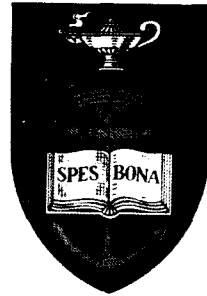


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An Alternative Model for Multivariate Stable Distributions



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in the Department of Statistical Sciences, University of Cape Town,
South Africa.

Supervisor: Professor Renkuan Guo

Declaration

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Abstract

As the title, “An Alternative Model for Multivariate Stable Distributions”, depicts, this thesis draws from the methodology of [J36] and derives an alternative to the sub-Gaussian alpha-stable distribution as another model for multivariate stable data without using the spectral measure as a dependence structure. From our investigation, firstly, we echo that the assumption of “Gaussianity” must be rejected, as a model for, particularly, high frequency financial data based on evidence from the Johannesburg Stock Exchange (JSE). Secondly, the introduced technique adequately models bivariate return data far better than the Gaussian model. We argue that unlike the sub-Gaussian stable and the model involving a spectral measure this technique is not subject to estimation of a joint index of stability, as such it may remain a superior alternative in empirical stable distribution theory. Thirdly, we confirm that the Gaussian Value-at-Risk and Conditional Value-at-Risk measures are more optimistic and misleading while their stable counterparts are more informative and reasonable. Fourthly, our results confirm that stable distributions are more appropriate for portfolio optimization than the Gaussian framework.

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Umbulelo

Ndibamba ngazibini kubazali bam, uBangile noNomvuyo, kunye nabantakwethu, uNomzekelo, ongasekhoyo uSisa, uNolitha, uYolokazi, uSisipho kunye noAvela ngenkxaso yabo de kube lelithuba. Umbulelo ongazenzisiyo kuwe sithandwa sam, Kanyisa, nawe nyana, Moyomhle. Ngaphandle kwenu ibingenakuba yimpumelelo le-thesis. Maz` enethole.

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Chapter 1. Introduction

1.1 Financial Modelling with Stable Distributions

Early evidence on the superiority of stable distributions in financial modelling includes [J38] and [J8]. Despite the lack of closed-form expressions for probability density functions, through advancement in computational power enormous research in the context of stable distributions has evolved over the past few decades. In this research evolution we make mention of

- Modelling the distributional form of security returns where evidence favours the stable to Gaussian distribution, [J19] and [J20].
- Given such evidence, the reformulation of the Black-Scholes option pricing problem, [J39], under stable distributions by [J40] concluding that the stable model explains the volatility smile effect close to reality.
- The development of the sub-Gaussian stable distribution as an alternative multivariate model to the spectral measure, [W13], thus partially reducing the challenge of working with stable distributions when applied to the portfolio selection problem, [J31]. These authors conclude that Markowitz portfolios are sub-optimal.
- Reformulation of risk measures Value-at-Risk and Conditional Value-at-Risk under stable distributions, [J28] and [W11], who respectively strongly recommend a stable approach to calculation of these measures.

1.2 Aims and Objective

In this endeavour we wish to extend a hand in research by extracting a formula in [J36] and use it to formulate another alternative model for multivariate stable distributions. Beyond this objective the aims include

- Testing suitability of stable distributions on the Johannesburg Securities Exchange.
- Comparing the proposed model to the multivariate Gaussian one.
- Further investigating the portfolio selection problem under the sub-Gaussian model compared to the Markowitz formulation.
- Investigating risk measures Value-at-Risk and Conditional Value-at-Risk under stable vs. Gaussian distribution.

1.3 Overview of the Thesis

In chapter 2 we provide a review of literature with respect to further developments on the portfolio selection problem of [J1] with emphasis on the distribution of asset returns and risk measures employed. An introductory treatment of stable distributions and the methodology of [J36] are given in chapter 3. Chapter 4 provides further evidence, from the Johannesburg Stock Exchange (JSE), on stable distributions being a far better reasonable choice in modelling high frequency return data. An empirical investigation on the goodness-of-fit of the proposed model is articulated in chapter 5. Chapter 6 reflects on risk measures, portfolio selection and derivative pricing under the stable model by using latest findings. In chapter 7 we discuss our findings and provide potential topics for further research. A conclusion is drawn in chapter 8.

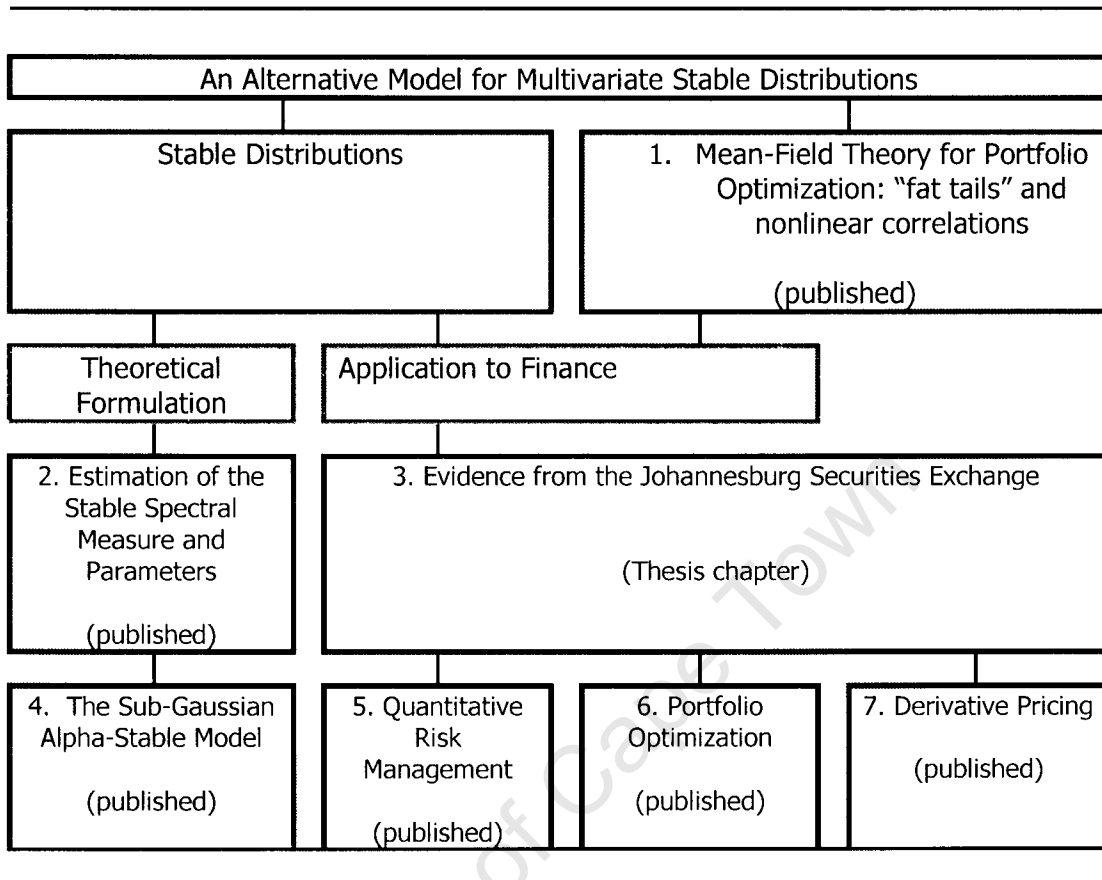


Figure 1.1 Overview of the Thesis.

Chapter 2. Background

2.1 Literature Review

In 1952 Portfolio Selection, [J1], made a major breakthrough in portfolio optimization via a Mean-Variance (MV) algorithm which assumes that asset returns normally distributed. The academic debate however has shown that returns often have non-normal features like heavy tails, extra-kurtosis and asymmetry, [J25]. This then implies that variance as a symmetric measure

- Under-estimates the risk of a portfolio whose returns are in fact asymmetric.
- May be infinite, a usual characteristic of heavy-tail distributions.

The academic debate has questioned the inclusion of desirable upside returns in the determination of portfolio risk. It has been argued that this is nonsensical and as a result downside risk measures like lower absolute deviation, lower semi-variance, also referred to as lower partial risk measures, were proposed, [W2]. In 1997 [W1] developed the notion of coherent risk measures which lead to a further notion of convex risk measures by [J4]. These measures possess special desirable properties which variance does not. Recently there have been further developments on these measures and their application to portfolio optimization which has proved better performance compared to the MV framework. A few of these include shortfall, expected shortfall (Conditional Value at Risk), spectral measures which [J7] declares belong to a class of polyhedral coherent risk measures. [J2] introduces the minimax optimization problem which has the benefit that it is a linear programming problem that can handle integer constraints. This problem is concerned with minimization of maximum portfolio loss given some target return. It is shown empirically that under multivariate normal assumptions the minimax portfolio optimization problem performs as well as the Mean-Variance one. It is argued that this method performs reasonably well when returns are non-normal and there are complexities in the constraint structure of the problem.

In light of such findings as stylized facts, researchers proposed the use of the stable family of distributions in modelling and simulation of return data. Following the pioneering stable Paretian hypothesis by [J38], [J8] elaborates on the importance of this family of distributions by making a comparison with the Gaussian distribution. Stable distributions are a very difficult family to model with moments, in some cases, infinite. Further sophistication in computer technology has enabled efficient modelling of univariate cases. There is still ongoing research with respect to the multivariate case because of the stable spectral measure of dependence being particularly challenging to estimate [W13]. As a result of this difficulty some texts give a polar representation of the distribution [B3]. The Gaussian, Cauchy and Levy distributions are the only laws with closed-form density and distribution functions in the stable family. Despite attempts by academics proposing the use of stable distributions and the power of computers today, practitioners are still sentimentally attached to the Gaussian distribution.

A debate has emerged from the Physics world where [J36] proposes mean-field theory to transform heavy-tailed returns via a nonlinear mapping from the supposed distribution into a standard normal one. This mapping also defines a measure of nonlinear dependence among variables being the covariance matrix of the new standard normal variables. Once again this approach is shown to outperform the Markowitz model. With regard to diversification a measure of linear dependence, classical correlation, has been popularly used. This measure underscores any nonlinear dependence between variables; this is another major defect of the MV framework. [J9] argue that an application of the random magnet problem proves a better measure for correlations of stock returns. A nonlinear measure of dependence proposed by [J21] would prove better than the classical measure as it captures both linear and nonlinear relations in a multivariate data set, to our knowledge this measure has not been applied to the MV problem.

In this paper we aim to provide an alternative approximation model for a multivariate stable distribution of a large dimension without considering the stable spectral measure, [J26]. This research obtains its inspiration from [J36]. We further reflect on the results

achieved by applying stable distributions to portfolio theory, quantitative risk management and derivative pricing.

2.2 Background and Datasets

Our research focuses on providing another alternative model for multivariate stable distributions without (1) a spectral measure and (2) estimation of a joint index of stability. While the sub-Gaussian model [W13] and [J26] use a dispersion matrix and a spectral measure respectively, both methods however require an estimate of a joint index of stability which is subject to error. Our proposed model is immune to the latter and intuitively this should be a benefit to financial modelling in general.

The data used comprises daily, weekly and monthly logarithmic returns of twenty five stocks, three indices i.e. Financials, Industrials and Top 40; and two exchange rates i.e. US Dollar /SA Rand and UK Pound/ SA Rand, in South African Rand terms over a period of twenty years i.e. 05-May-1988 to 05-may-2008. The data was obtained from McGregor database at University of Cape Town Library. Because this database fails to filter out public holidays on the South African calendar year, a VBA code was used to eliminate such dates. It must be noted that some data had to be shortened to minimize the influence of thin-trading, especially at early stages of a company listing on the Johannesburg Securities Exchange (JSE), on the estimation of parameters and hence the results.

2.3 Mathematical Theories

The mathematical theory used in this endeavour is founded from [J36] coupled with both stable distribution theory [B2] and the portfolio selection problem [J1]. An introductory treatment of the former two is given in Chapter 3. We emphasize reiteratively that this is far from a complete theory of the respective topics and that should interest arise the relevant references must be consulted.

2.4 Methodology

Using the computer program STABLE, developed by John P. Nolan, marginal stable distributions will be fitted to the individual series by Maximum Likelihood Estimation. The “marginals” will then be used to approximate joint distributions of exchange rates and stock returns using the method of [J36], herein called the SSA method. The abbreviation, SSA, refers to the initials of the last names of the authors, Sornette, Simonetti and Andersen. Stabilize p-p and density plots will be used to assess the goodness-of-fit for marginal stable distributions while bivariate density, contour and cross-sectional density plots will be used to assess the goodness-of-fit of the SSA method. Risk measures, Value-at-Risk and Conditional Value-at-Risk, will be calculated and compared under stable vs. Gaussian models. The portfolio selection problem is studied under both stable and Markowitz models [J1].

Chapter 3. Stable Distributions and Approximation

3.1 Monivariate Stable Distributions

A general stable distribution is explained by four parameters namely; the coefficient of stability $0 < \alpha \leq 2$, the coefficient of skewness $-1 \leq \beta \leq 1$, the scale parameter $\gamma > 0$ and the shift parameter $-\infty < \delta < +\infty$.

Definition 3.1 Suppose X_1, \dots, X_n are independent and identically distributed stable random variables then the random variable X called

1. Stable (stable in broad sense) if $X_1 + \dots + X_n \stackrel{d}{=} c_n X + d_n$
2. Strictly stable (stable in narrow sense) if $c_1 X_1 + \dots + c_n X_n \stackrel{d}{=} c X$
3. Symmetric stable if it is stable satisfies $X \stackrel{d}{=} -X$

For some positive constants c, c_1, \dots, c_n and real constant d_n

Definition 1.1 maintains that the distribution of X is preserved under addition and scalar multiplication.

Definition 3.2 A random variable is stable if it has a characteristic function of the following form, in the 0-parametrization in [B2] and not the standard one used in [B3].

$$\phi_X(t) = \begin{cases} \exp\left\{-\gamma^\alpha |t|^\alpha \left[1 + i\beta \operatorname{sign}(t) \tan\left(\frac{\pi\alpha}{2}\right) \left(|\gamma|^{1-\alpha} - 1\right)\right] + it\delta\right\} & \alpha \neq 1 \\ \exp\left\{-\gamma |t| \left[1 + i\beta \frac{2}{\pi} \operatorname{sign}(t) \ln(\gamma |t|)\right] + it\delta\right\} & \alpha = 1 \end{cases} \quad (3.1)$$

$$\text{where } \operatorname{sign}(t) = \begin{cases} -1 & t < 0 \\ 0 & t = 0 \\ 1 & t > 0 \end{cases}$$

Definition 3.3 All stable distributions are scales and shifts of the stable random variable Z with characteristic function

$$\phi_Z(t) = \begin{cases} \exp\left[-|t|^\alpha \left(1 - i\beta(\text{sign}(t))\tan\left(\frac{\pi\alpha}{2}\right)\right)\right] & \alpha \neq 1 \\ \exp\left[-|t| \left(1 + i\beta(\text{sign}(t))\frac{2}{\pi}\ln(t)\right)\right] & \alpha = 1 \end{cases} \quad (3.2)$$

In what follows we introduce the notation $S(\alpha, \beta, \gamma, \delta)$ to denote a stable random variable.

3.1.1 Properties of stable distributions

A. Standardization: If a random variable X is $S(\alpha, \beta, \gamma, \delta)$ then

$$\frac{X - \delta}{\gamma} \text{ is distributed as } S(\alpha, \beta, 1, 0)$$

B. If a random variable X is $S(\alpha, \beta, \gamma, \delta)$ with probability density function $f_X(x|\alpha, \beta, \gamma, \delta)$ then its probability density function has support

$$\text{support}(f_X(x|\alpha, \beta, \gamma, \delta)) = \begin{cases} \left[\delta - \gamma \tan\left(\frac{\pi\alpha}{2}\right), \infty \right) & \alpha < 1, \beta = 1 \\ \left(-\infty, \delta + \gamma \tan\left(\frac{\pi\alpha}{2}\right) \right] & \alpha < 1, \beta = -1 \\ (-\infty, \infty) & \text{elsewhere} \end{cases} \quad (3.3)$$

C. If X is $S(\alpha, \beta, \gamma, \delta)$ and $1 < \alpha \leq 2$ then

$$E[X] = \delta - \beta\gamma \tan\left(\frac{\pi\alpha}{2}\right) \quad (3.4)$$

D. If X is $S(\alpha, \beta, \gamma, \delta)$ then for any $a \neq 0$ and a real number b then;

$$aX + b \text{ is distributed as } S\left(\alpha, (\text{sign}(a))\beta, |a|\gamma, a\delta + b\right)$$

E. If X_k $k = 1, \dots, n$ are independent $S(\alpha, \beta_k, \gamma_k, \delta_k)$ variables then for any c_1, \dots, c_n it

follows that $\sum_{k=1}^n c_k X_k$ is distributed as $S(\alpha, \beta, \gamma, \delta)$ where $\gamma^\alpha = \sum_{k=1}^n |c_k \gamma_k|^\alpha$,

$$\beta = \frac{\sum_{k=1}^n (\text{sign}(c_k)) \beta_k |c_k \gamma_k|^\alpha}{\gamma^\alpha} \quad (3.5)$$

$$\delta = \begin{cases} \sum_{k=1}^n c_k \delta_k + \left(\beta \gamma - \sum_{k=1}^n \beta_k c_k \gamma_k \right) \tan\left(\frac{\pi\alpha}{2}\right) & \alpha \neq 1 \\ \sum_{k=1}^n c_k \delta_k + \frac{2}{\pi} \left(\beta \gamma \ln \gamma - \sum_{k=1}^n \beta_k c_k \gamma_k \ln |c_k \gamma_k| \right) & \alpha = 1 \end{cases} \quad (3.6)$$

F. Scaling: If X_k $k = 1, \dots, n$ are independent and identically distributed $S(\alpha, \beta, \gamma, \delta)$

then $\sum_{k=1}^n X_k$ is distributed as $S(\alpha, \beta, n^{1/\alpha} \gamma, \delta^*)$ where

$$\delta^* = \begin{cases} n\delta + \gamma\beta \left(n^{1/\alpha} - n \right) \tan\left(\frac{\pi\alpha}{2}\right) & \alpha \neq 1 \\ n\delta + \frac{2}{\pi} \gamma\beta n \ln(n) & \alpha = 1 \end{cases} \quad (3.7)$$

G. Reflection: For any α, β if X is symmetric stable then

$$f_X(x, \alpha, \beta, 1, 0) = f_X(-x, \alpha, -\beta, 1, 0)$$

H. Tail approximation: If X is $S(\alpha, \beta, \gamma, \delta)$ with $0 < \alpha < 2$ and $-1 < \beta \leq 1$ then

$$\lim_{x \rightarrow \infty} P(X > x) \approx \gamma^\alpha \sin\left(\frac{\pi\alpha}{2}\right) \Gamma(\alpha) \frac{(1+\beta)}{x^\alpha} \quad \text{and} \quad (3.8)$$

$$\lim_{x \rightarrow \infty} f(x|\alpha, \beta, \gamma, \delta) \approx \alpha \gamma^\alpha \sin\left(\frac{\pi\alpha}{2}\right) \Gamma(\alpha) \frac{(1+\beta)}{x^{\alpha+1}} \quad (3.9)$$

3.1.2 Estimation

Various estimation methodologies have been established for the purpose of estimating the parameters of both univariate and multivariate stable distributions. Early procedures include [J25] who consider the symmetric case. An early maximum likelihood method

was established by [J24] who found his estimates to be consistent. [J20] argues that asymmetric stable laws are “more appropriate than symmetric stable laws”. Three main procedures that seem prevalent include Sample Characteristic Function method, quantile method and Maximum Likelihood Estimation. Noting the comments by [J22] regarding a “confusion” with respect to the parameterization of stable distributions, only the Maximum Likelihood Estimation method is considered in this study following the findings of this author.

3.1.2.1 Maximum Likelihood Estimation and Diagnostics

Despite the lack of closed-form formulae for stable density functions J. P. Nolan has managed to develop a programme STABLE that reasonably estimates all the parameters via the three methodologies. This programme further enables one to assess how well the proposed stable density fits the given data. Two approaches have been proposed namely stabilized probability plots and stabilized quantile plots as well as observing the fitted distribution on the data. If the distribution is in fact stable the stabilized p-p and q-q plots must not deviate from the S-shaped reference plot. Visually the proposed distribution must show a reasonable fit to the data.

3.2 Multivariate Stable Distributions

In the review of literature on multivariate stable distributions we notice that the term “difficult” appears very often as a result of (1) working with multivariate data (2) the fact that, except for three cases mentioned earlier, there are no closed-form formulae for density and distribution functions of stable distributions and (3) the dependence structure of the component variables is measured by an infinite dimensional spectral measure. Most of the difficulty, as noted in [W16] and [W13], is a result of estimation of this spectral measure, Γ as it is denoted. With advances in research we are aware of the techniques in [J10] and [J26] regarding estimation of the spectral measure. However the introduction of the sub-Gaussian class, under the umbrella of multivariate stable distributions, there is no need for the spectral measure as it is replaced by a dispersion matrix, [W13]. The dispersion matrix has statistical advantages like (1) relevance in portfolio choice theory and (2) computation of Principal Components, to mention a few examples in [W13]. Another class of multivariate stable distributions is that of Operator-stable distributions which, for technical reasons, we do not elaborate on. Refer to [J30] for this class is stable distributions.

In this section we give without proof some of the properties of multivariate stable distributions. For a technical treatment on the entirety of stable distributions the reader can visit the books listed in the reference page.

Definition 3.4 A random vector X in R^d is defined as stable if for independent copies X_1, X_2 of X it is true that

$$aX_1 + bX_2 \stackrel{d}{=} cX + D \text{ where } a, b, c > 0 \text{ and } D \text{ in } R^d$$

stable and strictly stable are defined as in the monivariate case

Definition 3.5 A random vector X is defined as stable if definition 3.4 holds with

$$c = (a^\alpha + b^\alpha)^{1/\alpha} \tag{3.10}$$

3.2.1 Properties of multivariate stable distributions ([B3] section 2)

I. If X is a d -dimensional stable random vector, i.e. stable in any sense. Then there exists a constant α in $(0,2]$ such that (3.10) holds. More-over any linear combination of the components of X is stable in that sense.

J. Let X in R^d be a random vector

(a) If all linear combinations of X have strictly stable distributions, then X is strictly stable.

(b) If all linear combinations of X are symmetric stable, then X is symmetric stable.

(c) If all linear combinations of X are stable with $\alpha \geq 1$ then X is stable.

This property ascertains that if X is multivariate stable then a linear combination of its components is also stable. However the converse may not hold for $\alpha < 1$, see [B3] section 2.2

K. A random vector X in R^d is stable, $0 < \alpha < 2$, if and only if there exists a finite measure Γ defined on an R^{d-1} unit sphere $S_d = \{s \text{ in } R^{d-1} : \|s\| = 1\}$ and a shift vector δ in R^d such that

$$\begin{aligned} \phi_X(t) &= \exp(-I_X(t)) \text{ where} & (3.11) \\ I_X(t) &= \int_{S_d} \psi_\alpha(\langle t, s \rangle) \Gamma(ds) + i\langle \delta, t \rangle \\ \psi_\alpha(u) &= \begin{cases} |u|^\alpha \left(1 - i \operatorname{sign}(u) \tan \frac{\pi\alpha}{2} \right) & \alpha \neq 1 \\ |u| \left(1 + i \frac{2}{\pi} \operatorname{sign}(u) \ln(u) \right) & \alpha = 1 \end{cases} \end{aligned}$$

The pair (Γ, δ) is unique.

Example 3.1 In the case of $d = 1$, $S_1 = \{-1, 1\}$ and the spectral measure is concentrated on these two points. A solution to the characteristic function gives

$$\gamma = (\Gamma(-1) + \Gamma(1))^{\frac{1}{\alpha}} \text{ and } \beta = \frac{\Gamma(1) - \Gamma(-1)}{\Gamma(1) + \Gamma(-1)}$$

Clearly knowledge of Γ means knowledge of γ, β . Note that if Γ is symmetric then $\beta = 0$.

L. If a d -dimensional random vector X is multivariate stable, then any linear combination of the components of X , $Y = \sum_{i=1}^d b_i X_i$ is stable with parameters

$$\gamma_b = \left(\int_{S_d} |\langle b, s \rangle|^\alpha \Gamma(ds) \right)^{\frac{1}{\alpha}} \quad (3.12)$$

$$\beta_b = \frac{\int_{S_d} |\langle b, s \rangle|^\alpha \text{sign}(\langle b, s \rangle) \Gamma(ds)}{\int_{S_d} |\langle b, s \rangle|^\alpha \Gamma(ds)} \quad (3.13)$$

$$\delta_b = \begin{cases} \langle b, \mu \rangle + \left(\tan \frac{\pi\alpha}{2} \right) \beta_b \gamma_b & \alpha \neq 1 \\ \langle b, \delta \rangle - \frac{2}{\pi} \int_{S_d} \langle b, s \rangle \ln |\langle b, s \rangle| \Gamma(ds) & \alpha = 1 \end{cases} \quad (3.14)$$

This formulation of parameters is called the projection parameterization.

3.2.2 Estimation

There is still ongoing research with respect to estimation of multivariate stable distributions. J. P. Nolan has managed to develop a two dimensional estimation procedure using STABLE, which he articulates has limited accuracy. The difficulty arises with respect to definition of the infinite set S_d as the dimension gets larger and the spectral measure. [W17] and [W13], respectively, can be consulted regarding the estimation of stable distributions in the sub-Gaussian context

3.2.3 Simulation

As stated in the preceding section for a two-dimensional stable distribution can be simulated using the programme STABLE available in J. P. Nolan's webpage.

3.3 Approximation of Heavy-Tailed Unimodal Multivariate Distributions

The material in this section, as outlined in the original paper of [J36] herein re-introduced in the context of multivariate stable distributions lays the foundation of this thesis. An important note is that these authors argue that this theory is relevant for unimodal distributions to which, despite the lack any closed-form for the mode, stable distributions belong, [J37].

3.3.1 Monovariate Case

According to [J36] any unimodal distribution has the following representation

$$P(\delta x)d\delta x = C \frac{f'(\delta x)}{\sqrt{|f(\delta x)|}} \exp\left\{-\frac{1}{2} f(\delta x)\right\} d\delta x \quad (3.15)$$

Where the return δx of a security is non-centred

$$P(\delta x)d\delta x = C \frac{f'(\delta x)}{\sqrt{|f(\delta x)|}} \exp\left\{-\frac{1}{2} \left[\text{sign}(\delta x)\sqrt{|f(\delta x)|} - m\right]^2 f(\delta x)\right\} d\delta x \quad (3.16)$$

Where δx is centred at $m = E\left[\text{sign}(\delta x)\sqrt{|f(\delta x)|}\right]$ (3.17)

C is a normalizing constant and $f(\cdot)$ satisfies: $f(\delta x) \rightarrow +\infty$ as $|\delta x| \rightarrow \infty$. We define a new variable $y = \text{sign}(\delta x)\sqrt{|f(\delta x)|}$, where the sign function ensures that correlations are retained, such that it has a Gaussian probability distribution function with the property that the conservation, $P(y)dy = P(\delta x)d\delta x$ given the above transformation, of probability holds, [J36]. The later statement simply means that the probability of any δx should be

exactly the same as that of a corresponding $y_i(\delta x_i)$ under their respective probability density functions.

3.3.2 Multivariate case

Suppose that $\delta x'_1, \dots, \delta x'_n$ $t = 1, \dots, T$ denote the returns of n securities over a period of T time periods and that each $\delta x_i, i = 1, \dots, n$ follows a stable density function $P_i(\delta x_i)$ with distribution function $F_i(\delta x_i)$ approximated from STABLE. By conservation of probability the corresponding standard Gaussian variable y_i is such that

$$P_i(\delta x_i) d\delta x_i = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{y_i^2}{2}\right] dy_i \quad (3.18)$$

Integrating both sides of (3) gives

$$F_i(\delta x_i) = \frac{1}{2} \left[1 + \operatorname{erf}\left(\frac{y_i}{\sqrt{2}}\right) \right] \quad (3.19)$$

From which y_i is defined as

$$y_i = \sqrt{2} \operatorname{erf}^{-1} [2F_i(\delta x_i) - 1] \quad (3.20)$$

erf^{-1} denotes the inverse of the error function.

