COMBINATION OF THREE DIMENSIONAL GEODETIC NETWORKS USING SIMILARITY TRANSFORMATIONS

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ABSTRACT

Classical terrestrial (horizontal and vertical) networks and Doppler satellite derived networks are combined in a three dimensional transformation adjustment by solving for the external bias parameters using any of the three standard seven parameter similarity transformation models, namely the Bursa, Molodensky and Veis models.

The object of this combination may be merely to merge the systems or networks, but may additionally involve an attempt to assign physical meaning to the estimated bias parameters. These two aspects, and the influence of the a priori Variance-Covariance matrix of the observables on the parameters and their interpretation is studied in detail.

An in-depth conceptual, mathematical and numerical comparative assessment of the three standard models is made. The homogeneity of the classical terrestrial South African networks is investigated by comparing the transformation parameter sets derived for different regions and sub-regions of the country.

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CHAPTER 1

INTRODUCTION

1.1 STATEMENT OF THE PROBLEM

One of the major goals of Geodesy is the establishment and maintenance of national and global one-, two- and three dimensional geodetic control networks on land. These geodetic networks are used for absolute and relative positioning on land, at sea and in space.

In practice we may have networks of different kinds covering the same area. For various reasons it may be desirable to exploit the strength of each kind of network by merging them together.

Two types of networks are considered in this report. These are the classical terrestrial networks and the satellite Doppler networks which exist in South Africa. After studying the characteristics of each type, the advantages of combining satellite Doppler networks with the classical terrestrial networks can be clearly stated as follows:

Satellite networks are a source of independent, homogeneous data and can be used to strengthen existing classical networks, and to provide new control in areas where such does not exist, eg. in 'gaps' in the terrestrial network. The satellite networks are homogeneous and can therefore be used to control or model and remove the effects of systematic errors that exist in classical terrestrial networks. Satellite networks represent their geocentric datums to a very high degree of accuracy, and as such can be used both for positioning a new terrestrial datum and for relating various existing terrestrial datums.

Now, since the coordinates of network points usually come from different sources, the individual networks may refer to different systems or datums. This is certainly true for classical terrestrial and satellite networks. In order to use the data from one network as observables in another, one may use only that data which is invariant of the coordinate system (ie. spatial angles and spatial distances), or the datum differences must be modelled (Thomson, 1976). However, since the individual networks are usually computed separately, the coordinates of the network points of each network are generally available. In order to use these coordinates, the datum differences must then be modelled. It may therefore be neccessary to do various transformations in order to merge or combine the systems or datums. The opinions of researchers differ regarding the 'best' methods for achieving this merger.

A number of methods for achieving this merger have been described in the literature. Each of these have been developed with specific objectives in mind, and differ greatly in many respects. Some of these are the a priori assumptions regarding the state of the data and the coordinate systems involved, the number of data points available, the number and types of parameters involved and their physical or geometrical interpretation, if any. For the purposes of this report, these combination methods have been divided into two main groups. The first are those which have clear geometrical meaning, and the second those that do not. The selection of a particular method will in general depend on the specific objectives of the user, and therefore the 'best' method of merging two systems is subject to this qualification. *The problem then is one of finding a method of combining the terrestrial and Doppler satellite networks which is suitable for South Africa.*

1.2 OBJECTIVES OF THIS STUDY

A number of methods for merging three dimensional geodetic networks are considered. The main features and uses of each are studied and summarised in Tables 2.1 & 2.2 at the end of CHAPTER 2. Three of the "geometric" models are selected for detailed study to determine their usefulness and applicability for the combination of the terrestrial and satellite geodetic networks in South Africa. These three models are evaluated numerically using real data for this country.

It should be pointed out that this part of the study has *two* important aspects which need consideration. The *first* is the attempt to recover the actual (real) external bias parameters of the terrestrial geodetic system and network with respect to the satellite or Conventional Terrestrial (CT) system. These are the overall average position, orientation and scale as determined from a combined adjustment using a geometric 7-parameter conformal transformation. This process whereby two networks are combined through the estimation of such bias parameters in a least squares adjustment, is called a transformation adjustment (Harvey, 1985, 1986). It is therefore attempted to assign real physical meaning to the estimated parameters from such a transformation adjustment.

The use of different models for the transformation adjustment leads to the estimation of parameters that are sometimes fundamentally different. This fact must obviously be appreciated to avoid an improper comparison of parameter sets, and to avoid assigning false meanings to such estimated parameters.

It is stated at the outset that, in this report, the classical terrestrial geodetic network is viewed as distictly separate but intricately connected with the datum on which it is based. With this view in mind, it is clear that a distinction must be made between the datum and network bias parameters. This is done through the use of a weight model or variance-covariance (VCV) matrix of the observations designed to reflect the uncertainty of the network coordinates in representing physical reality, and thus the datum on which the network is supposedly based. The use of this VCV matrix leads to the recovery of what are termed here "datum transformation parameters", whilst the use of the identity matrix I as the VCV matrix of the observations leads to the "network transformation parameters".

When the parameters are assigned physical meaning, these then represent the overall average position, orientation and scale of the geodetic datum (or system) and geodetic network with respect to the satellite or CT system.

The *second* aspect concerns the merging or combination of the two networks, without any attempt to assign any meaning (physical or geometrical) to the resulting parameters. This set of parameters is simply used to achieve the neccessary merging through transformation.

These two aspects will be considered in this report.

The homogeneity of the data used here is investigated for two main reasons. The first concerns the attempt to model the residuals to the observables, as resulting from the transformation adjustment procedure, by an algebraic polynomial. The second reason is to demonstrate that the geodetic network is not homogeneous enough to justify the use of only one national set of transformation parameters when seeking the best possible transformation accuracy. The use of regional sets of parameters is investigated.

A number of conclusions and recommendations resulted from this study and are presented in CHAPTER 6.

CHAPTER 2

BACKGROUND

2.1 REFERENCE COORDINATE SYSTEMS AND FRAMES - GENERAL

A great many problems encountered in the earth sciences and especially in geodynamics depend on the spatial relationship of points and their temporal variations. These problems can conveniently be solved (at least partially) by the use of a well-defined reference system (Veis,1981, Mueller, 1985). In mathematics we work with abstract spaces, whereas in geodesy we work with physical space in which there are physical objects (which imply the physical space) and in which we can take our measurements. Further, as expressed by Vanicek (1975):

"these physical objects become geometrical objects ... once we transfer our problems from the physical space to the abstract space."

We shall assume that both the physical and abstract spaces we deal with here are Euclidean, in which a Cartesian reference coordinate frame with straight and mutually orthogonal coordinate axes can be defined (Vanicek,1975). Three mutually orthogonal unit vectors define a triad (Veis,1981). Such a triad will be called a reference frame (or datum) and the distances from the three unit vectors, the Cartesian coordinates. Various forms of curvilinear coordinates are also used in astronomy and geodesy, some of which will be discussed later.

2.1.1 Definition of a reference system

Once we have selected a reference frame, and the form or type of coordinates to be used, we need to do two more things before we can refer coordinates of objects to the reference frame.

Firstly, one has to define in detail the model that is to be used in the relationship between the basic reference frame and its coordinates. This model must include the description of the physical environment into which the reference frame is to be introduced, as well as the theories used (including the neccessary measurements, corrections and reductions to these, mathematical relations and computations) in the definition of the coordinates (Mueller, 1985). The model may therefore involve physical laws, accepted theories and parameters. A model as described above constitutes a conventional reference system.

Secondly, although the reference system and its coordinates are now completely defined, it must now be realized or materialized, so as to make the system available to its users. This is a complicated operation. It is usually done by assigning conventionally chosen or derived coordinates (parameters) to a number of stars or triangulation points. It is the catalogue of these selected stars or points and their coordinates that practically realizes or materializes the system.

A reference system and the associated frame are abstract concepts and are thus introduced in space (physical or abstract) by definition as mentioned above. This considerable freedom in the choice of the position and orientation of the reference frame, and the form of coordinates to be used allows for the selection of a reference system so as to optimize its use (Veis,1981).

2.1.2 Determination of nominal coordinates

There are two methods of determining the nominal coordinates of the basic set of control points, namely the geometric and dynamic methods.

Geometric (static) method: The reference systems that have traditionally been used in astronomy and geodesy were based on simple Euclidean geometry. Angular and (some) distance measurements were made between a selected set of points, and by a process involving reductions and adjustments, nominal coordinates for these selected points were obtained.

Dynamic method: Instead of using a geometric (static) method to determine nominal coordinates of selected points, one could use a moving object, such as an artificial earth satellite, the motion of which, expressed in some reference frame, is considered precisely known (Veis, 1981). Such a dynamic reference system is very complex to define, since the theory and computations used are very complicated indeed, and a very large number of parameters need to be known (mainly in order to model the earth's gravity field). However, this *dynamic satellite* method for defining a reference system has *two distinct advantages* (Veis, 1981). *Firstly,* it gives positions expressed in a dynamically defined (almost) inertial reference system, and *secondly,* the reference frame can be geocentric to a very high degree of accuracy.

2.1.3 Internal consistency of a reference system

The internal consistency of a reference system depends on the accuracy of the measurements and the correctness of the reductions applied to them, on the completeness of the theories used and the correctness of the constants, and on the precision of the computations. As stated by Veis (1981):

"Coordinates derived from two different reference systems will not agree if the measurements, the theories and the computations are not consistent. In order to relate two reference systems, it is neccessary to find the relation between the two theories and constants, the two sets of observations (and their corrections) and the two computations used for their definition."

The inter-comparison between and combination of geometrically derived geodetic reference systems and dynamically derived (satellite) reference systems forms the major theme of this report and will be discussed in more specific detail later.

2.1.4 <u>Coordinates, coordinate lines, coordinate surfaces, families of coordinate systems and inter-transformations</u>

Coordinate frames have associated coordinate lines and surfaces, the first being lines on which two of the three coordinates remain constant, and the latter being surfaces on which one of the coordinates remain constant. If the constant coordinate(s) equal zero, then we have the basic reference coordinate line(s) or axes, and surfaces. One can use not only cartesian coordinates but also a variety of curvilinear coordinates.

A family of co-axial systems may consist of one cartesian system and one or more curvilinear systems such that the systems share not only the zero point (0,0,0) or origin of coordinates, but also the basic coordinate lines of the curvilinear systems would lie in the basic coordinate surfaces of the Cartesian system (Vanicek,1975). An example of this would be a co-axial spherical, ellipsoidal and cartesian system. When considering transformations of coordinates within one family, we can in general express the curvilinear coordinates (u,v,w) as functions of the cartesian coordinates, by transformation equations such as $u = u(x,y,z)$; $v = v(x,y,z)$; $w = w(x,y,z)$, where these equations may have simple explicit solutions, or they may be iterative.

When considering inter-family transformations, we can assume that each family can be represented by its cartesian system. If the relative position and orientation of the two reference frames are known, the transformation reduces to a simple matrix operation as shown below. It is well known that any two cartesian systems $[XYZ]_1$ and $[XYZ]_2$ are related by the following transformation equation:

$$[XYZ]_2^T = [XYZ]_0^T + R_3(\varepsilon_z)R_2(\varepsilon_y)R_1(\varepsilon_x).[XYZ]_1^T$$

where $[XYZ]_0^T$ are the coordinates of the origin $[0,0,0]$ of system 1 in the frame of system 2, $R_3(\varepsilon_z)$, $R_2(\varepsilon_y)$ and $R_1(\varepsilon_x)$ are the three well known rotation matrices and $\varepsilon_z,\varepsilon_y,\varepsilon_x$ are the three rotation angles about the Z,Y,X axes of system 1 respectively.

In general, we assume that the scale of all the systems within one family are the same, but that it may vary between families, so that we may have to introduce a scale factor as $(1+k)$, where k is the scale difference from unity, into the transformation equation.

Hence we obtain the well known 7-parameter similarity (conformal) transformation:

$$[XYZ]_2^T = [XYZ]_0^T + (1+k)\ R_3(\varepsilon_z)R_2(\varepsilon_y)R_1(\varepsilon_x).[XYZ]_1^T$$

Now if we are to transform curvilinear coordinates from one family into another, say A to B, then we proceed naturally as follows: $(uvw)_A \rightarrow (XYZ)_A \rightarrow (XYZ)_B \rightarrow (uvw)_B$

2.2 POSITIONING

Position determination can be either absolute or relative.

2.2.1 Absolute positioning

Absolute position determination or point positioning as it is also called, is the determination of the position of a point on land, at sea and in space with respect to some implied coordinate frame. Observations are made to extra-terrestrial objects, being known points, eg. stars, the moon or artificial earth satellites, and the position of such points are computed from these known coordinates and the observations.

Since the positions of the stars, moon, artificial earth satellites and points on the earth are involved, there is a need for three distinct classes of reference coordinate systems.

Terrestrial reference systems that are earth-fixed, revolve around the sun and spin with the earth on its axis, are used for expressing the locations of points on the surface of the earth.

Celestial reference systems may revolve but do not spin with the earth, and are used for expressing the positions of stars.

Orbital reference systems, nominally geocentric, revolve but do not spin with the earth, and are used for expressing the positions of artificial earth satellites.

The effects of time deformations of the earth are not considered for the purposes of this report.

2.2.2 Relative positioning

Relative positioning is the determination of the position of one point with respect to another. Observations or measurements are made either directly between the two points involved, or from the two points to extra-terrestrial objects such as artificial earth satellites. Relative positions of points can be expressed in three-, two- or one dimensions, the choice of which depends on the purpose of such positioning and the nature of the observations.

Positions of points, both absolute and relative, may be expressed in any number of related reference systems, where the relations may or may not be known a priori. The transformations of coordinates from one system to another and the estimation of unknown transformation parameters between systems will be treated later.

2.2.3 Expressing positions of terrestrial points

The position of a point on the surface of the earth, at sea or in space can be represented in a number of ways. The conventional systems used are either natural (astronomic) or geometric (geodetic) (Mueller, 1974). The *natural* coordinates are defined in terms of some physical properties of the earth, eg. earth rotation, geopotential surfaces and gravity. These natural coordinates are the astronomic latitude (Φ), astronomic longitude (Λ), and orthometric (mean sea level) height, (H^0). They are determined from 'natural' observations namely astronomic, gravimetric and spirit levelling. The *geometric (geodetic)* coordinates, on the other hand, are the geodetic latitude (ϕ), the geodetic longitude (λ), and the ellipsoidal height (h). They are referenced to a rotational ellipsoid of arbitrary size, shape, position and orientation, and are determined from geometric observations (distances,directions).

2.3 REFERENCE COORDINATE SYSTEMS

2.3.1 Terrestrial systems

2.3.1.1 Conventional Terrestrial (CT) system (See Figures 2.1 & 2.2). The Conventional Terrestrial system is the closest approximation to the natural geocentric system. The term 'natural' implies a system dictated by some physical properties of the earth and is independent of any subjective preferences. The geocentric natural system is cartesian, right-handed and its axes coincide with the axes of the principal ellipsoid of inertia (Vanicek and Krakiwsky, 1982).

The CT system is by definition cartesian, geocentric, right-handed, its Z_{CT}-axis points to the Conventional International Origin or CIO (defined as the mean position of the instantaneous pole during the period 1900-1905), the XZ_{CT}-plane contains the Mean Greenwich Observatory and the Y_{CT}-axis completes the right-handed system.

2.3.1.2 Local Astronomic (LA) system (See Fig 2.1). This is the natural, topocentric (ie. the origin is at the surface observation point) system in which observations are made. The negative Z_{LA}-axis is defined by the local gravity vector, and together with a parallel to the conventional rotation axis (Z_{CT}-axis) of the earth, they define the XZ_{LA}-plane. The X_{LA}-axis thus points to the conventional astronomic north, and Y_{LA}-axis to conventional astronomic east, completing the lefthanded system. The natural angular astronomic observations are the astronomic azimuth, (A), the vertical angle, (V), and the zenith distance, (Z).

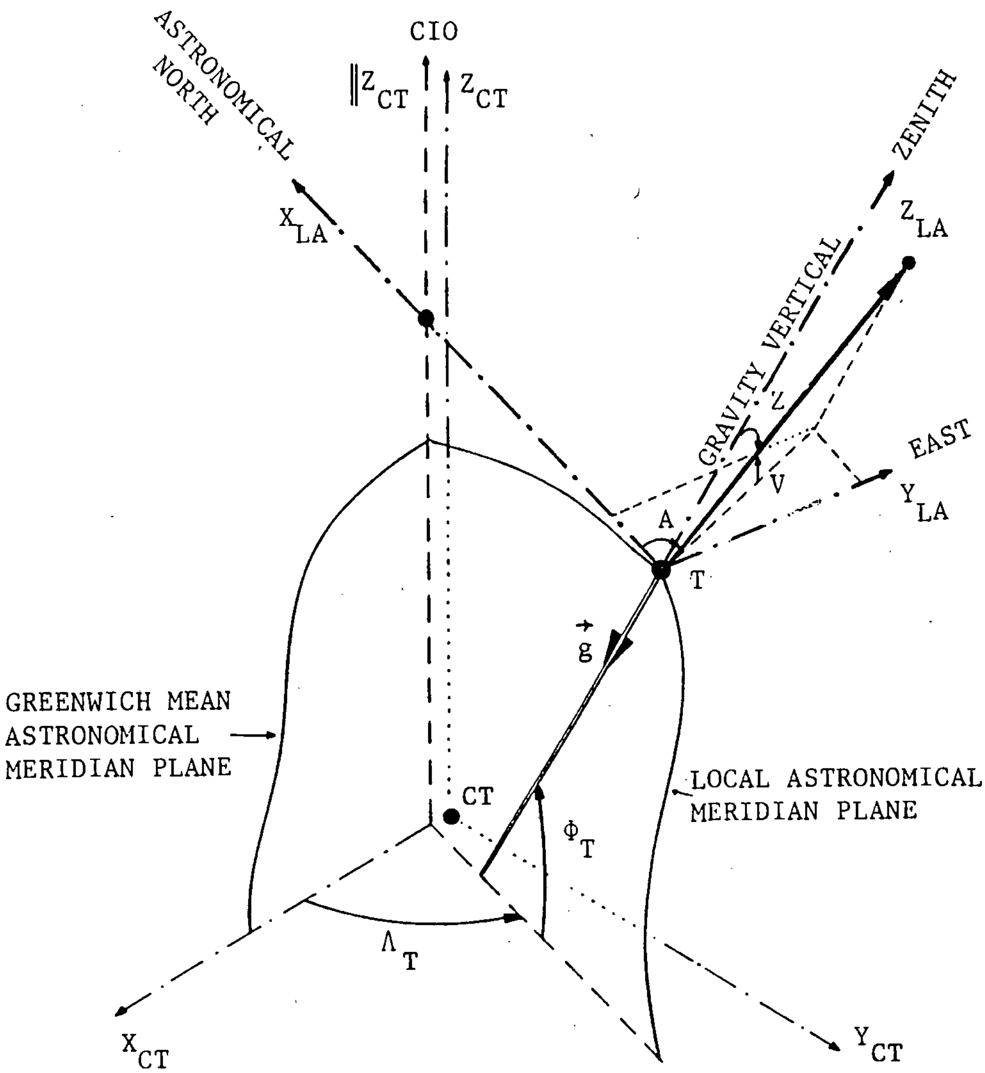


Figure 2.1 Conventional Terrestrial (CT) and Local Astronomical (LA) systems (after Vanicek and Krakiwsky, 1986)

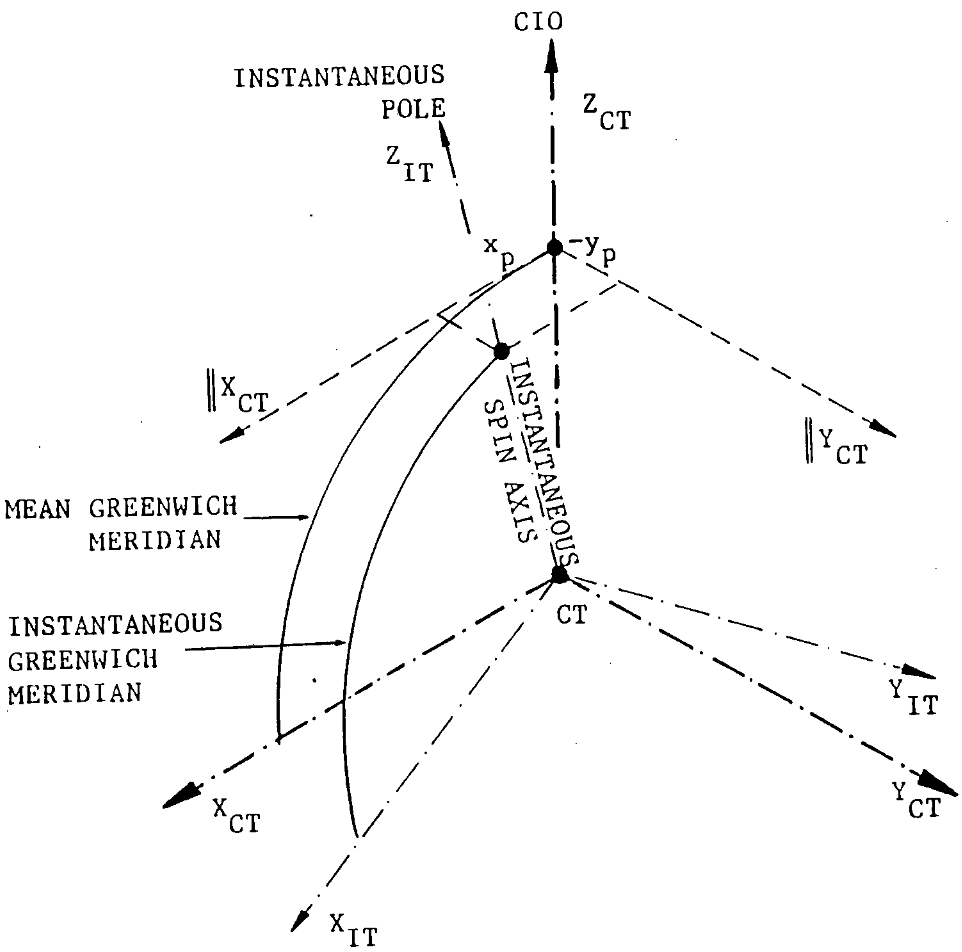


Figure 2.2 Instantaneous Terrestrial (IT) and Conventional Terrestrial (CT) systems

(after Vanicek and Krakiwsky, 1986)

2.3.1.3 Instantaneous Terrestrial (IT) system. (See Figure 2.2) The instantaneous terrestrial system differs from the CT system only in that the Z_{IT}-axis coincides with the instantaneous rotation axis of the earth and not the conventional rotation axis. Thus the Z_{IT}-axis wobbles around the Z_{CT}-axis, this motion being described by the parameters of polar motion as x_p, y_p in angular units (Vanicek and Krakiwsky, 1982). Note that the X_{IT}- and Y_{IT}-axes obviously then do not coincide with the equivalent CT-axes, and the XZ_{IT}-plane contains the instantaneous (ie. true) Greenwich meridian plane.

2.3.1.4 Geodetic (G) system: (See Fig 2.3). The geodetic system or family comprises a co-axial right-handed cartesian and curvilinear system. The curvilinear system is ellipsoidal and thus referenced to a rotational ellipsoid. The position of a point may be expressed in either geodetic cartesian (xyz) or geodetic ellipsoidal (ϕ,λ,h) coordinates. The location and orientation of a geodetic system, being a mathematical (geometrical) system, as opposed to a natural system, is arbitrary and may be located and oriented so as to optimise its use as mentioned before. However, it is generally approximately geocentric (to a few hundreds of metres), and oriented so as to be very nearly aligned (parallel) with the CT-system.

2.3.1.5 Local Geodetic (LG) system (See Fig 2.3). The system is topocentric, the Z_{LG}-axis is the outward ellipsoidal normal passing through the topocentric point P, and together with the minor axis (Z_G-axis) of the ellipsoid defines the XZ_{LG}-plane, and the Y_{LG}-axis completes the left-handed system. The X_{LG}-axis thus points to geodetic north and the Y_{LG}-axis to geodetic east.

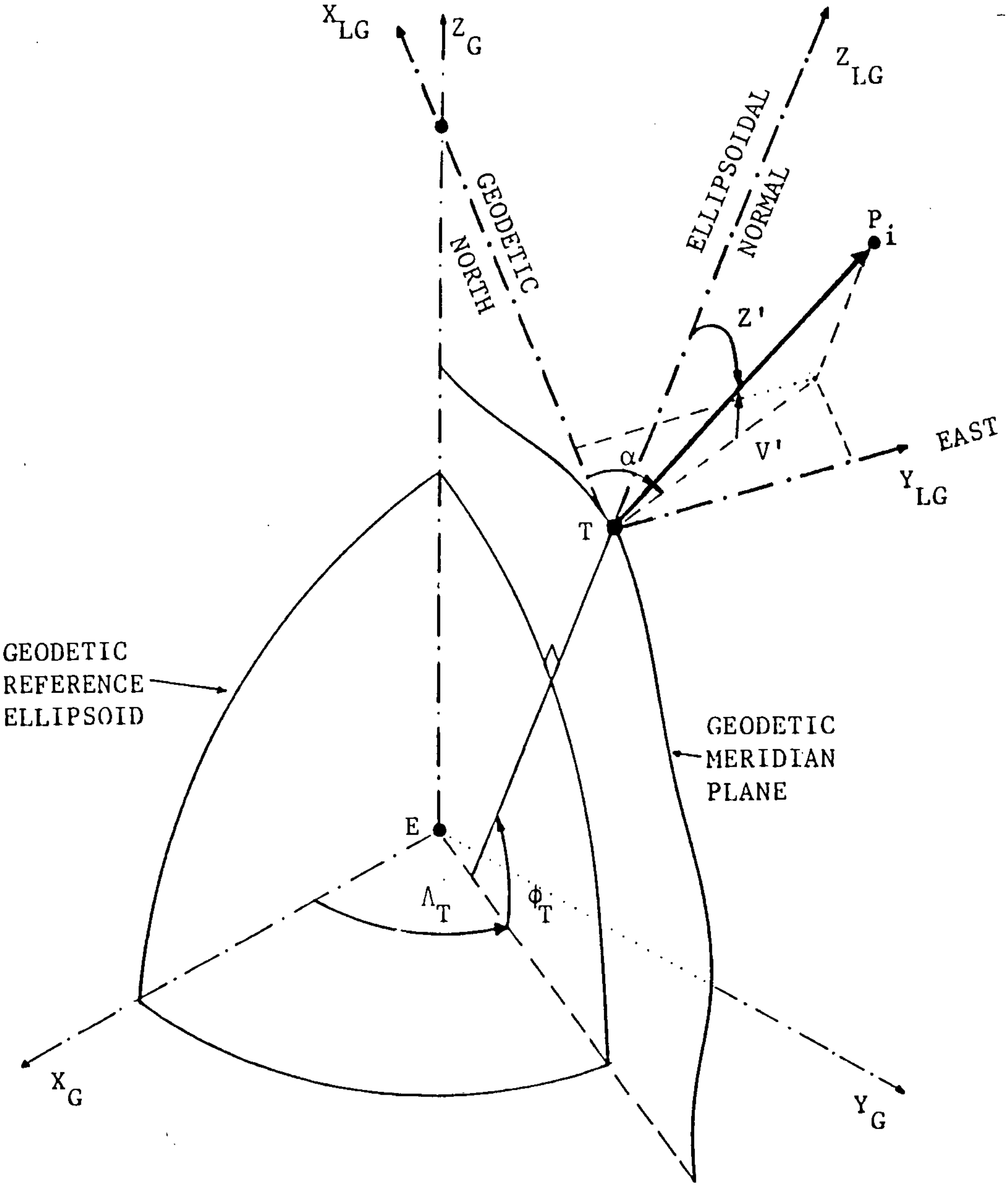


Figure 2.3 Geodetic (G) and Local Geodetic (LG) systems (after Vanicek and Krakiwsky, 1986)

Note that this system is mathematical (geometrical), and not natural. By analogy to the natural astronomical observables, one may define the geodetic (geometric) quantities namely the azimuth (α), vertical angle (V'), and zenith distance (Z').

2.3.2 Celestial systems

Fundamental to the definition of all celestial systems is the concept of the celestial sphere (Vanicek & Krakiwsky, 1982). All these systems will be considered spherical, and the position of a star on the celestial sphere is specified uniquely by two angles.

The Right Ascension (RA) system is the most important celestial system. The coordinates of stars expressed in this system are the angles right ascension (α) and declination (δ).

Suffice it to say here that there are various versions of the basic RA system, the most important for surveyors and geodesists being the Apparent Place RA system (AP). This system provides the link between the celestial (stars), orbital (satellites) and terrestrial reference systems and coordinates of stars in this system are available directly from the catalogue for the Apparent Places of Fundamental Stars (APFS).

2.3.2.1 Apparent Place Right Ascension system (AP).

(See Figures 2.4, 2.5A) This celestial system is geocentric, the Z_{AP}-axis coincides with the instantaneous rotation axis of the earth, the X_{AP}-axis points to the vernal equinox, and the Y_{AP}-axis completes the right-handed system. It should therefore be obvious that, since the Z_{AP}- and Z_{IT}-axes coincide, the only difference between these systems is a rotation angle around the common Z-axis.

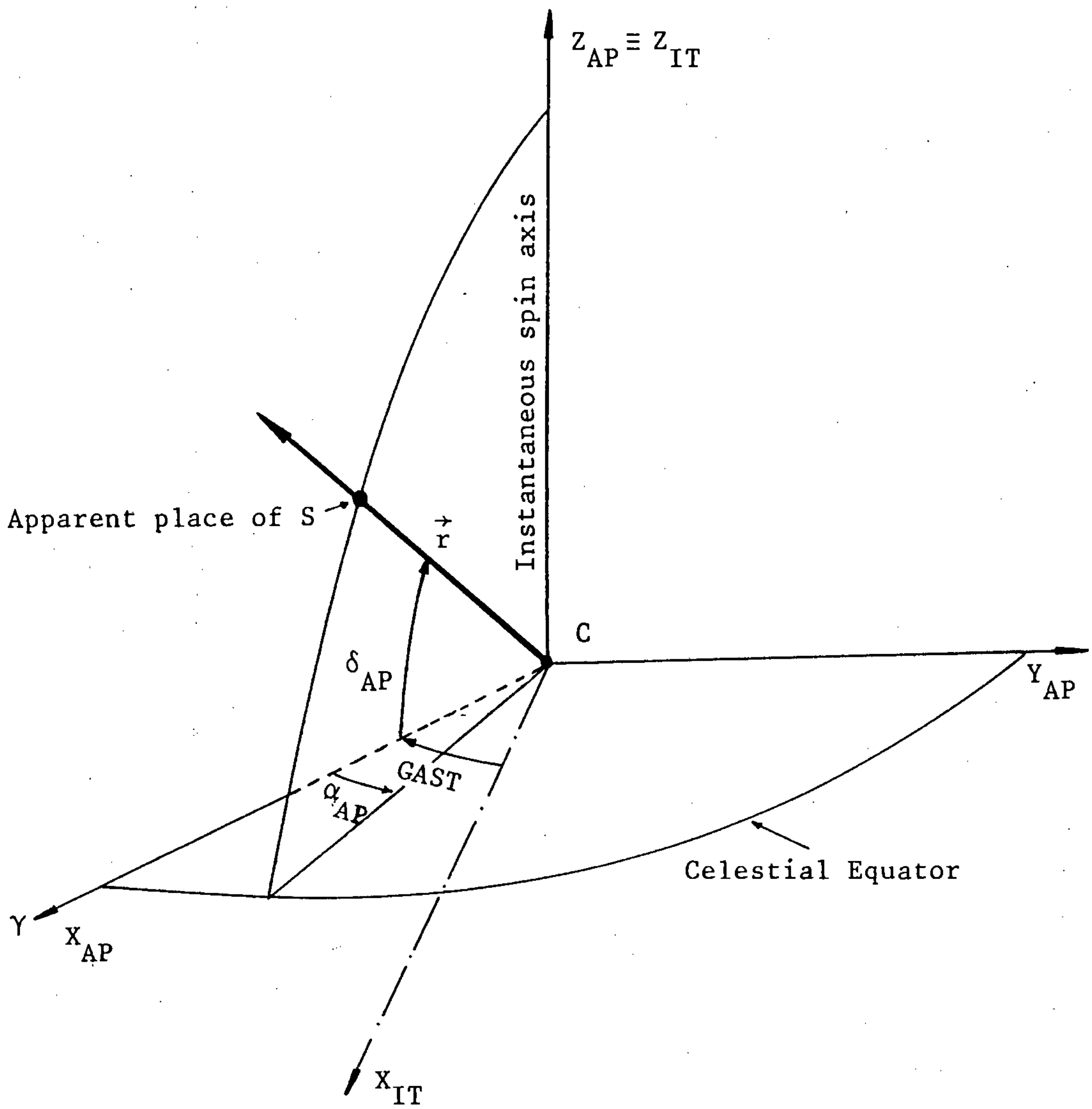


Figure 2.4 Apparent Place Right Ascension system (AP)

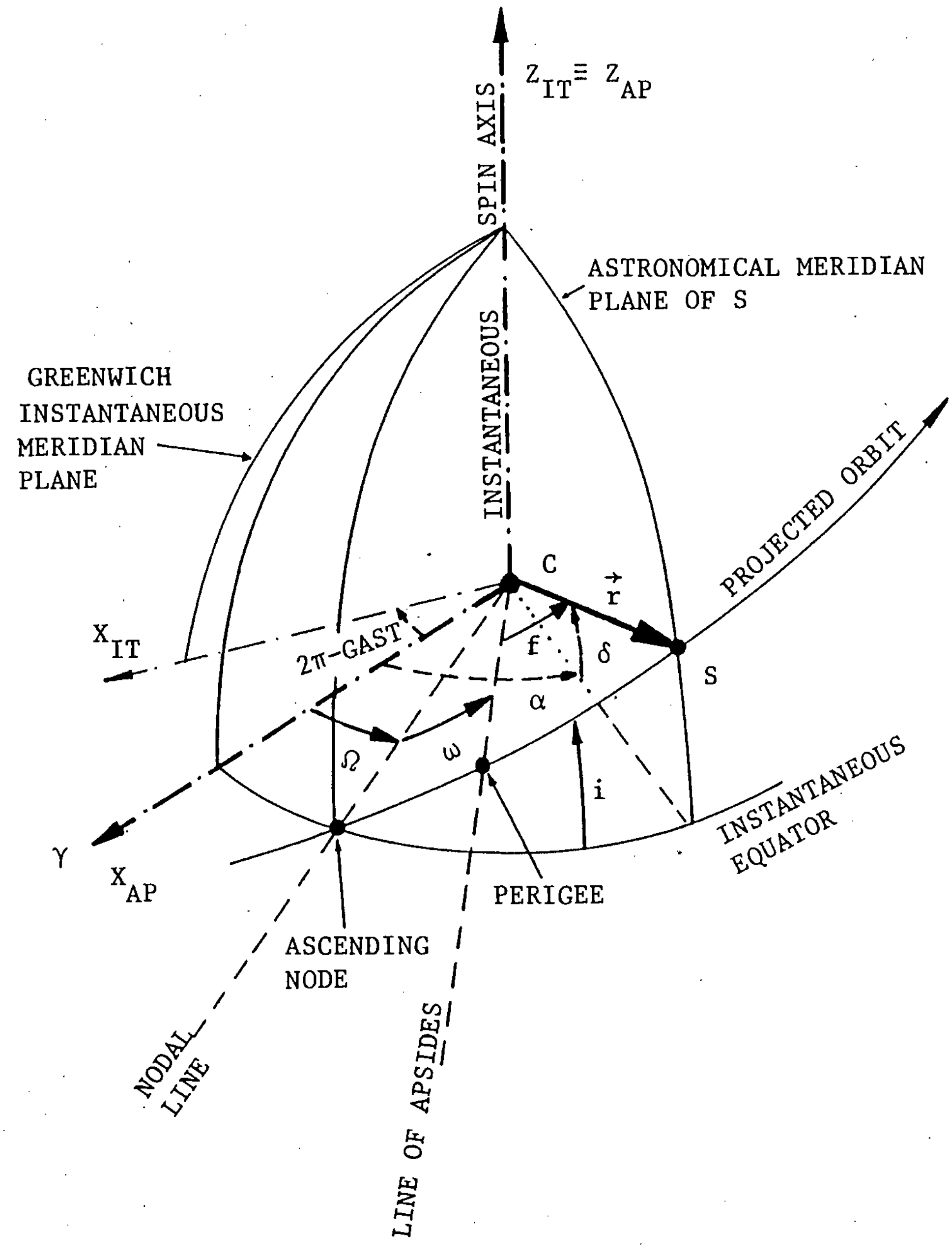


Figure 2.5A Keplerian orbital elements
(after Vanicek and Krakiwsky,1986)

This angle between the XZ_{AP}- and XZ_{IT}-planes is known as Greenwich Apparent Sidereal Time (GAST), and is practically obtained from Universal Time (UT) by applying a few reductions.

Astronomic observations are made in the topocentric LA system at the observation point. These observations are transformed to the AP system in which the coordinates (α,δ) of the stars are given. The mathematical models for point positioning are then formulated in this system and the coordinates of the observation station obtained by their solution.

The astronomic position of a point on the celestial sphere is defined by two angles, the astronomic latitude, Φ, and astronomic longitude, Λ. The XZ_{LA}-plane defines the local astronomic meridian plane of the topocentric observation point. The astronomic latitude, Φ, of a point is defined as the angle, measured in the local astronomic meridian plane, between the extension of the local gravity vector and the mean astronomical equator. The astronomical latitude is defined positive north and negative south of the equator. The XZ_{CT}-plane (defined as containing the mean Greenwich observatory) is the Greenwich Mean Astronomical Meridian plane. The astronomic longitude is defined as the angle between the Greenwich Mean Astronomical Meridian plane and the local astronomic meridian plane, measured positive eastwards, from the mean Greenwich meridian, in the plane of the mean astronomical equator.

Note that the local astronomical meridian plane of the observer contains both the local gravity vector and a parallel to the conventional rotation axis, but does not in general contain the geocentre (Vanicek & Krakiwsky, 1982).

2.3.3 The ORbital system (OR) (See Figures 2.5, 2.5A)

This system is used for expressing the position of orbiting artificial earth satellites. Such satellites move in nearly plane, elliptical orbits around the earth, the geocentre of which is located at one of the foci of the ellipse. This orbital ellipse can be viewed in the first approximation as describing exactly Keplerian motion. Any perturbations of this motion are then treated as temporal variations of the parameters which describe such Keplerian motion.

The orbital ellipse is specified in size and shape by two parameters, the semi-major axis (a) and the eccentricity (e). The position of the satellite in the orbital ellipse is given by the true anomaly (f), being the angle at the geocentre between the direction to perigee (line of apsides) and the radius vector to the satellite, measured counter-clockwise as seen from the North Celestial Pole (NCP) (Vanicek & Krakiwsky, 1982).

The orbital system (OR) is geocentric, the X_{OR}-axis coincides with the line of apsides, positive towards the perigee, the Y_{OR}-axis coincides with the direction for which the true anomaly (f) is 90^0, and the Z_{OR}-axis completes the right-handed system. The position vector of the satellite can now be given as (Vanicek and Krakiwsky, 1982):

$$\vec{r}_{OR} = [X,Y,Z]^T_{OR} = r.[\cos.f,\sin.f,0]^T = \begin{bmatrix} a.(\cos.E-e) \\ a\ (1-e^2)^{1/2}.\sin.E \\ 0 \end{bmatrix}$$

where E is the eccentric anomaly, the angle at the geometric centre of the ellipse between the line of apsides (towards perigee) and the line from the centre to the projection point, S′, of the satellite on the concentric circle of radius a, see Figure 2.5 (Vanicek and Krakiwsky 1982).

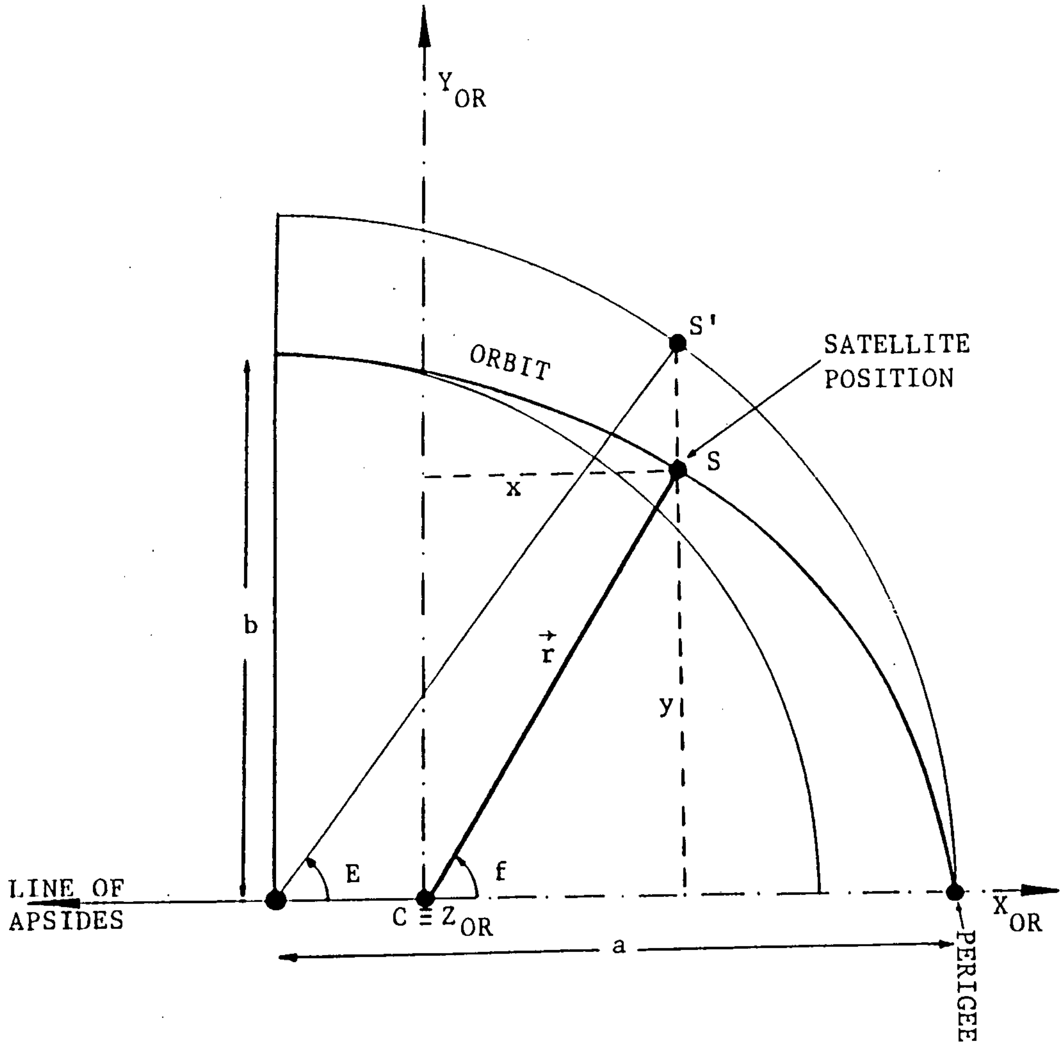


Figure 2.5 One quarter of a satellite orbital ellipse showing the ORbital (OR) system (after Vanicek and Krakiwsky, 1986)

The position and orientation of the orbital ellipse or equivalently the OR-system need to be specified with respect to the AP-system. The three angles known as the right ascension of the ascending node (Ω), the argument of perigee (ω), and the inclination of the orbit (i), specify this relation completely (See Figure 2.5A).

Note that the orbital ellipse does not rotate (spin) with the earth , but is (nearly) fixed in the AP-system.

2.3.3.1 Satellite ephemeris. Satellite point positioning requires both the position of the satellite and appropriate observations from the point to be fixed. The position of the satellite may be expressed in either Cartesian coordinates or equivalently, in Keplerian orbital elements. The list of coordinates expressing the position of the satellite as a function of time constitutes the ephemeris pertaining to that particular satellite. These coordinates are then transformed to a terrestrial system; usually the CT, in which the mathematical model is formulated and the coordinates of the observation point computed.

2.4 GEODETIC NETWORKS AND DATUMS

2.4.1 Geodetic datums

It has been shown that the shape of the earth is best represented by a surface called the geoid. This surface is that gravity equipotential surface which best approximates the (mean) sea level over the whole earth (Vanicek and Krakiwsky, 1982). The geoid plays a fundamental role in positioning. Mean sea level is practically realized through the use of tide gauges and as such is a first order approximation of the qeoid. A number of factors result in mean sea level not beinq completely coincident with the geoid on a qlobal scale, but mean sea level is nevertheless used as a practical realisation of the datum used for vertical networks, namely the qeoid.

A datum can be defined as "a real or assumed thing used as the basis for calculations" (Thomson, 1976).

The geometrical shape of the earth as represented by the geoid is for all practical purposes, closely approximated by a rotational reference ellipsoid of a certain size and shape. This ellipsoid is used as the datum for classical horizontal terrestrial qeodetic networks. Before such a datum can be used for computations its size and shape must be selected to best fit a reqion locally or the earth globally. Further, its position and orientation with respect to some earth fixed coordinate system must be specified. This task of *positioning* the *reference ellipsoid* in the earth body is known as the *establishment of a horizontal geodetic datum.* This is classically done by making some astronomical observations at a terrestrial initial point, and adhering to certain conditions for ensuring parallelism of the G and CT system axes.

The datum associated with satellite geodetic networks is treated in section 2.4.7. It is obvious that the *classical* terrestrial *horizontal* and *vertical* datums are quite *distinctly separate entities.* The horizontal datum is a geometrical and therefore conventional body, the ellipsoid, positioned and oriented by definition so as to best serve its purpose. The vertical datum, the qeoid or its first order approximation, mean sea level (MSL), on the other hand, is a natural surface realized through suitable tide gauge, gravity or other observations.

Classically the position of a point on the surface of the earth has been given in two dimensions as the astronomical latitude (Φ) and -longitude (Λ) or the geodetic (geometric) latitude (ϕ) and -longitude (λ). The third component, the height has been the orthometric height (H^0), ie. the geometric height above the geoid or rather MSL. Each of these refer to their respective datums, of course.

To establish the link between these two rather different types of datums, the geoidal height, N, has to be known (see Figure 2.6). The geoidal height, at a point P on the ellipsoid, is the geometric distance between the reference ellipsoid and the geoid, measured along the ellipsoidal normal through P. When the geoidal heights are known at a sufficiently large number of points, a geoid map of the area can be produced, showing the variation of the natural surface, the geoid, with respect to the artificial one, the ellipsoid. The geoid height is used to give the ellipsoidal height (h) as $h = H^0 + N$ (refer to Figure 2.6). This enables the position of a terrestrial point to be given in terms of its three dimensional curvilinear geodetic coordinates as latitude, longitude, and ellipsoidal height (ϕ, λ, h).

Note that the word *'datum'* is synonymous with the term 'reference frame' or 'triad'. When the surface of the reference ellipsoid is referred to as the datum for horizontal networks, what is meant of course is that it is this surface which is used as the basis on which horizontal networks are computed. This surface is simply one of the basic reference surfaces of the ellipsoid, namely that on which the ellipsoidal height (h) is zero, in this case. Since the geodetic (G) cartesian system is co-axial with the ellipsoid, these two 'datums' are of course just different forms of the same concept.

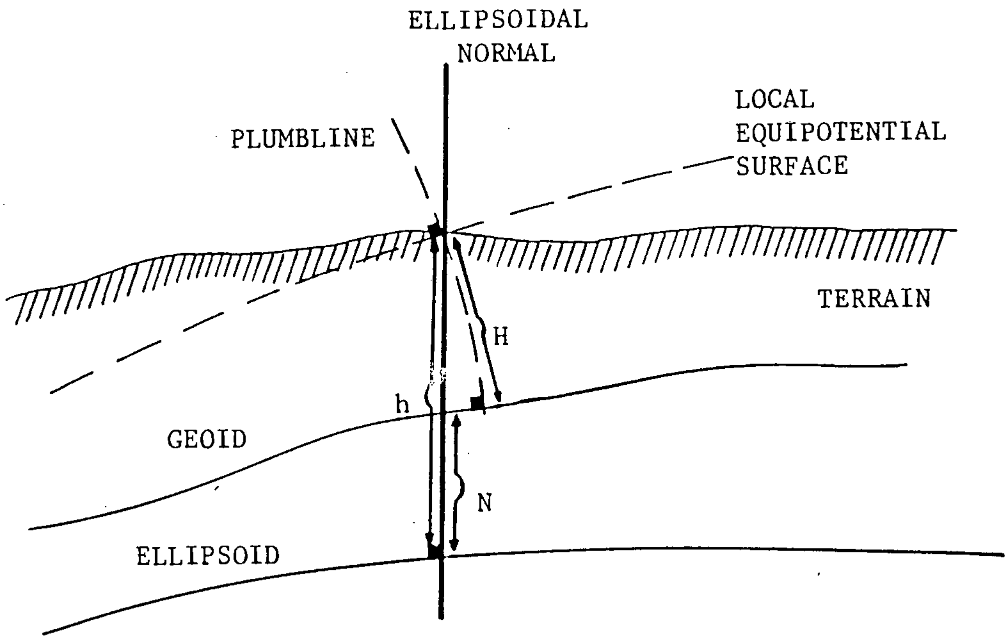


Figure 2.6 Reference surfaces and heights
(after Thomson, 1976)

2.4.2 Geodetic networks

"A geodetic network can be said to be a geometric object in which the various network points are uniquely defined by their coordinates" (Thomson, 1976).

