

**The Real Cost
of
the Government Mortgage Indemnity
Scheme: An Application
of
the Option Pricing Theory**

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ABSTRACT

The legacy of apartheid in the social and economic fabric of South Africa is pervasive. More than two million households, with an average of five persons per household, are living in shacks or in hostels. Thus, the South African Government of National Unity as its most urgent priority has endeavoured to find solutions to this disastrous housing crisis. Thus, the Government proposed--amongst other measures--to establish a Government-supported Mortgage Indemnity Scheme.

However, such loan-guarantees are not cost free. Moreover, since they are contingent liabilities, the contingency of which may be realised and thus impose a cost to the Government, it is important that such cost be known or estimated. Using the modified Merton's model of an analytic derivation of the cost of loan guarantees, this paper evaluates the potential cost that may be imposed to the Government.

While the paper recognised that there may be scope for some kind of the Government loan guarantees, the overriding theme is that the Government should charge a fee for its loan guarantee. Moreover, it has also been illustrated that the main beneficiaries of the MIS will be: (a) households at the upper end of the low-cost housing market, and (b) private financial institutions which will be indemnified by the terms of MIS. Accordingly, the mere fact that the main beneficiaries will be those two categories of end-users and not these at the lower segment of the low-cost housing market¹ suggests that the MIS may not attain its principal purpose--that of serving these in the lowest income group. Thus, there is no reason why the Government should bear the likely cost of the MIS. In contrast, the Government should charge a fee for its guarantee.

¹Based on the fact that the households at the upper-end of low-cost housing market could presumably afford home loan and that private financial institutions would enter this segment of the market regardless of the government loan guarantee.

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Ljubiša Šešlija,

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INTRODUCTION

The legacy of apartheid in the social and economic fabric of South Africa is pervasive. Half of the South African population is illiterate; thirty-five percent of the economically active population is unemployed; more than two million households, with an average of five persons per household, are living in shacks or in hostels. Moreover, the mere fact that almost half of these households earn less than R800 per month gives them very little hope--in short or medium terms--of acquiring even a modest home. At the same time, to those households which are earning more than R800 and which can afford to buy or build a house, access to the home-loan market has been largely foreclosed. Thus, in spite of democratisation of the country, at the present, there are still two major categories of the South African citizens: the first category are those citizens who have access to the home loan market and are reasonably able to obtain a residential dwelling; and second, which consists of almost sixty percent of the total South African population, and which can be sub-divided into further two classes; the first of which can afford to borrow and to obtain housing units on their own but to whom access to the home loan market is denied, and the second class which under the present conditions--earning less than R800 per month, high interest rates--can not afford to buy houses or obtain home loans.

There are several reasons why access to the home loan market was denied to the majority of the South African population. Before the 1980's, the most important factor was apartheid itself with its racially-based laws which banned the non-white population from owning land in "white areas" and residential units in the territory of South Africa, and--while after 1980's--the bond repayment default and the ever-increasing political violence which led financial institutions to completely abandon the lower end of the housing market. Thus, in the last years of apartheid, the housing "backlog", and in particular the housing "backlog" amongst the non-white population, has become even worse.

The South African Government of National Unity, elected in the first democratic elections of April 1994, as its most urgent priority has endeavoured to find solutions to this disastrous housing crisis. However, the Government was also aware of the limited resources that could be devoted to solve the housing crisis. The Government, faced with the State's limited resources and an unwillingness of the major private financial institutions to re-enter the low-cost housing market, proposed--amongst other measures--to establish a Government-supported Mortgage Indemnity Scheme. The purpose of this scheme is to induce the major financial institutions to serve the lower end of the home loan

market and to make it possible for 'qualified' low-income households to borrow money to build houses.

However, such loan-guarantees are not cost-free. Moreover, since they are contingent liabilities, the contingency of which may be realised and thus impose a cost to the Government, it is important that such cost be known or estimated. The problem is that in establishing the Mortgage Indemnity Scheme, the South African Government neglected to quantify the cost of its loan guarantee, i.e. the cost of the Mortgage Indemnity Scheme, which cost may be substantial. Thus, the first goal of the paper is to evaluate the *real* cost of the Government loan guarantees, i.e. the cost of the Mortgage Indemnity Scheme.

Another unanswered question is whether the South African Government, and ultimately the South African tax-payers, should bear all likely cost of such loan guarantees. Under the proposed present structure of the Mortgage Indemnity Scheme, the only actor who will bear the likely cost is the South African Government. This paper will attempt to demonstrate that the end-users of the Government loan guarantees--private financial institutions and home loan borrowers those who will ultimately benefit most of the government loan guarantees--should bear the major part of the likely cost of the Mortgage Indemnity Scheme, if not its entirety. Thus, the paper will attempt to present the case that the Government should charge a fee for its loan guarantee programme.

The paper is structured as follows: Chapter One reviews the general theory of the consequences of the government loan guarantees and presents four different models for evaluating the cost of such guarantees. The Second chapter gives an overview of the current situation in housing in South Africa. Chapter Three presents a housing model which will be used to evaluate the *real* cost of the proposed Mortgage Indemnity Scheme, while the various results obtained by this model will be presented and analysed in Chapter Four. Finally, the Conclusion presents the summary of the main findings of the paper.

CHAPTER 1

GOVERNMENT LOAN GUARANTEE PROGRAMMES

1.1. The Consequences of Loan Guarantees Programmes

Almost all Governments are involved, in one way or another way, in the process of credit allocation. According to Kane,¹ credit allocation involves a number of different techniques that are intentionally employed by the Government to influence lenders to discriminate (negatively or positively) against some groups of would-be borrowers in economically arbitrary, but politically preferred, ways. The explanations usually applied to justify such Government interventions are based on: (a) unintentional discriminatory consequences of macro-economic aggregate policies;² (b) alleged "market failures" due to the unique nature of credit markets and, in particular, of information friction in credit markets.³ The aim of such Government interventions is to neutralize the negative effects of aggregate policies and/or market failure thus "improving one or more dimensions of national economic performance".⁴

Two major forms of Government intervention in credit markets are direct loans and loan guarantees. In the case of direct loans, the loans are dispensed by a Government agency which acts as an intermediary between the Government and the final recipients (i.e., targeted borrowers) of the loans instead of through private financial institutions, e.g. banks. The Government agencies raise the necessary funds on the financial markets by issuing Treasury securities so that the interest cost to the ultimate borrowers will be lower than that on the funds raised by banks or individuals because of the greater creditworthiness of the Government. If it is assumed that Government agencies pass this reduction in the interest rate on to borrowers, then the targeted borrowers will be able to obtain loans at lower rates of interest.

In the case of Government loan guarantees, the Government-targeted borrowers will obtain their loans through the banking system in the same way as do other borrowers. The difference is that loans granted to the designated borrowers are fully guaranteed by the

¹Kane, E. "Good intentions and unintended evil: The case against selective credit allocation", Journal of Money, Credit and Banking, (1977, vol. 9, pp. 50-69).

²The most obvious example is housing, where an increase of the rate of interest will have a negative effect on the low-income household's ability to purchase a house due to a higher rate of payment.

³Williamson gives the case of credit rationing as an example of the special nature of credit markets. Credit rationing exist when some of would-be borrowers are denied credits even though they are prepared to pay the market interest rate (or even higher) while there are some similar borrowers who do obtain the credit. Williamson, S. "Do informational friction justify federal credit programs", Journal of Money, Credit and Banking, (1994, vol. 26, pp. 523-544)

⁴Kane, E. *op.cit.* pp. 51

Government against payment default. In such circumstances, and within the framework of the competitive banking environment, banks will presumably pass the economic value of the guarantee on to the borrowers. Thus banks will charge a lower interest rate on such loans than on loans without the Government guarantees.

To assess the overall effects of Government intervention on credit markets, two models of Government loans guarantees are presented: the first model is Fried's general model of portfolio choice,⁵ and the second is Williamson's model of informational friction in credit markets.

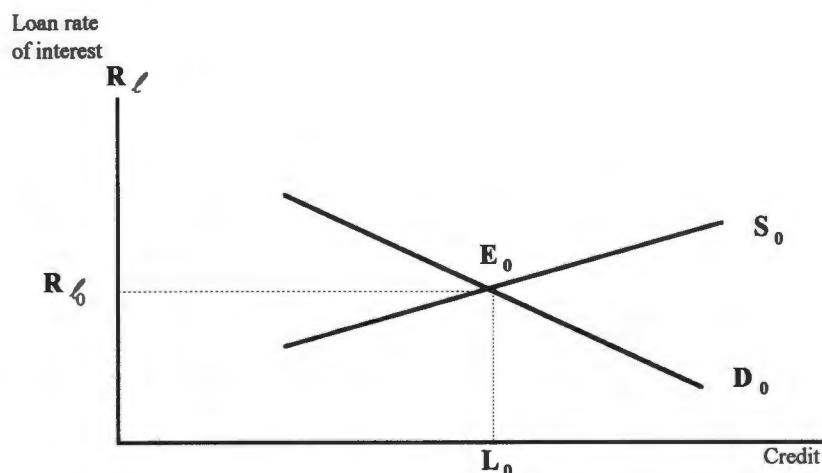
1.1.1. Fried's General Model of Portfolio Choice

In his assessment of the consequences of: (a) Government loan guarantees, and (b) Government direct loans programmes, on the rate of interest and aggregate demand, Fried employs "a general model of portfolio choice". According to Fried, the principal difference between the two programmes lies in a choice of the portfolios that banks and households will hold. The basic idea is that for any change in the amount of Government direct loans granted there is a change in "the amount of Government securities that must be held either by banks or households".⁶ The model is a two-period one: (a) Period 0--without any Government programmes; and (b) Period 1--with the Government-guaranteed loans programmes.

The loan market in Period 0 is presented in Figure 1-1, below. The demand for loans by the private sector is conventionally downward sloping ($=D_0$), and is a function of three variables: (1) the interest rate ($=R_\ell$); (2) rates of return on capital goods; and (3) rates of return on Government securities. Due to the increasing marginal cost of lending, the credit supply curve ($=S_0$) is upward sloping. It is a function of the rate of return on assets that banks can hold. The supply curve will shift up and inwards for an increase in the rate of return on these assets. The intersection of demand and supply curves at point E_0 depicts a loan market equilibrium. The point L_0 depicts the equilibrium level of loans, while $R_{\ell 0}$ depicts the equilibrium level of the loan rate of interest.

⁵Fried, J. "Government loan and guaranty programs", Federal Reserve Bank of St. Louis, (December 1983, pp. 22-31). In his article, Fried evaluates the consequences of both government direct loans and government loan guarantees on interest rate and aggregate demand. Throughout this paper the implications of government direct loans programs are ignored.

⁶Fried, J. *op. cit.* pp. 24



Source: Fried, (1983), pp. 25

FIGURE 1-1
The Loan Market, Excluding Government Loans Programmes

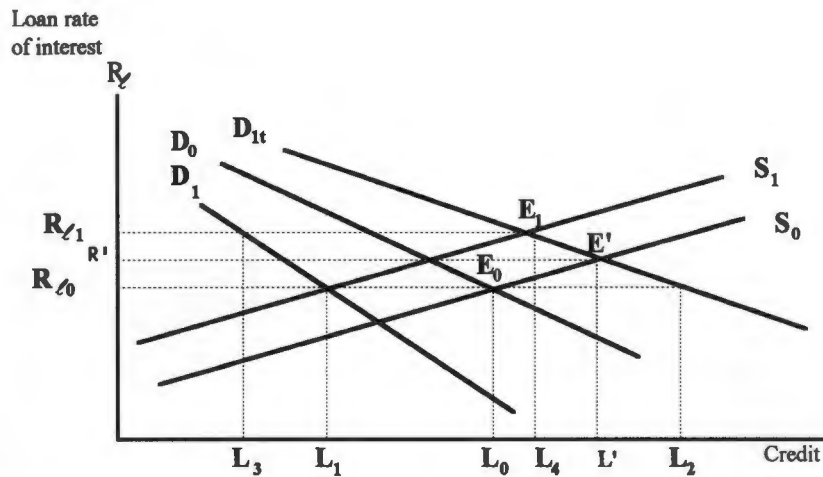
With the introduction of a Government-guaranteed loans programmes at the beginning of Period 1 the following two assumptions complete the model:

- (a) Government will provide designated borrowers with a fixed subsidy rate ($=R_s$); and
- (b) In a competitive banking environment, at the margin, the profits rate on guaranteed loans ($=R_g$) and on non-guaranteed loans ($=R_l$) are equalised, or

$$R_g = R_l - R_s. \tag{1.1}$$

This is shown in Figure 1-2, below, which presents the loan market with the Government-guaranteed loans programmes. This period can be divided into three stages. In the first stage, with the introduction of the loan guarantees, the total demand for loans will increase for a given interest rate R_{l_0} . The line D_{1t} presents the new total demand, while D_1 describes all potential borrowers who are not eligible for the guaranteed loans. At this stage, there is an excess demand for the loans given by $L_2 - L_0$.⁷ Thus, excess demand will force the loan rate of interest R_{l_0} to rise to a new level given by R . The new-quasi equilibrium ($=E'$) is given by the intersection between S_0 and D_{1t} at the point L' which depicts the quasi-equilibrium level of loans.

⁷Due to the introduction of loans guarantees, the total demand will become less inelastic, while demand for the non-guaranteed loans will become more elastic causing the demand curve for this loans to become steeper.



Source: Fried, (1983), pp. 26

FIGURE 1-2
The Loan Market, Including Government Guaranteed Loans

Secondly, a higher rate of interest ($=R'$) will induce banks to raise additional funds to satisfy the higher demand for loans. Thus, banks will sell Government securities causing their price to fall, and thus increase the rate of return on these securities, which will shift the supply curve up.⁸

In the capital goods market, the additional supply of loans ($=L'-L_0$) leads to an increase in demand for 'titles to capital goods' and pushes up the price of capital goods. However, a combination of a rise in price and a rise in interest rate leads to a decline in the rate of return on capital goods ($=R_k$). This decline in R_k causes the total demand for loans to decline but the amplitude of decline is assumed by Fried to be insufficient to shift the demand curve to its initial level at L_0 .⁹ Thus, at the end of this stage, the new equilibrium will be at the intersection of the new supply curve ($=S_1$) and the new total demand curve ($=D_{1t}$). The total supply of loans will be equal to an amount L_4 , consisting of L_3 non-guaranteed loans and L_4-L_3 Government-guaranteed loans. The new level of loans, L_4 , is lower than L_2 but it is still greater than L_0 . The equilibrium interest rate is R_{l1} and the rate of interest on Government-guaranteed loans, $R_g = R_{l1} - R_s$, will be less than R_{l0} .

In the third stage, as a result of a movement in the loan market, and, in particular, of an

⁸As mentioned above, the supply curve will shift up and inwards for an increase in the rate of return on assets that banks can hold.

⁹The demand of loans for capital goods is only a part of the total demand of loans.

increase in the price of capital goods,¹⁰ there is an increase in aggregate demand in general. Fried assumes that an increase in aggregate demand will lead to a rise in real income initially, and only after some time will it cause an increase in the price level. Thus, at the beginning of this stage, there is an increase in savings (as a result of an increase in the rate of interest), and an increase in investments. Over time, the price level will begin to rise, causing a decline in the real balances that are held by banks and households. At the end of this stage, banks, in an attempt to preserve the real value of their assets, will decrease the supply of the loans, thus forcing interest rates to increase again.