3.3.2.1 Joint distribution of security returns

Given that each y_i is a standard Gaussian variable, their covariance matrix $V = E[yy']$ is estimated using standard techniques. This covariance matrix defines a “more stable” nonlinear measure of dependence between the variables $\delta x_i, i = 1, \dots, n$. [J36] argues that given this covariance matrix of standard Gaussian variables, the best parametric representation of the joint distribution is the multivariate Gaussian distribution one

$$\hat{P}(y) = \frac{1}{(2\pi)^{\frac{n}{2}} |V|^{\frac{1}{2}}} \exp\left[-\frac{1}{2} y' V^{-1} y\right] \quad (3.21)$$

By construction the variances of the y_i are finite hence the covariances are finite, [B9]. [J36] argues that the multivariate Gaussian distribution is the most likely, although not exact, representation of the vector of y_i s based on the knowledge of the covariance matrix; [36] references [B9] on this matter. The joint distribution of Gaussian variables is not necessarily multivariate Gaussian; however the marginal distributions of a multivariate Gaussian vector are necessarily Gaussian. The last step required is then to retrieve the multivariate distribution of $\delta x_1, \dots, \delta x_n$ using the Jacobian $\frac{dy}{dx}$. From conservation of probability it follows that

$$P(x) = \hat{P}(y) \frac{dy}{dx} \quad (3.22)$$

The Jacobian is then defined as a “determinant of a diagonal matrix” whose i^{th} entry is given by

$$\frac{dy_i}{d\delta x_i} = \sqrt{2\pi} P_i(\delta x_i) \exp\left(\frac{y_i^2}{2}\right) \quad (3.23)$$

It follows that the multivariate stable distribution of asset returns $\delta x_1, \dots, \delta x_n$ is given by

$$P(\delta x) = \frac{1}{|V|^{\frac{1}{2}}} \exp\left\{-\frac{1}{2} y'(V^{-1} - I)y\right\} \prod_{i=1}^n P_i(\delta x_i) \quad (3.24)$$

I defines the $n \times n$ identity matrix. It must be noted that when the components of δx are independent then their joint distribution is a product of the marginal distributions, so is the joint characteristic function.

Chapter 4. Evidence of Stable Distributions on the Johannesburg Securities Exchange (JSE)

4.1 Methodology and Fit Diagnostics

Distribution fitting was done using STABLE with two choices (1) Maximum Likelihood Estimation and (2) Gaussian option and the reliability and accuracy of the results thus depends also on the reliability and accuracy of STABLE. An assessment of both distributions fitted is presented using density plots on all variables and stabilized p-p plots on selected variables. Using a stabilized p-p plot instead of the standard p-p plot gives a clear reference regarding the heavy tail nature of a general stable distribution [W16], as such it is “S” shaped. The diagnostic methods used are those presented in [W16] and include that a stable model is plausible if

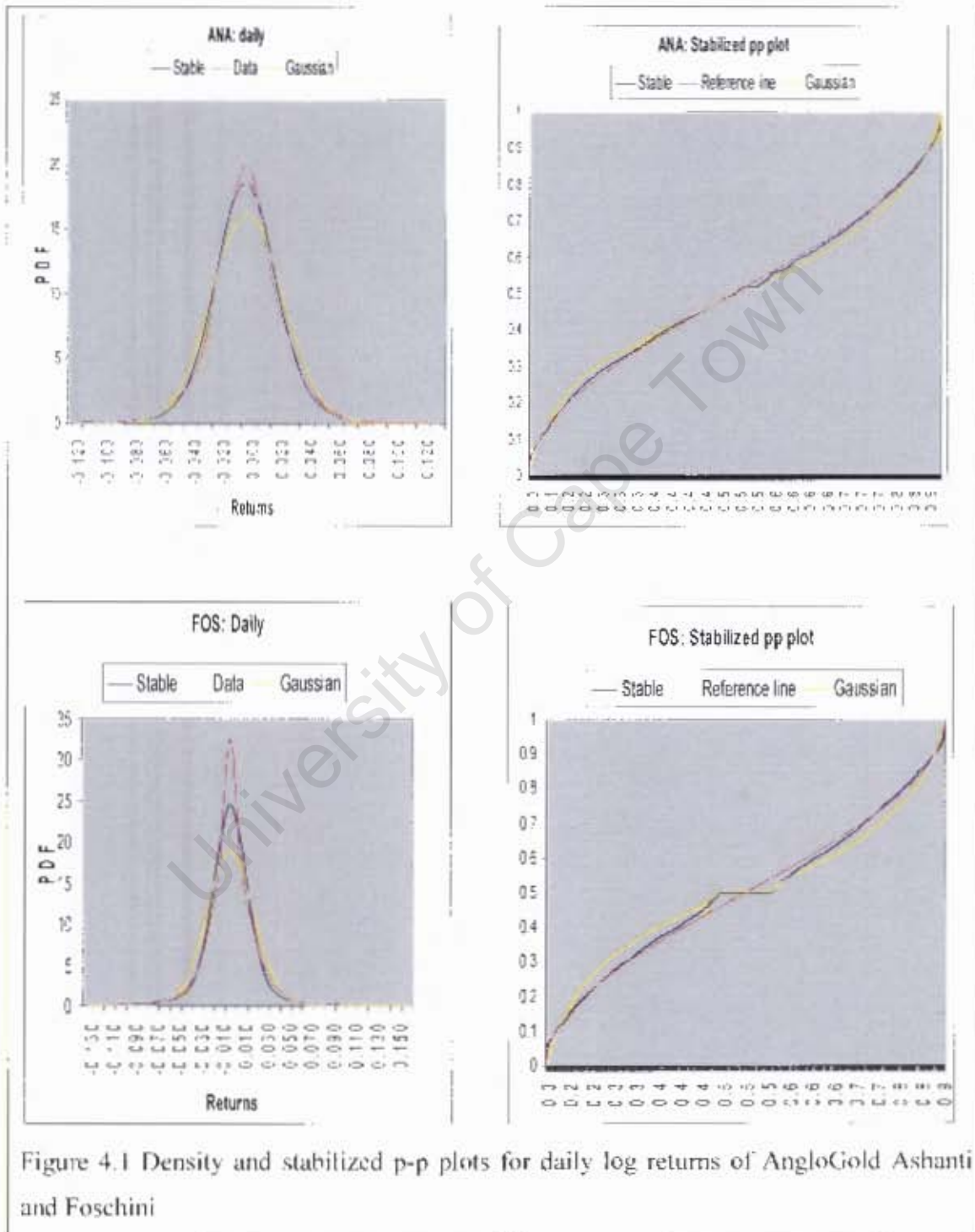
- The Kernel density plot should not show multimodality
- The thinned quantile-quantile (q-q) or stabilized p-p plots should remain very close to the reference line

In this text we will only investigate the nature of the Kernel density and stabilized p-p plots as our diagnostics. The results presented in this section are for illustration purposes. Appendix A gives full names of the abbreviations used. The rest of the results can be found in Appendices B and C.

Note: In all plots, the blue graph denotes a general stable distribution, the pink graph denotes a Gaussian Kernel fit, while the yellow graph denotes Gaussian fitting.

4.2 Results

4.2.1 Stocks



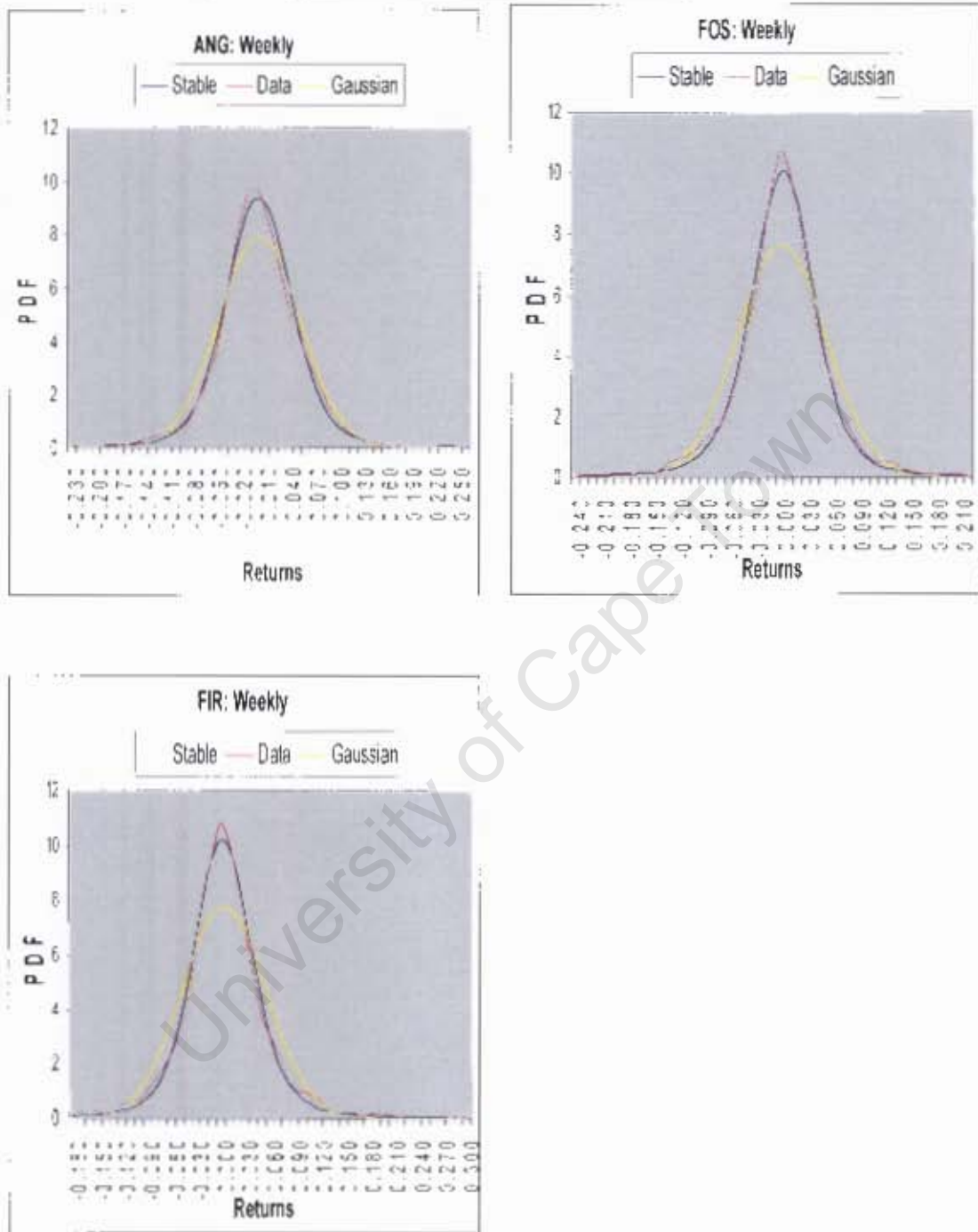
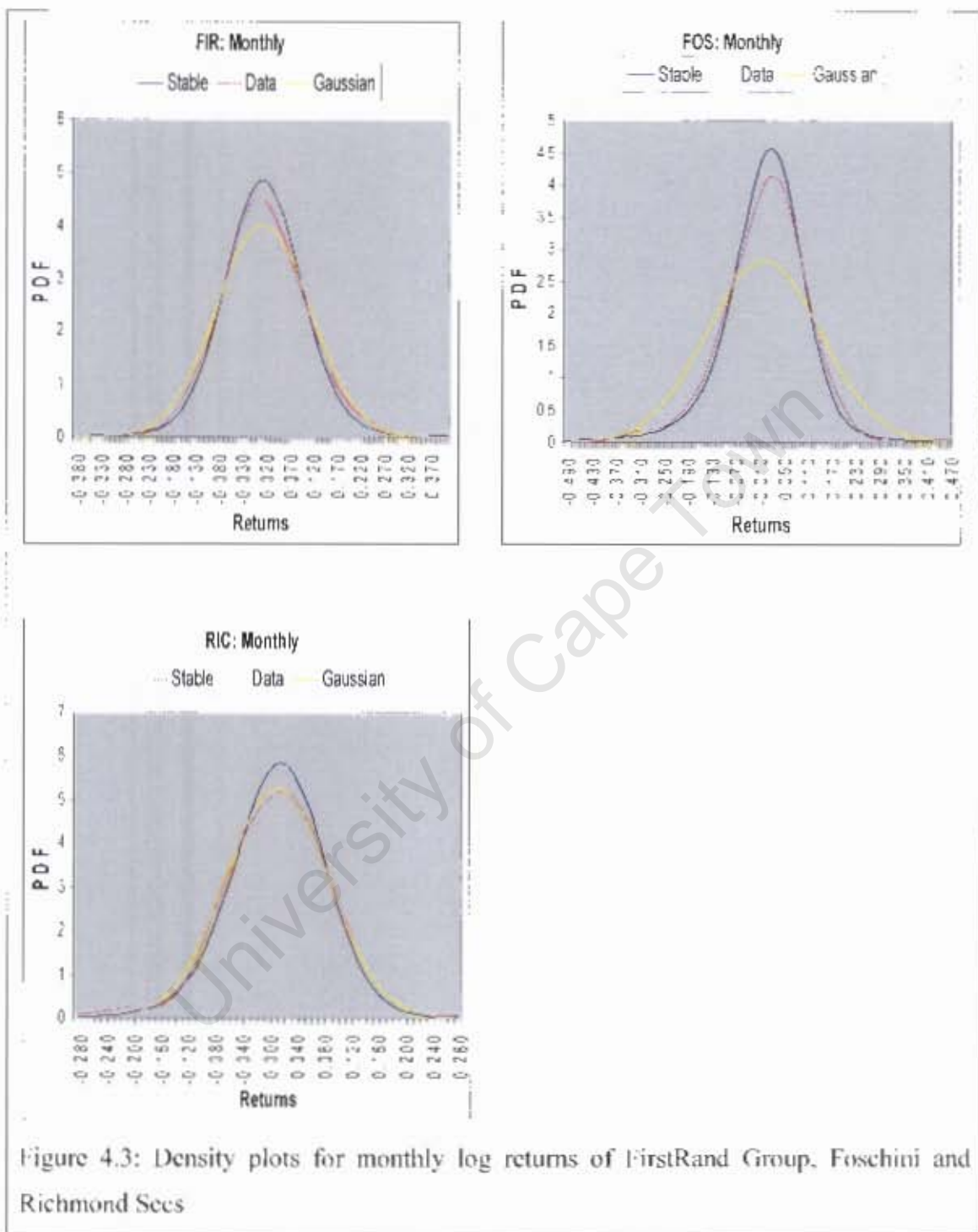


Figure 4.2: Density plots for weekly log returns of Anglo American, Foschini and FirstRand Group



4.2.2 Indices

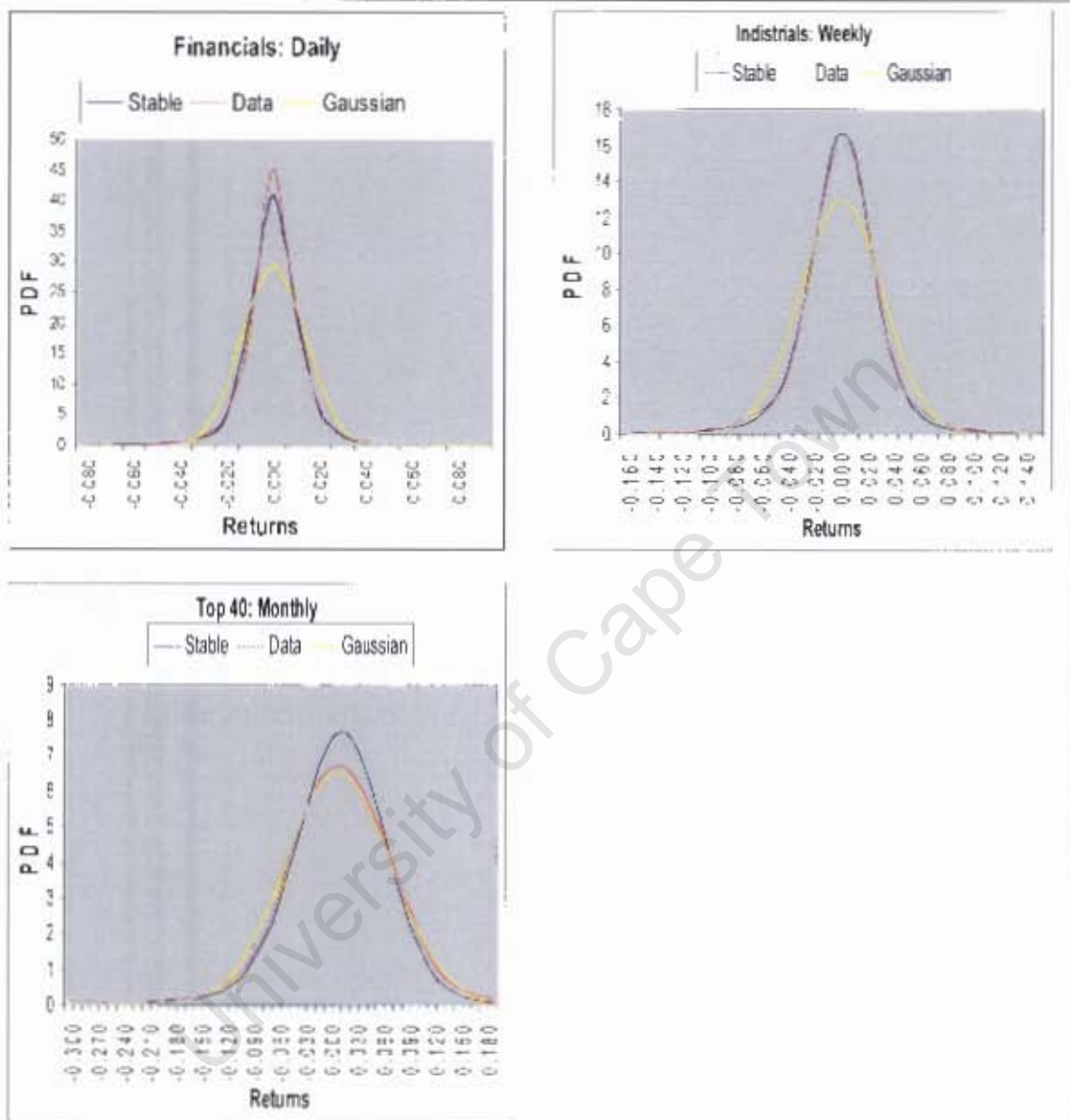


Figure 4.4: Density plots for daily, weekly and monthly log returns of Financials, Industrials and Top 40 indices respectively.

4.2.3 Exchange Rates

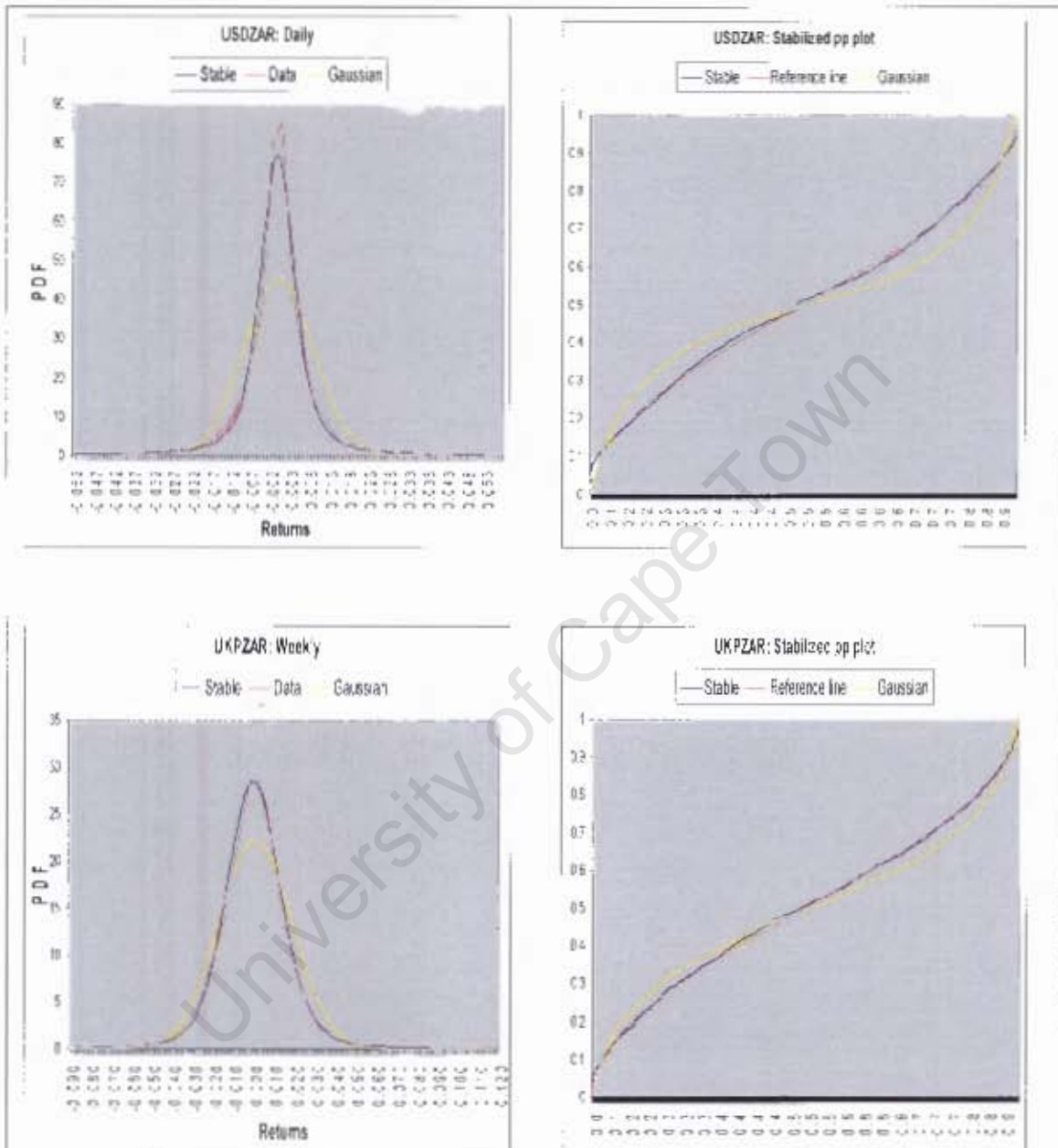


Figure 4.5: Density and stabilized p-p plots for daily and weekly log returns of the US Dollar-SA Rand and UK Pound-SA Rand respectively

4.3 Discussion

This section provides an overall word with respect to the results observed from fitting returns of selected variables with a Gaussian distribution compared with a general stable distribution that comprises four parameters. The results however fulfill what is expected in the sense that a four parameter distribution should provide a reasonably better fit than a distribution with fewer parameters. In this case the general stable distribution has parameters, alpha and beta, which capture kurtosis and skewness respectively. The latter are features observed in [J38] and [J8]. From the results the following have been observed.

- The stabilized p-p plot gives a clear picture of approximately how close a distribution is to fitting data that is of skew and heavy tail nature. In all instances we have observed that the Gaussian plot very much deviates from the reference line, throughout, while the general stable plot remains very close, see Figure 4.5 for an example. Further the Gaussian Kernel density plot exhibits unimodality. Thus a stable model is reasonable [W16].
- At higher frequencies, daily, the general stable distribution provides a distinctly better fit than the Gaussian distribution. This is attributed to the fact that at such frequencies the kurtosis of the data tends to be very high thus leaving the general stable distribution a better choice. However,
- At such higher frequencies the data may tend to have an extra kurtosis than the general stable model can explain. This is a common feature on all of the density plots from daily data. A reverse of this feature is a tendency in some of the results from monthly data, for an example Figure 4.4, the monthly returns of the Top 40 index.
- In conclusion, the log returns of stocks, indices and exchange rates are far better modelled using a general stable distribution than the popular Gaussian.

Chapter 5. An Alternative Model for Multivariate Stable Distributions

5.1 Overview

Literature asserts that the index of stability is invariant under linear transformations of a given multivariate stable random vector, see Chapter 3. A well known example is that of a multivariate Gaussian vector. In some texts [W13] what triggers a “reasonable” assumption for joint stability of a random vector is how close the individual indices of stability of the monovaraite projections of such a vector are to each other. With reference to Richmond and Rand Merchant Bank in appendix C and the argument above one would argue that their daily returns are jointly stable. However as the frequency lowers the corresponding indices of stability differ significantly. Clearly, from this observation, if we were to speak of a portfolio of these two stocks, a suggestion of a joint index of stability would be questionable. Under the SSA methodology it is not necessary to define a joint index of stability; the technique has the advantage that the components of a random vector can be stable with different indices of stability and one would still be able to define their multivariate stable density, see (3.24). Thus even the approximations we undertake in this chapter are defined using components that do not necessarily have “equal” indices of stability. Among other things, the disadvantage of the SSA transformation is that in the context of multivariate stable distributions, we are unable to define, in closed-form, a joint characteristic function if the components are not independent. On the approximation of multivariate stable densities recent methodology includes (1) estimation of a spectral measure [J26], (2) using a copula function, (3) the sub-Gaussian model [W13] to which we add the SSA methodology as an alternative.

5.2 Procedure and Fit Diagnostics

The data used comprises daily and weekly returns of four JSE listed stocks, i.e. IMP, INV, NAS and PIC, and two exchange rates, i.e. USDZAR and UKPZAR, whose codes are fully described in Appendix A. Below we explain the step-by-step procedure followed.

5.2.1 Procedure

Step 1: Use STABLE to approximate the parameters of each market variable. Compute the pdf and cdf at discrete points starting from -0.24 to 0.24 at increments of 0.001 for each set of parameters. The same is done for the exchange rates but the starting and ending values are -0.13 and 0.13. The extreme points are chosen as the most extreme values of the commensurate group of data. The pdfs and cdfs can be found in spreadsheets PDF and CDF of the workbooks SSA Stocks and SSA Exchange Rates in the attached data CD.

Step 2: Using (3.20) the discrete points are transformed to be standard Gaussian, see sheets GAU of the respective workbooks. Using the linear interpolation formula

$$f(x) = \frac{(b-x)f(a) + (x-a)f(b)}{b-a} \quad \text{for } a \leq x \leq b \quad (5.1)$$

which can be found in [W18] the standard Gaussian values of the actual realised returns for each variable are computed, using the VBA code in Appendix H, in the sheet TRA found in the workbooks in Step 1.

Step 3: The covariance matrices of the Gaussian values are calculated, with determinants, and (3.24) is used to compute the “exact” bivariate densities of the chosen pairs of variables.

Step 4: The bivariate vectors of returns with their corresponding pdf are imported from Excel to MatLab. The pdf is then plotted in 3-dimensions using the code in Appendix H.

