

**APPLICATION OF CEPSTRUM TECHNIQUES AND A
GUARD TUBE TO THE MEASUREMENT
OF THE NORMAL INCIDENT
SOUND POWER ABSORPTION COEFFICIENT
OF ROAD SURFACES IN-SITU.**

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SYNOPSIS

The work described in this thesis was directed towards studying signal processing techniques that could best be incorporated in an apparatus that was to measure the plane wave sound power absorption coefficient of road surfaces in-situ.

Road traffic noise has been identified as the greatest noise pollutant in the industrialised world with the tyre/road interaction being the major source of noise for traffic speeds in excess of 50 km/hr. Open pore bitumen asphalt material has been found to present a sound absorbing surface that is able to contribute to the mitigation of road traffic noise. This has generated research into the production of sound absorbing road surface materials which, in turn, has generated a need for an apparatus that is able to measure the sound power absorption coefficient of such materials both in the laboratory and in the field.

It was considered that the development of an easily transportable apparatus was needed which would enable a single, non-skilled operator to measure rapidly the normal incident sound power absorption coefficient, over a broad frequency band, of road surfaces, in-situ. This included, for example, the measurement of the absorption coefficient of open-pore asphalt materials developed in the laboratory, the measurement of newly laid surfaces, the comparison of different experimental surfaces, as well as determining the effect of contamination, over time, of the pores of open-pore asphalt by ingress of dust.

After considering various potential signal processing methods the work described in this thesis was primarily directed towards the study and application of cepstral signal processing techniques suitable for incorporation in the development of such a system. A prototype system for use in the laboratory was developed which generated a suitable sound signal which insonified the material under investigation, separated the reflected signal from the total signal received at a single microphone position and presented a complete spectrum of the sound power absorption coefficient within a short space of time.

Specifically, the sample impulse response was extracted from the power cepstrum of the combined signal containing the direct sound and the sound reflected from the sample. The extracted impulse response was Fourier transformed to produce the reflection coefficient from which the absorption coefficient over a frequency range of 100Hz to beyond 2000Hz was derived.

Laboratory measurements were conducted on several material samples with different sound absorbing properties. The results of these measurements correlated closely with theory and with measurements conducted on the same samples using a standard impedance tube method.

The work was directed at the adaptation of the well-known impedance tube method to permit the measurement of the material surface under investigation without disturbance of the surface. An important consideration was that field measurements were to be obtained without the removal of core specimens or the need to drive the tube into the surface which could otherwise alter the properties of the material. Measurements conducted with an outer, "Guard" tube surrounding the measuring tube indicated that such a condition could be met.

The prototype apparatus consisted of two 2 meter long concentric tubes with a loudspeaker located at one end and open at the other end for placement on the measurement surface. Sound from the loudspeaker propagated down the inner and outer tube. The purpose of the outer, "guard" tube was to provide an equilibrium of sound pressure on either side of the inner tube walls at the measurement surface thereby minimising the errors that would otherwise be introduced due to sound pressure leakage between the end of the inner, measuring tube and the material being measured.

Laboratory measurements performed with microphones placed within porous material placed at the end of the tubes showed there to be no significant difference in pressure amplitude and phase over the frequency range of interest. This provided confidence in the expectation that extended material surfaces could be measured with normal incident plane waves insonifying the material.

For the majority of experiments only the inner tube was used with the material to be measured being placed in a sample holder attached to the open end of the inner tube.

The results of the work conducted in this thesis showed that, of various methods studied, the cepstrum method was the most suitable for implementation in a portable, in-situ measurement system. The plane wave sound power absorption coefficient of a material could be accurately determined over a wide frequency range within a very short period of time. The measurement procedure was several orders of magnitude quicker than existing procedures. It was shown that this could be achieved automatically, with relatively small apparatus and without the need for calibration.

The thesis is organised in the following manner:

Chapter 1 presents the background to the need for the development of a portable measurement apparatus and the establishment of criteria to be met.

Chapter 2 surveys potential signal processing techniques that could be used in such an apparatus and establishes that the cepstrum method is the most suitable. The cepstrum theory is developed followed by a description of factors that will influence the size of the apparatus.

Chapter 3 describes the laboratory apparatus and the experimental procedure used to obtain the plane wave sound power absorption coefficient of material samples. Results of experiments using an outer, "guard" tube are included in this chapter.

Chapter 4 records the results of measurements on several samples displaying different sound absorbing properties. Comparisons of theoretical predictions and experimental results are presented and it is found that most features of the measured results can be accounted for.

The conclusions are presented in chapter 5. The main conclusion arrived at is that the cepstrum method is indeed suitable for inclusion in a portable measurement apparatus.

Several factors needing further investigation are presented in chapter 6.

Appendices are included in chapter 7.

Each chapter is self contained with a list of references referred to being appended at the end of each chapter.

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TABLE OF CONTENTS

SYNOPSIS	i
ACKNOWLEDGMENTS	iv
TABLE OF CONTENTS.....	v
LIST OF ILLUSTRATIONS	vii
LIST OF SYMBOLS.....	ix
1. INTRODUCTION.....	1
1.1. CRITERIA.....	4
1.2. REFERENCE.....	5
2. SURVEY OF POTENTIAL TECHNIQUES	6
2.1. TIME DELAY SPECTROMETRY, TDS.....	6
2.1.1 Principle	6
2.1.2 Advantage.....	7
2.1.3 Disadvantage	7
2.2. TRANSFER FUNCTION	
STANDARD METHOD: 2 microphones.....	8
2.2.1 Principle	8
2.2.2 Advantages.....	9
2.2.3 Disadvantages	9
2.3. TRANSFER FUNCTION	
ALTERNATIVE METHOD: 1 microphone	9
2.3.1 Principle	9
2.3.2 Advantages.....	9
2.3.3 Disadvantage	10
2.4. THE CEPSTRUM	10
2.4.1 Historical Background	10
2.4.2 Theory.....	12
2.4.3 Tube dimensions.....	18
2.4.4 Advantages.....	19
2.4.5 Disadvantage	19
2.5. DIRECTION OF PRESENT INVESTIGATION	19
2.6 REFERENCES.....	20
3. EXPERIMENTAL METHOD	22
3.1. DESCRIPTION OF APPARATUS.....	22
3.2. REDUCTION OF PRESSURE LEAKAGE BY USING A GUARD	
TUBE.....	24

3.3.	SYSTEM RESPONSE - INFLUENCE OF THE MEASUREMENT TUBE.....	25
3.4.	REDUCTION OF LOUDSPEAKER ECHO AND STANDING WAVES.....	26
3.5.	THE CHIRP SIGNAL USED IN THE CEPSTRUM EXPERIEMENTS.....	29
3.6.	CEPSTRUM SUBTRACTION	32
3.7.	INVERSE FILTERING	33
3.8.	LIFTERING OF ECHO FROM CEPSTRUM.....	37
3.9	REFERENCES.....	39
4.	MEASUREMENT OF DIFFERENT MATERIALS USING THE CEPSTRUM METHOD.....	40
4.1	ANECHOIC TERMINATION	41
4.2	SOUND REFLECTING 60MM THICK STEEL CYLINDER.....	42
4.3	60MM ROCKWOOL	46
4.4	POROUS ASPHALT SPECIMEN 7 - 3.....	50
4.5	POROUS ASPHALT SPECIMEN 9 - 5.....	53
4.6	DISCUSSION OF THE RESULTS.....	55
4.7.	REFERENCES.....	57
5.	CONCLUSIONS.....	58
6.	FURTHER WORK.....	60
7.	APPENDICES.....	62
7.1	APPENDIX I 3RD ORDER HIGH-PASS FILTER	62
7.2	APPENDIX II LISTING OF USER-DEFINABLE DISPLAY FUNCTIONS	63
7.2.1	CEPS	63
7.2.2	MODSWEEP.....	64
7.2.3	ALPHA3	65
7.2.4	TAPER.....	66
7.2.5	WIN3T.....	67

LIST OF ILLUSTRATIONS

FIG.	CAPTION	PAGE
1.1	Survey of seriously disturbed as a percentage of population in the Netherlands.	1
2.1	Sketch of a typical cepstrum.	14
2.2	Cepstra relating to three different materials.	15
2.3	Realisation of the absorption coefficient.	16
2.4	Frequency and impulse response of an 800 Hz high-pass filter.	17
2.5	Frequency and impulse response of a 200 Hz high-pass filter.	17
2.6	Absorption coefficient of a typical asphalt sample.	18
3.1	Sketch of apparatus used in the cepstrum experiments.	23
3.2	Relative sound pressure levels in tube with and without sound absorbing plug.	26
3.3	Insertion loss of single layers of three types of Rockwool mineral fibre.	27
3.4	Sound absorption coefficient of the 400 mm long anechoic termination.	28
3.5	Swept sine signal from B&K 3106 generator.	30
3.6	Cepstrum of 3.2 kHz sine signal.	31
3.7	Cepstrum of microphone signal and of microphone signal after subtraction of generator cepstrum.	32
3.8	Modified waveform and spectrum stored in user definable waveform generator.	35
3.9	Waveform and spectrum of modified sweep at the generator output.	36
3.10	Sketch of cepstrum showing rectangular and one-sided hanning window.	38
4.1	Two typical asphalt core specimens.	41
4.2	Absorption coefficient and windowed cepstrum of 400 mm long anechoic termination.	42
4.3	Absorption coefficient & cepstrum window of steel reflector.	43

4.4	Absorption coefficient & cepstrum window of steel reflector.	44
4.5	Comparison of absorption coefficient values of 60 mm thick steel termination determined using modified and unmodified chirp signals.	45
4.6	Cepstrum & absorption coefficient of 60 mm Rockwool - modified sweep; one-sided Hanning window.	46
4.7	Cepstrum & absorption coefficient of 60 mm Rockwool - unmodified sweep; one-sided Hanning window.	48
4.8	Cepstrum & absorption coefficient of 60 mm Rockwool - unmodified sweep; rectangular window.	48
4.9	Comparison of absorption coefficient values of 60 mm thick Rockwool type 201 measured by various cepstrum techniques & by impedance tube method.	49
4.10	Absorption coefficient & cepstrum window of asphalt sample specimen 3-7 using modified sweep & one-sided hanning window taper.	50
4.11	Absorption coefficient & cepstrum window of asphalt sample specimen 3-7 using normal sweep and one-sided hanning window taper.	51
4.12	Absorption coefficient & cepstrum window of asphalt sample specimen 3-7 using normal sweep and rectangular window.	51
4.13	Comparison of absorption coefficient values of asphalt specimen 7-3 measured by various cepstrum techniques & by impedance tube method.	52
4.14	Absorption coefficient & cepstrum window of asphalt sample specimen 9-5 using modified sweep & one-sided hanning cepstrum window taper.	53
4.15	Absorption coefficient & cepstrum window of asphalt sample specimen 9-5 using normal sweep & one-sided hanning cepstrum window taper.	54
4.16	Absorption coefficient & cepstrum window of asphalt sample specimen 9-5 using normal sweep & rectangular cepstrum window.	54
4.17	Comparison of absorption coefficient values of asphalt specimen 9-5 measured by various cepstrum techniques & by impedance tube method.	55
7.1	3rd order high-pass filter circuit.	62

LIST OF SYMBOLS

B	Filter bandwidth Hz
ΔT	Analysis time sec.
T	Travel time; maximum analysis time; sweep duration sec.
t_r	Filter delay time sec.
$H_{1,2}, H_{i,r}$	Acoustic transfer function
$p_t(t)$	Total sound pressure
$p(t)$	Direct sound pressure
$h(t)$	Sample impulse response
$P_t(\omega)$	Fourier transform of $p_t(t)$
$P(\omega)$	Fourier transform of $p(t)$
$H(\omega)$	Fourier transform of $h(t)$; reflection coefficient
\ln	naperian logarithm
τ	Delay time
$\hat{p}_t(t)$	Power cepstrum of the total sound signal
$\hat{p}(t)$	Power cepstrum of direct sound signal
n	Integer
N	Data length
α	Sound power absorption coefficient
α_r	Sound power reflection coefficient
ρ	Density kg/m^3
Ξ	Flow resistance Rayls
v_0	Voltage amplitude of swept sine signal
f_1	Start frequency Hz.
f_2	Stop frequency Hz.
$s(t)$	Swept sine signal
$s_m(t)$	Modified swept sine signal
$x(n)$	Discrete time function
$x(k)$	Discrete frequency function
$S(k)$	Discrete Fourier transform of $s(t)$
$c_o(n)$	Discrete cepstrum obtained from $S(k)$
$m(n)$	One-sided cepstrum obtained from $c_o(n)$

1. INTRODUCTION.

In virtually all industrialised countries the urban spread and the enormous increase in number of motor vehicles since the 2nd World War has resulted in road traffic becoming the Number 1 noise pollutant affecting the quality of life of almost everyone. An increasing number of residents in urban areas are consciously or unconsciously becoming disturbed by traffic noise. It affects the enjoyment of their property during the day and affects their sleep during night time. More and more health problems are being attributed to the stress that noise imposes on the human.

During the 1970's various countries commenced studying the effects of traffic noise in earnest and introduced legislation to reduce the levels of noise produced. Figure 1 shows the results of two extensive nation-wide surveys on the effects of traffic and other noise on the population conducted in The Netherlands in 1977 and again in 1987 [1]. A further survey is being conducted during 1993.

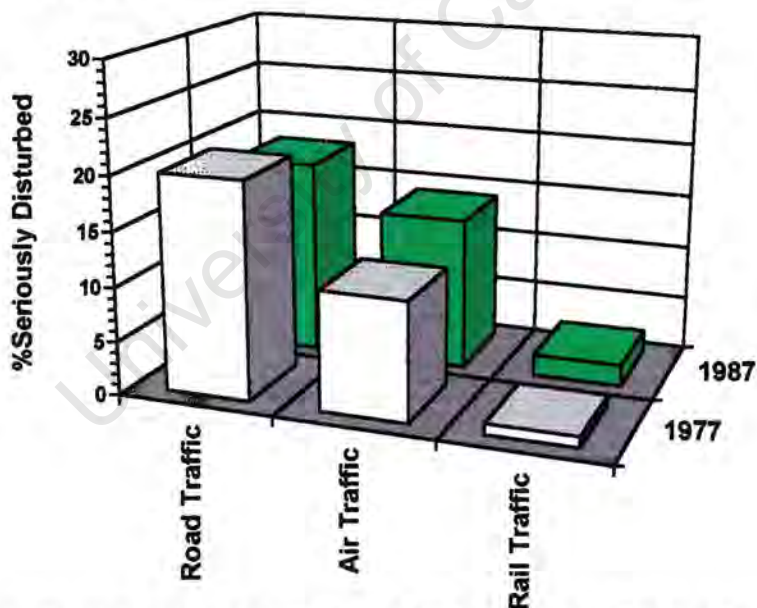


FIGURE 1.1 SURVEY OF SERIOUSLY DISTURBED AS A PERCENTAGE OF POPULATION IN THE NETHERLANDS.

It can be seen that the disturbance caused by road traffic noise was significantly higher than all other forms of traffic. It is interesting to note further that, although the number of road vehicles increased significantly between 1977 and 1987, the percentage of people seriously disturbed by road traffic noise decreased during the same period. This was in large part a result of

result of legislation having encouraged vehicle manufacturers to produce quieter vehicles.

Noise emission from road vehicles is composed of several different noise producing mechanisms. These can be divided into the following three main categories: [2,3]

1. Power-train - including engine, air intake, exhaust, gearbox and fan.
2. Tyre/road interaction - due to tyre vibration and air-pumping between the rolling tyre and road surface.
3. Aerodynamic noise - due to air turbulence around the vehicle.

The first two mechanisms are the primary sources of noise emission.

The power-train is the primary source of noise at low vehicle speeds especially during acceleration and deceleration at road intersections. At higher speeds the tyre/road interaction noise becomes increasingly dominant. In this regard it is interesting to note that early publications reported this to occur at speeds in excess of approximately 70 km/hr [4,5]. However, in a recent publication tyre/road interaction noise is reported to dominate at speeds in excess of 50 km/hr [2]. This phenomenon may be ascribed to reduced power-train noise in newer vehicles. Tyre/road interaction noise is thus increasingly becoming the dominant source of traffic noise. In order to reduce the overall road traffic noise emission increasing efforts are needed to seek ways of reducing tyre/road interaction noise.

The noise due to the interaction of the rolling vehicle tyre with the road surface produces a broadband noise spectrum with a peak between 500 Hz and 1500 Hz depending on the road surface texture [5].

It has been found that certain porous asphalt road surfaces can significantly reduce this "rolling noise" as it is termed by reducing the amount of sound reflected off the surface. Research is being conducted into altering material properties of asphalt, such as the stone size and shape, porosity and binding material to optimise the sound absorbing properties by producing a sound absorption spectrum coinciding with the noise spectrum.

This research has generated the need for a portable instrument capable of rapid, in-situ measurement of the sound absorption coefficient of the road surface material during the various stages of its life. Namely, during the design stage in the laboratory, during the measurement of small test surfaces, during control measurements of newly laid road surfaces, as well as later on during the maintenance stage of the road.

This need is presently being addressed by the International Organisation for Standardisation, ISO, which recently formed a new Working Group, ISO/TC 43/SC WG 38 with the task of producing an International Standard entitled: "Procedure for measuring sound absorption properties of road surfaces - In-situ method."

An interim report [6], describing much of the work contained in the present thesis, was submitted by the author to the convenor of Working Group 38, Dr. A. von Meier. In response, an invitation was extended to the author to become a member of the Working Group. At the first meeting of WG 38, held on 1st and 2nd June 1993, in Oslo, Norway, the method described in this thesis was presented by the author as a candidate method. Several other candidate methods were also presented by representatives from different countries in the world. At the meeting the method described here was considered the most likely to fulfil the requirements relating to the new standard.

An edited version of the thesis, describing the application of cepstrum signal processing techniques to in-situ sound absorption measurements was presented at a recent conference [7].

The work described in this thesis was directed at investigating the principle and application of potential techniques suitable for the subsequent development of an apparatus that could measure the sound absorption coefficient of road surfaces in-situ. The apparatus was to be easily transportable and to be placed on the road surface by a single, non-skilled operator, without the need for any preparation of the surface or any special consideration needing to be given to the measurement process such as contact between apparatus and the measurement surface. Associated signal generation and analysis electronics built into the apparatus was to be so designed that the operator could, by the press of a button, rapidly obtain a

complete spectrum of the absorption coefficient of the surface being measured.

It was considered that an adaptation of the well-known impedance tube method would best enable measurement of the material surface to take place without disturbance of the surface that could otherwise result in alteration of the properties of the material. This would permit results to be compared directly with measurements conducted by the standardised impedance tube method [8] but would replace the existing time consuming procedure whereby several 100mm diameter core specimens were removed from the road surface and subsequently taken to a laboratory for analysis.

At the commencement of the project it was considered that such an apparatus would need to satisfy certain criteria. These same criteria were subsequently contained in the requirements formulated by the ISO Working Group 38. These are set out in the following paragraph.

1.1. CRITERIA

- 1.1.1 Measurement of absorption coefficients is to be within 0.05 of that measured by standardised impedance tube method[8] in the frequency range 200 Hz to 2 kHz.
- 1.1.2 The measurement surface is to be insonified by plane, normal incident sound waves with constant pressure & phase across the measurement surface so as to be comparable with the standardised impedance tube method. The measurements are to be conducted without disturbing the surface and without the need for special preparation of the surface.
- 1.1.3 The apparatus is to be transportable by a single person hence,
 - weight to be ≤ 20 kg and,
 - length to be equal to, or less than a man's height, thus $\leq 1,7$ m,
- 1.1.4 The measurements are to be performed by a non-skilled person, thus,
 - operation is to be simple,
 - there is to be no need for manual adjustments or calibration,

- the measurement procedure and read-out is to be performed automatically,
- the apparatus is to be of robust construction.

1.1.5 Measurements must be able to be performed in a reasonably noisy environment.

1.2. REFERENCES.

1. van den Berg, M. "Het voorspellen van hinder door geluid van wegverkeer". Geluid, Nr 4, December 1992 147-151
2. Jabben, J. & von Meier, A. "Een nieuw geluidarm wegdek voor binnenstedelijke gebieden." Geluid, Nr 1 March 1992 19-22
3. Sandberg, U. Low noise Road surfaces. A state-of-the-art review. Swedish Road and Traffic Research Institute. 31 March 1992.
4. von Meier, A., & Heerkens, J. "Minder verkeerslawaaai." Kunstof en Rubber, Nr 3 1984 38-41
5. von Meier, A. "Het wegdek als absorberende laag." NAG publ. Nr 84 April 1987 39-53
6. Jongens, A.W.D. "Application of Cepstrum Techniques and a guard tube to the measurement of the normal incident sound absorption coefficient of road surfaces in-situ." Unpublished report, March 1993.
7. Jongens, A.W.D. "Application of Cepstrum Techniques and a guard tube to the measurement of the normal incident sound absorption coefficient of road surfaces in-situ." Proc. IEEE South African Symposium on Computation and Signal Processing, COMSIG 93 6 August 1993 25-30
8. Standard Test Method for Impedance and Absorption of Acoustical Materials by Impedance Tube Method. ASTM C384-90a

2. SURVEY OF POTENTIAL TECHNIQUES

Several potential techniques that could be used in a modified impedance tube and that could possibly meet some or all of the stated criteria were investigated. These were:

1. Time Delay Spectrometry - utilising 1 microphone
2. Transfer function - standard method utilising random noise and 2 microphones,
3. Transfer function - alternative method utilising pseudo-random noise (maximum length sequence) and 1 microphone.
4. Cepstrum - utilising 1 microphone,

A brief survey of alternatives 1, 2 and 3 are presented summarising advantages and disadvantages of each technique as perceived at the commencement of the project. The treatment of each is not in any way intended to be exhaustive but is considered sufficient in order to determine whether they would comply with the stated criteria. Each of these techniques is covered in depth in the literature. The more relevant references are included at the end of the chapter. This is followed by technique 4, the Cepstrum method. As this method is less well covered in the literature in so far as it pertains to the present application it is introduced in somewhat greater detail.

