OBSERVATIONS OF GALAXIES IN A COSMOLOGICAL CONTEXT

by

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Declaration

No portion of this thesis has been previously submitted in support of an application for any other degree or qualification in this or any other University.

Abstract

The basic theory of observations of galaxies in a cosmological context is reviewed and extended to include e.g. the point-spread effect of the atmosphere. From this the relation between the sources (objects) and the images of these sources is derived (the observational map). A program is developed to calculate this map and some results are given.

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1. Introduction

There are many different models of the universe in the large, proposed by cosmologists. In principle it is possible to use distant astronomical objects as beacons in space-time and so decide between different cosmological models [see e.g. Ellis 1980 or Sandage 1961]. The goal of observational cosmology is to use astronomical observations in order to select among this multitude. The difficulties in measuring faint astronomical objects and interpreting these observations are severe. We can only hope to be successful if we can make a clear statement of the observational method used and the resulting observational limits. Following the paper "Towards a 'correctionless' observational cosmology" [G.F.R.Ellis and J.J.Perry (1979)], a clear analysis of astronomical observations is possible without the introduction of "corrections". This thesis can be considered as a step towards this aim. It develops and presents both the details and some extensions to an observational method outlined by G.F.R.Ellis in "Limits to verification in cosmology" [Ellis 1980].

Consider a class of similar objects that can be specified by two parameters, take e.g. the class of elliptical galaxies of EO type. This class of objects define an OBJECT PLANE, whose axes are the object's intrinsic magnitude and its scaling radius (which is proportional to the diameter) [see Arp 1965]. Then, at any distance or redshift z, the observational method used results in an OBSERVATIONAL MAP from the object-plane to an IMAGE PLANE, whose axes are the apparent magnitude and apparent angle of the resulting images (in a Friedmann universe the distance can be characterized by z, the cosmological redshift of an astronomical object) [see Fig.1 and 2]. The corrections necessary to obtain the

absolute properties of astronomical sources, e.g. to invert the map, are contained in the theory, and the assumptions made can be stated explicitly.

By having a clear division between the object- and image-plane, it is evident that OBSERVATIONAL LIMITS are to be placed in the image-plane [see section 4]. It turns out that a limit on apparent magnitude m_{ap} may be of only secondary importance. The limits shown in Fig.13 were drawn for the Anglo Australian Sky Survey and it can be shown that a limit of $m_{ap} = 23$ is not the critical feature, but rather that the most important limit is the angular limit (limit of resolution; distinguishing galaxies from stars) and the limit on the apparent surface brightness. The resolution and detection limits expected for space telescope observations are also shown. It can be seen clearly how the expected increase in resolution by a factor of 10 increases the number of objects that can be observed in different sections of the object-plane [see Fig.12].

Light-rays from stars and galaxies are smeared out by the atmosphere ('seeing') and by the detecting instrument (e.g. diffraction). This POINT-SPREAD EFFECT may be represented by a point-spread function. For ground-based telescopes, seeing dominates and a good approximation to the point-spread function is a Gaussian distribution. The same point-spread function also represents, to an first approximation, the central diffraction image of a point-source (e.g. a star) and may be a reasonable approximation to the point-spread function of the space telescope [see J.Stock and A.D.Williams 1962]. Near the limit of resolution the point-spread effect dominates [see Fig.4] and may not be ignored in any realistic calculation. Particularly striking is the large drop in central surface brightness of galaxies due to

this effect [see Fig.15], which implies that many objects "detectable" without point-spread, fall below detection and are lost for observation.

In section 2 the basic theory of the observational map and its inverse is given and then applied to a particular simplified case used for numerical calculations (the relevant computer program is given in the appendix). In section 3 the numerical methods and their application in the program are detailed, and some results obtained by use of the program are shown in section 4. Finally, some suggestions for generalization of the observational map and how to apply it to a wider class of objects, are given in section 5. One of the most important applications of the observational map will be an investigation of the statistics of number-counts. The appropriate analysis and numerical methods are presently under development.

2. Theory: The Observational Map

2.1. Summary

The "Observational" map from the object- to the image-plane and its inverse is derived: Firstly only fairly general assumptions are made, and secondly, a more specific, simplified form is derived to be used in calculations.

The assumptions that have been made in the derivation are:

1)The source (e.g. galaxy) is spherically symmetric (or circular, lying perpendicular to the line of sight) with a surface brightness B_ν(at the object) at the metric radius r given by

$$B_\nu(r,z) = B(r,z)\, g(\nu,r,z) \; ; \; \int_0^\infty g(\nu,r,z)\, d\nu = 1 \, ,$$

where $g(\nu,r,z)$ is the spectral function defined by the above relations. Here the dependence on z, the redshift of the source, indicates a possible source evolution.
[The notation used here follows Ellis and Perry 1979.]

2) The observations are made in directions in which the images remain undistorted. This condition is satisfied in any Friedmann Universe in all directions. A metric radius r then subtends an angle α at the observer given by the relation

$$\alpha = r/r_o(z) \, ,$$

where r_o, the OBSERVER AREA DISTANCE, is only a function of z. The measured specific intensity, I_ν, is given by:

$$I_\nu(\alpha,z) = \frac{B(r,z)}{(1+z)^3}\, g[\nu(1+z), r, z],$$

assuming no emission or absorbtion, extinction or scattering into or out of the beam along the line of sight [see Ellis 1971, 1980; Ellis and Perry 1979].

3) The SPECTRUM of the emitted radiation does not change across the source, i.e.

$$g[\nu, r, z] = g[\nu, z]$$

and the radial variation of brightness is the product of an amplitude $B_o(z)$ and a normalized radial function $f(\beta)$ whose functional form does not evolve. Thus

$$B(r,z) = B_o(z)\, f(\beta), \qquad f(0) = 1,$$

here $\beta = r/a(z)$ and $a(z)$ is the scaling radius. Again dependence on z indicates source evolution ; the spectrum, central brightness and the radius of a galaxy may evolve (the radius of a galaxy is proportional to $a(z)$).

4) We represent the combined effects of the optical imaging system (e.g. diffraction, seeing and instrumental scattering) by a POINT-SPREAD FUNCTION (p.s.f.) in the focal plane. This p.s.f. is here taken to be a Gaussian with dispersion $\tilde{\sigma}$, i.e. to have the form:

$$S(s,\nu) = \frac{1}{2\pi\tilde{\sigma}^2}\, e^{-s^2/2\tilde{\sigma}^2}.$$

It is circularly symmetric and thus implies circularly symmetric images. In general, $\tilde{\sigma} = \tilde{\sigma}(\nu)$ but for simplicity we shall initially take $\tilde{\sigma}$ to be constant with frequency ν.

In the first section the fundamental concepts of the theory of the radiation field is outlined. Then it is shown that the point-spread effect can be represented by a radial function $\tilde{f}$ (the "new" brightness profile) given by a convolution of the profile f:

$$\tilde{f}(\beta) \equiv \tilde{f}(R,A) = \frac{1}{\pi} \int_0^{\infty} dy \int_0^{\pi} d\theta \; y \; e^{-(y^2/2 + R^2/2 - yR\cos\theta)} \; f(Ay)$$

such that the specific intensity is given by

$$\tilde{I}(\alpha,\nu,z) = \frac{B_0(z)}{(1+z)^3} \; g[\nu(1+z),z] \; \tilde{f}(\beta) \; .$$

Here $R = r/W$ and $A = W/a(z)$ where we have defined $W = r_0(z)\,\sigma$. $\sigma = \frac{\tilde{\sigma}}{d}$ is the dispersion in radians (**d** represents the focal length of the camera) and thus W is the radius of the disc(at the observed galaxy) corresponding to the angle σ.

Using the assumptions 1-3 the equations for the emitted radiation flux and luminosity for galaxies are derived. The LUMINOSITY is given by

$$L_\nu(z) = 4\pi \, B_0(z) \, g(\nu,z) \, a^2(z) \; g(\beta_m)$$

and the RADIATION FLUX at distance $R > a(z)\cdot\beta_m$

$$F_\nu(z) = \frac{L_\nu(z)}{4\pi R^2} = \frac{B_0(z)\, g(\nu,z)\, a^2(z)\, g(\beta_m)}{R^2} \; ,$$

where $g(\beta_m) = 2\pi \int_0^{\beta_m} \beta \, f(\beta) \, d\beta$.

Using these equations, the fundamental definitions for the absolute and apparent magnitude and assumption 4, the APPARENT MAGNITUDE $m_\nu(A_{ap})$ is derived. $2A_{ap}$ is the apparent diameter of

the observed galaxy given by a DETECTION LIMIT $S_L(\nu) = I_\nu(A_{ap}, z)$

on the measured intensity. The equations are

$$S_L(\nu) = \frac{B_0(z)}{(1+z)^3}\, g[\nu(1+z), z]\; e^{-p(\nu,z)}\, \tilde{f}(\beta_{ap}), \quad \text{— A}$$

$$A_{ap} = \frac{r_{ap}(1+z)^{p_1}}{r_0(z)\cdot 10^3} = \frac{r_{ap}}{a}\,\frac{a}{r_0(z)\cdot 10^3}(1+z)^{p_1} = \beta_{ap}\,\frac{a}{r_0(z)\cdot 10^3}(1+z)^{p_1}, \quad \text{— B}$$

$$m_\nu(A_{ap}) = M_{bol} + \Delta M - 2.5\log_{10}(1+z) - 2.5\log_{10}\frac{\tilde{g}(\beta_{ap})}{g_\infty} + 5\log_{10}\frac{r_0(z)(1+z)^2}{D_M} - (2.5p_2 + 5p_1)\log_{10}(1+z) + 25. \quad \text{— C}$$

Where $p(\nu, z)$ is the optical depth up to z,

$a(z) = a(1+z)^{p_1}$ gives the radial evolution,

$B_0(z) = B_0(1+z)^{p_2}$ gives the brightness evolution,

$\Delta M - 2.5\log(1+z)$ is the bolometric and K-correction and

$-2.5\log\frac{\tilde{g}(\beta_{ap})}{g_\infty}$ is the aperture correction.

As above: $\tilde{g}(\beta_{ap}) = 2\pi\int_0^{\beta_{ap}} \beta\,\tilde{f}(\beta)\,d\beta$

and $g_\infty = \lim_{\beta\to\infty} g(\beta) = \lim_{\beta\to\infty} \tilde{g}(\beta)$.

Finally M_{bol} is the bolometric absolute magnitude and $D_M = 10^6$ pc a scaling distance.

NOTE: r_0 is given in Mpc, a and r in kpc.

1pc or 1 parsec = 3.262 light years = $3.086\cdot 10^{13}$ km

The equations A,B and C determine the map from the object plane (a,Mbol) to the image plane (A-ap,m-ap) and its inverse. Further

assumptions are made in section 2.5; simple brightness profiles are chosen, a FRW cosmology is assumed and simple expressions for intergalactic absorbtion and the magnitude correction are derived, giving a more specialised form of the map, which is used for numerical calculations shown in subsequent sections.

2.2. The Radiation Field

The basic definitions and concepts of radiation theory are reviewed and then applied to a spherically symmetric radiation field. The specific intensity of radiation (received), valid for all redshifts, is stated and the effects of intergalactic absorption, point-spread and spectral sensitivity of the detector are shown and the resultant intensity derived. Finally, by making several simplifying assumptions about the spectral and radial form of the radiation emitted by galaxies, specific expressions for the radiation flux and the luminosity are given. The basic concepts can be found in references 1 to 6 or in any textbook on Astrophysics.

2.2.1. Basic Concepts

A. Specific Intensity: It is convenient to describe a radiation field by the specific intensity. In order to define this quantity consider a point P in a radiation field, an infinitesimal surface $d\sigma$ with normal $\underline{d\sigma}$ ($d\sigma = |\underline{d\sigma}|$) at point P and a unit vector $\underline{n}$ in direction (θ,φ) [see fig. 2-1]. Then let

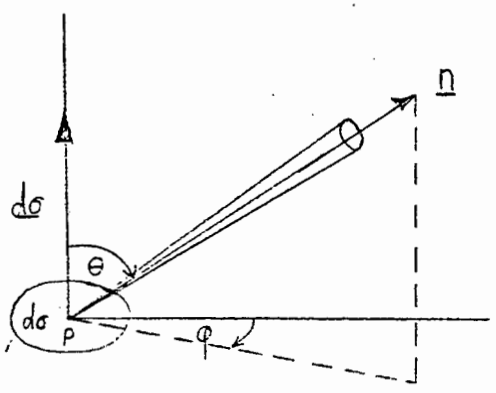
$dE_\nu(\theta,\varphi)$ = amount of radiation (energy) in the freqency interval $[\nu,\nu+d\nu]$, in the time interval $[t,t+dt]$ passing through $d\sigma$ in element of solid angle dw about $\underline{n}$ [erg].

Define the specific intensity by

$$I_\nu(\theta,\varphi) \equiv \frac{dE_\nu(\theta,\varphi)}{d\nu\, d\omega\, (\underline{d\sigma}\cdot\underline{n})\, dt} \quad [\text{erg}-\text{Hz}^{-1}-\text{ster}^{-1}-\text{cm}^{-2}-\text{sec}^{-1}]$$

if this angular limit exists.

Figure 2-1.



i.e. The specific intensity = energy travelling per unit time, per unit freqency, per unit surface ~~perpendicular to~~ in the direction $\underline{n}$, in a unit solid angle.

The total intensity or integrated intensity is given by

$$I(\theta,\varphi) = \int_0^\infty I_\nu(\theta,\varphi)\, d\nu \ .$$

B. Radiation Flux and Luminosity: The radiation flux is defined to be the radiant energy crossing the surface $d\sigma$ in all directions per unit frequency, per unit time and per unit area. Thus

$$F_\nu \equiv \int_0^{4\pi} d\omega\, I_\nu(\theta,\varphi)\cos\theta = \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta\, I_\nu(\theta,\varphi)\cos\theta\,\sin\theta \ , \qquad (1\ a)$$

where the element of solid angle $d\omega = \sin\theta\, d\varphi\, d\theta$.

Often the flux F^+_ν in the "upper half space" defined as:

$$F_\nu^+ \equiv \int_0^{2\pi} d\omega\, I_\nu(\theta,\varphi)\cos\theta = \int_0^{2\pi} d\varphi \int_0^{\pi/2} d\theta\, I_\nu(\theta,\varphi)\cos\theta\,\sin\theta \qquad (1\ b)$$

$$[F_\nu^+] : \ \text{erg} - \text{Hz}^{-1} - \text{cm}^{-2} - \text{sec}^{-1}$$

is used. The mean intensity $\bar{I}^+_\nu$ (i.e. the mean over all directions

for the "outgoing" radiation) is defined by:

$$\overline{I}_\nu^+ \equiv \frac{\int_0^{2\pi} d\varphi \int_0^{\pi/2} d\theta\, I_\nu(\theta,\varphi) \cos\theta \sin\theta}{\int_0^{2\pi} d\varphi \int_0^{\pi/2} d\theta \cos\theta \sin\theta} = \frac{F_\nu^+}{\pi}$$

so that

$$F_\nu^+ = \pi\, \overline{I}_\nu^+ \,. \qquad (1\,c)$$

The luminosity L_ν of a radiating body is the total energy emitted into exterior space by that body per unit frequency and per unit time. It is:

$$L_\nu = \int_\Sigma d\sigma\, F_\nu^+ \quad [\text{erg} - \text{Hz}^{-1} - \text{sec}^{-1}] \,, \qquad (2)$$

where Σ is a closed surface enclosing the radiating body. The quantities above are monochromatic quantities. Define the integrated flux and integrated or total luminosity by:

$F = \int_0^\infty F_\nu\, d\nu$ and $L = \int_0^\infty L_\nu\, d\nu$ respectively.

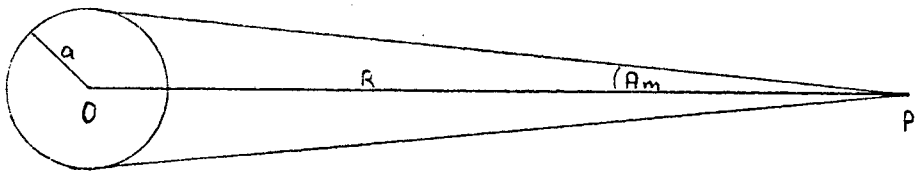
Note: In some textbooks the flux F is defined as $F = \mathcal{F} \cdot \pi$

2.2.2. Spherically Symmetric Radiation Field

Consider a spherical radiating body with radius a, center O at distance R from an observer at P. Choose polar coordinates whose origin is at the center of symmetry O, then

$$I_\nu(\theta,\varphi) = I_\nu(\theta).$$

Figure 2-2.



For any $R > a$, the flux F^+_ν is constant over the surface of the sphere with radius R, thus from eq 2:

$$L_\nu = \int d\sigma\, F^+_\nu = 4\pi R^2\, F^+_\nu \quad , \text{ and we can write}$$

and

$$F^+_\nu = \frac{L_\nu}{4\pi R^2}$$

$$\bar{I}^+_\nu = \frac{L_\nu}{4\pi^2 R^2} \, . \qquad (3)$$

Consider the case where $R >> a$ but R (and a) small enough such that $z \approx 0$

From eq. 1 b :

$$F^+_\nu = \int_0^{2\pi} d\varphi \int_0^{A_m} d\theta\; I_\nu(\theta) \cos\theta \sin\theta \quad ,$$

where A_m is the angle subtended by a at the observer at P.

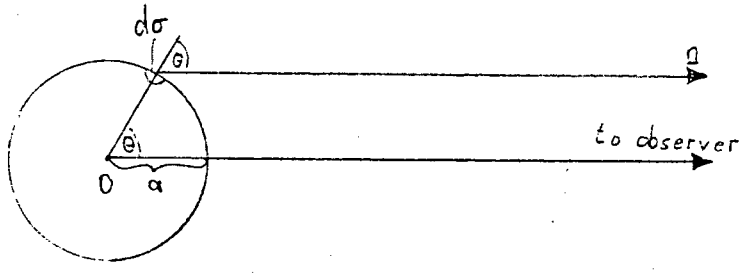
Now for $A_m << 1$: $\cos\theta \doteq 1$ and $\sin\theta \doteq \theta$, thus

$$F^+_\nu \doteq 2\pi \int_0^{A_m} I_\nu(\theta)\, \theta\, d\theta \, . \qquad (4)$$

The radiation in direction of the observer at P (direction $\underline{n}$ say) may be assumed to be plane parallel. A surface element on the sphere is given by [see Fig. 2-3]

$$d\sigma = a^2 \sin\theta\, d\theta\, d\varphi \, .$$

Figure 2-3.



Hence the mean intensity $\bar{I}_{\nu,n}$ over the disc with radius **a** in direction $\underline{n}$ is given by

$$\bar{I}_{\nu,n} = \frac{\int_0^{2\pi} d\varphi \int_0^{\pi/2} d\theta\, I_\nu(\theta) \cos\theta \sin\theta\, a^2}{\pi a^2}$$

$$= \frac{F^+_\nu}{\pi} .$$

by eq. 1b and the assumption of spherical symmetry.

Thus $\bar{I}^+_\nu = \bar{I}_{\nu,n} = F^+_\nu/\pi$, and the mean <u>at one point</u> over <u>all directions</u> is equal to the mean at <u>all points</u> in <u>one direction</u>.

So for a spherically symmetric radiator subtending an angle 2Am with Am << 1 we may project the angular variation in intensity into a radial variation across an effective object-plane and write:

$$I_\nu(\theta,\varphi) \doteq B_\nu(r,z) = B(r,z)\, g(\nu,r,z) : \int_0^\infty g(\nu,r,z)\, d\nu = 1 \quad . \qquad (5a)$$

This is the emitted specific intensity or <u>surface brightness</u>. The dependence on z reflects the possibility of brightness and

spectral evolution; r is the distance measured at the source (e.g.galaxy) from its center. r is called the metric or projected radius for galaxies. Equation 5a also defines the spectral function g. Note that $B_\nu(r,z)$ is not limited by this definition.

At the observer (z=0) the specific intensity of radiation, due to a source at redshift z with surface brightness given by eq.5a, is given by:

$$I_\nu(\alpha,z) = \frac{B(r,z)}{(1+z)^3}\, g[\nu(1+z), r, z] \quad , \tag{5b}$$

where α is the angle measured at the observer corresponding to a metric radius r. Here we exclude any emission or absorbtion, extinction or scattering into or out of the beam along the line of sight. (see Ellis and Perry 1979)

In an "ideal" cosmological model (null geodesics are distortion free) the metric radius r can be given by:

$$r = \alpha\, r_o(z). \tag{6}$$

Here $r_o(z)$ is the observer area distance (see e.g.Ellis 1971).

In an Friedmann Universe the observer area distance is given by

$$r_o(z) = \frac{c}{H_o q_o^2 (1+z)^2}\left\{q_o z + (q_o - 1)\left(\sqrt{2q_o z + 1} - 1\right)\right\} \quad [Mpc] \quad , \tag{7}$$

where c = speed of light in km/sec

H_o = Hubble constant in km/sec/Mpc

and q_o = the deacceleration parameter

Two assumptions are usually made to describe galaxies: [see Ellis and Perry 1979]

1) The frequency dependence of the emitted radiation does not vary across the face of the galaxy. Thus $g(\nu,r,z) \doteq g(\nu,z)$.

2) The radial variation of brightness is characterised by an amplitude which may evolve with z, $B_o(z)$, and a normalized radial function whose functional form does not evolve, $f(\beta)$. $\beta = r/a$ and a is a scaling radius which may evolve i.e. $a=a(z)$. Thus

$$B_\nu(r,z) = B_o(z)\, g(\nu,z)\, f\left(r/a(z)\right) \qquad (8a)$$

and

$$I_\nu(\alpha,z) = \frac{B_o(z)}{(1+z)^3}\, g\left[\nu(1+z),z\right]\, f\left(r/a(z)\right) . \qquad (8b)$$

2.2.3. The measured intensity of radiation

We consider the effect of intergalactic absorbtion, point-spread and detector spectral sensitivity in turn

A Intergalactic Absorbtion

Allowing for absorbtion by intervening matter we can rewrite eq.5b as:

$$I_\nu(\alpha,z) = \frac{B(r,z)}{(1+z)^3}\, g\left[\nu(1+z),r,z\right] e^{-p(\nu,z)} , \qquad (9)$$

where $p(\nu,z)$ is the optical depth up to redshift z given by:

$$p(\nu,z) = \int_0^z n_a(z')\, \sigma_a\left(z', \nu(1+z')\right) \frac{dl}{dz'}\, dz' .$$

n_a is the number density of absorbers with interaction cross-section σ_a along the light ray and dl is a proper length interval. Usually n_a is given in cm^{-3} and σ_a in cm^2.

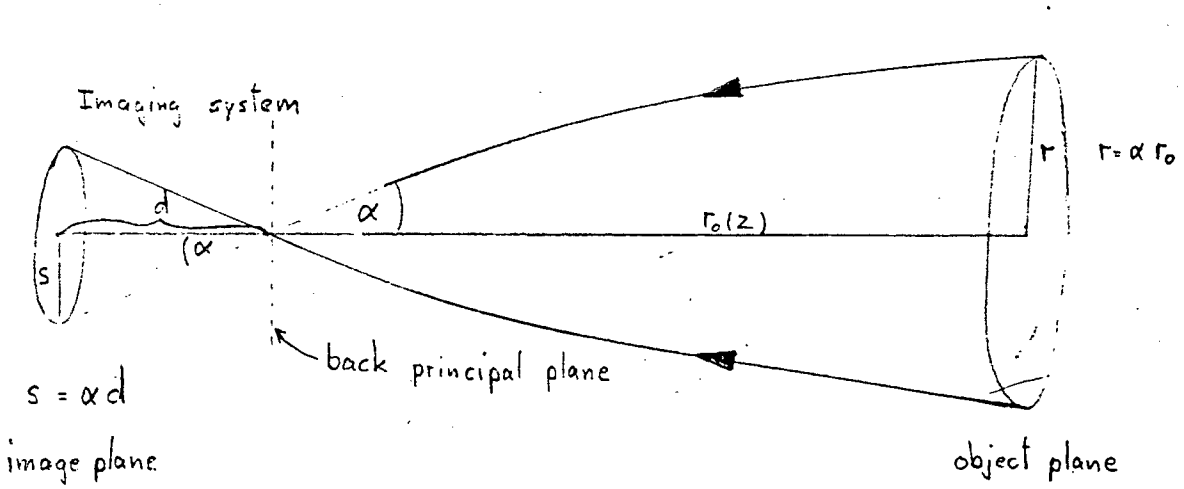
[see section 6'6.4 of Ellis 1971]

B Point-spread Effect

The image of a galaxy can be thought of as made up by a very large number of light rays. Each ray is slightly blurred by seeing, diffraction, scattering etc. i.e. the image of a galaxy has to be convolved with a point-spread function. At any point in the image all "point-spread" light rays contribute to the measured specific intensity, which is then represented as a sum (integral) of all these contributions.

The source-detector relation for the ideal imaging system can be represented as follows:

Figure 2-4.

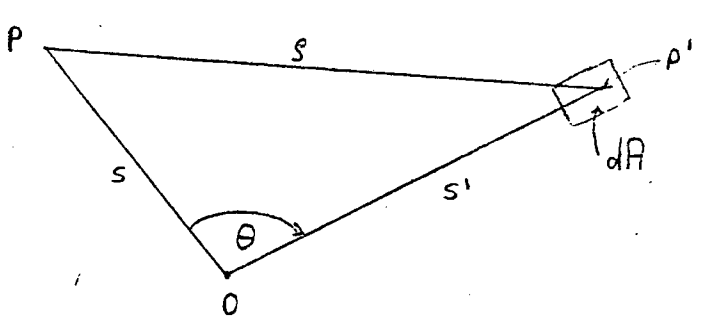


d is the distance from the back principal plane of the detector to the image plane. If the angle subtended by the object is α, then the angle subtended by the image from the back principal plane is also α and $s = \alpha d$.

Consider the image-plane: The center of the image is at O [see Fig.2-5]. We want to determine the intensity at the point P. Assume a spherically symmetric point-spread function $S(s, \nu)$, where s is the distance between point P and point P' of integration. Then the intensity dI at P due to the radiation incident on point P will be:

$$dI = I_\nu(\alpha', z)\, S(s, \nu)\, dA \,.$$

Figure 2-5.



Here $\alpha' = s'/d$ and dA is an infinitesimal area-element at P'. Then the intensity $\tilde{I}$ will be:

$$\tilde{I}_\nu(\alpha, z) = \iint_{\text{plate}} I_\nu(\alpha', z)\, \mathcal{S}(\rho, \nu)\, dA \quad , \qquad \text{where } \alpha = s/d$$

i.e. $\tilde{I}_\nu$ is a convolution of I_ν with a point-spread function $\mathcal{S}$.

When the atmosphere dominates, the most important part of the pointspread function has a Gaussian form [King 1971; Kormendy 1973; Schweizer 1979]. Then

$$\mathcal{S}(\rho, \nu) = \frac{1}{2\pi\tilde{\sigma}^2}\, e^{-\rho^2/2\tilde{\sigma}^2} \quad , \quad \tilde{\sigma} = \tilde{\sigma}(\nu) \, . \qquad (10)$$

Where $\tilde{\sigma}$ is called the <u>dispersion</u>, it is the value for ρ where $\mathcal{S}$ has dropped to $e^{-1/2}$ of its maximum value.

The Gaussian point-spread function also represents, to a first approximation, the central diffraction image of a focal stellar image. [see Stock and Williams 1962 pg 393]

The factor $1/2\pi\tilde{\sigma}^2$ normalises $\mathcal{S}$ since

$$\frac{1}{2\pi\tilde{\sigma}^2}\int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dy\; e^{-(x^2+y^2)/2\tilde{\sigma}^2} = \frac{1}{2\pi\tilde{\sigma}^2}\int_0^{\infty}dr\; 2\int_0^{\pi}d\theta\;\; e^{-r^2/2\tilde{\sigma}^2} = 1 \quad .$$

The specific intensity at α is given by

$$\tilde{I}_\nu(\alpha,z) = 2\int_0^{\infty}ds'\int_0^{\pi}d\theta\; I_\nu(\alpha',z)\frac{s'}{2\pi\tilde{\sigma}^2}\, e^{-(s^2+s'^2-2ss'\cos\theta)/2\tilde{\sigma}^2} \quad .$$

[Choosing polar coordinates (s,θ) with origin O and reference line OP, $dA = s'ds'd\theta$ and $\mathcal{S} = s^2 + (s')^2 - 2s\,s'\cos\theta$]

Changing variables to $y = s/\tilde{\sigma}$, and further put $R = s/\tilde{\sigma}$, we have

$$\tilde{I}_\nu(\alpha,z) = \frac{1}{\pi}\int_0^{\infty}dy\int_0^{\pi}d\theta\; I_\nu\left(y\frac{\tilde{\sigma}}{d},z\right)\, y\, e^{-(y^2/2 + R^2/2 - Ry\cos\theta)} \quad .$$

By eq.8b and putting $A = \frac{\tilde{\sigma}}{d}\,\frac{r_0(z)}{a(z)}$ we obtain

$$\tilde{I}_\nu(\alpha,z) = \frac{B_0(z)}{(1+z)^3}\,g[\nu(1+z),z]\,\frac{1}{\pi}\int_0^{\infty}dy\int_0^{\pi}d\theta\; y\, e^{-(y^2/2+R^2/2-Ry\cos\theta)}\, f(Ay) \quad . \qquad (11)$$

Note that $R = s/\tilde{\sigma} = \frac{s}{d}\,\frac{d}{\tilde{\sigma}} = \alpha\,\frac{d}{\tilde{\sigma}} = \frac{r}{r_0}\,\frac{d}{\tilde{\sigma}}$ so that $r = \frac{\tilde{\sigma}}{d}\,r_0 R$.

We define $\sigma = \frac{\tilde{\sigma}}{d}$, the dispersion in radians and $W = r_0\sigma$. W is the metric radius at the galaxy cooresponding to the angle σ subtended by the point-spread function. Now $R = r/W$ and $A = W/a$ so that $A\times R = \frac{r}{a} = \beta$.

By comparison of eq 8b and eq 11 we can define a point-spread profile $\tilde{f}$ as:

$$\tilde{f}(\beta) \equiv \tilde{f}(R,A) = \frac{1}{\pi}\int_0^{\infty}dy\int_0^{\pi}d\theta\; y\, e^{-(y^2/2+R^2/2-Ry\cos\theta)}\, f(Ay) \quad . \qquad (12)$$

It is easy to show that in the limit as $\sigma \to 0$, $\tilde{f} \to f$ and $\tilde{I} \to I$. If we write eq 11 in the form

$$\tilde{I}_\nu(\alpha, z) = \frac{B_0(z)}{(1+z)^3}\, g[\nu(1+z), z] \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy\; I(\alpha', z)\, \frac{1}{2\pi\hat{\sigma}^2}\, e^{-(x^2+y^2)/2\hat{\sigma}^2}$$

where cartesian coordinates (x,y) with origin P are chosen. By theorem I [see pg.2-I7] with $\frac{1}{2\hat{\sigma}^2} = n$ we have

$$\lim_{\hat{\sigma}\to 0} \tilde{I}_\nu(\alpha, z) = I_\nu(\alpha, z)\,. \qquad \text{(at } x=y=0,\ \alpha' = \alpha\text{)}$$

DEFINITION: A good function is one which is everywhere differentiable any number of times and such that it and all its derivatives are $O(|x|^N)$ as $x \to \infty$ for all N.

THEOREM I

The sequence eqivalent to $e^{-n(x^2+y^2)}(n/\pi)$ defines a generalised function $\delta(x,y)$ such that

$$\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy\ \delta(x,y)\ F(x,y) = F(0,0)$$

if F is a good function.

This theorem is proved in Lighthill (1960) pg 17 for one dimension. It is here rewritten for two dimensions.

