

WATER SURFACE PROFILES IN
SIDE CHANNEL SPILLWAYS

(Comparisons between computed and experimental values)

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A thesis submitted in partial fulfilment of the requirements for the
degree of Master of Science in Engineering

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September 1989

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SYNOPSIS

This thesis is an investigation into the water surface profiles found in side channel spillways. The classical method proposed by Hinds is investigated, with specific reference to the water surface slope equation. A literature review is given on the evolution of the theory, from the conceptual ideas of Hinds through to a systematic method of analysis. Experimental methods used over the years are also discussed.

A computer program called WSPISCS (Water Surface Profiles In Side Channel Spillways), is developed to calculate the water surface profile. This in turn alleviates the laborious and tedious hand calculations necessary in the past. An iterative, step by step calculation of the water surface profile from a known control point is conducted. A Runge Kutta fourth order algorithm is employed for the numerical integration.

The program is verified with previous hand calculated examples, including the classical example calculated by Hinds. A thorough sensitivity analysis is conducted with regard to the magnitude of the step length, and also to the location of the starting point. Comparisons are made with experimental results obtained over a number of years of undergraduate research. The correlation between the calculated and experimental profiles is generally good. The only exception is a horizontal receiving channel with a hydraulically steep outflow chute, where the experimental profile falls below the calculated one as the inflow rate increases.

ACKNOWLEDGEMENTS

The author wishes to express a few words of thanks to those who provided valuable information and technical assistance throughout the duration of this thesis.

Special thanks is given to the following :

Professor F.A. Kilner, Head of the Department of Civil Engineering, University of Cape Town, for guidance and expertise.

The Council for Scientific and Industrial Research for the award of a post-graduate bursary.

Mr A. Paterson, Department of Civil Engineering, University of Cape Town, for valuable assistance in the formulation of the computer program.

Mrs V. Atkinson, for the efficient typing of this thesis.

Mrs S. Cullen for draughting the required sketches.

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LIST OF SYMBOLS

<u>Sybmbo1</u>	<u>Description</u>	<u>Dimension</u>	<u>Unit</u>
A	cross sectional area	L^2	m^2
b	channel base width	L	m
B	channel surface width	L	m
E	specific energy at a section	L	m
f	Darcy-Weisbach friction factor	-	-
F	total force at a section	ML/T^2	N
F_a	net accelerating force on a water prism	ML/T^2	N
F_r	Froude number	-	-
	siggravitational acceleration	L/T^2	9,81 m/s^2
H_0	water depth at upstream wall	L	m
k	critical depth multiplier	-	-
k'	conveyance function	-	-
K_1	critical depth constant	-	-
K_2	pseudo normal depth constant	-	-
M	momentum flux	ML/T^2	N
n	Manning's roughness coefficient	$T/3\sqrt{L}$	$s/3\sqrt{m}$
p	wetted perimeter	L	m
P	static pressure	M/LT^2	N/m^2
	lateral inflow per metre length of channel	L^2/T	m^2/s
Q	discharge across a channel cross section	L^3/T	m^3/s
R	hydraulic radius	L	m
S_c	longitudinal bed slope in the outflow chute	-	-
S_f	energy line slope associated with boundary shear	-	-
S_o	longitudinal bed slope in the receiving channel	-	-
U	streamwise velocity component of the lateral inflow	L/T	m/s
V	flow velocity	L/T	m/s
w	specific weight of water	M/L^2T^2	9,81 kN/m^3

x	distance along the channel	L	m
y	typical water depth	L	m
z	submergence of the centre of area of a cross section	L	m
Z	variable to represent dy/dx at a gate control	-	-
Z_0	upstream channel depth (crest to floor)	L	m

<u>Greek symbol</u>	<u>Description</u>	<u>Dimension</u>	<u>Unit</u>
α	energy coefficient	-	-
β	Momentum correction coefficient (Farney & Marcus)	-	-
γ	elevation of water surface relative to horizontal datum	L	m
θ	side wall angle with horizontal	-	degrees
θ'	dissipated energy gradient	-	-
λ	Keulegan friction coefficient	-	-
ν	kinematic viscosity of water	L^2/T	m^2/s
τ	average shear stress on channel boundary surfaces	M/LT^2	Pa
ϕ	numerator of water surface slope equation	-	-
ψ	denominator of water surface slope equation	-	-

<u>Subscript</u>	<u>Description</u>
B	denotes bottom
c	denotes critical condition
chute	denotes the outflow chute
cr	denotes critical depth line
crchute	denotes the critical depth in the outflow chute
d	denotes downstream
f	denotes friction

HJBottom	denotes just upstream of the hydraulic jump
HJTop	denotes just downstream of the hydraulic jump
m	denotes an arbitrary section
n	denotes an arbitrary section
norm	denotes normal depth in the outflow chute.
o	denotes uniform conditions
pnd	denotes pseudo normal depth line
po	denotes pseudo normal conditions
u	denotes upstream
water	denotes the water surface profile.

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1 INTRODUCTION

Side channel spillways have been employed in many dams over the last century. Table 1.1 lists the names and construction dates of a number of dams with this particular type of spillway. Figure 1.1 illustrates a typical side channel spillway. From this diagram it can be seen that in essence a side channel spillway is one in which the receiving channel's axis lies parallel to the spillway crest. In effect, the spillway crest forms a side wall of the receiving channel. This geometric arrangement leads to spatially increasing discharge within the receiving channel.

Historically, the methods employed for determining the water surface profile within the receiving channel are laborious and tedious. The hydraulic theory does not lend itself to quick analytical solution. Calculations are based on an iterative procedure, starting at a control point and then implementing a step by step calculation to derive the water surface profile.

Since the advent of the computer, the daunting task of large iterative calculations is greatly simplified. Incorporated in this thesis is therefore the creation of a computer program capable of calculating the water surface profile within a side channel spillway.

This thesis follows the evolution of the hydraulic theory concerning side channel spillways, up until the development of the program WSPISCS (Water Surface Profiles In Side Channel Spillways). The program algorithms are then portrayed along with verification and operation examples. Finally this calculated water surface profile is compared with the profiles obtained in previous experiments conducted in the department of Civil Engineering at the University of Cape Town.

Table 1.1 Examples of Side Channel Spillways

<u>Name and Location</u>	<u>Construction Date</u>	<u>Design Discharge</u>
Solingen, Germany	1902	
Quielle, France	1904	
New Croton, New York City	1906	
Wachusett, Boston, Mass.	1906	
Rochebut, France	1909	
Coldsprings, Oregon	1910	
Concully, Washington	1910	
Croton Falls, New York City	1910	
Sweetwater, San Diego, Calif.	1911	
Ashokan, New York City	1912	
Mauer, Germany	1912	
Oakley, Idaho	1913	
Moehne, Germany	1913	
Klingenberg, Bohemia	1914	
Cedar Lake, Seattle, Washington	1914	
Morena, San Diego, California	1915	
Arrowrock, Idaho	1915	1 133 m ³ /s
Don Pedro, California	1915	424 m ³ /s
Burrinjuck, New South Wales	1916	
Teiton, Washington		
McKay, Oregon		
Boulder, Nevada	1933	11 310 m ³ /s
Dorena, Oregon		
Townsend, New England		560 m ³ /s
North Hartland, Vermont		
Nooitgedacht, Carolina, RSA	1963	1 700 m ³ /s
Nuane, Lobatsi, Botswana	1965	850 m ³ /s
Loerie, Cape, RSA	1968	736 m ³ /s
Ida's Valley, Stellenbosch, RSA	1968	45 m ³ /s
Spitskop, Cape, RSA	1974	1 700 m ³ /s
Mtata, Transkei, RSA	1978	2 430 m ³ /s
Middle Leteaba, Gazankulu, RSA	1984	850 m ³ /s
Oxkraal, Ciskei, RSA	1989	660 m ³ /s

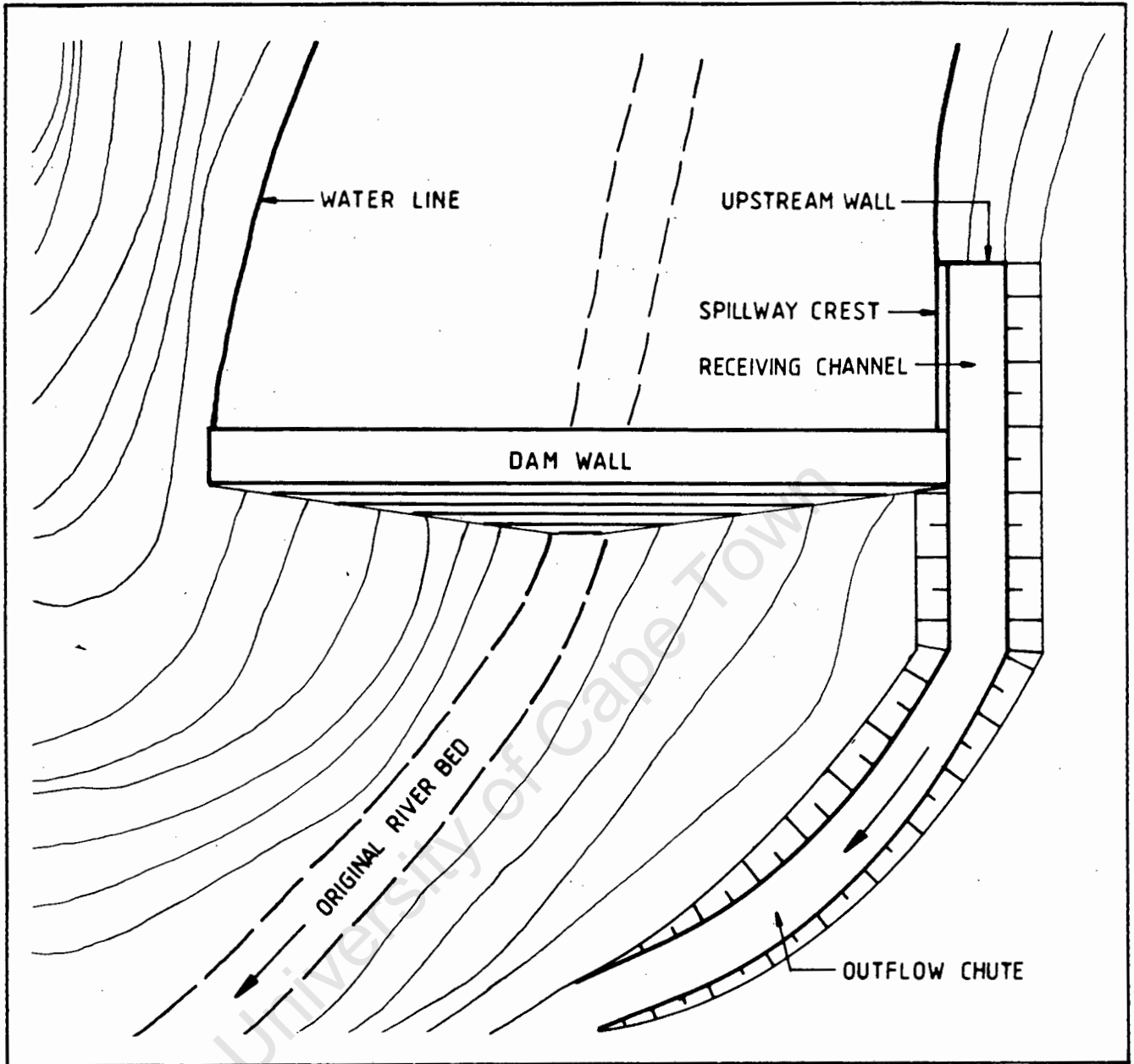


FIGURE 1.1 PLAN OF A TYPICAL SIDE CHANNEL SPILLWAY

2. EVOLUTION OF THE HYDRAULIC THEORY

In this chapter the historical progression of research relating to side channel spillways is explored. Particular attention is paid to the literature relating to the calculation of the water surface profile within the receiving channel. Before reviewing this accumulated knowledge, the concept of spatially varied discharge needs to be defined.

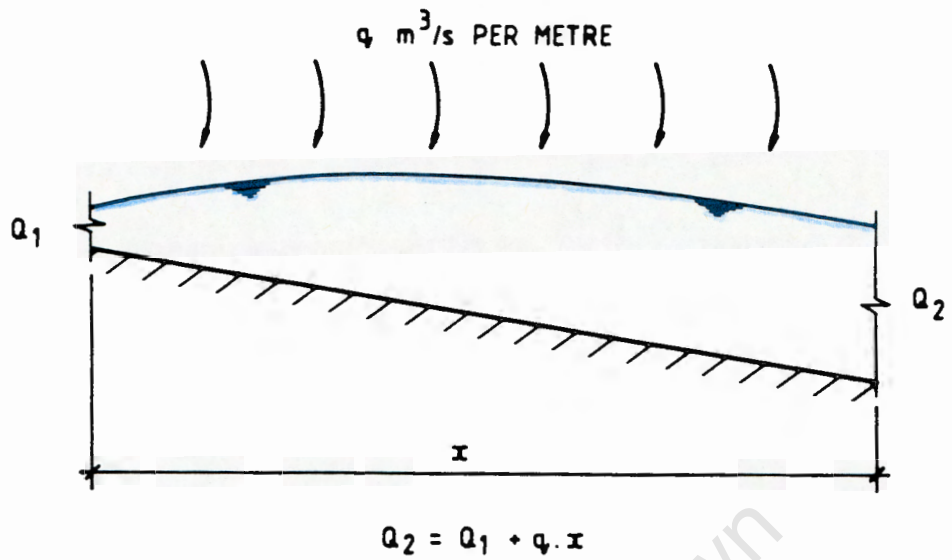
2.1 Spatially Varied Discharge

In essence, spatially varied discharge is a situation in which water depth, velocity and discharge vary with respect to space (ie. position along the channel), but are constant with respect to time. This is in contrast to spatially constant discharge where depth and velocity (but not discharge) may vary in respect to space and all are constant with respect to time. Figure 2.1 illustrates the principal differences between these two cases.

There are two types of spatially varied discharge, spatially increasing discharge and spatially decreasing discharge. A side channel spillway is an example of spatially increasing discharge. Roof gutters, wash water troughs in filters and effluent channels around sewage tanks also form part of this group. A main channel with side weirs is the most common example of spatially decreasing discharge.

In the case of a typical side channel spillway, the inflow per metre length of the receiving channel is constant. This simplifies the water surface profile calculation. In the following sections the evolution of this calculation is explored, along with the growth of understanding in the subject.

SPATIALLY VARIED DISCHARGE



BOTH CONSTANT
WITH TIME

SPATIALLY CONSTANT DISCHARGE

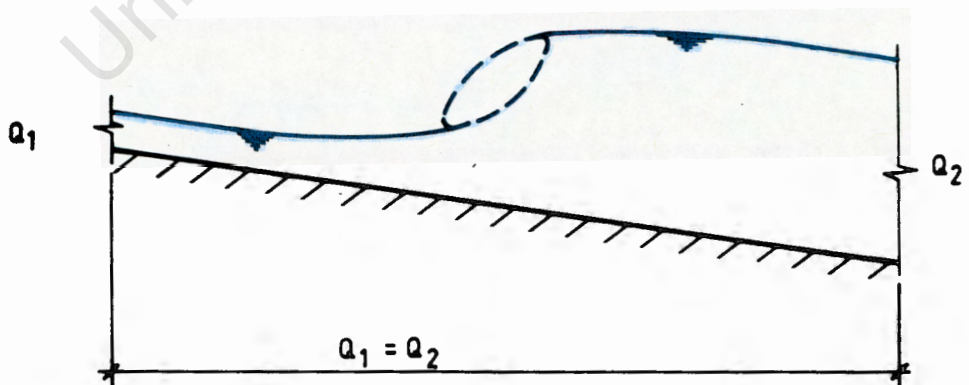


FIGURE 2.1 COMPARISON BETWEEN SPATIALLY
VARIED / CONSTANT DISCHARGE

2.2 Early Developments

Prior to 1926 the research into side channel spillways was almost exclusively experimental. These consisted mainly of spillway models, constructed to observe the expected behaviour of the full scale structure. Dare [1] in an article on the Burrinjuck Dam, highlighted the lack of theoretical understanding of the subject at that time. The scope of research on side channel spillways was specific to a particular dam and all results were of an empirical nature.

2.3 The water surface slope equation

An article published by the American Society of Civil Engineers in 1926, established the fundamental hydraulic theory associated with side channel spillways. This classical paper written by Hinds [2] forms the basis of all further research.

Hinds' derivation was based on the following assumptions :

- The velocity distribution over a given cross-section is uniform.
- The incoming water has no momentum component parallel to the receiving channel's axis.
- The pressure distribution in the receiving channel is hydrostatic.
- The effect of the channel friction is negligible.
- The principle of conservation of momentum is more applicable than that of the conservation of energy.

Hinds considered two sections, a small distance apart (Δx) in the receiving channel. The inflow rate is q m³/s per metre. Therefore the inflow between the sections is $q \Delta x$. The velocity and discharge at the upstream section are denoted by V and Q respectively. Similarly at the downstream section the velocity and discharge are $V + \Delta V$ and $Q + q \Delta x$.

The respective momentum flux equations are :

$$\text{Upstream} \quad M_u = \frac{w QV}{g} \quad (2.1)$$

$$\text{Downstream} \quad M_d = \frac{w}{g} (Q + q \Delta x)(V + \Delta V) \quad (2.2)$$

Subtracting eqn. (2.1) from (2.2) yields the change in momentum flux

$$\Delta M = \frac{w}{g} [Q \Delta V + q \Delta x (V + \Delta V)] \quad (2.3)$$

As Δx approaches zero, $\Delta V \cdot \Delta x$ becomes insignificant in comparison with V , leading to the differential equation.

$$\frac{dM}{dx} = \frac{w}{g} \left[Q \frac{dV}{dx} + Vq \right] \quad (2.4)$$

Using the relationship $\frac{dM}{dt} = \frac{dM}{dx} \cdot \frac{dx}{dt} = V \frac{dM}{dx}$, eqn. (2.4) becomes

$$\frac{dM}{dt} = \frac{w}{g} \left[V Q \frac{dV}{dx} + q V^2 \right] \quad (2.5)$$

The rate of change of momentum flux with respect to time is also equal to $w Q dy/dx$, therefore eqn. (2.5) divided by $w Q$ yields the water surface slope :

$$\frac{dy}{dx} = \frac{1}{g} \left[V \frac{dV}{dx} + \frac{qV^2}{Q} \right] \quad (2.6)$$

Later research by Favre [4] and subsequently Camp [6] led to the inclusion of a friction term. Camp's derivation is identical to that of Hinds, up to and including eqn. (2.5).

Figure 2.2 illustrates the channel geometry used by Camp. Consider the two sections m and n, a distance Δx apart. Equation (2.5) is related to the net force action on the water prism between sections m

and n , and this in turn is the difference between the static pressure force minus the frictional force.

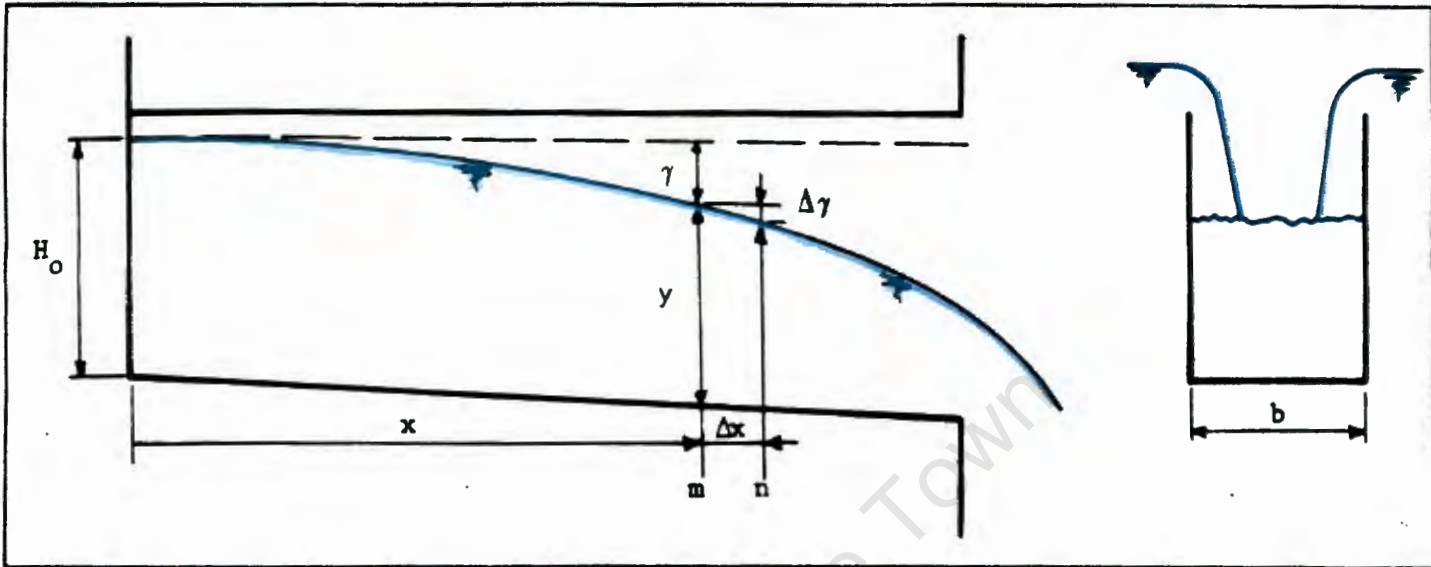


Figure 2.2 : Geometry of Camp's theoretical receiving channel

The static pressure force at section m is

$$P_m = \frac{b w y^2}{2} \quad (2.7)$$

and at section n is

$$P_n = \frac{bw}{2} (y - \Delta\gamma + S_o \Delta x)^2 = \frac{bw}{2} (y^2 - 2y \Delta\gamma + 2y S_o \Delta x) \quad (2.8)$$

The horizontal component of the static pressure on the bottom is

$$P_B = bw \left[\frac{2y - \Delta\gamma + S_o \Delta x}{2} \right] S_o \Delta x = w b y S_o \Delta x \quad (2.9)$$

The net static pressure force on the water prism between sections m and n is the summation of eqtns. (2.7), (2.8) and (2.9), in other words

$$P_m - P_n + P_B = w b y \Delta\gamma \quad (2.10)$$

The slope of the receiving channel has no effect upon the value of the net pressure. It may also be shown that changing the width b has no influence on the value of this force.

The frictional drag force is

$$P_f = \tau(b + 2y)\Delta x = \frac{\tau b y \Delta x}{R} \quad (2.11)$$

in which τ is the frictional drag per unit area of channel wall, and R is the hydraulic radius at section n . The equivalent water head on the area, corresponding to the friction drag P_f is :

$$\text{Head} = \frac{P_f}{w b y} = \frac{\tau b y \Delta x}{w b y R} = \frac{\tau}{w R} \Delta x \quad (2.12a)$$

and from the Darcy-Weisbach formula, this lost head is given by

$$\text{lost head} = \frac{f \Delta x V^2}{2gR} \quad (2.12b)$$

Hence by equating (2.12a) and (2.12b) :

$$\tau = \frac{w f V^2}{2g} \quad (2.13)$$

where f is the Darcy-Weisbach friction factor. From eqtns. (2.11) and (2.13)

$$P_f = \frac{w f V^2}{2g} \cdot \frac{b y}{R} \Delta x \quad (2.14)$$

Therefore the net acceleration force on the water prism between sections m and n , can be derived from eqtn. (2.10) and (2.14).

$$F_{a_{mn}} = w b y \Delta\gamma - \frac{w f V^2}{2g} \cdot \frac{b y}{R} \Delta x \quad (2.15)$$

Since the water volume between sections m and n is

$$b y \Delta x = \frac{\Delta x}{V} Q \quad (2.16)$$

the accelerating force on the water flow per second (Q), is

$$\frac{V}{\Delta x} \left[w b y \Delta \gamma - \frac{w f V^2}{2g} \cdot \frac{b y \Delta x}{R} \right] = w Q \frac{\Delta \gamma}{\Delta x} - \frac{w Q f V^2}{2g R} \quad (2.17)$$

Expressing eqtn. (2.17) in terms of differentials and equating to eqtn. (2.6) :

$$\frac{d\gamma}{dx} - \frac{f V^2}{2g R} = \frac{1}{g} \left[V \frac{dV}{dx} + \frac{q V^2}{Q} \right] \quad (2.18)$$

The only difference between this equation and the one derived by Hinds eqtn. (2.6), is the inclusion of a friction term.

Mathematical manipulation of eqtn. (2.18), with the following inclusions, leads to the more familiar and convenient form of expressing the water surface slope.

$$V = \frac{Q}{A} \quad (2.19)$$

$$\begin{aligned} \frac{dV}{dx} &= \frac{q}{A} - \frac{Q}{A^2} \frac{dA}{dx} \\ &= \frac{q}{A} - \frac{Q}{A^2} \frac{dA}{dy} \cdot \frac{dy}{dx} \\ &= \frac{q}{A} - \frac{QB}{A^2} \frac{dy}{dx} \end{aligned} \quad (2.20)$$

where A is the cross-sectional area and B is the water surface width, at a distance x from the upstream wall. The water depth from the bottom of the channel is y with H_0 as the depth at the upstream wall. Therefore

$$\eta = H_0 + S_0 x - y \quad (2.21)$$

$$\frac{d\eta}{dx} = S_0 - \frac{dy}{dx} \quad (2.22)$$

Substituting eqtns. (2.19), (2.20) and (2.22) into eqtn. (2.18) gives

$$S_0 - \frac{dy}{dx} = \frac{1}{g} \left[\frac{Q}{A} \left[\frac{q}{A} - \frac{QB}{A^2} \frac{dy}{dx} \right] + \frac{qQ}{A^2} \right] + \frac{f Q^2}{2g A^2 R} \quad (2.23)$$

This can be rearranged to give

$$\frac{dy}{dx} = \frac{S_0 - \frac{f Q^2}{2g A^2 R} - \frac{2qQ}{g A^2}}{1 - \frac{Q^2 B}{g A^3}} \quad (2.24)$$

In more general terms eqtn. (2.24) can be written as

$$\frac{dy}{dx} = \frac{S_0 - S_f - \frac{2qQ}{g A^2}}{1 - F_r^2} \quad (2.25)$$

where S_f = the friction slope or head loss per unit length and F_r is the Froude number. In numerical terms

$$S_f = \frac{f Q^2}{2g A^2 R}$$

and

$$F_r^2 = \frac{Q^2 B}{g A^3}$$

Research by Farney and Marcus [13] into a side channel spillway with inflow over the upstream wall as well as the side walls, led to a modification of eqtn. (2.25). This was due to the appreciable momentum in the direction of flow, of the water entering the channel over the upstream wall.

In turn, velocity variations near the upstream end of the receiving channel were evident. As a result a momentum correction factor β was introduced to eqtn. (2.25),

$$\frac{dy}{dx} = \frac{S_o - S_f - \frac{2q^2 x \beta}{gA^2} - \frac{V^2}{g} \frac{d\beta}{dx}}{1 - \frac{q^2 x^2 B \beta}{g A^3}} \quad (2.26)$$

The factor β was derived empirically from model studies. The value varied from unity at the downstream end of the receiving channel, to a maximum at the upstream end. In this study the inflow is limited to the side walls of the receiving channel. That is, no inflow over the upstream wall. This implies a value of unity to the momentum coefficient ($\beta = 1$), reducing eqtn. (2.26) to the original eqtn. (2.25).

Smith [19] utilised the principle of energy conservation to derive an equation for the slope of the water surface profile. In this derivation the energy to accelerate the flow has two components, namely the energy absorbed and the energy dissipated. An equation was derived similar to that of Farney and Marcus eqtn. (2.26),

$$\frac{dy}{dx} = \frac{S_o - S_f - \frac{2q^2 x \alpha}{gA^2}}{1 - \frac{q^2 x^2 \beta \alpha}{gA^3}} \quad (2.27)$$

where α is the energy coefficient. However this equation did not take into account the variation of the energy coefficient for different values of x that is for different positions along the receiving channel.

In the discussion of Smith's paper, Babb and Ross [19] modify Smith's equation to include a term that accounts for the variation of α with x . The alternative derivation presented by Babb and Ross leads to the following equation :

$$\frac{dy}{dx} = \frac{S_o - S_f - \frac{3}{2} \frac{q^2 x \alpha}{gA^2} - \theta' - \frac{V^2}{2g} \frac{d\alpha}{dx}}{1 - \frac{q^2 x^2 \beta \alpha}{gA^3}} \quad (2.28)$$

where θ' is the dissipated energy gradient. The momentum - correction eqn. (2.26) must then be compared to eqn. (2.28) in order to establish a value for θ' . However the numerical values of α and β are not usually known. Therefore assuming α and β are unity, then $\theta' = q^2 x / 2gA^2$. Figure 2.3 illustrates the nomenclature used.

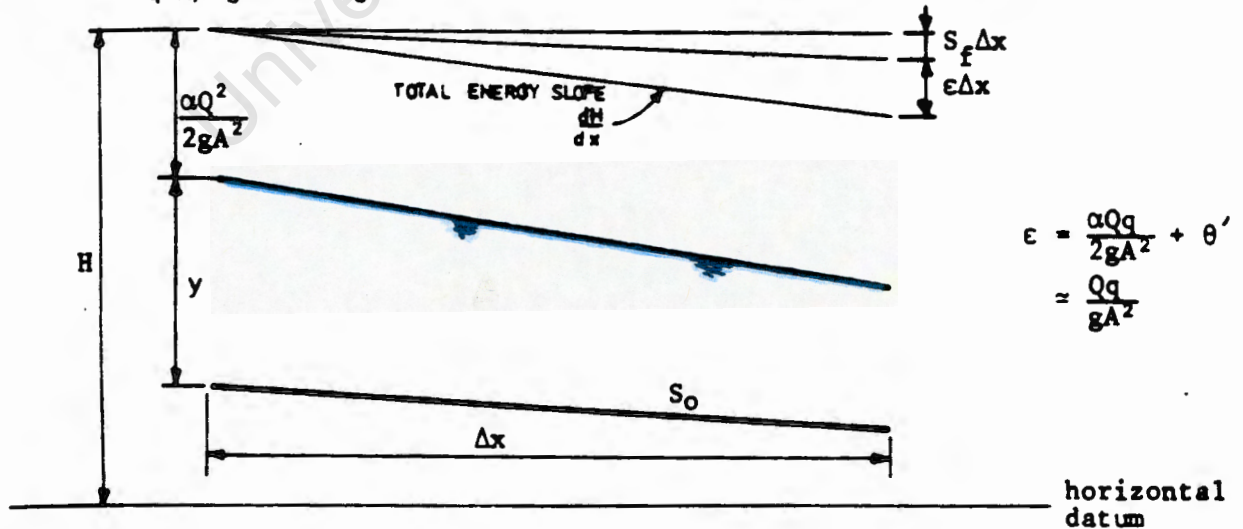


Figure 2.3 : Babb and Ross - definition sketch

Further experiments were conducted by Babb and Ross to determine values of α and β . Figure 2.4 shows a graphical representation of their results. These values were obtained from velocity profiles taken at intervals along the channel.

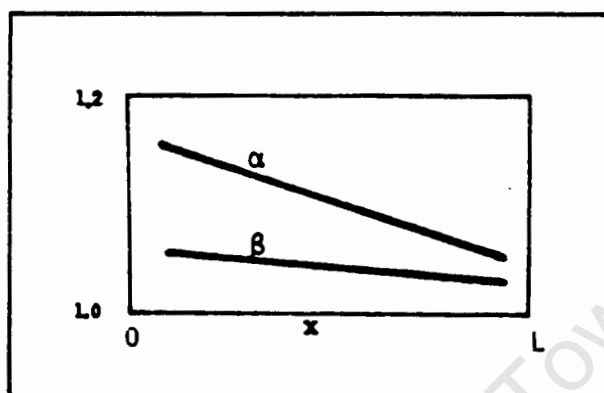


Figure 2.4 : Empirical values for α and β

After establishing these values, a comparison was made between the experimental results and the calculated profiles with α and β . These in turn were compared to calculated profiles with α and β equal to one. The difference between the two calculated profiles was negligible.

Hager [40] conducted a dimensionless analysis on steady flows in prismatic, trapezoidal side channel spillways. In this analysis the water surface slope equation was taken as equation (2.25). However in a later paper published by Bremen and Hager [42], this equation was extended to include the streamwise momentum contribution due to the lateral inflow. Assuming that the lateral inflow lies in a plane normal to the spillway, the water surface slope equation then reads

$$\frac{dy}{dx} = \frac{S_0 - S_f - \left[2 - \frac{U \cdot S_0}{V} \right] \frac{q^2 x^2}{g A^2}}{1 - F_r^2} \quad (2.29)$$

where $U \cdot S_0$ is the streamwise velocity component of the lateral inflow and V is the local velocity in the receiving channel. Equation (2.29) has been derived assuming a prismatic channel, hydrostatic pressure and uniform velocity distributions. The lateral inflow velocity U depends on the difference in elevation of the head on the spillway H_{spill} and the water surface profile $y(x)$, and is expressed as :

$$U = [2g (Z_0 + H_{spill} + S_0 \cdot x - y)]^{1/2} \quad (2.30)$$

The addition of the streamwise momentum component of the lateral inflow, leads to a shallower water surface profile than before. A series of numerical comparisons were conducted, involving bed slopes of up to 5%, with reasonable spillway heads and channel depths. In comparing the water surface profile generated with the inclusion of this term, to the one without, the average difference is less than 2%. The maximum difference in water depth was found to be 3.5%, located at the upstream wall of the receiving channel. This discrepancy was considered minor. The additional calculation complexity with the inclusion of this term, especially when a "gate" control within the receiving channel is involved, seems unnecessary when comparing the difference in water depths obtained for moderate slopes.

Therefore in all further discussion, the correct equation for the slope of the water surface profile shall be taken as eqn. (2.25). However this equation cannot be directly integrated to yield an explicit equation for the water depth. Numerical integration must be employed in a specific channel from a known starting point on the water surface profile. These known points are called control points.

2.4 Control Points

There are three types of control points, a critical depth control point, a normal depth control point and a "gate" control. The

location and type of control depends on the inflow and downstream conditions. These conditions are directly influenced by the channel geometry, slope and inflow rate.

The most common type of control point is the critical depth control point. This occurs when the inflow slope is considered "mild" and the outfall chute at the end of the receiving channel is "steep". Assuming that the channel is not drowned, the water depth at the end of the receiving channel will be critical, hence the term critical depth control.

To determine whether the inflow slope is hydraulically "mild" or "steep", the concept of indicator lines is used.

2.4.1 Indicator lines

The equations for the indicator lines are derived from eqn. (2.25). The numerator and denominator of this eqn. lead to the concepts of the pseudo normal depth line and critical depth line respectively, the equations of these lines being obtained by equating the numerator and denominator to zero separately.

In mathematical terms the pseudo normal depth line is given by :

$$S_o - S_f - \frac{2 q^2 x}{gA^2} = 0 \quad , \quad (2.31)$$

and the critical depth line by

$$1 - F_r^2 = 0 \quad . \quad (2.32)$$

The pseudo normal depth line is generally analogous to the normal depth line in spatially constant flow. The name pseudo normal depth line was coined by Kilner [39]. Prior to this Fox and Goodwill [22] stated that the concept of a normal depth line is meaningless in spatially varied discharge.

The line described by eqn. (2.31) was called the "energy balance line". Fox and Goodwill's claim is only true if the normal depth definition is limited to that depth to which the water will tend to in a long uniform channel. If one looks at the broader behaviour and interaction with the critical depth line, a good correlation can be found to that of spatially constant flow. So the prefix "pseudo" compensates for the fact that this is not a real normal depth line but a good indicator of the behaviour of the normal depth line with respect to the critical depth line. In future the abbreviation PNDL will denote pseudo normal depth line and CDL will denote critical depth line.

Applying eqn. (2.32) to a rectangular channel gives

$$1 - \frac{q^2 x^2}{g b^2 y_c^3} = 0 \quad , \quad (2.33)$$

where y_c is the critical depth. Therefore

$$y_c = K_2 x^{2/3} \quad (2.34)$$

with K_2 a constant equal to $3 \sqrt{\frac{q^2}{gb^2}}$. Similarly, from eqn. (2.31) neglecting frictional effects.

$$S_0 - \frac{2 q^2 x}{gb^2 y_0^2} = 0 \quad (2.35)$$

where y_0 is the pseudo normal depth, implying

$$y_0 = K_2 x^{\frac{1}{2}} \quad , \quad (2.36)$$

(Bottom width, 10 ft.; side slopes, 1 : 1.)

$$\Delta y = \frac{Q_1 (V_1 + V_2)}{g (Q_1 + Q_2)} \left\{ \Delta V + \frac{b V_2 \Delta x}{Q_1} \right\}$$

x	Δx	Q	Q ₁ +Q ₂	d _c	V _c	V ₁ +V ₂	ΔV	$\frac{b V_2 \Delta x}{Q_1}$	$\Delta V + \frac{b V_2 \Delta x}{Q_1}$	Δy	h _f ?	Δy+h _f
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
0												
10	10	400	400	3.4	10.0	10.0	10.0				0.05	
25	15	1 000	1 400	6.2	12.5	22.5	2.5	18.8	21.3	4.25	0.04	4.29
50	25	2 000	3 000	9.2	14.9	27.4	2.4	14.9	17.3	4.91	0.06	4.99
100	50	4 000	6 000	13.5	17.6	32.5	2.7	17.6	19.3	6.77	0.15	6.92
150	50	6 000	10 000	16.9	19.3	36.9	1.7	9.6	11.3	5.21	0.15	5.36
200	50	8 000	14 000	19.7	20.6	39.9	1.3	6.8	8.1	4.33	0.15	4.48
250	50	10 000	18 000	22.1	21.6	42.8	1.0	5.4	6.4	3.74	0.15	3.89
300	50	12 000	22 000	24.2	22.4	44.0	0.8	4.5	5.3	3.30	0.15	3.45
350	50	14 000	26 000	26.2	23.2	45.6	0.8	3.8	4.6	3.01	0.15	3.19
400	50	16 000	30 000	28.0	23.8	47.0	0.6	3.1	4.0	2.75	0.15	2.89
										33.08	1.17	39.45

* h_f = friction loss, computation not shown.

Table 2.2 : Computation of Control point [Hinds, 1926]

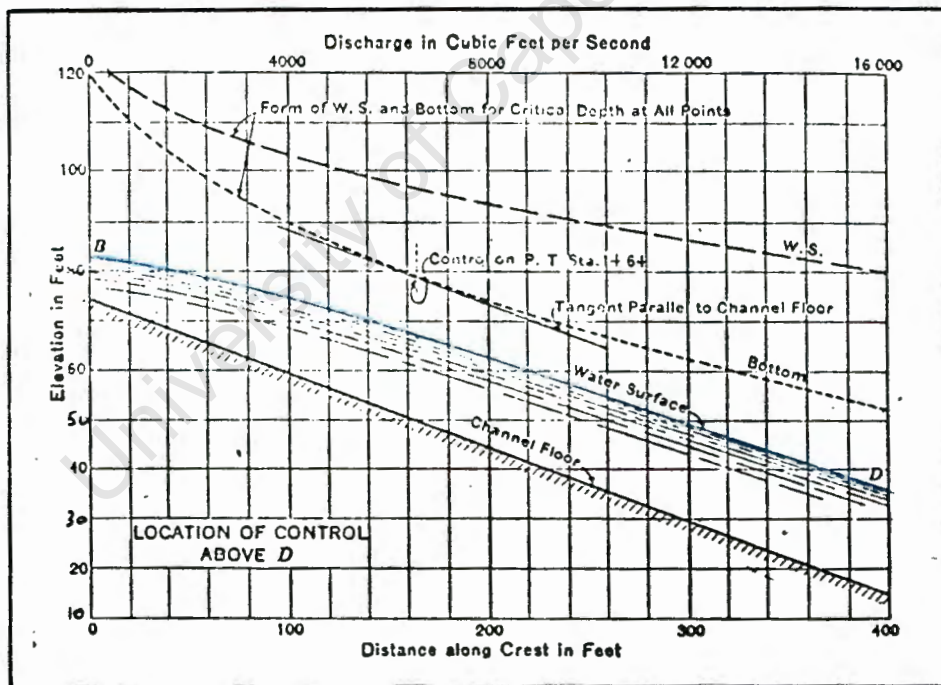


Figure 2.7 : Location of Control point [Hinds, 1926]

refine the present method of locating the control point. The term energy balance line was later changed to pseudo normal depth line by Kilner [39].

2.4.2 Types of control points

There are three types of control points. These are defined as follows:

- A "gate" control is one in which the pseudo normal depth line and critical depth line intersect within the receiving channel, and this permits the water surface to pass through this point, hence the concept of a "gate".
- A critical depth control is one in which the pseudo normal depth line and critical depth line intersect at the end of the receiving channel. The intersection occurs at the vertical transition from the pseudo normal depth line in the receiving channel to the normal depth line in the outflow chute. This vertical transition is due to the cessation of lateral inflow.
- A normal depth control is one in which the outflow chute's hydraulic slope is mild, leading to a known water depth at the end of the receiving channel.

Figures 2.5 and 2.6 illustrate these three types of control points. It should be noted that either a gate control or a critical depth control can occur with or without an additional normal depth control. A gate control occurs in a hydraulically steeper receiving channel than a critical depth control. The occurrence of a normal depth control is entirely dependent on the hydraulic slope of the outflow chute.

Both the gate control and the critical depth control present problems in determining the initial water surface slope. This is due to the fact that at both of these control points, the water surface passes through the critical depth. Therefore Froude number is equal to zero,

with K_2 a constant equal to $\sqrt{\frac{2 q^2}{S_0 g b^2}}$. Thus comparing y_0 to y_c

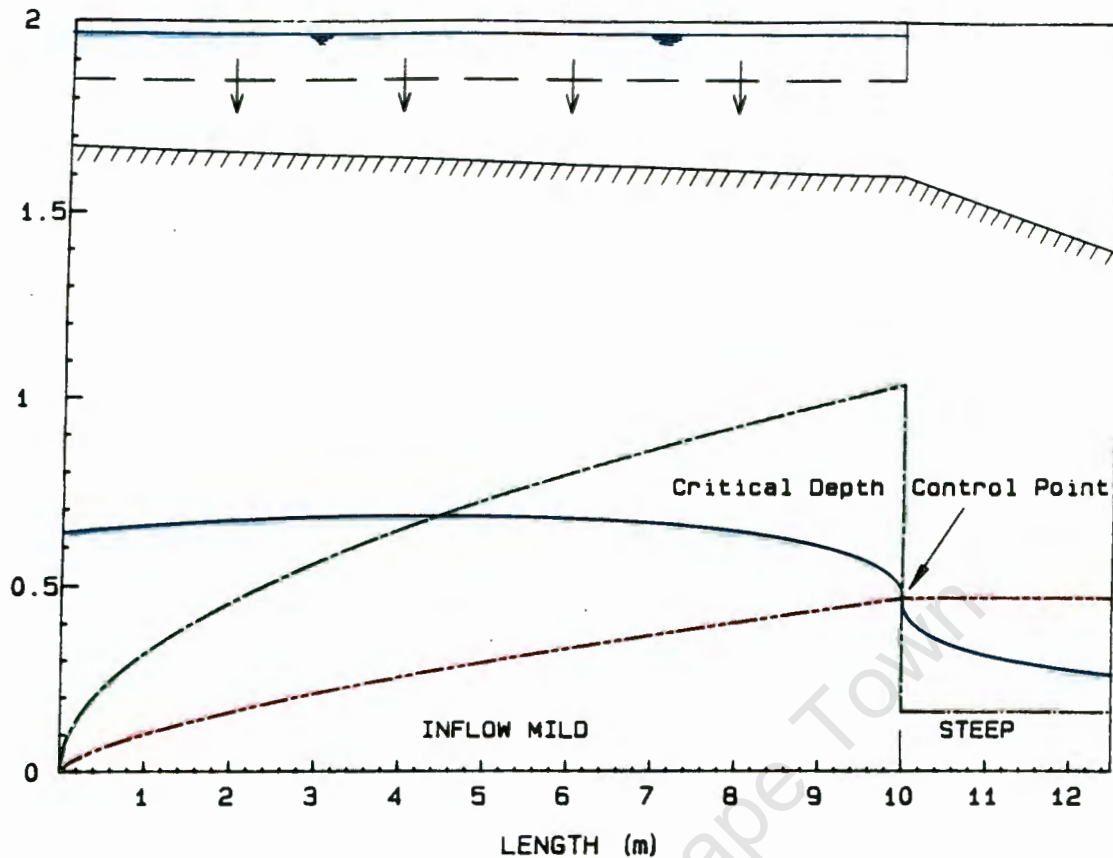
$$\frac{y_0}{y_c} = \frac{K_2 x^{\frac{1}{3}}}{K_1 x^{\frac{2}{3}}} \propto \frac{1}{x^{\frac{1}{6}}} \quad (2.37)$$

As x tends to zero, y_0/y_c tends to infinity. Therefore the pseudo normal depth is always greater than the critical depth near the upstream wall. When the pseudo normal depth is less than the critical depth, then the channel is hydraulically STEEP. When the pseudo normal depth is greater than the critical depth, then the channel is hydraulically MILD. Therefore in spatially varied discharge as x tends to zero, the channel is always hydraulically mild in slope. To differentiate the hydraulic slopes of spatially varied discharge and spatially constant discharge the word INFLOW is written as a prefix for the spatially varied discharge case.