Geodetic networks may be divided into terrestrial and satellite (space derived) networks. These networks may be one-, two- or three dimensional, as are the terrestrial networks. A vertical geodetic network is one dimensional, and a classical horizontal network, based on an ellipsoid, is two dimensional. Three dimensional terrestrial networks are treated in section 2.4.6. Satellite networks are inherently three dimensional.

Appropriate observations are made amongst network points on the surface of the earth, or between such points and artificial earth satellites, and after appropriate reductions, these are used in suitable models to compute a homogeneous set of network coordinates.

Terrestrial networks are those which are computed from classical astronomical and geodetic observations. The classical horizontal networks have an initial point at which the topocentric parallelism conditions are satisfied. The network is then extended in a sequential manner to cover the entire area for which it was designed. Due to the sequential nature of terrestrial network development, extensive classical networks (both horizontal and vertical) are very susceptible to the accumulation of unaccounted for systematic errors. This is regarded as the fundamental problem of classical terrestrial geodetic networks (especially the horizontal networks).

Satellite geodetic networks, dynamic or geometric (static), do not suffer from this problem to any great extent (eg. Thomson, 1976). They are therefore a valuable source of independent information, and as such can be used in combination with terrestrial networks in an attempt to model or control the accumulation of the systematic errors in these terrestrial networks. These networks are treated in section 2.4.7.

Geodetic networks can be of *regional* or *global extent*. Which they are depends on the purpose for which they are created. They could be used for a great variety of purposes such as for geophysical studies, boundary locations, demarcations, map making, exploration for natural resources, tracking of artificial satellites. The precision and homogeneity of the set of network point coordinates is dependent on the type and quality of observables used, the correctness of the reductions applied and of the constants used, and the completeness of the mathematical models employed to compute such coordinates.

2.4.2.1 <u>Classical terrestrial geodetic networks: Horizontal and vertical</u>. Horizontal and vertical positions have traditionally been determined separately, mainly because it was easier and more economical. Each requires quite different types of field observations and procedures, and they also affect each other weakly (Vanicek and Krakiwsky, 1982). Thomson (1976) gives the reasons for this traditional splitting as "psychological, historical, physical and mathematical" This practice resulted in the separate development of horizontal (two dimensional) and vertical (one dimensional) networks, each based on their respective, but unconnected datums. The reason for the continuation of this practice is largely a practical one.

Very few of the horizontal and vertical control points are coincident, due mostly to the distinct ways in which they were (are) obtained. Horizontal control points are on high ground, due to the need for intervisibility, often with difficult access, whereas vertical control points are established along lines of communication such as railways and roads since these points are established by the sequential method of differential levelling.

The horizontal network is obtained by projection of the observations made in the physical space, onto the mathematically defined reference ellipsoid surface. The vertical network is not projected but treated in the natural environment of the earth's gravity field where the natural observations are made.

In horizontal networks the height component of a point is only weakly determined, if at all, whereas in vertical networks the horizontal component need only be approximately known.

2.4.2.2 Classical horizontal geodetic networks. The methods used for the establishment of an extensive horizontal geodetic network are traditionally triangulation, trilateration, traversing and astronomic observations. The observables are the geometric ones, namely horizontal angles (directions), distances and (some) zenith distances, and the natural quantities, namely astronomic latitude, longitude, azimuth, and spirit levelled height. Each of these quantities must be reduced from the topography, where they are carried out, to the appropriate reference surfaces where they are used in the calculations (eg. Mueller, 1974). The geometric observations are reduced to the reference ellipsoid, and the natural ones to the geoid.

The corrections for these quantities are given briefly by Mueller (1974). The geometric corrections are distance corrections for instrumental, atmospheric, and geometrical factors, direction corrections for deflection of the vertical, for skewness of the normals, and for the correction from normal section to geodesic, and a zenith distance correction for deflection of the vertical.

The natural quantities namely astronomic latitude, longitude, and azimuth, are corrected for polar motion, for (UTC−UT1) (longitude only), for curvature of the plumbline (at both the observation and observed station) between the surface of the earth where the observations are made and the geoid, and the orthometric correction for spirit levelled height differences.

These reduced quantities are then used to compute the set of network point coordinates, using either semi-rigorous or rigorous methods, whichever is or was available.

However, the first approximation of a horizontal geodetic network can be computed by knowing only the geodetic coordinates of the network initial point, (ϕ_0, λ_0), and the geodetic azimuth (α_0) of only one line emanating from the network initial point (NIP). Neither the geoidal height (N_0) nor the components of the deflection of the vertical (ξ_0, η_0) are needed at the NIP, nor for that matter are they needed anywhere else. Heights above sea level are used to reduce the distances, and directions are not reduced for the effect of the deflection of the vertical. The initial network is then computed from the directions, distances and azimuths observed on the surface of the earth (Vanicek and Krakiwsky, 1982).

The geoidal height (N_0) and deflection components (ξ_0, η_0) are only needed at the NIP when the position of the datum is desired to be known in order to do transformations to another coordinate system. The task of positioning and orienting the ellipsoid within the earth body is known as the establishment of a datum and can be done in various ways. These will now be treated.

2.4.3 Establishment of horizontal geodetic datum

The size and shape of the ellipsoid is specified by two parameters, namely the semi-major axis (a) and the flattening (f). The positioning and orientation of the ellipsoid with respect to some 'fixed reference' tied to some physical properties of the earth, requires six more parameters to be specified, to eliminate the six degrees of freedom. The 'fixed reference' with respect to which the six datum parameters are specified, is the CT system, being the closest approximation of the geometric natural system. The object is always to align the G system (and the ellipsoid with it of course) and the CT system, since this condition then simplifies several geodetic equations.

If the ellipsoid is positioned such that its centre coincides wiht the centre of mass of the earth, and it is of the same size and shape as the equipotential ellipsoid which generates normal gravity, then the datum is geocentric or absolute (Mueller,1974, Vanicek and Krakiwsky, 1982), as opposed to non-geocentric, relative, or astrogeodetic. These terms are also used when speaking of geoidal heights and deflection components.

The six parameters are three positional and three rotational parameters. These can be specified at either the geocentre or the topocentric point. The *geocentric* parameters are the CT coordinates of the ellipsoid centre, called datum translation components (X_0, Y_0, Z_0), and the three rotation angles $(\varepsilon_x, \varepsilon_y, \varepsilon_z)$ required to define the misalignment between the G and CT axes (Vanicek and Krakiwsky, 1982).

The classical (topocentric) way of positioning a datum will be treated first, and then two other methods using (some or all) network points.

2.4.3.1 Classical positioning of a horizontal geodetic ellipsoid. The topocentric parameters that are selected at the NIP, should satisfy the topocentric conditions of parallelism of the G and CT system axes. These conditions can be expressed as either of two sets (Vanicek and Krakiwsky, 1982):

a. The Laplace azimuth and deflection conditions:

$$A - \alpha = (\Lambda_0 - \lambda_0).\sin\phi_0 - (\xi_0.\sin A - \eta_0.\cos A).\cot Z \qquad 2.1$$

$$= \eta_0.\tan\phi_0 - (\xi_0.\sin A - \eta_0.\cos A).\cot Z$$

$$\xi_0 = \Phi_0 - \phi_0 \qquad 2.2$$

$$\eta_0 = (\Lambda_0 - \lambda_0).\cos\phi_0 \qquad 2.3$$

OR

b. The Laplace azimuth and zenith distance conditions:

$$A - \alpha = \eta_0.\tan\phi_0 - (\xi_0.\sin A - \eta_0.\cos A).\cot Z \qquad 2.4$$

$$Z - Z' = -\xi_0.\cos A - \eta_0.\sin A \qquad 2.5$$

The theoretical requirements for datum positioning using this method will be treated first and then the method used in practice.

2.4.3.1.1 <u>Theoretical requirements</u>. *Considering set a (equations 2.1, 2.2, 2.3):* There are two options here. *Firstly,* select (ϕ_0, λ_0, h_0) and define them to be fixed, ie. error-less, or, *secondly,* select the components of the deflection of the vertical and the geoidal height (ξ_0, η_0, N_0) and define them to be fixed.

Since the latitude (ϕ_0) and longitude (λ_0) and the deflection components (ξ_0, η_0) are to be related by equations 2.1 - 2.3 above, and since $h_0 = H_0^0 + N_0$, error propagation in these equations will always result in uncertainties in the derived values, due to the uncertainties in the observed quantities appearing in these equations.

If the *first* option is used, then the datum is tied to the NIP, without error in position. However, due to errors propagating through the equations, the deflection components (ξ_0, η_0) and azimuth (α) are not error-free and thus the datum is not perfectly aligned with the CTS.

If the *second* option is used, then, even though the deflection components (ξ_0, η_0) are error-free, the positional parameters (ϕ_0, λ_0, h_0) and the azimuth (α) are not. Therefore the datum is not entirely fixed in space and the azimuth (α) introduces uncertainty in the alignment with the CTS.

Considering set b (equations 2.4, 2.5): Here the two deflection conditions have been replaced by a zenith distance condition. The Laplace azimuth condition has remained unchanged. Since we need to remove three angular degrees of freedom, we need to satisfy the azimuth and zenith distance conditions for two geodetic lines emanating from the NIP (Hotine, 1969, Thomson, 1976). Mueller (1974) quotes Hotine (1969) on the need for adherence to the two conditions at the NIP, but does not mention that these conditions have to be satisfied for two lines. Vanicek and Krakiwsky (1982) have similarly not expressed this requirement. However it is submitted that what is minimally required is that either the azimuth or the zenith distance condition be satisfied on the second line.

2.4.3.1.2 <u>Standard technique of horizontal datum positioning in practice (topocentrically):</u> The three positional parameters (ϕ_0, λ_0, h_0) and the three angular parameters (ξ_0, η_0, α) are selected to satisfy the first set of the parallelism conditions, as well as the equation $h_0 = H_0^0 + N_0$. This process then automatically ensures parallelism of the G & CT systems (theoretically at least) (Vanicek and Krakiwsky, 1982), and on this assumption the network is then computed. The Laplace equation is abreviated in practice by the omission of the term containing cot Z.

Further, the zenith distance condition is ignored completely. This condition is of course not needed if the deflection conditions have been adhered to as is invariably the case. However, the reason for originally neglecting zenith distances is that these cannot be observed with the required accuracy, and further that zenith distances are very nearly equal to 90^0 for geodetic lines (eg. Mueller, 1974).

Vanicek and Krakiwsky (1982) further state that, under these conditions, the Laplace equation is valid at any point of the network, and it can be used to compute the Laplace azimuth from observed astronomic azimuths. In practice the Laplace azimuth equation is applied regularly throughout the network at certain intervals. This repeated application of the truncated Laplace azimuth equation is generally assumed to be ensuring the parallelity of the datum (ellipsoid) axes with the CT axes.

However, opinions differ as to the validity of this claim (Thomson, 1976).

Thomson points out that the view one takes with regard to this claim, depends on how one views the relationship between the geodetic network and the datum on which it is based. If one views the network as a separate, geometric object, but intricately tied to the datum, then this claim cannot be supported. If however one assumes that the datum is represented by the network, then this claim may have some validity (Thomson, 1976). Vanicek and Krakiwsky (1982) have clearly stated that they view the network as a separate geometric object from its datum, and the following quotation confirms their view:

"These (Laplace azimuth observations) help in strengthening the network, but do not, as some scholars have believed, ensure parallelism of the geodetic coordinate system to the CT system. In fact, we see that parallelism is achieved without the need of a network of points."

The view expressed by these authors is accepted and used in this report.

2.4.3.2 Floating datum. Here, the whole geodetic network, that is all the control points indiscriminately, are considered as defining the position of the geodetic datum and thus the geodetic coordinate system. As explained by Vanicek and Krakiwsky (1982):

"The inherent problem with this approach is that, even though the geodetic coordinate system seems to be positioned with respect to the physical object, ie. the network of points described by the adopted coordinate values of the points, this is not really the case. The positioning is really done through the geometrical representation of the physical object. Therefore, any errors in the initial determination of these coordinate values are transmitted into the position of the datum, and thus possible corrective measures bring about a change in the position of the coordinate system with respect to the earth. In addition there is the rather unfortunate consequence of this definition in that the position of the geodetic datum with respect to the earth fluctuates (floates) with the addition of points to the network, with local readjustment, etc. This leaves us with a floating datum for which the transformation equations to another coordinate system are epoch dependent."

In the classical technique all errors in positions, both systematic and random, are associated only with the network, and thus the geometrical representation of reality is considered distorted, the datum being totally unaffected. In the floating datum concept, these errors are distributed evenly between the coordinate system and the network.

2.4.3.3 <u>Datum positioning by a set of selected points</u>. A compromise alternative to the above two is to select a set of well-distributed control points, and declare their coordinate values to define the position of the datum. Again, as explained by Vanicek and Krakiwsky (1982):

"The meaning of this definition is that the physical object consisting of the markers of the selected points is taken as the reference to which the coordinate system is then

positioned......It is usually understood that in any subsequent computations, adjustments, or additions to the network, the coordinate values of these selected points will not be changed. This is the positioning technique used nowadays when satellite networks are merged with terrestrial networks..., it is the natural technique to use if the datum is to be positioned geocentrically."

This procedure has been followed in the new definition of the North American Datum, 1983 (NAD 83) (Vanicek and Carrera, 1985).

2.4.4 Accuracy assessment of horizontal geodetic networks

Such an assessment is not a simple operation. As mentioned before, the accuracy of the final coordinates is dependent on a number of factors. These are the accuracy of the observations and the reductions; the accuracy and completeness of the parameters and the mathematical model used, both functional and stochastic, and of the computations.

This assessment is treated under two heads.

Firstly, the effect of random errors are fully characterised by the VCV matrix of the estimated coordinates (Vanicek and Krakiwsky, 1982). This is subject to the adjustment process being complete and rigorous, of course.

Secondly, the assessment of systematic distortions needs consideration. These distortions can be due to unknown systematic errors in the observations, and/or to their improper reduction. In geometric observations the effect of atmospheric refraction (both vertical and lateral) may be partially accounted for, but due to inadequate knowledge of atmospheric conditions there may be residual effects remaining, which may be systematic. The natural quantities are similarly affected by various natural and procedural factors.

The reduction of the observations may lead to errors due firstly to the incompleteness of such reductions and secondly, to the use of parameters in these reductions of which the values are not perfectly known. These parameters may themselves be observed quantities and therefore subject to both random and systematic errors, or (even more complicated) they may be computed from some other observables.

The quantities needed at each network point in order to completely reduce geodetic observations are the orthometric height (H^0), the geoidal height (N), and the two components of the deflection of the vertical (ξ, η). Classically, of these only the orthometric height, or some approximation thereof, has been available at these network points. Furthermore, in order to compute the geoid heights and components of the deflection of the vertical at each point, one needs very detailed gravity and/or astro-geodetic data.

Semi-rigorous computation methods used in the past may also introduce considerable systematic errors in the network coordinates.

However, unknown systematic errors would still be present in a completely rigorous solution due mainly to residual errors in the observations and in the quantities used in the reduction procedures. The existence of such systematic errors cannot be detected, modelled and removed using only terrestrial data. This is where independent data must be incorporated such as satellite derived network coordinates of common network points.

When considering an accuracy estimate of coordinates in a horizontal network, it is important to ascertain what this estimate really means. In horizontal networks the accuracy estimate of the coordinates of a point is generally regarded as being relative to the NIP of the network.

2.4.5 Vertical geodetic networks

Various height systems are in use around the world. The datum normally used is nominally the geoid, or its closest practical approximation, the mean sea level (MSL) as obtained from tide gauge observations. The specific height system used determines the exact nature of the height coordinate. Normally the orthometric height system is used, which is a function of the spirit levelled heights and of measured gravity.

2.4.5.1 Establishment of vertical networks. Precise spirit levelled height differences are used, in conjunction with measured gravity, to obtain orthometric height differences (ΔH^0). These height differences are adjusted, in a suitable model to give a homogeneous set of vertical coordinates, H_i^0, and a VCV matrix, Σ_{H^0}.

In practice, however, there are several problems associated with height networks. Some of these are as follows:

The use of MSL, as obtained from tide gauge observations, as the practical approximation of the vertical datum, leads to two main problems. The first is that the theoretical zero-height surface is the geoid, an equipotential surface. MSL is not an equipotential surface, and hence is not everywhere coincident with the geoid. Secondly, the accuracy with which MSL can be established using tide gauges is poor, and hence the stability of the vertical height datum used, is also poor.

Due to the sparsity of gravity observations in some areas, normal (theoretical) gravity is used instead of measured gravity.

The 'observed' orthometric height differences are subject to errors due to various sources. Errors in the spirit levelled height differences are dependent on various instrumental and atmospheric effects. The orthometric corrections are functions of reduced gravity values, which are of course influenced both by inadequate (or rather, not so good) data and reduction procedures.

Some of these errors may be eliminated entirely, whilst others may be modelled and accounted for in a rigorous computation of the vertical network. However, some residual errors will be present which cannot so be modelled and removed, for example residual refraction errors. The effect of these residual errors is of course to distort the network. If these are random, their effect is estimated by the VCV matrix. If the effect of these errors is systematic in nature, then the orientation of the vertical network with respect to its datum is affected, ie. a 'tilt' is introduced in the vertical network. The effect of these errors is small and will only be significant and detectable on a continental scale.

2.4.5.2 <u>Accuracy assessment of vertical geodetic networks</u>. This is again treated under two heads. The random errors are propagated by the square root law. One estimate of the accuracy of a height difference is given in the form $\sigma_{\Delta H} = c.K^{1/2}$, where K (in kilometers) is the length of the section for which the estimate is given, and c is a constant that depends on the quality of the network.

The effect of systematic errors on height networks is a much more difficult proposition (Vanicek and Krakiwsky, 1982). It can be done by either modelling the distortions, or by studying the misclosures of levelling circuits and the degree of statistical dependence of levelled height differences. However, the effect of these errors on height networks cannot be completely modelled and removed by using terrestrial observations alone, independent information is needed (Vanicek and Krakiwsky, 1982).

2.4.6 Three dimensional terrestrial networks

These are networks wherein the position of each point in space is completely defined by a triplet of cartesian $(XYZ)_G$ or curvilinear $(\phi\lambda h)_G$ coordinates, and the precision estimate given by their associated VCV matrices (eg. Thomson, 1976, Vanicek and Krakiwsky, 1982).

The purpose of establishing three dimensional (3D) terrestrial networks is now explained. Since the advent of artificial earth satellites, three dimensional positioning has become a reality, and many satellite derived geodetic networks have been established, also in South Africa. Now in order to make full use of the 3D satellite network we need a set of homogeneous 3D terrestrial network coordinates for points coincident with the satellite network points.

These types of networks are established in either of two ways. The first is through the use of all observations, as they are made (ie. without any reductions), in one integrated approach. The second is through the rigorous combination of the two dimensional (2D) horizontal networks with the one dimensional (1D) vertical networks and geoidal heights, as mentioned below.

In the *first* approach, these networks are established by using all the terrestrial observations made namely horizontal directions, slope distances, zenith distances, spirit levelled height differences, and astronomically observed latitudes, longitudes, and azimuths. The mathematical models for the development of these networks are available.

The advantages of these types of networks are given briefly by Thomson (1976). The observed quantities are not reduced to the ellipsoid. Fewer astronomical observations are needed. The degrees of freedom of the solution is increased by combining horizontal and vertical adjustments. The method is more rigorous and straightforward.

However, the disadvantages are very severe: The horizontal and vertical control points are at present hardly ever coincident. Zenith distances were, if observed at all, not observed with a three dimensional adjustment in mind, and the accuracy would probably be far too poor owing to refraction over long lines. Hence it is obvious that if a three dimensional network is to be developed, the program of observation must be suitably planned.

From these points made above it is clear that three dimensional networks are subject to many of the same unknown errors as the classical networks (2D and 1D). The only ones that are eliminated in the three dimensional adjustment are those due to the reduction of the observations to their respective datums.

In the *second* approach to obtaining a rigorous three dimensional terrestrial network as (ϕ,λ,h), the horizontal two dimensional network (ϕ,λ) is combined with the one dimensional height network (H^0) and the geoidal heights (N).

The stochastic model is suitably formulated. The VCV matrix for the horizontal network ($\Sigma_{\phi\lambda}$) is combined with that for the height component (Σ_h) to give the VCV matrix for the three dimensional network expressed in ellipsoidal form as $\Sigma_{\phi\lambda h}$. The matrix for the height component, Σ_h, is obtained from those for the orthometric heights (Σ_{H^0}) and geoidal heights (Σ_N) as $\Sigma_h = \Sigma_{H^0} + \Sigma_N$. The set of ellipsoidal coordinates $(\phi,\lambda,h)_G$ and their VCV matrix $\Sigma_{\phi\lambda h}$ are transformed into cartesian $(xyz)_G$ form with corresponding Σ_{xyz} matrix. This procedure is rigorous but incomplete since the statistical covariance between the horizontal and vertical components is not present (Thomson, 1976). *The details of these transformations are treated in* APPENDICES A & B.

This approach has up to the present time been the only viable method of obtaining such three dimensional networks of national extent by using classical horizontal and vertical networks, and is the method used in this report.

2.4.7 Satellite geodetic networks

The history of the development of artificial earth satellites is described in many excellent papers on the subject. Krakiwsky, Wells and Kirkham (1972) give a number of references to these. Suffice it here to give a very brief overview. The datum that is associated with satellite networks is usually regarded as an orthogonal triad, usually geocentric and an approximation to the CT system. The exact definition of this datum is dependent on the nature of the observations made and on their analysis. This analysis can be done in either the geometric or the dynamic mode.

The main characteristics of the *geometric* analysis can be summarised as follows (Thomson, 1976): The satellite is regarded as an active or passive target only. Its position is treated as unknown and independent of all other positions at other times and is solved for using the observations made at that instant of time only. In comparison with the dynamic analysis below, the computations of the position of the satellite are independent of errors in the models for the earth gravity field, the atmospheric drag and radiation pressure, etc.

The origin of the datum is dependent on definition from external sources. The orientation of the Cartesian axes of the system, as well as the scale, are dependent on the observation techniques used. An example is that of the optical networks which have orientation but no scale, and a range network which has no orientation information.

The types of observations used in the geometric solutions are simultaneous spatial directions by photography in the optical networks and simultaneous ranges (electronic or laser) in the range networks.

The *dynamic* analysis is characterised by the following: The satellite is treated in the physical environment subject to all the forces affecting its motion, its position is regarded as known, and successive positions are functionally related through the adopted equations of motion. Dynamic methods are considered statistically stronger due to the vast increase in the number of degrees of freedom.

The datum origin is forced to be geocentric through setting the first degree gravity field coefficients equal to zero when used in the orbit computations.

The orientation of the axes is treated in two parts. The direction of the X-axis or longitude origin is defined by external information, and the Z-axis or primary pole is usually defined by a combination of satellite and terrestrial data.

Scale is primarily introduced through the value of the earth gravitational constant (GM). In electronic range, range difference and laser range systems, the value of the velocity of light adopted together with the value for GM provide the scale of the system.

A number of other methods have been developed from the above two basic ones, eg quasi-geometric, semi-dynamic, short-arc, semi short-arc, and translocation. Each of these may have some special benefits for a particular situation.

The most important aspect of Doppler positioning is that the accuracy of point positioning is independent of location. Thus Doppler positioning has inherently homogeneous accuracy, and is essentially free of systematic errors. This aspect is characteristic not only of the Doppler networks, but also of the satellite triangulation networks (eg. Thomson, 1976).

The accuracy of Doppler networks have been extensiely tested against various external standards (eg. Hothem, et al, 1982).

2.4.7.1 Examples of satellite geodetic networks. A great number of satellite derived geodetic networks of various types have been established throughout the world. Examples of these are the World Geometric Satellite Triangulation (BC-4) network, and the North American densification of this network, the Geometric solutions eg.

WN-12, WN-14, the dynamic Doppler solution NWL-9D, and many others (eg. Anderle, 1974a).

The most important however, has been the Doppler networks that have been derived from observations to the U.S. Navy Navigation Satellite System (NNSS) or TRANSIT system. This system was released to the general public in 1967 (eg. Kouba, 1983). From its humble beginnings this system has developed to such an extent that today point positions are routinely determined with metre-level accuracy throughout the world. This very useful technique has been used to establish numerous Doppler networks in various countries and continents around the world.

An example of these are the extensive Doppler networks in North America. The purpose of these networks was to support the redefinition of the North American Datum and the associated terrestrial networks, and were included in the new North American Datum, 1983 (NAD83). The datum for these Doppler networks is nominally the CT system, but depends on a number of factors, *some of which are discussed in* section 2.4.7.6.

2.4.7.2 Uses of Doppler positioning. The high accuracy, both absolute and relative, of Doppler positioning, enables the method to be used for a large number of purposes. Some of these are briefly as follows:

Geodetic datum establishment: The inherent accuracy of absolute positions established through the independent or single point positioning technique using the precise ephemeris implies a position and orientation of a geodetic datum to much higher accuracy than possible using classical methods, ie. by astronomical and gravity observations. Hence precise Doppler positioning could be used for the realization of a geodetic datum, ie. its positioning and orientation in the earth body with respect to the CT system.

Establishment and densification of geodetic control: Doppler single point positioning with the precise ephemeris has extensively been used for establishing new geodetic control in areas where such do not exist. The high relative accuracy of special techniques eg. translocation, short arc and semi short arc have made it possible to use the method for densification of geodetic networks at spacings of less than 200 kilometre (Kouba, 1983).

Doppler levelling: Doppler positioning is inherently three dimensional, so that it provides a direct determination of the geometrical height above the reference ellipsoid, ie. the ellipsoidal height. These heights can be used in two ways. *Firstly,* if orthometric heights are available for the Doppler points, the geoidal heights are simply the difference between the ellipsoidal and orthometric heights as $N = h - H^0$. Such Doppler derived geoidal heights are used as constraints in continental geoid determinations. *Secondly,* if the geoid is known sufficiently accurately, the orthometric heights of points can be determined. This method could certainly be used very succesfully in remote and inaccessible regions.

2.4.7.3 Doppler positioning. The observable in Doppler positioning is the measurement of the Doppler shift of signals emitted by the satellites of the TRANSIT system. This Doppler shift is functionally related to range differences. The mathematical models for solving for the station coordinates and other parameters are well documented in the literature (eg. Anderle, 1974b), and will not be treated here.

Doppler positioning can be done in either an *absolute* or *relative* sense.

Absolute Doppler positioning can be done in various ways. Points can be determined independently from others in the so-called single point positioning mode (SPP), or in combination with others in a network approach. If the data is processed in the SPP mode, then these positions are obtained with no correlations between network points.

However, if the network approach is used in a multi-station solution, then the solution yields a homogeneous Doppler network, with full covariance. The satellite positions used in these computations can be either of the externally generated ephemerides, namely the Broadcast ephemeris, available in the satellite message, the post processed precise ephemeris, or from a specially computed short-arc ephemeris.

Precise Doppler single point positioning is based on many satellite passes with the hope that orbital, propagation and instrumental errors will average out. If the highest accuracy is required, this may well not be the case for the Broadcast ephemeris (BE).

In order to overcome these problems, the *relative* technique of translocation was developed. In this method the data from several simultaneously observing stations is used, and in the simplest case the simultaneous point positioning solutions are subtracted without any regard for commonality of passes or Doppler counts. Strict translocation involves careful preprocessing of data to ensure that such criteria are met.

The data from simultaneously occupied stations can also logically be used in a rigorous least squares multistation solution using again either the externally generated orbits (PE or BE), or a locally computed short arc ephemeris. The use of simultaneous observations from several stations eliminates, in the difference, many of the errors due to various sources, as these errors are common to all points. Specifically, most of the orbital error and part of the propagation error can be eliminated from relative positioning for short station separations (less than about 250 kilometres).

2.4.7.4 Broadcast Ephemeris (BE). This ephemeris, which is available in real-time in the satellite message, is computed by the U.S. Navy Astronautics Group (NAG) from Doppler data observed at the four OPNET stations located in California, Minnesota, Maine and Hawaii. Broadcast ephemerides are fitted to a thirty-hour Doppler data span every twelve hours, and resulting orbits are extrapolated for up to thirty hours into the future, and finally uploaded into the satelite memory every twelve hours. The extrapolated ephemeris is uploaded in the form of a precessing and osculating Keplerian ellipse defined by the so-called fixed parameters, with corrections to this ellipse at even minute intervals, the so-called variable parameters (Kouba, 1983). Although the BE is computed using comprehensive modelling, the BE errors can still approach 100 metres mainly due to uncertainty in the orbit computations.

2.4.7.5 Precise Ephemeris (PE). The U.S. Defense Mapping Agency (DMA) regularly computes the PE for at least one NNSS satellite. This PE is computed from forty eight hour data spans observed by the world wide TRANET stations.

The PE is a post mission computed ephemeris, and hence no extrapolation is involved. The ephemeris is in the form of earth fixed Cartesian coordinates (x y z) and their time derivatives ($\dot{x}$ $\dot{y}$ $\dot{z}$) at one minute intervals. It is available only to certain approved organisations.

2.4.7.6 <u>Comparison of coordinate systems of BE and PE</u>. The coordinate systems implied by using a satellite ephemeris is a function of, amongst other factors, the coordinates of the tracking stations used, the gravity field model used, and the software and procedures employed in the data reduction process. There has been a number of changes introduced into the tracking station coordinate sets, gravity field models and software used over the years. Kumar (1982) discusses and lists some of these changes.

The BE and PE were originally intended to be the same, and although the vast majority of the physics of the two systems are extremely close if not identical, there are some differences that result in systematic discrepancies between BE and PE derived station positions. Jenkins and Leroy (1979) made a detailed study of the software and reduction procedures, and discussed a number of these factors that are known to be causing such discrepancies. They also compared the results of SPP derived using the BE and PE for over fifty stations distributed world wide. This study showed that, although the differences were fairly small, they could not be neglected for metre level accuracies. This aspect of their results has been demonstrated by a number of similar comparisons made by other researchers, although the comparisons differed vastly regarding distribution of points used and their non-global extent. The quantitative aspects of the different investigations however, differ considerably. Meade (1982) suggested that a fairly simple three parameter transformation adequately describes the transformation from the BE to the PE. The parameters are a z-shift, a z-axis rotation and a scale change. This has, at least qualitatively, been suggested by other researchers.

The BE coordinate system is that implied by the NWL-10D coordinate set used for the four OPNET tracking stations (Jenkins and Leroy, 1979). These coordinates are within several metres of the NSWC 9Z-2 coordinate set used by the U.S. DMA for the computation of the PE. However these NWL-10D coordinates were rotated in longitude to preserve the old longitude reference of the BE before they were used by the NAG for production of the present BE (Jenkins and Leroy, 1979). Hence the coordinate system for the BE is really "modified" NWL-10D (Kumar, 1982).

Another point of confusion is that, when NAG implemented the WGS-72 gravity field model in 1975, many organisations mistakenly believed that the coordinates obtained from the BE were automatically in the WGS-72 system. This was not the case (Jenkins and Leroy, 1979).

The PE was computed by U.S.DMA using the NWL 9D coordinate set and the NWL-10E gravity field model prior to 15 June 1977. After that date, the coordinate set was designated NSWC 9Z-2 and the gravity field NWL-10E-1 (eg. Kumar, 1982).

2.4.8 African DOppler Survey (ADOS)

The ADOS project, initiated in 1981, is a multi-national effort to, among other things, establish zeroth order control over the African continent. Over 270 stations have been surveyed to date (May 1986) by survey teams from various countries (Knopp, 1986).

The main objectives of the ADOS campaign are given by Mueller (1982). These are to provide zero-order control for future geodetic networks which could support mapping, control for datum unification and/or strengthening, an improved geoid for Africa, and coastal calibration points for Geos C altimetry.

Knopp (1986) made a comparative study of the ADOS point positioning results computed using various softwares. The four computing centres participating in the ADOS project use three different programs namely GEODOP V, ORB-SPP and DOPL79, which programs have very different origins. Knopp investigated the differences in results when identical data sets were processed by the different programs when installed on the same computer. The average differences in coordinates reached the metre level in some cases. The greatest variations were observed in longitude and height, with the latitude variations being much smaller.

2.5 COMBINATION OF THREE DIMENSIONAL NETWORKS.

The combined adjustment of independent data will improve both the *accuracy* and *precision* of a network (Harvey, 1985). The accuracy is improved because the addition of independent data controls the systematic errors in the network. The precision is improved because additional data is included. If systematic errors in scale and orientation exist in a geodetic network, then the adjusted coordinates will be incorrect, and also their estimated accuracies, ie. their VCV matrices, will be optimistic. Hence the combination of independent data also produces more realistic accuracy estimates (Harvey, 1985).

Two main problems arise when combining data from different networks (Harvey, 1985). The first is to find the most *accurate estimates* of the *external bias parameters* (ie. the scale, rotation, translation and systematic error terms) between the systems. The second is achieving the *best internal combination* of the different systems, ie. to minimise the corrections to the observables. Unfortunately the solution which provides the best estimates of the external bias parameters is not neccessarily the solution which provides the best internal combination and vice versa (Harvey, 1985).

2.5.1 Combination of networks: general considerations

When combining two data sets, *a number of questions have to be answered.* It must be decided whether the combination should be done in two or three dimensions. The type of transformation to be used, as well as the number and type of bias parameters must be selected. The method, if any, which is used to model the systematic errors in the networks must be selected. The question of the assignment of a priori variances and correlations or alternatively, the structure of the VCV matrices of the observations, and their influence on the results, must be addressed. These points will be considered below.

2.5.1.1 Two or three dimensional combination.

Classical networks are traditionally split into horizontal and vertical components as stated above, whilst satellite networks are inherently three dimensional. Some researchers favour the use of two dimensional combinations for some purposes. For the purposes of this report the combination of three dimensional networks only will be considered.

2.5.1.2 Type of transformation. The type of transformation or combination procedure that is used depends, amongst other factors, on the purpose for which the transformation is to be made. Harvey (1985) discusses various ways of combining networks, and finally selects a combination procedure which solves for the external bias parameters, namely a transformation adjustment. Numerous researchers have addressed this combination problem, resulting in as many solutions. Some models have some geometric significance, whilst others are solely algebraic. Some of these models will be mentioned in a later section.

Some general aspects of transformations will now be discussed before treating specific models.

2.5.1.2.1 Transformations in general. Various types of transformations are available. A projection transformation in which the scale factor is a function of position in the net is useful when there are many common network points (Harvey, 1985) We are however concerned with the combination of networks in which the number of common points are relatively few, and therefore the projection transformation is not a practical proposition.

An affine transformation, in which the scale factor is a function of the orientation of the line, may be used. Since it requires twelve parameters, we need at least four common points.

The basic conformal or similarity transformation in which the scale is the same in all directions preserves shape, so that angles are not changed, but the lengths of lines and the positions of points may be changed. An orthogonal transformation is simply a conformal transformation with unit scale.

We can use a simple conformal transformation to describe the differences between the two sets of coordinates. It must however be remembered that a transformation is essentially an interpolation procedure, resulting in the smoothing of these coordinate differences. The scale and orientation parameters that are estimated represent, in a sense, some average for the whole network. Conformal transformations guard against undue deformations in small regions, but may distort local scale and orientation when used for large networks.

It is therefore obvious that, if a conformal transformation is to be used, then each system must have uniform scale. Whilst this may be true of Doppler satellite networks, it is hardly likely to be the case in terrestrial networks of continental extent where discontinuities exist due (partially) to the adjustment methods used. Hence the important question is whether such local distortions in scale and orientation are significant. If this is so, then it would be better to solve for parameters in local and regional areas rather than for one set of parameters for the whole of the continent.

Although a conformal transformation is a linear transformation of the coordinates, it cannot be expressed linearly in terms of the seven bias parameters. However, if the rotation angles are small, as expected in geodetic networks, then the equations are approximately linear. A single iteration of the least squares estimation routine is then generally sufficient.

2.5.1.2.1.1 <u>Rotation parameters</u>. There are three ways to rotate a network. The method used here uses the Cardanian angles which are rotations around the three axes of a cartesian coordinate frame (Harvey,1985). When the rotation angles are small, the order of rotation is not important. The following quotation from Harvey (1985) is illuminating in regard to the recovery of the rotation angles:

"The rotation angles depend on the baseline vectors (ie. relative positions) and not on the absolute coordinates. Thus it does not matter where the origin of coordinates is because the estimated rotation angles will be the same, provided care is taken to avoid round-off errors."

This point will be mentioned later when comparing different models.

2.5.1.2.1.2 Scale_factor. The application of a scale factor to a three dimensional cartesian coordinate system implies that the individual coordinates are multiplied by the scale factor. This is identical to multiplying the corresponding baseline lengths by the same scale factor. As expressed by Harvey (1985):

"So the scale factor can be determined from either the 3D site coordinates or from the baseline lengths. Thus, as in the case of rotation angles, the origin of coordinates has no effect on the results."

The scale factor in a geodetic network may be due to an error in the terrestrial distance scale as well as to systematic errors in the heights used for the reduction of distances.

2.5.1.2.1.3 Distribution_of_data. To ensure a stable solution it is neccessary to have an even geographical distribution of data points. If one point is some distance from the others, it will tend to destabilise the solution. Points should not be in a straight line as some components of the rotation angles could not then be estimated.

2.5.1.3 Type and number of parameters: modelling of systematic errors. A conformal transformation in three dimensions has seven parameters. Thus one needs at least seven equations or seven coordinates common to both networks in order to solve for the parameters. Parameters representing systematic errors can also be added to the basic similarity transformation. However the removal of systematic errors in the observations can be very difficult because either the source of error or its magnitude may be unknown.

Ideally what is required is a set of rotation matrices to relate each network with its datum (where required), and with the CT system. This is not practicable as some of the parameters will be indistinguishable from others. In practice only one set of rotation parameters is usually estimated. Some models have been specially developed to represent the systematic errors in the geodetic network by another set of rotations, but some special estimation techniques are required to separate the two sets of rotations.

If the model includes a large number of parameters the adjustment may lead to a poorly conditioned system of equations. Many parameters will usually fit the data better, ie. produce smaller residuals, than a few parameters will. However, the estimate of the parameters may not be accurate, and the degrees of freedom of the solution will be reduced, which causes the statistical tests to be less effective.

When the translation and rotation components are highly correlated, non-existing rotations may absorb part of the translation corrections. This happens when the network covers only a small part of the globe. However, "if it is only desired to obtain a good internal adjustment then the high correlations between the parameters and these errors in the adjusted values of the parameters, are of no concern provided the solution is stable." (Harvey, 1985).

2.5.1.4 VCV matrices of the observables; their influence on results of transformation. In semi-dynamic satellite networks, eq. the Doppler networks, the satellite datum is in reality implied by various factors, as discussed in section 2.4.7. It is generally accepted that, due to the high inherent accuracy and homogeneity of satellite Doppler positioning, the coordinates of the network points very closely represents physical reality and thus the datum on which they are supposedly based. Therefore, the datum is recoverable to a very high degree of certainty through the use of the coordinates of the network points. It is for this reason that we do not distinguish between a Doppler network and its datum.

The homogeneous character of these networks led to the use, in this report, of very simple VCV matrices to represent the accuracy estimates of network point coordinates. Average values were asigned to the variances of the coordinates of network points. These represent the measure of confidence or reliability in the coordinate values to correctly represent pysical reality, and therefore also the satellite datum.

The coordinates of the Doppler network points in South Africa were (effectively) determined by using the precise ephemeris, and although the absolute accuracy of these points is slightly inferior to the relative accuracy between network points, this point is not laboured. These estimates of absolute accuracy are regarded, when neccessary, as slightly pessimistic estimates of the accuracy of the relative positions.

In the case of classical terrestrial networks, the network, being the geometrical representation of physical reality, is intricately connected with the datum upon which it is based, but is viewed, in contrast with satellite networks, as a distinctly separate entity from the datum.

Now, it is accepted that the geodetic coordinates of the network initial point are, by definition, exactly representative of the datum and pysical reality at that particular point. The geodetic coordinates of this point are held fixed in all adjustments, and it is assumed that the datum has been aligned (theoretically anyway) with the CT system through the procedure of classical datum establishment described earlier. Due to the influence of systematic errors in the observables and in their reduction, as well as the methods of adjustment used, the network coordinates of points are subject to error. These errors are viewed as being errors in the position of points in the geometrical network as compared with physical reality.

The estimates of these errors are separately treated for the horizontal and vertical components, but both are regarded as a function of the distance from the network initial point. They are regarded as estimates of the accuracy with which the network coordinates represent the difference between network points and the initial point. Now if the view is taken, as here, that the datum is invariant of the geodetic network, then it is submitted that these accuracy estimates can also be viewed as estimates of the reliability of these network coordinates in representing physical reality and therefore the datum on which they are supposedly based.

It is therefore clear that all network points cannot be regarded as contributing equally, in a confidence sense, to the recovery of the datum position and orientation. Wells and Vanicek (1975) recognized this fact and stated that, when investigating a geodetic coordinate system, ie. the datum, the effect of systematic distortions in the network coordinates can be minimized by dealing only with stations near the network initial point. *This is the reasoning behind the terminology used in this report, where a distinction is made between the* "datum transformation parameters" *and the* "network transformation parameters".

When an attempt is made to recover the (overall average) position and orientation of the geodetic datum through the use of the coordinates of the network points a special weighting scheme is used. This scneme implies that the closer a network point is to the network initial point (NIP), the more significant is its contribution in terms of representing its datum accurately. These terrestrial geodetic coordinates of the NIP are held fixed in these adjustments. These parameters are then the "datum transformation parameters".

Where it is simply required to obtain the overall average position and orientation of the geometric network with respect to the CT frame, each point of the network (including the NIP) is assigned equal weight. This gives rise to the so-called "network transformation parameters". It is seen therefore that the *character of the VCV matrices is very important,* and may lead to rather *different interpretations of the resulting parameters.* A further discussion of this aspect is found in CHAPTER 4.

2.5.2 Description of combination models

A brief description of some of the models available for the combination of three dimensional networks will be given below. The models which have clear geometrical interpretation will be treated first, and some others later.

2.5.2.1 "Geometric" models. The models treated here will be divided into two classes. The first are those which include only one set of rotation parameters and the second are those which solve for more than one set of rotation parameters. Three models are considered in each class. The first three will be called the Bursa, Molodensky and Veis models, the second three the Hotine, Krakiwsky-Thomson and Vanicek-Wells models.

2.5.2.1.1 Models with one set of rotations only.

2.5.2.1.1.1 Bursa model: (See Figure 2.7). The Bursa model has been used by numerous authors (eg. Thomson, 1976). Since this model is the basic 7-parameter similarity transformation, many authors have used it without referring to the Bursa name used in this report.

Three translations, three rotations and a scale change are applied to the discordant Geodetic (G) system to bring it into conformity with the satellite system. The rotations are reckoned around the G system axes at the origin of the G system, ie. at the centre of the G ellipsoid. The rotations and scale change are regarded as referring to the G system, and not the G network. The reason for this is that the total position vectors $(\vec{r}_i)_2$ of the G system are rotated and scaled.

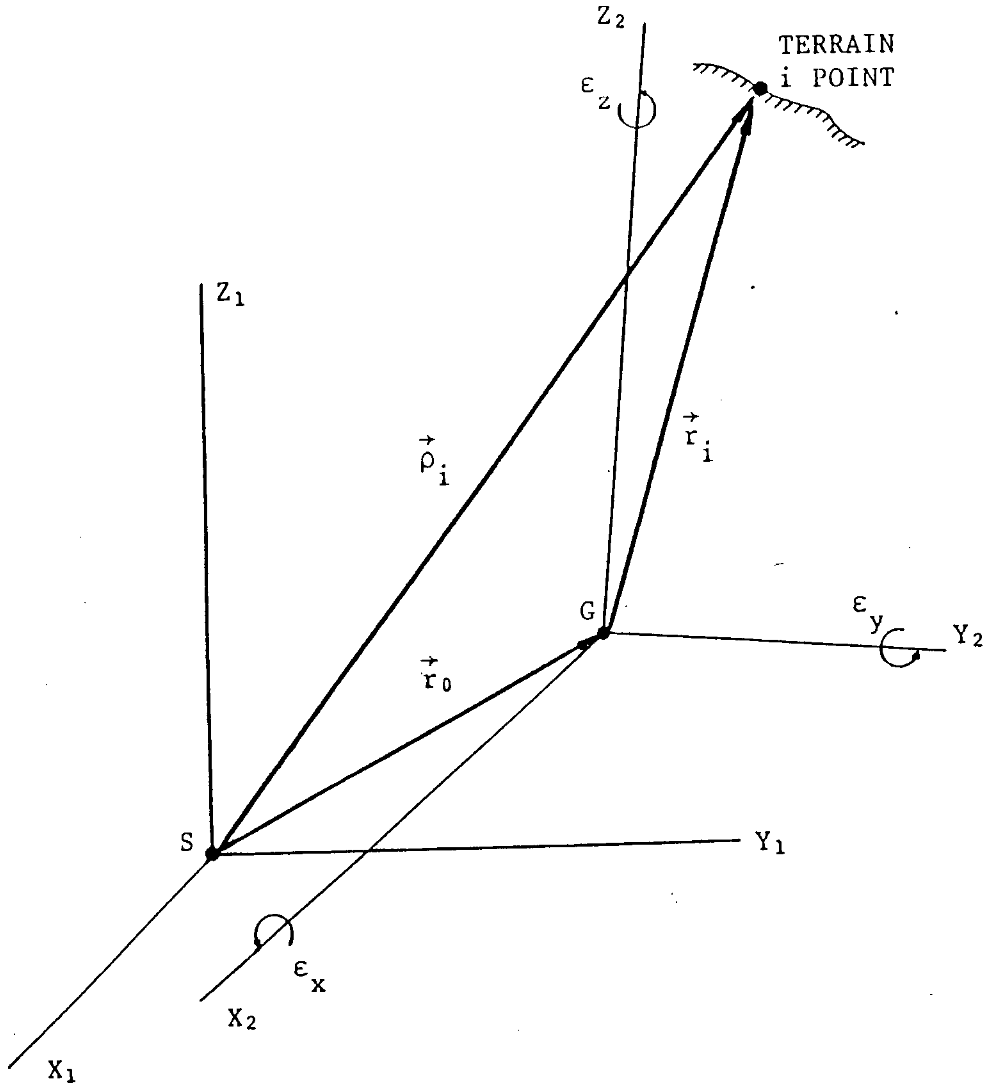


Figure 2.7 Bursa model

(after Thomson, 1976)

The model is given by (eg. Thomson, 1976)

$$\dot{F}_i = (\vec{r}_0)_1 + (1+k).R_\varepsilon.(\vec{r}_i)_2 - (\vec{\rho}_i)_1 = 0$$

where $(\vec{r}_i)_2$ and $(\vec{\rho}_i)_1$ are the position vectors of the point i in systems 2 (here the Geodetic or G system) and 1 (here the satellite or S system) respectively, and are the *observables* in this model, ie. the cartesian coordinates of the points i,

$(\vec{r}_0)_1$ is the translation vector between the origin of coordinate systems 1 and 2,

k is the scale difference from unity, representing the change of scale between systems 2 and 1.

The matrix $R_\varepsilon = R_1(\varepsilon_x).R_2(\varepsilon_y).R_3(\varepsilon_z)$ represents the rotations $(\varepsilon_x,\varepsilon_y,\varepsilon_z)$ around the second or Geodetic (G) system axes, situated at the origin (0,0,0) of the G system.

This model is treated in detail in CHAPTER 3.

2.5.2.1.1.2 Molodensky model: (See Figure 2.8). This model has been used by various authors in different forms. These will each be treated individually in CHAPTER 3. The basic form of the model will be discussed briefly here.

