The Effects of Dilutions and Payout Policy on Equity- and Stock-linked Call Options on a Firm with Leverage

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Declaration

I declare that this dissertation is my own, unaided work. It is being submitted for the Degree of Master of Philosophy at the University of Cape Town. It has not been submitted before for any degree or examination in any other University.

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Abstract

Capital-structure models are useful tools for pricing claims on equity. They provide insight into the effects of changing capital-debt structures on the value of options. Backwell *et al.* (2022) developed a capital structure framework which, in addition to a typical structural model, specifically considers the number of outstanding shares. This allows for the differentiation between options on total firm equity and options on share price. The extension by Backwell *et al.* (2022) also allows for the effects of dilutions and buyback policy on stock-linked options to be explored. Using the framework developed by Backwell *et al.* (2022) with asset value dynamics presented by Leland (1994), the capital-debt structure of a firm is modelled. Finite-difference methods utilising a generalised version of the Black-Scholes equation are then used to value and compare call options on total equity and call options on share price. Under the presented model, dilutions have little to no effect on stock-linked call option value in firms with low levels of leverage. However, dilutions clearly decrease the value of call options in firms with higher levels of leverage. Share buybacks significantly improve the value of stock-linked call options, particularly in lower leveraged firms where there is more available cash flow. This indicates that while shareholders are indifferent between cash dividends and share buybacks in a perfect market, holders of options on share price are not indifferent. In fact, option holders prefer payout policies that favour buybacks over dividends. Finally, the leverage effect is demonstrated by calculating implied volatilities under various levels of firm leverage.

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Chapter 1

Introduction

Structural models provide insight into how firms choose their capital structure and payout policies. Merton (1974) developed the inaugural capital-debt structure model in quantitative finance, which stimulated a field of research including valuing debt securities under various bankruptcy conditions, valuing equity as an option on the firm's assets and determining the impact of leverage on financial instruments. Capital-structure models provide a framework in which claims on equity can be priced and Geske *et al.* (2016) show that these models improve option pricing error. However, it appears that the literature on pricing options on equity within a capital-structure model is limited in its insights into the changing number of outstanding shares before expiry of the option. Backwell *et al.* (2022) recognises this shortcoming and develop a framework which differentiates between options on total firm equity, called equity options, and options on the share price, called stock options. The fundamental difference between options considered in the earlier capital-structure literature and the Backwell *et al.* (2022) stock option, is that the share price is not simply a proportional value of the firm's equity but is more specifically equity divided by the number of outstanding shares, which is dynamic. Therefore, in addition to typical features of a structural model, Backwell *et al.* (2022) considers the changing number of outstanding shares.

Dilutions cause the number of shares in issue to change. Consider a firm which faces bankruptcy because it cannot meet its debt obligations. To avoid bankruptcy, the firm can issue new shares (equity funding) or increase its debt (debt funding). Issuing new shares dilutes original shareholders' ownership and increases the number of shares in issue (Backwell *et al.*, 2022). This causes a decline in the share price which alters the value of an option on the stock. Share buybacks also cause the number of shares in the open market to change. Buybacks are becoming increasingly popular as a payout policy over cash dividends. Buybacks increase shareholder wealth, by decreasing the number of shares and increasing fractional ownership. In a perfect market, shareholders are indifferent between receiving cash dividends and increased fractional ownership (Backwell *et al.*, 2022). However, holders of stock options are likely not indifferent, since buybacks decrease the number of shares. This increases the share price relative to a situation without buybacks, and thus changes the value of the stock option. Therefore, due to variations in the number of shares, dilutions and payout policy arguably affect the value of options on the share price. Consequently, it is important to consider activities which change the number of shares in issue when pricing stock-linked options.

The firm's leverage also affects stock options. Changes in leverage tend to be linked to changes in volatility. The leverage effect usually refers to a negative correlation between changes in realised volatility and returns (Cont, 2001). As such, low (high) returns tend to be accompanied by high (low) volatility. Consider a call option on a stock. To end up in the money, high strikes require high returns. However, this tends to be accompanied by low volatilities given the negative correlation of the leverage effect. Since implied volatility is linked to realised volatility, options on firms with higher levels of leverage are likely to have larger implied volatilities than options on firms with lower leverages. Toft and Prucyk (1997) show this result under the Leland (1994) model, and Rathgeber *et al.* (2021) gives an example of this effect within the German stock option market.

In this dissertation, the framework developed by Backwell *et al.* (2022) is used in conjunction with the Leland (1994) model to define the capital-debt structure of a firm. Then, finite-difference methods are used to price equity- and stock-linked call options. The value of equity options depends on a single state variable, the firm's equity. Stock options, however, depend on the firm's equity and the number of shares, which is also modelled as a state variable. As such, a two-dimensional finite-difference scheme is needed to value stock options. The effects of dilutions and payout policy on stock call options are then explored by comparing option values under various firm conditions. Finally, the leverage effect is demonstrated by comparing implied volatilities of stock options on firms at various levels of leverage.

The dissertation proceeds as follows. In Chapter 2, the model used to define the firm's capital-debt structure is outlined. Implementation and numerical methods are described in Chapter 3. Pricing results are discussed in Chapter 4 and conclusions are made in Chapter 5.

Chapter 2

Model Description

2.1 Capital Structure Model

Consider a firm with asset value dynamics given by Leland (1994):

$$dV_t = (r - q)V_t dt + \sigma V_t dW_t, \tag{2.1}$$

where r is the risk-free rate, q is a fixed rate at which assets produce cash, σ is the volatility parameter and W_t is a standard Brownian motion under the risk neutral measure, $\mathbb{Q}$.

