Portfolio Optimisation with Quantitative and Qualitative Views

by

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Portfolio construction with quantitative and qualitative forecasts is described through the exposition of two asset allocation models. The two models are the Black-Litterman Asset Allocation Model and the Qualitative Forecasts Model developed by Herold Ulf. The models are developed theoretically and made intuitively accessible with real market data examples. Methodology is developed using the two models to transport alpha across benchmarks. The portable alpha technique is performed on a basket of 29 shares from the ALSI 40. The forecasts are inferred from existing unit trust data.

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Chapter 1

Introduction

One of the most relevant issues in asset management is that of asset allocation optimisation. Ever since Harry Markowitz [MH 1952] published his paper on portfolio selection in 1952, the asset allocation optimisation has been loosely interchanged with mean-variance optimisation. Markowitz set up a mathematical framework which deals with asset allocation in a risk-return context. Diversification of risk was possible through the combination of risky assets that behaved differently to each other in different market conditions. Mathematically that is to be interpreted as assets that have low correlation coefficients.

Although theoretically sound, Markowitz' theory was difficult to implement in practice. Most optimisation problems are highly sensitive to changes in inputs and Markowitz' mean-variance technique was no different. Elements of the financial markets that are required as inputs to the mean-variance optimisation, do not display continuous behaviour. Although it can be argued that we are able to trade in continuous time, asset prices do not follow a continuous path. A share may open at R80 and in the course of the day's trade end up at R85, but that does not imply that the price has smoothly traded through all the available prices between R80 and R85. Similarly asset volatilities and asset return forecasts are discrete variables that we try to smooth out in an effort to ease computation. These are the inputs that are required in the Markowitz mean-variance frame. It tries to optimise in a sensitive way a problem that has highly granular and discrete underlying variables. The result of the optimisation also yields highly concentrated (corner) solutions with large weights allocated to a few assets and no weights to the other assets. The portfolios, although efficient and optimal theoretically, are highly unintuitive and take on large tracking errors. In practice, asset managers do not like to use the techniques of mean-variance optimisation owing to the above mentioned problems. However, Markowitz' work need not be discarded, but merely improved upon.

In the recent past, asset allocation models that deal with the problems illustrated above have been developed. In 1990, Fisher Black and Robert Litterman [BL 1990] put forward a model, based on bayesian statistics, which processes quantitative inputs and generates a new vector of expected forecast returns which can be used in a mean-variance framework in a highly intuitive and robust way.

The vast majority of active portfolio managers use a fundamental investment approach. They do not rely only on quantitative models and they do not generate quantitative forecasts. Instead their investment process comprises a broad array of the different tools they employ and indicators they look at. Through this process they finally form qualitative forecasts. Herold Ulf [UH 2003] developed a model in 2003 that is able to process qualitative views from an alpha generating process and transport those views in an asset allocation model largely similar to the Black-Litterman model, differing only in the qualitative nature of the inputs.

My research explores the inner workings of the two models. I concentrated my attention on both the Black Litterman Model and the Qualitative Forecasts Model, because these models seek to answer the investor's asset allocation dilema. I have taken into account that managers are largely categorised into quantitative and fundamental styles. Quantitative managers are better able to benefit from the Black-Litterman Model and fundamental managers now have at their disposal a robust model that is able to process their qualitative views.

The strength of my work lies in the mathematical development of these models, since very little has been written about their theoretical foundations. Black-Litterman [BL 1992] and Herold Ulf [UH 2003] themselves give little indication of the mathematical development of their models, largely due to the competitive nature of the financial world. It must be understood that practitioners in the quantiative finance arena are torn between producing cutting edge research that promotes their recognition in academic circles and the need to divulge as little information as possible in order to allow their models to trade at an advantage to their competitors.

In my thesis I develop the models theoretically and provide very detailed demonstrations of their inner workings from an intuitive point of view using real market data. From a practical point of view I extend the application of the models to the problem of transporting alphas across benchmarks. I pick a basket of shares from the Alsi 40. I create my own index based on these shares, which I then use as a benchmark against which to manage my portfolios. I actively tilt my benchmark using views backed out of unit trust data. The qualitative and quantitative forecasts models are used to implement the views.

In Chapter 2 I develop the Black-Literman Asset Allocation Model. The model is revealed using a nine asset example from the economic sectors of the JSE All Share Index. The chapter ends with the mathematical derivation of the model. In Chapter 3 I follow the same recipe and develop the Qualitative Forecasts Model with the illustration based on the same nine assets used in Chapter 2. Chapter 4 reveals the ideas, methodology and the practice of transporting alpha across benchmarks. I discuss the creation of my own tracker funds and the methodology used in backing out views from unit trust data. The remaining chapters show the results of the portable alpha technique and I conclude with ideas for future research on these topics.

Chapter 2

The Black-Litterman Asset Allocation Model

2.1 Overview of the model

The Black-Litterman asset allocation model [BL 1992], developed by Fisher Black and Robert Litterman of Goldman, Sachs & Company, is a portfolio construction method which combines investor views with market equilibrium returns to provide optimal asset allocation in a novel way that overcomes the problem of unintuitive highly concentrated portfolios, input sensitivity and estimation error maximization.

Harry Markowitz [MH 1952] in 1952 provided a mathematical framework to be used in portfolio analysis in which return is maximised for a given level of risk. It would seem that one of the easiest and most accurate ways to optimise asset allocation in a portfolio is to use the methods developed by Markowitz in combination with our very powerful computers. However, quantitative asset allocation models have not played the important role they should in global portfolio management.

Here are some of the problems with the mean-variance optimisation technique, proposed by Markowitz and developed by William Sharpe [SW 1964].

- When investors impose no conditions, the model almost always ordains large short positions in many assets;
- When constraints rule out short positions, the model predicts "corner" solutions, namely zero weights in many asset classes and unreasonably large long positions in assets with small market capitalisations; and
- The result of mean-variance analysis is extremely sensitive to its inputs. (see Best and Grauer [BG 1991]).

The Black-Litterman methodology was developed to alleviate the input-sensitivity problem. It starts with equilibrium expected returns extracted inversely from observed market value weights, with a covariance matrix and a risk aversion factor given (see Best and Grauer [BG 1985]). When these equilibrium expected returns are the inputs, a mean-variance optimisation automatically leads to the market portfolio, which guarantees a high degree of diversification. The most innovative feature of the Black-Litterman method is that it uses a Bayesian approach to combine the subjective views of an investor regarding the expected returns of one or more assets with the market equilibrium vector of returns (the prior distribution) to form a new, mixed estimate of returns. The resulting new vector of returns (the posterior distribution), described as a complex, weighted average of investor's views and the market equilibrium (see Bevan and Winkelmann [BW 1998] and Lee [LW 2000]), leads to intuitive portfolios with sensible portfolio weights.

For the remainder of the chapter I shall follow closely the paper by Thomas Idzorek [IT 2003] to illustrate in detail the inner-working of the Black-Litterman model. I will provide a glossary of terms used in this chapter and describe the intuition and the method of actually combining market equilibrium expected returns with investor views. I will then provide a formal derivation of the Black-Litterman formula from two points of view. One is a straight application of Bayesian Analysis and Multivariate Normal Distribution theory and the other uses a Generalised Least Squares approach to solve for the new vector of returns.

A concise literature review for the Black-Litterman model is exposed in Idzorek [IT 2003]. The model was introduced in Black and Litterman [BL 1990], expanded in Black and Litterman [BL 1992], and discussed in greater detail in Bevan and Winkelmann [BW 1998], He and Litterman [HL 1999], and Litterman [LR 2003]. Good works on the model include Lee [LW 2000], Satchel and Scowcroft [SS 2000], and from a mathematical point of view, Christodoulakis [CG 2002] and Blamont and Firoozye [BF 2003]. The Black-Litterman model combines the CAPM (see Sharpe [SW 1964]), reverse optimisation (see Sharpe [SW 1974]), mixed estimation (see Theil [TH 1978]), and mean-variance optimisation (see Markowitz [MH 1952]).

2.2 Intuition behind the Black-Litterman Model

Before we discuss the inner-workings of the Black Litterman Model it is best to clarify the meaning of some of the most common terms used in our discussion.

- **Asset Excess Returns:** These are returns on the asset less the domestic short rate (the one period risk free rate).
- **Balance:** A measure of how close a portfolio is to the equilibrium portfolio.
- **Benchmark Portfolio:** The standard used to define the risk of other portfolios. If a benchmark is defined, the risk of a portfolio is measured as the standard deviation of the tracking error - the difference between the portfolio's returns and those of the benchmark.

- **Expected Excess Returns:** Expected values of the distribution of future excess returns.

- **Equilibrium:** The condition in which *expected excess returns* equilibrate the demand for assets with the outstanding supply.

- **Equilibrium or Market Portfolio:** The portfolio held in equilibrium, in this chapter, market capitalisation weights.

- **Neutral Portfolio:** An optimal portfolio given neutral views.

- **Normal portfolio:** The portfolio that an investor feels comfortable with when he has no views. He can use the normal portfolio to infer a benchmark when no explicit benchmark exists.

- **Risk Premiums:** Expected excess returns implied by the equilibrium model.

The terminology above is introduced in the paper by Black and Litterman. [BL 1992]

2.2.1 Implied Expected Excess Returns

The Black-Litterman Model overcomes the problem of input sensitivity by creating a stable, mean-variance efficient portfolio, based on an investor's subjective view. The model also "largely mitigates", according to Lee [LW 2000], the problem of estimation-error maximisation by spreading the errors throughout the vector of expected returns.

The most important input in the mean-variance optimisation is the *vector of expected returns*. A great deal of work has gone into establishing a reasonable starting point for expected returns. Black and Litterman [BL 1992], He and Litterman [HL 1999] and Litterman [LR 2003] showed that alternative forcasts such as : historical excess returns, equal expected excess returns for all assets, and risk adjusted equal excess returns, all lead to extreme portfolios.

The Black-Litterman Model does not assume that the world is always at CAPM equilibrium, but rather that when expected returns move away from their equilibrium values, "market forces" will tend to push them back. Black and Litterman [BL 1992] state "Equilibrium risk premiums provide a center of gravity for expected returns". In other words *expected equilibrium excess returns* are a good neutral starting point for expected returns. This is because optimally, a neutral investor with no views should hold the market portfolio or the benchmark portfolio. Intuitively we see this in the following scenario. Suppose an investor has no views. How does he go about defining his optimal portfolio? The answer to this question highlights the importance of the equilibrium risk premium. It is unrealistic to demand of an investor to state exact expected excess returns for every asset in his portfolio. The equilibrium is thus able to provide the investor with a neutral starting point.

2.2.2 Reverse Optimisation

Litterman [LR 2003][Ch 6] derives the global equilibrium expected returns from a multiple currency perspective but this is beyond the scope of this illustration. In our case the equilibrium returns are derived using a reverse optimisation technique in which the vector of implied expected excess equilibrium returns is extracted from known information. We can assume that in equilibrium, the average investor will hold assets in proprotion to their market capitalisations. Hence the investor maximises his utility function:

$$\max\left(\omega_{mkt}\pi - \frac{\lambda}{2}\,\omega'_{mkt}\Sigma\omega_{mkt}\right)\quad \text{with respect to } \omega_{mkt}$$

thus

$$\pi = \lambda\Sigma\omega_{\text{mkt}} \tag{2.1}$$

where

- π is the $(n \times 1)$ Implied Excess Equilibrium Return Vector;

- λ is the risk aversion coefficient

$$\lambda = \frac{E(r) - r_f}{\sigma^2} \tag{2.2}$$

  where

  - $E(r)$ is the expected market or benchmark total return;
  - r_f is the risk free rate; and,
  - $\sigma^2 = \omega_{mkt}^T \Sigma \omega_{mkt}$ is the variance of the market or benchmark excess returns.

- Σ is the $(n \times n)$ covariance matrix of excess returns; and,

- ω_{mkt} is the $(n \times 1)$ vector of market capitalisation weight.

In the reverse optimisation process the risk aversion coefficient acts as a scaling factor for the reverse optimisation estimate of excess returns. Grinold and Kahn [GK 1999] show that the *implied* risk aversion coefficient (λ) for a given portfolio can be estimated by dividing the expected excess returns by the variance of the portfolio. Idzorek [IT 2003] points out that literature on the Black-Litterman Model often refers to the reverse optimised Implied Excess Equilibrium Return Vector (π) as the CAPM returns. This is misleading. Unless the portfolio in question is a reasonable proxy for the "market" portfolio, the reverse optimised returns are not CAPM returns.

2.2.3 The Black-Litterman formula

Let us now introduce the Black-Litterman formula and provide a brief description of each of its elements. Throughout this chapter, k is used to represent the number of views and n is used to express the number of assets. The formula for the new Combined Return Vector

($E[R]$) is

$$E[R] = \underbrace{\left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1}}_{n\times n \quad \text{Matrix}} \overbrace{\left[(\tau\Sigma)^{-1}\pi + P'\Omega^{-1}Q\right]}^{n\times 1 \quad \text{Vector}} \tag{2.3}$$

where

- $E[R]$ is the new (posterior) *Combined Return Vector* ($n \times l$ column vector);
- τ is a scalar;
- Σ is the *Covariance Matrix of Excess Returns* ($n \times n$ matrix);
- P is the *View Projection Matrix* that identifies the assets involved in the views ($k \times n$ matrix);
- Ω is the diagonal *Covariance Matrix of Error Terms* from the expressed views representing the uncertainty in each view ($k \times k$ matrix);
- π is the old (prior) *Implied Equilibrium Return Vector* ($n \times 1$ column vector); and
- Q is the *View Vector* ($k \times 1$ column vector).

2.2.4 Investor Views

In order to illustrate the Black-Litterman model I am going to present a nine asset example. I have chosen asset classes from the JSE Economic Groups. Figure (2.1) gives a break down of the asset classes used together with their respective market capitalisations in millions of Rand. The Vector of Implied Equilibrium Excess Returns (π) was calculated using formula (2.1), with market capitalisation weights.

Usually, an investment manager has specific views regarding the expected return of some of the assets in the portfolio. These views differ from the Implied Equilibrium Excess Returns

Index All Share Economic Group	Code	Market Cap in R mil	Market Cap Weights	Implied Equil Vector
			ω_{mkt}	π
JSE:RESOURCES	J000	622881.159	42.8599%	0.830%
JSE:BASIC INDUSTRY	J010	51047.92	3.5126%	0.700%
JSE:GENERAL INDUSTRY	J020	40021.986	2.7539%	0.610%
JSE:CYC CONSUMER	J030	99462.509	6.8439%	0.580%
JSE:NCYC CONSUMER	J040	118767.343	8.1723%	0.550%
JSE:CYCLICAL SERVICES	J050	102818.938	7.0749%	0.550%
JSE:NON-CYC SERVICES	J060	62176.504	4.2783%	0.660%
JSE:FINANCIALS	J080	343053.378	23.6052%	0.590%
JSE:IT	J090	13065.189	0.8990%	0.860%
Total Market Cap		1453294.926	100%	

Figure 2.1: *Vector of Expected Excess Returns based on the Market Portfolio*

(π). The Black-Litterman Model allows these views to be expressed in either absolute or relative terms. I will center my exposition of the model based of the following three views. These views are exhaustive in terms of the type of views an investment manager might have regarding his portfolio.

- **View 1:** JSE Financials will have an absolute excess return of 0.65%
- **View 2:** JSE Cyclical Consumer will outperform JSE Cyclical Services by 10 basis points
- **View 3:** JSE IT and JSE Non-Cyclical Consumer will outperform JSE Basic Industrials and JSE General Industrials by 5 basis points.

View 1 is an example of an absolute view. From the final column of Figure (2.1), the Implied Equilibrium Return of JSE Financials is 0.59%. View 1 tells us the Black-Litterman Model to set the excess return of JSE Financials to 0.65%.

Views 2 and 3 represent relative views. This way of representing views is a lot closer to the way investors feel or express their views. View 2 tells us that the expected returns on

Cyclical Consumer are ten basis points higher than those of Cyclical Services. In order to interpret the effect this view has on the resulting weights of the portfolio, we must look at Figure (2.1) and see what the respective values are for the Implied Equilibrium Returns. We see that Cyclical Consumer has an implied equilibrium return of 0.58% and Cyclical Services one of 0.55%. Our view tells us to expect a ten basis points difference in the favour of Cyclical Consumer. This view difference is larger than that implied by the equilibrium returns so we expect the model to tilt the weight in the favour of Cyclical Consumer. The degree of the tilt depends on the confidence of the view. I shall address that issue in a later section. Note however, that should View 2 have stipulated a difference between the same assets of only two basis points, then the model would have tilted the weights towards the "underperforming" asset.

View 3 illustrates a view concerning multiple assets. The methodology used to deal with such a view is used uniformly for a view concerning three or more assets. The number of "outperforming" assets need not match the number of "underperforming" assets. In our situation we have two outperforming and two underperforming assets. Intuitively, it is difficult to assess the impact of a multiple view on the results of the model. Idzorek [2003], tells us that we have to separate the assets in the multiple view into two mini-portfolios, a long one and a short one. The relative weighting of each nominally outperforming asset is proportional to that asset's market capitalisation divided by the total market capitalisations of the assets that are nominally outperforming. Similarly, the relative weighting of each nominally underperforming asset is proportional to its market capitalisation divided by the total market capitalisation of the other nominally underperforming assets. The sum of the long positions and the short positions must equal zero. However, the mini-portfolio that receives the positive view may not be the nominally outperforming asset or assets from the expressed view.

From View 3 we see that the nominally "outperforming" assets are JSE IT and JSE Non-Cyclical Consumer and the nominally "underperforming" assets are JSE General Industri-

Outperforming Asset Class	Asset Nr	MRKT Cap in Rm	Relative Weight	Implied Eq Return	W'ted Excess Return
JSE:IT	9	13065.189	9.910%	0.8600%	0.0852%
JSE:NCYC CONS	5	118767.343	90.090%	0.5500%	0.4955%
Total		131832.532	100%	1.4100%	0.5807%
Underperforming Asset Class	**Asset Nr**	**MRKT Cap in Rm**	**Relative Weight**	**Implied Eq Return**	**W'ted Excess Return**
JSE:GEN IND	3	40021.986	43.946%	0.6100%	0.2681%
JSE:BAS IND	2	51047.92	56.054%	0.7000%	0.3924%
Total		91069.906	100%	1.3100%	0.6604%

Figure 2.2: *The two mini-portfolios as dictated by View 3. Note that the nominally outperforming assets have a smaller weighted excess equilibrium return than the nominally underperforming assets.*

als and JSE Basic Industrials. From Figure (2.2) we see that the weighted average Implied Equilibrium return of the mini-portfolio formed by JSE IT and JSE NCYC Consumer is 0.5807% and the weighted average Implied Equilibrium return of the mini-portfolio formed by JSE General Industry and JSE Basic Industry is 0.6604%. The weighted average Implied Equilibrium return differential is −0.0797%. View 3 tells us that we expect the differential to be 5 basis points. This happens to be a rather large view. The difference between the View and the weighted average Implied Equilibrium return differential is 0.1297%. This is double the magnitude of the expressed view.

2.2.5 Building the Inputs

The View Projection Matrix P

One of the more difficult aspects of the model is to translate the investor views into valid inputs for the Black-Litterman formula. It is important to note that the model does not require the investor to have views on all the assets. As it is shown later in the derivation of the model, no views at all simply reduces the model to the market portfolio. In our example, $k = 3$ and thus the View Vector (Q) is a 3×1 column vector. We can express views that differ from equilibrium with a certain degree of confidence. Namely,

$$P'E[R] = Q + \varepsilon$$

where

- ε is a $k \times 1$ vector of errors. $\varepsilon \sim MVN(0, \Omega)$; and
- Ω is a diagonal covariance matrix of error terms, discussed in detail at a later stage.

In general we have

$$P'E[R] = Q + \varepsilon = \begin{bmatrix} Q_1 \\ \vdots \\ Q_k \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_k \end{bmatrix}$$

Except in the unlikely event that an investor has 100% confidence in his views, the error term (ε) is a positive or negative value other than zero. The Error Term Vector (ε) does not enter the Black-Litterman formula directly. However, what does enter the formula is the variance of each error term (ω), which is the absolute difference from the error term's (ε) expected value of zero. The variances (ω) of the error terms form Ω, where Ω is a diagonal covariance matrix. That means that all the off-diagonal elements are zero because the model assumes the views to be independent. The variances (ω) of the error terms represent the uncertainty of the views. As the variance of the error terms increases so does the level of uncertainty in the views. In general

$$\Omega = \begin{bmatrix} \omega_1 & 0 & \cdots & 0 \\ 0 & \omega_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \omega_k \end{bmatrix} \tag{2.4}$$

One of the most important and most cumbersome aspect of the model is determining the individual variances of the error terms (ω). I will discuss this procedure at length.

Each of the expressed views in column vector Q are matched to the respective assets by the matrix P. Matrix P is sometimes intuitively referred to as the Projection Matrix as it reduces the space of all assets to the space of the assets upon which views are selected. However, I think a better name for it would be a *View Projection Matrix* because P is not mathematically a projection matrix ($P^2 \neq P$). Each expressed view results in a $1 \times n$ row vector, thus k views result in a $k \times n$ matrix. In our three-view example, in which there are 9 assets, P is a 3×9 matrix. In general

$$P = \begin{bmatrix} p_{1,1} & \cdots & p_{1,n} \\ \vdots & \ddots & \vdots \\ p_{k,1} & \cdots & p_{k,n} \end{bmatrix}$$

Two different methods have been presented in literature when we have an instance of views involving 3 or more assets. The two distinct methods are due to Satchel and Scowcroft [SS 2000] and He and Litterman [HL 1999].

Satchel and Scowcroft use an *equal weighting scheme* whereby weightings are proportional to 1 divided by the number of respective assets outperforming or underperforming. This scheme ignores the market capitalisations of any of the assets. One of the problems with this method is illustrated in the instance when we consider a view involving asset classes with very contrasting market capitalisations. Satchel and Scowcroft affect the respective weights equally thus causing large changes in the asset class with a small market capitalisation. This method may result in unnecessary or undesired tracking error.

The example based on Satchel and Scowcroft [SS 2000] takes the following form:

$$P = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & -0.5 & -0.5 & 0 & 0.5 & 0 & 0 & 0 & 0.5 \end{bmatrix} \tag{2.5}$$

Let us examine the rows of the View Projection Matrix. The first row represents View 1. It is an absolute view so we only pick out the asset involved. The 1 in the 7^{th} column represents the JSE Financials. Rows 2 and 3 represent Views 2 and 3 respectively. In the case of relative views, each row sums to zero. In Matrix P, the nominally outperforming assets receive a positive weighting and the nominally underperfoming assets a negative weighting.

On the other hand, He and Litterman use a *relative weighting scheme* of the assets entering the matrix P. The weightings are proportional to their market capitalisations. Moreover, the relative weighting of each individual asset is proportional to the asset's market capitalisation divided by the total market capitalisation of either the outperforming or underperforming assets in that view. I choose this method of evaluation the View Projection Matrix. Hence, P becomes

$$P = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & -0.5605 & -0.4395 & 0 & 0.9009 & 0 & 0 & 0 & 0.0991 \end{bmatrix} \quad (2.6)$$

Now that we have Matrix P defined we can begin to calculate the variance of each individual view. If we treat each view as a "mini-portfolio" (long the outperforming, short the underperforming) then the variance of an individual view portfolio is $p_k \Sigma p_k'$, where p_k is a single $1 \times n$ row vector from matrix P that corresponds to the k^{th} view and Σ is the covariance matrix of excess returns. The respective variance of each individual view portfolio is an important source of information regrading the degree of certainty of that view. This information will lead us naturally to define the variances of the error terms (ω) that form the diagonal elements of Ω.

