INVESTIGATION OF REFRACTION EFFECTS FOR SMALL GPS NETWORKS

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ABSTRACT

Using observations from the Global Positioning System (GPS) satellites to determine a three dimensional (3-D) geodetic control network are considered.

The repeatability of individual baselines and 3-D vector closures are examined, in order to investigate refraction effects on GPS networks. The effect on GPS baselines of a height bias in the reference point's coordinates is also investigated. A least squares adjustment program is developed and used to obtain a single consistent set of 3-D coordinates for the Tygerberg Test Network (TTN). The results of two GPS processing packages are compared by means of a conformal transformation.

It is concluded that single frequency measurements produce better results than the ionospheric free observable on short baselines. Furthermore, a standard atmospheric model shows an improvement over the Marini model to account for tropospheric refraction.

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H*.2 : -50m height bias
H*.3 : -100m height bias
Q*.L1: L1 processing with standard atmospheric model
Q*.L3: L3 processing with standard atmospheric model
Z*.L1: L1 processing with Marini model
Z*.L3: L3 processing with Marini model

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GLOSSARY

3-D	Three dimensional
A.D.E.	Automatic data editing
BDOP	Bias dilution of precision
BIH	Bureau International de l'Heure
c	Vacuum velocity of light
cm	Centimetre
CTP	Conventional terrestrial pole
df	Degree of freedom
DMA	Defence Mapping Agency
DoD	Department of Defence
DOY	Day of year (Julian date)
E	East
EDM	Electronic distance measurement
f	Frequency
GDOP	Geometric dilution of precision
GPS	Global Positioning System
HDOP	Horizontal dilution of precision
IfAG	Institut für Angewandte Geodäsie
km	Kilometre
L1	Microwave radio frequency (1575.42 megahertz)
L2	Microwave radio frequency (1227.60 megahertz)
m	Metre
mb	Millibar
MHz	Megahertz
mm	Millimetre
NASA	National Aeronautics and Space Administration
NGS	National Geodetic Survey
PC	Personal computer
PDOP	Position dilution of precision
ppm	Parts per million
PPS	Precise Positioning Service
PRN	Pseudo random noise

GLOSSARY (continued)

RH	Relative humidity
R.M.S.	Root mean square
RSS	Root sum square
S	South
SA	Selective availability
SAST	South African standard time
SPS	Standard Positioning Service
SV	Space vehicle
TI	Texas Instrument
TSH	Tropospheric scale height
TTA	Tygerberg Test Area
TTN	Tygerberg Test Network
UB	University of Bonn (Germany)
UCT	University of Cape Town (South Africa)
U.S.	United States
UTC	Universal Time Coordinated
VCV	Variance covariance
VDC	Volts direct current
VLBI	Very long baseline interferometry
WGS-84	World Geodetic System - 1984
WVR	Water vapour radiometer

CHAPTER ONE

INTRODUCTION

The classical approach to establishing three dimensional (3-D) Geodetic networks is to treat horizontal positions separately from height determinations. In this approach horizontal positions and heights are essentially referred to different datums namely a reference ellipsoid and the geoid respectively. For visibility and accessibility horizontal control were situated on hilltops and levelling benchmarks in valleys respectively [Cross and Sutisna, 1987].

With the advent of the Global Positioning System the two separate systems can be integrated, ie. a set of three dimensional positions is the result of the satellite positioning system. The signals from GPS satellites are more accurate and more economical than any previous method for geodetic control surveying [Bock *et al.*, 1985].

The Global Positioning System (GPS) was devised to provide three dimensional positions, velocity information, accurate time and frequency transfer on a global basis. GPS, originally developed by the U.S. DoD as a navigation system, is revolutionising surveying technology. It is a continuous, 24-hour, all-weather and a 3-D positioning system. Intervisibility between stations is no longer a constraint on the network design.

GPS satellites transmit coded information modulated on two L-band frequencies viz. L1 and L2, which allows for position determination [Langley, 1990].

Two fundamental observations are used in GPS for position determination. Pseudo ranges are used under dynamic and static conditions, while phase observations are mainly utilised for static applications.

1.1 POSITIONING WITH GPS INTERFEROMETRY

GPS interferometry is a method by which relative positions of several survey marks can be determined in three dimensions, in a global earth-fixed coordinate system [Engelis *et al.*, 1985]. At each station, an instrument receives signals from several satellites and the simultaneous observations are combined to determine the relative station coordinates. Many sources of error tend to affect simultaneous observations equally at any given time. Differencing algorithms were developed to reduce or eliminate such errors. It is that possibility with GPS, of observing multiple satellites at the same time, and of differencing the observations, that makes GPS interferometry so powerful for relative positioning.

1.2 OBJECTIVES OF THE PROJECT

The primary objectives of this thesis are:

(1) to process GPS data that were accumulated in the Tygerberg Test Network (TTN) during the summer of 1988,

(2) to investigate the effect of ionospheric and tropospheric refraction on a small GPS network,

and, (3) to develop software in order to estimate 3-D coordinates from a rigorous adjustment of GPS vectors and their associated variance covariance (VCV) matrices.

The secondary aims include a 3-D similarity transformation [Rens, 1988] of the results of (3) into the set processed by the University of Bonn (UB); and the investigation of the effect on GPS baselines with biased heights.

1.3 THE TYGERBERG TEST AREA (TTA)

The Tygerberg Test Area (TTA) is a research project of Professor Williams at the University of Cape Town [Williams, 1982]. The project was commenced in 1978 and funding is provided by the University of Cape Town (UCT) and the Council for Scientific and Industrial Research, Pretoria.

A relative point accuracy of 7mm standard deviations is anticipated in the three components, viz. x, y and z. Precise levelling, gravity measurements, angle observations and precise EDM were to be used to achieve the specified accuracy.

The primary objective of that project is to provide an accurate calibration range for positioning and inertial systems.

1.3.1 The Tygerberg Test Network (TTN)

The network was designed to achieve the following objectives [Williams *et al.*, 1989]:

(a) to gauge GPS results against precise terrestrial measurements,

(b) to obtain ellipsoidal heights for use in conjunction with orthometric heights to evaluate techniques for determining differential geoidal heights,

(c) to link the TTA to the national height datum,

and, (d) to connect the TTA indirectly to the World Geodetic System.

The TTN 3-D network consists of sixteen stations (Figure 3.1). The eastern stations 20, 30, 193, 202, 205, 213, 222, 234, 243, 415, 417, 421, 482 and 528 are located in the TTA of Professor Williams, whereas stations TG1 and TG2 are tide-gauge sites at Granger Bay and Hout Bay respectively. All station monuments consist of concrete pillars except the tide-gauge sites which are marks on a concrete slab and a bollard, and station 482 is a metal tripod on a reservoir.

The GPS observations were carried out over twelve days with four dual frequency receivers, which were kindly provided by the Institut für Angewandte Geodäsie (IfAG) - Germany. The raw carrier phase data were sent to Germany for reformatting. Data reduction was carried out at the Universities of Cape Town (UCT) and Bonn (UB).

1.4 GPS PROCESSING

Numerous private, academic and government organisations have developed computer programs for processing GPS phase data [Wanless and Lachapelle, 1988]. The OMNI software was developed at NGS in the U.S., DIPOP at the University of New Brunswick, NOVAS (Nortech Vector Adjustment Software) in Canada, the Bernese software was a joint effort between the Universities of Berne and New

Brunswick, POPS by Wild Heerbrugg and Magnavox California, etc.

The processing techniques, models and algorithms used in these different programs vary. Therefore, various levels of accuracy are obtained. Different levels of operator subjectivity in terms of data analysis for cycle slip detection and rejection of noisy data are required.

The models and techniques incorporated in a GPS processing program are based on the applications of the software and the required accuracy level. Precise static applications require the phase data in a relative mode to achieve an accuracy of one to a few parts per million, or better. This accuracy level is sufficient for applications such as the establishment of geodetic control and local deformation monitoring. For higher levels of precision required for monitoring regional crustal deformation the software must use precise ephemerides. Water vapour radiometers (WVR) may be required to reduce tropospheric refraction and dual frequency measurements are preferred in order to compensate for ionospheric delays.

1.5 GPS GEOMETRY

The accuracies of GPS positions are affected by the satellite configuration [Merminod *et al.*, 1988]. If the satellite configuration is bad - satellites all bunched together in the sky - our derived positions will be poor. However, if the satellites are spread out our position determinations will be better. The quantities GDOP and BDOP (geometric and bias dilution of precision) indicate the quality of the satellite constellation in instantaneous and relative positioning respectively.

Pseudo ranges are contaminated by a satellite clock error, therefore the normal matrix would contain two parts. One part pertaining to the instantaneous point position coordinates and the other to a receiver clock offset.

In relative positioning, the geodetic parameters are estimated along with the integer ambiguities. Hence, the quantities GDOP and BDOP are the trace of the normal matrix in each case.

1.6 ACCURACY CONSIDERATIONS

GPS vectors are not measured directly; they are derived from carrier beat phase observations and satellite orbital data which are adjusted by least squares. Some of the unknowns in the adjustment are station coordinates relative to the fixed station in the adjustment. This initial reduction provides the variances of all the unknowns and the covariances among them. If more than two stations participate in a session, correlations exist between baselines, such as those due to atmospheric conditions, ephemeris errors, etc. which are accommodated by the VCV matrix.

Accuracy measures of GPS solutions can be acquired by means of the following methods [Gurtner *et al.*, 1989]:

1.6.1 Formal errors

The uncertainties of the estimated coordinates only reflect the actual accuracy if the mathematical model strictly matches the truth, ie. if

(a) the weight matrix of the observations are known,

and, (b) if the model accounts for all systematic errors.

In reality these conditions are never fully met and formal errors are almost always considerably smaller than the actual errors of the unknowns.

The VCV matrix of a vector describes the dimensions and the orientation of the error ellipsoid of the position relative to the fixed station. It is known, however, that they are over-optimistic and do not describe the true error model adequately. This is partly due to hidden errors which are dependent on the baseline length and are not sensed by the initial reduction [Vincenty, 1987].

The formal standard deviations of baselines with fixed integer biases are scaled by a factor of ten [Leeman and Fletcher, 1985]. The *a posteriori* variance of unit weight can be tested against the *a priori* variance (assumed to be unity) of unit weight [Bock *et al.*, 1985]. The F or χ^2-test could be used for variance testing [Milford, 1983].

1.6.2 Consistency

A comparison of independent solutions e.g. solutions of the same data set processed with different processing programs, etc. gives insight into the possible variations of the solutions. Usually a similarity transformation yields the information for this comparison.

1.6.3 External comparison

If a coordinate set derived from precise terrestrial observations is available, the accuracy of the GPS solution can be gauged, by transforming them into the terrestrial network (treated as error-free). For small GPS networks covering $100km^2$, terrestrial networks may provide external assessment, whereas VLBI could be used for large scale applications provided satellite orbits are appropriately modelled [Gurtner *et al.*, 1989].

Accurate ephemerides for data reduction is essential for precise positioning with GPS [Stolz *et al.*, 1987]. The policy of the United States Department of Defence (U.S. DoD) has been implemented in which the accuracy of the transmitted signals and ephemerides are degraded. These considerations have motivated tracking and orbit computations of GPS satellites.

1.7 IMPROVING GPS ACCURACY

Carrier phase measurements are used for high precision applications where relative positions are determined between several static survey marks. To exploit the high precision of carrier phase measurements, all errors must be modelled down to the level of a few centimetres. Phase differencing methods are used to reduce or eliminate first order effects of these errors. In these algorithms, satellite and receiver clock errors are removed in the differencing, whereas, atmospheric and orbit errors are drastically reduced. The error reduction is more likely to be effective if the simultaneously observing receivers are close together than if they are far apart [Kleusberg and Langley, 1990].

1.8 ATMOSPHERIC REFRACTION

The first order effects in both tropospheric and ionospheric refraction are removed in the double differencing. However, the remaining residual effects are negligible or they can be reduced by means of models.

1.8.1 Tropospheric refraction

An important advantage of differencing seems to be that most of the tropospheric refraction effects cancel. However, a residual remains which will be negligible over short baselines (less than 10km) with height differences smaller than 100m [Beutler *et al.*, 1989]. Over longer baselines this refraction effect is usually reduced with a model e.g. Marini, Hopfield, etc. This refraction effect given by equation 2.7 in units of metres, must be scaled by f/c to convert to units of cycles, prior to correcting the input phase data.

There is also the option of processing with a standard atmospheric model. It is reported that accounting for tropospheric refraction with surface measurements in small (smaller than $225km^2$) networks, leads to worse results than using a standard model [Beutler *et al.*, 1989, Rothacher *et al.*, 1986]. This is because station weather data do not always represent the atmosphere above the sites. It would be advisable that when improved atmospheric models become available, the data be reprocessed to improve the baseline results.

1.8.2 Ionospheric refraction

Most of the effect due to ionospheric refraction cancels in the double differencing. Single frequency GPS receivers can make use of the coefficients of an ionospheric model which forms part of the broadcast ephemeris message. However, during a solar maximum this effect would not be satisfactorily removed.

By a suitable combination of dual frequency phase measurements it is possible to eliminate the bias of the signal caused by the ionospheric delay. In relative GPS positioning the so-called ionospheric free combination (L3) has proved advantageous for baselines longer than, say, 100km [Sjöberg, 1990]. On the other hand, for short baselines this combination will yield larger r.m.s. residuals than for single frequency (L1 or L2) solutions. In the L3 combination (short baselines), the disturbing effect of combining two data types, each with their own noise, is greater than the advantage gained by reducing the ionosphere noise. However, on longer baselines these effects will be reversed.

1.9 BIASES IN GPS RESULTS

GPS results may be biased by atmospheric effects, orbit errors, erroneous station coordinates kept fixed, and by physical parameters (e.g. GM-value) [Beutler *et al.*, 1989]. The influences of these biases on GPS results are summarised in Table 1.1.

TABLE 1.1 Influence of biases on GPS results.

Biases	Type	Influence (order of magnitude)
Troposphere (relative)	Height	A bias of 1mm in zenith direction of tropospheric correction causes a (relative) height bias of ~3mm.
Troposphere (absolute)	Scale	A bias of 1m in zenith direction of tropospheric correction causes a scale effect of 0.3ppm. (if troposphere is neglected, baselines are longer.)
Ionosphere (absolute)	Scale	Network shrinks by 0.6ppm if electron content in the zenith direction is neglected.
GM-value	Scale	If the gravity constant is increased by 1ppm, the scale of the network is increased by 0.05ppm.
Fixed station height	Scale	A height bias of 10m introduces a scale effect of 0.4ppm. (if the station height is too big, scale is too small.)
Fixed station horizontal coordinates	Rotation	An error of 1" in horizontal position causes the network to rotate 0.1 arc seconds about a horizontal axis perpendicular to the station bias vector.
Along track orbit error	Rotation	An along track orbit error of 1" rotates the GPS network 1" about an axis perpendicular to the orbit plane.

1.9.1 Atmospheric errors

Tropospheric effects are the most important biases in small GPS surveys with height differences in excess of 100m [Gurtner *et al.*, 1989]. The estimated station heights may be biased if standard atmospheric models are used without real weather data. In this case surface meteorological measurements must be used at each site to derive a tropospheric refraction correction as a function of height.

When networks of continental size are considered it is best to use the observed meteorological values and to solve for a tropospheric scale height correction. This allows that the tropospheric corrections may be in error by a scale factor. However, WVR's could be used to minimise tropospheric biases for large scale applications.

An unmodelled ionosphere causes a scale error of 0.6ppm which results in a network contraction [Georgiadou and Kleusberg, 1988]. The expansion of a network results if tropospheric refraction is completely neglected.

1.9.2 Erroneous coordinates of reference station

It was shown by Beutler *et al.* [1988] that a height error of the reference station in the phase adjustment causes a scale error in GPS networks. This means that a height bias of 10m causes a scale error of 0.3ppm. GPS scale biases are less than 0.04ppm, if the correct gravity constant is chosen, if the reference geocentric position is correct to approximately 1m, and if the weather parameters are measured at every site.

Orbit errors and biased horizontal coordinates of the reference station may be the reason for rotations of GPS networks. The influence of tropospheric biases on the horizontal positions is relatively small [Gurtner *et al.*, 1989].

CHAPTER TWO

THEORY

2.1 GPS OBSERVABLES

There are two important types of GPS observables for surveying and navigation applications: pseudo ranges and carrier phases. The pseudo range observable is generally used for navigation. The carrier phase is usually utilised where higher precision is needed, in which the pseudo range observable is restricted to determining initial point positions and receiver clock offsets. Various algorithms have been developed to process certain linear combinations, for example, double difference, etc. of the original carrier phase observation.

Only the theory of carrier beat phases will be covered in this thesis.

2.1.1 Carrier beat phases

Signals are transmitted at known frequencies by the satellites. A receiver oscillator generates replicas of these signals and then differences the generated signal in the receiver with the received signal transmitted by the satellite. The instantaneous difference in phase between the generated signal by the receiver clock or oscillator and the received signal is given by [Leick, 1990]:

$$\phi_k^p(t_r) = \phi^p(t_r) - \phi_k(t_r) + N_k^p(1) + \varepsilon \qquad (2.1)$$

In which, $\phi^p(t_r)$ is the transmitted signal generated by the satellite, $\phi_k(t_r)$, the receiver signal phase, t_r, the time of reception, $N_k^p(1)$ is the integer ambiguity and ε is the unmodelled noise. The transit time for a signal travelling from a satellite to a receiver consists of two parts. The main part can be solved from the topocentric range between the satellite and the receiver at the time of reception. The other part of the transit signal depends on the atmospheric conditions. The effect is to cause a phase delay or equivalently an increase in range.

However, the difference between the observed carrier phase and the topocentric range is biased. These biases are made up of an integer number (n) of wavelengths because of the cycle ambiguity, and non-integer biases such as clock instabilities and atmospheric refraction (Figure 2.1) [Merminod, 1988].

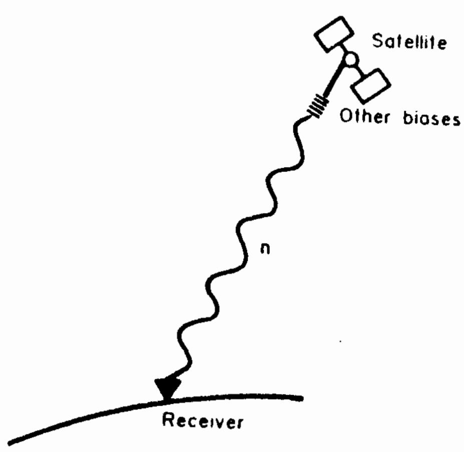


Figure 2.1 Range difference observations.

"When the carrier beat phase is measured in the receiver the measurement is ambiguous as to the number of integer cycles. This integer is set to some arbitrary value when the satellite signal is first acquired (e.g. zero). This will be a unique number for every satellite-receiver pair. It will be constant in time if the receiver tracks the integer number of cycles after the satellite signal has been acquired. This observation is then called integrated carrier beat phase. If the receiver fails to track the signal correctly and the ambiguity changes between epochs then a cycle slip has been introduced." [Leick, 1990]

"It follows that

$$\begin{aligned}\phi_k^p(t) = {} & a^p(t-t_o) + \tfrac{1}{2} b^p(t-t_o)^2 + \phi_T^p(t_o) \\ & - [a^p + b^p(t-t_o)]\tau + \tfrac{1}{2} b^p\tau^2 - f.\tau - \phi_k(t_o) \\ & - f.q_k - f.r_k(t-t_o) - \tfrac{1}{2} f.s_k(t-t_o)^2 + N_k^p(1)\end{aligned} \quad \textbf{(2.2)}$$

where the first three terms in equation 2.2 depend on the satellite clock frequency offset, satellite clock frequency drift and transmitted phase at reference epoch t_o. The next three terms are a function of the signal travel time, that is, the topocentric receiver-satellite distance. The fifth term, which is a function of the satellite clock frequency drift and the square of the signal travel time, is generally neglected. The terms remaining consist of the receiver phase at the reference epoch, the receiver clock error terms, and the initial integer ambiguity." [Leick, 1990] The nominal (scheduled) time of the observation and the reference time are defined by t and t_o respectively. a^p and b^p are the receiver clock frequency offset at t_o and the receiver clock frequency drift. The signal travel time is denoted by τ and the nominal satellite clock frequency (constant) by f. The satellite clock frequency offset, drift and drift rate are given by q_k, r_k and s_k respectively.

The equation for the fully developed expression, for the undifferenced carrier phase observation is given by

$$\phi_k^p(t_r) = a^p(t-t_o) + \frac{1}{2} b^p(t-t_o)^2 + \phi_T^p(t_o) - \frac{f}{c} \rho_k^p(t) - \frac{1}{c} [a^p + b^p(t-t_o)] \rho_k^p(t) - \frac{f}{c} \dot{\rho}_k^p dt_k - \phi_k(t_o) - f.dt_k + N_k^p(1) \qquad (2.3)$$

in which c is the vacuum velocity of light and dt_k the receiver clock error. $\rho_k^p(t)$ and $\dot{\rho}_k^p(t)$ are respectively the topocentric range and the rate of change of this quantity.

2.2 OBSERVATION DIFFERENCING

2.2.1 Double and triple differencing

At any given time, simultaneous observations at each end of a baseline, tend to be affected equally by many sources of error such as orbit bias, atmospheric refraction, etc. Differencing techniques were developed to reduce or eliminate the effect of these errors.

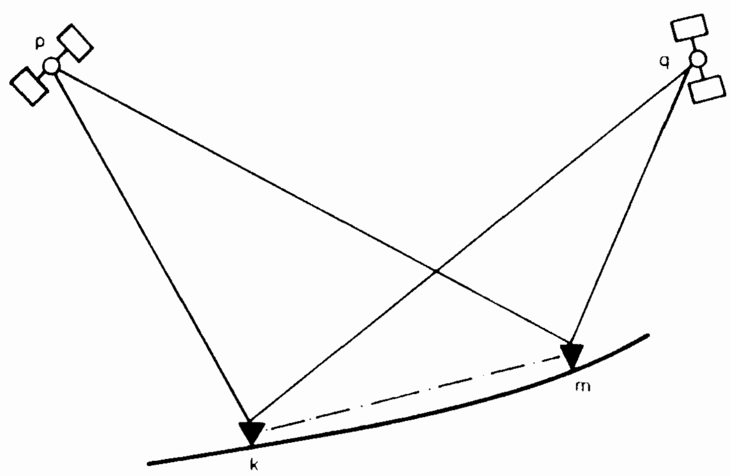


Figure 2.2 The double difference.

A double difference observation is obtained by differencing the four raw phases observed at two receivers from two satellites at the same epoch (Figure 2.2).

The double difference observation equation is given by [Leick, 1990]:

$$
\begin{aligned}
\Delta^{pq}_{km} &= \Delta^{p}_{km} - \Delta^{q}_{km} \\
&= -\frac{1}{c}\,[a^{p} + b^{p}(t-t_{o})][\rho^{p}_{k}(t) - \rho^{p}_{m}(t)] \\
&\quad + \frac{1}{c}\,[a^{q} + b^{q}(t-t_{o})][\rho^{q}_{k}(t) - \rho^{q}_{m}(t)] \\
&\quad - \frac{f}{c}\,[\rho^{p}_{k}(t) - \rho^{p}_{m}(t) + \dot{\rho}^{p}_{k}(t)dt_{k} - \dot{\rho}^{p}_{m}(t)dt_{m}] \\
&\quad + \frac{f}{c}\,[\rho^{q}_{k}(t) - \rho^{q}_{m}(t) + \dot{\rho}^{q}_{k}(t)dt_{k} - \dot{\rho}^{q}_{m}(t)dt_{m}] \\
&\quad + N^{pq}_{km}(1) \qquad\qquad \textbf{(2.4)}
\end{aligned}
$$

The effect of the satellite and receiver clock errors are largely reduced in the double differencing. Small clock terms remain due to the difference in Doppler shift between the satellites and stations.

