UNIVERSITY OF CAPE TOWN

DEPARTMENT OF MATHEMATICAL STATISTICS

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AN ECONOMETRIC ANALYSIS OF SOUTH AFRICAN MONETARY PHENOMENA

BY

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PREFACE

Monetarism, and more particularly, the Monetary approach to the balance of payments has been the subject of important theoretical debate and empirical research over the last twenty years, and has assumed particular relevance in the world inflationary impasse of the present day. Although of great relevance to South Africa, the theory has received comparatively little attention locally and statistical studies are rare. It was the purpose of this thesis to go some way towards remedying this lack of information, and it is hoped that in doing so a more active interest will be stimulated in what this author believes to be of fundamental economic importance, namely the sources and effects of the money supply.

Chapters Two and Three investigate the sources of the money supply. Chapter Two was written by Brian Kantor of the School of Economics, and is included in the work for completeness as it forms an economic introduction to all that follows, particularly to Chapter Three. Chapters Four and Five analyse the effects of the money supply, and use the most up-to-date estimation procedures currently available.

Rigour and generality have not been entirely sacrificed, and - where space permitted - additional material was included so that the Statistician or Economist, anxious to acquire a good understanding of econometric methodology, and who might not be particularly interested in Monetary Theory would still profit from a reading. This is particularly true of Chapters One and Four. In addition an extensive Appendix (viz. Appendiz A), covers in a logical order many techniques of econometrics that students new to the theory, will find of value. Most of the data used in the analysis has also been included so that results may, as far as possible, be reproduced and extended.

I should like to thank my Supervisors, Mr. Brian Kantor and Associate Professor A.H. Money most sincerely for their constant support and encouragement throughout, and also to Brian Kantor for allowing me to reproduce, as Chapter Two, an earlier paper of his own. Many thanks go to Professor C.G. Troskie for his friendly assistance, and for his help in standing in as Supervisor while Associate Professor Money was on sabbatical. To Dr. C.R. Wymer at the International Monetary Fund in Washington, my deepest appreciation for allowing me to make use of his suite of computer programs without which the estimations of Chapter Five would not have been possible.

I should also like to thank the staff of the University of Cape Town Computer Centre, particularly Mr. David Wright, for their invaluable computing advice, and to our secretary, Mrs. M.I. Cousins, my gratitude for her superb typing which transformed my illegible scrawl to something resembling a work of art! Finally I should like to say that it has been a pleasure and an honour to work with my colleagues in the Department of Mathematical Statistics during the time that I have been here.

I am indebted to the Council for Scientific and Industrial Research and the University of Cape Town for scholarships awarded which enabled me to undertake the research necessary for the completion of this thesis.

A.M. Hurwitz
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C O N T E N T S

CHAPTER ONE

THE NATURE OF ECONOMETRIC ANALYSIS

"Nature's action is complex : and nothing is gained in the long run by pretending that it is simple, and trying to describe it in a series of elementary propositions." (Marshall (1947)).

The construction, estimation, and interpretation of Economic models in terms of statistical analysis is fraught with many serious problems and drawbacks. It is the purpose of this chapter to outline some of these areas of difficulties, and is intended to serve as a guide to the interpretation not only of results presented in the following chapters, but also for Econometric analysis in general. It is assumed that the reader is generally familiar with such common terms as "linear model," "endogenous and exogenous variables," and so on.

1.1 PROBLEMS OF STATISTICAL INFERENCE - THE SAMPLE/ POPULATION PARADIGM

There exist two major schools of thought in the interpretation of statistical experiments, namely the "frequentists" and "Bayesian" viewpoints. It is the former that we adhere to in this work, although this should not be taken as reflecting any particular bias.

The prerequisites of the frequentist position are an underlying population that is well-defined and a statistical distribution associated with this population that is also well-defined and which accurately describes its probalistic behaviour. The unknown parameters of this distribution are called the "population parameters." On the basis of a sample that is chosen randomly from the population, inferences are made concerning the form of the population distribution. More specifically, for our type of analysis, a sample realization of a time series is chosen at random from the population of all possible realizations - this population being assumed to have certain distributional properties, for example: stationarity. When we consider, as we usually do, a situation involving a random variable that is continuous, then the number of different random samples that may be drawn from the population of all possible values of the random variable is *a priori* infinite. It is then assumed that with larger and larger samples our inferences concerning the population parameters become more accurate and, in the theoretical limit, converge to the "true", or population, values. This convergence cannot be defined in the same way as the concepts of mathematical convergence can, but is an intuitive construct. Although this approach works well in many real-world applications there are a number of serious obstacles to its correct application in an economic context. This will be discussed below. There are however conceptual difficulties associated with the frequentist argument which lead many statisticians to an alternative view of the meaning of probability namely the Bayesian, or "degree of belief," interpretation (Jeffreys (1922)).

In the view of the Bayesians, probability statements about population characteristics - for example, confidence intervals on parameter values - are to be looked on as reflecting the degree of belief that we are prepared to place in our assertions about population values. The odds that we are prepared to place on, say, a parameter lying in a certain range gives rise to a probability distribution - the "prior distribution" - of that parameter. This prior distribution is then combined with a likelihood function* based on a random sample from the population in containing the parameter question, and this gives rise to a "posterior distribution" on the parameter, which incorporates the additional information derived from the sample with our prior beliefs. The posterior distribution then becomes our prior in any new sampling experiment, and so on. It can be shown (Box and Tiao (1973)) that this procedure has the desirable property of allowing even initially "diffuse" prior distributions to converge to accurate statements concerning the population.

Both frequentist and Bayesian procedures allow us to make more accurate probability statements on the basis of

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*Jeffreys (1922) gives the likelihood function a degree-of-belief interpretation. See, however, Bulmer (1967) for an alternative view.

larger samples. There is, however, no watertight procedure for protecting us against false inferences, although we usually try to minimise this possibility in a "Type I and Type II Error" sense, (Mood and Graybill (1963)). So long as we do not have complete knowledge about the population there is always the danger that we are either rejecting a true hypothesis, or accepting one which is false. Further, so long as our statements about the population distribution are accurate our inferences about it will be, in the "long run" of statistical theory, also accurate. If, however, we have made errors of specification - if we have not allowed for all possibilities - our inferences will be either inconsistent, biased, or totally false.

In the physical sciences, noted for their success at achieving thoroughing consistency and predictive accuracy, all "laws" - even the fundamental ones of conservation of energy and of momentum - are nothing more than statements of degrees of belief about the "real" world that are held with high confidence. This confidence is based on countless observations of these laws operating without fail, and of the constant effort to weed out inconsistencies, and to reconcile apparent contradictions in the theory. Why then has Economic science not been able to apply the "Scientific Method" with any great degree of success in reconciling competing theories?

1.2 PROBLEMS OF ECONOMIC SCIENCE

The general methodology of econometric analysis is as follows: from economic theory, either received or obtained via direct observations as the "real world," a mathematical model is specified in an acceptable form and its equations are given statistical content usually by the inclusion of error terms with specified distributions. The parameters of the model, be they elasticities, marginal propensities, lag coefficients and so on are either unknown or specified a priori to have certain values. Observations on the economic variables appearing in the model, for example: consumption, the interest rate, money supply, etc., are obtained in the form of historical time-series data, and the unknown parameters of the model are estimated by a procedure such as Least-Squares Regression which gives values that are in some sense optimal. Inferences about the parameters may then be made with reference to their estimates and other concomitant statistics, for example their t-values.

The question is: "Where does this procedure, as applied, usually break down?" and the answer unfortunately is: "At almost every step of the way."

In the first place it may be asked: "What is Economic theory?" Is there one theory - the theory - or are there many different theories which may be expected to "converge" eventually into a single theory? Is theory just any possible interaction is of a dynamic nature. This point is made by Culbertson (1968) who points out that this, in itself, is a specification error which leads again to specious "structural" models.

The methodology of statistical hypothesis-testing - that of random sampling from a potentially infinite population is not achieved for two main reasons. Firstly it is impossible to set up controlled experiments (in the nature of the physical sciences) in a social situation. One cannot "stop the clock, turn it back, and then go over the same historical period - this time changing only one variable - and observing the effects on other variables." It is further not permissible to take two separate historical time periods as two random samples and to compare the results however close the time periods may be because it is intrinsic to the nature of a social system that it changes its structure continuously, and hence a model that might be accurate for an earlier period is not valid for the later one. A model that is valid at the start of a period will be an inaccurate reflection of the economic structure at the end of the period. This objection is usually overcome by estimating over relatively short periods when the structure is assumed to be changing only very slowly, or allowed for by dummy variables or other a priori changes in parameters. We are given only one set of historical data from which to make inferences. Secondly it is in the nature of students of economics to go on trying out different time series in their structural models until a "good fit" is achieved. This breaks the rules of the frequentist's random sampling although it might, dubiously, be explained on Bayesian grounds as representing increased prior knowledge about the suitability of different indicators. Instead of changing the model to fit the facts, the data is changed to fit the model.

The procedure whereby stochastic properties are introduced into a structural model by simply adding an error term on to each equation and then, as is usually done, assuming that it is Normally distributed, is not as arbitrary as it appears at first sight. It achieves the same simplification as did Langevin in connection with the Maxwell-Boltzmann diffusion equations (Fuller (1969)). There are certain well-known conceptual difficulties to this procedure in the form of the non-definition of derivatives arising from its use in a differential equation model, and which will be made clearer later in this thesis. As an idealization it is acceptable, and does go some way towards accommodating the effects of excluding variables from a model; the Central Limit Theorem (Mood and Graybill (1963)) makes the assumption of Normality more plausible, but we must be on guard against stretching these positive aspects too far. Certain variables, for example, may only take on positive values, and to describe their variation as random Normal is to make a specification error.

Lastly, there is a tendency in economics to introduce variables that are non-observable, for example, the "autonomous investment" of Keynes or the "expectations" of present day theory. With the use of these variables any economic behaviour may be explained ex post. A theory, or model, containing such variables is a non-theory, or non-model, for the basic reason that it cannot by any logical or empirical means be refuted. It is also true that the use of such variables is practically endemic to any area of applied economics. We must either define a concept quite explicitly or not refer to it at all. When the use of these variables is eliminated we will have come a long way towards the reconciliation of conflicting theories.

1.3 CAUSALITY

It is the purpose of this section to give an indication of some of the operational concepts attached to "causality" within the econometric framework.

> "......., the confrontation of theories raises basic issues of research methodology for which no firmly established answers seem to exist. Perhaps the central question..... is what to be taken as the explanatory variable, the independent variable of the theory. The basic question is, "What caused what?" What are the policy implications of the complex of interrelations among variables of the economic system?" (Culbertson (1968) p 84).

> "A central problem in macroeconomics for more than a century has been to explain why fluctuations occur. The problem can be separated into two parts: the initiating impulse and the adjustment process by which the impulse is transmitted from market to market and from short- to long-run equilibrium. Both topics have been disputed, hotly, at times."
> (Brunner and Metlzer (1976)).

The concept of causality is most fruitful when given a purely technical definition in terms of a mathematical model.

> "....... the term "cause," carefully scrubbed free of any undesirable philosophical adhesions, can perform a useful function and should be retained."
> (Simon (1953)).

Perhaps best known are the attempts by Strotz and Wold: (Wold (1956), (Wold (1960), Strotz (1960) and Strotz and Wold (1960)), to establish the approach of a "recursive system." In the view of these two authors we must distinguish between "descriptive" and "explanatory" models: the former being a "description of given observations as a random drawing from a joint conditional probability distribution," (Strotz and Wold (1960) p 418), and is represented (in the linear case) by

$$Ax' = u' \qquad (1.3.1)$$

where A is a constant matrix, x' is a vector of variables, and u' is a vector of either zeroes or stochastic variables.

Explanatory models, on the other hand, tell us something about the "directions of influence" among the variables. A causal relationship is in essence asymmetrical - if we can control y indirectly by controlling z, then z cannot be simultaneously controlled by controlling y. In probability terms, the probability distribution of y is "causally conditional" on z, but not vice versa.

Consider the system of linear equations

$$Dy' + \Gamma z' = u' \qquad (1.3.2)$$

where y', z', and u' are vectors - y' of endogenous, z' of exogenous, and u' of stochastic variables, D and Γ are matrices with D square. Suppose that D is upper right triangular with unit diagonal; then the system is called "recursive." If we are interested in the "causal effect" of the variable y_g, say, when we have gained control over it by the use of factors other than the z' variables, then we first delete the g^{th} equation (i.e. the one in which y_g appears with coefficient unity) - the remaining coefficients of the model (including those of y_g) remain unchanged. In this sense, according to Strotz and Wold, may we give causal interpretability to the variable y_g: its remaining non-unit coefficients describe the influence of the variable y_g (now exogenous) on the other endogenous variables.

If in (1.3.2) D is not triangular the model is "nonrecursive;" we may then either regard it as merely descriptive and estimate it as such, or we may say that the vector z "causes" the vector y. This does not, however, tell us anything about the interactions between the y variables and we are led into problems of "causal circles" and "bicausality" when we try to consider this problem. In the opinion of Strotz and Wold a nonrecursive system of this nature is in a sense a "reduced form" of an underlying recursive system, and only with reference to this may we give it a proper causal interpretation.

Thus the problem of apparent causal circles or bicausality arising out of the use of equilibrium systems may be seen as a further objection to comparative static analysis: the causal interpretation in an equilibrium model is to be sought for in the underlying dynamic model. (For further details and examples of this last point see Strotz (1960).)

Taking vector causality a step further, we may suppose that D is block triangular and then talk about causal relations between the corresponding subsets of endogenous variables. This idea is similar to that of Simon (1953). Within each subset, however, no causal relations are defined.

Lastly, and of relevance to a later chapter in this work, it is noted that differential equation systems are regarded as recursive with respect of infinitesimal time intervals; in the model

$$\frac{dy'}{dt} = f(y') + u' \qquad (1.3.3)$$

where $y' = y'(t)$ is a vector of (time-dependent) variables, $f(\cdot)$ is a vector function, and u' is as in (1.3.1), y' is assumed to precede dy'/dt. In this sense (1.3.3) can be given a causal interpretation.

For some objections to Wold's view see the discussion following Wold (1956). The implications of assuming that simultaneous equation models are limiting approximations to nonsimultaneous ones as time lags go to zero is discussed in Fischer (1970).

1.4 SUMMARY AND CONCLUSIONS

The above discussion, it is hoped, has amply demonstrated some of the many pitfalls of applied economic analysis. We are faced with the problem of trying to sort out cause and effect within the framework of a mathematical model of an economy. One may ask whether it is not merely consistent covariation that we are observing amongst a set of arbitrarily chosen economic time series, and whether it is at all meaningful in a "real world" sense to attach words such as "causal" to these empirical regularities. It seems clear that there are consistent and stable mechanisms at work within an economic system although Empiricism tells us that we cannot in any way perceive necessary conditions among events.

Mathematics can hardly be expected to clarify issues when our domains of definition are badly set out. If what is hoped for is some sort of "convergence" of our models akin to the convergence of mathematical iteration towards an economic theoretic "solution," then it must be remembered that even a mathematical iterative procedure will usually only approach its solution if it is started in some neighbourhood of that solution. Further, the existence of multiple solutions is always possible - especially if the functions involved are complicated.

Perhaps we should start out with no preconceived theory at all - or the barest minimum - and then treat the economic system as a "black box." We would then not attempt any economic justification of our results, but only try to establish the existence of empirical regularities in sets of data. It would be very tempting, of course, to give such regularities theoretical content - but should not the econometrician avoid any such speculation? The Box-Jenkins (Box and Jenkins (1970)) approach to forecasting time series is one such attempt which has met with fair success. A combination of such methods with those of conventional modelling has been attempted (Prothero and Wallis (1976)), but much work remains to be done.

The "causality-free" approach may be looked on as a type of "reduced form" analysis: The reduced form of equation (1.3.2) is

$$y' = -D^{-1}\Gamma z + D^{-1}u' \tag{1.4.1}$$

It is not possible in this situation to determine any interrelations between the endogenous variables. Such an approach was used by Friedman and Meiselman (1963) in their famous tests on the relative stability of monetary velocity and income multipliers. This has met with much criticism as has subsequent work in their tradition, for example the work of Anderson and Jordan (1968).

> "The second and major criticism is methodological. Should one rely on a single equation that relies on high correlation coefficients as the criterion for a model's predictive capacity? The alternative approach, of course, involves the full specification of the economy's interdependencies. Friedman and Meiselman reply that this is not the name of the game. If we find stable relationships we do not need to know what goes on inside the black box."
> (Johnson (1971))*

Another approach would be to avoid the aggregation biasses of conventional macroeconometric work, and build up macro-models only on a microtheoretic basis. These models would have to be very large however - larger even than the current largest. The drawback to very large models is that they become cumbersome to work with in the sense that they

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* See also the comments by Ando and Modigliani in Stein (1976).

require a team of econometricians to be handled and interpreted successfully, and it is difficult to trace through them the complex feedback effects of changes in some subset of their variables. This is an argument in favour of smaller models, and the work of this thesis is in fact based on the assumption that small, carefully worked out aggregative models are much more useful, informative and tractable than large ones.

> "The mass of allocative detail in large scale econometric models seems to us irrelevant for aggregative analysis. We assert that a small number of explicitly stated allocative patterns have observable aggregative consequences."
> (Brunner and Meltzer (1976)).

The best methodology is thus still very much an open question, and an answer seems hardly in sight. It is up to each individual researcher to choose that procedure he thinks in some way optimal, and perhaps in this way some "convergence" towards a satisfactory approach may be attained.

CHAPTER TWO

THE MONEY SUPPLY PROCESS IN SOUTH AFRICA*

2.1 INTRODUCTION

Recent Monetary Policy in South Africa has been highly inflationary. This is self evident on an inspection of the behaviour of the important monetary aggregates. The rate of growth of money and near money was roughly 23% per annum in 1973 and continued to grow at about 20% per annum to June 1976. Clearly the acceleration in the rate of price increases over the same period has not be coincidental.

It is not, however, the purpose of this paper to explain why rapid increases in the money supply are inflationary or why sharp fluctuations in its rate of growth are destabilising. This is taken for granted and substantial support for these surely uncontroversial propositions can be found in the recent literature. The object of this paper is to investigate closely the money supply process in order to explain how the present inflation in South Africa came about and what therefore we may hope to be able to do about inflation.

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*From: "The Money Supply Process in South Africa: Explanation, Verification, Implication" (A.M. Hurwitz and B. Kantor). (Paper presented at the 7th Kanstanzer Seminar on Monetary Theory and Monetary Policy - June 1976.)

The rate of growth of the money supply has not been regarded by the South African monetary authorities as an appropriate target for monetary policy. Instead, policy attention has been concentrated on a variety of credit market phenomena. Control has been exercised over the level and structure of interest and deposit rates. The authorities have also paid particular attention to the level of excess liquid assets of the commercial banks and have periodically varied the banks required liquid asset ratio. The Reserve Bank has indicated its concern with the supply of bank credit and has imposed and removed ceilings on bank lending to the private sector.*

The objects of monetary policy in South Africa have often been stated as the usual desire for internal and external stability. The acceleration in the rate of increase of prices after 1970 points to an obvious failure to satisfactorily realize at least one of the objectives of policy. It is the contention of this study that the recent South African inflation and the money supply growth that was the proximate cause of it was entirely consistent with external stability until 1975. It will be argued below that it was the monetary authority's primary concern for the balance of payments and their neglect of money supply aggregates that led South Africa to the inflationary impasse of the present time.

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* For details of policy interventions and official explanations of these see the Quarterly Bulletins of the South African Reserve Bank.

The importance attached to balance of payments considerations has long been emphasized in the official reviews of South Africa's monetary policy and policy intentions. Clearly particular regard has been held for the level of foreign exchange reserves as the means of safeguarding the preferred structure of foreign exchange rates. The 'monetary theory of the balance of payments' has recently rediscovered the implications of fixed exchange rates for monetary policy, and in particular the money supply. The theory has revived the traditional understanding of the relationship between the balance of payments and money, and maintains that an economy, particularly a small open economy, that aspires to fixed or controlled exchange rate links to trading and financial partners cannot operate, for anything but the short run, a money supply and interest rate policy that is independent of the balance of payments. In other words, domestic interest rates and domestic money supply growth must be consistent with money supply growth elsewhere, and any attempt to break this dependence of monetary policy on the balance of payments with given exchange rates will soon be aborted by a shortage or abundance of foreign exchange reserves. Variations in exchange of import controls, or indeed even adjustments to the exchange rate do not provide any fundamental correction. At best they may provide more time for domestic monetary policy to become consistent with monetary policy elsewhere.*

*For a convenient collection of recent contributions in this area of research, see Jacob A. Frenkel and Jarry G. Johnson (eds.) "The monetary approach to the Balance of Payments," London, George Allen and Unwin, 1976. See also Stephen P. Magee, "The Empirical Evidence on the Monetary Approach to the Balance of Payments and Exchange Rates," American Economic Review, May 1976, p. 163.

The all-important background to the recent South African inflation is that the United States dollar standard, to which we were linked and which had ultimately "looked after" our money supply, became increasingly suspect in the 1960's, and in 1971 collapsed.* This is not to say that our own inflation was unavoidable. Our inflation was avoidable, but it could not have been avoided without abandoning the previous neglect of the money supply. The consequences of this neglect are revealed with a vengeance in our present rate of inflation. The check on monetary expansion provided by normal balance of payments relationships and fixed exchange rates became inoperative after 1971, and direct control of the money supply should have been put in its place. Had this been done and our own monetary expansion limited to, say, about 10 percent per annum growth on average over the last few years, South African exchange rates would have strengthened substantially against the rest of the world and so have offset part of the effect of higher import prices.

The South African monetary authorities singularly failed to make this response. They carried on their monetary policy as if nothing fundamentally had changed with the result that because of highly favourable balance of payments developments after 1972 the money supply expanded very rapidly. Furthermore, even after the balance of payments ceased to be as

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* For a recent interpretation of U.S. monetary policy in the '60's see Jürg Niehans, "How to Fill an Empty Shell," The American Economic Review, May 1976, p.177.

strongly favourable, increases in the price of gold during 1974 increased the real value of the Reserve Bank's gold reserves. In the absence, therefore, of any effective balance of payments constraint, but with official concern for nominal interest rates, the money supply increased rapidly. It did so in response to extra demands for money and credit required to finance a higher level of real output, output that came increasingly to be valued at higher prices. It is worth noticing in Figure 4 the decline in South African interest rates relative to foreign interest rates, and the persistance of relatively low South African rates in 1972 and 1973.

International monetary conditions changed markedly in 1975. U.S. monetary growth and monetary growth else where slowed down; moreover slower monetary growth was accompanied by the severest recession experienced by the advanced industrial countries in the post war period. However, as previously indicated, monetary growth in South Africa continued at absolutely, and by then, relatively high rates. The implications of relatively fast South African monetary expansion and world wide recession revealed itself in lower prices and demands for South African exports and not least, of course, by a fall in the price of gold.

The rand which had been linked to a weighted basket of currencies was reattached to the dollar in June 1975. Soon after, the balance of payments and the exchange rate came under pressure, and the rand was devalued by 18% against the dollar in September 1975. The devaluation did not improve the balance of payments trends. The explanation for this state of affairs is to be found, as suggested, in the maintenance of relatively rapid monetary growth in South Africa. The excess supplies of money continued to spill over into higher prices, high levels of imports and high rates of domestic credit expansion, thus the need to borrow elsewhere declined, and so imports of capital were correspondingly reduced. In 1976 the capital account of the balance of payments was also affected by greater political uncertainty.

Since mid 1975 the balance of payments has itself had a contractionary influence on money supply and monetary policy. The state of the balance of payments is not, however, fully reflected by the level of gold and foreign exchange reserves held by the Reserve Bank. These have been supported by accommodating foreign borrowing by the Treasury and other public sector borrowers. Unfortunately for the achievement of balance of payments stability, government finance relied increasingly on money creation in 1975. Despite the balance of payments deficits, high rates of monetary growth were perpetuated by the unwillingness of government to avoid inflationary finance. The increase in the central governments demands for domestic credit is revealed in Figure 2, which shown how the earlier errors of monetary omission came to be replaced by classic inflationary commission.

The study provides an analysis of monetary developments in South Africa that differs from the official method.

Some attempt however will be made to reconcile the alternative approaches.

2.2 THE THEORETICAL MECHANISM

An all important monetary relationship is that between the money supply and what is called the money base, or more evocatively the high powered money of the system. The money base consists of the sum of coin, notes and other non-government deposit liabilities (mainly commercial bank deposits) with the Reserve Bank, and this constitutes the cash reserves of the monetary system. The money base-money supply relationship, the money multiplier, depends on a number of factors, for example, the cash reserves banks prefer to hold, and are required to hold, against their deposit liabilities will in part determine this multiplier. The preferences of the public to hold either bank deposits or alternatively Reserve Bank notes also influences the multiplier as do their preferences for current rather than time deposits.* The authorities therefore are not able to control the supply of money directly. The authorities are able to control the money base and therefore the supply of cash reserves available to the banking system. The authorities can also influence directly the banks demand for cash reserves by either altering the compulsory reserve requirements of banks, or by

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* See Karl Brunner and Alan H. Meltzer, "Liquidity Traps for money, Bank Credit and Interest Rates," The Journal of Political Economy January - February 1968.

changing those interests rates that influence the portfolio preference of the banks. These effects will be discussed more fully below.

Changes in the money base occur in what may be usefully defined as two alternative ways: either as a result of changes in the foreign assets (including gold) held by the Reserve Bank, or changes in what may be described as its Net Domestic Asset (NDA) position. However, as will be indicated, changes in the foreign and net domestic assets of the Reserve Bank are not in any way independent of each other.

A simplified balance sheet for the Reserve Bank would have notes and deposits on the liabilities side and foreign assets (including gold) and domestic assets (mostly government securities) on the other side. It would look as follows:

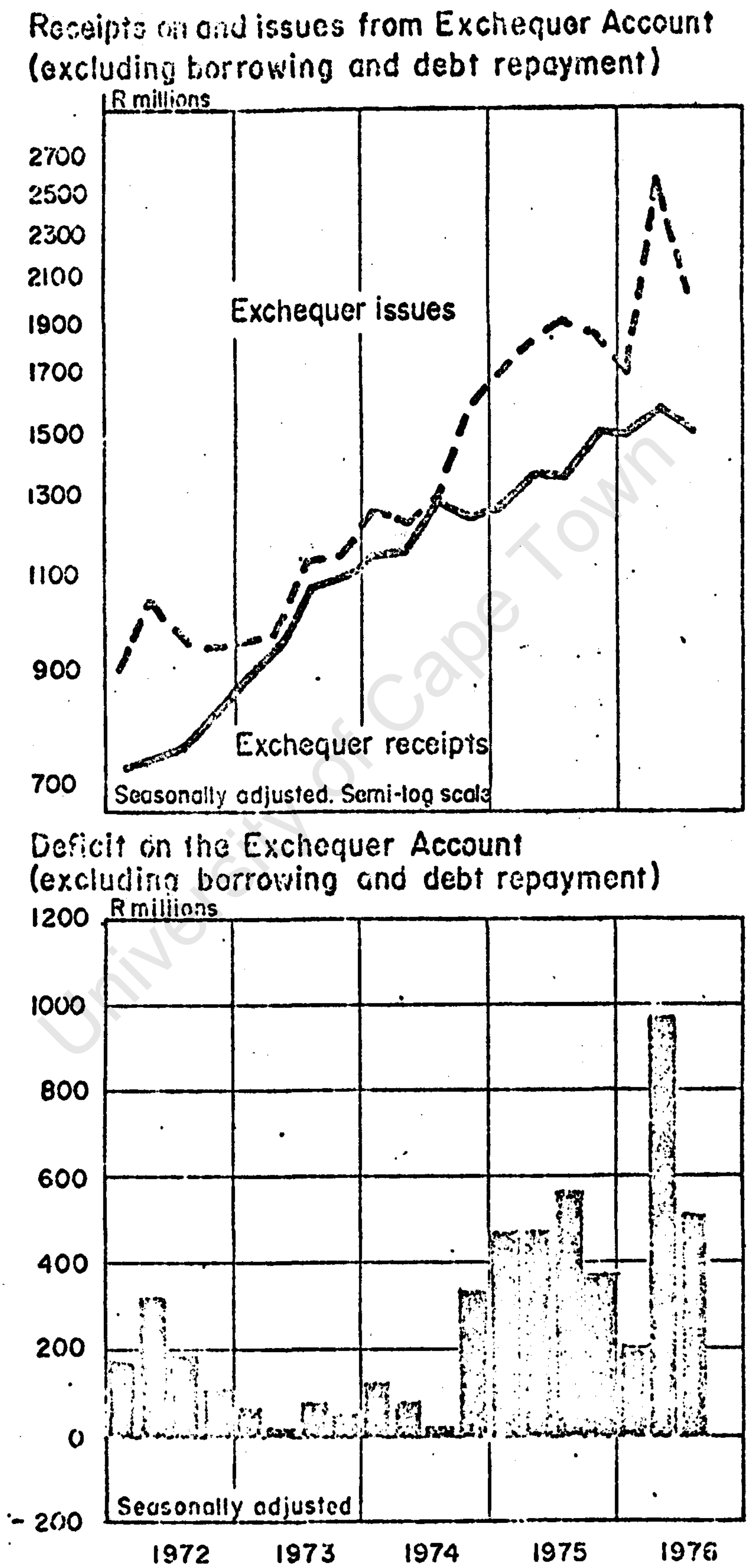
F I G U R E 1

S.A. RESERVE BANK

LIABILITIES	ASSETS
Notes	Gold and Foreign Assets
Bank Deposits Government Deposits	Domestic Assets

The notes and commercial bank and other private deposits constitute the money base. Changes in the money base are caused by seperate and interdependent changes in either foreign assets, domestic assets or government deposits. When the balance of payments is favourable commercial banks will be

FIGURE 2

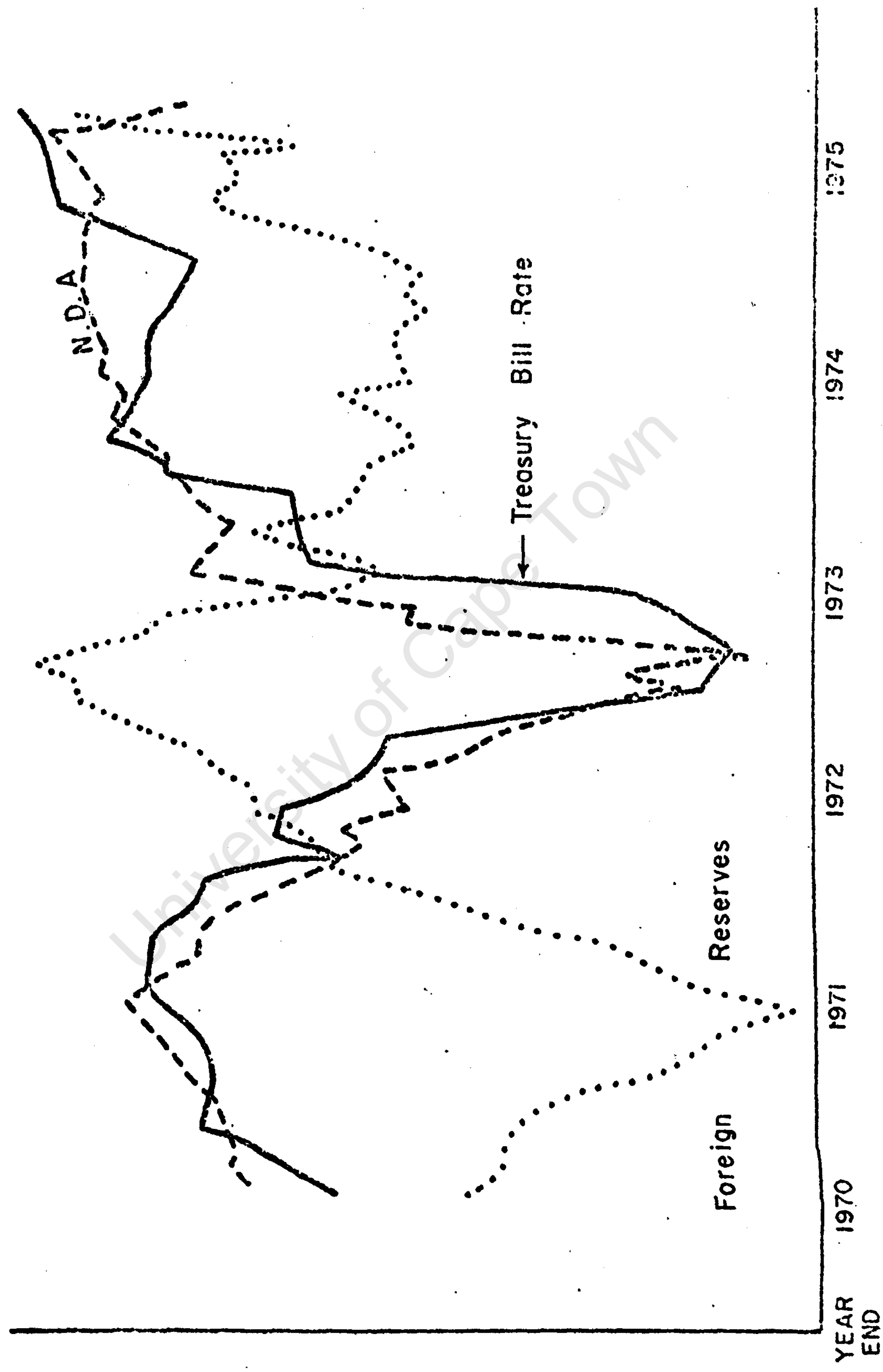


Source: S.A. Reserve Bank Quarterly Bulletin December 1976 selling foreign exchange to the Reserve Bank and their deposit accounts with it will be credited accordingly. The money base will have increased. When the balance of payments is unfavourable the banks will draw on the Reserve Bank for foreign exchange. The money base decreases. When the Reserve Bank buys securities from the public or the banks, the deposit accounts of the commercial banks with the Reserve Bank will be credited. When the Reserve Bank sells securities, commercial bank deposits will decrease. Similarly, when the Treasury receives taxes or the proceeds of loan issues the government balance with the Reserve Bank increases while those of the commercial banks decrease. Accordingly the money base decreases. The opposite results occur when government spends. If the government balance is subtracted from the domestic securities held by the Reserve Bank one gets what is called the Net Domestic Asset position of the Reserve Bank, i.e. Money Base (MB) = Foreign Assets (FA) + Net Domestic Assets (NDA). It has not been thought useful to distinguish between those cash reserves the banks or discount houses acquire from the Reserve Bank (borrowed reserves) and those acquired by running down excess liquid assets or via open market operations.