From Backwell *et al.* (2022), it is assumed that all debt is grouped together, and the firm pays a fixed coupon rate to service its debt obligations. The available cash flow rate to the firm after tax savings is given by

$$CFR_t = qV_t - (1 - \rho)c, \tag{2.2}$$

where ρ is the tax rate and c is a perpetual fixed annual debt payment rate. The parameter c can be used to control the level of leverage that the firm has. The larger the coupon payment is, the more leverage the firm has.

If the cash flow rate is positive, the firm has a cash surplus. The firm can pay out cash dividends to shareholders or buy back shares with the available cash. While cash dividends distribute monetary wealth to shareholders, buybacks increase original shareholders' ownership by decreasing the number of shares in issue. Therefore, assuming the absence of transaction costs, taxes and other frictions, shareholders are indifferent between dividends and buybacks.

If the cash flow rate is negative, the firm suffers a cash shortfall, and cannot finance its debt obligations with cash generated. To avoid bankruptcy, the firm is required to issue new shares to raise capital. This dilutes equity claims of original shareholders and increases the number of outstanding shares. Dilutions are preferred to bankruptcy, given future equity is sufficiently positive. In accordance with Backwell *et al.* (2022), it is assumed that the firm can only avoid bankruptcy by equity funding and not by debt funding.

In general, the firm's equity is computed as the discounted expectation (under risk-neutral $\mathbb{Q}$) of all future cash flows due to equity holders. Backwell *et al.* (2022) prove that the firm's equity value is given by

$$EQ_t = \sup_{\tau \geq t} \mathbb{E}_t^{\mathbb{Q}} \left[\int_t^{\tau} e^{-\int_t^u r_s ds} CFR_u du \right], \tag{2.3}$$

where $\mathbb{E}_t^{\mathbb{Q}}[\cdot]$ represents the time-t conditional expectation under the risk neutral measure and τ is the bankruptcy time of the firm. Note that equity holders can maximise the value of their position by deciding when to declare bankruptcy; this is reflected above as an optimisation over stopping times τ. The equity valuation includes both positive and negative future cash flows rates, calculated using equation (2.2). Positive cash flow rates contribute positively to equity as future cash flows received. Negative cash flows are included to account for future dilutions. The effect of future dilutions on equity is explained by Backwell *et al.* (2022) as synthetic negative dividends. Shareholders have limited liability and typically do not pay cash to the firm as negative dividends. However, when dilutions occur to cover cash shortfalls, original shareholders endure synthetic negative dividends because future earnings must be distributed among old and new shareholders. Therefore, synthetic negative dividends, as future negative cash flows, contribute unfavourably to the value of equity.

Leland (1994) presents the following analytical solution for the equity value of the firm. The equity value is given as a function of the asset value, with asset value dynamics given in equation (2.1).

$$EQ_t(V_t) = \left(V_t - \frac{(1-\rho)c}{r} + \left(\frac{(1-\rho)c}{r} - V_B \right) \left(\frac{V_t}{V_B} \right)^{-x} \right) \mathbb{I}_{\{V_t > V_B\}}, \tag{2.4}$$

where $\mu = r - q - \sigma^2/2$, $x = (\mu + \sqrt{\mu^2 + 2r\sigma^2})/\sigma^2$ and $V_B = (1-\rho)cx/(r(x+1))$. The variable V_B is a critical bankruptcy boundary. It is the asset value at which the equity of the firm becomes zero and bankruptcy is declared.

Debt is typically valued as the discounted expectation of all future debt payments. Backwell *et al.* (2022) presents the debt value of the firm as

$$CB_t = \mathbb{E}_t^{\mathbb{Q}} \left[\int_t^{\tau_b} e^{-\int_t^u r_s ds} c_u du + e^{-\int_t^{\tau_b} r_s ds} (1-b) V_{\tau_b} \right], \tag{2.5}$$

where τ_b is the optimal bankruptcy time determined in equation (2.3), and $b \in [0,1]$ is the proportional bankruptcy cost. Therefore, in the event of bankruptcy, debt holders expect to receive the discounted value of their asset minus the cost of bankruptcy.

The closed form solution for the value of debt under the Leland (1994) model is given by

$$CB_t(V_t) = \left(\frac{c}{r} + \left((1-b)V_B - \frac{c}{r}\right)\left(\frac{V_t}{V_B}\right)^{-x}\right)\mathbb{I}_{\{V_t>V_B\}}. \tag{2.6}$$

The closed form solutions for equity and debt presented by Leland (1994), given in equations (2.4) and (2.6), are used for the remaining parts of this dissertation.

Given the equity and bond values of the firm, the share price is computed as

$$S_t = \frac{EQ_t}{n_t}, \tag{2.7}$$

where n_t is the number of shares in issue at time t. The dynamics of n_t are given by Backwell *et al.* (2022) as

$$\frac{dn_t}{n_t} = \left(\frac{CFR_t^- - B_p CFR_t^+}{EQ_t}\right)dt. \tag{2.8}$$

In equation (2.8), CFR_t^- represents cash flow shortfalls. As discussed, when the firm faces a shortfall, new shares are issued as dilutions, thus increasing the number of shares in issue. CFR_t^+ represents cash surpluses. A proportion of this, B_p, is used for share buybacks and the remainder is paid out as cash dividends. Therefore, the payout policy is defined by the parameter B_p. Backwell *et al.* (2022) describes the proportion of cash dedicated to buybacks as synthetic positive dividends. Opposite to synthetic negative dividends, synthetic positive dividends benefit original shareholders by decreasing the number of shares in the market thus increasing ownership.