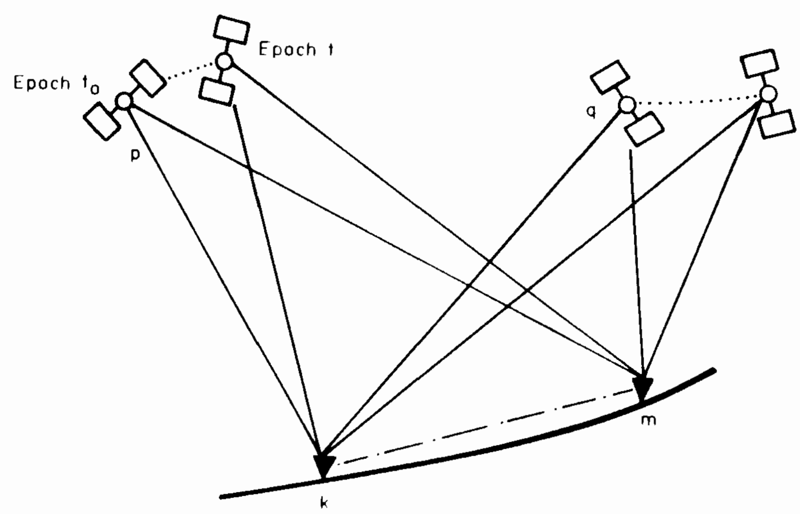


Figure 2.3 The triple difference.

The difference of two double differences for successive epochs is called a triple difference (Figure 2.3).

The triple difference observation

$$\Delta^{pq}_{km} = \Delta^{pq}_{km}(t+1) - \Delta^{pq}_{km}(t) \qquad (2.5)$$

eliminates the integer ambiguity, which is constant in time. It is useful for cycle slip detection, since cycle slips create spikes in the triple difference residuals. The triple difference residuals are used in the data editing to identify double differences where cycle slips occurred [Wells, 1986].

The most important features of differencing observations are tabulated below [Leick, 1990].

TABLE 2.1 Carrier phase differences used in relative positioning.

Observations	Effects eliminated	Effects reduced	Option
single difference	first order satellite clock	Ionosphere	constrain integer ambiguity
double difference	first order satellite and station clock	and	constrain integer ambiguity
triple difference	first order satellite and station clock	Troposphere (short lines)	integer ambiguity eliminated

2.2.2 Correlation

Only pure phase and single difference observables are independent. Double difference observables could be derived from single differences; similarly for triple differences. Errors inherent in the observed phase will occur in the double differenced phase in which they are used, thus, the double differences and similarly triple differences are correlated. Pure phase, single and double difference observables yield identical results provided the appropriate weight matrices are considered [Ashkenazi *et al.*, 1987].

2.3 MEASURE OF PRECISION

In position determination, the possibility exists that the recovered position error is greater than the error in the measurements [Goad, 1989]. For example, angle observations of an ill-conditioned triangle will yield an unsuitable horizontal position. Similarly, if the satellite geometry is unfavourable in pseudo range or carrier phase position determination, the results could be poor even though the measurements are good. This reduction in precision of recovered position is called dilution of precision. Position dilution of precision (PDOP) represents instantaneous point position recovery. Although PDOP is used to describe the satellite geometry in absolute positioning it can be used for the planning of static surveys.

For static relative positioning using carrier phase observations a quantity, BDOP (bias dilution of precision), is used to describe satellite geometry and whether the integer ambiguities can be resolved [Merminod, 1988]. This quantity (BDOP) and satellite visibility plots should be used when planning static GPS surveys.

2.4 AMBIGUITY RESOLUTION

The key to precise relative positioning is in the resolution of the integer ambiguities and removal of cycle slips (section 2.5) [Merminod, 1988].

Double differences are normally used to estimate the ambiguity biases along with the geodetic parameters. The ambiguity biases are then constrained to integers, if possible, and the geodetic parameters estimated again [King *et al.*, 1985].

The observation equations are

$$A_g x_g^o + A_b x_b^o = f^o + V^o$$

where x_g^o are the geodetic parameters, x_b^o are the ambiguity biases and f^o = (l-c) are the pre-fit residuals. Both x_g^o and x_b^o are estimated in the first iteration. Then x_b^o are constrained to integers and a second iteration is performed with

$$A_g x_g = f - V$$

where $f = (f^o - A_b x_b^1)$ and x_b^1 are the integer biases. x_g contains the most probable values of the geodetic parameters.

In order to resolve the ambiguities, significant changes in the satellite constellation are needed to have occurred. Hence, it is dependent on the duration of the sessions; usually one hour is required. In GPS data processing, the concept of selecting a base station and satellite is usually preferred, since the number of unknowns are reduced. The ambiguities are then determined with respect to the reference ambiguities of the reference station and satellite. Thus the non-integer biases that are not eliminated by the double differencing affect the integer nature of the ambiguities adversely. The largest of these are tropospheric delay, ionospheric delay, orbit errors and errors in the coordinates of the reference station.

The effect of these errors on the estimated ambiguities increase with an increase in baseline length. The ambiguities could normally easily be resolved for baselines up to 20-30km. Currently, no constraints are placed on longer baselines because unmodelled atmospheric effects and satellite orbits make it difficult, if not impossible, to distinguish the correct integer combinations [Leick, 1990].

2.5 CYCLE SLIP CORRECTION

The measured phase in the receiver consists of an integer and a fractional part. When a receiver loses lock on a satellite signal, a cycle slip occurs (Figure 2.4) [Murakami, 1989].

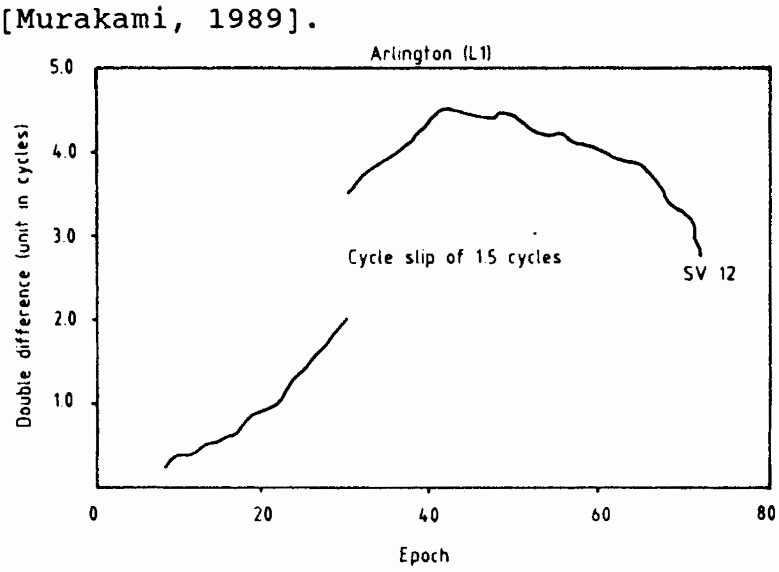


Figure 2.4 An example of a cycle slip.

In obtaining precise baseline estimates in GPS data processing, cycle slip detection is extremely important [Wanless and Lachapelle, 1987]. The double difference residuals are normally inspected to estimate the value which will restore a continuous time series plot. This observable is preferred, since oscillator errors cancel in the differencing which dominates one-way phases [King *et al.*, 1985]. Factors like the noise level of the data and the accuracy of the initial baseline estimate influence the ability to reliably detect and estimate the magnitude of the slip. The dependence on the initial baseline estimate requires the execution of a triple difference solution - which eliminates the cycle ambiguity - to obtain a more accurate estimate of the baseline.

The approach used in OMNI (GPS software used at UCT for data reduction) is as follows [Mader, 1989]:

A triple difference solution is executed whereby large cycle slips are fixed. Two phases at successive epochs are used to linearly extrapolate a third phase value which is compared with the observed value. An approximate correction is determined from the difference between the measured and extrapolated phase values. The observed phase is then corrected. Discontinuities in the double differenced phases are then isolated by searching the triple difference residuals for outliers. These large residuals are subsequently assumed to be cycle slips.

"If the base-station base-satellite concept is used, the cycle slips pertaining to the base-site or base-satellite are transferred to the subsequent data pertaining to all other sites and satellites. However, this incorrect transfer of the cycle slip to the other receiver channels cancels during double differencing or when the station-satellite clock offsets at each epoch are estimated." [King *et al.*, 1985]

Double difference residual plots are inspected for further editing. Then the double difference solution is performed once more to estimate the bias parameters.

2.6 TYPES OF ERROR IN GPS MEASUREMENTS

GPS measurements are affected by three categories of errors, namely errors introduced at the satellite where the signal is generated and transmitted, receiver related errors and errors caused as the signal travels from the satellite to the user [Kleusberg and Langley, 1990].

2.6.1 Satellite related errors

A satellite transmits coded information - that is superimposed on the carrier waves - which allows the satellite orbit to be computed. This information together with phase data and timing data are used to estimate geodetic parameters. The satellite positions computed from the broadcast information are contaminated by orbit errors because ephemeris predictions cannot be made with absolute certainty.

The timing data which are needed in position computations are essentially obtained by comparing a clock in the satellite and a clock in the receiver. Since the satellite clocks are not perfect, the travel-time measurement between the satellite and receiver will be contaminated by a satellite clock error.

GPS provides two types of positioning services, viz. standard positioning service (SPS) which is authorised for civilian users and precise positioning service (PPS) which was designed primarily for defence. The U.S. DoD implemented selective availability (SA) on 25 March 1990 which only affects the SPS users. SA is the intentional degradation of ephemerides and the clocks of GPS satellites. In static relative positioning the errors due to SA, orbits, receiver and satellite clocks are removed, or at least reduced, in the differencing.

2.6.2 Receiver related errors

Receiver clock error and measurement noise originate in the receiver.

In double differencing, the receiver clock error is treated as an unknown and solved with 3-D geodetic parameters. This requires observations from at least four satellites.

The measurement noise depends on whether code or carrier phase measurements are used. Code measurements permit range measurement precision between a few metres to a few decimetres, while phase measurements can be resolved within a few millimetres.

Another receiver-dependent effect is multipath errors caused by signal reflection from the ground or other objects before reaching the antenna. The magnitude of the effect is of the order of a few centimetres. It can be reduced to a few millimetres by some antenna designs and careful siting of the antenna during the observations. The period of multipath variations is roughly 10 minutes, so that some averaging occurs over long sessions [King *et al.*, 1985].

2.6.3 Propagation errors

This class of errors is introduced when the GPS signals travel through the earth's atmosphere. Refraction is a phenomenon caused by the atmosphere, by changing the speed at which the signal travels [Kleusberg and Langley, 1990]. For GPS observations the arrival times of carrier modulations and carrier phases of the satellite transmissions are of concern, and not the angular changes in the zenith angle [Leick, 1990]. The earth's atmosphere may be divided into two distinct layers, viz. the ionosphere and the troposphere, each of which affects the GPS signal differently [Hamfeldt, 1986]. The troposphere extends 50km above the earth's surface and the ionosphere lies roughly between 50km and 500km above the earth (Figure 2.5).

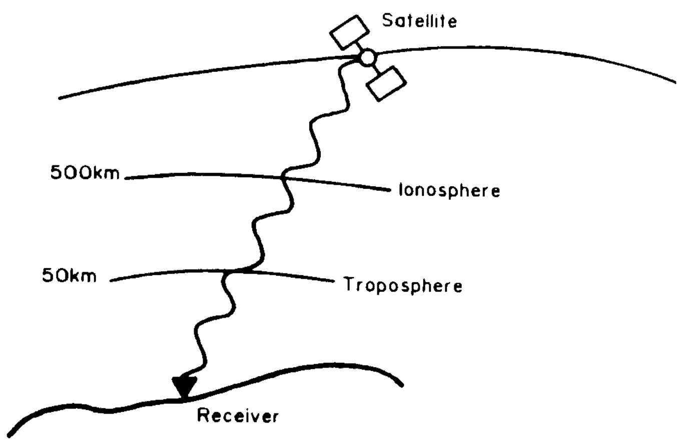


Figure 2.5 Distinct layers of the atmosphere.

2.6.3.1 Ionospheric refraction

The delay caused by the ionosphere is dependent on the frequency of the carrier wave and the total electron content of the ionosphere [Leick, 1990]. The ionosphere consists of a large number of electrically charged particles which is not constant throughout the medium. Hence, the refraction error varies daily and spatially. It can reach 30m in the zenith direction and typically increases by a factor of three near the horizon. The ionospheric refraction error (τ_{ion}) is proportional to the total electron content (Ne) and inversely proportional to the square of the signal frequency [King *et al.*, 1985]:

$$\tau_{ion} = \frac{-1.35 \times 10^{-7} Ne}{f^2} \tag{2.6}$$

Satellites transmit signals at two L-band frequencies, viz. L1 and L2. This dependency of the ionospheric error on the frequency allows for error correction. The geometric propagation delay (τ_g) can be expressed in terms of the observed phases (ϕ_1 and ϕ_2) for the L_1 and L_2 frequencies as:

$$\tau_g = \phi_1/f_1 + 1.35 \times 10^{-7} Ne/f_1^2$$

$$\tau_g = \phi_2/f_2 + 1.35 \times 10^{-7} Ne/f_2^2$$

which gives:

$$\phi_{ion(L2)} = (f_1/f_2)\phi_{ion(L1)} \tag{2.7}$$

The total phase pathlength at each frequency is the sum of the geometric phase pathlength and the ionospheric delay:

$$\phi_1 = f_1\tau_g + \phi_{ion(L1)} \tag{2.8}$$

$$\phi_2 = f_2\tau_g + \phi_{ion(L2)} \tag{2.9}$$

From combining equations 2.7, 2.8 and 2.9 and the factor R ($=f_1/f_2$) the ionospheric correction can be determined. Multiplying the result by f_2 yields:

$$f_1\phi_1 - f_2\phi_2 = f_1^2\tau_g - f_2^2\tau_g \qquad (2.10)$$

Making τ_g the subject of the formula in equation 2.10 gives:

$$\tau_g = \frac{1}{f_1}\left(\frac{\phi_1 - R\phi_2}{1 - R^2}\right) \qquad (2.11)$$

The dual frequency observable, ϕ_c, which is free of ionospheric effects, can be obtained as follows:

$$\phi_c = f_1\tau_g = \phi_1 - R(\phi_2 - \phi_1)/(1 - R^2)$$

Simplifying the above equation results in (with R=1227.60/1575.42):

$$\phi_c = \phi_1 - 1.984(\phi_2 - 0.779\phi_1) \qquad (2.12)$$

With single frequency measurements, the ambiguity terms are integers, while for dual frequency observations this integer nature of the ambiguities is lost. Equation 2.12 should be written as:

$$\phi_c = \phi_1 + n_1 - 1.984[\phi_2 + n_2 - 0.779(\phi_1 + n_1)] + \phi_e$$

$$= \phi_1 - 1.984(\phi_2 - 0.779\phi_1) + (2.546n_1 - 1.984n_2) + \phi_e$$

Where n and ϕ are respectively the integer and fractional phase values at the L_1 and L_2 frequencies. ϕ_e contains the errors in the models arising from sources other than the ionosphere e.g. troposphere, etc.

This equation only holds for double differenced and undifferenced carrier beat phase observations.

One error source that may inhibit ambiguity resolution over long baselines is the ionospheric delay. The ionospheric free observable has the unfortunate effect that the noise in the observations is amplified in this combination. However, for long baselines the ionospheric residual effect may be large enough to justify processing with this ionospheric free combination. For short baselines data processing on either L1 or L2 is preferred.

The effect of ionospheric refraction could be severe, even on short baselines, especially when ionospheric activity is at a high. Ionospheric activity is dependent upon sunspot activity which in turn has a cyclic nature with a period of about eleven years (Figure 2.6) [Cain, 1988].

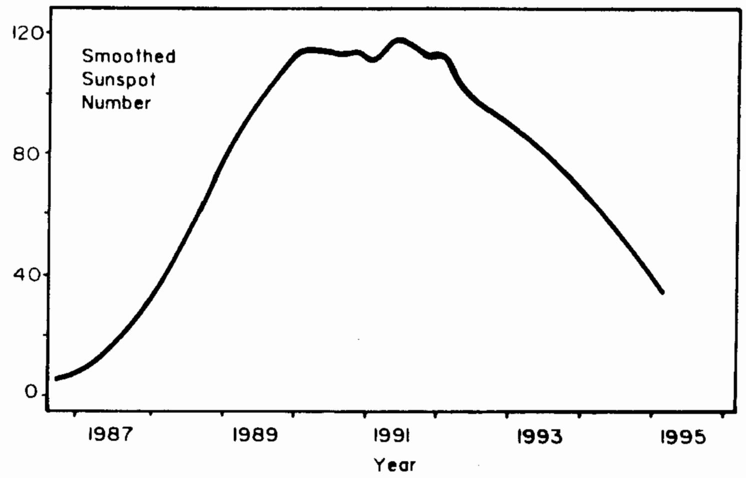


Figure 2.6 Sunspot activity.

GPS receivers with dual frequency capability will be a valuable asset during these periods.

Diurnal variations of the ionospheric range error can reach 15m in the zenith direction [Leick, 1990]. This effect would be at a low between midnight and before dawn, reaching a maximum at noon. "At night, the ionospheric delay is a factor of 5 or more less than for daytime observations." [King *et al.*, 1985] The range error would vary with the solar cycle, but the effect of ionospheric refraction could be reduced by scheduling night time observations.

2.6.3.2 Tropospheric refraction

The tropospheric refraction error occurs in the region where normal weather changes take place [Leick, 1990]. The effect is roughly two metres in the zenith direction and increases by a factor of 10 near the horizon. Measurements of weather conditions are usually recorded at receiver sites to correct for this delay.

The total refraction effect consists of two parts - a dry and a wet component - of which the dry part contributes 90% of the total effect. Using meteorological observations it can be modelled to 2.5%. The wet contribution is included in a model together with the dry component, to model the total effect of tropospheric refraction. For the highest possible precision in GPS measurements, the water vapour content along the line of transmission could be measured. In large scale projects water vapour radiometers (WVR) are used for this purpose. The wet component is about 5-30cm in continental mid-latitudes which can be modelled to about 2-5cm. However, with a WVR, a further reduction to 1-2cm is envisaged.

The MARINI model for the troposphere is used in OMNI to compute this correction in metres, ΔR_{Trop}, from [Mader, 1989]:

$$\Delta R_{Trop} = \frac{1}{f(\phi)} \left\{ \frac{A + B}{\sin E + \left[\frac{B/(A+B)}{\sin E + 0.015} \right]} \right\} \qquad (2.13)$$

Where E = elevation angle

ϕ = station latitude

H = station height above sea level in km.

$$f(\phi) = 1 - 0.0026(1 - 2\sin^2\phi) - 0.00031(H)$$

$$A = 0.002277(P + (1255/T + 0.05)P_{H_2O}$$

$$B = 0.002644e^{-0.14372H}$$

T = thermodynamic temperature in degrees Kelvin

P = pressure in millibars

$$P_{H_2O} = (6.11)RH.10^{7.5(T-273.16)/(T-35.86)} \quad \textbf{(2.14)}$$

RH = relative humidity

The height above mean sea level is required in this model and it is aproximated by the ellipsoidal height above the WGS-84 reference ellipsoid. Average meteorological values are used to compute a correction for the propagation delay due to the troposphere for every satellite-station combination at each epoch.

The relative humidity which is required in equation 2.14 above, can be computed from surface meteorological measurements. Relative humidity is expressed as a percentage ratio of the partial vapour pressure of a mixture to the saturation pressure at the same temperature and is derived from the following formulae [Penman, 1955]:

$$E = E_w - 0.66(T_d - T_w)$$

$$= 10(9.73 - 2450/T_w^k) - 0.66(T_d^c - T_w^c) \quad (2.15)$$

where T_d^k, T_w^k = dry and wet bulb temperatures in ^{0}Kelvin

T_d^c, T_w^c = dry and wet bulb temperatures in ^{0}Celsius

E = partial vapour pressure in millibars

E_w = saturation vapour pressure in millibars

E_d = saturation pressure at the dry bulb temperature.

$$RH = 100.\frac{E}{E_d} \%$$

$$= \left[\frac{100.E}{10(9.73 - 2450/T_d^k)} \right] \%$$

2.7 RELIABILITY OF GPS RESULTS

The following procedures are normally evaluated in order to investigate the internal and external reliability of the results.

2.7.1 Repeated baselines

A measure of repeatability is obtained by evaluating differences between single baseline solutions for each repeated baseline. When the same baseline has been observed several times the repeatability of baseline length can be expressed as two terms, ie. an offset A and B ppm [Wanless and Lachapelle, 1988].

A quantity, root sum square (RSS), is used to demonstrate baseline repeatability:

$$\text{Root sum square} = \frac{\text{discrepancy}}{\text{baseline length}} \qquad (2.16)$$

where:

$$\text{discrepancy} = \sqrt{R_{dx}^2 + R_{dy}^2 + R_{dz}^2}$$

such that R_{dx} = difference between the X components of two baselines, etc.

2.7.2 Loop closures

Independent baselines are combined to form loops, the closure of which provides a measure of the accuracy in the network [Shrestha, 1990].

The relative accuracy for a loop is computed by:

$$\text{Relative accuracy} = \frac{\text{loop closure error}}{\text{loop distance}} \qquad (2.17)$$

where:

$$\text{Loop closure error} = \sqrt{E_{dx}^2 + E_{dy}^2 + E_{dz}^2}$$

such that: E_{dx} = misclosure between the two X components of different baselines, etc.

2.7.3 Root mean square accuracy

Root mean square (r.m.s.) accuracy is used in order to gain an overall estimate of the precision of loops and repeated baselines in a network. For multiple baselines or loops a r.m.s. value is defined by [Spiegel, 1972]:

$$\text{R.M.S. accuracy} = \sqrt{\frac{1}{n} \sum (d_i^2)} \quad (i=1,n) \qquad (2.18)$$

in which n is the number of baselines or loops and d_i is the repeatability (equation 2.16) or relative accuracy (equation 2.17) of baselines or loops in a network.

2.7.4 Internal consistency

The internal consistency of the network is tested by performing a minimal or inner constraint adjustment of the whole network (section 2.8.3). The relative accuracy should be homogeneous throughout the entire network. The results from this adjustment will indicate the geometrical precision of the observed network and is not an indication of the final accuracy of the network points [Rapatz *et al.*, 1987]

2.7.5 External accuracy

Independent information is needed to assess GPS accuracies.

A comparison is made between the adjusted GPS baselines and spatial distances measured by methods expected to yield superior results to the GPS survey. The baseline residuals provide a measure of external accuracy. In modern techniques of measurement, e.g. GPS, VLBI, EDM, etc. scale is primarily defined by the measurement of time intervals. Hence, no scale difference should occur between them. However, because the scale of GPS determined baselines may be biased by atmospheric refraction, the adopted gravity constant (GM-value) and a biased height of the reference station there exist scale differences between GPS results and EDM measurements [Beutler *et al.*, 1989]. This scale difference could be as much as 1ppm.

2.8 ADJUSTMENT OF GPS BASELINES

2.8.1 GPS datum definition

Evaluation of GPS signals yield baseline vectors in a three dimensional coordinate system, viz. WGS-84. This reference system is earth-fixed and is defined such that the X-axis lies in the intersection of the equatorial and Greenwich meridian planes and the Z-axis coincides with the mean rotation axis of the earth. The Y-axis forms right angles with the X and Z axes in order to complete a right handed coordinate system (Figure 2.7) [Lohmar, 1986].

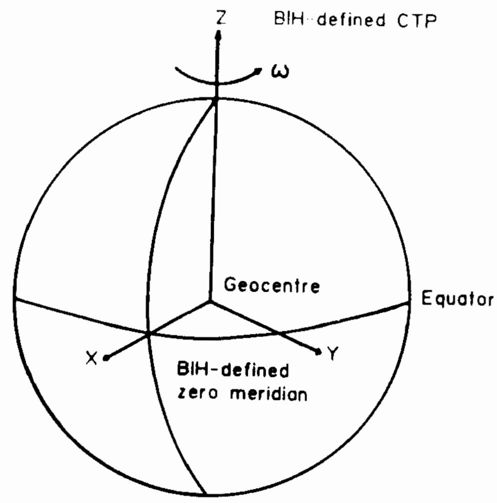


Figure 2.7 Definition of the WGS-84 coordinate system.

The defining parameters of this reference ellipsoid are tabulated below (Table 2.2):

TABLE 2.2 Defining parameters of the WGS-84 ellipsoid.

Parameters	Notation	Value
Semi-major axis	a	6378137 m
Second degree zonal harmonic coefficient of the geopotential	$\bar{C}_{2,0}$	$-484.16685 \times 10^{-6}$
Angular velocity of the Earth	ω	7292115×10^{-11} rad s^{-1}
The Earth's gravitational constant	GM	3986005×10^{8} $m^{3}s^{-2}$

A three dimensional datum (reference system) is defined by three properties [Papo and Perelmuter, 1988]:

(a) origin (three components),
(b) orientation (three components),
and, (c) scale (one component).

