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CANONICAL QUANTIZATION AND QUANTUM CHROMODYNAMICS IN A CAVITY

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ABSTRACT

The canonical quantization formalism is applied to the Lagrange density of chromodynamics in a general covariant gauge. The physical states are characterized by their BRS-invariance. We develop the quantum theory of the interacting fields in the Dirac picture, based on the Gell-Mann and Low Theorem and the Dyson expansion of the time evolution operator. Subsequently, confinement is introduced phenomenologically by imposing, on the quark, gluon and ghost field operators, the linear boundary conditions of the M.I.T. bag model at the surface of a spherically symmetric and static cavity. Based on this formalism, we calculate, in the Feynman gauge, all non-divergent Feynman diagrams of second order in the strong coupling constant g. Explicit values of the matrix elements are given for low-lying quark and gluon cavity modes.

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APPENDICES

2. THE CLASSICAL THEORY

2.1 Basics

In order to establish the notation, let us start with a brief introduction to chromodynamics, the classical field theory that is based on the nonabelian gauge group $SU(3)_{colour}$. This renormalizable field theory can be derived from the locally gauge invariant Lagrange density [2]

$$\mathcal{L}_{QCD} = \bar{\psi}(i\gamma_\mu D^\mu - M)\psi - 1/4\, \vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu} \quad . \tag{2.1}$$

The interactions between quarks and gluons are determined by the covariant derivative which depends on the strong coupling constant g

$$D^\mu\psi = (\partial^\mu - ig\vec{\lambda}/2\cdot\vec{A}^\mu)\psi \quad , \tag{2.2}$$

and the gluon self-interaction originates from the term containing the chromoelectromagnetic field $\vec{F}^{\mu\nu}$ which can be expressed in terms of the real gluon fields $\vec{A}^\mu$ as

$$\vec{F}^{\mu\nu} = \partial^\mu\vec{A}^\nu - \partial^\nu\vec{A}^\mu + g\vec{A}^\mu \times \vec{A}^\nu \quad . \tag{2.3}$$

The antisymmetric field strength tensor $\vec{F}^{\mu\nu}$ and the covariant derivative D^μ are connected by the Bianchi identity

$$[D^\mu, D^\nu] = -ig\,\frac{\vec{\lambda}}{2}\,\vec{F}^{\mu\nu} \quad . \tag{2.4}$$

In eqs. (2.1) to (2.4) we have made use of the eight-dimensional scalar and vector products operating in the colour space of the gluons

$$\vec{A}\cdot\vec{B} = \sum_{a=1}^{8} A_a B_a \tag{2.5}$$

and

$$(\vec{A} \times \vec{B})_a = \sum_{b,b'=1}^{8} f_{abb'} A_b B_{b'} \quad , \tag{2.6}$$

respectively. The indices a, b and b' describe the eight colour degrees of freedom of the gluon, the $f_{abb'}$ being the structure constants of $SU(3)_{colour}$, and the λ_a denote the eight Gell-Mann matrices.

The symbol ψ denotes a large column consisting of the complex quark fields $\psi_{cf\alpha}$, where the labels c, f and α stand for the various colour (c = 1,2,3), flavour (f = 1,...,6) and Dirac (α = 1,2,3,4) indices. The mass matrix M is diagonal in these labels and depends only on the flavour label of the quarks i.e.

$$M_{c,f,\alpha;c',f',\alpha'} = \delta_{cc'}\delta_{ff'}\delta_{\alpha\alpha'}m_f \quad , \tag{2.7}$$

where m_f is the mass of the quark with flavour f.

The Lagrange density (2.1) is invariant under the following local gauge transformation of the fields $\psi(x)$ and $\vec{A}^\mu(x)$:

$$\begin{aligned} \psi(x) &\to \psi'(x) &&= U(x)\psi(x) \\ \frac{\vec{\lambda}}{2}\cdot\vec{A}_\mu(x) &\to \frac{\vec{\lambda}}{2}\cdot\vec{A}'_\mu(x) &&= U(x)\frac{\vec{\lambda}}{2}\cdot\vec{A}_\mu(x)\, U^{-1}(x) + \frac{i}{g}\, U(x)\partial_\mu U^{-1}(x) \, , \end{aligned} \tag{2.8}$$

where U(x) is a $SU(3)_{\text{colour}}$-matrix (3 x 3 matrix with $U^{-1}(x) = U^+(x)$, det U(x) = 1) which can be chosen differently at different space-time points.

2.2 Gauge Fixing and BRS-Invariance

Our aim will be to quantize the classical theory of chromodynamics. It is, however, well known, that the Lagrange density (2.1) is not suitable for quantization. This is related to the gauge freedom (2.8) as can be seen either in the path integral formalism or, in the case of canonical quantization, by the vanishing of the conjugate momentum $\vec{\Pi}^0$ of $\vec{A}^0$. The gauge invariance (2.8) of the Lagrange density (2.1) has to be violated. We use the standard approach and add to (2.1) the so-called covariant gauge fixing term

$$\mathcal{L}_{\text{Fix}} = -\frac{\lambda}{2}(\partial_\mu\vec{A}^\mu)\cdot(\partial_\nu\vec{A}^\nu) \quad . \tag{2.9}$$

Here, λ is a real parameter characterising the gauge.

We now want to find a substitute for the broken local gauge invariance. This will turn out to be the so-called Becchi-Rouet-Stora (BRS) invariance,[11,12] which is usually introduced with the help of path integrals. The following derivation may seem more explicit.

Under a local gauge transformation with $U(x) = \exp[-i\varepsilon\frac{\vec{\lambda}}{2}\cdot\vec{\omega}(x)]$ where ε

2.3 The Lagrange and the Hamilton densities

Let us adopt as the new Lagrange density, $\mathcal{L}$, the following one that differs from (2.14) by a four divergence

$$\mathcal{L} = \bar{\psi}(i\gamma_\mu D^\mu - M)\psi - \tfrac{1}{2}i\partial_\mu(\bar{\psi}\gamma^\mu\psi) - 1/4\, \vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu} - \frac{\lambda}{2}(\partial_\mu\vec{A}^\mu)(\partial_\nu\vec{A}^\nu)$$
$$- i(\partial_\mu\vec{\chi})(\mathcal{D}^\mu\vec{\omega}) \quad . \tag{2.21}$$

The BRS-transformation of the fields, which we have found to be[12]

$$\delta_\varepsilon\psi = -i\varepsilon\,\frac{\vec{\lambda}}{2}\cdot\vec{\omega}\psi$$
$$\delta_\varepsilon\vec{A}_\mu = -\frac{\varepsilon}{g}\mathcal{D}_\mu\vec{\omega}$$
$$\delta_\varepsilon\vec{\omega} = \tfrac{1}{2}\varepsilon\,\vec{\omega}\times\vec{\omega}$$
$$\delta_\varepsilon\vec{\chi} = i\varepsilon\,\frac{\lambda}{g}\,\partial_\mu\vec{A}^\mu \tag{2.22}$$

changes $\mathcal{L}$ by

$$\delta_\varepsilon\mathcal{L} = \varepsilon\,\frac{\lambda}{g}\,\partial_\mu[(\partial_\nu\vec{A}^\nu)\cdot\mathcal{D}^\mu\vec{\omega}] \, , \tag{2.23}$$

thus leaving the action invariant.
The BRS-invariance of the action replaces the violated local gauge invariance. Even though it is a global symmetry (ε is space-time independent), it is sufficient to guarantee the validity of the Ward-Slavnov-Taylor identities[7,8] that are necessary for the renormalisability[9-12] of the corresponding quantum theory. The BRS parameter ε and the Faddeev-Popov ghost fields $\vec{\omega}$ and $\vec{\chi}$ are Grassmann numbers

$$\{\varepsilon,\varepsilon\} = \{\varepsilon,\omega_a\} = \{\varepsilon,\chi_a\} = 0$$
$$\{\omega_a,\omega_b\} = \{\omega_a,\chi_b\} = \{\chi_a,\chi_b\} = 0 \, . \tag{2.24}$$

Further, they commute with the real gluon fields $\vec{A}_\mu$ and anticommute with the quark fields ψ.

The factor i multiplying the ghost term in (2.21) ensures that the Lagrange density $\mathcal{L}$ is real in the classical theory (corresponding to a hermitean Lagrangean in the quantum theory), if we use the convention[13]

$$\omega_a{}^* = \omega_a \ , \qquad \chi_a{}^* = \chi_a$$

$$(\omega_a\chi_b)^* = \chi_b{}^*\omega_a^* = -\omega_a\chi_b \ , \tag{2.25}$$

i.e. ω_a and χ_a are real Grassmann-numbers. For consistency of the BRS-transformation with this hermicity assignement, the parameter ε in (2.22) is required to be an imaginary Grassmann-number

$$\varepsilon^* = -\varepsilon$$

$$(\varepsilon\omega_a)^* = \omega_a{}^*\varepsilon^* = \varepsilon\omega_a \ . \tag{2.26}$$

An important property of the BRS-transformation, worth noticing already at this stage, is its nilpotency. Using the relations (2.22), it is easily verified that for any field ϕ

$$\delta_{\varepsilon_1}(\delta_{\varepsilon_2}\phi) = 0 \ , \tag{2.27}$$

even though the product $\varepsilon_1\varepsilon_2$ does not vanish in general ($\varepsilon_1 \neq \varepsilon_2$).

The Lagrange density (2.21) can be separated in a g-independent or "free" part which describes the "free" quark, gluon and ghost fields

$$\mathcal{L}_0(\phi_i,\partial_\mu\phi_i) = \bar{\psi}(i/2\ \gamma_\mu \overleftrightarrow{\partial}^\mu - M)\psi - 1/4(\partial_\mu\vec{A}_\nu - \partial_\nu\vec{A}_\nu)\cdot(\partial^\mu\vec{A}^\nu - \partial^\nu\vec{A}^\mu)$$

$$- \lambda/2\ \partial_\mu\vec{A}^\mu\cdot\partial_\nu\vec{A}^\nu - i\partial_\mu\vec{\chi}\cdot\partial^\mu\vec{\omega} \ , \tag{2.28}$$

with $\overleftrightarrow{\partial}^\mu = \vec{\partial}^\mu - \overleftarrow{\partial}^\mu$, and a g-dependent or "interaction part

$$\mathcal{L}_{int}(\phi_i,\partial_\mu\phi_i) = g\bar{\psi}\gamma_\mu\frac{\vec{\lambda}}{2}\psi\cdot\vec{A}^\mu - \tfrac{1}{2}g(\partial^\mu\vec{A}^\nu - \partial^\nu\vec{A}^\mu)\cdot(\vec{A}_\mu \times \vec{A}_\nu)$$

$$- 1/4\ g^2(\vec{A}^\mu \times \vec{A}^\nu)\cdot(\vec{A}_\mu \times \vec{A}_\nu) - ig(\partial_\mu\vec{\chi})\cdot(\vec{A}^\mu \times \vec{\omega}) \tag{2.29}$$

which describes the 2-quark-1-gluon, 3-gluon, 4-gluon and 2-ghost-1-gluon couplings, respectively.

By applying the Euler-Lagrange equations to the Lagrange density (2.21) we readily arrive at the Dirac equations for the quark fields

$$(i\gamma_\mu D^\mu - M)\psi = 0$$

$$\bar{\psi}(i\gamma_\mu\overleftarrow{D}^{\mu *} + M) = 0 \ , \tag{2.30}$$

and for the corresponding field equations for the gluon field we obtain

$$\mathcal{D}_\nu \vec{F}^{\nu\mu} + g\bar{\psi}\gamma^\mu \frac{\vec{\lambda}}{2}\psi + \lambda\partial^\mu\partial_\nu\vec{A}^\nu + ig(\partial^\mu\vec{\chi}) \times \vec{\omega} = 0 \quad , \tag{2.31}$$

where the last two terms originate from the gauge fixing and ghost terms, respectively. By varying the Lagrange density (2.21) with respect to the ghost fields, we arrive at the field equations for the ghost fields

$$\partial_\mu \mathcal{D}^\mu \vec{\omega} = 0 \tag{2.32}$$

$$\mathcal{D}_\mu \partial^\mu \vec{\chi} = 0 \, . \tag{2.33}$$

In order to write down the Hamilton density, we need to evaluate the conjugate momenta of the interacting quark, gluon and ghost fields. For the quark fields we readily obtain

$$\pi = \frac{\partial \mathcal{L}}{\partial \dot{\psi}} = \frac{i}{2}\psi^+$$

$$\bar{\pi} = \frac{\partial \mathcal{L}}{\partial \dot{\bar{\psi}}} = -\frac{i}{2}\bar{\psi}^+ \tag{2.34}$$

and for the gluon fields we arrive at

$$\vec{\Pi}^k = \frac{\partial \mathcal{L}}{\partial \dot{\vec{A}}_k} = \vec{F}^{ko} \qquad k = 1,2,3$$

$$\vec{\Pi}^o = \frac{\partial \mathcal{L}}{\partial \dot{\vec{A}}_o} = -\lambda\partial_\nu\vec{A}^\nu \quad . \tag{2.35}$$

Thus, as we mentioned earlier, without the gauge fixing term present in the Lagrange density (2.21), the zeroth component of the canonical conjugate momentum would vanish and eventually lead to an ill-defined Hamilton density. The canonical conjugate momenta of the ghost fields are given by

$$\vec{X} = \frac{\partial \mathcal{L}}{\partial \dot{\vec{\chi}}} = -i\, \mathcal{D}_o\vec{\omega}$$

$$\vec{\Omega} = \frac{\partial \mathcal{L}}{\partial \dot{\vec{\omega}}} = i\dot{\vec{\chi}} \quad . \tag{2.36}$$

The positive sign in the last equation results from the anticommuting character of the Grassmann fields ω_a and χ_a and the definition of the derivative with respect to a Grassmann field. The dependence of the conjugate momenta on the coupling constant g in eqs. (2.35) and (2.36) arises from the derivative couplings in the original Lagrange density and is often a major source of confusion.

We now turn to the evaluation of the Hamilton density which is defined as

$$\mathcal{H} = \frac{\partial\mathcal{L}}{\partial\dot{\psi}}\dot{\psi} + \dot{\bar{\psi}}\frac{\partial\mathcal{L}}{\partial\dot{\bar{\psi}}} + \dot{\vec{A}}_\mu\cdot\frac{\partial\mathcal{L}}{\partial\dot{\vec{A}}_\mu} + \dot{\vec{\chi}}\cdot\frac{\partial\mathcal{L}}{\partial\dot{\vec{\chi}}} + \dot{\vec{\omega}}\cdot\frac{\partial\mathcal{L}}{\partial\dot{\vec{\omega}}} - \mathcal{L}. \tag{2.37}$$

$\mathcal{H}$ is a function of the fields, the spatial derivatives and the corresponding canonical momenta; it does not depend on the time derivatives of the fields any more. We must thus replace all time derivatives of the fields by the corresponding canonical momenta, some of which will depend on the strong coupling constant g. The Hamilton density can now be split in a part that does not depend on the coupling constant g explicitly

$$\begin{aligned}\mathcal{H}_0 = {} & \bar{\psi}\,(-\tfrac{i}{2}\gamma_k\overleftrightarrow{\partial}^k + M)\psi + \tfrac{1}{4}(\partial_k\vec{A}^\ell - \partial_\ell\vec{A}^k)\cdot(\partial_k\vec{A}^\ell - \partial_\ell\vec{A}^k) + \tfrac{1}{2}\vec{\Pi}^k\cdot\vec{\Pi}^k \\ & - \tfrac{1}{2\lambda}\vec{\Pi}^o\cdot\vec{\Pi}^o + \vec{\Pi}^k\cdot\partial_k\vec{A}^o - \vec{\Pi}^o\cdot\partial_k\vec{A}^k - i\vec{\Omega}\cdot\vec{X} - i\partial_k\vec{\chi}\cdot\partial_k\vec{\omega}\,,\end{aligned} \tag{2.38}$$

and an interaction term which depends linearly and quadratically on the coupling constant g

$$\begin{aligned}\mathcal{H}_{int} = {} & -g\bar{\psi}\gamma_\mu\frac{\vec{\lambda}}{2}\psi\cdot\vec{A}^\mu \\ & -\tfrac{1}{2}g(\partial_k\vec{A}^\ell - \partial_\ell\vec{A}^k)\cdot(\vec{A}^k \times \vec{A}^\ell) - g\vec{\Pi}^k\cdot(\vec{A}^k \times \vec{A}^o) \\ & + \frac{1}{4}g^2(\vec{A}^k \times \vec{A}^\ell)\cdot(\vec{A}^k \times \vec{A}^\ell) \\ & + g\vec{\Omega}\cdot(\vec{A}^o \times \vec{\omega}) + ig\partial_k\vec{\chi}\cdot(\vec{A}^k \times \vec{\omega})\,.\end{aligned} \tag{2.39}$$

The various terms in eq.(2.39) describe the 2-quark-1-gluon-, 3-gluon-, 4-gluon- and 2-ghost-1-gluon-interactions, respectively. Note that, due to the derivative couplings in the Lagrange density (2.29), $\mathcal{H}_{int}$ differs from $-\mathcal{L}_{int}$, and we have

$$\mathcal{H}_{int} = -\mathcal{L}_{int}(\phi_i,\partial_\mu\phi_i) - \tfrac{1}{2}g^2(\vec{A}^o \times \vec{A}^k)\cdot(\vec{A}^o \times \vec{A}^k)\,. \tag{2.40}$$

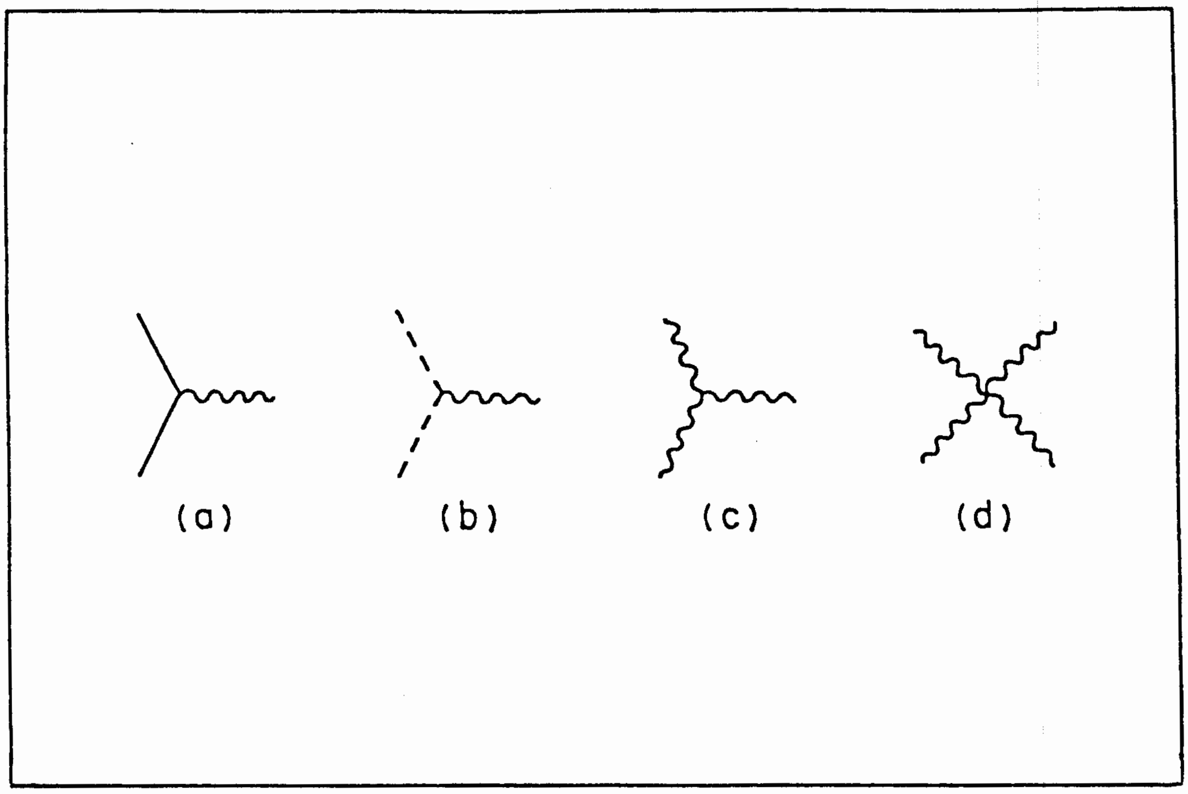


FIGURE 1

The vertices describing the emission (or absorption) of a gluon by (a) a quark, (b) a ghost and (c) a gluon and (d) the four-gluon vertex.

2.4 Conserved Currents

Let us now shortly discuss some of the invariances and conservation laws of the theory given by the Lagrange density (2.21). The usual (abelian) phase transformation of the quark fields alone leads to the conserved Noether current

$$j^\mu_Q = \bar{\psi}\gamma^\mu\psi \quad ; \quad \partial_\mu j^\mu_Q = 0\,, \tag{2.41}$$

thus ensuring the conservation of the quark number. The colour carrying quark current, however, is not conserved; instead it is covariantly (in the sense of the gauge group) conserved

$$\vec{J}^\mu_Q = \bar{\psi}\gamma^\mu\frac{\vec{\lambda}}{2}\psi \quad ; \quad \mathcal{D}_\mu \vec{J}^\mu_Q = 0\,. \tag{2.42}$$

The nonconservation of $\vec{J}^\mu_Q$ reflects the fact that quarks are not the only colour carrying particles of the theory.

Even though the Lagrangean (2.21) is not invariant under a local gauge transformation, a global (space-time independent) gauge transformation is still a good symmetry of the theory. The corresponding Noether current reads

$$\vec{J}^\mu_C = \vec{F}^{\mu\nu} \times \vec{A}_\nu + \lambda(\partial_\nu \vec{A}^\nu) \times \vec{A}^\mu + \vec{J}^\mu_Q + i(\partial^\mu\vec{\chi}) \times \vec{\omega} - i\vec{\chi} \times \mathcal{D}^\mu\vec{\omega}$$

$$\partial_\mu \vec{J}^\mu_C = 0\,, \tag{2.43}$$

and leads to colour-conservation. As we have demonstrated above, the BRS transformation leaves the action of chromodynamics invariant. Taking due care of eq.(2.23), the associated current is easily found to be

$$J^\mu_B = \vec{\omega}\cdot[\mathcal{D}_\nu\vec{F}^{\nu\mu} + g\vec{J}^\mu_Q + \tfrac{1}{2}ig(\partial^\mu\vec{\chi}) \times \vec{\omega}] + \lambda(\mathcal{D}^\mu\vec{\omega})\cdot(\partial_\nu\vec{A}^\nu)\,; \quad \partial_\mu J^\mu_B = 0\,. \tag{2.44}$$

Here, we have omitted a term of the form $\partial_\nu(\vec{\omega}\cdot\vec{F}^{\mu\nu})$ which is conserved by itself and does not contribute to the charge upon integration.[30] The resulting conserved charge Q_B

$$Q_B = \int d^3x\, J^0{}_B = \int d^3x\, \{\vec{\omega}\cdot[\mathcal{D}_k\vec{\Pi}^k + g\psi^+\frac{\vec{\lambda}}{2}\psi + \tfrac{1}{2}g\vec{\Omega} \times \vec{\omega}] - i\vec{X}\cdot\vec{\Pi}^0\} \tag{2.45}$$

is a real Grassmann number and will be called the BRS-charge. It will have a very important function in the quantized theory.