Proof: Let $I = \left| \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy\ e^{-n(x^2+y^2)} \left(\frac{n}{\pi}\right) F(x,y) - F(0,0) \right|$, then

$$I \leq \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy\ e^{-n(x^2+y^2)} \left(\frac{n}{\pi}\right) |F(x,y) - F(0,0)|$$

$$\leq \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy\ e^{-n(x^2+y^2)} \left(\frac{n}{\pi}\right) \max\left\{ \left|\frac{\partial F}{\partial x}\right| ; \left|\frac{\partial F}{\partial y}\right| \right\} |(x-0)+(y-0)|$$

$$\leq \max\left\{ \left|\frac{\partial F}{\partial x}\right| ; \left|\frac{\partial F}{\partial y}\right| \right\} \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy\ e^{-n(x^2+y^2)} \left(\frac{n}{\pi}\right) (|x| + |y|).$$

Now $\int_{-\infty}^{\infty} e^{-nx^2} \sqrt{\frac{n}{\pi}}\, |x|\, dx = \frac{1}{\sqrt{n\pi}}$

and $\int_{-\infty}^{\infty} e^{-nx^2} \sqrt{\frac{n}{\pi}}\, dx = \sqrt{\frac{n}{\pi}} \sqrt{\frac{\pi}{n}} = 1$ so that

$$I \leq \max\left\{ \left|\frac{\partial F}{\partial x}\right| ; \left|\frac{\partial F}{\partial y}\right| \right\} \frac{2}{\sqrt{n\pi}} \to 0 \quad \text{as } n \to \infty,$$

as the derivatives of F are bound (F is a good function)

C Detector Response

Given a image intensity $\tilde{I}_\nu(\alpha, z)$ at the focal plane, some detector will be used to measure this intensity. The measurement will depend on the instrument used and on the conditions under which it is used (e.g. for a photographic plate ~~it is~~ they are the exposure time, the development time and temperature, sensitivation procedure etc.) Denote the total of these conditions relevant to the observation by C.

The final detector output depends basically on two factors:

1) The detector spectral sensitivity:

Under conditions C only certain frequency ranges can effect the detector output. This can be discribed by a function ϕ, the spectral sensitivity [see e.g.Weaver 1962] such that the effective total or integrated intensity I_D is given by

$$I_D(\alpha,z) = \int_0^\infty \tilde{I}_\nu(\alpha,z)\ \phi(\nu,C)\, d\nu \ . \qquad (13)$$

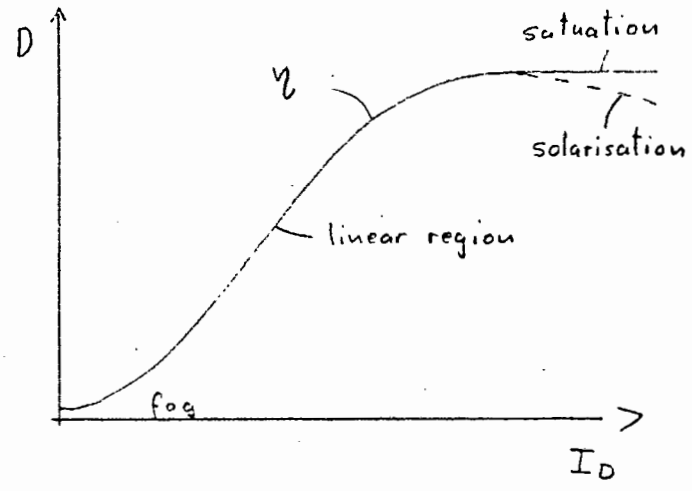
2) The detector characteristic curve:

This gives the total detector response D (also called darkening) to the image plane radiant energy distribution I_D [see Stock and Williams (1962)] Write

$$D(\alpha,z) = \eta\,(I_D, C)\ ,$$

where η is the detector response to I_D under conditions C. In general, this curve will have a form like that shown (Fig 2.6), for any specific C. It may turn over for large I_D, if the detector has a region of solarisation. [see Weaver 1962]

Figure 2-6.



In derivations that follow we do not consider the detector reponse function or night sky subtraction. We assume that the observational data are corrected for both effects and our starting point is eq.13.

2.2.4. The Flux and Luminosity for "nearby" Galaxies

Some "absolute" properties of the galaxy model are given here, assuming Euclidean space. Assume a galaxy at distance R with redshift $z \ll 1$, the metric radius r will correspond to an angle $\theta = r/R$ and $A_m = r_m/R$, where r_m is that value of r which includes "all" of the galaxy (the theoretical profile has to be truncated). Assuming further that $A_m \ll 1$, then eq.4 holds and from eq.5a

$$F_\nu = 2\pi \int_0^{A_m} B_\nu \, \theta \, d\theta ,$$

then from eq. 8a

$$F_\nu = 2\pi \int_0^{r_m} B_\nu(r,z) \, \frac{r}{R} \, \frac{dr}{R}$$

$$= 2\pi \, \frac{B_0(z) \, \mathcal{F}(\nu,z)}{R^2} \int_0^{r_m} f\left(\frac{r}{a(z)}\right) r \, dr .$$

Put $\frac{r}{a(z)} = \beta \Rightarrow \beta_m = \frac{r_m}{a(z)}$,

then

$$F_\nu = \frac{B_0(z)\, g(\nu,z)\, a^2(z)}{R^2} \int_0^{\beta_m} 2\pi f(\beta)\, \beta\, d\beta \quad ;$$

define $$g(\beta_m) = \int_0^{\beta_m} 2\pi \beta\, f(\beta)\, d\beta \,,$$

then $$F_\nu(z) = \frac{B_0(z)\, g(\nu,z)\, a^2(z)}{R^2}\, g(\beta_m) \,. \qquad (14)$$

The luminosity L_ν is given by eq 3, i.e.

$$L_\nu(z) = 4\pi\, B_0(z)\, g(\nu,z)\, a^2(z)\, g(\beta_m)$$

and

$$L(z) = 4\pi\, B_0(z)\, a^2(z)\, g(\beta_m) \,, \qquad (15)$$

since $$\int_0^\infty g(\nu,z)\, d\nu = 1 \,.$$

In later applications we replace $g(\beta_m)$ with $g_\infty \equiv \lim_{\beta\to\infty} g(\beta)$. It is not obvious that this is physically meaningful [see King 1978], but De Vaucouleurs gives for the De Vaucouleurs profile a range for β of $0.003 < \beta < 15$ and $\frac{g(15)}{g_\infty} = 0.98$. It seems thus reasonable to approximate $g(\beta_m)$ by g_∞. [see De Vaucouleurs 1959]

NOTE: It is assumed in this derivation that the flux at optical wavelengths is much larger than the X-ray or radio flux.

2.3. The Magnitude Scales

We first summarize the basic definitions for the absolute and apparent magnitudes together with the correct magnitude scales (taken from "Astrophysical Quantities" by C.W.Allen 3rd edition). [Allen 1973] We then give the absolute and apparent colour magnitudes for galaxies in terms of the flux and luminosity expressions of section 2.2. The detection limit on the received specific intensity is incorporated in these expressions. In order to compare our equations with those of the "standard" literature, the K-correction, aperture correction and bolometric correction are defined. Finally source evolution is introduced and we are in the position to write down the observational map.

2.3.1. Definitions

M = absolute magnitude D = 10pc

m = apparent magnitude D = 1kpc

d = luminosity distance D = 1Mpc

r_o = observer area distance

f_i = actual smoothed flux of radiation near effective wavelength

L = luminosity

1pc = $3.086\ 10^{18}$cm

and we have that $d = r_o(1+z)^2$ [Weinberg 1972]

The basic equations are

$$M = m - 5 \log_{10} \frac{d}{D} = m - 5 \log \frac{r_0 (1+z)^2}{D_M} - 25 , \tag{16a}$$

$$M = M_0 - 2.5 \log_{10} \left(L / L_0 \right) , \tag{16b}$$

$$m_1 - m_2 = -2.5 \log_{10} \left(f_1 / f_2 \right) . \tag{16c}$$

The above definitions assume measurement within an aperture $A > A_m$ and proper sky subtraction. Actual measurements are based on the use of "standard stars" of known magnitude.

Note: Write log for $\log_{10}$

2.3.2. Bolometric Absolute Magnitudes

Define the bolometric absolute magnitude Mb as:

$$Mb = M_\odot^b - 2.5 \log(L/L_\odot) \; ; \; M_\odot^b = 4.75$$

$$L_\odot = 3.826(8) \times 10^{33} \text{ erg/sec} .$$

The sun is a spherically symmetric radiator with mean surface brightness $B_\odot$ (say), then from eq. 3

$$L_\odot = 4\pi^2 R_\odot^2 B_\odot \; ; \; R_\odot \text{ the sun's radius}$$

"Near" a galaxy we have from eq 15

$$L(z) = 4\pi B_0(z) \, a^2(z) \, g_\infty .$$

Then $$M_b = M_0^b - 2.5 \log \left[\frac{B_0(z)\, a^2(z)\, g_\infty}{B_0\, R_0^2\, \pi} \right]$$

$$= M_0^b - 2.5 \log \left[\frac{B_0(z)}{B_0} \left(\frac{a}{D}\right) \frac{g_\infty}{\pi} \right] + 5 \log \frac{R_\odot}{D}.$$

$$\Rightarrow \quad M_b = -2.5 \log \left[\frac{B_0(z)}{B_0} \left(\frac{a}{D}\right)^2 \frac{g_\infty}{\pi} \right] - 38.48 \qquad (17)$$

and $B_\odot = 2.04\ 10^{10}$ erg-cm^{-1}-sec^{-1}-ster^{-1}.

Physical Units:

Definitions: We use units where

$$m_{bol} \equiv -2.5 \log f/f_0 \; ; \; f_0 \equiv 2.48 \cdot 10^{-5} \text{ erg-sec}^{-1}\text{-cm}^{-2}$$

It follows that for the absolute magnitude

$$M_{bol} = -2.5 \log (L/L_0) \; ; \; L_0 = 2.97 \cdot 10^{35} \text{ erg-sec}^{-1},$$

and, to measure the surface brightness in magnitudes per square arc-sec define

$$M_{bol} \equiv -2.5 \log (B/S_0) \; : \; S_0 = 2.48 \cdot 10^{-5} \text{ erg-sec}^{-1}\text{-cm}^{-2}\text{-(arc sec)}^{-2}$$

$$\text{or } S_0 = 1.055 \cdot 10^{6} \text{ erg-sec}^{-1}\text{-cm}^{-2}\text{-ster}^{-1}.$$

$$L_e \equiv 4\pi D_k^2 S_0 \pi = 3.966 \cdot 10^{50} \text{ erg-sec}^{-1}.$$

Then

$$M_{bol} = -2.5 \log \left(\frac{L}{L_e} \frac{L_e}{L_0} \right)$$

$$= -2.5 \log \frac{L_e}{L_0} - 2.5 \log \left[\frac{4\pi a^2 B_0 g_\infty}{4\pi D_k^2 S_0 \pi} \right].$$

$$\therefore \quad M_{bol} = -37.82 - 2.5 \log \left[\frac{B_0(z)}{S_0} \left(\frac{a}{D_k}\right)^2 \frac{g_\infty}{\pi} \right] \qquad (18a)$$

Note: There is a shift in scale between Mb and Mbol of .025. This shift in scale is not present if values from Allen 2nd edition are used.

2.3.3. Absolute Colour Magnitude

Definition

$$M_\nu = M_\nu^\odot - 2.5 \log \frac{\mathcal{L}_\nu(z)}{\mathcal{L}_\nu^\odot} \quad ; \mathcal{L}_\nu = L_\nu \Delta\nu$$

where $L_\nu \Delta\nu$ is the energy emitted per sec in $\Delta\nu = \nu_2 - \nu_1$

more correctly $\mathcal{L}_\nu = \int_{\nu_1}^{\nu_2} L_\nu d\nu$ [erg-sec^{-1}]

From eq.15

$$\mathcal{L}_\nu(z) = 4\pi B_o(z)\, a^2(z)\, g_\infty\, \mathcal{J}(\nu, z)\, \Delta\nu$$

and for the Sun

$$\mathcal{L}_\nu^\odot = 4\pi^2 B_\odot R_\odot^2 \mathcal{J}_\odot(\nu)\, \Delta\nu$$

from the definition

$$M_\nu = M_\nu^\odot - 2.5 \log \left[\frac{B_o(z)\, a^2(z)\, \mathcal{J}(\nu,z)\Delta\nu\, g_\infty}{B_\odot\, R_\odot^2\, \mathcal{J}_\odot(\nu)\Delta\nu\, \pi} \right]$$

$$= M_\nu^\odot - 2.5 \log \left[\frac{B_o(z)}{\mathcal{J}_o} \left(\frac{a}{D_{uc}}\right)^2 \frac{\mathcal{J}(\nu,z)}{\mathcal{J}_\odot(\nu)} \frac{g_\infty}{\pi} \right] - 2.5 \log \left[\frac{\mathcal{J}_o}{B_\odot} \left(\frac{D_{uc}}{R_\odot}\right)^2 \right]$$

Now from eq.18a with $B_o = B_\odot$; $a = R_\odot$ and for a spherical radiator we have $g_\infty = \pi$, thus:

$$-2.5 \log \left[\frac{\mathcal{J}_o}{B_\odot} \left(\frac{D_{uc}}{R_\odot}\right)^2 \right] = 2.5 \log \left[\frac{B_\odot}{\mathcal{J}_o} \left(\frac{R_\odot}{D_{uc}}\right)^2 \frac{\pi}{\pi} \right]$$

$$= -M_{bol}^\odot - 37.82$$

$$\therefore M_\nu = -37.82 - 2.5 \log \left[\frac{B_o(z)}{\mathcal{J}_o} \left(\frac{a}{D_{uc}}\right)^2 \frac{\mathcal{J}(\nu,z)}{\mathcal{J}_\odot(\nu)} \frac{g_\infty}{\pi} \right] + M_\nu^\odot - M_{bol}^\odot$$

The bolometric correction for the sun is given by

$$M_\nu^\odot - M_{bol}^\odot = m_\nu^\odot - m_{bol}^\odot$$

and since

$$m_\nu^0 - m_{bol}^0 \equiv -2.5 \log \frac{f_\nu^0}{f_{bol}^0} = -2.5 \log \frac{\mathcal{J}_0(\nu)\,\Delta\nu}{\int_0^\infty \mathcal{J}_0(\nu)\,d\nu}$$

$$= -2.5 \log \mathcal{J}_0(\nu)\,\Delta\nu$$

we have

$$M_\nu = -2.5 \log \left[\frac{B_0(z)}{\mathcal{J}_0} \left(\frac{a(z)}{D_L} \right)^2 \frac{g_\infty}{\pi} \mathcal{J}(\nu,z)\,\Delta\nu \right] - 37.82 \qquad (19a)$$

If the frequency band is not small enough for 19a to be accurate we write

$$M_\nu = -2.5 \log \left[\frac{B_0(z)}{\mathcal{J}_0} \left(\frac{a(z)}{D_L} \right)^2 \frac{g_\infty}{\pi} \int_{\nu_1}^{\nu_2} \mathcal{J}(\nu,z)\,d\nu \right] - 37.82 \qquad (19b)$$

2.3.4. Apparent Colour Magnitude

We measure the flux up to a detection limit $\mathcal{J}_L(\nu)$ in specific intensity corresponding to an aperture Aap. Thus from eq.9

$$\mathcal{J}_L(\nu) \equiv I_\nu(A_{ap}, z) = \frac{B(A_{ap}\, r_0(z), z)}{(1+z)^3} \mathcal{J}[\nu(1+z), A_{ap}\, r_0(z), z]\, e^{-p(\nu,z)}$$

and from eq's 11 and 12

$$\mathcal{J}_L(\nu) = \frac{B_0(z)}{(1+z)^3} \mathcal{J}[\nu(1+z), z]\, e^{-p(\nu,z)} \tilde{f}(\beta_{ap}) \quad \text{where } \beta_{ap} = \frac{A_{ap}\, r_0(z)}{a(z)} \qquad (20a)$$

Note: Since the spectral distribution of the night sky is not the same ~~than~~ as that of any particular galaxy, the apparent angle Aap depends on the measured frequency: Aap = Aap(ν), therefore $\beta = \beta(\nu)$.

We assume that this effect is not important in the derivation

that follows, restricting its validity (possibly) to narrow bandwidth filters.

Also $$\tilde{A}_{ap}(z) = \frac{a(z)}{r_0(z)} \left(\tilde{f}^{-1} \left(\frac{\mathcal{S}_L (1+z)^3}{B_0(z)\, g[\nu(1+z), z]} \right) \right) .$$

We measure the flux in an aperture $\tilde{A}ap$ at frequency ν. From eq's. 4, 11 and 12 we have that

$$\tilde{F}_\nu(\tilde{A}_{ap}) = \int_0^{\tilde{A}_{ap}} \tilde{I}_\nu(\alpha, z)\, 2\pi\alpha\, d\alpha$$

$$= \frac{B_0(z)}{(1+z)^3}\, g[\nu(1+z), z]\, e^{-p(\nu,z)} \int_0^{\tilde{A}_{ap}} \tilde{f}\left(\frac{\alpha r_0(z)}{a(z)}\right) 2\pi\alpha\, d\alpha .$$

put $\frac{\alpha r_0}{a} = \beta$ and $\tilde{\beta}_{ap} = \frac{\tilde{A}_{ap} r_0}{a}$

$$\therefore \tilde{F}_\nu(\tilde{A}_{ap}) = \frac{B_0}{(1+z)^3}\, g[\nu(1+z), z]\, e^{-p(\nu,z)} \left(\frac{a}{r_0}\right)^2 \int_0^{\tilde{\beta}_{ap}} \tilde{f}(\beta)\, 2\pi\beta\, d\beta .$$

Define $\tilde{g}(\beta) = \int_0^{\beta} \tilde{f}(x)\, 2\pi x\, dx$ and then

$$\tilde{F}_\nu(\tilde{A}_{ap}) = \frac{B_0}{(1+z)^3}\, g[\nu(1+z), z]\, e^{-p(\nu,z)} \left(\frac{a(z)}{r_0(z)}\right)^2 \tilde{g}(\tilde{\beta}_{ap}) . \qquad (20b)$$

Over the frequency band $\Delta\nu_i = [\nu_1, \nu_2]$ of the i-th filter (with spectral sensitivity $\phi_i(\nu, C)$) the flux $F_{\Delta\nu_i}$ is given by

$$\tilde{F}_{\Delta\nu_i}(\tilde{A}_{ap}) = \int_{\nu_1}^{\nu_2} \tilde{F}_\nu(\tilde{A}_{ap})\, \phi_i(\nu, C)\, d\nu .$$

Here C denotes the the conditions of observation [see section 2.2.3]. Now

$$\tilde{F}_{\Delta\nu_i}(\tilde{A}_{ap}) = \frac{B_0(z)}{(1+z)^3} \left(\frac{a}{r_0}\right)^2 \int_{\nu_1}^{\nu_2} \phi_i(\nu, C)\, g[\nu(1+z), z]\, e^{-p(\nu,z)}\, \tilde{g}(\tilde{\beta}_{ap})\, d\nu .$$

and if we make the additional assumption, that $\sigma \neq \sigma(\nu)$, we have

$$\left.\begin{aligned} \tilde{F}_{\Delta\nu_i}(\tilde{A}_{ap}) &= \frac{B_0(z)}{(1+z)^3}\left(\frac{a(z)}{r_0(z)}\right)^2 \tilde{g}(\tilde{\beta}_{ap})\ \Phi_i(z), \\ \text{where} \\ \Phi_i(z) &\equiv \int_{\nu_1}^{\nu_2} \phi_i(\nu, C)\ g[\nu(1+z), z]\ e^{-p(\nu,z)}\, d\nu . \end{aligned}\right\} \qquad (20c)$$

We measure

$$m_{\Delta\nu_i} - m^0_{\Delta\nu_i} = -2.5 \log\left(F_{\Delta\nu_i} / F^0_{\Delta\nu_i}\right)$$

$$= -2.5 \log\left[\frac{B_0(z)}{(1+z)^3}\left(\frac{a}{r_0}\right)^2 \tilde{g}^6(\tilde{\beta}_{ap})\ \Phi_i(z) \Big/ \frac{\pi R_\odot^2 B_\odot}{d_\odot^2}\int_{\nu_1}^{\nu_2} g_\odot(\nu)\, d\nu\right].$$

Since for the sun $L_\odot = 4\pi^2 R_\odot^2 B_\odot$, the flux at distance $d_\odot$ is given by $F_\odot = L_\odot/4\pi d_\odot^2 = \pi R_\odot^2 B_\odot / d_\odot^2$ at a particular wavelength.

Rewrite this as

$$m_{\Delta\nu_i} - m^0_{\Delta\nu_i} = -2.5 \log\left[\frac{B_0(z)}{g_0}\ \Phi_i(z)\left(\frac{a}{a_u}\right)^2 \frac{g_\infty}{\pi}\right] - 2.5 \log\left[\frac{g_0}{B_0}\left(\frac{D_u}{R_0}\right)^2\right]$$

$$+5 \log\left[\frac{r_0(z)\ (1+z)}{D}^{\nu_2}\right] - 5 \log\frac{d_0}{D} + 2.5 \log\int_{\nu_1}^{\nu_2} g_0(\nu)\, d\nu - 2.5 \log\frac{\tilde{g}(\tilde{\beta}_{ap})}{g_\infty}.$$

From section 2.3.3:

$$-2.5 \log\left[\frac{g_0}{B_0}\left(\frac{D_u}{R_0}\right)^2\right] = -M_{bol}{}^0 - 37.82 ,$$

further from eq.16a

$$m^0_{\Delta\nu_i} - 5 \log\frac{d_0}{D} = M_{\Delta\nu_i}{}^0$$

and the bolometric correction is

$$2.5 \log\int_{\nu_1}^{\nu_2} g_0(\nu)\, d\nu = -\left(M^0_{\Delta\nu_i} - M_{bol}{}^0\right)$$

so that

$$m_{\Delta\nu_i}(\tilde{A}_{ap}) = -2.5 \log\left[\frac{B_0(z)}{g_0}\left(\frac{a(z)}{D_u}\right)^2 \frac{g_\infty}{\pi}\int_{\nu_1}^{\nu_2} g(\nu, z)\, d\nu\right] - 2.5 \log\left[\Phi_i(z) \Big/ \int_{\nu_1}^{\nu_2} g(\nu, z)\, d\nu\right]$$

$$+ 5 \log\left[\frac{r_0(z)(1+z)}{10^{-5}\ \mathrm{pc}}^2\right] - 2.5 \log\left[\frac{\tilde{g}(\tilde{\beta}_{ap})}{g_\infty}\right] - 37.82 .$$

Finally from eq.19b

$$m_{\Delta\nu_i}(\tilde{A}_{ap}) = M_{\Delta\nu_i} - 2.5 \log \left[\Phi_i(z) \Big/ \int_{\nu_1}^{\nu_2} g(\nu, z)\, d\nu \right] + 5 \log \left[\frac{r_0(z)\,(1+z)^{3/2}}{D_M} \right] - 2.5 \log \frac{\tilde{g}(\tilde{\beta}_{ap})}{g_\infty} + 25 \qquad (21a)$$

which, in terms of the bolometric magnitude, is

$$m_{\Delta\nu_i}(\tilde{A}_{ap}) = M_{bol} - 2.5 \log \Phi_i(z) + 5 \log \left[\frac{r_0(z)\,(1+z)^{3/2}}{D_M} \right] - 2.5 \log \frac{\tilde{g}(\tilde{\beta}_{ap})}{g_\infty} + 25 \qquad (21b)$$

We can rewrite eq.21b as follows (for comparison with the literature)

$$m_{\Delta\nu_i}(\tilde{A}_{ap}) = M_{bol} - 2.5 \log (1+z) - 2.5 \log \frac{\int_{\nu_1}^{\nu_2} \phi_i(\nu, C)\, g[\nu(1+z), z]\, d\nu}{\int_{\nu_1}^{\nu_2} \phi_i(\nu, C)\, g[\nu, z]\, d\nu}$$

$$- 2.5 \log \left[\int_{\nu_1}^{\nu_2} \phi_i(\nu, C)\, g(\nu, z)\, d\nu \right] - 2.5 \log \frac{\tilde{g}(\tilde{\beta}_{ap})}{g_\infty}$$

$$+ 5 \log \left[\frac{r_0(z)}{D_M} (1+z)^2 \right] + 25$$

$$= M_{bol} + 5 \log \left[\frac{r_0(z)}{D_M} (1+z)^2 \right] + 25 + K_i(z) + \Delta m_{bol\,i} + A^{\sigma}(\tilde{\beta}_{ap}),$$

Where $K_i(z)$ = K-correction as given by [Oke and Sandage 1968 or De Vaucouleurs 1977]

$$= -2.5 \log (1+z) - 2.5 \log \left[\frac{\int_{\nu_1}^{\nu_2} \phi_i(\nu, C)\, g[\nu(1+z), z]\, d\nu}{\int_{\nu_1}^{\nu_2} \phi_i(\nu, C)\, g[\nu, z]\, d\nu} \right],$$

$\Delta m_{bol\,i}$ = Bolometric correction

$$= -2.5 \log \left[\int_{\nu_1}^{\nu_2} \phi_i(\nu, C)\, g(\nu, z)\, d\nu \right]$$

and $A^{\sigma}(\tilde{\beta}_{ap})$ = Aperture correction

$$= -2.5 \log \frac{\tilde{g}(\tilde{\beta}_{ap})}{g_\infty}.$$

Source Evolution:

Define evolution parameters p_1 and p_2 as

$$a(z) = a\,(1+z)^{p_1}$$

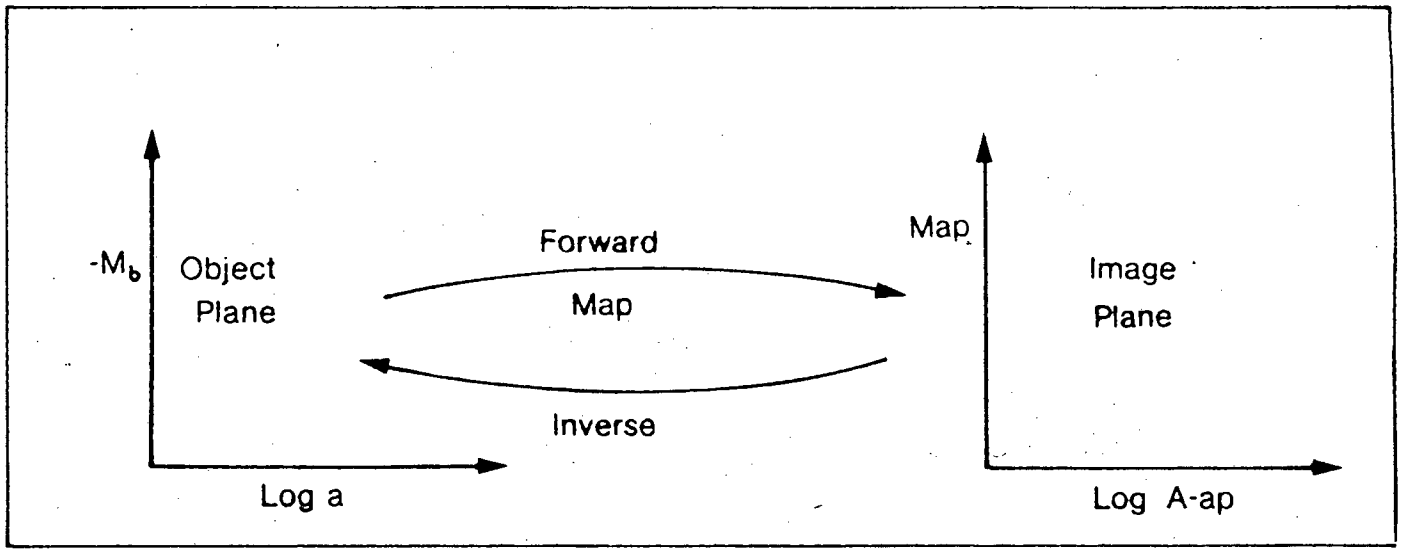
$$B_0(z) = B_0\,(1+z)^{p_2} \qquad (22)$$

thus replace Mbol by $Mbol-2.5\log(1+z)^{p_2}-5\log(1+z)^{p_1}$ and $a(z)$ in the equations derived above. p_1 is the radial evolution index and p_2 the luminosity evolution index.

2.4. The Observational Map

The map from the object to the image plane and its inverse is derived. The map is not defined on the whole of the object plane and the condition for its existence is given. Likewise for the inverse map.

Figure 2-7.



