Hedge Funds
and Higher Moment Portfolio Selection

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Abstract

Recent studies by Amin and Kat (2001) and Lo (2001) show that, notwithstanding the central limit theorem, the returns of several hedge fund indices exhibit distributional characteristics inconsistent with normality. In this context, this study empirically compares the Markowitz (1952) mean-variance optimisation technique with a higher moment methodology recently proposed by Davies, Kat and Lu (2005). It extends the methodology to optimise portfolios without a unity-variance constraint. In addition, this study augments the application of Davies et al (2005) beyond that of fund of hedge fund portfolio construction to also incorporate the traditional asset classes of equities, bonds and cash.

The descriptive statistics show that hedge fund strategies of Fixed Income Arbitrage and Event-Driven while displaying low volatility, also exhibit latent higher moment risk in their negative skewness and high kurtosis. These two higher moments collectively suggest an increase in the probability of extreme adverse returns to the investor that is not revealed in traditional mean-variance analysis. Confirming the findings of Amin and Kat (2001) and Lo (2002), Jarque-Bera tests find that only two out of the fourteen hedge fund indices used in this study are normal at the 5% level.

Applying Markowitz (1952) mean-variance portfolio selection to an array of published hedge fund indices produces portfolios with higher ex-post returns but naïve exposure to undesirable higher moment risks. When the higher moments of hedge fund index return distribution are accounted for in the portfolio optimisation algorithm, the resultant portfolios have improved diversification and higher moment statistics. This study confirms the findings of Davies, Kat and Lu (2003) and Feldman, Chen and Goda (2002) that Global Macro and Equity Market-Neutral strategies are crucial constituents in a fund of hedge funds portfolio. When comparing optimised multi-asset class portfolios including an allocation to hedge funds, the results show that mean-variance optimisation overallocates to the hedge fund class on the basis of its high reward to volatility ratio. The higher moment optimised portfolios all outperform the mean-variance comparatives when evaluated on an Omega function basis. More generally, the results suggest that when assembling portfolios that include hedge funds, higher-order optimisation makes a meaningful difference to portfolio composition.

Declaration

I, Gregory Bergh, hereby declare that the work on which this thesis is based is my original work (except where acknowledgements indicate otherwise) and that neither the whole work nor any part of it has been, is being, or is to be submitted for another degree in this or any other University. I empower the University to reproduce for the purpose of research either the whole or any portion of the contents in any manner whatsoever.

Signed by candidate

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Introduction

"A hedge fund is an actively managed investment fund that seeks attractive absolute return. In pursuit of their absolute return objective, hedge funds use a wide variety of investment strategies and tools. Hedge funds are designed for a small number of large investors, and the manager of the fund receives a percentage of the profits earned by the fund."

--ROBERT A. JAEGER, Chief Investment Officer – EACM
Selected Definitions of "Hedge Fund" – www.sec.gov

1.1 Introduction

As a result of the general contraction in value of global equity markets post the 1990s technology bubble, hedge funds have strengthened their position as a key investment opportunity. One of the key reasons behind this trend is that the extended methods of operation of hedge funds including short-selling and the use of derivatives can deliver diversification qualities beyond those offered by traditional investments.

Furthermore, in the context of a perceived bear market, the delivery of targeted absolute returns by hedge funds has made them an appealing choice for inclusion in portfolios. There is evidence of large institutions and endowments setting up specific hedge fund programs within their portfolios[1] for precisely this rationale. Over the last decade, research on hedge funds has increased dramatically, covering numerous questions such as performance analysis, style analysis as well as portfolio construction. Nevertheless, research on hedge funds remains thin by comparison with that conducted on traditional investments.

1 The California Public Employees Retirement System (CalPERS) and Yale and Harvard University endowment funds are examples.

1.2 A brief history of hedge funds

The first hedge fund was started in the winter of 1949 by Alfred Winslow Jones in the United States of America. Jones, Australian by birth, earned a Ph.D in sociology from the Columbia University in 1938. He worked as a journalist for both Fortune and Time and it was after spending time writing about the market that he switched to fund management to make his living. A.W. Jones & Co. was formed with four friends as a limited partnership and Jones was the first known investor to hedge his market exposure through a combination of long and short equity positions while utilising leverage to increase returns.

A small number of hedge fund practitioners operated in relative obscurity through the 1960s until a Fortune article about Jones raised interest in hedge funds. Many new managers moved to a more speculative model (i.e. long and leveraged) in the late sixties and were ruined in the market crash of 1969 (Ineichen, 2000). Only with the advent of famous managers like George Soros, Michael Steinhardt and Julian Robertson did growth in the number of hedge funds increase through the 1980s and 1990s. These managers were able to outperform major indices in bull and bear markets and attracted considerable attention from the press. At the same time, the concept became more mainstream in Europe and Asia (AIMA, 2002).

The late 1990's saw a dramatic increase in the number of hedge funds employing a multitude of strategies. It was during this era that the most infamous event in hedge fund history, the collapse of the giant Long Term Capital Management (LTCM) fund occurred. Despite this setback, the industry has continued to grow dramatically and it is currently estimated that there are more than 6000 hedge funds today.

1.3 Motivation and thesis structure

Two key tasks faced by industry practitioners are that of asset allocation and secondly, that of strategy allocation within the hedge fund universe. It is well documented (cf. Cvitani´c, Lazrak, Martellini, and Zapatero, (2002), Agarwal and Naik (2002), Amenc and Martellini (2002) and Amin and Kat (2001)) that hedge funds are marked by their heterogeneity and unusual statistical properties. This makes the use of conventional methods of portfolio construction subject to question and necessitates the investigation of a more sophisticated approach to achieve rational guidelines in the pursuit of appropriate and efficient portfolios.

This thesis compares and evaluates the results of two related optimisation procedures. Firstly, the classic mean-variance portfolio optimisation of Markowitz (1952) and secondly, a recent approach introduced by Davies, Kat and Lu (2005) utilising Polynomial Goal Programming (PGP) to optimise portfolios return distributions for higher moments to include mean, variance, skewness and kurtosis for a given set of investor preferences. Whilst mean-variance optimisation technology has broad acceptance amongst the financial community, the results obtained with its use with respect to hedge funds are not rational and practitioners need more realistic guides for asset allocation. Therefore, this study will investigate a comparison of mean-variance optimisation with the PGP approach of Davies et al (2005) to further optimise for higher moments. This comparison will be presented in the context of both a fund of hedge fund strategy allocation as well as an asset allocation decision of what proportion to allocate to hedge fund assets in a balanced portfolio.

The remainder of this thesis is organised as follows: Chapter 2 presents an overview of the theory of portfolio selection that is relevant to the analysis conducted in this thesis. Classic mean-variance methodology and utility theory are discussed as well as an overview of the properties of probability distributions. In addition, conventional and more recent performance appraisal measures are reviewed.

Chapter 3 provides a review of the prior literature relating to hedge funds and summarises how this interacts with traditional portfolio theory. The chapter is organised to allow a comparison between the various new allocation methodologies that prior research has proposed be utilised with respect to hedge funds. Prior research relating to the appropriateness of mean-variance optimisation is also highlighted.

Chapter 4 presents the hedge fund indices used as well as the proxies used for traditional equity and bond portfolios. A concise summary of the hedge fund styles corresponding to that of the various hedge fund indices is provided. Descriptive statistics and a discussion thereof ensue. The methodology used in the empirical research performed in Chapter 5 in this thesis is also outlined in detail.

Chapter 5 applies the techniques outlined in the previous chapter to investigate the differences between mean variance optimisation and that of the more sophisticated PGP methodology. This analysis will be applied to the two problems of (i) fund of hedge fund strategy allocation and (ii) the asset allocation decision with hedge funds.

Chapter Six concludes with a summary of the key points of the preceding chapters and a discussion of results in the context of the theory provided in Chapter Two and suggests areas for future research.

Contents – Theoretical Review

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Theoretical Review

2.1 Introduction

The process of optimal portfolio selection is a partnership of two areas in finance: utility theory and portfolio optimisation. Utility theory addresses how individuals make investment decisions in order to realise their objectives. Portfolio optimisation provides the means to realise these objectives in the most efficient manner for the investor. This chapter provides a theoretical overview of utility theory and portfolio optimisation theory.

The chapter is set out as follows: Section 2.2 introduces State-Preference Theory (SPT) and Expected Utility Theory (EUT) as a precursor to MVT which is presented in Section 2.3. Section 2.4 provides an alternative to these conventional theories and discusses a Behavioural framework under Prospect Theory. Section 2.5 presents the seminal Mean-Variance portfolio optimisation theory and is followed by an overview of analysis of probability distributions with higher moments contained in Section 2.6.

2.2 Utility Theory

2.2.1 State-Preference Theory

The SPT approach to uncertainty was first introduced by Arrow (1953). The SPT has a three step approach consisting of States, Choice of Action and the resultant Outcomes. This approach as outlined by Bailey (2001) is as follows:

1. States: each distinct state (s_k) is inferred as a marker for some event that could possibly happen. A further assumption is that only one unique event will occur from the alternatives in the set. Investors, however, do not know at the outset, which specific event will occur. In the application of SPT to finance, each mutually exclusive state specifies the payoffs of every asset.

2. Choice of Action: which show all applicable features of the decisions that are made before the state is realised. In the case of portfolio allocation, the action is described by the selection of a particular set of assets.

3. Outcomes: which reflect the results of an action analogous to each state of the world. In its application in the portfolio allocation case, the outcome of an action (the chosen portfolio of assets outlined above) is a set of terminal values of wealth.

According to SPT, each investor is assumed to have preferences defined by outcomes or, the investor has a utility function which value is determined by the attractiveness of the outcomes and therefore can be used to rank the results of the action taken. More specifically, the utility function can be expressed as:

$$U = U(f(s_1, a), f(s_2, a), \ldots, f(s_l, a)) \tag{2.1}$$

where the function $U(.,.,\ldots,.)$ is allowed to differ across investors and s denotes a unique state and a the choice of action.

2.2.2 Expected Utility Theory

The Expected Utility Theory (EUT) builds on the SPT framework. Since its introduction von Neumann and Morgenstern (1944), it has been subject to much debate by researchers. von Neumann-Morgenstern utility describes a utility function that has the expected utility property that: the investor is indifferent between receiving a given good or a gamble with the same expected value.

In other words, the EUT augments the SPT by associating a level of probability which each distinct state of the world. The SPT makes no provision for what investors believe will actually happen. This extension allows for a separation between what the investor's beliefs (denoted by probabilities) and the investor's preferences for a particular state Bailey (2001). The EUT approach then follows a similar rationale to the aforementioned SPT as summarised by Bailey (2001):

1. Evaluate an event and assess actions that differ in their outcomes for the nature of the event but have outcomes that are the same as one another for states not in the event. Under the EUT the investor's ranking of the actions is independent of the outcomes for states not in the event.

2. Evaluate the outcomes in any particular state (a set of states can constitute an event). Under the EUT the investor's preferences for a particular outcome are independent of the state in which they occur. This is a distinguishing feature of von Neumann-Morgenstern utility. The website "The History of Economic Thought"[1] uses a simple analogy of two states *"ice cream when it is sunny"* and *"ice cream when it is raining"*. The EUT assigns the same value to each outcome irrespective of conditions or "state" in which the ice cream is received. In contrast, under SPT, these goods would be seen as two distinct exclusive states with a differing value.

3. The final axiom of the EUT is the investor's degree of conviction about whether a state will occur is independent of the outcomes in the state.

[1] http://cepa.newschool.edu/het/essays/uncert/statepref.htm

The above three assumptions of the von Neumann and Morgenstern utility function simply infer that: (i) the investor acts as if probabilities are related to the states in which the outcomes happen; (ii) that the von Neumann-Morgenstern utility function is a function only of the outcomes; and (iii) the investor ranks the actions corresponding to the expected value of the von Neumann-Morgenstern utility function.

In a formal notation, the EUT implies:

$$U = U(W_1, W_2, \ldots, W_l) = \pi_1 u(W_1) + \pi_2 u(W_2) + \ldots + \pi_l u(W_l) \qquad (2.2)$$

where π_k is the probability that the investor assigns to state s_k. An important assumption is that $u'(W) > 0$ for all relevant levels of W, i.e. investors prefer more wealth to less. In conclusion, the EUT says that actions are chosen to maximise expected utility: This is summarised in the equation below:

$$\mathrm{E}\left[u(W)\right] = \pi_1 u(W_1) + \pi_2 u(W_2) + \ldots + \pi_l u(W_l) \qquad (2.3)$$

von Neumann and Morgenstern (1944) applied the EUT to lotteries and gambling and how investors should behave. In its translation to the financial realm, however, it is utilised as a positive theory in an attempt to show how investors actually behave with respect to gains and losses from an investment perspective. No reason exists to assume that investors are aware of the concept of probability or that they are consciously assigning probabilities to various states per se.

The SPT is too general a theory to apply to a topic such as the future of the global economy and its impact on the outcome of a particular asset selection. In terms of EUT, while investors may not consciously assign probabilities to states, a more complete knowledge of the return distribution of an asset would influence the probability assigned to various states. Therefore, more accurate awareness by an investor of the distribution of asset returns will impact resultant utility. This notion has important implications for hedge funds where distributions are often non-normal.

2.3 Mean-Variance Utility Theory

Assuming the EUT is taken as a fair representation of the investor's aim, it is, however, too general a theory to amount to any practical use in the portfolio selection realm. Thus, a more special form of the von Neumann-Morgenstern function is required. The outline of this theory is again sourced from Bailey (2001).

What is most commonly assumed in financial theory is that the function $u(\cdot)$ is quadratic[2] in nature. Expected utility can be expressed as a function of the expected value, or mean, of terminal wealth and the variance of terminal wealth, if the von Neumann-Morgenstern function is quadratic. This is the origin of "mean-variance" analysis in the context of financial theory and its application in portfolio selection. In the case of many financial assets, this assumption is not unreasonable.

More formally expressed, the expected value of terminal wealth is denoted by $E[W]$ and its variance by $\text{var}[W] = E[W - E(W)^2]$. Then, assuming the function $u(\cdot)$ is quadratic, the expected value of $u(W)$ is a function of $E[W]$ and $\text{var}[W]$, and can be expressed as:

$$E[u(W)] = F(E[W], \text{var}[W]) \tag{2.4}$$

MVT is by its construct restrictive in nature. Some critical features of probability distributions are not revealed in terms of means and variances. As will be shown in Chapter 4, covering data and methodology, hedge funds do not have probability distributions well described in terms of only means and variances. Higher moments of probability distribution are ignored completely. Disregarding these higher moments results in an incomplete analysis of the investor's true utility and therefore the application of MVT with hedge funds can result in faulty conclusions and analyses.

[2] A quadratic equation: "Pertaining to terms of the second degree; as, a quadratic equation, in which the highest power of the unknown quantity is a square."
(Source: http://www.hyperdictionary.com)

This exact topic is addressed by Fung and Hsieh (1997). They find that using the mean-variance methodology to rank investor actions with regard to portfolio selection is *"nearly correct"*. However, the authors also note that: *"It is important to point out that there are circumstances when mean-variance analysis is not appropriate. In particular, risk assessments cannot be done accurately using a second order (i.e. mean-variance) approach."*

The restrictive nature of MVT implies that it is only suited to analysis with random variables that are in fact, well described by means and variances. This may seem an innocuous point but MVT has been applied in instances in financial theory where it may not be wholly appropriate (e.g. hedge funds).

Most MVT models express the objective as a function of the expected value and variance of the rate of return to wealth rather than the absolute level of wealth.

The rate of return on wealth is defined as $r_p = \frac{(W-A)}{A}$,

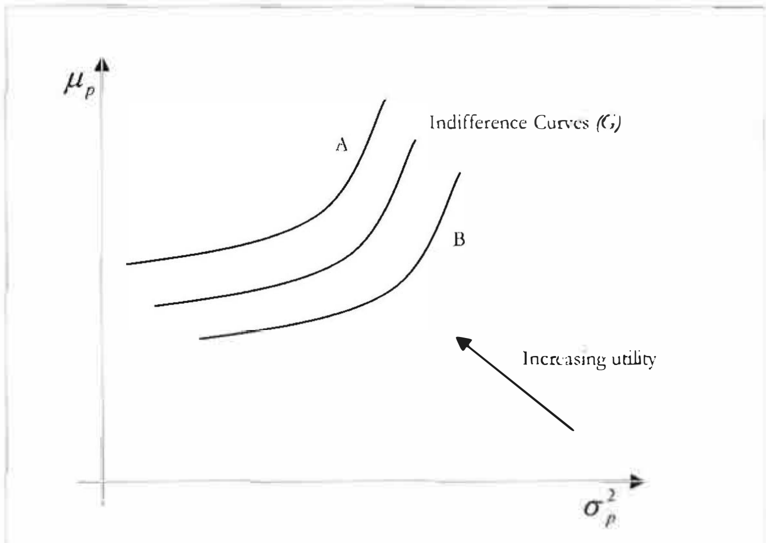
where W represents terminal wealth and A is initial wealth. The expectation and variance of r_p are written as µ$\mu_p = E[r_p]$ and $\sigma_p^2 = E[(r_p - \mu_p)^2]$, respectively (where the subscript P stands for 'Portfolio'). Interpreted in this manner the mean-variance criterion can be expressed as:

$$\text{Mean-variance criterion: } G(\mu_p, \sigma_p^2). \tag{2.5}$$

Conventionally, the variance of return, σ_p^2, (or standard deviation, σ_p) is expressed as the risk of the portfolio. In the case where a normal distribution accurately describes the full extent of an asset distribution, this is an appropriate interpretation. To understand the trade-off between expected return, denoted by μ_p, and risk, expressed by σ_p, it is assumed that $G(\mu_p, \sigma_p^2)$ is increasing in μ_p and decreasing in σ_p. More clearly, expected return is a 'good' and risk is a 'bad'. A further assumption is that the parallel curves (indifference curves) of $G(\mu_p, \sigma_p^2)$ in μ_p, σ_p space, are convex from below.

Figure 2.1: Indifference curves expressed on a μ_p, σ_p^2 plane.

Under a mean-variance framework, investor (or investor) preferences can be depicted by indifference curves in a, μ_p, σ_p^2 space. The portfolio efficient frontier depicts the minimum σ_p^2 for each level of μ_p. The indifference curves depict uniform levels of utility for different combinations of μ_p and σ_p^2. An investor will prefer point A to point B.



The utility theories above have been utilised as models for decision making with risk but have been criticised for assuming too strong a level of rationality from investors. An often cited example of this in markets is the existence of "bubbles" where prices escalate beyond rational value, only to collapse[3]. Ineichen (2001) states that *"bubbles occur when the consensus view with respect to expected returns increases and investors cuddle in the comfort of the consensus view and de-emphasise sound research, due diligence and logical economic reasoning."* This has lead to the development of alternative utility theories.

[3] Examples of bubbles include the South Sea bubble in Britain in 1720 and the technology bubble in the 1990's.

2.4 Behavioural Theory

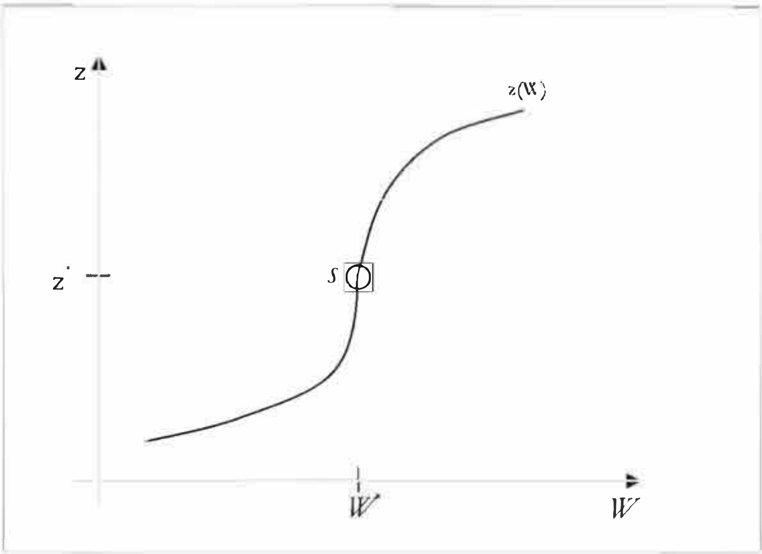
"Behavioural economics" substitutes the strong rationality assumptions in economic modeling (like the SPT, EUT and MVT) with assumptions that are consistent with evidence from human psychology, while maintaining an emphasis on mathematical rigour. Behavioural substitutes generally incorporate a concept known as 'bounded rationality', a term devised by Simon (1957). Bounded rationality suggests that investors cannot take cogniscence of all the information required to make a decision regarding their actions in accordance with rational theories, such as mean-variance theory.

Kahneman and Tversky (1979) proposed a cognitive psychology alternative to the EUT called "Prospect Theory". The key concept in prospect theory is that investors make decisions based on levels of pleasure and thus that utility is determined by gains and losses from some reference point (starting point), as opposed to overall terminal wealth. Prospect Theory assumes 'true' objective probabilities but that investors make errors.

Furthermore, in Prospect Theory the utility function is replaced by a 'value function', say $z(W)$, that is of the kind shown in Figure 2.2. The important features of the value function are: (a) it is a continuous, increasing function of wealth, (b) there exists a point of inflection at a 'reference point' or starting point, S, (which can be identified with the individual's initial wealth, so that $W^* = A$), (c) for wealth in excess of W^*, the individual is risk-averse ($z(\cdot)$ is concave from below, i.e. $u''(W) < 0$), (d) for wealth less than W^*, the individual is a risk-seeker ($z(\cdot)$ is convex from below, i.e. $u''(W) > 0$).

Figure 2.2: Prospect Theory value function.

Kahneman and Tversky devised the value function to express utility from a starting point of wealth (s). Furthermore, for levels of wealth in excess of (s), the investor is risk-averse and seeks to protect wealth, while, for levels of wealth lower than (s), the investor is risk-seeking and seeks higher risk to regain wealth in order to return to (s).



In terms of Prospect Theory as it relates to hedge funds, Figure 2.2 suggests that investors change behaviour based on their relative location from their wealth starting point s. More specifically, investors are thought to under-estimate low probabilities and to over-estimate high probabilities. In other words, they act as if rare events will never occur and as if highly probable events surely will. This is important from a utility perspective as analysis of higher moments may indicate the probability of large adverse returns in terms of their skewness and kurtosis. This is discussed further Section 2.6.

2.5 Mean-Variance Portfolio Optimisation

An optimal portfolio is defined as "*An efficient portfolio most preferred by an investor because its risk/reward characteristics approximate the investor's utility function. A portfolio that maximises an investor's preferences with respect to return and risk.*" (Campbell R. Harvey's Hypertextual Finance Glossary). The standard method for deriving the optimal portfolio is to use the now seminal methodology of Markowitz (1952). The theory in this section is drawn from Bailey (2001) and Bodie, Kane and Marcus (2002).

This model is a mean-variance framework reflecting the trade-off between risk and return as weightings between different assets are altered. Investors are assumed to have utility derived only from means and variance, and assets are assumed to have normally distributed returns. Investors who are acting according to a mean-variance goal select a portfolio that maximises the objective utility function *G:*

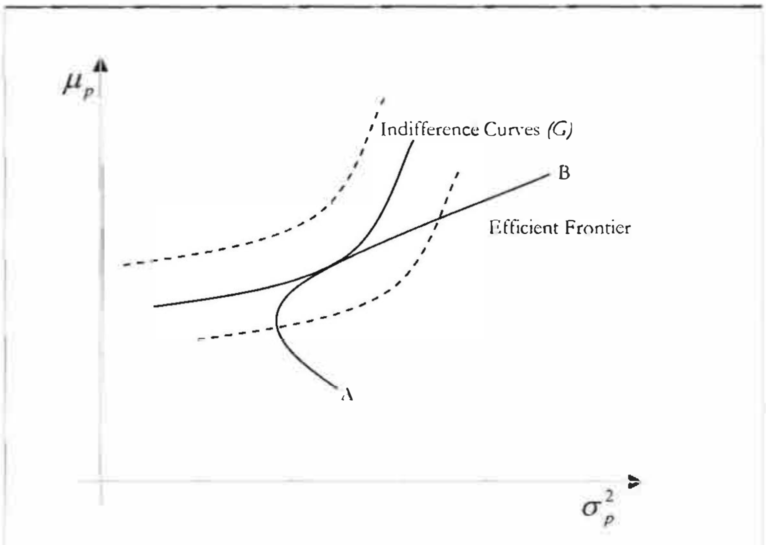
$$G = G(\mu_p, \sigma_p^2) \tag{2.6}$$

Where μ_p is he expected return on portfolio p and σ_p^2 is the variance of return on portfolio p.