At last, the Lagrangean (2.21) possesses a symmetry transformation which involves the ghost fields $\vec{\omega}$ and $\vec{\chi}$ only and is similar to the abelian phase transformation of the quark fields. Of course, since the ghost fields are real (Grassmann) numbers, we are not allowed to perform a complex phase transformation with them. However, the following scale transformation is admisible[13]

$$\vec{\omega} \to e^{\Theta}\vec{\omega} \ , \quad \vec{\chi} \to e^{-\Theta}\vec{\chi} \ , \tag{2.46}$$

where Θ is a real, space-time independent c-number. As a consequence, we find

$$J^{\mu}{}_{Gh} \ = \ i(\partial^{\mu}\vec{\chi})\cdot\vec{\omega} - i\vec{\chi}\cdot(\mathcal{D}^{\mu}\vec{\omega}); \quad \partial_{\mu}J^{\mu}{}_{Gh} \ = \ 0 \tag{2.47}$$

giving rise to the conservation of the ghost number

$$Q_{Gh} \ = \ \int d^3x \ J^{0}{}_{Gh} \ = \ \int d^3x \ \{\vec{\Omega}\cdot\vec{\omega} + \vec{\chi}\cdot\vec{X}\} \ . \tag{2.48}$$

3. THE QUANTIZED THEORY

3.1 Canonical Quantisation

In the preceding section, we have developed a Hamiltonian formulation of chromodynamics which is suitable for the quantization of the theory. Rather than relying on the usual path integral methods, we want to apply the canonical quantization formalism to the Hamilton densities $\mathcal{H}_0$ and $\mathcal{H}_{int}$ as given in eqs. (2.38) and (2.39). We thus impose the equal time anticommutation and commutation relations for the quark and the hermitean gluon field operators, respectively.

$$\{\psi_{cf\alpha}(\underset{\sim}{x},t),\ \psi_{c'f'\alpha'}{}^{+}(\underset{\sim}{y},t)\} = \delta_{cc'}\delta_{ff'}\delta_{\alpha\alpha'}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) \tag{3.1}$$

$$[A^{\mu}{}_{a}(\underset{\sim}{x},t),\ \Pi^{\nu}{}_{b}(\underset{\sim}{y},t)] = ig^{\mu\nu}\delta_{ab}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) \quad . \tag{3.2}$$

The hermitean ghost field operators must satisfy anticommutation relations, since they are described in the classical theory by real anticommuting colour octet and spin zero Grassmann fields

$$\{\omega_a(\underset{\sim}{x},t),\ \Omega_b(\underset{\sim}{y},t)\} = -i\delta_{ab}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) \tag{3.3}$$

$$\{\chi_a(\underset{\sim}{x},t),\ X_b(\underset{\sim}{y},t)\} = -i\delta_{ab}\delta^{(3)}(\underset{\sim}{x}-\underset{\sim}{y}) \quad . \tag{3.4}$$

Here it is understood that all commutators of the gluon field operators, all anticommutators of the quark and ghost field operators and all commutators involving two different types of field operators, which have not been written down explicitly, vanish.

As a consistency check, we can evaluate the commutation relations of the field operators and the corresponding canonical momenta with the Hamilton operator defined as

$$H = H_0(t) + H_{int}(t) = \int d^3x\ \mathcal{H}_0(\underset{\sim}{x},t) + \int d^3x\ \mathcal{H}_{int}(\underset{\sim}{x},t) \ . \tag{3.5}$$

Here $\mathcal{H}_0(\underset{\sim}{x},t)$ and $\mathcal{H}_{int}(\underset{\sim}{x},t)$ are formally given by (2.38) and (2.39) but now the fields must be interpreted as field operators. If the quantization rules are correct, these commutators and anticommutators must yield Heisenberg equations of motion that are equivalent to the Euler-Lagrange equations for the field operators. Indeed, using the anticommutation relations (3.1), the commutators for the quark field operators turn out to be

$$[\psi,H] \;=\; \gamma^{o}(-i\gamma^{k}\partial_{k}\psi + M\psi - g\gamma^{\mu}\vec{A}_{\mu}\cdot\frac{\vec{\lambda}}{2}\,\psi) \;=\; i\dot{\psi} \tag{3.6}$$

$$[\psi^{+},H] \;=\; -i\partial_{k}\bar{\psi}\gamma^{k} - \bar{\psi}M + g\bar{\psi}\gamma^{\mu}\vec{A}_{\mu}\cdot\frac{\vec{\lambda}}{2} \;=\; i\dot{\psi}^{+} \tag{3.7}$$

which are equivalent to the Dirac equations (2.30). Based on the commutation relations for the gluon field operators (3.2), we arrive at

$$[\vec{A}^{o},H] \;=\; i(-\partial_{k}\vec{A}^{k} - \frac{1}{\lambda}\vec{\Pi}^{o}) \;=\; i\dot{\vec{A}}^{o} \tag{3.8}$$

$$[\vec{A}^{k},H] \;=\; i(-\vec{\Pi}^{k} - \partial_{k}\vec{A}^{o} + g\vec{A}^{k}\times\vec{A}^{o}) \;=\; i\dot{\vec{A}}^{k} \;, \tag{3.9}$$

consistent with the definition of the canonical conjugate momenta (2.35). Moreover, the commutators of the canonical momenta with H yield

$$[\vec{\Pi}^{o},H] \;=\; i(-\mathcal{D}^{k}\vec{\Pi}^{k} + g\bar{\psi}\gamma^{o}\,\frac{\vec{\lambda}}{2}\,\psi + g\vec{\Omega}\times\vec{\omega}) \;=\; i\dot{\vec{\Pi}}^{o} \tag{3.10}$$

$$[\vec{\Pi}^{k},H] \;=\; i(\mathcal{D}^{\ell}\vec{F}^{k\ell} + \partial_{k}\vec{\Pi}^{o} - g\vec{A}^{o}\times\vec{\Pi}^{k} + g\bar{\psi}\gamma^{k}\,\frac{\vec{\lambda}}{2}\psi - ig(\partial_{k}\vec{\chi})\times\vec{\omega}) = i\dot{\vec{\Pi}}^{k} \tag{3.11}$$

which are equivalent to the field equations for the gluons as given in eqs.(2.31). Finally, using the anticommutators (3.3) and (3.4), the ghost field operators are easily shown to satisfy

$$[\vec{\omega},H] \;=\; -\vec{X} - ig\vec{A}^{o}\times\vec{\omega} \;=\; i\dot{\vec{\omega}} \tag{3.12}$$

$$[\vec{\chi},H] \;=\; \vec{\Omega} \;=\; i\dot{\vec{\chi}} \;, \tag{3.13}$$

consistent with the definition of the canonical conjugate momenta (2.36). Similarly, the commutators of the canonical momenta with the Hamilton operator turn out to be

$$[\vec{\Omega},H] \;=\; \mathcal{D}_{k}\partial^{k}\vec{\chi} + ig\vec{\Omega}\times\vec{A}^{o} \;=\; i\dot{\vec{\Omega}} \tag{3.14}$$

$$[\vec{X},H] \;=\; -\partial_{k}\mathcal{D}^{k}\vec{\omega} \;=\; i\dot{\vec{X}} \tag{3.15}$$

which are equivalent to the field equations for the ghosts as given in eqs. (2.32) and (2.33).

The BRS-transformation (2.22) can be realized, in the quantized theory, with the help of the corresponding generator: the BRS-charge, eq.(2.45). Using the canonical commutation and anticommutation relations (3.1) to (3.4), we can show that

$$[i\varepsilon Q_B, O] = g\delta_\varepsilon O \tag{3.16}$$

for any field operator O. Of course, $\delta_\varepsilon O$ is given by eqs.(2.22). Note that ε anticommutes with the operators $\psi, \vec{\omega}$ and $\vec{\chi}$.
In analogy with the decomposition of the Hamiltonian (3.5), let us separate Q_B into

$$Q_B = Q_o(t) + Q_1(t). \tag{3.17}$$

Here, $Q_o(t)$ is independent of the strong coupling constant g

$$Q_o(t) = \int d^3x[\vec{\omega}\cdot\partial_k\vec{\Pi}^k - i\vec{\chi}\cdot\vec{\Pi}^o] \tag{3.18}$$

and $Q_1(t)$ is proportional to g:

$$Q_1(t) = g\int d^3x\vec{\omega}\cdot[\vec{A}_k \times \vec{\Pi}^k + \psi^+\frac{\vec{\lambda}}{2}\psi + \tfrac{1}{2}\vec{\Omega}\times\vec{\omega}]. \tag{3.19}$$

The nilpotency (2.27) of the BRS-transformation is now reflected in the anticommutators

$$\begin{aligned}
\tfrac{1}{2}\{Q_o(t), Q_o(t)\} &= Q_o{}^2(t) = 0 \\
\{Q_o(t), Q_1(t)\} &= 0 \\
\tfrac{1}{2}\{Q_1(t), Q_1(t)\} &= Q_1{}^2(t) = 0 \\
\tfrac{1}{2}\{Q_B, Q_B\} &= Q_B{}^2 = 0
\end{aligned} \tag{3.20}$$

whereas the commutators with the Hamiltonians

$$\begin{aligned}
[Q_o(t), H_o(t)] &= 0 \\
[Q_o(t), H_{int}(t)] &= -[Q_1(t), H_o(t)] \\
[Q_1(t), H_{int}(t)] &= 0
\end{aligned} \tag{3.21}$$

assure the BRS-invariance of the Hamilton operator (3.5).

Evaluating the commutators of the ghost number operator Q_{Gh}, eg.(2.48), with the various field operators, we find that the only nonvanishing results are given by

$$[Q_{Gh},\vec{\omega}] = i\vec{\omega}\ , \quad [Q_{Gh},\vec{\chi}] = -i\vec{\chi}$$

$$[Q_{Gh},\vec{\Omega}] = -i\vec{\Omega}\ , \quad [Q_{Gh},\vec{X}] = i\vec{X}. \tag{3.22}$$

As a consequence, the eigenvalues of the hermitean operator Q_{Gh} will turn out to be purely imaginary. This rather strange fact is related to the indefinite metric of the Fock space. We will comment on this in section 3.3. The equations (3.22) imply that the fields $\vec{\omega}$ and $\vec{X}$ carry ghost number $N_{Gh} = -iQ_{Gh} = 1$ while $\vec{\chi}$ and $\vec{\Omega}$ carry ghost number $N_{Gh} = -iQ_{Gh} = -1$. In this notation, the BRS charge has ghost number $N_{Gh} = 1$

$$[Q_{Gh},Q_o(t)] = iQ_o(t)$$

$$[Q_{Gh},Q_1(t)] = iQ_1(t) \tag{3.23}$$

and the Hamilton operator satisfies $N_{Gh} = 0$

$$[Q_{Gh},H_o(t)] = [Q_{Gh},H_{int}(t)] = 0 \tag{3.24}$$

due to the conservation of the ghost number.

3.2 The Interaction Picture

With the quantization, the fields have become operators in the Heisenberg picture which satisfy the Heisenberg equations of motion with the full Hamilton operator H in the commutator, i.e.

$$i\frac{\partial}{\partial t}\,O(\underset{\sim}{x},t) = [O(\underset{\sim}{x},t),\ H]\ . \tag{3.25}$$

The state vectors that define the Fock space in which the operators act are time independent in the Heisenberg picture

$$i\frac{\partial}{\partial t}\,|\psi\rangle = 0. \tag{3.26}$$

It is useful to transform all the state vectors and the field operators into the interaction or Dirac picture using a unitary transformation U(t) in the Fock space. This transformation satisfies the differential equation

$$i\frac{\partial}{\partial t}\,U(t) = U(t)\,H_{int}(t). \tag{3.27}$$

Contrary to the unitary transformation $U(t)$, the time evolution operator acts completely in the Dirac picture. Using the time independence of the Heisenberg state vectors (3.26) and eq.(3.28), we arrive at

$$|\hat{\psi}(t)\rangle = U(t,t_0)|\hat{\psi}(t_0)\rangle , \tag{3.44}$$

justifying the name "time evolution operator" for $U(t,t_0)$. Of course, eqs.(3.44) and (3.42) lead together to the Schrödinger equation (3.31). The full quantum theory is now contained in the operator $U(t,t_0)$. The solution of the differential equation (3.42) with the initial condition (3.43) is given by Dyson's expansion in terms of n-dimensional integrals which involve timeordered T-products of $\hat{H}^{\varepsilon}_{int}(t)$

$$U^{\varepsilon}(t,t_0) = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{t_0}^{t} dt_1 \ldots \int_{t_0}^{t} dt_n T(\hat{H}^{\varepsilon}_{int}(t_1) \ldots \hat{H}^{\varepsilon}_{int}(t_n)). \tag{3.45}$$

Here we have introduced the usual adiabatic switching on of the interaction

$$\hat{H}^{\varepsilon}_{int}(t) = e^{-\varepsilon|t|}\, \hat{H}_{int}(t) , \tag{3.46}$$

where ε is a small positive quantity which makes it possible to study the limits of $U(t,t_0)$ for $t,t_0 \to \pm\infty$, as well.

We now want to determine the eigenstates and eigenvalues of the full Hamilton operator

$$\hat{H}(0) = \hat{H}_0 + \hat{H}_{int}(0) . \tag{3.47}$$

Let $|\hat{\Phi}_k\rangle$ be a complete and orthonormal set of eigenvectors of the non-interacting Hamiltonian $\hat{H}_0$ in the Dirac picture, $E_k^{(0)}$ being the eigenvalues, i.e.

$$\hat{H}_0|\hat{\Phi}_k\rangle = E_k^{(0)}|\hat{\Phi}_k\rangle . \tag{3.48}$$

If the state vector given by

$$|\hat{\Psi}_k\rangle = \lim_{\varepsilon\to 0} \frac{U^{\varepsilon}(0,-\infty)|\hat{\Phi}_k\rangle}{\langle\hat{\Phi}_k|U^{\varepsilon}(0,-\infty)|\hat{\Phi}_k\rangle} \tag{3.49}$$

exists to all orders, then, due to the Gell-Mann and Low theorem[14] $|\hat{\Psi}_k\rangle$ is an eigenstate of the full Hamilton operator $\hat{H}(0)$ with the energy E_k, i.e.

$$\hat{H}(0)|\hat{\Psi}_k\rangle = (\hat{H}_0 + \hat{H}_{int}(0))|\hat{\Psi}_k\rangle = E_k|\hat{\Psi}_k\rangle \quad , \tag{3.50}$$

Multiplying this equation with $\langle\hat{\Phi}_k|$ from the left, and using the hermiticity property of H_0, we immediately obtain

$$E_k - E_k^{(0)} = \frac{\langle \hat{\Phi}_k|\hat{H}_{int}(0)|\hat{\Psi}_k\rangle}{\langle \hat{\Phi}_k|\hat{\Psi}_k\rangle} \tag{3.51}$$

for the difference of the energy eigenvalues in the interacting and noninteracting systems, respectively. Moreover, introducing the eigenvectors (3.49) we easily arrive at

$$E_k - E_k^{(0)} = \lim_{\varepsilon\to 0} \frac{\langle \hat{\Phi}_k|\hat{H}_{int}(0)U^{\varepsilon}(0,-\infty)|\hat{\Phi}_k\rangle}{\langle \hat{\Phi}_k|U^{\varepsilon}(0,-\infty)|\hat{\Phi}_k\rangle} . \tag{3.52}$$

Similarly, we can expand the eigenvectors of the interacting system in terms of the eigenstates of the noninteracting system, i.e.

$$|\hat{\Psi}_k\rangle = \lim_{\varepsilon\to 0} \sum_{\ell=0}^{\infty} \frac{\langle \hat{\Phi}_\ell|U^{\varepsilon}(0,-\infty)|\hat{\Phi}_k\rangle}{\langle \hat{\Phi}_k|U^{\varepsilon}(0,-\infty)|\hat{\Phi}_k\rangle} |\hat{\Phi}_\ell\rangle \tag{3.53}$$

which also follows from eq.(3.49). Finally, using Dyson's expansion (3.45) we can readily write down the energy shifts due to the interaction as

$$E_k - E_k^{(0)} = \lim_{\varepsilon\to 0} \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{-\infty}^{0} dt_1 \ldots \int_{-\infty}^{0} dt_n$$

$$\langle \hat{\Phi}_k| T(\hat{H}^{\varepsilon}_{int}(0)\hat{H}^{\varepsilon}_{int}(t_1)\ldots\hat{H}^{\varepsilon}_{int}(t_n))|\hat{\Phi}_k\rangle_{\text{connected}}, \tag{3.54}$$

where, for convenience, $\hat{H}^{\varepsilon}_{int}(0)$ has been placed inside the timeordered T-product. The Wick decomposition of eq.(3.49) will eventually lead to an expansion in terms of Feynman diagrams in coordinate space. If we restrict this sum to the so-called connected diagrams, the denominator which was present in eq.(3.52) must be omitted.

3.3 The Physical States

It is well known that covariant canonical quantisation of gauge fields inevitably leads to a Fock space V with indefinite metric. This is most easily seen by realizing that, in a symbolic notation, the commutator

$$[A^{\mu}, \Pi^{\nu}] = ig^{\mu\nu} \tag{3.55}$$

Based mainly on the algebra

$$Q_B^2 = 0$$

$$[Q_{Gh},Q_B] = iQ_B \ , \tag{3.60}$$

V_P can be shown to be positive semi-definite.

Of course, any state vector $|\Psi'\rangle$ that can be expressed as

$$|\Psi'\rangle = Q_B|\Psi\rangle \tag{3.61}$$

with an arbitrary $|\Psi\rangle$ trivially satisfies the physicality condition (3.59) due to the nilpotency of the BRS-charge. However it has zero norm and is orthogonal to V_P. Thus, it cannot contribute to any measurable quantity. Moreover, the state $|\Psi'_{phys}\rangle$ which is obtained from the physical state $|\Psi_{phys}\rangle$ (3.59) by

$$|\Psi'_{phys}\rangle = O|\Psi_{phys}\rangle \tag{3.62}$$

where the operator O either commutes or anticommutes with the BRS-charge

$$[Q_B,O] = 0 \quad \text{or} \quad \{Q_B,O\} = 0 \ , \tag{3.63}$$

is again a physical state. An operator satisfying one of eqs.(3.63) is called an observable.

It is interesting to note that the field equation (2.31) for the gluons can also be expressed in the form[13]

$$\partial_\nu \vec{F}^{\nu\mu} + g\vec{J}_C^\mu + \{Q_B,\mathfrak{D}^\mu\vec{\chi}\} = 0 \tag{3.64}$$

where $\vec{J}_C^\mu$ is the colour carrying current (2.43). We deduce from this representation (3.64) that the gluon fields $\vec{A}^\mu$ obey, in the physical subspace V_P (3.59), the generalized Maxwell equations

$$\langle\psi'_{phys}|\partial_\nu \vec{F}^{\nu\mu} + g\vec{J}_C^\mu|\psi_{phys}\rangle = 0. \tag{3.65}$$

Let us now discuss the condition (3.59) in the Dirac picture, especially with regard to the perturbation expansion (3.45) to (3.54). Using the relations

$$i\frac{\partial}{\partial t}\hat{Q}_o = 0$$

$$[\hat{Q}_o,\hat{H}_{int}(t)] \;=\; -i\,\frac{\partial}{\partial t}\,\hat{Q}_1(t)$$

$$[\hat{Q}_1(t),\hat{H}_{int}(t)] \;=\; 0 \qquad (3.66)$$

that follow from (3.21), it can be shown that the BRS-charge inherits the adiabatic damping factor $e^{-\varepsilon|t|}$ from the Hamiltonian

$$\hat{Q}_B^{\varepsilon}(t) \;=\; \hat{Q}_o + e^{-\varepsilon|t|}\,\hat{Q}_1(t). \qquad (3.67)$$

Since the noninteracting or asymptotic states $|\hat{\Phi}\rangle$, eq.(3.48), correspond to the limit $t \to -\infty$, eq.(3.59) translates into

$$\hat{Q}_o|\hat{\Phi}_{phys}\rangle \;=\; 0. \qquad (3.68)$$

Moreover, the algebra (3.60) guarantees that the state vector $|\hat{\Psi}_{phys}\rangle$ which developes adiabatically from $|\hat{\Phi}_{phys}\rangle$ in (3.68) according to (3.49), cannot destroy the positive semi-definiteness of V_P.

Evaluating the eq.(3.18) in the Dirac picture, we obtain with the help of the field equation (3.34) and the definition of the conjugate momentum $\hat{\pi}^{\mu}$ (3.38)

$$\hat{Q}_o \;=\; \lambda\int d^3x\,\{(\partial^0\hat{\vec{\omega}})(\partial_\nu\hat{\vec{A}}^\nu) - \hat{\vec{\omega}}\cdot\partial^0(\partial_\nu\hat{\vec{A}}^\nu)\}. \qquad (3.69)$$

This form of the asymptotic BRS charge reflects a close relation of the definition of the asymptotic physical subspace V_P (3.68) with the Gupta-Bleuler condition (3.58).

with the two-dimensional surface element $d\Omega$. Hence, if the conserved current j^μ satisfies the boundary condition

$$n_\mu j^\mu(x) = 0 \qquad x \in \partial V, \tag{4.6}$$

the charge (4.4) will be time independent in the cavity.

Inspecting the currents given in section 2.4, it is readily verified that eq.(4.6) holds if we impose the following set of boundary conditions on the field operators in the Heisenberg picture

$$(in_\mu\gamma^\mu - 1)\psi|_{\partial V} = \bar{\psi}(in_\mu\gamma^\mu + 1)|_{\partial V} = 0 \tag{4.7}$$

$$n_\mu \vec{F}^{\mu\nu}|_{\partial V} = n_\mu \vec{A}^\mu|_{\partial V} = n_\mu \partial^\mu(\partial_\nu \vec{A}^\nu)|_{\partial V} = 0 \tag{4.8}$$

$$n_\mu \partial^\mu \vec{\omega}|_{\partial V} = n_\mu \partial^\mu \vec{\chi}|_{\partial V} = 0 \,. \tag{4.9}$$

Eq.(4.7) to (4.9) are essentially those introduced by the M.I.T. group.[16-19] Of course, one could think of more complicated boundary conditions than the above. However, the set (4.7) to (4.9) has the advantage of being linear in the field operators and independent of the strong coupling constant g.

The first of eqs. (4.8) implies that the space integral of $\partial_\mu \vec{F}^{\mu 0}$ over the cavity volume vanishes. Combining this result with the field equation for gluons in the form (3.64) we can express the colour generator $\vec{Q}_c$ as

$$g\vec{Q}_c = g\int_V d^3x\, \vec{J}_c^{\,0} = -\int_V d^3x\, [\partial_\mu \vec{F}^{\mu 0} + \{Q_B, \mathcal{D}^0\vec{\chi}\}] = -\{Q_B, \int_V d^3x\, \mathcal{D}^0\vec{\chi}\}. \tag{4.10}$$

This representation leads immediately to

$$\langle \Psi'_{phys}|\vec{Q}_c|\Psi_{phys}\rangle = 0. \tag{4.11}$$

The colour charge vanishes in the subspace of physical states in the cavity.

Localizing the field operators in a cavity, we are violating translational invariance of the theory. Consequently, the 3-momentum will not be conserved. If we restrict ourselves to a static cavity ($n^0 = 0$), the Hamiltonian will be time-independent and the energy will be conserved. In this static case with a time-independent surface ∂V, the boundary conditions (4.7) to (4.9), which are formulated in the Heisenberg picture, translate easily into the Dirac picture as follows

Using eqs.(3.3), (3.4) and (A35), we can easily show that the only non-vanishing anticommutators are

$$\{\hat{d}_{am},\hat{e}^{+}_{a'm'}\} = -\{\hat{d}^{+}_{am},\hat{e}_{a'm'}\} = i\delta_{aa'}\delta_{mm'} \quad . \tag{4.25}$$

The zero-particle state or the asymptotic vacuum $|\hat{0}\rangle$ is the state with norm one which is annihilated by all the quark, antiquark, gluon and ghost annihilation operators

$$\begin{aligned} \hat{a}_{cn}|\hat{0}\rangle &= \hat{b}_{cn}|\hat{0}\rangle = 0 \\ \hat{c}^{\Sigma}_{am}|\hat{0}\rangle &= \hat{d}_{am}|\hat{0}\rangle = \hat{e}_{am}|\hat{0}\rangle = 0 \\ \langle\hat{0}|\hat{0}\rangle &= 1. \end{aligned} \tag{4.26}$$

The Fock space of asymptotic states which is, by assumption, complete and contains thus all the states of the interacting theory, is obtained in the standard way by applying any combination of creation operators on the zero-particle state (4.26).