This model also uses three translations, three rotations and a scale change to model the coordinate differences between the two systems. A fundamental point (FP), m, is introduced which is some point in the second network, eg. the centre of gravity of the second network, or the NIP (k).

The rotations applied to the second network are reckoned around axes, nominally parallel to the G system axes, situated at the FP, m. The position vector, $(\vec{r}_m)_2$, is

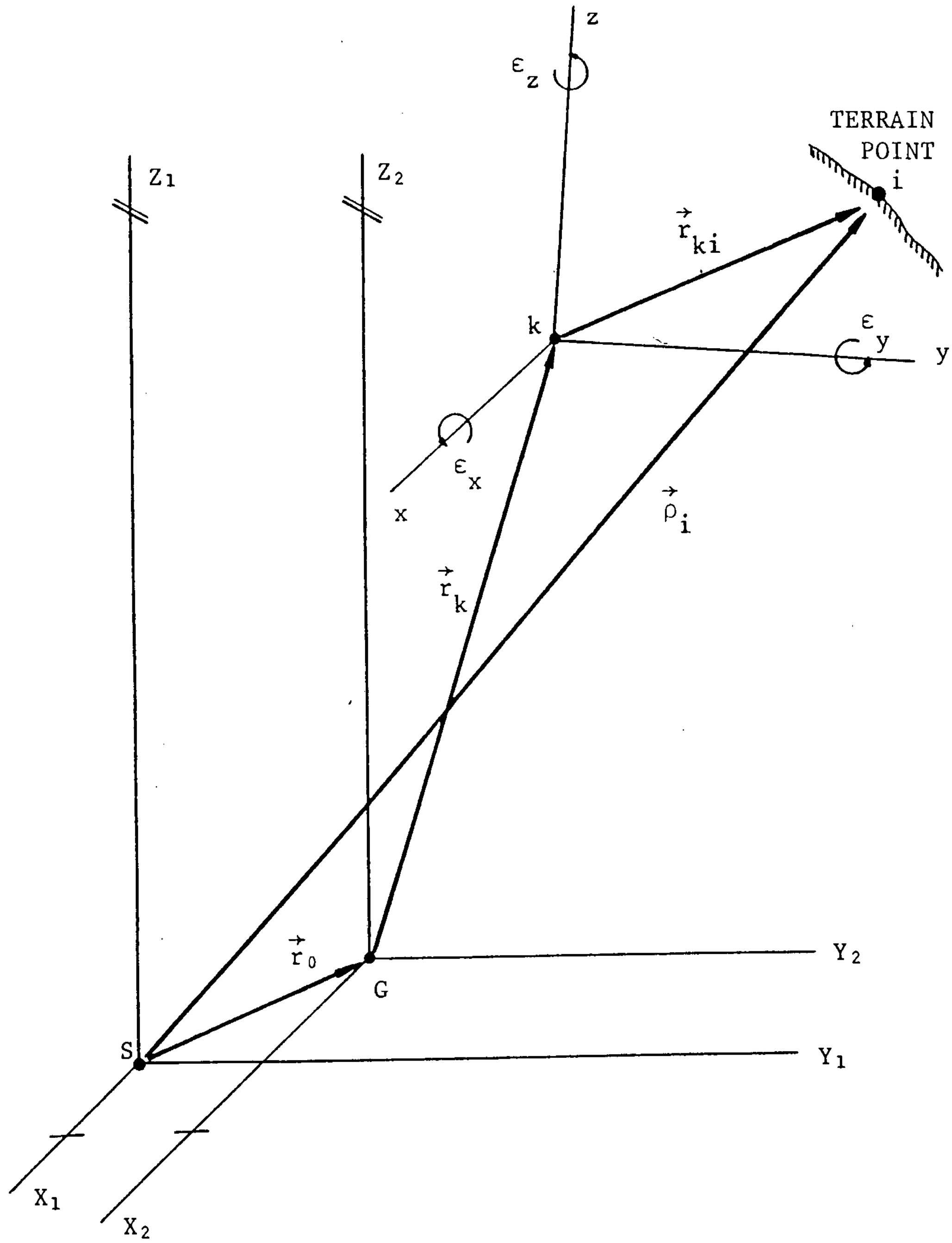


Figure 2.8 Molodensky model
(after Thomson, 1976)

introduced, and since this vector appears singly in the equation, there is the implicit assumption that the G and CT system axes are parallel. The rotations and scale change are applied to the difference vectors of the second network, ($(\vec{r}_{mi})_2$, the vector from the FP point m to the point i) and not the total position vectors, $(\vec{r}_i)_2$. Therefore the rotations and scale change are viewed as applicable to the G network, and not the system.

The model is given as (eg. Harvey, 1986).

$$\vec{F}_i = (\vec{r}_0)_1 + (\vec{r}_m)_2 + (1+k).R_\psi.(\vec{r}_{mi})_2 - (\vec{\rho}_i)_1 = 0$$

This model is treated in detail in CHAPTER 3.

2.5.2.1.1.3 <u>Veis model</u>: (See Figure 2.9). The Veis model is mathematically equivalent to the Molodensky model. It uses the initial point, k, of the second network as the FP, and the rotations and scale change are applied to the difference vectors of the second network, $(\vec{r}_{ki})_2$. The only difference is that the rotations are reckoned around the axes of the Local Geodetic (LG) system at the FP. These are dν and dμ, tilts in the prime vertical and meridian planes respectively, and dA, a rotation in azimuth.

Note that the (singular) use of the position vector $(\vec{r}_k)_2$ implies the same basic assumption of the parallelity of the G and CT system axes.

The model is given by (Thomson, 1976)

$$\vec{F}_i = (\vec{r}_0)_1 + (\vec{r}_k)_2 + (1+k).R_V.(\vec{r}_{ki})_2 - (\vec{\rho}_i)_1 = 0$$

where $(\vec{r}_0)_1$, $(\vec{r}_k)_2$, $(\vec{r}_{ki})_2$, $(\vec{\rho}_i)_1$, & k have similar

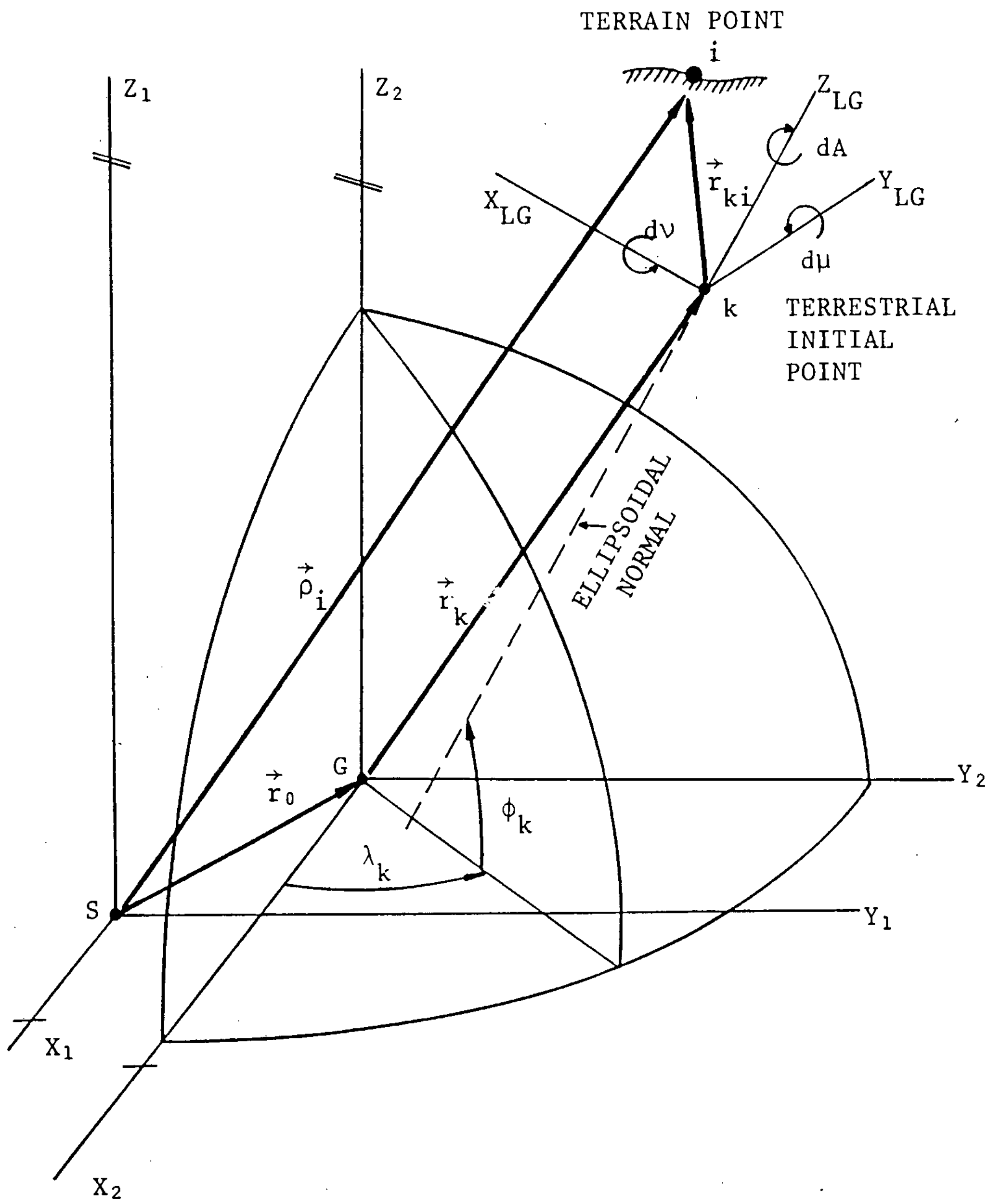


Figure 2.9 Veis model
(after Thomson, 1976)

meanings to those in the Molodensky model, and where

$$R_V = R_3(180-\lambda_k).R_2(90-\phi_k).P_2.R_1(d\nu).R_2(d\mu).R_3(dA).$$
$$.P_2.R_2(\phi_k-90).R_3(\lambda_k-180)$$

where (ϕ_k,λ_k) are the geodetic coordinates of the initial point k of the second network.

P_2 is a reflection matrix about the y-axis.

This model is treated in detail in CHAPTER 3.

2.5.2.1.2 Models with *more than one* set of rotations

2.5.2.1.2.1 Hotine model: (See Figure 2.10). Hotine (1969) mentions that, in addition to the coordinate system (datum) rotations, there may be systematic errors of scale and orientation present in the network itself. He proposes a model in which these systematic errors in the network may be modelled by two rotation parameters and a scale difference parameter. The rotations are a change in azimuth, $d\alpha$, and a change in zenith distance, $d\beta$. The first is a rotation around the z-axis of the local geodetic (LG) system at the network initial point (NIP), whilst the second is a constant applied to all lines radiating from the NIP.

This model is given as

$$\vec{F}_i = (\vec{r}_0)_1 + R_S.[(\vec{r}_k)_2+(1+k)R_3(180-\lambda_k).R_2(90-\phi_k).P_2.R_H.$$
$$.P_2.R_2(\phi_k-90).R_3(\lambda_k-180).(\vec{r}_{ki})_2]-(\vec{\rho}_i)_1=0$$

where the symbols have similar meanings as in the models

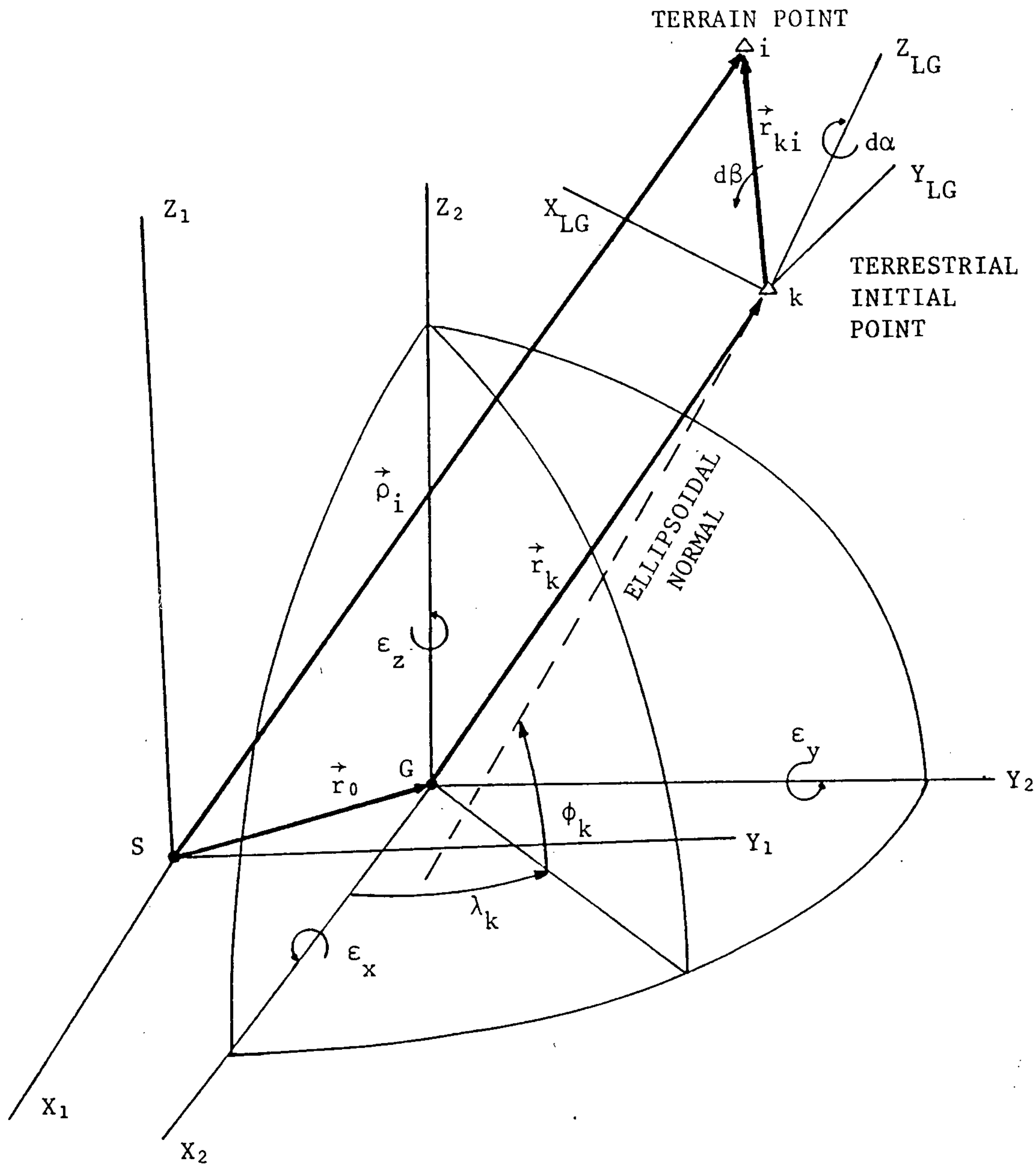


Figure 2.10 Hotine model
(after Thomson, 1976)

given above. The rotations $(\varepsilon_x, \varepsilon_y, \varepsilon_z)$ contained in the matrix R_ε refer to the discordant system (datum). The rotation angles, $d\alpha, d\beta$, contained in the matrix R_H given below, and the scale difference, k, refer to the second network. Hence there are eight parameters in this model.

The difference vector $(\vec{r}_{ki})_2$ is the position vector of the point i with respect to the NIP, k, and is given by

$$(\vec{r}_{ki})_2 = \begin{bmatrix} x_{ki}, y_{ki}, z_{ki} \end{bmatrix}^T = \begin{bmatrix} |r_{ki}| \sin.\beta_{ki} . \cos.\alpha_{ki} \\ |r_{ki}| \sin.\beta_{ki} . \sin.\alpha_{ki} \\ |r_{ki}| \cos.\beta_{ki} \end{bmatrix}$$

as expressed in the LG system at the NIP, k.

Thomson (1976) gives the matrix R_H as

$$R_H = \begin{bmatrix} 1+k & -d\alpha & d\beta . \cos.\alpha_{ki} \\ d\alpha & 1+k & d\beta . \sin.\alpha_{ki} \\ -d\beta/\cos.\alpha_{ki} & 0 & 1+k \end{bmatrix}$$

The expanded model is given by Thomson (1976).

There are only two networks, one terrestrial and one satellite, involved in this model. Hence this model requires a special estimation technique in order to separate the two sets of rotations. Hotine did not propose a solution, but Thomson (1976) stated that the estimation method used in the Krakiwsky-Thomson model could be used here. A disadvantage (Thomson, 1976) of this model is that the rotations $d\alpha$ and $d\beta$ cannot be split to give either the Molodensky or the Veis type rotations.

Note that this model can easily be modified to accommodate more than one satellite network (Thomson, 1976).

2.5.2.1.2.2 Krakiwsky-Thomson model. This model is used to combine one satellite and one terrestrial network. There are two versions of this model in the literature. These differ in the application of the scale difference, k, and in the second of the two rotation matrices used. In the first version, this scale difference is a system scale, and in the second version a network scale.

The model contains two sets of rotations. The first set, $(\varepsilon_x, \varepsilon_y, \varepsilon_z)$, contained in matrix R_ε, is for the misaligned geodetic system, and the second $[(\psi_x, \psi_y, \psi_z)$ in R_ψ or $(d\nu, d\mu, dA)$ in R_V], is for the misoriented network. The first version of this model used the matrix R_ψ, and the second version the matrix R_V. These two sets of rotations $[(\psi_x, \psi_y, \psi_z)$ and $(d\nu, d\mu, dA)]$ have identical meanings to those in the Molodensky and Veis models respectively.

The first version is given by (Krakiwsky and Thomson, 1974)

$$\vec{F}_i = (\vec{r}_0)_1 + (1+k).R_\varepsilon.[(\vec{r}_k)_2 + .R_\psi.(\vec{r}_{ki})_2] - (\vec{\rho}_i)_1 = 0$$

and the second (Figure 2.11) by (Thomson and Krakiwsky, 1975, Thomson, 1976)

$$\vec{F}_i = (\vec{r}_0)_1 + R_\varepsilon.[(\vec{r}_k)_2 + (1+k).R_V.(\vec{r}_{ki})_2] - (\vec{\rho}_i)_1 = 0$$

The model has ten unknown parameters, six of which are rotations. Hence four common points are minimally needed, but it is obvious that many more points will be needed to ensure realistic estimates of the parameters.

This model again needs a special estimation technique to separate the two sets of rotations. The data points are split into two groups (Thomson, 1976) . The first group, the so-called "innner zone", contains those points that are

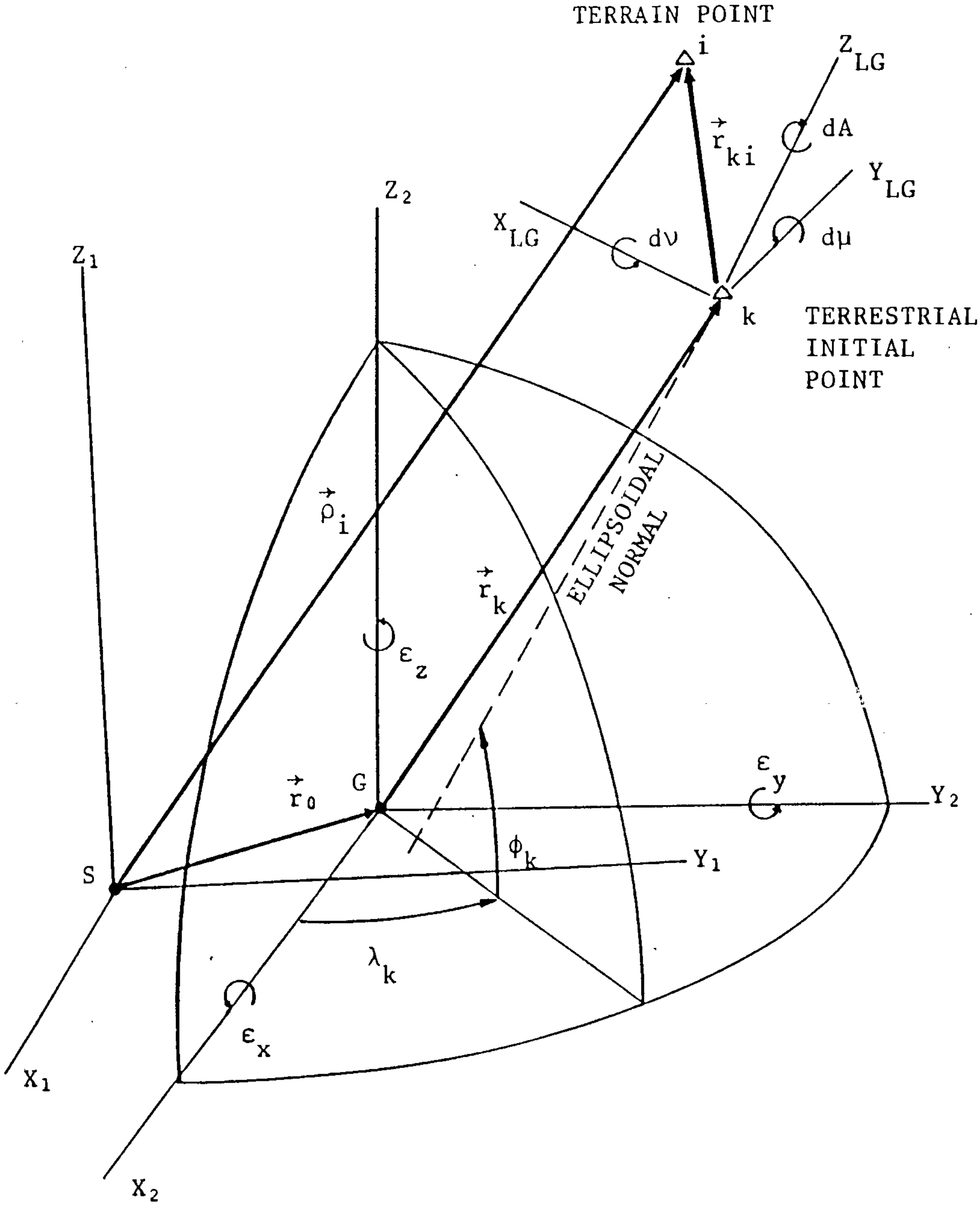


Figure 2.11 Krakiwsky-Thomson model
(after Thomson, 1976)

so close to the NIP that they are essentially unaffected by any systematic errors in the terrestrial network. The second group, the so-called "outer zone", contains the remaining points. The estimation method is essentially a combination of models as follows:

$F_1(\overline{X}_1, \overline{L}_1) = 0$ and $F_2(\overline{X}_1, \overline{X}_2, \overline{L}_2) = 0$

where $\overline{X}_1$ contains the system parameters, $(\vec{r}_0)_1 = [x_0, y_0, z_0]^T$ and $(\varepsilon_x, \varepsilon_y, \varepsilon_z)$,

$\overline{X}_2$ contains the rotation and scale difference parameters pertaining to the second network, $[k, \psi_x, \psi_y, \psi_z]$,

$\overline{L}_1$ contains the observables of the "innner zone", ie the coordinates and coordinate differences,

$\overline{L}_2$ contains the observables of the "outer zone".

The "inner zone" must contain sufficient observables to solve for the six unknown system parameters contained in $\overline{X}_1$.

The details of this model are given by Thomson (1976). Note that this model as formulated here is used for the combination of one terrestrial and one satellite (CT) network, but it may easily be modified to accommodate more than one satellite network (Thomson, 1976).

2.5.2.1.2.3 <u>Vanicek-Wells model</u>: (See Figure 2.12). The objective of this model is *"to examine numerically the parallelism of geodetic systems (based on terrestrial observations) and satellite systems (based on satellite observations) to the average terrestrial system"* (Wells and Vanicek, 1975). The average terrestrial system is the earlier term used for what is known as the conventional terrestrial (CT) system today. One satellite and two or more geodetic systems, each having at least two points common with the satellite system, are combined in one parametric least squares solution.

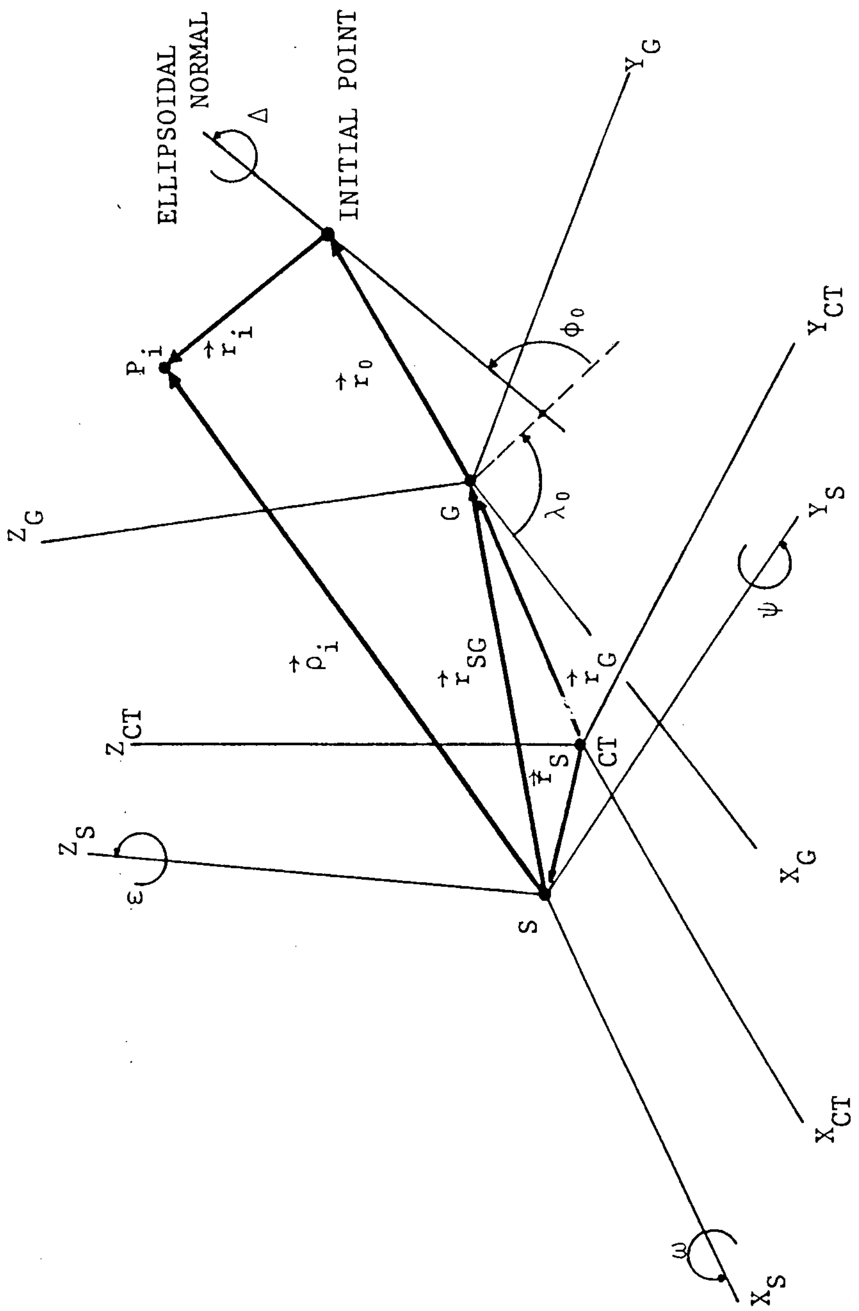


Figure 2.12 Vanicek-Wells model
(after Wells & Vanicek, 1975)

The authors proceeded under the assumption that "*a geodetic system is a fixed framework invariant with respect to geodetic network adjustment*".

Vanicek and Wells (1974) have shown that, under certain conditions of datum establishment, only four datum position and orientation parameters exist, namely three translations and one azimuth rotation. This condition exists when the datum has been established by the classical method, ie. when the position and orientation of the datum is fixed by definition at the NIP. The deflection conditions $\xi_k=\Phi_k-\phi_k$ and $\eta_k=(\Lambda_k-\lambda_k)\cos\phi_k$ are satisfied at the NIP by definition. These two equations are equivalent to the zenith distance condition, which is thus fulfilled by implication (Vanicek and Wells, 1974) The only orientation condition remaining is the azimuth condition, and the rotation matrix R_Δ, pertaining to the geodetic system (datum), contains only this azimuth orientation unknown, Δ.

The authors use the following reasoning: The transformation from a geodetic to a satellite system involves relating each system to the CT system. The satellite system is related to the CT system by three translations and three rotations $(\omega,\phi,\varepsilon)$, whilst the geodetic system requires only three translations and one rotation (Δ). The reason for this single rotation is given above, namely that we know the direction around which the rotation takes place ie. the ellipsoidal normal at the NIP.

This model (Wells and Vanicek, 1975) is given as follows, using their notation, and expressing all position vectors in the CT system:

$$\vec{r}_S + R_{\omega\psi\varepsilon}.\vec{\rho}_i = \vec{r}_G + R_\Delta.L.(\vec{r}_0 + \vec{r}_i)$$

where $\vec{r}_S$, $\vec{r}_G$ are the position vectors of the origins (0,0,0) of the geodetic and satellite systems respectively with respect to the CT system, $\vec{r}_0$ is the position vector of the (G) NIP, $\vec{r}_i$ is the difference vector from the NIP to the point i, $\vec{\rho}_i$ is the position vector of the point i in the satellite system, L is the scale ratio between the geodetic and satellite systems, $(\omega,\psi,\varepsilon)$ and (Δ) respectively are the rotations required to align the satellite and geodetic systems with the CT system.

The unit vector in the direction of the ellipsoidal normal at the NIP (ϕ_0,λ_0) around which the rotation (Δ) takes place, can be expressed as $[\cos\phi_0.\cos\lambda_0,\cos\phi_0.\sin\lambda_0,\sin\phi_0]^T$.

Hence the rotation Δ can be split into its x,y,z components as $[\omega_g,\psi_g,\varepsilon_g] = \Delta\ [\cos\phi_0.\cos\lambda_0,\cos\phi_0.\sin\lambda_0,\sin\phi_0]$

So the matrix R_Δ is then given by

$$R_\Delta = \begin{bmatrix} 1 & \Delta.\sin\phi_0 & -\Delta.\cos\phi_0.\sin\lambda_0 \\ -\Delta.\sin\phi_0 & 1 & \Delta.\cos\phi_0.\cos\lambda_0 \\ \Delta.\cos\phi_0.\sin\lambda_0 & -\Delta.\cos\phi_0.\cos\lambda_0 & 1 \end{bmatrix}$$

Now, there is no way of distinguishing between the vectors $\vec{r}_S$ and $\vec{r}_G$, so that we can solve only for their difference $\vec{r}_{SG}=\vec{r}_G- \vec{r}_S$ (Wells and Vanicek, 1975). The authors substitute $(1+\Delta L)$ for L, and the matrices $(I+Q)$ for R. Further, since the difference between $(\vec{r}_0+\vec{r}_i)$ and $\vec{\rho}_i$ is only of the order of a few hundred metres, the authors substitute $\vec{\rho}_i$ for $(\vec{r}_0+\vec{r}_i)$ wherever the latter is multiplied by the small quantities Q_Δ and ΔL, as this introduces an insignificant error.

The observation equation containing the eight unknown parameters, $[\omega,\psi,\varepsilon,\Delta,\Delta L,\vec{r}_{SG}]$, is finally given by

$$[Q_{\omega\psi\varepsilon} - Q_{\Delta} - \Delta L.I].\vec{\rho}_i - \vec{r}_{SG} \simeq \vec{r}_0 + \vec{r}_i - \vec{\rho}_i = \vec{a}_i$$

The first three rotations align the satellite system with the CT, the fourth aligns the geodetic system with the CT. The last four parameters represent the scale difference and translation components between the geodetic and satellite systems.

In order therefore to solve for the eight unknowns, we require at least three points of the geodetic system common with the satellite system. However, since the two sets of rotations are additive, they are indistinguishable unless more than one geodetic system is involved. Each additional geodetic system increases the number of unknowns by five, so that at least two stations are required on each additional geodetic system.

Wells and Vanicek (1975) computed a number of solutions using this model. Their conclusion was that, in order to detect these alignments or rotations with respect to the CT system, care must be taken both in the selection of the data points used, and in the datums (both satellite and geodetic) used. Since they are attempting to recover the geodetic datum alignment (not the network), they use only data points near the NIP in order to minimise the effect of systematic errors in the geodetic network point coordinates. The selection of the satellite system, the geodetic systems and the station configuration may all in general change the results obtained, hence their note of caution.

2.5.2.2 "Other" models. A method whereby the systematic errors in the terrestrial network may be modelled more completely, is to use the method of least squares approximation. Here the systematic errors are represented by a three dimensional polynomial of n^{th} order. As expressed by Thomson and Krakiwsky (1976):

"The coefficients of the polynomial are determined using least squares approximation in which the quantity to be minimised is the sum of the squares of the weighted discrepancies between a mathematically defined vector field and the vector field represented by the coordinate differences."

2.5.2.2.1 Total difference polynomial modelling. In this method the total coordinate difference $[(\vec{r}_i)_2-(\vec{\rho}_i)_1]$ is modelled by an algebraic polynomial (Thomson and Krakiwsky, 1976):

$$\vec{P}_n(\vec{r}) = \sum_{i,j,k=0}^{n} \vec{C}_{ijk}\, x^i y^j z^k = (\vec{r}_i)_2 - (\vec{\rho}_i)_1$$

and in expanded form

$$\vec{P}_n(\vec{r}) = \begin{bmatrix} P_n^x(\vec{r}) \\ P_n^y(\vec{r}) \\ P_n^z(\vec{r}) \end{bmatrix} = \begin{bmatrix} \sum_{i,j,k=0}^{n} C^x_{ijk}\, x^i y^j z^k \\ \sum_{i,j,k=0}^{n} C^y_{ijk}\, x^i y^j z^k \\ \sum_{i,j,k=0}^{n} C^z_{ijk}\, x^i y^j z^k \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_2 - \begin{bmatrix} x \\ y \\ z \end{bmatrix}_1$$

in which $\vec{C}_{ijk}$ (or $C^x_{ijk}, C^y_{ijk}, C^z_{ijk}$) are the unknown polynomial coefficients to be solved for. Once the mathematical vector field has been defined, it may be used to compute the expected (interpolated) vector difference $[(\vec{r}_i)_2 - (\vec{\rho}_i)_1]$ for any arbitrary point of the network.

In this model all the components of the transformation, ie. the translations and rotations of the geodetic system and systematic errors (rotations and scale difference) in the geodetic network, are lumped together. The result is that realistic geometrical interpretations are probably not possible.

2.5.2.2.2. Combination of Hotine, Krakiwsky-Thomson and polynomial modelling. Thomson and Krakiwsky (1976) give another model in which the concepts of Hotine, the Krakiwsky-Thomson model and the least squares approximation methodology are combined:

$$\vec{F}_i = (\vec{r}_0)_1 + R_\varepsilon.[(\vec{r}_k)_2 + (\vec{r}_{ki})_2 + \vec{P}_n(\vec{r}_{ki})_2] - (\vec{\rho}_i)_1 = 0$$

The translation and rotation parameters required to relate the discordant geodetic system to the satellite system appear explicitly as $(\vec{r}_0)_1$ and R_ε. The systematic errors in the geodetic network are modelled by the vector field represented by $\vec{P}_n(\vec{r}_{ki})_2$. This model also requires a special estimation routine, similar to that used for the Krakiwsky-Thomson model, in order to obtain a solution. This is given by

$F_1(\bar{X}_i, \bar{L}_1) = 0$ and $F_2(\bar{X}_1, \bar{C}, \bar{L}_2) = 0$

in which $\bar{C}$ are the coefficients of the three dimensional polynomial.

2.5.2.2.3 Geodetic datum transformation by multiple regression equations. Appelbaum (1982) used a least squares stepwise multiple regression procedure to derive polynomial equations for converting coordinates from one geodetic datum to another as a function of (normalized) latitude and longitude. The regression procedure uses reference coordinate differences throughout the datum when deriving the equations. Thus the resulting transformation equations exhibit sensitivity to regional variations in the coordinate differences ie. to the systematic distortions in the geodetic network.

The regression procedure ensures that an equation of a given precision contains a minimum number of terms. This is achieved by a procedure in which variables are sequentially incorporated and evaluated, and all previously incorporated variables are examined and removed if no longer significant. Hence the resulting transformation equations are relatively computer efficient (Appelbaum, 1982).

This method was used for the transformation of European Datum 1950 (ED50) coordinates to the World Geodetic System 1972 (WGS72), and vice versa. An example of the equations used are given on page 211 (Appelbaum, 1982), and take the form:

$$\Delta\phi'' = f_1(u,v) \qquad \Delta\lambda'' = f_2(u,v) \qquad \Delta H(m) = f_3(u,v)$$
$$\Delta x(m) = f_4(u,v) \qquad \Delta y(m) = f_5(u,v) \qquad \Delta z(m) = f_6(u,v)$$

where u,v are normalized latitudes and longitudes given here by the equations $u=3(\phi-0.87)$ and $v=3(\lambda-0.08)$ where 0.87 and 0.08 are approximate *mean* latitude and longitude values (in radians) respectively for the area concerned, and 3 is a convenient factor to inhibit large values of equation coefficients. The latitude and longitude values used in the equations can be those on either datum.

2.5.3 Summary of combination models.

Tables 2.1 & 2.2 below summarise the main features and uses of the different models mentioned in this report. These tables are taken from Thomson (1976) and expanded.

Table 2.1 General characteristics of the models (after Thomson, 1976, modified)

3 - D MODEL	DATUM TRANSFORMATION PARAMETERS (TER ↔ SAT)			PARAMETERIZATION OF SYSTEMATIC ERRORS IN THE TERRESTRIAL NETWORK		ORIENTATION OF DATUMS W.R.T. CT SYSTEM		ESTIMATION PROCEDURE
	TRANS-LATIONS	ROTA-TIONS	SCALE DIFFERENCE	ROTATIONS	SCALE DIFFERENCE	SATELLITE	TERRESTRIAL	
BURSA	3	3	1	-	-	-	-	COMBINED CASE LEAST SQUARES
MOLODENSKY	3	-	-	3	1	-	-	COMBINED CASE LEAST SQUARES
VEIS	3	-	-	3	1	-	-	COMBINED CASE LEAST SQUARES
HOTINE	3	3	-	1 azimuth rotation 1 zenith dist	1	-	-	STEPWISE LEAST SQUARES
KRAKIWSKY-THOMSON	3	3	-	3	1	-	-	STEPWISE LEAST SQUARES
VANICEK-WELLS	3	-	1	-	-	3	1	PARAMETRIC CASE LEAST SQUARES
POLYNOMIAL MODELLING	-	-	-	-	-	-	-	PARAMETRIC CASE LEAST SQ.
HOT &K-T & POL. MOD.	3	3	-	-	-	-	-	STEPWISE LEAST SQUARES
REGRESSION EQUATIONS	-	-	-	-	-	-	-	L.S. STEPWISE MULTIPLE REG.

USES OF MODEL	MODELS LISTED IN THIS REPORT								
COMBINATION OF: SAT ≡ satellite TER ≡ terrestrial	BUR	MOL	VEI	HOT	K-T	V-W	POL	COM	REG
2 SAT	x						x		x
1 TER & (1) 1+ SAT				x	x		(x)	(x)	(x)
2+ TER datums & 1 SAT datum						x			
1 TER & 1 SAT : systematic errors of TER modelled				x	x				
1 TER & 1 SAT : G and SAT system axes parallel. Scale /rotations model TER systematic errors.		x	x						

<u>Table 2.2</u> USES of the models

LEGEND:

BUR ≡ BURSA model

MOL ≡ MOLODENSKY model

VEI ≡ VEIS model

HOT ≡ HOTINE model

K-T ≡ KRAKIWSKY-THOMSON model

V-W ≡ VANICEK-WELLS model

POL ≡ Total difference polynomial modelling

COM ≡ Combination of HOT, K-T and polynomial modelling

REG ≡ regression equations

2+ ≡ 2 or **more** systems

(1) 1+ ≡ if marked (x), then only 1; if marked x, then 1 or **more**.

CHAPTER 3

THEORETICAL MODELS

Three of the models mentioned in the previous chapter have been selected for detailed study in this report. These are the Bursa, Molodensky and Veis models, which contain one set of rotation parameters only. The reasons for this choice are given below.

The initial aim of this study was to evaluate the transformation parameters (as real physical quantities) between the geodetic and satellite or Conventional Terrestrial (CT) datums through the use of common coordinated points. Since the geodetic datums and networks are geometric objects, it was decided to use a model which has geometric interpretation, from which could be inferred the relative position of the Geodetic and satellite or CT datums. It is for this reason that some of the "geometric" models were selected for further study and not the "non-geometric" models.

However, it became clear during the course of the study that these estimated transformation parameters are to be interpreted with a great deal of caution, and that it would be presumptious to "blindly" attach geometrical or real physical meaning to them.

The decision to use the models which have one set of rotations in preference to those which have more than one set, was based largely on the situation peculiar to South Africa. This is a small country and covers a very small portion of the globe. It is therefore very difficult to recover with any degree of accuracy even one set of rotation parameters by using our geodetic networks of national extent. The inclusion of a second set of rotation parameters to represent the overall systematic errors present in the geodetic network is therefore not regarded as a practical proposition in this case.

Further, the distortions in the horizontal geodetic network, which became apparent on comparison with the Doppler satellite network, are indeed considerable. There is no doubt that these distortions are caused by a number of factors, and cannot be attributed merely to systematic errors in the network. The particular method of adjusting the network in sections eg. gave rise to some rather distinct "blocks" of triangulation between which eg. the scale varies considerably. The regional variation of the transformation parameters between the geodetic network and the Doppler (CT) network is treated in section 5.6.2.

Therefore, in view of the considerable variation of the transformation parameters between these "blocks" of triangulation, it simply does not make much sense to attempt the recovery of an additional set of rotation parameters that represent the systematic errors in the network as a whole. The assumptions on which the use of the more complex models, which contain more than one set of rotations, are based, is that the geodetic networks have been computed homogeneously and rigorously. This is not the case for the South African networks.

It is for these reasons that the Bursa, Molodensky and Veis models were selected for a comparative study.

3.1 BURSA MODEL

This model is given as (eq. Thomson, 1976) (See Figure 2.7)

$$\vec{F}_i = (\vec{r}_0)^1_{x} + (1+k).R_\varepsilon.(\vec{r}_i)_2 - (\vec{\rho}_i)_1 = 0$$

where $(\vec{r}_i)_2$ and $(\vec{\rho}_i)_1$ are the position vectors of the point i in systems 2 and 1 respectively, and are the *observables* in this model, ie. the cartesian coordinates of the points i,

$(\vec{r}_{0,})_1$ is the translation vector between the origin of coordinate systems 1 and 2,

k is the scale difference from unity, representing the change of scale between system 2 and 1.

R_ε is the *rotation matrix* given by the matrix product below, which reduces to the matrix given for small angles of rotation. The rotations $\varepsilon_x, \varepsilon_y, \varepsilon_z$ are those required to align the axes of system 2 with those of system 1. The *point* about which the *rotations* are made is the *origin* of the *second system*.

$$R_\varepsilon = R_1(\varepsilon_x).R_2(\varepsilon_y).R_3(\varepsilon_z) = \begin{bmatrix} 1 & \varepsilon_z & -\varepsilon_y \\ -\varepsilon_z & 1 & \varepsilon_x \\ \varepsilon_y & -\varepsilon_x & 1 \end{bmatrix}$$

Harvey (1986) states that this approximation is good for rotation angles of up to about 3 seconds of arc, when rotating vectors of earth radius length, but when the vectors that are rotated are much shorter, considerably larger angles of rotation can be accommodated.

NOTE: Due to the way that the model is formulated, the *rotations* and *scale change* are regarded as referring to the *second system*. There are *no a priori assumptions* regarding parallelity of system axes in this model.

The expanded form of the equation is:

$$\begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}_1 + (1+k) \begin{bmatrix} 1 & \varepsilon_z & -\varepsilon_y \\ -\varepsilon_z & 1 & \varepsilon_x \\ \varepsilon_y & -\varepsilon_x & 1 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}_2 - \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}_1 = 0$$

The problem of high correlation amongst the estimated parameters has been mentioned by numerous authors, eg. Harvey (1985). The reason is that the geodetic network is generally only of regional and not global extent, hence the data covers only a very small portion of the earth's surface.

Thomson (1976) mentions that this model can be used for the combination of two networks, provided the coordinates of each network can be used to recover the datum to which each network refers. Since this is only true for satellite networks, he concludes that this Bursa model is not appropriate for the combination of one satellite and one terrestrial network. This point is also made by saying that the coordinates of the satellite network are representative of its datum, whilst those of the terrestrial network are not. This is also reflected in the character of the VCV matrix (used in this report) of each type of network, as seen in sections 2.5.1.4 and 4.2.3.

The model for the *inverse transformation* is derived simply as

$$\vec{r}_i = (1+k)^{-1} . R_\varepsilon^{-1} . [(\vec{\rho}_i)_1 - (\vec{r}_0)_1] - (\vec{r}_i)_2 = 0$$

$$= (1+k)^{-1} . R_\varepsilon^{T} . [(\vec{\rho}_i)_1 - (\vec{r}_0)_1] - (\vec{r}_i)_2 = 0$$

since the rotation matrix R_ε is orthogonal.

3.1.1 Estimation method

The theory of least squares estimation is treated in detail in various works, eg. Mikhail and Gracie (1981). Krakiwsky and Gagnon (1983) give a concise summary of the equations used for the different cases of the least squares solution.

The solution to this equation is obtained by the combined case least squares estimation procedure, expressed as :

$$F(\overline{X},\overline{L}) = \emptyset \qquad \ldots 3.1$$

where $\overline{X}$ represents the unknown parameters and $\overline{L}$ the observables.

The model is non-linear and linearization produces the expression

$$\overline{A}.\hat{X} + \overline{B}.\hat{V} + W^0 = \emptyset \qquad \ldots 3.2$$

where $\overline{A},\overline{B}$ are the design matrices

$$\overline{A} = \left.\frac{\partial F}{\partial X}\right|_{(X^0,L)} \qquad \overline{B} = \left.\frac{\partial F}{\partial L}\right|_{(X^0,L)}$$

$\hat{X}$ is the least squares estimate of the unknown parameters

$\hat{V}$ is the least squares estimate of the residuals of the observables

$W^0 = F(X^0,L)$ is the misclosure vector

X^0 is an initial estimate of the unknown parameters

L are the observed values of the observables.

The least squares solution to equation 3.2 is given by

$$\hat{X} = -[A^T(B.\Sigma_L.B^T)^{-1}.A]^{-1}.A^T(B.\Sigma_L.B^T)^{-1}.W^0$$

$\hat{V} = \Sigma_L.B^T.K$ where the correlate vector, K, is

$$K = -(B.\Sigma_L.B^T)^{-1}.(A.\hat{X}+W^0)$$

The least squares estimate of the unknown parameters are $\bar{X} = X^0 + \hat{X}$ and the adjusted observables $\bar{L} = L + \hat{V}$.

Further, $Q_{\bar{X}} = [A^T(B.\Sigma_L.B^T)^{-1}.A]^{-1}$ and

$$Q_{\bar{L}} = \Sigma_L - \Sigma_L.B^T.[(B.\Sigma_L.B^T)^{-1} - (B.\Sigma_L.B^T)^{-1}.A.Q_{\bar{X}}.A^T(B.\Sigma_L.B^T)^{-1}]B.\Sigma_L$$

where $Q_{\bar{X}}$ and $Q_{\bar{L}}$ are the weight coefficient matrices of the *estimated parameters* and the *adjusted observables* respectively. The associated VCV matrices are given by $\Sigma_{\bar{X}} = \bar{\sigma}_0^2.Q_{\bar{X}}$ and $\Sigma_{\bar{L}} = \bar{\sigma}_0^2.Q_{\bar{L}}$

where $\bar{\sigma}_0^2 = \dfrac{V^T.P.V}{n-m}$ is the a posteriori variance factor with n = number of observables, m = number of unknown parameters and $P = \Sigma_L^{-1}$ is the weight matrix. Σ_L is the (a priori) VCV matrix of the observables and in this case is a combination of the VCV matrices Σ_{xyz} of the terrestrial cartesian coordinates and Σ_{XYZ} of the satellite cartesian coordinates.