View	Formula	Variance
1	$p_1 \Sigma p_1'$	0.59%
2	$p_2 \Sigma p_2'$	0.72%
3	$p_3 \Sigma p_3'$	0.33%

Figure 2.3: *The Variance of the View Portfolios*

Setting the value of τ

In reality, "the Black-Litterman model is a complex weighted average of the Implied Equilibrium Return Vector (π) and the View Vector (Q), in which the relative weightings are a function of the scalar (τ) and the uncertainty in the views."(Idzorek [IT 2003]). This basically implies that as confidence in the views increases, the new return vector will approach the views. Similarly, as confidence in the views decreases, the new return vector approaches the Implied Equilibrium Return Vector (π). We can see τ as being approximately proportional to the relative weight given to the Implied Equilibrium Return Vector (π). The biggest problem we face is the fact that very little is said in the literature about ways to set the value for the scalar τ.

Both Black and Litterman [BL 1992] and Lee [LW 2000] have addressed this issue. Since the uncertainty in the mean is less than the uncertainty in the return, the scalar (τ) is close to zero. Idzorek explains this by saying that one would expect Equilibrium returns to be less volatile than the historical returns. Let us examine View 2, a relative view involving two assets of equal size. View 2 states that $p_2 \cdot E[R] = Q_2 + \varepsilon_2$, where $Q_2 = E[R_{\text{JSE CYC Consumer}}] - E[R_{\text{JSE CYC Services}}]$. View 2 is $N \sim (Q_2, \omega_2)$. In the absence of additional information, one can assume that uncertainty in the view is proportional to the covariance matrix Σ however, since the view is describing the mean return differential, the uncertainty of the view should be considerably less than the uncertainty of a single return (or return differential) represented by the covariance matrix (Σ). Therefore, the investor's views are represented by a distribution with a mean of Q and a covariance structure $\tau\Sigma$.

Lee sets the value of the scalar τ between 0.01 and 0.05 and then calibrates the model based on a target level of tracking error. Similarly, Satchel and Scowcroft say the value of τ is often set to 1. However, we know that the Black-Litterman Model only recommends a departure from an asset's market capitalisation if it is the subject of a view. For assets that are subject to a view, the ratio of the scalar τ to the variance of the error term (ω) controls the magnitude of their departure from their market capitalisation weights. The investor's confidence in a particular view is inversly related to the variance of the error term (ω). Thus complete certainty in a view results in a zero variance of the error term (ω). Idzorek [IT 2003] and Black and Litterman [BL 1990] explain that " placing 100% weight on the views, which corresponds to 0's in all the diagonal elements of Ω, does not cause the model to completely ignore the Implied Equilibrium Return Vector(π), unless the number of views is equal to the dimensionality of the expected return vector and 100% confidence is given to the views". The best way however to calibrate the Black-Litterman model is to make an assumption about the value of τ. He and Litterman [HL 1999] calibrate the confidence of a view so that the ratio ω/τ is equal to the variance of the view portfolio ($p_k \Sigma p_k'$). They reason as follows: If $\tau\Sigma$ is the covariance matrix of expected returns, then $P\tau\Sigma P'$ is the covariance matrix of $P \cdot E[R]$, so we can expect the covariance matrix Ω to be close to that of $P \cdot E[R]$. Equation (2.12) clarifies this assertion. It basically stems from the initial assumptions of the model. In other words:

$$\Omega \sim \tau P \Sigma P' \quad \rightarrow \quad \omega/\tau \sim p\Sigma p'$$

Thus the covariance matrix of the error term (Ω) has the following form:

$$\Omega = \begin{bmatrix} (p_1 \Sigma p_1') \times \tau & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & (p_k \Sigma p_k') \times \tau \end{bmatrix} \tag{2.7}$$

In the above set up, if we calculate the covariance matrix of error terms (Ω) by this method, we notice that the value of the scalar τ becomes irrelevant. What is of importance is the ratio ω/τ. For example, if we dramatically change the value of the scalar τ by a factor of 100

Assets	J000	J010	J020	J030	J040	J050	J060	J080	J090
J000	85	63	40	41	35	27	24	24	53
J010		82	39	33	31	30	38	38	35
J020			52	34	39	43	51	51	52
J030				70	27	28	34	34	61
J040					48	37	44	44	42
J050						58	64	64	63
J060							144	144	92
J080								57	60
J090									193

Figure 2.4: *The Covariance Matrix of Historical Excess Returns* ($\Sigma \times 10^{-4}$)

(say), then the diagonal elements of Ω will also change drastically, only the New Combined Return Vector $E[R]$ is unaffected. Thus assuming $\tau = 0.025$ and using the individual variances of the view portfolios ($p_k \Sigma p_k'$) as in Figure (2.3), the covariance matrix of error terms (Ω) is:

$$\Omega = \begin{bmatrix} 0.0001466 & 0 & 0 \\ 0 & 0.0001796 & 0 \\ 0 & 0 & 0.0000824 \end{bmatrix} \tag{2.8}$$

2.2.6 Calculating the New Combined Return Vector

Having specified the covariance matrix of the error terms (Ω), which implicitly also sets the value of the scalar τ, all the inputs are entered into the Black-Litterman Formula and the New Combined Return Vector($E[R]$) is derived.

The New Recommended Weights (w^*) are calculated by solving the unconstrained maximisation problem $w^* = (\lambda\Sigma^{-1})E[R]$. The covariance matrix of historical excess returns (Σ) is given in Figure (2.4). Even though the expressed views only involved 7 of the 9 asset classes, the individual returns of all the assets changed from their respective Implied Equilibrium returns (see column 4 of Figure (2.5)). Since each individual return is linked to the other returns via the covariance matrix, it is not uncommon for a single view to cause the return of every asset in the portfolio to change from its Implied Equilibrium return.

The New Recommended Weights (w^*) in column 5 of Figure (2.5) are based on the New Combined Return Vector $E[R]$. The final column of Figure (2.5) shows one of the features of the features of the Black-Litterman Model, namely that only the weights of the assets for which views were expressed had changed from their original market capitalisation weights. Moreover, the directions of the changes, for the assets where views were expressed, are intuitive.

From a general point of view, the new portfolio can be viewed as the sum of two portfolios. One portfolio is the original market portfolio, and the second portfolio is a series of long and short positions based on the views. The second portfolio can be subdivided into mini-portfolios, each according to the specific view. The mini-portfolios that result from relative views, have offsetting long and short positions that sum to zero. However, absolute views, such as our View 1, have no offsetting positions, and this results in portfolio weights that no longer sum to zero.

All Share Econ Group	Eq Vec Π	New Vec E[R]	Diff E[R] - Π	New Weight w*	MktCap wmkt	Diff w* -wmkt
JSE:Res	0.83%	0.82%	-0.01%	42.86%	42.86%	0.00%
JSE:Bas Ind	0.70%	0.65%	-0.05%	-3.91%	3.51%	-7.42%
JSE:Gen Ind	0.61%	0.62%	0.01%	-3.07%	2.75%	-5.82%
JSE:Cyc Cons	0.58%	0.61%	0.03%	11.49%	6.84%	4.65%
JSE:Ncyc Cons	0.55%	0.59%	0.04%	20.11%	8.17%	11.93%
JSE:Cyc Serv	0.55%	0.56%	0.013%	2.43%	7.07%	-4.65%
JSE:Ncyc Serv	0.66%	0.67%	0.015%	4.28%	4.28%	0.00%
JSE:Fin	0.59%	0.62%	0.03%	26.75%	23.61%	3.15%
JSE:IT	0.86%	0.91%	0.05%	2.21%	0.90%	1.31%
Total				103.15%	100.00%	3.15%

Figure 2.5: *Return Vectors and the Resulting Portfolio Weights*

Looking at Figure (2.6) we observe that the assets have been "tilted" by the model intu-

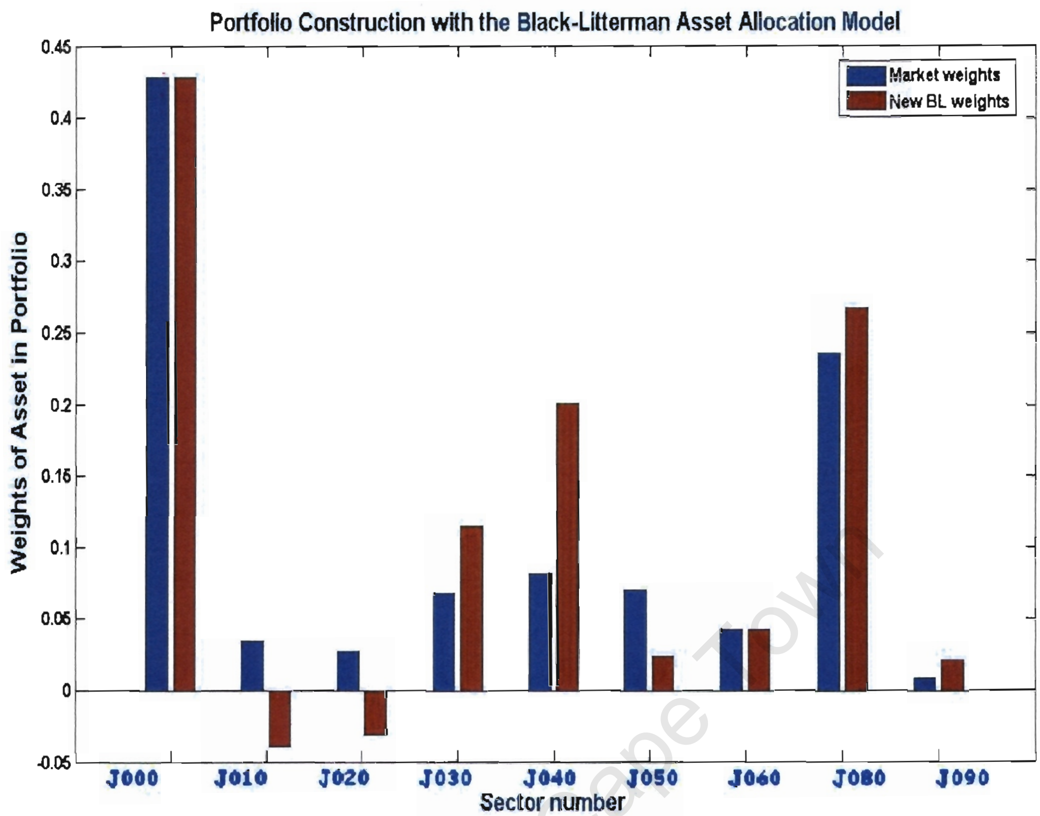

Figure 2.6: *Portfolio Allocations Based on the New Weights*

itively in the correct expected manner. Recall the third view that JSE Basic Industries (J010) and JSE General Industries (J020) will underperform versus JSE Non-Cyclical Consumer (J040) and JSE IT (J090). We now look at the graphical representaion and see that the weights for J010 and J020 have decreased and the weights for J040 and J090 have increased. The total amount by which the underperforming assets decrease is equal to the amount by which the overperforming assets increase. This is a direct consequence of the fact that the sum of the entries along any row of the View Projection Matrix add up to zero.

Similarly, in an earlier section we saw that the value of the scalar τ is arbitrary as far as the outcome of the optimisation is concerned. This is visualised in Figure (2.7) where several choices of the scalar τ resulted in identical weight distributions.

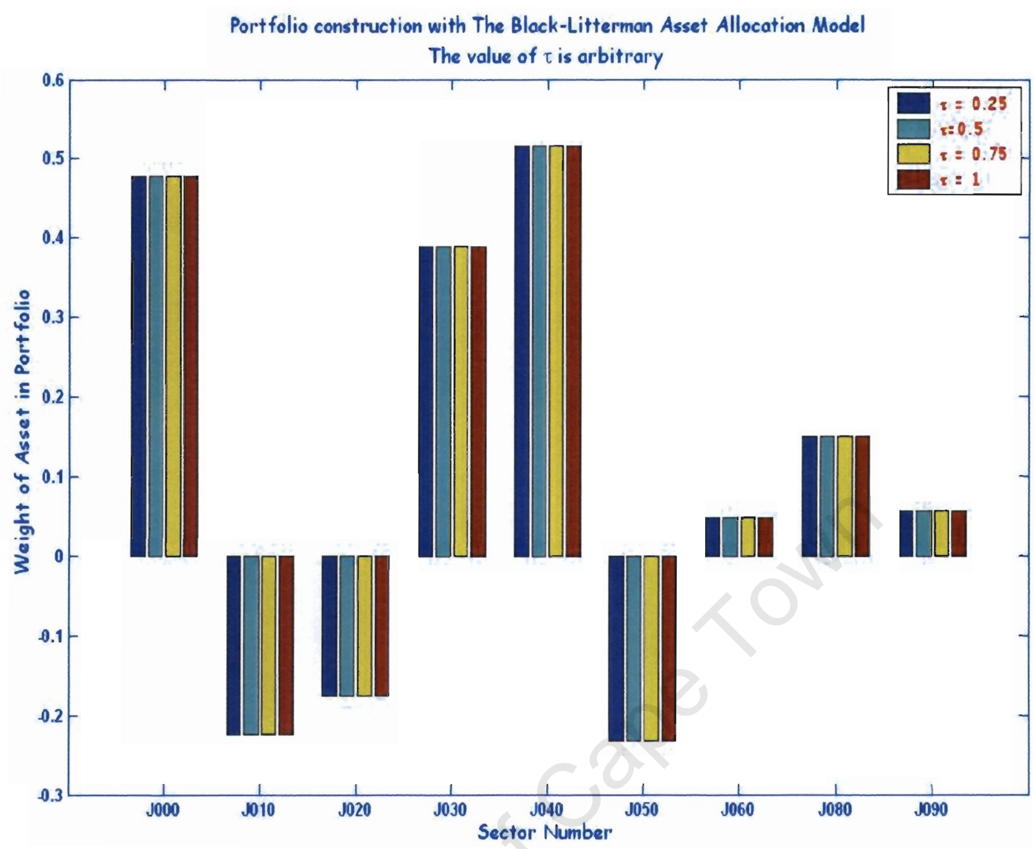

Figure 2.7: *Weight distributions for several values of* τ

However, variation of the coefficient of risk aversion, λ, does influence the distribution of weights. Firstly, when we calculate the Equilibrium Expected Return Vector π, the risk aversion of the manager comes into play. Secondly once the Black-Litterman formula has been applied and we need to reverse optimise weights from the New Return Vector $E[R]$, given a certain risk aversion. Thus the optimal weights vary more dramatically, i.e., they take on more tracking error as the coefficient of risk aversion is decreased. This is best illustrated in Figure (2.8) where λ is varied from 1 to 10, with 1 representing low risk aversion (high risk appetite) and 10 representing large risk aversion. Note that all the other parameters in the Black-Litterman formula have remained unchanged. Sectors J000 and J060 have had no views expressed on them, but they still vary because the Equilibrium Return Vector π is a function of λ. The rest of the assets diverge quite dramatically from π with varing λ, as more and more tracking error is taken on.

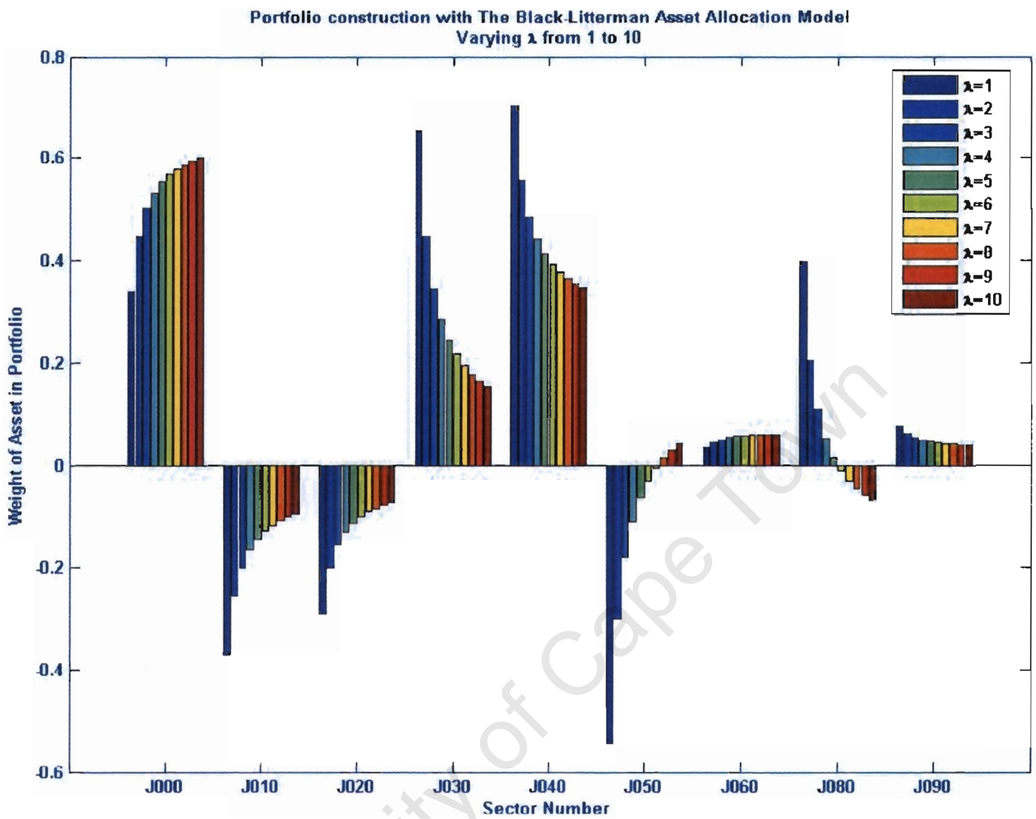

Figure 2.8: *The effect of λ on portfolio weights*

2.3 Derivation of the model

As seen in earlier sections, the Black-Litterman model can help to construct stable mean-variance efficient portfolios. We now seek to derive the the form of the New Combined Returns Vector (E[R]). We do this from two points of view. One point of view applies Bayesian Inference techniques and the other is a more accessible application of Generalised Least Squares Method. Christodoulakis [CG 2002] offers guidance with the Bayesian Inference derivation.

In the Bayesian context we consider a framework to assess the joint likelihood of investor's subjective views (or prior beliefs) and the empirical data (model based estimates). At the heart of Bayesian Statistics and Decision Theory is Bayes' Theorem. In its simplest form, if H is a hypothesis and E is evidence, then the theorem is

$$p(H|E) = \frac{p(E|H) \cdot p(H)}{p(E)} \tag{2.9}$$

provided $p(E) > 0$, so that $p(H|E)$ is the probability of belief in H after obtaining E, and $p(H)$ is the *prior* probability of H before considering E. The left hand side of the theorem, $p(H|E)$, is usually referred to as the *posterior* probability of H. The theorem thus supplies a solution to the general problem of inference or induction, giving us a mechanism for learning about a hypothesis H from data E.

Bayes' Theorem itself is merely an accounting identity that follows from the axiomatic foundations of probability linking joint, conditional and marginal probabilities. Bayes' Theorem is sometimes referred to as the *rule of inverse probability*, since it shows how a conditional probability B given A can be "inverted" to yield the conditional probability A given B.

In the continuous case (e.g., learning about a parameter θ given data y), Bayes' Theorem is often expressed as

$$p(\theta|y) \propto f(y;\theta)p(\theta) \tag{2.10}$$

where $f(y;\theta)$ is the *likelihood function.* The constant of proportionality $[\int f(y;\theta)p(\theta)d\theta]^{-1}$ does not depend on θ and ensures that the posterior density integrates to 1. This version of Bayes' Theorem is fundamental to Bayesian statistics. It shows how the likelihood function (the probability of the data, given θ) can be turned into a probability statement about θ, given data y. Note also that the Bayesian approach treats parameter θ as a random variable, and conditions on the data whereas the frequentist approache considers θ a fixed (but unknown) property of a population from which y is randomly sampled.

In our situation let us consider the probability density function (pdf) of the expected returns given the data equilibrium $p(E[R]|\pi)$. This is given by the product of the conditional pdf of the data equilibrium return $p(\pi|E[R])$ and the prior pdf $p(E[R])$ which summarises the investment manager's subjective views, in units of marginal probabilities $p(\pi)$ of equilibrium returns.

Thus, Bayes' Law provides a formal mechanism to synthesise subjective views with empirical realities. As new data arrives, the posterior density can play the role of a new prior, thus updating investor's beliefs in this set up.

2.3.1 Model Assumptions

I will state up-front the 5 key assumptions upon which the Black-Litterman model is built.

1. Excess returns are multivariate normal with unknown mean M and covariance Σ.
2. Beliefs about linear combinations of expected returns are provided by forecasters: $Q = PE[R]$.
3. Beliefs can be expressed as multivariate normal distributions for $PE[R]$ with mean Q and covariance Ω where the beliefs are independent and unrelated to eachother.

4. The current market portfolio, w_{mkt} implies a set of consensus views π such that $w_{mkt} \propto \Sigma^{-1}\pi$. The scaling factor is derived from the Sharpe Ratio.

5. The market view itself is random with mean M and covariance $\tau\Sigma$ for some scalar τ. Here τ is a measure of the strength of the forecasters private views relative to the consensus or market view.

We now use the necessary assumptions in order to construct the composite equation (2.9) which in our current notation we could write

$$p(E[R]|\pi) = \frac{p(\pi|E[R])p(E[R])}{p(\pi)} \tag{2.11}$$

We assume that the prior beliefs in $p(E[R])$ shall take the form of k linear constraints on the vector of n expected returns $E[R]$ which can be expressed with a $k \times n$ View Projection matrix P such that

$$P \cdot E[R] = Q + \varepsilon$$

where $\varepsilon \sim N(0, \Omega)$ and Ω is the $k \times k$ diagonal covariance matrix of error terms. It follows that

$$P \cdot E[R] \sim N(Q, \Omega) \tag{2.12}$$

The existence of an error vector ε signifies the existence of uncertain views. The normality assumption coupled with a diagonal Ω implies that the investment manager's subjective views are formed independently of each other. In this set up P, Q and Ω are known by the investor.

The probability density function of the data equilibrium returns conditional on the investor's prior beliefs, is assumed to be

$$p(\pi|E[R]) \sim N(E[R], \tau\Sigma) \tag{2.13}$$

The assumption of homogeneous views of all investors in a CAPM-type world is reflected in the fact that $E(\pi) = E[R]$.

The marginal density function of the data equilibrium returns, $p(\pi)$ is a constant that will be absorbed into the integrating constant of $p(E[R]|\pi)$.