2.2 Equity and Stock Options

Under the Leland (1994) model within the framework developed by Backwell *et al.* (2022), the time-t price of a call option on the total equity of the firm, struck at K and expiring at $T \geq t$ is given by the discounted risk neutral expectation of the payoff:

$$EO_t = \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^T r_s ds}(EQ_T(V_T)\mathbb{I}_{\{V_s>V_B\}} - K)^+\right], \tag{2.9}$$

$\forall s \in [t, T]$, where $EQ_T(\cdot)$ is the value of equity under the Leland (1994) model given in equation (2.4). The indicator in equation (2.9) accounts for the possibility of bankruptcy before expiry of the option. If bankruptcy occurs at any time between issuance and expiry of the option, then equity and the claim have a zero value at expiry.

The time-t price of a call option on a single share, struck at K and expiring at $T \geq t$ is similarly given by

$$SO_t = \mathbb{E}_t^{\mathbb{Q}} \left[e^{-\int_t^T r_s ds} \left(S_T \mathbb{I}_{\{V_s > V_B\}} - K \right)^+ \right] \tag{2.10}$$

$$= \mathbb{E}_t^{\mathbb{Q}} \left[e^{-\int_t^T r_s ds} \left(\frac{EQ_T(V_T)\mathbb{I}_{\{V_s > V_B\}}}{n_T} - K \right)^+ \right], \tag{2.11}$$

$\forall s \in [t, T]$.

In practise, options are usually linked to share price but options on equity are typically considered in the literature (Toft and Prucyk, 1997). By modelling the number of shares in equation (2.8), Backwell *et al.* (2022) develops a more relevant pricing approach. Now through the n_T term in equation (2.11), any issuance or buyback activities will have a clear effect on the stock option.

Chapter 3

Implementation and Numerical Methods

3.1 Finite-Difference Methods

Finite-difference methods provide flexible numerical solutions to partial differential equations (PDEs) (Crepey, 2013, Ch. 8). Here, finite-difference methods are used to price equity and stock options in the context of the capital-structure model in Chapter 2. This will quantify the effects of dilutions and payout policy on equity- and stock-linked call options.

The geometric Brownian motion dynamics in equation (2.1) ensure that V_t is Markov. Therefore, equity option value in equation (2.9) is given as a function of asset value:

$$EO_t = f(t, V_t), \tag{3.1}$$

while the stock option value can be given as a function of asset value and the number of shares:

$$SO_t = g(t, V_t, n_t), \tag{3.2}$$

with V_t and n_t being jointly Markov.

3.1.1 Equity Option

Typical, non-distressed firms will be considered in this dissertation, making the effect of the bankruptcy indicator in equation (2.9) minor. Furthermore, bankruptcy occurs at relatively low asset values. This corresponds to cases where call options are likely to be deep out-the-money, making the effect of the indicator function even more minor. As such, we omit the bankruptcy indicator in equation (2.9) and approximate the value of the equity option:

$$EO_t = f(t, V_t) \approx \mathbb{E}_t^{\mathbb{Q}} \left[e^{-\int_t^T r_s ds} (EQ_T(V_T) - K)^+ \right], \tag{3.3}$$

where $EQ_T(\cdot)$ is given in equation (2.4).

As is well known, the the Feynman Kac Theorem implies that the function in equation (3.1) satisfies a PDE (Crepey, 2013, Ch. 3.5), namely

$$\frac{\partial f}{\partial t} + (r-q)V\frac{\partial f}{\partial V} + \frac{1}{2}\sigma^2 V^2 \frac{\partial^2 f}{\partial V^2} - rf = 0, \tag{3.4}$$

as well as the terminal condition

$$f(T, V_T) = (EQ_T(V_T) - K)^+. \tag{3.5}$$

This is precisely the familiar Black-Scholes equation, since geometric Brownian motion is used for the asset value dynamics in equation (2.1).

Since the payoff specifes a terminal condtion, a time reversal is made to change the terminal condition into an initial condition. This is done by introducing a time-to-maturity variable, $\tau = T - t$. Then using the time-reversed Black-Scholes PDE,

$$\frac{\partial f}{\partial \tau} - (r-q)V\frac{\partial f}{\partial V} - \frac{1}{2}\sigma^2 V^2 \frac{\partial^2 f}{\partial V^2} + rf = 0, \tag{3.6}$$

with initial condition equal to the payoff, the value of an equity option on the firm can be computed using finite-difference methods.

The variable V is specified by a truncated domain, $[V_{\text{min}}, V_{\text{max}}]$, which is discretised into N_V-many steps of size δ_V. The time-to-maturity variable is also discretised with M-many time steps, of size δ_τ, from $\tau = 0$ to $\tau = T$. This creates a regular mesh defined by

$$\{(V_{\text{min}} + i\delta_V, m\delta_\tau) : 0 \leq i \leq N_V, 0 \leq m \leq M\}.$$

Using a backward difference estimate for the time-to-maturity derivative and central difference estimates for the asset value derivatives in equation (3.6), the following implicit finite-difference algorithm is obtained:

$$\frac{f^i_{m+1} - f^i_m}{\delta_\tau} - (r-q)V\frac{f^{i+1}_{m+1} - f^{i-1}_{m+1}}{2\delta_V} - \frac{1}{2}\sigma^2 V^2 \frac{f^{i+1}_{m+1} - 2f^i_{m+1} + f^{i-1}_{m+1}}{(\delta_V)^2} + rf^i_{m+1} = 0. \tag{3.7}$$

