Ex-ante Evaluation of Investment Performance Fees Using Spread Options

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A dissertation submitted to the Faculty of Commerce, University of Cape Town, in partial fulfilment of the requirements for the degree of Master of Philosophy.

October 12, 2017

MPhil in Mathematical Finance,
University of Cape Town.

Declaration

I declare that this dissertation is my own, unaided work. It is being submitted for the Degree of Master of Philosophy at the University of Cape Town. It has not been submitted before for any degree or examination to any other University.

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October 12, 2017

Abstract

This dissertation analyses ex-ante asymmetric performance fee structures used by South African Mutual Funds and estimates performance fees some time before the fees are paid. Certain parties might benefit from having a reasonable estimate of its value. We use spread option theory to value ex-ante performance fees. The data consist of monthly benchmark and fund gross returns from December 1999 to October 2014. The theoretical value of ex-ante performance fees is a function of spread volatility, therefore high spread volatilities give rise to high ex-ante performance fees. Ex-ante performance fee estimates are highly sensitive to the correlation between the fund and benchmark and a low positive correlation gives rise to a high ex-ante performance fee. The distribution of ex-ante performance fees is positively skewed because of the maximum function in the payoff. Ex-ante performance fee estimates obtained are lower than the actual performance fees paid.

Acknowledgements

I would like to thank my supervisor Dr Andrew van Biljon for his guidance and assistance during my research. I would also like to extend my appreciation to Alex Backwell and Professor Thomas McWalter for their assistance, time and support.

Finally my appreciation goes to Professor David Taylor for assisting me with funding and for his support throughout the year.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

Investment companies are compensated for managing their clients' assets through management fees and performance/incentive fees. These two types of fees form the total compensation paid to investment companies for managing their clients' assets. In South Africa about 20 percent of unit trust asset managers have a reward scheme that is comprised of both management fees and performance fees (Treasury, 2013). Performance fees are rare in the United States of America because of the Investment Amendment Act of 1970 (Thomas and Jaye, 2006). This Act only allows investment companies to charge a symmetric fee structure, therefore very few funds are willing to use a compensation scheme that penalises underperformance. Performance fees are quite common in Europe and a lot of research has been done in this field of study in papers by Elton, Gruber and Blake (2003), Drago, Lazzari and Navone (2010) and Pohjanpalo (2013) to mention a few.

Management fees are calculated as a fixed percentage of assets under management (AUM) and accrue regardless of the manager's performance, whereas performance fees only accrue when the fund manager outperforms a pre-set target. There are different ways in which performance fees are calculated but the simplest way is to take the difference between the portfolio return and a relative benchmark, scaled by a factor called the participation rate. These different ways of calculating incentive fees are referred to as performance fee structures. We shall list performance fee structures we consider in this dissertation in Chapter 2.1.

Since there are different methods of calculating performance fees, many research papers have questioned their appropriateness and also the way these fees are calculated because there is no standard way of calculating performance fees. In this dissertation we investigate the use of option pricing theory to estimate performance fees some time before the fee is paid, because certain parties might benefit from having a reasonable estimate of its value. Value here could simply mean the best estimate for the fee itself, given the information at that time, or perhaps the hypothetical market value of the right to receive the fee. We shall compare ex-ante performance fee values we obtain to the actual performance fees paid to determine whether these spread option theory valuation techniques are useful.

We estimate ex-ante performance fees under the real world measure $\mathbb{P}$ and the risk-neutral measure $\mathbb{Q}$.

1.1 Performance Fee Categories

Performance fees are classified into two broad categories: fulcrum fees (symmetrical fees) and bonus plan fees (asymmetrical fees), (Starks, 1987). Fulcrum fees are calculated simply as the difference between the fund and benchmark returns (outperformance), scaled by a participation rate. The payoff of a symmetric fee payoff is given as

$$TC = [b_0 + b_1(R_F - R_B)] \times AUM,$$

where TC is the total compensation payable to the manager, b_0 is the management fee, b_1 is the participation rate (a fraction of the full excess return that the manager gets), R_F is the fund return, R_B is the benchmark return and AUM is assets under management.

If the return of the benchmark is more than that of the fund, the amount is deducted from the management fee, therefore this can result in a lower management fee or a negative fee. We can avoid this scenario by specifying a floor, i.e., an amount A (the maximum amount they can lose, which can be negative) to the payoff in above to get

$$TC = [b_0 + b_1 \max(R_F - R_B, A)] \times AUM.$$

The Equation above looks more like an asymmetric (bonus) fee structure, except that the manager's loss is capped and the cap (A) can be a number less than zero. For examples if the difference between the fund and benchmark return is -6% and A is -2% the manager only loses -2%.

A bonus plan fee is calculated as the maximum between outperformance and zero, also scaled by a participation rate. Therefore in the case of a bonus plan fee, managers hold a free call option and are not penalised directly for underperforming. The payoff of an asymmetric fee payoff is given as

$$TC = [b_0 + b_1 \max(R_F - R_B, 0)] \times AUM.$$

In South Africa, a modified version of the bonus plan is generally used (Treasury, 2013). Fund managers get rewarded for outperforming a benchmark and in the case of underperformance, they should exceed previous trading period's losses before being able to charge performance fees. This is referred to as setting a high-water mark. These high-water marks are imposed to prevent fund managers from taking excessive risk at the expense of the investor. Incentive fees can encourage tournament behaviour, which is adjusting the risk of the portfolio depending on whether the fund manager has outperformed or underperformed a benchmark to maximise performance fees (Treasury, 2013). This might not be a big issue since fund managers are also concerned about their reputation (Chevalier and Ellison, 1998) which means there is a downside to taking excessive risk and in addition, fee structures that penalise managers from deviating from a pre-set risk level can be put in place.

1.2 Definitions

The different terms that we shall use in this dissertation are briefly explained below.

1. Ex-ante: It refers to, before the event. Ex-ante results are forecasts rather than actual results. Therefore in this research ex-ante performance fees are forecast based on historical data.

2. Relative Benchmark: This is the pre-set target that is used to measure outperformance. A good benchmark should show the most important risk and return drivers of the underlying fund, and it should be a tradable asset like an index such as the JSE All Share Index (Treasury, 2013).

3. Hurdle Rate: This is the return in excess of outperformance that needs to be achieved before performance fees accrue. Therefore a manager should beat a particular relative benchmark by a certain percentage, where the percentage above the benchmark reflects the hurdle rate. Most funds in our sample have a hurdle rate and we treat it as a strike rate K in our calculations.

4. High-Water Mark: A high-water mark is the highest value a fund reached in the previous periods. Funds with high-water marks should recover all benchmark-relative underperformance losses before they start charging performance fees (Whitelaw and O'Donnell, 2011). Hence in this dissertation, our fee structures have high-water mark constraints.

5. Participation Rate: This is the portion of excess return that managers get as performance fees. Therefore managers do not get the entire excess return as performance fees, but just a fraction of the excess return.

6. Crystallization Period: This is the interval in which performance fees is paid. We use a one year crystallization period because this is minimum interval in which genuine outperformance can be measured.

7. Capped Fee: This is the maximum fee that managers can charge. Therefore fee structures with caps prevent fund managers from charging performance fees above this pre-set cap.

8. Performance Fee Structure: Refers to the way in which expected performance fees are calculated, i.e., the fund's relative benchmark, its' participation rate, whether it has any high-water mark constraints or any caps and whether it has a hurdle rate.

9. NAV: The net asset value (NAV) per share is the traded price of the fund.

10. AUM: Refers to assets under management. AUM is equal to the NAV of the fund at time zero.

11. Monte Carlo simulation: It involves simulating random paths of a stochastic process used to describe the evolution of the underlying asset (i.e., randomly sampling changes in market variables). This kind of pricing is possible since derivatives can be valued by computing the expectation of the payoff as an integral (Glasserman, 2013).

1.3 Performance Fee Estimation using Spread Options

Past research papers by Drago, Lazzari and Navone (2008) and Pohjanpalo (2013) have used spread options to come up with an ex-ante estimate for performance fees. We discuss the contribution of these research papers in Chapter 3.

We shall use spread options in this dissertation because we are estimating the fee as an option on the difference between two underlying asset portfolios (i.e., the fund portfolio and the benchmark portfolio) at some time t, which is the definition of a spread option. The replicating portfolio technique of derivative pricing (Hull, 2006) does not work in this context since replication is not possible given that the underlying cannot be hedged. Therefore using option pricing is just a means of approximation, and it is one we accept because it is a well-developed method (consider the large literature following Black and Scholes (1973)) of pricing obligations of the kind we are interested in (the maximum of zero and the difference between fund and benchmark portfolios). We shall use spread options to estimate the value of performance fees under the risk-neutral world where the underlying fund and benchmark growth at a risk-neutral rate r and we also consider the value of performance under the real world measure, where the underlying fund and benchmark grow at a drift rate μ. We estimate the drift rate μ using a formula suggested in a research paper by Brewer, Feng and Kwan (2012).

Since spread options have a positive Vega, an increase in volatility increases the value of performance fees.

Spread Option Formula Parameters

There are a number of parameters that are useful in the spread option formula and these parameters also affect the value of the option or the value of expected performance fees.

1. Correlation: The extent to which the fund and benchmark returns fluctuate together. A positive low correlation between the fund and benchmark gives rise to a high expected performance fee value and a high positive correlation in-turn gives rise to a low expected performance fee value.

2. Historical Volatility ("volatility"): The volatility input into an option pricing model is a measure of expected fluctuations of the underlying over a given period. It is estimated as the standard deviation of fund and benchmark returns respectively. High volatilities give rise to high expected performance fees.

3. Spread Volatility: This is the standard deviation of the difference between fund and benchmark prices, (i.e., on the monthly fund and benchmark price differences over 5 years for example). It measures how closely a fund portfolio mimics its relative benchmark portfolio.

4. Risk-free Rate: The growth rate of assets under the risk-neutral measure $\mathbb{Q}$. We use The Johannesburg Interbank Average Rate ("jibar") as a proxy for the risk-free rate (jibar is a short-term average interest rate, which banks use in the interbank market to buy and sell their Negotiable Certificate of Deposits). Jibar can be used as a short term risk-free rate in South Africa (Oosthuizen and Van Rooyen, 2013). Therefore, since our option term is one year (i.e., short dated), we used jibar as a proxy for the risk-free rate.

5. Drift rate: The growth rate μ of assets under the real world measure $\mathbb{P}$. We shall estimate μ using a formula proposed in a research paper by Brewer, Feng and Kwan (2012).

6. Strike Rate: This is the hurdle rate, i.e., the return in excess of outperformance that needs to be achieved before performance fees accrue. We shall denote the strike rate by K.

1.4 Dissertation Structure

This dissertation is structured as follows: Chapter 2 outlines the objectives of this dissertation and the research question we want to answer. In the same chapter, we briefly discuss the method used to estimate performance fees and the assumptions we make.

In Chapter 3 we look at past research papers that have used spread options to calculate ex-ante performance fees. Chapter 3 also outlines different performance fee structures we shall consider in this dissertation in more detail. In this Chapter we shall provide information on how we create a performance fee model and the assumptions we make.

Chapter 4 illustrates how expected performance fees are evaluated or calculated. In this chapter we provide the different methods we shall use to estimate the numerical value of expected performance fees. We shall utilise; 1. the price of an exchange option for cases where the strike rate is equal to zero 2. Monte Carlo simulation for cases where the strike rate is greater than zero and 3. The spread option approximation formula proposed in a book by Haug (2007) and in a research paper by Alexander and Venkatramanan (2007) for cases where the strike rate is greater than zero. We shall look into the usefulness of the approximation formula, as its ease of calculation compared to Monte Carlo simulation, might be useful to interested parties like investment managers and potential clients. Lastly, we show how spread option parameters are estimated.

In Chapter 5 we analyse in detail the expected performance fees results we obtain under different performance fee structures.

Chapter 6 concludes the dissertation. We discuss the results obtained.

Chapter 2

Research Objectives

2.1 Objectives

In 2013, National Treasury released a discussion paper on charges in South African retirement funds (Treasury, 2013). In this discussion paper the question of performance fees and their appropriateness arose. The aim of this dissertation is to investigate the various performance fee mechanisms generally used in the South African investment management industry and to determine whether spread option pricing theory can be used to determine on an ex-ante basis the estimated value of the performance fee structures. If these valuation techniques are found to be appropriate, they could be useful in evaluating performance fee structures in the investment industry. A key output of the project is the determination of the ex-ante value of performance fee structures under the various historical market scenarios.

2.1.1 Research Questions

The research objective above can be summarised into four key questions, which are listed below;

1. What is the value of expected performance fees, when a plain fee structure is used. A plain performance fee structure is where expected performance fees are estimated simply as the difference between fund and benchmark returns. We would expect this fee structure to give the highest expected performance fee out of the four performance fee structures.

2. What is the value of expected performance fees when a fee structure with a strike rate K (hurdle rate) is used. Expected performance fees are estimated as the difference between fund and benchmark returns, minus a strike rate K. Expected performance in this instance will therefore be lower than in 1. above.

3. What is the value of expected performance fees when a fee structure with high-water mark constraints is used. Expected performance fees are estimated as

the difference between fund and benchmark returns minus a hurdle rate and a high-water mark constraint. We expect these high-water constraints to lower expected performance fees since managers have to recoup any trading period losses they previously incurred, before they get the incentive fee.

4. Lastly, we shall consider the value of expected performance fees with a capped fee structure. We expect the distribution of performance fees at T to be bimodal because performance fees will be equal to zero or the cap, most of the time.

We shall provide detailed information on performance fee structures in Chapter 3.2 and give equations for each performance fee structure. Expected performance fees shall be estimated in two ways:

- When the underlying price grows at the risk-neutral rate r.
- When the underlying price grows at the drift rate μ.