The overall result is, in the words of Fried, that "the distribution of loans would be such that recipients of Government-guaranteed loans would have a greater command over resources at the expense of borrowers ineligible for guarantees and the population at large who pays for the subsidies in the programmes".¹¹

1.1.2. Williamson's Model of a Credit With Costly State Verification

To evaluate the effects of Government loan guarantees on credit markets, Williamson uses "the costly state verification model".¹² The model is a two-period, partial equilibrium model of credit markets. The only agents ($=i$) in the credit market are lenders ($=\alpha$) and entrepreneurs ($=1-\alpha$). The agents ($=i$) are uniformly distributed over the interval $[0, 1]$, where $1/2 < \alpha < 1$.

In period 1, each lender has one unit of time. They have two choices regarding their use of the unit of time: (a) to consume it as leisure; or (b) to produce one unit of an investment good (i.e. savings) and to lend it to entrepreneurs. In the case of (b) consumption is postponed until period 2. Consumption in period 2 will depend on the expected return from investment in period 1, which is at least equal to the credit market expected return ($=r$).

In period 1, each entrepreneur can use one unit of savings to obtain technology that will yield return x in period 2. Thus, each entrepreneur will use return x to repay his debt in period 2. If x is less than the rate of promised payment ($=R$), the entrepreneur is in a state of bankruptcy. The value of the rate of promised payment is equal to zero and the lender

¹⁰The rise in the price of capital goods will stimulate the production of them, thus shifting the aggregate supply curve to the right.

¹¹Fried, J., *op. cit.* pp. 27

¹² Williamson, S., *op. cit.*

will incur a loss. On the assumption that x is known only to the entrepreneur, any lender who wants to verify that the entrepreneur is reporting a true x will incur a fixed verification cost ($=\gamma$) in the process of evaluating x . Thus, a financial debt contract is required to ensure that borrowers report correctly about their real returns while minimising the lender's verification cost.¹³

The expected return to the lender can be expressed as:

$$r = \pi'(R, \gamma) = \int_0^R (x - \gamma) dF(x) + R [1 - F(R)], \quad (1.2)$$

where $F(\dots)$ is the probability distribution function, with the corresponding probability density function $f(\dots)$. Simplifying, using partial integration, equation (1.2) can be expressed as

$$\pi'(R, \gamma) = R - \int_0^R F(x) dx - \gamma F(R). \quad (1.3)$$

For a constant γ , under the restriction on $F(x)$ such that

$$\pi''(R, \gamma) = -f(R) - \gamma f'(R) > 0, \quad (1.4)$$

the expected return to the lender is a concave function of R . The condition in equation (1.4) implies that there will be a sole maximum of expected return to entrepreneurs for some R^* and a constant γ , with $R = R^*$.

At equilibrium, the model may display two solutions: (a) if the quantity of the available loans at equilibrium is q , and q is equal to $1-\alpha$, then, all entrepreneurs will receive loans and rationing does not exist;¹⁴ (b) in the case when q is less than $1-\alpha$, in spite of the fact that all entrepreneurs are identical, some of them may be rationed out of the credit market while others very similar to them may receive the loans. In this case, the 'rationed' entrepreneurs are worse off than these entrepreneurs who received loans.¹⁵ And, this is the case that Government loan guarantees programmes attempt to correct.

¹³In spite of the existence of a financial debt contract the entrepreneur may always report that $x \leq 0$ which imply that $R = 0$. Therefore, lenders need to verify that entrepreneurs report a real value of x .

¹⁴A highly unrealistic assumption.

¹⁵The problem is that the rationed entrepreneurs can not offer a better debt contract to the lenders than existing ones. If they do, it will lead to higher interest rates, thus increasing the probability of bankruptcy, which in turn will increase the verification cost incurred by lenders, and ultimately that will lead to lower

On the assumptions that:

- (1) the Government guarantees are a fixed fraction of the promised payment on a loan to the lenders given by vR , where $0 < v < 1$, and R is the value of the promised payment;
- (2) lenders need to pay a fixed insurance premium given by P ;
- (3) verification of the entrepreneur's return is publicly observable;
- (4) the Government will not pay anything to the lender unless: (a) the borrower defaults on the promised payment; (b) the lender has verified the entrepreneur's return at some place in the world; (c) $x < vR$,

the expected return to the lender can be written as

$$\pi^l(R, \gamma) = \int_0^{vR} (vR - \gamma) dF(x) + \int_{vR}^R (x - \gamma) dF(x) + R [1 - F(R)] - P. \quad (1.5)$$

Simplifying, using partial integration, equation (1.5) can be expressed as

$$\pi^l(R, \gamma) = R - \int_{vR}^R F(x) d(x) - \gamma F(R) - P + R [1 - F(R)]. \quad (1.6)$$

With the insurance premium, P , set by the Government in such a way that the Government-guaranteed loan programme becomes self-financing, P can be expressed as

$$P = \int_0^{vR} (vR - \gamma) dF(x). \quad (1.7)$$

Substituting, the expected return

$$\pi'_1(R, \gamma, v) = 1 - F(R) + vF(vR) - \gamma f(R), \quad (1.8)$$

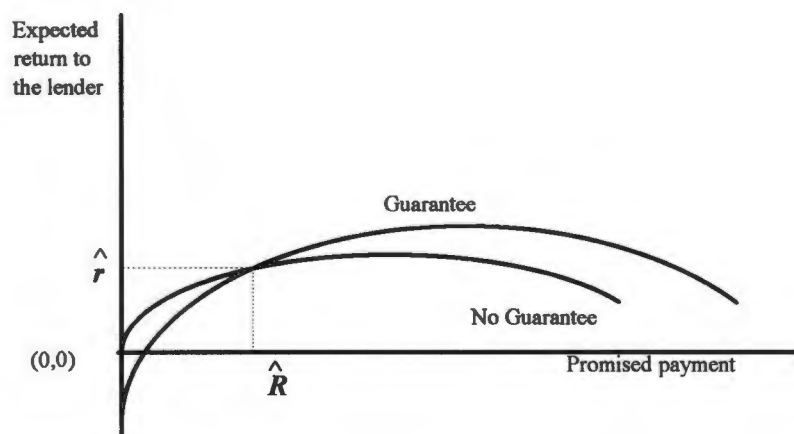
so that

$$\pi'_{13}(R, \gamma, v) = F(vR) + v^2 f(vR) > 0. \quad (1.9)$$

returns to the lenders.

Substituting for P (given by equation (1.7)) in equation (1.6) and differentiating with respect to the interest rate the result will be that the equilibrium rate of interest will be the same both without the Government loan guarantees programmes ($P=v=0$) and with no rationing ($q=1-\alpha$) and with the Government loan guarantees programmes on the credit market (P given by equation (1.6) and $v>0$), i.e. there will be no change in the interest rate due to the introduction of Government loan guarantees programmes.

This result is due to the fact that the equilibrium expected rate of return to the lenders is less than the maximum rate of returns ($=\hat{R}<R^*$), since the expected rate of return with the loan guarantee is $\pi'_{13}(R, \gamma, v)>0$. Figure 1-3, below, presents the situation with no rationing (a highly unrealistic situation) with and without the Government loan guarantees programme.



Source: Williamson (1994), pp. 532

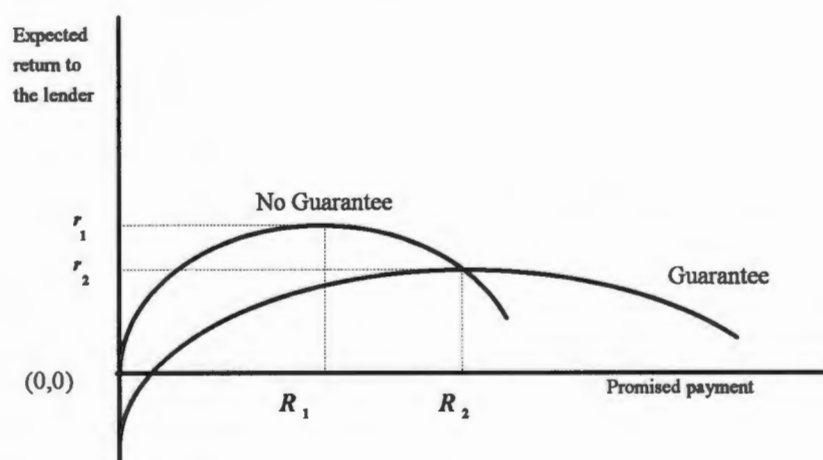
FIGURE 1-3
Effect of Loan Guarantee on the Lender's Expected Return Function

The equilibrium loan payment, ($=\hat{R}$), and the equilibrium risk-free market return, ($=\hat{r}$), is given by the intersection of the guarantee and non-guarantee curve.

In the situation where there is credit rationing equilibrium (a more realistic situation) and in the absence of the Government loan guarantees, there will be an initial equilibrium risk-free rate of return and loan payment given by r_1 and R_1 respectively. When the Government loan guarantees are introduced, there is a shift in the equilibrium rate of return and in the loan payment. The new equilibrium rates are given by r_2 and R_2 respectively. Figure 1.4, below, presents the equilibrium risk-free market return, (r_1 and r_2) and the equilibrium loan payment (R_1 and R_2) with and without loan guarantees on the

credit market.

Because the loan guarantee programme is self-financing, the new equilibrium rate must be at the point of intersection between "the perceived expected return function for the lender with a loan guarantee" and "the expected return function with no guarantee".¹⁶ Thus, r_2 has to be less than or equal to r_1 , which will lead to credit rationing. Since $\pi'_{13}(R, \gamma, \nu) > 0$ then R_2 is greater than R_1 and r_2 is less than r_1 . In this case, the consequences of the Government loan guarantees programmes are contradictory. All participants in the credit markets are 'worse off' with the introduction of loan guarantees. The lenders are confronted by a lower market rate of return thus causing the supply of the loans to be lower. Moreover, due to the lower supply of the loans, borrowers face a higher interest rate and the probability of credit rationing is increased.



Source: Williamson (1994), pp. 532

FIGURE 1-4
Effect of Loan Guarantee on the Lender's Expected Return Function

According to Williamson, the reason for the situation where "the Government loan guarantees have at best no effect, and at worse reduce welfare for everyone"¹⁷ lies in the fact that, with the introduction of loan guarantees, the optimal financial debt contract between lenders and would-be borrowers is deformed. His recommendation is that to prevent these harmful effects of loan guarantees the Government needs to introduce a

¹⁶Williamson, S., *op. cit.* pp. 532

¹⁷*Ibid.* pp. 532

premium "that reflects the higher risk inherent in higher loan interest rates; thus, loan guarantees will become neutral and that is the optimal situation".¹⁸

1.2. The Cost of Loan Guarantees Programmes

As has been shown by the above models, the costs of loan guarantees are implicit rather than explicit. The only explicit costs to the Government are the administrative costs of governing the programmes.¹⁹ All other costs are implicit in their appearance. They are in the form of inefficient uses of productive resources and affect the whole population. The feature of the loan guarantees--to appear to be cheaper than they are in reality-- has a huge appeal to politicians, thus leading to various loan guarantees programmes.

As mentioned above, under the loan guarantees programmes a Government guarantee to the lenders--usually at no charge--means that in the case of a borrower's default on his payment obligation, the Government will cover the value of the losses that lenders will incur in the process. That is similar to a Put option, where a Put option holder has a right, but not an obligation, to sell the underlying securities to the Put writer at a predetermined price. This feature of the loan guarantees--to behave as a put option--was first noted by Merton²⁰ who showed that option-pricing theory can be applied to determine the value of a Government loan guarantee. In the case of the Government loan guarantees, the Government is actually acting as a Put option writer and banks are acting as an option Put holder, whereas the underlying stock is equal to a different types of real holdings pledged as a security against the loans.

Subsequent to Merton's article, a number of models based, in one way or another, on the principles of option pricing theory, were developed to evaluate different types of loan guarantees. Therefore, it is useful to devote this section explaining very briefly the main characteristics of an option and the theoretical models of valuation of the option's "fair value".

1.2.1. Option Price Theory

According to Figlewski and Silber,²¹ a Call option is the right, but not the obligation to

¹⁸*Ibid.* pp. 532

¹⁹On the assumption of no default payments by the borrowers

²⁰Merton, C. "An analytic derivation of the cost of deposit insurance and loan guarantees; An application of modern pricing theory", *Journal of Banking and Finance*, (1977, vol. 1, pp 3-1).

²¹Figlewski, S., and Silber, W. "Options and options markets", in *Financial options: From theory to*

buy a specified quantity of some underlying assets/securities on or before an expiration date. A Put option is the right, but not the obligation, to sell a specified quantity of some underlying assets for a specified exercise price on or before an expiration date.

The main elements of an option contract are:

- (1) A writer or a grantor of an option and a buyer of the option. While a buyer has a right, but no obligation to undertake a specified transaction, a writer is under an obligation to perform a specified transaction chosen by the buyer.
- (2) The expiration date--which, in the case of an American option, is the time period within which a buyer has the right to exercise an option, and in the case of a European option, is the exact date when an option can be exercised.
- (3) The exercise price--a price at which the option buyer has the right to require performance of a specified transaction, and,
- (4) Underlying assets/securities--the assets/securities on which an option is written.

The holder of a Put option will exercise his right to sell the underlying assets to the writer of the Put only if the current value of the underlying assets is less than the option exercise price at the expiration date. If, at the expiration date, the current value of the underlying asset is higher than the exercise price, the option will expire unused. Thus, the upper and lower boundary value of the option, and the decision of the option holder, will depend on what happened to the value of the underlying assets. This can be expressed in mathematical form as:

$$\text{Put payoff at expiration} = \text{Max}(X - S, 0), \quad (1.10)$$

where X represents the exercise price and S is the current value of the underlying assets. Thus, a Put option payoff to the option holder will be whatever is greater within the boundaries given by $(X - S)$ or zero.

On the other hand, the profit to the writer, from writing a Put option, would have an upper limit given by the Put option value, while his losses could be as high as the value of the exercise price, if the value of the underlying assets falls to zero. Figure 1-5, below,

presents the profit and/or losses for a writer of a Put option. It can be seen from the illustration that in the case when the value of underlying assets falls from 100, the loss increases almost one unit for each unit of price, while on the opposite side, the profit is constant regardless of an increase in the value of the assets .

From Figure 1-5, below, it can be seen that any losses incurred by the buyer of the Put option, due to the fall in the value of the underlying assets, will be covered by the Put option writer. In the case that the value of underlying securities is equal to the exercise prices and a Put option expired unused the maximum loss is equal to the initial investment in buying a put option. Thus, buying a Put option in combination with a long position in the underlying securities is virtually the same as buying an insurance policy.