The variance acts as a proxy for the risk of owning the portfolio. Pairs of μ_p and σ_p^2 for which the objective utility function G is constant; delineate indifference curves for an investor as shown in Figure 2.3.

Figure 2.3: Efficient Frontier and Indifference Curves.

The portfolio efficient frontier depicts the minimum σ_p^2 for each level of μ_p. The indifference curves depict uniform levels of utility for different combinations of σ_p^2 and μ_p.



The selection of the optimal portfolio by a mean-variance investor has two stages. In the first stage, a portfolio minimum-variance frontier (efficient frontier) is constructed consisting of portfolios for which σ_p^2 is minimised for each μ_p. In the second stage, the utility function *G* is maximised within the feasible boundaries of the efficient frontier. It is assumed this objective utility function is characterised by an increasing μ_p with a decreasing σ_p^2.

Thus, only a segment of this minimum-variance frontier, i.e. the efficient portfolio, is relevant in the second stage. It is this portfolio chosen from the efficient frontier that satisfies the expressed individual preferences of an investor regarding μ_p and σ_p^2. The set of efficient portfolios depends only on expressions of means and variances, not on preferences with regard to expected return. Investors with different risk preferences select different portfolios but will always select a portfolio from the efficient frontier. In practice, observations of past values for returns and variances are often used to estimate

μ_p and σ_p^2. However, these estimates are prone to error and are not necessarily indicative of their future values.

Using a simple example of two risky assets A and B, in figure 1 the end-points of the efficient frontier indicate the expected return and variance of the two assets. Assuming the weight assigned to the assets in the portfolio sums to unity then the expected return of portfolio p is:

$$\mu_p = w\mu_A + (1-w)\mu_B \tag{2.7}$$

Where w is the weighting allocated to asset A. The general version for the expected return of portfolio p with multiple assets is:

$$\mu_p = \sum_{i=1}^{n} w_i \mu_i \tag{2.8}$$

And the variance of portfolio p is:

$$\sigma_p^2 = w^2\sigma_A^2 + (1-w)^2\sigma_B^2 + 2w(1-w)\rho_{AB}\sigma_A\sigma_B \tag{2.9}$$

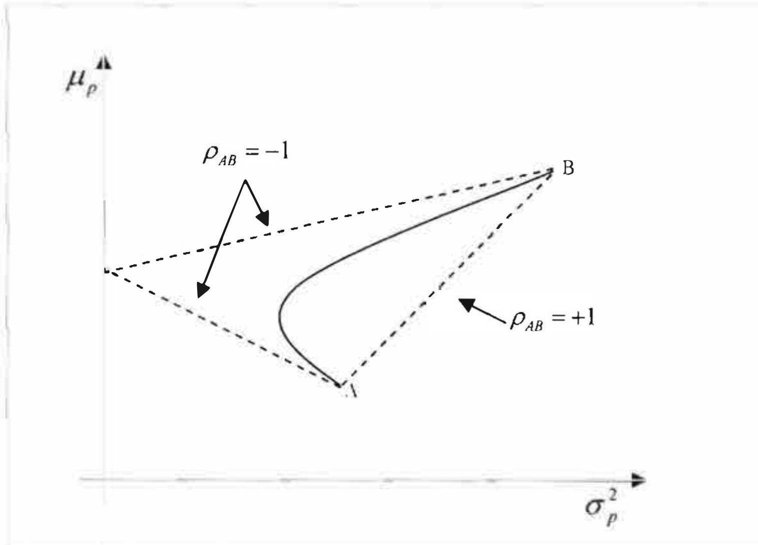
Where w is the weighting in the portfolio of asset A and ρ is the correlation co-efficient between asset A and asset B. The general version for the variance of portfolio p with multiple assets is:

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij} \tag{2.10}$$

From equation (2.9) it can be concluded that the greater (lower) the level of correlation between the two risky assets A and B – the less (more) efficient the minimum variance frontier will be. Furthermore, the less (more) efficient the frontier, the less (more) utility will be derived from such a portfolio. This is illustrated in Figure 2.4.

Figure 2.4: Efficient Frontier and Correlation.

The portfolio efficient frontier is partially dependent on the correlation between asset A and asset B. In extreme cases where ρ is strictly 1 or -1 – the efficient frontier is collinear.



It is this phenomenon of extending the efficient frontier by blending uncorrelated assets that has piqued the interest of practitioners. Hedge funds, with their extended methods of operation, often incorporate this "uncorrelated" attribute. Agarwal and Naik (2002) comment: *"Unlike mutual funds, hedge funds are not evaluated against a passive benchmark and therefore can follow more dynamic trading strategies. Moreover, they can take long as well as short positions in securities … As a result, hedge funds can offer exposure to risk-factors that traditional long-only strategies cannot."*

It is in this pursuit that practitioners have utilised factor analysis to better describe the risk profiles of the potential assets they have at their disposal, in order to effect efficient portfolio construction. Furthermore, factor analysis assists practitioners in understanding the source of the unsystematic (i.e. market or beta independent) returns. Factor analysis is a generic term for a group of statistical techniques, as it relates to the reduction of observable variables to a number of underlying factors. The concept that a number of unobserved latent variables (or "factors") describe the correlations among observed variables, is a key assumption of factor analysis.

An example of such a study as it relates to hedge funds is that of Schneeweis and Spurgin (1996) who construct a multi-factor model in order to explain the returns of managed futures, hedge fund and mutual fund portfolios. They found that hedge funds and managed futures have sources of return unique from traditional stock and bond funds and therefore are good diversifiers to traditional portfolios.

While it has become convention that mean-variance analysis is a cornerstone of modern portfolio selection – it is interesting to note that Markowitz himself understood the limitations of his methodology. In Harvey, Liechty, Liechty, and Muller (2003) it is noted that: *"In a less well known part of Markowitz (1952), he details a condition whereby mean-variance efficient portfolios will not be optimal - when an investor's utility is a function of more than two moments, e.g. mean, variance, and skewness."*

2.6 Moments of a Probability Distribution

A normal distribution is adequately characterised by its mean and variance and it is widely used in science due to its *"mathematical tractability"* NIST (2004). The normal distribution (or Gaussian distribution) has its theoretical foundation in the Central Limit Theorem. According to NIST (2004), this means that as the sample size increases, the following effect occurs: *"The sampling distribution of the mean becomes approximately normal regardless of the distribution of the original variable."*

The Central Limit Theorem states as the number of random variables grows large, the average of a number of independent variables approaches a normal distribution, if certain conditions are fulfilled. These conditions can be summarised as:

1. The mean and standard deviations or the processes generating the returns should be in general stationary and not a function of time.
2. The processes generating the returns should be independent of each other rather than a function of general systematic factors.

Johnson, Macleod and Thomas (2002) comment: *"It is fairly obvious that neither of these conditions is strictly true for hedge funds and it is in part for this reason that the "fat-tails" appear in the distributions of hedge fund strategy returns. For example, systematic trend-followers depend on the existence of trends in various financial markets so that the returns of managers operating this strategy will tend to exhibit a high degree of interdependence and notable time structure."*

A normal distribution is adequately described by its mean and standard deviation. Standard deviation is the second moment of a probability distribution. The standard formula for the calculation of the standard deviation is:

$$\text{Standard Deviation} = \sqrt{\frac{n\sum x^2 - \left(\sum x\right)^2}{n(n-1)}} \tag{2.11}$$

— Do this

Most economic and financial theory assumes that asset returns series are normally distributed. This is largely an accurate assumption when referring to traditional assets, such as equities and bonds. As will be shown in this thesis and already revealed in other research, hedge funds return series cannot be characterised by a normal distribution. Hedge fund data exhibit asymmetric returns series as well as leptokurtic tails[4] and are thus non-normal. Therefore, mean-variance analysis is inadequate a methodology for allocating capital among hedge funds or deciding how much capital to allocate in a balanced portfolio. It is in this vein that this thesis pursues an extension of mean-variance analysis to that of mean-variance-skewness-kurtosis.

Skewness is a measure of symmetry, or more precisely, the lack of symmetry. A distribution, or data set, is symmetric if it looks the same to the left and right of the center point. The standard formula for calculating skewness is:

$$\text{Skewness} = \frac{n}{(n-1)(n-2)}\sum\left(\frac{x_j - \bar{x}}{\sigma}\right)^3 \tag{2.12}$$

[4] Distributions with high excess kurtosis are described as *leptokurtic* while those with low excess kurtosis are described as *platykurtic*.

where $\bar{x}$ is the mean, σ is the standard deviation and n is the number of data points in the sample. The skewness of a normal distribution is zero. Negative values for skewness indicate data that are skewed left and positive skewness indicate data that are skewed right. From a portfolio perspective, investors prefer portfolios with higher (right-skewed) skewed distributions. Hall and Satchell (2004) explain: "*At a more practical level, liking positive skewness is a partial explanation ofwhy people buy lottery tickets. At the level of investments, positive skewness is present in the payouts of a portfolio long in (safe) bonds and long call options. These pay a guaranteed amount with an occasional large positive payout when the option expires in the money. Such products, often referred to as guaranteed products, have enjoyed tremendous success in the last decade, especially in the retail market. In behavioural finance terms, skewness strategies cater for risk aversion with respect to losses and risk affection with respect to gain.*"

Kurtosis is a measure of how the relative peakedness or flatness of a distribution compares with the normal distribution. Positive kurtosis indicates a relatively peaked distribution. Negative kurtosis indicates a relatively flat distribution. The standard formula for calculating excess portfolio kurtosis is:

$$\text{Kurtosis} = \left\{ \frac{n(n+1)}{(n-1)(n-2)(n-3)} \sum \left(\frac{x_j - \bar{x}}{\sigma} \right)^4 \right\} - \frac{3(n-1)^2}{(n-2)(n-3)} \tag{2.13}$$

where $\bar{x}$ is the mean, σ is the standard deviation and n is the number of data points in the sample. The kurtosis value of a normal distribution is three. Thus, a distribution with a high excess kurtosis would have a sharp peak rapidly declining to "fat" or "heavy" tails. From a portfolio perspective, Hall and Satchell (2004) show that as the number of assets in a portfolio increase, kurtosis and skewness both decline but that kurtosis disappears faster than skewness.

2.7 Performance Appraisal Measures

This section aims to outline three key performance appraisal measures utilised by industry practitioners in order to evaluate the attractive of one portfolio over another. The three measures covered are the Sharpe Ratio, Sortino Ratio and a relatively new measure called the Omega function.

2.7.1 Sharpe Ratio

The Sharpe Ratio, originally called the *"reward-to-variability"* ratio by its creator, Nobel Laureate William F. Sharpe, is a measure of excess return-to-risk. The ratio builds directly on the Markowitz mean-variance framework. In other words, the ratio assumes that the mean and standard deviation of a return series adequately describe the outcomes (return and risk respectively) of an investment.

Sharpe recognises the limitations of this measure, he states: *"Clearly, comparisons based on the first two moments of a distribution do not take into account possible differences among portfolios in other moments or in distributions of outcomes across states of nature that may be associated with different levels of investor utility. When such considerations are especially important, return mean and variance may not suffice, requiring the use of additional or substitute measures. Such situations are, however, beyond the scope of this article. Our goal is simply to examine the situations in which two measures (mean and variance) can usefully be summarised with one (the Sharpe Ratio)."* [5] The Sharpe ratio is defined as the excess portfolio return beyond the risk-free rate per unit of volatility defined by the standard deviation of returns:

$$\text{Sharpe Ratio} = \frac{E(r_p) - r_f}{\sigma_p} \tag{2.14}$$

where $E(r_p)$ denotes the expected return of the portfolio, r_f the risk-free rate and σ_p the standard deviation of the portfolio return series.

[5] Source of Sharpe quotes on this page: William F. Sharpe website - http://www.stanford.edu/~wfsharpe/art/sr/sr.htm

2.7.2 Sortino Ratio

The Sortino Ratio was introduced by Sortino and Price (1994) and is essentially a derivative of the Sharpe Ratio. The Sortino Ratio substitutes standard deviation for the downside deviation (or semi-standard deviation) statistic. This means that the measure does not penalise upside volatility. The downside deviation can be measured from any given point but are usually measured below either the risk-free rate or zero. Therefore, the Sortino Ratio is defined as the excess portfolio return beyond the risk-free rate per unit of downside volatility as measured by the semi-standard deviation.

$$\text{Sortino Ratio} = \frac{E(r_p) - r_f}{\sigma_{dp}} \quad (2.15)$$

where $E(r_p)$ denotes the expected return of the portfolio, r_f the risk-free rate and σ_{dp} the semi-standard deviation of the portfolio return series. And the semi-standard deviation is: $\sigma_{dp} = \frac{1}{n-1}\sum(E_{dr} - r_f)^2$, where E_{dr} denotes returns below the risk-free rate r_f

2.7.3 Omega Function

The Omega function was developed by Shadwick and Keating (2002) and incorporates all the higher moments into a performance evaluation. The function also takes into consideration a "threshold level" above which an investor would be satisfied with the absolute return and vice versa. The objective of the authors was to find a "universal performance measure". Unlike other performance measures, like Sharpe or Sortino (which only consider the volatility and downside volatility of returns respectively) the Omega function was designed to take the entire return distribution into account.

The Omega function is defined as follows:

$$\Omega(r) = \frac{\int_r^b [1 - F(x)]dx}{\int_a^r F(x)dx} \tag{2.16}$$

where x is the random one-period return on an investment, r is a threshold selected by the investor and a and b denote the upper and lower bounds of the return distribution respectively. The Omega ratio is effectively the area of the distribution above the threshold level divided by the area below the threshold level. This is an important measurement tool for portfolios that include hedge funds. From a risk-adjusted perspective, it is critical that performance is assessed in the context of the potential increased probability of large extreme losses in hedge funds.

In a paper extending the Omega ratio, Kazemi, Schneeweis and Gupta (2003) show that for ease of calculation, the Omega function can also be expressed as the ratio of the price of a long European call option on the investment divided by the price of a long European put option, where the strike price is the investor threshold level r.

2.8 Summary

Conventional utility theories make strong assumptions of investor rationality and may be too specialised to be practical in portfolio selection. However, utility theories that are either normative or positive in nature all focus on cases of investor risk aversion. The EUH and its more specialised case the MVT reference probability (directly or indirectly) of outcome as a partial determinant of utility. Moreover, in the context of behavioural utility theory where investors become risk-seeking in the face of large losses, an increased awareness as to the full risk of assets that are not necessarily characterised by a normal distribution is crucial. This means that investors balance possible returns and the probability of that return when forming decisions.

MVT is a very special utility case where variance is a proxy for risk and performs a dual function of incorporating the element probability. In certain cases where variance does not completely describe the probability of returns, variance may not adequately account for risk. In this area of inadequacy, higher moments should augment the utility and portfolio construction analysis.

As a counterpart to portfolio construction and investor utility, various performance appraisal measures have been devised. More recently in recognition of the role that higher moments play, performance measures like the Omega function have been developed to more sufficiently account for risk in order to better evaluate return. This better assessment of risk-adjusted returns is important for portfolios containing hedge funds or an allocation to hedge funds.

Contents – Literature Review

3

3

Literature Review

3.1 Introduction

This section provides a review of the relevant prior literature as a precursor to the empirical work in Chapters 4 to 5 and is set out as follows: Section 3.2 introduces portfolio selection under a mean-variance framework with regard to asset allocation with hedge funds. Section 3.3 surveys the literature relating to various alternative methods of portfolio construction. Section 3.4 reviews literature relating to the assembly of fund of hedge funds portfolios and Section 3.5 summarises and concludes.

3.2 Mean-Variance Portfolio Selection with Hedge Funds

The Markowitz mean-variance model is reviewed in Chapter 2. Mean-variance portfolio optimisation forms a cornerstone of portfolio theory. Mean-variance optimisation is a natural starting point for examining and modeling portfolios that include an allocation to hedge funds.

In a study of the risk and return benefits of traditional portfolios with a hedge fund allocation, Schneeweis, Spurgin and Karavas (2002) construct portfolios including hedge funds using the Markowitz mean-variance model. The authors use returns series data from hedge fund data provider EACM, the S&P500 large-cap equity index and the Salomon Brothers Government/Corporate Bond Index over the period 1990-1998. They find that under historical market conditions, a portfolio of hedge funds offers improved risk and return characteristics when pooled with traditional stock and as well as balanced (multi-asset) portfolios. The authors specifically point to *"…the low correlation* *between stock, bond markets, and a wide variety of alternative investments makes the results (improved risk and return opportunities) for the inclusion of various hedge fund strategies ... consistent across a wide variety of stock and bond portfolios.".* Their findings are supportive of the hypothesis that an inclusion of hedge funds in the investment opportunity set enhances the efficient frontier and resultant investor utility. Using this methodology with no allocation constraints imposed often leads to large allocations to hedge funds (i.e. in excess of 90%).

In a related work, Schneeweis and Georgiev (2002) conduct a similar study to Schneeweis, Spurgin and Karavas (2002). This study replaces the Salomon Brothers Government/Corporate Bond Index with the Lehman Brothers Bond Index and the data covers a longer period from 1990-2001. They conclude that hedge funds offer the opportunity to reduce portfolio variance and enhance portfolio returns in economic environments in which traditional stock and bond investments offer limited opportunities. They further conclude that hedge funds allow investors to participate in a wide variety of new financial products and markets not available in traditional investor products.

In addition, Schneeweis and Georgiev (2002) note that the allocation to hedge funds under this mean-variance framework may be *"...affected by the historical high returns achieved by hedge funds in the first half of the 1990's."* Amenc and Martellini (2002) caution that portfolio optimisation procedures are very sensitive to differences in expected returns. They caution that portfolio optimisers typically allocate the largest proportion of capital to the asset class for which the estimation error in the expected returns is the greatest.

The conventional mean-variance approach above is also criticised by numerous other investigations, including Cvitani´c, Lazrak, Martellini, and Zapatero, (2002), Agarwal and Naik (2002), Amenc and Martellini (2002) and Amin and Kat (2001). These studies observe that mean-variance portfolio optimisation makes the key assumption of normal asset return distributions.

Lo (2001) states that: *"hedge-fund returns are highly non-normal, i.e., they are asymmetrically distributed, highly skewed, often multi-modal, and with fat tails that imply many more tail events than the normal distribution would predict."* Research conducted by Amin and Kat (2001) finds that *"The return distribution of a number of hedge fund indices appears to be highly skewed."* Amin and Kat (2001) also find only 14.1% of the individual hedge fund returns are normal utilising a Jarque-Bera test for normality at the 5% significance level.[1]

Amenc and Martellini (2002) construct mean-variance optimised portfolios using an improved estimation of the covariance structure of hedge fund returns. While the authors state awareness that hedge fund returns may not be normally distributed, they justify their approach based on work by Fung and Hsieh (1997) that mean-variance analysis may still be applicable to hedge funds as it preserves the ranking of preferences in standard utility functions. The study is conducted using the S&P style indices as well the CSFB Tremont Hedge Fund indices. The authors conclude that the inclusion of hedge funds in an equity portfolio can *"generate a dramatic decrease in the portfolio volatility on an out-of-sample basis while maintaining a reasonable exposure to traditional investment styles."*

Amin and Kat (2002) conduct a study with the objective of examining the effects of diversification by adding hedge funds to a traditional stock and bond portfolio. More specifically, they study the change in the portfolio return distribution with the hedge fund augmentation. They find the similar results as Schneeweis and Georgiev (2002), as well as Schneeweis, Spurgin and Karavas (2002) that the inclusion of hedge funds significantly improves the portfolios mean-variance characteristics. However, they also find that portfolios constructed of equities and hedge funds do not combine well into truly low risk portfolios as it lowers the skewness and increases the kurtosis of the portfolio. The authors note that: *"...in terms of skewness hedge funds and equity do not mix very well. In economic terms, the data suggest that when things go wrong in the stock market, they also tend to*

[1] The Jarque-Bera normality test observes both the third and fourth moments of a probability density function. The statistic follows a χ^2 distribution with two degrees of freedom.

Jarque-Bera test statistic: $$JB = n\left[\frac{S^2}{6} + \frac{(K-3)^2}{24}\right]$$

where S is the skewness, K is the kurtosis and n is the number of observations.

go wrong for hedge funds. In a way, this makes sense. A significant drop in stock prices will often be accompanied by a widening of a multitude of spreads, a drop in market liquidity, etc."

Interestingly, in Harvey, Liechty, Liechty, and Muller (2003) it is noted that: *"In a less well known part of Markowitz (1952), he details a condition whereby mean-variance efficient portfolios will not be optimal - when an investor's utility is a function of more than two moments, e.g. mean, variance, and skewness."* As discussed in Chapter 2, it is clear that from a behavioural utility perspective that these higher moments play a significant role in the determination of investor utility. Based on this evidence, many researchers have chosen alternative methods of portfolio optimisation to account for the non-normality of hedge fund returns.

3.3 Other Approaches to Portfolio Selection with Hedge Funds

As a consequence of the findings presented in the previous section, indicating that hedge funds are not suitable for mean-variance optimisation due to their non-normal nature, other portfolio selection approaches have been developed. These developments aim to better estimate the risk implicit in hedge funds and to use the information embedded in their return distributions to create more efficient and robust portfolios. Furthermore, more progressive approaches with respect to investor utility have been adopted in order to provide more practical portfolios.

Amenc and Martellini (2002) also investigate the out-of-sample performance of a fund of hedge funds portfolio, with an improved estimator of the covariance structure of hedge fund index returns, as mentioned above. Results show a 7 times reduction in volatility of the optimal mean-variance portfolio, in comparison to the CSFB Tremont Hedge Fund Composite (an asset-weighted diversified hedge fund index, also used in this study). Conversely, the difference in returns between these portfolios is not statistically significant. These results suggest that the enhancement in volatility does not necessarily come at the cost of lower expected returns.

Cvitani´c, Lazrak, Martellini and Zapatero (2002) investigate what portion of a portfolio managers should allocate to hedge funds. They apply investor utility maximising model for optimal investment portfolios in the presence of uncertain abnormal returns to the MAR (Managed Account Reports) database of hedge funds. Utility is modeled as a function of final wealth. Cvitani´c et al (2002) find that the presence of model risk significantly decreases an investor's optimal allocation to hedge funds. Another useful discovery of this paper is that low beta hedge funds may serve as natural substitutes for a substantial portion of an investor's risk-free asset holdings.

Indjic (2002) investigates the benefits of introducing a fund of hedge funds to a portfolio that already contains large allocations to other alternative assets such as Real Estate and Private Equity. This is especially relevant for very wealthy individuals who often have time-horizons beyond their own lifetime. The author concludes that the inclusion of a customised fund of hedge funds can deliver a superior risk profile than a fund with pre-determined ("off-the-shelf") strategy allocations. A different strategy mix optimisation may be required to meet the objectives of reducing correlations and reducing risk.

Hagelin and Pramborg (2003) construct diversified portfolios including hedge funds using a model that focuses on growth in invested capital while allowing for all the higher moments of the return distribution to impact the analysis. The model has a utility formula based on the power and logarithmic functions. Using a combination of the S&P500, the MSCI World Equity as well as a sovereign bond and corporate bond indices, the authors compare diversified portfolios with and without an allocation to hedge funds. The HFR fund weighted index is used a proxy for a hedge fund asset. The authors find that an allocation to hedge funds brings about an increase in geometric returns for almost all strategies without an increase in volatility. Furthermore, the authors conclude that: *"our results show that allocations to hedge funds are substantial at times, and that hedge funds enter the risk-neutral portfolio as well as the most risk-averse portfolio."*

3.4 Fund of Hedge Fund Portfolios

Several recent studies investigate the construction of the optimal fund of hedge funds portfolio. Fund of hedge funds are often seen by investors as an efficient manner to access hedge fund manager capability. Industry data provider Hedge Fund Research (HFR) statistics show that Fund of Hedge Funds currently hold 30% of the estimated $650 billion invested in hedge funds globally as at December 2003.