2.3.2 Uncertain Prior Beliefs on Expected Returns

When the investor forms prior beliefs with a degree of uncertainty, it is signified in the non-zero value of the diagonal elements of the Ω matrix. A good exposition of the following result can be found in [SS 2000]. Using assumptions (2.12) and (2.13) in (2.11) we obtain the following results:

Theorem 1

The posterior probability density function (pdf) $p(E[R]|\pi)$ is multivariate normal with mean

$$\underbrace{\left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1}}_{n\times n} \overbrace{\left[(\tau\Sigma)^{-1}\pi + P'\Omega^{-1}Q\right]}^{n\times 1}$$

and variance

$$\left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1}.$$

Proof:

Assumptions (2.12) and (2.13) state respectively that:

$$p(P \cdot E[R]) = \frac{1}{\sqrt[n]{2\pi_c|\Omega|}} \exp\left(-\frac{1}{2}(P \cdot E[R] - Q)'\,\Omega^{-1}\,(P \cdot E[R] - Q)\right)$$

and

$$p(\pi|E[R]) = \frac{1}{\sqrt[n]{2\pi_c|\tau\Sigma|}} \exp\left(-\frac{1}{2}(\pi - E[R])' \ (\tau\Sigma)^{-1} \ (\pi - E[R])\right)$$

where π_c is the constant π. From (2.11) we know that

$$p(E[R]|\pi) = \frac{p(\pi|E[R])p(E[R])}{p(\pi)}$$

By combining the pdfs of the numerator in (2.11) the posterior density will be proportional to

$$\exp\left(-\frac{1}{2}(P \cdot E[R] - Q)' \ \Omega^{-1} \ (P \cdot E[R] - Q) - \frac{1}{2}(\pi - E[R])' \ (\tau\Sigma)^{-1} \ (\pi - E[R])\right)$$

Which can be written as

$$\begin{aligned}
&= \exp\left(-\frac{1}{2}\left[(E[R]'HE[R] - 2C'E[R] + A\right]\right) \\
&= \exp\left(-\frac{1}{2}\left[(E[R]'H'HH^{-1}E[R] - 2C'H^{-1}HE[R] + A\right]\right) \\
&= \exp\left(-\frac{1}{2}\left[(H\ E[R] - C)'H^{-1}(H\ E[R] - C) - C'H^{-1}C + A\right]\right) \\
&= \exp\left(-\frac{1}{2}[A - C'H^{-1}C]\right) \times \exp\left(-\frac{1}{2}(H\ E[R] - C)'\ H^{-1}(H\ E[R] - C)\right)
\end{aligned}$$

where

$$\begin{aligned}
H &= (\tau\Sigma)^{-1} + P'\Omega^{-1}P \\
C &= (\tau\Sigma)^{-1}\pi + P'\Omega^{-1}Q \\
A &= \pi'(\tau\Sigma)^{-1}\pi + Q'\Omega^{-1}Q
\end{aligned}$$

Thus the term $\exp\left(-\frac{1}{2}[A - C'H^{-1}C]\right)$ and the denominator pdf(π) which is not modeled will be absorbed into the integrating constant for the posterior pdf. The result follows immediately. We see the variance to be H^{-1} and the mean $H^{-1}C$.

2.3.3 Alternative Derivation

For clarity I refer to the Appendix Section "Notes of Generalised Least Squares". A good reference on GLS is [HJ 1994]. Let us consider assumptions (2.12) and (2.13) and re-write them in the following format

$$\begin{aligned} \pi &= E[R] + \epsilon \\ Q &= P \cdot E[R] + \varepsilon \end{aligned}$$

where $\epsilon \sim MVN(0, \tau\Sigma)$ and $\varepsilon \sim MVN(0, \Omega)$. Then we have the following model

$$Y = X(E[R]) + u$$

where $Y = \begin{pmatrix} \pi \\ Q \end{pmatrix}$, $X = \begin{pmatrix} I \\ P \end{pmatrix}$, $u \sim MVN(0, V)$ and $V = \begin{pmatrix} \tau\Sigma & 0 \\ 0 & \Omega \end{pmatrix}$

So using Generalised Least Squares

$$E[R] = (X'V^{-1}X)^{-1}X'V^{-1}Y$$

By matrix manipulation we get

$$\begin{aligned} E[R] &= \left[(I \;\; P') \begin{pmatrix} \tau\Sigma & 0 \\ 0 & \Omega \end{pmatrix}^{-1} \begin{pmatrix} I \\ P \end{pmatrix} \right]^{-1} (I \;\; P') \begin{pmatrix} \tau\Sigma & 0 \\ 0 & \Omega \end{pmatrix}^{-1} \begin{pmatrix} \pi \\ Q \end{pmatrix} \\ E[R] &= \left[\left((\tau\Sigma)^{-1} \;\; P'\Omega^{-1} \right) \begin{pmatrix} I \\ P \end{pmatrix} \right]^{-1} \left((\tau\Sigma)^{-1} \;\; P'\Omega^{-1} \right) \begin{pmatrix} \pi \\ Q \end{pmatrix} \\ E[R] &= \left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P \right]^{-1} \left[(\tau\Sigma)^{-1}\pi + P'\Omega^{-1}Q \right] \end{aligned} \tag{2.14}$$

2.3.4 Certain Prior Beliefs On Expected Returns

We now look at two particular cases where there is certainty in prior beliefs.

Firstly, a neutral investor, who has no views of his own, is required by the model to place 100% confidence in the equilibrium and 0% in his views. That implies a View Projection Matrix $P = 0$ or $\Omega^{-1} = 0$. If we put this into equation (2.14)

$$E[R] = \left[(\tau\Sigma)^{-1}\right]^{-1}\left[(\tau\Sigma)^{-1}\,\pi\right] = \pi$$

Secondly, if we look at an investor who has 100% certainty regarding his prior beliefs, then the error terms have zero standard deviation. In other words the diagonal matrix Ω is the zero matrix.

Thus the investor's views are expressed as an exact relationship which will simply form a constraint in an optimisation problem. In particular we need to determine the optimal estimate for $E[R]$ which minimises the variance of $E[R]$ around the equilibrium returns π:

$$\text{Min} \quad (w^* - w_{mkt})'\Sigma(w^* - w_{mkt})$$

Now using the result in equation (2.1) we get

$$\text{Min}\left(\frac{\Sigma^{-1}w^*}{\lambda} - \frac{\Sigma^{-1}w_{mkt}}{\lambda}\right)'\Sigma\left(\frac{\Sigma^{-1}w^*}{\lambda} - \frac{\Sigma^{-1}w_{mkt}}{\lambda}\right)$$

and finally

$$\text{Min}(E[R] - \pi)'\tau\Sigma(E[R] - \pi)$$

such that

$$P \cdot E[R] = Q$$

Theorem 2

The optimal predictor of $E[R]$ that minimises its variance around the equilibrium returns π and satisfies k exact linear belief constraints is given by:

$$\widehat{E[R]} = \pi + \Sigma^{-1}P'(P\Sigma^{-1}P')^{-1}(q - P\pi)$$

Proof:

This is a conventional linearly constrained least-squares problem and thus this admits a closed-form solution for $E[R]$. We can form a Lagrangian function

$$\begin{aligned} L &= (E[R]-\pi)'\tau\Sigma(E[R]-\pi)-\lambda(P\cdot E[R]-q) \\ &= E[R]'\tau\Sigma E[R]-E[R]'\tau\Sigma\pi-\pi'\tau\Sigma E[R]+\pi'\tau\Sigma\pi-\lambda P\cdot E[R]+\lambda Q \end{aligned}$$

the first order constraints will be $\frac{\partial L}{\partial E[R]}=0$ and $\frac{\partial L}{\partial \lambda}=0$. Explicitly:

$$\begin{aligned} \frac{\partial L}{\partial E[R]} &= \tau\Sigma' E[R]+\tau\Sigma E[R]-\tau\Sigma\pi-\tau\Sigma'\pi-P'\lambda=0 \\ &= 2\tau\Sigma E[R]-2\tau\Sigma\pi-P'\lambda=0 \end{aligned} \quad (2.15)$$

and

$$\frac{\partial L}{\partial \lambda}=-P\cdot E[R]+Q=0 \quad (2.16)$$

solving equation (2.15) with respect to $E[R]$ we obtain

$$\begin{aligned} 2\tau\Sigma E[R] &= P'\lambda+2\tau\Sigma\pi \\ E[R] &= \frac{1}{2\tau}\Sigma^{-1}P'\lambda+\pi \end{aligned} \quad (2.17)$$

Now put (2.17) into (2.16) to get λ

$$\lambda=(P\Sigma^{-1}P')^{-1}2\tau(Q-P\pi)$$

thus substituting λ back into (2.15) we obtain the optimal value for $E[R]$.

$$E[R]=\pi+\Sigma^{-1}P'(P\Sigma^{-1}P')^{-1}(Q-P\pi)$$

2.3.5 Interpretation of the Previous Result

In Theorem 1 we have proved that $\widehat{E[R]}|\pi$ has posterior mean

$$\left[(\tau\Sigma)^{-1}+P'\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\pi+P'\Omega^{-1}Q\right]$$

which can be written as

$$\left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1} \left[(\tau\Sigma)^{-1}\pi + (P'\Omega^{-1}\underbrace{P)(P'P)^{-1}P'Q}_{I\cdot Q}\right]$$

Also we know that

$$P \cdot \widehat{E[R]} = Q + v$$

or that

$$Q = P \cdot \widehat{E[R]} - v$$

which can be seen as a "regression" of Q on P, with $\widehat{E[R]}$ being the vector of unknown coefficients to be estimated. Then

$$(P'P)^{-1}P'Q$$

can be interpreted as the least squares estimate of expected returns, $\widehat{E[R]}$, according to investor's views

$$(P'P)^{-1}P'Q = \widehat{E[R]}$$

We can thus write the posterior mean in the form

$$\left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1} \left[(\tau\Sigma)^{-1}\pi + (P'\Omega^{-1}P)\widehat{E[R]}\right]$$

which makes clear how subjective views are combined with data-equilibrium. The term in the second square brackets is a weighted average of data equilibrium π and the least squares estimate of expected returns, $\widehat{E[R]}$, according to investors views, the (vector) weights being $(\tau\Sigma)^{-1}$ and $(P'\Omega^{-1}P)$ respectively. If the distribution of expected returns around the data equilibrium π is tight, i.e. $\tau\Sigma$ small, then $(\tau\Sigma)^{-1}$ will be large and more weight will be put to π. If the investor is confident about his views, then Ω is small, resulting in a large $(P'\Omega^{-1}P)$ which puts more weight on the least squares views $\widehat{E[R]}$.

Chapter 3

Portfolio Construction with Qualitative Forecasts

3.1 Overview of the model

The greater majority of active portfolio managers use an investment style approach that is of a fundamental nature. They do not soly rely on quantitative models and they also do not generate quantitative forecasts. Instead, their investment process is made up of several tools and indicators that they use to form qualitative forecasts. Such forecasts are often expresses as opinions such as "we are bullish on Financials and bearish on Rand Hedged Industrials". There is a great deal of effort spent in the investment industry to produce these forecasts, be it from econometric principles, from fundamental company analysis, from technical analysis, from economic forecasting or from guess work. This process is loosely called *alpha generation.* Surprisingly, very little effort has been spent on incorporating these views in an asset allocation model. Herold Ulf of Metzler Investment in Frankfurt, developed a portfolio construction model that is able to process purely qualitative forecasts [UH 2003]. The theoretical background of this model is largely similar to that of the Black-Litterman model. In the sections that follow I present a mathematical derivation of the model and then demonstrate the model using the same nine assets as in the Black-Litteman section. I conclude with some insight on the nature of the parameters used in the model.

3.2 Derivation of the model

3.2.1 Model Assumptions

What follows is an exposition of the model proposed by Ulf Herold [UH 2003]. Bayesian statistics is a natural setting for portfolio construction as it combines sample and prior information to get an improved estimate of the unknown parameters of a probability density function. This model is particularly set in an active management framework, thus specification of the prior distribution is relatively straightforward. In the absence of any views, the manager will hold the benchmark portfolio. Thus the prior distribution of alpha is centered around the zero vector. We define alpha to be the expected residual return, or in other words, the risk-adjusted expected excess return of an asset over the benchmark. It follows that, when using an alpha-tracking error optimiser, plugging in alphas with a value of zero would result in the benchmark portfolio.

Mathematically, the multivariate prior distribution of alpha is given by

$$\alpha \sim MVN(0, \Sigma) \tag{3.1}$$

where

- 0 is an $n \times 1$ vector of zeros;
- Σ is the $n \times n$ covariance matrix; and

- n is the number of assets in the benchmark portfolio

Note that equation (3.1) differs from equation (1) in [UH 2003] in that the scalar τ expresing the investor's confidence in the covariance matrix has been dropped. This is in light of the fact that we can control the certainty through the information coefficient. Through a clever construction identical to that used in the Black Litterman model we can eliminate the effect of τ on the outcome of the optimisation. Figure 2.7 shows the insensitivity of the weights to the scalar τ. Thus for simplicity and clarity I have chosen to set $\tau = 1$.

When managers expect alphas to be different from zero, they will diverge from the benchmark portfolio. The extent of the deviation from the benchmark portfolio will be determined by the level of confidence the manager has in his views. The views are expressed in a relative way, as linear combinations of assets. The views can be summarised in the following way

$$P\alpha \sim MVN(q, \Omega) \tag{3.2}$$

where

- P is the $k \times n$ View Projection Matrix storing the k views;
- q is the $k \times 1$ vector of expected returns on the views; and
- Ω is the $k \times k$ diagonal matrix of expected variances (forecast errors) on those views.

To make equation (3.2) useful, we note that each view can be stated as a long/short mini-portfolio: going long in the nominally bullish assets and going short in the nominally bearish assets. Each such mini-portfolio induces a tracking error. The tracking error variances (TEV) of the long/short portfolios can be found on the diagonal of the matrix $P\Sigma P'$.

The level of confidence in the views is captured in the information coefficient, IC. The information coefficient is defined as the correlation between forecasts and the subsequent realised returns. Hence the alpha of the long/short portfolio is equal to the information coefficient IC times its tracking error TE. To be more precise, alpha is defined by tracking error, TE, times information ratio, IR. For a single view portfolio, the information ratio is equal to the information coefficient because, according to the Fundamental Law of Active Management, the information ratio is defined as IC times the square root of the number of independent active bets. IR = IC $\sqrt{n}$, and with only one bet, IC = IR. The alphas are stored in the $q-$vector, whereas the conviction matrix Ω is defined as the diagonal matrix of $P\Sigma P'$

$$\Omega = \text{diag}(P\Sigma P') \tag{3.3}$$

where Ω is the diagonal matrix of forecast uncertainty. It can be interpreted that Ω gives guidelines to the amount of conviction one can place on a certain view given the current market covariance matrix Σ. A technical note: In calculations, we can hold Ω fixed and adjust the $q-$vector, or alternatively we can hold the $q-$vector fixed and increase or decrease the diagonal elements of the Ω matrix thus incorporating higher or lower forecast uncertainty. We choose the second alternative, by letting the information coefficient be a free parameter.

3.2.2 The Posterior Distribution of the Alphas

We combine the prior distribution of the alphas (3.1) and the views (3.2) to obtain the posterior distribution of the alphas.

Theorem

The mean vector of the posterior distribution of the alphas, denoted by α_{Bayes} is given by:

$$\alpha_{\text{Bayes}} = [\Sigma^{-1} + P'\Omega^{-1}P]^{-1}[P'\Omega^{-1}q] \tag{3.4}$$

Proof:

Let us consider assumptions (3.1) and (3.2) and re-write them in the following format

$$\begin{aligned} \alpha &= 0 + \epsilon \\ P \cdot \alpha &= q + v \end{aligned}$$

where $\epsilon \sim MVN(0, \Sigma)$ and $v \sim MVN(0, \Omega)$. Then we have the following model

$$Y = X(\alpha) + u$$

where $Y = \begin{pmatrix} 0 \\ q \end{pmatrix}$, $X = \begin{pmatrix} I \\ P \end{pmatrix}$, $u \sim MVN(0, V)$ and $V = \begin{pmatrix} \Sigma & 0 \\ 0 & \Omega \end{pmatrix}$

So using Generalised Least Squares

$$\alpha_{\text{Bayes}} = (X'V^{-1}X)^{-1}X'V^{-1}Y$$

By matrix manipulation we get

$$\begin{aligned} \alpha_{\text{Bayes}} &= \left[(I \quad P') \begin{pmatrix} \Sigma & 0 \\ 0 & \Omega \end{pmatrix}^{-1} \begin{pmatrix} I \\ P \end{pmatrix} \right]^{-1} (I \quad P') \begin{pmatrix} \Sigma & 0 \\ 0 & \Omega \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ q \end{pmatrix} \\ \alpha_{\text{Bayes}} &= \left[\left((\Sigma)^{-1} \quad P'\Omega^{-1} \right) \begin{pmatrix} I \\ P \end{pmatrix} \right]^{-1} \left((\Sigma)^{-1} \quad P'\Omega^{-1} \right) \begin{pmatrix} 0 \\ q \end{pmatrix} \\ \alpha_{\text{Bayes}} &= \left[(\Sigma)^{-1} + P'\Omega^{-1}P \right]^{-1} \left[P'\Omega^{-1}q \right] \end{aligned} \tag{3.5}$$

3.2.3 Optimal Scaling Factors

We are left to solve an optimisation problem, namely maximising the utility function below with respect to the active portfolio weights

$$\max \left(\alpha_P - \frac{\lambda_A}{2} \Psi_P^2 \right) = \max \left(w'_{PA} \alpha_{\text{Bayes}} - \frac{\lambda_A}{2} w'_{PA} \Sigma w_{PA} \right) \tag{3.6}$$

such that

$$w_{PA} \cdot \mathbf{1} = 0$$

where

- w_{PA} denotes the $n \times 1$ vector of active portfolio weights;
- λ_A is the investors aversion to active risk; and
- Ψ_P^2 is the tracking error variance of the active portfolio.

In other words we use the active weights as controls in the optimisation subject to the constraint that the active weights must sum to zero.

There is an alternative to maximising (3.6) in order to derive the optimal portfolio based on the refined α_{Bayes}. We do this by following the approach in He and Litterman [HL 1999] Appendix B. The solution presented there is to compute optimal scaling factor ϕ such that

$$w_{PA}^* = w_{eq} + P'\phi \qquad (3.7)$$

where

- w_{PA}^* is the solution to (3.6);
- w_{eq} are the benchmark active weights, note that $w_{eq} = 0$ since the manager holding the benchmark portfolio, has a risk-adjusted expected excess return of an asset over the benchmark of zero.

Since the columns of matrix P' are the long/short mini portfolios in the manager's view, the unconstrained optimal portfolio is the benchmark portfolio plus a weighted sum of the portfolios in the manager's views. The weights of these portfolios are given by the elements of the optimal scaling factor, the $k \times 1$ vector ϕ, where ϕ is given by the formula

$$\phi = \frac{\Omega^{-1}q}{\lambda_A} - [\Omega + P\Sigma P']^{-1} P\Sigma P' \frac{\Omega^{-1}q}{\lambda_A} \tag{3.8}$$

Hence, the optimal portfolio weights based on the refined alphas is given by

$$w^*_{PA} = P'\phi \tag{3.9}$$

with ϕ as in (3.8).

Equation (3.8) can be followed easily in He and Litterman [HL 1999] by setting τ to 1 and π to 0. That is because in active return space, equilibrium return vector is the zero vector and τ is arbitrarily set to 1 given that one should have strong confidence in one's covariance matrix, otherwise portfolio optimisation would be futile.

3.3 Demonstration of the Qualitative Forecasts Model

In order to illustrate the model, I will present the same nine asset example as the one I used in the Black-Litterman model. Figure (3.1) gives a breakdown of the asset classes used together with their respective market capitalisations in millions of Rand. The Qualitative Forecasts Model, takes into account the manager's views as a combinations of long/short portfolios and then performs the optimisation on the active weights. This is different from the Black-Litterman approach, which calculates a New Vector of Expected Returns from which we reverse optimise the portfolio weights. The "master" formula that we need use is given by equation (3.8). Let us examine in more detail the nature of the formula

$$\phi = \frac{\overbrace{\Omega^{-1}q}^{k\times 1}}{\lambda_A} - \underbrace{[\Omega + P\Sigma P']^{-1}}_{k\times k} \overbrace{P\Sigma P' \frac{\Omega^{-1}q}{\lambda_A}}^{k\times 1} \tag{3.10}$$

where we have k independent active bets in the form of k mini-long/short portfolios. The views thus can only be of the relative kind. We need to select bullish/bearish asset pairs.

Index All Share Economic Group	Code	Mkt cap in R mil	Mkt Cap Weights
JSE: Resources	J000	622,881.16	42.8599%
JSE:Basic Industries	J010	51,047.92	3.5126%
JSE: General Industry	J020	40,021.99	2.7539%
JSE: Cyclical Consumer	J030	99,462.51	6.8439%
JSE:Non-Cyclial Consumer	J040	118,767.34	8.1723%
JSE: Cyclical Services	J050	102,818.94	7.0749%
JSE:Non-Cyclical Services	J060	62,176.50	4.2783%
JSE:Financials	J080	343,053.38	23.6052%
JSE: IT	J090	13,065.19	0.8990%
Total		1453294.926	100%

Figure 3.1: *The nine assets used in the illustration of the qualitative forecasts model.*

- **View 1:** Bullish on JSE Financials and bearish on JSE Non-Cyclical Services.
- **View 2:** JSE Cyclical Consumer will outperform JSE Cyclical Services.
- **View 3:** Bullish on JSE IT and JSE Non-Cyclical Consumer vs JSE Basic and General Industry.

The *View Projection Matrix* is calculated as in the Black-Litterman model using the market capitalisation weighted method of He and Litterman (1999). Thus P becomes

$$P = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & -0.5605 & -0.4395 & 0 & 0.9009 & 0 & 0 & 0 & 0.0991 \end{bmatrix} \tag{3.11}$$

Notice that each of the rows of the P matrix sum to zero as we only allow relative views. The remainder of the inputs required for the formula in equation (3.10) are the covariance matrix Σ, the uncertainty matrix Ω and the alpha view vector q. Σ we calculate as in the Black-Litterman model, and Ω is the diagonal matrix defined by Herold Ulf to be $\Omega = diag(P'\Sigma P)$ which takes the form

$$\Omega = \begin{bmatrix} (p_1 \Sigma p_1') & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & (p_k \Sigma p_k') \end{bmatrix} \tag{3.12}$$

The difference between the uncertainty matrix here and the one used by Black-Litteman is that τ has been set to one because as I have shown in Chapter 2, the value of τ is in fact arbitrary. Lastly, the alpha view vector q is calculated by multiplying the Information Coefficient IC with the diagonal elements of the matrix $P'\Sigma P$. IC is kept a free parameter and it is an indication of the conviction of the manager. IC is defined to be the correlation between forecasts and subsequent realised returns, hence, a manager with no confidence will set an IC equal to zero and a manager with 100% confidence will set an IC equal to one. Depending on the size of IC one can take on more or less tracking error, so that could be another way of adjusting the value of IC. I have set a value of 0.2 for the subsequent calculation of ϕ and the optimal weights as calculated by the Qualitative Forecasts model, are illustrated in Figure (3.2).