As suggested previously, when the money base increases the money supply will tend to increase unless the banks prefer to hold excess cash reserves. However, the causation may well run the other way because of the opportunities the discount houses and banks have to borrow from the Reserve Bank. If the money supply expands independently of the money base the banks will require extra cash reserves to meet their required cash reserve ratios. To do so the banks may recall funds from the discount market, and the discount houses may in turn approach the Reserve Bank for additional assistance. If the Reserve Bank supplies additional reserves in this way, or if the banks and discount houses start reducing their holdings of Treasury Bills and other government securities, and by their doing so , redemptions of government debt exceed new debt issues, the government balances at the Reserve Bank will decrease and the money base will increase. In such circumstances, changes in the money supply will have led changes in the money base. Clearly, if the Reserve Bank did not provide extra reserves in the way indicated, or if the Treasury did not on balance redeem debt, or if the Reserve Bank provided additional reserves only at penal rates, the banks would be unable, or unwilling, to finance extra lending to either the private or public sectors. The potential increases in the money supply called for by extra demands for finance from the banks would soon be halted by higher interest rates unless the money base accommodated the money supply.

The relationship between the foreign assets held by the Reserve Bank and its net domestic asset position is an important and interesting one. As will be noted in Figure 3, these two variables generally move in opposite directions. The reason for this negative association is that when the money base increases,because of a change in the reserves, the banks have automatically acquired additional cash reserves.

FIGURE 3


N.D.A.
Treasury Bill Rate
Foreign Reserves
YEAR END
1970
1971
1972
1973
1974
1975

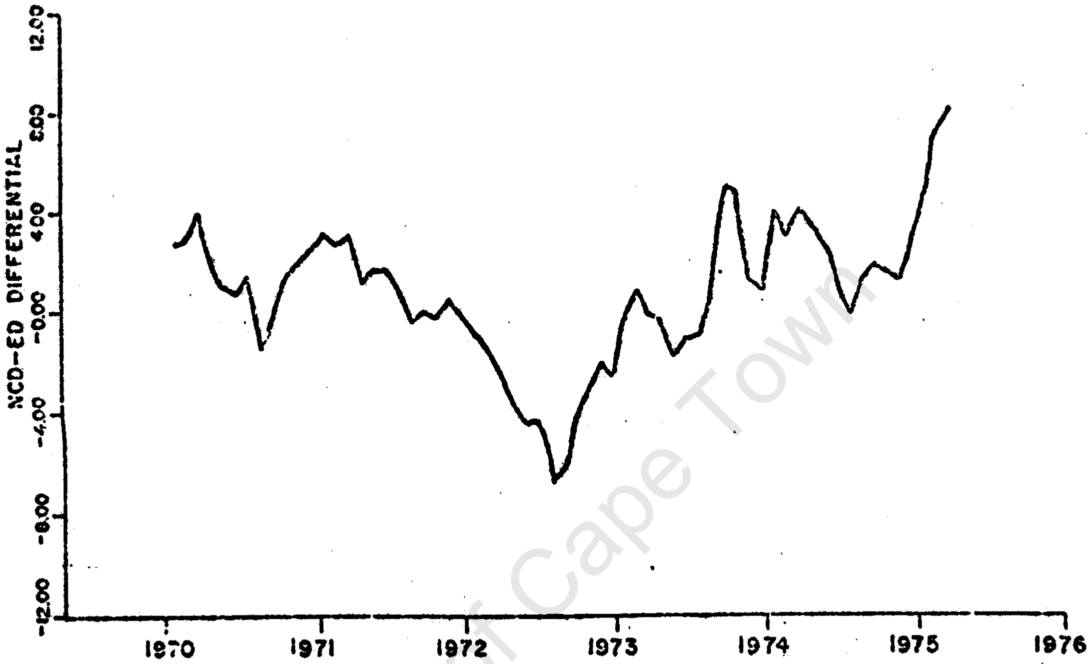
These additional cash reserves will almost immediately be placed with the discount houses and used to buy other liquid assets. The result is a net flow of funds to the Treasury or an equivalent repayment of debt to the Reserve Bank with the effect that the net domestic assets of the Reserve Bank decrease. At the same time, however, there may be competing demands from the private sector for the extra cash the banks have acquired through the balance of payments. If the demands for extra loans by the private sector are buoyant less funds will flow back towards the Reserve Bank and Treasury, and the money supply and the money base will tend to increase. The decrease in net domestic assets will then be less than the increase in reserves.

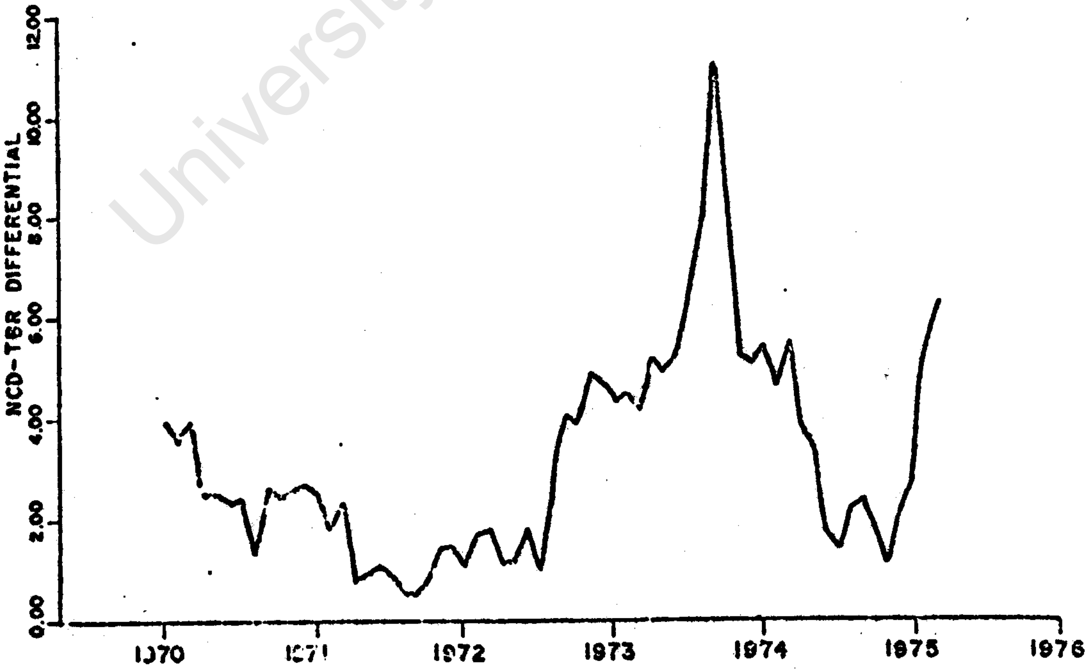
If the balance of payments turns into deficit obviously opposite effects will occur. The banks will be selling foreign exchange on balance, their cash reserves will be declining, and they will compensate for this by drawing on the discount houses and by running down their other liquid assets. The immediate pressure will be met by the Treasury and the Reserve Bank, and net domestic assets will automatically increase. Again the degree of pressure on the net domestic assets position will depend on the simultaneous buoyancy of demand for bank credit. The stronger the demand for bank overdrafts the higher the level of market interest rates and the greater the tendency for net domestic assets to expand. Again the increase in NDA may be smaller or greater than the decrease in reserves.

These are some of the automatic effects. On the other hand the impact of changes in reserves on net domestic assets will also depend, as intimated, on the relationship between the costs to the banks of acquiring or holding additional liquid reserves, and the returns to be gained from other kinds of bank lending. The costs to the banks of holding liquid reserves or acquiring additional reserves is reflected in the rate of interest obtained on liquid assets or implicitly by the rate charged by the Reserve Bank for accommodation. If the borrowing from the Reserve Bank is undertaken by the discount houses after the banks have recalled funds from them, this will be reflected in the discount houses call rate. The greater the differences between the cost of and the returns from additional bank lending to the private sector the faster, it may be assumed, will tend to be the rate of expansion of the money base.

These differences in interest rates will be maintained if the interest rates on liquid assets, controlled by the authorities, do not respond to changes in the conditions effecting market demands for and supplies of funds. In an upswing phase of the business cycle, associated as it will be with additional demands for credit, any tendency for these rates to lag behind will cause the money base to expand procyclically. Notice in Figure 4 how the Treasury Bill Rate came to lag behind the market determined, negotiable certificate of deposit rate in 1972 and 1973. In the downswing phase, as market rates approximate official rates more

FIGURE 4


NCD-ED DIFFERENTIAL
12.00
8.00
4.00
-0.00
-4.00
-8.00
-12.00
1970
1971
1972
1973
1974
1975
1976


NCD-TBR DIFFERENTIAL
12.00
10.00
8.00
6.00
4.00
2.00
0.00
1970
1971
1972
1973
1974
1975
1976

closely, the tendency to repay the Reserve Bank and to switch towards the Treasury will tend, again pro-cyclically, to accelerate the decline in the money base. The striking negative association in Figure 3 between the level of foreign reserves and the Treasury Bill Rate should be noted.

We have so far ignored the effects of interest rates themselves on the balance of payments. Clearly the tendency for South African firms and public corporations and even the Treasury itself to borrow abroad will depend on their planned expenditure, and also on the comparison they will make between the anticipated costs and availability of funds locally, and the costs and availability of funds abroad. If local borrowing is difficult and considered expensive after, of course, allowances for expected exchange rate movements, more will tend to be borrowed aborad and vice versa.

It is now possible to summarise the relationships between the balance of payments, the net domestic asset position, the money base and the relationships between interest rates, local, controlled and free market rates, and foreign rates. This will be done by way of an account of a stylised business-money supply cycle.

Let us begin the cycle where the balance of payments is favourable. Let us assume that this is because the relationship between foreign and local rates is such that, with increased local investments demands, larger than usual capital inflows have been attracted. The increase in reserves will be offset by opposite reductions in net domestic assets. However as indicated if the increase in reserves is also associated with an economic upswing, the decline in net domestic assets will be less than the increase in reserves and the money base will expand. The expansionary pressure on the money base will be still stronger if the opportunity cost of extra private bank lending falls because, perhaps, government rates lag behind market rates. Increases in reserves, and less-than-proportionate decreases in net domestic assets, and hence expansions of the money base could continue indefinitely with simultaneous increases in the money supply. This, however, would depend in turn on the concurrent balance of payments developments. One of the factors affecting the balance of payments will be the relationship between local and foreign interest rates. With the expansion of the money supply, and unless the demand for funds continued to increase proportionately (or more than proportionately), local interest rates would tend to fall. Depending then further on the trend in credit markets abroad, any relative decline in local interest rates will influence the capital account of the balance of payments. If conditions unattractive to foreign borrowing persist the balance of payments will sooner or later turn around.

Let us assume at a further stage in the cycle that the balance of payments then becomes unfavourable. As the foreign reserves decline the net domestic assets of the Reserve Bank increase and the ultimate effect on the money base will then depend on the inverse relationship between them. The net effects on the money base therefore will come to depend on the demand for funds in the economy and the opportunity costs to the banks of lending to the private sector. If the demands for funds remain bouyant and the official interest rate structure remains unchanged, net domestic assets will expand relatively quickly, local interest rates will remain low, and no adjustments to the changed balance of payments situation by way of reductions in the money supply will have been effected.

If the balance of payments deficit continues, sooner or later, the authorities, out of their concern for fixed exchange rates, will take action. They may tighten exchange and import controls. If such measures prove inadequate they would be forced to adjust their interest rate structure. Official interest rates will then be increased. Consequently local free market rates will tend to rise relative to foreign rates. If official rates rise relative to other local interest rates and local rates rise relative to foreign rates, these adjustments will serve to take the pressure off, firstly, the net domestic assets position, thereafter the money supply, and subsequently the balance of payments.

In this way, out of concern for the balance of payments and by way of adjustments made to interest rates, the money supply will have "looked after itself." A slower increase in money will directly help correct the balance of payments by way of lower demands for imports and increased demands for foreign capital. The money supply process described above implies considerable instability in the behaviour of the monetary aggregates, but it does not suggest any one way direction for their rate of growth.

The discussion has so far emphasized the interdependent effects of the balance of payments, economic activity and interest rates on the money base. It has not discussed the possible impact on net domestic assets of government expenditure, or more particularly, the difference between government expenditures and revenues and therefore the government borrowing requirement. Inflationary government borrowing may be defined as borrowing that has the effect of increasing the money base. As suggested above the obvious method for government to avoid inflationary financing techniques is by offering a relatively attractive return on issues of government securities. This has the effect of transferring the use of a greater proportion of a flow of savings from the private sector to the public sectors. However, higher nominal interest rates, if required, are not politically neutral. It is this that accounts for the authorities revealed preference for a structure of interest rates, and this may encourage resort to increases in the money base as a method of government finance. In addition, however, such concern may also help constrain government expenditure and hence borrowing requirements at an earlier stage in the budgetary process.

The South African authorities have employed a third non-inflationary alternative to both taxation and the payment of relatively attractive returns as methods of finance. They have been able to rely in part on the so called "captive market" for government securities. This technique is of course by no means peculair to South Africa. All financial intermediaries are obliged to hold minimum proportions of government and other special approved securities in their portfolios with the effect of guaranteeing a minimum demand for government securities independently of the rate of return offered. By calling for increased minimum holdings of liquid assets the authorities may avoid increases in the money base as a method of financing government expenditure.

Our monetary analysis has given comparatively little attention to the banks liquid asset ratio. The monetary authorities however, as has been mentioned, do pay particular attention to the liquid asset ratios of the banks of which the cash ratio is a part. We will now attempt to show how the effect of changes in required liquid asset ratios may be easily reconciled with the money base analysis.

Any extra demands for liquid assets by the banking system, whether induced voluntarily, or acquired under compulsion of increased liquid asset ratios, have the same effect on the net domestic asset position of the Reserve Bank, namely, an increase in the exchequer balance with the Reserve Bank and hence a decline in net domestic assets. The same results would follow from any net increased demands for government securities of whatever kind.

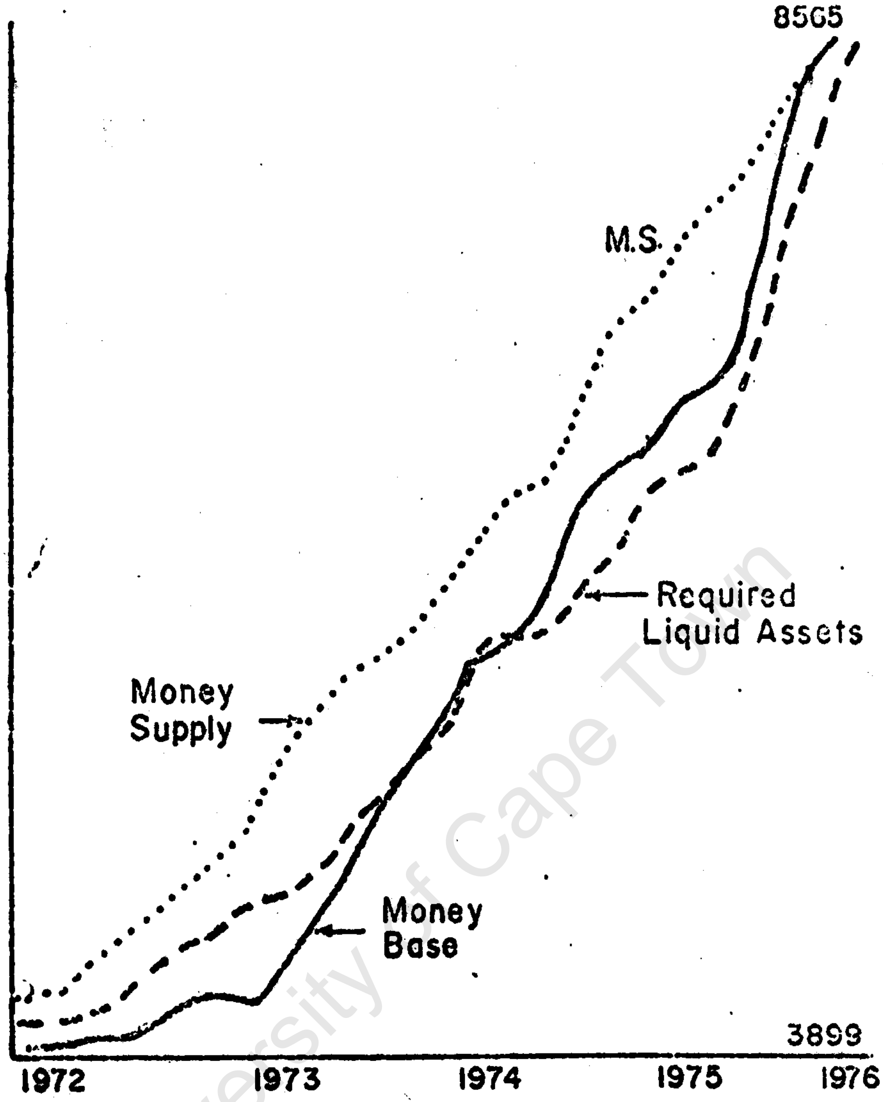
It is, however, very important to make the distinction, a distinction that is seldom made in the South African monetary context, between actual, required and desired liquid asset holdings. It should be noted that any required holdings of liquid assets are not really liquid in the proper sense of that term. Since the banks are required to hold them they cannot be sold and used for any other purpose. The banks are therefore locked into their required liquid asset holdings. If the bank wishes to preserve some portfolio flexibility it would have to hold genuinely liquid assets in excess of the required asset holding. The actual liquid asset holdings at any moment in time are usually in excess of required liquid assets and may be greater or less than desired liquid asset holdings. The banks portfolio preferences will depend on their assessment of the future demand for and supply of funds, and the opportunity costs of alternative asset and liability combinations.

One very good reason for holding excess liquid assets would be in anticipation of a call for extra required holdings of them. The alternative to keeping extra liquid assets for such a contingency would be a willingness to compete in the money market for surplus cash when the need arises. This competition for deposits may prove particularly expensive and is probably an option open only to the smaller banks who would not necessarily by so doing increase interest rates against themselves.

It seems clear that calls for extra holdings of liquid assets often follow extra bank holdings of them. If so, the authorities decision to raise the ratio may have no direct implications for the money base or the banks and the money supply. The developments that follow an increase in required liquid asset holdings may merely be a reclassification of some liquid assets from excess to required. Desired liquid assets and therefore total bank demands for government securities may be unaffected. If so, there would be no inflow of funds to the Treasury, and therefore no decrease in the money base. Figure 5 indicates a close association between the money base and required reserves. This would appear to confirm that increases in required reserves have not been an independent instrument used to control the money supply.

If the call for extra required asset holdings takes the banks by surprise then the banks may thereafter wish to rebuild their desired holdings of excess liquid assets to preserve a degree of portfolio flexibility. If so, there will be a net inflow of funds to the Treasury and a reduction in the money base. Furthermore, if the banks actually find themselves short of required liquid asset holdings then the net inflow of funds to the Treasury will be accompanied by more competition for cash and higher short term interest rates. In particular the rate on Negotiable Certificates of Deposits (NCD's) will rise.

FIGURE 5



3 Month moving average,

NOT TO SAME SCALE

Money supply R3899 in 1972
" " R8565 " March 1976

One important part of any explanation of the banks demand for excess liquid assets would be their consequent ability to satisfy extra demands for overdrafts. If such demands are met automatically the banks' liquid asset holding decline and the Treasury balance decreases. The money base accordingly increases. It is such a possibility, inherent in banks holdings of excess liquid asset reserves, that so excite the concern of the authorities. Nevertheless the authorities cannot prevent the banks holding such reserves. What the authorities are able to do is to make such excess holdings more attractive, either by periodically calling for extra required liquid assets or, less directly, by raising the rate of interest paid on government securities, including Treasury Bills. In addition the Reserve Bank may make its own borrowing and rediscounting facilities more costly so as to discourage reliance upon them. It should be understood therefore that excess liquid assets by no means necessarily imply inflationary increases in the money supply. It is only any excess of actual over desired liquid assets that have this implication. The banks excess liquid asset position indicates the degree of excess demand for bank credit. When excess liquid assets fall, demands for credit are abnormally high.

The role that needs to be played by appropriate interest rates to discourage any revealed preference for private sector rather than government sector lending is obvious. We have explained that it has usually been pressure from the balance of payments that has promoted a changed structure of interest rates with consequent effects for the money base and the money supply.

The discussion presented above clearly departs from the method of monetary analysis provided by the South African Reserve Bank. The Reserve Bank does not offer an account of the money base and its determination or of the money base - money supply relationship.

The official analysis takes the form of an aggregation of the balance sheets of the entire banking system including the Reserve Bank and attempts to draw conclusions from this. The net change in money and near-money over any period, that is the increase in short and medium term banking sector liabilities are described as having been caused by changes in different categories of banking assets or by changes in the composition of banking liabilities. For example "caused" by changes in the banks holdings of foreign assets or claims on the private sector or government sector or by a switch out of or into long term deposits which are not classified as money or near money.

This type of outcome is true by definition of the balance sheet identity, however, the weakness of the analysis is that it implies that the various components of the banking balance sheets are independent of each other. This, as we have tried to indicate in our alternative approach, is by

CHAPTER THREE

THE MONEY SUPPLY PROCESS IN SOUTH AFRICA : EMPIRICAL ANALYSIS*

3.1 INTRODUCTION

Following the extensive discussion of the previous chapter, a formal mathematical model is developed and estimated using the data given in Appendix C.

It must be emphasised that the perspective this model assumes is that of the monetary authorities, and we attempt to explain the policy reactions of the monetary authorities to developments outside their direct control. Briefly, the money base is examined with regard to its two components namely, the foreign and net domestic assets of the Reserve Bank. Further, the model endogenises officially controlled interest rates, and indicates the association between money base and money supply.

3.2 THE MODEL

The model consists of four behavioural equations and one identity.

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*from Part II of "The Money Supply Process in South Africa : Explanation, Verification, Implication" (A.M. Hurwitz and B. Kantor (1976)).

$$MB = R + NDA \tag{1}$$

$$R = f_1[(i-i^e), R_{-1}] \tag{2}$$

$$NDA = f_2[R\ gd, (i-i^t)] \tag{3}$$

$$i^t = f_3[R, i, (i-i^t)] \tag{4}$$

$$MS = f_4[i, i^t, Y].\ MB \tag{5}$$

where

MB = Money Base

R = Foreign Reserves of the Central Bank (including gold)

R_{-1} = Reserves at the end of the previous period

NDA = Net Domestic Assets

i = South African free market interest rates

i^e = foreign interest rates

gd = government deficit

i^t = South African target or controlled interest rates

MS = Money Supply

Y = the level of economic activity

These equations represent the general form of the model. After experimentation with alternative forms the specification below incorporating some simple one-period lags was adopted. The data used to estimate the model was very short term monthly data, which is consistent with the likely speed of the authorities' reactions.

The respective interest rates included in the model are meant to reflect relevant economic behaviour. The interest rates included are either differences or levels or, as in the case of the controlled interest rate equation, both levels and differences. The explanation for the inclusion of the foreign-domestic interest rate differential in the Reserve equation is of course for its effects on capital movements. The free market government interest differential in the NDA equation is included to indicate the relative interest rates upon which portfolios will be divided between government and private sector financial securities. Financial intermediaries will clearly not be concerned when choosing the composition of their assets with the level of interest rates. The authorities however are judged to concern themselves with both the level and the differences between interest rates. The money base - money supply multiplier is in turn assumed to be affected by the average return on the banks portfolio and therefore the level of interest rates is included as an explanation of it.

The Foreign Reserve equation is justified by the perspective of the model. The authorities of course do know the level of Reserves and may be concerned with effecting changes in that level, and the previous level of foreign reserves is accordingly included as an exogenous variable. The policy instrument at the disposal of the authorities is their influence over the domestic level of interest rates. The Reserve equation argues that the level of Reserves is positively influenced by differences between national and foreign interest rates, and it implicitly assumes a Koyck,*

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*See Appendix A

i.e. geometrically declining, lag in the $(i-i^e)$ variable. Further, linearity in the functional form is obtained via the assumption of constant elasticities. (More will be said on this point in the following section.) This specification is equivalent to the form

$$R = f\ [(i-i^e,\ (i-i^e)_{-1},\ (i-i^e)_{-2},\ldots]$$

The Net Domestic Asset position of the Reserve Bank is assumed, following the extensive discussion in Chapter Two, to be negatively related to foreign Reserves and directly influenced by changes in the government deficit. NDA is expected to increase with increases in the returns available on non government financial securities, that is with increases in the free market - official interest rate differential. Net domestic assets are, of course, presumed to increase with the governments borrowing requirement. In the equations we tested two measures of the government deficit, (gd). The first measure was simply that of the difference between government expenditures and taxation receipts. This proved to have no statistically significant effect on NDA. The other measure of the deficit attempted to allow for government borrowing from the financial intemediaries and the public. The difference between government spending, taxation and borrowing being presumed to require increases in NDA. The results for this measure of gd were statistically significant and so were included in the simultaneous equation system estimated below. It is important therefore to emphasize that we found no statistically significant relationship between NDA and the crude government deficit (see equation B3, Section 3.8)

The "target" or government controlled interest rates are assumed to vary inversely with the level of Reserves, positively with the level of free market rates, and negatively with the free market-controlled interest rate differential. We found a consistent and statistically significant negative relationship between the target interest rate and the target-free market interest rate differential, which perhaps requires some further explanation. There are two possible explanations for an increase in this differential: the differential may increase either because the private non government demands for finance have increased or because the supply of funds to satisfy private demands have been reduced. The latter would appear to be the correct explanation. An increase in the differential appears to reflect an unexpectedly large flow of funds to the Treasury and a consequent shortage of cash reserves for the banking system. This larger flow seems to encourage the Treasury to reduce the pattern of official rates. Similarly a reduction in the differential may reflect an unexpectedly large outflow from the Treasury balances and may therefore require increases in the official rates to restore the position.

The money supply equation is a modification of the standard money multiplier equations.

$$MS = K \cdot MB$$

where K is the "money multiplier."
In this specification, evolved by Brunner and Meltzer (1968), K is considered functionally related to interest rates, reserve requirement ratios, economic activity, etc. Our own investigations led us to adopt the simplified form $K = f(i,Y)$ which, as will be noticed, gives a highly satisfactory explanation. The final specification was

$$MS = f(i,Y,MB)$$

including MB in the argument for simplicity.

Normal assumptions would be for the money supply to increase with increases in the money base and in the level of economic activity and prices. Increases in prices and real output can be expected to lead to increases in demands for bank credit and to cause a reduction in the banks demand for cash reserves and an increase in the multiplier. It might be usually presumed that increases in interest rates - and so the average return on the banks portfolio - would also increase the money multiplier. In this statistical analysis we found however that increases in interest rates had the effect of reducing the supply of money. The explanation for this may be that the mechanism just described tends to be a longer-run phenomenon. In the short run an increased cost to the banks of borrowed funds may have a contractionary effect on the money supply as the banks, faced with penal rates, cut back on lending. In the longer term however, the desire by the authorities for low rates will lead to accomodation via an expansion of the money base, as outlined in Chapter Two, and money supply will increase as expected. Further analysis using quarterly data* gave the expected sign, and our above explanation therefore seems correct in the light of the evidence.

If banks avoided extra cash reserves, as seems to be the case in South Africa, one would not expect any direct effect of incomes on the money base-money supply multiplier. The direct impact of extra demands for funds would again be directly on the money base via net domestic assets. The relationship between income and the money supply should not therefore be looked on as causal but rather as one of descriptive association.

3.3 THE REGRESSIONS : RESULTS AND ANALYSIS

In deriving the ultimate structure of our model, in particular with respect to the lag structure, a large number of alternative regression specifications using different types of data were tried out. Some of these results are presented here, but most have been excluded on account of the lack of space available.** We have not given space to detailed discussions concerning the summary statistics (for example the Durbin-Watson statistic) or the regression techniques (for example, Two-Stage Least-Squares, etc.) used in

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* See Chapter Five

** More details, however, are abailable on request.

the present chapter. These are conveniently collected and presented in Appendix A together with further references for the interested reader.

We give now, however, an important note concerning the implication of the assumption of constant elasticities for the functional form of an economic relationship. This assumption is widely used (see, for example, Brunner and Meltzer (1968)), but is rarely explicitly set out.

Without loss of generality assume that an economic variable Z is functionally related to two other variables X and Y, i.e.

$$Z = f(X,Y)$$

Define the "x elasticity of z" to be

$$\epsilon(Z,X) = \frac{\partial Z}{\partial X} \cdot \frac{X}{Z} \qquad \text{(Chiang (1974))}$$

and similarly for the Y elasticity of Z.

If we further assume that both elasticities are constant, i.e.

$$\epsilon(Z,X) = K_1 \ , \qquad \epsilon(Z,Y) = K_2 \ ,$$

then we have arrived at two partial differential equations reducible to ordinary differential equations (Utz (1967) pp. 139-140). Hence, integrating the first equation as if Y were constant:

$$\int \frac{dZ}{Z} = K_1 \int \frac{dX}{X}$$

i.e. $\log_e Z = K_1 \log_e X + A(Y)$

where A is an arbitrary function. Similarly:

$$\log_e Z = K_2 \log_e Y + B(X) ,$$

and clearly:

$$B(X) = K_1 \log_e X + \log_e K$$

(where K is a constant of integration).

