# SUBSYNCHRONOUS RESONANCE IN SERIES COMPENSATED NETWORKS WITH HIGH PENETRATION OF RENEWABLE ENERGY SOURCES

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# Abstract

The global shift towards clean energy has seen a spiked rate of environmentally friendly renewable energy sources (RES) as utilities and regional energy blocks battle with rising demand. RES sites may be located in remote or populated areas with limited servitudes. To reduce investment costs, grid impact study assessments conducted by utilities often recommend new power plants to cut-in nearby existing lines for power evacuations after considering availability of wayleaves and other socio-economic concerns. There are foreseen and planned connections of RES evacuating power via series compensated lines in Africa. However, integrating RES through series compensated lines results in subsynchronous resonance (SSR) between generator rotors and the grid.

SSR is a phenomenon where there is exchange of electrical energy between the generator shaft system and the transmission system below the fundamental power frequency leading to electrical instability. The future trend in power systems entails deeper investments of the same highlighted SSR inducing sources such as series compensation. This brings uncertainty and misperceptions in respect to effective management of the SSR problem translated to a case of either retarding power system investment without SSR against reinforcing grid investments with SSR inducing consequences.

The dissertation dissects the SSR problem with emphasis on its management in a modern power system environment receiving rising RES and flexible alternating current transmission systems (FACTS) connections through series compensated lines.

The simulations performed in this study utilised the IEEE first benchmark model (FBM) within Digital Simulation and Electrical Network – PowerFactory (DIgSILENT) software. The unmodified FBM served as a base case representation of the power system without RES or STATCOM (static synchronous compensator) penetrations. The various modifications to the FBM captured multiple

scenarios representative of rising penetrations of wind power, solar power and static compensators. The generator, shaft, busbar terminals and series capacitor were monitored and analysed to derive SSR modulated torsional effects after applying a 3-phase fault to the system for 75 milliseconds.

From the perspective of conventional synchronous non–renewable generation, decrease in SSR probability is achievable by adding RES like solar PV power in the vicinity. Another output from the simulations confirmed that PV generation does not participate in SSR occurrence as seen by its pre and post fault conditions recovering immediately and remaining constant afterwards.

In addition, the results from the modal analysis comparison reinforced that as more RES or STATCOM devices are connected to the FBM, the overall system improves on oscillatory stability as seen by the oscillatory eigenvalue real parts becoming more negative and the damping ratios more positive.

The future work involves analysis of SSR in other RES and how SSR protection can be incorporated in protection relays using a filtering method based on the prony analysis.

## List of Abbreviations

| | |
|---|---|
| AC | Alternating current |
| ACCR | Aluminium conductor composite reinforced |
| ACCFR | Aluminium conductor composite carbon fibre reinforced |
| ACSR | Aluminium conductor steel reinforced |
| ACSS | Aluminium conductor steel supported |
| AVR | Automatic voltage regulator |
| CRAC | Composite reinforced aluminium conductor |
| DFIG | Doubly fed induction generator |
| DIgSILENT | Digital Simulation and Electrical Network GmbH software |
| EAPP | Eastern African Power Pool |
| EMTDC | Electromagnetic Transient Direct Current |
| EMTP | Electromagnetic Transients Program |
| FACTS | Flexible Alternating Current Transmission Systems |
| GCSC | Gate controlled series capacitor |
| GTACSR | Gap type heat resistant aluminium alloy conductor steel reinforced |
| GZTACSR | Gap type super heat resistant aluminium alloy conductor steel reinforced |
| GW | Gigawatts |
| IEDs | Intelligent Electronic Devices |

| | |
|---|---|
| IEEE | Institute of Electrical and Electronics Engineers |
| FBM | IEEE First Benchmark Model |
| HPAC | High Precision AC filter |
| HVDC | High voltage direct current |
| HTLS | High temperature, low sag |
| Hz | Hertz |
| IED | Intelligent electronic device |
| IGBTs | Insulated gate bipolar transistors |
| IGE | Induction generator effect |
| ms | milliseconds |
| MW | Megawatts |
| PMSG | Permanent magnet synchronous generator |
| PSCAD | Power System Computer Aided Design |
| PSS | Power system stabiliser |
| PV | Photovoltaic |
| RES | Renewable energy sources |
| RESISC | Renewable Energy Sources integrated through Series Compensated lines |
| SAPP | Southern Africa Power Pool |
| SG | Synchronous generator |
| SSCI | Subsynchronous control interaction |

| | |
|---|---|
| SCIG | Squirrel cage induction generator |
| SSDC | Subsynchronous damping controller |
| SSR | Subsynchronous resonance |
| STATCOM | Static synchronous compensator |
| SVC | Static var controller |
| TCSC | Thyristor controlled series capacitor |
| TA | Torque amplification |
| TI | Torsional interaction |
| THD | Total harmonic distortion |
| VSC | Voltage source converter |
| WRIG | Wound rotor induction generator |
| WRSG | Wound rotor synchronous generator |
| ZTACIR | Zirconium alloy aluminium conductor invar steel reinforced |

# Table of Contents

## List of Figures

## List of Tables

## Nomenclature

| | |
|---|---|
| $w_n$ | Natural angular frequency (radians/second) |
| f | Frequency (Hz) |
| i | Currrent (Amps) |
| $\theta$ | Rotor angle (Degrees) |
| P | Park's transformation |
| R | Resistance (Ohms) |
| $V_s$ | Sending-end phase voltage (Volts) |
| $V_r$ | Receiving-end phase voltage (Volts) |
| $X_L$ | Line reactance (Ohms) |
| $X_C$ | Capacitive reactance (Ohms) |
| $\delta$ | Phase angle (Degrees) |
| $S_{ssr}$ | Slip at subsynchronous frequency |
| $f_{er}$ | Frequency at electrical resonance (Hz) |
| $f_o$ | Natural frequency (Hz) |
| M | Rotational mass inertia |
| K | Shaft stiffness |
| D | Damping |
| $R_{eq}^{ssr}$ | Equivalent rotor resistance in subsynchronous range (Ohms) |
| $R_r$ | Rotor resistance (Ohms) |

| | |
|---|---|
| $w_B$ | System base frequency (Hz) |
| L | Inductance (H) |
| C | Capacitance (F) |
| tg_ex, tex_g | Exciter torque parameter |
| tg_lpb, tlpg_g | Shaft torque parameter |

# CHAPTER 1

## 1. INTRODUCTION

Series compensation is an effective way to increase the power transfer capability and transfer stability margin of long-distance transmission systems. Presence of series capacitors in transmission lines enables a decrease in the angular separation of the end voltages hence contributing towards an increase in the angular and voltage stability [1]. In simple terms, series compensation reduces the reactive part of line impedance to increase both the transient stability and maximum active power flow on the line.

According to a study performed by ABB, increasing the power transfer capability of an existing transmission line from 1300MW to 2000MW using series compensation proves 90% less costly than building a new transmission line [2]. However, series compensation exposes the turbo-generators to SSR risk [3]. Numerous utilities are either applying or considering applying series capacitors because of the associated financial benefits in capital outlay [4].

Wind power is one of the RES on the rise and due to large integration distances and capital requirements involved, series compensation is preferred to address evacuation capacity constraints. China, for example, has a forecasted 250GW of wind capacity by year 2020 as part of its government's pledge to produce fifteen percent (15%) of all electricity requirements from renewable resources by that year with series compensation at the core of such developments [5].

Generators connecting to a wind turbine are either asynchronous (induction) or synchronous. Squirrel cage induction generators (SCIG) and wound rotor induction generators (WRIG) fall under asynchronous generators whilst wound rotor generators (WRSG) together with permanent magnet generators (PMSG) come under synchronous generators. There is a migration from fixed speed to advanced variable speed wind turbines. In order to satisfy the modern grid codes, wind farms require voltage, frequency and reactive power support capability. In certain instances, the wind conversion system using PMSG is now being more preferred than DFIG for its wider speed range, elimination of gearbox, elimination of excitation and its yielding better power quality in terms of THD. DFIGs take less time to normalise since they possess an ability to support reactive power through the grid side converter or through the rotor side converter. There are observations that SSR is prevalent in wind farms, and in particular, there is introduction of SSCI in DFIGs integrated with series compensated lines [6].

In the past, conventional HVAC evacuation system has been adequate to transport power over regional and national grids. Now, vast extra high voltage transmission lines are high on the agenda of energy planners e.g. interconnections planned between Southern African Power Pool (SAPP), Eastern African Power Pool (EAPP) and West African Power Pool (WAPP) with the aim to diversify regional power networks and trading markets [7]. On the other hand, since 1964, HVDC transmission has been increasing due to its advantages over HVAC for long distance such as increasing overall stability of the power system, achieving better efficiency and reducing power losses [8] , [9]. Nevertheless, HVDC controllers under certain conditions give rise to subsynchronous resonance. At the same time, the introduction of associated flexible alternating current transmission systems (FACTS) like the gate controlled series capacitor (GCSC) may come with oscillatory damping abilities usable for mutual benefit [10].

Torsional oscillations principally affect conventional thermal or nuclear power plants and susceptibility to hydro plants is minimal due to the higher generator inertia and inherent mechanical turbine damping.

SSR has manifested itself broadly in terms of induction generator effect (IGE), torsional interaction (TI), torque amplification (TA), and most recently as Subsynchronous Control Interaction (SSCI).

This research has conveyed SSR root sources and the resultant impact to the power system network by monitoring the critical generator and shaft system changes. Both time domain and frequency analysis is utilised for different scenarios to assess and prove the hypothesis.

## 1.1 Background to the dissertation

SSR upsets radial networks connected through series compensated lines, but the risk or to what extend meshed compensated networks are prone to the SSR phenomenon still remains vague [8]. The compensation level is proportional to SSR fatality according to [11]. One attempt by [1] was to quantify SSR probability by way of coming up with risk matrix based on different combinations of network configurations, contingencies and compensation levels in a meshed network.

The ultimate hazard of SSR is generator shaft fracture albeit at rated speed. The history of SSR problem has had its twists and turns. In 2002, Joswig Frank and Stefan Kulig co-authored a different cause of SSR by faulty induction machines bringing to light that issue of faulty slip rings and faulty squirrel cages produce subsynchronous currents capable of exciting undesired oscillations in the generator shaft. They suggested localised approaches to isolate SSR by way of monitoring current and inserting torque sensors on the shaft [12].

The trio of Faried, Tang, and Edris [13] put it well that FACTS technology provides unprecedented way of controlling transmission grids dynamics and possess a capability of inherent SSR mitigation through use of supplemental control of an existing voltage source converter (VSC).

Power System Stabilizers (PSS) provide damping to power system oscillations at very low frequencies (0.1 - 2.0 Hz). The PSS in the automatic voltage regulator (AVR) excites subsynchronous oscillations if the field winding receives oscillatory injections corresponding to the shaft torsional modes (model diagram available in Appendix B). Electro-hydraulic governors control actions also excite turbine generators into subsynchronous oscillations because of their wide bandwidth [14].

Increasing levels of wind energy penetration utilising the conventional induction generators has moved researchers to express SSR influence and management thereof [15], [16], [17] [18].

Adrees and Milanovic contributed significantly by noting that a combination of voltage source converter based HVDC operating in parallel to thyristor-controlled series compensated line has an ability to eliminate the risk of SSR [22]. Examinations of the Akkuyu Nuclear Electric power plant in southern Turkey of different combinations of grid topologies, loading conditions and generation dispatch conditions revealed that SSR risk analysis during the planning stages prevents possible hazardous effects [28].

Installing varying STATCOM setups at strategic grid points has also been proven to contribute to upgraded SSR mitigation whilst optimising reactive compensation [3], [17], [18], [19], [21], and [34].

Researchers continued to present various countermeasures to SSR suggesting adding damping controllers [32], [37], [38], [39]. In particular, 11kV variable frequency drive auxiliary damping controllers according to [43] and a subsynchronous damping controller (SSDC) demonstrated in [30] & [44] have good SSR mitigation properties. The University of Manchester (UK) took to task to evaluate the impact of dynamic loads, for example, induction motors on the damping of SSR based on load location and aggregate load inertia for the IEEE 68 bus network model [45]. SSR suppression comes as an added advantage to a thyristor controlled series capacitor (TCSC) which finds use to increase the power transfer capability of the transmission line through closed loop current control [20], [47]. Reducing levels of series compensation or utilising a tuned excitation controller remains as effective ways to dampen SSR [29].

It has only been a few years ago when researchers started hinting installing protection relays for power plants exposed to SSR risk. Clark [11] cited that an economic decision was possible to adopt the protection approach to protect the more prone generator unit rather than employ the SSR mitigation methods.

The new awareness regarding SSR protection brought in a diversion to SSR management unlike the greater part of the 20th century. In year 2016, significant discussions like the one held in Sweden [27] assessed the effectiveness of equipping the protection numerical Intelligent Electronic Devices (IEDs) with functionality to detect and isolate SSR related faults.

Thus, the dissertation explores and pursues this new direction of handling SSR in the network, which embraces the modern trend of technological developments.

## 1.2 Objectives of the dissertation

The objectives of this research are the following:

- To discover the origin of subsynchronous frequency that leads to the SSR problem.
- To discuss the various SSR types with their analysis methods and identify possible SSR detection and protection methods.
- To classify why series compensation is attractive for the SAPP region.
- To build time domain simulation models based on the IEEE first benchmark model in DIgSILENT focussed on SSR performance within synchronous, wind and solar PV resources.
- To design and build modal analysis models to validate the correlation of SSR incidences with RES (wind and solar PV) connected through series compensated lines.

- To consider and assess if increasing RES penetrations to a series compensated network serves as an inherent technique to manage the SSR problem.

## 1.3 Research Questions

1. What has been the critical impact of SSR?
2. Is it justified to invest in SSR mitigation, looking at the increasing penetration of its very sources in the power system e.g. RES and transmission line compensation equipment?
3. How is SSR mitigated by introducing and optimising RES injection?
4. Are conventional power plants with rotating masses subjected to higher SSR risk with rising RES penetration levels?
5. What is the current trend of RES integrations in Southern Africa, and will the approach of incorporating SSR detection in IEDs be effective?

## 1.4 Hypothesis

The dissertation tests the following hypothesis:

"The integration of RES such as wind, solar PV power and FACTS devices like STATCOM in series compensated AC networks with increasing penetration of renewables inherently improves stability performance of the network in managing SSR".

## 1.5 Research Methodology

The research objective was set to conduct a thorough literature review on the history and status of the SSR problem in a dynamic power system with vast injections of RES and series compensation. The review covered SSR types, suppression techniques utilised in the past, available SSR analysis techniques and recently proposed management methods. The literature examination also considered participation of the Park's transformation in the SSR definition.

The research progressed to uncover RES penetration opportunities in Southern Africa and discussed the effectiveness of the proposed management method of using protection filters to isolate and curb SSR.

The objective at this stage then culminated in assessing and verifying the SSR impact on the modern power system with renewable energy resources connected by series compensated lines by way of time domain and frequency domain analysis in DIgSILENT.

The unmodified FBM represented the base case scenario. Modifications to the base case were done to model solar PV, wind and STATCOM study case scenarios particularly of 200MW, 400MW, 600MW and 800MW power ramps. The SSR effects of the varying power ramps on the conventional synchronous system were tabled and evaluated. Modal analysis was then conducted for oscillatory stability validation on the base case, 200MW, 800MW and STATCOM scenario instances.

## 1.6 Research scope and limitations

The research is limited to SSR incidence analysis on the IEEE first benchmark model comprising of a synchronous generator with RES additions of solar PV, wind and STATCOM resources in the same network. Other energy resources such as solar thermal energy or hydro plants are excluded from the study scenarios. The focus is to test (using time domain and modal analysis), the correlation of SSR on conventional configurations receiving connections of RES of wind and solar PV integrated through series compensated networks. The simulations serve to give an inherent SSR management strategy.

## 1.7 Contribution to knowledge

The research has contributed in the following:

1. The dissertation introduces a new defined acronym of RESISC, to name and describe the forthcoming renewable energy sources integrated through series compensated networks.
2. Based on the hypothesis, the research demystifies the SSR problem extent and gives ramped RES and FACTS integrations as an inherent passive solution and technique to manage SSR incidences in a conventional network configuration with high usage of series compensation.

## 1.8 Research Output

Conference paper:

T. E. Chikohora and D. T. O. Oyedokun, "*Sub-Synchronous Resonance (SSR) in Series Compensated Networks with High Penetration of Renewable Energy Sources*", accepted for publication in International SAUPEC/RobMech/PRASA Conference, Cape Town, South Africa, from 29 - 31 January 2020.

## 1.9 Dissertation Outline

This dissertation outline and chapter arrangement is as follows,

| | |
|---|---|
| Chapter 1 | Gives the introduction, objective, research methodology, hypothesis, report outline and research contribution of the dissertation. |
| Chapter 2 | Covers the literature review on subsynchronous resonance concepts inclusive of series compensation, in-depth study of the SSR analysis methods, SSR mathematical analysis and the prony analysis. |
| Chapter 3 | Is a short chapter summarising the Southern African power system status regarding penetration of renewable energy sources and series compensation. |
| Chapter 4 | Outlines the methodology and approach for the DIgSILENT PowerFactory stability simulation studies. |
| Chapter 5 | Presents the time domain transient stability and modal analysis simulation tests results. |
| Chapter 6 | Concludes the dissertation and provides future recommendations. |

# CHAPTER 2

## 2. LITERATURE REVIEW

### 2.1 Origin of subsynchronous frequency- stability perspective

In accordance with the definition of Institute of Electrical and Electronics Engineers (IEEE), SSR is an electrical power system condition where the electrical network exchanges energy with turbine generator at one or more of the natural frequencies of the combined system below the synchronous frequency of the system following a disturbance from an equilibrium point [23].

The studies on SSR phenomenon widened since the well-known incidents at the Mohave generator in Arizona in 1971 and 1972 where the shaft fractured due to a growing vibration [24].

Renewable energy integrations emanate with devices composed of power electronic gadgetry that does not contribute synchronous inertial response to the power system grid. SSR is a form of stability type and in this case, it affects the large inertia power plants shaft system connected via radial networks with series compensation. Table 2-1 gives the different contributions of key power plant equipment to transient and frequency stability.

Table 2-1: Stability classes and their key parameters

| **Stability Class** | **Key Parameters** |
|---|---|
| Transient Stability | Inertia & Exciter |
| Small Signal Stability | Exciter & Power System Stabilizer |
| Voltage Stability & Control | Reactive Limits, Control +Protection |
| Frequency Stability and Control | Inertia + Governor |

Table 2-2 presents categorised generation plant equipment affected by SSR in the power system.

Table 2-2: Power plant equipment stability classes and their key parameters

| **Equipment** | | **Study Type** | | | | | |
|---|---|---|---|---|---|---|---|
| | | **SSR** | **Transient** | **Small Signal** | **Islanded operation** | **Long term dynamic** | **Voltage** |
| Turbo-Generator | Generator | Y (Yes) | Y | Y | Y | Y | Y |
| | Shaft system | Y | N (No) | N | N | N | N |
| Excitation System | | | | | | | |
| | Automatic Voltage Regulator | N | Y | Y | Y | Y | Y |
| | Stabiliser | N | Y | Y | Y | N | Y |
| | Limiters | N | Y | N | Y | Y | Y |
| | PF/VAR | N | N | N | Y | Y | Y |
| Turbine Control | | | | | | | |
| | Speed governor | N | N | Y | Y | Y | N |
| | Over-speed Control | N | N | N | Y | Y | N |
| | Automatic Governor Control | N | N | N | N | Y | N |
| | Prime Mover | N | N | N | Y | Y | N |
| | Loads | N | N | Y | Y | Y | Y |

Transients in power systems are classifiable into short-term (electromagnetic), medium-term (electromechanical) and long term transients. SSR falls in the short-medium term transients of frequency range of 0.1 - 10 Hz or typical time constants of 10 seconds and 100 milliseconds respectively. The subsynchronous term implies that the mechanical torque is less than the electrical torque. Electrical torque is on the increase due to the ongoing reinforcements and manipulations in the grid network.

SSR exhibits itself as poles and zeros of the system impedance below the fundamental frequency. Dangerous resonances occur if the mechanical torsion modes of the turbine or generator shafts get near the zeros of the system impedance. High subsynchronous voltages due to impedance poles at subsynchronous frequencies lead to another problem of pushing transformers (or reactors) into saturation causes establishing an undamped energy exchange between the transformers magnetic circuits and the series capacitor at subsynchronous frequency range.

An uncompensated transmission system has positive electrical damping but adding series compensation causes negative electrical damping in the subsynchronous frequency range. Torsional interaction with the negative damping effect becomes excessive if the electrical and torsional resonance frequencies nearly coincide as fundamental frequency (50/60 Hz) complements and if the inherent mechanical damping is low. Figure 2.1 shows the pictorial view of the shaft torsional forces [58].

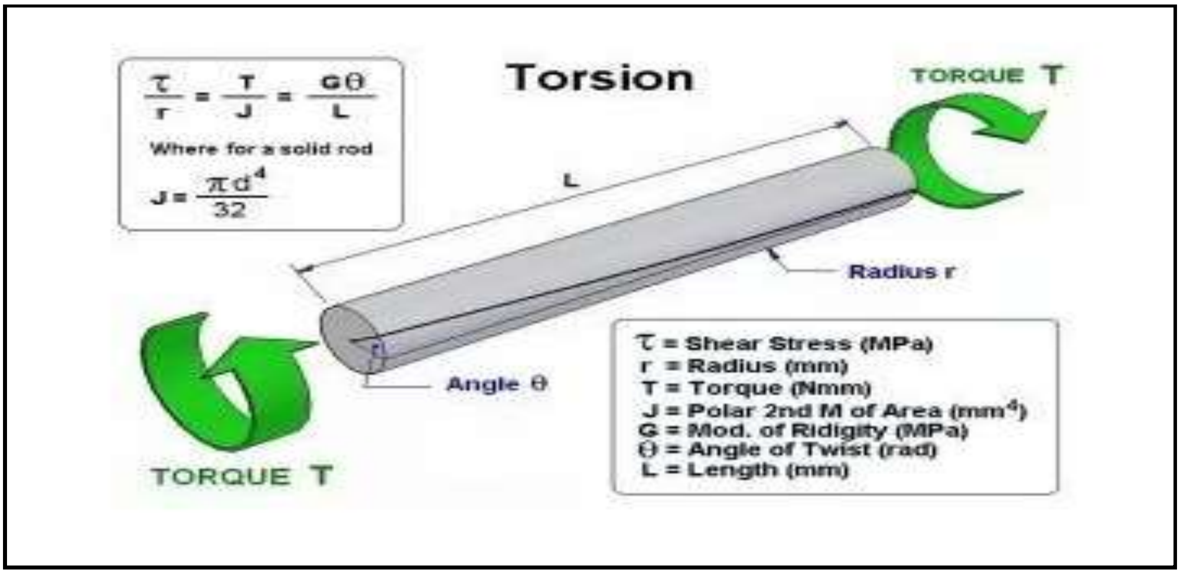


Figure 2.1: Torsional force imposed on turbo-generator shaft pictorial

## 2.2 Origin of subsynchronous frequency- from series compensation

Series compensation introduces inordinate presence of inductance and capacitance at strategic points along the transmission line and during disturbance oscillations the generator will be exposed to frequencies both below (subsynchronous) and above (supersynchronous) the nominal frequency. If the generator connection is through a series compensated electrical system, these natural modes in subsynchronous range when excited yield sustained or growing oscillations produced through air gap fluxes interaction. In a transmission system without series capacitors, these transients are always dc transients, which decay to zero with a time constant that depends on the ratio of inductance to resistance. For networks that contain series capacitors, the transient currents will contain one or more oscillatory frequencies that depend on the network capacitance as well as the inductance and resistance.

Fluxes at these frequencies will induce modulated voltages with corresponding frequencies that will result in currents in the grid. The subsynchronous and supersynchronous currents will interact with the subsynchronous and supersynchronous fluxes generating torques damping or amplifying the mechanical oscillations. The total torsional damping comes dominantly from subsynchronous electrical and mechanical damping. The supersynchronous electrical damping is negligible.

The effect of the disturbance on the rotor is to give rise to two voltages as follows,

$$(f_o + u) \qquad (2.1a)$$

$$(f_o - u) \qquad (2.1b)$$

The two voltages are of opposite sign. Furthermore, for the supersynchronous frequency the generator behaves like an induction motor, since the slip is positive $w_r < w_o$ .

On the other hand, for the subsynchronous frequency ($f_o - u$), the generator behaves similar to an induction generator, since the slip is negative;

$$w_r > w_o \qquad (2.1c)$$

The supersynchronous frequency poses no threat to the system stability, since an induction motor effect absorbs energy from the disturbance. Thus, its effect is similar to the action of a braking resistor. The subsynchronous frequency poses, however, a potential danger to the system stability, because a disturbance injects more energy into the system by its generator action. The induction generator action amplifies with time if the system net resistance is negative thus causing system instability through self-excitation.

If the frequency $(f_o - u)$ of the disturbing force whose resulting subsynchronous frequency is near or equal to one of the natural frequencies of the turbine mechanical system, resonance will occur. The frequency at electrical resonance is,

$$f_{er} = \pm\, f_s\sqrt{\frac{x_C}{x_L}} \qquad (2.3)$$

Where,

$f_s$ is the network's synchronous frequency,

$x_C$ is the series capacitor's reactance and

$x_L$ is the network's total reactance.

Now, since this is the resonant frequency, then any small disturbance in the network will excite modulated currents of $\pm f_{er}$. The positive-sequence component of this stator current induces stator flux at this frequency, which in turn produces rotor currents at subsynchronous frequency of,

$$f_r = f_o - f_{er} \qquad (2.4)$$

Where $fr$ is the electrical frequency of the rotor, $f_o$ is fundamental frequency and $f_{er}$ is the excited frequency at point of resonance [40].

## 2.3 Overhead lines new technology- high temperature, low sag conductors

The difficulty to obtain right of way corridors to construct new overhead lines together with escalating costs has necessitated a look at alternatives that increase the power transfer capacity utilising current wayleaves. Increased thermal rating of an existing line by approximately 50% is achievable by using a replacement conductor with twice the aluminium area or with an additional bundle. However, the increased load increases strain on suspension structures by about 40% necessitating structure reinforcement. High-temperature, low-sag (HTLS) conductors come as a remedy operate at temperatures above 100°C while exhibiting stable tensile strength and creep elongation properties. Replacing original ACSR conductors with HTLS conductors with approximately the same diameter is one method of increasing transmission line thermal rating. Among the choices available for HTLS conductors are aluminium conductor steel supported (ACSS), zirconium alloy aluminium conductor invar steel Reinforced (ZTACIR), gap type heat resistant aluminium alloy conductor steel reinforced (GTACSR), gap type super heat resistant aluminium alloy conductor steel reinforced (GZTACSR), aluminium conductor composite reinforced (ACCR), composite reinforced aluminium conductor (CRAC), aluminium conductor composite carbon fibre reinforced (ACCFR) [41]. The uptake of HTLS method in Southern Africa remains low since it proves to be more expensive than series compensation.

In western China, large-scale wind power is generally bundled with thermal power, and transmitted to eastern China by HVDC systems, in a new concept now known as the wind–thermal bundled system transmitted by HVDC (WTBH). Related analysis of WTBH showed that the integration of DFIG-based wind farms with SSDC could mitigate the TI or SSCI between the turbine and HVDC system [26] [30].

## 2.4 Advantages of series capacitors in transmission lines

a. The power transfer over a line is calculated as:

$$P = \frac{V_S.V_r}{X} * \sin\delta \qquad (2.5)$$

Where, P – Power transferred per phase

$V_s$– sending-end phase voltage

$V_r$ – receiving-end phase voltage

$X$ –Reactance of the line ($X$ =$X_L$ without series capacitor and $X$ = ($X_L$ -$X_C$) with series capacitor)

$\delta$ – phase angle

As shown in the equation, if a capacitor having capacitance reactance $X_C$ is connected in series with the line, $X$ = ($X_L$-$X_C$) becomes the new reduced reactance and it follows that the power transfer will increase.

b. Series compensation reduces δ for same power transfer, to yield better transient stability limits and increased transfer capacity.

c. Series capacitors facilitate the desired the load division between parallel circuits by enabling reactance variation to increase transfer capacity and reduce losses.

d. SC improves voltage regulation and power factor performance. For sudden load variations, series compensation allows an equally automatic reactive power surge giving instant voltage correction. It also increases the available short circuit fault contribution through the line and reduces voltage drop.

The world trend as in [19], [33] is that RES are coming with series compensated lines that offer cheaper evacuation means requiring less transmission investment and offering improved angle as well as voltage stability as shown in Figure 2.2.

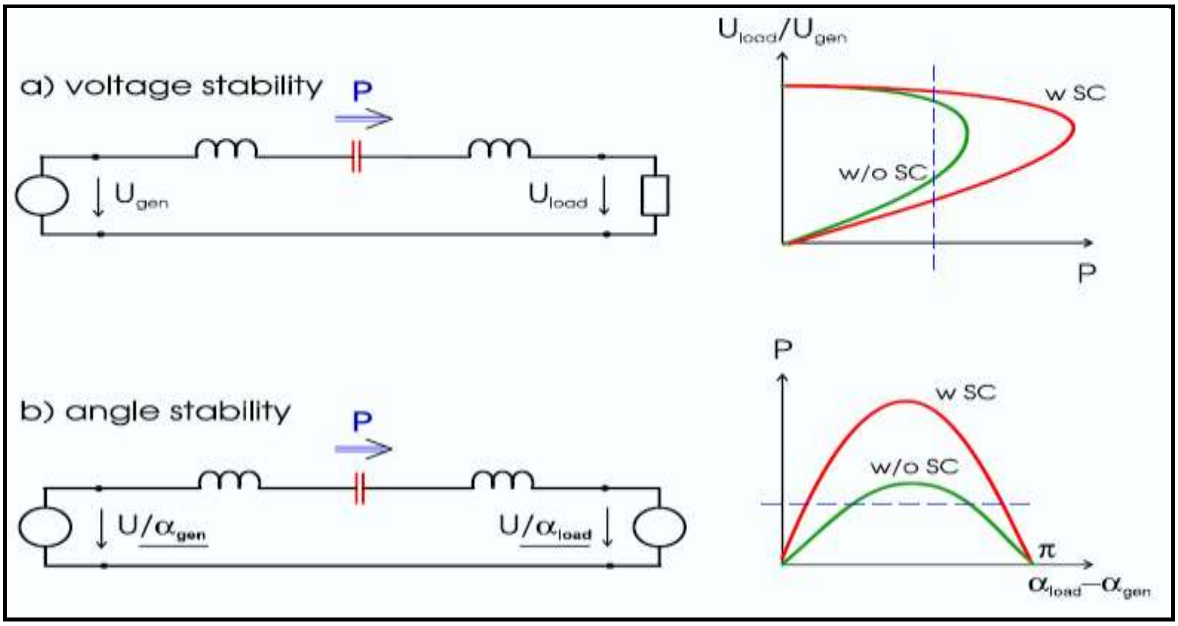


Figure 2.2: The impact of series compensation on voltage and angular stability [50]

## 2.5 Types of subsynchronous resonance

There exist four known types of classical SSR, namely: Induction Generator Effect (IGE), Torsional Interaction (TI), Torque Amplification (TA) and Subsynchronous Control Interaction (SSCI).

The steady-state SSR comprises of IGE and TI. The transient SSR involves TA. A recent manifestation of SSR now frequently referred to SSCI called Subsynchronous Control Interaction citing potential interaction between a series compensated line and the same fast acting controls used on HVDC systems, FACTS devices, and modern power electronics win turbine generators [11].

Subsynchronous frequency currents entering the generator terminals produce terminal voltage components that sustains the currents to produce the self-excitation effect. There are two types of self-excitation, one involving only rotor electrical dynamics (induction generator effect) and the other involving both rotor electrical and mechanical dynamics (torsional interaction).

### 2.5.1 Induction Generator Effect (IGE)

Induction generator effect is the effect of having subsynchronous positive sequence currents in the armature of a synchronously rotating generator [4], a phenomenon referred to as self-excitation. The rotor resistance to the subsynchronous current, viewed from the generator, is negative. The grid network will subject a positive a resistance to these same currents. If the negative resistance of the generator is greater in magnitude than the positive resistance of the network, there will be sustained subsynchronous currents at system natural frequency.

The subsynchronous current generated from the grid interactions flows in the stator of the generator creating a magnetic motive force (MMF) that rotates slower than the generator's MMF. This causes the synchronous generator to act as an induction generator in the subsynchronous frequency range. In this instance, the resistance of the rotor as viewed from the generator terminals, at subsynchronous current, is negative [25].

The slip at subsynchronous frequency is given as,

$$S_{ssr} = \frac{f_{er} - f_o}{f_{er}} \tag{2.6}$$

Since $f_{er} > f_o$ because we are in the subsynchronous range, below the fundamental frequency. From 2.1c, this leads to the corresponding resistance at resonance as a function of the slip to become negative as well,

$$R_{eq}^{ssr} = -\frac{R_r}{S_{ssr}} \tag{2.7}$$

Self-excitation happens when the magnitude of this rotor's negative resistance exceeds the sum of the armature and network resistances at a resonant frequency [4].

### 2.5.2 Torsional Interaction (TI)

TI is an electro-mechanical phenomenon that results in an energy exchange between the electrical system and the mechanical shaft of the generator unit. This occurs when the induced electrical torque setup by the subsynchronous current component in the generator is electrically close to one of the torsional natural frequency modes of the generator shaft at the same time the total damping of the system around this frequency is zero or negative [19]. Figure 2.3 depicts the oscillations that occurring between the turbo-generator and excitation system that results in increased armature voltage.