2.4.1. The Defining Equations

a) Define the detection limit S_L in magnitudes as

$$S_L = -2.5 \log \left[\frac{\int_{\nu_1}^{\nu_2} \phi_i(\nu, c)\, S_c(\nu)\, d\nu}{S_0} \right] \quad \text{and from eq. 20a and 22}$$

$$= -2.5 \log \left[\frac{B_0 (1+z)^{p_2}}{S_0} \frac{1}{(1+z)^3} \int_{\nu_1}^{\nu_2} \phi(\nu, z)\, g[\nu(1+z), z]\, e^{-p(\nu,z)}\, \tilde{f}(\beta_{ap})\, d\nu \right]$$

$$= -2.5 \log \left[\frac{B_0 (1+z)^{p_2}}{S_0} \left(\frac{a(1+z)^{p_1}}{D_L} \right)^2 \frac{g_\infty}{\pi} \right] + 5 \log \frac{a(1+z)^{p_1}}{D_L} + 2.5 \log \frac{g_\infty}{\pi}$$

$$+ 5 \log (1+z)^{3/2} - 2.5 \log \bar{\phi}_i(z) - 2.5 \log \tilde{f}(\beta_{ap}).$$

$$\therefore \quad S_L = M_{bol} + \Delta M_i + 5 \log \left(\frac{a}{D_L} \right) + 5 \log (1+z)^{3/2} - 2.5 \log \tilde{f}(\beta_{ap})$$

$$+ 2.5 \log \frac{g_\infty}{\pi} - 2.5 \log (1+z)^{p_2} + 37.82 \qquad (22a)$$

where $\Delta M_i \equiv -2.5 \log \underline{\phi}_i(z) = -2.5 \log \left[\int_{\nu_1}^{\nu_2} \phi_i(\nu, C)\, \vartheta[\nu(1+z), z]\, e^{-p(\nu,z)}\, d\nu \right]$

b) The apparent colour magmitude is given by

$$m_{\Delta\nu_i}(A_{ap}) = M_{bol} + \Delta M_i + 5 \log \frac{r_0(z)\,(1+z)^{3/2}}{D_M} - 2.5 \log \frac{\tilde{g}(\beta_{ap})}{g_\infty}$$

$$-(2.5\, p_2 + 5\, p_1) \log (1+z) + 25 . \qquad (22b)$$

c) The apparent angle in terms of the scaling radius is given by:

$$\log A_{ap} = \log \frac{a(1+z)^{p_1}}{r_0(z)} \beta_{ap} \quad , \text{ or}$$

$$\log A_{ap} = \log \frac{a}{D_k} - \log \frac{r_0(z)}{D_M} - 3 + \log \beta_{ap} + p_1 \log (1+z) . \qquad (22c)$$

2.4.2. Forward Map

$$\left(\log \frac{a}{D_k} \; ; \; M_{bol} \right) \rightarrow \left(\log A_{ap} \; ; \; m_{\Delta\nu}(A_{ap}) \right)$$

For each point in the object plane ($\log A_{ap}$, Mbol) at redshift z there is a one-to-one map to the image plane provided, that

$$I_\nu^6(0, z) > \mathcal{S}_L(\nu)$$

i.e.

$$-2.5 \log \tilde{F}(0) < S_L - (M_{bol} + \Delta M_i + 37.82) - 2.5 \log \frac{g_\infty}{\pi} - 5 \log \left(\frac{a}{D_k} (1+z)^{3/2} \right) + 2.5\, p_2 \log (1+z)$$

Solve for βap from eq.22a

$$-2.5 \log \tilde{F}(\beta_{ap}) = S_L - (M_{bol} + \Delta M_i + 37.82) - 2.5 \log \frac{g_\infty}{\pi} - 5 \log \left(\frac{a}{D_k} (1+z)^{3/2} \right) + 2.5\, p_2 \log (1+z)$$

$$\left.\begin{aligned} &\text{then} \quad \log A_{ap} = \log \frac{a}{D_{ik}} - \left[\log\left(\frac{r_0(z)}{D_M}\right) + 3\right] + \log \beta_{ap} + p_1 \log(1+z) \\ &\text{and} \quad m_{\Delta v_i}(A_{ap}) = M_{bol} + \Delta M_i - 2.5 \log \frac{\tilde{g}(\beta_{ap})}{g_\infty} + 5 \log \frac{r_0(z)(1+z)^{3/2}}{D_M} + 25 \\ &\qquad - (2.5 p_2 + 5 p_1) \log(1+z) \ . \end{aligned}\right\} \quad (23)$$

2.4.3. Inverse Map

$$\left(\log A_{ap} \ ; \ m_{\Delta v_i}(A_{ap})\right) \rightarrow \left(\log \frac{a}{D_{ik}} \ ; \ M_{bol}\right)$$

The inverse map exists only when

$$5 \log A_{ap} + m_{\Delta v}(A_{ap}) < S_L - 27.82 \qquad \text{[see pg.2-33].}$$

Substitude for Mbol and $\log \frac{a}{D_{ik}}$ in eq.22a from eq's 22b and 22c and regroup, then

$$-2.5 \log \left\{ \frac{\beta_{ap}^2 \tilde{f}(\beta_{ap})}{\tilde{g}(\beta_{ap})/g_\infty} \right\} = S_L - m_{\Delta v_i}(A_{ap}) - 5 \log A_{ap} - \left(27.82 + 2.5 \log \frac{g_\infty}{\pi}\right) . \quad (24a)$$

For any assumed profile $\beta^2 f(\beta)/g(\beta)$ is a well defined function which is constant along lines of constant surface brightness. Now solve for βap from eq.24a, the map is then defined by:

$$\log \frac{a}{D_{ik}} = \log A_{ap} + \log \frac{r_0(z)}{D_M} + 3 - \log \beta_{ap} - \log(1+z)^{p_1} \quad (24b)$$

and

$$M_{bol} = m_{\Delta v_i}(A_{ap}) - \Delta M_i + 2.5 \log \frac{\tilde{g}(\beta_{ap})}{g_\infty} - 5 \log \frac{r_0(z)(1+z)^{3/2}}{D_M} + (2.5 p_2 + 5 p_1) \log(1+z) - 25 \ . \quad (24c)$$

The limit of $\frac{\beta^2 f(\beta)}{g(\beta)}$ for $\beta \to 0$

This limit is indeterminate, thus use L'Hospital's rules

$$\therefore \lim_{\beta\to 0} \frac{\beta^2 f(\beta)}{g(\beta)} = \lim_{\beta\to 0} \frac{2\beta f(\beta) + \beta^2 f'(\beta)}{2\pi\beta f(\beta)} \quad \text{as } g(\beta) = 2\pi \int_0^\beta x f(x)\, dx$$

$$= \lim_{\beta\to 0} \frac{2f(\beta) + 2\beta f'(\beta) + 2\beta f'(\beta) + \beta^2 f''(\beta)}{2\pi f(\beta) + 2\pi\beta f'(\beta)} \quad , \; f'(\beta) = \frac{df}{d\beta}$$

$$= \frac{1}{\pi} \quad \text{if } \beta f'(\beta) \text{ and } \beta^2 f''(\beta) \to 0 \text{ as } \beta \to 0$$

define $G(\beta) \equiv -2.5 \log \frac{\beta^2 f(\beta)}{g(\beta)/g_\infty}$ then $\lim_{\beta\to 0} G(\beta) = -2.5 \log \frac{g_\infty}{\pi}$.

Note that for any finite value for β

$$\int_0^\beta x f(x)\, dx < \beta^2 f(\beta) \quad \text{for small enough } \beta$$

and the limit $\lim_{\beta\to 0} \frac{\beta^2 f(\beta)}{g(\beta)} = \frac{1}{\pi}$ is approached from below. Thus $G(\beta)$ has a minimum at $\beta = 0$ and by eq.24a we must have

$$G(0) < S_L - m_{av}(A_{ap}) - 5 \log A_{ap} - \left(27.82 + 2.5 \log \frac{g_\infty}{\pi}\right)$$

and as $G(0) = \lim_{\beta\to 0} G(\beta) = -2.5 \log \frac{g_\infty}{\pi}$ we have

$$m_{av}(A_{ap}) + 5 \log A_{ap} < S_L - 27.82$$

2.5. A Specialised Form of the Map

In order to compute the observational map, the following approximations have been made

2.5.1. Brightness Profiles for Elliptical Galaxies

A) de Vaucouleurs $r^{1/4}$-law [de Vaucouleurs 1959 or 1977]

Define B as the surface brightness at metric radius r, and let Bov be the central surface brightness (i.e. r=0), then

$$B/B_{ov} \equiv f(\beta) = e^{-[(\beta)^{1/4}]} \qquad \text{where } \beta = \frac{r}{a_{ov}}, a_{ov} \text{ a scaling radius} \qquad (25a)$$

The inverse profile f^{-1} is given by

$$f^{-1}(x) = (-\ln x)^4$$

From eq.14 $g(\beta)$ is given by

$$g_v(\beta) = 2\pi \int_0^\beta x f(x)\, dx = 2\pi \int_0^\beta x\, e^{-(x)^{1/4}}\, dx$$

put $z = x^{1/4}$ and integrate by parts

$$g_v(u) = 8!\pi \left\{ 1 - e^{-u}\left[1 + u + \frac{u^2}{2!} + \ldots + \frac{u^7}{7!}\right]\right\}$$

where $u = \beta^{1/4}$ and $g_v^\infty = 8!\pi$ so that

$$\frac{g_v(u)}{g_v^\infty} = 1 - e^{-u}\left[1 + u + \frac{u^2}{2!} + \ldots + \frac{u^7}{7!}\right]\Big\} \qquad (25b)$$

Note: In computing g_v/g_∞ eq.25b is numerically unstable for small values of u. Making use of the fact that

$$e^x = \underbrace{\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^7}{7!}\right)}_{A(x)} + \underbrace{\left(\frac{x^8}{8!} + \frac{x^9}{9!} + \ldots\right)}_{B(x)}$$

we may write $\frac{g_v(u)}{g_v\infty} = 1 - \frac{A(u)}{A(u)+B(u)} = \frac{B(u)}{A(u)+B(u)}$.

For u<0.8 approximate B(u) by

$$B(u) = \left\{ \frac{u^8}{8!} + \frac{u^9}{9!} + \frac{u^{10}}{10!} + \frac{u^{11}}{11!} \right\} = \frac{u^8}{8!}\left(1 + \frac{u}{9}\left(1 + \frac{u}{10}\left(1 + \frac{u}{11}\right)\right)\right).$$

$$\therefore \quad \frac{g_v(u)}{g_v\infty} = e^{-u}\left\{\frac{u^8}{40\,320}\left(1 + \frac{u}{9}\left(1 + \frac{u}{10}\left(1 + \frac{u}{11}\right)\right)\right)\right. \quad \text{for } u < 0.8,$$

$$= 1 - e^{-u}\left\{1 + u\left(1 + \frac{u}{2}\left(1 + \frac{u}{3}\left(1 + \frac{u}{4}\left(1 + \frac{u}{5}\left(1 + \frac{u}{6}\left(1 + \frac{u}{7}\right)\right)\right)\right)\right)\right)\right\}.$$

for $u \geq 0.8$

<u>An alternative form of the de Vaucouleurs law</u>

Define an "effective" scaling radius r_e such that $g_v(1) = \frac{1}{2} g\infty$ i.e. 1/2 of the total light of the galaxy is contained within a metric radius r_e. Solve the implicit equation $g(r/a_v) = \frac{1}{2} g\infty$ to find

$$r_e = 3459\, a_v.$$

Now $$B/B_{ov} = e^{-(r/r_e)^{1/4}\,(r_e/a_v)^{1/4}} = e^{-7.669\,(r/r_e)^{1/4}},$$

or $$\log \frac{B}{B_{ov}} = -3.331\left(\frac{r}{r_e}\right)^{1/4}.$$

Define the surface brightness at $r=r_e$ as $B_e = B(r_e)$. Then

$$\log \frac{B}{B_e} = -3.331\left\{\left(\frac{r}{r_e}\right)^{1/4} - 1\right\}$$

and in magnitude units $\mu \equiv -2.5 \log B$

$$-2.5 \log B/B_e = \mu - \mu_e = 8.326\left\{\left(\frac{r}{r_e}\right)^{1/4} - 1\right\}. \qquad (25c)$$

Note: This form of the de Vaucouleurs law has only one free fitting parameter. This is not a serious limitation, as the central surface brightness B_o is unknown for galaxies beyond the

local group (due to the effect of seeing and limited instrumental resolution the central cores of elliptical galaxies beyond the local group are unresolved [Schweizer 1979, de Vaucouleurs 1979]).

B) Abell-Mihalas profile [Abell and Mihalas 1966]

Define as for the de Vaucouleurs profile

$$\frac{B}{B_{oA}} \equiv f(\beta) = \begin{cases} (1+\beta)^{-2} & \text{for } \beta \leq 21.4 \\ 22.4(1+\beta)^{-3} & \text{for } \beta > 21.4 \end{cases} \qquad (26a)$$

where $\beta = r/a_A$, a_A a scaling radius. Integrating the profile as before and defining $\beta_c = 21.4$ we have

$$g_A(\beta) = \begin{cases} 2\pi \left[\ln(1+\beta) - \beta/(1+\beta) \right] & \text{for } \beta \leq \beta_c \\ 2\pi \left[\ln(1+\beta_c) + \frac{1}{2(1+\beta_c)} - (1+\beta)\left\{ \frac{1}{1+\beta} - \frac{1}{2(1+\beta)^2} \right\} \right] & \text{for } \beta > \beta_c . \end{cases} \qquad (26b)$$

$$g_A^{\infty} = 2\pi \left[\ln(1+\beta_c) + \frac{1}{2(1+\beta_c)} \right] = 19.675 .$$

The inverse profile f^{-1} is given by

$$f^{-1}(x) = \begin{cases} 1/\sqrt{x} - 1 & \text{for } x \geq 1.993 \cdot 10^{-3} \\ \left(\frac{22.4}{x}\right)^{1/3} - 1 & \text{for } 0 < x < 1.993 \cdot 10^{-3} . \end{cases}$$

Fitting the profiles

De Vaucouleurs gives [de Vauc.1959] a very good fit for $0.03 < r/r_e < 5$, perhaps even a good fit up to $r/r_e = 15$. Abell and Mihalas [Abell and Mihalas 1966] give a good fit for both profiles for $0.35 < r/a_A < 200$ ($0.03 < r/r_e < 18$; $r_e = 11 a_A$). Kormendy gives the relations $a_A = 0.093 r_e + 0.00$ kpc and $-2.5 \log B_{oA} = -2.5 \log B_e - 5.27$, obtained by fitting the brightness profiles to galaxies. [Kormendy 1977]

(Kormendy fits the Hubble profile, but the parameters a_A and B_{oA} will have the same values as those for the Abell-Mihalas profile. See eq's A2 and A3 in Kormendy 1977]

Mathematically we can compare the de Vaucouleurs and the Abell-Mihalas profiles as follows. Since the central surface brightness of galaxies is not known and both profiles are physically meaningless at the center (both profiles have a non-zero brightness gradient at r=0, this corresponds to an infinite space density at the center [King 1978]), assume that the metric radius that contains 1/2 of the light is the same for both profiles, as well as the luminosity L. From eq.15

$$L = 4\pi B_o a^2 g_\infty .$$

Now solve $\frac{g_A(\beta)}{g_{A\infty}} = \frac{1}{2}$ for the Abell-Mihalas profile, obtaining $\beta = 10.97$. By definition for the de Vauc. law $\beta = 1$

$$\therefore a_A = \frac{r_e}{10.97} = 0.09119\, r_e .$$

Comparing luminosities $B_{oV} a_V^2 g_{V\infty} = B_{oA} a_A^2 g_{A\infty}$ and by substituting $a_V = \frac{r_e}{3459}$ and $a_A = \frac{r_e}{10.97}$ as well as for $g_{V\infty}$ and $g_{A\infty}$ we obtain

$$B_{oV} = 15.45\, B_{oA} \quad \text{and} \quad -2.5 \log B_{oV} = -2.5 \log B_{oA} - 2.972 .$$

Thus at the center the de Vauc. law is nearly 3 magnitudes brighter. Now $\log \frac{B_e}{B_{oV}} = -3.331$

$$\therefore -2.5 \log B_{oA} = -2.5 \log B_e - 5.356$$

which differs from the relation found by Kormendy by less than 0.1 magnitude. Alternatively, we can compare the profiles by assuming that the mean surface brightness over a central region r_c is the same for both profiles.

2.5.2. Intergalactic Absorbtion

From section 2.2.3 the optical depth up to redshift z is given by

$$\rho(\nu, z) = \int_0^z n_a(z')\, \sigma_a(z', \nu(1+z'))\, \frac{dl}{dz'}\, dz' ,$$

where $n_a(z)$ = number density of absorbers at z

and $\sigma_a(z,\nu)$ = interaction cross-section of absorbers.

Following Gunn and Peterson [Gunn and Peterson 1965] we specialise to Thompson scattering by free electrons in an ideal cosmological model (FRW).

$$\therefore \quad \sigma_a(z,\nu) = \sigma_e = \text{constant} .$$

The probability of scattering of a photon in a proper length interval $dl = R(t)\, du$, at cosmic time t and dimensionless parameter distance u [Sandage 1961], is given by

$$dp = n_e(t)\, \sigma_e\, dl ,$$

where n_e and σ_e are the number density and cross-section for electrons respectively. Then the optical depth p is given by

$$p(z) = \sigma_e \int n_e\, dl = \sigma_e \int_0^z n_e(z')\, \frac{dl}{dz'}\, dz' ,$$

now for a FRW universe

$$\frac{dl}{dz} = R(t)\, \frac{du}{dz} = \frac{c\, H_0^{-1}}{(1+z)^2 (1+2q_0 z)^{1/2}}$$

and $$n_e(z) = n_e(0)\, (1+z)^3$$

$$\therefore \quad \rho(z) = \frac{\sigma_e\, n_e(0)\, c}{H_0} \int_0^z \frac{(1+z')}{(1+2q_0 z')^{1/2}}\, dz' .$$

which can be integrated to give

$$p(z) = \frac{\sigma_e n_e(0) c}{H_0} \frac{1}{3q_0^2}\left\{(3q_0 + q_0 z - 1)(1+2q_0 z)^{1/2} - (3q_0 - 1)\right\} \text{ when } q_0 \neq 0$$

and

$$p(z) = \frac{\sigma_e n_e(0) c}{H_0} \frac{1}{2}(1+z)^2 \quad \ldots\ldots\ldots\ldots \text{ when } q_0 = 0. \qquad (27)$$

Assuming that the universe is composed entirely out of Hydrogen and fully ionised we have for an Einstein-de Sitter universe ($q_0 = 0.5$) $n_e = 10^{-5}$ cm^{-3} and for the low-density universe ($q_0 = 0.01$) $n_e = 10^{-7}$ cm^{-3}. Taking H_0 = 100km-sec -Mpc , we have for electrons [Bahcall and Salpeter (1965)]:

$$\frac{\sigma_e n_e(0) c}{H_0} = 6.5 \times 10^3 n_e(0) ,$$

The number density for electrons is proportional to q_0 i.e. $n_e(0) \propto q_0$ [see Sandage 1961 or Bahcall and May 1968] and if f_0 represents the fraction of ionized hydrogen we can write:

$$n_e(0) = 2 \times 10^{-5} f_0 q_0$$

$$\therefore \quad \frac{\sigma_e n_e(0) c}{H_0} = \frac{13 f_0 q_0}{H_0} . \qquad (28)$$

2.5.3. The Magnitude Correction

A first approximation to the magnitude correction ΔM (a combination of the K-correction and bolometric correction see section 2.3.4) is given. A fit to known galaxies showed that the results are approximately correct. The maps are much more sensitive to the value of the detection limit S_L however than to the value of the magnitude correction.

From section 2.4.1 eq.22a :

$$\Delta M_i = -2.5 \log \Phi_i(z)$$

$$= -2.5 \log \left\{ \int_{\nu_1}^{\nu_2} \phi_i(\nu, C)\, \mathcal{J}[\nu(1+z), z]\, e^{-p(\nu,z)}\, d\nu \right\}.$$

We make the assumptions:

1) Thompson scattering [see section 2.5.2]

$$p(z) = \frac{\sigma_e n_e(0) c}{H_0} \frac{1}{3q_0^2} \left\{ (3q_0 + q_0 z - 1)(1 + 2q_0 z)^{1/2} - (3q_0 - 1) \right\} \quad ; q_0 \neq 0$$

$$= \frac{\sigma_e n_e(0) c}{H_0} \frac{1}{2} (1+z)^2 \quad ; q_0 = 0 .$$

2) Power law spectrum $\mathcal{J}(\nu, z) = C \left(\frac{\nu}{\nu_0}\right)^{-\alpha}$; α = spectral index

3) ϕ_i a stepfunction

$$\phi_i(\nu, C) = \begin{cases} 0 & \nu < \nu_1 \\ 1 & \nu_1 \leq \nu \leq \nu_2 \\ 0 & \nu > \nu_2 \end{cases} .$$

Consider the integral

$$\int_{\nu_1}^{\nu_2} \phi_i(\nu, C)\, \mathcal{J}[\nu(1+z), z]\, d\nu = \int_{\nu_1}^{\nu_2} C \left(\frac{\nu(1+z)}{\nu_0}\right)^{-\alpha} d\nu$$

$$= (1+z)^{-\alpha} \frac{C}{(1-\alpha)\nu_0^{-\alpha}} \left(\nu_2^{1-\alpha} - \nu_1^{1-\alpha}\right), \alpha \neq 1 .$$

$$\therefore \Delta M_i = -2.5 \log (1+z)^{-\alpha} + \frac{2.5}{\ln 10} p(z) - 2.5 \log \frac{C\left(\nu_2^{1-\alpha} - \nu_1^{1-\alpha}\right)}{(1-\alpha)\nu_0^{-\alpha}}$$

or

$$\left.\begin{aligned} \Delta M_i &= 2.5\, \alpha \log(1+z) + 1.086\, p(z) + \Delta M_{in}, \\ \text{where} \quad \Delta M_{in} &= -2.5 \log \frac{C\left(\nu_2^{1-\alpha} - \nu_1^{1-\alpha}\right)}{(1-\alpha)\nu_0^{-\alpha}} . \end{aligned}\right\} \qquad (29)$$

Monochromatic case: Measure the apparent magnitude in a narrow wavelength interval $\Delta\nu$ around ν and make the following approximations:

3. Numerical Methods

3.1. The Problem

In the absence of point-spread - the "idealised" map - both the forward and the inverse map can be calculated without any numerical approximation and is straight forward. In the presence of point-spread, the brightness profile is a 2-D convolution of the incident flux with a p.s.f. and has to be approximated numerically. In the forward map with point-spread, an implicit equation (section 2.4 eq.23) of the form

$$-2.5 \log \tilde{f}(\beta_{ap}) - \text{const.} = 0$$

is solved for every mapping.

Here

$$\tilde{f}(\beta_{ap}) = \frac{1}{\pi} \int_0^{\infty} dy \int_0^{\pi} d\theta \; y \; e^{-(y^2/2 + R_{ap}^2/2 - R_{ap} y \cos\theta)} f(Ay),$$

where

$$\beta_{ap} = R_{ap} A = \frac{r_{ap}}{W} \frac{W}{a} .$$

To complete the map, a triple integral has to be evaluated

$$\tilde{g}(\beta_{ap}) = 2\pi \int_0^{\beta_{ap}} \beta \, \tilde{f}(\beta) \, d\beta$$

put $\beta = RA$ and $\beta_{ap} = R_{ap}A$, then

$$\tilde{g}(\beta_{ap}) = 2A^2 \int_0^{R_{ap}} dR \int_0^{\infty} dy \int_0^{\pi} d\theta \; R y \; e^{-(y^2/2 + R^2/2 - Ry\cos\theta)} f(Ay) .$$

To compute the inverse map, an equation (eq.24a) of the form

$$\tilde{G}(\beta_{ap}) + \text{const.} = 0$$

has to be solved for a given mapping, where

$$\tilde{G}(\beta_{ap}) \equiv -2.5 \log \frac{\beta_{ap}^2 \tilde{f}(\beta_{ap})}{\tilde{g}(\beta_{ap})/g_\infty} .$$

As an first approximation for β_{ap}, a fifth order polynominal has been fitted to the inverse of $\tilde{G}$ for $\sigma=0$. A better approximation for β_{ap} is then found by an iterative procedure given in section 3.4. This procedure works well for the map with no point-spread ($\sigma=0$), but often fails to converge for $\sigma>0$. The program has to be extended to include a better initial approximation for β_{ap} to make the inverse map with point-spread reliable.

A double and triple integration routine (section 3.2 and 3.3) as well as a root-solving routine is thus needed to compute the observational map. It was not easy to make the program efficient: To map a single pair of points, often more than ten double integrals have to be evaluated.

3.2. The Integration Procedure

The seeing convolved profile is given by

$$\tilde{f}(R,A) = \frac{1}{\pi} \int_0^\infty dy \int_0^\pi d\theta \; y \, e^{-(y^2/2 + R^2/2 - Ry\cos\theta)} f(Ay) .$$

It was found empirically that when $R \gtrsim 15$, seeing is unimportant (i.e. the image of a galaxy has apparent diameter roughly 15 times the diameter of a star). No such restriction can be put on A, in fact A may become very large ($\sim 10^6$), so that the shape of the integrand can vary considerably over the range of R and A. No satisfactory analytical expression for an upper bound for the integration over y could be found. It was found by calculation that R + 5 is always a sufficiently large upper bound, but for very large values for A this bound could be a thousand

times larger than the least sufficient upper bound. This can result in very inaccurate results. Instead the integral is calculated in two parts; from zero to a suitable chosen, not too large, upper bound y_b and from y_b to infinity. The user may subdivide the interval $[0,y_b]$ at wish into equally spaced intervals (usually a division in two subintervals suffices). On each subinterval a 16-point Legendre-Gauss quadrature is performed. In case y_b is not a sufficiently large upper bound to give an sufficiently accurate approximation to the integral, a 16 point quadrature is done over the interval $[y_b, \infty)$, which is not further subdivided.

The bound y_b is found in the following way. Define the function

$$I(y, R, A) \equiv y\, e^{-(y^2/2 + R^2/2 - Ry + (Ay)^{1/4})},$$

where we take $\cos(\Theta)=1$ and $f(\beta) = e^{-(\beta)^{1/4}}$, the de Vaucouleurs profile for ellipticals. The function I has a maximum when

$$\frac{\partial I}{\partial y} = 0 \Rightarrow 1 - y^2 + Ry - \frac{1}{4}(Ay)^{1/4} = 0 \quad ;$$

define the function SLOPE by

$$\mathrm{SLOPE(Y)} = 1 - y^2 + Ry - \tfrac{1}{4}(Ay)^{1/4},$$

indicating whether the slope of the tangent to I in y is negative, zero or positive.

There are two extreme cases:

A) $R \gg A$

Approximate I by I(y,R) given by

$$I(y, R) = y\, e^{-(y^2/2 + R^2/2 - Ry)},$$

with a maximum y_1 for y satisfying

$$1 - y^2 + yR = 0 .$$

$$\therefore y_1 = R/2 + \sqrt{R^2/4 + 1} .$$

B) $R << A$

Approximate I by I(y,A) given by

$$I(y,A) \cdot y\, e^{-(Ay)^{1/4}}$$

with a maximum y_2 for y satisfying

$$1 - \tfrac{1}{4}(yA)^{\frac{1}{4}} = 0 .$$

$$\therefore y_2 = \frac{256}{A} .$$

An upper bound y_b was chosen in each of both cases such that the integrand drops a factor of ~100 from its maximum value

In case A : $y_{b_1} = R + 3$.

In case B : $y_{b_2} = 40\,000/A$.

The smaller of y_{b_1} and y_{b_2} is selected

$$y_m = \min \left\{ y_{b_1} ; y_{b_2} \right\}$$

and if SLOPE(Y_m) < 0, this value is chosen as y_b.

The same procedure is followed for the Abell-Mihalas profile where it is found that

$$y_{b_1} = R + 3 \quad \text{and} \quad y_{b_2} = 400/A .$$

3.3. The Integration Routines

The double and triple integration are both done with a Legendre-Gauss Quadrature (16-point). The 16-point quadrature is accurate for all polynominals with order up to 31 (2×16 - 1). The results were more accurate by an order of magnitude compared to an 5-point quadrature (used earlier).

The following is taken from [Hildebrand 1974]. Another useful reference is [Conte 1965].

n-point Legendre-Gauss Quadrature

$$\int_{-1}^{1} f(x)\,dx = \sum_{k=1}^{n} A_k f(x_k) + E ,$$

where

$$A_k = -\frac{2}{(n+1)\,P_{n+1}(x_k)\,P_n'(x_k)} = \frac{2}{n\,P_{n-1}(x_k)\,P_n'(x_k)} .$$

$P_n(x)$ is the n-th Legendre polynominal, $P_n'(x)$ its first derivative and x_k its k-th root. E is an error term given by

$$E = \frac{2^{2n+1}\,(n!)^4}{2n+1\,[(2n)!]^3}\,f^{(2n)}(\xi) \quad ; -1 \le \xi \le 1 .$$

The general case is given by

$$\int_a^b f(x)\,dx = \frac{b-a}{2}\int_{-1}^{1} f\left(y\,\frac{b-a}{2} + \frac{b+a}{2}\right) dy ,$$

if we make the transformation $x = y\,\frac{b-a}{2} + \frac{b+a}{2}$.

and $$\int_a^{\infty} f(x)\,dx = \int_{-1}^{1} \frac{2}{(1-y)^2} f\left(\frac{2}{1-y} + a - 1\right) dy ,$$

if we make the transformation $x = \frac{2}{1-y} + a - 1$.

Note: $$\int_a^{\infty} f(x)\,dx = 2\sum_{i=1}^{n} \frac{A_i}{(1-x_i)^2} f\left(\frac{2}{1-x_i} + a - 1\right)$$

$$= 2\sum_{i=1}^{n} \frac{A_i}{(1+x_i)^2} f\left(\frac{2}{1+x_i} + a - 1\right),$$

as the roots of P_n are symmetric with respect to zero i.e. $x_i = -x_{n-i+1}$.

Integrating over two dimensions it follows that:

$$\int_a^b dx \int_c^d dy\, f(x,y) = \frac{b-a}{2}\,\frac{d-c}{2} \sum_{i=1}^{n}\sum_{j=1}^{n} A_i A_j f\left(\frac{x_i(b-a)+(b+a)}{2}; \frac{x_j(d-c)+(d+c)}{2}\right),$$

and

$$\int_a^b dx \int_d^{\infty} dy\, f(x,y) = (b-a) \sum_{i=1}^{n}\sum_{j=1}^{n} A_i A_j/(1+x_j)^2 \, f\left(\frac{x_i(b-a)+(b+a)}{2}; \frac{2}{x_j+1} + d - 1\right).$$

And finally in three dimensions, deviding every interval in N equal subintervals e.g. $[x_a; x_b]$ is devided into NX equal subintervals $[x_{ah}; x_{bh}]$ with length $\Delta x = x_{bh} - x_{ah}$.

$$\int_{z_a}^{z_b} dz \int_{y_a}^{y_b} dy \int_{x_a}^{x_b} dx\, f(x,y,z) = \frac{\Delta x}{2}\,\frac{\Delta y}{2}\,\frac{\Delta z}{2} \sum_{h=1}^{NX}\sum_{i=1}^{NY}\sum_{j=1}^{NZ}\sum_{k=1}^{n} A_k \sum_{l=1}^{n} A_l \sum_{m=1}^{n} A_m \times$$

$$f\left[\frac{\Delta x}{2} x_k + \frac{x_{bh}+x_{ah}}{2}; \frac{\Delta y}{2} x_l + \frac{y_{bi}+y_{ai}}{2}; \frac{\Delta z}{2} x_m + \frac{z_{bj}+z_{aj}}{2}\right],$$

and

$$\int_{z_a}^{z_b} dz \int_{y_b}^{\infty} dy \int_{x_a}^{x_b} dx\, f(x,y,z) = \frac{x_b - x_a}{2}\,\frac{z_b - z_a}{2} \sum_{k=1}^{n} A_k \sum_{l=1}^{n} \frac{2A_l}{(1+x_l)^2} \sum_{m=1}^{n} A_m \times$$

$$f\left[\frac{x_b - x_a}{2} x_k + \frac{x_b + x_a}{2}; \frac{2}{1+x_l} + y_b - 1; \frac{z_b - z_a}{2} x_m + \frac{z_b + z_a}{2}\right].$$

TABLE: Weights and Abscissas for 16-point Quadrature

A_i ; weighting coefficients

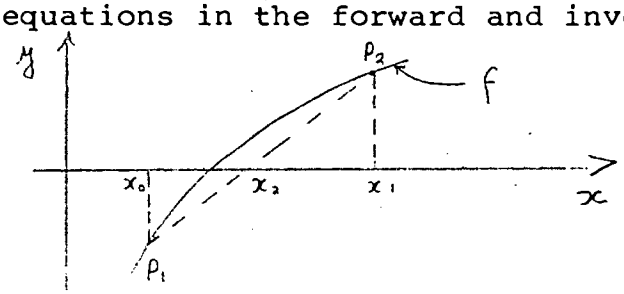
$A_1 = A_{16}$	1.894506105 E-1
$A_2 = A_{15}$	1.826034150 E-1
$A_3 = A_{14}$	1.691565194 E-1
$A_4 = A_{13}$	1.495959888 E-1
$A_5 = A_{12}$	1.246289713 E-1
$A_6 = A_{11}$	9.515851168 E-2
$A_7 = A_{10}$	6.225352394 E-2
$A_8 = A_9$	2.715245941 E-2

x_i ; abscissas

$x_1 = -x_{16}$	9.501250984 E-2
$x_2 = -x_{15}$	2.816035508 E-1
$x_3 = -x_{14}$	4.580167777 E-1
$x_4 = -x_{13}$	6.178762444 E-1
$x_5 = -x_{12}$	7.554044084 E-1
$x_6 = -x_{11}$	8.656312024 E-1
$x_7 = -x_{10}$	9.445750231 E-1
$x_8 = -x_9$	9.894009350 E-1

3.4. Root Solving Routines

The Secant Method (or "The Rule of False Position") proved to be a reliable and nearly quadratic converging algorithm to solve the implicit equations in the forward and inverse map.



We want to solve f(x) = 0. The equation of the straight line through P_1 and P_2 is given by

$$y - f(x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_1),$$

now if y=0 put $x=x_2$

$$\therefore \quad x_2 - x_1 = -f(x_1)\frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

or

$$x_2 = x_1 - f(x_1) \Big/ \frac{f(x_1) - f(x_0)}{x_1 - x_0}.$$

This relation gives the iterative scheme

$$x_{n+1} = x_n - f(x_n) \Big/ \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}} \quad \text{for } n = 1, 2, \ldots$$

i.e. given two approximations x_{n-1} and x_n to the root of f(x) = 0 a new approximation will be x_{n+1}.

This method has been implemented as follows:

A) Program SECA

A positive solution (root) of f(x) = 0 is required. The program assumes a monotonic increasing function and requires only one initial guess xl say. x2 is then chosen such that the root is

either enclosed by xl and x2, or x2 lies closer towards the root than xl. The program then starts to iterate. A search for a root is successful if:

1) The relative error $\left(\frac{x_n}{x_{n-1}} - 1\right) < \varepsilon$.

2) The function value $f(x_n) < 10^{-32}$ for n=2,3,4...

