

**AN ANALYSIS OF THE DYNAMIC MULTIPLIER
IN A TWO-REGION ECONOMY**

BY

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1. INTRODUCTION

In the literature on regional economics various models have been developed to study the causes of economic growth and income fluctuations within a region. One of the best known models is that of the export base. The validity and general applicability of this model was first emphasised by North (1955), though Tiebout (1956) subsequently refuted it by claiming that factors other than exports may have a strong effect on the growth of a region. These factors included private investment, government expenditure and productivity increases amongst local industries. The North-Tiebout debate focuses essentially on the difference between the long-run and the short-run sources of regional economic growth.

The North-Tiebout debate was followed by two main approaches: one based on the Keynesian income-expenditure approach and the other on input-output analysis. This essay is concerned with the application of the Keynesian approach within the context of a two-region economy. Section 2 provides a review of the literature on the export base and Keynesian approaches. This is followed, in section 3, by a discussion of Hartman and Seckler's application of dynamic analysis to the regional economy. Section 4 then shows how the Keynesian model can be adapted and applied to a two-region dynamic framework. Finally, in section 5, the stability conditions of the Keynesian model are examined, while final conclusions are drawn in section 6.

2. LITERATURE SURVEY

According to the export base model, the growth of a region depends on the growth of its exports. Adopted from conventional international trade theory, the model distinguishes between export activities and non-export ('service' or 'local') activities, where the former is assumed to be autonomous and the latter represents the endogenous component of total regional income. Hoyt (1937) utilised the idea of an economic base and the related multiplier principle to conduct research on the northern New Jersey housing market. In subsequent studies (1941, 1949), he introduced the concept of the "basic-service ratio" and defined it as the proportion of total employment of a city's basic activities, to that of its service activities. The multiplier is given by the level of (or change in) total employment in both basic and service activities divided by the level of (or change in) basic employment.

The North-Tiebout debate focused on the relative importance of exports in determining the growth of a region. This issue crucially depends on the definition and size of the region, its degree of industrial diversification and the time period involved in the analysis. They agreed that there is no 'ideal' region, and that it is useful to define a region according to its specialisation (e.g., export of machines, clothes, grain) rather than in spatial terms. Tiebout (1956, pp.257-259), however, argued that "the larger the region, the more the dynamic forces causing income change will be found inside its borders"; and that "the higher the incomes in the neighbouring areas, given the propensity to import, the higher the volume of their imports,...". These arguments suggest that the degree of industrial specialisation and the level of exports depend on market conditions within the region itself, and also in its neighbouring regions. In sum, Tiebout (1956, p. 260) maintained that "the concept of the export base is merely

one aspect of a general theory of short-run regional income determination". In reply, North (1955) argued that Tiebout's claims focused mainly on the "determination of income" of a region which is evidently a short-run phenomenon and does not explain the role played by exports in determining regional growth over the **long run**.

Weiss and Gooding (1968), following the economic base approach, constructed a partially disaggregated economic base model to estimate the differential employment multipliers of three independent export activities, namely, the Portsmouth Naval Shipyard (with multiplier equal to 1,55), the Pease Air Force Base (1,35) and private export industries (1,78) in the Portsmouth, New Hampshire, economy. As a consequence of the heterogeneity of the export activities, the resultant multiplier effects vary widely with the sector in which exports expand. These differences arise from variations in the inter-industry linkages associated with particular export activities (indirect effects) and variations in the consumption patterns of workers employed in these activities (induced effects). The results of this study reinforce the advantage of regional base analysis by focusing on the most volatile sectors that influence a region's growth and highlight the differences among the components of a region's exports.

The regional multiplier model has been further extended by several modifications to the multiplicand of the multiplier. This has been done by integrating the effects which arise from regional interactions such as injection leakages (Wilson, 1968; Brownrigg, 1971; Sinclair and Sutcliffe, 1978, 1982, 1983), induced expenditures on investment and import leakages (Fouraker, 1955; Archibald, 1967; Black, 1981), and trade repercussions (or feedback effects) (Brown et al., 1967; Wilson, 1968; Steele, 1969; Black, 1981, 1984). Following Brownrigg's (1971) review of the current literature, the modifications to the multiplicand of the conventional model include:

1. incorporation of injection leakages;
2. taking account of induced investment and induced leakages;
- and
3. accommodating repercussions from inter-regional trade.