Now in this case, we have the design matrices $\bar{A}$ and $\bar{B}$ given by Thomson (1976):

$$A_i = \left.\frac{\partial F_i}{\partial X_i}\right|_{X,L} \qquad B_i = \left.\frac{\partial F_i}{\partial L_i}\right|_{X,L} \qquad \text{where}$$

$$A_i = \left[\begin{array}{ccc:ccc:c} 1 & 0 & 0 & 0 & -(z_i+k^0z_i) & (y_i+k^0y_i) & (x_i+\varepsilon_z^0y_i-\varepsilon_y^0z_i) \\ 0 & 1 & 0 & (z_i+k^0z_i) & 0 & -(x_i^0+k\ \ x_i) & (y_i^0+\varepsilon_x z_i^0-\varepsilon_z x_i) \\ 0 & 0 & 1 & -(y_i+k^0y_i) & (x_i+k^0x_i) & 0 & (z_i+\varepsilon_y^0x_i-\varepsilon_x^0y_i) \end{array}\right]$$

$$B_i = \left[\begin{array}{ccc:ccc} (1+k^0) & (\varepsilon_z^0+k^0\varepsilon_z^0) & (\varepsilon_y^0+k^0\varepsilon_y^0) & -1 & 0 & 0 \\ -(\varepsilon_z^0+k^0\varepsilon_z^0) & (1+k^0) & -(\varepsilon_x^0+k^0\varepsilon_x^0) & 0 & -1 & 0 \\ -(\varepsilon_y^0+ k^0\varepsilon_y^0) & (\varepsilon_x^0+k^0\varepsilon_x^0) & (1+k^0) & 0 & 0 & -1 \end{array}\right]$$

The superscript ^{0} indicates initial approximate estimates of the unknown parameters.

The solution of this model for the seven parameters requires a minimum of three common points.

The forms given above are for the general case, but if the point of expansion is taken as zero for all parameters, ie.

$$(X^0)^T = (x_0^0, y_0^0, z_0^0, \varepsilon_x^0, \varepsilon_y^0, \varepsilon_z^0, k^0) = 0$$

then the matrices $\bar{A}$ and $\bar{B}$ reduce to

$$A_i = \left[\begin{array}{ccc:ccc:c} 1 & 0 & 0 & 0 & -z_i & y_i & x_i \\ 0 & 1 & 0 & z_i & 0 & -x_i & y_i \\ 0 & 0 & 1 & -y_i & x_i & 0 & z_i \end{array}\right]$$

$$B_i = \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{array}\right] = [\ I \ \vdots \ -I\ ]$$

The misclosure vector W^0 is given by

$$W_i^0 = F_i(X^0, L) = \begin{bmatrix} x_i - X_i \\ y_i - Y_i \\ z_i - Z_i \end{bmatrix}$$

3.2 MOLODENSKY MODEL (See Figure 2.8)

This model has been developed to overcome the problem of high correlation between the parameters by relating the scale and rotation parameters to some fundamental point m, and using diffference vectors from this point (Hoar,1982). This aspect of the model has been mentioned by numerous authors (eg. Thomson, 1976, Harvey, 1985). There are a few versions of this model given in the literature. The main differences lie in the fundamental point which is selected as the point of rotation.

The *first version* of this model uses the *centre of gravity (m) of the second (terrestrial) network* as the *fundamental point of rotation.* This version is the one recommended by the Defence Mapping Agency (DMA) (Hoar, 1982) and by various other authors (eg Harvey, 1985). *The implicit assumptions made in the formulation of this model are that the terrestrial geodetic and satellite (CT) system axes are aligned and of the same scale, as will be pointed out below.*

This model is given by

$$\vec{F}_i = (\vec{r}_0)_1 + (\vec{r}_m)_2 + (1+k).R_\psi.(\vec{r}_{mi})_2 - (\vec{p}_i)_1 = 0 \quad \text{where}$$

$$(\vec{r}_m)_2 = \left[x_m,\ y_m,\ z_m \right]_2^T = 1/n \,.\left[\sum_{i=1}^{n} x_i ,\ \sum_{i=1}^{n} y_i ,\ \sum_{i=1}^{n} z_i \right]_2^T$$

is the position vector of the centre of gravity (m) of the (second) terrestrial network, computed as shown above. Note that this vector enters the equation singly, ie. without coefficients. This implies that the axes of systems 1 & 2 are parallel, and since the scale factor applies to the network and not the system, the implication is that the terrestrial system is of the same scale as the satellite (CT) system.

$(\vec{r}_0)_1$ is the translation vector between the origin of coordinate systems 1 and 2,

$(\vec{r}_{mi})_2$ & $(\vec{\rho}_i)_1$ are the observables in this model,

$(\vec{r}_{mi})_2$ is the difference vector from the point m to the point i of the second (terrestrial) network,

$(\vec{\rho}_i)_1$ is the position vector of the point i in the satellite network.

The rotation matrix R_ψ is given by

$$R_\psi = R_1(\psi_x).R_2(\psi_y).R_3(\psi_z) = \begin{bmatrix} 1 & \psi_z & -\psi_y \\ -\psi_z & 1 & \psi_x \\ \psi_y & -\psi_x & 1 \end{bmatrix}$$

These rotations are reckoned around axes nominally parallel to those of systems 1 and 2 (and situated at the fundamental point), and k is the scale difference from unity.

The rotations and scale difference are considered to refer to the second network, since those difference vectors are the ones that are rotated and scaled.

The *second version* of this model uses the *initial point (k) of the second network* as the *fundamental point of rotation*. Thomson (1976) identified a few versions of this model. Thomsons' (1976) Version 1 of the Molodensky model is given by

$$\vec{F}_i = (\vec{r}_0)_1 + (\vec{r}_k)_2 + (1+k).R_\psi.(\vec{r}_{ki})_2 - (\vec{\rho}_i)_1 = \emptyset$$

where $(\vec{r}_0)_1$, $(\vec{\rho}_i)_1$, k, & R_ψ have the same meaning as above. Here the vector $(\vec{r}_k)_2$ is the position vector of the network initial point (fundamental point) of the second network, and the difference vector $(\vec{r}_{ki})_2 = (\vec{r}_i)_2-(\vec{r}_k)_2$ is the vector from the initial point (k) to the point i in the second system. The observables are the vectors $(\vec{\rho}_i)_1$ and $(\vec{r}_{ki})_2$.

Since the position vector of the fundamental point, the centre of gravity, $(\vec{r}_m)_2$, and the initial point of the second network, $(\vec{r}_k)_2$, in the two models above, enter the model singly, there is again the *implicit assumption that the coordinate axes of systems 1 (CT) and 2 (G) are parallel. The application of the scale factor to the network again implies a uniform scale for both the systems or datums. This is an important point that is not always expressly mentioned.*

Thomsons' (1976) Version 2 of the Molodensky model is given by

$$\vec{F}_i = (\vec{r}_0)_1 + (1+k).R_\psi.[(\vec{r}_k)_2 + (\vec{r}_{ki})_2] - (\vec{\rho}_i)_1 = 0$$

where each of the terms have the same meaning as in the models above.

In this version there is *no assumption of parallelity of the axes of systems 1 and 2,* but the implicit assumption here is that there is a local coordinate system at the initial point of network 2 which is parallel to the axes of system 2, since the position vector of the initial point and the difference vector are simply added. Using this interpretation *"it has been shown that it is equivalent to the Bursa model"* (Thomson and Krakiwsky, 1976) It is for this reason that this second version of Thomsons' Molodensky model will not be considered any further in this report.

The Molodensky model can be used for the combination of a satellite and a terrestrial network, but it must be born in mind that the assumption of the parallelity of axes may be incorrect. It can not be used for two satellite networks since a satellite network has no initial point associated with it.

The opinion has been expressed by various authors (eg. Thomson & Krakiwsky, 1976) that the Bursa model is best suited for the combination of networks that have global coverage as opposed to local or non-global networks, whereas the Molodensky model is more suitable for non-global networks.

The model for the *inverse transformation* is simply derived as follows (for the first version of the Molodensky model given above):

$$\vec{F}_i = (1+k)^{-1}.R_\psi^{-1}.[(\vec{p}_i)_1-(\vec{r}_0)_1-(\vec{r}_m)_2] - (\vec{r}_{mi})_2 = 0$$

$$= (1+k)^{-1}.R_\psi^{T}.[(\vec{p}_i)_1-(\vec{r}_0)_1-(\vec{r}_m)_2] - (\vec{r}_{mi})_2 = 0$$

since the rotation matrix R_ψ is orthogonal.

3.2.1 Comparison of geometry of Molodensky and Bursa models

It has been shown by various authors that the numerical values of the rotations and scale difference are identical for the Molodensky and Bursa models. The translations are however different.

The *following argument* is from *Harvey* (1985):

The Molodensky model can be written as follows:

$F = P + Q = 0$ where

$$P = (\vec{r}_m)_2 - (1+k).R_\psi.(\vec{r}_m)_2$$

$$Q = (\vec{r}_0)_1 + (1+k).R_\psi.(\vec{r}_i)_2 - (\vec{p}_i)_1$$

Q is the standard Bursa model, and P is simply a constant term which is the same for all points, and would obviously affect the translation terms. The values for the rotation parameters (represented by the matrix R_ψ) and the scale difference k, are determined by Q only, and therefore equal those from the Bursa model. The difference in the translation terms obtained from the two models is clearly due to the scaling and rotating of the fundamental point m.

The difference in the translation terms may also be considered by using the following development of the models similar to that given by Boucher (1979):

If the rotation matrices in the Bursa and Molodensky models, R_ε and R_ψ respectively, are replaced by $R_\varepsilon = I + Q_\varepsilon$, $R_\psi = I + Q_\psi$ where I = identity matrix, and Q_ε and Q_ψ are given by

$$Q_\varepsilon = \begin{bmatrix} 0 & \varepsilon_z & -\varepsilon_y \\ -\varepsilon_z & 0 & \varepsilon_x \\ \varepsilon_y & -\varepsilon_x & 0 \end{bmatrix} \qquad Q_\psi = \begin{bmatrix} 0 & \psi_z & -\psi_y \\ -\psi_z & 0 & \psi_x \\ \psi_y & -\psi_x & 0 \end{bmatrix}$$

and the models expanded as follows (the B and M superscripts denote the Bursa and Molodensky models respectively):

$$(\vec{r}_0)_1^B + (1+k^B).R_\varepsilon.(\vec{r}_i)_2 - (\vec{\rho}_i)_1 =$$

$$= (\vec{r}_0)_1^B + (1+k^B).(I+Q_\varepsilon).(\vec{r}_i)_2 - (\vec{\rho}_i)_1 = 0$$

Hence, the Bursa model produces (neglecting the second order product $k^B.Q_\varepsilon$)

$$(\vec{\rho}_i)_1 - (\vec{r}_i)_2 = (\vec{r}_0)_1^B + k^B.(\vec{r}_i)_2 + Q_\varepsilon.(\vec{r}_i)_2$$

The Molodensky model similarly produces (neglecting the second order product $k^M.Q_\psi$)

$$(\vec{\rho}_i)_1 - (\vec{r}_i)_2 = (\vec{r}_0)_1^M + k^M.[\vec{r}_i - \vec{r}_m]_2 + Q_\psi.[\vec{r}_i - \vec{r}_m]_2$$

From these equations it is again obvious that in the Bursa model all points i are treated in the same manner, since it is the position vectors $(\vec{r}_i)_2$ of all points that are rotated and scaled. In the Molodensky model however, it is only the difference vectors $[\vec{r}_i - \vec{r}_m]_2$ from the fundamental point m that are rotated and scaled, not the position vectors $(\vec{r}_i)_2$. Hence the *fundamental point m* is *not* rotated and scaled in the Molodensky model, whichever point is used as such.

Now if the rotations and scale difference from the two models are equal, ie. $Q_\varepsilon = Q_\psi = Q$ and $k^E = k^M = k$ then we have on subtraction of the first from the second equation :

$$(\vec{r}_0)_1^M - (\vec{r}_0)_1^E = k.(\vec{r}_m)_2 + Q.(\vec{r}_m)_2 = (k.I + Q).(\vec{r}_m)_2$$

which expression confirms Harvey's (1985) statement that the difference in the translations is due to the rotation and scaling of the position vector of the fundamental point $(\vec{r}_m)_2$ of the second network.

Hence the Bursa and Molodensky translation terms are equal $[(\vec{r}_0)_1^M = (\vec{r}_0)_1^E]$ if either one of the following two conditions are satisfied:

If $(\vec{r}_m)_2 = 0$, ie. the point of rotation is the centre of the geodetic ellipsoid, *OR*

if $k = 0$ *and* $Q = I$ ie. the scale difference and rotation parameters are restrained to zero.

The precisions of the Molodensky translations are generally an order of magnitude smaller than those from the Bursa model. However, it must be clearly stated that, in the light of what has been said previously, the Molodensky translations are *not better estimates* of the Bursa translations, they are *fundamentally different* (Harvey, 1986).

3.2.2 Estimation method:

This is done as in the Bursa case by the combined case least squares method, and the matrices $\bar{A}$ and W^0 are as follows:

$$A_i = \left[\begin{array}{ccc|ccc|c} 1 & 0 & 0 & 0 & -z_{ki} & y_{ki} & x_{ki} \\ 0 & 1 & 0 & z_{ki} & 0 & -x_{ki} & y_{ki} \\ 0 & 0 & 1 & -y_{ki} & x_{ki} & 0 & z_{ki} \end{array}\right]$$

$$W_i^0 = F_i(X^0, L_i) = \begin{bmatrix} x_k + x_{ki} - X_i \\ y_k + y_{ki} - Y_i \\ z_k + z_{ki} - Z_i \end{bmatrix} = \begin{bmatrix} x_i - X_i \\ y_i - Y_i \\ z_i - Z_i \end{bmatrix}$$

The B matrix is identical to the B matrix for the Bursa model namely $B_i = [\ I \mid -I\ ]$ of size 3x6 for each network point. Hence, since the W^0 misclosure vector is likewise identical, the only difference is in the structure of the $\bar{A}$ matrix.

To solve for the seven parameters from this model one needs the position vector of the initial point of the second system as well as the coordinates of at least three more common points.

3.3 VEIS MODEL (See Figure 2.9)

This model is given by Thomson (1976) as

$$\vec{F}_i = (\vec{r}_0)_1 + (\vec{r}_k)_2 + (1+k).R_V.(\vec{r}_{ki})_2 - (\vec{\rho}_i)_1 = 0$$

where $(\vec{r}_0)_1$, $(\vec{r}_k)_2$, $(\vec{r}_{ki})_2$, $(\vec{\rho}_i)_1$, & k have identical meanings to those in the Molodensky model, and where

$$R_V = R_3(180-\lambda_k).R_2(90-\phi_k).P_2.R_1(d\nu).R_2(d\mu).R_3(dA).$$
$$.P_2.R_2(\phi_k-90).R_3(\lambda_k-180)$$

where (ϕ_k, λ_k) are the geodetic coordinates of the initial point k of the second network. P_2 is a reflection matrix about the y-axis,

$$P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and the matrix product

$$R_1(d\nu).R_2(d\mu).R_3(dA) = \begin{bmatrix} 1 & dA & -d\mu \\ -dA & 1 & d\nu \\ d\mu & -d\nu & 1 \end{bmatrix}$$

The point of rotation is the initial point of the second network and the rotations are referred to the axes of the local geodetic (LG) system. The rotations are $d\nu$, a rotation about the x-axis ie. a tilt in the prime vertical plane, $d\mu$, a rotation about the y-axis ie. a tilt in the meridian plane, and dA, a rotation about the z-axis ie. a rotation in azimuth.

According to Thomson (1976), the product $P_2.R_2(\phi_k-90).R_3(\lambda_k-180)$ is needed to transform the difference vector $(\vec{r}_{ki})_2$ to the local geodetic system at k from the system 2 (G) in which it was formulated. The final set of rotations $R_3(180-\lambda_k).R_2(90-\phi_k).P_2$ is required to rotate the transformed difference vectors $(r_{ki})_2$ back to the second coordinate system (G).
The matrix R_V is given by Thomson (1976) as

$$R_V = \begin{bmatrix} 1 & -\sin\phi_k.dA-\cos\phi_k.d\nu & \sin\lambda_k.\cos\phi_k.dA+\cos\lambda_k.d\mu-\sin\phi_k.\sin\lambda_k.d\nu \\ \sin\phi_k.dA+\cos\phi_k.d\nu & 1 & -\cos\phi_k.\cos\lambda_k dA.+\sin\lambda_k.d\mu+\sin\phi_k.\cos\lambda_k d\nu \\ -\cos\phi_k.\sin\lambda_k.dA-\cos\lambda_k.d\mu+\sin\phi_k.\sin\lambda_k.d\nu & \cos\phi_k.\cos\lambda_k.dA-\sin\lambda_k.d\mu-\sin\phi_k.\cos\lambda_k d\nu & 1 \end{bmatrix}$$

This model has the same implicit assumptions of parallelity of the axes of systems 1 (CT) and 2 (G) as in the Molodensky model. The interpretation of the resulting parameters are similar, ie. the rotations and scale change refer to the second network. The translations are identical to those from the Molodensky model.

The Veis model is seen to be equivalent to the Molodensky model, and can therefore be used for the combination of a satellite and terrestrial network, but not for two satellite networks.

The model for the *inverse transformation* is simply derived as follows:

$$\vec{F}_i = (1+k)^{-1}.R_V^{-1}.[(\vec{\rho}_i)_1 - (\vec{r}_0)_1 - (\vec{r}_k)_2] - (\vec{r}_{ki})_2 = 0$$

$$= (1+k)^{-1}.R_V^{T}.[(\vec{\rho}_i)_1 - (\vec{r}_0)_1 - (\vec{r}_k)_2] - (\vec{r}_{ki})_2 = 0$$

since the rotation matrix R_V is orthogonal.

3.3.1 Relation between Molodensky and Veis rotations

The Molodensky rotations (ψ_x, ψ_y, ψ_z) can be derived from the Veis rotations $(d\nu, d\mu, dA)$ by the following relation

$$(\psi_x, \psi_y, \psi_z)^T = [R_3(180-\lambda_k).R_2(90-\phi_k).P_2].(d\nu, d\mu, dA)^T \quad ,$$

and the Veis rotations from the Molodensky ones by the inverse operation.

3.3.2 Estimation procedure

This is identical to the Bursa model, as are the matrices $\bar{B}$ and W^0.

The matrix $\bar{A}$ is given by Thomson (1976) as

$$A_i = \left[\begin{array}{ccc:c:c:c:c} 1 & 0 & 0 & -\sin\phi_k . y_{ki} + \sin\lambda_k . \cos\phi_k . z_{ki} & \cos\lambda_k . z_{ki} & -\cos\phi_k . y_{ki} - \sin\phi_k . \sin\lambda_k . z_{ki} & x_{ki} \\ 0 & 1 & 0 & \sin\phi_k . x_{ki} - \cos\phi_k . \cos\lambda_k . z_{ki} & \sin\lambda_k . z_{ki} & \cos\phi_k . x_{ki} + \sin\phi_k . \cos\lambda_k . z_{ki} & y_{ki} \\ 0 & 0 & 1 & -\cos\phi_k . \sin\lambda_k . x_{ki} + \cos\phi_k . \cos\lambda_k . y_{ki} & -\cos\lambda_k . x_{ki} - \sin\lambda_k . y_{ki} & \sin\phi_k . \sin\lambda_k . x_{ki} - \sin\phi_k . \cos\lambda_k . y_{ki} & z_{ki} \end{array}\right]$$

This model has seven parameters and therefore requires the coordinates of the Fundamental Point (FP) and at least three common points, similarly to the Molodensky model.

CHAPTER 4

DATA

4.1 STATE OF GEODETIC SURVEY IN SOUTH AFRICA

4.1.1 Horizontal network

The historical development of the geodetic networks in South Africa is described in various official publications. What is presented here is a very brief abstract from a paper by Wonnacott (1985).

Geodetic triangulation chains forming closed loops covering most of the country were established in the late nineteenth century as a result of the work of Sir David Gill and others. In the period 1900 to 1973 triangulation breakdown and densification was done to fill in the areas between the earlier triangulation chains. The methods of adjustment which treated sections separately resulted in a 'patchwork' style of adjustment, the results of which are now shown up by more modern surveying techniques.

With the introduction in the early 1970's of electronic distance measuring apparatus, it was decided to run a scheme of first order traverses. These were to be traverses with legs of about 40 kilometres in length, and about 2 degrees of longitude by 1 degree of latitude in extent. The intention was to include the triangulation inside these well controlled traverses in order to control the swing and scale of the triangulation. The swing in these traverses were to be controlled by the observation of astronomical azimuths at every third or fourth leg, and Laplace latitude and longitude was observed at the terminals of the azimuth legs.

Wonnacott (1985) explains how these traverses showed up problems in the triangulation by running across 'patches' of triangulation resulting from different adjustment stages. In order to confirm the traverse terminal points it was neccessary to establish some form of uniform control to cover the whole country.

This was the reason for the start of the systematic translocation Doppler survey of the country in 1980, as mentioned below. Suffice it to say here that Wonnacott has identified various shortcomings in the national network, some of which will be confirmed below (at least partially) using the available data.

4.1.2 Vertical network

The South African height system is described in an official publication (Anonymous, 1966) and summarised by Merry (1985). It is a spheroidal orthometric system in which normal gravity is used instead of measured gravity. The height network is based on the least squares adjustment of six first order circuits that were initially established. Later circuits were then adjusted onto these, and this work continues. However, some changes are contemplated in anticipation of the future readjustment of this network (Merry, 1985).

4.1.3 Geoidal heights

One result of the work done over many years by Merry and Van Gysen (eg. Merry and Van Gysen, 1987) at the University of Cape Town is that we have today a detailed geoid model of Southern Africa which is more than sufficient for the reduction of geodetic observations to the ellipsoid. The geoid heights that are required for combination with the spheroidal orthometric heights in order to obtain ellipsoidal heights, are obtainable from these authors.

4.1.4 Doppler survey of South Africa

The Control Surveys Branch of the Directorate of Surveys and Mapping in South Africa initiated a systematic Doppler translocation survey of the country in 1980 (Wonnacott, 1985). The result of this survey is a uniform network of high relative accuracy covering the whole country as well as South West Africa, comprising 22 stations in South Africa and 13 in South West Africa (Wonnacott, 1986). These Doppler stations were generally selected to coincide with existing or proposed Laplace stations on the primary traverse network, and with spacings of approximately 300 kilometres. The positions of 21 of the main Doppler stations in South Africa are shown in Figure 4.1.

In the period 1982-1983 six points in South Africa (see Figure 4.1) and three in South West Africa were selected to be occupied as part of South Africa's contribution to the ADOS campaign (Wonnacott,1986) These nine points were surveyed in accordance with the technical specifications for the ADOS project, and were computed in the SPP mode using the DMA precise ephemeris.

The translocation survey was done in sections. Initially three and later five stations were occupied simultaneously. Each of these simultaneously occupied group of stations (3-5) was treated as a subnet and reduced using the Broadcast ephemeris. These subnets were then adjusted to form a homogeneous continental network (Newling, personal communication, 1986). This translocation network was adjusted onto the ADOS points, which served as zero order points. These ADOS points acted as weighted constraints in the adjustment of the translocation network, and as a result, "the adjusted Doppler network is strongly biased towards the precise ephemeris reference frame, ie. NSWC 9Z-2." (Wonnacott, 1986)

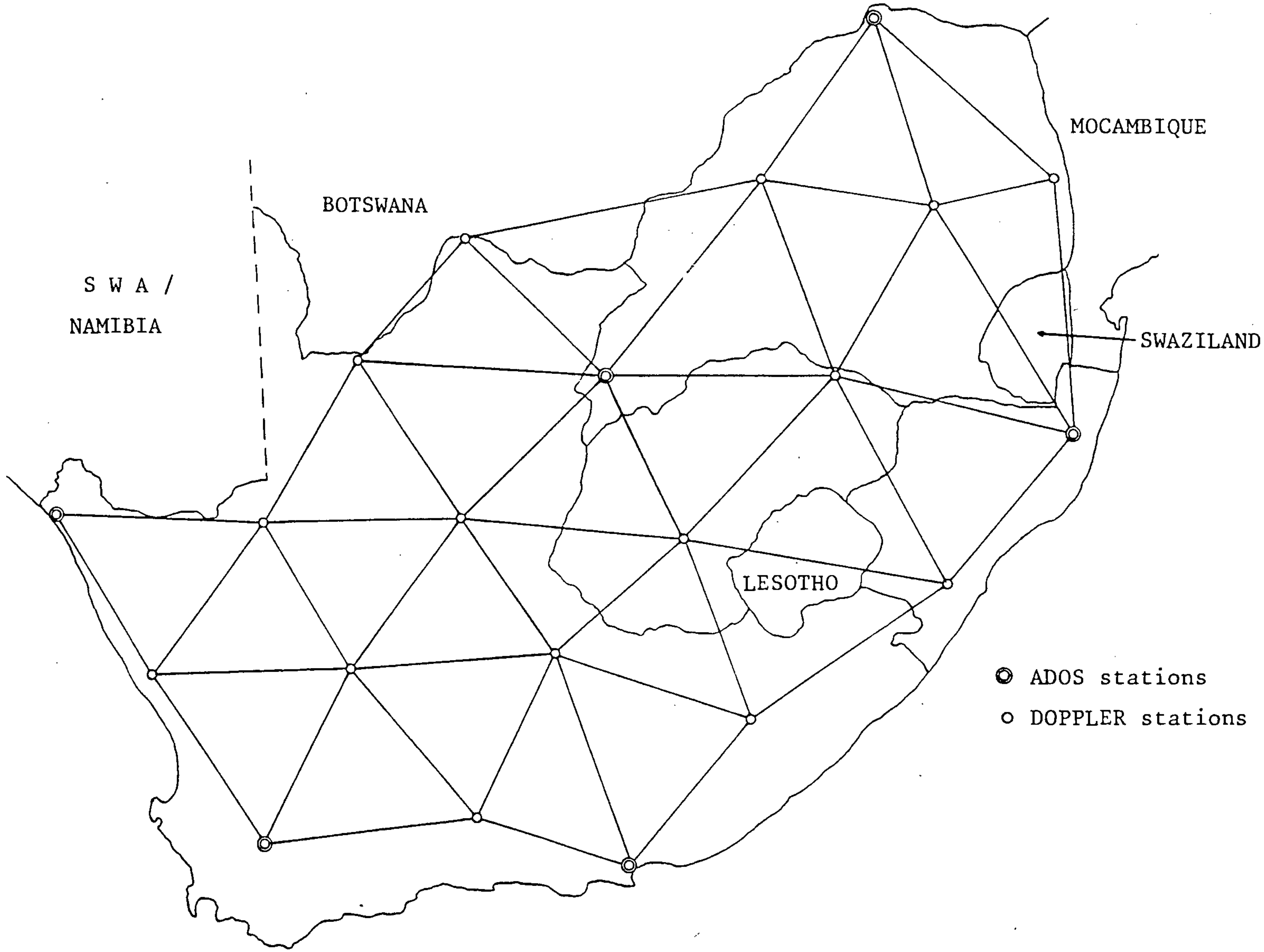


Figure 4.1 Doppler Translocation network in South Africa

4.2 DATA USED IN THIS REPORT

4.2.1 Source of data

The geoidal height data on the Cape datum was supplied by Merry (personal communication, 1986, 1987). The terrestrial and satellite data was supplied by Newling (personal communication, 1986, 1987). This comprised the data for the 21 main Doppler stations shown in Figure 4.1, as well as for 2 additional stations which are not shown. Data obtained from other sources will be referenced individually.

4.2.2 Data

The parameters of the Cape Datum are those for the modified Clarke 1880 ellipsoid, namely

semi major axis $a = 6378249.145326$ metres

semi minor axis $b = 6356514.966721$ metres

The *initial point* of the geodetic network (NIP) is Buffelsfontein near the city of Port Elizabeth. The adopted geodetic ellipsoidal coordinates of this point are

latitude $\phi_0 = -33^\circ\ 59'\ 32.''000$

longitude $\lambda_0 = +25^\circ\ 30'\ 44.''622$

The geoid height, although not explicitly defined, is implicitly taken as zero for Buffelsfontein, so that the ellipsoidal height becomes the speroidal orthometric height, ie. $h_0 = 282.00$ metres.

4.2.2.1 Terrestrial coordinate data. The horizontal coordinates, geodetic latitude (ϕ) and longitude (λ), are combined with the spheroidal orthometric (H^0) and geoidal heights (N) to obtain the ellipsoidal coordinates (ϕ,λ,h). These were then transformed to cartesian coordinates (xyz) on the Cape Datum using the well known formulae given in APPENDIX A. The geoidal data on the Cape Datum was obtained from the geoid model computed on the GRS80 ellipsoid . This geocentric geoid, shown in Figure 4.2, was transformed to the local Cape Datum by using approximate transformation parameters determined previously. The resulting geoid height at the Network Initial Point, Buffelsfontein, was zero, as defined implicitly.

4.2.2.2 Satellite Doppler coordinate data. The location of the points of the Doppler translocation network and the six points of the ADOS campaign are shown in Figure 4.1. As mentioned above, the translocation network was adjusted onto the six ADOS points which were fixed in SPP mode using the PE. Thus the whole Doppler network can be considered to be in the PE coordinate frame, ie. NSWC 9Z-2.

All known biases of the Doppler PE coordinate frame (NSWC 9Z-2) with respect to the CT system were removed before using such coordinate data (eg. Mueller, 1982). The transformation parameters used here for the transformation of the Doppler PE coordinates (NSWC 9Z-2) to the CT system, are those from Hothem, Vincenty and Moose (1982). These are:

a Z-shift of +4 metres,

a Z-axis rotation or longitude rotation of -0.8 arc seconds (ie. increasing longitude east),

a scale change of -0.5 ppm.

This transformation was applied to the Doppler coordinates throughout all computations in this report. Therefore all transformations computed here are for the transformation of the Geodetic network or datum onto the CT system as represented by the transformed (using the specific parameters mentioned above) Doppler NSWC 9Z-2 coordinates.

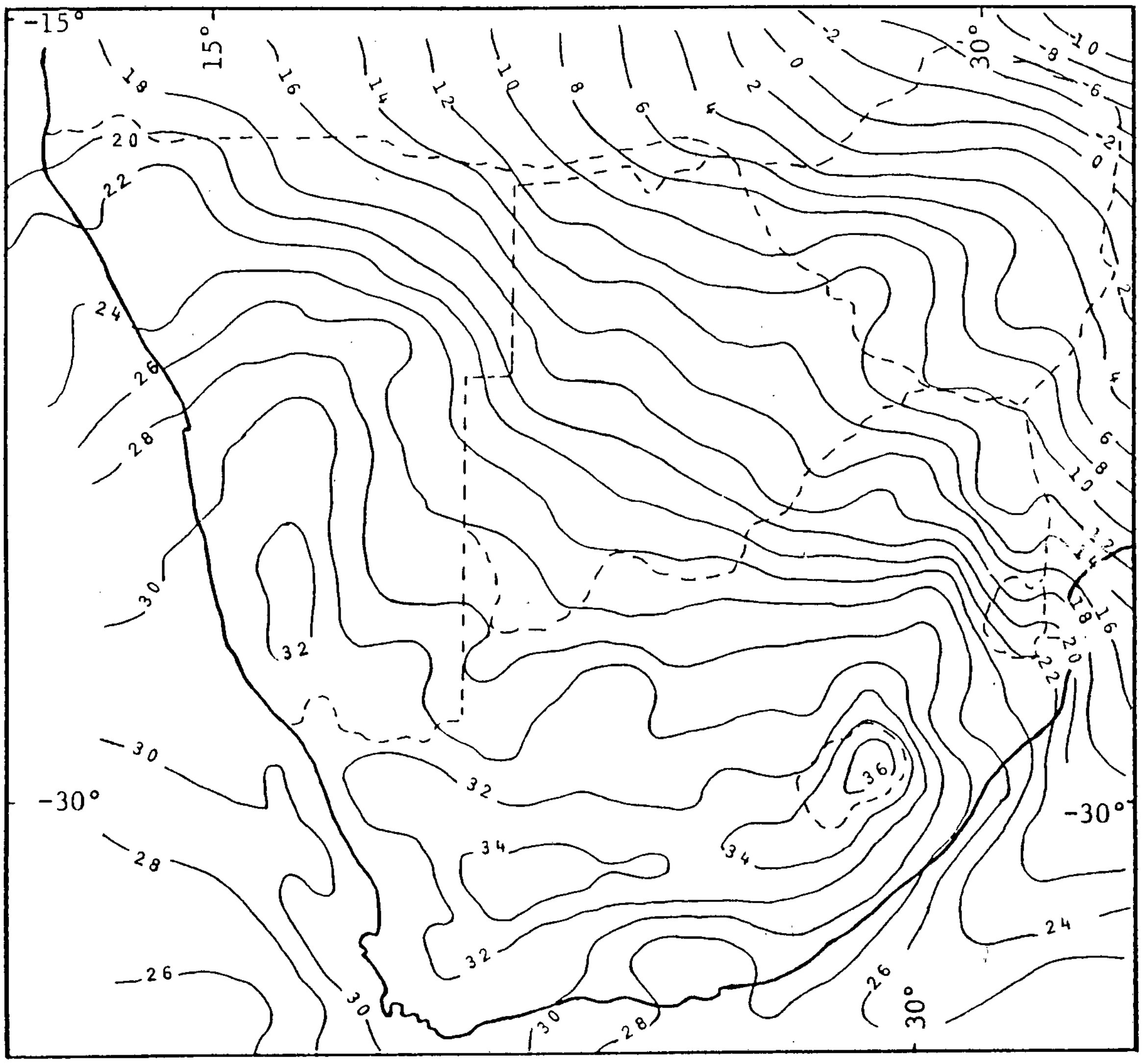


Figure 4.2 Geoid model of Southern Africa on the GRS80 ellipsoid (after Merry and Van Gysen, 1987)

4.2.3 VCV matrices of observables

If the coordinates of the networks had been produced by rigorous adjustment methods, then the associated VCV matrices would have been produced as byproducts of this process. However, this is seldom true of national networks that have been adjusted before the advent of modern computers, and one thus has no such information. Here we will attempt to form these matrices empirically, even though they will not be fully populated, ie. they may only be diagonal or block-diagonal at best.

4.2.3.1 Satellite network VCV_{XYZ} matrix.

The precision of satellite single point positioning is regarded as homogeneous as mentioned before. The following assumptions will be made in the formation of this matrix:

The precision of all network points is regarded as equal, as are the precision of the X, Y, and Z components of a point. Further all correlations are neglected, be they between network points or between coordinate components of one point The result of these assumptions is that the VCV matrix for the satellite network is strictly diagonal with the entries on the main diagonal being the variances σ^2 of a coordinate component. The VCV matrix for the point i is a 3x3 diagonal sub-matrix, hence the full matrix is of size 3nx3n where n represents the number of points.

The meaning which is here attached to this precision estimate is that it is the precision of the position vector with respect to the origin of the implied coordinate frame, here nominally the geocentre.

4.2.3.2 Terrestrial network VCV_{xyz} matrix. The 3D terrestrial (xyz) network is derived from the combination of the 2D horizontal (ϕ,λ) and the 1D vertical (H^0) networks, together with geoidal height (N) information for each network point. The VCV_{xyz} matrix is similarly derived from the $VCV_{\phi\lambda h}$ matrix by the well-known law for the propagation of VCV matrices, namely $VCV_{xyz} = J.(VCV_{\phi\lambda h}).J^T$ where J is the usual Jacobian matrix of partial differentials. The details of the formation of this matrix are treated in APPENDIX B.

4.2.3.3 Formation of the $VCV_{\phi\lambda h}$ matrix. The classical estimate of precision of the horizontal components (ϕ,λ) is derived from Simmons' rule (Simmons, 1950) which states that they are a function of the distance from the network initial point. This precision estimate is regarded as relative to the network initial point.

The ellipsoidal height h is the sum of the orthometric height H^0 and the geoidal height N. Since we are interested in heights of network points relative to the NIP, we will adopt some empirical formula similar to Simmons' rule, so that the relative precision estimate for the height is a funtion of distance from this initial point.

Now, in the classical method of datum establishment, the position of the ellipsoid is determined by the geodetic coordinates adopted for the initial point, and by the adherence to the topocentric parallelism conditions. In order for this ellipsoid to be fixed in space, we will not allow these coordinates of the initial point to vary at all. One exception to this rule is the case of the "network transformation parameters" when *all* coordinates of the network have equal variances.

This implies that the precision of the position vector (xyz) of any point in the terrestrial system is equal to the precision of the relative or difference vector from the NIP to that point. Hence the relative precision estimates that we compute for the network points (in ϕ,λ,h) will also be the precision estimates relative to the centre of the ellipsoid.

The correlations between points as well as between the components of a point will be neglected and therefore the $VCV_{\phi\lambda h}$ will be diagonal. Since the Jacobian matrix is not strictly diagonal, the resulting VCV_{xyz} matrix for the terrestrial xyz coordinates will be block-diagonal with each 3x3 block representing one point. This implies that the VCV_{xyz} matrix is of size 3nx3n as in the case of the satellite network.

4.2.3.4 Numerical values used. The formulae used here for the computation of the standard deviations of the ellipsoidal coordinates (ϕ,λ,h) are $\sigma_\phi = \sigma_\lambda = 0.05 \times K^{2/3}$ metres, where K is the distance in kilometres of the network point from the initial point (NIP). This yields the estimates $\sigma_\phi = \sigma_\lambda = 5.0$ metres for K = 1000 kilometres.

The variance of the ellipsoidal height (σ_h^2) is the sum of the variances of the spheroidal orthometric ($\sigma_{H^0}^2$) and geoidal heights (σ_N^2) as $\sigma_h^2 = \sigma_{H^0}^2 + \sigma_N^2$. The numerical forms used in this report were rather arbitrary estimates. The formulae for the estimation of the standard deviations of the spheroidal orthometric and geoidal heights are $\sigma_{H^0} = 0.015 \times K^{2/3}$ metres, and $\sigma_N = 0.001 \times K$ metres, where K has the same meaning as above. Thus the standard deviation for the ellipsoidal height is $\sigma_h = [\sigma_{H^0}^2 + \sigma_N^2]^{1/2} = [(0.015 \times K^{2/3})^2 + (0.001 \times K)^2]^{1/2}$ which yields $\sigma_h \simeq 1.8$ metres for K = 1000 kilometres.

The standard deviations assigned to the Doppler satellite (or CT) coordinates of the satellite network, were $\sigma_X = \sigma_Y = \sigma_Z = 0.5$ metres, and represent an average value.

4.2.3.5 <u>Formation of the combined Σ_L matrix of the observables</u>. Due to the way that the three standard models considered here, namely the Bursa, Molodensky and Veis models, are formulated, the Σ_L matrix represents a combination of the VCV_{XYZ} and VCV_{xyz} matrices. Since we do not need either of these two matrices individually, the Σ_L matrix is formed directly. It is of size 6nx6n, block-diagonal, with each 6x6 block representing one network point, being a combination of the terrestrial and satellite VCV 3x3 sub-matrices for that point.

CHAPTER 5

RESULTS AND ANALYSIS

A computer program was developed specifically for the purposes of this report. This program allows for the combination of a classical terrestrial network, using both horizontal and vertical networks, and a three dimensional Doppler satellite network by solving for the external bias or transformation parameters using any one of the three models described in CHAPTER 3. This program, which is described in Appendix C, was used for all computations.

The numerical results of a number of adjustments using the three different models, Bursa, Molodensky and Veis, are presented and analysed. These results are in the form of sets of transformation parameters and precision estimates for different adjustments. When viewed dispassionately, these sets of estimated bias parameters are simply a set of parameters to be used for the transformation of coordinates from one network or datum to another. However, an attempt is made to assign some physical meaning to these parameters in two ways.

The *first* represents the case where an attempt is made to recover the transformation parameters for the Geodetic (G) datum itself, the so-called "datum transformation parameters". The *second* represents the case where it is attempted to recover the transformation parameters for the G network, the so-called "network transformation parameters". This point is also discussed in general terms in section 2.5.1.4. The following discussion refers to the specific models used in this report, namely the Bursa, Molodensky and Veis models.

In this report the coordinates of common network points are used as observations. It was mentioned in an earlier section that the coordinates of network points can be used to recover the position and orientation of the datum on which the network is based, provided that the coordinates do accurately represent physical reality, and are thus representative of their datum. Whilst this is true of satellite networks, it is not true of classical terrestrial networks. In order then to use these coordinates to recover the datum, the weighting model discussed in section 2.5.1.4 is used in the adjustments. This model uses a differential weighting scheme whereby the weight of a network point is a function of the distance from the NIP, thus compensating for the effect of various errors in the terrestrial network. Points closer to the NIP thus contribute more to the recovery of the datum position and orientation than those further away, as expressed by Wells and Vanicek (1975). Therefore, it is submitted that the coordinates of network points do, when associated with this type of weighting model or VCV matrix, represent their datums to a greater or lesser degree of accuracy. This procedure then leads to the recovery of the "datum transformation parameters".

The case where all network points, including the NIP, are given equal weight and therefore $\Sigma_L = I$, represents the recovery of the "network transformation parameters".

Now, in the case of the Bursa model, both interpretations may be used without complications. Since the Bursa model makes no a priori assumptions in the formulation of the model, there is no violation of any such assumptions.

In the case of the Molodensky and Veis models, there is the implicit assumption that the G system is both parallel and of the same scale as the CT system, and that the estimated rotation and sale parameters refer to the G network. No complication arises when estimating the "network transformation parameters". There is however a complication when using these models to estimate the "datum transformation parameters". When attempting to use the above differential weight model in conjunction with network coordinates to recover "datum transformation parameters", there is the obvious difficulty in that these parameters can not then be attributed to the datum, since this interpretation violates the a priori assumption mentioned above. Therefore it does not seem to make much sense to speak of "datum transformation parameters" in the Molodensky and Veis cases.

However, it must be kept in mind that this parallelity assumption will only be strictly valid if the datum was initially perfectly aligned with the CT system. Since this is hardly ever likely, the above assumption will usually not be perfectly valid. With this fact in mind it is submitted that it may still be interesting to use this variable weight model in the case of the Molodensky and Veis models. However, to avoid an obvious difficulty, these estimated parameters shall be called "*quasi* datum transformation parameters" for the purposes of this report. These will be compared to the "datum transformation parameters" estimated in the Bursa model.

A number of general points concerning transformations, which apply equally to all three models, will be considered by using the Bursa model only.

A parameter that is useful in assessing the *quality* of a transformation will now be introduced.

5.1 THE "MEAN GOODNESS OF FIT" VALUE, Δ

The results of a transformation adjustment such as considered here, are two sets of adjusted cartesian coordinates that differ by the adjusted parameters. Now, the a priori variance factor, $\bar{\sigma}_0^2 = V^T.P.V/(n-m)$, being a function of the residuals, is some measure of the quality of the adjustment. However, since the residuals are multiplied by their weight matrix $P = \Sigma_L^{-1}$, the effect is to normalize them. Therefore, $\bar{\sigma}_0^2$ does not give a direct measure of the misclosure after transformation. Consider the following argument:

The derived set of transformation parameters is used (inversely) to transform the set of CT coordinates into the G system so as to produce what are called pseudo-geodetic (PG) coordinates. These coordinates will differ from the geodetic coordinates (for the same points of course). What is now required is a *single numerical value* that would indicate the "average linear difference" or the *"mean goodness of fit"* between the pseudo-geodetic and geodetic coordinates for all the points. The value that is adopted for this purpose in this report is called the *"mean goodness of fit" value after transformation, Δ*, and is computed as follows: (Burford (1985) used a similar (mean) value in his investigations).

The pseudo-geodetic coordinates are differenced from the geodetic ones, and these differences are expressed in component form as $x_{PG}-x_G$, $y_{PG}-y_G$, $z_{PG}-z_G$. The total

displacement is then computed for each point i (in metres) as displacement = $\left[(x_G - x_{PG})_i^2 + (y_G - y_{PG})_i^2 + (z_G - z_{PG})_i^2 \right]^{1/2}$.
The simple arithmetic mean, of these displacements for all points is computed and used to indicate the "mean goodness of fit" after transformation,

$$\Delta = \sum_{i=1}^{n} \left[(x_G - x_{PG})_i^2 + (y_G - y_{PG})_i^2 + (z_G - z_{PG})_i^2 \right]^{1/2},$$

where n is the number of common points. It will be appreciated that the absolute value of the total coordinate difference in each component after transformation, ie. $x_G - x_{PG}$, $y_G - y_{PG}$, $z_G - z_{PG}$, is simply the sum of the absolute values of the residuals to each of the coordinate components in each system, eg. $|x_G - x_{PG}| = |v_G| + |v_{CT}|$. Note that the CT coordinates become the PG coordinates after transformation.

However, an important remark should be made now. The value Δ is a funtion of the coordinate differences after transformation, and therefore depends on the parameter values used. These in turn depend on the specific weight or VCV matrix Σ_L used in the determination of these parameters. Hence, the value Δ is a function of the VCV matrix Σ_L used in the determination of the parameter set, and is therefore not to be used indiscriminately. If this fact is borne in mind, no problem should result from its use. The effect of the VCV matrix Σ_L on the value Δ is seen by comparing these values for similar cases (columns) from Tables 5.1 & 5.2. The 7-parameter cases in columns one of Tables 5.1 and 5.2 yield Δ values of 6.5 and 6.0 metres respectively, this difference being entirely due to the use of the different VCV matrices Σ_L in the solutions.

5.2 BURSA MODEL

5.2.1 Presentation of results

Table 5.1 summarises the results of five different cases (columns 1 to 5) of the transformation adjustment using the Bursa model, with the *VCV matrix* Σ_L *not equal to the identity matrix I*. This represents the cases where an attempt is made to recover the *datum transformation parameters* by using a weight model designed to reflect the fact that there are unknown and unaccounted for systematic errors present in the terrestrial network. The *correlation matrix* for the 7-parameter transformation, case (column) 1 of Table 5.1, is given in Table 5.1.1. Each of the five columns of Table 5.1 presents the results of one solution of the transformation adjustment. These solutions differ in that the number of parameters that are solved for, is varied. In column 1 of Table 5.1, all 7 parameters are solved for, ie. none are constrained to an a priori value of zero, and this is reflected by entries in all rows of that column. In the other four solutions, columns 2 to 5 inclusive, some of the parameters are constrained to zero a priori values, and are therefore not solved for in the solution. This is indicated in this report by blank spaces in the blocks corresponding to those parameters, rather than by inserting zero values which could be misleading.

Table 5.2 summarises the results of four different cases (columns 1 to 4) of the transformation adjustment using the Bursa model, where the *VCV matrix* Σ_L *is the identity matrix I*. In this case *all* coordinates have equal weight, ie. no attempt is made to reflect the effect of systematic errors in the terrestrial network, and the parameters are thus *network transformation parameters*. The correlation matrix for the 7-parameter case in column 1, Table 5.2, is given in Table 5.2.1.