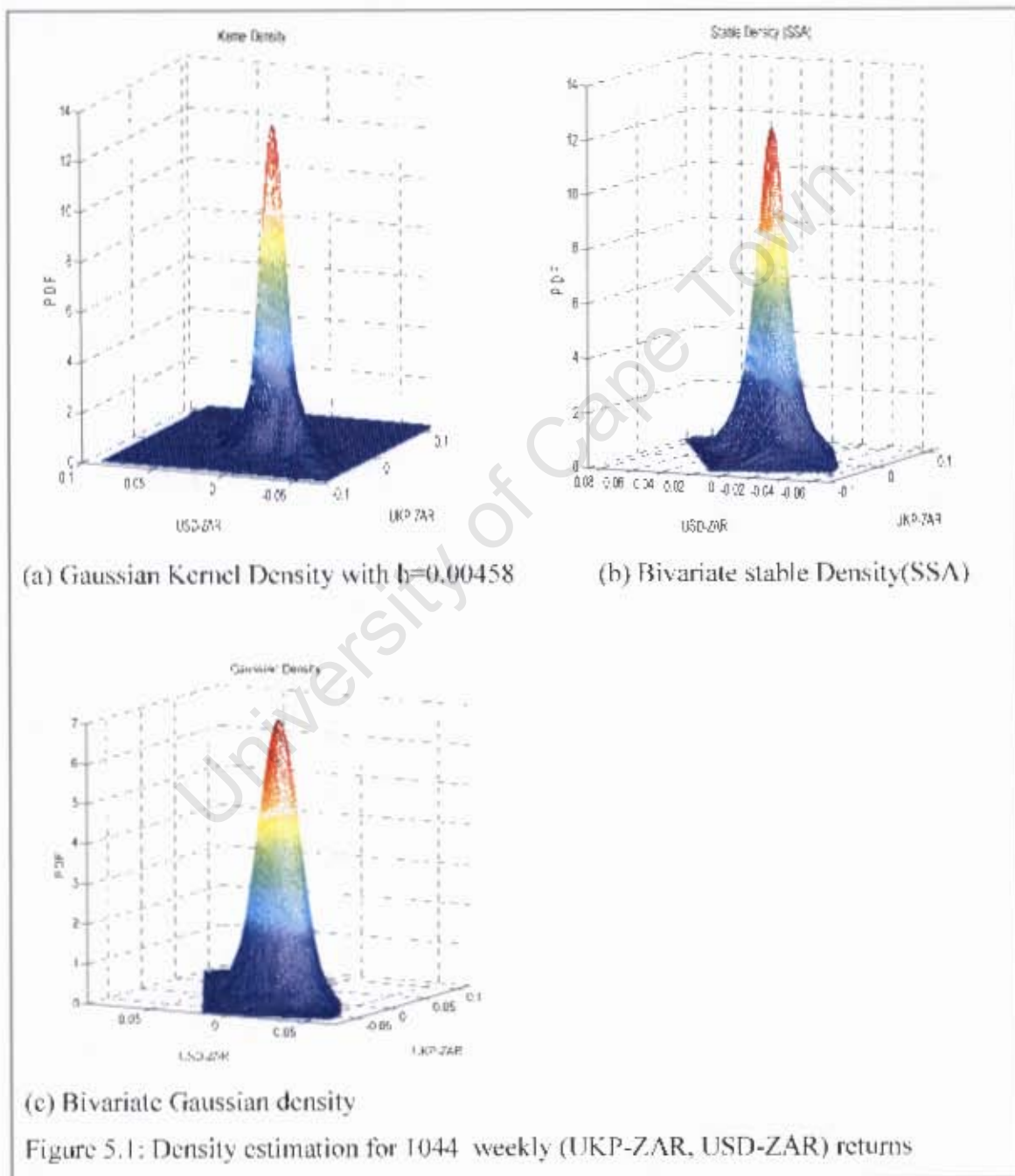
5.2.2 Fit Diagnostics

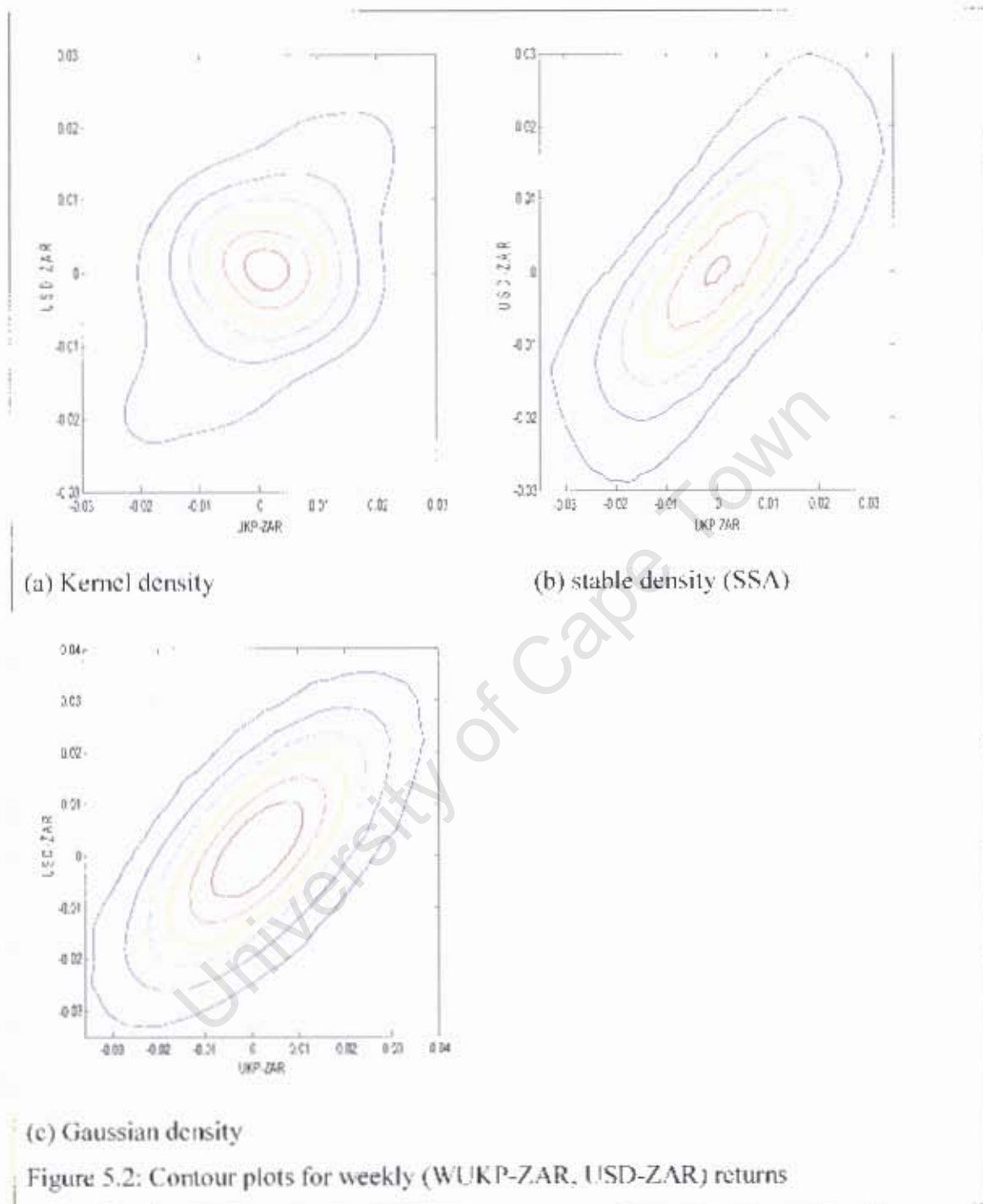
While herein we present results for bivariate cases, in the interest of time, the technique can be easily extended to any multivariate case. A Gaussian kernel density plot [J42], where the bandwidth, h , is obtained from MVSTABLE, is used as an empirical reference for how well the SSA method fits the data compared to its counterpart, the multivariate Gaussian distribution. In order to assess the appropriateness of the two fits we use density, contour, as well as cross-sectional density plots. Cross-sectional density plots give a two-dimensional view of the bivariate density plot and as such are important in assessing the kurtosis structure of the proposed methods relative to the data as explained by the kernel density. At this stage we are unable to use goodness-of-fit tests like the Kolmogorov-Smirnov, quantile plots etc, due entirely to the complex structure of the SSA stable density function. The plots presented were computed in MatLab. Due to computational time expensiveness we have chosen to use variables with shorter length of data.

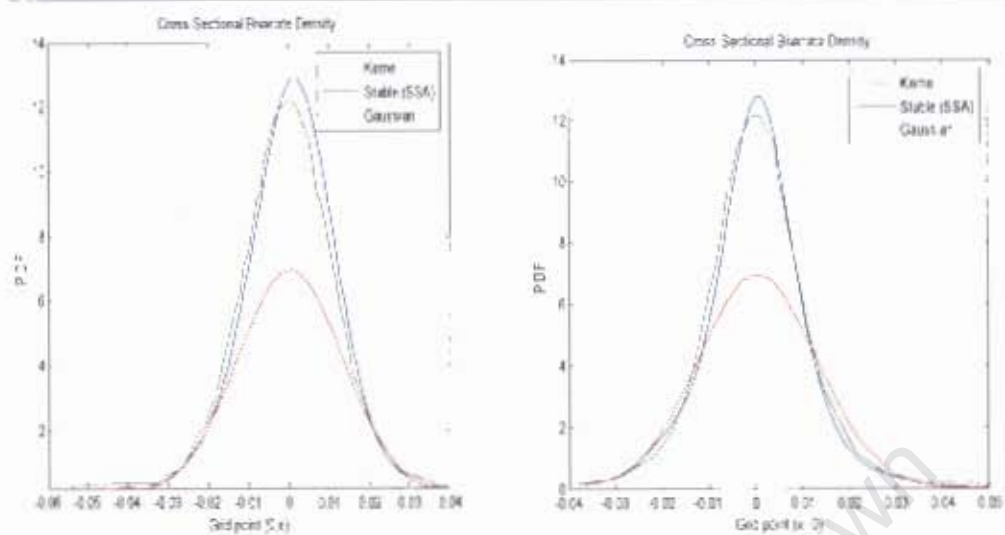
5.3 Results

Figures 5.1, 5.2 and 5.3 below present the density, contour and cross sectional density plots of weekly (UKP-ZAR, USD-ZAR) returns respectively. Results of a similar task are presented in Figures 5.4 to 5.6 for daily (IMP, INV) returns. The density plots show that the data has a much higher peak than explained by the Gaussian fit, which is approximately two times below the peak of the kernel density. However the SSA stable fit provides a much closer fit for the peak. The corresponding contour plots confirm that the data is concentrated around the mode as such the SSA stable fit becomes a choice over the Gaussian. Towards the tails the SSA stable fit is still a reasonable choice reflecting on possible extreme observations as shown by the kernel plot. Another interesting feature is the fact that these methods “agree” on the dependence structure between the variables as the contour plots indicate positive “correlations”, in the Gaussian language, with the SSA-stable contour plot assuming the shape of a

parallelogram towards the tails, [E1] Chapter 3. The cross-sectional densities clearly indicate the SSA stable density as a natural choice over the Gaussian distribution, satisfactorily being able to explain the kurtosis of the data. The same conclusion is drawn for the rest of the data in Appendix D.



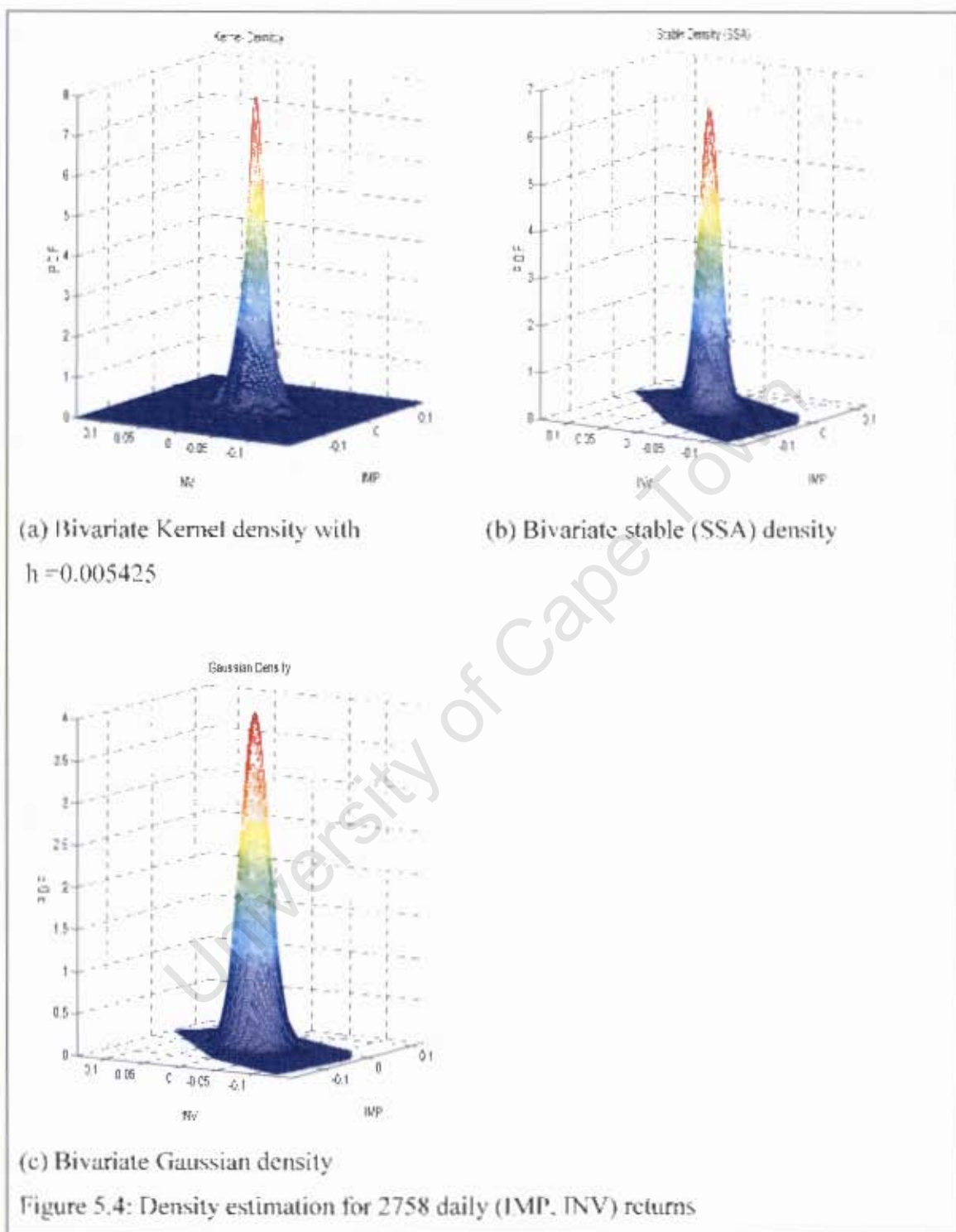


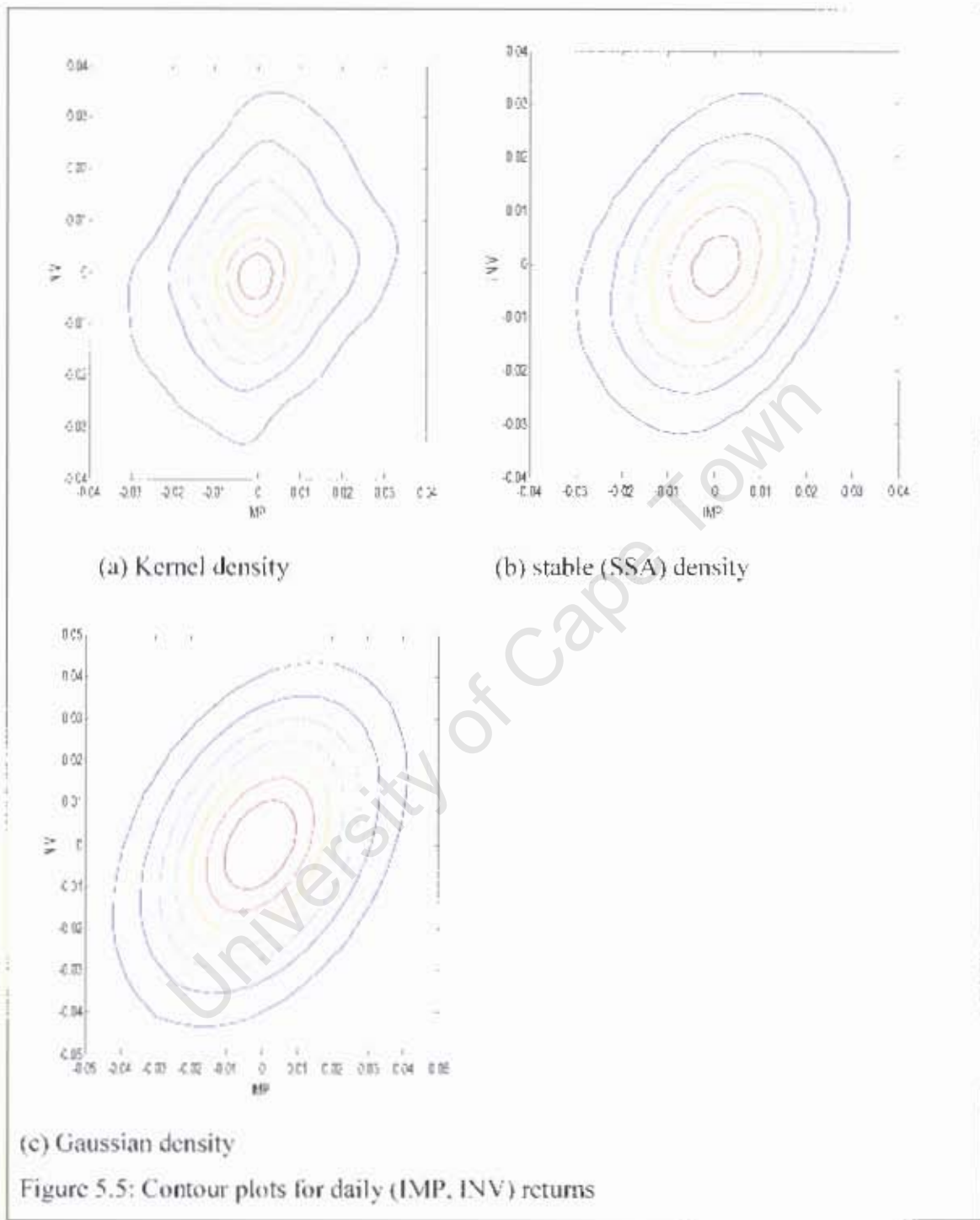


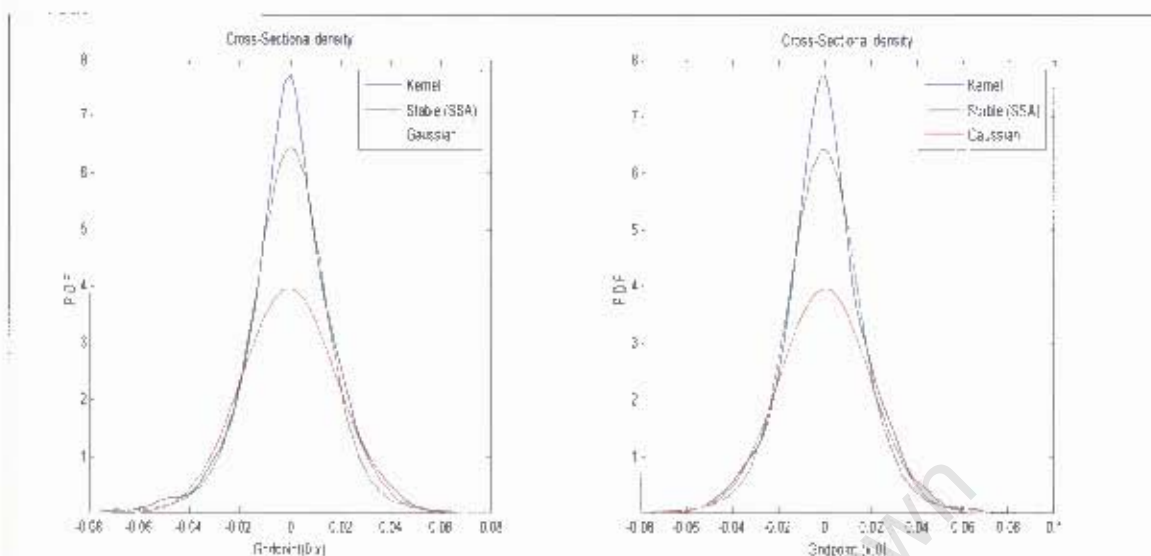
(a) Bivariate (0,USD-ZAR) density

(b) Bivariate (UKP-ZAR,0) density

Figure 5.3: Cross-sectional density plot for weekly (UKP-ZAR, USD-ZAR) returns







(a) Bivariate (0, INV) density

(b) Bivariate (IMP, 0) density

Figure 5.6: Cross-sectional density plot for daily (IMP, INV) returns

5.4 Discussion

We have used the SSA model to express bivariate stable distributions and have shown via no formal statistical tests that this method can be indispensable in the theory and applications of general stable distributions. This method provides an alternative to the spectral measure and the sub-Gaussian models.

The advantages of this technique include that it

- Does not require estimation of a joint index of stability, which under current methods is not immune to errors. That said,
- Provides a better choice to financial problems under which we mention portfolio optimization. This is because, as shown in the appendix, the indices of stability for daily returns of a particular stock can differ significantly from that of a different frequency.
- Further paves natural way of estimation of all classes, even the operator-stable, of stable distributions.

Thus far the disadvantages of the SSA model include

- Being computationally expensive,
- Difficulty in using standard statistical methods like goodness-of-fit tests and
- Dependence on the underlying complexity of univariate stable distributions.

Chapter 6. Quantitative Risk management, Portfolio Optimization and Derivative Pricing

6.1 Risk Tools

We reflect on the risk measure most commonly used in practice, Value-at-Risk (VaR), and its coherent counterpart popularly known as Conditional Value-at-Risk (CVaR), Expected Tail Loss (ETL) or Expected Shortfall (ES). A technical treatment on these measures can be found in [B4] while regarding coherency issues of the latter measure [W1] can be consulted.

6.1.1 Value-at-Risk (VaR)

Definition 6.1: Given a loss distribution with distribution function F and confidence level $0 < 1 - q < 1$, Value-at-Risk, $VaR(q)$, is defined to be the q^{th} quantile of the loss distribution. Often q is chosen to be 1% or 5%, [B4].

In this section we give parametric, i.e. Gaussian and stable, as well as empirical estimates of VaR measures using weekly returns of the variables concerned. Table 6.1 shows that a much more reasonable and informative estimate of VaR is the stable one as it is very close to the empirical estimate. We note however that the Gaussian VaR can be reasonable at 95% but is undoubtedly misleading at 99% confidence level as the estimates are about half the empirical ones. These results confirm those observed in [J28] however ours are computed using [J22]'s S_0 parameterization. We, too, strongly recommend that for high frequency market data the stable model should be used for more informative VaR calculations.

Table 6.1	5% VaR			1% VaR		
Variable	Empirical	Gaussian	Stable	Empirical	Gaussian	Stable
USD-ZAR	-0.027	-0.019	-0.026	-0.044	-0.027	-0.067
UKP-ZAR	-0.026	-0.019	-0.025	-0.046	-0.028	-0.050
Financials index	-0.051	-0.039	-0.048	-0.097	-0.056	-0.107
Industrials index	-0.045	-0.034	-0.044	-0.094	-0.049	-0.095
Top 40 index	-0.053	-0.035	-0.048	-0.097	-0.051	-0.096

6.1.2 Conditional Value-at-Risk (CVaR)

Definition 6.2 Given a loss distribution and a confidence level $0 < 1 - q < 1$, Conditional Value-at-Risk is defined to be the expectation of losses beyond VaR.

Given that CVaR fulfils the desirable properties of an ideal risk measure [W1] it is often preferred because it is more informative as an aggregate than the quantile, VaR, [W4]. We herein compare this risk measure when calculated assuming Gaussianity on one hand and Stability on the other relative to the empirical estimate. The stable CVaR is estimated as an average of VaRs calculated at a reasonably fine grid of increments of 0.0001 from zero to the desired confidence. Table 6.2 shows empirical, Gaussian and stable CVaR for selected stocks at confidence levels of 5% and 1% respectively. In both cases we observe the over optimism of the Gaussian measure. The stable estimate is undoubtedly the most reasonable and more informative risk measure. While at the 1% level the later estimates appear more pessimistic, more in Appendix E, this feature should be viewed more positively than the opposite because of the heavy tail nature of the data. Most importantly this should be viewed as the most reasonable caution against potential losses.

Table 6.2	5% CVaR			1% CVaR		
Variable	Empirical	Gaussian	Stable	Empirical	Gaussian	Stable
ANA	-0.122	-0.091	-0.128	-0.180	-0.119	-0.217
ANG	-0.112	-0.073	-0.127	-0.169	-0.097	-0.261
GOL	-0.128	-0.094	-0.130	-0.176	-0.123	-0.200
BID	-0.089	-0.059	-0.102	-0.138	-0.077	-0.215
EXX	-0.134	-0.086	-0.149	-0.209	-0.115	-0.296

6.2 Portfolio Optimization

6.2.1 Stable Portfolio Optimization Models

Criticism on the Markowitz frame work on portfolio selection focused on the risk measure used, the standard deviation, while still assuming that the return distribution is a multivariate Gaussian one. Using downside, [W2], and coherent risk measures, [W4], a similar conclusion is drawn i.e. criticism of the standard deviation and the under-performance of a Markowitz portfolio.

Over the past two decades, with advancement on the computation of multivariate stable distributions, and the evidence against “Gaussianity”, [J8], the Markowitz problem has been reformulated on a reasonable assumption of multivariate stability of the return vectors, [J31]. As alluded to in Chapter 3, the spectral measure introduces computational challenges in which case the multivariate α -stable sub-Gaussian class provides a good alternative. Using the sub-Gaussian multivariate distribution and providing an estimate of the dispersion matrix as a dependence structure, [J31] shows its superiority to the Gaussian distribution. In a comparison of asset allocation for different indices of joint stability, the authors further show that a smaller index incorporates more risk due to leptokurtosis and as such the model reduces exposure to assets with that particular characteristic. [J35] confirms the superiority of the sub-Gaussian model.

In [W11] a stable variant of expected tail loss, the stable Expected Tail Loss (SETL), is shown to be a “more informative” risk measure as it captures the feature of leptokurtosis. Further, an alternative measure for risk adjusted performance, stable Tail Adjusted Return Ratio (STARR), a variant of the Sharpe ratio is introduced and shown to be a superior performance measure. [W13] provides an estimate of the dispersion matrix and performs a stable principal components analysis and shows its superiority over the covariance one. In [J41] integral formulae for the calculation of stable Expected Tail Loss (SETL) are developed and pave a way to easily optimize portfolios using this measure. These measures are easily computed under the α -stable sub-Gaussian model. For ease of

computation we will follow the same approach bearing in mind the necessary requirements to impose the sub-Gaussian model. A mathematical formula for this computation is given below.

Suppose that w, r for denote the weight and return vectors of assets $1 \leq i \leq K$ over time and that the joint distribution of asset returns is sub-Gaussian $S(\alpha, 0, Q, \mu)$ where Q is the dispersion matrix and μ is the mean vector. Then [J41] asserts that the Expected Tail Loss of the portfolio, at confidence level p is given by

$$ETL_p(w'r) = \sqrt{w'Qw}ETL_p\left(\frac{w'r - \mu}{\sqrt{w'Qw}}\right) - w'\mu \quad (6.1)$$

where $\frac{w'r - \mu}{\sqrt{w'Qw}}$ is $S(\alpha, 1, 0, 0)$, a standard stable variable. In [J41] tables of Expected Tails Losses for $p \in \{0.01, 0.05\}$ are provided for various values α and β for the standard case.

We wish to emphasize that this model is only valid for the case where $\alpha > 1$.

6.2.2 Empirical Comparison of Stable to Gaussian portfolios

In this section we investigate some of the results found as argued above particularly with respect to

- Principal Component Analysis
- stable Portfolio Allocation

6.2.2.1 Principal Component Analysis

Using 2758 daily returns of twenty two, 22, of the stocks listed in appendix A i.e. all except EXX, RAI and RMB we computed both their covariance and dispersion matrices and performed principal component analysis on both. This, as argued in [W13], is an attempt to quantify and compare the variation explained by the first k principal components of each matrix. An important point to note is that a background check for the

suitability of a stable dispersion matrix was done using the steps outlined in [W13]. We note that

- The tail indices of these stocks do not differ much from each other,
- Their skewness coefficients are close to zero and
- Bivariate scatter plots of these stocks are elliptically contoured. We show some of these in appendix F.
- Thus it is just to use an average tail index to estimate a dispersion matrix, [W13].

Figure 6.1 below shows the results. The results confirm those in [W13] that the first k principal components of the dispersion matrix explain more variation compared to their covariance matrix counterparts. For an example the first two stable principal components explain about 60% while their counterparts explain about 45%. These findings trigger an obvious question of what this means for portfolio optimization.

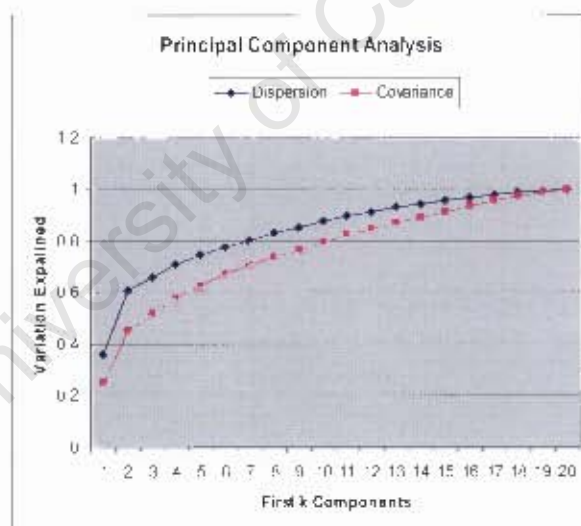


Fig. 6.1 Dispersion vs. Covariance PCA

6.2.2.2 Stable Portfolio Allocation

We have experienced that dealing with multivariate stable distributions is particularly cumbersome when there is no readily available software for computation. With this in mind in this subsection we choose only four stocks IMP, INV, NAS and PIC to

investigate the portfolio allocation problem using stable Expected Tail Loss, dispersion and covariance matrices as in [J41], [W13] and [J1] respectively. These stocks were not chosen in any fashion except to benefit from diversification across industries. Both dependence matrices, shown in appendix G, were calculated using daily data ending on 05 May 2006 and were then assumed constant until 05 May 2008.

Our analysis, performed by using Excel SOLVER, assumes that

- The portfolio manager has perfect forecasts for the returns of each of these stocks for the next day.
- The objective is to construct a portfolio that performs at least as much as a Naïve portfolio at minimum possible risk. The Naïve portfolio is an equally weighted one.
- There are no transaction costs.
- Thus each day the portfolio manager alters the weights according to these conditions.

While the assumption of perfect forecasts for returns may be too optimistic we firmly argue that our analysis is merely for comparison purposes and that the naïve portfolio is there as a benchmark. Figure 6.2 below shows 522 rolling one-year returns and corresponding standard deviations, which is common industry practice, to 05 May 2008 for these portfolios including the benchmark. The stable optimal portfolios beat their Markowitz counterparts by an average of +27% in both restricted and unrestricted cases at no significant difference in risk. Another interesting finding is that the stable optimal portfolios yield the “same” results. This is what one would expect given that they share the dispersion matrix as a common measure of dependence. The performance of the portfolios for the unrestricted case was not significantly different to that of the restricted one hence it suffice to show only one of this here. Further evidence is provided by examining the allocation statistics in table 6.3 below.

