downstand height for the EPL FLH case. There was an increase of 16.92kNm for the 5m span and 10.89kNm for the 6m span showing once again the shorter span increased more.

The 203x133x25 (**Figure 6.30**) and 533x210x82 (**Figure E.45**) sections showed slight changes around the centroid shear centre coincidence point whereas in this case there are no obvious changes around the 117mm downstand height. However, the critical elastic moment increases with each increase in downstand height.

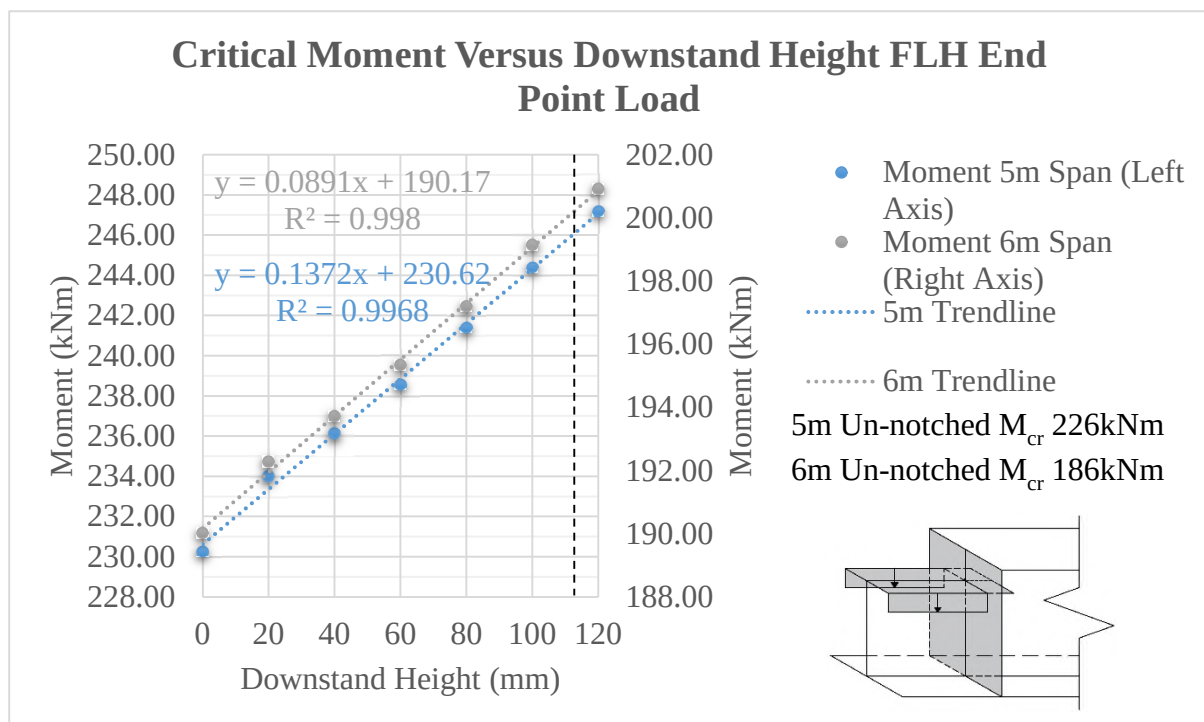


Figure E.71: Critical Moment Versus Downstand Height FLH Technique EPL Model 2 203x203x46

Figure E.72 shows the results for the MSL FLH case which is very similar to the EPL case (**Figure E.71**). There was an increase of 9.98kNm and 6.6kNm for the 5m and 6m spans (the shorter span increased more) and no obvious change around the 117mm downstand height.

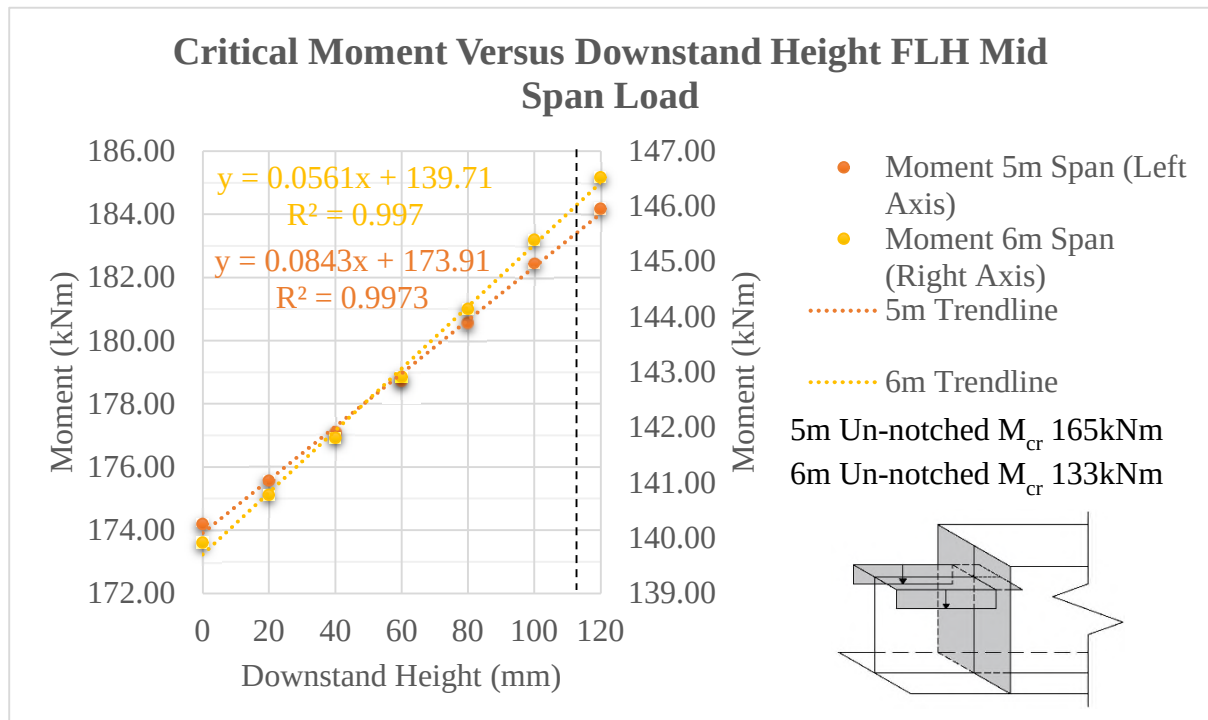


Figure E.72: Critical Moment Versus Downstand Height FLH Technique MSL Model 2 203x203x46

E.3.2.5 FLHFHV Technique

Figure E.73 shows the results of the FLHFHV technique for the EPL case. The section decreased its critical elastic moment moving from the upstands to the no downstand/upstands at stand height of 0mm by 15.35kNm for the 5m span. The introduction of downstands then increased the critical elastic moment by 3.38kNm. The 6m span also decreased its critical elastic moment (by 9.85kNm) then increased it with the downstands (2.29kNm). The trend is evident in both models for the 203x133x25, 305x102x25 and 533x210x82 sections, but they are unique to the FLHFHV technique.

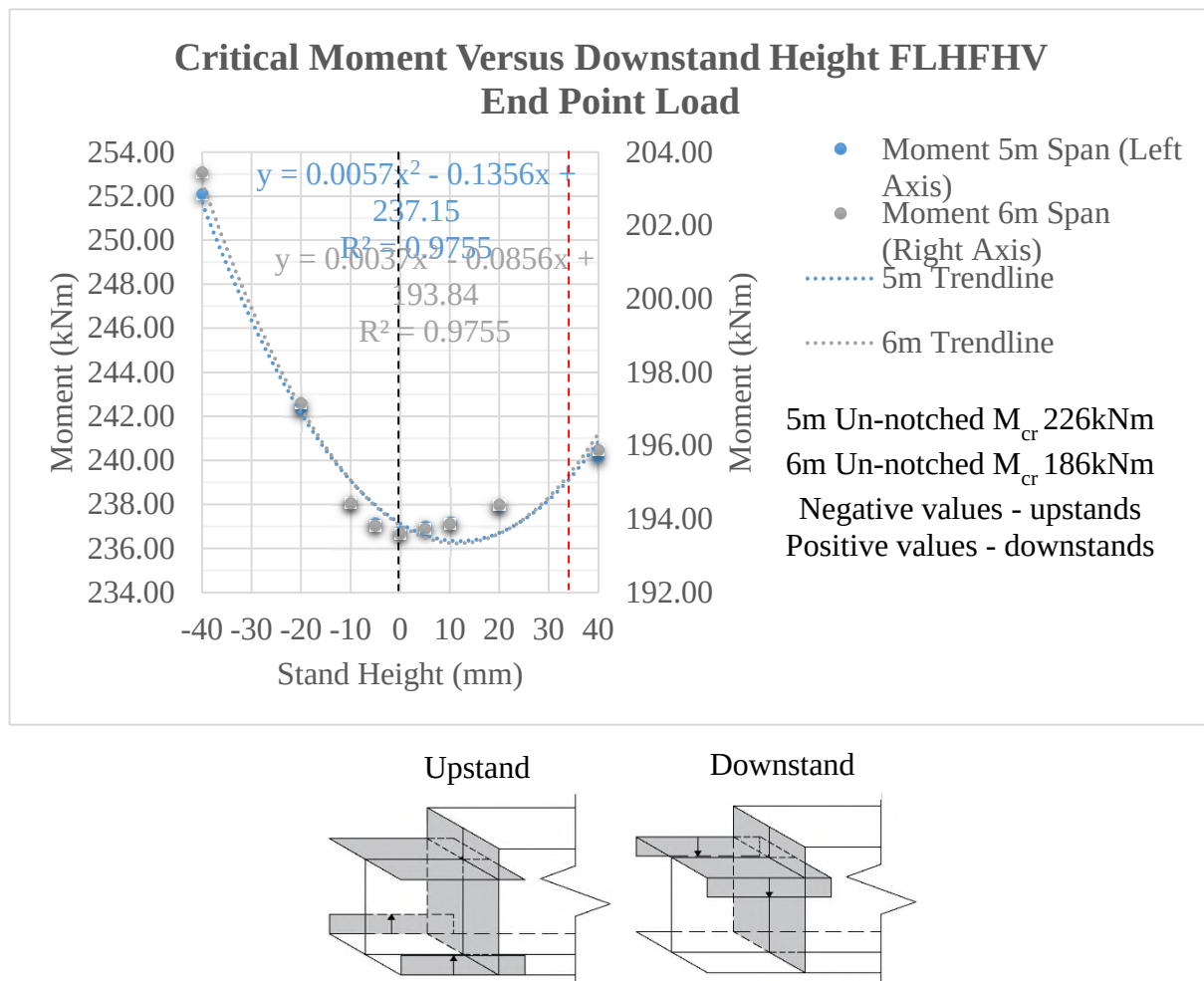


**Figure E.73: Critical Moment Versus Stand Height FLHFHV Technique EPL Model 2
203x203x46**

Figure E.74 shows the results of the FLHFHV technique for the MSL case. The section decreased its critical elastic moment moving from the upstands to the no downstand/upstands at stand height of 0mm by 9.70kNm for the 5m span. The introduction of downstands then increased the critical elastic moment by 2.58kNm. The 6m span also decreased its critical elastic moment (by 6.43kNm) then increased it with the downstands (1.77kNm). Again, just as in the EPL case (**Figure E.73**), Model 1 and in both models in the previous sections, the trends are unique to the FLHFHV technique.

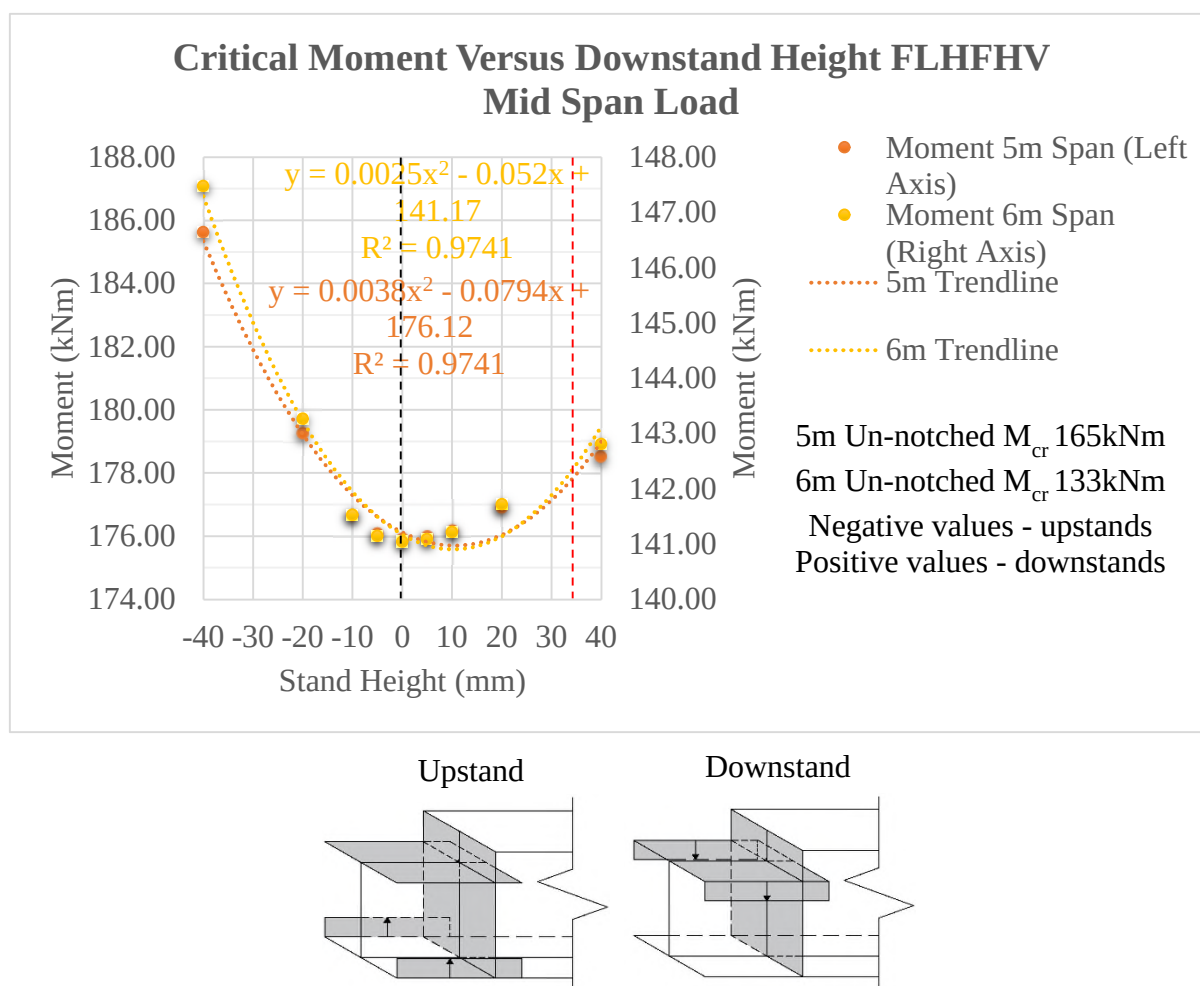


**Figure E.74: Critical Moment Versus Stand Height FLHFHV Technique MSL Model 2
203x203x46**

Just as in the previous sections: by isolating the downstand case of the FLHFHV technique it is clear the trend is similar to the other techniques. This is clear since with each increase in downstand height, there is an increase in the critical elastic moment as shown in **Figure E.75**.