MODEL: BURSA

Data set: South Africa

Number of common points: 23

Point of rotation: Centre of Geodetic ellipsoid

F.P. : $x_G = y_G = z_G = 0.00$

VCV-matrix of the observables $\Sigma_L \neq I$

Col	1	2	3	4	5	6
$\bar{\sigma}_0$	1.10	1.10	1.38	1.14	1.41	
T_x	-177.4	-181.2	-134.7	-176.0	-134.5	metres
σ	± 8.1	± 7.1	± 5.2	± 7.0	± 0.5	metres
T_y	-121.5	-118.7	-102.1	-130.3	-111.0	metres
σ	± 5.9	± 5.7	± 6.3	± 3.3	± 0.6	metres
T_z	-260.4	-263.8	-286.4	-263.6	-292.3	metres
σ	± 6.5	± 4.7	± 6.2	± 4.9	± 0.6	metres
ε_x	-0.31		-0.25			seconds
σ	± 0.24		± 0.30			seconds
ε_y	-0.29		-0.37			seconds
σ	± 0.21		± 0.27			seconds
ε_z	0.58	0.49	0.55			seconds
σ	± 0.22	± 0.20	± 0.28			seconds
k	8.4	8.4		8.5		ppm
σ	± 1.4	± 1.4		± 1.4		ppm
Δ	6.5	6.3	7.8	6.4	7.5	metres

Δ ≡ mean value of displacement after transformation

TABLE 5.1 BURSA MODEL DATUM TRANSFORMATION PARAMETERS

MODEL: BURSA

Data set: South Africa

Number of common points: 23

Point of rotation: Centre of Geodetic ellipsoid

F.P.: $x_G = y_G = z_G = 0.00$

VCV-matrix of the observables $\Sigma_L = I$

Col	1	2	3	4	5	6
$\bar{\sigma}_0$	2.91	3.93	2.89	3.87		
T_x	-198.5	-142.4	-189.9	-133.8		metres
σ	± 14.4	± 16.4	± 7.7	± 1.1		metres
T_y	-122.2	-96.1	-138.2	-112.1		metres
σ	± 12.6	± 16.3	± 3.7	± 1.1		metres
T_z	-259.8	-293.7	-257.9	-291.7		metres
σ	± 16.1	± 20.8	± 4.7	± 1.1		metres
ε_x	0.36	0.36				seconds
σ	± 0.34	± 0.45				seconds
ε_y	0.25	0.25				seconds
σ	± 0.58	± 0.79				seconds
ε_z	0.44	0.44				seconds
σ	± 0.45	± 0.61				seconds
k	11.1		11.1			ppm
σ	± 1.5		± 1.5			ppm
Δ	6.0	7.6	6.1	7.7		metres

$\Delta \equiv$ mean value of displacement after transformation

TABLE 5.2 BURSA MODEL NETWORK TRANSFORMATION PARAMETERS

TLN-X	1.0						
TLN-Y	0.2	1.0					
TLN-Z	-0.2	-0.4	1.0				
ROT-X	0.0	0.3	0.1	1.0			
ROT-Y	-0.5	0.2	-0.6	0.3	1.0		
ROT-Z	-0.3	0.6	-0.2	-0.4	0.0	1.0	
SCALE	-0.9	-0.5	0.7	0.0	0.1	0.0	1.0
	TLN-X	TLN-Y	TLN-Z	ROT-X	ROT-Y	ROT-Z	SCALE

TABLE 5.1.1 BURSA MODEL CORRELATION MATRIX [$u = 7, \Sigma_L \neq I$]
FOR THE DATUM TRANSFORMATION PARAMETERS

TLN-X	1.0						
TLN-Y	-0.4	1.0					
TLN-Z	0.6	-0.4	1.0				
ROT-X	0.1	0.4	0.4	1.0			
ROT-Y	-0.8	0.4	-0.9	-0.2	1.0		
ROT-Z	-0.7	0.9	-0.5	0.0	0.5	1.0	
SCALE	-0.5	-0.3	0.3	0.0	0.0	0.0	1.0
	TLN-X	TLN-Y	TLN-Z	ROT-X	ROT-Y	ROT-Z	SCALE

TABLE 5.2.1 BURSA MODEL CORRELATION MATRIX [$u = 7, \Sigma_L = I$]
FOR THE NETWORK TRANSFORMATION PARAMETERS

5.2.2 Analysis of results

5.2.2.1 Statistical tests: A number of statistical tests can be applied to the transformation adjustment, some of which will now be considered. Many variations of these tests can be found in the literature, and these usually differ in the degree of rigour involved. The tests used here are based on those given by Steeves and Fraser (1983):

5.2.2.1.1 Variance factor (VF) test. Given the degrees of freedom of the adjustment as ν, the a priori estimate of the variance factor (σ_0^2) as unity (1), compute the 95% (α=0.05) confidence region for the a priori VF (σ_0^2). This is given by the interval

$$\left[\ \nu.\hat{\sigma}_0^2 \ /\ \chi^2_{\nu,1-\alpha/2},\ \nu.\hat{\sigma}_0^2 \ /\ \chi^2_{\nu,\alpha/2}\ \right]$$

where $\hat{\sigma}_0^2$ is the a posteriori estimate of the VF, and $\chi^2_{\nu,1-\alpha/2}$, $\chi^2_{\nu,\alpha/2}$ are the chi-square values for the limits as indicated.

For the 7-parameter Bursa case in column 1 of Table 5.1 we have from tables the χ^2 values: $\chi^2_{62,0.975} \approx 85$; $\chi^2_{62,0.05} \approx 42$. So the interval (approximate) for σ_0^2 is [0.80, 1.62], and includes the a priori value of 1. Hence the variance factor passes the test at the 95% confidence level.

In Table 5.1, cases 1, 2 & 4 pass the VF test, but cases 3 & 5, in which the scale parameter is neglected, do not. This point is considered in more detail below. Not one of the cases in Table 5.2 passes the VF test, which is probably an indication that the weight model of Σ_L=I is not realistic.

5.2.2.1.2 Testing significance of parameter values. A method (though not quite rigorous since it neglects the correlation between parameters) that is often used for testing whether the least squares estimate of a parameter $(\bar{x})$ is significantly different from an a priori estimate (x), often taken as zero, is to compare it against its variance as follows:

$$y = (x-\bar{x})^2 / \bar{\sigma}_x^2 < F_{u,v,1-\alpha}$$

If this test is satisfied, then the least squares estimate $(\bar{x})$ is not significantly different from the a priori estimate (x) at the $(1-\alpha)\%$ level.

In the case given above for Bursa, Table 5.1, column 1, when testing for 1 parameter, $(u=1)$, we have from tables for the F-distribution the value $F_{1,62,0.95} \approx 4.0$. Hence if $(\bar{x}-x) < 4.0\ \bar{\sigma}_0^2$ then the parameter estimate $(\bar{x})$ is not significantly different from the initial estimate (x) at the 95% confidence level.

If we consider the seven parameters shown in Table 5.1, Column 1, we can see that the ε_x and ε_y parameters are not significantly different from zero (the initial estimates) at the 95% confidence level. All other parameters are numerically significant. These parameters $(\varepsilon_x, \varepsilon_y)$ are then constrained to zero for the next adjustment, the results of which are shown in Table 5.1, Column 2. All five remaining parameter estimates are significant in this adjustment, and the VF test is passed at 95% . $(\bar{\sigma}_0^2 = 1.10)$ These 5 remaining parameters are then used for subsequent transformations.

In Table 5.2, case 1, it is obvious that none of the rotation parameters are significant. These are neglected in case 3 (4-parameter solution).

5.2.2.2 Correlations amongst parameters. This information is contained in the correlation matrix of the estimated parameters. Some aspects of these correlations will now be discussed, but it must be pointed out that the correlations depend rather heavily on the VCV matrices (Σ_L) that are associated with the observables. This will be demonstrated by comparing the two cases, one where $\Sigma_L \neq I$ (Table 5.1.1), and one where $\Sigma_L = I$ (Table 5.2.1), corresponding to the two cases for the 7-parameter Bursa adjustments given in Tables 5.1 & 5.2. These correlation matrices are given in Tables 5.1.1 & 5.2.1.

With reference to Tables 5.1.1 and 5.2.1, the following points are noted: The existence of some correlations amongst the translations and the rotations themselves is perhaps surprising. In Table 5.1.1 the correlations between the translations and the rotations are fairly high (up to 0.6), but not as high as one might have expected. That this is due to the VCV matrix used here, is shown by comparing these results with those of Table 5.2.1, where the VCV matrix is the identity matrix I. Here the rotations are highly (up to 0.9) correlated with the translations. In Table 5.1.1 the scale corrrelates highly (0.5–0.9) with the translations, but in Table 5.2.1 these are much reduced. (0.3–0.5). The scale does not correlate with the rotations in either case.

5.2.2.3 Effect of correlations between parameters. When parameters are highly correlated, the values of the remaining parameters are affected when one or more are constrained to zero in the solution. This is not a desirable quality. This effect can be seen in the Bursa case in Table 5.1, when the 7-parameters solution is followed by a five parameter solution in which the ε_x and ε_y values are constrained to zero. Compare the values of the remaining parameters in column 2 with those of column 1. The variance factor $\bar{\sigma}_0^2$ remains virtually unchanged at 1.10. The translations change by a few (2-4) metres. The ε_z value changes from 0.58 to 0.49 seconds of arc, but the scale difference remains unchanged. These differences are not however significant at the 95% level. As mentioned above, this remaining set of 5 parameters is then used for subsequent transformations.

Table 5.2: Due to the much higher correlations between the translations and the rotations in this case, the changes in the translations when neglecting the insignificant rotation parameters, are much greater than those in Table 5.1. The translations in case 3 differ by about 9, 16 and 2 metres from those in case 1, while the scale remains unchanged.

A further point of interest in Table 5.2 is the marked changes in the precision estimates of the translations when the insignificant rotation parameters are neglected - compare cases 1 & 2 with cases 3 & 4. This does not occur in Table 5.1.

5.2.2.4 Effect of correlation of scale with translations. Due to the high correlation of scale with the translations, especially with the x-translation, the changes in these translations, when holding the scale fixed at zero as in columns 3 & 5 of Table 5.1, are very definitely significant. Note that these two adjustments (shown in columns 3 & 5, Table 5.1) do not pass the variance factor test at the 95% level. Since the scale is so large and significant, (8.4 ± 1.4 ppm) this parameter can obviously not be neglected, as this leads to large systematic discrepancies that cannot be modelled by any of the other parameters. This results in the failing of the variance factor test.

5.2.2.5 <u>Effect of number of parameters on "mean goodness of fit" after transformation</u>. From Table 5.1 it can be seen that the "mean goodness of fit" value Δ has the smallest value for the 5-parameter case in column 2 where $\varepsilon_x = \varepsilon_y = 0$. This is followed in ascending order (progressively larger values for Δ) by the 4-, 7-, 3- and 6-parameter cases (columns 4,1,5,3). This implies that the "mean goodness of fit" is not neccessarily improved by including more parameters in the adjustment. This is illustrated by the fact that the 6-parameter (3 translations, 3 rotations) case produces a poorer fit than any of the other cases, and the 7-parameter case is marginally poorer than the 5- and 4-parameter cases.

The implication seems to be that the best fit is produced by including only those parameters that are significant, and no more. The inclusion of insignificant parameters does not seem to make the fit much poorer, but the omission of a significant parameter, such as the scale factor in cases 3 & 5 (columns), leads to a relatively poor fit. Further, this may lead, as in this case, to the failing of the variance factor test.

The conclusion is again a confirmation of the following procedure: Compute a full 7-parameter adjustment . Test each parameter for significance. Re-run the adjustment constraining insignificant parameters to zero. If all remaining parameters are now significant, use this set of transformation parameters for subsequent transformations.

In Table 5.2, the best fit (marginally) is produced by the 7-parameter solution (case 1), followed closely by the 4-parameter solution (case 3). Cases 2 & 4, in which the scale parameter is neglected, again produces relatively poor fits.

In comparing Tables 5.1 & 5.2, it can be seen that the "mean goodness of fit" value (Δ) is smaller in Table 5.2 than in Table 5.1 for all similar cases (with the exception of the 3-parameter case – translation only). This point of interest confirms the relation between the "mean goodness of fit" and the VCV matrix used.

It can be shown that this "best fit" (in terms of the smallest value Δ) for this 7-parameter similarity transformation is achieved simply by assigning constant weights to all points of each network, irrespective of whether the two networks are weighted the same or not. The simplest way to achieve this (as above) is of course to use the identity matrix I as the VCV matrix Σ_L, ie. *all* coordinates of *both* networks given equal weight, hence solving for the "network transformation parameters".

5.2.2.6 Changes in correlation matrix. A few remarks will now be made regarding the changes in the correlation matrix when changing the number of parameters in the solution:

5.2.2.6.1 Translations. The correlations between the translation components are smallest in the 7- and 3-parameter (3 translations only) cases. These appear to increase for the other cases, and in the 4-parameter case (3 translations and scale) there is virtually complete correlation (0.98) between the translations. This is surprising, and could not be explained.

5.2.2.6.2 Translations and scale. The correlation between the scale and the translations appear to increase as the rotations are omitted, and in the 4-parameter solution (3 translations and scale) this correlation is complete (0.99).

5.2.2.6.3 Rotations. In the 6-parameter solution, (scale constrained to zero), the correlations between the y- and z-rotations and the translations increase as would be expected. The scale is not correlated with the rotations.

5.2.2.7 Comparison of *datum* transformation parameters (Table 5.1) with *network* transformation parameters (Table 5.2). The 7-parameter solutions differ as follows - cases 1 in both tables: The x-translations differ by about 20 metres, while the y- & z-translations differ by less than 1 metre. The x- and y-rotations have similar absolute magnitudes but they are of opposite signs. The z-rotations differ slightly (0."14). The scale changes from 8.4 ppm (Table 1) to 11.1 ppm (Table 2). The effect of using the VCV matrix $\Sigma_L = I$ for the "network tranformation parameters" is of course to give all points equal weights. Since the points that are far away from the NIP have a great influence on the scale and swing, these effects are particularly marked as shown here, particularly in the x-translation, x- and y-rotation and the scale. A further reason is that the terrestrial network is known to have rather poor scale in the North Western Cape region, and with this equal weighting, the effect thereof is not reduced.

When the two sets of parameters are tested individually for each parameter, not one of these differences are really significant. The (absolute) differences are generally less than the sum of the respective standard deviations (absolute values).

When comparing the parameter sets in which only significant parameters are present (case 2, Table 5.1 and case 3, Table 5.2), the following is noted:

The x- and z-translations, and the scale do not differ significantly. The y-translations differ by about 20 metres, which is more than twice the sum of the respective standard deviations. The z-rotation in case 2, Table 5.1 is significant, whereas it is not in case 3, Table 5.2. Hence it is submitted that these two sets are significantly different. It has however been pointed out that they do in fact represent entirely different approaches to the combination problem.

5.3 MOLODENSKY MODEL

5.3.1 Presentation of results

The results of a number of adjustments using the Molodensky model are summarised in the tables below. These are as follows: Tables 5.3 & 5.4 present the *quasi datum transformation parameters* ie. the VCV matrix $\Sigma_L \neq I$. In Table 5.3 the fundamental point (FP) of rotation is the (terrestrial) *network initial point* (NIP), whereas in Table 5.4, the FP is the *centre of gravity* (CG) of the terrestrial network. Tables 5.5 & 5.6 present the *network transformation parameters* ie. the VCV matrix $\Sigma_L = I$. In Table 5.5 the FP is the (terrestrial) *network initial point* (NIP) and in Table 5.6 the FP is the *centre of gravity* (CG) of the terrestrial network. A correlation matrix is associated with each table, and refers to the first solution or adjustment (column 1) in each case. Tables 5.3.1, 5.4.1, 5.5.1 & 5.6.1 are the correlation matrices respectively for the solutions in case (column) 1 of Tables 5.3, 5.4, 5.5 & 5.6.

These numerical results will be discussed under the same headings as in the Bursa case. Comparisons will be made and attention will be drawn to differences encountered.

MODEL: MOLODENSKY

Data set : South Africa

Number of common points: 23

Point of rotation: Network initial point (Geodetic)

F.P.: x_G= 4777935.98 y_G= 2280227.48 z_G= -3545622.42

VCV-matrix of the observables $\Sigma_L \neq I$

Col	1	2	3	4	5	6
$\bar{\sigma}_0$	1.10	1.10	1.38	1.14	1.41	
T_x	-135.9	-135.5	-134.9	-135.6	-134.5	metres
σ	± 0.5	± 0.4	± 0.6	± 0.5	± 0.5	metres
T_y	-110.5	-110.8	-110.6	-111.0	-111.0	metres
σ	± 0.6	± 0.5	± 0.7	± 0.5	± 0.6	metres
T_z	-293.5	-293.7	-292.1	-293.6	-292.3	metres
σ	± 0.6	± 0.5	± 0.6	± 0.6	± 0.6	metres
ψ_x	-0.31		-0.25			seconds
σ	± 0.24		± 0.30			seconds
ψ_y	-0.29		-0.37			seconds
σ	± 0.2		± 0.27			seconds
ψ_z	0.58	0.49	0.55			seconds
σ	± 0.22	± 0.20	± 0.28			seconds
k	8.4	8.4		8.5		ppm
σ	± 1.4	± 1.4		± 1.4		ppm
Δ	6.5	6.3	7.8	6.4	7.5	metres

Δ ≡ mean value of displacement after transformation

TABLE 5.3 MOLODENSKY MODEL QUASI DATUM TRANSFORMATION PARAMETERS [FP=NIP]

MODEL: MOLODENSKY

Data set: South Africa

Number of common points: 23

Point of rotation: Centre of gravity of the Geodetic network (Cartesian XYZ coordinates)

F.P.: x_G = 5049535.53 y_G = 2354738.96 z_G = −3045424.75

VCV-matrix of the observables $\Sigma_L \neq I$

Col	1	2	3	4	5	6
$\bar{\sigma}_0$	1.10	1.10	1.38	1.14	1.41	
T_x	-132.7	−133.1	−133.8	−133.3	−134.5	metres
σ	± 0.5	± 0.5	± 0.6	± 0.5	± 0.5	metres
T_y	−111.4	−110.8	−111.9	−110.4	−111.0	metres
σ	± 0.7	± 0.5	± 0.8	± 0.5	± 0.6	metres
T_z	−289.6	−289.5	−292.5	−287.4	−292.3	metres
σ	± 0.7	± 0.7	± 0.6	± 0.7	± 0.6	metres
ψ_x	−0.31		−0.25			seconds
σ	± 0.24		± 0.30			seconds
ψ_y	−0.29		−0.37			seconds
σ	± 0.21		± 0.27			seconds
ψ_z	0.58	0.49	0.55			seconds
σ	± 0.22	± 0.20	± 0.28			seconds
k	8.4	8.4		8.5		ppm
σ	± 1.4	± 1.4		± 1.4		ppm
Δ	6.5	6.3	7.8	6.4	7.5	metres

Δ ≡ mean value of displacement after transformation

TABLE 5.4 MOLODENSKY MODEL QUASI DATUM TRANSFORMATION PARAMETERS [FP=CG]

MODEL: MOLODENSKY

Data set: South Africa

Number of common points: 23

Point of rotation: Network initial point (Geodetic)

F.P.: x_G= 4777935.98 y_G= 2280227.48 z_G= -3545622.42

VCV-matrix of the observables Σ_L = I

Col	1	2	3	4	5	6
$\bar{\sigma}_0$	2.91	3.93	2.89	3.87		
T_x	-136.4	-133.3	-136.8	-133.8		metres
σ	± 1.6	± 2.1	± 0.9	± 1.1		metres
T_y	-113.2	-112.4	-112.9	-112.1		metres
σ	± 1.3	± 1.8	± 0.9	± 1.1		metres
T_z	-297.5	-291.9	-297.3	-291.7		metres
σ	± 1.4	± 1.6	± 1.1	± 1.1		metres
ψ_x	0.36	0.36				seconds
σ	± 0.34	± 0.45				seconds
ψ_y	0.25	0.25				seconds
σ	± 0.58	± 0.79				seconds
ψ_z	0.44	0.44				seconds
σ	± 0.45	± 0.61				seconds
k	11.1		11.1			ppm
σ	± 1.5		± 1.5			ppm
Δ	6.0	7.6	6.1	7.7		metres

Δ ≡ mean value of displacement after transformation

TABLE 5.5 MOLODENSKY MODEL NETWORK TRANSFORMATION PARAMETERS [FP=NIP]

MODEL: MOLODENSKY

Data set: South Africa

Number of common points: 23

Point of rotation: Centre of gravity of the Geodetic network (Cartesian XYZ coordinates)

F.P.: x_G = 5049535.53 y_G = 2354738.96 z_G = -3045424.75

VCV-matrix of the observables Σ_L = I

Col	1	2	3	4	5	6
$\bar{\sigma}_0$	2.91	3.93	2.89	3.87		
T_x	-133.8	-133.8	-133.8	-133.8		metres
σ	± 0.9	± 1.2	± 0.9	± 1.1		metres
T_y	-112.1	-112.1	-112.1	-112.1		metres
σ	± 0.9	± 1.2	± 0.9	± 1.1		metres
T_z	-291.7	-291.7	-291.7	-291.7		metres
σ	± 0.9	± 1.2	± 0.9	± 1.1		metres
Ψ_x	0.36	0.36				seconds
σ	± 0.34	± 0.45				seconds
Ψ_y	0.25	0.25				seconds
σ	± 0.58	± 0.79				seconds
Ψ_z	0.44	0.44				seconds
σ	± 0.45	± 0.61				seconds
k	11.1		11.1			ppm
σ	± 1.5		± 1.5			ppm
Δ	6.0	7.6	6.1	7.7		metres

Δ ≡ mean value of displacement after transformation

TABLE 5.6 MOLODENSKY MODEL NETWORK TRANSFORMATION PARAMETERS [FP=CG]

	TLN-X	TLN-Y	TLN-Z	ROT-X	ROT-Y	ROT-Z	SCALE
TLN-X	1.0						
TLN-Y	-0.1	1.0					
TLN-Z	0.3	0.1	1.0				
ROT-X	0.1	-0.5	0.0	1.0			
ROT-Y	0.5	0.0	-0.3	0.3	1.0		
ROT-Z	0.1	0.4	-0.1	0.4	0.0	1.0	
SCALE	-0.3	0.0	-0.4	0.0	0.1	0.0	1.0

<u>TABLE 5.3.1</u> MOLODENSKY MODEL CORRELATION MATRIX $[u=7, \Sigma_L \neq I]$
FOR THE QUASI DATUM TRANSFORMATION PARAMETERS
(Fundamental point = network initial point)

	TLN-X	TLN-Y	TLN-Z	ROT-X	ROT-Y	ROT-Z	SCALE
TLN-X	1.0						
TLN-Y	-0.3	1.0					
TLN-Z	0.4	0.2	1.0				
ROT-X	-0.3	0.6	-0.1	1.0			
ROT-Y	-0.4	0.3	0.2	0.3	1.0		
ROT-Z	0.3	-0.5	0.0	-0.4	0.0	1.0	
SCALE	0.4	0.1	0.7	0.0	0.1	0.0	1.0

<u>TABLE 5.4.1</u> MOLODENSKY MODEL CORRELATION MATRIX $[u=7, \Sigma_L \neq I]$
FOR THE QUASI DATUM TRANSFORMATION PARAMETERS
(Fundamental point = Centre of gravity)

	TLN-X	TLN-Y	TLN-Z	ROT-X	ROT-Y	ROT-Z	SCALE
TLN-X	1.0						
TLN-Y	0.3	1.0					
TLN-Z	-0.3	-0.2	1.0				
ROT-X	-0.1	-0.6	0.2	1.0			
ROT-Y	0.8	0.3	-0.6	-0.2	1.0		
ROT-Z	0.4	0.5	-0.3	0.0	0.5	1.0	
SCALE	-0.3	-0.1	-0.6	0.0	0.0	0.0	1.0

TABLE 5.5.1 MOLODENSKY MODEL CORRELATION MATRIX $[u=7,\Sigma_L=I]$
FOR THE NETWORK TRANSFORMATION PARAMETERS
(Fundamental point = network initial point)

	TLN-X	TLN-Y	TLN-Z	ROT-X	ROT-Y	ROT-Z	SCALE
TLN-X	1.0						
TLN-Y	0.0	1.0					
TLN-Z	0.0	0.0	1.0				
ROT-X	0.0	0.0	0.0	1.0			
ROT-Y	0.0	0.0	0.0	-0.2	1.0		
ROT-Z	0.0	0.0	0.0	0.0	0.5	1.0	
SCALE	0.0	0.0	0.0	0.0	0.0	0.0	1.0

TABLE 5.6.1 MOLODENSKY MODEL CORRELATION MATRIX $[u=7,\Sigma_L=I]$
FOR THE NETWORK TRANSFORMATION PARAMETERS
(Fundamental point = centre of gravity)

5.3.2 Analysis of results

From the comparison of equivalent solutions in Table 5.3 (network initial point used) & Table 5.4 (centre of gravity of network used) [and the Bursa model in Table 5.1], the following points are noted:

5.3.2.1 Numerical values of parameters. The rotation and scale parameters have identical values and precision estimates for the two versions of the Molodensky model, and equal those for the Bursa model. The translations differ by a few metres for the two versions, but their precision estimates are almost identical. These translations are however substantially different from the Bursa translations, and their precision estimates are about an order of magnitude smaller. In the 3-parameter (translations only) solution, the translations and their precision estimates are identical for the two versions and equal those for the Bursa model, as shown algebraically in section 3.2.1.

5.3.2.2 A posteriori variance factor ($\bar{\sigma}_0^2$). These values are identical (for equivalent solutions) for both versions and for the Bursa model.

5.3.2.3 "Mean goodness of fit" value after transformation. The mean difference value after transformation, Δ, is identical for both versions and for the Bursa model. The implication appears to be that the "goodness of the fit" after transformation is independent of any particular location of the fundamental point, and further, that the Molodensky (both versions) and Bursa models produce identical results after transformation, ie. the internal adjustments are identical.

5.3.2.4 Statistical tests.

5.3.2.4.1 Variance factor test. Since the $\bar{\sigma}_0^2$ values are identical for equivalent solutions (also to those from the Bursa model), it is clear that solutions 1, 2 & 4 of both Tables 5.3 & 5.4 pass this test, whereas solutions 3 & 5 (in which scale is omitted) do not.

5.3.2.4.2 Testing significance of parameter values. It is clear (on comparison with the Bursa model) from the 7-parameter solutions in columns 1 of Tables 5.3 & 5.4, that the x- and y-rotations are not significantly different from zero at the 95% level. These are neglected in the 5-parameter solutions in columns 2, in which all remaining parameters are significant.

5.3.2.5 Correlations amongst parameters. The comparison of the correlation matrices in Tables 5.3.1 & 5.4.1 [and the Bursa case in Table 5.1.1] for the *quasi datum transformation parameters* ($\Sigma_L \neq I$), shows the following:

That part of the matrix relating to the correlations between the rotation and the scale parameters, are identical. The remaining correlations in columns 1-3 are generally slightly smaller in Table 5.3.1 compared to Table 5.4.1.

The correlations between the rotations and the translations are less in Tables 5.3.1 & 5.4.1 compared to Table 5.1.1 (Bursa model) [but this difference (when $\Sigma_L \neq I$) is not nearly as marked as the difference between Tables 5.6.1 and Table 5.2.1 (Bursa model), when $\Sigma_L = I$, which again confirms the effect that the weight model has on the correlations].

The scale correlates considerably less with the translations in Tables 5.3.1 & 5.4.1 compared to Table 5.1.1 (Bursa).

Hence it appears from the correlation matrices that the Molodensky model reduces (to a certain extent) the correlations of the rotation- and scale parameters with the translations.

The comparison of Tables 5.5.1 & 5.6.1 (for the network parameters, ie $\Sigma_L=I$) with Tables 5.3.1 & 5.4.1 (datum parameters, $\Sigma_L \neq I$) respectively, shows the following:

Table 5.5.1 ($\Sigma_L=I$,) shows higher values for the correlations between the rotations and translations and between the scale and the translations compared to Table 5.3.1 ($\Sigma_L \neq I$). In both Tables the fundamental point used is the network initial point.

Table 5.6.1 ($\Sigma_L=I$; rotation point is the CG of terrestrial network)) shows some very interesting results that are markedly different from table 5.5.1 (also $\Sigma_L=I$; rotation point is the network initial point). All correlations between parameters in Table 5.6.1 are zero, with the exception of the x- and y-rotations (0.2) and the y- and z-rotations (0.5). This is thus the only case so far (see Veis case later) in which the correlations between parameters have been (virtually) completely removed. The reason for the existence of the two correlations mentioned here is not clear to this author, and no mention of this fact has been found in the literature thus far. This fact was confirmed by an independent transformation adjustment computed by Newling (unpublished memorandum,1986)

The marked changes in the correlation matrices of Tables 5.6.1 and 5.4.1, for which the same fundamental point (NIP) was used, shows the *effect of using different VCV matrices* (Σ_L). Similarly, the *effect* that the *selection of the fundamental point* has on the correlation matrices, is shown by the comparison of Tables 5.6.1 and 5.5.1, in which identical VCV matrices were was used.

The implications of these results appear to be as follows: When it is desirable to have the smallest possible correlations between parameters, it is not sufficient to have a balanced distribution of the data points about the fundamental point used (here the CG of the terrestrial network), but the fundamental point used should be the centre of gravity of the *weighted* data points. If all points of each network are weighted equally, then the centre of gravity of the terrestrial network data points gives this optimal solution regarding smallest correlations.

NOTE: It should just be stressed again here that there is an important conceptual difference between the (*quasi*) *datum transformation parameters* ($\Sigma_L \neq I$) and the *network* ($\Sigma_L = I$) *transformation parameters*. If the object is to obtain the "best" internal adjustment for the similarity transformation (ie. smallest value Δ) and to have smallest correlations between parameters, then the procedure suggested above is appropriate, ie. when solving for the *network* transformation parameters.

5.3.2.6 Effect of correlations between parameters.
The comparison of the different solutions (columns 1-5) in Tables 5.3, 5.4, 5.5 & 5.6 shows the favourable effect that the smaller correlations between parameters in the Molodensky model have on the stability of the remaining parameter values when neglecting some parameters. The most important effects are due to the reduced correlations between the rotation- and translation parameters and between the translation- and scale parameters, as compared to the Bursa model.

In Tables 5.3 & 5.4 ($\Sigma_L \neq I$), the translations change by a few metres (cf. Table 5.1, where the translations change by up to 40 metres), the rotations by less than 0.1 arc seconds, and the scale by 0.1 ppm between the different solutions. These rotation and scale changes are identical to those in the Bursa model. In Table 5.5 (Σ_L), the translations similarly change by a few metres, but the rotations are not affected by the omission of the (constant) scale parameter.

In Table 5.6 ($\Sigma_L = I$), where all remaining parameters are unchanged for all solutions, it is quite clear that none of the three groups of parameters, the translations, rotations and scale, affect one another at all. The translations are totally unaffected by the omission of either the rotations or the scale parameter, although their precision estimates are marginally affected. The rotation- and scale parameters are similarly unaffected by each other.

Table 5.6 ($\Sigma_L = I$) contrasts most markedly with Table 5.2 ($\Sigma_L = I$) for the Bursa model, in which the translations are substantially affected by the other parameters.

These results confirm that the *Molodensky model reduces* the *correlations between the translations and the rotation- and scale parameters* as stated above.

5.3.2.7 Changes in correlation matrix. A few remarks will now be made regarding the changes in the correlation matrix when changing the number of parameters in the solution. The most remarkable fact is that in the Molodensky model, when using $\Sigma_L \neq I$, the correlations between the translations themselves and between the scale and the translations, are much reduced (less than 0.6) in the *4-parameter solution* as compared to the Bursa model (0.98+).

In the Molodensky model, when using $\Sigma_L = I$, and the centre of gravity of the terrestrial network as the fundamental point, all these correlations are zero. This is not the case when using the network initial point as the fundamental point and $\Sigma_L = I$, in which case these correlations are similar to those of Molodensky when using $\Sigma_L \neq I$.

5.4 VEIS MODEL

5.4.1 Presentation of results:

The results of a number of adjustments using the Veis model are summarised in Tables 5.7 & 5.8 below. Table 5.7 presents the *quasi datum transformation parameters* (ie. $\Sigma_L \neq I$) for a number of different cases (columns 1-5). The correlation matrix corresponding to the 7-parameter solution in column 1 of Table 5.7 is presented in Table 5.7.1 below. Table 5.8 presents the *network transformation parameters* (ie. $\Sigma_L = I$) for a number of different cases (columns 1-4). The correlation matrix corresponding to the 7-parameter solution in column 1 of Table 5.8 is presented in Table 5.8.1 below. The *fundamental point* (FP) of rotation used in both Tables 5.7 & 5.8 is the *initial point* of the second (terrestrial) network (NIP).

Note that in the Veis model the *order of the rotations* are different, namely given as rotations around the z_{LG}-axis (dA), the y_{LG}-axis (dμ) and the x_{LG}-axis (dν).

These numerical results will be discussed and compared to the other two models used and attention drawn to any differences encountered. However, since the Veis model is so similar to the Molodensky model, many of the features of this model is the same as the Molodensky model and as such will not be treated in the same detail.

MODEL: VEIS

Data set: South Africa

Number of common points: 23

Point of rotation: Network initial point (Geodetic)

F.P.: x_G= 4777935.98 y_G= 2280227.48 z_G= -3545622.42

VCV-matrix of the observables $\Sigma_L \neq I$

Col	1	2	3	4	5	6
$\bar{\sigma}_0$	1.10	1.11	1.38	1.14	1.41	
T_x	-135.9	-135.8	-134.9	-135.6	-134.5	metres
σ	± 0.5	± 0.4	± 0.6	± 0.5	± 0.5	metres
T_y	-110.5	-110.4	-110.6	-111.0	-111.0	metres
σ	± 0.6	± 0.6	± 0.7	± 0.5	± 0.6	metres
T_z	-293.5	-293.5	-292.1	-293.6	-292.3	metres
σ	± 0.6	± 0.5	± 0.6	± 0.6	± 0.6	metres
dA	0.66	0.68	0.63			seconds
σ	± 0.28	± 0.29	± 0.36			seconds
dμ	0.13		0.23			seconds
σ	± 0.19		± 0.23			seconds
dν	-0.26		-0.24			seconds
σ	± 0.19		± 0.24			seconds
k	8.4	8.5		8.5		ppm
σ	± 1.4	± 1.4		± 1.4		ppm
Δ	6.5	6.6	7.8	6.4	7.5	metres

Δ ≡ mean value of displacement after transformation

TABLE 5.7 VEIS MODEL QUASI DATUM TRANSFORMATION PARAMETERS [FP=NIP]

MODEL: VEIS

Data set: South Africa

Number of common points: 23

Point of rotation: Network initial point (Geodetic)

F.P.: x_G = 4777935.98 y_G = 2280227.48 z_G = -3545622.42

VCV-matrix of the observables Σ_L = I

Col	1	2	3	4	5	6
$\bar{\sigma}_0$	2.91	3.93	2.89	3.87		
T_x	-136.4	-133.3	-136.8	-133.8		metres
σ	± 1.6	± 2.1	± 0.9	± 1.1		metres
T_y	-113.2	-112.4	-112.9	-112.1		metres
σ	± 1.3	± 1.8	± 0.9	± 1.1		metres
T_z	-297.5	-291.9	-297.3	-291.7		metres
σ	± 1.4	± 1.6	± 1.1	± 1.1		metres
dA	-0.12	-0.12				seconds
σ	± 0.32	± 0.43				seconds
dμ	-0.07	-0.07				seconds
σ	± 0.57	± 0.77				seconds
dν	-0.60	-0.60				seconds
σ	± 0.48	± 0.65				seconds
k	11.1		11.1			ppm
σ	± 1.5		± 1.5			ppm
Δ	6.0	7.6	6.1	7.7		metres

Δ ≡ mean value of displacement after transformation

TABLE 5.8 VEIS MODEL NETWORK TRANSFORMATION PARAMETERS [FP=NIP]

TLN-X	1.0						
TLN-Y	-0.1	1.0					
TLN-Z	0.3	0.1	1.0				
dA	-0.2	0.5	0.1	1.0			
dμ	-0.5	-0.2	0.3	0.0	1.0		
dν	-0.3	0.0	0.2	0.0	0.3	1.0	
SCALE	-0.3	0.0	-0.4	0.0	-0.1	0.0	1.0
	TLN-X	TLN-Y	TLN-Z	dA	dμ	dν	SCALE

TABLE 5.7.1 VEIS MODEL CORRELATION MATRIX [$u = 7, \Sigma_L \neq I$]
FOR THE QUASI DATUM TRANSFORMATION PARAMETERS
(Fundamental point = network initial point)

TLN-X	1.0						
TLN-Y	0.3	1.0					
TLN-Z	-0.3	-0.2	1.0				
dA	-0.2	0.6	0.0	1.0			
dμ	-0.8	-0.5	0.6	-0.1	1.0		
dν	-0.5	-0.2	0.3	-0.1	0.5	1.0	
SCALE	-0.3	-0.1	-0.6	0.0	0.0	0.0	1.0
	TLN-X	TLN-Y	TLN-Z	dA	dμ	dν	SCALE

TABLE 5.8.1 VEIS MODEL CORRELATION MATRIX [$u = 7, \Sigma_L = I$]
FOR THE NETWORK TRANSFORMATION PARAMETERS
(Fundamental point = network initial point)

5.4.2. Analysis of results

5.4.2.1 Comparison with Molodensky model. From the comparison of equivalent solutions in Table 5.7 and Table 5.3 (for the Molodensky model), *[with the very important exception of solution 2, about which more below]*, where the same fundamental point (network initial point) and VCV matrices ($\Sigma_L \neq I$) have been used in all solutions, the following are noted:

The translation- and scale parameters have identical values and precision estimates for the Veis and Molodensky models.

The rotations are of course different, since they are reckoned around the axes of two non-parallel systems. The Veis rotations are reckoned around the axes of the local geodetic (LG) system (at the topocentric network initial point), where as the Molodensky ones refer to a system which is nominally parallel to the geodetic system axes. The Veis rotations are given in the order dA, $d\mu$, $d\nu$ being rotations around the z_{LG}-, y_{LG}-, and x_{LG}-axes, and representing a rotation in azimuth and tilts in the meridian- and prime vertical planes respectively as mentioned above.

When the Veis rotations are rotated into the same system as that in which the Molodensky ones are given (by the use of the appropriate rotation matrices given in section 3.3.1), then identical values are obtained. This was confirmed numerically in this study.

The Veis model gives identical values for the a posteriori variance factor, ($\bar{\sigma}_0^2$), and for the mean "goodness of fit" value, (Δ), as the Molodensky model, and hence also equal those for the Bursa model.

5.4.2.2 Remarks on solution 2 in Table 5.7. The comparison of the results of the 5-parameter solutions in columns 2 of Table 5.7 and Table 5.3 (Molodensky model), illustrates an important point. Both these solutions retain only the significant parameters, the insignificant ones (the rotations around the x-axis and the y-axis, see column 1) have been constrained to zero. Note that in the Veis case (Table 5.7), it is the dμ & dν rotations that are not significant.

Now, it can be seen from the mean "goodness of fit" value, Δ, and the a posteriori variance factor, $\bar{\sigma}_0^2$, and more clearly from individual "goodness of fit" values for the common points, that in this particular case, the Veis and Molodensky models have *not* produced identical transformations. The reason is that, due to the different orientations of the rotation axes of the Veis and Molodensky models, the remaining (significant) z-rotation, dA, in the Veis model, cannot produce identical results to the remaining z-rotation in the Molodensky model. Hence these two particular solutions of each model are not equivalent. Note that the Molodensky model produced slightly smaller values for $\bar{\sigma}_0^2$ and Δ in this particular case, but this fact is not, singly, regarded as significant.

The point raised by the above solution was tested by using the centre of gravity of the terrestrial network as the fundamental point in the Veis model. The comparison of this case with that in which the network initial point was used (Table 5.7), led to the following results:

The two solutions produce identical transformations for all cases where all three rotation parameters, as a group, are either present or omitted. When only one or two of these rotation parameters are present, the transformations are not identical. The reason is that the *orientation of the rotation axes of the local geodetic system is a function of the position of the fundamental point used.*

These findings are contrasted with those of the Molodensky model in which *all* equivalent solutions produce identical transformations. The reason is that, in this model, the orientation of the rotation axes (nominally parallel to the CT axes) are constant and independent of any particular fundamental point used.

5.4.2.3 Summary. The results given above lead to the following generalisation:

The Bursa and Molodensky models produce identical (internal) transformations for all equivalent solutions. The Veis model will produce transformations identical to those of the Bursa and Molodensky models for all equivalent solutions in which all three rotation parameters are either present or omitted as a group. This fact is independent of the particular fundamental point used, and independent of the orientation of the axes around which the three rotation parameters are reckoned.

5.4.2.4 Statistical tests.

5.4.2.4.1 Variance factor test: Since the $\hat{\sigma}_0^2$ values are identical with the Molodensky model for equivalent solutions (with the exception of column 2, but this solution does still pass this test), it is obvious that solutions 1 and 4 of Table 5.7 ($\Sigma_L \neq I$) pass this test (at 95% level), whereas solutions 3 and 5 (in which scale is omitted) do not. It must be pointed out that not one of the solutions in Table 5.8, for the network parameters, passes this test, again (probably) indicating that the ($\Sigma_L = I$) weight model used was unrealistic.

5.4.2.4.2 Testing significance of parameter values. From the 7-parameter solution in column 1 of Table 5.7, it is seen that the rotation parameters, $d\mu$ & $d\nu$, are not significantly different from zero at the 95% level. Hence these are constrained to zero in the second solution, in which all remaining parameters are now significant.

5.4.2.5 Correlations amongst parameters. The Veis model produces a correlation matrix (eg. Table 5.7.1) which is similar in character to that of the Molodensky model (eg. Table 5.3.1) The correlations amongst the translations and between the translations and the scale is identical for both models. The correlations amongst the rotations and between the rotations and the other parameters are different to those in the Molodensky model, due obviously to the different orientation of the rotation axes.

When the centre of gravity of the terrestrial network is used as the fundamental point, and the VCV matrix $\Sigma_L = I_s$ (the results are not presented here) the Veis model again produces a correlation matrix, very similar to the Molodensky model, in which all correlations between parameters are zero, with the exception of that between the x-rotation ($d\nu$) and the y-rotation ($d\mu$), of 0.5. The reason for this correlation is not clear to the author. (cf Molodensky model)

5.4.2.6 Effect of correlations between parameters. The Veis model, similarly to the Molodensky model, removes the high correlations between the translations and the rotations and between the translations and the scale. The effect of this reduced correlations can be seen in Tables 5.7 & 5.8. The translations are affected much less by the omission of either the scale or the rotation parameters, as compared to the Bursa model.

Since this model is equivalent in most aspects to the Molodensky model, most of the remarks regarding the correlation matrices may be seen to be applicable to the Veis model as well. These will therefore not be repeated here.

5.5 SUMMARY & COMPARISON OF BURSA, MOLODENSKY AND VEIS MODELS

5.5.1 Descriptive

All three models use seven parameters namely three translations, three rotations and a scale difference, to model the differences between two sets of cartesian coordinates. The Bursa model is the standard seven parameter similarity transformation in which no a priori assumptions regarding parallelity of system axes are made. The Molodensky and Veis models are variations of this transformation in which there is the implicit a priori assumption of the parallelity and uniform scale of the geodetic (G) and Conventional Terrestrial (CT) system axes.

In the Bursa model, the rotations are reckoned around the discordant axes of the G system at the origin (0,0,0) of the G system (centre of the Geodetic ellipsoid). The position vectors of the common points of the second (terrestrial) system are rotated and scaled. Note that all points, including the fundamental point of the terrestrial network, are treated the same. In this model, the rotations and scale are therefore seen as applicable to the Geodetic *system*, and not the network.

In the Molodensky and Veis models, the implicit assumption is that the G system is both parallel and of the same scale as the CT system. Here it is the G network that is regarded as being discordant with the G (& CT) system. A fundamental point (FP), from which network difference vectors (of the terrestrial network) are reckoned, is used. These difference vectors are then scaled and rotated.

The Molodensky rotations are reckoned around axes at the FP which represent the orientation of the discordant network. These misalignment angles are very small, and therefore these axes are very nearly parallel to the G & CT system axes. In the Veis model, these rotations are around the axes of the Local Geodetic (LG) system. The rotation and scale parameters are regarded as applying to the network, and not the system.

5.5.2 Parameter values

5.5.2.1 7-parameter solutions. The Bursa and Molodensky models produce identical values and precision estimates for the rotation and scale parameters. The translations are different, and the precision estimates of the Molodensky translations are generally an order of magnitude smaller than the Bursa translations. This does not mean that the Molodensky model produces more precise estimates of the Bursa translations, the Molodensky translations are fundamentally different.

The Veis and Molodensky models are equivalent, and the translations and scale parameters (and their precision estimates) are identical. The Veis rotations, when rotated into the Molodensky system, equal those of the Molodensky model.

5.5.2.2. 3-parameter solutions. When solving only for three translations, all three models produce identical values for these.

5.5.3 Correlations between parameters

The Bursa model suffers from having very high correlations between the translations and the rotations and between the translations and the scale. The reason for this is that the data for a regional (non-global) network covers such a small portion of the globe. The Molodensky and Veis models reduce these correlations (and in some cases removes them completely). These points are discussed in great detail in CHAPTER 5. The main advantage is that the omission of some parameters has little or no effect on the remaining ones.

5.5.4 Transformations

5.5.4.1 7-parameter transformations. All three models produce identical (internal) transformations, ie. the coordinates produced after transformation are identical, provided the models are applied properly. It is obvious that the parameter set used must be the one that pertains to that particular model.

It needs to be stressed here that, in the *Molodensky and Veis* models, the *parameter set* derived from the solution is *dependent on the particular fundamental point used* in the solution. It is therefore *imperative* that the *coordinates* of the *fundamental point be published with the parameter set*, if it is intended to be used for subsequent transformations. This will *ensure* the *proper application* of these models. This has unfortunately not invariably been done in the past, and has on occasion led to the improper use of these models.

The a posteriori variance factors are identical, as are the residuals or corrections to the observables, in this case the coordinates, and therefore the adjusted coordinates (both the satellite and the terrestrial) are identical.

When using *fewer than seven parameters,* the Bursa and Molodensky models produce identical transformations for all equivalent solutions, due to the parallelity of the rotation axes used. The Veis model, however, will produce identical transformations for all solutions where the three rotation parameters are either all present or omitted.

5.5.5 Uses of models

Thomson (1976) recommended that the Bursa model be used for the combination of two satellite networks, but not for a satellite and terrestrial network. The Molodensky and Veis models would not be used for two satellite networks, but may be used for a satellite and terrestrial network. However, Thomson (1976) mentions that the assumption of the parallelity of the G and CT system axes may not be valid. He reasons that these three models are not adequate for the combination of terrestrial and satellite networks since *"they do not contain sufficient transformation unknowns to adequately describe the relationship between the two networks and their datums."* This was the reason why the more complex models containing more than one set of rotations were developed. Thomson (1976) notes that *"the latter three models [Hotine, Krakiwsky-Thomson, Vanicek-Wells] are far more flexible than the Bursa, Molodensky, and Veis models, and will thus more easily reflect physical reality."*

5.5.6 Physical significance of parameters

All three of these standard models suffer from the fact that they only have one set of rotation elements, and therefore the system and network rotations are confused. Therefore, caution is to be excercised when attempting to attach physical significance to the parameters that are estimated. This view is endorsed by the fact that these parameters depend on the model used, on the particular weighting scheme used (VCV matrices) and on the fundamental point used in the Molodensky and Veis models. The conclusions in this regard will be summarised in CHAPTER 6.

However, when the full 7-parameter sets pertaining to each model are simply used to transform a set of coordinates, the resulting transformed coordinates and their VCV matrices are identical, provided of course each model is used correctly.

It must be stressed here that such a parameter set is not to be used outside the area spanned by the common network points used in the derivation of the parameter set.

It can therefore be stated that the final transformed coordinates are independent of the particular model used, when the choice is between the Bursa, Molodensky and Veis models.

5.6 HOMOGENEITY OF DATA SET

The homogeneity of the data set used in this report is investigated for two main reasons. The first is to determine whether the residuals to the observables (ie. the coordinates of network points) from the adjustment can be modelled by an algebraic polynomial. This would obviously be advantageous as a means of interpolation. The second reason is to determine whether a single national set of transformation parameters can be used for (transformations for) the whole of South Africa, or whether regional (or sub-regional) sets are required to give satisfactory transformations.

5.6.1 <u>Modelling of residuals (resulting from a transformation adjustment)</u>

The residuals to the coordinates resulting from a transformation adjustment can be examined for any obvious trends. If such were to exist, they may be indicative of the effects of systematic errors in one or both the networks. Since we are concerned with the combination of a geodetic terrestrial and satellite or CT network, it is assumed that any systematic errors would pertain to the terrestrial network.

These trends could then possibly be modelled by low order algebraic polynomials, using the existing residuals as reference values.

This procedure was attempted for South Africa. The terrestrial G network and satellite Doppler or CT networks were combined (using the common points) in a transformation adjustment using the Bursa 7-parameter similarity transformation model, the results of which are shown in column 1, Table 5.9 given below under section 5.6.2. The VCV matrix used for the combined observables was the identity matrix, I. Residuals result for both the G and CT sets of cartesian coordinates, and these are of equal magnitude, but of opposite sign. These residuals in cartesian form are transformed to ellipsoidal form using differential formulae, which residuals are shown in Figures 5.1 & 5.2 respectively.

From Figure 5.1, the ranges in the cartesian residuals are [-2.3;3.3], ie. 5.6 m in x, [-6.1;2.9], ie. 10.0 m in y, and [-3.6;4.9], ie. 8.5 m in z. From Figure 5.2, these are in ellipsoidal form [-4.0;5.8], ie. 9.8 m in ϕ, [-5.8;2.2], ie. 8.0 m in λ, and [-0.6;0.8], ie. 1.4 m in h.