Thus: $Z = KX^{\epsilon(Z,X)} \; Y^{\epsilon(Z,Y)}$

The above derivation justifies our use of log-linear form (to base e) throughout this chapter. Variables with negative entries were adjusted by adding a constant positive factor making each series positive, e.g. interest rate differentials and government deficit. Multiple linear regression* was then undertaken on each equation using the University of Cape Town's Univac 1106 system and the Econometric Package "COMET."

Notes:

(1) "Short Term (S.T.) Data" in the regression refers to the interest-rates i, i^e and i^t, and are respectively Negotiable Certificates of Deposit (NCD), Eurodollar and Treasury Bill rates. Electricity Consumption (E) was used as an index for "economic activity" in all S.T. regressions. The S.T. time-period was either 1971 January to 1974 December (n = 50), or 1971 February to 1974 December (n = 49).**

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* See Appendix A

** See Appendix C

(2) "Long term (L.T.) Data" likewise refers to interest rates i, i^e and i^t in this case Company Debenture (COYDEB), Long-Term British Government Bond (LTUK) and Long-Term South African Government Bond (LTSA) Rates. Gross Domestic Product (GDP) at market prices (Y) was used in place of ELEC in the L.T. regressions.**

(3) t-Statistics are shown in brackets beneath each of the coefficients; (asymptotic t's in the case of Two-Stage Regressions). (In each regression the t-statistics have n-1 degrees of freedom.)*

(4) Other summary statistics are given for each regression,* viz.

R^2 = Coefficient of Determination

D.W. = Durbin-Watson Statistic

S.E. = Standard Error of the estimate

n = number of observation points used.

The following four regression equations show the Ordinary Least Squares (OLS) results for the four behavioural equations of the model. (Using S.T. data and with the t subscript omitted.)

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* see Appendix A

** see Appendix C

$$R = -0{,}35575669 + 0{,}0574984 \,.\, (i-i^e)_{-1} + 1{,}0366261 \,.\, R_{-1} \qquad \text{(1A)}$$
$$\quad [-0{,}992365] \qquad [2{,}0259682] \qquad [21{,}624329]$$

$(R^2 = 0{,}9447;\ SE = 0{,}0688;\ DW = 1{,}1546;\ n = 49)$

$$NDA = 13{,}3612878 - 1{,}1623271 \,.\, R + 0{,}086642 \,.\, gd_{-1} + 0{,}25270618 \,.\, (i-i^t) \qquad \text{(1B)}$$
$$\quad [17{,}1163604] \qquad [-10{,}88906] \qquad [2{,}518322] \qquad [6{,}2155237]$$

$(R^2 = 0{,}8291086;\ SE = 0{,}202233;\ DW = 0{,}37229;\ n = 49)$

$$i^t = 1{,}7565759 - 0{,}30801646 \,.\, R_{-1} + 1{,}0390595 \,.\, i_{-1} - 0{,}3010272 \,.\, (i-i^t)_{-1} \qquad \text{(1C)}$$
$$\quad [4{,}234038] \qquad [-6{,}3945106] \qquad [13{,}832981] \qquad [-10{,}280309]$$

$(R^2 = 0{,}93218;\ SE = 0{,}073001;\ DW = 1{,}40327;\ n = 49)$

$$MS = 3{,}2407805 + 0{,}61003587 \,.\, MB + 0{,}0059708 \,.\, Y_{-1} - 0{,}0798299 \,.\, i \qquad \text{(1D)}$$
$$\quad [9{,}57977] \qquad [10{,}049036] \qquad [18{,}934063] \qquad [-6{,}511457]$$

$(R^2 = 0{,}9912739;\ SE = 0{,}01862;\ DW = 1{,}70133;\ n = 49)$

Two-Stage Least Squares (TSLS) regressions (Appendix A) were carried out on the NDA and MS equations. This type of regression takes into account the fact that endogenous variables appear as regressors, viz. MB in the case of MS, and R in the NDA equation; it can be shown that the MS and NDA equations are identified (Appendix B).

$$NDA = \underset{[16{,}45679]}{13{,}2117310} \quad \underset{[-10{,}38956]}{-1{,}141860} \cdot R + \underset{[2{,}56269]}{0{,}0891296} \cdot gd_{-1}$$
$$+ \underset{[6{,}186178]}{0{,}2538543} \cdot (i-i^{t}) \qquad (1E)$$

(R^2 = 0,82997; SE = 0,2039497; DW = 0,3835; n = 49)

$$MS = \underset{[2{,}642775]}{2{,}2656250} + \underset{[5{,}0689005]}{0{,}7841798} \cdot MB$$
$$+ \underset{[6{,}73765]}{0{,}00512695} \cdot y_{-1} \underset{[-4{,}34995]}{-0{,}1040039} \cdot i \qquad (1F)$$

(R^2 = 0,986824; SE = 0,02278 DW = 1,18158; n = 49)

The one-sided 95% significance point for the t-statistic with 40 degrees of freedom is 1,684 (or -1,684 for a left-sided test). (For a two-sided test it is 2,021). It can be seen that all the above coefficients are significant, with the exception of the constant in the R equation. High R^2's throughout indicate a high degree of explanation in each equation and the Standard Errors are generally very small.

Considering the Durbin-Watson Statistics (Appendix A) we find that, (for n = 50, at the 95% significance level), the following significance points apply:

k = 2		k = 3		k = 4	
dL	du	dL	du	dL	du
1,46	1,63	1,42	1,67	1,38	1,72

where k = number of explanatory variables

dL = lower significance bound

du = upper significance bound

This indicates that equations (1A, 1B, 1C, 1E and 1F) show significant positive first-order autocorrelation, and that for equation (1D) the DW statistic lies in the non-significant region. However, for equation 1A, it should be noted that the Koyck lag used invalidates the strict use of the DW statistic). Application of the TSLS method does not alter appreciably the overall OLS results. An analysis of the autocorrelation present in the equations is undertaken below.

In arriving at the above equations we experimented with many different specifications. We started off using only the long term (LT) data, later changing to the short term (ST) data and arriving at the generally short term lag structure shown above.

Graphs of the actual and fitted values are provided below. The fitted equations are respectively 1A, 1E, 1C and 1F; (a fitted line is indicated by small circles).

We also ran the LT Data through the model for the "Short Period" (1971 to 1974). These results are shown below. In doing this we are ignoring the Short-Term lag structure and, as will be seen, the regression of coefficients do not compare very well, although a high degree of explanation is again achieved. All signs remain unchanged except for the constant of an i^t equation. Equations 2A, 2B, 2C and 2D are OLS regression results; 2E, 2F are TSLS results.

$$R = -0{,}073298 + 1{,}0021368 \cdot R_{-1} \quad (2A)$$
$$[-0{,}2684] \qquad [25{,}772]$$
$$(R^2 = 0{,}94412; \; SE = 0{,}06914; \; DW = 1{,}18657; \; n = 49)$$

$$NDA = 11{,}24142647 - 0{,}868292 \cdot R + 0{,}1181616 \cdot gd_{-1}$$
$$[9{,}89922] \qquad [-5{,}52509] \qquad [2{,}921543]$$
$$+ 0{,}4137846 \cdot (i - i^t) \quad (2B)$$
$$[3{,}591799]$$
$$(R^2 = 0{,}75445; \; SE = 0{,}24241; \; DW = 0{,}37218; \; n = 49)$$

$$i^t = -0{,}334616 - 0{,}008706 \cdot R_{-1} + 1{,}293709 \cdot i_{-1}$$
$$[-2{,}11702] \qquad [-0{,}517161] \qquad [16{,}0936]$$
$$-0{,}18585 \cdot (i - i^t)_{-1} \quad (2C)$$
$$[-9{,}045244]$$
$$(R^2 = 0{,}87146; \; SE = 0{,}02414; \; DW = 1{,}0899; \; n = 49)$$

$$MS = 7{,}1749206 + 0{,}106142 \,.\, MB + 0{,}000609 \,.\, Y_{-1}$$
$$[17{,}36724] \qquad [1{,}47526] \qquad [21{,}3117]$$
$$-0{,}113109 \,.\, i \qquad (2D)$$
$$[-2{,}84849]$$

(R^2 = 0,989163; SE = 0,02075; DW = 0,83196; n = 50)

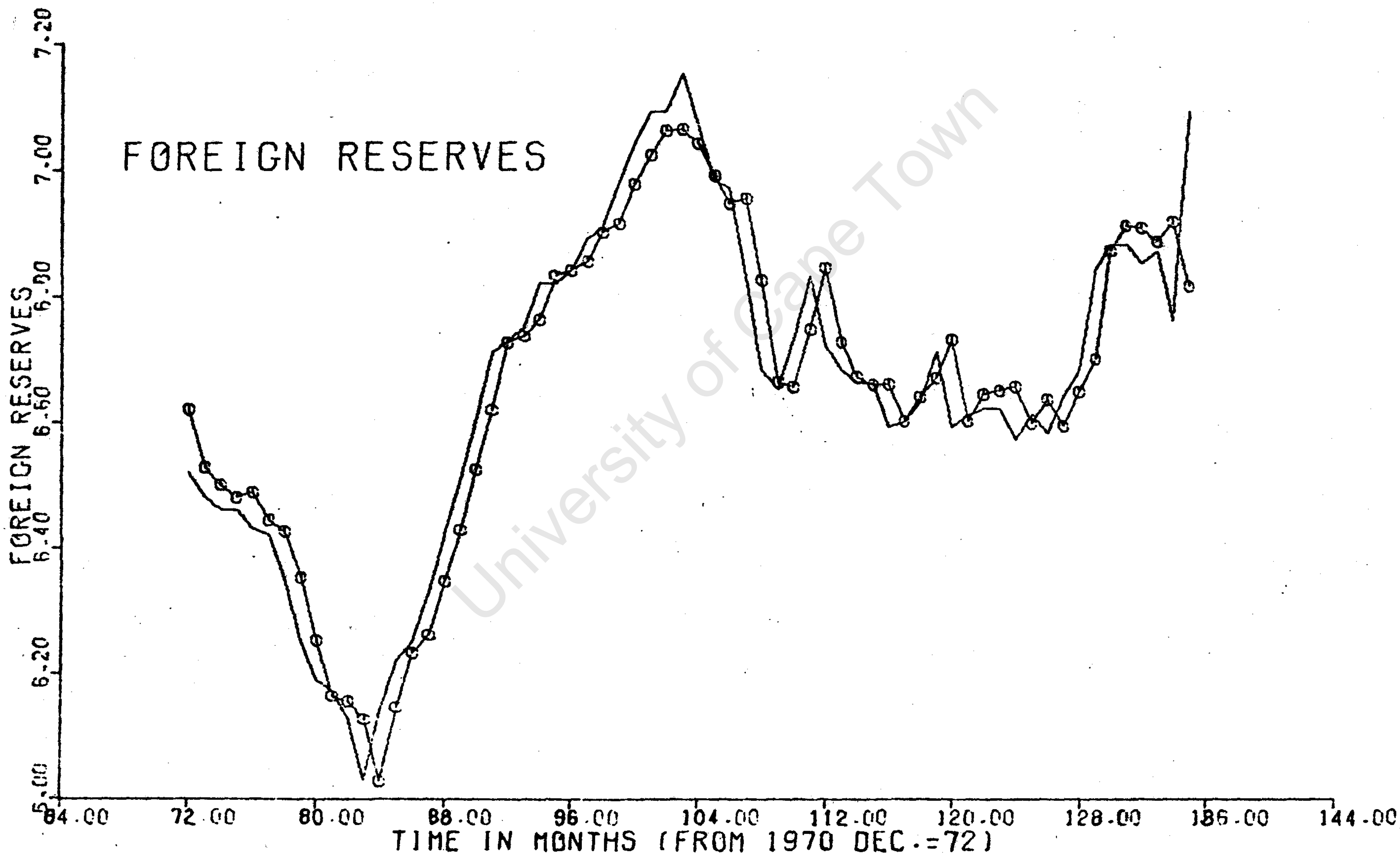
$$MS = 6{,}72265 + 0{,}18505 \,.\, MB + 0{,}000357 \,.\, Y_{-1}$$
$$[11{,}223] \qquad [1{,}7540] \qquad [8{,}750]$$
$$-0{,}13403 \,.\, i \qquad (2E)$$
$$[-2{,}86498]$$

(R^2 = 0,98794; SE = 0,021789; DW = 0,7345; n = 49)

$$NDA = 10{,}70227 \quad 10{,}79292 \,.\, R + 0{,}12208 \,.\, gd_{-1}$$
$$[9{,}00356] \qquad [-4{,}81520] \qquad [2{,}9863]$$
$$+ 0{,}451217 \,.\, (i-i^t) \qquad (2F)$$
$$[3{,}81052]$$

(R^2 = 0,75547; SE = 0,24458; DW = 0,40146; n = 49)

As an alternative specification the model was tested with the Bank's Acceptance Rate (BA) included as the free market interest rate in place of the NCD rate. The reason for including the NCD rather than the BA rate was that it was undoubtedly a market determined rate and are not directly influenced by captive market considerations. This is not true to the same extent of Bankers' acceptances which have been eligible for inclusion as liquid assets in required liquid asset holdings. The NCD rate represents in effect the marginal cost of bank lending. The Banks compete for negotiable


FOREIGN RESERVES
FOREIGN RESERVES
7.20
7.00
6.80
6.60
6.40
6.20
6.00
64.00
72.00
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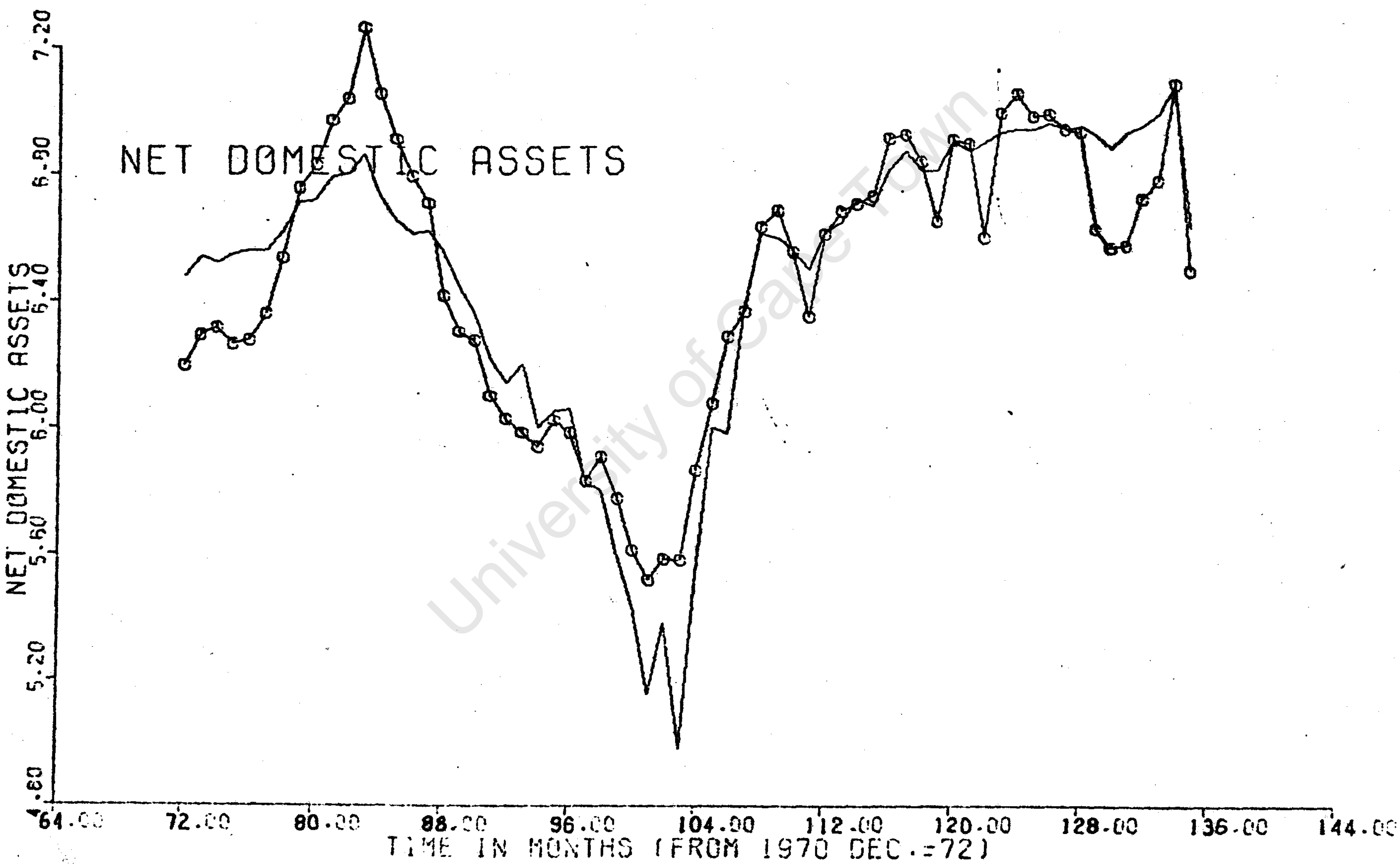
TIME IN MONTHS (FROM 1970 DEC.=72)


NET DOMESTIC ASSETS
NET DOMESTIC ASSETS
7.20
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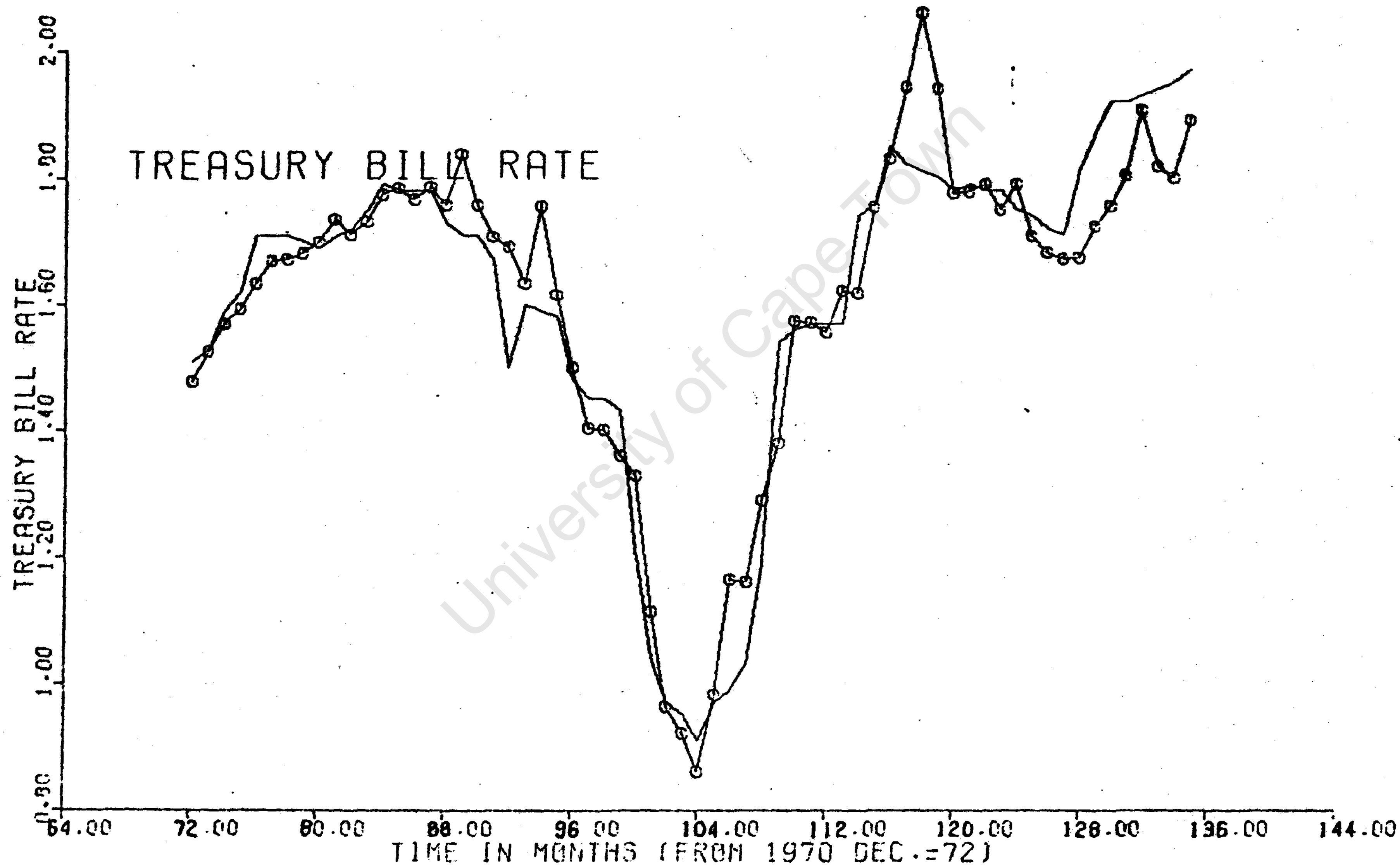
TIME IN MONTHS (FROM 1970 DEC.=72)



TREASURY BILL RATE
TREASURY BILL RATE
2.00
1.80
1.60
1.40
1.20
1.00
0.80
64.00
72.00
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104.00
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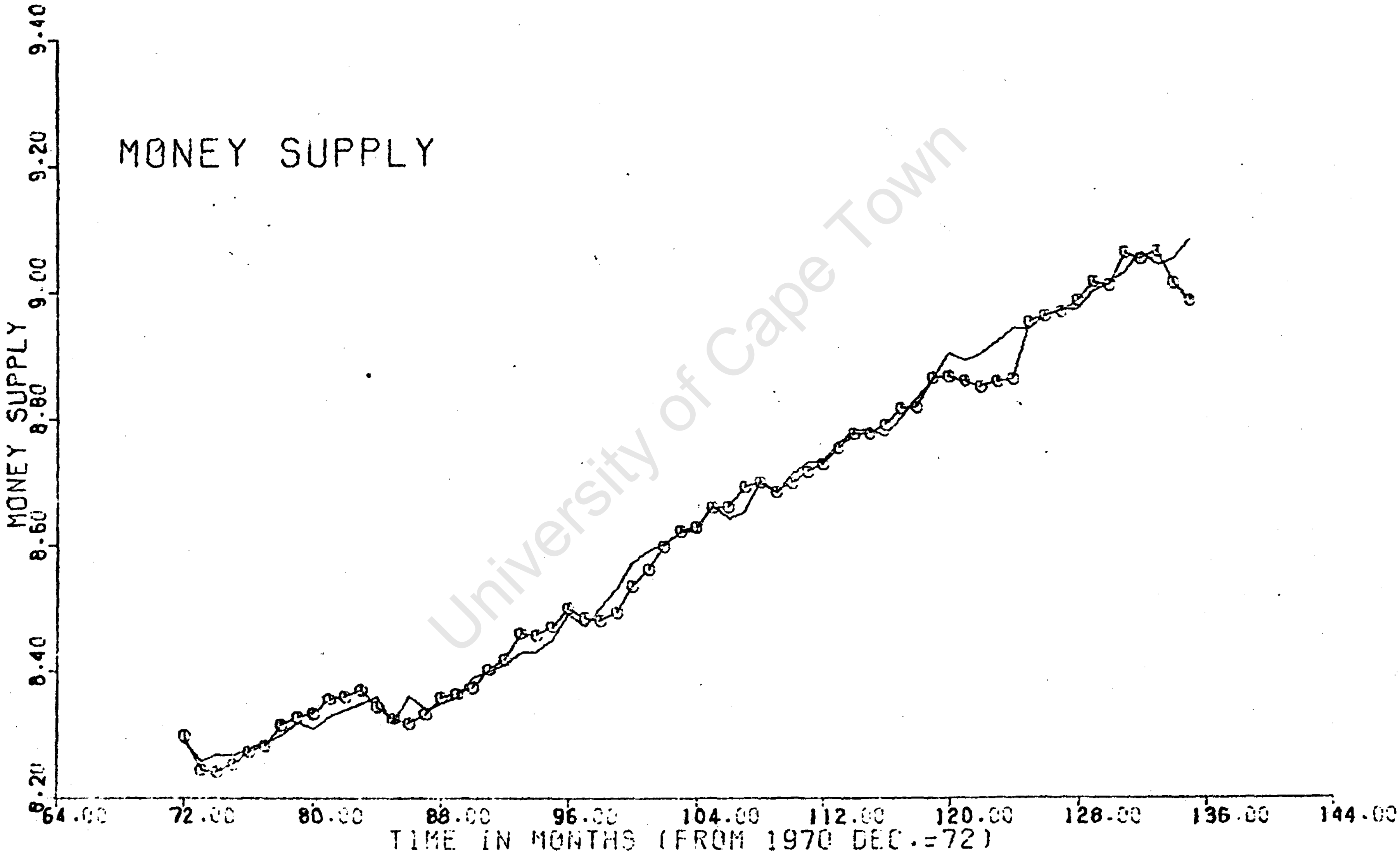
TIME IN MONTHS (FROM 1970 DEC.=72)


MONEY SUPPLY
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104.00
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144.00
TIME IN MONTHS (FROM 1970 DEC.=72)

deposits to match inflow. The BA rate is more obviously representative of the banks marginal revenue. It was thought that the inclusion of a marginal revenue rather than a marginal cost proxy might have changed the signs of the interest rate coefficients in the interest rate and money supply equations. The results are given below for the period from October 1971.

$$R = 0{,}254 + \underset{[17{,}889]}{0{,}950} \cdot R_{-1} + \underset{[1{,}906]}{0{,}0497} \cdot (i-i^{e})_{-1} \qquad (3A)$$

$$NDA = 13{,}587 - \underset{[-6{,}672]}{1{,}204} \cdot R + \underset{[2{,}549]}{0{,}119} \cdot gd_{-1} + \underset{[4{,}841]}{0{,}302} \cdot (i-i^{t}) \qquad (3B)$$

$$i^{t} = 1{,}628 \underset{[-3{,}927]}{-0{,}283} \cdot R_{-1} + \underset{[12{,}175]}{1{,}013} \cdot i_{-1} \underset{[-7{,}517]}{-0{,}229} \cdot (i-i^{t})_{-1} \qquad (3C)$$

$$MS = 3{,}861 + \underset{[6{,}503]}{0{,}507} \cdot MB + \underset{[10{,}226]}{0{,}0062} \cdot E_{-1} \underset{[-4{,}577]}{-0{,}0592} \cdot i_{-1} \qquad (3D)$$

It would seem that generally the use of the NCD rate gives the better results. There are no remarkable changes in the t statistics, all signs remain unchanged as do most coefficients.

3.4 STRUCTURAL TESTS ON THE MODEL

An additional 9 months of data readings have become available since the inception of research into our model, and it was of interest to determine whether or not the model, which seems to provide a consistent explanation of past events, would apply equally to the new series. In other words - are the model equations structurally stable throughout the whole period.

The appropriate test in this case is the "Chow" test for structural stability (Appendix A). In view of the limited extent of the new data, and the consequent loss of degrees of freedom when estimating over the new period, it was thought best to treat the test as if the degrees of freedom were inadequate and proceed with the modification of the Chow test in this type of situation.

The equations were first re-estimated for the whole period (i.e. including the new data under the assumption of first-order autocorrelation using a Cochrane-Orcutt Iterative procedure (see the section below on Autocorrelation Analysis); then the regression coefficients were re-estimated for the original data period - i.e. up to December 1974 - again under a first-order scheme (except in the case of the NDA equation), and the Chow tests were carried out as explained above. The four equations were then again estimated but this time forcing the autocorrelation coefficient to remain the same as for the "old" data.

The results were as follows:

(F = F statistic associated with the Chow test.)

1. <u>Money Supply</u>: $F_{9,42}$ = 0,992126 (non-forced)

   $F_{9,42}$ = 1,00364 (forced)

   (Significance level (5%) of F(9,42) = 2,86.)

2. <u>Treasury Bill Rate</u>: $F_{9,41}$ = 0,06231 (non-forced)

   $F_{9,41}$ = 2,31675 (forced)

   (Significance level (5%) of F(9,41) = 2,87.)

3. <u>Reserves</u>: $F_{9,43}$ = 1,048102 (non-forced)

   $F_{9,43}$ = 1,79043 (forced)

   (Significance level (5%) of F(9,43) = 2,85.)

4. <u>Net Domestic Assets</u>: $F_{9,40}$ = 36,823373 (forced)

   (Significance level (5%) of F(9,40) = 2,88.)

It proved impossible to re-estimate the NDA equation using the Cochrane-Orcutt tecniques. This was because the autocorrelation coefficient became very high (higher even than that previously) on the basis of the new data resulting in a nearly singular data matrix.

It is clear, however, that apart from the NDA equation all the other equations show no significant structural shift even when forced through a uniform autocorrelation scheme. This augurs well for their predictive abilities.

The NDA equation must however be treated with care, for, although none of the regression coefficients changed sign in the forced regression :
($NDA = 12{,}9508 - 1{,}1225 \cdot R + 0{,}1228 \cdot gd_{-1} + 0{,}2559 \cdot ((i-i^t))$),
the autocorrelation structure indicates lack of robustness for the NDA specification.

3.5 FURTHER INVESTIGATION OF THE MONEY BASE - MONEY SUPPLY RELATIONSHIP

The MS equations so far considered took into account MB only in the current period. It is conceivable that MB has a lagged effect on MS; in other words we may have:
$MS = f(i, Y_{-1}, MB, MB_{-1}, MB_{-2}, \ldots, MB_{-t})$ for some t.

In order to investigate more fully the extent of the lags, we undertook an Almon lag analysis. This type of lag is assumed to be approximated by a polynomial of degree s ($s \leq t$), and the lag weights are derived under this assumption (Appendix A).

Eight different (t,s) specifications were tried, viz. (2,2), (3,2), (3,3), (4,3), (5,3), (5,4), (6,3) and (6,4). The results are given in tabular form overleaf. Each regression has its t-statistic printed beneath. (R here =

t	s	Y_{-1}	i	MB	MB-1	MB-2	MB-3	MB-4	MB-5	MB-6	CONST.	R^2	S.E.	D.W	D.F
2	2	1.2082 12.886	-0.086 -5.645	0.476 4.886	0.066 0.597	0.168 1.536					-2.628 -11.959	0.9896	0.0189	1.92	38
3	2	1.1398 16.217	-0.112 -9.243	0.422 6.193	-0.017 -0.329	-0.008 -0.155	0.448 5.850				-3.133 -16.868	0.994	0.0140	1.77	38
3	3	1.140 15.862	-0.113 -9.028	0.427 5.772	-0.032 -0.379	0.007 0.087	0.444 5.468				-3.138 -16.497	0.994	0.0142	1.76	37
4	3	1.136 14.917	-0.113 -8.098	0.448 5.997	-0.098 -1.372	0.115 2.217	0.383 5.122	0.0016 0.0189			-3.133 -13.695	0.994	0.0147	1.86	37
5	3	1.122 12.872	-0.113 -6.877	0.400 4.733	0.0515 0.796	0.056 1.002	0.188 3.204	0.225 3.237	-0.058 -0.640		-3.154 -10.834	0.992	0.0168	2.21	37
5	4	1.125 13.981	-0.116 -7.613	0.450 5.620	-0.114 -1.347	0.166 2.553	0.297 4.405	0.059 0.672	0.010 0.120		-3.199 -11.899	0.993	0.016	2.08	36
6	3	1.128 12.307	-0.110 -6.114	0.358 4.042	0.138 2.685	0.076 1.311	0.099 2.344	0.134 2.244	0.106 1.952	-0.059 -0.635	-3.119 -8.998	0.991	0.018	2.13	37
6	4	1.121 13.436	-0.117 -7.077	0.432 5.090	-0.056 -0.699	0.108 2.019	0.268 3.978	0.160 2.908	-0.086 -1.066	0.051 0.553	-3.233 -10.186	0.992	0.016	2.15	36

R^2 adjusted for degrees of freedom (DF) : see Appendix A.)