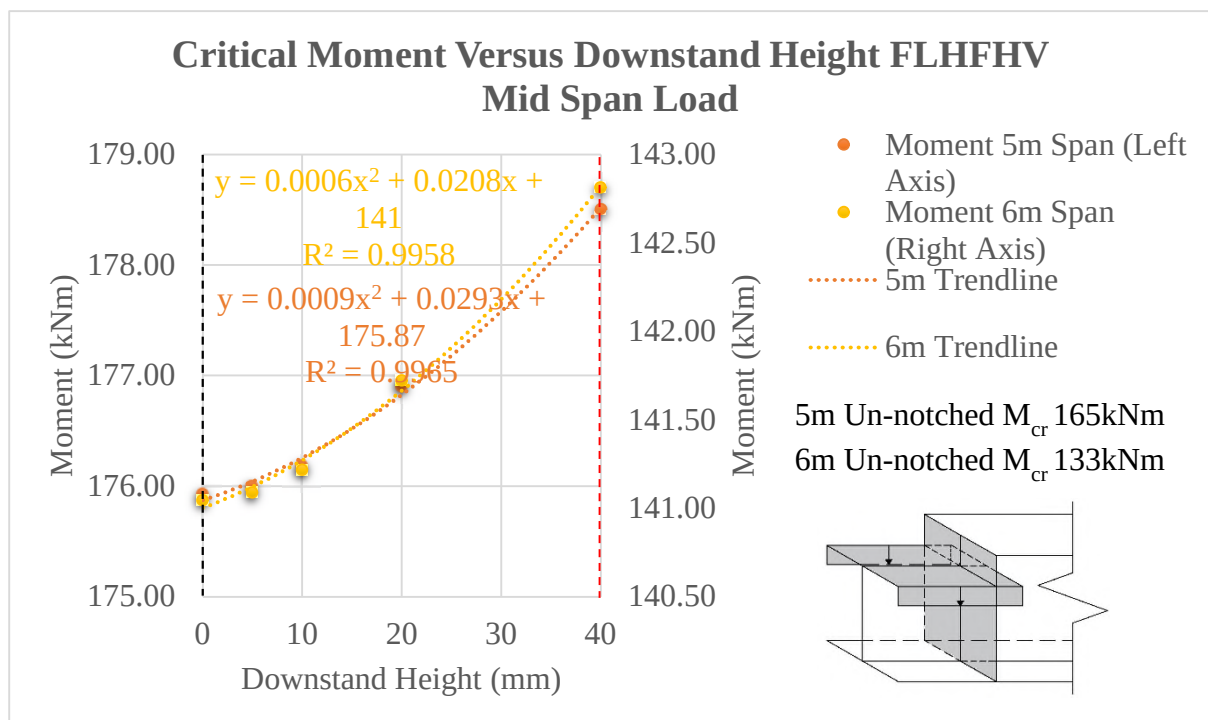
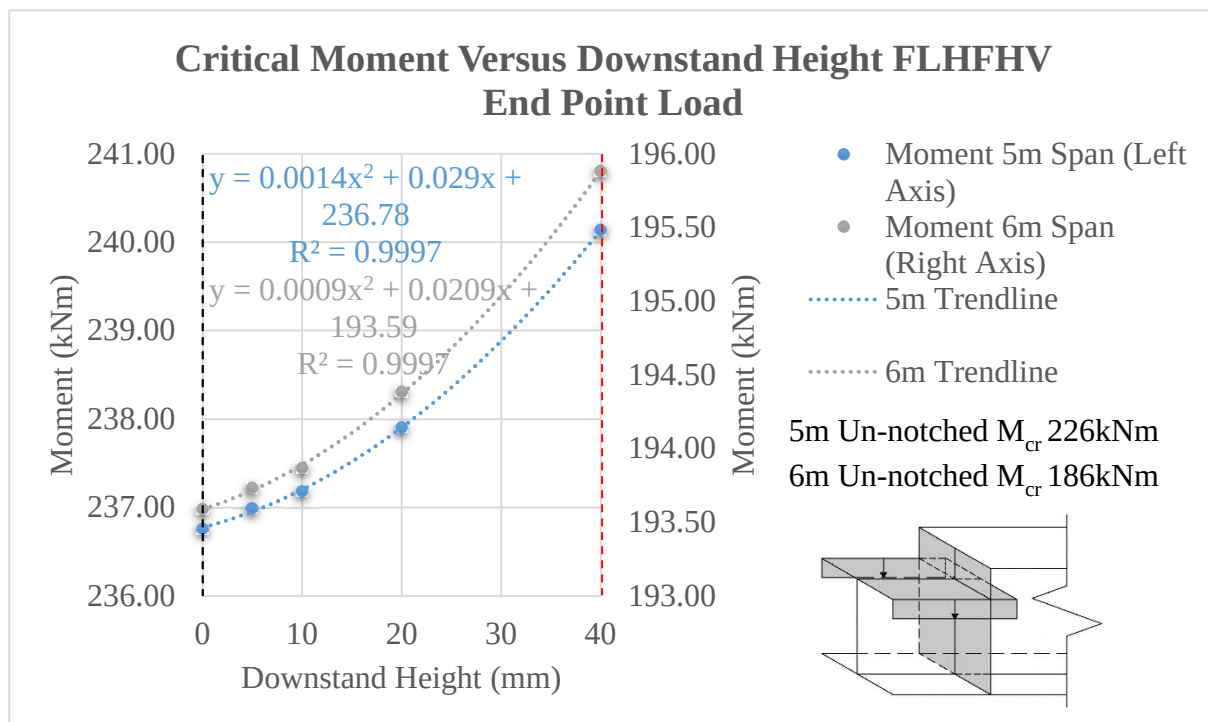


**Figure E.75: Critical Moment Versus Downstand Height FLHFHV Technique Model 2
203x203x46**

E.3 305 x 305 x 97 Section

E.3.1 Model 1

Figure E.76 shows the 1m span and the 3m span for the EPL case. It also shows the five stiffening techniques and by how much they restore the critical elastic moment to the notched section.

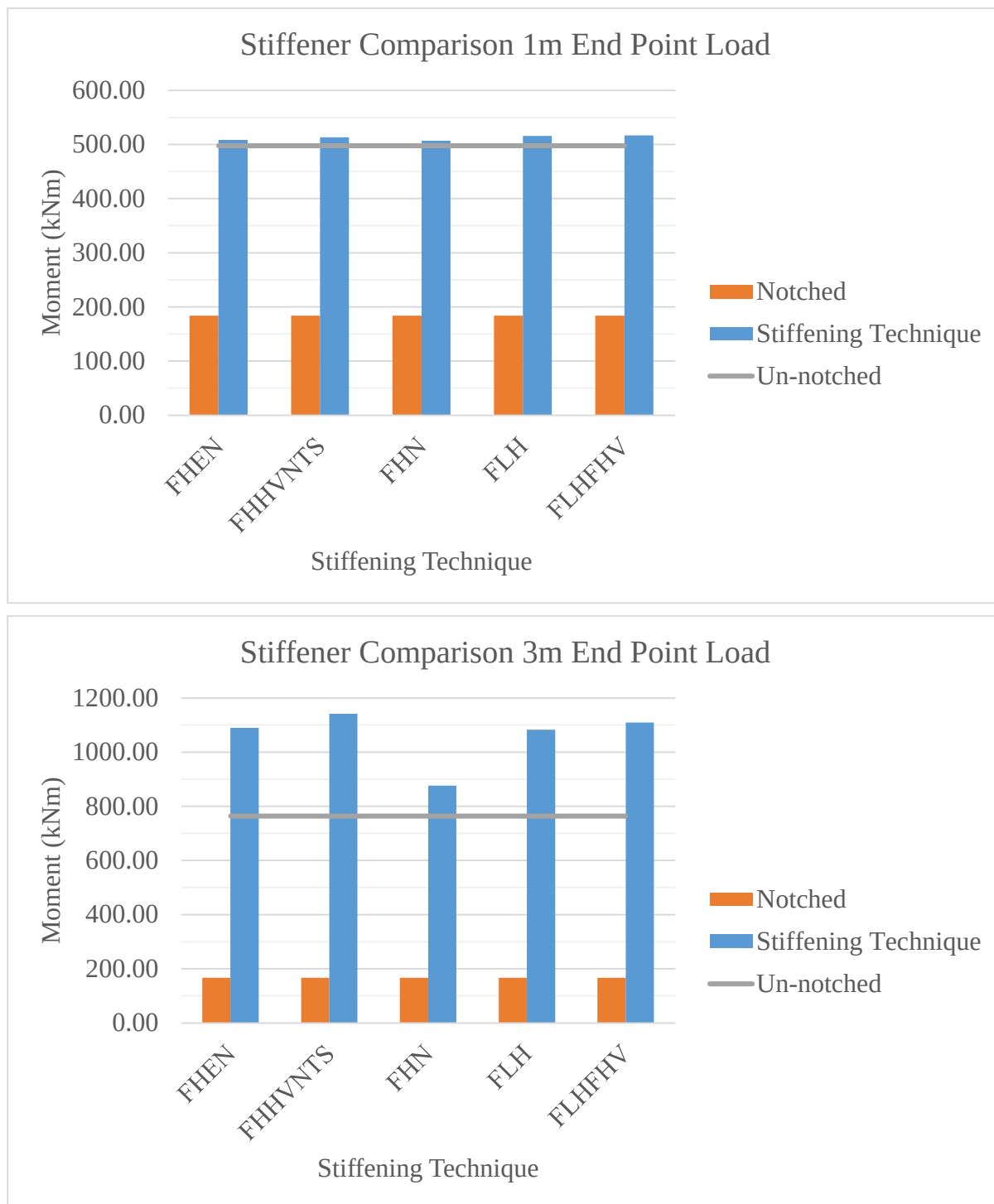


Figure E.76: Stiffener Comparison 1m, 3m EPL 305x305x97

Figure E.77 shows the 1m span and the 3m span for the MSL case. It also shows the five stiffening techniques and by how much they restore the critical elastic moment to the notched section.

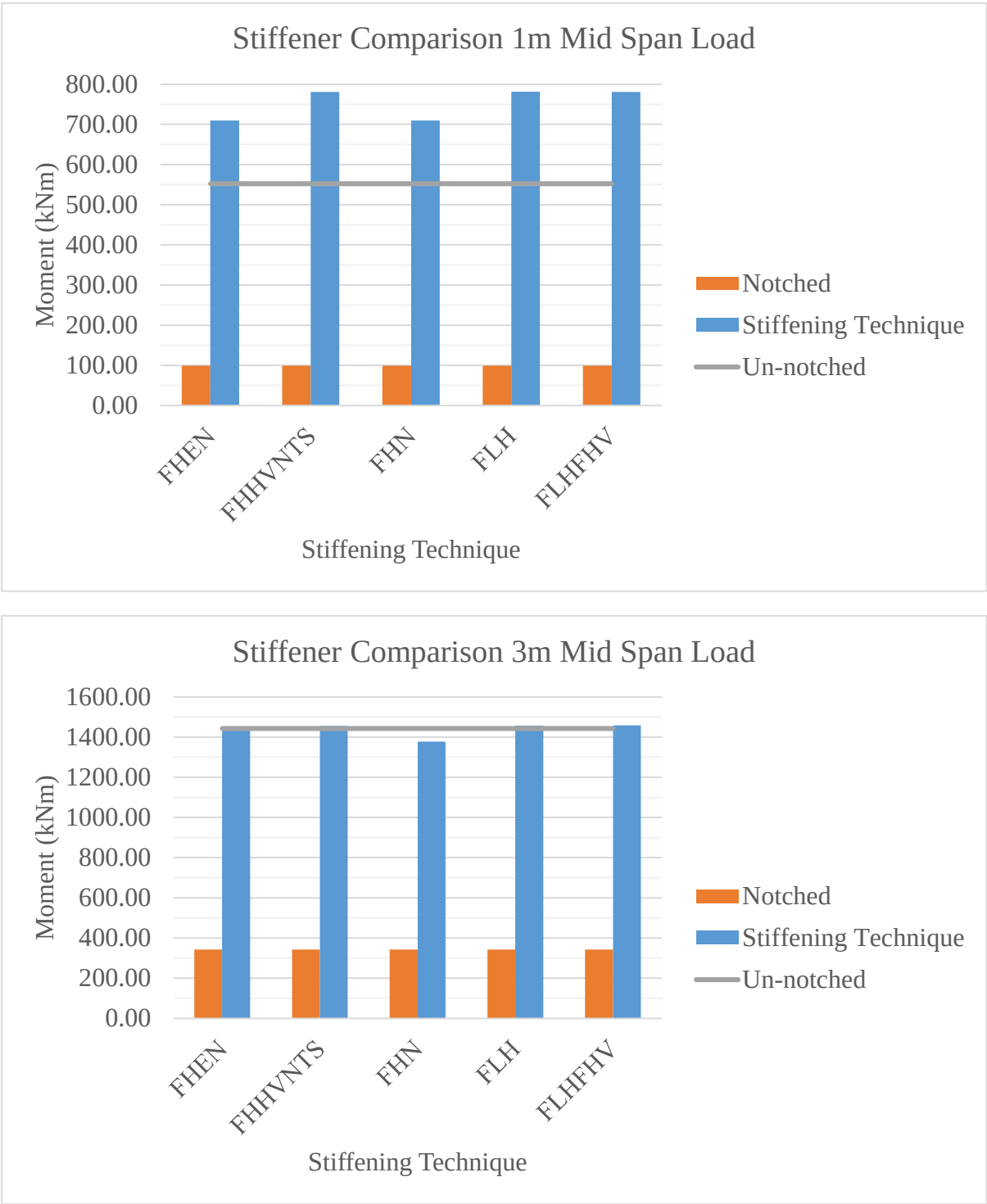


Figure E.77: Stiffener Comparison 1m, 3m MSL 305x305x97

E.3.1.1 FHEN Technique

Figure E.78 shows the critical elastic moment versus downstand height for the FHEN EPL case. There was an increase of 4.77kNm and 17.04kNm for the 1m and 3m spans (note the longer of the two spans increased more).

The trend is similar to the 533x210x82 FHEN case (**Figure E.27**) because both cases appear to approach a maximum value or asymptote. The curves start at a minimum value then increase parabolically to a maximum value. It is clear in both spans the maximum value occurs after the centroid shear centre coincidence point at 122mm downstand height but the turning points if these two spans are at different downstand heights. The longer span, 3m, occurs around the 125mm downstand height whereas the 1m span experiences a reduction in gradient with each increase in downstand height. This shows there is some influence on the trend due to the shifting centroid and shear centre.

Due to height restrictions this cannot be confirmed since 200mm is the largest height that can be fitted in the notched region.

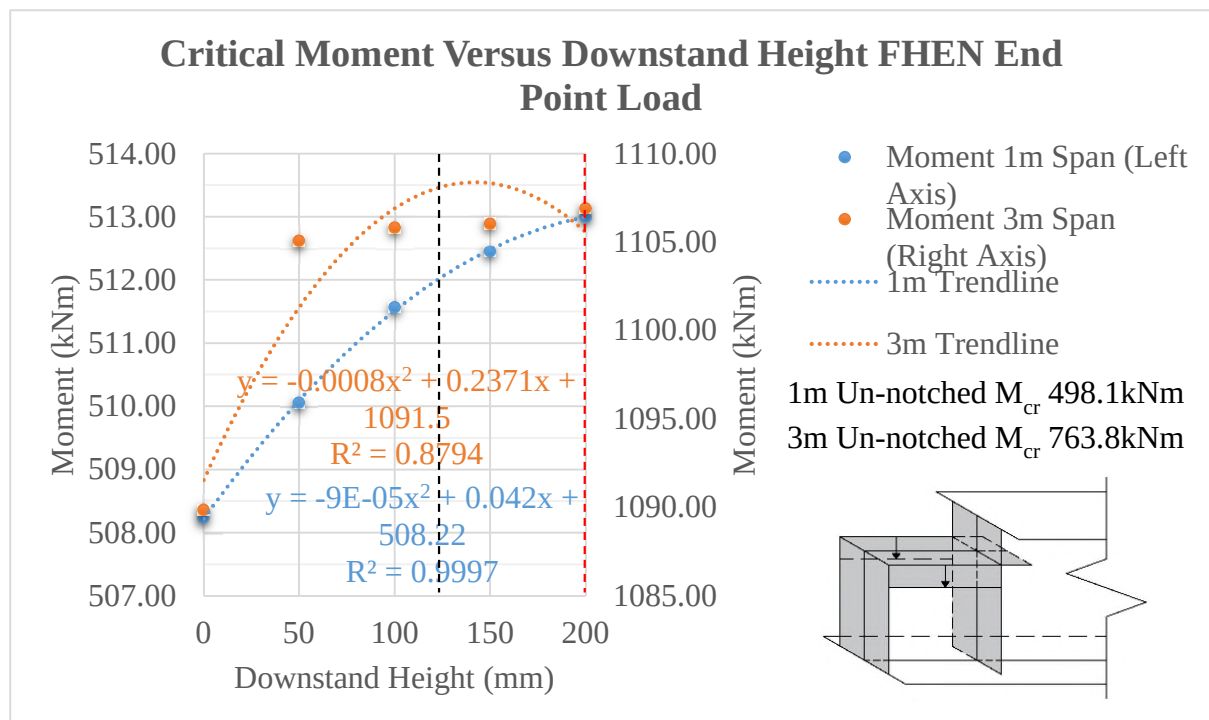


Figure E.78: Critical Moment Versus Downstand Height FHEN Technique EPL Model 1 305x305x97

Figure E.79 shows the results for the MSL FHEN case. There was an increase of 3.10kNm and 25.48kNm for the 1m and 3m spans and the longer span increased more than the shorter span which is opposite to the EPL case.