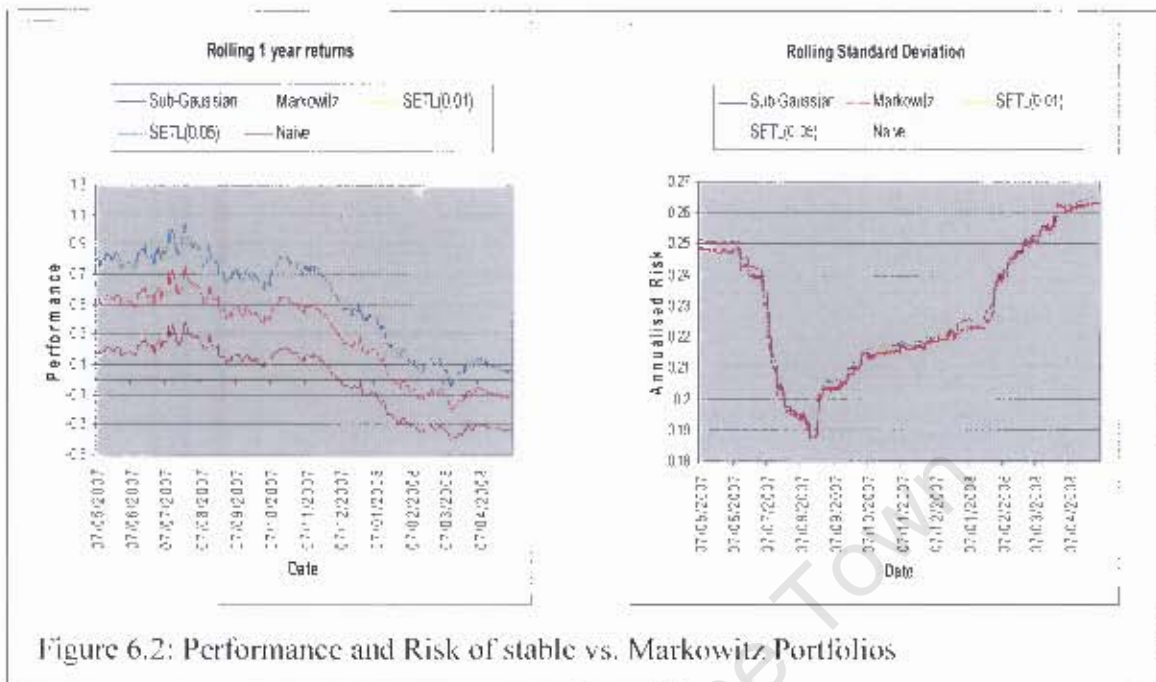


Figure 6.2: Performance and Risk of stable vs. Markowitz Portfolios

A further step of the investigation was to examine the allocations to assets that lead to this performance gap. Table 6.3 gives a summary of allocations for the restricted case; the summary of the unrestricted case is not significantly different to the former under both methods. Our immediate observations are that

- The stable allocations have a wider range which, given the performance results, indicates that the stable model is more suitable for dynamic weight allocation in active portfolio management.
- Given the riskiness of NAS, tail index equal to 1.5, the stable model allocates a minimum weight of zero and -2% in restricted and unrestricted cases respectively while the Markowitz model allocates a minimum of 10% under both cases. This confirms the results in [J31] that a small index of stability incorporates an extra risk due to leptokurtosis which only the stable model is able to capture.

Table 6.3	IMP	INV	NAS	PIC
<i>Alpha</i>	1.64	1.67	1.5	1.63
<i>stable Allocations</i>				
<i>Min</i>	0.12	0.12	0.00	0.19
<i>Average</i>	0.30	0.29	0.03	0.37
<i>Max</i>	0.46	0.43	0.25	0.46
<i>Stdev</i>	0.07	0.06	0.06	0.05
<i>Markowitz Allocations</i>				
<i>Min</i>	0.17	0.16	0.10	0.22
<i>Average</i>	0.28	0.26	0.12	0.33
<i>Max</i>	0.37	0.35	0.25	0.38
<i>Stdev</i>	0.04	0.03	0.04	0.03
<i>SETL(0.01) Allocations</i>				
<i>Min</i>	0.13	0.12	0.00	0.19
<i>Average</i>	0.31	0.28	0.04	0.37
<i>Max</i>	0.46	0.42	0.25	0.46
<i>Stdev</i>	0.07	0.05	0.06	0.05

Given these findings our last step, which is a popular academic and investment industry practice, is to compute and compare the Efficient Frontiers. Assuming a historical mean vector of

$(IMP, INV, NAS, PIC) = (0.00246, 0.00149, 0.0003, 0.00118)$, a zero risk-free rate and constant covariance and dispersion matrices, Fig. 6.3 below shows what we expected, that the results should conform with those reported in the previous paragraphs. Given that the risk values computed i.e. SETL vs. Markowitz standard deviation are not of the same scale, we decided to standardize them to make a fair comparison. This is why in Table 6.4 there are negative risk values. Most particularly the table shows that for the less risky portfolios, i.e. those with risk below average, SETL(0.05) portfolios provide an extra 3 basis points over the Markowitz model for a commensurate level of risk. This number decreases as the risk increases but this is no concern for risk-averse investors. Regarding weight allocation we still observe a significant difference between the two models and most notably that the SETL model generally under-weights the more risky asset, NAS, by at least 4%. The results of the restricted case are similar. Our results confirm those reported in [W11].

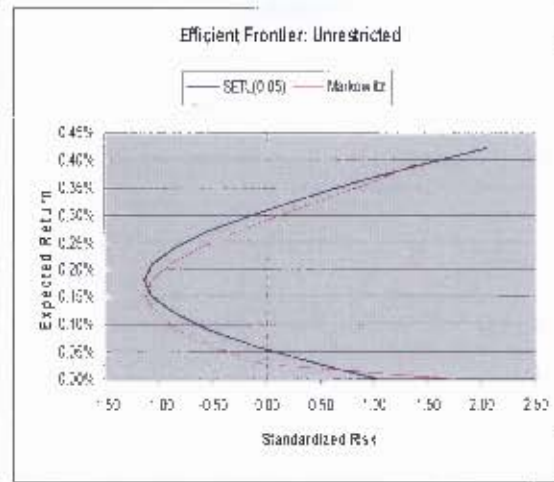


Figure 6.3: SETI vs. Markowitz efficient frontier

Table 6.4: Relative Returns

Standardized Risk	Relative Return
-1.13	0.03%
-0.85	0.03%
-0.54	0.03%
-0.52	0.03%
-0.13	0.03%
-0.09	0.03%

6.2.3 Performance Measures

Since the formulation of portfolio selection framework, [J1], under the assumption of “Gaussianity”, alternative techniques to the problem have evolved including ratios to assess the performance of a portfolio. In this regard we speak of the Sharpe ratio which depends on the standard deviation as a risk measure. Criticism evolved based on the argument that the standard deviation, as it is symmetric, penalises desirable upside returns. In this context the Mini-Max [J2], mean absolute deviation (MAD) and the VaR ratios surfaced, following the pioneering work [J8], that of [W1] on coherency, the standard deviation and VaR measures came under criticism and the CVaR ratio or STARR was invented as a measure that captures the fat-tail and asymmetry nature of

returns while possessing the desirable coherency properties. We are also aware of the Rachev generalised ratio. [W11] shows the performance of the latter two ratios compared to the [J1] formulation. [J27] articulates on the portfolio selection problem giving its “more realistic reformulation” under stable innovations and [W11] argues that the stable version outperforms the Markowitz Model.

6.3 Derivative Pricing

6.3.1 Stable Pricing Models

This section is intended to give some of the results presented thus far regarding option pricing since the surfacing of evidence of (1) stable distributions as a better choice in modelling stock returns [J8] and; (2) stylized facts is such return series. The popular Black-Scholes option pricing model, with an unrealistic assumption of constant volatility resulting to the “smile” phenomenon, has been found to misprice options [J32]. In an attempt to incorporate the clustering of volatility [J34] proposes a generalized autoregressive conditional heteroskedastic model with Gaussian innovations as a model for returns but fails to address the question of heavy tails by using such innovations.

In [J32] an improvement of the model [J34] is achieved by replacing the Gaussian innovations with symmetry stable innovations, thus achieving a GARCH-stable model that captures both volatility clustering and leptokurtosis. In a comparison of these models relative to the Black-Scholes one it is argued that the GARCH-stable model is “better suited” for the purpose of pricing derivatives in the context of a locally risk-neutral valuation relationship. A GARCH option pricing model with (1) the power to explain such anomalies as time varying volatility and leptokurtosis, (2) a finite second moment, unlike the general stable distribution and (3) that allows for any non-Gaussian innovations is introduced in [W12]. This model is termed a smoothly truncated stable distribution whose centre is stable and has Gaussian tails not necessarily described by the same parameters. The model is shown to outperform the Black-Scholes model. [J35] recommends the use of the stable option pricing model as the authors show its superiority

over the classical Black-Scholes model not only on pricing but also on reduction of hedging costs.

6.4 Extreme Value Theory

Extreme Value Theory is “most naturally developed as a theory of large losses, rather than small profits”, [W9]. We shall also adopt the notion that losses are positive and gains are negative simply because the Pareto distribution is defined on R^+ . In the area of risk management, the risk manager is concerned with modelling large operational, credit or market losses occurring at the tail of the supposed distribution. Because of the heavy tail nature of the GPD, it has proven a successful model for such random variables over the normal distribution.

6.4.1 Properties of GPD

A. Suppose that the random variable X denotes the losses, daily, weekly etc, incurred in a portfolio. The GPD of X is a two parameter model given by the distribution function:

$$G(X|\xi, \eta) = \begin{cases} 1 - \left(1 + \frac{\xi x}{\eta}\right)^{-1/\xi} & \xi \neq 0 \\ 1 - \exp\left(-\frac{x}{\eta}\right) & \xi = 0 \end{cases} \quad (6.2)$$

where $\eta > 0$, $\xi \leq 0$. ξ is a shape parameter and η is a scale parameter. $\xi < 0$, $\xi = 0$ and $\xi > 0$ gives a type II Pareto, an Exponential and Pareto distribution respectively. We consider the case $\xi > 0$, where the GPD is heavy tailed such that for $k \geq \frac{1}{\xi}$, $E[X^k]$ is infinite.

B. If $0 < \xi < 1$ then the mean of a GPD is given by

$$E[X] = \frac{\eta}{1-\xi} \quad (6.3)$$

C. If X has distribution function F_X then the distribution of excess random variables $Y = X - u$ given some threshold value u is given by

$$F_u(y) = P(X - u \leq y | X > u) = \frac{F_X(y + u) - F_X(u)}{1 - F_X(u)} \quad (6.4)$$

If X follows a GPD then $F_u(y) = G_{\xi, \eta(u)}$ where $\eta(u) = \eta + \xi u$. This means that the excess random variable follows a GPD with the same shift parameter as X but a different scale parameter $\eta(u) = \eta + \xi u$

D. Suppose X is a random variable that follows a GPD and that u is some threshold value, the expected value, *mean excess function*, of the excess random variable, $X - u$ is given by

$$E[X - u | X > u] = \frac{\eta(u)}{1 - \xi} = \frac{\eta + \xi u}{1 - \xi} \quad (6.5)$$

E. Property C shows that for $X > u$ it follows that

$$F_X(x) = (1 - F_X(u))G_{\xi, \eta}(x - u) + F_X(u) \quad (6.6)$$

F. Lemma 7.22 of [B4] asserts, given a new threshold; $v \geq u$, that the new *mean excess function* is given by

$$E[X - v | X > u] = \frac{\xi v}{1 - \xi} + \frac{\eta - \xi u}{1 - \xi} \quad (6.6)$$

with sample mean estimate given by:

$$\overline{X}_v = \frac{\sum_{i=1}^n (X_i - v) I_{\{X_i > v\}}}{\sum_{i=1}^n I_{\{X_i > v\}}} \quad (6.7)$$

where I denotes the indicator function.

G. From (6.4) and (6.6) [B4] argues that the mean excess function “should become increasingly linear for higher values” of u and v respectively. A plot of the mean excess function vs. the i^{th} order statistic of the observed data may reveal three scenarios; firstly a

linear upward trend indicates a GPD with $\xi > 0$; secondly, a horizontal trend indicates a GPD with $\xi = 0$; thirdly, a linear downward trend indicates a GPD with $\xi < 0$. As the threshold is increased this plot may become very volatile, hence [B4] recommends not to compute this function in such cases. A clear observation of this plot may indicate the “exact” value to be selected as threshold. In an example given in [B4], 7.23, the threshold is taken to be the value 10 where there is a “kink” or change in slope of the mean excess function.

6.4.2 Estimation of Tail Probability, VaR and ES

6.4.2.1 Tail Probability

Given that equation (6.6) is valid for $X > u$ we need to estimate $F_X(u)$ to be able to estimate tail probabilities. The usual choice as argued in [W9] is the historical estimate HS given by

$$HS = 1 - \frac{N_u}{n} \quad (6.8)$$

where n is the total number of data points and N_u is the number of points that exceed u , the threshold. Thus beyond u the tail probability is estimated as

$$\hat{F}_X(x) = 1 - \frac{N_u}{n} \left(1 + \hat{\xi} \frac{x - u}{\hat{\eta}} \right)^{-1/\hat{\xi}} \quad (6.9)$$

where the “hat” denotes estimate and the parameters are estimated according to (6.3). [W9] argues that the HS estimate becomes “too unreliable” for higher values of the threshold.

6.4.2.2 Value-at-Risk (VaR)

Given some threshold u , $VaR_q(X|q > F_X(u))$ is the q^{th} quantile estimate of the loss distribution obtained by inverting the (6.9) and is given by

$$VaR_q(X|q > F_X(u)) = u + \frac{\hat{\eta}}{\hat{\xi}} \left(\left(\frac{u}{N_u} (1-q) \right)^{-\hat{\xi}} - 1 \right) \quad (6.10)$$

6.4.2.3 Expected Shortfall (ES)

Given some threshold value, u , the $ES_q(X|q > F_X(u))$ is defined as the expected loss beyond VaR and these are related by:

$$ES_q(X|q > F_X(u)) = VaR_q(X) + E[X - VaR_q | X > VaR_q] \quad (6.11)$$

With a sample estimate of

$$E\hat{S}_q = \frac{Va\hat{R}_q}{1 - \hat{\xi}} + \frac{\hat{\eta} - \hat{\xi}u}{1 - \hat{\xi}} \quad (6.12)$$

6.5 Discussion

Value-at-Risk under the assumptions of ‘‘Gaussianity’’ remains a popular measure for financial and banking institutions as outlined in the Basel II risk capital framework. On the calculation of such a measure and its coherent counterpart, Conditional Value-at-Risk our results confirm the superiority of the stable model to the Gaussian one. We have also reflected on the results found by other authors with respect to the portfolio optimization problem and the pricing of derivatives. Our results confirm the superiority of the stable model to the Gaussian one. Based on our own findings we, too, echo that the Gaussian model should be discarded and the stable one be considered in financial engineering.

Chapter 7. Discussion

7.1 Summary and Critical Assessment

In this undertaking we have found

- Empirical evidence that financial asset returns can be modelled better using stable distributions than the popular Gaussian one, thus confirming the original work by [J8].
- That the proposed alternative model for multivariate stable distributions provides a far reasonable fit to bivariate asset return distribution than a Gaussian counterpart. We further, intuitively, suggest that this model, as it does not require estimation of a joint index of stability, may be superior to both the spectral measure and sub-Gaussian models. However this remains to be tested.
- In light of bullet point one, that stable risk measures are more informative and close approximations to empirical ones. Gaussian risk measures are overly optimistic.
- Using a dispersion matrix suggested by [W13], that such a dependence structure carries more information, evidence from Principal Component Analysis, and thus yields superior portfolios over its Markowitz counterpart.

Despite such impressive finds we note the following critical points

- That our findings are subject to some degree of error resulting from estimation of parameters of stable distributions.
- That our proposed model can be computationally expensive as it depends on estimation of single variable stable parameters.

- That scientifically accepted goodness-of-fit statistical tests, like Kolmogorov-Smirnov, were not used in testing our model. This solely because of computational expensiveness.
- That it remains a challenge to convince practitioners in the investment industry to reject the Markowitz model and accept the stable one.

7.2 Future Developments

Recognising that there is still a huge scope of research regarding stable distributions and their applications, regarding this research endeavour the following remain key future developments

- Using standard statistical goodness-of-fit tests to assess the appropriateness of the SSA model. Having done that one would be in position to test our intuitive claim that the model may be superior to the sub-Gaussian one both as a model for data and in the context of portfolio choice theory.
- Thus far we are not aware of any coherency issues raised regarding the dispersion parameter of an α -stable sub-Gaussian distribution. Thus what would be of particular interest would be to compare the asset allocation and portfolio performance as a result of this measure compared to our model.
- Given that the GARCH-stable [J32] model and STS-GARCH [W12] model provide better models of derivative prices it remains to be answered which of the two, when compared to each other, is superior.

Chapter 8. Conclusion

An investigation of a suitable model for the distributional behaviour of stock, index and exchange rate logarithmic returns revealed in Chapter 4 that the stable model is a far better reasonable choice over the sentimental Gaussian one. This evidence is undisputable at higher frequencies. In light of such findings we echo the call to reject the Gaussian distribution as a model for such data. Despite the lack of use of traditional formal statistical tests we have shown using bivariate density, contour and cross-sectional density plots that the SSA technique fits data far reasonable than its Gaussian counterpart. By comparing the two models in relation to the Gaussian Kernel there is strong evidence that our proposed model captures kurtosis as desired. The SSA model provides a reasonable alternative to existing methodology on the theory and applications of general multivariate stable distributions with, intuitively, desirable advantages. The results provided in this endeavour not only achieved an alternative model to for multivariate stable distributions but also give an intuitive suggestion about the model's superiority to existing ones given the non requirement of a joint index of stability. Our results in Chapter 6 confirm that the Gaussian risk measures Value-at-Risk and Conditional Value-at-Risk are more optimistic and misleading while their stable counterparts are far reasonable. We have further shown that Markowitz portfolios are sub-optimal compared to their superior stable counterparts at commensurate levels of risk and that there is no significant difference in optimal portfolios achieved by using stable Expected Tail Loss or simply the stable dispersion matrix. We have also confirmed that the principal components of a stable dispersion matrix explain more variation than their covariance counterparts. That said, it is wise for financial institutions to consider market risk tools and portfolio optimization methods based on the stable model. Though we acknowledge that a lot still has to be done we do hope that our model will be warmly received by the research community.

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Appendix A: Description of Stock, Index and Exchange Rate Codes

Table A1: Explanation of prefixes

DXXX	Daily log price changes of variable XXX
WXXX	Weekly log price changes of variable XXX
MXXX	Monthly log price changes of variable XXX

Table A2: Variables used

CODE	FULL NAME (JSE)
Stocks	
ANA	AngloGold Ashanti
ANG	Anglo American
BAR	Barloworld
BID	Bidvest Group
DRD	DRD Gold
EXX	Exxaro Resources
FIR	FirstRand Group
FOS	Foschini
GOL	Goldfields
GRO	Group Five
HAR	Harmony Gold Mng
IML	Impala Platinum
IMP	Imperial
INV	Investec
LIB	Liberty Group
MTN	MTN Group
NAM	Nampak
NAS	Naspers
PIC	Pick n Pay Stores
RAI	Rainbow Chicken
RIC	Richmond Secs
RMB	Rand Merchant Bank
SAS	Sasol
STA	Standard Bank
TIG	Tiger Brands
Indices	
FIN	Financials
IND	Industrials
T40	Top 40
Exchange Rates	
USDZAR	US Dollar-SA Rand
UKPZAR	UK Pound-SA Rand

Appendix B: Results from Stable Modelling of log Returns

2.1 Stocks

2.1.1 Daily Returns

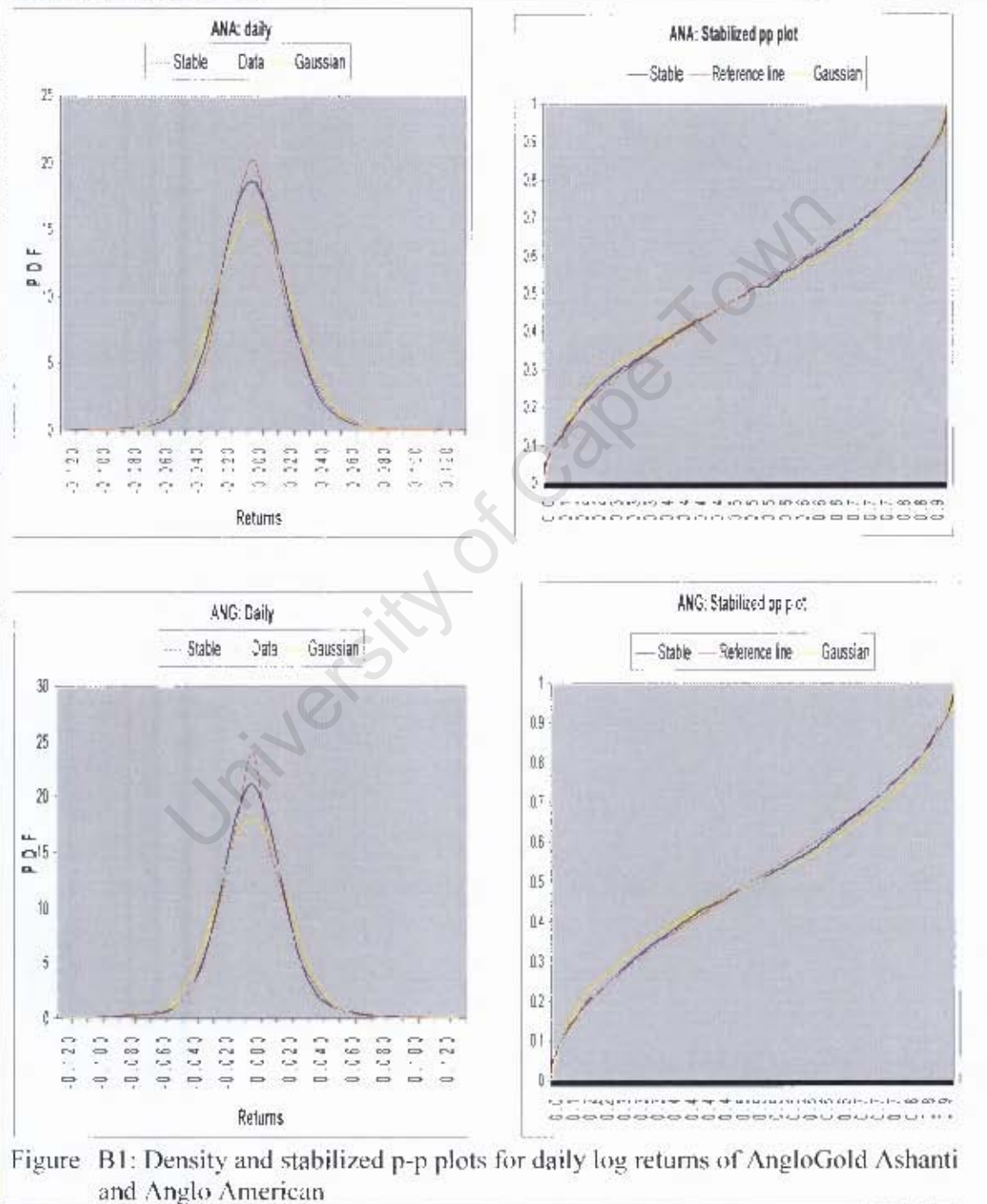
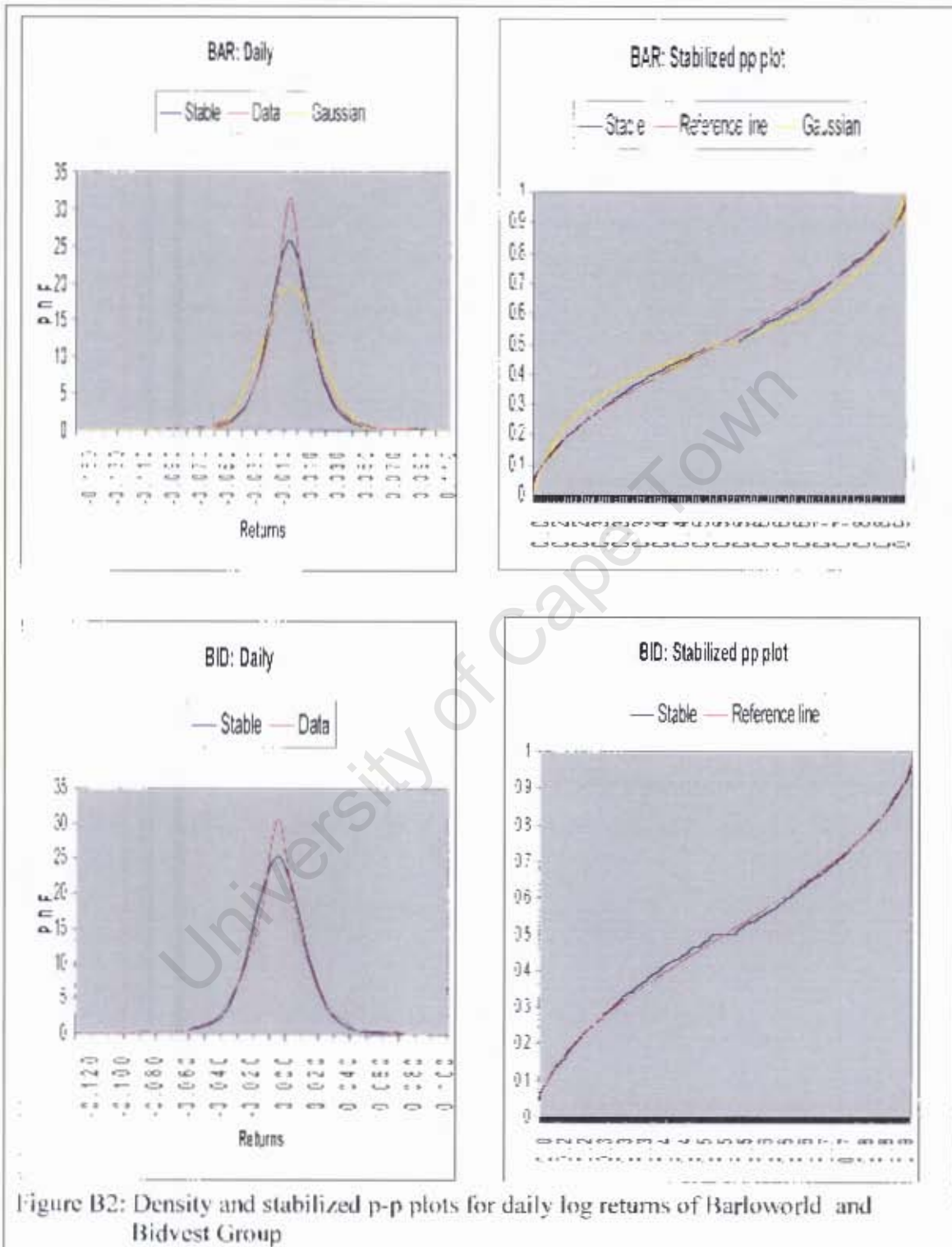


Figure B1: Density and stabilized p-p plots for daily log returns of AngloGold Ashanti and Anglo American