Looking at Figure (3.2) we see that the benchmark weight (market capitalisation weight) for sector J000 is the same as the new optimal weight because we had not placed a view on that sector. View 1 is Bullish on J080 and bearish on J060. w_{new} on J060 has decreased from w_{mkt} by the same amount that w_{new} has increased on J080 from its w_{mkt}. If we compare the results of the Black-Litterman Model on the same nine asset example, we notice immediately that we are short J060 in the qualitative forecasts model. This can be explained as follows: The Views expressed in the two models differ slightly. The Qualitative Forecasts Model does not allow absolute views so **View 1** in the Qualitative case pairs up a view on J080 vs J060. In contrast, the Black-Litterman Model views had no view placed on J060, hence the asset allocation was Benchmark weight. View 2 and View 3 are

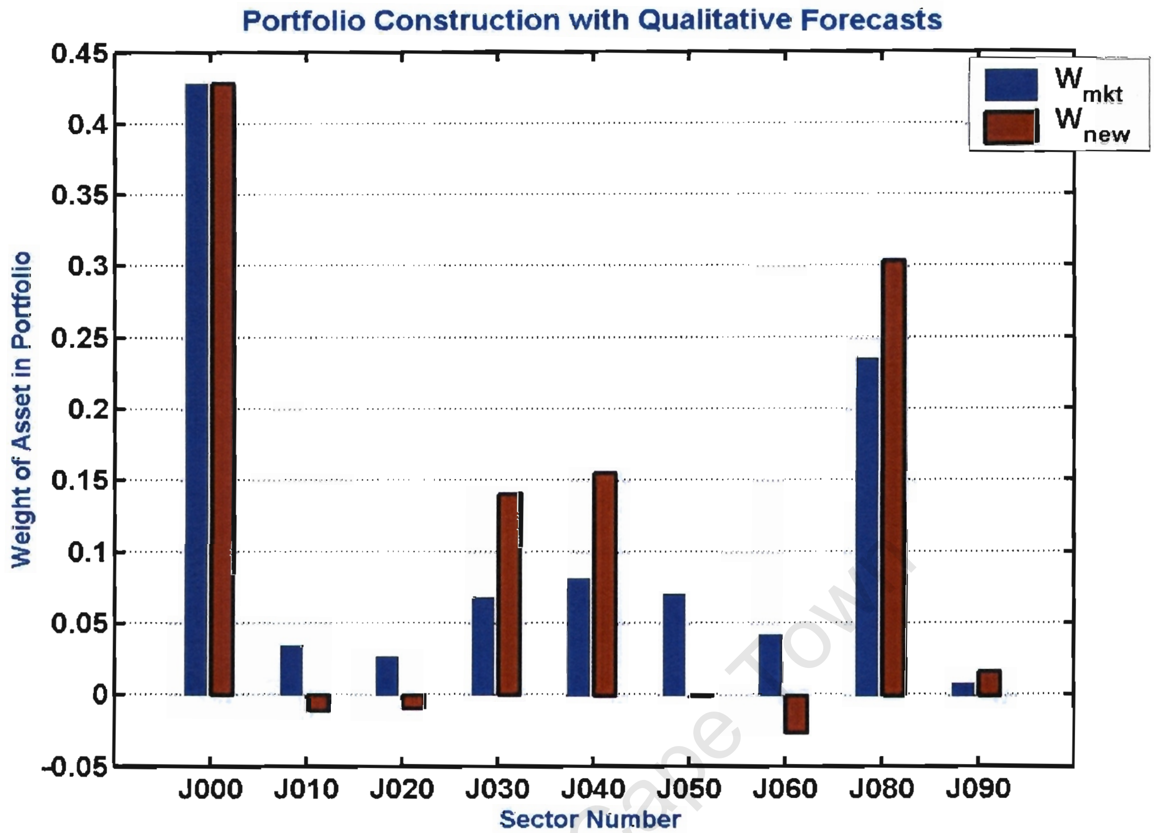

Figure 3.2: *Optimal portfolio weights as calculated by the Qualitative Forecasts model, with* $\lambda = 2$ *and* $IC = 0.2$

likewise, intuitively represented.

I have mentioned that it is possible to vary IC as a means to take on different tracking errors. I illustrate the effect of varing IC on the optimal portfolo weights in Figure (3.3). Note that the bigger the IC the bigger the departure from the benchmark weights. I use this idea when I construct tracker funds in the next chapter.

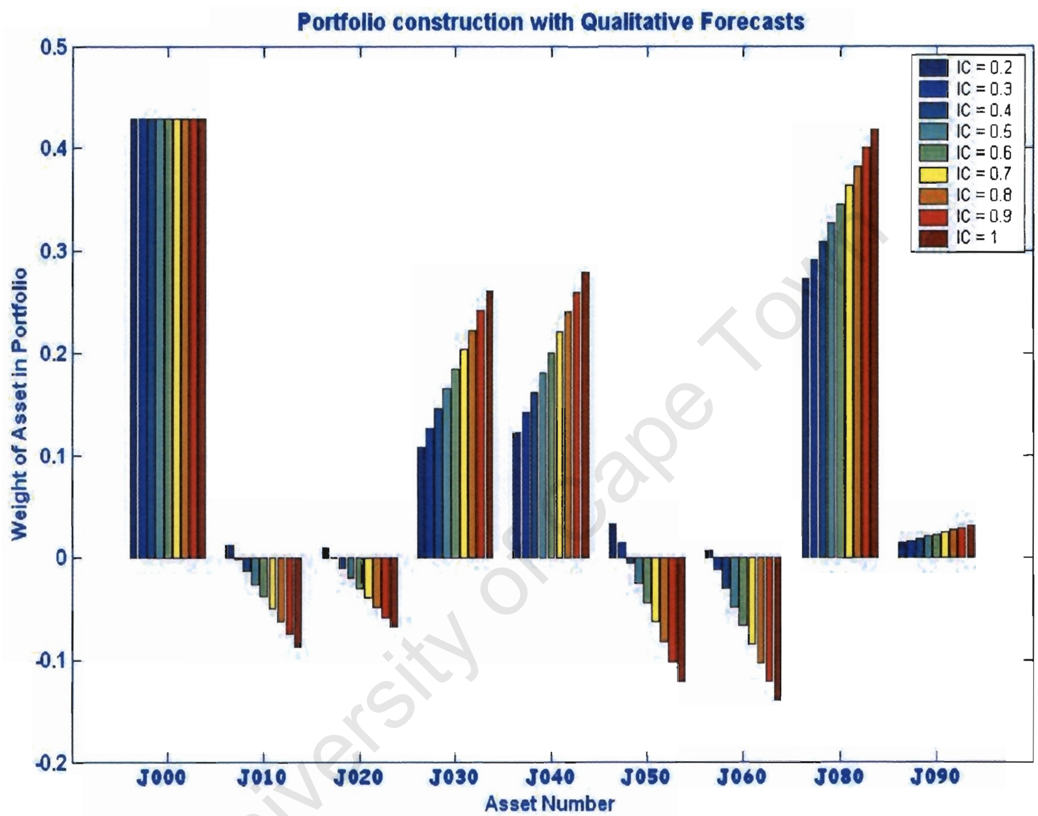

Figure 3.3: *The effect of varing IC on optimal portfolio weights*

Chapter 4

Transporting Alpha Across Benchmarks

4.1 Outline

We now have at our disposition two portfolio optimisation models, the Black-Litterman Model and the Qualitative Forecasts Model. They both allow the manager to input his views and re-calculate a set of optimal weights that take into account those views. The two models are similarly set in a Bayesian statistical framework but they differ in they way they process manager views. The Black-Litterman Model is a suitable portfolio construction tool for quantitative managers only, since the portfolio manager must attach a probability density function to each view. The Qualitative Forecasts Model, as the name suggests, is capable of processing purely qualitative inputs. This is useful to fundamental managers who only have directional views (bullish/bearish), and attach at best a level of conviction to them.

In this chapter I develop five Enhanced Index Tracker Funds that are based on assets taken from the ALSI Top40. An enhanced index tracking fund is a type of asset management strategy that is used to achieve superior returns. The majority of fund managers are assigned mandates which they have to manage with respect to a pre-defined benchmark. Their performance can be attributed foremost to good market bets and ability to maintain a low risk level. Fee structure, transaction costs, high turnover all contribute to drain away performance achieved through manager skill. Thus it is highly desirable for a manager to be able to successfully transport his views on the market into his portfolio in an optimal way that minimises overall active risk. The enhanced index tracker funds that I develop using my qualitative and quantitative asset allocation models are an ideal way to transport alpha across benchmarks. Suppose a manager is given a mandate which requires him to outperform a particular benchmark call it B. Suppose further that this particular manager has a good understanding of the market forces that drive the economy in his area of focus and he has performeed particularly well in that environment, i.e., he has been outperforming benchmark A. How does he translate those views that are generating positive alpha on benchmark A onto his new mandatory benchmark B? I outline one such answer in the work that follows in this chapter.

I use both the Black-Literman Model and the Qualitative Forecasts model to set up the enhanced tracker funds with the assets from the Top40 as benchmark. The demonstration is done over four years from March 2000 to March 2004. I discuss in detail the methodology required to set up such funds and analyse the performance of the five funds using standard portfolio diagnostic tools.

4.2 Data

The data consists of two main bodies

- Daily Closing Prices of the ALSI Top40 Assets from January 1997 till March 2004.

- Two sets of Quarterly Unit Trust Fund Data for the same period.

Details of the Unit Trust Data is proprietary to Cadiz Holdings and I am unable to discuss how such data is collected and calculated.

However, I use the Unit Trust Data to back out views for my enhanced tracker funds each quarter. It has been observed that one of the Unit Trusts Funds consistently outperforms the benchmark and so I reverse engineer quarterly expected returns and use that as inputs for my views. The data is manipulated in Excel and all the modelling is performed in Matlab.

4.2.1 Choice of Assets

The modelling to follow is intended to illustrate the strength of the two models given real market conditions. I could not include all the assets in the present Top40 in my portfolio, since I would have run into considerable difficulty with calculating a covariance matrix of expected returns. Firstly, some assets presently in the Top40, were not in the Top40 three or four years ago. The recent listing of Telkom is one such example. Secondly, some assets in the Top40 are strongly correlated and that leads to co-linearity between the columns of the covariance matrix. The covariance matrix is ill-conditioned. As a result we are no longer able to invert the covariance matrix, and also from a financial point of view, we lose diversification potential when we have strongly correlated assets. Hence I chose the 29 shares shown in Figure (4.1) from the Top40 as a basket of shares that will make up my benchmark and will be used to construct the five enhanced tracker funds.

Shares	Type	JSE Economic Group
ABI	Soft Drinks	Non Cyclical Consumer Goods
AGL	Other Mineral	Resources
AMS	Platinum	Resources
ANG	Gold Mining	Resources
ASA	Banks	Financials
AVG	Gold Mining	Resources
BAW	Diversified Industrials	General Industrials
BIL	Other Mineral	Resources
BVT	Business Support Services	Cyclical Services
FSR	Banks	Financials
GFI	Gold Mining	Resources
HAR	Gold Mining	Resources
IMP	Platinum	Resources
IPL	Diversified Industrials	General Industrials
NED	Banks	Financials
NPK	Business Support Services	Cyclical Services
NPN	Cable and Satellite	Cyclical Services
NTC	Hosp Mng LTC	Non Cyclical Consumer Goods
PIK	Food and Drug Retailers	Non Cyclical Services
PPC	Building and Construction Materials	Basic Industries
RCH	Household Appliances and Housewares	Cyclical Consumer Goods
RMH	Banks	Financials
SAB	Beverages Brewers	Non Cyclical Consumer Goods
SAP	Paper	Basic Industries
SBK	Banks	Financials
SHF	Furnishings and Floor Coverings	Cyclical Consumer Goods
SLM	Life Assuranc	Financials
SOL	Oil Integrated	Resources
TBS	Food Processors	Non Cyclical Consumer Goods

Figure 4.1: *The basket of shares used to develop enhanced tracker funds together with the repective economic sector.*

4.3 Methodology

The objective is to set up a benchmark index from the basket of shares,MyTop30, selected for my funds. I will observe the performance of this basket over the period modelled, namely March 2000 to March 2004. I will then set up five enhanced tracker funds, three based on the Qualitative Forecasts Model and two based on the Black-Litterman Model. These funds will take on tracking error of 1%, 2% and 3% for the Qualitative Model and 1% and 2% for the Black-Litterman Model. The funds will be named QM1, QM2, QM3, BL1 and BL2 respectively. For each of these funds I have to calculate the covariance matrix of returns Σ, the view projection matrix P, and then separately the view vector Q for each model and the uncertainty matrix Ω. With these inputs I run the optimisation in such a way that the tracking error is constrained at each re-balancing of the weights.

4.3.1 Benchmark Indexing

I invest R100 in the MyTop30 basket in such a way that the weights are market capitalisation weighted and the rebalancing occurs monthly. I calculate the number of shares I have at the beginning and I allow for fractional shares. At month end I calculate the value of the basket of shares and using the market cap weighs at that time I re-calculate the new number of shares that I can hold for the following month. This type of boot-strapping method indexes MyTop30 and provides me with the benchmark. Below I display the Matlab code I used to perform the indexing.

```
IV=100;
ITop30=[];
 for i=1:49
  MWmkt(:,i)=MMktCaps(:,i)./sum(MMktCaps(:,i));
 end
 for i=1:48
     ITop30(i)=IV;
     NS(:,i)=(MWmkt(:,i)*ITop30(i))./MCPrices(:,i);
     IV=NS(:,i)'*MCPrices(:,i+1);
 end
```

Where

- IV is the initial value of investment of R100;
- ITop30 is the value of the index;
- MWmkt are the monthly market cap weights normalised to 1. They are stored as a 29×49 matrix, with each column representing a month end starting at March 2000;

- NS is the number of shares held during this period. It is stored as a 29×48 matrix, again with each column representing the number of shares to be held till the following month end; and
- MCPrices is an array that stores the Monthly Closing Prices for these shares.

The Monthly Closing Prices and the Monthly Market Cap Weights are extracted from the Daily Closing Prices and Number of Shares in Issue data and manipulated in Excel using pivot tables. The data is then arranged to be imported into Matlab where all the calculations are done. The MyTop30's performance can be seen in Figure (4.2). Note that we started with R100; the minimum value was R87,89 and the maximum value was R167,50.

4.3.2 Calculation of Inputs

Let us recall the two operational formulas that we need in order to run our models: The Black-Litterman formula is

$$E[R] = \left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\pi + P'\Omega^{-1}Q\right] \tag{4.1}$$

and the Scaling Factor for the Qualitative Forecasts Model is

$$\phi = \frac{\Omega^{-1}q}{\lambda_A} - [\Omega + P\Sigma P']^{-1}P\Sigma P'\frac{\Omega^{-1}q}{\lambda_A} \tag{4.2}$$

The Black-Litterman formula has as its output the New Vector of Expected Returns $E[R]$ from which we reverse engineer optimal weights. The Qualitative Forecasts Model calculates a scaling factor such that the new optimal weights are given by the View Projection Matrix P multiplied by the Optimal Scaling Factor ϕ. Thus it is sufficient to compute equations (4.1) and (4.2) to be able to set up the enhanced tracker funds.

The common inputs to these formulas are Σ and P. The two models differ in the way that views are entered. The Black-Litterman Model requires a probability density function and

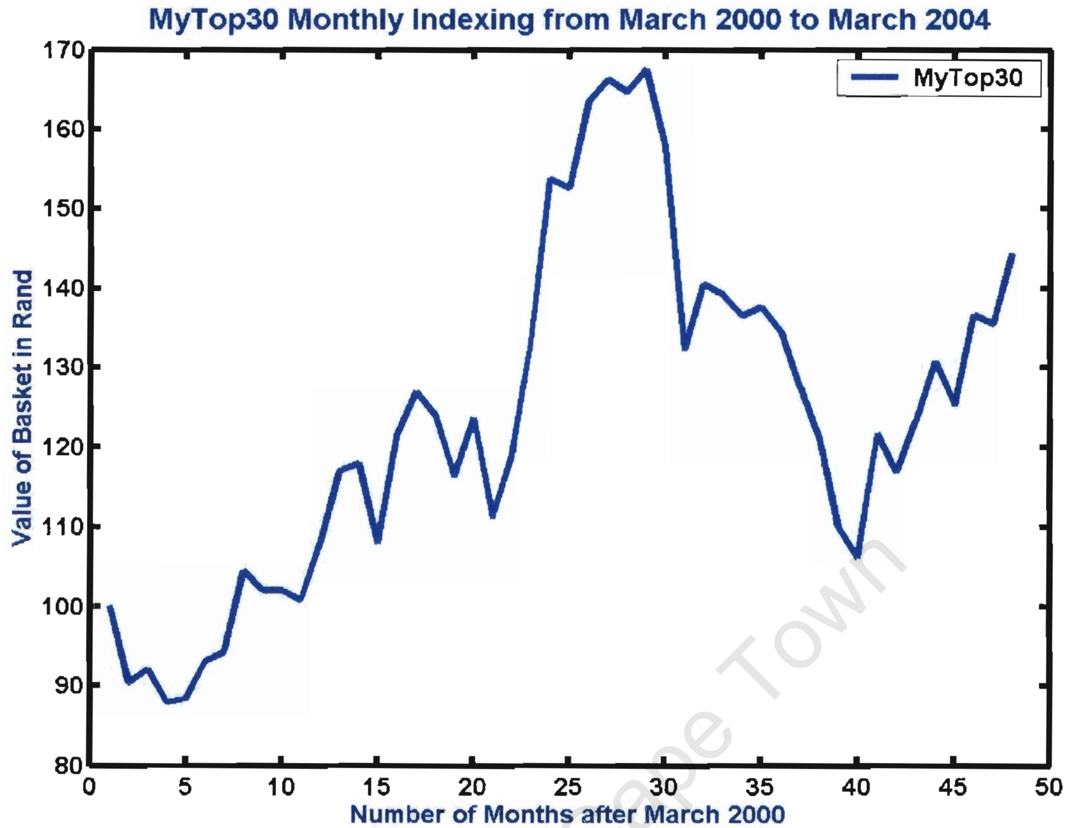

Figure 4.2: *Performance of the MyTop30 Basket of Shares*

a point forecast, to be stated for each view. The Qualitative Forecasts Model requires only directional views and they are entered as a product of the Information Coefficient multiplied by the tracking error. Hence all that is required of the manager is to give an estimate of his IC. This can be worked out via the Information Ratio, or in our case we constrain the tracking error so IC is automatically chosen. Thus in reality the manager need only set up long-short pairs of assets on which he has views. In the sections that follow I give a detailed demonstration of how the inputs are calculated and then how the models are run to set up the enhanced tracker funds.

Covariance Matrix of Returns Σ

The Covariance Matrix of Returns is calculated using Weekly Closing Prices. Since the optimisation is run every quarter, Σ is re-calculated every quarter. This is done by having a rolling window of 65 Weekly Closing Prices, and each quarter 12 weeks of data is added and the oldest 12 weeks of data is removed. I set up the data in Excel and import it into Matlab where the covariance matrix is calculated. The code below performs that function.

```
%This sets up a multi dimensional array holding the weekly closing prices for the basket of stocks

ClosingPrices=[];
for i = 1:65
    for j = 1:29

        ClosingPrices(i,j,1)=RetM00(i,j);
        ClosingPrices(i,j,2)=RetJ00(i,j);
        ClosingPrices(i,j,3)=RetS00(i,j);
        ClosingPrices(i,j,4)=RetD00(i,j);
        ClosingPrices(i,j,5)=RetM01(i,j);
        ClosingPrices(i,j,6)=RetJ01(i,j);
        ClosingPrices(i,j,7)=RetS01(i,j);
        ClosingPrices(i,j,8)=RetD01(i,j);
        ClosingPrices(i,j,9)=RetM02(i,j);
        ClosingPrices(i,j,10)=RetJ02(i,j);
        ClosingPrices(i,j,11)=RetS02(i,j);
        ClosingPrices(i,j,12)=RetD02(i,j);
        ClosingPrices(i,j,13)=RetM03(i,j);
        ClosingPrices(i,j,14)=RetJ03(i,j);
        ClosingPrices(i,j,15)=RetS03(i,j);
        ClosingPrices(i,j,16)=RetD03(i,j);
        ClosingPrices(i,j,17)=RetM04(i,j);

    end
end
%here we calculate the Covariance matrix relevant to each of the quarters
Returns = [];
Sigma=[];
Determinants=[];
for ii=1:17
    for i = 2:65
```

```
        for j = 1:29
            Returns(i-1,j,ii)=(ClosingPrices(i-1,j,ii)-ClosingPrices(i,j,ii))/ClosingPrices(i,j,ii);
        end
    end
    Sigma(:,:,ii)=cov(Returns(:,:,ii));
end
```

where

- Closing Prices are stored as a $65 \times 29 \times 17$ multiple array with each layer representing one of the 17 quarters, having 65 weekly closing prices for the 29 shares;
- Returns are calculated based on these Closing Prices and stored in a $64 \times 29 \times 17$ multiple array; and
- Finally, Sigma is calculated from these returns and stored as a $29 \times 29 \times 17$ multiple array with each layer representing the covariance matrix relevant to that quarter.

View Projection Matrix P

The calculation of the View Projection Matrix P is perhaps the most interesting. It relies heavily on our type of views. Since P is a $k \times n$ matrix where we have k views on n assets, the computation depends on how one decides to incorporate views on the basket of stocks. I will thus outline the method I used to back out views from the Unit Trust Fund Data and then the method used to incorporate these views into my choice of P.

Backing out Views from Unit Trust Fund Data

The Unit Trust Data is made up of two funds, A and B. Fund A is an outperforming fund and Fund B is a consensus fund. I have quarterly data which comprises the weights allocated to shares in each of these funds. I make the following assumptions

- First, since Fund A is not a static fund but each quarter it is made up of outperforming assets we can assume that such a fund will not deteriorate greatly in performance over a three month period. Proprietary reseach has been used in constructing these funds. In this study I used the funds as supplied by Cadiz Financial Strategists. The views we back out from Fund A are mostly valid for the next quarter; and

- Second, the published quarterly weight allocations in both Fund A and Fund B are assumed to be the weights that are in effect for the next quarter or at least next month.

Utilising these two assumptions I proceed in reverse engineering expected returns each quarter on the shares that I hold in MYTop30 basket. The unconstrained Markowitz optimisation (reference Markowitz) yields

$$\begin{aligned} E[R_A](t) &= \lambda\Sigma(t)(w_A)_t \\ E[R_B](t) &= \lambda\Sigma(t)(w_B)_t \end{aligned}$$

where

- $E[R_A](t)$ is the expected return on the assets in MyTop30 basket at time t;

- λ is the risk aversion coefficient;

- $\Sigma(t)$ is the covariance matrix of returns at time t; and

- $(w_A)_t$ is the weight distribution vector in Fund A at time t

and therefore the View Vector at time t is the difference between these two expected returns, namely

$$Q(t) = E[R_A](t) - E[R_A](t)$$

The code below illustrates the procedure.

```
% Back out views from Unit Trust Fund Data
ER1=[];
ER2=[];
% W1 and W2 are imported quarterly from Fund A and Fund B data
for i=1:16
    W1adj(:,i)=W1(:,i)./sum(W1(:,i)); % normalise to 1
    W2adj(:,i)=W2(:,i)./sum(W2(:,i));
end
% Calculate Expected Returns Quarterly
for i = 1:16
    ER1(:,i) = Lambda*Sigma(:,:,i)*W1adj(:,i);
    ER2(:,i) = Lambda*Sigma(:,:,i)*W2adj(:,i);
end
% Finally calculate View Vector Q - 29x16 matrix
% each column represents the view vector relevant for that quarter
for j=1:16
    que(:,j)= ER1(:,j)-ER2(:,j);
end
```

We now have the difference in expected return on each share in the MyTop30 basket every quarter. One now has to decide as to how best to incorporate these differences into the two models. I decided to group the shares into their economic sectors and thus have views on each sector. The shares in MyTop30 basket are grouped into eight economic sectors, thus each quarter I will have eight views. For the Black-Litterman Model the views have to be of a quantitative nature and I shall demonstrate the methodology in the section below. The Qualitative Forecasts Model only requires directional views and so we merely look at the relative views in each sector. This particular methodology is outlined in a subsequent section. The nature of the inputs of the View Vector affect the View Projection Matrix and I will show how I calculate P for the current scenario. The View Projection Matrix is the same for both models, because the views on sectors are expressed, either quantitatively or qualitatively but P remains a 8×29 market cap weighted matrix.