In the assessment of the techniques at the end of the chapter the decision is made to investigate the Cepstrum technique in greater depth.

2.1. TIME DELAY SPECTROMETRY, TDS

2.1.1 Principle

Heyser[1] provides a detailed explanation of time delay spectrometry and its application. Two [2,3] of several articles describing the techniques and application of TDS appear in the list of references at the end of this chapter.

A chirp signal is emitted by the loudspeaker at a sweep speed of S Hz/sec.

A band pass tracking filter with filter width of B Hz is synchronised with the frequency of the emitted frequency with a delay of t_r seconds after emission of the chirp signal so as to analyse the reflected signal received by the microphone over the same bandwidth.

The signal path length, d , between the direct signal and reflected signal and that due to any other reflecting surfaces, such as the loudspeaker, must be such that there is no overlap of the reflected signal within the same bandwidth as the direct sound received by the microphone.

The frequency resolution, B , which is the smallest bandwidth resolvable and also the lowest measurable frequency, is related to the analysis time, ΔT , by,

$$B = 1/\Delta T$$

For a desired resolution bandwidth of 200Hz, and thus also the lowest frequency, the travel time, $T = 1/B = 5\text{msec}$. Assuming a sound speed of 344 m/s the microphone will therefore need to be located no less than 0.86m from the sample.

T is also the maximum analysis time permitted to avoid overlap of the signal band and hence also the maximum filter delay time, t_r .

For smallest frequency resolution the microphone should be placed as far from the sample as possible, thus bringing it nearer to the loudspeaker. This location will, however, be closest to any rear reflecting surfaces and the sound reflected up the outer tube. Unless these are adequately attenuated the tube length will need to be extended to 1.72m with the microphone located midway between loudspeaker and sample.

2.1.2 Advantages

- Single microphone location,
- Relatively simple signal processing,

2.1.3 Disadvantage

- Relatively long tube length required.

2.2. TRANSFER FUNCTION STANDARD METHOD: 2 MICROPHONES

2.2.1 Principle

A broadband stationary random signal is decomposed into its incident and reflected components using a transfer-function relation between the acoustic pressures measured simultaneously at two locations in the tube by means of two microphones [4,5].

Decomposition of the wave leads to the determination of the complex reflection coefficient from which the acoustic impedance and absorption coefficient are derived. Thus,

$$\alpha = 1 - \left| \frac{H_{12} - H_i}{H_r - H_{12}} \right|^2$$

where,

H_{12} = acoustic transfer function evaluated from the pressures at the two microphone positions.

$H_{i,r}$ = acoustic transfer function associated with the incident and reflected wave component, respectively, evaluated at two microphone positions.

The potential of the method is limited by several interrelated factors which may produce unacceptably large errors especially when measuring highly reflective materials. An error analysis of the technique was first made by Seybert and Soenarko [6] followed by a more detailed analysis by Bodén and Åborn [7]. Besides being influenced by background noise, which is not negligible in the intended application, the method suffers from bias errors caused by inadequate spectral resolution unless the microphones are placed close together. However, choosing a smaller microphone separation leads to larger errors for low frequencies. Large errors also occur at a microphone separation approaching a half wavelength. Random errors can become significant unless high coherence is maintained between the acoustic source and the pressure at the microphones. The latter requires very careful calibration procedures.

2.2.2 Advantages

- Short tube length needed
- Broad frequency band measurements
- System commercially available.

2.2.3 Disadvantages

- Two microphones are required.
- Very close magnitude & phase tolerance required between microphones.
- Accurate location of the two microphones needed.
- Above aspects result in relatively high cost of the system.
- Microphones need to be switched for calibration purposes.

2.3. TRANSFER FUNCTION

ALTERNATIVE METHOD 1 MICROPHONE

2.3.1 PRINCIPLE

The elaborate calibrating procedure and errors [6,7] associated with phase-mismatching of two microphones may be eliminated by employing a single microphone and replacing a stationary random noise with a deterministic, pseudo-random, or maximum length sequence (MLS) signal [8,9]. The signal processing remains essentially the same as for the two microphone method.

The sound pressures at two locations no longer need to be measured simultaneously but can be measured sequentially using a single microphone. However, the microphone still needs to be moved from one location to another.

2.3.2 Advantages

- Reduced error potential compared to 2-microphone method.
- Reduced cost compared to 2-microphone method.
- Software relatively simple.

2.3.3 Disadvantage

- Single microphone still needs to be handled.

2.4. THE CEPSTRUM

2.4.1. Historical Background

Cepstrum analysis is a name given to a range of similar techniques involving functions which can be considered as a "spectrum of a logarithmic spectrum" [10]. It found its origin in the analysis of seismic waves when it was proposed as a better alternative to the autocorrelation function for the detection of echoes in seismic signals. It was first defined in 1963 by Bogert et. al. [11] as the "power spectrum of the logarithmic power spectrum" and a new set of terms were introduced of which all were anagrams of the equivalent frequency domain terms. The most commonly used are as follows:

CEPSTRUM	corresponds to	SPECTRUM
quefrequency		frequency
rahmonic		harmonic
saphe		phase
lifter		filter

Further derivations are "short-pass lifter" from "low-pass filter" and "long-pass lifter" from "high-pass filter".

Bogert and Ossanna [12] developed the heuristics of cepstrum analysis for extracting echoed signals from noise. A more general formalism of separating convolved signals and its relation with cepstrum analysis was treated by Oppenheim [13] and is treated in depth in a text book by Oppenheim and Schaffer [14].

Childers [15] defined the power cepstrum as the "inverse Fourier transform of the logarithmic power spectrum". This differs from the original definition in that the result of the second Fourier transformation is not modified by obtaining the amplitude squared at each quefrequency. It is thus reversible back to the logarithmic spectrum. This latter definition is now most commonly used and is the form used in this thesis.

A further definition is the "complex cepstrum" [16] which is the inverse Fourier transform of the complex logarithm of the complex spectrum. Despite its name it is a real-valued function of time, differing from the power cepstrum primarily in that it uses phase as well as logarithmic amplitude information at each frequency in the spectrum. It is thus reversible to a time function (from which the complex spectrum is obtained by direct Fourier transformation).

The present thesis is based on the work conducted by Bolton & Gold[17] and by Bolton[18] who consider in detail the application of the technique specifically to the measurement of the acoustical properties of material. In Bolton's experiments [18] an omnidirectional loudspeaker source and a microphone were located some distance above the plane surface to be measured. The reflected sound was assumed to appear to originate at a source image located below the plane. He assumed that the spherical waves propagating from the point source could be taken to reflect as plane waves save for an attenuation due to spherical spreading.

It is interesting to note that in his Introduction Bolton ruled out the use of the impedance tube as a basis for the cepstrum method. Amongst his reasons were the unacceptable disturbance of the material when samples are cut to fit the tube or cutting a ring into the material to fit the tube so as to minimise sound pressure leakage at the end of the tube in contact with the material. It will be shown in the present thesis that the guard tube overcomes these difficulties.

Bolton's use of a small loudspeaker to approximate a point source introduced an inherent limitation in his work in not providing sufficient low frequency energy. This limited his frequency response from 2 kHz to 10 kHz. With additional refinements he was able to extend this from 300 Hz to 19 kHz. Measurements of the sound absorption properties of materials are normally conducted in the frequency range 100 Hz to 4 kHz. Absorption properties at higher frequencies are less important, whether considering outdoor sound propagation or room acoustics, due to the predominance of atmospheric absorption. The frequency range of Bolton's work thus does not cover the range applicable to many audio phenomena including the application considered in the present thesis. It is shown that the low frequency response of the measurement system needs to extend down to at least 200 Hz and preferably an octave or more below this.

2.4.2. Theory

This section presents the cepstrum of a microphone located in a tube receiving a sound from a loudspeaker source at one end of the tube convolved with an echo received from a material located at the other end.

A microphone located in a tube, between a loudspeaker sound source at one end and a reflecting material sample at the other, receives a total acoustic pressure given by,

$$p_r(t) = p(t) + p(t)*h(t - \tau) \quad \dots\dots (1)$$

where,

- $p(t)$ is the contribution arriving directly from the loudspeaker,
- $h(t)$ is the impulse response of the reflecting material,
- τ is the delay resulting from the path length difference between the direct and reflected signals.

The two components of the signal may be separated by employing the fact that a Fourier transform converts a convolution in the time domain into a product in the frequency domain. The final result, the absorption coefficient, is a power related quantity therefore the squared modulus of the Fourier transform of the time history is used. Hence,

$$|P_r(\omega)|^2 = |P(\omega)|^2 [1 + H(\omega) e^{-j\omega\tau}] [1 + H^*(\omega) e^{+j\omega\tau}] \quad \dots\dots (2)$$

where ω is the radian frequency and the asterisk indicates the complex conjugate.

Taking the natural logarithm converts the product into an addition as follows,

$$\ln|P_r(\omega)|^2 = \ln|P(\omega)|^2 + \ln[1 + H(\omega) e^{-j\omega\tau}] + \ln[1 + H^*(\omega) e^{+j\omega\tau}] \quad \dots\dots (3)$$

The last two terms of this expression may be expanded by using the series representation of the naperian logarithm[19], namely,

$$\ln[1+z] = z - (z^2/2) + (z^3/3) - \dots \quad \dots (4)$$

This series is valid for the logarithm to the base e provided that $-1 < z \leq 1$ ie. $-1 < H(\omega) \leq 1$ where $H(\omega)$ is the reflection coefficient. It is thus valid for completely reflecting surfaces, when $H(\omega) = 1$, and for completely absorbent surfaces, when $H(\omega) = 0$.

An inverse Fourier transform produces a series representation of the power cepstrum, a time domain function,

$$\begin{aligned} \hat{p}_i(t) = & \hat{p}(t) + h(t-\tau) - h(t-\tau)*h(t-\tau)/2 + \dots \\ & + h(-t-\tau) - h(-t-\tau)*h(-t-\tau)/2 + \dots \quad \dots (5) \end{aligned}$$

Here $\hat{p}_i(t)$ and $\hat{p}(t)$ are the power cepstra of the total and direct signals, respectively. In practice the measured direct signal, $\hat{p}(t)$, is the result of the convolution of the electrical signal with the impulse response of the sound generation equipment, predominantly the loudspeaker, and the signal acquisition hardware such as the anti-aliasing filters.

An idealised sketch of a cepstrum is shown in Figure 1. At the left is shown the power cepstrum of the direct sound followed by $h(t-\tau)$ the impulse response of the sample occurring at a time $+\tau$ determined by the difference in path length between the arrival of the direct signal and that from the sample. This is followed by the second and subsequent harmonics separated by τ in each case as derived from equation (5). Also shown in the diagram is the impulse response of the loudspeaker which arrives at the microphone at a time slightly greater than 2τ after the direct arrival of sound; here occurring just after the second harmonic.

Equation (5) indicates that the power cepstrum is real and even and thus that the impulse response and harmonics appear at both positive and negative times. Most analysis and simulation software display the cepstrum with the positive time record on the left half of the plot from points 0 to $(N/2-1)$ and the negative time mirrored on the right side of the plot from points $N/2$ to $(N-1)$.

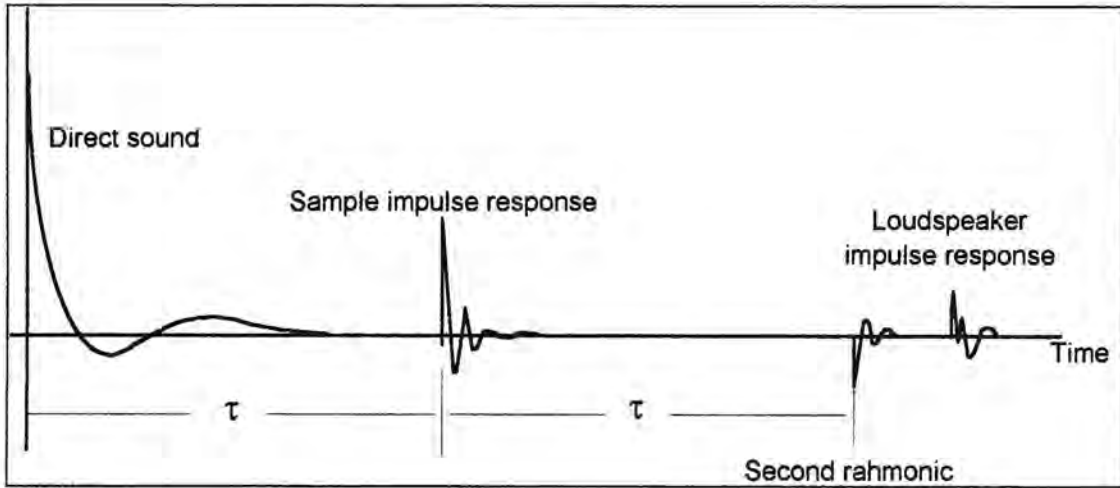


FIGURE 2.1 SKETCH OF A TYPICAL CEPSTRUM

Provided that the cepstrum amplitude of $\hat{p}(t)$ decays to an insignificant value with respect to $h(t)$ within the time delay, τ , it should be possible to extract the sample impulse response by means of a suitable windowing function - or bandpass lifter.

A further condition is that the series in equation (5) converges rapidly enough so that cepstral aliasing is insignificant, i.e., that the negative time rahmonics do not overlap the positive time rahmonics within the selected time window.

This is illustrated in Figure 2.2 which displays the cepstra from 0 to 32 ms. of three different material samples that will be described in Chapters 3 and 4. The Figure also displays the dependence of the convergence rate of the series on the absorption properties of the sample being measured.

The three traces are:

- Upper: A highly sound reflecting steel termination.
- Middle: An asphalt sample displaying absorption over limited frequency bands.
- Lower: Rockwool displaying high absorption at high frequencies.

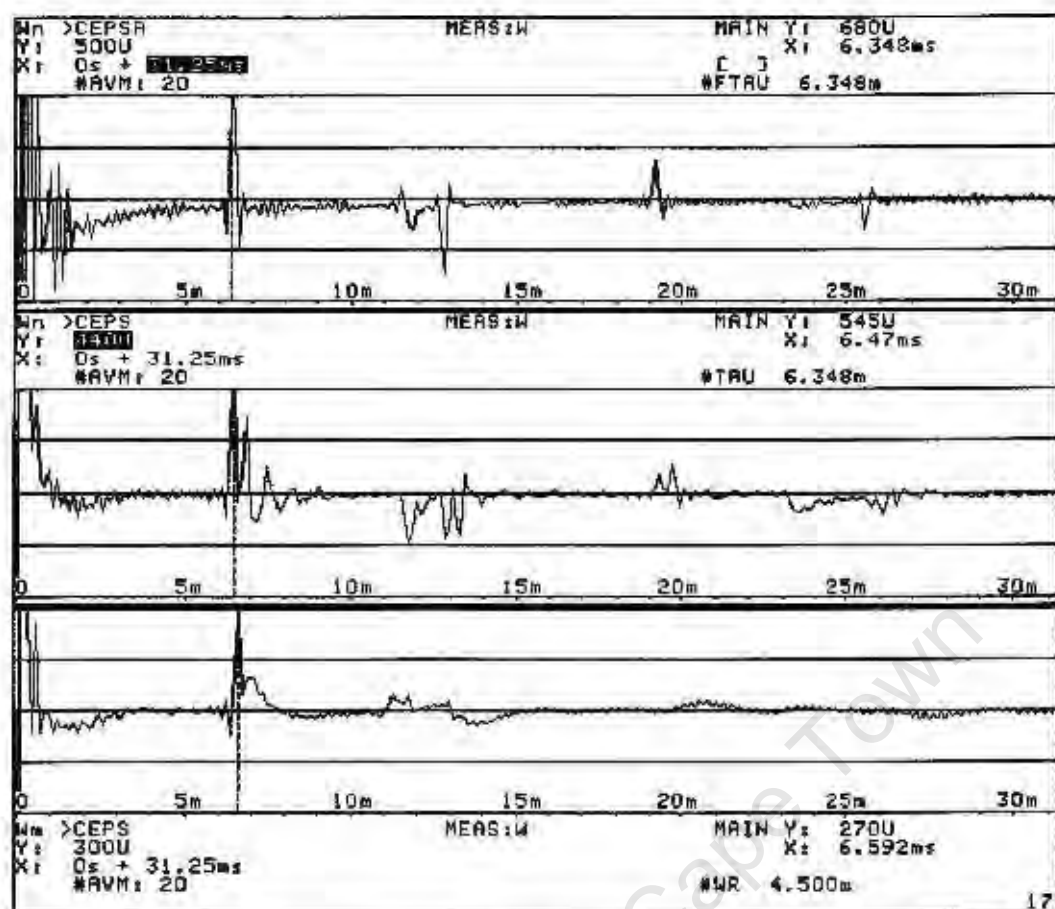


FIGURE 2.2 CEPSTRA RELATING TO THREE DIFFERENT MATERIALS.

Upper Trace:	Steel reflecting surface.	Vertical scale: 500U
Middle Trace:	Asphalt sample 9 - 5	Vertical scale: 440U
Lower Trace:	60mm thick Rockwool.	Vertical scale: 300U

In all cases the cepstrum of the direct sound is shown on the left decaying to an insignificant level prior to the sample impulse response which occurs at time $\tau = 6,348, 6,47$ and $6,592$ ms. respectively. The 2nd and higher harmonics occur at integer multiples, n , of these times. The impulse response of the loudspeaker echo is shown at $11,5$ ms. with its harmonics occurring at 23 ms.

For a highly reflecting material (upper trace) the amplitude of the harmonics are seen to decrease slowly with increasing n . The negative time cepstrum consists of a mirror image of the positive time cepstrum and extends contiguously to the right of the cepstrum shown. It is not difficult to appreciate that the negative time harmonic extending into the positive time will still have sufficient amplitude to contaminate the sample impulse response.

The middle and lower traces show a significantly more rapid decrease in amplitude with increasing n with associated decreased contamination. The reader should note the amplified vertical scale of the two lower traces compared with the upper trace.

The ramifications of cepstrum aliasing will be considered in more detail in chapter 4.

Once the sample impulse response has been successfully filtered Fourier transformation gives, directly, the sound power reflection coefficient, α_r , of the sample from which the sound power absorption coefficient, α , can readily be calculated.

The realisation of the absorption coefficient, α , is summarised in Figure 2.3.

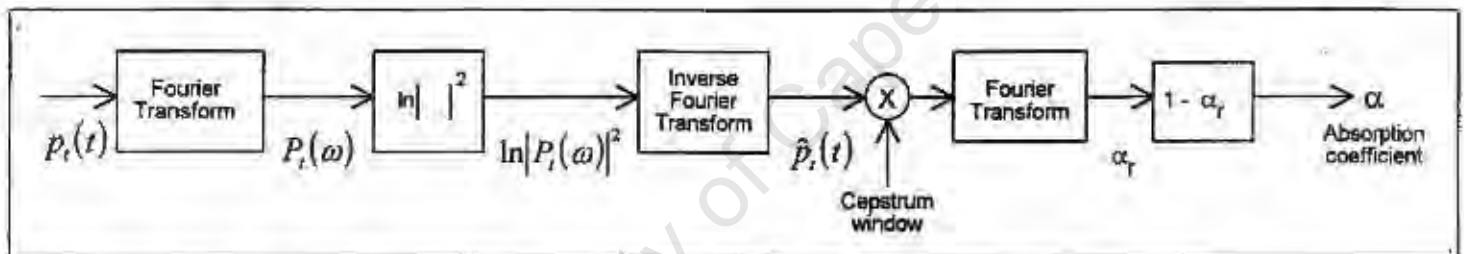


FIGURE 2.3 REALISATION OF THE ABSORPTION COEFFICIENT.

The decay rate of $\hat{p}(t)$ is dependent on the Nyquist frequency of the analyser provided that the log spectrum of the signal is flat from 0 Hz to beyond the Nyquist frequency. Any sharp variation from a smooth spectrum will produce slowly decaying oscillations in the cepstrum determined by the frequency at which the irregularity occurs. These may have sufficient amplitude to corrupt the impulse response at τ .

The frequency response of practical systems do not extend to 0 Hz but roll off below a cut-off frequency greater than 0 Hz. The cepstrum will thus contain an oscillation with a period determined by the cut-off frequency and whose cepstrum amplitude decay rate will depend on the roll-off rate in the spectrum. The minimum value of τ for insignificant contamination of $h(t)$ to take place in the cepstrum is thus dependent on the period of the low frequency cut-off and the roll-off rate below cut-off.

Insight into the effects of these two parameters can be gained by simulation of the impulse response of a high pass filter.

Figure 2.4 shows the effect of two different roll-off rates, of order 1 (dotted line) and 32 (continuous line), respectively, on the impulse response of a high-pass Butterworth filter with cut-off frequency of 800 Hz. A rapid frequency roll-off produces slowly decaying oscillations in the impulse response with a first zero-crossing at half a period of 0,625 msec. The amplitude of the first order filter decays to an insignificant value within this time.