Control points occur where the critical depth line and the pseudo normal depth line intersect. The hydraulic conditions in the outflow chute also affect the location and type of control point. It is therefore necessary to sketch the critical depth line and normal depth line as defined in spatially constant discharge, for the outflow chute.

Historically the critical depth line has been used as an indicator line since Hinds' classical paper [2]. However in these early papers (excluding Hinds) the control point was assumed to be a critical depth control (to be defined in the next section), located at the end of the receiving channel. Not until 1967 was an attempt made to locate the control point within the receiving channel. In these steeper channels Smith [19] employed the concept of a transition profile. The transition profile was plotted on a long section with the critical depth line. The intersection of the two lines is the location of the control point. Only in 1970 did Fox and Goodwill [22]

DEPTH (m)



Code:

Critical Depth Line

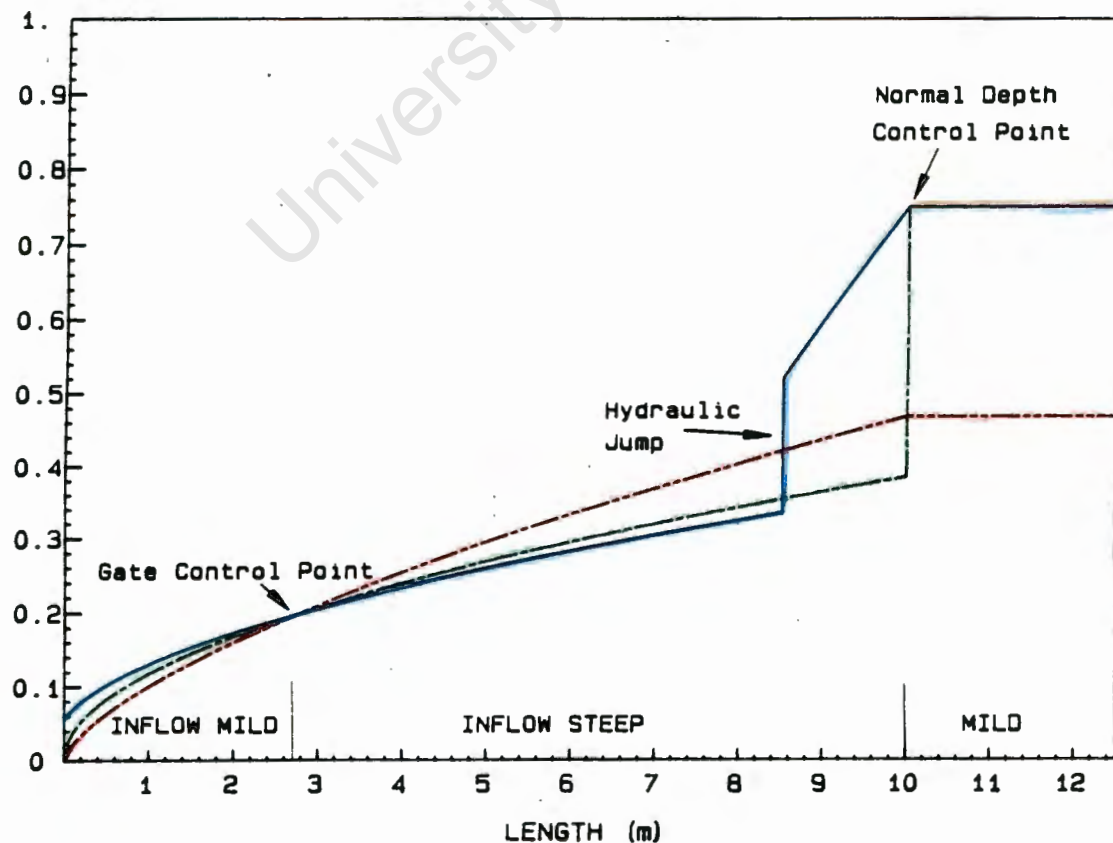
Pseudo Normal Depth Line

Normal Depth Line

Calculated Water Surface Profile

FIGURE 2.5 CRITICAL DEPTH CONTROL

DEPTH (m)



Code:

Critical Depth Line

Pseudo Normal Depth Line

Normal Depth Line

Calculated Water Surface Profile

FIGURE 2.6 GATE CONTROL PLUS NORMAL DEPTH CONTROL

thus making the denominator of eqn. (2.25) zero. This in turn makes dy/dx indeterminate in its present form.

The next step is to derive a method to determine the local water surface slope at these control points.

2.4.3 The location of the control point and the determination of the local water surface slope

At a critical depth control point the denominator of the water surface slope eqn. (2.25) is zero, while the numerator has a finite value. This leads to an infinite value for dy/dx . An approximation is therefore necessary for the slope of the water surface in this region. In the case of gate control, both the numerator and denominator are zero, leading to an apparently indeterminate value for dy/dx .

Early researcher's attempted to simplify the situation, to obtain quick numerical solutions. Hinds [2] adopted the following approach to locate the control point.

The critical velocities and discharges corresponding to a number of depths are calculated as shown in Table 2.1 . Hydraulic radii are calculated at the same time in order to estimate the friction losses. From Table 2.1 values of the critical depths and velocities are then substituted into Table 2.2 in order to locate the control point. Column 13 in Table 2.2 gives the drop in the water surface needed to maintain the flow at the critical depth throughout the full length of the channel. Starting from an arbitrary water surface elevation, a profile of the channel base can be plotted for critical flow at all points. This is illustrated in Figure 2.7 . A tangent drawn parallel to the actual channel slope may then be drawn to the channel profile for critical flow. The point of tangency is then the location of the control point. The water depth at the control point is the critical depth at that point. This is then taken as the starting point for the calculation of a backwater function.

(Bottom width, 10 ft.; side slopes, 1 : 1.)

$$\Delta y = \frac{Q_1 (V_1 + V_2)}{g (Q_1 + Q_2)} \left\{ \Delta V + \frac{b V_2 \Delta x}{Q_1} \right\}$$

x	Δx	Q	Q ₁ +Q ₂	d _c	V _c	V ₁ +V ₂	ΔV	$\frac{b V_2 \Delta x}{Q_1}$	$\Delta V + \frac{b V_2 \Delta x}{Q_1}$	Δy	h _f ?	Δy+h _f
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
0												
10	10	400	400	3.4	10.0	10.0	10.0				0.08	
25	15	1 000	1 400	6.2	12.5	22.5	2.5	18.8	21.3	4.25	0.04	4.29
50	25	2 000	3 000	9.2	14.9	27.4	2.4	14.9	17.3	4.91	0.08	4.99
100	50	4 000	6 100	13.5	17.6	32.5	2.7	17.6	19.3	6.77	0.15	6.92
150	50	6 000	10 000	16.9	19.3	38.9	1.7	9.6	11.3	5.21	0.15	5.36
200	50	8 000	14 000	19.7	20.6	39.9	1.3	6.8	8.1	4.33	0.15	4.48
250	50	10 000	18 000	22.1	21.6	42.2	1.0	5.4	6.4	3.74	0.15	3.89
300	50	12 000	22 000	23.2	22.4	44.0	0.8	4.5	5.3	3.30	0.15	3.45
350	50	14 000	26 000	23.2	23.2	45.6	0.8	3.8	4.6	3.01	0.15	3.16
400	50	16 000	30 000	23.0	23.8	47.0	0.6	3.4	4.0	2.73	0.15	2.88
										39.28	1.17	39.45

* h_f = friction loss, computation not shown.

Table 2.2 : Computation of Control point [Hinds, 1926]

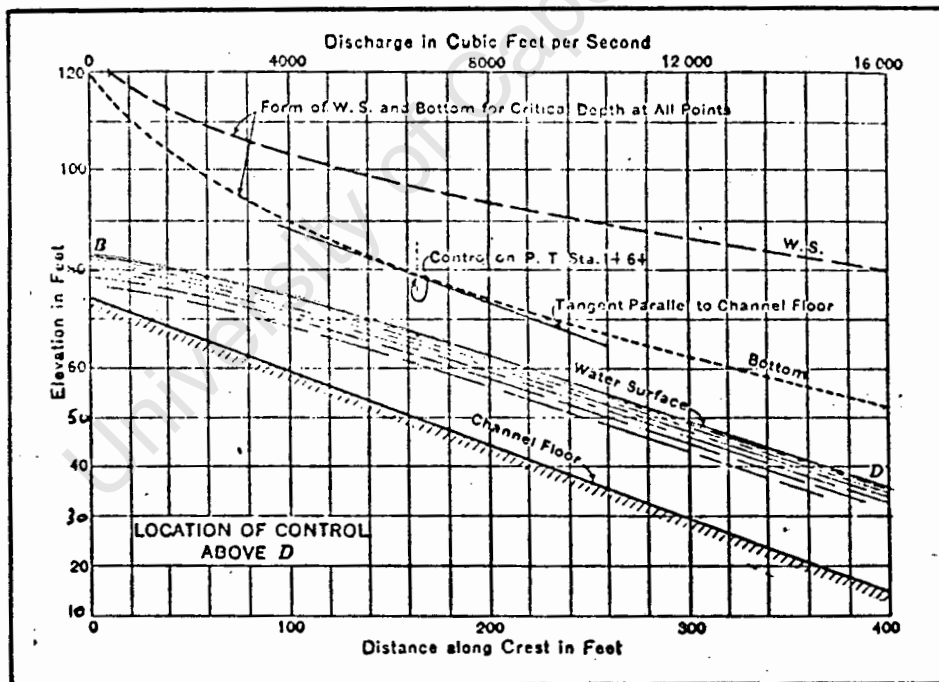


Figure 2.7 : Location of Control point [Hinds, 1926]

In the years following the publication of Hinds' paper, researchers concentrated on the calculation of the actual water surface profile, not the location of the control point. The common assumption was that the control point was a critical depth one located at the end of the receiving channel.

Data for Hinds' example, (in foot units as originally presented)

Receiving channel's length : $L = 400$ ft
width : $b = 10$ ft
side slope : $\theta = 2:1$ (trapezoid)
inflow rate : $q = 40$ cubic ft/sec/ft
Bed slope : $S_0 = 0.15$

(Bottom width, 10 ft.; side slopes, $\frac{1}{2}:1$)

Depth, in feet, d .	Area, in square feet, A .	Top width, in feet, T .	Velocity head, $\frac{A}{2T}$	Critical velocity, in feet per second.	Discharge, Q .	Hydraulic radius, in feet.
2	22	12	0.92	7.68	169	1.52
4	48	14	1.71	10.49	294	2.53
6	78	16	2.44	12.52	373	3.33
8	112	18	3.11	14.15	485	4.01
10	150	20	3.75	15.58	630	4.63
12	192	22	4.36	16.75	816	5.22
14	238	24	4.96	17.66	1032	5.76
16	288	26	5.54	18.33	1280	6.29
18	342	28	6.11	19.02	1560	6.82
20	400	30	6.67	20.71	1874	7.31
22	462	32	7.22	21.55	2220	7.81
24	528	34	7.76	22.34	2600	8.29
26	598	36	8.31	23.19	3010	8.77
28	672	38	8.84	23.84	3460	9.26

Table 2.1 : Computation of Critical depth [Hinds, 1926]

It was not until 1952 that further research was done on the location of the control point by Keulegan [10]. In this research Keulegan observed that the point where the water surface crosses the critical depth is only located at the exit of the receiving channel for gently sloping channels. In steeper channels this point moves upstream giving a control point within the receiving channel.

Due to the fact that the numerator and the denominator of the water surface slope eqn. (2.25) are zero at a control point within the receiving channel, an equation is derived for locating the position of this gate control.

$$x = \frac{8 q^2}{(S_o - \frac{\lambda}{2})^3 g} \quad (2.38)$$

where λ is a coefficient of friction derived by Beij [5]. This is given by the formula

$$\lambda = 800 \left[\frac{v R}{\nu} \right]^{-1} \quad (2.39)$$

where ν is the kinematic viscosity. The equation for the maximum slope which will still permit a critical depth control point at the end of the receiving channel is

$$S_o = \frac{\lambda}{2} + \left[\frac{8q^2}{gL} \right]^{1/3} \quad (2.40)$$

In 1968 Sharp and James [15] developed another equation for locating the control point. This equation ignores friction and is expressed as

$$x = \frac{8 \cos^2 \theta_{so} q^2}{q b^2 \sin^3 \theta_{so}} \quad , \quad (2.41)$$

where $\sin \theta_{so}$ is the bed slope of the channel. This was the first time that the water surface slope was derived at the control point by using de l' Hopital's rule.

$$\lim_{y \rightarrow y_c} \frac{dy}{dx} = \frac{4}{3} \frac{q}{Q} y_c \quad , \quad (2.42)$$

where $Q = q x$, x being the location of the control point and y_c the critical depth.

Smith [19] in 1967, devised a new method for determining the location of the control point. The control point is obtained graphically as the intersection of two lines, one of which is the critical depth line, the other being described as the transition profile. The equation given for the transition profile is

$$\frac{q}{Q} = \frac{1}{2} \left[\frac{S_o B}{A} - \frac{gA^2}{\sigma K^2} \right] \quad , \quad (2.43)$$

$$\text{where } K = \frac{A R^{2/3}}{0,015} \quad . \quad (2.44)$$

As an illustration of this technique, Smith re-calculated Hinds example. Figure 2.8 and Table 2.3 illustrate the calculation procedure.

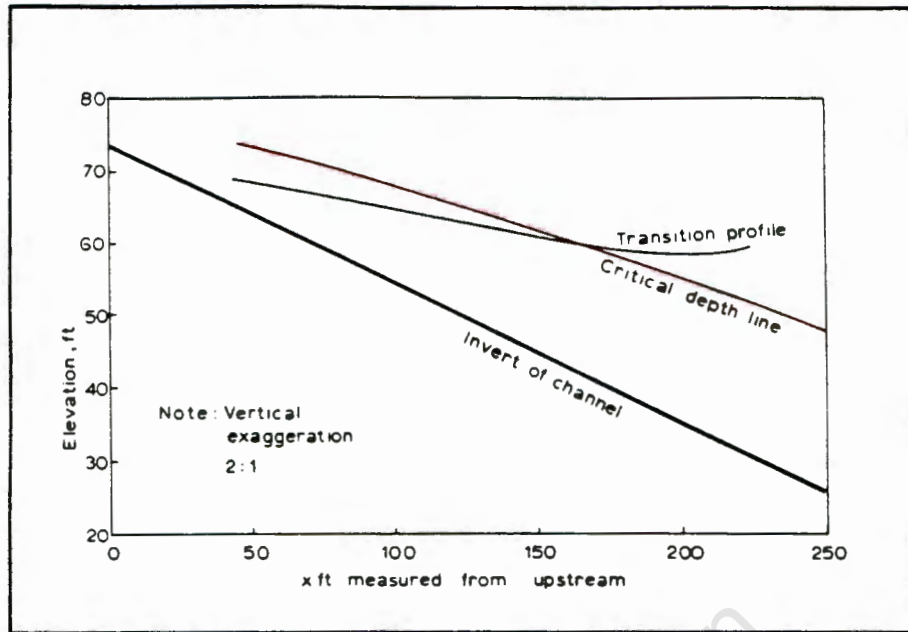


Figure 2.8 : Transition profile and critical depth line [Smith, 1967]

d	A	T	$\frac{A}{T} = d_m$	$V_c = \sqrt{g d_m}$	$Q_c = V_c A$	R	$R^{2/3}$	K	$\frac{1}{2} \left(\frac{S_0 T}{A} - \frac{g A^3}{K^2} \right)$	x
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
4	48	14	3.42	10.49	504	2.52	1.85	8,800	0.0215	46
8	112	18	6.22	14.15	1,585	4.01	2.53	28,100	0.0118	85
12	192	22	8.72	16.75	3,216	5.22	3.02	57,400	0.00843	119
16	288	26	11.08	18.68	5,440	6.29	3.40	97,200	0.00666	150
20	400	30	13.34	20.71	8,284	7.31	3.77	149,000	0.00553	181
24	528	34	15.52	22.34	11,800	8.29	4.10	212,000	0.00475	211
28	672	38	17.68	23.84	16,020	9.26	4.41	293,000	0.00418	239

Inflow = 40 cfs per ft of channel.

Table 2.3 : Computation of transition profile [Smith, 1967]

In 1970 Fox and Goodwill [22] formulated the indicator line procedure used at present. The term energy balance line was used instead of the pseudo normal depth line. The procedure has already been described in section 2.4.1 for determining the location of the control point. Smith [28] reinforced this approach in 1972. However no contribution

to the determination of the water surface slope at the control point, was made in either of these papers.

Kilner [39] addressed the problem of the determination of the water surface slope at the control point. In solving this problem two reference cases were developed, one for a critical depth control at the end of the receiving channel, the other for a gate control within the receiving channel. For the first of these reference cases the channel was considered horizontal with zero friction. The derivation for both of these reference cases is given in Appendix B. In the derivation a rectangular channel is considered initially then the theory is applied to a trapezoidal channel.

In the case of a critical depth control reference case, the theoretical basis is that in the vicinity of y_c the influence of the channel slope and friction are small and to some extent self cancelling. An initial point is taken as

$$Y = k y_c , \quad (2.45)$$

where k is a multiple of y_c , usually 1.01. This yields an initial water depth slightly greater than the critical depth. The x co-ordinate of this starting point is

$$\frac{x}{L} = \sqrt{\frac{k(3-k^2)}{2}} , \quad (2.46)$$

for a rectangular channel and

$$x^2 = \frac{gA}{q^2} \left[A_c \bar{z}_c + \frac{A_c^2}{B_c} - A \bar{z} \right] \quad (2.47)$$

for a trapezoidal channel. Subscript c denotes the critical depth parameters. The water surface slope at this starting point is given as

$$\frac{dy}{dx} = \frac{2 y_c \sqrt{2 k (3-k^2)}}{3 L (1-k^2)} \quad (2.48)$$

for a rectangular channel. The basic dy/dx eqn. (2.25) is used for a trapezoidal channel.

Consider a gate control within the receiving channel. The location is given by the intersection of the pseudo normal depth line and the critical depth line. The water surface slope eqn. (2.25) is indeterminate due to the numerator and the denominator being zero. Applying de l' Hopital's principle to this water surface slope equation gives a quadratic in Z , where Z is equal to dy/dx , $\phi(x,y)$ is the numerator and $\psi(x,y)$ is the denominator.

$$A Z^2 + B Z + C = 0 \quad (2.49)$$

$$\text{where } A = \frac{\delta\phi}{\delta y}, \quad B = \left[\frac{\delta\phi}{\delta x} - \frac{\delta\psi}{\delta y} \right] \text{ and } C = - \frac{\delta\psi}{\delta x} .$$

The values for the partial derivatives in a rectangular channel are :

$$\frac{\delta\phi}{\delta x} = - \frac{2q^2}{gb^2y^2} \left[1 + \frac{g x n^2}{R^{4/3}} \right]$$

$$\frac{\delta\psi}{\delta x} = - \frac{2q^2x}{gb^2y^3}$$

$$\frac{\delta\phi}{\delta y} = \frac{2q^2x}{3b^2y^3} \left[\frac{6}{g} + \frac{3xn^2}{R^{4/3}} + \frac{2xn^2}{yR^{1/3}} \right]$$

$$\frac{\delta\psi}{\delta y} = \frac{3q^2x^2}{gb^2y^4}$$

Through algebraic neatening, the quadratic is reduced to

$$\left[9x^2R^{4/3}\right] Z^2 - [(2x)\{9yR^{4/3} + gxn^2(3y + 2R)\}] Z + [(6y^2)(R^{4/3} + gxn^2)] = 0 \quad (2.50)$$

The values for the partial derivatives in a trapezoidal channel are:

$$\frac{\delta\phi}{\delta x} = 2 \left[\frac{q^2}{gA^2} - \frac{S_0}{x} \right] ,$$

$$\frac{\delta\phi}{\delta x} = -\frac{2}{x}$$

$$\frac{\delta\phi}{\delta y} = \frac{4q^2n^2x^2}{A^2R^{4/3}} \left[\frac{B}{2A} + \frac{1}{3R} \frac{dR}{dy} + \frac{BR^{4/3}}{xn^2gA} \right]$$

$$\frac{\delta\phi}{\delta y} = \frac{3B}{A} - \frac{1}{B} \frac{dB}{dy} ,$$

where the following geometrical statements apply :

θ = side slope angle with horizontal

B = $b + 2y \cot \theta$

p = $b + 2y \operatorname{cosec} \theta$

A = $y(b + y \cot \theta)$.

$$\text{Thus } \frac{dB}{dy} = 2 \cot \theta , \quad \frac{dp}{dy} = 2 \operatorname{cosec} \theta$$

$$\text{and } \frac{dR}{dy} = - \left[\frac{A}{p^2} \right] \frac{dp}{dy} + \frac{B}{p} = - 2 \operatorname{cosec} \theta \cdot \frac{A}{p^2} + \frac{B}{p} .$$

2.5 Water Surface Profiles

By examining the basic eqtn. (2.25) the general form of the various profiles can be deduced. These are similar to the deductions made for spatially constant discharge.

Initial attention is focused on the sign of dy/dx . This serves as an indicator as to whether the water depth is increasing or decreasing in the direction of flow. In Figure 2.9 the channel profile is divided into three zones. The labels 1, 2 and 3 indicate whether the water surface is above both lines, between the lines or below both lines respectively. The three zones are divided into the two inflow hydraulic slopes. These are labelled inflow mild (IM) and inflow steep (IS) to differentiate from spatially constant discharge.

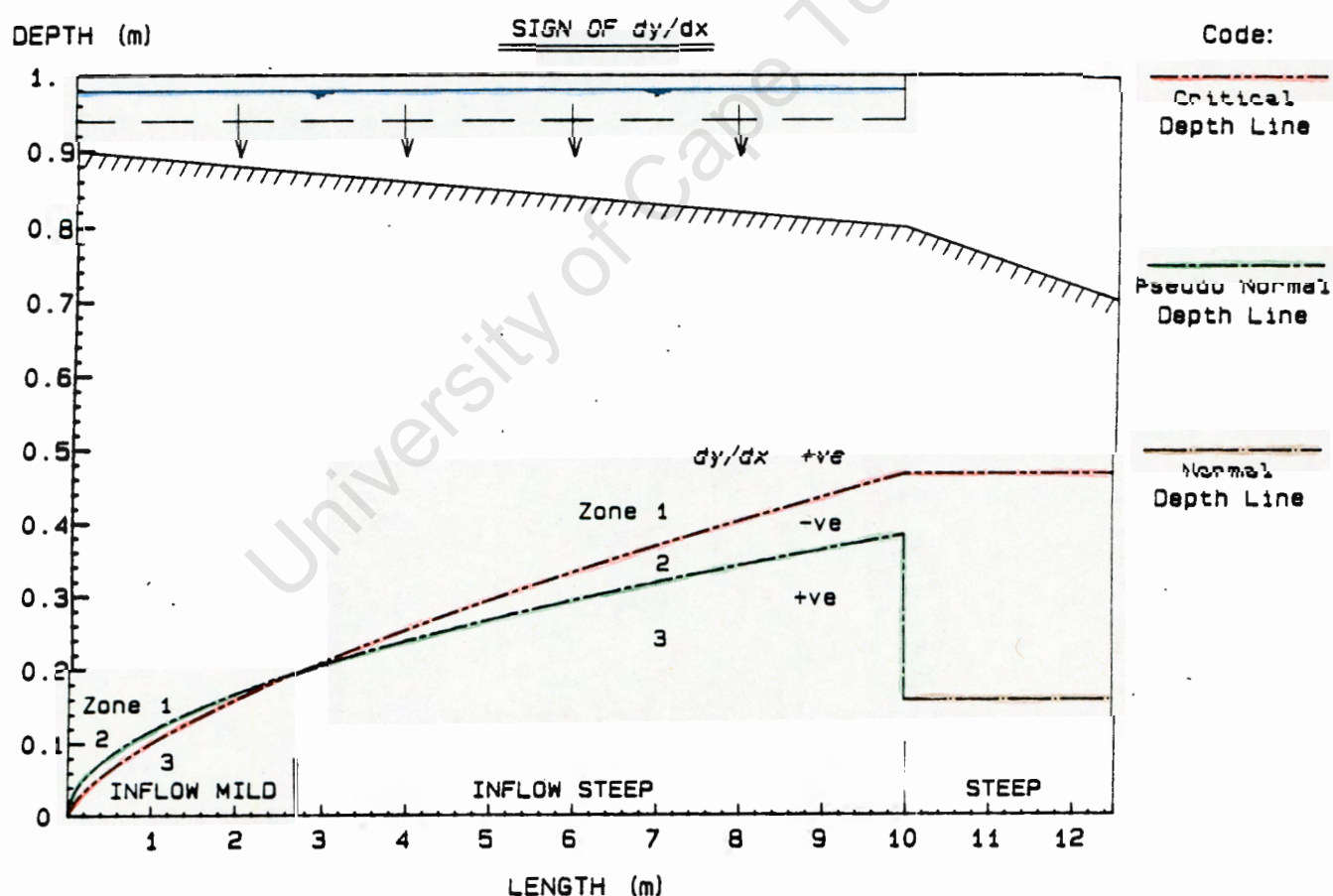


Figure 2.9 : Sign of dy/dx

Table 2.4 tabulates the different zone characteristics. Note the flow type and depth trend in each zone for the different inflow conditions. The profile labels used consist of the inflow condition followed by the zone number (eg. IM1 - inflow mild, zone 1).

<u>INFLOW MILD</u>			
	<u>ZONE 1</u>	<u>ZONE 2</u>	<u>ZONE 3</u>
Numerator	+	-	-
Denominator	+	+	-
Sign of dy/dx	+	-	+
Flow type	streaming	streaming	shooting
Profile label	IM 1	IM 2	IM 3
Depth trend	deepening	shallowing	deepening
<u>INFLOW STEEP</u>			
	<u>ZONE 1</u>	<u>ZONE 2</u>	<u>ZONE 3</u>
Numerator	+	+	-
Denominator	+	-	-
Sign of dy/dx	+	-	+
Flow type	streaming	shooting	shooting
Profile label	IS 1	IS 2	IS 3
Depth trend	deepening	shallowing	deepening

NOTE: At the upstream wall $x = 0$ $y = 0$ thus $dy/dx = S_0$ and the water surface is horizontal. Where the water surface crosses the PNDL, $dy/dx = 0$, i.e. y has a maximum value, unless the water surface crosses the CDL at the same point.

Table 2.4 Zone Characteristics

Figure 2.10 illustrates the general shape of the water surface profile types. There are only six profile types plus an inflow hydraulic jump that occurs within the receiving channel. Figures 2.11 to 2.19 illustrate all the possible combinations of water surface profile types associated with a side channel spillway.

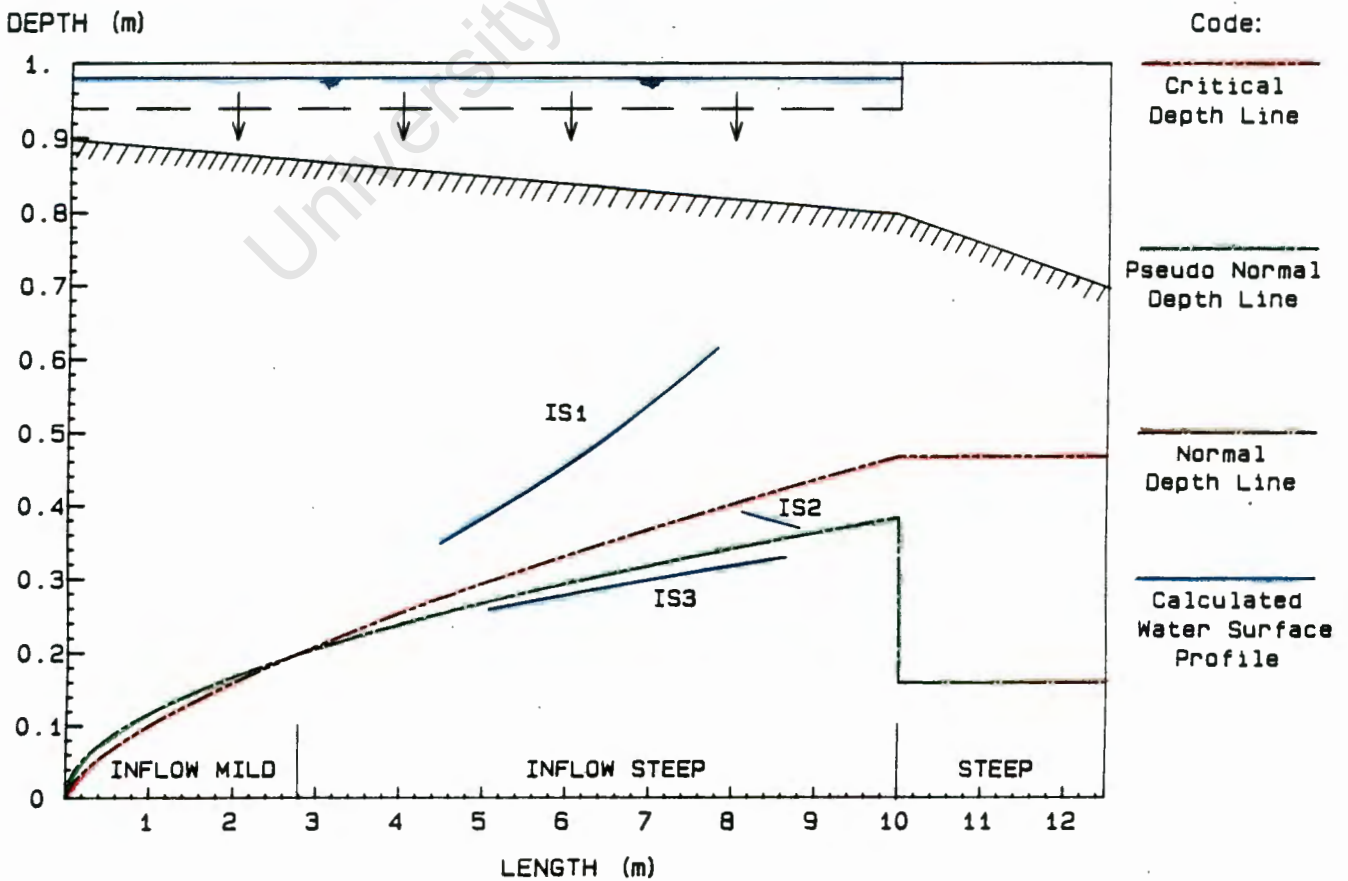
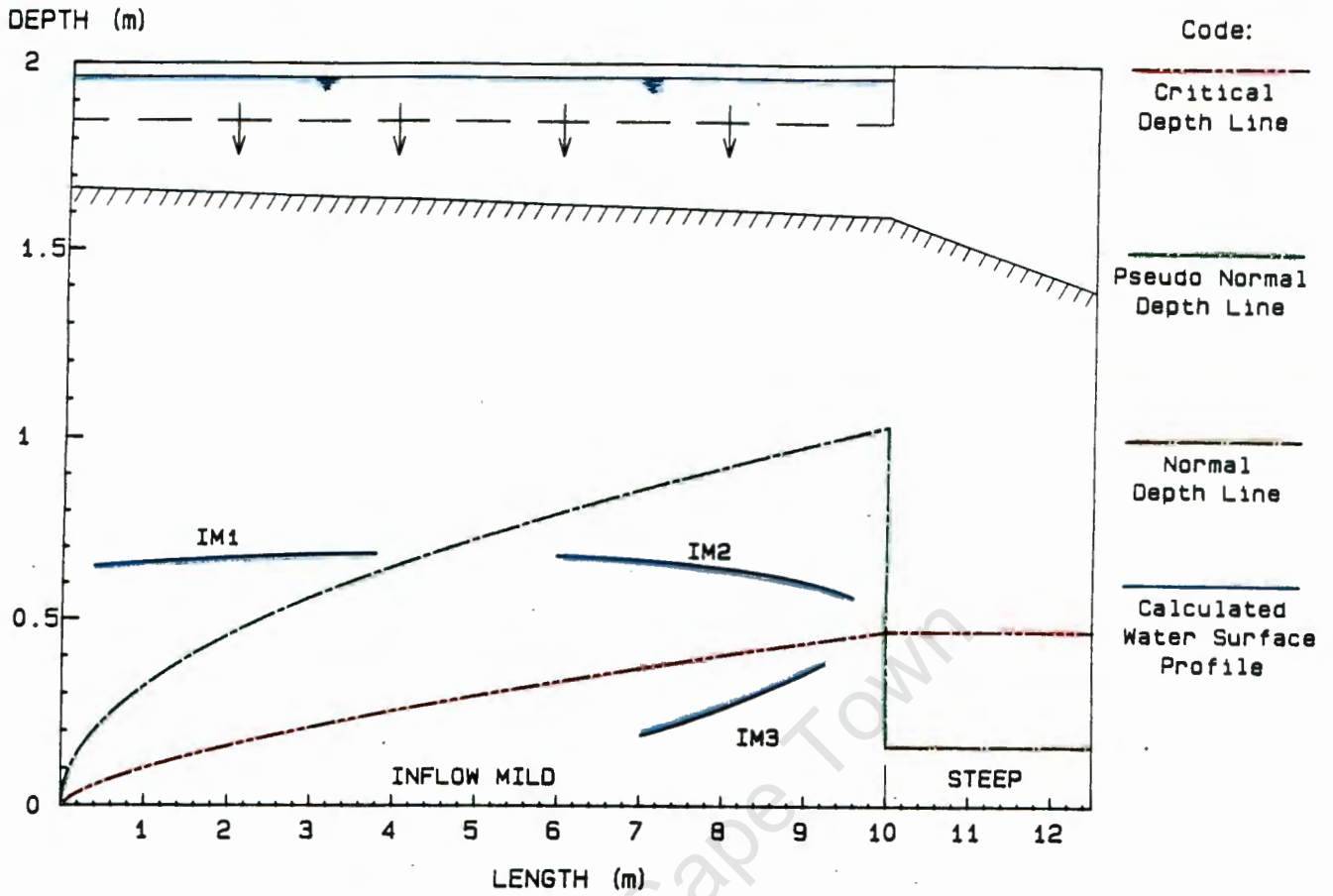
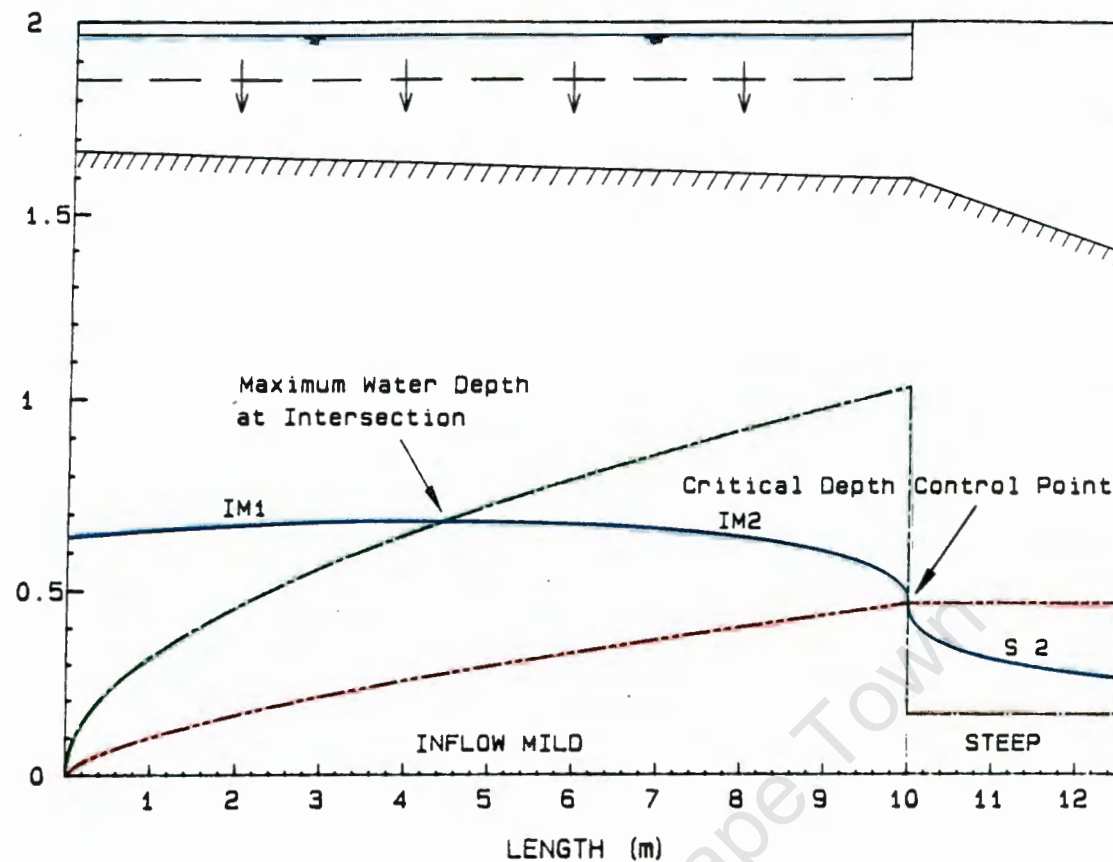


FIGURE 2.10 GENERAL SHAPE OF WATER SURFACE PROFILE TYPES

DEPTH (m)



Code:

Critical Depth Line

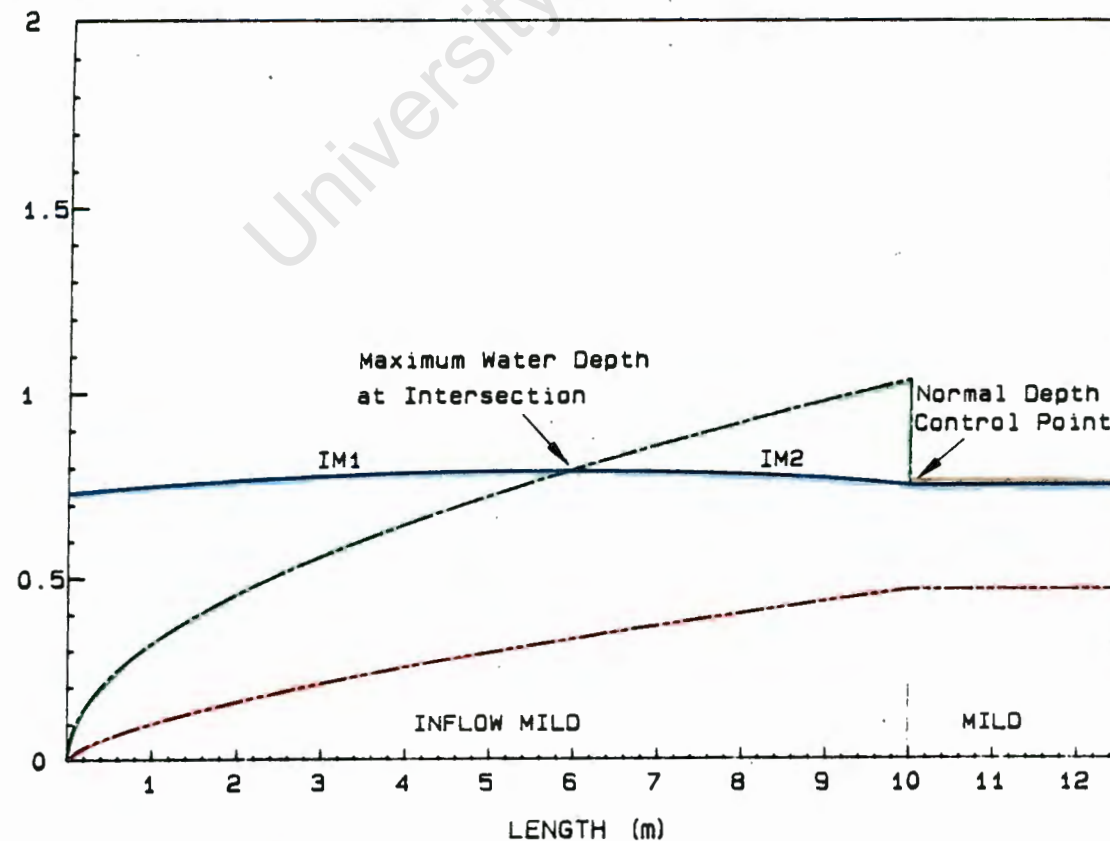
Pseudo Normal Depth Line

Normal Depth Line

Calculated Water Surface Profile

FIGURE 2.11 DEPTH CHANGES AND WATER SURFACE PROFILES

DEPTH (m)



Code:

Critical Depth Line

Pseudo Normal Depth Line

Normal Depth Line

Calculated Water Surface Profile

FIGURE 2.12 DEPTH CHANGES AND WATER SURFACE PROFILES

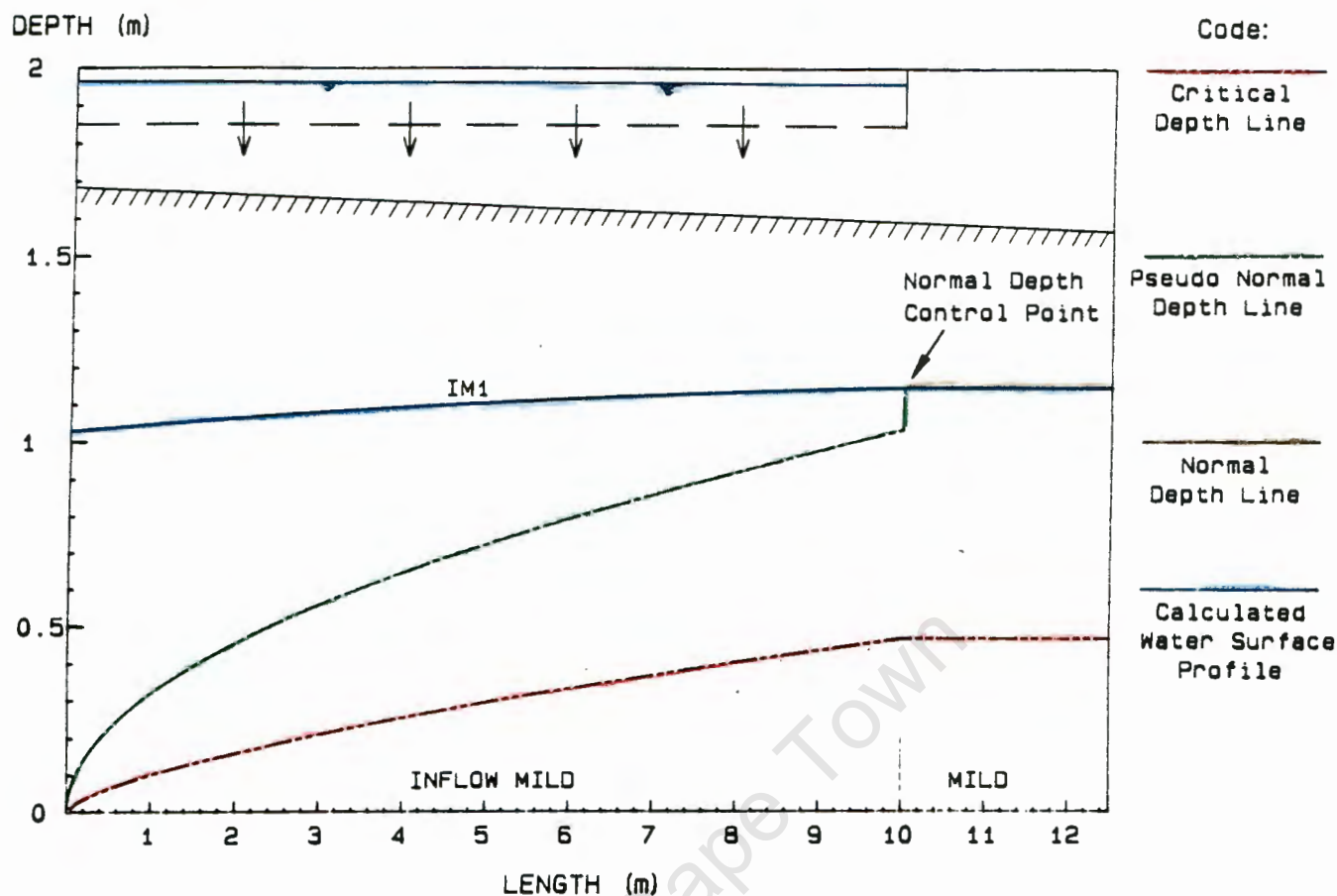


FIGURE 2.13 DEPTH CHANGES AND WATER SURFACE PROFILES

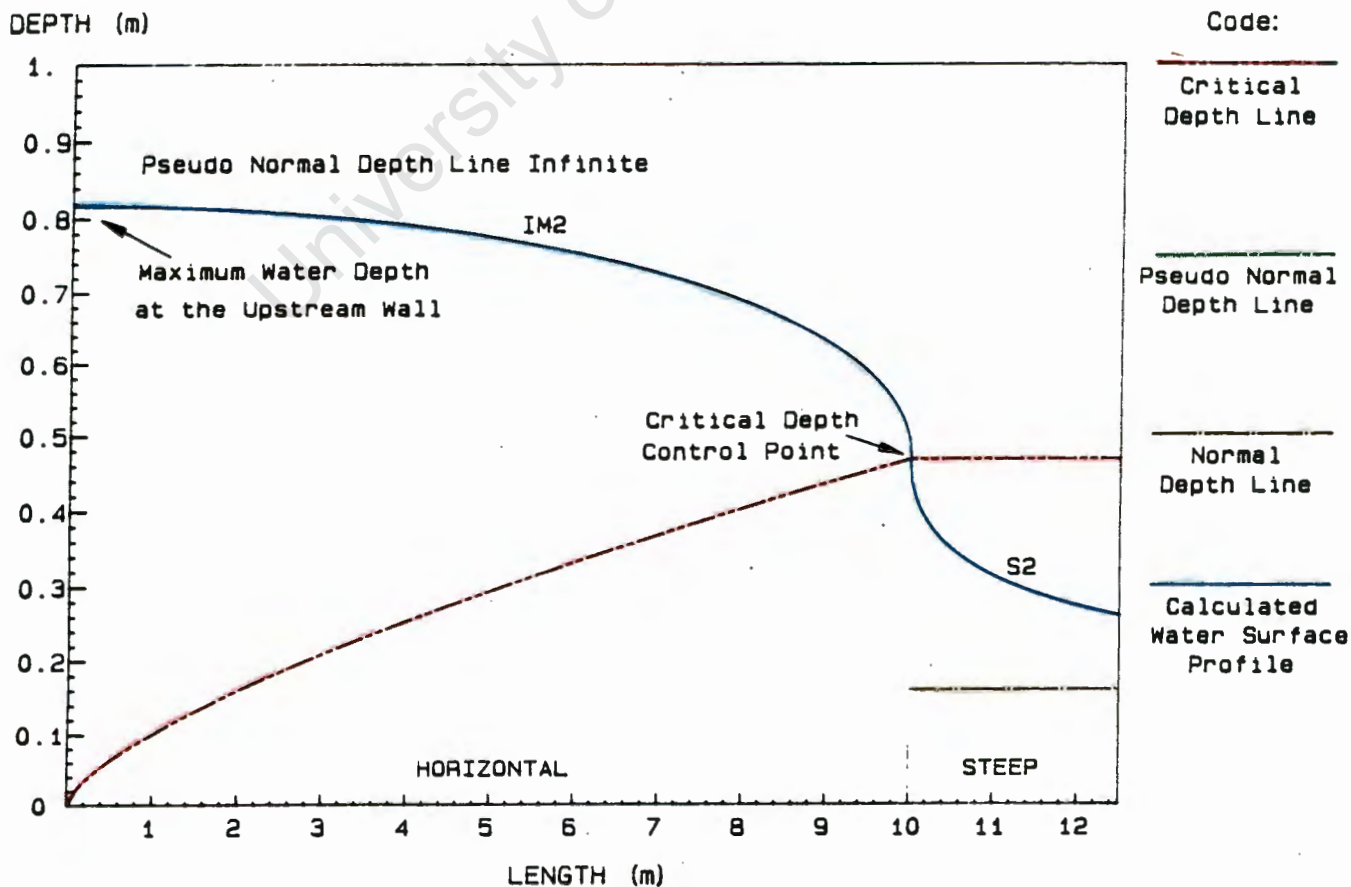


FIGURE 2.14 DEPTH CHANGES AND WATER SURFACE PROFILES

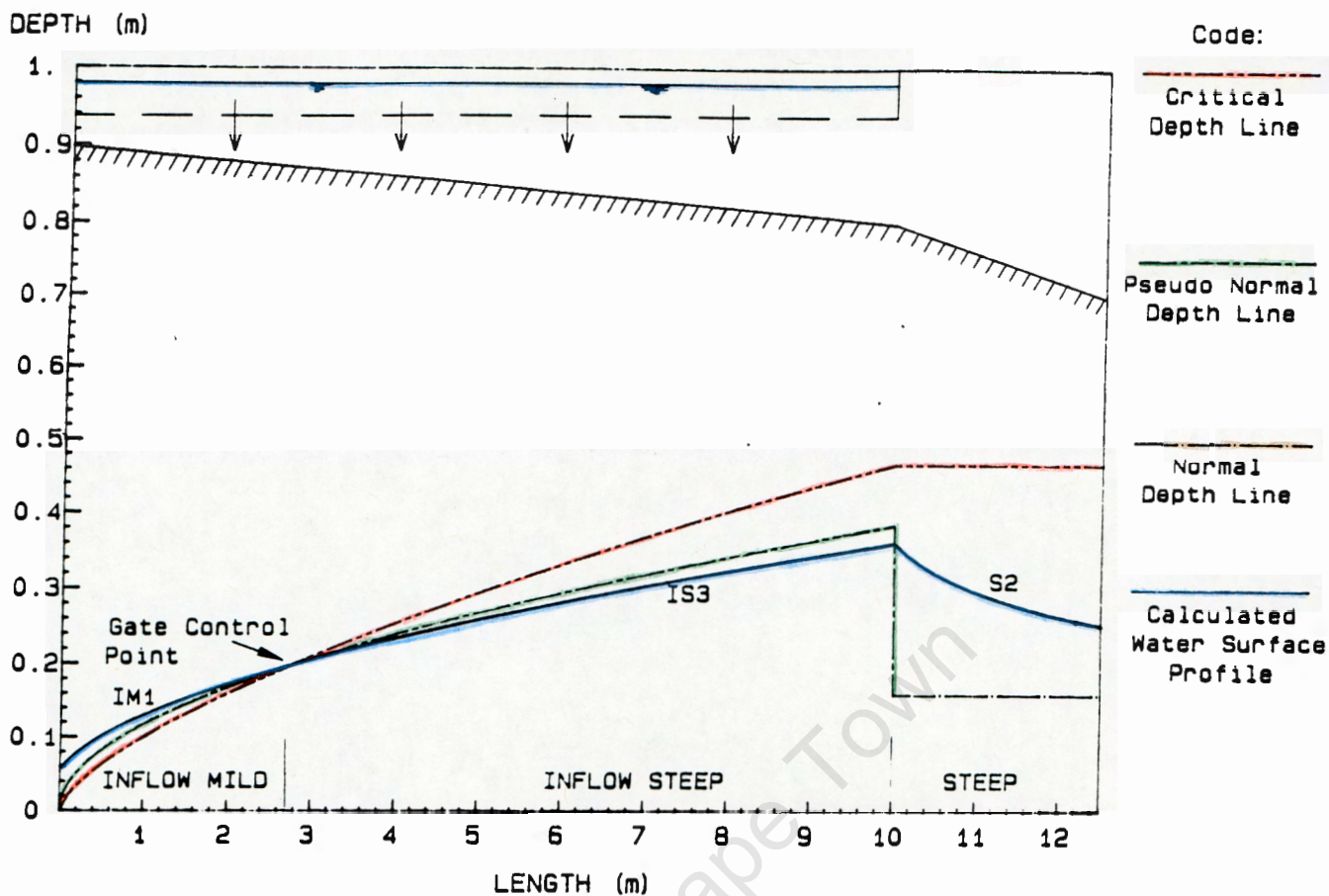


FIGURE 2.15 DEPTH CHANGES AND WATER SURFACE PROFILES

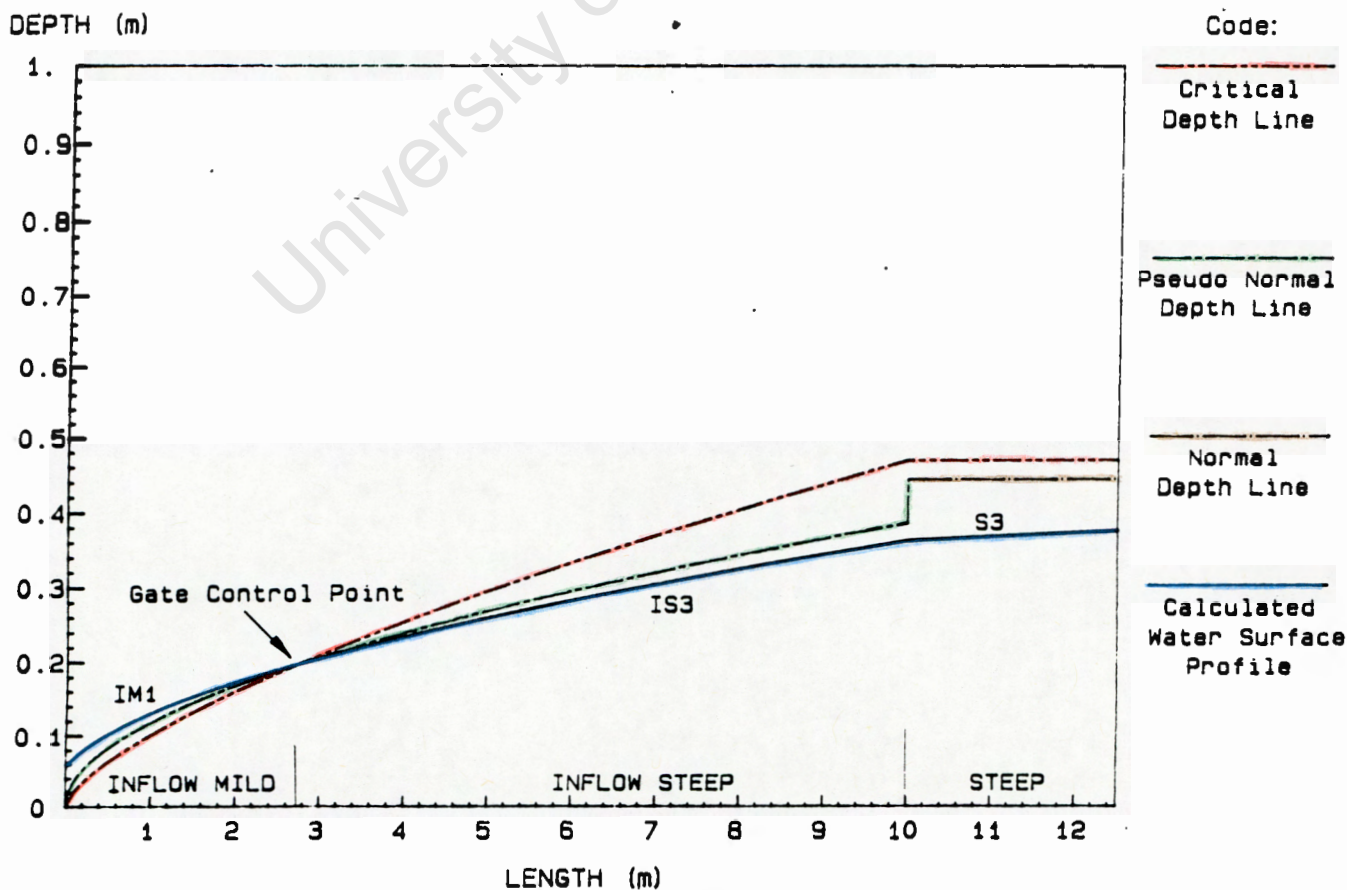


FIGURE 2.16 DEPTH CHANGES AND WATER SURFACE PROFILES

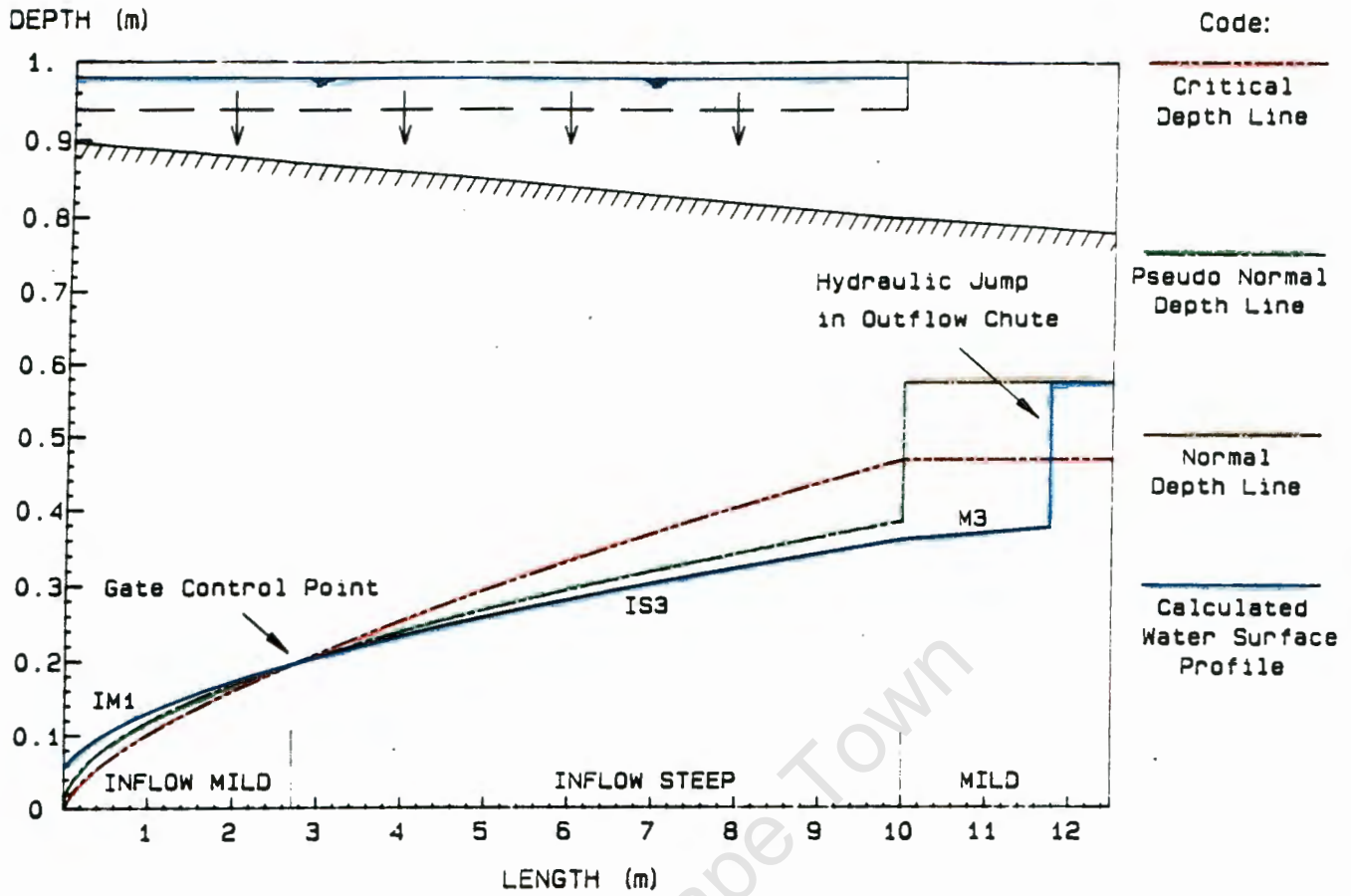


FIGURE 2.17 DEPTH CHANGES AND WATER SURFACE PROFILES

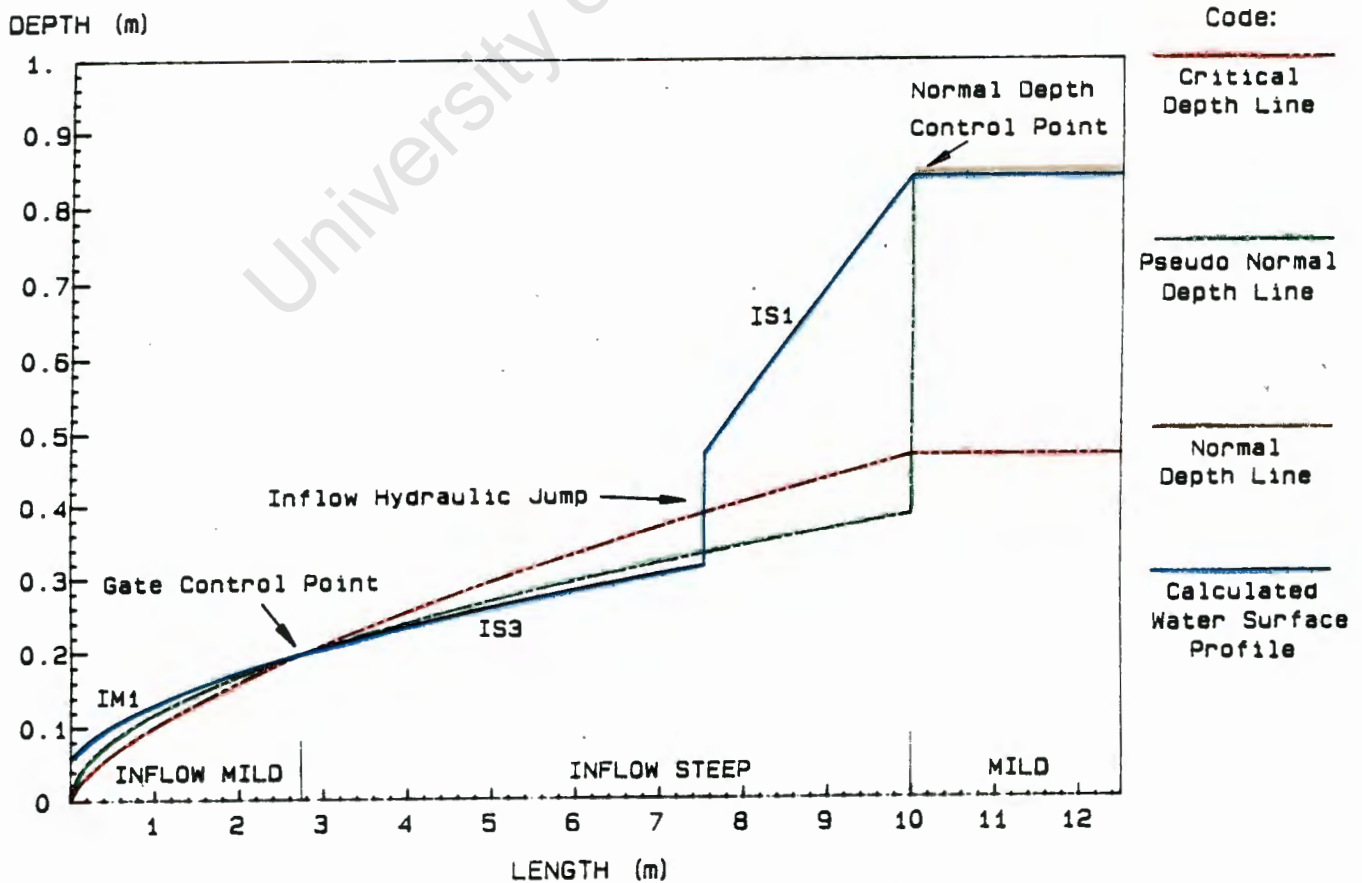


FIGURE 2.18 DEPTH CHANGES AND WATER SURFACE PROFILES

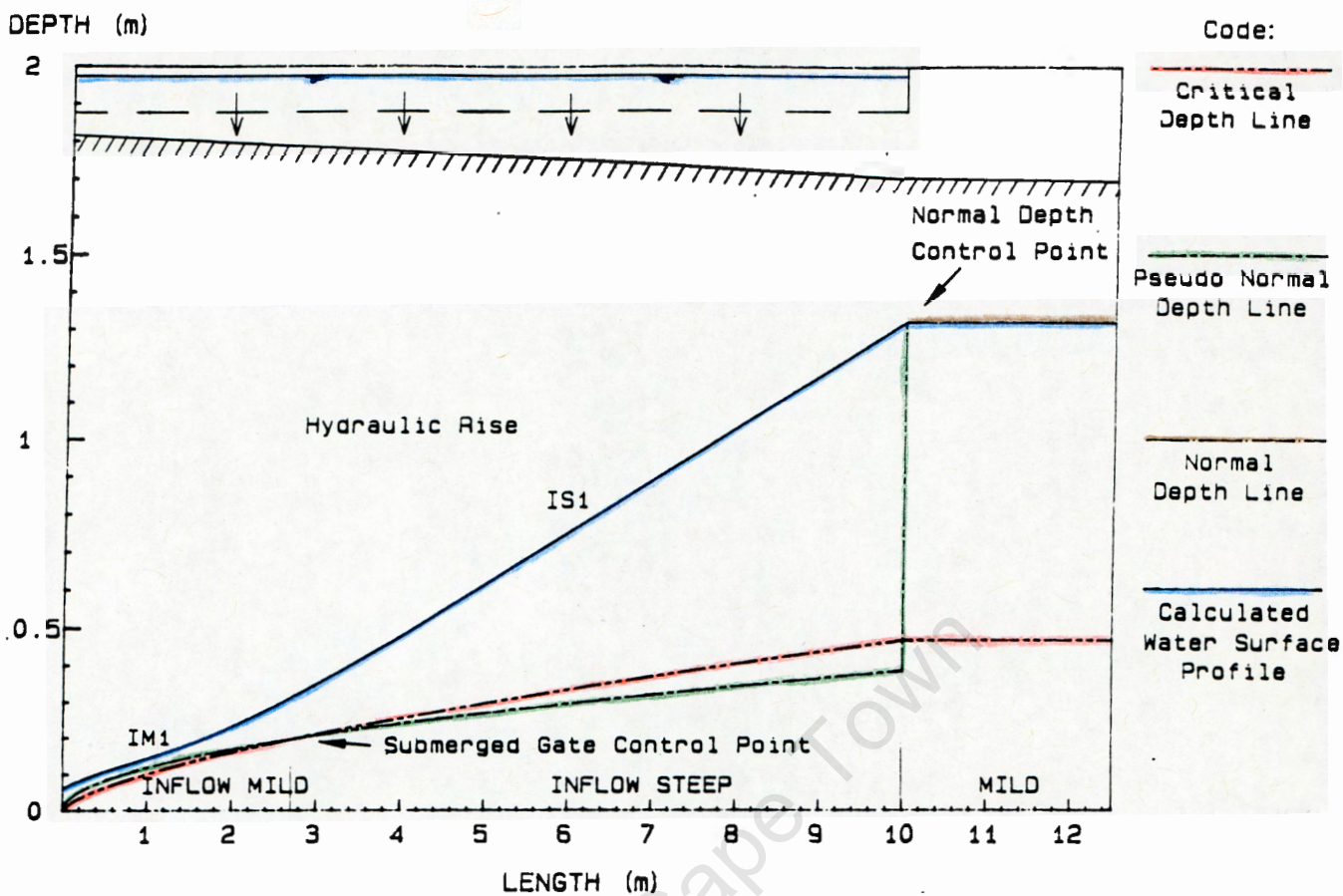


FIGURE 2.19 DEPTH CHANGES AND WATER SURFACE PROFILES

3. EXPERIMENTAL METHODS

The earliest paper written on a side channel spillway model was that of Dare [1]. The scale of the model was one twenty-fourth of the full size of Burrinjuck Dam. An illustration of the complete structure is given in Figure 3.1 . A plan view of the prototype for the curved spillway channel is given in Figure 3.2 .

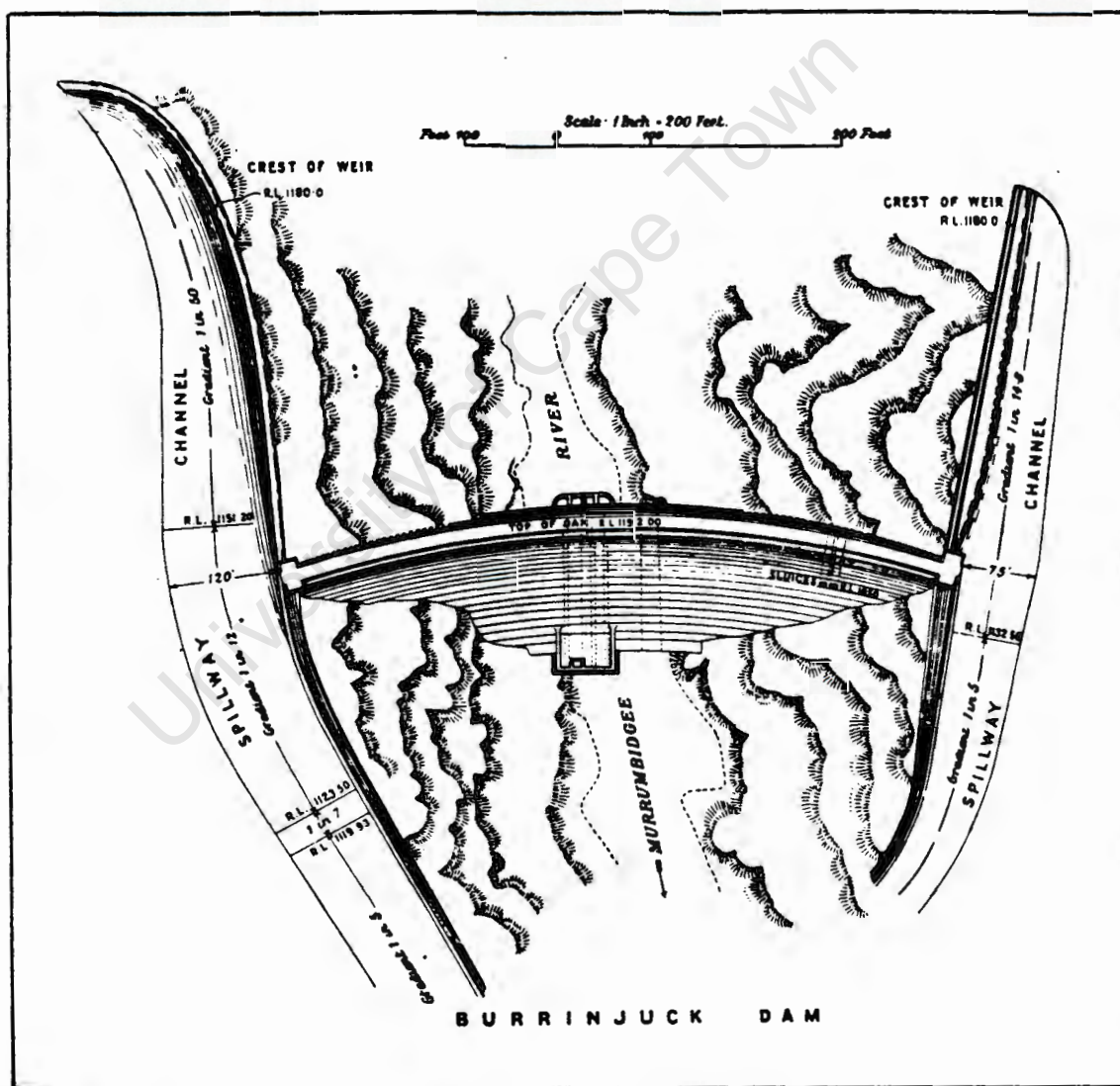


Figure 3.1 : Plan of Burrinjuck Dam [Dare, 1922]

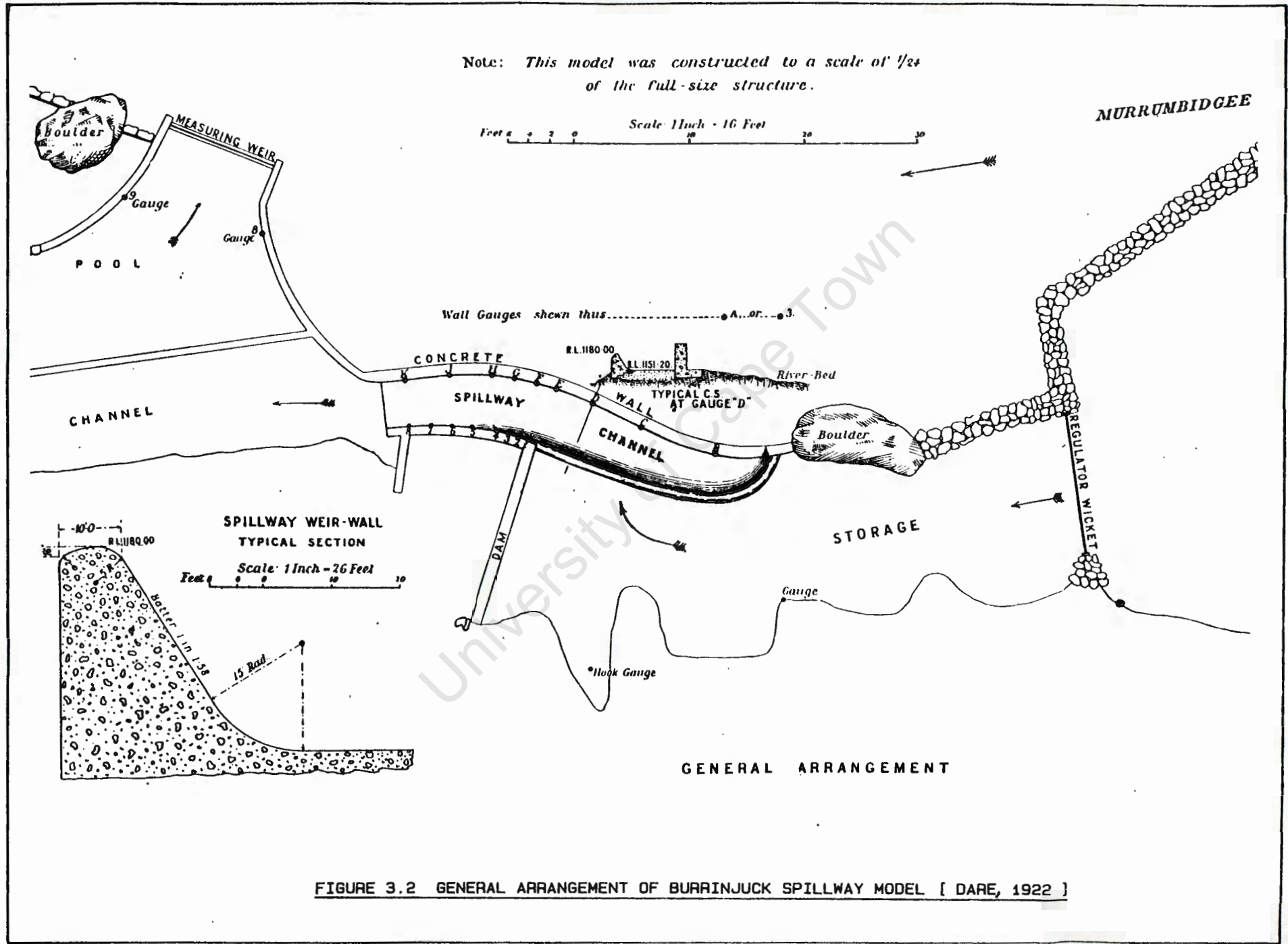


FIGURE 3.2 GENERAL ARRANGEMENT OF BURRINJUCK SPILLWAY MODEL [DARE, 1922]

A hook gauge was used to measure the water surface level of the storage pool. The velocity of the water approaching the measuring weir was determined with a Price current meter. Photographs were then taken of the spillway channel to determine the water surface profile under different discharges. The bed slope of the spillway channel was also varied and results tabulated.

Hinds [2] in the latter part of December 1923, conducted a model test at Bellvue laboratory. The aim of these experiments was to verify the theory proposed in the paper. Figure 3.3 illustrates the general arrangement of the model spillway. In these experiments timber boards were used for the spillway trough. Hook gauges placed in metal stilling wells were used for measuring the water depths at specific positions within the receiving channel. The stilling wells were connected to tappings in the walls of the spillway trough by means of flexible tubing. Five of these gauges were positioned along the length of the receiving channel.

The results were plagued by problems of aeration and freezing of pressure tappings. The high level of water disturbance led to the air entrainment. The icing problem was due to the tests taking place in winter. The experimental profile was generally above the theoretical one. This was attributed to the air entrainment and the horizontal vortex within the receiving channel.

A field study was conducted by Steward [2] on the Arrowrock dam spillway. To measure the water surface profile, wires were attached across the receiving channel to support a trolley to which a weighted cord was attached. By adjusting the length of the cord so that the end just touches the water surface, an estimate of the water surface profile was obtained. The surface velocity of the water within the receiving channel was determined by surface floats consisting of driftwood or bundles of kindling. Moderate agreement was obtained between experimental and theoretical profiles. However the observed water surface profile was always above the theoretical one.

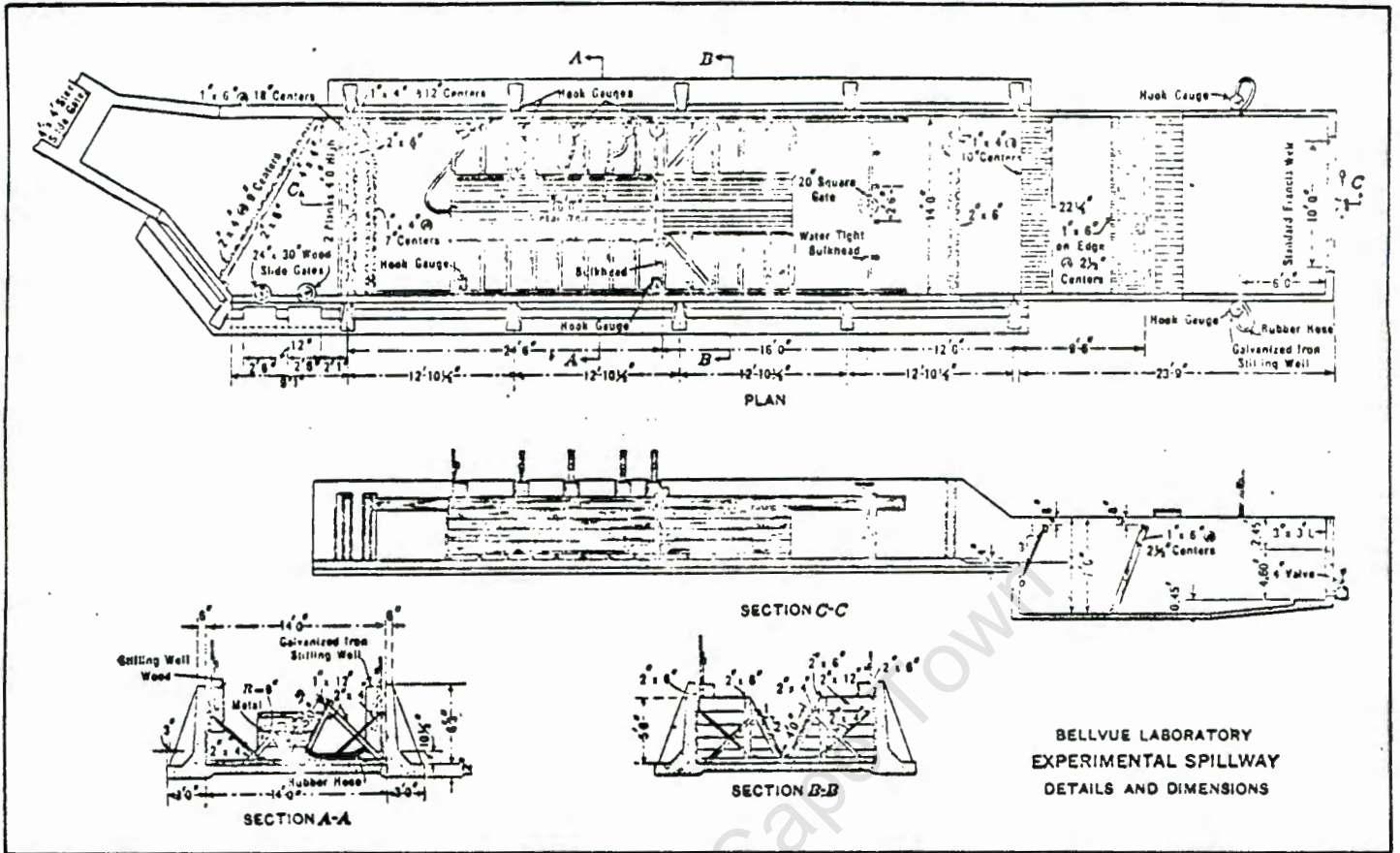


FIGURE 3.3 BELLVUE EXPERIMENTAL APPARATUS [HINDS, 1926]

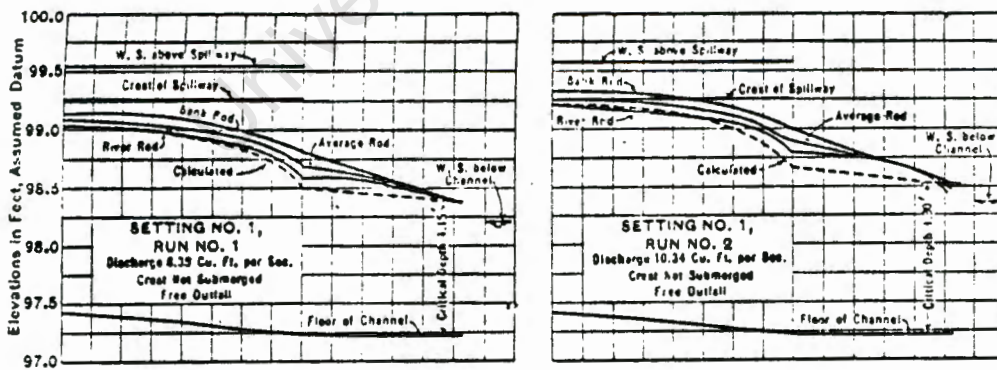
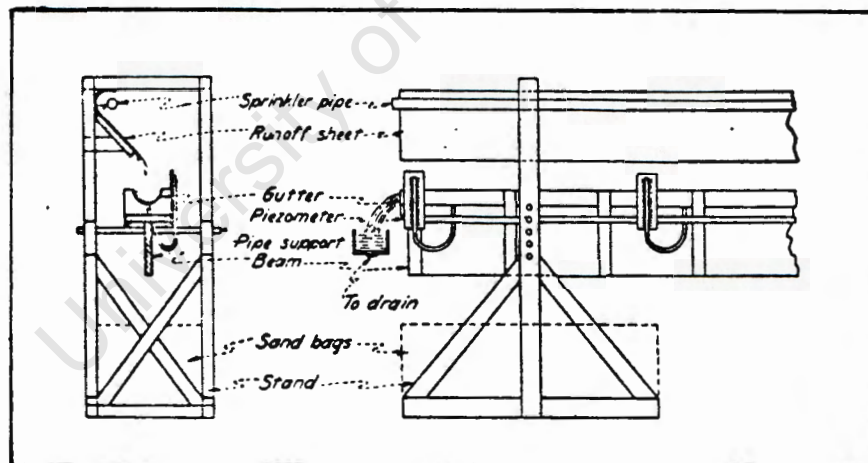


FIGURE 3.4 EXPERIMENTAL WATER SURFACE PROFILES [HINDS, 1926]

In 1935 a hydraulic model test of the Boulder dam spillway was conducted. A prototype to a scale of 1:20 was constructed. The main purpose of this prototype was to examine the behaviour of the spillway under the flood discharge. Special attention was paid to "smoothing" the flow pattern within the receiving channel. Minor geometric modifications were employed in order to achieve this result. No mention is made of the equipment used to compare different geometric arrangements, except for the use of photography.

The Boulder dam spillway was also analysed by Meyer-Peter and Favre [4] in 1934. In a model test to the scale of 1:150 water surface profiles were measured and compared to theoretical ones. How the water surface profile was measured at the various cross-sections is not mentioned.

Research by Beij [5] into the flow in roof gutters, used piezometers located at various positions along the test gutter to determine the water surface profile. Figure 3.5 shows the experimental apparatus used. The results obtained for different inflow rates were plotted as shown in Figure 3.6 .



Figures 3.5 : Experimental apparatus used in roof gutter tests [Beij, 1934]

Lund and Schwartz [17] conducted a model test on the Nuane dam in 1966. The scale selected for the model was 1:72 . In the model painted wax was chosen to represent smooth concrete and brush roughened mortar to simulate unlined

rock. These were in accordance with the similitude criterion based on Manning's "n". The main objective of this model test was to improve the hydraulic efficiency of the original design. A flip bucket on the spillway crest was used to deflect the water jet to the opposite side of the trough. This induced an anticlockwise vortex pattern as viewed in the direction of flow. The natural vortex action was clockwise. This led to a reduction in the pulsations within the channel. However no accurate method for measuring the true water surface profile was derived.