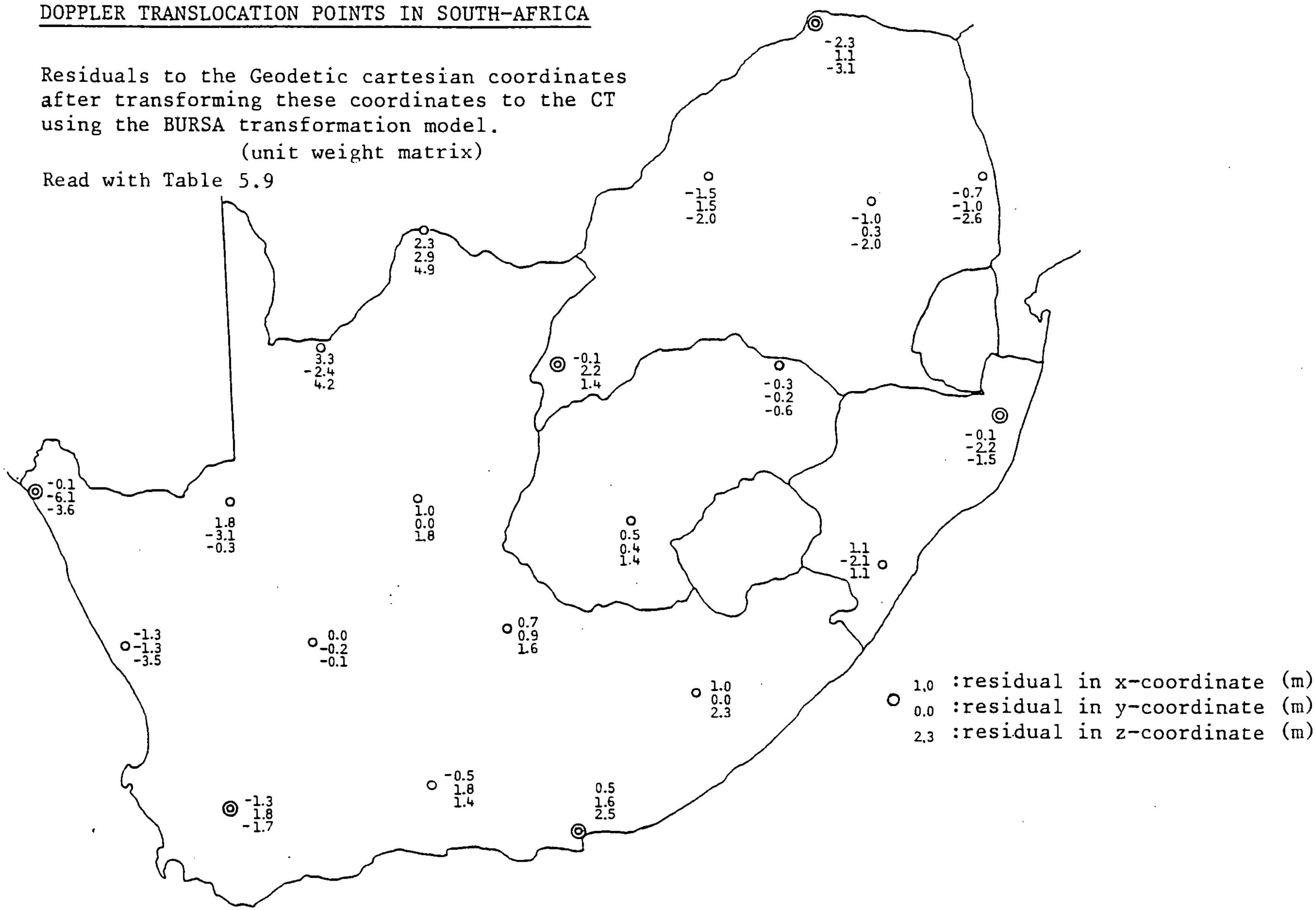
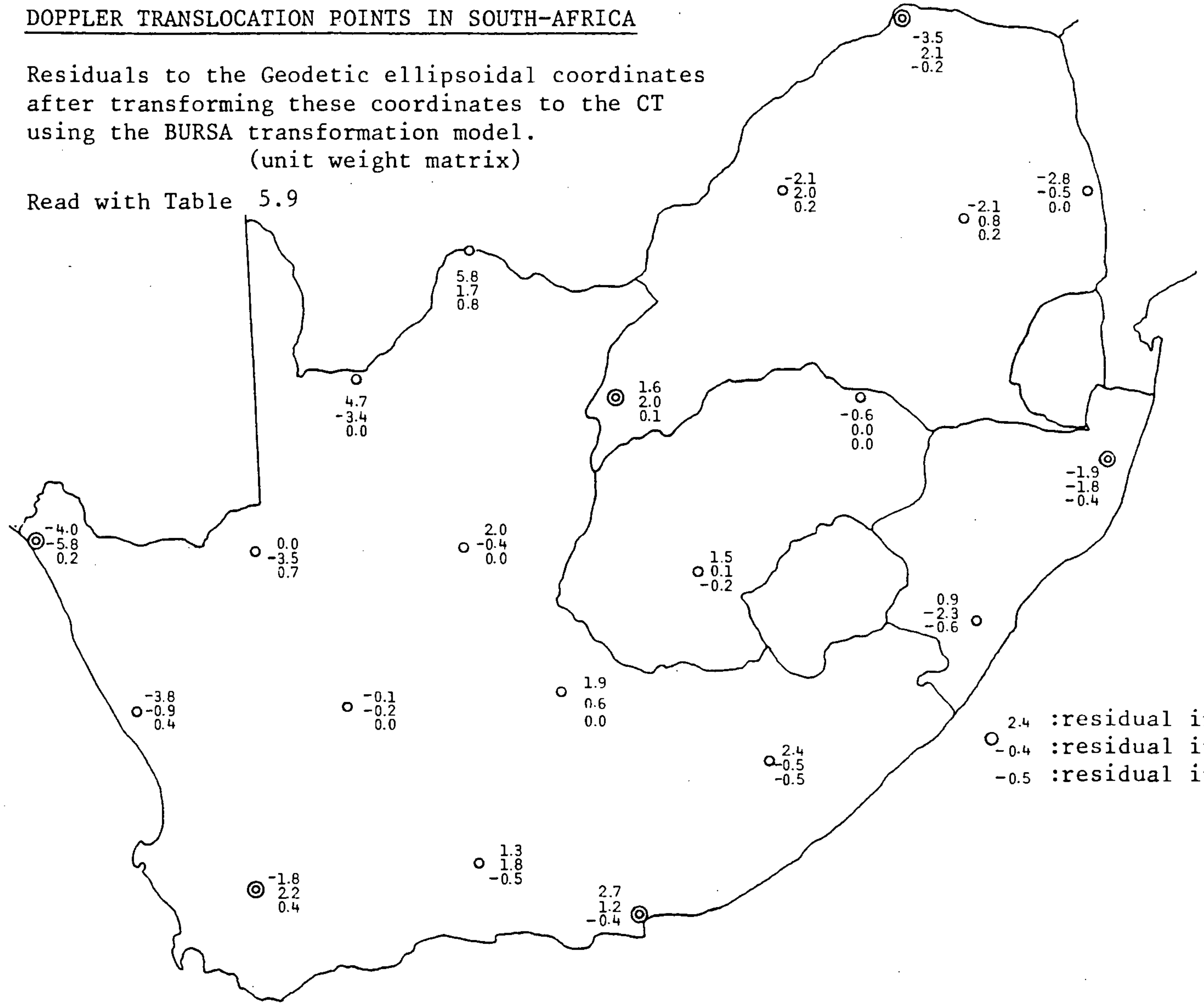


Figure 5.1 Geodetic cartesian residuals for Southern Africa

DOPPLER TRANSLOCATION POINTS IN SOUTH-AFRICA

Residuals to the Geodetic ellipsoidal coordinates after transforming these coordinates to the CT using the BURSA transformation model.
(unit weight matrix)

Read with Table 5.9



2.4 :residual in latitude, ϕ (m)
-0.4 :residual in longitude, λ (m)
-0.5 :residual in ell. height, h (m)

Figure 5.2 Geodetic ellipsoidal residuals for Southern Africa

An examination of the ellipsoidal residuals leads to the following observations: The latitude residuals display a ridge of highs running roughly NW-SE through the middle of the country, with lows on either side, ie. to the SW and NE. The longitude residuals are much more random and do not indicate any obvious pattern. The ellipsoidal height residuals are considerably smaller than either of the latitude or longitude, and appear to be positive over most of the country except for some 5 points along the southern and eastern coastline. In addition, these heights seem to indicate a slope from the NW down towards the SE.

From a consideration of the position of South Africa on the globe, it is obvious that the y-coordinate is strongly correlated with longitude, and that the x- and z-coordinates are correlated with both latitude and height. Hence the tendency is for these cartesian residuals to appear even more random than the ellipsoidal ones.

An initial attempt was made to model these residuals, in both forms, with low order algebraic polynomials. However, these attempts were only marginally successful for the latitudes, but of very little use for the others. Consequently these ideas were abandoned as it was felt that the residuals could not be modelled sufficiently well and that the use of such polynomials would therefore be misleading. It was decided to rather show these residuals on a small scale map of the area concerned, such as Figures 5.1 and 5.2, where an appreciation could be formed of the actual variation. These maps could then be used for a limited amount of (cautious) interpolation when required. The rather random variation of these residuals should be viewed as indicating (at least partially) the results of the "patchwork" style of adjustment of the geodetic terrestrial network, referred to earlier in this report.

The mean "goodness of fit"' value, Δ, indicates that an average accuracy of transformation is about 6 metres when using a single set of parameters for the whole country. This is to be compared with the results of the regional and sub-regional data sets used below.

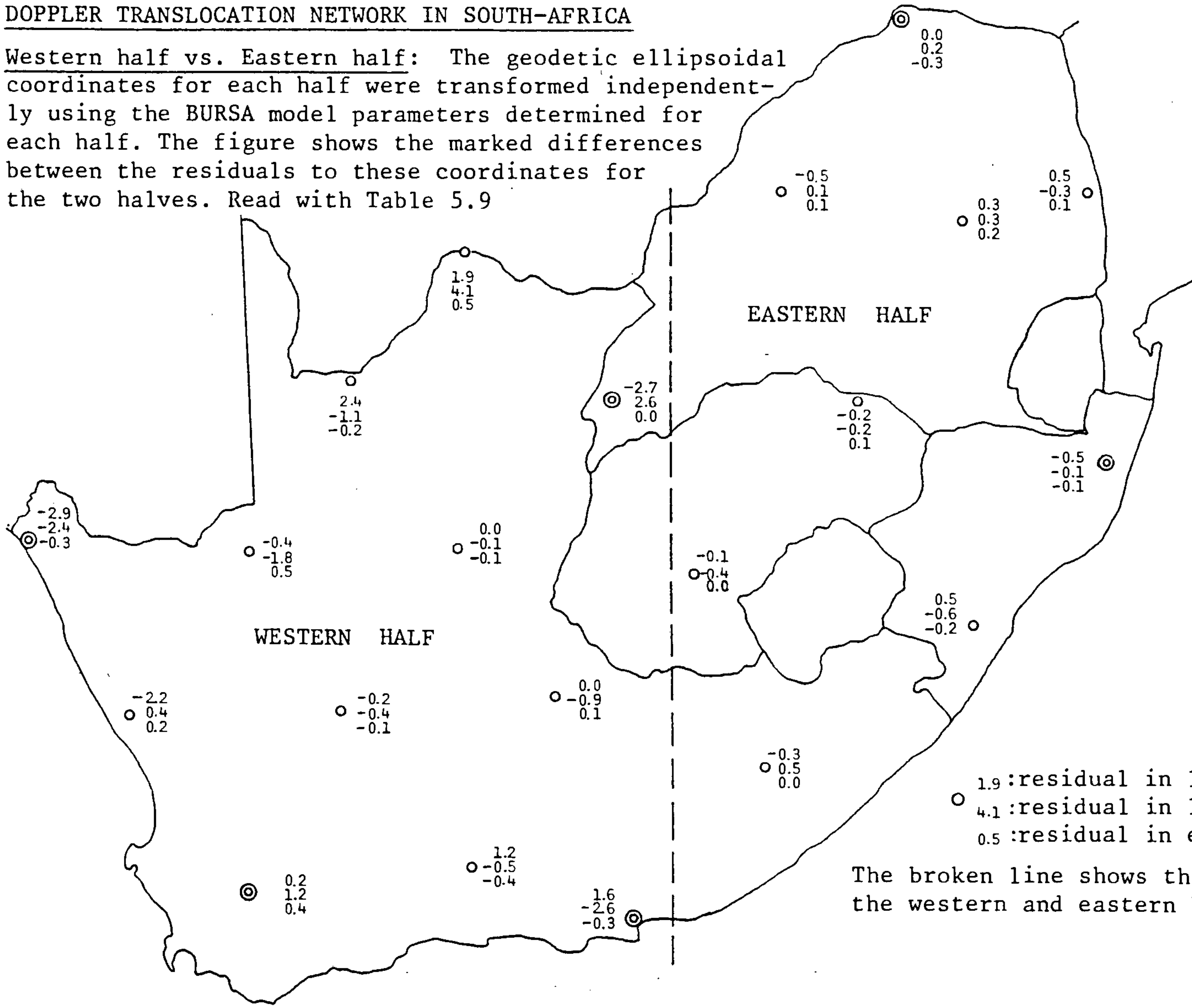
5.6.2 Variation of parameter sets (for different regions)

A set of transformation parameters can be used to transform coordinates from one system or network to another. If the area covered by the common points used for the derivation of the parameter set is very large, it must be ascertained with certainty that this parameter set is in fact representative of all regions of this area. Since a similarity transformation is basically an interpolation procedure, the estimated parameters will represent the overall average translations, rotations and scale difference. If the network is not homogeneous, parameter sets should be estimated for different regions and tested for consistency. Should these differences be significant, the use of a single parameter set for the whole area is not justified.

The procedure suggested above was applied to South Africa. The country was firstly divided into two halves along (approximately) the line of 26 degrees east longitude (refer to Figure 5.3). A 7-parameter set was estimated independently for each of the western (WH) and eastern halves (EH), and these are shown in columns 2 and 3 of Table 5.9 below. Thereafter the country was divided into four regions. These are shown in Figure 5.4 and will be referred to as the South Western Quarter (SWQ), the North Western Quarter (NWQ), the North Eastern Quarter (NEQ) and the South Eastern Quarter (NEQ) respectively. Seven-parameter sets were again estimated independently for each region, and are presented in columns 4-7 respectively of Table 5.9.

DOPPLER TRANSLOCATION NETWORK IN SOUTH-AFRICA

Western half vs. Eastern half: The geodetic ellipsoidal coordinates for each half were transformed independently using the BURSA model parameters determined for each half. The figure shows the marked differences between the residuals to these coordinates for the two halves. Read with Table 5.9



The broken line shows the division between the western and eastern halves.

Figure 5.3 Geodetic ellipsoidal residuals for the Western and Eastern halves

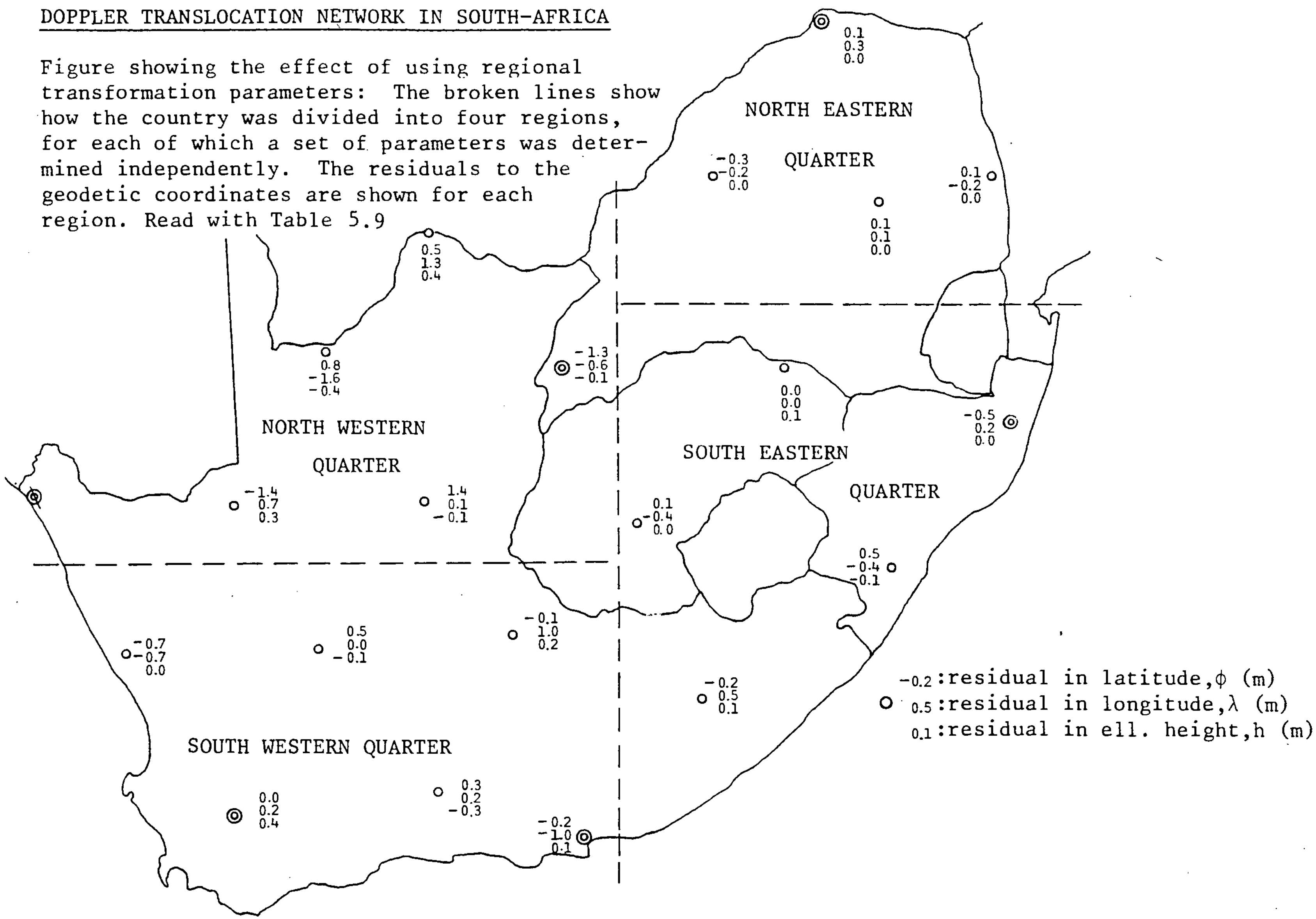


Figure 5.4 Geodetic ellipsoidal residuals for the four quarters

MODEL: BURSA

Dataset: South Africa

Point of rotation: Centre of geodetic ellipsoid

VCV-matrix of the observables' $\Sigma_L = I$

Col	1	2	3	4	5	6	7
DATA SET:	S.A.	WH	EH	SWQ	NWQ	NEQ	SEQ
$\bar{\sigma}_0$	2.91	2.27	0.49	0.74	1.70	0.36	0.55
T_x	-198.5	-242.3	-140.8	-187.7	-317.6	-142.9	-134.0
σ	± 14.4	± 17.0	± 5.5	± 11.1	± 40.7	± 9.7	± 11.8
T_y	-122.2	-144.9	-95.8	-131.0	-184.5	-94.5	-96.7
σ	± 12.6	± 21.3	± 6.9	± 11.0	± 38.4	± 10.4	± 10.5
T_z	-259.8	-225.4	-283.0	-254.0	-175.7	-307.3	-283.0
σ	± 16.1	± 17.8	± 4.5	± 14.7	± 47.6	± 14.2	± 12.5
ε_x	0.36	-1.55	1.59	-2.45	1.01	0.74	1.46
σ	± 0.34	± 0.54	± 0.15	± 0.36	± 1.04	± 0.35	± 0.29
ε_y	0.25	-0.64	0.59	-1.19	0.25	0.96	0.41
σ	± 0.58	± 0.61	± 0.16	± 0.50	± 1.69	± 0.44	± 0.45
ε_z	0.44	1.48	-0.38	1.87	-0.25	-0.06	-0.40
σ	± 0.45	± 0.67	± 0.24	± 0.33	± 1.32	± 0.32	± 0.37
k	11.1	20.5	0.2	10.7	35.9	-0.8	-0.2
σ	± 1.5	± 2.3	± 0.6	± 1.4	± 4.7	± 1.3	± 1.3
Δ	6.0	4.3	1.0	1.4	3.0	0.5	0.9

$\Delta \equiv$ mean value of displacement after transformation

TABLE 5.9 BURSA MODEL NETWORK TRANSFORMATION PARAMETERS FOR DIFFERENT REGIONS OF SOUTH AFRICA

5.6.2.1 <u>Western and Eastern halves</u>. With reference to columns 2 & 3 of Table 5.9, and Figure 5.3, the following points are noted:

The parameter values of the two sets differ considerably. It is noted that the difference is greater than the sum of the respective standard deviations for each of the parameters. In fact, when a statistical test is applied to the sets as units, it is confirmed that they are different at the 95% confidence level. Furthermore, these standard deviations are considerably smaller for the eastern than for the western region.

The $\bar{\sigma}_0$ value for the east (0.49) is much smaller than for the west (2.27) which indicates that the residuals for the eatern region are smaller, as confirmed by an examination of Figure 5.3. The range in the residuals for the two regions are as follows:

	WESTERN HALF	EASTERN HALF
Range in ϕ:	[-2.9;2.4] ie. 5.3 m	[-0.5;0.5] ie. 1.0 m
Range in λ:	[-2.6;4.1] ie. 6.7 m	[-0.6;0.5] ie. 1.1 m
Range in h:	[-0.4;0.5] ie. 0.9 m	[-0.3;0.2] ie. 0.5 m

The "mean goodness of fit" value Δ indicates that the average fit after transformation will be about 4.3 and 1.0 metres respectively for the Western and Eastern halves.

It should be noted that the scale in the WH is 20.5 parts per million (ppm) whilst that of the EH is a negligible 0.2 ± 0.6 ppm. A possible explanation is that scale in the EH is controlled by *seven* bases, whilst the WH has only *four*, excluding the *two* common bases (Wonnacott, 1985, 1986). A further point to note is that there are fewer latitude and longitude stations in the WH than in the EH in the original triangulation.

A consideration of the above results leads to the conclusion that the use of a single set of transformation parameters for the two halves of the country is not justified when attempting to obtain the best transformation with the available data.

5.6.2.2 The four quarters. With reference to columns 4-7 of Table 5.9 and Figure 5.4, the following points are noted:

The most western Doppler station, Witbank New, was rejected as shown in Figure 5.4. This point was excluded from the SWQ and NWQ since it does not really fall in either region. In addition, Wonnacott (1985) has identified an irregularity or uncertainty in the geodetic coordinates of this station, especially in the longitude. A possible reason for this is that this station is not on the main triangulation chain, but on some secondary triangulation on the far north west coast.

A comparison of the parameter values for the NEQ and SEQ shows that the differences in these values are much smaller than those for the two halves of the country treated above. These differences in the parameter values are in fact smaller than the sums of the respective standard deviations for all parameters except the rotation in x (R_x), and suggests that the two parameter sets are not significantly different. Since the number of points used here are so small (4-5), the statistical test used above is not particularly effective, and is not regarded as conclusive.

The $\bar{\sigma}_0$ values, the residuals and the "mean goodness of fit" values Δ for the two sets are of very similar magnitudes.

The ranges in the residuals are:

	NORTH EASTERN QUARTER	SOUTH EASTERN QUARTER.
Range in ϕ:	[-0.3;0.1] ie. 0.4 m	[-0.5;0.5] ie. 1.0 m
Range in λ:	[-0.2;0.3] ie. 0.5 m	[-0.4;0.5] ie. 0.9 m
Range in h:	[0.0;0.0] ie. 0.0 m	[-0.1;0.1] ie. 0.2 m

These residuals and the Δ values [0.5;0.9] show that a single parameter set for the whole Eastern region is justified, and would produce a transformation accuracy of about 1 metre.

A comparison of the parameter values for the SWQ and NWQ produces some interesting results. The differences in the parameter values are very large and indeed greater than the sums of the respective standard deviations for all parameters except R_y, which suggests that the parameter sets are significantly different. The statistical test, though not very effective, confirms that the sets are different at the 95% level. Note the very large scale difference of about 36 ppm for the NWQ as opposed to 11 ppm for the SWQ.

The ranges in the residuals are:

	SOUTH WESTERN QUARTER	NORTH WESTERN QUARTER
Range in ϕ:	[-1.4;1.4] ie. 2.8 m	[-0.7;0.5] ie. 1.2 m
Range in λ:	[-1.6;1.3] ie. 2.9 m	[-1.0;1.0] ie. 2.0 m
Range in h:	[-0.4;0.4] ie. 0.8 m	[-0.3;0.4] ie. 0.7 m

Although these residuals are quite similar (except for the latitude), the $\bar{\sigma}_0$ values of 0.7 (SWQ) and 1.7 (NWQ) and the Δ values of 1.4 (SWQ) and 3.0 (NWQ) metres seem to indicate that a transformation for the SWQ produces a better fit by half than for the NWQ.

5.6.2.3 <u>Conclusions on variations</u>. If independent transformation parameter sets are used for each of the North Western Quarter, the South Western Quarter and the Eastern half of South Africa, the mean accuracy of transformation between the terrestrial geodetic and Conventional Terrestrial coordinates will be in the order of 3.0, 1.5 and 1.0 metres respectively. This conclusion is however only based on the presently available data, and should be treated with caution.

Nevertheless, it can be stated conclusively that *the use of a single parameter set for the whole country is not justified (at present)* if the best transformation or fit possible is to be obtained with the presently available data.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 GENERAL

The combination of three dimensional networks of various types is a complicated operation. The methods studied in this report achieve this merger by solving for transformation parameters or external bias parameters between the two or more networks in a least squares solution using one of the "geometric" models. This procedure is called a transformation adjustment.

The selection of a particular model depends on the type and number of networks that are to be combined and on the purpose of the combination. These points are considered below.

Note that, before using the coordinates of a satellite network, all known biases of the satellite (S) system with respect to the Conventional Terrestrial (CT) are to be removed through the application of an appropriate transformation to the S system. This has invariably been done in this report, and since then the S and CT systems only differ by these known transformation parameters, the S and CT systems will be considered interchangebly in this section.

6.2 PURPOSE OF THE COMBINATION

Two or more three dimensional networks of different types are usually combined to exploit the strengths of each type. A number of advantages of combining S (CT) and terrestrial geodetic (G) networks were given in the Introduction.

The parameters estimated in a transformation adjustment do not neccessarily have real physical meaning. The parameter set may merely be a set of values used to transform one set of coordinates into another, ie. to merge the two networks. If however, one is additionally interested in recovering the relative position, orientation and scale differences of various networks (and datums) as real physical quantities, the matter requires careful investigation. This aspect will be treated first, and the merger or combination aspect thereafter.

6.2.1 Parameters as physical quantities

Since S networks truly represent their datums, no distinction is made between them. This is not so with classical G networks, and we distinguish between the network and the datum on which it is based. Thus when using the coordinates of common network points to combine G networks with other networks, the G network requires special treatment.

Two S networks can be combined using the standard Bursa model. Since this model also reflects physical reality in this case, the parameters should have real meaning. These parameters then are *datum* (and network) parameters.

The combination of a S and a G network should be done using the more complex models of Hotine and Krakiwsky-Thomson, since the additional set of rotation parameters enables these models to reflect physical reality more closely than any of the standard models, ie. Bursa, Molodensky or Veis, each of which have one set of rotations only. The more complex models use a second set of rotatons and a scale difference to model the systematic errors in the G network. These models thus estimate the position and alignment of the G and S (CT) systems (datums), as well as the alignment and scale difference between the G network and the G datum. Hence the parameters should have more realistic physical meaning than those from the standard models. However, these models require a special estimation technique, and a completely homogeneous terrestrial G network that covers a suitably large portion of the globe in order to realistically separate the two sets of rotations. As mentioned, this is a severe limitation in the case of the South Africa network.

When estimating the relative position, orientation and scale difference between the G network and the S (CT) network (or system), the coordinates of the G network are assigned equal variances, as are those of the S network, and in fact, the VCV matrix Σ_L used in this report was the identity matrix I. It is suggested that, even though the three standard models have only one set of rotations, they may quite adequately be used for this purpose. This was done in this report, and the resulting parameters identified as "network transformation parameters". Note that the three models yield parameters that are different (especially the Bursa ones as compared to the Molodensky and Veis ones), and should be interpreted cautiously in terms of the models used.

An attempt was made to use the G network coordinates to represent the G datum by adopting a weight model designed to reflect the effect of systematic errors in the G network. The Bursa model was used and the resulting parameters identified as "datum transformation parameters". If it were possible to know the correct VCV matrix for the G network, it is submitted that these "datum parameters" would give the proper relation between the G datum and the S (CT) system, even though the systematic errors in the G network will not be parameterised as in the Hotine and Krakiwsky-Thomson models.

As explained in Chapter 5, it does not make much sense to use this weight model with the Molodensky or Veis models as one is then violating the initial assumptions of these models. Nevertheless, this was done here as a matter of interest, and the resulting parameters identified as "quasi datum transformation parameters". These are compared to the Bursa parameters.

However, it should be stressed here that any attempt to assign physical meaning to any of the parameters estimated, irrespective of the model used, should be done with extreme caution.

6.2.2 Comparison of Bursa, Molodensky and Veis models.

The Veis model is mathematically equivalent to the Molodensky model, and the Veis rotations, given in terms of rotations around the axes of the Local Geodetic system, can be transformed to yield the Molodensky type rotations. With this in mind, it can be stated that the two models yield identical values and precision estimates for the translations, rotations and scale difference. These models are identical in all other aspects.

The Bursa rotations and scale difference are identical to those of the Molodensky (& Veis) model, and have identical precision estimates. However, the Bursa translations, being fundamentally different, are numerically different to those of the Molodensky (& Veis) model, and generally have precision estimates one order of magnitude greater.

One weakness of the Bursa model is the existence of very high correlations between the translations and the rotations and scale difference. The Molodensky (& Veis) model reduces these correlations, in some cases drastically, by estimating the rotations around a Fundamental Point (FP) of the G network, usually the centre of gravity (CG) or the network initial point (NIP). The correlations are, however, a function of the weight model used, and must be considered in this context. These correlations were treated in detail in CHAPTER 5.

6.3 COMBINATION OF GEODETIC AND SATELLITE (OR CT) NETWORKS

If the purpose of the combination is merely to merge the two networks without attaching any physical meaning to the estimated parameters, then the problem is simplified considerably. This merger can then be done by using any of the geometric models mentioned above, provided there are at least three translations, three rotations and a scale difference parameter present to model the network or datum differences, since these seven parameters completely model the differences between two perfect systems or networks. On the authority of Harvey (1985), there is nothing to be gained in this case by separating the rotations into components as is done in the Hotine and Krakiwsky-Thomson models.

Considering then the three standard models, it was shown in this report that they produce identical internal transformations, ie. the adjusted observations, their precision estimates and the a posteriori variance factor are identical. Thus these three models produce identical transformed coordinates when used to transform one set to another. If this is the only objective, it is immaterial which of the three are used. The high correlations between the parameters of the Bursa model are then entirely irrelevant. Since the Bursa model is the simplest, this would be the obvious choice.

6.4 NATIONAL / REGIONAL / SUB-REGIONAL PARAMETER SETS

The matter under consideration here is the use of a derived parameter set to transform coordinates of one system or network into another system or network. It was shown in this report that the G network in South Africa is not at present sufficiently homogeneous to justify the use of a single national parameter set for all transformations. Depending on the accuracy required from the transformation, it may be neccessary to derive a set of regional parameters, as shown in Chapter 5, or to derive parameter sets for smaller (sub-) regions. The residuals to the G coordinates resulting from such an adjustment may be presented as in Figures 5.1 - 5.4, and these may then be used for a limited amount of interpolation. However, due to the rather widespread data points and the existence of definite "blocks" of triangulation between which transformation parameters sometimes differ substantially, it would be unwise to do this for any large regions.

The results of a satellite survey could be transformed into the national G network by using any of the three standard models, provided that the coordinates of at least three known stations in that area are available in the satellite system to allow the computation of local transformation parameters.

It should be stressed here that, since a transformation of this nature is basically an interpolation procedure, a parameter set should not be used outside the area spanned by the common stations used for the derivation of that parameter set. However, since these models are geometric, the degredation of the transformation outside this area should be fairly graceful, and therefore a (rather) limited amount of "extrapolation" could perhaps be justified.

6.5 RESULTS OF TRANSFORMATION ADJUSTMENT

The results of a transformation adjustment are two sets of adjusted coordinates that differ by the adjusted set of parameters. The residuals to the coordinates, as well as the parameters, depend on the particular VCV matrix used in the adjustment. If therefore, the VCV matrix is realistic, then so will be the residuals and consequently also the adjusted coordinates (and the parameters).

The question then is which set of adjusted coordinates should be adopted. If the position, orientation and scale of the S (CT) system is desired, then those adjusted coordinates are adopted. If the orientation and scale of the S (CT) system is desired, but the position of the Geodetic system (network), then the adjusted Satellite (CT) coordinates are shifted by the values of the translations as estimated in the adjustment. In both cases these S (CT) coordinates will be used as weighted constraints in subsequent adjustments of the Geodetic network.

6.6 SUMMARY OF CONCLUSIONS

The *purpose* of the combination of a geodetic (G) and satellite (S) or CT network must be *clearly stated*. *Two satellite networks* can be combined with the *Bursa* model. A *geodetic* and *satellite* network should be combined with either the *Hotine* or the *Krakiwsky-Thomson* models. These models estimate the relative position and orientation of the G and S (CT) systems or datums, as well as the relative orientation and scale of the G network and its datum. The use of these models presupposes a homogeneous terrestrial network that covers a significant portion of the globe in order to obtain realistic estimates of the parameters.

The *Bursa, Molodensky* and *Veis* models give *identical transformed coordinates* when used to combine a G and S network even though the parameter sets are different. Interpretation of these parameters should however be done with great caution. The *Molodensky* and *Veis* models *reduce* the *high correlations,* between the translations and the rotation- and scale parameters, of the Bursa model. The *"datum transformation parameters"* and the *"network transformation parameters"* estimated in the Bursa model (section 5.2.2.7) were found to be *insignificantly different*. The distinction between these two sets lies in the *nature* of the *VCV matrix* of the observables.

The *homogeneity* of the South African terrestrial networks was investigated by estimating transformation parameters for different regions. When using parameter sets for each of the North Western Quarter, the South Western Quarter and the Eastern Half of the country, the average accuracy of transformation between the G and S networks was found to be 3.0, 1.5, 1.0 metres respectively using the presently available data. The *use of a single national parameter set* for the whole country is *not justified at present.*

6.7 RECOMMENDATIONS FOR THE FUTURE

It is quite obvious, even from a cursory reading of the writings of Rousseau (1986), Wonnacott (1985, 1986), Newling (1986) and others that a complete re-adjustment of the South African terrestrial geodetic networks is neccessary. Many of the hitherto unknown problem areas in the networks have come to light through comparison with the results of the first order network of traverses and the Doppler translocation survey. The variation in scale is but one of these problems that has to be resolved.

However glibly stated, such a re-adjustment of national networks, with all the associated ramifications, is a mammoth task, the enormity of which certainly cannot be fully appreciated by this author. Nevertheless, for the sake of completeness, the following remarks, although quite obvious, should be made regarding the re-adjustment:

A number of important questions need to be addressed amongst which are the size, shape, position and orientation of the new geodetic datum (ellipsoid) to be used (Newling, 1986), the dimensionality of the adjustment, the types of data to be incorporated, and many others.

The processing of all data presupposes that such data is in computer readable form, which is not the case for South Africa. This enormous task needs to be addressed.

Present and future survey projects need to be considered which should include survey data obtained from modern aids such as the Global Positioning System and Very Long Baseline Interferometry.

The re-adjustment of national/continental geodetic networks has been done in a number of countries such as North America and Australia. An in-depth study of the technical reports of such re-adjustments, produced by the National Survey Organisations of these countries, would obviously be of vital importance for gaining a proper appreciation of the complexities of the task. It is largely in this sphere that a recommendation for future studies must fall.

The task resting on the shoulders of the personnel of the Geodetic Branch, Surveys and Mapping Directorate, is indeed a great one.

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APPENDIX A

GEODETIC COORDINATE TRANSFORMATIONS

A.1 Transformation: $(\phi,\lambda,h) \rightarrow (x,y,z)$

The transformation from geodetic ellipsoidal coordinates (ϕ,λ,h) to geodetic cartesian coordinates (x,y,z) is given by the standard relations as follows:

$x = (N+h).\cos\phi.\cos\lambda$

$y = (N+h).\cos\phi.\sin\lambda$

$z = (N.b^2/a^2+h).\sin\phi = (N.(1-e^2)+h).\sin\phi = (N+h-e^2.N).\sin\phi$

where N is the prime vertical radius of curvature at the point on the ellipsoid, and is given by

$N = a^2 / [a^2.\cos^2\phi + b^2.\sin^2\phi]^{0.5}$ or

$N = a^2 . [a^2.\cos^2\phi + b^2.\sin^2\phi]^{-0.5}$

and where

x,y,z are the geodetic cartesian coordinates

ϕ,λ,h are the geodetic latitude, longitude and ellipsoidal height (ellipsoidal coordinates)

$a \equiv$ semi-major axis of ellipsoid

$b \equiv$ semi-minor axis of ellipsoid

$e^2 = (a^2-b^2)/a^2$ where e is the first eccentricity of the ellipsoid.

A.2 Transformation: $(x,y,z) \rightarrow (\phi,\lambda,h)$

The inverse transformation from ellipsoidal (ϕ,λ,h) to cartesian (x,y,z) coordinates is not as simple as the direct transformation and has received attention from a number of authors. A number of solutions will be mentioned briefly. The solution given by *Bowring,* treated here under section A.2.3, *is used in the transformation adjustment program* developed for this study.

A.2.1 Iterative solution:

The traditional solution has been an iterative one, as follows:

The longitude is given immediately by

$$\lambda = \arctan\ (y/x)$$

The latitude and height are solved by iterations as follows (eg. Vanicek and Krakiwsky, 1986). Given that

$$p = (x^2+y^2)^{0.5} = (N+h).\cos\phi \qquad \ldots(1)$$

$$z = (N.b^2/a^2+h).\sin\phi = (N.(1-e^2)+h).\sin\phi =$$

$$= (N+h-e^2.N).\sin\phi \qquad \ldots(2)$$

Dividing the final forms of equations (2) by (1), we get

$$z/p = [1-e^2.N/(N+h)].\tan\phi = [1-e^2.N.(N+h)^{-1}].\tan\phi \qquad \ldots(3)$$

Now, the iterations are usually initiated by solving first for ϕ from equation (3) by putting $h = 0$, which gives

$$z/p = [1-e^2.N/N].\tan\phi = (1-e^2).\tan\phi$$

from which follows $\tan\phi = z/p.(1-e^2)^{-1}$

So the first value for ϕ is given by

$$\phi^{(0)} = \arctan [z/p.(1-e^2)^{-1}]$$

and the iterative routine is then

$$N^{(k)} = N(\phi^{(k-1)}) = a^2.[a^2.\cos^2\phi^{(k-1)} + b^2.\sin^2\phi^{(k-1)}]^{-0.5}$$

$$h^{(k)} = h(\phi^{(k-1)}, N^{(k)}) = p/\cos\phi^{(k-1)} - N^{(k)}$$

$$\phi^{(k)} = \phi(N^{(k)}, h^{(k)}) =$$

$$= \arctan [z/p.\{1-e^2.N^{(k)}.(N^{(k)}+h^{(k)})^{-1}\}^{-1}]$$

The iterations are repeated until the following inequalities are satisfied:

$$| h^{(k)} - h^{(k-1)} | < a.\varepsilon \quad \text{and} \quad | \phi^{(k)} - \phi^{(k-1)} | < \varepsilon$$

for some a priorily chosen value of ε.
This then gives the final values for ϕ and h with λ from above.

A.2.2 Closed form solution

Vanicek and Krakiwsky (1986) mention that the closed solution of this inverse transformation is derived from the solution of a biquadratic equation in $\tan\phi$.. This biquadratic equation in $\tan\phi$ is derived from equation (3) above by a number of substitutions.

$$p.\tan\phi - z = e^2.N.\sin\phi \qquad \text{and substituting for N,}$$

$$p.\tan\phi - z = a.e^2.\sin\phi \;/\; [\cos^2\phi + (b^2/a^2).\sin^2\phi\,]^{0.5}$$

Dividing the numerator and denominator of the right hand side by $\cos\phi$ and squaring the whole equation leads to

$$p^2.\tan^4\phi \;-\; 2.p.z.\tan^3\phi \;+\; [z^2+(p^2-a^2.e^2)/(1-e^2)].\tan^2\phi \;-$$

$$- 2.p.z/(1-e^2).\tan\phi + z^2/(1-e^2) = 0$$

which is a biquadratic equation in $\tan\phi$ in which all values of the coefficients are known.

Vanicek and Krakiwsky (1986) do not give an explicit solution for this equation. However, Paul (1973) does derive an explicit solution for this equation. The solution is a complex one, and employs both a precise formula and an approximate formula. This is because the precise formula cannot be used for small values of z/a, when the approximate formula for ϕ must be used. This solution was investigated in this study, and a convenient change-over point from the precise to the approximate formula was found to be when arc tan $|z/a| < 1^0$. This formula then ensures that all values of ϕ would be correct to four decimals of an arc second ($0.''0001$), and the resulting ellipsoidal height be correct to three decimals of a metre (0.001 m).

A.2.3 The Bowring solution:

Bowring (1976, 1985) developed simple equations for the *latitude and height* which *eliminate* the usual need for *iteration*, and which are both *sufficiently accurate for outer space applications*. His solution is used in this report. The formulae which he proposes are as follows:

The solution for λ is as usual:

$$\lambda = \text{arc tan}(y/x) \qquad \text{....(4)}$$

The solution for ϕ is given by

$$\phi = \text{arc tan } [\{z+\varepsilon.b.\sin^3 u\}/\{p-e^2.a.\cos^3 u\}] \qquad \text{....(5)}$$

where: $\tan u = b.z/a.p.(1+\varepsilon.b/R)$

$\cos u = (1+\tan^2 u)^{-0.5}$ $\qquad$ $\sin u = (1-\cos^2 u)^{0.5}$

$R = (p^2+z^2)^{0.5} = (x^2+y^2+z^2)^{0.5}$

$e^2 = (a^2-b^2)/a^2$ $\qquad$ $\varepsilon = [(a^2-b^2)/b^2]^{0.5}$

and for h: $\quad h = p.\cos\phi + z.\sin\phi - a^2/N \qquad$ ....(6)

where N is the prime vertical radius of curvature,

$$N = a^2.[a^2.\cos^2\phi + b^2.\sin^2\phi]^{-0.5}$$

APPENDIX B

DERIVATION OF THE VCV MATRIX Σ_{XYZ}

The derivation of the variance-covariance matrix of the geodetic cartesian coordinates (Σ_{xyz}) from the variance-covariance matrix of the ellipsoidal coordinates ($\Sigma_{\phi\lambda h}$) is treated here.

The Σ_{xyz} matrix is computed from the $\Sigma_{\phi\lambda h}$ matrix using the covariance law $\Sigma_{xyz} = J.\Sigma_{\phi\lambda h}.J^T$ where J is the usual Jacobian matrix of partial differentials, composed of 3x3 submatrices, J_i of the form

$$J_i = \begin{bmatrix} \frac{\partial x_i}{\partial \phi_i} & \frac{\partial x_i}{\partial \lambda_i} & \frac{\partial x_i}{\partial h_i} \\ \frac{\partial y_i}{\partial \phi_i} & \frac{\partial y_i}{\partial \lambda_i} & \frac{\partial y_i}{\partial h_i} \\ \frac{\partial z_i}{\partial \phi_i} & \frac{\partial z_i}{\partial \lambda_i} & \frac{\partial z_i}{\partial h_i} \end{bmatrix}$$

$$J_i = \begin{bmatrix} -(M_i+h_i)\sin\phi_i.\cos\lambda_i & -(N_i+h_i)\cos\phi_i.\sin\lambda_i & \cos\phi_i.\cos\lambda_i \\ -(M_i+h_i)\sin\phi_i.\sin\lambda_i & -(N_i+h_i)\cos\phi_i.\cos\lambda_i & \cos\phi_i.\sin\lambda_i \\ (M_i+h_i)\cos\phi_i & 0 & \sin\phi_i \end{bmatrix}$$

where M_i is the meridian radius of curvature and N_i the prime vertical radius of curvature of the reference ellipsoid.

Now, the VCV matrix Σ_{xyz} is composed of 3x3 submatrices of the form

$$J_i = \begin{bmatrix}
\sigma^2_{\phi_i}\cdot(M_i+h_i)^2\cdot\sin^2\phi_i\cdot\cos^2\lambda_i + \sigma^2_{\lambda_i}\cdot(N_i+h_i)^2\cdot\cos^2\phi_i\cdot\sin^2\lambda_i + \sigma^2_{h_i}\cdot\cos^2\phi_i\cdot\cos^2\lambda_i & [\sigma^2_{\phi_i}\cdot(M_i+h_i)^2\cdot\sin^2\phi_i + \sigma^2_{\lambda_i}\cdot(N_i+h_i)^2\cdot\cos^2\phi_i + \sigma^2_{h_i}\cdot\cos^2\phi_i\cdot\cos^2\lambda_i]\cdot\sin\lambda_i\cdot\cos\lambda_i & \sin\phi_i\cdot\cos\phi_i\cdot\cos\lambda_i\cdot[\sigma^2_{h_i} - \sigma^2_{\phi_i}\cdot(M_i+h_i)^2] \\
[\sigma^2_{\phi_i}\,(M_i+h_i)^2\cdot\sin^2\phi_i - \sigma^2_{\lambda_i}\cdot(N_i+h_i)^2\cdot\cos^2\phi_i + \sigma^2_{h_i}\cdot\cos^2\phi_i]\cdot\sin\lambda_i\cdot\cos\lambda_i & \sigma^2_{\phi_i}\,(M_i+h_i)^2\cdot\sin^2\phi_i\cdot\sin^2\lambda_i + \sigma^2_{\lambda_i}\cdot(N_i+h_i)^2\cdot\cos^2\phi_i\cdot\cos^2\lambda_i + \sigma^2_{h_i}\cdot\cos^2\phi_i\cdot\sin^2\lambda_i & \sin\phi_i\cdot\cos\phi_i\cdot\sin\lambda_i\cdot[\sigma^2_{h_i} - \sigma^2_{\phi_i}\cdot(M_i+h_i)^2] \\
\sin\phi_i\cdot\cos\phi_i\cdot\cos\lambda_i\cdot[\sigma^2_{h_i} - \sigma^2_{\phi_i}\cdot(M_i+h_i)^2] & \sin\phi_i\cdot\cos\phi_i\cdot\sin\lambda_i\cdot[\sigma^2_{h} - \sigma^2_{\phi_i}\cdot(M_i+h_i)^2] & \sigma^2_{\phi_i}\cdot(M_i+h_i)^2\cdot\cos^2\phi_i + \sigma^2_{h_i}\cdot\sin^2\phi_i
\end{bmatrix}$$

APPENDIX C

NOTES ON THE TRANSFORMATION ADJUSTMENT PROGRAM, FLOWCHART AND PROGRAM LISTING, EXAMPLES OF OUTPUT

Purpose: This program has been developed to perform a three dimensional transformation adjustment between a classical terrestrial geodetic network and a satellite derived geodetic network by making use of the coordinates of common network points.

Computer system and language: The program was written in ASCII FORTRAN and used on the UNISYS 1100/81 main frame computer.

Model used: The program computes this adjustment using any one of the three standard 7-parameter models, Bursa, Molodensky or Veis. In addition, the different cases of the Molodensky (or Veis) models regarding the point of rotation used, are catered for.

Results of the adjustment: The results of the adjustment are two sets of adjusted coordinates which differ by the adjusted parameters. The program computes the least squares estimates of the seven transformation parameters, the residuals to the observables (coordinates of common network points) and hence the adjusted coordinates. Both the VCV-matrix and the correlation matrix of the adjusted parameters are computed, as is the a posteriori variance factor, to give the neccessary (internal) precision estimates.

Input data: The main input data consists of the coordinates of the common network points, each in its own system.

The terrestrial network coordinates are input in ellipsoidal form as geodetic latitude, longitude, orthometric height and geoidal height for each point (ϕ,λ,H^0,N). The parameters of the particular ellipsoid used, here the modified Clark 1880, are input, and the ellipsoidal coordinates are converted to geodetic cartesian form using the formulae given in APPENDIX A.

The 3D cartesian (xyz) satellite network coordinates are in the NSWC 9Z-2 system and are input directly in this form. However, since the desirability of using the Conventional Terrestrial coordinate system as a form of reference has been expressed by various authors, these satellite coordinates (NSWC 9Z-2) are transformed to the CT system by a three parameter transformation - a Z-translation, a rotation around the Z-axis of the satellite system, and a scale change. The numerical values used are given in section 4.2.2.2.