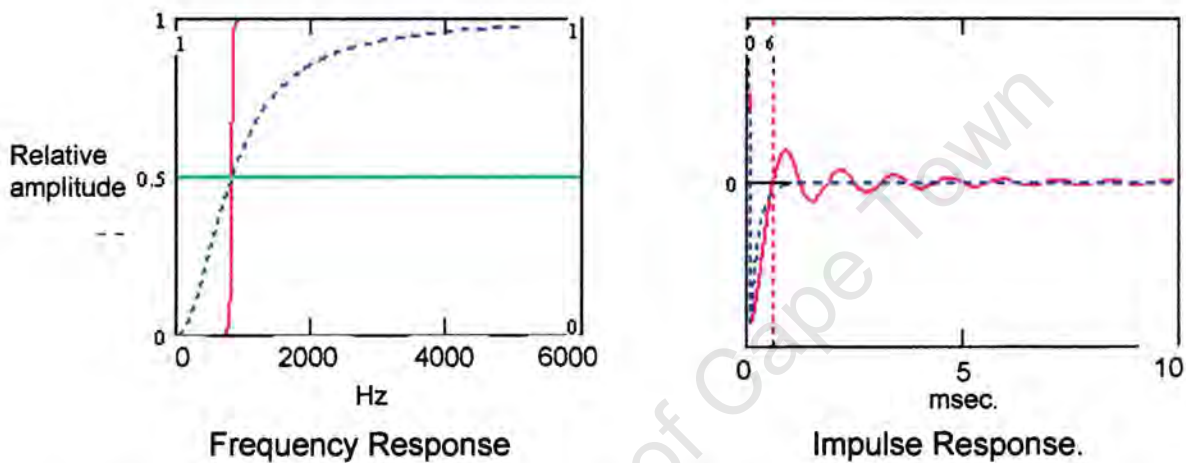


FIGURE 2.4 FREQUENCY AND IMPULSE RESPONSE OF AN 800 Hz HIGH-PASS FILTER OF ORDER 1 (dotted line) AND 32 (continuous line).

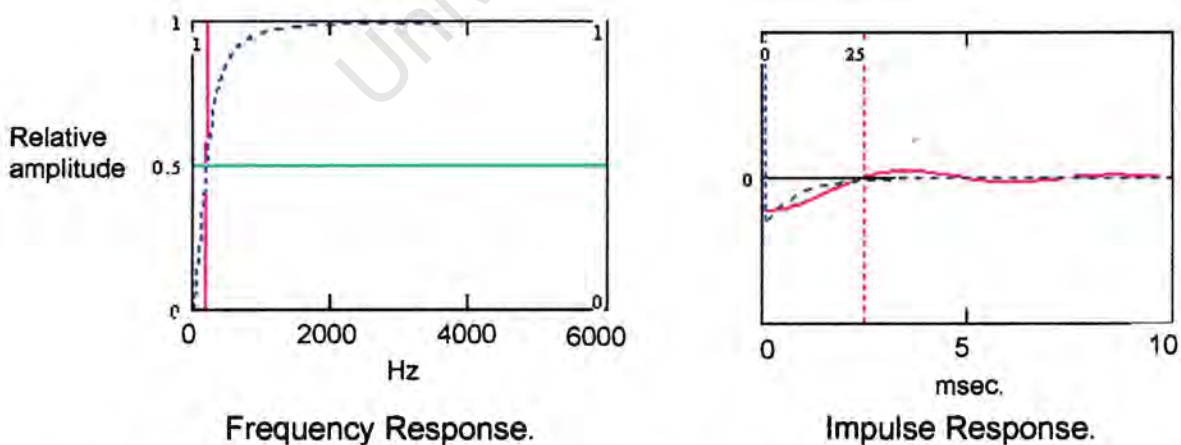


FIGURE 2.5 FREQUENCY AND IMPULSE RESPONSE OF A 200 Hz HIGH-PASS FILTER OF ORDER 1 (dotted line) AND 32 (continuous line).

Figure 2.5 displays the impulse response of two Butterworth filters of the same orders as previously but with the cut-off frequency reduced by a factor of four to 200 Hz. It can be seen that the impulse response amplitude of the first order filter again decays to insignificance within half a period but the actual time is increased by a factor of four to 2,5 msec.

2.4.3. Tube dimensions

Figure 2.6 displays the result of absorption coefficient measurements [20] conducted in an impedance tube on a typical asphalt sample. Each of the two absorption peaks display a bandwidth of approximately 200Hz. Similar results were obtained on other samples.

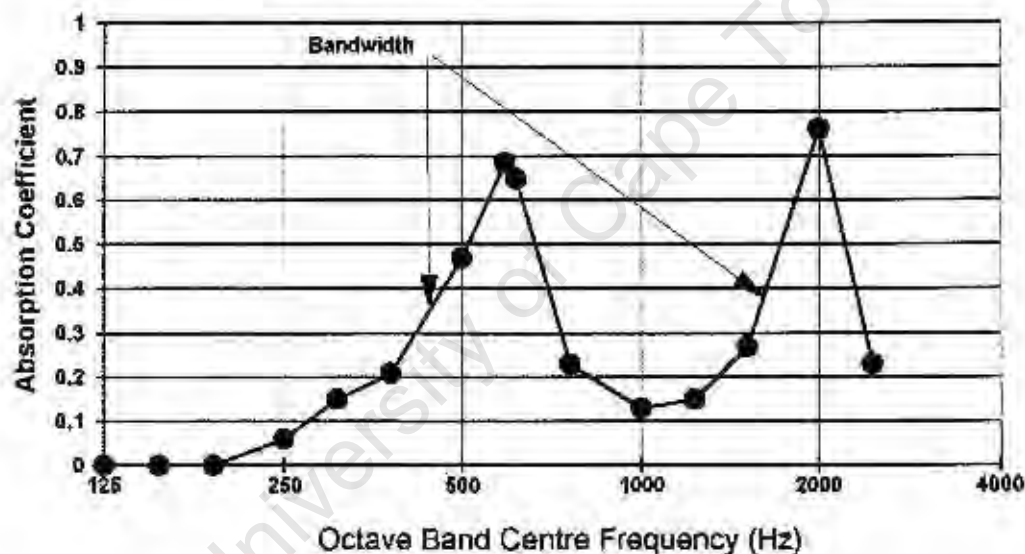


FIGURE 2.6 ABSORPTION COEFFICIENT OF A TYPICAL ASPHALT SAMPLE

Based on the results of such measurements it was considered that in order to display results with the same resolution the required bandwidth and hence the low-frequency cut-off of the measurement system would need to be equal to or smaller than 200 Hz. Provided that one can avoid the oscillations produced by a rapid low-frequency roll-off rate the minimum delay, τ , between pulse and sample echo must be no less than that equivalent to half a period at 200 Hz. This is 2,5 msec. as determined at the top of the page. Assuming a sound speed of 344 m/s the shortest path length between direct

pulse and echo must be 0,86 m. The length of the tube between microphone and test surface must consequently be no less than $\frac{1}{2} \times 0,86$, i.e. 0.43 m.

The impulse response due to echoes from other reflecting surfaces, such as the loudspeaker, occur after the sample impulse response at a time dependent on the path length. For successful lifting of the sample impulse response the amplitude of the impulse response of such later echoes must be insignificant compared with the sample impulse response or, alternatively, such response must occur after the sample impulse response at a further time $\geq \tau$.

The shortest permissible tube length is thus 2×0.43 namely, 0,86m, with the microphone located midway between loudspeaker and sample.

2.4.4 Advantages

- Single microphone at one location.
- No calibration procedure.
- Short tube length.

2.4.5 Disadvantage

- None foreseen.

2.5. DIRECTION OF PRESENT INVESTIGATION

The two transfer function methods were excluded from further consideration as the need for calibration and switching around of the microphone(s) violated one of the important criteria. The TDS method is available commercially and could thus be considered a likely candidate. However, initial considerations described in the section 2.4.3 indicated that the cepstrum method would require a tube length half that required for the TDS method thus providing a clear advantage for employing the cepstrum method.

For this reason it was decided to study the cepstrum method in greater depth.

2.6 REFERENCES.

1. Heyser, R.C. Acoustical measurements by time delay spectrometry. J. Audio Engineering Soc. Vol. 15 p. 370 (Oct 1967).
2. Biering, H. & Pedersen, O.Z. System analysis and time delay spectrometry. Brüel & Kjaer Technical Review parts 1 & 2 (1983).
3. Kok, B.H.M. Praktische toepassing van time delay spectrometry. NAG publikatie nr 78, 31-39 (1985).
4. Chung, J.Y. & Blaser, D.A. Transfer function method of measuring in-duct acoustic properties. I Theory. J. Acoust. Soc. Am. 68(3), 907-913 (1980).
5. Chung, J.Y. & Blaser, D.A. Transfer function method of measuring in-duct acoustic properties. II Experiment. J. Acoust. Soc. Am. 68(3), 914-921 (1980).
6. Seybert, A.F. & Soenarko, B. Error analysis of spectral estimates with application to the measurement of acoustic parameters using random sound fields in ducts. J. Acoust. Soc. Am. 69(4) 1190-1199 (1981).
7. Bodén, H. & Åborn, M. Influence of errors on the two-microphone method for measuring acoustic properties in ducts. J. Acoust. Soc. Am. 79(2) 541-549 (1986).
8. Chu, W.T. Transfer function techniques for impedance and absorption measurements in an impedance tube using a single microphone. J. Acoust. Soc. Am. 80(2) 555-560 (1986).
9. Chu, W.T. Further experimental studies on the transfer-function technique for impedance tube measurements. J. Acoust. Soc. Am. 83(6) 2255-2260 (1988).
10. Randall, R.B. Frequency Analysis. Brüel & Kjaer 3rd Edition. Sept. 1987. Chapter 8: Cepstrum Analysis.
11. Bogert, B.P., Healy, M.J.R. & Tukey, J.W. The quefrequency alanysis of time series for echoes: Cepstrum, pseuo-autocovariance, cross-cepstrum and saphe cracking. Proc. Symp. of the Time Series Analysis. M. Rosenblatt (Ed.) Wiley N.Y. 209 - 243 (1963).

12. Bogert, B.P. & Ossanna J.F. The heuristics of a stationary complex echoed gaussian signal in stationary gaussian Noise. IEEE Trans. Information Theory IT-12, No 3, 343 (1966)
13. Oppenheim, A.V. Nonlinear filtering of convolved signals. M.I.T. Quart. Progr. Rept. No 80, 168-175 (1966).
14. Oppenheim, A.V. & Schafer, R.W. Digital Signal Processing. Prentice Hall Int. 1975. Chapter 10.
15. Childers, D.G.P., Skinner, D.P. & Kemerait, R.C. Proc. IEEE, 65(10) 1428 (1977)
16. Oppenheim, A.V., Schafer, R.W. & Stockham, T.G. Non-linear filtering of multiplied and convolved signals. IEEE Trans. Audio & Electr-acoustics, AU-16 No 3 Sept. 1968.
17. Bolton, J.S. & Gold, E. The application of cepstral techniques to the measurement of transfer functions and acoustical reflection coefficients. J. Sound & Vibration 93(2) 217-233 (1984).
18. Bolton, J.S. Cepstral techniques in the measurement of acoustic reflection coefficients, with application to the determination of acoustic properties of elastic porous material. Doctoral thesis submitted to the Institute of Sound & Vibration Research, Faculty of Engineering and Applied Science, University of Southampton. July 1984.
19. Selby, S.M. CRC Standard Mathematical Tables. 1975 pg. 472.
20. von Meier, A., van Blokland, G.J. & van Bochove, G.G. Onderzoek Proefvak Welschap - Hoofdrapport. M+P Raadgevende Ingenieurs b.v. report submitted to the Netherlands Ministry of Housing, Physical Planning and Environment. 2 December. 1991

3. EXPERIMENTAL METHOD

This chapter describes the apparatus used for the experiments conducted in the laboratory. Brief experiments are described which were conducted to determine the feasibility of using an outer guard tube to eliminate pressure leakage between the tube lip and the material being measured. The influence of the inner measurement tube on the frequency response of the applied signal are described as well as attempts to reduce the amplitude of sound reflecting off the loudspeaker. Various aspects relating to the cepstrum method are then described commencing with the generation of a chirp signal with a flat frequency response over the measurement frequency range. This is followed by attempts at reducing contamination of the cepstrum and the process of extracting the impulse response of the sample from which the absorption coefficient is obtained.

3.1 DESCRIPTION OF APPARATUS

The experimental apparatus is shown in Figure 3.1.

All cepstrum related measurements were conducted using samples mounted in the end of a 2 meter long, 110mm external diameter and 2.2mm thick PVC tube mounted centrally within a 2meter long, 160mm diameter and 3.2mm thick PVC outer tube. The 105.6mm internal diameter of the inner, measuring, tube corresponded closely with the 105mm diameter of the asphalt sample holder used in conventional impedance tube measurements.

The outer tube was attached by means of a standard PVC plumbing flange fixed around the opening of a loudspeaker cabinet by means of four bolts. The cabinet was constructed from 15 mm thick plywood and comprised an internal volume of 14.6 litre loosely filled with mineral wool.

A Visaton type WSP 21 S 8 inch cone loudspeaker was mounted on the inside of the cabinet opening. The choice of loudspeaker was determined by the need for a flat frequency response from below the lowest measurement frequency to beyond the highest measurement frequency.

All samples were mounted inside sample holders comprising lengths of 110mm diameter PVC tube long enough to contain the sample and 60mm thick steel backing piece which formed a sound reflecting termination behind the sample. The back of each sample holder was closed off with a standard PVC pipe termination. The front of the sample holder was connected flush with the end of the main tube by means of a standard PVC sleeve.

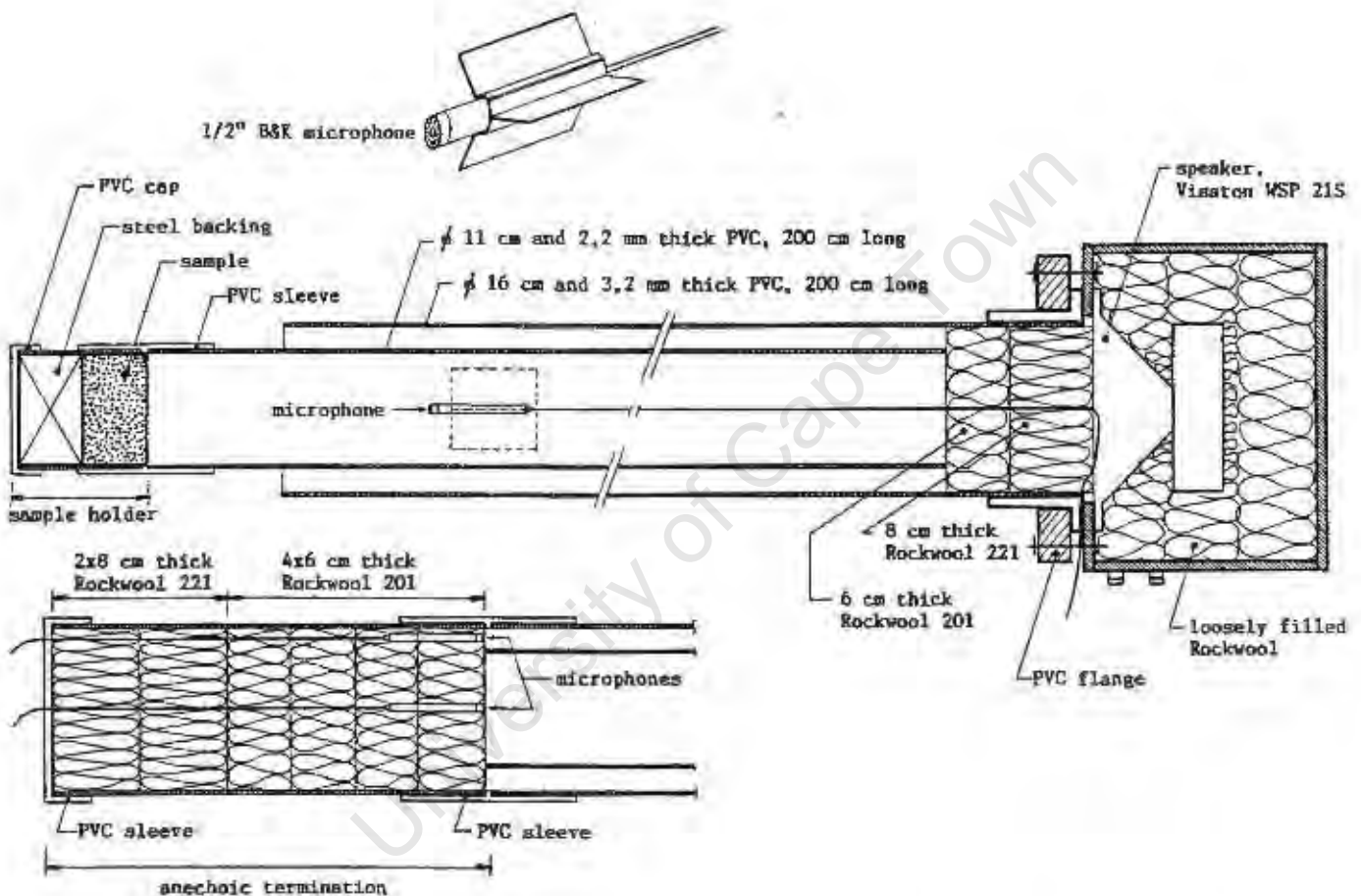


FIGURE 3.1 SKETCH OF APPARATUS USED IN THE CEPSTRUM EXPERIMENTS. Showing:

- Microphone in holder and centring device;
- Sample holder connected to the inner measuring tube and surrounded by the outer tube;
- anechoic termination with two microphones for pressure leakage tests;

Where necessary tape was wound around the sample in order to provide an air tight seal. In certain cases these measures appeared to be inadequate as will be discussed later in the relevant section.

The generation and analysis of the test signals was performed using a Brüel & Kjaer two channel type 3550 Analyzer System fitted with a Brüel & Kjaer type 3106 Generator & Sampling Module. The data records were 2048 points in length. The airborne signal in the inner, measuring, tube was received by a Brüel & Kjaer type 4165 ½ inch microphone connected to the input of the analyzer. The microphone was placed in a custom made holder to ensure central location in the tube.

The Brüel & Kjaer analyzer generator combination is a very versatile instrument. Besides containing a very large number of predefined signal processing routines it provides for the programming of User Definable Functions, UDF. Up to 60 programming lines can be contained in the internal memory. Numerous routines are provided on disc which can be down-loaded into memory. The Taper Function was one such routine incorporated to perform a Hanning window in the extraction of the cepstrum echo impulse response.

For control purposes chirp signals were also generated using an IEC Function Generator. Being an analog device the chirp sweep duration was infinitely variable whereas that of the Brüel & Kjaer 3106 generator could only be altered in discrete steps determined by the upper frequency limit of the sweep. Thus the duration of a 0Hz to 6.4kHz chirp was 125msec whilst that of a 0Hz to 3.2kHz chirp was 250msec.

3.2 REDUCTION OF PRESSURE LEAKAGE BY USING A GUARD TUBE.

Conventional impedance tube measurements require an air-tight seal between the sample to be tested and the surrounding tube so as not to introduce errors in the measurement. Placing the end of the tube onto an extended porous surface will result in pressure leakage at the pipe lip and within the depths of the material thereby presenting a different impedance to the end of the tube.

Kosten[1] initially proposed the use of an outer, guard tube surrounding the measuring tube. It was suggested that insonifying the outer tube with the same signal as the inner tube would ensure that there was no pressure

differential between inner and outer tube and hence no pressure leakage. The material surface would then appear to the inner tube as being contained or sealed within it.

Preliminary measurements were conducted with two identical, calibrated microphones, one being positioned in the centre and the other at the perimeter of the "anechoic" termination as shown in Figure 3.1.

Discrete sine waves from a manually adjusted Kenwood sine wave generator were radiated by the loudspeaker and propagated down both inner and outer tubes. The signal of the microphones at the end of both tubes was measured simultaneously by means of a dual-beam oscilloscope. Within the frequency range of 100Hz to 2000Hz the two traces on the oscilloscope remained virtually superimposed as one trace. There was thus no significant difference in amplitude or phase between the two measured signals.

The results gave confidence that the principle of a guard tube could be used to reduce or eliminate pressure leakage at the tube/sample interface. Extensive testing would need to be conducted on various porous surfaces once a prototype instrument had been constructed that could be used in the field.

3.3 SYSTEM RESPONSE - INFLUENCE OF THE MEASUREMENT TUBE.

The need for a flat frequency response within the measurement range of the system has been explained in Chapter 2. In practice this is not a trivial achievement. Cone loudspeakers are designed to provide a flat frequency response above the loudspeaker/cabinet resonant frequency. However, placing a tube in front of such a loudspeaker will, below a certain frequency determined by the pipe diameter, provide an increase in radiation resistance and consequently an increase in acoustic pressure with decreasing frequency[2,3].

This is shown by the upper, continuous curve in Figure 3.2 which displays the relative sound pressure levels in the frequency range 200Hz to 1kHz measured in the 160mm diameter tube fitted with the "anechoic" termination. The loudspeaker signal was generated by a Kenwood sine wave generator

with the frequency being manually adjusted and the microphone signal amplitude read off an oscilloscope screen. The amplitudes are displayed on a dB scale relative to a maximum value at 240Hz.

The curve shows that the sound pressure decreases by approximately 8dB per octave up to 1kHz as shown by the -8dB/octave curve with the broken line. This is due to the radiation loading of the tube. Above 1kHz the mean sound pressure level remained constant but the measured values are not shown so as to present a clearer graph. The lower two curves are discussed in section 3.4.

The superimposed "undulations" are due to the effect of phase interference between sound waves propagating past the microphone in opposite directions in the tube causing standing waves at multiples of the 85Hz fundamental resonance frequency of the tube[4]. This indicates that the amplitude of sound reflecting off the so called "anechoic" termination used was still of sufficient magnitude to produce standing waves in the tube.