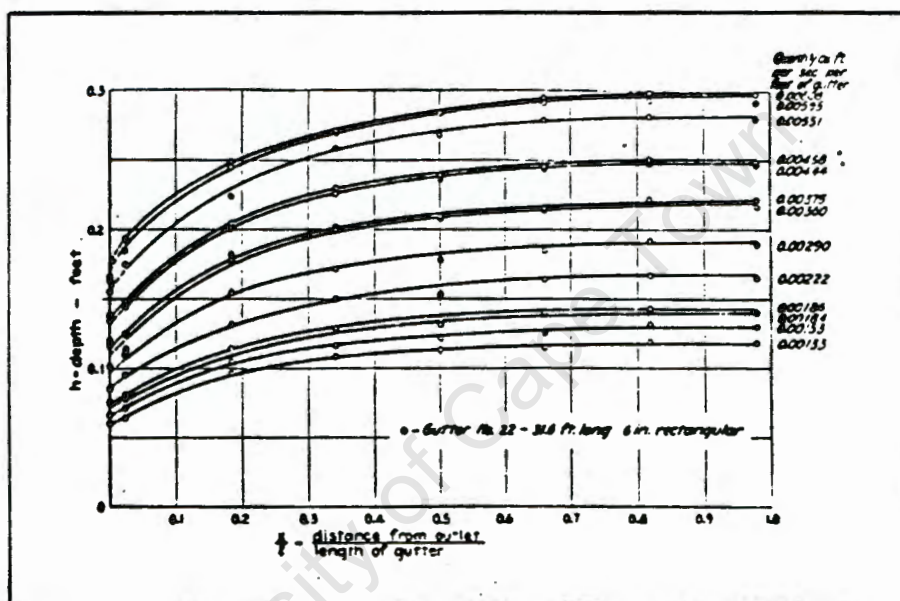


Figure 3.6 : Results of roof gutter tests [Beij, 1934]

Babb and Ross [19] in the discussion of Smith's [19] paper, briefly mentioned some experimental results. These included not only a water surface profile but also a velocity contour pattern within the receiving channel. Figure 3.7 shows the velocity contour pattern at three different locations along the receiving channel. This series of experiments illustrates the inherent error in the assumption of a uniform velocity profile.

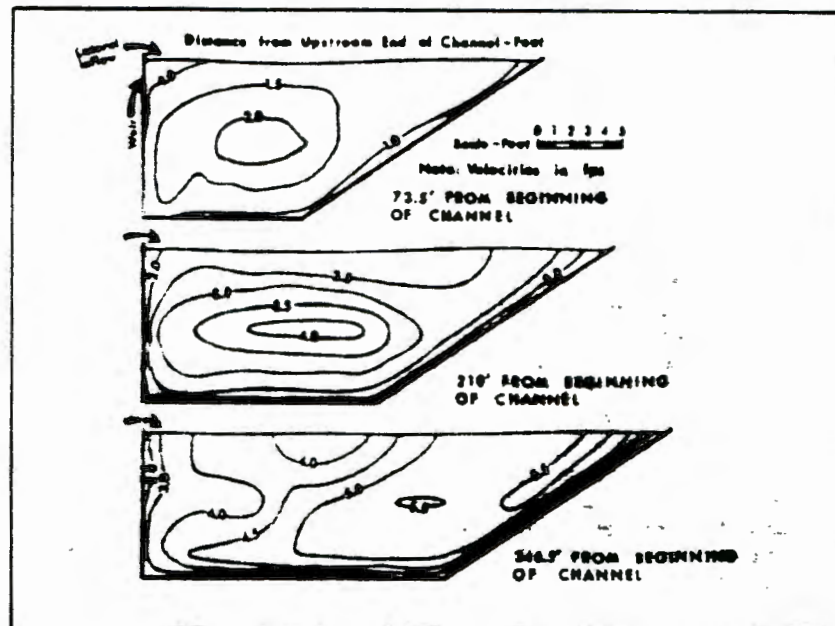


Figure 3.7 : Velocity contours within the receiving channel [Babb and Ross, 1968]

In order to verify the theory proposed by Fox and Goodwill [22] a series of spatially varied discharge experiments were performed. To alleviate the high level of aeration and water disturbance, the inflow was introduced through perforated side walls of the channel. Figure 3.8 illustrates the laboratory equipment used. Careful consideration was given to the even distribution of lateral inflow, so that the assumption of constant inflow per metre length of the channel was valid.

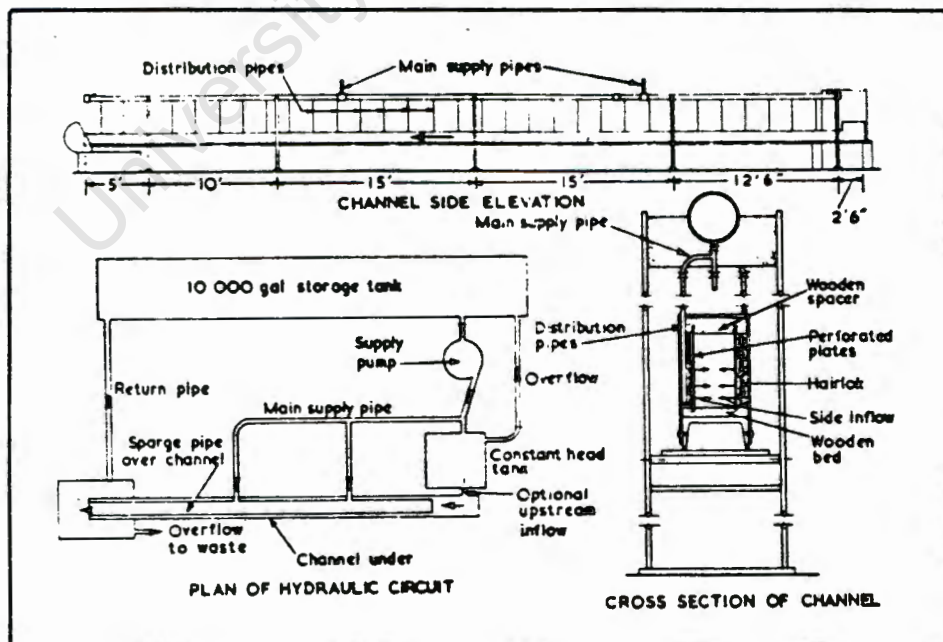


Figure 3.8 : Experimental apparatus used by Fox and Goodwill

A comparison between the theoretical and experimental water surface profiles is presented in Figure 3.9. Note the three different inflow conditions leading to three different profile types. The correlation between the theory and the experiments is good. However the effect of introducing the inflow through the side wall of the channel, on the channel friction and the velocity contours within the receiving channel, was not investigated.

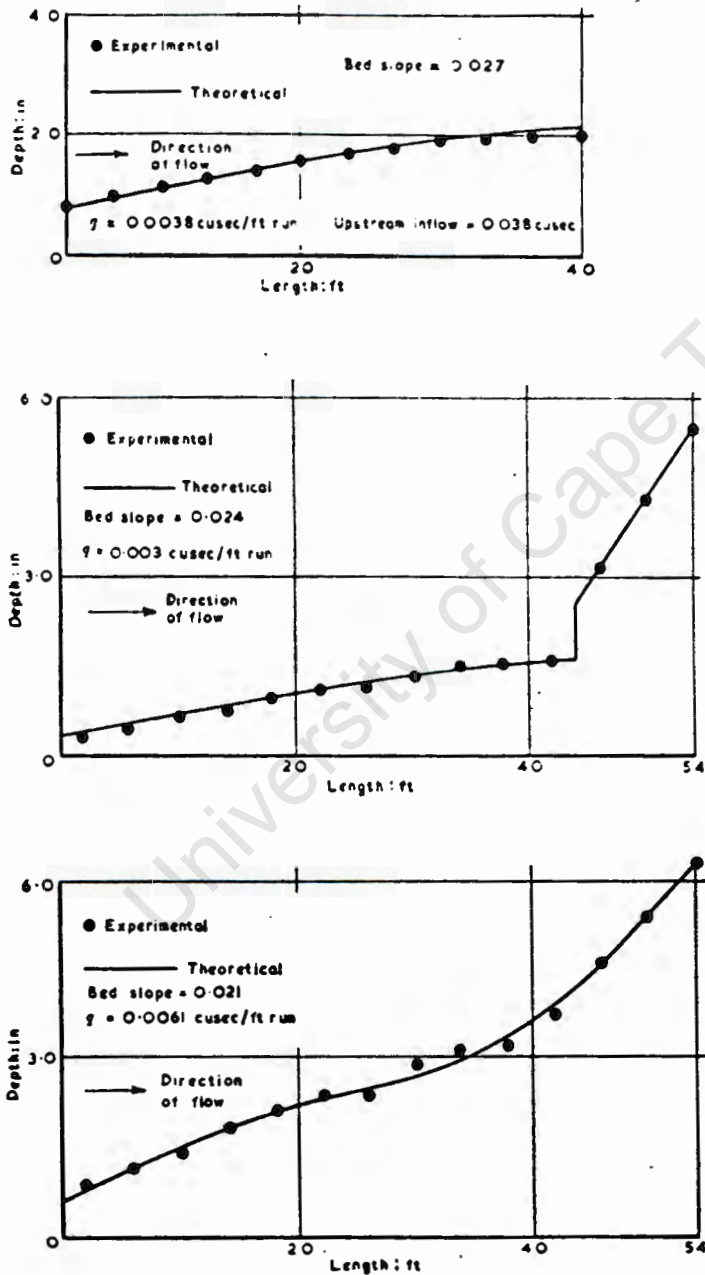


FIGURE 3.9 WATER SURFACE PROFILES [FOX & GOODWILL, 1970]

In 1977 Gill [34] conducted a series of laboratory experiments. In these a perforated perspex pipe was used to discharge uniformly along its length into the channel below. Tests show that for all practical purposes the discharge throughout the series of holes in the pipe was uniform. Figure 3.10 shows the general arrangement of the experimental equipment. Note the tailgate at the end of the receiving channel. This was used to regulate the water depth within the receiving channel.

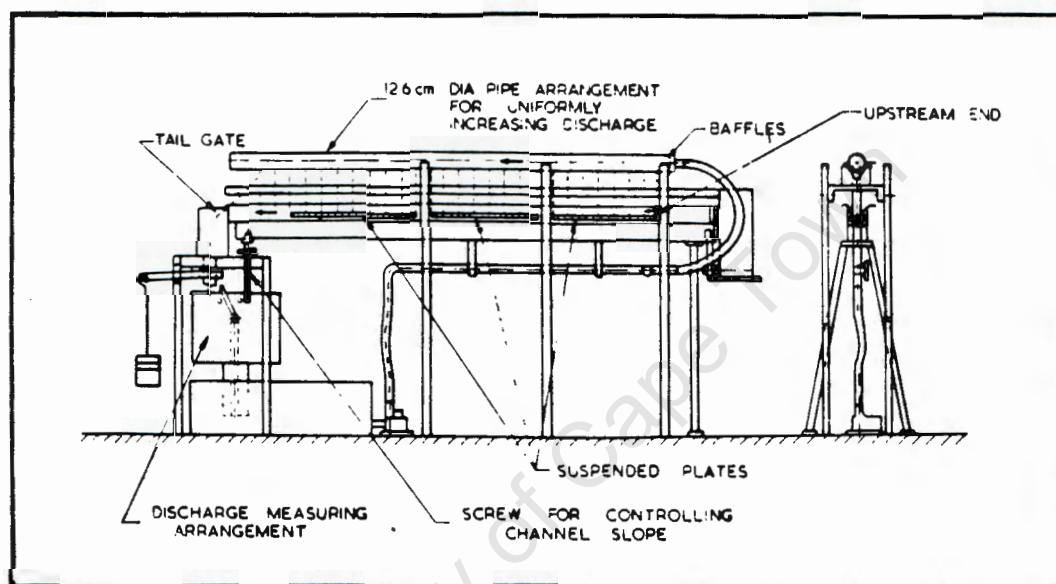


Figure 3.10 : General arrangement of Experimental equipment [Gill, 1977]

Comparing the experimental profile to the theoretical one reveals certain discrepancies. The theoretical profile lies below the experimental profile. This agrees with previous researcher's results. The discrepancy is attributed to the inflow water jets tending to oscillate at high discharge. Another reason is the uncertainty concerning the frictional resistance associated with spatially varied discharge. It appears that a higher frictional factor would reduce the difference between the theoretical and experimental profile. Figure 3.11 illustrates the difference between these two profiles.

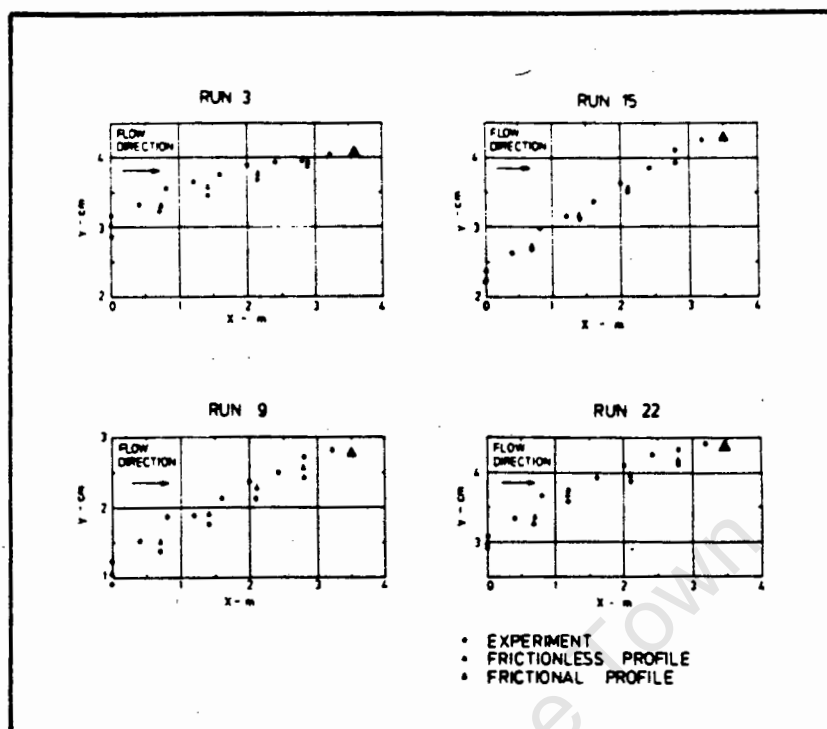


Figure 3.11 : Comparison of Experimental and theoretical profiles
[Gill, 1977]

In 1989, Bremen and Hager [42] conducted a series of experiments in a rectangular side channel spillway. These were carried out in a rectangular PVC channel, with a single sided spillway configuration fed by a reservoir. Figure 3.12 illustrates the experimental apparatus used. The spillway was subdivided by five piers, these piers carry rails on which a measurement trolley may be moved. The water surface profile was recorded with a conventional point gauge mounted on the trolley. The bottom slope of the receiving channel was 0,24% . By inserting a false floor, the bed slope could be increased to 5,24% . A false side wall, mounted on the side opposite the spillway, allowed for the investigation of flow conditions in a non prismatic channel. The facility existed for the introduction of super critical flow at the upstream end of a shortened receiving channel.

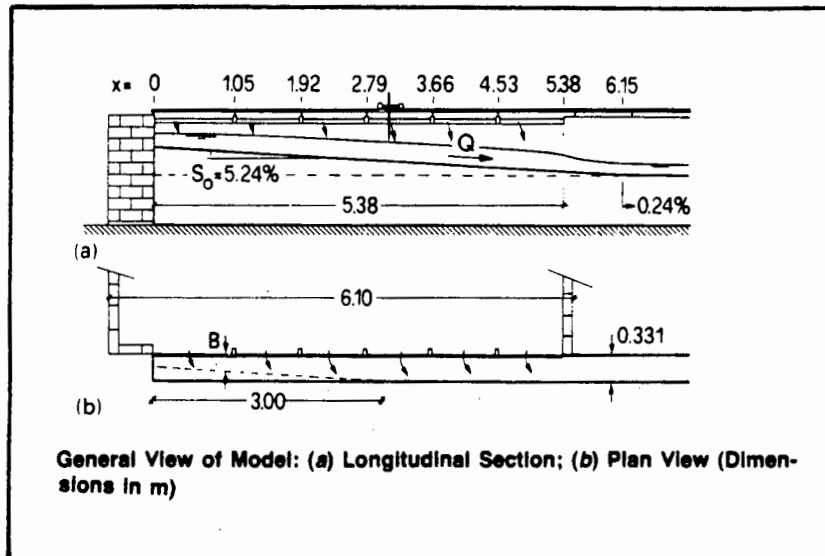


Figure 3.12 : Experimental Model [Bremen and Hager, 1989]

The experimental results revealed that the flow depths along the channel side wall opposite the spillway crest, were considerably higher than those along the channel axis. A series of tests were then conducted in which the lateral inflow length was progressively reduced, whilst recording both the axial and side wall water surface profiles. Evidently the axial water surface profile was found to continue straight into the stagnant upstream water level, while the side wall profile rises significantly above the axial flow depth along the inflow zone. In this way the axial water surface profile was deemed to be the average cross-sectional depth of flow.

Figure 3.13 illustrates typical axial water surface profiles obtained. Other factors observed were the horizontal vortex flow along the receiving channel. This was attributed to the free jet inflow used. The deviations between the predicted water surface profile and the observed one, increased with an increasing lateral inflow rate. This effect was attributed to streamline curvature, especially when a critical depth control point exists at the end of the receiving channel. A normal depth control at the end of the receiving channel, improved the correlation between the predicted and observed profiles. However the theoretical profile was still found to lie above the observed profile. It was found that for a given outflow chute width, the prismatic channel geometry normally yielded the best hydraulic flow features.

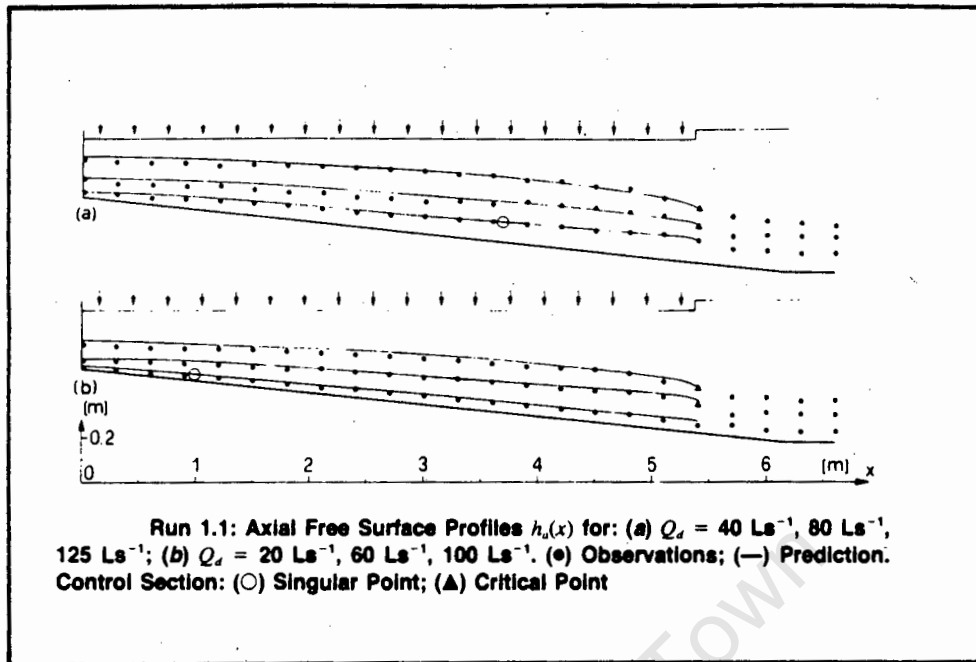


Figure 3.13 : Axial water surface profiles [Bremen and Hager, 1989]

4. THE COMPUTER PROGRAM

The number of iterative calculations involved in determining the water surface profile, make hand calculations laborious and tedious. For this reason a computer based solution is sought to provide a quick and accurate approximation of the water surface profile.

In this chapter the various components of the computer program are described. Initially the system documentation and design philosophy are dealt with. This is followed by detailed descriptions of each of the various program components. The documentation is discussed in the logistic order used by the program in deriving the solution. Finally an example of the program operation is given, along with a sensitivity analysis of the starting point and the step length.

4.1 Design Philosophy

The computer program WSPISCS (Water Surface Profiles In Side Channel Spillways) was designed to fulfill the following criteria :

- the program must be written for a micro computer, with as few hardware restrictions as possible
- the input be kept to a minimum
- the output be in graphical as well as tabular form
- the step length and starting point be optimised for speed and accuracy.

4.2 System Documentation

4.2.1 Computer equipment

The system used for the formulation of the original program, consisted of :

- A Sperry personal computer, double disk drive with 640 kilobytes of memory
- A Star radix 15 printer
- A Hewlett Packard 7475A graphics plotter

Other compatible configurations may be used for the operation of this program. The only restriction is on the memory size of the system. A memory size of 640 kilobytes or greater is essential.

4.2.2 Source Language

True Basic is used as the main program language. One of the main reasons for this choice is the machine independence of the True Basic language and associated graphics. The Hewlett Packard Graphic Language [HPGL] is used for plotter commands. These are relayed to the plotter through the use of the True Basic communications support library. The program has been compiled and bound with the True Basic runtime package. This creates a free standing .EXE file with the following advantages :

- A bound program requires less computer memory
- A bound program initialises faster than its unbound counterpart
- The program can now be run directly from DOS without the use of the True Basic language disk.

4.3 Program Documentation

4.3.1 General layout of Program

The flow chart in figure 4.1 illustrates the various subroutines that make up "WSPISCS". Each of the subroutines perform a specific function. These are briefly described as :

- DRIVER As the name suggests, this section effectively "drives" the program logic. Included in this section is the data input, library access codes and array dimensions.
- SUB Props A short routine for calculating the hydraulic properties at a specific water depth. This subroutine is frequently accessed by the other subroutines for recalculation of the hydraulic properties at different water depths.
- SUB CHUTE This routine calculates the normal and critical depths within the outflow chute. Newton's method is used to determine these finite values.
- SUB CDL To calculate the critical depth line within the receiving channel. Based on equating the denominator of the water surface slope equation to zero.
- SUB PNDL To calculate the pseudo normal depth line within the receiving channel. Based on equating the numerator of the water surface slope equation to zero.
- SUB WATERSURF In this subroutine, two functions are performed. Firstly, the location and type of control point is established. Secondly, the water surface profile is calculated for the receiving channel and the outflow chute.
- SUB GRAPHPLOT Plots the title and axis of the water surface profile on the monitor screen.
- SUB POINTPLOT Plotting routine for the CDL, PNDL and water surface profile on the monitor screen.
- SUB PLOTTER Routine used to obtain a plotter output of the graph.

A clear colour graph is obtained with the inflow data and maximum water depth highlighted. Allowance is made for additional notes and comparisons with experimental data.

In the following sections the subroutines are explained in more detail. Specific details are given of the actual method of calculation and the formulas used. Particular attention is paid to the decision structure used.

4.3.2 DRIVER

This section effectively controls the computational sequence of events for the whole program. Figure 4.2 portrays the essence of this routine, for the sake of clarity a brief narrative description is now given of DRIVER :

- Display title in a framed box on the monitor screen.
- Prompted input, showing the data required and the applicable metric units.
- Print option for tabulated values to be sent to printer. If this option is not utilised, then the tabulated values are displayed on the screen.
- Specification of the value of the tangent of 90° , that is set the value to the maximum number for that specific computer.
- Dimension the arrays used and display "Processing Data" in a framed box on the monitor screen.
- Call the subroutines CHUTE, CDL and PNDL to calculate the normal and critical depths in the outflow chute, the critical depth line and normal depth line in the receiving channel respectively.
- Display "calculations in progress" in a framed box on the monitor screen.
- Call the subroutine WATERSURF, to calculate the water surface profile.
- Prints the location and value of the maximum water depth.

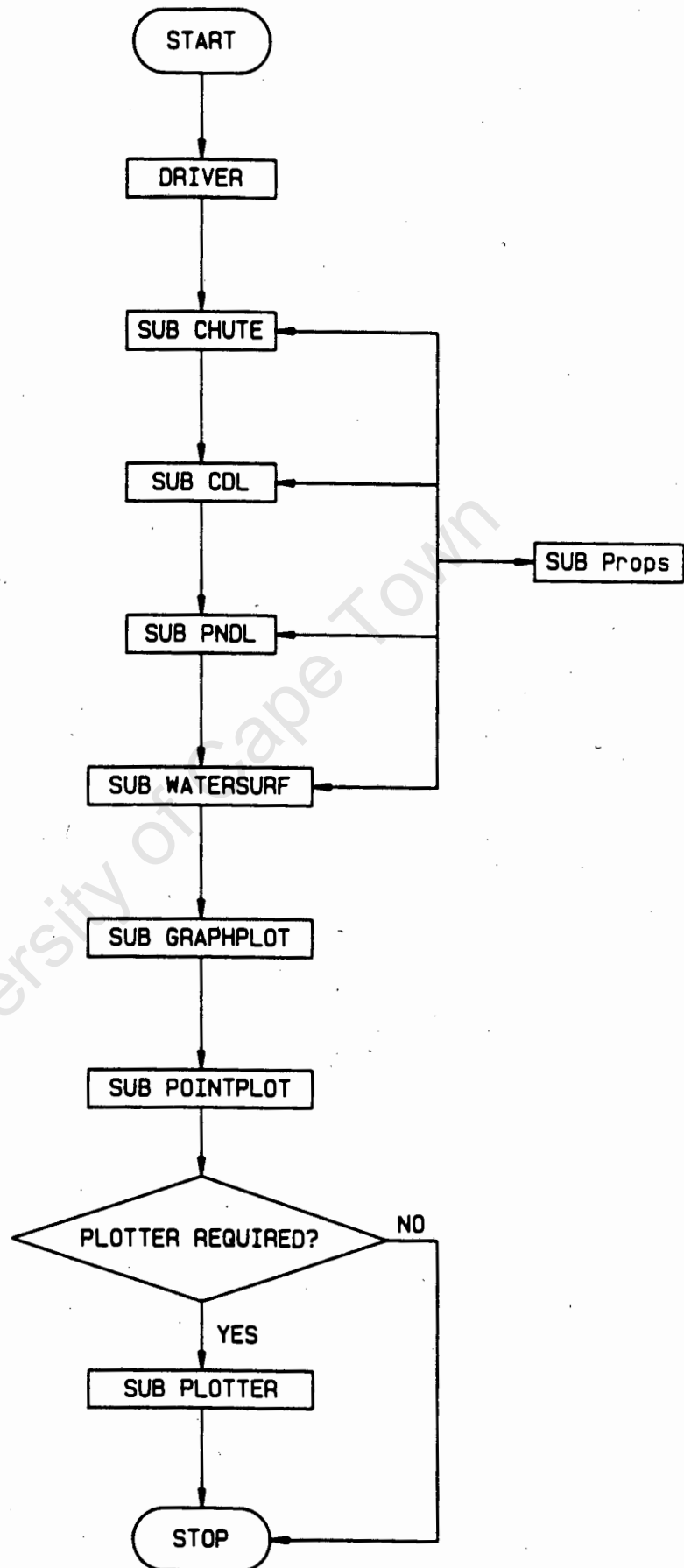


FIGURE 4.1 MACRO FLOW DIAGRAM FOR THE PROGRAM WSPISCS

- Calls the subroutine GRAPHPLOT and POINTPLOT for the critical depth line, pseudo normal depth line and the water surface profile.
- Plotter option plus relative information (Baud Rate, pen number/colour specification). Calls the subroutine PLOTTER if required.
- Displays "The End" in a framed box on the monitor screen.

4.3.3 SUB Props

This short routine is designed to calculate the hydraulic properties of trapezoidal as well as rectangular channels. The hydraulic properties calculated are the cross sectional area, the cross sectional area multiplied by the centre of submergence, the wetted perimeter, the hydraulic radius and the top width of the water surface in the channel. These are required at numerous stages within the other subroutines.

4.3.4 SUB CHUTE

In the outflow chute, spatially constant discharge conditions prevail. Hence the critical and normal depths have constant finite values. These are determined by equating the numerator of the water surface slope equation (4.1) to zero. These yield the normal and critical depths respectively.

In other words the water surface slope equation is

$$\frac{dy}{dx} = \frac{S_o - \frac{Q^2 n^2}{A^2 R^{4/3}}}{1 - \frac{Q^2 B}{g A^3}} \quad (4.1)$$

in the chute. Hence the critical depth equation is given as

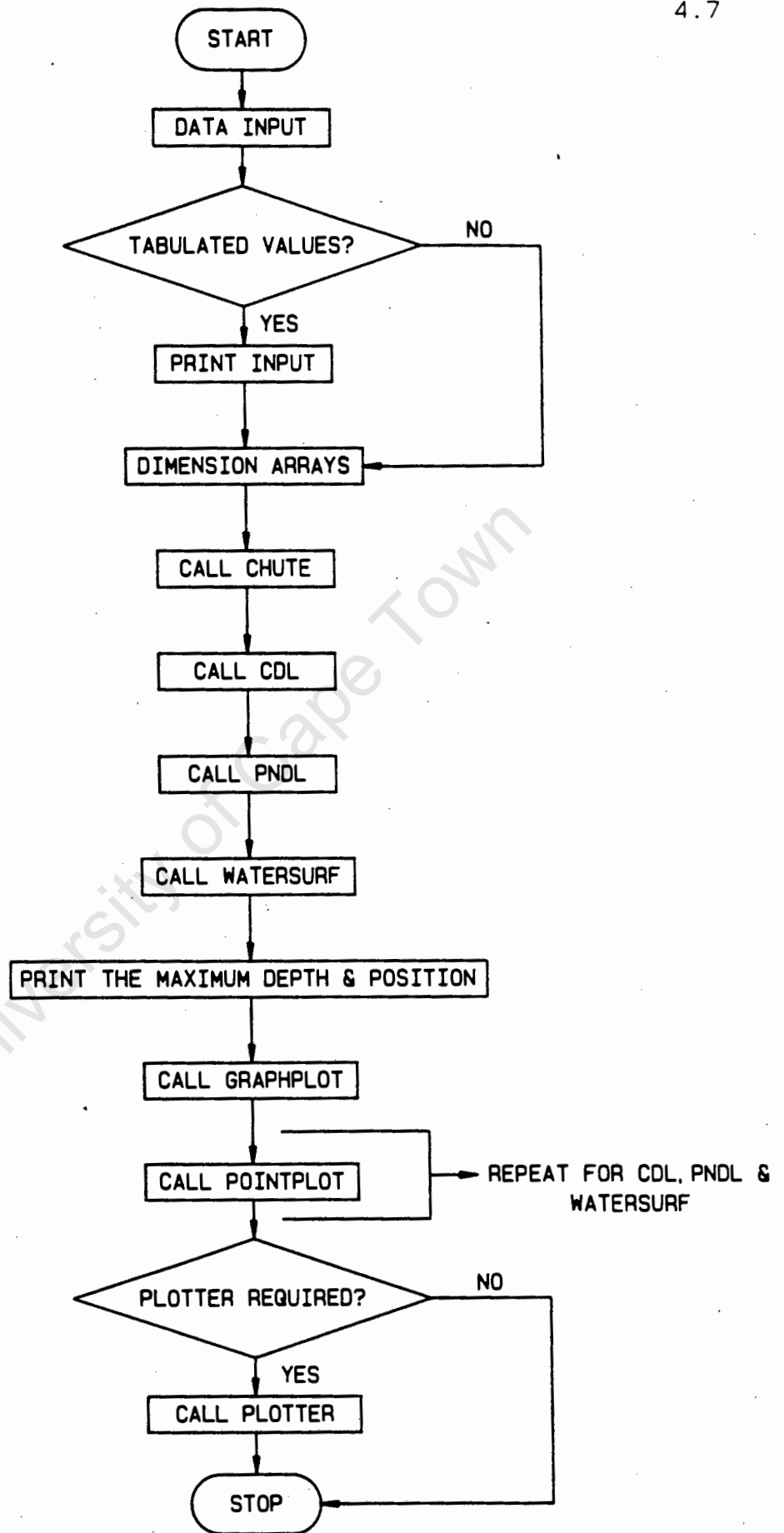


FIGURE 4.2 FLOWCHART FOR DRIVER

$$1 - \frac{Q^2 B}{g A^3} = 0 \quad (4.2)$$

and the normal depth equation by

$$S_0 - \frac{Q^2 n^2}{A^2 R^{4/3}} = 0 \quad (4.3)$$

Newton's method is the mathematical procedure used to determine the water depths that fulfill these two equations. From an initial approximation a converging iterative procedure is utilised to approximate these values to an error of less than $1/10^6$. In numerical terms, Newton's algorithm is expressed as :

$$Y_{(i+1)} = Y_i - \frac{f(Y_i)}{f'(Y_i)} \quad (4.4)$$

4.3.5 SUB CDL

The critical depth line in the receiving channel is defined by the formula

$$1 - F_r^2 = 0 \text{ given as equation (2.32) in Chapter 2}$$

where
$$F_r^2 = \frac{q^2 x^2 B}{g A^3}$$

The method chosen for obtaining x , y co-ordinates of this line, is to solve the equation

$$x_{cr} = \left[\frac{g A^3}{q^2 B} \right]^{1/2} \quad (4.5)$$

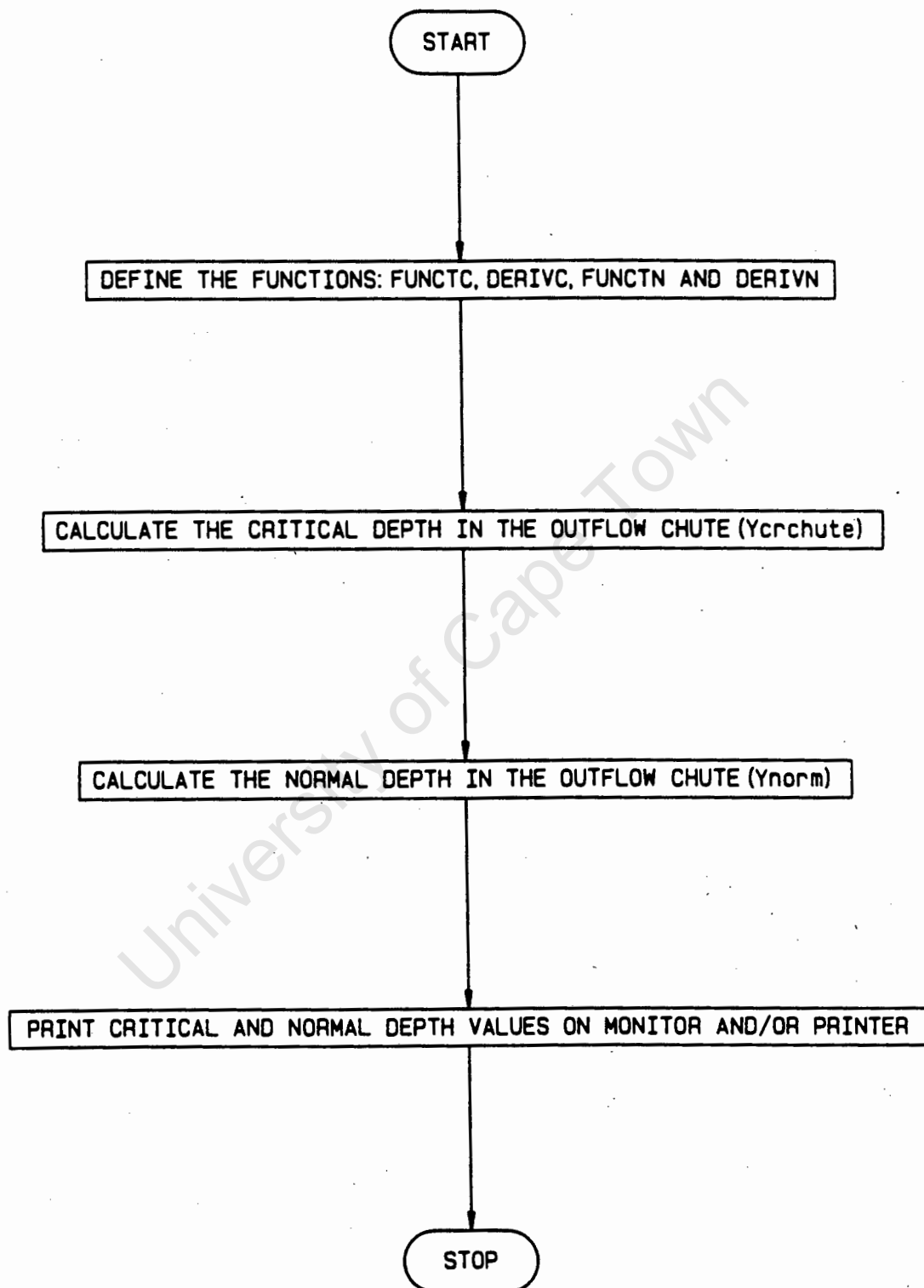


FIGURE 4.3 FLOWCHART FOR SUB CHUTE

for increments of y_{cr} . The increments are chosen as a fraction of $Y_{crchute}$. These increments are reduced as x_{cr} approaches L . This is to elevate the problem of "overshooting" the end of the receiving channel.

It must be noted that this line serves only as an indicator line at this stage programme, along with the pseudo normal depth line. These two lines are used to determine the location and type of control point along with the normal and critical depths in the outflow chute. If a gate control is located by the indicator lines, a Newtonian convergence is initiated to obtain the exact intersection point of the critical depth line and the pseudo normal depth line. This is within the subroutine WATERSURF and shall be explained fully at a later stage.

4.3.6 SUB_PNDL

The pseudo normal depth line within the receiving channel is defined by the formula

$$S_o - S_f - \frac{2 q^2 x}{g A^2} = 0 \quad [\text{equation (2.31)}]$$

where

$$S_f = \frac{q^2 n^2 x^2}{A^2 R^{4/3}}$$

The method used for obtaining x , y co-ordinates of this line is the same as that of the critical depth line. That is the equation

$$x_{pnd}^2 \left[\frac{q^2 n^2}{A^2 R^{4/3}} \right] + x_{pnd} \left[\frac{2 q^2}{g A^2} \right] - S_o = 0 \quad (4.6)$$

is solved for increments of y_{pnd} . As before the chosen increments are reduced as x_{pnd} approaches the end of the receiving channel (L).

Now that the two indicator lines have been established, it is possible to find the location and type of control point for the

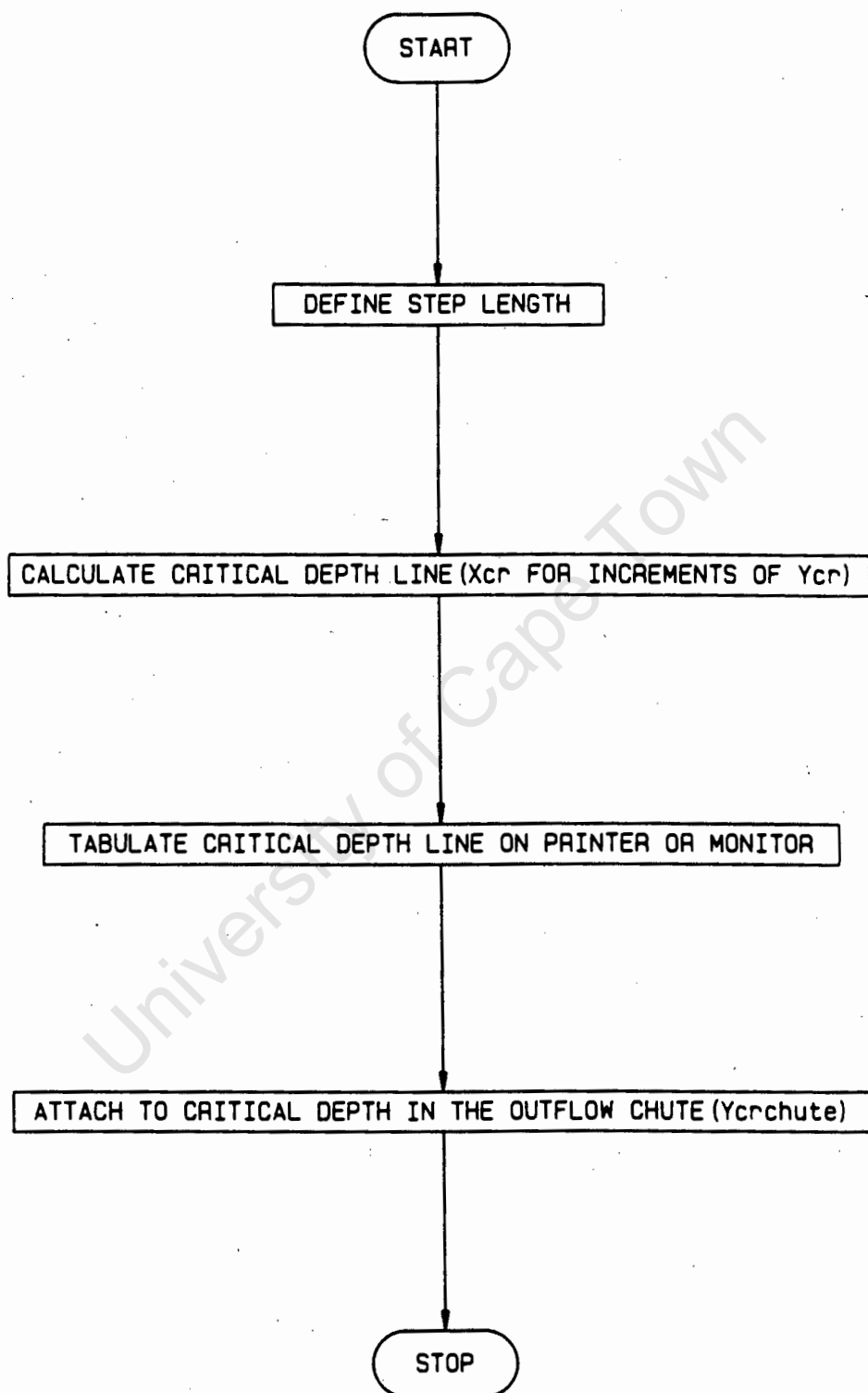


FIGURE 4.4 FLOWCHART FOR SUB CDL

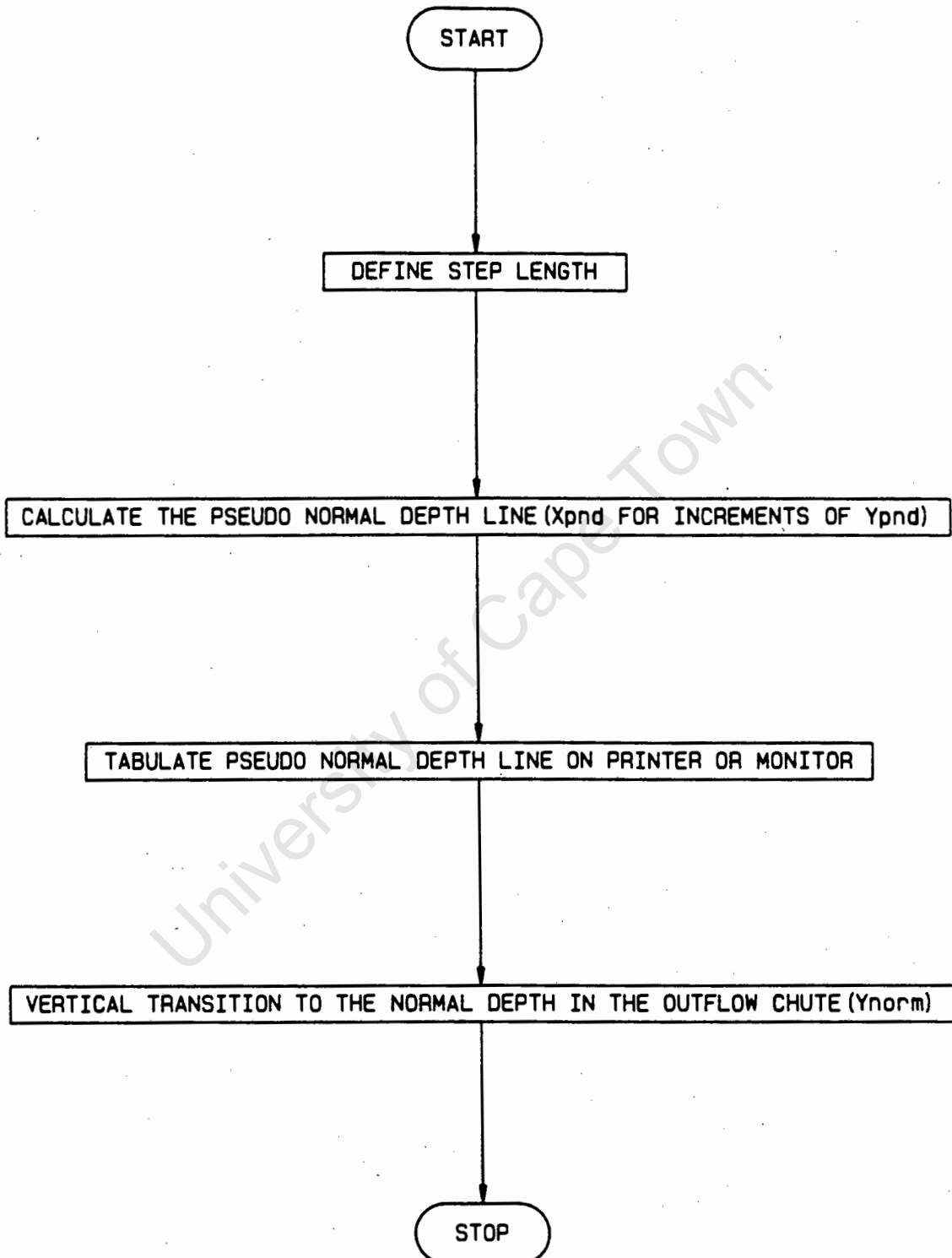


FIGURE 4.5 FLOWCHART FOR SUB PNDL

receiving channel. Once this has been done, the actual calculation of the water surface profile can commence. These procedures are incorporated in the subroutine WATERSURF.