<u>The VCV matrices of the observables</u>:
Due to the lack of information in this respect, these matrices were not available. The VCV matrices of each network are not used individually, hence the combined VCV matrix for all the observables together is constructed. The terrestrial VCV matrix ($\Sigma_{\phi\lambda h}$) is computed in ellipsoidal form using empirical formulae for the horizontal and vertical coordinates similar to Simmons' formula. These precision estimates are transformed to cartesian form using the law of propogation of VCV matrices, given in APPENDIX B. The precision estimate for the satellite coordinates is simply an average value for absolute positioning using the precise ephemeris. This constant value is used for all points and all three coordinate components. *The details of the formation of the VCV matrices are given in section 4.3.2.*

Other input data: A number of codes are input which control the following:

1. the number of common points
2. the model selected
3. the point of rotation selected
4. If the point of rotation is selected as the centre of gravity of the terrestrial network, this code determines whether this CG is computed in terms of ellipsoidal or geodetic cartesian coordinates.
5. whether the combined VCV matrix of the observables Σ_L is the Identity matrix I or not, ie. whether ALL cartesian coordinates (both geodetic and satellite) are given the same precision or not.
6. which of the 7 parameters are solved for, or in other words, which of the parameters are restrained to zero.

The coordinates, in ellipsoidal form, of the network initial point is input, as well as the coordinates of the point of rotation if this is not the CG of the terrestrial network.

Output: The following information is output on the respective pages.

Page 1: The parameters values and their standard deviations. If a parameter is restrained to zero, no value for it is printed, ie. it is neglected. The point of rotation is specified, and the coordinates given (in ellipsoidal and cartesian form.)

Page 2: The observations used in the adjustment, ie. the geodetic cartesian and transformed satellite or CT coordinates, and the residuals, both in ellipsoidal and cartesian form.

Page 3: The a posteriori variance factor and its square root, and the VCV and correlation matrices of the estimated parameters.

Pages 4 & 5: All the input data is output to facilitate checking. The VCV matrix Σ_L of the observations is output as well.

Page 6: The differences between the Geodetic (G) and CT coordinates are given as $(X_G - X_{CT})$, $(Y_G - Y_{CT})$, $(Z_G - Z_{CT})$. These differences are averaged for all points for each coordinate eg. $\Delta X_m = \sum_{i=1}^{n} (X_G - X_{CT})_i$. where ΔX_m is the mean value. The standard deviations of these means are computed as well as the individual deviations from these means.

Page 7: The CT coordinates are transformed into the G system by the inverse transformation to obtain the Pseudo-Geodetic (PG) coordinates. These PG coordinates are differenced from the G coordinates, and these differences are given in both cartesian and ellipsoidal form. The individual total displacements and the "mean goodness of fit" value, Δ, are given (See section 5.1).

SIMPLIFIED FLOWCHART

The simplified flowchart given below shows the main steps in the computation sequence. This is to be read with the preceding notes on the program (above) and the program listing which follows (below).

* INPUT includes the set of geodetic ellipsoidal and satellite Doppler cartesian coordinates of the common network points, the three parameters used to transform the satellite Doppler coordinates to the Conventional Terrestrial (CT) system, and a number of codes which are used for making a variety of decisions, eg. the selection of the particular model to be used, the structure of the VCV matrix of the observables, etc.

↓

* Read/compute the ellipsoidal and cartesian coordinates of the Fundamental Point of rotation used.

↓

* Transform satellite Doppler cartesian coordinates to CT coordinates using a three parameter transformation in SUBROUTINE DOPTRS.

↓

* Compute the standard deviations of the geodetic ellipsoidal coordinates using empirical formulae in SUBROUTINE SDELL.

↓

* Form the required design matrices and vectors for the combined case least squares solution. The design matrix A, the misclosure vector W and the VCV matrix of the observables Q are formed explicitly, whilst the design matrix B is not.

↓

* Computation of the desired quantities by using the least squares estimation procedure using the matrix inversion SUBROUTINE CHOLD.

↓

* Obtain pseudo-geodetic coordinates from the CT coordinates by applying the inverse transformation to them. The differences between the geodetic and pseudo-geodetic coordinates are then computed as well as the mean goodness of fit value, Δ.

↓

* OUTPUT the required quantities in a desirable format.

LIST OF SUBROUTINES

The following is a list of the subroutines used in this program and a brief description of what they are used for.

ELLCRT : computes 3D cartesian (xyz) coordinates from ellipsoidal ($\phi\lambda h$) coordinates (on a specified ellipsoid)

DOPTRS : transforms satellite Doppler cartesian coordinates to Conventional Terrestrial (CT) coordinates using a generally accepted three-parameter transformation.

SDELL : computes standard deviation estimates (σ_ϕ σ_λ σ_h) for the ellipsoidal coordinates using empirical formulae.

DIFCE : converts differential quantities in cartesian form to ellipsoidal form - here the residuals to the cartesian observables.

REVTFN : applies the inverse transformation to the CT coordinates, ie. the transformation changes from $\vec{\rho}_i = f(\vec{r}_i)$ to $\vec{r}_i = f^{-1}(\vec{\rho}_i)$, where $\vec{\rho}_i =$ CT position vector, $\vec{r}_i =$ geodetic cartesian position vector.

CE : computes ellipsoidal coordinates (on a specified ellipsoid) from 3D cartesian coordinates using the formulae of B.R.Bowring (see APPENDIX A)

CHOLD : matrix inversion routine using the Cholesky method.

```
@RUN,Z/N RENS,A0520-R003,RENS,5,70
@SYM PRINT$,,RMTENG
@FTN,IO TPF$.MAIN
C-----------------------------------------
C     TRANSFORMATION ADJUSTMENT PROGRAM:
C     ----------------------------------
C   PURPOSE: TO DETERMINE THE TRANSFORMATION  PARAMETERS  FOR A
C             3-D TRANSFORMATION BETWEEN TWO SETS OF 3-D CARTESIAN
C             COORDS, BY ONE OF 3 DIFFERENT MODELS: BURSA,MOLODENSKI,
C             VEIS. FURTHER, IN THE CASE OF THE LAST TWO MODELS, THERE
C            ARE ANY NUMBER OF POSSIBLITIES, SINCE THE POINT OF ROTATION
C            OF THE TERRESTRIAL SYSTEM IS EXPLICITLY SPECIFIED. IN THE
C            BURSA MODEL, THIS POINT OF ROTATION IS THE ORIGIN(0,0,0) OF
C            THE TERRESTRIAL SYSTEM, WHICH MUST BE SPECIFIED EXPLICITLY.
C ----------------------
C      INPUT: AM,BM : ELLIPSOID PARAMETERS
C             N: NUMBER OF POINTS IN THE ADJUSTMENT
C            NTYP: 1,2,3 FOR MODELS BURSA,MOLODENSKI,VEIS.
C            NROT: 1/0: USE 1 FOR BURSA MODEL ALWAYS, AND FOR MOLODENSKI
C                 AND VEIS MODELS IF THE POINT OF ROTATION IS THE C.G.OF
C                 THE TERRESTRIAL NETWORK. WHERE THE POINT OF ROTATION
C                 IS NOT THE ORIGIN (0,0,0) OR THE C.G. OF TERRESTRIAL
C                 NETWORK, USE NROT=0. THIS IMPLIES THE POINT OF
C                 ROTATION MUST BE TRANSFORMED FROM LAT,LONG,HT,N TO XYZ.
C             NCG: 1/2: THE CG. OF THE NETWORK CAN BE COMPUTED IN TWO
C                  WAYS: 1. CG OF THE CARTESIAN COORDS XYZ OF TERR. PTS
C                        2. CG OF THE ELLIPSOIDAL COORDS LAT,LONG,HT OF
C                           THE TERR. PTS
C          NQ: 1/0: NQ=1 IF 'Q-MX' MUST BE 'I-MX'.
C                   NQ=0 IF 'Q-MX' MUST NOT BE 'I-MX'.
C          (NP(I),I=1,7)=1/0: CODE FOR DETERMINING WHICH PARAMETERS TO
C                   SOLVE FOR, IN THE ORDER: TX,TY,TZ,RX,RY,RZ,SCALE.
C               EG. 1 1 1 0 0 0 1 SOLVES ONLY FOR TRANSLATIONS AND SCALE.
C          DOPTZ,DOPRZ,DOPS: TRANSFORMATION ON DOPPLER COORDS AS
C                            SHIFT IN Z, ROTATION AROUND Z, SCALE CHANGE.
C         RP0,RL0,RH0: LAT,LONG,ELL.HT      OF NETWORK INITIAL POINT(NIP)
C   NOTE: THE LAT/LONG OF NIP. IS NEEDED FOR THE COMPUTATION OF THE
C          STANDARD DEV. ESTIMATES FOR THE ELLIPSOIDAL COORDS IN SUB. SDELL.
C          NAME,ELL: NAMES OF POINTS, AND ELLIPOIDAL COORDS AS LAT,LONG,
C                    ORTHOMETRIC HEIGHT,GEOID HEIGHT. LAT,LONG INPUT AS
C                    DEG.MMSSS
C          SATIN,SDS: SATELLITE XYZ COORDS, AND STD.DEVS. OF EACH OF THESE
C        ROTLAT,ROTLON,ROTHT: LAT/LONG/ELL.HT OF ROTATION POINT IF NEEDED
C-------------------------------------------------------
      IMPLICIT REAL*8(A-H,O-Z)
      PARAMETER IQ=23
      CHARACTER NAME*11(IQ),HEAD*10(7)
      COMMON NP(7),NPSUM(7)

      DIMENSION ELL(IQ,5),SDE(IQ,3),CRT(IQ,9),SAT(IQ,9)
     C,SATIN(IQ,3),DDX(IQ),DDY(IQ),DDZ(IQ),
     CSDS(IQ,3),SDX(7),A(3*IQ,7),W(3*IQ),
     CX(7),V(6*IQ),V1(6*IQ),CORR(7,7),DIFVEC(IQ),
     CDELTA(IQ,7),Q(6*IQ,6*IQ),QX(7,7),PSAT(IQ,3),
     CBQB(3*IQ,3*IQ),ATB(7,3*IQ),ATBW(7),AX(3*IQ)
C THE ABOVE LINE OF DIMENSIONING IS NECCESSARY WHEN USING THE
C ALGEBRAIC MODELLING PROCEDURE  NOW STORED IN ELL. MOD
C------------
C  RAD(A): A IN DEG.MINSEC
      DEFINE RAD(A)=(DINT(A)+(DINT((A-DINT(A))*100D0))/60D0+
     C(A*100D0-DINT(A*100D0))/36D0)*3600/T
C DMS(A): A IN RADIANS
```

```
       DEFINE DMS(A)=DINT(A*T/3600D0)+
      CDINT((A*T/3600D0-DINT(A*T/3600D0))*60D0)/100D0+
      C((A*T/3600D0-DINT(A*T/3600D0))*60D0-
      CDINT((A*T/3600D0-DINT(A*T/3600D0))*60D0))*60D0/10000D0
C------------
      T=206264.80625D0
      READ(5,100) AM,BM
      READ(5,100) N,NTYP,NROT,NCG,NQ
      N3=3*N
      N6=6*N
       READ(5,100)(NP(I),I=1,7)
      NPSUM(1)=NP(1)
      DO 1400 I=2,7
      NPSUM(I)=NPSUM(I-1)+NP(I)
1400  CONTINUE
      NU=NPSUM(7)
      READ(5,100) DOPTZ,DOPRZ,DOPS
C    READ ELLIPSOIDAL COORDS OF THE N.I.P.(NETWORK INITIAL POINT)
      READ(5,100) RP0,RL0,RH0
      DO 1000 I=1,N
C READ ELL. CRDS AND CONVERT TO CARTESIAN
1100  READ(5,120)NAME(I),(ELL(I,J),J=1,4)
      ELL(I,5)=ELL(I,3)+ELL(I,4)
      CALL ELLCRT(AM,BM,ELL(I,1),ELL(I,2),ELL(I,5),CRT(I,1),CRT(I,2),
     CCRT(I,3))
1000  CONTINUE
C
C
      DO 2000 I=1,N
      READ(5,140)(SATIN(I,J),J=1,3),(SDS(I,K),K=1,3)
 2000 CONTINUE
      IF(NROT.EQ.0)READ(5,100) ROTLAT,ROTLON,ROTHT
C
C  COMPUTE CG. OF TERRESTRIAL NETWORK, AS CGX,CGY,CGZ,CGP,CGL,CGH.
      CGX=0D0
      CGY=0D0
      CGZ=0D0
      CGP=0D0
      CGL=0D0
      CGH=0D0
      IF(NCG.EQ.1) GOTO 10001
      IF(NCG.EQ.2) GOTO 11001
10001 DO 10000 I=1,N
      CGX=CGX+CRT(I,1)
      CGY=CGY+CRT(I,2)
      CGZ=CGZ+CRT(I,3)
10000 CONTINUE
      CGX=CGX/DFLOAT(N)
      CGY=CGY/DFLOAT(N)
      CGZ=CGZ/DFLOAT(N)
      CALL CE(AM,BM,CGX,CGY,CGZ,CGP,CGL,CGH)
10500 CONTINUE
      GOTO 11500
11001 DO 11000 I=1,N
      CGP=CGP+RAD(ELL(I,1))
      CGL=CGL+RAD(ELL(I,2))
      CGH=CGH+ELL(I,5)
11000 CONTINUE
      CGP=CGP/DFLOAT(N)
      CGL=CGL/DFLOAT(N)
      CGH=CGH/DFLOAT(N)
      CGP=DMS(CGP)
```

```
      CGL=DMS(CGL)
C CONVERT CGP,CGL,CGH TO CARTESIAN COORDS CGX,CGY,CGZ
      CALL ELLCRT(AM,BM,CGP,CGL,CGH,CGX,CGY,CGZ)
11500 CONTINUE
C
C   NOW  DETERMINE THE CARTESIAN COORDS OF THE "POINT OF ROTATION"
C  NOTE: IF NROT=1 THEN THE POINT OF ROTATION MUST BE COMPUTED
      IF(NROT.EQ.1) GOTO 3000
       CALL ELLCRT(AM,BM,ROTLAT,ROTLON,ROTHT,XROT,YROT,ZROT)
      GOTO 5000
3000  CONTINUE
      IF(NTYP.NE.1) GOTO 4000
      XROT=0D0
      YROT=0D0
      ZROT=0D0
      ROTLAT=0D0
      ROTLON=0D0
      ROTHT=0D0
      GOTO 5000
4000  XROT=CGX
      YROT=CGY
      ZROT=CGZ
      ROTLAT=CGP
      ROTLON=CGL
      ROTHT=CGH
5000  CONTINUE
C NOTE: WE HAVE THE CARTESIAN AND ELLIPSOIDAL COORDS OF THE ROTATION
C        POINT AS XROT,YROT,ZROT, AND ROTLAT,ROTLON,ROTHT. THE ELLIPSOIDAL
C        COORDS OF THE ROTATION POINT IS USED IN THE VEIS-MODEL.
C     *******************************************************************
C        MATRIX 'HEAD' CONTAINS THE NAMES OF THE UNKNOWNS SOLVED FOR
      IF(NP(1).EQ.1) HEAD(NPSUM(1))='TLN-X'
      IF(NP(2).EQ.1) HEAD(NPSUM(2))='TLN-Y'
      IF(NP(3).EQ.1) HEAD(NPSUM(3))='TLN-Z'
      IF(NP(4).EQ.1) HEAD(NPSUM(4))='ROT-X'
      IF(NP(4).EQ.1.AND.NTYP.EQ.3) HEAD(NPSUM(4))=' dA   '
      IF(NP(5).EQ.1) HEAD(NPSUM(5))='ROT-Y'
      IF(NP(5).EQ.1.AND.NTYP.EQ.3) HEAD(NPSUM(5))=' dU   '
      IF(NP(6).EQ.1) HEAD(NPSUM(6))='ROT-Z'
      IF(NP(6).EQ.1.AND.NTYP.EQ.3) HEAD(NPSUM(6))=' dV   '
      IF(NP(7).EQ.1) HEAD(NPSUM(7))='SCALE'

C   CONVERT: DOPPLER COORDS TO CTS BY Z-ROT,Z-SHIFT,SCALE
      DO 20000 I=1,N
      IF(DOPTZ.EQ.0.AND.DOPRZ.EQ.0.AND.DOPS.EQ.0) GOTO 19000
      CALL DOPTRS(DOPTZ,DOPRZ,DOPS,SATIN(I,1),SATIN(I,2),SATIN(I,3),
     CSAT(I,1),SAT(I,2),SAT(I,3))
      GOTO 20000
19000 SAT(I,1)=SATIN(I,1)
      SAT(I,2)=SATIN(I,2)
      SAT(I,3)=SATIN(I,3)
20000 CONTINUE
C  COMPUTE: STANDARD DEVIATION ESTIMATES FOR LAT,LONG,HEIGHT BY EMPIRICAL
C  FORMULAE
28005  DO 30000 I=1,N
       GOTO 29000
28000  SDE(I,1)=0.2D0/T
       SDE(I,2)=0.2D0/T
       SDE(I,3)=1D0
29000  CALL SDELL(RP0,RL0,ELL(I,1),ELL(I,2),SDE(I,1),SDE(I,2),SDE(I,3))
30000 CONTINUE
C-------------------------------------------------------------
```

```
C  NOW FORM DESIGN MATRIX A
       IF(NTYP.EQ.1.OR.NTYP.EQ.2) GOTO 31000
       IF(NTYP.EQ.3)GOTO 33000
C SUBROUTINE FOR A MATRIX IN BURSA AND MOLODENSKI MODELS
31000 CONTINUE
      DO 32000 I=1,N
C     FORM FIRST ROW OF A

      IF(NP(1).EQ.1) A(3*I-2,NPSUM(1))=1.D0
      IF(NP(2).EQ.1) A(3*I-2,NPSUM(2))=0.D0
      IF(NP(3).EQ.1) A(3*I-2,NPSUM(3))=0.D0

      IF(NP(4).EQ.1) A(3*I-2,NPSUM(4))=0.D0
      IF(NP(5).EQ.1) A(3*I-2,NPSUM(5))=-1.D0*(CRT(I,3)-ZROT)
      IF(NP(6).EQ.1) A(3*I-2,NPSUM(6))=(CRT(I,2)-YROT)

      IF(NP(7).EQ.1) A(3*I-2,NPSUM(7))=(CRT(I,1)-XROT)
C
C     FORM SECOND ROW OF A
      IF(NP(1).EQ.1) A(3*I-1,NPSUM(1))=0.D0
      IF(NP(2).EQ.1) A(3*I-1,NPSUM(2))=1.D0
      IF(NP(3).EQ.1) A(3*I-1,NPSUM(3))=0.D0

      IF(NP(4).EQ.1) A(3*I-1,NPSUM(4))=(CRT(I,3)-ZROT)
      IF(NP(5).EQ.1) A(3*I-1,NPSUM(5))=0.D0
      IF(NP(6).EQ.1) A(3*I-1,NPSUM(6))=-1.D0*(CRT(I,1)-XROT)

      IF(NP(7).EQ.1) A(3*I-1,NPSUM(7))=(CRT(I,2)-YROT)
C
C     FORM THIRD ROW OF A
      IF(NP(1).EQ.1) A(3*I,NPSUM(1))=0.D0
      IF(NP(2).EQ.1) A(3*I,NPSUM(2))=0.D0
      IF(NP(3).EQ.1) A(3*I,NPSUM(3))=1.D0

      IF(NP(4).EQ.1) A(3*I,NPSUM(4))=-1.D0*(CRT(I,2)-YROT)
      IF(NP(5).EQ.1) A(3*I,NPSUM(5))=(CRT(I,1)-XROT)
      IF(NP(6).EQ.1) A(3*I,NPSUM(6))=0.D0

      IF(NP(7).EQ.1) A(3*I,NPSUM(7))=(CRT(I,3)-ZROT)
32000 CONTINUE
      GOTO 34000
C SUBROUTINE FOR MATRIX A IN VEIS-MODEL CASE.
33000 RAT=ROTLAT
      RON=ROTLON
C      FORM FIRST ROW OF A
      DO 34000 I=1,N
      IF(NP(1).EQ.1) A(3*I-2,NPSUM(1))=1.D0
      IF(NP(2).EQ.1) A(3*I-2,NPSUM(2))=0.D0
      IF(NP(3).EQ.1) A(3*I-2,NPSUM(3))=0.D0
      IF(NP(4).EQ.1) A(3*I-2,NPSUM(4))=-SIN(RAD(RAT))*(CRT(I,2)-YROT)+
     CSIN(RAD(RON))*COS(RAD(RAT))*(CRT(I,3)-ZROT)
      IF(NP(5).EQ.1) A(3*I-2,NPSUM(5))=COS(RAD(RON))*(CRT(I,3)-ZROT)
      IF(NP(6).EQ.1) A(3*I-2,NPSUM(6))=-COS(RAD(RAT))*(CRT(I,2)-YROT)-
     CSIN(RAD(RAT))*SIN(RAD(RON))*(CRT(I,3)-ZROT)
      IF(NP(7).EQ.1) A(3*I-2,NPSUM(7))=(CRT(I,1)-XROT)
C     FORM SECOND ROW OF A
      IF(NP(1).EQ.1) A(3*I-1,NPSUM(1))=0.D0
      IF(NP(2).EQ.1) A(3*I-1,NPSUM(2))=1.D0
      IF(NP(3).EQ.1) A(3*I-1,NPSUM(3))=0.D0
      IF(NP(4).EQ.1) A(3*I-1,NPSUM(4))=SIN(RAD(RAT))*(CRT(I,1)-XROT)-
     CCOS(RAD(RAT))*COS(RAD(RON))*(CRT(I,3)-ZROT)
```

```
      IF(NP(5).EQ.1) A(3*I-1,NPSUM(5))=SIN(RAD(RON))*(CRT(I,3)-ZROT)
      IF(NP(6).EQ.1) A(3*I-1,NPSUM(6))=COS(RAD(RAT))*(CRT(I,1)-XROT)+
     CSIN(RAD(RAT))*COS(RAD(RON))*(CRT(I,3)-ZROT)
      IF(NP(7).EQ.1) A(3*I-1,NPSUM(7))=(CRT(I,2)-YROT)
C
C     FORM THIRD ROW OF A
      IF(NP(1).EQ.1) A(3*I,NPSUM(1))=0.D0
      IF(NP(2).EQ.1) A(3*I,NPSUM(2))=0.D0
      IF(NP(3).EQ.1) A(3*I,NPSUM(3))=1.D0
      IF(NP(4).EQ.1) A(3*I,NPSUM(4))=-COS(RAD(RAT))*SIN(RAD(RON))*
     C(CRT(I,1)-XROT)+COS(RAD(RAT))*COS(RAD(RON))*(CRT(I,2)-YROT)
      IF(NP(5).EQ.1) A(3*I,NPSUM(5))=-COS(RAD(RON))*(CRT(I,1)-XROT)-
     CSIN(RAD(RON))*(CRT(I,2)-YROT)
      IF(NP(6).EQ.1) A(3*I,NPSUM(6))=SIN(RAD(RAT))*SIN(RAD(RON))*
     C(CRT(I,1)-XROT)-SIN(RAD(RAT))*COS(RAD(RON))*(CRT(I,2)-YROT)
      IF(NP(7).EQ.1) A(3*I,NPSUM(7))=(CRT(I,3)-ZROT)
34000 CONTINUE
C       PRINT OUT A-MATRIX.
      GOTO 199
198   WRITE(6,190)
 190  FORMAT(1H1,20X,'A-MATRIX',/)
      WRITE(6,195) (HEAD(I),I=1,NU)
 195  FORMAT(18X,7(A10,5X))
      DO 199 I=1,N3
      WRITE(6,191) (A(I,J),J=1,NU)
 191  FORMAT(/,5X,7(F12.2,3X))
199   CONTINUE
C-------------------------------------------------------
C  FORM  MISCLOSURE VECTOR W
      DO 42000 I=1,N
      W(3*I-2)=CRT(I,1)-SAT(I,1)
      W(3*I-1)=CRT(I,2)-SAT(I,2)
      W(3*I)  =CRT(I,3)-SAT(I,3)
42000 CONTINUE
C-------------------------------------------------------------------
C  FORM THE VCV-MATRIX OF THE OBSERVATIONS(APRIORI): Q
      DO 50000 I=1,N6
      DO 50000 J=1,N6
      Q(I,J)=0D0
      Q(I,I)=1D0
50000 CONTINUE
      IF(NQ.EQ.1) GOTO 52000
C    THIS 'GOTO' IS USED IF THE VCV-MATRIX  'Q' MUST BE THE IDENTITY
C    MATRIX 'I', ie. DIAGONAL WITH 1'S ON THE MAIN DIAGONAL.
50005 E=1D0-BM**2/AM**2
      DO 52000 I=1,N
      DEN=1D0-(E*SIN(RAD(ELL(I,1))))**2
      RMPH=AM*(1D0-E)/DEN**1.5D0+ELL(I,5)
      RNPH=AM/DEN**0.5D0+ELL(I,5)
      SINP=SIN(RAD(ELL(I,1)))
      COSP=COS(RAD(ELL(I,1)))
      SINL=SIN(RAD(ELL(I,2)))
      COSL=COS(RAD(ELL(I,2)))
C     GOTO 51000
C THIS GOTO IS USED IF THE VCV-MATRIX MUST BE DIAGONAL ONLY,IE NO
C OFF-DIAGONAL TERMS FOR THE TERRESTRIAL SECTION.(3,3)
      Q(6*I-5,6*I-4)=((SDE(I,1)*RMPH*SINP)**2-
     C(SDE(I,2)*RNPH*COSP)**2+(SDE(I,3)*COSP)**2)*SINL*COSL
      Q(6*I-5,6*I-3)=SINP*COSP*COSL*(SDE(I,3)**2-(RMPH*SDE(I,1))**2)
      Q(6*I-4,6*I-5)=Q(6*I-5,6*I-4)
      Q(6*I-4,6*I-3)=SINP*COSP*SINL*(SDE(I,3)**2-(SDE(I,1)*RMPH)**2)
      Q(6*I-3,6*I-5)=Q(6*I-5,6*I-3)
```

```
      Q(6*I-3,6*I-4)=Q(6*I-4,6*I-3)
51000 CONTINUE
      Q(6*I-5,6*I-5)=(SDE(I,1)*RMPH*SINP*COSL)**2+
     C(SDE(I,2)*RNPH*COSP*SINL)**2+(SDE(I,3)*COSP*COSL)**2
      Q(6*I-4,6*I-4)=(SDE(I,1)*RMPH*SINP*SINL)**2+
     C(SDE(I,2)*RNPH*COSP*COSL)**2+(SDE(I,3)*COSP*SINL)**2
      Q(6*I-3,6*I-3)=(SDE(I,1)*RMPH*COSP)**2+(SDE(I,3)*SINP)**2
      Q(6*I-2,6*I-2)=SDS(I,1)**2
      Q(6*I-1,6*I-1)=SDS(I,2)**2
      Q(6*I,6*I)=SDS(I,3)**2
52000 CONTINUE
208   WRITE(6,209)
209   FORMAT(2X,'VCV-MATRIX  "Q" OF THE OBSERVATIONS:  Q(N6,N6)',/)
       DO 53000 I=1,N
       WRITE(6,204) Q(6*I-5,6*I-5),Q(6*I-5,6*I-4),Q(6*I-5,6*I-3)
       WRITE(6,204) Q(6*I-4,6*I-5),Q(6*I-4,6*I-4),Q(6*I-4,6*I-3)
       WRITE(6,204) Q(6*I-3,6*I-5),Q(6*I-3,6*I-4),Q(6*I-3,6*I-3)
       WRITE(6,204) Q(6*I-2,6*I-2),Q(6*I-2,6*I-1),Q(6*I-2,6*I)
       WRITE(6,204) Q(6*I-1,6*I-2),Q(6*I-1,6*I-1),Q(6*I-1,6*I)
       WRITE(6,204) Q(6*I,6*I-2),Q(6*I,6*I-1),Q(6*I,6*I)
204   FORMAT(3(2X,D12.6)/)
53000 CONTINUE
C------------------------------------------------------------------
C--------------------------------------------------------------
C   SUBROUTINE FOR LEAST SQUARES SOLUTION FOR X,QX,V,QL,SIGMA 0
C   ARGUMENTS : N: NUMBER OF POINTS USED IN TRANSFORMATION ADJUSTMENT
C               NU=NUMBER OF UNKNOWNS/PARAMETERS (IN VECTOR X)
C               A(N3,NU): DESIGN MATRIX FOR PARAMETERS X
C               B(N3,N6): DESIGN MATRIX FOR OBSERVATIONS L
C               W(N3,1) : MISCLOSURE VECTOR
C               Q(N6,N6):VCV-MATRIX OF THE OBSERVATIONS
C              X(NU,1): PARAMETERS
C              QX(NU,NU):VCV-MATRIX OF PARAMETERS
C              V(N6,1) : RESIDUALS TO THE OBSERVATIONS
C              QL(N6,N6):VCV-MATRIX OF OBSERVATIONS (OR RESIDUALS)
C              AVF:APOSTERIORI VARIANCE FACTOR
C FORM BQB' DIRECTLY FROM Q:
      DO 60020 I=1,N
      BQB(3*I-2,3*I-2)=Q(6*I-5,6*I-5)+Q(6*I-2,6*I-2)
      BQB(3*I-1,3*I-1)=Q(6*I-4,6*I-4)+Q(6*I-1,6*I-1)
      BQB(3*I,3*I)=Q(6*I-3,6*I-3)+Q(6*I,6*I)
      BQB(3*I-2,3*I-1)=Q(6*I-5,6*I-4)
      BQB(3*I-2,3*I)=Q(6*I-5,6*I-3)
      BQB(3*I-1,3*I)=Q(6*I-4,6*I-3)
      BQB(3*I-1,3*I-2)=BQB(3*I-2,3*I-1)
      BQB(3*I,3*I-2)=BQB(3*I-2,3*I)
      BQB(3*I,3*I-1)=BQB(3*I-1,3*I)
60020 CONTINUE
      GOTO 60022
207   DO 60022 I=1,N
      WRITE(6,204)BQB(3*I-2,3*I-2),BQB(3*I-2,3*I-1),
     CBQB(3*I-2,3*I)
      WRITE(6,204)BQB(3*I-1,3*I-2),BQB(3*I-1,3*I-1),
     CBQB(3*I-1,3*I)
      WRITE(6,204)BQB(3*I,3*I-2),BQB(3*I,3*I-1),
     CBQB(3*I,3*I)
60022 CONTINUE
      CALL CHOLD(BQB,N3,N3,0)
C       NOW BQB IS THE INVERSE OF THE "OLD" BQB
C     CALL AB(IQ,A,N3,NU,1,BQB,N3,N3,0,ATB)
      DO 60030 I=1,NU
      DO 60030 J=1,N3
```

```
      SUM=0D0
      DO 60032 K=1,N3
      SUM=SUM+A(K,I)*BQB(K,J)
60032 CONTINUE
      ATB(I,J)=SUM
60030 CONTINUE
C     CALL AB(IQ,ATB,NU,N3,0,A,N3,NU,0,QX)
      DO 60040 I=1,NU
      DO 60040 J=1,NU
      SUM=0D0
      DO 60042 K=1,N3
      SUM=SUM+ATB(I,K)*A(K,J)
60042 CONTINUE
      QX(I,J)=SUM
60040 CONTINUE
      CALL CHOLD(QX,7,NU,0)
C     CALL AB(IQ,ATB,NU,N3,0,W,N3,1,0,ATBW)
      DO 60050 I=1,NU
      SUM=0D0
      DO 60052 K=1,N3
      SUM=SUM+ATB(I,K)*W(K)
60052 CONTINUE
      ATBW(I)=SUM
60050 CONTINUE
C     CALL AB(IQ,QX,NU,NU,0,ATBW,NU,1,0,X)
      DO 60060 I=1,NU
      SUM=0D0
      DO 60062 K=1,NU
      SUM=SUM+QX(I,K)*ATBW(K)
60062 CONTINUE
      X(I)=-SUM
60060 CONTINUE
C   NOW FOR CORRELATES V1:
C     CALL AB(IQ,A,N3,NU,0,X,NU,1,0,AX)
      DO 60070 I=1,N3
      SUM=0D0
      DO 60072 K=1,NU
      SUM=SUM+A(I,K)*X(K)
60072 CONTINUE
      AX(I)=SUM
60070 CONTINUE
      DO 62000 I=1,N3
      AX(I)=AX(I)+W(I)
62000 CONTINUE
C     CALL AB(IQ,BQB,N3,N3,0,AX,N3,1,0,V1)
      DO 60080 I=1,N3
      SUM=0D0
      DO 60082 K=1,N3
      SUM=SUM+BQB(I,K)*AX(K)
60082 CONTINUE
      V1(I)=-1D0*SUM
60080 CONTINUE
C    NOW FOR V:
C     CALL AB(IQ,QB,N6,N3,0,V1,N3,1,0,V)
C COMPUTE V DIRECTLY FROM Q AND V1(CORRELATES)
      DO 64000 I=1,N
      V(6*I-5)=Q(6*I-5,6*I-5)*V1(3*I-2)+Q(6*I-5,6*I-4)*
     CV1(3*I-1)+Q(6*I-5,6*I-3)*V1(3*I)
      V(6*I-4)=Q(6*I-4,6*I-5)*V1(3*I-2)+Q(6*I-4,6*I-4)*
     CV1(3*I-1)+Q(6*I-4,6*I-3)*V1(3*I)
      V(6*I-3)=Q(6*I-3,6*I-5)*V1(3*I-2)+Q(6*I-3,6*I-4)*
     CV1(3*I-1)+Q(6*I-3,6*I-3)*V1(3*I)
```

```
      V(6*I-2)=-Q(6*I-2,6*I-2)*V1(3*I-2)
      V(6*I-1)=-Q(6*I-1,6*I-1)*V1(3*I-1)
      V(6*I)=-Q(6*I,6*I)*V1(3*I)
64000 CONTINUE
C  NOW FOR AVF=V(T)*Q*V/DOF, WHERE Q=INVERSE(Q)
      CALL CHOLD(Q,N6,N6,0)
C     CALL AB(IQ,V,N6,1,1,Q,N6,N6,0,V1)
      DO 60170 J=1,N6
      SUM=0D0
      DO 60172 K=1,N6
      SUM=SUM+V(K)*Q(K,J)
60172 CONTINUE
      V1(J)=SUM
60170 CONTINUE

C     CALL AB(IQ,V1,1,N6,0,V,N6,1,0,AVF)
      SUM=0D0
      DO 60182 K=1,N6
      SUM=SUM+V1(K)*V(K)
60182 CONTINUE
      AVF=SUM/DFLOAT(3*N-NU)
60180 CONTINUE
      DO 70000 I=1,NU
      DO 70000 J=1,NU
      QX(I,J)=QX(I,J)*AVF
70000 CONTINUE
C-----------------------------------------------------------------
C   TRANSFER THE CARTESIAN RESIDUALS TO THE RESPECTIVE OBSERVATION
C   MATRICES CRT AND SAT
      DO 80000 I=1,N
      CRT(I,4)=V(6*I-5)
      CRT(I,5)=V(6*I-4)
      CRT(I,6)=V(6*I-3)
      SAT(I,4)=V(6*I-2)
      SAT(I,5)=V(6*I-1)
      SAT(I,6)=V(6*I)
80000 CONTINUE
C   THE STANDARD DEVIATIONS(SDX) OF THE PARAMETERS X ARE THE
C     ROOTS OF THE DIAGONAL ELEMENTS OF THE VCV MATRIX QX:
      DO 82000 I=1,NU
      SDX(I)=DSQRT(QX(I,I))
82000 CONTINUE
C FORM CORRELATION MATRIX(CORR) FROM THE VCV MATRIX QX:
      DO 84000 I=1,NU
      DO 84000 J=1,NU
      CORR(I,J)=QX(I,J)/DSQRT(QX(I,I)*QX(J,J))
84000 CONTINUE
C   TRANSFORM THE RESIDUALS INTO ELLIPSOIDAL RESIDUALS
      DO 86000 I=1,N
      CALL DIFCE(ELL(I,1),ELL(I,2),CRT(I,4),CRT(I,5),CRT(I,6),
     CCRT(I,7),CRT(I,8),CRT(I,9))
      CALL DIFCE(ELL(I,1),ELL(I,2),SAT(I,4),SAT(I,5),SAT(I,6),
     CSAT(I,7),SAT(I,8),SAT(I,9))
86000 CONTINUE
C   APPLY REVERSE TRANSFORMATION TO THE DOPPLER SATELLITE COORDS,AND
C   OBTAIN PSEUDO-GEODETIC COORDS.
      DELVEC=0D0
      TX=DFLOAT(NP(1))*X(NPSUM(1))
      TY=DFLOAT(NP(2))*X(NPSUM(2))
      TZ=DFLOAT(NP(3))*X(NPSUM(3))
      RX=DFLOAT(NP(4))*X(NPSUM(4))
      RY=DFLOAT(NP(5))*X(NPSUM(5))
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      RZ=DFLOAT(NP(6))*X(NPSUM(6))
      SCA=DFLOAT(NP(7))*X(NPSUM(7))+1D0
C   THE NP(I):1/0 WILL CANCEL THE MULTIPLICATION IF THE PARAMETER I
C   DOES NOT EXIST; NPSUM(I) LOCATES THE POSITION OF THE PARAMETER
C   IN THE VECTOR X.
C
      IF(NTYP.NE.3) GOTO 88100
C COMPUTE MOLODENSKI ROTATIONS FROM THE VEIS ROTATIONS USING THE
C FORMULAE ON PAGE 101 OF THOMSON'S THESIS.
      BR=RAD(ROTLAT)
      EN=RAD(ROTLON)
      THETAX=(-COS(BR)*COS(EN)*RX+SIN(EN)*RY+SIN(BR)*COS(EN)*RZ)*T
      THETAY=(-COS(BR)*SIN(EN)*RX-COS(EN)*RY+SIN(BR)*SIN(EN)*RZ)*T
      THETAZ=(-SIN(BR)*RX-COS(BR)*RZ)*T
      WRITE(6,88001)THETAX,THETAY,THETAZ
88001 FORMAT(5X,'MOLODENSKI ROTATIONS COMPUTED FROM THE VEIS ONES:',
     C/,5X,3(2X,F8.3),/)
88100 CONTINUE
      DO 88000 I=1,N
      CALL REVTFN(NTYP,TX,TY,TZ,RX,RY,RZ,SCA,XROT,YROT,ZROT,
     CROTLAT,ROTLON,SAT(I,1),SAT(I,2),SAT(I,3),
     CPSAT(I,1),PSAT(I,2),PSAT(I,3))
88000 CONTINUE
C   OBTAIN COORD-DIFFERENCES, AND TRANSFORM TO ELLIPSOIDAL DIFFERENCES
      DELVEC=0D0
      DO 90000 I=1,N
      DELTA(I,1)=CRT(I,1)-PSAT(I,1)
      DELTA(I,2)=CRT(I,2)-PSAT(I,2)
      DELTA(I,3)=CRT(I,3)-PSAT(I,3)
      DELTA(I,7)=DSQRT(DELTA(I,1)**2+DELTA(I,2)**2+DELTA(I,3)**2)
      DELVEC=DELVEC+DELTA(I,7)
      CALL DIFCE(ELL(I,1),ELL(I,2),DELTA(I,1),DELTA(I,2),DELTA(I,3),
     CDELTA(I,4),DELTA(I,5),DELTA(I,6))
90000 CONTINUE
      DELVEC=DELVEC/DFLOAT(N)
C     CONVERT ROTATION ANGLES AND STD DEV. FROM RADIANS TO SECONDS OF ARC
      IF(NP(4).EQ.1) X(NPSUM(4))=X(NPSUM(4))*T
      IF(NP(4).EQ.1) SDX(NPSUM(4))=SDX(NPSUM(4))*T
      IF(NP(5).EQ.1) X(NPSUM(5))=X(NPSUM(5))*T
      IF(NP(5).EQ.1) SDX(NPSUM(5))=SDX(NPSUM(5))*T
      IF(NP(6).EQ.1) X(NPSUM(6))=X(NPSUM(6))*T
      IF(NP(6).EQ.1) SDX(NPSUM(6))=SDX(NPSUM(6))*T
C      CONVERT SCALE TO PPM
      IF(NP(7).EQ.1) X(NPSUM(7))=X(NPSUM(7))*1.D6
      IF(NP(7).EQ.1) SDX(NPSUM(7))=SDX(NPSUM(7))*1.D6
C
C      COMPUTE MEANS OF THE COORD DIFFERENCES(GEOD.CRT-TRANSFORMED DOPPLER)
C      THESE COORD DIFFERENCES ARE CONTAINED IN THE W-VECTOR FORMED ABOVE.
C      THEN COMPUTE RESIDUALS (DDX,DDY,DDZ) FOR EACH OF THESE MEANS.
      DXM=0.D0
      DYM=0.D0
      DZM=0.D0
      SDEVX=0D0
      SDEVY=0D0
      SDEVZ=0D0
      VEC=0D0
      DO 91000 I=1,N
      DXM=DXM+W(3*I-2)
      DYM=DYM+W(3*I-1)
      DZM=DZM+W(3*I)
91000 CONTINUE
      DXM=DXM/DFLOAT(N)
```

```
      DYM=DYM/DFLOAT(N)
      DZM=DZM/DFLOAT(N)
      DO 92000 I=1,N
      DDX(I)=W(3*I-2)-DXM
      DDY(I)=W(3*I-1)-DYM
      DDZ(I)=W(3*I)-DZM
      DIFVEC(I)=DSQRT(DDX(I)**2+DDY(I)**2+DDZ(I)**2)
      VEC=VEC+DIFVEC(I)
      SDEVX=SDEVX+DDX(I)*DDX(I)
      SDEVY=SDEVY+DDY(I)*DDY(I)
      SDEVZ=SDEVZ+DDZ(I)*DDZ(I)
92000 CONTINUE
      SDEVX=DSQRT(SDEVX/DFLOAT(N*(N-1)))
      SDEVY=DSQRT(SDEVY/DFLOAT(N*(N-1)))
      SDEVZ=DSQRT(SDEVZ/DFLOAT(N*(N-1)))
      VEC=VEC/DFLOAT(N)
C::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
C      ***********************************************************
C
C     NOW DO ALL WRITING
C      INPUT FORMATS FOLLOW
 100  FORMAT()
 120  FORMAT(A11,2F12.7,F8.2,F7.2)
 140  FORMAT(T12,2(F10.2,1X),F11.2,3F4.0)
C      ***********************************************************
C AAA
      WRITE(6,200)
200   FORMAT(1H1,5X,'PAGE 1',/)
      WRITE(6,210)
210   FORMAT(6X,6('-'),/)
      IF(NTYP.EQ.1)WRITE(6,220)
      IF(NTYP.EQ.2)WRITE(6,222)
      IF(NTYP.EQ.3)WRITE(6,224)
 220  FORMAT(20X,'DETERMINATION OF DATUM TRANSFORMATION PARAMETERS USING
     C "BURSA'S" MODEL',/,20X,70('-'),/)
 222  FORMAT(20X,'DETERMINATION OF DATUM TRANSFORMATION PARAMETERS USING
     C "MOLODENSKI'S" MODEL',/,20X,75('-'),/)
 224  FORMAT(20X,'DETERMINATION OF DATUM TRANSFORMATION PARAMETERS USING
     C "VEIS'S" MODEL',/,20X,69('-'),/)
      WRITE(6,240)
 240  FORMAT(20X,'FOR THE TRANSFORMATION : CAPE DATUM TO CONVENTIONAL ',
     C'TERRESTRIAL (C.T.)',/,20X,70('-'),/)

C
      WRITE(6,280) DOPTZ,DOPRZ,DOPS
 280  FORMAT(/,20X,'TRANSFORM DOPPLER SATELLITE SYSTEM (NSWC 9Z-2)',
     C' TO CONVENTIONAL TERRESTRIAL (C.T.) SYSTEM BY :',
     C//,20X,'Z-TRANSLATION =',F4.1,' METRES',5X,'Z-ROTATION =',F4.1,
     C' ARC SECONDS',5X,'SCALE =',F4.1,' PPM',//)
C
      WRITE(6,300) N
 300  FORMAT(20X,'NUMBER OF COMMON POINTS =',I3,//)
      WRITE(6,320)
 320  FORMAT(30X,'TRANSLATION COMPONENTS (METRES)')
      WRITE(6,330)
 330  FORMAT(30X,31('-'),/)
      IF(NP(1).EQ.1) WRITE(6,340) X(NPSUM(1)),SDX(NPSUM(1))
 340  FORMAT(30X,'X0=',F8.1,2X,'+/-',2X,F5.1,/)
      IF(NP(2).EQ.1) WRITE(6,360) X(NPSUM(2)),SDX(NPSUM(2))
 360  FORMAT(30X,'Y0=',F8.1,2X,'+/-',2X,F5.1,/)
      IF(NP(3).EQ.1) WRITE(6,380) X(NPSUM(3)),SDX(NPSUM(3))
 380  FORMAT(30X,'Z0=',F8.1,2X,'+/-',2X,F5.1,/)
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      WRITE(6,400)
  400 FORMAT(//,30X,'SCALE FACTOR (COORD. SYSTEM)')
      WRITE(6,410)
  410 FORMAT(30X,28('-'),/)
      IF(NP(7).EQ.1) WRITE(6,420)  X(NPSUM(7)),SDX(NPSUM(7))
  420 FORMAT(30X,F5.1,3X,'+/-',1X,F5.1,3X,'PPM'/)

      WRITE(6,440)
  440 FORMAT(//,30X,'ROTATION ANGLES (SECONDS OF ARC)')
      WRITE(6,450)
  450 FORMAT(30X,32('-'),/)
      IF(NP(4).EQ.1.AND.NTYP.NE.3) WRITE(6,460) X(NPSUM(4)),
     CSDX(NPSUM(4))
      IF(NP(4).EQ.1.AND.NTYP.EQ.3) WRITE(6,462) X(NPSUM(4)),
     CSDX(NPSUM(4))
  460 FORMAT(30X,'RX=',F7.3,3X,'+/-',1X,F7.3,/)
  462 FORMAT(30X,'DA=',F7.3,3X,'+/-',1X,F7.3,/)
      IF(NP(5).EQ.1.AND.NTYP.NE.3) WRITE(6,480) X(NPSUM(5)),
     CSDX(NPSUM(5))
      IF(NP(5).EQ.1.AND.NTYP.EQ.3) WRITE(6,482) X(NPSUM(5)),
     CSDX(NPSUM(5))
  480 FORMAT(30X,'RY=',F7.3,3X,'+/-',1X,F7.3,/)
  482 FORMAT(30X,'DU=',F7.3,3X,'+/-',1X,F7.3,/)
      IF(NP(6).EQ.1.AND.NTYP.NE.3)WRITE(6,500) X(NPSUM(6)),
     CSDX(NPSUM(6))
      IF(NP(6).EQ.1.AND.NTYP.EQ.3)WRITE(6,502) X(NPSUM(6)),
     CSDX(NPSUM(6))
  500 FORMAT(30X,'RZ=',F7.3,3X,'+/-',1X,F7.3,/)
  502 FORMAT(30X,'DV=',F7.3,3X,'+/-',1X,F7.3,/)
      IF(NTYP.EQ.1) WRITE(6,510)
      IF(NTYP.EQ.2.AND.NROT.EQ.1.AND.NCG.EQ.1) WRITE(6,512)
      IF(NTYP.EQ.2.AND.NROT.EQ.1.AND.NCG.EQ.2) WRITE(6,513)
      IF(NTYP.EQ.3.AND.NROT.EQ.1.AND.NCG.EQ.1) WRITE(6,512)
      IF(NTYP.EQ.3.AND.NROT.EQ.1.AND.NCG.EQ.2) WRITE(6,513)
      IF(NTYP.EQ.2.AND.NROT.EQ.0.OR.NTYP.EQ.3.AND.NROT.EQ.0)
     CWRITE(6,514)
  510 FORMAT(/,30X,'THE POINT OF ROTATION IS THE CENTRE OF GEODETIC',
     C' ELLIPSOID',/)
  512 FORMAT(/,30X,'THE POINT OF ROTATION IS THE CENTRE OF GRAVITY'
     C,/,30X,'  OF THE TERRESTRIAL NETWORK.(CARTESIAN CRDS)',/)
  513 FORMAT(/,30X,'THE POINT OF ROTATION IS THE CENTRE OF GRAVITY'
     C,/,30X,'  OF THE TERRESTRIAL NETWORK.(ELLIPSOIDAL CRDS)',/)
  514 FORMAT(/,30X,'THE POINT OF ROTATION IS THE NETWORK INITIAL',
     C' POINT.',/)

      WRITE(6,516) XROT,YROT,ZROT,ROTLAT,ROTLON,ROTHT
  516 FORMAT(/,30X,'CARTESIAN COORDS OF ROTATION POINT:',3(2X,F12.2)/
     C,30X,'ELLIPSOIDAL COORDS OF ROTATION POINT:',2F15.7,F12.2)
C     *****************************************************************
      WRITE(6,520)
  520 FORMAT(1H1,5X,'PAGE 2',/)
      WRITE(6,210)
      WRITE(6,530)
  530 FORMAT(25X,'OBSERVATIONS AND RESIDUALS',/,25X,26('-'),/,
     C27X,'GEODETIC CARTESIAN',24X,'RESIDUALS',15X,'ELLIPSOIDAL',/)
      WRITE(6,822)
      DO 95000 I=1,N
      WRITE(6,830)NAME(I),(CRT(I,J),J=1,9)
95000 CONTINUE
      WRITE(6,560)
  560 FORMAT(1H1,12X,'TRANSFORMED DOPPLER SATELLITE COORDINATES',
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     C14X,'RESIDUALS',/)
      WRITE(6,822)
 822  FORMAT(19X,'X',14X,'Y',11X,'Z',16X,'X',7X,'Y',7X,'Z',
     C8X,'LAT',5X,'LONG',5X,'HT',/)
      DO 95100 I=1,N
      WRITE(6,830)NAME(I),(SAT(I,J),J=1,9)
95100 CONTINUE
C     ***********************************************************************
      WRITE(6,600)
 600  FORMAT(1H1,5X,'PAGE 3',/)
      WRITE(6,210)
      WRITE(6,610) AVF,DSQRT(AVF)
 610  FORMAT(30X,  'REFERENCE VARIANCE =',2X,F8.4,//,
     C30X,   'SIGMA NOUGHT =',2X,F8.4,//)
C
      WRITE(6,630)
 630  FORMAT(//,40X,17HCOVARIANCE MATRIX,/)
      WRITE(6,195) (HEAD(I),I=1,NU)
      DO 95200 I=1,NU
      WRITE(6,650)HEAD(I), (QX(I,J),J=1,NU)
 650  FORMAT(/,5X,A10,7(2X,E12.6))
95200 CONTINUE
C
      WRITE(6,670)
 670  FORMAT(//,30X,'CORRELATION MATRIX',/)
      WRITE(6,675) (HEAD(I),I=1,NU)
 675  FORMAT(15X,7(2X,A5))
      DO 95300 I=1,NU
      WRITE(6,680)HEAD(I),(CORR(I,J),J=1,NU)
 680  FORMAT(/,5X,A10,7(2X,F5.2))
95300 CONTINUE
C     ***********************************************************************
C     PRINT ALL INPUT DATA

      WRITE(6,700)
 700  FORMAT(1H1,5X,'PAGE 4',30X,'INPUT DATA',/,6X,6('-'),30X,10('-'),/)
      WRITE(6,720)  AM,BM
 720  FORMAT(5X,'ELLIPSOID PARAMETERS',7X,'AM= ',F15.6,5X,
     C'BM=',F15.6,/)
      WRITE(6,730) N,NTYP,NROT,NCG,NQ
 730  FORMAT(5X,'N,NTYP,NROT,NCG,NQ:',5(2X,I2),/)
      WRITE(6,740)(NP(I),I=1,7)
 740  FORMAT(5X,'CODE FOR IDENTIFYING UNKNOWNS :',3X,
     C'TRANSLATIONS :',3I3,5X,'ROTATIONS :',3I3,5X,'SCALE :',I3,/)
      WRITE(6,750)DOPTZ,DOPRZ,DOPS
 750  FORMAT(5X,'TRANSFORM DOPPLER SAT.COORDS TO C.T. COORDS :',3X
     C,'Z-TRANSLATION DOPTZ=',F4.1,3X,'Z-ROTATION DOPRZ=',F4.1,3X,
     C'SCALE DOPS=',F4.1,/)
      WRITE(6,770)
 770  FORMAT(/,38X,'GEODETIC ELLIPSOIDAL',20X,'STANDARD DEVIATIONS',
     C/,25X,'LATITUDE',7X,'LONGITUDE',6X,'ELL HT',3X,
     C'GEOID HT',9X,'"',5X,'"',5X,'M',/)
      DO 95400 I=1,N
C     CHANGE S.D. OF ELLIPSOIDAL COORDS(LAT,LONG) FROM RADIANS TO SECONDS.
      SDE(I,1)=SDE(I,1)*T
      SDE(I,2)=SDE(I,2)*T
      WRITE(6,790)NAME(I),(ELL(I,J),J=1,4),(SDE(I,K),K=1,3)
 790  FORMAT(/,12X,A11,2(F12.7,3X),2F9.2,10X,2(F3.2,3X),F3.1)
95400 CONTINUE
      WRITE(6,810)
 810  FORMAT(1H1,28X,'DOPPLER SATELLITE ',18X,'STANDARD DEVIATIONS',/)
      WRITE(6,820)
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 820  FORMAT(19X,'X',14X,'Y',11X,'Z',16X,'X',7X,'Y',7X,'Z',/)
      DO 95500 I=1,N
      WRITE(6,830)NAME(I),(SATIN(I,J),J=1,3),(SDS(I,K),K=1,3)
 830  FORMAT(/,2X,A11,3(F12.2,2X),5X,3(F6.2,2X),2X,3(F6.2,2X))
95500 CONTINUE
      WRITE(6,835) RP0,RL0,RH0
 835  FORMAT(5X,/,'NETWORK INITIAL POINT COORDS:',//,5X,'LATITUDE=',
     CF13.7,//,5X,'LONGITUDE=',F13.7,//,5X,'ELL. HT  =',F8.2,//)
C     ****************************************************************
      WRITE(6,840)
 840  FORMAT(1H1,5X,'PAGE 5',/)
      WRITE(6,210)
      WRITE(6,850)
 850  FORMAT(28X,'GEODETIC CARTESIAN',18X,'STANDARD DEVIATIONS',/)
      WRITE(6,820)
      DO 25000 I=1,N
      WRITE(6,830)NAME(I),(CRT(I,J),J=1,3)
25000 CONTINUE
      WRITE(6,860)
 860  FORMAT(1H1,10X,'COORDINATE DIFFERENCES : GEODETIC CARTESIAN ',
     C' - TRANSFORMED DOPPLER',/,11X,66('-'),/)
      WRITE(6,870)

 870  FORMAT(30X,'DX',6X,'RES',7X,'DY',6X,'RES',7X,'DZ',6X,'RES',
     C11X,'VECTOR',/)
      DO 95600 I=1,N
      WRITE(6,880) NAME(I),W(3*I-2),DDX(I),W(3*I-1),DDY(I),W(3*I),DDZ(I)
     C,DIFVEC(I)
 880  FORMAT(12X,A11,3X,3(F8.2,1X,F7.2,2X),5X,F8.2,/)
95600 CONTINUE
      WRITE(6,890)DXM,SDEVX,DYM,SDEVY,DZM,SDEVZ,VEC
 890  FORMAT(/,10X,'MEANS :',9X,3(F8.2,1X,'+/-',F4.2,2X),5X,F8.2,//)
C     ****************************************************************
      WRITE(6,945)
945   FORMAT(1H1,5X,'PAGE 7',5X,'"PSEUDO-GEODETIC" COORDS = ',
     C'REVERSE TRANSFORMATION * (CTS COORDS)',/,6X,6('-'),5X,
     C64('-'),/)

      WRITE(6,955)
955   FORMAT(5X,'DIFFERENCES: GEODETIC - "PSEUDO-GEODETIC"',/,
     C5X,41('-'),//,22X,'DX',9X,'DY',9X,'DZ',10X,'VECTOR',11X,
     C'DLAT',8X,'DLONG',7X,'DHT',/)
      DO 95800 I=1,N
      WRITE(6,940)NAME(I),(DELTA(I,K),K=1,3),DELTA(I,7),
     C(DELTA(I,L),L=4,6)
 940  FORMAT(3X,A11,3(3X,F8.2),5X,F8.2,10X,3(F8.2,3X),/)
95800 CONTINUE
      WRITE(6,941) DELVEC
       CALL ELLCRT(AM,BM,CGP,CGL,CGH,CGX1,CGY1,CGZ1)
 941  FORMAT(/,20X,'MEAN VECTOR OF DISPLACEMENT:',6X,F6.2,/)
      WRITE(6,943) CGX,CGY,CGZ,CGP,CGL,CGH,CGX1,CGY1,CGZ1
943   FORMAT(5X,'CG. OF NETWORK',3(2X,F12.2),/,20X,2(F13.7),F10.2
     C,/,20X,3(2X,F12.2)/)
C ****************************************************************
      STOP
      END
C-----------------------------------------------------------
C-------------------------------------------------------------
      SUBROUTINE  ELLCRT(A,B,PHI,RL,H,XO,YO,ZO)
      IMPLICIT REAL*8(A-H,O-Z)
C DMS(A): A IN RADS
      DEFINE DMS(A)=DINT(A*T/3600D0)+
```

```
     CDINT((A*T/3600D0-DINT(A*T/3600D0))*60D0)/100D0+
     C((A*T/3600D0-DINT(A*T/3600D0))*60D0-
     CDINT((A*T/3600D0-DINT(A*T/3600D0))*60D0))*60D0/10000D0
C  RAD(A): A IN DEG.MINSEC
      DEFINE RAD(A)=(DINT(A)+(DINT((A-DINT(A))*100D0))/60D0+
     C(A*100D0-DINT(A*100D0))/36D0)*3600/T
      T=206264.80625D0
      E2=1.D0-B**2/(A**2)
      RN=A/DSQRT((COS(RAD(PHI)))**2+(1.D0-E2)*(SIN(RAD(PHI)))**2)
      XO=(RN+H)*COS(RAD(PHI))*COS(RAD(RL))
      YO=(RN+H)*COS(RAD(PHI))*SIN(RAD(RL))
      ZO=(RN*(1.D0-E2)+H)*SIN(RAD(PHI))
      RETURN
      END
C-----------------------------------------------------------------
C SUBROUTINE FOR CARTESIAN COORD CONVERSION BY Z-SHIFT,Z-ROT,SCALE
C AS IN TRANSFORMING DOPPLER COORDS TO CTS COORDS

C    ARGUMENTS: TZ: SHIFT IN Z
C               RZ: ROT AROUND Z AXIS IN SECONDS OF ARC
C               S : SCALE CHANGE FROM UNITY IN PPM
C               DX,DT,DZ: OLD (DOPPLER) COORDS TO BE TRANSFORMED
C               X,Y,Z:  NEW (TRANSFORMED) COORDS
      SUBROUTINE DOPTRS(TZ,RZ,S,DX,DY,DZ,X,Y,Z)
      IMPLICIT REAL*8(A-H,O-Z)
      T=206264.80625D0
      DS=1D0+S*1D-6
      X=(DX*COS(RZ/T)+DY*SIN(RZ/T))*DS
      Y=(DY*COS(RZ/T)-DX*SIN(RZ/T))*DS
      Z=(DZ*DS)+TZ
      RETURN
      END
C-----------------------------------------------------------------
C  SUBROUTINE FOR COMPUTING STANDARD DEVIATIONS FOR ELLIPSOIDAL
C  COORDS ACCORDING TO SOME EMPIRICAL FORMULAE (EG SIMMONS)
C  THE FORMULAE FOR SDP,SDL ARE MODIFIED FROM THOSE USED BY DON THOMSON.
C  THE FORMULA FOR SDH IS ENTIRELY IMAGINARY
C NOTE:::: THESE FORMULAE ARE SUPPOSED TO GIVE ACCURACY ESTIMATES FOR
C THE ELLIPSOIDAL COORDS  R E L A T I V E  TO THE NETWORK INITIAL PT.
C   ARGUMENTS: PH,RL LAT,LONG OF N I P.
C              PHN,RLN : LAT,LONG OF STATION
C              SDP,SDL,SDH: STANDARD DEVIATIONS OF LAT,LONG,HT
      SUBROUTINE SDELL(PH,RL,PHN,RLN,SDP,SDL,SDH)
      IMPLICIT REAL*8(A-H,O-Z)
C  RAD(A): A IN DEG.MINSEC
      DEFINE RAD(A)=(DINT(A)+(DINT((A-DINT(A))*100D0))/60D0+
     C(A*100D0-DINT(A*100D0))/36D0)*3600/T
      T=206264.80625D0
      DPH=RAD(PHN)-RAD(PH)
      DRL=RAD(RLN)-RAD(RL)
      DIST=6370D0*DSQRT(DPH**2+(DRL*COS((RAD(PHN)+RAD(PH))/2D0))**2)
      SDP=(0.05D0*DIST**(2D0/3D0))/6378000D0
      SDL=SDP
      SDH=0.015D0*DIST**(2D0/3D0)
      SDHN=0.001D0*DIST
      SDH=DSQRT(SDH**2+SDHN**2)
C NOTE THAT ELLIPSOIDAL HEIGHT H=H'+N, WHERE H'=ORTHOMETRIC HT,AND
C N=GEOIDAL HT. THEN BY PROPOGATION OF ERRORS, VAR(H)=VAR(H')+VAR(N).
C HERE THE ESTIMATE FOR S.DEV(N)=0.5 METRE, HENCE VAR(N)=0.5**2, AS ABOVE.
C NOTE THAT THE STANDARD DEVIATIONS FOR LAT,LONG ARE EXPRESSED IN
C RADIANS OF ARC, AND FOR HEIGHT IN METRES, SINCE IN THE FORMATION
C OF THE VCV MATRIX ONE NEEDS THESE IN RADIANS FOR LAT,LONG AND METRES
```

```
C FOR HEIGHT TO GET THE UNITS (AND SCALE FACTORS) RIGHT.
      RETURN
      END
C=================================================================
       SUBROUTINE DIFCE(RP,RL,DX,DY,DZ,DP,DL,DH)
C  ARGUMENTS: RP,RL: LAT,LONG OF POINT
C             DX,DY,DZ: DIFFERENTIAL CARTESIAN COORDS TO BE TRANSFORMED
C INTO        DP,DL,DH: DIFFERENTIAL ELLIPSOIDAL COORDS
      IMPLICIT REAL*8(A-H,O-Z)
C  RAD(A): A IN DEG.MINSEC
      DEFINE RAD(A)=(DINT(A)+(DINT((A-DINT(A))*100D0))/60D0+
     C(A*100D0-DINT(A*100D0))/36D0)*3600/T
      T=206264.80625D0
      DP=COS(RAD(RP))*DZ-SIN(RAD(RP))*COS(RAD(RL))*DX-
     CSIN(RAD(RP))*SIN(RAD(RL))*DY
      DL=COS(RAD(RL))*DY-SIN(RAD(RL))*DX
      DH=COS(RAD(RP))*COS(RAD(RL))*DX+COS(RAD(RP))*SIN(RAD(RL))*
     CDY+SIN(RAD(RP))*DZ
      RETURN
      END
C**********************************************************************
      SUBROUTINE REVTFN(NTYP,TX,TY,TZ,RX,RY,RZ,SCA,XROT,YROT,ZROT,
     CRLA,RLO,X,Y,Z,PSX,PSY,PSZ)
C      ARGUMENTS: TX,TY,TZ,RX,RY,RZ,SCA- SHIFT,ROTATION,SCALE PARAMETERS
C                 NTYP: MODEL TYPE BURSA(1),MOLOD(2),VEIS(3).
C                XROT,YROT,ZROT: TERRESTRIAL CART. COORDS OF ROTATION PT.
C     NOTE: IN BURSA MODEL, XROT=YROT=ZROT=0, AND IN MOLOD. AND VEIS
C           MODELS, THE ROTATION PT. CAN BE THE N.I.P. OR THE CG. OF NETW.
C                RLA,RLO: LATITUDE/LONGITUDE OF ROTATION POINT FOR USE
C                           IN THE ROTATION MATRIX IN THE VEIS MODEL.
C                 X,Y,Z: INPUT COORDS TO BE REVERSELY TRANSFORMED
C                 PSX,PSY,PSZ: TRANSFORMED COORDS
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION R(3,3)
C  RAD(A): A IN DEG.MINSEC
      DEFINE RAD(A)=(DINT(A)+(DINT((A-DINT(A))*100D0))/60D0+
     C(A*100D0-DINT(A*100D0))/36D0)*3600/T
      T=206264.80625D0
      IF (NTYP.EQ.3) GOTO 10000
C     FORM 'R' MATRIX FOR BURSA/MOLODENSKI MODELS.
      R(1,1)=1D0
      R(1,2)=RZ
      R(1,3)=-1D0*RY
      R(2,1)=-1D0*RZ
      R(2,2)=1D0
      R(2,3)=RX
      R(3,1)=RY
      R(3,2)=-1D0*RX
      R(3,3)=1D0
      GOTO 20000
C  FORM 'R' MATRIX FOR VEIS MODEL:
10000 RP=RAD(RLA)
      RL=RAD(RLO)
      DA=RX
      DU=RY
      DV=RZ
      R(1,1)=1D0
      R(1,2)=-1D0*SIN(RP)*DA-COS(RP)*DV
      R(1,3)=SIN(RL)*COS(RP)*DA+COS(RL)*DU-SIN(RP)*SIN(RL)*DV
      R(2,1)=SIN(RP)*DA+COS(RP)*DV
      R(2,2)=1D0
      R(2,3)=-1D0*COS(RP)*COS(RL)*DA+SIN(RL)*DU+SIN(RP)*COS(RL)*DV
```

```
      R(3,1)=-1D0*COS(RP)*SIN(RL)*DA-COS(RL)*DU+SIN(RP)*SIN(RL)*DV
      R(3,2)=COS(RP)*COS(RL)*DA-SIN(RL)*DU-SIN(RP)*COS(RL)*DV
      R(3,3)=1D0
20000 CONTINUE
      X1=(X-TX-XROT)/SCA
      Y1=(Y-TY-YROT)/SCA
      Z1=(Z-TZ-ZROT)/SCA
      PSX=R(1,1)*X1+R(2,1)*Y1+R(3,1)*Z1+XROT
      PSY=R(1,2)*X1+R(2,2)*Y1+R(3,2)*Z1+YROT
      PSZ=R(1,3)*X1+R(2,3)*Y1+R(3,3)*Z1+ZROT
C   NOTE: THE TERR.CART.COORDS OF ROTATION PT (XROT,YROT,ZROT) IS ADDED
C   IN THE REVERSE TRANSFORMATION, SINCE THE RESULTING VECTORS AFTER
C   APPLYING THE REVERSE TRANSFORMATION TO THE SATELLITE COORDS ARE
C   RELATIVE TO THE ROTATION POINT; IN BURSA CASE, THIS IS THE POINT
C   (0,0,0), BUT IN THE MOLOD/VEIS CASES, THIS COULD BE THE CG. OR NIP.
      RETURN
      END
C------------------------------------
C    CONVERSION CARTESIAN(X,Y,Z) TO ELLIPSOIDAL(LAT,LONG,H) USING
C    B.R.BOWRING FORMULAE (SURVEY REVIEW VOL 28, 218, OCTOBER 1985
C    PAGES 202-206)

C INPUT DATA: A,B -SEMI-MAJOR/MINOR AXES OF EARTH ELLIPSOID
C               X,Y,Z : GEODETIC CARTESIAN COORDS
C INTERMEDIATES: E1/E2: SQUARE OF FIRST/SECOND ECCENTRICITIES
C                 V: RADIUS OF CURVATURE IN PRIME VERTICAL PLANE
C   A AS INPUT FOR DMS(A) IS IN RADIANS
      SUBROUTINE CE(A,B,X,Y,Z,PHI,RL,H)
      IMPLICIT REAL*8(A-H,O-Z)
      DEFINE DMS(A)=DINT(A*RHO/3600D0)+
     CDINT((A*RHO/3600D0-DINT(A*RHO/3600D0))*60D0)/100D0+
     C((A*RHO/3600D0-DINT(A*RHO/3600D0))*60D0-
     CDINT((A*RHO/3600D0-DINT(A*RHO/3600D0))*60D0))*60D0/10000D0
C   A AS INPUT FOR RAD(A) IS IN DEG.MINSEC
      DEFINE RAD(A)=(DINT(A)+(DINT((A-DINT(A))*100D0))/60D0+
     C(A*100D0-DINT(A*100D0))/36D0)*3600/RHO
      RHO=206264.80625D0
      E1=1.D0-B**2/(A**2)
      E2=(A**2-B**2)/B**2
      P=DSQRT(X**2+Y**2)
      SZ=SIGN(1.D0,Z)
      R=DSQRT(P**2+Z**2)
      TANU=B/A*Z/P*(1.D0+E2*B/R)
      COSU=1.D0/DSQRT(TANU**2+1.D0)
      SINU=SZ*DSQRT(1.D0-COSU**2)
      TANB=(Z+E2*B*SINU**3)/(P-E1*A*COSU**3)
      PHI=DATAN(TANB)
      V=A/DSQRT(1.D0-E1*(SIN(PHI))**2)
      H=P*COS(PHI)+Z*SIN(PHI)-A**2/V
      RL=DATAN2(Y,X)
      PHI=DMS(PHI)
      RL=DMS(RL)
C CONVERTING LAT/LONG IN FROM RADIANS TO DEG.MMSSS
      RETURN
      END
C ------------------------------------
C
@MAP,ISE TPF$.MAP,.MAIN
 IN TPF$.MAIN,RENS.CHOLD
@XQT TPF$.MAIN
```