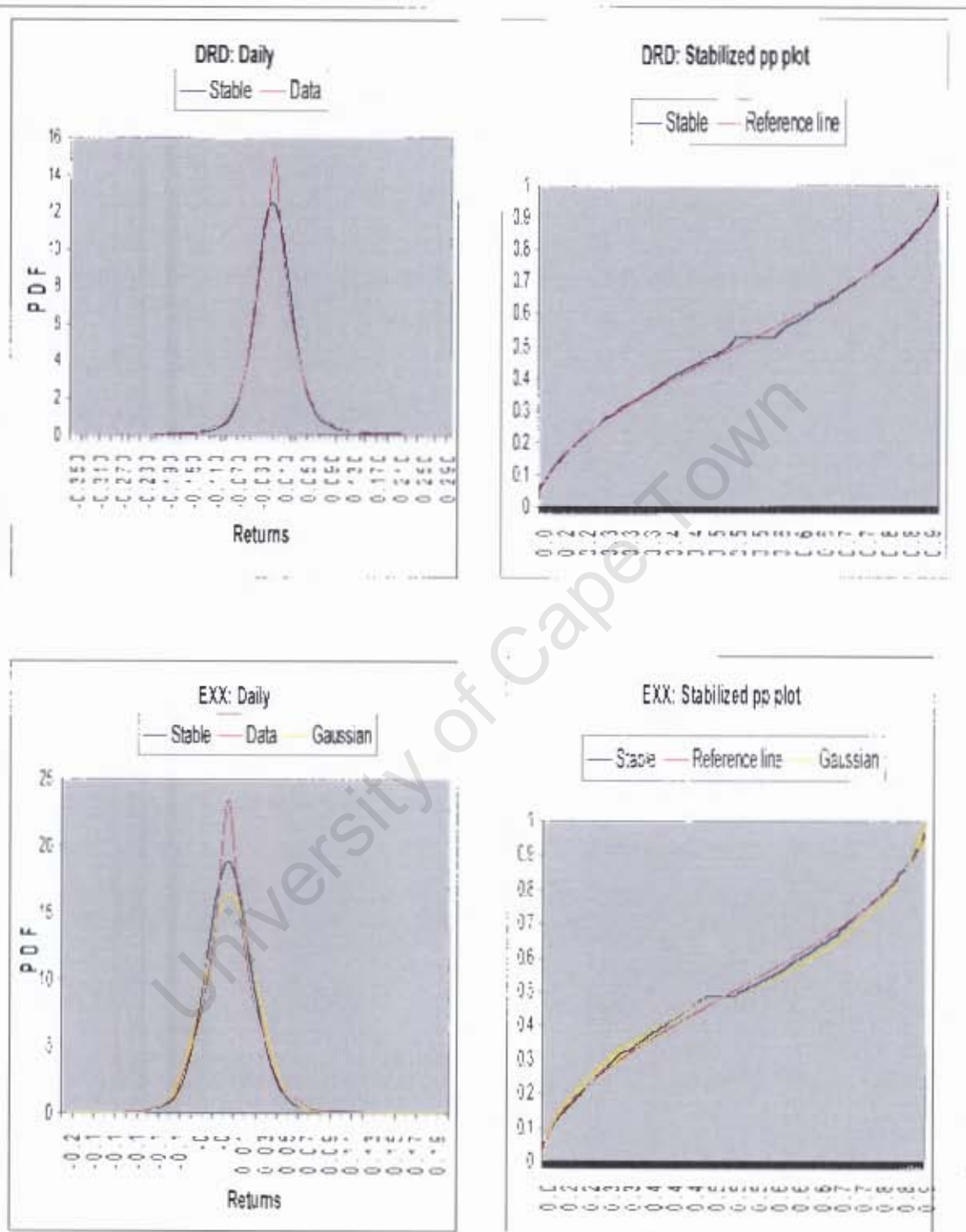


Figure B3: Density and stabilized p-p plot for daily log returns of DRD Gold and Exxaro Resources

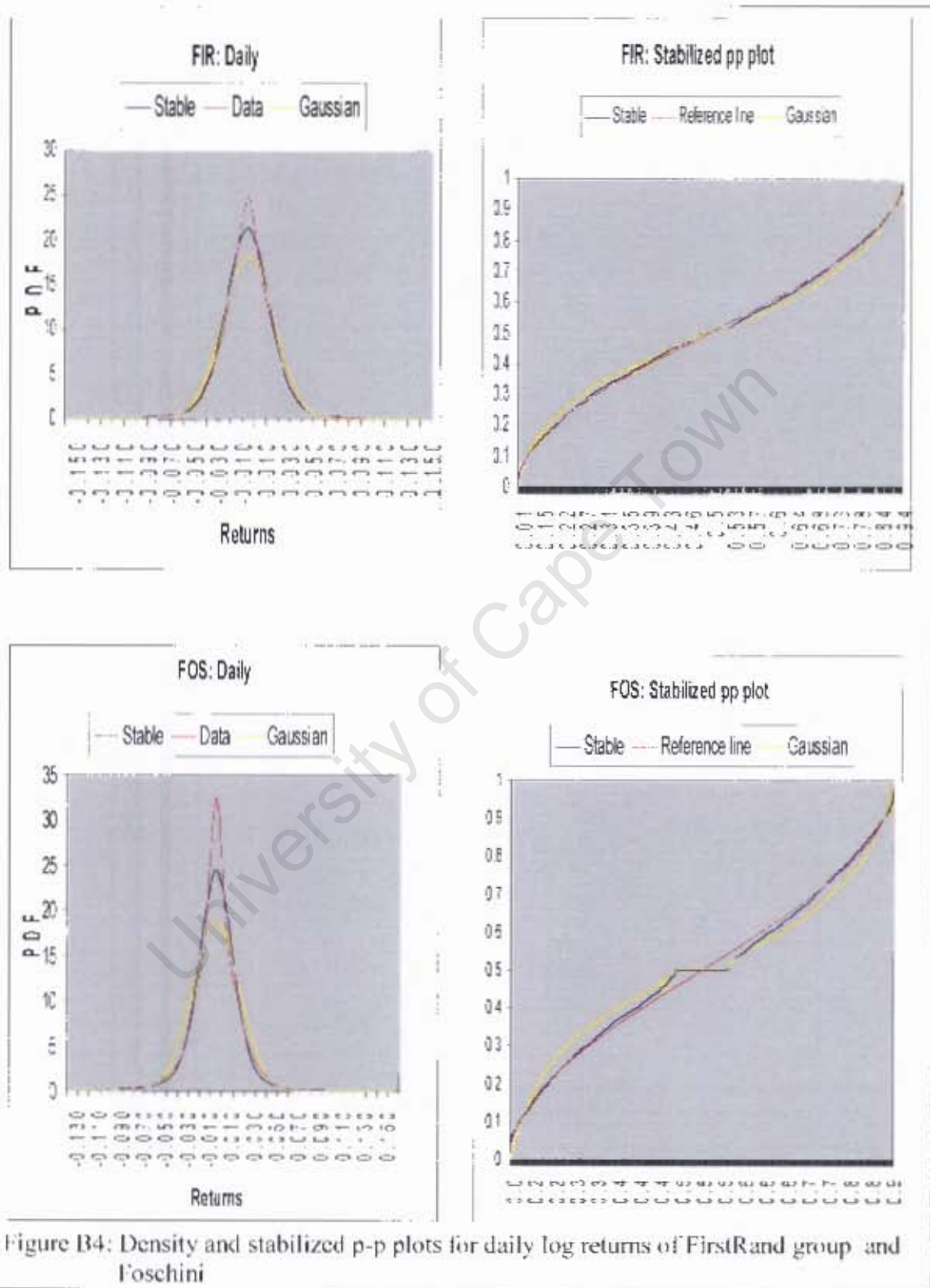


Figure B4: Density and stabilized p-p plots for daily log returns of FirstRand group and Foschini

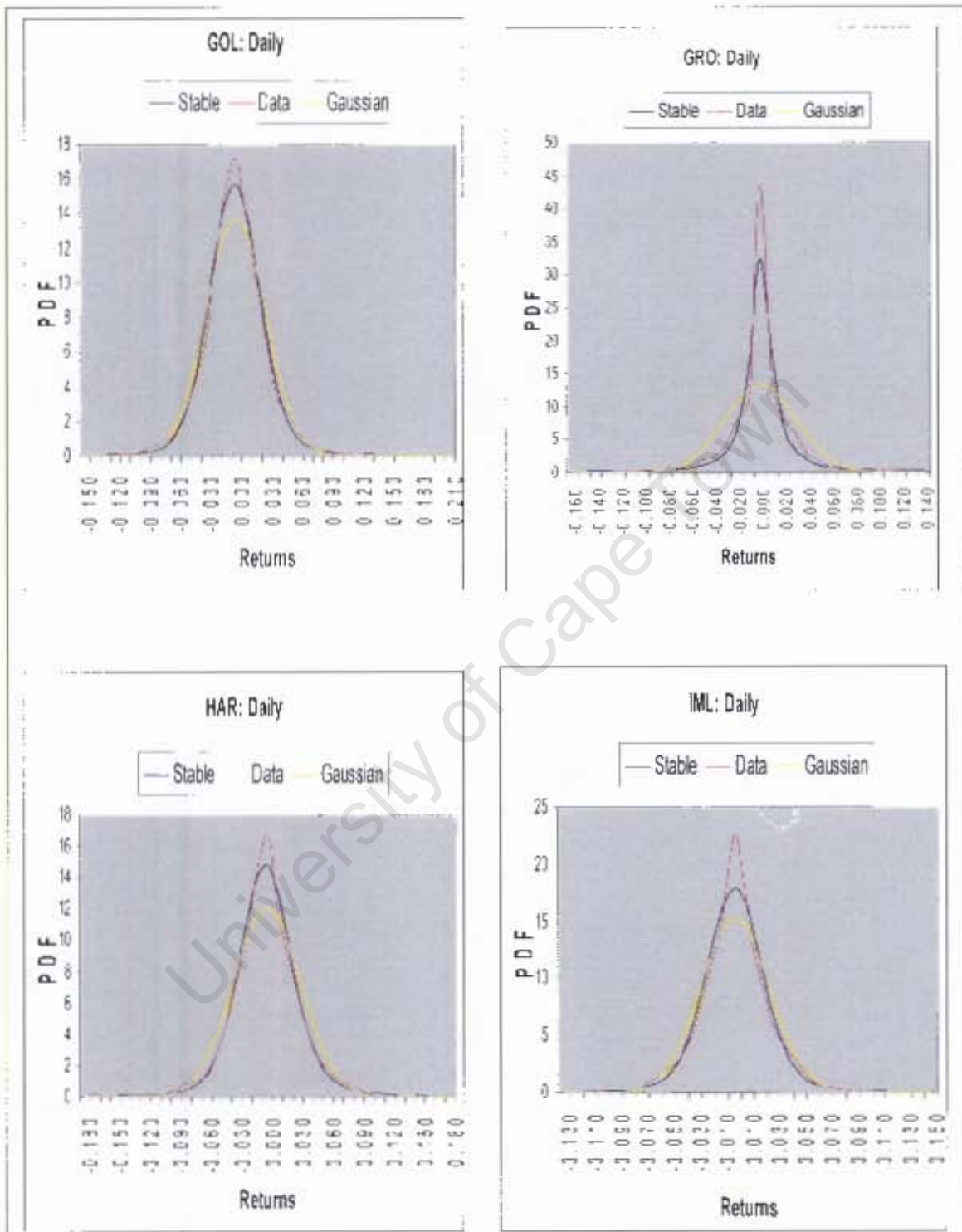
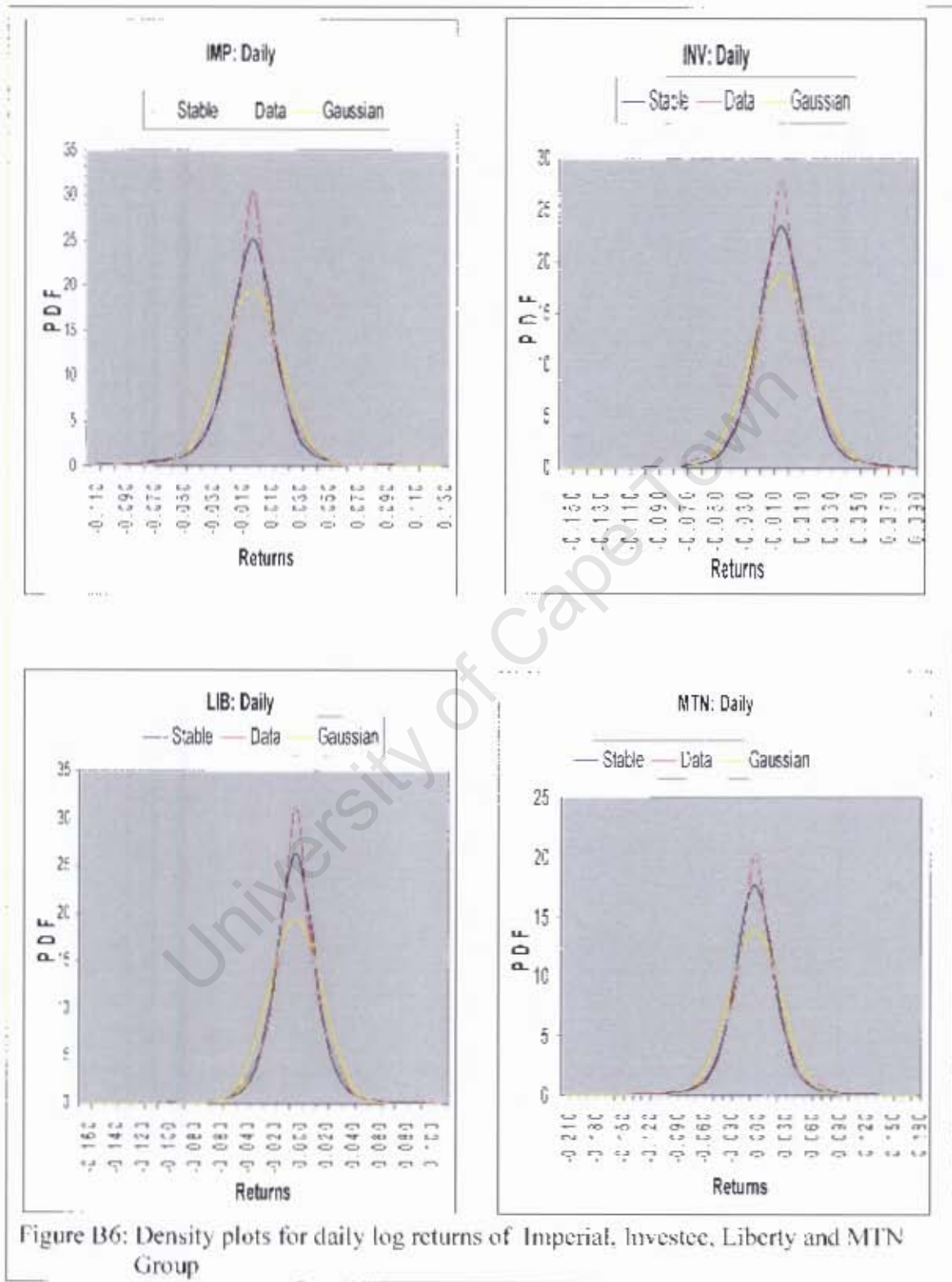
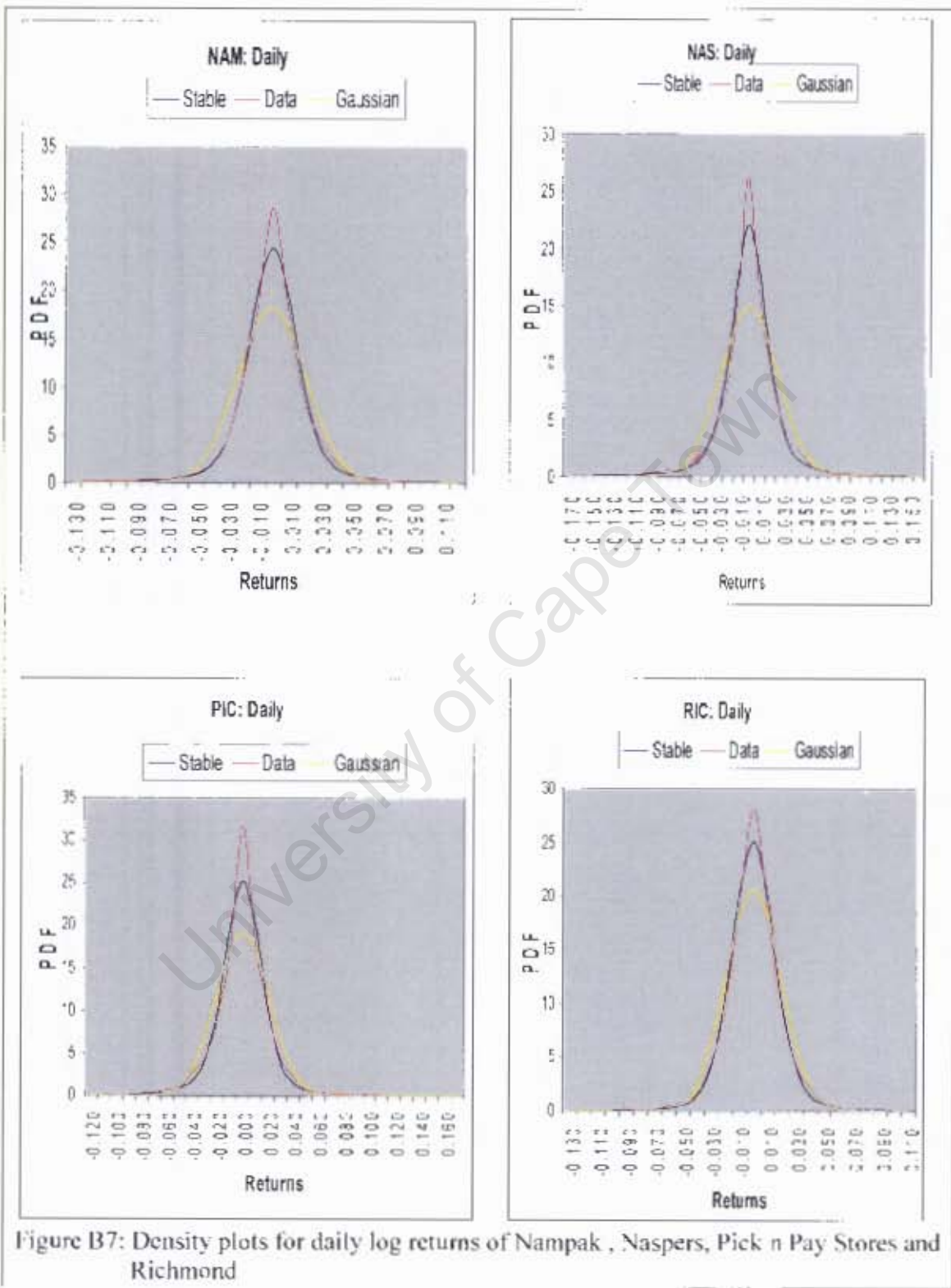


Figure B5: Density plots for daily log returns of Goldfields , Group Five, Harmony Gold and Impala Platinum





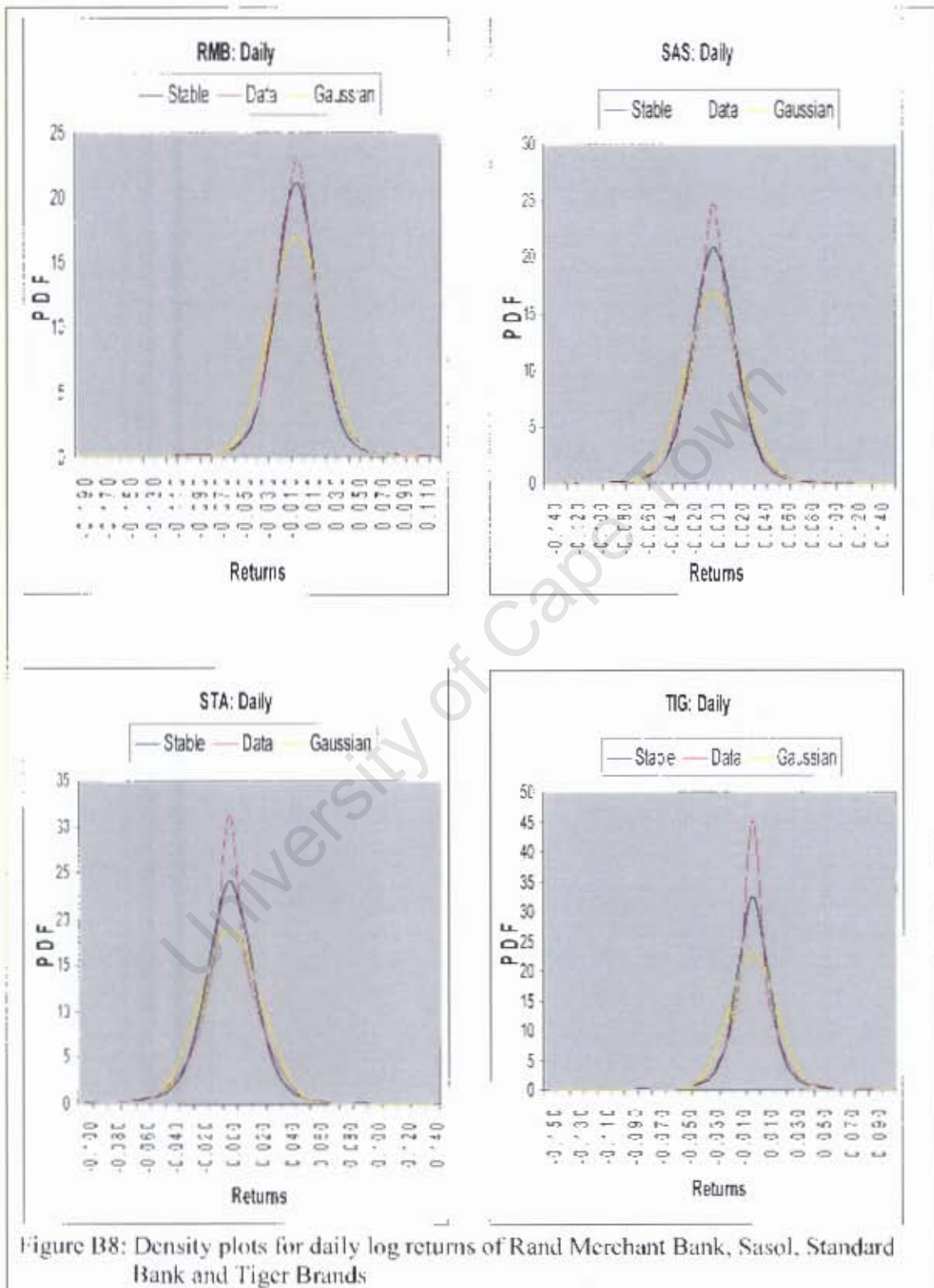
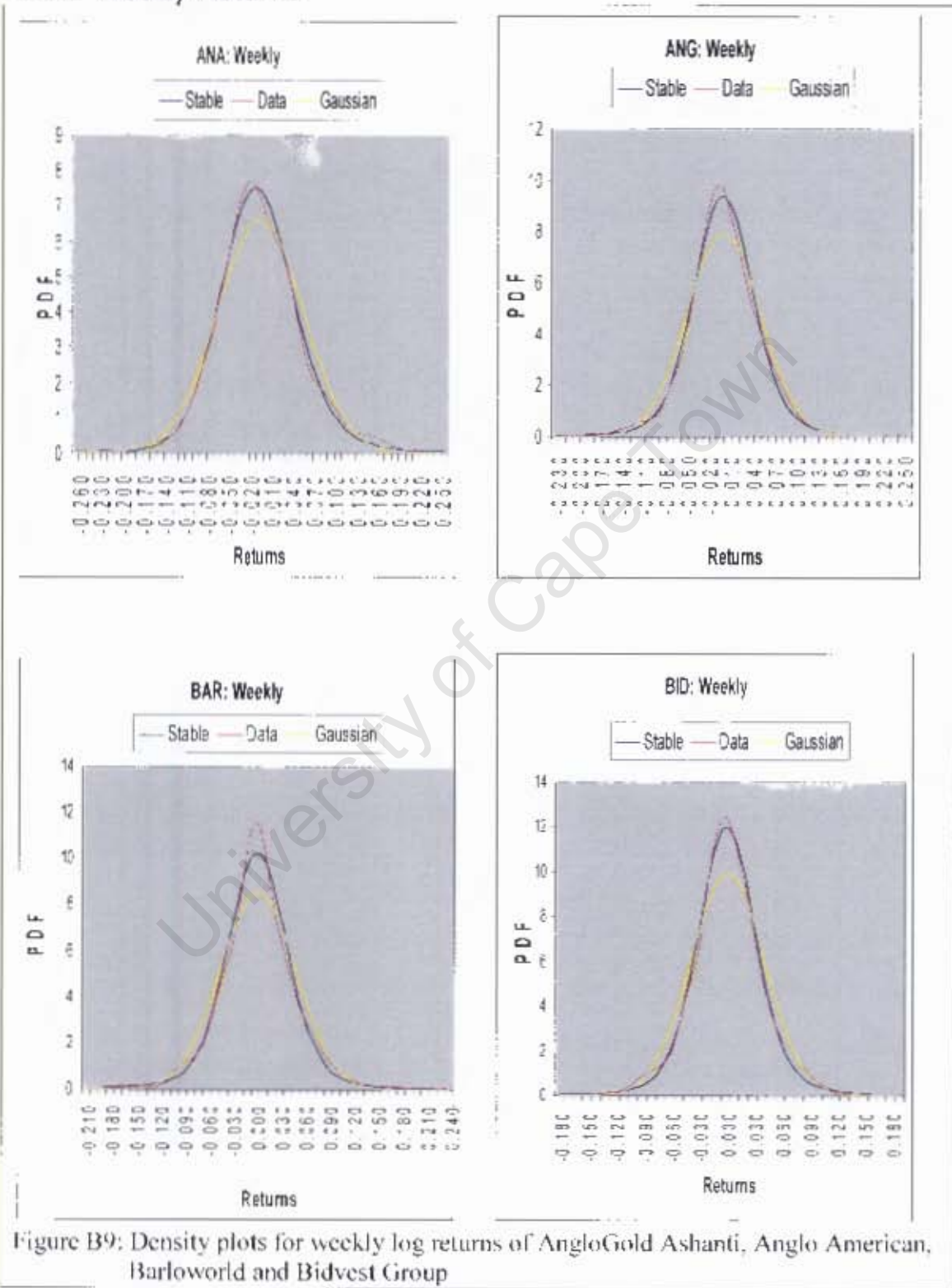
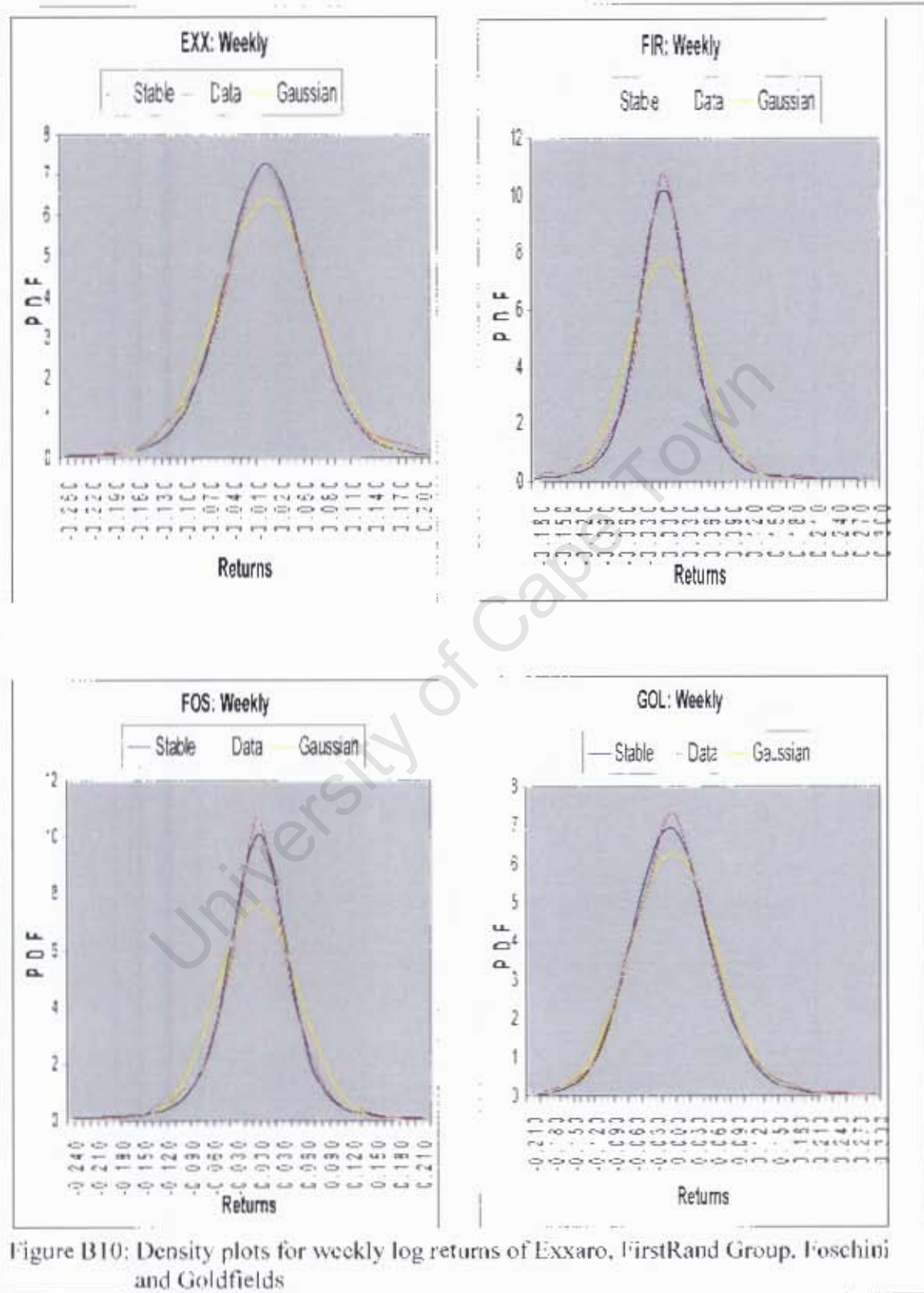