Equation (3.7) can be written in a matrix form, for all values of i, as

$$\mathbf{G}\mathbf{f}_{m+1} = \mathbf{f}_m, \tag{3.8}$$

where $\mathbf{f}_{m+1}$ and $\mathbf{f}_m$ are column vectors of size $1 \times (N_V + 1)$ containing the values of the equity option for all asset values at time steps $m+1$ and m respectively. The matrix $\mathbf{G}$ is given by

$$\mathbf{G} = 2\mathbf{I} - \mathbf{F}, \tag{3.9}$$

where $\mathbf{I}$ is the identity matrix and

$$\mathbf{F} = (1 - r\delta_\tau)\mathbf{I} + \frac{1}{2}(r-q)\delta_\tau \mathbf{D_1}\mathbf{T_1} + \frac{1}{2}\sigma^2\delta_\tau \mathbf{D_2}\mathbf{T_2}.$$

The matrices $\mathbf{D_1}$, $\mathbf{D_2}$, $\mathbf{T_1}$ and $\mathbf{T_2}$ are

$$\mathbf{D_1} = \text{diag}\left(\frac{V_{\text{min}}}{\delta_V} + [0, 1, ..., N_V]\right), \quad \mathbf{D_2} = \mathbf{D_1}^2,$$

$$\mathbf{T_1} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ -1 & 0 & 1 & \ddots & \vdots \\ 0 & -1 & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 1 \\ 0 & \dots & 0 & -1 & 0 \end{bmatrix} \text{ and } \mathbf{T_2} = \begin{bmatrix} -2 & 1 & 0 & \dots & 0 \\ 1 & -2 & 1 & \ddots & \vdots \\ 0 & 1 & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 1 \\ 0 & \dots & 0 & 1 & -2 \end{bmatrix}.$$

Matrices $\mathbf{G}$, $\mathbf{I}$, $\mathbf{F}$, $\mathbf{D_1}$, $\mathbf{D_2}$, $\mathbf{T_1}$ and $\mathbf{T_2}$ have dimensions $(N_V + 1) \times (N_V + 1)$.

Before solving the system of equations in (3.8), Neumann boundary conditions are applied at the boundaries of the truncated V domain (see (Crepey, 2013, Ch. 8)). At the boundaries, the option under consideration is typically either deeply in- or out-the-money. As such, the behaviour beyond the boundary is well approximated by extrapolating the behaviour near the boundary. The behaviour of the derivative is typically zero for out-the-money boundary values, or upward-sloping getting further into-the-money boundary values. To enforce the Neumann boundary conditions, the first three entries in the first row, and the last three entries of the last row of matrix $\mathbf{G}$ are overwritten with $[1, -2, 1]$. Furthermore, the first and last entries of column vector $\mathbf{f}_m$ are overwritten to zero at each time step m. This ensures that, for $0 < m \leq M$,

$$f_m^0 - f_m^1 = f_m^1 - f_m^2, \tag{3.10}$$

and

$$f_m^{N_V-2} - f_m^{N_V-1} = f_m^{N_V-1} - f_m^{N_V}. \tag{3.11}$$

To solve the system of equations in (3.8), the initial condition, $\mathbf{f}_0$, is specified as follows. Each entry in the column vector, $\mathbf{f}_0$, is given by:

$$f_0^i = (EQ_T(V_{\text{min}} + i\delta_V) - K)^+ \quad \text{for} \quad 0 \leq i \leq N_V, \tag{3.12}$$

where $EQ_T(\cdot)$ is the equity function given in equation (2.4).

3.1.2 Stock Option

For the same reasons given in subsection 3.1.1, we omit the bankruptcy indicator in equation (2.11) and approximate the value of the stock option as

$$SO_t \approx \mathbb{E}_t^{\mathbb{Q}} \left[e^{-\int_t^T r_s ds} \left(\frac{EQ_T(V_T)}{n_T} - K \right)^+ \right]. \tag{3.13}$$

The stock option (equation (3.2)) is valued using an extension of the time reversed Black Scholes PDE (equation (3.6)) to include a term from the dynamics of n_t:

$$\frac{\partial g}{\partial \tau} - (r-q)V\frac{\partial g}{\partial V} - \frac{1}{2}\sigma^2 V^2 \frac{\partial^2 g}{\partial V^2} - \left(\frac{CFR_t^-(V) - B_p CFR_t(V)^+}{EQ_t(V)} \right) n \frac{\partial g}{\partial n} + rg = 0. \tag{3.14}$$

The $\frac{\partial g}{\partial n}$ term accounts for the effects of n_t on the option price, given the dynamics of n_t in equation (2.8).

Similarly to in the equity option algorithm, n and V are specified by truncated domains, $[n_{\text{min}}, n_{\text{max}}]$ and $[V_{\text{min}}, V_{\text{max}}]$ respectively. The variables τ, V and n are discretised, with step sizes δ_τ, δ_V and δ_n respectively, to form the mesh:

$$\{(V_{\text{min}} + i\delta_V, n_{\text{min}} + j\delta_n, m\delta_\tau) : 0 \le i \le N_V, 0 \le j \le N_n, 0 \le m \le M\}.$$

Central difference estimates are made for the asset value derivatives as well as the number of shares derivative, and a backward difference estimate is made for the time-to-maturity derivative in equation (3.14). This results in a finite-difference algorithm which is implicit in V and explicit in n:

$$\frac{g_{m+1}^{i,j} - g_m^{i,j}}{\delta_\tau} - (r-q)V\frac{g_{m+1}^{i+1,j} - g_{m+1}^{i-1,j}}{2\delta_V} - \frac{1}{2}\sigma^2 V^2 \frac{g_{m+1}^{i+1,j} - 2g_{m+1}^{i,j} + g_{m+1}^{i-1,j}}{(\delta_V)^2} - \left(\frac{CFR_t^-(V) - B_p CFR_t(V)^+}{EQ_t(V)} \right) n \frac{g_m^{i,j+1} - g_m^{i,j-1}}{2\delta_n} + rg_{m+1}^{i,j} = 0. \tag{3.15}$$