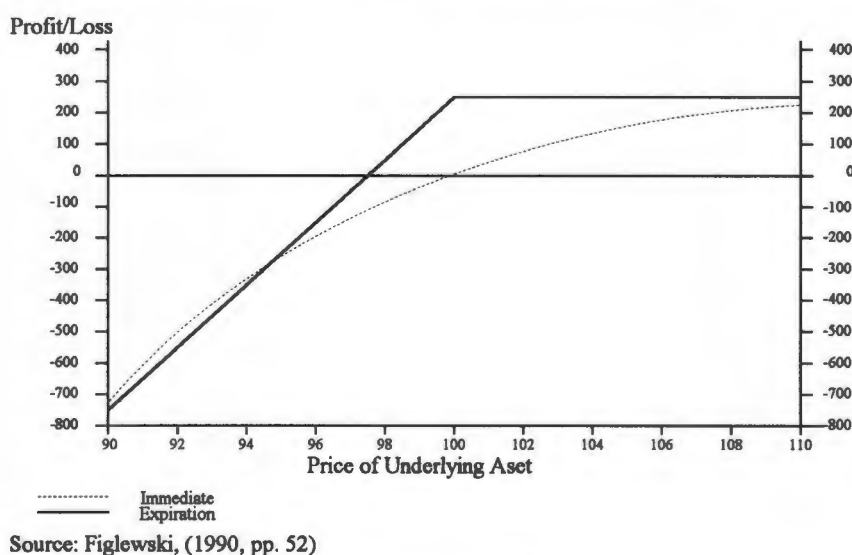


FIGURE 1-5
Short Put

The question arises as to how much an option buyer should pay for an option. Two most widely used theoretical valuation models for the purpose of calculating an option's fair value are the Black-Scholes and the Binomial option pricing models. Both models are based on the principle of arbitrage and on the option price relationship known as the "Put-Call parity" relationship. An arbitrage is a simultaneous buying at a lower price and selling the same thing, i.e. securities or portfolio of securities, at a higher price in order to obtain a riskless profit. In theory an arbitrage profit should not exist. A Put-Call parity is a relationship between the price of a Call and a Put option with the equal exercise price on one side, and on the other side, the current value of the underlying securities and the

present value of the option exercise price at the expiration date. In mathematical form the Put-Call parity can be expressed as: $C - P = S - PV(X)$, where C and P are the prices of a Call and a Put option respectively and $PV(X)$ is present value discounted from the expiration date.

1.2.1.1. The Black-Scholes Option Pricing Model

The model's five basic assumptions are as follows:

- (1) There exists a risky asset that can be bought and sold freely on the market at some current price S . The risky assets pays no dividends or other dispersal before the option's expiration date. There are no restrictions to short selling.
- (2) The rate of interest at which one can lend or borrow without risk is fixed and interest accrues on a continuous basis at a rate r .
- (3) The price of the underlying risky assets is continuous in time, and follows a random walk. The instantaneous mean and variance of the price change are proportional to the current price of the security, so that the mean and variance of the (continuously compounded) rate of return per unit time both have constant values. This implies that the distribution of the security's current price over any period will be log normal.²²
- (4) There are no factors such as taxes, transaction costs, or margin requirements that can affect a rate of return on a risky assets.
- (5) The option can be exercised only at the time of expiration T , i.e. the option is a European option.

Thus, the Black-Scholes option pricing model can be expressed in mathematical form as:

$$C = S N [D] - X e^{-rT} N [D - \sigma\sqrt{T}], \quad (1.11)$$

and

$$P = X e^{-rT} N [-D + \sigma\sqrt{T}] - S N [-D], \quad (1.12)$$

²²The feature of the log normal distribution is that price can not be negative and that the size of the average price change is larger at higher price level.

where:

- C = a call option fair price,
 P = a put option fair price,
 S = the price of the underlying assets,
 X = the exercise price,
 T = time to option expiration,
 r = the instantaneous riskless interest rate,

$$D = \frac{\ln\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}},$$

- σ = volatility of the underlying assets,
 $N(\dots)$ = the cumulative normal distribution function.

Although assumption (5) states that the option under consideration is a European option, equations (1.11) and (1.12) can be also used to calculate the fair value of an American option.²³

1.2.1.2. The Binomial Model

In addition to the Black-Scholes option pricing model, yet another model for determination of 'correct' option value has been derived. This is the Binomial model. The model is based on the 'two price' approach, where the current price of an underlying asset (S) can move, over the relevant interval of time, only in two directions: (1) up to a value uS , or, (2) down to a value dS .

Other assumptions are:

(a) There is a one-period Put option on the assets, with an exercise price of X . When the Put option expires in the next period, it pays the greater of the exercise price minus the asset's value or 0. P denotes the price of the Put option; and

(b) there is a riskless asset available that returns a total of R Rands in the next period for each one unit invested today. The gross interest rate R is one plus the interest rate, and it

²³The problem is that in the case of an American put option, the equation does not take into consideration whether it is economically optimal to exercise the option before the expiration date. In the case of an American call option, except in the case of payment of dividend, it is economically non-optimal to exercise an American option before expiration, in which case the value of the European and the American call option are equal.

is possible to lend or borrow freely at that rate.

Figure 1-6, below, presents both possible assets prices and its corresponding put price respectively over one period.

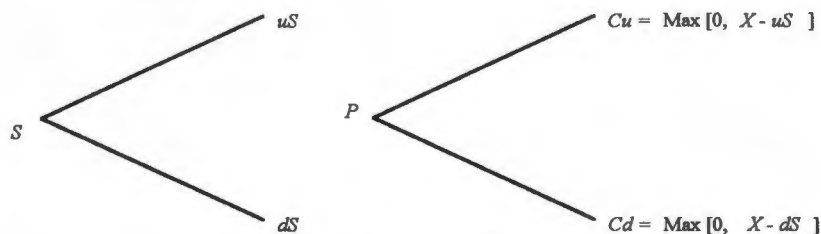


FIGURE 1-6
Possible Asset Prices and Corresponding Put Prices Over One Period

Under principles of arbitrage, the cost of a portfolio that consists only of some quantity of an underlying assets plus riskless borrowing and lending and has an equal payoff as the option, should be equal to the cost of the option, otherwise, there is an arbitrage opportunity. Thus the mathematical expressions of a portfolio that consists of h units of the risky assets and B -amount of Rands invested in the riskless asset that is equal to the option value in both the up-state and the down-state are:

Portfolio Payoff	Option Value
$h \times uS + RB$	$= X - uS$ (up-state),

(1.13)

$h \times dS + RB$	$= 0$ (down-state),
--------------------	---------------------

(1.14)

Solving for the system of two equations with two unknowns the result is:

$$h = \frac{X - uS}{S(u - d)} \tag{1.15}$$

$$B = \frac{d(X - uS)}{-R(u - d)} \tag{1.16}$$

Since, the portfolio of h units of assets plus B Rands of riskless borrowing costs is the same as a one period put option, so, $P=hS+B$,

or,

$$P = \frac{(R - d)(X - uS)}{R(u - d)} \quad (1.17)$$

Equation (1.17) is the Binomial model's formula for the fair value of a one-period put option.²⁴

1.2.2. The Application of the Option Pricing Theory in the Models of Valuation of Loan Guarantees

In general, two basic types of valuation models of loan guarantees have been developed: (1) continuous-time models and (2) discrete-time models. Continuous-time models are based on an assumption that investors are able to trade the underlying assets continuously in time and therefore to maintain a continuously hedged riskless portfolio. Since many loans and loan guarantees are not traded at all, and even if they were traded, would induce some very real cost, the assumption of continuous trading is rather too restrictive.

Discrete-time models relax the assumption of continuous trading by restricting investors' preferences. Both types of models have their advantages and limitations, and pending on the particular application one type of model may be more appropriate than the other. Below, four different models are presented in some more details.

1.2.2.1. Merton's Model of an Analytic Derivation of the Cost of Loan Guarantees

As mentioned above, the first attempt to evaluate the cost of loan guarantees to a Government was that of Merton. His model is a continuous-time model, based essentially on the same assumptions as the Black-Scholes model (see section 1.2.1.1.) The model is constructed around a firm that--in order to raise funds--issues a discounted bond with the face value of B dollars at maturity date. Merton uses two scenarios in his model: (a) Scenario 1, in which the firm will be able to meet its payment obligation at the maturity date; and (b) Scenario 2, in which the firm will not be able to meet its payment obligations at maturity.

At the maturity date in Scenario 1, the value of a firm's assets ($=V$) will be greater than its promised payment on the bond issues ($=B$). Thus, it will be worthwhile for the

²⁴The formula for the call option is almost the same one. The only difference is that instead of $(X - uS)$ in the above formula there is $(uS - X)$.

management of the firm to repay the firm's debt even if that means selling the assets of the firm. The value of the firm's debt will be equal to B and the value of the firm's assets will be equal to $V-B$. In Scenario 2, the value of the firm's debt is greater than the value of the firm's assets. Thus, the value of debt will be equal to V and the value of the firm's assets will be equal to zero. In an abbreviated form the firm's debt can be expressed as $\text{Min}[V, B]$ and the value of the asset as $\text{Max}(0, V-B)$.

When a third-party guarantor (a Government) is introduced into the model the outcomes are slightly different. In scenario 1, the result will be the same as without Government involvement. The management will repay the firm's debt, equal to B , and the value of the firm's assets will be equal to $(V-B)$. In Scenario 2, the bondholders will receive the value of the debt that is equal to B , the firm's assets will be equal to zero and the Government will incur a net payment or loss equal to $(B-V)$, the difference between its payment to bondholders and the value of the firm's assets.

Thus, at the date of expiration, in an abbreviated form, the value of the firm's assets is the same with or without the Government loan guarantee, or $\text{Max}(0, V-B)$; the value of the debt is always B , which implies that the debt is riskless, while the value of the loan guarantee (G) is $\text{Max}(0, B-V)$.

Thus, the equation for the value of the loan guarantee²⁵ can be expressed as:

$$G(T) = Be^{-rT}\Phi(x_2) - V\Phi(x_1), \quad (1.18)$$

where:

$$x_1 \equiv \frac{\log\left(\frac{B}{V}\right) - \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$x_2 \equiv x_1 + \sigma\sqrt{T}$$

where

σ^2 is the variance rate per unit time of the logarithmic changes in the value of the assets.

²⁵This is the same as the modified equation from the Black-Scholes model, section 1.2.1.1.

On the assumption that $Be^{-R(T)T}$ is the market value of the debt without the loan guarantee and $R(T)$ is the promised bond yield, then the market value of the debt with the loan guarantee will be Be^{-rT} . Thus,

$$G(T) + Be^{-R(T)T} = Be^{-rT} \quad (1.19)$$

or,

$$\frac{G(T)}{Be^{-rT}} = 1 - e^{-(R(T) - r)T}. \quad (1.20)$$

Equation (1.20) gives the cost of the loan guarantees as the fraction of the firm's debt covered by the guarantee.

1.2.2.2. Sosin's Model of the Valuation of Federal Loan Guarantees to Corporations

Sosin's model²⁶ estimates the value of loan guarantees in the presence of two types of debt, senior and junior debt. The model is based on the same general assumptions already mentioned under the Black-Scholes model (section 1.2.1.1.) with additional assumptions as follows: (a) throughout the existence of the firm, stockholders will receive constant proportional dividends ($=\delta$); and (b) at the expiration date T , the bondholders will receive the full face value of the outstanding bond, while the stockholder will receive any residual moneys.

The model is constructed around a firm that issued common stocks and bonds at time $t = t_0$. The value of the bonds as a fraction of the firm's assets is given by $100(1-s)\%$, where s ($0 \leq s \leq 1$) is the fraction of equity in the firm's capital structure. Thus, the aggregate market value of the firm, ($=V(t_0)$) can be expressed as: $V(t_0) \equiv S(t_0) + B(t_0)$. $S(t_0)$ is the market value of the firm's common stock, while $B(t_0)$ is the market value of the bonds, where $B(t_0) \equiv (1-s)V(t_0)$.

At time T , the stockholders have a right to repurchase the face value of outstanding debt from the bondholders. Thus, the position of the stockholders ($=S(t)$) is equal to that of a holder of a European call option, with the face value of the outstanding debt equal to the

²⁶Sosin, B. "On the valuation of federal loan guarantees to corporations", The Journal of Finance, (1980, vol. 35, pp. 1209-1221).

exercise price X . Thus, if $C(V(t_0), X, \tau)$ denotes the value of the European call option at time $t=t_0$, where τ is the term of the option ($\tau \equiv T-t_0$), then the exercise price can be determined implicitly from the following equation:

$$S(t_0) \equiv sV(t_0) = C(V(t_0), X, \tau) + (1 - e^{-\delta\tau})V(t_0), \quad (1.21)$$

where the term $(1-e^{-\delta\tau})V(t_0)$ is the present value of the future proportional dividend stream. The value of a European call option that pays constant proportional dividends can be expressed as:

$$C(V(t_0), X, \tau) = V(t_0)e^{-\delta\tau}N(d_1) - Xe^{-r\tau}N(d_2) \quad (1.22)$$

where

$$d_1 = [\ln(V(t_0)/X + (r - \delta + \sigma^2/2)\tau)/\sigma\sqrt{\tau}], \text{ and } d_2 = d_1 - \sigma\sqrt{\tau}.$$

In the second period, the firm undertakes an investment project where the costs of the project ($=I$) are greater than the project's market value ξI , where ξ ($0 \leq \xi \leq 1$) is a present value profitability index of the project.²⁷

The firm will finance the project with the subordinated (junior) debt γ , ($0 \leq \gamma \leq 1$) and with a fraction ($=1-\gamma$) of equity. At time $t = t_0$, after the project has been financed,

can be expressed as:

$$V_A(t_0) \equiv V(t_0) + \xi I = B_A(t_0) + D(t_0) + S_A(t_0) \quad (1.23)$$

where, under the assumption of the competitive market, $D(t_0) = \gamma I$ and $D(t_0)$ denotes the initial market value of the junior debt.²⁸

Because the firm is undertaking an investment project for which the market value is less than the investment costs, the firm will incur a loss at time T . Due to assumption (b) the loss, $(1-\xi)I$, will be borne by pre-existing stockholders, while the inclusion of the junior debt will ensure that senior debt holders will be paid in full at time T . Thus, the value of the firm's bond before the project will not be influenced by the project, or $B_A(t_0) = B(t_0)$. The market value of the firm's common stock after financing the project can be expressed as

²⁷Given the choice, a value maximising firm will invest only in projects where $\xi > 1$

²⁸Subscript A denotes values after financing.

the call option to buy the firm at time T for the sum of the exercise prices of the junior ($=X_D$) and senior ($=X_A$) debt.

In spite of its financial un-feasibility, on the assumption that the project is politically and socially beneficial, the Government would provide the firm with a loan guarantee for a fraction of the junior debt equal to α , ($0 \leq \alpha \leq 1$), at zero cost. As in the case of the senior debt without loan guarantees, the value of the senior debt will be unaffected with the inclusion of loan guarantee in the model, thus, $B_G(t_0) = B(t_0)$. Subscript G denotes values under the loan guarantee. The value of the common stock will be

$$S_G(t_0) = C(V_A(t_0), X_A + X_D^G, \tau) + (1 - e^{-\delta\tau})V_A(t_0). \quad (1.24)$$

The firm's assets value under a loan guarantee will be: $V_G(t_0) \equiv B(t_0) + D(t_0) + S_G(t_0)$. The theoretical value of the loan guarantee and the potential cost to a Government can be expressed as $G(\dots) = V_G(t_0) - V_A(t_0)$. According to Sosin, this will be the cost to a Government if the Government were to buy the guarantee in a competitive market.

At the end of his paper, Sosin presents his simulation results. The results suggest that the cost of loan guarantee moves in the same direction as the variance of the rate of return on the firm's assets.

