# *Risk Parity and other Risk Based Portfolio Allocation Approaches in South African and International Equity Markets*

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By

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# Abstract

Risk parity, a portfolio allocation technique based on the equalization of constituent risk contributions, has garnered significant attention in academic circles over the past decade. This study employs back-tests to explore the empirical performance of the approach relative to other prominent heuristic and risk based allocation techniques on South Africa's All Share Index (ALSI) and 12 auxiliary international equity indices. We find that the technique discharges its core risk contribution equalization objectives well in out of sample testing but appears to lag other risk based allocation techniques in terms of risk and return performance. We also establish links between the approaches' performance and leverage aversion theory and find some evidence that levels of market concentration may impact the performance of risk parity portfolios across equity indices.



Keywords: Portfolio Allocation, Risk Parity, Minimum Variance Portfolio, Most Diversified Portfolio, Back-testing, South Africa, International Equity Markets, Leverage Aversion Theory, Market Concentration.

# CONTENTS

# SECTION 1: PREAMBLE

## 1.1 AIM

Risk parity is a portfolio allocation technique based on equalizing risk contributions across portfolio constituents. This study explores the performance of the approach in South African equity markets; with an emphasis on opportunities and constraints that are representative of typical South African institutional equity investments.

In particular this study aims to address the following considerations with respect to risk parity:

i. Empirical performance compared to other risk based allocation techniques.
ii. Consistency of performance within market regimes of particular interest e.g. financial crises.
iii. The effects of investment universe selection on the approaches' performance across time.

## 1.2 APPROACH

This study constructs various heuristic and risk based portfolios in order to assess the economic value of using risk parity in South African equity portfolio allocation. Back-tests are conducted using equity data from diverse economic periods and performance evaluated with comparison to a benchmark that is likely to be representative of those applied to major institutional investors. Thereafter portfolio performance is considered in the context of market regimes and the characteristics of the universe employed in order to produce inferences about the usefulness of the risk parity approach relative to others commonly employed in portfolio allocation.

# SECTION 2: THE EMPIRICAL AND THEORETICAL FRAMEWORK

## 2.1 INTRODUCTION

This section provides an exposition of the theoretical and practical aspects of the portfolio allocation decision. Sections 2.2 and 2.3 explore the academic foundations of portfolio allocation with a discussion of Modern Portfolio Theory and other fundamental results. Section 2.4 explores the practical implementation of portfolio allocation and discusses some popular risk based and heuristic approaches. Section 2.5 focuses on Risk Parity and discusses its characteristics, theory, empirics and implementation.

## 2.2 MODERN PORTFOLIO THEORY

Markowitz's (1952) seminal paper, *Portfolio Selection,* outlined an approach to portfolio allocation that traded off risk and return. Markowitz posited that portfolio allocation is segmented into two parts, the first of which processes available information and uses it to create investable insight, and a latter part which involved transforming this insight into actual portfolios. Focusing in on the latter part of the process, Markowitz suggested that maximizing returns is not a sufficient objective for investors in portfolio allocation as it does not ensure that they will attain a satisfactorily diversified portfolio in the process. Attainment of the secondary diversification objective can be achieved if investors seek to conduct portfolio allocation in a mean variance (MV) framework that assumes investors are risk averse and hence seek to maximize risk adjusted returns. Markowitz demonstrated how the MV framework allows investors to build on their insights to achieve an allocation that maximizes the expected return to the portfolio for a given level of risk and suggested that this could prove a useful tool in assessing investor risk preferences and conducting portfolio allocation.

Markowitz developed some key insights about the MV approach. Firstly, he observed that the framework is not only well geared towards achieving diversification but that it also tends to achieve the right type of diversification. The approach favours portfolios that not only allocate resources to a variety of capital assets, but also consider the correlation between the constituents and diversify the effective bets investors make on factors common to subsets of their opportunity set. He also noted that the optimality of the approach is strongly dependent on the quality of the investment insight (surrounding the expected risk and return profiles of investment opportunities) that the investor possesses.

Roy (1952) also developed the MV approach independently, albeit with a more probabilistic focus. He outlined an approach called the Safety First Criterion which recommends that investors should maximize the ratio of their expected return (over some minimum required threshold) to the volatility of the expected return. Roy reasoned that the holdings that maximize this criterion also have the least likelihood of reaching or falling below a given threshold (e.g. the risk free return level) and hence satisfy the investor's desire to minimize risk of loss. Hence his version of the MV approach also shares Markowitz's assumption of investor risk aversion and the resultant objective of maximizing expected return with consideration for the level of risk. Roy, like his contemporary, also underlined the importance of using reliable estimates for the return and risk inputs if the approach is to be useful. However Roy's approach recommends one portfolio instead of an efficient set the investor can select from and also allowed the possibility of short-selling whereas Markowitz initially assumed a long-only approach (Markowitz, 1999.)

Prior to the work of Markowitz and Roy, investment rationale was largely limited to work on security valuation techniques (Shipway, 2009.) Markowitz (1999) acknowledges the topical relevance of work by some academics in the 1930's onward to 1952 but observes that, although they hinted at the notion of preferences in the MV framework, they did not adequately formalize the concept. By contrast the notion of diversification was already well established by 1952, however investors typically selected securities on an individual basis without regard for how they behaved as a collective (Markowitz, 1999). Hence rigorous portfolio theory was absent prior to the work of Markowitz and Roy and the move towards a theory of portfolio allocation under conditions of uncertainty represented significant progress. However it would be some years until investors had access to the optimization tools necessary to make the approach practical (Perold, 2004)

Tobin (1958) extended portfolio theory by incorporating an allocation decision between risky assets and a risk-free asset. Tobin demonstrated that, given the optimal set of portfolios under the MV framework, the best allocation for an investor with a given level of risk aversion is some combination of a risk free asset and a risky, efficient portfolio which is tangential to the risk free rate. This combination rests on a line segment between the risk free rate and the unique tangency portfolio on the efficient frontier and, on an aggregated level, is referred to as the Capital Market Line (CML.) Throughout the rest of this study any such a portfolio is referred to as the Mean Variance Portfolio (MVP) and is optimal given the investors level of risk aversion. Tobin theorized that assets such as cash, although subject to some level of risk e.g. inflation, are *relatively* risk free compared to other capital assets and hence investors who have a higher degree of risk aversion will exhibit higher liquidity preference and skew their portfolios more towards such assets. Tobin's result is referred to as the Tobin Separation Theorem.

The works of Markowitz (1952), Roy (1952), and Tobin (1958) shaped Modern Portfolio Theory (MPT) by providing a framework for using investment insights to form efficient portfolios tailored to the investor's levels of risk averseness.

## 2.3 THE CAPITAL ASSET PRICING MODEL

The Capital Asset Pricing Model (CAPM) provides a useful characterization of expected returns in terms of market and firm specific factors, hence creating an invaluable paradigm for investors to use in formulating the essential inputs or investment insights whose reliability both Markowitz (1952) and Roy (1952) emphasized. Markowitz (1999) implies a link to the MPT by suggesting Tobin's Separation theorem can be considered an early version of the Capital Asset Pricing Model (CAPM) as the difference between Tobin's findings and the CAPM of Sharpe(1964), Treynor (1961), Mossin (1966) and Lintner (1965a,b) boils down to relatively minor, although non- trivial, differences in assumptions.

The CAPM also quantifies the sensitivity of an individual asset or portfolio to the systematic factor of interest (e.g. the benchmark) and hence provides a sensitivity parameter useful for forming estimates surrounding measures of dispersion such as variance and covariance.

## 2.4 PORTFOLIO ALLOCATION IN PRACTICE

### 2.4.1 INSTITUTIONAL INVESTORS AND RETURN OBJECTIVES

Actual investment practice has deviated significantly from its roots in MPT. Levell (2010) notes that MPT is rarely utilized in its pure form and that the practice of asset allocation has evolved to meet institutional investor return requirements. The typical optimal portfolio allocation based on the MVP is often associated with inadequate expected return given the return targets of such investors. Within the MPT framework the optimal recourse would be for the investor to leverage up the tangency portfolio along the CML by shorting the risk free asset. However investors are typically unwilling to do this for several reasons.

The first is due to leveraging restrictions typically enforced on institutional investors. Pension funds for instance may be restricted by legislation with regards to their use of borrowings to fund their investments. Similarly there may be mandate or legal restrictions on short selling as a means of generating funds for such investors.

In some cases some investors may not have explicit mandate or legal restrictions imposed on them in terms of leveraging. However they may still voluntarily restrain themselves from using leverage to achieve an optimal portfolio allocation. This may be due to several factors such as the potential costs of forced deleveraging or "maverick" risk from engaging in leveraging when other competing funds abstain (Levell, 2010.) A more comprehensive discussion of leveraging is offered in section 2.5.6.1.

Given that investors require higher absolute return than the MVP can provide them but are unwilling to use the intuitive remedies, their only recourses are to sacrifice diversification and incur more beta exposure or invest in alpha generating strategies such as hedge funds. However the former approach is enacted at the cost of additional risk uptake and the latter approach is more expensive and may only produce fleeting returns (Levell, 2010.)

The implication of investors being unable or unwilling to hold a portfolio that is located on the CML is that any alternative portfolio that they form using the risk free-risky asset combination will be suboptimal. This is because such a portfolio is, by definition, different from the MVP and hence must lie somewhere within the efficient frontier and/or be dominated by the MVP combination that yields the same level of risk.

Hence, despite the theoretical integrity of MPT and its long standing status as the core philosophy of portfolio allocation, an overwhelming portion of investors deviate from its recommendations and employ other approaches in deciding their allocations. The next section discusses other reasons for deviations from MPT before considering approaches employed by investment managers to circumvent some of these concerns.

### 2.4.2 REAL WORLD ADJUSTMENTS, NOISY ESTIMATES AND MEAN VARIANCE EFFICIENCY

The other major deterrent preventing investors from investing in a mean variance efficient fashion is the problem of noisy estimates. Michaud (1989) notes that, despite the relatively elegant nature of MPT and the CAPM, investors may decide not to invest in optimal portfolios as they are financially unintuitive. Michaud concludes that the pure MV approach is not suited for practical use as it magnifies errors in risk and return estimates and lacks the refinements necessary to cope with the investment landscapes most investors face.

As previously mentioned, Markowitz (1952) and Roy (1952) emphasized the importance of reliable estimates being utilized in the MV framework if investors are to obtain truly optimal portfolios. It is unsurprising then that this proves to be one of the most significant challenges facing the real world implementation of MPT. Investors who attempt to create ex-ante estimates of risk and return often fall victim to estimation error. Although models such as the CAPM provide a useful framework for deriving risk and return characteristics they are inexact and ex-post realizations of risk and return will often vary from expectation, making real world mean variance optimization (MVO) a similarly inexact science. Jobson and Korkie (1981) demonstrate that such error on an individual asset level is maximized within the portfolio context through MVO and can lead to false optimum portfolios. Michaud notes that such error maximization is linked to the fact that the MV approach overweighs securities with large returns which are in fact those that are most prone to estimation errors.

In addition to a propensity to maximize estimation errors, Michaud (1989) criticizes the MPT framework's inflexibility with respect to incorporating the practical constraints faced by investors. He notes that, in addition to the borrowing and short selling constraints previously discussed, many investors consider the liquidity of securities and the relative quality of their information about various investment opportunities. Liquidity considerations are relevant as price impact often limits the ability to exploit expected return opportunities. Additionally many investors often have differential levels of certainty about their estimates but have no straightforward means of incorporating this reality into the MV framework. Consequently they depart from MPT in order to allow themselves to hold portfolios that are more financially intuitive than the unconstrained MVP.

Finally Michaud (1989) observes that the efficient portfolio under MPT is very sensitive to the values of estimates used. Hence, even though optimizers will specify the exact composition of the efficient portfolios, there is actually a relatively wide confidence interval surrounding such estimates and the portfolios output by the optimizers are not statistically unique. Consequently Michaud notes that they are in fact a number of other portfolios that do not lie on the computed efficient frontier but cannot be said to be inefficient. In some instances the investor's existing holdings may well fall within this margin of error. In any case this lack of uniqueness provides additional motivation for investors to deviate from conducting portfolio allocation in the exact spirit of MPT. He concludes that portfolios based on mean variance efficiency must often be subjected to constraints and skilful tailoring of inputs if they are to provide financially meaningful results.

Corrections put forward to help alleviate problems with MVP sensitivity and adherence to constraints include the portfolio resampling method of Michaud and Michaud (2008) and the robust allocation technique of Tuntucu and Koenig (2004) However these techniques are not without their own drawbacks. Maillard, Roncalli and Teiletche (2008) observe that these alternatives do not fare significantly better in out of sample testing and most investors instead opt for more heuristic approaches to address the issues raised and generate financially meaningful portfolios.

### 2.4.3 HEURISTIC AND RISK BASED APPROACHES TO PORTFOLIO ALLOCATION

There are a number of heuristic and risk based approaches popular in the academic literature and in investment practice. We consider briefly the characteristics of the traditional 60/40, Global Minimum Variance, Most Diversified and Equal Weighted portfolios in the rest of this section before considering the Risk Parity approach in depth in Section 2.5.

#### 2.4.3.1 THE TRADITIONAL 60/40 PORTFOLIO (60/40)

The 60/40 approach is common amongst institutional pension fund investors and is geared towards achieving the 8-9% target returns that such pension funds typically require to fund their obligations (Chaves, Hsu, Li & Shakernia, 2011). This approach allocates approximately 60% of holdings to equity investments and the remainder to bonds. There have been moves recently towards giving such investors more leeway to invest in other asset classes. Chavez et al observe that, despite such liberalization, the majority of institutional investors still allocate the greater part of their holdings between equity and bonds. This leaves such investors relatively undiversified.

Lee (2011) notes that the 60/40 approach only targets returns and does not necessarily lead to diversified holdings; the fundamental reason behind Markowitz's (1952) criticism of return maximization as the sole objective of portfolio allocation. Qian (2011) echoes these sentiments, stating that the 60/40 approach is contrary to the principle of diversification as it over-weights the relatively high risk equity component. Hence the 60/40 portfolio represents the industry's attempt to generate adequate returns and form financially meaningful portfolios; however this leads to portfolio holdings that do not adhere to the principles of mean variance efficiency. Our equity focus precludes consideration of the 60/40 but it remains one of the most popular heuristic approaches utilized in portfolio allocation nonetheless.

#### 2.4.3.2 THE EQUAL WEIGHTED PORTFOLIO (EWP)

The equal weighted (EW) approach accords equal weighting to portfolio constituents in order to achieve diversification. Lee (2011) suggests that this is the quintessential heuristic portfolio as it does not involve specification of any objective function or make an attempt to allocate weights with consideration of constituent risk and return profiles.

Lee (2011) evaluates the EW approach and finds that it is the optimal portfolio if asset returns, volatilities and correlations are all identical. He also notes that work by the likes of DeMiguel, Garlappi & Uppal (2009) and Kritzman, Page and Turkington(2010) has shown that the EW approach is not consistently bested by other more theoretically sound approaches; although the latter authors did suggest that use of longer term estimates may help better theoretically backed techniques to outperform the EW.

Lee also observes that the equally weighted approach may not achieve true diversification. This is because it is highly sensitive to the investment universe under consideration. The implication is that equally weighting certain sectors for instance may not achieve sufficient diversification as some have disproportionately more securities; leading to higher concentrations of risk in select sectors in the overall portfolio. Lee observes that this problem is likely to persist even in the absence of such differences in sector membership numbers since the EW approach will still accord the riskiest and least risky assets the same weight.

Finally Dash and Loggie (2008) suggest that the equal weighted approach can be significantly more expensive to implement compared to a market capitalization weighted approach as transaction costs necessarily incurred to establish equal weighting for a wide variety of assets can be quite significant. Lee (2011) suggests that the associated cost of such activity can be crippling when operating in a relatively illiquid environment.

#### 2.4.3.3 THE GLOBAL MINIMUM VARIANCE PORTFOLIO (GMVP)

The Global Minimum Variance Portfolio (GMVP) is the portfolio on the efficient frontier that has the lowest risk. Formally, it is the portfolio on the frontier that is tangential to the ray orthogonal to the risk axis in the MV plane. Studies by Clark, de Silva and Thorley (2006) and Behr, Guttler and Miebs (2008) demonstrate that minimum variance portfolios outperformed the ex-post MVP in out of sample testing.

Lee (2011) observes that this portfolio is the only portfolio on the efficient frontier that is uniquely determined by the covariance matrix. Additionally the GMVP equalizes marginal risk contributions of its constituents but does not eliminate the possibility of uneven contributions to total risk; meaning that it can still be a relatively concentrated portfolio. Lee (2011) shows that the MVP overweighs low risk assets which may make it overly sensitive to estimates of covariance. He notes that other studies such as Clark, De Silva and Thorley (2006) apply weight limitations to the MVP in order to avoid such predilections. He also observes that, since the GMVP is unavoidably dominated by portfolios that lie on the CML, the only possible incentive for investors to hold the MVP is its empirical performance.

#### 2.4.3.4 THE MOST DIVERSIFIED PORTFOLIO (MDP)

Choueifaty and Coignard (2008) describe an approach geared towards forming the best diversified portfolio given the investor's opportunity set. They first use portfolio algebra to define the ratio of the weighted average of constituent volatilities to the volatility of their resultant portfolio as a measure of diversification. Thereafter they define the Most Diversified Portfolio (MDP) as the portfolio that maximizes this diversification metric subject to the investment universe and applicable constraints.

Choueifaty and Coignard proceed to describe the theoretical properties of the MDP, noting that the MDP will be the tangency portfolio when asset returns within the universe are proportional to their volatilities. They also find that the MDP has the same correlation with any one of its constituents and hence can equivalently be defined in these terms. Finally they show that the approach will be equivalent to the GMVP in cases where all assets have the same volatility.

In terms of testing, they compared the MDP to the EW, GMVP and simple market capitalization weighted approaches to find that the MDP attained a superior Sharpe ratio relative to the other portfolios. They hence concluded that the MDP displays less size bias than the market cap weighted approach and is usually about 1.5 times more diversified than the benchmark on average - although this can vary with shifting correlations.

The pair compared the market capitalization approach and the MDP further and observed that the distinction between the two lies in the MDP's relative pricing of correlations in asset selection. They suggested that markets may in fact require such correlation to be correctly priced and the MDP may gain its outperformance from this fact. Accordingly they concluded that the MDP is likely to dominate investment landscapes where investors are rational in seeking higher return for risk, markets are efficient on an individual asset level, estimates of volatility are reliable and other estimates are unreliable or are not priced in equilibrium.

Lee (2011) provides a critique of their findings around the MDP by showing that the diversification measure that Choueifaty and Coignard derived may allow the MDP to be more concentrated than other portfolios depending on how it is computed. Hence it may, in select situations, prove to be less diversified than the benchmark. Lee also notes that the optimality requirement that all assets have the same Sharpe ratio has the implication that, within this universe, an investor could form a synthetic asset combining any such two assets to obtain holdings with the same expected return and a lower volatility. This being the case, he suggests that, from an MV perspective, we could conclude that the investor is trying to maximize his benefit in a world of equalized Sharpe ratios and hoping to profit from arbitrage opportunities.

## 2.5 THE RISK PARITY PORTFOLIO (RPP)

Risk parity, the central focus of this study, is a risk based approach premised on the idea of equalizing risk contributions across portfolio constituents. The next subsections provide a thorough exposition of risk parity, its characteristics, empirical performance, theoretical framework, variants and implementational considerations.

### 2.5.1 EARLY STUDIES

Risk parity is a special case of risk contribution based allocation. Qian (2006) observes that the use of risk contribution analysis was already popular in risk and investment management towards the turn of the century-providing examples such as Litterman (1996), Lee and Lam (2001), Wander, Silva and Clarke (2002) and Winkelmann (2004.) Significant focus was accorded to establishing a meaningful measure of risk and Qian notes that Sharpe (2002) and Chow and Kritzman (2001) both criticized existing measures of risk contribution that relied on marginal risk contributions of assets. Their objections related to the fact that risk expressed in terms of Value at Risk (VaR) or standard deviation is not additive and the marginal characterization of risk contribution is often posited without theoretical justification. However Chow and Kritzman (2001) asserted that there was still value in using a VaR approach as it has financial meaning. They also demonstrated how such an approach could enable investors to make more efficient use of leverage once they had established acceptable VaR levels for their portfolio with reference to the MV framework.

### 2.5.2 CHARACTERISTICS OF RISK PARITY PORTFOLIOS

Risk parity portfolios seek to provide equalization of risk contributions across portfolio constituents ex post. Qian (2006) defines an individual asset's risk contribution as its weighted marginal contribution to risk. He uses the properties of conditional expectations to show that that expected loss contributions of individual assets are closely linked to percentage contributions to risk. In fact Qian shows that the two are identical in three cases; the first two being the trivial cases where the portfolio is composed of a single asset and unconditional expected returns are zero and the third being when constituent risk contributions are equivalent to their return contribution in the portfolio context. Hence ex-ante risk contributions are a good proxy for expected loss contributions when return budgets are conformed to their risk counterparts. Qian notes however that this relationship will only hold for optimal mean variance portfolios and may prove inexact for real world portfolios that may deviate from optimality. In such cases he demonstrates that risk contributions will still prove to be a good indicator of loss contribution in crisis type scenarios where losses are relatively large.

Qian tests these theoretical results and finds that they hold up well under normality assumptions. However they seem to exhibit drops in performance when losses become larger suggesting that the return distributions has fat tails. For such cases he finds that a VaR approach that employs a Cornish Fisher expansion augmenting the z-score for higher moments proves fairly reliable at equalizing risk contributions with losses. Overall Qian reaches the notable conclusion that risk parity portfolios generally fulfil their ex ante risk equalization objectives ex post.

In an earlier work, Qian (2005) also established that the RPP is MV efficient when Sharpe Ratios are equal and asset class correlations are zero. He compares the 60/40 with the risk parity approach in a bond and stock investment universe and finds the risk parity approach does better on a risk adjusted basis. He finds however that it does not generate adequate total return to satisfy typical requirements and requires leverage in order to dominate on both total and risk adjusted return terms. The approaches' general superiority in that study leads Qian to suggest that it may be safe to assume that risk and return are approximately commensurate and correlations are near zero (as may be applicable in the short term with bonds and equities).

### 2.5.3 RISK PARITY PORTFOLIO VARIANTS

Four distinctions within the risk parity approach are present in the literature on risk parity; the Naïve, Dynamic Risk Parity, 'Optimal' Risk Parity and Diversified Risk Parity approaches. The Naïve approach is the most elementary. It relies on the observation that the principles of risk parity will necessarily lead to riskier assets receiving relatively smaller allocations than less risky ones in order to equalize overall risk contribution. Using this fact, the Naïve approach sets portfolio weights proportional to the inverse of the individual asset's volatility. Chavez et al (2012) note that this approach is MV efficient when all asset correlations are equivalent; however it is unlikely to achieve true diversification in situations where this assumption is invalid.