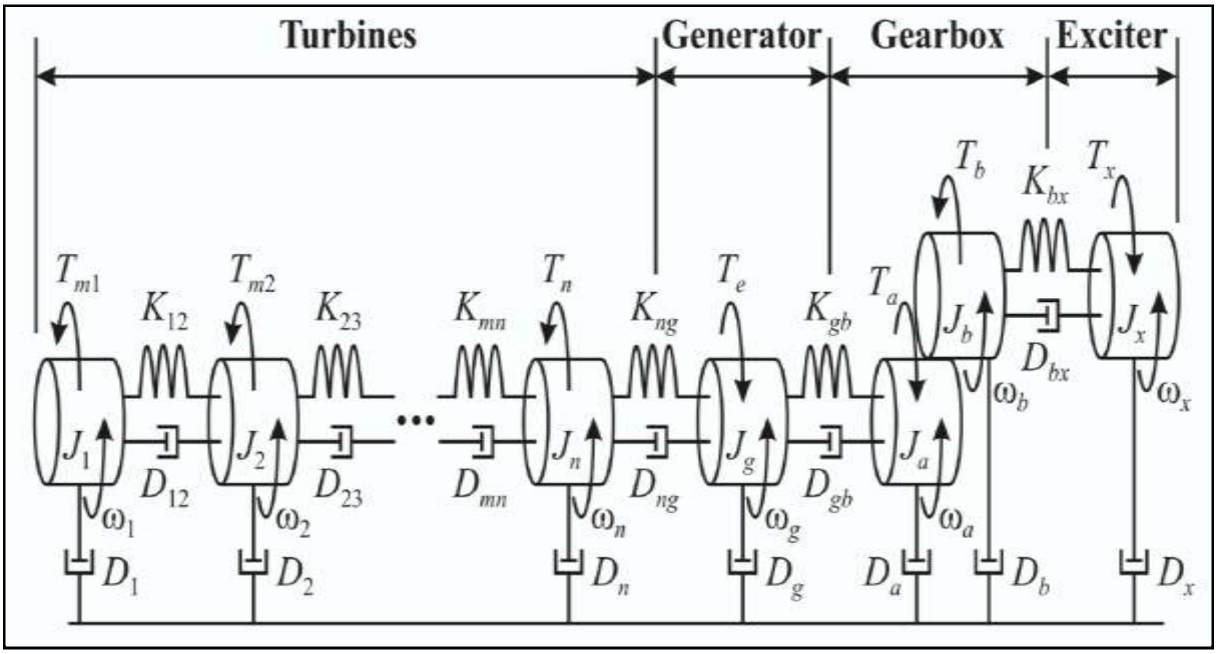


Figure 2.3: Torsional oscillations created under TI

Another view is to treat TI as a condition when the (negative) damping of the series compensated transmission system exceeds the inherent mechanical damping of the shaft at a specific subsynchronous torsional frequency [11]. According to [19], even the interaction of the turbine's system with a neighbouring power electronics controller is TI.

### 2.5.3 Torque Amplification (TA)

System disturbances impose electromagnetic torques on generator rotors, subjecting shaft segments to torsional stresses. Following a significant system disturbance in a series capacitor compensated network, the resulting electromagnetic torque oscillates at the natural frequencies of the network. If this frequency is near of any shaft section, the resulting shaft torques could be much larger than those produced by a three-phase fault in a system without series capacitors [4].

Unlike IGE and TI, the growing rate of TA is high with the oscillating shaft torque reaching damaging level within a short time (0.1 seconds) [12]. In addition, as the non-linearity of the system comes into play, analysis using conventional linearized model will not feasible.

Transient torque amplification (TA) studies carried out in [58] revealed the near effectiveness of blocking filters to reduce torsional stress on the generator shaft.

### 2.5.4 Subsynchronous Control Interaction (SSCI)

SSCI involves the oscillations between a series compensated line and a power electronics controller. These controllers all operate in subsynchronous range. According to [19], destructive energy exchange occurs between turbine or HVDC power converters and series compensated networks. Probable SSR in variable speed wind turbines received attention 10 years ago following SSR conducive contingencies in the power system network. Researchers in [33] investigated the emerging SSR control interaction problem within a DFIG-based offshore wind farm. The recent development of insulated gate bipolar transistors (IGBTs), and other self-commutating high-voltage high-current semiconductor switches, has led to the rise of VSC-HVDC. These devices control both switch turn-on and turn-off allowing a direct current voltage source (hence the name VSC) to be switched between phases. Opportunities exist for integration of the highly flexible VSC HVDC directly as part of the smart grid architecture, harnessing VSCs current and voltage capabilities, especially in unbundled markets with a high penetration of renewables, where operators value new ancillary services to manage variability in power flows [31]. An analysis was done in [35] of the susceptibility of the different wind types to subsynchronous interactions, showing that type one & type four were immune to such phenomena but problems could exist with type two & type three turbines. The related simulation case studies also indicated that wind power plants terminated to series compensated lines experience subsynchronous interaction problems involving purely electrical (SSCI) caused by fast action of rotor-side current controller, or electro-mechanical phenomenon involving turbine rotating masses (SSTI).

## 2.6 Review of SSR analysis methods

SSR occurs when a natural frequency of a series compensated transmission system coincides with the complement of one of the torsional modes of the turbine-generator shaft system. Under such circumstances, the turbine-generator shaft system oscillates at a frequency corresponding to the torsional mode frequency and unless one takes corrective action, the torsional oscillations grow within a few seconds resulting in shaft damage.

The trio of Ankita Singh, Neeraj Verma, and Atul Kumar used an IEEE study framework, comprising of a steam turbine driven synchronous generator supplying power to a series compensated transmission line as shown in

Figure 2.4 [4].

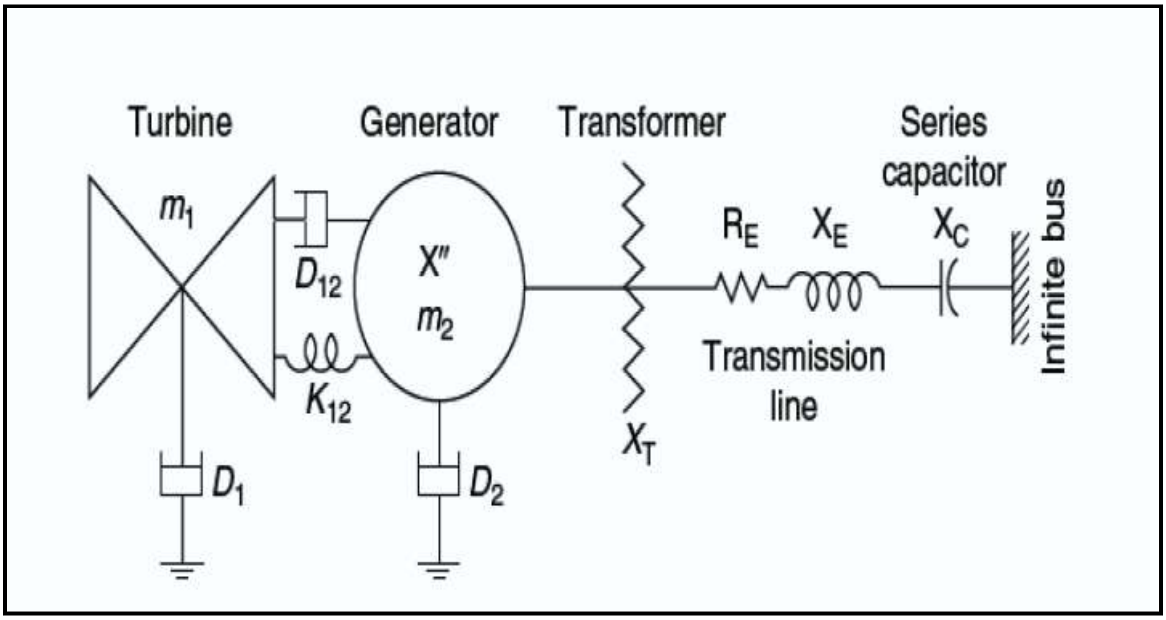


Figure 2.4: IEEE study framework model

Where, M = rotational mass inertia, K = shaft stiffness, D = damping

Several analytical tools in the form of digital computer programs are available for studying SSR problems as outlined below.

### 2.6.1 Frequency Scan Technique

Frequency scans, harmonic penetration, and harmonic power flow are the most popular analysis methods in the frequency domain. Results of frequency scanning studies aid in identifying potential induction generator effect, torsional interaction and transient torque amplification problems. The frequency scan technique determines the system impedance, as a function of frequency, viewed from the neutral of the generator under study. It gives information about the natural frequencies of the system and the tendency toward self-excitation and SSR. This approach is particularly suited for preliminary analysis of SSR problems [4]. The frequency scanning program basic output is the SSR impedance as viewed from behind the generator rotor. In such a case, both the real and the imaginary part of the SSR impedance play an important role in the analysis of SSR problems [42].

For self-excitation, the focus is on the lightly damped modes of the mechanical system that cause instability depending on the amount of negative damping due to torsional interactions. For transient torque, the frequency scanning program provides an indication of potential transient torque problems.

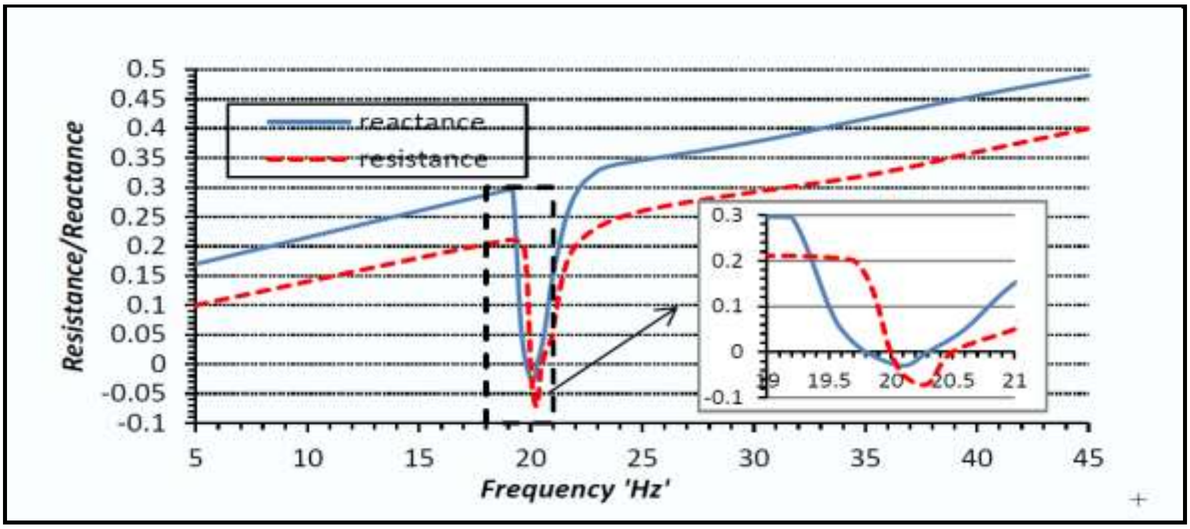


Figure 2.5: Standard frequency scan snapshot

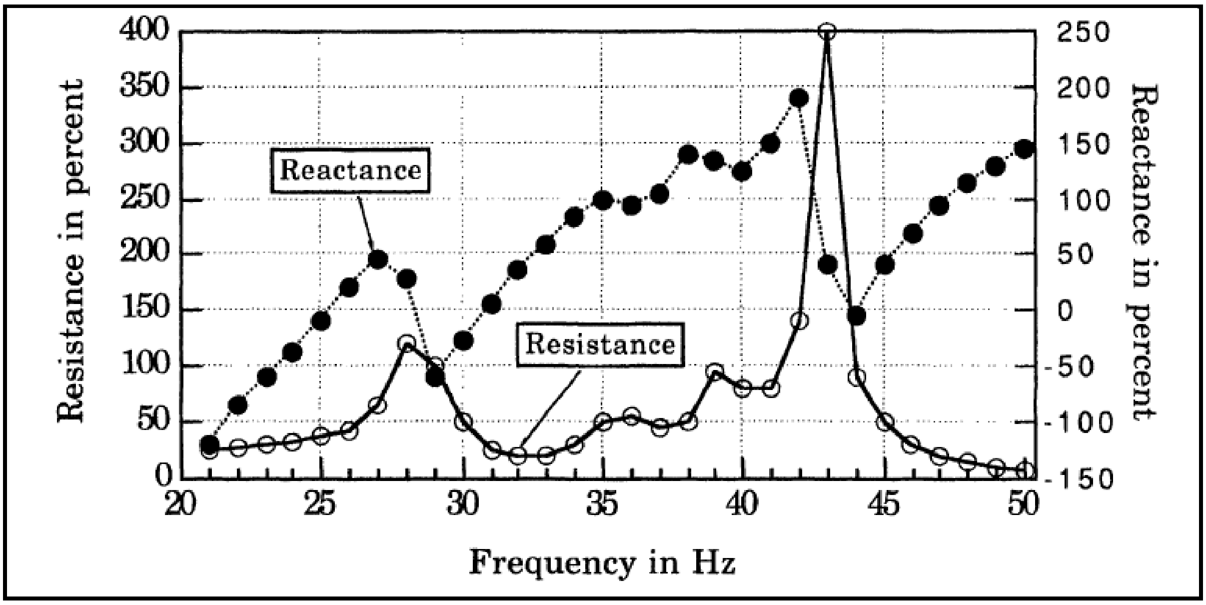


Figure 2.6: Standard frequency scan snapshot

Figure 2.5 & Figure 2.6 show examples of frequency scan shots representing the impedance seen looking into a grid network from the generator. Assessment of many conditions is necessary since the frequency scan results change with different system conditions and on line generators quantity [42].

### 2.6.2 Eigen value Technique

Modal analysis is available only for balanced conditions and in steady state but has an advantage over the frequency scanning method as it provides oscillatory frequencies and damping behaviour information. The analysis gets attractive more by providing the frequencies and the damping at each frequency for the entire system in a single calculation. The only major downside with eigenvalue analysis is the bulkiness especially for a large system.

There are two methods for calculating the eigenvalues:

a) QR or QZ method: calculates all the eigenvalues. This method is preferred for small systems. Commonly used for systems up to 1000 state variables.

b) Arnoldi or Lanczos method: it is a selective modal analysis, suitable for large power systems up to 20 000 busbars. It calculates only a selected number of eigenvalues in the selected area.

The modal analysis solution gives the s-plane insight into the generator's oscillations stability, speed, damping and frequency. The original system is asymptotically stable when the eigenvalues have negative real parts, and unstable when at least one of the eigenvalues has a positive real part. The damping ratio is the eigenvalue decay rate of the amplitude. Power system oscillations fall into local or machine modes (swinging of generator units with respect to the rest of the power system), inter-area modes (swinging of several machines in one part of the power system against machines in other parts), control modes (relating to the generator units and controllers) and torsional modes (linked to turbine-generator shaft system oscillations). State space representation and linearization are other available methods to analyse power system small signal stability [36].

For every eigenvalue, there are associated left and right orthogonal eigenvectors. The right eigenvector identifies the "observability" since it gives the degree of activity of a state variable when a particular mode is excited. The left eigenvector identifies the "controllability" since it gives the contribution of a variable to the activity of a particular mode.

### 2.6.3 Digital Time Simulation Technique

Digital time simulation programs analyse a broad range of machine-network interaction problems, including SSR using a systematic numerical integration to solve the set of differential equations (linear and nonlinear) representing the overall system under study. These techniques allow detailed modelling of machines and system controllers as well as circuit breaker action, transient faults, and other types of switching event. The ability to model nonlinear devices accurately is a distinct advantage of this analytic tool. However, it is not well suited principally for the study of self-excitation [4].

### 2.6.4 Electromagnetic Transients Program (EMTP)

Electromagnetic Transients Program (EMTP) is for numerical integration of the system differential equations. Unlike a transient stability program, which usually models only positive sequence quantities representing a perfectly balanced system, EMTP is a full three-phase model of the system with much more detailed models of transmission lines, cables, machines, and special devices such as series capacitors with complex bypass switching arrangements. Moreover, the EMTP permits nonlinear modelling of complex system components. It is therefore well suited for analysing the transient torque SSR problems [59].

## 2.7 SSR mathematical concepts

### 2.7.1 SSR mathematical concept definition

As already defined, SSR refers to an electrical condition in the power system whereby the electric network exchanges energy with the mechanical system (turbine generator system) at one or more of the natural frequencies of the combined system below the synchronous frequency. The definition inherently includes any power system condition that provides the opportunity for energy exchange at any subsynchronous frequency [59].

Series capacitor compensated lines are a perfect example that lead to natural modes of subsynchronous oscillations in the power system network as follows,

$$w_n = \sqrt{\frac{1}{LC}} = w_B\sqrt{\frac{X_C}{X_L}} \qquad (4.1)$$

Where $w_n$ is the associated natural frequency,

$w_B$ is the system base frequency,

$X_C$ is the equivalent per-unit capacitive reactance,

and $X_L$ is the equivalent per-unit (p.u.) inductive reactance.

These frequencies appear to the generator rotor as modulations of the base frequency, giving both subsynchronous and supersynchronous rotor frequencies. The subsynchronous frequency normally interacts with one of the natural torsional modes of the turbine-generator shaft, thereby setting up the conditions for an exchange of energy at a subsynchronous frequency, with possible torsional fatigue damage to the turbine-generator shaft [59].

### 2.7.2 SSR link with Park's transform

For SSR studies, we are interested in the prime mover (turbines) and generators and their primary controls, the speed governors and excitation systems.

The computation of eigenvalues and eigenvectors for the study of SSR is an excellent method of providing crucial information about the nature of the power system [59].

A general expression for the current in a simple series R-L-C network is,

$$i_{t} = K[A\sin(w_1 + u_1) + B\,e^{-\alpha w_2 t}\sin(w_2 t + u_2) \tag{4.2}$$

Where $i$ is current, A & B are constants, $w_1$ is the driving voltage and $w_2$ is the resultant frequency determined by the network configuration.

Park's transformation describes the reflection of currents flowing in the stator windings end up in the generator rotor. This transformation is very fundamental to SSR studies. It reflects how the SSR problem links between the grid and the synchronous machines explaining part of the problem roots.

This transformation converts vectors in balanced two-phase orthogonal stationary (stator) system into orthogonal rotating (rotor) reference frame. The transformation basis on the two-axis theory and uses a frame of reference on the rotor as follows,

$$P = \frac{2}{3}\begin{bmatrix} \cos(\theta) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ \sin(\theta) & \sin(\theta - \frac{2\pi}{3} & \cos\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{4.3}$$

Where;

$P$ = Park's transformation, $\theta$ = angle between one the rotor current phases and reference horizontal axis (q).SSR detection and protection by high precision a/c filters (HPAC)

### 2.7.3 Overview - system protection intelligent electrical devices

As alluded to earlier in the introduction, there is a new view to SSR management largely because the power system is in transition. Electricity generation, transmission and distribution practise emphasizes on least cost planning and robust system security. This requires major technical input, and the attitude of engineers initially from the 18th century onwards was to mitigate and eliminate the probability of SSR events.

Now, the rising technology in power system intelligent electronic devises (IEDs) allows for discriminatory handling of SSR through conventional protection schemes. SSR

Many protection functions are now available in numeric relays. The list includes but not limited to high impedance differential, line distance protection, under-impedance protection, backup protections , stator ground fault protection, pole slip function, loss and under excitation, reverse or low forward power function, Negative phase sequence functions, accidental energizing, thermal overload, overfluxing, various frequency protections, basic protection functions, metering, programmable logic and disturbance recording and recently SSR protection [17].

The approach of using IEDs to detect, suppress and isolate SSR detrimental incidences will become important [46].

Only a few papers such as [48], [49] have highlighted on the new approach to the SSR problem but are lacking on mathematical background having largely referenced manufacturer's information guides. The available literature is not specific and shies away from explaining the particular philosophy behind the algorithm that drives this new SSR management approach. This dissertation highlights that gap and positions the SSR phenomenon in the rising RES connected to series compensated constellations (RESISC).

Inevitably, the power system exposure to SSR will remain a concern for heavily loaded interconnected networks.

### 2.7.4 SSR management module in protection relays

One protection method was introduced in [54] of a centralized protection coordinator and several distributed protection relays (DPRs) for SSCI protection in wind farms. One other protection scheme was suggested for a nuclear plant in [48]. Rezaei [55] [56] and [57] raised interesting facts on impact of SSR on protection relays in the power system due to the ferroresonance problem as a result of the nonlinear increase of voltage or current. SSR causes the overcurrent, differential and distance relays to pick up and operate on erroneous currents resulting in current imbalances and distortions of relay protective zones. It possible to develop a relay adaptive algorithm based on sub-harmonic measurements and ferroresonance analysis to discriminate SSR events.

### 2.7.5 HPAC Filter – Prony Analysis

Gaspard Riche de Prony developed the algorithm for extracting phasors from an AC waveform in 1795. Until now, the filter is more precise than the Discrete Fourier Transform (DFT) by avoiding sampling rate adjustment errors. Extraction of desired information from a uniformly sampled signal allows for the estimation of frequency, amplitude, phase and damping components of a signal, by solving a set of linear equations for the coefficients of the recurrence equation that the signals satisfy [60].

The algorithm involves four critical settings, with an example of the graphical representation for the filter length set out in Figure 2.7 & Figure 2.8 .

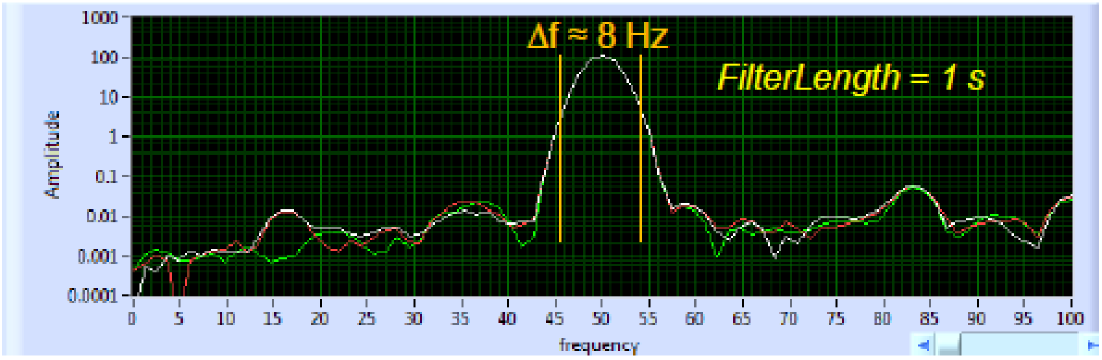


Figure 2.7: Filter length analysis and visual impacts comparison - 1 second

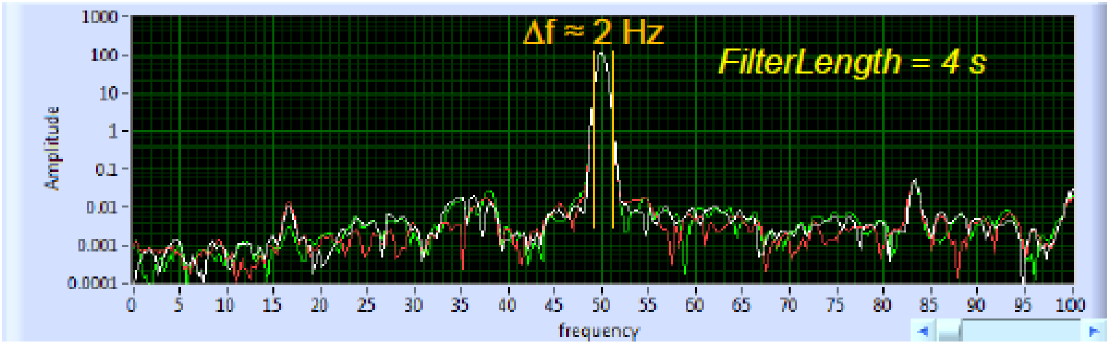


Figure 2.8: Filter length analysis and visual impacts comparison - 4 seconds

In this case, set-frequency is the required frequency extracted from the input signal, frequency bandwidth allows broadening the bandwidth around the set-frequency, filter length defines the natural pass band and overlap specifies the percentage previous filter data introduced in the next successive calculation.

### 2.7.6 Case study on the use of HPAC Filter in Swedish Grid [49] and other approaches

Relay manufacturers are on the narrow path spearheading SSR protection filters, which explains why there is scarce literature amongst scholars on development of the HPAC algorithm. This dissertation closes this gap.

Sweden has been crucial in development of the SSR protection functionality in the 670 relay series. They did pilot test in their network to evaluate the phenomena offline. Two 670 series relays (IEDs) were installed in their 400kV network which would record and store files when triggered from existing SSR protection. One at 400 kV overhead line going from SS Stackbo towards the north, and of cause one on a 1.17GW generator at NPP Forsmark3 as shown in Figure 2.9.


Figure 2.9: Sweden network under SSR pilot tests

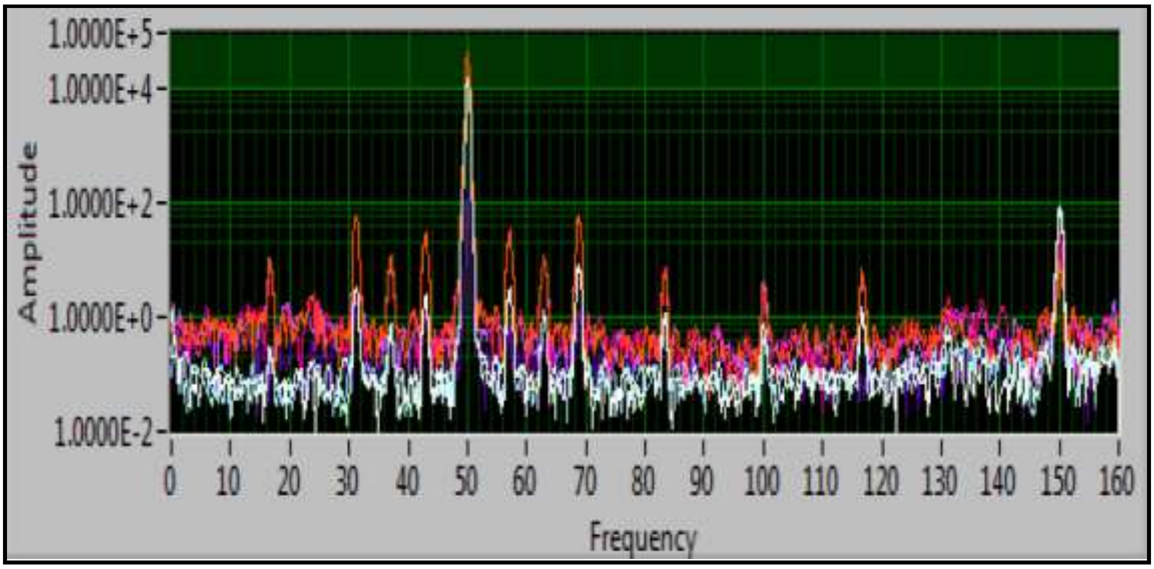


Figure 2.10: SSR signals recorded in the pilot distance relays

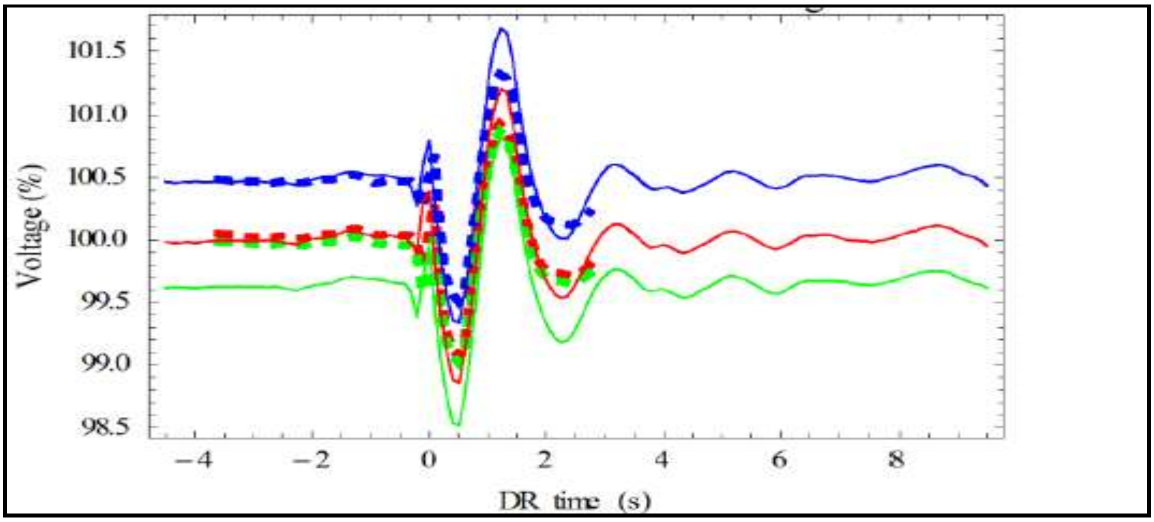


Figure 2.11: Voltage performance at Forsmark (full lines) and Stackbo (dashed lines)

The observation was that SSR transient events trigger SSR but the HPAC filter is able to detect SSR activity by software. Both relays at Stackbo and Forsmark shown in Figure 2.9: Sweden network under SSR pilot tests saw the event caused by a quick 300 MW ramp down of Fennoskan link.

At Stackbo, through an 8s analysis they noticed three frequency pairs as shown in Figure 2.10;
32 & 68 Hz, with a shaft mechanical frequency of 18 Hz,

38 & 62 Hz, with a shaft mechanical frequency of 12 Hz,

42 & 58 Hz, with a shaft mechanical frequency of 8 Hz.

Figure 2.11 is a snapshot of the voltage performance at fundamental frequency with full lines representing Forsmark and dashed lines representing Stackbo.

# CHAPTER 3

## 3. SSR & PENETRATION OF RENEWABLES IN SAPP

There has been an increase in renewable energy projects in Southern African Power Pool (SAPP) with South Africa leading the pack with approximately 1.98 GW in wind and 1.474GW in solar. South Africa's recent integrated resource plan (IRP) proposed that by 2030 the installed PV and wind capacity in South Africa will be 7.0GW and 9.6GW respectively [51]. Naturally, these inevitable changes bring about a different grid dynamics.

### 3.1 SAPP RES penetration

RES grid penetrations will reduce reliance on fossil fuels thereby reducing emissions. As already mentioned, series compensation is likely to accompany these new generation injections. Subsynchronous resonance studies focus on transmission systems with series compensated overhead lines near or connected to a synchronous generation plant. Series capacitors interact with generator mechanical systems, exciting torsional vibration modes that potentially lead to severe shaft system damage. SAPP is yet to comprehend the SSR problem.

Currently in SAPP, the installed generation is 67,190 MW, with an operating capacity of 60,719MW against a demand and reserve of 57,762MW [52]. Meanwhile in the SAPP region, a similar increase in renewable energy penetration is evident as shown in Figure 3.1 & Figure 3.2.