We do this to determine how expected performance fee estimates generated by the two methods compare to the actual performance fees paid. We expect incentive fees generated using an underlying the grows at μ, to have expected performance fee estimates that are more comparable to the actual fees paid.

Chapter 3

Performance Fee Structures

3.1 Subject Review

Past research on ex-ante evaluation of performance fees has been done using spread options. A spread option can be defined as an option written on the difference of the two underlying assets, whose values at time t we denote by S_t^1 and S_t^2 (Hurd and Zhou, 2010). Spread options are normally used in markets where traders wish to isolate basis risk (Carmona and Durrleman, 2003). Spread options are useful in our research because their payoff is similar to that used in calculating performance fees.

We looked at a number of past research papers, focusing on the problem, method used and the result obtained in each paper in order to understand how performance fees have been modelled thus far. Below we discuss the contribution of each paper in detail and the fee structure used.

Drago, Lazzari and Navone (2008) use mutual funds and benchmark returns data obtained from Datastream for their analysis. They evaluated the ex-ante cost of performance fees using spread options adapting the payoff to a huge variety of fee structures. When the strike price in the spread option is set to zero, it becomes an *exchange option* which has a closed form solution, i.e., the Margrabe formula (Margrabe, 1978). Their value then reduces to a function of the tracking error (which is the volatility of the difference between the fund and the benchmark returns data), since the strike price is no longer an input because it's zero. Drago, Lazzari and Navone (2008) evaluate the ex-ante cost of different compensation schemes as the premium of a spread option on the active return of the fund. In the Italian market, investment companies can adopt either a fulcrum (symmetric) or a bonus (asymmetric) fee structure. They investigate the rationale of bonus incentive fee structures in the Italian market, which has the following payoff

$$\text{Payoff} = \text{Max}(S_T^1 - S_T^2 - K, 0), \tag{3.1}$$

where S_T^1 and S_T^2 are the fund and benchmark prices respectively.

They used the above formula to evaluate performance fees and found evidence that this fee structure (which comprises of the participation rate, strike price and benchmark) gives rise to high investment fees which are difficult to forecast without a proper technique. The results obtained suggest that this ex-ante value is sensitive to market conditions. They discover that there is lack of transparency on the cost of the incentive fee since the investors are giving a free call option (on the difference between the benchmark and fund return) to the fund manager without receiving a premium for it. This paper suggests the use of the ex-ante performance fee estimates as part of the information given to investors and such forecasted fees should be part of a fund's performance evaluation.

The rationale behind why and how performance fees are charged is investigated by Drago, Lazzari and Navone (2010). They use a logistic regression model to determine significant factors that give rise to a firm charging performance fees. The presence of performance fees is modelled as the dependent variable, which takes the value one if the fund charges performance fees, and zero otherwise. Various explanatory variables are used, for example, the size of the investment firm or whether the fund is a hybrid fund.

Drago, Lazzari and Navone (2010) investigate the possibility of fund managers taking excessive risks to increase returns, since the asymmetric fee structure used in Italy gives an option-like payoff. Their results show that this is false, because such positions can result in great losses which are not good for the fund's reputation. This can lead to the fund losing clients and thereby causing a reduction in assets under management. They also discover that performance fees are not used by good managers as a signal to separate themselves from bad managers.

Pohjanpalo (2013) looks at the structure of performance fees in Finnish mutual funds. He exposes the impact of performance fees on the portfolio risk-return profile and on obtaining a theoretical value for performance fees. It highlights different approaches of performance fee estimation and different regulatory approaches in selected European countries. There is evidence to suggest that funds that charge performance fees have a better risk-return profile than funds which charge only management fees. His findings show that funds that charge performance fees are as risky as funds that do not charge performance fees. Funds that charge performance fees have a higher tracking error than funds that do not charge performance fees, which shows that such funds take more active risk. Their results suggest that funds that charge performance fees have a lower management fee, at the same time extra costs associated with the incentive fees makes these funds more expensive on an annual basis. The theoretical ex-ante performance average estimate was 1.35% per annum of assets under management at the beginning of a calculation period. This estimate they obtained was highly sensitive to key parameters such as volatility and correlation.

This dissertation seeks to calculate an ex-ante performance fee value for South African mutual funds. The next section looks at different types of performance fee structures we consider.

3.2 Performance Fee Structures

A performance fee structure is made up of the following elements:

- The relative benchmark of the fund in question.
- A participation rate.
- High-Water mark constraints, if the fee structure has any high-water constraints.
- A hurdle rate, if part of the fund's fee structure.
- A performance fee cap, if the fee structure has a cap.

We shall also assume the following in our calculations:

1. The payment period equals the crystallization period and performance fees are paid at the end of the period.
2. The crystallization period is a year.
3. There are no purchases or sales of shares before the end of the crystallization period. All subscriptions and redemptions are done at the beginning or end of the crystallization period.
4. We shall do all calculations using NAV per share, i.e., as if there was one investor in the fund. This is done to simplify expected performance fee calculations.

3.2.1 Plain Vanilla Payoff

The total compensation payable to a fund manager charging both management and performance fees is

$$TC = [b_0 + b_1 \max(R_F - R_B, 0)] \times AUM, \tag{3.2}$$

where b_0 is the management fee, b_1 is the participation rate (a fraction of the full excess return that the manager gets), R_F is the fund return, R_B is the benchmark return and AUM is assets under management.

Since we do not have a strike rate K, our payoff becomes the payoff of an exchange option, with a long position in the fund and a short position in the benchmark.

3.2.2 Payoff with a Hurdle Rate

The total compensation payable to a manager with a performance fee structure that has a hurdle rate is

$$TC = [b_0 + b_1 \max(R_F - R_B - K, 0)] \times AUM, \tag{3.3}$$

where K is the hurdle rate.

K represents the minimum excess return above the relative benchmark that needs to be achieved before performance fees accrue.

3.2.3 Payoff with a High-Water Mark

An additional constraint to the payoff with a hurdle rate above, is a high-water mark

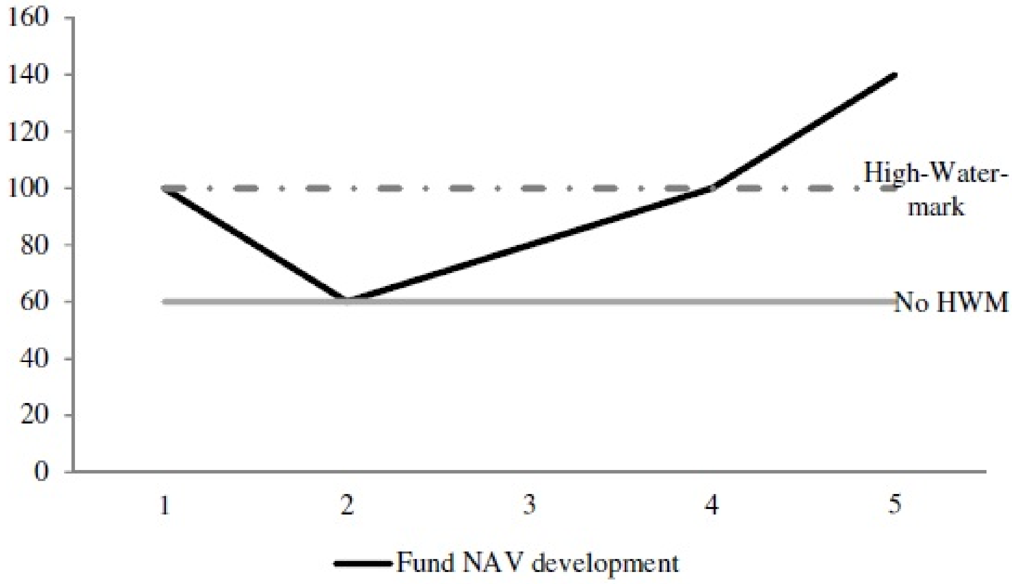
$$TC = \left[b_0 + b_1 \mathbb{I}_{\{NAV_{\text{Fund}} > NAV_{\text{HWM}}\}} \max(R_F - R_B - K, 0)\right] \times AUM, \tag{3.4}$$

where $\mathbb{I}$ is an indicator function that takes 1 if $NAV_{\text{Fund}} > NAV_{\text{HWM}}$ at time T and takes the value 0 otherwise.

Therefore managers only get a performance fee if the value of the fund at T is above the maximum historical value of the fund over a particular measurement period. Managers must therefore recoup any trading period losses before performance fees start accruing. Example, for a one year performance fee, the historical maximum will be observed over a one year period. Figure 3.1 illustrates the high-water mark scenario as in a paper by Pohjanpalo (2013).

We observe from Figure 3.1 that fund managers can only charge performance fees in the fourth quarter when the NAV per share of the fund is above the high-water mark.

Fig. 3.1: High-Water Mark: The vertical axis is the NAV of the fund and the horizontal axis represents time in years.



3.2.4 Payoff with Performance Fee Caps

In this section we consider a capped performance fee payoff. If the excess return scaled by a participation rate is above the cap, performance fees will be equal to the cap. The total compensation payable to a fund manager with a capped performance fee structure is

$$TC = [b_0 + b_1 \min(\max(R_F - R_B - K, 0), \mathbf{c})] \times AUM, \tag{3.5}$$

where $\mathbf{c}$ is the cap.

3.3 Modelling the Fee Structure

Spread options are suitable for modelling fee structures in Section 3.2 because the payoffs are basically evaluating the positive part of the spread between the fund and benchmark portfolio which is the definition of a spread option (Carmona and Durrleman, 2003). Since the payoffs in Section 3.2 can be written in terms of fund and benchmark prices we have

$$\begin{aligned} \text{Payoff} &= \max(R_F - R_B - K, 0) \\ &= \max\left(\frac{S_T^F - S_0}{S_0} - \frac{S_T^B - S_0}{S_0} - K, 0\right) \\ &= \frac{1}{S_0}\max(S_T^F - S_T^B - KS_0, 0), \end{aligned} \tag{3.6}$$

where S_T^F and S_T^B are the fund and benchmark prices at T (i.e., the NAV per share of the fund and benchmark). S_0 is the initial price of the fund and benchmark observed at time 0. This initial price is the same for both the fund and benchmark since we want to capture relative performance.

The returns R are over the period $[0, T]$. We model the fund and benchmark prices using geometric Brownian motion (GBM), described below.

3.3.1 Model Assumptions

When pricing with the Margrabe Formula, Monte Carlo simulation and the spread option approximation formula we consider the Black-Scholes-Merton framework and the usual conditions for Equivalent Martingale Pricing Theory (Zhang, 1997). The evolution of the fund and benchmark prices under the real-world measure $\mathbb{P}$ may be described by the following stochastic differential equations (SDEs):

$$\begin{aligned} dS_t^B &= \mu_B S_t^B dt + \sigma_B S_t^B dW_t^B \\ dS_t^F &= \mu_F S_t^F dt + \sigma_F S_t^F \left[\rho dW_t^B + \sqrt{1-\rho^2} dW_t^F \right], \end{aligned} \tag{3.7}$$

where μ_F and μ_B are the drifts, i.e., return of the fund and benchmark respectively, W_t^F and W_t^B are independent Brownian motions and ρ is the correlation between the fund and benchmark.

The drift and volatility parameters are assumed to be constant.

Using Girsanov's theorem, the price processes of the underlying assets satisfy the following SDEs under $\mathbb{Q}$, the risk-neutral measure:

$$\begin{aligned} dS_t^B &= r S_t^B dt + \sigma_B S_t^B d\tilde{W}_t^B \\ dS_t^F &= r S_t^F dt + \sigma_F S_t^F \left[\rho d\tilde{W}_t^B + \sqrt{1-\rho^2} d\tilde{W}_t^F \right], \end{aligned} \tag{3.8}$$

where $\tilde{W}_t^F$ and $\tilde{W}_t^B$ are Brownian motions under $\mathbb{Q}$ and r is the risk free rate.

We use jibar taken at a specific point in time as r (i.e., as a proxy of the risk-free rate).

Chapter 4

Evaluation of Performance Fees

We use the SDEs in the previous chapter to get formulas for generating performance fee values. If the strike price is greater than zero then a closed form solution does not exist, therefore Monte Carlo simulation and the spread option approximation formula is used to value the option. When the strike price is equal to zero, the spread option becomes an exchange option, which has a closed form solution (Margrabe, 1978). When we incorporate high-water marks, closed form solutions are not possible to obtain (Drago, Lazzari and Navone, 2008) therefore numerical method techniques like Monte Carlo simulation can be utilised.

4.1 Closed Form Solutions $K = 0$

Performance fee structures in Chapter 3 can be viewed as a long position in the fund portfolio and a short position in the benchmark portfolio. We utilise Equation 3.6 to estimate performance fees. Since we want to capture relative performance, the price of the benchmark and fund at the beginning of the period are set to be equal, $S_0^F = S_0^B = S_0$. The strike price $K = 0$ therefore, we utilise the Margrabe Formula (Margrabe, 1978). The payoff of the option follows from Equation 3.6 and is given by

$$\frac{1}{S_0}\max(S_T^F - S_T^B, 0). \tag{4.1}$$

Therefore the price of the option at time zero $V(S_0^F, S_0^B, \sigma)$ is given by

$$V(S_0^F, S_0^B, \sigma) = \frac{1}{S_0}\left[S_0^F N(d_1) - S_0^B N(d_2)\right] \tag{4.2}$$

where

$$\begin{aligned} d_1 &= \frac{\ln\left(\frac{S_0^F}{S_0^B}\right) + (\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \\ d_2 &= d_1 - \sigma\sqrt{T} \end{aligned} \tag{4.3}$$

$$\sigma = \sqrt{\sigma_F^2 + \sigma_B^2 - 2\rho\sigma_F\sigma_B}$$

Since we use gross returns data which includes dividends, we do not adjust for dividends in our calculations. Since $S_0^F = S_0^B = S_0$, the log part of Equation 4.3 becomes zero and the equation reduces to

$$d_1 = \frac{\frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}, \tag{4.4}$$

and

$$d_2 = d_1 - \sigma\sqrt{T},$$

therefore

$$V(\sigma) = N(d_1) - N(d_2). \tag{4.5}$$

We calculate an estimate for expected performance fees using the above formula $V(\sigma)$. The next sections show how expected performance fee values are calculated, in cases where the fee structure has a strike rate greater than zero $K > 0$.