DETERMINATION OF DATUM TRANSFORMATION PARAMETERS USING "BURSA'S" MODEL

FOR THE TRANSFORMATION : CAPE DATUM TO CONVENTIONAL TERRESTRIAL (C.T.)

TRANSFORM DOPPLER SATELLITE SYSTEM (NSWC 9Z-2) TO CONVENTIONAL TERRESTRIAL (C.T.) SYSTEM BY :

Z-TRANSLATION = 4.0 METRES Z-ROTATION = -.8 ARC SECONDS SCALE = -.5 PPM

NUMBER OF COMMON POINTS = 23

TRANSLATION COMPONENTS (METRES)

X0=	-198.5	+/-	14.4
Y0=	-122.2	+/-	12.6
Z0=	-259.8	+/-	16.1

SCALE FACTOR (COORD. SYSTEM)

11.1 +/- 1.5 PPM

ROTATION ANGLES (SECONDS OF ARC)

RX=	.363	+/-	.335
RY=	.243	+/-	.584
RZ=	.436	+/-	.451

THE POINT OF ROTATION IS THE CENTRE OF GEODETIC ELLIPSOID

CARTESIAN COORDS OF ROTATION POINT:	.00	.00	.00
ELLIPSOIDAL COORDS OF ROTATION POINT:	.0000000	.0000000	.00

OBSERVATIONS AND RESIDUALS

	GEODETIC CARTESIAN			RESIDUALS			ELLIPSOIDAL		
	X	Y	Z	X	Y	Z	LAT	LONG	HT
PONT	5157893.78	2378293.94	-2400431.26	-2.31	1.10	-3.14	-3.47	2.09	-.18
KRANZBERG	5149397.07	2691037.17	-2626422.06	-1.54	1.45	-2.00	-2.11	2.00	.19
THAMAKOOSH	5019377.00	2903697.05	-2651973.76	-.96	.33	-1.96	-2.06	.77	.21
MANNERHEIM	5291622.88	2255083.54	-2740304.51	2.33	2.88	4.91	5.84	1.73	.83
HAOIDA	5294743.90	2085821.71	-2872769.70	3.28	-2.38	4.19	4.72	-3.41	.04
MORGENZON	5138865.54	2424474.51	-2890855.24	-.13	2.18	1.36	1.59	2.02	.10
MOOIDAM	4983839.20	2732706.71	-2837742.11	-.29	-.15	-.60	-.68	.00	-.01
MAGWAZA	4786233.73	3006513.75	-2946617.37	-.05	-2.16	-1.50	-1.88	-1.80	-.36
WITBANK N	5361121.05	1605173.39	-3050291.37	-.07	-6.05	-3.61	-4.03	-5.78	.16
GRASKOP	5246359.55	1904123.93	-3079065.05	1.75	-3.06	-.32	.01	-3.48	.68
WITWATER	5137640.02	2135077.40	-3078166.97	1.04	.01	1.81	2.05	-.40	-.04
LEEUKOP.SA	4986898.49	2475851.86	-3104547.73	.45	.37	1.40	1.50	.13	-.19
INKOMINKULU	4776617.80	2809322.79	-3149351.18	1.14	-2.05	1.08	.91	-2.34	-.59
FRANSMANKOP	4988162.54	2267479.43	-3256556.98	.67	.94	1.59	1.87	.58	.04
LOUISFTN	5205455.86	1690219.94	-3265501.32	-1.26	-1.34	-3.50	-3.83	-.89	.42
POTLOER	5101378.09	1977632.65	-3269927.02	-.01	-.23	-.11	-.14	-.21	-.02
LUBISI	4815770.00	2506293.13	-3339947.63	.96	-.02	2.26	2.37	-.46	-.47
HEXRIVIER	5022168.52	1795165.38	-3490357.75	-1.30	1.83	-1.71	-1.77	2.16	.43
BLYDEBERG	4917299.12	2106514.48	-3463660.17	-.47	1.77	1.36	1.28	1.81	-.52
COEGAKOP	4785924.06	2295261.42	-3524985.75	.51	1.55	2.49	2.70	1.13	-.44
M'PUMBE N	4943188.45	3076434.84	-2596398.32	-.68	-1.04	-2.56	-2.79	-.52	.02
KARSRIVIER	4944384.20	1811092.50	-3587431.02	-1.92	2.72	-.38	-1.21	3.21	-.22
BRIT 44	5036495.71	2657589.54	-2767924.39	-1.14	1.36	-.56	-.67	1.73	-.09

	TRANSFORMED DOPPLER SATELLITE COORDINATES			RESIDUALS					
	X	Y	Z	X	Y	Z	LAT	LONG	HT
PONT	5157756.90	2578190.79	-2400772.91	2.31	-1.10	3.14	3.47	-2.09	.18
KRANZBERG	5149761.51	2690982.25	-2626713.54	1.54	-1.45	2.00	2.11	-2.00	-.19
THAMAKOOSH	5019241.55	2903592.48	-2652266.05	.96	-.33	1.96	2.06	-.77	-.21
MANNERHEIM	5291495.38	2264981.24	-2741092.60	-2.33	-2.83	-4.91	-5.84	-1.73	-.83
HADIDA	5294613.60	2085701.67	-2873050.37	-3.28	2.38	-4.19	-4.72	3.41	-.04
MORGENZON	5133732.45	2424367.64	-2891142.55	.13	-2.13	-1.36	-1.59	-2.02	-.10
MOOIDAM	4983705.32	2732598.93	-2888034.05	.29	.15	.60	.68	.00	.01
MAGWAZA	4755093.17	3006405.33	-2946912.43	.05	2.16	1.50	1.88	1.90	.36
WITBANK N	5550989.00	1605040.21	-3050578.69	.07	6.05	3.61	4.03	5.78	-.16
GRASKOP	5246210.54	1904000.25	-3079556.77	-1.75	3.06	.32	-.01	3.48	-.68
WITWATER	5137508.95	2182963.19	-3078455.04	-1.04	-.01	-1.81	-2.05	.40	.04
LEEUKOP.SA	4986755.22	2475721.88	-3104837.61	-.45	-.37	-1.40	-1.50	-.13	.19
INKOMINKULU	4776484.34	2809212.05	-3149643.05	-1.14	2.05	-1.08	-.91	2.34	.59
FRANSMANKOP	4985029.47	2267368.02	-3256847.81	-.67	-.94	-1.59	-1.87	-.58	-.04
LOUISFTN	5205320.12	1690097.08	-3268801.59	1.26	1.34	3.50	3.83	.89	-.42
POTLOER	5101244.35	1977515.43	-3270220.74	.01	.23	.11	.14	.21	.02
LUBISI	4815636.21	2506182.66	-3340238.67	-.96	.02	-2.26	-2.37	.46	.47
HEXRIVIER	5022031.23	1795050.02	-3490656.90	1.30	-1.83	1.71	1.77	-2.16	-.43
BLYDEBERG	4917152.90	2106502.72	-3463953.56	.47	-1.77	-1.36	-1.28	-1.81	.52
COEGAKOP	4785785.81	2295151.49	-3525278.06	-.51	-1.55	-2.49	-2.70	-1.18	.44
M'PUMBE N	4943053.11	3076329.71	-2596691.59	.68	1.04	2.56	2.79	.52	-.02
KARSRIVIER	4944244.90	1810979.09	-3587729.71	1.92	-2.72	.88	1.21	-3.21	.22
BRIT 44	5086360.38	2667484.06	-2768214.69	1.14	-1.36	.56	.67	-1.73	.09

REFERENCE VARIANCE = 8.4574

SIGMA NOUGHT = 2.9082

COVARIANCE MATRIX

	TLN-X	TLN-Y	TLN-Z	ROT-X	ROT-Y	ROT-Z	SCALE
TLN-X	.207367+003	-.719404+002	.131152+003	.224453-005	-.320258-004	-.210895-004	-.117711-004
TLN-Y	-.719404+002	.158604+003	-.698869+002	.782140-005	.141770-004	.239871-004	-.548920-005
TLN-Z	.131152+003	-.698869+002	.258143+003	.976913-005	-.421393-004	-.164215-004	.709928-005
ROT-X	.224453-005	.782140-005	.976913-005	.263626-011	-.705297-012	-.410221-013	-.177542-028
ROT-Y	-.320258-004	.141770-004	-.421393-004	-.705297-012	.801629-011	.323295-011	.163537-027
ROT-Z	-.210895-004	.239871-004	-.164215-004	-.410221-013	.323295-011	.477510-011	.946411-028
SCALE	-.117711-004	-.548920-005	.709928-005	-.177542-028	.163537-027	.946411-028	.233113-011

CORRELATION MATRIX

	TLN-X	TLN-Y	TLN-Z	ROT-X	ROT-Y	ROT-Z	SCALE
TLN-X	1.00	-.40	.57	.10	-.79	-.67	-.54
TLN-Y	-.40	1.00	-.35	.38	.40	.87	-.29
TLN-Z	.57	-.35	1.00	.37	-.93	-.47	.29
ROT-X	.10	.38	.37	1.00	-.15	-.01	.00
ROT-Y	-.79	.40	-.93	-.15	1.00	.52	.00
ROT-Z	-.67	.87	-.47	-.01	.52	1.00	.00
SCALE	-.54	-.29	.29	.00	.00	.00	1.00

DETERMINATION OF DATUM TRANSFORMATION PARAMETERS USING "MOLODENSKI'S" MODEL

FOR THE TRANSFORMATION : CAPE DATUM TO CONVENTIONAL TERRESTRIAL (C.T.)

TRANSFORM DOPPLER SATELLITE SYSTEM (NSWC 9Z-2) TO CONVENTIONAL TERRESTRIAL (C.T.) SYSTEM BY :
Z-TRANSLATION = 4.0 METRES   Z-ROTATION = -.3 ARC SECONDS   SCALE = -.5 PPM

NUMBER OF COMMON POINTS = 23

TRANSLATION COMPONENTS (METRES)

X0= -136.4 +/- 1.6
Y0= -113.2 +/- 1.3
Z0= -297.5 +/- 1.4

SCALE FACTOR (COORD. SYSTEM)

11.1 +/- 1.5 PPM

ROTATION ANGLES (SECONDS OF ARC)

RX= .363 +/- .335
RY= .248 +/- .534
RZ= .436 +/- .451

THE POINT OF ROTATION IS THE NETWORK INITIAL POINT.

CARTESIAN COORDS OF ROTATION POINT: 4777935.98 2280227.48 -3545622.42
ELLIPSOIDAL COORDS OF ROTATION POINT: -33.5932000 25.3044622 282.00

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OBSERVATIONS AND RESIDUALS

	GEODETIC CARTESIAN			RESIDUALS			ELLIPSOIDAL		
	X	Y	Z	X	Y	Z	LAT	LONG	HT
PONT	5157893.78	2878293.94	-2400431.26	-2.31	1.10	-3.14	-3.47	2.09	-.18
KRANZBERG	5149397.07	2691087.17	-2626422.08	-1.54	1.45	-2.00	-2.11	2.00	.19
THAMAKOOSH	5019377.00	2903697.05	-2651973.76	-.95	.33	-1.96	-2.06	.77	.21
MANNERHEIM	5291622.88	2265088.54	-2740804.51	2.33	2.88	4.91	5.84	1.73	.83
HADIDA	5294743.90	2085821.71	-2872769.70	3.28	-2.38	4.19	4.72	-3.41	.04
MORGENZON	5138865.54	2424474.51	-2890855.24	-.13	2.13	1.36	1.59	2.02	.10
MOOIDAM	4933839.80	2732706.71	-2887742.11	-.29	-.15	-.60	-.68	.00	-.01
MAGWAZA	4786233.73	3006513.75	-2946617.37	-.05	-2.16	-1.50	-1.83	-1.80	-.36
WITBANK N	5361121.05	1605173.39	-3050281.37	-.07	-6.05	-3.61	-4.03	-5.78	.16
GRASKOP	5246339.55	1904123.93	-3079065.05	1.75	-3.06	-.32	.01	-3.48	.68
WITWATER	5137640.02	2183077.40	-3078166.97	1.04	.01	1.81	2.05	-.40	-.04
LEEUKOP.SA	4986398.49	2475831.86	-3104547.73	.45	.37	1.40	1.50	.13	-.19
INKOMINKULU	4776617.80	2809322.79	-3149351.18	1.14	-2.05	1.08	.91	-2.34	-.59
FRANSMANKOP	4988162.54	2267479.43	-3256556.98	.67	.94	1.59	1.87	.58	.04
LOUISFTN	5203455.86	1590219.94	-3268501.32	-1.26	-1.34	-3.50	-3.83	-.89	.42
POTLOER	5101372.09	1977632.65	-3269927.02	-.01	-.23	-.11	-.14	-.21	-.02
LUBISI	4815770.00	2506293.13	-3339947.63	.96	-.02	2.26	2.37	-.46	-.47
HEXRIVIER	5022168.62	1795165.38	-3490357.75	-1.30	1.83	-1.71	-1.77	2.16	.43
BLYDEBERG	4917299.12	2106614.40	-3463660.17	-.47	1.77	1.36	1.28	1.81	-.52
COEGAKOP	4735924.06	2295261.42	-3524985.75	.51	1.55	2.49	2.70	1.18	-.44
M'PUMBE N	4943168.45	3076434.84	-2596398.32	-.68	-1.04	-2.56	-2.79	-.52	.02
KARSRIVIER	4944384.20	1811092.50	-3587431.02	-1.92	2.72	-.88	-1.21	3.21	-.22
BRIT 44	5036495.71	2667539.54	-2767924.39	-1.14	1.36	-.56	-.67	1.73	-.09

	TRANSFORMED DOPPLER SATELLITE COORDINATES			RESIDUALS					
	X	Y	Z	X	Y	Z	LAT	LONG	HT
PONT	5157756.90	2278190.79	-2400772.91	2.31	-1.10	3.14	3.47	-2.09	.18
KRANZBERG	5149761.51	2690982.25	-2626713.64	1.54	-1.45	2.00	2.11	-2.00	-.19
THAMAKOOSH	5019241.63	2903592.43	-2652266.05	.96	-.33	1.96	2.06	-.77	-.21
MANNERHEIM	5291495.88	2264981.24	-2741082.60	-2.33	-2.88	-4.91	-5.84	-1.73	-.83
HADIDA	5294618.60	2035701.67	-2873050.37	-3.28	2.38	-4.19	-4.72	3.41	-.04
MORGENZON	5138732.45	2424367.64	-2891142.55	.13	-2.18	-1.36	-1.59	-2.02	-.10
MOOIDAM	4985705.32	2732598.93	-2888034.05	.29	.15	.60	.68	.00	.01
MAGWAZA	4786098.17	3006405.33	-2946912.48	.05	2.16	1.50	1.88	1.80	.36
WITBANK N	5360939.00	1605040.21	-3050578.69	.07	6.05	3.61	4.03	5.78	-.16
GRASKOP	5246210.54	1904000.25	-3079356.77	-1.75	3.06	.32	-.01	3.48	-.68
WITWATER	5137508.95	2182963.19	-3078455.04	-1.04	-.01	-1.81	-2.05	.40	.04
LEEUKOP.SA	4985765.22	2475721.88	-3104837.61	-.45	-.37	-1.40	-1.50	-.13	.19
INKOMINKULU	4776484.34	2809212.05	-3149643.05	-1.14	2.05	-1.08	-.91	2.34	.59
FRANSMANKOP	4988029.47	2267368.02	-3256847.81	-.67	-.94	-1.59	-1.87	-.58	-.04
LOUISFTN	5203320.12	1690097.08	-3268801.69	1.26	1.34	3.50	3.83	.89	-.42
POTLCER	5101244.33	1977515.43	-3270220.74	.01	.23	.11	.14	.21	.02
LUBISI	4815636.21	2506182.66	-3340238.67	-.96	.02	-2.26	-2.37	.46	.47
HEXRIVIER	5022031.28	1795050.02	-3490656.90	1.30	-1.83	1.71	1.77	-2.16	-.43
BLYDEBERG	4917162.90	2106502.72	-3463953.56	.47	-1.77	-1.36	-1.28	-1.81	.52
COEGAKOP	4785788.81	2295151.49	-3525273.06	-.51	-1.55	-2.49	-2.70	-1.18	.44
M'PUMBE N	4943053.11	3076329.71	-2596691.59	.68	1.04	2.56	2.79	.52	-.02
KARSRIVIER	4944244.90	1810979.09	-3587729.71	1.92	-2.72	.88	1.21	-3.21	.22
BRIT 44	5086360.38	2667484.06	-2768214.69	1.14	-1.36	.56	.67	-1.73	.09

REFERENCE VARIANCE = 8.4574

SIGMA NOUGHT = 2.9082

COVARIANCE MATRIX

	TLN-X	TLN-Y	TLN-Z	ROT-X	ROT-Y	ROT-Z	SCALE
TLN-X	.269857+001	.564633+000	-.732981+000	-.349731-006	.376884-005	.126132-005	-.633133-006
TLN-Y	.564633+000	.177134+001	-.346503+000	-.132979-005	.123086-005	.131743-005	-.173696-006
TLN-Z	-.732981+000	-.346503+000	.195319+001	.387990-006	-.222977-005	-.881125-006	-.116603-005
ROT-X	-.349731-006	-.132979-005	.387990-006	.263626-011	-.705297-012	-.410221-013	-.695236-031
ROT-Y	.376884-005	.123086-005	-.222977-005	-.705297-012	.801629-011	.323295-011	.790193-030
ROT-Z	.126132-005	.131743-005	-.881125-006	-.410221-013	.323295-011	.477510-011	.318683-030
SCALE	-.633133-006	-.173696-006	-.116603-005	-.695236-031	.790193-030	.318683-030	.233113-011

CORRELATION MATRIX

	TLN-X	TLN-Y	TLN-Z	ROT-X	ROT-Y	ROT-Z	SCALE
TLN-X	1.00	.26	-.32	-.13	.81	.35	-.25
TLN-Y	.26	1.00	-.19	-.62	.33	.45	-.09
TLN-Z	-.32	-.19	1.00	.17	-.56	-.29	-.55
ROT-X	-.13	-.62	.17	1.00	-.15	-.01	.00
ROT-Y	.81	.33	-.56	-.15	1.00	.52	.00
ROT-Z	.35	.45	-.29	-.01	.52	1.00	.00
SCALE	-.25	-.09	-.55	.00	.00	.00	1.00

DETERMINATION OF DATUM TRANSFORMATION PARAMETERS USING "VEIS'S" MODEL

FOR THE TRANSFORMATION : CAPE DATUM TO CONVENTIONAL TERRESTRIAL (C.T.)

TRANSFORM DOPPLER SATELLITE SYSTEM (NSWC 9Z-2) TO CONVENTIONAL TERRESTRIAL (C.T.) SYSTEM BY :
Z-TRANSLATION = 4.0 METRES   Z-ROTATION = -.8 ARC SECONDS   SCALE = -.5 PPM

NUMBER OF COMMON POINTS = 23

TRANSLATION COMPONENTS (METRES)

X0=	-136.4	+/-	1.6
Y0=	-113.2	+/-	1.3
Z0=	-297.5	+/-	1.4

SCALE FACTOR (COORD. SYSTEM)

11.1 +/- 1.5 PPM

ROTATION ANGLES (SECONDS OF ARC)

DA=	-.117	+/-	.317
DU=	-.067	+/-	.567
DV=	-.604	+/-	.434

THE POINT OF ROTATION IS THE NETWORK INITIAL POINT.

CARTESIAN COORDS OF ROTATION POINT: 4777935.98 2280227.48 -3545622.42
ELLIPSOIDAL COORDS OF ROTATION POINT: -33.5932000 25.3044622 282.00

OBSERVATIONS AND RESIDUALS

	GEODETIC CARTESIAN			RESIDUALS			ELLIPSOIDAL		
	X	Y	Z	X	Y	Z	LAT	LONG	HT
PONT	5157393.73	2878293.94	-2400481.26	-2.31	1.10	-3.14	-3.47	2.09	-.18
KRANZBERG	5149897.07	2691087.17	-2626422.08	-1.54	1.45	-2.00	-2.11	2.00	.19
THAMAKOOSH	5019377.00	2903697.05	-2651973.76	-.96	.33	-1.96	-2.06	.77	.21
MANNERHEIM	5291622.83	2265088.54	-2740804.51	2.33	2.88	4.91	5.84	1.73	.83
HADIDA	5294743.90	2035821.71	-2872769.70	3.28	-2.38	4.19	4.72	-3.41	.04
MORGENZON	5138865.54	2424474.51	-2890855.24	-.13	2.18	1.36	1.59	2.02	.10
MOOIDAM	4983839.80	2732706.71	-2887742.11	-.29	-.15	-.60	-.68	.00	-.01
MAGWAZA	4786233.73	3006513.75	-2946617.37	-.05	-2.16	-1.50	-1.88	-1.80	-.36
WITBANK N	5361121.05	1605173.39	-3050281.37	-.07	-6.05	-3.61	-4.03	-5.78	.16
GRASKOP	5246339.55	1904123.93	-3079065.05	1.75	-3.06	-.32	.01	-3.48	.68
WITWATER	5137640.02	2183077.40	-3078166.97	1.04	.01	1.81	2.05	-.40	-.04
LEEUKOP.SA	4986898.49	2475831.86	-3104547.73	.45	.37	1.40	1.50	.13	-.19
INKOMINKULU	4776617.80	2809322.79	-3149351.18	1.14	-2.05	1.08	.91	-2.34	-.59
FRANSMANKOP	4988162.54	2267479.43	-3256556.98	.67	.94	1.59	1.87	.58	.04
LOUISFTN	5203455.86	1690219.94	-3268501.82	-1.26	-1.34	-3.50	-3.83	-.89	.42
POTLOER	5101378.09	1977632.65	-3269927.02	-.01	-.23	-.11	-.14	-.21	-.02
LUBISI	4815770.00	2506293.13	-3339947.63	.96	-.02	2.26	2.37	-.46	-.47
HEXRIVIER	5022168.62	1795165.38	-3490357.75	-1.30	1.33	-1.71	-1.77	2.16	.43
BLYDEBERG	4917299.12	2106614.43	-3463660.17	-.47	1.77	1.36	1.28	1.81	-.52
COEGAKOP	4785924.06	2295261.42	-3524985.75	.51	1.55	2.49	2.70	1.18	-.44
M'PUMBE N	4943188.45	3076434.84	-2596398.32	-.68	-1.04	-2.56	-2.79	-.52	.02
KARSRIVIER	4944384.20	1811092.50	-3587431.02	-1.92	2.72	-.88	-1.21	3.21	-.22
BRIT 44	5086495.71	2667589.54	-2767924.39	-1.14	1.36	-.56	-.67	1.73	-.09

	TRANSFORMED DOPPLER SATELLITE COORDINATES			RESIDUALS					
	X	Y	Z	X	Y	Z	LAT	LONG	HT
PONT	5157756.90	2878190.79	-2400772.91	2.31	-1.10	3.14	3.47	-2.09	.18
KRANZBERG	5149761.51	2690932.25	-2626713.64	1.54	-1.45	2.00	2.11	-2.00	-.19
THAMAKOOSH	5019241.63	2903592.48	-2652266.05	.96	-.33	1.96	2.06	-.77	-.21
MANNERHEIM	5291495.38	2264981.24	-2741082.60	-2.33	-2.88	-4.91	-5.84	-1.73	-.83
HADIDA	5294618.60	2035701.67	-2873050.37	-3.23	2.38	-4.19	-4.72	3.41	-.04
MORGENZON	5138732.45	2424367.64	-2891142.55	.13	-2.18	-1.36	-1.59	-2.02	-.10
MODIDAM	4983705.32	2732598.93	-2888034.05	.29	.15	.60	.68	.00	.01
MAGWAZA	4726095.17	3006405.33	-2946912.43	.05	2.16	1.50	1.86	1.80	.36
WITBANK N	5360989.00	1605040.21	-3050578.69	.07	6.05	3.61	4.03	5.75	-.16
GRASKOP	5246210.54	1904000.25	-3079356.77	-1.75	3.06	.32	-.01	3.48	-.68
WITWATER	5137508.95	2182963.19	-3079455.04	-1.04	-.01	-1.81	-2.05	.40	.04
LEEUKOP.SA	4926765.22	2475721.88	-3104237.61	-.45	-.37	-1.40	-1.50	-.13	.19
INKOMINKULU	4776484.34	2809212.05	-3149643.05	-1.14	2.05	-1.08	-.91	2.34	.59
FRANSMANKOP	4988029.47	2267368.02	-3256847.81	-.67	-.94	-1.59	-1.87	-.58	-.04
LOUISFTN	5203320.12	1690097.08	-3268801.69	1.26	1.34	3.50	3.83	.89	-.42
POTLOER	5101244.33	1977515.43	-3270220.74	.01	.23	.11	.14	.21	.02
LUBISI	4815636.21	2506182.65	-3340232.67	-.96	.02	-2.26	-2.37	.46	.47
HEXRIVIER	5022031.28	1795050.02	-3490656.90	1.30	-1.83	1.71	1.77	-2.16	-.43
BLYDEBERG	4917162.90	2106502.72	-3463953.56	.47	-1.77	-1.36	-1.26	-1.81	.52
COEGAKOP	4785783.81	2295151.49	-3525278.06	-.51	-1.55	-2.49	-2.70	-1.16	.44
M'PUMBE N	4943053.11	3076329.71	-2596691.59	.68	1.04	2.56	2.79	.52	-.02
KARSRIVIER	4944244.90	1810979.09	-3587729.71	1.92	-2.72	.88	1.21	-3.21	.22
BRIT 44	5036360.38	2667484.06	-2768214.69	1.14	-1.36	.56	.67	-1.73	.09

REFERENCE VARIANCE = 8.4574

SIGMA NOUGHT = 2.9082

COVARIANCE MATRIX

	TLN-X	TLN-Y	TLN-Z	dA	dU	dV	SCALE
TLN-X	.269857+001	.564683+000	-.732981+000	-.378998-006	-.355198-005	-.177684-005	-.633133-006
TLN-Y	.564683+000	.177134+001	-.346503+000	.129205-005	-.168359-005	-.717724-006	-.173696-006
TLN-Z	-.732981+000	-.346503+000	.195319+001	.133211-007	.217946-005	.107171-005	-.116603-005
dA	-.378998-006	.129205-005	.133211-007	.235736-011	-.275031-012	-.274378-012	-.344692-029
dU	-.355198-005	-.168359-005	.217946-005	-.275031-012	.756656-011	.335493-011	-.536422-029
dV	-.177684-005	-.717724-006	.107171-005	-.274378-012	.335493-011	.550375-011	-.372080-029
SCALE	-.633133-006	-.173696-006	-.116603-005	-.344692-029	-.536422-029	-.372080-029	.233113-011

CORRELATION MATRIX

	TLN-X	TLN-Y	TLN-Z	dA	dU	dV	SCALE
TLN-X	1.00	.26	-.32	-.15	-.79	-.46	-.25
TLN-Y	.26	1.00	-.19	.63	-.46	-.23	-.09
TLN-Z	-.32	-.19	1.00	.01	.57	.33	-.55
dA	-.15	.63	.01	1.00	-.07	-.08	.00
dU	-.79	-.46	.57	-.07	1.00	.52	.00
dV	-.46	-.23	.33	-.08	.52	1.00	.00
SCALE	-.25	-.09	-.55	.00	.00	.00	1.00