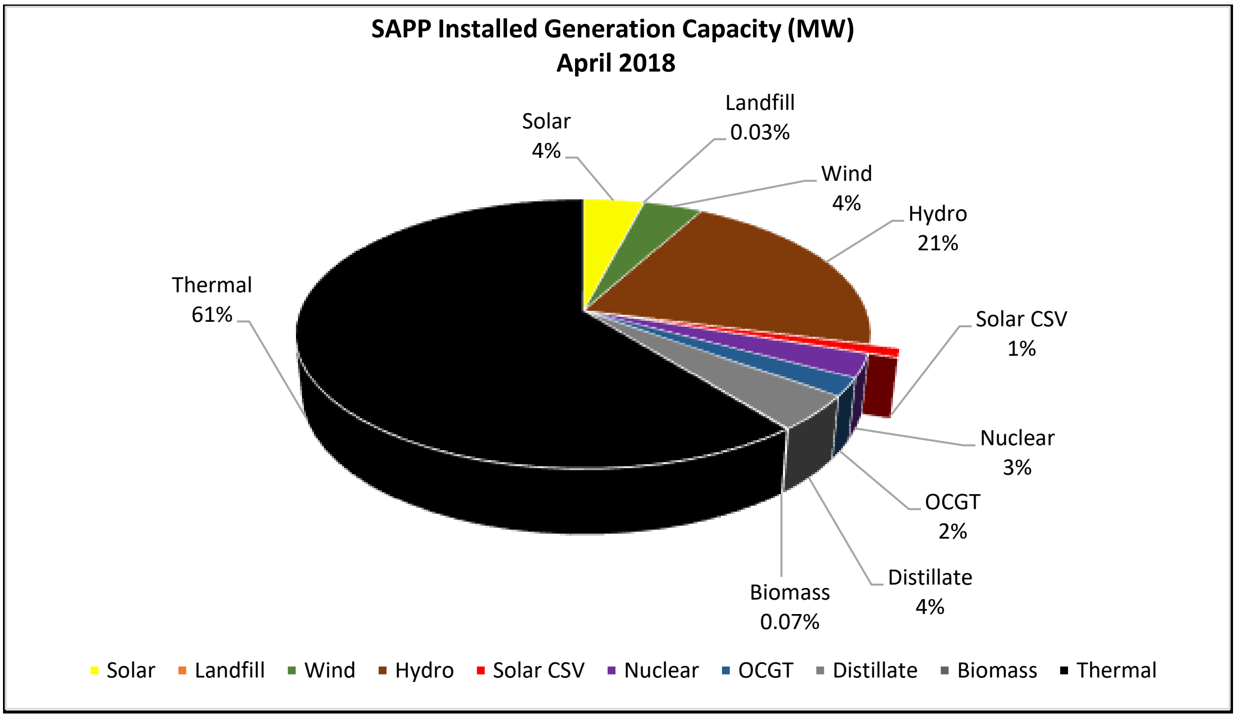


Figure 3.1: SAPP (current) installed generation capacity – April 2018 [52]

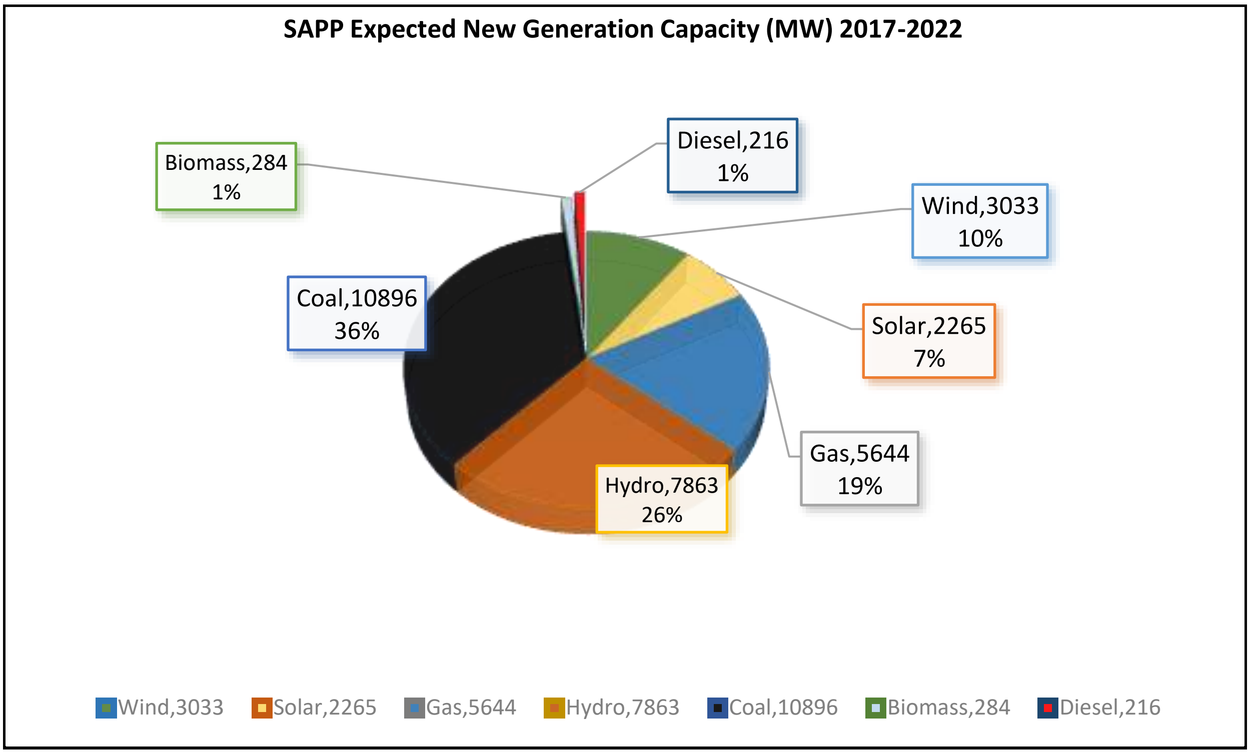


Figure 3.2: SAPP mid-term expected generation capacity: 2017- 2022 [53]

Power utility transmission planners and operators are being forced by the rising demand to look into enhancing generation and transmission transfer capacities at minimum costs. Series compensation is a readily available solution for power evacuation.

### 3.2 Challenges in SAPP RES penetration [52]

Investment in electricity infrastructure remains a challenge for most African countries. This is because generation and transmission technologies are expensive, and in most cases, require imported engineering consultancy and external funding. Vast intercontinental distances render the procured goods or services costly. African utilities more often than not find themselves locked in slow or non-performing engineering-procurement-construction (EPC) + funding contracts. Some projects are included within bilateral memorandum of understandings or grant proposals that never reach financial closure. Procurement of external financing, equipment or intellect has its lead-time challenges that stall project developments. Environmental, legal and social compliance are part of the reasons why big projects delay in their implementation. Project sponsors demand mandatory project disclosures and strict environmental and social management planning without which they pull out funding. Besides weighing heavily on a utility's own capital budget, power projects delivery and related risks are complex and volatile. Oftentimes, feasibility studies bankability dictate the need for technical assistance or that the engineering services be provided externally by experienced consultants. Without new infrastructure investment or uprates, RES will have to connect into aged constricted systems prone to outages. The rate of infrastructure replacement is slower than equipment decay of remaining life, hence the foreseen uptake of cheaper technologies like series compensation utilising existing setups.

### 3.3 SAPP peak demands overview

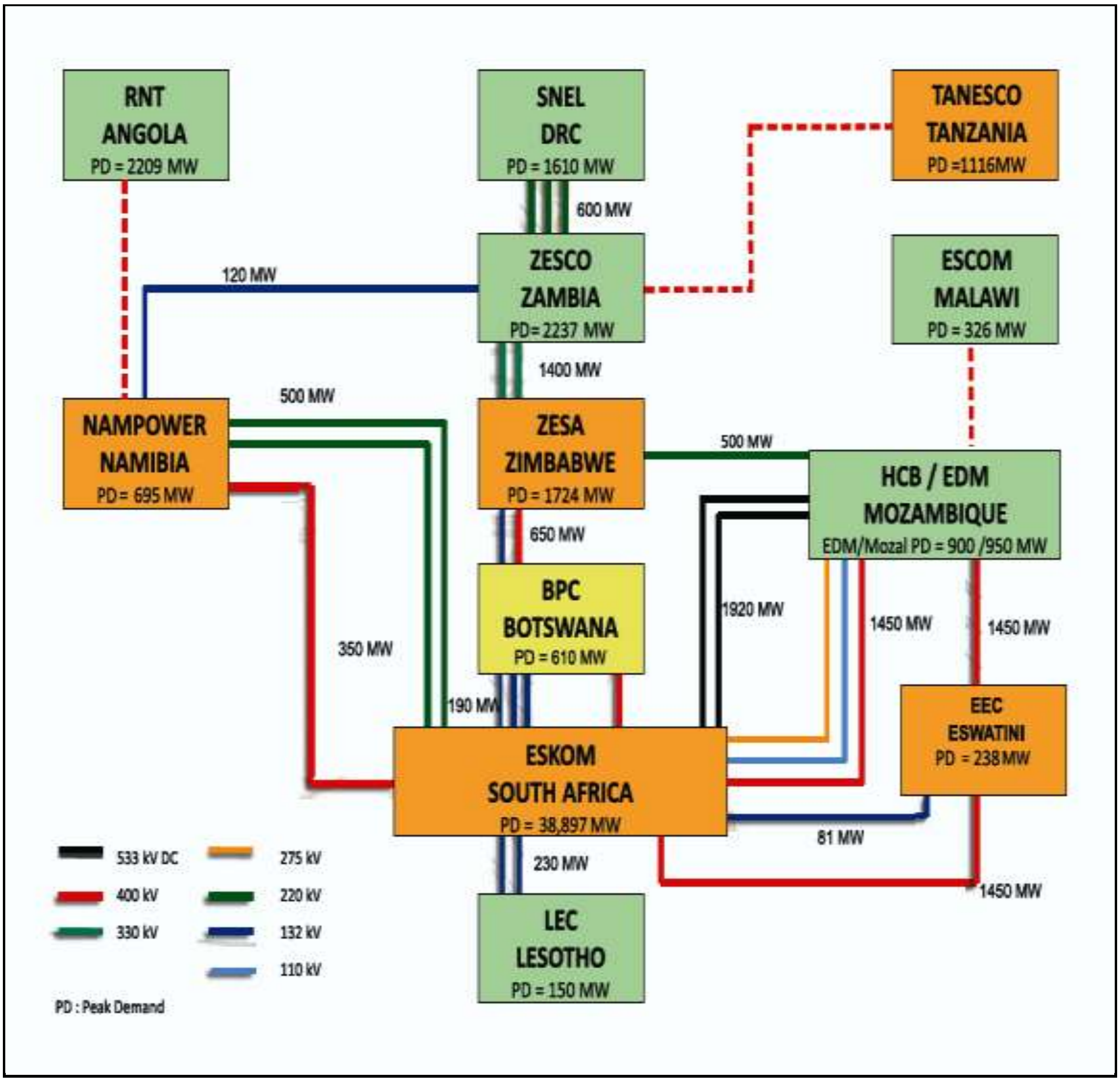


Figure 3.3: SAPP peak demand overview [52]

Electromechanical oscillations are natural to interconnected power systems and the resultant oscillatory frequencies depend on the network system configuration.

As highlighted in Figure 3.3, regional interconnections are the basis for trading corridors that support industrial development and enhance energy security. Potential power flow corridors have been identified to facilitate renewable energy trade between SAPP and EAPP member countries. Series compensation technology assists to unlock transmission paths bottlenecks thereby supporting growing penetrations of RES. This dissertation is key for such an environment where the SSR problem will need proper management.

# CHAPTER 4

## 4. METHODOLOGY AND APPROACH

The IEEE First Benchmark Model (FBM) is subjected to both dynamic time domain transient stability analysis and frequency domain modal analysis.

### 4.1 Simulation model

The IEEE Working Group on Subsynchronous Resonance created the FBM in 1977 for applicable simulations purposes. As shown in Figure 4.1, the system consists of a single generator rated 803.16MW connected to a 500kV voltage source infinite bus through a single 500kV series compensated line operating at 60 Hz.

Most of the new RES devices integrate to the grid utilizing the voltage source converter that quickly adjusts to keep constant current. This makes them better off than conventional generation in terms of transient stability performance. However, this ability of fast active and reactive power exchange with the grid makes wind and solar in particular of interest to the SSR problem

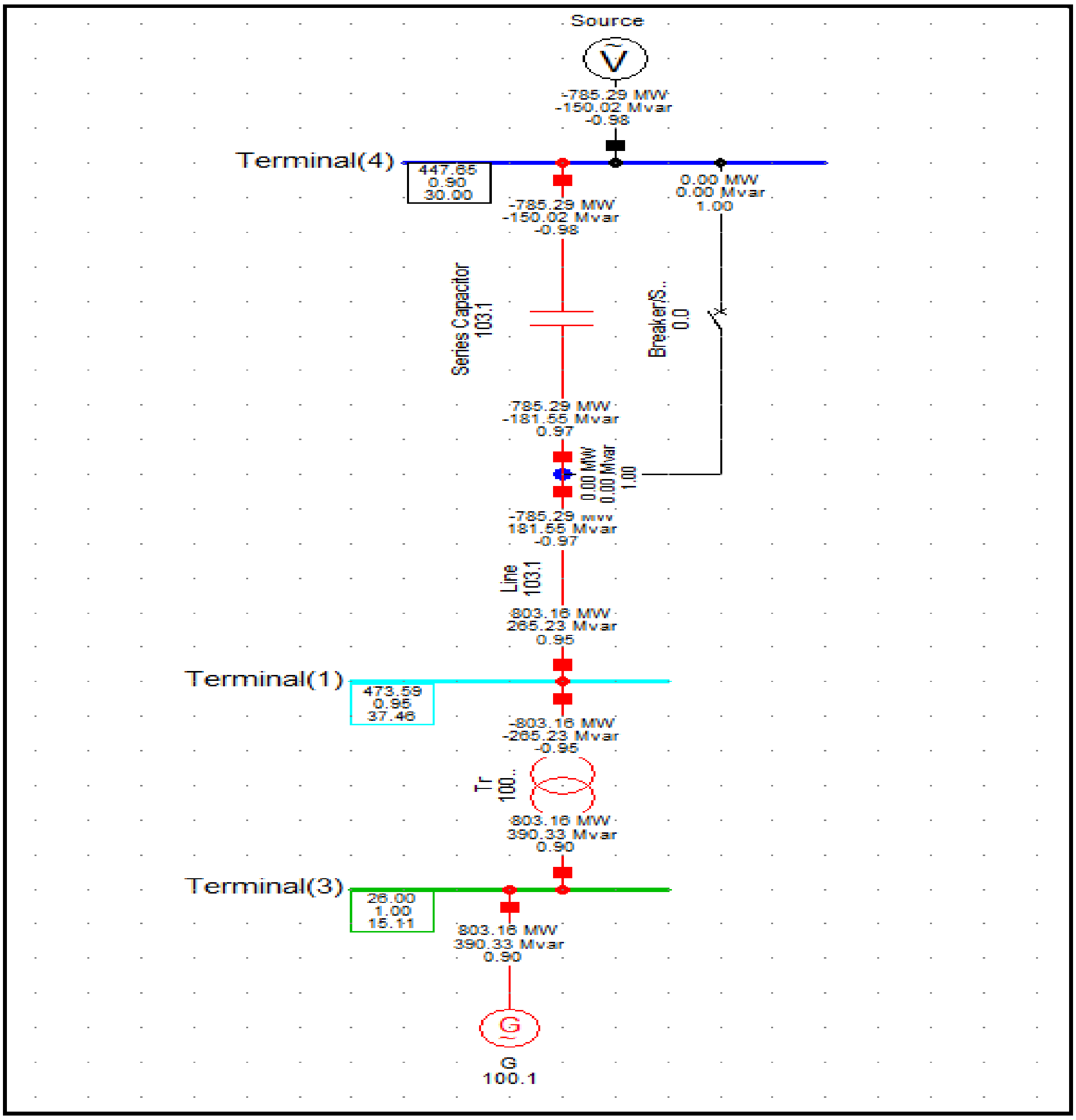


Figure 4.1: Unmodified IEEE FBM with aggregated generation

## 4.2 Procedure for DIgSILENT time domain transient stability simulations

The FBM is the basis of the transmission topology in this study. In this system, different generators types connect to the infinite bus through a 500kV series compensated transmission line. Penetration levels vary by changing the active power output of the connected generator in individual or aggregated configurations, which undergo comparison for effective evaluation.

In order to test whether subsynchronous resonance has a direct proportional impact on increasing levels of renewable energy integrating to the grid, the following configurations will be simulated;

1. Synchronous machine generator – (aggregated conventional generators e.g. thermal – 200MW, 400MW, 600, 800MW).
2. Wind power system (aggregated wind farm of different magnitudes – 200MW, 400MW, 600MW, 800MW).
3. PV solar power (aggregated PV system of different magnitudes – 200MW, 800MW).
4. STATCOM integration
5. Hybrid scenarios.

The procedure followed is to connect the different studied renewable energies to the FBM in different scenarios. A temporary three phase fault with a duration of 0.075 seconds is applied at terminal four at t = 0 seconds in order to excite induction generator effect (IGE) and torque amplification (TA) SSR effects at subsynchronous frequencies. The 75 milliseconds comes inherently defined in the FBM. A fault duration of 100 milliseconds stands as a separate scenario. The fault duration impact requires evaluation since circuit breakers have different extra margin allowance to break asymmetrical currents from fault initiation, breaker initiation, primary contacts parting and final arc extinction.

Table 4-1: Comparison of signal amplitudes after applying 75ms & 100ms fault durations

| **Equipment** | **Level** | **Base Case – 75 ms fault** | **Base Case – 100 ms fault** |
|---|---|---|---|
| **Generator torques(p.u.)** | Max | 1.384 | 1.639 |
| | Min | 0.505 | 0.388 |
| **Shaft torques(p.u.)** | Max | 1.495 | 1.772 |
| | Min | -1.495 | -1.772 |

As demonstrated by the amplitudes presented in Table 4-1, the maximum and minimum amplitudes for the generator torques differ by 25.5% and 11.7% per unit for the 75ms and 100ms cases respectively. For the shaft torque, the maximum and minimum amplitudes all differ by 27.7%

for both the 75ms case and the 100ms case. This analysis shows there is no significant difference in applying the 75 ms or 100 ms and Appendix D captures the simulation plot outs.

### 4.3 Procedure for the eigenvalue modal analysis (small signal stability).

Small signal stability refers to the ability of a power system to maintain synchronism when subjected to small disturbances such as continuous and small variations in load and generation. Instability arises due to lack of synchronising torque, which manifest as widened generator rotor angle or lack of sufficient damping torque seen as rotor angle oscillations [36].

From the literature review, eigenvalue analysis is a recommended exercise to validate and verify the SSR modes behaviour after transient disturbances. Modal analysis in DIgSILENT subjects the FBM to small signal stability analysis regarding SSR.

The modal analysis simulation protocol entails,

1. Running the eigenvalue analysis for the base case i.e. the unmodified IEEE FBM.

2. Running the eigenvalue analysis for the hybrid scenario having ramped up PV power in parallel to the synchronous generation (200MW and 800MW simulation cases)

3. Running the eigenvalue analysis for the hybrid scenario having ramped up wind power in parallel to the synchronous generation (200MW and 800MW simulation cases).

4. Running the eigenvalue analysis for the STATCOM integration into the FBM

The decision to select the particular cases as outlined from number 1- 3 above was to achieve definite, clear and effective comparison of the synchronous, PV and wind generator in terms of small signal stability when excited with SSR. The cases with power ramps in between 200MW and 800MW were not included intentionally to avoid visual graphic impairments for easier comparison.

# CHAPTER 5

# 5. SIMULATIONS RESULTS AND DISCUSSIONS

## 5.1 Introduction

This chapter presents the simulation results for the base case, photovoltaic, wind power and FACTS scenarios. It looks at the comparison between the performance of the unmodified FBM and performance of the modified system after integrating PV power and wind power in ramps ranging from 200MW to 800MW. The SSR mitigation impact of incorporating a FACT devise is also evaluated. The assessment of these scenarios is through transient stability analysis and validation by modal analysis to determine the impact of RES injections into the FBM system in suppressing SSR oscillations under various configurations (case 1 to case 5).

## 5.2 Base case simulation results

### 5.2.1 Case 1a: Base Case- synchronous generator scenario

Figure 4.1 shows the unmodified FBM in DIgSILENT where a synchronous generator acts as the power source with a voltage source connected at the remote terminal end as stated in chapter 4. This acts as the base case, the unmodified FBM. The synchronous generator will deliver 803.16MW at 26kV generation voltage into a step-up generator transformer that connects to a 500kV busbar terminal. At the remote end there is an AC voltage source connected via a series compensated transmission line. A temporary three phase fault with a duration of 0.075 seconds is applied at terminal four at t = 0 seconds in order to excite IGE and TA subsynchronous effects. This is the similar fault applied for all the other following study case scenarios involving wind and solar PV modified configurations.

The responses of the synchronous generator parameters and shaft torques are as shown in Figure 5.1 & Figure 5.2 respectively. Exporting the simulation results from DIgSILENT and plotting in

Microsoft Excel provides better representation quality. As an example, Appendix C captures a detailed Excel file for the base case.

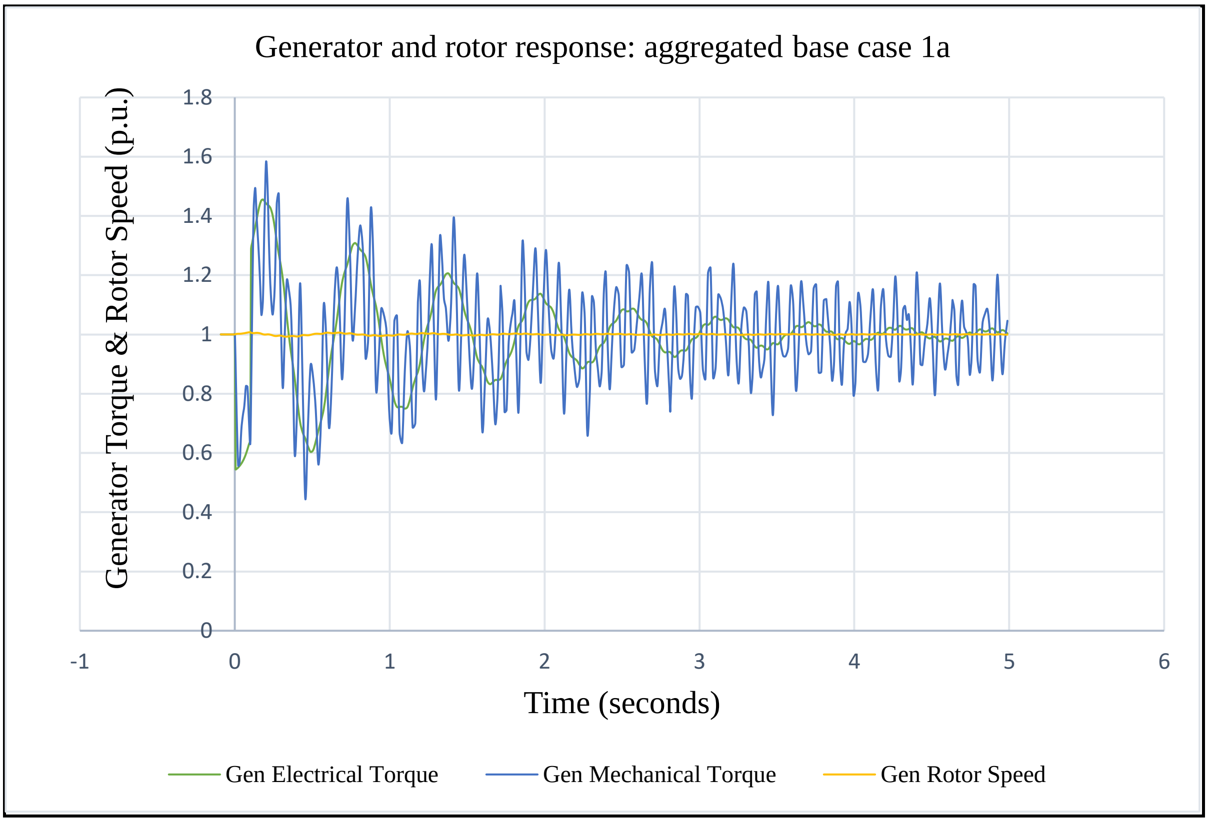


Figure 5.1: Generator response for unmodified IEEE FBM with aggregated generation.

Figure 5.1 captures a constant rotor speed of one (1) per unit. The electrical and mechanical torque oscillatory behaviour is similar and remain undamped beyond the measured 5 seconds undulating between amplitudes of 0.4 to 1.6 per unit.

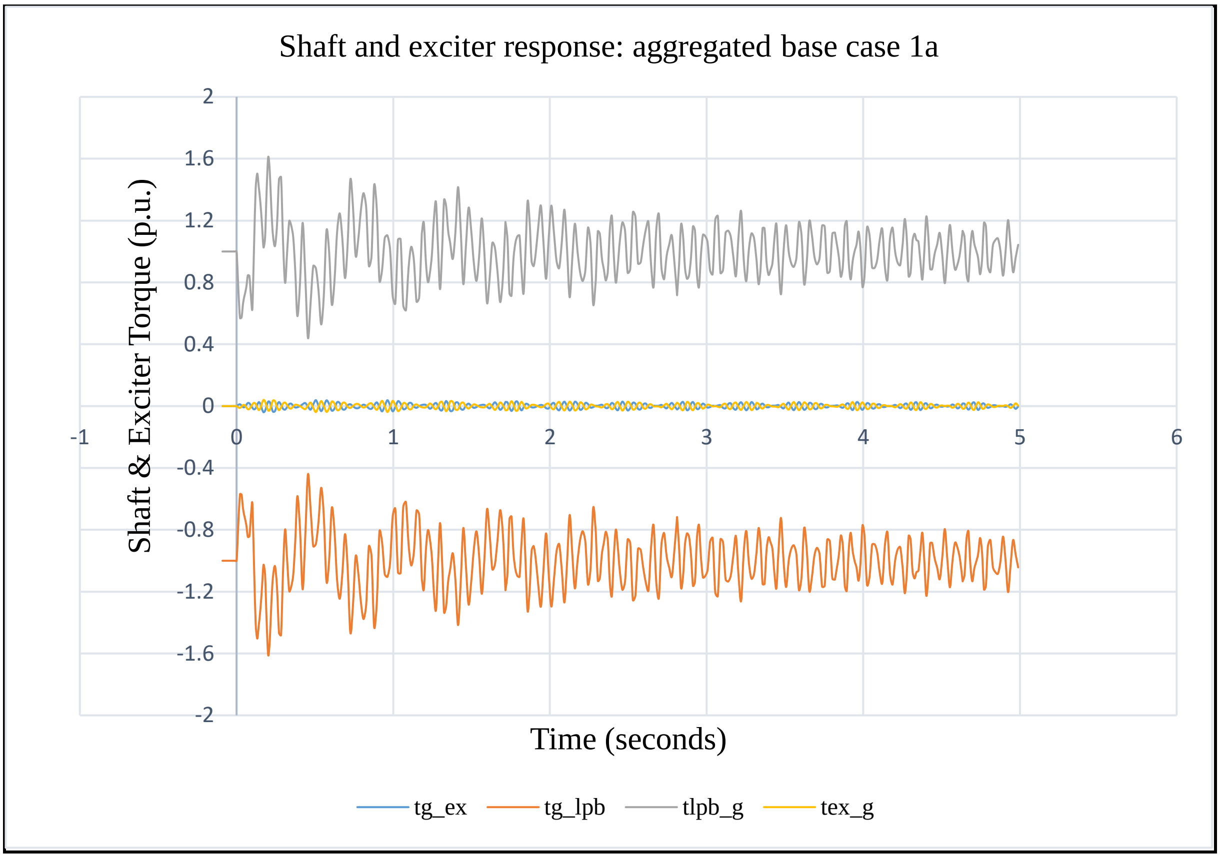


Figure 5.2: Shaft and exciter response for unmodified FBM with aggregated generation

In Figure 5.2, the shaft parameters i.e. tg_lpb & tlpg_g (shaft torques under the turbine's low-pressure stage b) had similar amplitudes oscillating within 1.2 per unit, and the exciter parameters i.e. tg_ex & tex_g (exciter torques under the turbine's low-pressure stage b) were almost similarly flat with small undershoots and overshoots about the x –axis. More detail of the exciter and shaft parameters is given in Appendix A.

It is also imperative, at this point, to assert whether the aggregated set of generators with setup as in Figure 4.1 will vary from a multiple generator setup as depicted in Figure 5.3.

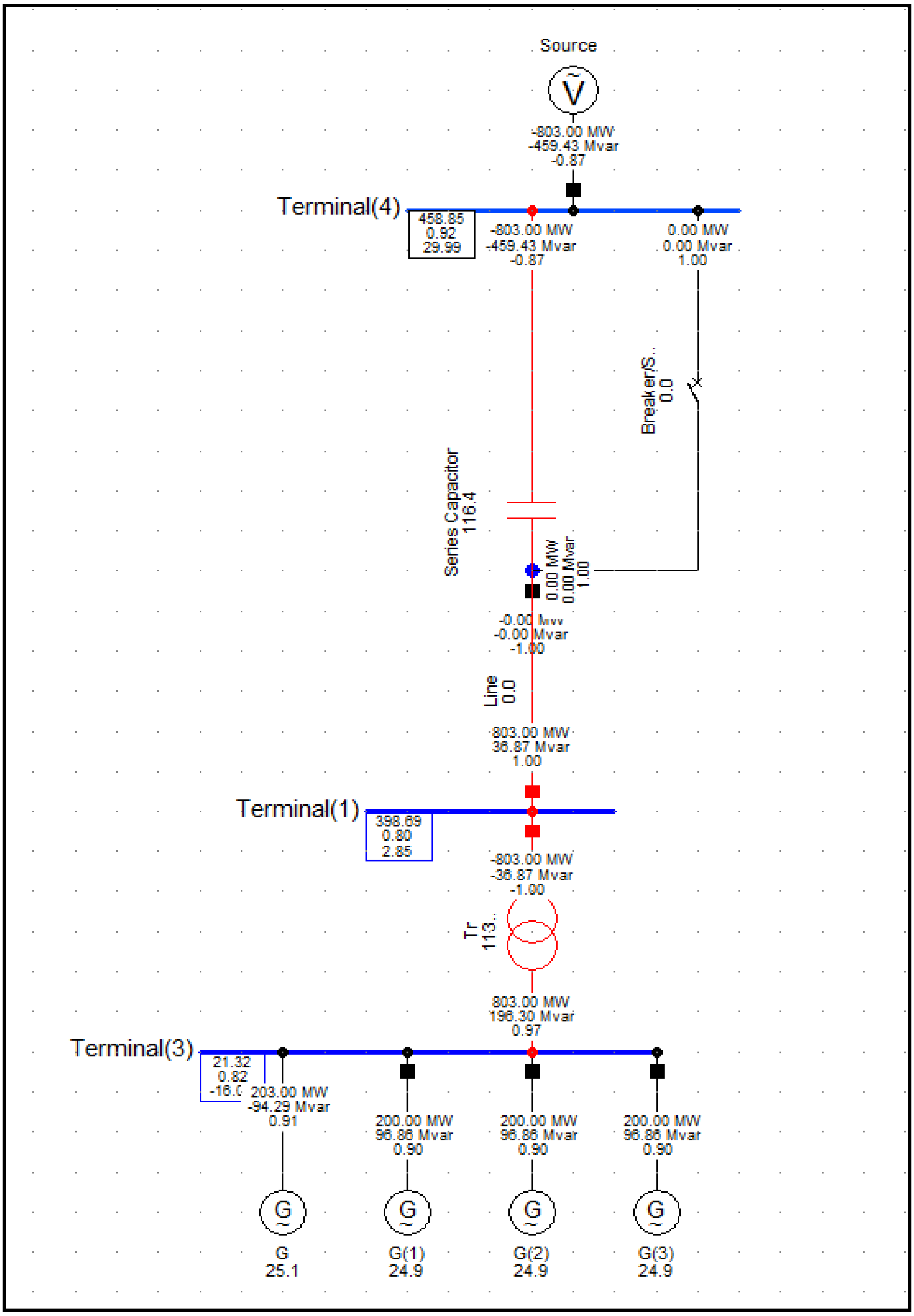


Figure 5.3: Modified IEEE FBM with multiple set of generation

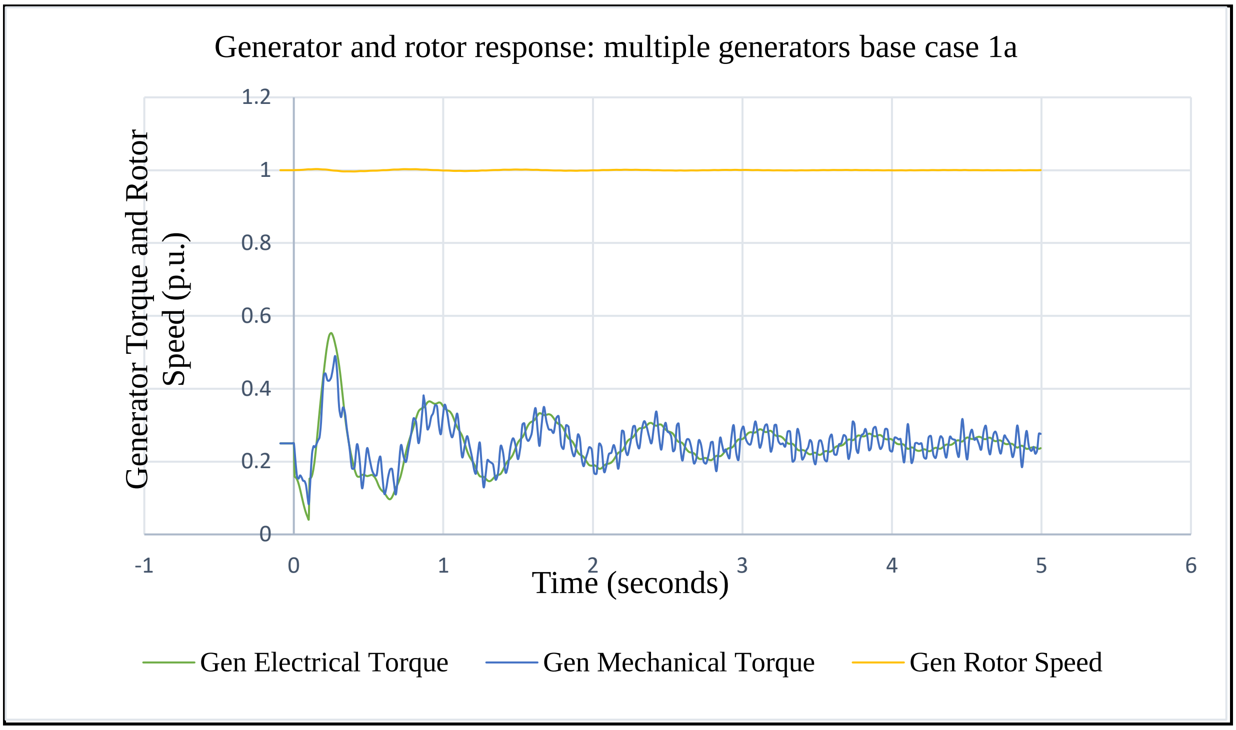


Figure 5.4: Generator and rotor response for unmodified FBM with multiple generators

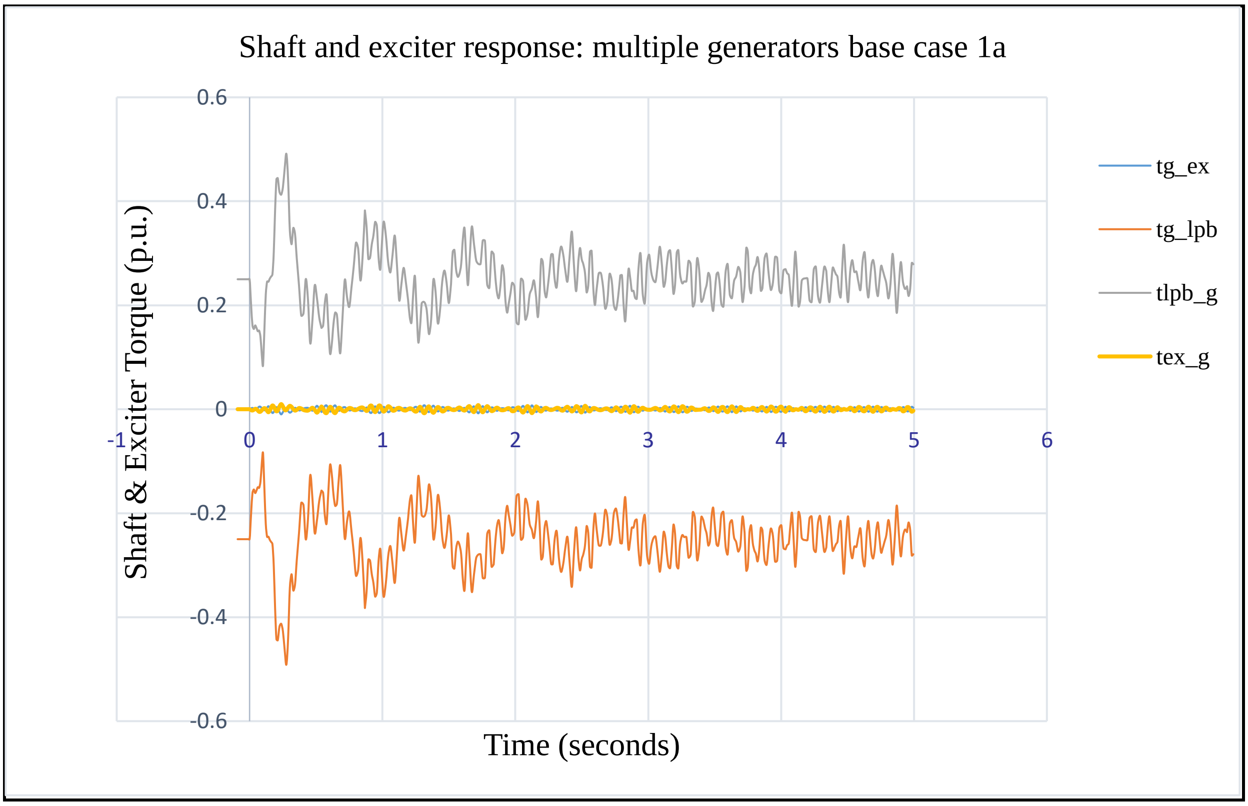


Figure 5.5: Shaft and exciter response for unmodified FBM with multiple generators

The comparison between using an aggregated setup or multiple generator setup was necessary to evaluate the faster way of data input in the simulation modelling process. Such clarity is always helpful when running multiple scenarios as above and helps the simulator not to waste time. As such, comparing Figure 5.1 versus Figure 5.4 and Figure 5.2 versus Figure 5.5 clearly illustrates that the generator and shaft responses curves are quite similar for both the aggregated and the multiple individual generators.