4.2 The Spread Option Approximation Formula when $K > 0$

We consider the spread option approximation formula proposed by Kirk (1996) and presented in a paper by Alexander and Venkatramanan (2007). Spread option approximations are a better and simpler way of estimating fees than Monte Carlo simulation. We shall test whether the formula works well with our dataset. Since we want to capture relative performance, the price of the benchmark and fund at the beginning of the period are set to be equal, $S_0^F = S_0^B = S$. The payoff of the option at time T is given by

$$\begin{aligned} &\max(S_T^F - S_T^B - K, 0) \\ &= \max\left(\frac{S_T^F}{S_T^B + K} - 1, 0\right)\left(S_T^B + K\right). \end{aligned} \tag{4.6}$$

The approximate value of the option at time zero $V(S_0^F, S_0^B, \sigma, K)$ is given by

$$V(S_0^F, S_0^B, \sigma, K) \approx \left(S_0^B + Ke^{-rT}\right)\left[S^* N(d_1) - e^{(-r+\tilde{r})T} N(d_2)\right] \tag{4.7}$$

where

$$\begin{aligned} d_1 &= \frac{\ln(S^*) + \left(r - \tilde{r} + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}, \\ d_2 &= d_1 - \sigma\sqrt{T}, \\ S^* &= \frac{S_0^F}{S_0^B + Ke^{-rT}}, \end{aligned} \tag{4.8}$$

where volatility σ is then approximated using the following equation

$$\sigma \approx \sqrt{\sigma_F^2 + (\sigma_B F^*)^2 - 2 \times \rho \times \sigma_F \times \sigma_B \times F^*} \tag{4.9}$$

where

$$F^* = \frac{S_0^B}{S_0^B + Ke^{-rT}}$$
$$\tilde{r} = rF^*.$$

K is the strike price, σ_F and σ_B are the volatilities of the fund and the benchmark respectively, ρ is the correlation between the fund and the benchmark, T is the time to expiration and σ is the spread volatility.

4.3 Monte Carlo Simulation $K > 0$

In addition to the approximation formula in Section 4.2, Monte Carlo simulation methods can be used to price a spread option with strike rate greater than zero ($K > 0$). Monte Carlo simulation methods are known for their slowness but variance reduction techniques can be used to mitigate this problem and to improve the accuracy of Monte Carlo simulation estimates.

We utilise the equations below, as given by Glasserman (2013) to simulate price paths for both the fund and benchmark portfolios:

$$S_{t_i}^F = S_{t_{i-1}}^F e^{(r-\frac{1}{2}\sigma_F^2)T+\sigma_F\sqrt{T}X^F} \tag{4.10}$$

$$S_{t_i}^B = S_{t_{i-1}}^B e^{(r-\frac{1}{2}\sigma_B^2)T+\sigma_B\sqrt{T}X^B}, \tag{4.11}$$

where S_t^F and S_t^B are the prices of the fund and benchmark portfolios respectively, r is the risk-free rate, σ_F and σ_B are the volatilities of the fund and benchmark portfolios respectively, X^F and X^B are standard normal random numbers, correlated with the Cholesky decomposition and T is the length of time between time nodes, which can be 1/12 for monthly performance fees and 1 for annual performance fees.

Using Martingale pricing theory, the spread option price (performance fee) is given by the expected discounted payoff:

$$\begin{aligned} &V(S_0^F, S_0^B, r, \sigma_F, \sigma_B, T, K) \\ &= e^{-rT}\mathbb{E}^{\mathbb{Q}}\left[H(T)|F_t\right] \\ &= e^{-rT}\mathbb{E}^{\mathbb{Q}}\left[\left(S_T^F - S_T^B - K\right)^+\right] \\ &= e^{-rT}\mathbb{E}^{\mathbb{Q}}\left[\left(S_0 e^{(r-\frac{1}{2}\sigma_F^2)T+\sigma_F\sqrt{T}X^F} - S_0 e^{(r-\frac{1}{2}\sigma_B^2)T+\sigma_B\sqrt{T}X^B} - K\right)^+\right]. \end{aligned} \tag{4.12}$$

where $\mathbb{E}^{\mathbb{Q}}$ is the expectation under the risk-neutral world $\mathbb{Q}$, F_t is the filtration, i.e., the history of the fund and benchmark prices up until time t and $H(T)$ is the spread option payoff.

4.3.1 Law of Large Numbers

For completeness we shall provide some results from probability theory. The law of large numbers (LLNs) shows the results of doing the same experiment a number times. Therefore the sample average converges in the limit to the expected value (Robert, 2004)

$$\lim_{n\to\infty} \frac{1}{n}\sum_{j=0}^{n} X_j = \mathbb{E}(X).$$

It follows therefore

$$e^{-rT}\mathbb{E}^{\mathbb{Q}}\left[H(T)|F_t\right] = e^{-rT}\lim_{n\to\infty}\left(\frac{1}{n}\sum_{j=0}^{n} H_j(T)\right).$$

We can estimate the expectation fairly accurately by the sample mean, if we insure that n is big enough. We shall simulate a large number of fund and benchmark price paths in order to calculate the spread option payoff and then take the sample mean of the payoff to estimate the expectation in Equation 4.12. Lastly we then discount this expected payoff to get the option price (expected performance fees).

4.4 Estimation of Parameters

Our dataset is made up of monthly gross fund and benchmark returns from 31 December 1999 to 31 October 2014. Our sample contains funds which use numbers as names, for example, fund 102, fund 106 or fund 1102. The benchmarks in our sample include the All Share Index (ALSI), All Bond Index (ALBI), Shareholder Weighted Index (SWIX), Short-term Fixed Interest Index (SteFi) and a combination of indices.

The parameters our model uses as mentioned in Chapter 1 include, fund and benchmark volatility, correlation between the fund and the benchmark, and the spread volatility.

4.4.1 Volatility

Fund and Benchmark Volatility

The volatility of the fund and benchmark are estimated simply as the standard deviation of the fund and benchmark returns respectively. Since we are concerned with a one year expected performance fee, we have to convert monthly returns in our data sample to annual returns:

$$R_A = (1+R_1)(1+R_2)(1+R_3)(1+R_4)......(1+R_{12}) - 1.$$

where R_A, is the annual return and $R_1 ... R_{12}$, are the monthly returns, from the first month to the twelfth month respectively.

We then use these annual returns to calculate the annual volatilities. Firstly, the sample period spanning from 31 December 1999-31 December 2000 is used as the in-sample period to calculate the standard deviation of the fund and benchmark. Then we use different in-sample periods on a monthly and yearly rolling window basis.

Spread Volatility

The spread volatility measures how closely the fund portfolio mimics its benchmark. If a fund manager deviates more from the benchmark, i.e., takes more active risk, spread volatility widens. Pohjanpalo (2013) finds evidence that funds which offer performance fees are more inclined to take more active risk when the fund is underperforming to get back in-the-money, i.e., start outperforming again. We calculate the spread volatility as the standard deviation of the difference in fund and benchmark annualised returns:

$$\sqrt{\frac{1}{n-1}\sum_{t=1}^{n}(x_t - \bar{x})^2}. \tag{4.13}$$

where n is the sample size , i.e., the length of the fund and benchmark returns vector, x_t represents the active return, which is the difference between the fund and benchmark annualised returns and $\bar{x}$ is the mean of the active return.

4.4.2 Correlation

Correlation measures the extent to which the fund and benchmark portfolio fluctuate together. We will observe in our computation of performance fees that a positive low correlation between the fund and benchmark gives rise to high performance fees. A positive low correlation can be as a result of managers taking more active risk, i.e., deviating more from the benchmark in anticipation of higher returns. We use the standard correlation formula (Cohen, Cohen, West and Aiken, 2013) to estimate the correlation between the fund and benchmark returns.

4.4.3 Drift Rate μ

In this Subsection we consider the procedure we are going to use to estimate the drift rate μ. The equation we utilize for generating price paths is

$$S_T = S_0 e^{\left(\mu - \frac{1}{2}\sigma^2\right)\Delta t + \sigma \Delta W}.$$

Then by taking logs and simplifying we obtain

$$\log\left(\frac{S_T}{S_0}\right) = \left(\mu - \frac{1}{2}\sigma^2\right)\Delta t + \sigma \Delta W. \tag{4.14}$$

Taking the expectation of Equation 4.14, we get the following

$$\mathbf{E}\left(\log\left(\frac{S_T}{S_0}\right)\right) = \mathbf{E}\left(\left(\mu - \frac{1}{2}\sigma^2\right)\Delta t\right) + \mathbf{E}\left(\sigma \Delta W\right).$$

The Brownian Motion increment has a mean of zero, therefore

$$\hat{\mu} = \frac{\log\left(\frac{S_T}{S_0}\right)}{\Delta t} + \frac{1}{2}\sigma^2.$$

where $\hat{\mu}$ is the drift rate estimate, i.e., the expected return (annualised) earned by an investor over a short period of time Δt, S_T is the stock price at terminal time T, S_0 is the stock price at the beginning of the period and σ is the standard deviation of the Fund and Benchmark returns, respectively.

4.5 Sensitivity Analysis

We shall introduce a short sensitivity analysis section. We follow the method in a paper by Pohjanpalo (2013), where the author calculated sensitivities with respect to volatility and correlation between the fund and benchmark portfolio. We shall also calculate the sensitivity of performance fees to changes in the strike price and risk metrics in this dissertation, which are not included in our reference research paper. This sensitivity analysis section will assist interested parties to have an idea on how expected performance fee estimates change as these parameters fluctuate.

4.5.1 Sensitivity with Respect to Strike

In this section we analyse how sensitive ex-ante performance fee values are to changes in the strike price K. We derive the sensitivity using a Monte Carlo simulation backward-difference approximation (Glasserman, 2013).

The purpose of carrying out such analysis is to help potential clients to make better decisions on which fee structure to choose, from a pool of fee structures, (i.e., strike price, participation rate, benchmark and high-water marks).

Chapter 5

Ex-ante Cost of Performance Fees in South Africa

In this chapter, we consider the numerical values of expected performance fees under different fee structures. To recap the calculation procedure; we utilise the methods in Chapter 4, to estimate ex-ante performance fee values depending on the fee structure of the fund. Our sample consist of 76 mutual funds, with some funds having data over the whole time span and other funds with data over shorter time periods. The word *mean* in the tables that follow in this chapter, is just an average figure across a particular asset class. All funds in our sample use stock market, money market and bond market indices or a combination of indices as relative benchmarks.

We shall categorise our results based on the four performance fee structures we consider in this dissertation. Since we are estimating ex-ante performance fees, i.e., performance fees that will be paid at a future date, we do not know the actual performance fees paid at that future time. Therefore we only compare the actual historical performance fees paid with respective ex-ante performance fees in the appendices. We do this to see how well the spread option model has estimated the actual performance fees paid in the past. Figure 5.1 shows the average volatility of funds relative to their respective benchmarks for bonds (ALBI), equity (ALSI) and money market funds (SteFi).

Fig. 5.1: Average Volatility of Equity Funds Relative to ALSI Benchmark.

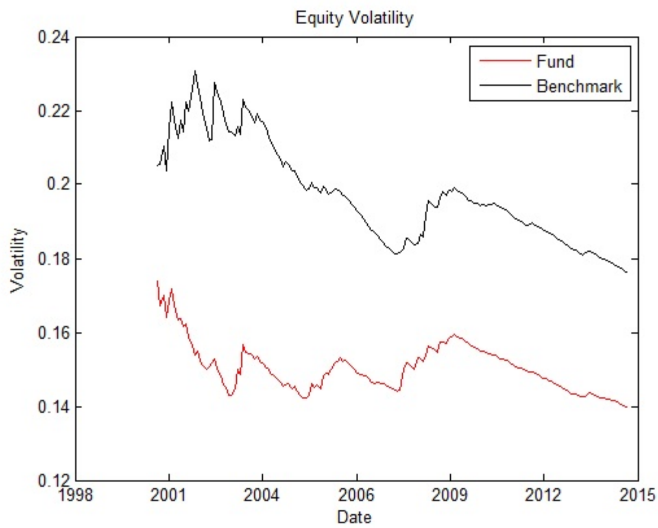


Fig. 5.2: Average Volatility of Fixed Income Funds Relative to ALBI Benchmark.