A search is unsuccessful if

1) No convergence after 20 iterations .

2) If $f(x_{n+1}) = f(x_n)$ to eight decimal places for n=1,2,3,...

In the event that $x_n < 0$, then $x_n = x_{n-1} + x_{n-1}/4$ for n=3,4,5,...

B) Program SECANT

The same program as in SECA, but a positive root of a monotonic decreasing function is required. In the event that $x_n < 0$ for any iteration n=3,4,5,..., then $x_n = x_{n-1}/4$.

3.5. Fitting a Polynominal

A 5th order polynominal has been fitted to the function G in section 2.4.3 to obtain an first approximation to the inverse map. G is fitted independently over several intervals.

A) De Vaucouleurs profile

Define:

$$\beta_e = \frac{r}{r_e} \; ; \; r_e = 3459 \; a_v \; . \qquad \text{(see section 2.5.1)}$$

1) For $\beta_e < 0.0005$ an analytical approximation could be used

$$\beta_e(G) = \left(\sqrt{36288\, 10^{0.4G - 6.5}} - 5\right)^4 .$$

2) For $0.0005 < \beta_e < 0.5$ the polynominal

$$\beta_e(G) = +41.875150\ x^5 - 450.97870\ x^4 - 873.34070\ x^3 + \\ +2816.3958\ x^2 + 7315.3342\ x + 4282.6471\ ,$$

where $x = G + 10$

3) For $0.5 < \beta_e < 200$ the polynominal

$$\beta_e(G) = -0.70328080\ x^5 + 21.785877\ x^4 - 111.20230\ x^3 + \\ +3521.4592\ x^2 + 7626.6401\ x + 4330.0641\ .$$

4) For $\beta_e > 200$ the program returns and prints a message.

B) Abell Mihalas profile

1) For $0 < \beta < 5$ the polynominal

$$\beta(G) = -28.999653\ G^5 - 225.19037\ G^4 - 684.14948\ G^3 + \\ -1009.9881\ G^2 - 715.31806\ G - 188.83001\ .$$

2) For $5 < B < 895$

$$\beta(G) = -2.6711450\ G^5 + 15.395485\ G^4 - 9.5759662\ G^3 + \\ -14.928260\ G^2 + 42.047339\ G + 36.346761\ .$$

3) For $\beta > 895$ the program returns and prints a message.

4. Application of the Program

In this section the observational map is calculated for several examples and the results are shown in a series of figures. The computations are done with the program listed in the appendix. The parameters that determine the map are taken from the current literature to make the calculation more realistic. Throughout the calculations a Hubble constant of 50-km-sec-Mpc, a spectral gradient of 0.7 and the De Vaucouleurs brightness law for elliptical galaxies have been used. [see section 2.5]

In figure 1 the object-plane is shown with a grid that is mapped into the image-plane in figures 2 to 7. The x-coordinate is taken to be log r_e, where r_e is the radius in kpc that contains one-half of the light of the De Vaucouleurs brightness model [section 2.5.1]. The y-coordinate is the absolute bolometric magnitude. A typical bright cluster galaxy will have coordinates of r_e=10kpc and Mbol=-22 and is indicated by a dot (e.g. Kormendy gives for M87 in the Virgo cluster r_e=7.3kpc and Mbol=-21.64 [Kormendy 1977]). The crosses mark points that are mapped into the image-plane.

Figure 2 shows the grid from fig.1 at a redshift z=0.01. The map is taken to be the "ideal" map with no point-spread, no intergalactic absorbtion and no angular or brightness evolution. A low density FRW-universe is assumed with q_o=0.01 and the detection limit on the surface brightness S_L is 25 mag per square arc sec. The detection limit S_L is indicated by a line. The x-coordinate is the log(Aap), Aap the apparent angle in radians. Some values for Aap in arc sec are also indicated. The y-coordinate is the apparent magnitude measured through an aperture with an angle of

2Aap. The large distortion of the grid "against" the detection limit is immediately apparent.

In figure 3 the grid is shown at a cosmological redshift z=0.2. The same parameters are taken for the map as in fig.2. Comparing fig.3 and fig.2 a slight change in shape of the grids can be noted, although the major effect is a displacement of the grid as a whole along the line of constant surface brightness S_L.

Figure 4 shows the grid mapped with point-spread. all other parameters in the map have the same values ~~than~~ as in fig.3. Searching the literature [see e.g. R.S.Ellis (1977), King (1978), de Vaucouleurs (1979) or Schweizer (1979)] we found that a seeing disk of diameter 2-3 seconds of arc is fairly common corresponding to a σ of 1 arc sec in the p.s.f. The huge additional distortion can be seen immediately when fig.4 and fig.3 are compared. The general effect is that images are shifted closer to the detection limit S_L. This shift is large for images with very small or very large r_e (for a given absolute magnitude). Images with small r_e are enlarged drastically and the apparent magnitude does not change very much, images with large r_e are shifted towards and along the line of constant surface brightness S_L [see fig.7] or fall below the detection limit resulting from the large drop in central surface brightness of the image [see fig.15].

In figure 5 the effect of intergalactic absorbtion is shown for a FRW universe with $q_o = 1.0$. The cross + marks the grid from fig.1 at redshift z=0.2 for no absorbtion and the cross x marks it for full absorbtion f=1 [see section 2.5.2]. Again $S_L=25$, no evolution and no pointspread (p.s.) is assumed. For a low density universe with $q_o=0.01$ the map was nearly identical with and without absorbtion (this remains true for a redshift of 0.7 !).

In figure 6 the grid in figure 1 is mapped assuming $S_L=27$ mag

per square arc sec. The remaining parameters of the map are the same as those used in fig.3. Comparing fig.6 and 3 it can be seen that the grid is shifted to the right in fig.6 (as expected). Noteworthy is the change in shape of the grid: images tend to be further away from the brightness limit S_L, therefore the "distortion" of the grid is less in fig.6 than in fig.3.

In order to show the effect of p.s. more clearly only part of the grid in fig.1 is shown in figure 7 (Mbol=-21, -23, and -25). Triangles mark images mapped without p.s. and circles mark those mapped with p.s. The displacement resulting from p.s. is marked by arrows. All parameters in the map are the same as in fig.3 and 4.

In figures 8 to 10 (redshift, apparent angle) graphs and (redshift, apparent magnitude) graphs are shown for a galaxy with r_e=10kpc and Mbol=-22. The detection limit S_L is assumed to be 25 mag per square arc sec. In figure 8 the effect of galactic evolution is shown. The radial evolution is taken to be $r_e(z)=r_e(1+z)^{p_1}$ and the central surface brightness is kept constant. The magnitude evolution is taken to be an evolution of the central surface brightness $B_o(z)=B_o(1+z)^{p_2}$ [see the end of section 2.3.4]. A low density universe with q_o=0.01, no p.s. and no absorbtion is assumed. Log cz is drawn against log Aap or m-ap where c is the speed of light.

Even more drastic is the change of the limiting redshift (the max redshift for which the galaxy remains detectible) if we introduce p.s. shown in figure 9. As labelled in the fig. the curves are drawn for no p.s. and a point spread of σ = 1". In both cases the curves for full absorbtion and for no absorbtion are shown for a universe with q_o=1. Notice that the limiting redshift is reached much more suddenly if the map is done with p.s. than without, the curves flatten rather abruptly, as the central region

of the p.s. profile is flat [see fig.15].

In figure 10 curves with different values for q_o are shown as marked in the fig.(q_o=0.02 and 1.0). Again the curves are shown with and without p.s. As in fig.9 the p.s. effect is dominant and makes it very difficult to distinguish curves with different values for q_o. The limiting redshift is the same for curves with different q_o but no p.s. This is not so if the map is done with p.s., as the limiting redshift is reached much earlier.

In figure 11 the image plane is shown with several observational limits. Firstly the detection limit on the surface brightness S_L is shown, then the limits m-ap=23 and A-ap=1 arc sec are drawn, beeing approximately the magnitude limit and the limit of resolution for the Anglo Australian Sky Survay. Finally a magnitude limit of A-ap=0.1 arc sec are taken to represent the observational limits of the Space Telescope [Longair (1979)]. The observable region is in each case the region below the brightness limit S_L, below the magnitude limit, and to the right of the limit of resolution. These limits are mapped into the object plane in fig's 12, 13 and 14.

Figure 12 shows both, the limit for the sky survey and for the space telescope at a redshift of z=0.05. No point spread, no evolution and no absorbtion and q_o=0.01 are assumed. The observational region is above the curves showing the magnitude limit, brightness limit and limit of resolution. Clearly the magnitude limit is not important in both cases, and instead the resolution limit and the surface brightness limit determine the area in the object plane that can be observed. The line parallel to the brightness limit is a line of constant surface brightness of 14.8 mag per square arc sec and corresponds to the average central surface brightness for bright elliptical galaxies [Disney

(1976)]. This line may be close to that shown by Halton Arp in fig.3 [Arp (1965)].

In figure 13 the limits are shown again for the sky survey at redshift z=0.05 and z=0.2. The observable region shifts towards the top of the figure as the redshift increases. The same is done in figure 14 for the limits for the space telescope.

In figure 15 the brightness profiles for a galaxy with r_e=10kpc is shown for different redshifts (corresponding to different σ in the p.s.f. as the seeing disk increases relatively to the image of the galaxy). The profiles are drawn for z=0.0034, 0.01, 0.05, 0.1, 0.2, 0.4 and 0.6.

Figure Captions

Symbols:

r_e = effective radius in de Vaucoulers brightness law

A-AP = apparent angle

M BOL = absolute magnitude

M-AP = apparent magnitude

z = cosmological redshift

S_L = surface brightness limit mag per $(\text{arc sec})^2$

$H_\circ$ = Hubble constant

$q_\circ$ = deacceleration parameter

σ = dispersion in p.s.f. in arc sec

p_1 = radial evolution index

p_2 = brightness evolution index

f = amount of intergalactic absorbtion

$H_\circ$ = 50 km-sec^{-1}-Mpc^{-1}, a spectral gradient of .7 and the De Vaucouleurs profile for ellipticals assumed in all calculations.

Figure I : Grid in the object-plane.

Figure 2 : Image-plane: $z = 0.01$; $S_L = 25$; $q_\circ = 0.01$; $\sigma = 0$
$p_1 = 0$, $p_2 = 0$, $f = 0$.

Figure 3 : Image-plane: $z = 0.2$; $S_L = 25$; $q_\circ = 0.01$; $\sigma = 0$
$p_1 = 0$; $p_2 = 0$; $f = 0$.

Figure 4 : Image-plane: $z = 0.2$; $S_L = 25$; $q_\circ = 0.01$; $\sigma = 0$ and I"
$p_1 = 0$; $p_2 = 0$; $f = 0$.

Figure 5 : Image-plane: $z = 0.2$; $S_L = 25$; $q_o = 1.0$; $\sigma = 0$
$p_1 = 0$; $p_2 = 0$; $f = 0$ and 1.0.

Figure 6 : Image-plane: $z = 0.2$; $S_L = 27$; $q_o = 0.01$; $\sigma = 0$
$p_1 = 0$; $p_2 = 0$; $f = 0$.

Figure 7 : Image-plane: $z = 0.2$; $S_L = 25$; $q_o = 0.01$; $\sigma = 0$ and $1''$
$p_1 = 0$; $p_2 = 0$; $f = 0$.

Figure 8 : Projection: $S_L = 25$; $q_o = 0.01$; $\sigma = 0$
$p_1 = -1, 0, +1$; $p_2 = -1, 0, +1$; $f = 0$.

Figure 9 : Projection: $S_L = 25$; $q_o = 1.0$; $\sigma = 0$ and $1''$
$p_1 = 0$; $p_2 = 0$; $f = 0$ and 1.0.

Figure 10: Projection: $S_L = 25$; $q_o = 0.02$ and 1.0; $\sigma = 0$ and $1''$
$p_1 = 0$; $p_2 = 0$; $f = 0$.

Figure 11: Image-plane with Observational Limits.

Figure 12: Object-plane, Limits: $z = 0.05$; $S_L = 25$; $q_o = 0.01$;
$\sigma = 0$; $p_1 = 0$; $p_2 = 0$; $f = 0$.

Figure 13: Object-plane, Limits: $z = 0.05$ and 0.2 "

Figure 14: Object-plane, Limits: $z = 0.05$ and 0.2 "

Figure 15: Brightness profiles convoluted with a p.s.f

Figure 1

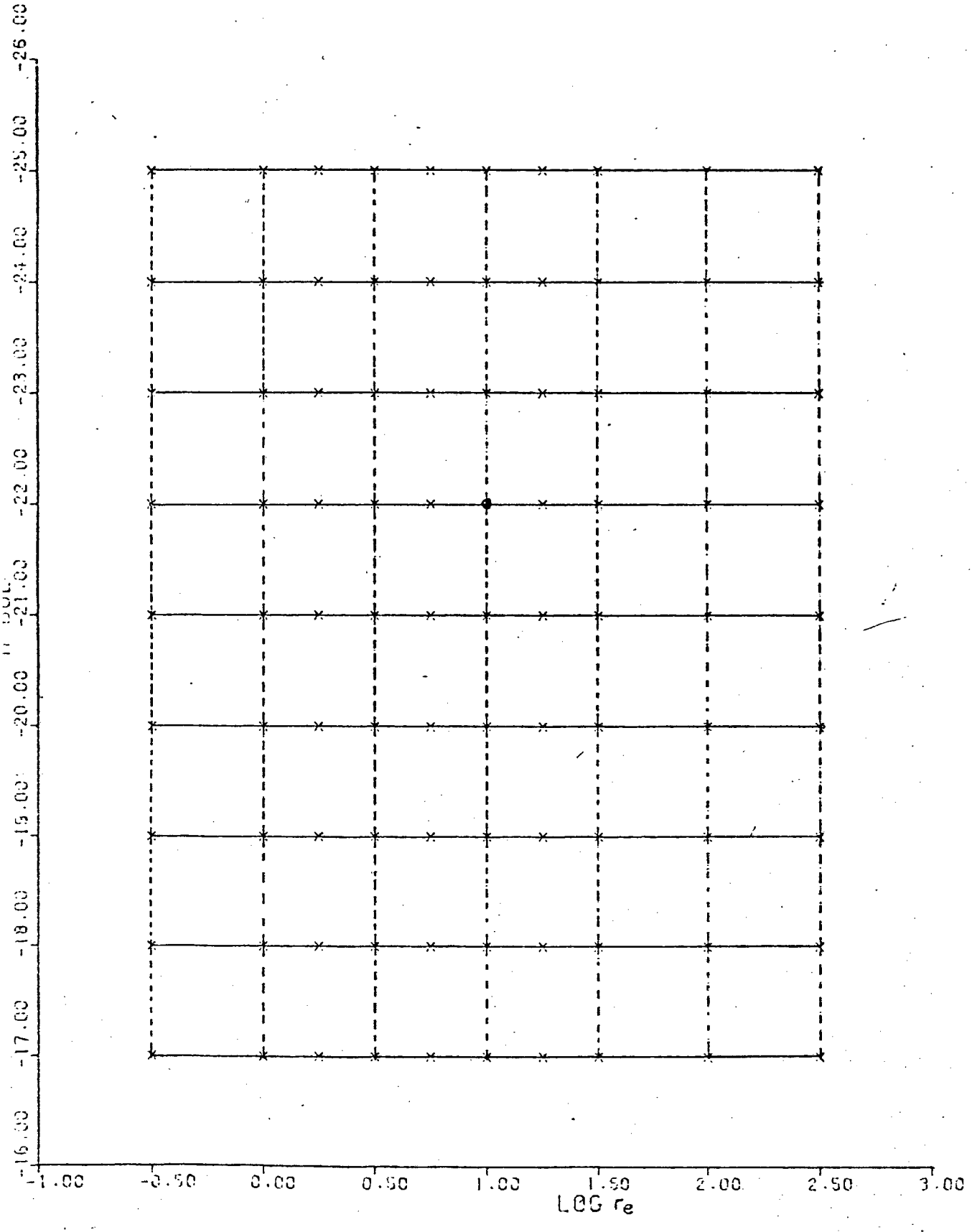

-26.00
-25.00
-24.00
-23.00
-22.00
-21.00
-20.00
-19.00
-18.00
-17.00
-16.00
-1.00
-0.50
0.00
0.50
1.00
1.50
2.00
2.50
3.00
LOG r_e

Figure 2

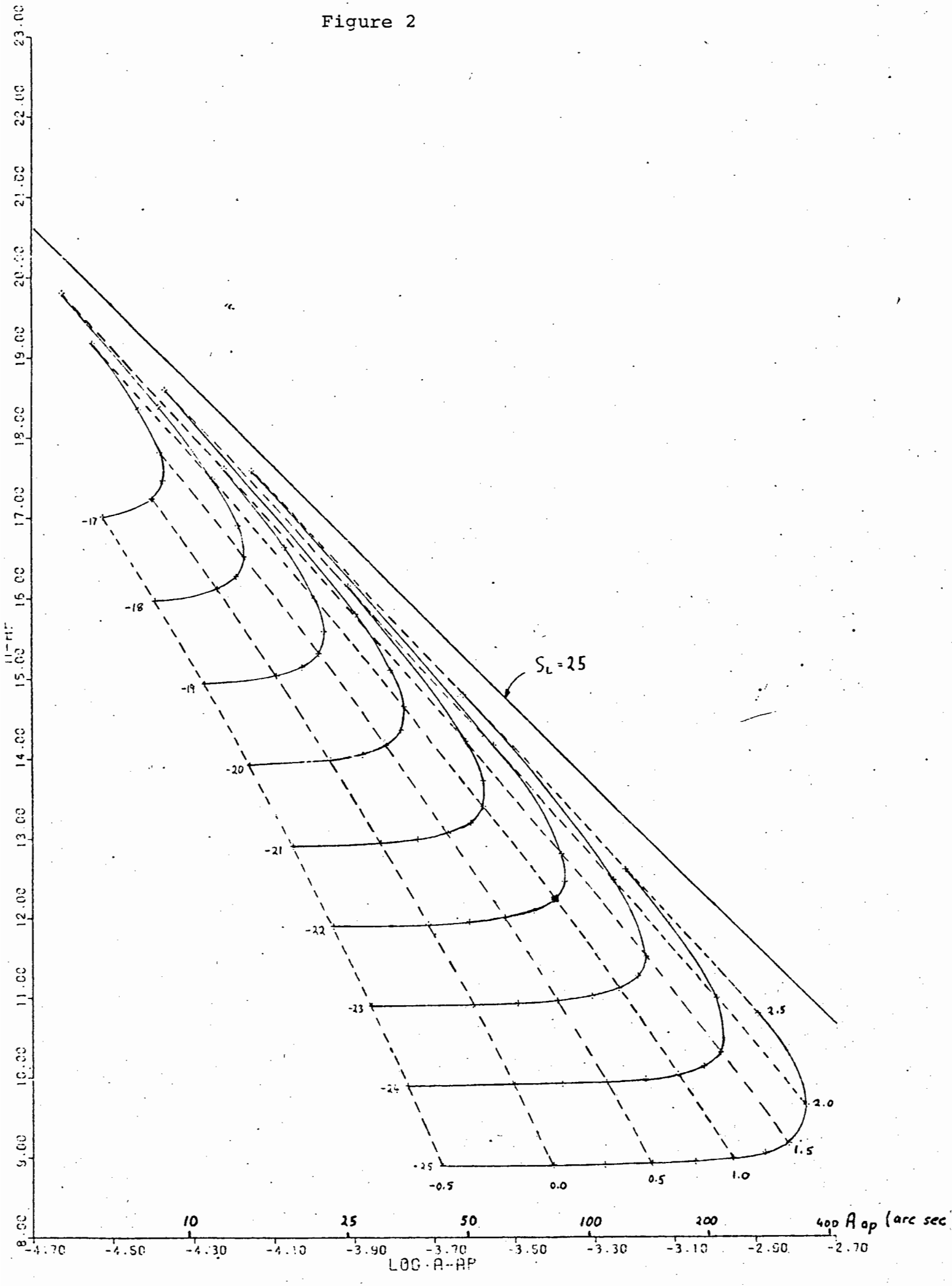

$S_L = 25$
-17
-18
-19
-20
-21
-22
-23
-24
-25
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
10
25
50
100
200
400 A_{ap} (arc sec)
-4.70
-4.50
-4.30
-4.10
-3.90
-3.70
-3.50
-3.30
-3.10
-2.90
-2.70
LOG·A-AP

Figure 3

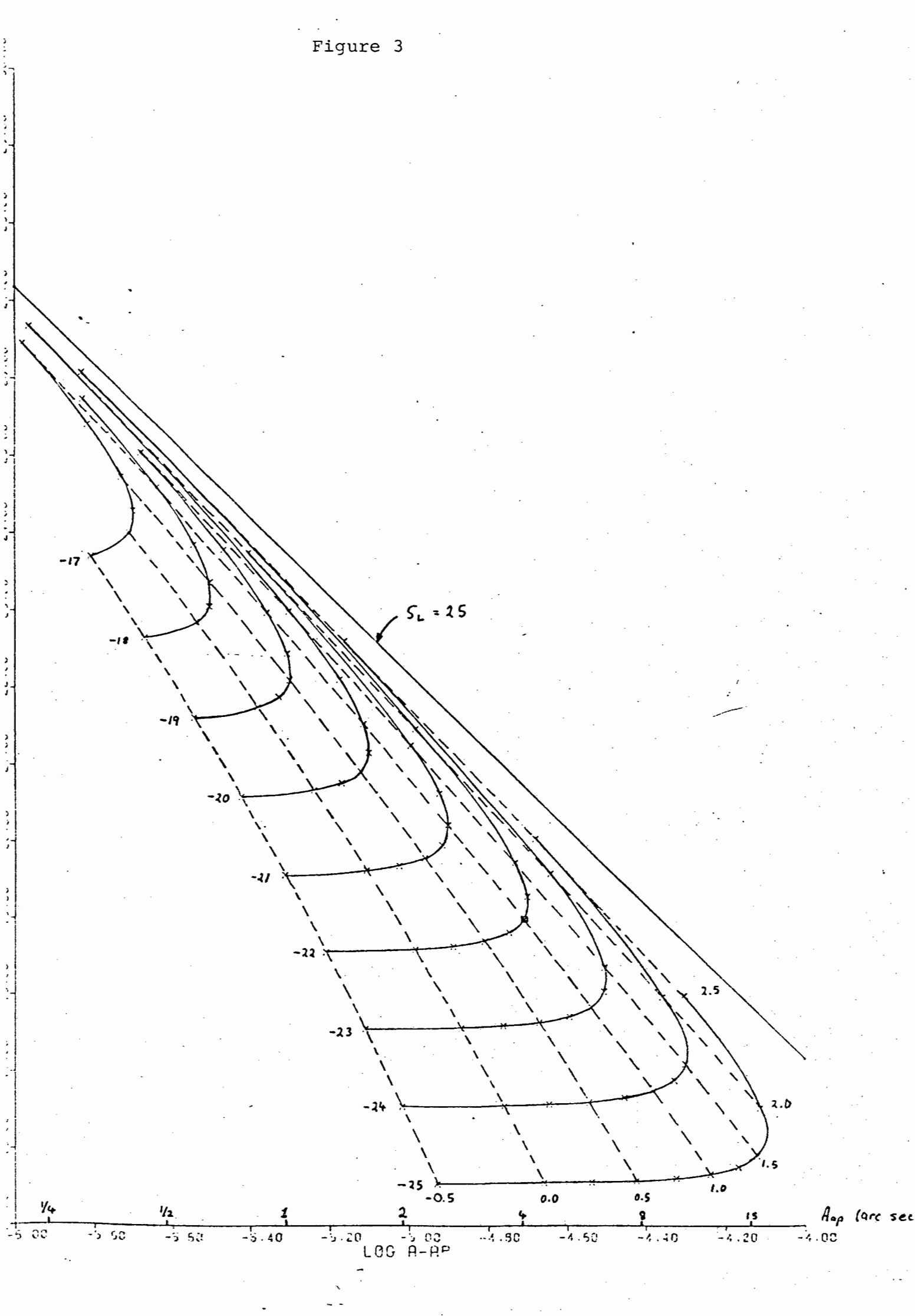

$S_L = 25$
-17
-18
-19
-20
-21
-22
-23
-24
-25
2.5
2.0
1.5
1.0
-0.5
0.0
0.5
1/4
1/2
1
2
4
8
15
A_{ap} (arc sec
-5.00
-5.00
-5.60
-5.40
-5.20
-5.00
-4.80
-4.60
-4.40
-4.20
-4.00
LOG A-AP

Figure 4

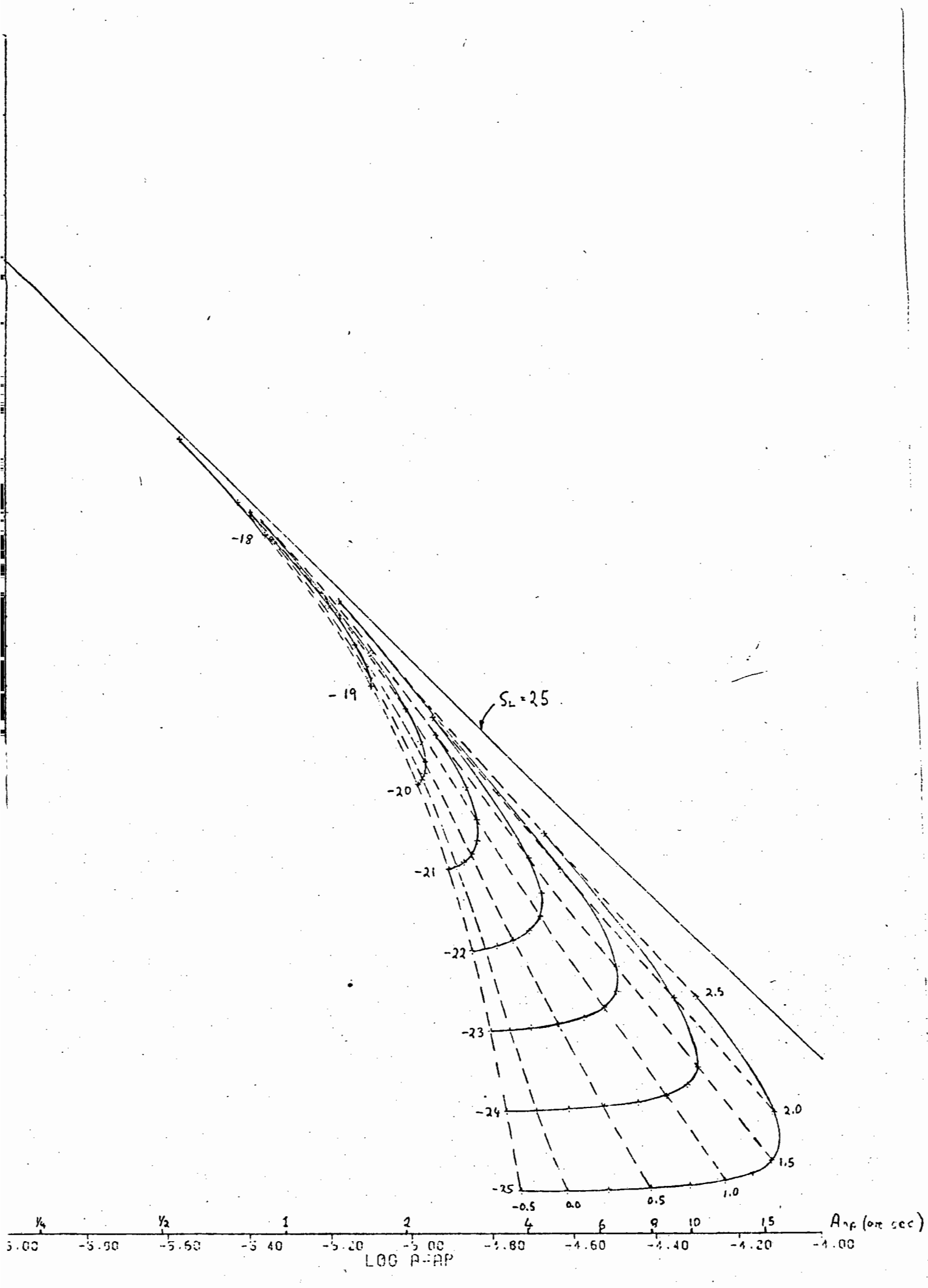

-18
-19
$S_L = 25$
-20
-21
-22
-23
-24
-25
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
¼
½
1
2
4
6
8
10
15
A_{ap} (on sec)
-5.00
-5.80
-5.60
-5.40
-5.20
-5.00
-4.80
-4.60
-4.40
-4.20
-4.00
LOG A-AP

Figure 5

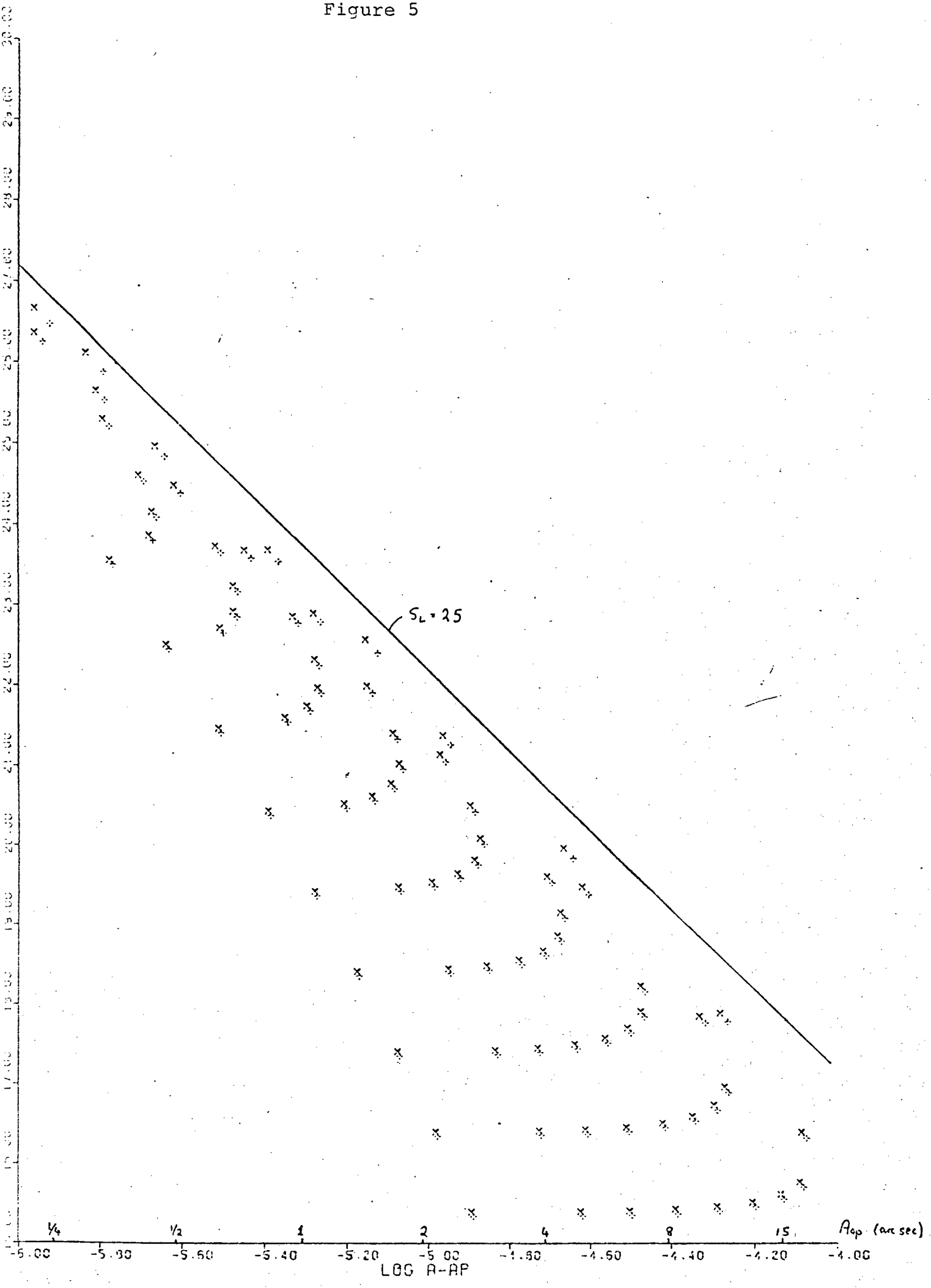

$S_L = 2.5$
1/4
1/2
1
2
4
8
15
A_{ap} (arc sec)
-5.00
-5.00
-5.60
-5.40
-5.20
-5.00
-4.80
-4.60
-4.40
-4.20
-4.00
LOG A-AP