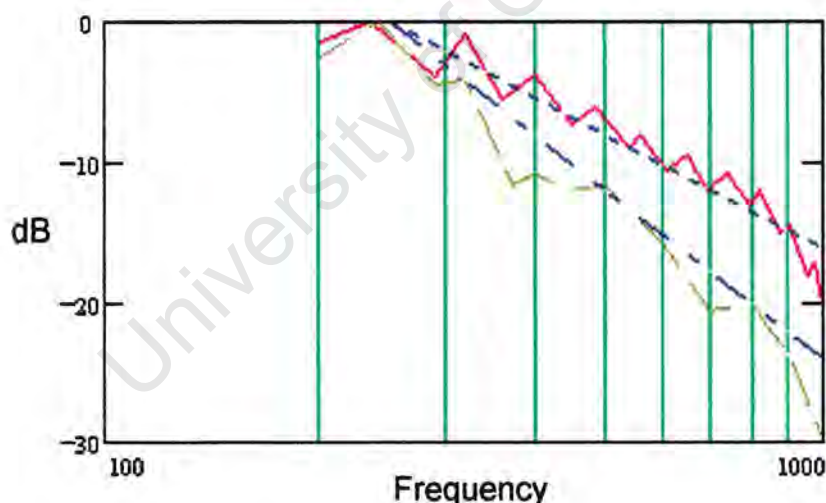


FIGURE 3.2 RELATIVE SOUND PRESSURE LEVELS IN TUBE WITH AND WITHOUT SOUND ABSORBING PLUG

3.4 REDUCTION OF LOUDSPEAKER ECHO AND STANDING WAVES.

The desirability of reducing the amplitude of the loudspeaker echo response in the cepstrum has been explained. Consideration was given to inserting a

sufficiently thick sound absorbing plug, or "Drosseldämpfer"[5] in the tube immediately in front of the loudspeaker in order to reduce the loudspeaker echo response in the cepstrum to a level where it would not significantly influence the sample impulse response. It was initially estimated that an echo reduction of at least 40dB and hence a single-pass insertion loss of at least 20dB would be required.

Sound insertion loss measurements were conducted within the 160mm diameter tube in the frequency range of 200Hz to 1000Hz on three different types of Rockwool mineral fibre with differing density, ρ , and flow resistance, Ξ . The values of ρ and Ξ were obtained from the Rockwool manufacturer's brochure "Geluidabsorptie met Rockwool" dated 5-9-1989. An Ξ value for type 123 could not be found. The results of measurements on each sample are displayed in Figure 3.3. These correlate closely to the results presented by Schmidt[5] and support the well known fact that it becomes increasingly more difficult to attenuate sound at low frequencies with attenuators of small dimensions relative to the wavelength here occurring below 400 Hz.

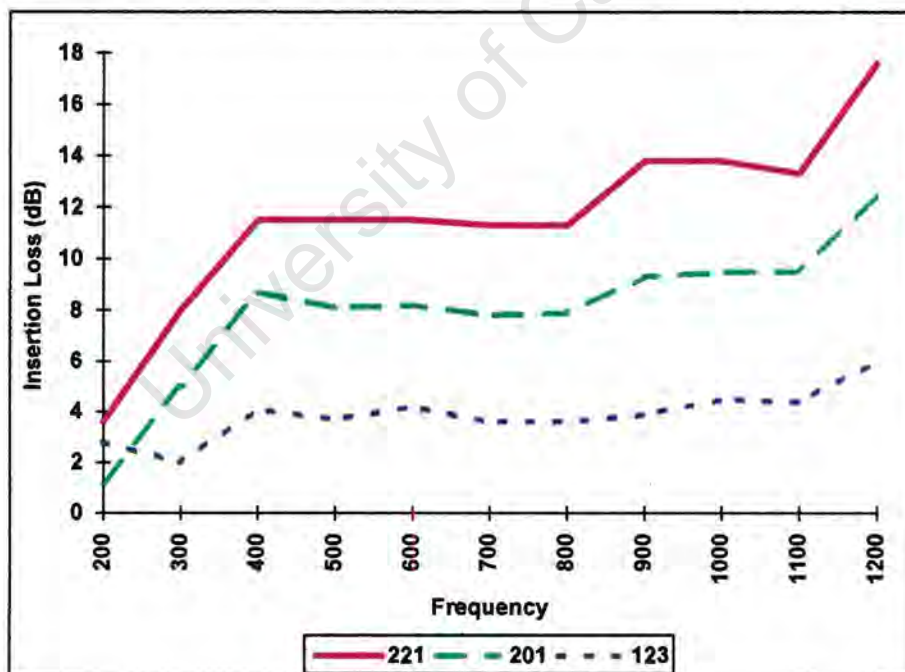


FIGURE 3.3 INSERTION LOSS OF SINGLE LAYERS OF THREE TYPES OF ROCKWOOL MINERAL FIBRE.

	TYPE	ρ kg/m ³	Ξ Rayls	Thickness mm
—	221	55	94	80
- -	201	35	72	60
- - -	123	23	not avail.	60

Figure 3.3 indicates that at least five layers, totaling 300mm thick, of type 221 Rockwool would be required to provide an attenuation of 20dB at 200Hz. However, such a layer would present an attenuation approaching 100dB for frequencies above 1kHz. Such a dynamic range is not feasible for a cone driven loudspeaker and is very difficult to realise with all but the most sophisticated instrumentation.

It is thus clear that other means would be required to eliminate the loudspeaker echo response. The possibility of employing inverse filtering or other signal processing techniques remains a topic for further investigation.

Figure 3.4 displays the sound absorption coefficient of the 400mm long "anechoic" termination used in the experiments and measured by the cepstral method. The sound absorption coefficient is seen to be very close to 1 at all frequencies up to approximately 1600Hz where the values drop to the order of 0.98. This is equivalent to an echo reduction of 17dB.

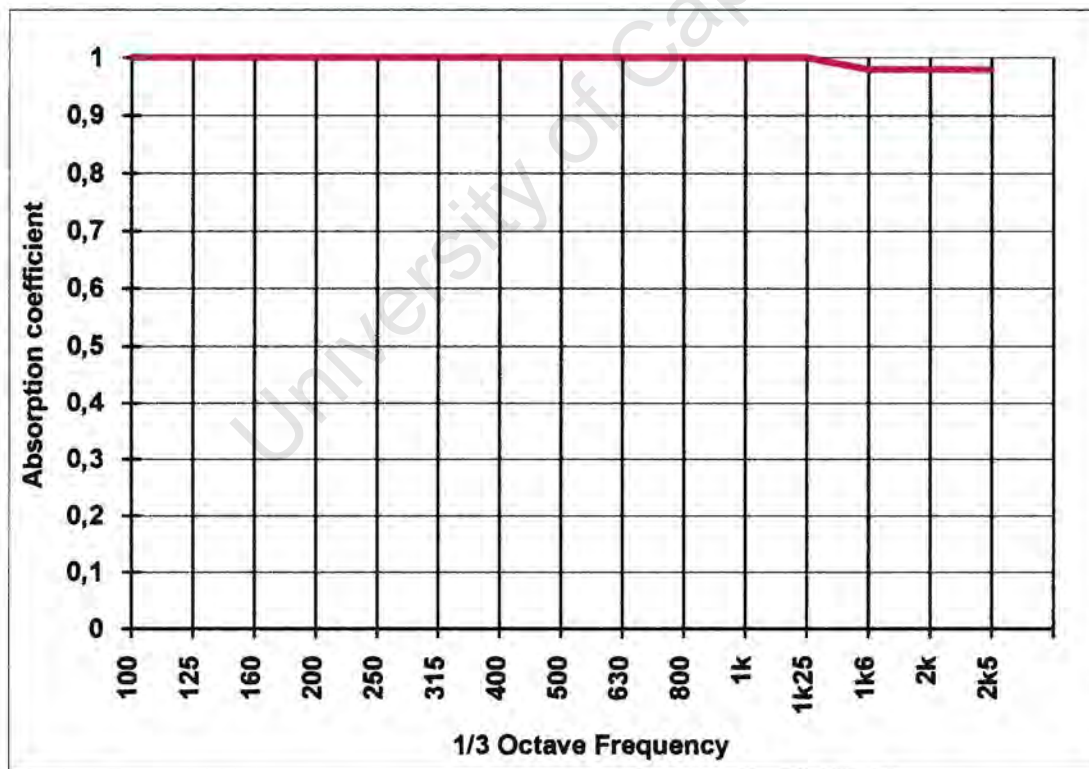


FIGURE 3.4 SOUND ABSORPTION COEFFICIENT OF THE 400mm LONG ANECHOIC TERMINATION.

The insertion of a single 80mm thick layer of type 221 Rockwool (55 kg/m³ density) immediately in front of the loudspeaker did slightly reduce the effects of standing waves in the tube but also increased the slope below 1kHz to approximately 12dB per octave as shown by the lower two curves in Figure 3.2.

In order to offset the increasing sound pressure with decreasing frequency a simple 3rd order RC high-pass electrical filter, with a cut-off frequency of 5.3kHz was connected between the signal generator and the power amplifier. Provision was made to select a 1st or 2nd order filter so as to enable altering of the slope below cut-off as needed. Details of the filter are shown in Appendix I.

3.5 THE CHIRP SIGNAL USED IN THE CEPSTRUM EXPERIMENTS

The procedure closely follows that described by Bolton & Gold[6] and by Bolton[7]. Bolton stresses the need for a signal with a very smooth, ideally flat frequency response up to the Nyquist frequency of the analyser. For successful extraction of $h(t)$ from the power cepstrum the contribution from the cepstrum due to the direct signal must be concentrated at low times so that it is negligible in comparison to the sample impulse response in the vicinity of τ . This necessitates a rapid decay of the impulse response of the direct signal.

Any sharp variation from a flat spectrum will introduce contamination of the cepstrum in the form of slowly decaying oscillations which may mask or distort the sample impulse response at τ .

The chirp, or swept sine wave is a very suitable signal to use[7]. It has a low crest factor which limits the dynamic range required of the loudspeaker. It is causal making time-domain editing relatively easy. Also, the duration of the sweep and the bandwidth are easily controllable.

Subsequent work will consider the suitability of using a pseudo-random signal or Maximum Length Sequence, MLS

The chirp, or swept sine wave is given by the following expression:

$$v(t) = v_0 \sin \left\{ 2\pi \left[\left(\frac{f_2 - f_1}{2T} \right) t + f_1 \right] t \right\}$$

where,

f_1 = start frequency,

f_2 = stop frequency,

T = duration of the sweep.

This, however, displays characteristic sharp cut-off frequencies and spectral ripple which may cause serious contamination throughout the cepstrum. An example of the waveform of a 0Hz to 3200Hz chirp signal generated by the Brüel & Kjaer 3106 generator and the corresponding frequency spectrum is shown in the upper and lower half of Figure 3.5, respectively. Each vertical division in the lower half is 1dB.

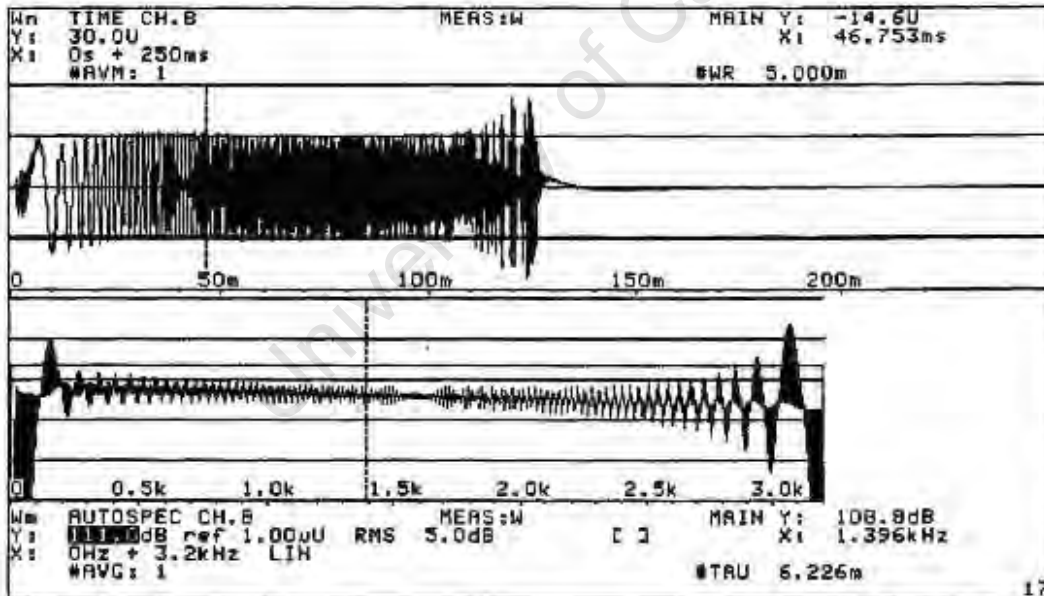
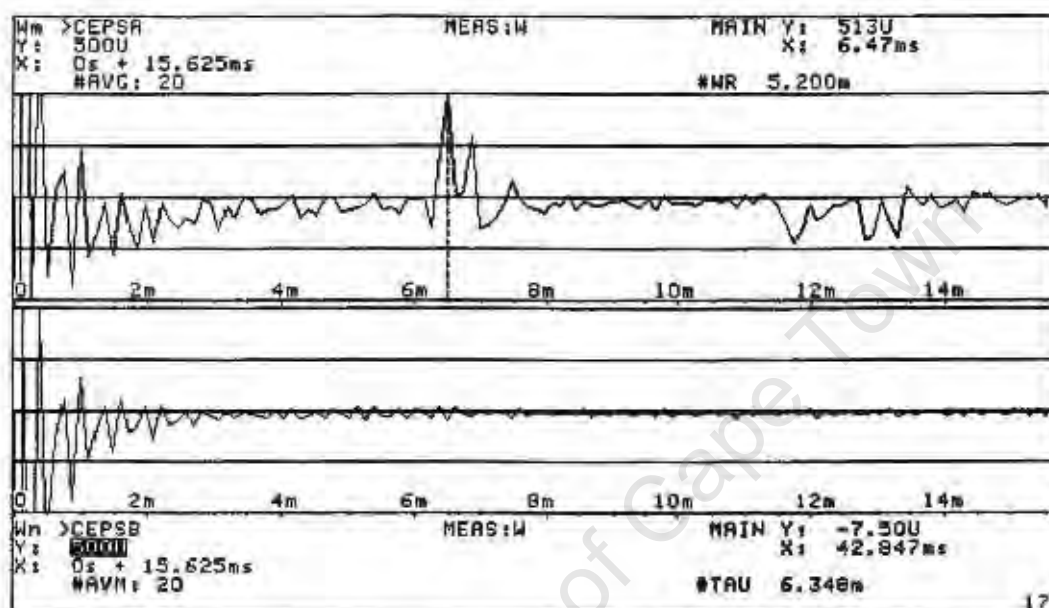


FIGURE 3.5 SWEPT SINE SIGNAL FROM B & K 3106 GENERATOR

Upper Trace: Waveform
Lower Trace: Spectrum

The influence of the spectral ripple due to the upper and lower cut-off frequencies on the cepstrum are shown in Figure 3.6.

The upper half of figure 3.6 is a typical cepstrum of the microphone signal of incident and reflected sound off an asphalt sample. The sample echo response occurs at a time, $\tau = 6.47\text{ms}$. The harmonic of this impulse occurs at 12.94ms . The echo from the loudspeaker occurs just before this at 11.5ms . This condition would not permit optimum filtering of the sample impulse response.



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FIGURE 3.6 CEPSTRUM OF 3.2kHz SWEEPED SINE SIGNAL

Sample: Asphalt 9 - 5 Analyzer frequency span: 3.2kHz
 Upper Trace: Cepstrum of microphone signal
 Lower Trace: Cepstrum of electrical generator output signal

The lower half of figure 3.6 displays the cepstrum corresponding to the generator output signal. The decay of the signal impulse response can be seen to the left. Superimposed high frequency oscillations can clearly be seen throughout the cepstrum.

It can be seen further that the contamination of the cepstrum of the generator signal is contained within that of the microphone signal.

3.6 CEPSTRUM SUBTRACTION

Fortunately, this contamination can, to some extent, be removed by simple subtraction of the generator cepstrum from the microphone cepstrum. This is shown for the same sample in Figure 3.7. The lower half of the figure shows the same microphone cepstrum as in Figure 3.6. The upper half shows the greatly improved result after subtracting the generator cepstrum from the microphone cepstrum.

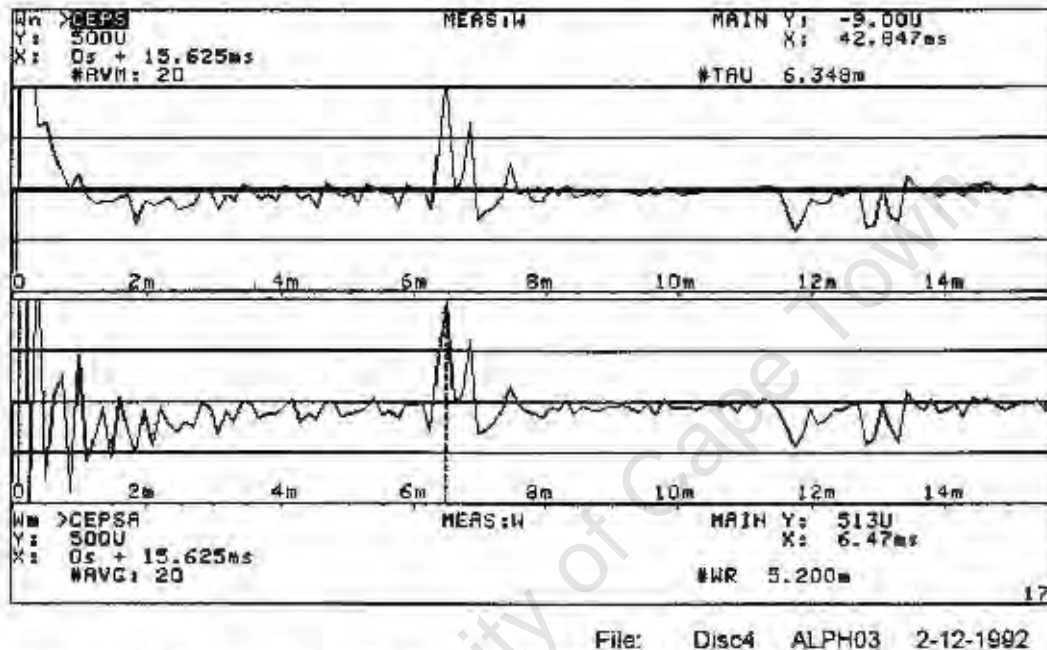


FIGURE 3.7 CEPSTRUM OF MICROPHONE SIGNAL (lower half) AND OF MICROPHONE SIGNAL AFTER SUBTRACTION OF GENERATOR CEPSTRUM (upper half).

This modified cepstrum was obtained with the B&K Analyzer using the User Definable Display Function: CEPS shown in Appendix II.

Low- and high-frequency contamination is still present however. The high-frequency component, occurring at the Nyquist frequency, was found to be inherent in the B & K analyzer processing and could not be removed despite various attempts at filtering this out. Fortunately, being outside the measuring frequency range, this had no deleterious effect in obtaining the correct absorption coefficient. It did, however, make it more difficult to study the cepstrum visually for other effects.

The cepstrum of the generator signal could be measured separately and could thus be subtracted from the total cepstrum. There was, however, no

way of obtaining the loudspeaker impulse response separately from the sample echo unless an infinitely long, echo free tube was used. Thus, after subtraction of the generator signal cepstrum that of the loudspeaker still remained in the resultant spectrum. Even the use of the highly absorbent "anechoic" termination described in section 3.4 did not provide a sufficiently echo-free termination.

3.7 INVERSE FILTERING

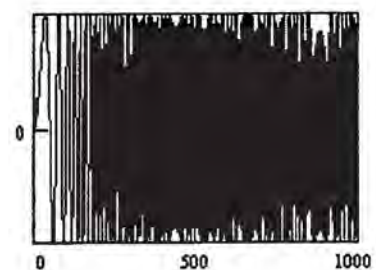
The low-frequency component of the cepstrum contamination was due to spectral ripple characteristic of the chirp signal and the low frequency roll-off of the loudspeaker.

Bolton[7] describes the design of a minimum phase filter that, when convolved with a sampled swept-sine, would cause the result to have a spectral magnitude that was flat from 0Hz to the Nyquist frequency. Bolton's method was repeated in the present investigation. First the cepstrum, a function very similar to the power cepstrum, was formed, namely,

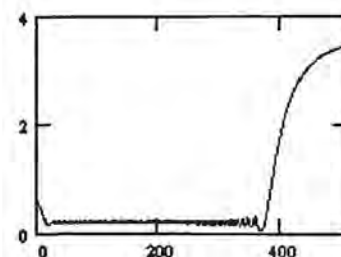
$$c_p(n) = \text{IFFT}\{ \ln|S(k)|^{-1} \}$$

The procedure is outlined on the next few pages. The result of each step is illustrated next to each paragraph using a Mathcad simulation.

- The 0Hz to 6400Hz swept sine signal from the B&K internal generator, $s(t)$, was Fourier transformed to give $S(k)$.
- The modulus of this function was formed, inverted and the natural logarithm taken to produce $\ln | S(k) |^{-1}$
- This was inverse Fourier transformed to obtain $c_p(n)$.