Based on the expansions (4.20), (4.23) and (4.24) and the orthogonality of the cavity modes (A35), the g-independent part Q_o of the BRS-charge, eq. (3.69), is readily found to be

$$\hat{Q}_o = -\sum_{am} \Omega^{o}_{m} \left[(\hat{c}^{\mathcal{L}}_{am} - \hat{c}^{o}_{am})^{+} \hat{d}_{am} + \hat{d}^{+}_{am}(\hat{c}^{\mathcal{L}}_{am} - \hat{c}^{o}_{am})\right] . \tag{4.27}$$

Thus, sufficient conditions that an asymptotic state $|\hat{\Phi}\rangle$ fulfills the physicality criterion (3.69) can be stated as

$$(\hat{c}^{\mathcal{L}}_{am} - \hat{c}^{o}_{am})|\hat{\Phi}_{phys}\rangle = 0; \quad \hat{d}_{am}|\hat{\Phi}_{phys}\rangle = 0. \tag{4.28}$$

Here, we see the close relation of eq.(3.69) with the Gupta-Bleuler condition (3.58). Of course, the asymptotic vacuum $|\hat{0}\rangle$ is a physical state due to eq.(4.26).

Similarly we can write down the normal-ordered noninteracting part of the Hamiltonian (2.38) in the Dirac picture as

$$:\hat{H}_o: = \sum_{cn} \varepsilon_n \left[\hat{a}^{+}_{cn}\hat{a}_{cn} + \hat{b}^{+}_{cn}\hat{b}_{cn}\right] + \sum_{\substack{am \\ \Sigma=\mathcal{M},\mathcal{E}}} \Omega^{\Sigma}_{m}\, \hat{c}^{\Sigma+}_{am}\hat{c}^{\Sigma}_{am} + \{\hat{Q}_o,\hat{K}\} \tag{4.29}$$

where the fermionic operator $\hat{K}$ is given by

$$\hat{K} = \frac{i}{2}\sum_{am} [\hat{e}^{+}_{am}(\hat{c}^{\ell}_{am} + \hat{c}^{o}_{am}) - (\hat{c}^{\ell}_{am} + \hat{c}^{o}_{am})^{+}\,\hat{e}_{am}]. \tag{4.30}$$

The last term in eq.(4.28) which represents the contribution to H_0 from the unphysical longitudinal and scalar gluon fields and the ghost fields has been cast into a form which shows explicitly that its matrix elements, taken between the physical states (3.69), vanish.

states with nonvanishing ghost number, since the norm of such a state is zero, and the nominator in eqs.(3.49) to (3.53) vanishes. To satisfy all requirements of eq.(5.2), we take the asymptotic state $|\hat{\Phi}_k\rangle$ to contain only quarks, antiquarks and the two physical degrees of freedom of the gluon: the transverse electric and magnetic polarisation modes.

In general, $E_k^{(o)}$ is degenerate and therefore several orthogonal eigenvectors $|\hat{\Phi}_k\rangle$, $|\hat{\Phi}_{k'}\rangle$, ... belong to this eigenvalue. A linear combination of these vectors is again an eigenvector of $:\hat{H}_o:$ with the same eigenvalue and can be used in the Gell-Mann and Low formula (3.49). We are thus lead to consider the off-diagonal matrix elements of the right hand side of (5.1), as well,

$$V_{k'k} = \langle\hat{\Phi}_{k'}|\hat{H}_{int}(o)|\hat{\Phi}_k\rangle - i \lim_{\varepsilon\to 0_+} \int_{-\infty}^{o} dt \langle\hat{\Phi}_{k'}|T(\hat{H}_{int}(o)\, \hat{H}^{\varepsilon}_{int}(t))|\hat{\Phi}_k\rangle_{connected} \, . \tag{5.3}$$

The energy shifts and the corresponding eigenstates are then obtained by diagonalizing the matrix $V_{k'k}$.

As an example, let us now, for a two-quark-system, calculate the energy shift due to the one-gluon-exchange interaction. The eigenstates of $:\hat{H}_o:$ are given by

$$|\hat{\Phi}_k\rangle = \hat{a}^{+}_{c_1 n_1}\, \hat{a}^{+}_{c_2 n_2}|\hat{0}\rangle \, . \tag{5.4}$$

The part of $\hat{H}_{int}(t)$ that describes the 2-quark - 1-gluon vertex can be obtained from the first term in the Hamilton density (2.39) integrating over the volume

$$\hat{H}_{int}(t) = -g \int d^3x\, \hat{\bar{\psi}}(x)\, \gamma_\mu(\frac{\lambda^a}{2})\hat{\psi}(x)\, \hat{A}^\mu_a(x) + \text{other terms.} \tag{5.5}$$

Inserting this operator into eq.(5.3) and using Wick's theorem to expand the time-ordered into normal-ordered products, we arrive at the matrix element

$$\begin{aligned} V_{k'k} = {} & -ig^2(\gamma_\mu)_{\alpha'\alpha}\, (\delta_\nu)_{\beta'\beta}\, (\frac{\lambda^a}{2})_{c'c}\, (\frac{\lambda^b}{2})_{d'd}\, \delta_{f'f}\, \delta_{g'g} \\ & \lim_{\varepsilon\to 0_+} \int_{-\infty}^{o} dt\, e^{-\varepsilon|t|} \int d^3x\, d^3y \langle\hat{0}|T(\hat{A}^\mu_a(x)\, \hat{A}^\nu_b(y))|\hat{0}\rangle \\ & \langle\hat{\Phi}_{k'}|:\hat{\bar{\psi}}^{(+)}_{c'f'\alpha'}(x)\hat{\psi}^{(+)}_{cf\alpha}(x)\hat{\bar{\psi}}^{(+)}_{d'g'\beta'}(y)\hat{\psi}^{(+)}_{dg\beta}(y):|\hat{\Phi}_k\rangle \, . \end{aligned} \tag{5.6}$$

Here we have picked out the term where the gluon fields are contracted and introduced the coordinates $x = (\underset{\sim}{x},0)$ and $y = (\underset{\sim}{y},t)$. As usual, a summation over all indices occurring repeatedly is understood. Expanding the quark field operators into cavity modes, as given in eqs.(4.15) and (4.18), and using the explicit form of the gluon propagator (B7), we arrive at the matrixelement

$$V_{k'k} = -g^2(\frac{\lambda^a}{2})_{c'c}(\frac{\lambda^a}{2})_{d'd}\delta_{f'f}\delta_{g'g}\ \frac{g^{\Sigma\Sigma}}{2\Omega_m^\Sigma}\ \frac{1}{\Omega_m^\Sigma + \varepsilon_{p'} - \varepsilon_p}$$

$$\int d^3x\bar{u}_{n'}(\underset{\sim}{x})\ \gamma_\mu u_n(\underset{\sim}{x})\ a_m^{\mu\Sigma}(\underset{\sim}{x})\quad \int d^3y\bar{u}_{p'}(\underset{\sim}{y})\gamma_\nu u_p(\underset{\sim}{y})\ a_m^{\nu\Sigma}(\underset{\sim}{y})^*$$

$$\langle \hat{\Phi}_{k'}|\hat{a}^+_{c'n'}\ \hat{a}^+_{d'p'}\ \hat{a}_{cn}\ \hat{a}_{dp}|\hat{\Phi}_k\rangle \quad , \tag{5.7}$$

where the energy denominator arises from the time integration. Thus $V_{k'k}$ can be interpreted as the matrixelement of the two-body operator

$$\hat{V} = g^2(\frac{\lambda^a}{2})_{c'c}(\frac{\lambda^a}{2})_{d'd}\delta_{f'f}\delta_{g'g}\ \frac{g^{\Sigma\Sigma}}{2\Omega_m^\Sigma}\ \frac{Q^\Sigma_{n'nm}\tilde{Q}^\Sigma_{p'pm}}{\Omega_m^\Sigma+\varepsilon_{p'}-\varepsilon_p}\ \hat{a}^+_{c'n'}\ \hat{a}^+_{c'p'}\ \hat{a}_{cn}\ \hat{a}_{dp} \tag{5.8}$$

which describes the one-gluon-exchange interaction between two quarks. Here we have made use of the definition of the quark-gluon vertex functions (C1) and (C1').

In order to describe a complex many-quark system, it is convenient to have the two-body operator (5.8) expressed in first instead of second quantization. The Fock space can be embedded in the space given by the direct product wavefunctions of the quarks. The Pauli principle is taken care of by restricting the Hilbert space to the subspace defined by the antisymmetrized product wavefunctions of the quarks. The two-body operator V_{12} corresponding to $\hat{V}$ and acting on the quantum numbers of the first and second quark can be written as

$$V_{12} = \frac{g^2}{4\pi R}\vec{F}_1\cdot\vec{F}_2\ \sum_J \mu_{12}(J)\ K_{12}(J)\ . \tag{5.9}$$

Here J describes the angular momentum exchanged between the quarks and $\vec{F}_i$ (i=1,2) denotes the colour generator in the fundamental representation. The operators $\mu_{12}(J)$ and $K_{12}(J)$ are defined as two-body operators that act on the radial and angular part of the two-body wavefunction, respectively. These are readily defined in terms of their matrixelements. Using eqs.(C6) and (C7) to decompose the vertex functions $Q^\Sigma_{nn'm}$ in eq.(5.8) into radial and angular parts, we arrive at

$$\langle n_1', n_2' | K_{12}(J) | n_1, n_2 \rangle = (2J+1) \sum_M (-1)^M F_{JM}(n_1', n_1) \, F_{J-M}(n_2', n_2) \ . \tag{5.10}$$

Here $|n_1,n_2\rangle$ denotes the direct product of the Dirac spinors (A2) and the factor $F_{JM}(n',n)$ arises from the angular integration (C8)

$$F_{JM}(n',n) = (-1)^{\mu'+\frac{1}{2}} \hat{j}'\hat{J}\hat{j} \begin{pmatrix} j' & J & j \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} j' & J & j \\ -\mu' & M & \mu \end{pmatrix} \ . \tag{5.11}$$

Similarly we can define

$$\langle n_1', n_2' | \mu_{12}(J) | n_1, n_2 \rangle = \sum_{\substack{N>0 \\ \Sigma}} \frac{g^{\Sigma\Sigma} \eta_\Sigma}{2(2J+1)} S^\Sigma_{n_1' n_1 m} S^\Sigma_{n_2' n_2 m}$$

$$\frac{1}{\Omega^\Sigma_m} \left[\frac{1}{\Omega^\Sigma_m + \varepsilon_{n_1'} - \varepsilon_{n_1}} + \frac{1}{\Omega^\Sigma_m + \varepsilon_{n_2'} - \varepsilon_{n_2}} \right], \tag{5.12}$$

where the matrixelement $S^\Sigma_{nn'm}$ is related to the radial integrals $R^\Sigma_{nn'm}$ defined in eqs.(C9) to (C12)

$$S^\Sigma_{nn'm} = R^\Sigma_{nn'm} \frac{1 - g^{\Sigma\Sigma} \eta_\Sigma (-1)^{\ell+J+\ell'}}{2} \ . \tag{5.13}$$

The second factor in eq.(5.12) governs the parity selection rule, and $g^{\Sigma\Sigma}$ and η^Σ are defined in (A37) and (A43), respectively.

Let us now consider the case where the quarks in the initial and final state carry total angular momentum $j = \frac{1}{2}$, which means that the Dirac quantum number κ takes the values +1 or -1. Using the Wigner-Eckart theorem one can easily show that the only non-vanishing $K_{12}(J)$ are

$$K_{12}(0) = \mathbb{1}_1 \mathbb{1}_2 \quad \text{and} \quad K_{12}(1) = 4 \, \underset{\sim}{S}_1 \cdot \underset{\sim}{S}_2 \ , \tag{5.14}$$

where $\mathbb{1}_i$ and $\underset{\sim}{S}_i$ denote the unit and the spin operator, respectively, acting on the i-th quark. Thus for quarks in the angular momentum state $j = \frac{1}{2}$, we arrive at the well-known expression[18,19]

$$V_{12} = \frac{g^2}{4\pi R} \vec{F}_1 \cdot \vec{F}_2 \left[\mu_{12}(0) + 4\mu_{12}(1) \, \underset{\sim}{S}_1 \cdot \underset{\sim}{S}_2 \right] \tag{5.15}$$

which is very convenient for the calculation of the properties of many-quark systems.

In a similar way we can determine the two-body operators describing the interactions corresponding to all other non-divergent Feynman graphs of order g^2 that do not involve the ghosts. The resulting two-body operators in first or second quantization are presented in appendix D.

5.2 Lowest cavity modes interactions

We now discuss the various two-body interactions between the quarks and gluons. For simplicity, we consider here massless up and down quarks and gluons occupying the lowest cavity modes. The corresponding quantum numbers are $(p,q)J^{\pi}I = (1,0)\frac{1}{2}^{+}\frac{1}{2}$ for the quarks and $(p,q)J^{\pi}I = (1,1)1^{-}0$ for the gluons. Here, the two integers (p,q) characterize the irreducible representation of $SU(3)_{colour}$ and are connected to the dimension and the quadratic Casimir operator in this representation by

$$\begin{aligned} SU(3):\quad N(p,q) &= \tfrac{1}{2}(p+1)(q+1)(p+q+2) \\ C(p,q) &= \frac{1}{3}(p^2+pq+q^2)+p+q. \end{aligned} \tag{5.16}$$

The irreducible representations of $SU(2)_{spin}$ and $SU(2)_{isospin}$ are described by the spin J and the isospin I, respectively. Of course, the dimension and the Casimir operator are given by

$$\begin{aligned} SU(2):\quad N(J) &= 2J+1 \\ C(J) &= J(J+1). \end{aligned} \tag{5.17}$$

The two particles will, in general, be embedded into a larger many-body system. This system must be in a singlet representation of the colour group $SU(3)_{colour}$, in order not to obtain a contradiction with eq.(4.11) which essentially says that physical states in the cavity are colourless. However, this restriction does not apply to the constituents of the many-body system. Thus, the two particles can be coupled to any of the possible quantum numbers. Of course, identical particles must obey the Fermi-Dirac statistics for fermions and the Bose-Einstein statistics for bosons.

In Table 1 we have collected for all diagrams of Fig.2 the corresponding dimensionless interaction operators Δ_{12} which are related to V_{12} by

$$V_{12} = \frac{g^2}{4\pi R}\,\Delta_{12}\quad . \tag{5.18}$$

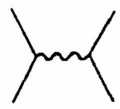
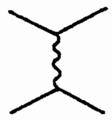
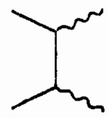
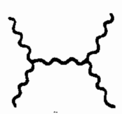
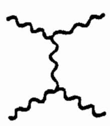
<table>
<tr><th>DIAGRAM</th><th>$\Delta_{12} = \frac{4\pi R}{g^2} V_{12}$</th></tr>
<tr><td></td><td>$F_{12}[\mu_{12}(0) + 4\mu_{12}(1)\ S_{12}]$</td></tr>
<tr><td></td><td>$(\frac{1}{4} - T_{12})(F_{12} + \frac{4}{3})[\mu_{12}(0)(\frac{1}{4} - S_{12}) + \mu_{12}(1)(\frac{3}{4} + S_{12})]$</td></tr>
<tr><td></td><td>$\frac{1}{18}(4F_{12}^2 - 1)[\mu_{12}(1)(\frac{1}{2} - S_{12}) + \mu_{12}(2)(1 + S_{12})]$</td></tr>
<tr><td></td><td>$\frac{1}{18}(4F_{12}^2 + 6F_{12} - 1)[\mu_{12}(1)(\frac{1}{2} + S_{12}) + \mu_{12}(2)(1 - S_{12})]$</td></tr>
<tr><td></td><td>$F_{12}[\mu_{12}(0) + \mu_{12}(1)\ S_{12}]$</td></tr>
<tr><td></td><td rowspan="3">$F_{12}[\rho_{12}(0) + \rho_{12}(1)\ S_{12} + \rho_{12}(2)\ S_{12}^2]$</td></tr>
<tr><td></td></tr>
<tr><td></td></tr>
</table>

TABLE 1

The dimensionless two-body interaction operators Δ_{12} for the various diagrams of Fig. 2. We use the abbreviations $F_{12} = \vec{F}_1 \cdot \vec{F}_2$, $S_{12} = \underset{\sim}{S}_1 \cdot \underset{\sim}{S}_2$ and $T_{12} = \underset{\sim}{T}_1 \cdot \underset{\sim}{T}_2$ for the product of colour, spin and isospin generators, respectively. The matrix elements of the operators $\mu_{12}(J)$ and $\rho_{12}(k)$ are given in Tables 2, 3 & 4.

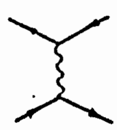
DIAGRAM	(κ_1,κ_2)	$\mu_{12}(0)$	$\mu_{12}(1)$
	$(-,-)$	0,0098 (0)	-0,1770 (-0,1770)
	$(-,+)_+$	0,0353 (0)	-0,1432 (-0,2082)
	$(-,+)_-$	0,0352 (0)	-0,0796 (-0,0146)
	$(+,+)$	0,1321 (0)	-0,1173 (-0,1173)
	$(-,-)$	0 (0)	0,1875 (-0,0124)
	$(-,+)_+$	0 (0)	0,0494 (0,0494)
	$(-,+)_-$	0,1806 (0)	0 (0)
	$(+,+)$	0 (0)	0,1055 (0,0135)

TABLE 2

Matrix elements of the operators $\mu_{12}(J)$ for the interactions via gluon exchange between two quarks, two antiquarks or a quark-antiquark pair and via annihilation into a gluon for the quark-antiquark system. The values are given for massless quarks in the lowest energy cavity state. The numbers in brackets are the contributions from the transverse polarizations of the virtual gluon. The mixed states correspond to $(-,+)_{\pm} = \frac{1}{\sqrt{2}}\left[(-,+) \pm (+,-)\right]$.

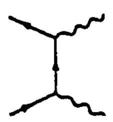
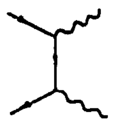
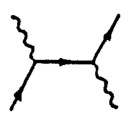
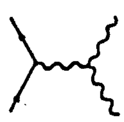
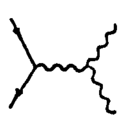
DIAGRAM	(κ,Σ)	$\mu_{12}(1)$	$\mu_{12}(2)$
	$(-,\mathcal{M})$	0,5616	0,0632
	$(-,\mathcal{E})$	0,4240	0,0346
	$(+,\mathcal{M})$	0,3175	0,0411
	$(+,\mathcal{E})$	0,4915	0,0286
	$(-,\mathcal{M})$	-0,3298	0,2101
	$(-,\mathcal{E})$	-0,0116	0,0913
	$(+,\mathcal{M})$	0,1130	0,0673
	$(+,\mathcal{E})$	-0,3076	0,0617
		$\mu_{12}(0)$	$\mu_{12}(1)$
	$(-,\mathcal{M})$	-0,0053 (0)	-0,4900 (-0,4900)
	$(-,\mathcal{E})$	0,0336 (0)	-0,4009 (-0,4009)
	$(+,\mathcal{M})$	-0,0214 (0)	-0,3204 (-0,3204)
	$(+,\mathcal{E})$	0,1347 (0)	-0,3900 (-0,3900)

TABLE 3

Matrix elements of the operators $\mu_{12}(J)$ for the various diagrams of the quark (antiquark)-gluon interaction. The numbers in brackets represent the contributions to the gluon exchange from the transverse polarizations of the virtual gluon.

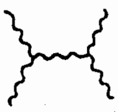
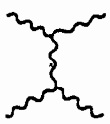
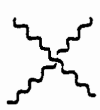
DIAGRAM	(Σ_1,Σ_2)	$\rho_{12}(0)$	$\rho_{12}(1)$	$\rho_{12}(2)$
	$(\mathcal{M},\mathcal{M})$	-0,1045 (0)	-0,2992 (-0,3399)	0,0813 (0)
	$(\mathcal{M},\mathcal{E})_+$	-0,0220 (0,0069)	-0,2968 (-0,4440)	-0,0020 (-0,0052)
	$(\mathcal{M},\mathcal{E})_-$	-0,0358 (-0,0069)	-0,2720 (-0,1280)	0,0084 (0,0052)
	$(\mathcal{E},\mathcal{E})$	0,1382 (0)	-0,3205 (-0,3277)	0,0143 (0)
	$(\mathcal{M},\mathcal{M})$	-0,1963 (0)	0,0981 (0)	0,0981 (0)
	$(\mathcal{M},\mathcal{E})_+$	-0,3241 (-0,0340)	0,1621 (0,0170)	0,1621 (0,0170)
	$(\mathcal{M},\mathcal{E})_-$	-0,0011 (0)	-0,0017 (0)	-0,0005 (0)
	$(\mathcal{E},\mathcal{E})$	-0,2073 (0)	0,1036 (0)	0,1036 (0)
	$(\mathcal{M},\mathcal{M})$	-0,1549	0,1549	0,0774
	$(\mathcal{M},\mathcal{E})_+$	-0,0555	0,0748	0,0513
	$(\mathcal{M},\mathcal{E})_-$	0,0469	-0,0278	-0,0941
	$(\mathcal{E},\mathcal{E})$	-0,1616	0,1616	0,0807

TABLE 4
Matrix elements of the $\rho_{12}(k)$ operators for the gluon-exchange, the annihilation and the four-gluon vertex interactions in a two-gluon system. The numbers in brackets (where given) show the contributions from the transverse polarizations of the virtual gluon. Note that $(\mathcal{M},\mathcal{E})_{\pm} = \frac{1}{\sqrt{2}}\left[(\mathcal{M},\mathcal{E}) \pm (\mathcal{E},\mathcal{M})\right]$.

Tables 2, 3 and 4 contain the matrix elements of the two-body operators μ_{12} or ρ_{12}. For the following discussion, the relevant entries in these tables are the ones with $(\kappa_1,\kappa_2) = (-,-)$ for the quarks and antiquarks and $(\Sigma_1,\Sigma_2) = (\mathcal{M},\mathcal{M})$ for the gluons.