View Vector Q and the View Projection Matrix P for the Black-Litterman Model

I will demonstrate the methodology for assigning views to sectors by considering data for the first quarter of our simulation. The views are backed out using unit trust data available at the 31 March 2000. The views Q, the view projection matrix P and the covariance matrix Σ are calculated with data available on the 31 March 2000 and are used to set up the tracker funds for the next quarter until 30 June 2000. In the interim period, closing price data is used to track the performance of the funds but Q, P, and Σ are only adjusted quarterly, as new unit trust fund data becomes available.

Figure (4.3) contains information that is backed out of the two funds in March 2000. The shares present in the basket are numbered and allocated to their economic sector. However, in order to impose views on each sector we have to sort the information according to that sector. Therefore Figure (4.4) will be the starting point of the demonstration.
Let us now consider an even smaller subset of Figure (4.4), namely the Resources Sector together with the corresponding market capitalisations. The Market Capitalisation and Relative Market Capitalisation columns are used for the View Projection Matrix and the Vector of Expected Returns column (E[R]) is used for the View Vector Q. Note that we are now looking at data from the first quarter (March 2000), sector eight, thus what we are calculating is row eight of the first P matrix and the $(8, 1)$ entry of the View Vector Q. Figure (4.5) tabulates the eight shares that fall in the resources sector. The second column shows us the asset number of each share in the MyTop30 basket. The ordering is important because the P matrix has to be standardised and also the covariance matrix Σ is calculated with the assets in the order shown in Figure (4.3). Standardising the P matrix means that the asset order remains constant so that the Matlab routine that picks

Share	Asset Nr	Sector	Sector name	E[R] Diff	Mkt Cap
ABI	1	6	Non Cyclical Consumer Goods	-1.3463E-05	6.3439E+09
AGL	2	8	Resources	1.6302E-04	1.2319E+11
AMS	3	8	Resources	-4.4127E-06	3.7599E+10
ANG	4	8	Resources	2.3068E-04	3.3468E+10
ASA	5	4	Financials	-3.0614E-04	1.5500E+10
AVG	6	8	Resources	3.3015E-04	1.8250E+09
BAW	7	5	General Industrials	-1.9304E-04	9.2796E+09
BIL	8	5	General Industrials	1.6716E-04	6.5851E+10
BVT	9	3	Cyclical Services	-1.3841E-04	1.4474E+10
FSR	10	4	Financials	-7.1519E-05	4.3018E+10
GFI	11	8	Resources	5.2649E-04	1.0334E+10
HAR	12	8	Resources	5.2189E-04	3.0524E+09
IMP	13	8	Resources	-1.0879E-05	1.4959E+10
IPL	14	5	General Industrials	-1.6189E-04	1.1722E+10
NED	15	4	Financials	-1.6043E-04	3.1592E+10
NPK	16	3	Cyclical Services	-4.0542E-05	7.7367E+09
NPN	17	3	Cyclical Services	-3.4817E-04	1.1647E+10
NTC	18	6	Non Cyclical Consumer Goods	-9.7806E-05	1.3725E+09
PIK	19	7	Non Cyclical Services	-6.5071E-05	4.7009E+09
PPC	20	1	Basic Industries	-2.9142E-05	2.5990E+09
RCH	21	2	Cyclical Consumer Goods	1.0785E-05	8.5608E+10
RMH	22	4	Financials	-3.6122E-05	1.0553E+10
SAB	23	6	Non Cyclical Consumer Goods	-9.5172E-05	3.8684E+10
SAP	24	1	Basic Industries	1.6702E-05	1.2097E+10
SBK	25	4	Financials	-7.2186E-05	3.7138E+10
SHF	26	2	Cyclical Consumer Goods	9.7113E-06	4.5428E+09
SLM	27	4	Financials	-1.6844E-04	2.2059E+10
SOL	28	8	Resources	-6.6430E-06	2.4755E+10
TBS	29	6	Non Cyclical Consumer Goods	-1.4464E-04	1.1248E+10

Figure 4.3: *The table contains the vector of expected returns for the basket of shares in March 2000*

up the views is the same for any period of analysis. Columns three and four are redundant for the immediate discussion but they show the sector number and the sector name. The sector number dictates the view number. Column five indicates the difference in expected returns for the eight shares in the resources sector backed out from unit trust fund data for March 2000. Column six shows the market capitalisations of each of the shares in March 2000. Column seven displays the relative market capitalisations of the shares with respect to the relevant subtotals. It can be seen that the table in Figure (4.5) separates the shares that have a positive expected return differential from those with a negative expected return differential. This is because the construction on the P matrix demands that each row that contains a relative view (nominally outperforming assets vs nominally underperforming assets) must add up to zero. Hence the relative market capitalisations in column seven are for the nominally outperforming shares and nominally underperforming relative to their

Share	Asset Nr	Sector	Sector name	E[R] Diff	Mkt Cap
PPC	20	1	Basic Industries	-2.9142E-05	2.5990E+09
SAP	24	1	Basic Industries	1.6702E-05	1.2097E+10
RCH	21	2	Cyclical Consumer Goods	1.0785E-05	8.5608E+10
SHF	26	2	Cyclical Consumer Goods	9.7113E-06	4.5428E+09
BVT	9	3	Cyclical Services	-1.3841E-04	1.4474E+10
NPK	16	3	Cyclical Services	-4.0542E-05	7.7367E+09
NPN	17	3	Cyclical Services	-3.4817E-04	1.1647E+10
ASA	5	4	Financials	-3.0614E-04	1.5500E+10
FSR	10	4	Financials	-7.1519E-05	4.3018E+10
NED	15	4	Financials	-1.6043E-04	3.1592E+10
RMH	22	4	Financials	-3.6122E-05	1.0553E+10
SBK	25	4	Financials	-7.2186E-05	3.7138E+10
SLM	27	4	Financials	-1.6844E-04	2.2059E+10
BAW	7	5	General Industrials	-1.9304E-04	9.2796E+09
BIL	8	5	General Industrials	1.6716E-04	6.5851E+10
IPL	14	5	General Industrials	-1.6189E-04	1.1722E+10
ABI	1	6	Non Cyclical Consumer Goods	-1.3463E-05	6.3439E+09
NTC	18	6	Non Cyclical Consumer Goods	-9.7806E-05	1.3725E+09
SAB	23	6	Non Cyclical Consumer Goods	-9.5172E-05	3.8684E+10
TBS	29	6	Non Cyclical Consumer Goods	-1.4464E-04	1.1248E+10
PIK	19	7	Non Cyclical Services	-6.5071E-05	4.7009E+09
AGL	2	8	Resources	1.6302E-04	1.2319E+11
AMS	3	8	Resources	-4.4127E-06	3.7599E+10
ANG	4	8	Resources	2.3068E-04	3.3468E+10
AVG	6	8	Resources	3.3015E-04	1.8250E+09
GFI	11	8	Resources	5.2649E-04	1.0334E+10
HAR	12	8	Resources	5.2189E-04	3.0524E+09
IMP	13	8	Resources	-1.0879E-05	1.4959E+10
SOL	28	8	Resources	-6.6430E-06	2.4755E+10

Figure 4.4: *The same content as in the previous table, only now the shares are sorted according to their economic sectors*

market capitalisation subtotal. Note that each of the relative subtotals add up to one. Thus the work to evaluate row eight of the first P matrix is done. The entries have the numerical value of the relative market capitalisation column. The entries are positive for the nominally outperforming shares, namely AGL, ANG, AVG, GFI, HAR and negative for the nominally underperforming shares AMS, IMP, SOL. The value of the asset number found in column two indicates the position of the entry in the eighth row of the first P matrix. Thus $P(8,2,1)$ where the 8 indicates the eighth row, the 2 indicates the second column and the 1 indicates that this is the first P matrix, i.e., the one that is relevant for the first quarter of our simulation, March 2000 is the entry for AGL from column seven, namely 0.716765. This is best illustrated in Figure (4.6) where I have transposed row eight of the first P matrix to show how the entries are placed. Note the position of the non-zero entries. The assets used in view eight are in bold face. I have boxed the

Share	Asset Nr	Sector	Sector name	E[R] Diff	Mkt Cap	Rel Mkt Caps
AGL	2	8	Resources	1.6302E-04	1.2319E+11	0.716765
ANG	4	8	Resources	2.3068E-04	3.3468E+10	0.194730
AVG	6	8	Resources	3.3015E-04	1.8250E+09	0.010618
GFI	11	8	Resources	5.2649E-04	1.0334E+10	0.060127
HAR	12	8	Resources	5.2189E-04	3.0524E+09	0.017760
Subtotal					**1.7187E+11**	**1.000000**
AMS	3	8	Resources	-4.4127E-06	3.7599E+10	0.486323
IMP	13	8	Resources	-1.0879E-05	1.4959E+10	0.193490
SOL	28	8	Resources	-6.6430E-06	2.4755E+10	0.320187
Subtotal					**7.7314E+10**	**1.000000**

Figure 4.5: *The Resources Sector with the corresponding market capitalisations*

nominally underperforming assets (numbers 3, 13, and 28) for easy comparison with the table in Figure (4.5).

The $(8, 1)$ entry of the View Vector Q remains to be calculated for the eighth view. That entry is simply the straight sum of the entries in column five of Figure (4.5). In other words the expected return differentials for the assets are added and combined to get an expected return differential for the sector. The Q vector for the first quarter is illustrated in Figure (4.7) and contains the entry calculated in the eighth row.

The method outlined so far is repeated quarterly for each economic sector, producing sixteen 8×29 View Projection Matrices and an 8×16 matrix Q containing the View Vector. The code below that achieves this task is performed in Matlab with data imported from Excel.

```
%  View Projection Matrix P and View Vector Q are now Assembled   %

n=length(Asset_Nr); % scales the matrix according to number of assets
% n=29 in this case
k=max(Sector); % gets the number of views to be had
% k = 8
P=zeros(k,n); %Creates an 8 by 29 matrix to stand in for the
% View Projection Matrix
Q=[];
```

Shares	Asset Nr	Row 8 transposed
ABI	1	0
AGL	**2**	**0.716764544**
AMS	**3**	**-0.486323239**
ANG	**4**	**0.19472971**
ASA	5	0
AVG	**6**	**0.010618323**
BAW	7	0
BIL	8	0
BVT	9	0
FSR	10	0
GFI	**11**	**0.0601274**
HAR	**12**	**0.017760023**
IMP	**13**	**-0.193489626**
IPL	14	0
NED	15	0
NPK	16	0
NPN	17	0
NTC	18	0
PIK	19	0
PPC	20	0
RCH	21	0
RMH	22	0
SAB	23	0
SAP	24	0
SBK	25	0
SHF	26	0
SLM	27	0
SOL	**28**	**-0.320187135**
TBS	29	0

Figure 4.6: *The Eighth Row of the First P matrix, transposed*


```
Perf=[];
MktRet=[];
Perf1=[];
MktRet1=[]; % Preamble
for j = 1:16 %j stand for number of quarters
    QU=que(:,j);    % que the expected return differential on each asset
     % as calculated earlier
    Mkt_Cap=Mkt_Cap_Final(:,j); % selects the Mkt Cap applicable to that quarter
% calculate each row of the P matrix and each entry on the Q vector view by view
for i = 1:k
        View=[];
        V=[];
        Mcap=[];
```

View Nr	Q Vector for the First Quarter
1	-1.24395E-05
2	2.04958E-05
3	-5.27122E-04
4	-8.14838E-04
5	-1.87774E-04
6	-3.51080E-04
7	-6.50714E-05
8	1.75029E-03

Figure 4.7: *View Vector Q , for the first quarter*

```
        Wmkt=[];
        blah=[];
        Mkt_Cap1=[];
        Mkt_Cap2=[];
        Summ1=[];
        signcheckneg=[];
        signcheckpos=[];
        p=[];
        View=find(Sector==i);
        V=Asset_Nr(find(Sector==i));  %finds the relevant assets involved in the view
        signcheckneg=find(QU(View)<0); %finds relevant Exp Returns pos or neg
        signcheckpos=find(QU(View)>0);
        Vneg=V(signcheckneg);
        Vpos=V(signcheckpos);
        Mcapneg=Mkt_Cap(Vneg);
        Mcappos=Mkt_Cap(Vpos);
        Wmktneg=Mcapneg./sum(Mcapneg); %Calculates relative Mkt Caps
        Wmktpos=Mcappos./sum(Mcappos);
        p=zeros(1,n);
        p(V(signcheckneg))=-Wmktneg; %Puts in the negative entries
        p(V(signcheckpos))=Wmktpos;  % and teh positive ones
        P(i,:,j)=p ;  % Row i is now complete
        findque=QU(View);
        Q(i,j)=sum(findque); %Q entry is calculated
    end
end
```

View Vector Q for the Qualitative Forecasts Model

The View Projection matrix for the Qualitative Forecasts Model is the same as for the Black-Litterman Model, except in the way the views are entered. The Qualitative Forecasts Model requires as inputs directional views such as bullish on AVG and bearish on HAR and stores the views in a column vector. The column vector is calculated as being the information coefficient IC times the diagonal elements of the matrix $P'\Sigma P$. The projection matrix selects the views, bullish or bearish, by simply looking at the sign of the expecter return differential forecast for each asset. Thus the qualitative views are only entered in the P matrix and IC simply selects the amount of tracking error that the manager wishes to take on. At this stage there is no difference in the method used to calculate the necessary inputs for the two models. In the sections to follow I construct enhanced tracker funds using the inputs calculated so far and the formulae in equations (4.1) and (4.2).

4.3.3 Building Enhanced Tracker Funds

Tracking Error

Portfolio performance is usually evaluated against a prespecified benchmark portfolio. In our case the benchmark is the MyTop30 basket of shares. One of the most frequently used measures is tracking error (TE), defined as the standard deviation of the difference between portfolio returns and the benchmark portfolio returns. Two common sources of TE arise from attempts to outperform the benchmark and the passive replication of the benchmark by a sampled portfolio. There are two different measures of tracking error: *ex ante* and *ex post*. Ex post tracking error is usually larger than ex ante tracking error. The first measure for TE is simply the standard deviation of difference between portfolio returns and the benchmark portfolio returns. Satchell and Hwang argue the case that ex ante and ex post tracking errors must necessarily differ, since portfolio weights are ex post stochastic in nature. Their results imply that fund managers always have a higher ex post tracking error than their planned tracking error. When I constructed the enhanced tracker funds to take on a certain (ex ante) tracking error, the realised tracking error (ex post) was slightly larger as described above.

Satchell and Hwang [SH 2001] show that when the difference between portfolio weights and the benchmark portfolio weights is stochastic, ex ante tracking error is on average downward biased. The results imply that again the realised tracking error is typically larger than the planned tracking error.

Formally, let r_t be a vector of rates of return at time t with mean vector θ and covariance matrix Σ. Let the active portfolio weights at time t be the vector a_t and the benchmark weights be the vector b_t. Then

$$\begin{aligned} TE_t &= \sqrt{\text{var}(a_t'r_t - b_t'r_t)} \\ &= \sqrt{(a_t - b_t)'\text{cov}(r_t)(a_t - b_t)} \\ &= \sqrt{(a_t - b_t)'\Sigma(a_t - b_t)} \end{aligned}$$

Let the portfolio weight be w_t, where $w_t = a_t - b_t$ then

$$TE_t = \sqrt{w_t'\Sigma w_t} \tag{4.3}$$

It is well understood that w_t, which is assumed non-stochastic ex ante, will be stochastic ex post. Since ex post TE is computing TE from the actual portfolio returns, r_{p_t}, where $r_{p_t} = w_{t-1}'r_t$, then a time series calculation of TE would involve, over a period from $t = 1, \ldots, T$, the terms $w_0'r_1, w_1'r_2, \ldots, w_{T-1}'r_T$. Conclusions about forecast failure arrise when we compare ex ante TE given in (4.3) with ex post TE given by

$$\widehat{TE} = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(r_{p_t} - \bar{r}_p)^2} \tag{4.4}$$

Satchell and Hwang show that calculations based on treating portfolio weights as fixed will, on average, underestimate ex post tracking error over a historical period if the weights are not kept fixed. For example, if we take a particular fund, which in our case would be the enhanced tracker fund that I constructed, and compute its monthly rate of return r_{p_t} and *then* calculate the tracking error/variance over a a period, say T $(t = 1, ..., T)$, where the weights have not been rebalanced monthly prior to reporting the returns, we should expect underestimation of the actual tracking error.

Enhanced Index Tracking Funds

I have constructed five enhanced tracker funds which use the MyTop30 Basket as the benchmark. The active weights for the different enhanced tracker funds I calculate using both the Black-Litterman Model and the Qualitative Forecasts Model. The views are calculated quarterly for the period March 2000 to March 2004 using the method outlined earlier. The funds are constrained to take on different tracking errors of 1%, 2% and 3% in the case of the Qualitative Forecasts model and 1% and 2% for the Black-Litterman Model. The enhanced tracker funds are called QM1, QM2, QM3, BLM1 and BLM2 respectively. The Matlab code that implements the tracking error constraint while optimising the portfolio weights for the Black-Literman model is shown below.


```
TETarget=input('please input target tracking error TE = '); % Set the target tracking error

for i=1:16
    Wmkt(:,i) = Mkt_Cap_Final(:,i)./sum(Mkt_Cap_Final(:,i));

    delta = 0.001;
    while abs(TETarget-TEBL(i))>0.001
        delta=delta+0.001;
    OmegaDiag=diag(P(:,:,i)*Sigma(:,:,i)*P(:,:,i)')*tau*delta;
     for jj=1:k
            Omega(jj,jj,i)=OmegaDiag(jj);
     end
L(:,i) = tau*inv(Omega(:,:,i))*Q(:,i)/Lambda-inv(Omega(:,:,i)/tau

         +P(:,:,i)*Sigma(:,:,i)*P(:,:,i)')*P(:,:,i)*Sigma(:,:,i)*Wmkt(:,i)
```

```
        -inv(Omega(:,:,i)/tau

        +P(:,:,i)*Sigma(:,:,i)*P(:,:,i)')*P(:,:,i)*Sigma(:,:,i)*P(:,:,i)'

        *tau*inv(Omega(:,:,i))*Q(:,i)/Lambda;

    Wblraw(:,i) =  Wmkt(:,i)+P(:,:,i)'*L(:,i);
    Summ2=sum(Wblraw(:,i));
    Wbl(:,i)=Wblraw(:,i)./Summ2;

TEBL(i)=sqrt((Wbl(:,i)-Wmkt(:,i))'*Sigma(:,:,i)*(Wbl(:,i)-Wmkt(:,i)));
    end
end
```

The algorithm works by re-adjusting the Uncertainty matrix Ω until the (ex ante) tracking error is achieved to the prespecified tolerance level. This is done quarterly because the views and the covariance matrix change oever that interval. Thereafter the optimal portfolio weights are calculated using the Black-Litterman model. For the case where the target tracking error is set to be 1% we get the BLM1 enhanced tracker fund.

In a similar fashion the Qualitative Forecasts model funds QM1 to QM3 are calculated by varing λ until the target tracking error is achieved. This is done quarterly so that each quarter the funds take on the prespecified tracking error. The Matlab Code below achieves this task.

```
TETarget=input('please input target tracking error TE = ');
% Set the target tracking error
for j=1:16
    Wmkt(:,j) = Mkt_Cap_Final(:,j)./sum(Mkt_Cap_Final(:,j));
    IC=0.001;
        while abs(TETarget-TEQ(j))>0.0001
        % Adjust IC until the tracking error falls within limits
        IC=IC+0.001;
        Q=IC*OmegaDiag;
        Summ2=[];
% Work out the new weights via the Optimal scaling factor Phi
Phi(:,j)=(inv(Omega(:,:,j))*Q)/Lambda - inv(Omega(:,:,j)
```

```
        + P(:,:,j)*Sigma(:,:,j)*P(:,:,j)')*(P(:,:,j)
    *Sigma(:,:,j)*P(:,:,j)'*inv(Omega(:,:,j))*Q/Lambda);
    Wactive(:,j) = P(:,:,j)'*Phi(:,j);
    Wadj(:,j)=(Mkt_Cap_Final(:,j)./sum(Mkt_Cap_Final(:,j)))+Wactive(:,j);
        % Normalise the weights to 1

Summ2=sum(Wadj(:,j));
Wquali(:,j)=Wadj(:,j)./Summ2;
    TEQ(j)=sqrt((Wquali(:,j)-Wmkt(:,j))'*Sigma(:,:,j)*(Wquali(:,j)-Wmkt(:,j)));
end
end
```

Chapter 5

Results and Discussion of the Portable Alpha Technique

5.1 Results

The results for the portable alpha technique are presented in this section. Figures (5.1) to (5.6) display the results for the QM1 index tracker fund. The table in Figure (5.1) summarises the performance of the five funds. The funds are ranked according to their period absolute return. Note that I started with R100 invested in the fund on 31 March 2000. All the funds had positive returns, but I did not include transaction costs or portfolio turnover figures. The table lists Sharpe ratios, annualised tracking errors and monthly tracking error and annualised total risk. The results for the remaining funds, QM2, QM3, BLM1 and BLM2 are displayed in Appendix C. I do this because the methodology has remained the same for all 5 funds and their inclusion would result in redundancy in the discussion section. Appendix B lists the Matlab source code that I used to construct my own benchmark and also to transport the views from the unit trusts fund data to my own basket of shares.

5.2 Discussion

5.2.1 QM1 vs MyTop30 Performance

I begin the discussion of the results with Figure (5.2). Figure (5.2) displays the absolute performance of the benchmark index MyTop30 and the portable alpha fund QM1. The date in months is allocated to the horizontal axis. There are 48 months in total representing the 4 years of data from March 2000 to February 2004. The fund value in Rand is represented on the vertical axis. The model starts with R100 invested in both funds at 31 March 2000. The monthly ex-ante tracking error of the QM1 fund is 1%. Such ex-ante tracking error is notable high, leading to an annualised ex-post tracking error of 5.08%. The absolute return of the QM1 fund for the period is $R140.23$.