On examining the table we note first that in all specifications a high R^2 and low SE are achieved. The Durbin-Watson statistics seem to indicate non-significant autocorrelation except perhaps for the 2nd and 3rd equations.

It should be noted that the regression coefficient of Y_{-1} increases (above that for the specifications not involving lagged MB, as given in a previous section). The t-statistic nevertheless remains high, and the sign does not change; the sign and significance of i likewise. The sign of the constant term does change.

Looking again at the MB variables it seems quite clear that MB in the current period is consistently significant; for MB_{-1} and MB_{-2} it is either not or just significant, with an inconsistent sign pattern. MB_{-3} emerges as significant throughout, but with the size of the regression coefficient altering. MB_{-4} has significance in equations 5, 7 and 8 only. Thereafter the size and significance of lagged MB tails off.

These results point to some interesting, if tentative, conclusions. Firstly MB has an immediate effect of MS, and this effect is strongly positive. MS is also positively effected by MB in the previous quarter (i.e. MB_{-3}, MB_{-4}).

Furthermore, the sum of the MB_{-3} and MB_{-4} coefficients exhibits a fair degree of consistency (lying between 0,3 and 0,5); if we sum the t-values for MB_{-3} and MB_{-4} we get a similar result.

It appears, therefore, that apart from the immediate effect of MB or MS, MB has a lagged effect on MS that is spread over the quarter previously.

3.6 AUTOCORRELATION ANALYSIS*

The OLS and TSLS results (equations 1A to 1F) have indicated significant (or indeterminate) autocorrelation with respect to the MS, R, TBR and NDA specifications. It is the purpose of this section to examine some possible models of autocorrelation and to try and eliminate these effects from the regressions. (The effect of autocorrelation is to give inefficient, though unbiased, estimates of the coefficients.)

Using the computer package AUTO (Appendix D) we first assumed a "first-order" scheme for the residuals (i.e. assumed that, for the regression equation $Y_t = BX_t + u_t$, the u_t followed: $u_t = \rho u_{t-1} + \varepsilon_t$, where ρ = (constant) autocorrelation coefficient, and ε_t was distributed as a Normal $(0,\sigma_\varepsilon^2)$ random variable. A Cochrane-Orcutt iterative technique (employed by AUTO) was then used to estimate the coefficients and the results are given for MS, R, TBR

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*See Appendix A for further technical details.

and NDA; ($\bar{R}^2 = R^2$ adjusted for degrees of freedom; $\hat{\rho}$ = estimate of ρ used in the regression; n = degrees of freedom).

Considering the MS result we note that, under the first order scheme (with $\hat{\rho} = 0,18$) the DW statistic indicates absence of residual autocorrelation. By comparing the (original) OLS regression we note that the constant term is changed (in sign and magnitude), although still highly significant; the coefficients (signs, magnitudes and significance levels) of i and of MB remain largely unaltered, whereas the size of the Y_{-1} increases substantially (from 0,00596 to 1,2127). When we compare this with the Y_{-1} coefficients given as part of the Almon Lag analysis (previous section) it is immediately obvious that 1,2127 seems the better estimate of the two. It also suggests that it is the lagged MB effects that are producing whatever autocorrelation is present in the OLS results. A similar argument applies to the constant term.

In the R equation we again have a DW statistic lying in the non-significant region, with an estimated ρ of 0,447. The constant term, (comparing with the OLS results), has become small and non-significant. The R_{-1} coefficient has altered slightly and so has that of $(i-i^e)_{-1}$ but, whereas R_{-1} remains highly significant, $(i-i^e)-1$ has a t-statistic that lies slightly below the 5% significance level.

No	R	gd_{-1}	$(i-i^t)$	CONST.	$\bar{R}^2$	S.E.	D.W.	n'	ρ_1	ρ_2
1	-1.1852 -10.484	0.0834 2.279	0.2574 6.017	13.5351 16.405	0.8467	0.193	0.432	42	0.10	0.30
2	-1.1941 -10.343	0.0743 2.231	0.2546 5.921	13.6475 16.455	0.8709	0.177	0.507	42	0.20	0.30
3	-1.2035 -10.1146	0.0662 2.2128	0.2495 5.768	13.760 16.343	0.8923	0.162	0.606	42	0.30	0.30
4	-1.2136 -9.766	0.0592 2.236	0.2410 5.540	13.874 15.995	0.9107	0.147	0.739	42	0.40	0.30
5	-1.2255 -9.269	0.0542 2.3171	0.228 5.219	13.996 15.339	0.926	0.134	0.914	42	0.50	0.30
6	-1.2415 -8.610	0.0507 2.466	0.2098 4.801	14.1419 14.336	0.939	0.122	1.137	42	0.60	0.30
7	-1.3042 -6.981	0.0476 2.939	0.1617 3.789	14.6424 11.559	0.955	0.105	1.743	42	0.80	0.30
8	-1.3523 -6.351	0.0472 3.165	0.1406 3.364	15.0355 10.374	0.958	0.101	2.065	42	0.90	0.30

The original (OLS) regression on NDA gave a DW statistic (of 0,3722) that indicated a high degree of (positive) autocorrelation in the residuals. It is therefore not surprising to find a very high estimated ρ (of 0,933). The DW statistic now lies in the non-significant (negative) autocorrelation region, and we have an improvement in R^2 and in the SE. (R^2 for the OLS result was 0,9044.)

$$MS = -2,2927 + 1,2127 \,.\, E_{-1} - 0,072 \,.\, i + 0,651 \,.\, MB$$
$$[-11,405] \quad [15,6605] \quad [-4,922] \quad [9,534] \qquad (4A)$$
$$(\bar{R}^2 = 0,9902;\ SE = 0,0191;\ DW = 1,959;\ n' = 45;\ \hat{\rho} = 0,18)$$

$$R = -0,082 + 0,9948 \,.\, R_{-1} + 0,0601 \,.\, (i-i^e)_{-1}$$
$$[-0,166] \quad [14,6417] \quad [1,723] \qquad (4B)$$
$$(\bar{R}^2 = 0,9538;\ SE = 0,0622;\ DW = 1,875;\ n' = 45;\ \hat{\rho} = 0,447)$$

$$NDA = +15,1361 - 1,3607 \,.\, R + 0,047 \,.\, gd_{-1} + 0,136 \,.\, (i-i^t)$$
$$[10,232] \quad [16,299] \quad [3,255] \quad [3,319] \qquad (4C)$$
$$(\bar{R}^2 = 0,958;\ SE = 0,0995;\ DW = 2,1454;\ n' = 43;\ \hat{\rho} = 0,933)$$

$$i^t = 2,1212 - 0,3407 \,.\, R_{-1} + 0,9562 \,.\, i_{-1} - 0,279.(i-i^t)_{-1}$$
$$[3,6266] \quad [-4,9652] \quad [9,1749] \quad [-6,968] \qquad (4D)$$
$$(\bar{R}^2 = 0,935;\ SE = 0,0701;\ DW = 1,9505;\ n' = 44;\ \hat{\rho} = 0,356)$$

It is remarkable therefore that the regression coefficients show no change in sign and, although the coefficients and t-values alter, the changes are not as drastic as might have been expected.

Because of the high estimated ρ and because the NDA specification has been the most problematic with regard to autocorrelation, it was decided to make further investigations into the nature of the residual correlation. The AUTO package provides a facility for specifying a "second-order" scheme (i.e. $u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \varepsilon_t$) with the use of a "scanning" technique, viz. a range of ρ_1 and ρ_2 values are specified, and a Generalised Least Squares (GLS) regression is performed for each (ρ_1, ρ_2) combination.

To begin with we assume that ρ_1 is positive; this is suggested by the "first-order" analysis. ρ_1 was varied between 0,1 and 0,9 in steps of 0,1 and ρ_2 likewise. The results for $\rho_2 = 0,3$ are given below; (all other ρ_2 combinations were closely similar).

It is immediately obvious from the table that as ρ_1 increases a steady improvement in all associated statistics (R^2, SE, DW) results. (Varying ρ_2, with $\rho_2 > 0$ hardly changed this picture.) Specification No. 8 is clearly optimal within the table, although there is not much to choose between it and the "first-order" result given above; (the DW statistic is marginally improved). All coefficients and their associated significance levels are in broad agreement, and therefore, if a choice is to be made, it seems better to accept the "first-order" result on grounds of its simplicity.

An identical 2nd order analysis was also done, this time using negative ρ_2's; the above conclusions were not altered at all.

Finally, the TBR (i^t) coefficients do not differ substantially from their OLS counterparts, nor do the t-statistics. The DW statistic is now in the non-significant region with an estimated first-order autocorrelation coefficient of 0,356.

3.7 MULTICOLLINEARITY

A feature of concern is the evidence of possible multicollinearity in equations 1C, 1D and 1F, as evidenced by the following correlations:

(i) between i_{-1} and $(i-i^t)_{-1}$ = 0,906

(ii) between MB and Y_{-1} = 0,935

It is possible that this collinearity between explanatory variables is producing unstable estimates (see Appendix A). With this in mind the coefficients of equations 1C and 1D were re-estimated, this time using a technique known as Explicit Ridge Regression (Hemmerle (1975)) that eliminates the effects of multicollinearity. A computer program developed to undertake this type of estimation was applied,* and the results were as follows:

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*Explicit Ridge Estimation Program written by Patrick Wong Fung, Department of Mathematical Statistics, University of Cape Town (1976), (as part of an M.Sc. thesis project).

$$i^t = 0,4031687 - 0,1648 \,.\, R_{-1} + 1,28744 \,.\, i_{-1}$$
$$-0,3864 \,.\, (i-i^t)_{-1}$$

$$MS = 0,4960655 + 0,4484 \,.\, MB + 0,6926 \,.\, Y_{-1}$$
$$-0,03237 \,.\, i$$

It is immediately obvious that the coefficients do not differ materially from the OLS coefficients; (t-statistics were not available). It is not clear whether the TSLS coefficients of 1F are being adversely affected by multicollinearity, but because the R^2, DW and SE statistics are more favourable in the case of 1D than in 1F it was decided at this stage to accept the OLS results as being more accurate.

3.8 OTHER REGRESSION RESULTS

A number of other regression results, obtained during our research, that were thought of interest are presented here, but without any descriptive analysis. We leave it to the reader to decide what, if any, conclusions may be drawn from them.

3.8.1 FOREIGN RESERVES (R)

$$R = 4,75562 + \underset{[0,9]}{1,98006 \,.\, Y_{-2}} + \underset{[0,47]}{1,07 \,.\, Y_{-3}} \underset{[-0,44]}{-1,04 \,.\, Y_{-4}}$$

$$\underset{[-0,58]}{-1,27 \,.\, Y_{-5}} \underset{[-2,43]}{-1,04 \,.\, (i-i^e)_{-1}} + \underset{[0,08]}{0,054 \,.\, (i-i^e)_{-2}} \qquad \text{(A1)}$$

$$\underset{[-0,151]}{-0,09 \,.\, (i-i^e)_{-3}} + \underset{[0,20]}{0,133 \,.\, (i-i^3)_{-4}} + \underset{[0,156]}{0,104 \,.\, (i-i^e)_{-5}}$$

$$+ \underset{[0,223]}{0,12 \,.\, (i-i^e)_{-6}}$$

(LT. Data: $R^2 = 0,439$)

$$R = 1{,}782 + \underset{[3{,}7]}{1{,}009} \cdot E \quad \underset{[-6{,}01]}{-0{,}3} \cdot (i-i^e) \qquad \text{(A2)}$$

(ST. Data: $R^2 = 0{,}5962$)

$$R = \underset{[0{,}051]}{+0{,}031} + \underset{[0{,}44]}{0{,}012} \cdot (i-i^e) + \underset{[0{,}06]}{0{,}008} \cdot E$$

$$+ \underset{[15{,}751]}{0{,}985} \cdot R_{-1} \qquad \text{(A3)}$$

ST. Data: $R^2 = 0{,}9357$)

$$R = -0{,}1186 \quad \underset{[-1{,}65]}{-0{,}065} \cdot (i-i^e) + \underset{[2{,}635]}{0{,}1002} \cdot (i-i^e)_{-1}$$

$$+ \underset{[20{,}04]}{1{,}007} \cdot R_{-1} \qquad \text{(A4)}$$

ST. Data: $R^2 = 0{,}947$)

3.8.2 NET DOMESTIC ASSETS (NDA)

$$NDA = 0{,}4442 + \underset{[6{,}608]}{1{,}8909} \cdot i^t + \underset{[0{,}4582]}{0{,}044} \cdot gd$$

$$+ \underset{[4{,}71384]}{0{,}4601} \cdot gd_{-1} \qquad \text{(B1)}$$

(ST. Data: $R^2 = 0{,}6741$)

$$NDA = 3,757 \quad -1,03494 \,.\, R + 1,35 \,.\, i^t + 0,568 \,.\, (i-i^t)$$
$$[-2,1398] \qquad [3,425] \qquad [3,141]$$
$$-2,964 \,.\, (i-i^t)_{-1} \; -2,373 \,.\, (i-i^t)_{-2} + 0,855 \,.\, E_{-1}$$
$$[-1,373] \qquad [-1,2142] \qquad [0,782] \qquad \text{(B2)}$$
$$+ 0,4593 \,.\, g_{-1}$$

$$NDA = 13,8603 - 1,2008 \,.\, R + 0,0502 \,.\, gd_{-1}$$
$$[-8,009] \qquad [1,0199]$$
$$+ 0,3076 \,.\, (i-i^t) \qquad \text{(B3)}$$
$$[5,2727]$$

(R^2 = 0,6573; SE = 0,2923; DW = 0,22; $\hat{\rho}$ = 0,9329)

3.8.3 THE "CONTROLLED" OR "TARGET" INTEREST RATE (i^t)

$$i^t = 1,7397 \quad -0,306 \,.\, R_{-1} + 0,0007 \,.\, gd_{-1} + 1,039 \,.\, i_{-1}$$
$$[4,035] \quad [-6,269] \qquad [0,053] \qquad [13,8]$$
$$-0,301 \,.\, (i-i^t)_{-1} \qquad \text{(C1)}$$
$$[-10,248]$$

(ST. Data: R^2 = 0,92)

$$i^t = 6,20 + 0,003 \,.\, gd + 0,077 \,.\, (i-i^t) + 0.73 \,.\, R$$
$$[9,9] \quad [0,1] \qquad [2,13] \qquad [1,98]$$
$$-1,457 \,.\, R_{-1} \qquad \text{(C2)}$$
$$[-3,966]$$

(ST. Data: R^2 = 0,64)

$$i^t = 1{,}7 \quad -0{,}3 \;.\; R_{-1} \quad + \quad 1{,}04 \;.\; i_{-1}$$
$$[-6{,}4] \qquad\qquad [13{,}8]$$
$$+ \quad 0{,}02 \;.\; (i-i^t) \quad 0{,}328 \;.\; (i-i^t)_{-1} \qquad \text{(C3)}$$
$$[1{,}01] \qquad\qquad [-8{,}212]$$

(ST. Data: $R^2 = 0{,}933$)

3.8.4 MONEY SUPPLY (MS)

$$MS = -0{,}85476948 \quad + \quad 1{,}370181 \;.\; MB$$
$$[-1{,}527] \qquad\qquad [16{,}736] \qquad\qquad \text{(D1)}$$

($R^2 =$ 0,8506; DW = 0,29236; n = 48; ST. Data)

$$k \left(= \frac{MS}{MB}\right) = -3{,}33306 \quad -1{,}82005 \;.\; i \quad -0{,}06118 \;.\; i^e$$
$$[-11{,}233] \qquad [-2{,}332] \qquad [-1{,}592]$$
$$+ \; 0{,}082638 \;.\; i^t \; + \; 0{,}023625 \;.\; (i-i^t)$$
$$[1{,}326] \qquad\qquad [1{,}008]$$
$$+ \; 0{,}00635 \;.\; (i-i^t)_{-1} - 0{,}0059 \;.\; (i-i^t)_{-2} - 0{,}119 \;.\; (i-i^e)$$
$$[0{,}429] \qquad [-0{,}484] \qquad [-0{,}456] \qquad \text{(D2)}$$
$$-0{,}0103 \;.\; (i-i^e)_{-1} \; -0{,}0009 \;.\; (i-i^e)_{-2} + 2{,}5558 \;.\; E$$
$$[-0{,}425] \qquad [-0{,}049] \qquad [1{,}008]$$
$$+ \; 0{,}75231 \;.\; E_{-1}$$
$$[3{,}001]$$

($R^2 =$ 0,9179; DW = 1,4304; n = 50; ST. Data)

$$MS = -0{,}91588213 + 0{,}2899 \, . \, MB_{-1} + 1{,}0898 \, . \, MB$$
$$[-1{,}567] \qquad [0{,}807] \qquad [3{,}182] \qquad \text{(D3)}$$
$$(R^2 = 0{,}8526; \quad DW = 0{,}18552; \quad n = 49; \quad \text{ST. Data})$$

$$\Delta MS = 54{,}4183 + 1{,}67528 \, . \, \Delta MB + 26{,}8569 \, . \, \Delta(i - i^t)$$
$$[3{,}3135] \qquad [0{,}5964]$$
$$+ \; 2{,}89503 \, . \, \Delta E_{-1} - 55{,}01975 \, . \, \Delta i \qquad \text{(D4)}$$
$$[0{,}5464] \qquad [-1{,}41648]$$
$$(R^2 = 0{,}21717; \quad SE = 88{,}982; \quad DW = 2{,}3905; \quad n = 48)$$

Note: the above regression did not use the logarithms of the variables.

In a preliminary attempt to endogenize the "Free Market" Interest Rate (in this case NCD) we regressed NCD as various lag specifications of "Excess Reserves of Commercial Banks" (as classified in the South African Reserve Bank Quarterly Bulletins).

The results are shown overleaf for three of the specifications. It will be seen that some of the coefficients are significant at the 90% level ($t_{0,9} = -1{,}303$ at 40 d.o.f.). The signs (-) of the coefficients are as expected, although R^2's are not very high and the DW statistic in equation (G3) indicates significant positive autocorrelation.

$$i\ (\equiv NCD) = \underset{[8,404]}{2,4561} \quad \underset{[-1,4910]}{-0,072704} \cdot EXR \qquad (G1)$$

$(R^2 = 0,044;\ SE = 0,317;\ n = 49)$

$$i\ (\equiv NCD) = \underset{[7,794]}{2,7210} \quad \underset{[-0,7779]}{-0,041218} \cdot EXR \quad \underset{[-1,4355]}{-0,07635} \cdot EXR_{-1} \qquad (G2)$$

$(R^2 = 0,08440;\ SE = 0,31414;\ n = 49)$

$$i\ (\equiv NCD) = \underset{[8,2385]}{3,7477} \quad \underset{[-1,1402]}{10,051869} \cdot EXR \quad \underset{[-1,11129]}{-0,054206} \cdot EXR_{-1}$$

$$\underset{[-0,90702]}{-0,041990} \cdot EXR_{-2} \quad \underset{[-0,56235]}{-0,02325} \cdot EXR_{-3} \qquad (G3)$$

$$\underset{[-2,9638]}{-0,12557} \cdot EXR_{-4}$$

$(R^2 = 0,3142;\ SE = 0,26769;\ DW = 0,59974;\ n = 46)$

3.9 RESULTS OBTAINED AFTER JUNE 1976

We continued to update and reanalyse the model during November 1976, by which time a total of 64 monthly observations on the "short-term" data had become available. These results were incorporated into an updated version of the Hurwitz-Kantor (1976) Konstanzer paper.

Most of the regression results associated with the earlier paper remained largely unchanged* except for the

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* details available upon request.

in the second period (from negative to positive), and remained significant. This suggests a reverse effect from Reserves to controlled interest rate after October 1974. This association may be explained with reference to the increasing reliance on accommodating foreign borrowing by the Reserve Bank after this period. The level of Reserves the authorities were concerned with were no longer measured reserves. The negative association between net Reserves and interest rates over this period would surely be sustained.

The instability of the NDA equation has always been problematic. The disadvantage of having to estimate NDA as a residual, (i.e. MB - R), rather than measure it directly means that we are excluding from it factors which may be important, e.g. "Other Liabilities" in the Reserve Bank balance sheet (see Appendix C). ("Other Liabilities form on average about 10% of Total Liabilities.)

The Money Supply equation is, as mentioned above, a highly simplified form of the Brunner-Meltzer type model; implying causation from MB to MS; as suggested previously the causation may well be the other way.

The model that we have presented has the advantage of simplicity, and high explanatory power. As we have noted, however, this specification should be looked upon as a very useful first approximation to any future attempt to build a more realistic monetary model. A model incorporating dynamic partial adjustment mechanisms would be a more realistic approach.

An additional improvement would be gained by an appropriate integration of the demands for and supplies of money base, money supply and bank credit.

3.10 CONCLUSION

The degree of independence of South African monetary policy from balance of payments consideration has been nominal despite the panoply of exchange and import controls with which we have attempted to insulate ourselves. The South African economy has remained an extremely open one which is inevitably given the extent to which the economy engages in foreign trade and depends on imports of foreign capital. The study indicates that official preferences for relatively stable exchange rates cannot necessarily prevent inflationary monetary developments. That, on the contrary, international monetary developments to which fixed exchange rates provide a direct link may prove highly inflationary as indeed they have done in other periods of South African monetary history, particularly during the World Wars (Kantor (1971)). The only way in which South Africa could have avoided the post 1972 inflation would have been to have adopted a strictly independent monetary policy with the closest attention being paid to limiting the growth in monetary aggregates. Not only did South African monetary policy fail in this, but also allowed more purely domestic influences on the money base to perpetuate rapid rates of monetary expansion long after the balance of payments itself had ceased to be an expansionary influence. This indeed was a measure of monetary independence but led inevitably, given the retained official preferences for exchange rates, to formal devaluations and extensions of the exchange and import control mechanisms.

The crucial issue exposed by the study is the question of the appropriate monetary standard. There are essentially two options available. These are either for South Africa to exercise preferences for exchange rates, or to adopt a monetary policy independently of balance of payments considerations. It is not within the scope of this paper to argue the theoretical advantages of alternative monetary standards (Mundell (1968)). Both a dependent and independent monetary policy could under certain circumstances provide a high degree of absolute and relative price stability. It will all depend in the case of fixed exchange rates on the rate of growth of international money, and for any independent monetary policy on the rate of growth of domestic money.

CHAPTER FOUR

MODELLING LINEAR STOCHASTIC DIFFERENTIAL EQUATION SYSTEMS

4.1 INTRODUCTION

The idea that an economic system may be represented as a simultaneously interacting system of linear differential equations is not new. We must however be sure what we mean by "linear", and also what the implications for stability, etc., of such a system are. Further, the econometrician, in common with an engineer modelling any physical process, has eventually to take into serious consideration the error processes associated with his model; that is, those uncontrolled or uncontrollable forces acting on the system which give rise to departures from complete determinancy. These are out "stochastic" effects and they too must be the subject of our investigation. Sections 4.2 and 4.3 develop these two aspects in some detail, and thereafter we discuss a particular approach (that of Bergström (1966) and others) towards the estimation of such systems.

4.2 LINEAR FILTERS*

It is of interest to establish solutions and stability

* The treatment given here largely follows that of James, Nichols and Phillips (1947).

criteria for a linear system of differential equations with constant coefficients of the form $(D = d/dt)$:

$$a_n D^n x_1(t) + \ldots + a_1 D x_1(t) + a_0 x_1(t)$$
$$= b_m D^m x_2(t) = b_{m-1} D^{m-1} x_2(t) + \ldots + b_0 x_2(t) \qquad (4.2.1)$$

i.e. $\alpha_{12}(D)x_1(t) = \beta_{12}(D)x_2(t)$ (4.2.2)

where α_{12} and β_{12} are polynomials of degree n and m respectively).

This equation may be regarded as the governing differential equation of a system with "input $x_2(t)$ and "output" (or "solution") $x_1(t)$, (James, et al (1947), Phillips (1959)). The general solution to (4.2.1) is the sum of the general solution to the "homogenous" equation

$$\alpha_{12}(D)x_1(t) = 0 \qquad (4.2.3)$$

plus any solution of the "nonhomogenous" equation (4.2.2), (Utz (1967), Theorem 8).

The output $x_1(t)$ of the system (4.2.1) during any period in which the input $x_2(t)$ is identically zero is a solution of (4.2.3), and may be regarded as the "transient response" of the filter to the earlier history of its input. If, after some time t_0, the input $x_2(t)$ takes on a "steady state" form, the transient response then becomes the difference between the actual output for $t > t_0$ and the "asymptotic form" that it approaches; this asymptotic form is the solution of (4.2.2).

Unit-Impulse (or "Dirac Delta Function") $\delta(t-t_0)$ is a singular function defined to be zero everywhere except at $t = t_0$ in such a way that it posesses the following properties $(t_1 < t_2)$:

$$\int_{t_1}^{t_2} f(t)\delta(t-t_0)dt = 0 \quad \text{if} \quad t_1 > t_0 \quad \text{or} \quad t_2 < t_0$$

$$\int_{t_1}^{t_2} f(t)\delta(t-t_0)dt = \tfrac{1}{2}f(t_0) \quad \text{if} \quad t_0 = t_1 \quad \text{or} \quad t_2 = t_0$$

$$\int_{t_1}^{t_2} f(t)\delta(t-t_0)dt = f(t_0) \quad \text{if} \quad t_1 < t_0 < t_2$$

and $$\int_{-\infty}^{\infty} \delta(t-t_0)dt \equiv 1.$$

We define the Normal Response of a linear system to a unit impulse applied at time $t = 0$ to be $W(t)$, called the "Weighting Function" of the system. The determination of $W(t)$ is most conveniently achieved by the use of the Laplace Transforms. This method is duscussed below. At this point it is in order to clarify the notion of "linearity" with regard to linear systems such as (4.2.1). The following are characteristics of linear systems:

(a) The Normal Response is a linear function of the input; that is, if $y_1(t)$ is the Normal Response of the system to the input $x_1(t)$, and $y_2(t)$ to $x_2(t)$, then the Normal Response to the input $x(t) = c_1x_1(t) + c_2x_2(t)$, ($c_1, c_2$ arbitrary constants), is $y(t) = c_1y_1(t) + c_2y_2(t)$.

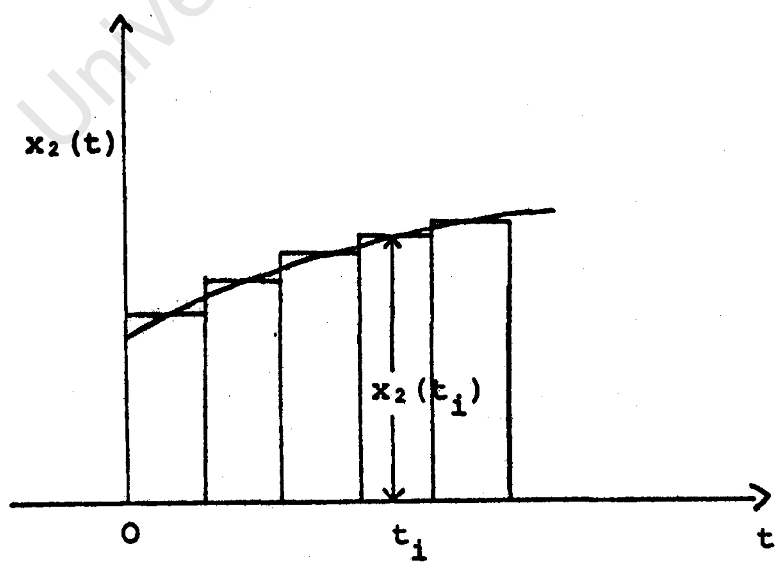
(b) The Normal Response at any time depends only on past values of the input.

(c) The Normal Response is independent of the time origin.

A system that is linear in the sense described above we call a "Linear Filter."

4.2(c) NORMAL RESPONSE OF A LINEAR FILTER TO AN ARBITRARY INPUT

The Normal Response of a linear filter to an arbitrary bounded input $x_2(t)$ can be conveniently expressed in terms of the response to a unit-impulse input. We assume $x_2(t) = 0$ for $t < 0$. The function $x_2(t)$ may be approximated by a set of n narrow rectangles of width Δt_i and height $x_2(t_i)$:



At time t_i the input represented by such a rectangle may be approximately written as:

$$x_2(t)\Delta t_i \delta(t-t_i).$$

As $\Delta t_i \to 0$ for all i this approximation becomes more accurate, and we may thus write:

$$x_2(t) = \Sigma_n x_2(t_i)\Delta t_i \delta(t-t_i).$$

The Normal Response of a linear filter to a succession of impulse-inputs will, by the definition of linearity, be the sum of the responses to each of these inputs. Since, by our definition of $W(t)$, we have that the filter response to:

$$x_2(t_i)\Delta t_i \delta(t-t_i)$$

is: $x_2(t_i)\Delta t_i W(t-t_i)$, we may write the filter response to the input

$$\Sigma_n\ x_2(t_i)\Delta t_i \delta(t-t_i)$$

as: $$\Sigma_n\ x_2(t_i)\Delta t_i W(t-t_i).$$

In the limit, as $\Delta t_i \to 0$ for all i, this becomes

$$\int_0^\infty x_2(t_i)W(t-t_i)dt_i \qquad (4.2.5)$$

Since $W(t-t_i) = 0$ for $(t-t_i) < 0$ we may write this as:*

$$\int_0^t x_2(t_i)W(t-t_i)dt_i \qquad (4.2.6)$$

* One may handle delta function terms in the Weighting function by modifying the limits of integration: (James, et al (1949) pp 34-35).

4.2(d) THE LAPLACE TRANSFORM

Definition: If $g(t)$ is a function defined for $t \geq 0$ then it may be shown (subject to certain boundedness conditions*) that

$$\mathbb{L}[g(t)] \equiv \int_0^\infty g(t)e^{-pt}dt$$

exists, and is called the "Laplace Transform" of $g(t)$. It may also be shown that $\mathbb{L}$ is linear, and:

$$\mathbb{L}[\frac{dg(t)}{dt}] = p\mathbb{L}[g(t)].$$

Hence, for (4.2.1), we obtain:

$$\alpha_{12}(p)\mathbb{L}[x_1(t)] = \beta_{12}(p)\mathbb{L}[x_2(t)].$$

Writing $$Y(p) = \frac{\beta_{12}(p)}{\alpha_{12}(p)}, \qquad (4.2.7)$$

we get: $$\mathbb{L}[x_1(t)] = Y(p)\mathbb{L}[x_2(t)] \qquad (4.2.8)$$

Thus, to obtain $x_1(t)$, calculate $Y(p)\mathbb{L}[x_2(t)]$ and take the inverse Laplace transform.**

It may be shown (James et al (1949) p 60) that the linear filter (4.2.1) is stable*** only if $m < n$, (otherwise the $W(t)$ may also contain delta functions).

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* see Fadell (1968) "Vector Calculus and Differential Equations." D. Von Nostrand N.Y.

** It can be shown that $Y(p)$ is, in fact, the Laplace Transform of the Weighting Function, and is called the "Transfer Function" of the system.

*** In the sense that a bounded input produces a bounded output.

4.2(e) SIMULTANEOUS SYSTEMS

If we have N interrelated variables $x_i(t)$, $(i = 1,\ldots,N)$, we may in most cases write their relationships as differential equation systems of the type (4.2.1), i.e.

$$\alpha_{ij}(D)x_i^j(t) = \beta_{ij}(D)x_j(t) \qquad (i,j = 1,\ldots,N; \quad i \neq j) \tag{4.2.2b}$$

$x_i^j \equiv$ response of x_i to input x_j.