Mailard et al (2008) and subsequently Chavez et al (2012) demonstrate how RPP weight formation can be modified to incorporate more flexible correlation assumptions, providing the 'Optimal' Risk Parity approach. This flexibility enables more realistic and efficient use of the covariance matrix. The approach is discussed further in Section 2.5.6.2.

In a similar vein, Qian (2011) observes that, although the equalized Sharpe ratio requirement for RPP to be efficient is intuitively attractive, real world application with a more active approach may make it desirable to tailor risk parity portfolios based on forecasts that assume different Sharpe ratios. Hence Qian introduces a risk parity variant he dubs Dynamic Risk Parity to allow investors to maximize the portfolio Sharpe ratio using varying constituent Sharpe Ratios. Peters (2011) indicates why an approach of this nature may prove useful by showing that the RPP dynamically allocates based on volatility and assigns more/less of the holdings to bonds as equities become more/less volatile. He suggests high equity volatility typically coincides with relative underperformance with respect to bonds and hence may provide a dynamic allocation approach based on the RPP an advantage.

The final variant, Diversified Risk Parity, alters the risk premia entailed and is based on diversification with respect to risk factors as opposed to asset classes. Bender, Briand, Nielsen and Stefek (2010) demonstrate that diversification by established risk premia such as asset class, style and fundamental factors could be beneficial even if done on a simple return basis. Meucci (2009) also takes a similar approach and extends it to risk contributions by performing a Principle Components Analysis (PCA) based risk decomposition and attempting to diversify based on the orthogonal risks identified. Lohre, Opfer and Orszag (2012) adopt Meucci's (2009) approach and apply risk parity principles to form a portfolio diversified according to such underlying risk factors. They find that the Diversified Risk Parity approach achieves ex post risk equalization. Its comparison to asset class based risk parity shows that the asset class based approach demonstrates a propensity towards becoming less well diversified with time and suffers from a need to heuristically allocate to sectors before calculating weights. These are all problems that the Diversified Risk Parity approach is largely free from.

Page and Taborsky (2011) add to the arguments in favour of the approach by linking increased asset correlations in crisis periods to factor effects and suggesting these remain latent during bull markets as their associated returns are typically attributed to alpha but become more evident in bear markets. They hence recommend a factor based diversification approach in order to protect investors from the macro risks that typically dominate returns in crisis periods and drive up correlations. However Kowara and Idzorak (2011) contend that a factor based approach to diversification may provide additional ways to view the asset allocation decision but does not add enough to the process to be truly ground breaking in its own right.

### 2.5.4 EMPIRICAL PERFORMANCE

This section provides an overview of relative performance studies involving the RPP. Firstly, as broached in a Section 2.5.2, Qian (2005) demonstrated the superiority of the RPP over the 60/40 in an investment universe of bonds and equities. The author examined a test period from 1983-2004 and found that risk parity achieves genuine diversification through equalization of risk contributions across portfolio constituents.

Maillard et al (2008) back tested the risk parity approach and compared it to GMV, MD and EW portfolios using data from 1973-2008. They implemented a series of back tests employing the S&P500, agricultural commodities, and a global diversified portfolio respectively to find that the RPP is intermediate to the GMVP and EWP in terms of risk. Maillard et al found that the GMVP approach generally dominates the RPP in terms of risk adjusted return but observed that the latter leads to less concentrated portfolio allocations and may be less prone to drawdowns. They recommend it on this basis as a good approach to achieving true diversification.

Lee (2011) employed data from 1990-2010 across ten sectors formed from the Russell 1000 equity index to compare simple market capitalization weighted, GMVP and EW portfolios to the RPP. Lee found that the RPP performs best in terms of equalizing risk contributions amongst the various portfolios and notes that the GMVP and EW are actually no more diversified than the market capitalization weighted approach. Lee also reached some theoretical conclusions that are discussed in the next section.

Chavez et al (2012) compared the MVP, RPP, EW, GMVP and two 60/40 variants. They used a variety of asset classes (including several bond and equity classes, commodities and real estate) from a dataset spanning 1980-2010 to find that the RPP portfolio provides the best diversification amongst the various approaches in terms of distributing risk. However they also observed that the RPP does not consistently dominate the alternatives in terms of risk adjusted returns and its outperformance may be dependent on the window employed. Additionally they demonstrated that the RPP seems to exhibit a significant level of sensitivity to the choice of asset classes used to form the portfolio and its performance may hinge significantly on the availability of low volatility assets with high Sharpe ratios. Finally, they note that the RPP and EW portfolio Sharpe ratios are a more reliable indicator of future risk adjusted performance than those formed using Sharpe ratios from the other approaches.

Asness, Frazzini and Pedersen (2012) compared the simple market value weighted portfolio, the 60/40 and a realized volatility scaled RPP portfolio and found that the risk parity approach is closest to the ex-post MVP. Their comparison employed three asset classes formed from U.S stocks, bonds, global stocks, credit and commodities and spanned periods from 1926-2010. They found that, in the long only case, the traditional portfolio does better on a total return basis but the risk parity approach does better on a risk adjusted basis. Relaxation of the long only constraint leads to clear outperformance for the RPP and they found these results to be reasonably robust even when leveraging costs are accounted for. Asness et al also sought to address concerns that the outperformance of the RPP in contemporary periods is largely a consequence of the presence of a bond bull market in the last few decades. They showed that the RPP portfolio still performs well over the period from 1926-2010 in which equities have actually generally outperformed bonds; thus suggesting such concerns may be unwarranted.

More recently Kaya and Lee (2012) sought to discover how the RPP may be expected to perform by using controlled simulations. They established that the RPP may not dominate over the short term but can be expected to demonstrate some superiority over the long term. They also explored the sensitivity of the approach to estimates of the covariance matrix and found that it is less sensitive than the GMVP to inputs although it does exhibit more sensitivity to estimates of variance than covariance. Their back-tests also showed that it seemed to outperform the GMVP and the MVP in cases where estimates were noisy. Hence they note that the RPP should perhaps be seen as a way of avoiding an over reliance on noisy estimates and that investors who have good information should not need to diversify their risks to the extent the RPP does. They also found that the RPP maintains its efficiency relatively well over time and may outperform the ex-post MVP because it overweighs low idiosyncratic risk assets with low beta, a result also linked to the GMVP in other studies such as Scherer (2010) and Clarke, de Silva and Thorley(2011).

There are also a number of results concerning the Diversified Risk Parity approach. Bender et al (2010) established that a simple factor return diversification approach using MSCI indices produced superior results to simple asset allocation approach. Lohre et al (2012) used a variety of international indices including the MSCI for developed and emerging market stocks and a bond index amongst other asset classes. They found relevant factors to be equities, equity spreads, interest rates and credit spreads and also discovered that the Diversified Risk Parity approach achieves its risk contribution objects well even in the face of a long only constraint.

Overall the majority of studies suggest that the RPP has some tangible financial value and the approach can outperform other common approaches on a risk adjusted basis and on a total return basis as well (assuming leverage). These results may be more evident if the approach is employed over the long term. The Diversified Risk parity approach offers another interesting dimension for implementing the risk contribution approach. However there are indications, such as the findings of Chavez et al (2012), which suggest that the overall RPP approach may not be infallible in the face of the universe inclusion decision and selection of time period.

### 2.5.5 THEORETICAL FRAMEWORK AND CRITICISM

Notwithstanding the generally positive empirical results that the RPP has generated there has been considerable controversy over its rationale and theoretical worth; causing it to attract a significant amount of criticism. Schachter and Thaigaran (2011) suggested that the Risk Parity approach is intuitively appealing but provides little theoretical backing. They observed that the RPP is attractive since it is intuitive for investors to think that the purpose of holding a portfolio is to achieve diversification and equalizing risk contributions is an intuitive way of achieving this.

Inker (2011) questions the assumptions that the RPP makes, highlighting concerns that standard deviation may not be a complete measure of risk and assumed risk premia for certain asset classes may be illusory. He points out that use of standard deviation is problematic as risk may be time variant and it is difficult to obtain good estimates within specific market regimes as those of vital interest, such as crises, do not occur often enough to obtain robust estimates. This is problematic as the RPP will often leverage asset classes based on low levels of historical volatility but there is little reason to believe that volatilities and correlations will stay the same in the event of another market regime change. Hence the timing of leverage is a significant consideration. Inker also contends that the equity risk premium exists because corporates are willing to offer it to investors in return for funding on favourable terms. However other markets (such as bond and commodities markets) may lack such obvious providers of premia and there has been an influx of price insensitive investors in these markets who may have eroded any historic instances of risk premia within them. Consequently the RPP may at times be optimizing using an inadequate risk measure and hoping for rewards to risk premia that are non-existent.

Lee (2011) also noted the dearth of theoretical backing behind the RPP and stated that the establishment of a sound theoretical reason for it would be profound as it would have the implication that investors who do not attempt to uncover special return information can expect to do better than those who do. This would call into doubt the information content of asset prices.  Lee also highlighted that one of the key theoretical issues facing the RPP is the objective investors are presumed to have in the portfolio allocation process. The RPP seeks to achieve diversification and appears to largely ignore estimates of return but investors do not diversify for its own sake but rather as a means of achieving higher returns. Assuming reliable estimates, investors using the RPP should not fare better than those who consider both return and risk attributes in forming portfolios; thus achieving a higher Sharpe ratio. Lee concludes that the theoretical issues that the RPP and other risk based approaches (see section 2.4.3.) face mean that, although they may exhibit superior performance in cases, they cannot be considered new portfolio theories and are just aspects of MPT that prove optimal in specific market regimes.

Qian (2011) attempts to more accurately define the investment objectives of the RPP as seeking higher diversification and total returns. He notes that the RPP needs to be levered up to achieve the latter objective but will still have a superior Sharpe ratio compared to a traditional 60/40 approach with the same level of risk.

Asness et al (2011) propose a formal theoretical framework for the RPP. They suggest that the RPP is in fact anything but agnostic on returns since it’s approach to equalizing risk contributions necessarily means it assumes that the equity risk premium (in the simple bond stock universe) is not significant enough to justify granting equities a higher risk budget. They observe that this is fundamentally in conflict with CAPM which says the market portfolio is optimal given the structure of risk premia and the impact of market forces. Asness et al invoke the findings of Frazzini and Pedersen[1] (2010) to conclude that leverage aversion may cause the standard CAPM relationship to break down and hence the highest risk adjusted return can be obtained by overweighting less risky assets. They conclude that risk parity can be considered an aspect of leverage aversion theory. They note that leverage aversion has also been show to hold worth in other diverse applications such as the Law of One Price (Garleanu and Pedersen, 2011) and liquidity dynamics (Brunnermeier and Pedersen, 2009).

Consequently, although the risk parity approach came to prominence with a focus on its empirical performance, there has been greater consideration of its theoretical backing in recent years. Criticism about a lack of clear investment objectives and rationale for outperformance has been met by contextualization in the existing MV framework (Qian,2011) and theoretical explanations based on the theory of leverage aversion (Asness et al, 2011).

## 2.5.6 IMPLEMENTATION

The RPP is in many ways an approach intended to bridge the gap between the theoretical objectives of the MPT and actual practice. To this end there are several pertinent considerations in terms of its implementation. This section discusses leveraging, computation and transitioning to the approach.

### 2.5.6.1 LEVERAGING

Section 2.4.1 explained that institutional investors have typically been unwilling to use leverage in order to achieve MV optimality due to legal or self-imposed constraints stemming from concerns about costs of forced deleveraging, deviation from the practices of their peers etc. These concerns are similarly relevant when considering the RPP as, although yielding higher risk adjusted returns, it provides relatively inferior total returns compared to the traditional 60/40.

Leveraging has been a mainstay of risk contribution methods. As early as the turn of the century, Chow and Kritzman (2001) discussed using risk budgeting to derive VaR figures that would allow investors to employ leverage within these allocated risk budgets and hence enable greater capital flexibility. Qian (2005) notes that the return objectives of typical institutional investors would necessitate leveraging up the RPP. However he also observes that leveraging is already a natural part of equities (via corporate borrowings) and suggests that, although there are short term risks to its use, it may be relatively safe over the long term.

Levell (2010) notes the similarities between the MVP and the RPP in terms of requiring leveraging and suggested that the two approaches are essentially the same (i.e. the RPP is a special case of the tangency portfolio) and hence also requires leveraging up the CML until investors can achieve their required return. Levell observes that the risk of losing more than 100% of their investment, costs of margin or prime brokerage in excess of risk free and “maverick risk” tend to dissuade investors from leveraging up but this is an overcautious approach as leverage is only a problem when combined with other factors. He suggests some of these factors include asset illiquidity, the likelihood of distressed sales and pronounced kurtosis in return distributions leading to higher risk of extreme losses and margin calls. Hence he acknowledges the role of prudence in the use of leverage but asserts it is still a useful tool that

[1] ) Frazzini and Pedersen(2010) demonstrate that high beta assets often have lower risk adjusted returns than their low beta counterparts; possibly due to a higher level of liquidity chasing them as leverage constrained/averse investors seek higher total returns,

should be given fair consideration on a risk/reward basis before investors decide to summarily exclude its use.

Schachter and Thaigaran (2011) point out that the possibility of changes in markets based on market regimes such as suggested by Inker (2011) make it inadvisable to lever up bonds in situations where the prospective risk premium may not be as significant as assumed due to structural shifts. They suggest that there are three significant challenges to applying leverage; the cost of leverage relative to the expected return of the levered asset, timing risk associated with leveraging up on an asset in an adverse market environment and tail risk during crises. They however highlight that preliminary studies exploring transactions cost such as Maillard et al (2008) suggest that RPP may have lower turnover than the MV approach and hence could possibly counterbalance the costs of leverage. They note the preliminary nature of the lower turnover results however and suggest further evidence is necessary.

Evidently leverage is a key requirement for the use of the RPP by the average institutional investor given their levels of required return. Many investors exhibit pronounced leverage aversion but those for whom leverage is not precluded by legislative or other restrictions have scope to relax their stance and evaluate usage of leverage based on its relative risks and benefits. The literature is strongly suggestive of the fact that leveraging the RPP can lead to superior Sharpe outcomes for the investor. Leverage risks are undeniably present but indications are that considered use of leverage, even with its short term risks, may prove relatively safe over the long run and allow investors to achieve better risk adjusted returns from their portfolio holdings.

#### 2.5.6.2 COMPUTATION

Chavez et al (2012) observe that under the standard RPP assumptions of identical Sharpe ratios and zero correlations, the Naïve RPP (setting portfolio weights to the inverse of risk contribution) approach is optimal within the MV framework. However Maillard et al (2008) introduced a higher degree of flexibility about the correlation assumption and demonstrated how to calculate optimal RPP weights in its absence. Chavez et al (2012) dub this approach 'Optimal Risk Parity' and note that the optimization process outlined by Maillard et al can be complicated, require specialist software and entail long calculation times. All these factors could hamper the practical use of RPP in the investment environment.

The optimization process for the RPP in the general case is not a straightforward one. This is due to the fact that the process encounters problems with endogeneity within the weight calculation process since the optimal weight in the RPP for any asset is a function of its portfolio beta which in turn is dependent on the asset weight. The problem simplifies in the case where a constant correlation is assumed between assets; yielding the naïve version of the RPP portfolio with asset weights proportional to inverse volatility. In the general case though, where correlations are variable, this results in a need for more complicated optimization procedures. Maillard et al (2008) discussed two approaches to finding optimal RPP weights, one of which minimizes the sum of squared differences between asset risk contributions subject to some linear constraints and another more complicated approach based on nonlinear programming.

However Chavez et al (2012) present algorithms that do not require any optimization and can be deployed using simple matrix programming or spread sheets. They proffer two algorithms; one based on the Newton's method and another based on the power method commonly used to calculate matrix eigenvalues. They note a preference for the first approach as it is more robust and converges faster. They demonstrated that these two approaches yield results reflecting those of Maillard et al (2008) in terms of risk equalization and provide results that also hold up well ex-post. They also found that implementations based on such algorithms provided superior results to those that used the conventional Naïve approach.

Qian (2011) also provides a general formula that can be employed to provide more on-going rebalancing based on changing correlations and Sharpe ratios. The Dynamic Risk Parity approach he describes does not require optimization and provides unlevered portfolio weights (albeit only for the bond equity universe) that can be employed to form the RPP and hence leveraged to meet return targets.

Any of these approaches could, in principle, be employed to implement Diversified Risk Parity with use of appropriate risk factors.

### 2.5.6.3 TRANSITIONING TO RISK PARITY

Hence there are practical, cost effective approaches to calculating the RPP in existence. Most investors are likely to be using some 60/40 variant in their portfolio allocation process and the literature also provides some guidance about avenues for making the transition from this approach to the RPP.

Ruban and Melas (2010,2011) examined three approaches to creating a risk parity portfolio; rebalancing the existing portfolio to reflect RPP weights, leveraging up the low risk asset (e.g. the bond component) in the portfolio in order to achieve the desired relative weightings or using a combination of the rebalancing and leveraging approaches. They note that the rebalancing approach should lead to a higher Sharpe ratio but inferior total returns. The leveraging approach however will only reduce portfolio volatilities if correlations between the assets are adequately negative. In fact, they show that these correlations need to become increasingly negative as the amount of leverage employed increases, relative equity holdings fall and the proportion of stock to bond volatility increases. Hence the risk equalization benefits of the selective leveraging approach are less assured since there are a number of restrictions surrounding when this method can achieve the desired risk equalization objective. An intermediate approach to achieve risk parity is through a combination of leveraging and rebalancing. Ruban and Melas (2011) discuss this hybrid mix in more detail.

Their earlier paper notes that we cannot assume that the Sharpe ratio is linear with regards to leverage. If the second approach is employed and only the low risk/bond component is leveraged then (depending on the relative risk, Sharpe ratios and correlation of the levered and unlevered portfolios) the RPP derived may in fact be suboptimal. Consequently, in order to maximize the likelihood of achieving equalized risk contributions through leverage, it is likely to prove more useful to first determine the RPP weights and rebalance the whole portfolio accordingly prior to applying leverage to attain portfolio return targets.

Qian (2011) suggests that the RPP approach can be deployed at the portfolio level (as the main allocation philosophy) or to a subset such as the alternatives investment allocations. He notes that stocks have been underperforming in more recent history and there are also concerns about inflationary pressures eroding some low risk returns. These considerations make some investors unwilling to transition to the RPP and overweight bonds in the prospective environment. Qian acknowledges the relevance of concerns surrounding the timing decision but asserts that managing the associated short term risk can allow investors to reap long term benefits from the transition. Accordingly, Qian proposes dollar averaging to help alleviate the risk and suggests that including a significant allocation to real assets in the RPP can help alleviate some of the inflation concerns. Finally he suggests that using the Dynamic Risk parity approach can better enable investors to adjust their portfolios to changing market conditions.

## SECTION 3: A NOTE ON THE SOUTH AFRICAN INVESTMENT REGULATORY ENVIRONMENT

A comprehensive discussion of the investment regulatory landscape in South Africa is out of the scope of the current study but an indicative consideration of the constraints facing institutional investors is useful in order to qualify our analysis.

Regulation of the domestic investment environment falls under the purview of the Financial Services Board (FSB) which enforces a number of legal acts covering various aspects of financial markets; including the investment practices of significant market players. The FSB oversees a generally comprehensive legal framework for investments; however there are differential levels of regulation and restrictions imposed on various aspects of the institutional investment community. For instance, under the Collective Investment Schemes Control Act (CISCA), collective investment schemes, such as mutual funds, are prohibited from use of derivatives or leveraging and also restricted to borrowing up to 10% of fund value for short term liquidity management purposes (PLC, 2013.) Additionally weights are limited to 5% if the market capitalization of a candidate equity security is less than R 2 billion (PSG, 2014).

Insurers are subject to relatively less stringent restrictions on their investment methods. The Long and Short Term Insurance acts (FSB, 2013a) (FSB, 2013b) generally require that such funds ensure that they meet solvency requirements at all times; both on an aggregate basis and also with respect to domestic assets and liabilities. The two acts define assets relatively liberally, allowing investment in domestic and foreign listed securities and imposing no explicit upper bounds on specific types of investments. However, other types of investments, such as derivatives and property, are limited to domestic holdings; somewhat restricting the available investment universe. Generally insurers are free to engage in borrowing and short selling- provided that their overall actuarially determined balance sheets (valued in compliance to these acts) remain solvent.

Another notable piece of legislation, Regulation 28 of the Pension Fund Act (1956), governs the investment practices of domestic pension funds. Amongst other provisions, it limits equities to 75% of holdings with further limitations applied based on market capitalization (Cameron, 2011). Additionally South African pension funds are only permitted to engage in borrowing for bridging finance purposes (Bennett & Loubser, 2012) and hence are largely precluded from direct use of leveraging. However privately marketed hedge funds are currently not subject a formal regulatory environment creating such restrictions (Bennett & Loubser, 2012). This is likely to change for hedge funds in the near future as new draft regulation is under consideration (FSB, 2012) but, despite introducing some limitations, this legislation will not fundamentally change the general strategic approaches synonymous with such investment vehicles. Consequently pension funds can gain limited, indirect leveraging ability using such investment vehicles within their alternative investment allocations under Regulation 28.

This analysis will make partial deference to the leveraging, borrowing and allocation considerations facing such institutional investors but only to the extent that it can still provide an unencumbered, generalizable view of the performance of the approaches under consideration. We take particular note of the leverage/short selling constraints and attempt to account for these to make our analysis more relevant. Additionally we act to prevent levels of concentration that are not representative of typical institutional investment practices. Investors subject to more stringent constraints can consider the results provided in subsequent sections to be merely indicative with respect to their own investment opportunities and constraints.

# SECTION 4: DATA AND METHODOLOGY

Section 4.1 discusses the datasets employed and section 4.2 provides an in depth overview of the methodology employed to create this analysis.

## 4.1 DATA

Our dataset, in its entirety, consists of daily price and market capitalization data for the constituents of 12 equity indices. The ALSI naturally forms the core dataset of this analysis although additional equity indices are employed to provide further context. These additional equity indices are representative of diverse financial markets, including those in the USA, Japan, France, India, Australia, Switzerland, Hong Kong, United Kingdom, Taiwan, China and Israel (Countries selected solely based on the availability of equity index market data for this study.) It is our hope that providing additional context via these indices will allow us to infer what themes of performance are generalizable and which are more unique to portfolios formed using the investment universe comprising ALSI constituents. Table 1 summarizes the equity datasets.