4.3.7 SUB_WATERSURF

This subroutine forms the nucleus of the program WSPISCS. All the other subroutines may be seen as either preprocessing or post-processing the information for the subroutine WATERSURF.

The first function of this subroutine is to establish the location and type of control point. The indicator lines derived in SUB CDL and SUB PNDL, along with the normal and critical depths within the outflow chute form the basis of the decision structure. From the location and type of control point/s, the starting point and the initial water surface slope are derived. Chapter 2 describes the problems associated with establishing the initial water surface slope, especially when a critical depth control exists at the end of the receiving channel. Therefore only the formulae used for the different scenarios are stated.

In the case of a critical depth control at the end of the receiving channel, the starting point has co-ordinates :

$$Y_{\text{water}} = k \cdot y_{\text{crchute}} \quad (4.7)$$

$$\text{and } x_{\text{water}} = \sqrt{\frac{k(3-k^2)}{2}} \cdot L \quad (4.8)$$

for a rectangular channel or

$$(x_{\text{water}})^2 = \frac{g A}{q^2} \left[A_c \bar{z}_c + \frac{A_c^2}{B_c} - A \bar{z} \right] \quad (4.9)$$

for a trapezoidal channel, where k is a multiple of y_c (e.g. 1,01).

The initial water surface slope at this starting point is given as

$$\frac{dy}{dx} = \frac{2 y_{crchute} \sqrt{2 k (3-k^2)}}{3 L (1 - k^2)} \quad (4.10)$$

for a rectangular channel. For a trapezoidal channel the basic water surface slope equation is used.

$$\frac{dy}{dx} = \frac{S_o - \frac{q^2 x^2 n^2}{A^2 R^{4/3}} - \frac{2 q^2 x}{g A^2}}{1 - \frac{q^2 x^2 B}{g A^3}} \quad (4.11)$$

In the case of a gate control within the receiving channel, a quadratic in terms of Z is given, where Z represents dy/dx . For the sake of brevity, the equations expressed in Chapter 2 for this scenario shall not be repeated. Reference is therefore made to the latter half of section 2.4.

For a normal depth control at the end of the receiving channel, the initial water surface slope is determined directly from the basic water surface slope equation. However if a normal depth control occurs along with a gate control, the possibility of a hydraulic jump complicates the situation. The procedure adopted in such a scenario, incorporates the use of the total force principle. That is the hydraulic jump is idealised as a vertical step in the water surface profile. Thus the total force just downstream of the hydraulic jump (F_{Top}) must equal the total force just upstream of the hydraulic jump (F_{Bottom}). Figure 4.7 illustrates the principle involved.

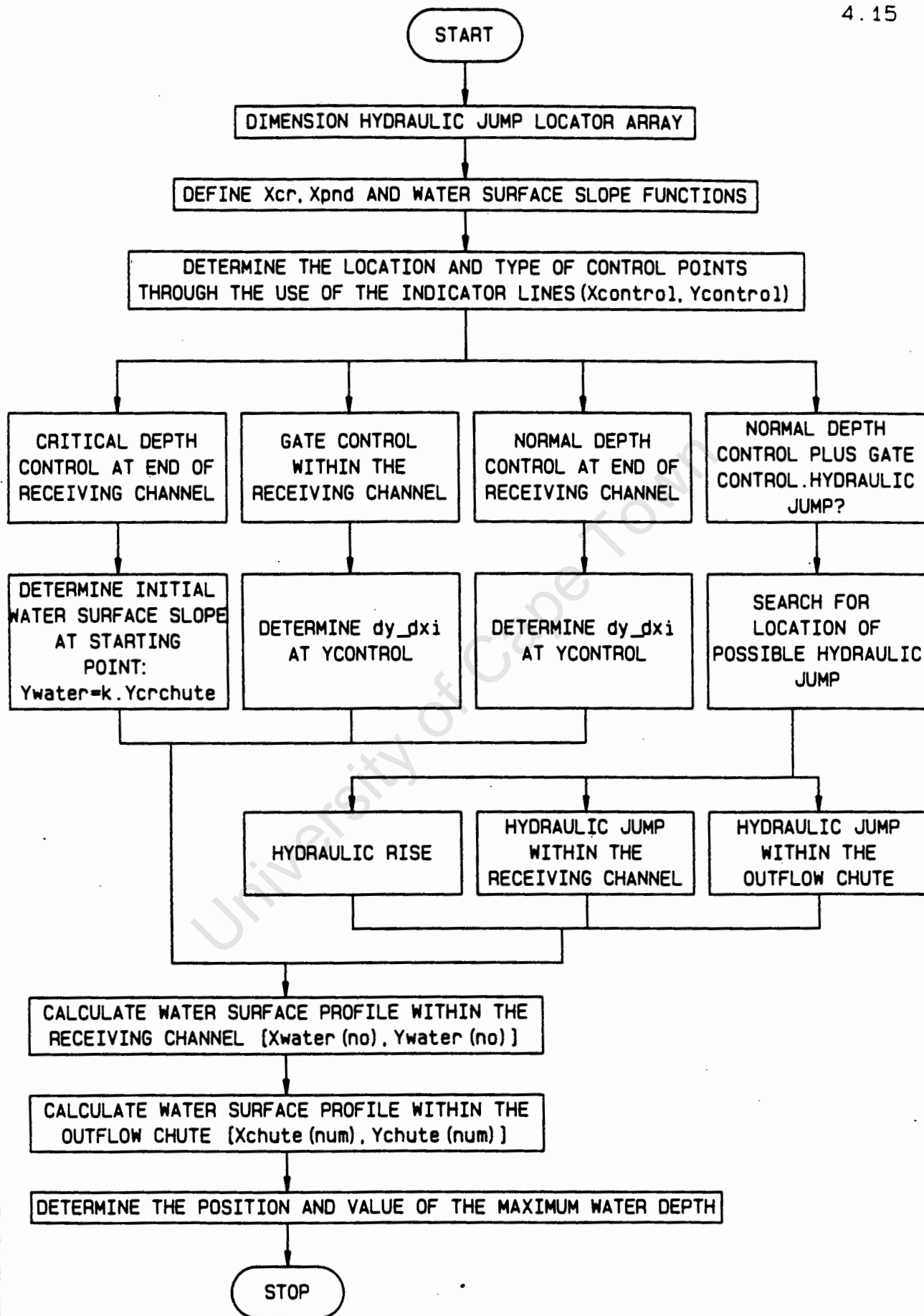


FIGURE 4.6 FLOWCHART FOR SUB WATERSURF

The procedure adopted when two control points are present, is to calculate the water surface profile from the gate control to the end of the receiving channel under the scenario of supercritical flow. That is assuming a hydraulic jump has not occurred. Then the water surface profile is calculated back from the normal depth control to the gate control under the scenario of subcritical flow. During the subcritical flow profile calculation, the total force is calculated. This calculation is done for both the supercritical (F_{Bottom}) and subcritical (F_{Top}) profiles simultaneously at corresponding positions along the channel. The calculation is iterative, working along the channel from the end of inflow to the gate control, until the difference in the total force values converge to zero (ie. $F_{\text{Bottom}} = F_{\text{Top}}$). This is then the location of the hydraulic jump.

If the difference in the total force values is converging but no hydraulic jump can be located, then a hydraulic rise is present. Two forms of a hydraulic rise can be found. In the one the normal depth control dominates, effectively submerging the gate control. The other is a unique profile in which the water surface profile passes through the gate control as a smooth transition from supercritical to subcritical profile. This correlates to the second root of the quadratic for the initial water surface slope at a gate control.

However, if the difference in the total force values is diverging, then the hydraulic jump lies within the outflow chute. In this case the supercritical profile is extended along the chute until the total force values converge (ie. $F_{\text{Bottom}} = F_{\text{norm}}$).

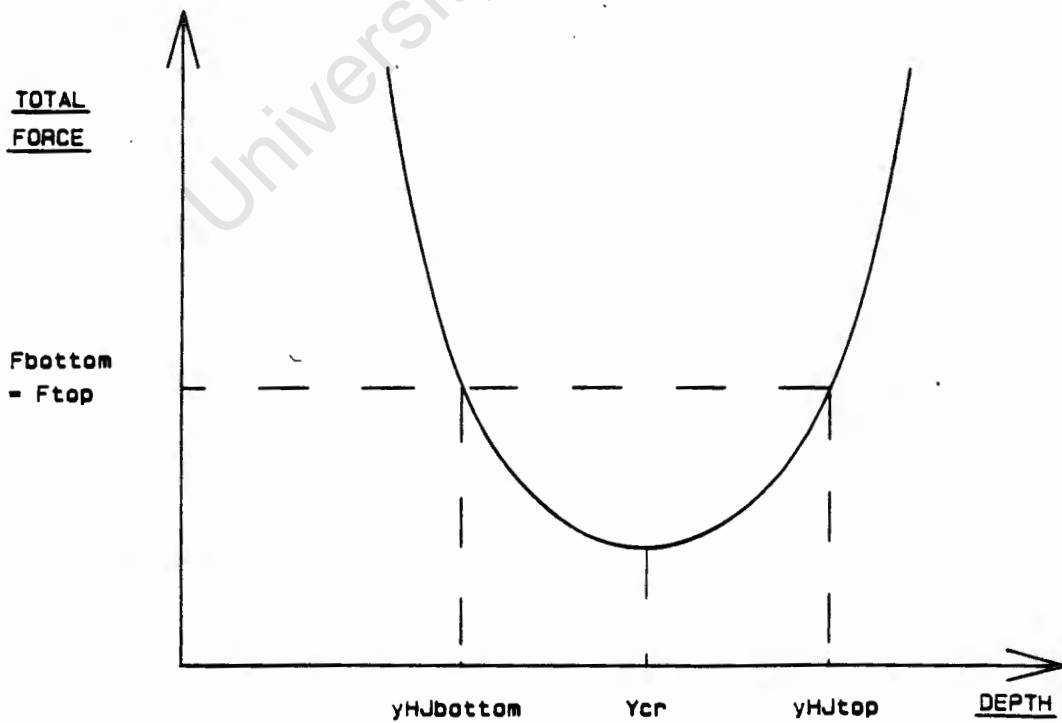
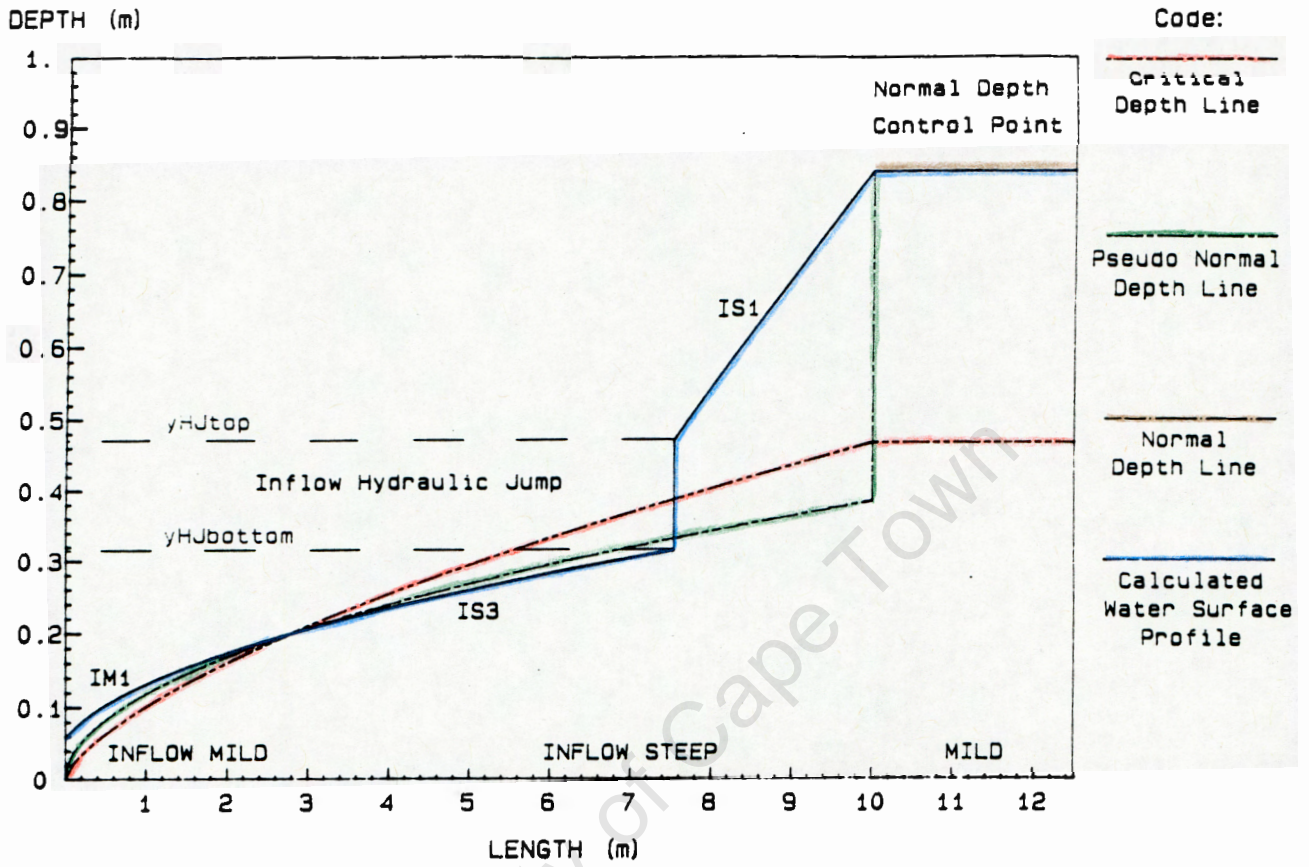


FIGURE 4.7 HYDRAULIC JUMP IDEALISATION

Now that the starting point and the initial water surface slope has been established, the actual calculation of the whole water surface profile can commence. The first section considered is the section upstream from the control point. A backwater calculation of the water surface is initiated, by calculating the water surface slope over a finite step along the channel, multiplying it by the step length and adding it to the initial water depth. In other words

$$y_{\text{water}}(\text{no}+1) = y_{\text{water}}(\text{no}) + \text{step} \cdot \frac{dy}{dx} \quad (4.12)$$

where step is the finite step length. The dependency of the accuracy of the water surface profile on the step length and the variation of dy/dx across the step length, is obvious. For this reason the Runge-Kutta fourth order algorithm is employed for a high degree of accuracy in the numerical integration.

Expressing the differential equation for the water surface slope as

$$\frac{dy}{dx} = f(x, y) \quad ; \quad x(\text{no}), y(\text{no}) \text{ given.}$$

The fourth order Runge-Kutta estimate of $y(\text{no}+1)$

at $x(\text{no}+1) = x(\text{no}) + \text{Step}$ is given by

$$y(\text{no}+1) = y(\text{no}) + (k_1 + 2k_2 + 2k_3 + k_4)/6 \quad (4.13)$$

where $k_1 = \text{Step} \cdot f(x(\text{no}), y(\text{no}))$

$$k_2 = \text{Step} \cdot f(x(\text{no}) + 0,5 \cdot \text{Step}, y(\text{no}) + 0,5 k_1)$$

$$k_3 = \text{Step} \cdot f(x(\text{no}) + 0,5 \cdot \text{Step}, y(\text{no}) + 0,5 k_2)$$

$$k_4 = \text{Step} \cdot f(x(\text{no}) + \text{Step}, y(\text{no}) + k_3)$$

This procedure is then applied to the section downstream of the control point. The water surface slope equation changes once the inflow ceases. Thus the equation governing the water surface slope in the outflow chute is the equation for spatially constant discharge.

The final function of this subroutine is to find the location and value of the maximum water depth. A simple greater than decision structure is used for this purpose.

Now all the water surface profile calculations are complete. The following routines are all post-processing facilities, to present the results in a convenient format.

4.3.8 SUB GRAPHPLOT

This subroutine devotes itself to creating a graph on the monitor. In fulfilling this task, the graph range must first be defined. This is to ensure that all the data fits within the picture window. The picture window must also include the title and axis labels within its boundaries.

Once the picture window is defined, the axes are drawn and incremented. These graphics are hardware independent, designed for a monochrome monitor.

4.3.9 SUB POINTPLOT

This short subroutine plots the required lines on the graph framework assembled in SUB GRAPHPLOT. There is a choice of plotting points, lines or points and lines. The points may be either a dot or a symbol. An iterative loop is used to plot the critical depth line, the pseudo normal depth line and the water surface profile on the monitor.

Unfortunately the different lines cannot be easily distinguished on the monitor, due to the graphics limitation. However, this must be seen only as a crude illustration of the clearly discernible graph produced by the plotter.

4.3.10 SUB PLOTTER

As the name suggests, this routine yields a graphical form of the water surface profile through the use of the plotter. A communications package is used to convey the Hewlett Packard graphics

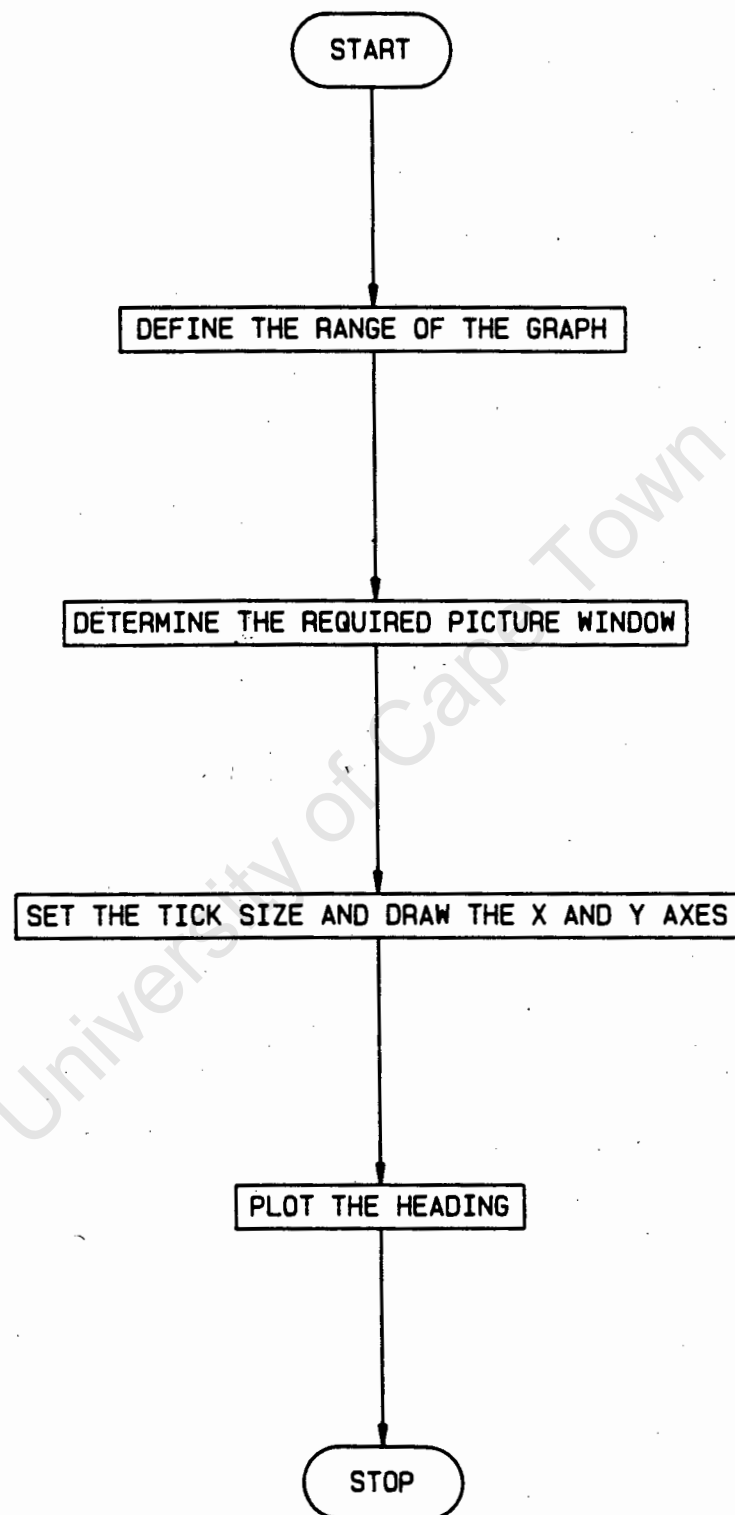


FIGURE 4.8 FLOWCHART FOR SUB GRAPH PLOT

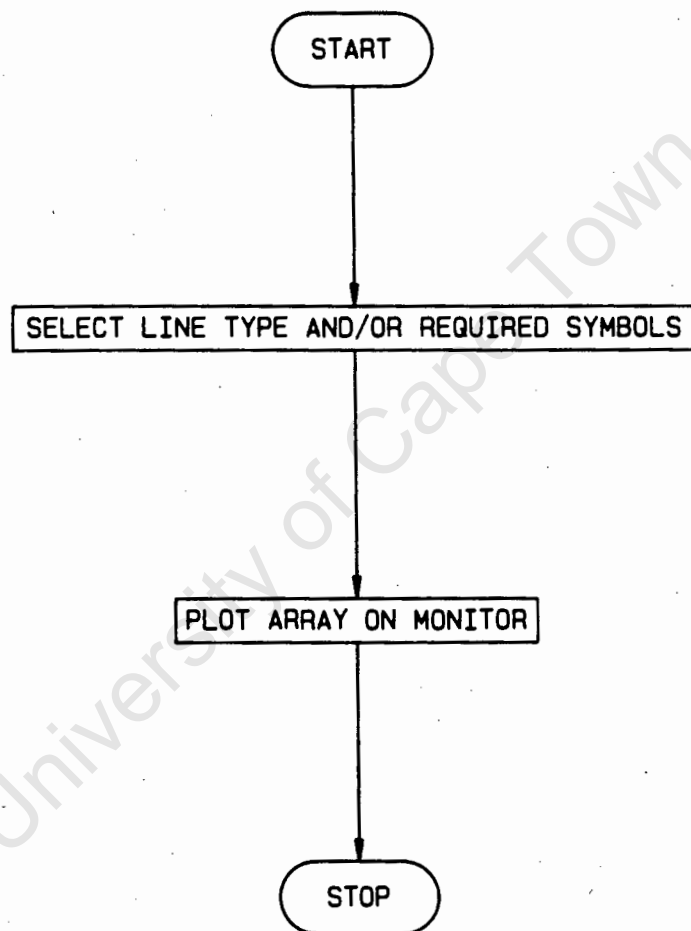


FIGURE 4.9 FLOWCHART FOR SUB POINTPLOT

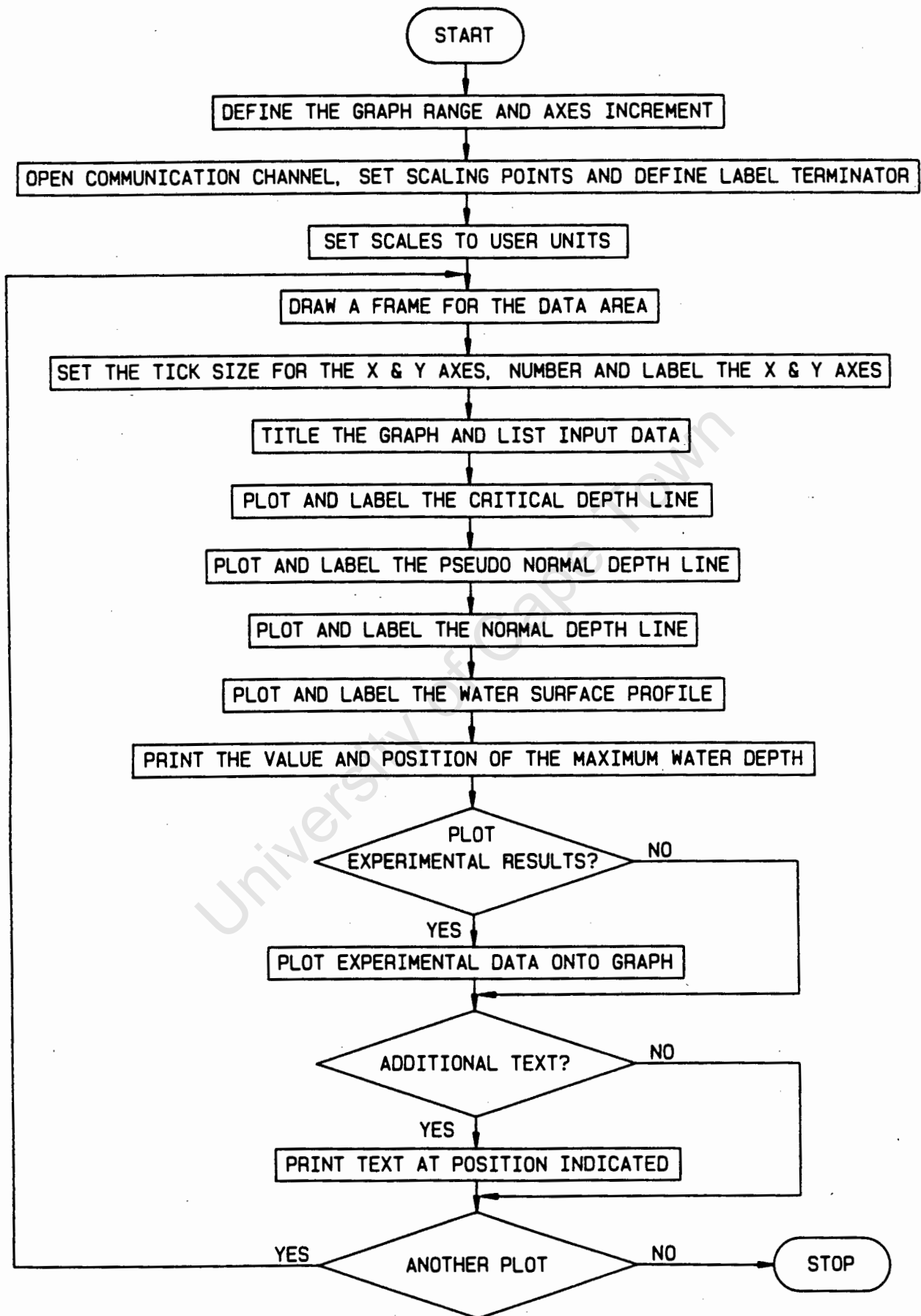


FIGURE 4.10 FLOWCHART FOR SUB PLOTTER

language commands across to the plotter. The procedure adopted is similar to that of SUB GRAPHPLOT and SUB POINTPLOT. However the output is in a more refined form, with the input data listed and the maximum water depth noted.

The use of colour pens enhances the plot. The option of adding additional text to the graph as well as plotting experimental data onto the theoretical profile exists. Figure 4.10 illustrates the plotting sequence used in this subroutine.

Unfortunately the data transmission rate (BAUD rate) has had to be reduced to 600 from the default value of 9600. This is due to the way in which the information is sent across in "packages". Too fast a transmission rate leads to an overflow in the plotter's buffer, which in turn leads to an incomplete plot.

4.4 PROGRAM OPERATION

The simplest way of illustrating the operating procedure for WSPISCS is by means of an example. For this purpose a rectangular channel of dimensions shown in Figure 4.11 is used.

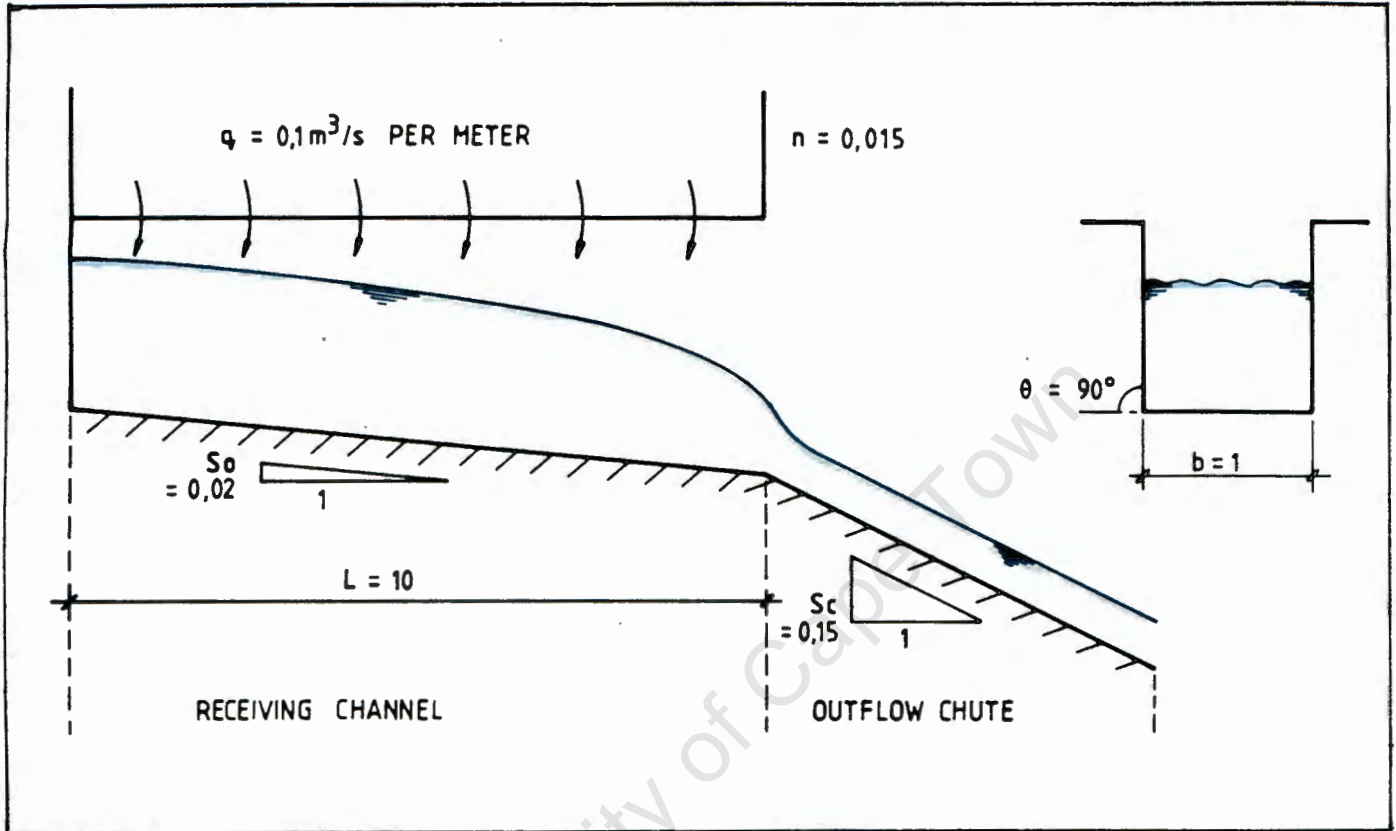


Figure 4.11 : Example 1 Geometric arrangement

As previously stated WSPISCS exists in .EXE file form. The program can therefore be run directly from DOS. Simply type in WSPISCS once the disk containing WSPISCS.EXE is placed in the current drive. Within a few seconds the program is activated.

Initially the title Side Channel Spillway Simulation is displayed on the monitor, along with the subtitle Data required. This is followed by a number of questions concerning the geometry and inflow conditions. Take note of the units required.

At this stage the monitor should display the following :

SIDE CHANNEL SPILLWAY SIMULATION

Data Required:

Test no. or reference string : EXAMPLE 1
Receiving Channel Width in m : 1
Crest Length of Spillway in m : 10
Channel's side slope angle with horizontal (degrees) : 90
Inflow Rate in cubic metres / second per metre : 0.1
Bed Slope of Receiving Channel : 0.02
Manning Coefficient for Channel : 0.015
Bed Slope of Chute : 0.15

Do you want tabulated values as well as the graph ? (Y/N)

The last question refers to the printer output option. Tabulated results are then printed by the printer. Note that if this option is not utilised, the tabulated values are displayed on the monitor. Now the actual calculation process begins. Firstly the critical and normal depths within the outflow chute are calculated. These are then displayed on the monitor along with a flashing wait message to show that the program is still running :

The Critical Depth in the Chute is (m) : 0.467
The Normal Depth in the Chute is (m) : 0.159

WAIT : PROCESSING DATA

The next stage is the location of the control point. Once this is done, the following message is displayed:

WAIT : CALCULATIONS IN PROGRESS

Critical Depth control point at the end of the receiving channel

Depth at the control point : 0.467

Position of control point from the start of the channel : 10.000

Other messages are displayed for the different types of control points. These are :

"Normal Depth control point at the end of the receiving channel"

"Gate Control point within the receiving channel where PNDL and CDL intersect"

"Normal Depth control point at the end of the receiving channel plus a gate control within the receiving channel"

"Either a Hydraulic Rise or a Hydraulic Jump present"

The last message is displayed in conjunction with one of the following :

"Hydraulic Jump within the chute"

"Hydraulic Jump lies outside the chute length under consideration"

"Hydraulic Rise within the receiving channel"

"Hydraulic Jump within the receiving channel"

Once the control point is located, the program goes on to calculate the water surface profile. On the monitor a graphical presentation of the water surface profile is displayed. First the critical depth line is plotted, followed by the pseudo normal depth line and the normal depth line. Finally the water surface profile is plotted along with the maximum water depth and its location along the channel. This is illustrated in Figure 4.12.

Maximum water depth: 0.683
 at X = 4.454
 Press any key to continue
 SIDE CHANNEL SPILLWAY SIMULATION

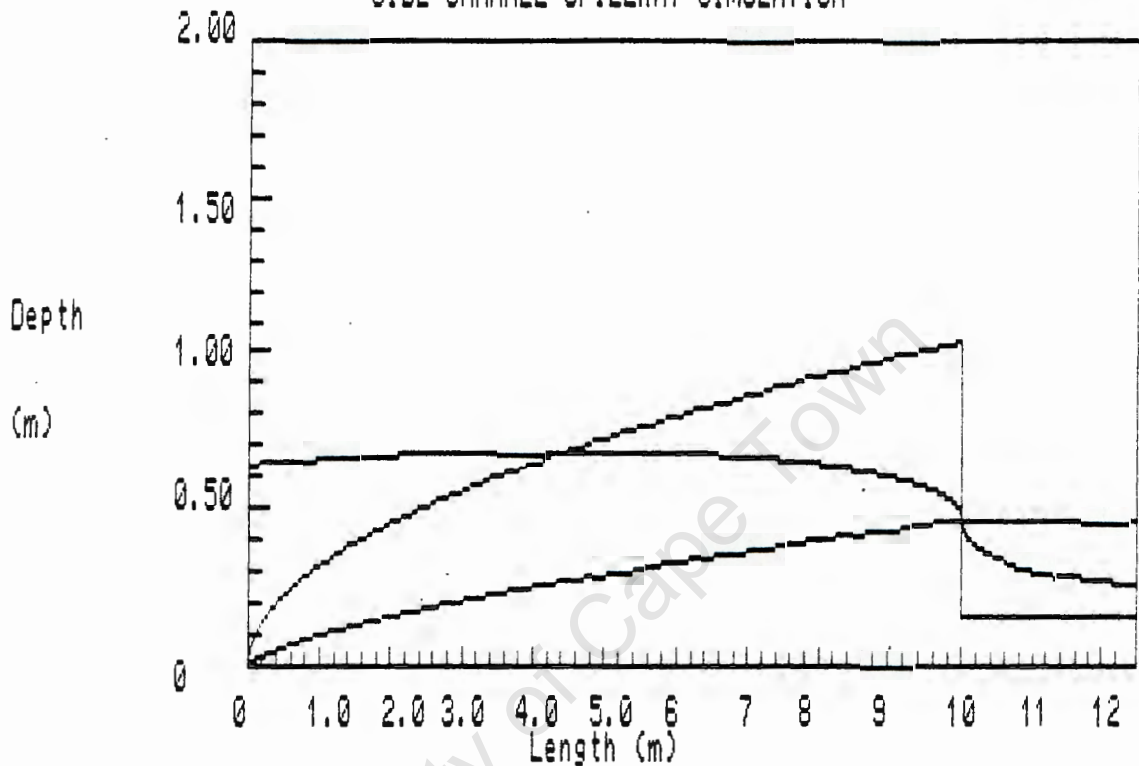


Figure 4.12 : Screen plot of water surface profile

After pressing any key, the plotter option is displayed, along with the required Baud Rate and pen selection. This stage is optional bearing in mind that the plotter routine is written for a Hewlett Packard Plotter 7475A.

Do you want a plot of the graph ? (Y/N & Return) Y

SET THE BAUD RATE TO 600 !

Load the plotter pens : Pen 1 = .3 BLACK
Pen 2 = .7 BLACK
Pen 3 = .3 RED
Pen 4 = .3 GREEN
Pen 5 = .3 BLUE
Pen 5 = .3 BROWN

Press Y & Return when ready !

Within the plotter routine is an option to plot experimental data onto the theoretical profile. The facility also exists for the positioning and plotting of additional text onto the profile. An example of the plotter output is given in Figure 4.13 . To terminate the program the following message is displayed :



THE END

4.5 PROGRAM VERIFICATION

In order to verify the mathematical accuracy of the calculated water surface profile, known examples were processed. The first, classical example checked was that of Hinds (Ref. 2). The correlation between the profile obtained by Hinds and that of the program WSPISCS is good. Only minor discrepancies are evident, due to a difference in the friction calculation. Two further examples calculated by Kilner (Ref. 39) confirm the mathematical accuracy of the program. The profiles in Figures 4.14 and 4.15 illustrate the correlation achieved.