Amin and Kat (2002) analyse the performance of baskets of hedge funds ranging in size from 1 to 20 funds. Using 1721 hedge funds (drawn from the Tremont TASS database) from June 1994 to May 2001, they show that increasing the number of funds can be expected to lead not only to a lower volatility but also, and less attractive, to lower skewness and increased correlation with the S&P500. Most of this change occurs for relatively small portfolios while holding more than 15 hedge funds changes little. Their efficiency test indicates that an investor only needs to combine a small number of funds to obtain a substantially more efficient risk-return profile than that offered by the average individual hedge fund.

Lhabitant and Learned (2002) investigate the same question using a naïve diversification (equal weighted) Monte Carlo simulation on a database of 6985 hedge funds. They find that increasing the number of hedge funds (from 1 to 50 funds) in a portfolio reduces the return distribution symmetry and increases kurtosis. The authors find that most of the diversification benefits are delivered with a small number of hedge funds (5-10 funds).

In Morton, Popova and Popova (2003), the authors consider portfolio construction where the underlying investment instruments are hedge funds. Benchmarks and conditional-value-at-risk measures are employed alongside utility functions involving the probability of outperforming a benchmark and the expected shortfall from another benchmark. They model portfolios using the normal-to-anything (NORTA) method – which accounts for non-normal distributions. The proposed framework can be used to construct a fund of funds matched to a given investor objective. It can also be used to actively manage a portfolio of hedge funds with the goal of systematically maintaining performance above a given benchmark.

A wholly different approach is developed by Rosenberg, Tomeo, and Chung (2004) who categorise hedge fund strategies into "convergent" and "divergent" in nature. Convergent strategies rely heavily on markets being efficient and assets being accurately priced. These strategies seek to benefit from small mispricings in the market. Convergent strategies include Equity Market Neutral, Event-driven and Arbitrage-type strategies[2]. Divergent strategies are based on the idea that markets are occasionally inefficient providing opportunities that can be exploited by technically oriented managers. Examples of Divergent strategies are Global Macro and Managed Futures. It is explained explain that Convergent strategies tend to be "short volatility" and Divergent strategies are "long volatility". Their research shows that Divergent strategies experience higher performance during periods of increasing market uncertainty. The authors state that: *"Despite its popularity among practitioners, academics have historically dismissed the utility of divergent strategy because it is inconsistent with one of the most fundamental theories in traditional finance – the theory of market efficiency."* and conclude: *"Our study, however, shows the time-varying validity of the divergent strategy and its potential benefits as a portfolio component...provides increased return and reduced risk opportunities..."*

Feldman, Chen and Goda (2002) develop a simulation-based optimisation method for the construction of optimal fund of hedge fund portfolios that is based on the skewness and kurtosis of returns. Vector autoregression (VAR) methods are used to model the relations among asset returns. Investor preferences are represented by a group of utility functions that integrate both risk and loss aversion. Results suggest that the returns to market-neutral and global macro funds have disttribution characteristics that make them attractive investment vehicles for risk and loss-averse investors.

2 These hedge fund strategies are explained in Chapter 4.

There is substantial research on portfolio selection extending mean-variance analysis to the third moment – more recently researchers have begun to extend this framework to include the fourth moment as well. Using the TASS hedge fund database, Davies, Kat and Lu (2003) explore the interaction of the higher order co-moments and their impact on portfolio construction and their basic findings are outlined below. Co-moments are defined as the interaction between up to four assets, within the framework of a portfolio.

They specifically focus on the higher co-moments between various hedge fund strategies, particularly co-skewness and co-kurtosis. They explicitly note: *"...diversification deteriorates skew and improve kurtosis in most strategies. Skewness in all strategies, kurtosis in all but distressed securities and merger arbitrage funds are reduced when moving from the individual fund level to the portfolio level. This implies a tradeoff between variance-skewness-kurtosis in hedge fund portfolios. Thus, mean-variance optimal criteria can lead to sub-optimal portfolios in the presence of skewness and kurtosis."* Davies et al (2003) draw a similar conclusion as Feldman et al (2002) that market neutral funds and global macro funds have a key role in optimal hedge fund portfolios. In addition the authors conclude that market neutral funds are kurtosis reducers while global macro funds are skewness enhancing.

The above study assembles single-strategy fund of hedge fund portfolios. They find that as more funds are included, portfolio volatility (standard deviation) and skewness fall. Davies et al (2003) note: *"Risk and skewness reduction both occur at a decreasing rate, with the reduction in portfolio skewness occurring at a much slower speed. Since positive skewness is generally a desirable trait, there is a clear trade-off between skewness and risk."*

This finding concurs with that of Lhabitant and Learned (2002) and is the rationale behind their conclusion to limit the number of funds within a fund of hedge fund portfolio to 5-10 funds. Davies et al (2003) find that as the number of funds contained in the fund of hedge funds increases *"portfolio expected skewness depends only on the coskewness between three different funds."* and that following the same rationale: *"portfolio expected standard deviation depends only on covariance and portfolio expected kurtosis depends only on the cokurtosis between four different funds. The influence from individual fourth central moment, cokurtosis between two different funds and three different funds on expected portfolio kurtosis tends to zero"* Both studies agree that as the number of Event-driven type strategies are included, the kurtosis of the portfolio will increase (the fund of hedge funds becomes more likely to be affected by a systematic shock e.g. LTCM, failure of a mega-merger).

In a related work, Davies, Kat and Lu (2005) utilise a polynomial goal programming technique to construct fund of hedge fund portfolios adjusting for investor preferences with respect to competing objectives in terms of mean, variance, skewness and kurtosis. The findings from this study confirm their earlier work above and provide a useful framework for optimising hedge fund portfolios. It is this methodology which is used in this study to investigate optimal allocations to hedge funds within a traditional portfolio as well as allocation among hedge fund strategies within a fund of hedge funds.

3.5 Summary

Mean-variance portfolio optimisation is a cornerstone of portfolio theory. Researchers have used the work of Markowitz (1952) to analyse what effect the addition of hedge funds would have on the broad characteristics of a balanced portfolio. Mean-variance results indicate improved risk and return diagnostics. Of concern, is the fact that without the imposition of allocation constraints, mean-variance allocations to hedge funds can be very large (i.e. above 90%).

Nevertheless, this approach has been criticised by other researchers for being unsophisticated, given that research shows that hedge funds are not compatible with mean-variance optimisers due to non-normal return series as found by Amin and Kat (2001) and Lo (2001). It is in this vein, that researchers have attempted new methods of portfolio construction with hedge funds. Methodologies have been developed to make use of these irregular distribution features to the advantage of the investor.

Contents – Data and Methodology

4

4

Data and Methodology

4.1 Introduction

This chapter introduces the data, discusses the descriptive statistics and outlines the methodology used in the empirical section of this study. Section 4.2 provides an overview of hedge fund strategies. Section 4.3 reviews the various data sources and section 4.4 discuss the problem of survivorship bias in hedge fund and hedge fund index data. Section 4.5 reviews the data with an emphasis on the various moments of the probability distribution and how they differ across strategies. This section concludes with a test for normality on all data series. Section 4.6 discusses the two portfolio construction methodologies utilised in this thesis, namely, mean-variance optimisation and a more recent framework introduced by Davies, Kat and Lu (2005) that optimises for mean, variance, skewness and kurtosis based on investor preferences. Section 4.7 summarises and concludes.

4.2 Hedge Fund Strategies

It is worthwhile given the alternative nature of hedge fund strategies to discuss the diversity and distinguishing attributes of the different hedge fund strategies. One of the most discussed and debated points contained in hedge fund research is whether hedge funds constitute a separate asset class. According to Ineichen (2003): *"Viewing hedge funds as a separate asset class is probably the consensus view among institutional investors as of April 2002."* However, a growing view in the opinion of the author is that stated by Terhaar, Staub and Singer (2003) when they note: *"We generally distinguish between alternative assets and alternative strategies. The returns of alternative assets are primarily a function of passive or systematic* *market characteristics. Alternative strategies, on the other hand, produce returns that are largely a function of active management; that is, they are hedge funds."* Thus, hedge funds are not really a new asset class but a different style of active portfolio management not unlike that of value or growth in equity investing.

UBS Warburg classifies hedge funds into three broad categories of Relative-Value, Event-Driven and Opportunistic.

Table 4.1: Hedge fund strategies

This table is organised so that the directional bias of the strategies increases from left to right

Relative-Value	Event-Driven	Opportunistic
Convertible arbitrage	Risk arbitrage	Macro
Fixed income arbitrage	Distressed securities	Short selling
Equity market-neutral		Emerging markets
Statistical arbitrage		Long/Short equity
Fundamental arbitrage		

Source: UBS Warburg

4.2.1 Relative-Value strategies

Relative-Value strategies are also known as market-neutral strategies and seek to exploit mispricings between related securities with the objective of not assuming any directional bias with respect to the underlying market in which the securities trade. Relative-Value strategies can be based on either fundamental or statistical analyses which have discovered that a particular relationship has deviated from its historical norm or fair value. *"These strategies are engineered to profit if and when a particular instrument or spread returns to its theoretical or fair value."* Ineichen (2003). Market risk is hedged by assuming both long and short positions in the related securities.

4.2.1.1 Convertible Arbitrage

This strategy involves investing in convertible securities (usually convertible bonds) and valuing each underlying analytic within the instrument independently to determine a fair overall valuation. Using a convertible bond as an example, the instrument is composed of a straight bond combined with either a long call or short put option on the underlying firm's equity. Valuing a straight bond is a straightforward exercise and hence most of the value lies in the valuation of the optionality included in the synthetic convertible bond. A common strategy to extract value from an undervalued convertible bond would be to buy the convertible bond and short treasuries against it to hedge duration risk. This effectively leaves the fund with a cheap option. A key risk that remains is that of credit risk.

4.2.1.2 Fixed Income Arbitrage

These funds seek to exploit mispricings between and within world fixed income markets. Market risk is hedged by the fund taking offsetting long and short positions in order for the overall duration to be zero. Common strategies include: being long and short "on-the-run" and "off-the-run" bonds (a bond is said to be "on-the-run" if it is the most current offering of its maturity, e.g. 30-year treasuries. An "off-the-run" 30-year treasury would have been issued slightly earlier and have shorter maturity, the market tends to pay a premium for the most recently issued bond); arbitrage between physical bonds and futures (basis trading) and credit (being long corporate debt and short sovereign debt in order to hedge duration and earn additional yield). As the pricing anomalies tend to be small, leverage utilised within fixed income arbitrage strategies tends to be very high.

4.2.1.3 Equity Market-Neutral

Equity Market-Neutral is the original Alfred Winslow-Jones model, utilising a combination of both long and short equity positions to extract value from a relationship while bearing very little exposure to the market as a whole. According to CSFB-Tremont: *"Market neutral portfolios are designed to be either beta or currency neutral, or both. Well designed portfolios typically control for industry, sector, market capitalisation, and other exposures."*

Equity Market-Neutral strategies can be either quantitative or qualitative in nature categorising managers into either statistical arbitrage or fundamental arbitrage respectively.

4.2.1.3.1 Statistical Arbitrage

Quantitative market-neutral funds apply statistical techniques to historical data to discover systematic mispricings between related securities. Ineichen (2003) states: "*The traditional discipline entails hypothesising the existence of a particular type of systematic opportunity for unusual returns, and then back-testing the hypothesis.*"

4.2.1.3.2 Fundamental Arbitrage

These funds invest long and short based on the fundamental attractiveness of an equity security. These funds would be simultaneously long and short stocks in the same sector sharing similar economic exposures, thus creating a portfolio with a beta of zero or no market exposure.

4.2.2 Event-Driven

Event-Driven strategies invest and trade based on analysis of securities whose value should benefit or deteriorate from the incidence of extraordinary events. These events can be mergers, unbundlings, bankruptcies, corporate restructurings or buy-backs.

4.2.2.1 Risk Arbitrage

Risk Arbitrage involves exploiting the activity around securities involved in a merger. This often involves longing the stocks of the company to be acquired and shorting the stock of the acquirer. Once a deal has been announced, a manager will consider fair value for the company to be acquired and judge the likelihood of the transaction taking place. This strategy depends on shareholders and regulators approving the deal in order for value to be realised.

4.2.2.2 Distressed Securities

Managers base their strategies on the actual or probable occurrence of a particular event such as an insolvency announcement or company reorganisation as a result of difficult operating or financial difficulties. AIMA (2002) states: "*Distressed or high yield securities are generally below investment grade and require a high level of due diligence to take advantage of the inexpensive prices at which they are trading.*"

4.2.3 Opportunistic Strategies

Opportunistic hedge funds involve the manager implementing a portfolio based on their current view of economic conditions and market opportunities. These strategies tend to be more volatile than either Relative-Value or Event-Driven strategies and this is generally a result of them being more directional in nature.

4.2.3.1 Macro

Macro (also called Global Macro) is the most flexible of investment strategies, with the manager often taking a top-down or investment theme approach. The strategy allocates capital between regions, markets and instruments based on the manager's forecasts of changes in factors such as interest rates, exchange rates and liquidity. Various trading strategies are utilised depending on the opportunities identified.

4.2.3.2 Short-Selling

The manager here runs a portfolio that has a net-short bias and seeks to profit from securities they believe are overvalued and will fall in price. Short-bias funds also benefit from the interest they earn on the cash received from the proceeds of the short sales.

4.2.3.3 Emerging Markets

These funds are involved in investing in emerging markets around the world. As many emerging markets do not allow short selling, nor offer viable futures or other derivative products with which to hedge, emerging market investing often employs a long-only strategy.

4.2.3.4 Long/Short Equity

Managers employing this strategy will hold both long and short positions with a net long exposure. The objective is not to be market neutral. Managers have the ability to shift from value to growth and from small to medium to large capitalisation stocks. Managers may use futures and options to hedge.

4.3 Data

The data consists of hedge fund index data and conventional long-only market index data. The data series utilised in this study is monthly in periodicity and covers the period from January 1994 until end of June 2004[1]. Hedge fund index data is provided by CSFB Tremont and the market index data by Morgan Stanley Capital International (MSCI) and Lehman Brothers. The hedge fund data below is organised according to the UBS Warburg classifications with corresponding weightings in the Composite index. The Event-Driven category has three sub-indices for which no weightings are available[2].

CSFB Tremont Hedge Fund Composite	100%
Relative Value Indices:	
CSFB Tremont Hedge Fund Convertible Arbitrage	4.7%
CSFB Tremont Hedge Fund Equity Market Neutral	4.4%
CSFB Tremont Hedge Fund Fixed Income Arbitrage	7.8%
Event-Driven Indices:	
CSFB Tremont Hedge Fund Event Driven	21.6%
CSFB Tremont Hedge Fund Distressed Securities	
CSFB Tremont Hedge Fund Event Driven Multi-Strategy	
CSFB Tremont Hedge Fund Risk Arbitrage	
Opportunistic Indices:	
CSFB Tremont Hedge Fund Managed Futures	5.8%
CSFB Tremont Hedge Fund Global Macro	12.7%
CSFB Tremont Hedge Fund Long/Short Equity	26.3%
CSFB Tremont Hedge Fund Dedicated Short Bias	0.5%
CSFB Tremont Hedge Fund Emerging Markets	3.7%

[1] This window of time includes the Southeast Asian and Russian debt defaults of 1998 which coincides with the LTCM crisis. It also covers the technology bubble of the late nineties, its subsequent collapse and the September 11, 2001 tragedy. This is a meaningful period over which to evaluate hedge funds as various market cycles and events have occurred.

[2] The above data can be viewed at: http://www.hedgeindex.com

Other:

CSFB Tremont Hedge Fund Multi-Strategy	12.6%

CSFB Tremont is a major provider of hedge fund data and compiled the first asset weighted hedge fund indices. These hedge fund indices use the TASS database as the source of the individual hedge fund data. TASS is one of the leading providers of individual hedge fund data.

The CSFB Tremont indices also have minimum criteria for inclusion into the index: a minimum of US $10 million assets under management, a minimum one-year track record, and current audited financial statements. The index is calculated and rebalanced monthly. Funds are reselected on a quarterly basis as necessary. To minimise survivorship bias, funds are not removed from the index until they are fully liquidated or fail to meet the financial reporting requirements.

The long-only market index data is provided by MSCI who has been a leading provider of market data since 1969. MSCI provides the following index data:
MSCI World Equity Index
MSCI World Sovereign Bond Index

The MSCI World Index is a free float-adjusted market capitalisation index that measures global developed market equity performance. As of December 2003 the MSCI World Index consisted of the following 23 developed market country indices: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hong Kong, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, the United Kingdom and the United States.

The MSCI World Sovereign Bond Index is a market capitalisation index that is designed to measure global developed market fixed interest performance. Over 50% of the index consists of debt originating from Japan (28.7%), United States (19.5%) and Germany (9.8%). Cash returns are proxied using the Lehman Brothers Cash Composite. Long-only index data is sourced from the Bloomberg database. All indices in this study are denominated in US dollars.

4.4 Descriptive Statistics

As this thesis focuses on hedge fund portfolio optimisation incorporating four moments[3], all the data series analysed have been ranked in tables 4.1 through 4.4. A full table of descriptive statistics for all the aforementioned data has been calculated using EViews software and is displayed in Appendix A. The rationale for the ranking follows that of Athayde and Flores (2001) who propose that investors *"like odd moments and dislike even ones"*. Therefore, the mean and skewness sorted tables (Tables 4.2 and 4.4 respectively) are ranked in descending order while standard deviation and kurtosis sorted tables (Tables 4.3 and 4.5 respectively) are ranked in ascending order.

It can be seen from the statistics in Table 4.2 that most hedge fund strategies (with the exception of the Dedicated Short-Bias strategy) have a higher monthly mean return than either global equities or bonds over the sample period. The top three performing hedge fund strategies in absolute terms over the period are Global Macro, Distressed Securities and Long/Short Equity. The worst performing hedge fund strategy is that of Dedicated Short-Bias. All of these strategies can be categorised as opportunistic under the UBS Warburg classification discussed earlier in this chapter.

3 A probability distribution can be described by its moments. The first four moments of a probability distribution are the mean (1), variance (2), skewness (3) and kurtosis (4).

Table 4.2: Series distribution moments ranked by Mean

This table is ranked on the Mean value in descending order. The mean is calculated by taking the sum of all values in a data series and dividing by the total number of values:

	Probability distribution moments ranked by Mean			
	Mean	Std. Dev.	Skewness	Kurtosis
CSFB Tremont Hedge Fund Global Macro	1.17%	3.42%	-0.02	5.07
CSFB Tremont Hedge Fund Distressed Securities	1.07%	1.97%	-2.75	19.13
CSFB Tremont Hedge Fund Long/Short Equity	0.98%	3.11%	0.24	6.38
CSFB Tremont Hedge Fund Event Driven	0.92%	1.71%	-3.46	25.83
CSFB Tremont Hedge Fund Composite	0.89%	2.40%	0.10	4.77
CSFB Tremont Hedge Fund Event Driven Multi-Strategy	0.83%	1.80%	-2.68	19.83
CSFB Tremont Hedge Fund Equity Market Neutral	0.83%	0.88%	0.25	3.21
CSFB Tremont Hedge Fund Convertible Arbitrage	0.81%	1.37%	-1.47	6.60
CSFB Tremont Hedge Fund Multi-Strategy	0.73%	1.27%	-1.27	6.45
CSFB Tremont Hedge Fund Emerging Markets	0.68%	5.03%	-0.57	6.71
CSFB Tremont Hedge Fund Risk Arbitrage	0.67%	1.26%	-1.31	9.19
CSFB Tremont Hedge Fund Managed Futures	0.58%	3.54%	0.04	3.41
CSFB Tremont Hedge Fund Fixed Income Arbitrage	0.56%	1.12%	-3.24	19.35
MSCI World Equity	0.54%	4.16%	-0.57	3.47
MSCI World Sovereign Bond	0.52%	1.91%	0.37	3.49
Lehman Brothers Cash Composite	0.34%	0.15%	-0.69	2.06
CSFB Tremont Hedge Fund Dedicated Short Bias	-0.13%	5.10%	0.92	5.17

From a volatility perspective, the least volatile hedge fund strategy is that of Equity Market Neutral and the only series less volatile is that of US cash, as displayed in Table 4.3. This is quite intuitive given that these portfolios are largely devoid of market risk, having roughly equal long and short positions in related securities. The most volatile strategy is Dedicated Short-Bias. Table 4.3 also roughly approximates the UBS Warburg categorisation of hedge fund strategies in Table 4.1. It is notable and logical that the "opportunistic" strategies are the most volatile: Dedicated Short-Bias, Emerging Markets, MSCI World Equity, Managed Futures, Global Macro and Long/Short Equity are all directional strategies. This group is followed by mostly "event-driven" strategies and is then followed by the "relative value" type strategies.

Table 4.3: Series distribution moments ranked by Standard Deviation

This table is ranked on the Standard Deviation value in ascending order. Higher values of the Standard Deviation indicate greater volatility of a distribution around the mean. The Standard Deviation is given by:

$$\sigma = \left(\frac{1}{n-1} \sum (x_i - \overline{x})^2 \right)^{\frac{1}{2}}$$

	Probability distribution moments ranked by Std. Dev.			
	Mean	Std. Dev.	Skewness	Kurtosis
Lehman Brothers Cash Composite	0.34%	0.15%	-0.69	2.06
CSFB Tremont Hedge Fund Equity Market Neutral	0.83%	0.88%	0.25	3.21
CSFB Tremont Hedge Fund Fixed Income Arbitrage	0.56%	1.12%	-3.24	19.35
CSFB Tremont Hedge Fund Risk Arbitrage	0.67%	1.26%	-1.31	9.19
CSFB Tremont Hedge Fund Multi-Strategy	0.73%	1.27%	-1.27	6.45
CSFB Tremont Hedge Fund Convertible Arbitrage	0.81%	1.37%	-1.47	6.60
CSFB Tremont Hedge Fund Event Driven	0.92%	1.71%	-3.46	25.83
CSFB Tremont Hedge Fund Event Driven Multi-Strategy	0.83%	1.80%	-2.68	19.83
MSCI World Sovereign Bond	0.52%	1.91%	0.37	3.49
CSFB Tremont Hedge Fund Distressed Securities	1.07%	1.97%	-2.75	19.13
CSFB Tremont Hedge Fund Composite	0.89%	2.40%	0.10	4.77
CSFB Tremont Hedge Fund Long/Short Equity	0.98%	3.11%	0.24	6.38
CSFB Tremont Hedge Fund Global Macro	1.17%	3.42%	-0.02	5.07
CSFB Tremont Hedge Fund Managed Futures	0.58%	3.54%	0.04	3.41
MSCI World Equity	0.54%	4.16%	-0.57	3.47
CSFB Tremont Hedge Fund Emerging Markets	0.68%	5.03%	-0.57	6.71
CSFB Tremont Hedge Fund Dedicated Short Bias	-0.13%	5.10%	0.92	5.17

The descriptive statistics become more remarkable when looking at the third and fourth moments (i.e. skewness and kurtosis). Most of the hedge fund return series are negatively skewed and all strategies exhibit some level of excess kurtosis (i.e. greater than 3). The most positively skewed strategy is that of Dedicated Short-Bias. This is most likely due to the funding mechanics of these portfolios[4].

The most negatively skewed is that of the Event Driven category. This has been noted in prior research by Agarwal and Naik (2002) who state: "*We find … the Event Arbitrage index showing significant factor loading on risk factor corresponding to writing at OTM put option on S&P 500 index … this result is intuitive as Event Arbitrage strategy involves the risk of deal failure. A larger fraction of deals fail when markets are down and the Event Arbitrage strategy incurs losses. In contrast, when markets are up a larger proportion of deals go through and the strategy makes profits. But*

[4] In a Dedicated Short-Bias portfolio, all short positions generate cash which in turn earn interest – this creates a positive drift to the performance of the portfolio.

the profits are unrelated to the extent by which the market goes up. Thus, the payoff to Event Arbitrage strategy resembles that obtained by writing a naked put option on the market."

From a kurtosis point of view, the strategy with the highest kurtosis is that of Event-Driven, followed by its substrategy Event Driven Multi-Strategy and then Fixed Income Arbitrage.