(i) The quark-quark interaction

Since the antiquark-antiquark interaction is, with trivial changes of the quantum numbers, identical to the quark-quark interaction, we will discuss here only the latter. The diquark can occupy the following irreducible representations of the various symmetry groups

$$SU(3)_{\text{colour}}: \quad (1,0) \otimes (1,0) = (0,1)_A \oplus (2,0)_S$$

$$SU(2)_{\text{spin}} \;: \quad \tfrac{1}{2} \otimes \tfrac{1}{2} = 0_A \oplus 1_S$$

$$SU(2)_{\text{isospin}}: \quad \tfrac{1}{2} \otimes \tfrac{1}{2} = 0_A \oplus 1_S \,. \tag{5.19}$$

We have indicated with a subscript whether the representation is symmetric (S) or antisymmetric (A) with respect to the interchange of the two quarks. The two-quark states that are consistent with the Fermi-Dirac statistics have the quantum numbers

$$\begin{array}{lll} (p,q)J^{\pi}I = & (0,1)0^{+}0 & (2,0)0^{+}1 \\ & (0,1)1^{+}1 & (2,0)1^{+}0 \end{array} \tag{5.20}$$

The two-body operator describing the interaction via one-gluon exchange between two quarks or antiquarks, Fig. 2a or 2b respectively, is given by

$$\Delta_{12} = \vec{F}_1\cdot\vec{F}_2(\mu_{12}(0) + 4\mu_{12}(1)\underset{\sim}{S}_1\cdot\underset{\sim}{S}_2) \,. \tag{5.21}$$

This operator is diagonal in the two-particle space and has the matrix elements

$$\Delta = \langle (p,q)J^{\pi}I|\Delta_{12}|(p,q)J^{\pi}I\rangle = \tfrac{1}{2}[C(p,q)-\tfrac{8}{3}]\,[\mu(0) + 2\mu(1)(J(J+1)-\tfrac{3}{2})] \tag{5.22}$$

where $C(p,q)$ is the Casimir operator of $SU(3)_{\text{colour}}$, eq. (5.16). The matrix elements of Δ_{12} are shown in Fig. 3 and also in Table 5.
Note that the colour $\{\bar{3}\} = (0,1)$ interaction is twice as strong as the colour $\{6\} = (2,0)$ interaction. Since the ordinary baryons are usually

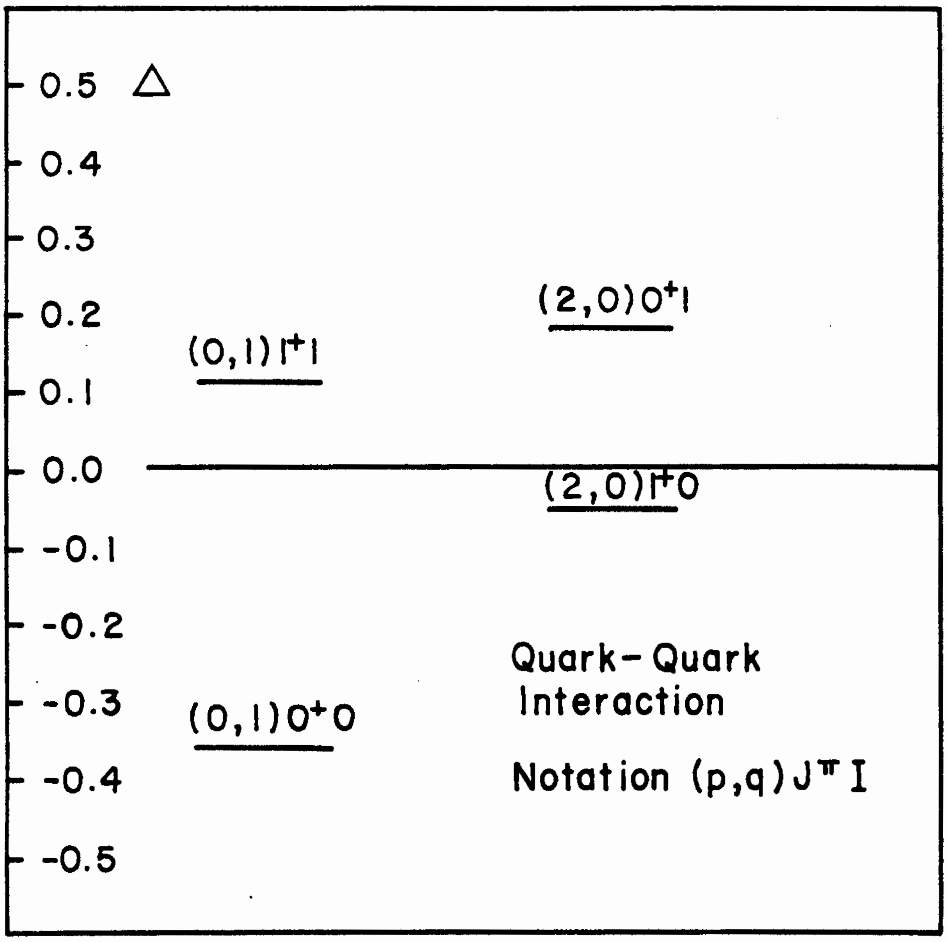


FIGURE 3

Matrix elements Δ of the dimensionless two-body interaction operator Δ_{12} for the quark-quark (antiquark-antiquark) system. See also Table 5.

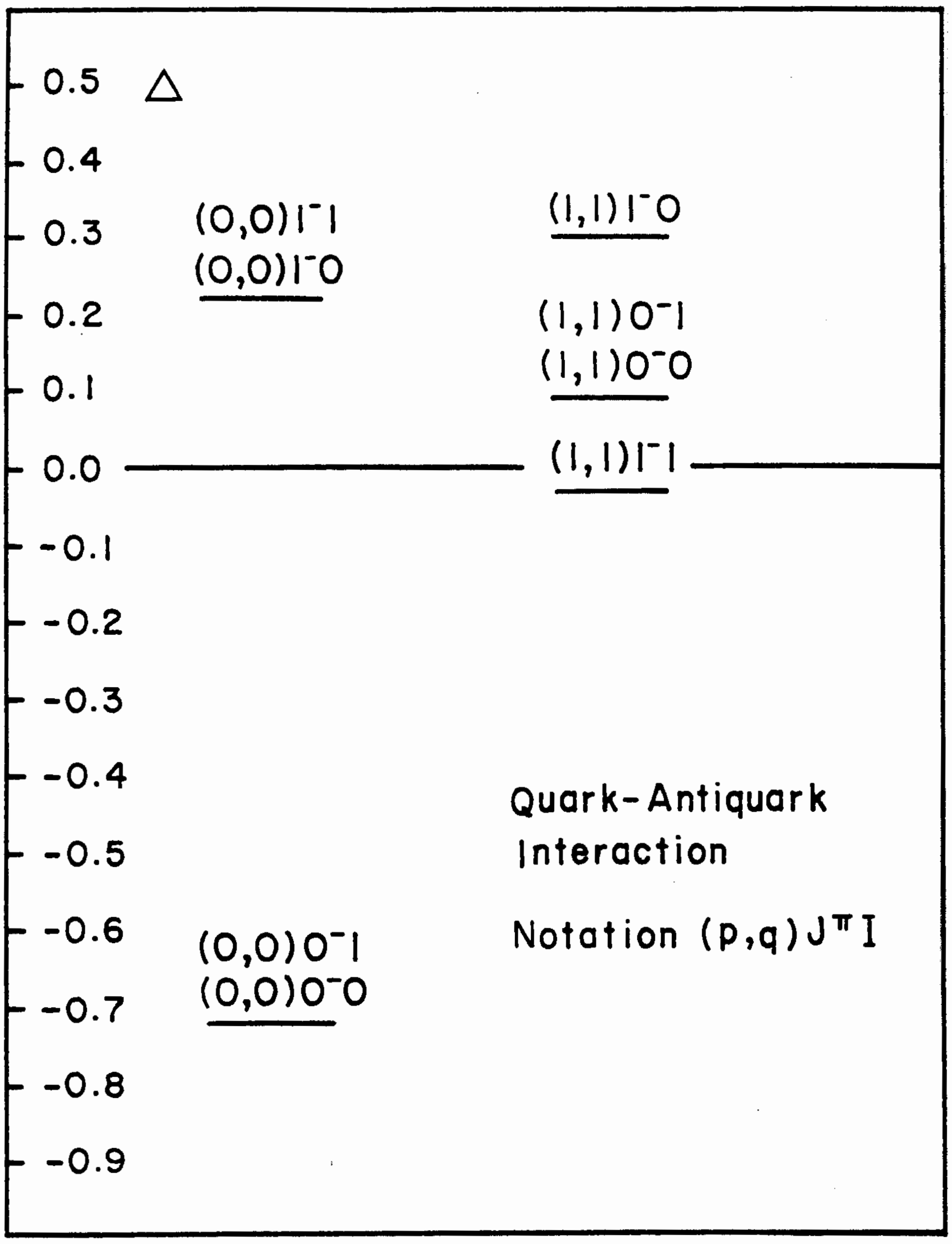


FIGURE 4

Matrix elements Δ of the dimensionless two-body interaction operator Δ_{12} for the quark-antiquark system. See also Table 6.

$(p,q)J^{\pi}I$	
$(0,1)0^{+}0$	-0,3605
$(0,1)1^{+}1$	0,1115
$(2,0)0^{+}1$	0,1803
$(2,0)1^{+}1$	-0,0557

TABLE 5

The dimensionless energy shifts Δ for the quark-quark (antiquark-antiquark) system. See also Fig. 3.

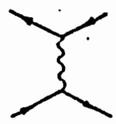
$(p,q)J^{\pi}I$			TOTAL
$(0,0)0^{-}0$	-0,7211	0	-0,7211
$(0,0)0^{-}1$			
$(0,0)1^{-}0$	0,2229	0	0,2229
$(0,0)1^{-}1$			
$(1,1)0^{-}0$	0,0901	0	0,0901
$(1,1)0^{-}1$			
$(1,1)1^{-}0$	-0,0279	0,2813	0,2534
$(1,1)1^{-}1$		0	-0,0279

TABLE 6

The dimensionless energy shifts Δ for the quark-antiquark system. We have separated the contributions from the gluon exchange and the annihilation diagrams. See also Fig. 4.

taken to be colourless three quark states, only the colour $\{\bar{3}\}$ interaction is accessible to experimental tests so far.

(ii) The quark-antiquark interaction

The quark-antiquark system can have the following colour quantum numbers

$$SU(3)_{colour}: \quad (1,0) \otimes (0,1) \; = \; (0,0) \oplus (1,1) \tag{5.23}$$

whereas the spin and isospin products correspond to the quark-quark case (5.19). Since the antiquark is distinguishable from the quark, the exclusion principle does not apply and the eight possible states are

$$\begin{aligned} (p,q)J^{\pi}I \; = \; & (0,0)0^{-}0 \qquad (1,1)0^{-}0 \\ & (0,0)0^{-}1 \qquad (1,1)0^{-}1 \\ & (0,0)1^{-}0 \qquad (1,1)1^{-}0 \\ & (0,0)1^{-}1 \qquad (1,1)1^{-}1 \end{aligned} \tag{5.24}$$

The diagrams contributing to the quark-antiquark interaction are the one-gluon exchange (Fig. 2c) and the virtual annihilation into a gluon (Fig. 2d). The corresponding two-body operator is

$$\Delta_{12} \; = \; \vec{F}_1\cdot\vec{F}_2[\mu_{12}(0) + 4\mu_{12}(1)\underset{\sim}{S}_1\cdot\underset{\sim}{S}_2] + \nu_{12}(1)[\tfrac{1}{4} - \underset{\sim}{T}_1\cdot\underset{\sim}{T}_2][\vec{F}_1\cdot\vec{F}_2+\tfrac{4}{3}][\underset{\sim}{S}_1\cdot\underset{\sim}{S}_2 + \tfrac{3}{4}] \tag{5.25}$$

with the matrix elements

$$\begin{aligned} \Delta \; = \; \langle (p,q)J^{\pi}I|\Delta_{12}|(p,q)J^{\pi}I\rangle \; &= \; \tfrac{1}{2}[C(p,q) - \tfrac{8}{3}][\mu(0) + 2\mu(1)(J(J+1) - \tfrac{3}{2})] \\ &+ \tfrac{1}{4}\nu(1)C(p,q)J(J+1)[1 - \tfrac{1}{2}I(I+1)] \; . \end{aligned} \tag{5.26}$$

In order to avoid misunderstanding, we have denoted with $\nu(1)$ the nonvanishing coefficient of the annihilation diagram, so that $\nu(1) = 0{,}1875$. The matrix elements Δ are shown in Fig. 4. Table 6 contains the contributions to Δ from the gluon exchange and the annihilations diagrams.

Note that the second term in eqs. (5.25) or (5.26) which represents the annihilation diagram, vanishes unless the quark-antiquark pair carries the quantum number of the gluon $(p,q)J^{\pi}I = (1,1)1^{-}0$. For this case we find that the annihilation diagram is in magnitude ten times bigger than the one-gluon exchange diagram and of opposite sign. Without this annihilation diagram the two states $(p,q)J^{\pi}I = (1,1)1^{-}0$ (colour octet ω-meson) and $(p,q)J^{\pi}I = (1,1)1^{-}1$ (colour octet ρ-meson) would be degenerate. The possibility of virtually decaying into a gluon makes the ω^8 heavier than the ρ^8. However, the pion-like state $(p,q)J^\pi I = (0,0)0^-1$ is still degenerate with the eta-like state $(p,q)J^\pi I = (0,0)0^-0$ in "contradiction" to experiment. Of course, we know that the π- and η-mesons are, in real world, more complex objects than quark-antiquark systems consisting of up and down quarks.

(iii) The quark-gluon interaction

As already in the quark-quark case, the antiquark-gluon interaction leads to the same matrix elements for corresponding quantum numbers as the quark-gluon interaction, so that we consider here only the latter. The quark-gluon system can couple to the following quantum numbers

$$\begin{aligned} SU(3)_{colour} &: \quad (1,0) \otimes (1,1) = (1,0) \oplus (0,2) \oplus (2,1) \\ SU(2)_{spin} &: \quad \tfrac{1}{2} \otimes \tfrac{1}{2} = \tfrac{1}{2} \oplus \tfrac{3}{2} \\ SU(2)_{isospin} &: \quad \tfrac{1}{2}. \end{aligned} \tag{5.27}$$

Of course, we do not have to symmetrize or antisymmetrize the direct product states; the possible quantum numbers of the quark-gluon system are

$$\begin{aligned} (p,q)J^\pi I : \quad & (1,0)\tfrac{1}{2}^-\tfrac{1}{2} \qquad (0,2)\tfrac{1}{2}^-\tfrac{1}{2} \qquad (2,1)\tfrac{1}{2}^-\tfrac{1}{2} \\ & (1,0)\tfrac{3}{2}^-\tfrac{1}{2} \qquad (0,2)\tfrac{3}{2}^-\tfrac{1}{2} \qquad (2,1)\tfrac{3}{2}^-\tfrac{1}{2} \end{aligned} \tag{5.28}$$

The interaction between these two particles receives contributions from the direct and exchange Compton diagrams and the one-gluon exchange that are shown in Figs. 2e, 2f and 2g respectively. The corresponding two-body operators can be read off Table 1, the matrix elements of their sum are

$$\begin{aligned} \Delta = & \langle (p,q)J^\pi I|\Delta_{12}|(p,q)J^\pi I\rangle = \\ & \tfrac{1}{18}[4F^2-1]\,[\tfrac{1}{2}\mu^D(1)(\tfrac{15}{4} - J(J+1)) + \tfrac{1}{2}\mu^D(2)(J(J+1) - \tfrac{3}{4})] \\ & + \tfrac{1}{18}[4F^2 + 6F - 1]\,[\tfrac{1}{2}\mu^E(1)(J(J+1) - \tfrac{7}{4}) + \tfrac{1}{2}\mu^E(2)(\tfrac{19}{4} - J(J+1)] \\ & + F[\mu(0) + \tfrac{1}{2}\mu(1)(J(J+1) - \tfrac{11}{4})] \;. \end{aligned} \tag{5.29}$$

Here, F is related to the Casimir operator of $SU(3)_{colour}$ by

$$F = \langle (p,q)|\vec{F}_1\cdot\vec{F}_2|(p,q)\rangle = \tfrac{1}{2}\left[C(p,q) - \tfrac{13}{3}\right] . \tag{5.30}$$

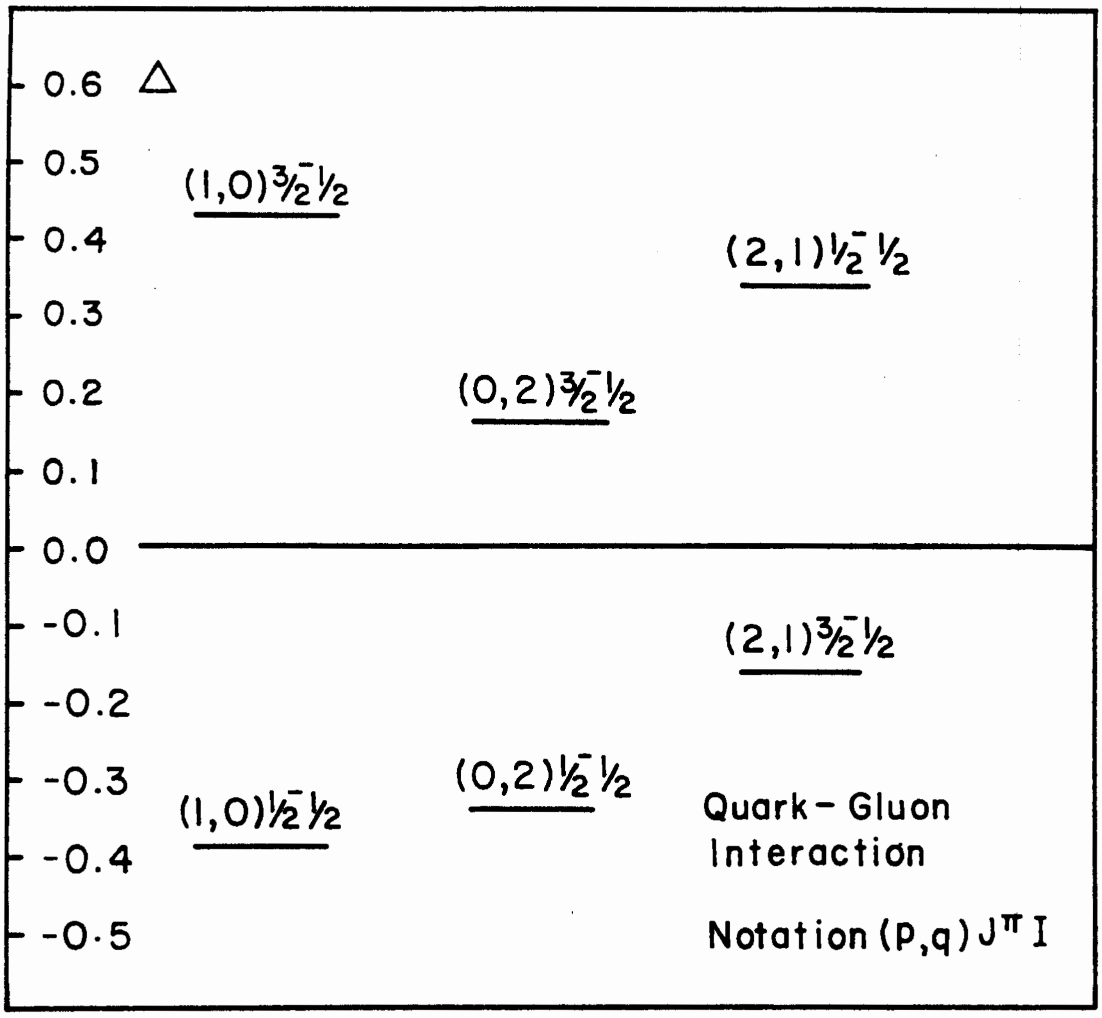


FIGURE 5

Matrix elements Δ of the dimensionless two-body operator Δ_{12} for the quark (antiquark)-gluon system. See also Table 7.

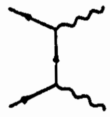
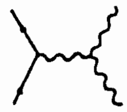
$(p,q)J^{\pi}I$				TOTAL
$(1,0)\frac{1}{2}^{-}\frac{1}{2}$	0,3744	-0,0325	-0,7271	-0,3852
$(1,0)\frac{3}{2}^{-}\frac{1}{2}$	0,0421	0,0125	0,3755	0,4301
$(0,2)\frac{1}{2}^{-}\frac{1}{2}$	0	-0,0975	-0,2424	-0,3399
$(0,2)\frac{3}{2}^{-}\frac{1}{2}$	0	0,0375	0,1252	0,1627
$(2,1)\frac{1}{2}^{-}\frac{1}{2}$	0	0,0975	0,2424	0,3399
$(2,1)\frac{3}{2}^{-}\frac{1}{2}$	0	-0,0375	-0,1252	-0,1627

TABLE 7

The dimensionless energy shifts Δ for the quark(antiquark)-gluon system. We show the contributions from the various Feynman diagrams. See also Fig. 5.

In eq. (5.29), the second, third and fourth line correspond to the direct and the exchange Compton diagrams and the gluon exchange respectively. The shifts of the two-particle energy levels due to these three interactions may be found in Fig. 5. As can be seen in Table 7, the energy shifts are dominated by the contribution from the gluon exchange between the quark and the gluon.

(iv) The gluon-gluon interaction

Two gluons can be accommodated in the following irreducible representations of the groups $SU(3)_{colour}$ and $SU(2)_{spin}$

$$SU(3)_{colour} : \ (1,1) \otimes (1,1) = (0,0)_S \oplus (1,1)_S \oplus (1,1)_A \oplus (3,0)_A \oplus (0,3)_A \oplus (2,2)_S$$

$$SU(2)_{spin} : \ 1 \otimes 1 = 0_S \oplus 1_A \oplus 2_S \ . \tag{5.31}$$

We have again indicated the symmetry property of the representations with respect to the interchange of the two particles. The gluons being bosons, the Bose-Einstein statistics allow only the following nine totally symmetric two-particle states

$$\begin{array}{lllll} (p,q)J^\pi : & (0,0)0^+ & (1,1)0^+ & (3,0)1^+ & (2,2)0^+ \\ & (0,0)2^+ & (1,1)1^+ & (0,3)1^+ & (2,2)2^+ \\ & & (1,1)2^+ & & \end{array} \tag{5.32}$$

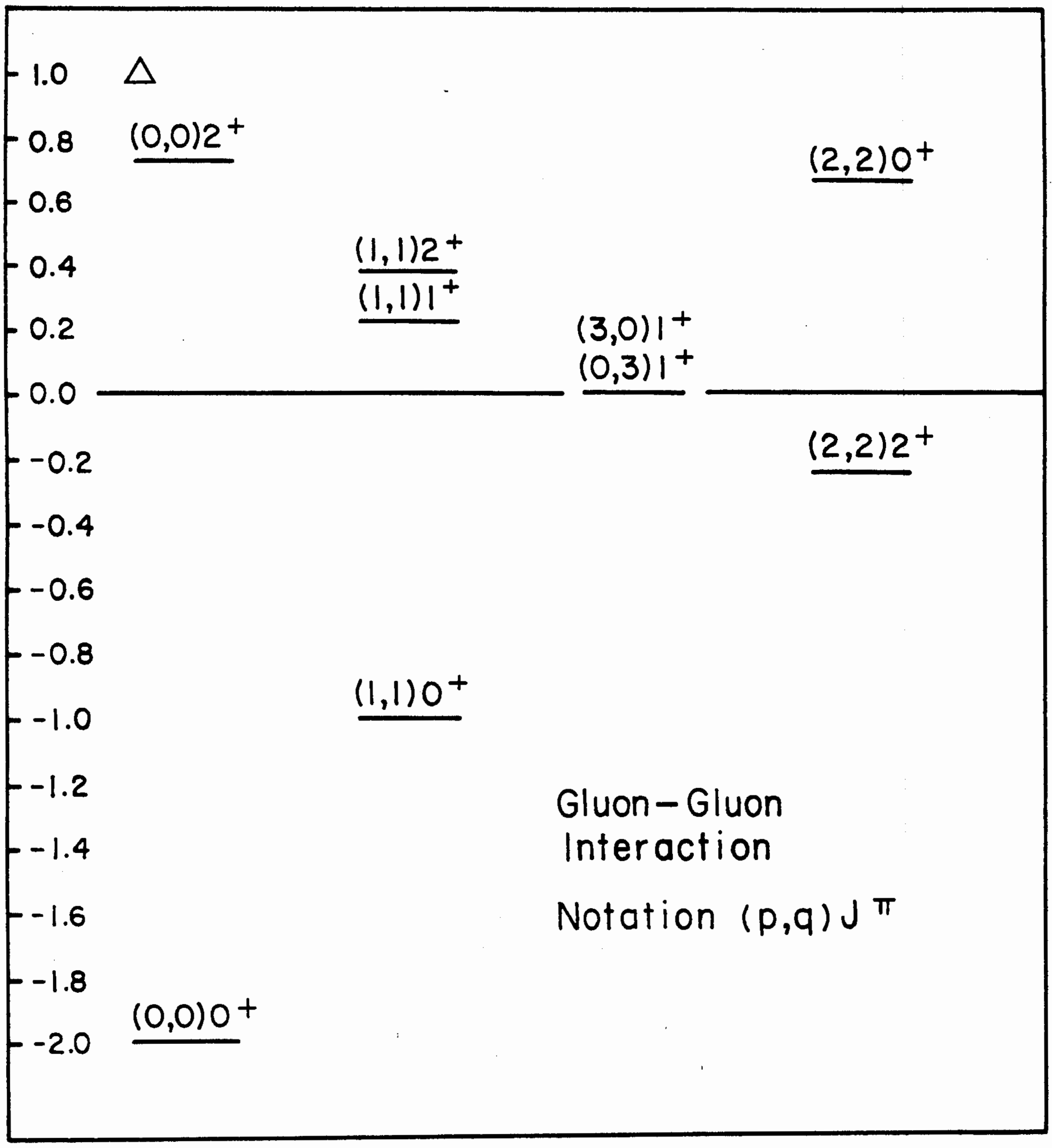
The two-body operators which describe the gluon-gluon interactions through gluon exchange, annihilation and the four gluon vertex are given in all three cases by

$$\Delta_{12} = \vec{F}_1\cdot\vec{F}_2[\rho_{12}(0) + \rho_{12}(1)\underset{\sim}{S}_1\cdot\underset{\sim}{S}_2 + \rho_{12}(2)(\underset{\sim}{S}_1\cdot\underset{\sim}{S}_2)^2] \tag{5.33}$$

and have the matrix elements

$$\Delta = \langle(p,q)J^\pi|\Delta_{12}|(p,q)J^\pi\rangle = \tfrac{1}{2}[C(p,q) - 6][\rho(0) + \tfrac{1}{2}\rho(1)(J(J+1) - 4)$$
$$+ \frac{1}{4}\rho(2)(J(J+1) - 4)^2] \ . \tag{5.34}$$

The coefficients $\rho(k)$ can be read off in Table 4 for the various diagrams. The resulting matrix elements Δ are shown in Fig. 6, Table 8 indicates again the contribution to Δ from the individual diagrams. As compared to the previous cases (i), (ii) and (iii), the two-particle energy shifts due to the interactions in second order perturbation theory are much bigger for



<u>FIGURE 6</u>

Matrix elements Δ of the dimensionless two-body operator Δ_{12} for the gluon-gluon system. See also Table 8.