1.2.2.3. Jones and Masson's Model of Valuation of Loan Guarantees

In their article, Jones and Mason²⁹ evaluate four types of debt: (a) a fully guaranteed issue of non-callable coupon debt; (b) a partially guaranteed issue of non-callable coupon debt; (c) a junior and a senior issue of non-callable coupon debt with guarantees and (d) a callable coupon debt with guarantees. They use contingent claim valuation models similar to that of Black-Scholes and Merton. Their assumptions are as follows:

- (1) There is continuous trading with no cost and no restriction on lending and borrowing at a risk-free interest rate. Short sales are allowed, and proceeds from such sale can be reinvested;
- (2) The risk-free interest rate, r , is known and is constant over time;
- (3) The price changes of the firm's common stock are continuous over time.

²⁹Jones, P., Masson, S. "Valuation of loan guarantees", Journal of Banking and Finance, (1980, vol. 34, pp 89-107)

- (4) The instantaneous variance of return, σ^2 , on the assets value, V , is constant over time;
- (5) Total cash payout, P , to all claimants depends at most on the asset value of the firm.

The valuation of a contingent claim such as a non-guaranteed debt $D(V, \tau)$ or loan guarantee $G(V, \tau)$ is based on the partial differential equations given by Merton.³⁰ The partial differential for a non-guaranteed debt $D(V, \tau)$ is

$$\frac{1}{2} \sigma^2 V^2 D_{VV} + (rV - P) D_V - D_\tau - rD + p = 0, \quad (1.25)$$

and the partial differential equation for a loan guarantee $G(V, \tau)$ is

$$\frac{1}{2} \sigma^2 V^2 G_{VV} + (rV - P) G_V - G_\tau - rG = 0, \quad (1.26)$$

where τ is time to maturity, P is the cash payout per unit time to the claim and subscripts VV , V , and τ denotes partial differentiation. Thus, the valuation of the contingent claims given by the above equations depends only on observable variables--or if the variables are not observable--ones that can be easily estimated.

To determine a unique solution of the contingent claims from equation (1.25) and (1.26) respectively there is a need for terminal and boundary conditions. The terminal condition will determine the value of the contingent claims at maturity, $\tau=0$, as a function of the firm's asset value. There are two boundary conditions: a lower boundary condition and an upper boundary condition. The lower boundary condition will determine the value of the contingent claim in the case of the firm defaulting on its coupon payment obligation before maturity date. The upper boundary condition will determine the value of the contingent claim in the case when the assets is growing in value, $V \rightarrow \infty$.

On the assumption that at the maturity date, $\tau=0$, the asset value ($=V$) is equal to or greater than the face value of the bond ($=B$) or $V \geq B$, the bondholders will receive a full amount of the bond's face value and $D(V, 0)=B$. If the asset value is less than the face value of the firm's bond, $V < B$, then the debt can be worth only as much as the value of the firm's assets, or $D(V, 0)=V$. Thus, the terminal condition of a fully guaranteed non-callable coupon debt is

³⁰*Ibid.* pp. 91

$$D(V, 0) = \min(B, V) \quad (1.27)$$

which implies that at the maturity date, the debt will be worth the lesser of the face value of the bond or the firm's asset value.

If, at maturity date, $\tau=0$, the asset value ($=V$) is greater than the face value of the bond ($=B$), $V>B$, the value of the guarantee is equal to zero. If the asset value is less than the face value of the bond, $V<B$, then the value of the guarantee is equal to the difference between two, $G(V, 0) = B-V$. Thus, the terminal condition of the guarantee is

$$G(V, 0) = \min(0, B-V), \quad (1.28)$$

which implies that at the maturity date, the debt will be worth zero or the difference between the principal and the assets value, whichever is lower.

Should the firm's asset value decrease to zero, $V=0$, at any time prior to maturity, the debt will also be worth zero. The lower boundary condition for a fully guaranteed non-callable coupon debt is

$$D(0, \tau) = 0 \quad (1.29)$$

Under the same circumstances, but with the additional assumption that a Government would undertake on principal payment, a lower boundary condition for the value of the guarantee will be

$$G(0, \tau) = B. \quad (1.30a)$$

In many loan guarantees programmes, a Government will undertake not only to repay the principal but also to repay the present value of coupons, discounted at the risk-free interest rate. Thus, if there is a promised coupon of c per unit time, the total liability of a Government will be equal to $R(\tau)$, or $R(\tau) = \frac{c}{r}(1-e^{-r\tau}) + Be^{-r\tau}$. Thus, an alternative lower boundary condition for the guarantee will be

$$G(0, \tau) = R(\tau). \quad (1.30b)$$

In the case when the firm's assets value is growing, $V \rightarrow \infty$, an upper boundary condition of a fully guaranteed non-callable coupon debt is

$$D(\infty, \tau) = R(\tau), \quad (1.31)$$

which implies that the value of the debt will approach the value of a risk-free bond. The value of a guarantee will be equal to zero.

$$G(\infty, \tau) = 0. \quad (1.32)$$

If a firm pays dividends ($=d$) per unit time over the life of the non-callable coupon debt, with c representing promised payments, B the principal, and τ the number of time periods, the value of the non-guaranteed debt can be calculated from equation (1.25). Now, $P=c+d$, and $p=c$. The terminal condition is given by equation (1.27). The lower and the upper boundary conditions are given by equation (1.29) and (1.31) respectively.

The value of the guarantee³¹ can be calculated from equation (1.26) with $P=c+d$. The terminal condition is given by equation (1.28). The lower boundary condition will be either equation (1.30a) or (1.30b) and the upper boundary condition are given by equation (1.32).

In the case where the loan guarantee covers only a fraction ($=\delta$) of the principal debt, equation (1.26) can be used to calculate the value of the partial guarantee. The terminal condition will be

$$G(V, 0) = \max(0, \delta B - V), \quad (1.33)$$

The lower boundary is given by

$$G(0, \tau) = \delta B, \quad (1.34)$$

while the upper boundary will be given by equation (1.32). The value of the non-guaranteed part of the debt can be calculated by equation (1.25), with the terminal and boundary conditions given by equations (1.27), (1.29) and (1.31) respectively.

The third case considered by Jones and Mason is that of a firm issuing two types of non-callable coupon bonds, junior and senior. The junior bond pays coupons with value equal to c' per unit of time and with the bond's principal equal to B' . Similarly, the senior bond pays coupons with value equal to c per unit of time and with principal equal to B . The maturity date for both bonds is the same and the firm will pay dividends d per unit of time.

On the assumptions that only the senior debt is fully guaranteed and the junior debt is non-

³¹This situation is highly unlikely, i.e. that a firm under a loan guarantee will pay dividends.

guaranteed, the value of the fully guaranteed debt is given by the sum of the values of the non-guaranteed debt and the loan guarantee. The values of non-guaranteed debt and of loan guarantee can be calculated using equations (1.25) and (1.26) respectively, with $P=c+c'+d$, and $p=c$. Conditions for the debt's value are given by equations (1.27), (1.29) and (1.31), while conditions for the value of the loan guarantee are given by equations (1.28), (1.30a) and (1.32).

The value of the junior debt, when it is fully guaranteed and the senior debt is non-guaranteed, is the sum of the values of non-guaranteed junior debt and loan guarantees, given by equations (1.25) and (1.26), with $P=c+c'+d$, and $p=c'$. However, based on the assumption that the junior debt is subordinated to the senior debt, implying that the non-guaranteed junior debt will receive only the difference between the firm's assets value and the face value of the senior bond, the terminal condition of the value of the non-guaranteed junior debt is given by

$$D(V, 0) = \min (B', \max (0, V - B)), \quad (1.35)$$

Based on the same argument, if the value of the firm's assets should be less than the sum of the face values of junior and senior bonds, the guarantor must pay any differences between those two values up to the face value of the junior debt. Thus, the terminal condition of the value of the loan guarantee is given by

$$D(V, 0) = \min (B', \max (0, B' + B - V)). \quad (1.36)$$

The boundary conditions are the same as for the case of the fully guaranteed senior debt described above.

The last case evaluated in the paper is the case of a callable coupon debt with guarantees. Again, equations (1.25) and (1.26) can be used to evaluate the value of the non-guaranteed callable debt and its guarantee, where $P=c+d$, and $p=c$. However, in contrast to the non-callable bond, depending on the value of the firm's assets ($=\bar{V}(\tau)$) at time τ -- if $V(\tau) \geq K(\tau)$, where $K(\tau)$ is the value of callable debt at time τ --it may be optimal for the stockholders to call the debt. Thus, the upper boundary condition of the value of the callable debt is given by

$$D(\bar{V}(\tau), \tau) = K(\tau). \quad (1.37)$$

If the stockholders recall the debt, there is no need for a guarantee. Thus, the value of the

loan guarantee will be equal to zero. The upper boundary condition of the value of the loan guarantee is given by

$$D(\bar{V}(\tau), \tau) = 0. \quad (1.38)$$

The terminal and lower boundary conditions of the value of the loan guarantee is given by equations (1.27) and (1.29) respectively. The terminal and lower boundary conditions of the value of the guarantee is given by equations (1.28) and (1.30a) respectively. The value of the guaranteed debt is given by the sum of the values of the non-guaranteed callable debt and its guarantee.

As in the case of Sosin's model, the result of the simulation presented at the end of the Jones and Masson's paper suggest that the value of the guarantees moves together with the instantaneous variance of return on the asset's value.

1.2.2.4. The Model of Chen, Chen and Sears: the Case of Chrysler Corporation

A rare example of the use of discrete time models to evaluate Government loan guarantees is the model of Chen, Chen and Sears³². A theoretical model, based on a risk-neutral valuation relationship (RNVR) in discrete time,³³ is presented in the first part of their paper, while the second part of their paper presents an empirical analysis of the case of Chrysler corporation.³⁴

The two-period model is constructed around the company (in this case Chrysler corporation) which in the first period issues a combination of common stock (=E) and discounted bonds/senior debt (=S) with face value (=D₁) at maturity date and in the second period issues a junior bond under the Government loan guarantee. The firm's cash flow at maturity date, (=V̄) is normally distributed.

³²Chen, A., Chen, K., Sears, R. "The value of loan guarantees: The case of Chrysler corporation", Research in Finance, (1986, vol. 6, pp. 101- 117)

³³According to Brennan, "a valuation relationship is a formula relating the value of contingent claim, or its derivatives, to the value of the underlying asset and other exogenous parameters". Brennan, M. J., "The pricing of contingent claims in discrete time models", The Journal of Finance, (1977, vol. 34, pp. 56-68)

³⁴At the end of 1979, Chrysler corporation incurred deficit of US\$ 1 billion. Thus, without help, the corporation would have become bankrupt. With bankruptcy came the prospect of unemployment of thousands of workers and almost US\$ 1.8 billion in liabilities faced by the US government in pension and unemployment compensation. At the beginning of 1980, American President Carter provided US\$ 1.5 billion to the corporation in government loan guarantees. This made it possible for Chrysler to obtain new loans on the credit market and to restructure itself.

If, at maturity date in the first period, the company's cash flow is greater than its debt, $\bar{V}_1 > D_1$, the payment to the stock holders (\bar{Y}_E) will be $\bar{V}_1 - D_1$. If the value of the company's cash flow at maturity is less than or equal to D_1 , the stockholders will receive nothing. Thus, according to the model and based on RNVR, the current market value of the company ($=V_E$) can be expressed as

$$V_E = (V_0 - D_1 R^{-1}) N(-K_1) + R^{-1} \sigma n(K_1) \quad (1.39)$$

and the current market value of the senior debt ($=V_S$) is

$$V_S = D_1 R^{-1} N(-K_1) + V_0 [N(-K_0) - N(-K_1)] + R^{-1} \sigma [n(-K_0) - n(K_1)], \quad (1.40)$$

where subscript 0 denotes current values, R is 1 plus the risk-free interest rate, σ is the standard deviation of the cash flow at the maturity date of period one, $N(\dots)$ is the cumulative standard normal distribution, $n(\dots)$ is the standard normal density function, and $K_0 = \frac{-V_0 R}{\sigma}$, $K_1 = \frac{D_1 - V_0 R}{\sigma}$, thus $V_0 = V_E + V_S$.

At the beginning of the second period, the company, in order to finance a new risky project, issues a junior debt with a face value of D_2 that is guaranteed by the Government. The post-investment total cash flow ($=\bar{V}_3$) will be $\bar{V}_3 = \bar{V}_1 + \bar{V}_2$. The cash-flow variance is equal to σ_p^2 . If, at maturity date in the second period, the company's cash flow is greater than its total debt ($=B$), where $B = D_1 + D_2$, the payment to the stock holders ($=\bar{Y}_E^P$) will be $\bar{V}_3 - B$. If the value of the company's cash flow at maturity is less than or equal to B , the stockholders will receive nothing and senior bondholders will receive \bar{V}_3 , and if the post-investment value of the cash flows is less than zero, both of them will receive nothing.

The case of junior bondholders is different. Due to the existence of the Government guarantee at maturity date they will receive the full face value of their bonds. Expressed in abbreviated form the final payoff to junior bondholders ($=\bar{Y}_J^P$) will be

$$Y_J^P = \begin{cases} D_2 & \text{if } \bar{V}_3 \geq B \\ D_2 & \text{if } D_1 < \bar{V}_3 < B \\ D_2 & \text{if } \bar{V}_3 \leq 0 \end{cases}$$

Thus, according to the model, the market value of the common stock and senior debt

under the Government guarantee can be expressed as:

$$V_E^P = (V_0^P - BR^{-1})N(-K_1) + R^{-1}\sigma_F n(K'_B) \quad (1.41)$$

and

$$V_S^P = D_1R^{-1}N(-K'_1) + V_0^P [N(-K'_0) - N(-K'_1)] + R^{-1}\sigma_F [n(-K'_0) - n(-K'_1)] \quad (1.42)$$

where V_0^P is the post-investment current value of the company, and $K'_B = \frac{B - V_0^P R}{\sigma_F}$,

$$K'_1 = \frac{D_1 - V_0^P R}{\sigma_F}, K'_0 = \frac{-V_0^P R}{\sigma_F}.$$

The market value of the junior debt, without ($=V_J$) and with ($=V_J^P$) the Government guarantee, is given by

$$V_J = D_2R^{-1}N(-K'_2) + [V_0^P - D_1R^{-1}][N(-K'_1) - N(-K'_B)] + R^{-1}\sigma_F [n(-K'_1) - n(-K'_B)], \quad (1.43)$$

and

$$V_J^P = D_2R^{-1} \quad (1.44)$$

where $K'_1 = \frac{D_2 - V_0^P R}{\sigma_F}$.

Subtracting equation (1.43) from equation (1.44) gives the value of the Government loan guarantee (V_G) as $V_G = V_J^P - V_J$

or

$$V_G = D_2R^{-1}[1 - N(-K'_2)] - [V_0^P - D_1R^{-1}][N(-K'_1) - N(-K'_B)] - R^{-1}\sigma_F [n(-K'_1) - n(-K'_B)] \quad (1.45)$$

As mentioned, the second part of this paper is an empirical study of the Chrysler corporation. In order to analyse the effect of the Government loan guarantee on the value of Chrysler's common stock and debt, three hypotheses were developed. The hypotheses were:

- H₁: The Government loan guarantee should bring benefits to senior bondholders.
- H₂: The Government loan guarantee should bring benefits to shareholders of Chrysler.
- H₃: The risk level of Chrysler's common stock should be lower with the Government loan guarantee.