GPS measurements are essentially three dimensional ranges measured from satellites to the ground. These ranges contain information concerning the orientation and scale of the network. The missing origin components are defects in the datum, in this case three. In a GPS network adjustment (section 2.8.3), these missing components must be provided by some constraints on the coordinates, for example, by holding one point fixed [Lindlohr and Wells, 1985].

2.8.2 Independent baselines

Consider three baselines forming a triangle. Given difference observations from two sides of the triangle, observations from the remaining side are redundant. Hence, any two baselines are independent. In general n(n-1)/2 baselines can be formed if simultaneous observations on n stations are carried out, in which (n-1) baselines would be independent as long as they do not close in a loop.

GPS observations made in the network mode can be processed in two ways:

(a) baseline by baseline
Any selection of independent baselines as long as they do not form a closed loop.
(b) all independent baselines in the network mode
Normally, processing is based on a base-station

base-satellite concept. It involves the estimation of vectors radially from the base station.

In the network adjustment only independent baselines are chosen as observations, in order to obtain an overdetermined network. The vector network should not contain unconnected vectors whose endpoints are not tied to other parts of the network.

In case (a) where a full correlation matrix is absent, a rigorous network adjustment cannot be performed, since the binding agent (interbaseline correlations), is missing.

Baselines observed in the network mode (b) are combined with a covariance matrix (which accounts for the correlations between the baselines and the vector components) in a rigorous network adjustment. The quality of the observations can be assessed, internal and external reliability computations can be carried out and blunders possibly be discovered and removed. For example, a blunder in measuring the antenna height cannot be discovered in the GPS data processing but will be revealed in the network solution, neglecting to fix the integer ambiguities when it was possible to fix them, etc.

2.8.3 Minimal and inner constraint adjustment

Two kinds of adjustment can be performed on GPS baselines [Bock *et al.*, 1985, Leick, 1990, Vanicek and Krakiwsky, 1986].

(a) Inner constraint adjustment
    The least squares estimates for the inner constraint solution are based on the pseudo-inverse

$$N^{+} = (A^{T}PA + E^{T}E)^{-1} - E^{T}(EE^{T}EE^{T})^{-1}E \qquad (2.19)$$

of the normal matrix, in which E consists of 3x3 identity matrices for as many stations as there are in the adjustment. This solution is useful for analyzing the standard deviations of the adjusted coordinates or the standard ellipsoids. They reflect the true geometry of the network and the satellite constellation. The origin of this coordinate system is defined by the centroid of the coordinates.

(b) Minimal constraint adjustment

Minimal constraints are imposed simply by holding one station fixed to complete the definition of the datum. In this case the confidence ellipsoids depict the uncertainty of the relative position of the points with respect to the origin.

2.9 FUNCTIONAL MODEL

The mathematical model is

$$L_a = F(X_a)$$

in which L_a contains the adjusted vector observations and X_a denotes the adjusted station coordinates.

The observation equations for the network adjustment are

$$DX_{ij} = X_{jo} - X_{io} + dX_j - dX_i + V_{ij}$$

where

$$DX_{ij} = \begin{pmatrix} x_j - x_i \\ y_j - y_i \\ z_j - z_i \end{pmatrix}$$

which contains the baseline vector components, X_{jo} and X_{io} are provisional coordinates of stations j and i, dX_j and dX_i are corrections to these coordinates, and V_{ij} are the associated residuals.

Assigning, for a particular baseline ij,

$$L_{ij} = DX_{ij} - (X_{jo} - X_{io})$$

$$X_{ij} = dX_j - dX_i$$

the observation equations for the observations can be written as

$$L = Ax + V$$

where the elements of the design matrix are 1, -1 or 0, similar to a levelling network.

$$\begin{array}{cc} & \begin{array}{cccccc} x_i & y_i & z_i & x_j & y_j & z_j \end{array} \\ A_{ij} = & \begin{pmatrix} -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

For n observed baselines there would be 3xn number of rows in the design matrix.

The *a posteriori* variance of unit weight, is computed by

$$\sigma_o^2 = \frac{V^T PV}{df}$$

in which

$$df = n - u$$

where n and u are the number of observations and unknowns respectively.

The GPS determined coordinates refer to the coordinate system in which the satellite positions are given. This is now the WGS-84 geodetic datum (section 2.8.1). These positions can be transformed into another coordinate system for external comparison. Alternatively, GPS baselines can be compared (after reduction to the plane) to terrestrial distances.

CHAPTER THREE

FIELDWORK AND SOFTWARE

3.1 FIELDWORK

During the summer of 1988, four dual frequency GPS receivers (2 Mini-mac 2816 and 2 TI-4100) participated in the observation of Professor Williams's Tygerberg Test Network (Figure 3.1). All GPS receivers collected data in the static mode. The receivers were provided by IfAG in Germany and the project was sponsored by Professors Williams and Merry of the University of Cape Town. The extent of the local network is twenty by fifteen kilometres with a height range of 500 metres and most baselines are shorter than ten kilometres. This network was connected to the national levelling datum by including two tide-gauge sites in the observing schedule. The longest baseline is forty kilometres. One session was carried out each day in which four stations were occupied.

The aims of the GPS project were [Williams *et al.*, 1989]:

(a) to do a comparison of GPS results with precise terrestrial measurements,

(b) to obtain ellipsoidal heights for use in combination with orthometric heights to evaluate techniques for determining differential geoidal heights,

and, (c) to connect the Tygerberg Test Area indirectly to the World Geodetic System via the VLBI station at Hartebeeshoek.

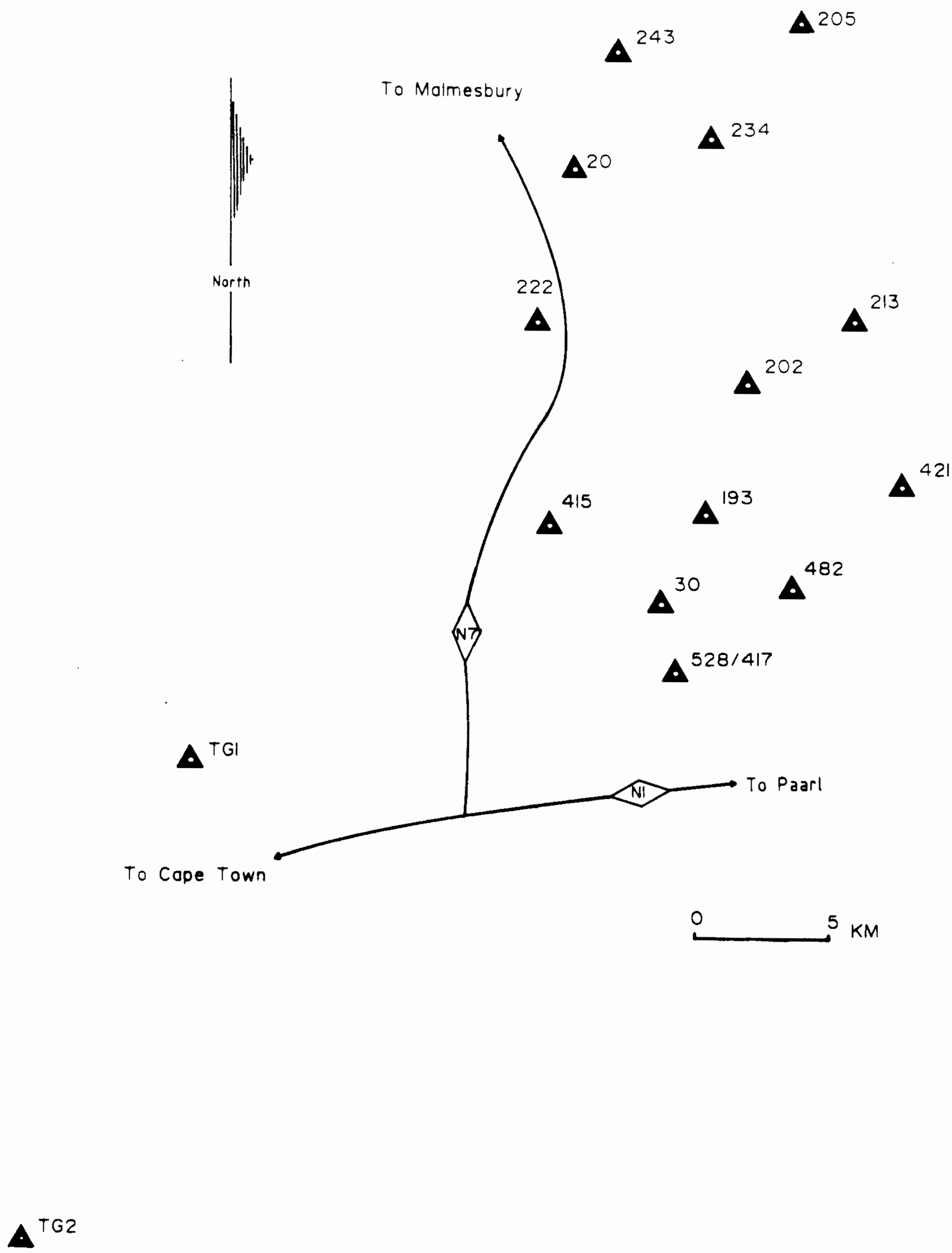


Figure 3.1 Locality sketch of the TTN

In the GPS campaign, a typical observing day went as follows: The equipment were driven to their assigned stations, 20 minutes were allowed for warming up the oscillators of the receivers, the antennae were positioned over the station marks, and their heights were measured. The observations of the satellites then proceeded according to the predetermined schedule (Table 3.1).

TABLE 3.1 Satellite observing schedule.

Proposed schedule for Tygerberg test campaign. (20.11.1988 - 30.11.1988)					
Time	Visible satellites	Chosen geometry	GDOP	PDOP	HDOP
17:40	6 8 11				
17:50	6 8 9 11	6 8 9 11	427	332	—
18:00	6 8 9 11	6 8 9 11	169	129	—
18:10	6 8 9 11	6 8 9 11	97	73	58
18:20	6 8 9 11	6 8 9 11	65	49	36
18:30	6 8 9 11	6 8 9 11	49	37	23
18:40	6 8 9 11	6 8 9 11	41	30	16
18:50	6 8 9 11	6 8 9 11	36	27	11
19:00	6 8 9 11	6 8 9 11	33	26	8
19:10	6 8 9 11 13	6 8 9 11	32	25	5
19:20	6 8 9 11 12 13	8 9 11 12	39	29	23
19:30	6 8 9 11 12 13	8 9 11 12	14	11	8
19:40	6 8 9 11 12 13	8 9 11 12	9	7	5
19:50	6 8 9 11 12 13	8 9 11 12	7	6	4
20:00	6 8 9 11 12 13	8 9 11 12	6	5	3
20:10	6 8 9 11 12 13	8 9 11 12	6	5	2
20:20	6 8 9 11 12 13	8 9 11 12	5	5	2
20:30	6 8 9 11 12 13	8 9 11 12	5	5	2
20:40	8 9 11 12 13	8 9 11 12	5	5	2
20:50	8 9 11 12 13	8 9 11 12	5	5	2
21:00	8 9 11 12 13	8 9 11 12	5	5	2
21:10	8 9 11 12 13	8 9 11 12	5	5	3
21:20	8 9 11 12 13	8 9 11 12	6	5	3
21:30	3 8 9 11 12 13	3 8 11 12	4	3	2
21:40	3 8 11 12 13	3 8 11 12	4	3	2
21:50	3 8 11 12 13	3 8 11 12	4	3	2
22:00	3 8 11 12 13	3 8 11 12	4	3	2
22:10	3 11 13				
Station : Cape Town University 20.11.1988					

A combination of satellites 3, 6, 8, 9, 11, 12 and 13 (PRN numbers) were tracked during each session. The NASA launch numbers for these satellites are 11, 3, 4, 6, 8, 10 and 9. The TI-4100 and Mini-mac receivers can track a maximum of four and eight satellites respectively, hence the satellite combinations with the lowest PDOP values were selected (Table 3.1).

This is not necessarily the best approach, but it was the only tool at our disposal for the pre-analysis, prior to the 1988 observations. A sample skyplot (Figure 3.2) shows satellite visibility during the observations for the vicinity of Cape Town [Merry, personal communication 1989].

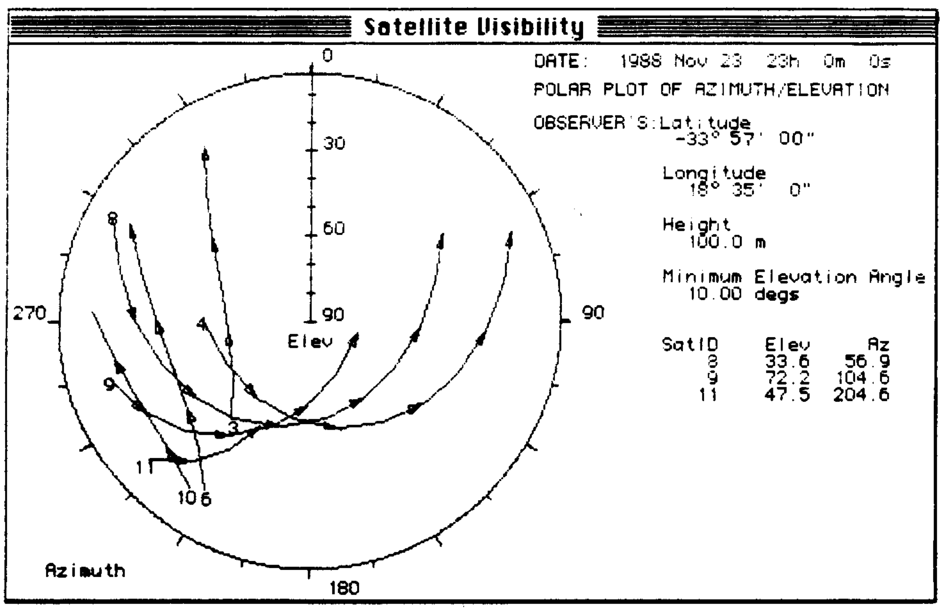


Figure 3.2 Skyplot of satellite visibility

From day to day during the GPS observations the satellite traces did not change considerably. The station occupation plan is given in Table 3.2. The observations took twelve days to complete and a typical session duration was 4^h30^m (Figure 3.3).

TABLE 3.2 Station occupation plan.

Date (1988)	Start time (SAST)	DOY	GPS receivers			
			TI (Cas)	TI (PC)	M/mac1	M/mac2
22.11	19:42	327	417	20	TG1*	TG2
23.11	19:38	328	528	20*	205	421
24.11	19:34	329	234*	20	205	243
25.11	19:30	330	234*	202	205	213
26.11	19:26	331	222	20*	202	213
27.11	19:22	332	222	415	202*	193
28.11	19:18	333	482	421	202	213*
29.11	19:14	334	528	482*	30	193
30.11	19:10	335	528*	415	30	193
1.12	19:06	336	482	202*	30	193
2.12	19:02	337	528	20	TG1*	TG2
3.12	18:58	338	222	234*	202	243

* Reference station in the phase adjustment.

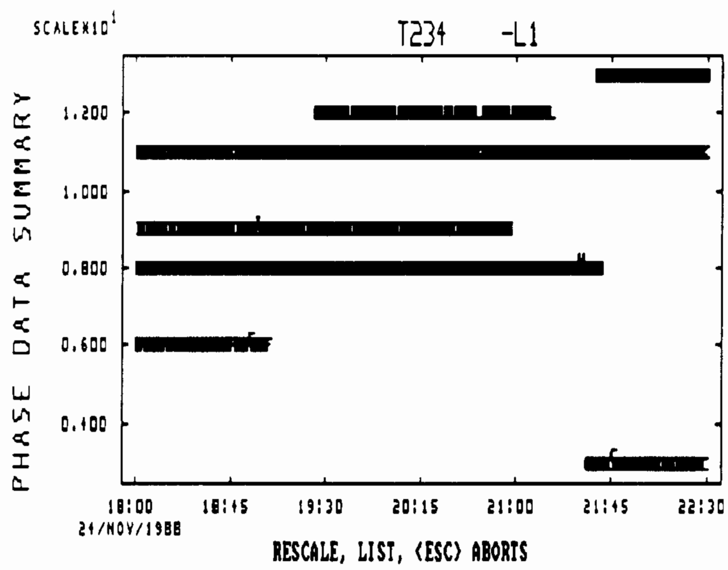


Figure 3.3 Phase data summary

Meteorological observations were taken at hourly intervals, to correct for the delay due to tropospheric refraction. The averages of the atmospheric measurements were computed for every session and relative humidities were computed using equation 2.9 (Table 3.3).

TABLE 3.3 The field observations.

Station	DOY	Temperature(°C) dry	wet	R.H. (%)	Pressure (mb)	Antenna offset (m)
417	327	15.0	15.0	100	964	0.228
20		14.3	13.6	92	969	0.132
TG1		17.7	16.1	85	1015	1.331
TG2		18.0	15.4	76	1017	0.516
528	328	15.2	13.8	85	967	0.215
20		14.0	12.5	84	970	0.132
205		15.0	13.8	87	1002	0.184
421		15.4	13.8	84	1002	0.180
234	329	15.9	14.0	81	1007	0.227
20		14.0	12.0	79	973	0.132
205		15.0	13.4	83	1006	0.184
243		15.2	13.8	85	999	0.196
234	330	17.6	16.0	85	1007	0.228
202		14.8	13.7	88	967	0.129
205		17.0	15.3	83	1005	0.182
213		15.8	14.8	90	983	0.181
222	331	16.9	15.2	83	1007	0.232
20		14.8	12.6	77	971	0.132
202		14.3	13.1	87	984	0.189
213		15.8	13.6	78	981	0.181
222	332	17.3	14.9	77	1010	0.231
415		17.6	13.5	62	1012	0.132
202		14.2	12.5	82	982	0.188
193		14.1	12.6	84	971	0.186
482	333	15.8	13.0	72	994	0.232
421		16.1	11.4	55	1007	0.132
202		13.4	10.4	68	986	0.190
213		13.7	11.1	72	987	0.181
528	334	13.8	11.2	72	975	0.232
482		14.9	11.0	61	995	0.132
30		12.8	10.3	73	984	0.180
193		12.8	10.9	79	974	0.186
528	335	14.9	12.5	75	971	0.221
415		17.3	12.9	59	1009	0.134
30		13.9	11.2	71	986	0.180
193		14.5	12.0	74	969	0.185
482	336	16.6	14.5	79	983	0.227
202		15.5	12.5	70	963	0.132
30		15.3	13.2	78	984	0.182
193		15.5	13.3	78	966	0.187
528	337	16.1	14.3	82	966	0.229
20		16.4	13.5	72	968	0.130
TG1		21.4	15.6	53	1013	1.347
TG2		18.4	15.0	69	1017	0.692
222	338	18.4	15.9	77	1005	0.229
234		19.5	15.0	61	1006	0.132
202		16.1	13.6	75	983	0.190
243		18.2	14.6	67	996	0.197

Some stations were occupied more than once and several baselines were repeated to check the validity of the data. The data processing could not be done after each session, since a preprocessor was not available to format the raw carrier phase data. In practice the GPS data processing should be done immediately after each session, in case re-observing of a session is required.

Special attention must be paid to centreing the GPS antenna over the observed point and the measurement of the antenna height. A centreing device was used to centre and stabilise each antenna on a pillar. It is reported that the real weakness in GPS surveys appears to be in the connection between the GPS antenna and the survey mark. It is advisable to record the antenna height in two units of measurement and checked by the observer to agree within a millimetre. The antenna height above a station marker was referred to the basal plane of the antenna as for the Mini-mac (Figure 3.4) and the TI-4100 (Figure 3.5) antennae.

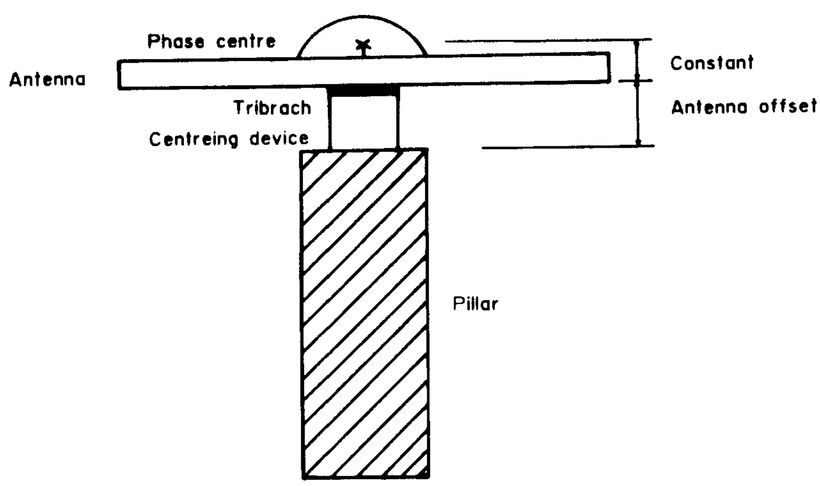


Figure 3.4 Measurement of the Mini-mac antenna height.

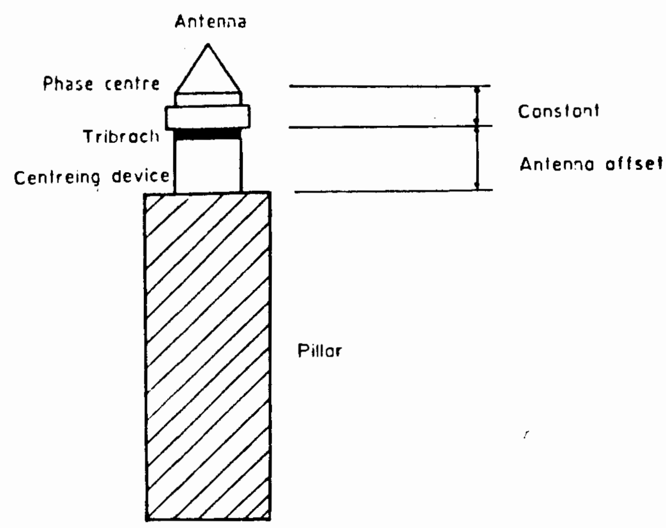


Figure 3.5 Measurement of the TI-4100 antenna height.

Possible eccentricity due to antenna phase centre ambiguity can be avoided, if the antennae are oriented to magnetic north. Only the TI antennae were oriented with respect to magnetic north, since the Mini-mac antennae are not azimuth-sensitive.

Due to interference from a microwave repeater at 417 (DOY 327), station 528 (DOY 337) was used in a later session (Table 3.2). Power supply always seems to be a problem in GPS surveys, but extra 12 VDC batteries were kept to eliminate this problem.

3.2 SOFTWARE

GPS processing can be divided into three stages namely:
Stage 1 - Preprocessing
Stage 2 - Processing
Stage 3 - Post processing.

3.2.1 Preprocessing

"The first stage in obtaining precise static baseline solutions from GPS carrier phase data is preprocessing of the raw data. In general, this step involves decoding the raw data from the storage medium (e.g. floppy disk, etc.), applying any desired corrections to the observations, and reformatting the data for input to the phase adjustment program.

The output from the preprocessing program consists of time tagged phase data, broadcast ephemerides for all tracked satellites and a station coordinate file. The coordinate file contains the single-point pseudo range solution that can be used as the fixed coordinates in the final phase processing, and measured meteorological values." [Wanless and Lachapelle, 1988]

When different receiver types are mixed special problems occur. One is that each receiver outputs its observations with a different time tag. In the field the TI-4100's and Mini-macs were set to sample at 10 and 6 second intervals respectively. The data were combined to give one observation every 30 seconds.

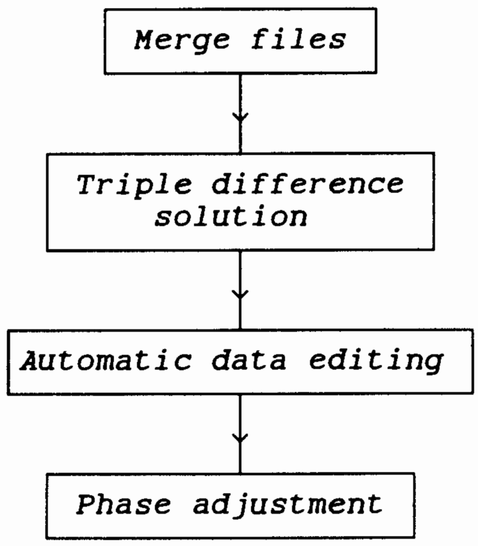
A clock offset of 1 milli-second exists between the TI and Mini-mac receivers [Breuer and Seeger , personal communication 1989]. This correction was applied to the Mini-mac time tags. The OMNI software requires the time tagged phase and broadcast ephemerides to be in NGS-format [Mader, 1989]. Most of the preprocessing was done by IfAG in Germany. The received formatted data was not in the correct NGS-format, hence further reformatting was required. The identification and correction of this problem took some time.