The rotor speed in both cases remains stable at 1.0 p.u. in the recorded time of 5 seconds.

The generator's electrical and mechanical torques for the aggregated set in Figure 5.2have bigger amplitude in overshoots and undershoots ranging from 0.44 – 1.58 p.u. than that of the multiple generators setup with 0.04 – 0.52 p.u. in Figure 5.5. This displays a maximum amplitude difference of 106%.

The shaft response (specifically the tg_lpb mode) has a negative trough of -1.610 p.u. for the aggravated case and -0.49 p.u. for the multiple case scenario. However, the rotor speed in both cases remains flat at 1.0 p.u. All this variations are expected and attributable to the larger amount of generator power at time of contingency for the aggregated case whilst there is less power from each contributing generator in the multiple case scenario.

The indication from such analysis is that, for analysis purposes, both methods are implementable without affecting the results but the aggregated case will be preferable since it is easier and more convenient to implement in the software. There is no effect on the behaviour trend on by solving the simulation iterations with the local voltage controller mode under power factor or voltage control mode. This is possibly because the voltage controller does not influence the SSR problem.

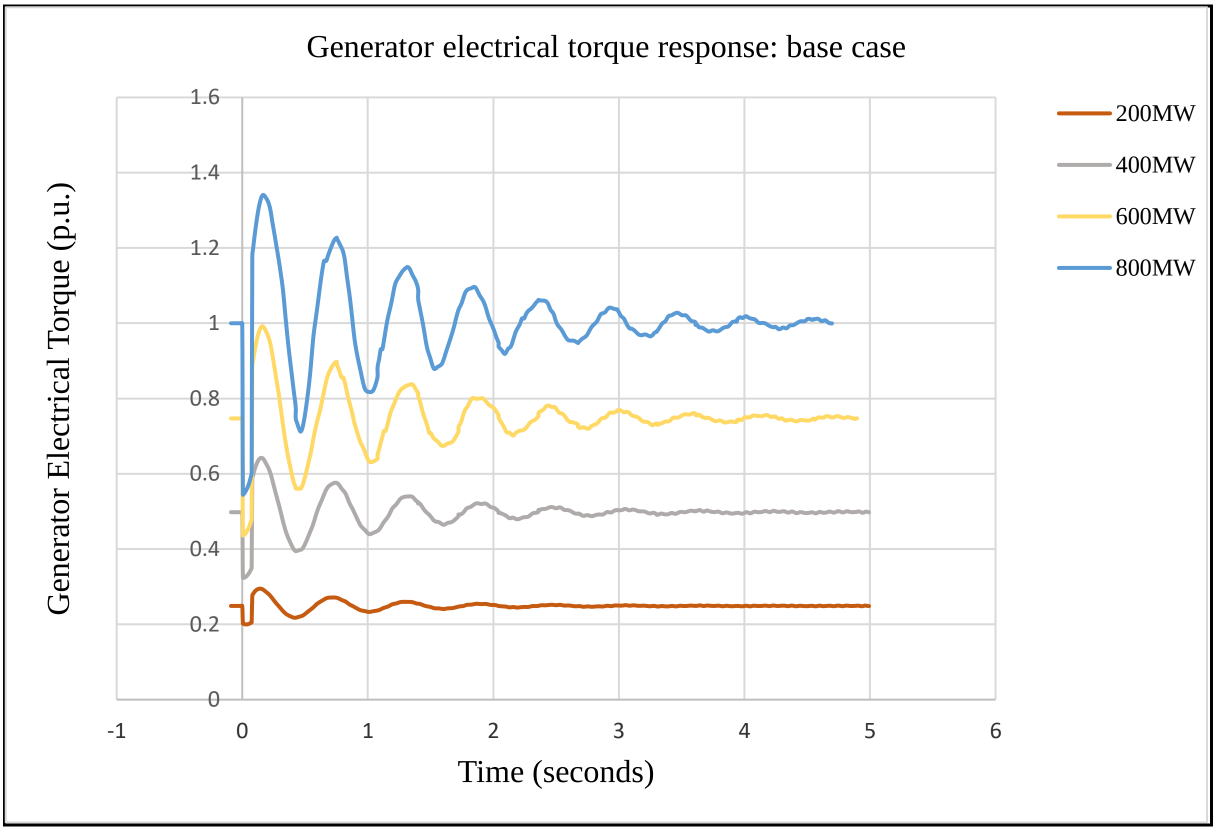


Figure 5.6: Synchronous generator response on ramping up power from 200MW – 800MW

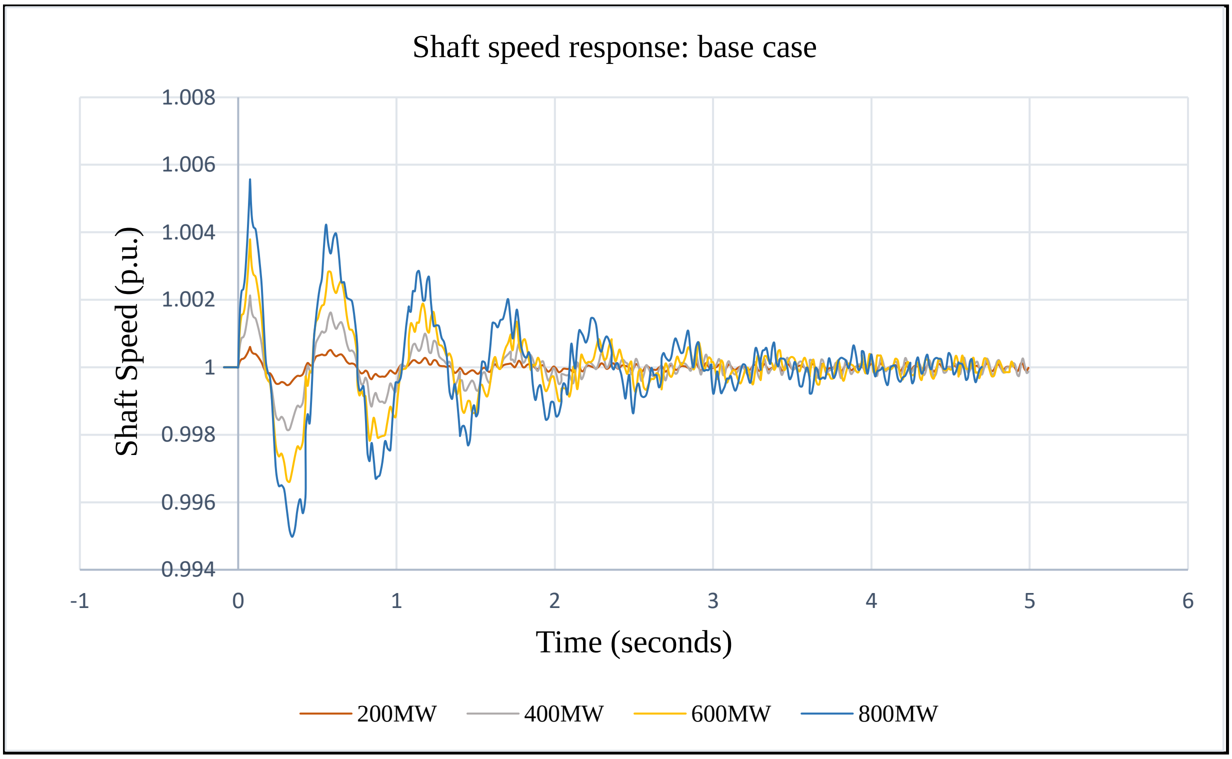


Figure 5.7: Shaft speed on ramping up power from 200MW – 800MW

Figure 5.6 & Figure 5.7 illustrate the generator electrical torque and shaft speed response curves respectively for different power loadings at time of fault.

In Figure 5.7, the 800MW ramp indicates larger overshoots and undershoots of 1.005 p.u. to 0.995 p.u. respectively. The settling time beyond 5 seconds is similar for the all the power ramps from 200MW to 800MW. Increasing the integrated power from 200MW upwards has a direct effect on the amplitude of the electrical torque and shaft speed. Torque amplification affects the same shaft with increasing power output.

This is attributable to the increase in power flows that enhances the resonance problem. When the system is carrying more power, it takes a longer time for the system to stabilise. At the same time carrying more power i.e. when network is heavily loaded, at that instance there is a certain buoyancy and friction that tends to dampen synchronous oscillations and improve system stability.

This is expected because SSR involves matching of the shaft resonances of a specific turbine generator and the electrical resonances of a series compensated transmission network. The rotor is a spring-mass system where coupling shafts act as torsional springs and the moment of inertia of each rotating section acts as mass. It has intrinsic modes of torsional vibration. The electrical torque of the generator in this case acts as a forcing function to excite the torsional vibration modes.

### 5.2.2 Simulation results: Case 1b: Base Case- parallel machines configuration

There is provision to select the number of parallel machines on the generator settings within DIgSILENT. By using this method, you save on time and having to build the multiple generators on terminal 3 bus. Figure 5.8 to Figure 5.9 display the settings configuration pages within DIgSILENT.

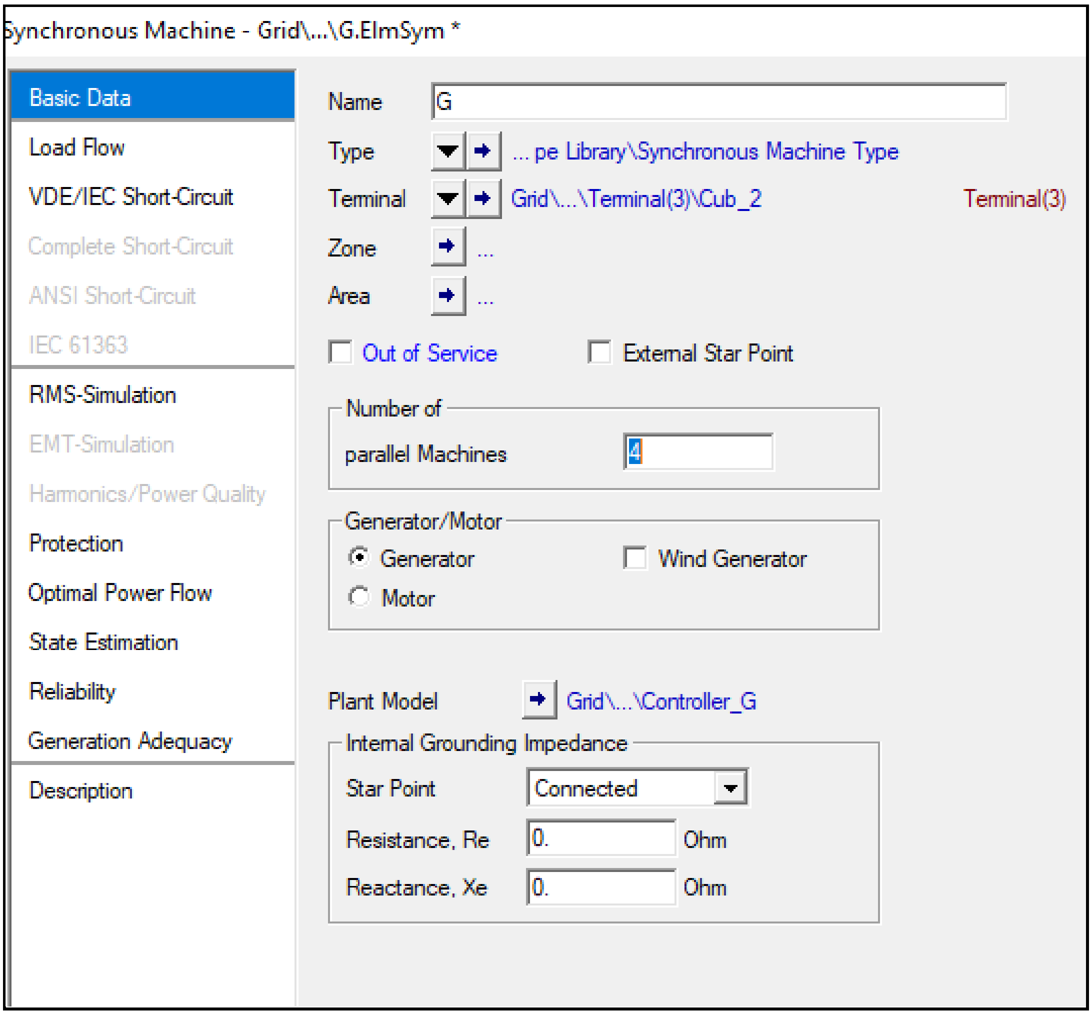


Figure 5.8: Settings page 1 in DIgSILENT showing setup a set of parallel machines

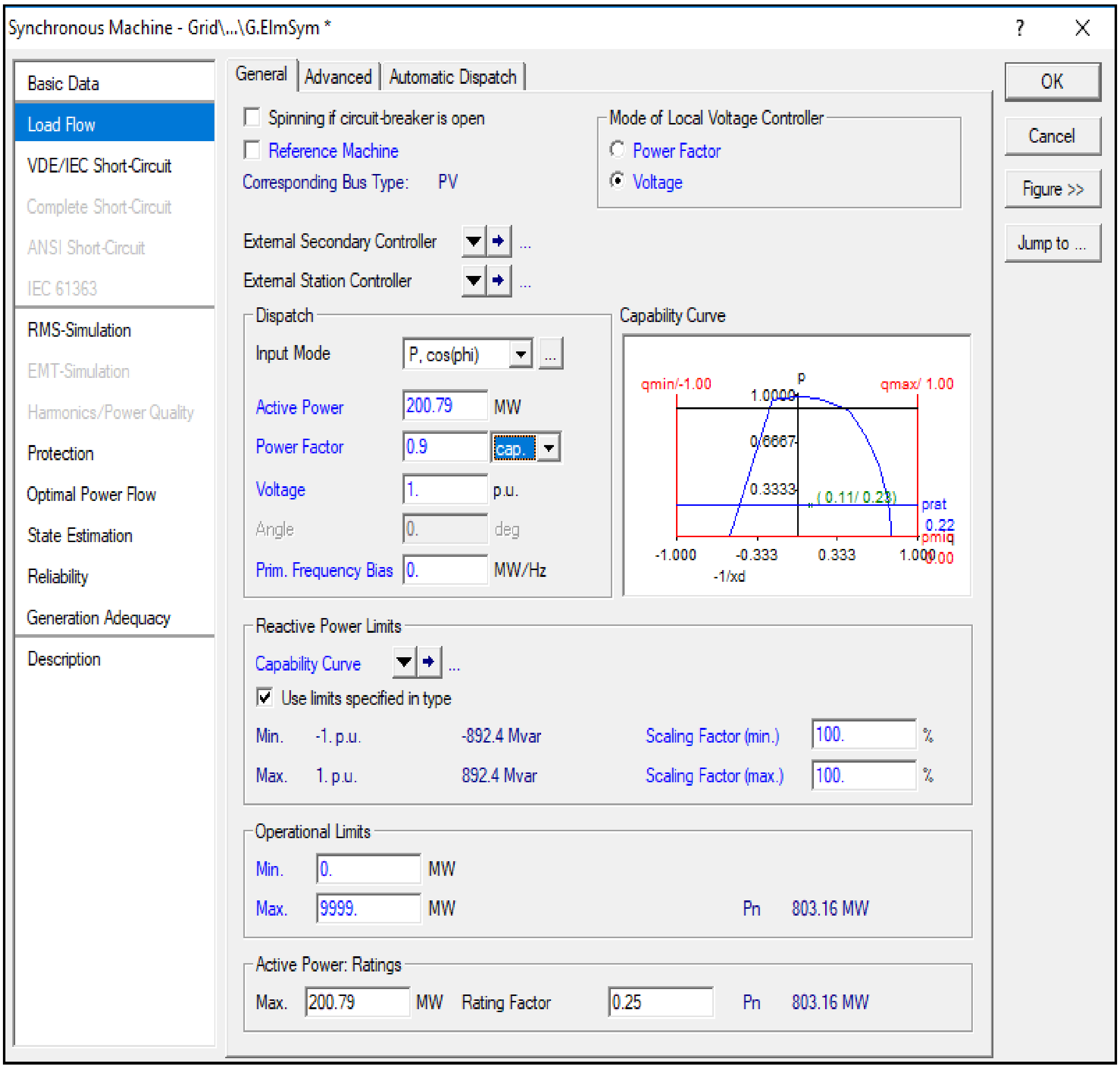


Figure 5.9: Setting page 2 in DIgSILENT showing setup a set of parallel machines

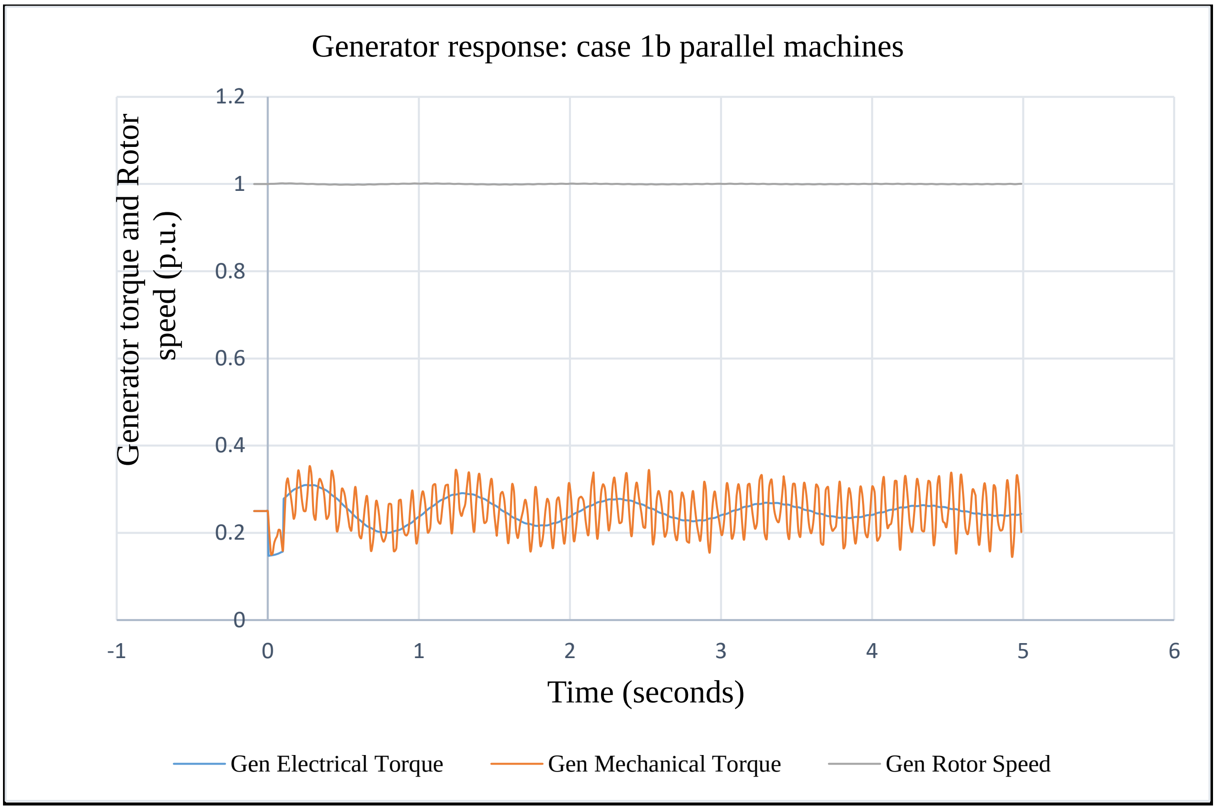


Figure 5.10: Generator and rotor response responses after defining parallel machines

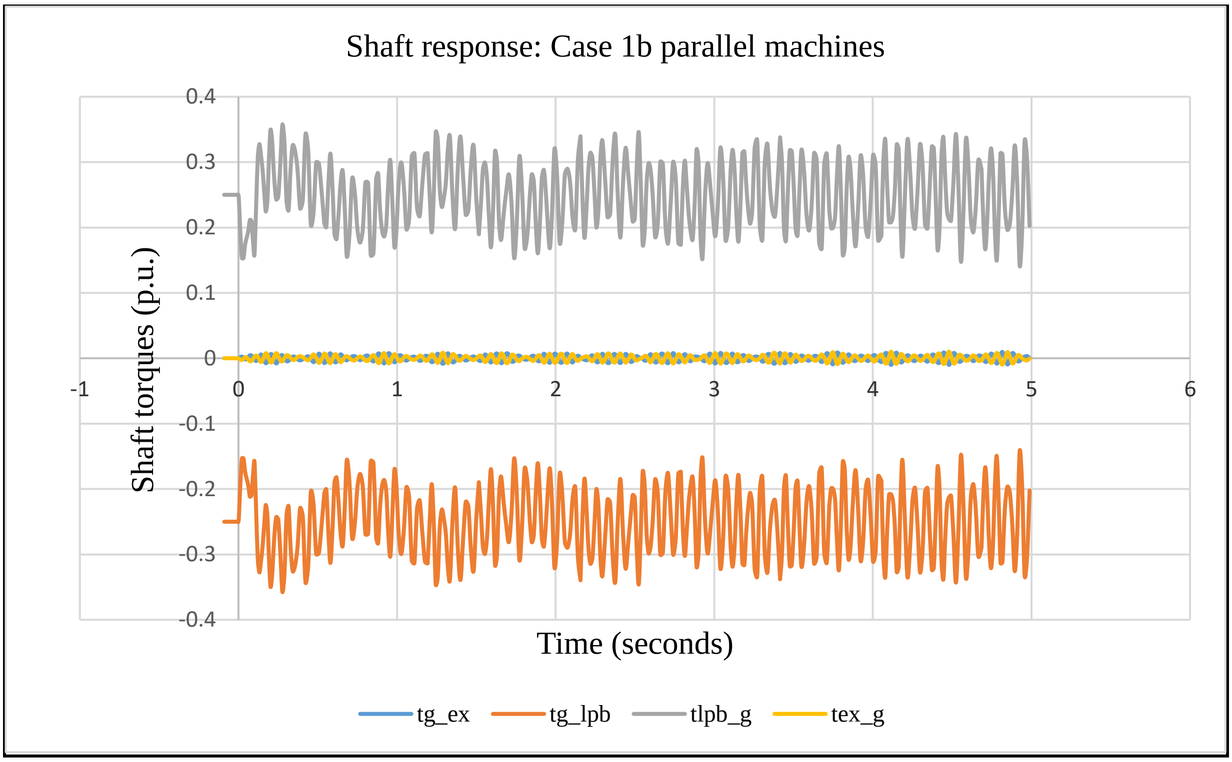


Figure 5.11: Shaft responses after defining parallel machines within DIgSILENT in case 1b.

The objective for simulating this case 1b was to evaluate and determine if different data entry methods in DIgSILENT affected the outcome of the same configuration.

The results presented in Figure 5.10 & Figure 5.11 (DIgSILENT parallel machine generator setting) when analysed in conjunction with the results in Figure 5.4 & Figure 5.5 (physical multiple generators) indicate that the two methods of splitting generators give almost the same results.

It is now clear from the results comparison that, since the results are fundamentally the same, it is simpler and faster to use the method of specifying number of parallel machines for analysis within DIgSILENT. One only has to be careful to match total power from the parallel machines with the stipulated total power output.

### 5.2.3 Modal analysis - base case

Table 5-1: Eigenvalues from Case 1a; Unmodified FBM

| **Modes** | **Real part (1/s)** | **Imaginary part (rad/s)** | **Magnitude (1/s)** | **Damped Frequency (Hz)** | **Period (s)** | **Damping Ratio** |
|---|---|---|---|---|---|---|
| Mode 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| Mode 2 | 0 | 298.1678 | 298.1678 | 47.45488 | 0.021073 | 0 |
| Mode 3 | 0 | -298.1678 | 298.1678 | 47.45488 | 0.021073 | 0 |
| Mode 4 | 0 | 187.2593 | 187.2593 | 29.80324 | 0.033553 | 0 |
| Mode 5 | 0 | -187.2593 | 187.2593 | 29.80324 | 0.033553 | 0 |
| Mode 6 | 0 | 148.403 | 148.403 | 23.61907 | 0.042339 | 0 |
| Mode 7 | 0 | -148.403 | 148.403 | 23.61907 | 0.042339 | 0 |
| Mode 8 | 0 | 64.54062 | 64.54062 | 10.27196 | 0.097352 | 0 |
| Mode 9 | 0 | -64.54062 | 64.54062 | 10.27196 | 0.097352 | 0 |
| Mode 10 | 0 | 124.64 | 124.64 | 19.83707 | 0.050411 | 0 |
| Mode 11 | 0 | -124.64 | 124.64 | 19.83707 | 0.050411 | 0 |
| Mode 12 | -33.39289 | 0 | 33.39289 | 0 | 0 | 1 |
| Mode 13 | -1.128281 | 19.45305 | 19.48575 | 3.096049 | 0.322992 | 0.05790288 |
| Mode 14 | -1.128281 | -19.45305 | 19.48575 | 3.096049 | 0.322992 | 0.05790288 |
| Mode 15 | -0.3256534 | 0 | 0.3256534 | 0 | 0 | 1 |
| Mode 16 | -3.677803 | 0 | 3.677803 | 0 | 0 | 1 |
| Mode 17 | -21.42747 | 0 | 21.42747 | 0 | 0 | 1 |

The eigenvalues of the modes in Table 5-1 show that the six relevant modes (mode 12-mode 17) are stable as depicted by the negative real parts and positive damping ratios except for Mode 13 & 14. Modes 13 & 14 are oscillatory modes with a damping ratio of 6%. The damped frequency of 3.1 Hz shows that it is around the local area mode. The rest of the modes i.e. 12, 15, 16 and 17 are stable and critically damped with damping ratio of one. Figure 5.12 shows the eigenvectors overview for the base case.

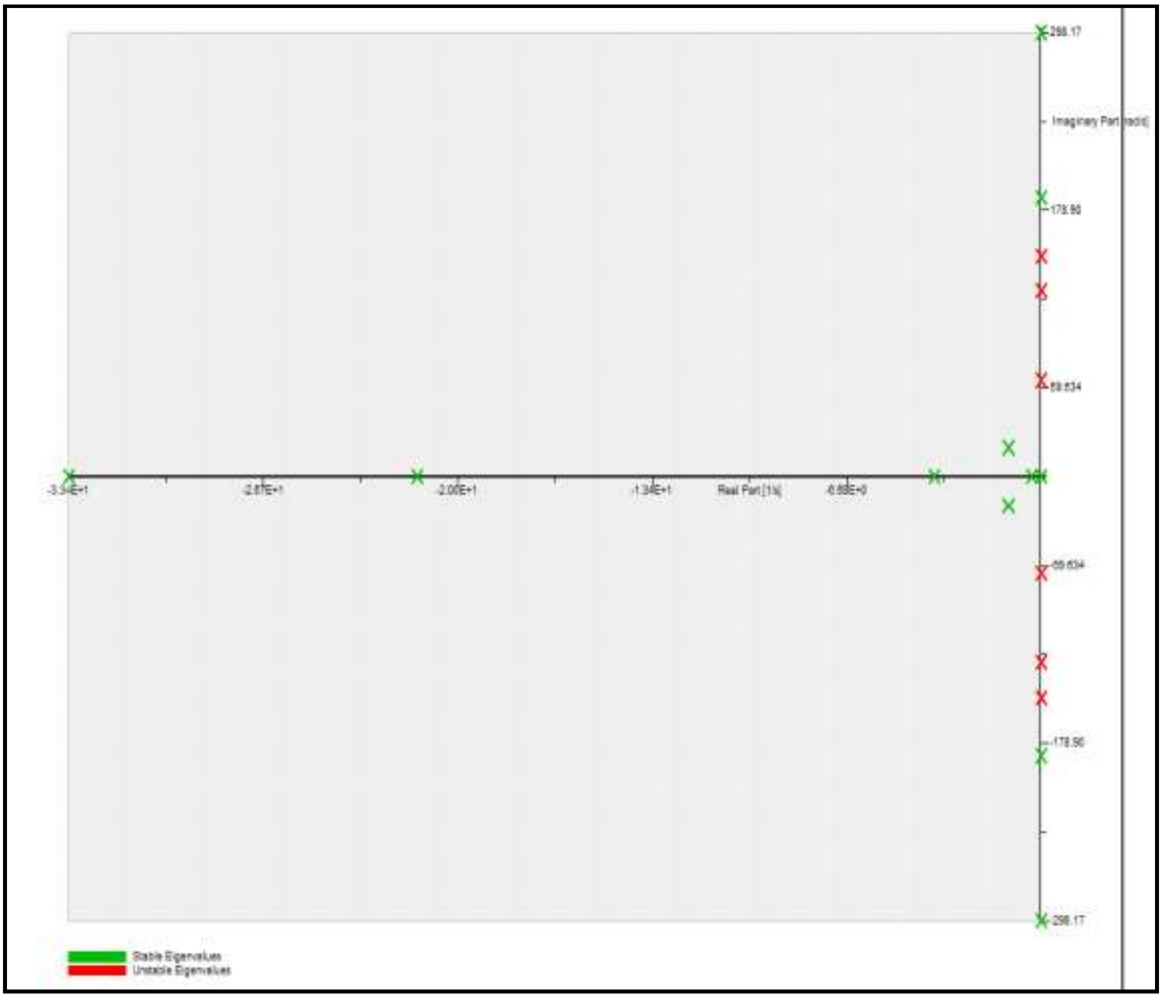


Figure 5.12: Eigenvalue plot overview for the base case.

Appendix E – Appendix G contains additional plots showing additional case files mode phasor and bar plots.