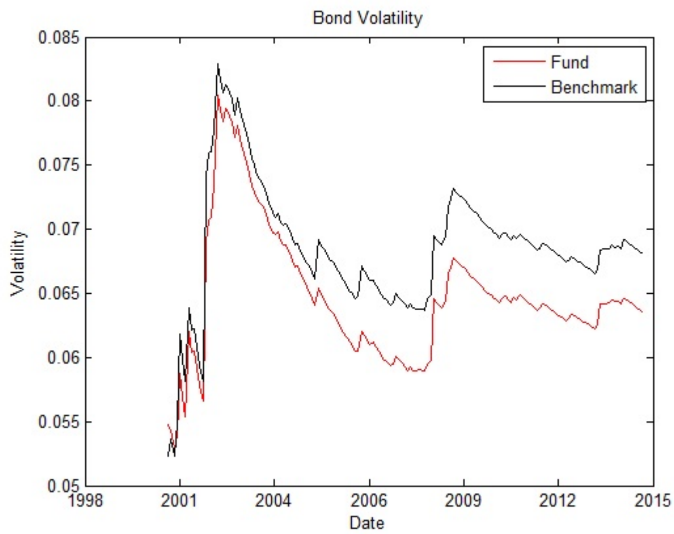
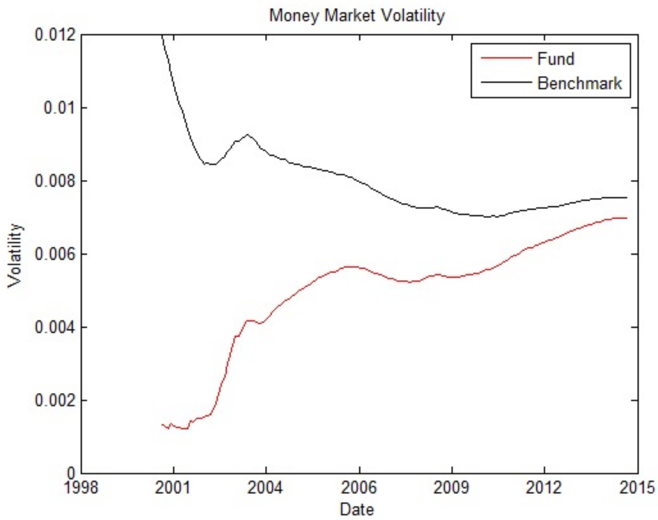


Fig. 5.3: Average Volatility of Money Market Funds Relative to SteFi Benchmark.



We can observe from Figure 5.1 that fund managers try to match the risk profile of their respective benchmarks and they always keep the volatility of the fund below that of the benchmark. Although fund portfolios and their respective benchmark portfolios are highly correlated, the weighting of holdings in the portfolios are different, which gives rise to the difference between the fund and benchmark portfolio volatilities we observe in Figure 5.1. We can observe that fixed income funds have a risk profile that closely matches that of its benchmark. Equity funds have the highest volatility as expected, with the fund portfolio being less volatile than its benchmark portfolio mainly because of the weightings of the holdings in the portfolio and how diversified it is. Money market funds exhibit the lowest volatilities of the three asset classes as expected, mainly because they are held over a short time horizon, i.e., less than a year. The huge difference between the fund and benchmark in the lower end of the money market graphs, may be as a result of a low sample size. We can observe that the graph become more stable as the number of years increase and in-turn as the sample size increases.

5.1 Numerical Value of Performance Fees

In the subsections that follow we look at the value of ex-ante performance fees calculated using spread options. We shall analyse ex-ante performance fees obtained using closed form solutions, the spread option approximation formula and using Monte Carlo simulation. Under Monte Carlo simulation, various variance reduction techniques such as Control Variates and Antithetic Monte Carlo simulation shall be utilised in order to improve the accuracy of expected performance fees generated using Monte Carlo simulation (Glasserman, 2013). We perform 100 000 Monte Carlo simulations to estimate expected performance fees in each of the sections that follow. In each section we shall consider performance fees calculated when the price process is driven by r, the risk-free rate and when its driven by a drift rate μ.

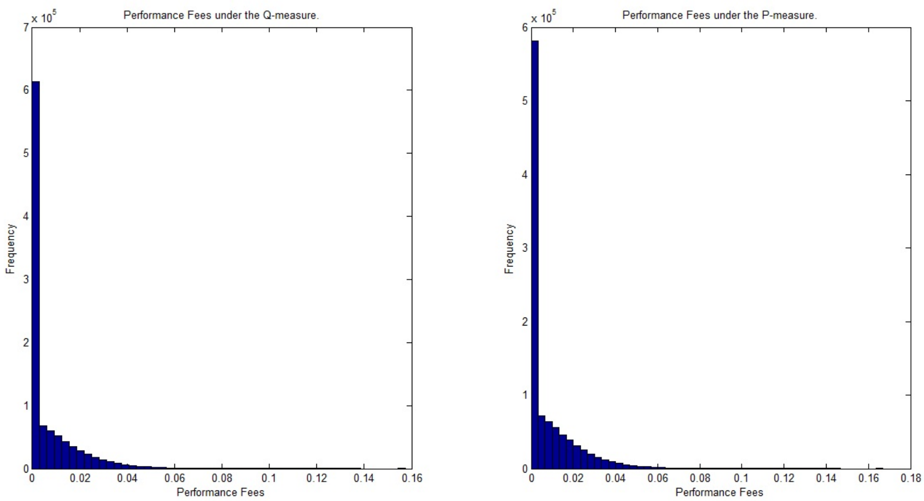
5.1.1 Distribution of Performance Fees at T

In this subsection we show the distribution of performance fees at T, i.e., performance fees generated by the payoff function.

We sampled from the returns data and randomly extracted a Fund and Benchmark return twelve times to generate monthly prices of the fund and benchmark respectively. From these monthly prices we then calculated performance fees and drew a histogram of the payoff.

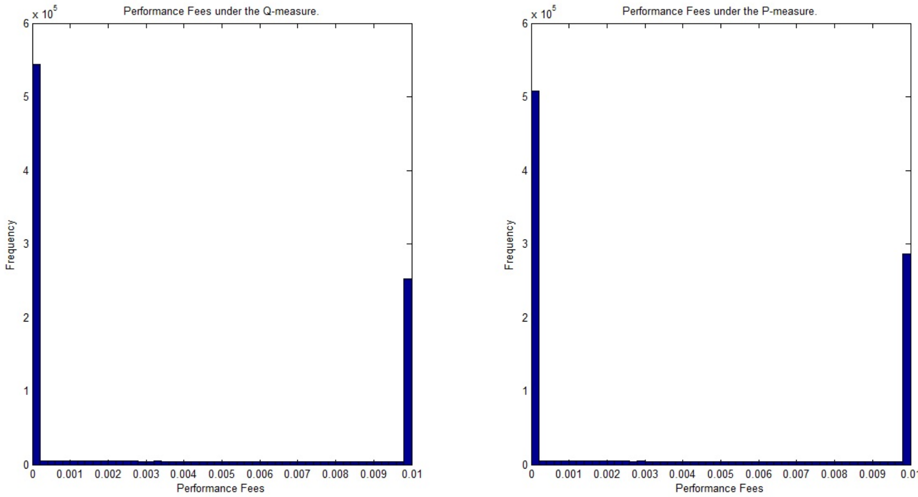
One of the reasons we do this, is to illustrate that an expected value we intend to estimate, might not accurately approximate the value of performance fees.

Fig. 5.4: Distribution of Performance Fees.



We can observe that most of the time performance fees are zero, since there is a lot of mass at zero. We can also observe from the graphs that the distribution of performance fees is positively skewed as expected because of the maximum function in our payoff. An expected value for performance fees will therefore lie somewhere in the distribution above, for a performance fee structure with a strike rate. In Figure 5.5, we show the distribution of performance fees for a performance fee payoff with a cap.

Fig. 5.5: Distribution of Capped Performance Fees.



The distribution of capped performance fees is bimodal, therefore most of the time performance fees are either zero or one. The expected performance fee value will likely be either zero or one, since there is a lot of mass at zero and at one.

5.1.2 Plain Vanilla Ex-ante Performance Fees

We first consider a simple payoff, where expected performance fees are calculated as the difference between the fund and benchmark portfolio. The payoff is that of an exchange option with a closed form solution, the Margrabe formula (Margrabe, 1978). Expected performance fees are calculated over a 1 year period, using Equation 4.5. Our dataset stretches from 31 December 1999 to 31 October 2014. We calculate the fee fourteen times. Parameters such as correlation and volatility are estimated using an in-sample period of the most recent observations of historical data. Table 5.1 shows the average cost of performances fees across different asset classes. We use the whole sample space to estimate parameters and we use an arbitrary participation rate of 15%. We calculate expected performance fees as at 31 October 2015. Since we are calculating an expected performance fee value to be paid in the future, i.e., at 31 October 2015, we cannot compare it to an actual performance fee value since the fee has not been paid as yet.

Tab. 5.1: Plain Vanilla Performance Fees in Percentage.

	Number-of-Funds	Mean [1]	Min [2]	Max
Equity	36	0.3358 %	0 %	0.8038 %
Fixed Income	22	0.0868 %	0 %	0.3733 %
Money Market	18	0.0177 %	0 %	0.0605 %

[1] *Mean performance fees are just an average figure across all equity, bonds and money market funds in our sample.*
[2] *The minimum performance fee is zero as expected. This is because our formula has a maximum function and takes zero if the difference between the fund and benchmark is negative.*

Tab. 5.2: Plain Vanilla Performance Fees in Basis Points.

	Number-of-Funds	Mean [1]	Min [2]	Max
Equity	36	33.58 Bps	0 Bps	80.38 Bps
Fixed Income	22	8.68 Bps	0 Bps	37.33 Bps
Money Market	18	1.77 Bps	0 Bps	6.05 Bps

As expected the mean expected performance fees for equity is higher than that of bonds and money market funds in Table 5.1 and Table 5.2. This is as a result of high volatilities in equity funds, which in turn produce high performance fee estimates.

Money market instruments are investments for less than 12 months, therefore they have low volatility and as a result we expect them to have low performance fee estimates as listed in the Table 5.1 and Table 5.2.

5.1.3 Ex-ante Performance Fees with a Hurdle Rate

In this subsection we consider what the ex-ante fees are when the fee structure contains a hurdle rate K. Like in the previous subsection, we use the whole sample size as the in-sample period, an arbitrary participation rate of 15%, an arbitrary strike rate of 1% and performance fees are calculated as at 31 October 2015. As expected these performance fee estimates are lower than those in Table 5.1, which have no hurdle rates. In the tables that follow, we shall consider what expected performance fees are when the price process is driven by risk-free rate r and when the price process is driven by μ the drift rate.

In Table 5.3 MC stands for Monte Carlo simulation and in this table we consider performance fees generated using the spread approximation formula, Monte Carlo simulation, Antithetic Monte Carlo simulation and Control Variates. We use these variance reduction techniques to improve Monte Carlo simulation estimates for expected performance fees.

Tab. 5.3: Performance Fees with Hurdle Rates. Prices are generated using a risk-free rate r.

	Approximation-Formula	MC	MC-(Antithetic)	MC-(Control Variate) [1]
Equity	0.2968 %	0.2758 %	0.2959 %	0.2960 %
Fixed Income	0.0470 %	0.0425 %	0.0456 %	0.0457 %
Money Market	0.0011 %	0.0009 %	0.00094 %	0.00095 %

[1] *We can observe that the variance reduction techniques have better performance fee estimates that are close to the approximation formula results. This shows that these variance reduction methods improve the accuracy of performance fee estimates in cases where expected performance fees can only be calculated using Monte Carlo simulation.*

Tab. 5.4: Performance Fees with Hurdle Rates. Prices are generated using a drift rate μ.

	MC [1]	MC-(Antithetic)
Equity	0.2555 %	0.2741 %
Fixed Income	0.0381 %	0.0409 %
Money Market	0.00092 %	0.00098 %

[1] *In cases where the price process is driven by a drift rate μ, we shall only use Monte Carlo simulation and Antithetic Monte Carlo simulation.*

In Table 5.4, we generate expected performance fees using prices that evolve or are generated using a drift rate μ. We expect performance fee estimates in Table 5.3, to relate more to the actual fees paid because they are being modelled under real world dynamics and risk preferences.

5.1.4 Ex-ante Performance Fees with High-Water Mark (HWM)

In this subsection we consider what the ex-ante fees are when the fee structure contains a high-water mark. When the period-end NAV is lower the HWM (i.e., the highest value the NAV has reached in the measurement period), the fund manager does not get a performance fee. Which means that fund managers must recover all losses incurred in a particular measurement period, before performance fees start accruing. Therefore if managers do not recover all their losses, ex-ante performance fees become zero. We expect performance fee structures with high-water marks, to have lower expected performance fees than those of the other two fee structures we have discussed earlier. We use the whole sample size as the in-sample period, an arbitrary participation rate of 15%, an arbitrary strike rate of 1% and performance fees are calculated as at 31 October 2015.

Tab. 5.5: Performance Fees with HWMs.

	Number-of-Funds	Mean [1]	Min	Max
Equity	36	0.2084 %	0 %	0.7215 %
Fixed Income	22	0.0396 %	0 %	0.2740 %
Money Market	18	0.001 %	0 %	0.00501 %

[1] *Mean performance fees are just an average figure across all equity, bonds and money market funds in our sample.*

As expected, performance fee estimates in Table 5.5 are the lowest of all the performance structures we have discussed so far. We are of the opinion that, such performance fee structures are fair since managers do not get compensated when they perform badly in a particular measurement period. Some research papers have looked at the rationale of using such fee structures (Ruckes and Sevostiyanova, 2011).

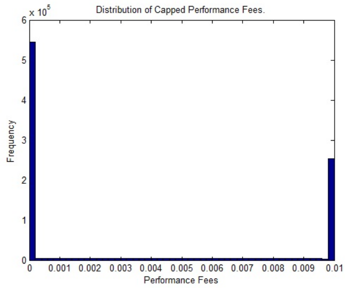
Lastly we consider the values of expected performance fees when the fee structure has a cap of 1%.

5.1.5 Ex-ante Performance Fees with a Capped Payoff

In this subsection we consider a performance fee structure that caps expected performance fees at 1%, i.e., the maximum performance fee that a manager can get. Such structures can prevent managers from taking unnecessary active risk, in pursuit of higher expected performance fees, since performance fees above 1% are reduced to a cap of 1%. The distribution of capped performance fees is bimodal as displayed in Figure 5.6, since performance fees will either be zero or one most of the times. The graph in Figure 5.6, is the payoff of a single equity fund, i.e., Specialist Domestic Equity Fund 102.