3.2.2 Processing

The data processing was undertaken by the Universities of Bonn and Cape Town (see Q*.* and Z*.*-files on attached disk provided inside back cover).

The processing steps explained here refers specifically to the OMNI software [Mader, 1989].

FLOWCHART - Main program options for the OMNI software.



Files containing the observations at each station in the session are merged. Session duration times, the sampling interval, satellite PRN numbers, elevation limit, field measurements and provisional station coordinates can be entered at this stage.

A base station is selected and a triple difference solution carried out. Large cycle slips are edited (section 2.5) and station clock corrections determined from undifferenced L1 pseudo ranges.

The next step is automatic; the data is searched for more cycle slips and bad phase data. A reference station and satellite are specified with respect to which the differencing is done. Plot files of corrected double difference phase residuals can be inspected to manually edit remaining cycle slips. Manual inspection of these plot files is a tedious process.

Finally, a double difference phase adjustment is executed. Post-fit double difference phase residual plots are generated for final inspection. The computed bias values are constrained to integers by inspection and the phase adjustment carried out once more.

The output of this adjustment consists of the adjusted station coordinates and their associated standard errors. Variance covariance matrices and GPS vectors are now available for input to a post processing program.

3.2.3 Post processing

The computed baselines and their associated VCV matrices for each session are combined in a least squares adjustment to test the internal consistency of the network. Normally a minimal or inner constraint adjustment (section 2.8.3) is performed to demonstrate the internal consistency. The variance covariance matrix allows a rigorous network adjustment to be carried out, since correlations are accounted for.

In order to assess GPS accuracies, a 3-D transformation into a terrestrial network is made for external comparison.

CHAPTER FOUR

DATA PROCESSING

Relative positions between stations were analysed by means of double differencing. The three baseline components are estimated along with (n-1) phase bias parameters (for n satellites) [Bock *et al.*, 1985]. If the baselines are short then atmospheric refraction, orbit biases, etc. cancel in the differencing. On longer baselines these effects are usually modelled or satellite orbits adjusted together with geodetic parameters, etc.

It is recommended to keep one point fixed - for small networks - where satellite orbits are not adjusted [Beutler *et al.*, 1989]. The scale of GPS results is affected if the reference station's coordinates are biased. Therefore, good provisional coordinates are required for the starting point. Normally, an initial point positioning is performed with the pseudo range data. However, the appropriate software was not available to carry out this task.

Station 234 (DOY 329) was selected to define the origin for the entire TTN. The reason for choosing this particular station - to define the origin - is simply because the session on DOY 329 was processed first and station 234 was held fixed in the phase adjustment. The reference station for each session in subsequent processing are shown in Table 3.2. Geocentric provisional coordinates for all the stations were obtained by transforming their respective geodetic coordinates to the WGS-84 datum (Table 4.1). For this purpose a three dimensional conformal transformation was used [Merry, personal communication 1988]. The provisional coordinates are accurate to ±5m.

TABLE 4.1 Provisional coordinates.

Name	Approximate ellipsoidal height (m)	Cartesian coordinates (WGS-84)		
		X (m)	Y (m)	Z (m)
20	416	5035018.33	1690405.11	-3520438.00
30	451	5026062.02	1690343.31	-3533227.16
193	494	5027183.52	1692467.46	-3530708.52
202	465	5029114.45	1694708.69	-3526848.66
205	153	5034812.93	1699259.58	-3516026.94
213	379	5028999.94	1698575.69	-3525007.28
222	104	5032346.10	1688083.06	-3524774.99
234	105	5033760.95	1694982.58	-3519484.59
243	226	5036496.65	1692448.72	-3517027.61
415	95	5028491.66	1687153.78	-3530668.54
417	426	5024606.86	1690582.68	-3535138.38
421	171	5025274.89	1699048.01	-3529689.14
482	279	5024706.05	1694666.74	-3532777.04
528	428	5024597.30	1690458.86	-3535201.18
TG1	29	5028158.28	1673902.28	-3537281.55
TG2	29	5021338.86	1665250.14	-3550933.77

Antenna heights were measured with respect to the basal planes of the antennae. The heights of the L1 and L1-L2 phase centres are tabulated (Table 4.2) below [Sims, 1985, Mader, personal communication 1990]:

TABLE 4.2 L1 and L1-L2 phase centre offsets.

GPS receivers	Phase centre offsets	
	L1 (m)	L1-L2 (m)
Mini-mac 2816 (crossed dipole antenna)	0.107	0.015
TI-4100 (spiral antenna)	0.227	0.025

Hence, the antenna height entered at the triple difference stage is obtained by adding the constant to the respective field measured value (Figures 3.4 and 3.5).

The GPS data were processed (section 3.2.2) in a 386 PC using the OMNI software developed at NGS [Mader, 1989]. The strategies of processing the GPS data are [Hollmann *et al.*, 1990]:

Program	: OMNI
Cut-off angle	: 15^0
Sampling rate	: 30 seconds
Cycle slips and bad data elimination	: Automatic or manual
Ambiguities	: Fixed to integers
Frequencies	: L1 and L3
Orbits	: Broadcast ephemerides
Tropospheric model	: Marini (TSH)
Station clock	: MOD, RNG or SLV.

OMNI outputs GPS-derived coordinates (with standard deviations) and vectors. VCV information is also available, allowing rigorous post adjustment.

A sampling rate of 30 seconds was adopted to increase the number of redundancies (Table 4.3).

TABLE 4.3 Session durations of usable data.

Session (DOY)	Available phase data (0^h00^m)
327	3:15
328	4:58
329	4:29
330	4:58
331	3:54
332	1:30
333	5:01
334	5:49
335	3:49
336	1:40
337	4:18
338	4:59

Various plot files are generated by the software in the different stages of the processing e.g. station clock correction plots (CLK) during triple differencing, double difference residual plot files before (DDA) and after (DDB) data editing, and double difference residual plots before (DDR) and after (PFR) the static solution. The graphics capability of the software is extensively used to aid the processing. Although the automatic editor is reliable, the post-fit double difference residual plots should be carefully inspected to determine if further editing is required.

There are two differences between the DDA/DDB and the DDR plots [Mader, personal communication 1990].

First, the double difference residuals in the DDA or DDB plots do not make use of station clock corrections. However, during the static solution, station clock corrections are solved for or determined from pseudo range observations. Hence, double difference residuals in the DDR plots are corrected for the station clock corrections. Sometimes clock jumps or anomalous values will show up for the first time in the DDR plots. The usual procedure is then to inspect the clock plot files to edit bad pseudo range values.

Secondly, a fixed reference satellite is used in DDA or DDB while in DDR the reference satellite may change. In the DDA or DDB plots, an epoch is dropped if the reference satellite is not present for that particular epoch. When the user specified reference satellite is not present during the static solution, another satellite will be selected by the program until the specified satellite reappears. This also often causes jumps in the DDR residuals which will go away in the PFR residual plots.

In Mini-mac receivers cycle slips can occur down to one cycle, while for TI-4100's half cycle slips can occur on the L1 phase data.

Prior to L3 processing, L1 and L2 solutions are carried out, after which bias values are constrained to integers. However, the L2 data are squared in the Macrometer and hence half cycle bias values are permitted. Due to half cycle biases which may occur on the L2 phase data, a half cycle should be added to or subtracted from the data for that satellite, frequency and station.

The usual procedure in fixing bias terms to integers is to inspect a file containing the bias values, and then rounding them to the nearest integers. Sometimes the solution may have to be improved by iterating with some of the biases fixed before attempting to fix the remainder.

Broadcast ephemerides were used to compute satellite positions. Precise ephemerides are not available to South Africans and the size of the network does not allow for the adjustment of satellite orbits.

The cut-off angle specifies the elevation angle below which data will be ignored. A 15^0 cut-off was selected since data below this altitude is contaminated with excessive refraction effects. The Marini model (equation 2.7) is used to correct observations for the effect due to the tropospheric delay. There is no option for another model, except for a standard atmospheric model (T_{dry}= 20^0C, RH = 50% and $P_{sealevel}$= 1013mb). During the static solution, a tropospheric scale height correction can be solved for. This allows that the tropospheric corrections applied to the data may be in error by a scale factor. However, this can only be done whenever the stations are sufficiently separated that the tropospheric refraction effects at each station are likely to be uncorrelated.

The double differencing removes the first order station clock effect. There is a remaining residual effect due to the Doppler shift between the satellite and receiver [Mader, 1989]. This residual effect can be entirely eliminated if the station clock corrections can be independently determined (equation 2.4). This may be done using the pseudo range observations (RNG) corrected for satellite clock offset, a second order polynomial describing the station clock (MOD) or solving for the station clock offset (SLV). The RNG option uses the pseudo range clock recomputed each epoch to adjust the phase. The Mini-mac phase data is already corrected for clock variation in the receiver. Hence, the usual procedure in OMNI is to replace the Mini-mac clock coefficients - determined during the triple differencing - with zeroes and then use the MOD option to model the receiver clock offset.

There are three types of clock errors:

(a) an epoch offset from UTC [King *et al.*, 1985],

(b) an epoch offset between the receivers,

and, (c) a rate difference between the receivers.

When different receiver types are mixed, it is advisable to solve for the station clock corrections to account for all types of clock errors. The OMNI software does not solve for a rate difference between the receivers, since a rate offset is better determined from single differencing [Bock *et al.*, 1985]. The RNG and MOD options were used in the data processing because the SLV option is not suitable for both absolute and relative clock determination on short baselines. These options also yield more redundancies in the adjustment because estimating extra parameters considerably weakens the solution.

The problems which were encountered while processing the data are as follows:

(1) The data of DOY 336 yielded triple difference plots with a broken nature (perhaps due to the jumps in the observed epochs), hence the only usable part of the data was from 18^h00^m to 19^h30^m (Figure 4.1). This led to a poor determination of baseline 202-193 for which phase data from only three SV's (6, 8 and 9) were analysed.

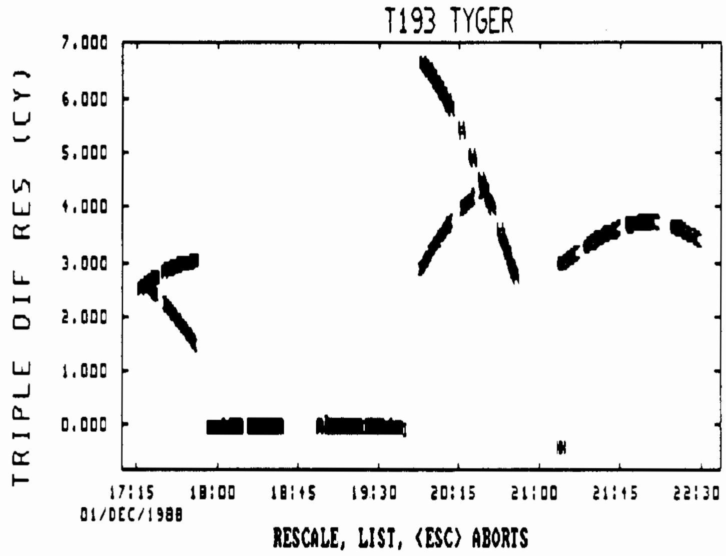


Figure 4.1 Triple difference residuals (DOY 336)

(2) Due to interference from a microwave repeater at station 417, the data contained a lot of gaps, therefore the bias term of SV 12 could not be fixed on baseline TG1-417 (Figure 4.2). SV 12 is at a low altitude throughout the observing campaign (Figure 3.2), hence a refraction bias may inhibit ambiguity resolution.

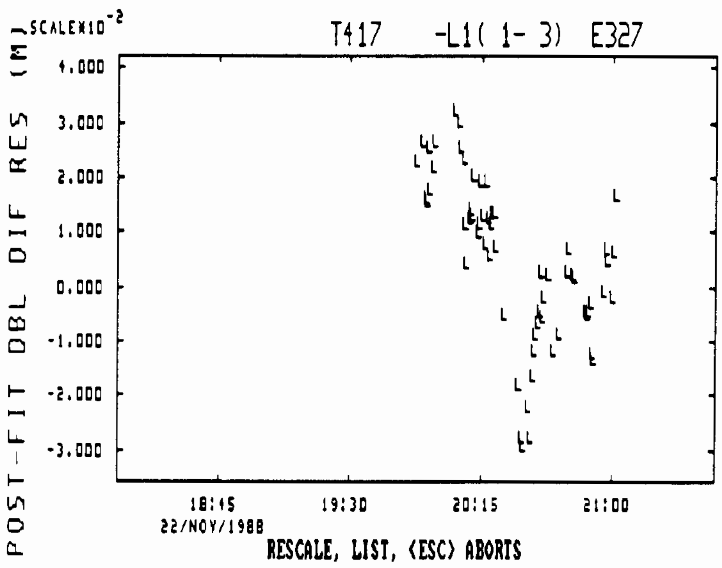


Figure 4.2 Gaps in the phase data of SV 12

(3) Jumps of 3 seconds occurred in the epochs of the TI receivers at DOY 332, 333 and 335. Only baseline 213-482 could be determined on DOY 333. On DOY 332 and 335 the coordinates of station 415 could not be determined. The usable data on DOY 332 for station 202 is from 17^h30^m - 21^h00^m (Figure 4.3), at station 193 from 19^h00^m - 21^h00^m (Figure 4.4) and station 222 between 17^h30^m - 18^h50^m (Figure 4.5).

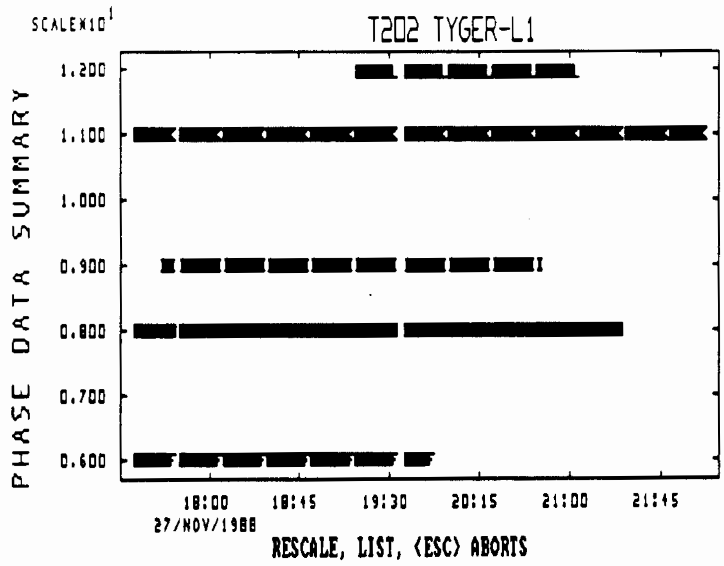


Figure 4.3 Phase data at the reference station

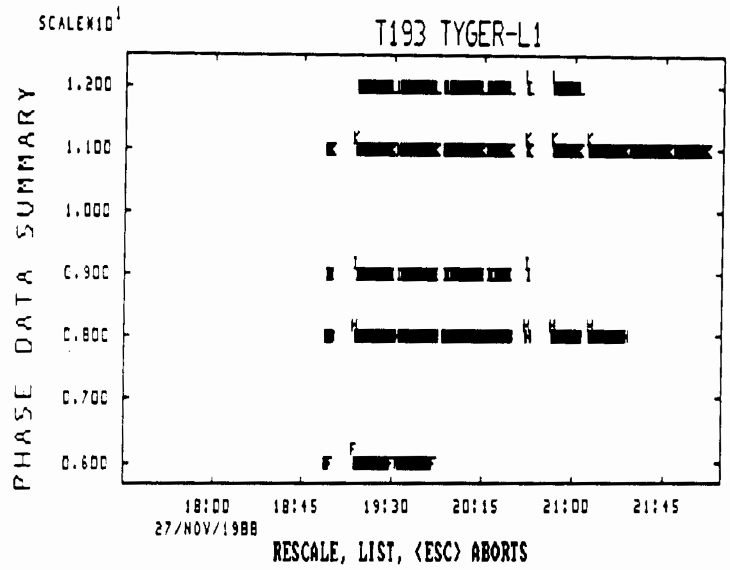


Figure 4.4 Phase data at station 193

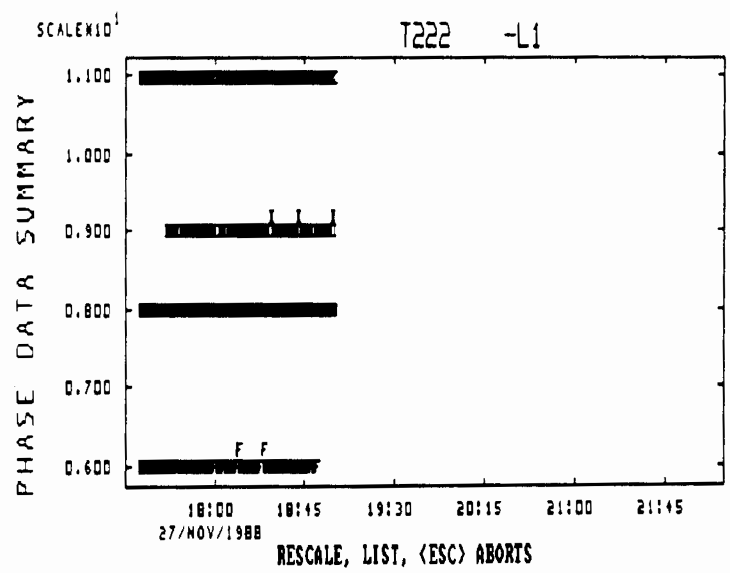


Figure 4.5 Phase data at station 222

The spread of the data justifies station 202 as the reference during the phase adjustment. No ephemeral data was available for SV 3 and 13, which reduced the session duration by approximately one and a half hours (DOY 332). The ambiguities could not be resolved as integers in the L2 processing.

The importance of processing GPS data during the course of the observations cannot be overemphasised. The reason for this being that problems with the data will be discovered at an early stage, and further observations of a session can be carried out if the need arises.

CHAPTER FIVE

DISCUSSION AND ANALYSIS OF THE RESULTS

The determination of the 3-D coordinates of the stations of the TTN was basically a three-step process.

First, geocentric coordinates of the starting point (origin of network) are usually determined from code observations. The software to carry out an initial point position was not available at the time of the processing, hence geocentric coordinates for the starting point was obtained by transforming the coordinates of this point (local datum) to the WGS-84 datum [Merry, personal communication 1988].

The second step was to derive the relative positions of the remaining points with respect to this initial point. The manufacturers' accuracy specifications for the TI-4100 and Mini-mac receivers are 5mm + 1ppm and 1.7 to 3.5mm + 1.7 to 3.5ppm respectively, if orbits accurate to 1ppm or better are used [Hothem, 1986 & 1990]. However, to obtain these accuracies simultaneous observations over at least one hour from at least four satellites are needed, with the geometry of the satellites being an important consideration. Since three or less satellites were visible for much of the observing period (Figure 3.3) it is natural to assume that these accuracies would not be achieved in subsequent data processing.

The final step was to perform a minimal constraint adjustment (Appendix B gives a program listing) of all the independent vectors and their associated VCV matrices in order to obtain a single consistent set of coordinates for the stations.

The design of the project allowed for a high degree of redundancy to provide good network strength and valuable statistical testing of the adjusted results. However, due to receiver malfunctioning during the observations some of the GPS data were corrupted. Weak areas resulted in the well-designed network and the coordinates of one station (415) could not be determined.

The comparative results for the baseline solutions are subdivided into two distinct categories:

(a) short (less than 10km),

and, (b) medium (from 10 to 30km).

Fifteen sample loops of independent GPS vectors are shown in Figure 5.1. All baselines in the TTN are less than 18km long, except the vectors which include a tide-gauge site as a terminal.

5.1 REFRACTION

In this investigation of refraction effects on a small GPS network, data accumulated in the TTN were used. The data consisted of dual frequency phase measurements and broadcast ephemerides; and phase adjustments were carried out for each session.

The observations were carried out at night when the effect of the ionosphere is normally less than that during daytime. The reason for scheduling the observations at night was not one of reducing the effect of the ionosphere, but rather so because of available satellites of the 1988 GPS constellation. The baseline results were evaluated by comparison of repeat measurements (Tables 5.3, 5.4 and 5.7) and the computation of loop closures (Tables 5.1, 5.2, 5.5 and 5.6). The analyses were carried out using the vector components.

TABLE 5.1 VECTOR CLOSURES - L1 with standard model.

Loop	Loop dist (km)	Survey dates (DOY 1988)	Misclosures Δx (mm)	Δy (mm)	Δz (mm)	Accuracy (ppm)
1	13.5	338,329	8.7	-1.2	-0.4	0.6
2	17.2	329,338,330	11.5	-12.5	-9.5	1.1
3	27.4	330,338	-8.5	0.4	-9.3	0.5
4	28.0	330,331	10.9	-2.5	-17.1	0.7
5	25.2	338,331	-0.4	4.0	10.5	0.4
6	19.2	329,338,331	-5.7	-7.0	8.2	0.6
7	21.4	333,336,331	-2.6	-9.0	15.7	0.9
8	16.2	336,332	11.5	22.0	19.9	2.0
9	21.5	332,331	22.5	-8.4	-20.9	1.5
10*	11.9	334,336	-0.3	0.3	6.4	0.5
10*	11.9	335,336	6.0	-1.3	-1.3	0.5
11*	11.9	334,336	-32.9	2.2	20.7	3.3
11*	11.9	334,335,336	-7.4	5.1	12.2	1.3
12	52.2	330,33,34,328	18.7	5.1	-14.3	0.5
13f	89.8	337,328	-74.8	-16.6	4.8	0.9
14f	89.9	337,327	-215.8	-261.9	-140.5	4.1
15	83.9	337,327	-25.4	-39.7	74.1	1.0

* Common baselines from different sessions were used to form these loops.

f Some of the bias terms could not be fixed for baselines in this loop.

TABLE 5.2 VECTOR CLOSURES - L3 with standard model.

Loop	Loop dist (km)	Survey dates (DOY 1988)	Misclosures Δx (mm)	Δy (mm)	Δz (mm)	Accuracy (ppm)
1	13.5	338,329	5.3	0.0	8.6	0.7
2	17.2	329,338,330	2.2	-11.6	4.6	0.7
3	27.4	330,338	7.9	-8.8	12.3	0.6
4	28.0	330,331	9.0	-5.3	-21.8	0.9
5	25.2	338,331	6.4	15.9	-0.7	0.7
6	19.2	329,338,331	-5.7	-5.4	10.8	0.7
7	21.4	333,336,331	-0.7	-6.5	-8.9	0.5
8 f	16.2	336,332	-62.5	48.1	30.9	5.2
9 f	21.5	332,331	63.9	1.8	-41.1	3.5
10	11.9	334,336	-2.1	-1.9	16.9	1.4
10 *	11.9	335,336	-1.9	4.2	10.2	0.9
11	11.9	334,336	-30.6	3.1	21.4	3.2
11 *	11.9	334,335,336	6.7	-2.4	6.7	0.8
12	52.2	330,33,34,328	13.4	4.9	-17.1	0.4

* Common baselines from different sessions were used to form these loops.

f Some of the bias terms could not be fixed for baselines in this loop.

TABLE 5.3 REPEATABILITY - L1 with standard model.

Baseline	Length (km)	Sessions	Misclosures Δx (mm)	Δy (mm)	Δz (mm)	Repeat-ability (ppm)
234-205	5.6	329,330	2.8	-11.3	-9.1	2.6
234-243	4.5	329,338	-8.7	1.2	0.4	2.0
234-202	8.7	330,338	8.5	-0.5	9.3	1.4
202-222	7.6	331,332	-23.2	8.0	23.6	4.4
202-222	7.6	331,338	-0.3	-4.2	-7.7	1.2
202-222	7.6	332,338	22.9	-12.2	-31.3	5.3
202-213	4.3	330,331	-11.0	2.1	17.7	4.9
202-193	4.9	332,336	11.5	-22.0	19.9	6.5
482-193	3.9	334,336	32.6	-1.9	-14.3	9.1
482- 30	4.6	334,336	32.9	-2.2	-21.0	8.6
30-193	3.5	334,335	-6.3	1.6	6.8	2.7
30-528	2.5	334,335	-25.5	-2.9	8.5	11.0
30-193	3.5	335,336	6.0	-1.3	-0.5	1.8
20-205	9.9	329,328	0.9	-25.5	13.2	2.9
20-528	18.1	328,337	-74.8	16.6	4.7	4.2
TG1- 20	24.6	337,327	241.2	301.6	66.4	16.0
TG1-TG2	17.5	337,327	25.3	39.8	-74.2	5.0

TABLE 5.4 REPEATABILITY - L3 with standard model.