	Absolute					**Relative**		
Funds	**Return for Period**	**Ann. Total Risk**	**Sharpe Ratio**	**Sharpe Ratio Rank**	**Beta**	**Ann. Tracking Error**	**M'ly TE**	**Hit Rate**
QM3	357.55	29.50	0.32	(1)	1.07	14.68	4.24	68.75%
QM2	233.19	27.03	0.26	(2)	1.05	9.85	2.84	68.75%
Avg	**193.14**	**26.71**	**0.21**		**1.02**	**10.16**	**2.93**	**68.8%**
QM1	140.23	25.26	0.17	(3)	1.03	5.08	1.47	79.17%
BLM2	126.57	26.77	0.15	(4)	0.95	14.09	4.07	62.50%
BLM1	108.16	24.98	0.13	(5)	1.00	7.10	2.05	64.58%

Figure 5.1: *Summary Performance Analysis of the five Portable Alpha Funds: Last 47 months*

5.2.2 Relative returns for the QM1 Fund

Figure (5.3) is a bar graph representing the monthly relative returns of the QM1 fund. The date in months is displayed on the horizontal axis and the percentage relative return on the vertical axis. The hit-rate for the QM1 fund is 79.19%. The hit-rate is defined as the percentage number of months with positive relative return. The high hit-rate is easily visualised in Figure (5.3). Such results clearly indicate success with with regard to transporting alpha across benchmarks. The views, which ultimately become alpha, are inferred from unit trust data with a Peer Mean unit trust fund as benchmark. The QM1 fund replicated the superior performance of the outperforming unit trust fund on a different benchmark.

The high hit-ratio can also be represented by a positively skewed histogram of relative monthly returns. This is illustrated in Figure (5.5).

The difference in ex-post and ex-ante tracking error is shown in Figure (5.4). The monthly ex-ante tracking error of the QM1 fund is set to 1% and, the realised, ex-post tracking error is 1.47%.

Lastly, Figure (5.6) shows a scatter plot of the monthly returns of the QM1 fund relative to the benchmark. Several diagnostic analyses are shown in the figure. The horizontal axis represents the benchmark's excess returns and the vertical axis represents the fund's excess returns. The top right quadrant represents a selection of times where the benchmark and the fund had positive returns. Superb performance is noted in the top left quadrant with positive returns from the fund with negative returns from the benchmark. Terrible performance, likewise, is displayed in the bottom right quadrant. The beta of the ideal tracker fund is 1 and is denoted with the dashed line. The realised beta of the QM1 fund is 1.03. A positive alpha of 1.12 is noteworthy. Afterall, the purpose of this entire methodology is to try and transport alpha in an efficient way across different benchmarks. My arbitrarily chosen basket of shares were used to construct a market weighted index. That index, serving as a new benchmark, was actively tilted based on views inferred on a different benchmark, the Unit Trust Peer Mean, and a positive alpha was delivered.

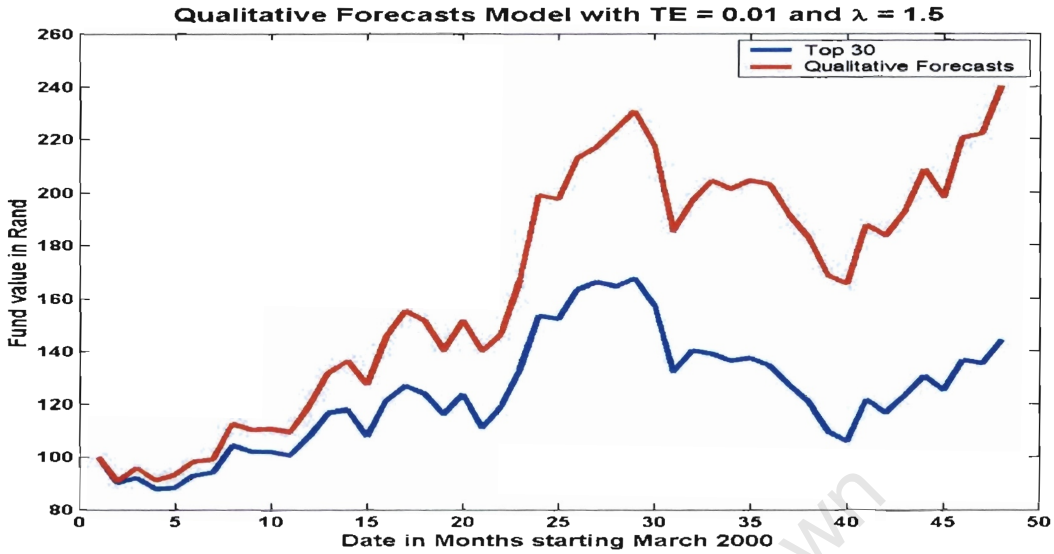

Figure 5.2: *Performance of the QTE1 Fund*

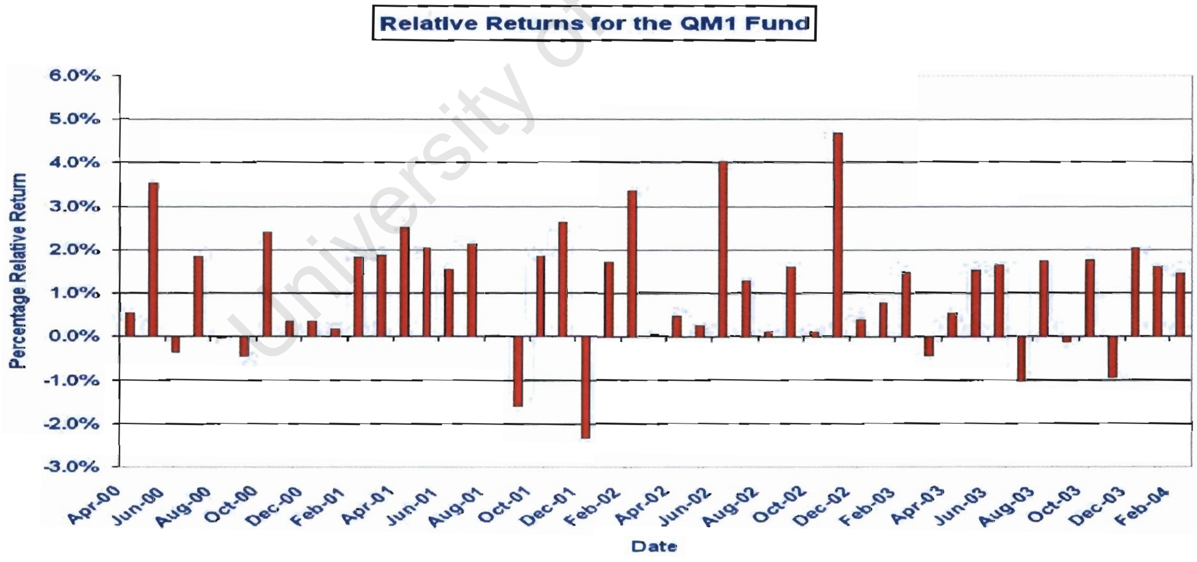

Figure 5.3: *Relative monthly returns for the QM1 Fund*

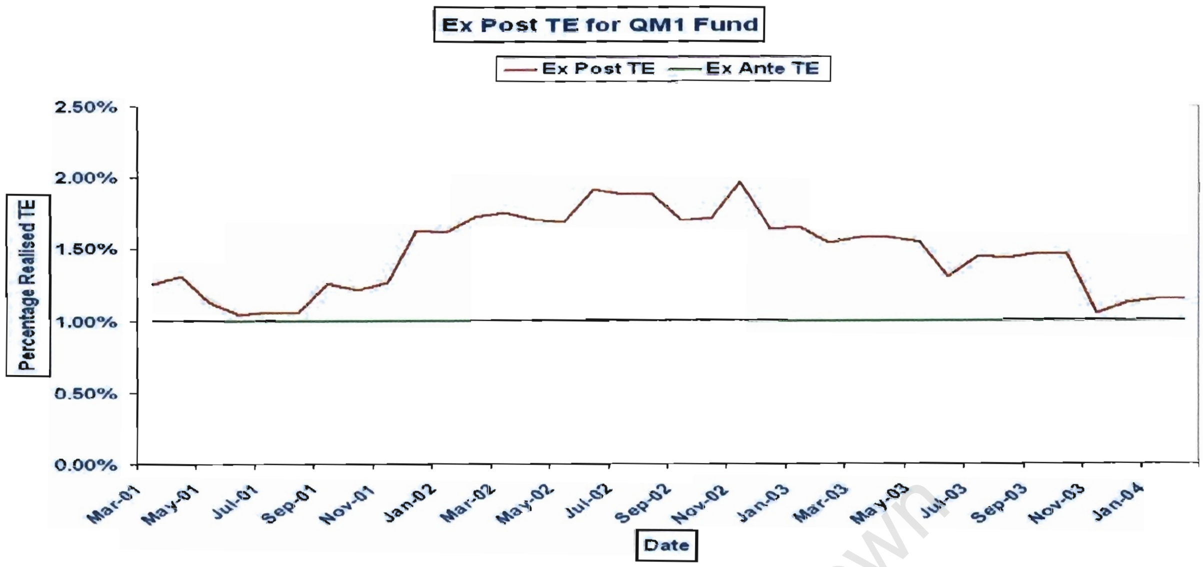

Figure 5.4: *Analysis of Ex Post vs Ex Ante TE for the QM1 Fund*

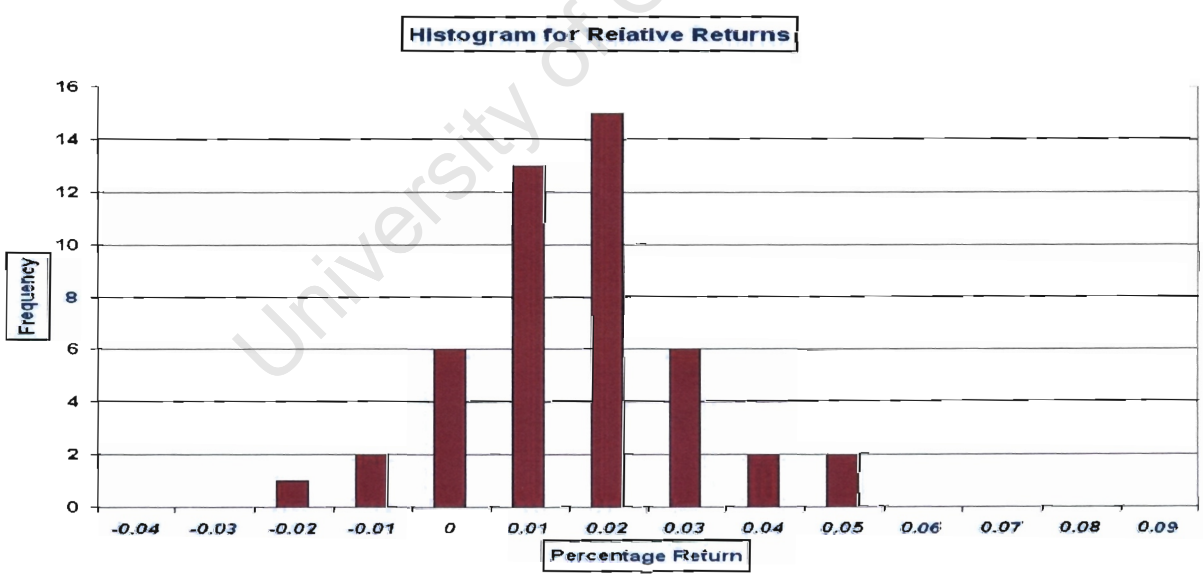

Figure 5.5: *Histogram showing the positive skewness of returns from the QM3 Fund*

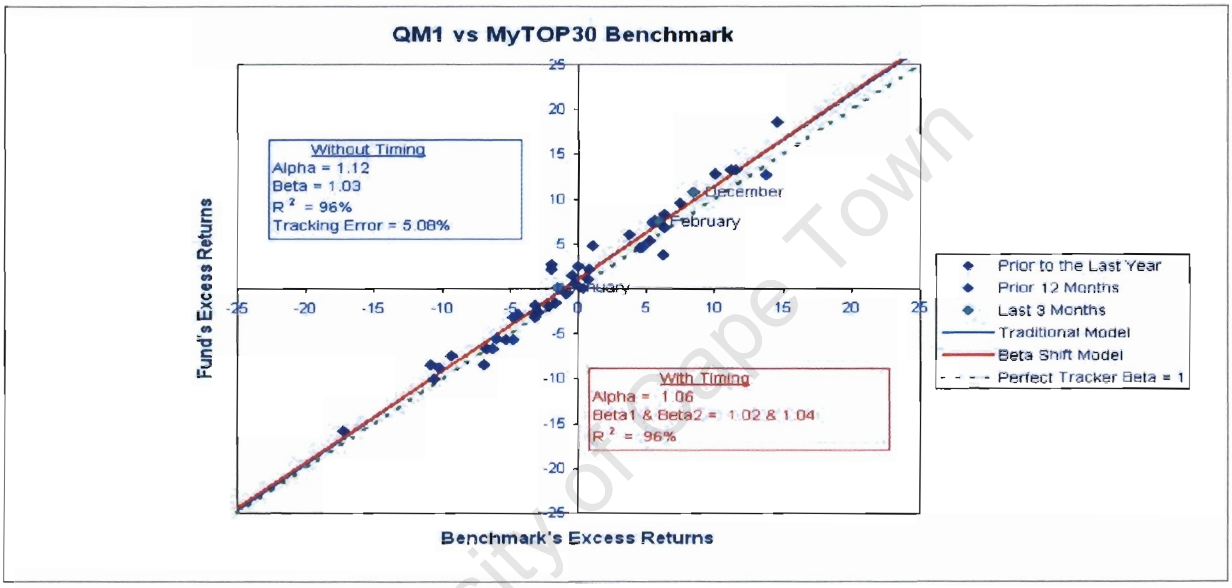

Figure 5.6: *Scatter Plot and analysis of the performance of the QM1 Fund. Several manager diagnostic analyses are shown, including the positive* α.

Chapter 6

Conclusion and Further Research

In my research I had set out to achieve three main goals. The first goal was to set the Black-Litterman Asset Allocation Model on sound theoretical footing and then provide insight on its inner workings via a step by step implementation on a nine asset example. The second goal was to develop a Qualitative Forecasts Asset Allocation Model from a theoretical and a practical point of view and then demonstrate its inner workings on the same nine asset example. Lastly, I set out to implement the two models on a basket of shares. I used this basket of shares to create my own index , the MyTop30 Index, and I inferred views on the basket from existing unit trust fund data. I thus developed a mechanism to transport alpha accross benchmarks.

In retrospect I have managed to meet the goals set out initially. The chapter on the Black-Litterman Model, deals extensively with insights in the model's parameters and the theoretical derivation is achieved from two points of view. Chapter 3 addresses the Qualitative Forecasts Model with a derivation adapted from the Black-Litterman chapter and a demonstration of the inner workings based on the same nine assets as used to demonstrate the Black-Litterman model. Lastly, the methodology developed in Chapter 4 successfully manages to transport alpha across benchmarks. The methodology can certainly be improved by allowing for more sophisticated ways to back out views and also by using less turnover in the portfolios but as an initial starting point, I have shown that both the qualitative and quantitative forecasts models are capable of achieving this goal.

The research has produced some interesting insight on the nature of the inputs used in the two models. Fundamentally the View Projection Matrix ultimately scales the importance that is placed on views. If an equally weighted P matrix is used then all the nominally outperforming assets have the same degree of importance placed upon them. If a market capitalisation weighting scheme is used then relative importance is placed on the assets in the same view, based on their respective relative market capitalisations. However, one can bring in factors such as liquidity or free floating number of shares in issue to use as weighting schemes in the View Projection Matrix. To an even greater extent, one could incorporate strength of conviction in the P matrix by further scaling up the weights on a view level by a confidence factor. Further research in the effects of different choices of P matrix weighting schemes would yield very useful results.

The scalar τ used in the Black-Litterman to scale the covariance matrix of excess returns turns out to be arbitrary when we incorporate τ as a ratio in the diagonal matrix of error terms Ω. This result enforces my suggestion that more work needs to be done on the P matrix in order to incorporate confidence in the views at that level.

More research can be done on the Qualitative Forecasts model to incorporate a system capable of converting several forecasts into a meaningful value for the Information Coefficient , IC, that is used as an input in the model. In that way, a fundamental manager can look at several indicators and analysts forecasts and use them to compute a reasonable IC given his choice of indicators and the current state of the market. My research has shown that varying the information coefficent produces varying deviations from the benchmark weights.

Further work could include attempts to have forecasts on the expected volatility and not only on the expected returns. That way the covariance martix would be a variable input rather than a calculated status quo variable outlining the current market volatilities and cross correlations.

Chapter 7

Appendix

A1 Notes on the Multivariate Normal Distribution

The multivariate normal distribution (MVN) is one of the most important examples of multivariate distributions. It is a direct generalization of the univariate normal and shares many of its properties. Besides from being analytically tractable the MVN often arises as the limiting distribution in many multidimensional Central Limit Theorems.

A1.1 Definition of the Multivariate Normal Distribution

The following are equivalent definitions of the MVN. Given a vector $\mu \in \mathbb{R}^n$ and a positive semi-definite $(n \times n)$ matrix $\mathbf{\Sigma}$, $Y \sim \mathbf{N_n}(\mu, \Sigma)$ if:

Definition 1:

For positive definite $\mathbf{\Sigma}$, the density function of $\mathbf{Y}$ is

$$f_Y(y) = (2\pi)^{-n/2}|\Sigma|^{-1/2}\exp\left[-\frac{1}{2}(y-\mu)'\Sigma^{-1}(y-\mu)\right]$$

Definition 2:

The moment generating function (m.g.f) of $\mathbf{Y}$ is

$$M_Y(t) \equiv E\left[e^{t'Y}\right] = e^{\mu' t + \frac{1}{2}t'\Sigma t}$$

Definition 3:

$\mathbf{Y}$ has the same distribution as $\mathbf{AZ}+\ \mu$, where $\mathbf{Z} = (Z_1, ..., Z_k)$ is a random sample from $N(0,1)$ and $\mathbf{A_{n\times k}}$ satisfies $\mathbf{AA'} = \mathbf{\Sigma}$.

Note that Definitions 1, 2 and 3 are equivalent for det$\mathbf{\Sigma}\ \geq 0$, and Definitions 2 and 3 are equivalent for det$\mathbf{\Sigma}\ > 0$.

A1.2 Properties of the Multivariate Normal Distribution

- **General Properties**:

  1. $E[\mathbf{Y}] = \mu$, cov$(\mathbf{Y}) = \mathbf{\Sigma}$
  2. If $(Z_1, ..., Z_k)$ is a random sample from $N(0,1)$ then $\mathbf{Z}$ has the $N_n(0_n, I_{n\times n})$ distribution.
  3. If $\mathbf{\Sigma}$ is not positive definite, then $\mathbf{Y}$ has a singular MVN distribution and no density function exists.

- **General Transformations of MVN Vectors**:

  1. If $Y \sim \mathbf{N_n}(\mu, \mathbf{\Sigma})$ and $\mathbf{C_{p\times n}}$ is a constant matrix of rank p, then $\mathbf{CY} \sim \mathbf{N_n}(\mathbf{C}\mu, \mathbf{C\Sigma C'})$.
  2. $\mathbf{Y}$ is **MVN** iff $\mathbf{a'Y}$ is normally distributed for all non zero constant vectors $\mathbf{a}$.

- **Orthogonal Transformations of MVN Vectors**:

Let $\mathbf{Y} \sim \mathbf{N_n}(\mu, \sigma^2\mathbf{I})$, and let $\mathbf{T_{n\times n}}$ be an orthogonal matrix of constants. Then $\mathbf{TY} \sim \mathbf{N_n}(\mathbf{T}\mu, \sigma^2\mathbf{I})$.

Interpretation: Mutually independent normal random variables with common variance remain mutually independent with common variance under orthogonal transformations. Orthogonal matrices correspond to rotations and reflections about the origin and are thus isometries.

- **Partitioned MVN Distributions**:

Let $\mathbf{Y} \sim \mathbf{N_n}(\mu, \boldsymbol{\Sigma})$ be partitioned as

$$\mathbf{Y} = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}$$

where $\mathbf{Y_1}$ is $(p \times 1)$ and $\mathbf{Y_2}$ is $(q \times 1)$ such that $p + q = n$. Then the mean and covariance matrix are correspondingly partitioned as

$$\mu = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}$$

and

$$\boldsymbol{\Sigma} = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix} = \begin{pmatrix} \mathrm{cov}(\mathbf{Y_1}) & \mathrm{cov}(\mathbf{Y_1}, \mathbf{Y_2}) \\ \mathrm{cov}(\mathbf{Y_2}, \mathbf{Y_1}) & \mathrm{cov}(\mathbf{Y_2}) \end{pmatrix}$$

Properties:

1. Marginal distributions: $\mathbf{Y_1} \sim \mathbf{N_p}(\mu_1, \boldsymbol{\Sigma_{11}})$, $\mathbf{Y_2} \sim \mathbf{N_q}(\mu_2, \boldsymbol{\Sigma_{22}})$.
2. Uncorrelated implies independent: $\mathbf{Y_1}$ and $\mathbf{Y_2}$ are independent iff $\boldsymbol{\Sigma_{11}} = \boldsymbol{\Sigma}'_{\mathbf{21}} = 0$.
3. Conditional distributions: If $\boldsymbol{\Sigma}$ is positive definite then the conditional distribution of $\mathbf{Y_1}$ given $\mathbf{Y_2}$ is

$$\mathbf{Y_1}|\mathbf{Y_2} = \mathbf{y_2} \sim \mathbf{N_p}\left(\mu_1 + \boldsymbol{\Sigma_{12}}\boldsymbol{\Sigma_{22}^{-1}}(\mathbf{y_2} - \mu_2), \boldsymbol{\Sigma_{11}} - \boldsymbol{\Sigma_{12}}\boldsymbol{\Sigma_{22}^{-1}}\boldsymbol{\Sigma_{21}}\right).$$

A2 Notes on Generalised Least Squares

Consider the linear regression model

$$Y = X\beta + u$$

where

- Y is a $n \times 1$ vector of observations on a dependent variable;
- X is a $n \times k$ matrix of independent variables of full column rank;
- β is a k vector of parameters to be estimated, and
- u is a n vector of disturbances.

Via the Gauss-Markov Theorm, if

G-M 1 $E[u|X] = 0$ (i.e., the disturbances have conditional mean zero, and

G-M 2 $E[uu'|X] = \sigma^2\Omega$, where $\Omega = I_n$, a $n \times n$ identity matrix (i.e., conditional on the X the disturbances are independent and identically distributed of "iid" with conditional variance σ^2),

then the *ordinary least squares* estimator $\hat{\beta}_{OLS} = (X'X)^{-1}X'Y$ with covariance martix $V(\hat{\beta}_{OLS}) = \sigma^2(X'X)^{-1}$ is

1. the best linear unbiased estimator (BLUE) of β, in the sense of having smallest sampling variability in the class of linear unbiased estimators;
2. a consistent estimator of β (i.e., as $n \to \infty, \mathbb{P}[|\hat{\beta}_{OLS} - \beta| < \varepsilon] = 1$ for any $\varepsilon > 0$, or $\hat{\beta}_{OLS} = \beta$ in the probability limit).