### 5.3 Case 2 simulation results – solar photovoltaic only scenario

#### 5.3.1 Simulation results: Case 2- Photovoltaic Generator Scenario

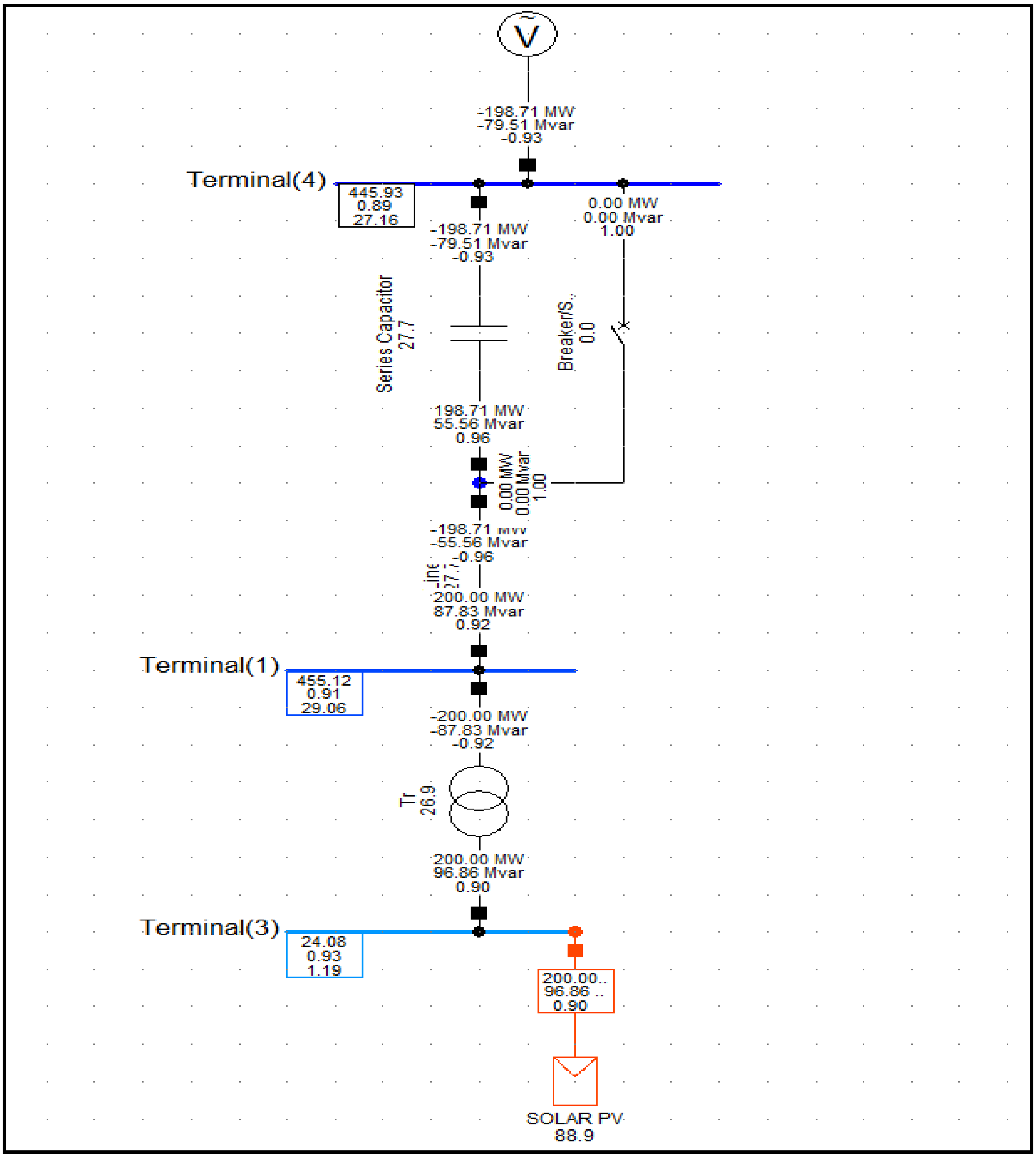


Figure 5.13: PV power connection arrangement for case 2

The real link between high PV penetration and SSR was evaluated case 2 as shown in Figure 5.13.

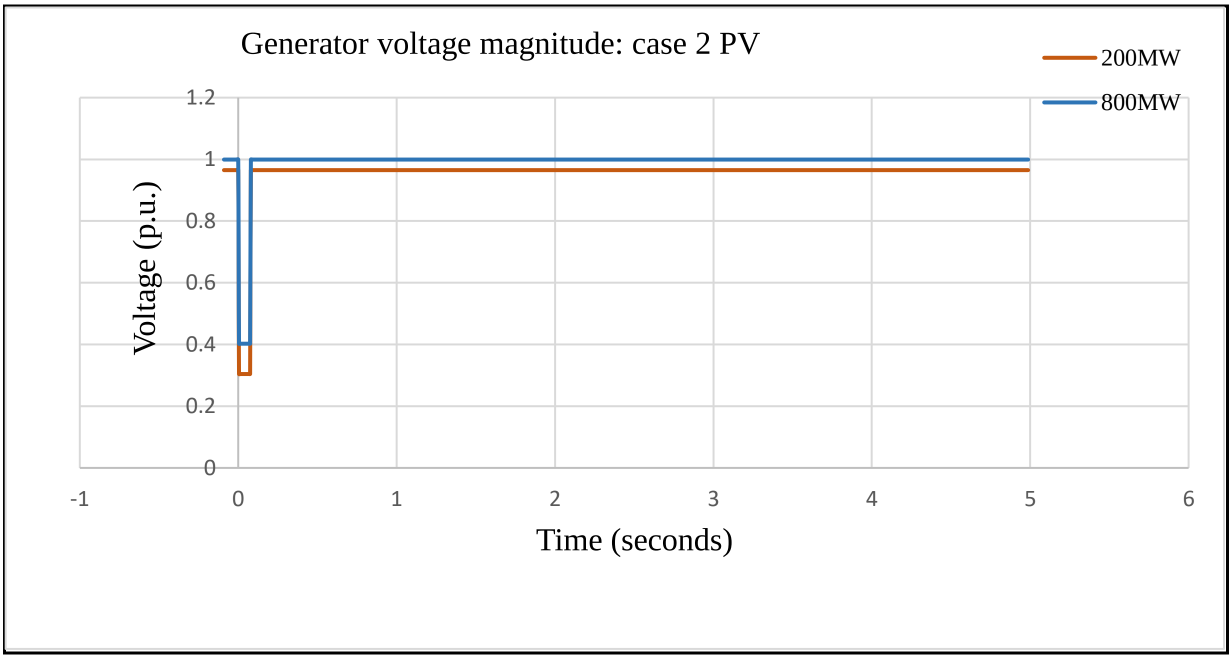


Figure 5.14: Voltage step response comparison between the 200MW & 800MW PV ramps

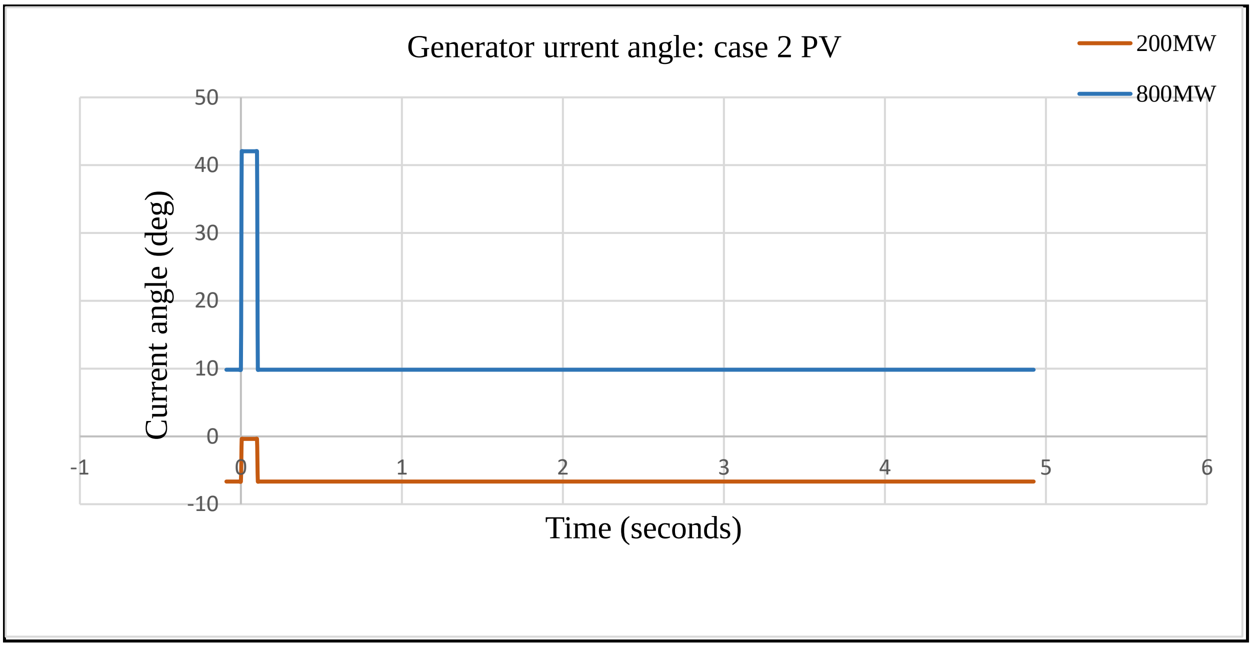


Figure 5.15: Current angle comparison between the 200MW & 800MW PV ramps

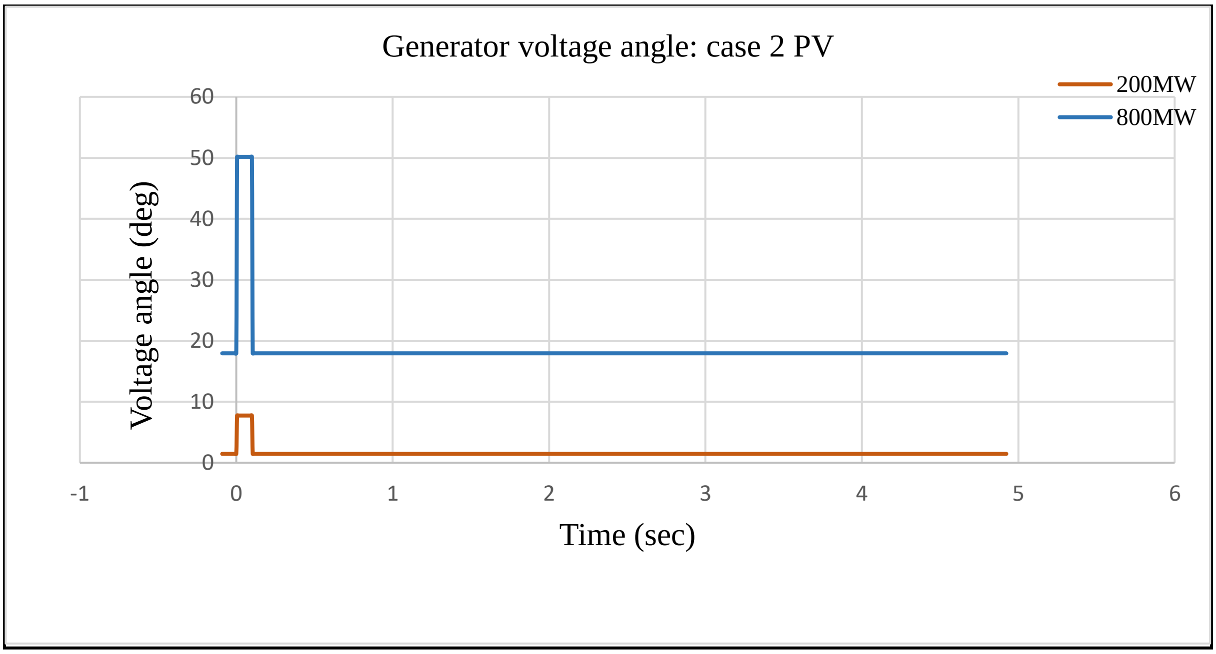


Figure 5.16: Voltage angle comparison between the 200MW & 800MW PV ramps

The results in monitoring the voltage and angle of the PV system in Figure 5.14, Figure 5.15 and Figure 5.16 reveal that SSR does not affect PV plants significantly as seen by the lack of modulation or amplification in the step signals.

The voltage magnitude and voltage or current angle show no signs of oscillations for either 200MW or 800MW PV injection. In Figure 5.14, as expected, the voltage dips after applying a three 3-phase fault at 0 seconds and recovers almost immediately within seven milliseconds for both the 200MW and 800MW cases. In Figure 5.15, the current angle shoots from 18 degrees to 42 degrees for the 800MW and normalise quickly after the same seven milliseconds back to 18 degrees. Similarly, for the 200MW PV ramp, the current angle step change is from -7 degrees to 0 degrees and back again to -7 degrees within eight milliseconds. In Figure 5.16, the voltage angle behaviour is similar to the current angle behaviour, soaring to 50 degrees before normalising immediately to the pre-fault angle of 18 degrees for the 800MW PV injection.

The need to understand the effect of high inverter based generation on the connected conventional plants with rotating masses brought about the hybrid scenarios outlined from section 5.4.

## 5.4 Simulation Results: Case 3- Increasing PV generation, dropping SG

### 5.4.1 Time domain transient stability analysis results – case 3, PV case

In this case, a solar PV plant with capacity to ramp up to 800 MW is placed on bus terminal 3 that is currently is connected to a 800MW synchronous generator. The penetration of the solar PV plant increases in steps of 200MW, as shown in Figure 5.17, replacing the active power of the synchronous generator to zero. The transient stability are analysed in terms of the synchronous generator behaviour and overall system parameters.

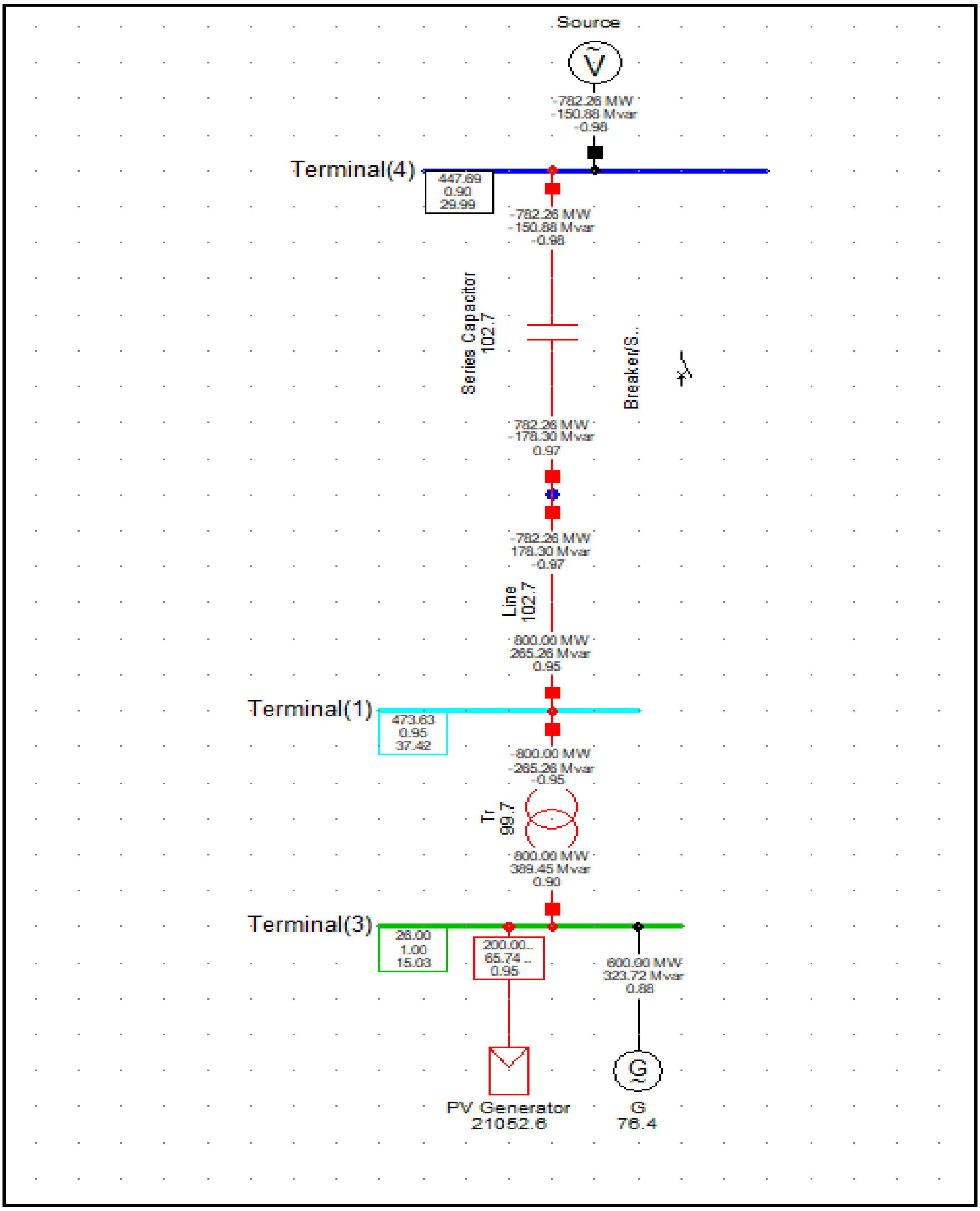


Figure 5.17: Solar PV and SG hybrid configuration

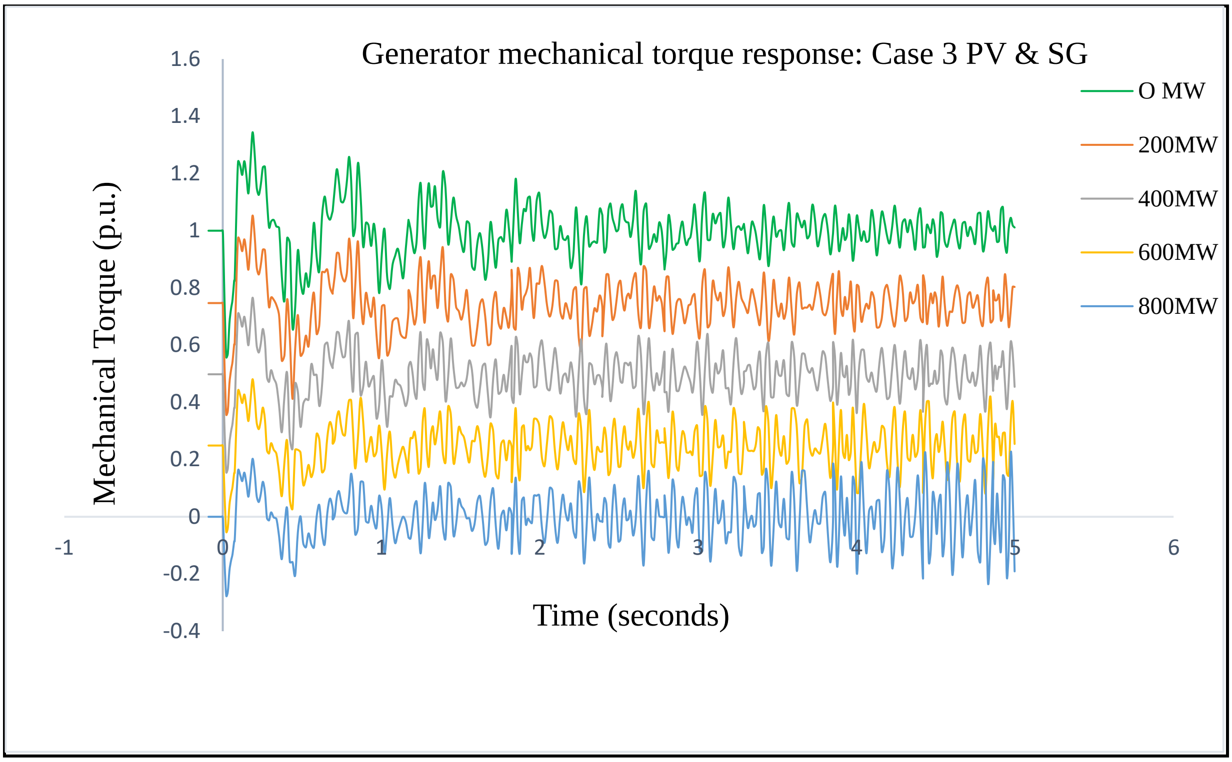


Figure 5.18: Synchronous generator response as PV power penetration to the FBM increases

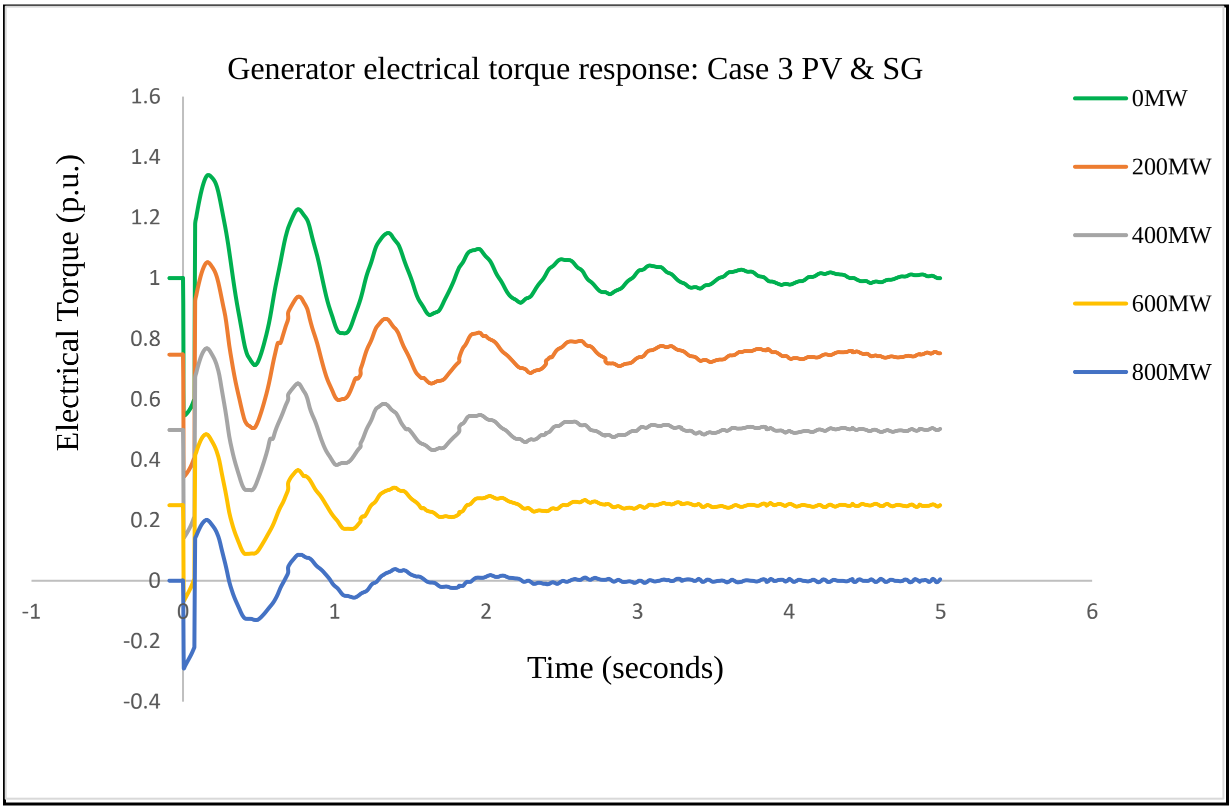


Figure 5.19: Synchronous generator response as PV power penetration to the FBM increases

The graphs in Figure 5.18 & Figure 5.19show the results after integrating a PV plant on the same FBM model. In Figure 5.18, the mechanical torque oscillations maximum overshoot values grow from 20% for the 800MW ramp to 134% for the 0MW PV ramp where there is no injection of PV power in the FBM. This constitutes an increase of 114% on the shaft torque i.e. transient torque SSR. In Figure 5.19, the blue curve for the case where PV power is ramped up to 800MW resonate mildly around the x - axis and subside to zero after about 2.5 seconds. In contrast, the curves for PV ramps of 600MW, 400MW, 200MW and 0MW resonate about the midline values of 0.2 p.u., 0.5 p.u., 0.8 p.u. and 1.0 p.u. respectively. The contribution of 800MW of PV power results in the fastest settling time of approximately 2.5 seconds whilst the scenario with no PV power has a settling time of more than 5 seconds. The observation from this analysis is that PV power integration reduces severity of the mechanical torque oscillations in the synchronous generator. Slight differences transpire between the graphs of the mechanical torque and electrical torque for PV power. PV generation does not have any rotating parts. The mechanical torque resonance remains undamped for all power ramps above 200MW whilst the electrical torque subsides significantly after 3 seconds. In this case, since PV has no moving parts, the electrical torque provides for the most sensible explanation of IGE traces albeit quickly dampened by inherent controllers.

### 5.4.2 Modal analysis results – case 3, PV case

The studied system configuration remains similar as in Figure 5.17. Presentation of the modal results for the PV case is given in Table 5-2 & Table 5-3 and in Figure 5.20 & Figure 5.21.

Table 5-2: Eigenvalues from Case 3 - 200MW PV power penetration to the FBM

| **Modes** | **Real part (1/s)** | **Imaginary part (rad/s)** | **Magnitude (1/s)** | **Damped Frequency (Hz)** | **Period (s)** | **Damping Ratio** |
|---|---|---|---|---|---|---|
| Mode 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| Mode 2 | 0 | 298.1678 | 298.1678 | 47.45488 | 0.021073 | 0 |
| Mode 3 | 0 | -298.1678 | 298.1678 | 47.45488 | 0.021073 | 0 |
| Mode 4 | 0 | 187.2593 | 187.2593 | 29.80324 | 0.033553 | 0 |
| Mode 5 | 0 | -187.2593 | 187.2593 | 29.80324 | 0.033553 | 0 |
| Mode 6 | 0 | 148.403 | 148.403 | 23.61907 | 0.042339 | 0 |
| Mode 7 | 0 | -148.403 | 148.403 | 23.61907 | 0.042339 | 0 |
| Mode 8 | 0 | 64.54062 | 64.54062 | 10.27196 | 0.097352 | 0 |
| Mode 9 | 0 | -64.54062 | 64.54062 | 10.27196 | 0.097352 | 0 |
| Mode 10 | 0 | 124.64 | 124.64 | 19.83707 | 0.050411 | 0 |
| Mode 11 | 0 | -124.64 | 124.64 | 19.83707 | 0.050411 | 0 |
| Mode 12 | -33.51364 | 0 | 33.51364 | 0 | 0 | 1 |
| Mode 13 | -1.164997 | 19.42484 | 19.45974 | 3.091559 | 0.323461 | 0.05986702 |
| Mode 14 | -1.164997 | -19.42484 | 19.45974 | 3.091559 | 0.323461 | 0.05986702 |
| Mode 15 | -0.400228 | 0 | 0.400228 | 0 | 0 | 1 |
| Mode 16 | -3.258505 | 0 | 3.258505 | 0 | 0 | 1 |
| Mode 17 | -21.37748 | 0 | 21.37748 | 0 | 0 | 1 |
| Mode 18 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 5-2 outlines 18 eigenvalue modes. Mode 1 – 11 & 18 have zero real parts. Mode 12- 17 are stable with negative real parts. Mode 13 & 14 are poorly damped with a damping ratio of 6% whilst Mode 12, 15, 16 and 17 are critically damped with a ratio of 100%. Figure 5.20 shows the plot for the 200MW case whilst Figure 5.21 illustrates the eigenvalue plot for the 800MW PV case.

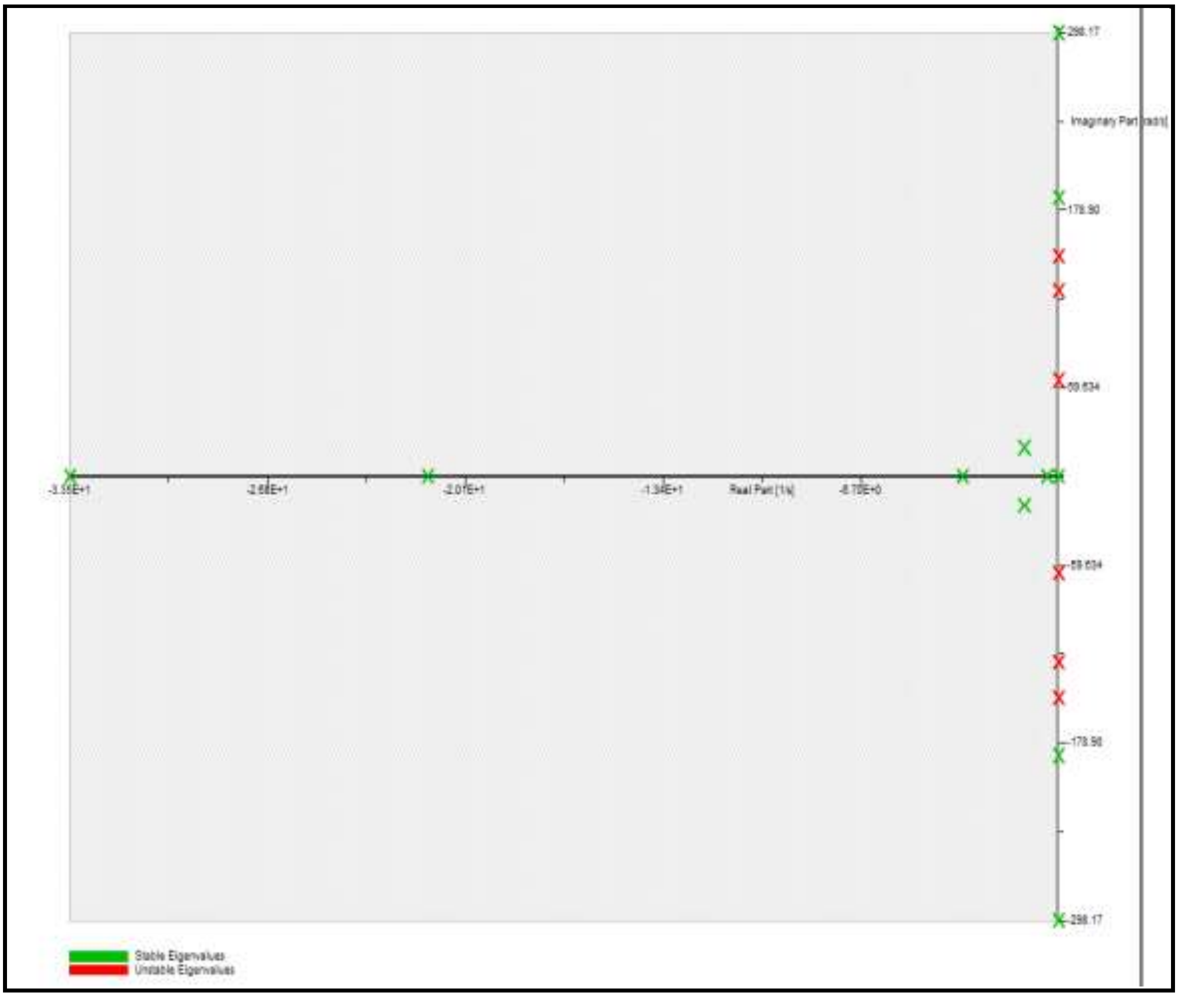


Figure 5.20: Graph showing the eigenvalue plot overview for case 3- 200MW PV scenario

Table 5-3: Eigenvalues from Case 3 - 800MW PV power penetration to the FBM

| Modes | Real part (1/s) | Imaginary part (rad/s) | Magnitude (1/s) | Damped Frequency (Hz) | Period (s) | Damping Ratio |
|---|---|---|---|---|---|---|
| Mode 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| Mode 2 | 0 | 298.1678 | 298.1678 | 47.45488 | 0.021073 | 0 |
| Mode 3 | 0 | -298.1678 | 298.1678 | 47.45488 | 0.021073 | 0 |
| Mode 4 | 0 | 187.2593 | 187.2593 | 29.80324 | 0.033553 | 0 |
| Mode 5 | 0 | -187.2593 | 187.2593 | 29.80324 | 0.033553 | 0 |
| Mode 6 | 0 | 148.403 | 148.403 | 23.61907 | 0.042339 | 0 |
| Mode 7 | 0 | -148.403 | 148.403 | 23.61907 | 0.042339 | 0 |
| Mode 8 | 0 | 64.54062 | 64.54062 | 10.27196 | 0.097352 | 0 |
| Mode 9 | 0 | -64.54062 | 64.54062 | 10.27196 | 0.097352 | 0 |
| Mode 10 | 0 | 124.64 | 124.64 | 19.83707 | 0.050411 | 0 |
| Mode 11 | 0 | -124.64 | 124.64 | 19.83707 | 0.050411 | 0 |
| Mode 12 | -33.74245 | 0 | 33.74245 | 0 | 0 | 1 |
| Mode 13 | -1.527487 | 18.96752 | 19.02893 | 3.018775 | 0.33126 | 0.080272 |
| Mode 14 | -1.527487 | -18.96752 | 19.02893 | 3.018775 | 0.33126 | 0.080272 |
| Mode 15 | -21.21962 | 0 | 21.21962 | 0 | 0 | 1 |
| Mode 16 | -1.134408 | 0 | 1.134408 | 0 | 0 | 1 |
| Mode 17 | -0.973135 | 0 | 0.973135 | 0 | 0 | 1 |
| Mode 18 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 5-3 outlines 18 eigenvalue modes corresponding to the 800MW PV case. Mode 1 – 11 & 18 have zero real part. Mode 12- 17 are stable with negative real parts. Mode 13 & 14 are similarly poorly damped with a damping ratio of 8% whilst Mode 12, 15, 16 and 17 are critically damped with a ratio of 100%.