Figure B8: Density plots for daily log returns of Rand Merchant Bank, Sasol, Standard Bank and Tiger Brands

2.1.2 Weekly Returns





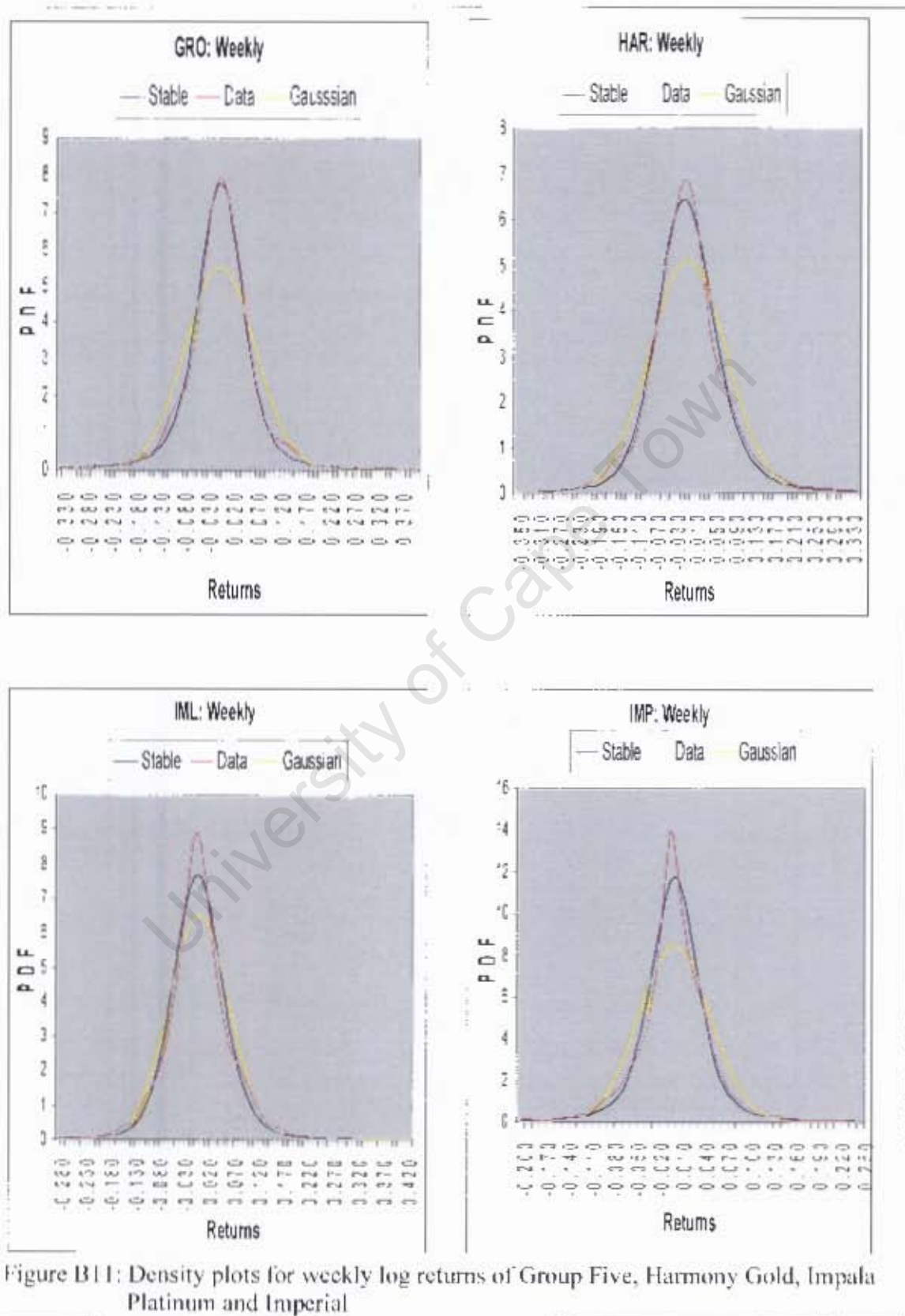
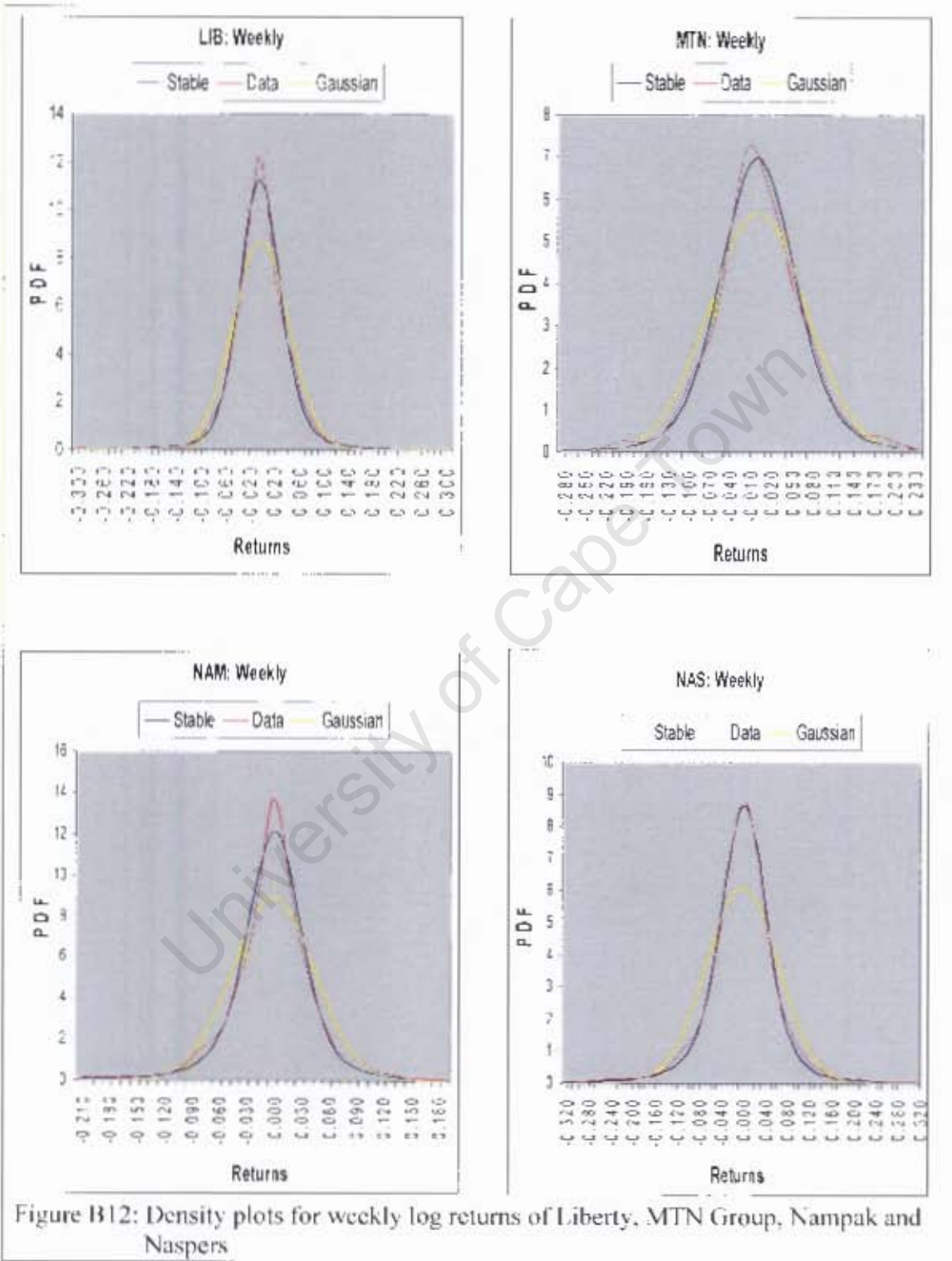


Figure B11: Density plots for weekly log returns of Group Five, Harmony Gold, Impala Platinum and Imperial



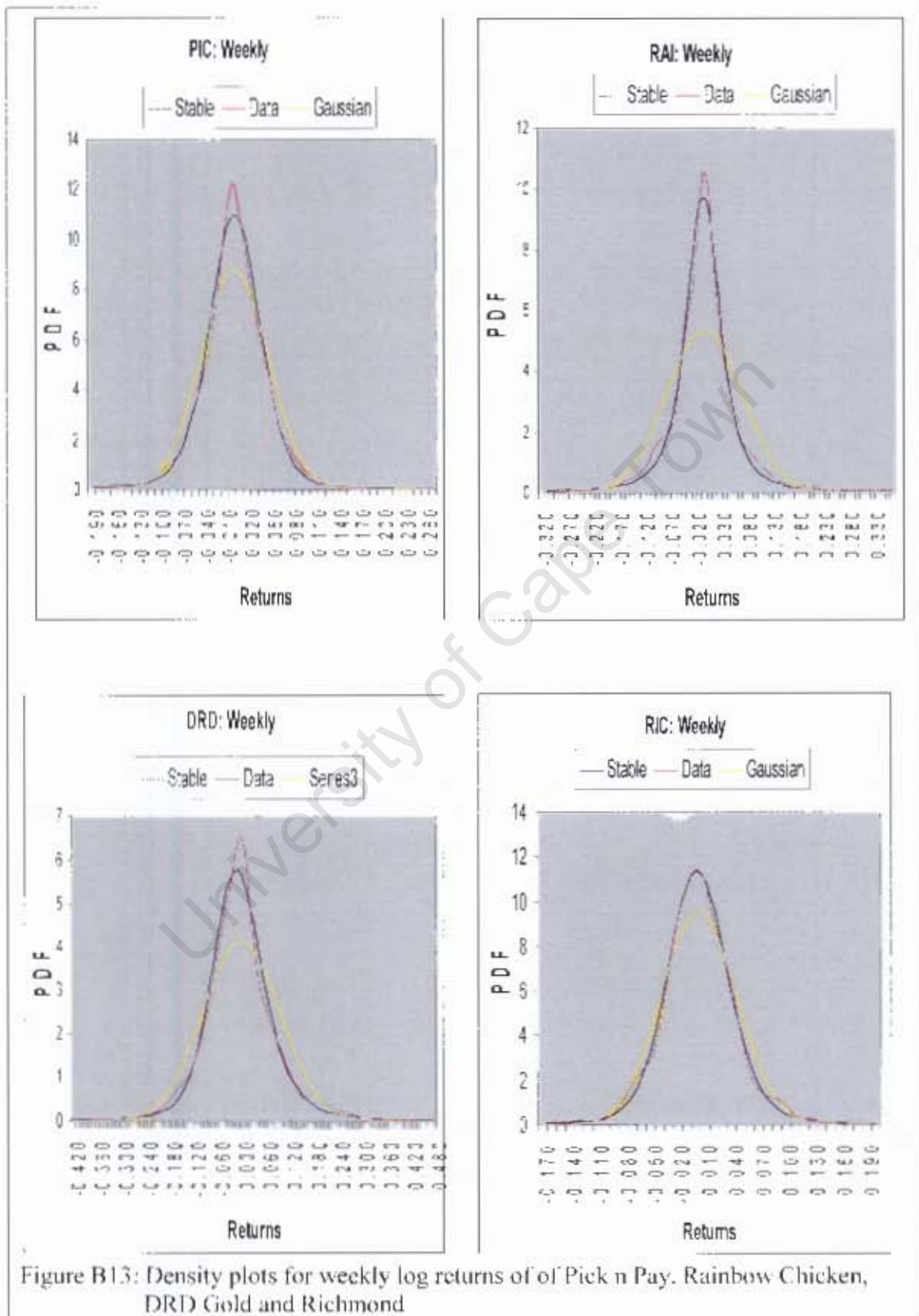


Figure B1.3: Density plots for weekly log returns of of Pick n Pay, Rainbow Chicken, DRI Gold and Richmond

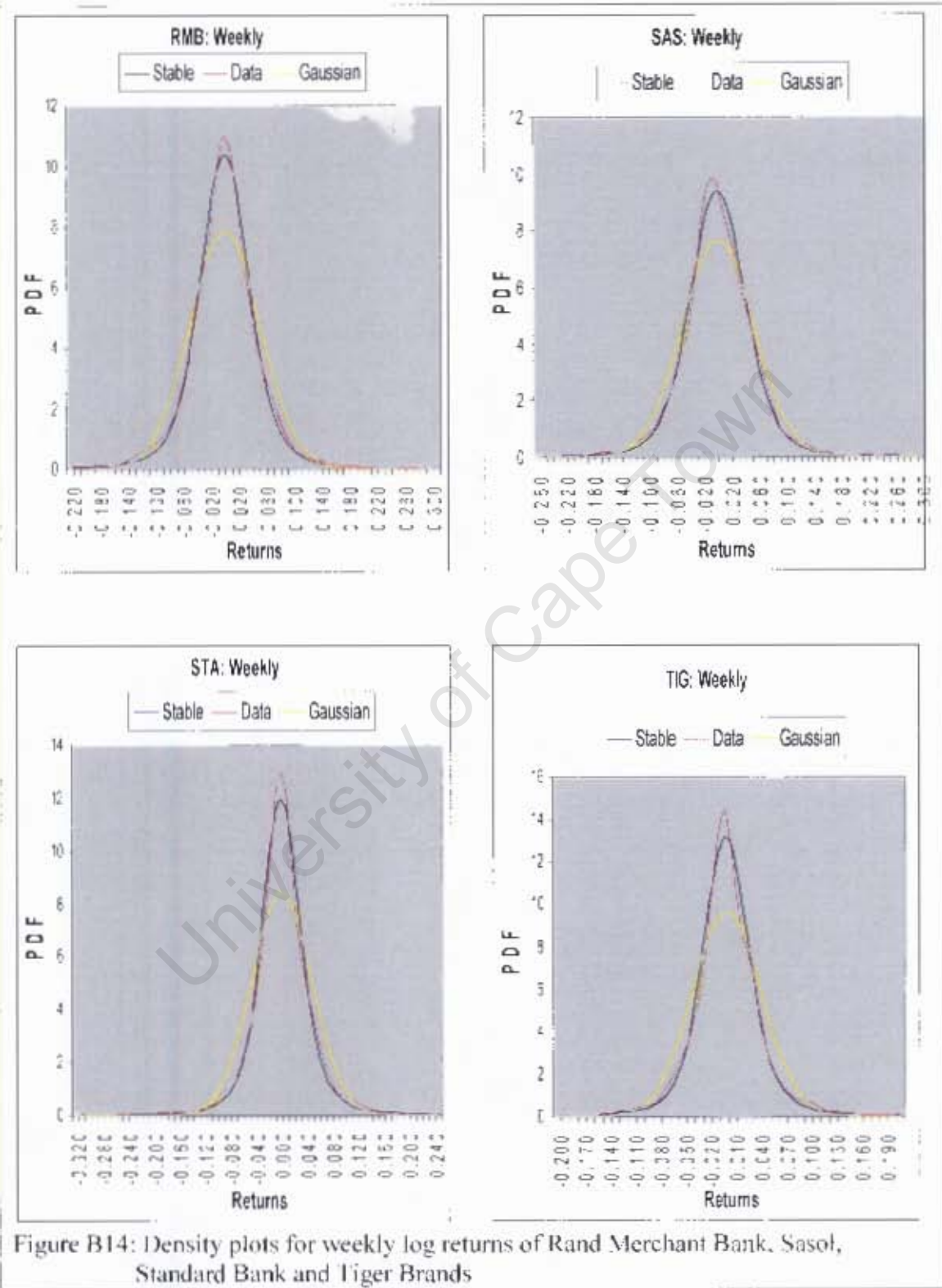


Figure B14: Density plots for weekly log returns of Rand Merchant Bank, Sasol, Standard Bank and Tiger Brands

2.1.3 Monthly Returns

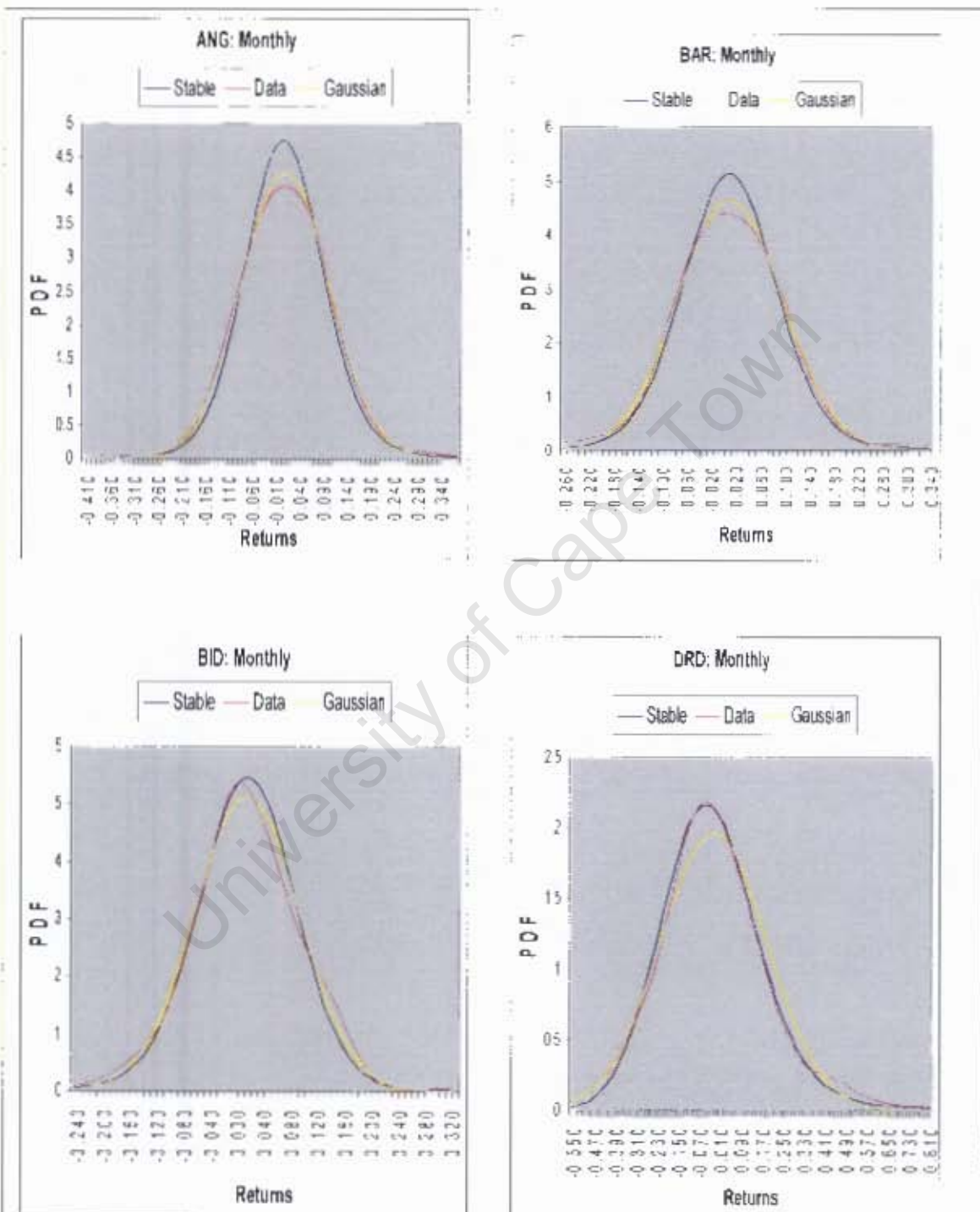
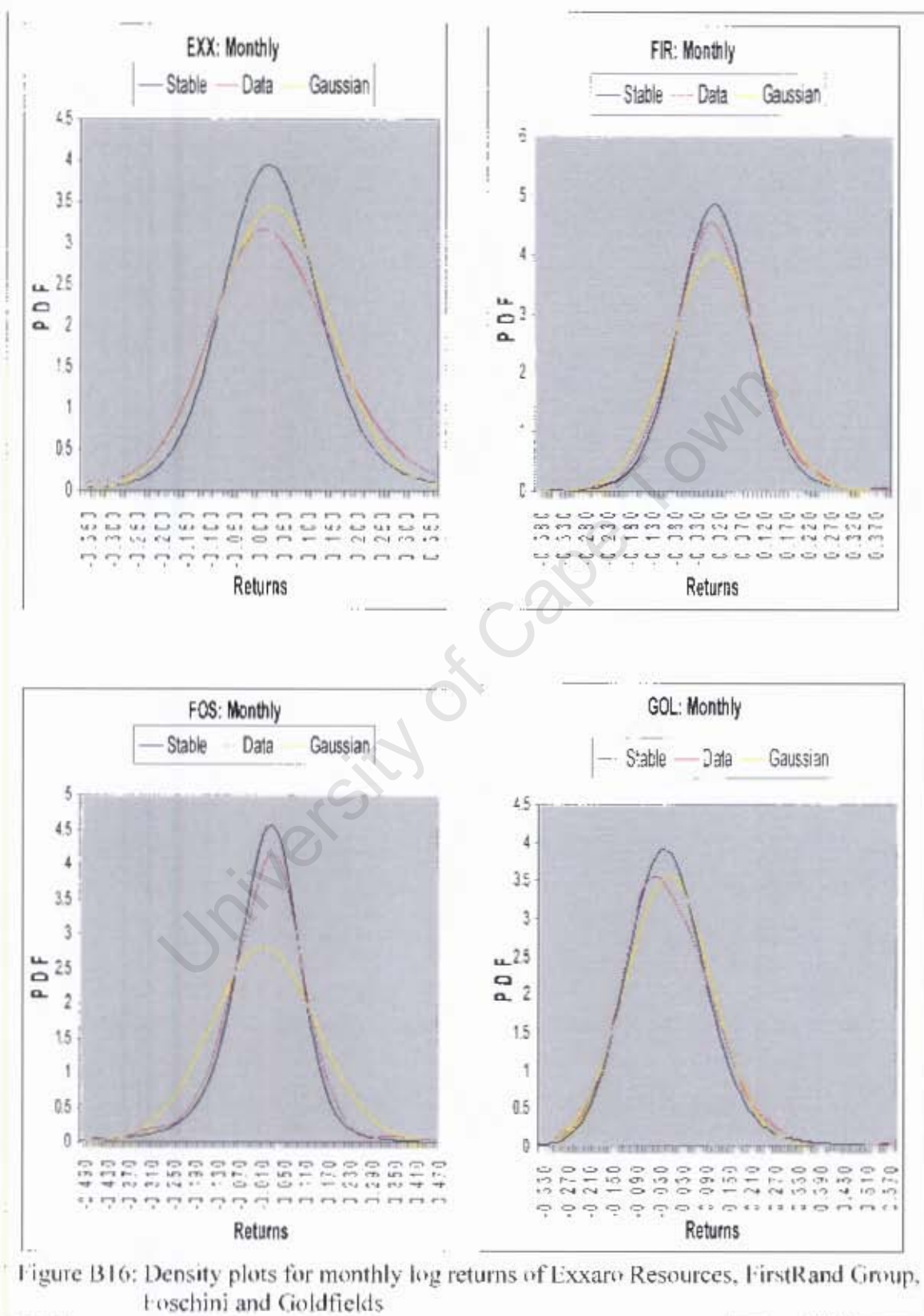


Figure B15: Density plots for monthly log returns of Anglo American, Barloworld, Bidvest Group and DRD Gold



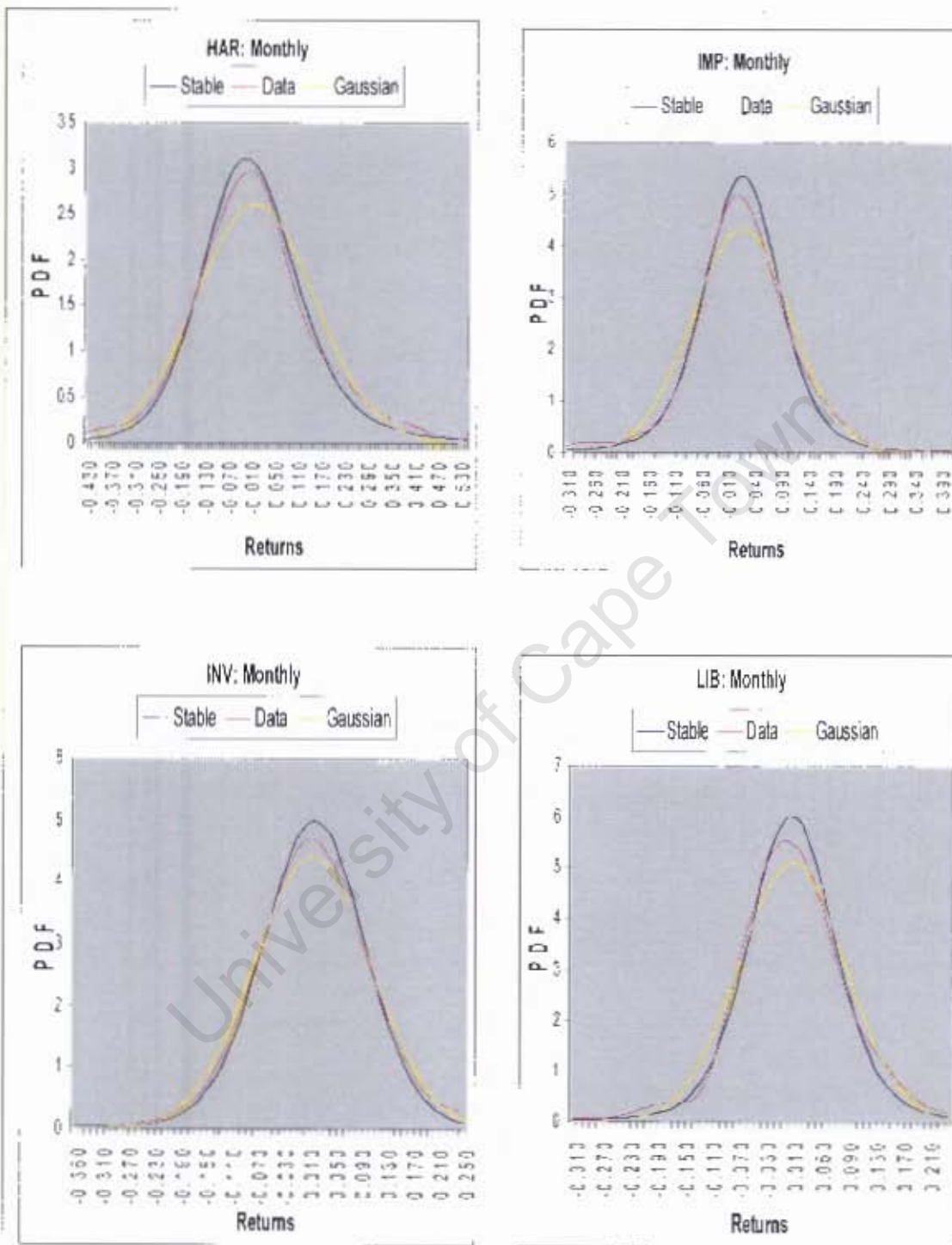


Figure B17: Density plots for monthly log returns of Harmony Gold Mng, Impala Platinum, Investec and Liberty Group