TABLE 1: LIST OF EQUITY INDICES

| Index | Countries | Constituents | Start Date | Observations |
|---|---|---|---|---|
| S&P100 | USA | 100 | 1/2/1995 | 4937 |
| TOPIX100 | Japan | 100 | 1/2/1995 | 4937 |
| EURONEXT100 | France, Portugal, Belgium,Netherlands,Luxembourg | 100 | 1/2/1995 | 4937 |
| S&PBSE100 | India | 100 | 1/2/1995 | 4937 |
| S&PASX100 | Australia | 100 | 1/2/1995 | 4937 |
| ALSI | South Africa | 165 | 1/2/1995 | 4937 |
| UBS100 | Switzerland | 100 | 1/2/1995 | 4937 |
| HANGSENG100 | Hong Kong | 100 | 1/1/2003 | 2850 |
| FTSE100 | United Kingdom | 101 | 1/2/1995 | 4937 |
| TWSE100 | Taiwan | 100 | 1/2/1995 | 4937 |
| SZSE100 | China | 100 | 1/1/1998 | 4154 |
| TA 100 | Israel | 99 | 1/3/2000 | 3632 |

The datasets all terminate on 29 November 2013 but in some cases have different start dates in order to ensure a sufficient number of securities are available to meet index size and minimum trading frequency specifications discussed later on in this section. Nonetheless the majority of indices span 2 January 1995-29 November 2013 which enables us to examine the effects of possible structural shifts in the risk and return profile of the investment universe and hence allows analysis of performance during significant market periods such as the Asian Crisis of 1997, Russian Financial Crisis of 1998, the Subprime Mortgage Crisis (2007-2010) and more recently, the European Sovereign Debt Crisis. Note that the start dates in Table 1 represent start dates for index time series data and not the initial point at which out of sample performance is generated. The date on which initial performance is recorded depends on the number of in-sample observations employed to form initial estimates for use in optimization routines as discussed in Section 4.2.2.

## 4.2 METHODOLOGY

Our aim is to analyse candidate portfolios formed within the investment universes represented by the equity indices in Table 1. Pursuant to this we employ a back-testing approach. This involves the use of an initial sample selection to form portfolios adhering to the various heuristic and risk based philosophies and the subsequent computation of out of sample performance for analysis.

A 6 month lag sample is used in order to estimate the covariance matrix. Thereafter we create our candidate portfolios based on these estimates and subject them to out of sample testing with monthly rebalancing. Finally performance evaluation is conducted by generating reports along several dimensions including realized returns, volatility, turnover, drawdowns, and risk contributions.

The rest of this section delves into specific aspects of the back testing process in greater detail. Section 4.2.1 provides a formal overview of the portfolio optimization routines, Section 4.2.2 discusses the structure of the performance generation process and Section 4.2.3 describes the range of performance and statistical measures that are employed to analyse out of sample results.

### 4.2.1 OPTIMIZATION ROUTINES

Our analysis considers five approaches to portfolio allocation; the Naïve Risk Parity (NRPP), Risk Parity (RPP), Equal Weighted (EWP), Most Diversified (MDP), and Global Minimum Variance (GMVP) portfolios. Together they represent perhaps the most notable heuristic and risk based approaches applicable to equity portfolio allocation. This subsection adds to the expository overview in Section 2.4.3 by providing a more rigorous discussion of the methodology used to compute the various candidate portfolios.

#### 4.2.1.1 NAÏVE RISK PARITY PORTFOLIO (NRPP)

Analytically, the Naïve approach is based on a simple inversion of asset volatility and subsequent re-scaling of the raw weights to achieve full investment i.e. $\sum_{i=1}^{n} w_i = 1$. Formally, for any given security within the relevant $n$-asset investment universe, the relevant portfolio weight ($w_i$) is:

$$(1) \qquad w_{i_{naive}} = \frac{1}{\sigma_i} \times \frac{1}{\left(\sum_{i=1}^{n} \frac{1}{\sigma_i}\right)}$$

Here (as in the rest of the paper) risk ($\sigma$) is measured using standard deviation. The naïve approach is naturally unconstrained, it is also long only as variance is strictly non-negative by nature.

#### 4.2.1.2 RISK PARITY PORTFOLIO (RPP)

Risk parity is similarly predicated on the contents of the covariance matrix. However it makes more complete use of the information; accounting for both asset variances and covariances. As discussed in Section 2.4.3, this approach aims to equalize risk contributions. Chavez et al (2012) demonstrate that, if portfolio return and risk are characterized as shown in (2) and (3) below, then the total risk in the portfolio is equal to the weighted average marginal risk contributions of the constituent securities.

$$(2) \qquad r_p = \sum_{i=1}^{n} w_i \cdot r_i$$

$$(3) \qquad \sigma_p = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} w_i \cdot w_j \cdot \sigma_{i,j}}$$

These marginal risk contributions are represented by the partial derivative of portfolio risk with respect to the specific asset weights. However, since the summation is taken over the entirety of portfolio constituents, the end result simplifies to a consideration of the covariance between the asset and the portfolio as in (4). Subsequently, each asset's total risk contribution is merely its actual contribution to total portfolio risk i.e. its weighted marginal risk contribution as expressed in (5.) Here $r_i$ and $r_p$ represent individual asset and portfolio returns respectively.

$$(4) \qquad MRC_i = \frac{\partial \sigma_p}{\partial w_i} = \sum_{j=1}^{n} w_j \cdot \sigma_{i,j} = cov(r_i, r_p)$$

$$(5) \qquad TRC_i = w_i \cdot \frac{\partial \sigma_p}{\partial w_i} = \sum_{i=1}^{n} w_i \cdot w_j \cdot \sigma_{i,j} = w_i \cdot cov(r_i, r_p)$$

Consequently the objective of equalizing risk contributions across all portfolio constituents can be restated in the form of (6), where the problem is reduced to that of trying to ensure that total risk contributions are equal to some yet undetermined constant $\lambda$.

$$(6) \qquad w_i \cdot \frac{\partial \sigma_p}{\partial w_i} = w_j \cdot \frac{\partial \sigma_p}{\partial w_j} = \lambda \quad \forall i, j$$

However, as also discussed by Chavez et al (2012), attempting to solve for risk parity weights as in (6) analytically is not possible since the weights are a function of asset beta with respect to the portfolio. This beta is itself a function of the portfolio weights by virtue of the fact that it references the portfolio variance in its denominator. Consequently both Maillard et al (2008) and Chavez et al (2012) present other ways to calculate risk parity weights in the face of such endogeneity.

We employ the first of Chavez et al's (2012) proposed solutions to the endogeneity problem to calculate risk parity weights. The authors note that (6) can be rewritten as (7) if we observe that **Ω·w** is equal to a vector of asset covariances.

$$(7) \qquad \mathbf{\Omega} \cdot \mathbf{w} = \lambda \cdot \frac{1}{\mathbf{w}}$$

The authors point out that this form resembles the eigenvector-eigenvalue equation in form and is hence amenable to solving using iterative procedures commonly employed to derive solutions within that context. We employ the first of these, an algorithm based on Newton's method, to solve for risk parity weights. Chavez et al (2012) remind us that in trying to find the solution to *F(y) = 0* we can use a Taylor expansion to write a linear approximation for the system of equations as follows:

$$(8) \qquad F(y) \approx F(c) + J(c) \cdot (y - c)$$

Where *J(c)* represents the Jacobian matrix of the system of equations evaluated at point *c*. We set *F(y) = 0* in order to find the root of the system of equations-hence enabling us to solve for *y* as in (9)*:*

$$(9) \qquad y = c - [J(c)]^{-1} \cdot F(c)$$

We expect that repeated iterations of equation (9) will yield estimates of $y$ that are closer to the true value. Hence, given an initial approximation $y^n$, we can compute a better approximation to the solution as in equation (10) and approximations will tend towards the true value if the method converges.

$$(10) \qquad y^{(n+1)} = y^{(n)} - \left[J(y^{(n)})\right]^{-1} \cdot F(y^{(n)})$$

Consequently if we make the risk parity problem the subject of the system of linear equations and include a full investment constraint then we obtain equation (11.)

$$(11) \qquad F(y) = F(\mathbf{w}, \lambda) = \begin{bmatrix} \Omega \cdot \mathbf{w} - \lambda \cdot \dfrac{1}{\mathbf{w}} \\ \sum_{i=1}^{n} w_i - 1 \end{bmatrix} = 0$$

The Jacobian of this system of equations is shown in equation (12) where $\iota'$represents a vector of ones and $diag(\frac{1}{w^2})$ represents a square matrix with diagonal elements equal to the constituents of $\frac{1}{w^2}$. Note that the apostrophe in $\iota'$ denotes matrix transposition.

$$(12) \qquad J(y) = J(\mathbf{w}, \lambda) = \begin{bmatrix} \Omega + \lambda \cdot diag\left(\dfrac{1}{\mathbf{w}^2}\right) & -\left(\dfrac{1}{\mathbf{w}}\right) \\ \iota' & 0 \end{bmatrix}$$

Consequently Chavez et al (2012) formally describe the algorithm to compute risk parity weights as follows:

1. Begin with an initial guess $w^{(0)}$ for the portfolio weights, $\lambda^{(0)}$ (any positive number on the unit scale) and a stopping threshold $\varepsilon$. Create an initial estimate $y^{(0)} = [w^{(0)'} \; \lambda^{(0)}]'$
2. Compute $F\left(y^{(n)}\right), J\left(y^{(n)}\right)$ *and* $y^{(n+1)}$ as outlined above.
3. If the condition in equation (13) is satisfied i.e. the change in approximated solution is less than the stopping threshold, then the iterations cease. Otherwise return to step 2.

$$(13) \qquad \| \, y^{(n+1)} - y^{(n)} \, \| \; < \varepsilon$$

The authors present two algorithms but recommended this approach over the alternative as it converges faster and is supposedly more robust. Consequently we primarily employ this technique to compute risk parity weights.

Note that the only constraint imposed in the Newton method approach to calculating risk parity weights is the full investment constraint. Consequently it is possible, depending on the covariance information, to have negative weights within the weight vector. Where the default estimation yields negative weights we resort to using a Sequential Quadratic Programming (SQP) algorithm to run the optimization program in (14) as suggested by Maillard et al (2008).

$$(14) \qquad \min \qquad F(\mathbf{w}) = \sum_{i=1}^{n}\sum_{j=1}^{n}(TRC_i - TRC_j)^2$$

$$s.t.: \qquad \sum_{i=1}^{n} w_i = 1$$

$$w_i \geq 0$$

Hence the Newton method of Chavez et al (2012) and Maillard et al's (2008) SQP optimization problem represent the two approaches we employ to calculate risk parity weights. Both RPP implementations typically provide maximum weights to the magnitude of 5-8% across equity universes and hence we do not impose a formal maximal weight constraint on the optimization routines although it is applied to subsequent techniques that exhibit excessive concentration.

#### 4.2.1.3 GLOBAL MINIMUM VARIANCE PORTFOLIO (GMVP)

Deriving an analytical expression for the GMVP without the imposition of a non-negativity constraint is relatively simple. However given our desire to consider long only portfolios we implement the following optimization program to calculate minimum variance weights.

$$(15) \qquad \min: \qquad \sigma_p = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} w_i \cdot w_j \cdot \sigma_{i,j}}$$

$$s.t \qquad \sum_{i=1}^{n} w_i = 1$$

$$w_i \geq 0$$

$$w_i \leq 0.1$$

Hence the minimum variance portfolio is calculated by minimizing portfolio variance subject to full investment, non-negativity and maximal weight constraints. We impose the additional maximal weight constraint as its absence sometimes yields individual asset weights in excess of 70%. Consequently we find that the imposition of a maximal weight constraint of 10% is necessary to avoid excessive levels of portfolio concentration that are atypical of most other candidate portfolios and the real world practices of institutional investors who tend to avoid employing excessive active weights relative to their benchmarks.

#### 4.2.1.4 MOST DIVERSIFIED PORTFOLIO (MDP)

Choueifaty and Coignard (2008) present the Most Diversified Portfolio as the portfolio that exhibits the greatest improvement in portfolio risk due to diversification. Let $\mathbf{w}$ be the portfolio weight vector, $\boldsymbol{\Omega}$ represent the covariance matrix and $\boldsymbol{\Psi}$ represent a vector of asset volatilities. The authors define the diversification ratio, $D(P)$, of a given portfolio P as follows:

$$(16) \qquad D(P) = \frac{\mathbf{w}' \cdot \boldsymbol{\Psi}}{\mathbf{w}' \cdot \boldsymbol{\Omega} \cdot \mathbf{w}}$$

They subsequently define the Most Diversified Portfolio as the portfolio that maximizes the diversification ratio under a set of linear constraints. We use full investment, non-negativity and maximal weight constraints (10%) to define the MDP. The maximal weight constrained is also warranted here as its absence leads to extreme levels of concentration akin to those of the (maximally unconstrained) GMVP. MDP weights are calculated using the optimization program in (17) below.

$$(17) \qquad \max: \quad D(P) = \frac{\mathbf{w}' \cdot \mathbf{\Psi}}{\mathbf{w}' \cdot \mathbf{\Omega} \cdot \mathbf{w}}$$

$$s.t \qquad \sum_{i=1}^{n} w_i = 1$$

$$w_i \geq 0$$

$$w_i \leq 0.1$$

#### 4.2.1.5 EQUALLY WEIGHTED PORTFOLIO (EWP)

The equally weighted portfolio is computationally trivial but, in keeping with the formal exposition provided for each preceding portfolio philosophy, is also defined here. For an *n* asset portfolio the portfolio weight for any given asset *i* within the EWP is:

$$(18) \qquad w_i = \frac{1}{n}$$

The equally weighted portfolio is fully invested and employs non-negative weights by construction.

### 4.2.2 PERFORMANCE GENERATION AND THE BACK-TESTING PROCESS

This section provides a comprehensive overview of portfolio performance generation through the back-testing process. Ultimately our aim is to provide an analysis of the various candidate portfolios discussed in the last section and contrast their performance with each other and a market determined benchmark-we implement this process accordingly.

Let $t_0$ be the date on which the initial portfolios are to be formed, $\delta_{t_i}$ portfolio the *i'th* portfolio rebalancing date, $\Delta_{t_i}$ re-indexing date *i* (i.e. the point at which benchmark constituents are revised), and *h* the number of observations employed to form estimates of the covariance matrix. Further define **p** and **mc** as matrices containing price and market capitalization panel data respectively for all stocks within the investment universe (i.e. the constituents of the relevant equity index in Table 1). We define the initial information set at $t_0$ ; $\mathrm{I}_{initial,t_0}$ as the set $\{\boldsymbol{p}_{(t_0,t_0-h)}, \boldsymbol{mc}_{(t_0,t_0-h)}\}$. The information set represents price and market capitalization data for all *n* assets in the investment universe from the start date over the required lag necessary to ensure each asset has *h* observations for use in estimation.

We seek to conduct our analysis based on a market capitalization weighted benchmark and hence we restrict ourselves to a smaller subset of *m* assets with the highest market capitalization. This smaller subset of *m* assets serves as the market benchmark's constituents and the effective universe available to our candidate portfolios. Assuming $\mathrm{I}_{initial,t_0}$, we need to reduce the number of assets to *m* using the market value criterion. However we find it prudent to first apply a thin trading filter at this stage of benchmark formation. In particular, certain market related benchmarks in some equity universes (especially before the turn of the millennium) sometimes include assets that do not trade at all between rebalancing periods over the given lag *h.* This gives rise to zero covariances, introducing singularity in the covariance matrix and compromising the integrity of the weight estimation process for approaches that rely on matrix inversion to compute portfolio weights. Consequently we specify a minimum trading frequency $\gamma$ such that all candidate assets for benchmark inclusion trade with at least this frequency between rebalancing dates until index constituents are next revised. Once this thin trading filter is applied, we use market values as of $t_0$ to enact the market value criterion in order to reduce the number of securities in the information set from *n* to the top *m* securities by market capitalization.

Once the *m* asset subset is established we calculate log returns for each of the assets within the subset to create the terminal information set for portfolio formation. We can define the updated (*m* asset) information set $\mathrm{I}_{terminal,t_0} = \{\boldsymbol{r}_{(\boldsymbol{t_0},\boldsymbol{t_0}-(\boldsymbol{h}-\boldsymbol{1}))}, \boldsymbol{mc}_{(\boldsymbol{t_0},\boldsymbol{t_0}-(\boldsymbol{h}-\boldsymbol{1}))}\}$ where **p** is replaced by **r,** the matrix of log returns. Subsequently we calculate the covariance matrix $\boldsymbol{\Omega}_{t_0}$ based on the returns of the assets in $\mathrm{I}_{terminal,t_0}$. We compute the EWP weights at this point and use $\boldsymbol{\Omega}_{t_0}$ to calculate optimal weights for the NRPP, GMVP, MDP and RPP. In addition to the heuristic and risk based portfolios of interest we also calculate the market value weighted benchmark portfolio (MKT.) MKT is a fully invested market value weighted portfolio based on the *m* assets in $\mathrm{I}_{terminal,t_0}$ and uses the most current market values as of time step $t_0$. Formation of portfolio weights yields the full set of weights $\boldsymbol{w}_{t_0} = \{\boldsymbol{w}_{rpp,t_0}, \boldsymbol{w}_{gmvp,t_0}, \boldsymbol{w}_{mdp,t_0}, \boldsymbol{w}_{nrpp,t_0}, \boldsymbol{w}_{ewp,t_0}, \boldsymbol{w}_{mkt,t_0}\}$ and provides us with our portfolio holdings for $t_{(0)} \rightarrow t_{(0+\tau)}$; the time period until the portfolio is next rebalanced.

Rebalancing the portfolio proceeds intuitively; we revise the initial information set to include the latest price and market capitalization data i.e. $\mathrm{I}_{initial,t_{(0+\tau)}} = \{\boldsymbol{p}_{(\boldsymbol{t_{0+\tau}},\boldsymbol{t_{0+\tau}}-\boldsymbol{h})}, \boldsymbol{mc}_{(\boldsymbol{t_{0+\tau}},\boldsymbol{t_{0+\tau}}-\boldsymbol{h})}\}$. Effectively, as time progresses, we implement a rolling window of observations that incorporates the most recent information available for use in estimation whilst maintaining its size at *h.* However, assuming that $\Delta_{t_{i+1}} - \Delta_{t_i}$ (the time between index constituent revision dates) is greater than $\delta_{t_{i+1}} - \delta_{t_i}$ (the number of trading days between portfolio rebalancing dates), re-indexing only occurs after the portfolio has been rebalanced $\Delta_{t_{i+1}} - \Delta_{t_i}/\delta_{t_{i+1}} - \delta_{t_i}$ times. Naturally the condition $\Delta_{t_{i+1}} - \Delta_{t_i} = c \cdot (\delta_{t_{i+1}} - \delta_{t_i})\ for\ \forall c \in \mathbb{N}$ must be enforced in order to ensure that rebalancing and re-indexing dates coincide. If $c > 1$ then rebalancing and market value based re-indexing don't occur at the same frequency. However, for the purposes of consistency, we still wish to keep the benchmark reference portfolio, MKT, current based on new market information and hence we continue to rebalance it every $\delta_{t_i}$ using the most recent asset market values in $\mathrm{I}_{terminal,t_i}$ i.e. within the reduced *m* asset universe.

Re-indexing date $\Delta_{t_i}$ must coincide with some $\delta_{t_i}$ as discussed in the last paragraph and we always ensure that re-indexing occurs before portfolio rebalancing. On re-indexing dates we revise the *m* asset universe to reflect the latest market values as at $t_i$ by considering $\mathrm{I}_{initial,t_i}$; the information set for all *n* assets at $t_i$. As usual we first apply the thin trading filter based on minimum trading frequency $\gamma$ before forming a new *m* asset portfolio based on the market value criterion. Once the benchmark constituents are revised we conduct portfolio optimization based on the new *m* asset sub-universe and generate the set of weights $\boldsymbol{w}_{t_i}$.

Portfolio rebalancing and re-indexing are conducted on a regular basis to generate a comprehensive weight database. Over the cycle we form comprehensive weight and return matrices for performance generation. In particular we define overall weights over the entire sample period as $\boldsymbol{w} = \{\boldsymbol{w}_{rpp}, \boldsymbol{w}_{gmvp}, \boldsymbol{w}_{mdp}, \boldsymbol{w}_{nrpp}, \boldsymbol{w}_{ewp}, \boldsymbol{w}_{mkt}\}$ where any particular portfolio weight matrix is hence referred to as $\boldsymbol{w}_{port}$ . Additionally if we let $\boldsymbol{r}_{m-assets}$ denote the overall return matrix for the assets within the *m* asset benchmark and $\boldsymbol{r}_{m-assets_{\Delta_{(i-1)} \to \Delta_{(i)}}}$ represent the matrix of individual asset returns over the period between indexing dates then the final weight matrix for a given portfolio style and the overall return matrix for the *m* asset universe are simply the vertical concatenation of intra indexing period weight or return estimate matrices as in (19.)

$$(19) \qquad \mathbf{w}_{port} = \begin{bmatrix} \mathbf{w}_{port_{\Delta_{(i-1)} \to \Delta_i}} \\ \mathbf{w}_{port_{\Delta_{(i-2)} \to \Delta_{(i-1)}}} \\ . \\ . \\ . \\ . \\ \mathbf{w}_{port_{\Delta_{(i-k)} \to \Delta_{(i-(k-1))}}} \end{bmatrix} \qquad \mathbf{r}_{m-assets} = \begin{bmatrix} \mathbf{r}_{m-assets_{\Delta_{(i-1)} \to \Delta_i}} \\ \mathbf{r}_{m-assets_{\Delta_{(i-2)} \to \Delta_{(i-1)}}} \\ . \\ . \\ . \\ . \\ \mathbf{r}_{m-assets_{\Delta_{(i-k)} \to \Delta_{(i-(k-1))}}} \end{bmatrix}$$

Defining $\boldsymbol{w}_{port}$ and $\boldsymbol{r}_{m-assets}$ in this manner enables us to account for possible revisions to the investment universe on re-indexing dates and ensure weight-asset return correspondence when calculating $\boldsymbol{r}$ , the set of out-of-sample portfolio returns. We define the set of portfolio returns as $\boldsymbol{r} = \{\boldsymbol{r}_{rpp}, \boldsymbol{r}_{gmvp}, \boldsymbol{r}_{mdp}, \boldsymbol{r}_{nrpp}, \boldsymbol{r}_{ewp}, \boldsymbol{r}_{mkt}\}$ where any given $\boldsymbol{r}_{port}$ is defined as in (20).

$$(20) \qquad \mathbf{r}_{port} = \begin{bmatrix} \mathbf{w}_{port_{t_i}} \cdot \mathbf{r}'_{m-assets_{t_i}} \\ \mathbf{w}_{port_{t_{(i-1)}}} \cdot \mathbf{r}'_{m-assets_{t_{(i-1)}}} \\ . \\ . \\ . \\ . \\ \mathbf{w}_{port_{t_{(i-k)}}} \cdot \mathbf{r}'_{m-assets_{t_{(i-k)}}} \end{bmatrix}$$

Ultimately sets **w** and **r** represent the key data for this analysis. Having generated weight and return estimates for the different portfolios we can proceed to analyse different aspects of portfolio performance along several different dimensions including risk, total returns, risk adjusted returns, portfolio concentration, drawdowns, turnover and beta profiles. We describe these performance analysis techniques in Section 4.2.3. Before commencing that discussion however we provide values for the key parameters employed in the back testing process as described in this section in Table 2.