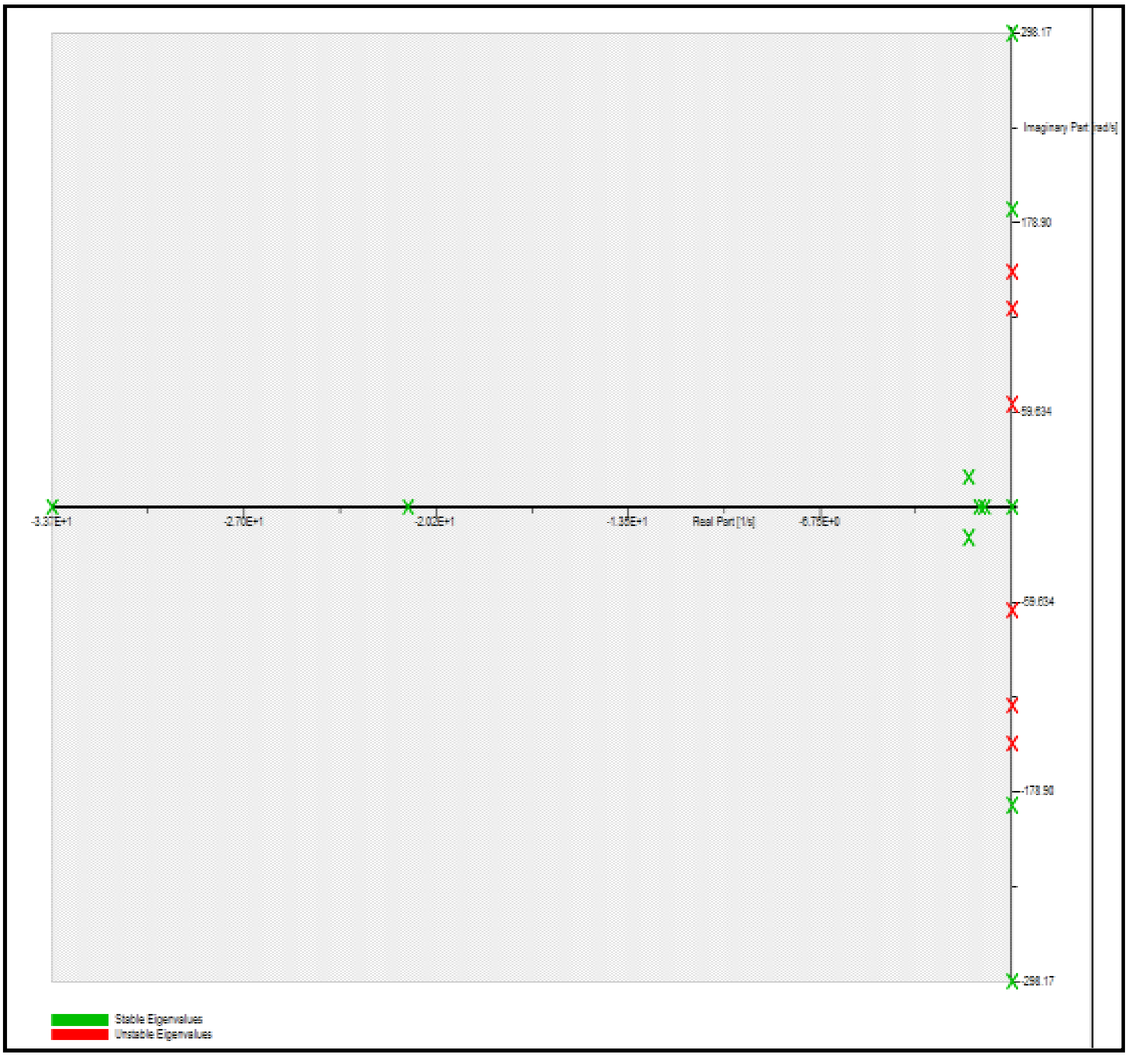


Figure 5.21: Eigenvalues from Case 3 - 800MW PV power penetration to the FBM

## 5.5 Simulation results: Case 4 - Increasing wind generation, dropping SG

### 5.5.1 Time domain transient stability analysis results – Case 4, wind case

DIgSILENT has a stand-alone doubly fed induction generator model available for analysis considered for the wind generation scenario as shown in Figure 5.22.

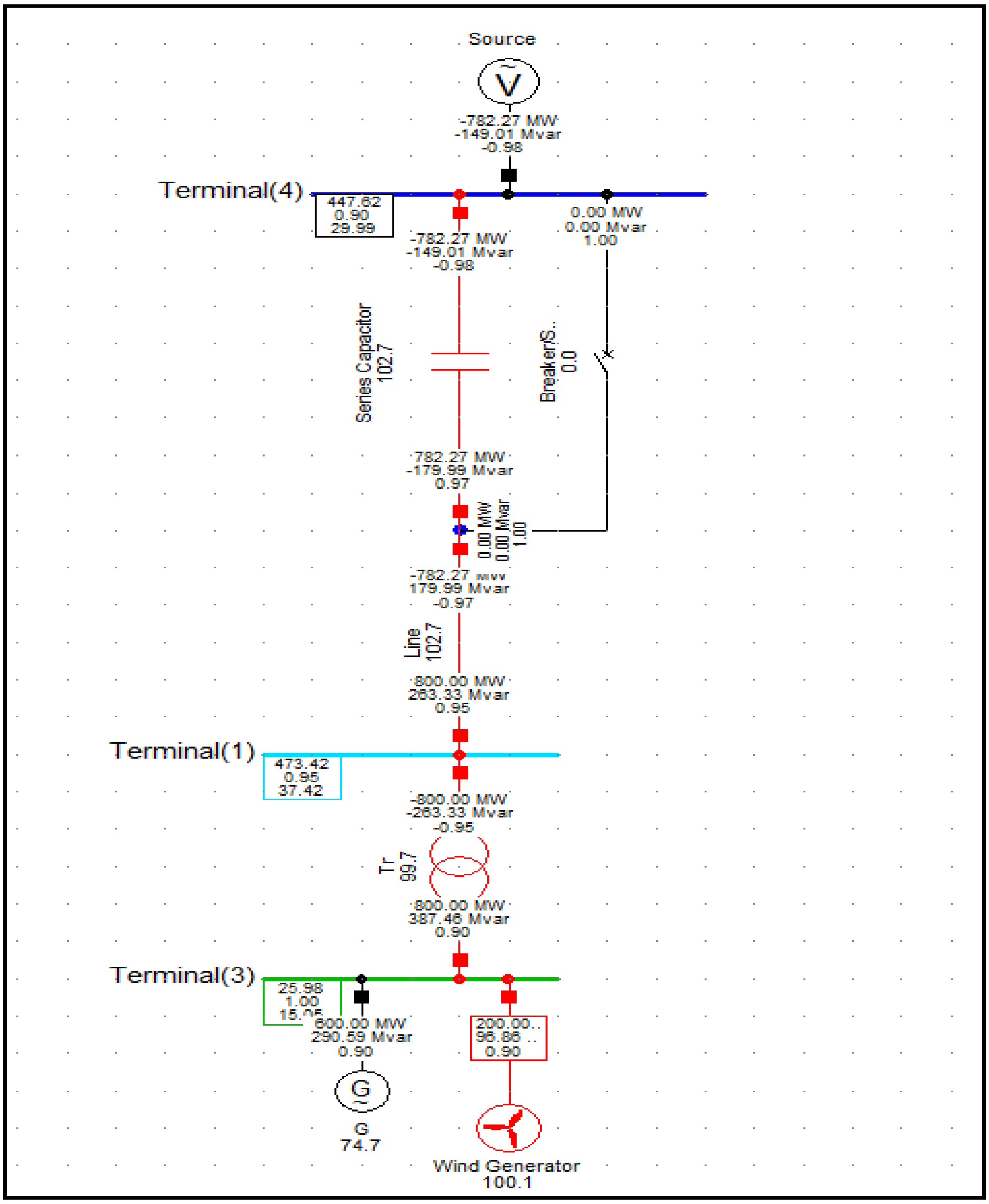


Figure 5.22: Wind and SG hybrid configuration

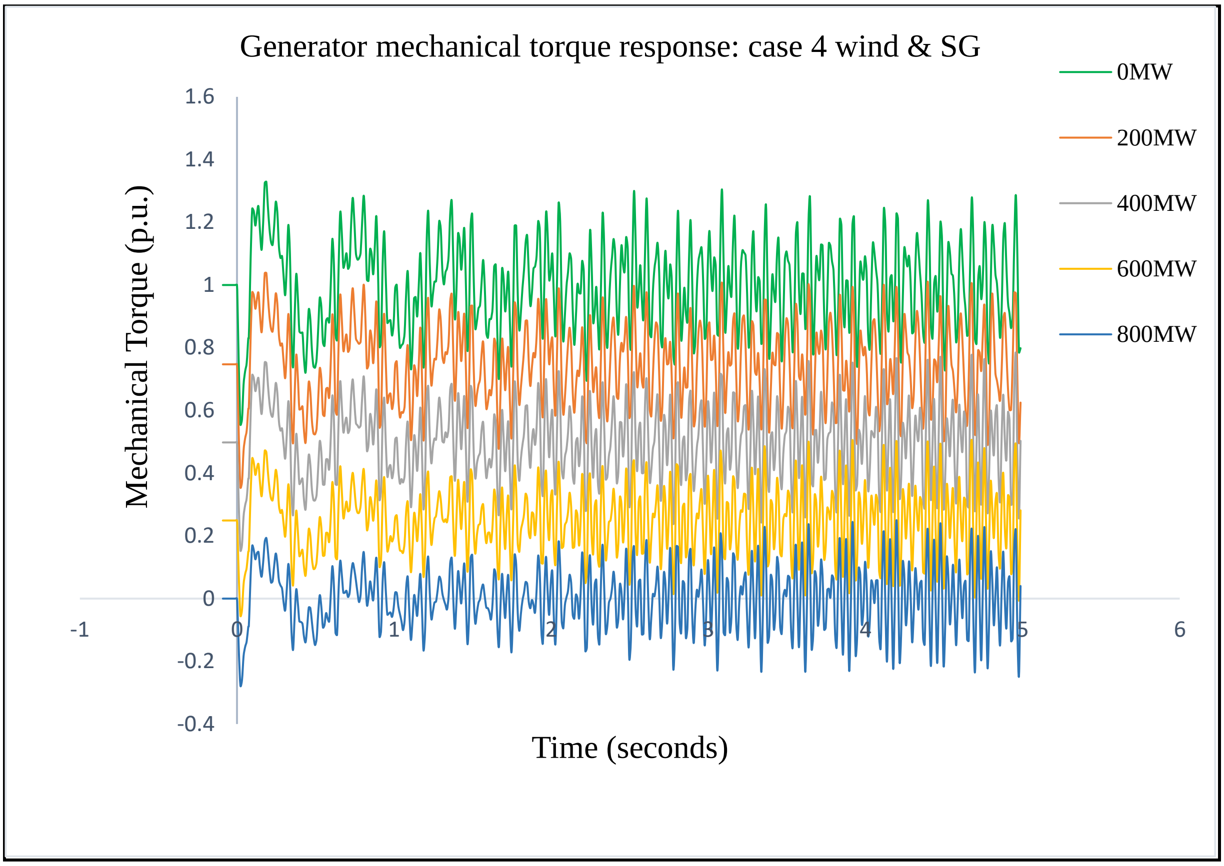


Figure 5.23: Generator mechanical torque as wind power penetration to the FBM increases

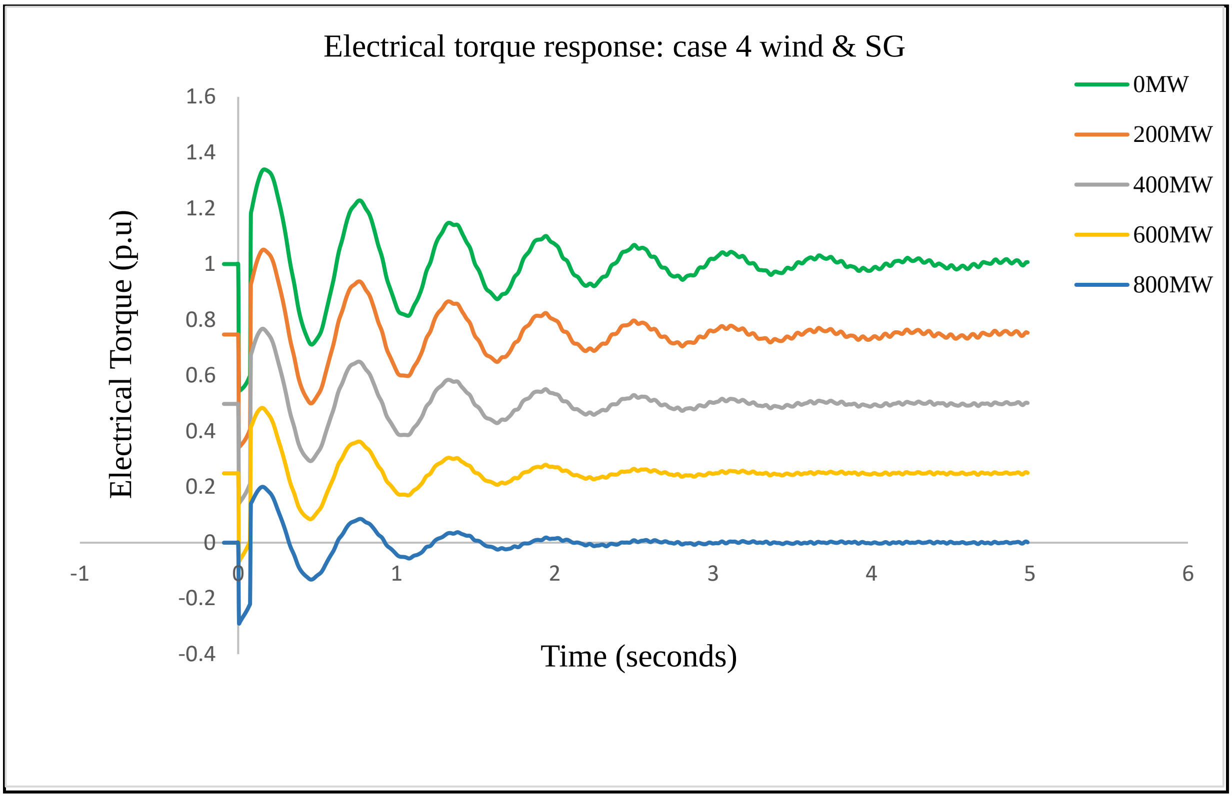


Figure 5.24: Generator electrical torque as wind power penetration to the FBM increases

The transient SSR responses of the mechanical torque and electrical torque under ramped up wind generation are as shown in Figure 5.23 & Figure 5.24. In Figure 5.23, the amplitude of the mechanical torque, though sustained, are restricted to between -0.3 to 0.2 p.u. for the 800MW wind power case. The lower the wind generation, the higher impact on amplitudes of the mechanical and electrical torque. In Figure 5.24, electrical torque stabilises from approximately 2 seconds to 4 seconds for all power ramps. The 0MW case where there is no wind injection has larger undershoots and overshoots subsiding to a flat slope after 4.5 seconds. The cases where wind power ramps rise from 200MW display graphs that become less oscillatory with transient decay within 2.5 seconds (800MW case). This illustrates that as wind power comes in to replace conventional thermal power, there is a direct reduction on the generator mechanical or electrical torque.

The general analysis is in two ways, i.e. stability study in the frequency domain and the transient torque study in the time domain. The critical measure of the transient torque is the magnitude of the shaft vibration excited during each network transient event.

Torsional interaction becomes unstable and excessive if the electrical and torsional resonance frequencies nearly coincide as fundamental frequency (50/60 Hz) complements and when the inherent mechanical damping is lower than the negative damping effect of series capacitor.

### 5.5.2 Modal analysis results – Case 4 - wind case

The studied system configuration remains similar as in Figure 5.22 and modal analysis results are captured the results in Table 5-4 & Table 5-5 and Figure 5.25 & Figure 5.26.

Table 5-4: Eigenvalues from Case 4 - 200MW wind power penetration to the FBM

| Modes | Real part (1/s) | Imaginary part (rad/s) | Magnitude (1/s) | Damped Frequency (Hz) | Period (s) | Damping Ratio |
|---|---|---|---|---|---|---|
| Mode 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| Mode 2 | 0 | 298.1678 | 298.1678 | 47.45488 | 0.021073 | 0 |
| Mode 3 | 0 | -298.1678 | 298.1678 | 47.45488 | 0.021073 | 0 |
| Mode 4 | 0 | 187.2593 | 187.2593 | 29.80324 | 0.033553 | 0 |
| Mode 5 | 0 | -187.2593 | 187.2593 | 29.80324 | 0.033553 | 0 |
| Mode 6 | 0 | 148.403 | 148.403 | 23.61907 | 0.042339 | 0 |
| Mode 7 | 0 | -148.403 | 148.403 | 23.61907 | 0.042339 | 0 |
| Mode 8 | 0 | 64.54062 | 64.54062 | 10.27196 | 0.097352 | 0 |
| Mode 9 | 0 | -64.54062 | 64.54062 | 10.27196 | 0.097352 | 0 |
| Mode 10 | 0 | 124.64 | 124.64 | 19.83707 | 0.050411 | 0 |
| Mode 11 | 0 | -124.64 | 124.64 | 19.83707 | 0.050411 | 0 |
| Mode 12 | -33.5136 | 0 | 33.51364 | 0 | 0 | 1 |
| Mode 13 | -1.16500 | 19.42484 | 19.45974 | 3.091559 | 0.323461 | 0.05986702 |
| Mode 14 | -1.16500 | -19.42484 | 19.45974 | 3.091559 | 0.323461 | 0.05986702 |
| Mode 15 | -0.40023 | 0 | 0.400228 | 0 | 0 | 1 |
| Mode 16 | -3.25851 | 0 | 3.258505 | 0 | 0 | 1 |
| Mode 17 | -21.3775 | 0 | 21.37748 | 0 | 0 | 1 |
| Mode 18 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 5-4: Eigenvalues from Case 4 - 200MW wind power penetration to the FBM outlines 18 eigenvalue modes corresponding to the 200MW wind case. Mode 1 – 11 & 18 have zero real part. Mode 12- 17 are stable with negative real parts. Mode 13 & 14 are poorly damped with a damping ratio of 6% whilst Mode 12, 15, 16 and 17 are critically damped with a ratio of 100%.

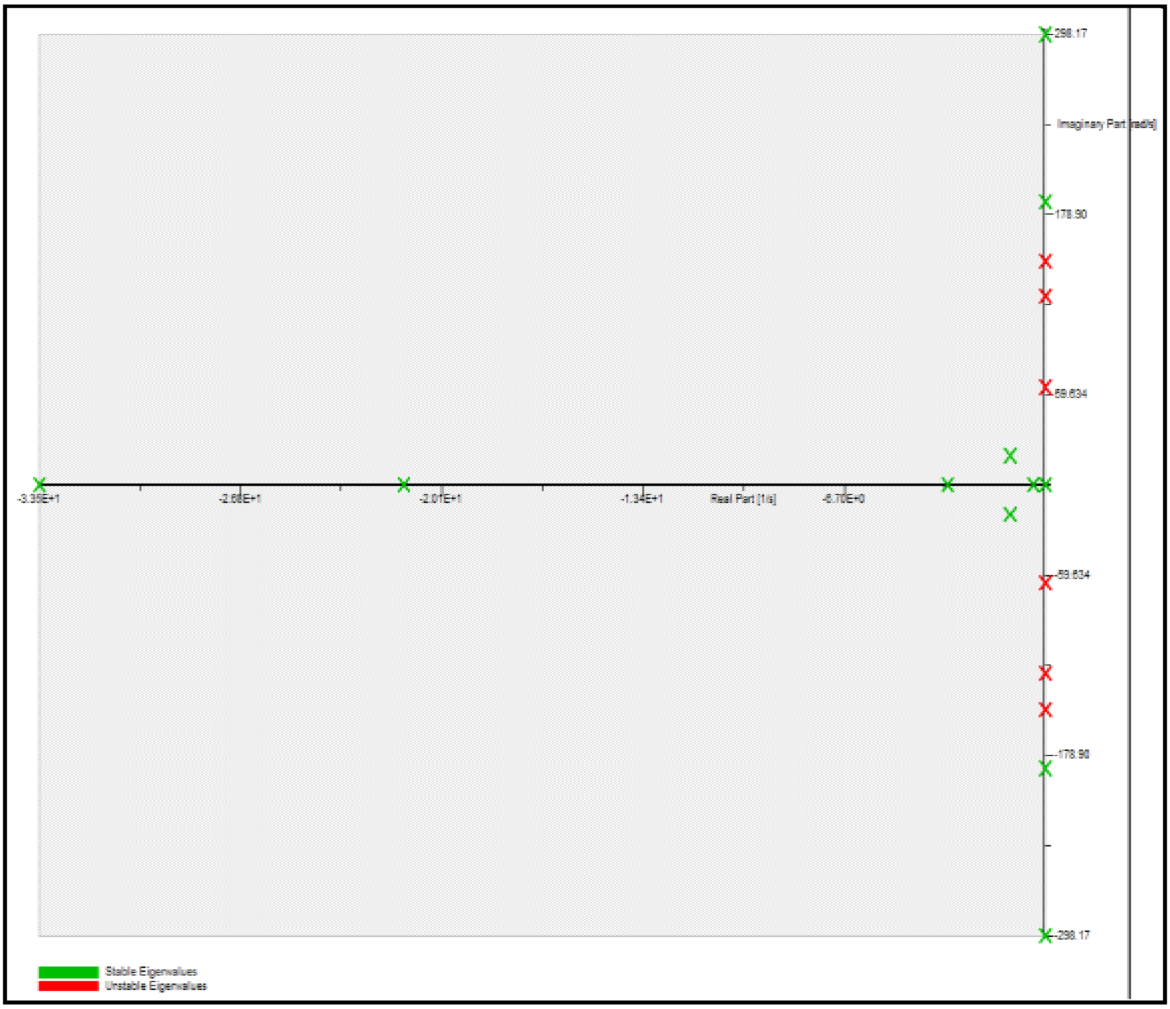


Figure 5.25: Graph showing the eigenvalue plot overview for case 4- 200MW wind scenario

Table 5-5: Eigenvalues from Case 4 - 800MW wind power penetration to the FBM

| **Modes** | **Real part (1/s)** | **Imaginary part (rad/s)** | **Magnitude (1/s)** | **Damped Frequency (Hz)** | **Period (s)** | **Damping Ratio** |
|---|---|---|---|---|---|---|
| Mode 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| Mode 2 | 0 | 298.1678 | 298.1678 | 47.45488 | 0.021073 | 0 |
| Mode 3 | 0 | -298.1678 | 298.1678 | 47.45488 | 0.021073 | 0 |
| Mode 4 | 0 | 187.2593 | 187.2593 | 29.80324 | 0.033553 | 0 |
| Mode 5 | 0 | -187.2593 | 187.2593 | 29.80324 | 0.033553 | 0 |
| Mode 6 | 0 | 148.403 | 148.403 | 23.61907 | 0.042339 | 0 |
| Mode 7 | 0 | -148.403 | 148.403 | 23.61907 | 0.042339 | 0 |
| Mode 8 | 0 | 64.54062 | 64.54062 | 10.27196 | 0.097352 | 0 |
| Mode 9 | 0 | -64.54062 | 64.54062 | 10.27196 | 0.097352 | 0 |
| Mode 10 | 0 | 124.64 | 124.64 | 19.83707 | 0.050411 | 0 |
| Mode 11 | 0 | -124.64 | 124.64 | 19.83707 | 0.050411 | 0 |
| Mode 12 | -33.74245 | 0 | 33.74245 | 0 | 0 | 1 |
| Mode 13 | -1.527487 | 18.96752 | 19.02893 | 3.018775 | 0.33126 | 0.080272 |
| Mode 14 | -1.527487 | -18.96752 | 19.02893 | 3.018775 | 0.33126 | 0.080272 |
| Mode 15 | -21.21962 | 0 | 21.21962 | 0 | 0 | 1 |
| Mode 16 | -1.134408 | 0 | 1.134408 | 0 | 0 | 1 |
| Mode 17 | -0.973135 | 0 | 0.973135 | 0 | 0 | 1 |
| Mode 18 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 8 outlines 18 eigenvalue modes corresponding to the 800MW wind case. Mode 1 – 11 & 18 have zero real part. Mode 12- 17 are stable with negative real parts. Mode 13 & 14 are poorly damped with a damping ratio of 8% whilst Mode 12, 15, 16 and 17 are critically damped with a ratio of 100%.

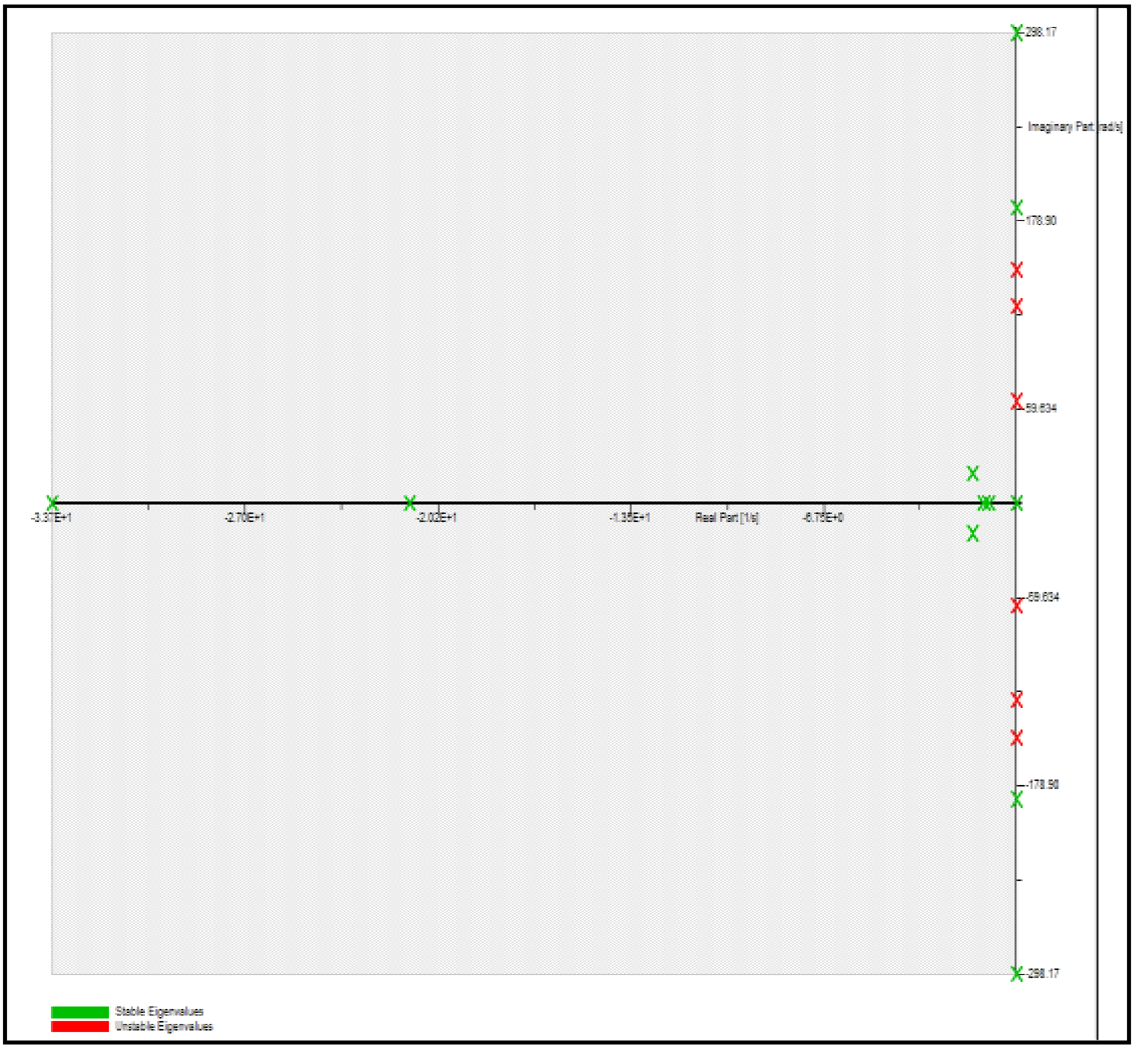


Figure 5.26: Graph showing the eigenvalue plot overview for case 4 -800MW wind scenario

The general results outlook looks more like that of case 3. As in the PV case, we also have two oscillatory modes and four critically damped modes for the wind generator cases. This indicates that the modal analysis is sensitive to the amount of power injected rather than the type of generation. This is because modal analysis emphasises on determination of the vibration characteristics (natural frequencies and mode shapes) of the mechanical structure i.e. turbo-generator under certain dynamic loading conditions.

Therefore, in this case what is under consideration is the output or loading aspect and not the generator technology. This further explains why the eigenvalue plots are identical for the respective power injections in the wind and PV case scenarios. As such, one would utilise the time domain transient analysis for deeper analysis.

## 5.6 Case 5- modified base case with STATCOM

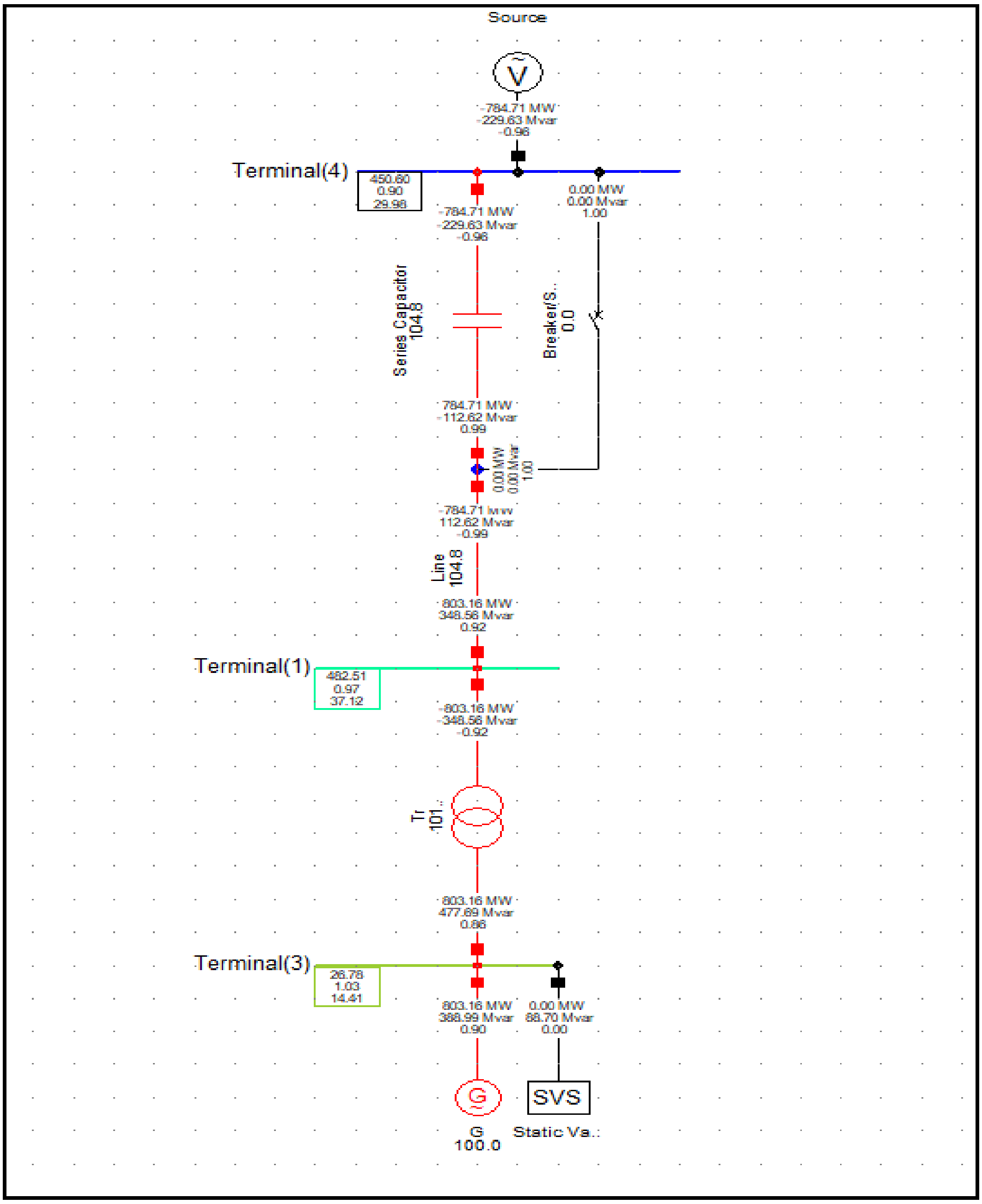


Figure 5.27: STATCOM configuration in the FBM for case 5

### 5.6.1 Simulation results case 5 – STATCOM effect

The configuration for case 5 was done according to Figure 5.27. The time domain analysis for the modified base case with STATCOM could not indicate a significant difference brought about by connection of the static compensator. On further analysis using modal analysis, there was a significant observation of increased oscillatory stability brought by the STATCOM.