The trend is different between the spans: both spans increase parabolically however the 1m span starts at a minimum value with the gradient increasing with each increase in downstand height. The 3m span approaches a maximum value like the EPL case and the 533x210x82 section (**Figure E.28**). The trend is similar to the MSL FHN technique section (**Figure E.32** and the FLH EPL case (**Figure E.33**) for the 533x210x82.

It is interesting to see both spans have a change in the trend around the 125mm downstand height, close to the coincidence point of the centroid and shear centre at 122mm, despite the different behaviours of the two trends. The 3m span, just as in the EPL case (**Figure E.78**), may be approaching an asymptote and not a turning point but this cannot be confirmed due to height limitations in the notched region. What can be taken from these results is the shifting centroid and shear centre in the notched region does influence the trend.

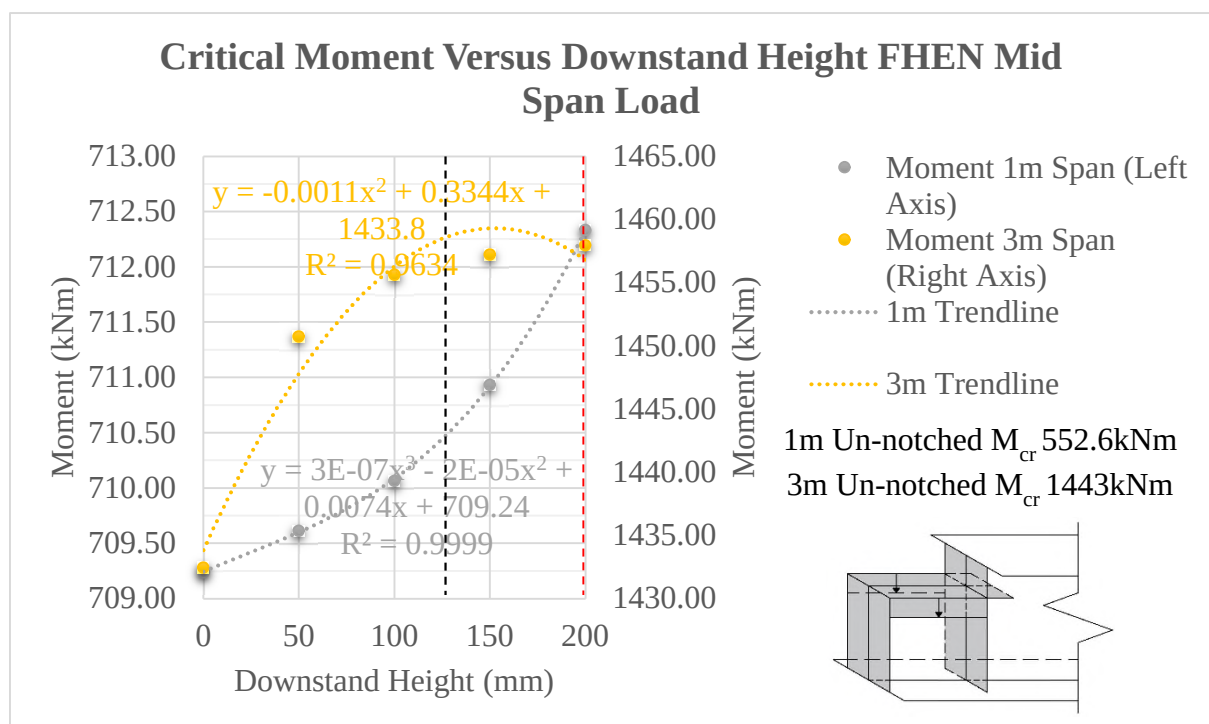


Figure E.79: Critical Moment Versus Downstand Height FHEN Technique MSL Model 1 305x305x97

E.3.1.2 FHHVNTS Technique

Figure E.80 shows the results for the EPL FHHVNTS technique. There was a negligible increase of 0.39kNm for the 1m span and a slight increase of 1.01kNm for the 3m span. The 3m span trend is slightly parabolic but it is difficult to see any change around the 18mm downstand height which is the downstand height at which the centroid and shear centre in the notched region coincide. The only change could be a change in gradient of the trendline at the 15mm downstand height since the first few downstand heights are approximately linear and the trend after the 15mm downstand height is also linear.

The parabolic trend is consistent across all span and loading configurations for all the different sections tested. This is very interesting since the sections are all very different. This shows the shifting centroid and shear centre have an impact regardless of the different sections, loading and span configurations when using the FHHVNTS technique.

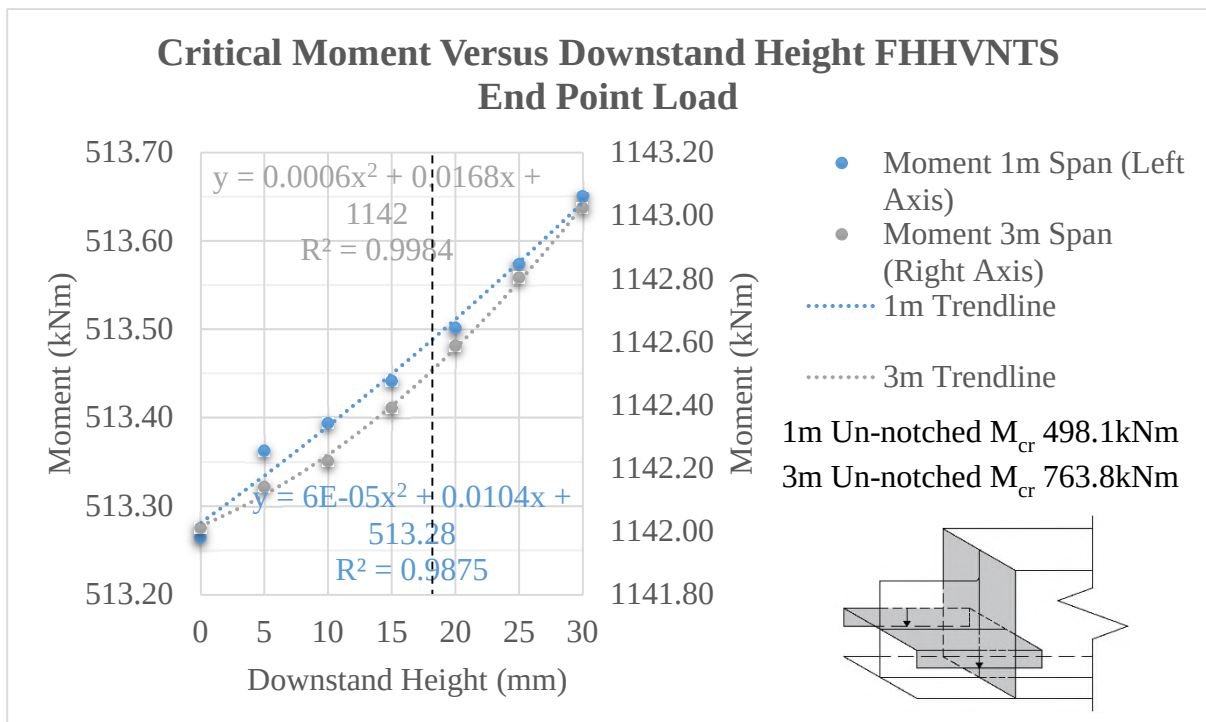


**Figure E.80: Critical Moment Versus Downstand Height FHHVNTS Technique EPL Model 1
305x305x97**

Figure E.81 shows the results for the MSL FHHVNTS case. There was a negligible increase of 0.33kNm for the 1m span and 1.11kNm for the 3m span which is the same as the EPL case. The 3m span is parabolic starting at a minimum value and increasing in gradient with each increase in downstand height. The trend is consistent with the EPL case. It is difficult to see if there is any change at the 18mm, but it can be said there is change in the curve type or gradient before and after the 18mm downstand height.

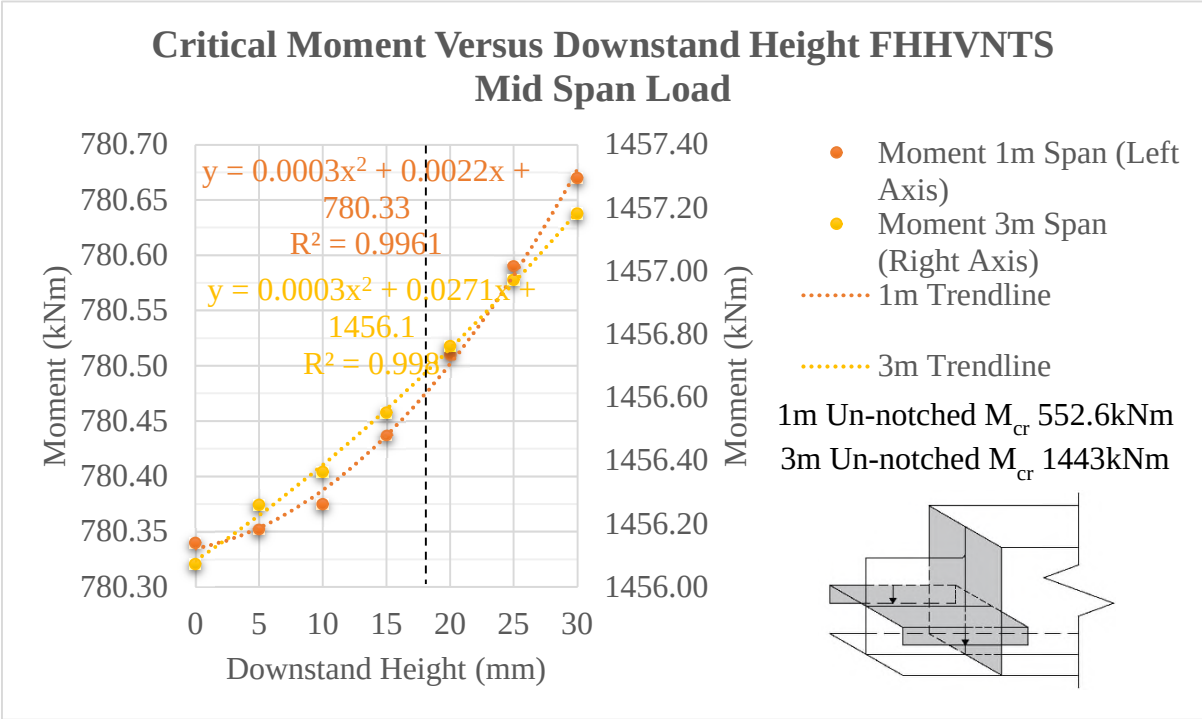


Figure E.81: Critical Moment Versus Downstand Height FHHVNTS Technique MSL Model 1 305x305x97

E.3.1.3 FHN Stiffening Technique

Figure E.82 shows the results for the FHN EPL case. The 1m and 3m span increased their critical elastic moment by 2.53kNm and 72.49kNm respectively. The trend is similar to the:

- FHHVNTS 203x133x25 section (**Figure 6.13**)
- 3m span FHN in the 533x210x82 section (**Figure E.31**)
- FHN 203x203x46 section (**Figure E.56**)

The curves start at a minimum value and increase parabolically with each increase in downstand height. There is no clear change around the centroid shear centre coincidence point at the 122mm downstand height. The trend does appear more parabolic after this downstand height or there is a change in gradient if the data is split into linear trends as “before the 122mm downstand” and “after the 122mm downstand”. It is clear that as the shear centre approaches and passes the centroid in the notched region the critical elastic moment increases more rapidly with each increase in downstand height which is evident through the steepening of the gradient of the curves.

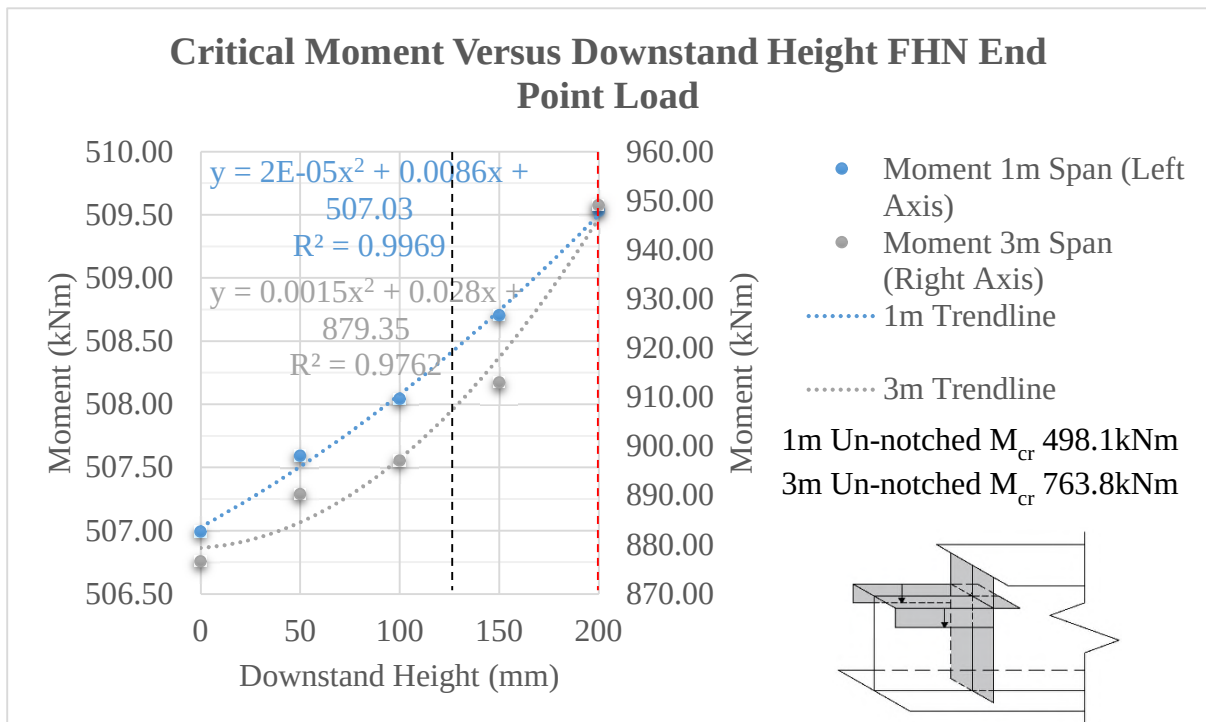


Figure E.82: Critical Moment Versus Downstand Height FHN Technique EPL Model 1 305x305x97

Figure E.83 shows the results for the MSL FHN case. There was an increase of 3.66kNm and 67.99kNm for the 1m and 3m spans respectively. Notice the longer span increased its critical elastic moment more than the shorter span.

The trend is not similar to the other sections with the FHN technique applied to them loaded at mid span. The other sections were parabolic either approaching a maximum value or starting at a minimum value with an ever-increasing gradient. They were never linear. With the trend being linear it is clear there is no changes around the 122mm downstand height. It can be said the influence of the shifting centroid and shear centre is still present because there is an increase in the critical elastic moment with each increase in downstand height.