In Table 5.6, we show expected performance fee values that have a capped perfor-

Fig. 5.6: Distribution of Capped Performance Fees at time T.



mance fee structure. As expected, these expected performance fees are lower than those of the fee structures in Subsections 5.1.1 and 5.1.2. We use the whole sample size as the in-sample period, a participation rate of 15%, a strike rate of 1% and performance fees are calculated as at 31 October 2015.

Tab. 5.6: Performance Fees with Caps.

	Number-of-Funds	Mean	Min	Max
Equity	36	0.2782 %	0 %	0.6542 %
Fixed Income	22	0.0441 %	0 %	0.2944 %
Money Market	18	0.001 %	0 %	0.0621 %

Table 5.7, is a summary of all the performance fee structures we have considered in this section. In this table we display all the mean expected performance fees for all the four performance fee structures for comparison. These fees have price processes that are generated using the risk-free rate r. PF is the abbreviation for performance fees, thus *PF-PlainVanilla* is the performance fee structure in Subsection 5.1.1, *PF-HurdleRate* Subsection 5.1.2 and so on.

Tab. 5.7: Summary Table.

[1]	PF-PlainVanilla	PF-HurdleRate	PF-HWMs	PF-Caps
Equity	0.3358%	0.2968 %	0.2084 %	0.2782 %
Fixed Income	0.0868 %	0.0470 %	0.0396 %	0.0441 %
Money Market	0.0177 %	0.0011 %	0.001 %	0.001 %

[1] *We can observe from the Table the effect of performance fee structures on expected performance fee values.*

5.2 Sensitivities

We conduct sensitivity analysis in order to observe how expected performance fee estimates change as some key parameters (i.e., correlation, volatility and participation rate.) change. We shall use the same in-sample period from 31 December 1999 to 31 October 2014, an arbitrary small strike rate of 1% and a participation rate of 15%. Performance fees calculated are payable at a future date, 31 October 2015.

5.2.1 Performance Fees as Correlation and Volatility Change

We first consider how performance fees change as we vary the fund and benchmark portfolio correlation and volatilities. In Chapter 4.4 we estimated correlation and volatility but in this section we use arbitrary correlation and volatility values. Table 5.8 shows how ex-ante performance fees change as the correlation and volatility vary. In Table 5.8, we notice that ex-ante performance fees increase when the correlation between the fund and benchmark portfolio is low. A positive low correlation implies that managers are taking more active risk by deviating from the benchmark in pursuit of higher returns. This in-turn increases the volatility of the payoff.

In Table 5.8, funds that are highly correlated with their respective benchmarks and also having low volatilities have very low performance fees. Which is the case with our data sample, hence the low expected performance fees in Section 5.1.

Tab. 5.8: The Effect of Correlation and Volatility on Expected Performance Fees.

[1]	5%	10%	15%	18%	20%	22%	25%	30%
0.1 [2]	0.3741 %	0.9278 %	1.2332 %	1.4905 %	1.7808 %	1.8328%	2.0886 %	2.6289 %
0.2	0.3572 %	0.8724 %	1.1585 %	1.4012 %	1.6760 %	1.7241 %	1.9656 %	2.4766 %
0.3	0.3220 %	0.8136 %	1.0791 %	1.3062 %	1.5644 %	1.6085 %	1.8346 %	2.3141 %
0.4	0.2932 %	0.7507 %	0.9937 %	1.2040 %	1.4445 %	1.4841 %	1.6938 %	2.1392 %
0.5	0.2620 %	0.6826 %	0.9007 %	1.0928 %	1.3144 %	1.3486 %	1.5402 %	1.9489 %
0.6	0.2276 %	0.6078 %	0.7980 %	0.9699 %	1.1706 %	1.1988 %	1.3704 %	1.7380 %
0.7	0.1887 %	0.5237 %	0.6815 %	0.8303 %	1.0077 %	1.0287 %	1.1774 %	1.4986 %
0.8	0.1432 %	0.4261 %	0.5435 %	0.6649 %	0.8153 %	0.8269 %	0.9484 %	1.2146 %
0.9	0.0855 %	0.3045 %	0.3642 %	0.4497 %	0.5680 %	0.5640 %	0.6498 %	0.8458 %

[1] *On the top horizontal axis, 5%-30% represent fund volatilities.*

[2] *On the extreme left vertical axis, numbers from 0.1-0.9 represent correlation between the fund and benchmark.*

5.2.2 Dual Delta

We consider the sensitivity of expected performance fees to changes in the strike rate K, in this subsection. Table 5.9 uses the backward-difference formula to calculate the sensitivity of the spread option with respect to a 1% change in the strike price (i.e., the definition for dual delta). There is less chance of the spread option ending in the money as the strike increases, therefore our dual delta values should be negative. This means that the spread option values (ex-ante performance fees) decreases as the strike increases. Just as in previous subsections we categorize our results in terms of asset class.

Tab. 5.9: Dual Delta as K varies

	Equity	Money Market	Fixed Income
$K = 0.04$	-0.0461 %	0%	-0.003%
$K = 0.03$	-0.0531 %	0 %	-0.0058 %
$K = 0.02$	-0.0605 %	-0.0005 %	-0.0143%
$K = 0.01$	-0.0682 %	-0.0260 %	-0.0406 %

As the strike increases and the options are pulled further away from the money, there is a lower (negative) sensitivity to the strike price. The dual delta for the money market asset class is zero for high strike rates. This implies that as we increase the strike rate the fee estimate becomes less sensitive to this strike price as we mentioned above. Equity is more sensitive to movements in the strike rate than the other two asset classes, as expected.

5.2.3 Performance Fees as Participation Rate Changes

This section investigates how fee estimates change as we vary the participation rate, (i.e., the fraction of performance fees that the manager gets). We want to observe whether fee estimates in all of the three asset classes react the same to changes in the participation rate. We classified our results according to asset class as we did in other sections. We take the mean across fund in each asset class.

We can observe from Table 5.10 that performance fee estimates for Equity, Fixed Income (i.e., bonds) and the Money Market Funds are equally affected by changes in the participation rate. Therefore a 20% increase in the participation rate will also result in a 20% increase in the ex-ante performance fees.

Tab. 5.10: Performance Fee Estimates as the Participation Rate Changes.

[1]	Equity	Fixed Income	Money Market
0.1 [2]	0.1837%	0.0283%	0.0006%
0.15	0.2758%	0.0425%	0.0009%
0.2	0.3678%	0.0568%	0.0012%

[1] *The table has mean values for Equity, Money Market and Fixed Income funds.*
[2] *The numbers 0.1-0.2 represent different participation rates being used to calculate the value of performance fees.*

5.2.4 Performance Fees as Spread Volatility and Boundary Condition Change

In this Subsection we consider the value of performance fees as we alter the spread volatility between the Fund and Benchmark portfolio. We are also going to examine how performance fee structures react to changes in the spread volatility. We shall simulate ex-ante performance fees for every spread volatility level using the spread option approximation formula. A one year crystallisation period, a participation rate of 15% and benchmark volatility of 20%, shall be used in calculations.

Tab. 5.11: The Effect of Correlation and Volatility on Expected Performance Fees.

Spread Volatility	8.9048%	12.5919%	14.9245%	16.3198%	20.5715%	22.8531%	28.5846%
Fund Volatility	20%	20%	22%	22%	25%	25%	30%
Correlation	0.90	0.80	0.75	0.70	0.60	0.50	0.40
Plain Vanilla	0.5743%	0.8119%	0.9604%	1.0503%	1.3218%	1.4685%	1.8330%
Hurdle Rate	0.5072%	0.7440%	0.8938%	0.9834%	1.2562%	1.4024%	1.7688%
Cap at 1%	0.4543%	0.4864%	0.5199%	0.5629%	0.6580%	0.6941%	0.7938%

[1] *Plain Vanilla as mentioned in Subsection 5.1.1, refers to a payoff without a Hurdle rate, HWM and Cap.*
[2] *We use arbitrary fund volatilities and correlation coefficients.*

In the appendices we look at ex-ante performance fees for each fund in our dataset. We display the values of ex-ante performance fees when the price process is generated by a risk-free rate r and when the price process is generated by a drift rate μ. As a basis for comparison, we provide a historical table with the actual performance fees paid.

Chapter 6

Conclusion

The aim of this dissertation was to estimate performance fees some time before the fee is paid, in order to have an idea of the value of this fee under different performance fee structures used in the South African investment industry. Certain parties might also benefit from having a reasonable estimate of this value.

We calculated these ex-ante performance fee estimates using the Margrabe formula when the strike rate was equal to zero and we utilised the spread option approximation formula and Monte Carlo simulation when the strike rate was greater than zero. We utilised variance reduction techniques under Monte Carlo simulation. These variance reduction techniques increase the accuracy of ex-ante performance fee estimates.

Ex-ante performance fee estimates we obtained have a positively skewed distribution, mainly because of the maximum function on the payoff. These ex-ante fees are highly sensitive to the correlation between the fund and benchmark respectively and to the spread volatility. Which implies that a low positive correlation between the fund and benchmark gives rise to a high ex-ante performance fee and likewise high spread volatilities also give rise to high ex-ante performance fees.

We can observe from the tables in the appendix that during the 2007 to 2008 financial market crisis when markets tumbled down increasing the volatility of the holdings in the fund and benchmark portfolios (i.e., leverage effect). The value of ex-ante performance fees increased in subsequent years, since we are forecasting ex-ante performance fees using historical data.

Performance fee structures with high-water marks are more neutral to the investor since performance fees are only paid after the manager recovers any loss they incurred. In addition caps help to deter fund managers from taking excess risk (i.e., increasing the spread volatility in pursuit of higher performance fees), since the maximum performance fee they can earn is capped.

The sensitivity analysis results suggest that changes in key parameters like the strike price, correlation and participation rate, do not have the same effect on all asset classes, for example equity is more sensitive to movements in the strike than fixed income and money market instruments.

Our results also suggest that risky funds do not always have the highest performance fees. Therefore its only funds with high spread volatilities (volatility of the difference between the fund and the benchmark returns data), that also exhibit high performance fee estimates.

Ex-ante performance fees obtained are lower than the actual performance fees paid. This might mean that fund managers are over charging their clients or spread option valuation underestimates the value of performance fees. Lastly, we do not do any analysis on Hedge Funds because we do not have sufficient data in our sample.

Bibliography

Alexander, C. and Venkatramanan, A. (2007). Analytic approximations for spread options, *Available at SSRN 1012521* .

Black, F. and Scholes, M. (1973). The pricing of options and corporate liabilities, *The Journal of Political Economy* pp. 637–654.

Brewer, K. D., Feng, Y. and Kwan, C. C. (2012). Geometric Brownian motion, option pricing, and simulation: Some spreadsheet-based exercises in financial modeling, *Spreadsheets in Education (eJSiE)* **5**(3): 4.

Carmona, R. and Durrleman, V. (2003). Pricing and hedging spread options, *Siam Review* **45**(4): 627–685.

Chevalier, J. and Ellison, G. (1998). Career concerns of mutual fund managers, *Technical report*, National Bureau of Economic Research.

Cohen, J., Cohen, P., West, S. G. and Aiken, L. S. (2013). *Applied multiple regression/correlation analysis for the behavioral sciences*, Routledge.

Drago, D., Lazzari, V. and Navone, M. (2008). The real cost of asymmetric incentive fees, *EFMA 2005 annual meetings, Italy.*

Drago, D., Lazzari, V. and Navone, M. (2010). Mutual fund incentive fees: determinants and effects, *Financial management* **39**(1): 365–392.

Elton, E. J., Gruber, M. J. and Blake, C. R. (2003). Incentive fees and mutual funds, *The Journal of Finance* **58**(2): 779–804.

Glasserman, P. (2013). *Monte Carlo methods in financial engineering*, Vol. 53, Springer Science & Business Media.

Haug, E. G. (2007). *The complete guide to option pricing formulas*, McGraw-Hill Companies.

Hull, J. C. (2006). *Options, futures, and other derivatives*, Pearson Education India.

Hurd, T. R. and Zhou, Z. (2010). A Fourier transform method for spread option pricing, *SIAM Journal on Financial Mathematics* **1**(1): 142–157.

Kirk, E. (1996). Correlation in energy markets, *In: V. Kaminski (Ed.), Managing Energy Price Risk* pp. 71–78.

Margrabe, W. (1978). The value of an option to exchange one asset for another, *The Journal of Finance* **33**(1): 177–186.

Oosthuizen, A. and Van Rooyen, J. (2013). Risk-free assets: Are they truly risk-free? a comparative study of south african rates and instruments.

Pohjanpalo, T. (2013). Anatomy of performance fees in Finnish mutual funds.

Robert, C. P. (2004). *Monte carlo methods*, Wiley Online Library.

Ruckes, M. and Sevostiyanova, M. (2011). Time to wind down: Closing decisions and high water marks in hedge fund management contracts, *Technical report*, unpublished working paper, Karlsruhe Institute of Technology.

Starks, L. T. (1987). Performance incentive fees: An agency theoretic approach, *Journal of Financial and Quantitative Analysis* **22**(01): 17–32.

Thomas, F. S. and Jaye, J. C. (2006). Compensating mutual fund advisers: a return to the basics of properly structured performance fees, *Journal of Investment Compliance* **7**(4): 28–37.

Treasury, N. (2013). Charges in South African retirement funds.

Whitelaw and O'Donnell (2011). Best practices in managed fund performance fees.

Zhang, P. (1997). *Exotic Options: A Guide to the Second Generation Options*, World Scientific Publishing Company, Incorporated.
URL: *http://books.google.co.za/books?id=M4OYQgAACAAJ*

Appendix A

Ex-ante Performance Fees, where the Price Process is Generated by r

Tab. A.1: Change in Performance Fees from one Period to Another for Equity Funds.