TABLE 2: BACKTESTING BASE SPECIFICATION

| Parameter | Symbol | Value |
|---|---|---|
| Benchmark Size | $m$ | 40 Assets |
| Rolling Sample Size | $h$ | 126 Days |
| Thin Trading Filter | $\gamma$ | 2 Times/Period |
| Rebalancing Frequency | $\delta_{t_{i+1}} - \delta_{t_i}$ | 21 Days |
| RPP Stopping Tolerance | $\varepsilon$ | 1.0E-15 |
| Index Constituent Revision Frequency | $\Delta_{t_{i+1}} - \Delta_{t_i}$ | 252 Days |

Our choice of index and re-indexing/constituent revision frequency allows us to simulate a Top 40 style benchmark with constituents revised annually. We use 6 months of observations to form portfolios that are rebalanced on a monthly basis and a stopping tolerance of $10 \times 10^{-15}$ (chosen here since testing demonstrated that TRC's are generally well equalized for this value) for RPP weight calculations using Newton's method. Finally we impose a thin trading filter of two for benchmark inclusion in order to avoid problems with singularity without being too heavy handed in excluding assets based on this criterion. Deviations from this specification are disclosed as and when they occur.

### 4.2.3 PERFORMANCE AND STATISTICAL ANALYSIS

Assuming the availability of portfolio weights (**w**) and return series (**r**) once the back-testing process is complete, we conduct performance analysis by considering portfolio risk, cumulative returns, risk-adjusted returns, concentration, drawdowns, turnover and beta profiles. In addition we conduct statistical analysis at a later stage in order to examine the significance of certain relationships and we also outline these tests in this section. We begin with risk and return.

#### 4.2.3.1 RISK AND RETURN

Given that the elements of set **r** are all log returns we calculate cumulative performance by merely adding the contents of each $\boldsymbol{r}_{port}$ vector as in (21).

$$(21) \qquad Cum\left(\mathbf{r}_{port_{(n:n+h)}}\right) = \sum_{i=n}^{i=n+h} \mathbf{r}_{port_{i,1}}$$

Here $h$, the interval between the start and the end of the period, is variable. For instance if we set $h$=21 then we calculate monthly returns and if $h$ is equal to the number of rows in $\boldsymbol{r}_{port}$ we compute returns over the entire sample period. Hence we employ this definition to describe returns over any given frequency $h$ although we will primarily restrict ourselves to monthly frequencies and the entire sample period.

We define portfolio volatility in a similarly flexible manner to accommodate varying time horizons. We formally define portfolio volatility $\sigma_{port_{(n:n+h)}}$ over the period $n$ to $n+h$ in (22). It is also worth noting at this stage that, where risk information is depicted alone or in conjunction with return figures, we annualize the figures as appropriate for standardized representation.

$$(22) \qquad \sigma_{port_{(n:n+h)}} = \sqrt{\frac{1}{(h-1)}\sum_{i=n}^{n+h}\left(\mathbf{r}_{i,1} - E\left(\mathbf{r}_{port_{(n:h,1)}}\right)\right)^2}$$

Finally we consider risk-adjusted performance by computing the ratio of average portfolio return over a certain period to average volatility over the same interval. Naturally this stops short of providing consideration based on excess returns (i.e. in excess of a risk free asset) akin to the Sharpe ratio. However our primary intent is to compare candidate portfolio performance to the *m* asset benchmark and obtaining complete and reliable time series data for an appropriate risk free asset across the multiple equity universes considered proved impractical. Consequently we posit that the lack of comparison to a risk free asset is not only enforced but also not exceptionally material to our consideration of the reward to an investor's assumption of risk as portfolio risk adjusted rank performance is certainly captured by this metric; notwithstanding the absence of a comparison to the risk free asset. In essence, ranking performance using this metric will be analogous to that computed using Sharpe ratios and we formally define it as the return to volatility ratio (RVR) below.

$$(22) \qquad RVR = \frac{\mu\left(\mathbf{r}_{port_{(n:n+h)}}\right)}{\sigma_{port_{(n:n+h)}}}$$

The RVR measure represented above is computed based on annualized risk and mean return figures.

#### 4.2.3.2 CONCENTRATION AND RISK CONTRIBUTION

A consideration of portfolio concentration and risk contribution is fundamental to any discussion of risk parity given the stated objectives of the portfolio approach. Consequently we present two measures of portfolio concentration and a measure of risk contribution in this section.

We adopt the approach of Kruger and van Rensburg (2008) and measure portfolio concentration using Effective Shares (EFS) and the Richard Roll measure of concentration (RRC). The EFS measure is perhaps the more intuitive of these two. Given an *n* asset portfolio and a set of associated weights $\boldsymbol{w}_{port_t}$ at a particular time step *t*, the EFS measure is defined as in equation (24) below and provides the effective number of shares for $\boldsymbol{w}_{port_t}$ taking into account its degree of concentration.

$$(24) \qquad EFS = \frac{1}{\sum_{i=1}^{n} w^2_{port_t,i}}$$

An equally weighted portfolio represents the least possible concentration attainable and hence we expect an equally weighted portfolio to have the same number of effective shares as the applicable investment universe i.e. *n.* Otherwise we expect that portfolios that deviate more from equal weights will have a lower number of effective shares than those with weights that are relatively closer to equal weighting.

The second measure of portfolio concentration, the RRC, calculates portfolio concentration as (25) and provides a number within the interval [0,1] with the lower bound representing concentration equivalent to the equally weighted portfolio and the upper bound representing a 100% investment in a single asset within the applicable *n* portfolio investment universe. In calculating both the EFS and RRC we set n=m i.e. we consider portfolio concentration relative to the number of assets in the *m* asset benchmark.

$$(25) \qquad RRC = \frac{n}{n-1} \cdot \left( \sum_{i=1}^{n} w^2{}_i - \frac{1}{n} \right)$$

Finally we consider the analysis of portfolio constituent risk contributions. We assess portfolio risk contributions by calculating ex post total risk contributions (TRC's) for each asset and subsequently calculating the gini coefficient for the TRCs of portfolio constituents over the appropriate sample period. Formally, if $\mathbf{\Omega}_{t_{(i-1)} \to t_i}$ is the covariance matrix of the *m* benchmark constituents over the time step $t_{(i-1)} \to t_i$ and the total risk contribution of asset *i* at $t_i$ is as in equation (5) and is taken with respect to $\mathbf{\Omega}_{t_{(i-1)} \to t_i}$, then we assess the degree of equality in asset risk contributions by considering the measure $\tilde{G}_{port,t_1}$ in (26) which is the gini coefficient of the Lorenz curve that plots cumulative TRCs against cumulative asset index numbers for the relevant $\boldsymbol{w}_{port,t_i}$. Gini coefficients lie in the interval [0,1], where the lower bound signifies perfect equality and the upper bound maximal inequality.

$$(26) \qquad \tilde{G}_{port,t_i} = \frac{2}{m} \cdot \sum_{i=1}^{m} i\left(TRC_i - T\tilde{R}C\right)$$

#### 4.2.3.3 TURNOVER AND DRAWDOWNS

The practical costs and risk of prolonged underperformance of any given portfolio approach are material considerations for any investment manager. Consequently we also present measures for assessing these two characteristics. To begin with we define portfolio turnover. The degree of turnover for a given portfolio at rebalancing date $t_i$ is $\zeta_{port,t_i}$ and is defined as in equation (27.) Essentially, we calculate portfolio turnover by considering the change in weights for each asset *j* within the portfolio between rebalancing dates and summing up the total weight revision across all *j* assets.

$$(27) \qquad \zeta_{port,t_i} = \sum_{j=1}^{n} \left( w_{j,t_i} - w_{j,t_{(i-1)}} \right) \quad for \quad w_{j,t_i} \in \mathbf{w}_{port,t_i}, \quad w_{j,t_{(i-1)}} \in \mathbf{w}_{port,t_{(i-1)}}$$

Naturally $\zeta_{port,t_i}$ has its limitations as a measure of turnover as it does not account for differences in asset prices that may render the transaction costs associated with the same $\zeta_{port,t_i}$ figure between portfolios materially different in actual monetary value terms. However it does provide a good sense of the extent to which a portfolio requires constant revision relative to its peers and hence provides a sense of the degree of churn required to adhere to a given portfolio approach.

In addition to turnover we also assess the portfolio's susceptibility to drawdowns in out of sample performance. A drawdown is defined as at least two instances of negative return for consecutive time steps. We calculate drawdowns by cumulating consecutive negative returns until a zero or positive return is realized and recording the number of days that any given drawdown lasts. Drawdowns are calculated based on daily, weekly and monthly return frequencies.

### 4.2.3.4 BETA PROFILES

As discussed in section 2.5.5, arguably the most significant theoretical framework to emerge to explain the performance of the RPP in other studies has been leverage aversion theory. In particular Frazzini and Pedersen (2010) suggested that the RPP's generally positive empirical performance may be a consequence of the fact that higher beta assets have lower risk adjusted returns than lower beta assets. We seek to assess the validity of this theory by computing 'beta profiles' for all candidate portfolios over longer time scales.

At each re-indexing point $\Delta_i$ the back-testing process provides $\boldsymbol{r}_{mkt,\Delta_{(i-1)}\rightarrow\Delta_i}$ and the set of weights $\boldsymbol{w}_{\Delta_{(i-1)}\rightarrow\Delta_i}$. We calculate the beta of every asset return series $\boldsymbol{r}_{i,\Delta_{(i-1)}\rightarrow\Delta_{(i)}} \in \boldsymbol{r}_{m-assets_{\Delta_{(i-1)}\rightarrow\Delta_{(i)}}}$ with $\boldsymbol{r}_{mkt,\Delta_{(i-1)}\rightarrow\Delta_i}$. This provides us with $\boldsymbol{\beta}_{\Delta_{(i-1)}\rightarrow\Delta_i}$, the vector of betas for the $m$ asset universe with respect to the benchmark portfolio as of $\Delta_i$. Consequently we first average $\boldsymbol{w}_{\Delta_{(i-1)}\rightarrow\Delta_i}$ and cumulate $\boldsymbol{r}_{m-assets_{\Delta_{(i-1)}\rightarrow\Delta_{(i)}}}$ to find $\mu_{w,\Delta_i}$ and $Cum_{r,\Delta_i}$; the average weights (in each candidate portfolio) and matrix of cumulative returns for each asset $i$ respectively. Then we sort $\mu_{w,\Delta_i}$ in order of ascending weights to obtain the hierarchy of average asset weights within each portfolio. The sorting order for $\mu_{w,\Delta_i}$ is also applied to $Cum_{r,\Delta_i}$ and $\boldsymbol{\beta}_{\Delta_{(i-1)}\rightarrow\Delta_i}$ so that elements with identical co-ordinates within the sorted average weight, average return and average beta vectors all correspond to the same asset.

The beta profile is formed by combining the average weight, cumulative return and beta estimates calculated at each $\Delta_i$. We average beta and weight estimates across inter-temporal estimates of $\boldsymbol{\beta}_{\Delta_{(i-1)}\rightarrow\Delta_i}$ and $\mu_{w,\Delta_i}$ whilst return estimates are cumulated over all $Cum_{r,\Delta_i}$. The upshot of this process is that we obtain representative weight, beta and cumulative return figures for the *i*th largest asset in each of the candidate portfolios under consideration for the overall sample period. This allows us to gauge what weight, beta and total return is associated with the *i*th largest asset in each portfolio. Hence the beta profile for the relevant portfolio is essentially the set $\{\boldsymbol{\mu_w}, \boldsymbol{\mu_\beta}, \boldsymbol{Cum_r}\}$ and we present this information by plotting average betas and cumulative returns for each representative asset against its average weight on the Cartesian plane.

### 4.2.3.5 STATISTICAL ANALYSIS

The main thrust of our statistical analysis is geared towards considering structural breaks in volatility and assessing correlation and causation across some key time series variables. Consequently we briefly discuss the various statistical tests used to conduct these analyses, namely the Brown-Forsythe and Levene's tests, Spearman rank and Pearson correlation and the Granger causality test.

Our analysis of structural breaks is used to motivate the selection of subsamples from our full sample. Different market regimes are typically characterized by varying levels of volatility so testing to see if such heterogeneity is present in our benchmark return series is necessary to motivate our sample compartmentalization. Consequently we use events in global financial market history to divide the return series $\boldsymbol{r_{mkt}}$ into $k$ smaller subsets $\{\boldsymbol{r}_{mkt,1}, \boldsymbol{r}_{mkt,2} \dots \dots r_{mkt,k}\}$. We then conduct tests for homogeneity of variance between these $k$ groups of returns to test the theory that these groupings do in fact represent different market regimes as evinced by differences in volatility. The Brown-Forsythe test is one such test for homogeneity of variance between groups. The test is based on a one way ANOVA and uses transformations of each of the $k$ return sub-series to calculate the F statistic. More precisely, the transformation conducted involves calculating the absolute value of each return observation's deviation from the series' median to form a new set of groups $\boldsymbol{z_{mkt}} =$

$\{\boldsymbol{z}_{mkt,1}, \boldsymbol{z}_{mkt,2} \dots \dots \boldsymbol{z}_{mkt,k}\}$ which are used to calculate the F statistic in (28.) Within this characterization, $N$ is the total number of observations across all groups, p is the number of groups and $n_j$ is the number of observations in group $j$.

$$(28) \qquad F = \frac{(N-p) \cdot \sum_{i=1}^{p} n_j \left(\mu_{z_j} - \mu_z\right)^2}{(p-1) \cdot \sum_{j=1}^{p} \sum_{i=1}^{n_j} n_j \left(z_{i,j} - \mu_{z_j}\right)^2}$$

The Brown-Forsythe test statistic has (*N-p, p-1)* degrees of freedom and is reported along with its critical and p-values. In addition we also report the results of Levene's test for homogeneity of variance. This is similar to the Brown-Forsythe test except that it calculates the $z_{i,j}$ by considering deviations from the mean of the series as opposed to the median. We choose to provide prominence to the Brown-Forsythe test however as it is relatively more robust to possible instances of non-normality within datasets.

We assess correlation between variables using Pearson and Spearman rank correlation. Given two sample series $X$ and $Y$ of equal length, the sample Pearson correlation $r$ of the two series is estimated as in (29) where $\tilde{x}$ and $\tilde{y}$ represent the means of the two variables.

$$(29) \qquad r = \frac{\sum_{i=1}^{n} (x_i - \tilde{x})(y_i - \tilde{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \tilde{x})^2} \sqrt{\sum_{i=1}^{n} (y_i - \tilde{y})^2}}$$

Spearman rank uses the same formulation of $r$ as Pearson correlation but is conducted using the *ranks* of the observations within $X$ and $Y$ given their relative magnitudes instead of using the values of the actual observations themselves. We report both Pearson and Spearman rank correlation statistics along with their p-values.

Demonstrating causation naturally requires a higher burden of proof than showing the existence of correlation through the two tests described above. In order to determine possible causal links between variables we employ the Granger Causality test. Given $k$ time series variables $\{y_1, y_2, \dots, y_k\}$, we test Granger causality by estimating a Vector Autoregressive Regressive (VAR) model with the $y_k$ and then using the structure of the VAR model to test the forecasting ability of $y_{i,}\ for\ i \neq j$ (over $p$ lags) on $y_j$ .

More completely, we first estimate the number of lags $p$ to employ by testing different lags to establish which lag selections minimize Akaike, Hannan Quinn and Schwarz information criteria and Final Prediction Error (FPE). Then we proceed to estimate the VAR *(p)* model as indicated in (30) below. Here **C** is a vector of constants, **u** is a vector of zero mean errors and $\boldsymbol{\delta_i}$ is a square matrix of coefficient estimates at the appropriate lag.

$$(30) \qquad Y_{(t)} = \mathbf{C} + \delta_{(1)}\mathbf{Y}_{(\mathbf{t-1})} + \delta_{(2)}\mathbf{Y}_{(\mathbf{t-2})} + \dots \delta_p \mathbf{Y}_{(\mathbf{t-p})} + u$$

$$for$$

$$\mathbf{Y}_{(\mathbf{t})} = \left[y_{(1,t)}, y_{(1,t)} \dots, y_{(k,t)}\right]'$$

$$\boldsymbol{\delta}_{\mathbf{i}} = \begin{bmatrix} \delta^i_{1,1} & \delta^i_{1,2} & \dots & \delta^i_{1,k} \\ \delta^i_{2,1} & \delta^i_{2,2} & \dots & \delta^i_{2,k} \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ \delta^i_{k,1} & \delta^i_{k,2} & \dots & \delta^i_{k,k} \end{bmatrix}$$

$$\mathbf{C} = \left[c_{(1)}, c_{(1)} \dots, c_{(k)}\right]'$$

$$\mathbf{u} = \left[u_{(1)}, u_{(1)} \dots, u_{(k)}\right]'$$

Having estimated the VAR $(p)$ model we conduct the Granger Causality test which essentially tests if the distribution of $y_{i,t} \mid \{y_{1,(t-1),(t-2),..(t-p)}, y_{2,(t-1),(t-2),..(t-p)} \cdots y_{n,(t-1),(t-2),..(t-p)}\}$ is different from $y_{i,t} \mid \{y_{1,(t-1),(t-2),..(t-p)}, y_{2,(t-1),(t-2),..(t-p)} \cdots y_{(n-1),(t-1),(t-2),..(t-p)}\}$; the distribution of $y_{i,t}$ conditioned on an information set that excludes $y_n$. If excluding $y_n$ does not significantly alter the distribution of $y_{i,t}$ then we conclude that the time series variable $y_n$ does not Granger cause $y_{i,t}$. We present the results of the Granger Causality test, reporting coefficients, critical and p-values as appropriate.

# SECTION 5: RESULTS

This section discusses our core findings. We roughly compartmentalize our findings into two parts; the first of which examines the applicability of findings elsewhere to the domestic equity market and the latter part considering what the impact of increased market concentration is likely to be on RPP performance. We focus on presenting results for the ALSI investment universe in this section with results from international investment universes in the Appendices.

## 5.1 PERFORMANCE: RISK AND RETURN

We present results for the ALSI investment universe dataset graphically overleaf with performance figures available in Appendix 1 along with descriptive statistics for each candidate portfolio. Note that actual performance generation begins at the end of July 1995; 126 trading days (6 months) after the sample start date.

We find that the RPP lags MDP and GMVP formulations in terms of cumulative returns over the sample period 2 January 1995-29 November 2013 within the core ALSI dataset. The GMVP performs best followed by the MDP, RPP, NRPP, MKT and EWP respectively. Hence all but one of the candidate portfolios outperform the market weighted benchmark over the period; suggesting that there has been historical value in having been invested in heuristic and risk based portfolios on the ALSI over the last one and a half decades. An examination of the spread between the GMVP and MDP suggests that they are close analogues of each other, moving more in tandem with each other than competing portfolios. This is perhaps unsurprising given that each of these techniques is roughly premised on portfolio diversification; the MDP through its maximization of the diversification ratio and the GMVP through its minimization of portfolio variance. By contrast the NRPP seems to be a closer analogue of the EWP as opposed to the RPP for much of the sample period; suggesting that asset covariance information is priced and is a significant determinant of performance within this market context.

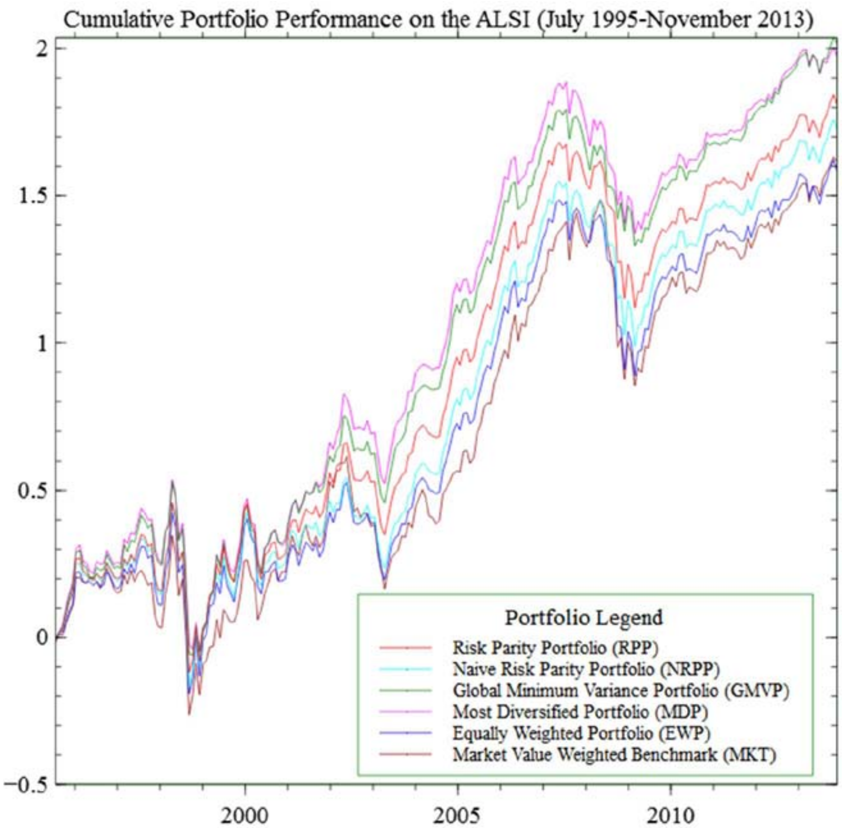



FIGURE 1: ALSI INVESTMENT UNIVERSE CUMULATIVE HEURISTIC AND RISK BASED PORTFOLIO RETURN PERFORMANCE

Relative portfolio rankings in terms of cumulative returns seem to vary across equity indices however. In particular the RPP ranks second and first in our S&P100 and TOPIX100 datasets respectively, the former's results underlining previous findings within the American financial market context. Performance across the other nine datasets is more mixed and is presented in Appendix 1.1. The mixed fortunes in equity universes outside of our USA market proxy raises the prospect that RPP performance relative to other common allocation approaches in this particular market context may not be as readily extrapolated to others.

A consideration of portfolio volatility over the same period suggests that the various candidate portfolios are generally less volatile than ALSI. The GMVP and MDP again provide the best volatility levels with near identical risk figures. The RPP and NRPP offer the next best volatility levels (also reasonably closely matched) and the EWP comes in just ahead of the market portfolio. These findings are roughly in line with Maillard et al's (2008) contention that the natural hierarchy of volatilities between the RPP, GMVP and EWP is $\sigma_{GMVP} < \sigma_{RPP} < \sigma_{EWP}$. Maillard et al do not consider the other variants examined here or provide an analysis across international equity universes but our findings intimate that this hierarchy may be extended to state that portfolios based on traditional portfolio diversification (GMVP,MDP) achieve the lowest volatility followed by equal risk contributions (RPP,NRPP) and equal weight portfolio variants. This order of volatility ranking is consistently upheld across the equity indices considered.