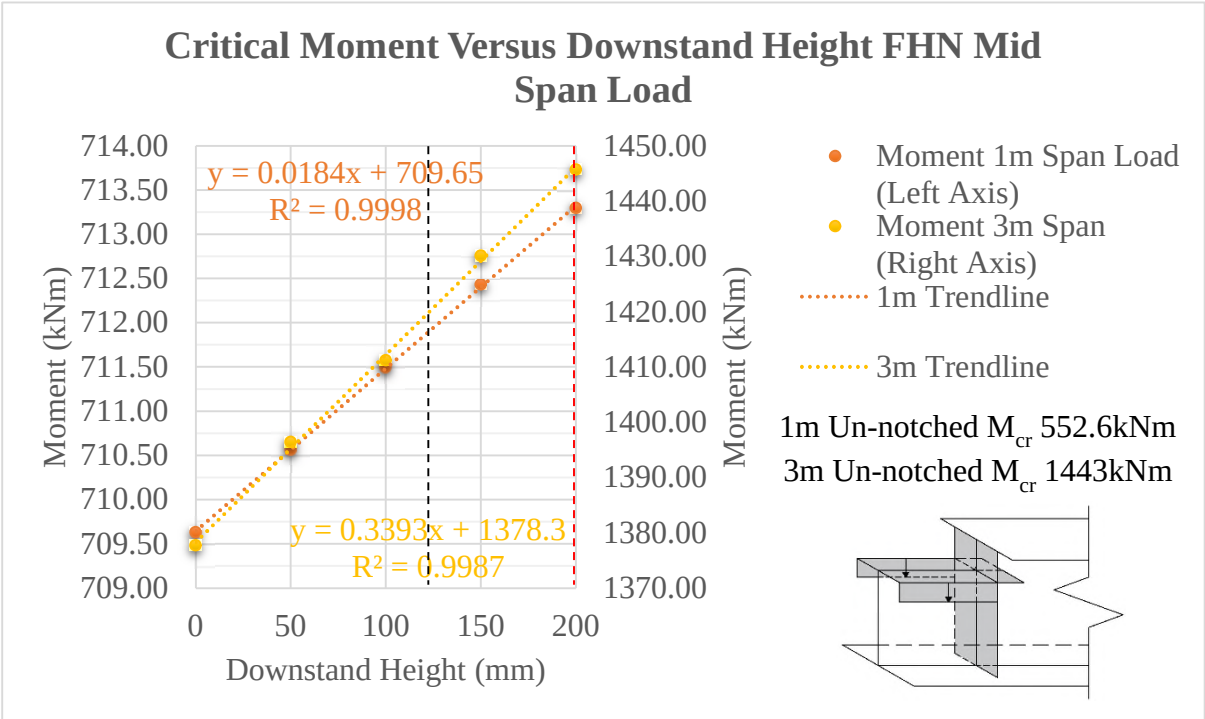


Figure E.83: Critical Moment Versus Downstand Height FHN Technique MSL Model 1 305x305x97

E.3.1.4 FLH Technique

Figure E.84 shows the results for the EPL FLH case. The 1m span increased its critical elastic moment by 0.62kNm whereas the 3m span, which is considered an outlier since it is very different to all other FLH cases, increased its critical elastic moment by 12.33kNm with the introduction of the 50mm downstand but decreased its critical elastic moment by 21.95kNm with the increase in the downstand heights.

The 1m span is similar to the 1m EPL case of the 533x210x82 section (**Figure E.33**) because it seems to approach a maximum value in a disjointed parabolic manner. No clear change occurs around the 122mm downstand height for either span.

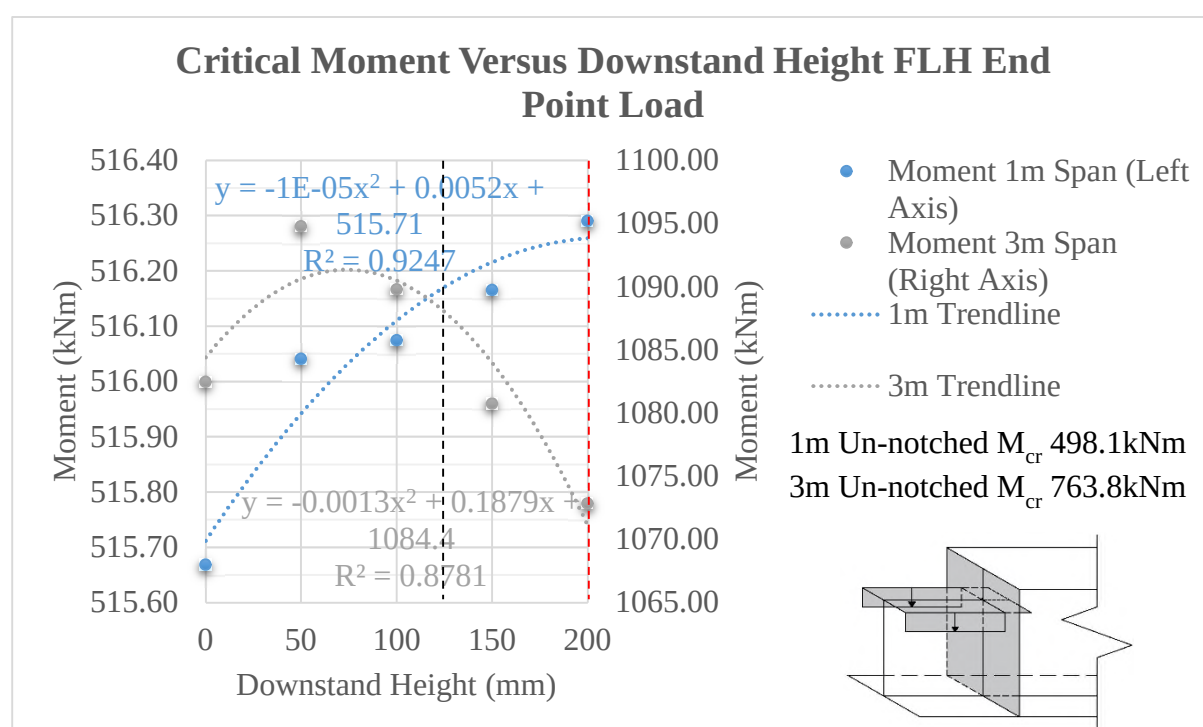


Figure E.84: Critical Moment Versus Downstand Height FLH Technique EPL Model 1 305x305x97

Figure E.85 shows the results for the MSL FLH case. There was an increase in the critical elastic moment for the 1m and 3m spans of 0.56kNm and 2.78kNm respectively. The trends are similar to the 203x203x46 section (**Figure E.59**) but here the 1m span has a marginal increase whereas the 203x203x46 had a negligible increase. This trend is unique to the H sections.

The 1m span approaches a minimum value at the 100mm downstand but increases thereafter. It is unclear if the turning point of the parabola is at a downstand height of 122mm, the downstand height at which the centroid and shear centre in the notched region coincide, because that downstand height was not tested. It is clear after the 125mm downstand height, the gradient is steeper than moving backwards from 100mm to 0mm. The 3m span is also similar to the 3m span of the 533x210x82 section (**Figure E.34**) since it starts at a minimum value and appears to approach a maximum value.

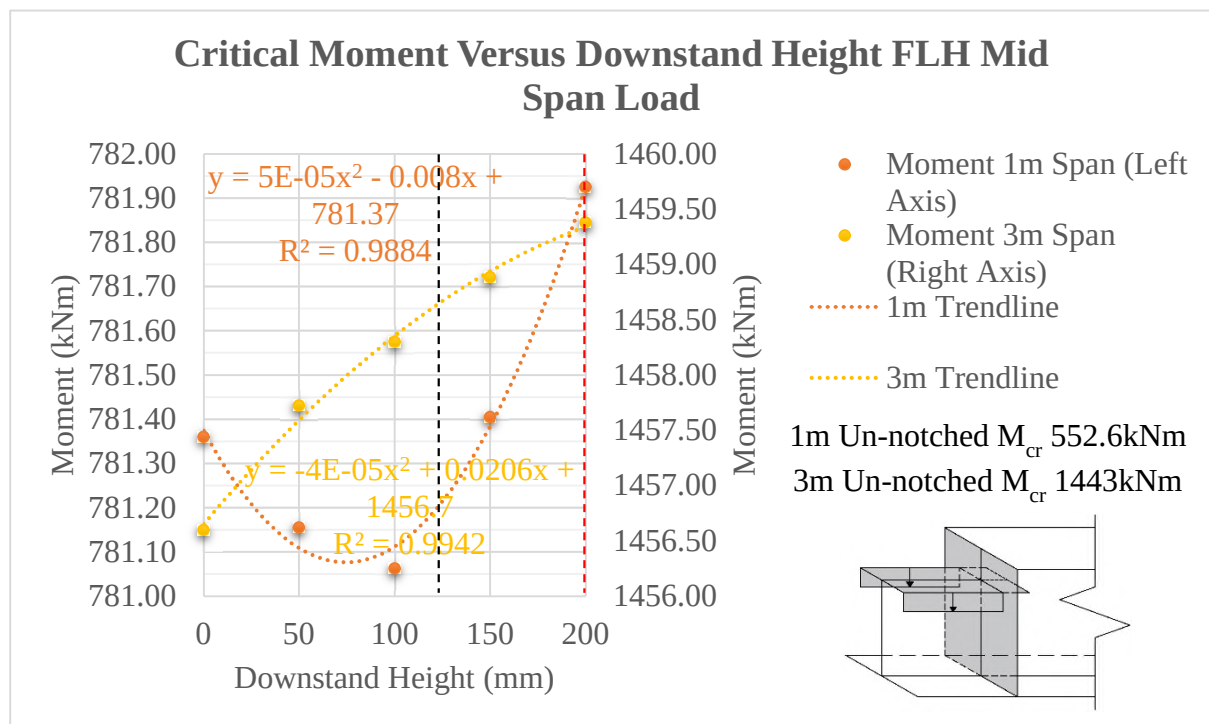


Figure E.85: Critical Moment Versus Downstand Height FLH Technique MSL Model 1 305x305x97

E.3.1.5 FLHFHV Technique

Figure E.86 shows the results of the FLHFHV technique for the EPL case. The section decreased its critical elastic moment moving from the upstands to the no downstand/upstands at stand height of 0mm by 0.86kNm for the 1m span. The introduction of downstands then increased the critical elastic moment by a negligible 0.06kNm.

The 3m span also decreased its critical elastic moment (by 26.06kNm) then negligibly decreased with the downstands (0.48kNm). The behaviour here is unlike any of the trends in the other techniques previously discussed and is only similar to the FLHFHV technique for the other sections. As the shear centre approaches the centroid in the notched section by reducing the height of the upstands, the critical elastic moment reduces and only increases, if by a negligible amount, once downstands are introduced. This stiffening technique could be considered an outlier to the other techniques.

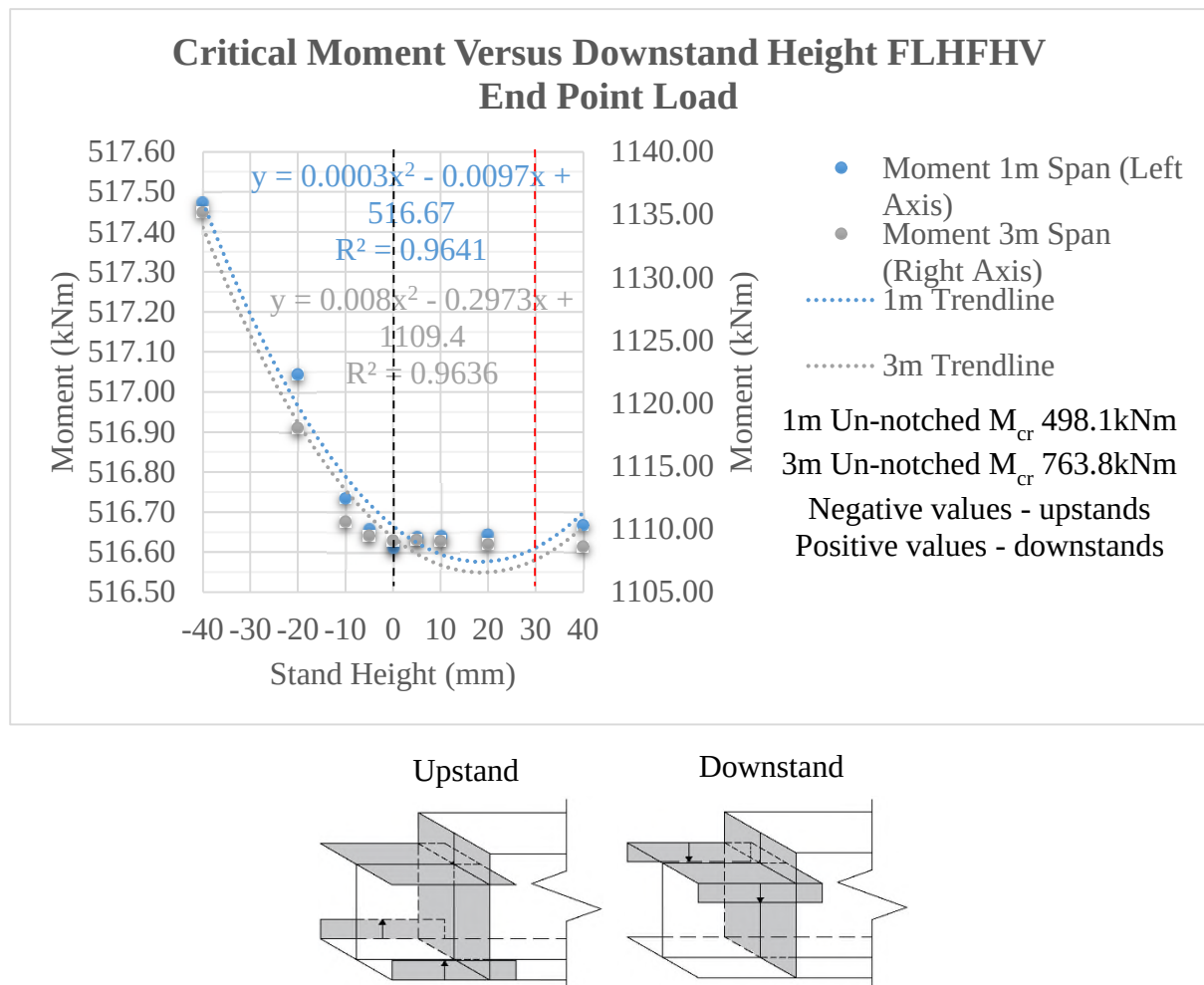


**Figure E.86: Critical Moment Versus Stand Height FLHFHV Technique EPL Model 1
305x305x97**

Figure E.87 shows the results of the FLHFHV technique for the EPL case. The section decreased its critical elastic moment moving from the upstands to the no downstand/upstands at stand height of 0mm by 4.32kNm for the 1m span. The introduction of downstands then decreased the critical elastic moment by a negligible 0.08kNm.

The 3m span also decreased its critical elastic moment (by 2.09kNm) then increased with the downstands (0.25kNm which is negligible). The behaviour here is unlike any of the trends in the other techniques previously discussed and is only similar to the FLHFHV technique for the other sections. As the shear centre approaches the centroid in the notched section by reducing the height of the upstands, the critical elastic moment reduces and only increases, if by a negligible amount, once downstands are introduced. This stiffening technique could be considered an outlier to the other techniques.

The graphs showing the isolated downstand are not shown since the total change in the critical elastic moment is less than 0.5kNm.