By our above discussion this may be written as ($T_i \equiv$ the transient response):

$$x_i^j(t) = T_i + \int_0^\infty W_{ij}(h)x_j(t-h)dh \tag{4.2.9}$$

Because of the linearity of such systems the total response of x_i to simultaneous inputs of x_j $(i \neq j)$ is:

$$\begin{aligned} x_i(t) &= \Sigma_{i\neq j}\, x_i^j(t) \\ &= \Sigma T_i + \Sigma_{i\neq j} \int_0^\infty W_{ij}(h)x(t-h)dh \end{aligned} \tag{4.2.10}$$

If we assume that the system is stable (i.e. $m < n$ in all cases), and that the initial conditions are at $t = -\infty$ (i.e. $T_i = 0$ for all i)*, then:

$$x_i(t) = \Sigma_{i\neq j} \int_0^\infty W_{ij}(h)x_j(t-h)dh \tag{4.2.11}$$

(4.2.11) is then the <u>general</u> solution to (4.2.2b), which may be written as:

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* Which will be true if the roots of (4.2.4) have negative real parts.

$$x_i(t) = \Sigma_{i\neq j}\frac{\beta_{ij}(D)}{\alpha_{ij}(D)}\, x_j(t) \qquad (4.2.12)$$

4.3 STOCHASTIC CONSIDERATIONS

In order to accommodate the fact that (1) the system may not be exactly linear and (2) the input may contain "noise" elements we include stochastic distrubance terms into the equations as:

$$x_i(t) = \Sigma_{i\neq j} \int_o^\infty W_{ij}(h)x_j(t-h)dh + \Pi_i(t) \qquad (4.3.1)$$

If we suppose that, as with other inputs, the "noise" is a linearly additive input we may write in analogous fashion to (4.2.11):

$$x_i(t) = \Sigma_{i\neq j} \int_o^\infty W_{ij}(h)x_j(t-h)dh + \textstyle\sum_{k=1}^{N} \displaystyle\int_o^\infty r_{ik}(h)\pi_k(t-h)dh \qquad (4.3.2)$$

or $$x_i(t) = \Sigma_{i\neq j} \frac{\beta_{ij}(D)}{\alpha_{ij}(D)}\, x_j(t) + \sum_{k=1}^{N} \frac{\eta_{ik}(D)}{\gamma_{ik}(D)}\, \pi_k(t) \qquad (4.3.3)$$

We suppose that $\eta_{ik}(s)$ is of lower order than $\gamma_{ik}(s)$, and that $\pi_k(t)$ is a stochastic process. The $r_{ik}(s)$ have Laplace Transforms (cf. (4.2.7)):

$$R_{ik}(s) = \frac{\eta_{ik}(s)}{\gamma_{ik}(s)} \quad .$$

This approach is discussed in detail in Phillips (1959). We however choose to deal with the simplified form:

$$\alpha_i(D)x_i(t) = \sum_{i\neq j}\beta_{ij}(D)x_j(t) + u_i(t) \qquad (4.3.4)$$
$$(i,j = 1,\ldots,n)$$

or, rewriting in matrix form* (Wymer (1972)):

$$D^r y(t) = \sum_{k=1}^{r} A_k D^{k-1} y(t) + Bz(t) + u(t) \qquad (4.3.5)$$

where we have split the vector $x(t)$ into a vector $y(t)$ of n endogenous variables and a vector $z(t)$ of m exogenous variables. $u(t)$ is a disturbance vector and is assumed to be generated by a "white noise" process (see Section 4.4). A_k are matrices of order n, and B is a matrix of order $(n\times m)$. (4.3.5) is our basic system, and may be written as a first-order system (Hirsch and Smale (1974)) as follows:

$$Dy^*(t) = A^*y^*(t) + B^*z(t) + u^*(t) \qquad (4.3.6)$$

where $y^*(t) = \begin{bmatrix} y_1^*(t) \\ \cdot \\ \cdot \\ \cdot \\ y_r^*(t) \end{bmatrix}$, $A^* = \begin{bmatrix} O & \cdot & I \\ \cdot & \cdot\cdot\cdot & \cdot \\ & \cdot & \\ & \cdot & \\ A_1 & \cdot & A_r \end{bmatrix}$, $B^* = \begin{bmatrix} O \\ B \end{bmatrix}$, $u^*(t) = \begin{bmatrix} O \\ u(t) \end{bmatrix}$

and $Dy_i^*(t) = y_{i+1}^*(t) \quad (i = 1,\ldots,r-1; \quad y_1^*(t) = y(t))$

$$Dy_r^*(t) = \sum_{k=1}^{r} A_k y_k^*(t) + Bx(t) + u(t) \ .$$

(4.3.6) is a "nonhomogenous and nonautonomous" linear differential equation system (Hirsch and Smale (1974)) of the form

$$Dy^*(t) = A^*y^*(t) + C(t) \ .$$

To find a solution to such a system we first assume that such a solution, if it exists, is of the form ($f\colon R\to R^n$ a

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* We change the usage of the letters n, m, k, etc., here.

$$y_t^* = e^{\delta A^*} y_{t-1} = \psi_t^* + \omega_t^* \qquad (4.5.1)$$

This is called the "exact discrete model."* If there are a priori restrictions on the matrices A^*, and B^* (as are likely in econometric work), it is seen that these determine complicated restrictions on $\exp(\delta A^*)$ and ψ_t^*, which makes (4.5.1) difficult to estimate directly. An approximate discrete model has accordingly been developed to obtain estimates of A^* and B^*.

4.6 THE APPROXIMATE DISCRETE MODEL

Making the approximations (Houthakker, Taylor (1966)):

$$\left(\frac{dy}{dt}\right)_{t=r\delta-\frac{1}{2}\delta} \approx \frac{1}{\delta}(y_r - y_{r-1})$$

and $\quad y(r\delta-\tfrac{1}{2}\delta) \approx \tfrac{1}{2}(y_r+y_{r-1}),$

we integrate (4.3.6) over the interval $(t-\delta,t)$ and obtain:

$$\int_{t-\delta}^{t} Dx(s)\,ds = \Delta x_t, \quad \int_{t-\delta}^{t} x(s)\,ds = Mx_t$$

(where $\Delta = (1-L)$,

$M = \tfrac{1}{2}\delta(1+L)$,

$Lx_t = x_{t-1}$)

We assume that all variables in (4.3.6) are observable at a point in time (i.e. they are all "stock" variables). The analysis must be modified if the model includes "flow" variables, and details of this are given below.

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* "exact discrete" in the sense that the continuous model (4.3.8) generates the same observations at the discrete time points $t = 1,2,\ldots$ as does (4.5.1).

Setting $\Delta y^*_{1t} = My_{i+1,t} + u_{it}$,

$$\Delta y^*_{rt} = \sum_{k=1}^{r} A_k M y^*_{kt} + BMz_t + u_{rt} ,$$

$$y_t = y^*_{1t} \quad , \quad \text{and}$$

$$v_t = \sum_{i=1}^{r} (M^{i-1}\Delta^{r-1}) u_{it},$$ (and assuming terms in A_j and u_{jt} negligible), we may write (Wymer (1972)) the approximate model as:

$$\Delta^r y_t = \sum_{i=1}^{r} A_i (M^{r-i+1}\Delta^{i-1}) y_t + M^r Bz_t = v_t \qquad (4.6.1)$$

This may be rewritten in first-order form as:

$$\Delta y^*_t = A^*(My^*_t) + B^*Mz_t + v^*_t \qquad (4.6.2)$$

where $v^{*\prime}_t = (u'_{1t}, u'_{2t}, \ldots, u'_{rt})$ y^*_t, A^*, and B^* correspond to those given for (4.3.6).

(4.6.2) (or (4.6.1)) may be regarded as an approximate equation, and v^*_r as an arbitrary random error. We may however regard (4.6.2) as <u>defining</u> v^*_t in conjunction with (4.5.1) in the following way: Rewrite (4.6.2) as:

$$\frac{1}{\delta}(I-\tfrac{1}{2}\delta A^*)y^*_t - \frac{1}{\delta}(I+\tfrac{1}{2}\delta A^*)y^*_{t-1} = B^*Mz_t + v^*_t$$

and (4.5.1) as:

$$\frac{1}{\delta}(I-\tfrac{1}{2}\delta A^*)y^*_t - \frac{1}{\delta}(I-\tfrac{1}{2}\delta A^*)\, e^{\delta A^*}\, y^*_{t-1}$$

$$= \frac{1}{\delta}(I-\tfrac{1}{2}\delta A^*)\psi^*_t + \frac{1}{\delta}(I-\tfrac{1}{2}\delta A^*)\omega^*_t \quad .$$

Then we may equate*:

$$v^*_t = \frac{1}{\delta}(I-\tfrac{1}{2}\delta A^*)\omega^*_t$$

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* Sargan (1974) gives the errors involved in making this type of approximation, and shows that they are small for small δ.

To establish the covariance generating function (L the lag operator):

$$F(L) = \sum_{-\infty}^{\infty} \phi_k L^k$$

(where $\phi_k = E(v_t v_{t+k}')$) of the process v_t write, without loss of generality, the error process generating v_t^* as a "moving average" process of the form (Quenouille (1957), Kendall and Stuart (1966)):

$$v_t^* = U(L)\varepsilon_t \tag{4.6.3}$$

where $U(L)$ is a matrix of polynomials in the lag operator, and ε_t is a vector of independent random elements with the same variance σ^2 (say), zero mean, and are uncorrelated. By our assumptions on the ω_t^*, u_{it}, and v_t^* above it may be shown (Wymer (1972)) that the v_t^* are serially uncorrelated with contemporaneous covariance matrix*

$$E(v_t^* v_t^{*\prime}) = \Omega_v^* = \Gamma_r \otimes \Omega = \begin{pmatrix} \gamma_{r-1} & & 0 \\ & \ddots & \\ 0 & & \gamma_0 \end{pmatrix} \otimes \Omega$$

(with $\gamma_k = \dfrac{\delta^{2k-1}}{(k!)^2(2k+1)(2k-1)} \{(2k-1) + \tfrac{1}{4}(2k+1)$

$- \tfrac{1}{2}(2k+1)(2k-1)\}$

for $k = 1,2,\ldots,$ and $\gamma_0 = \frac{1}{\delta}$;

Ω as in Section 4.4).

This implies that $U(L)U'(L^{-1}) = \Gamma_r \otimes \Omega$.

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* $\otimes$ is the Krönecker product.

Further, v_t is generated by an "autoregressive" process of the form:

$$V(L)v_t = v_t^* ,$$

where $V(L)$ is again a matrix of polynomials in L. By referring to the definitions of v_t and v_t^* it may be seen that:

$$V^{-1}(L) = (p'(L) \otimes I_n)$$

where $p(L)$ is a vector of length r whose i^{th} element is:

$$(\tfrac{1}{\delta}(1-L)^{r-1}(\tfrac{1}{2}(1+L))^{i-1} .$$

This immediately implies (Kendall and Stuart (1966)):

$$\begin{aligned} F(L) &= V^{-1}(L)U(L)U'(L^{-1})V'^{-1}(L^{-1}) \\ &= (p'(L)\otimes I_n)\Gamma_r \otimes \Omega(p(L^{-}\ )\otimes I_n) \\ &= p'(L)\Gamma_r p(L^{-1})\Omega \end{aligned} \qquad (4.6.4)^*$$

---

* Since, by the rules of the Krönecker product:

$$\begin{aligned} &(p'(L)\otimes I_n)\Gamma_r \otimes \Omega(p(L^{-})\otimes I_n) \\ &= (p'(L)\Gamma_r(\otimes(I_n\Omega)(p(L^{-1})\otimes I_n) \\ &= (p'(L)\Gamma_r p(L^{-1}))\otimes(I_n\Omega I_n) \\ &= (p'(L)\Gamma_r p(L^{-1}))\otimes\Omega \\ &= p'(L)\Gamma_r p(L^{-1})\Omega , \end{aligned}$$ the last step

because $p'(L)\Gamma_r p(L^{-1})$ is (1×1) .

For example, if $r = 2$ (4.6.4) gives:

$$\delta F(L) = L^{-1}\{-\tfrac{1}{12}(1-L)^2 + \tfrac{1}{4}(1+L) + \tfrac{1}{4}(1+L)^2\}\Omega$$

$$\propto (1 + 0{,}268L)(1 + 0{,}268L^{-1})\Omega$$

(We need only determine $F(L)$ up to a constant of proportionality).

In order therefore to "prewhiten" the elements of v_t we merely multiply them by the inverse of the moving average process $(1 + 0{,}268L)$ (truncated after a suitable number of terms). This transformation must then also apply to all the variables in the model (4.6.2) in order that Full-Information Maximum-Likelihood estimation techniques* (which require a serially uncorrelated error process), may be performed to yield (approximate) estimates of A* and B*.

It may, in a similar way, be seen that $r = 3$ gives rise to a moving average process of order 2, and in general an r^{th}-order system of stock variables gives rise to a moving average error process of order $(r-1)$, and must be perwhitened accordingly.

In the above we have considered a model involving only stock variables. If our model also contains "flow" variables we must, in deriving the approximate discrete model, integrate <u>twice</u> over the interval $(t-\delta,t)$. This procedure

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* Appendix A

has the effect of raising the degree of the moving average error process.

<u>Theorem 1</u>: The approximate model of order r has moving average error of order r if some flow variables are determined in equations of order r, and has moving average error of order $(r-1)$ if all flow variables are determined in equations of order $(r-1)$ or less.

<u>Proof</u>: (i) This is true for $r = 1$. (Proofs may be found in Wymer* (1974, 1976)).

(ii) Assume that it is true for r up to k.

(iii) Consider a mixed (stock/flow) system or order $r = (k+1)$:

(a) If only stock variables are determined in equations of order $(k+1)$, and all flow variables in equations of order k or less, then we may re-write the system as one of order k by defining new variables $p = DP$ where P is a stock variable determined in an equation of order $(k+1)$. By (ii) this system has moving average error process of order k.

(b) If some flow variables are determined in equations of order $(k+1)$, then we may transform the system to a $(k+2)^{th}$ order system by writing

* An explicitly worked-out proof is given in "Static and Dynamic Modelling of the South African Real Sector," G. Barr (1977), unpublished M.Sc. thesis (available: Jagger Library, University of Cape Town).

all flow variables x as Dy, where y is the corresponding "stock" variable. By the extensive discussion given above, this stock system has moving average error of order (k+1), hence proof by induction.

4.7 SUMMARY

Starting with the notion of a Linear Filter, we showed that the stability of such a system is ensured given certain restrictions on the degrees of the polynomials involved in their specification. These restrictions correspond to the concepts of causality developed in Chapter One, although an alternative interpretation of such a system as a transformation of a model of the type (4.2.11) could be given. It was also shown (Section 4.3) how stochastic considerations may be introduced within this analytical framework.

The narrower form (4.3.5) however, was adopted involving a somewhat different error mechanism, (Section 4.4). This was then rewritten in a first-order differential equation form and solved (Sections 4.3, 4.5) to obtain the so-called "exact discrete model." The exact model, involving complex restrictions on the coefficient matrices, proves difficult to estimate, and an alternative "approximate discrete model" (Section 4.6) was developed.

The error process of the approximate model was then analysed, and it was found that an r^{th}-order model involving only "stock" variables produced an error process of order $(r-1)$, whereas a mixed "stock-flow" (or pure flow) model gave an error process of order r if flow variables were determined in equations of order r, and $(r-1)$ otherwise (Theorem 1, Section 4.6).

We proceed, in the next chapter, to use the foregoing theory as a basis for estimating a differential-equation model of the balance-of-payments mechanism in South Africa.

CHAPTER FIVE

TESTING THE MONETARY THEORY OF THE BALANCE OF PAYMENTS

5.1 INTRODUCTION

Chapters Two and Three concerned themselves largely with an analysis of the sources of money in South Africa. This final chapter takes the theory a step further, specifying a model embodying the effects of money on prices, bank credit and interest rates.

The model, along with the accompanying theory, is first presented. This discussion follows that in Hurwitz (1977). A more extensive version of the same theory may be found in Kantor (1977). Estimation methods and a discussion of the parameter estimates is given, and a number of suggestions for future research follows a note on stability analysis.

5.2 MONETARY THEORY

"In democratic societies, central bank behaviour is really endogenous to a model of the political process ... [this] ... suggests that economists should not pretend that central banks can pursue freely their own preferences or those of their advisers ..... .

The extent to which central banks can influence high-powered money can also be questioned. Work on the monetary approach to the balance of payments has reversed the whole chain of influence, at least for a small country with fixed exchange rate .... Attempts by the central bank to supply more or less than the desired amount of money will simply result in deficits or surpluses in the balance of payments."
(Brainard and Cooper (1975).)

This study is concerned with the explanation and empirical estimation of important monetary transmission mechanisms for South Africa. Monetary effects are analysed within the framework of a small, open economy with a preference for fixed exchange rates. Demand and supply pressure in the money and bank credit markets are shown to affect the price level and the balance of payments, and the interactions between money, prices, interest rates and the b.o.p. are indicated.

Money is assumed to be a substitute for real goods and services, and of course domestic goods and services are substitutes for foreign goods and services. An excess of actual over desired real money balances is therefore seen to stimulate extra demands for imports and discourage exports. Domestic prices also tend to increase in response to excess supplies of money, which also leads to increased supplies of domestic credit and hence less demand for foreign loans.

The essentially simultaneous nature of an economic system is therefore seen to lie at the heart of any serious attempt to analyse changes in economic variables. For this very reason the model was specified and estimated as a simultaneously interacting system of differential equations,* using the techniques of Full-Information-Maximum-Likelihood.** The more detailed assumptions were as follows:

THE PRICE EQUATION

An increase in the supply of money leads to inflationary pressure via a "partial equilibrium adjustment," that is - a lagged adjustment to equilibrium (see below). If the supply of real money balances is in excess of the desired demand for money, the rate of change of prices will increase. Domestic prices are assumed to follow world prices via standard arbitraging mechanisms. The open economy assumption implies that domestic and foreign goods are good substitutes or that, in other words, given fixed exchange rates, prices change in the direction of purchasing power parity.

EXPORTS

The supply of exports depends on the growth in desired supplies and on relative prices. Exports adjust with a lag to eliminate differences between desired and actual supplies.

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* Chapter Four

** Appendix A

PRIVATE CAPITAL IMPORTS

Capital is assumed to flow in response to differences between desired and actual levels of private bank credit, and to differences between local and foreign rates of interest. The desired demand for bank credit depends on real income; income is taken as exogenously determined.

PRIVATE CREDIT

The level of private credit responds, as in the argument involving capital inflow, to differences between desired and actual levels of credit, as well as to the difference between the rate of interest and its equilibrium, or "steady-state," level.

TREASURY BILL INTEREST RATE

This interest rate is understood to be set by the authorities in response to the state of the balance of payments and the level of demand in the credit markets. A decrease in interest rates will follow easier credit conditions, but a seriously deteriorating balance of payments position will invariably oblige the authorities to adjust the rate upward even if credit conditions are tight.

NET DOMESTIC ASSETS (NDA)

Deposits, at the Reserve Bank, of commercial banks - that is, the Domestic Asset position of the central bank, net of government deposits - will change as banks satisfy their demands for cash by borrowing from the Reserve Bank. NDA will also depend on whether or not the commercial banks are able to obtain cash via the balance of payments, i.e. through the reserve position. It is assumed therefore that NDA is a function of the relationship between the total demand for bank credit and the supply of Foreign Exchange Reserves. The model assumes that the supply of cash accomodates itself to the demands for credit; this assumption follows from the concern for nominal interest rates revealed by the authorities.

THE MONEY SUPPLY

The sources of changes in the Money Base - the "high-powered" money of the system - are seen as two-fold: Money Base may change with changes in either the NDA position of the Reserve Bank or with its Foreign Reserves. An increase in the base leads to an increase in the Money Supply via a "Brunner-Meltzer" type multiplier , which is itself a function of a range of interest rates, reserve requirement ratios, income, wealth and prices.

5.3 PARTIAL EQUILIBRIUM ADJUSTMENT : A NOTE

Much of the subsequent formulation is put into the form of "partial equilibrium adjustment." A very simple example of this would be:

$$Dy(t) = \beta\{\hat{y}(t) - y(t)\} \qquad (5.3.1)$$

where $\hat{}$ denotes an equilibrium level. In other words, the rate of change in $y(t)$ is proportional to the difference between the current value of y and its (partial) equilibrium level $\hat{y}$, also at time t.

We call $|\frac{1}{\beta}|$ the "mean time lag to equilibrium." (5.3.1) may be rewritten in the (inverse operator) form:

$y = \frac{\beta}{D+\beta}\hat{y}$, and this has solution (Utz (1967), p 45):

$$y = \int_0^\infty \beta e^{-\beta s}\,\hat{y}(t-s)ds.$$

Here $\beta e^{-\beta s}$ is the Weighting function $W(s)$, (or "lag distribution"); this follows from the fact that:

$\int_0^\infty W(s)\,\hat{y}(t-s)ds$ is the general solution to $(D+\beta)y = \beta\hat{y}$ (under certain assumed conditions mentioned in Chapter 4), and because the Transfer function of the system is,

$Y(s) = \frac{\beta}{s+\beta}$, and also because the weighting function is the inverse Laplace transform of the Transfer function (Spiegel (1968), formula 32.24), i.e:

$$W(s) = \mathbb{L}^{-1}[\frac{\beta}{s+\beta}] = \beta e^{-\beta s}.$$

Note also that:

$\int_0^\infty W(s)ds = 1$, and:

$\int_0^{|\frac{1}{\beta}|} W(s)ds = 0{,}63$, that is $|\frac{1}{\beta}|$ is the time taken for 63% of the discrepancy between $\hat{y}$ and y to be removed by changes in y.

The Export equation given below (Section 5.4) has the form (5.3.1); for the others a modification of the above analysis would be necessary.

5.4 THE MODEL EQUATIONS

The following model, similar in its approach to those of R. Bergström and C.R. Wymer (1974), P.D. Jonson (1976, 1977a, 1977b) is set out as a system comprising five first-order differential equations, two zero-order equations and three identities - a total of ten simultaneous equations applicable to South African data. (Stochastic variables are omitted here, but are implicitly assumed in the non-identity equations.)

1. THE PRICING EQUATION

$$D \log P = \gamma_1 [\log(MS/P) - \log \hat{m}] + \gamma_2 \log(P_m/P)$$

$$\hat{m} = m_{00}\, Y^{\beta_7} (i^t)^{\beta_3}$$

($\hat{}$ denotes a desired level; $D = d/dt$).

Corresponding to any level of real income and interest rates there is assumed to be a partial equilibrium level of desired demand for real balances $(\hat{m})$. The proportional rate of change in the price index is assumed to be an increasing function of the proportional increase in actual over desired real balances: an increase of say, one percent in the log of actual real balances over desired will increase the rate of increase of prices by γ_1 percent per unit time period. Further, following our above discussion, the variable (P_m/P) is included to represent the effect of arbitraging mechanisms that maintain purchasing power parity.

2. REAL EXPORTS

$$D \log X = \gamma_3[\log x_0 e^{\lambda_1 t}(P_m/P) - \log X]$$

Desired supply of South African exports is assumed to depend on relative prices as well as an exponential time trend, which represents the growth, or decline, in the level of world demand. The proportional rate of change of exports is assumed to depend upon the excess of desired supply over actual supply. The supply of exports adjusts, with a lag, to this partial equilibrium level of desired supply.

3. PRIVATE CAPITAL IMPORTS

$$D \log K_p = \gamma_4[\log \hat{TP} - \log TP] + \gamma_{10} \log(i^t/i^e)$$
$$\hat{TP} = c_0 Y^{\beta_2}$$

The desired level of private credit is taken to be a function of real income, and a function of the ratio of desired and actual levels of credit influences the rate of increase in private capital inflows. The ratio of local and foreign interest rates is also assumed to affect this rate of increase.

4. DEMAND FOR PRIVATE CREDIT

$$D \log TP = \gamma_5[\log \hat{TP} - \log TP] - \beta_4 \log(i^t/i^{**})$$

$$i^{**} = i^*$$

Changes in the level of private credit is assumed to be a function of desired-actual differences as well as a decreasing function of the ratio between the rate of interest and its equilibrium, or "steady state," level i^{**}. In the steady-state interest rates are constant. This is consistent with the assumption of a fixed exchange rate in an inflationary world with prices growing at a constant exponential rate.

5. TREASURY BILL INTEREST RATE

$$D \log i^t = -\gamma_6 \log (EXR/EXR^{**}) + \gamma_7 \log (NDA/NDA^{**})$$

$$EXR^{**} = EXR^* \exp \lambda_2 t$$

$$NDA^{**} = NDA^* \exp \lambda_3 t$$

A level of commercial bank excess reserves (EXR) in excess of its steady-state level is indicative of easy domestic credit conditions, and is thus thought to lead to a decrease in interest rates. On the other hand a level of NDA in excess of its steady-state value is taken to indicate unsatisfactory balance of payments developments requiring correction in terms of higher interest rates. It may also, not perhaps simultaneously, suggest a high level of government spending relative to taxation, and therefore the necessity for government to adjust the rate upwards in order to attract more bank credit, or to impose additional reserve requirements which may lead to tighter credit conditions.

The interest adjustment mechanism appeared to be subject to variable lags, and good estimates of changes in interest rates in these terms were not obtained. A previous study (Hurwitz and Kantor (1976)) found a satisfactory explanation for the level of interest rates in terms of the level of reserves and credit market conditions.

6. NET DOMESTIC ASSETS (OF THE RESERVE BANK)

$$\log NDA = \gamma_8 \log TRG + \gamma_9 \log i^t$$

It also proved to be extremely difficult to find a satisfactory explanation for changes in the level of NDA, and so, following the earlier discussion, the level of NDA is taken to be an increasing function of interest rates and the ratio of total commercial bank credits to total foreign reserves of the Reserve Bank.

Higher interest rates could be expected to lead to more profitable bank lending and therefore an increase in borrowed bank reserves. TRG = TBC/RG indicates the more-or-less automatic adaption of banks cash reserves to the level of bank credit given the controlled structure of interest rates.

7. THE MONEY SUPPLY

$$\log MS = \log m_0 + \beta_5 \log i^t + \beta_6 \log MB$$

This function is a modified and simplified "Brunner-Meltzer" multiplier relationship: the Money Supply is assumed to be a multiple of the Money Base where the multiplier is taken to be a function of the rate of interest - the interest elasticity of the multiplier being β_5.

8. MONEY BASE IDENTITY

$$MB \equiv RC + NDA$$
$$\equiv R + FL + NDA$$

9. TOTAL BANK CREDIT IDENTITY

$$TBC \equiv TP + TGL$$

10. CHANGE IN FOREIGN RESERVES IDENTITY

$$DR \equiv DK_p + P(X-M)$$

i.e. $DRG \equiv DK_p + DFL + P(X1M)$

$$\equiv DKG + P(X-M)$$

P = Price Index

X = Exports

K_p = Private Foreign Capital

TP = Private Credit (of Commercial Banks)

i^t = Interest rate

NDA = Net Domestic Assets (of Reserve Bank)

MS = Money Supply

MB = Money Base

TBC = Total (Commerical) Bank Credit

R = Foreign Reserves (of Reserve Bank) net of Government Foreign Borrowing

RG = Total Foreign Reserves (of Reserve Bank)

t = Time trend

p_m = Foreign Price Index

Y = Real Income

i^e = Foreign Interest Rates

EXR = Excess Reserves (of the Commercial Banks)

TRG = TBC/RG

FL = Foreign Borrowing of the Government

TGL = Total Credit to Government (of the Commerical Banks)

M = Real Imports

KG = K_p + FL

(m_{00}, x_0, c_0, m_0, i*, EXR*, NDA* : constants)

See Appendix C for a list of the data series used.

5.5 THE STEADY STATE GROWTH RATES

The model system may, in general, be written as:

$$DY(t) = F(Y(t), Z(t), \theta) + u(t)$$

This represents a linear or non-linear system of endogenous (Y) and exogenous (Z) variables, and a vector (θ) of parameters which have to be estimated.

The "Steady State" or "equilibrium growth path" of of this system is obtained by assuming behaviour for the Z variables - usually assume that the Z_i are either constant, or that

$$Z_i(t) = Z_i^* e^{\lambda_i t} \quad \text{for all } i,$$

that is, the exogenous variables grow at constant exponential rate.

We then solve the system with out assumed Z_i to obtain a "particular solution:"

$$Y_i(t) = Y_i^* e^{\rho_i t} \quad \text{for all } i,$$

where $\rho_i = g_i(\lambda,\theta)$, $Y_i^* = G_i(\lambda,\theta,Z^*)$.

For example, in our pricing equation we first assume that the exogenous variables Y and P_m have steady-state forms:

$$Y^{**} = Y^* \exp(\rho_y t)$$

$$P_m^{**} = P_m^* \exp(\rho_{p_m} t)$$

$(Y^*, P_m^*, \rho_y, \rho_{p_m}$ constant).

Similarly, the endogenous variables P, MS and i^t have steady-state forms:

$$P^{**} = P^* \exp(\ _p t)$$

$$MS^{**} = MS^* \exp(\ _{MS} t)$$

$$i^{t**} = i^{t*} \quad \text{(constant - see footnote)}$$

---

* For any interest rate, $i = D \log P + i_0$, where i_0 is a constant rate of return (on real capital), and $D \log P$ adjusts for inflation; thus $i^{**} = D \log P^{**} + i_0 = \rho_p + i_0$, a constant.

Substituting these in the pricing equation, and equating coefficients, obtain:

$$\rho_p = (\gamma_1+\gamma_2)^{-1}(\gamma_1\rho_{ms}-\gamma_1\beta_7\rho_y+\gamma_2\rho_{p_m})$$

Doing the same for all the other equations:

$$\rho_x = \lambda_1 + \rho_{p_m} - \rho_p$$

$$\rho_{K_p} = \gamma_4 \log c_0 + \gamma_4\beta_2 \log Y^* - \gamma_4 \log TP^* + \gamma_{10} \log i^{t*} - \gamma_{10} \log i^{e*}$$

$$\rho_{TP} = \beta_2\rho_Y$$

$$\rho_{NDA} = \gamma_8\rho_{TRG}$$

$$\rho_{ms} = \beta_6\rho_{MB}$$

The equation for MB, has to be suitably log-linearised using Taylor Series expansions about means (Spiegel (1968)). This yields the results (bars refer to mean values):

$$\rho_{MB} \simeq c_2c_3\rho_{RG} + c_2c_4\rho_{NDA}$$

where: $c_2 = \log \{\exp(\overline{\log RG}) + \exp(\overline{\log NDA})\}^{-1}$

$c_3 = \exp(\overline{\log RG})$

$c_4 = \exp(\overline{\log NDA})$

Similarly, (all c_i's being constant functions of means):

$$\rho_{TBC} \simeq c_6c_7 \log \rho_{TP} + c_6c_8\rho_{TGL}$$

After some rather lengthy and tedious algebra the DR equation also yields the simple linear form:

$$D \log R \approx c_9 + c_{10} D \log K_p + c_{11} \log X + c_{12} \log M + c_{13} \log P,$$

and hence:

$$\rho_R \approx c_9 + c_{10}\rho_{K_p} + c_{11} \log X^* + c_{12} \log M^* + c_{13} \log P^*.$$

It is of interest to note a few points concerning the above functions: the rate of growth of prices in the steady-state is proportional to the growth rates of the money supply and foreign prices, which reflect the domestic and purchasing-power-parity/fixed-exchange-rate effects respectively, and is negatively related to the growth in real income. The growth rate of exports depends positively on that of foreign prices and negatively on local price growth as may be expected. Private demand for credit grows at a rate proportional to the growth of real income. Money base growth is approximately linearly related to that of gross Reserves and NDA, and the money supply grows at an exponential rate that is proportional to that of the money base. Similar comments apply to the other growth rates apart from those of private capital inflow and Reserves which grow at constant rates depending on other steady-state levels.