The first modification to the conventional multiplier model involves consideration of the leakage effect associated with the initial injection. Wilson (1968) indicated that the injection itself was subject to leakages before undergoing multiplier expansion. If the initial injection in a region takes the form of a business investment (I), its multiplier effect will be reduced by the initial import leakage. Apart from the leakage effect, there is a case where allowance is made for the diversion of expenditure by local inhabitants in favour of the new investment (e.g. a new tourist attraction); accordingly, the portion attributable to local inhabitants should be netted out from the autonomous expenditure. (Johnson and Thomas, 1990; Black et al., 1991)

If we restrict ourselves to import leakages only, the final multiplier is obtained by netting out the import portion from the initial injection:

$$Y = k I (1-m) \tag{1}$$

where Y is the change in the level of the region's income, I the initial injection, k is the value of the regional multiplier and m is the direct leakage resulting from imported capital goods.

This modification shows that, if the value of m is taken to be 0,4, the final multiplied expansion of income $Y = k I (1-m)$ could be less than the original expenditure on investment (I) that gave rise to it.

In attempting to complete the existing regional multiplier model, Brownrigg (1971) maintained that there are different effects associated with different types of injection, e.g., the setup of the new University of Stirling. He distinguished between different types of injection, namely, "the construction expenditure involved in setting up and equipping the project" and the "continuing flow of income arising from the employment given by the project". This distinction can be expressed as follows:

$$Y = k\{ I [p(1 - m) + (1 - p)] \} \quad (2)$$

where p is the proportion representing construction expenditure of the investment project and to which the initial import leakage is applied; and $(1 - p)$ denotes the proportion representing disbursement to those who are employed to maintain the operation of that new project.

Fouraker (1955), in his development of a Keynesian-type two-region model, assumed that a region's imports of consumption goods, imports of investment goods, savings, taxes and induced investment are each a linear function of the region's gross product. He differentiated autonomous investment from induced investment and worked out the corresponding multipliers. Archibald (1967) examined induced investment in a situation where the initial injection took the form of wages and salaries paid to immigrant employees in a new project. He maintained that investment would be induced by the immigrants' expenditure. Wilson (1968) argued that as income and expenditure rise through the multiplier, some additional investment is likely to be induced. These modifications can be expressed as

$$N = n Y \quad (3)$$

where N denotes the induced investment resulting from the change in income of the region under analysis.

Taking induced investment into consideration, equation (2) becomes

$$\begin{aligned}
 Y &= k \{ I [p(1 - m) + (1-p)] + N \} \\
 &= k \{ I [p(1 - m) + (1-p)] + n Y \} \\
 &= k \{ I [p(1 - m) + (1-p)] \} / (1 - kn) \quad (4)
 \end{aligned}$$

We now turn to a two-region framework and assume that the exports of region i (X_i) are the imports of region j (M_j), which are in turn equal to a proportion m_j , of region j 's income; i.e. $X_i = M_j = m_j Y_j$. The repercussion from inter-regional trade is provided by the extra imports of j from region i which arise from i 's extra imports from j . Brown et al., (1967), analysing the secondary boost from increased exports to other regions, concluded that the interregional trade repercussion effect only marginally raises the magnitude of the multiplier. A similar study of Steele (1969) found that the feedback effects resulting from inter-regional trade in some regions are far from insignificant and should not be ignored when considering the multiplier effect of particular projects and policies.

According to the export base model, i 's income level is a multiple of its export base, i.e. $Y_i = k_i X_i$. The exports of region i are now also regarded as the imports of region j , i.e., $Y_i = k_j m_j Y_j$. Furthermore, in Black's (1981) formulation of an economic base type regional multiplier, the economic base of region i , A_i , consists of investment

and exports, e.g., $Y_i = k_i A_i = k_i (I_i + X_i)$, in which the modified multiplicand can be expressed as,

$$Y_i = k_i \{ I_i [p_i(1 - m_i) + (1 - p_i)] + X_i + N_i \} \quad (5)$$

$$\text{Since } X_i = M_j = m_j Y_j$$

$$\text{and } Y_j = k_j X_j = k_j m_j Y_j$$

$$X_i = m_j k_j m_i Y_i$$

and given that,

$$N_i = n_i Y_i$$

substitute into (5) and rearrange of terms give

$$Y_i = k_i \{ I_i [p_i(1 - m_i) + (1 - p_i)] \} / [1 - k_i(m_i k_j m_j + n_i)] \quad (5')$$