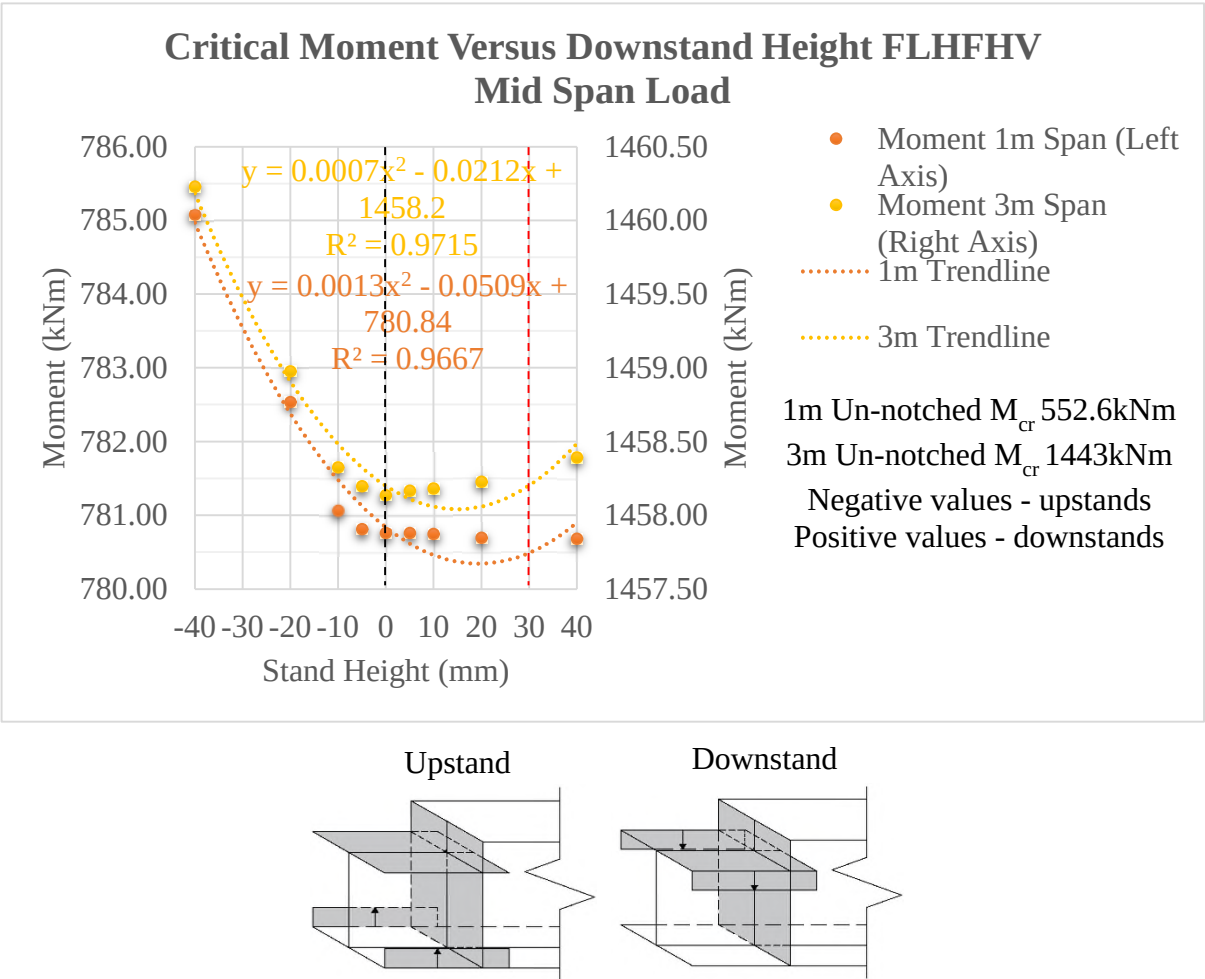


Figure E.87: Critical Moment Versus Stand Height FLHFHV Technique MSL Model 1 305x305x97

E.3.2 Model 2

Figure E.88 shows the 5m span and the 6m span for the EPL case. It also shows the five stiffening techniques and by how much they restore the critical elastic moment to the notched section.

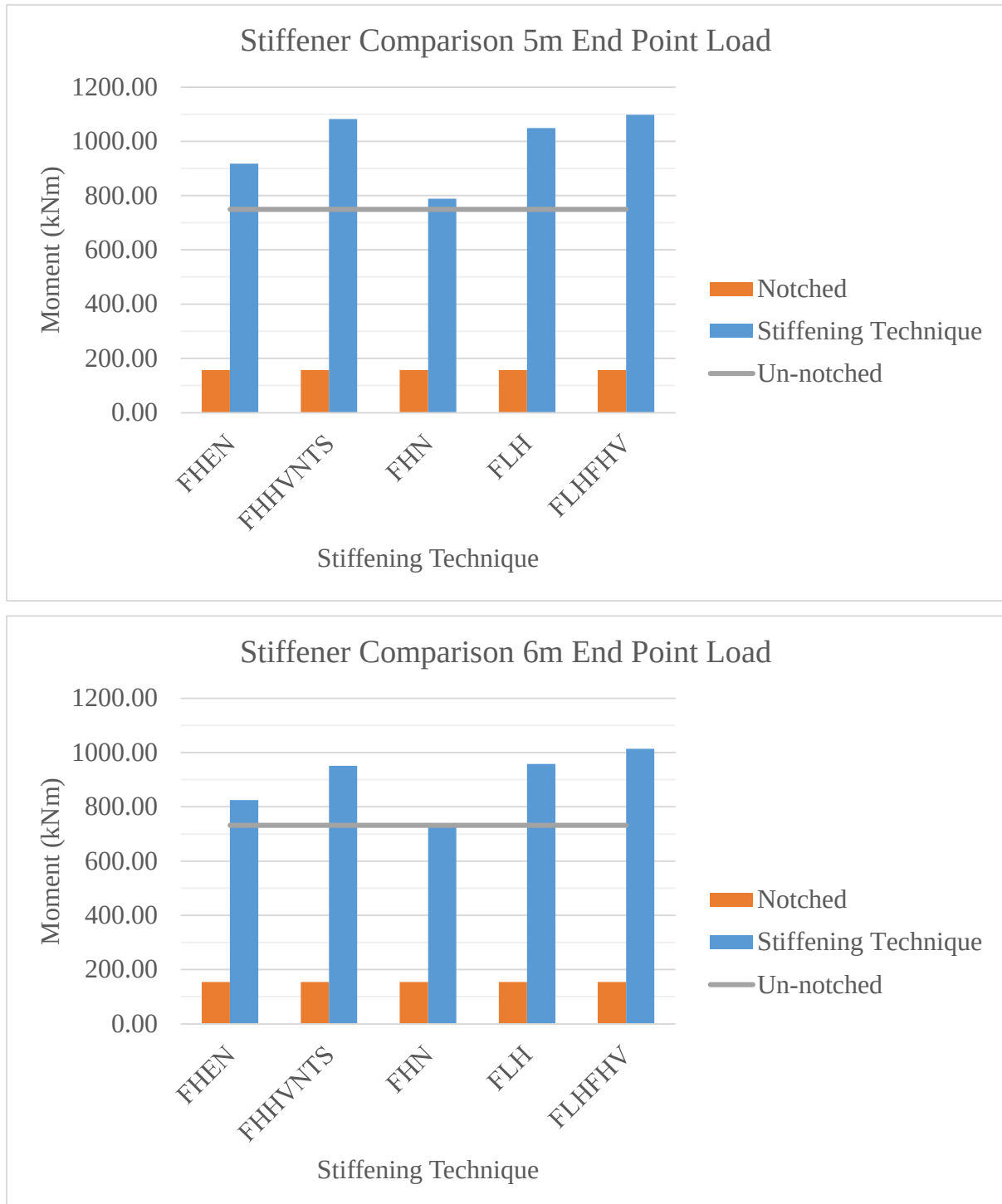


Figure E.88: Stiffener Comparison 5m, 6m EPL 305x305x97

Figure E.89 shows the 5m span and the 6m span for the MSL case. It also shows the five stiffening techniques and by how much they restore the critical elastic moment to the notched section.



Figure E.89: Stiffener Comparison 5m, 6m MSL 305x305x97

E.3.2.1 FHEN Technique

Figure E.90 shows the critical elastic moment versus downstand height for the EPL FHEN case. The 5m span increased by 243.42kNm and the 6m span increased by 284.79kNm (note the longer span increased more).

The trend is similar to the 533x210x82 section (**Figure 6.39**): the curves start at a minimum value and increase parabolically with each increase in downstand height. It is also similar because the change in the curve appear around the centroid shear centre coincidence point at a downstand height of 122mm. The trend is strong between the spans and they appear to approach either turning point or asymptote.

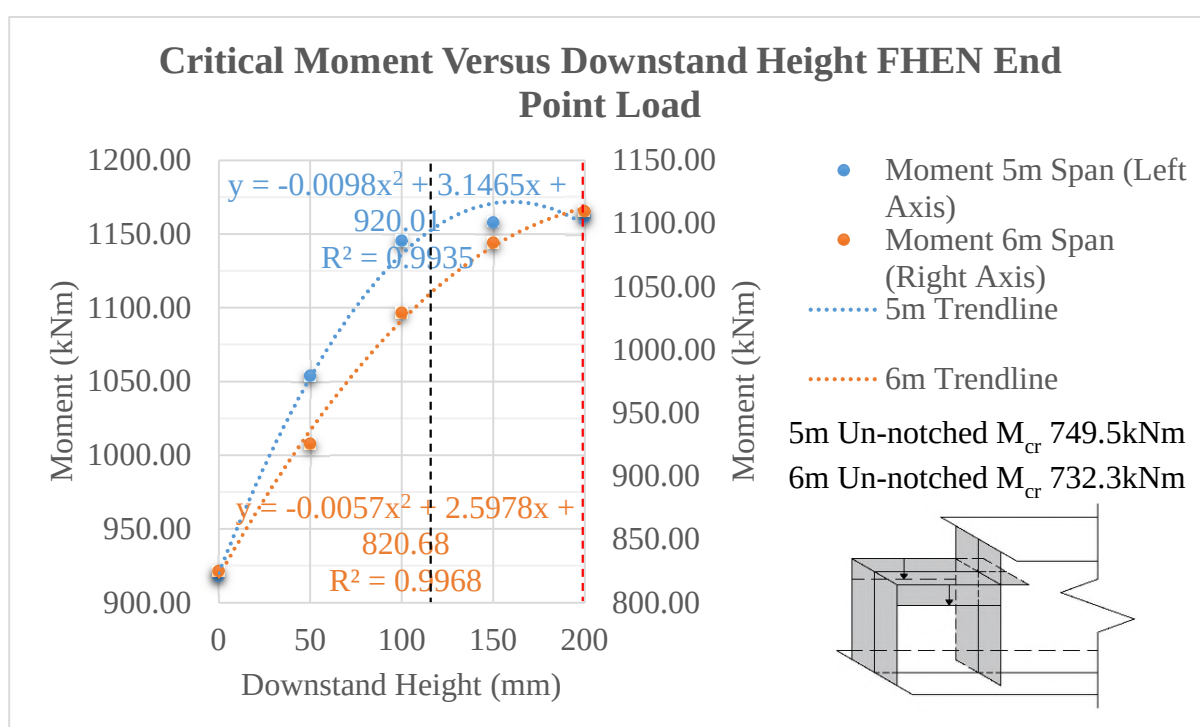


Figure E.90: Critical Moment Versus Downstand Height FHEN Technique EPL Model 2 305x305x97

Figure E.91 shows the results for the MSL FHEN case. The 5m and 6m spans increased their critical elastic moment by 145.92kNm and 90.27kNm respectively. The shorter span increased its critical elastic moment more than the longer span (which is opposite to the EPL case).

The trend is not as parabolic as the 533x210x82 section (**Figure E.40**), but there appears to be a slight decrease in gradient with each increase in downstand height. It is not clear if this occurs at the 122mm downstand height, the coincidence point of the centroid and shear centre in the notched region. The trend is strong between the two spans and just as in the EPL case, it appears the curves approach a turning point or asymptote after the 122mm downstand height.

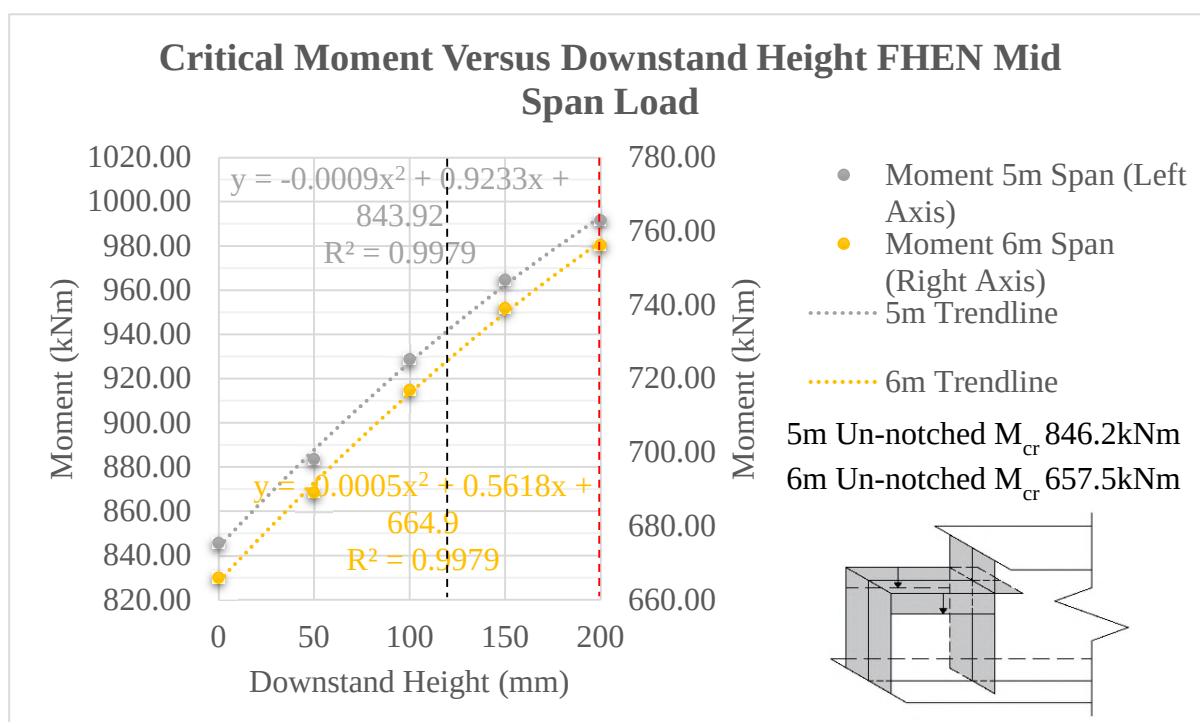


Figure E.91: Critical Moment Versus Downstand Height FHEN Technique MSL Model 2 305x305x97

E.3.2.2 FHHVNTS Technique

Figure E.92 shows the results for the EPL FHHVNTS case. There was an increase of 28.99kNm for the 5m span and 22.58kNm for the 6m span (notice the shorter span increased more).

The trend is similar to all the other sections: the curves start at a minimum value and increase parabolically (here the parabolic curve is slight) with each increase in downstand height. There is no clear change in trend around the 18mm downstand height, the downstand height at which the centroid and shear centre in the notched region coincide, but it is evident the gradient of the curves is increasing as the shear centre approaches and passes the centroid. This trend is evident in almost all the FHHVNTS cases.

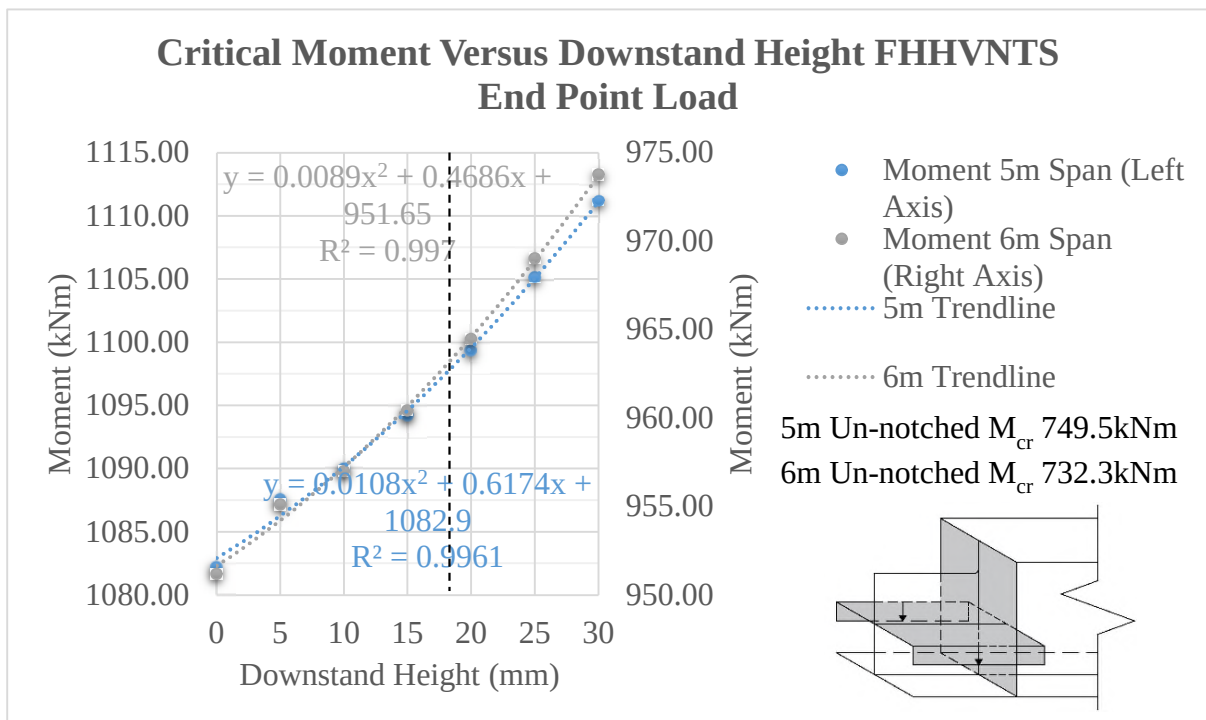


Figure E.92: Critical Moment Versus Downstand Height FHHVNTS Technique EPL Model 2 305x305x97

Figure E.93 shows the critical elastic moment versus downstand height for the MSL FHHVNTS case. There was an increase of 10.38kNm and 6.43kNm for the 5m and 6m spans respectively. Once again, the shorter of the two spans increased its critical elastic moment more. The trend is strong between the two spans, there are no outliers, and is similar to the EPL case. There is no obvious change in behaviour around the 18mm downstand height, just as in the EPL case, but the influence of the shifting centroid and shear centre is evident as the curves start at a minimum value and increase parabolically with each increase in downstand height.