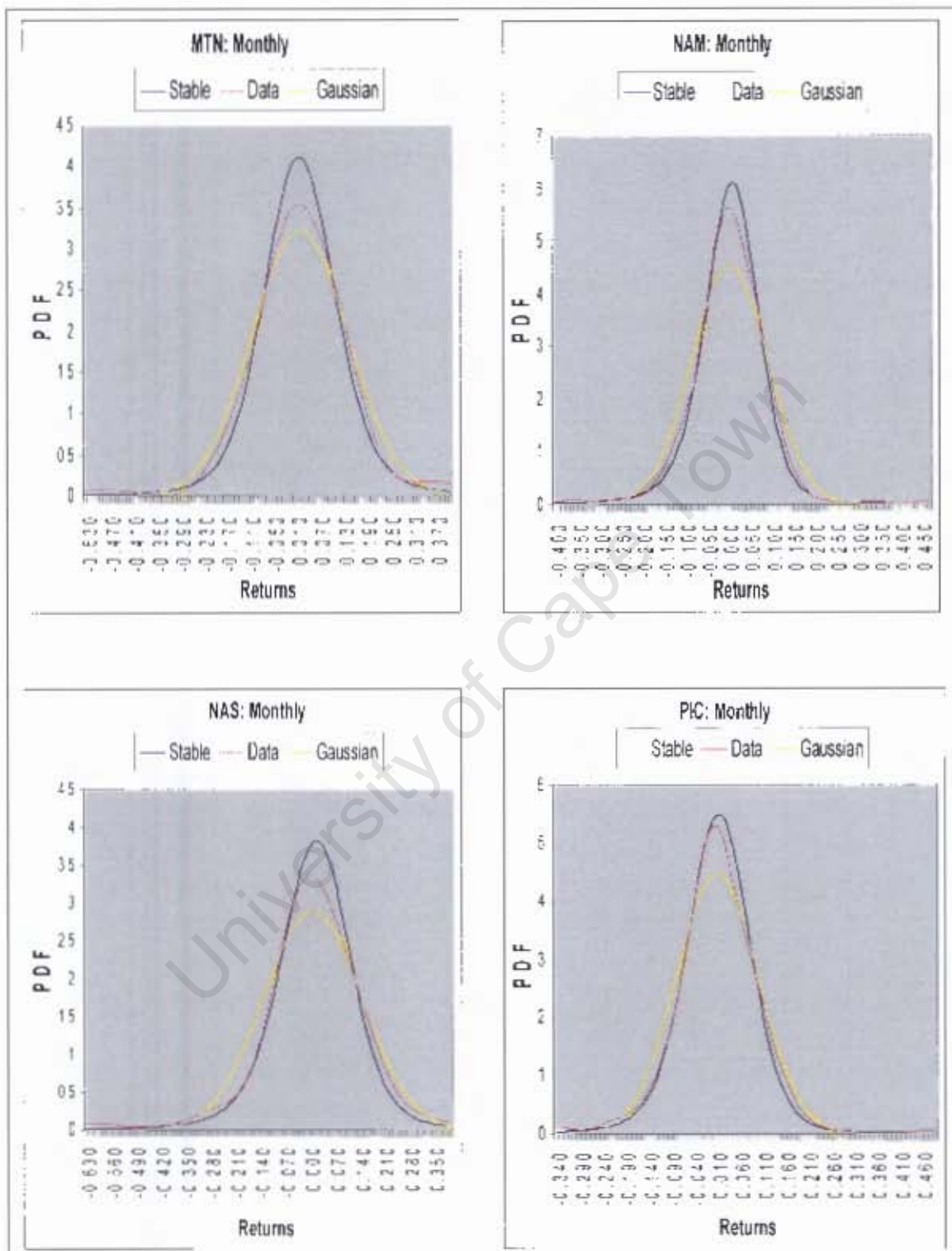


Figure B18: Density plots for monthly log returns of MTN Group, Nampak, Naspers and Pick n Pay

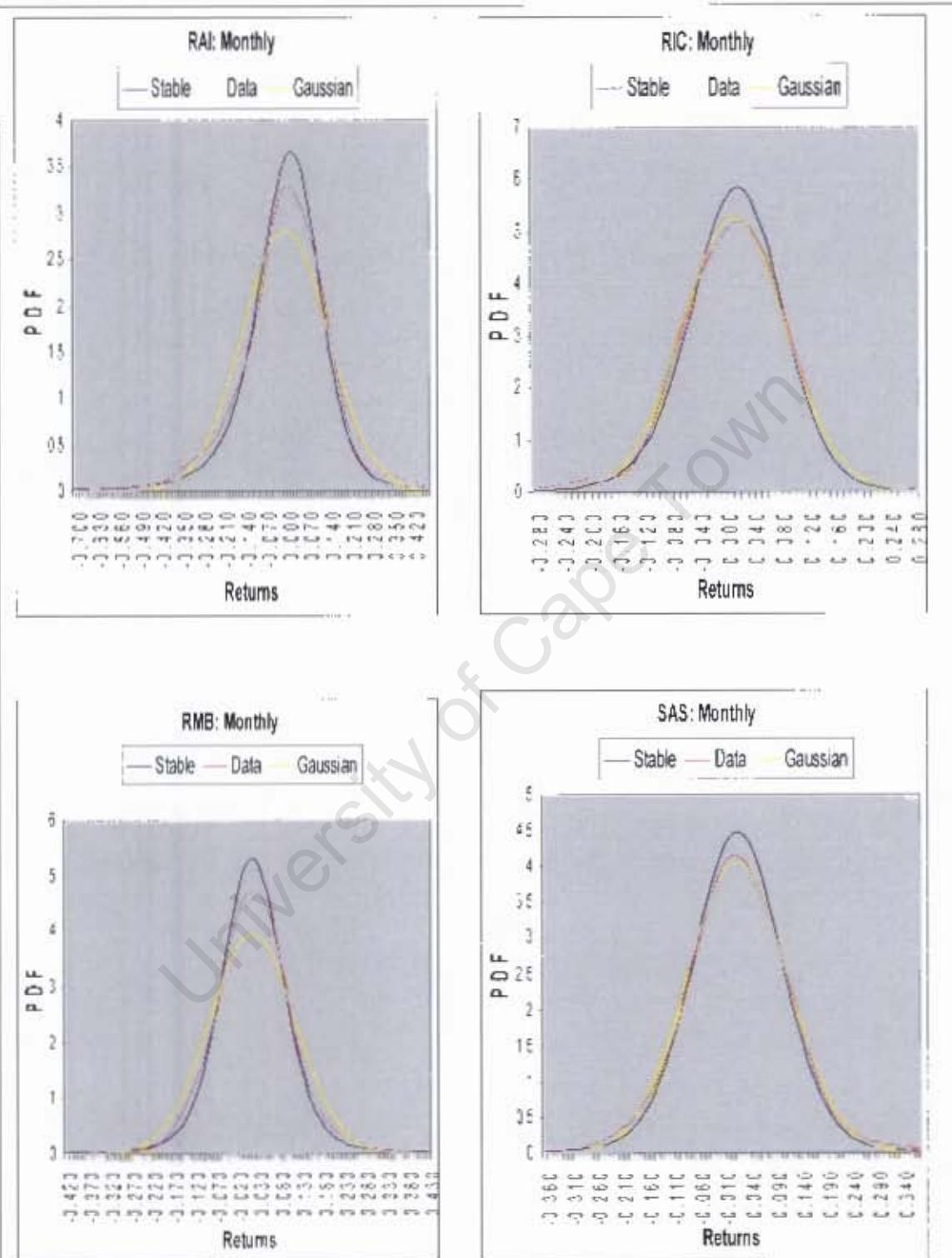


Figure B19: Density plots for monthly log returns of Rainbow Chicken, Richmond Secs, Rand Merchant Bank and Sasol

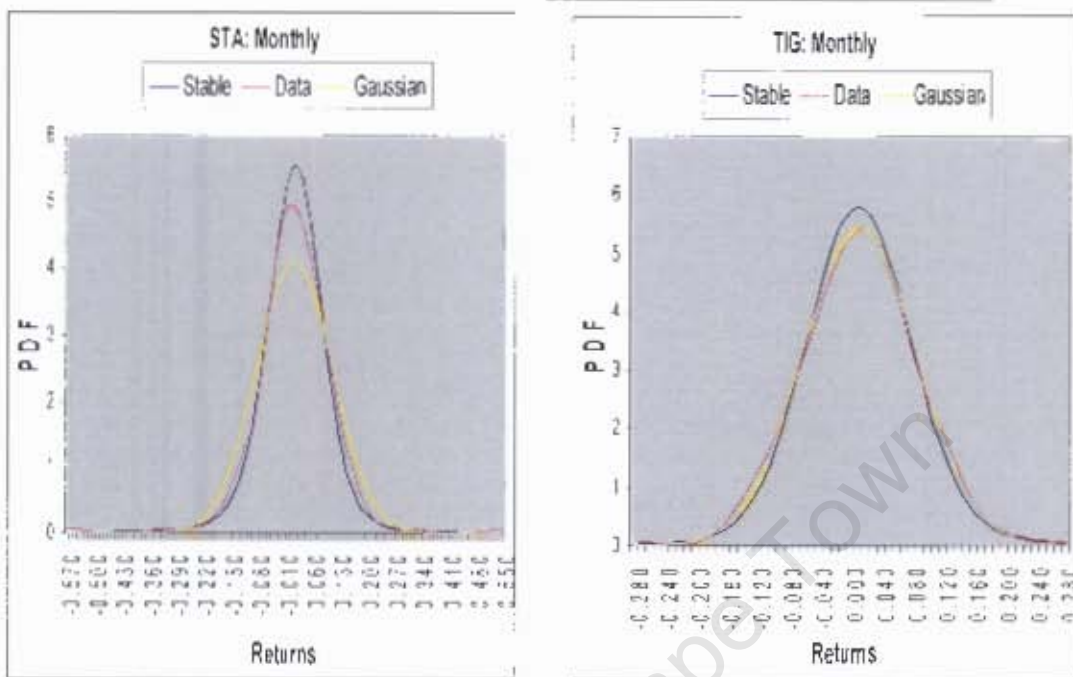


Figure B20: Density plots for monthly log returns of Standard Bank and Tiger Brands

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2.2 Indices

2.2.1 Daily Returns

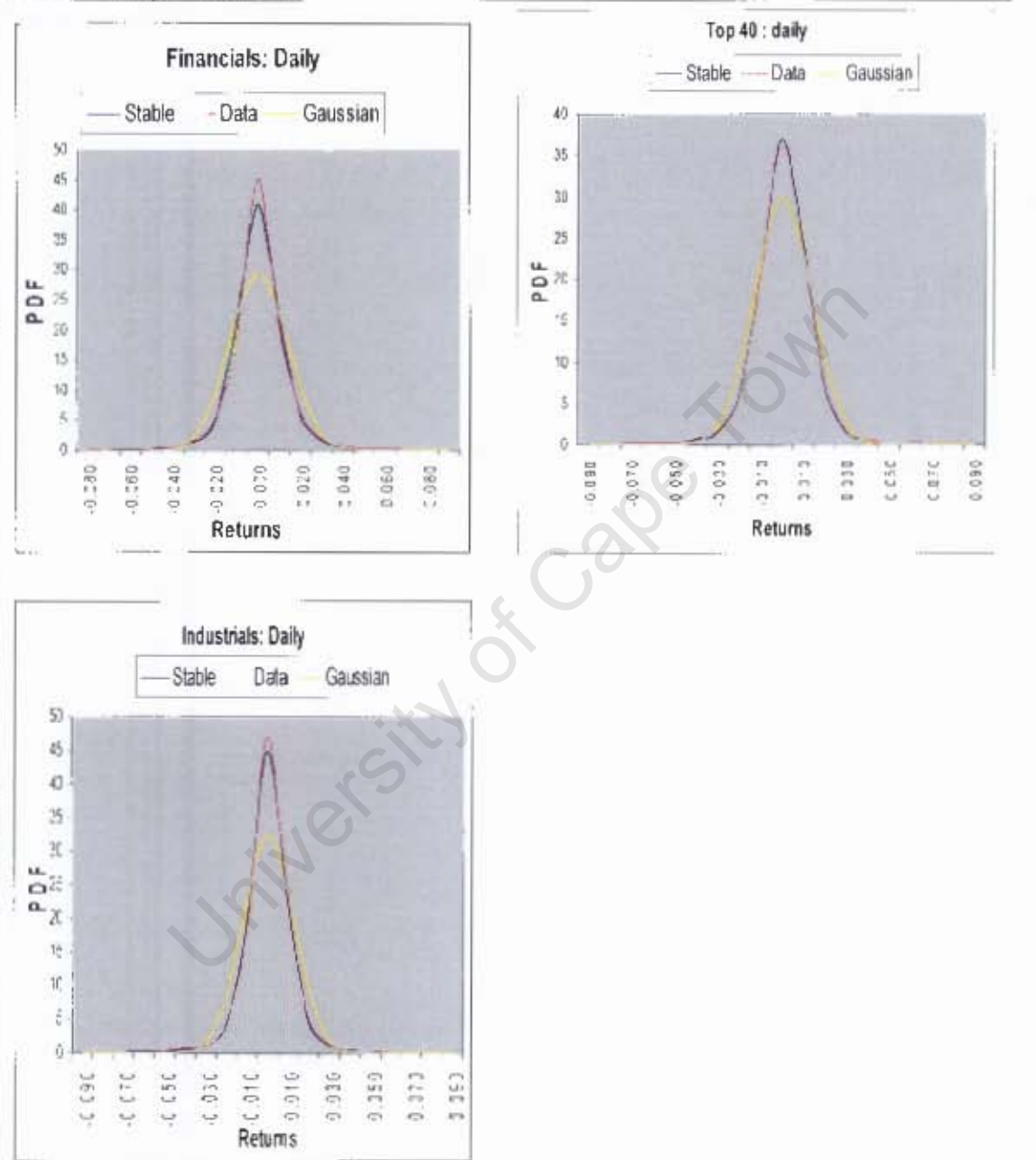


Figure B21: Density plots for daily log returns of Financials, Top 40 and Industrials

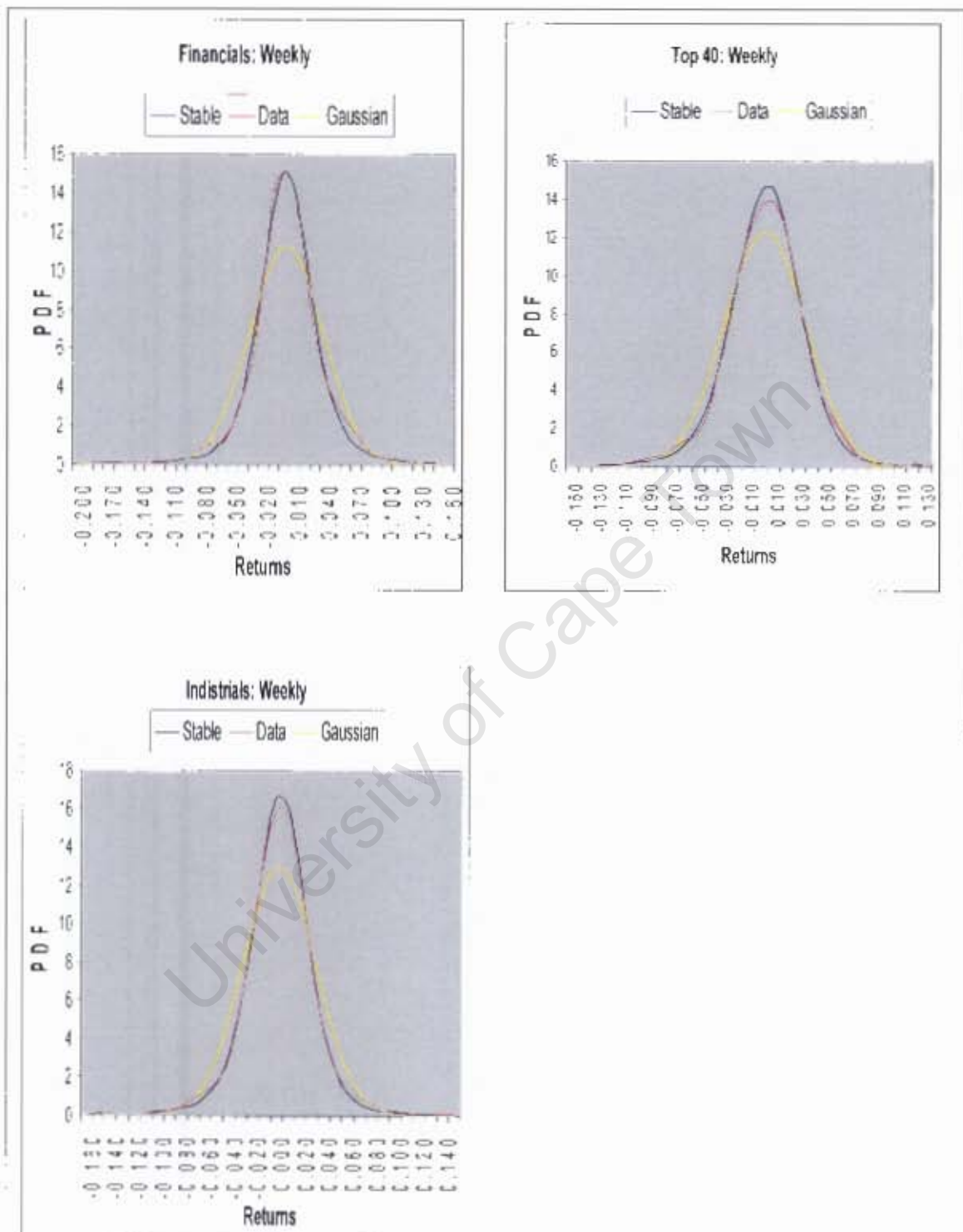


Figure B22: Density plots for weekly log returns of Financials, Top 40 and Industrials

2.2.3 Monthly Returns

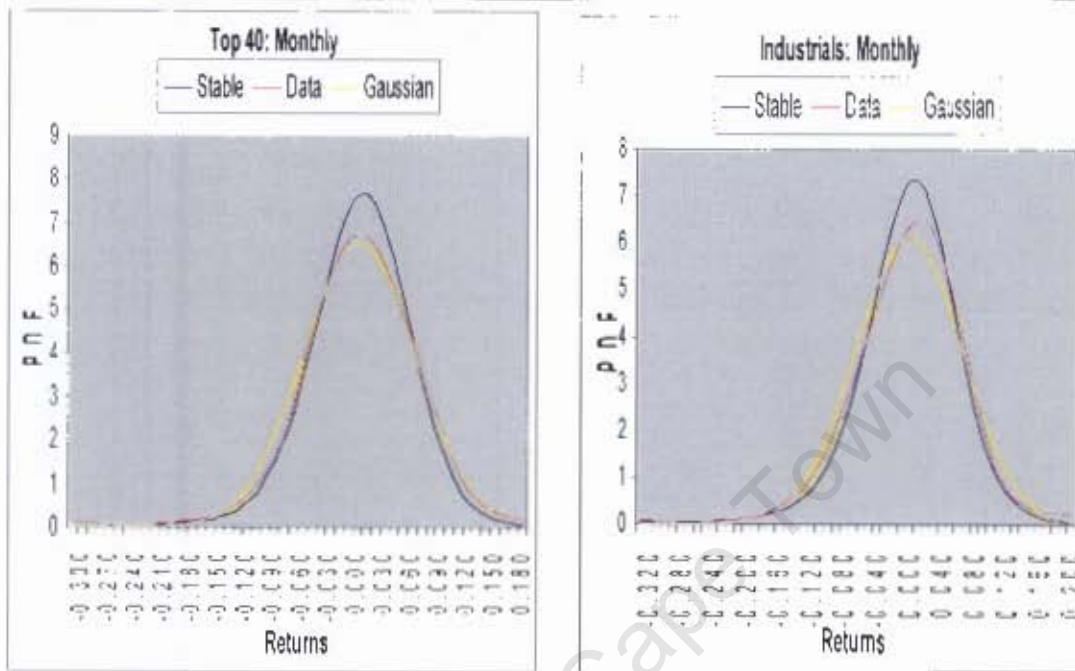
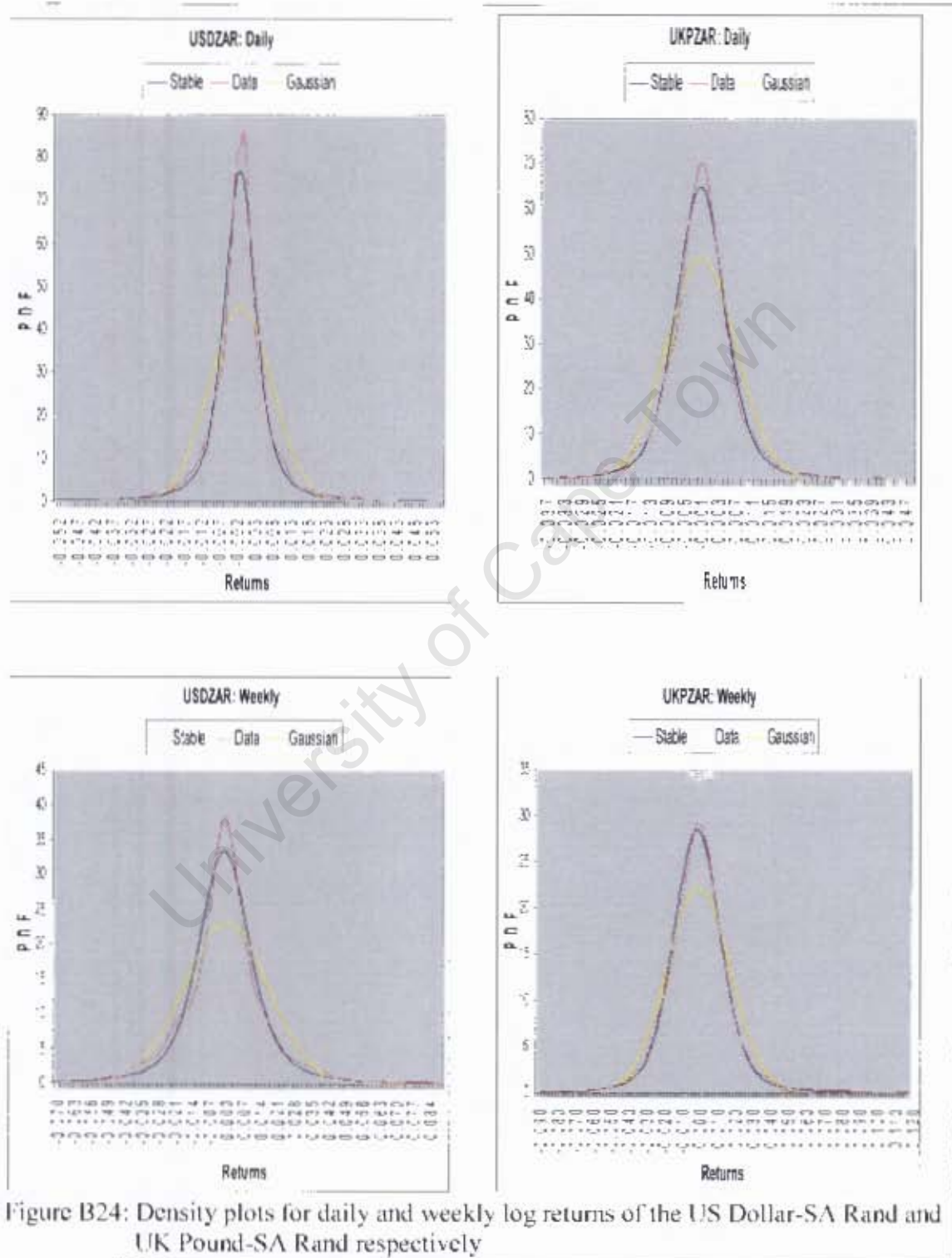


Figure B23: Density plots for monthly log returns of Top 40 and Industrials

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2.3 Exchange Rates



Appendix C: Maximum Likelihood Estimates of Parameters of Stable Distributions

Table C1: MLE Estimates from Daily Stock Returns

Stocks	General Stable				Gaussian	
	Alpha	Beta	Gamma	Delta	Gamma	Delta
ANA	1.7863	0.1474	0.0152	-0.0017	0.0175	-0.0010
ANG	1.7766	0.0927	0.0135	-0.0005	0.0157	-0.0002
BAR	1.6414	0.0925	0.0111	-0.0005	0.0144	-0.0001
BID	1.6923	-0.0472	0.0111	0.0003		
DRD	1.6243	0.1894	0.0228	-0.0038	0.0300	-0.0016
EXX	1.8008	0.2417	0.0151	0.0007	0.0172	0.0018
FIR	1.7849	0.0869	0.0132	-0.0005	0.0155	-0.0002
FOS	1.6505	-0.0594	0.0117	0.0002	0.0151	-0.0003
GRO	1.1103	0.0199	0.0095	0.0005	0.0207	-0.0010
GOL	1.8057	0.1449	0.0180	-0.0018	0.0214	0.0000
HAR	1.7014	0.1646	0.0192	-0.0016	0.0234	-0.0005
IML	1.7649	0.0258	0.0158	0.0001	0.0185	0.0001
IMP	1.6453	-0.0957	0.0113	0.0003	0.0144	-0.0004
INV	1.6770	0.0598	0.0122	-0.0002	0.0151	0.0000
LIB	1.6531	0.0401	0.0108	-0.0003	0.0144	-0.0004
MTN	1.6792	0.0420	0.0161	0.0004	0.0205	0.0005
NAM	1.6165	-0.0670	0.0117	-0.0001	0.0155	-0.0008
NAS	1.5046	-0.0047	0.0131	0.0004	0.0189	0.0001
PIC	1.6350	-0.0561	0.0113	0.0003	0.0147	0.0001
RIC	1.7134	0.0687	0.0113	-0.0002	0.0137	0.0000
RMB	1.7177	0.0766	0.0134	-0.0007	0.0166	-0.0004
SAS	1.7070	-0.0028	0.0136	0.0002	0.0168	0.0000
STA	1.6725	-0.0217	0.0118	0.0003	0.0149	0.0000
TIG	1.5349	-0.0022	0.0088	-0.0001	0.0123	-0.0002

Table C2: ML Estimates from weekly stock returns

	General Stable				Gaussian	
	Alpha	Beta	Gamma	Delta	Gamma	Delta
ANA	1.7866	0.5070	0.0377	-0.0044	0.0428	0.0008
ANG	1.7380	0.0790	0.0303	0.0030	0.0358	0.0036
BAR	1.7278	0.0753	0.0277	0.0024	0.0331	0.0030
BID	1.7127	0.1189	0.0237	0.0018	0.0286	0.0028
EXX	1.7852	0.0244	0.0390	0.0084	0.0442	0.0085
FIR	1.5990	0.0718	0.0280	0.0031	0.0364	0.0044
FOS	1.6000	-0.1163	0.0284	0.0037	0.0370	0.0017
GOL	1.8567	0.5086	0.0407	-0.0030	0.0446	0.0012
GRO	1.5467	-0.0689	0.0368	0.0039	0.0513	0.0024
HAR	1.6488	0.2630	0.0442	-0.0048	0.0552	0.0010
IML	1.7445	0.1678	0.0370	0.0024	0.0433	0.0045
IMP	1.5523	-0.0227	0.0244	0.0042	0.0333	0.0040
LIB	1.6975	0.1658	0.0253	0.0011	0.0325	0.0024
MTN	1.7248	0.1035	0.0408	0.0051	0.0497	0.0055
NAM	1.5614	-0.0677	0.0237	0.0028	0.0317	0.0017
NAS	1.5637	-0.1199	0.0330	0.0051	0.0462	0.0023
PIC	1.6848	-0.0998	0.0259	0.0041	0.0322	0.0029
RAI	1.3269	-0.0121	0.0301	-0.0013	0.0527	-0.0001
DRD	1.5821	0.2815	0.0495	-0.0099	0.0685	-0.0010
RIC	1.7330	0.1460	0.0249	0.0021	0.0295	0.0034
RMB	1.6383	-0.0172	0.0274	0.0037	0.0360	0.0038
SAS	1.7100	0.0586	0.0303	0.0033	0.0369	0.0040
STA	1.5682	0.0678	0.0239	0.0035	0.0337	0.0038
TIG	1.5741	0.0562	0.0216	0.0020	0.0293	0.0028

Table C3: ML Estimates from monthly stock returns

	General Stable				Gaussian	
	Alpha	Beta	Gamma	Delta	Gamma	Delta
ANG	1.8658	0.3720	0.0597	0.0121	0.0663	0.0155
BAR	1.8541	0.0489	0.0551	0.0122	0.0608	0.0131
BID	1.8880	-0.2122	0.0517	0.0195	0.0551	0.0183
EXX	1.7566	0.3591	0.0717	0.0308	0.0822	0.0367
FIR	1.8558	0.2178	0.0584	0.0193	0.0706	0.0198
FOS	1.6620	-0.4476	0.0622	0.0199	0.1006	0.0043
GOL	1.8699	0.8838	0.0721	-0.0064	0.0794	0.0049
HAR	1.6832	0.4738	0.0915	-0.0128	0.1083	0.0048
IMP	1.6838	0.0048	0.0532	0.0189	0.0654	0.0174
INV	1.8760	-0.1785	0.0567	0.0202	0.0642	0.0169
LIB	1.7220	0.1238	0.0472	0.0099	0.0553	0.0104
MTN	1.7034	0.0748	0.0691	0.0243	0.0878	0.0249
NAM	1.6788	-0.3391	0.0466	0.0132	0.0625	0.0072
NAS	1.6630	-0.1849	0.0745	0.0210	0.0978	0.0102
PIC	1.7383	-0.2148	0.0517	0.0166	0.0635	0.0128
RAI	1.6125	-0.4296	0.0779	0.0164	0.1007	-0.0003
RIC	1.8561	-0.4680	0.0483	0.0193	0.0536	0.0146
RMB	1.7210	-0.0076	0.0536	0.0186	0.0716	0.0167
SAS	1.8530	-0.0984	0.0631	0.0183	0.0696	0.0172
STA	1.7330	-0.1394	0.0512	0.0190	0.0691	0.0160
TIG	1.8946	0.1429	0.0488	0.0115	0.0519	0.0122

Table C4: ML Estimates from indices

	General Stable				Gaussian	
	Alpha	Beta	Gamma	Delta	Gamma	Delta
Daily returns						
FIN	1.5937	0.0285	0.0070	0.0005	0.0096	0.0005
IND	1.6262	-0.0960	0.0063	0.0008	0.0087	0.0005
T40	1.7341	-0.1102	0.0077	0.0010	0.0095	0.0006
Weekly returns						
FIN	1.6120	0.0937	0.0189	0.0022	0.0251	0.0024
IND	1.6701	-0.0964	0.0170	0.0032	0.0219	0.0023
T40	1.7475	-0.2386	0.0193	0.0045	0.0230	0.0028
Monthly returns						
FIN	1.7853	-0.3617	0.0398	0.0160	0.0549	0.0104
IND	1.7444	-0.4175	0.0386	0.0154	0.0461	0.0099
T40	1.8518	-0.3992	0.0369	0.0162	0.0432	0.0120