TABLE 3: VOLATILITY AND RETURN/VOLATILITY RATIOS (ALSI INVESTMENT UNIVERSE) JULY 1995-NOVEMBER 2013

| Portfolio | Volatility | RVR |
|---|---|---|
| EWP | 18.53% | 0.45 |
| GMVP | 15.67% | 0.67 |
| MDP | 15.87% | 0.65 |
| MKT | 20.66% | 0.41 |
| NRPP | 17.82% | 0.51 |
| RPP | 17.14% | 0.55 |

The Global Minimum Variance (GMVP) and Most Diversified (MDP) portfolios provide the highest ratio of average return to volatility (RVR) over the period followed by risk parity implementations and the Equally Weighted Portfolio (EWP).

Consistent with their superior performance in terms of both risk and return, the MDP and GMVP portfolios exhibit the highest average return per unit of risk (RVR) amongst the candidate approaches, significantly superior to the RPP which in turn leads its naïve counterpart. Generally the RVR performance is consistent with the volatility rankings. Ranking consistency in terms of cumulative returns and volatilities is strongly suggestive of the fact that portfolios that emphasize portfolio diversification relatively more are likelier to perform better in terms of both risk and return than those that don't-at least in the domestic financial market.

These findings are perhaps most notable in that they imply strong dominance of the more risk-conservative portfolio approaches over the market benchmark and other competing portfolios. In particular we find that the GMVP and MDP outperform the other candidate portfolios in 10 of the 12 equity universes considered in this study. This is a continuation of the low volatility anomaly documented by in other studies and further calls into question the notion of commensurate compensation for the assumption of risk as put forth by the CAPM.

## 5.2 STRUCTURAL SHIFTS

We are interested in exploring if performance across the entire sample period is reflected across subsamples. Pursuant to this we divide the sample period in order to reflect significant events in global market history as outlined in the table overleaf. Here we select 3 subsamples, the first of which spans a period that roughly captures the impact of the Asian and Russian financial crises. Subsequent subsamples relate to the Sub-prime mortgage and Eurozone crises. These subsamples are contrasted to each other and the overall sample period used to consider cumulative returns in Section 5.1 in order to determine if structural breaks in volatility are present.

TABLE 4: SUB-SAMPLES EMPLOYED FOR INTRA SAMPLE PERIOD TESTING

| Crisis | Period |
|---|---|
| Asian Crisis/Russian Crisis | 1 July 1997-31 December 1998 |
| Subprime Mortgage Crisis | 1 March 2007-31 December 2009 |
| Eurozone Crisis | 1 April 2010-31 December 2012 |

Appendix 2.3 provides the results of tests for homogeneity of variance conducted to verify that these subsamples represent different market regimes. We find that the extent of the differences in volatility present within the subsamples is highly significant (even at the 1% confidence level) and hence proceed to evaluate candidate portfolio performance within these different periods. It is worth noting however that, although the first two market environments did induce

higher volatility than average on the ALSI, the Eurozone crisis period does not have the same impact, in fact average volatility during this period was *lower* than that of the overall sample period (see Appendix 2.2 for sub-sample descriptive statistics)

We find that no portfolio approach exhibits entirely consistent ranking performance across the different subsamples. Tables 5-7 exhibit cumulative return, volatility and return/volatility information across the different subsamples.

TABLE 5: ASIAN/RUSSIAN CRISIS SUBSAMPLE RESULTS (JULY 1997-DECEMBER 1998)

| Portfolio | Cumulative Return | Volatility | RVR |
|---|---|---|---|
| EWP | -37.74% | 27.15% | -0.89 |
| GMVP | -52.24% | 24.77% | -1.36 |
| MDP | -52.22% | 24.64% | -1.36 |
| MKT | -39.03% | 29.69% | -0.84 |
| NRPP | -39.85% | 26.99% | -0.95 |
| RPP | -41.50% | 25.54% | -1.04 |

The Equally Weighted Portfolio (EWP) surprisingly provides the highest degree of protection during the Asian/Russian crisis sub-sample with the Global Minimum Variance (GMVP) and Most Diversified (MDP) portfolios significantly underperforming.

The EWP performs best within the first subsample in terms of return, indeed it is the only portfolio that outperforms the market on these terms. The GMVP and MDP are the worst performers within the subsample and significantly lag their risk parity counterparts. The RVR is not meaningful in this case as negative returns are present. However the hierarchy of volatilities is still consistent with Maillard et al's (2008) assertion and all candidate portfolios are less volatile than the benchmark over the period.

TABLE 6: SUBPRIME CRISIS RESULTS (MARCH 2007-DECEMBER 2009)

| Portfolio | Cumulative Return | Volatility | RVR |
|---|---|---|---|
| EWP | -11.76% | 25.27% | -0.16 |
| GMVP | -2.82% | 19.59% | -0.05 |
| MDP | -4.89% | 19.79% | -0.08 |
| MKT | -9.71% | 29.35% | -0.11 |
| NRPP | -9.22% | 23.79% | -0.13 |
| RPP | -13.20% | 23.08% | -0.19 |

The Global Minimum Variance (GMVP) and Most Diversified (MDP) portfolios lead the way in loss minimization over the subprime crisis period.

The Subprime crisis presents an entirely different hierarchy of portfolios in terms of performance. The EWP is only surpassed in its degree of underperformance by the RPP whilst the GMVP and MDP perform relatively strongly. Once more all portfolios incur losses (albeit of lesser magnitude) over this period rendering the RVR measure meaningless. However the hierarchy of portfolio volatility rankings is once again upheld.

TABLE 7: EUROZONE CRISIS RESULTS (APRIL 2010 -DECEMBER 2012)

| Portfolio | Cumulative Return | Volatility | RVR |
|---|---|---|---|
| EWP | 31.76% | 15.88% | 0.70 |
| GMVP | 43.54% | 12.27% | 1.25 |
| MDP | 44.61% | 12.27% | 1.28 |
| MKT | 29.45% | 16.02% | 0.65 |
| NRPP | 35.08% | 15.19% | 0.81 |
| RPP | 35.39% | 14.36% | 0.87 |

The Global Minimum Variance (GMVP) and Most Diversified (MDP) portfolios again provide the best return performance the Eurozone crisis period.

The GMVP and MDP perform best within our Eurozone crisis sample period as well although they do interchange rankings compared to the previous subsample. The risk parity techniques provide the next best return performance whilst the EWP edges the market in performance. The volatility rankings again adhere to the expected hierarchy.

Consequently performance within subsamples of interest on the ALSI suggests our candidate heuristic and risk based portfolios may generally be expected to perform better than the market weighted benchmark-but not without exception. Additionally the lack of risk parity dominance across the sub-samples and within the dataset as a whole suggests that risk parity implementations may not be the best options for an equity portfolio style that emphasizes total returns. Naturally this is distinct from considerations of leveraging raised by authors such as Ruban and Melas (2010, 2011) as our consideration suffers no impedance due to overweighting fixed income investments. Additionally, given Maillard et al's (2008) hierarchy of volatilities we expect that risk parity will generally be dominated on RVR terms (and consequently on Sharpe ratio terms) as well given that returns to the RPP tend to lag those of less volatile risk based portfolio such as the GMVP and MDP. Comprehensive portfolio performance figures for all three subsamples are included in Appendix 2.1.

## 5.3 TURNOVER AND DRAWDOWNS

The RPP's average performance in terms of risk and return does not extend to our consideration of turnover. Unsurprisingly, the EWP experiences the least amount of portfolio churn as weights are not revised based on new return or covariance information. Nonetheless it does experience the baseline turnover required at annual re-indexing points as benchmark constituents are revised.

TABLE 8: AVERAGE TURNOVER (ALSI INVESTMENT UNIVERSE) FOR OVERALL SAMPLE AND SUB-SAMPLES

| Portfolio | Overall | Asian/Russian Crisis | Subprime Crisis | Eurozone Crisis |
|---|---|---|---|---|
| EWP | 0.08% | 0.08% | 0.08% | 0.03% |
| GMVP | 1.91% | 2.12% | 1.73% | 1.45% |
| MDP | 2.36% | 2.50% | 2.05% | 1.76% |
| MKT | 0.80% | 0.77% | 0.77% | 0.48% |
| NRPP | 0.43% | 0.53% | 0.36% | 0.27% |
| RPP | 0.71% | 0.88% | 0.59% | 0.48% |

The two risk parity approaches only generate about a third (or less) of the turnover of the Global Minimum Variance (GMVP) and Most Diversified (MDP) portfolio approaches.

The NRPP's simple weight allocation rule based on asset volatility also leads to a small degree of portfolio churn-only surpassed by the EWP. The more complete risk parity implementation generates noticeably more turnover; suggesting that incorporating covariance information introduces more variability in optimal weights. The RPP's level of turnover is roughly in line with that of the market proxy however which suggests that employing this approach can be just as cost effective as or even more so than employing a passive market weighted style. The GMVP and MDP however have levels of churn that are at roughly three times that of the RPP in each of subsamples and the overall period. Evidently the runaway return performance of these risk based approaches is likely to be tempered by higher transaction costs once the level of portfolio churn is accounted for. We also note that the MDP consistently generates more turnover than the GMVP which may make the latter a more attractive option for a portfolio style that emphasizes traditional diversification given their close return performance on most other performance metrics.

The results for the ALSI dataset are generally upheld in our other equity universes. It is also worth mentioning that consideration of our subsamples does not suggest that periods of increased volatility coincide with increased turnover. In fact, turnover was lower for most portfolios styles in the Subprime Crisis period relative to the overall sample period, suggesting that increased market volatility (see Appendix 2.2) does not always imply increased portfolio churn.

No portfolio style exhibits total consistency in terms of drawdowns. However we find that the MDP and GMVP loosely exhibit the best performance in terms of drawdown duration and loss minimization at daily, weekly and monthly frequencies. All return figures presented in Table 9 overleaf are annualized for standardized representation. These results are upheld in both the ALSI dataset and the other equity universes but are perhaps most evident at monthly frequencies; which are likely the greatest concern for most investment managers as they typically report on this basis. Portfolio styles outside of the MDP and GMVP do not exhibit any real consistency in terms of ranking performance. However we do find that average drawdown durations are quite similar for most of the techniques.

TABLE 9: ALSI INVESTMENT UNIVERSE DRAWDOWN STATISTICS FOR THE OVERALL SAMPLE PERIOD (JULY 1995-NOVEMBER 2013)

| Portfolio | Avg. Length Days | Avg. Length Weeks | Avg. Length Months | Avg. Loss(Daily) | Avg. Loss(Weekly) | Avg. Loss (Monthly) |
|---|---|---|---|---|---|---|
| EWP | 2.93 | 3.04 | 2.90 | 2.67% | 316.33% | 168.71% |
| GMVP | 2.94 | 2.99 | 2.65 | 2.29% | 280.02% | 141.45% |
| MDP | 2.97 | 2.98 | 2.89 | 2.34% | 287.27% | 161.01% |
| MKT | 2.97 | 2.86 | 2.55 | 2.98% | 317.41% | 157.53% |
| NRPP | 2.92 | 3.04 | 2.67 | 2.56% | 307.00% | 146.18% |
| RPP | 2.91 | 2.97 | 2.80 | 2.46% | 291.85% | 150.52% |

Most heuristic portfolios have average drawdown duration lower than the market at daily frequency. However this does not extend to longer time periods. Except for the Equal Weighted Portfolio (EWP), candidate portfolios generally incur lower average losses in drawdown periods than the market at the various frequencies.

The hierarchy of average losses at different reporting frequencies is evidently more variable. Note that average (annualized) weekly losses are of much higher magnitudes than average monthly losses. This is because such short term loss severity rarely lasts long enough to reflect in monthly drawdown returns at such high levels. We find that the tenor of these results is maintained across the other portfolio styles.

## 5.4 CONCENTRATION AND EX POST RISK CONTRIBUTIONS

Equalization of total risk contributions is a central tenet of the risk parity approach. However before considering how the different portfolios perform in terms of equalizing risk contributions we explore how concentrated each portfolio is in the traditional sense. The table below present's average EFS data for the different portfolios formed using the ALSI across the overall period and different subsamples. We also present RRC figures in Appendices 1.1 and 2.1.

TABLE 10: EFFECTIVE SHARES (EFS) DATA FOR CANDIDATE (JULY 1995-NOVEMBER 2013)

| Portfolio | Overall | Asian/Russian Crisis | Subprime Crisis | Eurozone Crisis |
|---|---|---|---|---|
| EWP | 40.00 | 40.00 | 40.00 | 40.00 |
| GMVP | 16.75 | 16.26 | 15.04 | 14.64 |
| MDP | 14.12 | 13.49 | 13.16 | 12.26 |
| MKT | 17.15 | 20.14 | 16.48 | 17.26 |
| NRPP | 37.65 | 37.68 | 37.91 | 38.08 |
| RPP | 33.58 | 33.34 | 34.60 | 33.03 |

The hierarchy of portfolio concentration is largely invariant to sub sample selection with the two risk parity approaches leading the Global Minimum Variance (GMVP) and Most Diversified (MDP) portfolios.

As previously discussed, the EWP necessarily has the least concentration. Beyond that however we find that the NRPP provides the second highest level of evenness in weight distribution followed by the RPP. Our benchmark, the MKT is third leaving the GMVP and the MDP as the most concentrated portfolio approaches considered here. The GMVP is marginally less concentrated than its counterpart on average. These results are generally consistent across equity universes except for some cases where the market benchmark MKT is so concentrated as to render the GMVP and MDP weight allocations relatively less concentrated.

We also find that the RPP and NRPP perform well in terms of equalizing total risk contributions in out of sample testing. Across all equity universes and our ALSI subsamples the RPP invariably provides the best equalization of total risk contributions (TRCs) as evinced by the low gini coefficients generated by our evaluation of out of sample TRCs.

TABLE 11: GINI COEFFICIENTS FOR EX POST TOTAL RISK CONTRIBUTIONS (TRCS) (JULY 1995-NOVEMBER 2013)

| Portfolio | Overall | Asian/Russian Crisis | Subprime Crisis | Eurozone Crisis |
|---|---|---|---|---|
| EWP | 0.22 | 0.21 | 0.20 | 0.19 |
| GMVP | 0.52 | 0.53 | 0.57 | 0.57 |
| MDP | 0.67 | 0.68 | 0.69 | 0.73 |
| MKT | 0.57 | 0.54 | 0.59 | 0.53 |
| NRPP | 0.17 | 0.19 | 0.14 | 0.13 |
| RPP | 0.10 | 0.13 | 0.07 | 0.06 |

Risk parity based techniques consistently provide the best diversification of out of sample total risk contributions, yielding the lowest gini coefficients amongst candidate portfolios.

It is evident that the hierarchy of portfolios in terms of ex post equalization of TRCs sees the RPP take prominence followed by the NRPP. The EWP provides the next best equalization performance followed by the market and the GMVP and MDP prove to be the most concentrated in terms of risk contribution. Evidently the MDP is on average more concentrated than the GMVP which may once more encourage investors who seek to implement portfolios based on traditional diversification to choose the latter over the former. We find that this hierarchy of performance is upheld in other equity universes too with the exception that, in some cases, the GMVP and MDP achieve greater risk equalization than the market proxy.

Consequently risk parity based approaches seem to perform well both in terms of traditional measures of concentration and the equalization of ex post risk contributions. In this regard, it is safe to say that both the full-fledged and naïve approach are just as able to achieve equalization of risk contributions within the domestic market context as they have been shown to do abroad. We also note the high level of concentration in risk contributions for the GMVP and MDP and recognize the former's dominance over the latter when considering both concentration and ex post TRC equalization.

## 5.5 LEVERAGE AVERSION THEORY TESTS

As previously discussed, a mainstay of leverage aversion theory is that the outperformance of the RPP can be attributed to the overweighting of low beta assets. The subsequent outperformance of these low beta assets, and the portfolio, is attributed to the fact that most institutional investors tend to avoid these assets-thus keeping their returns from being competed down as aggressively as would ordinarily be the case. This leaves room for higher risk adjusted returns (considering the lower betas associated with these assets) which risk parity portfolios can benefit from. If the theory holds some validity we expect that (i) the RPP will accord the highest weights to assets that have low beta. (ii) Markets where the RPP does well relative to the market value weighted benchmark will be characterized by higher returns for lower beta assets.

We present the beta profile for the RPP on the ALSI overleaf by plotting representative returns and betas of the i'th smallest asset against the same asset's representative weight. For instance, in Figure 2 below, the representative largest holding within the portfolio is the 40th asset. This asset has a representative weight of 5.8%, cumulative return to the order of 362% and representative beta of 0.31%. Consequently we can use this depiction to provide an overview of the RPP portfolio on the ALSI across the entire sample period.

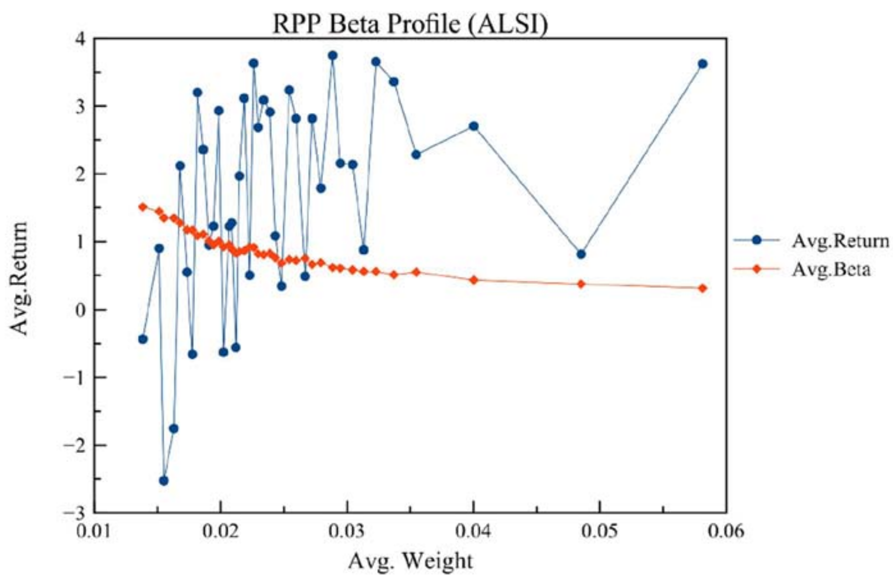



FIGURE 2: RPP BETA PROFILE ON THE ALSI (JULY 1995-NOVEMBER 2013)

Evidently the RPPs most heavily weighted asset on the ALSI tends to represent under 6% of the portfolio; underlining the RPPs low concentration credentials. Additionally the average beta curve is convex; signifying that average beta does in fact tend to decrease with increased weight allocation within the portfolio. This would seem to confirm our first hypothesis aimed at evaluating leverage aversion theory as applied to the RPP within the domestic equity context. Secondly we note that the more heavily weighted, relatively low beta assets are associated with higher returns. The relatively high beta assets further left are also the only assets associated with negative returns over this period. Thus there seems to be a general trend towards improving return performance as average asset beta decreases and this would appear to confirm leverage aversion theorists' contentions that RPP performance is predicated on overweighting low volatility assets and exploiting their subsequent outperformance.

Hence it would appear that RPP benchmark outperformance on the ALSI is likely driven by leverage aversion principles. However, it may bear argument that a consideration of the ALSI in isolation is not adequate basis for generalizing the validity of our two hypotheses to the wider equity context. In addition we would like to explore if the same forces that seem to drive excess returns with respect to the benchmark also impact the rank performance of the RPP relative to other risk based portfolios. Consequently we also consider the beta profiles for the TOPIX100 (where the RPP ranked first) and the TWSE100 (where it ranked last) to support the insights gleaned from considering the beta profile on the ALSI. Understanding how asset returns and betas interact within these markets should be instructive in generalizing the validity of our core hypotheses and inferring possible extensions of leverage aversion theory to performance rankings of our candidate portfolios.

We present the RPP beta profile for the TOPIX100 in Figure 3 below. Evidently the RPP within this equity universe also over-weights low beta assets as evinced by the convex average beta curve; confirming the general tenor of our findings on the ALSI. This appears to support our hypotheses about the drivers of RPP performance relative to the benchmark. However a consideration of the portfolio's ranking relative to our other candidate portfolios proves less straightforward. This is because the portfolio's biggest competitors; the MDP and GMVP, also exhibit some evidence of leverage aversion (see Appendix 3). In particular they also possess convex average beta curves- although their maximum weights also tend to correspond to lower betas as compared to those associated with the maximum weight of the corresponding RPP portfolio.

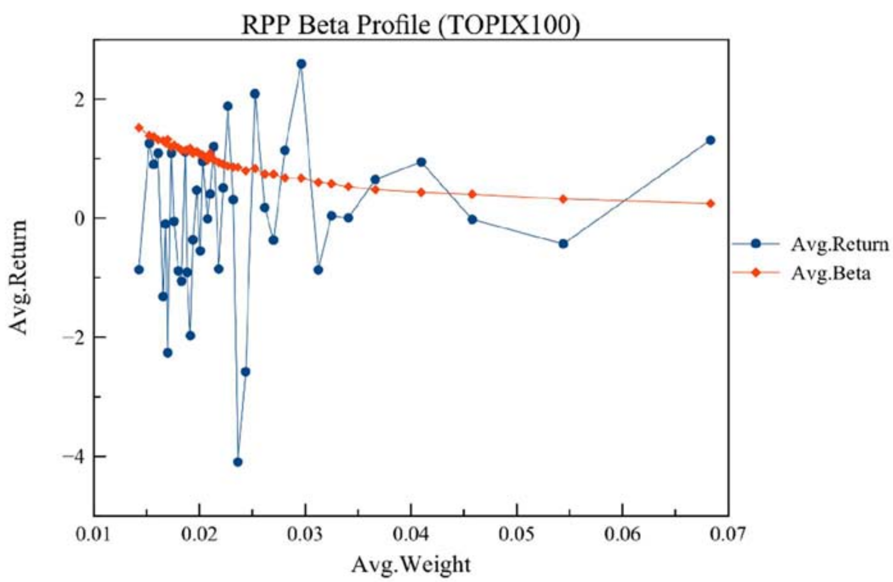



FIGURE 3: RPP BETA PROFILE ON THE TOPIX100 (JULY 1995-NOVEMBER 2013)

Given that these three risk based portfolio approaches are seemingly geared towards exploiting returns that accrue to low beta assets, the main differentiator between them becomes their relative aggressiveness in overweighting assets from the low beta end of the spectrum. We already established that the RPP tends to be the least concentrated amongst these three approaches. This is also evident from the beta profiles as the largest weights for the representative RPP portfolios on any given index are invariably smaller than those of their corresponding GMVP and MDP portfolios. The implication of the fact that all three portfolios overweight low beta assets but the GMVP and MDP exhibit higher levels of concentration is that the latter two tend to overweight lower beta assets *relatively more* than the RPP by virtue of being more concentrated.