Baseline	Length (km)	Sessions	Misclosures Δx (mm)	Δy (mm)	Δz (mm)	Repeat-ability (ppm)
234-205	5.6	329,330	-3.1	-11.5	-4.1	2.3
234-243	4.5	329,338	-5.3	0.1	-8.6	2.3
234-202	8.7	330,338	7.9	8.7	-12.3	2.0
202-222	7.6	331,332	-64.6	-2.2	43.1	10.2
202-222	7.6	331,338	-7.1	-16.3	3.4	2.4
202-222	7.6[f]	332,338	57.5	-14.1	-39.7	9.3
202-213	4.3	330,331	-9.2	5.1	21.5	5.6
202-193	4.9[f]	332,336	62.5	-48.0	-30.3	17.4
482-193	3.9	334,336	28.5	5.0	-4.5	7.5
482- 30	4.6	334,336	30.7	-3.1	-21.4	8.2
30-193	3.5	334,335	-8.8	0.5	10.2	3.9
30-528	2.5	334,335	-28.7	-1.1	11.2	12.5
30-193	3.5	335,336	6.6	-2.4	6.7	2.8
20-205	9.9	329,328	0.5	-19.4	18.1	2.7
20-528	18.1	328,337	-49.1	11.6	5.2	2.9

f Some of the bias terms were not fixed to integers for these baselines.

TABLE 5.5 VECTOR CLOSURES - L1 with Marini model.

Loop	Loop dist (km)	Survey dates (DOY 1988)	Misclosures Δx (mm)	Δy (mm)	Δz (mm)	Accuracy (ppm)
1	13.5	338,329	3.7	-1.7	2.0	0.3
2	17.2	329,338,330	73.2	11.4	-51.9	5.3
3	27.4	330,338	-43.1	-12.2	11.9	1.7
4	28.0	330,331	52.6	15.0	41.8	2.4
5	25.2	338,331	-114.9	-40.8	85.7	5.9
6	19.2	329,338,331	-88.7	-35.1	66.0	6.0
13	89.8	337,328	-57.1	-16.0	0.6	0.7
14[f]	89.9	337,327	-71.3	4.4	-10.4	0.8
15	83.9	337,327	38.2	-12.1	30.7	0.6

f Some of the bias terms could not be fixed for baselines in this loop.

TABLE 5.6 VECTOR CLOSURES - L3 with Marini model.

Loop	Loop dist (km)	Survey dates (DOY 1988)	Misclosures Δx (mm)	Δy (mm)	Δz (mm)	Accuracy (ppm)
13	89.8	337,328	-29.1	-10.7	-18.7	0.4
14[f]	89.9	337,327	-33.7	10.4	-43.9	0.6
15	83.9	337,327	30.2	-11.8	26.0	0.5

f Some of the bias terms could not be fixed for baselines in this loop.

TABLE 5.7 REPEATABILITY - L1 and L3 with Marini model.

Baseline	Length (km)	Sessions	Misclosures Δx (mm)	Δy (mm)	Δz (mm)	Repeatability (ppm)
L1 Processing						
TG1- 20	24.6	337,327	33.1	7.7	-20.3	1.6
TG1-TG2	17.5	337,327	-38.2	12.1	-30.7	2.9
528- 20	18.1	328,337	-57.0	-16.0	0.6	3.3
L3 Processing						
TG1- 20	24.6	337,327	3.5	1.4	17.9	0.7
TG1-TG2	17.5	337,327	-30.2	11.8	-26.0	2.4
528- 20	18.1	328,337	-29.1	-10.7	-18.8	2.0

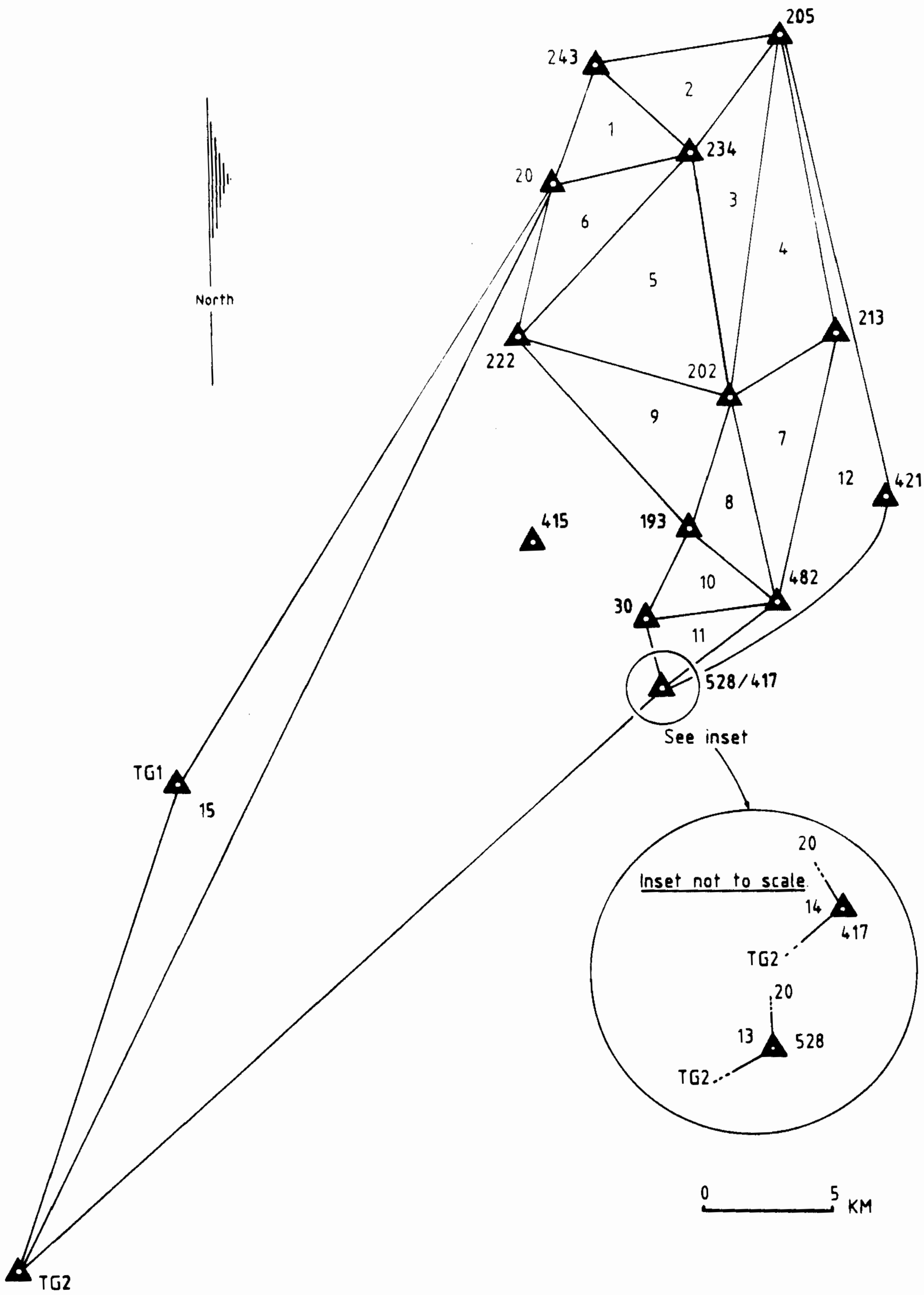


Figure 5.1 Fifteen sample loops of independent GPS vectors.

5.1.1 Ionospheric refraction

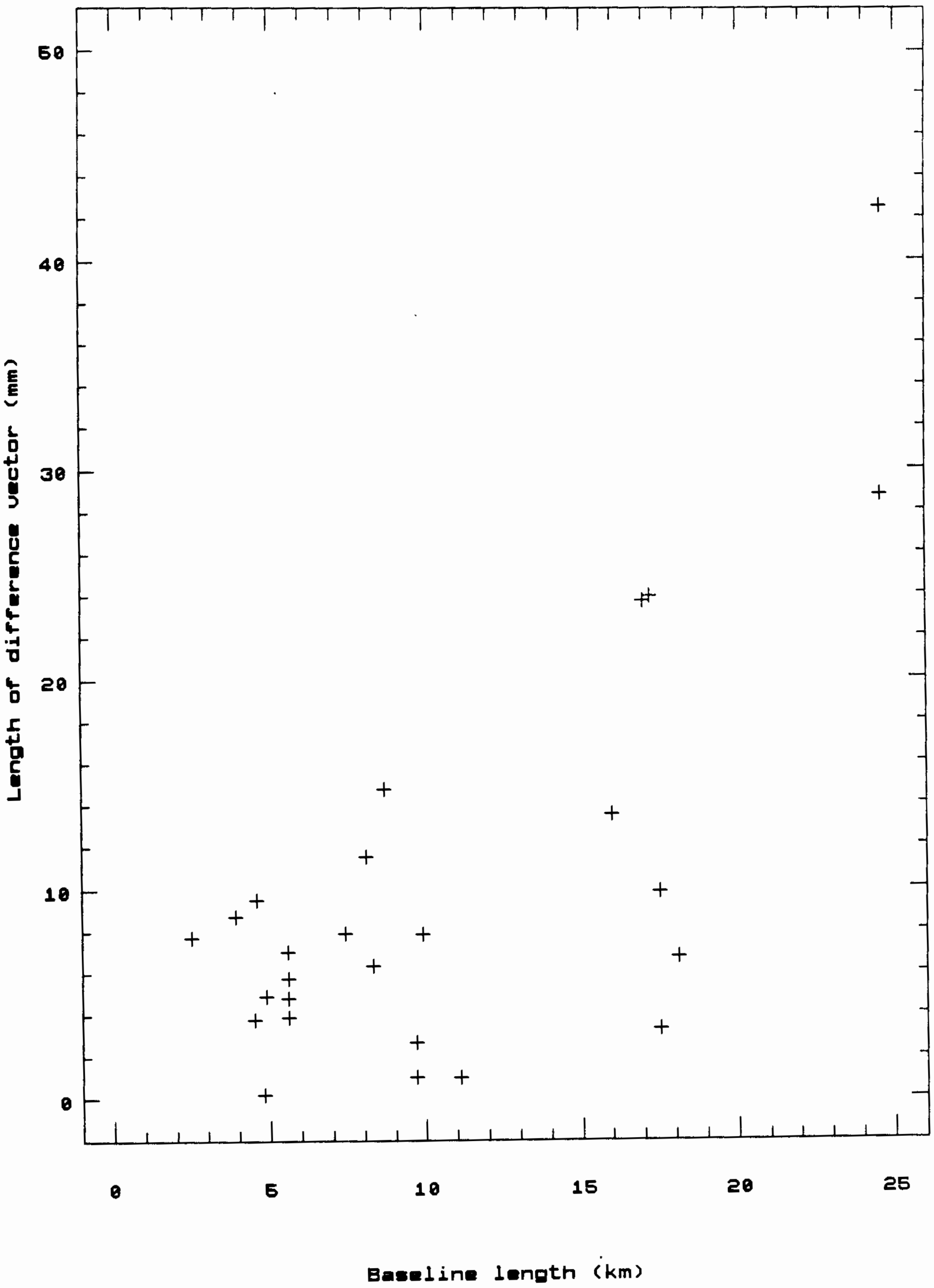
Ionospheric refraction is the most severe limiting factor for single frequency measurements [Beutler *et al.*, 1984]. Neglecting to compensate for ionospheric refraction causes a network contraction [Georgiadou and Kleusberg, 1988] by as much as 0.6ppm. The ionospheric effect was modelled by the L3 combination of dual frequency measurements for the entire network, in order to investigate the effect of this observable on short and medium length baselines.

The effect of the ionosphere can readily be seen by comparing single and dual frequency results (Figure 5.2). The differences between the L1 and L3 solutions appear to be random, perhaps due to the noise in the ionosphere free combination on short baselines. When neglecting to account for ionospheric refraction, baseline errors ranging from 0 to 0.2ppm may be induced for baselines ranging from 2km to 25km.

The main reason why GPS satellites transmit coded information on a second frequency is to correct the observations for the delay due to the ionosphere. The r.m.s. accuracies of repeatability for the single frequency (Table 5.3) and ionosphere free (Table 5.4) observables (short, common baselines with fixed ambiguities) are 5.5 and 5.7ppm respectively. Therefore, the L3 combination of the two L-band observations does not improve the accuracy of baseline determinations over short ranges (less than 10km).

For baselines in excess of 18km (Figure 5.2), a significant difference is apparent between L1 and L3 baseline determinations. Several researchers showed

Figure 5.2 L1 versus L3 processing.



that the L3 combination should be used for long baselines [Cain, 1988, King, *et al.*, 1985, Sjöberg, 1990, etc.]. However, due to a limited number of baselines in excess of 10km, their results could not be confirmed with absolute certainty.

5.1.2 Tropospheric refraction

Correcting for the tropospheric delay was done using weather observations in the Marini model; and a standard model. A sample of seven sessions were processed with the standard and Marini models on the L1 frequency to investigate tropospheric refraction. The vector closures are given in Tables 5.1 and 5.5.

The loop distances of six loops are under 28km and three are between 83 and 90km long. Figure 5.3 gives the results of the vector closures as a function of the average baseline length.

The maximum height difference in a session with respect to the reference station is given in Table 5.8.

Figure 5.3 Standard versus Marini model.

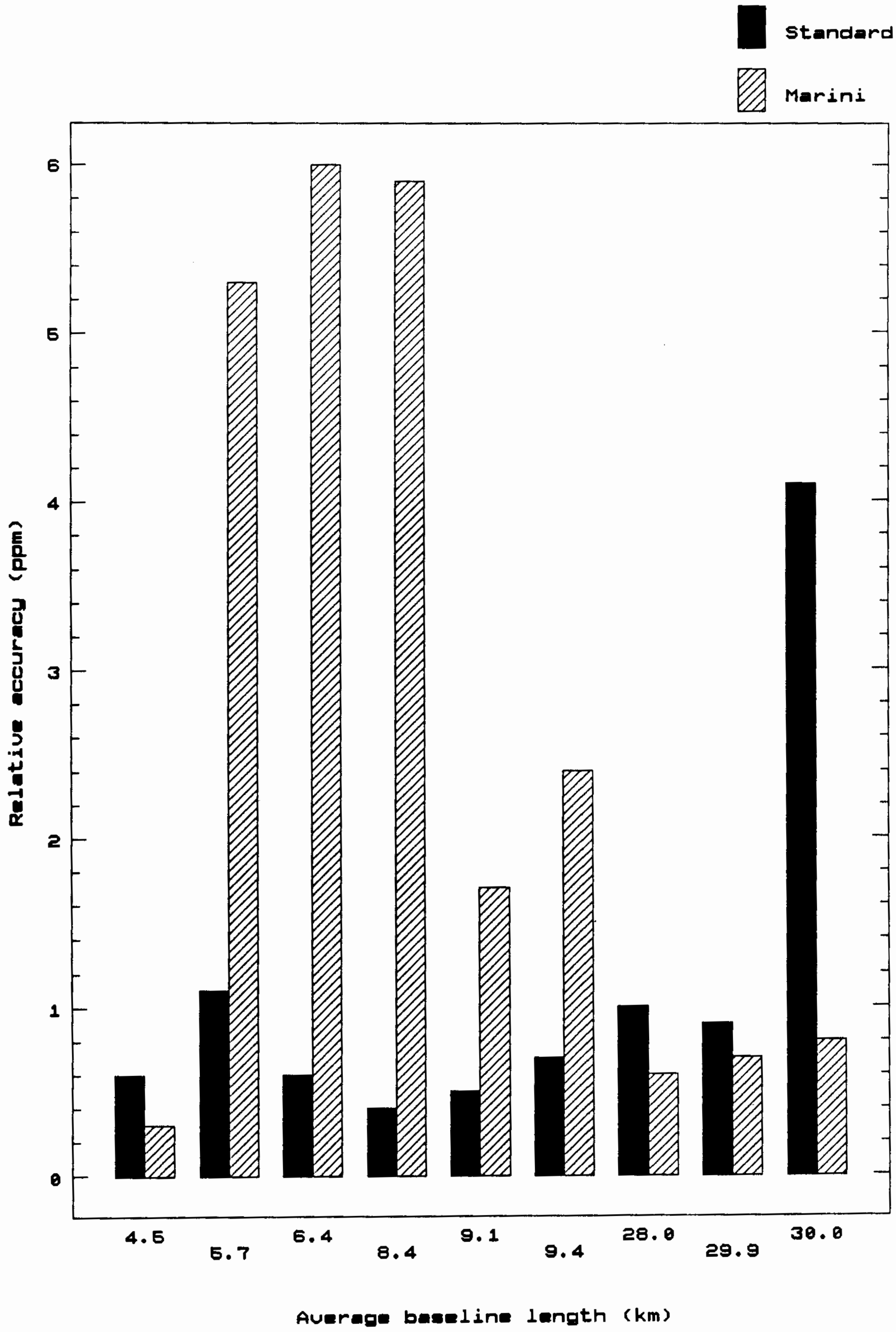


TABLE 5.8 Maximum height differences.

Session	Height difference (m)
327	397
328	262
329	311
330	361
331	311
332	361
333	208
334	215
335	66
336	186
337	399
338	361

It is evident from the results that processing of short baselines yields better results when a standard atmospheric model is used, provided the height differences are small (<400m). This is so because perhaps the standard atmospheric values are a better representation of the actual atmospheric conditions.

However, a considerable improvement is apparent as baselines lengthen (Figure 5.3). It shows that a tropospheric model with surface meteorological measurements, produce better results than the standard atmospheric model on longer baselines (medium length) with height differences less than 400m.

5.2 INTERNAL ACCURACY

The OMNI software outputs a formal statistical summary of each measured baseline which includes adjusted baselines, adjusted network coordinates and VCV information. Standard errors of the adjusted parameters are derived from the VCV matrix. Generally, the formal standard deviations seem to be unduly optimistic, especially in those cases where the integer ambiguities were fixed. In these cases, r.m.s. standard errors for vectors of 10km are of the order of 0.1 to 0.3ppm.

5.2.1 Formal accuracy

The formal accuracy of the solutions is given by the post adjustment estimates of the r.m.s. errors of the unknowns [Beutler *et al.*, 1984]. Formal precision estimates are small. However, they are based on presumptions that are not valid here because our modelling of the atmospheric refraction is far from being perfect. A more realistic check of different baseline solutions (OMNI solutions with and without an ionospheric model) may be found in Tables 5.1, 5.2, 5.3 and 5.4.

Internal accuracy is the measure as processed by a GPS processing package [Hollmann *et al.*, 1990]. Table 5.9 displays the phase adjustment statistics resulting from L1 and L3 processing (with a standard atmospheric model and the Marini model) of the adjusted coordinates.

TABLE 5.9 R.M.S. standard errors of adjusted coordinates

Analysis	Processing method	R.M.S. accuracies		
		σ_x (mm)	σ_y (mm)	σ_z (mm)
UCT	L1	4.1	1.6	2.5
	L3	5.8	3.9	3.3

From these results it can be seen that the uncertainty of the X-component is worse than the Y or Z components. This is perhaps so due to e.g. atmospheric biases, unfavourable satellite geometry (Figure 3.2), etc. which degrade the accuracy to which this component can be determined.

5.2.2 Vector closures

"A powerful check on precision is to establish how well the vectorial addition of sides of a polygon agree in closure." [Goad and Remondi, 1983]

This provides an estimate of the relative accuracies between stations in the network. Only independent baselines are used to compute loop closures. A set of 15 loops is shown in Figure 5.1 and each loop consists of 3 to 5 baselines with a loop distance of between 11 to 90km. Although the loop distances (loops 1 to 12 in Tables 5.1 and 5.2) range from 11km to 52km, single baselines are mostly less than 10km except a few that are between 10 and 18km.

With reference to the data processing the ambiguities could not be resolved for all satellites on some baselines (DOY 327 and 332), mainly because of limited data. The session durations of Table 4.3 refers to the start and stop times of a session, which seems to be long enough for reliable resolution of the ambiguities. In Figure 4.2 it can be seen that the sessions are actually shorter due to gaps in the data which make ambiguity resolution difficult, if not impossible.

The degradation in the accuracy of these baselines is revealed in the loop closures involving Days 327 and 332 (Tables 5.1 and 5.2). The other excessively high vector closures are perhaps due to refraction biases and bad satellite geometry, since orbit biases for baselines under 10km would be eliminated in the differencing. Large triangle closures may be due to systematic errors such as atmospheric effects and ephemeris uncertainties which do not cancel because of different weather conditions and satellite geometry at both ends of the baseline. Multipath effects could be ruled out, since some averaging occurs over long sessions [King *et al.*, 1985].

A r.m.s. relative accuracy of 1ppm was anticipated among stations in the TTN. Figure 5.4 shows the relative accuracies of stations in the TTN as a function of the loop distance (L1 processing). The r.m.s. accuracies from the vector closure analysis indicate that the internal precisions of the network for the L1 (Table 5.1) and L3 (Table 5.2) solutions are 1.6 and 2.0ppm respectively.

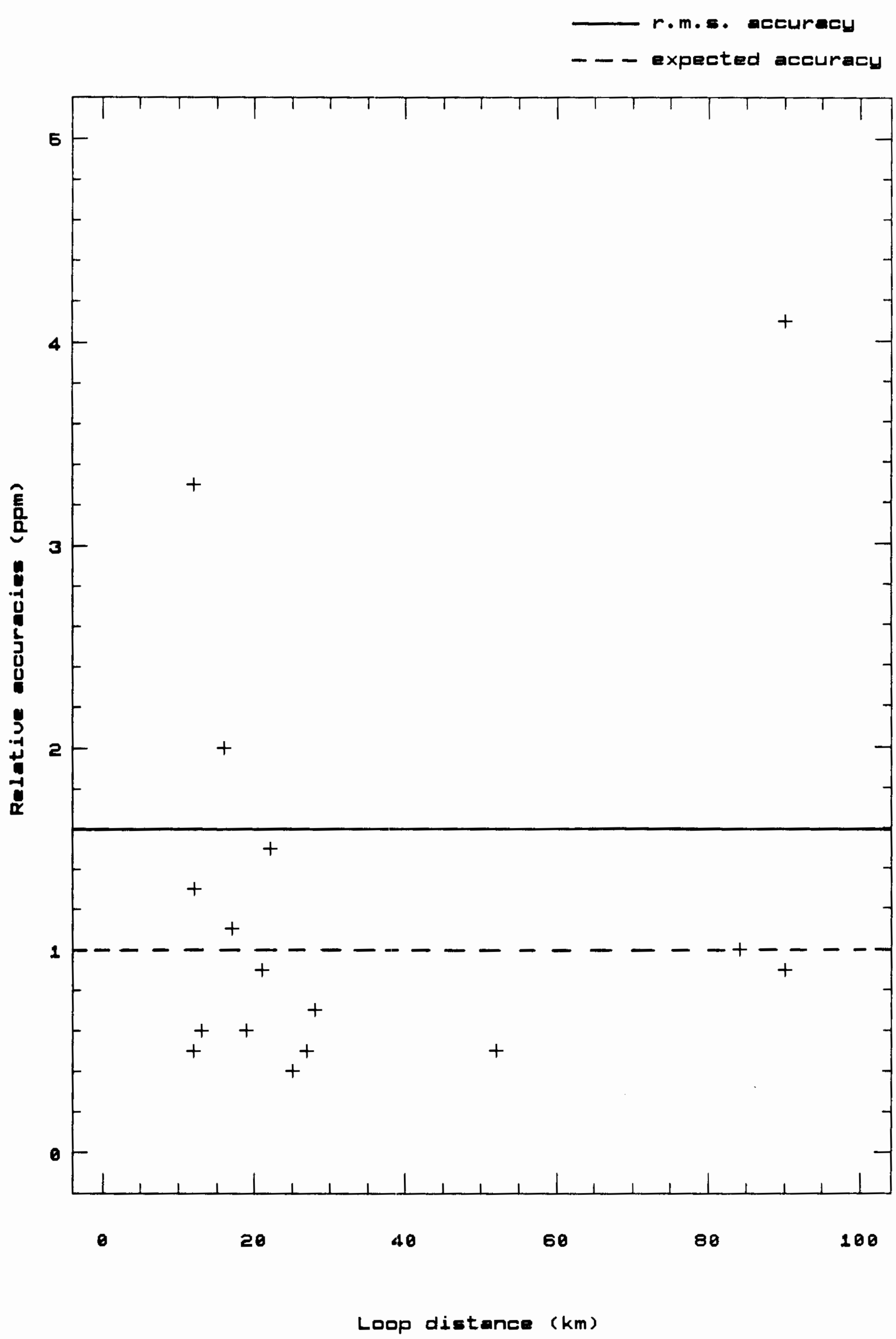
The obtained r.m.s. accuracies (Table 5.8) are unrealistic as the following investigation concerning the precision proves (section 5.2.3).