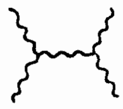
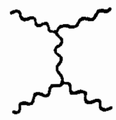
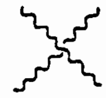
$(p,q)J^{\pi}$				TOTAL
$(0,0)0^{+}$	-2,4575	0	0,4646	-1,9929
$(0,0)2^{+}$	0,9672	0	-0,2323	0,7349
$(1,1)0^{+}$	-1,2289	0	0,2323	-0,9966
$(1,1)1^{+}$	-0,4140	0,2944	0,3485	0,2288
$(1,1)2^{+}$	0,4836	0	-0,1161	0,3675
$(3,0)1^{+}$ $(0,3)1^{+}$	0	0	0	0
$(2,2)0^{+}$	0,8191	0	-0,1549	0,6642
$(2,2)2^{+}$	-0,3224	0	0,0774	-0,2450

TABLE 8

The dimensionless energy shifts Δ for the gluon-gluon system. We show the contributions from the various Feynman diagrams. See also Fig. 6.

the gluon-gluon system. In particular, the state carrying the quantum numbers of the vacuum, $(p,q)J^{\pi} = (0,0)0^{+}$, is so much lowered in energy that it could become degenerate with the vacuum. Of course, whether this degeneracy is present depends on the value of the strong "hyperfine" constant α_S and the (up to now not reliably calculated) gluon self-energies amongst others. Due to the vanishing of $\vec{F}_1\cdot\vec{F}_2$ in the $SU(3)_{colour}$ representations $(p,q) = (3,0)$, $(0,3)$, the two gluons do not interact in the decuplet and anti-decuplet cases.

Our results are consistent with those of Carlson, Hanson and Peterson (CHP) in ref. (26) for their gluon exchange diagram denoted by 3g, corresponding to the diagram of Fig. 2 in this work if only the transverse polarization modes of the virtual gluon are taken into account. Using the transformation of the coefficients as given in ref. (26), we can further show agreement of our four-gluon vertex with CHP for the cases $J = 0$ and $J = 2$, but not for $J = 1$ (J being the spin of the gluon pair). (Note the different naming of the gluon polarization modes). As for the Coulomb interaction, which is the gluon exchange between two gluons mediated by the scalar and longitudinal polarizations, we disagree with CHP on the value of their coefficient $\tilde{c}$, even as corrected in ref. (27). The $\tilde{c}$ corresponds to $-\rho(0)-\rho(2)$ in this article. However, as noted in ref. (26), the Coulomb interaction without the inclusion of gluon self-energies is gauge dependent. Since the gluon self-energies are proportional to $\vec{F}_1\cdot\vec{F}_2$, they will manifest themselves in a change of $\rho(0)$. (Note that for colour singlet g^2 glueballs this corresponds to an overall shift of the spectrum). This explains the difference of the results in the case of the Coulomb interaction.

6. CONCLUSIONS AND OUTLOOK

In this work, we have formulated the field theory of quantum chromodynamics in a finite volume. Using the Feynman gauge, we have discussed the predictions of this theory to lowest nontrivial order in perturbation theory. In most cases, we have found agreement with previous calculations which were performed in other gauges. However, there still remains a lot to be done before one can decide whether quantum chromodynamics in a cavity is a consistent physical theory. A few of the most important problems which have to be addressed in the near future are

- The renormalization of quantum chromodynamics in a finite volume, in particular in a static, spherically symmetric cavity. The formalism as presented here, i.e. in the Feynman gauge including the Faddeev-Popov ghost fields, is best suited for renormalization. Due to locality, the short distance singularities are softer than in other formulations, e.g. the Coulomb gauge. Note that the short distance singularities are not affected by the boundary conditions and that there are no long distance singularities in a field theory in a cavity. An encouraging feature is that the boundary conditions of the M.I.T. group preserve the BRS-symmetry of the theory. It must be clarified whether the lack of translational invariance destroys renormalizability.
- Will the shape of the cavity be affected by renormalization? Does the so-called vacuum pressure term, which is needed in fitting hadronic spectra in order to stabilize the energy of the cavity, emerge from the renormalization procedure (e.g. by dimensional transmutation)?
- The self-energies of the quarks, gluons and ghosts have to be determined reliably.
- Higher order terms in the perturbative expansions have to be calculated in order to estimate their importance. Note that it is not the value of the strong "hyperfine" constant α_s alone, which, in certain fitting schemes, can be of the order 2 - 3, that determines the convergence of the perturbation series. Rather, it is a combination of α_s, energy denominators and overlap integrals which may decrease rapidly with increasing order.
- Quantum chromodynamics, whether in a finite or infinite volume, still needs to predict in the low energy region some numbers that can be verified in experiment to at least three or four digits (compare to quantum electrodynamics and the g-2 value or the Lamb shift). The hope is that for certain quantities

it will not matter in which way confinement was achieved so that one could use quantum chromodynamics in the cavity for calculations.

- The final and, from the theoretical point of view, the most important problem in quantum chromodynamics is to discover how the original (infinite volume) theory confines the colour carrying particles. Is confinement achieved dynamically, are some, as yet undiscovered, topological features of the theory responsible or is it still something different?

APPENDIX A. THE CAVITY MODES

A.1 Quarks

We now turn to the derivation of the cavity modes of the quarks in a spherically symmetric and static cavity. The explicit solutions of the time independent Dirac equation

$$(-i\gamma\cdot\nabla + m_f)\, u_n(r) = \varepsilon_n \gamma^0\, u_n(r) \,, \tag{A1}$$

where ε_n is the energy and m_f the mass of the quark, are given by the Dirac spinors

$$u_n(r) = \begin{pmatrix} g_n(r)\chi^\mu_\kappa(\hat{r}) \\ if_n(r)\chi^\mu_{-\kappa}(\hat{r}) \end{pmatrix} \,. \tag{A2}$$

Of course, the adjoint spinor is defined as

$$\bar{u}_n(r) = u_n^+(r)\gamma^0 \,. \tag{A3}$$

Here $\chi^\mu_\kappa(\hat{r})$ denote the usual two-component spherical spinors and n stands for the flavour, radial, Dirac quantum numbers and sometimes it also includes the magnetic quantum number, as well, i.e.

$$n = \{f,\nu,\kappa,(\mu)\} \,. \tag{A4}$$

The radial functions, $g_n(r)$ and $f_n(r)$, are given in terms of the spherical Bessel functions by

$$g_n(r) = \frac{\mathcal{N}_n}{R^{3/2}}\, j_\ell(p_n r) \tag{A5}$$

$$f_n(r) = \frac{\mathcal{N}_n p_n \operatorname{sgn}\kappa}{R^{3/2}(\varepsilon_n+m_f)}\, j_{\bar{\ell}}(p_n r) \,, \tag{A6}$$

R being the radius of the cavity. Here, the total and orbital angular momenta, j,ℓ and $\bar{\ell}$ are defined as functions of the quantum number κ

$$j(\kappa) = |\kappa| - \tfrac{1}{2} \tag{A7}$$

$$\ell(\kappa) = j(\kappa) + \tfrac{1}{2}\mathrm{sgn}\,\kappa \tag{A8}$$

$$\bar{\ell}(\kappa) = j(\kappa) - \tfrac{1}{2}\mathrm{sgn}\,\kappa\ . \tag{A9}$$

Finally, the symbol ν denotes the radial quantum number with $\nu > 0$ for positive and $\nu < 0$ for negative energies, respectively. Of course, a complete set of Dirac eigenfunctions must include the negative energy states as well. The quark momenta p_n are determined by the linear boundary condition of the M.I.T. bag model

$$(i\gamma\cdot\hat{r} + 1)u_n(r)\Big|_{r=R} = 0\ , \tag{A10}$$

or, equivalently, by the solutions of the equation

$$j_\ell(x_n) + \frac{x_n\ \mathrm{sgn}\,\kappa}{\omega_n + \zeta_f}\, j_{\bar{\ell}}(x_n) = 0\ . \tag{A11}$$

Here, we have introduced the dimensionless energy, momentum and mass parameters, respectively

$$\omega_n = \varepsilon_n R = \mathrm{sgn}\,\nu \underset{+}{\sqrt{x_n^2 + \zeta_f^2}} \tag{A12}$$

$$x_n = p_n R \tag{A13}$$

$$\zeta_f = m_f R\ . \tag{A14}$$

The normalization constant $\mathcal{N}_n$ is given by

$$\mathcal{N}_n^{-2} = [2\omega_n(\omega_n + \kappa) + \zeta_f]\left[\frac{j_\ell(x_n)}{x_n}\right]^2\ . \tag{A15}$$

The solutions of the Dirac equation (A1) satisfying the boundary condition (A10) represent a complete and orthonormal set of Dirac spinors in the cavity i.e.

$$\int u_n^+(r)\ u_{n'}(r)\ d^3r = \delta_{nn'} \tag{A16}$$

$$\sum_{\nu\kappa\mu} u_{n\alpha}^*(r)\ u_{n\beta}(r') = \delta_{\alpha\beta}\ \delta^{(3)}(r-r')\ , \tag{A17}$$

where $u_{n\alpha}(r)$ denotes the component α of the Dirac spinor $u_n(r)$.

A.2 Gluons and Ghosts

Let us now determine, in the Feynman gauge (λ = 1), the cavity modes of the gluon in a spherically symmetric and static cavity. The explicit solutions of the time independent d'Alembert equations

$$(\Delta + \Omega_m^{0^2})\, a_m^0(\underset{\sim}{r}) = 0 \tag{A18}$$

$$(\Delta + \Omega_m^{\Sigma\,2})\, \underset{\sim}{a}_m^{\Sigma}(\underset{\sim}{r}) = 0 \;; \quad \Sigma = \mathcal{L}, \mathcal{M}, \mathcal{E} \;, \tag{A19}$$

where Ω_m^{Σ} denotes the energy eigenvalue of the gluon, are given by the Hansen functions which include the scalar

$$a_m^0(\underset{\sim}{r}) = \frac{\mathcal{N}_m^0}{R^{3/2}}\, i j_J(\Omega_m^0 r)\, Y_{JM}(\hat{\underset{\sim}{r}}) \;, \qquad J \geq 0 \tag{A20}$$

and longitudinal multipole fields

$$\underset{\sim}{a}_m^{\mathcal{L}}(\underset{\sim}{r}) = \frac{\mathcal{N}_m^{\mathcal{L}}}{R^{3/2}} \frac{1}{\Omega_m^{\mathcal{L}}}\, \underset{\sim}{\nabla} j_J(\Omega_m^{\mathcal{L}} r)\, Y_{JM}(\hat{\underset{\sim}{r}}) \;, \qquad J \geq 0 \;. \tag{A21}$$

Here, m = {N,J,(M)} denotes a complete set of radial and angular momentum quantum numbers and Σ stands for the polarization of the gluon. The transverse magnetic and electric modes are given by

$$\underset{\sim}{a}_m^{\mathcal{M}}(\underset{\sim}{r}) = \frac{\mathcal{N}_m^{\mathcal{M}}}{R^{3/2}} \frac{\underset{\sim}{L}}{\sqrt{J(J+1)}} (j_J(\Omega_m^{\mathcal{M}} r)\, Y_{JM}(\underset{\sim}{r})) \;, \quad J \geq 1 \tag{A22}$$

$$a_m^{\mathcal{E}}(\underset{\sim}{r}) = \frac{\mathcal{N}_m^{\mathcal{E}}}{R^{3/2}} \frac{1}{i\Omega_m^{\mathcal{E}}}\, \underset{\sim}{\nabla} \times \frac{\underset{\sim}{L}}{\sqrt{J(J+1)}} (j_J(\Omega_m^{\mathcal{E}} r)\, Y_{JM}(\hat{\underset{\sim}{r}})) \;, \quad J \geq 1 \;, \tag{A23}$$

respectively. The eigenvalues of the gluon Ω_m^{Σ} are determined by the linear boundary conditions of the M.I.T. bag model

$$\hat{\underset{\sim}{r}} \cdot \underset{\sim}{\nabla} a_m^0(\underset{\sim}{r}) \Big|_{r=R} = 0 \tag{A24}$$

$$\hat{\underset{\sim}{r}} \cdot \underset{\sim}{a}_m^{\Sigma}(\underset{\sim}{r}) \Big|_{r=R} = 0 \tag{A25}$$

$$\hat{\underset{\sim}{r}} \times (\underset{\sim}{\nabla} \times \underset{\sim}{a}_m^{\Sigma}(\underset{\sim}{r})) \Big|_{r=R} = 0 \;; \qquad \Sigma = \mathcal{L}, \mathcal{M}, \mathcal{E} \tag{A26}$$

For the various multipole fields, we then arrive at three eigenvalue equations

$$\frac{d}{dr} j_J(\Omega_m^0 r)\Big|_{r=R} = 0 \tag{A27}$$

$$\frac{d}{dr}\left[r j_J(\Omega_m^{\mathcal{M}} r)\right]\Big|_{r=R} = 0 \tag{A28}$$

$$j_J(\Omega_m^{\mathcal{E}} r)\Big|_{r=R} = 0 \tag{A29}$$

since the scalar and longitudinal multipole fields satisfy the same eigenvalue equation, i.e. $\Omega_m^0 = \Omega_m^{\ell}$. The corresponding normalization constants are

$$[\mathcal{N}_m^0]^{-2} = \tfrac{1}{2} j_J^2(y_m^0)\left[1 - \frac{J(J+1)}{(y_m^0)^2}\right] = [\mathcal{N}_m^{\ell}]^{-2} \tag{A30}$$

$$[\mathcal{N}_m^{\mathcal{M}}]^{-2} = \tfrac{1}{2} j_J^2(y_m^{\mathcal{M}})\left[1 - \frac{J(J+1)}{(y_m^{\mathcal{M}})^2}\right] \tag{A31}$$

$$[\mathcal{N}_m^{\mathcal{E}}]^{-2} = \tfrac{1}{2} j_{J+1}^2(y_m^{\mathcal{E}}) \quad , \tag{A32}$$

where we have introduced the dimensionless energy parameters

$$y_m^{\Sigma} = \Omega_m^{\Sigma} R \; ; \qquad \Sigma = 0, \ell, \mathcal{M}, \mathcal{E} . \tag{A33}$$

For compactness of the notation we introduce the functions

$$a_m^{\mu\Sigma}(\underset{\sim}{r}) = \begin{cases} a_m^0(\underset{\sim}{r}) & \text{for } \mu=\Sigma=0 \\ [\underset{\sim}{a}_m^{\Sigma}(\underset{\sim}{r})]^{\mu} & \text{for } \mu=1,2,3 \text{ and } \Sigma=\ell,\mathcal{M},\mathcal{E} \\ 0 & \text{for all other } \mu \text{ and } \Sigma \text{ values.} \end{cases} \tag{A34}$$

The solutions of the d'Alembert equations (A18) and (A19) which satisfy the boundary conditions (A24) to (A26) are orthonormal, i.e.

$$\int a_{\mu m}^{\Sigma}(\underset{\sim}{r})^* \, a_{m'}^{\mu\Sigma'}(\underset{\sim}{r}) \, d^3r = g^{\Sigma\Sigma'} \delta_{mm'} \tag{A35}$$

and they also satisfy the completeness relation

$$\sum_{\Sigma m} g^{\Sigma\Sigma} a_m^{\mu\Sigma}(\underset{\sim}{r})^* \, a_m^{\nu\Sigma}(\underset{\sim}{r}') = g^{\mu\nu} \delta^{(3)}(\underset{\sim}{r}-\underset{\sim}{r}') . \tag{A36}$$

Here we have introduced the metric tensor

$$g^{00} = -g^{\mathcal{L}\mathcal{L}} = -g^{\mathcal{M}\mathcal{M}} = -g^{\mathcal{E}\mathcal{E}} = 1$$

$$g^{\Sigma\Sigma'} = 0 \text{ if } \Sigma \neq \Sigma' \ . \tag{A37}$$

The cavity modes $\mathbf{a}_m^{\Sigma}(\mathbf{r})$ can also be written in terms of the spherical Bessel functions and the vector spherical harmonics as

$$\mathbf{a}_m^{\Sigma}(\mathbf{r}) = \frac{\mathcal{N}_m^{\Sigma}}{R^{3/2}} \frac{1}{\sqrt{2J+1}} \sum_{L=|J-1|}^{J+1} \alpha_{JL}^{\Sigma}\, j_L(\Omega_m^{\Sigma} r)\, \mathbf{Y}_{JLM}(\hat{\mathbf{r}}) \ . \tag{A38}$$

A similar expansion holds for the curl of the cavity modes

$$\nabla \times \mathbf{a}_m^{\Sigma}(\mathbf{r}) = -i\, \frac{\Omega_m^{\Sigma}\mathcal{N}_m^{\Sigma}}{R^{5/2}} \frac{1}{\sqrt{2J+1}} \sum_{L=|J-1|}^{J+1} \beta_{JL}^{\Sigma}\, j_L(\Omega_m^{\Sigma} r)\, \mathbf{Y}_{JLM}(\hat{\mathbf{r}}) \ . \tag{A39}$$

The only nonvanishing coefficients α_{JL}^{Σ} and β_{JL}^{Σ} are given by

$$\alpha_{J,J+1}^{\mathcal{L}} = \sqrt{J+1}\,,\quad \alpha_{J,J-1}^{\mathcal{L}} = \sqrt{J}$$

$$\alpha_{J,J}^{\mathcal{M}} = \sqrt{2J+1}\,,\quad \beta_{J,J+1}^{\mathcal{M}} = \sqrt{J}\,,\quad \beta_{J,J-1}^{\mathcal{M}} = -\sqrt{J+1} \tag{A40}$$

$$\alpha_{J,J+1}^{\mathcal{E}} = -\sqrt{J}\,,\quad \alpha_{J,J-1}^{\mathcal{E}} = \sqrt{J+1}\,,\quad \beta_{J,J}^{\mathcal{E}} = \sqrt{2J+1} \ .$$

Under complex conjugation the gluon modes $a_m^{\mu\Sigma}(x)$ transform according to

$$a_m^{\mu\Sigma}(x)^* = \eta_{\Sigma}(-1)^M\, a_{m^*}^{\mu\Sigma}(x) \quad , \tag{A41}$$

where the set of quantum numbers m* is defined by

$$m^* = \{N, J, (-M)\} \tag{A42}$$

and the phase η_{Σ} stands for

$$\eta_{\Sigma} = \begin{cases} +1 & \text{for } \Sigma = \mathcal{L}, \mathcal{E}, \\ -1 & \text{for } \Sigma = 0, \mathcal{M} \ . \end{cases} \tag{A43}$$

Finally the cavity modes and eigenvalues for the ghost fields in a spherically symmetric and static cavity are also given by the scalar modes $a_m^o(\underset{\sim}{r})$ and the scalar or longitudinal eigenvalues Ω_m^o, since they satisfy the d'Alembert equation (A18) and the boundary condition (A24).

APPENDIX B. THE FEYNMAN PROPAGATORS

B.1 The Quark Propagator

We now turn to the evaluation of the Feynman propagators which are given in terms of vacuum expectation values of timeordered products of two field operators. There are two nonvanishing Feynman propagators that involve the quark field operators. Using the expansions (4.15) and (4.18) and the anti-commutation relations (4.19), we easily obtain

$$\langle \hat{0}|T(\hat{\bar{\psi}}^{+}_{cf\alpha}(x)\,\hat{\psi}_{c'f'\alpha'}(y))|\hat{0}\rangle =$$

$$\delta_{cc'}\delta_{ff'}\sum_{\substack{\kappa\mu\\ \nu>0}}[\bar{u}_{-n\alpha}(\underset{\sim}{x})u_{-n\alpha'}(\underset{\sim}{y})\Theta(x^{\circ}-y^{\circ}) - \bar{u}^{*}_{n\alpha}(\underset{\sim}{x})u_{n\alpha'}(\underset{\sim}{y})\Theta(y^{\circ}-x^{\circ})]e^{-i\varepsilon_n|x^{\circ}-y^{\circ}|}\,. \tag{B1}$$

The other nonvanishing vacuum expectation value is

$$\langle \hat{0}|T(\hat{\psi}_{cf\alpha}(x)\hat{\bar{\psi}}^{+}_{c'f'\alpha'}(y))|\hat{0}\rangle =$$

$$\delta_{cc'}\delta_{ff'}\sum_{\substack{\kappa\mu\\ \nu>0}}[u_{n\alpha}(\underset{\sim}{x})\bar{u}_{n\alpha'}(\underset{\sim}{y})\Theta(x^{\circ}-y^{\circ}) - u_{-n\alpha}(\underset{\sim}{x})\bar{u}_{-n\alpha'}(\underset{\sim}{y})\Theta(y^{\circ}-x^{\circ})]e^{-i\varepsilon_n|x^{\circ}-y^{\circ}|}\,. \tag{B2}$$

In eqs.(B1) and (B2) we have made use of the step function which is defined by

$$\Theta(x^{\circ}) = \begin{cases} 0 & x^{\circ}<0 \\ 1 & x^{\circ}>0 \end{cases}\,. \tag{B3}$$

B.2 The Ghost Propagator

Based on the expansions (4.23) and (4.24) and the anticommutation relations (4.25) we arrive at Feynman propagators involving the ghost field operators which are given by

$$\langle \hat{0}|T(\hat{\omega}_a(x)\hat{\chi}_b(y))|\hat{0}\rangle = -\langle \hat{0}|T(\hat{\chi}_a(x)\hat{\omega}_b(y))|\hat{0}\rangle =$$

$$i\delta_{ab}\sum_{\substack{JM\\ N>0}}\frac{1}{2\Omega^{\circ}_m}\,a^{\circ}_m(\underset{\sim}{x})\;a^{\circ}_m(\underset{\sim}{y})^{*}\;e^{-i\Omega^{\circ}_m|x^{\circ}-y^{\circ}|}\,. \tag{B4}$$

These two propagators are the only nonvanishing vacuum expectation values we can build of two ghost field operators. The Feynman propagators involving the derivative of $\bar{\chi}_a$ are easily evaluated by calculating the derivatives of eq.(B4) with respect to x and y, respectively, i.e.

$$\langle\hat{0}|T(\frac{\partial}{\partial x_\mu}\hat{\chi}_a(x)\hat{\omega}_b(y))|\hat{0}\rangle = \frac{\partial}{\partial x_\mu}\langle\hat{0}|T(\hat{\chi}_a(x)\hat{\omega}_b(y))|\hat{0}\rangle \tag{B5}$$

$$\langle\hat{0}|T(\hat{\omega}_a(x)\frac{\partial\hat{\chi}_b(y)}{\partial y_\mu})|\hat{0}\rangle = \frac{\partial}{\partial y_\mu}\langle\hat{0}|T(\hat{\omega}_a(x)\hat{\chi}_b(y))|\hat{0}\rangle\ . \tag{B6}$$

B.3 The Gluon Propagators

Based on the expansion (4.20) and the commutation relations (4.22) we arrive at the Feynman propagator for the gluon which is given by

$$\langle\hat{0}|T(\hat{A}^\mu_a(x)\hat{A}^\nu_b(y))|\hat{0}\rangle =$$

$$-\delta_{ab}\sum_{\substack{JM\Sigma\\N>0}}\frac{g^{\Sigma\Sigma}}{2\Omega^\Sigma_m}\, a^{\mu\Sigma}_m(\chi)\; a^{\nu\Sigma}_m(\chi)^*\; e^{-i\Omega^\Sigma_m|x^\circ-y^\circ|}\ , \tag{B7}$$

where the metric tensor $g^{\Sigma\Sigma}$ is defined by eq.(A37).
We also need to know Feynman propagators involving derivatives of the field operators. In the case of a single derivative, these can be written as

$$\langle\hat{0}|T(\frac{\partial\hat{A}^\mu_a(x)}{\partial x_\rho}\;\hat{A}^\nu_b(y))|\hat{0}\rangle = \frac{\partial}{\partial x_\rho}\langle\hat{0}|T(\hat{A}^\mu_a(x)\;\hat{A}^\nu_b(y))|\hat{0}\rangle\ . \tag{B8}$$

In the case of two derivatives, however, we obtain

$$\langle\hat{0}|T(\frac{\partial\hat{A}^\mu_a(x)}{\partial x_\rho}\;\frac{\partial\hat{A}^\nu_b(y)}{\partial y_\sigma})\;|\hat{0}\rangle = \frac{\partial^2}{\partial x_\rho\,\partial y_\sigma}\langle\hat{0}|T(\hat{A}^\mu_a(x)\;\hat{A}^\nu_b(y))|\hat{0}\rangle$$

$$+\, i\delta_{\rho o}\delta_{\sigma o}\; g^{\mu\nu}\;\delta^{(4)}(x-y)\quad . \tag{B9}$$

Thus here also we can in general take the derivatives of the vacuum expectation value of the timeordered product of the field operators except for the case $\rho=\sigma=0$, where an additional delta term appears.