5.6 ESTIMATION PROCEDURES

Prior to the estimation of the present model, an earlier version incorporating a Traded/Non-traded Goods theory of the pricing mechanism, and given in Hurwitz (1976), was estimated using monthly data. These estimates were however disappointing in the sense of the low significance and often incorrect signs of the parameters in general.* Much work was subsequently put into finding a more satisfactory specification, and the results presented below show the most promising to date although the model should not be regarded as being fully developed. We discuss further possible modifications in the concluding section of the chapter.

It was also found, in work on the earlier model, that linearizations of the type given in Section 5.5 were quite unsatisfactory - producing large estimation errors. For this reason we exclude the identities and consider only the first seven equations. This implies that, in the reduced model, MB, TBC, and DR are exogenously determined although they are endogenous to the complete model.

The data used to estimate the model is quarterly, and was calculated by averaging monthly data. In order to obtain a model that is defined in terms of measurable quantities it is necessary to integrate it over the observation period so that the flow variables X and Y become:**

$$X^0 = \int_{t-1}^{t} X(s)ds, \quad Y^0 = \int_{t-1}^{t} Y(s)ds$$

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* Details available on request

** We have, in fact, set $\delta = 1 = \frac{1}{4}$ year (see Chapter 4). The published figures actually give PX^0 and PY^0.

Although all the other variables in the model are stock variables (i.e. observable at an instant in time), we are taking their 3-month averages. This will give a model with moving average error process of order 1 (Wymer (1975)), which would be true anyway since at least one flow variable is determined in an equation of order 1 (viz. in the Export equation).

Observations on the variables in the integrated model are given by:

$$y_t^0 = \int_{t-1}^{t} \log y(s)ds \simeq M \log y_t \quad ,$$

(where $M = \frac{1}{2}(1+L)$, $Lx_t = x_{t-1}$).

In view of the above comments (and footnote), the quarterly data at hand is already in the form y_t^0 for stock variables and My_t^0 for flow variables, $(M \simeq \frac{1}{3}(1+L+L^2))$. All model variables have now to be prewhitened by the inverse of the first-order moving average process, (suitably truncated*):

$$\alpha(L) = 1,0 + 0,268L$$

The model integrated a second time gives the approximate discrete model** $(\Delta = (1-L))$:

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* In this case, after the 4^{th} term; the sum of the coefficients of the inverse is: 0,785.

** It will be noted that we are omitting the steady-state parameters for interest rate, excess reserves and NDA (see Section 5.7).

1. $\Delta P^0 = \gamma_1 M(MS^0) - \gamma_1 MP^0 - 0{,}785\ \gamma_1 \log m_{00}$
$\qquad -\gamma_1 \beta_7 MY^0 - \gamma_1 \beta_3 Mi^{t^0} + \gamma_2 MP_m^0 - \gamma_2 MP^0$

2. $\Delta X^0 = 0{,}785\ \gamma_3 \log x_0 + 0{,}785\ \gamma_3 \lambda_1 (t-\bar{t})$
$\qquad + \gamma_3 MP_m^0 - \gamma_3 MP^0 - \gamma_3 MX^0$

3. $\Delta K_p^0 = 0{,}785\ \gamma_4 \log c_0 + \gamma_4 \beta_2 MY^0 - \gamma_4 M(TP^0)$
$\qquad + \gamma_{10} Mi^{t^0} - \gamma_{10} Mi^{e^0}$

4. $\Delta TP^0 = 0{,}785\ \gamma_5 \log c_0 + \gamma_5 \beta_2 MY^0 - \gamma_5 M(TP^0)$
$\qquad - \beta_4 Mi^{t_0}$

5. $\Delta i^{t^0} = -\gamma_6 M(EXR^0) + \gamma_7 M(NDA^0)$

6. $M(NDA^0) = \gamma_8 M(TRG^0) + \gamma_9 Mi^{t^0}$

7. $M(MS^0) = 0{,}785 \log m_0 + \beta_5 Mi^{t^0} + \beta_6 M(MB^0)$

The model must be rewritten in the form:

$$(I-\tfrac{1}{2}A)\Delta y - ALy - BMz = 0$$

for estimation by the program RESIMUL (Appendix D). This form is obtained by replacing all My by $(\tfrac{1}{2}\Delta+L)y$ for those <u>endogenous</u> variables determined in first-order equations (Wymer (1975) pp 21-24). This gives:

1. $f_{14}x_1 - \theta_1 x_7 + f_2 x_{13} - \theta_2 x_{14} + f_{22} x_8 + f_{11} x_5$
$\qquad + f_{15} x_{12} + f_{18} x_{20} = 0$

2. $f_3 x_2 + \theta_3 x_9 + f_4 x_1 + f_5 x_8 - f_7 x_{19} - f_5 x_{14}$
$\qquad - f_{19} x_{20} = 0$

TABLE

Continuous Variables	Discrete Variables	Variables for RESIMUL	Parameters	Functions
Endogenous			$\theta_1 = \gamma_1$	$f_1 = c_1\theta_{15}$
			$\theta_2 = \gamma_2$	$f_2 = \theta_1\theta_{18}$
DP^0	ΔP^0	$x_1 = \Delta P^0$	$\theta_3 = \gamma_3$	$f_3 = c_1+c_2\theta_3$
DX^0	ΔX^0	$x_2 = \Delta X^0$	$\theta_4 = \gamma_4$	$f_4 = c_2\theta_3\theta_{12}$
DK_p^0	ΔK_p^0	$x_3 = \Delta K_p^0$	$\theta_5 = \gamma_5$	$f_5 = \theta_3\theta_{12}$
DTP^0	ΔTP^0	$x_4 = \Delta TP^0$	$\theta_6 = \gamma_6$	$f_6 = c_2\theta_9$
Di^{t^0}	Δi^{t^0}	$x_5 = \Delta i^{t^0}$	$\theta_7 = \gamma_7$	$f_7 = \theta_3\theta_{11}$
NDA^0	$M(NDA^0)$	$x_6 = M(NDA^0)$	$\theta_8 = \gamma_8$	$f_8 = \theta_4\theta_{13}$
MS^0	$M(MS^0)$	$x_7 = M(MS^0)$	$\theta_9 = \gamma_9$	$f_9 = c_2\theta_4$
			$\theta_{10} = \gamma_{10}$	$f_{10} = c_2\theta_{10}$
Predetermined			$\theta_{11} = \lambda_1$	$f_{11} = c_2\theta_1\theta_{14}$
			$\theta_{12} = \beta_1$	$f_{12} = c_2\theta_{16}$
P^0	MP^0	$x_8 = LP^0$	$\theta_{13} = \beta_2$	$f_{13} = c_1+c_2\theta_5$
X^0	MX^0	$x_9 = LX^0$	$\theta_{14} = \beta_3$	$f_{14} = c_1+c_2\theta_1+c_2\theta_2$
K_p^0	MK_p^0	$x_{10} = LK_p^0$	$\theta_{15} = \beta_4$	$f_{15} = \theta_1\theta_{14}$
TP^0	$M(TP^0)$	$x_{11} = LTP^0$	$\theta_{16} = \beta_5$	$f_{16} = c_2\theta_{15}$
i^{t^0}	Mi^{t^0}	$x_{12} = Li^{t^0}$	$\theta_{17} = \beta_6$	$f_{17} = \theta_5\theta_{13}$
			$\theta_{18} = \beta_7$	$f_{18} = \theta_1\theta_{19}$
Exogenous			$\theta_{19} = \log m_{00}$	$f_{19} = \theta_3\theta_{20}$
Y^0	MY^0	$x_{13} = MY^0$	$\theta_{20} = \log x_0$	$f_{20} = \theta_4\theta_{21}$
P_m^0	MP_m^0	$x_{14} = MP_m^0$	$\theta_{21} = \log c_0$	$f_{21} = \theta_5\theta_{21}$
i^{e^0}	Mi^{e^0}	$x_{15} = Mi^{e^0}$	$\theta_{22} = \log m_0$	$f_{22} = \theta_1+\theta_2$
EXR^0	$M(EXR^0)$	$x_{16} = M(EXR^0)$		
TRG^0	$M(TRG^0)$	$x_{17} = M(TRG^0)$		
MB^0	$M(MB^0)$	$x_{18} = M(MB^0)$		
t	$0{,}785(t-\bar{t})$	$x_{19} = 0{,}785(t-\bar{t})$		
1,0	0,785	$x_{20} = 0{,}785$		

$c_1 = 1{,}0$

$c_2 = 0{,}5$

dence can be placed in these results as neither of the coefficients were significant.

The significant negative coefficient of the time-trend parameter in the Export equation is rather interesting, and is also consistent with earlier models tried out by the author. It suggests that the real level of world demand for South African exports has been declining over the period of the investigation (viz. 1965/I to 1975/I). The actual value of λ_1 is however open to some doubt, and another investigation (Barr (1977)) showed a value of λ_1 close to zero.

Numerically, $\gamma_1 = 0{,}13$ implies that a one percent change in log of real money balances over desired will increase the rate of increase of price changes by 0,13% per quarter (or 0,52% per year).

$\gamma_2 = 0{,}32$ implies that a one percent increase in log of the ratio of foreign to local prices will raise the rate of increase in local prices by 0,32 percent per quarter. $\gamma_3 = 1{,}48$ shows that the supply of exports takes only two months on average to move (in an average-lag/partial-adjustment sense) towards meeting extra desired supplies. The parameters γ_4 to γ_{10} and β_4 to β_6 have similar interpretations.

The value 11,41 for the relative price elasticity of desired supply of exports suggests a marked effect of local price changes (relative to foreign prices) on supply. β_7 and β_2, both income elasticities - the former for real balances, the latter for desired credit level - are both significant, β_7 being greater than one implying a greater-than-proportional change in real balance demand in response to changes in Y. β_3 is not, however, significant in the real-balance function, and β_2 is less than one.

5.8 STABILITY

In models of the type discussed above it is often possible to reduce then to the form:

$$Dy(t) = Ay(t) + f(y,t) .$$

This is usually accomplished by defining $y(t)$ to be a vector consisting of the ratio of the model variables to their steady state paths. If, in this case, the matrix A has eigenvalues whose roots all have negative real parts, and if

$$\frac{f(y,t)}{|y|}$$

tends to zero uniformly in t, $t \geq 0$, then (Coddington and Levinson (1955), ch 13) $y(t) = 0$ is an "asymptotically stable solution."

If the model is completely endogenised $f(y,t)$ usually turns out to be a function of t only, and this simplifies the uniform convergence considerations (for examples of these types of models see Bergström (1967) and Bergström and Wymer

(1974)). If however the model contains exogenous variables then we usually have to assume uniform convergence for f(y,t), which is not very satisfactory. On the other hand it is possible to examine the endogenous part of the sub-model containing only the differential equations. This subsystem had eigenvalues

1. -0,449395
2. -1,475427
3. 0,00000
4. -0,027515
5. -0,581574

(all imaginary parts equal zero). This would imply stable behaviour for the "endogenous model" - apart from the 3rd equation (Private Capital Imports), which would need further investigation.

5.9 THE DIRECTION OF FURTHER RESEARCH

The above model should be seen as a first step towards a more complete representation of the functioning of the economy as a whole. The real sector has been taken as exogenously determined, and the effects of changes in price, interest rates and money on aggregate output needs to be explained. Further, imports must be endogenised to complete the picture of the balance of payments mechanism. The unsatisfactory nature of the interest rate equation suggests that it might be better to concentrate on explaining levels rather than changes in the rate.

Policy reaction functions, quantifying as far as possible the actions of government, are vital if a realistic model is to be had, and the role of expectations, particularly in a situation of highly visible inflationary conditions should also be incorporated.

Given the limitations - self imposed - on the present analysis, it is pleasing to find that the model stands up well to the data, and gives parameter estimates that are, on the whole, significant and realistic.

A P P E N D I X   A

NOTES ON LINEAR REGRESSION ANALYSIS

INTRODUCTION

Although this appendix directly serves the foregoing chapters, an attempt has been made to present the material in a logical sequence so that any reader unfamiliar with econometric analysis might gain some expertise on a read-through. References to textbooks and papers are also given, these will contain further references; in this way the sketchy nature of the material here presented may be suitably "filled out" by those interested. A familiarity with basic statistical concepts is assumed, as is a knowledge of elementary matrix algebra.

A.1 MULTIPLE LINEAR REGRESSION (ORDINARY LEAST SQUARES) AND ASSOCIATED TEST STATISTICS

Consider the General Linear Model (Graybill (1961))

$$Y_t = \beta_1 + \beta_2 X_{2t} + \ldots + \beta_k X_{kt} + u_t \quad (t = 1,\ldots,n)$$

where Y_t is the dependent variable, the X_j are independent variables, the β_j are (constant) regression coefficients and u_t is an error term. The t subscript refers to time: Y_t is the value of Y at time t, etc. (We have n concurrent observations on the Y and X_j variables.)

This may be written in matrix notation as:

$$Y = X\beta + u \qquad \text{(A.1.1)}$$

where $Y = \begin{pmatrix} Y_1 \\ \cdot \\ \cdot \\ \cdot \\ Y_n \end{pmatrix}$, $X = \begin{pmatrix} 1 & X_{21} & \cdots & X_{k1} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ 1 & X_{2n} & \cdots & X_{kn} \end{pmatrix}$, $\beta = \begin{pmatrix} \beta_1 \\ \cdot \\ \cdot \\ \cdot \\ \beta_k \end{pmatrix}$, $u = \begin{pmatrix} u_1 \\ \cdot \\ \cdot \\ \cdot \\ u_n \end{pmatrix}$

The following assumptions are made:

1) u is a real, Normally distributed, (vector) random variable with

$$E(u) = 0, \quad E(u\,u') = \sigma^2 I$$

where I is the $(n\times n)$ identity matrix, and σ^2 is a constant - the variance of the u_t. (Note: this implies $E(u_t u_s) = 0 \quad \forall\, t \neq s$.)

2) u_t and X_{jt} are independent $\forall\, j = 1,\ldots,k$, and any two of the X_j are not perfectly collinear, i.e. $X_i \neq a_0 + a_1 X_j$ $(i \neq j,\ a_i$ constants), and X is of rank k.

3) The equation is correctly specified in an economic context (i.e. no important variables have been excluded or spurious ones included, and the linear form is correct).

4) The n observations on each (X_{it}, Y_t) variable have been given without measurement error.

It can then be shown (Graybill (1961) , Johnston (1963)) that the Best (i.e. minimum variance) Linear Unbiased Estimate (B.L.U.E.) of β is given by:

$$\hat{\beta} = (X'X)^{-1}X'Y \qquad \text{(A.1.2)}$$

(and is Normally distributed, being a linear combination of Normal variables).

To see this, multiply (A.1.2) through by X':

$$X'Y = X'X\beta + X'u \ ,$$

then $\quad \beta = (X'X)^{-1}X'Y - (X'X)^{-1}X'u$

$\therefore \quad E(\beta) = E(\hat{\beta}) - E((X'X)^{-1}X'u)$

$\qquad\qquad = E(\hat{\beta}) - 0$ (by assumption (2) above).

(The minimum variance proof is omitted.)

We now have estimates $\hat{\beta}_j$ of the k β's, and may write, as an estimate of Y_t,

$$\hat{Y}_t = \hat{\beta}_1 + \hat{\beta}_2 X_{2t} + \ldots + \hat{\beta}_k X_{kt} \qquad \text{(A.1.3)}$$

u_t is not observed, but an estimate of u_t is given by: $e_t = (Y_t - \hat{Y}_t)$ - this is the part of Y_t 'unexplained' by the regression equation. It may be shown (Johnston (1963)) that an unbiased estimate of σ^2 is given by:

$$s^2 = (n-k)^{-1}\sum_{t=1}^{n} e_t^2 \ .$$

Further, the variance-covariance matrix of the $\hat{\beta}_j$ is $\sigma^2(X'X)^{-1}$, and this is estimated as $s^2(X'X)^{-1}$ - the diagonal giving the variances.

Thus $\hat{\beta} \sim N(\beta, \sigma^2(X'X)^{-1})$, and so to test the hypothesis $H_0: \beta_j = 0$ against the alternative hypothesis $H_1: \beta_j \neq 0$ the statistic

$$t = \hat{\beta}_j / \hat{\sigma}_{\beta_j} \quad \text{applies.}$$

$(\hat{\sigma}_{\beta_j}) = \sqrt{\widehat{var}(\hat{\beta}_j)}$ is the estimate of the standard error of β_j, and is obtained from $s^2(X'X)^{-1}$.

By the Normality of $\hat{\beta}$, t has a t-distribution with (n-k) degrees of freedom.

A measure of the variation in the dependent variable "explained" by a linear regression is given by:

$$R^2 = \sum_{t=1}^{n}(\hat{Y}-\bar{Y})^2 / \sum_{t=1}^{n}(Y-\bar{Y})^2$$

where $\bar{Y} = n^{-1}\sum_{t=1}^{n} Y_t$.

R^2 is called the Coefficient of Multiple Determination, and its square root, R, is called the Multiple Correlation Coefficient.

As we add more and more X_j's to a regression equation as explanatory variables, R^2 automatically increases although the additional explanation brought about by the extra X_j's may be minimal and /or non-significant. In order to compensate for this we may calculate R^2 "adjusted for degrees of freedom" (Johnston (1963)), i.e. $\bar{R}^2 = 1 - [\frac{n-1}{n-k}(1-R^2)]$.

Multiple Linear Regression is most conveniently undertaken with the use of a computer package (such as the COMET program used to obtain most of the results of Chapter 3). Difficulties arise, however, when some or all of the assumptions (1) to (4) above break down. We shall discuss some of these areas of difficulty, and ways which may be used in detecting and overcoming them to some extent, in the following two sections.

A.2 AUTOCORRELATION, GENERALISED LEAST SQUARES AND THE DURBIN-WATSON STATISTIC

One of the crucial assumptions of the General Linear Model (A.1.1) that we have been discussing so far has been that of zero covariance for the distrubance terms (assumption (2)). This assumption breaks down if $E(u_t u_{t-s}) \neq 0$ ($\forall\ s \neq 0$). Such a situation arises if the successive terms u_t exhibit "serial correlation" (i.e. $E(u_t u_{t-1}) \neq 0$). In this case the Ordinary Least Squares (OLS) formula (A.1.2) will yield an unbiased estimate of β, but the variance-covariance matrix for $\hat{\beta}$ will be incorrect.

<u>Proof</u>: (1) OLS <=> $\hat{\beta} = (X'X)^{-1}X'Y$

$$\therefore E(\hat{\beta}) = (X'X)^{-1}X'X\beta + (X'X)^{-1}X'E(u)$$

$$= \beta$$

but (2) OLS <=> $var(\hat{\beta}) = \sigma^2(X'X)^{-1}$

when in actual fact $\text{var}(\hat{\beta})$ is, in this case,

$$E(\hat{\beta}-\beta)(\hat{\beta}-\beta) = \sigma^2(X'X)^{-1}X'\Omega X(X'X)^{-1}$$

if $$E(uu') = \sigma^2\Omega \qquad (A.2.1)$$

If Ω (and σ^2) are known, a B.L.U.E. of β may be constructed according to the theory of <u>Generalised Least Squares</u> (GLS) (Johnston (1963), p 210), and is

$$b = (X'\Omega^{-1}X)^{-1}X'\Omega Y \qquad (A.2.2)$$

(b is also called "Aitken's estimator" after A.C. Aitken.)

In this case the variance-covariance matrix of b is

$$\text{var}(b) = \sigma^2(X'\Omega^{-1}X)^{-1} \qquad (A.2.3)$$

and b, being a "best" estimator has smaller variance than $\hat{\beta}$ above.

Autocorrelation usually arises when we omit (an) important explanatory variables from the regression equation. It may also occur if the model is mis-specified or if the data used is inappropriate or measured with error.

We are rarely given exact knowledge of Ω and σ^2, and usually construct a theoretical model of the autocorrelation process and hence an estimate of $\sigma^2\Omega$. One such simple model is that of <u>first-order autocorrelation process</u>, i.e.

$u_t = \rho u_{t-1} + \varepsilon_t$, $|\rho| < 1$ and ε_t satisfies:

$E(\varepsilon_t) = 0$

$$E(\varepsilon_t \varepsilon_{t+s}) = \sigma_\varepsilon^2 \quad s = 0 \\ = 0 \quad s \neq 0$$

It may be then shown that

$$E(u_t u_{t-s}) = \rho^s \sigma^2, \quad (\sigma^2 \text{ as above})$$

ρ is called the autocorrelation coefficient. (Johnston (1963), pp. 244-246.)

Similarly we may define a second-order autocorrelation scheme as

$$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \varepsilon_t \qquad (A.2.4)$$

and so on for higher-order processes.

If, for (A.1.1), we suspect a first-order process we may test for it using the Durbin-Watson Statistic (DW) (Durbin and Watson (1950, 1951))

$$DW = \frac{\sum_{t=1}^{n} (e_t - e_{t-1})^2}{\sum_{t=1}^{n} e_t^2}$$

where e_t are the calculated residuals (see Section A.1). It may be shown that DW lies between 0 and 4.

If $DW < 2$ we test for positive autocorrelation as follows: looking at the table of the DW statistic at (say) the 5% level and (say) $n = 20$, we are given two points, viz. $dL = 1,00$ and $dU = 1,68$. If $DW > 1,68$ we reject the hypothesis of positive 1st order autocorrelation. If $DW < 1,00$ we accept the hypothesis. If $1,00 < DW < 1,68$ the test is indeterminate, (but see Durbin (1970) for a more conclusive test). For a test of negative autocorrelation

(DW > 2), compute 4 - DW and proceed as before. One must remember that, in order for the test using DW to be strictly valid, the model (A.1.) must hold accurately together with the first-order assumption. Tests using DW <u>when lagged dependent variable are present</u> as explanatory variables (as well as other non-standard assumptions) have been analysed and it was found (see Johnston (1963)) that the test still works fairly well.

If we knew ρ of a first-order shceme we then have

$$\sigma^2\Omega = \sigma^2 \begin{pmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{n-1} \\ \rho & 1 & \rho & \cdots & \rho^{n-2} \\ \vdots & \vdots & & & \vdots \\ \rho^{n-1} & \rho^{n-2} & \rho^{n-3} & \cdots & 1 \end{pmatrix}$$

and we may apply Aitken's estimator to find b. We are usually never given ρ, and resort to an iterative procedure to estimate its value from the Least-squares residuals, starting from an arbitrary initial value, estimating b, the e_u, and then another estimate of ρ. A modified version of this is the <u>Cochrane-Orcutt</u> procedure (Cochrane and Orcutt (1949)).

When second-order autocorrelation is suspected we resort to a <u>scanning technique</u> to find values for the ρ_i (i = 1,2,): A grid of (ρ_1,ρ_2) values is specified and, combining (A.1.1) with A.2.4) we obtain:

$$Y_t = \rho_1 Y_{t-1} + \rho_2 Y_{t-2} + \beta_1 X_{1t} + \ldots + \beta_k X_{kt}$$
$$- \rho_1\beta_1 X_{1,t-1} - \ldots - \rho_1\beta_k X_{k,t-1}$$
$$- \rho_2\beta_1 X_{1,t-2} - \ldots - \rho_2\beta_k X_{k,t-2} + \varepsilon_t \qquad (A.2.5)$$

Ordinary Least squares is then undertaken on (A.2.5) for each of the (ρ_1,ρ_2) points, the data now being transformed as $(Y_t - \rho_1 Y_{t-1} - \rho_2 Y_{t-2})$, $(X_{it} - \rho_1 X_{i,t-1} - \rho_2 X_{i,t-2}$, $i = 1,\ldots,k;$ $t = 3,\ldots,n)$, and estimates of the β are obtained. The DW test is then applied to the estimated residuals, and the regression giving the DW statistic closest to 2 is taken. This technique is also known as the Hildreth-Lu Scanning method (Hildreth and Lu (1960)). (See Grilliches and Rao (1969) for references to other techniques.)

A.3 MULTICOLLINEARITY AND THE METHOD OF RIDGE REGRESSION

For Multiple Linear Regression to be correctly applied to the system (A.1.1) we require that the rank of X (and hence the rank of $X'X$) to equal k. If this is not so - if condition (2) of Section A.1 is violated - it will be impossible to obtain $(X'X)^{-1}$, and thus $\hat{\beta}$ of (A.1.2) will not exist. A less extreme but equally serious case of this is that of near-perfect collinearity between two or more of the X variables. Johnston (1963 p. 60) lists three consequences of such a situation, viz.

"1) The precision of estimation falls so that it becomes very difficult, if not impossible to disentangle the relevant influences of the various X variables. This loss of precision has three aspects: specific estimates may have large errors; these errors may be highly correlated, one with another; and the sampling variances of the coefficients will be very large.

2) Investigators are sometimes led to drop variables incorrectly from an analysis because their coefficients are not significantly different from zero, but the true situation may not be that a variable has no effect but simply that the sample data has not enabled us to pick it up.

3) Estimates of coefficients become very sensitive to particular sets of sample data, and the addition of a few more observations can sometimes produce dramatic shifts in some of the coefficients."

The above three consequences are mainly a result of the near-singularity of X'X. An extensive test for the presence of multicollinearity has been proposed by Farrar and Glauber (1967). This in essence calls for an examination of the coefficient of multiple determination, R_i^2, for each X_i regressed against the remaining (k-1) X_j variables. It can be immediately seen that such a procedure is time-consuming, and a very simple alternative would be to examine the correlation matrix of the X variables, which is usually a standard option on computer packages. "High" correlations (i.e. those near to ± 1) will suggest possible collinearities.

It is clear that a situation of multicollinearity arises more often than not in econometric work, as we often regress together X variables that are lagged values of one another, or interest rates that move closely, etc. The problem is: how to estimate in such a situation. A technique recently proposed is that of <u>Ridge Regression</u> (Hoerl and Kennard (1970)). This technique is based on the observation that if $\hat{\beta}$ is given by (A.1.2) then

$$E[(\hat{\beta}-\beta)'(\hat{\beta}-\beta)] = \sigma^2 \sum_1^k (1/\lambda_i) > \sigma^2/\lambda_k$$

where the λ_k are the ranked (largest to smallest) eigenvalues of X'X. The further the vectors (columns) of X deviate from orthogonality the smaller becomes λ_k and $\hat{\beta}$ may be expected to be further from β. Ridge estimation replaces $\hat{\beta}$ by

$$\hat{\beta}^* = [X'X + H]^{-1} X'Y \qquad (A.3.1)$$

where H is a diagonal matrix of non-negative constants h_i $(i = 1,\ldots,p)$. Hoerl and Kennard suggest replacing H with hI_p, $h \geq 0$, and choosing an 'optimal' h value by visually inspecting the two-dimensional plot of $\hat{\beta}^*(h)$ (and the residual sum of squares $(n-k)\hat{\sigma}^2(h)$ against h for a number of values of h in $[0,1]$. This gives the <u>Ridge Trace</u>.

This technique works quite well in practice (Marquart and Snee (1975))*, but relies on the discretion of the investigator in visually choosing that h value he thinks optimal. An explicit method for obtaining an "optimal" value for the H Matrix (in the sense of minimum mean square error for the corresponding $\hat{\beta}^*$) is given by Hemmerle (1975). The argument used to derive this estimate is fairly complex and readers are referred to Hemmerle's article. A program has been written to obtain these explicit estimates by Patrick Wong Fung of the Department of Mathematical Statistics, and this was used to obtain the results mentioned in Chapter 3.

A.4 LAG STRUCTURES : KOYCK AND ALMON METHODS

It is (economically) plausible to believe that the effect of a change in an explanatory variable is not limited to the period (day, week, month, etc.) in which it occurs, but has repercussions - usually of a diminishing nature - in several subsequent periods. This is good reason for including lagged variables in the regression equation, for example

$$Y_t = \beta_0 + \beta_1 X_{t-1} + \ldots + \beta_s X_{t-s} + u_t \qquad (A.4.1)$$

There are, however, two reasons for not estimating the above equation as it stands, namely (1) the explanatory variables are likely to be highly correlated with each other and the problem of multicollinearity arises (see Section A.3), and

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*and a program that produces estimates in this way has been written by C. Barr of the Department of Mathematical Statistics - details available on request.

(2) s may be large and if we have only a limited amount of data we lose out on degrees of freedom in estimating the β_i.

We may circumvent these two difficulties if we are prepared to make some assumptions concerning the lag structure. The Koyck Lag (Koyck (1954)) assumes that the β_i decline geometrically in the model

$$Y_t = \beta_0 X_t + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \ldots + u_t \qquad (A.4.2)$$

i.e. $\beta_i = \beta(1-\lambda)\lambda^i$, where β and λ are constants, and $0 < \lambda < 1$. Rewriting (A.4.2) we obtain

$$Y_t = \beta(1-\lambda)[X_t+\lambda X_{t-1}+\lambda^2 X_{t-2}+ \ldots] + u_t \qquad (A.4.3)$$

Doing the same for Y_{t-1}, multiplying by λ, and then subtracting the resulting expression from (A.4.3) we get

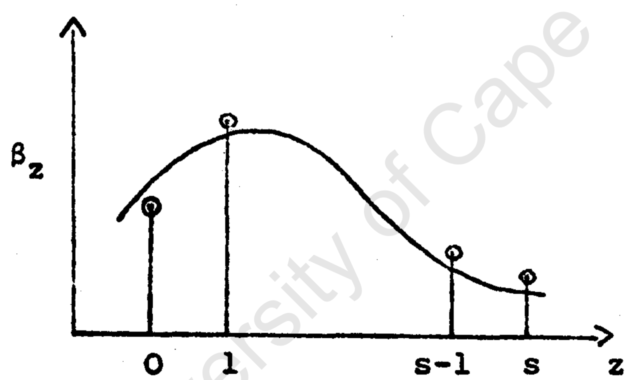
$$Y_t = (1-\lambda)X_t + \lambda Y_{t-1} + v_t \qquad (A.4.4)$$

(Note: similar forms are obtained for certain Stock Adjustment and Adaptive Expectations models - Johnston (1963) pp. 300-303.)

(A.4.4) may now, it appears, be estimated in OLS fashion. Care must, however, be exercised because (a) the v_t are not serially uncorrelated even if the u_t were ($v_t = u_t - \rho u_{t-1}$), (b) the Durbin-Watson test for autocorrelation (Section A.3) will be biased by the presence of a lagged dependent variable (Y_{t-1}), (c) Y_{t-1}, an explanatory variable, will be correlated with v_t (Y_{t-1} is correlated with u_{t-1}).

A somewhat more satisfactory procedure is that of Almon Lags (Almon (1965)) which makes use of the assumption that the lag structure may be approximated by a polynomial of degree r, and the lag coefficients may then be indirectly estimated by Lagrangian Polynomial Interpolation. This method tends to be fairly computationally burdensome and a simplified procedure given by Johnston (1963) is outlined below:

Suppose that we are asked to estimate equation (A.4.1). We suppose that the $\beta_0, \beta_1, \ldots, \beta_s$ look something like:



This suggests approximating the β_i's by a smooth curve $f(z)$ (shown). Note that $f(z) \approx \beta_z$, $(z = 0, ., \ldots, s)$. $f(z)$ may, in turn, be approximated by a polynomial of degree r in z,

i.e. $$f(z) \approx a_0 + a_1 z + \ldots + a_r z^r \qquad (A.4.5)$$

(this follows from a mathematical theorem by Weierstrass which states that a continuous function on a closed interval may be approximated over the interval by a polynomial of suitable degree (Dugundji (1970)). The higher is r the more accurate is the approximation; we choose $r < s$ to save on degrees of freedom. For example, let $r = 3$, $s = 7$, then

$$\begin{aligned}\beta_0 &\approx f(0) \approx a_0 \\ \beta_1 &\approx f(1) \approx a_0 + a_1 + a_2 + a_3 \\ \beta_2 &\approx f(2) \approx a_0 + 2a_1 + 4a_2 + 8a_3 \\ &\vdots \\ \beta_7 &\approx f(z) \approx a_0 + 7a_1 + 49a_2 + 343a_3\end{aligned} \qquad (A.4.6)$$

Substituting these β_i approximations into the (A.4.1) we obtain:

$$\begin{aligned}Y_t \approx\ & a_0(X_y + X_{t-1} + X_{t-2} + \dots + X_{t-7}) \\ & + \dots + a_3(X_{t-1} + 8X_{t-2} + \dots + 343X_{t-7}) \\ & + u_t\end{aligned} \qquad (A.4.7)$$

This is a simple three-explanatory-variable linear equation, and we may perform OLS on it to estimate a_i $(i = 0,1,2,3)$, which are then substituted into (A.4.6) to obtain $\tilde{\beta}_i$ $(i = 0,1,\dots,7)$, the Almon Lag β estimates.