5.2.3 PRECISION

If the results of repeated observations are compared, a measure of repeatability can be derived from the residuals. This may be called precision [Hollmann *et al.*, 1990]. Seventeen baselines had been remeasured during the course of the GPS survey to give an indication of repeatability.

The r.m.s. values of Tables 5.1, 5.2, 5.3 and 5.4 are much higher and probably closer to reality than the results of the internal accuracies of Table 5.9.

Figure 5.4 Relative accuracies of loops.
(L1 processing)



The RSS quantity defined in equation 2.10 demonstrates the baseline repeatability in terms of all the vector components [Wanless and Lachapelle, 1988]. Normally, baseline repeatability can be expressed as a constant and a term which is a function of the baseline length. The constant term usually accommodates antenna centreing errors, phase centre variations, multipath errors, etc. which often dominate on short baselines. The effect of this constant on short baselines is in the order of 10mm and the remainder is usually expressed in ppm to indicate baseline repeatability.

The TTN adjustment (section 5.5) was carried out with a combination of L1 and L3 processed baselines. In order to gain an estimate of the precision of GPS baselines in the TTN, the results from Tables 5.3 and 5.7 are used to derive values for A and B. A simple linear regression of baseline lengths versus misclosures yields 17.6mm ±1.1mm per km.

5.3 HEIGHT ERROR

Normally, the coordinates of one receiver are held fixed when GPS processing is done in the relative mode [Beutler *et al.*, 1988]. The subsequent investigation is carried out when an error is introduced in the height component of the reference station. The reason for this investigation is that UB carried out an initial point position (code observations) solution for the starting point, while at UCT the initial position was obtained by transforming from the local datum to the WGS-84 datum.

This error source is insignificant if a reliable geocentric position is available e.g. VLBI station coordinates. According to Beutler *et al.* [1989], the scale error introduced due to a height bias of Δh in the reference position is:

$$\frac{\Delta \ell}{\ell} = 0.20 \frac{\Delta h}{R}$$

where, $\Delta \ell$ is the change in the baseline length, ℓ is the baseline length, and R is the radius (6378km) of the earth. Therefore, a height bias of 10m introduces a scale factor of 0.3ppm in GPS baselines.

The geographical coordinates in the vicinity where the observations were carried out is roughly (34°S,18°E). Hence, the X coordinate of a point approximately represents the height component. Height errors of -10, -50 and -100 metres (see H•.•-files on attached disk provided inside back cover) were introduced in the reference station and the results are listed in Table 5.10.

TABLE 5.10 Scale error due to a height bias.

DOY	Baseline	Length (m)	Scale effect on height bias (ppm)		
			-10m	-50m	-100m
329	234- 20	4841.6812	-0.43	-1.03	-1.74
	234-243	4465.5493	-0.31	-1.19	-2.24
	234-205	5599.5649	-0.12	-0.52	-1.00
330	234-205	5599.5786	-0.12	-0.55	-1.09
	234-213	8128.7647	-0.25	-1.17	-2.32
	234-202	8711.6443	-0.25	-1.15	-2.25
331	20-213	11129.0552	-0.21	-1.03	-2.06
	20-202	9719.6168	-0.26	-1.29	-2.56
	20-222	5598.4416	-0.18	-0.91	-1.82
338	234-202	8711.6566	-0.35	-1.18	-2.22
	234-222	8808.6788	-0.11	-0.57	-1.14
	234-222	4465.5551	-0.09	-0.98	-2.08
333	213-482	9699.8766	-0.49	-1.06	-1.76
336	202- 30	8310.2310	-0.34	-1.49	-2.94
	202-482	7388.0225	-0.35	-1.58	-3.11
	202-193	4863.2206	-0.35	-1.56	-3.06
332	202-222	7657.7455	-0.22	-0.94	-1.84
	202-193	4863.2105	-0.23	-1.05	-2.06
334	482-528	4857.4113	-0.12	-0.58	-1.15
	482- 30	4553.3774	-0.15	-0.77	-1.42
	482-193	3905.5772	-0.23	-1.10	-2.20
		Averages :	-0.27	-1.03	-1.98

It was assumed that the trend of the scale effect on a height bias would be linear, ie. that a scale effect of -0.2ppm per 10m would be seen in the results, in other words a negative bias would cause an increase in the baseline length. However, the accuracy of the provisional coordinates is ±5m (section 4.0), hence the scale effect due to a height bias is insignificant. This stresses the importance of good *a priori* geocentric coordinates for the starting point.

5.4 TTN ADJUSTMENT

The quality of the TTN survey is described by a minimal constraint least squares solution (Appendix D provides a listing of the program output).

The TTN survey consists of 32 independent baselines which determine the relative positions of the fifteen station network [Collins and Leick, 1985]. Correlation between baselines due to similar atmospheric conditions, satellite and receiver related errors are fully accounted for during the initial reduction. The vectors are called independent since they are geometrically uncorrelated. The degree of freedom in this adjustment is (32-15-1)=16, since one station is held fixed in the adjustment. Hence, 16 redundant vectors are used in the least squares adjustment to strengthen the network and to discover blunders.

The observations consist of combined L1 and L3 processed data (see Q*.L1 and Z*.L3-files on attached disk provided inside back cover). The included L1 data (short baselines) were processed with a standard atmospheric model to correct for the tropospheric delay, while the L3 data (medium length baselines) were corrected for tropospheric refraction using the Marini model.

When selecting a particular reference station in the phase adjustment, the choice is justified by the difficulties encountered with ambiguity resolution which is perhaps due to a limited amount of phase data and gaps in the data. Independent adjusted vectors and their associated VCV information are derived with respect to this reference station. For a few sessions it was necessary to derive vectors and their associated VCV information with respect to another reference station (of the same session), such that each point in the network may be connected by at least two vectors (Figure 5.5). This is an important consideration in obtaining a well determined network.

The VCV matrix allows for correlations between vector components as well as correlations among baselines. A rigorous network adjustment is possible with VCV matrices and GPS vectors. The VCV matrix is too optimistic and is scaled in order to derive more realistic precision estimates [Vincenty, 1987]. A value of ten is suggested by researchers Leeman and Fletcher (1985), for scaling the standard deviations of each vector. For the TTN adjustment a factor of eleven was used to scale the standard deviations of each baseline determination. The weight matrix of a session is obtained by inverting the respective VCV matrix.

The *a posteriori* variance of unit weight indicates the quality of the adjustment. The computations yield a sigma nought *a posteriori* value of 1.3 which passes the χ^2-test at the 95% confidence level.

Relative error ellipsoids provide information on the relative precision of two stations with respect to each other. Small geodetic networks are relatively flat three dimensional networks, therefore the accuracy of the spatial distance reflects the accuracy of the relative horizontal positioning between the respective stations [Collins and Leick, 1985]. The position accuracies of some of the baselines are more than 4.0ppm which might be an indication that the VCV matrices of the respective observed vectors are somewhat pessimistic

(Table 5.11). The r.m.s. position accuracy of the TTN network is 3.3ppm.

The residual vector is inspected to see whether any systematic effects are present in the observations. Inspection of the residuals reveals that systematic errors are present in two or three baselines of sessions 327, 329, 330, 331, 332, 335, 336 and 338. The baselines of these sessions (except for DOY 327 - Marini model) were evaluated with a standard atmospheric model to account for the tropospheric delay.

Receiver malfunctioning was experienced on Days 332, 333 and 335. Therefore, the systematic effects are more likely to have been caused by receiver related errors and atmospheric biases. Systematic errors may have been introduced by atmospheric effects which do not cancel in the differencing because of, for example, different weather conditions at each end of the baseline.

Blunder detection is also based on the analysis of the residuals. In least squares, the adjustment tends to hide (reduce) the impact of blunders and distribute their effects more or less throughout the whole network [Leick, 1990]. A set of redundant observations is a prerequisite for detecting blunders. The absolute values of all the residuals are less than 3-sigma (3σ) (Table 5.11). Although this test is very crude, it is concluded that no gross errors are present in the observations.

The results of the minimal constraint solution are summarised in Tables 5.11 and 5.12.

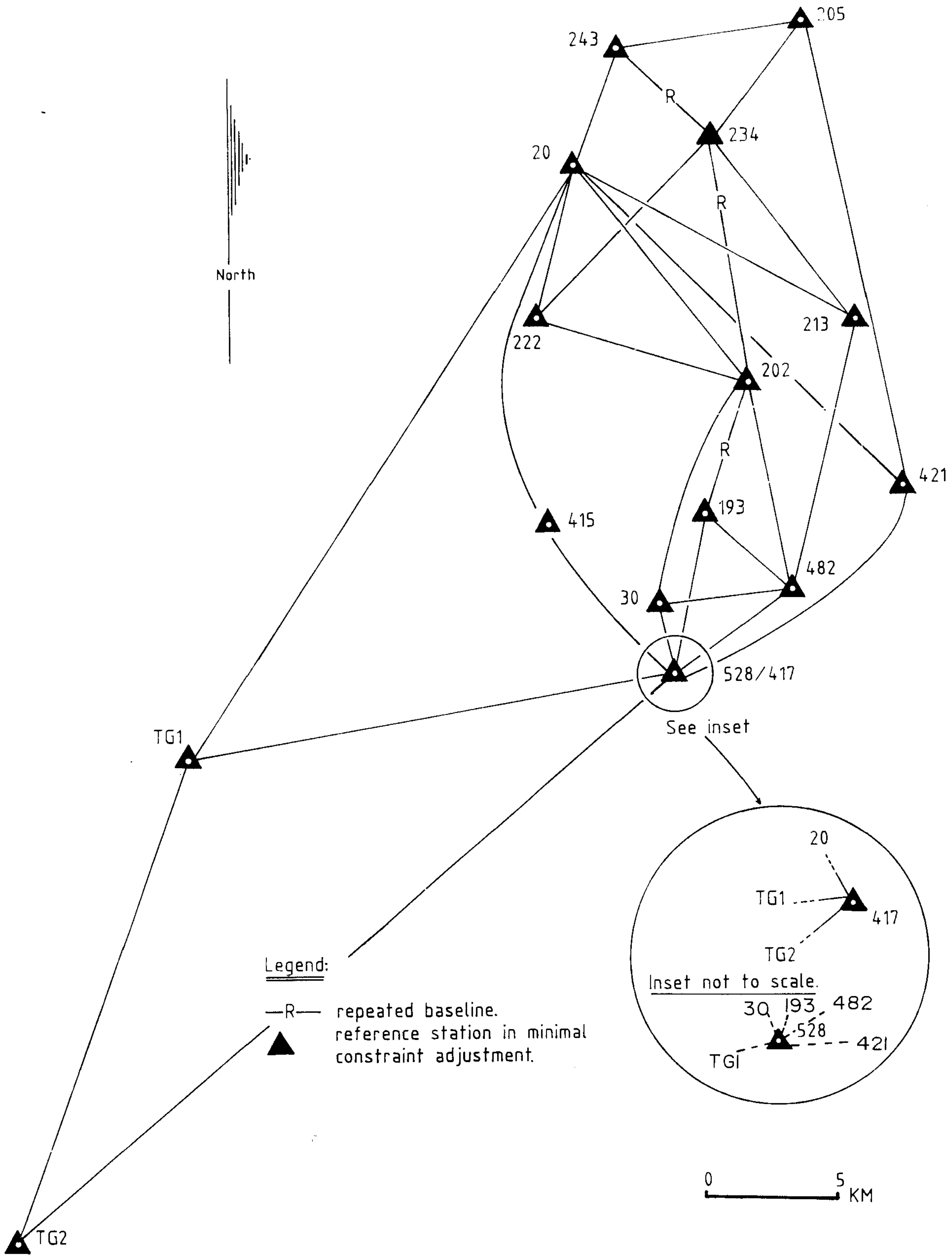


Figure 5.5 Configuration of baselines used in network adjustment.

TABLE 5.11 Minimal constraint GPS baseline adjustment.

From To	Residual (m)	Adjusted baseline (m)	Vector sigma (m)	Sigma/S (ppm)
20 202 (331)	0.0032 0.0052 -0.0013	9719.6137	0.0169	1.7
20 213	0.0096 0.0036 -0.0093	11129.0524	0.0178	1.6
20 222	0.0036 0.0025 -0.0045	5598.4400	0.0175	3.1
213 482 (333)	0.0136 0.0018 -0.0078	9699.8801	0.0193	2.0
202 193 (332)	0.0165 -0.0118 0.0038	4863.2131	0.0195	4.0
202 222	0.0229 -0.0110 -0.0241	7657.7334	0.0161	2.1
202 30 (336)	0.0083 0.0091 -0.0136	8310.2284	0.0201	2.4
202 482	0.0270 0.0072 -0.0291	7388.0151	0.0199	2.7
202 193	0.0050 0.0102 -0.0161	4863.2131	0.0195	4.0
528 193 (335)	-0.0042 -0.0022 0.0012	5559.3953	0.0107	1.9
528 30	-0.0070 -0.0019 0.0040	2460.8006	0.0108	4.4
482 528 (334)	-0.0045 -0.0012 -0.0008	4857.4099	0.0143	2.9
482 30	0.0140 -0.0003 -0.0053	4553.3724	0.0142	3.1
482 193	0.0106 0.0011 -0.0013	3905.5718	0.0140	3.6
421 20 (328)	0.0009 -0.0055 0.0027	15975.5456	0.0432	2.7
421 205	0.0007 0.0090 -0.0071	16663.5821	0.0443	2.7

From To	Residual (m)	Adjusted baseline (m)	Vector sigma (m)	Sigma/S (ppm)
421 528	-0.0004 -0.0030 0.0032	10228.1900	0.0441	4.3
243 234 (329)	0.0058 -0.0002 0.0029	4465.5545	0.0220	4.9
243 20	0.0059 0.0041 -0.0013	4241.7338	0.0232	5.5
243 205	0.0066 -0.0070 0.0023	7086.8879	0.0254	3.6
234 202 (338)	-0.0019 -0.0015 -0.0077	8711.6490	0.0182	2.1
234 222	-0.0020 -0.0002 -0.0005	8808.6780	0.0191	2.2
234 243	0.0029 -0.0010 -0.0033	4465.5545	0.0220	4.9
234 202 (330)	0.0066 -0.0020 0.0016	8711.6490	0.0182	2.1
234 205	-0.0020 0.0045 0.0085	5599.5703	0.0202	3.6
234 213	0.0020 -0.0010 0.0107	8128.7736	0.0198	2.4
417 TG1 (327)	-0.0098 -0.0006 0.0054	17185.7685	0.0848	4.9
417 20	-0.0008 0.0016 -0.0193	18014.8929	0.0881	4.9
417 TG2	-0.0057 -0.0134 0.0194	30030.4907	0.0856	2.9
TG1 528 (337)	-0.0432 -0.0135 0.0140	17059.8898	0.0399	2.3
TG1 20	0.0124 0.0036 -0.0068	24557.6245	0.0387	1.6
TG1 TG2	-0.0262 -0.0009 -0.0120	17542.6744	0.0356	2.0

TABLE 5.12 TTN adjusted coordinates - UCT.

Station	Geodetic datum(WGS-84)			
	X (m)	Y (m)	Z (m)	Flag
20	5035017.8578 0.0138	1690405.0792 0.0051	-3520437.8089 0.0110	0
213	5028999.6559 0.0148	1698575.7906 0.0056	-3525006.8931 0.0118	0
202	5029114.0532 0.0136	1694708.7540 0.0049	-3526848.2978 0.0110	0
222	5032345.6968 0.0143	1688083.1544 0.0054	-3524774.9452 0.0115	0
482	5024705.6912 0.0178	1694667.1109 0.0065	-3532776.8201 0.0141	0
193	5027183.0365 0.0179	1692467.6841 0.0066	-3530708.3006 0.0140	0
30	5026061.4771 0.0181	1690343.6388 0.0066	-3533226.9688 0.0144	0
528	5024596.6780 0.0179	1690459.2353 0.0065	-3535200.9345 0.0139	0
421	5025274.7026 0.0337	1699048.3805 0.0129	-3529688.8806 0.0269	0
205	5034812.9884 0.0152	1699259.5290 0.0059	-3516026.8350 0.0120	0
234	5033760.9510 0.0000	1694982.5750 0.0000	-3519484.5900 0.0000	1
243	5036496.6148 0.0165	1692448.6707 0.0061	-3517027.6305 0.0133	0
417	5024606.1008 0.0714	1690582.9866 0.0315	-3535138.1501 0.0442	0
TG2	5021341.7830 0.0362	1665254.5546 0.0134	-3550938.3125 0.0210	0
TG1	5028161.0953 0.0343	1673906.6865 0.0129	-3537286.0958 0.0201	0

Flag

1 : Fixed coordinates
0 : Estimated coordinates

Note

The standard errors are listed below the respective coordinate values.

TABLE 5.13 TTN adjusted coordinates - University of Bonn

Station	Geodetic datum (WGS-84)			
	X (m)	Y (m)	Z (m)	Flag
20	5035005.1010 0.0010	1690394.6693 0.0004	-3520437.2896 0.0008	0
213	5028986.8561 0.0011	1698565.3447 0.0004	-3525006.3584 0.0008	0
202	5029101.3265 0.0011	1694698.3482 0.0004	-3526847.8073 0.0008	0
222	5032332.7204 0.0013	1688072.6535 0.0005	-3524774.2806 0.0010	0
482	5024692.8572 0.0013	1694656.6533 0.0005	-3532776.2526 0.0010	0
193	5027170.3355 0.0015	1692457.2812 0.0005	-3530707.8269 0.0010	0
30	5026048.7556 0.0014	1690333.2336 0.0005	-3533226.4792 0.0010	0
528	5024583.9405 0.0012	1690448.8245 0.0005	-3535200.4389 0.0010	0
421	5025261.7537 0.0013	1699037.8770 0.0005	-3529688.2454 0.0009	0
205	5034800.0284 0.0000	1699249.0321 0.0000	-3516026.1875 0.0000	1
234	5033747.9496 0.0011	1694972.0635 0.0004	-3519483.9146 0.0009	0
243	5036483.6986 0.0013	1692438.2030 0.0005	-3517027.0172 0.0009	0
TG2	5021329.0327 0.0018	1665244.1420 0.0007	-3550937.7832 0.0014	0
TG1	5028148.2889 0.0019	1673896.2640 0.0007	-3537285.5099 0.0015	0
415	5028477.8107 0.0019	1687143.3435 0.0007	-3530667.5328 0.0014	0

Flag

1 : Fixed coordinates
0 : Estimated coordinates

Note

The standard errors are listed below the respective coordinate values.

After the adjustment of 32 independent vectors by minimal constraints, the r.m.s. accuracies (Table

5.14) of the network adjustment are considerably larger than the results of Table 5.9. This is due to the scaling of the VCV information in order to derive precision estimates which represent their actual values more closely.

TABLE 5.14 Network adjustment statistics.

Analysis	Processed vectors	R.M.S. accuracies		
		σ_x (mm)	σ_y (mm)	σ_z (mm)
Minimal con-straints	L1 & L3	28.4	11.5	19.2

5.5 COMPARISON OF THE RESULTS

The University of Bonn (UB) in Germany also produced 3-D cartesian coordinates for the TTN (Table 5.13). Data processing at UB was carried out with the Bernese software [Breuer, personal communication 1990]. Most baselines were evaluated using the L1 frequency only, while longer baselines were solved using the L3 combination with integer biases fixed for all baselines of the network.

An insight into the differences between the solutions may be taken from the comparison of coordinates. A comparison was made between the independent results of the two GPS processing packages, which used the identical raw observation material. This comparison was carried out to evaluate the consistency of the data processed at UCT with the OMNI software.

The UCT (Table 5.12) set was transformed into the set of coordinates obtained by UB (Table 5.13). The seven parameters of the transformation and the residuals are listed in Table 5.15.

TABLE 5.15 Results of 3-D transformation.

3-D Transformation (UCT to UB)

Transformation parameters

Shifts (metres) : X = -41.3 +/- 14.1
Y = 0.7 +/- 16.4
Z = -42.0 +/- 12.9

Rotation angles (arcseconds) : R_x = -0.1 +/- 0.3
R_y = 1.6 +/- 0.5
R_z = 0.5 +/- 0.5

Scale factor (ppm) : -0.6 +/- 1.4

Station	Residuals		
	X (m)	Y (m)	Z (m)
20	0.07	0.03	-0.04
213	0.02	0.00	-0.02
202	0.05	0.02	-0.04
222	-0.05	-0.02	0.04
482	-0.02	-0.01	0.02
193	0.05	0.02	-0.04
30	0.04	0.02	-0.03
528	0.02	0.01	-0.02
421	-0.07	-0.03	0.05
205	-0.02	-0.01	0.02
234	-0.05	-0.02	0.04
243	0.00	0.00	0.00
TG2	-0.02	-0.01	0.01
TG1	0.00	0.00	0.02

Inspection of the transformation parameters reveals an insignificant Y-translation, X and Z-rotations and a scale factor. The biases (Table 1.1) introduced into GPS baselines by e.g. erroneous reference station coordinates, etc. could not be investigated because of the insignificance of some of the transformation parameters. The insignificance of these parameters can be attributed to the highly correlated nature among these parameters in small networks.

The variability of GPS solutions are dependent on many factors [Li *et al.*, 1990]:

(a) the satellite constellation;
(b) the equipment used;
(c) environmental effects;
(d) orbit errors;
(e) centreing errors;
(f) phase adjustment software;
(g) operator, etc.

The differences between the UCT and UB GPS solutions can be attributed mainly to factors like: different GPS processing packages, modelling the atmospheric effects and the operator subjectivity. It is apparent from the inspection of the residuals that there are systematic errors present in one of the GPS solutions. Stations 193, 202, 205, 222, 234 and 482 appear to be contaminated with systematic errors.

5.6 EXTERNAL RELIABILITY

Results from extraterrestrial observations are usually gauged (external comparison) against a terrestrial standard.

A good and homogeneous internal reliability as yielded by the results of Table 5.9 does not automatically guarantee reliable coordinates [Leick, 1990]. Network adjustment analysis provides confirmation of the internal reliability of the coordinates ie. if no systematic or gross errors present in the observations are apparent from the inspection of the residuals.

Unfortunately, there is inadequate scale in the available terrestrial network, hence an external comparison could not be made [Riessner, 1985]. However, terrestrial data will be available in the near future and the results of the TTN adjustment can then be gauged against this terrestrial standard [Williams, personal communication 1991].

CHAPTER SIX

CONCLUSIONS

Interferometry with the Global Positioning System (GPS) is a method of determining absolute and relative positions in 3-D with respect to a global, earth-fixed coordinate system. Relative positioning with GPS is more accurate than absolute positioning. For the highest possible positioning accuracies, carrier beat phase observations must be obtained at the same time from several satellites and for at least an hour. The time limit on the session duration is mainly for resolving the integer ambiguities.

Atmospheric refraction is one of the most important biases in the results of GPS observations. In the differencing techniques employed in processing GPS data, most of the biases due to atmospheric refraction cancel over short baselines. The residual errors due to the atmosphere and the satellite orbits are approximately proportional to the distance between the stations. Hence, these errors are a function of baseline length and they are not easily modelled.

Weather parameters are used (in a model) to eliminate or reduce tropospheric biases, whereas dual frequency measurements are combined to model the ionospheric bias.

In this study of atmospheric effects on a small network the results of several reseachers were confirmed, namely:

(a) The importance of the second frequency which is transmitted by the GPS satellites, is mainly to model ionospheric refraction effects on GPS measurements. Single frequency results are combined to form the ionosphere free combination ie. L3. However, any noise in the phase observations e.g. multipath errors will be amplified in the L3 combination. Therefore, it is not desirable to form this combination on short baselines where ionospheric (and other) errors cancel in the differencing of the carrier beat phases. In this case, it is better to treat L1 as an independent observable. For long baselines, on which ionospheric (and other) errors are uncorrelated between ends of the baseline, it is preferable to form the ionosphere free combination and remove or reduce the ionospheric effects.