Table 4.4: Series distribution moments ranked by Skewness

Skewness is a measure of symmetry, or more precisely, the lack of symmetry. A distribution, or data set, is symmetric if it looks the same to the left and right of the center point. The standard formula for calculating skewness is:

$$\frac{n}{(n-1)(n-2)}\sum\left(\frac{x_j-\bar{x}}{\sigma}\right)^3$$

	Probability distribution moments ranked by Skewness			
	Mean	Std. Dev.	Skewness	Kurtosis
CSFB Tremont Hedge Fund Dedicated Short Bias	-0.13%	5.10%	0.92	5.17
MSCI World Sovereign Bond	0.52%	1.91%	0.37	3.49
CSFB Tremont Hedge Fund Equity Market Neutral	0.83%	0.88%	0.25	3.21
CSFB Tremont Hedge Fund Long/Short Equity	0.98%	3.11%	0.24	6.38
CSFB Tremont Hedge Fund Composite	0.89%	2.40%	0.10	4.77
CSFB Tremont Hedge Fund Managed Futures	0.58%	3.54%	0.04	3.41
CSFB Tremont Hedge Fund Global Macro	1.17%	3.42%	-0.02	5.07
CSFB Tremont Hedge Fund Emerging Markets	0.68%	5.03%	-0.57	6.71
MSCI World Equity	0.54%	4.16%	-0.57	3.47
Lehman Brothers Cash Composite	0.34%	0.15%	-0.69	2.06
CSFB Tremont Hedge Fund Multi-Strategy	0.73%	1.27%	-1.27	6.45
CSFB Tremont Hedge Fund Risk Arbitrage	0.67%	1.26%	-1.31	9.19
CSFB Tremont Hedge Fund Convertible Arbitrage	0.81%	1.37%	-1.47	6.60
CSFB Tremont Hedge Fund Event Driven Multi-Strategy	0.83%	1.80%	-2.68	19.83
CSFB Tremont Hedge Fund Distressed Securities	1.07%	1.97%	-2.75	19.13
CSFB Tremont Hedge Fund Fixed Income Arbitrage	0.56%	1.12%	-3.24	19.35
CSFB Tremont Hedge Fund Event Driven	0.92%	1.71%	-3.46	25.83

What is interesting to note about these two strategies (Event Driven and Fixed Income Arbitrage) is that they are also marked by the most extreme cases of negative skewness and positive kurtosis. As observed by Davies, Kat and Lu (2003): *"Compounded by a high kurtosis (leptokurtosis), a negative skewed return distribution produces much higher possibilities for extreme events. ...that in most strategies, negative expected skewness goes with leptokurtosis on both individual fund and portfolio levels. Thus, it is preferable to analyse these two moments in tandem."*

Table 4.5: Series distribution moments ranked by Kurtosis

Kurtosis is a measure of how the relative peakedness or flatness of a distribution compares with the normal distribution. Positive kurtosis indicates a relatively peaked distribution. Negative kurtosis indicates a relatively flat distribution. A normal distribution has a kurtosis value of 3. The standard formula for calculating excess portfolio kurtosis is:

$$\left\{\frac{n(n+1)}{(n-1)(n-2)(n-3)}\sum\left(\frac{x_j-\bar{x}}{\sigma}\right)^4\right\}-\frac{3(n-1)^2}{(n-2)(n-3)}$$

	Probability distribution moments ranked by Kurtosis			
	Mean	Std. Dev.	Skewness	Kurtosis
Lehman Brothers Cash Composite	0.34%	0.15%	-0.69	2.06
CSFB Tremont Hedge Fund Equity Market Neutral	0.83%	0.88%	0.25	3.21
CSFB Tremont Hedge Fund Managed Futures	0.58%	3.54%	0.04	3.41
MSCI World Equity	0.54%	4.16%	-0.57	3.47
MSCI World Sovereign Bond	0.52%	1.91%	0.37	3.49
CSFB Tremont Hedge Fund Composite	0.89%	2.40%	0.10	4.77
CSFB Tremont Hedge Fund Global Macro	1.17%	3.42%	-0.02	5.07
CSFB Tremont Hedge Fund Dedicated Short Bias	-0.13%	5.10%	0.92	5.17
CSFB Tremont Hedge Fund Long/Short Equity	0.98%	3.11%	0.24	6.38
CSFB Tremont Hedge Fund Multi-Strategy	0.73%	1.27%	-1.27	6.45
CSFB Tremont Hedge Fund Convertible Arbitrage	0.81%	1.37%	-1.47	6.60
CSFB Tremont Hedge Fund Emerging Markets	0.68%	5.03%	-0.57	6.71
CSFB Tremont Hedge Fund Risk Arbitrage	0.67%	1.26%	-1.31	9.19
CSFB Tremont Hedge Fund Distressed Securities	1.07%	1.97%	-2.75	19.13
CSFB Tremont Hedge Fund Fixed Income Arbitrage	0.56%	1.12%	-3.24	19.35
CSFB Tremont Hedge Fund Event Driven Multi-Strategy	0.83%	1.80%	-2.68	19.83
CSFB Tremont Hedge Fund Event Driven	0.92%	1.71%	-3.46	25.83

Another point of interest is that under a mean-variance framework, volatility (variance or standard deviation) is used as a proxy for risk in circumstances where the return series can be characterised by a normal distribution. Where this is not the case, risk is better accounted for by the standard deviation in conjunction with the higher moments of the probability distribution. Where series have low standard deviations – they are often combined with low/negative skewness and high kurtosis.

To properly address the question of normality in hedge funds, a more structured approach is required. All return series are subjected to the Jarque-Bera test statistic. The formula for the Jarque-Bera statistic is as follows:

$$\text{Jarque-Bera} = N\left(\frac{S^2}{6}+\frac{(K-3)^2}{24}\right) \quad 4.1$$

where N is the number of observations, S is the skewness and K is the kurtosis of the series. This statistic examines only the skewness and kurtosis of a given series. A normal distribution has a skewness value of 0 and a kurtosis value of 3.

Table 4.6: Jarque-Bera test statistic

The Jacque-Bera statistic is distributed as a χ^2 distribution with two degrees of freedom. Its critical values at the 5% and 1% confidence levels are 5.991 and 9.210, respectively. Therefore, the null hypothesis of normality is rejected when the Jacque-Bera statistic has a higher value than the corresponding critical value at the respective confidence level:

Jarque-Bera

	Mean	Std. Dev.	Skewness	Kurtosis	Jarque-Bera	Normal at 5%	Normal at 1%
MSCI World Equity	0.54%	4.16%	-0.57	3.47	8.12	NO	YES
MSCI World Sovereign Bond	0.52%	1.91%	0.37	3.49	4.15	YES	YES
Lehman Brothers Cash Composite	0.34%	0.15%	-0.69	2.06	14.73	NO	NO
CSFB Tremont Hedge Fund Composite	0.89%	2.40%	0.10	4.77	16.76	NO	NO
CSFB Tremont Hedge Fund Convertible Arbitrage	0.81%	1.37%	-1.47	6.60	113.52	NO	NO
CSFB Tremont Hedge Fund Dedicated Short Bias	-0.13%	5.10%	0.92	5.17	42.37	NO	NO
CSFB Tremont Hedge Fund Distressed Securities	1.07%	1.97%	-2.75	19.13	1525.22	NO	NO
CSFB Tremont Hedge Fund Event Driven	0.92%	1.71%	-3.46	25.83	2988.13	NO	NO
CSFB Tremont Hedge Fund Event Driven Multi-Strategy	0.83%	1.80%	-2.68	19.83	1637.60	NO	NO
CSFB Tremont Hedge Fund Emerging Markets	0.68%	5.03%	-0.57	6.71	79.04	NO	NO
CSFB Tremont Hedge Fund Fixed Income Arbitrage	0.56%	1.12%	-3.24	19.35	1624.54	NO	NO
CSFB Tremont Hedge Fund Managed Futures	0.58%	3.54%	0.04	3.41	0.94	YES	YES
CSFB Tremont Hedge Fund Global Macro	1.17%	3.42%	-0.02	5.07	22.48	NO	NO
CSFB Tremont Hedge Fund Long/Short Equity	0.98%	3.11%	0.24	6.38	61.10	NO	NO
CSFB Tremont Hedge Fund Multi-Strategy	0.73%	1.27%	-1.27	6.45	96.04	NO	NO
CSFB Tremont Hedge Fund Equity Market Neutral	0.83%	0.88%	0.25	3.21	1.56	YES	YES
CSFB Tremont Hedge Fund Risk Arbitrage	0.67%	1.26%	-1.31	9.19	236.98	NO	NO

Generally, most hedge fund index data is not normally distributed with the exception of Managed Futures and Equity Market Neutral. In contrast, both the MSCI World Equity and Sovereign Bond indices are normal under the Jarque-Bera test at the 1% level. This finding provides support for the use of mean-variance theory (MVT) as a methodology to construct portfolios that are allocated among assets that are normal in character. Congruent with this assessment, is that MVT is not adequate a framework to assess hedge funds.

4.5 Survivorship Bias

Survivorship bias occurs when data samples exclude markets or investment funds or individual securities that have disappeared. Campbell R. Harvey's Hypertextual Finance Glossary defines Survivorship Bias as follows: *"Usually pertaining to fund manager or individual investor performance. Suppose we examined the performance over the last ten years of a group of managers that exist today. This performance is biased upwards because we are only considering those that survived for 10 years. That is, some dropped out because of poor performance. Hence, in evaluating performance, one has to be careful to include both the current and the managers that dropped out of the sample due to poor performance."* The concern from researchers is that the data sample of survivors describes an environment that overstates the actual return and understates the actual risk.

Liang (1999) finds survivorship bias in hedge fund return data from January 1992 through to December 1996. However, the author concludes that, on a risk-adjusted basis, the average hedge fund outperformed the average mutual fund and that the outperformance cannot be explained by survivorship bias. Amin and Kat (2003) find that concentrating on surviving funds only will overestimate the mean return on individual hedge funds by approximately 2% as well as introduce significant biases in estimates of the standard deviation, skewness and kurtosis. Specifically, they point to: *"…significant underestimation of the standard deviation and kurtosis as well as overestimation of the skewness of individual hedge fund returns."*

Studies relating to survivorship bias with respect to hedge fund returns have not been extended to that of hedge fund indices. Ineichen (2000) addresses this issue as follows: *"For hedge funds, it is unclear if survivorship bias inflates returns of hedge fund indices. Poor, as well as stellar performing hedge funds, exit the database. Poor hedge funds exit because of poor performance. Stellar hedge funds can close to new partners and, as a result of good performance, stop reporting returns to the data vendor. Hedge funds report their performance on a voluntary basis. This self-selection bias may partially offset the survivorship bias caused by the disappearance of poorly performing funds."*

Survivorship bias in hedge fund index data is beyond the scope of this thesis. And, as there is a lack of any conclusive research on the matter; as well as efforts by CSFB-Tremont to minimise the impact of survivorship bias in their index data, all empirical research will use the published data in its original format.

4.6 Methodology

Two portfolio construction approaches are applied in this thesis. First, a conventional Markowitzian mean-variance optimisation is employed, and second, a more recent mean-variance-skewness-kurtosis (MVSK) optimisation. Both frameworks are applied in two objectives, firstly, in the formulation of optimal fund of hedge fund portfolios. And secondly, in asset allocation question, of how much to allocate to traditional asset classes (equities, bonds and cash) and hedge fund assets. Mean variance portfolio optimisation is covered in Chapter Two.

The second portfolio construction approach utilised in this thesis closely follows the methodology of Davies, Kat and Lu (2005). In their research, Davies et al (2005), make use of a Polynomial Goal Programming (PGP) approach. PGP facilitates the incorporation of both investor preferences beyond the mean-variance space (to higher moments) as well as a more complete representation of the probability distribution to effect efficient portfolio construction with hedge funds. This study is distinguished by using hedge fund index data as opposed to the single manager data used in Davies et al (2005). In addition, this study augments this work by also addressing what proportion of a balanced portfolio (i.e. a portfolio consisting of equities, bonds and cash) should be invested in the hedge fund class.

PGP is useful to solve problems where multiple and competing objectives are present. In this case, investors desire a highest return possible. However, investors also wish to assume as little risk as possible for their return. As can be seen from the Tables 4.2 through 4.5, even within the various risk attributes there is conflict. For example, a strategy such as Fixed-Income Arbitrage has a low variance but shows negative skewness and high excess kurtosis. It appears there is trade-off between risk attributes. As stated by Athayde and Flores (2001), investors would like to maximise the first and third moments (mean and skewness) and minimise the second and fourth (variance and kurtosis).

Davies et al (2005) formulate the portfolio construction question as a multiple objective programming problem[5]:

Maximise $$Z_1 = X^T \bar{R} \qquad 4.2$$

maximise $$Z_3 = \frac{T}{(T-1)(T-2)} \Sigma \left[\frac{(X^T(R - \bar{R}))}{\sqrt{X^T V X}} \right]^3 \qquad 4.3$$

minimise $$Z_4 = \left\{ \frac{T(T+1)}{(T-1)(T-2)(T-3)} \Sigma \left[\frac{(X^T(R - \bar{R}))}{\sqrt{X^T V X}} \right]^4 \right\} - \frac{3(T-1)^2}{(T-2)(T-3)} \qquad 4.4$$

subject to $$X^T V X = A \qquad 4.5$$ [6]

where, $X^T = (x_1, x_2, \cdots, x_n)$ and x_i is the capital weight percentage of the portfolio invested in the ith asset. The asset can be a risky asset or risk-free. The T superscript denotes the transpose of the array in a matrix formula. T is the number of observations in the time series (in this thesis, all series have 126 observations). Z_1 is the formula for portfolio mean return, $X^T V X$ is portfolio variance, Z_3 is portfolio skewness and Z_4 is excess kurtosis. A denotes the level of variance pre-specified in the optimisation.

[5] It must be noted that the skewness (Z_3) and kurtosis (Z_4) formulas differ from the original formulas applied by Davies, Kat and Lu (2005). The author contacted Sa Lu, a co-author of the Davies et al (2005) paper and she confirmed the appropriateness of the formulas applied above.

[6] A further difference is that Davies et al (2005) solve for optimal fund of hedge fund portfolios subject to a unity variance constraint.

Combining the objectives in 4.1; 4.2; 4.3 and 4.4 into a single objective statement, a PGP can be expressed as:

Minimise $$Z = (1 + d_1)^{\alpha} + (1 + d_3)^{\beta} + (1 - d_4)^{\gamma} \qquad 4.6$$

subject to $$X^T \bar{R} + d_1 = Z_1^* \qquad 4.7$$

$$\frac{T}{(T-1)(T-2)} \Sigma \left[\frac{(X^T(R - \bar{R})}{\sqrt{X^T V X}} \right]^3 + d_3 = Z_3^* \qquad 4.8$$

$$\left\{ \frac{T(T+1)}{(T-1)(T-2)(T-3)} \Sigma \left[\frac{(X^T(R - \bar{R})}{\sqrt{X^T V X}} \right]^4 \right\} - \frac{3(T-1)^2}{(T-2)(T-3)} + d_4 = Z_4^* \qquad 4.9$$

$$d_1, d_3 \geq 0 \qquad 4.10$$

$$d_4 \leq 0 \qquad 4.11$$

$$X^T V X = A \qquad 4.12$$

where α, β and γ are the nonnegative investor preferences for the mean, skewness and kurtosis of the portfolio return series. Z_1^* is the mean return for the optimal mean-variance portfolio with a specified variance; Z_3^* is the skewness value of the optimal skewness-variance portfolio with specified variance and Z_4^* is the kurtosis value of the optimal kurtosis-variance portfolio with specified variance.

By construction, the mean return for an optimal MVSK portfolio will be lower than the mean return for an optimal mean-variance portfolio. Similarly, skewness for an optimal MVSK portfolio will be lower than that of an optimal skewness-variance portfolio. Therefore, d_1 and d_3 represent positive deviations from Z_1^* and Z_3^*. Similarly for kurtosis, d_4 represents the negative deviation from Z_4^*.

Davies et al (2005) use their model to solve for optimal fund of hedge fund portfolios under the further constraint of optimising for a variance of one. This study extends their work by comparing the outcome of the MVSK optimisation with the mean-variance methodology for varying levels of volatility.

Solving the PGP is a two-step process. Firstly, the optimal values for Z_1^* (expected return), Z_3^* (skewness) and Z_4^* (kurtosis), respectively are solved for a pre-specified level of variance. Secondly, these optimal values are substituted into restrictions 4.7, 4.8 and 4.9 and a minimum value is found for the objective formula 4.6.

4.7 Summary

This chapter introduces the hedge fund index data and other data used in this study. Hedge funds can be categorised into three broad strategies: Relative-Value; Event-Driven and Opportunistic. It has been shown using a Jarque-Bera test that almost all hedge fund strategies do not exhibit characteristics compatible with a normal distribution. Conversely, the equity and bond indices do appear to be normal in nature and thus suitable for mean-variance optimisation.

Furthermore, certain strategies (viz. all Event-Driven strategies as well as Fixed Income Arbitrage) exhibit large negative skewness combined with leptokurtic tails resulting in a distribution that increases the probability of extreme adverse events. Often these strategies can show a low level of volatility. In these cases, total risk is not visible under mean-variance analysis and is a latent hazard to naïve investors.

Finally, the PGP MVSK methodology of Davies et al (2005) is presented. This framework seeks to maximise the odd moments and minimise the even moments subject to predetermined preferences in terms of the first four moments of the distribution.

Contents – Empirical Results

5

5

Empirical Results

5.1 Introduction

The empirical results of this chapter are presented in two sections. The first section reports the fund of hedge fund optimisation results. The second section presents the results with respect to an optimal asset allocation including a traditional assets and a hedge fund asset. The sections are also divided into results obtained under a mean-variance framework and results under the PGP MVSK methodology. The chapter concludes with a comparative performance evaluation.

5.2 Fund of hedge funds optimisation

This section presents results using data from the CSFB Tremont hedge fund indices. All CSFB Tremont indices are included in this analysis except the Composite index as the objective of this section is to derive an optimal composite.

5.2.1 Mean-variance optimisation

As stated in the previous chapter, a variance minimisation technique was used in this procedure. Minimum variance portfolios are found for 21 reference points of return in this hedge fund set. The points are derived by creating 20 equidistant points between the minimum average monthly return[1] and the maximum average monthly return[2] of all the indices in the set. The results are presented in Table 5.1 and Figure 5.1.

[1] Minimum average monthly return of all the series is -0.13% from the Dedicated Short-Bias index
[2] Maximum average monthly return of all the series is 1.17% from the Global Macro index

Table 5.1: Optimal fund of hedge fund portfolios under a mean-variance framework

Panel A presents the portfolio mean return, standard deviation and the higher moments while Panel B shows the detailed allocation to the individual hedge fund strategies.

Panel A: Portfolio mean return and risk statistics						Panel B: Percentage allocation to hedge fund strategy in fund of hedge fund portfolio													
Portfolio	Mean Return	Variance	Standard Deviation	Skewness	Kurtosis	Convertible Arbitrage	Dedicated Short-Bias	Distressed Securities	Emerging Markets	Equity Market-Neutral	Event Driven	Event Driven Multi-Strategy	Fixed Income Arbitrage	Global Macro	Multi-Strategy	Risk Arbitrage	Long/Short Equity	Managed Futures
1	-0.13%	0.26%	5.08%	0.93	2.30	0%	100%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
2	-0.07%	0.20%	4.46%	0.83	1.80	0%	92%	0%	8%	0%	0%	0%	0%	0%	0%	0%	0%	0%
3	0.0[illegible]%	0.15%	3.87%	0.67	1.18	0%	84%	0%	16%	0%	0%	0%	0%	0%	0%	0%	0%	0%
4	0.06%	0.11%	3.33%	0.46	0.48	0%	76%	0%	24%	0%	0%	0%	0%	0%	0%	0%	0%	0%
5	0.13%	0.08%	2.86%	0.18	-0.14	0%	68%	0%	32%	0%	0%	0%	0%	0%	0%	0%	0%	0%
6	0.19%	0.[illegible]6%	2.45%	0.10	-0.10	0%	58%	0%	27%	0%	0%	0%	15%	0%	0%	0%	0%	0%
7	0.26%	0.04%	2.06%	-0.02	0.10	0%	48%	0%	22%	0%	0%	0%	27%	0%	0%	3%	0%	0%
8	0.32%	0.03%	1.68%	-0.15	0.38	0%	39%	0%	17%	0%	0%	0%	31%	0%	0%	13%	0%	0%
9	0.39%	0.02%	1.32%	-0.36	1.07	0%	31%	0%	12%	0%	0%	0%	34%	0%	0%	23%	0%	0%
10	0.45%	0.01%	1.01%	-0.67	2.24	0%	22%	0%	7%	0%	0%	0%	37%	0%	0%	33%	0%	1%
11	0.52%	0.01%	0.78%	-0.97	2.86	0%	14%	0%	3%	0%	0%	0%	37%	0%	3%	39%	0%	4%
12	0.58%	0.[illegible]%	0.64%	-0.77	1.58	0%	10%	0%	1%	17%	0%	0%	29%	0%	6%	32%	0%	3%
13	0.65%	0.00%	0.57%	-0.39	-0.27	0%	8%	0%	0%	36%	0%	0%	21%	0%	8%	24%	1%	1%
14	0.71%	0.00%	0.59%	-0.16	-0.28	0%	7%	8%	0%	46%	0%	0%	11%	0%	10%	14%	1%	0%
15	0.78%	0.00%	0.66%	-0.09	0.53	0%	7%	17%	0%	55%	0%	0%	1%	1%	12%	5%	1%	0%
16	0.85%	0.01%	0.78%	-0.18	2.10	0%	4%	22%	0%	60%	0%	0%	0%	4%	9%	0%	0%	0%
17	0.91%	0.01%	0.98%	-0.57	4.45	0%	0%	26%	0%	62%	0%	0%	0%	8%	4%	0%	0%	0%
18	0.98%	0.02%	1.26%	-1.18	7.59	0%	0%	41%	0%	45%	0%	0%	0%	14%	0%	0%	0%	0%
19	1.04%	0.03%	1.60%	-1.57	9.04	0%	0%	59%	0%	21%	0%	0%	0%	21%	0%	0%	0%	0%
20	1.11%	0.04%	2.01%	-1.18	6.15	0%	0%	64%	0%	0%	0%	0%	0%	36%	0%	0%	0%	0%
21	1.17%	0.12%	3.40%	-0.02	2.20	0%	0%	0%	0%	0%	0%	0%	0%	100%	0%	0%	0%	0%

Figure 5.1: Optimal fund of hedge fund portfolios under a mean-variance framework

The stacked chart shows graphically the composition of the optimal fund of hedge fund portfolios. In addition, the risk moments of standard deviation, skewness and kurtosis are plotted against the right-hand side y-axis. Examining portfolios 16 to 19 clearly shows when optimising on a naive mean-variance level can lead to unfavourable portfolio construction from a higher moment perspective.