The technique used by the authors was that of 'intervention analysis'. Daily stock return data for Chrysler and the value-weighted stock index for the period from July 1977 to the end of September 1980 were used in the analysis. The results of their analysis confirm the first two hypotheses as correct. Based on the results of the analysis, the conclusion was that investors reacted positively to the announcement of the Government support to Chrysler, and that both shareholders and senior bondholders benefited from the guarantee. This confirmed the theoretical results obtained in the first part of the paper. Hypothesis number 3 was rejected. Chrysler behaved as any profit maximising firm. The total level of risk was increased after the introduction of the Government loan guarantee. The authors concluded that, based on their results, there was need for the Government intervention. Moreover, the Government should have charged a fee in exchange for the guarantee, since the loan guarantee provided value to the firm's owners and creditors. This was also the general conclusion of Merton, Sosin and Jones and Masson.

SUMMARY

Chapter One of this paper has attempted to give an broad picture of the theoretical framework which is employed herein to explain some of the consequences of loan guarantees and to present some of the theoretical models which can be used to evaluate the likely cost of the proposed Mortgage Indemnity Scheme. As was illustrated by Fried's general model of portfolio choice, and by Williamson's model of credit with costly state verification, with the introduction of Government loan guarantees in situations where there is credit rationing in the loan market--as it is the case in the South African home-loan market--the end result could also involve an increase in the overall interest rates paid by the borrowers. However, not all borrowers will be affected equally, with such likely movements of the interest rates. Due to the existence of the Government loan guarantee, which discriminates amongst different categories of the borrowers, the targeted borrowers will pay lower interest rates--which will, nevertheless, still be higher than interest rates before introduction of the Government loan guarantees--while other borrowers will pay the market interest rates of the day.

As far as the value of the Government loan guarantee to both bondholders and shareholders is concerned, the argument has been that loan guarantee has a value to both lenders and borrowers. Thus, Merton--with his model of an analytic derivation of the cost of loan guarantees--has illustrated that in the case when the company is faced by the inability to meet its debt obligation at the maturity date, where there is a Government loan guarantee the bondholders will receive the full face value of the bond and the Government will incur a net payment, or loss, equal to the difference between the payment to bondholders and the value of the firm's assets.

As far as Sosin's model is concerned, it has been illustrated that--in the case of the company's senior and junior bond and on the assumption that the Government would provide the company with a loan guarantee for a fraction of the junior debt equal to α at zero cost because the project is politically and socially beneficial in spite of its financial unfeasibility--the value of the senior debt will be unaffected with the introduction of loan guarantee in the model. At time of maturity, the payment of the senior bond is secured by the introduction of the junior bond, while the payment of the junior bond is secured by the Government guarantee. The theoretical value of the loan guarantee and the potential cost to a Government is the difference of the aggregate market value of the firm with, and without, the Government guarantee. According to Sosin, this will be the cost to the Government if the Government were to buy the guarantee in a competitive market. Based on his simulation results, Sosin concluded that the cost of loan guarantee moves in the same direction as the variance of the rate of return on the firm's assets.

Jones and Masson have evaluated four types of debt: (a) a fully guaranteed issue of non-callable coupon debt; (b) a partially guaranteed issue of non-callable coupon debt; (c) a junior and a senior issue of non-callable coupon debt with guarantees; and (d) a callable coupon debt with guarantees. They illustrated that, irrespective of the company's asset value at the maturity date, the bondholders will receive--in the presence of the Government loan guarantee--the full face value of the bond. However, the value of the guarantee moves together with the instantaneous variance of return on the asset's value.

Chen, Chen and Sear's model is a rare example of the use of discrete time models to evaluate Government loan guarantees. The two-period model is constructed around the company--Chrysler corporation--that in the first period issues a combination of common stock and discounted senior bond and in the second period issues a junior bond under the Government loan guarantee. They argued that at the maturity date if the company's cash flow is greater than its debt, the senior bondholder will receive the full face value of their

bonds, while in the opposite case, they will receive nothing. The case of junior bondholders is different. Due to the existence of the Government guarantee at maturity date they will receive the full face value of their bonds irrespective of the value of the company's cash flows.

Moreover, based on the results of their empirical analysis, they concluded that investors who invested in Chrysler's bonds reacted positively to the announcement of the Government's support to Chrysler, and that both shareholders and senior bondholders benefited from the guarantee. Thus the authors concluded that the Government should have charged a fee in exchange for its guarantee, since the loan guarantee provided value to the firm's owner and creditors.

Thus, in spite of some differences between the models, the general conclusion is that the Government loan guarantee has a positive value to both lenders and borrowers and that--based on these positive values of the Government loan guarantees--the Government should charge a fee in exchange for its guarantee. Moreover, the general conclusion is that the value of the Government loan guarantee is positively related to the standard deviation of the changes of the market value of the underlying security and the time to maturity of the debt.

CHAPTER 2

SOUTH AFRICAN HOUSING CRISIS

2.1. Housing Crisis

Amongst the disastrous consequences of apartheid, the housing crisis is one of the worst. Although there are presently no comprehensive statistical data regarding housing, some estimates suggest that in 1995, of approximately 7.9 million South African households with an average of 4.97³⁵ people per household, 1.7 million live in shacks on unsecured land and about 620,000 in shacks on serviced sites. About 2.1 million individuals live in hostels.³⁶ The size of the housing backlog varies with geographic area as indicated by Table 2-1, below.³⁷

TABLE 2-1
The Estimated Present and Future Urban Housing Backlog
of the Former South African Provinces

	PWV	E. Cape	W. Cape	Natal	Total SA
Urban population as % of total	81	45	90	51	61
Income as % of total (1993 prices)					
R 0-500	6	14	4	10	9
R 500-1500	18	33	15	30	26
R 1500+	76	13	81	60	65
Housing backlog 1990					
No of dwellings if standard 4-room houses ('000)	337	188	94	448	1,398
Future shortage					
1990 - 2010 ('000)	1,162	384	339	726	3,100

Source: Walker, N. (1993, pp. 31).

As it is true that not everyone was equally affected by apartheid, so it is also true that the housing backlog is not equally spread over all population groups. The largest housing backlog is amongst the African population, and the second largest is that of the Coloured population. Table 2-2 presents some estimates regarding the present and the future housing demand in Western Cape province, as an illustration of the different size of

³⁵"White Paper; A new housing policy and strategy for South Africa", *Government Gazette*, (23 December 1994, vol. 354, no. 16178, pp. 4), [hereafter cited as White Paper on Housing].

³⁶Barnard, D. "Housing, the reconstruction challenge", *Prodder Newsletter*, (November 1994, vol. 6, No.4) The estimate given by the White Paper on Housing is a slightly different. There are 8.3 million households with 4.7 persons per household, and with 1.06 million living in shacks. The estimated housing backlog for 1995 is 1.5 million units. White Paper on Housing, *op.cit.*

³⁷Walker, N. "A new approach to housing delivery: Some ideas for discussion", Research Report No. 1, (1993, Institute for Local Governance and Development, University of Western Cape). The numbers for the rural population are similar, with the estimated backlog for the period 1990-2010 equal to 1,341,000 units.

housing backlog amongst different population groups.³⁸

TABLE 2-2
Projected Housing Demand for the Western Province
(1990 to 2005 year)

Backlog	Whites	Coloured	Africans	Asians	Total
1990	-	59,881	91,294	-	151,175
1995	11,192	30,405	66,239	849	108,685
2000	7,969	28,390	90,890	764	128,013
2005	3,734	23,524	114,264	609	142,131
Total:	22,895	142,201	362,687	2,222	530,005

Source: du Plessis, (1992, pp. 4)

2.2. Housing Backlog: Historical Background

The differences in the size of the housing backlog presented in Table 2-2 are importantly influenced by the State's approach towards housing. For a long period, from roughly the late 1940's to the late 1970's and early 1980's, the State's approach toward the social 'necessities' of the population was what may be denoted as the "welfare approach". That is, it was widely accepted that the State should provide housing for low-income households. However, the State's approach toward the social requirements of the non-white population, which was controlled by the apartheid laws, was entirely different from that toward the white population. Under the policy of racial segregation, in order to control and influence the movements and economic activity of the non-white population, and in particular that of the African population in the urban areas, the State took charge of almost all housing for Africans. Thus, for a time, the State was producing, administering, controlling and allocating almost all the African housing. The result was an artificial creation of housing shortages and consequently a massive backlog. Such policy also created a widespread mentality of dependence amongst a large part of the South African population.³⁹

Since the late 1970's and early 1980's, the State's approach towards housing has changed. It has shifted, increasingly, towards the privatisation of housing supply and ownership.

³⁸du Plessis, S. "Housing in Working Papers on the Economy of the Western Cape", (WESGRO, Cape Town, March, 1992.) One needs to bear in its mind that the coloured population is a majority in the Western Cape province (approximately 55% of the total population).

³⁹The long waiting lists for the residential units independently of the race are the proof.

The emphasis has shifted from the State as provider, to the State as 'facilitator' for a number of housing-related functions. As a result of the new policy, the Urban Areas Act was amended in 1978. The amendments introduced the new 99-year leasehold right, alongside the already available 30-years leasehold, for the non-white population. These amendments have been directed toward the African population in particular. For an initial period of five years the response was very poor, with only 1,727 leases being registered. The reason was that the cost of establishing a leasehold was substantially higher than for renting a comparable property. In 1983, as the next step towards greater private ownership, the State initiated the sale of almost 500,000 housing units. Once more, the response was poor for a variety of reasons.

2.3. The Current Approaches Towards Housing

According to Walker,⁴⁰ the various forms of the current approach to housing can be divided into "four broad housing delivery options", which may be sub-divided further into a number of sub-options. Table 2-3 presents four delivery options with these related sub-options.

TABLE 2-3
The Current Housing Options

Primary options	Sub-options
Unaided self-help	- informal settlements - illegal sub-divisions - backyard shacks - floating squatters
Supported self-help	- emergency service provision - roll-over upgrading - community-sensitive in-site upgrading
Projected-initiated self-help	- self-help projects - IDT sponsored projects
Conventional housing	- State rental sector - public rental sector - State and private sector developed ownership units

Source: Walker, N. (1993, pp. 18)

An explanation of the primary options of Table 2-3 is in order:

⁴⁰Walker, N. *op. cit.* pp. 17-21

- **Unaided self help**--This delivery option refers, in general, to informal settlements. Such settlements are only quasi-legal. In general, the occupied sites are non-serviced and without any infrastructure. The erected dwellings units are mostly built from scrap materials.
- **Supported self-help**--In general, this form of housing refers to an upgraded informal settlement, described above. Usually, upgrading is initiated by the settlers themselves. The upgrading is in the form of basic services such as electricity and communal water taps, and very seldom takes the form of upgrading the dwellings themselves. In recent years, the State and various parastatal and private organisations have also been involved in upgrading the informal settlements.
- **Project-initiated self-help**--This approach is very similar to that of supported self-help. The difference is that the sites are already serviced with some basic infrastructure before the settlers begin occupying the sites. In many cases the State, or one of the parastatals, i.e. Independent Development Trust (which has been subsidised by the State), are the providers and facilitators of the serviced sites. The level of services and infrastructure can vary greatly from the one place to the other place; however, in general, the level of services are limited to water and electricity supply and the removal of wastage.
- **Conventional housing**--This delivery option refers to completed dwellings units in various forms, i.e. free standing houses, semi-detached housing or multi-story blocks of flats. In the past few years, the State was--to a lesser degree--involved in this delivery option.

2.4. Current Constraints to the Solution of Housing Crisis

With the recent democratisation of the country and with the 1994 general election, apartheid has been abandoned completely. Thus, the major factor that previously influenced the housing crisis no longer exists. Nonetheless, there are still numerous other factors that act as constraints to the solution of the housing crisis. Some of the major constraints are as follows:

- **Non-existence of a single, coherent State policy towards housing:**
Previous policies resulted in fragmentation of the housing function amongst a number of different Government departments as well as fragmentation at the provincial and regional level. This fragmentation has been reflected in the inability

of the State to offer or to initiate a solution to housing crisis in the past.

- Access to finance:

At present, access to mortgage loans is largely denied to the majority of the South African population. The major reasons for this denial are that such mortgages are largely not affordable⁴¹ and are of limited profitability to financial institutions.⁴² However, and in particular during 1980's, due to the continuous boycott of repayments for home-loans, almost all financial institutions cancelled their involvement with the low-cost housing market. The African population has been particularly severely adversely affected by this move.

- Affordability of housing:

The majority of homeless households can afford only houses under R35,000,⁴³ and there are limited number of such low-cost houses on the market. This is due to the withdrawal of the State as a provider of such housing, and to the reluctance of the private sector to enter this market due to a low profit margins in this market.⁴⁴ Another problem is high infrastructure cost related to the development of the sites for such low-cost housing.⁴⁵

- Availability of the land;

As a legacy of apartheid, the majority of the South African cities are divided on a racial basis. In addition, the majority of the work force must travel long distances from their homes to their place of work. In order to make cities less racially divided and to put a majority of the population closer to their place of work, new

⁴¹Due to the low level of income earned by the majority of African population, it is almost impossible for them to afford high monthly mortgage payments or to receive substantial amount of credit.

⁴²The administrative cost is not especially different between the loans of differing sizes. Taking in consideration the higher risk involved with small loans, the real costs are however even greater. Here "small loan" refer to the loans under R30,000. However, according to John Smale, managing director of NBS Bank "The low income housing market will be one of the fastest growing markets in the years ahead. We will go into it because it is profitable and it is right thing to do", (Emphasis added). Sunday Times, "Business Times", May 7, 1995, n.p.

⁴³According to the White Paper on Housing, almost 40-55% of households in need of housing will be dependent only on the state subsidisation and/or on their own resources, if any, to solve their basic housing needs. White Paper on Housing, *op. cit.* pp. 20.

⁴⁴Additionally, the question can be raised about the ability of the construction industry to met the targeted 300,000 units of low-cost housing in the consequent years.

⁴⁵The cost of tarred roads per site is in the range of R1,500-2,000. (du Plessis, C. *op.cit.* pp. 8.) The average cost of plan approval incurred by the developers in private sector in Western Cape for the house under R50,000 is R512 while water connection is R617. (Walker, N. *op.cit.* pp. 31)

residential units/areas could be developed within existing (white) residential areas or close to existing industrial areas. The difficulties with such 'solutions' is that there is very little space left for major housing developments and that land which does exist is too expensive for low income housing. Thus, the only *real* solution is to acquire available land around big cities, which then does not solve the problem of racial division nor the problem of the long distance travel to the workplace.

2.5. The Proposed Government Housing Policy

Recognising housing as one of its priorities, in its first year in power, the Government of National Unity has given much attention to the housing crisis. As a result at the end of 1994, the Government published its proposed housing policy in a White Paper on Housing.

2.5.1. The White Paper on Housing

In general, the Government's 'White Paper on Housing' may be divided into two parts: the first part of the paper provides a general analysis of the current South African housing situation, while the second part spells out a number of strategies and policies aimed at a long run solution to this crisis.