Using the matrix $\mathbf{G}$, defined in equation (3.9), equation (3.15) is given in matrix form as

$$\mathbf{G}\mathbf{g}_{m+1}^{\cdot,j} = \mathbf{g}_m^{\cdot,j} + \frac{1}{2}\frac{\delta_\tau}{\delta_n}(n_{\text{min}} + j\delta_n) \left(\frac{CFR_t^-(\cdot) - B_p CFR_t(\cdot)^+}{EQ_t(\cdot)} \right) \left(\mathbf{g}_m^{\cdot,j+1} - \mathbf{g}_m^{\cdot,j-1} \right), \tag{3.16}$$

for $0 < j < N_n$. The expressions $CFR_t(\cdot)^-$, $CFR_t(\cdot)^+$ and $EQ_t(\cdot)$ in equation (3.16) are column vectors, calculated using equations (2.2) and (2.4), for all asset values $V_{\text{min}} + (0 : N_V)\delta_V$. The term $\mathbf{g}_m^{\cdot,\cdot}$ is a matrix with dimensions $(N_V + 1) \times (N_n + 1)$. This matrix consists of option values for each possible $V - n$ combination along the discretised domains of V and n at time step m. By way of explanation, each $g_m^{i,j}$, for $0 \leq i \leq N_V, 0 \leq j \leq N_n$ and $0 \leq m \leq M$, represents the option value when the asset value is $V_{\text{min}} + i\delta_V$ and the number of shares is $n_{\text{min}} + j\delta_n$ at reversed time $m\delta_\tau$. Furthermore, $g_m^{i,j}$ is the entry in the i-th row and j-th column of the matrix $\mathbf{g}_m^{\cdot,\cdot}$. Therefore, $\mathbf{g}_m^{\cdot,j-1}$, $\mathbf{g}_m^{\cdot,j}$ and $\mathbf{g}_m^{\cdot,j+1}$ in equation (3.16) are column vectors of length $(N_V + 1)$. These column vectors contain the time-$(T - m\delta_\tau)$ option values for all asset values in the mesh and a particular number of shares. At each time step, $0 < m \leq M$, the matrix $\mathbf{g}_m^{\cdot,\cdot}$ is updated based on the PDE in equation (3.14).

Neumann boundary conditions are applied at the boundaries of the $\mathbf{g}_m^{\cdot,\cdot}$ matrix. Similarly to in the equity option case, this is achieved by setting the derivatives at the boundaries equal to the derivatives at points near the boundaries (Crepey, 2013, Ch. 8). To do this, the first three entries of the first row and the last three entries of the last row of matrix $\mathbf{G}$ are overwritten to $[1, -2, 1]$. In addition, the first and last entries of the column vectors $\mathbf{g}_m^{\cdot,j}$, $\mathbf{g}_m^{\cdot,j+1}$ and $\mathbf{g}_m^{\cdot,j-1}$ are overwritten to zero at each time step.

Finally, to solve the system in equation (3.16), the initial condition is specified by the matrix $\mathbf{g}_0^{\cdot,\cdot}$ with entries:

$$g_0^{i,j} = \left(\frac{EQ_T(V_{\text{min}} + i\delta_V)}{(n_{\text{min}} + j\delta_n)} - K \right)^+ \quad \text{for} \quad 0 \leq i \leq N_V, 0 \leq j \leq N_n, \tag{3.17}$$

where $EQ_T(\cdot)$ is the equity function given in equation (2.4).

Chapter 4

Option Pricing

4.1 Equity and Stock Options

Equity and stock options are priced using a set of parameters describing a typical scenario of a firm with leverage. The claims are valued for various initial asset values and number of shares in issue. Using the finite-difference algorithms described in Chapter 3, call options are priced for the risk neutral asset dynamics presented by Leland (1994), with parameters $r = 0.08$ and $\sigma = 0.2$. The tax rate is set to $\rho = 0.35$, and the annual fixed coupon payment to $c = 4$. The value $q = \frac{(1-\rho)c}{100} + 0.03 = 0.056$. This value of q is chosen to replicate the value of q in Toft and Prucyk (1997). It implies a 3% dividend yield on equity when $V = 100$. At first, $B_p = 0$ so that no buybacks are allowed, and dilutions occur if and only if the cash flow rate in equation (2.2) is negative. The finite-difference pricing surface of a call option on total equity, with strike $K = 40$ and expiry of $T = 1.5$ years, is given in the left panel of Figure 4.1. This surface shows the value of the claim for various asset values over time. The final pricing matrix of a call option on share price, also with strike $K = 40$ and an expiry of $T = 1.5$ years, is given in the right panel of Figure 4.1. The pricing matrix consists of the time-0 price of the stock option for various initial asset values and initial number of outstanding shares.

Figure 4.2 shows equity and stock call option prices, for the same parameters mentioned above. The left panel of Figure 4.2 shows the time-0 prices of the equity option for various initial equity values. Toft and Prucyk (1997) provide a closed form solution for a call option on total equity based on the Leland (1994) model. The left panel of Figure 4.2 shows this closed form solution, as well as Monte Carlo price estimates. For each Monte Carlo point in the figure, 10000 asset value paths are simulated using the log-normal distribution. Taking note of whether the firm reaches bankruptcy in each path, 10000 realisations of the time-T asset value are obtained. These realisations are used to calculate the payoff in equation (2.9). Then by appropriately discounting the payoff values, a Monte Carlo price is estimated.