	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2104
Fund102	-	0.5490 %	0.8594%	0.7577%	0.3351%	0.3216%	0.2039%	0.2764%	0.4146%	0.7327%	0.4273%	0.4209%	0.2031%	0.1442%	0.1976%
Fund1013	-	-	-	-	-	-	0.3021%	0.2907%	0.2794%	0.3658%	0.1993%	0.2392%	0.0745%	0.0852%	0.0743%
Fund1018	-	-	-	-	-	0.0715%	0.1499%	0.1438%	0.0635%	0.2478%	0.0728%	0.0614%	0.0446%	0.1167%	0.2229%
Fund1024	-	-	-	-	-	0.6847%	0.4848%	0.4439%	0.5316%	0.7226%	0.5383%	0.1091%	0.1789%	0.1683%	0.3250%
Fund1050	-	-	-	-	-	-	-	0.0789%	0.0666%	0.1522%	0.0426%	0.0941%	0.0419%	0.1115%	0.0478%
Fund1051	-	-	-	-	-	-	-	0.1302%	0.1893%	0.3451%	0.0850%	0.1290%	0.1113%	0.3450%	0.2019%
Fund1052	-	-	-	-	-	-	-	-	-	0.3819%	0.1179%	0.0183%	0.1216%	0.1345%	0.1841%
Fund1053	-	-	-	-	-	-	-	0.1480%	0.3286%	0.2447%	0.0812%	0.1191%	0.1094%	0.2213%	0.1257%
Fund1054	-	-	-	-	-	-	0.0758%	0.0057%	0.0265%	0.0904%	0.1369%	0.0405%	0.0657%	0.1160%	0.2199%
Fund1055	-	-	-	-	-	-	-	0.0374%	0.1024%	0.2547%	0.1683%	0.0814%	0.1500%	0.3473%	0.2375%
Fund1059	-	-	-	-	-	-	-	-	0.0823%	0.0509%	0.0085%	0.0256%	0.0367%	0.0188%	0.1175%
Fund1060	-	-	0.0529%	0.0219%	0.0258%	0%	0%	0%	0.0017%	0.0426%	0.0697%	0%	0.1931%	0.0532%	0.0454%
Fund1063	-	-	-	-	-	0.1053%	0.0785%	0.0713%	0.2462%	0.2000%	0.0924%	0.0721%	0.0541%	0.1534%	0.0365%
Fund1072	-	-	-	-	-	-	-	-	-	0.0509%	0.0877%	0.0297%	0.0424%	0.0965%	0.1819%
Fund1076	-	-	-	-	-	-	-	0.2505%	0.6715%	0.8054%	0.3514%	0.2434%	0.3952%	0.2768%	0.1573%
Fund1077	-	-	-	-	-	-	-	-	0.1228%	0.2037%	0.2081%	0.1268%	0.1486%	0.2656%	0.2006%
Fund1078	-	-	-	-	-	0.1654%	0.2524%	0.2103%	0.2186%	0.1651%	0.0807%	0.0842%	0.0612%	0.2695%	0.2298%
Fund122	-	0.1539%	0.1689%	0.3074%	0.1935%	0.1546%	0.1821%	0.2410%	0.2183%	0.6284%	0.5253%	0.1655%	0.1373%	0.1438%	0.2116%
Fund234	-	0.2799%	0.1767%	0.2387%	0.1988%	0.1149%	0.1206%	0.0845%	0.0792%	0.2084%	0.2236%	0.0656%	0.0411%	0.0432%	0.1130%
Fund313	-	-	-	1.2670%	0.6067%	0.3954%	0.2420%	0.1703%	0.2305%	0.5606%	0.3106%	0.0841%	0.1857%	0.0764%	0.1145%
Fund322	-	-	-	1.1536%	0.3885%	0.4219%	0.2770%	0.2713%	0.1590%	0.6849%	0.7117%	0.4281%	0.1766%	0.3724%	0.3628%
Fund323	-	1.3312%	1.9826%	1.0693%	0.4402%	0.5150%	0.2747%	0.1350%	0.2515%	0.3685%	0.6116%	0.2763%	0.2139%	0.2130%	0.1707%
Fund384	-	-	0.9735%	0.8306%	0.5371%	0.3205%	0.2668%	0.1765%	0.2632%	0.4759%	0.6343%	0.3652%	0.3755%	0.2292%	0.1550%
Fund385	-	-	-	0.0626%	0.2631%	0.1229%	0.0462%	0.0641%	0.0557%	0.2004%	0.1602%	0.0636%	0.0490%	0.1176%	0.1184%
Fund433	-	-	-	-	-	0.0846%	0.1957%	0.2056%	0.1505%	0.3886%	0.1718%	0.0659%	0.0561%	0.1995%	0.1960%
Fund434	-	-	-	0.1394%	0.2989%	0.1428%	0.1942%	0.2191%	0.1250%	0.2666%	0.1491%	0.0331%	0.0414%	0.2054%	0.2029%
Fund435	-	-	-	0.2414%	0.1280%	0.2242%	0.2253%	0.1791%	0.3525%	0.5944%	0.3739%	0.1994%	0.2823%	0.3284%	0.5460%
Fund439	-	-	-	0.1483%	0.2839%	0.3213%	0.0726%	0.0953%	0.1230%	0.3205%	0.2571%	0.1817%	0.0590%	0.18%	0.2292%
Fund441	-	0.4605%	0.9547%	0.8357%	0.8147%	0.6095%	0.4503%	0.4894%	0.6160%	1.4880%	0.6133%	0.3223%	0.2707%	0.2980%	0.4635%
Fund444	-	0.3650%	0.8102%	0.4575%	0.3234%	0.3970%	0.2253%	0.2537%	0.2372%	0.5605%	0.4593%	0.2121%	0.1917%	0.1584%	0.0820%
Fund480	-	-	-	0.0947%	0.4822%	0.1446%	0.1426%	0.1553%	0.0805%	0.1023%	0.0616%	0.0135%	0.02%	0.0445%	0.0343%
Fund652	-	-	-	-	0.3816%	0.5404%	0.6282%	0.3975%	0.9%	0.9787%	0.4569%	0.2616%	0.4212%	0.4256%	0.3909%
Fund694	-	-	-	0.1230%	0.4562%	0.1824%	0.1218%	0.1645%	0.1034%	0.0775%	0.0360%	0.0309%	0.0479%	0.0528%	0.0642%
Fund696	-	0.6219%	1.0899%	0.5486%	0.6712%	0.5434%	0.5043%	0.2543%	0.6548%	0.5819%	0.3218%	0.1978%	0.22%	0.2689%	0.5001%
Fund760	-	-	-	0.0756%	0.4771%	0.1208%	0.1495%	0.1112%	0.1695%	0.1842%	0.0693%	0.0710%	0.0606%	0.0432%	0.0842%
Fund911	-	-	-	0.4210%	0.2102%	0.4138%	0.3307%	0.1818%	0.2233%	0.4131%	0.2487%	0.1928%	0.0691%	0.1843%	0.1262%

[1] *The symbol "-", shows that the Fund did not exist at that point in time.*

[2] *We can observe that ex-ante performance fees increase a bit after the 2007-2008 financial crisis. Which is plausible since we are using historical data to forecast performance fees. Therefore high volatilities observed in 2007-2008, are only put into the model as an input in 2008 and 2009. As expected ex-ante performance fees increase, since spread options have a positive vega.*

Tab. A.2: Change in Performance Fees from one Period to Another for Fixed Income Funds.

	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2104
Fund1015	-	-	-	-	-	-	-	0.0026%	0.0014%	0.0042%	0.0123%	0.0517%	0.0134%	0%	0.0150%
Fund1058	-	-	-	-	-	-	-	-	0%	0.0081%	0%	0.0003%	0.0015%	0.0065%	0.0072%
Fund107	-	0.0133%	0.0203%	0.0089%	0.01%	0.0167%	0.0554%	0.0167%	0.0034%	0.0162%	0.0017%	0%	0.0032%	0.0001%	0.0007%
Fund1083	-	-	-	-	-	-	-	-	0.0876%	0.1149%	0.0007%	0.0001%	0%	0.0024%	0.0431%
Fund1185	-	-	-	-	-	-	-	-	-	0.0072%	0.0033%	0.0031%	0.0008%	0.0039%	0.0060%
Fund1188	-	-	-	-	-	-	-	-	0.1774%	0.5319%	0.1453%	0.3042%	0.2198%	0.2737%	0.1670%
Fund162	-	0.0043%	0.0058%	0.0019%	0%	0.0023%	0.0361%	0.0182%	0.0245%	0.1487%	0.0527%	0.0085%	0.0307%	0.0335%	0.0873%
Fund236	-	0%	0.0005%	0.0005%	0.0007%	0.0003%	0%	0.0003%	0%	0.0063%	0.0094%	0.0009%	0%	0%	0.0002%
Fund2512	-	-	-	-	-	-	0.0627%	0.0053%	0.0010%	0.0267%	0.0001%	0.0018%	0.0102%	0.0157%	0.0044%
Fund326	-	0.0161%	0.0180%	0.0007%	0.0215%	0.0144%	0.0034%	0.0022%	0.0097%	0.0337%	0.0273%	0.0043%	0.0150%	0.0025%	0.0209%
Fund327	-	0.0228%	0.0545%	0.0306%	0.0015%	0.0032%	0.0145%	0.0070%	0.0337%	0.0212%	0.02%	0.0137%	0.0674%	0.0055%	0.0397%
Fund343	-	0.0003%	0.0001%	0%	0%	0%	0%	0.0006%	0.0005%	0.0399%	0.0001%	0.0001%	0.0007%	0.0014%	0.0021%
Fund344	-	0.0034%	0.0343%	0.0074%	0.0029%	0.0698%	0.0044%	0.0001%	0.0053%	0.0029%	0.0019%	0.0009%	0.0011%	0.0054%	0.0009%
Fund347	-	0.0225%	0.0960%	0.0179%	0.0030%	0.0058%	0.0003%	0.0004%	0.0135%	0.0062%	0.0001%	0.0018%	0.0001%	0.0001%	0.0016%
Fund373	-	-	-	0.0109%	0.0012%	0.0035%	0.0099%	0.0011%	0.0021%	0.0038%	0.0165%	0.0096%	0.0683%	0.0083%	0.0509%
Fund375	-	0.0002%	0.0065%	0.0006%	0.0002%	0.0059%	0.0013%	0.0018%	0.0016%	0.0322%	0%	0.0030%	0.0030%	0.0041%	0.0154%
Fund377	-	0%	0.0068%	0.0167%	0.0023%	0%	0.0001%	0%	0%	0.0062%	0.0009%	0.0002%	0.0011%	0.0004%	0.0195%
Fund380	-	0.0001%	0.0145%	0.0092%	0.0069%	0.0065%	0%	0.0017%	0%	0.0196%	0.0003%	0.0008%	0%	0.0005%	0.0043%
Fund381	-	0.0086%	0.0215%	0.0285%	0.0053%	0.0011%	0.0004%	0.0041%	0%	0.0037%	0.0024%	0.0005%	0.0007%	0.0001%	0.0041%
Fund4133	-	-	-	-	-	-	-	-	-	-	-	0.2929%	0.2328%	0.2366%	0.4560%
Fund645	-	-	-	0.0006%	0.0012%	0%	0.0039%	0.0038%	0.1044%	0.0197%	0.015%	0.0360%	0.0018%	0.0031%	0.0220%
Fund768	-	-	-	-	-	0.1821%	0.0289%	0.1851%	0.0765%	0.1532%	0.0096%	0.0078%	0.0006%	0.0836%	0.0022%

[1] *Fixed Income funds have ex-ante performance fee values that are a bit lower than equity funds, as expected.*

Tab. A.3: Change in Performance Fees from one Period to Another for Money Market Funds.

	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2104
Fund1033	-	-	-	-	-	-	-	0.0032%	0%	0%	0%	0%	0%	0%	0%
Fund1042	-	-	-	-	-	-	0%	0.0069%	0.0008%	0.1489%	0.0018%	0%	0%	0%	0%
Fund1049	-	-	-	-	-	0.0001%	0%	0%	0%	0.04287%	0%	0%	0%	0%	0%
Fund1056	-	-	-	-	-	0%	0%	0%	0%	0.0002%	0.0015%	0%	0%	0%	0%
Fund106	-	0.0203%	0.0001%	0%	0.0014%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Fund1102	-	0%	0.0077%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Fund1490	-	0.52%	0.68%	0.83%	1.00%	1.02%	1.17%	1.49%	0.37%	0.92%	1.23%	1.49%	1.78%	1.83%	2.08%
Fund167	-	-	-	-	-	-	-	-	0.0038%	0.0066%	0%	0%	0%	0%	0%
Fund2230	-	0.0219%	0%	0%	0%	0%	0%	0%	0%	0.0008%	0%	0%	0%	0%	0%
Fund229	-	-	-	-	-	-	-	-	0.0032%	0.0029%	0.0279%	0.0070%	0.0082%	0.0018%	0.0386%
Fund280	-	-	-	-	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Fund282	-	-	-	0.0032%	0%	0%	0%	0%	0.004%	0%	0%	0%	0.0010%	0.0021%	0.0068%
Fund283	-	0.0091%	0%	0.0009%	0%	0%	0%	0%	0.0010%	0.006%	0%	0%	0%	0%	0.0006%
Fund285	-	0.0054%	0%	0.0065%	0%	0%	0%	0%	0%	0.0002%	0%	0%	0%	0.0011%	0%
Fund341	-	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0.0042%
Fund490	-	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0.0074%
Fund589	-	0%	0.0089%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Fund71	-	0.0152%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%

[1] *We can observe that Money Market funds have very low performance fees since they also have very low volatilities.*

Appendix B

Ex-ante Performance Fees, where the Price Process is Generated by a drift rate μ

Tab. B.1: Change in Performance Fees from one Period to Another for Equity Funds.