The general idea which undergirds the Paper, is that "housing is a basic human right" and almost the entire Government approach towards housing is based on this particular principle. However, there is a strong recognition of the limited State's financial, human and material resources. Thus, despite the Government's long run intention to act only as a "facilitator" rather than a "deliverer", it is aware that, in the short run, there is still a strong need for direct Government involvement in the low-income housing market.

Proposed Government direct involvement is largely limited to devising a framework for State subsidies for first-time home buyers. However, the Government is also aware that--at least in the short run--there is still the need for the Government to act as a direct deliverer of residential units.⁴⁶ The vehicle the Government of National Unity has chosen for doing so is to give to first-time buyers a subsidy which may be used as a down-payment to the provider of a home. Table 2-4, below, describes the proposed State subsidy scheme for first-time home buyers.

⁴⁶This is particularly true for the lowest end of the housing market which consists of households that earn

TABLE 2-4
The Conditions and the Value of the State Subsidies
for the First Time Buyers

Joint spouse monthly income	The value of the State subsidy
1	2
0 - 800	15,000
801 - 1,500	12,500
1501 - 2,500	9,500
2501 - 3,500	5,000
> 3,501	0

Source: The White Paper, 1994, pp. 46

On the other hand, to fulfil its role as a 'facilitator', the Government's intention is to:

- (1) establish a number of housing-related institutions; and,
- (2) to stabilise the housing environment.

Amongst the proposed housing-related institutions to be established are:

- The National and Provincial Housing Ministries;⁴⁷
- The National and Provincial Housing Boards. These boards will act mainly as a advisors to the Ministry of Housing for the variety of questions related to housing:
- The National Housing Finance Corporation. In light of the Government's desire for a long-run solution, the establishment of the Corporation will be one of the most important parts of the solution. The major function of the proposed Finance Corporation will be to release housing finance at the wholesale level through the policies such as:
 - partial underwriting of loans made by authorised financial institutions to the lower end of housing market;
 - securisation of the housing loans; and,
 - issuing own financial papers for the purpose of funding or the partial underwriting of paper issued by the banks.

less than R800 per month

⁴⁷These ministries have already been in the place at the time of writing this paper.

The Government's purpose with its policy of stabilisation of the housing environment is to induce major South African banks to re-enter the low-income housing market, which they earlier abandoned. However, the most important factors that influenced the banks' decisions to leave this market were (a) the introduction of the boycott of bond and other services repayments and (b) the ever-increasing level of violence in non-white residential areas that led to an almost completely break-down in law and order.⁴⁸ Thus, in order to induce the residents of these residential areas to pay their bond and for service obligation, at the beginning of 1995 the Government has launched a massive housing education campaign (*Masakhane*) aimed to change this alleged 'culture of non-payment' toward a 'culture of payments'. Simultaneously with this mass-campaign, the Government has proposed to establish the Mortgage Indemnity Scheme, which is described in more details below.

Some of the other proposals which are presented in the White Paper are related to the availability of land for low-cost housing and to consumer protection. As far as land is concerned, the Government's proposal is that public land owned by the State, and in particular, the land in the vicinity of urban areas, needs to be considered for the purpose of facilitating the low-income housing first.

2.5.2. The Government Indemnity Scheme Proposal⁴⁹

To a significant extent, the proposed Government housing solution, at least in its initial phase, depends on the willingness of the major South African financial institutions to lend money to the lower end of the housing market. Currently only the major banks have the necessary infrastructure and sufficient financial resources to support the proposed Government housing policy. However, the problem is that--given their negative experience and substantial losses caused by forces beyond their control⁵⁰--banks are not prepared to enter the market without some preconditions. One of these preconditions is that they be indemnified against payment default.

In order to meet this pre-conditions, the Government has proposed the establishment of the Mortgage Indemnity Scheme (MIS), with the purpose of protecting the banks from incurring losses due to forces beyond their control.

⁴⁸The payment's boycott has been introduced as a part of the struggle against the apartheid.

⁴⁹The White Paper on Housing *op.cit.* pp. 48-49.

⁵⁰The payment's boycott, the high level of violence in townships, the inability or unwillingness of the government of the day to force the laws and to vacate these residential units.

The most important characteristics of the Mortgage Indemnity Scheme are as follows:

- Only accredited banks will be covered by the MIS;
- The circumstances under which the banks will be indemnified will be mutually agreed in advance;
- In granting home loans, the banks will have to follow procedures established in consultation with those in charge of the MIS programme; and,
- The initial period of the MIS will be three years.⁵¹

As part of a separate agreement, however, though in much the same way as part of the MIS, the Government has also agreed to extend the coverage of the MIS to approximately 16,000 residential units already under court orders for repossession. The reason for this extension of the MIS programme is a recognition by the State of its inability to effectively execute court orders for repossessions. The Government's proposal is that, during the initial three years of the duration of the MIS, it will make additional efforts to enforce those court orders. Should, during those three years, the Government be unsuccessful in its effort to enforce the law, then it will purchase these residential units under the terms of the MIS.

Actually, with the Mortgage Indemnity Scheme, the accredited banks will be given--without cost to them--an "insurance policy" against default of payment, with the Government acting as the policy writer. However, as has been shown by Merton, this is equivalent to buying and writing a Put option, and this feature of the Government's Mortgage Indemnity Scheme will be exploited in the consequent chapter to calculate the 'real' value of the scheme to the South African Government and, ultimately, to the South African tax-payers.

⁵¹The very practical question is who will then lend money for a period longer than the initial three years. The possible solution to this problem can be found in the proposed establishment of the National Housing Finance Corporation which could take over the guarantee of the loans for the rest of the repayment period of the loan.

CHAPTER 3

DERIVATION OF A MODEL OF THE COST OF THE SOUTH AFRICAN GOVERNMENT PROPOSED MORTGAGE INDEMNITY SCHEME

3.1. The Model of Estimation of the Cost of the Indemnity Scheme to the South African Government

As it has been showed by the models presented in Chapter 1, Option Pricing Theory can be successfully used to evaluate the cost of Government guarantees of loans granted to firms. With modest modification, the same general models can also be used to evaluate the cost of the Mortgage Indemnity Scheme proposed by the South African Government as a part of its solution for housing crisis. The model which will be used here in to calculate the likely cost of the indemnity scheme is similar to Merton's formulation for the analytic derivation of the cost of loan guarantees. As the model has already been presented above (section 1.2.2.1.), only its major features are presented, and then but briefly, below.

3.1.1. Merton's Model

The five basic assumption of Merton's model are:⁵²

- (1) There exists a risky asset that can be bought and sold freely on the market at some current price S . The risky asset pays no dividends nor is there other income dispersal before the option's expiration date. There are no restrictions to short selling.
- (2) The rate of interest at which one can lend or borrow without risk is fixed and interest accrues on a continuous basis at a rate r .
- (3) The price of the underlying risky asset is continuous in time, and follows a random walk. The instantaneous mean and variance of the price change are proportional to the current price of the security, so that the mean and variance of the (continuously compounded) rate of return per unit time both have constant values. This implies that the distribution of the security's current price over any given period will be log normal.⁵³
- (4) There are no factors--such as taxes, transaction costs, or margin requirements--that can affect a rate of return on a risky assets.

⁵²The assumptions are the Black-Scholes assumptions presented in section 1.2.1.1.

⁵³The feature of the log normal distribution is that price can not be negative and that the size of the average price change is larger as price levels are higher.

(5) The option can be exercised only at the time of expiration T , i.e. the option is a European option.

Thus, according to Merton's model, the value of the loan guarantee is given by:

$$G(T) = Be^{-rT}\Phi(x_2) - V\Phi(x_1), \quad (3.1)$$

and the value of the loan guarantee as a fraction of the value of the loan covered by the guarantee, i.e. the insurance premium, is given by:

$$\frac{G(T)}{Be^{-rT}} = 1 - e^{-(R(T)-r)T}. \quad (3.2)$$

where:

$G(T)$ = the value of the loan guarantee;

B = promised payment on the bond issues;

V = the market value of a firm's assets at time of maturity;

r = the instantaneous risk-free interest rate;

T = time to bond maturity;

R = the promised bond yield;

$$x_1 \equiv \frac{\log\left(\frac{B}{V}\right) - \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$x_2 \equiv x_1 + \sigma\sqrt{T}$$

σ^2 = the variance rate per unit time of the logarithmic changes in the value of the assets.

Although the above assumptions are too restrictive, in order to solve the model, assumptions (1), (2)⁵⁴ and (3) have been taken as given. As far as assumption (4) is

⁵⁴Based on fact that the Reserve Bank changed interest rate four times during 1994, and it has already changed interest rate one more time in 1995.

concerned, this assumption needs to be relaxed so as to allow for the influence of a range of default rates on bank returns. Assumption (5) is presumably a realistic one, taking into consideration that the households affected by the indemnity scheme, have low levels of income, which implies that the probability of loan pre-payment is quite unlikely.

As the model was originally constructed to evaluate the value and cost of the guarantees related to loans granted to firm, some necessary modifications must to be made before the model can be used to evaluate the likely cost of the indemnity scheme.

3.1.2. The Model's Modifications

Modifications of Merton's equations (3.1) and (3.2) must be made for use in the housing model. Thus, these modified variables may be defined as follows:

- B = the total value of housing loans granted within the period covered by the indemnity scheme;
- V = the total market value of housing at the end of the period covered by the indemnity scheme;
- σ^2 = the variance rate per unit time of the logarithmic changes in the market value of housing;
- T = the period within which the indemnity scheme is exercised; and
- R = the interest rate on housing loans granted; while the variables r , x_1 , x_2 and $G(T)$ are the same as above.

In order to calculate the value of the Government mortgage indemnity scheme (MIS), some additional assumptions about the model have been made. These additional assumptions are as follows:

A1: The duration of the Mortgage Indemnity Scheme is 3 years;

A2: The amortisation period of the loans is 15 years;⁵⁵

⁵⁵A difficulty is that the initial period of indemnity is 3 years. On the other hand, the usual repayment period of housing loans is 15 to 20 years, which means that financial institutions would be uncovered for at least 12 years. The solution may be, as suggested by the Government White Paper, for the Finance Corporation to take over the loan guarantee for the rest of the repayment period.

- A3: In the first year there will be 50,000 loans granted, and in each of the second and third years the number of loans granted will increase to 100,000;
- A4: The instantaneous risk-free interest rate is equal to 16.67% proxied by the returns on the South African government bond R150;⁵⁶
- A5: The rate of default ($=d_r$) is a positive function of the interest rate;
- A6: The rate of depreciation ($=d_d$) is a function of the time and is invariant to the value of the house.

3.2. Data Used for the Calculations

The advantage and usefulness of financial models based on Option Pricing Theory is that the data for the variables used in the models are readily observable and easily obtained. In cases where some variables can not be observed directly such variables may be easily estimated. As far as the variables used in the housing model are concerned, most must be estimated. This is due to the fact that the Government housing programme is still in its initial phase and most of the relevant data does not yet exist.

Thus, we have here assumed that the value of the home loans granted is a function of the household's income. The value of a home loan is determined on the assumption that the household's monthly payment of the bond will be approximately 25% of its monthly income with the interest rate on home loans equal to the current interest rate prevailing on the market at the time of writing this paper and it is equal to 17.25%.⁵⁷

Table 3-1, below, presents the estimated ranges of home loans according to the joint spousal monthly income and with an interest rate of 17.25% per annum. Table 4.1 also presents an estimation of the possible range of the initial value of residential units, which is the sum of the value of the home loans granted, the value of the State's subsidies and the value of the individual's contributory payment.

⁵⁶Financial Mail, 21 April 1995, n.p.

⁵⁷Financial Mail, 21 April 1995, n.p.

TABLE 3-1
Data Used in Calculation of the Cost of
the Mortgage Indemnity Scheme

Joint spousal monthly income	Monthly repayment of the loan ⁵⁸	Range of Loans ⁵⁹	State Subsidy	Individual contribution ⁶⁰	Initial value of an individual house (3+4+5)
1	2	3	4	5	6
0 - 800	0 - 157	0 - 10,000	15,000	0 - 1,000	15,000 - 26,000
801 - 1,500	157 - 394	10,000 - 25,000	12,500	1,000 - 2,500	23,500 - 40,000
1,501 - 2,500	394 - 630	25,000 - 40,000	9,500	2,500 - 4,000	37,000 - 53,500
2,501 - 3,500	630 - 866	40,000 - 55,000	5,000	4,000 - 5,500	49,000 - 65,500
3,501 - 4,500	866 - 1,090	55,000 - 70,000	0	5,500 - 7,000	60,500 - 77,000
4,501 - 5,500	1,090 - 1,323	70,000 - 85,000	0	7,000 - 8,500	77,000 - 93,500
5,501 - 6,500	1,323 - 1,557	85,000 - 100,000	0	8,500 - 10,000	93,500 - 110,000
> 6,501	> 1,575	> 100,000	0	> 10,000	> 110,000

Note: All amounts are in Rand terms.

The cost of the MIS is estimated for the range of the constant change of the market value of the residential units, ($=\sigma$). Thus, σ ranges from 5% to 50%.⁶¹ The duration of the proposed Mortgage Indemnity Scheme is given by the White Paper on Housing and is assumed fixed at 3 years. As mentioned, the risk-free interest rate is assumed to be equal to the returns on RSA 150 bond and is equal to 16.67%.

As data for market value of low income housing is non-existent, it can not be observed directly or estimated by comparison with similar data. In the solution presented here, this data is simulated and, using time series so derived, the value of the relevant variables has been calculated. Thus, these simulated variables can be used in the model of estimation of the cost of the indemnity scheme to the South African Government. The simulation process used to simulate data for this variable is a Monte Carlo process, and is described in some more details below.

⁵⁸The value of the loan is amortised over a period of 15 years at interest of 17.5% per annum.

⁵⁹The range of loans are based on assumption that household's monthly repayment is around 25% of its monthly income.

⁶⁰Based on 10% of the value of the loan

⁶¹The Rode's Report estimates that the average price change of the residential units at the upper end of the market was 23.4% in 1994, 13.7% for the middle end of the market and 11.4% for the lower-priced homes. Cape Times, (17 April 1995, pp. 18)

3.3. Monte Carlo Simulation

According to Newman and Odell⁶² the term 'Monte Carlo' denotes various sampling techniques that are "used to study probabilistic and systems simulation problems, as well as deterministic problems".⁶³ The most common use of the Monte Carlo techniques is in cases when--for various reasons--some equation can not be solved by the usual numerical methods--or in other words--it can not be expressed explicitly through its parameters,⁶⁴ thus than "there may exist a stochastic process with a distribution or parameters which [do] satisfy the equation"⁶⁵ and make it possible to solve the equation for given values of the parameters. However, this requires estimating the numerical value of the parameter by generating a series (of size n) of random numbers ($=x_i$).

Associated with such generated series of random numbers ($=x_i$) is a cumulative distribution function ($=F(y)$), which is defined as the probability that the random numbers ($=x_i$) thus generated has a particular value not exceeding a prescribed value of y .⁶⁶ The frequency of the random numbers is given by the frequency function $f(x)$, which may also be called a *probability-density-function*.