$s(t)$

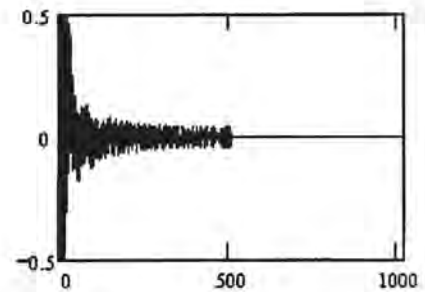


$\ln |S(k)|^{-1}$

- $c_p(n)$ was then windowed in the following manner to produce a one-sided record,

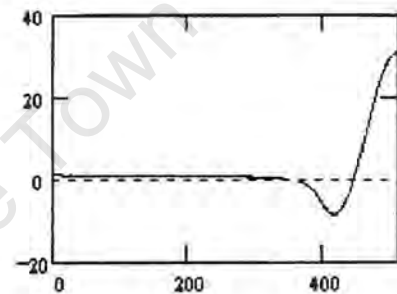
$$\begin{aligned} m(n) &= c_p(n), & n = 0, N/2, \\ &= 2c_p(n), & 1 \leq n < N/2, \\ &= 0 & N/2 < n \leq N - 1. \end{aligned}$$

The vertical scale of the graph is magnified for greater clarity.



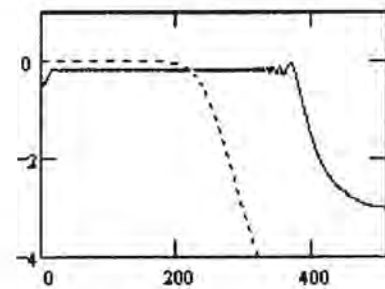
$m(n)$

- The resulting expression was Fourier transformed and exponentiated thereby producing the transfer function of the minimum phase inverse filter, $S_{mp}^{-1}(k)$.



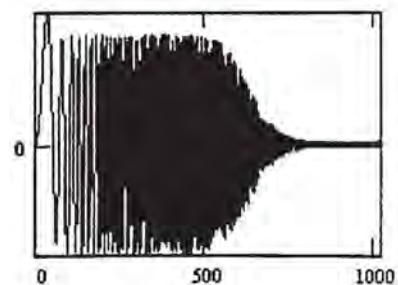
$S_{mp}^{-1}(k)$

- After multiplying this and the original signal spectrum, $S(k)$, together the product was then multiplied by a low-pass Butterworth filter with cut-off frequency of 4200Hz, $W(k)$, to produce the modified sweep spectrum. This is shown by means of the broken line. $S(k)$ is shown by means of the continuous line.



$S(k).S_{mp}^{-1}(k).W(k)$

- This was inverse Fourier transformed to produce the modified time history $s_m(t)$

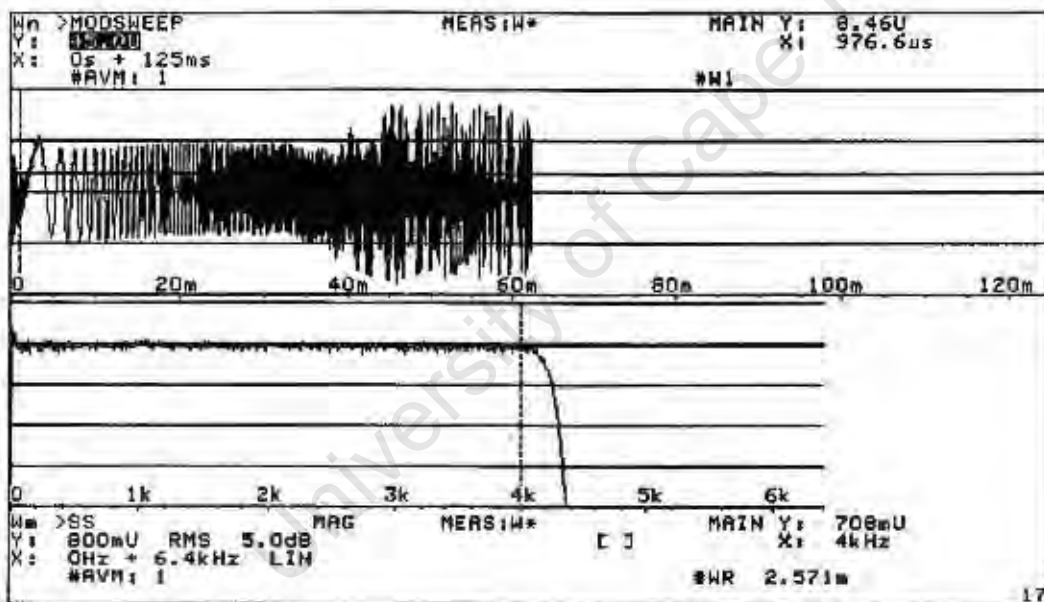


$s_m(t)$

Note that the time and frequency functions are shown as functions of n and k, respectively to indicate that the operations are being performed on sampled data of transform length N

A 6.4kHz sweep of 125ms duration from the internal generator was modified in this way using the User Definable Display Function: MODSWEEP and stored in the User Definable Waveform, UDW, register of the generator. MODSWEEP is listed in Appendix II

Figure 3.8 displays the modified waveform (upper half) and the resultant spectrum (lower half). It is noted that the frequency is flat from 0Hz till the filter cut-off frequency of 4200Hz. Even on a vertical scale of 1dB per division there is no indication of any ripple. It is further noted that the sweep duration has been halved to 62ms.



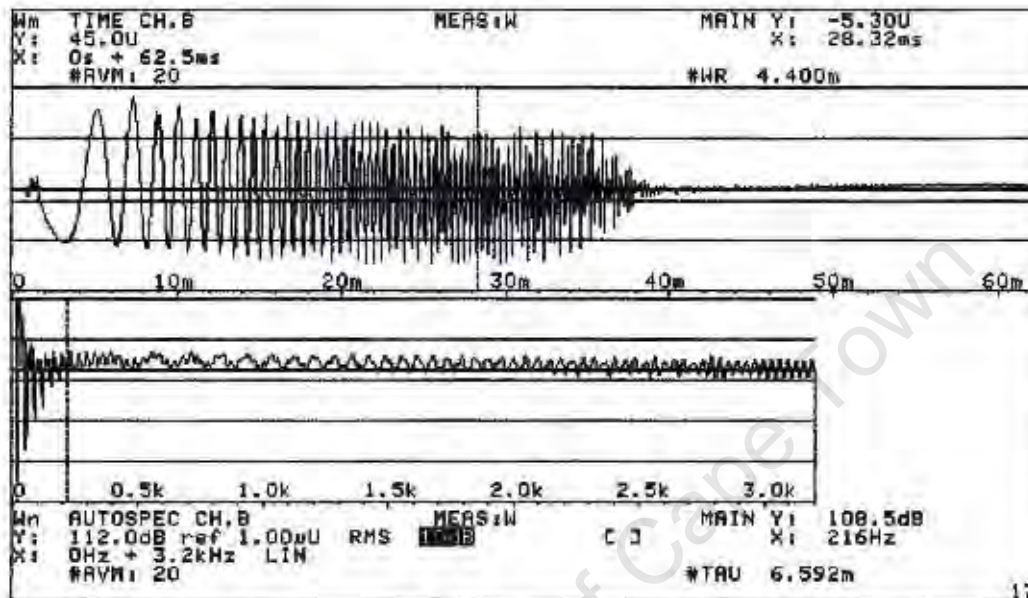
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FIGURE 3.8 MODIFIED WAVEFORM AND SPECTRUM STORED IN USER DEFINABLE WAVEFORM REGISTER.

In order to resolve a 200Hz bandwidth the sweep rate may not exceed 200Hz in 5ms. The duration of a linear sweep from 0Hz to 2000Hz must therefore be no less than $10 \times 5\text{ms} = 50\text{ms}$. The modified sweep duration of 62ms fulfills this requirement.

The upper half of Figure 3.8 shows evidence of some unexplained instability in the generator between 30ms and 62ms. This appears, however, to have no influence on the spectrum.

Figure 3.9 displays the modified waveform and spectrum measured at the generator output when using a stored User Definable Waveform.



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FIGURE 3.9 WAVEFORM AND SPECTRUM OF MODIFIED SWEEP MEASURED AT THE GENERATOR OUTPUT.

It can be seen that some ripple has been reintroduced but less than the original signal especially at the lower and upper frequency regions. This is possibly caused by windowing in the analyser. A further point to note is that the sweep duration has been reduced to approximately 40 msec. although in seeming contradiction to the above reasoning the frequency response is flat from approximately 200Hz to 4 kHz. These observations still need explanation.

The effect of using the modified sweep will become clear when studying the experimental results discussed in Chapter 4.

The inverse filtering, as described, removed the spectral ripple inherent in the chirp signal. However, it excluded effects contributed by the loudspeaker response. As explained in section 3.6 it would not be possible to determine this in isolation from the sample echo.

3.8 LIFTERING OF ECHO FROM CEPSTRUM

The sample impulse response, $h(t-\tau)$, occurring at time τ , is causal and is therefore zero for $t < \tau$. Provided that there is no contamination in the vicinity of the sample impulse response this may be liftered with a suitable lifter extending from the initial pulse until just prior to the 2nd rahmonic.

This was performed on the Brüel & Kjaer Analyzer by means of a window function. With no contamination present in the cepstrum the start time of the window would not be very critical. However, due to the fact that some contamination remained in the experiments performed, great care needed to be taken to commence the window one sample element prior to the maximum value of the sample impulse response.

Initially a rectangular window was applied. The algorithm, WIN3, is shown commencing in the fourth line of the User Definable Display Function: ALPHA3 listed in Appendix II. This program uses variables TAU and WR, displayed on the screen, to permit user choice of the window start and length, respectively.

Once the sample impulse response has been extracted by means of the window function the absorption coefficient is then readily obtained by subtracting from 1 the squared magnitude of the Fourier transform of windowed cepstrum. The User Definable Display Function: ALPHA3 was used to obtain a graph of the absorption coefficient, α , of the measured sample.

The use of a rectangular window resulted in ripples being superimposed on the α - graph. This is shown in chapter 4.

In order to eliminate the ripple a one-sided Hanning window was subsequently used. The TAPER program supplied by B&K and listed in Appendix II was incorporated in a modified version of the ALPHA3 program. The WIN function commencing in line four of the Function ALPHA3 was replaced with the User Definable Display Function: WIN3T listed in Appendix II.

The one-sided Hanning window was rectangular at the low time end and cosine squared at the high time end. The variable, TAPER_T_WIDTH,

determined the length, in number of sample elements, of the cosine-squared section of the window. This is shown in Figure 3.10.

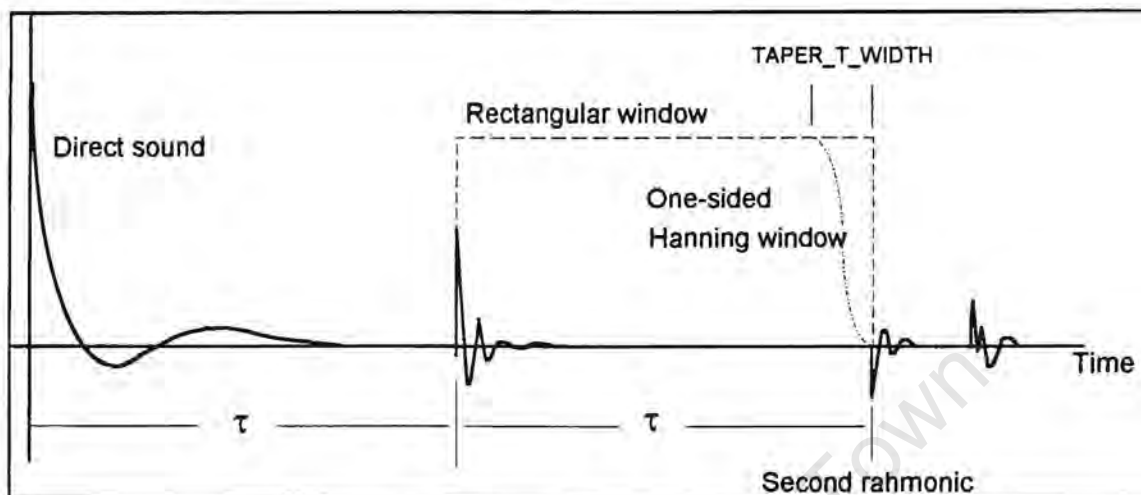


FIGURE 3.10 SKETCH OF CEPSTRUM SHOWING RECTANGULAR AND ONE-SIDED HANNING WINDOW.

This chapter described the method of generating a signal with a flat frequency response used to insonify material samples; the techniques used to obtain an acceptable cepstrum; the process of extracting the sample impulse response from which, finally, the absorption coefficient was obtained.

During each measurement process 10 cepstra were averaged in order to reduce random noise prior to extracting the sample impulse response.

Chapter 4 describes the material samples used and the results of the plane wave absorption coefficient measurements obtained using the cepstrum technique.

3.9 REFERENCES.

1. Kosten, C.W. A method for measuring sound absorption in the field. *Acustica*, 4 (1954) pp 108-110
2. Kinsler, L.E. et. al Fundamentals of Acoustics. 3rd Ed. John Wiley & Sons 1982. Chapter 14.
3. Beranek, L.L. Acoustics. Am. Inst. Physics. 1986. Chapter 9.
4. Kinsler, L.E. et. al Fundamentals of Acoustics. 3rd Ed. John Wiley & Sons 1982. Chapter 9.
5. Schmidt, H. Schalltechnisches Taschenbuch 4^e auflage VDI-Verlag 1989. pg 67.
6. Bolton, J.S. & Gold, E. The application of cepstral techniques to the measurement of transfer functions and acoustical reflection coefficients. J. Sound & Vibration 93(2) 1984 pp 217-233.
7. Bolton, J.S. Cepstral techniques in the measurement of acoustic reflection coefficients, with application to the determination of acoustic properties of elastic porous material. Doctoral thesis submitted to the Institute of Sound & Vibration Research, Faculty of Engineering and Applied Science, University of Southampton. July 1984.

4. MEASUREMENT OF DIFFERENT MATERIALS USING THE CEPSTRUM METHOD.

This chapter records the results of using the cepstrum method to measure the plane wave sound power absorption coefficient of five different samples. These were:

1. A highly sound absorbent, "anechoic" termination as shown in Figure 3.1,
2. A highly sound reflecting termination consisting of a 60 mm thick solid steel cylinder,
3. 60 mm thick Rockwool mineral wool type 201,
4. Porous Asphalt specimen 7 - 3,
5. Porous Asphalt specimen 9 - 5

The first two samples were chosen to test the method for two extreme conditions; a highly sound absorbent and a highly sound reflecting surface, respectively. Sample 3 was chosen as it is a familiar material and is a typical example of a fibrous, porous absorber often used in various acoustical applications.

Samples 4 and 5 were porous asphalt specimens, which, together with the results of careful measurement had been obtained from an experimental project[1] investigating the sound absorbing properties of different types of open pore asphalt. Nine different types of asphalt of differing consistency, texture and depth had been laid in contiguous 80 metre long by 3.5 metre wide strips on a disused runway of the Welschap military airfield near Eindhoven, The Netherlands.

Several specimens were bored from each of these strips. Sample 4 is thus specimen 3 of asphalt type 7. This is a "TWIN-lay" material comprising two layers of different size stone displaying two absorption maxima within the 2000 Hz frequency range of interest. Sample 5 is specimen 5 of asphalt type 9 made up of a single layer displaying a single absorption maximum within the frequency range.

Two typical asphalt core specimens are shown in Figure 4.1 with the type 7 "TWIN lay" shown on the left. A detailed description of the specimens and experiments are contained in reference [1].

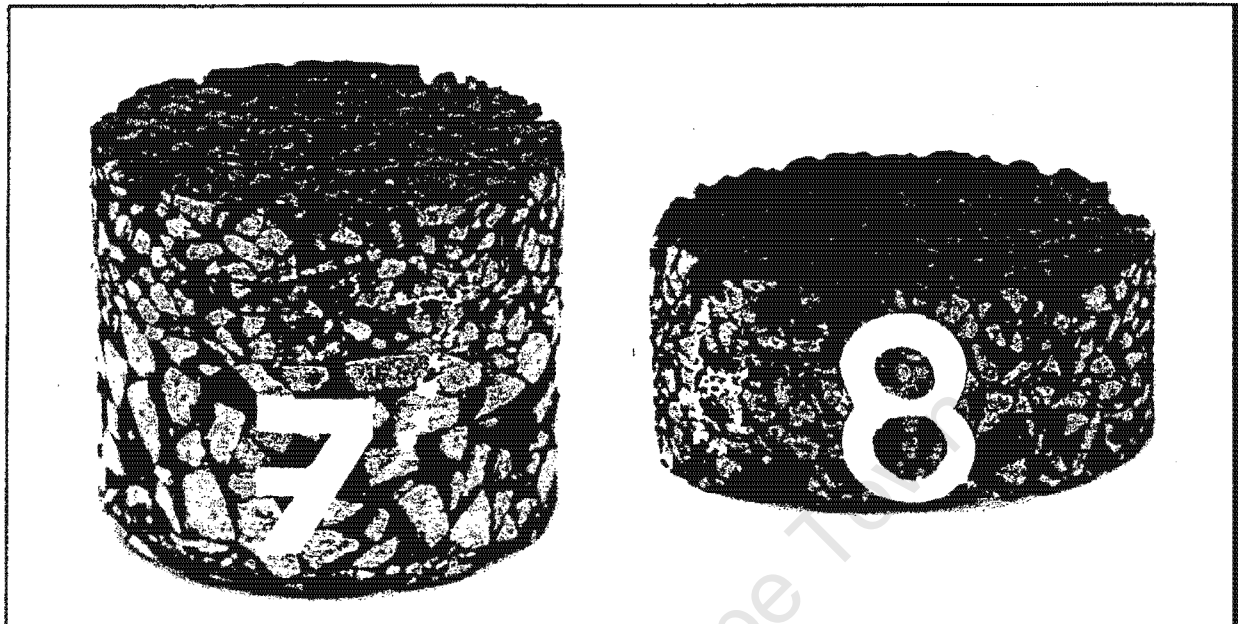


FIGURE 4.1 TWO TYPICAL ASPHALT CORE SPECIMENS.

Sample 4 - Specimen Type 7 "TWIN lay" is shown on the left.

4.1 ANECHOIC TERMINATION

The result of measuring the absorption coefficient of the 400 mm long "anechoic" termination shown in Figure 3.1 by means of the cepstrum method is shown in Figure 4.2.

A modified sweep was used together with a 4 millisecond, one-sided Hanning cepstrum window. The corresponding low-frequency limit to be expected was 250 Hz.

The upper half of the figure displays the absorption coefficient over the frequency range of 10 Hz to 3200 Hz. The vertical axis is the absorption coefficient 0 to 1 in 0.2 steps per division.

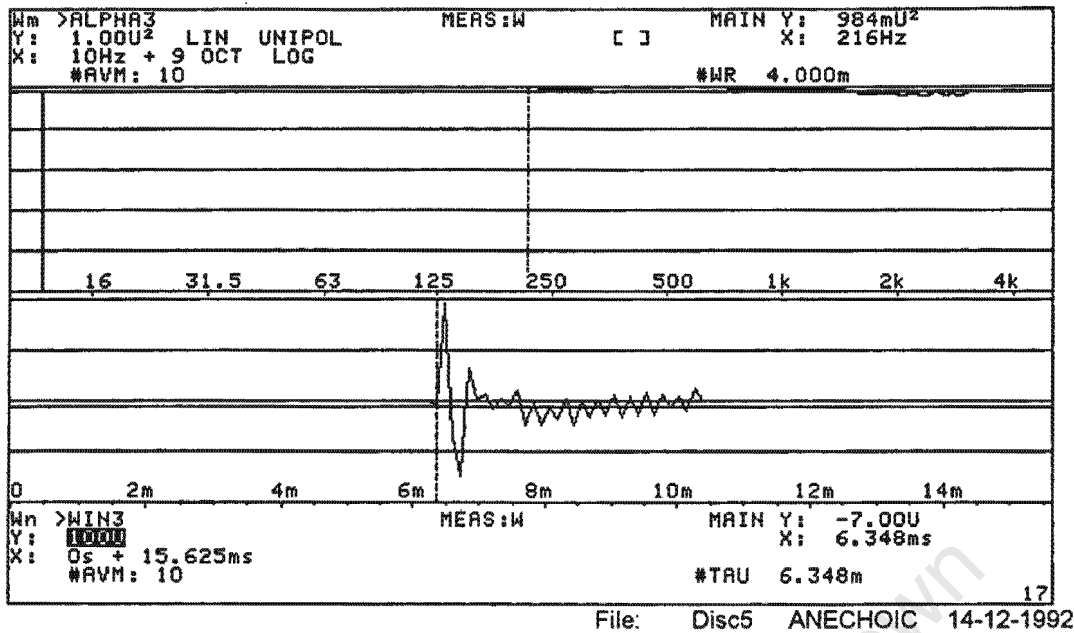


FIGURE 4.2 ABSORPTION COEFFICIENT AND WINDOWED CEPSTRUM OF 400mm LONG ANECHOIC TERMINATION

In the cepstrum window shown in the lower half of Figure 14 it can be seen that there still exists a significant cepstrum impulse response corresponding to approximately 1600 Hz. Also visible is high frequency contamination. Comparing the maximum amplitude of the impulse response in Figure 4.2 with that for a highly reflecting surface shown in the lower half of Figure 4.3, taking due note of the different scales, it is seen that the former is approximately 17 dB below that of the latter. This correlates with the absorption coefficient of 0,98 at 1600 Hz and higher frequencies displayed in Figure 3.4.

4.2 SOUND REFLECTING 60 MM THICK STEEL CYLINDER

The sound reflecting surface comprised a 60 mm thick solid steel cylinder wrapped with tape in order to obtain an air tight fit in the sample holder. An ideal reflector would display zero values of absorption coefficient over the entire frequency range.