4.6 SENSITIVITY ANALYSIS

The accuracy of the iterative calculation of the water surface profile, can be affected by two factors. Namely the starting point with its initial water surface slope and the step length. A series of

SIDE CHANNEL SPILLWAY SIMULATION

=====

Receiving Channel Width in m : 1.000
 Crest Length of Spillway in m : 10.000
 Channel's side slope angle with horizontal (degrees) : 90.000
 Inflow Rate in cubic meters / second per meter : 0.100
 Bed Slope of Receiving Channel : 0.020
 Manning Coefficeint for Channel : 0.015
 Bed Slope of Chute : 0.020

The Critical Depth in the Chute is (m): 0.467

The Normal Depth in the Chute is (m) : 0.317

CRITICAL DEPTH LINE

=====

Xcr value (m)	Ycr value (m)	noc
0.682	0.078	40
1.967	0.158	80
3.637	0.238	120
5.617	0.318	160
7.864	0.398	200
9.931	0.465	240
12.500	0.467	243

PSEUDO NORMAL DEPTH LINE

=====

Xpnd value (m)	Ypnd value (m)	nop
0.072	0.086	11
0.302	0.176	21
0.689	0.266	31
1.230	0.356	41
1.923	0.446	51
2.767	0.536	61
3.758	0.626	71
4.894	0.716	81
6.171	0.806	91
7.587	0.896	101
9.136	0.986	111
9.691	1.017	121
9.784	1.022	131
9.876	1.027	141
9.970	1.032	151
10.000	0.317	155
10.000	0.317	156
12.500	0.317	157

Critical Depth control point at the end of the receiving channel

Depth at the control point : 0.467

Position of control point from the start of the channel : 10.000

WATER SURFACE PROFILE

=====

In the Receiving Channel :

Xwater (m)	Ywater (m)	dy_dx	no
9.999	0.472	-3.099	1
9.494	0.570	-0.086	102
8.994	0.604	-0.053	202
8.494	0.626	-0.038	302
7.994	0.643	-0.028	402
7.494	0.655	-0.022	502
6.994	0.665	-0.016	602
6.494	0.672	-0.012	702
5.994	0.677	-0.009	802
5.494	0.680	-0.006	902
4.994	0.682	-0.003	1002
4.494	0.683	-0.000	1102
3.994	0.683	+0.002	1202
3.494	0.681	+0.004	1302
2.994	0.678	+0.007	1402
2.494	0.674	+0.009	1502
1.994	0.669	+0.011	1602
1.494	0.663	+0.013	1702
0.994	0.656	+0.015	1802
0.494	0.648	+0.018	1902

In Chute :

Xchute (m)	Ychute (m)	dy/dx	num
10.000	0.462	-3.130	1
10.005	0.447	-3.130	2
10.010	0.446	-0.085	3
10.510	0.423	-0.032	103
11.010	0.410	-0.021	203
11.510	0.401	-0.016	303
12.010	0.393	-0.013	403

Maximum water depth : 0.683
at X = 4.459

SIDE CHANNEL SPILLWAY

Water Surface Profile

$b = 1 \text{ m}$ $L = 10 \text{ m}$ $\theta = 90 \text{ degrees}$

DEPTH (m)

$q = 0.1 \text{ m}^3/\text{s per metre}$ $So = 0.02$ $n = 0.015$ $Sc = 0.15$

Code:

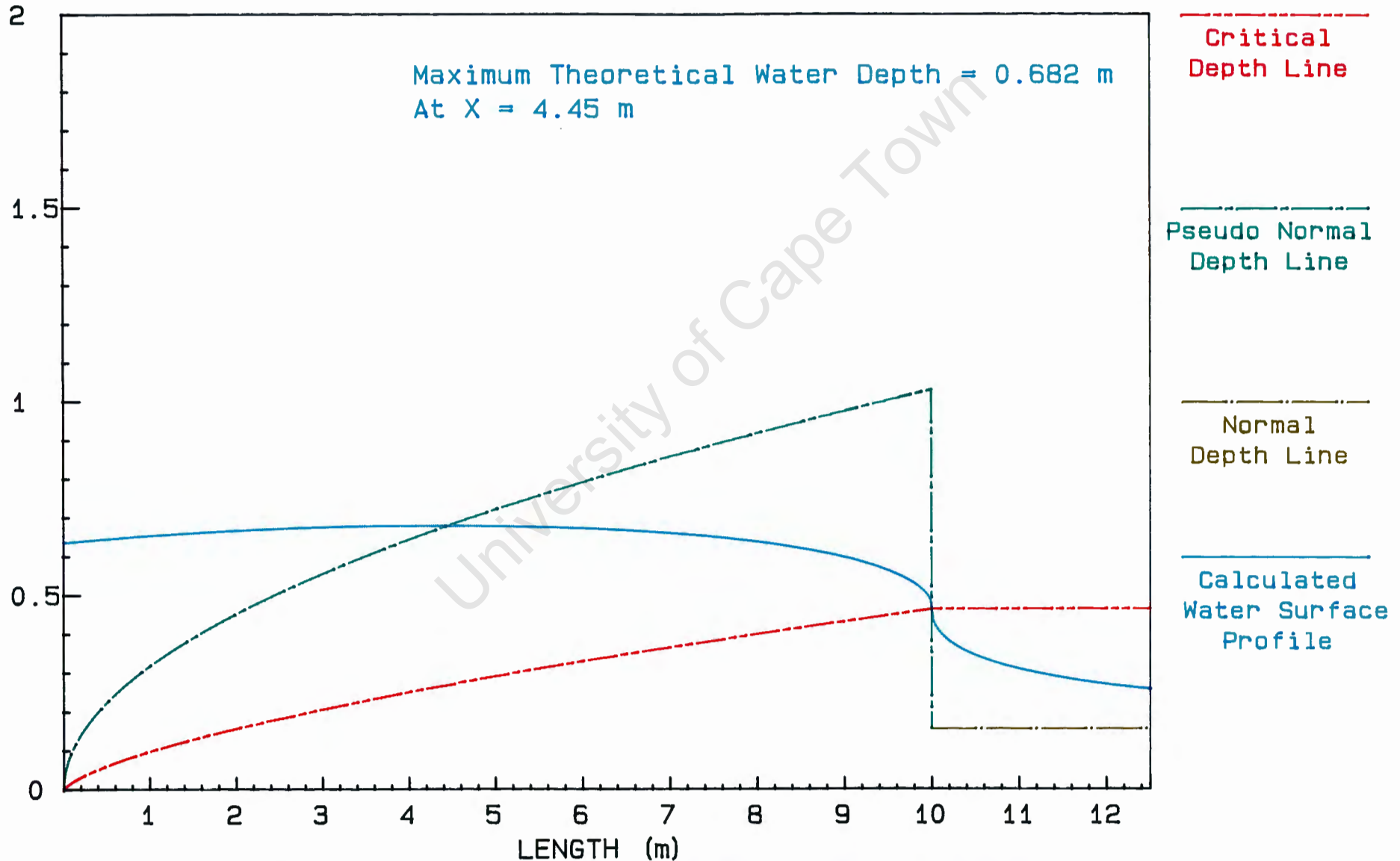


FIGURE 4.13 PLOTTER OUTPUT OF WATER SURFACE PROFILE

SIDE CHANNEL SPILLWAY

Verification Examples

$b = 3.048 \text{ m}$ $L = 121.92 \text{ m}$ $\theta = 63.4349 \text{ degrees}$

DEPTH (m) $q = 3.71612 \text{ m}^3/\text{s per metre}$ $So = 0.15$ $n = 0.015$ $Sc = 0.15$ Code:

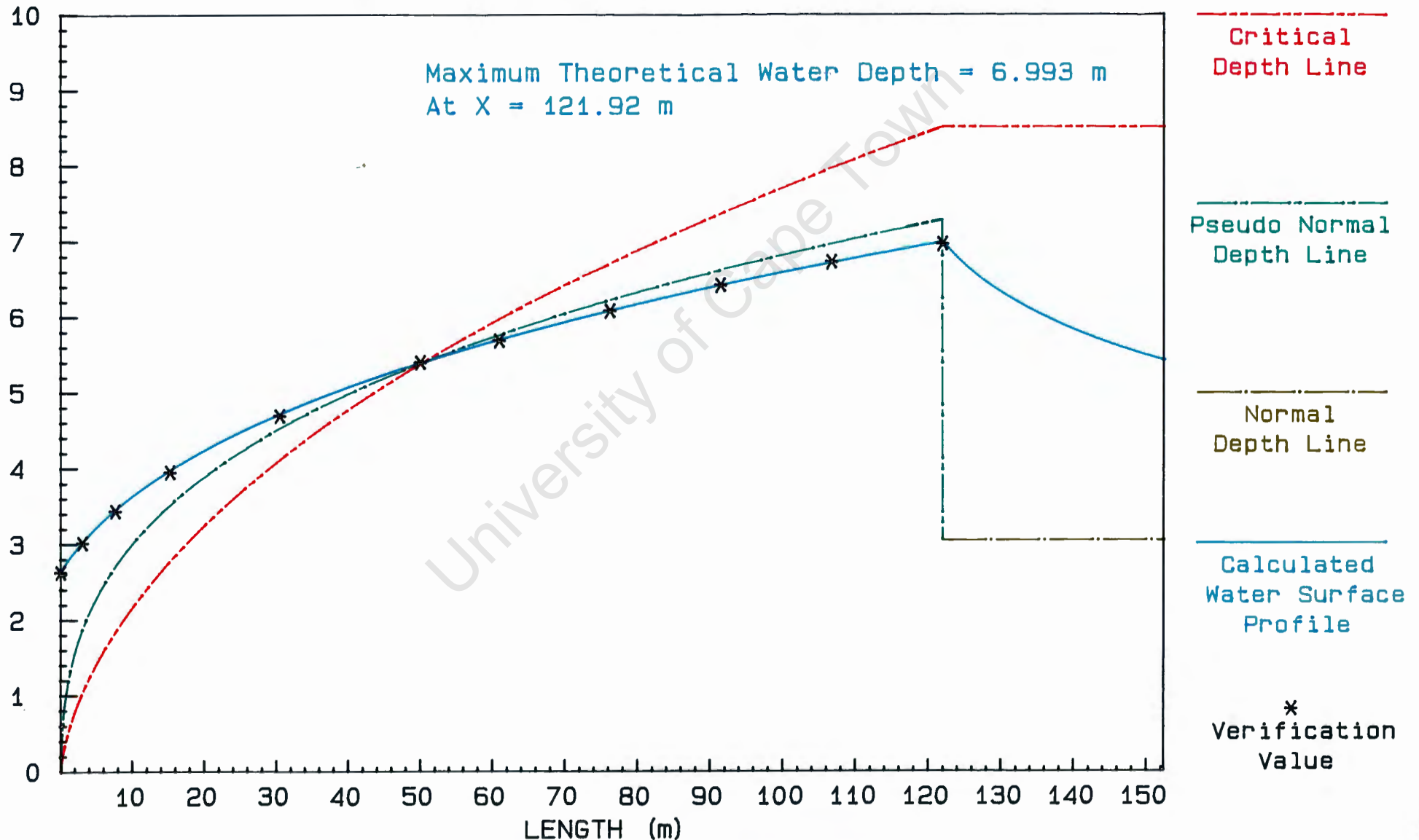


FIGURE 4.14 HINDS' EXAMPLE FOR PROGRAM VERIFICATION

SIDE CHANNEL SPILLWAY

Verification Examples

$b = 1 \text{ m}$ $L = 10 \text{ m}$ $\theta = 90 \text{ degrees}$

DEPTH (m)

$q = 0.1 \text{ m}^3/\text{s per metre}$ $So = 0.02$ $n = 0.015$ $Sc = 0.02$

Code:

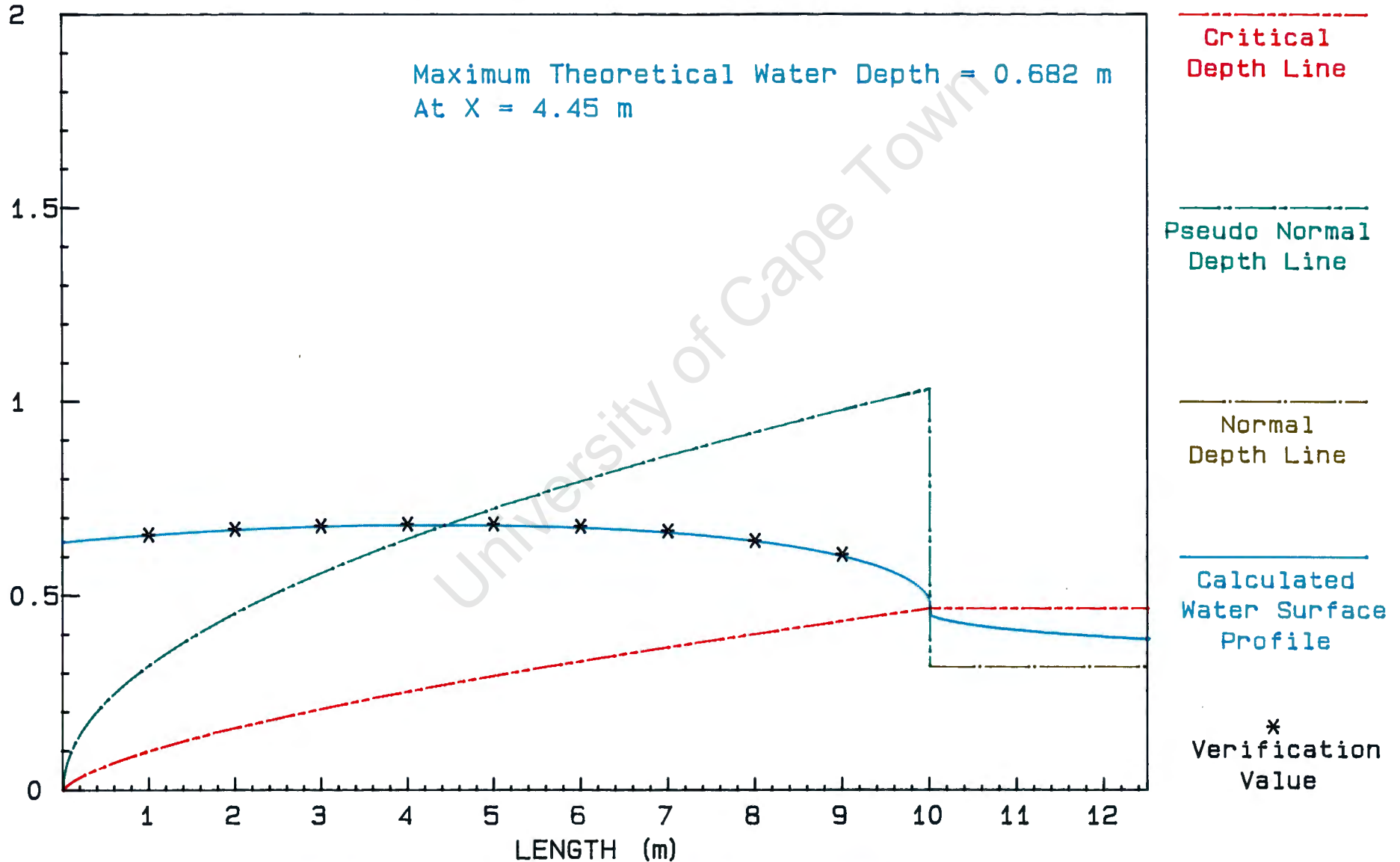


FIGURE 4.15 KILNER'S EXAMPLE FOR PROGRAM VERIFICATION

tests were conducted in order to determine the sensitivity of the program to the above two factors.

In the case of a critical depth control at the end of the receiving channel, the starting point has co-ordinates :

$$y_{\text{water}} = k \cdot y_{\text{crchute}} \quad (4.7)$$

$$\text{and } x_{\text{water}} = \sqrt{\frac{k(3-k^2)}{2}} \cdot L \quad (4.8)$$

for a rectangular channel. It is evident that as k tends to 1, x_{water} tends to L and the initial water surface slope given as :

$$\frac{dy}{dx} = \frac{2 y_{\text{crchute}} \sqrt{2k(3-k^2)}}{3L(1-k^2)} \quad (4.10)$$

increases. In other words the closer the starting point is to the critical depth, the greater the initial water surface slope. Therefore a very small step length is needed in order to obtain an accurate approximation of the water surface profile. Figure 4.16 illustrates the effect of too small a k value.

On the other hand, if the starting point is a fair distance from the critical depth (ie. $k = 1,1$), a larger step length may be used (ie. $\text{step} = L/500$) to obtain a good approximation. This is illustrated in Figure 4.17. Through this process the optimum step length of $L/2000$ and a starting point with $k = 1,01$ was selected for the program. This gives a smooth transition through the critical depth. Figure 4.18 is an illustration of the final step length and starting point.

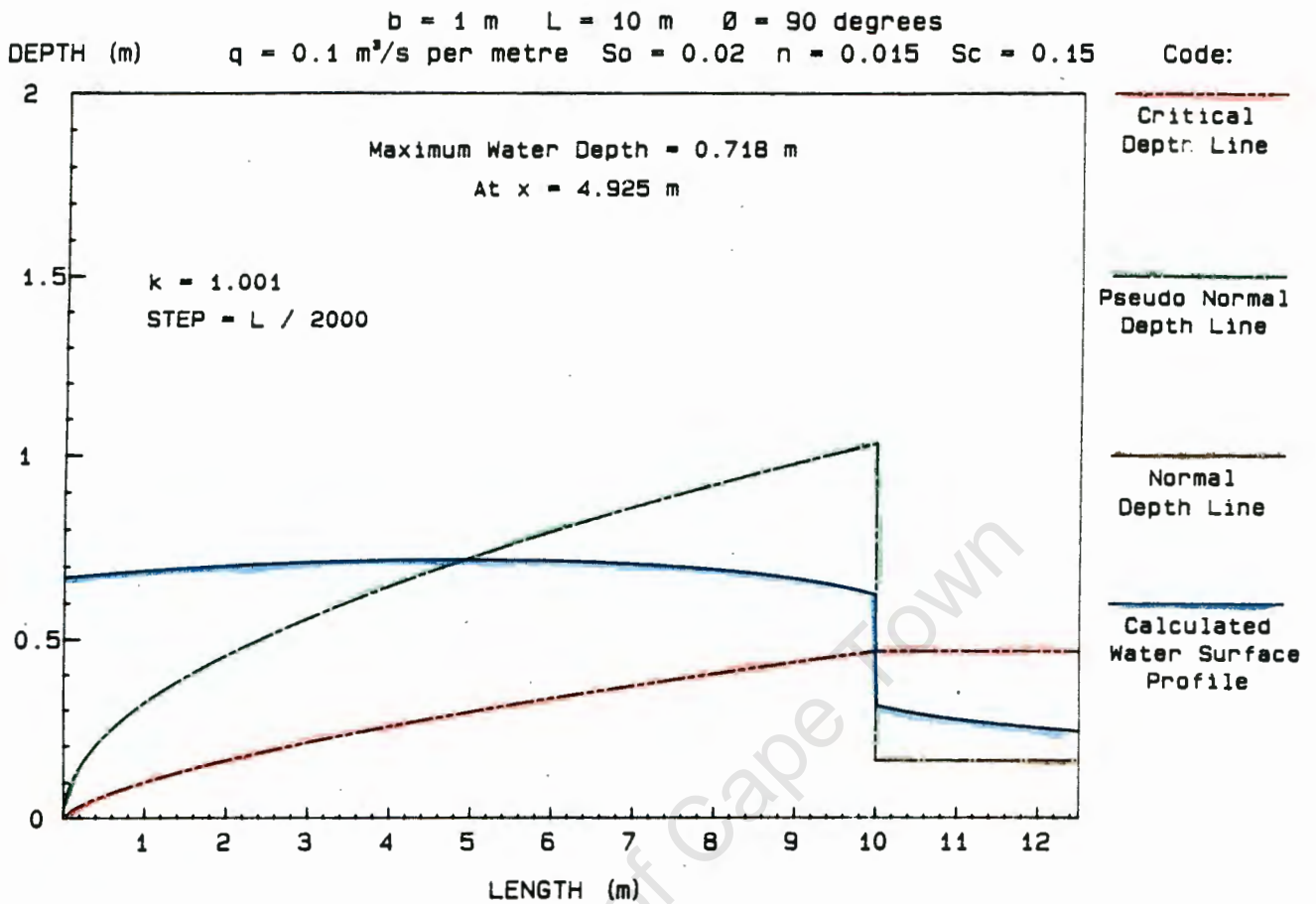


FIGURE 4.16 SENSITIVITY ANALYSIS

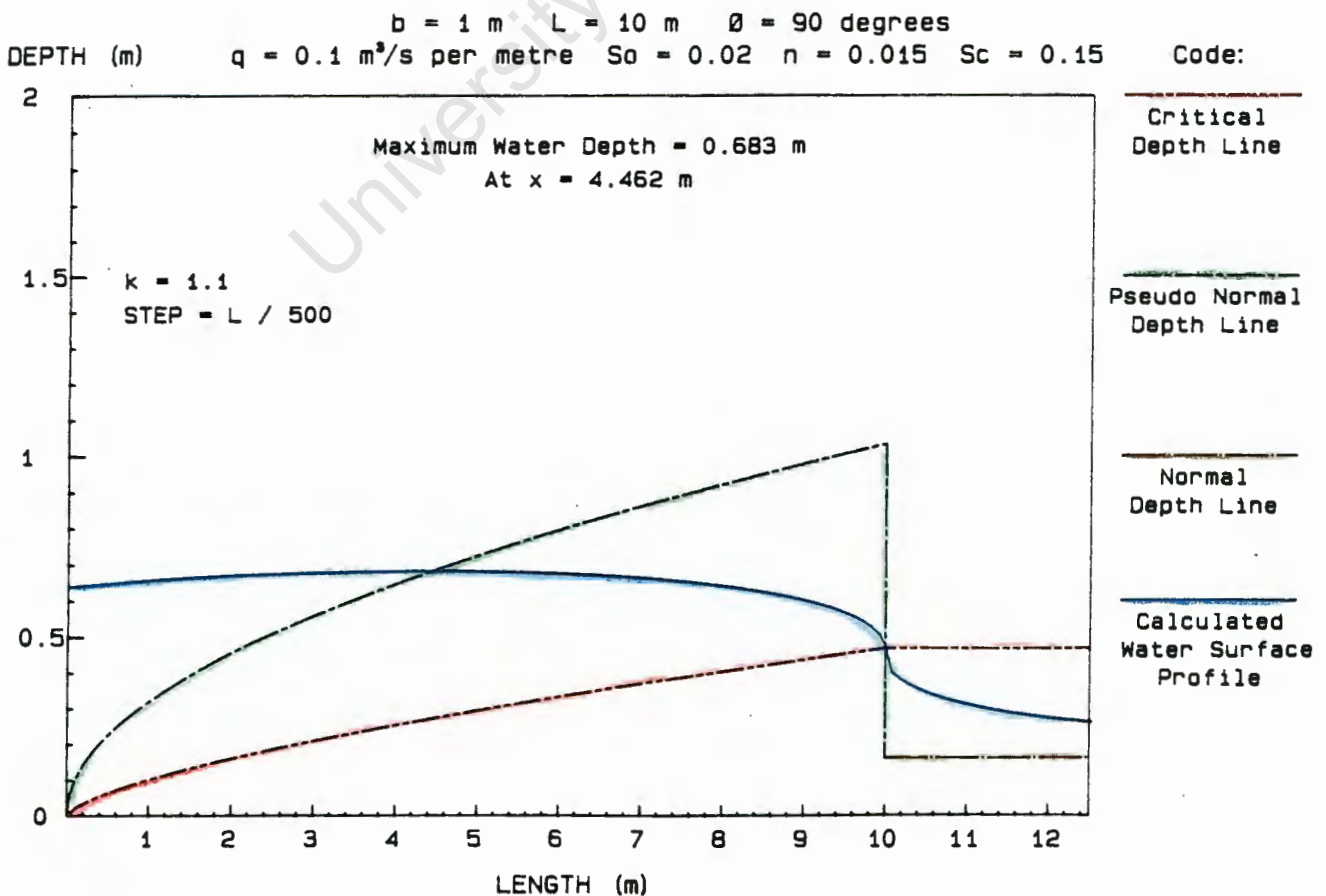


FIGURE 4.17 SENSITIVITY ANALYSIS

SIDE CHANNEL SPILLWAY

Sensitivity Analysis

$b = 1 \text{ m}$ $L = 10 \text{ m}$ $\theta = 90 \text{ degrees}$

DEPTH (m)

$q = 0.1 \text{ m}^3/\text{s per metre}$ $So = 0.02$ $n = 0.015$ $Sc = 0.15$

Code:

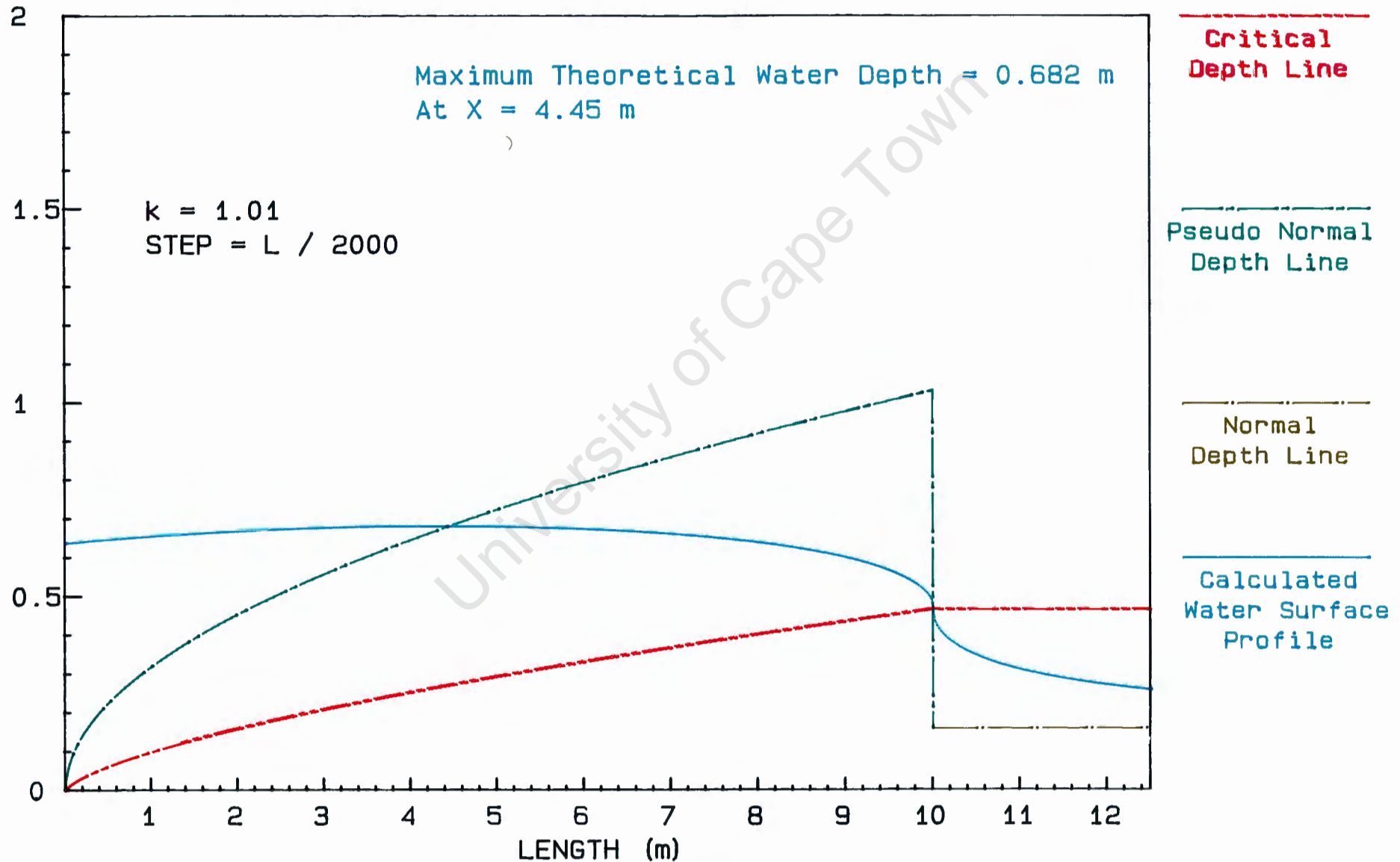


FIGURE 4.18 FINAL STEP LENGTH AND k VALUE

5. COMPARISON WITH PREVIOUS EXPERIMENTS

5.1 EXPERIMENTAL DATA

In order to determine the accuracy of the calculated water surface profile, a number of comparisons are made with available experimental data. This data has been extracted from undergraduate research conducted at the University of Cape Town, under the guidance of Professor F A Kilner. Unfortunately these experiments were conducted a number of years prior to this thesis. Exact details regarding the experimental procedure and accuracy of the measurements are unknown. This in turn leads to speculation as to the confidence placed in the results.

Two types of experimental apparatus were used. The first was a double crested side channel spillway, constructed in the 0,6 m laboratory flume. The two ogee shaped spillway crests discharge into a rectangular receiving channel. The floor of the receiving channel was supported by four adjustable foot screws. In this way a variety of receiving channel bed slopes could be tested. No outflow chute was employed in these experiments. A sluice gate at the end of the receiving channel was used to simulate the effect of hydraulically mild flow conditions in the outflow chute. A micrometer point gauge was used to obtain the water depth at specific points along the receiving channel. Maximum and minimum water depths were measured, due to variations in water depth across the channel, as shown in Figure 5.1.

The second type of apparatus used was similar to that used by Fox and Goodwill (Ref. 22). In their experiments the inflow was introduced into the receiving channel through perforated side walls (Figure 3.8). This greatly reduced the level of turbulence and air entailment associated with introducing the inflow from above. In the experiments considered in this thesis, the inflow was introduced into the receiving channel through a perforated floor. To assure uniformity of

inflow along the channel, three distinct inflow compartments were used. Each of the compartments were individually supplied with a gate valve and a rotameter for each compartment. The individual flows were regulated to give uniform spatially varied discharge in the receiving channel. Figures 5.2 and 5.3 illustrate the equipment utilised.

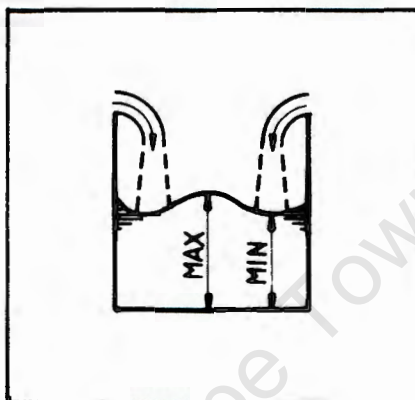


Figure 5.1 : Variation in Water depth across the receiving channel

The model was supported on four foot screws. These were adjustable, in order to vary the slope of the receiving channel. To simulate hydraulically steep flow conditions in the outflow chute, the water discharged freely under gravity from the receiving channel. A sluice gate is utilised at the end of the receiving channel, to simulate mild hydraulic conditions within the outflow chute. Figure 5.3 illustrates the water surface profile for hydraulically mild inflow conditions and steep conditions in the outflow chute.

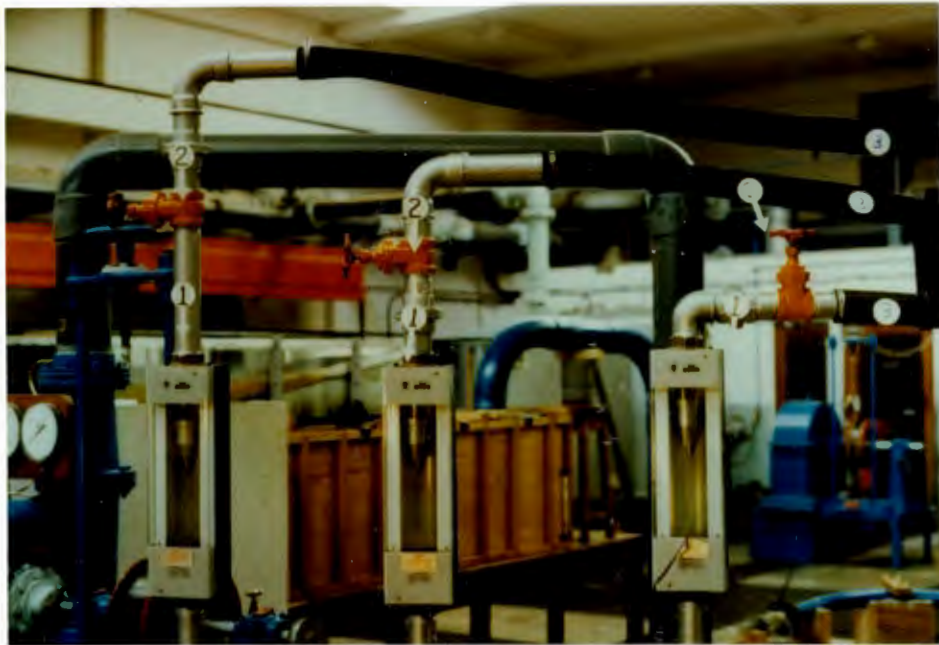


FIGURE 5.2 EXPERIMENTAL EQUIPMENT [HOLDEN, 1978]

KEY

- 1 ROTAMETERS
- 2 CONTROL VALVES
- 3 SUPPLY PIPES

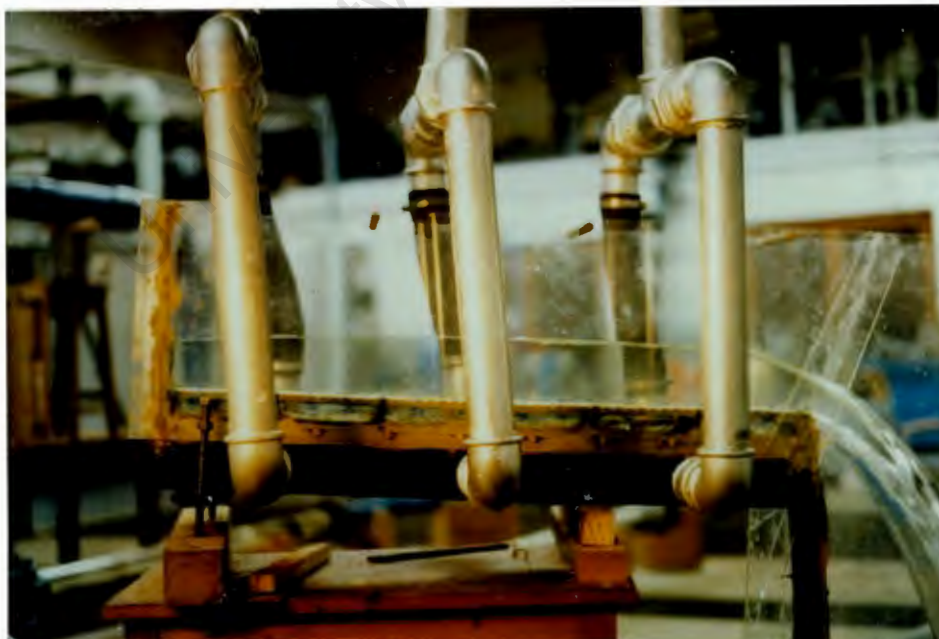


FIGURE 5.3 EXPERIMENTAL EQUIPMENT [HOLDEN, 1978]

5.2 CALCULATED AND EXPERIMENTAL PROFILES

To give clear illustration of the correlation between the calculated and experimental profiles, the experimental results are plotted onto the calculated profile. The * symbol is used to represent the experimental profile points. At this stage it must be reiterated that these experiments were not conducted under the direct control of the author, but rather some years previously. Therefore the degree of accuracy of these results is unknown.

In order to differentiate between the two experimental methods, a small diagram is given in the top left hand corner of each profile. Figure 5.4 illustrates the two diagrams used and their meaning.

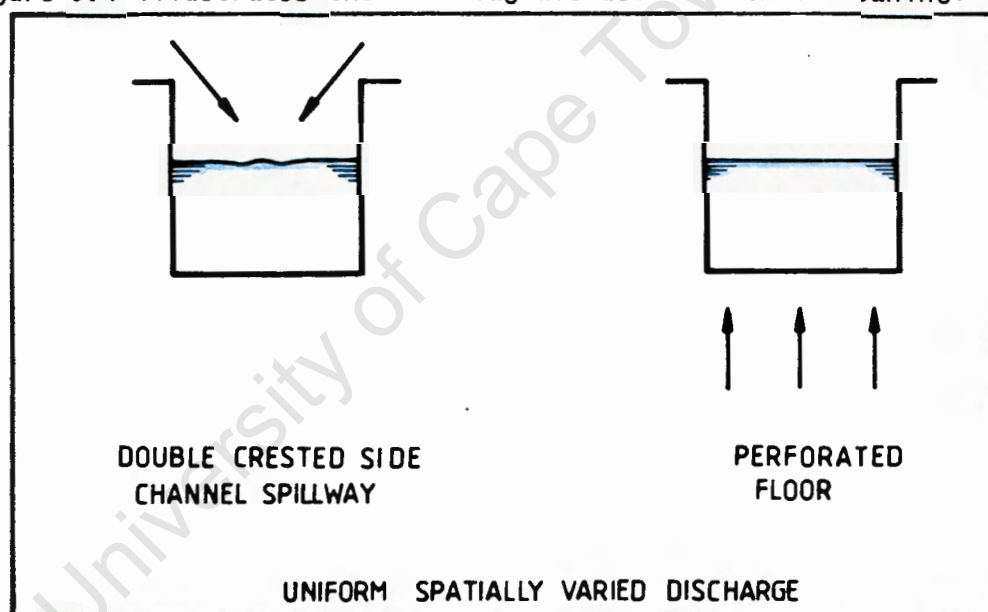


Figure 5.4 : Sketch description for inflow conditions

Note that the profiles given for a double crested side channel spillway, the maximum and minimum water depths are given at specific points. This is due to the variation in water depth across the channel as shown in Figure 5.1 . The average experimental water surface profile lies between these two points at any given section.

Only a limited selection of the water surface profiles are presented, to avoid repetition. The ones chosen illustrate the various trends

and anomalies. Figures 5.5 to 5.7 are concerned with a horizontal receiving channel. In Figure 5.5 the experimental results correlate well with the theoretical profile until near the end of the receiving channel. In this region the points dip distinctly below the calculated profile. In the author's opinion, the reasons for this is the phenomenon called streamline curvature. The outflow chute in these experiments was omitted, leaving the water to fall freely under gravity. In such circumstances the critical depth drops to 71,5% of its value calculated by the normal hydrostatic method. The experimental profile drops below the calculated profile to accommodate this reduced critical depth value at the end of the receiving channel.

Figure 5.6 illustrates the same geometric arrangement as that in Figure 5.5. The only difference is that the inflow rate has been tripled. In this case the experimental profile lies distinctly below the calculated one. The shape of the profiles is however almost identical. This phenomenon was observed in a number of cases. The trend seems to be, that with a horizontal receiving channel with an abrupt end, the experimental profile tends to drop below the calculated profile as the inflow rate increases.

To simulate the mild hydraulic conditions in the outflow chute, as portrayed in Figure 5.7, a sluice gate is employed. This sluice gate is positioned at the end of the receiving channel, and acts as a normal depth control point. The experimental and calculated profile correlation is good for the horizontal receiving channel. However in Figure 5.8 when the receiving channel is inclined, the correlation is not as good. The problems of a local rise in the water surface on the upstream face of the sluice gate seems to influence the water surface profile. The calculated profile utilises the upstream depth at the sluice gate as the starting point for the water surface profile calculation. This in turn leads to the calculated profile lying above the experimental one.

In Figures 5.9 and 5.10 good correlations are illustrated. Note that the maximum water depth is located at the point where the water surface profile crosses the pseudo normal depth line. In these two examples the receiving channel is considered as hydraulically mild.

The scenario portrayed in Figures 5.11 and 5.12 are those of a gate control located within the receiving channel. At the gate control the flow regime changes from hydraulically mild to hydraulically steep. However the bed slope is constant throughout the receiving channel. Undulations are observed in the water surface profile of these steeper slopes. The reasons for these undulations are not apparent, however the correlation between experimental and calculated profiles is still good.

Figure 5.13 illustrates the phenomenon called a hydraulic rise. This occurs when a normal depth control and a gate control exist. The normal depth control dominates the water surface profile. The gate control is effectively submerged and does not act as a control point. This unusual profile was only generated successfully once in all the previous experiments. There is a fair correlation between the experimental and calculated profiles. However the large discrepancy between the experimentally derived maximum and minimum water depths at specific points along the channel serves as a good indicator of the degree of turbulence within the receiving channel.

SIDE CHANNEL SPILLWAY

Calculated & Experimental comparisons

$b = 0.15 \text{ m}$ $L = 0.9 \text{ m}$ $\theta = 90 \text{ degrees}$

DEPTH (X 10^{-4} m) $q = 0.00961 \text{ m}^3/\text{s per metre}$ $So = 00$ $n = 0.01$ $Sc = 0.2$

Code:

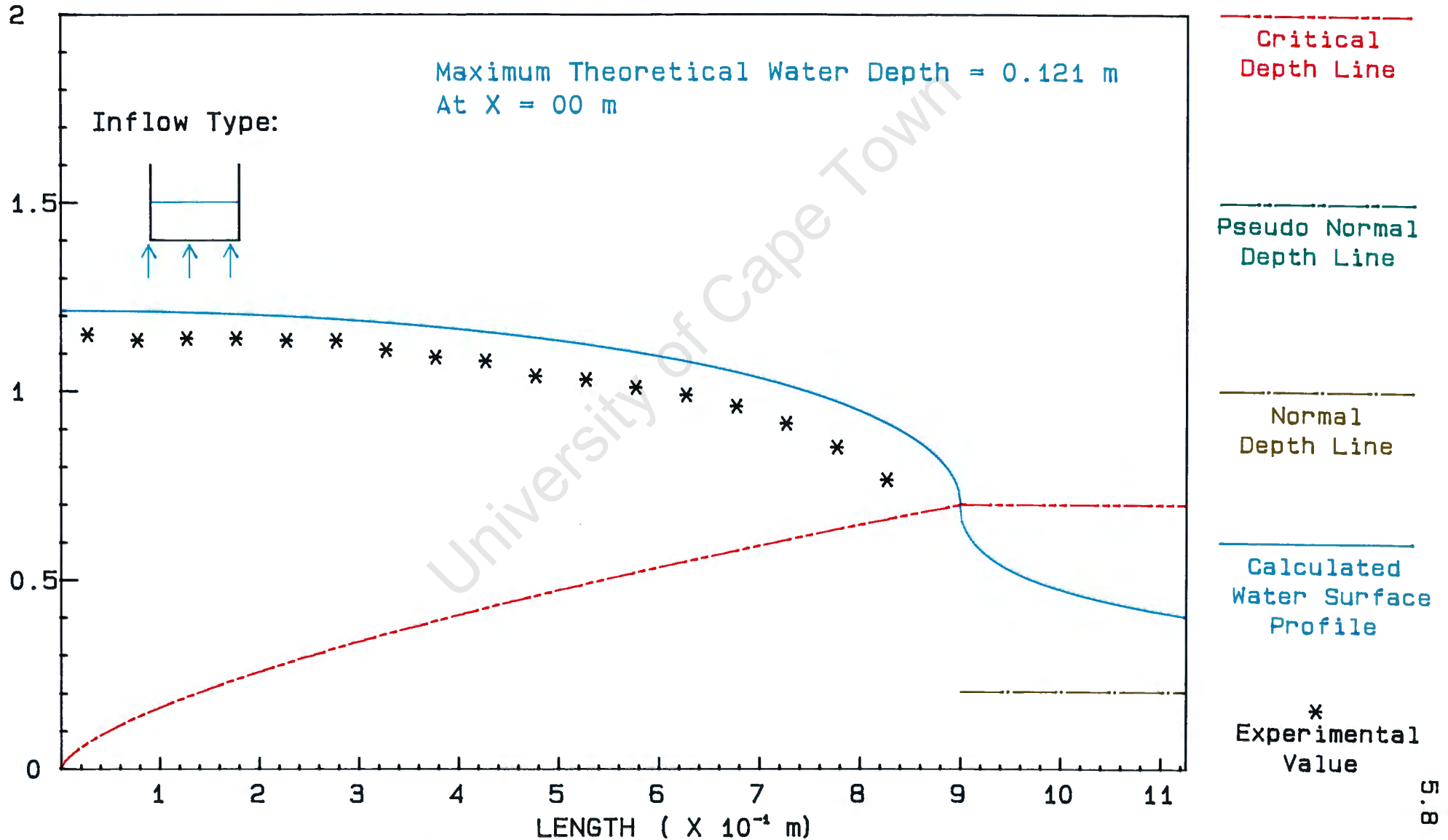


FIGURE 5.6 EXPERIMENTAL AND CALCULATED PROFILES

SIDE CHANNEL SPILLWAY

Calculated & Experimental comparisons

$b = 0.127 \text{ m}$ $L = 1.2192 \text{ m}$ $\theta = 90 \text{ degrees}$

DEPTH (X 10^{-4} m) $q = 0.01381 \text{ m}^3/\text{s per metre}$ $So = 00$ $n = 0.01$ $Sc = 0.003595$ Code:

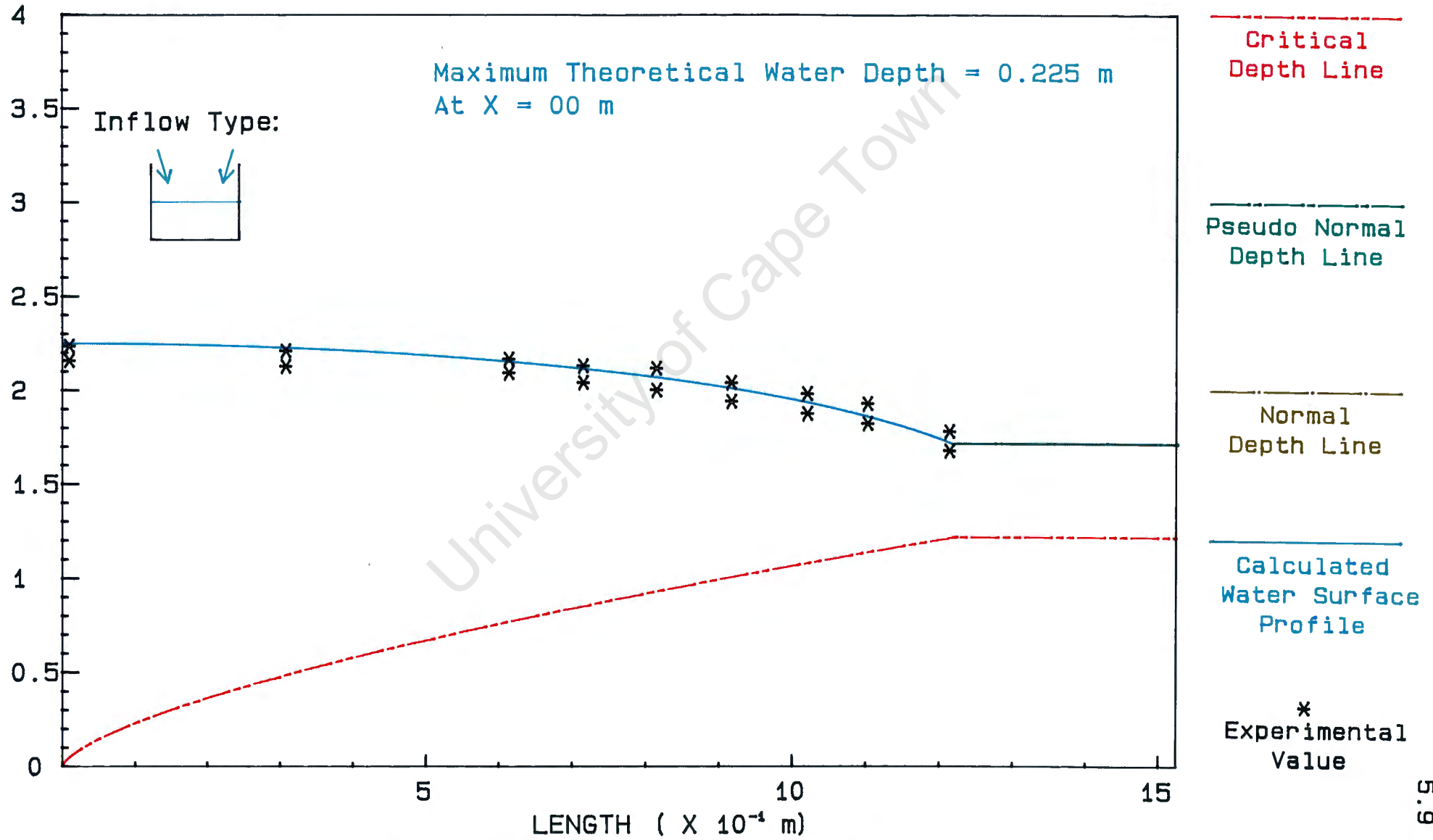


FIGURE 5.7 EXPERIMENTAL AND CALCULATED PROFILES

SIDE CHANNEL SPILLWAY

Calculated & Experimental comparisons

$b = 0.152 \text{ m}$ $L = 0.785 \text{ m}$ $\theta = 90 \text{ degrees}$

DEPTH (X 10^{-4} m) $q = 0.01064 \text{ m}^3/\text{s per metre}$ $So = 0.0405$ $n = 0.01$ $Sc = 0.00067$