	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2104
Fund102	-	0.5608%	0.6917%	0.8411%	0.3005%	0.3167%	0.2382%	0.3070%	0.3935%	0.6810%	0.3348%	0.3356%	0.1928%	0.0966%	0.1833%
Fund1013	-	-	-	-	-	-	0.2925%	0.3180%	0.2261%	0.3115%	0.1788%	0.1749%	0.0620%	0.0738%	0.0990%
Fund1018	-	-	-	-	-	0.0693%	0.1238%	0.1507%	0.0608%	0.2385%	0.0870%	0.0738%	0.0421%	0.1244%	0.2591%
Fund1024	-	-	-	-	-	0.7872%	0.4773%	0.4575%	0.4829%	0.5723%	0.4813%	0.0911%	0.1699%	0.1615%	0.2679%
Fund1050	-	-	-	-	-	-	-	0.0715%	0.0645%	0.1383%	0.0321%	0.0907%	0.0312%	0.1156%	0.0573%
Fund1051	-	-	-	-	-	-	-	0.1357%	0.1673%	0.4425%	0.0704%	0.1037%	0.0697%	0.3569%	0.1781%
Fund1052	-	-	-	-	-	-	-	-	-	0.3141%	0.1235%	0.0083%	0.0933%	0.1009%	0.1728%
Fund1053	-	-	-	-	-	-	-	0.1336%	0.2842%	0.2582%	0.0650%	0.0948%	0.07%	0.2127%	0.1035%
Fund1054	-	-	-	-	-	-	0.0720%	0.0051%	0.0275%	0.0803%	0.1490%	0.0325%	0.0593%	0.0961%	0.2555%
Fund1055	-	-	-	-	-	-	-	0.0411%	0.0982%	0.2712%	0.1468%	0.0773%	0.1139%	0.3692%	0.1896%
Fund1059	-	-	-	-	-	-	-	-	0.0908%	0.0563%	0.0087%	0.0218%	0.0305%	0.0111%	0.0964%
Fund1060	-	-	0.0397%	0.0197%	0.0275%	0%	0%	0%	0.0019%	0.0369%	0.0786%	0%	0.1878%	0.0466%	0.0358%
Fund1063	-	-	-	-	-	0.0987%	0.0641%	0.0618%	0.2184%	0.1671%	0.0923%	0.0780%	0.0565%	0.1776%	0.0333%
Fund1072	-	-	-	-	-	-	-	-	-	0.0424%	0.1091%	0.0353%	0.0351%	0.0751%	0.1618%
Fund1076	-	-	-	-	-	-	-	0.2201%	0.5124%	0.7018%	0.3202%	0.1973%	0.3369%	0.2335%	0.1316%
Fund1077	-	-	-	-	-	-	-	-	0.0985%	0.2598%	0.2219%	0.1140%	0.1350%	0.2840%	0.1701%
Fund1078	-	-	-	-	-	0.1910%	0.2773%	0.2238%	0.2739%	0.1482%	0.0740%	0.0700%	0.0644%	0.2241%	0.2267%
Fund122	-	0.2094%	0.1924%	0.3145%	0.1394%	0.1590%	0.1577%	0.2380%	0.1910%	0.5919%	0.4400%	0.1606%	0.1360%	0.1615%	0.2146%
Fund234	-	0.2613%	0.1425%	0.2161%	0.1810%	0.1252%	0.1018%	0.0702%	0.0720%	0.1824%	0.1789 %	0.0653%	0.0462%	0.0442%	0.1457%
Fund313	-	-	-	1.2493%	0.6909%	0.4046%	0.2074%	0.1561%	0.1523%	0.5350%	0.2989%	0.0710%	0.1790%	0.0523%	0.0988%
Fund322	-	-	-	1.0767%	0.3436%	0.4182%	0.2397%	0.2077%	0.1125%	0.5322%	0.6065%	0.3150%	0.1172%	0.2893%	0.2837%
Fund323	-	1.2848%	1.9096%	1.1184%	0.4230%	0.4697%	0.2040%	0.1190%	0.2586%	0.2873%	0.4405%	0.2298%	0.2043%	0.1814%	0.1512%
Fund384	-	-	0.6452%	0.7151%	0.2838%	0.2751%	0.2108%	0.2933%	0.4287%	0.6559%	0.2770%	0.2944%	0.2047%	0.1296%	0.1594%
Fund385	-	-	-	0.0548%	0.2729%	0.1242%	0.0348%	0.0565%	0.0450%	0.1808%	0.1219%	0.0506%	0.0362%	0.0985%	0.1131%
Fund433	-	-	-	-	-	0.0930%	0.1650%	0.2077%	0.1346%	0.3575%	0.2015%	0.0926%	0.0565%	0.1926%	0.2420%
Fund434	-	-	-	0.1347%	0.3143%	0.1482%	0.1803%	0.2214%	0.1084%	0.2664%	0.1579%	0.0414%	0.03%	0.2147%	0.2384%
Fund435	-	-	-	0.3176%	0.1272%	0.2138%	0.2359%	0.1388%	0.2857%	0.5246%	0.2924%	0.1398%	0.1905%	0.2757%	0.4601%
Fund439	-	0.35%	0.8713%	0.9755%	0.7740%	0.7311%	0.3791%	0.5%	0.5975%	1.4762%	0.6098%	0.2753%	0.2253%	0.1808%	0.3819%
Fund441	-	0.3749%	0.7171%	0.4956%	0.2750%	0.4140%	0.16%	0.2336%	0.2321%	0.4946%	0.3471%	0.1710%	0.1765%	0.1354%	0.0564%
Fund444	-	-	-	0.0812%	0.5379%	0.1347%	0.1505%	0.1503%	0.0936%	0.0916%	0.0517%	0.0098%	0.0197%	0.0466%	0.0383%
Fund480	-	-	-	0.3781%	0.5327%	0.5580%	0.4035%	0.7543%	0.8823%	0.3999%	0.2027%	0.3041%	0.3383%	0.3082%	0.3008%
Fund652	-	-	-	0.0994%	0.5030%	0.1871%	0.1342%	0.1418%	0.1231%	0.0743%	0.0251%	0.0294%	0.0470%	0.0513%	0.0573%
Fund694	-	0.5908%	0.9532%	0.4995%	0.7016%	0.4675%	0.4416%	0.3559%	0.5510%	0.5223%	0.2680%	0.1464%	0.1866%	0.2203%	0.4138%
Fund696	0.08%	0.30%	0.36%	0.44%	0.56%	0.56%	0.64%	0.84%	0.37%	0.92%	1.23%	1.49%	1.78%	1.83%	2.08%
Fund760	-	-	-	0.0841%	0.5389%	0.1074%	0.1567%	0.1123%	0.1933%	0.1265%	0.0615%	0.0813%	0.0673%	0.0544%	0.0605%
Fund911	-	-	-	0.4674%	0.2302%	0.4720%	0.2731%	0.1600%	0.1778%	0.4106%	0.1984%	0.1629%	0.0724%	0.1747%	0.1524%

[1] *Ex-ante performance fees calculated using prices generated by a drift rate μ, are not that different from ex-ante fees in Table A.1.*

Tab. B.2: Change in Performance Fees from one Period to Another for Fixed Income Funds.

	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2104
Fund1015	-	-	-	-	-	-	-	0.0023%	0.0009%	0.0027%	0.0095%	0.0413%	0.0114%	0%	0.0159%
Fund1058	-	-	-	-	-	-	-	-	0%	0.0090%	0%	0.0004%	0.0013%	0.0056%	0.0065%
Fund107	-	0.0088%	0.0162%	0.0091%	0.009%	0.0130%	0.0460%	0.0185%	0.0026%	0.0131%	0.002%	0%	0.0023%	0.0001%	0.0007%
Fund1083	-	-	-	-	-	-	-	-	0.0790%	0.0987%	0.0011%	0.0001%	0%	0.00260.0354%	2.08%
Fund1185	-	-	-	-	-	-	-	-	-	0.0068%	0.0048%	0.0034%	0.0007%	0.0051%	0.0068%
Fund1188	-	-	-	-	-	-	-	-	0.1572%	0.2065%	0.4421%	0.1468%	0.2857%	0.1922%	0.2482%
Fund162	-	0.0063%	0.0065%	0.0015%	0%	0.0014%	0.0330%	0.0148%	0.0224%	0.1199%	0.0602%	0.0062%	0.0250%	0.0219%	0.0792%
Fund236	-	0%	0.0006%	0.0004%	0.0008%	0.0005%	0%	0.0003%	0%	0.0068%	0.0091%	0.0014%	0%	0%	0.0003%
Fund2512	-	-	-	-	-	-	0.0638%	0.0061%	0.0009%	0.0304%	0.0001%	0.002%	0.008%	0.0129%	0.0041%
Fund326	-	0.0127%	0.0104%	0.0003%	0.0181%	0.0143%	0.0029%	0.0013%	0.0076%	0.0342%	0.0245%	0.0045%	0.0124%	0.0026%	0.0188%
Fund327	-	0.0247%	0.0495%	0.0234%	0.0013%	0.0020%	0.0138%	0.0071%	0.0391%	0.0182%	0.0236%	0.0181%	0.0653%	0.0092%	0.0399%
Fund343	-	0.0005%	0.0001%	0%	0%	0%	0%	0.0004%	0.0005%	0.0314%	0.0001%	0%	0.0004%	0.0007%	0.0021%
Fund344	-	0.0030%	0.0312%	0.0061%	0.0031%	0.0534%	0.0046%	0.0001%	0.0034%	0.0035%	0.0018%	0.0008%	0.0013%	0.0058%	0.0010%
Fund347	-	0.0128%	0.0769%	0.0158%	0.0025%	0.0048%	0.0002%	0.0003%	0.0128%	0.0072%	0.0001%	0.0015%	0.0001%	0%	0.0011%
Fund373	-	-	-	0.0092%	0.0017%	0.0027%	0.0077%	0.0011%	0.0022%	0.0042%	0.0160%	0.0122%	0.0679%	0.0117%	0.0510%
Fund375	-	0.0002%	0.005%	0.0004%	0.0001%	0.0042%	0.0011%	0.0013%	0.0016%	0.0243%	0%	0.0024%	0.0022%	0.0026%	0.0162%
Fund377	-	0%	0.0052%	0.0109%	0.0022%	0%	0%	0%	0%	0.0052%	0.0008%	0.0002%	0.0008%	0.0005%	0.0173%
Fund380	-	0.0001%	0.0159%	0.0100%	0.0075%	0.0049%	0%	0.0019%	0%	0.0187%	0.0002%	0.0013%	0%	0.0004%	0.0041%
Fund381	-	0.0072%	0.0234%	0.0349%	0.0073%	0.0019%	0.0008%	0.0039%	0%	0.0030%	0.0022%	0.0006%	0.0010%	0.0002%	0.0038%
Fund4133	-	-	-	-	-	-	-	-	-	-	-	0.2514%	0.1912%	0.1927%	0.4573%
Fund645	-	-	-	0.0007%	0.0012%	0%	0.0028%	0.0028%	0.0786%	0.0162%	0.0106%	0.0305%	0.0012%	0.0017%	0.017%
Fund768	-	-	-	-	-	0.1316%	0.0255%	0.1941%	0.0894%	0.1141%	0.0061%	0.0052%	0.0003%	0.0645%	0.0015%

Tab. B.3: Change in Performance Fees from one Period to Another for Money Market Funds.

	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2104
Fund1033	-	-	-	-	-	-	-	0.0022%	0%	0%	0%	0%	0%	0%	0%
Fund1042	-	-	-	-	-	-	-	0.0040%	0.0006%	0.1821%	0.0022%	0%	0%	0%	0%
Fund1049	-	-	-	-	-	0.0001%	0%	0%	0%	0.006208%	0%	0%	0%	0%	0%
Fund1056	-	-	-	-	-	0%	0%	0%	0%	0.0002%	0.0917%	0%	0%	0%	0%
Fund106	-	0.0222%	0.0001%	0%	0.0013%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Fund1102	-	0.0066%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Fund1490	-	-	-	-	-	-	-	-	0.0024%	0.0068%	0%	0%	0%	0%	0%
Fund167	-	0.0251%	0%	0%	0%	0%	0%	0%	0%	0%	0.0010%	0%	0%	0%	0%
Fund2230	-	-	-	-	-	-	-	-	0.0027%	0.0024%	0.0288%	0.0118%	0.0097%	0.0021%	0.0348%
Fund229	-	-	-	-	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Fund280	-	-	-	0.0040%	0%	0%	0%	0%	0.0052%	0%	0%	0%	0.0043%	0.0055%	0.0084%
Fund282	-	0.0010%	0%	0.0009%	0%	0%	0%	0%	0.0020%	0.0013%	0%	0%	0%	0%	0.002%
Fund283	-	0.0032%	0%	0.0036%	0%	0%	0%	0%	0%	0.0002%	0%	0%	0%	0.0098%	0%
Fund285	-	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0.006%	0%
Fund341	-	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0.0064%
Fund490	-	0.30%	0.36%	0.44%	0.56%	0.56%	0.64%	0.84%	0.37%	0.92%	1.23%	1.49%	1.78%	1.83%	2.08%
Fund589	-	-	-	-	-	0.0011%	0.0026%	0.%	0%	0%	0%	0%	0%	0%	0%
Fund71	-	0.0169%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%

Appendix C

Actual Performance Fees Paid

Tab. C.1: Change in Actual Performance Fees from one Period to Another for Equity Funds.