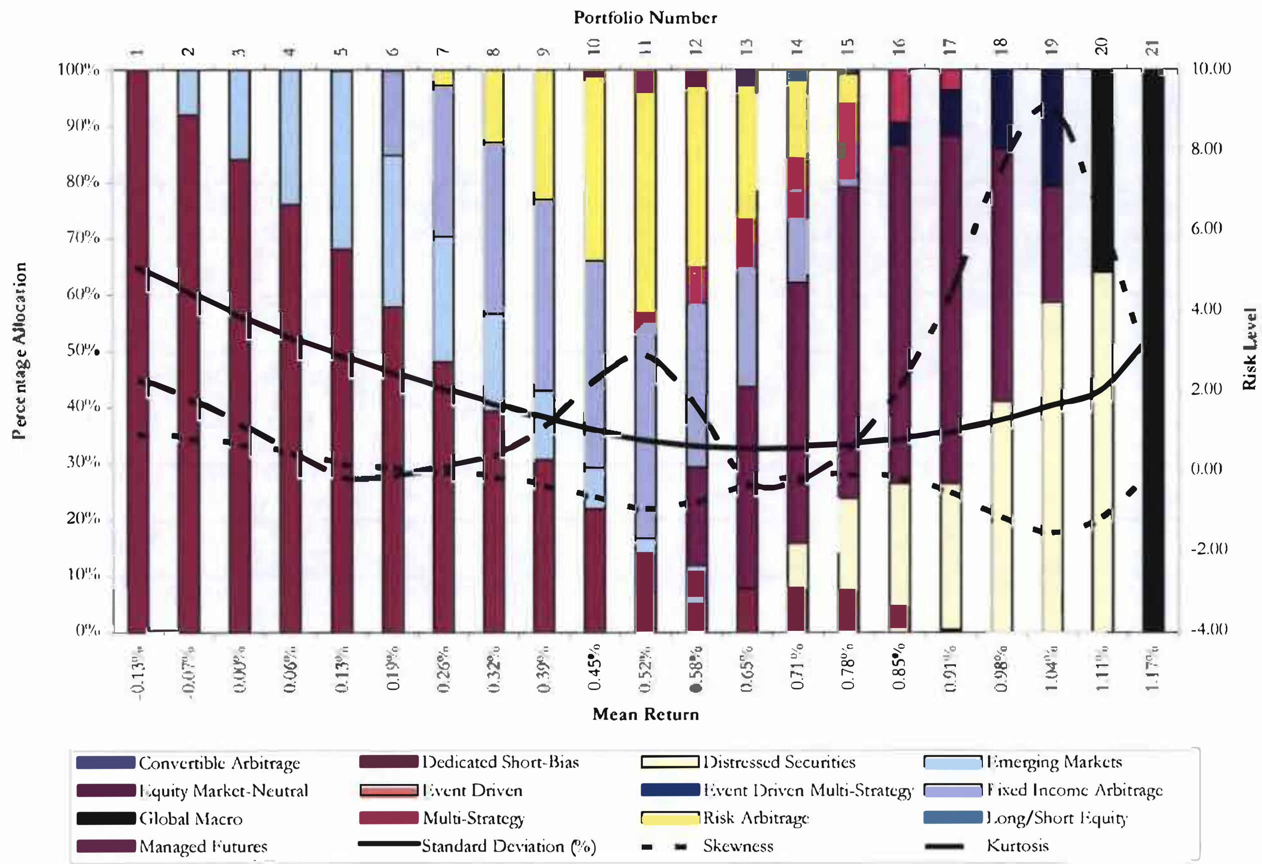


Table 5.1 splits the output into two panels, Panel A shows descriptive statistics of the output while Panel B presents the allocation in portfolios 1 through 21. From Table 5.1 and Figure 5.1 it can be seen that only portfolios 12 through 21 are part of the efficient frontier. For portfolios 1 through 11 there exists a portfolio on the minimum-variance frontier for which there is a point of higher return for the same quantum of volatility (standard deviation). This means that portfolios 1 to 11 are not an element of the efficient frontier set.

In the inefficient portfolios 1 through 5, the mean-variance optimisation initially allocates capital to Dedicated Short-Bias and Emerging Markets. The mean-variance model uses Dedicated Short-Bias as a means of initially reducing portfolio return. These two strategies have a correlation coefficient value of -0.63 (all correlation data is presented in Appendix B) and thus the Emerging Markets exposure reduces portfolio volatility. From portfolios 6 to 11, these two allocations are reduced in favour of Fixed Income Arbitrage and Risk Arbitrage (with small allocations to Multi-Strategy and Managed Futures).

Examining the efficient portfolios 12 to 21, shows almost all these portfolios contain Equity Market-Neutral allocations as well as Distressed Securities. Only at the extreme levels of expected return do Global Macro funds play a role. By construction, the efficient frontier is increasing in volatility from the minimum variance point to the point of maximum return. However, the mean-variance model does not evaluate the impact of higher moments on portfolio design. It must be noted from the Figure 5.1 that from the minimum-variance point that skewness initially increases (from portfolio 12 to 15) and then decreases. Portfolio kurtosis initially falls from the minimum-variance point (portfolio 12) and then increases (portfolio 15).

Under a mean-variance regime, portfolio 12 is the minimum-variance portfolio. This can also be expressed as the lowest risk portfolio under this framework. Taking higher moments into account, may yield a slightly different result, as portfolios 13 and 14 have more favourable third and fourth moments (higher skewness and lower kurtosis). Thus, it can be argued that based on particular investor preferences, either portfolio 13 or 14 could be the minimum risk portfolio.

Examining the more volatile portfolios (16 to 19), an investor would expect to assume a higher level of risk with these investments. However, the mean-variance regime does not provide the full risk picture. In these portfolios, it can be seen that skewness decreases and kurtosis increases. As noted in Chapter 4, this is an unfavourable combination as this increases the likelihood for more severe negative returns.

5.2.2 PGP optimisation for mean-variance-skewness-kurtosis

Upon calculating the mean-variance efficient frontier, 20 equidistant standard deviation points along the frontier are used as "anchors" to enable comparison with the PGP regime. Furthermore, these particular anchor points are along the section of the efficient frontier beyond the minimum variance portfolio and before the maximum expected return portfolio. The standard deviation anchors are: 0.59%; 0.66%; 0.78%; 0.98%; 1.26%; 1.60% and 2.01%.

Utilising these anchor points, PGP optimised mean-variance-skewness-kurtosis (MVSK) portfolios are modeled for 5 different profiles of investor preferences with respect to expected return, skewness and kurtosis. α denotes investor preference over expected return, while β and γ denote preference for skewness and kurtosis respectively. The 5 modeled profiles (labeled A to E) for each anchor point are displayed in Table 5.2.

Table 5.2: Preference scenarios in PGP mean-variance-skewness-kurtosis optimisation. Under this methodology, α, β and γ denote investor preferences for mean return, skewness and kurtosis respectively. These preferences form part of the objective function Z discussed in Chapter 4. 3 denotes that a high level of investor utility is derived from this moment, 2 a medium level and 1 a low level. 0 indicates no preference.

	A	B	C	D	E
α	1	3	1	3	2
β	3	1	0	2	3
γ	0	0	3	1	1

The full output of this PGP modeling can be found in Appendix 3. Figure 5.2 presents a section of the original mean-variance frontier calculated in section 5.1.

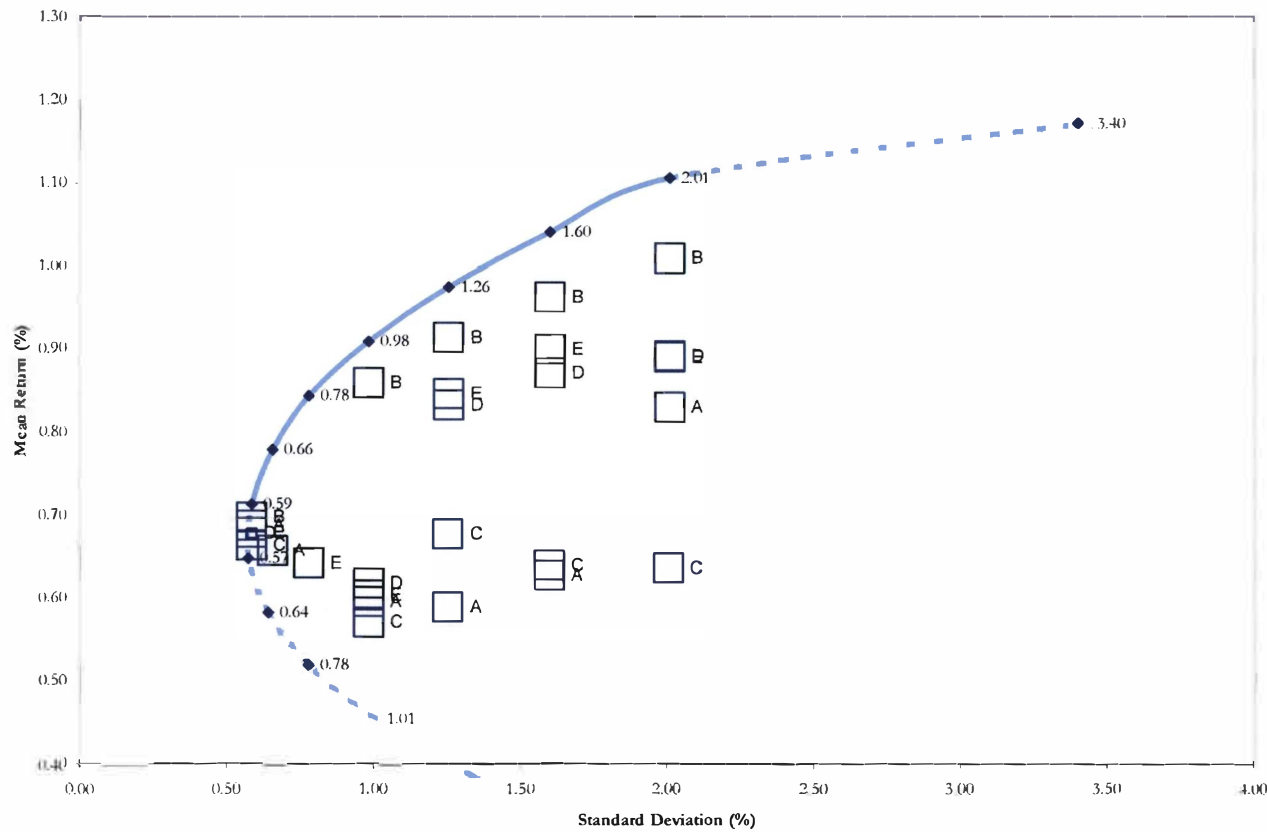
The frontier is augmented by the addition of the square indicators which show the expected return/standard deviation point for a MVSK optimised portfolio. The square indicators are marked by their labels (A to E), which denote the respective profiles to which they refer. It can be noted from figure 5.2 that the MVSK optimised portfolios differ substantially in return from the mean-variance efficient frontier portfolios. It is also clear that scenarios B and E generally map closer to the efficient frontier, while those of A and C map further away. These results are intrinsic to the MVSK model, for under scenarios B and E, greater preference is placed on portfolio return (α (mean or expected return preference) values of 3 and 2 respectively). On the other hand, scenarios A and C have maximum preference for one of the higher moments (β (skewness preference) value of 3 and γ (kurtosis preference) value of 3 respectively).

A number of notable observations can be made from Figure 5.2. First, all MVSK portfolios appear below the mean-variance efficient frontier. This shows that the optimisation of a fund of hedge fund portfolio in the MVSK space is one of competing objectives. Therefore, there is a consistent trade-off between the four moments. This finding is the same as that of Davies, Kat and Lu (2005). In other words, holding variance constant at a prespecified level and optimising for the other three moments must lead to deterioration in the expected portfolio return. If this was not the case, optimising for skewness and kurtosis would be at no cost to the investor and would effectively constitute a "free lunch".

Second, as the standard deviation increases along the efficient frontier, the divergence between the mean-variance optimised portfolio and the MVSK optimised portfolios increases. The reason for this is two-fold. As the volatility of the portfolio increases an offsetting large reduction in expected return must be sacrificed in order to improve the skewness and kurtosis of the portfolio. Furthermore, under the mean-variance regime, the portfolios optimised beyond the minimum variance portfolio initially have improving higher moment risk statistics. From portfolio 15 onwards, the optimal mean-variance portfolios have deteriorating skewness and kurtosis values. Thus, in order to improve these attributes, an ever larger return forfeit is required.

Figure 5.2: Optimal fund of hedge fund portfolios minimum-variance frontier with comparative MVSK portfolios

The solid line indicates the segment of the frontier for which MVSK portfolios are modeled, while the broken line denotes the remainder of the minimum-variance frontier. The squares plot the expected mean return and standard deviation for the MVSK portfolios. It is clear that optimising for higher moments while holding variance (or standard deviation) constant, results in a deterioration of mean return. This confirms that the optimisation of moments is a competing objective.



The portfolios also differ substantially in composition. Using the standard deviation anchor point of 1.26% as an example, the output from the model is displayed in Table 5.3.

Table 5.3: Comparison of optimal portfolios under a mean-variance regime and those under a mean-variance-skewness-kurtosis framework.

The portfolios are constructed under the MVSK PGP model depending on the investor preferences specified. A simple mean-variance portfolio can be run as a special case with the preferences of maximising return with no preference for either skewness or kurtosis.

Investor Profile	A	B	C	D	E	Mean-variance
α	1	3	1	3	2	3
β	3	1	0	2	3	0
γ	0	0	3	1	1	0
Mean	0.59	0.91	0.68	0.83	0.85	0.98
Variance	1.58	1.58	1.58	1.58	1.58	1.58
Skewness	0.75	0.72	-0.01	0.53	0.61	-1.18
Kurtosis	1.88	1.47	-0.65	0.21	0.49	7.59
Standard Deviation	1.26	1.26	1.26	1.26	1.26	1.26
Convertible Arbitrage	0%	0%	10%	10%	2%	0%
Dedicated Short-Bias	21%	0%	2%	0%	0%	0%
Distressed Securities	0%	0%	1%	0%	0%	41%
Emerging Markets	0%	0%	12%	0%	0%	0%
Equity Market-Neutral	49%	69%	7%	28%	36%	45%
Event Driven	0%	0%	3%	0%	0%	0%
Event Driven Multi-Strategy	0%	0%	7%	0%	0%	0%
Fixed Income Arbitrage	0%	0%	17%	0%	0%	0%
Global Macro	1%	27%	0%	9%	12%	14%
Multi-Strategy	16%	0%	7%	14%	15%	0%
Risk Arbitrage	0%	0%	3%	0%	0%	0%
Long/Short Equity	0%	0%	0%	20%	19%	0%
Managed Futures	14%	4%	28%	19%	16%	0%

All portfolio optimisations appear in Appendix C. Firstly, it must be noted that the MVSK portfolios A to E all have higher levels of skewness and lower levels of kurtosis than the mean-variance portfolio. Secondly, the MVSK are substantially different in their composition. With the exception of the 1% allocation in portfolio C, all the MVSK award no weighting to the Distressed Securities category. This is in stark contrast with the mean-variance portfolio which has a 41% holding.

Furthermore, with the exception of some inconsequential allocations, almost none of the optimal portfolios include strategies which exhibit the hazardous combination of negative skewness and high kurtosis. These strategies, Distressed Securities, Event

Driven, Event Driven Multi-Strategy, Fixed Income Arbitrage and Risk Arbitrage all bear an element of systematic risk. For example: Distressed Securities by their very nature imbue a high probability of bankruptcy while Fixed Income Arbitrage bears credit risk – so well borne out by the LTCM disaster[3].

All portfolios include an allocation to the Equity Market Neutral category. As shown in Chapter 4, the Equity Market Neutral index exhibits relatively low levels of volatility and kurtosis as well as benign skewness (close to zero). By pairing off similar long and short positions, systematic risks are reduced yielding a truly low-risk strategy. Davies et al (2005) note that: *"specifically, equity market neutral funds are risk and kurtosis reducers…"*

It is also of interest to examine the portfolio under MVSK scenario C. This scenario optimises primarily on the kurtosis moment ($\gamma = 3$) and does exhibit the lowest kurtosis value for the optimal portfolio for the anchor point of 1.26%. However, it is the only scenario to allocate capital to the Fixed Income Arbitrage strategy[4] which exhibits high levels of kurtosis on an individual index level. This indicates that the cokurtosis between the assets selected must be low in order to reduce the overall portfolio skewness. Davies et al (2003) show that: *"as portfolio size grows sufficiently large… portfolio expected kurtosis depends only on the cokurtosis between four different funds. The influence from the individual fourth central moment, cokurtosis between two different funds and three different funds on expected portfolio kurtosis tends to zero…"*

[3] LTCM was a hedge fund managed by ex-Salomon Brothers bond traders and Nobel laureates where the fund was highly leveraged and followed a strategy of being long high-yield debt and short sovereign treasuries. During the events of the Russian debt default and the Asian crisis of 1998 – the fund collapsed and was rescued by a group of Wall Street investment banks.

[4] The CSFB Tremont Fixed Income Arbitrage index has a skewness and kurtosis values of -3.24 and 19.35 respectively.

5.2.3 Performance evaluation

If MVSK portfolios are in fact more efficient than mean-variance portfolios then performance appraisal measures should reflect this. For the optimised portfolios under the anchor point 1.26%, three performance functions (Sharpe and Sortino ratio as well as the Omega function) are calculated and presented in Table 5.4.

Table 5.4: Performance measures for optimal fund of hedge fund portfolios.

The performance measures are presented below for different investor preference scenarios for the anchor point of 1.26% standard deviation. The Sharpe Ratio shows excess return (above the risk-free rate) per unit of volatility. The Sortino Ratio shows excess return (above the risk-free rate) per unit of downside volatility. The Omega function is a ratio of the area above to the area below a threshold level, of a probability distribution. The threshold level used is that of the risk-free rate to ensure comparability with the other ratios. The Omega rank refers to the ranking of the Omega function in descending order with 1 indicating the most preferred portfolio.

Investor Profile	A	B	C	D	E	Mean-variance
Sharpe Ratio	0.20	0.45	0.27	0.39	0.40	0.50
Sortino Ratio	0.41	1.15	0.59	1.10	1.11	0.50
Omega	1.010996695	1.010996695	1.010996694	1.010996695	1.010996689	1.010909474
Omega rank	1	3	4	2	5	6

By construction, the mean-variance optimised portfolio has a superior Sharpe ratio to the MVSK portfolios. Under the Sharpe ratio, where only excess return and volatility are considered, a mean-variance optimiser essentially also maximises the Sharpe ratio. As stated earlier, the MSVK portfolios are further optimised for higher moments and will all have a lower level of return for a given level of volatility (and therefore a lower Sharpe ratio).

Examining the performance under the Sortino ratio, all MVSK portfolios are superior with the exception of portfolio A. Given that the "risk denominator" in the Sortino ratio is downside deviation, additionally optimising for the higher moments of skewness and kurtosis should provide some benefit. The MVSK portfolios outperform the mean-variance portfolio in all cases but that of portfolio A. Portfolio A optimises heavily on the skewness preference and this appears to have lowered the overall return substantially below that of the other portfolios resulting in a lower Sortino score.

The Omega function observes the mass of a probability density function above a pre-determined threshold level. The threshold level defined in the function above is that of the risk-free rate for which the Lehman Brothers US Cash Composite is a proxy. Omega was specifically designed rate performance by using all the information contained in the probability distribution. In this case, the MVSK portfolios all outperform the mean-variance portfolio on this ex-post basis.

5.3 Asset allocation with hedge funds optimisation

This section attempts to resolve what proportion of their assets investors should allocate in a traditional portfolio that includes an allocation to hedge funds. The model utilises the MSCI World Equity and World Sovereign Bond indices to proxy for diversified global equity and bond portfolios. Furthermore, the data series from the CSFB Tremont Composite Hedge Fund Index is employed as a proxy for a well-diversified fund of hedge funds.

5.3.1 Mean-variance optimisation

The same technique as applied in the previous section is utilised with the above data set. Once more, minimum variance portfolios are found for 20 equidistant points of return among the 4 assets[5]. The results of this mean-variance model are presented in Table 5.5 and graphically in Figure 5.3. Examining these portfolios shows that unlike the previous minimum-variance frontier, this frontier has no inefficient segment.

The optimal mean-variance portfolios make no allocation to the World Equity index at all. This may be due to the World Equity and World Sovereign Bond indices having very similar average monthly returns over the sample period but World Sovereign Bond having a much lower level of volatility than World Equities[6]. Under a framework that rewards expected returns per unit of volatility, this is an intuitive explanation.

[5] Minimum average monthly return of all the series is 0.34% from the Lehman Brother US Cash Composite and the maximum average monthly return of all the series is 0.89% from the Hedge Fund Composite index

[6] The MSCI World Equity index has a monthly standard deviation of 4.16% and monthly mean return of 0.54%, while the MSCI World Sovereign Bond index has a monthly standard deviation of 1.91% and monthly mean return of 0.52%.

The optimal portfolios (numbered 1 to 21 in Figure 5.3) initially allocate capital to cash and gradually increase exposure to the World Sovereign Bond index and the Hedge Fund Composite. The Sovereign Bond allocation increases in proportion until portfolio 16 and then declines to zero. This indicates that beyond portfolio 16, the correlation or covariance benefit from a volatility reduction perspective is surpassed by the return offered by the hedge fund composite. The US Cash Composite allocation decreases consistently until portfolio 16 where it reaches zero.

Table 5.5: Optimal diversified portfolios under a mean-variance framework.

Panel A reflects portfolio expected return and the higher moment risk statistics while Panel B shows the detailed allocation to individual assets.

Panel A: Portfolio mean return and risk statistics						**Panel B: Percentage allocation to investment in diversified portfolio**			
Portfolio Number	Mean Return	Variance	Standard Deviation	Skewness	Kurtosis	Hedge Fund Composite	World Equity	World Sovereign Bond	US Cash Composite
1	0.34%	0.00%	0.15%	-0.70	-0.93	0%	0%	0%	100%
2	0.37%	0.00%	0.18%	-0.15	-0.51	4%	0%	3%	93%
3	0.39%	0.00%	0.25%	0.11	-0.05	8%	0%	6%	86%
4	0.42%	0.00%	0.35%	0.12	0.25	12%	0%	9%	79%
5	0.45%	0.00%	0.44%	0.09	0.41	16%	0%	11%	73%
6	0.48%	0.00%	0.54%	0.05	0.51	20%	0%	14%	66%
7	0.50%	0.00%	0.65%	0.02	0.57	25%	0%	16%	59%
8	0.53%	0.01%	0.75%	0.00	0.61	29%	0%	19%	52%
9	0.56%	0.01%	0.85%	-0.02	0.64	33%	0%	21%	46%
10	0.59%	0.01%	0.96%	-0.03	0.66	37%	0%	24%	39%
11	0.62%	0.01%	1.06%	-0.04	0.68	41%	0%	27%	32%
12	0.64%	0.01%	1.16%	-0.05	0.69	45%	0%	29%	26%
13	0.67%	0.02%	1.27%	-0.06	0.70	49%	0%	32%	19%
14	0.70%	0.02%	1.37%	-0.07	0.71	54%	0%	34%	12%
15	0.73%	0.02%	1.48%	-0.08	0.72	58%	0%	37%	6%
16	0.75%	0.03%	1.58%	-0.08	0.76	62%	0%	38%	0%
17	0.78%	0.03%	1.71%	-0.04	1.01	70%	0%	30%	0%
18	0.81%	0.03%	1.85%	0.00	1.29	77%	0%	23%	0%
19	0.84%	0.04%	2.02%	0.04	1.54	85%	0%	15%	0%
20	0.86%	0.05%	2.20%	0.08	1.74	92%	0%	8%	0%
21	0.89%	0.06%	2.39%	0.10	1.90	100%	0%	0%	0%

Examining the "risk moments" of variance, skewness and kurtosis, it can be seen that volatility rises across the frontier while the skewness and kurtosis values are more dynamic. Skewness initially increases across portfolios 1 to 3 and then declines until portfolio 16 then increases to the maximum return portfolio. Kurtosis generally increases with variance across this optimisation but increases more rapidly from portfolio 16 onwards.

Figure 5.3: Optimal diversified portfolio under a mean-variance framework

The stacked chart shows graphically the composition of the optimal diversified portfolio. Also plotted are the risk moments of standard deviation, skewness and kurtosis. The chart clearly shows when optimising on a naive mean-variance level can lead to unfavourable portfolio construction from a higher moment perspective. In terms of statistical theory, it must be said none of these portfolios would be considered to be leptokurtic or "fat-tailed" as all the kurtosis values are less than 3.