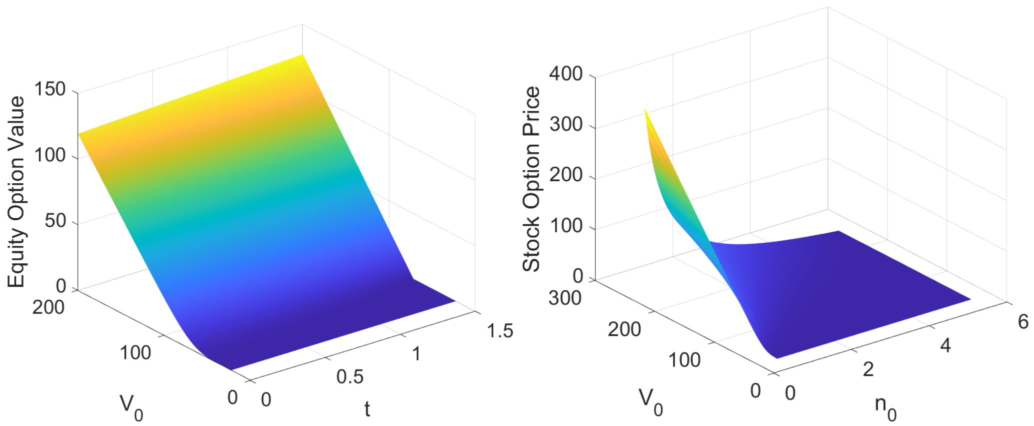

Fig. 4.1: Finite-difference pricing surfaces of an equity option (left) and a stock option (right).

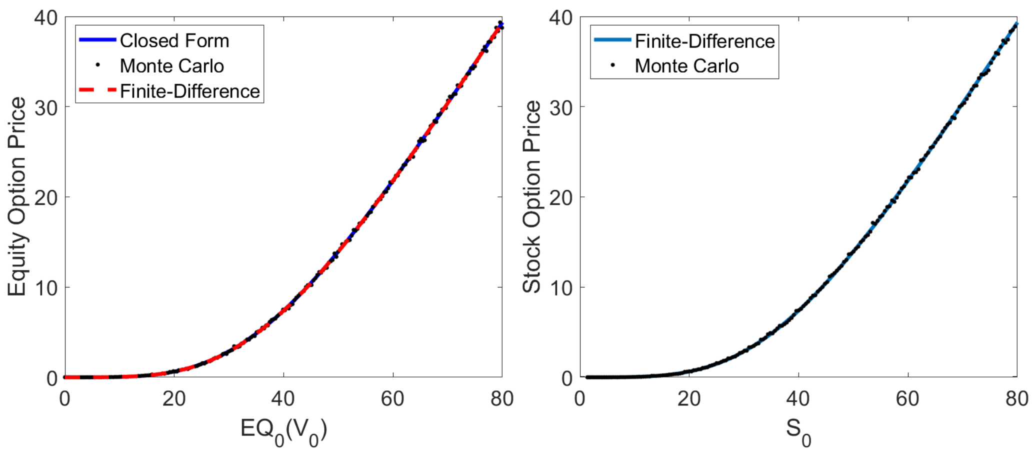

Fig. 4.2: Time $t = 0$ price of an equity option (left) and a stock option with $n_0 = 1$ (right).

Finite-difference prices are virtually the same as the closed form. In addition, the Monte Carlo estimates are scattered closely around both the finite-difference prices and the closed form, thus verifying that the equity option algorithm outlined in Chapter 3 has a working solution. The closed form prices presented by Toft and Prucyk (1997) and the Monte Carlo estimates capture the value of an option with payoff given in equation (2.9). However, the finite-difference approach uses the approximated option payoff given in equation (3.3). Since the differences in value between these amounts are negligible, the reasoning behind the omission of the indicator function in equation (2.9) is sound.

The right panel of Figure 4.2 shows the time-0 prices of the stock option for various initial stock values where the initial number of shares in issue is $n_0 = 1$. The plot also gives Monte Carlo price estimates. Similarly to the Monte Carlo estimates for the equity option, 10000 asset value paths are simulated using the log-normal distribution. In addition to simulating paths of V_t, paths of n_t, the number of outstanding shares, are also simulated by discretising the dynamics of n_t. Payoff values are then calculated by inserting the time-T realisations, V_T and n_T, into equation (2.11). Finally, by appropriately discounting the payoffs, Monte Carlo values are obtained. These values are scattered closely around the finite-difference solution, thus validating the finite-difference method used to price the stock option. The negligible differences between the finite-difference prices and Monte Carlo prices in the right panel of Figure 4.2 prove that the approximation made from equation (2.11) to equation (3.13) is sound.

4.2 The Effects of Dilutions and Buyback Policy

Now consider a firm, with an initial asset value of $V_0 = 150$, initial number of shares in the open market $n_0 = 1$, and varying coupon amounts c. Setting $n_0 = 1$ ensures that stock options and equity options are comparable. With parameters $r = 0.06$, $q = 0.04$, $\sigma = 0.25$, $\rho = 0.2$, the effects of dilutions and buybacks on call option prices under various levels of leverage are investigated. Using Leland (1994) closed-form formulas given in equations (2.4) and (2.6), the bankruptcy boundaries, equity values and debt values of the firm at various levels of leverage under these parameter specifications are computed. Table 4.1 shows these values, and defines the capital-debt structure of the firm at various levels of leverage. Table 4.1 illustrates how, evidently, increasing leverage increases the bankruptcy boundary and decreases the value of equity. Table 4.1 also shows that for all levels of leverage considered, the bankruptcy boundary, V_B, is significantly lower than the initial asset value, $V_0 = 150$, making it unlikely for the firm to reach bankruptcy in the short term. All claims in this subsection have an expiry time of $T = 3$ years. Figure 4.3 shows at-the-money call stock option values at various levels of firm leverage and under various buyback proportions. The figure also shows an equity option under the same parameter specifications. The equity option acts as a benchmark, as it does not consider dilutions or buybacks. The stock option with $B_p = 0$ in Figure 4.3 also acts as a benchmark, as it only allows for dilutions.