Besides having saved on degrees of freedom we also assume that the explanatory variables in (A.4.7), being different linear combinations of the X_{t-j}, $(j = 0,1,\dots,7)$, are less highly collinear than the X_{t-j} are between themselves. In this way the problem of multicollinearity is seen to be reduced.

The Almon technique may be easily extended to any number of explanatory variables in a single equation. A computer package edited by the author is available for this type of analysis (see Appendix D).

A.5 TESTS ON STRUCTURAL STABILITY: THE 'CHOW' TEST

In econometric regression analysis we often wish to divide the data series into two parts and then test a given model (A.1.1) to determine whether or not there has been a significant change in the set of β coefficients between the two periods, that is, we wish to test for a structural shift in the underlying workings of the model. Such a test has been proposed by Chow (1960), although his deviation is quite complex. A simpler exposition resulting in the same tests has been given by Fischer (1970), and our notes follow his method.

(a) TESTS OF EQUALITY OF THE FULL SET OF REGRESSION CO-EFFICIENTS IN TWO REGRESSIONS WHEN THERE ARE ALWAYS POSITIVE DEGREES OF FREEDOM

We have two samples with T_1 and T_2 observations respectively. Our model is

$$Y_i = X_i\beta_i + u_i \qquad (i = 1,2)$$

(u_i distributed as u_t in Section A.1, etc.), and we wish to test $H: \beta_1 = \beta_2$. To do this write the two regressions together as:

$$Y = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} = \begin{pmatrix} X_1 & 0 \\ 0 & X_2 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} + \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \equiv X^*\beta + u$$

Restricting β_1 to equal β_2 gives the model:

$$Y = \begin{pmatrix} X_1 \\ X_1 \end{pmatrix} \beta_1 + u \equiv X\beta_1 + u.$$

Let the number of parameters (β's) to be estimated be k, and assume $T_1 > k < T_2$. Observing that:

$$X = X^* \begin{pmatrix} I \\ I \end{pmatrix}, \quad \text{define}$$

$$M = I - X(X'X)^{-1}X', \quad M^* = I - X^*(X^{*\prime}X^*)^{-1}X^{*\prime},$$

and let $v = Mu$, $v^* = M^*u$, then it may be shown (Fisher (1970) Lemmas 2.2 and 2.3) that the statistic

$$F = \frac{(v'v - v^{*\prime}v^*)/k}{v^{*\prime}v^*/(T_1+T_2-2k)}$$

is distributed as an $F(k, T_1+T_2-2k)$. In other words the difference (adjusted for degrees of freedom) between the sum of squares of the calculated residuals for the restricted model (i.e. $v'v$) and the unrestricted model (i.e. $v^{*\prime}v^*$) divided by the sum of squares of the unrestricted model (also adjusted for degrees of freedom) has an F distribution, and may be used to test H.

(b) TESTS OF EQUALITY OF THE FULL SET OF REGRESSION CO-EFFICIENTS IN TWO REGRESSIONS WHEN DEGREES OF FREEDOM ARE INADEQUATE

This is the same problem as that of (a), except that

$T_2 \leq k$. Let v be as above, and let

$$v_1 = (I-X_1(X'X)^{-1}X_1')u_1 \equiv M_1u_1 .$$

Writing $M^* = \begin{pmatrix} M_1 & 0 \\ 0 & 0 \end{pmatrix}$ and $v^* = \begin{pmatrix} u_1 \\ 0 \end{pmatrix}$, equivalent results

to the above follow, and

$$F = \frac{(v'v-v^{*\prime}v^*)/T_2}{v^{*\prime}v^*/(T_1-k)}$$

has an $F(T_2,T_1-k)$ distribution, and we may then test $H: \beta_1 = \beta_2$ as before.

A.6 TWO-STAGE LEAST SQUARES REGRESSION

In all of the above five sections we have been considering a model consisting of the single equation (A.1.1). A more informative system is that of G Simultaneous linear equations of the form (Goldberger (1964))

$$\Gamma y + Bx = u \qquad (A.6.1)$$

where y is a $(G\times 1)$ vector of endogenous variables, x is a $(K\times 1)$ vector of exogenous and lagged endognous (i.e. predetermined) variables, u is a $(1\times M)$ matrix of unobserved distrubances, Γ is a $(G\times G)$ matrix of the coefficients of the jointly dependent variables (y), and B is a $(K\times M)$ coefficient matrix for the predetermined variables. If we have n contemporaneous observations on the G y's and M x's we may rewrite (A.6.1) as

$$Y\Gamma + XB = U \qquad (A.6.2)$$

where Y is now the $(n\times G)$ matrix of the n (vector) observations on y, and B is the $(n\times M)$ matrix of corresponding observations on x. U is an $(n\times G)$ matrix of errors. Our task is to estimate Γ and B.

Consider now only the j^{th} equation of (A.6.1), which may be written as

$$y_j = Z_j\delta_j + u_j \qquad \text{(A.6.3)}$$

where $Z_j = [Y_j : X_j]$ is an $(n\times N_j)$ matrix of n observations on all endogenous and predetermined variables that appear in the j^{th} equation <u>besides</u> y_j, and $\delta_j' = [\gamma_j' : \beta_j']$ is the $(N_j\times 1)$ vector of parameters that appear in the j^{th} equation (implicitly, we have normalised the j^{th} equation by setting the coefficient of y_j (viz. γ_{jj}) equal to 1). Leaving aside for the moment problems of <u>identification</u> (see Appendix B), we assume that (A.6.3) is estimable, the Two-Stage-Least-Squares method (TSLS) proceeds as follows:

$$X'y_j = X'Z_j\delta_j + X'u_j \qquad \text{(A.6.4)}.$$

Applying Aitken's method (GLS - see A.2) we obtain an estimate of δ_j as

$$d_j = [Z_j'X(X'X)^{-1}X'Z_j]^{-1}Z_j'X(X'X)^{-1}X'y_j$$

(here, $\Omega = E[X'u_ju_j'X] = \sigma_{jj}(X'X)$) .

Note that Aitken's method is not fully applicable because the $X'Z_j$ matrix contains random (i.e. y) elements. It can be shown, however, (Theil (1971)) that the d_j estimator is asymptotically unbiased with the (asymptotic) covariance matrix

$$S_{jj}[Z_j' X(X'X)^{-1}X'Z_j]^{-1}$$

For a comparison of the TSLS method with that of Instrumental Variables, and also for an alternative (Two-Stage) derivation of d_j, see Theil (1971).

A further, simultaneous, method - that of Three-Stage-Least-Squares Estimation - is not here discussed. For details see Zellner and Theil (1962).

A.7 FULL INFORMATION MAXIMUM LIKELIHOOD ESTIMATION (FIML)

The F.I.M.L. method, originally introduced by Koopmans (Hood and Koopmans (1953)), is a way of estimating the parameter matrices of the model (A.6.2). Considering* the G linear equations (A.6.3), we stack the y_j, δ_j and u_j, and, writing $diag(Z_1, Z_2, \ldots, Z_G)$ as Z (i.e. a "diagonal" matrix with the Z_j matrices down the diagonal) we may set

$$y = Z\delta + u \qquad (A.7.1)$$

(where $y' = (y_1, y_2, \ldots, y_G)'$, etc.). The following assumptions then apply:

(a) Each equation is identified (Appendix B) by virtue of the a priori restrictions on the matrices Γ and B (of (A.6.2)).

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* This discussion is taken largely from Rothenberg and Leenders (1964).

$$S_{jj}[Z_j' X(X'X)^{-1}X'Z_j]^{-1}$$

For a comparison of the TSLS method with that of Instrumental Variables, and also for an alternative (Two-Stage) derivation of d_j, see Theil (1971).

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$$y = Z\delta + u \tag{A.7.1}$$

(where $y' = (y_1, y_2, \ldots, y_G)'$, etc.). The following assumptions then apply:

(a) Each equation is identified (Appendix B) by virtue of the *a priori* restrictions on the matrices Γ and B (of (A.6.2)).

---

* This discussion is taken largely from Rothenberg and Leenders (1964).

(b) (i) plim* $(1/n)X'u_j = 0$

(ii) plim $(1/n)Z'Z_j < \infty \quad \forall\ i,j$

(iii) rank $(X) = M$, rank $(Z) = N$ where $N = \sum_{j=1}^{G} N_j$

(c) u is Normally distributed with

$$E(u) = 0, \quad E(uu') = \Psi = \Sigma \otimes I$$

($\otimes$ is the Kronecker product, Σ is a positive definite matrix; the nonsingularity of Σ implies that the model contains no identities; this may however be generalised.)

It then follows that the joint density of the Gn disturbances is:

$$f(u) = (2\pi)^{-\frac{1}{2}Gn}\det{}^{-\frac{1}{2}}\Psi\exp\{-\tfrac{1}{2}u'\Psi^{-1}u\} \qquad (A.7.2)$$

The Logarithmic Likelihood function is then

$$L(u,\Psi) \equiv \frac{1}{n}\log f(u)$$

$$\equiv K + \frac{1}{2n}\log\det\Psi^{-1} - \frac{1}{2n}u'\Psi^{-1}u \qquad (A.7.3)$$

Transforming from u to y (with Jacobian

$J = \det\left(\frac{\partial u_i}{\partial y_j}\right) = \det{}^n\,\Gamma)$ obtain:

$$L(y,\Psi,\delta,Z) = k + \log|\det\Gamma| + \frac{1}{2n}\log\det\Psi^{-1}$$

$$= \frac{1}{2n}(y-Z\delta)'\Psi^{-1}(y-Z\delta) \qquad (A.7.4)$$

---

*If $\lim_{n\to\infty} \mathrm{Prob}\{|Y_n - c| > \varepsilon\} = 0$ for any $\varepsilon > 0$ where Y_n is a sequence of random variables posessing distribution functions, and c is a constant, then we say $\operatorname{plim} Y_n = c$ (Theil (1971)).

To obtain Maximum Likelihood estimates, first differentiate (A.7.4) with respect to Σ^{-1}, obtaining

$$\frac{\partial L}{\mu\Sigma^{-1}} = \tfrac{1}{2}\Sigma - \frac{1}{2n}(y-Z\delta)'(y-Z\delta).$$

Thus at the maximum

$$\Sigma = \frac{1}{n}(y-Z\delta)'(y-Z\delta) \equiv S \quad \text{must hold.}$$

Substituting S into L (Rothenberg and Leenders (1964), equation 3.4)

$$L^* = k' + \log|\det \Gamma| - \tfrac{1}{2}\log\det S$$

L^* is called the "<u>concentrated likelihood function</u>."

Let

$$p(\delta) = \frac{\partial \log|\det \Gamma|}{\partial\delta}$$

$$q(\delta) = -\tfrac{1}{2}\,\frac{\partial \log\det S}{\partial\delta}$$

Then the estimating equations are

$p(\delta) + q(\delta) = 0$, which are nonlinear in δ and cannot be easily solved. Durban (19**) has proposed a simpler, iterative method for computing the FIML estimates, and which allows for the inclusion of identities in the system. A Newton-Raphson (A.8) procedure is used.

Sargan (1964) has shown that for the FIML estimator given above, if Σ is unrestricted the estimates differ from the Three-Stage estimates of Zellner and Theil (see A.6) by order (1/n)*.

---

*Mann and Wald (1943) define the order symbol 0_p as follows: $x_T = 0_p(f(T))$ if $\exists\ \theta_\varepsilon > 0$ for each $\varepsilon > 0$ such that

$$P(|x_T| \leq \varepsilon f(T)) \geq 1-\varepsilon \quad \forall\, T.$$

**Maximum Likelihood estimation of the parameters of a system of simultaneous regression equations, (unpublished).

It is also shown (Rothenberg and Leenders (1964)) that if Z contains current endogenous variables then the FIML estimator is efficient (in the sense of being "less positive definite") compared to the Three-Stage estimate in the presence of restrictions on Σ.

A.8 THE NEWTON-RAPHSON ITERATIVE PROCEDURE

Let $f(x)$ be a continuous function - called the "objective function" - with x a vector of length n. The (nonlinear) problem can be stated as: Minimise $f(x)$, $x \in \mathbb{R}^n$, subject to m linear and/or nonlinear equality constraints $h_j(x) = 0$, $(j = 1,\ldots,m)$, and $(p-m)$ linear and/or nonlinear inequality constraints $g_i(x) \geq 0$, $(i = m+1,\ldots,p)$.*

Let $$\nabla f(x^k) \equiv \begin{pmatrix} \frac{\partial f(x^k)}{\partial x_1} \\ \cdot \\ \cdot \\ \cdot \\ \frac{\partial f(x^k)}{\partial x_n} \end{pmatrix}$$. i.e. the gradient of $f(x)$

evaluated at the point x^k.

Newton's method involves the approximation of $f(x)$ by neglecting the third- and higher-order terms in the Taylor series expansion, i.e.

---

* For details consult Himmelblau (1972). In what follows we ignore constraints.

$$f(x) \simeq f(x^k) + \nabla' f(x^k)(x-x^k)$$

$$+ \tfrac{1}{2}(x-x^k)'\nabla^2 f(x^k)(x-x^k) \qquad \text{(A.8.1)}$$

($\nabla^2 f(x)$ is called the <u>Hessian</u> matrix of $f(x)$).

Replacing $(x-x^k)$ in (A.8.1) by

$\nabla x^k = (x^{k+1}-x^k)$, we obtain

$$f(x^{k+1}) \simeq f(x^k) + \nabla' f(x^k)\Delta x^k$$

$$+ \tfrac{1}{2}(\Delta x^k)'\nabla^2 f(x^k)(\Delta x^k) \qquad \text{(A.8.2)}$$

Differentiating $f(x)$ with respect to each of the components of Δx and equating the resulting expressions to zero, gives

$$\Delta x^k = -[\nabla^2 f(x^k)]^{-1}\nabla f(x^k)$$

which follows from the rules:

(1) $\frac{\partial}{\partial x}(x'Ax) = 2Ax$ (Anderson (1958) p. 347)

(2) $\frac{\partial}{\partial x}\gamma' x = \gamma$ (may be directly verified)

Thus $x^{k+1} = x^k - [\nabla^2 f(x^k)]^{-1}\nabla f(x^k)$ gives an iteration procedure which may be followed until some termination criterion is achieved.

APPENDIX B

IDENTIFICATION

It is not the purpose of this Appendix to define "Identification," or to give a general discussion of the Identification Problem. A good reference for the case of linear model identification is Johnston (1972). We consider a particular model - that of Chapter 3 - and show that it is identified.

The model is (excluding the identity):

1. $i^t = f(i_{-1}, R_{-1}, (i-i^t)_{-1})$
2. $R = f(R_{-1}, (i-i^e)_{-1})$
3. $NDA = f(R, g_{d-1}, (i-i^t))$
4. $MS = f(i, Y_{-1}, (R+NDA))$

This may be written in log-linear form under the assumption of constant elasticities (Section 3.3). The problem arises in equation 4, since log (R+NDA) is a nonlinear function of the variables R and NDA. Equations 1 and 2 cause no problems as they are both identified, being in reduced form (Goldberger (1964) p 311)).

The log-linear form is:

$$A\, q(\underline{x}) = u$$

where A is $M \times N^0$ and $q(\underline{x})$ is $N^0 \times 1$:

$$\underline{x}' = (i^t, R, NDA, MS \vdots i_{-1}, R_{-1}, \ldots, Y_{-1}, 1)$$
$$\quad\longleftarrow M \longrightarrow \quad \longleftarrow \Lambda \longrightarrow$$

$$q'(\underline{x}) = (\ell n, i^t, \ell nR, \ell n\ NDA, \ell n\ MS, \ell n(R+NDA) \vdots \ell n\ i^t_{-1}, \ldots, \ell nY_{-1}, e)$$
$$\longleftarrow M^0 \longrightarrow \quad \longleftarrow \Lambda^0 \longrightarrow$$

Fischer (1966) has shown that an equation in such a model is identified if and only if:

$$\text{rank}(A^*\phi) = M^*-1$$

where ϕ is a known, constant matrix of prior restrictions (e.g. exclusion restrictions) on the equation, and A^* is the $M^*\times N$ matrix of A augmented by the M^*-M constant vectors $h^1, h^2, \ldots, h^{M^*-M}$, i.e:

$$A^* = \begin{bmatrix} A \\ \cdots \\ h^1 \\ \cdot \\ \cdot \\ \cdot \\ h^{M^*-M} \end{bmatrix}$$

where h^i can be taken to lie in the row kernel of $Q'(\underline{x})$, the Jacobian matrix of $q(\underline{x})$ with rows corresponding to elements of $\underline{x}$, (and where we have excluded from h^i the trivial vector $(0,0,\ldots,0,\alpha)$ which arises from the fact that the last row of $Q'(\underline{x})$ is $(0,\ldots,0)$).

In the present case we have (excluding the last row of zeroes; $RN \equiv R+NDA$):

$$Q'(\underline{x}) = \begin{bmatrix}
i^{t-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & R^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & NDA^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & MS^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & RN^{-1} & RN^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & i_{-1}^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & R_{-1}^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & (i-i^{e})^{-1} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & (i-i^{t})_{-1}^{-1} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & g_{-1}^{-1} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (i-i^{t})^{-1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & i^{-1} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Y_{-1}^{-1}
\end{bmatrix}$$

Considering $hQ'(\underline{x})$, $h = (h_1, \ldots, h_{13})$, we see that:

$$h_1 * i^t = 0, \Rightarrow h_1 = 0$$

Similarly: $h_4 = \ldots = h_{13} = 0$

and
$$\left.\begin{aligned} h_2/R + h_5/RN &= 0 \\ h_3/NDA + h_5/RN &= 0 \end{aligned}\right\}$$

this implies:
$$\left.\begin{aligned} h_2 + h_5 R/RN &= 0 \\ h_3 + h_5 NDA/RN &= 0 \end{aligned}\right\}$$

If RN ($\equiv R+NDA$) remained a constant, k (say), then $h_2 = -kh_5$, but this impossible because if, in the model, we vary g_{d-1} then NDA will vary whereas R remains constant; hence $h_2 = 0 = h_5$, and thus $h_3 = 0$ also. This gives: $h = (0, \ldots, 0)$.

So $A^* = \begin{pmatrix} A \\ \cdots \\ h \end{pmatrix}$, $M^* = M+1$, and we must show that

rank $(A\phi) = M^* - 1$. Writing the model as:

$$By + \Gamma x = u$$

with:

$B = (\beta_{ij})$ $(i = 1,\ldots,4;\ j = 1,\ldots,5)$

$\Gamma = (\gamma_{k\ell})$ $(k = 1,\ldots,4;\ \ell = 1,\ldots,9)$

$(\gamma_{k9}$ constant $\forall\ k)$,

where y is a vector of length M^0 consisting of the first M^0 elements of $q(\underline{x})$ and x the vector of length Λ^0 of the last Λ^0 elements of $q(\underline{x})$, we find, for equation 3, that:

$$(A^*\phi_3) = \begin{pmatrix} \beta_{11} & 0 & 0 & \gamma_{11} & \gamma_{12} & \gamma_{13} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \gamma_{24} & 0 & 0 \\ \beta_{31} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_{41} & \beta_{44} & 0 & 0 & 0 & 0 & 0 & \gamma_{47} & \gamma_{48} \end{pmatrix}$$

Thus: rank $(A^*\phi_3) = 4 = M^*-1$, which implies that equation 3 is identified.

Similarly, for equation 4 rank $(A^*\phi_3) = 4$.

APPENDIX C

THE DATA

Unless otherwise stated, the following data series are taken from the South African Reserve Bank Quarterly Bulletins (SARBQB), and their supplements. All index series have 1965 as base, and all monetary aggregates are expressed in Rand millions.

(a) DATA SERIES USED IN CHAPTER THREE*

1. Money Base (MB) : obtained by considering the Reserve Bank balance sheet Liabilities and excluding all Government and Provincial Administration deposits, Capital and Reserves, and "Other" Liabilities, and "Foreign Loans" (as they will be reflected in Reserves).

2. Foreign Reserves (R) : Total gold and foreign reserves of the Reserve Bank.

3. Net Domestic Assets (NDA) : Calculated as a residual (MB-R) : (this follows the model identity).

4. Money Supply (MS) : Total Money and Near Money.

5. "Target" Interest Rate (i^t) : Two rates were tried:
   (a) a long-term (L.T.) rate, viz. Long Term S.A. government stock rate,
   (b) a short-term (S.T.) rate : the Treasury Bill rate.

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* Monthly data was used in estimating the models of Chapter Three

6. "Free Market" Rate (i) : as in 5. above:
   (a) (L.T.) : Rate on new issues of Company Debentures.
   (b) (S.T.) : The Negotiable Certificate of Deposits (NCD).

7. "Foreign" Interest Rate (i.e.) : as in 5. and 6. :
   (a) (L.T.) : Long-Term British government stock rate.
   (b) (S.T.) : Euro-Dollar Rate (Source of (a) and (b) : International Monetary Fund, "International Financial Statistics."

8. Economic Activity (Y) : Two indicators were tried:
   (a) "Electric Current Generated" (SARBQB 1965 = 100 deseasonalised).
   (b) Gross Domestic Product (GDP) at Market Prices (non-seasonally adjusted).

9. Government Deficit (gd) : We define "true government deficit" to equal the difference between government issues and the sum of government borrowing and government tax receipts.

   In order to estimate the size of current government borrowing it was necessary to consider the change in the total holdings of government debt by

   1. Permanent Building Societies,
   2. Insurers (Long and Short Term)
   3. Private Pension and Provident Funds
   4. Banks (Commercial, Merchant and Hire Purchase), as well as by the Public Debt Commissioners and the Loan Levy Account.

      For Permanent Building Societies we took (total) government borrowing equal to "Total Prescribed investments less Coin, Banknotes and Money at Call" ;

      For the Insurers, (we aggregate the Long and Short term Insurers), total government borrowing is taken as equal to "Government Stock + Local Authority Stock + Public

Corporation Stock + (approximated) Loans to Local Authorities" (where the last-named amount is assumed equal to ½ of "Other Loans").

Pension Funds holdings are taken equal to "Government Stock + Local Authority Stock + Public Corporation Stock + Loans to Local Authorities and to Public Corporations." For Unit trusts, take "(Total) Approved Securities."

In the Banking Sector, we take Commercial Bank holdings equal to ("Total other Prescribed Investments," less NCD's, plus "Other", plus "Treasury Bills, Bills and Advances to the Land Bank, Short-Term government stock and Short-Term debentures of the Land Bank") (up to October 1972, thereafter "Prescribed Investments excluding Liquid Assets").

Merchant Bank holdings are assumed equal to "Reserve Bank Balances" plus "S.T. government stock" plus "Other" liquid assets plus "Other" liquid assets plus (after October 1972) "Other" prescribed investments."

Hire-Purchase, Savings and General Banks holdings assumed equal to "Reserve Bank Balances" plus "S.T. government stock" plus "S.T. Land Bank debentures" plus "Other government stock" plus "Local Authority and Public Corporation Stock."

Notes:

1. With respect to monetary banking institutions, after October 1972 "Liquid Assets" were not included with "Prescribed Investments."

2. We have excluded the Discount Houses in the above consideration in view of their purely intermediary role between financial institutions.

(b) DATA SERIES USED IN CHAPTER FIVE+*

(m_{00}, m_0, c_0, x_0 are constants)

1. P = (Total) Wholesale Price Index (Seasonally Adjusted; 1963 = 1,0)

2. MS = (Total) Money and Near Money

3. Y = G.D.P. (at Market Prices) deflated by P

4. i^t = Treasury Bill Rate

5. P_m = (Laspeyres) Export Price Index for Industrial Countries (1963 = 1,0) (Source: IMF International Financial Statistics)

6. X = Exports deflated by P (including Re-exports and Gold Sales) (Source: S.A. Financial Mail)

7. t = time trend

8. K_p = Cumulated changes in the Capital Account (net of changes in Government Foreign Loans)

9. TP = Total Bank (excluding Reserve Bank) Loans to the Private Sector

10. i^e = United States Treasury Bill Rate (Source: IMF International Financial Statistics)

11. EXR = Excess Liquid Assets of Commercail Banks (i.e. Actual-Required)

12. NDA = Net Domestic Assets (calculated as MB-RG)

13. TRG = Total Bank Credit/Gross Reserves

14. MB = The Money Base (Notes and Coin in Circulation plus Commercial Bank Deposits at the R.B. (excluding "Other Assets"))

15. RG = "Gross Reserves" (total gold and foreign reserves of the Reserve Bank

16. FL = Foreign Borrowing (including SDR position) of the Reserve Bank

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+ I should like to thank Mr. J. Affleck-Graves of the Department of Mathematical Statistics, University of Cape Town, for making available his series for imports and exports.

* Quarterly data, usually obtained by either cumulating or averaging monthly data was used.

17. TBC = Total Bank Credit (calculated as TP + TGL)

18. TGL = Total Bank Loans to the Government (excluding Reserve Bank Lending)

19. M = Imports

20. DKG = "Gross Capital Inflow" (Private plus Government Foreign Borrowing)

(c) DATA LISTINGS

The following symbols apply for the data series given under (a) above:

MS : Money Supply
MB : Money Base
R : Foreign Reserves
ELEC : (or E) : Electricity Consumption
Y : (or y) : Gross Domestic Product at Market Prices
NCD : (or i) : 90 day Negotiable Certificate of Deposit Rate
TBR : (or i^t): Treasury Bill Rate
ED : (or i^e): Eurodollar Rate
LTSA : (i^t in LT Results) : Long-Term S.A. Bond Rate
LTUK : (i^e in LT Results) : Long-Term U.K. Bond Rate
COYDEB : (i in LT Results) : South Africa Company Debenture Rate
GO : Government Issues
TO : Government (Tax) Receipts
TG : Total Government Borrowing
TC : Total Captive Market (not used in present analysis)
PDCLL : Public Debt Commissioners and Loan Levy Accounts
GDEF : (gd) : Government Deficit

(Note : Zeroes are inserted where figures were not available.)

Symbols used for the quarterly data correspond to those given in (b) above.

TABLE

		HS	HB	R	ELEC	Y
1965	1	2107.	429.0	451.0	103.7	630.0
	2	2107.	430.0	430.0	113.7	630.0
	3	2107.	457.0	425.0	117.5	640.0
	4	2179.	438.0	382.0	116.9	650.0
	5	2193.	440.0	354.0	116.7	670.0
	6	2213.	449.0	328.0	118.6	670.0
	7	2243.	450.0	332.0	116.6	670.0
	8	2249.	441.0	327.0	118.5	670.0
	9	2286.	457.0	320.0	119.8	670.0
	10	2357.	450.0	338.0	120.8	670.0
	11	2403.	457.0	346.0	121.9	670.0
	12	2488.	477.0	383.0	120.5	670.0
1966	1	2415.	470.0	416.0	122.9	670.0
	2	2413.	479.0	435.0	119.9	670.0
	3	2372.	473.0	450.0	123.3	680.0
	4	2422.	474.0	456.0	122.9	690.0
	5	2443.	477.0	483.0	124.1	700.0
	6	2522.	486.0	506.0	120.6	710.0
	7	2553.	498.0	536.0	126.2	720.0
	8	2535.	497.0	538.0	126.7	730.0
	9	2596.	508.0	542.0	128.6	730.0
	10	2597.	508.0	539.0	128.7	730.0
	11	2587.	512.0	527.0	128.3	730.0
	12	2644.	533.0	521.0	131.5	730.0
1967	1	2587.	504.0	500.0	133.4	730.0
	2	2546.	496.0	481.0	135.9	730.0
	3	2559.	532.0	468.0	131.1	740.0

	4	2577.	505.0	440.0	137.2	750.0
	5	2552.	502.0	407.0	135.6	770.0
	6	2602.	514.0	411.0	136.0	780.0
	7	2635.	529.0	418.0	134.1	790.0
	8	2653.	522.0	414.0	137.7	830.0
	9	2675.	540.0	420.0	137.1	820.0
	10	2725.	525.0	441.0	138.8	810.0
	11	2774.	535.0	464.0	140.6	800.0
	12	2857.	563.0	485.0	140.6	800.0
1968	1	2807.	534.0	518.0	140.4	800.0
	2	2812.	529.0	554.0	142.9	600.0
	3	2928.	545.0	615.0	143.5	810.0
	4	2920.	536.0	664.0	144.9	820.0
	5	2959.	553.0	737.0	146.5	830.0
	6	3037.	561.0	769.0	149.1	840.0
	7	3027.	566.0	884.0	149.1	850.0
	8	3085.	561.0	906.0	149.3	870.0
	9	3198.	577.0	902.0	149.3	870.0
	10	3283.	593.0	906.0	149.7	870.0
	11	3500.	605.0	930.0	152.5	870.0
	12	3448.	647.0	985.0	152.4	870.0
1969	1	3373.	674.0	1035.	152.4	870.0
	2	3517.	662.0	1081.	152.8	670.0
	3	3379.	703.0	1121.	156.2	890.0
	4	3408.	696.0	1175.	154.2	910.0
	5	3471.	742.0	1096.	158.6	930.0
	6	3468.	758.0	1103.	158.8	940.0
	7	3402.	750.0	1003.	160.4	950.0
	8	3469.	699.0	966.0	162.4	970.0
	9	3590.	695.0	916.0	165.2	980.0

	10	3619.	695.0	916.0	164.2	990.0
	11	3675.	622.0	915.0	164.1	1000.
	12	3775.	707.0	881.0	162.7	990.0
1970	1	3647.	700.0	914.0	169.4	980.0
	2	3632.	678.0	879.0	169.3	970.0
	3	3701.	722.0	856.0	170.4	990.0
	4	3771.	726.0	845.0	174.5	1010.
	5	3808.	770.0	824.0	174.5	1030.
	6	3902.	790.0	902.0	175.1	1030.
	7	3903.	808.0	866.0	177.0	1030.
	8	3864.	791.0	805.0	175.8	1030.
	9	3933.	805.0	807.0	176.9	1020.
	10	3934.	827.0	794.0	180.8	1100.
	11	3931.	850.0	745.0	179.4	1100.
	12	3983.	856.0	681.0	181.9	1020.
1971	1	3873.	858.0	649.0	183.6	1020.
	2	3923.	842.0	641.0	182.4	1030.
	3	3902.	859.0	637.0	183.8	1060.
	4	3963.	843.0	619.0	183.7	1090.
	5	3999.	838.0	611.0	186.0	1130.
	6	4034.	843.0	570.0	187.8	1140.
	7	4088.	858.0	519.0	193.6	1150.
	8	4082.	838.0	486.0	194.3	1170.
	9	4151.	889.0	477.0	193.6	1180.
	10	4202.	876.0	460.0	196.8	1190.
	11	4219.	893.0	415.0	195.0	1200.
	12	4276.	816.0	464.0	197.6	1190.
1972	1	4113.	796.0	505.0	195.6	1180.
	2	4253.	785.0	520.0	197.6	1170.
	3	4185.	833.0	562.0	196.7	1190.