Figure 6

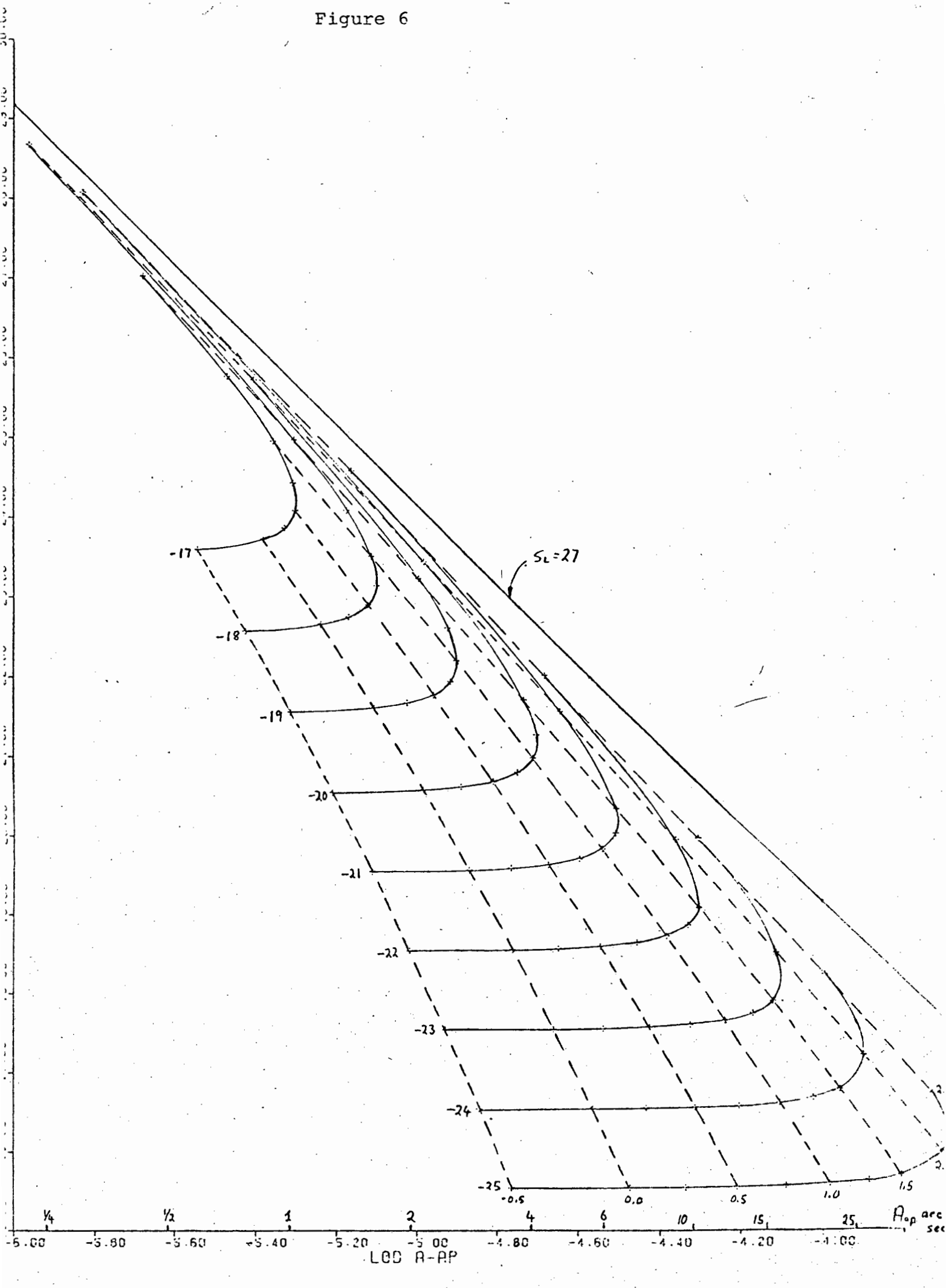

S_L=27
-17
-18
-19
-20
-21
-22
-23
-24
-25
-0.5
0.0
0.5
1.0
1.5
¼
½
1
2
4
6
10
15
25
A_{op} arc sec
-5.00
-5.80
-5.60
-5.40
-5.20
-5.00
-4.80
-4.60
-4.40
-4.20
-4.00
LOG A-AP

Figure 7

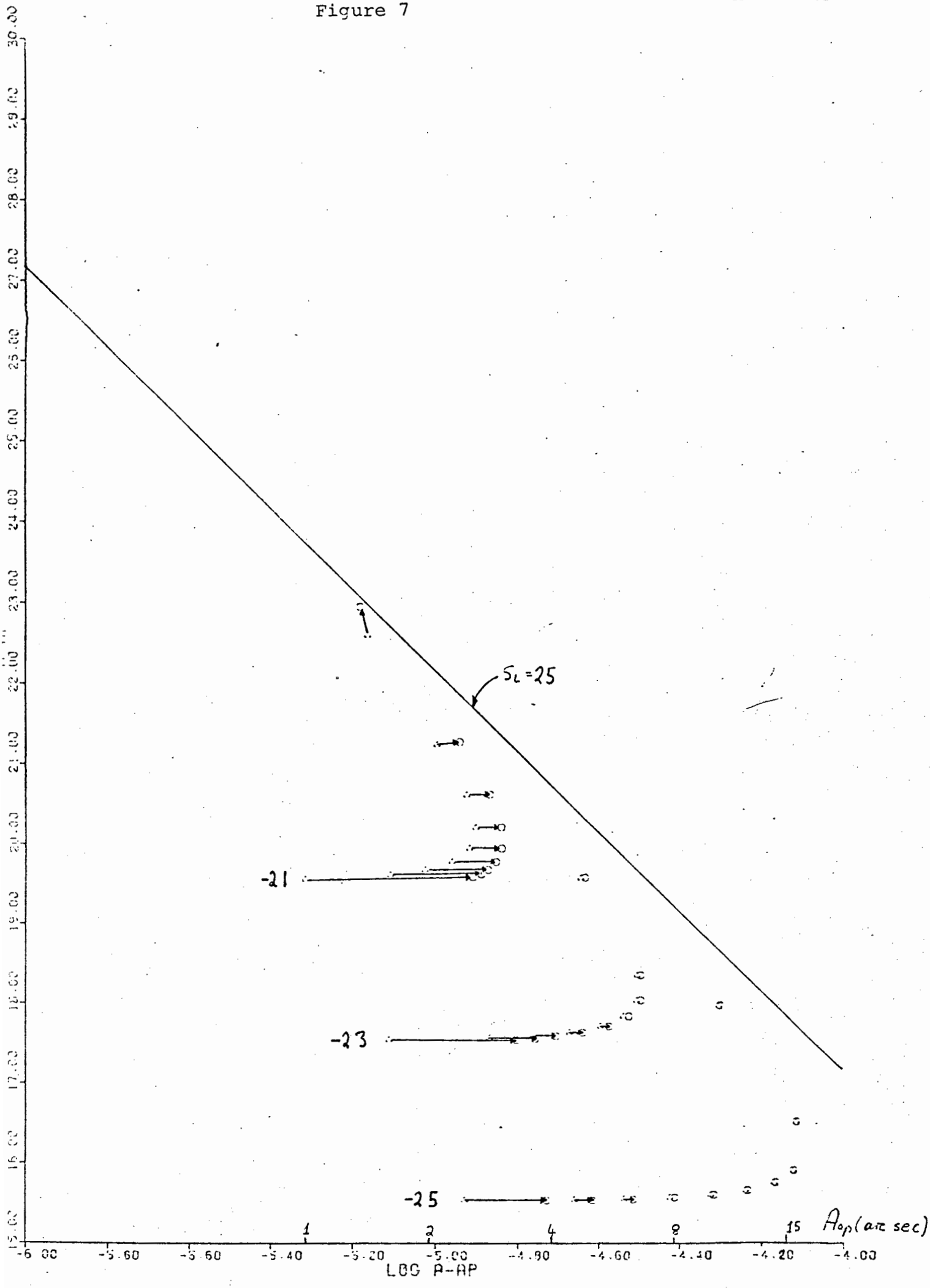
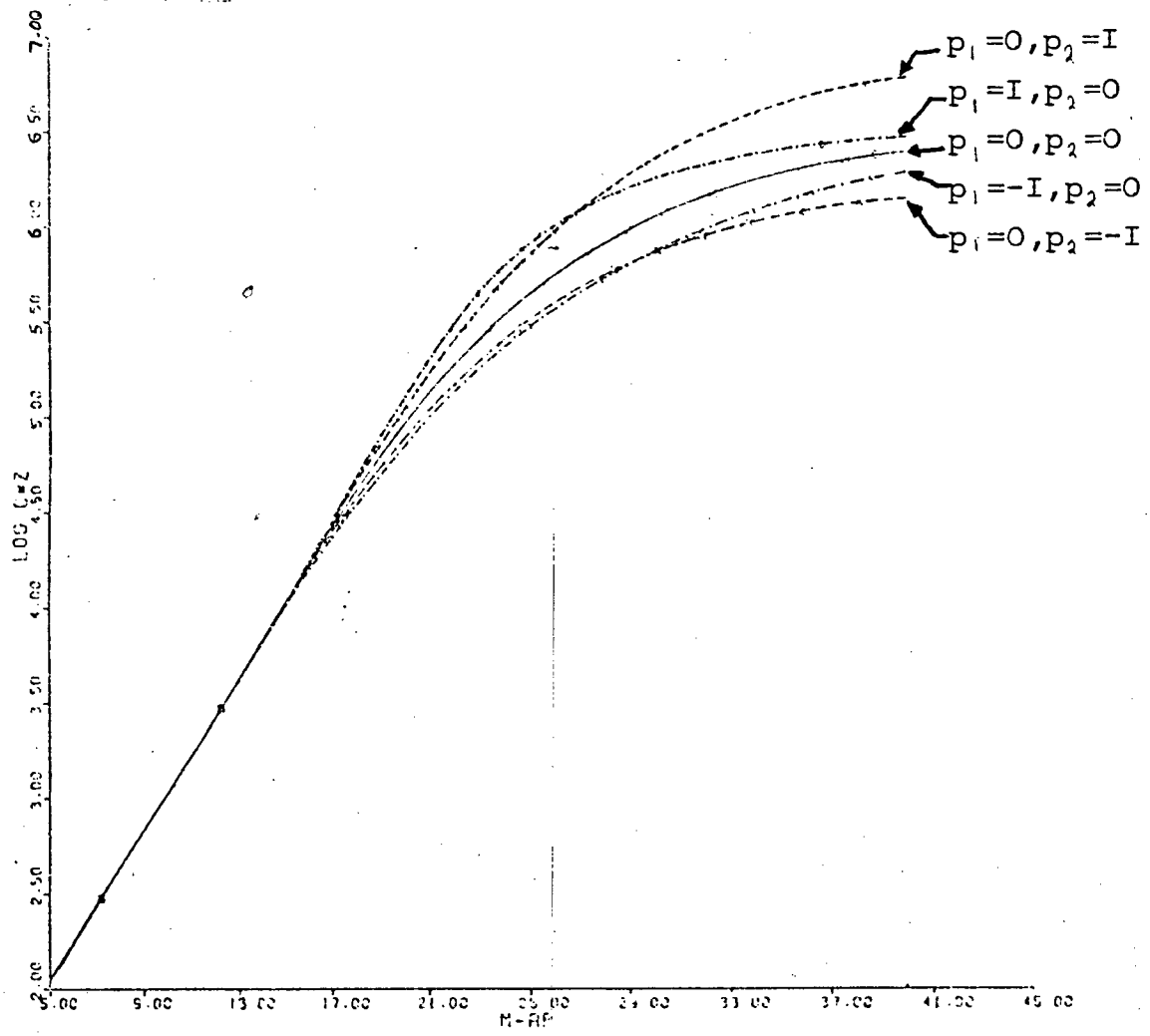

$S_L = 25$
-21
-23
-25
1
2
4
8
15
A_{op} (arc sec)
-6.00
-5.80
-5.60
-5.40
-5.20
-5.00
-4.80
-4.60
-4.40
-4.20
-4.00
LOG P-AP

Figure 8


$p_1=0, p_2=I$
$p_1=I, p_2=0$
$p_1=0, p_2=0$
$p_1=-I, p_2=0$
$p_1=0, p_2=-I$
LOG C=Z
M-AP

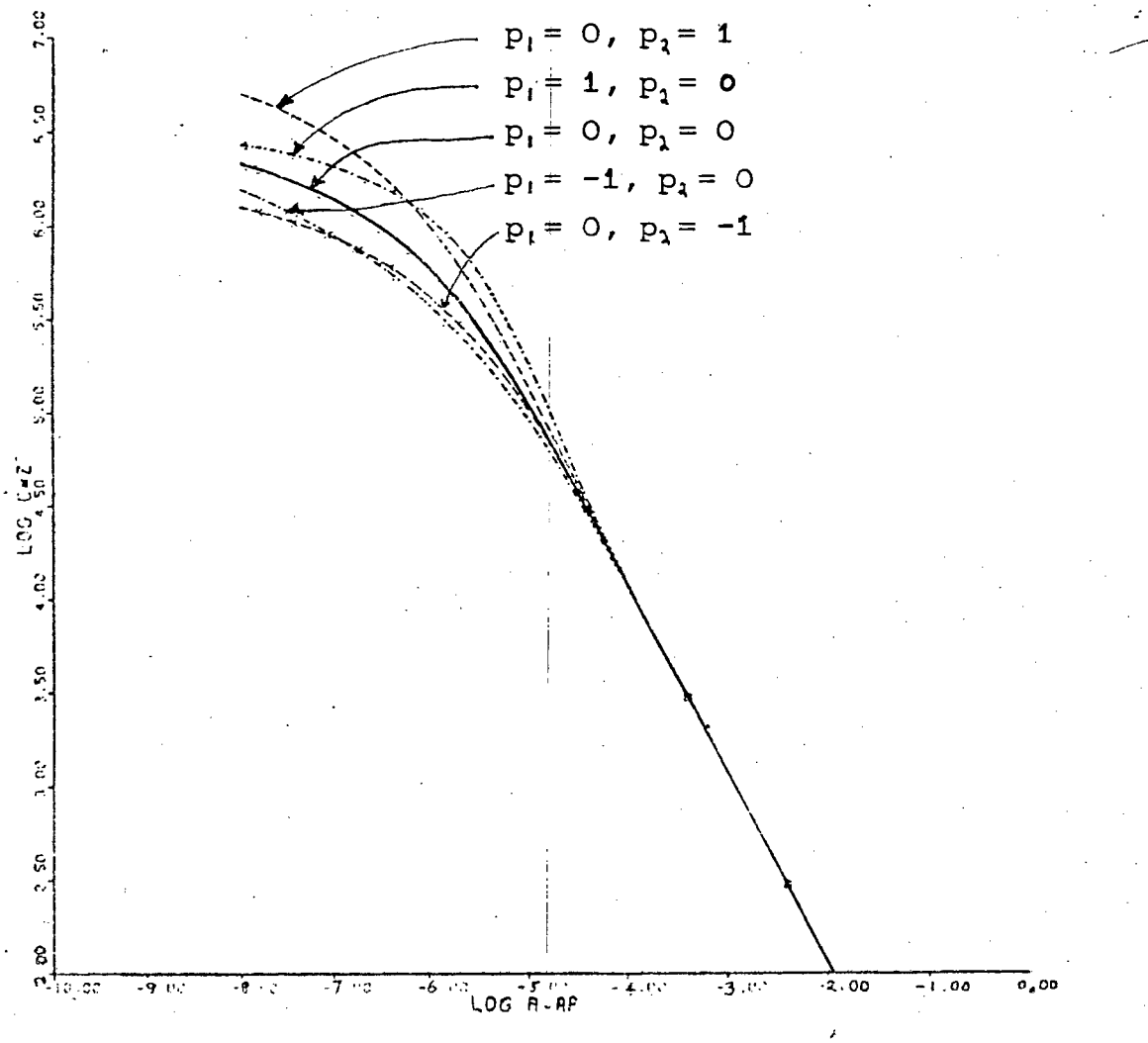
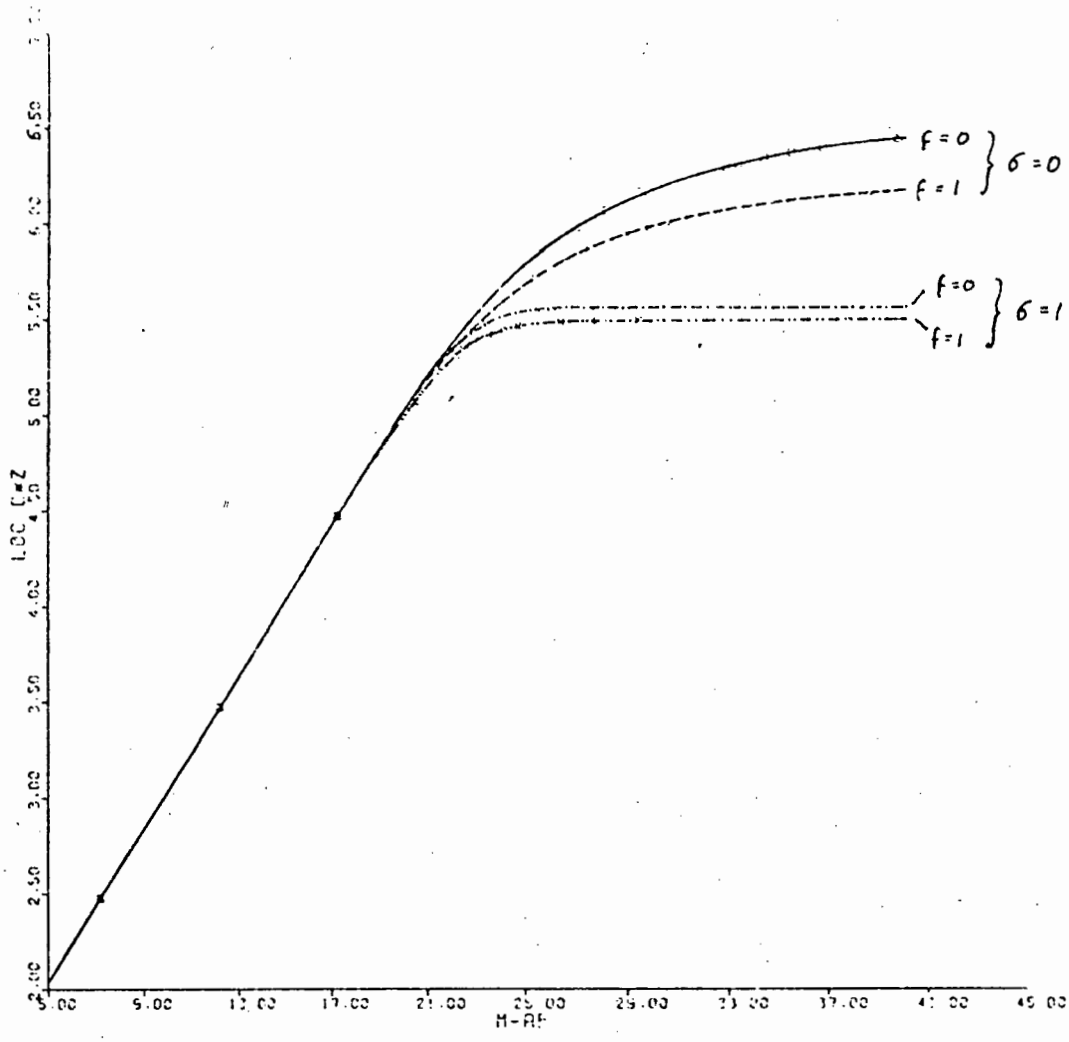

$p_1 = 0, p_2 = 1$
$p_1 = 1, p_2 = 0$
$p_1 = 0, p_2 = 0$
$p_1 = -1, p_2 = 0$
$p_1 = 0, p_2 = -1$
LOG C=Z
LOG A-AP

Figure 9



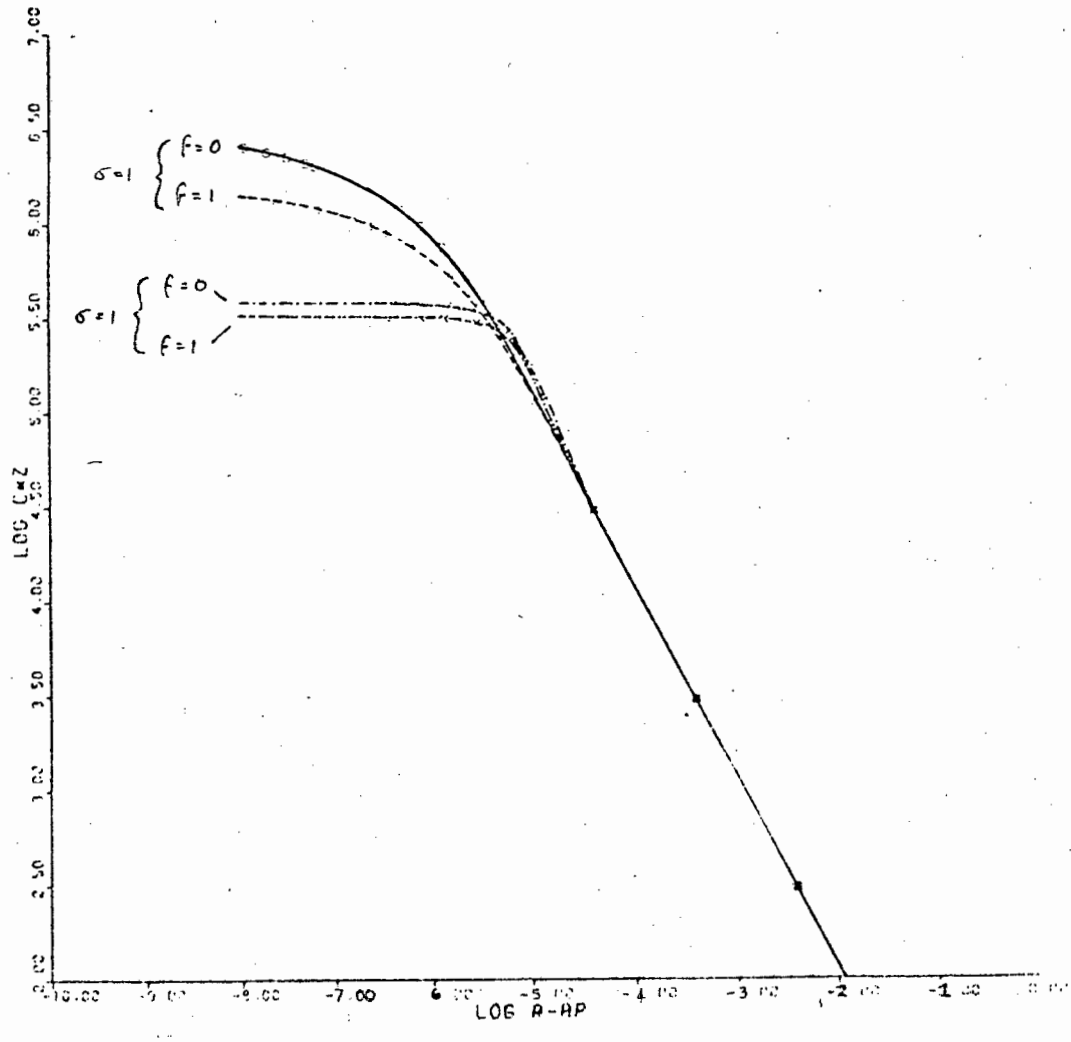
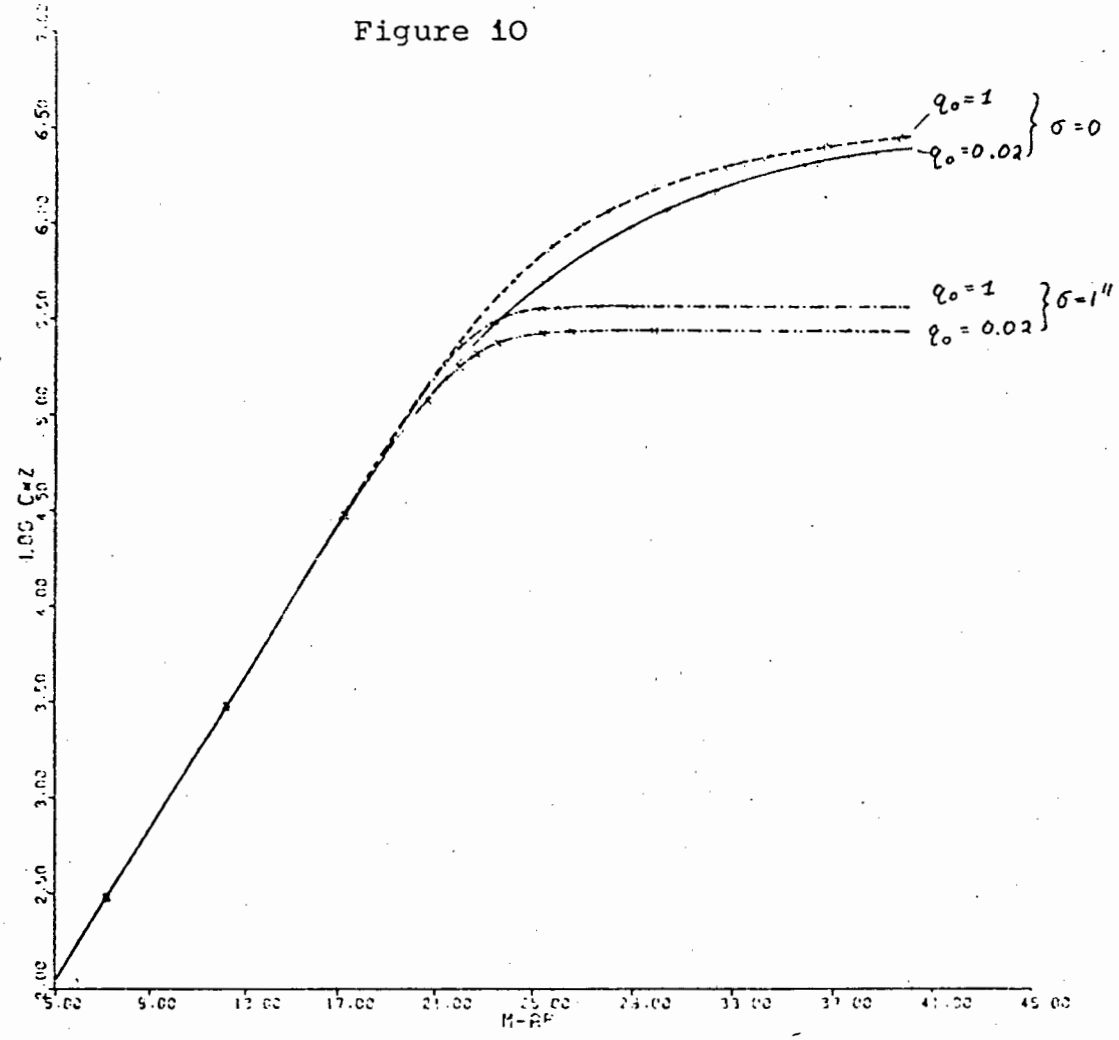


Figure 10


$q_o = 1$
$q_o = 0.02$
$\sigma = 0$
$q_o = 1$
$q_o = 0.02$
$\sigma = 1''$
LOG C*Z
M-AP

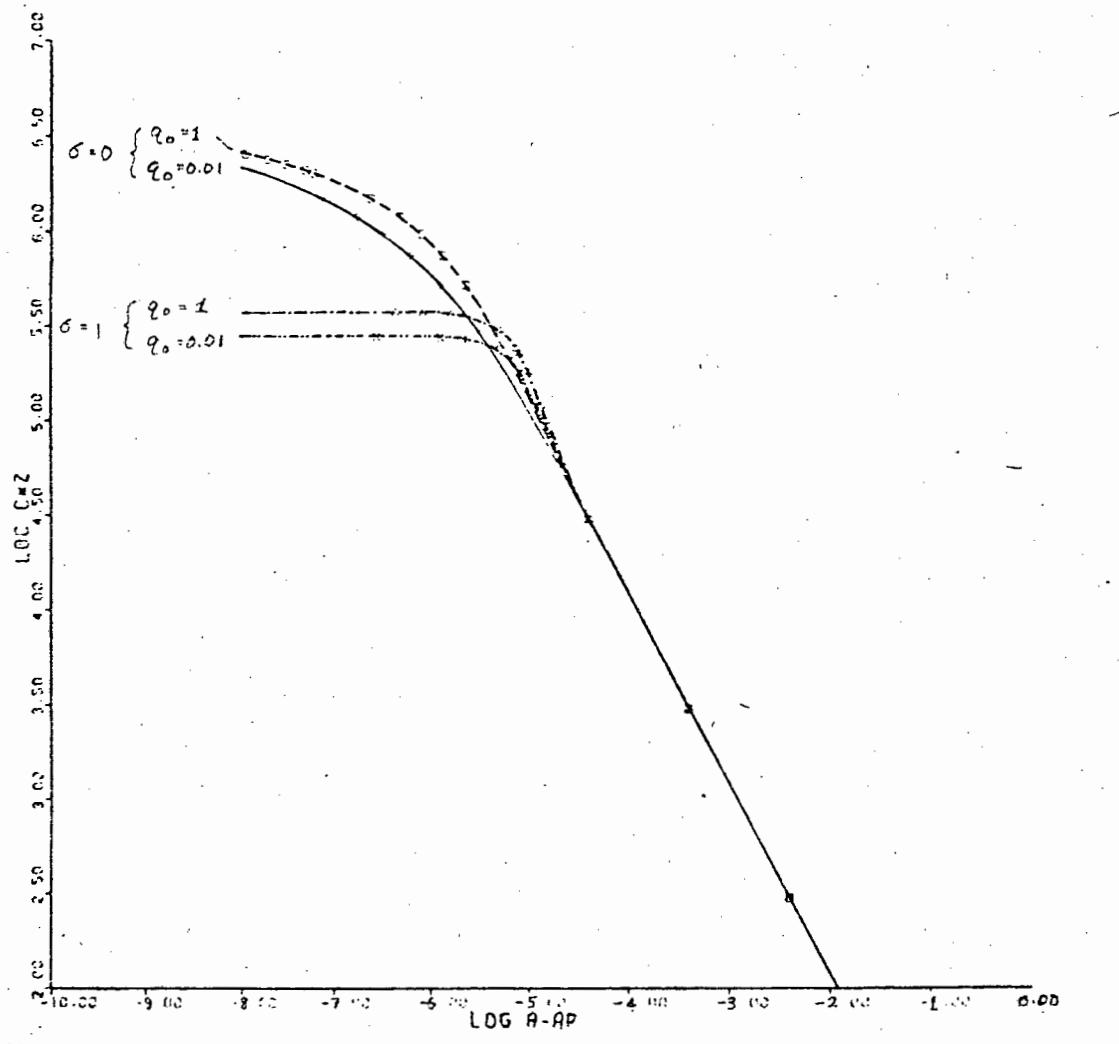

$\sigma = 0$
$q_o = 1$
$q_o = 0.01$
$\sigma = 1$
$q_o = 1$
$q_o = 0.01$
LOG C*Z
LOG A-AP