If **[G-M 2]** fails to hold (i.e., Ω is a positive definite matrix but not equal to I_n), then $\hat{\beta}_{OLS}$ remains unbiased but no longer "best", and remains consistent. Relying on $\hat{\beta}_{OLS}$ when **[G-M 2]** does not hold risks faulty inferences. Without **[G-M 2]**, $\sigma^2(X'X)^{-1}$ is a biased and inconsistent estimator of $V(\hat{\beta}_{OLS})$, meaning that the estimated standard errors for $\hat{\beta}_{OLS}$ are wrong, invalidating inferences and the results of hypothesis tests. **[G-M 2]** fails to hold if we have heteroskedasticity or serially correlated disturbances, such as when we have a time series that generates disturbances that are not conditionally independent.

When **[G-M 2]** does not hold, it may be possible to implement a *Generalised Least Squares* (GLS) estimator that is BLUE (at least asymptotically). For instance, if we know the exact form of the departure from **[G-M 2]** (i.e., we know Ω) then the GLS estimator $\hat{\beta}_{GLS} = (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}Y$ is BLUE, with covariance matrix $\sigma^2(X'\Omega^{-1}X)^{-1}$. Note that when **[G-M 2]** holds, $\Omega = I_n$ and $\hat{\beta}_{GLS} = \hat{\beta}_{OLS}$ (i.e., OLS is a special case of the more general estimator).

A3 Notes on the Information Ratio

The information ratio is a measure that seeks to summarise in a single number the mean-varianceproperties of an active portfolio. It builds on the Markowitz mean-Variance paradigm, which states that the mean and the variance (or mean and standard deviation) of returns are sufficient statistics for characterising an investment portfolio. Calculation of an information ratio is based on the standard statistical formulas for the mean and the standard deviation. If R_{P_t} is the return of an active portfolio in period t and R_{B_t} is the return on a benchmark portfolio over the same period, then $E[R_t]$, the excess return, is the difference:

$$E[R_t] = R_{P_t} - R_{B_t} \tag{7.1}$$

$\overline{E[R]}$ is the arithmetic average of excess returns over the historical period from $t = 1$ through T:

$$\overline{E[R]} = \frac{1}{T}\sum_{t=1}^{T} E[R_t] \tag{7.2}$$

and $\hat{\sigma}_{E[R]}$ is the standard deviation of excess returns from the benchmark, or *Tracking Error*, for the same period:

$$\hat{\sigma}_{E[R]} = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(E[R_t] - \overline{E[R]})^2} \tag{7.3}$$

The Information Ration (IR) based on historical data is simply the ratio of the return and the standard deviation:

$$IR = \frac{\overline{E[R]}}{\hat{\sigma}_{E[R]}} \tag{7.4}$$

B1 Matlab Source Code

Here I have outlined the Matlab code that I used to construct the index tracking funds, back-out views from unit trust data and ultimately transport those views onto a new benchmark, the MyTop30.

B1.1 Source Code for the Black-Litterman Model

```
%This M-File  calculates Index Tracker Funds generated by the Black-Litterman Model.
%Inputs required are the Daily closing prices of the assets in the basket, together with
%the Unit Trust Fund holdings weights for those assets at the relevant times.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%This sets up a multi dimensional array holding the weekly closing prices for the basket of stocks
tic
ClosingPrices=[];
for i = 1:65
    for j = 1:29
```

```
        ClosingPrices(i,j,1)=RetM00(i,j);
        ClosingPrices(i,j,2)=RetJ00(i,j);
        ClosingPrices(i,j,3)=RetS00(i,j);
        ClosingPrices(i,j,4)=RetD00(i,j);
        ClosingPrices(i,j,5)=RetM01(i,j);
        ClosingPrices(i,j,6)=RetJ01(i,j);
        ClosingPrices(i,j,7)=RetS01(i,j);
        ClosingPrices(i,j,8)=RetD01(i,j);
        ClosingPrices(i,j,9)=RetM02(i,j);
        ClosingPrices(i,j,10)=RetJ02(i,j);
        ClosingPrices(i,j,11)=RetS02(i,j);
        ClosingPrices(i,j,12)=RetD02(i,j);
        ClosingPrices(i,j,13)=RetM03(i,j);
        ClosingPrices(i,j,14)=RetJ03(i,j);
        ClosingPrices(i,j,15)=RetS03(i,j);
        ClosingPrices(i,j,16)=RetD03(i,j);
        ClosingPrices(i,j,17)=RetM04(i,j);

    end
end

%here I calculate the Covariance matrix relevant to each of the quarters
Returns = [];
Sigma=[];
Determinants=[];
for ii=1:17
    for i = 2:65
        for j = 1:29
            Returns(i-1,j,ii)=(ClosingPrices(i-1,j,ii)-ClosingPrices(i,j,ii))/ClosingPrices(i,j,ii);
        end
    end
    Sigma(:,:,ii)=cov(Returns(:,:,ii));
    Determinants(:,ii)=det(Sigma(:,:,ii));
end

%% Here I am Backing out Views from the Unit trust funds

Lambda = 1.5;     %Coefficient of risk aversion
ERsf=[];
ERpm=[];
for i=1:16
    Wsfadj(:,i)=Wsf(:,i)./sum(Wsf(:,i));
```

```
    Wpmadj(:,i)=Wpm(:,i)./sum(Wpm(:,i));
end

for i = 1:16
    ERsf(:,i) = Lambda*Sigma(:,:,i)*Wsfadj(:,i);
    ERpm(:,i) = Lambda*Sigma(:,:,i)*Wpmadj(:,i);
end

for j=1:16
    que(:,j)= ERsf(:,j)-ERpm(:,j);
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  View Projection Matrix P and View Vector Q are now Assembled   %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

n=length(Asset_Nr);
k=max(Sector);
P=zeros(k,n);
Q=[];
Perf=[];
MktRet=[];
Perf1=[];
MktRet1=[];
for j = 1:16
    QU=que(:,j);
    Mkt_Cap=Mkt_Cap_Final(:,j);
for i = 1:k
        View=[];
        V=[];
        Mcap=[];
        Wmkt=[];
        blah=[];
        Mkt_Cap1=[];
        Mkt_Cap2=[];
        Summ1=[];
        signcheckneg=[];
        signcheckpos=[];
        p=[];
        View=find(Sector==i);
        V=Asset_Nr(find(Sector==i));

        signcheckneg=find(QU(View)<0);
```

```
        signcheckpos=find(QU(View)>0);
        Vneg=V(signcheckneg);
        Vpos=V(signcheckpos);
        Mcapneg=Mkt_Cap(Vneg);
        Mcappos=Mkt_Cap(Vpos);
        Wmktneg=Mcapneg./sum(Mcapneg);
        Wmktpos=Mcappos./sum(Mcappos);
        p=zeros(1,n);
        p(V(signcheckneg))=-Wmktneg;
        p(V(signcheckpos))=Wmktpos;
        P(i,:,j)=p ;
        findque=QU(View);
         Q(i,j)=sum(findque);
    end
end


%%%%%%% We now set up the Diagonal Uncertainty matrix Omega %%%%%%%

tau=0.5;  %turns out to be arbitrary so any value would do...

%  for i = 1:16
% OmegaDiag=diag(P(:,:,i)*Sigma(:,:,i)*P(:,:,i)')*tau;
%            for jj=1:k
%              Omega(jj,jj,i)=OmegaDiag(jj);
%            end
%  end


Wmkt=[];
TEBL=TEQ;
TETarget=input('please input target tracking error TE = ');
% Set the target tracking error

for i=1:16
    Wmkt(:,i) = Mkt_Cap_Final(:,i)./sum(Mkt_Cap_Final(:,i));

    delta = 0.001;
    while abs(TETarget-TEBL(i))>0.0001
        delta=delta+0.0001;
    OmegaDiag=diag(P(:,:,i)*Sigma(:,:,i)*P(:,:,i)')*tau*delta;
    for jj=1:k
            Omega(jj,jj,i)=OmegaDiag(jj);
```

```
          end
    L(:,i)=tau*inv(Omega(:,:,i))*Q(:,i)/Lambda-inv(Omega(:,:,i)/tau+P(:,:,i)
         *Sigma(:,:,i)*P(:,:,i)')*P(:,:,i)*Sigma(:,:,i)*Wmkt(:,i)-inv(Omega(:,:,i)/tau+P(:,:,i)
*Sigma(:,:,i)*P(:,:,i)')*P(:,:,i)*Sigma(:,:,i)*P(:,:,i)'*tau*inv(Omega(:,:,i))
*Q(:,i)/Lambda;

    Wblraw(:,i) =  Wmkt(:,i)+P(:,:,i)'*L(:,i);
    Summ2=sum(Wblraw(:,i));
    Wbl(:,i)=Wblraw(:,i)./Summ2;

    TEBL(i)=sqrt((Wbl(:,i)-Wmkt(:,i))'*Sigma(:,:,i)*(Wbl(:,i)-Wmkt(:,i)));
end
end
TEBL
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Summ2=100;
ITop30=[];
 for i=1:49
 MWmkt(:,i)=MMktCaps(:,i)./sum(MMktCaps(:,i));
end
 for i=1:48
     ITop30(i)=Summ2;
     NS(:,i)=(MWmkt(:,i)*ITop30(i))./MCPrices(:,i);

         Summ2=NS(:,i)'*MCPrices(:,i+1);

 end
 close all

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
IBLM=[];
 for i=1:16
     for j = (i-1)*3+1: i*3
         MBL(:,j)=Wbl(:,i);
     end
 end
  Summ2=100;
 for i=1:48
     IBLM(i)=Summ2;
     NSBLM(:,i)=(MBL(:,i)*IBLM(i))./MCPrices(:,i);
         Summ2=NSBLM(:,i)'*MCPrices(:,i+1);
 end
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

Summ2=100;
IBLQ=[];

 for i=1:15
     IBLQ(i)=Summ2;
     NSBLQ(:,i)=(Wbl(:,i)*IBLQ(i))./QCPrices(:,i);
         Summ2=NSBLQ(:,i)'*QCPrices(:,i+1);

 end



 Summ2=100;
 ITop30Q=[];

 for i=1:15
     ITop30Q(i)=Summ2;
     NSQ(:,i)=(Wmkt(:,i)*ITop30Q(i))./QCPrices(:,i);
         Summ2=NSQ(:,i)'*QCPrices(:,i+1);
 end
hold
% ResultBL=[ITop30' IBLM'];
% bar(ResultBL)
 x=linspace(1,48,48);
 plot(x,ITop30,'b-',x,IBLM,'r-')
title('Black Litterman Model with  \lambda = 1.5')
xlabel('Date in Months starting March 2000')
ylabel('Fund value in Rand')
 toc
```

B1.2 Source Code for the Qualitative Forecasts Model

```
 %This M-File  calculates Index Tracker Funds generated by the Qualitative Forecasts Model.
%Inputs required are the Daily closing prices of the assets in the basket, together with
%the Unit Trust Fund holdings weights for those assets at the relevant times.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%This stes up a multi dimensional array holding the weekly closing prices for the basket of stocks
tic
```

```
ClosingPrices=[];
for i = 1:65
    for j = 1:29

        ClosingPrices(i,j,1)=RetM00(i,j);
        ClosingPrices(i,j,2)=RetJ00(i,j);
        ClosingPrices(i,j,3)=RetS00(i,j);
        ClosingPrices(i,j,4)=RetD00(i,j);
        ClosingPrices(i,j,5)=RetM01(i,j);
        ClosingPrices(i,j,6)=RetJ01(i,j);
        ClosingPrices(i,j,7)=RetS01(i,j);
        ClosingPrices(i,j,8)=RetD01(i,j);
        ClosingPrices(i,j,9)=RetM02(i,j);
        ClosingPrices(i,j,10)=RetJ02(i,j);
        ClosingPrices(i,j,11)=RetS02(i,j);
        ClosingPrices(i,j,12)=RetD02(i,j);
        ClosingPrices(i,j,13)=RetM03(i,j);
        ClosingPrices(i,j,14)=RetJ03(i,j);
        ClosingPrices(i,j,15)=RetS03(i,j);
        ClosingPrices(i,j,16)=RetD03(i,j);
        ClosingPrices(i,j,17)=RetM04(i,j);

    end
end
%here we calculate the Covariance matrix relevant to each of the quarters
Returns = [];
Sigma=[];
Determinants=[];
for ii=1:17
    for i = 2:65
        for j = 1:29
            Returns(i-1,j,ii)=(ClosingPrices(i-1,j,ii)-ClosingPrices(i,j,ii))/ClosingPrices(i,j,ii);
        end
    end
    Sigma(:,:,ii)=cov(Returns(:,:,ii));
    Determinants(:,ii)=det(Sigma(:,:,ii));
end

Lambda = input('please input coefficient of risk aversion lambda = ');

%Back out views from Unit Trust Fund Data
ERsf=[];
ERpm=[];
```

```
for i=1:16
    Wsfadj(:,i)=Wsf(:,i)./sum(Wsf(:,i));
    Wpmadj(:,i)=Wpm(:,i)./sum(Wpm(:,i));
end

for i = 1:16
    ERsf(:,i) = Lambda*Sigma(:,:,i)*Wsfadj(:,i);
    ERpm(:,i) = Lambda*Sigma(:,:,i)*Wpmadj(:,i);
end

for j=1:16
    que(:,j)= ERsf(:,j)-ERpm(:,j);
end


% Here we start calculating the View Projection Matrix P and the View Vector Q
n=length(Asset_Nr);
k=max(Sector);
P=zeros(k,n);
Q=[];
Perf=[];
MktRet=[];
Perf1=[];
MktRet1=[];
for j = 1:16
    QU=que(:,j);
    Mkt_Cap=Mkt_Cap_Final(:,j);
for i = 1:k
        View=[];
        V=[];
        Mcap=[];
        Wmkt=[];
        blah=[];
        Mkt_Cap1=[];
        Mkt_Cap2=[];
        Summ1=[];
        signcheckneg=[];
        signcheckpos=[];
        p=[];
        View=find(Sector==i);
        V=Asset_Nr(find(Sector==i));

        signcheckneg=find(QU(View)<0);
```

```
        signcheckpos=find(QU(View)>0);
        Vneg=V(signcheckneg);
        Vpos=V(signcheckpos);
        Mcapneg=Mkt_Cap(Vneg);
        Mcappos=Mkt_Cap(Vpos);
        Wmktneg=Mcapneg./sum(Mcapneg);
        Wmktpos=Mcappos./sum(Mcappos);
        p=zeros(1,n);
        p(V(signcheckneg))=-Wmktneg;
        p(V(signcheckpos))=Wmktpos;
        P(i,:,j)=p ;
    end

  %Calculate the Matrix of error terms Omega

    OmegaDiag=diag(P(:,:,j)*Sigma(:,:,j)*P(:,:,j)');

    for jj=1:k
    Omega(jj,jj,j)=OmegaDiag(jj);
    end
end

MCTEv=[];
TETarget=input('please input target tracking error TE = '); % Set the target tracking error
for j=1:16
    Wmkt(:,j) = Mkt_Cap_Final(:,j)./sum(Mkt_Cap_Final(:,j));
    IC=0.001;
        while abs(TETarget-TEQ(j))>0.0001    % Adjust IC until the tracking error falls within limits
        IC=IC+0.001;
        Q=IC*OmegaDiag;
        Summ2=[];
% Work out the new weights via the Optimal scaling factor Phi
Phi(:,j)=(inv(Omega(:,:,j))*Q)/Lambda -
 inv(Omega(:,:,j)+P(:,:,j)*Sigma(:,:,j)*P(:,:,j)')
*(P(:,:,j)*Sigma(:,:,j)*P(:,:,j)'*inv(Omega(:,:,j))*Q/Lambda);
Wactive(:,j) = P(:,:,j)'*Phi(:,j);
Wadj(:,j)=(Mkt_Cap_Final(:,j)./sum(Mkt_Cap_Final(:,j)))+Wactive(:,j);
% Normalise the weights to 1
Summ2=sum(Wadj(:,j));
Wquali(:,j)=Wadj(:,j)./Summ2;

TEQ(j)=sqrt((Wquali(:,j)-Wmkt(:,j))'*Sigma(:,:,j)*(Wquali(:,j)-Wmkt(:,j)));
end
```

```
        MCTEv(:,j)=(P(:,:,j)*Sigma(:,:,j)*P(:,:,j)'*Phi(:,j))/TEQ(j);
       % PCTEv(:,j)=MCTEv(:,j).*Phi(:,j)/TEQ(j);
end
TEQ;
MCTEv
%PCTEv
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Here we index the Qualitative Portfolio Quarterly %%%%%%
Summ2=100;
IQualiQ=[];

 for i=1:15
     IQualiQ(i)=Summ2;
     NSQualiQ(:,i)=(Wquali(:,i)*IQualiQ(i))./QCPrices(:,i);
     Summ2=NSQualiQ(:,i)'*QCPrices(:,i+1);
 end
 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

 % Here I index the Top 30 Monthly %%%%%%%%%%%%%%%%%%%%%%
ITop30=[];
  for i=1:49
 MWmkt(:,i)=MMktCaps(:,i)./sum(MMktCaps(:,i));
  end
  Summ2=100;
 for i=1:48
     ITop30(i)=Summ2;
     NS(:,i)=(MWmkt(:,i)*ITop30(i))./MCPrices(:,i);
         Summ2=NS(:,i)'*MCPrices(:,i+1);
 end
 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
 % Here I Index the Qualitative Portfolio Monthly %%%%%%%%%%%%%%%%
 IQualiM=[];
 for i=1:16
     for j = (i-1)*3+1: i*3
         MWquali(:,j)=Wquali(:,i);
     end
 end
  Summ2=100;
 for i=1:48
     IQualiM(i)=Summ2;
     NSQualiM(:,i)=(MWquali(:,i)*IQualiM(i))./MCPrices(:,i);
         Summ2=NSQualiM(:,i)'*MCPrices(:,i+1);
 end
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Here I index the Top30 Quarterly %%%%%%%%%%%%%%%%%%%%%%%%%
Summ2=100;
ITop30Q=[];

for i=1:15
    ITop30Q(i)=Summ2;
    NSQ(:,i)=(Wmkt(:,i)*ITop30Q(i))./QCPrices(:,i);
    Summ2=NSQ(:,i)'*QCPrices(:,i+1);
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
close all
% Here we plot the results with either a line graph or a bar graph
ResultQuali=[ITop30' IQualiM'];
%  bar(ResultQuali)
 x=linspace(1,48,48);
 plot(x,ITop30,'b-',x,IQualiM,'r-')
title('Qualitative Forecasts Model with TE = 0.03 and \lambda = 1.5')
xlabel('Date in Months starting March 2000')
ylabel('Fund value in Rand')
toc
```

C1 Results of the Remaining Tracker Funds

C1.1 Index Tracker Funds QM2, QM3, BLM1 and BLM2

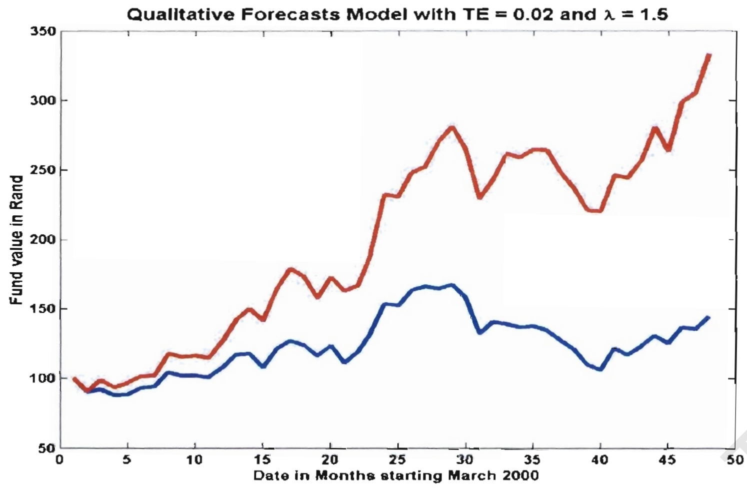

Figure C.1: *Performance of the QM2 Fund*

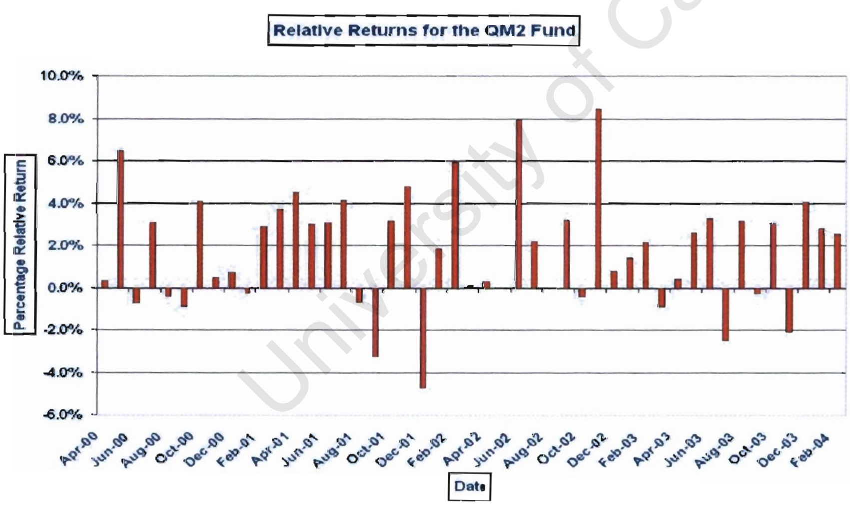

Figure C.2: *Relative monthly returns for the QM2 Fund*

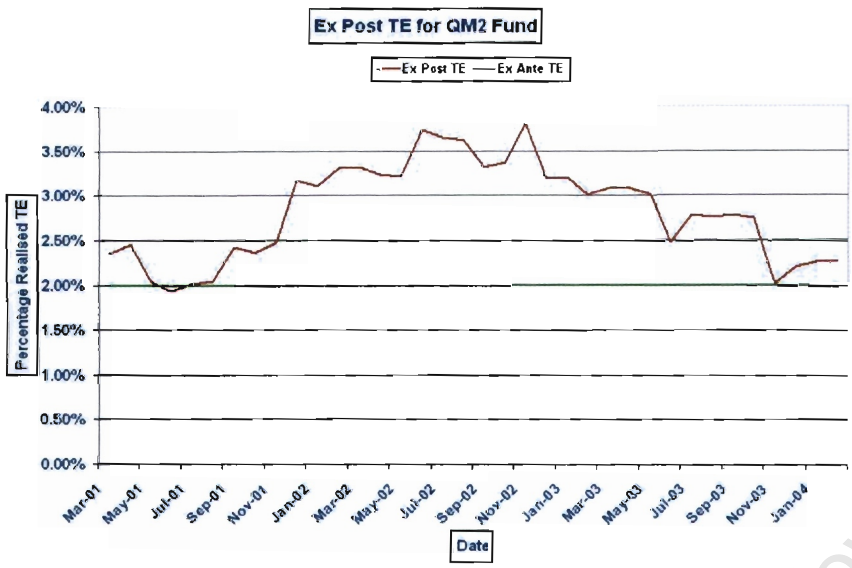

Figure C.3: *Analysis of Ex Post vs Ex Ante TE for the QM2 Fund*

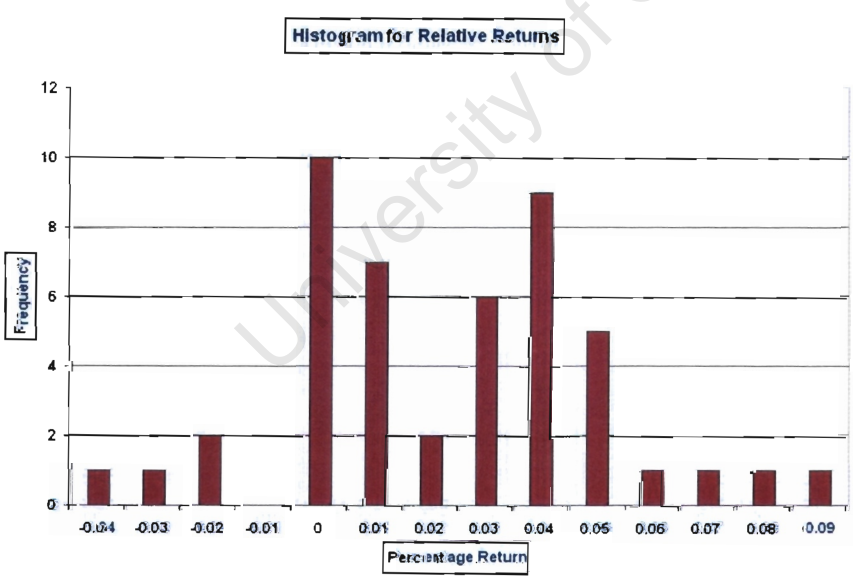

Figure C.4: *Histogram showing the positive skewness of returns from the QM2 Fund*