In the structural model described in Chapter 2, dilutions occur when a firm faces negative cash flows, i.e. when equation (2.2) is negative. All else equal, more leveraged firms with higher coupon payments have lower cash flow rates. As such, higher leveraged firms tend to experience future dilutions more frequently than lower leveraged firms. Figure 4.3 shows how equity options, which are not math-

Leverage	c	0	2	4	6	8
Bankruptcy Boundary	V_B	0	14.64	29.28	43.92	58.56
Equity Value	EQ_0	150	128.04	99.96	78.09	58.64
Debt Value	CB_0	0	32.02	60.55	84.96	104.88

Tab. 4.1: Bankruptcy boundary, equity and debt values for various levels of firm leverage.

ematically affected by dilutions, in highly leveraged firms have larger values than stock options, which are affected by dilutions. Therefore, likely future dilutions cause the value of options on share price to decrease when leverage is high. For lower coupon payments, the stock option with $B_p = 0$ and the equity option have similar values in Figure 4.3. This is because firms with low coupon payments are unlikely to experience negative cash flows, and hence dilutions. As such, stock call options (with initial number of shares $n_0 = 1$ and no buyback policy) and equity call options are indistinguishable in firms with low levels of leverage.

By adjusting the variable B_p, the effect of buyback policy on stock call option prices is analysed. Figure 4.3 shows that increasing the proportion of buybacks, B_p, increases the value of the option at all levels of leverage considered. This effect is larger in lower leveraged firms, where cash flow rates are typically larger. All else equal, buybacks increase the share price by decreasing the number of shares in issue. Therefore, buybacks shift call options to more in-the-money positions, making the claims more valuable. As a result, holders of call options on share price in a perfect market are not indifferent between firms paying cash dividends to shareholders and buybacks. In fact, holders of call stock options prefer buybacks over cash dividends.

4.3 Leverage and Implied Volatility

The leverage effect reveals that changes in leverage accompanied by changes in realised volatility, is negatively correlated with stock returns (Cont, 2001). Since implied volatility of stock options is directly linked to realised volatility, implied volatility should similarly depend on the level of the firms leverage. Consider a call option on share price. To end up in the money, high strikes require high returns. However, this tends to be accompanied by low volatilities given the negative correlation of the leverage effect. In agreement with the leverage effect, Toft and Prucyk (1997) find that implied volatilities of equity options are greater in highly leveraged companies. Toft and Prucyk (1997) also show that a skew implied volatility curve

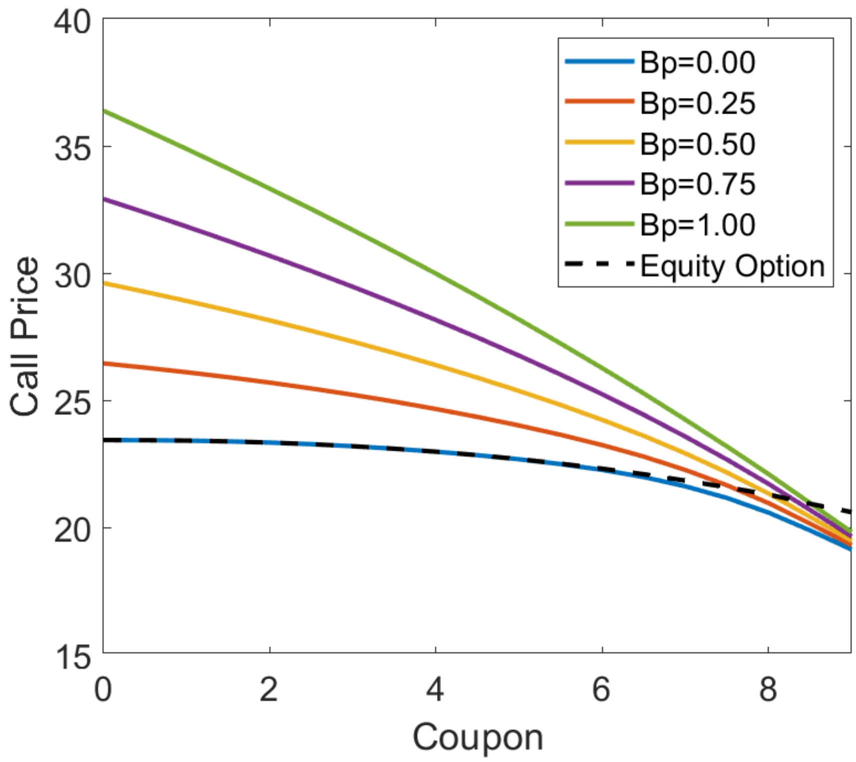

Fig. 4.3: Effects of dilutions and buyback policy on call stock option prices of firms with differing leverages.

results in a significantly leveraged firm. Rathgeber *et al.* (2021) demonstrate similar results in the context of the German stock option market.