Table C5: ML Estimates for Exchange rates

	General Stable				Gaussian	
	Alpha	Beta	Gamma	Delta	Gamma	Delta
	Daily returns					
USDZAR	1.3993	0.0073	0.0038	0.0001	0.0062	0.0003
UKPZAR	1.6457	0.0486	0.0044	0.0001	0.0057	0.0003
	Weekly returns					
USDZAR	1.4774	0.0375	0.0086	0.0005	0.0123	0.0012
UKPZAR	1.6872	0.0820	0.0100	0.0005	0.0128	0.0013
	Monthly returns					
USDZAR	1.5219	-0.0802	0.0204	0.0051	0.0279	0.0051

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Appendix D: Bivariate Density, Contour and Cross-Sectional Density Plots

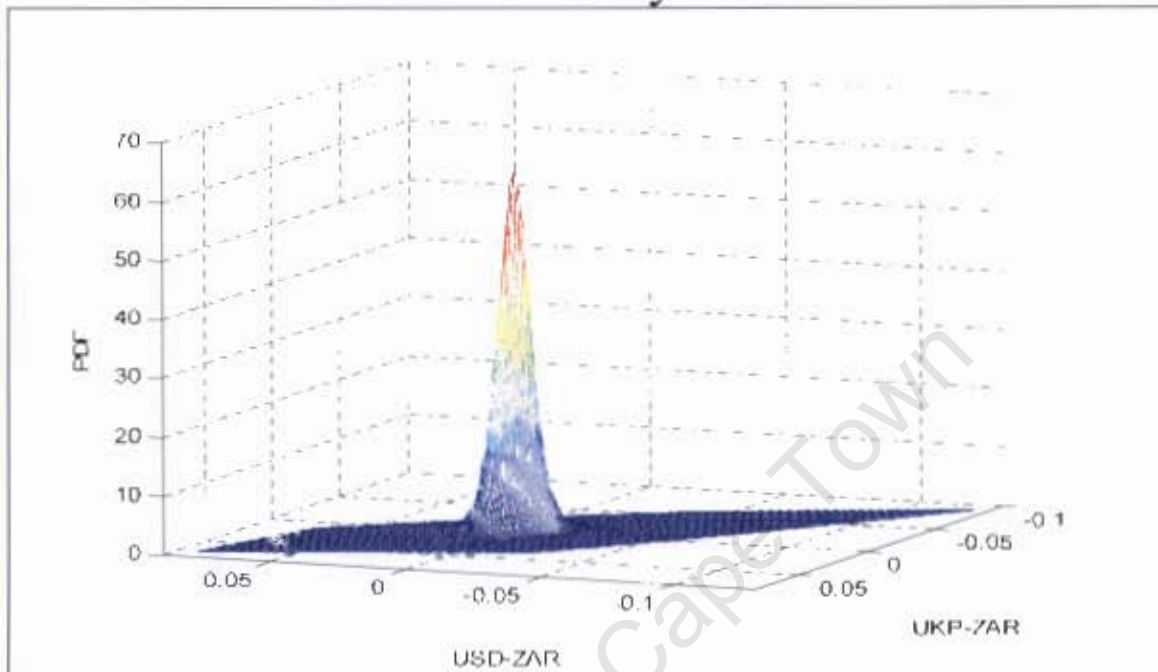


Figure D1(a): Bivariate stable density plot for 4941 daily (USD-ZAR, UKP-ZAR) returns

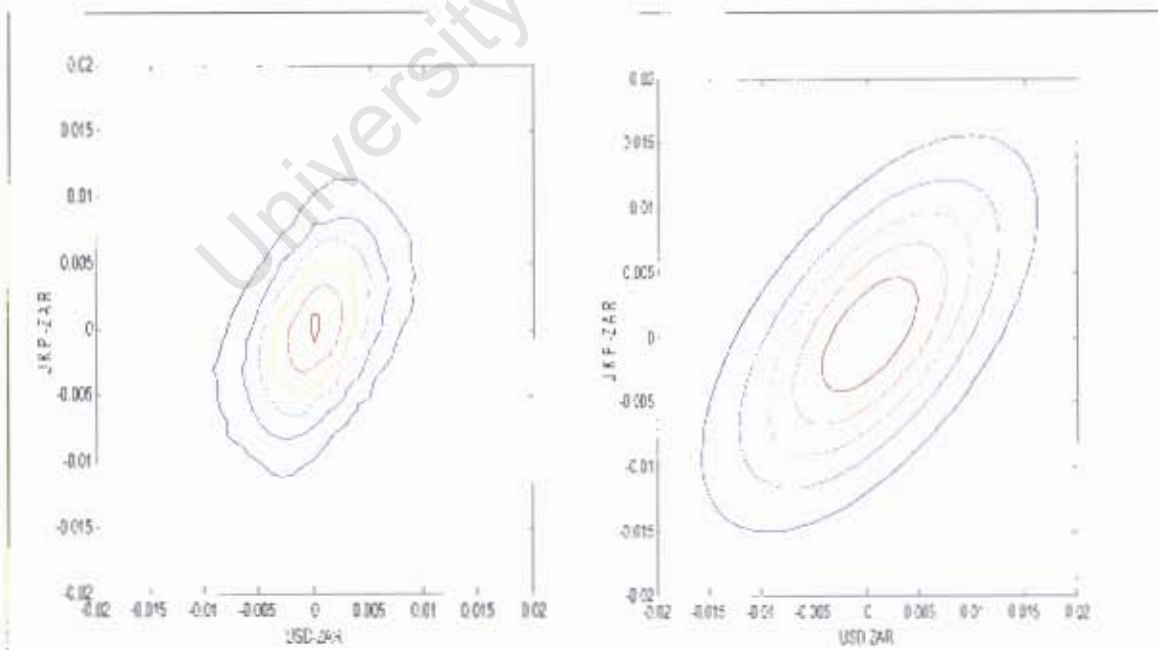
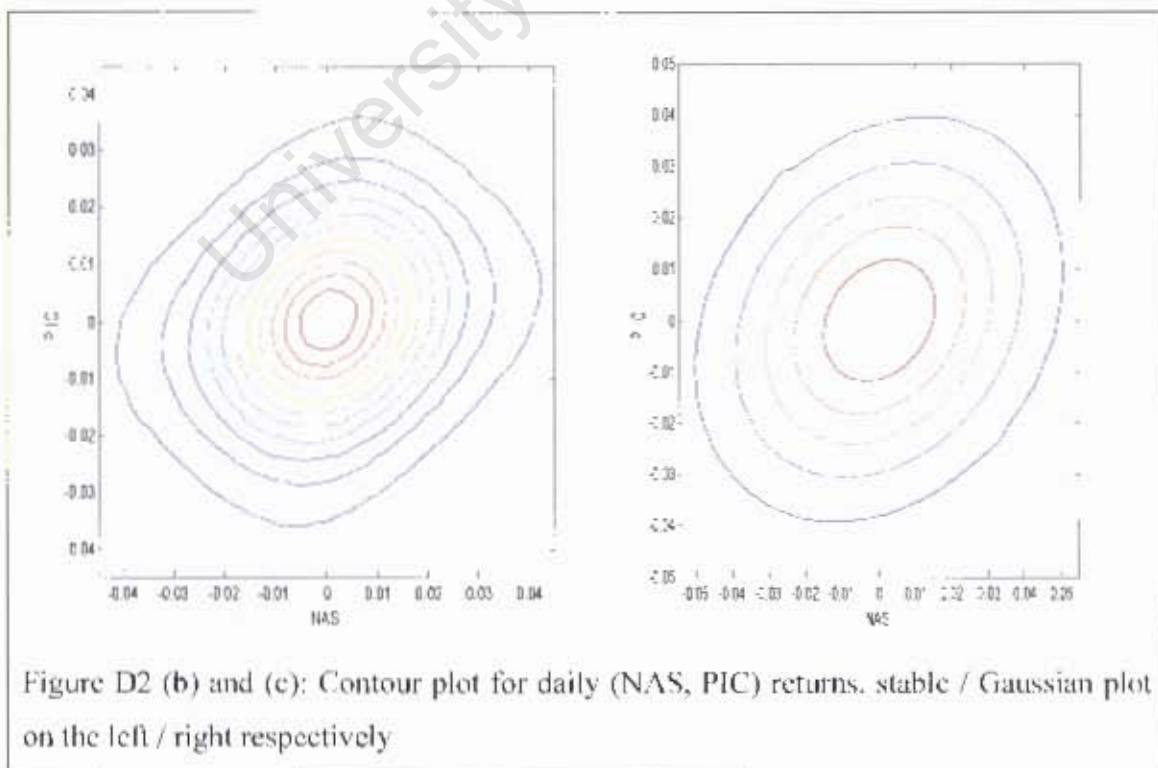
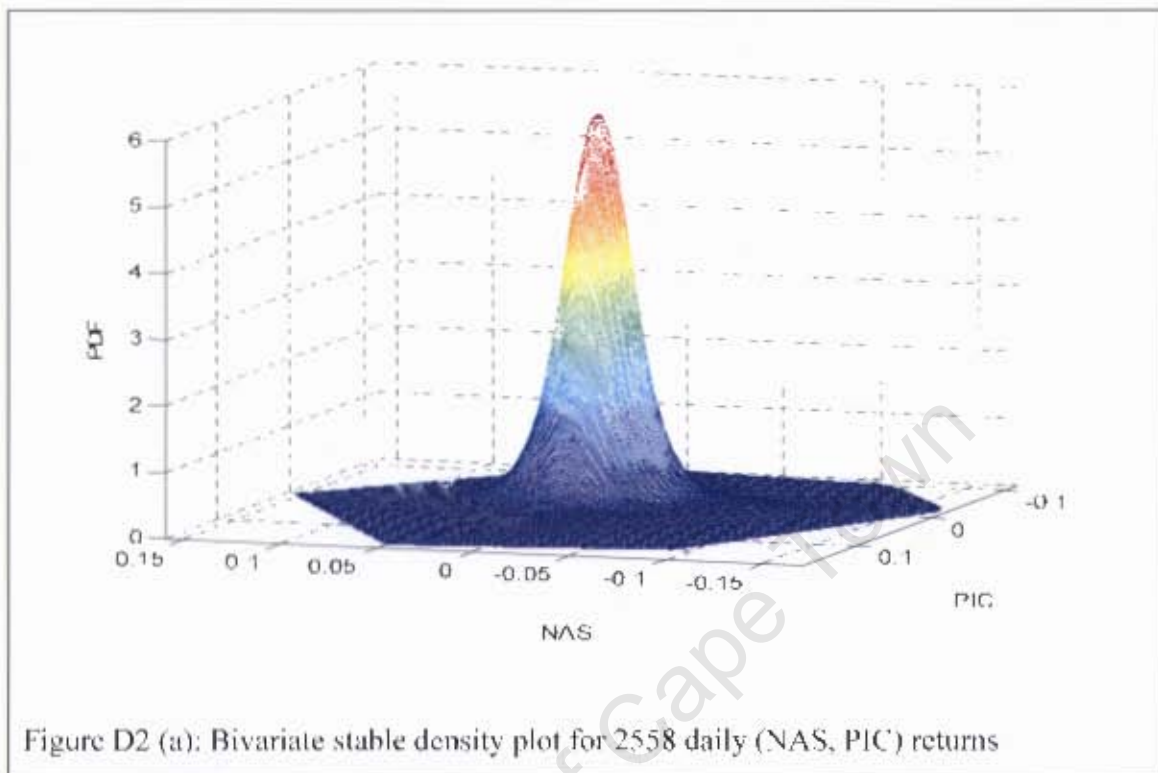


Figure D1(b) and (c): Contour plot for daily (USD-ZAR, UKP-ZAR) returns, stable / Gaussian plot on the left / right respectively



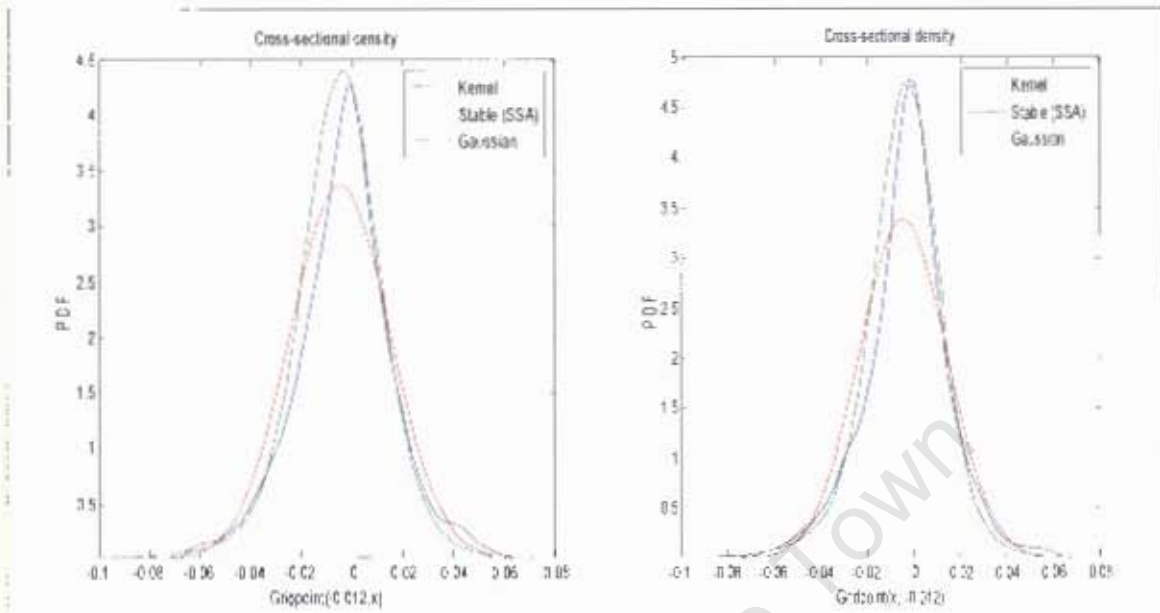


Figure D3: Cross-sectional density plots for daily (IMP, INV) returns

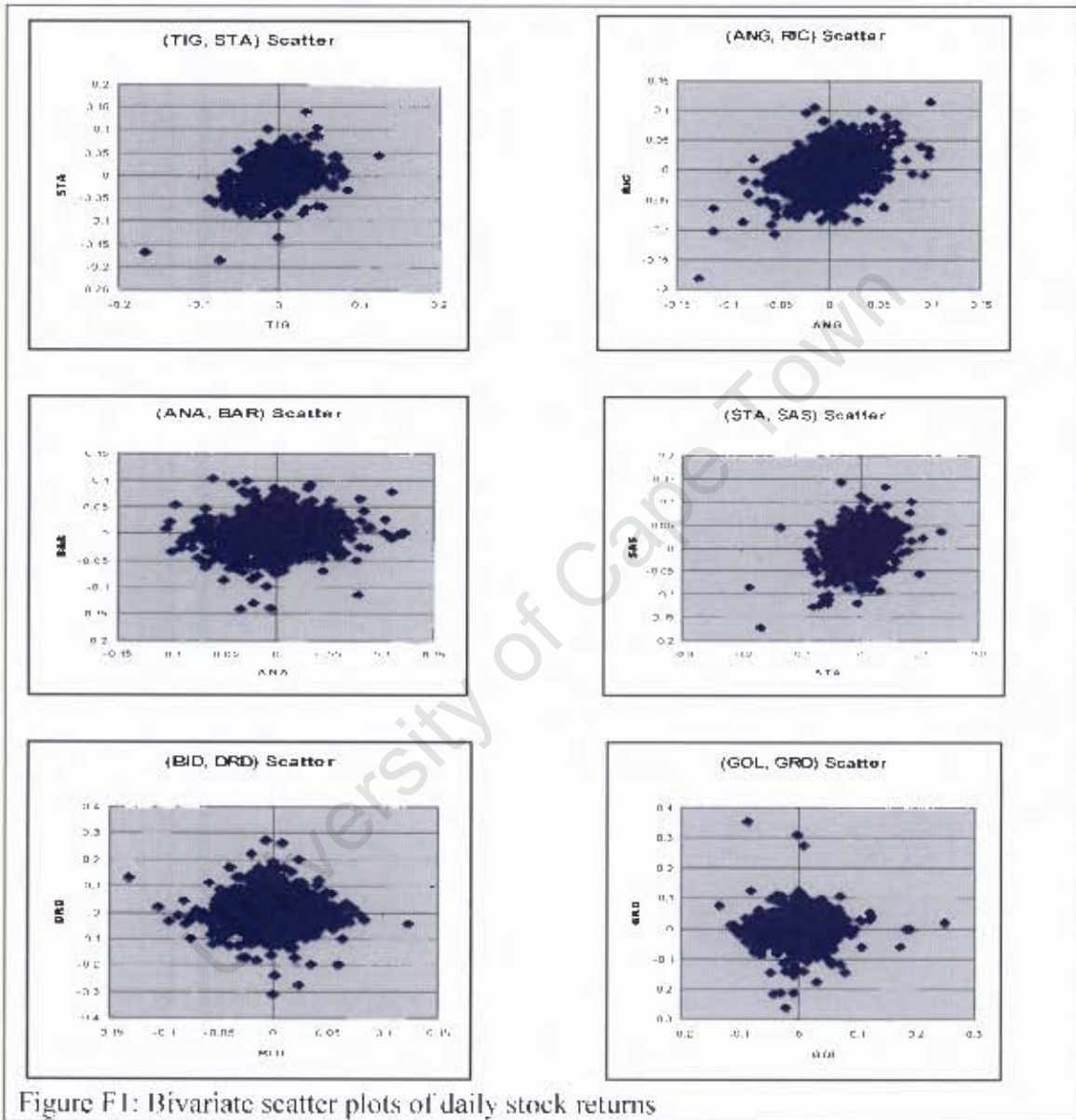
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Appendix E: Value-at-Risk and Conditional Value-at-Risk for Weekly Returns

Table E1	5% VaR			1% VaR		
Stock	Empirical	Gaussian	Stable	Empirical	Gaussian	Stable
ANA	-0.089	-0.070	-0.089	-0.147	-0.099	-0.137
ANG	-0.072	-0.055	-0.074	-0.139	-0.080	-0.137
BAR	-0.073	-0.051	-0.068	-0.137	-0.074	-0.128
BID	-0.061	-0.044	-0.058	-0.101	-0.064	-0.110
EXX	-0.100	-0.064	-0.089	-0.180	-0.094	-0.161
FIR	-0.079	-0.055	-0.073	-0.153	-0.080	-0.165
FOS	-0.086	-0.059	-0.080	-0.166	-0.084	-0.187
GOL	-0.093	-0.072	-0.096	-0.156	-0.103	-0.141
GRO	-0.109	-0.060	-0.108	-0.262	-0.095	-0.266
HAR	-0.129	-0.090	-0.113	-0.203	-0.127	-0.217
IML	-0.094	-0.067	-0.089	-0.152	-0.096	-0.159
IMP	-0.070	-0.051	-0.068	-0.139	-0.073	-0.167
LIB	-0.066	-0.051	-0.062	-0.117	-0.073	-0.118
MTN	-0.099	-0.076	-0.098	-0.192	-0.110	-0.185
NAM	-0.074	-0.050	-0.068	-0.140	-0.072	-0.165
NAS	-0.100	-0.074	-0.096	-0.225	-0.105	-0.235
PIC	-0.070	-0.050	-0.067	-0.144	-0.072	-0.141
RAI	-0.109	-0.087	-0.113	-0.241	-0.123	-0.351
DRD	-0.146	-0.114	-0.133	-0.240	-0.160	-0.273
RIC	-0.064	-0.045	-0.060	-0.121	-0.065	-0.110
RMB	-0.077	-0.055	-0.072	-0.133	-0.080	-0.157
SAS	-0.079	-0.057	-0.075	-0.149	-0.082	-0.145
STA	-0.065	-0.052	-0.063	-0.137	-0.075	-0.150
TIG	-0.063	-0.045	-0.058	-0.118	-0.065	-0.136

<i>Table E2</i>	<i>5% CVaR</i>			<i>1% CVaR</i>		
	<i>Empirical</i>	<i>Gaussian</i>	<i>Stable</i>	<i>Empirical</i>	<i>Gaussian</i>	<i>Stable</i>
ANA	-0.122	-0.091	-0.128	-0.180	-0.119	-0.217
ANG	-0.112	-0.073	-0.127	-0.169	-0.097	-0.261
BAR	-0.104	-0.068	-0.119	-0.166	-0.090	-0.247
BID	-0.089	-0.059	-0.102	-0.138	-0.077	-0.215
EXX	-0.134	-0.086	-0.149	-0.209	-0.115	-0.296
FIR	-0.112	-0.074	-0.153	-0.163	-0.097	-0.364
FOS	-0.126	-0.078	-0.172	-0.198	-0.102	-0.413
GOL	-0.128	-0.094	-0.130	-0.176	-0.123	-0.200
GRO	-0.144	-0.086	-0.246	-0.290	-0.119	-0.615
HAR	-0.168	-0.117	-0.204	-0.240	-0.152	-0.441
IML	-0.129	-0.088	-0.148	-0.184	-0.116	-0.297
IMP	-0.107	-0.068	-0.155	-0.163	-0.089	-0.387
LIB	-0.102	-0.067	-0.110	-0.185	-0.089	-0.233
MTN	-0.158	-0.101	-0.172	-0.279	-0.133	-0.359
NAM	-0.107	-0.066	-0.153	-0.174	-0.087	-0.377
NAS	-0.132	-0.097	-0.217	-0.273	-0.127	-0.537
PIC	-0.105	-0.066	-0.129	-0.162	-0.087	-0.289
RAI	-0.174	-0.113	-0.346	-0.280	-0.147	-1.011
DRD	-0.201	-0.146	-0.257	-0.320	-0.191	-0.588
RIC	-0.089	-0.060	-0.102	-0.140	-0.079	-0.209
RMB	-0.113	-0.074	-0.145	-0.167	-0.097	-0.335
SAS	-0.115	-0.075	-0.135	-0.181	-0.099	-0.287
STA	-0.112	-0.069	-0.139	-0.207	-0.091	-0.340
TIG	-0.093	-0.060	-0.126	-0.146	-0.079	-0.306

Appendix F: Bivariate Scatter Plots of Daily Returns



Appendix G: Covariance and Dispersion Matrices

<i>F1</i>	<i>Covariance Matrix: Using Daily Data</i>			
	IMP	INV	NAS	PIC
IMP	0.000416348	0.000166	0.000182	0.000112
INV	0.000166468	0.000454	0.000192	0.000105
NAS	0.000181641	0.000192	0.000716	0.000134
PIC	0.000112064	0.000105	0.000134	0.000434

<i>F2</i>	<i>Dispersion Matrix: Using Daily Data</i>			
	IMP	INV	NAS	PIC
IMP	4.47E-05	2.49E-05	3.3E-05	1.99E-05
INV	2.49E-05	4.91E-05	3.25E-05	1.8E-05
NAS	3.3E-05	3.25E-05	7.4E-05	2.56E-05
PIC	1.99E-05	1.8E-05	2.56E-05	4.59E-05

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Appendix H: MatLab and VBA Code used

H1 MatLab Code

Example: Plotting the bivariate stable distribution of weekly returns of USDZAR, UKPZAR.

```
>> t = -0.05:0.001:0.09;
>> [XI,YI] = meshgrid(t,t);
>> ZI = griddata(WUSDZAR,WUKPZAR,PDF,XI,YI);
>> mesh(XI,YI,ZI)
>> contour(XI,YI,ZI)
```

H2 VBA Code

Option Explicit

Option Base 1

'Define variables in vector form

'The following variables are declared public

Public WSF '

Public DUSDZAR(4941), DUKPZAR(4941), WUSDZAR(1044), WUKPZAR(1044)

Public DUSDZARGAU(261), DUKPZARGAU(261), WUSDZARGAU(261),

WUKPZARGAU(261), X(261)

Public DUSDZARCDF(261), DUKPZARCDF(261), WUSDZARCDF(261),

WUKPZARCDF(261)

Public DUSDZARPDF(261), DUKPZARPDF(261), WUSDZARPDF(261),

WUKPZARPDF(261)

Function Interpolate(u, X(), AF()) *'Linear interpolation formula in [W18]*

'Returns a linear interpolated value AF(.) of u given a corresponding vector X

Dim lowerx, upperx, lpos, upos

upos = LBound(X) *'Position of the first number larger than u in X*

lpos = UBound(X) *'Position of the first number smaller than u in X*

If (u >= X(LBound(X))) And (u <= X(UBound(X))) Then

Do

Do While u < X(lpos)

lpos = lpos - 1

Loop

Loop Until u >= X(lpos)

lowerx = X(lpos - 1) *'The first number smaller than u*

Do

Do While u > X(upos)

upos = upos + 1

```

Loop
Loop Until u <= X(upos)
upperx = X(upos + 1) 'The first number larger than u
Interpolate = ((upperx - u) * AF(lpos) + (u - lowerx) * AF(upos)) / (upperx - lowerx)
Else
If u < X(LBound(X)) Then
Interpolate = AF(LBound(AF))
Else
If u > X(UBound(X)) Then
Interpolate = AF(UBound(AF))
End If
End If
End If
End Function

```

Sub ReadDataIn() *'Reads data from Excl Sheets into the corresponding vectors*

```

Set WSF = Application.WorksheetFunction
Dim i, j
For i = 1 To 4941
DUSDZAR(i) = Sheets("Data").Range("B2").Cells(i, 1).Value
DUKPZAR(i) = Sheets("Data").Range("C2").Cells(i, 1).Value
Next i
For i = 1 To 1044
WUSDZAR(i) = Sheets("Data").Range("D2").Cells(i, 1).Value
WUKPZAR(i) = Sheets("Data").Range("E2").Cells(i, 1).Value
Next i
For i = 1 To 261
X(i) = Sheets("GAU").Range("A2").Cells(i, 1).Value
DUSDZARGAU(i) = Sheets("GAU").Range("B2").Cells(i, 1).Value
DUKPZARGAU(i) = Sheets("GAU").Range("C2").Cells(i, 1).Value
WUSDZARGAU(i) = Sheets("GAU").Range("D2").Cells(i, 1).Value
WUKPZARGAU(i) = Sheets("GAU").Range("E2").Cells(i, 1).Value
DUSDZARCDF(i) = Sheets("CDF").Range("B2").Cells(i, 1).Value
DUKPZARCDF(i) = Sheets("CDF").Range("C2").Cells(i, 1).Value
WUSDZARCDF(i) = Sheets("CDF").Range("D2").Cells(i, 1).Value
WUKPZARCDF(i) = Sheets("CDF").Range("E2").Cells(i, 1).Value
DUSDZARPDF(i) = Sheets("PDF").Range("B2").Cells(i, 1).Value
DUKPZARPDF(i) = Sheets("PDF").Range("C2").Cells(i, 1).Value
WUSDZARPDF(i) = Sheets("PDF").Range("D2").Cells(i, 1).Value
WUKPZARPDF(i) = Sheets("PDF").Range("E2").Cells(i, 1).Value
Next i
End Sub

```

Sub Transform() *Interpolates and returns standard Gaussian values of returns for each vector*

ReadDataIn

Dim i, j

For i = 1 To 4941

Sheets("TRA").Range("B4").Cells(i, 1).Value =
Interpolate(Interpolate(DUSDZAR(i), X, DUSDZARCDF), DUSDZARCDF,
DUSDZARGAU)

Sheets("TRA").Range("C4").Cells(i, 1).Value =
Interpolate(Interpolate(DUKPZAR(i), X, DUKPZARCDF), DUKPZARCDF,
DUKPZARGAU)

Sheets("TRA").Range("G4").Cells(i, 1).Value = Interpolate(DUSDZAR(i), X,
DUSDZARPDF)

Sheets("TRA").Range("H4").Cells(i, 1).Value = Interpolate(DUKPZAR(i), X,
DUKPZARPDF)

Next i

For i = 1 To 1044

Sheets("TRA").Range("D4").Cells(i, 1).Value =
Interpolate(Interpolate(WUSDZAR(i), X, WUSDZARCDF), WUSDZARCDF,
WUSDZARGAU)

Sheets("TRA").Range("E4").Cells(i, 1).Value =
Interpolate(Interpolate(WUKPZAR(i), X, WUKPZARCDF), WUKPZARCDF,
WUKPZARGAU)

Sheets("TRA").Range("I4").Cells(i, 1).Value = Interpolate(WUSDZAR(i), X,
WUSDZARPDF)

Sheets("TRA").Range("J4").Cells(i, 1).Value = Interpolate(WUKPZAR(i), X,
WUKPZARPDF)

Next i

End Sub

-----END-----