Two analysis methods were used on the identical sample and mounting. Figure 4.3 displays the result of using a modified 4 kHz swept sine signal and with the sample impulse response lifiered using a 4.1 millisecond, one-sided,

Hanning window with the length of the taper being 3.66 msec. Figure 4.4 displays the results when using an unmodified 6.4 kHz sweep.

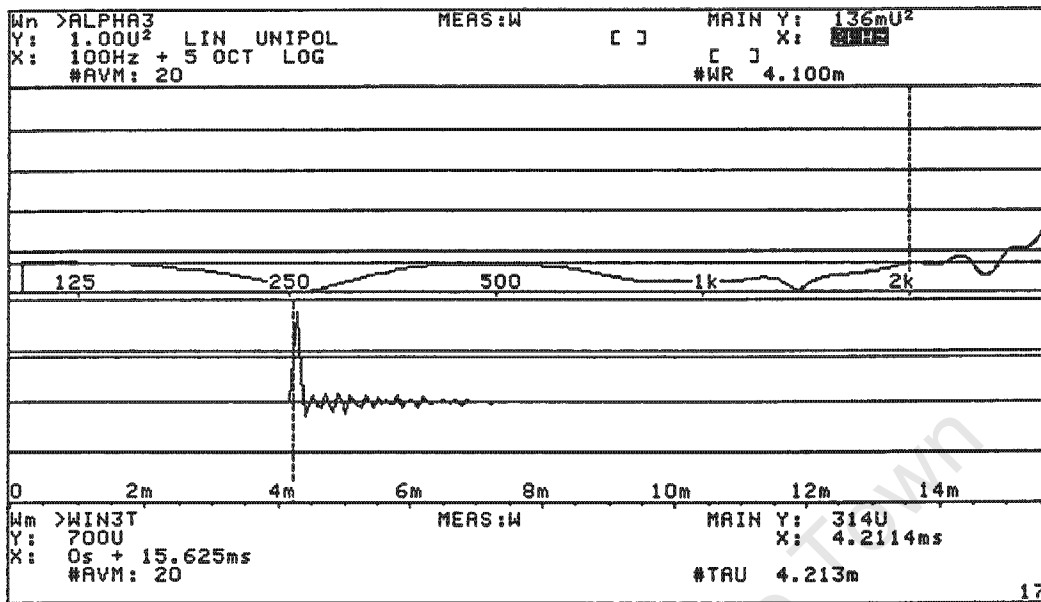


FIGURE 4.3 ABSORPTION COEFFICIENT & CEPSTRUM WINDOW OF STEEL REFLECTOR

Modified Sweep Fspan: 6.4kHz TAU: 4.213ms WR: 4.1ms
 One-sided Hanning window applied to cepstrum

It is to be noted that the absorption coefficient trace of Figure 4.3 and subsequent traces display the magnitude of the results. In certain of the results the values became negative over certain frequency ranges. The B&K analyser displays negative values by means of a brighter line which is not shown in the print-out. In Figure 4.3 the curve is negative between 250 Hz and approximately 1200 Hz.

The reason for this was explained in section 2.4.2. The cepstrum of highly reflecting surfaces comprises higher order convolution terms or rahmonics with amplitudes that decay only slowly with increasing order. The higher order convolution terms of equation 5 of Chapter 2 have not decayed to negligible values in one half of the record length. Cepstral aliasing occurs with the negative-time rahmonic that coincides with the positive-time sample impulse response having sufficient amplitude to cause significant distortion and hence producing the negative absorption coefficient values shown.

Above approximately 1200 kHz finite values of absorption coefficient of the order of 0.15 are displayed. It is possible that the tape wrapped around the metal cylinder did not provide an air tight seal or that the tape itself provided some absorption at these higher frequencies. Beyond approximately 2 kHz the results can no longer be accepted with confidence due to the existence of cross modes in the tube and the wave travelling in the tube no longer being a pure plane wave.

For a 4.1 millisecond window the expected low-frequency limit is 122 Hz. The apparent increase in absorption below 250 Hz may possibly be due to the low frequency roll-off of the modified chirp signal, still apparent in Figure 3.9, together with that of the loudspeaker causing a low frequency ripple in the cepstrum to interfere with the sample impulse response. This would influence the low frequency values of absorption coefficient.

Figure 4.4 displays the results of using an unmodified sweep signal. Comparison with Figure 4.3 shows a significant difference in results below 250 Hz caused by effects of the increased roll-off in frequency of the generator signal, seen in Figure 3.5, on the low-frequency cepstrum ripple. The loudspeaker low-frequency response was the same in all cases.

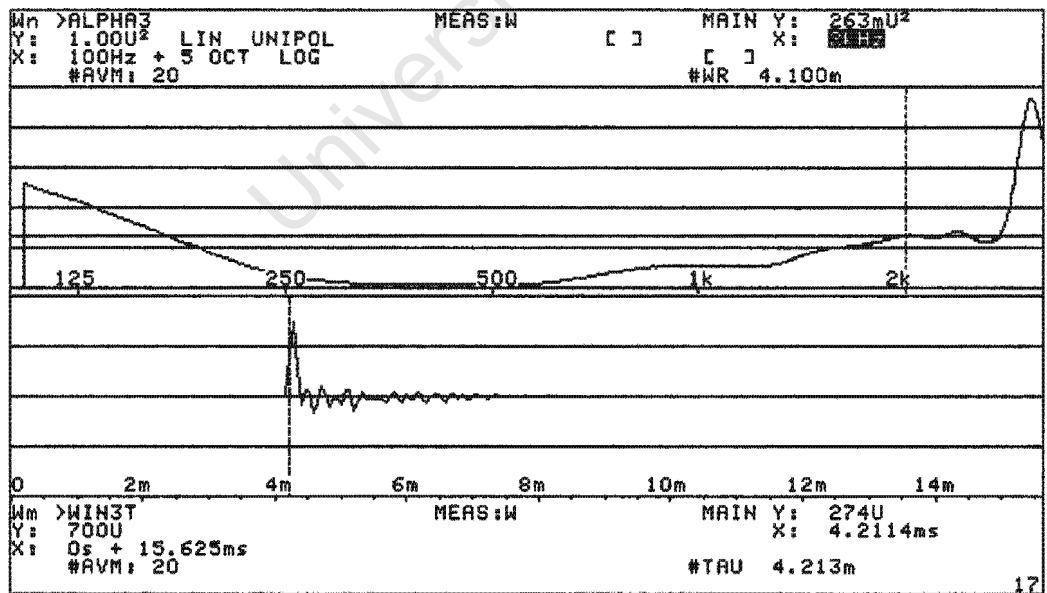


FIGURE 4.4 ABSORPTION COEFFICIENT & CEPSTRUM WINDOW OF STEEL REFLECTOR

Sweep: 6.4kHz Fspan: 6.4kHz TAU: 4.213ms WR: 4.1ms
 One-sided Hanning window applied to cepstrum

A clearer comparison between the two results was obtained by using the B&K analyser to read the absorption coefficient values at each 1/3rd octave frequency. These are plotted on the graph of Figure 4.5. The negative values of Figure 4.3 were plotted as zero in Figure 4.5.

Figure 4.5 clearly shows the larger errors obtained both at low and at high frequencies using the unmodified sweep compared to the modified sweep .

The effects of aliasing may be reduced by increasing the original time record. The same effect could also be achieved by zero padding the original time record[2].

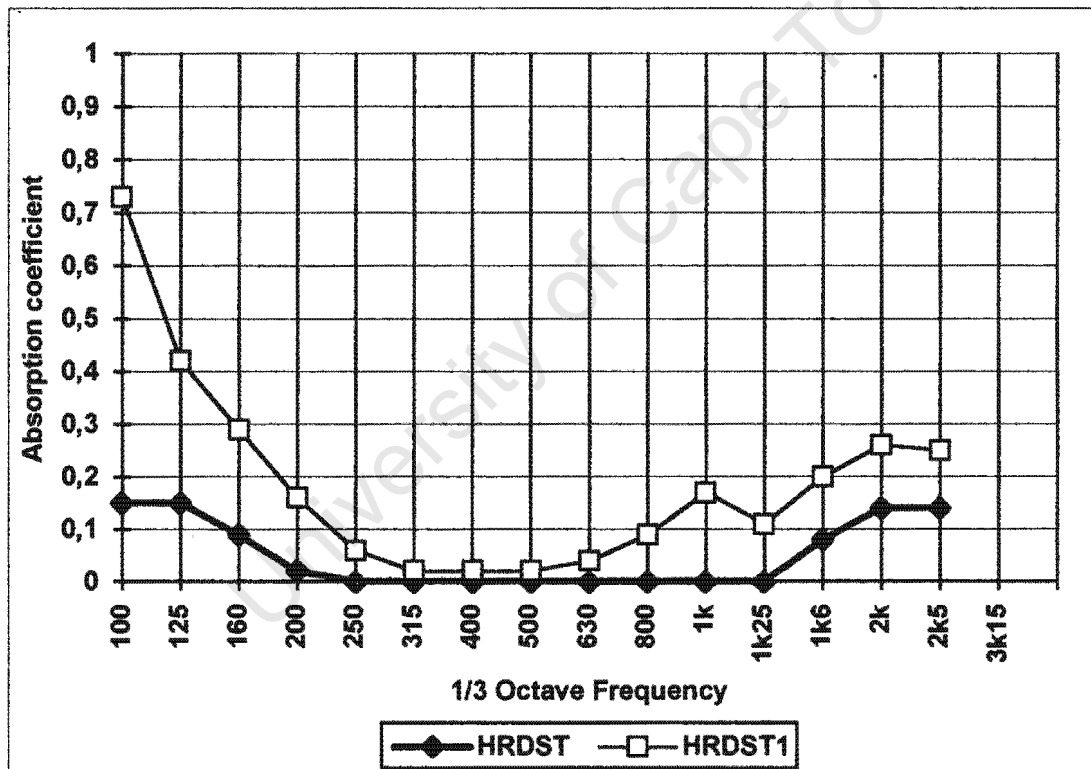


FIGURE 4.5 COMPARISON OF ABSORPTION COEFFICIENT VALUES OF 60 mm THICK STEEL TERMINATION DETERMINED USING MODIFIED AND UNMODIFIED CHIRP SIGNALS.

- HRDST Modified sweep One-sided Hanning cepstrum window
- HRDST1 6.4 kHz sweep One-sided Hanning cepstrum window

4.3 60 MM ROCKWOOL

A 60 m thick Rockwool type 201 mineral wool batt was measured in three different ways:

1. Using a modified 6.4 kHz swept sine signal of 62 msec duration and the sample impulse response liltered using a one-sided Hanning window with a 1.2 msec. taper. The results are displayed in Figure 4.6.
2. Using an unmodified 6.4 kHz swept sine signal of 124 msec duration and the sample impulse response liltered using a one-sided Hanning window with a 1.2 msec. taper. The results are displayed in Figure 4.7.
3. Using an unmodified 6.4 kHz swept sine signal of 124 msec duration and the sample impulse response liltered using a rectangular window. The results are displayed in Figure 4.8.

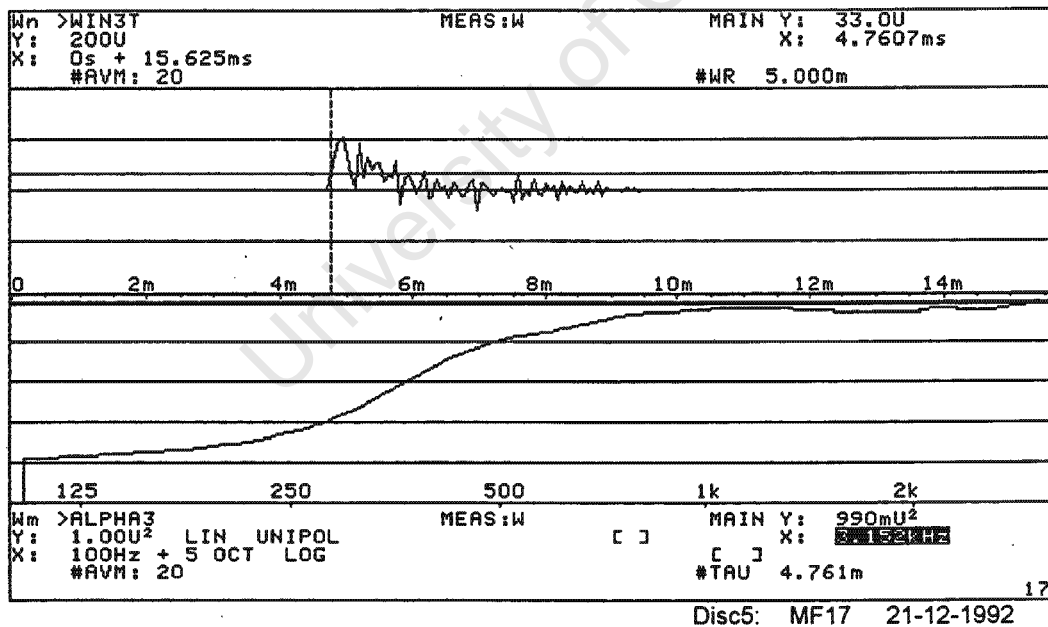


FIGURE 4.6 CEPSTRUM & ABSORPTION COEFFICIENT OF 60 mm ROCKWOOL

Modified Sweep Fspan: 6.4kHz TAU: 4.761msec WR: 5msec
 Microphone/sample distance: 835mm
 One - sided Hanning window applied to cepstrum.

In all three cases a cepstrum window length of 5 milliseconds is shown. This was the maximum length that could be used without interference from the 2nd

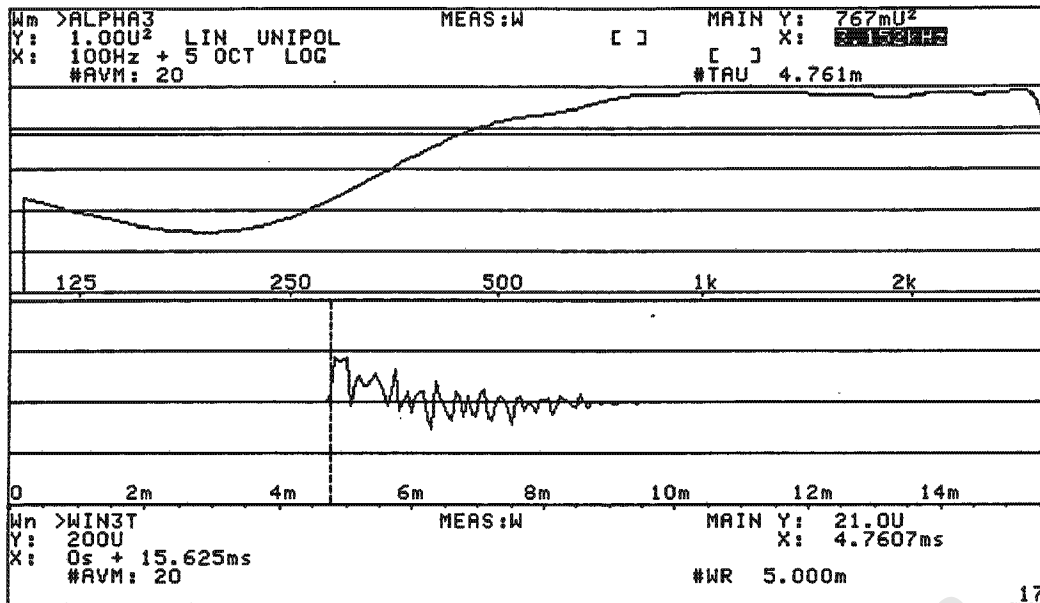
rahmonic. The anticipated low frequency cut-off was therefore 100 Hz. The absorption coefficient results are shown extending down to 100 Hz.

The effect of the Hanning window in modifying the cepstrum can be seen by comparing the cepstrum amplitudes between 8.5 msec. and 9.76 msec. of Figures 4.6 and 4.7 with that of Figure 4.8. The latter cepstrum was not tapered.

A clearer comparison between the three results was obtained by using the B&K analyser to read the absorption coefficient values at each 1/3rd octave frequency. These are plotted on the graph of Figure 4.9 together with the results of measurements conducted on the same sample using the impedance tube method. In positioning the sample in the impedance tube sample holder the 60 mm thick sample was inadvertently compressed to 57 mm.

Figure 4.9 shows that in the frequency range from 200 Hz to 3150 Hz a very close correlation exists between the results obtained when using the modified sweep and Hanning window with those obtained with the impedance tube method. The difference in absorption coefficient values at 315 Hz and 500Hz were within 0.08 of each other whilst all other values in the range were within 0.03. Considering that the sample was removed from one sample holder and slightly compressed when placed in another for the impedance tube measurements much of the variations could be attributed to experimental error. Below 200 Hz the variation increased monotonically to 0.9 at 100 Hz. This may have been due to a remaining low frequency contamination in the cepstrum as a result of the low-frequency roll-off of the loudspeaker. However, this still needs to be confirmed. It is to be noted that smaller differences were obtained with both of the asphalt samples to be described hereafter.

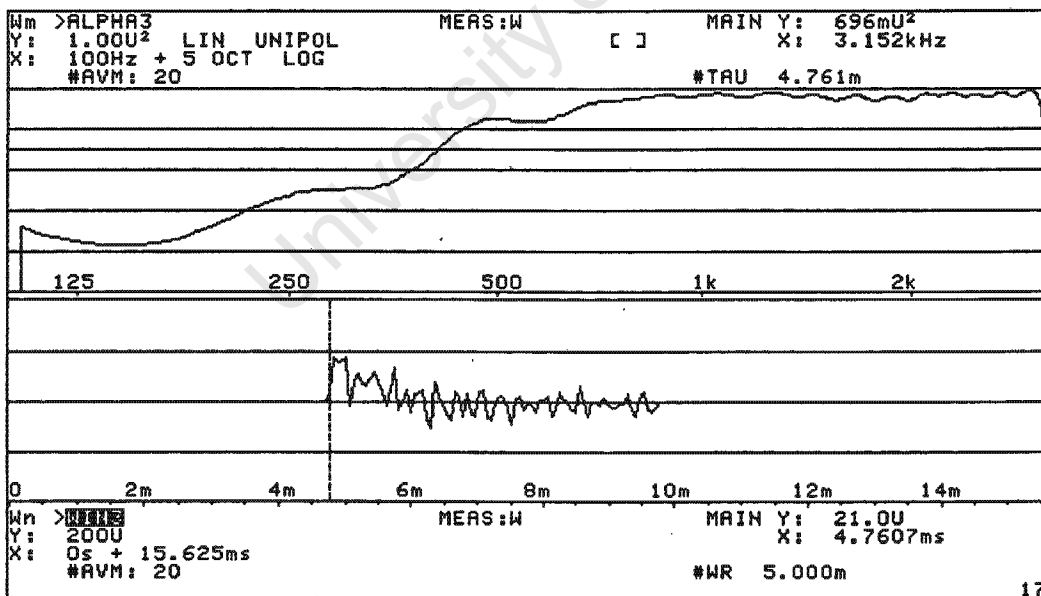
In the frequency range from 200 Hz to 3150 Hz an even closer correlation existed between the results of the two cepstrum methods employing the Hanning window. Other than at 315 Hz and 400 Hz for which the difference was 0.05 and 0.04, respectively, all values were within 0.02 of each other. Below 200 Hz the difference increased monotonically to 0.24. It is probable that this was due to low frequency contamination in the cepstrum.



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FIGURE 7.7 CEPSTRUM & ABSORPTION COEFFICIENT OF 60 mm ROCKWOOL

Sweep: 6.4 kHz Fspan: 6.4kHz TAU: 4.761msec WR: 5msec
 Microphone/sample distance: 835mm
 One - sided Hanning window applied to cepstrum.



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FIGURE 7.8 CEPSTRUM & ABSORPTION COEFFICIENT OF 60 mm ROCKWOOL

Sweep: 6.4kHz Fspan: 6.4kHz TAU: 4.761msec WR: 5msec
 Microphone/sample distance: 835mm
 Rectangular window applied to cepstrum.

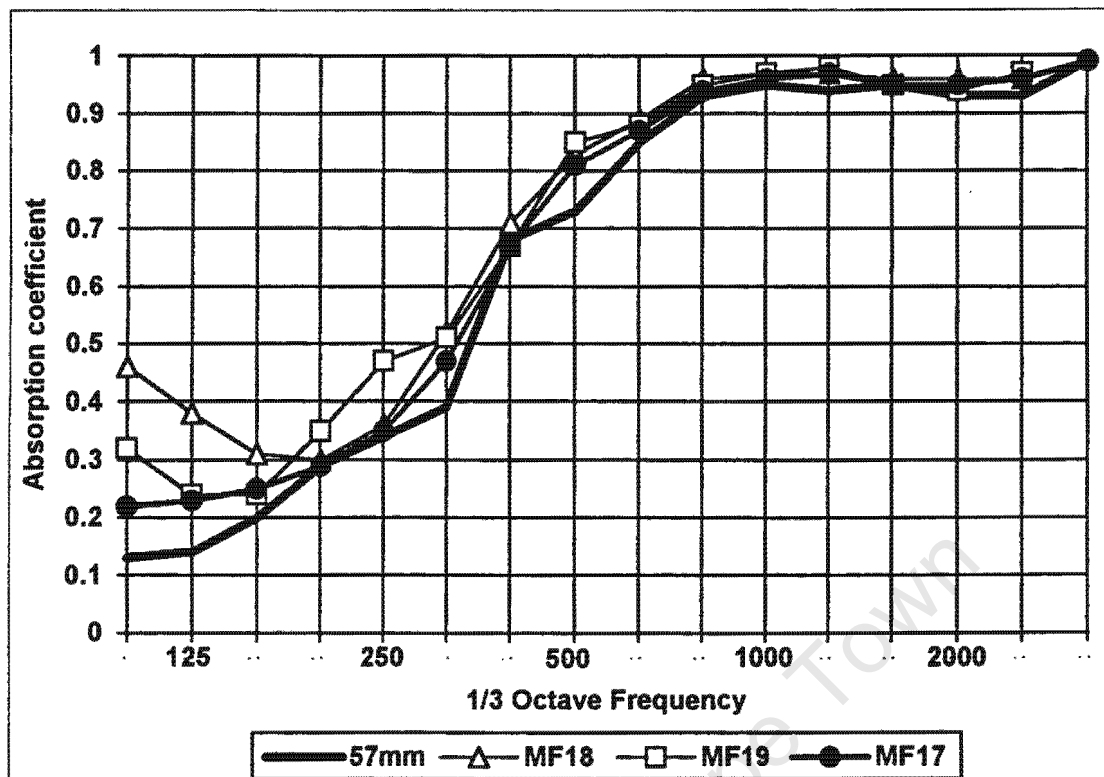


FIGURE 4.9 COMPARISON OF ABSORPTION COEFFICIENT VALUES OF 60 mm THICK ROCKWOOL TYPE 201 MEASURED BY VARIOUS CEPSTRUM TECHNIQUES & BY IMPEDANCE TUBE METHOD

- MF17 Modified sweep One-sided Hanning cepstrum window
- △ MF18 6.4kHz sweep One-sided Hanning cepstrum window
- MF19 6.4kHz sweep Rectangular cepstrum window
- 57 mm Identical sample compressed to 57mm measured by conventional impedance tube method.