(b) The OMNI software has two options for removal or reduction of tropospheric refraction effects, viz. a standard atmospheric model and the Marini model. The largest height difference between stations in a session with respect to the reference station is 400m. However, the use of a standard atmospheric model for baselines less than 10km (height range less than 400m) is recommended. Baselines longer than 10km should be processed using surface meteorological measurements in the Marini model.

Due to a limited number of baselines in excess of 10km, it was not possible to investigate the cut-off point at which it becomes necessary to process data using the ionosphere free combination (L3) and, to use a tropospheric refraction model with observed meteorological measurements.

The manufacturers of GPS receivers claim high accuracies obtainable with their equipment. However, these accuracies are indeed possible if a minimum of four satellites are observed for at least one hour provided the satellite geometry is good. The baselines in the Tygerberg Test Network (TTN) adjustment consist of L1 and L3 processed solutions. Hence, repeat measurements of these solutions are used in the repeatability analysis. The measure of precision of the baselines in the TTN as yielded by the repeatability analysis is 17.6mm ±1.1mm per km.

The following factors: a lot of gaps in the data perhaps due to receiver related errors, and the availability of three or less satellites for some parts of each session with one satellite consistently at a low elevation (less than 21°), might have influenced the obtained repeatability estimate.

During the TTN GPS observations in the summer of 1988, only seven satellites were available. As more GPS satellites are launched, there would be greater flexibility in selecting the most favourable constellation. Therefore, this will improve the confidence in achieving the desired accuracies.

Good and homogeneous internal reliability as yielded by loop closure and repeatability analysis do not automatically guarantee reliable coordinates. The results from a least squares network adjustment provides confirmation of the internal reliability of the coordinates, when no gross or systematic errors are revealed from the inspection of the residuals.

In order to assess GPS accuracies, independent information is required. Terrestrial measurements were used for this comparison. The GPS solutions from UCT and UB were compared to the terrestrial standard. An external comparison with the UB solution yield consistent results. It is apparent from the results that some of the baseline determinations of the UCT solution are corrupted, because the external comparison yield excessively large discrepancies with respect to the terrestrial measurements. Systematic errors are revealed by the TTN least squares adjustment.

The height differences between stations with respect to a reference station are relatively large (60-400m). In the initial reduction a standard atmospheric model was used to account for the tropospheric delay, hence the results may have been biased by tropospheric effects. These biases are not eliminated in the differencing because of different weather conditions that prevail at each end of a baseline, hence the inconsistencies in the UCT solution may be perhaps due to tropospheric biases.

The precision estimates obtained by GPS processing packages are too optimistic. However, in order to derive formal errors which are closer to their actual values, the standard deviations of the parameters are scaled by a factor of eleven.

Provisional coordinates are needed, prior to derivation of highly precise relative positions from phase data and satellite positions. However, these very precise position estimates may be biased by, for example, erroneous station coordinates, etc. A scale effect of -0.2mm per kilometre for a height error of 10m is yielded by this investigation. This means that a negative height error causes baselines to lenghten. Hence, good *a priori* geocentric coordinates for a reference station is of utmost importance. The starting point's coordinates are accurate to ±5m, hence the scale error due to a height bias is negligible.

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APPENDIX A

APPENDIX A

NOTES ON THE 3-D NETWORK ADJUSTMENT PROGRAM (TTA)

PURPOSE

The purpose of the network adjustment program is to rigorously adjust 3-D GPS vectors with their associated variance covariance information in order to obtain a single set of consistent coordinates for a network.

COMPUTER SYSTEM AND SOURCE CODE

The program was developed on the VAX mainframe and written in FORTRAN source code.

MATHEMATICAL MODEL

Minimal constraints were applied in the mathematical model. The theory is described in sections 2.8.3 and 2.9. A program listing is given in Appendix B.

INPUT

Prior to the execution of the adjustment program (TTA), a data file must be set up and matrices (see appendix B for variable descriptions) dimensioned in the program. Provisional coordinates for all points are needed and a reference point must be specified for the least squares adjustment. A reference station is necessary to remove the datum defect, explained in section 2.8.1. The observations consist of three dimensional vector components and their associated variance covariance information. The VCV matrix for every session is derived from the correlation matrix of the initial reduction (see Q*.* and Z*.*-files on the attached disk).

A listing of the data file is given in Appendix C. A description of the format for this file is presented as follows [**NOTE:** Most numbers are read in free format (F.F.) and other numbers are entered as integers (I); A - represents variables (ie. comments in the respective lines)]:

A69
A69
A69
A69
Blank line
A69
A69
Blank line
A69
A69
Blank line
A69
A69
I2,A50 No of stations
Free format   Number, station, X, Y, Z coords., flag(0/1)
⋮ ⋮
[n times (no. of stations)]
Blank line
A69
A69
A69
Blank line
* A69
Blank line
Free format Session no, from,to, Dx,Dy,Dz, no of baselines

Free format to, Dx,Dy,Dz

⋮ ⋮

[n times (no. of observations)]

Blank line

A69

Blank line

A69

F.F. Enter 1 if integers are fixed, else 0

F.F. Enter lower triangular VCV matrix

Blank line

[Repeat as from * above]

OUTPUT

The results of the adjustment consist of the adjusted coordinates, adjusted observations and their respective residuals (Appendix D). A variance covariance matrix and variance factor are computed to obtain the necessary precision estimates for the adjustment. The precision estimates obtained from the initial reduction are too optimistic, hence the weight matrix of the observations is scaled by a factor of eleven to obtain estimates which are closer to reality. After scaling of the weight matrix, the χ^2-test on the variance is passed at the 95% confidence level. The observation parameters e.g. number of fixed and adjusted stations, degrees of freedom, number of observations, etc. form part of the output.

APPENDIX B

```
*******************************************************************
*                                                                 *
*       This program adjusts GPS vectors and their associated     *
*       variance-covariance matrices in a least squares ad=       *
*       justment.                                                 *
*                                                                 *
*       The program is setup to  adjust  twelve   sessions with   *
*       a maximum of 3 independent baseline  per session and a    *
*       total number of unknowns of not more than 45 parameters,  *
*       ie. nc more than 15 stations.                             *
*                                                                 *
*                                                                 *
*       TTA - adjustment one                                      *
*                                                                 *
*                                                                 *
*       Date : January 1991                                       *
*                                                                 *
*       Code : Fortran                                            *
*                                                                 *
*******************************************************************

        implicit real*8 (a-h, o-z)

        dimension no(15),istn(15),x(15),y(15),z(15),iref(15),u(45)
        dimension ito(3),dx(3),dy(3),dz(3),P(9,9),a(45,45)
        dimension AtPA(45,45),AtPf(45),d(9,45),f(9),ne(40)
        dimension w(96),E(96,45),Qpp(96,96),spat(32),ifo(32,2)
        dimension ifn(32,2),sig(80),wo(9),ob1(32),ob2(32),ob3(32)
        dimension va(32)

        character*70 cc0
        character*70 cc1
        character*70 cc2
        character*70 cc3

*******************************************************************
*                                                                 *
*       DESCRIPTION OF VARIABLES                                  *
*                                                                 *
*******************************************************************
*                                                                 *
*       no(i)                                                     *
*       istn(i)                                                   *
*       x(i),y(i),z(i)                                            *
*       iref(i)    : Station parameters where i=number of stations *
*                    in the adjustment.                           *
*                    This parameter should also be entered in the *
*                    datafile.                                    *
*       ito(j)                                                    *
*       dx(j),dy(j),dz(j) : Baseline information per session, ie. *
*                           maximum number of baselines in a session.
*       a(k,k),AtPA(k,k)                                          *
*       AtPf(k) : Inversion, normal matrix, etc., where k=number of*
*                 unknowns, ie. 3xi.                              *
*       d(3*l,3*i)                                                *
*       f(3*l),wo(3*l) : Matrices d and f are the design matrix and*
*                        l-terms respectively. l=max. number of   *
*                        baselines in a session.                  *
*                        The design mx. for each session is formed.*
*       w(3*m),E(3*m,3*i)                                         *
*       ob1(m),ob2(m),ob3(m),va(m)                                *
*       Qpp(3*m,3*m) : m=total number of baselines, ie. number of *

*                      observations.                              *
*                      E (contains the complete design matrix)    *
*                      w (residuals)                              *
*                      ob(1-3) (x,y and z baseline components)    *
*                      Qpp (precision estimates)                  *
*                      va (corrected observations)                *
*       spat(m),ifo(m,2)                                          *
*       ifn(m,2) : The matrix spat contains the adjusted spatial  *
*                  distances computed from adjusted coordinates.  *
*                                                                 *
*       Note : The dimensions must be repeated in the subroutines. *
*                                                                 *
*******************************************************************
```

```
26      format(' From To    Residuals  Adjusted spatial  Distance
     &    Ratio',/
     & 26x,'  distance          sigma     sigma/S',/,15x,'(m)',
     & 11x,'(m)',11x,'(m)',6x,'(ppm)')

35      format(1x,I4,I3,5x,f13.4)
55      format(A70)
78      format( a69)
65      format(I2,A50)
45      format(3f12.4)
95      format(' THREE-DIMENSIONAL NETWORK  ADJUSTMENT  OF
     &INDEPENDENT',/,' GPS-BASELINES AND THEIR
     & ASSOCIATED VARIANCE-COVARIANCE',/,' MATRICES.',
     &/,1x,54('='), //)
115     format(//' Fixed stations         :', i3, /)
126     format(' Adjusted stations      :', i3, /)
135     format(' Independent baselines :', i3, /)
155     format(' Minimal constraint adjustment')
555     format(' -----------------------------')
165     format(' Constraint network adjustment', /)
178     format(///,' ADJUSTED COORDINATES',/,1x,20('='),/)
185     format(' No Station       X          sx            Y        sy
     &    Z        sz       Flag')
505     format('                 (m)        (m)           (m)       (m)
     &              (m)        (m)')
195     format(I3,I4,3(f14.4,f8.4),I5)

        open (unit=1,file='tta_local_11.dat',status='old')
        open (unit=2,file='tta_local_11.out',status='new')

*       initialisation

*       sigma (0) apriori has a value of 1.00

        apriori=1.00

*       scale factors for the Variance-Covariance matrices
*       afix - double difference with integers fixed.
*       afloat - d.d. with integers floating.

        afix=11d0
        afloat=5d0

        epsilon=1e-5
        ns=0
        mine=100
        count=0

        iteration=1
        iobs=0
        istop=1
        mn=1
        itel=1
        var=0
        ja=0
        jb=0
        ks=0
        ijan=1
        hy=0
        ma=1
        hg=0

        Do i=1,45
          U(i)=0d0
        end do
        Do i=1,9
          wo(i)=0d0
        end do
        Do ms=1,96
          w(ms)=0d0
        end do

        Do while (iteration.eq.1)
          jk=0
          nfix=0
          count=count+1
          if (iteration .eq. 1) then
            Do i=1,45
              Do j=1,45
                AtPA(i,j)=0d0
              end do
              AtPf(i)=0d0
            end do
          end if

          read(1,55) cc0
          read(1,55) cc1
          read(1,55) cc2
```

```
      read(1,55) cc3
      read(1,*)
      read(1,55) a0
      read(1,55) a0
      read(1,*)
      read(1,55) a0
      read(1,55) b0
      read(1,*)
      read(1,55) b1
      read(1,55) b2
      read(1,65) n1,b6

      i=1
      j=1
      do while (i .le. n1)
        read(1,*) no(i),istn(i),x(i),y(i),z(i),iref(i)
        if (iteration .eq. 1) then
          x(i)=x(i)+U(j)
          y(i)=y(i)+U(j+1)
          z(i)=z(i)+U(j+2)
        end if
        j=j+3

        i=i+1
      end do

      read(1,*)
      read(1,55) b3
      read(1,55) b4
      read(1,55) a0

*     the limit in this Do-loop = (total number of sessions)

      Do i=1,12

*       initialising the design matrix

        Do ii=1,9
          Do j=1,3*n1
            d(ii,j)=0.d0
          end do
        end do

        call readfile(mine,iobs,ifrom,ito,dx,dy,dz,n2,
     &  P,ifn,ks,ne,istop,afix,afloat)

        if (mine .eq. 100) then
          Do iu=1,n2
            ob1(ma)=dx(iu)
            ob2(ma)=dy(iu)
            ob3(ma)=dz(iu)
            ma=ma+1
          end do
        end if

        if (count .eq. 1) then
          ns=ns+1
          ne(ns)=n2
        end if

        call lterms(jk,dx,dy,dz,x,y,z,n2,ifrom,ito,f,w)

*       setup design matrix

        n=0
        Do j=1,3*n2,3
          n=n+1
          kl=3*(ifrom-1)+1
          km=3*(ito(n)-1)+1
          d(j,kl) = -1.d0
          d(j+1,kl+1) = -1.d0
          d(j+2,kl+2) = -1.d0
          d(j,km) = 1.d0
          d(j+1,km+1) = 1.d0
          d(j+2,km+2) = 1.d0

*         forming the full design matrix for precision estimate
*             computations during the first iteration.

          if (count .eq. 1) then
            E(mn,kl) = -1.d0
            E(mn+1,kl+1) = -1.d0
            E(mn+2,kl+2) = -1.d0
            E(mn,km) = 1.d0
            E(mn+1,km+1) = 1.d0

            E(mn+2,km+2) = 1.d0
            mn=mn+3
          end if
        end do
```

```
          if (iteration .eq. 1) then
            ia=3*n2
            call Pinverse(P,ia,ia,10)
            call matrix(d,f,P,AtPA,AtPf,n1,n2)
            if (count .ge. 2) then
              Do it=1,3*n2
                ja=ja+1
                wo(it)=w(ja)
              end do
              call variance(wo,P,var,n2)
            end if
          end if
        end do

        mine=200

        rewind 1

        ma=1

        if (iteration .eq. 1) then
          j=0
          Do i=1,3*n1,3
            j=j+1
            if (iref(j) .eq. 1) then
              AtPA(i,i)=1e20
              AtPA(i+1,i+1)=1e20
              AtPA(i+2,i+2)=1e20
              nfix=nfix+1
            end if
          end do

          ll=3*n1

          call chold(AtPA,ll,ll,10)
          call mxmul(AtPA,AtPf,u,ll,ll,1)
        end if

        if (iteration .eq. 1) then
          Do ir=1,nobs
            Do is=1,nobs
              Qpp(ir,is)=0.d0
            end do
          end do
          call fullQpp(AtPA,E,Qpp,n1,nobs)
        end if

        mm=0
        Do i=1,3*n1
          if (U(i) .lt. epsilon) then
            mm=mm+1
          end if
          if (mm .eq. 3*n1) then
            iteration=0
          end if
        end do

        if (count .eq. 1) then
          nobs=iobs
        end if

      end do

*     computing the spatial distances using coordinates

      Do lm=1,ks
        spat(lm)=dsqrt((x(ifn(lm,2))-x(ifn(lm,1)))**2+
     &(y(ifn(lm,2))-y(ifn(lm,1)))**2+(z(ifn(lm,2))-z(ifn(lm,1)))**2)
      end do

*     Global test

      kh=1
      Do hu=1,nobs/3
        va(hu)=(ob1(hu)-w(kh))**2+(ob2(hu)-w(kh+1))**2
     &+(ob3(hu)-w(kh+2))**2
        va(hu)=sqrt(va(hu))
        kh=kh+3
      end do

      do hu=1,nobs/3
        if ((va(hu)-spat(hu)) .ne. 0) then
          print*, 'Global check not passed'
          PAUSE
        end if
      end do
```

```
*       computing precision estimates for spatial distances

        j=0
        Do i=1,nobs,3
          j=j+1
          som=0d0
          sig(j)=0d0
          Do k=i,i+2
            som=som+Cpp(k,k)
          end do
          sig(j)=sqrt(som)
        end do

*       a-posteriori variance

        idof=(nobs/3)-(n1-nfix)
        apost=sqrt(var/idof)

*       chi-squared test on sigma nought a posteriori
*       for 95% confidence interval

        c0=2.515517d0
        c1=0.802853d0
        c2=0.010328d0
        d1=1.432788d0
        d2=0.189269d0
        d3=0.001308d0
        T=dlog(1.d0/(0.026d0**2))
        T=dsqrt(T)
        anorm=T-(c0+c1*T+c2*(T**2))/(1.d0+d1*T+d2*(T**2)+d3*(T**3))
        anorm=abs(anorm)
        aw=9.d0*idof
        chu=idof*(1.d0-(2.d0/aw) + anorm*(sqrt(2.d0/aw)))**3
        chl=idof*(1.d0-(2.d0/aw) - anorm*(sqrt(2.d0/aw)))**3
        chl=((apost**2)*idof)/chl
        chu=((apost**2)*idof)/chu

205     format(2x,I2,2x,I2,5x,f7.4,4x,f12.4,4x,f8.4,3x,f5.1)
215     format(13x,f7.4)
226     format(42x,'Average :',f5.1)
305     format(' Degrees of freedom     :',I3,/)
314     format(' A-priori sigma(0)      :',f4.1,/)
315     format(' A-posteriori sigma(0)  :',f4.1,/)
326     format(' VtPV                   :',f7.1,///)
335     format(' Chi-squared test on the variance :',//)
355     format(' ',A69,/,' ',A69,/,' ',A69,/,' ',A69,/)
356     format(' ','Scale factor - VCV matrix for the double differencing',
     & ':  ',f5.1,//)
365     format(/,' Flag',/,1x,4('-'),/,' C : Estimated coordinates',/,
     &' 1 : Fixed coordinates')
455     format('        ',f5.2,' =< ',f4.2,' =< ',f5.2,/)
465     format('  => Test passed at the 95% confidence interval.')
478     format('  => Test failed at the 95% confidence interval.')

        j=1
        i=1
        ik=0
        im=1

        write(2,95)
        write(2,26)

        ave=0
        Do while (i .le. ns)
          Do ii=1,ne(i)
            ik=ik+1
            write(2,205), ifn(ik,1),ifn(ik,2),w(j),spat(ik),sig(ik),
     &(sig(ik)/spat(ik))*1e6
            ave=ave+(sig(ik)/spat(ik))*1e6
            Do jj=1,2
              j=j+1
              write(2,215), w(j)
            end do
            j=j+1
          end do
          i=i+1
        end do

        write(2,226), ave/ik
        write(2,115), nfix
        write(2,126), n1-nfix
        write(2,135), nobs/3
        write(2,305), idof
        write(2,314), apriori
        write(2,315), apost
        write(2,326), var
        write(2,335)
        write(2,455), chu,apriori,chl
```

```
      if(apriori .le. chl .and. apriori .ge. chu)then
        write(2,465)
      else
        write(2,478)
      end if

      write(2,178)

      if (nfix .eq. 1) then
        write(2,155)
        write(2,555)
      else if (nfix .gt. 1) then
        write(2,165)
        write(2,555)
      end if

      write(2,355), cc0,cc1,cc2,cc3
      write(2,356), afix
      write(2,185)
      write(2,505)

      i=0
      Do j=1,3*n1,3
        i=i+1
        sx=sqrt(AtPA(j,j))
        sy=sqrt(AtPA(j+1,j+1))
        sz=sqrt(AtPA(j+2,j+2))
        write(2,195), i,istn(i),x(i),sx,y(i),sy,z(i),sz,iref(i)
      end do

      write(2,365)

      close (2)
      END

*************************************************************************
*                                                                       *
*       SUBROUTINES START HERE                                          *
*                                                                       *
*************************************************************************

      subroutine fullQpp(AtPA,E,Qpp,n1,nobs)

      implicit real*8 (a-h, o-z)

      dimension AtPA(45,45),E(96,45),Qpp(96,96)

      Do i=1,nobs
        Do k=1,nobs
          Do j=1,3*n1
            Do l=1,3*n1
              Qpp(i,k)=Qpp(i,k)+E(i,l)*AtPA(l,j)*E(k,j)
            end do
          end do
        end do
      end do

      return
      end

      subroutine matrix (d,f,p,AtPA,AtPf,n1,n2)

      implicit real*8 (a-h, o-z)

      dimension AtPA(45,45),AtPf(45),d(9,45),P(9,9)
      dimension f(9)

c     forming the normal equation matrix

      Do i=1,3*n1
         Do k=i,3*n1
            Do j=1,3*n2
               Do l=1,3*n2
                  AtPA(i,k)=AtPA(i,k)+d(l,i)*P(l,j)*d(j,k)
                  AtPA(k,i)=AtPA(i,k)
               end do
            end do
         end do
      end do
      Do i=1,3*n1
        Do j=1,3*n2
          Do l=1,3*n2
            AtPf(i)=AtPf(i)+d(l,i)*P(l,j)*f(j)
          end do
        end do
      end do

      return
      end
```

```
      subroutine variance(wo,P,var,n2)

      implicit real*8 (a-h, o-z)

      dimension P(9,9),wo(9)

      Do i=1,3*n2
        Do j=1,3*n2
          var=var+wo(i)*wo(j)*P(j,i)
        end do
      end dc
      return
      end

      subroutine lterms (jk,dx,dy,dz,x,y,z,n2,ifrom,ito,f,w)

      implicit real*8 (a-h, o-z)

      dimension x(15),y(15),z(15),ito(3),f(9)
      dimension dx(3),dy(3),dz(3),w(96)

*     initialising the L-vector

      Do m=1,3*n2
        f(m)=0.d0
      end do
      j=0
      Do i=1,3*n2,3
        j=j+1
        f(i)=dx(j)-(x(ito(j))-x(ifrom))
        f(i+1)=dy(j)-(y(ito(j))-y(ifrom))
        f(i+2)=dz(j)-(z(ito(j))-z(ifrom))
      end do
      Do j=1,3*n2
        jk=jk+1
        w(jk)=f(j)
      end dc
      return
      end

      subroutine chold(a,irda,na,ich)
****************************************************************************
*                                                                          *
*     matrix inversion using choleski decompksition                        *
*                                                                          *
*     inputs:  a = array containing pksitive definite matrix               *
*           irda = row dimension if array a                                *
*             na = size if matrix in a                                     *
*            ich = return code (=1, then decompksition only, >1 then       *
*                        full inversion)                                   *
*     outputs:  a = array containing inverse if input matrix               *
*                                                                          *
****************************************************************************

      implicit real*8 (a-h, o-z)

      dimension a(irda,na)

*     choleski decompksition into triangular matrix

      a(1,1)=dsqrt(a(1,1))
      do 10C i=2,na
100   a(i,1)=a(i,1)/a(1,1)
      do 500 j=2,na
      sum=C.d0
      j1=j-1
      do 20C k=1,j1
200   sum=sum+a(j,k)*a(j,k)
      a(j,j)=dsqrt(a(j,j)-sum)
      if(j.eq.na) goto500
      j2=j+1
      do 40C i=j2,na
        sum=0.d0
        do 300 k=1,j1
300     sum=sum+a(i,k)*a(j,k)
400     a(i,j)=(a(i,j)-sum)/a(j,j)
500   contirue
      if(ich.eq.1) return

*  inversion if lower triangular matrix

      do 6CC i=1,na
600   a(i,i)=1.d0/a(i,i)
      n1=na-1
      do 8CC j=1,n1
      j2=j+1
```

```
        do 8CC i=j2,na
          sum=0.d0
          i1=i-1
          do 700 k=j,i1
700         sum=sum+a(i,k)*a(k,j)
800         a(i,j)=-a(i,i)*sum

*   construction if inverse if input matrix

            dc 1300 j=1,na
            if(j.eq.1) goto1000
              j1=j-1
              do 900 i=1,j1
900             a(i,j)=a(j,i)
1000            do 1200 i=j,na
                  sum=0.d0
                  do 1100 k=i,na
1100                sum=sum+a(k,i)*a(k,j)
1200                a(i,j)=sum
1300        continue
        return
        end

        subroutine readfile (mine,iobs,ifrom,ito,dx,dy,dz,n2,
     &  P,ifr,ks,ne,istop,afix,afloat)

        implicit real*8 (a-h, o-z)

        dimension ito(3),dx(3),dy(3),dz(3),P(9,9),ifo(32,2)
        dimension ifn(32,2),ne(40)

95      format(A70)
36      format(f4.1,2f4.1,3f4.1,4f4.1,5f4.1,6f4.1,7f4.1,8f4.1,9f4.1)

*       initialising the covariance matrix

        Do m=1,9
          Do k=1,9
            P(m,k)=0.d0
          end do
        end do

        if (mine .ne. 10C .and. istop .ne. 1000) then
          do ip=1,ks
            ifn(ip,1)=ifc(ip,1)
            ifn(ip,2)=ifo(ip,2)
          end do
          istcp=1000
        end if

        read(1,*)
        read(1,95) b5
        read(1,*)
        i=1
        read(1,*) isession,ifrom,ito(i),dx(i),dy(i),dz(i),n2

*         weighting of 6.Cppm of baseline lengths

c          z0=sqrt(dx(i)**2+dy(i)**2+dz(i)**2)
c         z1=z0*6.0e-6
c         z2=z1**2
c         P(i,i)=1d0/z2
c         P(i+1,i+1)=1d0/z2
c         P(i+2,i+2)=1d0/z2

          if (mine .eq. 1C0) then
            ks=ks+1
            ifo(ks,1)=ifrom
            ifo(ks,2)=ito(i)
          end if

        do while (i .le. n2-1)
          i=i+1
          read(1,*) ito(i),dx(i),dy(i),dz(i)
          if (mine .eq. 1C0) then

            ks=ks+1
            ifo(ks,1)=ifrom
            ifo(ks,2)=ito(i)
          end if
        end dc
        ma=ma+1

        iobs=iobs+3*n2
        read(1,*)
        read(1,95) b6
        read(1,*)
```

```
*        this entry in the  data file specify whether or not
*             integers were fixed in the GPS solution.

         read(1,95) b7
         read(1,*) intgr

         tel=0
         Do i=1,3*n2
            tel=tel+1
            read(1,*) (P(i,j),j=1,tel)
*
*           relative weighting of the covariance matrices
*
*           scale factor of "afix" for fixed double difference solution
*                          "afloat"    float
*

            Do j=1,tel
              if (intgr .eq. 1) then
                P(i,j)=afix*afix*P(i,j)
                P(j,i)=P(i,j)
              else if (intgr .eq. 0) then
                P(i,j)=afloat*afloat*P(i,j)
                P(j,i)=P(i,j)
              end if
            end do
         end dc
      return
      end

      subroutine mxmul(AtPA,AtPf,U,i1,i2,i3)
**********************************************************************
*                                                                    *
* s/r to multiply  two matrices ie. A * B = U                        *
*                                                                    *
**********************************************************************

         implicit real*8 (a-h, o-z)

      real*8 AtPA(i1,i1),AtPf(i1),U(i1)

      do 210 i = 1, i1
         do 220 j = 1, i3
            U(i) = 0.d0
            do 230 k = 1, i2
               U(i) = U(i) + AtPA(i,k)*AtPf(k)
230         continue
220      continue

210   continue
      return
      end

      subroutine Pinverse(P,irda,na,ich)
***************************************************************************
*                                                                         *
*        matrix inversion using choleski decompksition                    *
*                                                                         *
*        inputs:  p = array containing pksitive definite matrix           *
*              irda = row dimension if array a                            *
*                na = size if matrix in a                                 *
*               ich = return code (=1, then decompksition only, >1 then   *
*                              full inversion)                            *
*        outputs:  p = array containing inverse if input matrix           *
*                                                                         *
***************************************************************************

         implicit real*8 (a-h, o-z)

         dimension p(9,9)

c        choleski decompksition into triangular matrix

         p(1,1)=dsqrt(p(1,1))
         do 10C i=2,na
100      p(i,1)=p(i,1)/p(1,1)
         do 500 j=2,na
         sum=C.d0
         j1=j-1
         do 2CC k=1,j1

200      sum=sum+p(j,k)*p(j,k)
```

```
      p(j,j)=dsqrt(p(j,j)-sum)
      if(j.eq.na) goto500
      j2=j+1
      do 400 i=j2,na
        sum=0.d0
        do 300 k=1,j1
300     sum=sum+p(i,k)*p(j,k)
400     p(i,j)=(p(i,j)-sum)/p(j,j)
500   continue
      if(ich.eq.1) return
c   inversion if lower triangular matrix
      do 600 i=1,na
600   p(i,i)=1.d0/p(i,i)
      n1=na-1
      do 800 j=1,n1
      j2=j+1
      do 800 i=j2,na
        sum=0.d0
        i1=i-1
        do 700 k=j,i1
700       sum=sum+p(i,k)*p(k,j)
800       p(i,j)=-p(i,i)*sum
c   construction if inverse if input matrix
          do 1300 j=1,na
          if(j.eq.1) goto1000
            j1=j-1
            do 900 i=1,j1
900           p(i,j)=p(j,i)
1000          do 1200 i=j,na
                sum=0.d0
                do 1100 k=i,na
1100              sum=sum+p(k,i)*p(k,j)
1200              p(i,j)=sum
1300      continue
      return
      end
```