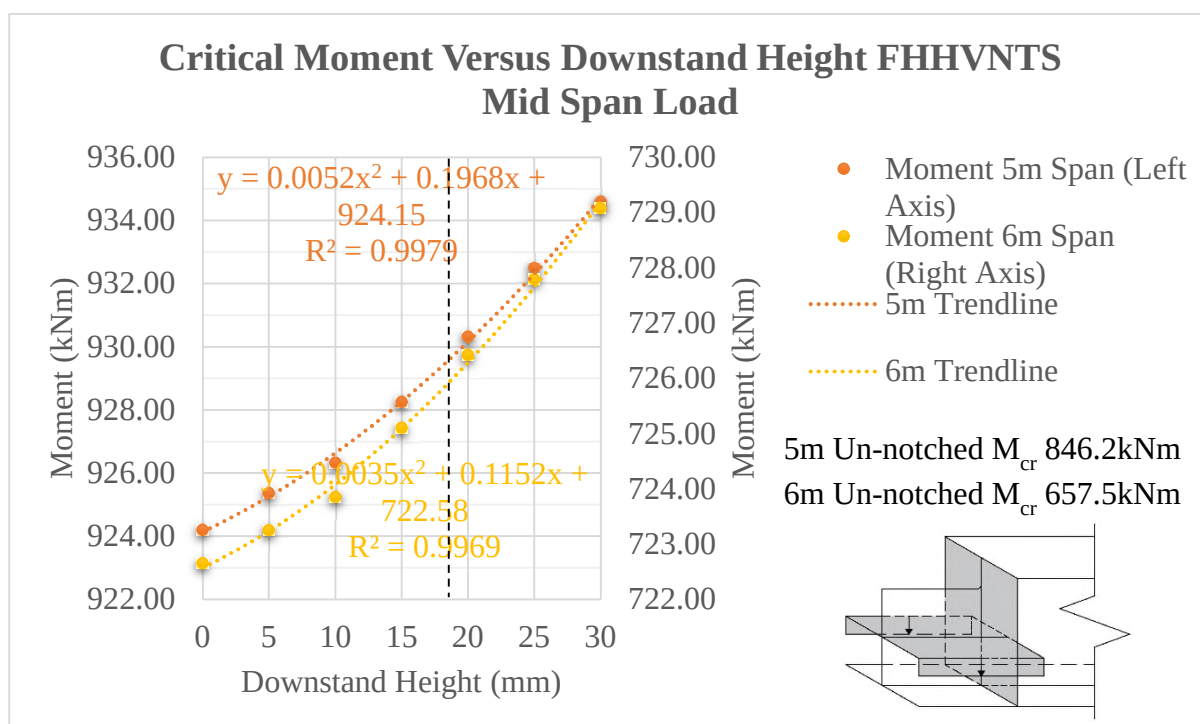


**Figure E.93: Critical Moment Versus Downstand Height FHHVNTS Technique MSL Model 2
305x305x97**

E.3.2.3 FHN Technique

Figure E.94 shows the results for the EPL FHN case. The 5m and 6m spans increased their critical elastic moment by 94.36kNm and 89.06kNm (note the shorter of the two spans increased its critical elastic moment more).

The trend is similar to the other sections (and the FHHVNTS technique): the curves start at a minimum value and increase parabolically with each increase in downstand height. There is no obvious change around the 122mm downstand height, but it appears the gradient increases with each increase in downstand height. This is how the other sections responded which is very interesting because it shows the influence of the shifting centroid and shear centre in the notched region influences the critical elastic moment regardless of the type of section, notch details, loading and span configurations.

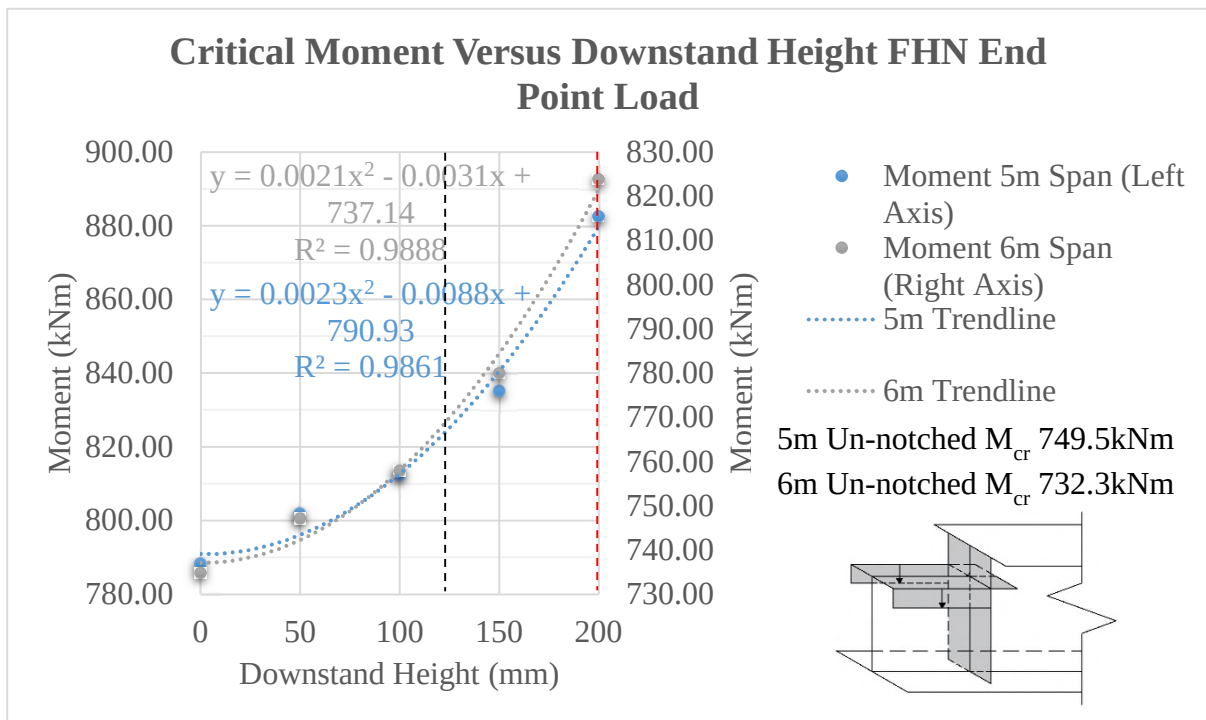


Figure E.94: Critical Moment Versus Downstand Height FHN Technique EPL Model 2 305x305x97

Figure E.95 shows the results for the FHN MSL case. There was an increase in the critical elastic moment of 44.20kNm and 27.46kNm for the 5m and 6m spans. Once again, the shorter of the two spans increased its critical elastic moment more than the longer span.

The trend is very similar to the other sections and the EPL case in **Figure E.94**. The trend is parabolic starting at a minimum value. There is no obvious change in the trend at the 122mm downstand height, but the influence of the shifting centroid and shear centre is clear: as the shear centre approaches and passes the centroid in the notched region the critical elastic moment also increases.

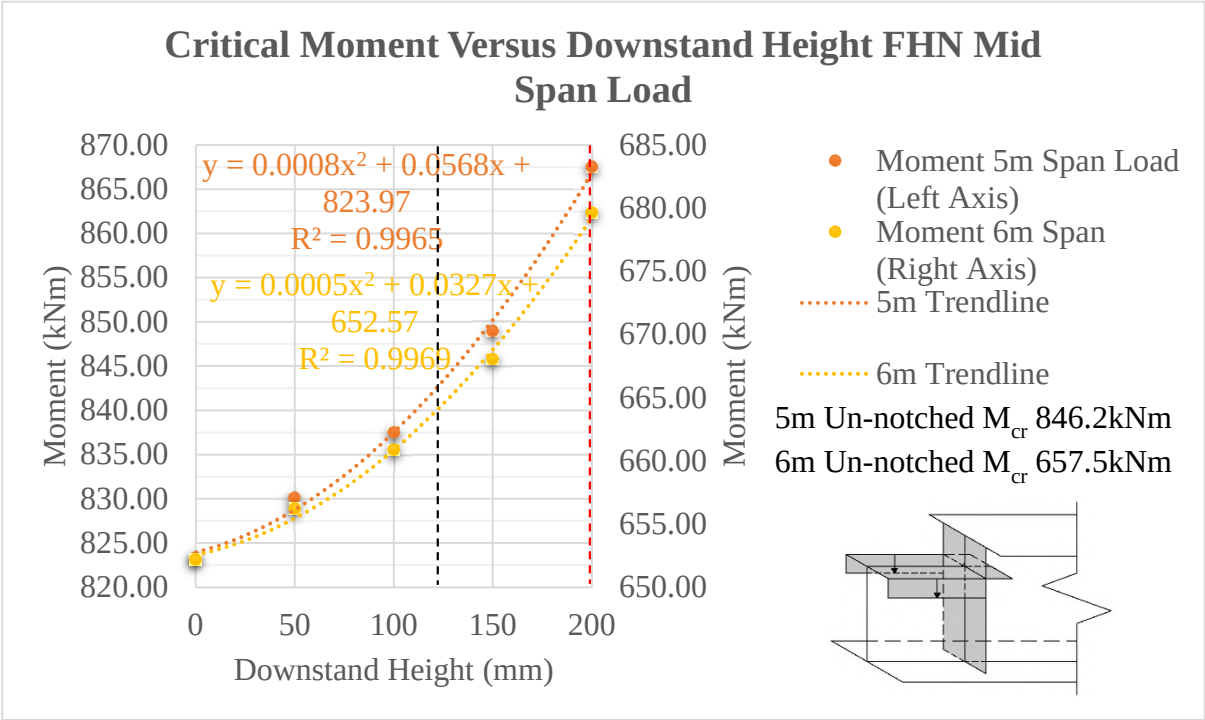


Figure E.95: Critical Moment Versus Downstand Height FHN Technique MSL Model 2 305x305x97

E.3.2.4 FLH Technique

Figure E.96 shows the results for the EPL FLH case. There was an increase of 20.58kNm with the introduction of the 50mm downstand but then a total decrease of 6.20kNm at the 200mm downstand for the 5m span. The 5m trend is only similar to the 3m spans tested in Model 1 (**Figure E.84**) so it can be considered unique to the 305x305x97 section and is an outlier.

There was an increase of 54.15kNm for the 6m. The trend is parabolic starting at a minimum value and increasing with each increase in downstand height. The trend seems to approach a maximum value or asymptote after the 122mm downstand height.

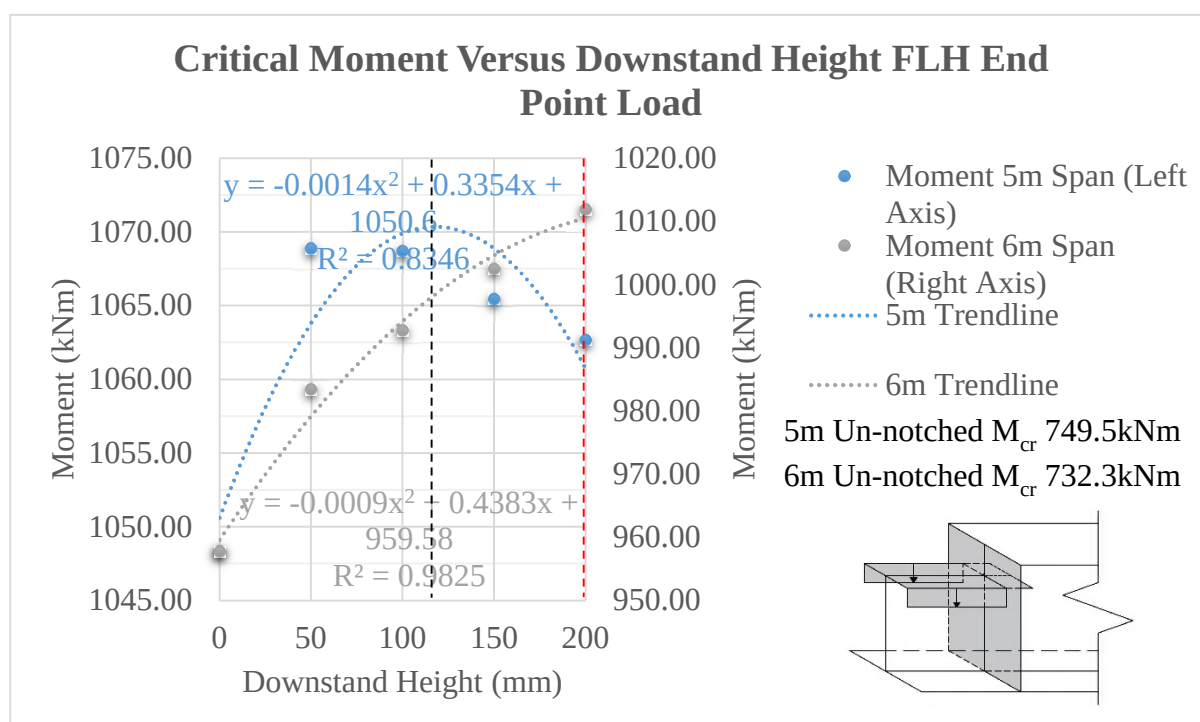


Figure E.96: Critical Moment Versus Downstand Height FLH Technique EPL Model 2 305x305x97

Figure E.97 shows the results for the MSL FLH technique. The 5m and 6m spans increased their critical elastic moment by 47.15kNm and 31.69kNm respectively (the shorter of the two spans increased more). The trend is similar to the other sections and unlike the EPL case: it is not similar to Model 1. The trend is similar to the EPL case (**Figure E.45**) and MSL case (**Figure 6.46**) of the 533x210x82 section.

The trend is almost perfectly linear with a slight change in gradient when moving from the 150mm downstand to the 200mm downstand. Due to height restrictions downstand heights beyond 200mm cannot be tested to see if this trend continues. It is also difficult to see any changes around the 122mm downstand height, but it is clear with each increase in downstand height which causes the shear centre to move closer and eventually passed the centroid in the notched region, the critical elastic moment increases.

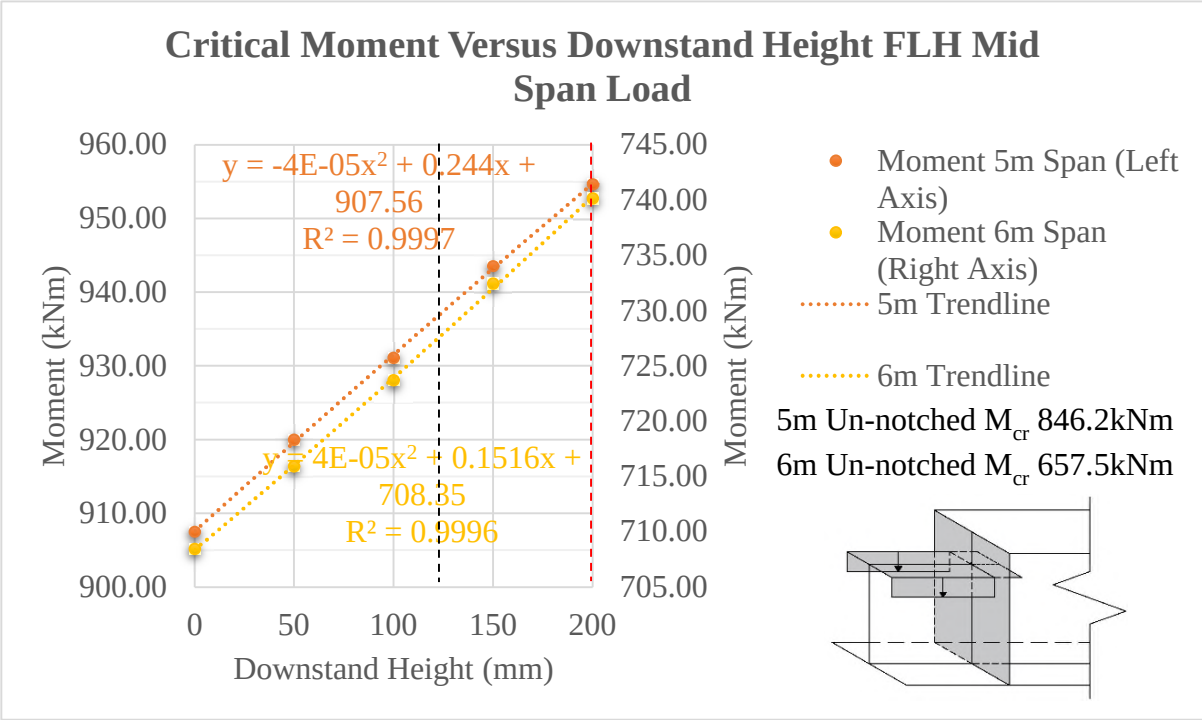


Figure E.97: Critical Moment Versus Downstand Height FLH Technique MSL Model 2 305x305x97

E.3.2.5 FLHFHV Technique

Figure E.98 shows the results of the FLHFHV technique for the EPL case. The section decreased its critical elastic moment moving from the upstands to the no downstand/upstands at stand height of 0mm by 47.57kNm for the 5m span. The introduction of downstands then increased the critical elastic moment by a negligible 0.25kNm. The 6m span also decreased its critical elastic moment by 60.73kNm then increased it with the downstands by 6.22kNm. The trend is evident in both models for all other sections, but they are unique to the FLHFHV technique.