APPENDIX C. THE VERTEX INTEGRALS

C.1. The Quark-Gluon Vertex

Here we evaluate the vertex integrals which describe the absorption (or emission) of a gluon by a quark (Fig 1a). These integrals are defined as

$$Q^{\Sigma}_{nn'm} = i \int \bar{u}_n(\underset{\sim}{r})\, \gamma_\mu\, u_{n'}(\underset{\sim}{r})\, a^{\mu\Sigma}_m(\underset{\sim}{r})\, d^3r \ . \tag{C1}$$

In Appendix D we will also use (see (A41) - (A43))

$$\tilde{Q}^{\Sigma}_{nn'm} = i \int \bar{u}_n(\underset{\sim}{r})\, \gamma_\mu\, u_{n'}(\underset{\sim}{r})\, a^{\mu\Sigma *}_m(\underset{\sim}{r})\, d^3r = (-1)^M\, \eta_\Sigma\, Q^{\Sigma}_{nn'm^*} = -Q^{\Sigma}_{n'nm} \ , \tag{C1a}$$

where n and m stand for the quark and gluon quantum numbers, respectively, e.g.

$$n = \{f,\nu,\kappa,(\mu)\} \tag{C2}$$

$$m = \{N,J,(M)\} \qquad m^* = \{N,J,(-M)\} \tag{C3}$$

and Σ for the polarization of the gluon

$$\Sigma = 0, \mathcal{L}, \mathcal{M}, \mathcal{E} \ . \tag{C4}$$

The scalar and longitudinal matrix elements, which transmit the instantaneous "Coulomb" interaction between two quarks, are related by current conservation, i.e.

$$Q_{nn'm} = \frac{\varepsilon_{n'} - \varepsilon_n}{\Omega^o_m}\, Q^o_{nn'm} \ . \tag{C5}$$

We can easily separate the radial and angular integrations yielding

$$Q^{\Sigma}_{nn'm} = R^{\Sigma}_{nn'm} \int \chi^{\mu +}_{\kappa}(\hat{\underset{\sim}{r}}) Y_{JM}(\hat{\underset{\sim}{r}})\, \chi^{\mu'}_{\kappa'}(\hat{\underset{\sim}{r}})\, d\Omega \ ; \qquad \Sigma = 0, \mathcal{L}, \mathcal{E}, \tag{C6}$$

$$Q^{\mathcal{M}}_{nn'm} = R^{\mathcal{M}}_{nn'm} \int \chi^{\mu +}_{\kappa}(\hat{\underset{\sim}{r}}) Y_{JM}(\hat{\underset{\sim}{r}})\, \chi^{\mu'}_{-\kappa'}(\hat{\underset{\sim}{r}})\, d\Omega \ , \tag{C7}$$

where the dependence on the magnetic quantum numbers is now contained in the angular matrix element. Integrating over the angular variables and using the abbreviation $\hat{J} = \sqrt{2J+1}$ one arrives, in terms of 3j-symbols, at

$$\int \chi_\kappa^{\mu^+}(\hat{r})\, Y_{JM}(\hat{r})\chi_{\kappa'}^{\mu'}(\hat{r})\, d\Omega \;=\; (-1)^{\mu+\frac{1}{2}}\, \frac{\hat{j}\hat{J}\hat{j}'}{\sqrt{4\pi}}$$

$$\times \begin{pmatrix} j & J & j' \\ -\mu & M & \mu' \end{pmatrix} \begin{pmatrix} j & J & j' \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{pmatrix} \frac{(-1)^{\ell+J+\ell'}+1}{2} \tag{C8}$$

which governs the angular momentum and parity selection rules. After some straightforward but tedious algebra, we obtain for the scalar and longitudinal radial matrix elements

$$R^{0}_{nn'm} \;=\; -\mathcal{N}^{0}_{m}\, R^{-\frac{3}{2}} \int_0^R j_J(\Omega^0_m r)\, S_{nn'}(r) r^2\, dr \tag{C9}$$

$$R^{\ell}_{nn'm} \;=\; \frac{-\mathcal{N}^{0}_{m}}{\Omega^0_m}\, R^{-\frac{3}{2}} \int_0^R [\{\Omega^0_m r\, j_{J+1}(\Omega^0_m r) - J j_J(\Omega^0_m r)\}\, U_{nn'}(r)$$

$$+ (\kappa-\kappa') j_J(\Omega^0_m r)\, T_{nn'}(r)]\, r dr\ . \tag{C10}$$

The transverse magnetic and electric radial matrix elements, turn out to be of the form

$$R^{\mathcal{M}}_{nn'm} \;=\; \frac{\kappa'+\kappa}{\sqrt{J(J+1)}}\, \mathcal{N}^{\mathcal{M}}_{m} R^{-\frac{3}{2}} \int_0^R j_J(\Omega^{\mathcal{M}}_m r)\, T_{nn'}(r)\, r^2 dr \tag{C11}$$

and

$$R^{\mathcal{E}}_{nn'm} \;=\; \frac{\mathcal{N}^{\mathcal{E}}_{m} R^{-\frac{3}{2}}}{\sqrt{J(J+1)}\,\Omega^{\mathcal{E}}_m} \int_0^R [J(J+1) j_J(\Omega^{\mathcal{E}}_m r)\, U_{nn'}(r)$$

$$+ (\kappa-\kappa')\, \{J j_J(\Omega^{\mathcal{E}}_m r) - \Omega^{\mathcal{E}}_m r\, j_{J-1}(\Omega^{\mathcal{E}}_m r)\}\, T_{nn'}(r)]\, r dr\ . \tag{C12}$$

Here we have introduced the radial functions

$$S_{nn'} \;=\; g_n g_{n'} + f_n f_{n'}$$

$$T_{nn'} \;=\; g_n f_{n'} + f_n g_{n'}$$

$$U_{nn'} \;=\; g_n f_{n'} - f_n g_{n'} \tag{C13}$$

which are given in terms of the radial functions of the quarks in the initial and final state.

C.2 The Ghost-Gluon Vertex

The vertex integrals that describe the absorption (or emission) of a gluon by a ghost (Fig.1b) are given by integrals of the type

$$T_{mm'm''} = i \int a^{o}_{m}(\underset{\sim}{r})\, a^{o}_{m'}(\underset{\sim}{r})\, a^{o}_{m''}(\underset{\sim}{r})\, d^3r \tag{C14}$$

and

$$T^{\Sigma\Sigma'}_{mm'm''} = -i \int \underset{\sim}{a}^{\Sigma}_{m}(\underset{\sim}{r}) \cdot \underset{\sim}{a}^{\Sigma'}_{m'}(\underset{\sim}{r})\, a^{o}_{m''}(\underset{\sim}{r})\, d^3r \quad , \tag{C15}$$

where m, m' and m" stand for the gluon and ghost quantum numbers, e.g.

$$m = \{N,J,(M)\} \; . \tag{C16}$$

The integral (C14) can be separated into a radial and an angular part giving

$$T_{mm'm''} = \frac{\mathcal{N}^{o}_{m}\, \mathcal{N}^{o}_{m'}\, \mathcal{N}^{o}_{m''}}{R^{9/2}}\, R_{JJ'J''}(\Omega^{o}_{m},\Omega^{o}_{m'},\Omega^{o}_{m''})$$

$$\int Y_{JM}(\hat{\underset{\sim}{r}})\, Y_{J'M'}(\hat{\underset{\sim}{r}})\, Y_{J''M''}(\hat{\underset{\sim}{r}})\, d\Omega \; . \tag{C17}$$

The radial integrals are defined by

$$R_{jj'J''}(\Omega,\Omega',\Omega'') = \int_0^R j_J(\Omega r) j_{J'}(\Omega' r)\, j_{J''}(\Omega'' r)\, r^2 dr \tag{C18}$$

and the angular integration yields

$$\int d\Omega\, Y_{J'M'}(\hat{r})\, Y_{J''M''}(\hat{r})\, Y_{JM}(\hat{r}) =$$

$$= \begin{pmatrix} J' & J'' & J \\ M' & M'' & M \end{pmatrix} \begin{pmatrix} J' & J'' & J \\ 0 & 0 & 0 \end{pmatrix} \frac{\hat{J}'\hat{J}''\hat{J}}{\sqrt{4\pi}} \; . \tag{C19}$$

Separating the radial and angular integration in the vertex function (C15) we obtain

$$T^{\Sigma\Sigma'}_{mm'm''} = \frac{\mathcal{N}^{\Sigma}_{m}\,\mathcal{N}^{\Sigma'}_{m'}\,\mathcal{N}^{o}_{m''}}{\hat{J}\hat{J}'\,R^{9/2}} \sum_{LL'} \alpha^{\Sigma}_{JL}\,\alpha^{\Sigma'}_{J'L'}$$

$$R_{LL'J}(\Omega^{\Sigma}_{m},\Omega^{\Sigma'}_{m'},\Omega^{o}_{m''}) \int \underset{\sim}{Y}_{JLM}(\hat{\underset{\sim}{r}})\cdot\underset{\sim}{Y}_{J'L'M'}(\hat{\underset{\sim}{r}})\; Y_{J''M''}(\hat{\underset{\sim}{r}})\; d\Omega\ . \tag{C20}$$

The angular integral is given in terms of 3j- and 6j-symbols, as

$$\int \underset{\sim}{Y}_{JLM}(\hat{\underset{\sim}{r}})\cdot\underset{\sim}{Y}_{J'L'M'}(\hat{\underset{\sim}{r}})\; Y_{J''M''}(\hat{\underset{\sim}{r}})\; d\Omega \; = \tag{C21}$$

$$(-1)^{L+J}\;\frac{\hat{J}\hat{J}'\hat{J}''\hat{L}\hat{L}'}{\sqrt{4\pi}} \begin{pmatrix} J & J' & J'' \\ M & M' & M'' \end{pmatrix} \begin{pmatrix} L & J'' & L' \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} J' & J'' & J \\ L & 1 & L' \end{Bmatrix}\ .$$

C.3 The Three-Gluon Vertex

We now turn to the evaluation of the three-gluon-vertex integral which describes the absorption (or emission) of a gluon by another gluon (Fig.1c). These integrals can be formulated in terms of the $T_{mm'm''}$ and $T^{\Sigma\Sigma'}_{mm'm''}$ which have been defined previously, and an additional integral of the type

$$T^{\Sigma\Sigma'\Sigma''}_{mm'm''} = \int (\underset{\sim}{a}^{\Sigma}_{m}(\underset{\sim}{r}) \times \underset{\sim}{a}^{\Sigma'}_{m'}(\underset{\sim}{r}))\cdot(\underset{\sim}{\nabla} \times \underset{\sim}{a}^{\Sigma''}_{m''}(\underset{\sim}{r}))\ . \tag{C22}$$

Separating the integration in a radial and an angular part, we arrive at

$$T^{\Sigma\Sigma'\Sigma''}_{mm'm''} = \frac{\Omega^{\Sigma''}_{m''}\,\mathcal{N}^{\Sigma}_{m}\,\mathcal{N}^{\Sigma'}_{m'}\,\mathcal{N}^{\Sigma''}_{m''}}{JJ'J''\;R^{11/2}} \sum_{LL'L''} \alpha^{\Sigma}_{JL}\alpha^{\Sigma'}_{J'L'}\beta^{\Sigma''}_{J''L''}\; R_{LL'L''}(\Omega^{\Sigma}_{m},\Omega^{\Sigma'}_{m'},\Omega^{\Sigma''}_{m''})$$

$$(-i) \int (\underset{\sim}{Y}_{JLM}(\hat{\underset{\sim}{r}}) \times \underset{\sim}{Y}_{J'L'M'}(\hat{\underset{\sim}{r}}))\cdot\underset{\sim}{Y}_{J''L''M''}(\hat{\underset{\sim}{r}})\; d\Omega\ , \tag{C23}$$

where the angular integral is given in terms of 3j- and 9j-symbols by

$$(-i) \int (\underset{\sim}{Y}_{JLM}(\hat{\underset{\sim}{r}}) \times \underset{\sim}{Y}_{J'L'M'}(\hat{\underset{\sim}{r}}))\cdot\underset{\sim}{Y}_{J''L''M''}(\hat{\underset{\sim}{r}})\; d\Omega$$

$$= \sqrt{\frac{3}{2\pi}}\;\hat{J}\hat{J}'\hat{J}''\hat{L}\hat{L}'\hat{L}'' \begin{pmatrix} L & L' & L'' \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} J & J' & J'' \\ 1 & 1 & 1 \\ L & L' & L'' \end{Bmatrix} \begin{pmatrix} J & J' & J'' \\ M & M' & M'' \end{pmatrix}. \tag{C24}$$

C.4 The Four-Gluon-Vertex

The four-gluon-vertex which describes the elementary interaction of four gluons (Fig.1d) is given by integrals of the type

$$F^{\Sigma\Sigma'}_{mm'm''m'''} = \int \underset{\sim}{a}^{\Sigma}_{m}(\underset{\sim}{r})\cdot\underset{\sim}{a}^{\Sigma'}_{m'}(\underset{\sim}{r})\; a^{0}_{m''}(\underset{\sim}{r})\; a^{0}_{m'''}(\underset{\sim}{r})\; d^3r \tag{C25}$$

and

$$F^{\Sigma\Sigma'\Sigma''\Sigma'''}_{mm'm''m'''} = \int \underset{\sim}{a}^{\Sigma}_{m}(\underset{\sim}{r})\cdot\underset{\sim}{a}^{\Sigma'}_{m'}(\underset{\sim}{r})\; \underset{\sim}{a}^{\Sigma''}_{m''}(\underset{\sim}{r})\cdot\underset{\sim}{a}^{\Sigma'''}_{m'''}(\underset{\sim}{r})\; d^3r \quad . \tag{C26}$$

For the first integral (C26) we obtain

$$F^{\Sigma\Sigma'}_{mm'm''m'''} = \frac{\mathcal{N}^{\Sigma}_{m}\;\mathcal{N}^{\Sigma}_{m'}\;\mathcal{N}^{0}_{m''}\;\mathcal{N}^{0}_{m'''}}{\hat{J}\hat{J}'\;R^6} \sum_{LL'} \alpha^{\Sigma}_{JL}\alpha^{\Sigma'}_{J'L'}\; R_{JJ'J''J'''} \tag{C27}$$

$$(\Omega^{\Sigma}_{m},\Omega^{\Sigma'}_{m'},\Omega^{0}_{m''},\Omega^{0}_{m'''}) \int \underset{\sim}{Y}_{JLM}(\hat{\underset{\sim}{r}})\cdot\underset{\sim}{Y}_{J'L'M'}(\hat{\underset{\sim}{r}})\; Y_{J''M''}(\hat{\underset{\sim}{r}})\; Y_{J'''M'''}(\hat{\underset{\sim}{r}})\; d\Omega \;.$$

Here the radial integral is defined as

$$R_{JJ'J''J'''}(\Omega,\Omega',\Omega'',\Omega''') = \int_0^R j_J(\Omega r) j_{J'}(\Omega' r) j_{J''}(\Omega'' r) j_{J'''}(\Omega''' r)\; r^2 dr, \tag{C28}$$

and the angular integration yields

$$\int \underset{\sim}{Y}_{JLM}(\hat{\underset{\sim}{r}})\cdot\underset{\sim}{Y}_{J'L'M'}(\hat{\underset{\sim}{r}})\; Y_{J''M''}(\hat{\underset{\sim}{r}})\; Y_{J'''M'''}(\hat{\underset{\sim}{r}})\; d\Omega \tag{C29}$$

$$= \sum_{k\kappa} \begin{pmatrix} J & J' & k \\ M & M' & \kappa \end{pmatrix} \begin{pmatrix} J'' & J''' & k \\ M'' & M''' & -\kappa \end{pmatrix} \frac{(-1)^{k+\kappa+J+L'}}{4\pi}$$

$$(2k+1)\; \hat{J}\hat{J}'(2J''+1)(2J'''+1)\hat{L}\hat{L}' \begin{pmatrix} J'' & J''' & k \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} L & L' & k \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} J & J' & k \\ L' & L & 1 \end{Bmatrix} \;.$$

The second integral (C26) also separates in a radial and an angular part yielding

$$F^{\Sigma\Sigma'\Sigma''\Sigma'''}_{mm'm''m'''} = \frac{\mathcal{N}^{\Sigma}_{m}\;\mathcal{N}^{\Sigma'}_{m'}\;\mathcal{N}^{\Sigma''}_{m''}\;\mathcal{N}^{\Sigma'''}_{m'''}}{\hat{J}\hat{J}'\hat{J}''\hat{J}'''\;R^6} \sum_{LL'L''L'''} \alpha^{\Sigma}_{JL}\alpha^{\Sigma'}_{J'L'}\alpha^{\Sigma''}_{J''L''}\alpha^{\Sigma'''}_{J'''L'''} \tag{C30}$$

$$R_{JJ'J''J'''}(\Omega^{\Sigma}_{m},\Omega^{\Sigma'}_{m'},\Omega^{\Sigma''}_{m''},\Omega^{\Sigma'''}_{m'''}) \int \underset{\sim}{Y}_{JLM}(\hat{\underset{\sim}{r}})\cdot\underset{\sim}{Y}_{J'L'M'}(\hat{\underset{\sim}{r}})\; \underset{\sim}{Y}_{J''L''M''}(\hat{r})\cdot\underset{\sim}{Y}_{J'''L'''M'''}(\hat{\underset{\sim}{r}})\; d\Omega \;,$$

Here one obtains for the angular integration

$$\int \chi_{JLM}(\hat{r})\cdot\chi_{J'L'M'}(\hat{r})\,\chi_{J''L''M''}(\hat{r})\cdot\chi_{J'''L'''M'''}(\hat{r})\,d\Omega$$

$$= \sum_{k\kappa} \begin{pmatrix} J & J' & k \\ M & M' & \kappa \end{pmatrix} \begin{pmatrix} J'' & J''' & k \\ M'' & M''' & -\kappa \end{pmatrix} \frac{(-1)^{\kappa+J+L'+J''+L'''}}{4\pi} \tag{C31}$$

$$(2k+1)\hat{J}\hat{J}'\hat{J}''\hat{J}'''\hat{L}\hat{L}'\hat{L}''\hat{L}''' \begin{pmatrix} L & L' & k \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} L'' & L''' & k \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} J & J' & k \\ L' & L & 1 \end{Bmatrix} \begin{Bmatrix} J'' & J''' & k \\ L''' & L'' & 1 \end{Bmatrix}.$$

APPENDIX D. INTERACTING TWO-PARTICLE SYSTEMS

D.1. Systems Consisting of Fermions

A system consisting of two quarks, two antiquarks or a quark-antiquark pair which interact through the exchange of a gluon can be represented by the Feynman diagrams Figs.(2a), (2b) or (2c) respectively. The energy shifts are obtained from eqs.(5.1) or (5.3) where $|\hat{\Phi}_k\rangle$ denotes a 2-particle eigenstate of the noninteracting Hamiltonian $:\hat{H}_0:$, i.e.

$$|\hat{\Phi}_k\rangle = \hat{a}^+_{c_1 n_1} \hat{a}^+_{c_2 n_2} |\hat{0}\rangle \tag{D1a}$$

$$|\hat{\Phi}_k\rangle = \hat{b}^+_{c_1 n_1} \hat{b}^+_{c_2 n_2} |\hat{0}\rangle \tag{D1b}$$

$$|\hat{\Phi}_k\rangle = \hat{a}^+_{c_1 n_1} \hat{b}^+_{c_2 n_2} |\hat{0}\rangle . \tag{D1c}$$

After a straightforward calculation we arrive at the two-body operators

$$\hat{V} = g^2 \left(\frac{\lambda^a}{2}\right)_{c'c} \left(\frac{\lambda^a}{2}\right)_{d'd} \delta_{f'f}\, \delta_{g'g}\, \frac{g^{\Sigma\Sigma}}{2\Omega^\Sigma_m}\, \frac{Q^\Sigma_{n'nm}\, \tilde{Q}^\Sigma_{p'pm}}{\Omega^\Sigma_m + \varepsilon_{p'} - \varepsilon_p}\, \hat{a}^+_{c'n'}\, \hat{a}^+_{d'p'}\, \hat{a}_{cn}\, \hat{a}_{dp} \tag{D2a}$$

$$\hat{V} = g^2 \left(-\frac{\lambda^a}{2}\right)_{cc'} \left(-\frac{\lambda^a}{2}\right)_{dd'} \delta_{f'f}\, \delta_{g'g}\, \frac{g^{\Sigma\Sigma}}{2\Omega^\Sigma_m}\, \frac{Q^\Sigma_{-n-n'm}\, \tilde{Q}^\Sigma_{-p-p'm}}{\Omega^\Sigma_m + \varepsilon_{p'} - \varepsilon_p}\, \hat{b}^+_{c'n'}\, \hat{b}^+_{d'p'}\, \hat{b}_{cn} \hat{b}_{dp} \tag{D2b}$$

$$\hat{V} = g^2 \left(\frac{\lambda^a}{2}\right)_{c'c} \left(-\frac{\lambda^a}{2}\right)_{dd'} \delta_{f'f}\, \delta_{g'g}$$

$$\cdot \frac{g^{\Sigma\Sigma}}{2\Omega^\Sigma_m}\, Q^\Sigma_{n'nm}\, \tilde{Q}^\Sigma_{-p-p'm} \left[\frac{1}{\Omega^\Sigma_m + \varepsilon_{p'} - \varepsilon_p} + \frac{1}{\Omega^\Sigma_m + \varepsilon_{n'} - \varepsilon_n}\right] \hat{a}^+_{c'n'}\, \hat{b}^+_{d'p'}\, \hat{a}_{cn}\, \hat{b}_{dp} . \tag{D2c}$$