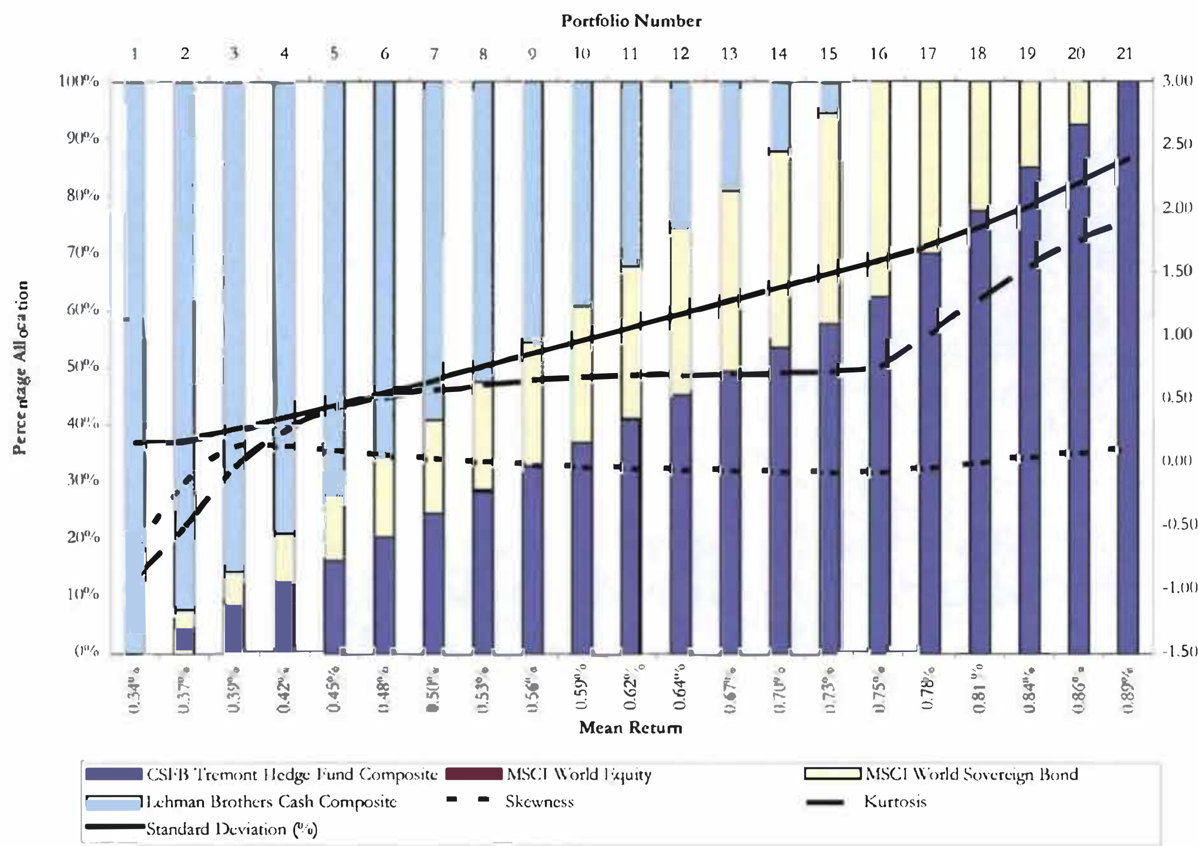
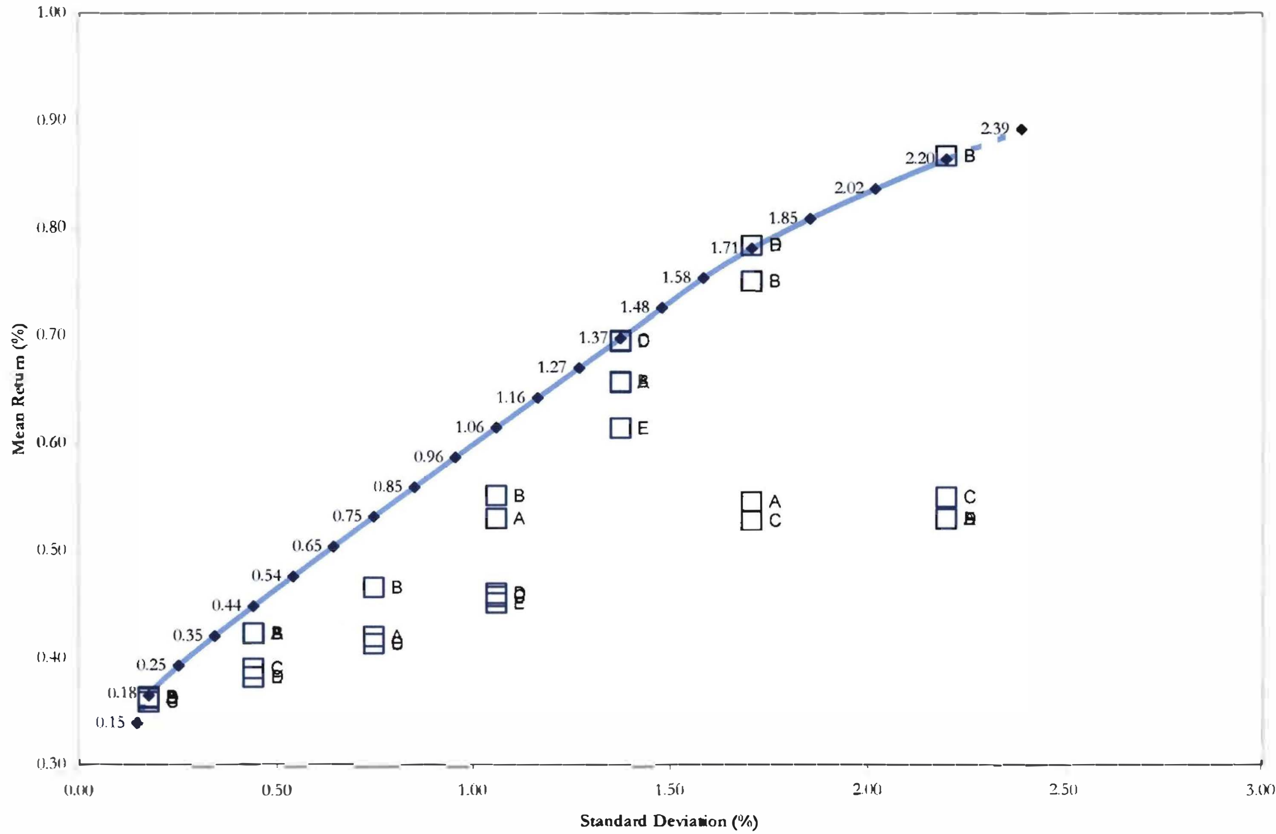


Figure 5.4: Optimal diversified portfolios under a mean-variance framework with comparative MVSK portfolios

The solid line indicates the segment of the frontier for which MVSK portfolios are modeled, while the broken line denotes the remainder of the minimum-variance frontier. The squares plot the expected mean return and standard deviation for the MVSK portfolios. It is clear that optimising for higher moments while holding variance (or standard deviation) constant, results in a deterioration of mean return. This confirms that the optimisation of moments is a competing objective.



In terms of portfolio composition, the MVSK and mean-variance portfolios are substantially different. Using the standard deviation anchor point of 1.06% as an example, the output of which is presented in Table 5.6.

Table 5.6: Comparison of optimal portfolios under a mean-variance regime and those under a mean-variance-skewness-kurtosis framework.

The portfolios are constructed under the MVSK PGP model depending on the investor preferences specified. A simple mean-variance portfolio can be run as a special case with the preferences of maximising return with no preference for either skewness or kurtosis.

Investor Profile	A	B	C	D	E	Mean-variance
α	1	3	1	3	2	3
β	3	1	0	2	3	0
γ	0	0	3	1	1	0
Mean	0.53	0.55	0.46	0.46	0.45	0.62
Variance	1.12	1.12	1.12	1.12	1.12	1.12
Skewness	0.21	0.15	-0.27	-0.01	0.21	-0.06
Kurtosis	1.08	1.08	-0.08	0.27	0.68	0.72
Standard Deviation	1.06	1.06	1.06	1.06	1.06	1.06
Hedge Fund Composite	16%	21%	5%	2%	0%	41%
World Equity	0%	0%	20%	15%	6%	0%
World Sovereign Bond	54%	52%	26%	41%	53%	27%
US Cash Composite	30%	28%	48%	41%	41%	33%

All the MVSK portfolios have a lower level of return than the mean-variance portfolio. For the MVSK portfolios that have no preference for either higher moment (viz. scenarios A, B and C), that particular moment is worse than the mean-variance portfolio. It is of interest to note that some of the MVSK portfolios do allocate to the World Equity asset unlike the mean-variance portfolio which holds no World Equity. This shows that its inclusion in a diversified portfolio has a benefit of either improving skewness or reducing kurtosis when blended with these other investments.

Another point of interest is the fact that the MVSK portfolios all allocate substantially lower weightings to hedge funds than the mean-variance portfolio. In fact, scenario E makes no allocation to the Hedge Fund Composite at all. This is in stark contrast to the findings above as well as those by prior research covered in Chapter 3, Literature review where mean-variance portfolios allocate large amounts of capital to hedge funds in balanced diversified portfolios.

5.3.2 Performance evaluation

Using the same format as before, Table 5.7 presents the three performance criteria for the anchor point 1.06%.

Table 5.7: Performance measures for optimal diversified portfolios.

The performance measures are presented below for different investor preference scenarios for the anchor point of 1.26% standard deviation. The Sharpe Ratio shows excess return (above the risk-free rate) per unit of volatility. The Sortino Ratio shows excess return (above the risk-free rate) per unit of downside volatility. The Omega function is a ratio of the area above to the area below a threshold level, of a probability distribution. The threshold level used is that of the risk-free rate to ensure comparability with the other ratios. The Omega rank refers to the ranking of the Omega function in descending order with 1 indicating the most preferred portfolio.

Investor Profile	A	B	C	D	E	Mean-variance
Sharpe Ratio	0.18	0.20	0.11	0.11	0.10	0.26
Sortino Ratio	0.31	0.33	0.18	0.22	0.18	0.41
Omega	1.015485507	1.015485497	1.015485535	1.015485499	1.015485526	1.015383107
Omega rank	3	5	1	4	2	6

The performance criteria are not as clear in expressing support for the MVSK portfolios. As in the previous section, the mean-variance portfolio should always have the best Sharpe ratio by construction. However, the mean-variance portfolio also shows a superior Sortino ratio when compared with the MVSK portfolios.

The Omega function was once more calculated using the risk-free cash return as a threshold level in order to make it comparable with the Sharpe and Sortino ratios. The Omega function ranks all the MVSK portfolios above that of the mean-variance portfolio. This is the case for all anchor points across the mean-variance frontier.

It is remarkable that scenario B displays the most attractive Sharpe and Sortino ratios within the MVSK portfolios but also ranks lowest with evaluated on the Omega function criterion. Scenario C which has the highest Omega score in fact has the most negative skewness but compensates by also having the lowest kurtosis.

5.4 Summary

This chapter has presented the comparative results of two distinct portfolio optimisation models with hedge fund assets. Portfolios are optimised using a conventional mean-variance framework and a new PGP MVSK model proposed by Davies et al (2005). The results show that the descriptive statistics of the optimal portfolios modeled between the two regimes are substantively dissimilar. Due to the competing moment nature of the return distributions, MVSK portfolios exhibit lower levels of return relative to mean-variance portfolios. Concomitantly, the MVSK portfolio is rewarded with a lower aggregate level of risk. Furthermore, it has been shown that mean-variance optimised fund of hedge funds can naively bear latent higher moment risks.

The results also show that portfolio composition is considerably different when optimising for higher moments when compared to mean-variance optimisation. In the case of the fund of hedge funds optimisation, the MVSK portfolios are more diversified and take advantage of the unusual distribution properties of the hedge fund indices. In line with findings by Davies et al (2005) on a single fund level that *"...equity market neutral funds and global macro funds have predominant roles in optimal fund of hedge funds portfolios."*, this study indicates the same outcome for hedge fund indices.

The same result is true for the problem concerning asset allocation with hedge funds. Mean-variance models allocate large weightings to hedge funds based their high return to volatility ratios. MVSK portfolios, accounting more fully for the non-normal probability distributions of hedge funds lower this allocation on a relative scale. It is noted that both models make extensive use of hedge funds in the diversified asset allocation decision.

On a performance appraisal level, the mean-variance portfolios outperform the MVSK portfolios on a Sharpe Ratio basis in terms of fund of hedge fund construction. However, the MVSK fund of hedge funds portfolio show improved performance under a Sortino measure and always rank higher using the Omega function. In terms of diversified portfolios, the MVSK portfolios clearly outperform on an Omega basis.

Contents - Conclusion

6

6

Conclusion

Hedge funds are gaining acceptance among the investment community as a mainstream form of investment. The diversification qualities of hedge funds are well-documented and investors are increasing their allocations to this category of alternative investments. Concurrent with this increase in interest is a requirement from practitioners to determine how to allocate to hedge funds and amongst the diversity of strategies within the hedge fund class.

From an investor utility perspective, the EUT and behavioural alternatives like Prospect Theory show that downside risk matters. Further, implicit in their hypotheses, the higher moments of skewness and kurtosis are partial factors from which utility is derived. However, the elegant and intuitive MVT has become the conventional model for utility measurement and portfolio selection.

As stated in Chapter One, this thesis has two objectives. The first objective is to confirm the findings of previous studies that hedge funds exhibit non-normal probability distributions and thus whether conventional portfolio construction under MVT is appropriate. The second and primary objective is to address the portfolio selection problem. The approach taken is to evaluate and compare MVT with the more recent PGP MVSK framework. This comparison is in the context of fund of hedge fund assembly as well as the asset allocation decision.

The remainder of the chapter is set out as follows. Section 6.1 provides a summary of the main findings with regard to the first objective of data analysis. Section 6.2 outlines the contrast between the PGP MVSK and mean-variance approaches to portfolio selection. Section 6.3 concludes by placing some of the main findings in the context of the theory reviewed in Chapter Two and suggests several areas for further research.

6.1 Data

This thesis has shown that hedge fund strategies are dissimilar from traditional investments with respect to their probability distributions. Almost all hedge fund strategies have non-normal distributions. Conversely, the Jarque-Bera test for the normality of the equity and bond indices was unable to reject the null hypothesis of normality at the 5% level. This indicates that MVT may be an appropriate model for optimising portfolios containing only equities and bonds. However, the MVT does not boast the sophistication (especially cognisance of higher moments) required to deal with complex investments like hedge funds.

Analysing the descriptive statistics of the hedge fund data provides more detail as to how certain strategies deviate from normality. This study has shown that many hedge fund strategies (especially the Event-Driven category as well as Fixed Income Arbitrage) have negatively skewed returns as well as leptokurtosis ("fat-tails"). Through analysis of these two higher moments together, it can be concluded that these strategies have a substantially higher probability of extreme losses than suggested by a normal distribution. As an example, research by Agarwal and Naik (2002) has shown that the payoff to the broad category of Event-Driven hedge fund strategies are akin to short put option positions on the market. This means that this strategy will yield low volatility consistent returns until the occurrence of an extreme adverse event, whereupon the strategy will make substantial losses. Based on the features of the probability distribution of this strategy, more specifically, significant negative skewness and a high level of kurtosis, these extreme adverse events occur more often than suggested by a normal distribution. A similar rationale can be made for other hedge fund strategies and Fixed Income Arbitrage in particular. Therefore, it can be concluded that in many cases, variance is an inadequate descriptor of risk for hedge funds.

6.2 Portfolio Selection

Due to this dichotomy in normality between hedge funds and traditional assets, this thesis utilises a recent approach pioneered by Davies, Kat and Lu (2005) to determine optimal portfolio assembly with hedge funds. This approach uses the mean and variance in conjunction with the higher moment information embedded in the probability distributions of the hedge fund data. More specifically, the PGP model facilitates interaction between the competing nature of these moments to derive portfolios based on predetermined investor preferences.

This study has shown that the results of an optimisation under a conventional mean-variance regime and that of the MVSK framework are substantially different. Comparing portfolios with the same level of volatility shows that the MVSK portfolios deliver lower returns than mean-variance optimised portfolios on a mean-variance plane. This illustrates that further optimising for higher moments has a cost and that measuring the returns of portfolios that include hedge funds on a purely mean-variance level is deceptive. Similarly, it has been shown that optimising on a naïve mean-variance basis can introduce the hazard of reducing skewness and increasing kurtosis in a portfolio further raising the probability of extreme adverse events.

The composition of the mean-variance portfolios and MVSK portfolios are also substantially different. The MVSK portfolios tend to be more diversified than the mean-variance portfolios. Further, the MVSK portfolios generally reduce allocations to the Event-Driven category of hedge funds. It has been shown that when assembling optimal fund of hedge funds portfolios, a substantial reduction in risk from changes in the higher moments can be achieved with relatively little decrease in mean return. This thesis concurs with previous studies of Feldman et al (2002) and Davies et al (2005) that Global Macro and Equity Market-Neutral strategies are key building blocks in fund of hedge fund construction. From an asset allocation perspective, the MVSK optimisation indicates that hedge fund allocations should not be as high as those suggested by mean-variance optimisation.

From a performance perspective, new measures have been developed that more completely account for the full return distribution of portfolios. Where variance is inadequate as a descriptor of risk, the Sharpe ratio is an inadequate measure of performance. In the case of portfolios that include hedge funds, MVSK portfolios outperform mean-variance portfolios on a risk-adjusted basis using measures that take the full return distribution into consideration. This shows that while mean returns for MVSK portfolio may be lower than that of mean-variance portfolios, resultant investor utility levels should be higher.

6.3 Summary

In summary, this thesis has shown that on the whole hedge fund returns are not normal. Furthermore, for portfolios with hedge funds, whether in a fund of hedge funds or asset allocation environment, MVT is an inadequate tool for portfolio construction. MVT underestimates the aggregate risk and thus overestimates the risk-return benefits of hedge funds. It is suggested that construction techniques like the MVSK PGP approach that account for higher moments are required to build portfolios that are fully matched to investor risk preferences.

Areas for future research that are suggested include optimising portfolios in order to maximise the recently devised Omega function. This could potentially provide an undemanding method for practitioners to build portfolios optimised for higher moments. Performing similar research using the so-called "investable indices" once enough data is present would make for an interesting comparison. The empirical results of this study are all ex-post in nature. An evaluation of the ex-ante results of higher-moment portfolio optimisation with hedge funds is a logical extension for portfolio selection research.

References

AIMA and ASSIRT (2002): Hedge Fund Booklet, www.aima.org, Alternative Investment Management Association.

Amin, G. and Kat, H.(2003): Hedge Fund Performance 1990-2000. Do the "money machines" really add value?, Journal of Financial and Quantitative Analysis 38(2): 251-274.

Amin, G. and Kat, H. (2002): Portfolios of Hedge Funds, *working paper*, Alternative Investment Research Centre, Cass Business School, City University

Amin, G. and Kat, H. (2002): Diversification and Yield Enhancement with Hedge Funds, *working paper*, Alternative Investment Research Centre, Cass Business School, City University

Amin, G. and Kat, H. (2003): Welcome to the dark side: Hedge fund attrition and survivorship bias over the period 1994-2001, Journal of Alternative Investments, Summer 2003.

Agarwal, V. and Naik, N. (2001): Characterizing Hedge Fund Risks with Buy-and-Hold and Option-Based Strategies, *working paper*, Georgia State University

Agarwal, V. and Naik, N.Y. (2004): Risks and Portfolio Decisions involving Hedge Funds, Review of Financial Studies 17(1): 63-98

Amenc, N. and Martellini, L. (2002): Portfolio Optimisation and Hedge Fund Style Allocation Decisions, Journal of Alternative Investments 5 (2): 7-20

Arrow, K. (1953): The Role of Securities in the Optimal Allocation of Risk-Bearing, Econometrie, as translated and reprinted in 1964, Review of Economic Studies

Athayde, G. and Flores, R. (2001): Finding a maximum skewness portfolio – a general solution to three-moments portfolio choice, Journal of Economics, Dynamic and Control (28): 1335-52

Bailey, R. (2001): Economics of Financial Markets, Cambridge University Press

Bodie, Z., Kane, A. and Marcus, A. (2002): Investments, 5th edition, McGraw-Hill/Irwin

Brown, S., Goetzmann, W. and Park, J. (1999): Conditions for Survival: changing risk and the performance of hedge fund managers and CTAs, *working paper*, Yale University

Cvitani´c, J., Lazrak, A., Martellini, L. and Zapatero, F. (2002): Optimal Allocation to Hedge Funds : An Empirical Analysis, Quantitative Finance 3: 1-12

Davies, R., Kat, H. and Lu, S. (2003): Higher moment portfolio analysis with hedge funds, *discussion paper*, ISMA Centre, University of Reading

Davies, R., Kat, H. and Lu, S. (2005): Fund of Hedge Funds Portfolio Selection: A Multiple Objective Approach, *discussion paper*, ISMA Centre, University of Reading

Economics New School: The History of Economic Thought Website, http://cepa.newschool.edu/het/essays/uncert/statepref.htm

Feldman, B., Chen, P. and Goda, C. (2002): Portfolio with hedge funds, *working paper,* Ibbotson Associates

Fung, W. and Hsieh, D. (1999): Is Mean-Variance Analysis Applicable to Hedge Funds?, Economic Letters 62: 53-58

Fung, W. and Hsieh, D. (2001): The Risk in Hedge Fund Strategies: Theory and Evidence from Trend Followers, The Review of Financial Studies 14: 313-341

Hall, A. and Satchell, S. (2004): The Anatomy of Portfolio Skewness and Kurtosis, *working paper*, University of Cambridge

Hagelin, N. and Pramborg, B. (2003): Evaluating gains from diversifying into hedge funds using dynamic investment strategies, *working paper*, Stockholm University

Harvey, C., Liechty, J.,Liechty, M. and Muller, P. (2003): Portfolio Selection with Higher Moments, *working paper*, Duke University

Indjic, D. (2002): Strategic Asset Allocation with Portfolios of Hedge Funds, AIMA Journal

Ineichen, A. (2000): In Search of Alpha, Investing in hedge funds, UBS Warburg

Ineichen, A. (2001): The Search for Alpha Continues, Do Fund of Hedge Fund Managers Add Value?, UBS Warburg

Johnson, D., Macleod, N. and Thomas, C. (2002): Modeling the Return Structure of a Fund of Hedge Funds, AIMA Journal

Kahneman, D. and Tversky, A. (1979): "Prospect Theory: An Analysis of Choice Under Risk," Econometrica 47(2): 263-292

Kat, H. (2004): In Search of the Optimal Fund of Hedge Funds, Journal of Wealth Management 6(4): 43-51

Kazemi, H. and Schneeweis, T. and Gupta, R. (2003): Omega as a Performance Measure, *working paper*, University of Massachusetts, Amherst

Lhabitant, F. and Learned, M. (2002): Hedge Fund Diversification: How Much Is Enough, Journal of Alternative Investments 5 (3): 23-49

Liang, B. (1999): On the Performance of Hedge Funds, Financial Analysts Journal 55: 72-85

Lo, A. (2001): Risk Management for Hedge Funds: Introduction and Overview, Financial Analysts Journal 57 (6): 16-33

Loomis, C. (1966): The Jones Nobody Keeps Up With, Fortune Magazine

Lowenstein, R. (2001): When Genius Failed, 4th Estate

Morton, D., Popova, E. and Popova, I. (2002): Optimal Hedge Fund Allocation with Asymmetric Preferences and Distributions, *working paper*, University of Texas at Austin

Morton, D., Popova, E. and Popova, I. (2003): Efficient Fund of Hedge Funds Construction Under Downside Risk Measures, *working paper*, University of Texas at Austin

Markowitz, H. (1952): Portfolio Selection, Journal of Finance 7 (1): 77-91.

NIST/SEMATECH, e-Handbook of Statistical Methods, http://www.itl.nist.gov/div898/handbook/

Rosenberg, M., Tomeo, J. and Chung, S. (2004): Hedge Fund-of-Funds Asset Allocation Using a Convergent and Divergent Strategy Approach, Journal of Alternative Investments, Summer 2004

Schneeweis, T. and Georgiev, G. (2002): The Benefits of Hedge Funds, CISDM, University of Massachusetts.

Schneeweis, T. and Spurgin, R. (1996): Multi-Factor Model in Managed Futures, Hedge Fund and Mutual Fund Return Estimation, *working paper*, CISDM, University of Massachusetts

Schneeweis, T., Spurgin, R. and Karavas, V. (2002): Alternative Investments in the Institutional Portfolio, Alternative Investment Management Association (AIMA)

Siegmann, A. and Lucas, A. (2002): Explaining hedge fund investment styles by loss aversion: A rational alternative, ***discussion paper***, Tinbergen Institute, Vrije Universiteit Amsterdam

Shadwick, W. and Keating, C. (2002): A Universal Performance Measure, ***working paper***, Finance Development Centre, London

Sharpe, W.: William F. Sharpe homepage, Stanford University, http://www.stanford.edu/~wfsharpe/home.htm

Simon, H. (1957): Models of Bounded Rationality, MIT Press

Sortino, F. and Price, L. (1994): "Performance Measurement in a Downside Risk Framework," Journal of Investing, Fall 1994: 59-65

Sortino, F. and Van der Meer, R. (1991): Downside risk, The Journal of Portfolio Management, Summer: 27-31

Terhaar, K., Staub, R. and Singer, B. (2003): Appropriate Policy Allocation for Alternative Investments, Journal of Portfolio Management 29 (3): 101-110

Vaughan, D.A. (2003): Selected Definitions of "Hedge Fund", U.S. Securities and Exchange Commission, http://www.sec.gov/spotlight/hedgefunds/hedge-vaughn

Von Neumann, J. and Morgenstern, O. (1944): Theory of Games and Economic Behavior, Princeton University Press

Appendix A

A.1 Descriptive Statistics.