In examining the leakage-induced repercussionary (LIR) effect which arises from the initial injection leakages between regions i and j via the multiplier-accelerator process, Black (1981) modified the individual export function under analysis. Provided that an investment project is initiated in region i , the increase in region j 's exports and income is induced by three leakages from the imports of region i , i.e., the leakage associated with the initial injection in region i , $I_i p_i m_i$; the multiplier-induced increase in region i 's imports of consumer goods, $m_i c_i Y_i$; and the initial leakage applicable to induced

investment in region i , $m_i p_i n_i Y_i$. Thus, the exports of region j (X_j) are equal to the sum of $I_i p_i m_i$, $m_i c_i Y_i$ and $m_i p_i n_i Y_i$. On the other hand, if induced investment in region j is given by $N_j = n_j Y_j [p_j (1 - m_j) + (1 - p_j)]$, where n_j is the induced investment coefficient for region j , then the amount leaking to region i is $n_j Y_j p_j m_j$. Hence, region i 's exports (X_i) are equal to the sum of $n_j Y_j p_j m_j$ and region j 's imports of consumer goods from region i , $m_j c_j Y_j$. This modified formulation of the multiplicand of both regions can be expressed as follows:

$$\begin{aligned}
 Y_i &= k_i \{ I_i [p_i (1 - m_i) + (1 - p_i)] + X_i + N_i [p_i (1 - m_i) + (1 - p_i)] \} \\
 &= k_i \{ I_i [p_i (1 - m_i) + (1 - p_i)] + (n_j Y_j p_j m_j + m_j c_j Y_j) + n_i Y_i [p_i (1 - m_i) + (1 - p_i)] \} \\
 &= k_i \{ I_i [p_i (1 - m_i) + (1 - p_i)] + (n_j Y_j p_j m_j + m_j c_j Y_j) \} / \{ 1 - k_i n_i [p_i (1 - m_i) + (1 - p_i)] \} \quad (6)
 \end{aligned}$$

$$\begin{aligned}
 Y_j &= k_j (X_j + N_j) \\
 &= k_j [I_i p_i m_i + m_i c_i Y_i + m_i p_i n_i Y_i + n_j Y_j [p_j (1 - m_j) + (1 - p_j)]] \\
 &= k_j (I_i p_i m_i + m_i c_i Y_i + m_i p_i n_i Y_i) / 1 - k_j n_j [p_j (1 - m_j) + (1 - p_j)] \quad (7)
 \end{aligned}$$

Black (1981), in a Brown-Steele-type analysis, calculated the size of the regional multiplier in three contexts, namely, the Archibald-Wilson multiplier (with no repercussionary effect), the Brown-Steele multiplier (with partial repercussionary effect), and his k'' multiplier (with total repercussionary effect). He found that the last, in which the LIR is taken into consideration, is greater than those having no repercussions or having merely partial repercussions. In particular, the estimated

values of LIR were shown to be greater than those of the Brown-Steele repercussions (Black, 1981, pp. 232-234), and he concluded that "the repercussionary effect associated with injection leakages is likely to raise the value of the regional multiplier to a considerable extent". Black (1981) also argued that "for the large and open regional economy at least, injection leakages may play an important role in determining the magnitude of interregional trade repercussions and hence of the regional multiplier too".

Sinclair and Sutcliffe (1983, pp. 275-280), when commenting on Black's (1981) article, argued that:

1. "one-shot and continuing injections should be considered as separate multiplicand" since induced investment might not be a continuous function of income;
2. Black's (1981) conclusions depend upon the way in which LIR is defined: the value of the regional multiplier varies according to the particular definition of income chosen; and
3. the feedback effects of allowing for a negative injection should also be taken into account, e.g. the relocation of a factory from one area to another involves an injection in one area and a withdrawal from the other.

They conducted a sensitivity analysis of the regional multipliers in the U.K. by investigating different types of feedback to changes in the propensity values. They questioned Black's assertion that the size of the LIR effect is proportional to the value of the propensity to leak from the initial injection. (p.280) They suggested that the definition of the income to be measured and the form in which the initial injection

takes place should be specified in order to compute the multiplier effect of any given injection into a region.