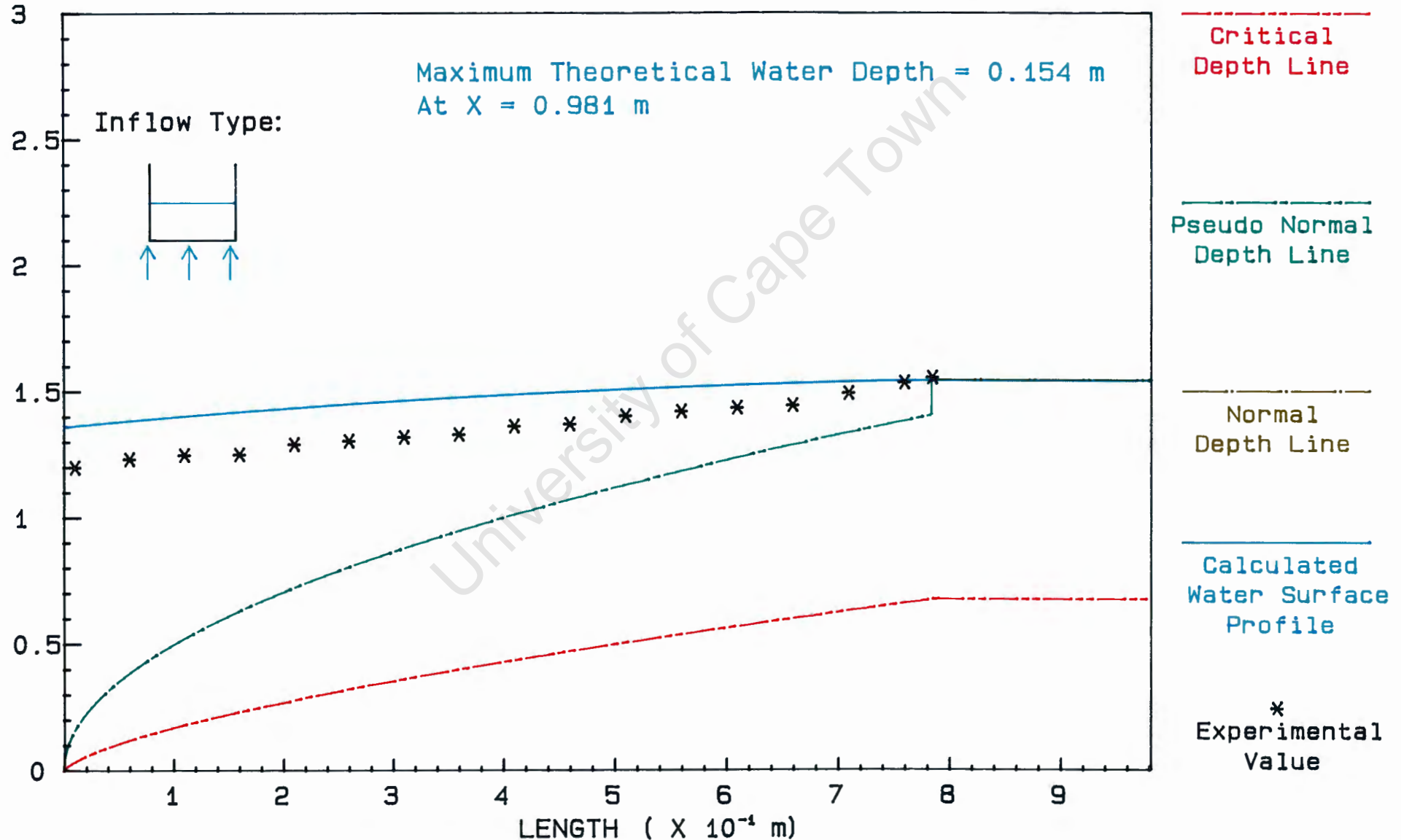


FIGURE 5.8 EXPERIMENTAL AND CALCULATED PROFILES

SIDE CHANNEL SPILLWAY

Calculated & Experimental comparisons

$b = 0.15 \text{ m}$ $L = 0.9 \text{ m}$ $\theta = 90 \text{ degrees}$

DEPTH ($\times 10^{-1} \text{ m}$) $q = 0.00961 \text{ m}^3/\text{s per metre}$ $So = 0.04143$ $n = 0.01$ $Sc = 0.013$ Code:

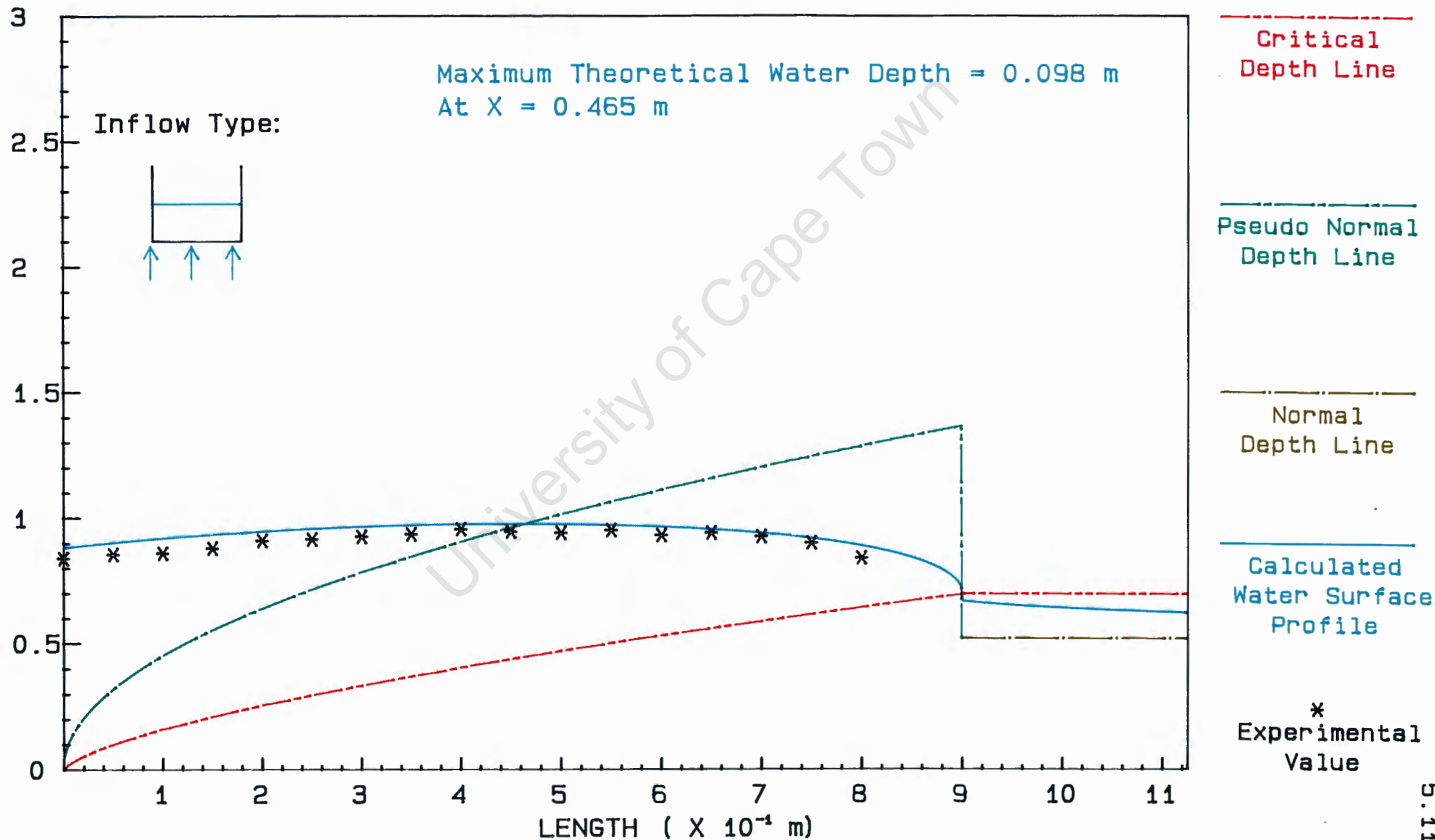


FIGURE 5.9 EXPERIMENTAL AND CALCULATED PROFILES

SIDE CHANNEL SPILLWAY

Calculated & Experimental comparisons

$b = 0.15 \text{ m}$ $L = 0.9 \text{ m}$ $\theta = 90 \text{ degrees}$

DEPTH (X 10^{-4} m) $q = 0.00961 \text{ m}^3/\text{s}$ per metre $So = 0.067$ $n = 0.01$ $Sc = 0.00384$ Code:

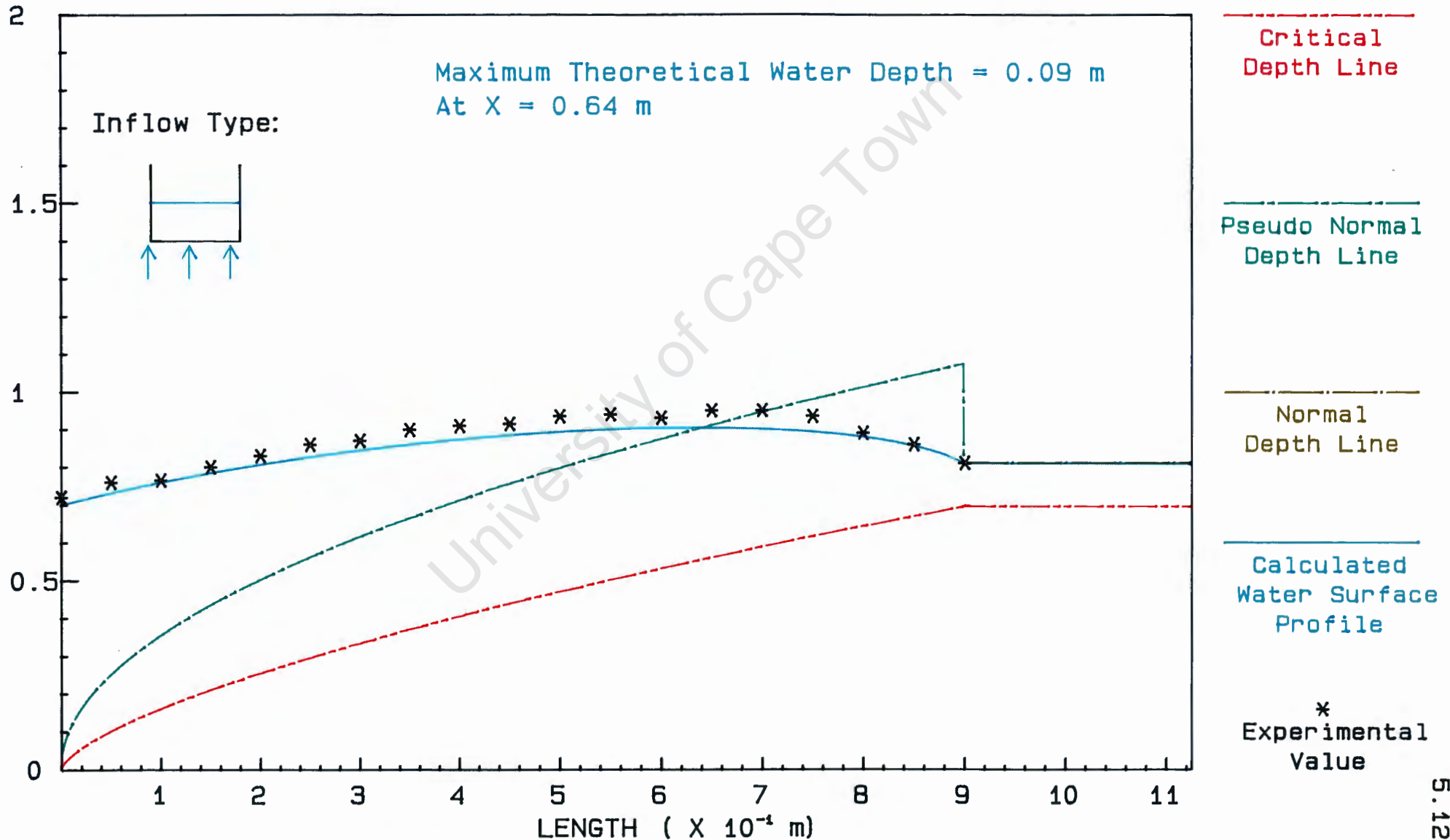


FIGURE 5.10 EXPERIMENTAL AND CALCULATED PROFILES

SIDE CHANNEL SPILLWAY

Calculated & Experimental comparisons

$b = 0.15 \text{ m}$ $L = 0.9 \text{ m}$ $\theta = 90 \text{ degrees}$

DEPTH (X 10^{-1} m) $q = 0.005 \text{ m}^3/\text{s per metre}$ $So = 0.11$ $n = 0.01$ $Sc = 0.2$

Code:

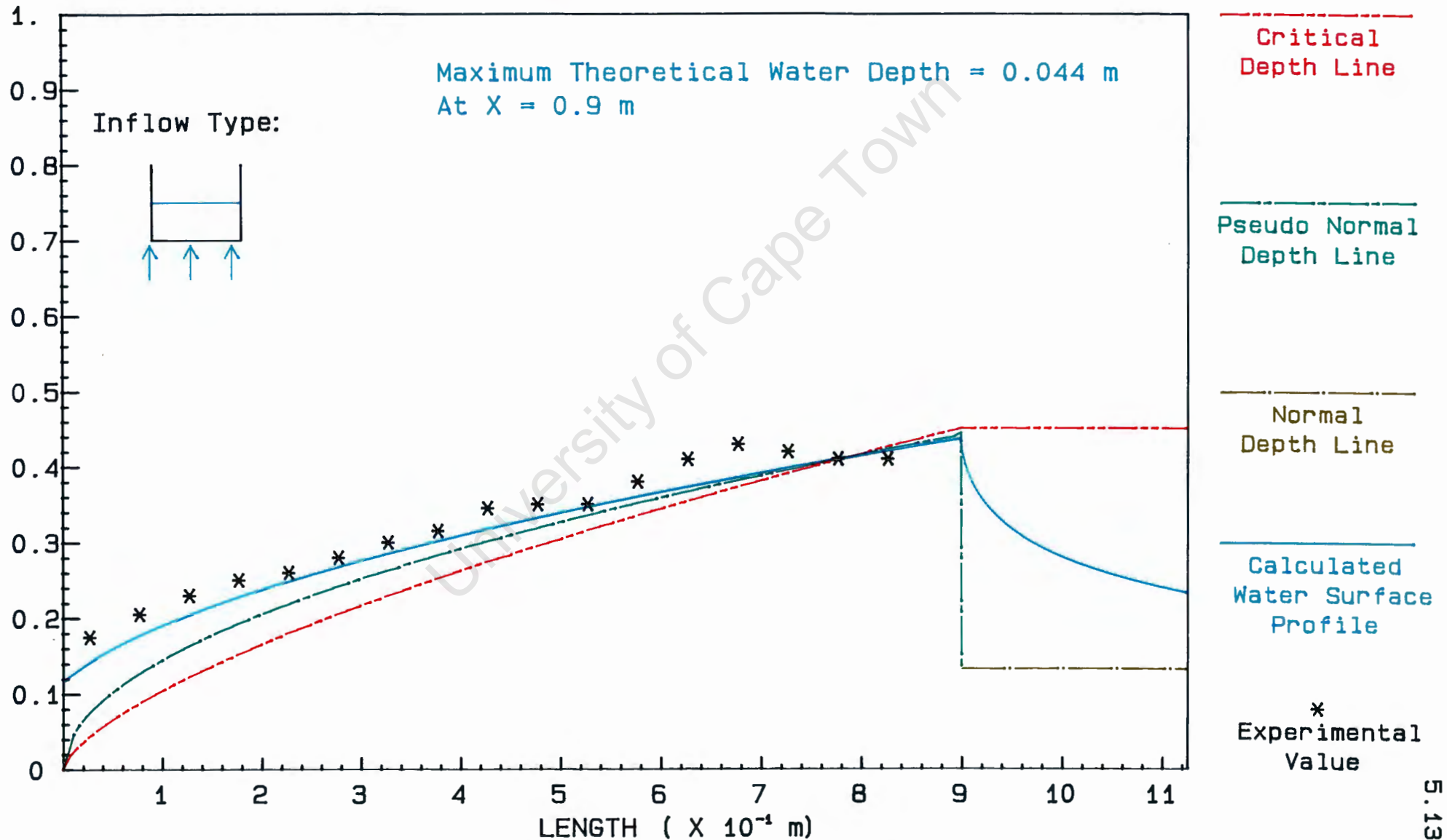


FIGURE 5.11 EXPERIMENTAL AND CALCULATED PROFILES

SIDE CHANNEL SPILLWAY

Calculated & Experimental comparisons

$b = 0.15 \text{ m}$ $L = 0.9 \text{ m}$ $\theta = 90 \text{ degrees}$

DEPTH (X 10^{-1} m) $q = 0.00961 \text{ m}^3/\text{s per metre}$ $So = 0.2144$ $n = 0.01$ $Sc = 0.3$

Code:

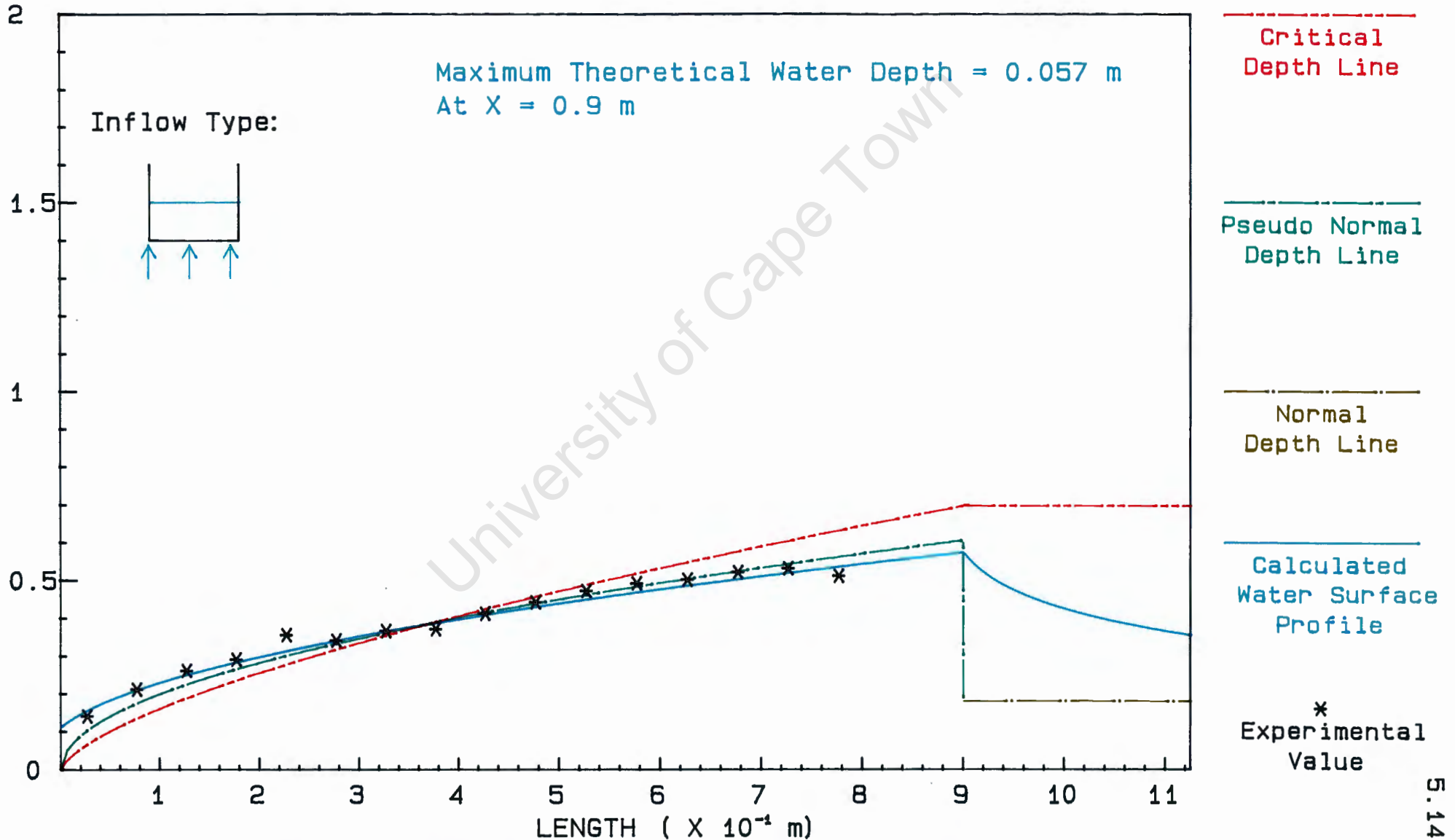


FIGURE 5.12 EXPERIMENTAL AND CALCULATED PROFILES

SIDE CHANNEL SPILLWAY

Calculated & Experimental comparisons

$b = 0.127 \text{ m}$ $L = 1.2192 \text{ m}$ $\theta = 90 \text{ degrees}$

DEPTH ($\times 10^{-4} \text{ m}$) $q = 0.00469 \text{ m}^3/\text{s per metre}$ $So = 0.1188$ $n = 0.01$ $Sc = 0.001176$

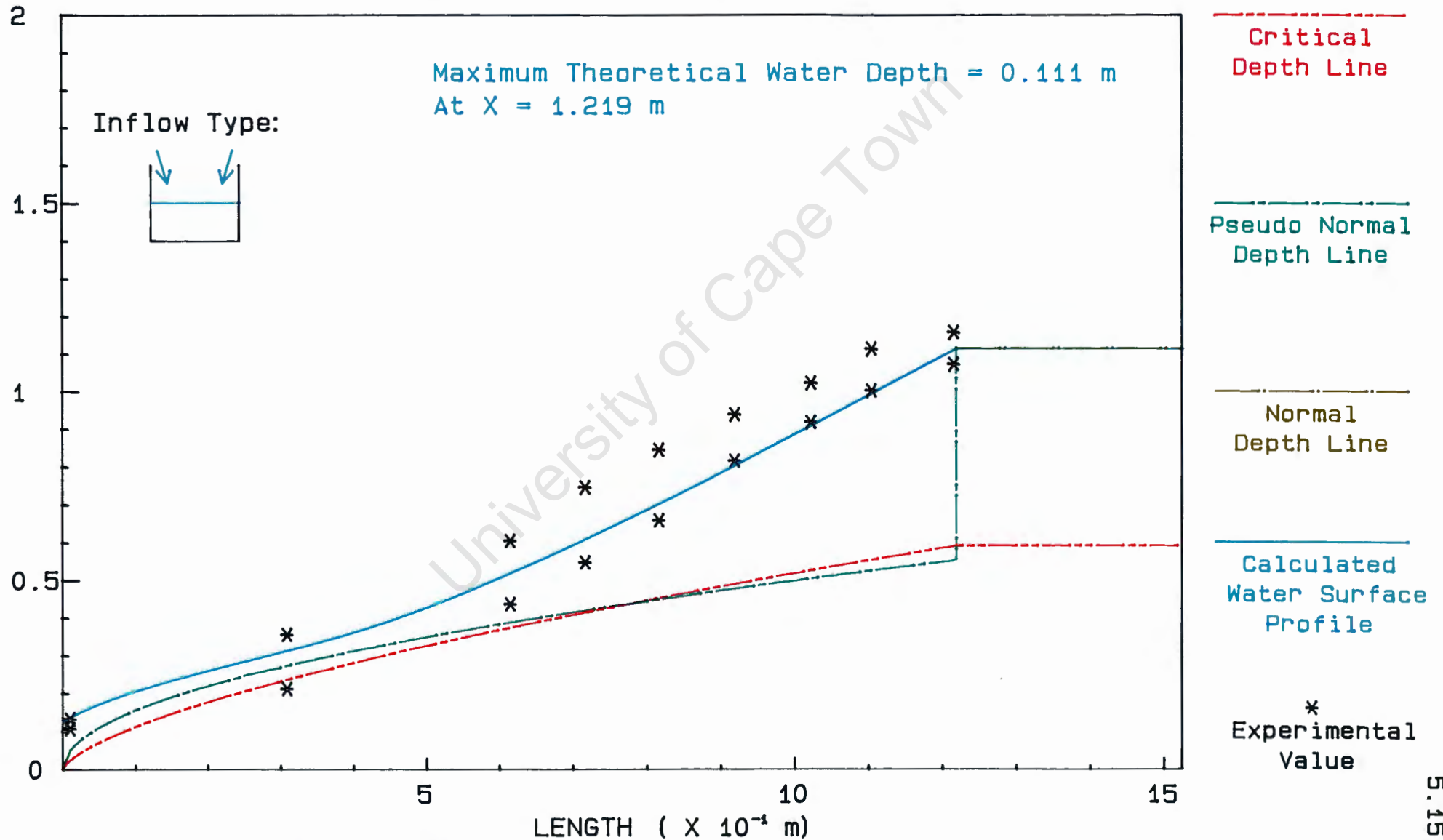


FIGURE 5.13 EXPERIMENTAL AND CALCULATED PROFILES

6. CONCLUSIONS

The theory initially proposed by Hinds [2] is successfully developed over the years into a complete systematic method of analysis. This method is successfully incorporated in the program WSPISCS, to compute an approximation of the water surface profile in a side channel spillway.

The Runge-Kutta fourth order algorithm provides an accurate method of numerical integration. This is verified by excellent correlations with previous calculation examples, including the classical example produced by Hinds. Comparisons with experimental results obtained over a number of years of undergraduate research is good. The correlation between the calculated and experimental profiles reveals one deviation. That is a horizontal receiving channel and a hydraulically steep outflow chute. In this case the experimental profile falls distinctly below the calculated one as the inflow rate increases.

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Note: The chronological style of Bibliography is deliberately used to emphasise the evolution of Side Channel Spillway concepts over a considerable period of time.

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APPENDIX A

Examinations written by the author to complete the requirements of the degree.

<u>Examination</u>		<u>Credit Rating</u>
CIV 509F	Structural Loading (July 1988)	3
CIV 516F	Coastal Hydraulics (July 1988)	5
CIV 592Z	Project Management in Civil Engineering (July 1988)	3
CIV 536S	Coastal Engineering Practice (November 1988)	5
CIV 525S	Contract Law (November 1988)	3
CIV 543S	Airport Design (November 1988)	3
CIV 522Z	Introduction to Finite Elements (July 1989)	3
Thesis:	"Water Surface profiles in Side Channel Spillways"	20
	TOTAL	<u>45</u>
Credits required for degree		<u>40</u>

UNIVERSITY OF CAPE TOWN
DEPARTMENT OF CIVIL ENGINEERING

CIV 509F - STRUCTURAL LOADING

Time : 3 hours

Date: 18 June 1988

*Answers should be concise, but must show understanding of the subject.
Approximately equal marks will be awarded for each question.*

- 1.(a) List the probability functions best suited to each of the following loads : dead, imposed office floor, wind, flood, earthquake.
 - (b) Explain how the overall probability of failure of a structural element is derived from the probability functions of load effect and of resistance.
 - 2.(a) For imposed loading on floors of buildings, describe the relationship between load intensity and floor area.
 - (b) For multi-storey buildings, describe the relationship between floor loading magnitude and the number of storeys.
 3. List the most likely combinations of loads on a steel railway bridge according to BS 5400. Discuss the relevance of each combination.
 4. Explain the need for specifying three different types of road bridge traffic loadings : NA, NB and NC in TMH 7, instead of just one single loading system.
 5. Describe the procedure for obtaining the displacement spectrum for a flexible tower from records of the wind gust velocity.
 6. Describe one example of a restraint action responsible for structural failure, that was discussed in the seminars in this course - other than the one that you might have submitted as a project.
 7. Discuss how each relevant property of a structural system can contribute to minimize earthquake damage.
 8. Discuss the major factors influencing the pressure distribution inside a tall circular grain silo filled from the top and emptied at the bottom.
 9. Sliding formwork is used to construct a tall concrete chimney. Discuss all the factors likely to affect the loading on the formwork and on the concrete just below the formwork.
-

[4 PAGES]

UNIVERSITY OF CAPE TOWN
DEPARTMENT OF CIVIL ENGINEERING

M.Sc. in CIVIL ENGINEERING

CIV 516F : COASTAL HYDRAULICS

UNIVERSITY EXAMINATION : JULY 1988

ALL question may be attempted.

TIME: 4 hours +

(OPEN BOOK EXAMINATION)

Constants

Sea water density = 1025 kg/m^3

Sea water weight = 10 kN/m^3

QUESTION 1

The standard alignment chart is attached and a new blank line has been inserted at the bottom of the page. This line is to be used for determining values of U_{\max} , the maximum horizontal orbital velocity at the water

surface, according to the Airy theory. If $U_{\max}^* = \frac{U_{\max}}{\pi H/T}$ is to be the dimensionless form of the variable on this line, mark off the correct positions of the U_{\max}^* values given in the following list.

U_{\max}^*	=	1,01	2	6
		1,10	3	
		1,40	4	

Note that H is the local wave height throughout. Suggest a small change in the line label which would permit the scale to be used for maximum horizontal surface acceleration values. Use the new line to solve the following problem.

A swell of 10 second period with a deep water wave height $H_0 = 1,59$ m approaches a beach with the wave crests parallel to the shore. Plot the value of u_{\max} at the water surface for the following selected water depths.

65 m ; 34,4 m ; 15,9 m ; 6,8 m ; 2,86 m .

Use these calculations to estimate the water depth when the U_{\max} value first reaches 1,5 m/s and check that the wave has not broken.

QUESTION 2

A sea platform consists of a square concrete slab positioned horizontally on four cylindrical vertical piles, each placed at a corner, the slab side being parallel to the local wave crest. The pile diameter is 1 m, the total pile height above sea bed is 6,4 m, and the slab dimensions are sides of 5 m with a thickness of 200 mm. The local wave characteristics are height 2 m, length 100 m, and period 12 s, the local water depth being 8 m.

- (a) Considering the central 1 m high slice of any pile, calculate the horizontal forces per metre due to velocity and acceleration and by plotting these throughout one wave period or otherwise, identify the maximum force and the timing of its occurrence. Check that the velocity and acceleration distributions over the height of the pile are reasonably constant and thus estimate the total force on one pile.

Take $C_D = 1,2$ and $C_M = 2,0$.

QUESTION 2 (continued)

- (b) Estimate the maximum vertical force on the slab due to wave action.

Take $C_D = 1,0$ and $C_M = 1,8$.

QUESTION 3

In a study of wave penetration into a bay, the 10 m, 9 m and 8 m sea bed contours are approximated by three straight lines with contained angles of 12 degrees as shown on the attached page. An incoming wave orthogonal, 10 second period, impinges on the 10 m contour at an angle of 50 degrees as shown. With the usual approximations obtain by trial the angle at which the emerging orthogonal cuts the 8 m contour. Take the step lines on the 9,5 m and 8,5 m lines.

[1 diagram attached]

QUESTION 4

A train of waves is approaching a shore line, of regular bed slope 1 in 80, the wave crests being essentially parallel to the shore. Two aerial still photographs are taken 8 seconds apart. On the first photograph, two successive crests are identified as being 247 m apart. A comparison between the two photographs indicates that the trough between the two crests has advanced forward a distance of 153 m. Further, stereo photographs taken at the same time as the first exposure indicate that the wave height in the vicinity of the trough is close to 3 m. Retrace the history of this wave as it came in from deep water, and further trace the progress of the wave as it moves towards the beach, including the following calculations:

- (i) the wave length and celerity in deep water ;
- (ii) the wave length, celerity and height for water depths at 20 m intervals from deep to the 10 m depth, and at 1 m intervals inshore from this to a depth of 3 m ;
- (iii) the depth of water in which the wave breaks, the type of breaker, and the wave height at breaking. Set up and down may be ignored ;
- (iv) the energy flow in W/m in two water depths outside the breakers, and one depth in the breaker zone (depths at your choice) and compare.

QUESTION 5

- (a) A storm at sea generates waves with a period range of 6 to 12 seconds. The resulting swell travels towards a harbour 400 km away. Estimate the time interval between the arrival of the shortest and longest waves, assuming deep water throughout.
- (b) A refraction diagram is constructed for a bay, and the spacing between a particular pair of adjacent orthogonals doubles in travelling from deep water to the 10 m depth zone, the wave period being 7 seconds. Estimate the percentage change in wave height occurring between these two zones on the assumption that no breaking waves are present in-between.
- (c) In a zero damage design calculation for the armour protection of a rubble mound breakwater, 3 tonne and 5 tonne dolosse are specified for the trunk and head respectively, the slope of the breakwater face being $\cot \alpha = 2$. Estimate the block masses and block heights if tetrapods had been used in the same design. If the design wave height was 3 m, and a storm causes damage of the order 20 - 30 per cent to the tetrapod scheme, estimate the storm wave height (concrete density 2245 kg/m^3).
- (d) An incoming swell has crests parallel to a straight beach with a deep water wave height of 2 m. Estimate the horizontal force (per metre along the beach) acting on the beach inside the refraction zone, due to the dynamic action of the waves.
- (e) In an area where the sea bed is horizontal, and the water depth is 3 m, a wave has a period of 7 s, a wave length of 38 m, and a wave height of 1.5 m. Estimate the drift velocity at bed level, and indicate the direction. Compare this velocity with the maximum orbital velocity at the same level, and indicate the influence on bed drift of a strong onshore wind.
- (f) A coastal model is to be constructed to explore wave action in a sea area 1 km offshore by 2 km along shore. The laboratory area available is 20 m wide and of considerable length. Suggest a linear scale suitable for this and calculate the wave period of the model paddles to duplicate a 12 second wave in nature. Discuss which of the following characteristics are accurately modelled in the laboratory:
- (i) wave refraction pattern ;
 - (ii) wave heights before breaking ;
 - (iii) wave heights after breaking ;
 - (iv) settlement of fine sands.

UNIVERSITY OF CAPE TOWN

DEPARTMENT OF CIVIL ENGINEERING

CIV 592F POSTGRADUATE EXAMINATION

PROJECT MANAGEMENT IN CIVIL ENGINEERING

15 JUNE 1988

TOTAL MARK 100

NOTE

- * The examination is three (3) hours
 - * Attempt all questions
 - * All writing to be in ink or ballpoint pen
-
1. Refer to Annexure A . Complete and hand in with the answer book.
 2. Discuss the functions of a Project Manager. [20]
 3. Describe the different types of information required by a Project Manager to plan a project. [20]
 4. Discuss the problems which may arise when a Project Manager underestimates the cost of a Contract in his motivation to the Client for acceptance. [10]
 5. Indicate , with comment , the cost items which may be affected in a contractor's claim as also the reasons for a claim . [10]

Name.....

15 June 1988

ATTEMPT ALL THE FOLLOWING QUESTIONS WITH SHORT PRECISE ANSWERS

TOTAL MARK 40

a) What is the purpose of organising. [1]

.....
.....

b) Give four reasons why organisational charts are useful. [2]

- 1).....
- 2).....
- 3).....
- 4).....

c) For what reason are people motivated. [1]

.....
.....

d) Name two reasons why a Client would be motivated to undertake a project. [1]

- 1).....
- 2).....

e) Time and cost is interrelated but can be in conflict ,is this true or false ? [1]

True / false (circle the correct reply)

d) Indicate which of the following are procedural constraints : [3]

- | | |
|-------------------------------|--------|
| availability of local funding | yes/no |
| tendering | yes/no |
| detail design | yes/no |
| conditions of contract | yes/no |
| contractual incentives | yes/no |
| arbitration | yes/no |

e) Give ten benefits of good planning :

[5]

- 1).....
- 2).....
- 3).....
- 4).....
- 5).....
- 6).....
- 7).....
- 8).....
- 9).....
- 10).....

f) Indicate which of the following can assist with project control . [3]

personal commitment	yes/no
programme coordination	yes/no
review meetings	yes/no
project reports	yes/no
management support	yes/no
communication	yes/no

g) List six project management monitoring actions which should be excercised during construction. [3]

- 1).....
- 2).....
- 3).....
- 4).....
- 5).....
- 6).....

h) Indicate four normal reactions to which a person may resort if faced with a risk situation . [2]

1).....

2).....

3).....

4).....

i) Give three types of critical path charts for programming a project . [3]

1).....

2).....

3).....

j) Give eight reasons for providing a client with an estimate for a project . [4]

1).....

2).....

3).....

4).....

5).....

6).....

7).....

8).....

k) What should be considered when compiling a Schedule of Quantities for a project . [3]

1).....

2).....

3).....

4).....

5).....

6).....

1) Is the cumulative monthly payments made to a contractor a straight line , if not what shape is it ? [2]

.....
.....

m) Who takes the risk in a fixed priced contract. [1]

.....

n) On which cost elements in a contract are the inflation index values applied. [2]

1).....

2).....

3).....

4).....

o) What is the normal maximum percentage variation of a payment item that can be accepted before a Contractor may request a revision of the pay item rate , and for what reason. [2]

.....
.....
.....

p) What are statutory increases. [1]

.....

UNIVERSITY OF CAPE TOWN

DEPARTMENT OF CIVIL ENGINEERING

UNIVERSITY EXAMINATION : 29 OCTOBER 1988

CIV 536 - COASTAL ENGINEERING PRACTICE

Time Allowed: 3 hours

Answer ALL Questions

There is a potential of 158 marks

SECTION 1 is to be handed in at the end of the first hour - CLOSED BOOK

SECTION 2 is "OPEN BOOK"

Name:

QUESTION 1

Briefly explain, in words and by means of annotated sketches, the meaning of the following terms:

1.1 Cope level [2]

1.2 Pendant fender [2]

1.3 Tidal prism [2]

1.4 Seiche [2]

University of Cape Town

- 1.5 Clinometer [2]
- 1.6 Show on a sketch plan of Hout Bay where you would expect to observe the effects of wave diffraction and refraction. Clearly indicate the physical cause of each effect and the form of the wave orthogonals and crests. [2]
- 1.7 Explain, by means of a sketch, the basic physical elements of airborne (single channel) linescan apparatus that could be used for remote sensing of the ground. [2]
- 1.8 Explain by means of a sketch how a dredger may be positioned using sextant resection. [2]

- 1.9 Explain the principle of subtense ranging using a sextant. Indicate the practical distance limit. [2]
- 1.10 Explain by means of a sketch the principle of echo sounding for seabed profiling. [2]
- 1.11 Give a sketch of the components and the arrangement that is used for tide recording at Granger Bay. [2]
- 1.12 Explain the term "tidal residual". What is the cause of tidal residuals? (2)

1.13 Explain the principle of the "Wave rider" accelerometer buoy. [2]

1.14 Show by means of simple sketches how field measurements of the following may be graphically presented in reports :

wind speed and direction [2]

Radioactive tracers [2]

Beach profiles [2]

1.15(a) Give a typical section of a rubble mound breakwater (annotate). [2]

1.15(b) Show the sequence of how this would be constructed in an exposed situation. [2]

1.16 Comment very briefly (with a simple sketch) on the adequacy or inadequacy of the following :

position of the slipway at Granger Bay [2]

boat "access" situation at harbour entrance in Hout Bay [2]

boat "access" situation at Kalk Bay harbour entrance [2]

a boat ramp at 1:6 [2]

a boat ramp at 1:15

[2]

1.17 Explain what you would look for in an aerial photograph of the coast to discern the direction of littoral drift.

[2]

Total for SECTION 1 = 48 marks.

University of Cape Town

SECTION 2 - OPEN BOOK

- 2.1 Assuming that you are a Consulting Engineer specialising in coastal matters, reporting to the local authority responsible for the coast, write advisory notes to the responsible Committee on the following :
- (a) It is September and the beach has steepened and eroded sufficiently for an adjacent parking area to appear to be in danger of being totally eroded.
- Outline your proposed method of investigation, your preliminary advice as to what the Council should instruct you or Contractors to do, what alternatives measures are likely to be appropriate after completing the investigation. Give a staged breakdown of costs with time/construction expense justification. Assume the total beach length is 1 km and that the situation is as occurs at Fish Hoek. [15]
- (b) It is proposed that the harbour at Granger Bay be improved. Write a memorandum to the responsible Committee outlining the problems that occur at present and the approach you would take to improve the situation. Provide an approximate cost for the investigation and the development of a new construction plan. Give a breakdown of the work required. (Give sketch plans as needed). [15]
- (c) Describe the present situation and outline the approach you would take to investigate the cause of the tilt on the breakwater at Hermanus. Indicate two possible alternative causes, and how you would remedy the situation for each case. (In a sketch show the type of construction). [10]
- (d) Briefly explain how you would determine the directions of nett littoral drift and how you would estimate the littoral drift quantity at Hout Bay beach. [10]
- (e) Hout Bay harbour is to be extended to provide for an additional 500 floating berths for small craft. Present a breakwater and mooring layout, and show in plan details of boat ramps, harbour control and other infrastructure requirements that should be considered at a preliminary stage. State all assumptions. [60]

Total for SECTION 2 = 110 marks

UNIVERSITY OF CAPE TOWN

DEPARTMENT OF CIVIL ENGINEERING

UNIVERSITY EXAMINATION - NOVEMBER 1988

COURSE CIV 5255 - CONTRACT LAW

OPEN BOOK EXAMINATION

Time : 150 Minutes

PLEASE ANSWER ALL QUESTIONS, BEARING IN MIND THE NUMBER OF
QUESTIONS, IT IS SUGGESTED THAT YOUR ANSWERS BE KEPT AS
BRIEF AS POSSIBLE.

TOTAL NUMBER OF MARKS

: 100 MARKS

External Exam
Marked 10/11/88
Intent to use
Exam
External Exam

QUESTION 1

"It must be conceded that the phraseology of Clause 54 (of the Standard Engineering Contract) is capable of bearing the construction placed upon it by the Court a quo. But in my opinion it is also open to a different interpretation".

(Per Van Heerden JA in Melmoth Town Board v Marius Mostert (Pty) Limited 1984 (3) 718 at 728 F).

Comment on the above statement and deal with the powers of the engineer in terms of the said Clause 54.

10 Marks

University of Cape Town

QUESTION 2

You are a director of a construction company.

The construction company applies to an insurance company for the issue of a performance bond and the insurance company requires you to sign a suretyship for the obligations of the construction company in respect of that contract. However, in terms of the suretyship you bind yourself "as surety and co-principal debtor in solidum for the due and faithful performance by the construction company to the insurance company of all and whatsoever obligations undertaken by it on behalf of the construction company in connection with any matter whatsoever".