The figures represent analysis of monthly return data of all indices included in this study. The data series start at 31 January 1994 and run until 30 June 2004.

	Descriptive Statistics								
	Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis	Jarque-Bera	Observations
MSCI World Equity	0.54%	1.00%	8.91%	-13.45%	4.16%	-0.57	3.47	8.12	126
MSCI World Sovereign Bond	0.52%	0.28%	6.02%	-4.27%	1.91%	0.37	3.49	4.15	126
Lehman Brothers Cash Composite	0.34%	0.39%	0.56%	0.02%	0.15%	-0.69	2.06	14.73	126
CSFB Tremont Hedge Fund Composite	0.89%	0.80%	8.53%	-7.55%	2.40%	0.10	4.77	16.76	126
CSFB Tremont Hedge Fund Convertible Arbitrage	0.81%	1.10%	3.57%	-4.68%	1.37%	-1.47	6.60	113.52	126
CSFB Tremont Hedge Fund Dedicated Short Bias	-0.13%	-0.41%	22.71%	-8.69%	5.10%	0.92	5.17	42.37	126
CSFB Tremont Hedge Fund Distressed Securities	1.07%	1.20%	4.10%	-12.45%	1.97%	-2.75	19.13	1525.22	126
CSFB Tremont Hedge Fund Event Driven	0.92%	1.02%	3.68%	-11.77%	1.71%	-3.46	25.83	2988.13	126
CSFB Tremont Hedge Fund Event Driven Multi-Strategy	0.83%	0.90%	4.66%	-11.52%	1.80%	-2.68	19.83	1637.60	126
CSFB Tremont Hedge Fund Emerging Markets	0.68%	1.17%	16.42%	-23.03%	5.03%	-0.57	6.71	79.04	126
CSFB Tremont Hedge Fund Fixed Income Arbitrage	0.56%	0.78%	2.02%	-6.96%	1.12%	-3.24	19.35	1624.54	126
CSFB Tremont Hedge Fund Managed Futures	0.58%	0.21%	9.95%	-9.35%	3.54%	0.04	3.41	0.94	126
CSFB Tremont Hedge Fund Global Macro	1.17%	1.20%	10.60%	-11.55%	3.42%	-0.02	5.07	22.48	126
CSFB Tremont Hedge Fund Long/Short Equity	0.98%	0.79%	13.01%	-11.44%	3.11%	0.24	6.38	61.10	126
CSFB Tremont Hedge Fund Multi-Strategy	0.73%	0.80%	3.61%	-4.76%	1.27%	-1.27	6.45	96.04	126
CSFB Tremont Hedge Fund Equity Market Neutral	0.83%	0.81%	3.26%	-1.15%	0.88%	0.25	3.21	1.56	126
CSFB Tremont Hedge Fund Risk Arbitrage	0.67%	0.63%	3.81%	-6.15%	1.26%	-1.31	9.19	236.98	126

Appendix B

B.1 Correlation Matrix.

The figures represent analysis of monthly return data of all indices included in this study. The data series start at 31 January 1994 and run until 30 June 2004.

Correlation Statistics

	CSFB Tremont Composite	CSFB Tremont Convertible Arbitrage	CSFB Tremont Dedicated Short Bias	CSFB Tremont Distressed Securities	CSFB Tremont Event Driven	CSFB Tremont Event Driven Multi-Strategy	CSFB Tremont Emerging Markets	CSFB Tremont Fixed Income Arbitrage	CSFB Tremont Managed Futures	CSFB Tremont Global Macro	CSFB Tremont Long/Short Equity	CSFB Tremont Multi-Strategy	CSFB Tremont Equity Market Neutral	CSFB Tremont Risk Arbitrage	MSCI World Equity	MSCI World Sovereign Bond	Lehman Brothers Cash Composite
CSFB Tremont Hedge Fund Composite	1.00	0.40	-0.48	0.57	0.65	0.34	0.66	0.68	0.45	0.86	0.19	0.38	0.78	0.10	0.47	-0.11	0.10
CSFB Tremont Hedge Fund Convertible Arbitrage		1.00	-0.23	0.51	0.32	0.32	0.58	0.59	0.53	0.29	0.39	0.40	0.26	-0.19	0.10	-0.11	0.21
CSFB Tremont Hedge Fund Dedicated Short Bias			1.00	-0.63	-0.57	-0.35	-0.63	-0.54	-0.08	-0.13	-0.06	-0.48	-0.72	0.23	-0.75	0.04	0.10
CSFB Tremont Hedge Fund Distressed Securities				1.00	0.59	0.35	0.94	0.76	0.31	0.31	0.12	0.56	0.58	-0.16	0.56	-0.07	0.02
CSFB Tremont Hedge Fund Event Driven					1.00	0.23	0.68	0.68	0.29	0.41	-0.01	0.43	0.59	-0.11	0.53	-0.18	-0.09
CSFB Tremont Hedge Fund Event Driven Multi-Strategy						1.00	0.38	0.36	0.09	0.22	0.23	0.31	0.35	0.15	0.37	0.08	0.28
CSFB Tremont Hedge Fund Emerging Markets							1.00	0.93	0.39	0.37	0.17	0.67	0.65	-0.22	0.58	-0.14	0.04
CSFB Tremont Hedge Fund Fixed Income Arbitrage								1.00	0.44	0.42	0.20	0.65	0.63	-0.25	0.50	-0.21	0.04
CSFB Tremont Hedge Fund Managed Futures									1.00	0.45	0.29	0.13	0.21	-0.08	0.03	-0.14	0.05
CSFB Tremont Hedge Fund Global Macro										1.00	0.14	0.12	0.43	0.25	0.18	-0.14	0.10
CSFB Tremont Hedge Fund Long/Short Equity											1.00	0.07	0.16	-0.03	0.09	0.14	0.09
CSFB Tremont Hedge Fund Multi-Strategy												1.00	0.50	-0.22	0.45	-0.06	0.20
CSFB Tremont Hedge Fund Equity Market Neutral													1.00	-0.05	0.61	0.04	0.09
CSFB Tremont Hedge Fund Risk Arbitrage														1.00	-0.16	0.37	-0.04
MSCI World Equity															1.00	0.06	0.00
MSCI World Sovereign Bond																1.00	-0.05
Lehman Brothers Cash Composite																	1.00

Appendix C

C.1 Comparison of optimal fund of hedge fund portfolios under a mean-variance regime and scenarios of investor preference under the MVSK framework. All portfolios optimised for a monthly standard deviation level of 0.59%

The portfolios are constructed under the MVSK PGP model depending on the investor preferences specified. A simple mean-variance portfolio can be run as a special case with the preferences of maximising return with no preference for either skewness or kurtosis.

Investor Profile	A	B	C	D	E	Mean-variance
α	3	3	1	3	2	3
β	2	1	0	2	3	0
γ	1	0	3	1	1	0
Mean	0.69	0.70	0.67	0.68	0.68	0.71
Variance	0.34	0.34	0.34	0.34	0.34	0.34
Skewness	0.01	0.00	-0.14	-0.03	-0.01	-0.16
Kurtosis	-0.59	-0.53	-0.92	-0.80	-0.75	-0.28
Standard Deviation	0.59	0.59	0.59	0.59	0.59	0.59
Sharpe Ratio	0.59	0.61	0.55	0.57	0.58	0.63
Sortino Ratio	2.04	1.85	1.86	2.13	2.11	1.48
Omega	1.051423965	1.051423965	1.051423968	1.051423965	1.051423965	1.05105844
Omega rank	2	5	1	3	4	6
Convertible Arbitrage	0%	0%	0%	0%	0%	0%
Dedicated Short-Bias	8%	8%	10%	9%	9%	7%
Distressed Securities	4%	5%	1%	3%	3%	8%
Emerging Markets	1%	1%	0%	1%	1%	0%
Equity Market-Neutral	48%	48%	36%	43%	45%	46%
Event Driven	0%	0%	0%	0%	0%	0%
Event Driven Multi-Strategy	0%	0%	0%	0%	0%	0%
Fixed Income Arbitrage	13%	12%	11%	10%	11%	11%
Global Macro	0%	0%	0%	0%	0%	0%
Multi-Strategy	6%	6%	8%	7%	6%	10%
Risk Arbitrage	17%	16%	26%	21%	20%	14%
Long/Short Equity	1%	1%	5%	4%	3%	1%
Managed Futures	2%	2%	2%	2%	2%	0%

C.2 Comparison of optimal fund of hedge fund portfolios under a mean-variance regime and scenarios of investor preference under the MVSK framework. All portfolios optimised for a monthly standard deviation level of 0.66%

The portfolios are constructed under the MVSK PGP model depending on the investor preferences specified. A simple mean-variance portfolio can be run as a special case with the preferences of maximising return with no preference for either skewness or kurtosis.

Investor Profile	A	B	C	D	E	Mean-variance
α	3	3	1	3	2	3
β	2	1	0	2	3	0
γ	1	0	3	1	1	0
Mean	0.66	0.66	0.66	0.66	0.66	0.78
Variance	0.43	0.43	0.43	0.43	0.43	0.43
Skewness	-0.04	-0.04	-0.04	-0.04	-0.04	-0.09
Kurtosis	-1.18	-1.18	-1.18	-1.18	-1.18	0.53
Standard Deviation	0.66	0.66	0.66	0.66	0.66	0.66
Sharpe Ratio	0.48	0.48	0.48	0.48	0.48	0.66
Sortino Ratio	2.03	2.03	2.03	2.03	2.03	1.13
Omega	1.040765731	1.040765731	1.040765731	1.040765731	1.040765731	1.040461385
Omega rank	1	1	1	1	1	6
Convertible Arbitrage	7%	7%	7%	7%	7%	0%
Dedicated Short-Bias	14%	14%	14%	14%	14%	7%
Distressed Securities	0%	0%	0%	0%	0%	17%
Emerging Markets	2%	2%	2%	2%	2%	0%
Equity Market Neutral	28%	28%	28%	28%	28%	55%
Event Driven	0%	0%	0%	0%	0%	0%
Event Driven Multi-Strategy	0%	0%	0%	0%	0%	0%
Fixed Income Arbitrage	0%	0%	0%	0%	0%	1%
Global Macro	0%	0%	0%	0%	0%	1%
Multi-Strategy	9%	9%	9%	9%	9%	12%
Risk Arbitrage	25%	25%	25%	25%	25%	5%
Long/Short Equity	12%	12%	12%	12%	12%	1%
Managed Futures	3%	3%	3%	3%	3%	0%

C.3 Comparison of optimal fund of hedge fund portfolios under a mean-variance regime and scenarios of investor preference under the MVSK framework. All portfolios optimised for a monthly standard deviation level of 0.78%

The portfolios are constructed under the MVSK PGP model depending on the investor preferences specified. A simple mean-variance portfolio can be run as a special case with the preferences of maximising return with no preference for either skewness or kurtosis.

Investor Profile	A	B	C	D	E	Mean-variance
α	3	3	1	3	2	3
β	2	1	0	2	3	0
γ	1	0	3	1	1	0
Mean	0.64	0.64	0.64	0.64	0.64	0.85
Variance	0.61	0.61	0.61	0.61	0.61	0.61
Skewness	0.11	0.11	0.11	0.11	0.11	-0.18
Kurtosis	-0.97	-0.97	-0.97	-0.97	-0.97	2.10
Standard Deviation	0.78	0.78	0.78	0.78	0.78	0.78
Sharpe Ratio	0.39	0.39	0.39	0.39	0.39	0.64
Sortino Ratio	1.31	1.31	1.31	1.31	1.31	0.88
Omega	1.028683266	1.028683311	1.028683311	1.028683311	1.028683311	1.028459026
Omega rank	5	1	1	1	1	6
Convertible Arbitrage	6%	6%	6%	6%	6%	0%
Dedicated Short-Bias	20%	20%	20%	20%	20%	4%
Distressed Securities	0%	0%	0%	0%	0%	22%
Emerging Markets	5%	5%	5%	5%	5%	0%
Equity Market-Neutral	51%	51%	51%	51%	51%	60%
Event Driven	0%	0%	0%	0%	0%	0%
Event Driven Multi-Strategy	0%	0%	0%	0%	0%	0%
Fixed Income Arbitrage	0%	0%	0%	0%	0%	0%
Global Macro	0%	0%	0%	0%	0%	4%
Multi-Strategy	4%	4%	4%	4%	4%	9%
Risk Arbitrage	0%	0%	0%	0%	0%	0%
Long/Short Equity	14%	14%	14%	14%	14%	0%
Managed Futures	0%	0%	0%	0%	0%	0%

C.4 Comparison of optimal fund of hedge fund portfolios under a mean-variance regime and scenarios of investor preference under the MVSK framework. All portfolios optimised for a monthly standard deviation level of 0.98%

The portfolios are constructed under the MVSK PGP model depending on the investor preferences specified. A simple mean-variance portfolio can be run as a special case with the preferences of maximising return with no preference for either skewness or kurtosis.

Investor Profile	A	B	C	D	E	Mean-variance
α	3	3	1	3	2	3
β	2	1	0	2	3	0
γ	1	0	3	1	1	0
Mean	0.60	0.86	0.57	0.62	0.61	0.91
Variance	0.97	0.97	0.97	0.97	0.97	0.97
Skewness	0.53	0.78	0.08	0.35	0.44	-0.57
Kurtosis	0.72	1.46	-0.80	-0.48	-0.08	4.45
Standard Deviation	0.98	0.98	0.98	0.98	0.98	0.98
Sharpe Ratio	0.26	0.53	0.23	0.28	0.27	0.57
Sortino Ratio	0.61	1.55	0.63	0.86	0.74	0.64
Omega	1.017960037	1.01796005	1.017960017	1.017960036	1.017960026	1.017817212
Omega rank	2	1	5	3	4	6
Convertible Arbitrage	0%	0%	1%	13%	2%	0%
Dedicated Short-Bias	19%	3%	28%	21%	21%	0%
Distressed Securities	0%	3%	0%	1%	0%	26%
Emerging Markets	0%	0%	10%	3%	4%	0%
Equity Market-Neutral	44%	74%	47%	59%	59%	62%
Event Driven	0%	0%	0%	0%	0%	0%
Event Driven Multi-Strategy	0%	0%	0%	0%	0%	0%
Fixed Income Arbitrage	0%	0%	0%	0%	0%	0%
Global Macro	0%	18%	0%	0%	0%	8%
Multi-Strategy	18%	0%	0%	0%	10%	4%
Risk Arbitrage	12%	0%	0%	0%	0%	0%
Long/Short Equity	0%	0%	15%	0%	0%	0%
Managed Futures	6%	2%	0%	3%	4%	0%

C.5 Comparison of optimal fund of hedge fund portfolios under a mean-variance regime and scenarios of investor preference under the MVSK framework. All portfolios optimised for a monthly standard deviation level of 1.26%

The portfolios are constructed under the MVSK PGP model depending on the investor preferences specified. A simple mean-variance portfolio can be run as a special case with the preferences of maximising return with no preference for either skewness or kurtosis.

Investor Profile	A	B	C	D	E	Mean-variance
α	1	3	1	3	2	3
β	3	1	0	2	3	0
γ	0	0	3	1	1	0
Mean	0.59	0.91	0.68	0.83	0.85	0.98
Variance	1.58	1.58	1.58	1.58	1.58	1.58
Skewness	0.75	0.72	-0.01	0.53	0.61	-1.18
Kurtosis	1.88	1.47	-0.65	0.21	0.49	7.59
Standard Deviation	1.26	1.26	1.26	1.26	1.26	1.26
Sharpe Ratio	0.20	0.45	0.27	0.39	0.40	0.50
Sortino Ratio	0.41	1.15	0.59	1.10	1.11	0.50
Omega	1.010996695	1.010996695	1.010996694	1.010996695	1.010996689	1.010909474
Omega rank	1	3	4	2	5	6
Convertible Arbitrage	0%	0%	10%	10%	2%	0%
Dedicated Short-Bias	21%	0%	2%	0%	0%	0%
Distressed Securities	0%	0%	1%	0%	0%	41%
Emerging Markets	0%	0%	12%	0%	0%	0%
Equity Market-Neutral	49%	69%	7%	28%	36%	45%
Event Driven	0%	0%	3%	0%	0%	0%
Event Driven Multi-Strategy	0%	0%	7%	0%	0%	0%
Fixed Income Arbitrage	0%	0%	17%	0%	0%	0%
Global Macro	1%	27%	0%	9%	12%	14%
Multi-Strategy	16%	0%	7%	14%	15%	0%
Risk Arbitrage	0%	0%	3%	0%	0%	0%
Long/Short Equity	0%	0%	0%	20%	19%	0%
Managed Futures	14%	4%	28%	19%	16%	0%

C.6 Comparison of optimal fund of hedge fund portfolios under a mean-variance regime and scenarios of investor preference under the MVSK framework. All portfolios optimised for a monthly standard deviation level of 1.60%

The portfolios are constructed under the MVSK PGP model depending on the investor preferences specified. A simple mean-variance portfolio can be run as a special case with the preferences of maximising return with no preference for either skewness or kurtosis.

Investor Profile	A	B	C	D	E	Mean-variance
α	3	3	1	3	2	3
β	2	1	0	2	3	0
γ	1	0	3	1	1	0
Mean	0.63	0.96	0.64	0.87	0.90	1.04
Variance	2.56	2.56	2.56	2.56	2.56	2.56
Skewness	-0.05	0.55	-0.05	0.49	0.58	-1.57
Kurtosis	-0.77	1.56	-0.77	0.29	0.70	9.04
Standard Deviation	1.60	1.60	1.60	1.60	1.60	1.60
Sharpe Ratio	0.18	0.39	0.19	0.33	0.35	0.43
Sortino Ratio	0.37	0.74	0.39	0.79	0.78	0.43
Omega	1.006754896	1.006754893	1.006754892	1.006754893	1.006754896	1.006701566
Omega rank	2	4	5	3	1	6
Convertible Arbitrage	10%	0%	15%	14%	0%	0%
Dedicated Short-Bias	0%	0%	0%	0%	0%	0%
Distressed Securities	0%	0%	0%	0%	0%	59%
Emerging Markets	19%	0%	19%	0%	0%	0%
Equity Market-Neutral	0%	57%	0%	21%	39%	21%
Event Driven	0%	0%	0%	0%	0%	0%
Event Driven Multi-Strategy	0%	0%	0%	0%	0%	0%
Fixed Income Arbitrage	36%	0%	31%	0%	0%	0%
Global Macro	0%	41%	0%	19%	25%	21%
Multi-Strategy	0%	0%	0%	0%	0%	0%
Risk Arbitrage	0%	0%	0%	0%	0%	0%
Long/Short Equity	0%	0%	0%	22%	17%	0%
Managed Futures	35%	3%	35%	25%	19%	0%

C.7 Comparison of optimal fund of hedge fund portfolios under a mean-variance regime and scenarios of investor preference under the MVSK framework. All portfolios optimised for a monthly standard deviation level of 2.01%

The portfolios are constructed under the MVSK PGP model depending on the investor preferences specified. A simple mean-variance portfolio can be run as a special case with the preferences of maximising return with no preference for either skewness or kurtosis.

Investor Profile	A	B	C	D	E	Mean-variance
α	3	3	1	3	2	3
β	2	1	0	2	3	0
γ	1	0	3	1	1	0
Mean	0.83	1.01	0.64	0.89	0.89	1.11
Variance	4.03	4.03	4.03	4.03	4.03	4.03
Skewness	0.67	0.39	-0.03	0.41	0.45	-1.18
Kurtosis	3.01	1.58	-0.63	0.27	0.42	6.15
Standard Deviation	2.01	2.01	2.01	2.01	2.01	2.01
Sharpe Ratio	0.24	0.33	0.15	0.27	0.27	0.38
Sortino Ratio	0.45	0.54	0.29	0.59	0.59	0.41
Omega	1.004287328	1.004287328	1.00428733	1.004287325	1.004287327	1.004253785
Omega rank	2	3	1	5	4	6
Convertible Arbitrage	0%	0%	9%	11%	9%	0%
Dedicated Short-Bias	8%	0%	0%	0%	0%	0%
Distressed Securities	0%	0%	0%	0%	0%	64%
Emerging Markets	0%	0%	26%	0%	0%	0%
Equity Market-Neutral	0%	34%	0%	3%	3%	0%
Event Driven	0%	0%	0%	0%	0%	0%
Event Driven Multi-Strategy	0%	0%	0%	0%	0%	0%
Fixed Income Arbitrage	0%	0%	20%	0%	0%	0%
Global Macro	0%	47%	0%	27%	23%	36%
Multi-Strategy	9%	0%	0%	0%	0%	0%
Risk Arbitrage	0%	0%	0%	0%	0%	0%
Long/Short Equity	71%	15%	0%	27%	33%	0%
Managed Futures	13%	4%	45%	32%	31%	0%

Appendix D

D.1 Comparison of optimal diversified multi-asset portfolios between a mean-variance and MVSK framework. All portfolios optimised for a monthly standard deviation level of 0.18%

The portfolios are constructed under the MVSK PGP model depending on the investor preferences specified. A simple mean-variance portfolio can be run as a special case with the preferences of maximising return with no preference for either skewness or kurtosis.

Investor Profile	A	B	C	D	E	Mean-variance
α	1	3	1	3	2	3
β	3	1	0	2	3	0
γ	0	0	3	1	1	0
Mean	0.37	0.37	0.36	0.36	0.36	0.37
Variance	0.03	0.03	0.03	0.03	0.03	0.03
Skewness	0.01	0.01	-0.10	0.01	0.01	-0.16
Kurtosis	-0.79	-0.78	-0.83	-0.80	-0.80	-0.42
Standard Deviation	0.18	0.18	0.18	0.18	0.18	0.18
Sharpe Ratio	0.13	0.14	0.11	0.13	0.13	0.16
Sortino Ratio	N/A	N/A	N/A	N/A	N/A	0.66
Omega	1.717553898	1.717554961	1.71755806	1.717560388	1.71755785	1.717542787
Omega rank	5	4	2	1	3	6
Hedge Fund Composite	4%	4%	3%	4%	4%	4%
World Equity	0%	0%	1%	0%	0%	0%
World Soveriegn Bond	1%	1%	0%	0%	0%	4%
US Cash Composite	95%	95%	96%	96%	96%	93%

D.2 Comparison of optimal diversified multi-asset portfolios between a mean-variance and MVSK framework. All portfolios optimized for a monthly standard deviation level of 0.25%

The portfolios are constructed under the MVSK PGP model depending on the investor preferences specified. A simple mean-variance portfolio can be run as a special case with the preferences of maximising return with no preference for either skewness or kurtosis.

Investor Profile	A	B	C	D	E	Mean-variance
α	1	3	1	3	2	3
β	3	1	0	2	3	0
γ	0	0	3	1	1	0
Mean	0.39	0.39	0.37	0.39	0.39	0.40
Variance	0.06	0.06	0.06	0.06	0.06	0.06
Skewness	0.45	0.45	-0.21	0.45	0.45	0.10
Kurtosis	0.24	0.24	-0.42	0.24	0.24	0.03
Standard Deviation	0.25	0.25	0.25	0.25	0.25	0.25
Sharpe Ratio	0.18	0.18	0.11	0.18	0.18	0.22
Sortino Ratio	0.57	0.57	0.28	0.57	0.57	0.81
Omega	1.306914982	1.306914982	1.306912413	1.306912709	1.306912709	1.306911652
Omega rank	1	1	5	3	3	6
Hedge Fund Composite	8%	8%	2%	8%	8%	8%
World Equity	0%	0%	4%	0%	0%	0%
World Soveriegn Bond	0%	0%	4%	0%	0%	6%
US Cash Composite	92%	92%	90%	92%	92%	86%

D.3 Comparison of optimal diversified multi-asset portfolios between a mean-variance and MVSK framework. All portfolios optimized for a monthly standard deviation level of 0.35%

The portfolios are constructed under the MVSK PGP model depending on the investor preferences specified. A simple mean-variance portfolio can be run as a special case with the preferences of maximising return with no preference for either skewness or kurtosis.