3. THE APPLICATION OF A DYNAMIC ANALYSIS TO THE REGIONAL ECONOMY

In an attempt to extend the previous static model, Hartman and Seckler (1967) investigated the question of whether a region, within a larger economy, is capable of strictly endogenous, self-sustained growth. This involves the incorporation in the model of a lagged investment function so as to examine the necessary conditions for endogenous, self-sustained growth. They formulated an investment function:

$$I_t = K [(C_t - M_t) - (C_{t-1} - M_{t-1}) + (X_t - X_{t-1})] \quad (8)$$

which depends on the change in consumption net of imports of consumer goods and the change in exports. Based on a Keynesian-type regional income equation, they derived the functional form of a region's income:

$$Y_t = X_t + (1-m) K (X_t - X_{t-1}) + b(1-c)[1 + (1-m)K] Y_{t-1} - [b(1-c)(1-m)K] Y_{t-2}. \quad (9)$$

By solving the second-order difference equation, they obtained

$$Y_t = X_t / [1 - b(1-c) + K(1-m)(X_t - X_{t-1}) / (1 - b(1-c) + a_1(R_1)^t + a_2(R_2)^t] \quad (10)$$

where X_t is autonomous exports; b is the marginal propensity to consume, c is the marginal propensity to consume imported goods, m is the marginal propensity to import capital goods. These propensities and K , the investment multiplier, are parameter values; a_1, a_2 are two constants; and R_1, R_2 are roots of the quadratic equation. They concluded that:

1. The export base approach is inadequate as an explanation of regional growth because of the appearance of R_1 and R_2 in equation (10).
2. Based on equation (8), the final income level, Y_t , is generated from exports and induced investment in a dynamic state.
3. the algebraic relation between parameters b, c, m and K , i.e. $b(1-c) \geq 4K(1-m) / [1+K(1-m)]^2$ in this model reveals the unrealistic claim for endogenous, positive growth. Hence, 'a region may not be able to generate endogenous growth because income effects are leaked out to other regions'. (p.102)

Accordingly, positive growth requires that the continuous growth of the export sector outweigh the income effects that are leaked out to other regions; in turn, this reaffirms North's argument on the importance of the export-base.

4. A TWO-REGION GROWTH MODEL

It is the objective of this essay to expand the Hartman-Seckler (1967) model within the context of a two-region economy by admitting interregional trade.

To begin with, the regional income identity in both regions is specified below:

$$Y_i = C_i + I_i + X_i - M_i \quad (11)$$

$$Y_j = C_j + I_j + X_j - M_j \quad (12)$$

where Y_i , Y_j , C_i , C_j , I_i , I_j , X_i , X_j , M_i and M_j are, respectively, income, consumption, investment, exports and imports in region i and region j .

Consumption is assumed to be a direct proportion of income

$$C_i = c_i Y_i \quad (13)$$

$$C_j = c_j Y_j \quad (14)$$

where c_i and c_j are the respective marginal propensities to consume in region i and region j .

It is assumed further that a regional investment project (e.g. the setup of an economic export zone) is launched in region i . A proportion, p_i , of the initial total expenditure, A_i , is spent in setting up the hardware and facilities and the rest, $1-p_i$, is used to hire workers for the operation of this project. As before, the proportion p_i is associated with an initial leakage resulting from the use of imported equipment. Obviously, the induced investment, N_i , which results from the change in income over time can also fall into the abovementioned partition. Thus, the investment function of region i is expressed as:

$$I_i = (A_i + N_i) [p_i (1-m_i) + (1-p_i)] \quad (15)$$

$$N_i = n_i(Y_{i,t} - Y_{i,t-1}) \quad (16)$$

On the other hand, the investment function in region j results merely from the change in income over time,

$$I_j = N_j [p_j (1-m_j) + (1-p_j)] \quad (17)$$

$$N_j = n_j(Y_{j,t} - Y_{j,t-1}) \quad (18)$$

Imports of region i consist of:

1. the import leakage associated with the initial expenditure, $A_i p_i m_i$;
2. current imports of consumer goods as well as capital goods in region i , $m_i Y_i$;
and
3. the import leakage applicable to induced investment from the change in income level, $n_i(Y_{i,t} - Y_{i,t-1}) p_i m_i$.