APPENDIX C

```
This datafile consists of data for the Tygerberg GPS
network which was observed in 1988.
******Local net. without (tropo) on (L1). (DOY 328 - 336)
***Medium size net. with (tropo) on (L3). (DOY 327 & 337)

GPS DATA FILE :
=============

1.  Provisional coordinates (WGS-84 reference ellipsoid)
    -----------------------

no stn          X            Y           Z      ref.(0=float)
                                                    (1=fixed)
15 stations
 1  20   5035017.858 1690405.084 -3520437.813    0
 2 213   5029007.122 1698573.365 -3525005.958    0
 3 202   5029121.513 1694706.330 -3526847.354    0
 4 222   5032353.157 1688080.728 -3524774.004    0
 5 482   5024706.549 1694667.405 -3532777.365    0
 6 193   5027184.141 1692468.062 -3530708.985    0
 7  30   5026061.069 1690344.106 -3533227.713    0
 8 528   5024596.251 1690459.701 -3535201.674    0
 9 421   5025274.700 1699048.390 -3529688.887    0
10 205   5034812.988 1699259.548 -3516026.849    0
11 234   5033760.951 1694982.575 -3519484.590    1
12 243   5036496.609 1692448.671 -3517027.633    0
13 417   5024603.007 1690578.596 -3535133.384    0
14 TG2   5021338.661 1665250.172 -3550933.501    0
15 TG1   5028157.970 1673902.313 -3537281.309    0

2.  Sessionwise computed baselines and their associated
    variance-covariance matrices.
    ---------------------------

session  from to         DX          DY          DZ    indep.b/lines

  331       1  3    -5903.8013   4303.6800   -6410.4901       3
               2    -6018.1923   8170.7150   -4569.0935
               4    -2672.1574  -2321.9223   -4337.1408

covariance matrix (only the lower triangular part)

integers fixec in GPS solution ?
1
 2.5600e-6
 3.2640e-7   3.6000e-7
-1.4144e-6   3.9000e-8   1.6900e-6
 1.1040e-6   1.4400e-7  -6.2400e-7   2.2500e-6
 1.5360e-7   1.6560e-7   1.5600e-8   3.1500e-7   3.6000e-7
-6.6560e-7   1.5600e-8   7.9430e-7  -1.3455e-6   3.1200e-8   1.6900e-6
 1.0800e-6   1.3500e-7  -5.8500e-7   1.0125e-6   1.4400e-7  -6.0450e-7   2.2500e-6
 1.5360e-7   1.6200e-7   1.5600e-8   1.4400e-7   1.6560e-7   1.5600e-8   3.2400e-7
 3.6000e-7
-6.4480e-7   1.5600e-8   7.7740e-7  -6.0450e-7   1.5600e-8   8.9700e-7  -5.3040e-7
 3.1200e-8   1.6900e-6

session  from to         DX          DY          DZ    indep.b/lines

  333       2  5    -4293.9511  -3908.6779   -7769.9348       1

covariance matrix (only the lower triangular part)

integers fixed in GPS solution ?
1
             3.2400e-6
             4.7880e-7   4.9000e-7
            -1.5372e-6  -1.2740e-7   1.9600e-6

session  from to         DX          DY          DZ    indep.b/lines

  332       3  6    -1931.0002  -2241.0817   -3859.9990       2
               4     3231.6664  -6625.6106    2073.3284

covariance matrix (only the lower triangular part)

integers fixed in GPS solution ?
1
             1.1560e-5
             3.1824e-6   1.6900e-6
            -4.1616e-6  -7.2540e-7   3.2400e-6
             0          -4.8100e-8   0            1.3690e-5
             0           2.3400e-8   0           -2.8638e-6   3.2400e-6
             0           4.4200e-8   0           -1.0945e-5   3.4272e-6   1.1560e-5

session  from to         DX          DY          DZ    indep.b/lines

  336       3  7    -3052.5679  -4365.1061   -6378.6847       3
               5    -4408.3350    -41.6359   -5928.5514
               6    -1931.0117  -2241.0597   -3860.0189
```

```
covariance matrix (only the lower triangular part)

integers fixed in GPS solution ?
1
 1.5210e-5
 1.4352e-6  6.4000e-7
-1.4079e-5 -9.7280e-7  1.4440e-5
 6.2322e-6  5.6400e-7 -5.8938e-6  2.2090e-5
 5.6160e-7  2.5920e-7 -3.7620e-7  2.7495e-6  8.1000e-7
-5.7018e-6 -3.7840e-7  5.8824e-6 -1.9402e-5 -2.0124e-6  1.8490e-5
 7.1955e-6  7.2160e-7 -6.6994e-6  5.9737e-6  5.1660e-7 -5.4653e-6  1.6810e-5
 7.3710e-7  2.9520e-7 -4.7880e-7  3.8070e-7  2.2680e-7 -2.3220e-7  1.9557e-6
 8.1000e-7
-6.5403e-6 -4.6800e-7  6.8172e-6 -5.4990e-6 -3.5100e-7  5.7018e-6 -1.5031e-5
-1.1934e-6  1.5210e-5

session  from to        DX          DY          DZ   indep.b/lines

  335       8  6    2586.3544   2008.4466   4492.6351      2
               7    1464.7921   -115.5985   1973.9697

covariance matrix (only the lower triangular part)

integers fixed in GPS solution ?
1
            8.1000e-7
            1.4760e-7  1.6000e-7
           -3.7800e-7  2.8000e-9  4.9000e-7
            3.7260e-7  6.8400e-8 -1.7010e-7  8.1000e-7
            6.8400e-8  7.0400e-8  0          1.5120e-7  1.6000e-7
           -1.7010e-7  0          2.2050e-7 -3.7170e-7  0          4.9000e-7

session  from to        DX          DY          DZ   indep.b/lines

  334       5  8     -109.0177  -4207.8768  -2424.1152      3
               7     1355.7999  -4323.4724   -450.1540
               6     2477.3559  -2199.4257   2068.5182

covariance matrix (only the lower triangular part)

integers fixed in GPS solution ?
1
 2.2500e-6
 2.4750e-7  2.5000e-7
-5.6700e-7 -1.2150e-7  8.1000e-7
 7.4100e-7  9.7500e-8 -2.8080e-7  1.6900e-6
 1.0500e-7  1.0250e-7 -3.6000e-8  2.5350e-7  2.5000e-7
-2.8350e-7 -3.1500e-8  3.4020e-7 -6.7860e-7 -9.9000e-8  8.1000e-7
 7.4100e-7  1.0400e-7 -2.9250e-7  7.6050e-7  1.1700e-7 -3.0420e-7  1.6900e-6
 1.0500e-7  1.0500e-7 -3.6000e-8  1.1700e-7  1.1000e-7 -4.0500e-8  2.6000e-7
 2.5000e-7
-2.8350e-7 -3.1500e-8  3.4020e-7 -3.0420e-7 -4.5000e-8  3.5640e-7 -6.7860e-7
-9.9000e-8  8.1000e-7

session  from to        DX          DY          DZ  indep.b/lines

  328       9  1    9743.1561   -8643.3068   9251.0743      3
              10    9538.2865     211.1574  13662.0385
               8    -678.0250   -8589.1482  -5512.0507

covariance mx. (lower triangular part)

ints fixed ?
1
 1.1560e-5
 1.7680e-6  1.6900e-6
-5.7834e-6 -5.9670e-7  7.2900e-6
 6.0860e-6  9.4900e-7 -3.0429e-6  1.2862e-5
 9.7240e-7  9.1260e-7 -3.1590e-7  2.0189e-6  1.8252e-6
-3.0226e-6 -3.0550e-7  3.8880e-6 -6.5541e-6 -6.7990e-7  8.3260e-6
 6.2866e-6  9.5290e-7 -3.1995e-6  6.1256e-6  9.2950e-7 -3.1183e-6  1.1903e-5
 9.9280e-7  9.5680e-7 -3.6990e-7  9.2980e-7  8.9700e-7 -3.4750e-7  1.7617e-6
 1.6636e-6
-3.2198e-6 -3.6010e-7  4.0608e-6 -3.1183e-6 -3.4970e-7  3.9348e-6 -6.0413e-6
-6.3250e-7  7.5916e-6

session    from to   DX         DY         DZ       indep.b/lines

  329        12 11  -2735.6580  2533.9041  -2456.9566     3
                 1  -1478.7511 -2043.5874  -3410.1797
                10  -1683.6198  6810.8513   1000.7978

covariance mat. (lower trian. part)
```

```
ints. fixed ?
1
 5.7600e-6
 1.1760e-6  1.0000e-6
-2.3256e-6 -1.9000e-7  3.6100e-6
 3.9816e-6  8.9100e-7 -1.4231e-6  5.8132e-6
 9.0720e-7  7.4100e-7 -9.6900e-8  1.1808e-6  9.7200e-7
-1.4256e-6 -8.5000e-8  2.4985e-6 -2.3471e-6 -1.9140e-7  3.6370e-6
 3.9360e-6  8.7600e-7 -1.3756e-6  3.9056e-6  8.7320e-7 -1.3456e-6  6.1120e-6
 8.6880e-7  7.1200e-7 -8.3600e-8  8.7260e-7  7.1620e-7 -8.6600e-8  1.2408e-6
 1.0640e-6
-1.3656e-6 -7.8000e-8  2.4244e-6 -1.3751e-6 -8.5700e-8  2.4169e-6 -2.4636e-6
-2.2760e-7  3.7988e-6

session   from to   DX           DY          DZ        indep.b/lines

   338      11  3  -4646.8997   -273.8225  -7363.7155     3
                4  -1415.2562  -6899.4208  -5290.3557
               12   2735.6667  -2533.9053   2456.9562

vcv (lower triang. part)

ints. fixed ?
1
 4.4100e-6
 4.4100e-7  4.9000e-7
-2.4990e-6 -8.3300e-8  2.8900e-6
 1.9320e-6  1.9600e-7 -1.0880e-6  4.0000e-6
 2.0580e-7  2.2540e-7 -4.7600e-8  4.4800e-7  4.9000e-7
-1.0752e-6 -4.4800e-8  1.2512e-6 -2.2400e-6 -1.2320e-7  2.5600e-6
 2.0286e-6  2.0580e-7 -1.1424e-6  1.8900e-6  2.0580e-7 -1.0752e-6  4.4100e-6
 2.0580e-7  2.2540e-7 -4.7600e-8  1.9600e-7  2.2540e-7 -3.3600e-8  4.7040e-7
 4.9000e-7
-1.1424e-6 -4.7600e-8  1.3294e-6 -1.0880e-6 -3.5700e-8  1.2512e-6 -2.4633e-6
-9.5200e-8  2.8900e-6

session  from to   DX          DY              DZ   indep.b/lines

   330     11  3  -4646.8912    -273.8230    -7363.7062     3
              10   1052.0354    4276.9585     3457.7635
               2  -4761.2931    3593.2146    -5522.2924

vcv mx. (lower triang. part)

ints. fixed ?
1
 6.2500e-6
 8.2500e-7  1.0000e-6
-3.5700e-6  6.3000e-8  4.4100e-6
 2.3000e-6  3.2200e-7 -1.1109e-6  5.2900e-6
 3.3750e-7  3.6000e-7 -7.5600e-8  7.6590e-7  8.1000e-7
-1.1250e-6 -9.0000e-8  1.3986e-6 -2.6496e-6 -1.9440e-7  3.2400e-6
 2.2425e-6  3.2200e-7 -1.0626e-6  2.2747e-6  3.3120e-7 -1.1178e-6  5.2900e-6
 3.1500e-7  3.6000e-7 -7.5600e-8  3.3120e-7  3.5640e-7 -9.7200e-8  8.0730e-7
 8.1000e-7
-1.0800e-6 -9.0000e-8  1.3608e-6 -1.1178e-6 -9.7200e-8  1.3932e-6 -2.6082e-6
-2.2680e-7  3.2400e-6

session   from to    DX             DY         DZ      indep.b/lines

   327      13 15    3554.9847  -16676.3007   -2147.9403     3
                1   10411.7561    -177.9058   14700.3219
               14   -3264.3235  -25328.4454  -15800.1430

vcv mx. (lower triang. part)

ints. fixed ?
1
 3.9690e-5
 1.2172e-5  7.8400e-6
-1.7048e-5 -2.7552e-6  1.6810e-5
 3.3944e-5  1.1769e-5 -1.3702e-5  5.1239e-5
 1.1454e-5  7.0952e-6 -2.5994e-6  1.7070e-5  9.9604e-6
-1.2751e-5 -2.1476e-6  1.2489e-5 -1.5358e-5 -1.9918e-6  1.7777e-5
 3.1752e-5  1.0408e-5 -1.3260e-5  3.3264e-5  1.1046e-5 -1.2869e-5  4.1454e-5
 1.0566e-5  6.6024e-6 -2.3370e-6  1.1550e-5  6.9881e-6 -2.2037e-6  1.2372e-5
 8.2548e-6
-1.3167e-5 -2.2848e-6  1.3481e-5 -1.3585e-5 -2.5546e-6  1.2631e-5 -1.8081e-5
-2.9138e-6  1.7992e-5

session    from to       DX          DY           DZ       indep.b/lines

   337       15  8   -3564.4605  16552.5353    2085.1754     3
                 1    6856.7749  16498.3963   16848.2801
                14   -6819.3384  -8652.1329  -13652.2287

vcv mx. (lower triang. part)
```

```
ints. fixed ?
1
 1.8490e-5
 5.2976e-6  2.5600e-6
-3.7023e-6 -7.0560e-7  4.4100e-6
 7.9335e-6  2.3040e-6 -1.5120e-6  2.0250e-5
 2.2704e-6  1.0752e-6 -3.3600e-7  5.6160e-6  2.5600e-6
-1.5824e-6 -3.3120e-7  1.7871e-6 -5.2785e-6 -9.9360e-7  5.2900e-6
 6.7080e-6  1.8096e-6 -1.1466e-6  6.6690e-6  1.8096e-6 -1.1661e-6  1.5210e-5
 1.6770e-6  7.9040e-7 -3.0030e-7  1.6965e-6  7.9040e-7 -2.9900e-7  3.8532e-6
 1.6900e-6
-1.1610e-6 -3.1680e-7  1.3986e-6 -1.2150e-6 -3.1680e-7  1.4076e-6 -2.8080e-6
-7.2540e-7  3.2400e-6
```

APPENDIX D

THREE-DIMENSICNAL NETWORK ADJUSTMENT OF INDEPENDENT GPS-BASELINES AND THEIR ASSOCIATED VARIANCE-COVARIANCE MATRICES.
==

From	To	Residuals (m)	Adjusted spatial distance (m)	Distance sigma (m)	Ratio sigma/S (ppm)
1	3	0.0032 0.0052 -0.0013	9719.6137	0.0169	1.7
1	2	0.0096 0.0036 -0.0093	11129.0524	0.0178	1.6
1	4	0.0036 0.0025 -0.0045	5598.4400	0.0175	3.1
2	5	0.0136 0.0018 -0.0078	9699.8801	0.0193	2.0
3	6	0.0165 -0.0118 0.0038	4863.2131	0.0195	4.0
3	4	0.0229 -0.0110 -0.0241	7657.7334	0.0161	2.1
3	7	0.0083 0.0091 -0.0136	8310.2284	0.0201	2.4
3	5	0.0270 0.0072 -0.0291	7388.0151	0.0199	2.7
3	6	0.0050 0.0102 -0.0161	4863.2131	0.0195	4.0
8	6	-0.0042 -0.0022 0.0012	5559.3953	0.0107	1.9
8	7	-0.0070 -0.0019 0.0040	2460.8006	0.0108	4.4
5	8	-0.0045 -0.0012 -0.0008	4857.4099	0.0143	2.9
5	7	0.0140 -0.0003 -0.0053	4553.3724	0.0142	3.1
5	6	0.0106 0.0011 -0.0013	3905.5718	0.0140	3.6
9	1	0.0009 -0.0055 0.0027	15975.5456	0.0432	2.7
9	10	0.0007 0.0090 -0.0071	16663.5821	0.0443	2.7
9	8	-0.0004 -0.0030 0.0032	10228.1900	0.0441	4.3
12	11	0.0058 -0.0002 0.0029	4465.5545	0.0220	4.9
12	1	0.0059 0.0041 -0.0013	4241.7338	0.0232	5.5
12	10	0.0066 -0.0070 0.0023	7086.8879	0.0254	3.6
11	3	-0.0019 -0.0015 -0.0077	8711.6490	0.0182	2.1
11	4	-0.0020 -0.0002 -0.0005	8808.6780	0.0191	2.2
11	1[illegible]	0.0029 -0.0010 -0.0033	4465.5545	0.0220	4.9
11	3	0.0066 -0.0020 0.0016	8711.6490	0.0182	2.1
11	10	-0.0020 0.0045 0.0085	5599.5703	0.0202	3.6
11	2	0.0020 -0.0010 0.0107	8128.7736	0.0198	2.4

13	15	-C.0098 -C.0006 C.0054	17185.7685	0.0848	4.9
13	1	-C.0008 C.0016 -C.0193	18014.8929	0.0881	4.9
13	14	-C.0057 -C.0134 C.0194	30030.4907	0.0856	2.9
15	8	-C.0432 -C.0135 C.0140	17059.8898	C.0399	2.3
15	1	C.0124 C.0036 -C.0068	24557.6245	0.0387	1.6
15	14	-C.0262 -C.0009 -C.0120	17542.6744	C.0356	2.0
				Average :	3.1

Fixed stations : 1

Adjusted stations : 14

Independent baselines : 32

Degrees of freedom : 18

A-priori sigma(0) : 1.0

A-posteriori sigma(0) : 1.3

VtPV : 28.2

Chi-squared test on the variance :

0.90 =< 1.00 =< 3.41

=> Test passed at the 95% confidence interval.

ADJUSTED COORCINATES

Minimal constraint adjustment

This datafile consists of data for the Tygerberg GPS network which was observed in 1988.
******Local net. without (tropo) on (L1). (DOY 328 - 336)
***Medium size net. with (tropo) on (L3). (DOY 327 & 337)

Scale factor - VCV matrix for the double differencing: 11.0

No.	Station	X (m)	sx (m)	Y (m)	sy (m)	Z (m)	sz (m)	Flag
1	20	5035017.8578	0.0138	1690405.0792	0.0051	-3520437.8C89	0.0110	0
2	213	5028999.6559	0.0148	1698575.7906	0.0056	-3525C06.8931	0.0118	0
3	202	5029114.C532	0.C136	1694708.754C	0.0049	-3526848.2978	0.0110	0
4	222	5032345.6968	0.0143	1688083.1544	0.0054	-3524774.9452	0.0115	0
5	482	50247C5.6912	0.0178	1694667.1109	0.0065	-3532776.8201	0.0141	0
6	193	5027183.C365	0.C179	1692467.6841	0.0066	-3530708.3006	0.0140	0
7	30	5026C61.4771	0.C181	1690343.6388	0.0066	-3533226.9688	0.0144	0
8	528	5024596.6780	0.0179	1690459.2353	0.0065	-3535200.9345	0.0139	0
9	421	5025274.7026	0.0337	1699048.3805	0.0129	-3529688.8806	0.0269	0
10	205	5034812.9884	0.0152	1699259.529C	0.0059	-3516026.8350	0.0120	0
11	234	5033760.9510	0.0C00	1694982.5750	0.00CC	-3519484.5900	0.0000	1
12	243	5036496.6148	0.0165	1692448.6707	0.0061	-3517C27.6305	0.0133	0
13	417	50246C6.1008	0.0714	1690582.9866	0.0315	-3535138.1501	0.0442	0
14	TG2	5021341.7830	0.0362	1665254.5546	0.0134	-3550938.3125	0.0210	0
15	TG1	5023161.0953	0.0343	1673906.6865	0.0129	-3537286.C958	0.0201	0

Flag

0 : Estimated coordinates
1 : Fixed coordinates