Thus, if $g(x_i)$ is a function of x_i ,⁶⁷ so the expectation (or mean value($=\bar{x}$)) of g is given by

$$\bar{x} = \int_A g(x) dF(y), \quad (3.3)$$

or, if $F(y)$ has the derivative $f(x)$, then \bar{x} can be expressed as

$$\bar{x} = \int_A g(x) f(x) dx, \quad (3.4)$$

where $\int_A f(x) dx = 1$, and A denotes a range of integration.

⁶²Newman, T., and Odell, P., The generation of random variates, (Charles Griffin & Co. Ltd, London, 1971)

⁶³*Ibid.*, pp. 1

⁶⁴As it is case with the housing model above because of the non-existence of the real world data for the market value of the residential units.

⁶⁵Newman, T., and Odell, P., *op.cit.*, pp. 4

⁶⁶Hammersley, J., and Handscomb, D. Monte Carlo Methods, (Methuen & Co. Ltd, London, 1965)

⁶⁷*Ibid.*

Boyle, P. "Options: A Monte Carlo approach", Journal of Financial Economics, (vol. 4, 1977, pp. 323 - 338);

In order to estimate the value of \bar{x} , a number ($=n$) of sample values ($=x_i$) is generated at random from the probability density function $f(x_i)$. Thus, the estimate ($=\hat{x}$) of \bar{x} is given by

$$\hat{x} = \frac{1}{n} \sum_{i=1}^n g(x_i), \quad (3.5)$$

and the standard error ($=\sigma$) of \hat{x} is given by

$$\sigma = \frac{\hat{s}}{\sqrt{n}} \quad (3.6)$$

where

$$\hat{s}^2 = \frac{1}{(n-1)} \sum_{i=1}^n (g(x_i) - \hat{x})^2. \quad (3.6a)$$

The factor \sqrt{n} in the denominator in equation (3.6) implies that, in order to reduce the standard error of the estimate by a given factor n , the number of observations needs to be increased by the factor n^2 .

An alternative method of reducing the standard error of the estimate is the 'control variate' method. Some other variance-reducing techniques are 'importance sampling', 'antithetic variate' and 'orthonormal' methods. According to Hammersley and Handscomb,⁶⁸ the common characteristic and usefulness of these techniques is that, as they do not introduce bias into the estimation, the results are more precise and the estimation requires less labour.

The 'control variate' method is based on the idea of changing the more complex model under consideration to a similar, but simpler, one that has an analytic solution. Thus, on the assumption that $g(x_i)$ is a function of x_i , with the mean value G given by

$$G = \int_A g(x)h(x) \quad (3.7)$$

where the integral can be evaluated analytically and $h(x)$ is a probability density function, solving equations (3.4) and (3.7) as a system of two equations with \bar{x} as unknown, then

⁶⁸Hammersley, J., and Handscomb, D. Monte Carlo Methods, *op.cit.*

$$\bar{x} = G + \int_A g(x)[f(x) - h(x)]dx. \quad (3.8)$$

Using the crude Monte Carlo method to evaluate the integral on the right hand side of equation (3.8), the new revised estimate ($=x^*$) of \bar{x} can be obtained, where h is the 'control variable'. Thus, x^* is given by

$$x^* = G + (\hat{x} - \hat{G}), \quad (3.9)$$

which is an unbiased estimate with the variance equal to $\text{var}(\hat{x}) + \text{var}(\hat{G}) - \text{cov}(\hat{x}, \hat{G})$,

and as long as $\text{cov}(\hat{x}, \hat{G}) > \frac{\text{var}(\hat{G})}{2}$, the variance of the x^* will be less than the variance of \hat{x} , or

$$\text{corr}(\hat{x}, \hat{G}) > \frac{1}{2} \sqrt{\frac{\text{var}(\hat{G})}{\text{var}(\hat{x})}}. \quad (3.10)$$

Equation (3.10) states that the efficiency gain is a function of the relationship between f and h .

3.3.1. Application of the Monte Carlo Simulation

The results of the crude Monte Carlo simulation of the average market values of housing is presented in Table 3-2 below where the last column presents 95% confidence interval limits for the actual mean (μ_x) of the range.

TABLE 3-2
The Estimated Average Market Value of Housing Using
the Crude Monte Carlo Simulation

No	Assumed range of the values of the residential units	Mid point	Crude Monte Carlo estimate (300 trials)	Sample standard deviation	95% confidence interval
1	23,500 - 40,000	31,750	32,121	8,023	$31,213 \leq \mu_x \leq 32,028$
2	37,000 - 53,500	45,250	45,273	7,977	$44,370 \leq \mu_x \leq 46,175$
3	49,000 - 65,500	57,250	57,224	7,898	$56,330 \leq \mu_x \leq 58,117$
4	60,500 - 80,000	70,250	68,664	7,856	$67,775 \leq \mu_x \leq 69,552$
5	77,000 - 93,500	85,250	85,473	7,809	$84,589 \leq \mu_x \leq 86,356$
6	93,500 - 110,000	101,750	101,606	7,423	$100,766 \leq \mu_x \leq 102,445$

CHAPTER 4

THE *REAL* COST OF THE MORTGAGE INDEMNITY SCHEME

4.1. The Calculation of the Cost

The cost of the proposed Mortgage Indemnity Scheme (MIS) was calculated for each year of the proposed MIS and for each of the different categories of loans as approximated by the midpoint of the range of the loan values ($=\dot{B}$). Table 4-1, below, presents the average, and total, value of the loans, while Table 4-2 presents the average, and total, market value of housing used in the calculation of the total cost of the MIS.

TABLE 4-1
Average and Total Value of Loans Granted

No	The midpoint value of the loan range (Rand)	Probability of occurrence P_0	Number of loans n			Total value of loans granted (Rand)		
			T=3	T=2	T=1	T=3	T=2	T=1
(1)	(2)	(3)	(4a)	(4b)	(4c)	(5a)	(5b)	(5c)
1	17,500	0.54	27,000	81,000	135,000	472,500,000	1,417,500,000	2,362,500,000
2	32,500	0.15	7,500	22,500	37,500	243,750,000	731,250,000	1,218,750,000
3	47,500	0.09	4,500	13,500	22,500	213,750,000	641,250,000	1,068,750,000
4	62,500	0.08	4,000	12,000	20,000	250,000,000	750,000,000	1,250,000,000
5	77,500	0.08	4,000	12,000	20,000	310,000,000	930,000,000	1,550,000,000
6	92,500	0.06	3,000	9,000	15,000	277,500,000	832,500,000	1,387,500,000
Total:		1.00	50,000	150,000	250,000	1,767,500,000	5,302,500,000	8,837,500,000

TABLE 4-2
Average and Total Market Value of Housing

No	The average value of a single unit ⁶⁹ (Rand)	Probability of occurrence P_0	Number of units n			Total market value of housing (Rand)		
			T=3	T=2	T=1	T=3	T=2	T=1
(1)	(2)	(3)	(4a)	(4b)	(4c)	(5a)	(5b)	(5c)
1	32,121	0.54	27,000	81,000	135,000	867,267,000	2,515,074,300	4,700,587,140
2	45,273	0.15	7,500	22,500	37,500	339,547,500	984,687,750	1,840,347,450
3	57,224	0.09	4,500	13,500	22,500	257,508,000	746,773,200	1,395,693,360
4	68,664	0.08	4,000	12,000	20,000	274,656,000	796,502,400	1,488,635,520
5	85,473	0.08	4,000	12,000	20,000	341,892,000	991,486,800	1,853,054,640
6	101,606	0.06	3,000	9,000	15,000	304,818,000	883,972,200	1,652,113,560
Total:		1	50,000	150,000	250,000	2,385,688,500	6,918,496,650	12,930,431,670

⁶⁹Obtained by a Monte Carlo simulation, Chapter 3, Section 3.1.3.

As can be seen from the tables above, the total number of the loans in the first year, $T=3$, is assumed to be 50,000. It has been also assumed that in the second year an additional 100,000 loans will be granted, thus the total number of the loans at the end of the year two, $T=2$, is equal to 150,000; and, that in the third year another 100,000 loans will be granted, so that the total number of the loans at the end of the year three, $T=1$, will be 250,000 loans.

The distribution of loans amongst the different categories of loans is given by the probability of occurrence ($=P_0$) in column 3 in the tables above. The probability of occurrence is invariant with respect to Time. Thus, the number of loans ($=n$) for each of the category of the loans within a particular year is a product of the value of P_0 for the particular midpoint value of the loans, and the total number of the loans for that year, and is given in columns 4a, 4b and 4c in Tables 4-1 and 4-2.

The total value of loans granted during the first, second and third years of the MIS is given in the last three columns of Table 4-1, i.e. columns 5a, 5b and 5c, respectively. It is the product of the midpoint of the loans and of related quantity of loans.

The total market value of housing is given in columns 5a, 5b and 5c of Table 4-2. While the total value of housing in the first year is a product of the number of residential units and the average market value of a single unit ($=\bar{V}$), provision has been made for depreciation of the residential units at 10% per year in order to calculate the total market value of housing during the second and the third years of the MIS. In addition, the cost of the MIS has also been calculated for a wide range of possible percentage changes in the market price of residential units as given by the standard deviation ($=\sigma$) of those percentage changes, which ranges involve from 0.05 to 0.50 percentage points.

4.2. The Real Value of the MIS

The complete results of the evaluation of the possible values of the Mortgage Indemnity Scheme using the housing model presented in Chapter 3 are given in Table 4-3, Table 4-4 and Table 4-5 below, for the each year of the MIS. The value of the MIS has been calculated on the assumption of a default rate ($=d_1$) equal to 100% so as to illustrate the maximum value of the MIS which can occur; any other default rate will, of course, yield lower values of the MIS.

TABLE 4-3**The Cost of the Mortgage Indemnity Scheme at Three years to Maturity**

σ	$\dot{B} = R17,500$		$\dot{B} = R32,500$		$\dot{B} = R47,500$		$\dot{B} = R62,500$		$\dot{B} = R77,500$		$\dot{B} = R92,500$		C
	A	B	A	B	A	B	A	B	A	B	A	B	
0.05	0	0.000	0	0.000	0	0.000	0	0.000	0	0.000	0	0.000	0.04
0.10	0	0.000	0	0.000	0	0.000	0	0.000	0	0.000	0	0.000	0.04
0.15	0	0.000	0	0.000	0	0.000	242,891	0.002	281,680	0.001	270,362	0.002	0.04
0.20	0	0.000	0	0.000	455,672	0.003	1,500,578	0.010	1,802,492	0.009	1,667,875	0.010	0.04
0.25	0	0.000	162,141	0.001	1,927,997	0.014	3,991,166	0.025	4,850,308	0.025	4,433,974	0.025	0.04
0.30	0	0.000	1,854,180	0.012	4,345,302	0.032	7,484,152	0.048	9,147,951	0.047	8,312,468	0.048	0.04
0.35	0	0.000	4,564,858	0.030	7,504,920	0.056	11,696,441	0.074	14,345,798	0.074	12,989,074	0.074	0.04
0.40	1,056,750	0.004	8,093,012	0.053	11,205,826	0.083	16,401,911	0.104	20,162,521	0.103	18,212,832	0.104	0.04
0.45	6,576,029	0.022	12,239,242	0.080	15,287,342	0.114	21,435,597	0.136	26,392,181	0.135	23,800,682	0.136	0.04
0.50	13,569,108	0.046	16,836,895	0.110	19,628,007	0.146	26,678,626	0.170	32,886,110	0.169	29,620,726	0.170	0.04

TABLE 4-4**The Cost of The Mortgage Indemnity Scheme at Two years to Maturity**

σ	$\dot{B} = R17,500$		$\dot{B} = R32,500$		$\dot{B} = R47,500$		$\dot{B} = R62,500$		$\dot{B} = R77,500$		$\dot{B} = R92,500$		C
	A	B	A	B	A	B	A	B	A	B	A	B	
0.05	0	0.000	0	0.000	0	0.000	0	0.000	0	0.000	0	0.000	0.027
0.10	0	0.000	0	0.000	0	0.000	77,019	0.000	76,261	0.000	86,246	0.000	0.027
0.15	0	0.000	0	0.000	0	0.000	2,112,869	0.004	2,472,616	0.004	2,350,972	0.004	0.027
0.20	0	0.000	0	0.000	2,466,224	0.005	8,084,037	0.015	9,707,515	0.014	8,985,438	0.015	0.027
0.25	0	0.000	128,526	0.000	8,035,819	0.017	17,416,564	0.032	21,135,532	0.031	19,350,037	0.032	0.027
0.30	0	0.000	5,581,333	0.010	16,100,363	0.034	29,057,801	0.053	35,462,518	0.052	32,275,920	0.053	0.027
0.35	0	0.000	13,826,018	0.026	25,959,965	0.055	42,215,293	0.077	51,700,781	0.076	46,883,658	0.077	0.027
0.40	0	0.000	24,213,357	0.045	37,069,039	0.079	56,360,709	0.102	69,188,161	0.101	62,587,076	0.102	0.027
0.45	8,044,333	0.008	36,188,537	0.067	49,036,972	0.104	71,143,205	0.129	87,483,717	0.128	78,996,959	0.129	0.027
0.50	26,618,174	0.026	49,320,617	0.092	61,586,563	0.131	86,323,589	0.157	106,286,399	0.156	95,847,978	0.157	0.027

TABLE 4-5**The Cost of the Mortgage Indemnity Scheme at One year to Maturity**

σ	$\dot{B} = R17,500$		$\dot{B} = R32,500$		$\dot{B} = R47,500$		$\dot{B} = R62,500$		$\dot{B} = R77,500$		$\dot{B} = R92,500$		C
	A	B	A	B	A	B	A	B	A	B	A	B	
0.05	0	0.000	0	0.000	10,147	0.000	316,748	0.000	30,141	0.000	34,777	0.000	0.014
0.10	0	0.000	0	0.000	5,387,306	0.004	19,672,551	0.013	4,906,120	0.003	4,788,603	0.003	0.014
0.15	0	0.000	0	0.000	23,999,347	0.019	55,480,209	0.037	21,142,324	0.013	19,858,567	0.014	0.014
0.20	0	0.000	0	0.000	48,257,567	0.038	91,991,847	0.061	44,529,742	0.028	41,171,346	0.029	0.014
0.25	0	0.000	103,235	0.000	74,173,489	0.058	126,945,016	0.085	71,650,184	0.045	65,693,333	0.046	0.014
0.30	0	0.000	11,069,325	0.009	100,553,308	0.078	160,504,328	0.107	100,813,828	0.063	91,957,910	0.064	0.014
0.35	0	0.000	26,163,918	0.021	127,014,095	0.099	193,014,128	0.129	131,156,429	0.082	119,221,669	0.084	0.014
0.40	0	0.000	44,246,693	0.035	153,414,575	0.120	224,730,525	0.150	162,197,571	0.102	147,073,013	0.103	0.014
0.45	0	0.000	64,483,825	0.051	179,693,533	0.140	255,822,516	0.171	193,648,000	0.122	175,264,499	0.123	0.014
0.50	3,146,678	0.001	86,281,092	0.069	205,818,869	0.160	286,399,022	0.191	225,320,914	0.141	203,636,317	0.143	0.014

In the tables above, A (=G(T)), is the real value of the Mortgage Indemnity Scheme in Rands; $B \left(= \frac{G(T)}{B e^{-rT}} \right)$, is the value of the loan guarantee as the fraction of the value of the loan covered by the guarantee, and $C (= 1 - e^{-(R(T)-r)T})$, is the value above the risk-free interest rate which banks will charge to cover the additional risk involved in lending funds to a home-loan customer.