To more closely investigate the leverage effect within this structural framework, consider Figure 4.4. This figure plots implied volatility skews of call stock options of a firm with model parameters defined in subsection 4.1, i.e. model parameters are given by $r = 0.08$, $\sigma = 0.2$ and $\rho = 0.35$. The value of q varies depending on the level of leverage with $q = \frac{(1-\rho)c}{100} + 0.03$. This firm has an initial asset value $V_0 = 100$, initial number of shares $n_0 = 1$, no buyback policy ($B_p = 0$) and it is considered under various levels of leverage. Implied volatility is presented as a function of moneyness. Moneyness is defined to be the strike price divided by the spot share price, making anything to the left of one on the moneyness axis in-the-money, and anything to the right out-the-money (since we are considering call options). There are a number of ways to quantify moneyness (see Häfner (2004)), but this definition is chosen as an adequate simple definition. The blue line in Figure 4.4 represents a firm with no leverage, which exactly recovers a flat Black-Scholes volatility structure in line with the 20% input volatility. This acts as a benchmark to compare leveraged firms against.

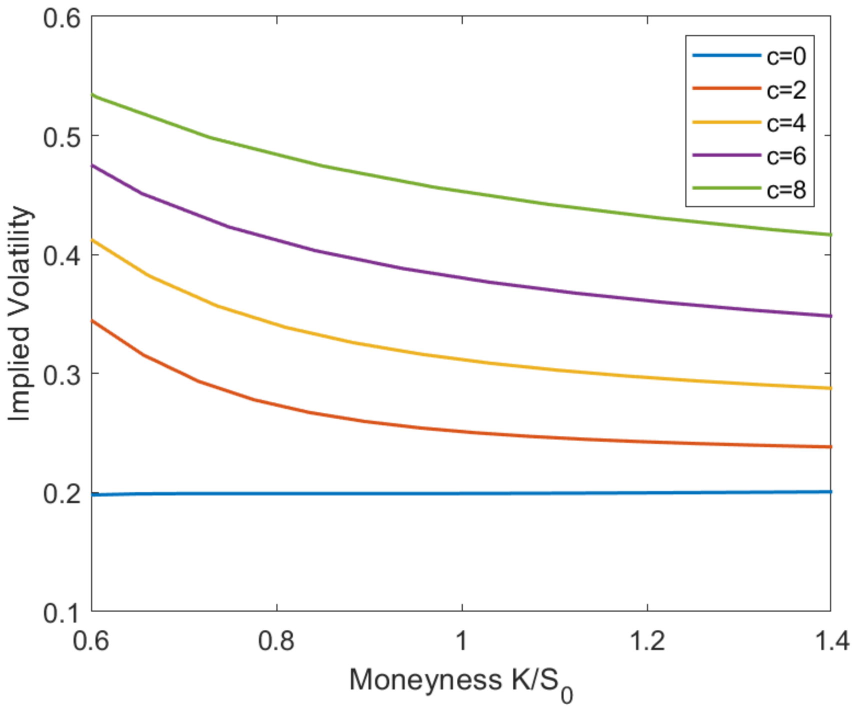

Fig. 4.4: Implied volatility skews as a function of moneyness.

Figure 4.4 clearly shows that increasing the firm's leverage, increases implied volatility of call options on stock. In agreement with the leverage effect, call options with lower (higher) strikes present higher (lower) implied volatilities. Low-strike calls depend on the lower side of the returns distribution, unlike high-strike calls. When asset values are lower, equity devalues relative to debt, and the firm becomes more leveraged. Smaller equity is then exposed to the same amount of asset volatility causing volatility on equity to increase. Due to the direct link between volatility on equity and implied volatility on calls, implied volatility also increases. Figure 4.4 also shows that implied volatility curves in significantly leveraged firms have a skewed shape. Therefore, the leverage effect is demonstrated in this figure, and these results agree with those of Rathgeber *et al.* (2021) and Toft and Prucyk (1997).

Chapter 5

Conclusion

Structural models provide insight into how firms choose their capital structure and payout policies. These models can be used to price claims on total equity and on share price given the capital-debt structure of a firm. In addition, they are tools for determining the impact of leverage on claims. Option pricing within a structural model typically does not consider the number of shares in the open market at any given time. Backwell *et al.* (2022) recognises this shortcoming in the literature and developed a structural framework which differentiates between claims on total equity and claims on share price. In addition to the common features of a structural model, Backwell *et al.* (2022) models the number of outstanding shares based on dilutions and share buybacks.

In this dissertation, the framework developed by Backwell *et al.* (2022), with asset value dynamics presented by Leland (1994), was used to define the structure of a firm. Finite-difference methods were then used to price call options on total equity and call options on share price under various capital-debt conditions. Monte Carlo price estimates and Toft and Prucyk (1997) closed form solutions were used to verify the solutions produced by the finite-difference schemes. After validating the finite-difference solutions, equity options and stock options were compared to determine the effects of dilutions and buybacks on option value.

Dilutions had an insignificant effect on stock call option value in low leveraged firms. However, all else equal, dilutions decreased the value of call options on share price in higher leveraged firms. Increasing the buyback parameter, B_p, increased the value of the call option. As such, buyback policy had a positive effect on the value of call stock options, particularly in lower leveraged firms, where there is more cash available to pursue buybacks. Therefore, while shareholders in a perfect market are indifferent between cash dividends and buybacks, holders of call options on share price are not indifferent and prefer buybacks over cash dividends.

Finally, the leverage effect was demonstrated by analysing implied volatilities of call options on share price in firms with various levels of leverage and no buyback policy. Higher levels of leverage led to greater implied volatilities. In addition, firms with significant leverage had skew shaped implied volatility curves. These results are consistent with those presented by Toft and Prucyk (1997) and Rathgeber *et al.* (2021).

The parameters used to generate the results in this dissertation represented a stable firm, far from bankruptcy. More interesting results may arise when considering firms which are closer to bankruptcy. Extensions to this research would include valuing put options on total equity and on share price. Pricing exotic options within this capital-structure model and exploring the effects that dilutions and payout policy have on exotic claims may also provide interesting results.

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