Consequently, assuming the existence of a consistent relationship between asset betas and levels of return, if the RPP outperforms/underperforms the benchmark due to overweighting low beta assets, the corresponding GMVP and MDP can be expected to outperform/underperform *both* the RPP and the benchmark by virtue of exploiting these low beta assets more aggressively than the RPP.

However, the RPP outperforms the benchmark, the GMVP and MDP on the TOPIX100. Given this outcome, and in light of the preceding discussion, we would expect to find that (i) the three risk based portfolios all overweight low beta assets (ii) the RPP does so less aggressively than the other two candidate portfolios considered (iii) the RPP's rank performance cannot hail from overweighting low beta assets alone and hence must be linked to an inconsistent relationship between beta and asset returns (e.g. not all representative low beta assets produce above average returns.) Such inconsistency in the relationship between average return and beta is necessary as it is the only means by which the natural ascendancy of the GMVP and MDP within this context can be overcome to the extent necessary to allow RPP outperformance.

A comparison of beta profiles for the RPP, GMVP and MDP portfolios (see Appendix 3) confirms the truth of these conjectures. All three portfolios demonstrate convex average beta curves and the RPP exhibits lower maximal weights than the other two portfolio approaches. We observe the expected inconsistency in the relationship between beta and asset returns in the assets within the top (by weight ranking) quartiles of the MDP and the GMVP. Each of these two portfolios has some top quartile representative assets associated with significantly negative average returns whereas all of the RPP's top quartile assets are associated with positive returns. Essentially the GMVP and MDP overweight some low beta assets that deviate from the generally inverse relationship between beta and return observed on the market. This then acts as the main differentiator of performance levels between the GMVP and MDP portfolios and the RPP within the context of the TOPIX100. Thus the RPP, which is less aggressive in this regard, is able to outperform the other two formulations and gain the ascendancy by virtue of being less concentrated in the low beta assets that do display such inconsistency.

We can return to the beta profiles on the ALSI to find results in keeping with this analysis-albeit within a context where a more consistent relationship between average beta and average return is present. The RPP ranked third behind the GMVP and MDP on the ALSI and once more was less concentrated on the low beta end of the spectrum. However there is a more consistent inverse relationship between low beta assets and average returns on the ALSI than the TOPIX100 as evinced by consistently positive returns in the top quartiles of the three portfolios. This allows the GMVP and MDP to exploit their more aggressive overweighting of low beta assets to outperform the RPP on the ALSI.

Hence we have established that the outperformance of the RPP, GMVP and MDP relative to the market benchmark can generally be attributed to overweighting low beta assets and the RPP can be usually be expected to be intermediate in performance to the benchmark on one extreme and the GMVP and MDP on the other due to the impact of relative portfolio concentrations. Given this theory, we would expect that the RPP's poor performance on the TWSE100 to be linked to the poor performance of low beta assets or an inconsistent relationship between average beta and return. The beta profile for this index is depicted in Figure 4 below and an examination of this and the corresponding MDP and GMVP profiles reveals that the RPP likely lags other candidate portfolios because the GMVP and MDP again assign much higher weights to the low beta section of the average beta curve than the RPP does. For instance, the RPP assigns weights of 2% or less to ten high beta assets and assigns weights in excess of 5% to only two representative assets. By contrast the GMVP (See Appendix 3.2) assigns weights of 2% or less to a total of 23 assets on the higher beta section of the average beta curve and assigns weights in excess of 5% to 8 assets on the low beta segment of the average beta curve. Returns associated with low beta assets are not consistently positive however as is evinced by instances of negative returns for some of the representative assets in the top quartiles of the MDP and GMVP portfolios. Consequently the RPP underperforms the market by just under 3% in this context since the inconsistent average beta/average return relationship means it has no marked advantage over the benchmark by virtue of its attempt to exploit low beta assets.

We find that these results are generally upheld across the various equity universes although we limit ourselves to presenting RPP, MDP and GMVP beta profiles for three equity universes here (additional beta profiles are available on request.). The RPP generally falls behind the GMVP and MDP but leads the market in performance. However where the RPP does outperform the GMVP and MDP (or underperforms these portfolios and the market benchmark), its performance can be attributed to the fact that such investment universes do not exhibit an entirely consistent relationship between levels of average asset beta and asset return. Our finding that the GMVP and the MDP also seem to be significantly predicated on overweighting low beta assets is also interesting in that it suggests that many risk based portfolios may owe their performance to the fundamentals of leverage aversion theory. This is roughly in keeping with the findings of Scherer (2010) and Clarke, de Silva and Thorley (2011) who also found evidence that the GMVP over-weights low beta assets.

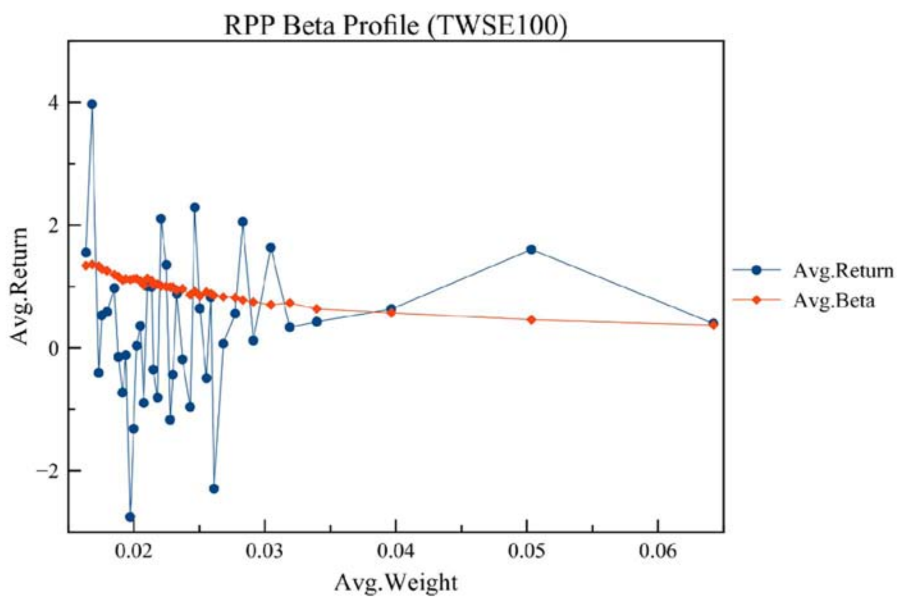



FIGURE 4: RPP BETA PROFILE ON THE TWSE100 (JULY 1995-NOVEMBER 2013)

Hence our findings on the ALSI and across our various equity universes suggest that the RPP does overweight low beta assets which is in keeping with leverage aversion theory as applied to the approach. However this relationship is more readily applied to benchmark comparisons than those involving other risk based portfolios as the latter also share the RPPs low beta predilections; thus first necessitating a consideration of relative concentrations between portfolios and the consistency of the relationship between beta and return within the relevant equity universe. However if (as appears to be the case here) RPP performance is evidence of the validity of leverage aversion theory, then this holds strong ramifications for the analytical form of the CAPM and adds to the debates surrounding its validity and the need for an improved theory about the return generating processes of marketable securities.

## 5.6 THE IMPACT OF MARKET CONCENTRATION ON RETURN PERFORMANCE

The last section discussed the impact of differential levels of portfolio concentrations in low beta assets on RPP performance relative to other portfolio approaches. However a cursory examination of the RPP's returns across the different equity universes suggests that higher levels of benchmark concentration may also be broadly linked to poorer performance of the RPP relative to other candidate portfolios. Given the ALSI's history of sectorial concentration (especially with respect to resource shares) we explore if the level of benchmark concentration has material effect on the performance of the RPP's philosophy relative to those of our other candidate approaches. We conduct an initial exploration of the association between benchmark concentration and RPP performance by comparing the average EFS and RRC of the market benchmark with the rank, return and excess benchmark return performance of the RPP within that universe. Calculating Spearman and Pearson rank correlation (see Appendix 4.1 and 4.2) for this equity universe data suggests that a significant association between benchmark concentration and the RPPs excess return is present at the 5% confidence level. More specifically, higher levels of benchmark EFS are associated with greater excess return (relative to the market benchmark portfolio) for the RPP thus confirming our early impression of the nature of this relationship, albeit not with respect to portfolio rank as initially theorized.

This finding may well play a part in explaining the relatively lukewarm performance of the RPP within the context of the ALSI as opposed to the S&P100. The US equity context, which has garnered the bulk of research consideration with regards to the RPP, is significantly less concentrated than its domestic counterpart-as is evident from benchmark concentration figures for each of these indices in Appendix 1.1. Consequently, inasmuch as the RPP does deliver in terms of equalizing risk contributions, the positive return performance observed in the US market does not automatically extend to other contexts and this is likely partly a consequence of certain factors that are at least linked to the higher level of concentration in other equity markets as compared to the S&P100.

Naturally this associative result applies to a comparison of final concentration figures to overall period cumulative returns *across* different equity universes. We proceed to assess if the evolution of benchmark concentration over time has an *ongoing* effect on future returns on the ALSI. Accordingly we conduct a Granger causality test using a time series of monthly benchmark EFS and RRC figures and monthly time series of excess and total returns for our candidate portfolios. Initial analysis indicated that the optimal number of lags for our analysis are 1, 5 and 9 months and we proceeded to estimate a VAR equation before generating results for the Granger Causality test (as outlined in Appendix 4.3.) Our consideration of total and excess return impact suggests that market benchmark EFS and RRC show no statistically significant (5% level) causal effects on the returns of the RPP and our other four candidate portfolios.

Consequently it would appear that, inasmuch as benchmark concentration across equity universes may correlate with levels of RPP excess return across benchmarks, the intra-benchmark evolution in concentration over time does not necessarily affect RPP performance in consistent, ongoing fashion within the single equity universe context. This raises the prospect that the significance of the Spearman rank and Pearson correlations relates to wider constructs that are merely linked to overall benchmark concentration on aggregate and do not necessarily manifest in benchmark concentration at more discrete points in time.

A possible explanation for this finding is that the significance of the association between concentration and RPP excess return is due to the fact that more concentrated benchmarks force the RPP to take on a greater active weight with respect to the sectors that account for most of this concentration. Consequently, in markets where such sectors subsequently account for a significant portion of positive returns within the equity universe, the RPP tends to lag the benchmark and other candidate portfolios due to being relatively underweight in these key sectors. Conversely if the market is relatively un-concentrated then the extent to which the RPP employs active weights relative to the benchmark is more limited and it's equalization of total risk contributions may serve a more prominent role in determining its performance relative to other candidate approaches and the market benchmark.

This may account for the lack of ongoing return influence as such influence would only be evident over sub periods when key sectors perform significantly differently from the rest of the index. Consequently causal influences may not manifest consistently enough to be significant within the context of the Granger causality test although their significance across the sample period as a whole is pronounced. It may be instructive to consider this hypothesis in subsequent research, possibly also exploring any possible links to our discussion of leverage aversion theory that this behaviour, if present, may possess.

## SECTION 6. CONCLUSION

This study has employed a back testing approach to assess the performance of the risk parity portfolio approach relative to that of several other heuristic and risk based portfolios. We compare the performance of these portfolios along several different dimensions and generate a number of insights regarding the performance of these portfolios with special consideration for the domestic context.

We find that the return performance of the RPP, although in excess of the market weighted benchmark, has lagged that of the GMVP and MDP domestically. This also seems to be the case within the bulk of the equity universes considered. Additionally we confirm the empirical validity of Maillard et al's (2008) suggestion about the ranking of volatilities of the GMVP, RPP and EWP portfolios and contend that this hierarchy can be further extended to include the NRPP and MDP.

We demonstrate that, notwithstanding the return performance of the RPP, the approach does in fact provide excellent management of portfolio concentration and equalization of total risk contributions in out of sample testing. Additionally the approach generates significantly lower portfolio turnover than the GMVP and MDP which may render its return after accounting for transaction costs significantly more competitive than our initial analysis may suggest. This finding is perhaps tempered by the finding that the GMVP and MDP generally tend to provide better drawdown performance when assessed at daily, weekly and monthly frequencies.

Our consideration of the theoretical reasons for the performance of the RPP appears to lend support to Frazzini and Pedersen's (2010) leverage aversion theory. We find that the RPP assigns the bulk of its weights to low beta assets which appear to generally outperform other assets in the markets where the RPP performs well. We also uncover an inverse association between the degree of benchmark concentration and the excess return of the RPP relative to the benchmark-although we stop short of suggesting a direct causal relationship. We suggest that this association is possibly a consequence of the RPP employing more active weights with respect to sectors that account for the bulk of index concentration. Subsequently, where such sectors also account for a significant portion of positive index performance at key points in time, the RPPs relative underweight may affect its relative return performance adversely.

In addition to testing this last hypothesis, there is scope for further research to determine if these findings are robust to changes in back-testing specification. Possible deviations include employing quarterly (as opposed to monthly) rebalancing, using a different in-sample rolling window size or increasing the size of the benchmark. Additionally it may be worth considering other risk based allocation procedures; for instance, an approach with weights based on inverse asset beta with respect to the benchmark should also be able to exploit equity contexts where leverage aversion causes higher returns to accrue to lower beta assets. Naturally our examination of beta profiles suggests that many of our risk based approaches already tacitly employ a similar weighting but it would be instructive to see if their performance is substantially different from that of the simple inverse beta weighting approach in the face of comprehensive performance evaluation.

To surmise, our consideration suggests the RPP is a suitable approach for investors who primarily seek to equalize total risk contributions across assets and minimize levels of portfolio turnover and related transaction costs. However risk and return performance may lag that of other portfolios and practitioners may want to consider the desirability of employing a portfolio approach that is skewed towards low beta assets and may provide increasingly inferior returns relative to the benchmark in the face of increased market concentration.

# APPENDICES

## APPENDIX 1: OVERALL SAMPLE PERIOD RESULTS

### 1.1 PERFORMANCE FIGURES

ALSI

| Portfolio | Cumulative Return | Volatility | RVR | Turnover(Avg) | EFS | RRC | Ex Post TRC Gini Coeff. | Drawdown Avg Days | Drawdown Avg Weeks | Drawdown Avg Months | Drawdown Avg Loss(Daily) | Drawdown Avg Loss(Weekly) | Drawdown Avg Loss (Monthly) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| EWP | 158.21% | 18.53% | 0.45 | 0.08% | 40.00 | 0.00 | 0.22 | 2.93 | 3.04 | 2.90 | 2.67% | 316.33% | 168.71% |
| GMVP | 201.53% | 15.67% | 0.67 | 1.91% | 16.75 | 0.04 | 0.52 | 2.94 | 2.99 | 2.65 | 2.29% | 280.02% | 141.45% |
| MDP | 197.08% | 15.87% | 0.65 | 2.36% | 14.12 | 0.05 | 0.67 | 2.97 | 2.98 | 2.89 | 2.34% | 287.27% | 161.01% |
| MKT | 161.47% | 20.66% | 0.41 | 0.80% | 17.15 | 0.03 | 0.57 | 2.97 | 2.86 | 2.55 | 2.98% | 317.41% | 157.53% |
| NRPP | 172.33% | 17.82% | 0.51 | 0.43% | 37.65 | 0.00 | 0.17 | 2.92 | 3.04 | 2.67 | 2.56% | 307.00% | 146.18% |
| RPP | 180.79% | 17.14% | 0.55 | 0.71% | 33.58 | 0.01 | 0.10 | 2.91 | 2.97 | 2.80 | 2.46% | 291.85% | 150.52% |

EURONEXT100

| Portfolio | Cumulative Return | Volatility | RVR | Turnover(Avg) | EFS | RRC | Ex Post TRC Gini Coeff. | Drawdown Avg Days | Drawdown Avg Weeks | Drawdown Avg Months | Drawdown Avg Loss(Daily) | Drawdown Avg Loss(Weekly) | Drawdown Avg Loss (Monthly) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| EWP | 43.65% | 22.79% | 0.10 | 0.06% | 40.00 | 0.00 | 0.23 | 2.87 | 2.93 | 2.58 | 3.20% | 397.94% | 166.98% |
| GMVP | 95.30% | 16.78% | 0.30 | 1.68% | 15.52 | 0.04 | 0.55 | 2.85 | 3.00 | 2.55 | 2.35% | 298.82% | 121.08% |
| MDP | 97.12% | 16.84% | 0.30 | 2.05% | 12.67 | 0.06 | 0.71 | 2.83 | 3.02 | 2.68 | 2.30% | 302.83% | 112.93% |
| MKT | 36.86% | 22.42% | 0.09 | 0.66% | 22.90 | 0.02 | 0.44 | 2.82 | 2.86 | 2.77 | 3.04% | 371.41% | 182.45% |
| NRPP | 66.01% | 20.76% | 0.17 | 0.38% | 37.34 | 0.00 | 0.14 | 2.86 | 2.90 | 2.64 | 2.91% | 360.21% | 153.74% |
| RPP | 73.76% | 19.91% | 0.19 | 0.57% | 34.04 | 0.00 | 0.07 | 2.84 | 3.10 | 2.74 | 2.74% | 375.11% | 151.74% |

FTSE100

| Portfolio | Cumulative Return | Volatility | RVR | Turnover(Avg) | EFS | RRC | Ex Post TRC Gini Coeff. | Drawdown Avg Days | Drawdown Avg Weeks | Drawdown Avg Months | Drawdown Avg Loss(Daily) | Drawdown Avg Loss(Weekly) | Drawdown Avg Loss (Monthly) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| EWP | 77.64% | 18.86% | 0.22 | 0.05% | 40.00 | 0.00 | 0.23 | 2.88 | 2.66 | 2.48 | 2.62% | 272.30% | 119.82% |
| GMVP | 95.94% | 14.59% | 0.34 | 1.68% | 16.99 | 0.04 | 0.50 | 2.88 | 2.84 | 2.37 | 1.95% | 225.07% | 96.05% |
| MDP | 93.69% | 14.89% | 0.33 | 2.19% | 13.66 | 0.05 | 0.68 | 2.88 | 2.84 | 2.39 | 2.03% | 232.01% | 103.63% |
| MKT | 44.02% | 19.24% | 0.12 | 0.60% | 20.28 | 0.03 | 0.50 | 2.81 | 2.80 | 2.48 | 2.57% | 261.93% | 118.36% |
| NRPP | 84.29% | 17.34% | 0.25 | 0.36% | 37.49 | 0.00 | 0.15 | 2.88 | 2.66 | 2.41 | 2.37% | 245.69% | 105.87% |
| RPP | 91.86% | 16.83% | 0.29 | 0.56% | 34.97 | 0.00 | 0.07 | 2.85 | 2.75 | 2.25 | 2.28% | 255.90% | 100.66% |

HANGSENG100

| Portfolio | Cumulative Return | Volatility | RVR | Turnover(Avg) | EFS | RRC | Ex Post TRC Gini Coeff. | Drawdown Avg Days | Drawdown Avg Weeks | Drawdown Avg Months | Drawdown Avg Loss(Daily) | Drawdown Avg Loss(Weekly) | Drawdown Avg Loss (Monthly) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| EWP | 133.26% | 28.93% | 0.43 | 0.12% | 40.00 | 0.00 | 0.19 | 2.89 | 2.88 | 2.92 | 4.03% | 519.34% | 238.33% |
| GMVP | 160.11% | 22.52% | 0.66 | 1.85% | 14.43 | 0.05 | 0.60 | 2.89 | 3.10 | 2.31 | 3.13% | 441.55% | 180.03% |
| MDP | 168.07% | 22.51% | 0.69 | 2.05% | 12.70 | 0.06 | 0.70 | 2.89 | 2.97 | 2.77 | 3.25% | 445.84% | 176.50% |
| MKT | 146.67% | 29.10% | 0.47 | 0.93% | 8.27 | 0.11 | 0.65 | 2.76 | 2.76 | 2.55 | 3.73% | 450.01% | 222.64% |
| NRPP | 133.34% | 27.92% | 0.44 | 0.46% | 37.68 | 0.00 | 0.15 | 2.89 | 2.96 | 2.75 | 3.91% | 511.90% | 236.49% |
| RPP | 139.86% | 26.70% | 0.48 | 0.65% | 34.68 | 0.00 | 0.07 | 2.87 | 3.00 | 2.83 | 3.75% | 500.23% | 225.34% |

S&P100

| Portfolio | Cumulative Return | Volatility | RVR | Turnover(Avg) | EFS | RRC | Ex Post TRC Gini Coeff. | Drawdown Avg Days | Drawdown Avg Weeks | Drawdown Avg Months | Drawdown Avg Loss(Daily) | Drawdown Avg Loss(Weekly) | Drawdown Avg Loss (Monthly) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| EWP | 103.40% | 19.97% | 0.27 | 0.07% | 40.00 | 0.00 | 0.22 | 2.75 | 2.86 | 2.52 | 2.50% | 278.58% | 133.05% |
| GMVP | 94.90% | 15.16% | 0.33 | 1.63% | 16.44 | 0.04 | 0.52 | 2.77 | 2.79 | 2.44 | 1.94% | 216.48% | 91.95% |
| MDP | 91.52% | 15.50% | 0.31 | 2.14% | 13.19 | 0.05 | 0.69 | 2.76 | 2.72 | 2.48 | 2.00% | 219.87% | 92.18% |
| MKT | 84.79% | 19.83% | 0.22 | 0.59% | 28.49 | 0.01 | 0.38 | 2.78 | 2.96 | 2.42 | 2.56% | 292.93% | 124.21% |
| NRPP | 112.20% | 18.19% | 0.32 | 0.40% | 37.10 | 0.00 | 0.14 | 2.74 | 2.84 | 2.43 | 2.26% | 260.12% | 122.85% |
| RPP | 110.18% | 17.82% | 0.32 | 0.56% | 35.30 | 0.00 | 0.07 | 2.76 | 2.83 | 2.38 | 2.22% | 260.81% | 108.06% |

S&PASX100

| Portfolio | Cumulative Return | Volatility | RVR | Turnover(Avg) | EFS | RRC | Ex Post TRC Gini Coeff. | Drawdown Avg Days | Drawdown Avg Weeks | Drawdown Avg Months | Drawdown Avg Loss(Daily) | Drawdown Avg Loss(Weekly) | Drawdown Avg Loss (Monthly) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| EWP | 83.35% | 15.02% | 0.29 | 0.06% | 40.00 | 0.00 | 0.21 | 2.84 | 2.85 | 2.68 | 2.02% | 231.67% | 150.85% |
| GMVP | 98.84% | 12.25% | 0.42 | 1.58% | 18.10 | 0.03 | 0.45 | 2.86 | 2.90 | 2.50 | 1.64% | 192.90% | 93.64% |
| MDP | 95.06% | 12.45% | 0.40 | 2.12% | 14.34 | 0.05 | 0.64 | 2.81 | 2.85 | 2.68 | 1.66% | 191.23% | 95.70% |
| MKT | 79.67% | 16.26% | 0.26 | 0.62% | 15.95 | 0.04 | 0.58 | 2.86 | 2.99 | 2.85 | 2.22% | 258.89% | 130.83% |
| NRPP | 87.66% | 14.19% | 0.32 | 0.37% | 37.19 | 0.00 | 0.15 | 2.89 | 2.89 | 2.68 | 1.92% | 218.73% | 139.16% |
| RPP | 91.08% | 13.80% | 0.35 | 0.59% | 34.98 | 0.00 | 0.08 | 2.87 | 2.86 | 2.68 | 1.88% | 207.81% | 135.32% |