In addition to the estimates presented in the tables above, different values of the MIS can be very easily calculated for any other default rate or any other distribution of the total quantity of the loans amongst a various categories of the loans. Thus, for example, to calculate the cost of the MIS at two years to maturity for that category of loans with a midpoint value of R32,500, on assumption that this category of the loans accounts for 65% of the total quantity of the loans distributed, i.e. $P_1=0.65$; and with value of the standard deviation of the percentage changes in the market values of residential units equal to 0.35, we need merely to refer to Table 4-4 ($\hat{B}=R32,500$, column A) to read the value of $G(T)$ and then multiply that value by $\frac{P_1}{P_0} d_r$, where P_1 is the new probability of occurrence, P_0 is the probability of occurrence given in Table 4-1 (or Table 4-2, column 3), and d_r is the new default rate. This, of course, yields the following values: $G_1=R13,826,018$; $P_1=0.65$; $P_0=0.15$; and $d_r=0.6$. Thus $G_2 = 13,826,018 \frac{0.65}{0.15} 0.6$ or $G_2=R35,947,647$ which is value of the MIS under these particular assumed values.

As can be seen from the above results, the 'real' value of the proposed Mortgage Indemnity Scheme depends crucially on the value of σ , the value of the loans, and the time to maturity. Moreover, it can be seen that there is a positive relationship between the value of the Government guarantee and the value of the standard deviation. Specifically, as σ becomes larger, the value of the Government guarantee also becomes larger. However, it is interesting to note that the value of $G(T)$, the Government guarantee, increases much faster than does the value of σ . Thus, for instance, if $\hat{B}=R62,500$, (in Table 4-3), the value of σ has been increased almost four-fold, from 0.15 to 0.50, while the value of $G(T)$ has increased fully 110 times, from R242,891 to R26,678,626 and the value of the loan guarantee as a fraction of the value of the loan covered by the guarantee rises from 0,002 to fully 0.170 percentage points. Similarly, as the value of loans, and their time to maturity, changes, we can see that if the value of the loan were equal to R17,500, (Table 4-3, $T=3$ years), the guarantee takes on a value greater than zero only at $\sigma \geq 0.4$, while if the value of the loans were equal to or greater than R47,500 the guarantee has a positive value of $\sigma \geq 0.15$. With a time to maturity of one year, (Table 4-5), for loans of

R17,500 the values of the guarantee becomes greater than zero at $\sigma \geq 0.5$, while for those loans of R47,500 or greater, the value of the guarantee becomes greater than zero at a value of σ as low as 0,05. This clearly shows the existence of the negative relationship between the standard deviation and the time to maturity on the one hand, and the value of the guarantee on the other.

That the value of the guarantee becomes greater than zero at lower values of σ for higher value of the loans (regardless of time to maturity) results from the initial ratio between the value of the loans and the initial value of the residential units. Table 4-6, below, illustrates this.

TABLE 4-6
The Value of the Loan as the Fraction of the
Initial Value of the Residential Unit

\dot{B}	17,500	R32,500	R47,500	R62,500	R77,500	R92,500
$\frac{\dot{B}}{\bar{V}}$	0.54	0.72	0.83	0.90	0.91	0.91

Source: Calculated from Table 4-1 and Table 4-2

As mentioned earlier, the initial value of the residential unit is the sum of the value of the loan, plus the value of the State subsidy, plus the individual's contribution. Since almost half of the initial value of the lower priced residential units is financed via subsidy and the individual down-payment, the lower priced residential units require a much greater reduction in price than do the higher priced units. In other words, there must be a much greater reduction in the market value of the lower priced units than in the market value of higher priced units for the outstanding balances to become greater than the value of housing--in which case, of course, the value of the guarantee also becomes greater.

It should be noted that the rate of depreciation of 10% per year importantly affects the value of the *loan-value-of-housing ratio* regardless of changes in the market value of housing. Thus, with a shorter time to maturity, smaller changes in values of σ are needed to affect the value of the Government guarantee.⁷⁰

⁷⁰For periods longer than three years, due to the repayment of the home loans, the value of the loans will decrease proportionally and this will not be the case any more. However the initial duration of the MIS is only three years, in which period home owner will pay only the interest on the loan while the principal of

As the value of the guarantee, $G(T)$, gives an indication of the absolute value of the Mortgage Indemnity Scheme, the value of the loan guarantee as a fraction of the value of the loan covered by the guarantee, $\frac{G(T)}{Be^{-rT}}$, (which results are given in column B of the above tables), gives an indication of the relative value of the guarantee to the financial institutions. Thus, when this value is greater than the home loan premium, $(=1-e^{-(R(T)-r)})$, as can be seen in column C in the above tables, then financial institution should--in principle--be willing to pay to obtain the Government guarantee as additional insurance to limit possible loss. In cases where the insurance premium is less than or equal to the home loan premium, financial institutions would not be prepared to pay to obtain the Government guarantee as additional insurance since it would merely increase their cost by the amount paid for the guarantee while not reducing risk.

To calculate the total value of the MIS for a chosen distribution of the total quantity of home loans amongst a different categories of the loans, we need merely sum the individual values of MIS for the different categories of the loans. Table 4-7 illustrate how to calculate the total value of the MIS for values of σ and P as given in the table.

TABLE 4-7
The Values of the MIS for the Individual
Categories of Loans
(for $T = 1, n = 250,000$)

\hat{B}	P	σ	$G(T)$
17,500	0.54	0.15	0
32,500	0.15	0.15	0
47,500	0.09	0.20	48,257,567
62,500	0.08	0.25	126,945,016
77,500	0.08	0.30	100,813,828
92,500	0.06	0.30	91,957,910

Source: Calculated from Table 4-5, above

The six values of $G(T)$ for a given σ was obtained from Table 4-5. Thus, the total value of the MIS is equal to fully **R367,974,321**. To calculate the total value of MIS for any other values of σ , the time to maturity ($=T$), P and d_r ⁷¹ we need merely replace the above values

the loan will not be affected, it will be the same.

⁷¹ $0.05 \leq \sigma \leq 0.5, 1 \leq T \leq 3, 0 \leq P \leq 1$ and $0 \leq d_r \leq 100$

of $G(T)$ with the new ones from the above tables and sum.

4.3. Summary of the Results

If the 'low income' housing market is divided yet further into two segments--an upper and a lower one, with R47,500 as the lower boundary of the upper segment--then Table 4-8 clearly shows that the higher risk, and ultimately a higher value of the MIS, is more likely at higher segments of the market than at lower ones.

TABLE 4-8
The Cut off Values of σ for which Insurance Premium
Becomes Greater than Risk Premium

\dot{B}	σ (T = 3)	$\frac{G(T)}{Be^{-rt}}$	σ (T = 2)	$\frac{G(T)}{Be^{-rt}}$	σ (T = 1)	$\frac{G(T)}{Be^{-rt}}$
17,500	0.50	0.046	/	/	/	/
32,500	0.40	0.053	0.40	0.045	0.35	0.021
47,500	0.35	0.056	0.30	0.034	0.15	0.019
62,500	0.30	0.048	0.25	0.032	0.15	0.037
77,500	0.30	0.047	0.25	0.031	0.20	0.028
92,500	0.30	0.048	0.25	0.032	0.20	0.029

Source: Calculated from Table 4-3, Table 4-4, and Table 4-5, above

Note: The corresponding values of the home loans premium are: 0.004, 0.027 and 0.014 respectively.

As mentioned above, according to Rode's Report,⁷² price variation of those residential units at the upper segment of the market was 23.4%, while the price variability for residential units in the lower segment of the market was 11.4% for 1994. Thus, according to the complete results presented in Table 4.3, Table 4-4, and Table 4-5, the value of the Government loan guarantee scheme--the MIS--could be substantial in the case of high price volatility and high default rates, and these amounts will ultimately be paid by the South African Government, and thus by South African tax-payers. However, the same results clearly show that there is a potentially large benefit for those financial institutions covered by the MIS since they can shift, completely, the risk of home loans to the Government.⁷³

⁷²Cape Times, 17 April 1995, n.p.

⁷³According to Business Day, the default rate at the low end of the housing market, (average loan of R15,000), was 40% for 1993 and 50% for 1994. Thus the SA Housing Trust is running related losses of approximately R60 mill. Business Day, (24 April 1995), n.p.

Thus, the crucial questions are whether South Africa really needs a Mortgage Indemnity Scheme, and if so, whether guarantees need to be given to financial institutions, and to home loan borrowers, at zero cost to them?

It is not unreasonable to suggest that, based on the political uncertainty which surrounds the low-cost housing market and the Government's unwillingness, or inability, to fully enforce the law, there is a strong political case to be made for some kind of Government loan guarantee. In this regard, then, the role of the MIS is positive. Without it, access to the loan market would be impossible for a majority of the South African population's, yet with it, the time needed to rebuild investors' confidence in this segment of the housing market may be shortened.

This noted, it is, however, difficult to make a case for 'free' insurance for financial institutions and home-loan borrowers. As far as financial institutions are concerned, they will clearly benefit from the MIS, and especially so in the segment of the home loans above R47,500⁷⁴ where risk is greater. On the other hand, if financial institutions require some additional incentives to re-enter the lower segment of the market, then it may be not unreasonable for the Government, for a given period of time, to offer loan guarantees without cost. In such a case, however, then there should be strict monitoring of all mutually agreed procedures regarding lending and strict enforcement of repossession and eviction court orders. For example, in the case of residential units which are repossessed for any reason, the Government could use those repossessed units as replacements for its financial subsidy, and the new home owner could merely take over repayment of the home loan, which may--in turn--reduce the overall cost of the MIS.

As far as the upper segment of the housing market are concerned, there is no need for such incentives. The value of the loans is sufficient to act as an incentive to financial institutions to re-enter this segment of the market. Combined with the Government guarantees and possible substantial benefits from the MIS, there is no necessity that Government give loan guarantee to financial institutions and to home loan borrowers without cost. Thus, it would be rational for the Government to charge a price for its loan guarantee on those loans above certain amount, which--in our model--are for those loans equal to or above R47,000.

Tables 4-3, 4-4 and 4-5 show that the difference between the value of the loan guarantee

⁷⁴As the lower boundary in our model

as a fraction of the value of the loan covered by the guarantee and the home loan premium can be substantial value and can range from as low as 0.005 to as high as 0.191 percentage points. By way of comparison, for instance, the very similar United States Government home loan guarantee scheme charges 0.005% per month on the outstanding amount of the loans for its services. However, these services also include the securisation of the home loans, which was one of the major instruments to attract non-traditional investors in the home loan market.⁷⁵

Thus, if the South African Government successfully proceeds with its apparent plans regarding securisation of home loans, then charging for loan guarantees will be yet more desirable. Risk-averse investors would have incentive to invest in such secured bonds, which will be virtually risk-free and would yield yet a higher return than would similar assets available in current markets.⁷⁶

⁷⁵Dunn, K., and McConnell, J. "Valuation of GNMA mortgage-backed securities", The Journal of Finance, (vol. 36, 1981, pp. 599 - 615)

⁷⁶The assumptions of the higher return is based on the proposed fixed 22,5% home loan interest rate for the low-cost housing. However this will be the case only if the future inflation could be sustained at present or below the present level.

CONCLUSION

This paper has evaluated the *real* cost of the proposed South African Government sponsored Mortgage Indemnity Scheme. This evaluation has been based on standard financial techniques, and, in particular, on Put option pricing theory. Therefore, in the first part of the paper, some of the standard models of the evaluation of the loan guarantees using the Put option method, i.e. Merton's analytic derivation of the cost of loan guarantees, Sosin's model of the valuation of federal loan guarantees to corporations, Jones and Masson's model of valuation of loan guarantees and the model of Chen, Chen and Sears, have been presented. The general conclusion of the models presented is that the value of a Government loan guarantee is positively related to the standard deviation of the changes of the market value of the underlying security or securities and to the time-to-maturity of debt, and that a Government loan guarantee has a positive value to both lenders and borrowers. Thus, the Government should charge a fee in exchange for the benefits received.

However, given the specific structure of the proposed Mortgage Indemnity Scheme,⁷⁷ the above models could not be directly applied in their original forms to evaluate the cost of the MIS. Therefore, in order to develop a housing model to estimate this cost, it was necessary to make some modification to Merton's model. The empirical results obtained by the modified Merton's model--presented in Table 4-3, Table 4-4 and Table 4-5--were consistent with the general conclusion mentioned above that (a) there is a positive relationship between the value of the MIS, on the one hand, and the value of the standard deviation, the time-to-maturity and the value of the loans, on the other, and (b) the Government loan guarantees, represented by the MIS, have a value to both lenders and borrowers. Thus, as mentioned above, the Government should charge a fee for the insurance benefits received by lenders and borrowers.

As far as the *real* cost of the MIS is concerned, it was illustrated that the cost of the MIS to the South African Government may be as low as zero and to as much as one billion Rand when the total value of the loans granted is approximately eight billion Rand⁷⁸, and

⁷⁷The coverage offered by the MIS is limited to only three years, while housing loans are usually granted to 15-20 years.

⁷⁸For example, the cost of the MIS for the lowest category of loans ($\dot{B} = R17,500$) is zero for almost all values of σ , regardless of time to maturity. In contrast, the cost of MIS of the highest category of loans ($\dot{B} = R92,500$) has a positive value of at $\sigma = 0.10$.

thus the Government (read: taxpayers) might be constrained to bear these potential cost.

Moreover, it has also been illustrated that the main beneficiaries of the MIS will be: (a) those households at the upper end of the low-cost housing market, and (b) private financial institutions which will be indemnified by the terms of MIS. This conclusion is obtained when the low-cost housing market is divided into two segments with R47,500 value of the loan as the border between the lower and the upper segment. Thus, it may be illustrated that the overall risk of lending related to the lower end (segment) of the market is far lower than the risk related to the upper end (segment) of the market.⁷⁹

Accordingly, the mere fact that the main beneficiaries will be those two categories of end-users and not these at the lower segment of the low-cost housing market⁸⁰ suggests that the MIS may not attain its principal purpose--that of serving these in the lowest income group. Therefore, it does not make sense for the Government to fail to charge a fee for its guarantee if the main beneficiaries are not the ones for whom the programme was devised in the first place.

Thus, if the Government wants--for political reasons--to establish some kind of loan guarantees, then it must distinguish between these two segments of the low-cost housing market, and must provide its loan guarantee without cost to the end users only for the lower segment of the market, which in our model are those households which can afford home loans less than R47,500. For those loans above R47,500, private financial institutions which will lend funds and the households which will borrow funds should be charged fees in return for the Government loan guarantee. We can conclude that there is no reason why *the Government should bear the likely cost of the MIS. In contrast, the Government should charge a fee for its guarantee.*

⁷⁹The empirical results illustrate that the home loan premium charged by financial institutions above the risk-free interest rate is sufficiently large for financial institutions to cover additional risk involved with the lending of the funds to the lower end of the low-cost housing market. On the other hand, the same results illustrate that at the upper end of the low-cost housing market, that difference is not large enough to cover additional risk involved with these segment

⁸⁰Based on the fact that the households at the upper-end of low-cost housing market could presumably afford home loan and that private financial institutions would enter this segment of the market regardless of the government loan guarantee.

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