Figure 11

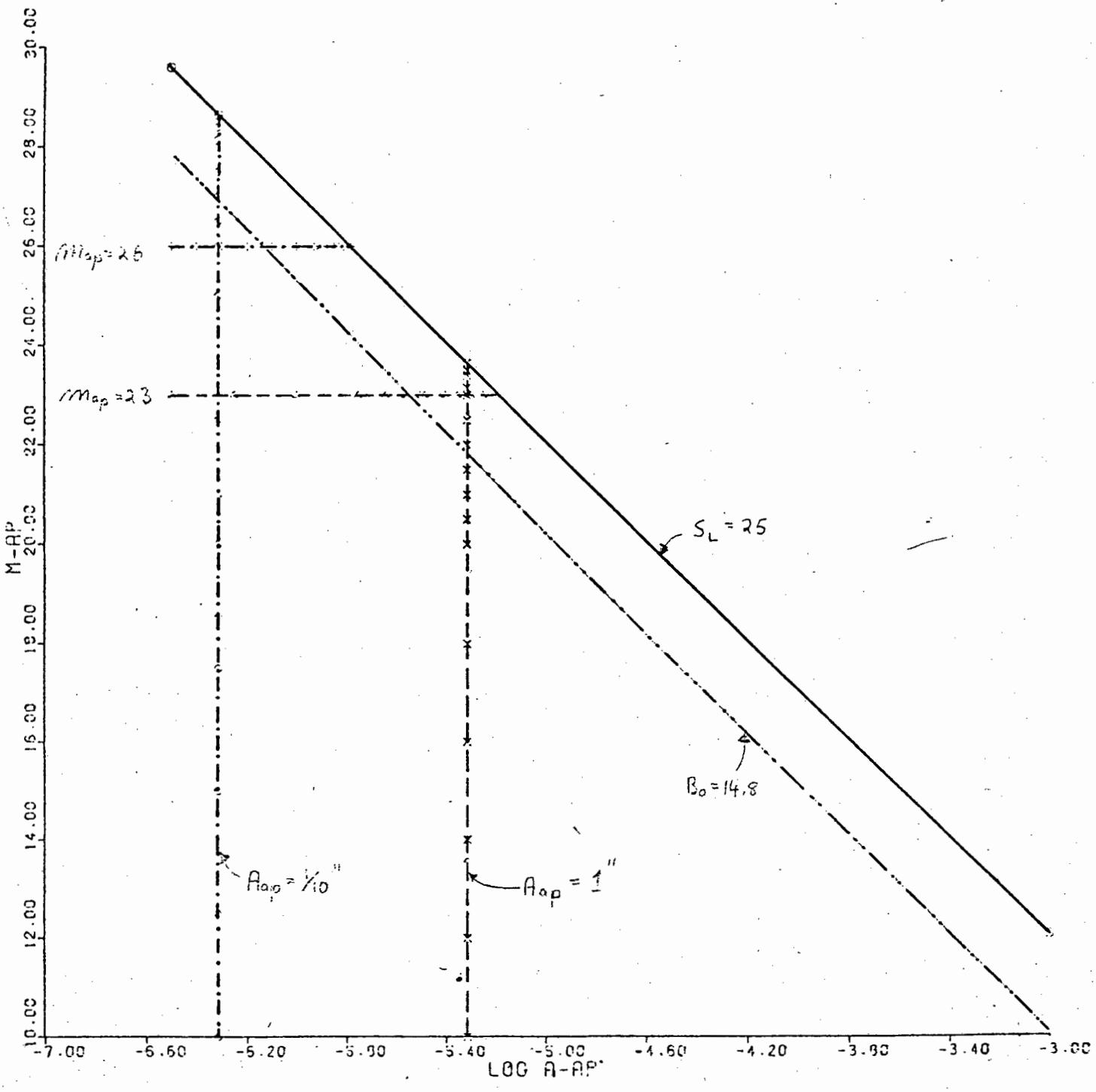

M-AP
LOG A-AP
30.00
28.00
26.00
24.00
22.00
20.00
19.00
16.00
14.00
12.00
10.00
-7.00
-6.60
-5.20
-5.00
-5.40
-5.00
-4.60
-4.20
-3.80
-3.40
-3.00
M_{ap}=26
M_{ap}=23
S_L = 25
B_o=14.8
A_{ap} = 1/10"
A_{ap} = 1"

Figure 12

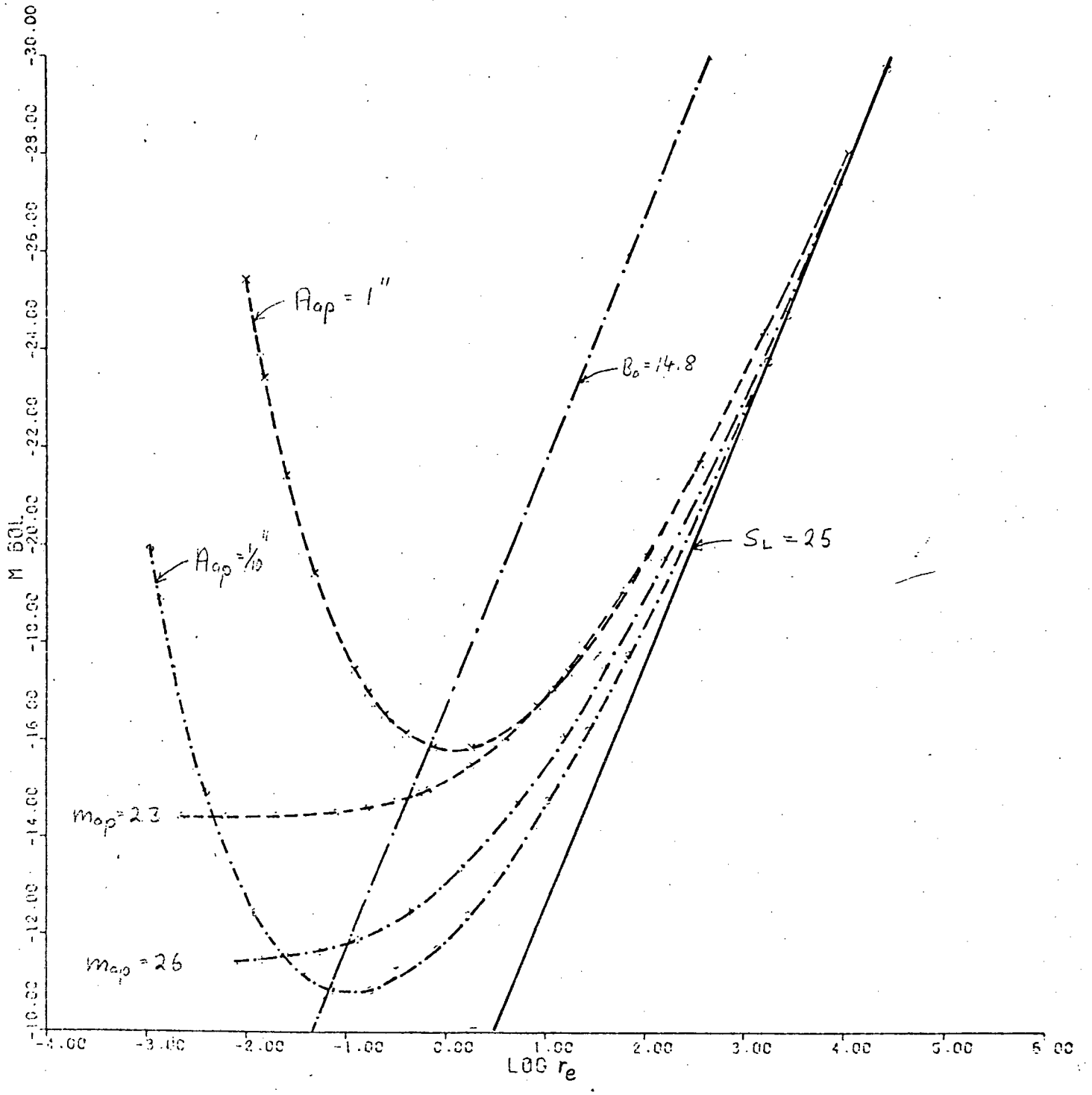

M 601
-30.00
-28.00
-26.00
-24.00
-22.00
-20.00
-18.00
-16.00
-14.00
-12.00
-10.00
$A_{ap} = 1''$
$A_{ap} = 1/10''$
$B_o = 14.8$
$S_L = 25$
$m_{ap} = 23$
$m_{ap} = 26$
-4.00
-3.00
-2.00
-1.00
0.00
1.00
2.00
3.00
4.00
5.00
5.00
LOG r_e

Figure 13

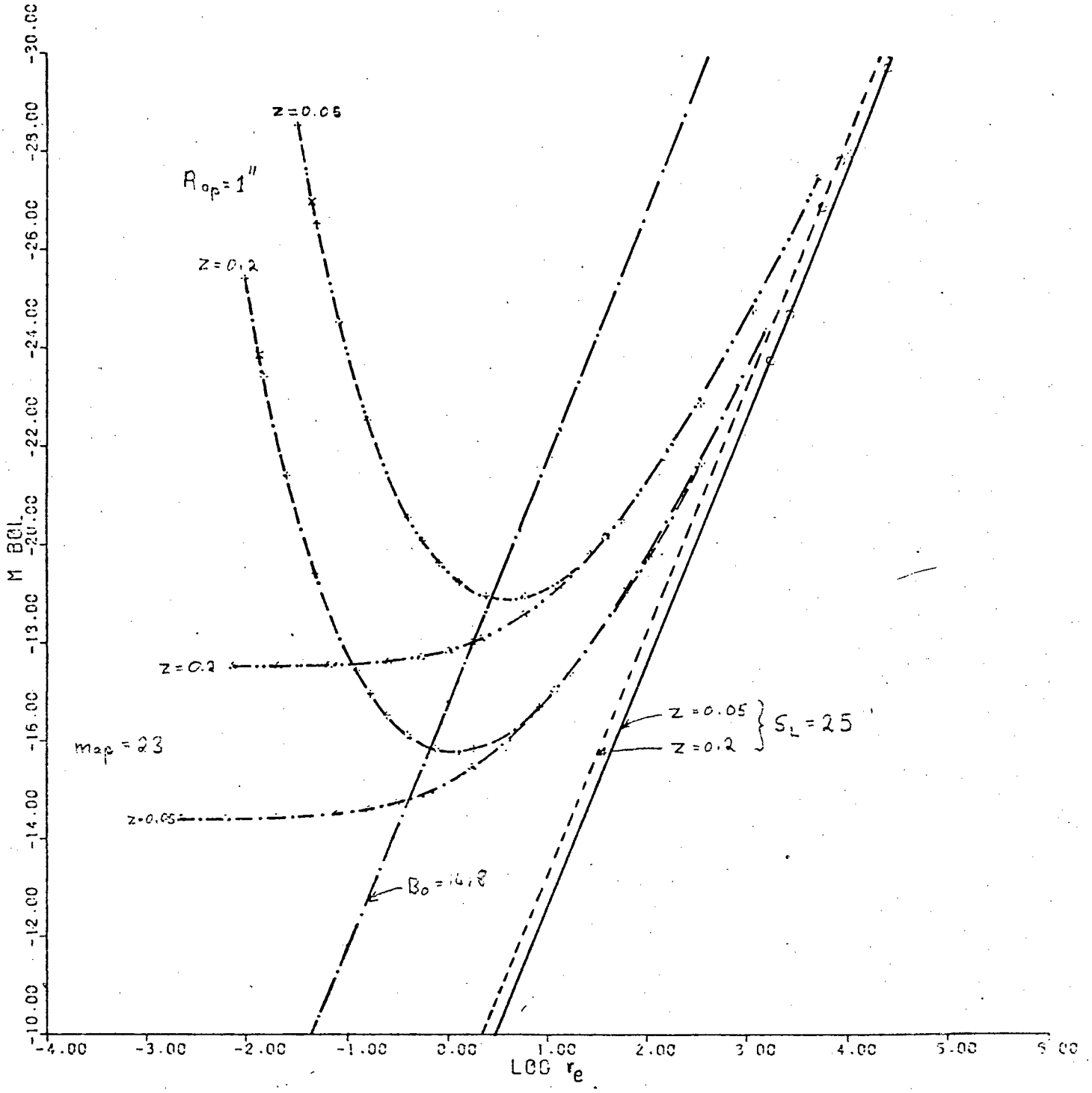
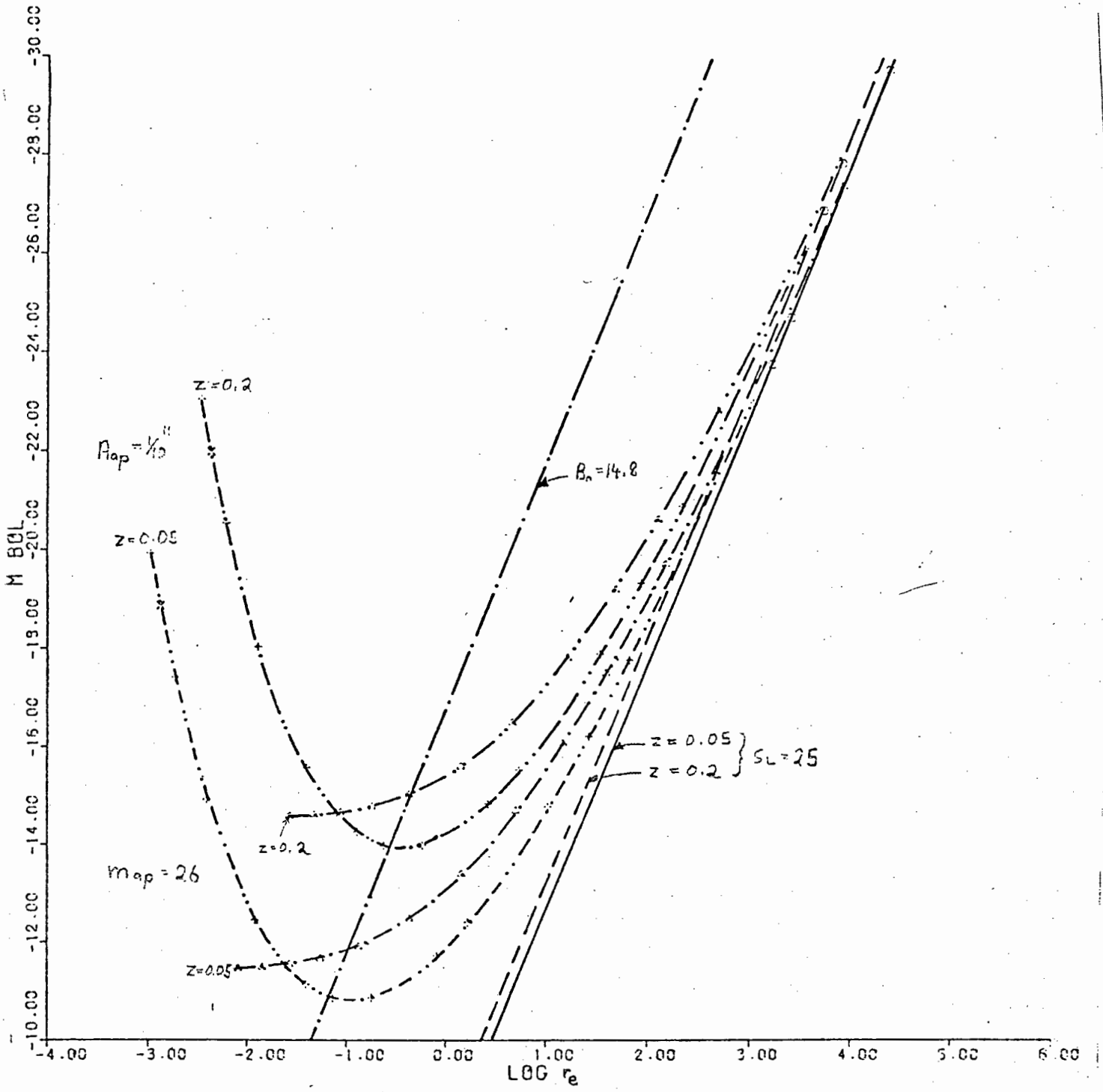

z=0.05
$R_{ap}=1''$
z=0.2
z=0.2
$m_{ap}=23$
z=0.05
$B_o=14.8$
z=0.05
z=0.2
$S_L=25$
M BGL
LGG r_e
-30.00
-28.00
-26.00
-24.00
-22.00
-20.00
-18.00
-16.00
-14.00
-12.00
-10.00
-4.00
-3.00
-2.00
-1.00
0.00
1.00
2.00
3.00
4.00
5.00

Figure 14


M BOL
LOG r_e
$\theta_{ap} = 1.5''$
$m_{ap} = 26$
z = 0.2
z = 0.05
$B_n = 14.8$
$S_L = 25$
-30.00
-28.00
-26.00
-24.00
-22.00
-20.00
-18.00
-16.00
-14.00
-12.00
-10.00
-4.00
-3.00
-2.00
-1.00
0.00
1.00
2.00
3.00
4.00
5.00
6.00

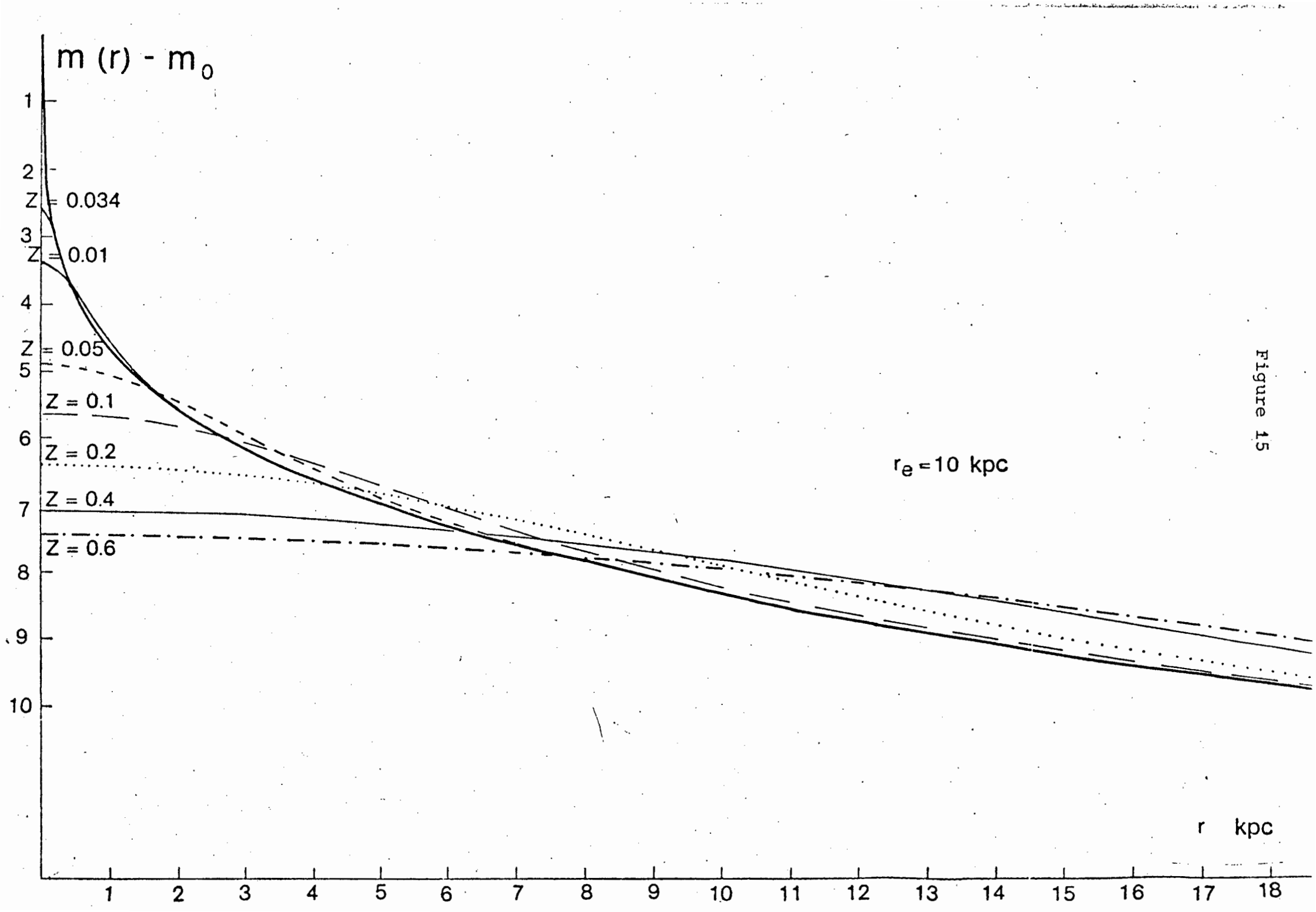

m (r) - m0
Z = 0.034
Z = 0.01
Z = 0.05
Z = 0.1
Z = 0.2
Z = 0.4
Z = 0.6
re = 10 kpc
r kpc
1
2
3
4
5
6
7
8
9
10
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18

Figure 15

5. Conclusion

A simplified theory of observation has been given. The main purpose is to demonstrate an approach to analysing galactic observations without the use of 'corrections' and to showing qualitatively the effect of change of various cosmological and astrophysical quantities on the observational map. For comparison with actual observations the observational method may be changed to include a wider class of objects and a more complex map. Several suggestions can be made:

1) The objects are considered to be spherical. When ELLIPTICITY is significant we can carry out an analysis similar to that in section 1.2. with $r = [x^2/a^2 + y^2/b^2]^{1/2}$ instead of $r = (x^2 + y^2)^{1/2}$ where a is the major and b the minor axes of the image [see Ellis and Perry 1979].

2) In order to provide for a colour gradient in the source one could write e.g.

$$B_\nu(r,z) = B_0^a(z)\, g^a(\nu,z)\, f^a(r/a_a(z)) + B_0^b(z)\, g^b(\nu,z)\, f^b(r/a_b(z))$$

for a galaxy model with two different populations of stars.

3) More detailed brightness profiles including profiles for SO and spiral galaxies can be used to model a more realistic galaxy population. Does the function f evolve with increasing redshift z?

4) A more detailed point-spread function may be used. E.g. F.Schweizer gives in his paper: "Effects of Seeing on the Light Distribution in the Cores of Elliptical Galaxies" [Schweizer 1979]:

$$S(r) \begin{cases} = S_0 \, e^{-(r^2/2\sigma^2)} & \text{for } 0 \leq r \leq 2.25\sigma \\ = S_1 \, e^{-(\beta r)} & \text{for } 2.25\sigma \leq r \leq 10\sigma \\ = 0 & \text{for } r > 10\sigma \end{cases}$$

This point-spread function accounts for the exponential wings in a star profile. In this paper Schweizer also points out that the seeing convolved profile is not very sensitive to the exponential wings.

5) The calculation of the magnitude correction (K-correction and bolometric correction) is only provisional: Galaxies do not have a power-law spectrum in the visible region of the spectrum, and the spectral response function is seldom close to a step function. For more accurate calculations tables of K-corrections could be used given by Oke and Sandage (1968); Oke (1971) or Pence (1976).

6) Different functional forms for the observer area distance to test the predictions of different cosmological models may be included.

This analysis is developed for observations in the visible light made by e.g. photographic plates. Does the same analysis hold for other wavelengths in the electromagnetic spectrum? Certainly the spectrum and the brightness profile differ for radio galaxies as well as the point-spread function. In different parts of the spectrum the detection limits may differ. In the radiospectrum the angular limit may be of little importance and the most important limits are the surface brightness limit and the magnitude limit.

It would be more realistic to give the detection limits as a probability function, which may include the effects of clustering.

There are many extensions ~~to~~ of the observational method given in this thesis, ~~nevertheless~~ through which important qualitative results may be obtained. In particular the effect of seeing has not been investigated in e.g. the analysis of number counts or magnitude-redshift diagrams, these will be applications of the observational map.

6. Appendix

A) Short Manual for the Program

A.W.SIEVERS APPLIED MATHEMATICS 26 OCT 1981

```
**********************************
*                                *
*   USERS MANUAL FOR GALAXY-MAP  *
*                                *
**********************************
```

FORWARD AND INVERSE MAP WITH AND WITHOUT POINT-SPREAD

NOTE: THE INVERSE MAP WITH POINT-SPREAD IS NOT RELIABLE
IF THE IMAGE IS JUST ABOVE DETECTION LIMIT

INTRODUCTION:

AN OBSERVATIONAL MAP OF GALAXYS FROM THE OBJECT-PLANE TO THE IMAGE-PLANE (OR ITS INVERSE) IS CALCULATED.THE OBJECT-PLANE IS LABELED (LOG A OR A,M-BOL) AND THE IMAGE-PLANE (LOG A-AP OR A-AP,M-AP),WHERE:

- A = SCALING RADIUS OF THE BRIGHTNESS-PROFILE IN KPC
- M-BOL = BOLOMETRIC ABSOLUTE MAGNITUDE
- A-AP = APPARENT ANGLE OF IMAGE IN SEC OF ARC (MEASURED FROM CENTRE OF IMAGE : IF LOG A-AP THEN A-AP IN RADIANS)
- M-AP = APPARENT MAGNITUDE

EFFECTS THAT ARE CONSIDERED ARE:

A) VALUE OF H0,Q0 IN A FRW-UNIVERSE
B) VALUE OF DETECTION LIMIT FOR SURFACE BRIGHTNESS SL
C) SCALE AND BRIGHTNESS EVOLUTION
D) INTERGALACTIC ABSORBTION
E) TWO DIFFERENT PROFILES
F) ATMOSPHERIC BLURRING REPRESENTED BY A POINT-SPREAD FUNCTION OF THE GAUSSIAN TYPE

DETAILS

THE PARAMETERS USED IN THE MAP ARE INITIALISED TO A GALAXY LIKE M 87 AND A MAP WITHOUT POINT-SPREAD(PS),EVOLUTION OR ABSORBTION IS DONE IF NO PARAMETER IS CHANGED.THE INITIALISATION IS PRINTED AT THE BEGINNING OF A RUN.THE PROGRAM THEN PROMPTS FOR AN INSTRUCTION WHICH HAS TO BE TYPED STARTING FROM COLUMN ONE.IT DETERMINES WHICH TASK HAS TO BE DONE(WHICH PART OF THE MAIN PROGRAM IS TO BE EXECUTED) IF REQIRED THE PROGRAM PROMPTS FOR DATA INPUT.DATA IS TYPED IN FREE FORMAT(ITEMS SEPARATED BY BLANKS OR COMMAS)

LIST OF INSTRUCTIONS:

SETPR1
******

H0 Q0 P1 P2 FRAC X10 X20IN GRAD

- H0 = HUBBLE CONSTANT IN KM/SEC/MPC

```
  Q0 = DEACCELERATION(DECELERATION) PARAMETER
  P1 = RADIAL EVOLUTION
  P2 = BRIGHTNESS EVOLUTION
  FRAC = FRACTION OF INTERGALACTIC GAS IONISED
  X10 = SCALING OF A (SEE NOTE 1)
  X20IN = CORRECTION IN MAGNITUDE DUE TO A FINITE SPECTRAL
          WINDOW AND INSTRUMENTAL EFFECTS

SETPR2
******
IF SL SIGMA IIN
  IF = 1 FOR DE VAUCOULEURS PROFILE FOR ELLIPTICALS
  IF = 2 FOR THE ABELL MIHALAS PROFILE
  SL = DETECTION LIMIT ON THE SURFACE BRIGHTNESS IN MAG.
       PER SQUARE ARC.SEC.
  SIGMA = DISPERSION IN GAUSSIAN POINT-SPREAD FUNCTION IN SEC OF ARC
          1 ARC.SEC. = 4.85E-6 RAD
  INN = 0 X1=LOG10 A , Y1 = LOG10 A-AP  (FOREWARD MAP)
  IIN = 1 X1= A      , Y1 = A-AP        (     "      )
       FOR THE INVERSE MAP X1,X2 AND Y1,Y2 ARE INTERCHANGED

ACURAT
******
XAPMAX EPS IX IY IZ IXD IYD
  XAPMAX    = MAXIMUM VALUE FOR XAP(R/W) FOR WHICH POINT-SPREAD IS
              IMPORTANT
  EPS       = CONVERGENCE CHECK IN SECANT AND SECA
  IX,IY,IZ  = NO.OF SUBINTERVALS ON THE X-,Y-AND Z-AXIS IN TRIPLE
  IXD,IYD   = NO.OF SUBINTERVALS ON THE X- AND Y-AXIS IN DOUBLE

INPUT
*****
X1 X2 Z NX1 NX2 NZ X1INC X2INC ZINC
  X1     = A(OR A-AP) FOR IIN = 1 (INITIAL VALUES FOR X1)
         = LOG10(A OR A-AP) FOR IIN = 0
  X2     = M-BOL OR M-AP
  Z      = INITIAL VALUE FOR THE REDSHIFT
  NX1    = NO OF X1 VALUES TO BE MAPPED (LIMITED TO 98)
  NX2    = SAME FOR X2
  NZ     = SAME FOR Z
  X1INC  = INCREMENT IN X1
  X2INC  =     "     IN X2
  ZINC   =     "     IN Z

MAP       FORWARD MAP IS DONE (SEE NOTE 2)
***
IMAP      INVERSE MAP IS DONE (     "      )
****
TABLE     TABLE OF RESULTS IS PRINTED
*****
NEWARR    COUNTER N1 SET TO ZERO (SEE NOTE 2)
******
STDATA    N1 ELEMENTS OF OUTPUT ARRAYS AND VALUES OF PARAMETERS
******
          WRITTEN TO FILE (UNIT 11) (SEE NOTE 4)
RDATA     SAME BUT ITEMS READ FROM FILE
*****
```

```
PSTART
******
  TITLE FOR X-AXIS
  TITLE FOR Y-AXIS
  TITLE FOR GRAPH
  NXTI NYTI NTI FIX LAX FIY LAY FACT
  NXTI,NYTI, NTI = NO OF CHARACTERS IN TITLES FOR X-,Y- AND
                   GRAPH
  FIX,FIY        = VALUE AT FIRST TICK ON X-,Y-AXIS
  LAX,LAY        = VALUE AT LAST TICK ON X-,Y-AXIS
  FACT           = RATIO OF DESIRED PLOT TO NORMAL PLOT SIZE
  NOTE: TO DO MORE THEN ONE GRAPH CALL PSTART AGAIN.

PLOT
****
LTYP INTEQ J
  LTYP DETERMINES TYP OF LINE DRAWN
       LTYP .GT. 0 POINTS ARE CONNECTED WITH STRAIGHT LINES,FOR EVERY
                   LTYP'TH POINT A SYMBOL IS DRAWN
       LTYP .EQ. 0 SAME BUT NO SYMBOL IS DRAWN
       LTYP .LT. 0 ONLY SYMBOL DRAWN AT EVERY LTYP'TH POINT,POINTS
                   NOT CONNECTED
  INTEQ = INTEGER EQUIVALENT OF PLOTTING SYMBOL DRAWN (SEE CALCOMP OR
          GDP INSTRUCTIONS)
  J     DETERMINES WHICH ARRAYS ARE PLOTTED
       J = 1 ARRAYS (Y1,Y2) (SEE NOTE 2+3)
       J = 2 ARRAYS (Y1,Z)        "
       J = 3 ARRAYS (Y2,Z)        "
       J = 4 ARRAYS (X1,X2) (INPUT ARRAYS)

PRJECT    (LOG A-AP OR A-AP,LOG C*Z) AND (M-AP,LOG C*Z) ARRAYS
******
           ARE PRINTED (NOTE 3)
STOP OR A BLANK LINE   PROGRAM TERMINATES
****


NOTE 1
------
      THE SCALING RADIUS A IN THE BRIGHTNESS PROFILE IS SCALED BY
X10. THE RELATION IS A'=X10*A.IF X10 = 1 THE SCALING RADIUS IS
THE ONE IN THE PROFILE B/B0 WHERE B0 = CENTRL SURFACE BRIGHTNESS
AND B = SURFACE BRIGHTNESS AT DISTANCE R FROM CENTRE.
X10 IS INITIALISED TO X10 = 3459 SO THAT THE SCALING RADIUS IS
THE RADIUS THAT CONTAINS 1/2 OF THE LIGHT OF THE DE VAUCOULEURS
MODLE.

NOTE 2
------
      THE PROGRAM SETS UP ARRAYS X1,X2 AND Z AND STORES THE
OUTPUT IN ARRAYS Y1 AND Y2. A COUNTER N1 COUNTS THE NUMBER OF
POINTS MAPPED. IT IS INITIALISED TO ZERO AND IS ONLY SET IF ONE
OF THE INSTRUCTIONS NEWARR OR RDATA ARE GIVEN.THE OBSERVER-
AREA-DISTANCE R0(Z) AND THE COLOUR-CORRECTION ARE PRINTED FOR
EVERY REDSHIFT.
FORWARD MAP:
            X1  = LOG A (IIN=0) OR A (IIN=1) A IN KPC
            X2  = M-BOL IN MAGNITUDES
            Z   = Z THE REDSHIFT
```

```
            Y1 = LOG A-AP(IIN=0,A-AP IN RAD) OR A-AP (IIN=1,IN ARC SEC)
            Y2 = M-AP IN MAGNITUDES
INVERSE MAP: X AND Y ARE INTERCHANGED

NOTE 3
------
      THE REDSHIFT IN ARRAY Z IS REPLACED BY LOG10 C*Z ,
C = SPEED OF LIGHT IN KM PER SECOND

NOTE 4
------
      A DATAFILE HAS TO BE ASSIGNED TO THE RUN. THE PROGRAM CAN
READ AND WRITE TO THIS FILE ON UNIT 11. DATA IS WRITTEN TO OR
READ FROM AS FOLLOWS:
  N1       INVERS
  HQ       Q0       P1       P2
  FRAC     SIGMA    X10      X20IN
  SL       GRAD     IF       IIN
  X1(1)    X2(1)    Z(1)     Y1(1)     Y2(1)
  X1(2)    X2(2)    Z(2)     Y1(2)     Y2(2)
   *        *        *
  X1(N1)   X2(N1)   Z(N1)    Y1(N1)    Y2(N1)
RDATA CAN ONLY BE USED IF A DATAFILE IN THIS FORMAT IS ASSIGNED
TO THE RUN. N1(AND ALL OTHER PARAMETERS GIVEN ABOVE) IS SET EACH
TIME RDATA IS USED.
```