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2104
Fund102	1.0919%	3.3126%	2.4191%	0.2682%	1.0179%	0.7610%	0.1187%	1.3600%	0%	0.1284%	0.4065%	0%	0%	0%
Fund1013	-	-	-	-	-	-	0.5501%	0%	0.1829%	0.0393%	0%	0%	0%	0.6588%
Fund1018	-	-	-	-	0%	0%	0%	1.0825%	0.3941%	0.5221%	0%	0.1061%	0.8455%	0%
Fund1024	-	-	-	-	1.1839%	0.1214%	0%	0%	0%	0%	0.4617%	0.3687%	0%	0%
Fund1050	-	-	-	-	-	-	0%	0%	0%	0%	0%	0.0246%	0.3509%	0%
Fund1051	-	-	-	-	-	-	0%	0%	0%	0%	0%	0%	0.1518%	0%
Fund1052	-	-	-	-	-	-	-	-	0%	0%	0%	0%	0%	0%
Fund1053	-	-	-	-	-	-	0%	1.1823%	0%	0%	0%	0%	0%	0%
Fund1054	-	-	-	-	0%	0.0518%	0.2478%	0.2771%	0%	0.0220%	0%	0.3842%	0%	0%
Fund1055	-	-	-	-	-	-	0.0711%	0.9304%	0.1133%	0%	0%	0.6402%	0%	0%
Fund1059	-	-	-	-	-	-	-	0%	0.1251%	0.0084%	0%	0%	0%	0%
Fund1060	-	0.0503%	0%	0%	0%	0%	0.0311%	0.0559%	0.1797%	0%	0%	0%	0%	0%
Fund1063	-	-	-	-	0%	0%	0%	0.3216%	0%	0.0814%	0.2396%	0.7306%	0.1180%	0%
Fund1072	-	-	-	-	-	-	-	-	0.3343%	0%	0%	0%	0%	0%
Fund1076	-	-	-	-	-	-	0%	4.0304%	0.6548%	0%	0.2406%	0%	0%	0%
Fund1077	-	-	-	-	-	-	-	0.6721%	0%	0%	0%	0%	0%	0%
Fund1078	-	-	-	-	0.9333%	0.3474%	0.5917%	0%	0.0837%	0%	0.6320%	0%	0%	0%
Fund122	1.1023%	1.5766%	0%	0.7613%	0%	0%	0%	0.8693%	0%	1.6569%	0.2250%	0.9145%	0.8819%	0%
Fund234	0.8913%	0.0657%	0.3086%	0.8458%	0%	0%	0%	0.4903%	0%	0%	0.0067%	0.0045%	0.6524%	0%
Fund313	-	-	2.5996%	0.9677%	0%	0%	0%	3.6779%	0.1302%	0.0140%	0.8805%	0%	0%	0%
Fund322	-	2.1783%	1.9295%	0%	0%	0%	0.9229%	0.4960%	0%	0%	0%	0%	0%	0%
Fund323	2.9677%	4.0404%	1.8224%	0.4244%	0%	0%	0%	0%	0%	0.0689%	0.3731%	0.3341%	0%	0.0076%
Fund384	-	5.4144%	1.1095%	0.8919%	1.1451%	1.1062%	0%	2.8214%	0%	0%	0.5553%	0%	0.0455%	0%
Fund385	-	-	0.1858%	0.7616%	0%	0%	0%	0.5105%	0%	0%	0%	0%	0.0991%	0.0687%
Fund433	-	-	-	-	0%	0%	0%	0.8385%	0.8730%	0.9084%	0%	0%	1.3801%	0%
Fund434	-	-	0.5886%	1.1317%	0.0028%	0.4201%	0%	0.9157%	0.6610%	0.3642%	0%	0.3016%	1.2149%	0.1120%
Fund435	-	-	1.0508%	0%	1.6425%	0%	0%	1.6149%	0%	0%	0%	0%	0%	0.0671%
Fund439	-	-	1.9121%	2.1778%	0%	0%	0.8236%	0%	0%	0%	0%	0%	0%	0%
Fund441	4.9631%	5.6961%	3.5458%	4.6352%	0%	0%	0%	1.1589%	0.3861%	0.9295%	0%	0%	0%	0%
Fund444	3.0293%	3.0325%	1.0893%	1.4985%	0%	0%	0.0383%	0.5809%	0%	0.3466%	0.0600%	0.0502%	0%	0%
Fund480	-	-	0%	0%	0%	0.0277%	0.0578%	0%	0%	0%	0%	0.0914%	0.0317%	0%
Fund652	-	-	-	0.8803%	0%	0%	0%	4.2587%	0.2742%	0%	0%	0%	0%	0%
Fund694	-	-	0%	0%	0.0890%	0%	0.5685%	0%	0%	0%	0%	0.3033%	0%	0%
Fund696	0%	2.8014%	1.1335%	0%	0%	2.9493%	0.3111%	0.0081%	0%	0%	0%	0%	0%	0%
Fund760	-	-	0.1808%	0%	0%	0.5544%	0%	0%	0%	0.0534%	0.3055%	0.6459%	0%	0%
Fund911	-	-	1.1755%	2.7437%	0%	0.6887%	0%	0.1745%	0%	0.1168%	0.3407%	0%	0.5283%	0%

[1] *Actual performance fees are zero, most of the time. We therefore expect the distribution of the these fees to have a lot of mass at zero. The histogram of the distribution of these fees is in the next section.*

[2] *These performance fees are a bit higher than the ex-ante performance fees calculated using spread options. But we can observe that in 2007, performance fees were either low or equal to zero.*

Tab. C.2: Change in Actual Performance Fees from one Period to Another for Fixed Income Funds.

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2104
Fund1015	-	-	-	-	-	-	0%	0%	0%	0%	0%	0.2231%	0.2775%	0%
Fund1058	-	-	-	-	-	-	-	0.0666%	0.0265%	0.0992%	0%	0%	0.0658%	0%
Fund107	0%	0.0866%	0%	0%	0%	0.1840%	0%	0.0458%	0.0448%	0.0012%	0%	0%	0.0702%	0%
Fund1083	-	-	-	-	-	-	-	0%	0.2427%	0%	0.0088%	0.1625%	0%	0%
Fund1185	-	-	-	-	-	-	-	-	0.1687%	0.0577%	0%	0.1878%	0.1682%	0%
Fund1188	-	-	-	-	-	-	-	0%	0.4604%	0.2156%	0.2199%	0%	0.2332%	0%
Fund162	0.1469%	0%	0%	0%	0.0597%	0%	0.0078%	0.2569%	0.2943%	0%	0%	0%	0.1070%	0%
Fund236	0.0734%	0%	0.0528%	0.0026%	0%	0%	0%	0%	0%	0.2609%	0.1045%	0.0165%	0.0305%	0%
Fund2512	-	-	-	-	-	0.1154%	0.0284%	0.0956%	0.0579%	0.0113%	0%	0%	0.0477%	0%
Fund326	0%	0%	0%	0%	0%	0%	0%	0.0653%	0%	0.0604%	0%	0.0317%	0.0472%	0%
Fund327	0%	0%	0%	0%	0%	0.1315%	0.3080%	0.0761%	0.2026%	0.3744%	0.2676%	0.3607%	0.3552%	0%
Fund343	0%	0%	0%	0%	0%	0%	0.0098%	0.0135%	0%	0%	0%	0%	0%	0%
Fund344	0%	0%	0.1296%	0%	0.0303%	0.0126%	0%	0%	0.0542%	0%	0%	0.2044%	0.1192%	0%
Fund347	0%	0%	0%	0%	0%	0%	0.0734%	0.2138%	0.0416%	0%	0%	0.0133%	0%	0%
Fund373	0.1061%	0%	0%	0.0167%	0.0501%	0.2080%	0.0324%	0.2493%	0.3624%	0.2862%	0.3477%	0.2013%	0%	0%
Fund375	0.0196%	0%	0%	0%	0%	0%	0.0189%	0%	0.0074%	0%	0%	0%	0.1840%	0%
Fund377	0%	0%	0%	0%	0%	0%	0%	0%	0%	0.0671%	0%	0.0114%	0.1118%	0%
Fund380	0.0399%	0%	0%	0%	0.0169%	0.0114%	0%	0%	0%	0.1496%	0%	0%	0%	0%
Fund381	0%	0%	0.2609%	0.1026%	0.1699%	0%	0%	0%	0.0033%	0.0059%	0.1299%	0.1079%	0.0127%	0%
Fund4133	-	-	-	-	-	-	-	-	-	-	0%	0%	1.1133%	0%
Fund645	-	-	0.0243%	0%	0%	0%	0%	0%	0.0770%	0%	0%	0%	0%	0%
Fund768	-	-	-	-	0%	0.6332%	0.6007%	0%	0%	0%	0%	0%	0%	0%

Tab. C.3: Change in Actual Performance Fees from one Period to Another for Money Market Funds.

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2104
Fund1033	-	-	-	-	-	-	0%	0%	0%	0%	0%	0%	0%	0%
Fund1042	-	-	-	-	-	0%	0%	0.0436%	0%	0.0229%	0.0065%	0%	0.0582%	0%
Fund1049	-	-	-	-	0%	0%	0%	0.0735%	0%	0%	0%	0%	0%	0%
Fund1056	-	-	-	-	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Fund106	0%	0%	0%	0%	0%	0%	0%	0.0127%	0%	0%	0%	0%	0%	0%
Fund1102	0%	0%	0%	0%	0%	0%	0%	0.0011%	0.0274%	0%	0%	0%	0%	0%
Fund1490	-	-	-	-	-	-	-	0.0882%	0.0122%	0.1233%	0.1322%	0.0522%	0.1119%	0%
Fund167	0%	0%	0%	0%	0%	0%	0%	0.0459%	0.0053%	0.0509%	0.0202%	0.0275%	0.0164%	0%
Fund2230	-	-	-	-	-	-	-	0%	0.2503%	0.3884%	0.2805%	0.2158%	0.0744%	0.0069%
Fund229	-	-	-	0%	0.0159%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Fund280	-	-	0%	0%	0.0111%	0%	0.0098%	0.0576%	0%	0.0333%	0.2190%	0.1390%	0.0816%	0%
Fund282	0.0531%	0%	0%	0.0333%	0%	0%	0.0096%	0.1501%	0.0585%	0.1185%	0.0553%	0.0759%	0.0959%	0%
Fund283	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Fund285	0%	0%	0%	0%	0%	0%	0%	0.0093%	0%	0.0913%	0.0517%	0.2316%	0.0625%	0%
Fund341	0%	0%	0%	0%	0%	0%	0.0018%	0.0287%	0.0137%	0.0392%	0.0407%	0.0489%	0.0390%	0.0308%
Fund490	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Fund589	-	-	-	-	0%	0%	0%	0%	0.0259%	0%	0%	0%	0%	0%
Fund71	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%

Appendix D

Distribution of Performance Fees

We consider the difference between the actual performance fees paid and ex-ante performance fees estimated. We do this to observe whether the variations between the two fees are consistent over time or whether they are time dependent and assess the extent expected performance fees differ from the actual perfromance fees paid.

We can observe from Figure D.1 to Figure D.3 that the differences between the actual performance fees paid and the ex-ante performance fee estimates is time dependent. Market conditions have an effect on the difference between ex-ante and actual performance fees.

Fig. D.1: Difference between Actual and Ex-ante Performance Fees for Equity Funds.

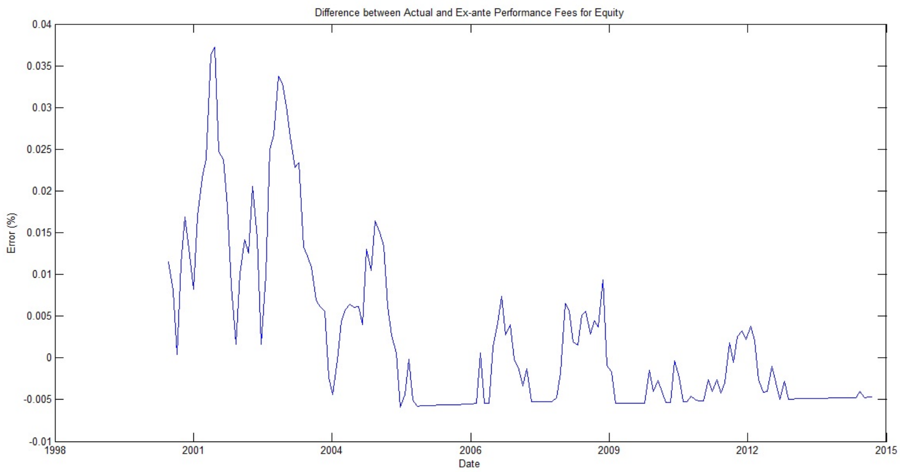


Fig. D.2: Difference between Actual and Ex-ante Performance Fees for Fixed Income Funds.

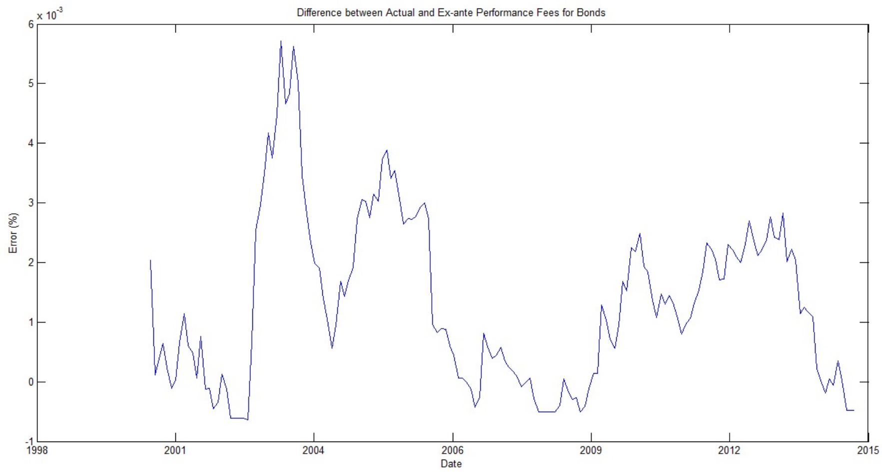


Fig. D.3: Difference between Actual and Ex-ante Performance Fees for Money Market Funds.