Shortly afterwards you resign from the construction company, and a year later you receive a letter from the insurance company advising you that in connection with another project carried out by the construction company after you had left its employ, the construction company was indebted to the insurance company, who are now looking to you for payment in terms of the suretyship signed by you.

You are very alarmed because you had not envisaged that you would be liable for obligations which were incurred after you had left the employ of the construction company. You decide to take legal advice.

What are you likely to be told?

10 Marks

QUESTION 3

You are a director of a construction company.

In terms of the construction contract, certain work was to be sub-contracted to a sub-contractor nominated by the architect.

The architect obtained a tender for this work from the sub-contractor and instructed your firm to accept the work. You sent an order on your standard printed form which contained on its reverse side printed conditions which included a clause reading as follows :-

"Payment of the amount due in terms of this order will only be effected after we (the main contractor) have received payment from the employer".

The sub-contractor wrote back to you thanking you for the order and the work was carried out.

The employer became insolvent before having paid for all the work.

The sub-contractor calls on you to obtain payment for all the work done by him.

What would you reply?

10 Marks

QUESTION 4

You are a director of a construction firm.

You contract with a development company (X Developers (Pty) Limited) to build houses on separate erven which are owned separately by individual owners who are clients of the development company, and with whom the the development company had entered into agreements for the building of these houses.

As the work progressed from time to time, each of the owners paid X Developers (Pty) Limited the amount required from them.

Unfortunately, X Developers (Pty) Limited find fault with your work and advise that they are not prepared to pay you any further sums until you have completed what they regard as the remedial work.

You dispute that in fact any remedial work is necessary, but you are not prepared to continue work until the amount due to you in terms of your contract with X Developers (Pty) Limited has been paid.

The individual owners need their accommodation urgently and decide to contract with another firm to build for them and finish off the work.

You do not want to give up possession of the building sites and the work thereon until you have been paid in full, and the individual owners institute action against your firm for an order claiming possession of the various sites. Your Board asks you for an outline of what your rights are in this matter.

Draw a short memorandum setting out your rights.

10 Marks

QUESTION 5

You are a director of a construction company.

It appears that your firm has failed to comply fully with performance of work undertaken in terms of the contract, but you wish to claim for the work which you have done.

You decide to take legal advice on your rights, and the attorney whom you consult says "Ah, this is a B K tooling case", and proceeds to give you certain advice.

On the basis that he knows what he is talking about, write a short memorandum for your managing director setting out what your rights are.

10 Marks

University of Cape Town

QUESTION 6

You are a director of a construction company.

Your firm has entered into a contract to build certain road works, and the contract is in terms of the General Conditions of Contract of 1982 as issued by the South African Institution of Civil Engineers.

Times are difficult - interest rates are rising - and your managing director advises you that it looks like it may be necessary to enter into an arrangement with the company's creditors.

He asks you to prepare a short memorandum for the Board setting out the rights of the employer under your contract, should your company decide to follow this course.

Prepare the memorandum.

10 Marks

QUESTION 7

You are a director of a construction company.

In terms of the contract your company has undertaken to pay your employer R10 000,00 a day for every day by which delivery of the completed works is delayed.

Due to internal disputes in the construction company, delivery of the completed works is delayed for a period of 3 months and your company's accountants make provision for the sum of R90 000,00 as being due by your company to the employer.

The managing director asks you to write a short memorandum for the Board setting out whether this is a liability, whether it can be reduced, and whether there would be any defence to a claim.

Write it.

10 Marks

QUESTION 8

In what circumstances may an extension of time be granted in terms of the General Conditions of Contract 1982.

10 Marks

University of Cape Town

QUESTION 9

You are a director of a construction company.

It is clear that a dispute is arising between the employer and your company, and the Board intends discussing whether the matter should go to arbitration or litigation.

You are asked to prepare a short memorandum discussing the relative merits of these methods of dispute settlement.

10 Marks

University of Cape Town

QUESTION 10

You are in practise as a consultant engineer.

You are appointed as the engineer in regard to a particular contract entered into in accordance with the General Conditions of Contract 1982.

One of the nominated sub-contractors complains bitterly to you that the main contractor has not paid him for work already done by such sub-contractor, for which you know the employer has already paid the contractor, and the nominated sub-contractor advises you that he is still working under the sub-contract and he would like you to protect him insofar as regards payment in the future.

What could you do?

Would your answer be any different if the sub-contractor was not a nominated sub-contractor?

10 Marks

UNIVERSITY OF CAPE TOWN

DEPARTMENT OF CIVIL ENGINEERING

CIV 543S AIRPORT DESIGN

POSTGRADUATE EXAMINATION

9 November, 1988

Total Mark 100

NOTE

- * Examination time 3 hours
- * Attempt all questions
- * All writing to be in ink or ballpoint

- 1 Refer to Annexure A . Complete and hand in with the answer book . [30]
- 2 Discuss fully all aspects which affect the determination of a runway length . [20]
- 3 Discuss the environmental impact a new airport could have on the status quo . [20]
- 4 Discuss fully the various aspects to be considered in the design of the terminal building relating to the passenger activities and the physical facilities . [20]
- 5 Provide a specification for both rigid and flexible pavements , with regard to the design layers and the surface treatment [10]

Name : Civ 543S Airport Design

a) On what date did Orville Wright make his first flight
..... (1)

b) For what reason did the Corps of Engineers decide to establish a design method for airport pavements during the last war.
.....
..... (1)

c) Give four factors that influence a passenger to choose air travel as a travel mode
.....
.....
.....
..... (2)

d) Which aircraft has steadily increased its seat/kilometres since its introduction to commercial aviation
..... (1)

e) What nature of demand variation affects a passenger airline in arranging its flight schedules
.....
..... (2)

f) What is the "standard busy rate "
..... (1)

g) Give eight factors which influence choosing air cargo as a means of transport

.....
.....
.....
.....
.....
.....
.....
.....

(4)

h) Indicate what is meant by the following terms :

Turboprop
Turbojet
Turbofan (3)

i) What is the most important function governing an aircraft's turning radius

.....

(1)

j) What percentage of the load is normally assumed on the nose gear

.....

(1)

k) Name six factors which have a bearing on the runway length

.....
.....
.....
.....
.....
.....

(3)

l) Define the following :

VFR

IFR

(2)

m) Give four points of information which the Air Traffic Control gives to a pilot for his landing

.....

.....

.....

.....

(2)

n) Give four points which are likely to cause strained relationships between airport administration and the residents of local neighbourhoods

.....

.....

.....

.....

(2)

o) Name the four different runway configurations

.....

.....

.....

.....

(2)

p) What is the accepted deceleration for a landing aircraft

.....

(1)

q) On approaching the runway what are the most important visual aspects to a pilot

.....

.....

.....

.....

(2)

UNIVERSITY OF CAPE TOWN

FRD/UCT CENTRE FOR RESEARCH IN
COMPUTATIONAL AND APPLIED MECHANICS

UNIVERSITY EXAMINATION : 11 MAY 1989

END 522Z : AN INTRODUCTION INTO FINITE ELEMENTS

PAPER 1

ANSWER ALL QUESTIONS

HAND IN : 12.00 am , 15 MAY 1989

TOTAL 60 MARKS

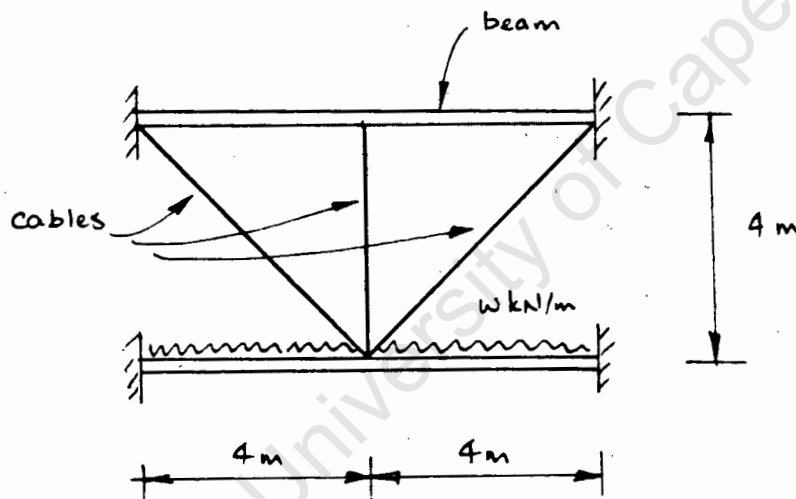
END 522Z - AN INTRODUCTION INTO FINITE ELEMENTS

Question 1

The structure shown below consists of two horizontal beams rigidly fixed at the ends. The bottom beam is supported by three cables which can only take axial forces. A uniformly distributed load of w kN/m is applied on the bottom beam.

Calculate by hand the vertical displacement at the centre of the bottom beam when $w = 25$ kN/m. Use the Euler-Bernoulli beam element. Show all your calculations, including the element stiffness matrices and the assembled global stiffness matrix and load vector. (Do not formulate the problem by minimizing the potential energy, but start with the equilibrium equations $\mathbf{Ku} = \mathbf{f}$.) Determine the bending moment distribution in the beam and the forces in the cables.

[20marks]



$$E = 200 \text{ GPa}$$

Beams:

$$I = 600 \times 10^6 \text{ m}^4$$

$$A = 45 \times 10^3 \text{ m}^2$$

Cables:

$$A = 1.2 \times 10^3 \text{ m}^2$$

END 522Z - AN INTRODUCTION INTO FINITE ELEMENTS

Question 2

The structure shown in Figure 2(a) consists of a girder supported vertically at points A, B and C, and horizontally at A. The girder is a H section with the details given in Figure 2(b). A uniformly distributed load of 5 kN/m is applied between B and C. The support at B consists of a column which can be considered as infinitely stiff. A load of 300 kN is applied at B by a solid column which covers the full width of the H section's flange.

This is a rather special structure in that this particular H section has to be used. The designer wants you to perform a finite element analysis to determine the maximum stresses in the H section at support B. He is concerned that plastic yielding may occur in the section. He wants the results to within 10% accuracy.

Your answer should be presented in the form of a report. State all your assumptions clearly, including justifications. Do not perform a 3-D analysis. Include your final computer printouts with your report.

[40marks]

END 522Z - AN INTRODUCTION INTO FINITE ELEMENTS

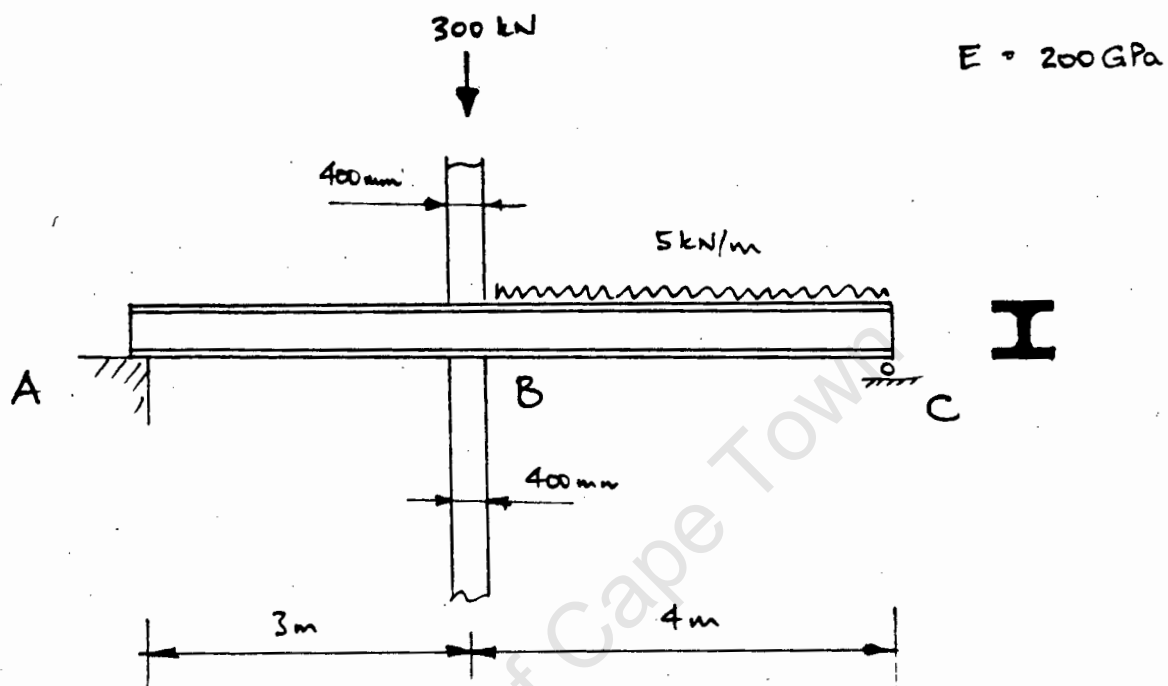
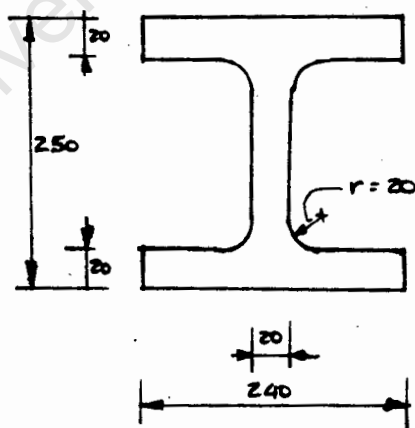


Figure 2(a)



All dimensions mm

Figure 2(b)

UNIVERSITY OF CAPE TOWN

**FRD/UCT CENTRE FOR RESEARCH IN
COMPUTATIONAL AND APPLIED MECHANICS**

UNIVERSITY EXAMINATION : 12 MAY 1989

END 522Z : AN INTRODUCTION INTO FINITE ELEMENTS

PAPER 2

ANSWER ALL QUESTIONS

Time : 1 Hour

TOTAL 40 MARKS

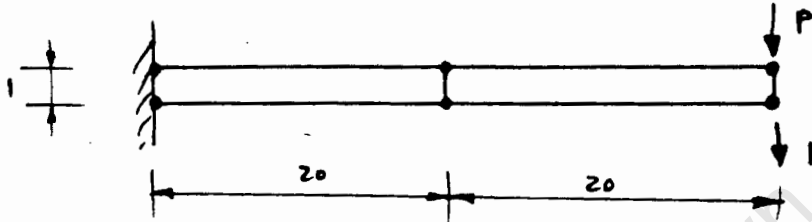
END 522Z - AN INTRODUCTION INTO FINITE ELEMENTS

Question 1

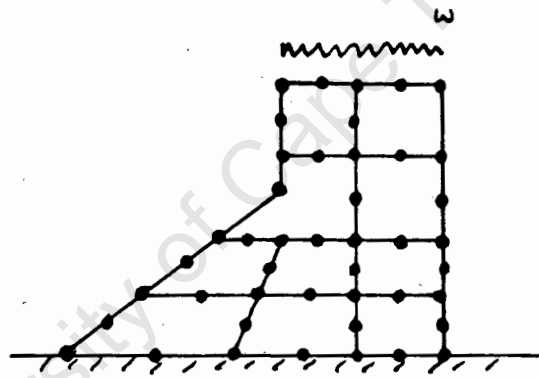
Discuss the problems associated with each of the following finite element meshes. Suggest ways to improve or correct the models.

[16 marks]

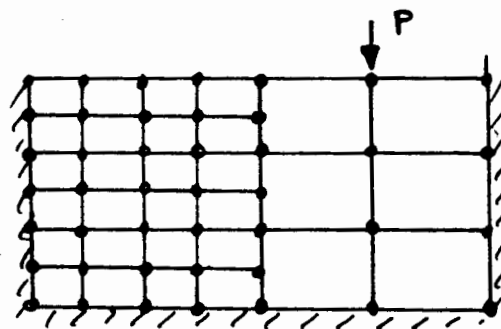
(a)



(b)

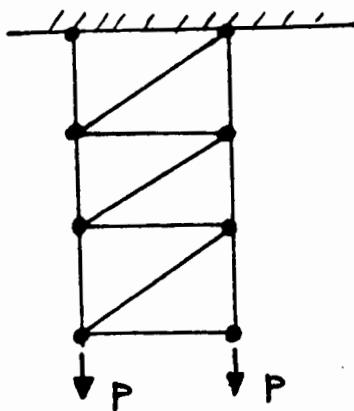


(c)

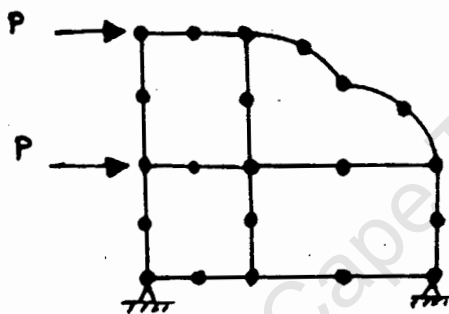


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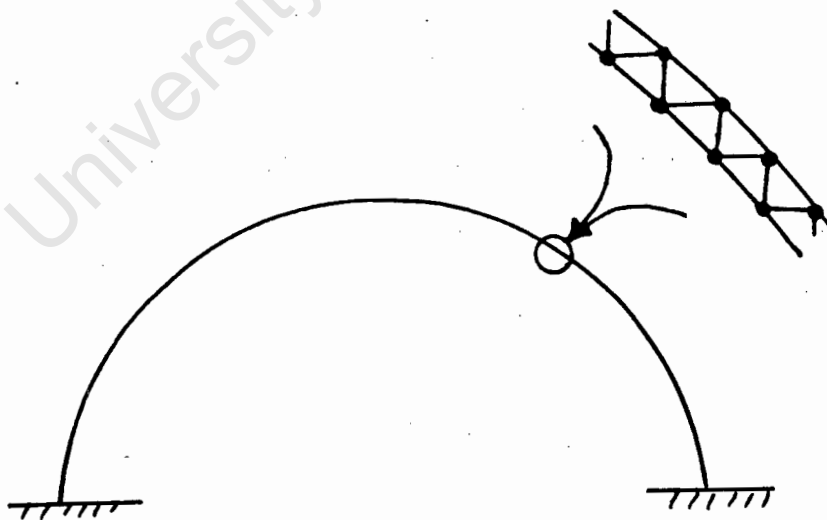
(d)



(e)



(f)



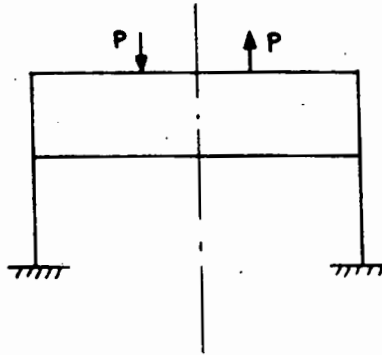
END 522Z - AN INTRODUCTION INTO FINITE ELEMENTS

Question 2

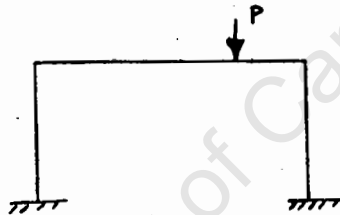
Show how symmetric and/or antisymmetric loading can be used for each of the following structures. The objective is to model only part of the structure in each case. Clearly illustrate the boundary conditions that must be used.

[12 marks]

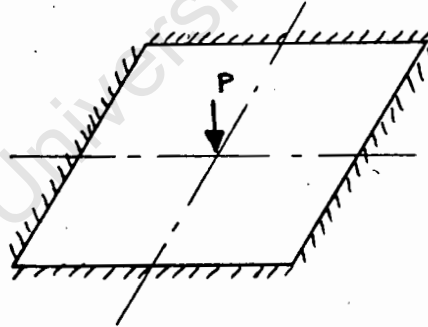
(a)



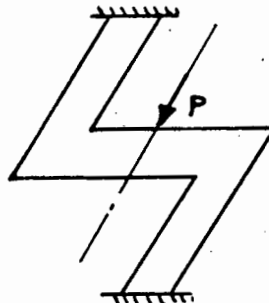
(b)



(c)



(d)



END 522Z - AN INTRODUCTION INTO FINITE ELEMENTS

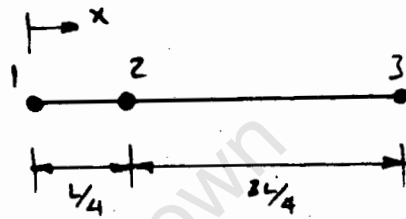
Question 3

Consider the three-node quadratic isoparametric truss element shown in Figure 3(a) below. Show that if node 2 is specified to be at the quarter point, as shown in Figure 3(b), the stress has a singularity of $\frac{1}{\sqrt{x}}$ at node 1. (Stress is given by $\sigma = DBu$)

[12 marks]



3(a)



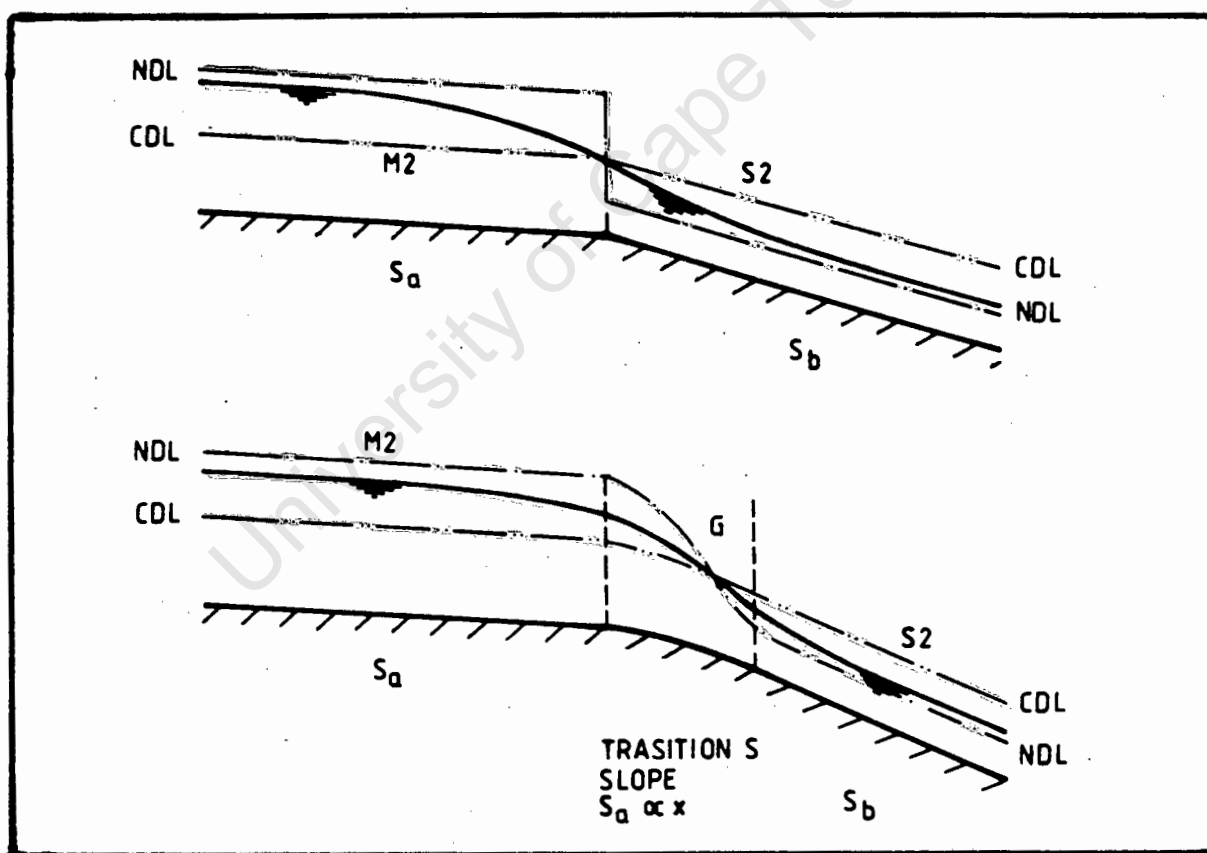
3(b)

APPENDIX B

Critical depth control points

In all water surface profile problems, a starting section must be chosen at which y , x , and dy/dx are known or calculable, and with this information a step by step procedure may be followed. In effect this procedure replaces the actual curved profile with a series of straight lines, and therefore involves some approximation but this effect may be reduced by varying the step length in x . In both side channel spillway channels and constant discharge channels, the expression for dy/dx goes to infinity when $y = y_c$ the local critical depth.

Consider for example the case of a slight change in grade, geometrically quite slight, but hydraulically sufficient to change the channel slope from mild to steep. Assuming unimpeded discharge upstream and downstream the appropriate profiles are the M2 and S2 curves :-



The conventional presentation is as in the top diagram, the water path is probably more as presented in the bottom diagram. In the latter the NDL is a continuous line changing over the transition slope and crossing CDL at a gate point G. At such a point if the water surface traverses it, both the

numerator and the denominator are zero, and the dy/dx value is finite. If the water surface crosses the CDL anywhere but G the dy/dx value is infinite, having no physical meaning. Therefore the water surface does in fact cross at G. When an expression in the form of a fraction appears to have an indeterminate form in the sense that both top and bottom of the fraction are zero simultaneously, the indeterminacy is usually apparent rather than real, and a procedure is available from calculus to obtain the correct value. This is known as the de l'Hopital principle.

Thus if the value of

$$\gamma(x) = \mu(x)/\eta(x)$$

is required at some point x_a where

$$\mu(x_a) = \eta(x_a) = 0$$

then $\gamma(x_a)$ is apparently indeterminate.

The original expression may be written as

$$\gamma(x) \cdot \eta(x) = \mu(x)$$

Differentiating

$$\gamma'(x) \cdot \eta(x) + \gamma(x) \cdot \eta'(x) = \mu'(x)$$

At the point $x = x_a$

$$\gamma'(x_a) \cdot 0 + \gamma(x_a) \cdot \eta'(x_a) = \mu'(x_a)$$

Therefore $\gamma(x_a) = \mu'(x_a)/\eta'(x_a)$

and this is the de l'Hopital principle. In other words differentiate the numerator and denominator separately, form the new ratio and insert the required x value. If the second ratio is also indeterminate the procedure may be continued.

In the side channel spillway both parts of the fraction contain x and y , ie. are functions of two variables. In this case the limit depends on the ratio of y to x as the limit point is approached. The link in the side channel spillway problem is a differential one viz. that the dy/dx is itself the ratio required.

There is one simple case of side channel inflow which yields the water surface directly without recourse to calculations of dy/dx at a critical depth section accompanied by step by step procedures. This case called the reference case, occurs where the assumption of zero slope and zero friction is made. The resulting profile gives information particularly concerning the shape of the profile in the vicinity of critical depth.

REFERENCE CASE [rectangular channel]

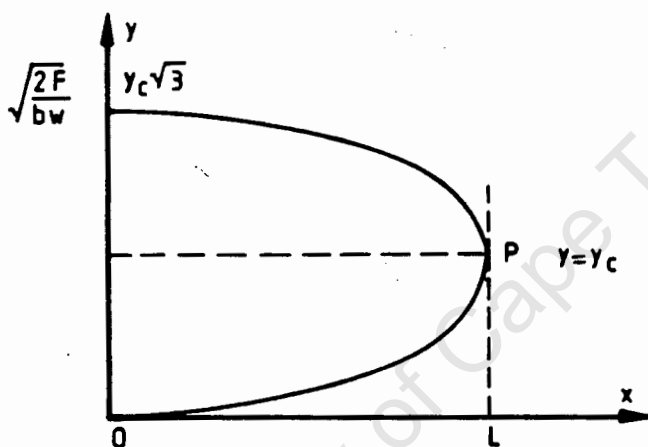
Consider a horizontal channel with zero friction, the second condition being an idealised state.

In such a receiving channel $F = \text{constant}$

$$\frac{F}{w} = \frac{Q v}{g} + \frac{by^2}{2} = \frac{q^2 x^2}{gby} + \frac{by^2}{2}$$

Thus for given F , q and b , y varies with x

In fact $q^2 x^2 + \frac{gb^2 y^3}{2} = \frac{Fgby}{w}$ (1)



When $x = 0$, either $y = 0$ or is given by

$$\frac{gb^2 y^3}{2} = \frac{Fgby}{w} \quad \text{or} \quad y_0^2 = \frac{2F}{bw}$$

as could be deduced from the fact that F is hydrostatic at the upstream wall.

At point P , $dx/dy = 0$

ie. $2q^2 x \frac{dx}{dy} + \frac{3gb^2 y^2}{2} = \frac{Fgb}{w}$

Thus $y = \sqrt{\frac{2F}{3bw}}$ and this may be recognised from spatially constant flow as being the local value for y_c , the critical depth.

If the x co-ordinate of P is taken as L then y_c is also given by

$$y_c^3 = \frac{q^2 L^2}{b^2 g}$$

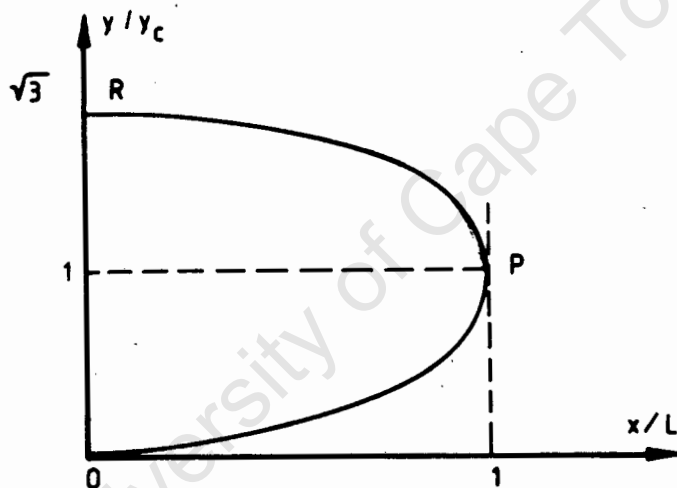
Thus equation (1) may be written in the form

$$q^2 x^2 + \frac{g b^2 y^3}{2} = \frac{3 g b^2 y_c^2 y}{2}$$

which simplifies to

$$\frac{2x^2}{L^2} + \frac{y^3}{y_c^3} = \frac{3y}{y_c} \quad (2)$$

which permits the dimensionless diagram below :



Let a general point be $y = k y_c$, then

$$\frac{x}{L} = \sqrt{\frac{k(3-k^2)}{2}}$$

thus a general point on the water surface profile is determinate. Differentiating eqn. (2) gives

$$\frac{4x}{L^2} \frac{dx}{dy} + \frac{3y^2}{y_c^3} = \frac{3}{y_c}$$

which simplifies to

$$\frac{dy}{dx} = \frac{2y_c \sqrt{2k(3-k^2)}}{3L(1-k^2)} \quad \text{Note that } \frac{dy}{dx} \text{ at wall is zero ie. same as bed slope}$$

The use of the reference case is based on the idea that in the vicinity of y_c the influence of bed slope and friction are small and to some extent self cancelling. Thus a convenient starting point say ($k = 1,01$) is chosen from the reference profile and used as the starting point for a step by step method in the case of a finite slope, finite friction receiving channel.

For example, if $k = 1,01$ then $x/L = 0,9999247$ and $dy/dx = -66,33(y_c/L)$

EQUATION FOR dy/dx AT CONTROL POINT WITHIN THE RECEIVING CHANNEL

[rectangular channel]

The following analysis is specifically for the rectangular channel. The case of general channel shape will be handled later. If a control point exists within the receiving trough, it occurs at the intersection of the PNDL and the CDL. At this point the equation for dy/dx is apparently indeterminate in that both the numerator and denominator are zero [value is determined not by substitution, but by calculus].

Let the numerator be $\phi(x, y)$ where

$$\phi(x, y) = s_0 - \frac{2q^2x}{gb^2y^2} - \frac{q^2x^2n^2}{b^2y^2R^{4/3}}$$

and the denominator be $\psi(x, y)$ where

$$\psi(x, y) = 1 - \frac{q^2x^2}{gb^2y^3}$$

and $\phi(x_c, y_c) = \psi(x_c, y_c) = 0$ at the control section.

Let Z be required value of dy/dx at the control section ie. $Z = \left. \frac{dy}{dx} \right|_c$.

By de L' Hopital principle

$$Z = \frac{\frac{\delta\phi}{\delta x} + \frac{\delta\phi}{\delta y} \cdot \frac{dy}{dx}}{\frac{\delta\psi}{\delta x} + \frac{\delta\psi}{\delta y} \cdot \frac{dy}{dx}} = \frac{\frac{\delta\phi}{\delta x} + \frac{\delta\phi}{\delta y} \cdot Z}{\frac{\delta\psi}{\delta x} + \frac{\delta\psi}{\delta y} \cdot Z}$$

Multiplying up gives a quadratic in Z, dimensionless

$$\text{ie. } AZ^2 + BZ + C = 0 \quad (A, B, C \text{ all have same dimensions})$$

$$\text{where } A = \frac{\delta\psi}{\delta y}$$

$$\text{and } B = \left[\frac{\delta\psi}{\delta x} - \frac{\delta\phi}{\delta y} \right]$$

$$\text{and } C = -\frac{\delta\phi}{\delta x}$$

These partial differentials may be evaluated as follows :

$$\frac{\delta\phi}{\delta x} = -\frac{2q^2}{gb^2y^2} - \frac{2q^2xn^2}{b^2y^2R^{4/3}} = -\frac{2q^2}{gb^2y^2} \left[1 + \frac{gxn^2}{R^{4/3}} \right]$$

$$\frac{\delta\psi}{\delta x} = -\frac{2q^2x}{gb^2y^3}$$

$$\frac{\delta\phi}{\delta y} = \frac{4q^2x}{gb^2y^3} - \frac{q^2x^2n^2}{b^2} \left[-\frac{2}{y^3R^{4/3}} - \frac{4}{3} \frac{1}{y^2R^{7/3}} \frac{dR}{dy} \right]$$

$$\text{Now } R = \frac{by}{b+2y} ; \quad \therefore \frac{dR}{dy} = \frac{b^2}{(b+2y)^2} = \frac{b^2R^2}{b^2y^2} = \frac{R^2}{y^2}$$

$$\therefore \frac{\delta\phi}{\delta y} = \frac{q^2x}{b^2y^3} \left\{ \frac{4}{g} + \frac{2xn^2}{R^{4/3}} + \frac{4}{3} \frac{xn^2}{yR^{1/3}} \right\}$$

$$= \frac{2q^2x}{3b^2y^3} \left\{ \frac{6}{g} + \frac{3xn^2}{R^{4/3}} + \frac{2xn^2}{yR^{1/3}} \right\}$$

$$\text{and } \frac{\delta\psi}{\delta y} = \frac{3q^2x^2}{gb^2y^4}$$

Thus the coefficients of the quadratic equation are

$$A = \frac{3q^2x^2}{gb^2y^4} ; \quad C = \frac{2q^2}{gb^2y^2} \left\{ 1 + \frac{gxn^2}{R^{4/3}} \right\}$$

$$\text{and } B = -\frac{2q^2x}{3b^2y^3g} \left\{ 9 + \frac{3xn^2g}{R^{4/3}} + \frac{2xn^2g}{yR^{1/3}} \right\}$$

Common factors may be cancelled in the coefficients and algebraic neatening gives the Z quadratic as

$$\left[9x^2R^{4/3} \right] Z^2 - \left[(2x) \left[9yR^{4/3} + gxn^2 (3y + 2R) \right] \right] Z + \left[(6y^2) (R^{4/3} + gxn^2) \right] = 0$$

In this above equation the values to be inserted for x , y and R are those pertaining to the control point. Note that both roots of the quadratic are positive, that is the water depth is increasing through the control point in the downstream direction in two alternative ways, the choice depends on the downstream conditions present.

Critical depth control point for a prismatic channel of general shape

The treatment is similar to that for a rectangular channel previously discussed. Let the lateral inflow be q (m^3/s per metre length of channel), channel geometry defined by area A and wetted perimeter p , the channel longitudinal slope S_o , distance along channel x measured from the section of commencement of inflow, the discharge and water depth at a general section being Q and y respectively. The total force principle will be used to establish the rate of change of depth with distance, ie. the magnitude of dy/dx .

Now

$$\frac{F}{w} = \frac{Qv}{g} + AZ = \frac{Q^2}{gA} + AZ$$

Differentiating w.r.t. x gives

$$\frac{1}{w} \frac{dF}{dx} = \frac{2Qq}{gA} - \frac{Q^2 B}{gA^2} \frac{dy}{dx} + A \frac{dy}{dx}$$

But from considerations of weight and shear forces on the element

$$\frac{1}{w} \frac{dF}{dx} = A (S_o - S_f)$$

Equating the two values for dF/dx gives

$$A (S_o - S_f) = \frac{2qQ}{gA} - \frac{Q^2 B}{gA^2} \frac{dy}{dx} + A \frac{dy}{dx}$$

rearranging gives

$$\frac{dy}{dx} = \frac{S_o - S_f - \frac{2Qq}{gA^2}}{1 - \frac{Q^2 B}{gA^3}} = \frac{S_o - S_f - \frac{2q^2 x}{gA^2}}{1 - \frac{q^2 x^2 B}{gA^3}}$$

and the equation for the rectangular channel may now be recognised as a special case of the above. The value to be substituted for S_f is

$$S_f = \frac{q^2 n^2 x^2}{A^2 R^{1/3}}$$

The value for dH/dx being the rate of change of the main stream total head is given by

$$\frac{dH}{dx} = S_f + \frac{qQ}{gA^2}$$

and the physical interpretation remains unchanged, viz that the latter term is equally divided between energy transferred to the lateral stream and energy dissipated in the mixing process.

REFERENCE CASE (trapezoidal channel)

Consider a horizontal channel with zero friction, the channel cross section being defined as base width b , and the side angle θ with the horizontal. Following the procedure used for the rectangular channel, $F = \text{constant}$ where

$$\frac{F}{w} = \frac{q^2 x^2}{gA} + A\bar{z} \quad (1)$$

Thus for given F , q and channel geometry y varies with x . In fact

$$\frac{FA}{w} = \frac{q^2 x^2}{g} + A^2 \bar{z}$$

Thus when $x = 0$, either $A = 0$ (i.e. $y = 0$)

or $F = wA\bar{z}$

as could be deduced from the fact that F is hydrostatic at the upstream wall. At a point where $dx/dy = 0$, differentiating eqn (1) gives

$$0 = -\frac{q^2 x^2}{gA^2} \frac{dA}{dy} + \frac{d}{dy} (A\bar{z})$$

or $\frac{Bq^2 x^2}{gA^2} = A$

which is recognised as the condition for the water depth being locally critical i.e. $y = y_c$.

When the depth is critical

$$\frac{F}{w} = A_c \left[\bar{z}_c + \frac{A_c}{B_c} \right]$$

thus the original eqtn (1) may be rewritten in the form

$$A_c \left[\bar{z}_c + \frac{A_c}{B_c} \right] = \frac{q^2 x^2}{gA} + A\bar{z}$$

or

$$x^2 = \frac{gA}{q^2} \left[A_c \bar{z}_c + \frac{A_c^2}{B_c} - A\bar{z} \right]$$

and this yields the general eqtn of the water surface x dependent on y once critical depth at the end of the channel is known. Because there is no explicit expression for y_c in the trapezoidal, a general expression is not possible for x versus y and individual cases have to be handled numerically. To illustrate, the trapezoidal section of Hinds example will be used, viz

$$b = 3 \text{ m} ; \quad q = 3,75 \text{ m}^2/\text{s} ; \quad L = 120 \text{ m} ; \quad \cot\theta = 0,5.$$

Thus the discharge at the end of the receiving channel is $450 \text{ m}^3/\text{s}$, and at this point critical depth is to be found from the implicit equation

$$\frac{A^3}{B} = \frac{Q^2}{g}$$

which yields $y_c = 8,530153253 \text{ m}$

$$A_c = 61,972217 \text{ m}^2$$

$$B_c = 11,530153 \text{ m}$$

$$Z_c = 3,43045 \text{ m} .$$

Thus the surface profile equation is

$$x^2 = \frac{gA}{q^2} [545,6806 - A\bar{z}] .$$

If y is chosen as $1,01 y_c$

$$x = 119,980658$$

and this is the starting point for the step by step solution to the general profile within the inflow zone.

EQUATION FOR dy/dx AT CONTROL POINT WITHIN THE RECEIVING CHANNEL

[trapezoidal channel]

The following analysis is specifically for the general shape channel. However the concepts are identical to the section previously discussed for the rectangular channel. Thus if a control exists within the receiving trough, it occurs at the intersection of CDL and the PNDL. At this point the

equation for dy/dx is apparently indeterminate in that both the numerator and the denominator are zero. If the numerator is $\phi(x, y)$ and the denominator is $\psi(x, y)$ and Z is the required value of dy/dx at the control section, i.e. $Z = (dy/dx)_c$ then Z is given by a quadratic equation

$$A Z^2 + B Z + C = 0$$

where

$$A = \partial\psi/\partial y \quad ; \quad B = \left[\frac{\partial\psi}{\partial x} - \frac{\partial\phi}{\partial y} \right] \quad \text{and} \quad C = -\frac{\partial\phi}{\partial x}$$

The remainder of this section is concerned with the valuation of these partial differential coefficients for the general prismatic channel

$$\text{Now} \quad \phi = S_0 - \frac{q^2 n^2 x^2}{A^2 R^{4/3}} - \frac{2q^2 x}{gA^2}$$

$$\begin{aligned} \therefore \quad \frac{\partial\phi}{\partial y} &= q^2 n^2 x^2 \left[\frac{2}{A^3 R^{3/4}} \frac{dA}{dy} + \frac{4}{3R^{1/3} A^2} \frac{dR}{dy} \right] + \frac{4q^2 x}{gA^3} \frac{dA}{dy} \\ &= \frac{4q^2 n^2 x^2}{A^2 R^{4/3}} \left[\frac{B}{2A} + \frac{1}{3R} \frac{dR}{dy} + \frac{B R^{1/3}}{x n^2 g A} \right] \end{aligned}$$

$$\begin{aligned} \frac{\partial\phi}{\partial x} &= -\frac{2q^2 n^2 x}{A^2 R^{4/3}} - \frac{2q^2}{gA^2} = \frac{4q^2}{gA^2} - \frac{2S_0}{x} - \frac{2q^2}{gA^2} \\ &= 2 \left[\frac{q^2}{gA^2} - \frac{S_0}{x} \right] \end{aligned}$$

$$\text{and} \quad \psi = 1 - \frac{q^2 x^2 B}{gA^3}$$

$$\begin{aligned} \therefore \quad \frac{\partial\psi}{\partial y} &= \frac{q^2 x^2}{g} \left[\frac{3B}{A^4} \frac{dA}{dy} - \frac{1}{A^3} \frac{dB}{dy} \right] \\ &= \frac{q^2 x^2}{gA^3} \left[\frac{3B^2}{A} - \frac{dB}{dy} \right] = \frac{3B}{A} - \frac{1}{B} \frac{dB}{dy} \end{aligned}$$

$$\text{and} \quad \frac{\partial\psi}{\partial x} = -\frac{2q^2 x B}{gA^3} = -\frac{2}{x}$$

The dimensions of all the partial coefficients will be found to be L^{-1} or m^{-1} .

A typical shape of channel encountered is the trapezoidal cross section where the following geometrical statements apply :

$$B = b + 2y \cot \theta$$

$$p = b + 2y \operatorname{cosec} \theta$$

$$A = y (b + y \cot \theta)$$

Thus

$$dB/dy = 2 \cot \theta$$

$$dp/dy = 2 \operatorname{cosec} \theta$$

$$\begin{aligned} dR/dy &= -(A/p^2) dp/dy + B/p \\ &= -2 \operatorname{cosec} \theta \cdot A/p^2 + B/p \end{aligned}$$

and these values would be inserted in the values for the differential coefficients. Note that all numerical values are calculated at the control point, previously located as an intersection.