****

B) The Program

```
C
C                MAIN PROGRAM FOR GALAXY MAP
C                ***************************
C
C
C       THIS PROGRAM CALCULATES AN OBSERVATIONAL MAP OF
C       GALAXIES FROM THE "OBJECT-" TO "IMAGE-PLANE"
C       OR THE INVERSE
C       PROGRAM WRITTEN IN FORTRAN(ASCII) LEVEL 9R1
C       AT MOST 98 POINTS CAN BE MAPPED AT ONCE
C       OUTPUT CAN BE PLOTTED ON CALCOMP PLOTTER
C       THE GRAPHICS DISPLAY PACKAGE AT UCT
C       IS USED FOR PLOTTING
C       ----------------------------------------
C
C       DISCRIPTION OF VARIABLES USED:
C
C       H0=HUBBLE PARAMETER IN (KM/SEC/MPC)
C       Q0=DEACCELERATION PARAMETER IN A FRIEDMANN UNIVERSE
C       OBJECT PLANE  (X1,X2):
C          X1=A=(SCALING RADIUS) IF IIN=1 , A IN KPC
C          X1=ALOG10(A)          IF IIN=0 , A IN KPC
C          X2=ABSOLUTE BOLOMETRIC MAGNITUDE=M-BOL
C       IMAGE PLANE (Y1,Y2)
C          Y1=A-AP=APPARENT ANGLE IF IIN=1 , A-AP IN ARC SEC
C          Y1=ALOG10(A-AP)        IF IIN=0 , A-AP IN RADIANS
C          Y2=M-AP=APPARENT MAGNITUDE
C       P1=  RADIAL EVOLUTION INDEX
C       P2=  BRIGHTNESS EVOLUTION INDEX
C       FRAC=  FRACTION OF INTERSTELLAR GAS IONISED
C       GRAD=  SPECTRAL GRADIENT IN POWER LAW SPECTRUM
C       X10=  RESCALING OF A
C       X20IN=  INSTRUMENTAL PART OF BOLOMETRIC CORRECTION X20
C       SIGMA=  DISPERSION IN GAUSSIAN POINT-SPREAD FUNCTION IN ARC-SEC
C       SIGMAD= SIGMA IN RADIANS (1 ARC SEC = 4.8481E-6 RAD)
C       Z=  REDSHIFT IN A FRIEDMANN UNIVERSE
C       SL=  DETECTION LIMIT ON SURFACE BRIGHTNESS IN (MAG/(ARC SEC)**2)
C       IF=  1 FOR DE VAUCOULEURS PROFILE FOR ELLIPTICALS
C       IF=  2 FOR ABELL MIHALAS PROFILE
C       GINF=  PROFILE INTEGRATED UP TO INFINITY
C       NX1,X1INC =   NO OF X1-VALUES,INCREMENT IN X1
C       NX2,X2INC =   NO OF X2-VALUES,INCREMENT IN X2
C       NZ ,ZINC  =   NO OF  Z-VALUES,INCREMENT IN  Z
C       EPS =   ACURACY WITH WHICH ROOT IS FOUND IN SECANT AND SECA
C       IXD,IYD  = NO OF SUBINTERVALS USED IN DOUBLE
C       IX,IY,IZ = NO OF SUBINTERVALS USED IN TRIPEL
C       INVERS = .TRUE.(INVERSE MAP) = .FALSE.(FORWARD MAP)
C       XAPMAX =  IS THE MAXIMUM VALUE FOR XAP = R/W FOR WHICH
C                 POINT-SPREAD IS IMPORTANT
C          R = APPARENT RADIUS IN KPC
C          W = HALF WIDTH OF POINT-SPREAD DISK IN KPC
C       PLDONE =.TRUE.(PLOTTING IS DONE) =.FALSE.(NO PLOTTING)
C       N1 = GIVES NO. OF POINTS IN OUTPUT ARRAYS
C
C       PROGRAM WRITTEN BY A.W.SIEVERS DEPARTMENT OF APPLIED MATHS
C       JULY 1981
```

```
C
      REAL GINF(3),H0,Q0,X1,X2,Z,P1,P2,FRAC,GRAD,SIGMAD,X10,X20IN,
     +     X1INC,X2INC,ZINC,EPS,XAPMAX,SIGMA
      INTEGER I,IF,IIN,NX1,NX2,NZ,IX,IY,IZ,IXD,IYD,N1
      CHARACTER INSTR*6,ARRAY*6(20)
      LOGICAL INVERS,PLDONE
C
C        VARIABLES INITIALISED
C        COMMON BLOCKS DEFINED
C
      COMMON /CGINF/GINF/CPAR1/H0,Q0,FRAC,GRAD,X20IN/CCPAR1/P1,P2,X10
     +       /CIF/IF/CFLAG/IIN,INVERS/CPAR2/SL,SIGMAD
     +       /CACURA/IX,IY,IZ/CACURB/XAPMAX,EPS,IXD,IYD
     +       /CINP/X1,X2,Z,NX1,NX2,NZ,X1INC,X2INC,ZINC
     +       /CMAP2/N1
      DATA H0,Q0,P1,P2,FRAC,SIGMA,X10,X20IN,X1,X2,Z,IF,IIN/50.0,0.01,
     +     4*0.0,3.459E3,0.0,7.3,-21.64,0.34E-2,1,1/,
     +     X1INC,X2INC,ZINC,NX1,NX2,NZ/3*0.0,3*1/,
     +     GINF/126669.0,2*19.64/,
     +     EPS,IX,IY,IZ/1E-4,2,2,2/,
     +     SL,GRAD/24.0,0.7/,IXD,IYD/2,2/,INVERS/.FALSE./,XAPMAX/15./,
     +     N1/0/
      SIGMAD = 4.8481E-6 * SIGMA
C
C        USER MAY CHOOSE AMONG FOLLOWING COMMANDS
C
      ARRAY(1)=6HSETPR1
      ARRAY(2)=6HSETPR2
      ARRAY(3)=6HACURAT
      ARRAY(4)=6HINPUT
      ARRAY(5)=6HMAP
      ARRAY(6)=6HIMAP
      ARRAY(7)=6HTABLE
      ARRAY(8)=6HNEWARR
      ARRAY(9)=6HPSTART
      ARRAY(10)=6HPLOT
      ARRAY(11)=6H
      ARRAY(12)=6H
      ARRAY(13)=6HSTDATA
      ARRAY(14)=6HRDATA
      ARRAY(15)=6HPRJECT
      ARRAY(16)=6H
      ARRAY(17)=6H
      ARRAY(18)=6H
      ARRAY(19)=6H
      ARRAY(20)=6HSTOP
      PLDONE = .FALSE.
      WRITE(5,2000)
 2000 FORMAT(1H1,10X,'INITIALISATION OF GALAXY MAP'/
     +       1H ,10X,28(1H*))
C
C        INITIALISATION IS PRINTED
C
      PRINT *,'X1=',X1,'X2=',X2,'Z=',Z
      PRINT *,'H0=',H0,'Q0=',Q0,'P1=',P1,'P2=',P2
      PRINT *,'FRAC=',FRAC,'SIGMA=',SIGMA,'X10=',X10
      PRINT *,'SL=',SL,'  GRAD=',GRAD,'  X20IN=',X20IN
      PRINT *,'IF=',IF,'IIN=',IIN,'EPS=',EPS
      PRINT *,'IX=',IX,'IY=',IY,'IZ=',IZ,'IXD=',IXD
```

```
      PRINT *,'IYD=',IYD,'XAPMAX=',XAPMAX
      WRITE(5,2005)
 2005 FORMAT(1H0,'TYPE INSTRUCTION STARTING IN COLUMN 1')
   10 WRITE(5,2010)
 2010 FORMAT(1H ,'INSTRUCTION?')
      READ(8,1000)INSTR
 1000 FORMAT(A6)
      WRITE(5,2015)INSTR
 2015 FORMAT(1H ,A6)
      DO 20 I=1,20
         IF(INSTR.EQ.ARRAY(I)) GO TO 5
   20 CONTINUE
      PRINT *,'DID YOU SPELL THE INSTRUCTION CORRECTLY?',INSTR
      GO TO 10
    5 GO TO (101,102,103,104,105,106,107,108,109,110,
     +       120,120,113,114,115,120,120,120,120,120) I
C
C        DATA INPUT FOR H0,Q0,P1,P2,FRAC,X10,X20IN,GRAD
C
  101 CALL PARA1
      GO TO 10
C
C        DATA INPUT FOR IF,SL,SIGMA,IIN
C
  102 CALL PARA2
      GO TO 10
C
C        DATA INPUT FOR XAPMAX,EPS,IX,IY,IZ,IXD,IYD
C
  103 CALL ACURAT
      GO TO 10
C
C        DATA INPUT FOR X1,X2,Z,NX1,NX2,NZ,X1INC,X2INC,ZINC
C
  104 CALL INPUT
      GO TO 10
C
C        FORWARD MAP FOR POINTS READ BY INPUT
C
  105 INVERS = .FALSE.
      CALL MAP
      GO TO 10
C
C        INVERSE MAP FOR POINTS READ BY INPUT
C
  106 INVERS = .TRUE.
      CALL MAP
      GO TO 10
C
C        TABLE OF RESULTS PRINTED
C
  107 CALL TABLE
      GO TO 10
C
C        COUNTER OF OUTPUT ARRAYS RESET
C
  108 N1 = 0
      GO TO 10
C
```

```
C        PLOT INITIALISED
C
  109 CALL PSTART
      PLDONE = .TRUE.
      GO TO 10
C
C        PLOTTING OF OUTPUT ARRAYS
C
  110 CALL GRAPH
      GO TO 10
C
C         DATA STORED IN A FILE (UNIT 11)
C
  113 CALL STDATA
      PRINT *,'DATA WRITTEN TO FILE:N1 =',N1
      GO TO 10
C
C         DATA READ FROM A FILE
C         UNIT 11
C
  114 CALL RDATA
      PRINT *,'DATA READ FROM FILE:N1 =',N1
      GO TO 10
C
C        PROJECTIONS PRINTED
C
  115 CALL PRJECT
      GO TO 10
C
C        PROGRAM TERMINATES
C
  120 PRINT *,'PROGRAM TERMINATES'
      IF(PLDONE) THEN
         PRINT *,'END OF PLOT'
         CALL PENDE
      END IF
      STOP 9999
      END
      SUBROUTINE PARA1
C     ****************
C
C        DATA INPUT FOR H0,Q0,P1,P2,FRAC,X10,X20IN,GRAD
C        ----------------------------------------------
C        DATA IS READ IN FREE FORMAT
C        ITEMS SEPARATED BY BLANKS OR COMMAS
C
      REAL H0,Q0,FRAC,GRAD,P1,P2,X10,X20IN
      COMMON/CPAR1/H0,Q0,FRAC,GRAD,X20IN
     +      /CCPAR1/P1,P2,X10
      WRITE(5,2000)
 2000 FORMAT(1H0,'DATA INPUT : H0,Q0,P1,P2,FRAC,X10,X201N,GRAD?')
      READ(8,2001)H0,Q0,P1,P2,FRAC,X10,X20IN,GRAD
 2001 FORMAT()
      PRINT *,'H0=',H0,'Q0=',Q0,'P1=',P1,'P2=',P2
      PRINT *,'FRAC=',FRAC,'X10=',X10,'X20IN=',X20IN,'GRAD=',GRAD
      RETURN
      END
```

```
      SUBROUTINE PARA2
C     ****************
C
C
C        DATA INPUT FOR IF,SL,SIGMA,IIN
C        ------------------------------
C        ITEMS IN FREE FORMAT SEPARATED BY BLANKS OR COMMAS
C
      REAL SL,SIGMAD,SIGMA
      INTEGER IF,IIN
      COMMON /CFLAG/IIN/CIF/IF/CPAR2/SL,SIGMAD
      WRITE (5,2000)
 2000 FORMAT(1H0,'DATA INPUT FOR : IF,SL,SIGMA,IIN?'/
     +       1H ,'IF=1(DE VAUC),=2(ABELL-MIHALAS)***IIN=0(LOG A),=1(A)')
      READ (8,2001) IF,SL,SIGMA,IIN
 2001 FORMAT()
      PRINT *,'IF=',IF,'SL=',SL,'SIGMA=',SIGMA,'IIN=',IIN
      SIGMAD = 4.8481E-6 * SIGMA
      RETURN
      END
      SUBROUTINE ACURAT
C     *****************
C
C
C        DATA INPUT FOR XAPMAX,EPS,IX,IY,IZ,IXD,IYD
C        ------------------------------------------
C        ITEMS IN FREE FORMAT SEPARATED BY BLANKS OR COMMAS
C
      COMMON/CACURA/IX,IY,IZ/CACURB/XAPMAX,EPS,IXD,IYD
      WRITE(5,2000)
 2000 FORMAT(1H0,'DATA INPUT FOR : XAPMAX,EPS,IX,IY,IZ,IXD,IYD?')
      READ(8,2005) XAPMAX,EPS,IX,IY,IZ,IXD,IYD
 2005 FORMAT()
      PRINT *,'XAPMAX=',XAPMAX,' EPS=',EPS
      PRINT *,'IX,IY,IZ=',IX,IY,IZ,'IXD,IYD=',IXD,IYD
      RETURN
      END
      SUBROUTINE INPUT
C     ****************
C
C
C        INITIAL VALUES FOR X1,X2,Z AND
C        VALUES FOR NX1,NX2,NZ,X1INC,X2INC,ZINC
C        --------------------------------------
C        ITEMS IN FREE FORMAT SEPARATED BY BLANKS OR COMMAS
C
      REAL X1,X2,Z,X1INC,X2INC,ZINC
      INTEGER NX1,NX2,NZ
      COMMON /CINP/X1,X2,Z,NX1,NX2,NZ,X1INC,X2INC,ZINC
      WRITE (5,2000)
 2000 FORMAT(1H0,'DATA INPUT FOR:X1,X2,Z,NX1,NX2,NZ,X1INC,X2INC,ZINC?')
      READ (8,1000) X1,X2,Z,NX1,NX2,NZ,X1INC,X2INC,ZINC
 1000 FORMAT()
      PRINT *,'X1=',X1,'X2=',X2,'Z=',Z
      PRINT *,'NX1=',NX1,'NX2=',NX2,'NZ=',NZ,'X1INC=',X1INC,
     +        'X2INC=',X2INC,'ZINC=',ZINC
      RETURN
      END
```

```
      SUBROUTINE TABLE
C     ****************
C
C
C        DATA OUTPUT ROUTINE
C        -------------------
C        TABLE WILL BE PRINTED BY PRINTING THE FIRST N1
C        POINTS OF THE ARRAYS X1ARR,X2ARR,...,Y2ARR
C
      REAL X1ARR(100),X2ARR(100),ZARR(100),Y1ARR(100),Y2ARR(100)
      INTEGER N1
      LOGICAL INVERS
      COMMON /CMAP2/N1,X1ARR,X2ARR,ZARR,Y1ARR,Y2ARR
     +       /CFLAG/IIN,INVERS
      IF (IIN.EQ.0) THEN
         WRITE(5,2000)
 2000    FORMAT(1H0,4X,'LOG(A)',7X,'M-BOL',9X,'Z',8X,'LOG(A-AP)',
     +          9X,'M-AP')
      ELSE
         WRITE(5,2001)
 2001    FORMAT(1H0,6X,'A',10X,'M-BOL',9X,'Z',11X,'A-AP',8X,'M-AP')
      END IF
      IF(INVERS)THEN
      WRITE(5,2002)(Y1ARR(I),Y2ARR(I),ZARR(I),X1ARR(I),X2ARR(I),I=1,N1)
      ELSE
      WRITE(5,2002) (X1ARR(I),X2ARR(I),ZARR(I),Y1ARR(I),Y2ARR(I),I=1,N1)
      END IF
 2002 FORMAT(1H ,1X,E11.4,3X,F7.3,6X,F8.4,6X,E11.4,3X,F7.3)
      RETURN
      END
      SUBROUTINE STDATA
C     *****************
C
C
C        DATA WRITTEN TO A DATA-FILE
C        ---------------------------
C        N1 POINTS OF ARRAYS X1,X2,Z,Y1,Y2
C        ARE WRITTEN TO A FILE (UNIT 11)
C        AS WELL AS PARAMETERS THAT DETERMINE THE MAP
C        FREE FORMATTING
C        N1 APPEARS ON THE FIRST RECORD IN FILE
C
      REAL X1(100),X2(100),Z(100),Y1(100),Y2(100)
      INTEGER N1
      LOGICAL INVERS
      COMMON /CMAP2/N1,X1,X2,Z,Y1,Y2
     +       /CPAR1/H0,Q0,FRAC,GRAD,X20IN/CCPAR1/P1,P2,X10
     +       /CIF/IF/CFLAG/IIN,INVERS/CPAR2/SL,SIGMAD
      SIGMA = SIGMAD/4.8481E-6
      WRITE(11,*) N1,INVERS
      WRITE(11,*)H0,Q0,P1,P2
      WRITE(11,*)FRAC,SIGMA,X10,X20IN
      WRITE(11,*)SL,GRAD,IF,IIN
      DO 10 I=1,N1
         WRITE(11,*) X1(I),X2(I),Z(I),Y1(I),Y2(I)
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE RDATA
C     ****************
C
C
C        READS DATA FROM DATA-FILE(WRITTEN TO BY STDATA)
C        -----------------------
C        UNIT 11
C        N1 POINTS READ INTO X1,X2,Z,Y1,Y2
C        N1 APPEARS ON FIRST RECORD
C
      REAL X1(100),X2(100),Z(100),Y1(100),Y2(100)
      LOGICAL INVERS
      COMMON /CMAP2/N1,X1,X2,Z,Y1,Y2
     +       /CPAR1/H0,Q0,FRAC,GRAD,X20IN/CCPAR1/P1,P2,X10
     +       /CIF/IF/CFLAG/IIN,INVERS/CPAR2/SL,SIGMAD
      READ(11,*) N1,INVERS
      READ(11,*)H0,Q0,P1,P2
      READ(11,*)FRAC,SIGMA,X10,X20IN
      READ(11,*)SL,GRAD,IF,IIN
      DO 10 I=1,N1
         READ(11,*) X1(I),X2(I),Z(I),Y1(I),Y2(I)
   10 CONTINUE
      SIGMAD = 4.8481E-6 * SIGMA
      RETURN
      END
      SUBROUTINE PRJECT
C     *****************
C
C
C        (LOG A-AP,LOC C*Z) AND (M-AP,LOG C*Z) CURVES
C        ARE COMPUTED AND PRINTED
C        ----------------------------------------
C        C = SPEED OF LIGHT IN KM/SEC
C
      REAL X1(100),X2(100),Z(100),Y1(100),Y2(100),C
      COMMON /CMAP2/N1,X1,X2,Z,Y1,Y2
      C = 299792.5
      DO 10 I=1,N1
         Z(I) = ALOG10(C*Z(I))
   10 CONTINUE
      WRITE(5,2000) (Y1(J),Z(J),Y2(J),J=1,N1)
 2000 FORMAT(1H0,4X,'LOG(A-AP)',4X,'LOG(C*Z)',5X,'MAP'/
     +       (1H0,3X,F8.3,5X,F8.3,4X,F8.3))
      RETURN
      END
      REAL FUNCTION F(X)
C     ******************
C
C
C        BRIGHTNESS PROFILES CALCULATED
C        ------------------
C        IF DETERMINES WHICH PROFILE IS EVALUATED
C
      REAL X,U
      INTEGER IF
      COMMON /CIF/IF
C
C        PROFILE IS SELECTED
C
      GO TO (10,20,20) IF
C
C        DE VAUC PROFILE
```

```
C
   10 U = X**.25
      F = EXP(-U)
      RETURN
C
C        ABELL MIHALAS PROFILE
C
   20 XP1 = X+1.
      IF (X.LE.21.4) THEN
         F = 1./(XP1*XP1)
      ELSE
         F = 22.4/(XP1*XP1*XP1)
      END IF
      RETURN
      END
      REAL FUNCTION FINV(X)
C     **********************
C
C        INVERSE OF BRIGHTNESS PROFILES EVALUATED
C        ----------------------------
C        FOR DIFFERENT IF
C
      REAL X
      INTEGER IF
      COMMON /CIF/IF
C
C        INVERSE IS SELECTED
C
      GO TO (10,20,20) IF
C
C        DE VAU PROFILE
C
   10 FINV = (-ALOG(X))**4
      RETURN
C
C        ABELL MIHALAS PROFILE
C
   20 IF (X.GE.1.99298E-3) THEN
         FINV = 1./SQRT(X) - 1.
      ELSE
         FINV = (22.4/X)**0.33333333 - 1.
      END IF
      RETURN
      END
      REAL FUNCTION R0(Z)
C     *******************
C
C        OBSERVER AREA DISTANCE IN MPC
C        ----------------------
C        FRIEDMANN UNIVERSE : C = SPEED OF LIGHT
C        H0 = HUBBLE CONSTANT : Q0 = DEACCELERATION PARAMETER
C        Z = REDSHIFT
C
      REAL Z,H0,Q0,C,K1,K2
      COMMON /CPAR1/H0,Q0
      C = 299792.5
      K1 = Z*C / (H0 * (1.+Z)**2)
      K2 = SQRT(1. + 2.*Q0*Z) + 1.+Q0*Z
      R0 = K1*(1. + Z*(1.-Q0)/K2)
```

```
      RETURN
      END
      REAL FUNCTION MAGCOR(Z)
C     ***********************
C
C
C        BOLOMETRIC CORRECTION
C        ---------------------
C        ASSUMPTIONS
C           1.POWER LAW SPECTRUM : SPECTRAL GRADIENT GRAD
C           2.ABSORBTION BY ELECTRON SCATTERING
C              FRAC = FRACTION OF HYDROGEN IONISED
C           3.FRIEDMANN UNIVERSE : TAU = OPTICAL DEPTH * 2.5*ALOG10(E)
C              E = BASE OF NATURAL LOGARITHM ; Q0 > 0
C           4.SQUARE SPECTRAL WINDOW,CORRECTION  WINDOW
C
      REAL H0,Q0,FRAC,GRAD,WINDOW,K1,K2,K3,TAU
      COMMON /CPAR1/H0,Q0,FRAC,GRAD,WINDOW
      IF (FRAC .GT. 0.0) THEN
         K1 = 3.*Q0 - 1.
         K2 = Q0*Z
         K3 = (K1+K2)*SQRT(1. + K2+K2) - K1
         TAU = 4.7049*FRAC*K3/(Q0*H0)
      ELSE
         TAU = 0.0
      END IF
      MAGCOR = 2.5*GRAD*ALOG10(1.+Z) + TAU + WINDOW
      RETURN
      END
      REAL FUNCTION BOUND(X0,A)
C     *************************
C
C
C        THE INTEGRATION BOUND YB FOR DOUBLE AND TRIPLE
C        ----------------------
C        IS CALCULATED
C        SIGN GIVES THE SIGN OF THE FIRST DERIVATIVE OF
C        THE INTEGRAND
C
      COMMON /CIF/IF
C
C        PROFILE SELECTED
C
      GO TO (10,20) IF
C
C        DE VAUC PROFILE
C
   10 B1 = X0 + 3.
      B2 = 40000./A
      SIGN = (X0-B2)*B2 - .25*(B2*A)**.25 + 1.
      IF(SIGN.LT.0.0 .AND. B2.LT.B1) THEN
         BOUND = B2
      ELSE
         BOUND = B1
      END IF
      RETURN
C
C        ABELL MIHALAS PROFILE
C
   20 B1 = X0 + 3.
      B2 = 400./A
```

```
      SIGN = B2*(X0-B2) - 2.*B2*A/(1.+A*B2) + 1.
      IF(SIGN.LT.0.0 .AND.B2.LT.B1) THEN
         BOUND = B2
      ELSE
         BOUND = B1
      END IF
      RETURN
      END
      SUBROUTINE MAP
C     **************
C
C
C        MAP SETS UP INPUT AND OUTPUT ARRAYS
C        AND COMPUTES THE FORWARD,INVERSE MAP WITH PSMAP,INVMAP
C        ------------------------------------------------------
C        OPTIONS:
C              "      X1=LOG(A),Y1=LOG(A-AP) IF IIN=0
C                     X1=A     ,Y1=A-AP      IF IIN=1
C        INVERSE MAP: X AND Y ARE INTERCHANGED
C        A HEADING IS PRINTED TO IDENTIFY SUBSEQENT OUTPUT
C        A MAXIMUM OF A 98 POINTS CAN BE MAPPED
C        NO OUTPUT IS PRINTED
C
      REAL X1ARR(100),X2ARR(100),ZARR(100),Y1ARR(100),Y2ARR(100),
     +      X1,X2,Z,X1INC,X2INC,ZINC,R0Z,X20,P1,P2,STOX1,STOX2,
     +      P1Z,P2LOG,MAGCOR,X1I,X2I,ZI
      INTEGER N1,I1,I2,I3,NX1,NX2,NZ
      LOGICAL FLAG,INVERS
      COMMON /CMAP1/R0Z,X20/CMAP2/N1,X1ARR,X2ARR,ZARR,Y1ARR,Y2ARR
     +       /CCPAR1/P1,P2,X10/CINP/X1I,X2I,ZI,NX1,NX2,NZ,
     +       X1INC,X2INC,ZINC
     +       /CFLAG/IIN,INVERS
C
C        NO. OF SECONDS OF ARC IN ONE RADIAN
C
      ASEC = 2.0627E05
      IF(INVERS) THEN
        WRITE(5,3000)
      ELSE
         WRITE(5,2000)
      END IF
 2000 FORMAT(1H0,5X,'FORWARD MAP FROM THE OBJECT TO IMAGE PLANE'/
     +       1H+,5X,'_____________________________________________')
 3000 FORMAT(1H0,5X,'INVERSE MAP FROM THE IMAGE TO OBJECT PLANE'/
     +       1H+,5X,'_____________________________________________')
C
C        N1 COUNTS NO OF POINTS MAPPED
C        INITIAL VALUES FOR X1,X2 AND Z PRESERVED
C
      X1 = X1I
      X2 = X2I
      Z  = ZI
C
C        Z-LOOP BEGINS
C
      DO 10 I1=1,NZ
C
C        OBSERVER AREA DISTANCE AND COLOUR CORRECTION
C
         R0Z=R0(Z)
```

```
      X20=MAGCOR(Z)
      PRINT *,'R0= ',R0Z,'MPC   COLOUR CORRECTION= ',X20,'MAG'
C
C     X1-LOOP BEGINS
C
      DO 20 I2=1,NX1
         STOX1 = X1
C
C        X1 CONVERTED TO LINEAR SCALE
C
         IF (IIN.EQ.0) X1=10**X1
C
C        X1 IS CONVERTED TO RADIANS : ONLY FOR LIN.INVERSE MAP
C
         IF (IIN.EQ.1 .AND. INVERS) X1=X1/ASEC
C
C        X2-LOOP BEGINS
C
         DO 30 I3=1,NX2
C
C           OUTPUT ARRAYS X1ARR,X2ARR,ZARR SET UP
C
            N1 = N1 + 1
            X1ARR(N1)=STOX1
            X2ARR(N1)=X2
            ZARR(N1)=Z
C
            IF(INVERS) THEN
C
C              INVERSE MAP IS DONE WITH INVMAP
C
               CALL INVMAP(X1,X2,Z,Y1ARR(N1),Y2ARR(N1),FLAG)
C
            ELSE
C
C              FORWARD MAP IS DONE WITH PSMAP
C
               CALL PSMAP(X1,X2,Z,Y1ARR(N1),Y2ARR(N1),FLAG)
C
            END IF
C
C           GALAXY COULD NOT BE MAPPED
C
            IF(.NOT.FLAG) THEN
               Y1ARR(N1) = 0.0
               Y2ARR(N1) = 0.0
            END IF
C
C           IF OUTPUT IS ON LOG SCALE
C
            IF(IIN.EQ.0.AND.FLAG) Y1ARR(N1) = ALOG10(Y1ARR(N1))
C
C           CONVERT TO SEC OF ARC (FORWARD MAP AND LIN SCALE ONLY)
C
            IF(IIN.EQ.1.AND..NOT.INVERS) Y1ARR(N1) = ASEC*Y1ARR(N1)
            X2=X2+X2INC
   30    CONTINUE
         X2 = X2I
         X1=STOX1+X1INC
```

```
   20    CONTINUE
         X1 = X1I
         Z=Z+ZINC
   10 CONTINUE
      RETURN
      END
      SUBROUTINE PSMAP(X1I,X2I,Z,Y1,Y2,FLAG)
C     ****************
C        FORWARD MAP
C        ------------
C           X1=A(SCALING RADIUS IN KPC) : X2=M-BOL(BOLOMETRIC MAGNITUDE)
C           Y1=A-AP(APPARENT ANGLE IN RAD) : Y2=M-AP(APPARENT MAGNITUDE)
C           FOR REDSHIFT Z
C
      REAL X1,X2,Z,Y1,Y2,GINF(3),SL,SIGMAD,EPS,Q1,ALFA,XAP,R0,
     +     X20,K1,AQ1,BAP,AAP,XAPMAX,
     +     P1,P2,X10
      INTEGER IF
      LOGICAL TEST,FLAG
      EXTERNAL PROFIL
      COMMON /CPAR2/SL,SIGMAD/CGINF/GINF/CACURB/XAPMAX,EPS
     +       /CPSMAP/Q1,ALFA,XAP/CMAP1/R0,X20/CIF/IF
     +       /CCPAR1/P1,P2,X10
      FLAG = .FALSE.
      PI = 3.141593
      ZLOG = ALOG10(1.+ Z)
      K1 = SL-X20-7.5*ZLOG - 37.82
      X1 = X1I/X10*(1.+ Z)**P1
      X2 = X2I - 2.5*P2*ZLOG - 5.*P1*ZLOG
      AQ1=0.4*(X2-K1)
      Q1=(GINF(IF)/PI)*X1*X1*10**AQ1
      ALFA=SIGMAD*R0*1000./X1
C
C        TEST:DOES IMAGE EXIST (NO POINT-SPREAD)
C
      IF (Q1.GT.1.) THEN
         WRITE (5,2000)
 2000    FORMAT(1H0,'GALAXY BELOW DETECTION LIMIT (NO POINT-SPREAD)')
         RETURN
      ELSE
C
C        BETA-APPARENT (BAP) AND A-AP COMPUTED FOR NO POINT-SPREAD
C
         BAP=FINV(Q1)
         AAP=.001*BAP*X1/R0
C
         IF (SIGMAD.GT.0.0) THEN
C
C           POINT-SPREAD ON
C
C           INITIAL GUESS FOR XAP
C
            XAP=AAP/SIGMAD
C
C           MAP IS COMPLETED WITHOUT POINT-SPREAD IF XAP.GE.XAPMAX
C
            IF (XAP.GE.XAPMAX) GO TO 10
C
C           TEST: DOES IMAGE EXIST WHITH POINT-SPREAD ON ?
```

```
C
            PR=PROFIL(0.0)
            IF (PR.LE.0.0) THEN
               WRITE (5,2010)
 2010          FORMAT(1H0,'GALAXY IMAGE BELOW DETECTION (WITH POINT-SPRE
     +AD)')
               RETURN
            ELSE
C
C              MAP WITH POINT-SPREAD
C
C              XAP IS DETERMINED IN SECANT
C
               CALL SECANT(PROFIL,XAP,EPS,TEST)
C
C              XAP IS FOUND IF TEST = .TRUE.
C
               IF (TEST) THEN
                  Y1=XAP*SIGMAD
                  BAP=1000.*R0*Y1/X1