As usual, a summation over all indices occuring repeatedly is understood. The two-body operators V_{12} have, for all three cases under consideration, the form

$$V_{12} = \frac{\alpha_s}{R} \vec{F}_1 \cdot \vec{F}_2 \sum_{J=0}^{\infty} \mu_{12}(J)\, K_{12}(J) , \tag{D3}$$

where α_s is the "fine structure constant" of the strong interaction

$$\alpha_s = \frac{g^2}{4\pi} \tag{D4}$$

and R is the radius of the cavity. Here, the vector $\vec{F}$ stands for the eight generators of $SU(3)_{colour}$ in the apropriate representation, i.e.

$$\vec{F} = \begin{cases} \frac{1}{2}\vec{\lambda} & \text{for quarks} \\ -\frac{1}{2}\vec{\lambda}^T & \text{for antiquarks .} \end{cases} \tag{D5}$$

The two-body operators $K_{12}(J)$ determine the angular momentum structure of the interaction V_{12}. Their matrix elements are most easily given in terms of the function $F_{JM}(n,n')$

$$F_{JM}(n,n') = (-1)^{\mu+\frac{1}{2}} \hat{j}\hat{J}\hat{j}' \begin{pmatrix} j & J & j' \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} j & J & j' \\ -\mu & M & \mu' \end{pmatrix} . \tag{D6}$$

This is, up to the parity selection rule and a factor of $(4\pi)^{-\frac{1}{2}}$ the angular integral contained in the vertex function $Q^{\Sigma}_{nn'm}$. Now, we have

$$\langle n_1',n_2'|K_{12}(J)|n_1,n_2\rangle = (2J+1) \sum_{M=-J}^{J} (-1)^M F_{JM}(n_1',n_1)\, F_{J-M}(n_2',n_2) \tag{D7a}$$

$$\langle \bar{n}_1',\bar{n}_2'|K_{12}(J)|\bar{n}_1,\bar{n}_2\rangle = (2J+1) \sum_{M=-J}^{J} (-1)^M F_{JM}(-\bar{n}_1,-\bar{n}_1')\, F_{J-M}(-\bar{n}_2,-\bar{n}_2') \tag{D7b}$$

$$\langle n_1',\bar{n}_2'|K_{12}(J)|n_1,\bar{n}_2\rangle = (2J+1) \sum_{M=-J}^{J} (-1)^M F_{JM}(n_1',n_1)\, F_{J-M}(-\bar{n}_2,-\bar{n}_2') \tag{D7c}$$

for the quark-quark, antiquark-antiquark and quark-antiquark interactions, respectively. As an example, the ket $|n,\bar{n}\rangle$ describes here the direct product state which is built up of a quark with quantum numbers n and an antiquark with quantum numbers $\bar{n}$. The operators $\mu_{12}(J)$ originate in the radial integral in the vertex function $Q^{\Sigma}_{nn'm}$. In addition, they carry the parity selection rule which we have omitted in the definition of $K_{12}(J)$. With the help of the modified radial integrals

$$S^{\Sigma}_{nn'm} = \frac{1-\eta_{\Sigma} g^{\Sigma\Sigma}(-1)^{\ell+J+\ell'}}{2} R^{\Sigma}_{nn'm} , \tag{D8}$$

the matrix elements of $\mu_{12}(J)$ are expressed as follows

$$\langle n_1', n_2' | \mu_{12}(J) | n_1, n_2 \rangle = \sum_{N\Sigma} \frac{\eta_\Sigma g^{\Sigma\Sigma}}{2(2J+1)} S^\Sigma_{n_1' n_1 m} S^\Sigma_{n_2' n_2 m}$$

$$\frac{1}{R^2 \Omega^\Sigma_m} \left[\frac{1}{\Omega^\Sigma_m + \varepsilon_{n_1'} - \varepsilon_{n_1}} + \frac{1}{\Omega^\Sigma_m + \varepsilon_{n_2'} - \varepsilon_{n_2}} \right] \tag{D9a}$$

$$\langle \bar{n}_1', \bar{n}_2' | \mu_{12}(J) | \bar{n}_1, \bar{n}_2 \rangle = \sum_{N\Sigma} \frac{\eta_\Sigma g^{\Sigma\Sigma}}{2(2J+1)} S^\Sigma_{-\bar{n}_1 - \bar{n}_1' m} S^\Sigma_{-\bar{n}_2 - \bar{n}_2' m}$$

$$\frac{1}{R^2 \Omega^\Sigma_m} \left[\frac{1}{\Omega^\Sigma_m + \varepsilon_{\bar{n}_1'} - \varepsilon_{\bar{n}_1}} + \frac{1}{\Omega^\Sigma_m + \varepsilon_{\bar{n}_2'} - \varepsilon_{\bar{n}_2}} \right]$$

$$\langle n_1', \bar{n}_2' | \mu_{12}(J) | \bar{n}_1, \bar{n}_2 \rangle = \sum_{NJ} \frac{2}{3} \frac{\eta_\Sigma g^{\Sigma\Sigma}}{(2J+1)} S^\Sigma_{n_1' n_1 m} S^\Sigma_{-\bar{n}_2 - \bar{n}_2' m}$$

$$\frac{1}{R^2 \Omega^\Sigma_m} \left[\frac{1}{\Omega^\Sigma_m + \varepsilon_{n_1'} - \varepsilon_{n_1}} + \frac{1}{\Omega^\Sigma_m + \varepsilon_{\bar{n}_2'} - \varepsilon_{\bar{n}_2}} \right] . \tag{D9c}$$

The quark-antiquark system can, in addition to the one gluon exchange discussed above, interact through the annihilation into a gluon which is represented by the Feynman diagram in Fig.(2d). Of course, this interaction is also contained in the part of the interaction Hamiltonian shown in eq.(5.5). The corresponding two-body operator is given by

$$\hat{V} = g^2 \left(\frac{\lambda^a}{2}\right)_{c'd'} \left(\frac{\lambda^a}{2}\right)_{dc} \delta_{f'g'} \delta_{fg} \tag{D10}$$

$$\frac{g^{\Sigma\Sigma}}{2\Omega^\Sigma_m} Q^\Sigma_{n'-p'm} \tilde{Q}^\Sigma_{-pnm} \left[\frac{1}{\Omega^\Sigma_m - \varepsilon_p - \varepsilon_n} + \frac{1}{\Omega^\Sigma_m + \varepsilon_{p'} + \varepsilon_{n'}} \right] \hat{a}^+_{c'n'} \hat{b}^+_{d'p'} \hat{a}_{cn} \hat{b}_{dp} .$$

Before writing down the two-body operator in first quantization, we have to cast the colour- and flavour-factors into a suitable form. Using the completeness and trace-orthogonality of SU(N) generators, the following identities are easily derived

$$\left(\frac{\lambda^a}{2}\right)_{c'd'} \left(\frac{\lambda^a}{2}\right)_{dc} = \frac{4}{9} \delta_{d'd} \delta_{c'c} + \frac{1}{3} \left(\frac{\lambda^a}{2}\right)_{c'c} \left(-\frac{\lambda^a}{2}\right)_{dd'} \tag{D11}$$

for $SU(3)_{colour}$ and, assuming $SU(2)_{isospin}$ for the flavour group,

$$\delta_{f'g'} \delta_{fg} = 2 \left[\frac{1}{4} \delta_{f'f} \delta_{g'g} - \left(\frac{\tau^\ell}{2}\right)_{f'f} \left(-\frac{\tau^\ell}{2}\right)_{gg'} \right] . \tag{D12}$$

The last term in eq.(D12) contains the $SU(2)_{isospin}$ generators in the representations which correspond to quarks and antiquarks, respectively

$$\underset{\sim}{I} = \begin{cases} \frac{1}{2}\underset{\sim}{\tau} & \text{for quarks} \\ -\frac{1}{2}\underset{\sim}{\tau}^T & \text{for antiquarks .} \end{cases} \tag{D13}$$

With the help of eqs.(D11) and (D12), the two-body operator in first quantization takes the form

$$V_{12} = \frac{\alpha_s}{R}\left[\frac{1}{4} - \underset{\sim}{I}_1\cdot\underset{\sim}{I}_2\right]\left[\vec{F}_1\cdot\vec{F}_2 + \frac{4}{3}\right]\sum_{J=0}^{\infty} \mu_{12}(J)\ K_{12}(J)\ . \tag{D14}$$

The angular momentum dependence of V_{12} is given by

$$\langle n_1',\bar{n}_2'|K_{12}(J)|n_1,\bar{n}_2\rangle = \frac{1}{2}(2J+1)\sum_{M=-J}^{J} (-1)^M\ F_{JM}(-\bar{n}_2,n_1)\ F_{J-M}(n_1',-\bar{n}_2') \tag{D15}$$

and $\mu_{12}(J)$ contains the radial integrals and the parity selection rule

$$\langle n_1',\bar{n}_2'|\mu_{12}(J)|n_1,\bar{n}_2\rangle = \sum_{N\Sigma}\frac{2}{3}\frac{\eta_\Sigma g^{\Sigma\Sigma}}{2J+1}\ S^{\Sigma}_{-\bar{n}_2 n_1 m}\ S^{\Sigma}_{n_1'-\bar{n}_2'm}$$
$$\frac{1}{R^2\Omega^{\Sigma}_m}\left[\frac{1}{\Omega^{\Sigma}_m-\varepsilon_{n_1}-\varepsilon_{\bar{n}_2}} + \frac{1}{\Omega^{\Sigma}_m+\varepsilon_{n_1'}+\varepsilon_{\bar{n}_2'}}\right]\ . \tag{D16}$$

D.2 Systems Consisting of Fermions and Gluons

The fermion-gluon interaction in second order perturbation theory has two sources in the interaction Hamiltonian. The contribution which we shall discuss first comes again from the part of $H_{int}(t)$ shown in eq.(5.5). It gives rise to the Compton diagrams in Figs.2e,2f,2h and 2i. As opposed to the fermion-fermion case, the Compton interaction is obtained by contracting two quark fields in the Wick decomposition of the time-ordered product. There are two different, non-vanishing possibilities of contracting two of the four quark fields in eq.(5.3). They lead to the direct (Fig.2e or 2h) and the exchange (Fig.2f or 2i) Compton diagrams.

Evaluating the matrix elements $V_{k'k}$, eq.(5.3), with the state vectors of the form

$$|\hat{\Phi}_k\rangle = \hat{c}^{\Sigma+}_{a_1 m_1}\ \hat{a}^{+}_{c_2 n_2}|\hat{0}\rangle \tag{D17a}$$

$$|\hat{\Phi}_k\rangle = \hat{c}^{\Sigma+}_{a_1 m_1}\ \hat{b}^{+}_{c_2 n_2}|\hat{0}\rangle \tag{D17b}$$

for the quark-gluon or antiquark-gluon systems, respectively, we arrive at the two-body operators

$$\hat{V} = -g^2 \left(\frac{\lambda^{a'}}{2} \cdot \frac{\lambda^{a}}{2}\right)_{c'c} \delta_{f'f} \frac{1}{2\sqrt{\Omega^{\Sigma}_{m} \Omega^{\Sigma'}_{m'}}} \cdot \left[\frac{Q^{\Sigma}_{-pnm} \tilde{Q}^{\Sigma'}_{n'-pm'}}{\varepsilon_p + \varepsilon_{n'} + \Omega^{\Sigma'}_{m'}} - \frac{Q^{\Sigma}_{pnm} \tilde{Q}^{\Sigma'}_{n'pm'}}{\varepsilon_p - \varepsilon_n - \Omega^{\Sigma}_{m}} \right]$$

$$\cdot\, \hat{c}^{\Sigma'+}_{a'm'} \hat{a}^{+}_{c'n'} \hat{c}^{\Sigma}_{am} \hat{a}_{cn} \tag{D18a}$$

$$\hat{V} = g^2 \left(\frac{\lambda^{a}}{2} \cdot \frac{\lambda^{a'}}{2}\right)_{cc'} \delta_{f'f} \frac{1}{2\sqrt{\Omega^{\Sigma}_{m} \Omega^{\Sigma'}_{m'}}} \cdot \left[\frac{Q^{\Sigma}_{-n-pm} \tilde{Q}^{\Sigma'}_{-p-n'm'}}{\varepsilon_p - \varepsilon_n - \Omega^{\Sigma}_{m}} - \frac{Q^{\Sigma}_{-npm} \tilde{Q}^{\Sigma'}_{p-n'm'}}{\varepsilon_p + \varepsilon_{n'} + \Omega^{\Sigma'}_{m'}} \right]$$

$$\cdot\, \hat{c}^{\Sigma'+}_{a'm'} \hat{b}^{+}_{c'n'} \hat{c}^{\Sigma}_{am} b_{cn} \tag{D18b}$$

for the direct diagrams in Figs.2e and 2h, and

$$\hat{V} = -g^2 \left(\frac{\lambda^{a}}{2} \cdot \frac{\lambda^{a'}}{2}\right)_{c'c} \delta_{f'f} \frac{1}{2\sqrt{\Omega^{\Sigma}_{m} \Omega^{\Sigma'}_{m'}}} \cdot \left[\frac{Q^{\Sigma}_{n'-pm} \tilde{Q}^{\Sigma'}_{-pnm'}}{\varepsilon_p + \varepsilon_{n'} - \Omega^{\Sigma}_{m}} - \frac{Q^{\Sigma}_{n'pm} \tilde{Q}^{\Sigma'}_{pnm'}}{\varepsilon_p - \varepsilon_n + \Omega^{\Sigma'}_{m'}} \right]$$

$$\cdot\, \hat{c}^{\Sigma'+}_{a'm'} \hat{a}^{+}_{c'n'} \hat{c}^{\Sigma}_{am} \hat{a}_{cn} \tag{D19a}$$

$$\hat{V} = g^2 \left(\frac{\lambda^{a'}}{2} \cdot \frac{\lambda^{a}}{2}\right)_{cc'} \delta_{f'f} \frac{1}{2\sqrt{\Omega^{\Sigma}_{m} \Omega^{\Sigma'}_{m'}}} \cdot \left[\frac{Q^{\Sigma}_{-p-n'm} \tilde{Q}^{\Sigma'}_{-n-pm'}}{\varepsilon_p - \varepsilon_n + \Omega^{\Sigma'}_{m'}} - \frac{Q^{\Sigma}_{p-n'm} \tilde{Q}^{\Sigma'}_{-npm'}}{\varepsilon_p + \varepsilon_{n'} - \Omega^{\Sigma}_{m}} \right]$$

$$\cdot\, \hat{c}^{\Sigma'+}_{a'm'} \hat{b}^{+}_{c'n'} \hat{c}^{\Sigma}_{am} \hat{b}_{cn} \tag{D19b}$$

for the exchange Compton diagrams as shown in Figs 2f and 2i. In order to define the two-body operators V_{12} in first quantization, we still have to rearrange the colour factors in eqs.(D18) and (D19). This is achieved with the help of the identity

$$\frac{\lambda^{a}}{2} \cdot \frac{\lambda^{b}}{2} = \frac{1}{6}\left[4\, \frac{\lambda^{c}}{2} \cdot \frac{\lambda^{d}}{2}\; f_{aec}\, f_{bed} - \delta_{ab} \mathbb{1}\right] . \tag{D20}$$

Now, we easily arrive at

$$V_{12} = \frac{\alpha_s}{18R} \left[4(\vec{F}_1 \cdot \vec{F}_2)^2 - 1\right] \sum_{p=1}^{\infty} \mu^{D}_{12}(p)\, K^{D}_{12}(p) \tag{D21}$$

$$\langle \Sigma_1' m_1', \bar{n}_2' | \mu^{D}_{12}(p) | \Sigma_1 m_1, \bar{n}_2 \rangle = \sum_{\substack{\mu \\ \kappa=\pm p}} \frac{\eta_{\Sigma_1'}}{R^2 \sqrt{\Omega^{\Sigma_1}_{m_1} \Omega^{\Sigma_1'}_{m_1'}}} \left[\frac{S^{\Sigma_1}_{-\bar{n}_2 - n m_1} \, S^{\Sigma_1'}_{-n-\bar{n}_2' m_1'}}{\varepsilon_n - \varepsilon_{\bar{n}_2} - \Omega^{\Sigma_1}_{m_1}} - \frac{S^{\Sigma_1}_{-\bar{n}_2\, n m_1} \, S^{\Sigma_1'}_{n-\bar{n}_2' m_1'}}{\varepsilon_n + \varepsilon_{\bar{n}_2'} + \Omega^{\Sigma_1'}_{m_1'}} \right] \quad \text{(D25b)}$$

$$\langle \Sigma_1' m_1', n_2' | \mu^{E}_{12}(p) | \Sigma_1 m_1, n_2 \rangle = \sum_{\substack{\mu \\ \kappa=\pm p}} \frac{\eta_{\Sigma_1'}}{R^2 \sqrt{\Omega^{\Sigma_1}_{m_1} \Omega^{\Sigma_1'}_{m_1'}}} \left[\frac{S^{\Sigma_1}_{n_2' n m_1} \, S^{\Sigma_1'}_{n n_2 m_1'}}{\varepsilon_n - \varepsilon_{n_2} + \Omega^{\Sigma_1'}_{m_1'}} - \frac{S^{\Sigma_1}_{n_2' -n m_1} \, S^{\Sigma_1'}_{-n n_2 m_1'}}{\varepsilon_n + \varepsilon_{n_2'} - \Omega^{\Sigma_1}_{m_1}} \right] \quad \text{(D26a)}$$

$$\langle \Sigma_1' m_1', \bar{n}_2' | \mu^{E}_{12}(p) | \Sigma_1 m_1, \bar{n}_2 \rangle = \sum_{\substack{\mu \\ \kappa=\pm p}} \frac{\eta_{\Sigma_1'}}{R^2 \sqrt{\Omega^{\Sigma_1}_{m_1} \Omega^{\Sigma_1'}_{m_1'}}} \left[\frac{S^{\Sigma_1}_{-n-\bar{n}_2' m_1} \, S^{\Sigma_1'}_{-\bar{n}_2 - n m_1'}}{\varepsilon_n - \varepsilon_{\bar{n}_2} + \Omega^{\Sigma_1'}_{m_1'}} - \frac{S^{\Sigma_1}_{n-\bar{n}_2' m_1} \, S^{\Sigma_1'}_{-\bar{n}_2 n m_1'}}{\varepsilon_n + \varepsilon_{\bar{n}_2'} - \Omega^{\Sigma_1}_{m_1}} \right] . \quad \text{(D26b)}$$

Throughout eqs.(D23) to (D26), the cases a and b correspond to the quark-gluon and antiquark-gluon systems, respectively. The ket $|\Sigma m,n\rangle$ denotes the direct product state consisting of a gluon with polarization Σ and quantum numbers m and a quark with quantum numbers n.

We now turn to the second contribution to the fermion-gluon interaction which is transmitted via the exchange of a virtual gluon and is depicted in Fig.2i and 2j. This process cannot take place in a theory based on an abelian gauge group (e.g. quantum electrodynamics), where the structure constants f_{abc} vanish.

Before we evaluate the two-body operator $\hat{V}$, it is convenient to introduce the functions $U^{\Sigma_1\Sigma_2\Sigma_3}_{m_1m_2m_3}(\sigma_1,\sigma_3)$, that describe the three-gluon vertex. They are given by

$$V_{12} = \frac{\alpha_s}{R} \vec{F}_1\cdot\vec{F}_2 \sum_{J=0}^{\infty} \mu_{12}(J)\, K_{12}(J) . \tag{D30}$$

The generators in the scalar product $\vec{F}_1\cdot\vec{F}_2$ are again understood to act in the representations of $SU(3)_{colour}$ which are appropriate to the different particles (see eqs.(D5) and (D23)).
The operator $K_{12}(J)$ has the matrix elements

$$\langle \Sigma_1'm_1',n_2'|K_{12}(J)|\Sigma_1m_1,n_2\rangle =$$
$$(-1)^{J+1}\sqrt{\tfrac{3}{2}}\sqrt{J+2}\sum_M (-1)^{M+M'} F_{JM}(n_2',n_2)\begin{pmatrix} J' & J & J \\ -M' & -M & M \end{pmatrix} \tag{D31a}$$

$$\langle \Sigma_1'm_1',\bar{n}_2'|K_{12}(J)|\Sigma_1m_1,\bar{n}_2\rangle =$$
$$-\sqrt{\tfrac{3}{2}}\sqrt{J+2}\sum_M (-1)^{M+M'} F_{JM}(-\bar{n}_2,-\bar{n}_2')\begin{pmatrix} J' & J & J \\ -M' & -M & M \end{pmatrix} \tag{D31b}$$

and the operator $\mu_{12}(J)$ is given by

$$\langle \Sigma_1'm_1',n_2'|\mu_{12}(J)|\Sigma_1m_1,n_2\rangle = \frac{(-1)^{J+1}}{\sqrt{24(J+2)}}\sum_{N\Sigma}\frac{\eta_{\Sigma_1'}\eta_\Sigma\, g^{\Sigma\Sigma}}{R^3\Omega^\Sigma_m\sqrt{\Omega^{\Sigma_1}_{m_1}\Omega^{\Sigma_1'}_{m_1'}}}$$
$$S^\Sigma_{n_2'n_2m}U^{\Sigma_1\Sigma_1'\Sigma}_{m_1m_1'm}(-,+)\left[\frac{1}{\Omega^\Sigma_m-\Omega^{\Sigma_1}_{m_1}+\Omega^{\Sigma_1'}_{m_1'}}+\frac{1}{\Omega^\Sigma_m-\varepsilon_{n_2}+\varepsilon_{n_2'}}\right] \tag{D32a}$$

$$\langle \Sigma_1'm_1',\bar{n}_2'|\mu_{12}(J)|\Sigma_1m_1,\bar{n}_2\rangle = \frac{(-1)}{\sqrt{24(J+2)}}\sum_{N\Sigma}\frac{\eta_{\Sigma_1'}\eta_\Sigma g^{\Sigma\Sigma}}{R^3\Omega^\Sigma_m\sqrt{\Omega^{\Sigma_1}_{m_1}\Omega^{\Sigma_1}_{m_1'}}}$$
$$S^\Sigma_{-\bar{n}_2-\bar{n}_2'm}U^{\Sigma_1\Sigma_1'\Sigma}_{m_1m_1'm}(-,+)\left[\frac{1}{\Omega^\Sigma_m-\Omega^{\Sigma_1}_{m_1}+\Omega^{\Sigma_2}_{m_2'}}+\frac{1}{\Omega^\Sigma_m-\varepsilon_{\bar{n}_2}+\varepsilon_{\bar{n}_2'}}\right] \tag{D32b}$$

Here, we refer with a and b to the quark-gluon and antiquark-gluon systems respectively.

D.3. Systems Consisting of Gluons

There are three different contributions to the gluon-gluon interaction in second order perturbation theory: the one-gluon exchange, the annihilation into a gluon and the four-gluon vertex which are shown in Figs. 2k, 2ℓ and 2m, respectively. The first two of these interactions emerge from the part of the interaction Hamiltonian which is cubic in the gluon field operators. Evaluating the matrix elements $V_{k'k}$ in eq.(5.3) with state vectors of the form

$$|\hat{\Phi}_k\rangle = \hat{c}^{\Sigma_1+}_{a_1m_1}\,\hat{c}^{\Sigma_2+}_{a_2m_2}|\hat{0}\rangle \tag{D33}$$

and considering terms with one contraction of the gluon fields in the Wick expansion of the timeordered product, we are still left with the sum of the gluon exchange and the annihilation interaction.