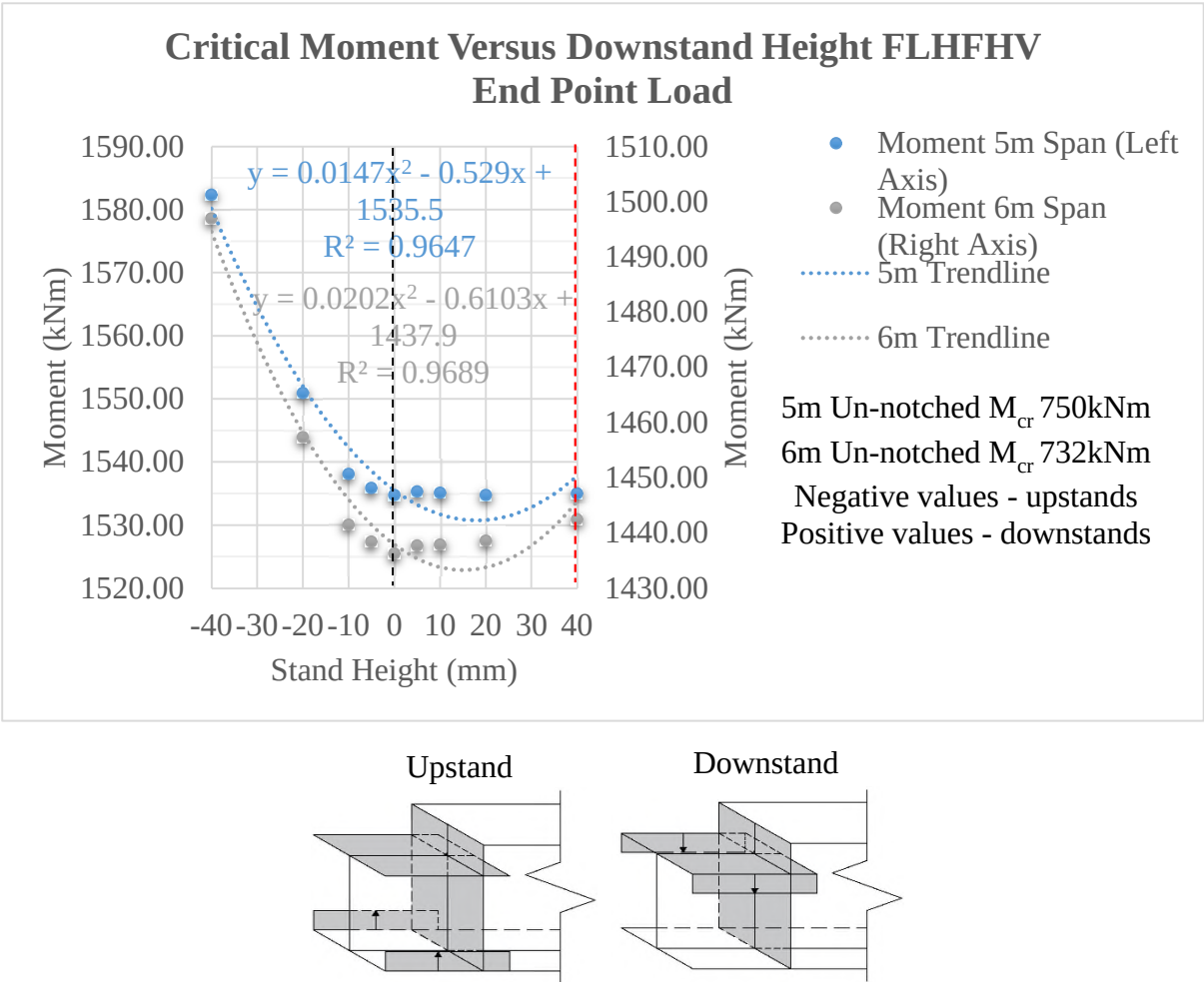


Figure E.98: Critical Moment Versus Stand Height FLHFHV Technique EPL Model 2
305x305x97

Figure E.99 shows the results of the FLHFHV technique for the MSL case. The section decreased its critical elastic moment moving from the upstands to the no downstand/upstands at stand height of 0mm by 34.51kNm for the 5m span. The introduction of downstands then increased the critical elastic moment by 5.97kNm. The 6m span also decreased its critical elastic moment by 24.25kNm then increased it with the downstands by 4.42kNm. Again, just as in the EPL case (**Figure E.98**, Model 1 and in all models in the previous sections, the trends are unique to the FLHFHV technique.

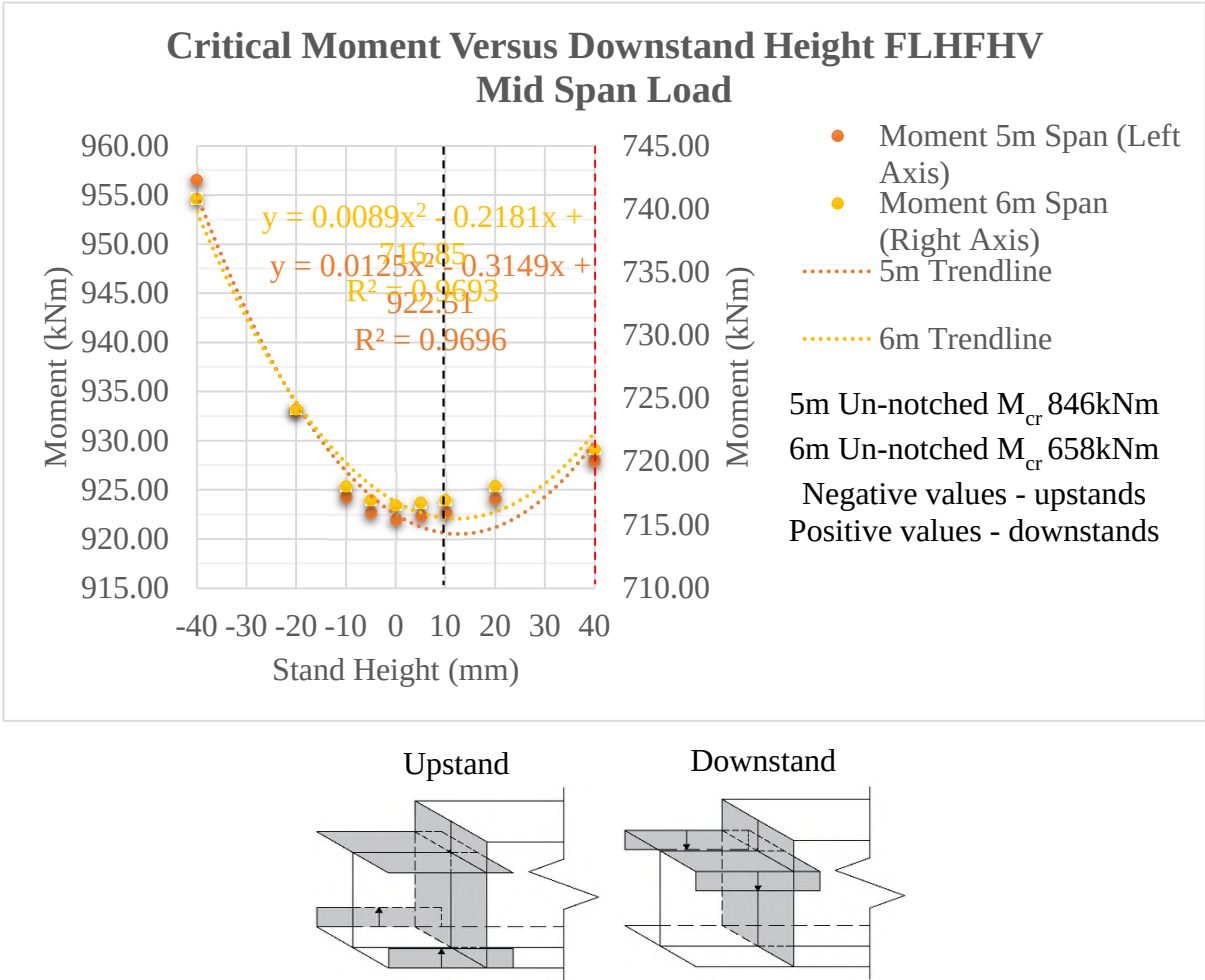


Figure E.99: Critical Moment Versus Stand Height FLHFHV Technique MSL Model 2 305x305x97

Just as in the 203x133x25 section, by isolating the downstand case it is clear the trend when the shear centre is above the centroid is similar to all the other sections as shown in **Figure E.100**. That is: there is an increase in the critical elastic moment as the shear centre moves further above the centroid. The EPL graph showing the isolated downstand does not show the 5m span since the total change in the critical elastic moment is less than 0.5kNm.

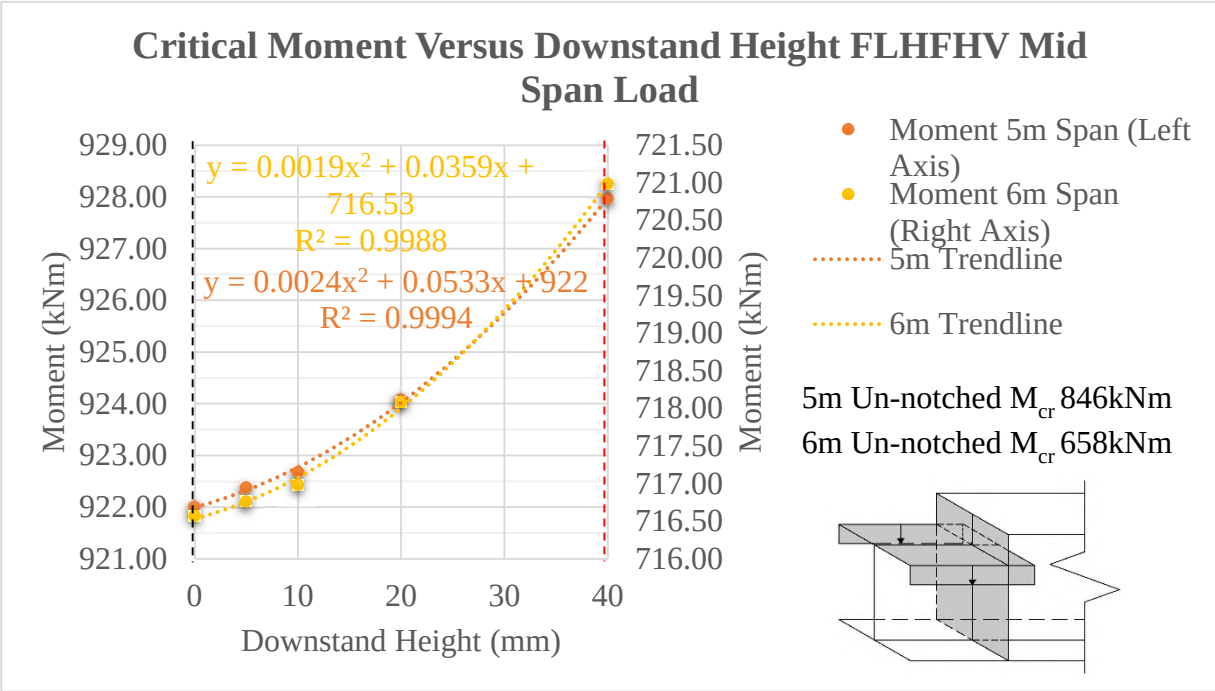
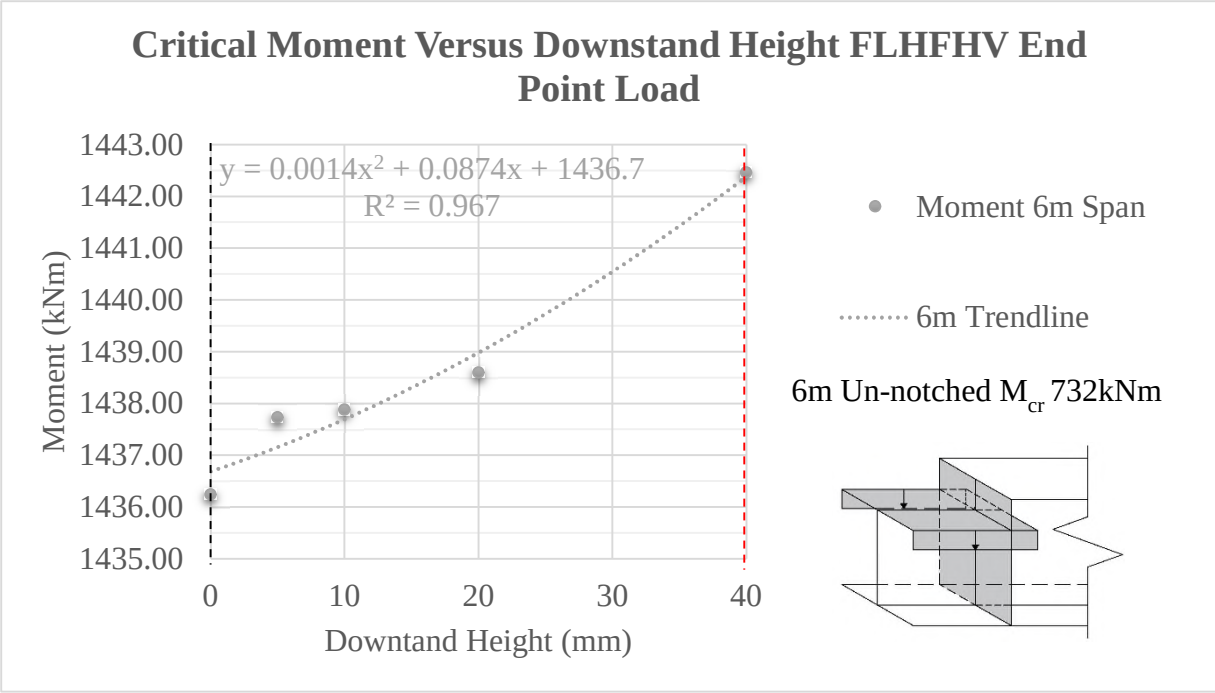


Figure E.100: Critical Moment Versus Downstand Height FLHFHV Technique Model
 2 305x305x97

Appendix F: Buckled Shapes

Below are images of the failure of the 533x210x82 section for the 250mm and 350mm downstand heights from the FHEN technique 5m span EPL case from Model 2.