Investor Profile	A	B	C	D	E	Mean-variance
α	1	3	1	3	2	3
β	3	1	0	2	3	0
γ	0	0	3	1	1	0
Mean	0.37	0.37	0.37	0.37	0.37	0.42
Variance	0.12	0.12	0.12	0.12	0.12	0.12
Skewness	0.13	0.13	0.13	0.13	0.13	0.11
Kurtosis	0.63	0.63	0.63	0.63	0.63	0.31
Standard Deviation	0.35	0.35	0.35	0.35	0.35	0.35
Sharpe Ratio	0.09	0.09	0.09	0.09	0.09	0.24
Sortino Ratio	0.18	0.18	0.18	0.18	0.18	0.61
Omega	1.155778216	1.155778216	1.155778216	1.155778216	1.155778216	1.155778181
Omega rank	1	1	1	1	1	6
Hedge Fund Composite	0%	0%	0%	0%	0%	12%
World Equity	0%	0%	0%	0%	0%	0%
World Soveriegn Bond	17%	17%	17%	17%	17%	9%
US Cash Composite	83%	83%	83%	83%	83%	79%

D.4 Comparison of optimal diversified multi-asset portfolios between a mean-variance and MVSK framework. All portfolios optimized for a monthly standard deviation level of 0.44%

The portfolios are constructed under the MVSK PGP model depending on the investor preferences specified. A simple mean-variance portfolio can be run as a special case with the preferences of maximising return with no preference for either skewness or kurtosis.

Investor Profile	A	B	C	D	E	Mean-variance
α	1	3	1	3	2	3
β	3	1	0	2	3	0
γ	0	0	3	1	1	0
Mean	0.42	0.42	0.39	0.38	0.38	0.42
Variance	0.20	0.20	0.20	0.20	0.20	0.20
Skewness	0.14	0.14	-0.25	-0.02	0.20	-0.05
Kurtosis	1.09	1.09	-0.21	0.13	0.60	0.54
Standard Deviation	0.44	0.44	0.44	0.44	0.44	0.44
Sharpe Ratio	0.19	0.19	0.11	0.10	0.10	0.25
Sortino Ratio	0.30	0.30	0.24	0.23	0.18	0.46
Omega	1.0919926	1.0919926	1.09199251	1.091992498	1.091992494	1.091681463
Omega rank	1	1	3	4	5	6
Hedge Fund Composite	14%	14%	3%	0%	0%	8%
World Equity	3%	3%	8%	7%	0%	3%
World Soveriegn Bond	0%	0%	10%	15%	23%	18%
US Cash Composite	83%	83%	79%	78%	77%	71%

D.5 Comparison of optimal diversified multi-asset portfolios between a mean-variance and MVSK framework. All portfolios optimized for a monthly standard deviation level of 0.54%

The portfolios are constructed under the MVSK PGP model depending on the investor preferences specified. A simple mean-variance portfolio can be run as a special case with the preferences of maximising return with no preference for either skewness or kurtosis.

Investor Profile	A	B	C	D	E	Mean-variance
α	1	3	1	3	2	3
β	3	1	0	2	3	0
γ	0	0	3	1	1	0
Mean	0.46	0.46	0.48	0.48	0.47	0.48
Variance	0.30	0.30	0.30	0.30	0.30	0.30
Skewness	0.38	0.38	-0.02	0.09	0.33	0.04
Kurtosis	1.55	1.55	0.49	0.67	1.42	0.56
Standard Deviation	0.54	0.54	0.54	0.54	0.54	0.54
Sharpe Ratio	0.22	0.22	0.25	0.25	0.23	0.25
Sortino Ratio	0.34	0.34	0.44	0.44	0.34	0.43
Omega	1.060210596	1.060210596	1.06021059	1.060210576	1.060210568	1.060210366
Omega rank	1	1	3	4	5	6
Hedge Fund Composite	22%	22%	19%	21%	22%	20%
World Equity	0%	0%	0%	0%	0%	0%
World Soveriegn Bond	0%	0%	17%	12%	2%	14%
US Cash Composite	78%	78%	64%	67%	76%	66%

D.6 Comparison of optimal diversified multi-asset portfolios between a mean-variance and MVSK framework. All portfolios optimized for a monthly standard deviation level of 0.65%

The portfolios are constructed under the MVSK PGP model depending on the investor preferences specified. A simple mean-variance portfolio can be run as a special case with the preferences of maximising return with no preference for either skewness or kurtosis.

Investor Profile	A	B	C	D	E	Mean-variance
α	1	3	1	3	2	3
β	3	1	0	2	3	0
γ	0	0	3	1	1	0
Mean	0.40	0.44	0.40	0.40	0.40	0.51
Variance	0.42	0.42	0.42	0.42	0.42	0.42
Skewness	0.28	0.19	0.28	0.28	0.28	0.01
Kurtosis	0.58	0.97	0.58	0.58	0.58	0.62
Standard Deviation	0.65	0.65	0.65	0.65	0.65	0.65
Sharpe Ratio	0.10	0.16	0.10	0.10	0.10	0.26
Sortino Ratio	0.18	0.29	0.18	0.18	0.18	0.42
Omega	1.042315439	1.042315394	1.042315512	1.042315512	1.042315512	1.042087844
Omega rank	4	5	1	1	1	6
Hedge Fund Composite	0%	7%	0%	0%	0%	24%
World Equity	0%	0%	0%	0%	0%	0%
World Soveriegn Bond	34%	33%	34%	34%	34%	16%
US Cash Composite	66%	59%	66%	66%	66%	59%

D.7 Comparison of optimal diversified multi-asset portfolios between a mean-variance and MVSK framework. All portfolios optimized for a monthly standard deviation level of 0.75%

The portfolios are constructed under the MVSK PGP model depending on the investor preferences specified. A simple mean-variance portfolio can be run as a special case with the preferences of maximising return with no preference for either skewness or kurtosis.

Investor Profile	A	B	C	D	E	Mean-variance
α	1	3	1	3	2	3
β	3	1	0	2	3	0
γ	0	0	3	1	1	0
Mean	0.42	0.47	0.41	0.41	0.41	0.51
Variance	0.56	0.56	0.56	0.56	0.56	0.56
Skewness	0.29	0.20	0.29	0.29	0.29	0.03
Kurtosis	0.64	1.01	0.58	0.58	0.58	0.90
Standard Deviation	0.75	0.75	0.75	0.75	0.75	0.75
Sharpe Ratio	0.11	0.17	0.10	0.10	0.10	0.26
Sortino Ratio	0.20	0.30	0.18	0.18	0.18	0.41
Omega	1.031307898	1.03130791	1.031307926	1.031307926	1.031307926	1.031124399
Omega rank	5	4	1	1	1	6
Hedge Fund Composite	1%	10%	0%	0%	0%	19%
World Equity	0%	0%	0%	0%	0%	0%
World Soveriegn Bond	39%	38%	39%	39%	39%	34%
US Cash Composite	60%	52%	61%	61%	61%	48%

D.8 Comparison of optimal diversified multi-asset portfolios between a mean-variance and MVSK framework. All portfolios optimized for a monthly standard deviation level of 0.85%

The portfolios are constructed under the MVSK PGP model depending on the investor preferences specified. A simple mean-variance portfolio can be run as a special case with the preferences of maximising return with no preference for either skewness or kurtosis.

Investor Profile	A	B	C	D	E	Mean-variance
α	1	3	1	3	2	3
β	3	1	0	2	3	0
γ	0	0	3	1	1	0
Mean	0.53	0.53	0.56	0.56	0.56	0.56
Variance	0.73	0.73	0.73	0.73	0.73	0.73
Skewness	0.26	0.26	-0.06	-0.05	0.08	-0.03
Kurtosis	1.80	1.80	0.59	0.61	1.12	0.69
Standard Deviation	0.85	0.85	0.85	0.85	0.85	0.85
Sharpe Ratio	0.23	0.23	0.26	0.26	0.25	0.26
Sortino Ratio	0.34	0.34	0.42	0.41	0.38	0.40
Omega	1.02407635	1.02407635	1.02407634	1.024076293	1.02407635	1.024076289
Omega rank	1	1	4	5	3	6
Hedge Fund Composite	35%	35%	30%	31%	35%	33%
World Equity	0%	0%	0%	0%	0%	0%
World Soveriegn Bond	0%	0%	27%	25%	12%	21%
US Cash Composite	65%	65%	43%	44%	53%	46%

D.9 Comparison of optimal diversified multi-asset portfolios between a mean-variance and MVSK framework. All portfolios optimized for a monthly standard deviation level of 0.96%

The portfolios are constructed under the MVSK PGP model depending on the investor preferences specified. A simple mean-variance portfolio can be run as a special case with the preferences of maximising return with no preference for either skewness or kurtosis.

Investor Profile	A	B	C	D	E	Mean-variance
α	1	3	1	3	2	3
β	3	1	0	2	3	0
γ	0	0	3	1	1	0
Mean	0.50	0.52	0.45	0.44	0.44	0.59
Variance	0.91	0.91	0.91	0.91	0.91	0.91
Skewness	0.24	0.17	-0.27	-0.01	0.24	-0.04
Kurtosis	1.03	1.08	-0.09	0.25	0.68	0.71
Standard Deviation	0.96	0.96	0.96	0.96	0.96	0.96
Sharpe Ratio	0.17	0.19	0.11	0.11	0.10	0.26
Sortino Ratio	0.29	0.33	0.19	0.22	0.18	0.41
Omega	1.019079184	1.019079188	1.019079182	1.019079177	1.01907915	1.018956442
Omega rank	2	1	3	4	5	6
Hedge Fund Composite	12%	17%	5%	1%	0%	37%
World Equity	0%	0%	18%	14%	4%	0%
World Soveriegn Bond	49%	48%	24%	37%	48%	24%
US Cash Composite	39%	36%	53%	48%	47%	39%

D.10 Comparison of optimal diversified multi-asset portfolios between a mean-variance and MVSK framework. All portfolios optimized for a monthly standard deviation level of 1.06%

The portfolios are constructed under the MVSK PGP model depending on the investor preferences specified. A simple mean-variance portfolio can be run as a special case with the preferences of maximising return with no preference for either skewness or kurtosis.

Investor Profile	A	B	C	D	E	Mean-variance
α	1	3	1	3	2	3
β	3	1	0	2	3	0
γ	0	0	3	1	1	0
Mean	0.53	0.55	0.46	0.46	0.45	0.62
Variance	1.12	1.12	1.12	1.12	1.12	1.12
Skewness	0.21	0.15	-0.27	-0.01	0.21	-0.06
Kurtosis	1.08	1.08	-0.08	0.27	0.68	0.72
Standard Deviation	1.06	1.06	1.06	1.06	1.06	1.06
Sharpe Ratio	0.18	0.20	0.11	0.11	0.10	0.26
Sortino Ratio	0.31	0.33	0.18	0.22	0.18	0.41
Omega	1.015485507	1.015485497	1.015485535	1.015485499	1.015485526	1.015383107
Omega rank	3	5	1	4	2	6
Hedge Fund Composite	16%	21%	5%	2%	0%	41%
World Equity	0%	0%	20%	15%	6%	0%
World Soveriegn Bond	54%	52%	26%	41%	53%	27%
US Cash Composite	30%	28%	48%	41%	41%	33%

D.11 Comparison of optimal diversified multi-asset portfolios between a mean-variance and MVSK framework. All portfolios optimized for a monthly standard deviation level of 1.16%

The portfolios are constructed under the MVSK PGP model depending on the investor preferences specified. A simple mean-variance portfolio can be run as a special case with the preferences of maximising return with no preference for either skewness or kurtosis.

Investor Profile	A	B	C	D	E	Mean-variance
α	1	3	1	3	2	3
β	3	1	0	2	3	0
γ	0	0	3	1	1	0
Mean	0.56	0.58	0.47	0.48	0.46	0.65
Variance	1.35	1.35	1.35	1.35	1.35	1.35
Skewness	0.20	0.13	-0.27	-0.01	0.20	-0.06
Kurtosis	1.10	1.06	-0.07	0.29	0.67	0.73
Standard Deviation	1.16	1.16	1.16	1.16	1.16	1.16
Sharpe Ratio	0.19	0.21	0.11	0.12	0.11	0.26
Sortino Ratio	0.32	0.34	0.18	0.23	0.18	0.41
Omega	1.012816487	1.012816498	1.012816488	1.012816483	1.012816497	1.012729948
Omega rank	4	1	3	5	2	6
Hedge Fund Composite	20%	25%	6%	4%	0%	45%
World Equity	0%	0%	22%	16%	8%	0%
World Soveriegn Bond	58%	56%	29%	46%	57%	29%
US Cash Composite	22%	20%	43%	34%	35%	26%

D.12 Comparison of optimal diversified multi-asset portfolios between a mean-variance and MVSK framework. All portfolios optimized for a monthly standard deviation level of 1.27%

The portfolios are constructed under the MVSK PGP model depending on the investor preferences specified. A simple mean-variance portfolio can be run as a special case with the preferences of maximising return with no preference for either skewness or kurtosis.

Investor Profile	A	B	C	D	E	Mean-variance
α	1	3	1	3	2	3
β	3	1	0	2	3	0
γ	0	0	3	1	1	0
Mean	0.59	0.61	0.48	0.50	0.48	0.67
Variance	1.61	1.61	1.61	1.61	1.61	1.61
Skewness	0.18	0.10	-0.27	-0.02	0.18	-0.07
Kurtosis	1.10	1.04	-0.07	0.32	0.65	0.74
Standard Deviation	1.27	1.27	1.27	1.27	1.27	1.27
Sharpe Ratio	0.19	0.21	0.11	0.13	0.11	0.26
Sortino Ratio	0.33	0.36	0.18	0.24	0.18	0.41
Omega	1.010780851	1.010780853	1.010780845	1.010780853	1.010780867	1.010706862
Omega rank	4	2	5	3	1	6
Hedge Fund Composite	23%	29%	6%	6%	0%	49%
World Equity	0%	0%	24%	17%	10%	0%
World Soveriegn Bond	63%	59%	32%	51%	61%	32%
US Cash Composite	14%	12%	38%	26%	29%	19%

D.13 Comparison of optimal diversified multi-asset portfolios between a mean-variance and MVSK framework. All portfolios optimized for a monthly standard deviation level of 1.37%

The portfolios are constructed under the MVSK PGP model depending on the investor preferences specified. A simple mean-variance portfolio can be run as a special case with the preferences of maximising return with no preference for either skewness or kurtosis.

Investor Profile	A	B	C	D	E	Mean-variance
α	1	3	1	3	2	3
β	3	1	0	2	3	0
γ	0	0	3	1	1	0
Mean	0.66	0.66	0.70	0.70	0.61	0.70
Variance	1.88	1.88	1.88	1.88	1.88	1.88
Skewness	0.17	0.17	-0.09	-0.09	0.17	-0.08
Kurtosis	1.90	1.89	0.64	0.64	1.10	0.75
Standard Deviation	1.37	1.37	1.37	1.37	1.37	1.37
Sharpe Ratio	0.23	0.23	0.26	0.26	0.20	0.26
Sortino Ratio	0.34	0.34	0.41	0.41	0.33	0.41
Omega	1.009193369	1.009193361	1.009193381	1.009193358	1.009193366	1.009129444
Omega rank	2	4	1	5	3	6
Hedge Fund Composite	57%	57%	49%	49%	27%	53%
World Equity	0%	0%	0%	0%	0%	0%
World Soveriegn Bond	0%	0%	44%	45%	67%	34%
US Cash Composite	43%	43%	7%	6%	6%	12%

D.14 Comparison of optimal diversified multi-asset portfolios between a mean-variance and MVSK framework. All portfolios optimized for a monthly standard deviation level of 1.48%

The portfolios are constructed under the MVSK PGP model depending on the investor preferences specified. A simple mean-variance portfolio can be run as a special case with the preferences of maximising return with no preference for either skewness or kurtosis.

Investor Profile	A	B	C	D	E	Mean-variance
α	1	3	1	3	2	3
β	3	1	0	2	3	0
γ	0	0	3	1	1	0
Mean	0.62	0.62	0.50	0.55	0.50	0.62
Variance	2.18	2.18	2.18	2.18	2.18	2.18
Skewness	0.16	0.16	-0.27	-0.02	0.16	0.16
Kurtosis	1.06	1.06	-0.06	0.40	0.62	1.06
Standard Deviation	1.48	1.48	1.48	1.48	1.48	1.48
Sharpe Ratio	0.19	0.19	0.11	0.14	0.11	0.26
Sortino Ratio	0.32	0.32	0.17	0.24	0.18	0.41
Omega	1.007931755	1.007931755	1.007931743	1.007931748	1.007931751	1.007876014
Omega rank	1	1	5	4	3	6
Hedge Fund Composite	25%	25%	7%	11%	0%	25%
World Equity	3%	3%	28%	18%	14%	3%
World Soveriegn Bond	72%	72%	37%	61%	70%	72%
US Cash Composite	0%	0%	28%	11%	17%	0%

D.15 Comparison of optimal diversified multi-asset portfolios between a mean-variance and MVSK framework. All portfolios optimized for a monthly standard deviation level of 1.58%

The portfolios are constructed under the MVSK PGP model depending on the investor preferences specified. A simple mean-variance portfolio can be run as a special case with the preferences of maximising return with no preference for either skewness or kurtosis.

Investor Profile	A	B	C	D	E	Mean-variance
α	1	3	1	3	2	3
β	3	1	0	2	3	0
γ	0	0	3	1	1	0
Mean	0.58	0.58	0.52	0.58	0.52	0.58
Variance	2.51	2.51	2.51	2.51	2.51	2.51
Skewness	0.15	0.15	-0.27	-0.02	0.15	0.15
Kurtosis	0.83	0.83	-0.05	0.46	0.61	0.83
Standard Deviation	1.58	1.58	1.58	1.58	1.58	1.58
Sharpe Ratio	0.15	0.15	0.11	0.15	0.11	0.26
Sortino Ratio	0.26	0.26	0.17	0.26	0.18	0.41
Omega	1.006908876	1.006908876	1.006908871	1.006908871	1.00690887	1.006859845
Omega rank	1	1	3	4	5	6
Hedge Fund Composite	14%	14%	7%	15%	1%	14%
World Equity	10%	10%	30%	18%	15%	10%
World Soveriegn Bond	76%	76%	40%	66%	74%	76%
US Cash Composite	0%	0%	22%	2%	10%	0%

D.16 Comparison of optimal diversified multi-asset portfolios between a mean-variance and MVSK framework. All portfolios optimized for a monthly standard deviation level of 1.71%

The portfolios are constructed under the MVSK PGP model depending on the investor preferences specified. A simple mean-variance portfolio can be run as a special case with the preferences of maximising return with no preference for either skewness or kurtosis.

Investor Profile	A	B	C	D	E	Mean-variance
α	1	3	1	3	2	3
β	3	1	0	2	3	0
γ	0	0	3	1	1	0
Mean	0.55	0.75	0.53	0.78	0.78	0.78
Variance	2.91	2.91	2.91	2.91	2.91	2.91
Skewness	0.14	0.09	-0.27	-0.05	-0.05	-0.05
Kurtosis	0.64	1.74	-0.05	1.04	1.04	1.04
Standard Deviation	1.71	1.71	1.71	1.71	1.71	1.71
Sharpe Ratio	0.12	0.24	0.11	0.26	0.26	0.26
Sortino Ratio	0.20	0.35	0.17	0.40	0.40	0.40
Omega	1.005945495	1.005945494	1.005945495	1.005945491	1.005945491	1.005902696
Omega rank	1	3	2	4	4	6
Hedge Fund Composite	5%	71%	8%	70%	70%	70%
World Equity	16%	0%	33%	0%	0%	0%
World Soveriegn Bond	80%	7%	43%	30%	30%	30%
US Cash Composite	0%	22%	16%	0%	0%	0%

D.17 Comparison of optimal diversified multi-asset portfolios between a mean-variance and MVSK framework. All portfolios optimized for a monthly standard deviation level of 1.85%

The portfolios are constructed under the MVSK PGP model depending on the investor preferences specified. A simple mean-variance portfolio can be run as a special case with the preferences of maximising return with no preference for either skewness or kurtosis.

Investor Profile	A	B	C	D	E	Mean-variance
α	1	3	1	3	2	3
β	3	1	0	2	3	0
γ	0	0	3	1	1	0
Mean	0.53	0.79	0.54	0.54	0.53	0.81
Variance	3.43	3.43	3.43	3.43	3.43	3.43
Skewness	0.01	0.07	-0.27	-0.07	0.01	-0.01
Kurtosis	0.29	1.68	-0.04	0.15	0.29	1.32
Standard Deviation	1.85	1.85	1.85	1.85	1.85	1.85
Sharpe Ratio	0.10	0.24	0.11	0.11	0.10	0.25
Sortino Ratio	0.17	0.36	0.17	0.18	0.17	0.38
Omega	1.005038022	1.005038018	1.005038019	1.005038018	1.005038021	1.005001249
Omega rank	1	5	3	4	2	6
Hedge Fund Composite	0%	78%	9%	4%	0%	77%
World Equity	28%	0%	36%	30%	28%	0%
World Sovereign Bond	72%	10%	47%	66%	72%	23%
US Cash Composite	0%	12%	9%	0%	0%	0%

D.18 Comparison of optimal diversified multi-asset portfolios between a mean-variance and MVSK framework. All portfolios optimized for a monthly standard deviation level of 2.02%

The portfolios are constructed under the MVSK PGP model depending on the investor preferences specified. A simple mean-variance portfolio can be run as a special case with the preferences of maximising return with no preference for either skewness or kurtosis.

Investor Profile	A	B	C	D	E	Mean-variance
α	1	3	1	3	2	3
β	3	1	0	2	3	0
γ	0	0	3	1	1	0
Mean	0.53	0.84	0.56	0.53	0.53	0.84
Variance	4.07	4.07	4.07	4.07	4.07	4.07
Skewness	-0.11	0.04	-0.27	-0.11	-0.11	0.03
Kurtosis	0.05	1.62	-0.04	0.05	0.05	1.58
Standard Deviation	2.02	2.02	2.02	2.02	2.02	2.02
Sharpe Ratio	0.09	0.24	0.11	0.09	0.09	0.25
Sortino Ratio	0.15	0.36	0.17	0.15	0.15	0.37
Omega	1.004245277	1.004245274	1.004245277	1.004245276	1.004245276	1.004213914
Omega rank	1	5	2	3	3	6
Hedge Fund Composite	0%	85%	9%	0%	0%	85%
World Equity	37%	0%	39%	37%	37%	0%
World Soveriegn Bond	63%	14%	51%	63%	63%	15%
US Cash Composite	0%	2%	1%	0%	0%	0%

D.19 Comparison of optimal diversified multi-asset portfolios between a mean-variance and MVSK framework. All portfolios optimized for a monthly standard deviation level of 2.20%

The portfolios are constructed under the MVSK PGP model depending on the investor preferences specified. A simple mean-variance portfolio can be run as a special case with the preferences of maximising return with no preference for either skewness or kurtosis.

Investor Profile	A	B	C	D	E	Mean-variance
α	1	3	1	3	2	3
β	3	1	0	2	3	0
γ	0	0	3	1	1	0
Mean	0.53	0.87	0.55	0.53	0.53	0.87
Variance	4.83	4.83	4.83	4.83	4.83	4.83
Skewness	-0.20	0.07	-0.28	-0.20	-0.20	0.07
Kurtosis	-0.01	1.78	-0.03	-0.01	-0.01	1.78
Standard Deviation	2.20	2.20	2.20	2.20	2.20	2.20
Sharpe Ratio	0.09	0.24	0.09	0.09	0.09	0.24
Sortino Ratio	0.13	0.35	0.15	0.13	0.13	0.35
Omega	1.003579247	1.003579247	1.003579245	1.003579245	1.003579245	1.003552531
Omega rank	2	1	3	4	4	6
Hedge Fund Composite	0%	92%	5%	0%	0%	92%
World Equity	45%	0%	45%	45%	45%	0%
World Sovcriegn Bond	55%	8%	50%	55%	55%	8%
US Cash Composite	0%	0%	0%	0%	0%	0%