There is noticeably less contamination to be seen in the cepstrum when using the modified sweep as shown in Figure 4.6 compared to that when using the unmodified sweep as shown in Figures 4.7 and 4.8.

Figure 4.8 shows clearly the superimposed ripple on the absorption coefficient curve due to high frequency contamination in the cepstrum caused by using a rectangular cepstrum window. Below 200 Hz this ripple appeared to offset the difference produced by the low frequency contamination in the cepstrum.

When comparing the results of the three cepstra techniques the difference was within 0.01 for frequencies in the range of 400 Hz to 3150 Hz excepting for a difference of 0.04 at 500 Hz.

4.4 POROUS ASPHALT SPECIMEN 7 - 3

The tests performed on the asphalt samples were similar to those performed on the Rockwool sample. The three different methods were:

1. Using a modified 6.4 kHz swept sine signal of 62 msec duration and the sample impulse response liltered using a one-sided Hanning window with a 1.2 msec. taper. The results are displayed in Figure 4.10.
2. Using an unmodified 6.4 kHz swept sine signal of 124 msec duration and the sample impulse response liltered using a one-sided Hanning window with a 1.2 msec. taper. The results are displayed in Figure 4.11.
3. Using an unmodified 6.4 kHz swept sine signal of 124 msec duration and the sample impulse response liltered using a rectangular window. The results are displayed in Figure 4.12.

The longest cepstrum window that could be used without interference from the second rahmonic was 4.76 msec. This corresponds to a low-frequency cut-off of 105 Hz. These results together with those obtained using the impedance tube method are shown in Figure 4.13.

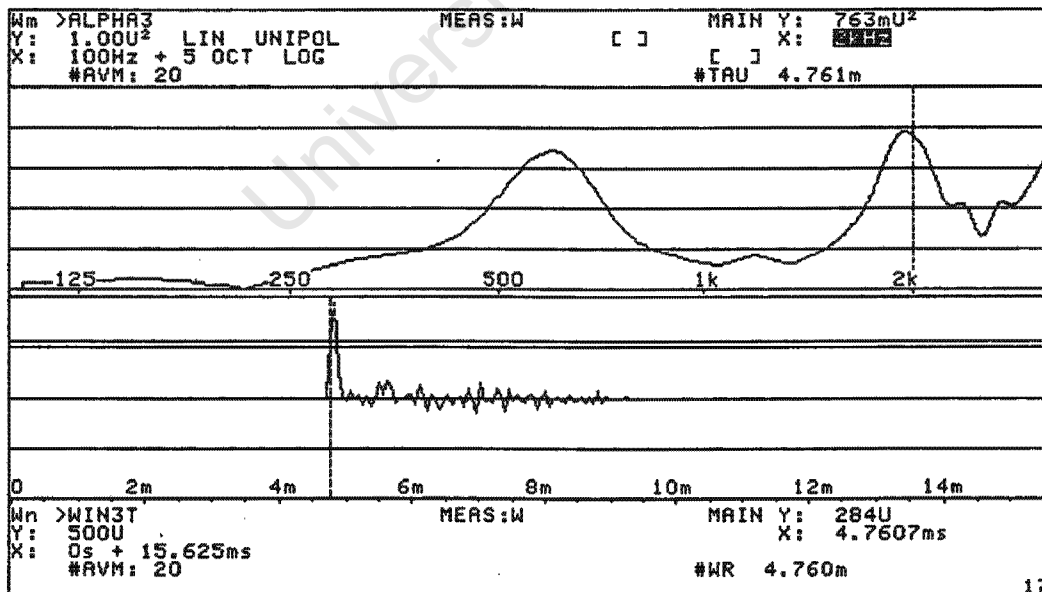
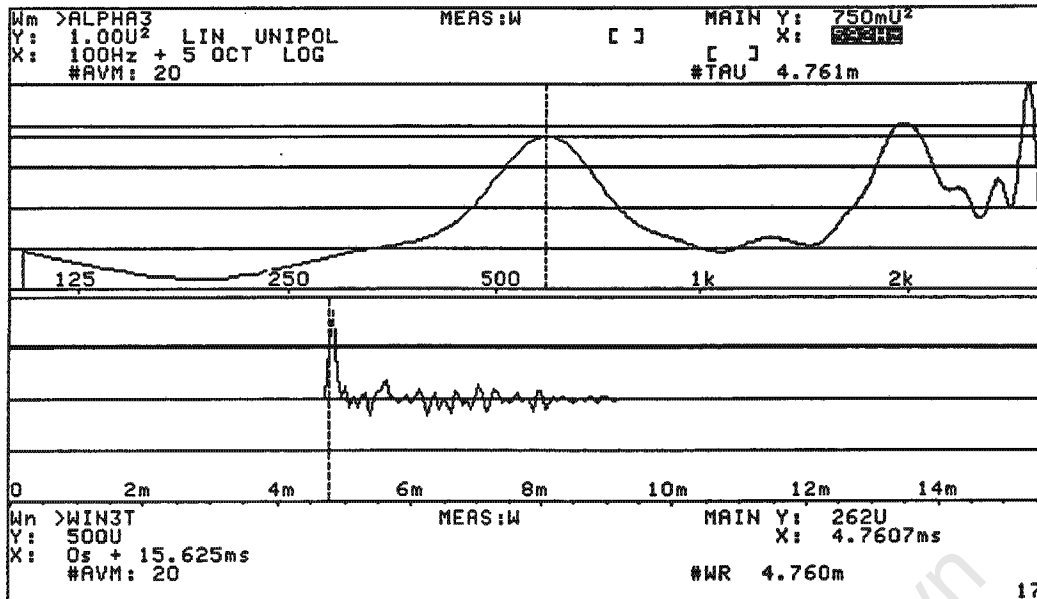


FIGURE 4.10 ABSORPTION COEFFICIENT & CEPSTRUM WINDOW OF ASPHALT SPECIMEN 7 -3 USING MODIFIED SWEEP AND ONE-SIDED HANNING WINDOW TAPER.

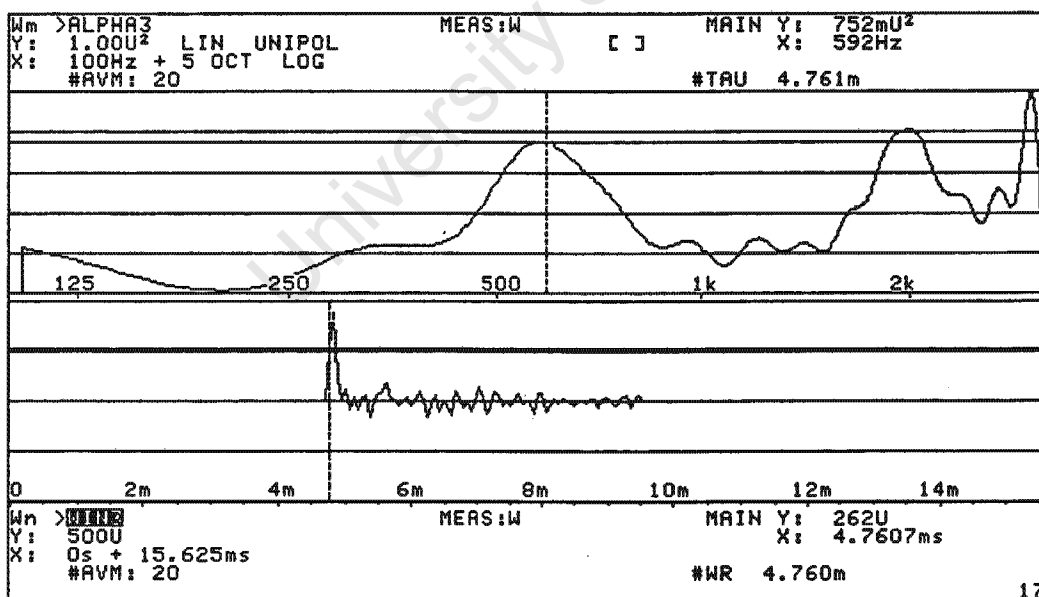
Fspan: 6.4kHz TAU: 4.761msec WR: 4.76msec



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FIGURE 4.11 ABSORPTION COEFFICIENT & CEPSTRUM WINDOW OF ASPHALT SPECIMEN 7 -3 USING NORMAL SWEEP AND ONE-SIDED HANNING WINDOW TAPER

Sweep: 6.4kHz Fspan: 6.4kHz TAU: 4.761msec WR: 4.76msec



Disc5: ASPH733 21-12-1992

FIGURE 4.12 ABSORPTION COEFFICIENT & CEPSTRUM WINDOW OF ASPHALT SPECIMEN 7 -3 USING NORMAL SWEEP AND RECTANGULAR WINDOW

Sweep: 6.4kHz Fspan: 6.4kHz TAU: 4.761msec WR: 4.76msec

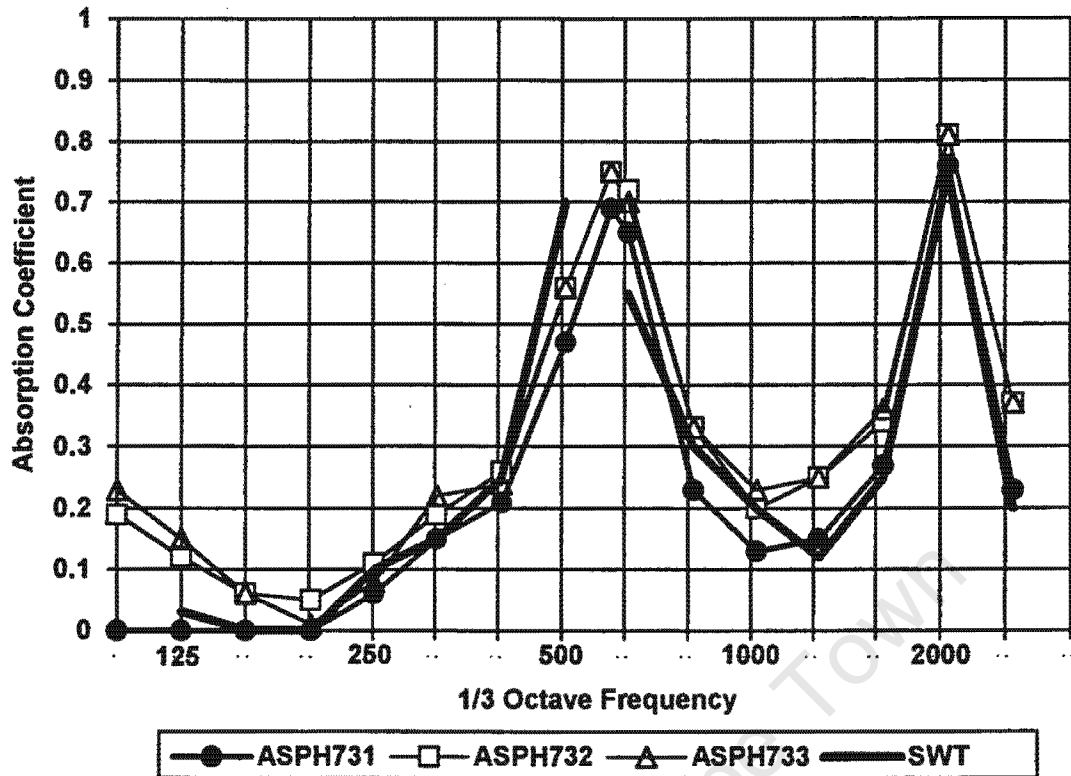


FIGURE 4.13 COMPARISON OF ABSORPTION COEFFICIENT VALUES OF ASPHALT SPECIMEN 7 - 3 MEASURED BY VARIOUS CEPSTRUM TECHNIQUES & BY IMPEDANCE TUBE METHOD

- ASPH731 Modified Sweep One-sided Hanning cepstrum window
- ASPH732 6.4kHz Sweep One-sided Hanning cepstrum window
- △ ASPH733 6.4kHz Sweep Rectangular cepstrum window
- Impedance Tube Method

In Figure 4.10 the absorption values were negative in the frequency range from 100 Hz to 200 Hz but are recorded in Figure 4.13 as being equal to zero. The reason for negative values has been discussed in paragraph 4.1.2.

Use of the modified sweep and one-sided Hanning window produced results which correlate closest with those obtained by the impedance tube method although the variation was typically 0.05 at most frequencies. The greatest variation in magnitude between cepstrum and impedance tube methods occurred at the first absorption maximum. The close correlation between the three cepstrum methods suggests that the variation was due to experimental differences in mounting of the sample. The ripple effects caused by using a rectangular window and magnitude variations below 200 Hz are as before.

4.5 POROUS ASPHALT SPECIMEN 9 - 5

Identical measurements were performed on asphalt specimen 9 - 5 as those performed on asphalt specimen 7 - 3. The results are displayed in Figures 4.14 through 4.16 and summarised in Figure 4.17 together with impedance tube results on the same sample.

In Figure 4.14 the absorption values are negative between 125 Hz and 250 Hz but are positive at 100 Hz.

Study of Figure 4.17 shows close correlation between the results using the modified sweep and the impedance tube method as before with variations of the order of 0.05 at most frequencies. It is of interest to note that, notwithstanding the negative values recorded, the values below 250 Hz remain closely correlated down to 100 Hz. This is an octave below the 200 Hz established in the criteria of section 1.1.

The increase in variation at low frequencies and the ripple due to the use of a rectangular window are similar to that recorded for the previous samples. An increase in absorption around 400 Hz displayed by all cepstrum methods suggests experimental error due to differences in sample mounting.

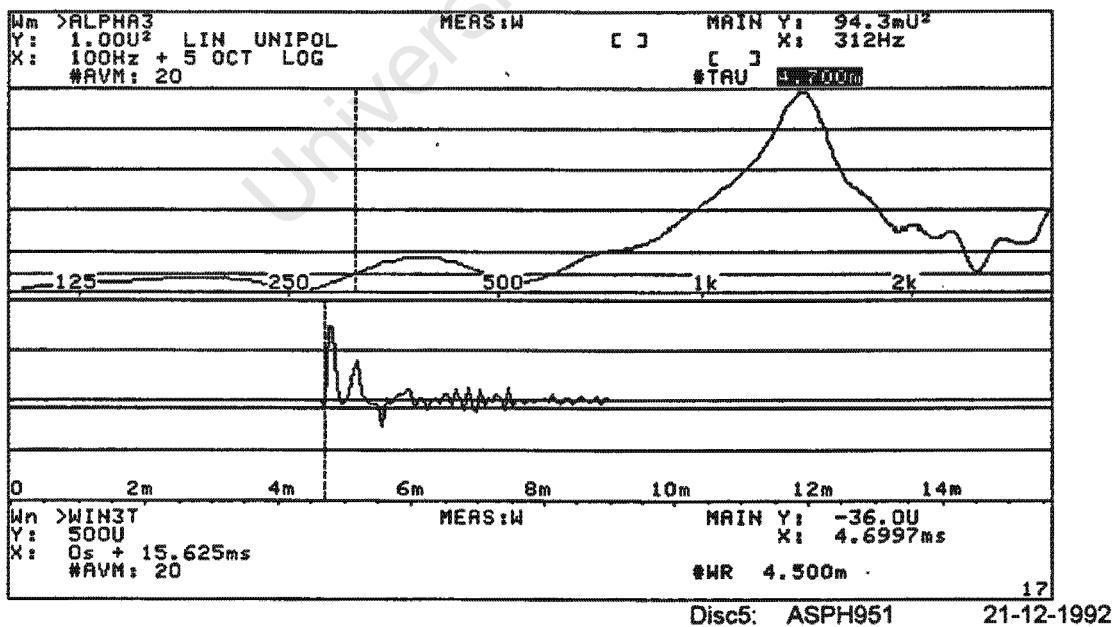
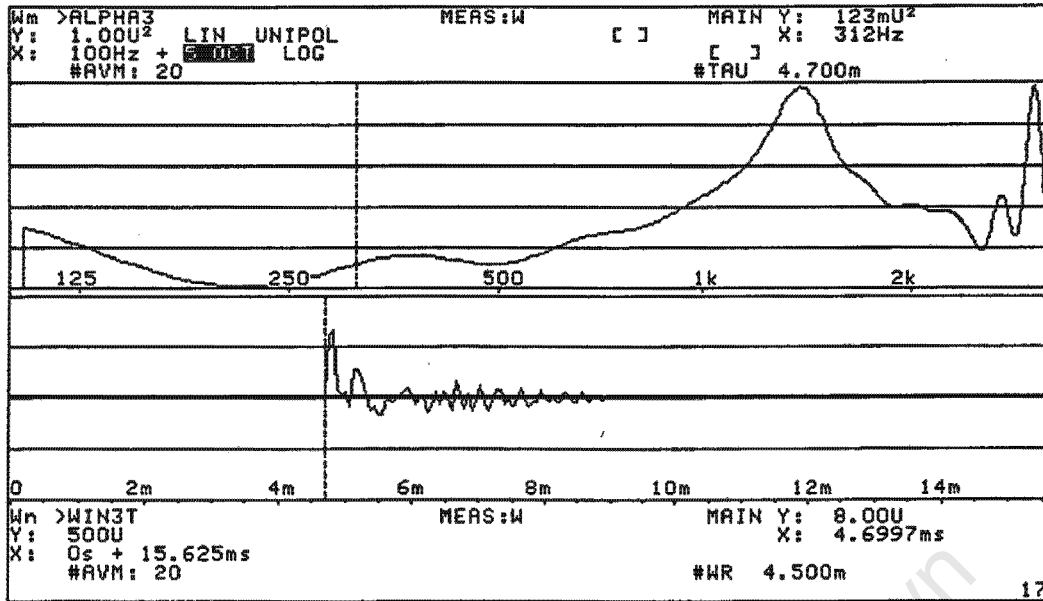


FIGURE 4.14 ABSORPTION COEFFICIENT & CEPSTRUM WINDOW OF ASPHALT SPECIMEN 9-5 USING MODIFIED SWEEP AND ONE-SIDED HANNING CEPSTRUM WINDOW TAPER

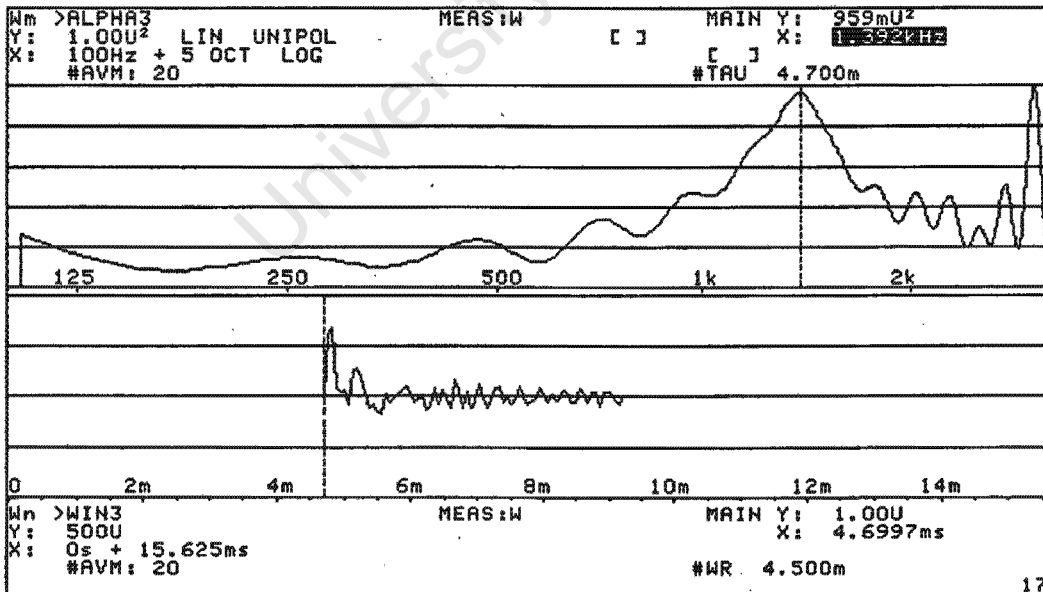
Fspan: 6.4kHz TAU: 4.7msec WR: 4.5msec



Disc5: ASPH952 21-12-1992

FIGURE 4.15 ABSORPTION COEFFICIENT & CEPSTRUM WINDOW OF ASPHALT SPECIMEN 9-5 USING NORMAL SWEEP AND ONE-SIDED HANNING CEPSTRUM WINDOW TAPER