Thus, the import function of region i has the form:

$$M_i = X_j = A_i p_i m_i + m_i Y_i + n_i (Y_{i,t} - Y_{i,t-1}) p_i m_i \quad (19)$$

and that of region j is:

$$M_j = X_i = m_j Y_j + n_j (Y_{j,t} - Y_{j,t-1}) p_j m_j \quad (20)$$

By substituting equations (13),(15),(16) and (19) into equation (11); and (14),(16), (18) and (20) into (12) we get

$$Y_{i,t} = \{A_i r_i - n_i r_i Y_{i,t-1} + (m_j + n_j p_j m_j) Y_{j,t} - (n_j p_j m_j) Y_{j,t-1}\} / (1 - c_i + m_i - n_i r_i) \quad (21)$$

$$Y_{j,t} = \{A_i p_i m_i - n_j r_j Y_{j,t-1} + (m_i + n_i p_i m_i) Y_{i,t} - (n_i p_i m_i) Y_{i,t-1}\} / (1 - c_j + m_j - n_j r_j) \quad (22)$$

where

$$r_i = q_i - p_i m_i = 1 - 2p_i m_i \quad (23)$$

$$q_i = [p_i(1 - m_j) + (1 - p_i)] \quad (24)$$

$$r_j = q_j - p_j m_j = 1 - 2p_j m_j \quad (25)$$

$$q_j = [p_j(1 - m_j) + (1 - p_j)] \quad (26)$$

By solving the above simultaneous first-order difference equations (21) and (22), we get (for details see appendix):

$$Y_{i,t} = A_1 (b_1)^t + A_2 (b_2)^t + D \quad (27)$$

$$Y_{j,t} = A_3 (b_1)^t + A_4 (b_2)^t + E \quad (28)$$

where

$$b_1, b_2 = \{-(e + e' + f'g + fg') \pm [(e + e' + f'g + fg')^2 - 4(1 - ff')(ee' - gg')]\}^{1/2} / 2(1 - ff');$$

$$e = n_i r_i / (1 - c_i + m_i - n_i r_i)$$

$$e' = n_j r_j / (1 - c_j + m_j - n_j r_j)$$

$$f = (m_j + n_j p_j m_j) / (1 - c_i + m_i - n_i r_i)$$

$$f' = (m_i + n_i p_i m_i) / (1 - c_j + m_j - n_j r_j)$$

$$g = n_j p_j m_j / (1 - c_i + m_i - n_i r_i)$$

$$g' = n_i p_i m_i / (1 - c_j + m_j - n_j r_j)$$

$$D = A_i r_i (1 + e') - A_i p_i m_i (g - f) / (1 + e)(1 + e') - (g - f)(g' - f')$$

$$E = A_i p_i m_i (1 + e) - A_i r_i (g' - f') / (1 + e)(1 + e') - (g - f)(g' - f')$$

A_1, A_2, A_3 and A_4 are constants obtained from solving the simultaneous equations given the initial values of $A_i, Y_{i,0}$ and $Y_{j,0}$; while

$$0 < c, e, e', f, f', g, g', m, n, p, q, r < 1 \text{ for } i \text{ and } j;$$

$$0 < (1 - c_i + m_i - n_i r_i) < 1$$

and $0 < (1 - c_j + m_j - n_j r_j) < 1.$

5. GENERAL RESULTS

The characteristic roots, b_1 and b_2 , emerge in three forms, namely,

1. distinct real roots, when $(e + e' + fg' + f'g)^2 > 4(1 - ff')(ee' - gg')$;
2. repeated real roots, when $(e + e' + fg' + f'g)^2 = 4(1 - ff')(ee' - gg')$; and
3. complex roots, when $(e + e' + fg' + f'g)^2 < 4(1 - ff')(ee' - gg')$

In terms of the Samuelson model (1939), the relationships between $(e + e' + fg' + f'g)^2$ and $4(1 - ff')(ee' - gg')$ can be examined in order to determine the corresponding conditions for convergence and divergence of the time paths. Samuelson (1939, pp. 75-78) provided a graphical presentation of all possible situations under different

solution forms. In view of the large number of parameters in the present model, however, such a presentation is not feasible here. Nevertheless, the analysis of the time paths of $Y_{i,t}$ and $Y_{j,t}$ can be undertaken by examining the relationships between different values of $(e+e'+fg'+f'g)^2$ and $4(1-ff')(ee'-gg')$. Specifically, the two characteristic roots b_1 and b_2 are related to each other by the following two equations:

$$b_1 + b_2 = -(e+e'+fg'+f'g)/(1-ff') < 0 \quad (29)$$

$$b_1 b_2 = (ee'-gg')/(1-ff') \in (0, 1) \quad (30)$$

Based on the two equations, we observe that

$$(1-b_1)(1-b_2) = 1-(b_1+b_2)+b_1 b_2 \quad (31)$$

$$= 1 + [(e+e'+fg'+f'g) + (ee'-gg')]/(1-ff') \quad (32)$$

$$= (1-ff' + e+e'+fg'+f'g + ee'-gg')/(1-ff') > 0 \quad (33)$$

Now consider Case 1, where the two roots b_1 and b_2 are real and distinct. If this happens then the product $b_1 b_2$ is positive and hence no oscillation can occur. From (33), one can infer that neither b_1 nor b_2 can be equal to one; for otherwise $(1-b_1)(1-b_2)$ would be zero, in violation of the inequality indicated under Case 1. Hence, the only possibility is that both b_1 and b_2 are positive so that the time paths of $Y_{i,t}$ and $Y_{j,t}$ are convergent.

The analysis of Case 2 is similar. By practically identical reasoning, one can conclude that the repeated root b can only turn out to be a positive fraction in this model; that

is, $0 < b < 1$. The time paths of $Y_{i,t}$ and $Y_{j,t}$ are again nonoscillatory and convergent.

In Case 3, with complex roots, we have stepped fluctuation since the characteristic roots are conjugate complex and take the form

$$b_1, b_2 = u \pm vi \quad (34)$$

where $u = -(e+e'+f'g+fg')/2(1-ff')$

and $v = [4(1-ff')(ee'-gg')-(e+e'+f'g+fg')^2]^{1/2} / 2(1-ff')$.

The absolute value of the complex roots, $R = (ee'-gg') / (1-ff')$, is also less than one. Thus the time paths of $Y_{i,t}$ and $Y_{j,t}$ are also convergent, although characterized by stepped fluctuation.

Since the time paths of $Y_{i,t}$ and $Y_{j,t}$ have identical b_1 and b_2 , they must both either diverge or converge according to the values of b_1 and b_2 . In the present situation, the conditions for convergence or divergence should be considered in terms of the values of parameters such as $p_i, m_i, n_i, c_i, p_j, m_j, n_j$ and c_j in that b_1 and b_2 are both expressed in terms of these parameters.

The three principal results of the model are as follows:

1. The business cycle of income of both regions exhibits the same periodicity,

2. The amplitude of fluctuation depends on the value of the parameters p_i , m_i , n_i , c_i , p_j , m_j , n_j and c_j ; while the level to which Y_i and Y_j converge depends on $Y_{i,0}$, $Y_{j,0}$, A_i , p_i , m_i , n_i , c_i , p_j , m_j , n_j and c_j ; and
3. the model is applicable to the business cycle of regional income, irrespective of whether short-run or long-run analysis is conducted.

6. CONCLUSION

Following Hartman and Seckler (1967), a model of a two-region economy in a dynamic framework is constructed here. Unlike in Hartman and Seckler, the effects of induced investment and inter-regional trade on regional income are accommodated. These additions, on the one hand, extend and improve upon Hartman & Seckler's study of the regional multiplier; on the other, they change the dynamic forces applicable to regional economies. The present model derives the regional multiplier from autonomous investment which in turn leads to induced changes in regional exports. This contrasts with the Hartman-Seckler model in which the multiplier effect is derived from an autonomous change in exports. Furthermore, the model developed here includes consideration of the fact that both the initial and the induced import leakages to other regions will lead to induced increases in exports in the region under consideration. Given these extensions to the model, the multiplier-accelerator effect proves to be greater than that in the Hartman-Seckler model.

The stability conditions of the improved framework are also investigated in this essay, and it is found that, as is the case with Hartman-Seckler, the time paths of both regions are convergent. This reinforces their argument that regions may not be able to achieve "endogenous, self-sustained growth". Although the extended model allows for induced investment and trade repercussions, the additional leakages account for the fact that regional economies are highly unlikely to achieve endogenous growth. An expansion of this study to business cycle analysis is also a noteworthy avenue for further research.

Notwithstanding the importance of the above two-region framework, the ultimate aim in this literature should be its extension to one of multi-regions in a general equilibrium setting. The subtle questions of existence and stability of general equilibrium should be the first two objectives towards the construction of a general theory of regional economic growth. The study of various economic features (e.g., Pareto optimality in trading and economic policy) can then be seriously undertaken within such a broad framework.