### 5.6.2 Modal analysis results case 5 – STATCOM effect

Table 5-6: Eigenvalues from case 5-STATCOM integration to the FBM

| **Modes** | **Real part (1/s)** | **Imaginary part (rad/s)** | **Magnitude (1/s)** | **Damped Frequency (Hz)** | **Period (s)** | **Damping Ratio** |
|---|---|---|---|---|---|---|
| Mode 1 | 0. | 0. | 0. | 0. | 0 | 0. |
| Mode 2 | 0.2999078 | 298.1623 | 298.1625 | 47.454 | 0.021073 | -0.00100585 |
| Mode 3 | 0.2999078 | -298.1623 | 298.1625 | 47.454 | 0.021073 | -0.00100585 |
| Mode 4 | 0.1846469 | 187.2534 | 187.2534 | 29.8023 | 0.033553 | -0.00098608 |
| Mode 5 | 0.1846469 | -187.2534 | 187.2534 | 29.8023 | 0.033553 | -0.00098608 |
| Mode 6 | 0.7149498 | 148.4054 | 148.4072 | 23.61946 | 0.042339 | -0.00481749 |
| Mode 7 | 0.7149498 | -148.4054 | 148.4072 | 23.61946 | 0.042339 | -0.00481749 |
| Mode 8 | 0.111458 | 64.5419 | 64.542 | 10.27216 | 0.097352 | -0.00172691 |
| Mode 9 | 0.111458 | -64.5419 | 64.542 | 10.27216 | 0.097352 | -0.00172691 |
| Mode 10 | 0. | 124.64 | 124.64 | 19.83707 | 0.050411 | -0. |
| Mode 11 | 0. | -124.64 | 124.64 | 19.83707 | 0.050411 | -0. |
| Mode 12 | **-33.42576** | 0. | 33.42576 | 0. | 0 | 1. |
| Mode 13 | -1.087571 | 19.37593 | 19.40642 | 3.083774 | 0.322992 | 0.05604181 |
| Mode 14 | -1.087571 | -19.37593 | 19.40642 | 3.083774 | 0.322992 | 0.05604181 |
| Mode 15 | -21.4281 | 0. | 21.4281 | 0. | 0 | 1. |
| Mode 16 | -0.3756412 | 0. | 0.3756412 | 0. | 0 | 1. |
| Mode 17 | -3.674226 | 0. | 3.674226 | 0. | 0 | 1. |

Mode 12 has a more negative real part with a difference of 3% as compared to the same mode in the base case showing improved oscillatory stability.

## 5.7 Overall summary of modal analysis simulation results

The results from the modal analysis dictates that comparison be at the loading level only and not the type of generation.

Table 5-1 shows the eigenvalues corresponding to the analysis done on the base case.

Tables 5-2 & 5-3 show eigenvalues corresponding to the PV case that has 200MW and 800MW ramps injected to the FBM respectively

Tables 5-4 & 5-5 show eigenvalues corresponding to the wind case that has 200MW and 800MW ramps injected to the FBM respectively.

Table 5-6 captures the modal results involving the STATCOM.

It is also observed that there is an additional mode i.e. mode 18 that gets introduced after the PV integration as captured in Table 5-2 & 5-3, but unfortunately it does not carry any participatory information

The oscillatory set for the 800MW case in Table 5-5 has a more negative real part of -1.523 that makes it more stable than 200MW and base cases that have real part values of -1.165 (Table 5-4) and -1.128 (Table 5-1)respectively. Likewise, the damping ratios exhibit a definite increase for base case, 200MW & 800MW cases with damping ratios of 5.8%, to 6% and 8% respectively.

The more negativity of the real part observed in mode 12 in Table 5-6 indicates that the STATCOM penetration comes with better stability performance.

The measure of SSR that increases with RES integration is the increased modulation of the generator/shaft electrical and mechanical torques. As expected, we observe that the 800MW PV and wind or PV power scenarios reflect more oscillatory and transient stability compared to the 200MW wind or PV power scenarios. This reinforces the negative correlation of SSR with increased RES power quantums.

# CHAPTER 6

# 6. CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORKS

The main aim of this dissertation research was to assess the subsynchronous resonance problem in a typical network environment populated with typical FACTS and renewable energy sources connected with series compensated lines. This chapter captures the related conclusions and outlines future work scope.

## 6.1 Conclusions

### 6.1.1 SSR management with increasing RES and FACTS penetration

Series compensation is one cheaper solution to overcome the RES related long distance transmission and space availability drawbacks. SC poses as an economic, environmentally friendly and faster way to increase transmission capacity without burdening power utilities capital budgets. Chapter 3 revealed that there is growing appetite and space for RES penetration in the SAPP region. However, the dispensation of RES integrated by lines with Series Compensation (RESISC) comes with the SSR problem.

The results in chapter 5 after rigorous DIgSILENT simulations of different FBM configurations have revealed that displacement of conventional power sources such as thermal generators and replacing them with RES has a direct consequence of reducing the SSR effect in series compensated networks.

Literature review herein and modal analysis in case 5 supports that FACTS devises lower the SSR probability and provide SSR mitigation options usable by power engineers.

Whereas possibilities of having subsynchronous frequency interactions are high in the modern power system, the impact thereof is still an area requiring ongoing research.

### 6.1.2 RES integration impacts on SSR

Photovoltaic power does not contribute to SSR occurrence since its generation mechanism has no moving parts. In large disturbance analysis, the monitored generator voltage magnitude for both the 200MW and 800MW cases recovered well to 1.0 p.u. after the disturbance. Similarly, the same results monitoring the current and voltage angles did not exhibit any oscillatory behaviour in transient stability analysis.

Through modal analysis, it was evident that increasing RES loading on the FBM network (from 200MW to 800MW) had a direct improvement of small signal stability on the synchronous generator. The 800MW PV injection case offers about 2.2% more damping than the base case. Similarly, the 200MW PV case improved in stability as shown by a marginal 0.2% increase in damping ratio variance with the base case.

The results analysis confirms that the average PV power plants (less than 800MW) are immune to risk of SSR save for the controller interactions-SSCI for large PV plants (greater than 800MW). The frequencies around PV based generation are high frequencies that would not fall under the SSR frequencies and the PV case simulation results show similar pre and post disturbance conditions.

The results in this study led to the conclusion that as we introduce more RES into the network the risk of SSR declines and there is enhanced SSR management capability.

### 6.1.3 SSR with series compensation

The SSR problem is very aggressive for fixed series compensated networks and controllable for series FACTS compensated lines. In Africa, SAPP in particular, as new generation integrations increase to meet rising demand, the penetration of series compensation is certain as a cheaper option to increase grid evacuation levels. This renders the SSR problem and its management very critical. FACTS are now preferred ahead of fixed compensation.

### 6.1.4 Validity of Hypothesis

The background and motivation to this research resulted initially at a hypothesis that states that: **"The integration of RES such as wind, solar PV power in series compensated AC networks with increasing penetration of renewables inherently improves stability performance of the network in managing SSR".**

The results in chapter 5 prove that the hypothesis is true and valid. In this view, the hypothesis has concisely addressed the overstated nature of the SSR problem.

### 6.1.5 Answers to Research Questions

This section summarises the answers to the six (6) research questions tabled in chapter 1.

1. **What has been the critical impact of SSR, what are the recently introduced sources of SSR in the power system?**

A known case of shaft failure occurred in Arizona in 1972 caused by unsuppressed SSR vibrations. RES, SCs, RESISCs, HVDC, FACTS are SSR ingredients being introduced in the power system.

2. **Is it justified to invest in SSR mitigation, looking at the increasing penetration of its very sources in the power system e.g. RES and transmission line compensation equipment?**

Incorporation of SSR protection via IEDs, utilising the HPAC filter is encouraged in this work as a reasonable and effective mitigation method. IEDs provide versatile SSR monitoring, detection, and protection functionalities with optimised performance for transmission overhead cables.

3. **How is SSR mitigated by introducing and optimising RES or STATCOM integrations?**

RESISC is becoming a reality in the power system. The simulations studies conducted herein have shown that increasing RES and STATCOM penetration inherently comes with SSR mitigation effects.

4. **Are conventional power plants with rotating masses subjected to higher SSR risk with rising RES penetration levels?**

The SSR risk percentage of plants with rotating masses decreases with rising RES penetration as revealed in the simulation results in chapter 5. The results also point that as RES displace thermal power, there is enhancement in the power system's ability to reduce SSR risk.

5. **What is current trend of RES integrations in Southern Africa, and will the approach of incorporating SSR detection in IEDs be effective?**

RES penetration in SAPP is growing as discussed in chapter 3. SSR protection method within IEDs becomes critical. ABB in Sweden has tested the effectives of such IEDs with good feedback. SAPP utilities can embrace the same SSR management method to their benefit.

## 6.2 Recommendations for Future Works

Future works and recommendations are to model other renewable energy sources and test the correlation with SSR incidence. It is also imperative to pursue and test by way of simulation models the active SSR management technique utilising the prony analysis. A practical test-bed setup can also be built, and the HPAC filter can be tested online for SSR mitigation on a transmission network system to validate the simulation models.

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# APPENDICES

## Appendix A: DIgSILENT shaft model

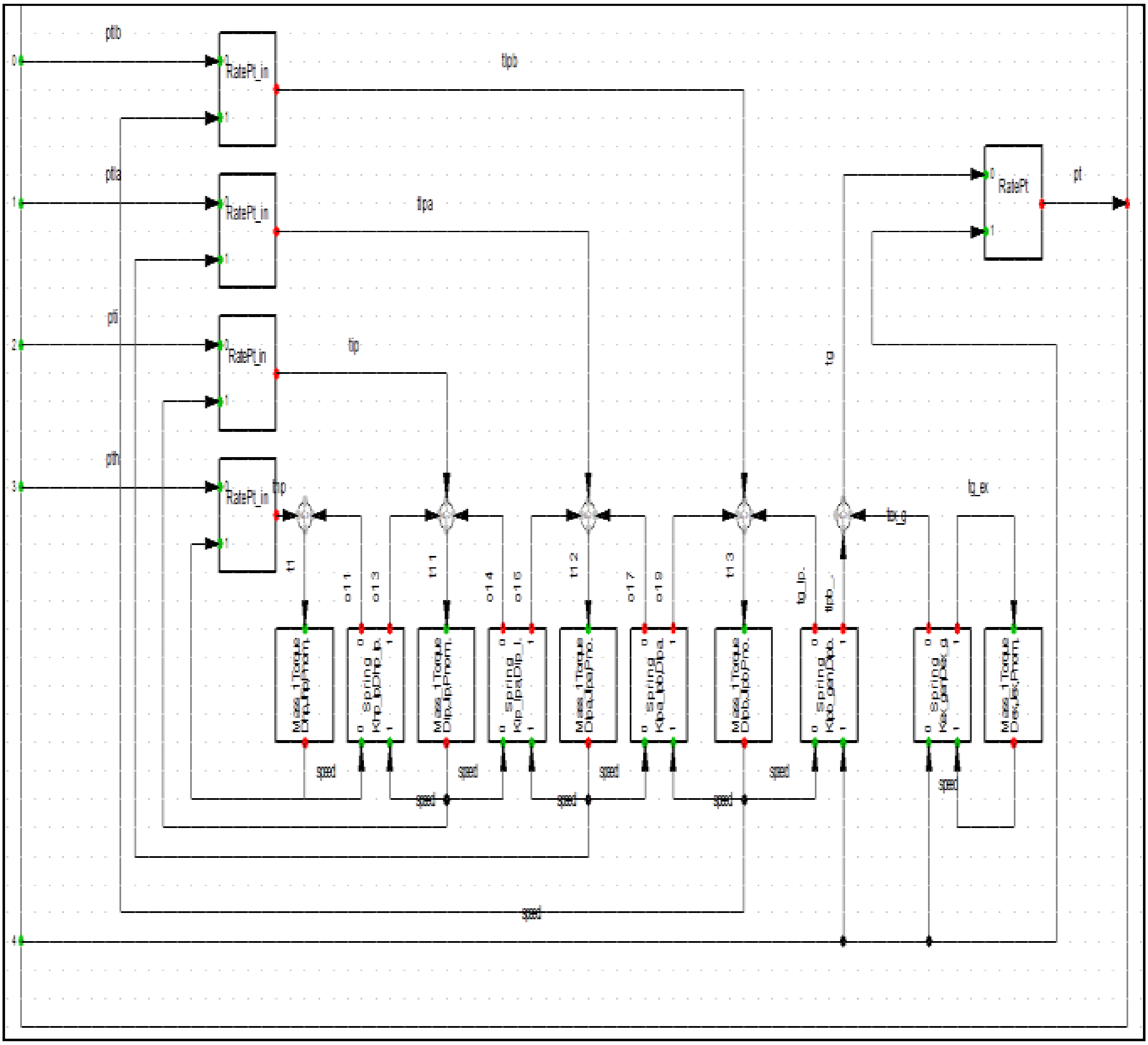


Fig. A-1. DIgSILENT 6 masses shaft model (11A- SHF)

**Appendix B: DIgSILENT linear model**

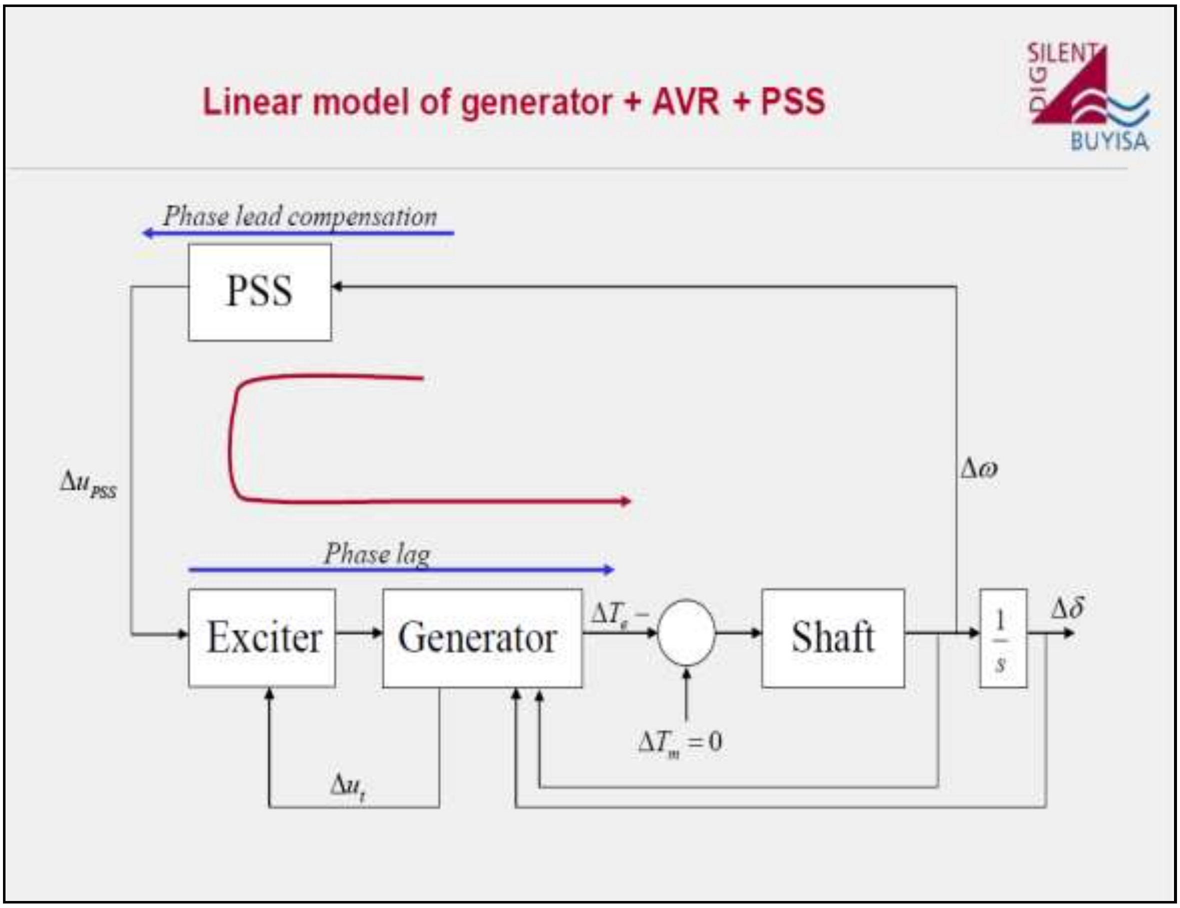


Fig. B-1. Linear model of generator, AVR and PSS in DIgSILENT

**Appendix C: Result output file from base case (unmodified FBM) simulation**

*Table C-1: DIgSILENT result output file for unmodified FBM scenario*

| Time | Exciter | Shaft | Shaft | Exciter | Time | Generator | Generator | Generator |
|---|---|---|---|---|---|---|---|---|
| Time in s | tg_ex in p.u. | tg_lpb in p.u. | tlpb_g in p.u. | tex_g in p.u | time | Electrical Torque in p.u. | Mechanical Torque in p.u. | Speed in p.u. |
| -0.09 | 0 | -0.25 | 0.25 | 0 | -0.09 | 0.25 | 0.25 | 1 |
| -0.08 | 0 | -0.25 | 0.25 | 0 | -0.08 | 0.25 | 0.25 | 1 |
| -0.07 | 0 | -0.25 | 0.25 | 0 | -0.07 | 0.25 | 0.25 | 1 |
| -0.06 | 0 | -0.25 | 0.25 | 0 | -0.06 | 0.25 | 0.25 | 1 |
| -0.05 | 0 | -0.25 | 0.25 | 0 | -0.05 | 0.25 | 0.25 | 1 |
| -0.04 | 0 | -0.25 | 0.25 | 0 | -0.04 | 0.25 | 0.25 | 1 |
| -0.03 | 0 | -0.25 | 0.25 | 0 | -0.03 | 0.25 | 0.25 | 1 |
| -0.02 | 0 | -0.25 | 0.25 | 0 | -0.02 | 0.25 | 0.25 | 1 |
| -0.01 | 0 | -0.25 | 0.25 | 0 | -0.01 | 0.25 | 0.25 | 1 |
| 0 | 0 | -0.25 | 0.25 | 0 | 0 | 0.25 | 0.25 | 1 |
| 0.00505 | 0.000736 | -0.23038 | 0.230379 | -0.00074 | 0.00505 | 0.14774 | 0.229639 | 1.000212 |
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|---|---|---|---|---|---|---|---|---|
| 0.676761 | 0.002215 | -0.23672 | 0.236724 | -0.00222 | 0.676761 | 0.218868 | 0.234504 | 0.999441 |
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| 1.436761 | -0.00143 | -0.24039 | 0.240386 | 0.001428 | 1.436761 | 0.267283 | 0.241806 | 0.999433 |
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| 1.496761 | -0.00359 | -0.29027 | 0.290274 | 0.003587 | 1.496761 | 0.257502 | 0.293857 | 0.999211 |
| 1.506761 | -0.00174 | -0.28453 | 0.284529 | 0.001739 | 1.506761 | 0.255856 | 0.286267 | 0.999384 |
| 1.516761 | 0.002858 | -0.23062 | 0.230621 | -0.00286 | 1.516761 | 0.254443 | 0.227766 | 0.999393 |
| 1.526761 | 0.003696 | -0.22348 | 0.22348 | -0.0037 | 1.526761 | 0.252798 | 0.219783 | 0.999239 |
| 1.536761 | -0.00034 | -0.25868 | 0.258683 | 0.000342 | 1.536761 | 0.25101 | 0.259036 | 0.999174 |
| 1.546761 | -0.00396 | -0.26067 | 0.260672 | 0.003956 | 1.546761 | 0.249119 | 0.26463 | 0.999235 |

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| 1.556761 | -0.00305 | -0.23805 | 0.238054 | 0.003052 | 1.556761 | 0.247335 | 0.241101 | 0.999259 |
| 1.566761 | 0.001206 | -0.24557 | 0.245573 | -0.00121 | 1.566761 | 0.245677 | 0.244364 | 0.999239 |
| 1.576761 | 0.00439 | -0.26862 | 0.268615 | -0.00439 | 1.576761 | 0.244022 | 0.26423 | 0.999288 |
| 1.586761 | 0.003176 | -0.25675 | 0.256748 | -0.00318 | 1.586761 | 0.242476 | 0.253569 | 0.999369 |
| 1.596761 | -0.00114 | -0.21568 | 0.215684 | 0.001143 | 1.596761 | 0.241094 | 0.216846 | 0.999335 |
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| 1.646761 | 0.003344 | -0.24553 | 0.245529 | -0.00334 | 1.646761 | 0.233508 | 0.242187 | 0.999416 |
| 1.656761 | -0.00217 | -0.25514 | 0.255143 | 0.002168 | 1.656761 | 0.232443 | 0.257316 | 0.999502 |
| 1.666761 | -0.00476 | -0.22982 | 0.229822 | 0.004759 | 1.666761 | 0.231456 | 0.234575 | 0.999575 |
| 1.676761 | -0.00216 | -0.20107 | 0.201067 | 0.002164 | 1.676761 | 0.230518 | 0.203225 | 0.999512 |
| 1.686761 | 0.00237 | -0.22069 | 0.220691 | -0.00237 | 1.686761 | 0.22947 | 0.218324 | 0.999413 |
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| 1.706761 | 0.002199 | -0.26283 | 0.262834 | -0.0022 | 1.706761 | 0.22739 | 0.260636 | 0.999623 |
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| 1.816761 | 0.000408 | -0.23373 | 0.233725 | -0.00041 | 1.816761 | 0.225569 | 0.233301 | 1.000025 |
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| 1.846761 | 0.001014 | -0.22863 | 0.22863 | -0.00101 | 1.846761 | 0.226712 | 0.22761 | 1.000091 |
| 1.856761 | 0.003501 | -0.26567 | 0.265666 | -0.0035 | 1.856761 | 0.227224 | 0.262175 | 1.000183 |
| 1.866761 | 0.003064 | -0.25419 | 0.254185 | -0.00306 | 1.866761 | 0.227942 | 0.251118 | 1.000334 |
| 1.876761 | -0.00034 | -0.21897 | 0.218972 | 0.000342 | 1.876761 | 0.228903 | 0.219307 | 1.000369 |
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| 1.946761 | -0.00442 | -0.2564 | 0.256395 | 0.004423 | 1.946761 | 0.236095 | 0.260815 | 1.000563 |
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| 1.966761 | 0.003522 | -0.22849 | 0.228492 | -0.00352 | 1.966761 | 0.238935 | 0.22497 | 1.000516 |
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| 1.986761 | 0.000149 | -0.25178 | 0.251776 | -0.00015 | 1.986761 | 0.24146 | 0.251628 | 1.000569 |

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| 1.996761 | -0.00346 | -0.22461 | 0.224612 | 0.003457 | 1.996761 | 0.242933 | 0.228084 | 1.000557 |
| 2.006761 | -0.00341 | -0.23706 | 0.237064 | 0.003405 | 2.006761 | 0.24426 | 0.240472 | 1.000508 |
| 2.016761 | 0.000356 | -0.27178 | 0.271779 | -0.00036 | 2.016761 | 0.245517 | 0.27143 | 1.000566 |
| 2.026761 | 0.004215 | -0.26259 | 0.26259 | -0.00422 | 2.026761 | 0.246916 | 0.258371 | 1.000663 |
| 2.036761 | 0.003443 | -0.23415 | 0.234154 | -0.00344 | 2.036761 | 0.248435 | 0.230704 | 1.000646 |
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| 2.106761 | -0.00351 | -0.26821 | 0.268214 | 0.003513 | 2.106761 | 0.258019 | 0.271725 | 1.000607 |
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| 2.156761 | 0.000074 | -0.23065 | 0.230652 | -7.4E-05 | 2.156761 | 0.264013 | 0.230567 | 1.00053 |
| 2.166761 | -0.00408 | -0.23833 | 0.238327 | 0.004083 | 2.166761 | 0.265036 | 0.242413 | 1.000385 |
| 2.176761 | -0.004 | -0.27671 | 0.276705 | 0.004004 | 2.176761 | 0.265828 | 0.280719 | 1.000365 |
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| 2.196761 | 0.004445 | -0.24883 | 0.248829 | -0.00445 | 2.196761 | 0.267352 | 0.244377 | 1.000391 |
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| 2.246761 | 0.002926 | -0.23369 | 0.233687 | -0.00293 | 2.246761 | 0.270442 | 0.230761 | 1.000132 |
| 2.256761 | 0.002175 | -0.28639 | 0.286392 | -0.00218 | 2.256761 | 0.270494 | 0.284214 | 1.000064 |
| 2.266761 | -0.00096 | -0.28643 | 0.286429 | 0.000959 | 2.266761 | 0.270617 | 0.287389 | 1.000143 |
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| 2.286761 | -0.00182 | -0.2573 | 0.2573 | 0.001815 | 2.286761 | 0.270893 | 0.259116 | 1.000063 |
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| 2.346761 | -0.00133 | -0.29791 | 0.297909 | 0.001333 | 2.346761 | 0.269548 | 0.299241 | 0.999933 |
| 2.356761 | 0.004604 | -0.2421 | 0.242103 | -0.0046 | 2.356761 | 0.269291 | 0.237502 | 0.999928 |
| 2.366761 | 0.004799 | -0.24532 | 0.24532 | -0.0048 | 2.366761 | 0.268791 | 0.240522 | 0.999772 |
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| 2.386761 | -0.00561 | -0.26616 | 0.266155 | 0.005612 | 2.386761 | 0.267375 | 0.271763 | 0.999799 |
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| 2.416761 | 0.005097 | -0.29524 | 0.295238 | -0.0051 | 2.416761 | 0.264845 | 0.290154 | 0.99959 |
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| 2.436761 | -0.00154 | -0.24135 | 0.241348 | 0.001535 | 2.436761 | 0.263053 | 0.242874 | 0.999702 |
| 2.446761 | -0.00567 | -0.23592 | 0.235919 | 0.005665 | 2.446761 | 0.262213 | 0.241584 | 0.999597 |
| 2.456761 | -0.00377 | -0.26267 | 0.262665 | 0.00377 | 2.456761 | 0.261206 | 0.266441 | 0.999557 |
| 2.466761 | 0.002201 | -0.2604 | 0.260398 | -0.0022 | 2.466761 | 0.260057 | 0.258195 | 0.999565 |
| 2.476761 | 0.00531 | -0.24059 | 0.240587 | -0.00531 | 2.476761 | 0.258895 | 0.235272 | 0.999499 |
| 2.486761 | 0.00198 | -0.25804 | 0.258044 | -0.00198 | 2.486761 | 0.257624 | 0.256062 | 0.999434 |
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| 2.516761 | -0.00036 | -0.22384 | 0.223836 | 0.000362 | 2.516761 | 0.254393 | 0.224189 | 0.999594 |
| 2.526761 | 0.003161 | -0.22792 | 0.227924 | -0.00316 | 2.526761 | 0.253315 | 0.224763 | 0.999442 |
| 2.536761 | 0.002678 | -0.26377 | 0.263765 | -0.00268 | 2.536761 | 0.251997 | 0.261096 | 0.999392 |
| 2.546761 | -0.00071 | -0.26672 | 0.266722 | 0.00071 | 2.546761 | 0.250617 | 0.267435 | 0.999459 |
| 2.556761 | -0.00285 | -0.23312 | 0.23312 | 0.002854 | 2.556761 | 0.249351 | 0.235993 | 0.999468 |
| 2.566761 | -0.00197 | -0.23958 | 0.239575 | 0.001973 | 2.566761 | 0.248148 | 0.241549 | 0.999416 |
| 2.576761 | 0.001015 | -0.27973 | 0.279732 | -0.00102 | 2.576761 | 0.246883 | 0.278728 | 0.999482 |
| 2.586761 | 0.003529 | -0.26679 | 0.26679 | -0.00353 | 2.586761 | 0.245794 | 0.263258 | 0.999609 |
| 2.596761 | 0.002147 | -0.21791 | 0.217911 | -0.00215 | 2.596761 | 0.244847 | 0.215754 | 0.999579 |
| 2.606761 | -0.00225 | -0.21455 | 0.214554 | 0.002252 | 2.606761 | 0.243809 | 0.216807 | 0.999434 |
| 2.616761 | -0.00458 | -0.2633 | 0.263299 | 0.004578 | 2.616761 | 0.242515 | 0.267873 | 0.99943 |
| 2.626761 | -0.00115 | -0.27636 | 0.276359 | 0.001152 | 2.626761 | 0.241482 | 0.277514 | 0.999589 |
| 2.636761 | 0.004324 | -0.23614 | 0.236143 | -0.00432 | 2.636761 | 0.240604 | 0.231809 | 0.999659 |
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| 2.716761 | -0.00204 | -0.24532 | 0.245316 | 0.002041 | 2.716761 | 0.234411 | 0.247354 | 0.999773 |
| 2.726761 | -0.0042 | -0.22618 | 0.226175 | 0.004201 | 2.726761 | 0.234025 | 0.230372 | 0.999798 |
| 2.736761 | -0.00159 | -0.23594 | 0.235943 | 0.001588 | 2.736761 | 0.233711 | 0.237527 | 0.999798 |
| 2.746761 | 0.002941 | -0.24307 | 0.243066 | -0.00294 | 2.746761 | 0.233363 | 0.240123 | 0.999825 |
| 2.756761 | 0.004309 | -0.23077 | 0.230768 | -0.00431 | 2.756761 | 0.233127 | 0.226455 | 0.999826 |
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| 2.786761 | -0.00315 | -0.25135 | 0.251348 | 0.003149 | 2.786761 | 0.232485 | 0.254498 | 0.999967 |
| 2.796761 | 0.000988 | -0.22927 | 0.229274 | -0.00099 | 2.796761 | 0.232562 | 0.22828 | 1.000013 |
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| 2.816761 | 0.002528 | -0.24479 | 0.244793 | -0.00253 | 2.816761 | 0.232776 | 0.24227 | 0.999964 |
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| 2.846761 | -0.00151 | -0.22445 | 0.224446 | 0.001506 | 2.846761 | 0.233494 | 0.225956 | 1.000059 |
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| 2.876761 | 0.002213 | -0.22417 | 0.224174 | -0.00221 | 2.876761 | 0.234946 | 0.221951 | 1.000271 |
| 2.886761 | -0.00225 | -0.20844 | 0.208435 | 0.002254 | 2.886761 | 0.235617 | 0.210686 | 1.000173 |
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| 2.916761 | 0.003721 | -0.24612 | 0.246123 | -0.00372 | 2.916761 | 0.237293 | 0.242397 | 1.000305 |
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| 2.966761 | 0.001235 | -0.2187 | 0.218699 | -0.00124 | 2.966761 | 0.241734 | 0.217459 | 1.000371 |
| 2.976761 | 0.00368 | -0.23594 | 0.235942 | -0.00368 | 2.976761 | 0.242602 | 0.232266 | 1.000282 |
| 2.986761 | 0.001542 | -0.26656 | 0.266556 | -0.00154 | 2.986761 | 0.243283 | 0.264992 | 1.000311 |
| 2.996761 | -0.00183 | -0.26367 | 0.263668 | 0.001833 | 2.996761 | 0.244237 | 0.265501 | 1.000422 |
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| 3.026761 | 0.00127 | -0.26456 | 0.264562 | -0.00127 | 3.026761 | 0.247265 | 0.263275 | 1.000418 |
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| 3.116761 | -0.00229 | -0.23985 | 0.239854 | 0.002293 | 3.116761 | 0.255828 | 0.242141 | 1.00043 |
| 3.126761 | 0.000152 | -0.23328 | 0.233277 | -0.00015 | 3.126761 | 0.256646 | 0.233123 | 1.000334 |
| 3.136761 | 0.002137 | -0.26816 | 0.268164 | -0.00214 | 3.136761 | 0.257385 | 0.266036 | 1.000295 |
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| 3.316761 | 0.001138 | -0.26203 | 0.262027 | -0.00114 | 3.316761 | 0.264052 | 0.260906 | 1.000084 |
| 3.326761 | -0.00169 | -0.22628 | 0.226277 | 0.001693 | 3.326761 | 0.26405 | 0.227962 | 0.999982 |
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| 3.416761 | 0.002569 | -0.26735 | 0.267354 | -0.00257 | 3.416761 | 0.26064 | 0.264766 | 0.99976 |
| 3.426761 | 0.001809 | -0.27601 | 0.276008 | -0.00181 | 3.426761 | 0.260023 | 0.274202 | 0.999807 |
| 3.436761 | -0.00023 | -0.23285 | 0.232854 | 0.000234 | 3.436761 | 0.25951 | 0.233113 | 0.999776 |
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| 3.466761 | 0.000563 | -0.28421 | 0.284211 | -0.00056 | 3.466761 | 0.257164 | 0.283648 | 0.999795 |
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| 3.756761 | 0.004349 | -0.22931 | 0.229309 | -0.00435 | 3.756761 | 0.238922 | 0.224952 | 0.999937 |
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| 3.856761 | 0.001768 | -0.26033 | 0.260332 | -0.00177 | 3.856761 | 0.23928 | 0.258542 | 1.00007 |
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| 4.196761 | 0.002713 | -0.23589 | 0.235891 | -0.00271 | 4.196761 | 0.257072 | 0.233169 | 1.000254 |
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| 4.376761 | 0.002304 | -0.26018 | 0.260183 | -0.0023 | 4.376761 | 0.258481 | 0.257885 | 0.999869 |
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| 4.456761 | -0.00349 | -0.27036 | 0.270358 | 0.003487 | 4.456761 | 0.255498 | 0.273854 | 0.999766 |
| 4.466761 | -0.00112 | -0.27258 | 0.272583 | 0.001121 | 4.466761 | 0.254985 | 0.273704 | 0.999862 |
| 4.476761 | 0.002661 | -0.24409 | 0.244088 | -0.00266 | 4.476761 | 0.254607 | 0.241419 | 0.999876 |
| 4.486761 | 0.00319 | -0.23951 | 0.239507 | -0.00319 | 4.486761 | 0.254236 | 0.236316 | 0.999796 |
| 4.496761 | -0.0004 | -0.27088 | 0.270877 | 0.000395 | 4.496761 | 0.253737 | 0.271247 | 0.999795 |
| 4.506761 | -0.00318 | -0.26716 | 0.267161 | 0.003181 | 4.506761 | 0.253358 | 0.270342 | 0.999884 |
| 4.516761 | -0.00194 | -0.22695 | 0.226954 | 0.001939 | 4.516761 | 0.252992 | 0.228884 | 0.999866 |
| 4.526761 | 0.000929 | -0.22991 | 0.229914 | -0.00093 | 4.526761 | 0.252574 | 0.228984 | 0.999742 |
| 4.536761 | 0.002443 | -0.26536 | 0.265355 | -0.00244 | 4.536761 | 0.25197 | 0.262921 | 0.999709 |
| 4.546761 | 0.001402 | -0.26988 | 0.26988 | -0.0014 | 4.546761 | 0.251332 | 0.26848 | 0.999782 |
| 4.556761 | -0.00072 | -0.23396 | 0.233964 | 0.000719 | 4.556761 | 0.250814 | 0.234703 | 0.999785 |
| 4.566761 | -0.00234 | -0.23363 | 0.233626 | 0.002344 | 4.566761 | 0.250219 | 0.23597 | 0.999706 |
| 4.576761 | -0.00174 | -0.27177 | 0.271774 | 0.001738 | 4.576761 | 0.249623 | 0.273523 | 0.999731 |
| 4.586761 | 0.001303 | -0.26871 | 0.26871 | -0.0013 | 4.586761 | 0.249077 | 0.267406 | 0.99984 |
| 4.596761 | 0.003031 | -0.23397 | 0.233969 | -0.00303 | 4.596761 | 0.248678 | 0.230929 | 0.999842 |
| 4.606761 | 0.001025 | -0.23052 | 0.23052 | -0.00103 | 4.606761 | 0.248263 | 0.229495 | 0.999747 |
| 4.616761 | -0.00273 | -0.26991 | 0.269908 | 0.002727 | 4.616761 | 0.247681 | 0.272605 | 0.999763 |
| 4.626761 | -0.00287 | -0.26892 | 0.268922 | 0.002872 | 4.626761 | 0.247346 | 0.271794 | 0.999891 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4.636761 | 0.001131 | -0.21988 | 0.219878 | -0.00113 | 4.636761 | 0.24713 | 0.21875 | 0.999881 |
| 4.646761 | 0.003283 | -0.22305 | 0.223045 | -0.00328 | 4.646761 | 0.246685 | 0.219762 | 0.999738 |
| 4.656761 | 0.001238 | -0.26495 | 0.264951 | -0.00124 | 4.656761 | 0.246135 | 0.263724 | 0.999714 |
| 4.666761 | -0.00189 | -0.26497 | 0.264971 | 0.001887 | 4.666761 | 0.245563 | 0.266859 | 0.999814 |
| 4.676761 | -0.00265 | -0.23537 | 0.235369 | 0.002648 | 4.676761 | 0.245166 | 0.23801 | 0.999851 |
| 4.686761 | -0.00046 | -0.23853 | 0.238528 | 0.000463 | 4.686761 | 0.244908 | 0.23899 | 0.999817 |
| 4.696761 | 0.002413 | -0.26396 | 0.263964 | -0.00241 | 4.696761 | 0.244546 | 0.261557 | 0.999846 |
| 4.706761 | 0.002939 | -0.2561 | 0.256098 | -0.00294 | 4.706761 | 0.244271 | 0.253157 | 0.999913 |
| 4.716761 | 0.00015 | -0.22581 | 0.225814 | -0.00015 | 4.716761 | 0.244059 | 0.225658 | 0.999888 |
| 4.726761 | -0.00316 | -0.22772 | 0.227724 | 0.003163 | 4.726761 | 0.243794 | 0.230889 | 0.999807 |
| 4.736761 | -0.003 | -0.25685 | 0.256853 | 0.002999 | 4.736761 | 0.243439 | 0.25986 | 0.999816 |
| 4.746761 | 0.000886 | -0.26204 | 0.262038 | -0.00089 | 4.746761 | 0.243131 | 0.261153 | 0.999906 |
| 4.756761 | 0.004368 | -0.23402 | 0.234021 | -0.00437 | 4.756761 | 0.242965 | 0.229671 | 0.999918 |
| 4.766761 | 0.002777 | -0.23511 | 0.235106 | -0.00278 | 4.766761 | 0.24277 | 0.23233 | 0.999856 |
| 4.776761 | -0.00222 | -0.26348 | 0.263479 | 0.002215 | 4.776761 | 0.242576 | 0.265703 | 0.999889 |
| 4.786761 | -0.00426 | -0.25323 | 0.25323 | 0.004263 | 4.786761 | 0.242438 | 0.25749 | 0.999988 |
| 4.796761 | -0.00099 | -0.21373 | 0.213729 | 0.00099 | 4.796761 | 0.242497 | 0.214742 | 0.999955 |
| 4.806761 | 0.002889 | -0.22824 | 0.228242 | -0.00289 | 4.806761 | 0.242266 | 0.225355 | 0.99984 |
| 4.816761 | 0.002842 | -0.28473 | 0.284726 | -0.00284 | 4.816761 | 0.241983 | 0.281879 | 0.999899 |
| 4.826761 | 0.000464 | -0.27258 | 0.27258 | -0.00046 | 4.826761 | 0.242043 | 0.272113 | 1.00008 |
| 4.836761 | -0.0016 | -0.21401 | 0.214006 | 0.001599 | 4.836761 | 0.242342 | 0.215609 | 1.000089 |
| 4.846761 | -0.00227 | -0.21853 | 0.218533 | 0.002269 | 4.846761 | 0.242442 | 0.220803 | 0.999964 |
| 4.856761 | -0.00072 | -0.26683 | 0.266834 | 0.000723 | 4.856761 | 0.242487 | 0.267569 | 0.999973 |
| 4.866761 | 0.002051 | -0.26255 | 0.262549 | -0.00205 | 4.866761 | 0.24254 | 0.260496 | 1.000084 |
| 4.876761 | 0.002593 | -0.22353 | 0.223532 | -0.00259 | 4.876761 | 0.242734 | 0.22093 | 1.000074 |
| 4.886761 | -0.0001 | -0.22904 | 0.229036 | 0.000104 | 4.886761 | 0.242918 | 0.229142 | 0.999982 |
| 4.896761 | -0.00258 | -0.26907 | 0.269065 | 0.002581 | 4.896761 | 0.243027 | 0.271658 | 1.00002 |
| 4.906761 | -0.00149 | -0.26982 | 0.269816 | 0.001489 | 4.906761 | 0.243247 | 0.271305 | 1.000167 |
| 4.916761 | 0.001659 | -0.23176 | 0.231764 | -0.00166 | 4.916761 | 0.243679 | 0.230095 | 1.000205 |
| 4.926761 | 0.002652 | -0.22072 | 0.22072 | -0.00265 | 4.926761 | 0.244125 | 0.218065 | 1.000102 |
| 4.936761 | 0.000299 | -0.24623 | 0.246231 | -0.0003 | 4.936761 | 0.244388 | 0.245938 | 1.000039 |
| 4.946761 | -0.00235 | -0.25835 | 0.258349 | 0.002351 | 4.946761 | 0.244555 | 0.260704 | 1.000084 |
| 4.956761 | -0.00198 | -0.24321 | 0.243205 | 0.001978 | 4.956761 | 0.244765 | 0.245193 | 1.000127 |
| 4.966761 | 0.000716 | -0.24554 | 0.245544 | -0.00072 | 4.966761 | 0.245163 | 0.244828 | 1.000128 |
| 4.976761 | 0.002663 | -0.26365 | 0.263653 | -0.00266 | 4.976761 | 0.245567 | 0.260994 | 1.000167 |
| 4.986761 | 0.00193 | -0.25168 | 0.251675 | -0.00193 | 4.986761 | 0.245962 | 0.249743 | 1.000216 |