Sweep: 6.4kHz Fspan: 6.4kHz TAU: 4.7msec WR: 4.5msec



Disc5: ASPH954 21-12-1992

FIGURE 4.16 ABSORPTION COEFFICIENT & CEPSTRUM WINDOW OF ASPHALT SPECIMEN 9-5 USING NORMAL SWEEP & RECTANGULAR CEPSTRUM WINDOW

Sweep: 6.4kHz Fspan: 6.4kHz TAU: 4.7msec WR: 4.5msec

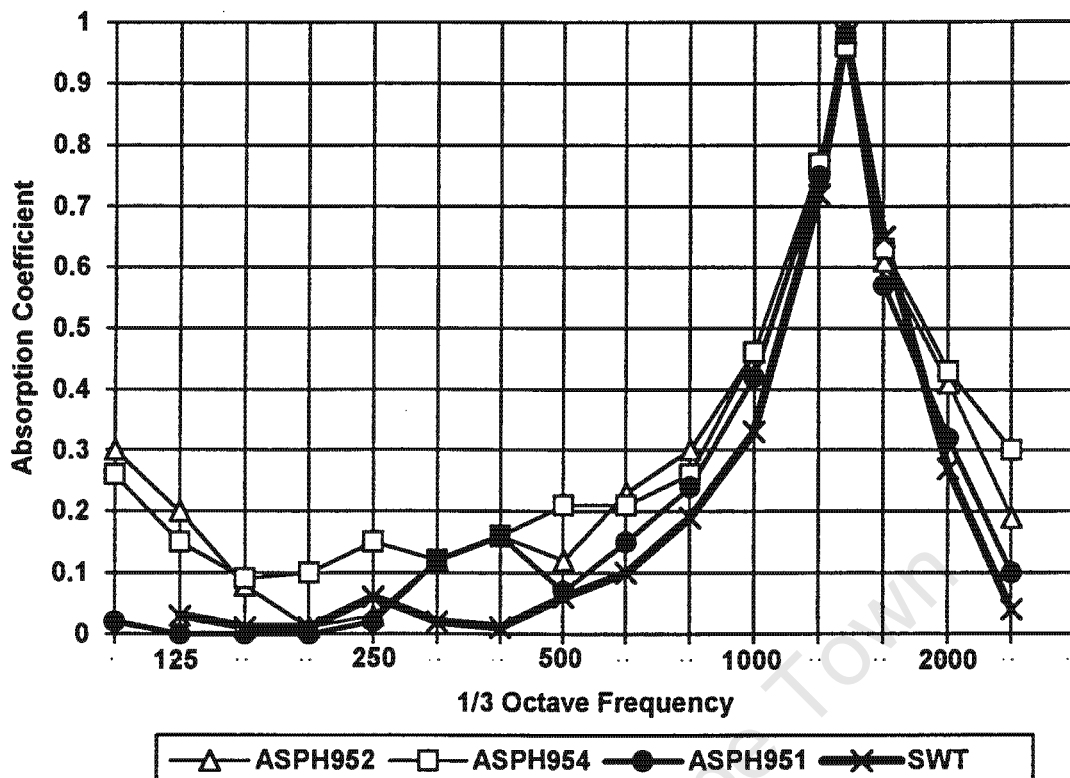


FIGURE 4.17 COMPARISON OF ABSORPTION COEFFICIENT VALUES OF ASPHALT SPECIMEN 9 - 5 MEASURED BY VARIOUS CEPSTRUM TECHNIQUES & BY IMPEDANCE TUBE METHOD

- ASPH951 Modified Sweep One-sided Hanning cepstrum window
- ASPH954 6.4kHz Sweep One-sided Hanning cepstrum window
- △ ASPH952 6.4kHz Sweep Rectangular cepstrum window
- Impedance Tube Method

4.6 DISCUSSION OF THE RESULTS

In all cases use of the modified chirp signal extended the low frequency resolution compared to the use of an unmodified sweep and produced results which correlated most closely to results obtained by means of the standardised impedance tube method at both the low and high frequency ends of the measured spectra. Within the frequency range of 200 Hz to 2000 Hz the correlation was within 0.05 when allowing for experimental error as discussed.

Considering that the Rockwool sample was physically altered when transferring from the cepstrum tube to the standing wave tube and that the asphalt samples were measured by different people at different times the correlation of results can be considered to be very good.

The modified chirp signal produced by the B&K arbitrary waveform generator displayed a frequency response which was not flat down to 0 Hz. This, together with the roll-off in frequency response of the loudspeaker at low frequencies may have been the cause of monotonically increasing variation in the results obtained by the two methods. Further work would be needed to gain insight into the cause of this variation.

At the time of writing a new cepstrum tube was being constructed with a single sample holder that could be used in both the cepstrum tube and the standing wave tube. By eliminating effects due to transfer of the samples between sample holders it was anticipated that closer correlation, to within 0.02, would be achieved when the experiments were repeated.

The results clearly showed the need for using a one-sided tapered cepstrum window such as a Hanning or cosine squared function to extract the sample impulse response.

The greatest errors were obtained for the highly sound reflecting steel surface with negative absorption coefficient values occurring over a broad frequency range and values greater than 0,1 at low and at high frequencies. Although some absorption may have occurred at high frequencies due to a small gap around the edge of the sample it was considered more likely that the cause was due to cepstral aliasing. This was discussed in section 2.4.2. Negative absorption values also occurred for asphalt sample 7 - 3 at low frequencies where the sample could be expected to display little absorption.

These results indicated that attention would need to be given to reducing or eliminating the effects of cepstral aliasing in order to measure materials displaying low values of sound absorption.

The tube used in the experiments was longer than needed to be able to meet the criteria of measuring down to 200 Hz. Indeed accurate results were achieved for all samples down to 100 Hz, other than the steel termination, using a cepstrum window length ranging between 4,7 ms. and 5 ms. This confirmed the theory in section 2.4.3 relating frequency resolution to minimum delay time, τ , and minimum distance between microphone and sample.

Figure 2.2 displays cepstra of three of the samples measured. During all the measurements the microphone was positioned such that the sample impulse response occurred at approximately 6,5 ms. From Figure 2.2 it is clear that the microphone could have been positioned closer to the sample, reducing the time to approximately 4 ms., while still permitting extraction of the sample impulse response without contamination due to the direct cepstrum.

A delay time of $\tau = 4$ ms. would permit measurements down to 125 Hz. within a tube less than 1,4 metres long. This would comfortably meet, not only the minimum frequency criteria, but also the length criteria in section 1.1.

The complete measurement process, including displaying the continuous absorption spectrum on the B&K analyser screen, as illustrated in this thesis, plus copying this to disc could be achieved within 10 seconds. Using the standard impedance tube method would require approximately 20 minutes to measure absorption values at each of 14 third-octave intervals within the same frequency range. The cepstrum method was thus capable of presenting more information in a fraction of the time compared with the existing standardised method.

4.7 REFERENCES

1. von Meier, A., van Blockland, G.J. & van Bochove, G.G. Onderzoek Proefvak Welschap. Hoofdrapport: Vermindering verkeersgeluid door wegdek ontwerp en afstemming band-wegdek. 2 December 1991.
2. Bolton, J.S. Cepstral techniques in the measurement of acoustic reflection coefficients, with application to the determination of acoustic properties of elastic porous material. Doctoral thesis submitted to the Institute of Sound & Vibration Research, Faculty of Engineering and Applied Science, University of Southampton. July 1984.

5. CONCLUSIONS

This thesis investigated the principle of applying cepstral techniques to the measurement of the plane wave sound power absorption coefficient of materials using an adaptation of the impedance tube.

Overall objectives were established in the form of criteria that a portable apparatus to be used in the field would need to meet. Five main criteria were established as set out in chapter 1.

The results of this thesis proved that the cepstrum method could provide an accurate and rapid method of obtaining a continuous spectrum of the absorption coefficient over a wider frequency range than the 200 Hz to 2000 Hz stipulated in criteria 1.1.1. It was shown that these results could be obtained in a tube considerably shorter than the 1,7 metre stipulated in criteria 1.1.3

Sound absorption measurements were performed on a wide variety of material samples using the cepstrum methods and the standard impedance tube method. There was a close correlation between the results of the cepstrum method utilising a modified sweep and Hanning window and the impedance tube method in the frequency range from 100 Hz to above 2000 Hz other than a steel termination. In the latter case the results were compared with theory.

It was established that accurate and repeatable results required the use of a signal with a flat frequency response beyond the measurement frequency bandwidth and the employment of a one-sided Hanning window to extract the sample impulse response from the cepstrum prior to obtaining the absorption coefficient.

Cepstral aliasing introduced significant contamination of the cepstra of materials exhibiting low values of sound absorption such as the steel termination. It was considered that this, combined with the effects of frequency roll-off of both the generator signal and the loudspeaker at low frequencies, were the cause of errors in the absorption coefficient values of these materials.

Results of experiments conducted using an outer guard tube surrounding the measurement tube provided confidence that the measurement of an extended surface placed at the end of the measuring tube could be insonified by a plane wave without the need for an air tight seal between sample and measuring tube as stipulated in criteria 1.1.2

With digital signal processing hardware currently available no difficulty could be foreseen in developing a dedicated signal generation and processing system based on that used in this thesis and that could form part of a single apparatus that was portable and simple to operate by a single person as stipulated in criteria 1.1.4.

During the process of experimentation there was no indication that the cepstral method was susceptible to external noise. With the presence of the guard tube it was anticipated that the system would present high immunity to external noise and that it would therefore be able to comply with criteria 1.1.5.

The results of the thesis provided confidence that cepstrum techniques could be applied to the development of a portable apparatus to measure the plane wave sound power absorption coefficient of road surfaces in-situ.

6. FURTHER WORK

At various stages in the body of the thesis factors were identified that would need further investigation.

Modifying the generator signal by means of inverse filtering extended the low frequency response of the measurement system and reduced, but did not eliminate, low frequency contamination in the cepstrum of the microphone signal. This was partly due to the reintroduction of ripple in the spectrum of the modified signal at the generator output as shown in Figure 3.9 and partly due to the low frequency roll-off of the loudspeaker. Although the electrical signal could be isolated and modified, what really was needed was to be able to measure and modify the system response comprising the generator signal plus the loudspeaker impulse response functioning as a sound source. There was no direct way that the impulse response of the system could be isolated from the combined sample echo. Further work is needed to seek ways of improving the signal radiated by the loudspeaker primarily to increase the decay rate in the cepstrum of the direct signal. This will permit a shorter delay time, τ , between direct signal and echo and hence lead to an even shorter tube length.

The delay time, τ , is also the separation time between the sample impulse response and the 2nd harmonic. The frequency resolution of the reflection coefficient and hence the absorption coefficient is proportional to the inverse of the length of the data extracted from the cepstrum. Reducing τ will decrease the length of the cepstrum window and hence increase the low frequency limit of the analysis unless a means exists to suppress the higher order convolution terms.

In his thesis Bolton described a means of modifying the cepstrum whereby the higher convolution terms could indeed be removed thus removing this as a limiting feature in the cepstrum window length. Time constraints did not permit this to be investigated and this remains a topic for further work. Successful implementation of such processing will then result in the impulse response of the loudspeaker (functioning as a passive reflector) in the cepstrum forming the limit to the length of the cepstrum window and hence the minimum tube length. The possibility of eliminating this constraint by signal processing techniques also needs further investigation.

Mention was made of the construction of a new tube and a sample holder that could be used in the new cepstrum tube as well as the standard impedance tube without the need for moving the sample from one holder to another. Once construction is completed the tests reported in this thesis will need to be repeated to ascertain the degree of correlation of results of the two methods in the absence of mounting errors.

Once the signal processing techniques have been perfected practical implementation of these techniques will need to be conducted by means of field trials with the prototype equipment. At that stage the influence of the outer guard tube on reducing pressure leakage at the tube/sample interface can be extensively tested. The results of the latter will form a key issue in the potential success of the apparatus.

The use of a signal with a flat frequency response was shown to be essential in permitting measurements over a wide frequency range. The chirp signal was successfully implemented in this thesis albeit with the need for additional, relatively complex, signal conditioning. It is considered prudent to include in further work the investigate of alternative signals such as the pseudo random, or Maximum Length Sequence, MLS.

7. APPENDICES

7.1 APPENDIX I

3rd ORDER HIGH PASS FILTER

A simple 3rd order R-C high-pass filter was connected in circuit between the Brüel & Kjaer signal generator and the power amplifier driving the loudspeaker in order to offset the effect of the tube mounted in front of the loudspeaker. With the filter shown in the diagram below a flat frequency response was obtained down to the cut-off frequency of the system. Refer to section 3.4.

Assuming a zero source resistance and a high load resistance the ratio of output of the filter, v_o , to the input voltage, v_i , is given by,

$$\left| \frac{v_o}{v_i} \right| = \frac{R}{\sqrt{R^2 + X^2}} \quad \text{where} \quad X = \frac{-1}{2\pi f_c} \quad \text{and } C = \text{capacitance in Farads}$$

The cut-off frequency is given by,

$$f_c = \frac{1}{2\pi RC} \text{ Hz.}$$

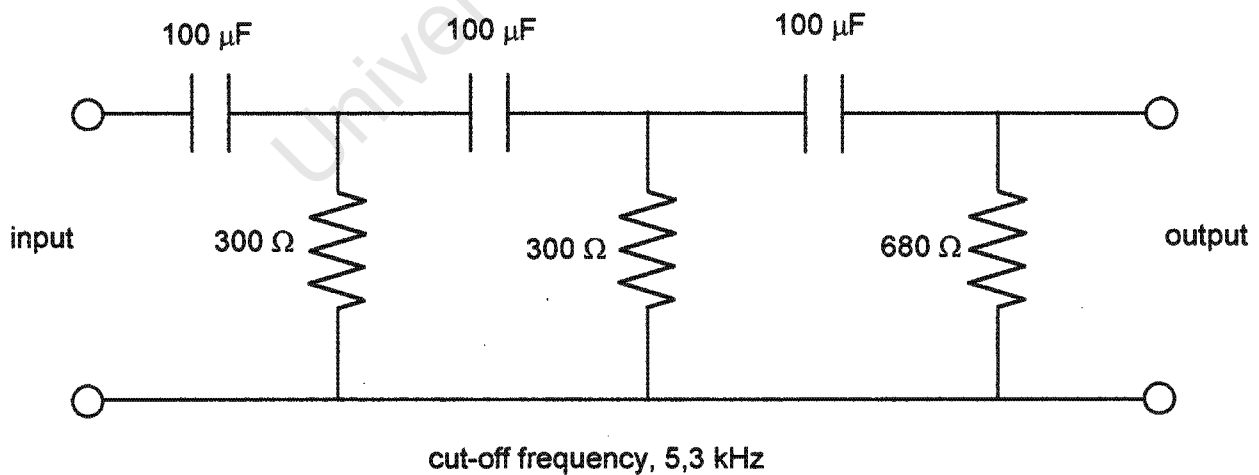


FIGURE 7.1 3rd ORDER HIGH-PASS FILTER CIRCUIT

7.2 APPENDIX II

LISTING OF THE USER-DEFINABLE DISPLAY FUNCTIONS

This appendix lists all the user-definable display functions used in the generation of the modified sweep signal and the analysis of the microphone signal by the Brüel & Kjaer analyser and built-in signal generator. For inclusion in the report the information on the screen of the B&K analyser was stored on disc as a Tagged Image File, TIF. It was subsequently imported into this document as a TIF picture. For use in the B&K analyser it was stored as a B&K UDD file.

7.2.1 CEPS

Uses the power spectrum, "\$:gaa" of the microphone signal in channel (a) and "\$:gbb" of the generator output at channel (b) as starting point. In each case the natural logarithm of the magnitude is obtained and inverse Fourier transformed so producing the cepstrum. The cepstrum of channel (b) is then subtracted from channel (a). Refer to section 3.6. CEPS forms part of User Definable Display Functions CEPST and WIN3T.

USER-DEFINABLE DISPLAY FUNCTION: CEPS

dB reference: 1.00

Graph Displayed as: TIME (Real, V)

Unit: V

```
ceps=cepsa-cepsb
cepsa=ifftr(ln(mag($:gaa)))
cepsb=ifftr(ln(mag($:gbb)))
```

7.2.2 MODSWEEP

Uses the waveform of the sweep signal from the internal generator as starting point. The transfer function of the minimum phase inverse filter, $S_{mp}^{-1}(k)$, is calculated. This is multiplied by a low-pass Butterworth filter and the Fourier transform of the original waveform. The result is inverse Fourier transformed to produce the modified sweep signal, $s_m(t)$. Refer to section 3.7.

USER-DEFINABLE DISPLAY FUNCTION: MODSWEEP

Graph Displayed as: TIME (Real, V)

Unit: V

```
modswEEP=real(iffT(ss)) ;modified linear sweep sm(t)
ss=(s*smp)*l_p         ;frequency response of modified sweep
smp=(exp(ffT(mn)))
l_p=1/(1+(f/4500)^64)  ;Butterworth low-pass filter
mn=(w1+w2+w3)-5
w1=window(cpn,0,1)
w2=window(cpn,1,(no_of_samples-1)/2-1)*2
w3=window(cpn,(no_of_samples-1)/2-1,1)
cpn=(iffT(ls))
ls=ln(mag(1/s))
s=(ffT(b_time))       ;frequency response of sweep s(w)
b_time=$:b            ;Generator sweep signal s(t)
```

7.2.3 ALPHA3

Uses the program CEPS to obtain the cepstrum of the microphone signal after subtraction of the cepstrum of the generator signal. The operator selects by means of the variable, TAU, the starting time of the rectangular cepstrum window and the window length by means of variable, WR, both in milliseconds. These are converted into number of data elements. The windowed cepstrum is Fourier transformed, squared and subtracted from 1 to obtain the absorption coefficient as a function of frequency.

Refer to section 3.8.

USER-DEFINABLE DISPLAY FUNCTION: ALPHA3

dB reference: 1.00

Graph Displayed as: SOUND INTENSITY (Real, V²)

Unit: V

```
alpha3=1-sq(mag(ft3))
sq(x) = x * x
ft3=fftr(win3)
win3=window(ceps,tau/dt,wr/dt)
ceps=cepsa-cepsb
TAU = 4.700m ;      Time in secs - variable
WR = 4.500m ;      ded window width - variable
cepsa=ifftr(ln(mag($:gaa)))
cepsb=ifftr(ln(mag($:gbb)))
```

7.2.4 TAPER

This program is supplied by Brüel & Kjaer as part of a software library included with the B&K analyser.

It permits the windowing of data by means of a Hanning, or cosine squared window with user adjustable length of taper at leading and tail edge using the variables TAPER_L_WIDTH and TAPER_T_WIDTH respectively in number of data elements. The commencement of the window is determined by the variable TAPER_START. The length of the window between tapers is determined by the variable TAPER_WIDTH. Refer section 3.8.

USER-DEFINABLE SUB-EXPRESSION: TAPER

```
taper(x) = x * taper_lead(x) * taper_trail(x)
taper_lead(x) = taper_2( x, taper_l_n0, taper_l_n )
taper_trail(x) = taper_trail_2( x, taper_t_n0, taper_t_n )
taper_l_n0 = taper_start - taper_l_width
taper_l_n = taper_l_width
taper_trail_2(x,n0,n) = mirror( taper_2( x, size(x) - n0 - 1, n ))
taper_2(x,n0,n) = taper_3( int( window( ones[x], n0, taper_min_1( n )) )
taper_t_n0 = taper_start + taper_width + taper_t_width
taper_t_n = taper_t_width
taper_3(x) = mag_squared( sin( (pi/2) * x / extract_element(x,9999)))
taper_min_1(x) = select( x-1, 1, x )
TAPER_START = 0.0 ;
TAPER_L_WIDTH = 0.0 ;
TAPER_WIDTH = 70.0 ;
TAPER_T_WIDTH = 4.0 ;
```


7.2.5 WIN3T

The program ALPHA3 was modified to make use of a one-sided Hanning window in the cepstrum to eliminate ripples in the cepstrum caused by a rectangular window. WIN3 in the fourth line of ALPHA3 was replaced with WIN3T which incorporated the TAPER program supplied by Brüel & Kjaer.

The window was used without taper at the low time end and cosine squared at the high time end. Refer to section 3.8.

USER-DEFINABLE DISPLAY FUNCTION: WIN3T

Graph Displayed as: TIME (Real, V)

Unit: 

```
win3t=taper(win3)
taper(x) = x * taper_lead(x) * taper_trail(x)
win3>window(ceps,tau/dt,wr/dt)
taper_lead(x) = taper_2( x, taper_l_n0, taper_l_n )
taper_trail(x) = taper_trail_2( x, taper_t_n0, taper_t_n )
ceps=cepsa-cepsb
taper_l_n0 = taper_start - taper_l_width
taper_l_n = taper_l_width
taper_trail_2(x,n0,n) = mirror( taper_2( x, size(x) - n0 - 1, n ))
taper_2(x,n0,n) = taper_3( int( window( ones[x], n0, taper_min_1( n ))) )
taper_t_n0 = taper_start + taper_width + taper_t_width
taper_t_n = taper_t_width
cepsa=ifftr(ln(mag($:gaa)))
cepsb=ifftr(ln(mag($:gbb)))
taper_3(x) = mag_squared( sin( (pi/2) * x / extract_element(x,9999)) )
taper_min_1(x) = select( x-1, 1, x )
TAPER_START = 0;tau/dt-2
TAPER_L_WIDTH = 0.0 ;
TAPER_WIDTH=(tau+wr)/dt-20
WR = 4.500m ; ded window width - variable
TAU = 4.700m ; Time in secs - variable
TAPER_T_WIDTH = 20.0 ;
```