A. APPENDIX

The simultaneous first-order difference equations are the following:

$$Y_{i,t} = \{A_i r_i - n_i r_i Y_{i,t-1} + (m_j + n_j p_j m_j) Y_{j,t} - (n_j p_j m_j) Y_{j,t-1}\} / (1 - c_i + m_i - n_i r_i) \quad (1)$$

$$Y_{j,t} = \{A_j p_j m_j - n_j r_j Y_{j,t-1} + (m_i + n_i p_i m_i) Y_{i,t} - (n_i p_i m_i) Y_{i,t-1}\} / (1 - c_j + m_j - n_j r_j) \quad (2)$$

where

$$r_i = q_i - p_i m_i = 1 - 2p_i m_i$$

$$q_i = [p_i(1 - m_i) + (1 - p_i)]$$

$$r_j = q_j - p_j m_j = 1 - 2p_j m_j$$

$$q_j = [p_j(1 - m_j) + (1 - p_j)]$$

$$e = n_i r_i / (1 - c_i + m_i - n_i r_i)$$

$$e' = n_j r_j / (1 - c_j + m_j - n_j r_j)$$

$$f = (m_j + n_j p_j m_j) / (1 - c_i + m_i - n_i r_i)$$

$$f' = (m_i + n_i p_i m_i) / (1 - c_j + m_j - n_j r_j)$$

$$g = n_j p_j m_j / (1 - c_i + m_i - n_i r_i)$$

$$g' = n_i p_i m_i / (1 - c_j + m_j - n_j r_j)$$

$$k = A_i r_i / (1 - c_i + m_i - n_i r_i)$$

$$l = A_j p_j m_j / (1 - c_j + m_j - n_j r_j)$$

Hence, equations (1) and (2) may be rewritten as,

$$Y_{i,t+1} + e Y_{i,t} - f Y_{j,t+1} - g Y_{j,t} = k \quad (3)$$

$$Y_{j,t+1} + e' Y_{j,t} - f' Y_{i,t+1} + g' Y_{i,t} = l \quad (4)$$

The general solution of these difference equations consists of two components: a particular integral, which is any solution of the complete nonhomogenous equations and a complementary function which is the general solution of the reduced equations (e.g. Chiang (1984)). Thus,

$$Y_{i,t+1} + e Y_{i,t} - f Y_{j,t+1} - g Y_{j,t} = 0 \quad (5)$$

$$Y_{j,t+1} + e' Y_{j,t} - f' Y_{i,t+1} + g' Y_{i,t} = 0 \quad (6)$$

Let us start with the complementary function. One can attempt a solution of the form

$$Y_{i,t} = m b^t; Y_{j,t} = n b^t$$

and obtain,

$$m b^{t+1} + e m b^t - f n b^{t+1} - g n b^t = 0 \quad (7)$$

$$n b^{t+1} + e' n b^t - f' m b^{t+1} + g' m b^t = 0 \quad (8)$$

which yields

$$(1 - f f') b^2 + (e + e' + f' g + f g') b + e e' - g g' = 0$$

with solutions of b_1 and b_2 ,

$$b_1, b_2 = \{-(e + e' + f'g + fg') \pm [(e + e' + f'g + fg')^2 - 4(1 - ff')(ee' - gg')]\}^{1/2} / 2(1 - ff')$$

$$\text{Let } Y_{i,t+1} = Y_{i,t} = D; \quad Y_{j,t+1} = Y_{j,t} = E$$

and rearrange equations (5) and (6),

$$(1+e) Y_{i,t} + (g-f) Y_{j,t} = k$$

$$(g'-f') Y_{i,t} + (1+e') Y_{j,t} = l$$

Following the Cramer's rule,

$$D = k(1+e') - l(g-f) / (1+e)(1+e') - (g-f)(g'-f')$$

$$E = l(1+e) - k(g'-f') / (1+e)(1+e') - (g-f)(g'-f')$$

When the complementary function and the particular integral are combined,

$$Y_{i,t} = A_1(b_1)^t + A_2(b_2)^t + D \quad (9)$$

$$Y_{j,t} = A_3(b_1)^t + A_4(b_2)^t + E \quad (10)$$

where A_1 , A_2 , A_3 and A_4 are coefficients obtained from solving the above simultaneous equations given the initial values of $Y_{i,0}$, $Y_{j,0}$ and A_0 .

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