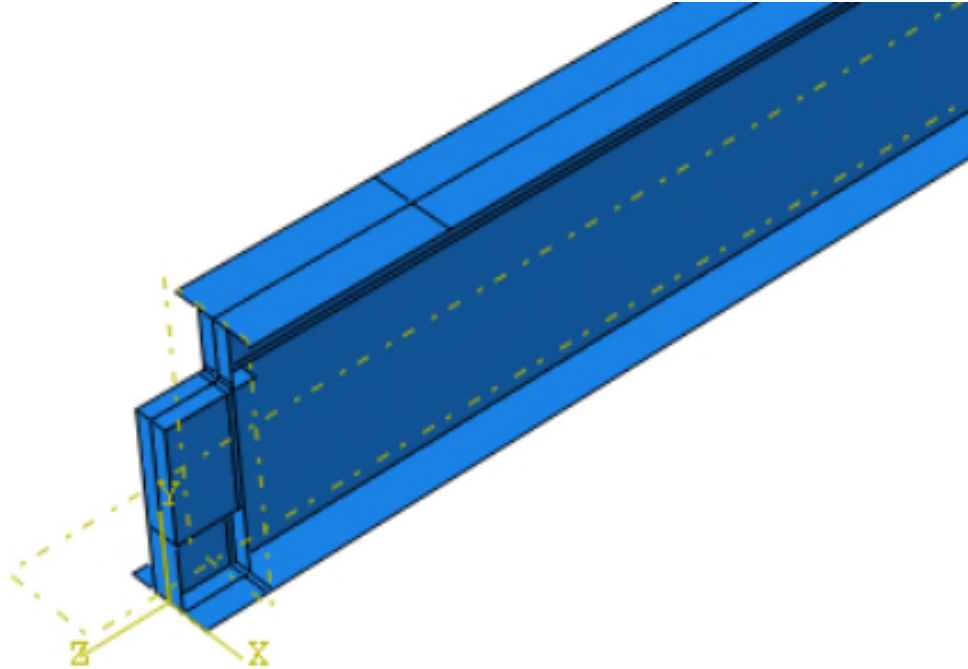


Figure F.1 Closeup of Stiffening Technique for the 250mm Downstand

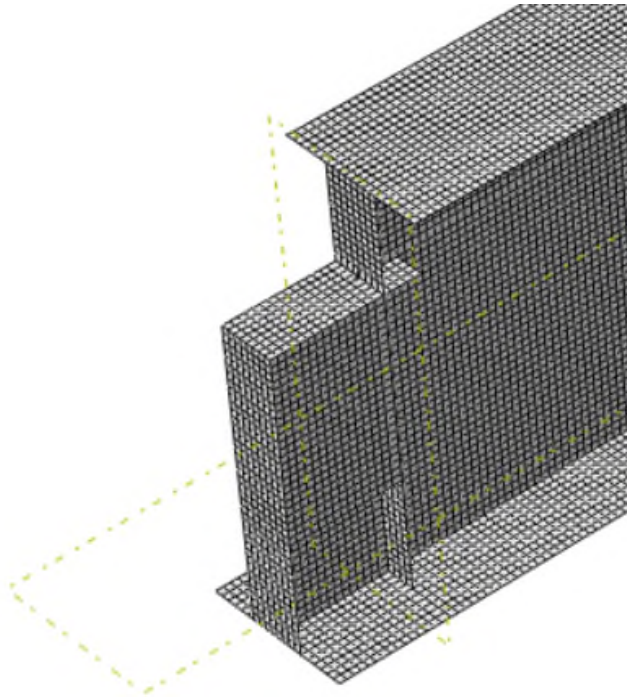


Figure F.2 Closeup of Mesh for the 250mm Downstand

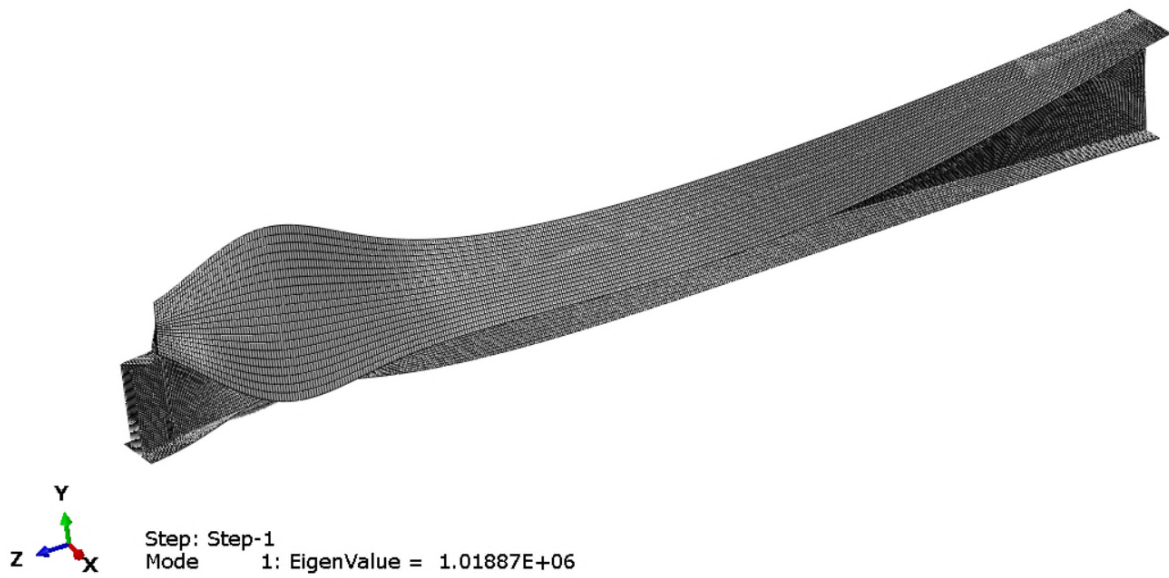
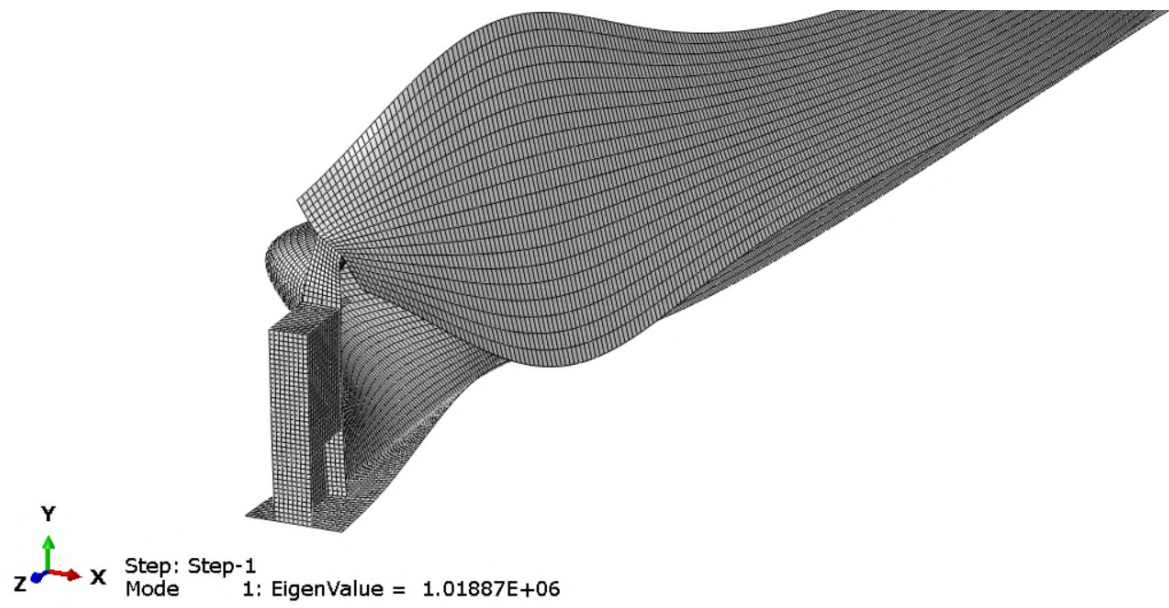


Figure F.3 250mm Downstand Height Buckled Shape

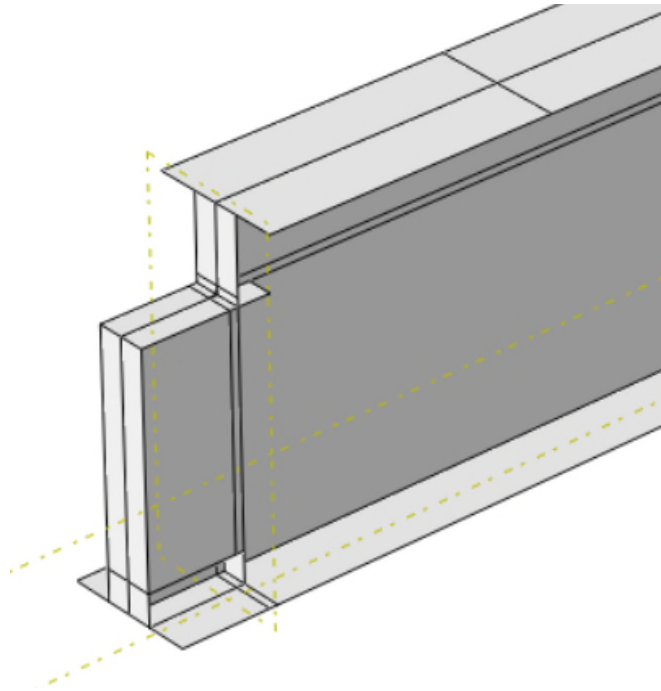


Figure F.4 Closeup of Stiffening Technique for the 350mm Downstand

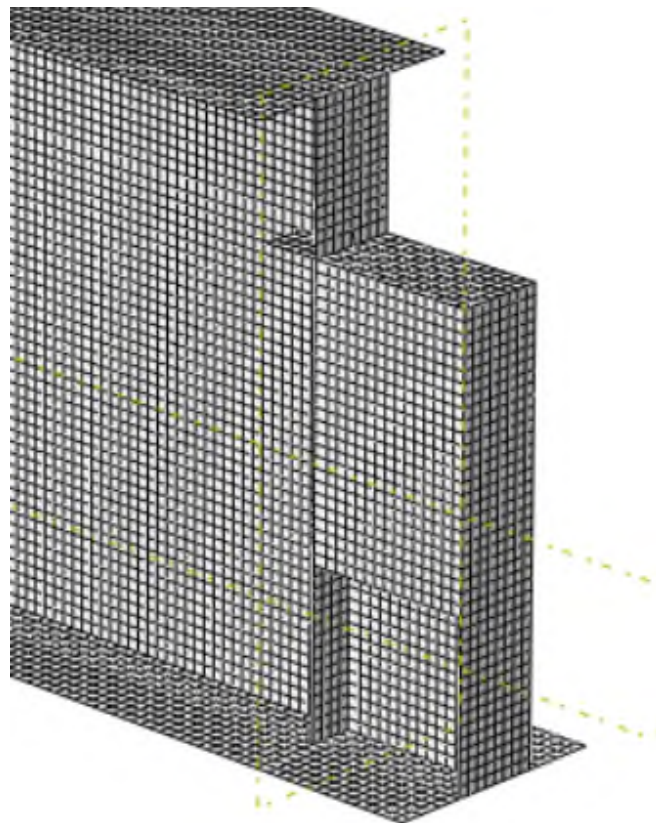
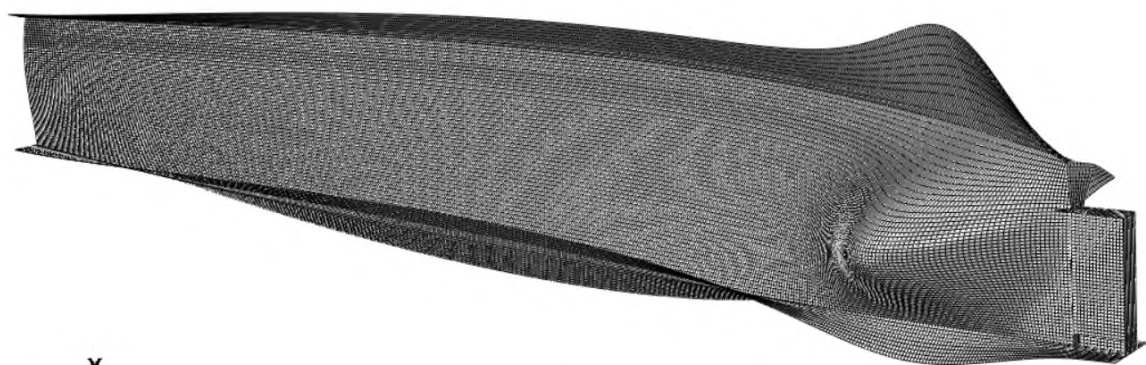
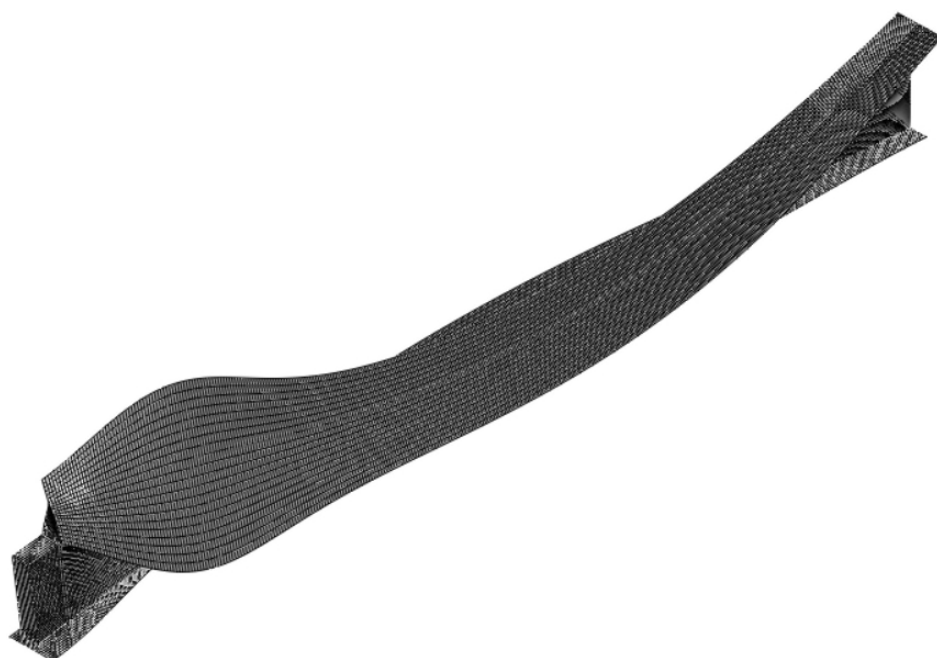


Figure F.5 Closeup of Mesh for the 350mm Downstand



Y
X
Z
Step: Step-1
Mode 1: EigenValue = 1.02044E+06



Y
X
Z
Step: Step-1
Mode 1: EigenValue = 1.02044E+06

Figure F.6 350mm Downstand Height Buckled Shape

Appendix G: Ethics Form

ETHICS APPLICATION FORM

Please Note:

Any person planning to undertake research in the Faculty of Engineering and the Built Environment (EBE) at the University of Cape Town is required to complete this form **before** collecting or analysing data. The objective of submitting this application *prior* to embarking on research is to ensure that the highest ethical standards in research, conducted under the auspices of the EBE Faculty, are met. Please ensure that you have read, and understood the **EBE Ethics in Research Handbook** (available from the UCT EBE, Research Ethics website) prior to completing this application form: <http://www.ebe.uct.ac.za/ebe/research/ethics1>

APPLICANT'S DETAILS	
Name of principal researcher, student or external applicant	
Department	
Preferred email address of applicant:	
If Student	Your Degree: e.g., MSc, PhD, etc.
	Credit Value of Research: e.g., 60/120/180/360 etc.
	Name of Supervisor (if supervised):
If this is a research contract, indicate the source of funding/sponsorship	
Project Title	

I hereby undertake to carry out my research in such a way that:

- there is no apparent legal objection to the nature or the method of research; and
- the research will not compromise staff or students or the other responsibilities of the University;
- the stated objective will be achieved, and the findings will have a high degree of validity;
- limitations and alternative interpretations will be considered;
- the findings could be subject to peer review and publicly available; and
- I will comply with the conventions of copyright and avoid any practice that would constitute plagiarism.

APPLICATION BY	Full name	Signature	Date
Principal Researcher/ Student/External applicant			
SUPPORTED BY	Full name	Signature	Date
Supervisor (where applicable)			

APPROVED BY	Full name	Signature	Date
HOD (or delegated nominee) Final authority for all applicants who have answered NO to all questions in Section 1; and for all Undergraduate research (Including Honours).	Dyllon Randall		1/4/2020
Chair: Faculty EIR Committee For applicants other than undergraduate students who have answered YES to any of the questions in Section 1.			