                  BY2=G(BAP,SIGMAD)/(R0*R0*(1.+Z)**3)
                  Y2=X2+X20-2.5*ALOG10(BY2)+25.0
                  FLAG = .TRUE.
               ELSE
                  WRITE(5,2020)
 2020             FORMAT(1H ,'NO ZERO FOUND OF PROFILE BY SECANT')
                  RETURN
               END IF
            ENDIF
         ELSE
C
C           MAP WITHOUT POINT-SPREAD
C
   10       Y1=AAP
            BY2=G(BAP,0.0)/(R0*R0*(1.+Z)**3)
            Y2 = X2 + X20 - 2.5*ALOG10(BY2) + 25.0
            FLAG = .TRUE.
         ENDIF
      ENDIF
      RETURN
      END
      SUBROUTINE INVMAP(X1,X2,Z,Y1,Y2,FLAG)
C     ******************
C
C        INVERSE MAP
C        -----------
C           X1 = A-AP(APPARENT ANGLE) : X2 = M-AP(APPARENT MAGNITUDE)
C           Y1 = A(SCALING RADIUS)    : Y2 = M-B (BOLOMETRIC MAG.)
C        FOR REDSHIFT Z
C        NOTE:Y1 IS IN KPC AND SCALED BY X10
C             X1 IS IN RADIANS
C
      REAL X1,X2,Z,Y1,Y2,GINF(3),SL,R0,X20,BD(12),BA(12),PI
     +     GA,GB,P1,P2,X10
      INTEGER IF
      LOGICAL FLAG,TEST
      EXTERNAL GBAP
      COMMON /CPAR2/SL,SIGMAD/CGINF/GINF/CCPAR1/P1,P2,X10
     +       /CMAP1/R0,X20/CIF/IF/CINV/GB/CPSMAP/Q1,ALFA,XAP
```

```
     +       /CACURB/XAPMAX,EPS
      DATA BD/41.87515,-450.9787,-873.3407,2816.3958,7315.3342,4282.6471
     +     ,-.7032808,21.785877,-111.2023,3521.4592,7626.6401,4330.0641/
     +     ,BA/-28.999653,-225.19037,-684.14948,-1009.9881,-715.31806,
     +        -188.83001,-2.671145,15.395485,-9.5759662,-14.92826,
     +        42.047339,36.346761/,
     +     PI/3.1415927/
      GB = SL-X2-5.*ALOG10(X1)-27.82-2.5*ALOG10(GINF(IF)/PI)
C
C        B-AP IS APPROXIMATED BY A POLYNOMINAL
C
      GO TO (10,20) IF
C
C        DE VAUCOULEURS PROFILE
C
   10 IF(GB.LT.-11.5138) THEN
C
C        NO INVERSE POSSIBLE
C
         FLAG = .FALSE.
         PRINT *,'GALAXY BELOW DETECTION (INVERSE MAP)'
         RETURN
      END IF
      IF (GB.GT.2.113) THEN
         FLAG = .FALSE.
         PRINT *,'APPROXIMATION FOR B-AP BREAKS DOWN'
         PRINT *,'B-AP.GT.200 (DE VAUCOULEURS PROFILE)'
         RETURN
      END IF
      IF (GB.LT.-11.37) THEN
C
C        ANALYTIC APPROXIMATION FOR BAP < 1.7
C
         BAP = (SQRT(3628800.*10**(.4*GB) - 65.) - 5.)**4
         GO TO 30
      END IF
      GA = GB + 10.
      IF(GB.LT.-10.4) THEN
C
C        BAP(BETA APPARENT) APPROXIMATED BY POLYNOMINAL
C
         BAP=BD(6)+GA*(BD(5)+GA*(BD(4)+GA*(BD(3)+GA*(BD(2)+GA*BD(1)))))
      ELSE
         BAP=BD(12)+GA*(BD(11)+GA*(BD(10)+GA*
     +           (BD(9)+GA*(BD(8)+GA*BD(7)))))
      END IF
      GO TO 30
C
C        ABELL MIHALAS PROFILE
C
   20 IF(GB.LT.-1.991918) THEN
         FLAG = .FALSE.
         PRINT *,'GALAXY BELOW DETECTION (INVERSE MAP)'
         RETURN
      END IF
      IF (GB.GT.4.0) THEN
         FLAG = .FALSE.
         PRINT *,'APPROXIMATION FOR B-AP BREAKS DOWN'
         PRINT *,'B-AP.GT.895 (ABELL MIHALAS PROFILE)'
```

```
          RETURN
        END IF
        IF (GB.LT.-.8897) THEN
          BAP=BA(6)+GB*(BA(5)+GB*(BA(4)+GB*
     +          (BA(3)+GB*(BA(2)+GB*BA(1)))))
        ELSE
          BAP=BA(12)+GB*(BA(11)+GB*(BA(10)+GB
     +     *(BA(9)+GB*(BA(8)+GB*BA(7)))))
        END IF
C
C       A BETTER APPROXIMATION FOR BAP FOUND IN SECA
C
   30 IF (SIGMAD.EQ.0.0) THEN
C
C       INVERSE WITHOUT POINT-SPREAD
C
        CALL SECA(GBAP,BAP,EPS,TEST)
        IF (TEST) THEN
C
C          INVERSE MAP WITHOUT POINT-SPREAD IF TEST =.TRUE.
C
          Y1 = X1*1000.*R0*X10/(BAP*(1.+ Z)**P1)
          Y2 = X2 - X20 + 2.5*ALOG10(G(BAP,0.0)/(R0*R0*(1.+ Z)**3))
     +          + 2.5*P2*ALOG10(1.+ Z) + 5.*P1*ALOG10(1.+ Z) - 25.
          FLAG = .TRUE.
        ELSE
          PRINT *,'NO BAP FOUND IN SECA:BAP=',BAP,'A-AP=',X1,'MAP=',X2
          FLAG = .FALSE.
        END IF
      ELSE
C
C       INVERSE WITH POINT-SPREAD
C
        XAP = X1/SIGMAD
        S = SIGMAD
        IF (XAP.GT.XAPMAX) THEN
C
C          INVERSE COMPLETED WHITHOUT POINT-SPREAD
C
          SIGMAD = 0.0
          GO TO 50
        END IF
C
C       NO INVERSE DONE IF B-AP IS SMALLER BY A FACTOR MORE THAN 100
C
        GBMIN = GBAP(BAP/100)
        IF (GBMIN.GT.0.0) THEN
          FLAG = .FALSE.
          PRINT *,'GALAXY BELOW DETECTION WITH POINT-SPREAD'
          RETURN
        END IF
   50   CALL SECA(GBAP,BAP,EPS,TEST)
        IF (TEST) THEN
C
C          INVERSE MAP WITH POINT-SPREAD IF TEST =.TRUE.
C
          Y1 = X1*1000.*R0*X10/(BAP*(1.+ Z)**P1)
          Y2 = X2 - X20 + 2.5*ALOG10(G(BAP,SIGMAD)/(R0*R0*(1.+ Z)**3))
     +          + 2.5*P2*ALOG10(1.+ Z) + 5.*P1*ALOG10(1.+ Z) - 25.
```

```
            FLAG = .TRUE.
            SIGMAD = S
         ELSE
            PRINT *,'NO BAP FOUND IN SECA:BAP=',BAP,'A-AP=',X1,'MAP=',X2
            FLAG = .FALSE.
         END IF
      END IF
      RETURN
      END
      SUBROUTINE SECA(F,ROOT,EPS,TEST)
C     ******************
C
C
C        THE SECANT METHOD IS USED TO FIND A POS.ROOT OF
C        THE FUNCTION F (MONOTONIC INCREASING)
C
      LOGICAL TEST
      TEST = .FALSE.
      X1 = ROOT
      F1 = F(X1)
      IF (F1.GT.0.0)THEN
         X2 = ROOT/2.
      ELSE
         X2 = ROOT + ROOT/2.
      END IF
      F2 = F(X2)
      IF (ABS(F2).LT.1E-32) GO TO 10
      DO 100 J1 = 1,20
         IF (F2.EQ.F1) RETURN
         DIRF = (F2-F1)/(X2-X1)
         X3 = X2-F2/DIRF
         IF(X3.LE.0.0) X3 = X2 + X2/4.
         F3 = F(X3)
         X1 = X2
         X2 = X3
         F1 = F2
         F2 = F3
         IF(ABS(X2/X1-1.).LE.EPS.OR.ABS(F3).LT.1E-32)GO TO 20
  100 CONTINUE
      RETURN
   10 ROOT = X2
      TEST = .TRUE.
      RETURN
   20 ROOT = X3
      TEST = .TRUE.
      RETURN
      END
      REAL FUNCTION GBAP(B)
C     **********************
      COMMON /CINV/GB/CPAR2/SL,SIGMAD
      IF (SIGMAD.EQ.0.0) THEN
         GBAP = -2.5*ALOG10(B*B*F(B)/G(B,0.0)) - GB
      ELSE
         GBAP = -2.5*ALOG10(B*B*PSPROF(B)/G(B,SIGMAD)) - GB
      END IF
      RETURN
      END
```

```
      SUBROUTINE SECANT(F,ROOT,EPS,TEST)
C     ******************
C
C
C        THE SECANT METHOD IS USED TO FIND A POS.ROOT OF
C        THE FUNCTION F
C
      LOGICAL TEST
      TEST = .FALSE.
      X1 = ROOT
      F1 = F(X1)
      IF (F1.LT.0.0)THEN
         X2 = ROOT/2.
      ELSE
         X2 = ROOT + ROOT/2.
      END IF
      F2 = F(X2)
      IF (ABS(F2).LT.1E-32) GO TO 10
      DO 100 J1 = 1,20
         IF (F2.EQ.F1) RETURN
         DIRF = (F2-F1)/(X2-X1)
         X3 = X2-F2/DIRF
         IF(X3.LE.0.0) X3 = X2/4.
         F3 = F(X3)
         X1 = X2
         X2 = X3
         F1 = F2
         F2 = F3
         IF(ABS(X2/X1-1.).LE.EPS.OR.ABS(F3).LT.1E-32)GO TO 20
  100 CONTINUE
      RETURN
   10 ROOT = X2
      TEST = .TRUE.
      RETURN
   20 ROOT = X3
      TEST = .TRUE.
      RETURN
      END
      REAL FUNCTION PROFIL(Z)
C     ***********************
C
C
C        ZERO OF PROFIL IS FOUND IN BISECA
C        TO INVERT POINT-SPREAD PROFILE
C
      REAL Z,X0,Q1,BOUND,PI
      INTEGER IX,IY
      COMMON /CPROF/X0/CACURB/X,E,IX,IY/CPSMAP/Q1,ALFA
      EXTERNAL DOUBIN
C
C        THE ARGUMENT OF PROFIL IS SAVED SO THAT DOUBIN
C        CAN BE EVALUATED
C
      X0 = Z
C
C         BOUND FOR Y-INTEGRATION CALCULATED
C
      B = BOUND(X0,ALFA)
      PI = 3.141593
C
C        DOUBLE INTEGRATION ROUTINE DOUBLE IS USED
```

```
C         RESULT OF INTEGRATION = V
C
      CALL DOUBLE(DOUBIN,V,0.0,PI,0.0,B,IX,IY)
      PROFIL = V/PI - Q1
      RETURN
      END
      REAL FUNCTION PSPROF(B)
C     ***********************
C
C         POINT-SPREAD PROFILE AS FUNCTION OF B=XAP*ALFA
C         FOR INVERSE MAP WITH POINT-SPREAD
C
      REAL B,X0,BOUND,PI,ALFA,XAP,BO,V
      INTEGER IX,IY
      COMMON /CPROF/X0/CACURB/X,E,IX,IY/CPSMAP/Q1,ALFA,XAP
      EXTERNAL DOUBIN
C
C         ALFA,X0 CACULATED SO THAT DOUBIN
C         CAN BE EVALUATED
C
      X0 = XAP
      ALFA = B/XAP
C
C         BOUND FOR Y-INTEGRATION CALCULATED
C
      BO = BOUND(X0,ALFA)
      PI = 3.141593
C
C         DOUBLE INTEGRATION ROUTINE DOUBLE IS USED
C         RESULT OF INTEGRATION = V
C
      CALL DOUBLE(DOUBIN,V,0.0,PI,0.0,BO,IX,IY)
      PSPROF = V/PI
      RETURN
      END
      SUBROUTINE DOUBLE(F,SUM,XA,XB,YA,YB,INX,INY)
C     ********************
C
C         DOUBLE-INTEGRATION ROUTINE
C         --------------------------
C         A GAUSSIAN 16-POINT QUADRATURE IS DONE
C         ON EVERY SUBINTERVAL
C            F   =  FUNCTION TO BE INTEGRATED
C            SUM   =  RESULT OF INTEGRATION
C            XA,XB   =  INTERVAL ON X-AXIS
C            YA,YB   =     "      "  Y- "
C            INX,INY   =  NO OF SUBINTERVALS ON THE X-,Y-AXIS
C
      REAL NI,NJ
      DIMENSION A(16),T(16)
      DATA (T(I),I=1,16)/9.501250984E-2,-9.501250984E-2,2.816035508E-1,
     +    -2.816035508E-1,4.580167777E-1,-4.580167777E-1,6.178762444E-1,
     +    -6.178762444E-1,7.554044084E-1,-7.554044084E-1,8.656312024E-1,
     +    -8.656312024E-1,9.445750231E-1,-9.445750231E-1,9.894009350E-1,
     +    -9.894009350E-1/,
     +    (A(I),I=1,16)/2*1.894506105E-1,2*1.826034150E-1,
     +    2*1.691565194E-1,2*1.495959888E-1,2*1.246289713E-1,
     +    2*9.515851168E-2,2*6.225352394E-2,2*2.715245941E-2/
C
```

```
C         INTEGRATION FROM YA TO YB
C
C         XC = (XBI-XAI)/2 : I STANDING FOR I-TH SUBINTERVAL
C         XD = (XBI+XAI)/2
C              SIMILAR FOR Y
C
      SUM = 0.0
      S1  = 0.0
      S2  = 0.0
      XN  = FLOAT(INX)
      YN  = FLOAT(INY)
      XC  = (XB-XA)/(2.*XN)
      YC  = (YB-YA)/(2.*YN)
      DO 20 I=1,INX
         NI = FLOAT(I)
         XD = XA + 2.*XC*(NI-.5)
         DO 30 J=1,INY
            NJ = FLOAT(J)
            YD = YA + 2.*YC*(NJ-.5)
            DO 40 K=1,16
               XIK = XC*T(K) + XD
               DO 50 L=1,16
                  YJL = YC*T(L) + YD
                  S1  = S1 + A(L)*F(XIK,YJL)
   50          CONTINUE
               S2 = S2 + S1*A(K)
               S1 = 0.0
   40       CONTINUE
   30    CONTINUE
   20 CONTINUE
C
C         INTERMEDIATE SUM
C
      SUM = SUM + XC*YC*S2
C
C         INTEGRATION FROM YB TO INFINITY
C
      S2  = 0.0
      XC  = (XB-XA)/2.
      XD  = (XB+XA)/2.
      DO 60 K=1,16
         XIK = XC*T(K) + XD
         DO 70 L=1,16
            TL  = T(L) + 1.
            YJL = 2./TL + YB - 1.
            S1  = S1 + A(L)/(TL*TL)*F(XIK,YJL)
   70    CONTINUE
         S2 = S2 + S1*A(K)
         S1 = 0.0
   60 CONTINUE
C
C         FINAL RESULT
C
      SUM = SUM + S2*XC*2.
      RETURN
      END
```

```
      REAL FUNCTION DOUBIN(X,Y)
C     *************************
C
C
C        FUNCTION INTEGRATED IN DOUBLE
C        -----------------------------
C        TO COMPUTE A POINT ON THE POINT-SPREAD PROFILE
C
      REAL X,Y,ALFA,Q1,X0,U,B,F,A1,B1
      INTEGER IF
      COMMON /CIF/IF/CPROF/X0
     +       /CPSMAP/Q1,ALFA
      B = Y*ALFA
C
C        PROFILE SELECTED
C
      GO TO (10,20,20) IF
C
C        DE VAU PROFILE
C
   10 U = B**.25
      F = EXP(-U)
      GO TO 100
C
C        ABELL MIHALAS PROFILE
C
   20 B1 = B + 1.
      IF (B .LE. 21.4) THEN
         F = 1.0/(B1*B1)
      ELSE
         F = 22.4/(B1*B1*B1)
      END IF
C
C        INTEGRANT COMPUTED
C
  100 A1 = X0*Y*COS(X) - Y*Y/2. - X0*X0/2.
      DOUBIN = Y*EXP(A1)*F
      RETURN
      END
      REAL FUNCTION G(B,SIGMAD)
C     *************************
C
C
C        G(BETA,SIGMAD)/G-INFINITY
C        -------------------------
C        G(B,SIGMAD) = INTEGRAL OF 2*PI*PROFILE FROM 0 TO B
C
      REAL GINF(3),B,U,G1,RESULT,XAP,BOUND,SIGMAD,PI,BCRIT,GINFIN
      INTEGER IF,IX,IY,IZ
      COMMON /CPSMAP/Q1,ALFA,XAP/CACURA/IX,IY,IZ
     +       /CGINF/GINF/CIF/IF
      EXTERNAL TRIPIN
      PI = 3.1415927
      BCRIT = 21.4
      IF (SIGMAD .GT. 0.0) THEN
C
C        POINT SPREAD ON
C
         YB = BOUND(XAP,ALFA)
         CALL TRIPLE(TRIPIN,RESULT,0.0,PI,0.0,YB,0.0,XAP,IX,IY,IZ)
         G = RESULT * ((B/XAP)**2) * 2.0/GINF(IF)
```

```
         RETURN
       ELSE
C
C         POINT SPREAD OFF
C         CHOICE BETWEEN DIFFERENT PROFILES MADE
C
          GO TO (10,20,20) IF
C
C         DE VAU PROFILE
C
   10     U = B**0.25
          IF (U .LE. 0.8) THEN
             G1 = U**8/40320 * (1.+ U/9.*(1. + U/10.*(1. + U/11.)))
             G = G1*EXP(-U)
          ELSE
            G1 = 1.+U*(1.+U/2.*(1.+U/3.*(1.+U/4.*(1.+U/5.*(1.+U/6.*
     +           (1.+ U/7.))))))
             G = 1. - EXP(-U) * G1
          END IF
          RETURN
C
C         ABELL MIHALAS PROFILE
C
   20     GINFIN = ALOG(1.+BCRIT) + 1./((1.+BCRIT)*2.)
          IF (B .LE. BCRIT) THEN
             G = (ALOG(1.+ B) + 1./(1.+ B) - 1.)/GINFIN
          ELSE
             G = (GINFIN - (1.+ BCRIT)*(1./(1.+ B) - 1./(2.*(1.+ B)**2)))
     +           /GINFIN
          END IF
       ENDIF
C
C         NOTE GINFIN = GINF/2.*PI
C
       RETURN
       END
       SUBROUTINE TRIPLE(F,ANS,XA,XB,YA,YB,ZA,ZB,NX,NY,NZ)
C      *****************
C
C         TRIPLE-INTEGRATION ROUTINE
C         --------------------------
C         A GAUSSIAN 16-POINT QUADRATURE IS DONE ON
C         EVERY SUBINTERVAL
C            F   =  FUNCTION TO BE INTEGRATED
C            ANS   =  RESULT OF INTEGRATION
C            XA,XB   =  INTEVAL ON X-AXIS
C            YA,YB   =      "     "  Y- "
C            ZA,ZB   =      "     "  Z- "
C            NOTE:INTEGRATION FROM YA TO YB TO INFINITY
C            NX,NY,NZ   =  NO OF SUBINTERVALS ON X-,Y-,Z-AXIS
C         IN THE FIRST 3 DO-LOOPS A SUBINTERVAL IS
C         SELECTED AND SOME QUANTITIES EVALUATED
C         RELATED TO THIS INTERVAL
C         IN THE INNER THREE DO-LOOPS THE SUM IS DONE
C         OVER THIS SUBINTERVAL
C         NO CONVERGENCE CHECK IS DONE
C
       DIMENSION A(16),T(16)
       DATA (T(I),I=1,16)/9.501250984E-2,-9.501250984E-2,2.816035508E-1,
```

```
     +    -2.816035508E-1,4.580167777E-1,-4.580167777E-1,6.178762444E-1,
     +    -6.178762444E-1,7.554044084E-1,-7.554044084E-1,8.656312024E-1,
     +    -8.656312024E-1,9.445750231E-1,-9.445750231E-1,9.894009350E-1,
     +    -9.894009350E-1/,
     +    (A(I),I=1,16)/2*1.894506105E-1,2*1.826034150E-1,
     +    2*1.691565194E-1,2*1.495959888E-1,2*1.246289713E-1,
     +    2*9.515851168E-2,2*6.225352394E-2,2*2.715245941E-2/
C
C         INTEGRATION FROM YA TO YB
C
C         XC = (XBI-XAI)/2 : I STANDS FOR THE I-TH SUBINTERVAL
C         XD = (XBI+XAI)/2 : "    "    "   "   "        "
C              SIMILAR FOR Y AND Z
C
      S1 = 0.0
      S2 = 0.0
      S3 = 0.0
      XN = FLOAT(NX)
      YN = FLOAT(NY)
      ZN = FLOAT(NZ)
      XC = (XB-XA)/(2.*XN)
      YC = (YB-YA)/(2.*YN)
      ZC = (ZB-ZA)/(2.*ZN)
      DO 10 IX = 1,NX
         XI = FLOAT(IX)
         XD = XA + 2.*XC*(XI-.5)
         DO 20 IY = 1,NY
            YI = FLOAT(IY)
            YD = YA + 2.*YC*(YI-.5)
            DO 30 IZ = 1,NZ
               ZI = FLOAT(IZ)
               ZD = ZA + 2.*ZC*(ZI-.5)
               DO 40 K=1,16
                  XIK = XC*T(K) + XD
                  DO 50 L=1,16
                     YIK = YC*T(L) + YD
                     DO 60 M=1,16
                        ZIK = ZC*T(M) + ZD
                        S1 = S1 + A(M)*F(XIK,YIK,ZIK)
   60                CONTINUE
                     S2 = S2 + S1*A(L)
                     S1 = 0.0
   50             CONTINUE
                  S3 = S3 + S2*A(K)
                  S2 = 0.0
   40          CONTINUE
   30       CONTINUE
   20    CONTINUE
   10 CONTINUE
C
C         INTERMEDIATE ANSWER
C
      ANS = XC*YC*ZC*S3
C
C         INTEGRATION FROM YB TO INFINITY
C
      S3  = 0.0
      XC  = (XB-XA)/2.
      ZC  = (ZB-ZA)/2.
```

```
      XD  = (XB+XA)/2.
      ZD  = (ZB+ZA)/2.
      DO 70 K=1,16
         XIK = XC*T(K) + XD
         DO 80 L=1,16
            TL = T(L) + 1.
            YIK = 2./TL + YB - 1.
            DO 90 M=1,16
               ZIK = ZC*T(M) + ZD
               S1 = S1 + A(M)*F(XIK,YIK,ZIK)
90          CONTINUE
            S2 = S2 + A(L)/(TL*TL)*S1
            S1 = 0.0
80       CONTINUE
         S3 = S3 + A(K)*S2
         S2 = 0.0
70    CONTINUE
C
C        FINAL ANSWER
C
      ANS = ANS + 2.*XC*ZC*S3
      RETURN
      END
      REAL FUNCTION TRIPIN(X,Y,Z)
C     ********************
C
C        FUNCTION INTEGRATED IN TRIPLE
C        -----------------------------
C        TO CALCULATE G(BETA) FOR SIGMAD > 0
C
      REAL X,Y,Z,ALFA,U,F,A1,B,B1
      INTEGER IF
      COMMON /CIF/IF/CPSMAP/Q1,ALFA
      B = Y*ALFA
C
C        PROFILE SELECTED
C
      GO TO (10,20,20) IF
C
C        DE VAUCOULEURS PROFILE
C
   10 U = B**.25
      F = EXP(-U)
      GO TO 100
C
C        ABELL MIHALAS PROFILE
C
   20 B1 = 1.+ B
      IF (B .LE. 21.4) THEN
         F = 1.0/(B1*B1)
      ELSE
         F = 22.4/(B1*B1*B1)
      END IF
C
C        INTEGRAND COMPUTED
C
  100 A1 = Z*Y*COS(X) - Y*Y/2. - Z*Z/2.
      TRIPIN = Y*Z*EXP(A1)*F
      RETURN
```

```
      END
      SUBROUTINE PSTART
C     *****************
C
C
C        PLOT IS INITIALISED
C        -------------------
C        ORIGIN IS DEFINED
C        AXES AND TITLE ARE DRAWN
C        XTI,NXTI(.LE.40) = TITLE,NO OF CHARACTERS IN TITLE FOR X-AXIS
C        YTI,NYTI(.LE.12) = SAME FOR Y-AXIS
C        TITLE,NTI(.LE.40)= SAME FOR GRAPH TITLE
C        FIX,FIY          = VALUE FOR X,Y AT FIRST TICK OF AXES
C        LAX,LAY          = VALUE FOR X,Y AT LAST TICK OF AXES
C        DELX,DELY        = NO.OF DATA UNITS PER CM
C        FACT             = RATIO OF PLOT TO NORMAL PLOT SIZE
C
      REAL FIX,FIY,LAX,LAY,DELX,DELY,FACT,SIZEX,SIZEY,TITX,TITY
      INTEGER NXTI,NYTI,NTI
      CHARACTER XTI*40,YTI*12,TITLE*40
      COMMON /CPSTA/FIX,DELX,FIY,DELY
      PRINT *,'INPUT FOR : TITLE FOR X-,Y-AXIS AND GRAPH'
      PRINT *,'            ON SEPERATE RECORDS'
      READ(8,1000) XTI,YTI,TITLE
 1000 FORMAT(A40/A12/A40)
      PRINT *,'INPUT FOR : NO OF CHAR.IN TITLE FOR X-,Y-AXIS AND GRAPH'
      PRINT *,'            FIX,LAX,FIY,LAY,FACT'
      READ(8,1001) NXTI,NYTI,NTI,FIX,LAX,FIY,LAY,FACT
 1001 FORMAT()
      DELX = FACT*(LAX-FIX)/20.
      DELY = FACT*(LAY-FIY)/20.
C
C        PLOT IS INITIATED,NEW PLOT-PAGE OPENED
C
      CALL NEWPAG
C
C        NEW ORIGIN DEFINED
C
      CALL PLOT(1.5,1.5,-3)
C
C        AXES DRAWN
C        FACT GIVES RATIO OF NEW PLOT TO OLD PLOT

      CALL FACTOR(FACT)
      SIZEX = 20./FACT
      SIZEY = 20./FACT
      NXTI  = -NXTI
      CALL AXIS(0.0,0.0,XTI,NXTI,SIZEX,0.0,FIX,DELX)
      CALL AXIS(0.0,0.0,YTI,NYTI,SIZEY,90.,FIY,DELY)

         TITLE FOR PLOT DRAWN

      TITX = 6./FACT
      TITY = 21./FACT
      CALL SYMBOL(TITX,TITY,.25,TITLE,0.0,NTI)
      RETURN
      END
```

```
      SUBROUTINE GRAPH
C     ****************
C        PLOTS N1 POINTS FROM ARRAYS Y1 Y2 Z
C        -----------------------------------
C        LTYP DETERMINES TYP OF LINE DRAWN
C        LTYP.GT.0 POINTS CONECTED BY STRAIGHT LINES,AT EVERY
C                     LTYP, SYMBOL IS PLOTTED
C             = 0  SAME BUT NO SYMBOLS PLOTTED
C            .LT.0 SYMBOLS PLOTTED BUT POINTS NOT CONNECTED
C        INTEQ = INTEGER EQUIVALENT OF SYMBOL
C        J DETERMINES WITCH ARRAYS ARE PLOTTED
C        J = 1 ARRAYS (Y1,Y2)
C        J = 2 ARRAYS (Y1,Z)
C        J = 3 ARRAYS (Y2,Z)
C        J = 4 ARRAYS (X1,X2)
C        NOT MORE THAN 98 POINTS CAN BE PLOTTED
      REAL X1(100),X2(100),Z(100),Y1(100),Y2(100)
      INTEGER N1,LTYP,INTEQ
      COMMON /CPSTA/FIRSTX,DELTAX,FIRSTY,DELTAY
     +       /CMAP2/N1,X1,X2,Z,Y1,Y2
      PRINT *,'DATA INPUT FOR : LTYP,INTEQ,J'
      PRINT *,'J=1 FOR (Y1,Y2):J=2 FOR (Y1,Z):J=3 FOR (Y2,Z):J=4 FOR (X1
     +,X2)'
      READ(8,1001) LTYP,INTEQ,J
 1001 FORMAT()
      PRINT *,'LTYP=',LTYP,' INTEQ=',INTEQ,' J=',J
      GO TO (10,20,30,40) J
   10 Y1(N1+1) = FIRSTX
      Y1(N1+2) = DELTAX
      Y2(N1+1) = FIRSTY
      Y2(N1+2) = DELTAY
      CALL LINE(Y1,Y2,N1,1,LTYP,INTEQ)
      GO TO 50
   20 Y1(N1+1) = FIRSTX
      Y1(N1+2) = DELTAX
      Z(N1+1) = FIRSTY
      Z(N1+2) = DELTAY
      CALL LINE(Y1,Z,N1,1,LTYP,INTEQ)
      GO TO 50
   30 Y2(N1+1) = FIRSTX
      Y2(N1+2) = DELTAX
      Z(N1+1) = FIRSTY
      Z(N1+2) = DELTAY
      CALL LINE(Y2,Z,N1,1,LTYP,INTEQ)
      GO TO 50
   40 X1(N1+1) = FIRSTX
      X1(N1+2) = DELTAX
      X2(N1+1) = FIRSTY
      X2(N1+2) = DELTAY
      CALL LINE(X1,X2,N1,1,LTYP,INTEQ)
   50 RETURN
      END
      SUBROUTINE PENDE
C     ****************
C        PLOT IS TERMINATED
C
      CALL ENDPLT
      RETURN
      END
```

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