S&PBSE100

| Portfolio | Cumulative Return | Volatility | RVR | Turnover(Avg) | EFS | RRC | Ex Post TRC Gini Coeff. | Drawdown Avg Days | Drawdown Avg Weeks | Drawdown Avg Months | Drawdown Avg Loss(Daily) | Drawdown Avg Loss(Weekly) | Drawdown Avg Loss (Monthly) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| EWP | 136.11% | 25.13% | 0.28 | 0.08% | 40.00 | 0.00 | 0.22 | 3.16 | 3.15 | 2.70 | 3.73% | 468.19% | 233.14% |
| GMVP | 191.61% | 19.88% | 0.51 | 1.85% | 15.11 | 0.04 | 0.57 | 3.08 | 3.06 | 2.61 | 2.86% | 354.18% | 177.39% |
| MDP | 207.21% | 20.01% | 0.54 | 2.16% | 12.94 | 0.05 | 0.70 | 3.03 | 3.14 | 2.58 | 2.84% | 361.72% | 171.04% |
| MKT | 125.92% | 25.91% | 0.25 | 0.77% | 19.54 | 0.03 | 0.50 | 3.09 | 3.17 | 2.71 | 3.81% | 490.74% | 242.70% |
| NRPP | 140.47% | 23.84% | 0.31 | 0.44% | 37.63 | 0.00 | 0.16 | 3.16 | 3.16 | 2.73 | 3.55% | 450.40% | 223.73% |
| RPP | 148.03% | 22.69% | 0.34 | 0.73% | 32.90 | 0.01 | 0.09 | 3.18 | 3.19 | 2.59 | 3.38% | 430.43% | 208.85% |

SZSE100

| Portfolio | Cumulative Return | Volatility | RVR | Turnover(Avg) | EFS | RRC | Ex Post TRC Gini Coeff. | Drawdown Avg Days | Drawdown Avg Weeks | Drawdown Avg Months | Drawdown Avg Loss(Daily) | Drawdown Avg Loss(Weekly) | Drawdown Avg Loss (Monthly) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| EWP | 53.38% | 27.61% | 0.12 | 0.13% | 40.00 | 0.00 | 0.15 | 2.97 | 3.27 | 3.36 | 3.86% | 466.65% | 255.42% |
| GMVP | 75.60% | 23.46% | 0.20 | 2.02% | 14.10 | 0.05 | 0.62 | 2.93 | 2.97 | 3.41 | 3.21% | 362.73% | 217.72% |
| MDP | 60.00% | 23.81% | 0.16 | 2.37% | 12.39 | 0.06 | 0.71 | 2.96 | 3.04 | 3.57 | 3.29% | 388.45% | 243.37% |
| MKT | 23.44% | 27.18% | 0.05 | 0.67% | 27.80 | 0.01 | 0.33 | 2.97 | 3.40 | 3.65 | 3.79% | 484.57% | 259.65% |
| NRPP | 40.54% | 26.84% | 0.09 | 0.46% | 37.93 | 0.00 | 0.11 | 2.97 | 3.32 | 3.43 | 3.72% | 461.70% | 257.70% |
| RPP | 16.97% | 26.20% | 0.04 | 0.65% | 34.70 | 0.01 | 0.07 | 2.99 | 3.31 | 3.50 | 3.71% | 464.33% | 270.65% |

TA100

| Portfolio | Cumulative Return | Volatility | RVR | Turnover(Avg) | EFS | RRC | Ex Post TRC Gini Coeff. | Drawdown Avg Days | Drawdown Avg Weeks | Drawdown Avg Months | Drawdown Avg Loss(Daily) | Drawdown Avg Loss(Weekly) | Drawdown Avg Loss (Monthly) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| EWP | 6.76% | 20.97% | 0.02 | 0.06% | 40.00 | 0.00 | 0.21 | 2.99 | 3.09 | 3.53 | 3.07% | 396.18% | 217.27% |
| GMVP | 54.06% | 15.79% | 0.25 | 1.75% | 15.30 | 0.04 | 0.57 | 2.95 | 3.15 | 2.73 | 2.33% | 299.72% | 126.58% |
| MDP | 56.11% | 15.89% | 0.25 | 2.07% | 12.78 | 0.06 | 0.70 | 2.95 | 3.19 | 2.86 | 2.37% | 309.99% | 129.71% |
| MKT | 37.13% | 18.15% | 0.15 | 0.91% | 7.61 | 0.11 | 0.55 | 2.84 | 2.95 | 2.60 | 2.53% | 315.24% | 138.07% |
| NRPP | 24.19% | 19.59% | 0.09 | 0.37% | 37.64 | 0.00 | 0.15 | 2.99 | 3.07 | 3.24 | 2.93% | 369.83% | 194.55% |
| RPP | 24.90% | 18.53% | 0.10 | 0.58% | 34.10 | 0.00 | 0.07 | 3.01 | 3.06 | 3.17 | 2.77% | 347.13% | 176.95% |

TWSE100

| Portfolio | Cumulative Return | Volatility | RVR | Turnover(Avg) | EFS | RRC | Ex Post TRC Gini Coeff. | Drawdown Avg Days | Drawdown Avg Weeks | Drawdown Avg Months | Drawdown Avg Loss(Daily) | Drawdown Avg Loss(Weekly) | Drawdown Avg Loss (Monthly) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| EWP | 29.96% | 23.85% | 0.07 | 0.07% | 40.00 | 0.00 | 0.16 | 2.85 | 3.03 | 2.43 | 3.32% | 429.32% | 183.34% |
| GMVP | 53.60% | 19.59% | 0.14 | 1.73% | 14.36 | 0.05 | 0.59 | 2.92 | 3.05 | 3.25 | 2.78% | 344.65% | 197.97% |
| MDP | 47.45% | 19.83% | 0.13 | 2.08% | 12.47 | 0.06 | 0.71 | 2.81 | 2.98 | 3.14 | 2.73% | 345.11% | 192.25% |
| MKT | 32.02% | 23.81% | 0.07 | 0.70% | 16.46 | 0.04 | 0.49 | 2.94 | 2.84 | 2.70 | 3.30% | 396.96% | 201.12% |
| NRPP | 31.92% | 22.78% | 0.07 | 0.39% | 37.26 | 0.00 | 0.11 | 2.88 | 2.94 | 2.58 | 3.17% | 401.56% | 184.03% |
| RPP | 29.75% | 22.12% | 0.07 | 0.52% | 33.78 | 0.01 | 0.06 | 2.87 | 3.00 | 2.56 | 3.07% | 399.50% | 184.63% |

TOPIX100

| Portfolio | Cumulative Return | Volatility | RVR | Turnover(Avg) | EFS | RRC | Ex Post TRC Gini Coeff. | Drawdown Avg Days | Drawdown Avg Weeks | Drawdown Avg Months | Drawdown Avg Loss(Daily) | Drawdown Avg Loss(Weekly) | Drawdown Avg Loss (Monthly) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| EWP | 3.38% | 22.00% | 0.01 | 0.06% | 40.00 | 0.00 | 0.21 | 2.90 | 3.17 | 3.04 | 3.04% | 377.62% | 186.07% |
| GMVP | -4.43% | 16.81% | -0.01 | 1.54% | 14.81 | 0.04 | 0.54 | 2.82 | 3.01 | 2.96 | 2.23% | 258.84% | 126.41% |
| MDP | 6.54% | 17.07% | 0.02 | 1.93% | 12.66 | 0.06 | 0.71 | 2.82 | 3.10 | 2.81 | 2.26% | 271.94% | 125.08% |
| MKT | -26.76% | 23.07% | -0.06 | 0.52% | 22.73 | 0.02 | 0.44 | 2.94 | 3.01 | 2.93 | 3.23% | 373.26% | 189.98% |
| NRPP | 0.56% | 20.44% | 0.00 | 0.37% | 36.83 | 0.00 | 0.15 | 2.90 | 3.06 | 3.25 | 2.80% | 327.58% | 174.48% |
| RPP | 7.82% | 19.32% | 0.02 | 0.58% | 31.74 | 0.01 | 0.07 | 2.87 | 2.98 | 3.13 | 2.62% | 301.15% | 162.81% |

UBS100

| Portfolio | Cumulative Return | Volatility | RVR | Turnover(Avg) | EFS | RRC | Ex Post TRC Gini Coeff. | Drawdown Avg Days | Drawdown Avg Weeks | Drawdown Avg Months | Drawdown Avg Loss(Daily) | Drawdown Avg Loss(Weekly) | Drawdown Avg Loss (Monthly) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| EWP | 81.47% | 18.12% | 0.24 | 0.06% | 40.00 | 0.00 | 0.27 | 2.90 | 2.97 | 3.05 | 2.73% | 365.22% | 203.21% |
| GMVP | 105.87% | 12.35% | 0.45 | 1.48% | 16.09 | 0.04 | 0.47 | 2.89 | 2.94 | 2.83 | 1.75% | 227.02% | 144.11% |
| MDP | 99.94% | 12.25% | 0.43 | 1.87% | 13.17 | 0.05 | 0.68 | 2.89 | 3.02 | 2.88 | 1.76% | 233.11% | 149.06% |
| MKT | 90.35% | 18.93% | 0.25 | 0.79% | 9.39 | 0.09 | 0.70 | 2.85 | 2.98 | 2.95 | 2.67% | 336.74% | 160.06% |
| NRPP | 79.03% | 16.06% | 0.26 | 0.49% | 33.85 | 0.01 | 0.19 | 2.90 | 3.02 | 3.11 | 2.39% | 332.03% | 182.26% |
| RPP | 79.83% | 14.99% | 0.28 | 0.78% | 29.45 | 0.01 | 0.10 | 2.95 | 2.97 | 3.00 | 2.27% | 309.95% | 165.53% |

## 1.2 DESCRIPTIVE STATISTICS FOR HEURISTIC AND RISK BASED PORTFOLIOS (ALSI INVESTMENT UNIVERSE) 1995-2013

| | RPP | NRPP | MKT | MDP | GMVP | EWP |
|---|---|---|---|---|---|---|
| Mean | 0.04% | 0.04% | 0.03% | 0.04% | 0.04% | 0.03% |
| Median | 0.06% | 0.04% | 0.02% | 0.05% | 0.06% | 0.03% |
| Maximum | 6.37% | 6.99% | 8.89% | 6.00% | 5.67% | 7.01% |
| Minimum | -11.03% | -12.13% | -13.84% | -8.66% | -9.57% | -11.88% |
| Std. Dev. | 1.08% | 1.12% | 1.30% | 1.00% | 0.99% | 1.17% |
| Skewness | -0.66875 | -0.66371 | -0.49561 | -0.66784 | -0.72228 | -0.59852 |
| Kurtosis | 9.942701 | 10.47115 | 10.19498 | 9.529681 | 10.35788 | 9.624206 |
| | | | | | | |
| Jarque-Bera | 10014.67 | 11535.22 | 10567.64 | 8898.96 | 11263.77 | 9077.701 |
| Probability | 0 | 0 | 0 | 0 | 0 | 0 |
| | | | | | | |
| Sum | 1.807875 | 1.72332 | 1.614679 | 1.970791 | 2.015291 | 1.582057 |
| Sum Sq. Dev. | 0.560385 | 0.605821 | 0.814141 | 0.480684 | 0.468406 | 0.655094 |

Sample Size: 4937 observations

# APPENDIX 2: SUB-SAMPLE RESULTS

## 2.1 PERFORMANCE INFORMATION FOR SELECTED SUB-SAMPLES (ALSI)

ASIAN AND RUSSIAN CRISES

| Portfolio | Cumulative Return | Volatility | RVR | Turnover(Avg) | EFS | RRC | Ex Post TRC Gini Coeff. | Drawdown Avg Days | Drawdown Avg Weeks | Drawdown Avg Months | Drawdown Avg Loss(Daily) | Drawdown Avg Loss(Weekly) | Drawdown Avg Loss (Monthly) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| EWP | -37.74% | 27.15% | -0.89 | 0.08% | 40.00 | 0.00 | 0.21 | 3.22 | 3.40 | 3.50 | 4.49% | 633.95% | 497.84% |
| GMVP | -52.24% | 24.77% | -1.36 | 2.12% | 16.26 | 0.04 | 0.53 | 3.39 | 3.00 | 2.75 | 4.40% | 516.78% | 294.77% |
| MDP | -52.22% | 24.64% | -1.36 | 2.50% | 13.49 | 0.05 | 0.68 | 3.28 | 3.08 | 2.75 | 4.33% | 526.15% | 292.67% |
| MKT | -39.03% | 29.69% | -0.84 | 0.77% | 20.14 | 0.03 | 0.54 | 3.21 | 3.00 | 3.00 | 5.11% | 623.01% | 291.02% |
| NRPP | -39.85% | 26.99% | -0.95 | 0.53% | 37.68 | 0.00 | 0.19 | 3.19 | 3.50 | 3.50 | 4.39% | 630.79% | 504.04% |
| RPP | -41.50% | 25.54% | -1.04 | 0.88% | 33.34 | 0.01 | 0.13 | 3.61 | 3.50 | 3.00 | 4.84% | 624.24% | 373.02% |

SUBPRIME CRISIS

| Portfolio | Cumulative Return | Volatility | RVR | Turnover(Avg) | EFS | RRC | Ex Post TRC Gini Coeff. | Drawdown Avg Days | Drawdown Avg Weeks | Drawdown Avg Months | Drawdown Avg Loss(Daily) | Drawdown Avg Loss(Weekly) | Drawdown Avg Loss (Monthly) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| EWP | -11.76% | 25.27% | -0.16 | 0.08% | 40.00 | 0.00 | 0.20 | 2.82 | 2.63 | 3.67 | 3.75% | 412.96% | 285.89% |
| GMVP | -2.82% | 19.59% | -0.05 | 1.73% | 15.04 | 0.04 | 0.57 | 2.90 | 2.82 | 4.00 | 2.78% | 264.70% | 176.66% |
| MDP | -4.89% | 19.79% | -0.08 | 2.05% | 13.16 | 0.05 | 0.69 | 2.77 | 2.90 | 4.50 | 2.71% | 290.73% | 254.01% |
| MKT | -9.71% | 29.35% | -0.11 | 0.77% | 16.48 | 0.04 | 0.59 | 3.02 | 2.74 | 3.67 | 4.67% | 472.67% | 331.88% |
| NRPP | -9.22% | 23.79% | -0.13 | 0.36% | 37.91 | 0.00 | 0.14 | 2.80 | 2.65 | 3.67 | 3.56% | 364.43% | 257.36% |
| RPP | -13.20% | 23.08% | -0.19 | 0.59% | 34.60 | 0.00 | 0.07 | 2.82 | 2.65 | 3.67 | 3.48% | 360.50% | 257.58% |

EUROZONE CRISIS

| Portfolio | Cumulative Return | Volatility | RVR | Turnover(Avg) | EFS | RRC | Ex Post TRC Gini Coeff. | Drawdown Avg Days | Drawdown Avg Weeks | Drawdown Avg Months | Drawdown Avg Loss(Daily) | Drawdown Avg Loss(Weekly) | Drawdown Avg Loss (Monthly) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| EWP | 31.76% | 15.88% | 0.70 | 0.03% | 40.00 | 0.00 | 0.19 | 2.57 | 2.37 | 2.00 | 2.10% | 185.52% | 41.83% |
| GMVP | 43.54% | 12.27% | 1.25 | 1.45% | 14.64 | 0.05 | 0.57 | 2.66 | 2.67 | 2.00 | 1.63% | 178.34% | 12.94% |
| MDP | 44.61% | 12.27% | 1.28 | 1.76% | 12.26 | 0.06 | 0.73 | 2.59 | 2.58 | 2.00 | 1.63% | 169.65% | 16.86% |
| MKT | 29.45% | 16.02% | 0.65 | 0.48% | 17.26 | 0.03 | 0.53 | 2.53 | 2.56 | 2.00 | 2.11% | 229.82% | 40.50% |
| NRPP | 35.08% | 15.19% | 0.81 | 0.27% | 38.08 | 0.00 | 0.13 | 2.58 | 2.38 | 2.00 | 2.06% | 199.03% | 46.61% |
| RPP | 35.39% | 14.36% | 0.87 | 0.48% | 33.03 | 0.01 | 0.06 | 2.55 | 2.29 | 2.00 | 1.95% | 173.66% | 39.81% |

## 2.2 DESCRIPTIVE STATISTICS FOR CANDIDATE PORTFOLIOS WITHIN SELECTED SUB-SAMPLES (ALSI)

Descriptive Stats for the Asian/Russian Crisis Period 1997-1999

| | RPP | NRPP | MKT | MDP | GMVP | EWP |
|---|---|---|---|---|---|---|
| Mean | -0.11% | -0.10% | -0.10% | -0.13% | -0.13% | -0.10% |
| Median | 0.00% | 0.00% | 0.02% | 0.00% | 0.00% | 0.04% |
| Maximum | 6.44% | 7.07% | 8.89% | 4.96% | 5.42% | 7.04% |
| Minimum | -10.84% | -11.96% | -13.79% | -8.35% | -9.26% | -11.70% |
| Std. Dev. | 1.61% | 1.70% | 1.87% | 1.55% | 1.56% | 1.71% |
| Skewness | -1.19829 | -1.26667 | -1.24614 | -0.93134 | -1.11422 | -1.14105 |
| Kurtosis | 10.14299 | 11.30224 | 12.74328 | 7.734835 | 8.788605 | 10.42772 |
| Jarque-Bera | 927.175 | 1230.635 | 1652.002 | 422.8409 | 628.4073 | 986.1913 |
| Probability | 0 | 0 | 0 | 0 | 0 | 0 |
| Sum | -0.41497 | -0.39851 | -0.3903 | -0.52222 | -0.52241 | -0.37744 |
| Sum Sq. Dev. | 0.101225 | 0.112994 | 0.136811 | 0.094239 | 0.095216 | 0.114354 |

Sample Size: 393 Observations

Descriptive Stats for the Subprime Crisis Period 2007-2010

| | EWP | GMVP | MDP | MKT | NRPP | RPP |
|---|---|---|---|---|---|---|
| Mean | -0.02% | 0.00% | -0.01% | -0.01% | -0.01% | -0.02% |
| Median | 0.00% | 0.00% | 0.00% | 0.00% | 0.01% | 0.01% |
| Maximum | 5.64% | 5.36% | 5.65% | 7.40% | 5.15% | 5.10% |
| Minimum | -8.23% | -6.34% | -6.44% | -8.47% | -7.83% | -7.48% |
| Std. Dev. | 1.59% | 1.23% | 1.25% | 1.85% | 1.50% | 1.45% |
| Skewness | -0.25866 | -0.09118 | -0.15018 | -0.15542 | -0.20892 | -0.23957 |
| Kurtosis | 5.075414 | 5.540354 | 5.766802 | 5.271778 | 5.005159 | 5.117318 |
| Jarque-Bera | 141.0613 | 200.0051 | 238.8166 | 162.1093 | 129.3537 | 145.3056 |
| Probability | 0 | 0 | 0 | 0 | 0 | 0 |
| Sum | -0.11758 | -0.02816 | -0.04887 | -0.09715 | -0.09221 | -0.13203 |
| Sum Sq. Dev. | 0.187325 | 0.112499 | 0.114873 | 0.252688 | 0.165915 | 0.156258 |

Sample Size: 741 Observations

Descriptive Stats for the Eurozone Crisis Period 2010-2013

| | EWP | GMVP | MDP | MKT | NRPP | RPP |
|---|---|---|---|---|---|---|
| Mean | 0.04% | 0.06% | 0.06% | 0.04% | 0.05% | 0.05% |
| Median | 0.06% | 0.05% | 0.05% | 0.04% | 0.07% | 0.07% |
| Maximum | 4.00% | 3.01% | 3.08% | 4.14% | 3.85% | 3.74% |
| Minimum | -4.05% | -3.01% | -2.80% | -3.86% | -3.94% | -3.73% |
| Std. Dev. | 1.00% | 0.77% | 0.77% | 1.01% | 0.96% | 0.90% |
| Skewness | -0.11024 | -0.27562 | -0.24817 | -0.08695 | -0.12389 | -0.16598 |
| Kurtosis | 4.605684 | 4.466187 | 4.364583 | 4.562715 | 4.694361 | 4.707118 |
| Jarque-Bera | 78.47667 | 73.30047 | 62.98985 | 73.86047 | 87.60118 | 90.3553 |
| Probability | 0 | 0 | 0 | 0 | 0 | 0 |
| Sum | 0.317578 | 0.435409 | 0.446093 | 0.294502 | 0.350802 | 0.353902 |
| Sum Sq. Dev. | 0.07169 | 0.042742 | 0.042769 | 0.072902 | 0.065553 | 0.058597 |

Sample Size: 718 Observations

## 2.3 TESTS FOR HOMOGENEITY OF VARIANCE

Test for Equality of Variances Between Series
Date: 02/04/14 Time: 21:45
Sample: 1 4809
Included observations: 4809

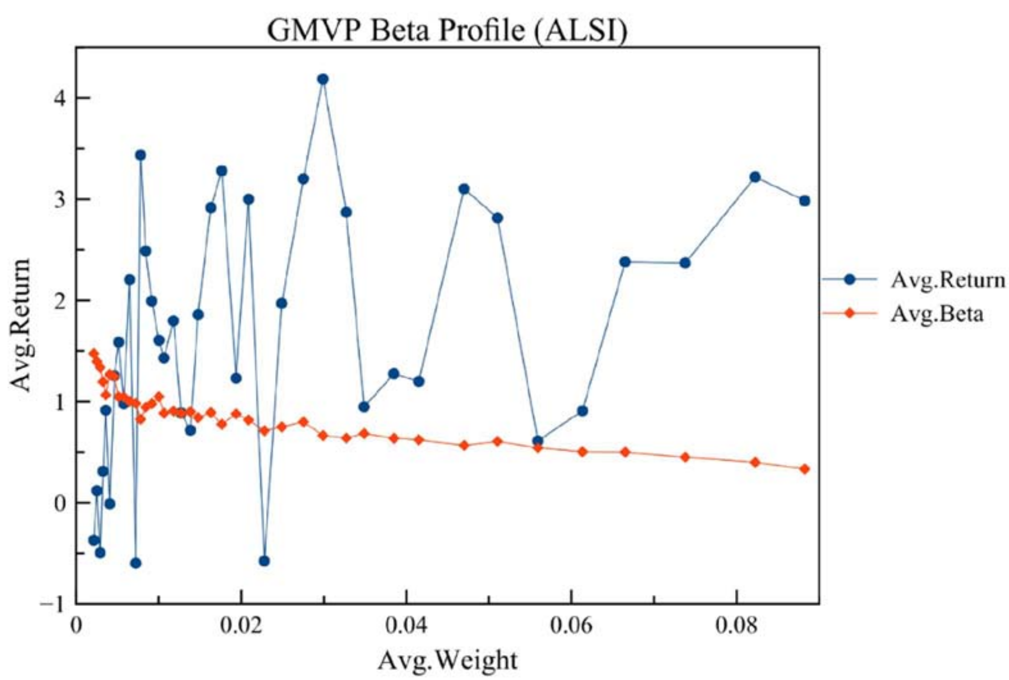
| Method | df | Value | Probability |
|---|---|---|---|
| Levene | (3, 6657) | 58.36794 | 0.0000 |
| Brown-Forsythe | (3, 6657) | 57.15346 | 0.0000 |