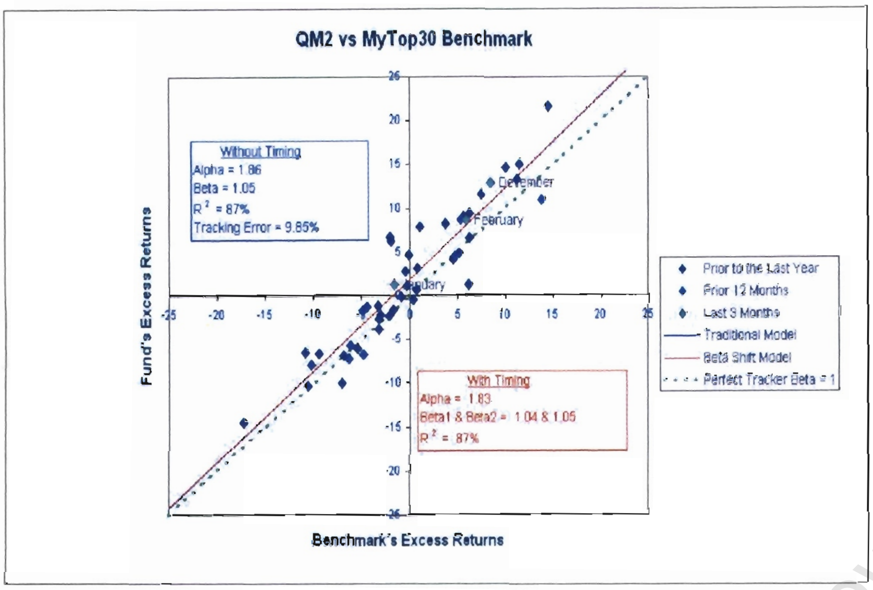

Figure C.5: *Scatter Plot and analysis of the performance of the QM2 Fund. Several manager diagnostic analyses are shown, including the positive α which is what the model tries to achieve*

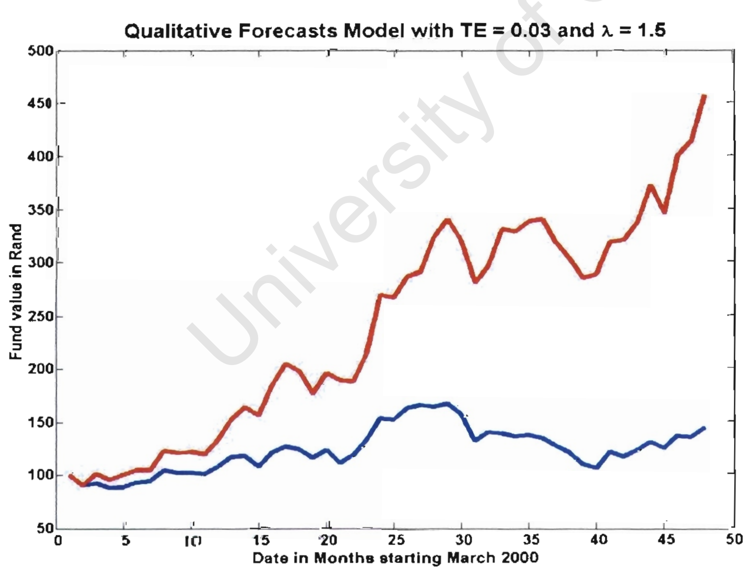

Figure C.6: *Performance of the QTE3 Fund*

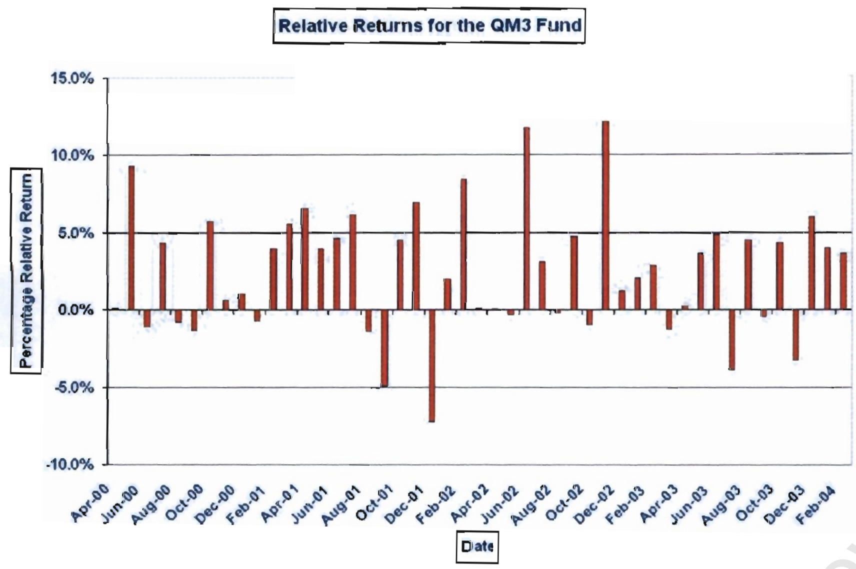

Figure C.7: *Relative monthly returns for the QM3 Fund*

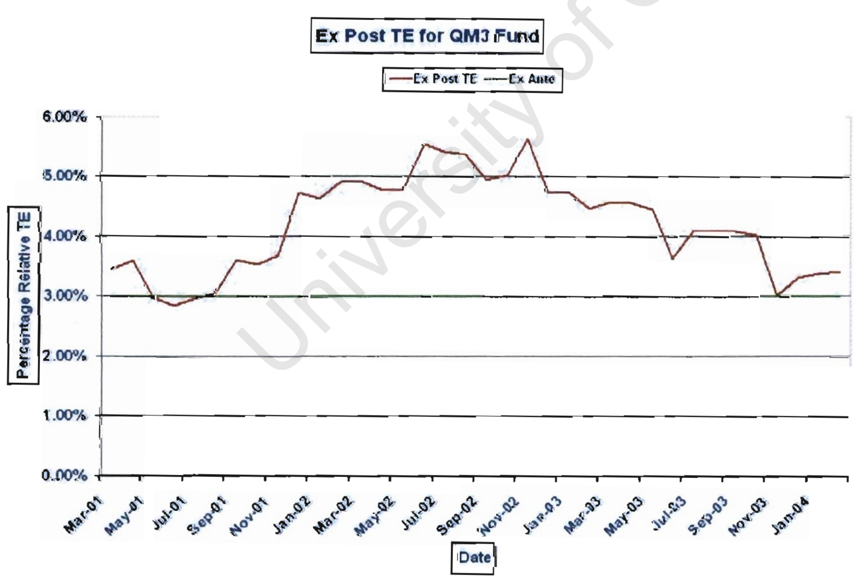

Figure C.8: *Analysis of Ex Post vs Ex Ante TE for the QM3 Fund*

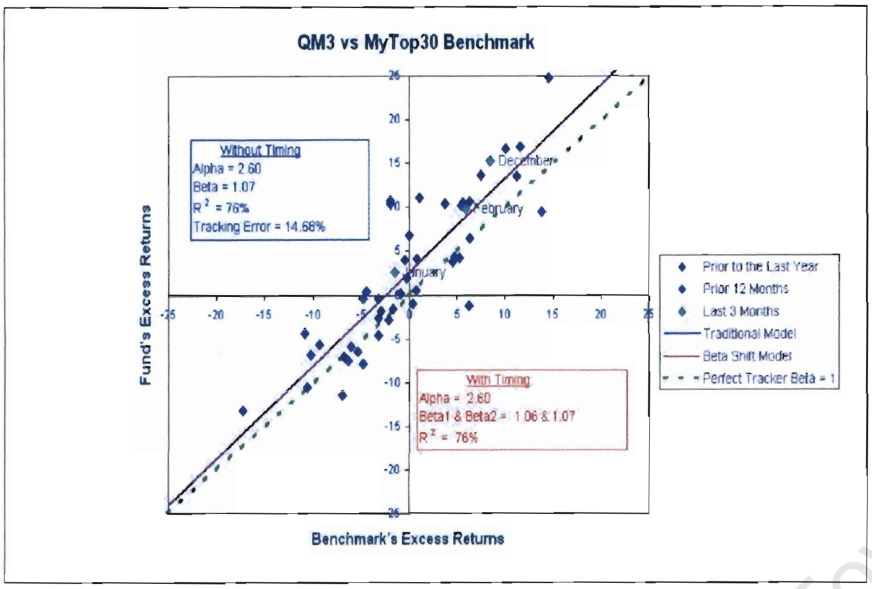

Figure C.9: *Scatter Plot and analysis of the performance of the QM3 Fund. Several manager diagnostic analyses are shown, including the positive α which is what the model tries to achieve*

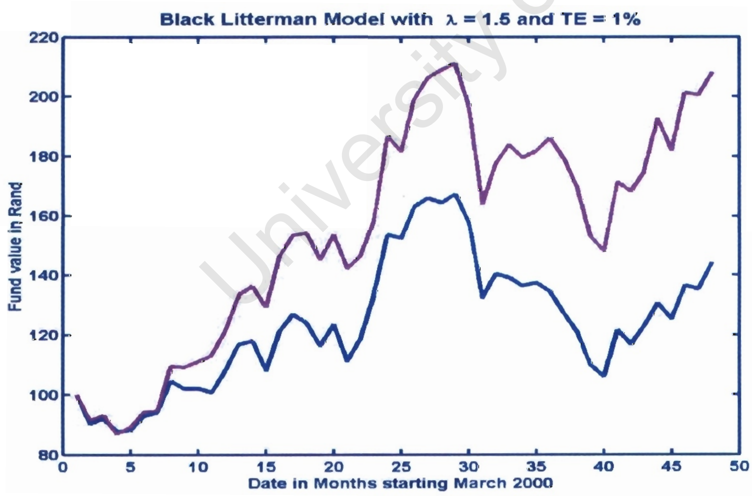

Figure C.10: *Performance of the BLM1 Fund*

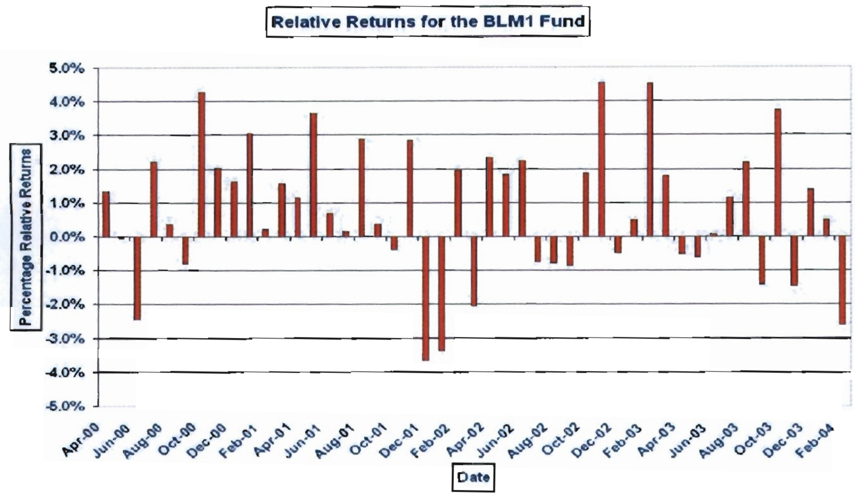

Figure C.11: *Relative monthly returns for the BLM1 Fund*

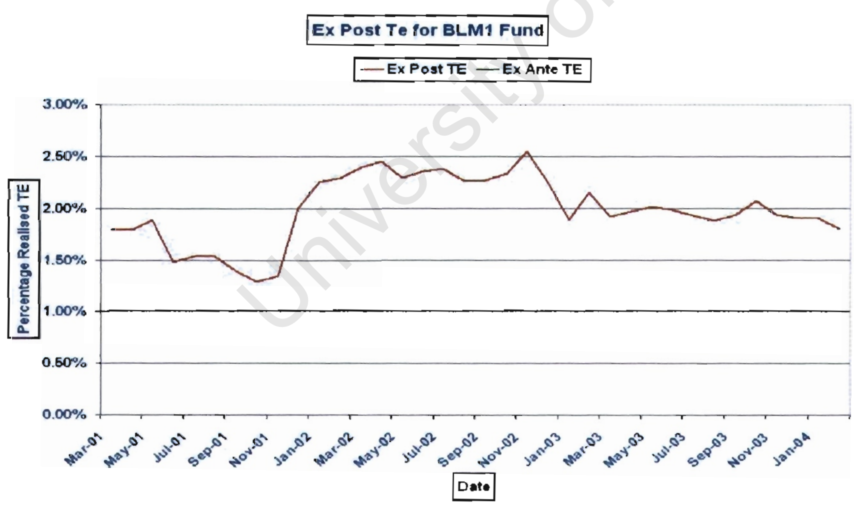

Figure C.12: *Analysis of Ex Post vs Ex Ante TE for the BLM1 Fund*

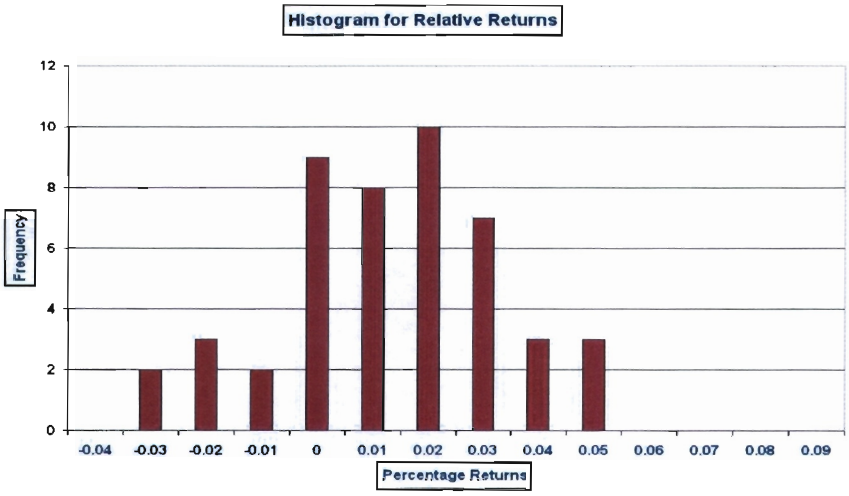

Figure C.13: *Histogram showing the positive skewness of returns from the BLM1 Fund*

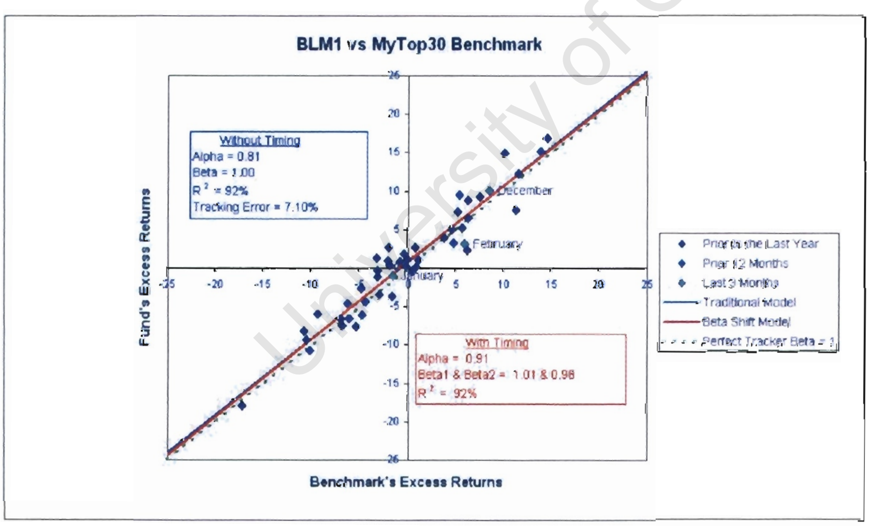

Figure C.14: *Scatter Plot and analysis of the performance of the BLM1 Fund. Several manager diagnostic analyses are shown, including the positive* α *which is what the model tries to achieve*

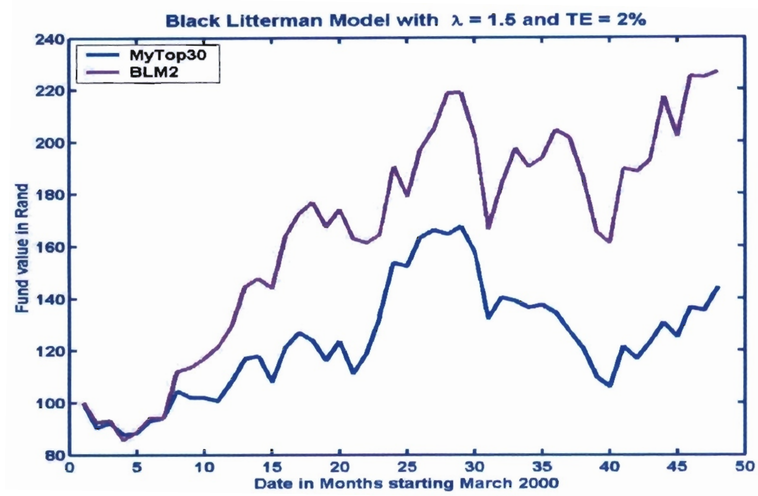

Figure C.15: *Performance of the BLM2 Fund*

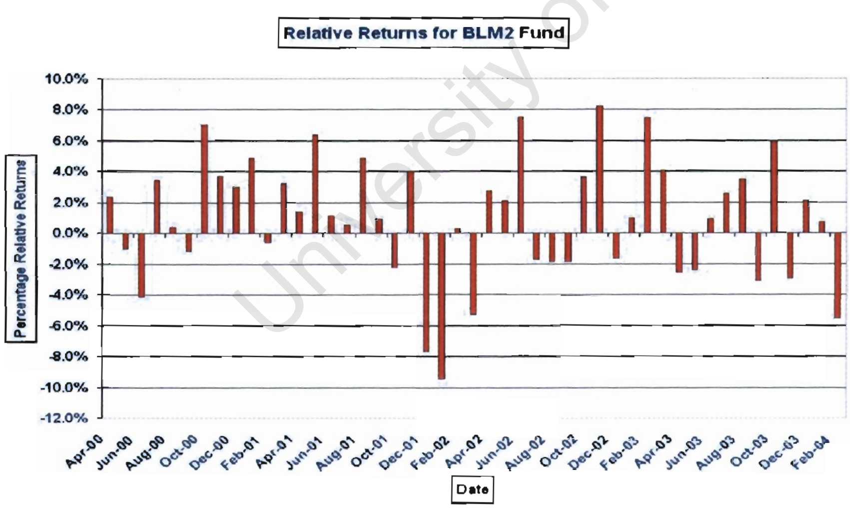

Figure C.16: *Relative monthly returns for the BLM2 Fund*

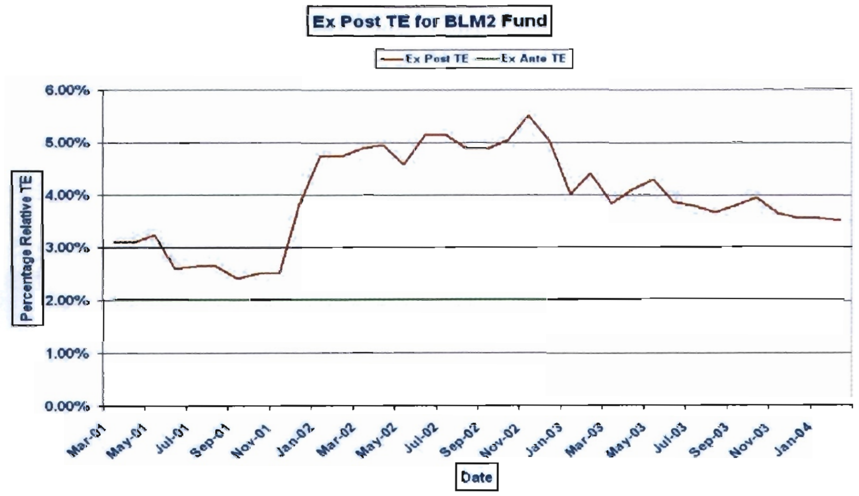

Figure C.17: *Analysis of Ex Post vs Ex Ante TE for the BLM2 Fund*

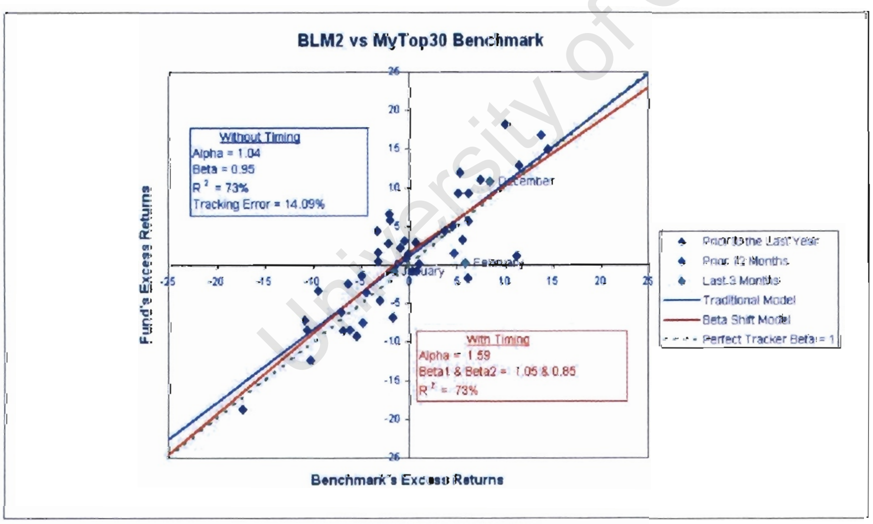

Figure C.18: *Scatter Plot and analysis of the performance of the BLM2 Fund. Several manager diagnostic analyses are shown, including the positive* α *which is what the model tries to achieve*

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