The contribution due to the gluon exchange is easily separated, yielding the two-body operator

$$\hat{V} = \frac{g^2}{4\pi R}(-if_{ba'''a'})(-if_{ba''a})\,\frac{\eta_{\Sigma'''}\eta_{\Sigma''}\eta_{\tilde{\Sigma}}\,g^{\tilde{\Sigma}\tilde{\Sigma}}}{8R^4\Omega^{\tilde{\Sigma}}_{\tilde{m}}\sqrt{\Omega^{\Sigma}_{m}\Omega^{\Sigma'}_{m'}\Omega^{\Sigma''}_{m''}\Omega^{\Sigma'''}_{m'''}}}\,(-1)^{M'''+M''+\tilde{M}} \tag{D34}$$

$$\begin{pmatrix} J''' & \tilde{J} & J \\ -M''' & \tilde{M} & M \end{pmatrix}\begin{pmatrix} J'' & \tilde{J} & J \\ -M'' & -\tilde{M} & M \end{pmatrix}\frac{U^{\Sigma'''\tilde{\Sigma}\Sigma'}_{m'''\tilde{m}m'}(-,+)\;U^{\Sigma''\tilde{\Sigma}\Sigma}_{m''\tilde{m}m}(-,+)}{\Omega^{\tilde{\Sigma}}_{\tilde{m}}+\Omega^{\Sigma''}_{m''}-\Omega^{\Sigma}_{m}}\;\hat{c}^{\Sigma'''+}_{a'''m'''}\,\hat{c}^{\Sigma''+}_{a''m''}\,\hat{c}^{\Sigma'}_{a'm'}\,\hat{c}^{\Sigma}_{am}\,.$$

For the annihilation, we arrive in a similar fashion at

$$\hat{V} = \frac{g^2}{4\pi R}(-if_{ba'''a''})(-if_{ba'a})\,\frac{\eta_{\Sigma'''}\eta_{\Sigma''}\eta_{\tilde{\Sigma}}\,g^{\tilde{\Sigma}\tilde{\Sigma}}}{32R^4\Omega^{\tilde{\Sigma}}_{\tilde{m}}\sqrt{\Omega^{\Sigma}_{m}\Omega^{\Sigma'}_{m'}\Omega^{\Sigma''}_{m''}\Omega^{\Sigma'''}_{m'''}}}\,(-1)^{M'''+M''+\tilde{M}} \tag{D35}$$

$$\begin{pmatrix} J''' & \tilde{J} & J'' \\ -M''' & -\tilde{M} & -M'' \end{pmatrix}\begin{pmatrix} J' & \tilde{J} & J \\ M' & \tilde{M} & M \end{pmatrix}U^{\Sigma'''\tilde{\Sigma}\Sigma''}_{m'''\tilde{m}m''}(-,-)\;U^{\Sigma'\tilde{\Sigma}\Sigma}_{m'\tilde{m}m}(+,+)$$

$$\left[\frac{1}{\Omega^{\tilde{\Sigma}}_{\tilde{m}}+\Omega^{\Sigma'''}_{m'''}+\Omega^{\Sigma''}_{m''}}+\frac{1}{\Omega^{\tilde{\Sigma}}_{\tilde{m}}-\Omega^{\Sigma'}_{m'}-\Omega^{\Sigma}_{m}}\right]\hat{c}^{\Sigma'''+}_{a'''m'''}\,\hat{c}^{\Sigma''+}_{a''m''}\,\hat{c}^{\Sigma'}_{a'm'}\,\hat{c}^{\Sigma}_{am}\;.$$

Here, we have made use of the function $U^{\Sigma_1\Sigma_2\Sigma_3}_{m_1m_2m_3}(\sigma_1,\sigma_2)$ defined in eqs. (D27) and (D28). In first quantization, the corresponding two-body operator V_{12} is given by

$$V_{12} = \frac{\alpha_s}{R}\,\vec{F}_1\cdot\vec{F}_2\sum_{J=0}^{\infty}\mu_{12}(J)\,K_{12}(J). \tag{D36}$$

This form applies to both the gluon exchange and the annihilation interaction. Of course, $\vec{F}_i$ (i=1,2) denotes the vector of the eight generators of $SU(3)_{colour}$ in the adjoint representation which are connected to the structure constants by eq.(D23). The angular momentum structure of V_{12} is contained in $K_{12}(J)$. For the gluon exchange, $K_{12}(J)$ is given by

$$\langle \Sigma_1'm_1',\Sigma_2'm_2'|K_{12}(J)|\Sigma_1m_1,\Sigma_2m_2\rangle = 2(2J+1)\sum_M (-1)^{M_1'+M_2'+M} \begin{pmatrix} J_1' & J & J_1 \\ -M_1' & M & M_1 \end{pmatrix} \begin{pmatrix} J_2' & J & J_2 \\ -M_2' & -M & M_2 \end{pmatrix}, \qquad \text{(D37)}$$

while for the annihilation we have

$$\langle \Sigma_1'm_1',\Sigma_2'm_2'|K_{12}(J)|\Sigma_1m_1,\Sigma_2m_2\rangle = 2(2J+1)\sum_M (-1)^{M_1'+M_2'+M} \begin{pmatrix} J_1' & J & J_2' \\ -M_1' & -M & -M_2' \end{pmatrix} \begin{pmatrix} J_1 & J & J_2 \\ M_1 & M & M_2 \end{pmatrix}. \qquad \text{(D38)}$$

Here, the ket $|\Sigma_1m_1,\Sigma_2m_2\rangle$ denotes the direct product state of two gluons with polarizations Σ_1 and Σ_2 and quantum numbers m_1 and m_2, respectively.
The matrix elements of the operators $\mu_{12}(J)$ are

$$\langle \Sigma_1'm_1',\Sigma_2'm_2'|\mu_{12}(J)|\Sigma_1m_1,\Sigma_2m_2\rangle = \frac{1}{16(2J+1)} \frac{\eta_{\Sigma_1'}\eta_{\Sigma_2'}}{R^4\sqrt{\Omega^{\Sigma_1}_{m_1}\Omega^{\Sigma_2}_{m_2}\Omega^{\Sigma_1'}_{m_1'}\Omega^{\Sigma_2'}_{m_2'}}} \sum_{N\Sigma} \frac{\eta_\Sigma g^{\Sigma\Sigma}}{\Omega^\Sigma_m} U^{\Sigma_1'\Sigma\Sigma_1}_{m_1'mm_1}(-,+)\, U^{\Sigma_2'\Sigma\Sigma_2}_{m_2'mm_2}(-,+) \left[\frac{1}{\Omega^\Sigma_m+\Omega^{\Sigma_1'}_{m_1'}-\Omega^{\Sigma_1}_{m_1}} + \frac{1}{\Omega^\Sigma_m+\Omega^{\Sigma_2'}_{m_2'}-\Omega^{\Sigma_2}_{m_2}}\right] \qquad \text{(D39)}$$

for the gluon exchange and

$$\langle \Sigma_1'm_1',\Sigma_2'm_2'|\mu_{12}(J)|\Sigma_1m_1,\Sigma_2m_2\rangle = \frac{1}{16(2J+1)} \frac{1}{R^4\sqrt{\Omega^{\Sigma_1}_{m_1}\Omega^{\Sigma_2}_{m_2}\Omega^{\Sigma_1'}_{m_1'}\Omega^{\Sigma_2'}_{m_2'}}} \sum_{N\Sigma} \frac{\eta_\Sigma g^{\Sigma\Sigma}}{\Omega^\Sigma_m} U^{\Sigma_1'\Sigma\Sigma_2'}_{m_1'mm_2'}(-,-)\, U^{\Sigma_1\Sigma\Sigma_2}_{m_1mm_2}(+,+) \left[\frac{1}{\Omega^\Sigma_m+\Omega^{\Sigma_1'}_{m_1'}+\Omega^{\Sigma_2'}_{m_2'}} + \frac{1}{\Omega^\Sigma_m-\Omega^{\Sigma_1}_{m_1}-\Omega^{\Sigma_2}_{m_2}}\right] \qquad \text{(D40)}$$

for the annihilation.

We now turn to the gluon-gluon interaction which is described by the elementary four-gluon vertex, Fig. 2m. The source of this process is the part of the interaction Hamiltonian which is proportional to g^2. Therefore, the contribution in second order perturbation theory to the matrix element $V_{k'k}$ is found in the first term of eq.(5.3). Using state vectors of the form (D33), we arrive at the two-body operator

$$\hat{V} = \frac{1}{8}g^2 \frac{1}{\sqrt{\Omega^{\Sigma}_{m}\Omega^{\Sigma'}_{m'}\Omega^{\Sigma''}_{m''}\Omega^{\Sigma'''}_{m'''}}} (-if_{ba'''a'})(-if_{ba''a}) \cdot \eta_{\Sigma'''}\eta_{\Sigma''}(-1)^{M'''+M''}$$

$$(2F^{*\,\Sigma'''\Sigma\Sigma''\Sigma'}_{m'''m\overset{*}{m}''m'} - F^{*\,\Sigma'''\Sigma'\Sigma''\Sigma}_{m'''m'\overset{*}{m}''m} - F^{*\,\Sigma'''\Sigma''\Sigma'\Sigma}_{m'''\overset{*}{m}''m'm})\ \hat{c}^{\Sigma'''+}_{a'''m'''}\ \hat{c}^{\Sigma''+}_{a''m''}\ \hat{c}^{\Sigma'}_{a'm'}\ \hat{c}^{\Sigma}_{am} \; . \qquad \text{(D41)}$$

The integral $F^{\Sigma\Sigma'\Sigma''\Sigma'''}_{mm'm''m'''}$ is given in Appendix C, eq.(C26).

In order to obtain a factorization of the angular momentum contribution in $\hat{V}$, we decompose $F^{\Sigma\Sigma'\Sigma''\Sigma'''}_{mm'm''m'''}$ as follows

$$F^{\Sigma\Sigma'\Sigma''\Sigma'''}_{mm'm''m'''} = \frac{1}{R^3}\sum_{\ell k}\frac{(2\ell+1)(-1)^k}{4\pi}\begin{pmatrix} J & \ell & J' \\ M & k & M' \end{pmatrix}\begin{pmatrix} J'' & \ell & J''' \\ M'' & -k & M''' \end{pmatrix} T^{\Sigma\Sigma'\Sigma''\Sigma'''}_{mm'm''m'''}(\ell) \; . \qquad \text{(D42)}$$

The form of $T^{\Sigma\Sigma'\Sigma''\Sigma'''}_{mm'm''m'''}(\ell)$ is easily determined from eqs.(C30) and (C31). In first quantization, the corresponding two-body operator V_{12} is given by

$$V_{12} = \frac{\alpha_s}{R}\vec{F}_1\cdot\vec{F}_2 \sum_{\ell=0}^{\infty}\left[\mu^A_{12}(\ell)\ K^A_{12}(\ell) + \mu^B_{12}(\ell)\ K^B_{12}(\ell) + \mu^C_{12}(\ell)\ K^C_{12}(\ell)\right]. \qquad \text{(D43)}$$

The $\vec{F}$ is again the vector of the $SU(3)_{colour}$ generators in the eight-dimensional adjoint representation. The matrix elements of the $K_{12}(\ell)$-operators are

$$\langle \Sigma_1'm_1',\Sigma_2'm_2'|K^A_{12}(\ell)|\Sigma_1m_1,\Sigma_2m_2\rangle = 2(2\ell+1)\sum_M(-1)^{M_1'+M_2'+M}$$
$$\begin{pmatrix} J_1' & \ell & J_1 \\ -M_1' & M & M_1 \end{pmatrix}\begin{pmatrix} J_2' & \ell & J_2 \\ -M_2' & -M & M_2 \end{pmatrix} \qquad \text{(D44a)}$$

$$\langle \Sigma_1'm_1',\Sigma_2'm_2'|K^B_{12}(\ell)|\Sigma_1m_1,\Sigma_2m_2\rangle = 2(2\ell+1)\sum_M(-1)^{M_1'+M_2'+M}$$

$$\begin{pmatrix} J_1' & \ell & J_2' \\ -M_1' & -M & -M_2' \end{pmatrix}\begin{pmatrix} J_1 & \ell & J_2 \\ M_1 & M & M_2 \end{pmatrix} \qquad \text{(D44b)}$$

$$\langle \Sigma_1'm_1',\Sigma_2'm_2'|K^C_{12}(\ell)|\Sigma_1m_1.\Sigma_2m_2\rangle = 2(2\ell+1)\sum_M (-1)^{M_1'+M_2'+M} \begin{pmatrix} J_1' & \ell & J_2 \\ -M_1' & M & M_2 \end{pmatrix}\begin{pmatrix} J_2' & \ell & J_1 \\ -M_2' & -M & M_1 \end{pmatrix}. \quad \text{(D44c)}$$

The $\mu_{12}(\ell)$-operators are given in terms of the T-function of eq.(D42) as

$$\langle \Sigma_1'm_1',\Sigma_2'm_2'|\mu^A_{12}(\ell)|\Sigma_1m_1,\Sigma_2m_2\rangle = -\frac{\eta_{\Sigma_1'}\eta_{\Sigma_2'}}{8R^2\sqrt{\Omega^{\Sigma_1}_{m_1}\Omega^{\Sigma_2}_{m_2}\Omega^{\Sigma_1'}_{m_1'}\Omega^{\Sigma_2'}_{m_2'}}} T^{\Sigma_1'\Sigma_1\Sigma_2'\Sigma_2}_{m_1'm_1m_2'm_2}(\ell) \quad \text{(D45a)}$$

$$\langle \Sigma_1'm_1',\Sigma_2'm_2'|\mu^B_{12}(\ell)|\Sigma_1m_1,\Sigma_2m_2\rangle = -\frac{\eta_{\Sigma_1'}\eta_{\Sigma_2'}}{8R^2\sqrt{\Omega^{\Sigma_1}_{m_1}\Omega^{\Sigma_2}_{m_2}\Omega^{\Sigma_1'}_{m_1'}\Omega^{\Sigma_2'}_{m_2'}}} T^{\Sigma_1'\Sigma_2'\Sigma_1\Sigma_2}_{m_1'm_2'm_1m_2}(\ell) \quad \text{(D45b)}$$

$$\langle \Sigma_1'm_1',\Sigma_2'm_2'|\mu^C_{12}(\ell)|\Sigma_1m_1,\Sigma_2m_2\rangle = -\frac{\eta_{\Sigma_1'}\eta_{\Sigma_2'}}{8R^2\sqrt{\Omega^{\Sigma_1}_{m_1}\Omega^{\Sigma_2}_{m_2}\Omega^{\Sigma_1'}_{m_1'}\Omega^{\Sigma_2'}_{m_2'}}} T^{\Sigma_1'\Sigma_2\Sigma_2'\Sigma_1}_{m_1'm_2m_2'm_1}(\ell). \quad \text{(D45c)}$$

D.4. Special Cases

Let us now consider the interactions between particles with the lowest possible angular momentum, i.e. $\kappa = \pm 1$ ($j = \frac{1}{2}$) for quarks and $J = 1$ for the (transverse) gluons. Here, the two-body operators K_{12} which were given in sections D.1, D.2 and D.3 can easily be expressed in terms of products of one-body spin operators $\underset{\sim}{S}$. Using the Wigner-Eckart theorem, the matrix elements of these one-body operators in spherical (instead of cartesian) coordinates are

$$(S^k)_{m'm} = \begin{cases} (-1)^{m'-\frac{1}{2}}\sqrt{\frac{3}{2}}\begin{pmatrix} \frac{1}{2} & 1 & \frac{1}{2} \\ -m' & k & m \end{pmatrix} & \text{for quarks} \quad \text{(D46a)} \\ -(-1)^{m+\frac{1}{2}}\sqrt{\frac{3}{2}}\begin{pmatrix} \frac{1}{2} & 1 & \frac{1}{2} \\ m & k & -m' \end{pmatrix} & \text{for antiquarks} \quad \text{(D46b)} \end{cases}$$

and

$$(S^k)_{M'M} = (-1)^{M'-1}\sqrt{6}\begin{pmatrix} 1 & 1 & 1 \\ -M' & k & M \end{pmatrix} \text{ for gluons .} \quad \text{(D46c)}$$

The scalar product in spherical coordinates is given by

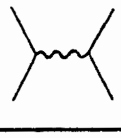
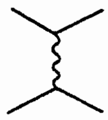
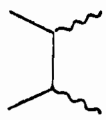
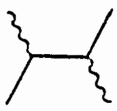
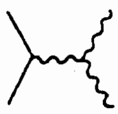
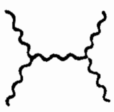
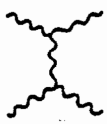
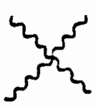
DIAGRAM		$K_{12}(0)$	$K_{12}(1)$	$K_{12}(2)$
		1	$4S_{12}$	0
		$\frac{1}{4} - S_{12}$	$\frac{3}{4} + S_{12}$	0
		-	$\frac{1}{2} - S_{12}$	$1 + S_{12}$
		-	$\frac{1}{2} + S_{12}$	$1 - S_{12}$
		1	S_{12}	0
		$\frac{2}{3}$	S_{12}	$-\frac{8}{3} + S_{12} + 2S_{12}^2$
		$-\frac{2}{3} + \frac{2}{3}S_{12}^2$	$-2 + S_{12} + S_{12}^2$	$\frac{2}{3} + S_{12} + \frac{1}{3}S_{12}^2$
	A	$\frac{2}{3}$	S_{12}	$-\frac{8}{3} + S_{12} + 2S_{12}^2$
	B	$-\frac{2}{3} + \frac{2}{3}S_{12}^2$	$-2 + S_{12} + S_{12}^2$	$\frac{2}{3} + S_{12} + \frac{1}{3}S_{12}^2$
	C	$-\frac{2}{3} + \frac{2}{3}S_{12} + \frac{2}{3}S_{12}^2$	$2 - S_{12}^2$	$\frac{2}{3} - \frac{2}{3}S_{12} + \frac{1}{3}S_{12}^2$

TABLE 9

The two-body operators $K_{12}(J)$ (see Appendix D) which contain the spin structure of the various diagrams. The in- and out-going quarks and gluons are assumed to carry lowest possible angular momentum ($\kappa = \pm 1$ for quarks, $J = 1$ for gluons). Note that $S_{12} = \underset{\sim}{S}_1 \cdot \underset{\sim}{S}_2$.

$$(-,+)_{\pm} = \frac{1}{\sqrt{2}}\left[(-,+) \pm (+,-)\right] \tag{D52}$$

for the quarks and

$$(\mathcal{M},\mathcal{E})_{\pm} = \frac{1}{\sqrt{2}}\left[(\mathcal{M},\mathcal{E}) \pm (\mathcal{E},\mathcal{M})\right] \tag{D53}$$

for the gluons in an obvious notation.

APPENDIX E. CONVENTIONS AND UNITS

We use the flat Minkowski space metric $g^{\mu\nu}$ with the signature

$$g^{\mu\nu} = g_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$$

$$g^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu} \; . \qquad \text{(E1)}$$

Greek indices ($\mu,\nu,\ldots$) can take the values 0,1,2 and 3, latin indices ($k,\ell,\ldots$) 1,2 and 3 when they refer to space-time. They are raised or lowered with $g^{\mu\nu}$

$$x^{\mu} = g^{\mu\nu}x_{\nu} \; , \qquad x_{\mu} = g_{\mu\nu}x^{\nu}$$

$$x_{o} = x^{o} \qquad x_{k} = -x^{k} = -[\underset{\sim}{x}]^{k} \; . \qquad \text{(E2)}$$

The 4 x 4 Dirac γ-matrices which satisfy the Clifford algebra

$$\{\gamma^{\mu},\gamma^{\nu}\} = 2g^{\mu\nu} \qquad \text{(E3)}$$

may be represented as follows

$$\gamma^{o} = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix} \quad \gamma^{k} = \begin{pmatrix} 0 & \sigma^{k} \\ -\sigma^{k} & 0 \end{pmatrix} , \qquad \text{(E4)}$$

where the σ^{k} are the 2 x 2 Pauli matrices

$$\sigma^{1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma^{2} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma^{3} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} . \qquad \text{(E5)}$$

Under hermitean conjugation, we have

$$(\sigma^{k})^{+} = \sigma^{k} \; , \qquad (\gamma^{\mu})^{+} = \gamma^{o}\gamma^{\mu}\gamma^{o} = \gamma_{\mu} \; . \qquad \text{(E6)}$$

We employ the usual Condon and Shortly phase-convention for the Clebsch-Gordon coefficients.

Throughout the text, we use "natural" units with

$$\hbar = c = 1 \, . \qquad \text{(E7)}$$

The correct factors of $\hbar$ and c, which have to be supplied in the various formulas, are easily found by considering the dimensions. As an example, eqs.(A12) and (A14) should be replaced by

$$\omega_n = \frac{\varepsilon_n R}{\hbar c} \quad \text{and} \quad \zeta_f = m_f R \frac{c}{\hbar} \quad . \tag{E8}$$

Some useful numerical relations are

$$\hbar c = 197{,}3285851 \text{ MeV fm}$$

$$1\text{GeV} = 5{,}06768963 \text{ hc fm}^{-1} \quad . \tag{E9}$$

BOOKS

Classical texts on quantum field theory are

- J.D. Bjorken, S.D. Drell, *Relativistic quantum Mechanics*, McGraw-Hill (1964)
  Relativistic quantum Fields, McGraw-Hill (1965)
- A.L. Fetter, J.D. Walecka, *Quantum Theory of Many-Particle Systems*, McGraw-Hill (1971)

A more recent book is

- C. Itzykson, J.-B. Zuber, *Quantum Field Theory*, McGraw-Hill (1980)

In the last few years, there have appeared many books which address quantum Chromodynamics directly. Some of these are

- F.E. Close, *An Introduction to Quarks and Partons*, Academic Press (1979)
- K. Huang, *Quarks, Leptons and Gauge Fields*, World Scientific (1982)
- F.J. Yndurain, *Quantum Chromodynamics; an Introduction to the Theory of Quarks and Gluons*, Springer (1983)
- T.-P. Chang, L.-F. Li, *Gauge Theory of Elementary Particle Physics*, Oxford University Press (1984)

An account on group theory may be found in

- M. Hamermesh, *Group Theory and its Application to Physical Problems*, Addison-Wesley (1962)
- R. Gilmore, *Lie Groups, Lie Algebras and Some of their Applications*, John Wiley & Sons (1974)

The Lie group SU(2) (angular momentum) is extensively discussed in

- A.R. Edmonds, *Angular Momentum in Quantum Mechanics*, Princeton University Press (1957)

The geometrical approach to gauge field theory is covered in

- C. Nash, S. Sen, *Topology and Geometry for Physicists*, Academic Press (1983)

There are many texts which show the connection between geometry and the BRS-transformation. One of these is

- B. Zumino, *Anomalies, Cocycles and Schwinger Terms*, in *Symposium on Anomalies, Geometry, Topology*, edited by W.A. Bardeen, A.R. White, World Scientific (1985)

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[30] Even though $\partial_\nu(\vec{\omega}\cdot\vec{F}^{\mu\nu})$ is conserved due to the antisymmetry of $\vec{F}^{\mu\nu}$, it does not necessarily lead to a well-defined charge in the quantized theory. In the general case, it should therefore be kept in the BRS-current in order to obtain a well-defined BRS-charge. The existence of this charge is assured by the assumption that the BRS-symmetry remains unbroken in the quantized theory, However, in view of the boundary conditions (4.8), the charge corresponding to $\partial_\nu(\vec{\omega}\cdot\vec{F}^{\mu\nu})$ is well-defined and integrates to zero in the finite volume theory.