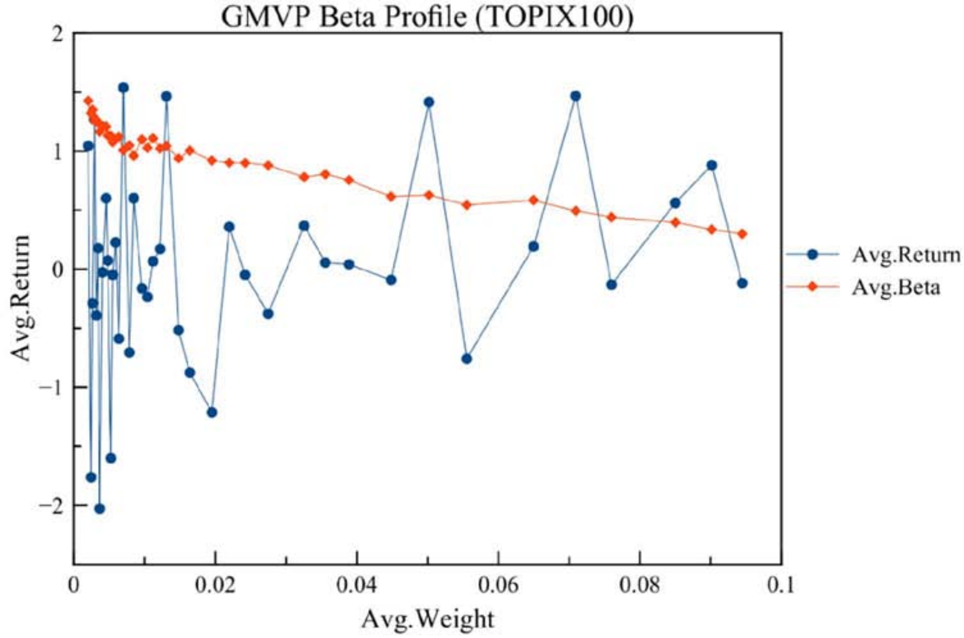
Category Statistics

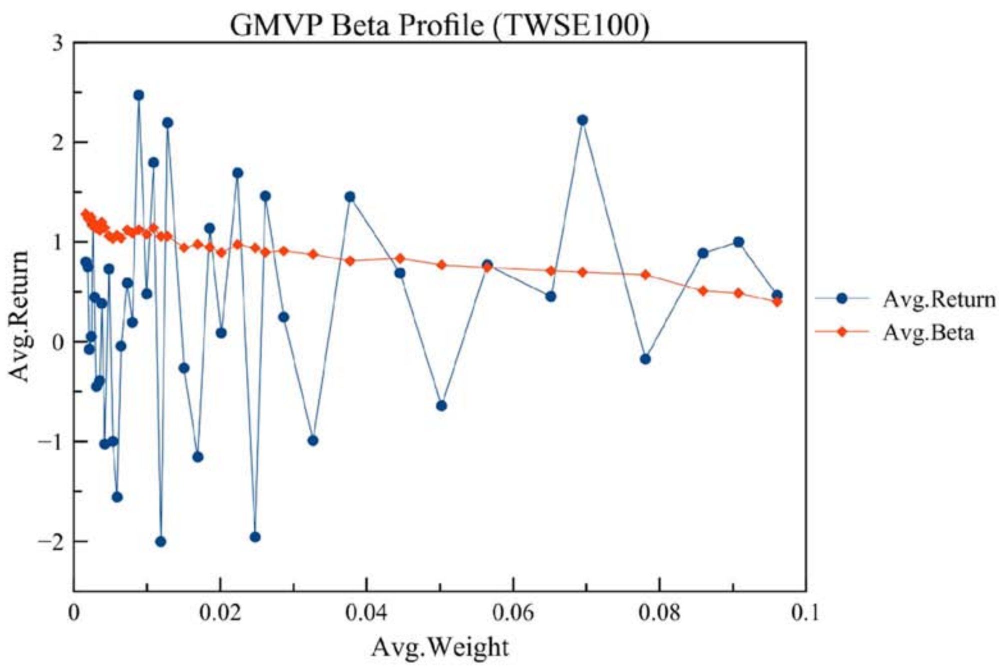
| Variable | Count | Std. Dev. | Mean Abs. Mean Diff. | Mean Abs. Median Diff. |
|---|---|---|---|---|
| ASIAN_RUSSIAN | 393 | 0.018828 | 0.012285 | 0.012178 |
| SUBPRIME | 741 | 0.018119 | 0.013130 | 0.013115 |
| MKT_OVERALL | 4809 | 0.013008 | 0.008986 | 0.008984 |
| EUROZONE | 718 | 0.010152 | 0.007409 | 0.007409 |
| All | 6661 | 0.013809 | 0.009471 | 0.009462 |

# APPENDIX 3: BETA PROFILES (1995-2013)

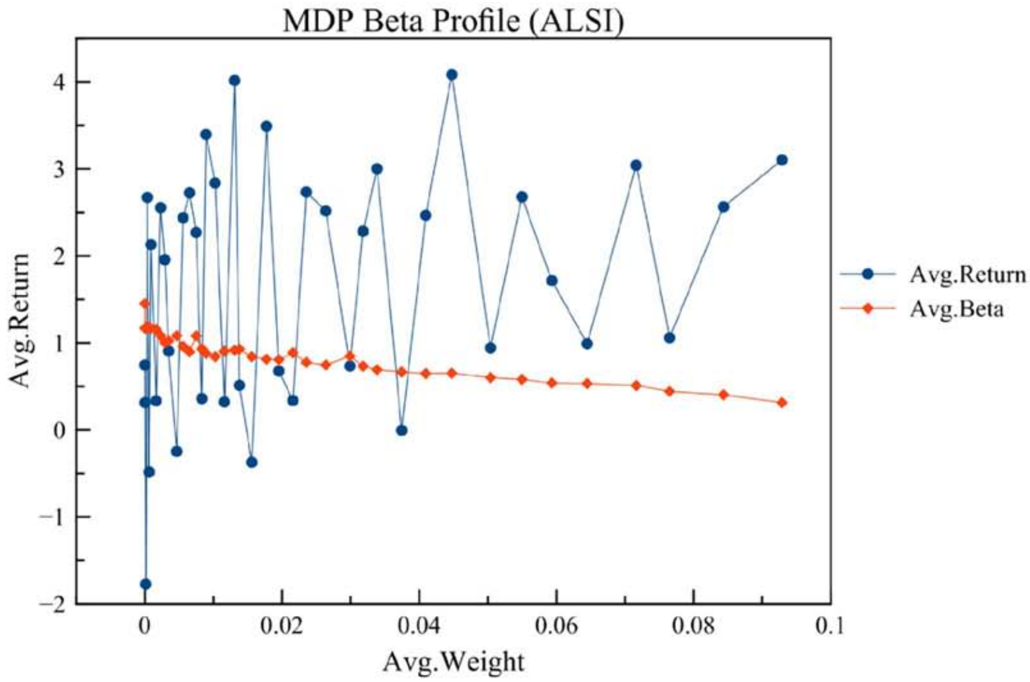
## 3.1 SELECTED GMVP BETA PROFILES

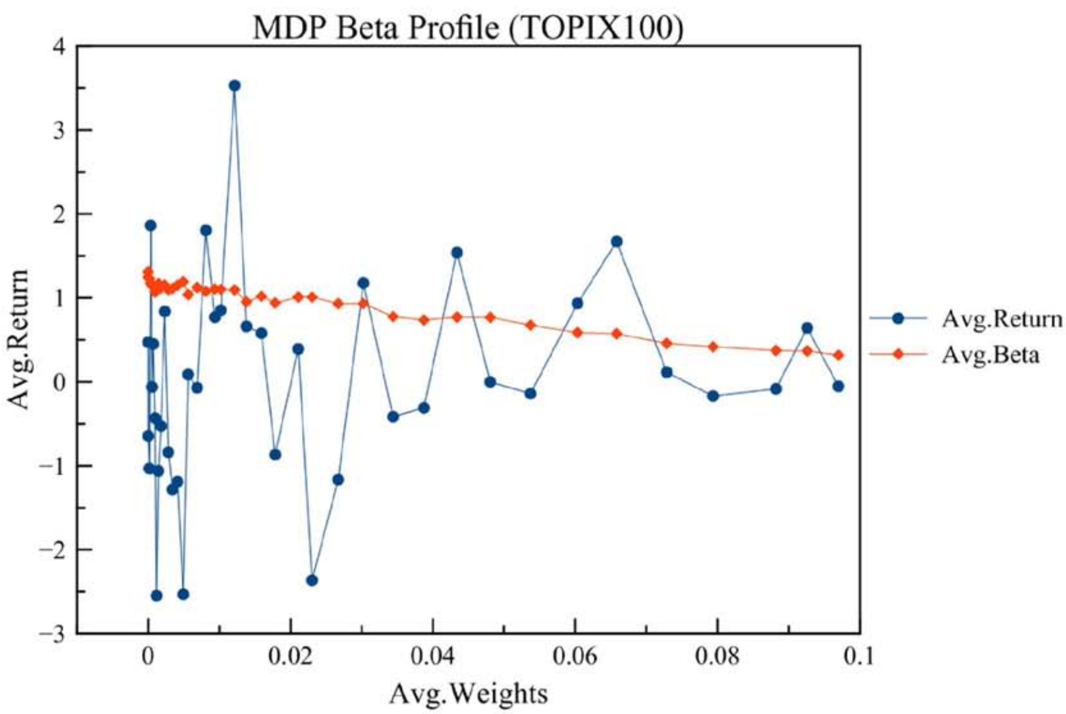


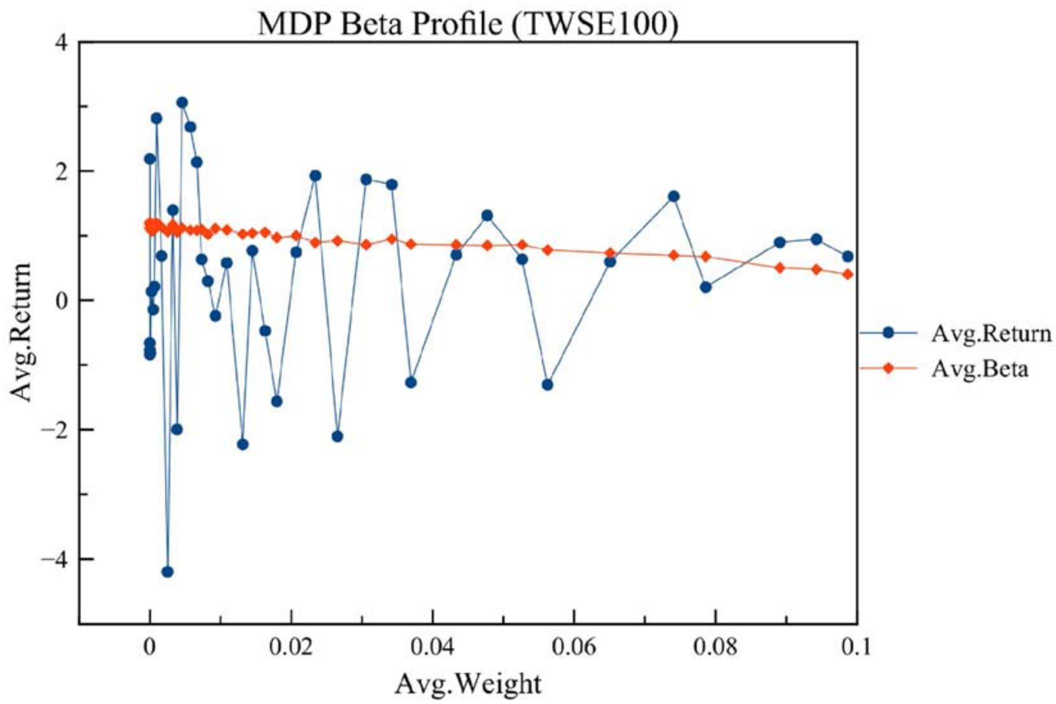

GMVP Beta Profile (TOPIX100)
Avg.Return
Avg.Weight
Avg.Return
Avg.Beta


GMVP Beta Profile (TWSE100)
Avg.Return
Avg.Weight
Avg.Return
Avg.Beta

## 3.2 SELECTED MDP BETA PROFILES






## APPENDIX 4: CONCENTRATION ANALYSIS

### 4.1 SPEARMAN RANK CORRELATION RESULTS

Covariance Analysis: Spearman rank-order
Date: 02/07/14 Time: 18:58
Sample: 1 12
Included observations: 12

| Correlation<br>Probability | EXCESS_RETURN | EFS | RANK | RETURN | RRC |
|---|---|---|---|---|---|
| EXCESS_RETURN | 1.000000<br>----- | | | | |
| EFS | 0.629371<br>0.0283 | 1.000000<br>----- | | | |
| RANK | -0.834510<br>0.0007 | -0.483137<br>0.1116 | 1.000000<br>----- | | |
| RETURN | 0.146853<br>0.6488 | -0.139860<br>0.6646 | -0.204967<br>0.5228 | 1.000000<br>----- | |
| RRC | -0.629371<br>0.0283 | -1.000000<br>NA | 0.483137<br>0.1116 | 0.139860<br>0.6646 | 1.000000<br>----- |

## 4.2 PEARSON CORRELATION RESULTS

Covariance Analysis: Ordinary
Date: 02/07/14 Time: 19:04
Sample: 1 12
Included observations: 12

| Correlation<br>Probability | EXCESS_RETURN | EFS | RANK | RETURN | RRC |
|---|---|---|---|---|---|
| EXCESS_RETURN | 1.000000<br>----- | | | | |
| EFS | 0.577944<br>0.0490 | 1.000000<br>----- | | | |
| RANK | -0.733401<br>0.0066 | -0.342453<br>0.2759 | 1.000000<br>----- | | |
| RETURN | 0.233162<br>0.4658 | -0.133290<br>0.6796 | -0.158112<br>0.6236 | 1.000000<br>----- | |
| RRC | -0.687083<br>0.0136 | -0.936944<br>0.0000 | 0.394806<br>0.2040 | 0.034361<br>0.9156 | 1.000000<br>----- |

## 4.3 GRANGER CAUSALITY TEST RESULTS

### 4.3.1 GRANGER CAUSALITY TEST ON EXCESS RETURNS

VAR Granger Causality/Block Exogeneity Wald Tests
Date: 02/09/14 Time: 00:38
Sample: 7/26/1996 11/02/2014
Included observations: 219

Dependent variable: RRC

| Excluded | Chi-sq | df | Prob. |
|---|---|---|---|
| RPP | 15.92480 | 9 | 0.0685 |
| NRPP | 24.14153 | 9 | 0.0041 |
| MDP | 6.677434 | 9 | 0.6707 |
| GMVP | 8.415196 | 9 | 0.4929 |
| EWP | 15.59479 | 9 | 0.0758 |
| EFS | 18.77259 | 9 | 0.0272 |
| All | 87.32164 | 54 | 0.0028 |

Dependent variable: RPP

| Excluded | Chi-sq | df | Prob. |
|---|---|---|---|
| RRC | 10.51648 | 9 | 0.3103 |
| NRPP | 12.42490 | 9 | 0.1904 |
| MDP | 4.419252 | 9 | 0.8817 |
| GMVP | 9.507673 | 9 | 0.3918 |
| EWP | 10.24970 | 9 | 0.3307 |
| EFS | 9.284432 | 9 | 0.4114 |
| All | 70.24501 | 54 | 0.0679 |

Dependent variable: NRPP

| Excluded | Chi-sq | df | Prob. |
|---|---|---|---|
| RRC | 12.89494 | 9 | 0.1674 |
| RPP | 9.666689 | 9 | 0.3781 |
| MDP | 4.074919 | 9 | 0.9064 |
| GMVP | 8.647016 | 9 | 0.4705 |
| EWP | 9.972951 | 9 | 0.3527 |
| EFS | 10.87905 | 9 | 0.2841 |
| All | 74.77297 | 54 | 0.0321 |

Dependent variable: MDP

| Excluded | Chi-sq | df | Prob. |
|---|---|---|---|
| RRC | 10.99788 | 9 | 0.2759 |
| RPP | 11.70466 | 9 | 0.2305 |
| NRPP | 13.21321 | 9 | 0.1532 |
| GMVP | 7.445864 | 9 | 0.5908 |
| EWP | 8.980697 | 9 | 0.4391 |
| EFS | 10.23812 | 9 | 0.3316 |
| All | 70.42510 | 54 | 0.0660 |

Dependent variable: GMVP

| Excluded | Chi-sq | df | Prob. |
|---|---|---|---|
| RRC | 13.29796 | 9 | 0.1496 |
| RPP | 14.20975 | 9 | 0.1151 |
| NRPP | 18.99663 | 9 | 0.0252 |
| MDP | 8.219610 | 9 | 0.5122 |
| EWP | 11.54587 | 9 | 0.2401 |
| EFS | 11.94799 | 9 | 0.2163 |
| All | 75.83523 | 54 | 0.0266 |

Dependent variable: EWP

| Excluded | Chi-sq | df | Prob. |
|---|---|---|---|
| RRC | 13.79834 | 9 | 0.1297 |
| RPP | 10.15219 | 9 | 0.3383 |
| NRPP | 12.45885 | 9 | 0.1887 |
| MDP | 3.997965 | 9 | 0.9115 |
| GMVP | 9.725035 | 9 | 0.3732 |
| EFS | 12.81171 | 9 | 0.1713 |
| All | 82.45792 | 54 | 0.0076 |

| Dependent variable: EFS | | | |
|---|---|---|---|
| Excluded | Chi-sq | df | Prob. |
| RRC | 15.01232 | 9 | 0.0906 |
| RPP | 19.52085 | 9 | 0.0211 |
| NRPP | 22.95304 | 9 | 0.0063 |
| MDP | 7.106265 | 9 | 0.6261 |
| GMVP | 7.674470 | 9 | 0.5673 |
| EWP | 13.14764 | 9 | 0.1560 |
| All | 81.02381 | 54 | 0.0101 |

## 4.3.2 GRANGER CAUSALITY TEST ON TOTAL RETURNS

VAR Granger Causality/Block Exogeneity Wald Tests
Date: 02/14/14 Time: 02:52
Sample: 7/26/1996 11/02/2014
Included observations: 223

| Dependent variable: RRC | | | |
|---|---|---|---|
| Excluded | Chi-sq | df | Prob. |
| RPP | 15.44516 | 5 | 0.0086 |
| NRPP | 16.38313 | 5 | 0.0058 |
| MDP | 3.111318 | 5 | 0.6828 |
| GMVP | 1.127796 | 5 | 0.9516 |
| EWP | 12.01823 | 5 | 0.0345 |
| EFS | 23.38911 | 5 | 0.0003 |
| All | 68.94892 | 30 | 0.0001 |

| Dependent variable: RPP | | | |
|---|---|---|---|
| Excluded | Chi-sq | df | Prob. |
| RRC | 8.561910 | 5 | 0.1279 |
| NRPP | 1.517400 | 5 | 0.9111 |
| MDP | 2.864638 | 5 | 0.7208 |
| GMVP | 7.031786 | 5 | 0.2183 |
| EWP | 3.732377 | 5 | 0.5886 |
| EFS | 7.477585 | 5 | 0.1875 |
| All | 46.81333 | 30 | 0.0259 |

| Dependent variable: NRPP | | | |
|---|---|---|---|
| Excluded | Chi-sq | df | Prob. |
| RRC | 10.49310 | 5 | 0.0624 |
| RPP | 9.137923 | 5 | 0.1037 |
| MDP | 2.578702 | 5 | 0.7646 |
| GMVP | 6.400566 | 5 | 0.2692 |
| EWP | 3.381975 | 5 | 0.6413 |
| EFS | 9.307886 | 5 | 0.0974 |
| All | 47.78524 | 30 | 0.0208 |

Dependent variable: MDP

| Excluded | Chi-sq | df | Prob. |
|---|---|---|---|
| RRC | 5.644760 | 5 | 0.3423 |
| RPP | 6.985285 | 5 | 0.2217 |
| NRPP | 1.943063 | 5 | 0.8570 |
| GMVP | 9.451705 | 5 | 0.0923 |
| EWP | 4.211796 | 5 | 0.5193 |
| EFS | 4.823390 | 5 | 0.4378 |
| All | 42.48491 | 30 | 0.0650 |

Dependent variable: GMVP

| Excluded | Chi-sq | df | Prob. |
|---|---|---|---|
| RRC | 6.384251 | 5 | 0.2706 |
| RPP | 6.861808 | 5 | 0.2311 |
| NRPP | 2.400055 | 5 | 0.7915 |
| MDP | 2.725483 | 5 | 0.7422 |
| EWP | 4.702733 | 5 | 0.4532 |
| EFS | 5.649626 | 5 | 0.3418 |
| All | 41.21829 | 30 | 0.0833 |

Dependent variable: EWP

| Excluded | Chi-sq | df | Prob. |
|---|---|---|---|
| RRC | 10.41285 | 5 | 0.0643 |
| RPP | 8.971505 | 5 | 0.1102 |
| NRPP | 1.834226 | 5 | 0.8716 |
| MDP | 3.251079 | 5 | 0.6613 |
| GMVP | 6.137358 | 5 | 0.2931 |
| EFS | 9.102562 | 5 | 0.1050 |
| All | 48.51661 | 30 | 0.0176 |

Dependent variable: EFS

| Excluded | Chi-sq | df | Prob. |
|---|---|---|---|
| RRC | 19.56779 | 5 | 0.0015 |
| RPP | 16.06357 | 5 | 0.0067 |
| NRPP | 14.30845 | 5 | 0.0138 |
| MDP | 2.353701 | 5 | 0.7983 |
| GMVP | 0.829794 | 5 | 0.9751 |
| EWP | 10.79919 | 5 | 0.0555 |
| All | 61.40214 | 30 | 0.0006 |

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