**Appendix D: Results output file comparing 75 ms and 100 ms fault duration**

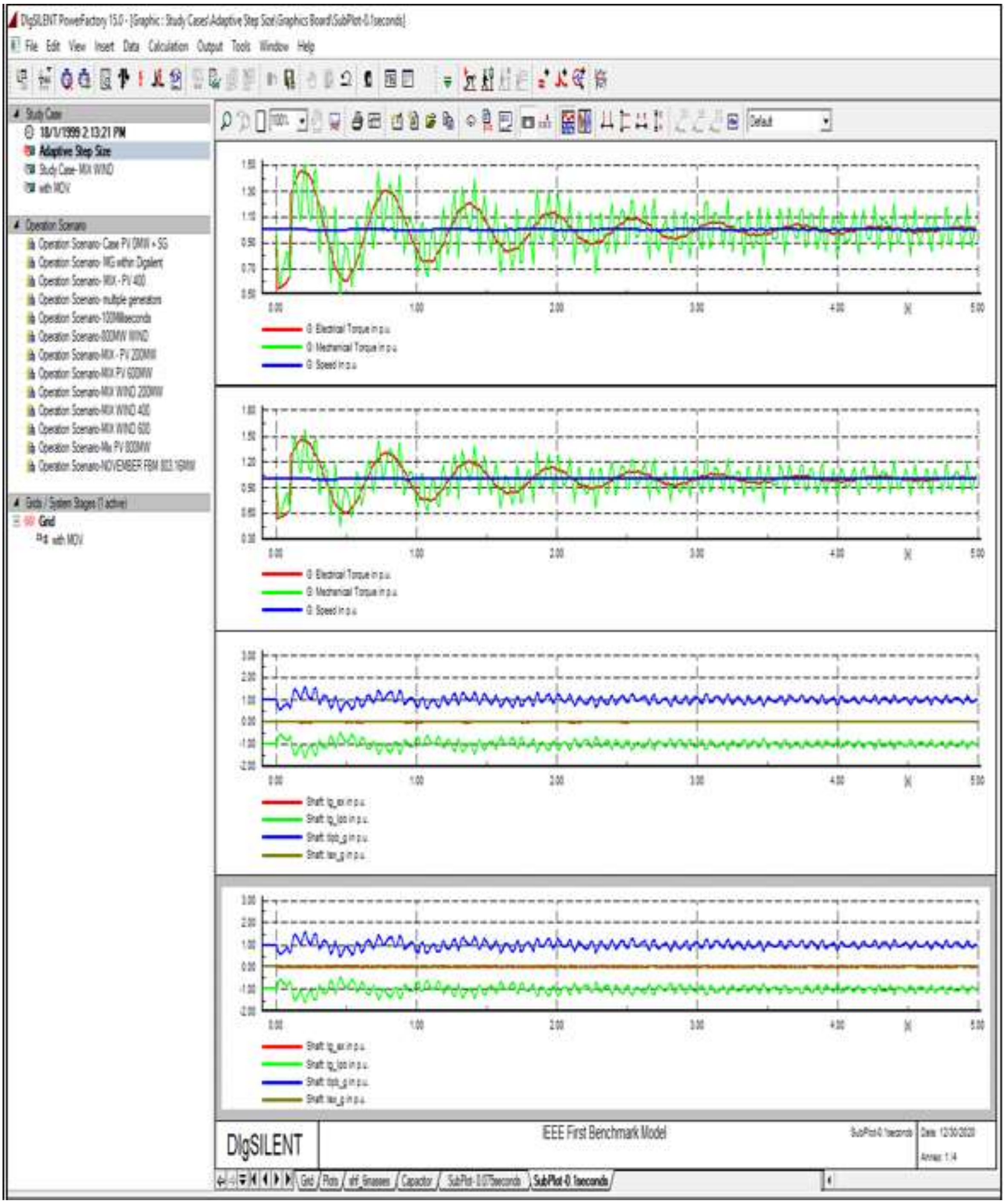



Fig. D-1. Comparison of 75ms & 100ms of simulation time length plots.

## Appendix E: Modal analysis - base case scenario phasor and bar plots

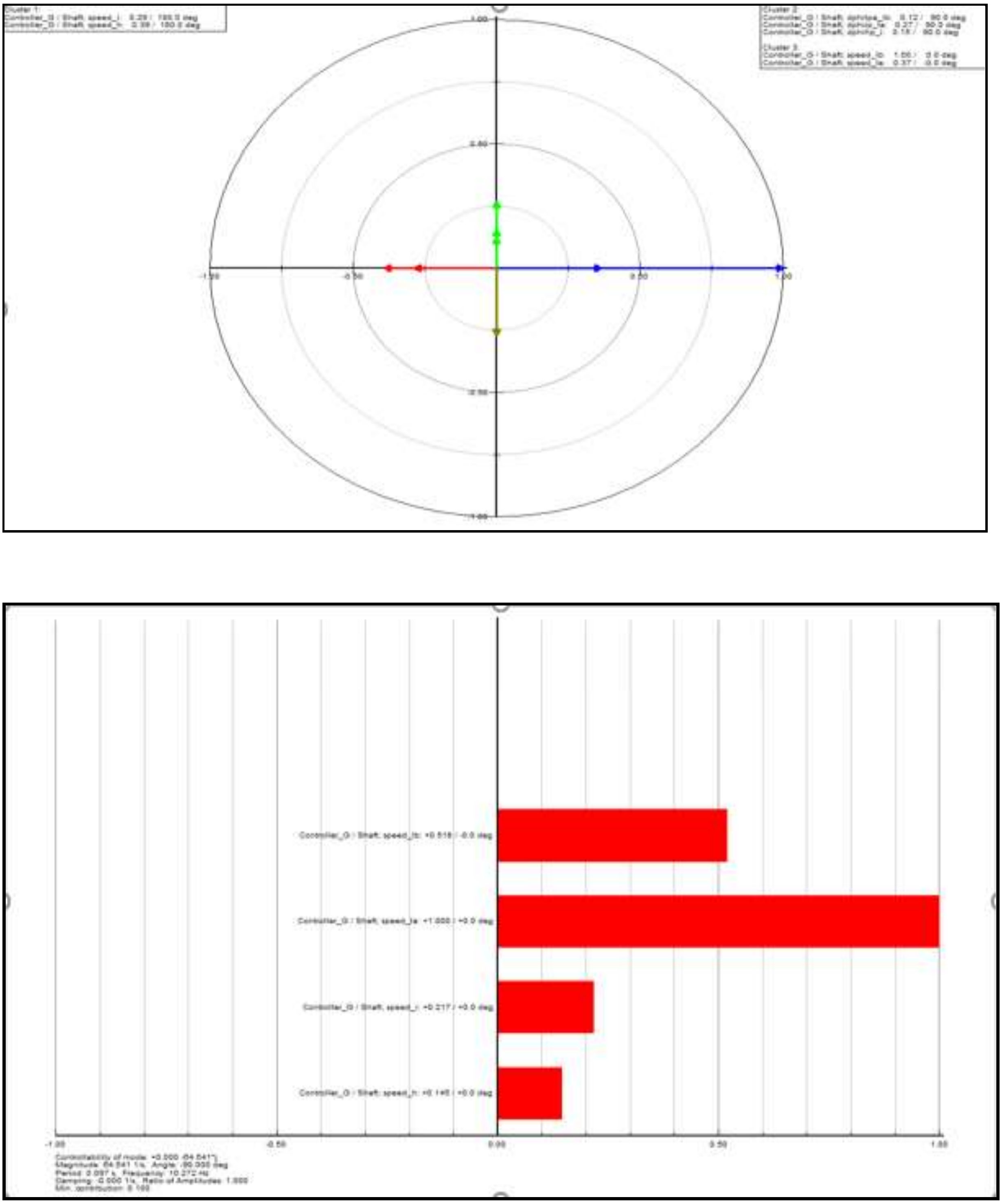
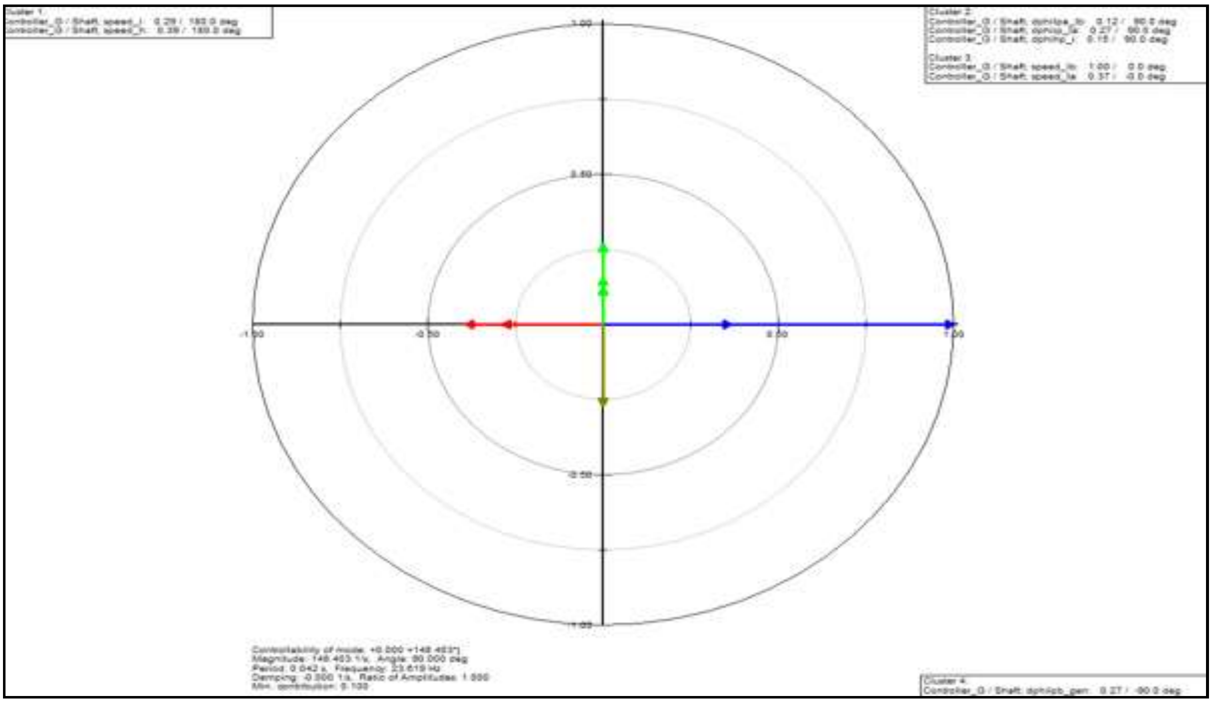



Fig. E-1. Modal analysis - base case phasor and bar plots

Appendix F: Modal analysis – case 3, PV case phasor and bar plots



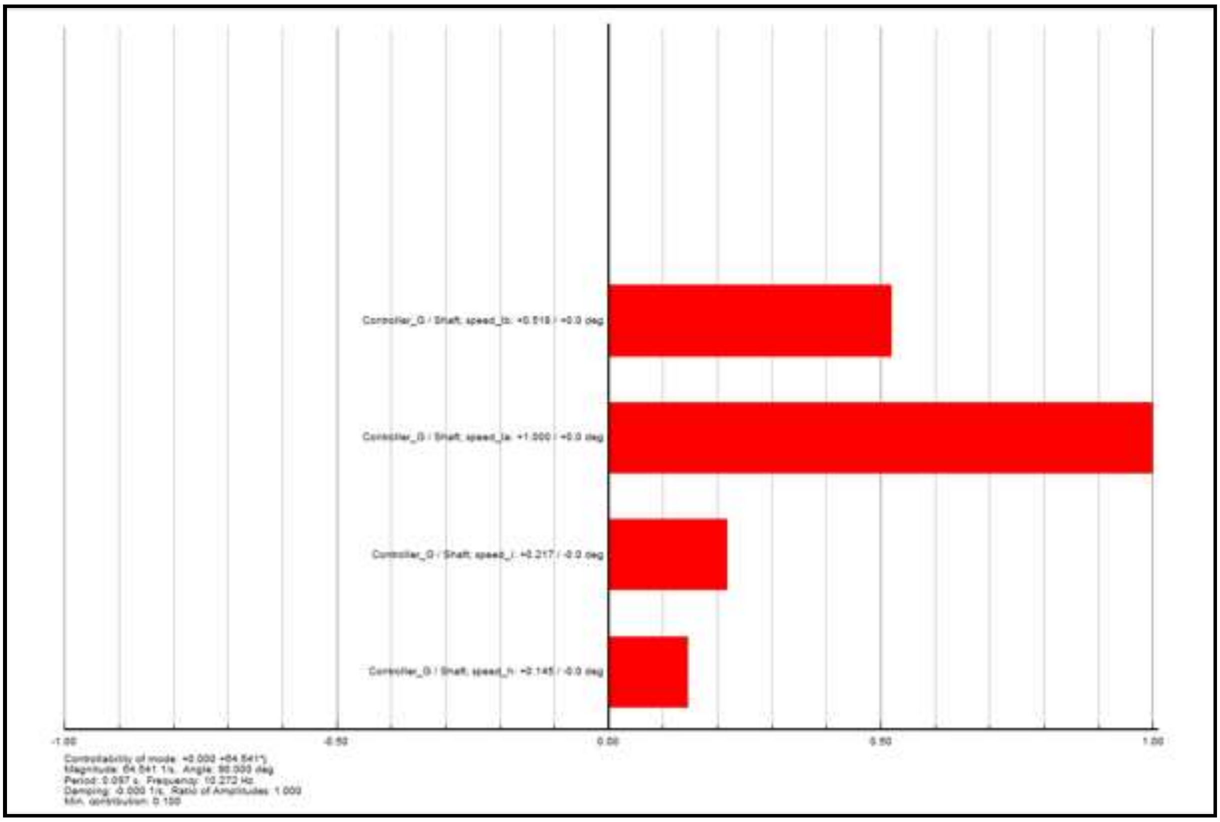


Fig. F-1. Modal analysis- Case 3-200MW PV phasor and bar plots

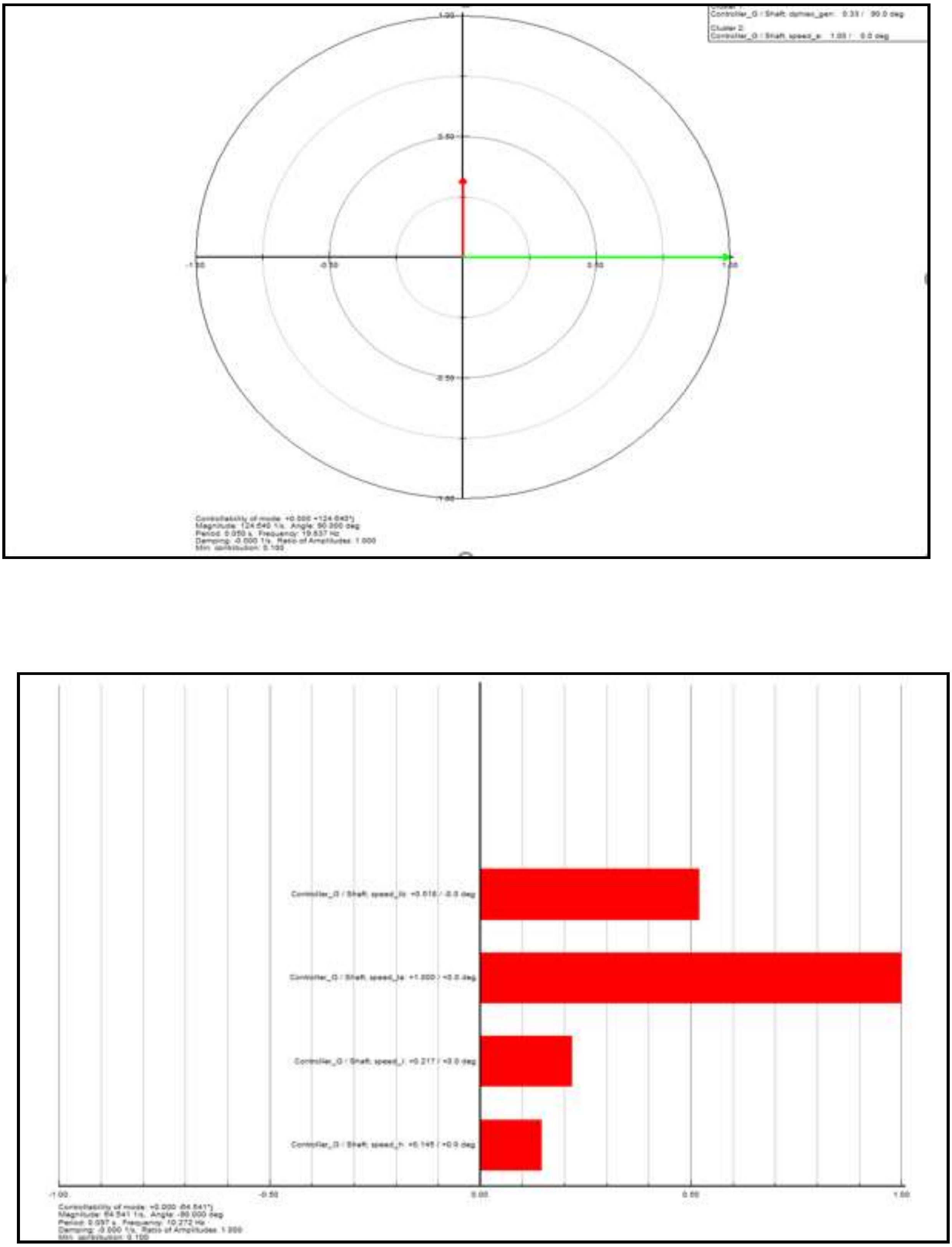


Fig. F-2. Modal analysis- Case 3-800MW PV phasor and bar plots

## Appendix G: Modal analysis – case 4, wind case phasor and bar plots

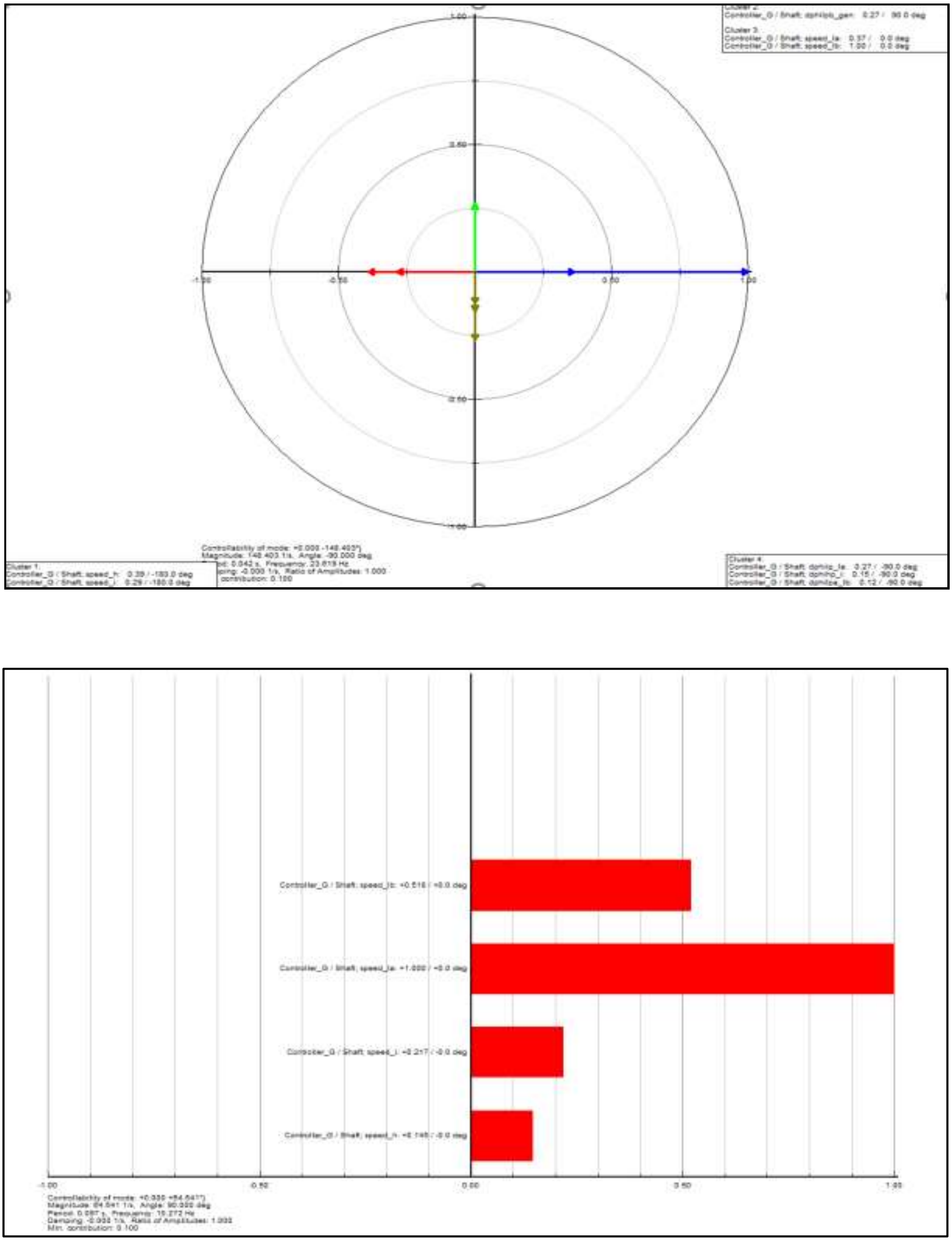


Fig. G-1. Modal analysis- Case 4-200MW wind phasor and bar plots

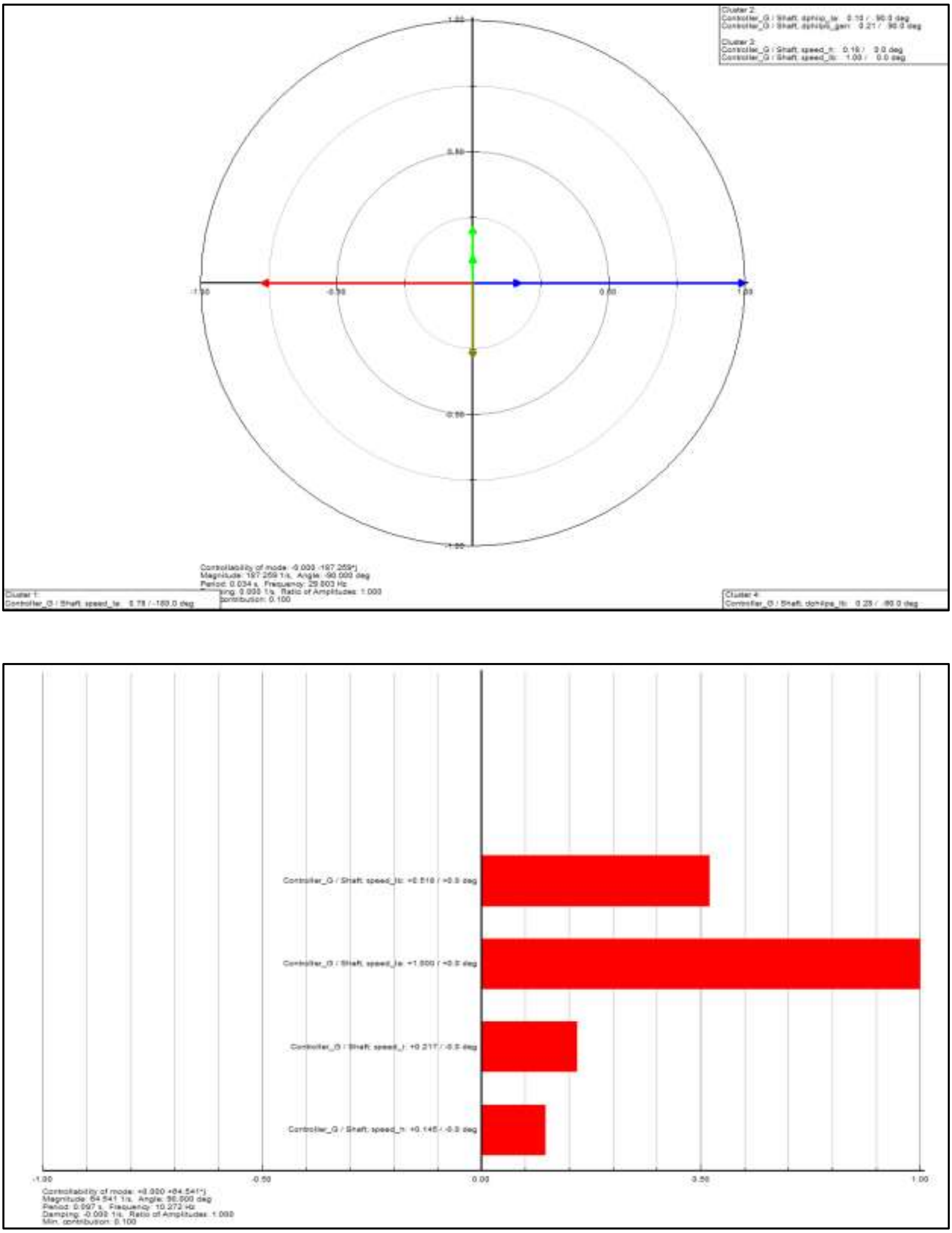



Fig. G-2. Modal analysis- case 4-800MW wind phasor and bar plot