	4	4243.	835.0	416.0	198.6	1210.
	5	4276.	818.0	673.0	201.5	1230.
	6	4407.	829.0	732.0	200.3	1260.
	7	4440.	839.0	823.0	204.6	1290.
	8	4509.	823.0	839.0	200.6	1330.
	9	4605.	870.0	858.0	205.5	1350.
	10	4580.	841.0	916.0	207.9	1370.
	11	4691.	859.0	916.0	207.7	1400.
	12	4863.	886.0	936.0	214.3	1400.
1973	1	4759.	842.0	984.0	215.2	1400.
	2	4893.	840.0	998.0	217.2	1400.
	3	5062.	847.0	1080.	220.5	1440.
	4	5272.	887.0	1138.	223.4	1480.
	5	5364.	888.0	1195.	220.1	1530.
	6	5446.	933.0	1196.	216.5	1560.
	7	5542.	933.0	1268.	223.0	1590.
	8	5558.	950.0	1176.	225.2	1630.
	9	5748.	1005.	1080.	226.9	1660.
	10	5656.	980.0	1066.	227.1	1690.
	11	5725.	1010.	924.0	229.9	1700.
	12	5983.	1058.	796.0	228.7	1700.
1974	1	5881.	1027.	772.0	232.5	1700.
	2	6038.	1061.	835.0	232.8	1700.
	3	6161.	1106.	923.0	236.7	1760.
	4	6191.	1095.	820.0	240.2	1820.
	5	6391.	1090.	800.0	238.6	1870.
	6	6497.	1122.	777.0	241.8	1890.
	7	6501.	1119.	784.0	242.1	1910.
	8	6438.	1152.	729.0	248.3	1930.
	9	6657.	1218.	737.0	254.3	1960.

10	6815.	1179.	754.0	245.6	1990.
11	7049.	1292.	817.0	249.2	2000.
12	7317.	1241.	729.0	249.1	2000.

TABLE

		HCD	TBR	ED	LTSA	LTUK	COYDER
1965	1	.0000	3.830	.0000	5.000	6.280	6.500
	2	.0000	3.930	.0000	5.000	6.300	6.500
	3	.0000	3.940	.0000	5.250	6.320	6.500
	4	.0000	3.900	.0000	5.250	6.440	6.500
	5	.0000	3.900	.0000	5.500	6.570	6.500
	6	.0000	3.980	.0000	5.500	6.650	6.500
	7	.0000	4.190	.0000	5.500	6.670	7.000
	8	.0000	4.250	.0000	6.000	6.540	7.000
	9	.0000	4.240	.0000	6.000	6.240	7.250
	10	.0000	4.170	.0000	6.000	6.260	7.250
	11	.0000	4.170	.0000	6.000	6.310	7.250
	12	.0000	4.260	.0000	6.000	6.440	7.250
1966	1	.0000	4.430	.0000	6.000	6.440	7.250
	2	.0000	4.360	.0000	6.000	6.500	7.250
	3	.0000	4.340	.0000	6.000	6.660	7.250
	4	.0000	4.160	.0000	6.000	6.690	7.250
	5	.0000	4.120	.0000	6.000	6.750	7.250
	6	.0000	3.840	.0000	6.000	6.900	7.250
	7	.0000	4.080	.0000	6.500	7.040	7.500
	8	.0000	4.080	.0000	6.500	7.170	7.500
	9	.0000	4.250	.0000	6.500	7.120	7.500
	10	.0000	4.160	.0000	6.500	6.900	7.500
	11	.0000	4.260	.0000	6.500	6.780	7.500
	12	.0000	4.560	.0000	6.500	6.760	7.500
1967	1	.0000	4.810	.0000	6.500	6.610	7.500
	2	.0000	4.840	.0000	6.500	6.400	8.000
	3	.0000	5.000	.0000	6.500	6.390	8.000

	4	.0000	4.760	.0000	6.500	6.340	8.000
	5	.0000	4.820	.0000	6.500	6.510	8.000
	6	.0000	4.940	.0000	6.500	6.680	8.250
	7	.0000	4.970	.0000	6.500	6.870	8.250
	8	.0000	4.750	.0000	6.500	6.800	8.250
	9	.0000	4.820	.0000	6.500	6.790	8.500
	10	.0000	4.890	.0000	6.500	6.870	8.500
	11	.0000	4.980	.0000	6.500	7.020	8.500
	12	.0000	5.010	.0000	6.500	7.130	8.500
1948	1	.0000	5.000	.0000	6.500	7.130	8.500
	2	.0000	5.050	.0000	6.500	7.180	8.500
	3	.0000	5.000	.0000	6.500	7.260	8.500
	4	.0000	5.030	.0000	6.500	7.180	8.500
	5	.0000	4.850	.0000	6.500	7.260	8.500
	6	.0000	4.780	.0000	6.500	7.460	8.500
	7	.0000	4.840	.0000	6.500	7.550	8.500
	8	.0000	4.590	.0000	6.500	7.440	8.500
	9	.0000	4.500	.0000	6.500	7.430	8.500
	10	.0000	4.570	.0000	6.500	7.440	8.500
	11	.0000	4.760	.0000	6.500	7.630	8.500
	12	.0000	4.660	.0000	6.500	7.990	8.500
1949	1	.0000	4.690	.0000	6.500	8.240	8.500
	2	.0000	4.680	.0000	6.500	8.560	8.500
	3	.0000	4.680	.0000	6.500	8.660	8.500
	4	.0000	4.660	.0000	6.500	8.780	8.500
	5	.0000	4.680	.0000	6.500	9.080	8.500
	6	.0000	4.690	.0000	6.500	9.460	8.500
	7	.0000	4.740	.0000	6.500	9.060	8.500
	8	.0000	4.660	.0000	6.500	9.180	8.500
	9	.0000	4.600	.0000	6.500	9.220	8.750

	10	.0000	4.400	.0000	6.500	8.760	8.750
	11	.0000	4.120	.0000	6.500	8.290	8.750
	12	.0000	4.230	.0000	6.500	8.940	8.500
1970	1	.0000	4.300	9.420	6.500	8.820	8.500
	2	.0000	4.440	9.420	6.500	8.530	8.500
	3	.0000	4.440	9.420	6.500	8.640	9.400
	4	.0000	4.400	8.350	6.500	8.810	9.500
	5	.0000	4.380	8.350	7.000	9.370	9.500
	6	.0000	4.360	9.400	7.000	9.510	9.500
	7	.0000	4.340	8.690	7.000	9.270	9.800
	8	.0000	4.340	8.160	7.750	9.180	9.800
	9	.0000	4.360	8.050	7.750	9.390	9.750
	10	.0000	4.400	7.940	7.750	9.290	9.750
	11	7.650	4.460	7.170	7.750	9.780	10.00
	12	7.900	4.530	7.290	7.750	9.690	10.00
1971	1	8.600	4.640	5.930	7.750	9.510	10.25
	2	8.400	4.880	5.600	7.750	9.330	10.25
	3	9.050	5.040	5.100	7.750	9.070	10.25
	4	8.030	5.510	5.950	7.750	9.070	10.25
	5	8.000	5.520	7.080	7.750	9.030	10.25
	6	7.850	5.510	7.160	7.750	9.080	10.25
	7	7.900	5.480	6.460	7.750	8.840	10.00
	8	6.650	5.420	8.210	7.750	8.820	10.00
	9	8.200	5.540	8.460	7.750	8.450	10.00
	10	8.000	5.600	6.600	7.750	8.230	10.25
	11	8.350	5.770	6.280	7.750	8.070	10.25
	12	8.700	5.960	6.110	8.500	8.060	10.25
1972	1	8.450	5.950	5.370	8.500	7.820	10.25
	2	7.750	5.940	5.150	8.500	7.790	10.25
	3	8.300	5.940	5.280	8.500	8.030	10.25

	4	6.400	5.620	5.270	8.500	8.120	10.25
	5	6.500	5.550	4.820	8.500	8.440	10.25
	6	6.650	5.530	5.060	8.500	9.060	10.25
	7	6.200	5.320	5.580	8.500	8.990	9.500
	8	5.000	4.500	5.490	8.500	9.140	9.500
	9	5.450	4.960	5.420	8.500	9.330	9.500
	10	5.700	4.920	6.000	8.500	9.260	9.500
	11	6.300	4.840	5.770	8.500	9.450	9.500
	12	5.850	4.380	6.040	8.130	9.620	9.500
1973	1	5.300	4.260	6.170	8.130	9.680	9.500
	2	6.000	4.280	7.450	8.130	9.750	9.500
	3	6.000	4.190	8.500	8.130	10.13	9.500
	4	4.500	3.340	8.160	8.130	10.03	9.380
	5	4.000	2.840	8.430	8.130	10.12	9.380
	6	4.500	2.640	8.810	8.130	10.15	9.250
	7	3.600	2.500	10.37	8.130	10.60	9.250
	8	5.300	2.480	11.46	8.130	11.30	9.130
	9	6.750	2.650	11.13	8.130	11.55	9.200
	10	6.600	2.700	9.930	8.130	11.28	9.250
	11	7.700	2.900	9.820	8.130	12.00	9.250
	12	8.000	3.280	10.63	8.000	12.50	9.500
1974	1	9.000	4.680	9.370	8.250	12.89	9.750
	2	9.250	4.760	8.500	8.250	13.50	9.750
	3	9.000	4.820	9.230	8.250	13.66	9.750
	4	10.00	4.820	10.53	8.250	14.21	9.750
	5	9.750	4.820	11.67	8.250	13.80	9.750
	6	11.00	5.720	12.11	8.500	14.38	11.20
	7	12.50	5.840	13.49	9.250	14.88	11.75
	8	14.25	6.360	13.56	9.750	14.95	11.75
	9	17.25	6.100	12.34	9.750	15.68	13.25

10	14.75	6.000	10.13	9.750	15.09	13.25
11	11.25	6.040	10.13	9.750	16.75	13.25
12	11.35	5.920	10.31	9.500	17.18	13.25

TABLE

		GO	TO	TG	TC	PDCLL	GDEF
1965	1	118.9	102.3	1696.	919.0	28.50	.0000
	2	89.90	130.8	1696.	919.0	-15.10	-25.80
	3	290.5	112.2	1696.	919.0	131.2	47.10
	4	95.50	76.50	1530.	927.0	.0000	185.3
	5	94.10	75.10	1536.	934.0	.0000	-7.100
	6	111.9	81.90	1553.	976.0	.0000	12.90
	7	133.8	121.3	1579.	968.0	12.90	-26.10
	8	135.2	142.2	1517.	969.0	-10.60	65.60
	9	136.1	108.1	1521.	950.0	-16.10	39.90
	10	130.8	90.40	1556.	1124.	9.700	-4.400
	11	161.3	108.5	1608.	957.0	22.20	-21.60
	12	116.1	110.0	1676.	1011.	.1000+00	-62.00
1966	1	134.4	117.8	1704.	989.0	17.60	-28.60
	2	106.3	111.3	1653.	1001.	-16.80	62.90
	3	222.7	140.2	1776.	1004.	32.10	-73.00
	4	110.8	71.40	1743.	999.0	7.800	64.30
	5	119.1	84.50	1808.	1116.	18.70	-48.90
	6	139.5	106.7	1893.	1051.	12.40	-69.20
	7	154.4	138.2	1923.	1037.	14.20	-23.80
	8	149.4	142.4	1915.	1041.	6.900	6.100
	9	149.8	119.8	1966.	1072.	5.200	-26.10
	10	142.3	95.00	1990.	1073.	.0000	23.70
	11	123.4	124.6	2040.	1076.	24.30	-75.40
	12	118.2	119.0	2120.	1127.	13.60	-94.90
1967	1	142.5	152.7	2114.	1115.	12.00	-15.90
	2	164.1	124.2	2096.	1128.	13.40	-15.10
	3	322.3	156.9	2165.	1135.	77.20	18.60

	4	124.3	89.90	2093.	1119.	22.40	84.40
	5	131.6	133.2	2115.	1123.	18.40	-12.00
	6	128.9	150.9	2178.	1167.	6.800	-91.80
	7	159.0	156.5	2135.	1155.	11.50	33.40
	8	166.4	171.8	2227.	1167.	10.50	-107.1
	9	147.3	134.5	2260.	1185.	3.300	-24.10
	10	152.6	105.9	2390.	1196.	26.90	-110.4
	11	139.0	124.0	2401.	1207.	18.20	-15.00
	12	122.3	126.9	2421.	1220.	3.000	-27.10
1968	1	154.3	185.1	2452.	1238.	33.90	-96.00
	2	129.7	155.5	2411.	1245.	13.80	1.700
	3	430.6	150.4	2455.	1255.	34.70	201.2
	4	128.0	108.7	2617.	1273.	43.10	-185.2
	5	149.6	106.3	2527.	1278.	9.300	123.4
	6	164.4	124.2	2633.	1338.	14.70	-80.00
	7	192.2	204.0	2649.	1325.	23.10	-50.20
	8	187.3	127.5	2654.	1338.	45.80	8.600
	9	161.7	139.2	2644.	1373.	-27.10	60.00
	10	144.8	130.2	2731.	1398.	9.600	-82.30
	11	147.0	136.7	2701.	1452.	4.600	36.00
	12	115.1	144.2	2881.	1524.	22.60	-232.4
1969	1	171.0	185.1	2859.	1522.	22.00	-13.20
	2	135.3	120.9	2814.	1538.	40.50	18.20
	3	375.7	157.9	2912.	1536.	11.30	108.8
	4	147.7	108.8	3049.	1568.	163.5	-261.7
	5	148.0	133.9	2932.	1620.	29.80	200.9
	6	161.3	166.7	2961.	1647.	.2000	-135.3
	7	220.7	251.3	2992.	1598.	34.20	-96.40
	8	200.2	173.5	2887.	1584.	-16.20	148.0
	9	216.9	158.5	2902.	1640.	-13.40	57.10

10	476.0	348.0	4841.	2941.	-11.00	152.0
11	530.0	442.0	4847.	2962.	24.00	58.00
12	504.0	343.0	4914.	2995.	57.00	35.00

		P	X	TP	TBR	MS	Y	ED	EXR	TGL	MB	FL	RG
1965	1	104.7	65,60	2141.	3,830	2107.	630.0	3,830	93,00	480,0	429,0	7.100	451.0
	2	105.0	81.40	2141.	3,930	2107.	630,0	3,930	93,00	480,0	430,0	7.100	430.0
	3	104.9	106.3	2141.	3.940	2107.	640.0	3.940	160.0	480,0	457,0	7.100	425,0
	4	105.2	91.40	2186.	3.900	2179.	650.0	3.930	99,00	450,0	438,0	7.100	382,0
	5	105.5	77,00	2212.	3.900	2190.	670.0	3.900	121.0	458,0	440.0	7.100	354.0
	6	105.5	95,70	2213.	3,980	2213.	670.0	3,810	81,00	470,0	444,0	14,30	328.0
	7	105.9	68,40	2213.	4,190	2243.	670.0	3,830	38,00	468,0	450,0	21,40	332,0
	8	105.8	96,40	2207.	4.250	2249.	670.0	3.840	86,00	452.0	441.0	28,60	327.0
	9	105.8	85,00	2175.	4.240	2286.	670.0	3,910	39,00	446.0	457,0	35,70	320.0
	10	105.8	97,10	2161.	4.170	2357.	670.0	4.030	66,00	492,0	450.0	30,70	338.0
	11	105.8	91,60	2131.	4,170	2400.	670.0	4.080	120.0	500,0	457,0	30,20	346.0
	12	106.5	84,70	2113.	4,260	2488.	670.0	4,360	123,0	483,0	477,0	30,60	383.0
1966	1	107.2	70,40	2080.	4.430	2415.	670,0	4,600	65,00	582,0	470,0	32,70	416.0
	2	108.6	85,60	2058.	4,360	2413.	670.0	4,670	87,00	592,0	479,0	36,00	435.0
	3	107.9	110.4	2033.	4,340	2372.	680,0	4,630	88,00	607,0	473,0	40,00	450.0
	4	108.6	87,80	2026.	4.160	2422.	690,0	4,610	129,0	664,0	474,0	40,00	456.0
	5	109.0	99,40	2008.	4.120	2445.	700,0	4,640	175.0	688,0	477,0	54,30	483,0
	6	109.4	106.6	2068.	3,840	2522.	710.0	4.540	220.0	754,0	486,0	54,30	506.0
	7	109.5	85,20	2093.	4,080	2558.	720,0	4,860	163.0	768,0	498,0	60,50	536.0
	8	110.1	119.3	2093.	4,080	2535.	730.0	4,930	243.0	811,0	497,0	49,30	538.0
	9	110.9	95,00	2134.	4,250	2596.	730.0	5,360	195,0	836,0	508,0	46,80	542.0
	10	112.0	114,8	2146.	4.160	2597.	730,0	5,390	217.0	807,0	508,0	43,20	539.0
	11	111.2	112.6	2154.	4,260	2587.	730.0	5,340	226.0	808.0	512,0	43,20	527.0
	12	111.3	99,20	2218.	4,560	2644.	730,0	5.010	233.0	807,0	533,0	46,10	521.0
1967	1	111.5	89,80	2336.	4,810	2587.	730.0	4,760	139.0	810.0	504,0	33,90	500.0
	2	111.5	105.9	2314.	4,840	2546.	730.0	4.550	121.0	740.0	496,0	33,90	481.0
	3	111.9	118.1	2323.	5.000	2559.	740.0	4.290	119.0	675,0	532.0	33,90	468.0

4	112.1	107.3	2350.	4.760	2577.	750.0	3.850	73.00	652.C	505.0	33.90	440.0
5	112.1	109.2	2357.	4.820	2552.	770.0	3.640	125.0	670.0	502.Q	34.10	407.0
6	112.9	110.7	2413.	4.940	2682.	780.0	3.480	167.0	680.0	514.0	30.50	411.0
7	112.8	115.4	2428.	4.970	2635.	790.0	4.310	154.0	689.0	529.0	69.00	418.0
8	112.7	120.4	2444.	4.750	2655.	830.0	4.280	247.0	715.0	522.0	69.00	414.0
9	112.5	116.5	2434.	4.820	2675.	820.0	4.450	239.0	697.0	548.0	42.60	420.0
10	112.4	126.3	2428.	4.890	2725.	810.0	4.590	315.0	860.0	525.0	49.70	441.0
11	112.3	123.5	2468.	4.980	2774.	800.0	4.760	285.0	820.0	535.0	49.70	464.0
12	112.7	118.3	2497.	5.010	2857.	800.0	5.010	359.0	780.0	563.0	67.50	485.0
1968 1	113.1	115.8	2500.	5.000	2807.	800.0	5.080	203.0	775.0	534.0	67.50	518.0
2	113.1	137.1	2533.	5.050	2812.	800.0	4.970	200.0	808.0	529.0	71.00	554.0
3	113.2	132.1	2557.	5.080	2828.	810.0	5.140	193.0	789.0	545.0	71.00	615.0
4	113.2	141.7	2546.	5.030	2920.	820.0	5.360	238.0	794.0	536.0	57.60	664.0
5	113.3	129.7	2537.	4.850	2959.	830.0	5.620	278.0	835.0	553.0	23.30	737.0
6	113.3	109.8	2568.	4.780	3037.	840.0	5.540	337.0	880.0	561.0	23.30	769.0
7	113.3	135.7	2573.	4.840	3027.	850.0	5.390	275.0	943.0	566.0	23.30	884.0
8	113.5	124.5	2605.	4.590	3085.	870.0	5.100	277.0	930.0	561.0	59.00	906.0
9	113.5	117.4	2648.	4.500	3198.	870.0	5.200	293.0	954.0	577.0	59.00	902.0
10	113.8	130.8	2660.	4.570	3283.	870.0	5.330	276.0	960.0	593.0	59.00	906.0
11	114.2	120.5	2821.	4.760	3500.	870.0	5.490	246.0	954.0	605.0	59.00	930.0
12	115.1	108.5	2744.	4.660	3448.	870.0	5.920	300.0	948.0	647.0	59.00	985.0
1969 1	115.0	111.6	2730.	4.690	3370.	870.0	6.180	203.0	934.0	674.0	59.00	1035.
2	115.1	128.7	2839.	4.680	3417.	870.0	6.160	209.0	936.0	662.0	55.40	1081.
3	115.3	134.2	2875.	4.680	3379.	890.0	6.080	153.0	927.0	703.0	55.40	1121.
4	116.1	132.4	2835.	4.640	3408.	910.0	6.150	133.0	954.0	696.0	55.40	1175.
5	116.7	116.0	2936.	4.680	3471.	930.0	6.080	104.0	944.0	742.0	55.40	1096.
6	115.8	129.2	2950.	4.690	3468.	940.0	6.490	144.0	943.0	758.0	55.40	1103.
7	115.8	106.8	2986.	4.740	3402.	950.0	7.000	12.00	873.0	750.0	55.40	1003.
8	116.1	134.4	3013.	4.660	3468.	970.0	7.010	123.0	862.0	699.0	65.40	966.0
9	116.5	127.9	3132.	4.600	3590.	980.0	7.130	146.0	880.0	695.0	55.40	916.0

	4	132.7	165.5	3833.	5.620	4293.	1210.	3.700	148.0	855.0	835.0	100.9	616.0
	5	133.5	168.6	3823.	5.550	4276.	1230.	3.680	186.0	1010.	818.0	85.30	673.0
	6	134.8	169.3	3940.	5.530	4407.	1260.	3.870	241.0	1047.	829.0	54.00	732.0
	7	135.7	153.4	3915.	5.320	4440.	1290.	4.060	262.0	1078.	839.0	37.30	823.0
	8	137.5	179.0	3982.	4.500	4509.	1330.	4.010	266.0	1158.	823.0	47.30	839.0
	9	138.8	166.9	4096.	4.960	4605.	1350.	4.650	253.0	1217.	870.0	47.30	858.0
	10	140.4	180.0	4110.	4.920	4560.	1370.	4.720	233.0	1176.	841.0	47.60	916.0
	11	141.2	180.0	4186.	4.840	4691.	1400.	4.770	217.0	1185.	859.0	47.40	916.0
	12	142.3	168.6	4302.	4.380	4863.	1400.	5.060	322.0	1211.	886.0	47.40	936.0
1973	1	144.4	159.5	4360.	4.260	4759.	1400.	5.310	204.0	1275.	842.0	47.40	984.0
	2	145.8	170.6	4454.	4.280	4893.	1400.	5.560	156.0	1310.	848.0	39.10	998.0
	3	149.1	220.9	4663.	4.190	5062.	1440.	6.050	143.0	1267.	867.0	39.10	1080.
	4	149.0	208.6	4711.	3.340	5272.	1480.	6.290	210.0	1363.	887.0	21.50	1138.
	5	151.0	215.6	4805.	2.840	5364.	1530.	6.350	245.0	1439.	868.0	4.700	1195.
	6	152.5	206.4	5026.	2.640	5446.	1560.	7.190	160.0	1426.	933.0	4.700	1196.
	7	153.8	208.3	5142.	2.580	5542.	1590.	8.020	253.0	1475.	933.0	.0000	1268.
	8	156.4	208.6	5360.	2.480	5558.	1630.	8.670	145.0	1443.	950.0	.0000	1174.
	9	157.8	174.4	5496.	2.650	5748.	1660.	8.480	166.0	1251.	1005.	.0000	1080.
	10	159.5	203.6	5500.	2.700	5656.	1690.	7.160	174.0	1237.	980.0	.0000	1066.
	11	161.6	187.1	5401.	2.800	5725.	1700.	7.870	81.00	1231.	1040.	.0000	924.0
	12	163.7	163.1	5830.	3.280	5983.	1700.	7.360	104.0	1234.	1058.	.0000	796.0
1974	1	165.5	163.4	6010.	4.680	5881.	1700.	7.760	-119.0	1227.	1027.	.0000	772.0
	2	167.9	195.0	6163.	4.760	6038.	1700.	7.060	-239.0	1267.	1061.	.0000	835.0
	3	169.7	257.0	6185.	4.820	6161.	1760.	7.960	-299.0	1277.	1104.	.0000	923.0
	4	172.0	249.6	6270.	4.820	6191.	1820.	8.330	-74.00	1322.	1095.	.0000	828.0
	5	176.4	256.5	6320.	4.820	6391.	1870.	8.230	5.000	1233.	1090.	.0000	800.0
	6	181.1	249.5	6583.	5.720	6497.	1890.	7.900	20.00	1205.	1122.	.0000	777.0
	7	183.5	279.0	6622.	5.840	6501.	1910.	7.750	95.00	1230.	1119.	.0000	784.0
	8	188.5	295.3	6752.	6.360	6438.	1930.	8.960	-16.00	1227.	1152.	46.00	729.0
	9	189.9	268.8	6724.	6.180	6657.	1960.	8.060	104.0	1224.	1218.	46.00	737.0

	10	191.2	279.6	6777.	6.080	6815.	1990.	7.460	219.0	1289.	1179.	46.00	754.0
	11	193.9	252.0	6835.	6.040	7044.	2000.	7.470	224.0	1347.	1242.	25.00	817.0
	12	195.3	264.3	6934.	5.920	7317.	2000.	7.150	358.0	1441.	1241.	25.00	729.0
1975	1	200.5	286.3	7194.	5.980	7258.	1908.	6.260	164.0	1527.	1222.	53.00	740.0
	2	202.3	243.4	7293.	5.920	7321.	1902.	5.500	123.0	1477.	1255.	28.00	747.0
	3	203.4	262.7	7449.	5.940	7482.	2092.	5.490	149.0	1587.	1292.	27.00	751.0
	4	204.8	273.3	7446.	5.760	7663.	2092.	5.610	270.0	1613.	1269.	89.00	714.0
	5	207.2	296.6	7421.	5.670	7638.	2092.	5.230	283.0	1707.	1294.	138.0	744.0
	6	209.1	248.7	7637.	5.580	7778.	2092.	5.340	490.0	1835.	1297.	203.0	720.0
	7	212.9	273.5	7773.	5.540	7831.	2189.	5.400	466.0	1859.	1314.	364.0	763.0
	8	216.1	244.8	7854.	6.120	7849.	2189.	5.400	262.0	1833.	1358.	431.0	794.0
	9	217.9	260.0	8031.	6.480	8124.	2189.	5.400	444.0	1947.	1466.	507.0	933.0
	10	221.9	267.2	7978.	6.800	8150.	2289.	5.400	262.0	2051.	1469.	551.0	972.0
	11	225.4	314.6	8090.	6.820	8338.	2289.	5.400	221.0	2022.	1510.	551.0	972.0
	12	228.5	375.0	8336.	6.890	8591.	2289.	5.500	238.0	2071.	1499.	599.0	940.0
1976	1	229.6	369.9	8480.	6.930	8456.	2173.	4.960	89.00	1922.	1561.	632.0	960.0
	2	232.4	328.7	8584.	7.040	8496.	2173.	4.850	9.000	2023.	1569.	675.0	859.0
	3	234.2	267.9	8526.	7.170	8743.	2173.	5.050	132.0	1990.	1482.	678.0	1205.
	4	237.2	329.2	8541.	7.210	8954.	2397.	4.880	103.0	2153.	.0000	678.0	1069.
	5	240.8	416.2	8447.	7.190	8766.	2397.	5.190	61.00	2286.	.0000	660.0	941.0
	6	.0000	.0000	.0000	.0000	.0000	.0000	.0000	407.0	.0000	.0000	.0000	.0000
	7	.0000	.0000	.0000	.0000	.0000	.0000	.0000	82.00	.0000	.0000	.0000	.0000
	8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	192.0	.0000	.0000	.0000	.0000
	9	.0000	.0000	.0000	.0000	.0000	.0000	.0000	135.0	.0000	.0000	.0000	.0000

TABLE

		DKG	PH	G
1965	1	28.00	104.0	190.0
	2	29.00	104.0	190.0
	3	83.00	104.0	195.0
	4	95.00	105.0	200.0
1966	1	22.00	105.0	196.0
	2	59.00	105.0	189.0
	3	-12.00	105.0	193.0
	4	73.00	106.0	190.0
1967	1	31.00	107.0	192.0
	2	54.00	106.0	197.0
	3	33.00	106.0	194.0
	4	44.00	106.0	189.0
1968	1	116.0	105.0	178.0
	2	145.0	106.0	197.0
	3	57.00	105.0	195.0
	4	128.0	106.0	199.0
1969	1	33.00	107.0	216.0
	2	-10.00	108.0	190.0
	3	11.00	109.0	216.0
	4	146.0	112.0	225.0
1970	1	86.00	114.0	192.0
	2	139.0	115.0	214.0
	3	107.0	116.0	217.0
	4	209.0	117.0	214.0
1971	1	285.0	119.0	205.0
	2	157.0	120.0	227.0
	4	170.0	126.0	251.0
1972	1	146.0	129.0	257.0
	2	137.0	130.0	274.0
	3	62.00	131.0	317.0
	4	157.0	132.0	314.0
1973	1	74.00	139.0	362.0
	2	34.00	150.0	437.0
	3	-214.0	165.0	505.0
	4	-12.00	168.0	465.0
1974	1	143.0	175.0	602.0
	2	-39.00	194.0	643.0
	3	244.0	201.0	606.0
	4	428.0	212.0	714.0
1975	1	383.0	225.0	638.0
	2	467.0	225.0	615.0
	3	427.0	215.0	644.0
	4	620.0	213.0	643.0
1976	1	483.0	214.0	598.0
	2	89.00	217.0	603.0

APPENDIX D

COMPUTER PROGRAMS

The Computer Programs AUTO and COMET were used to produce the results of Chapter Three, RESIMUL and TRANSF of Chapter Five.

AUTO is a program that gives Almon Lag coefficient estimates (see Appendix A), and also performs Cochrane-Orcutt, and one- and two-stage scanning regression estimates in the presence of autocorrelated errors. AUTO was written by M.R. Norman of the Wharton School of Finance and Commerce, University of Pennsylvania, and was edited for use on the University of Cape Town's UNIVAC 1106A computer by the writer. COMET is a UNIVAC package that gives linear least-squares estimates of regression equation coefficients.

Dr. C.R. Wymer of the London School of Economics kindly made available his suite of programs that enabled the exact and approximate discrete models (see Chapter Four) to be estimated and analysed. A complete list of these programs is given below. Of these the subprograms RESIMUL and TRANSF (and their attendant subroutine system LIBRARY) were edited by the writer. TRANSF is a data transformation program that, among other things, enables the data to be prewhitened by the appropriate moving average error process.

RESIMUL estimates the approximate discrete model. At present we are editing the stability analysis program CONTINEST. It is hoped that all the programs of Dr. Wymer will eventually be made available to researchers at the University of Cape Town. (AUTO is currently in operation, and its manual : "ECON and AUTO" is available at the University's Computer Centre Library.)

PROGRAM LISTINGS FOR CHAPTER FOUR

(a) TRANSF

```
@D.TA,L TRANI.
DATA T7 RL70-5 05/17-11:21:51
        @ASG,UP T2.,F14///5
        @USE 12,T2.
        @ASG,A  TRANSF.
        @XQT TRANSF.ABSOLUTE
        START,3                  B.O.P. (2)
             1   14   29   46   45    1    1    0    1    2 .
              8    1    9    2   10    3   11    4   12    5   13    6   29    7   22   14   23   15   24   16
            25   17   26   18   27   19   28   20
        (F8.2)
        B.O.P. (2)
        PRICE INDEX
        @ADD,P P.
        EXPORTS
        @ADD,P X.
        PRIVATE K(CAPITAL)
        @ADD,P KP.
        TOTAL PRIVATE LENDING
        @ADD,P TP.
        TREASURY BILL RATE
        @ADD,P TBR.
        NET DOMESTIC ASSETS
        @ADD,P NDA.
        MONEY SUPPLY
        @ADD,P M.
        G.D.P.(MP)
        @ADD,P Y.
        FOREIGN PRICE INDEX
        @ADD,P PM.
        U.S. TREAS. BILL RATE
        @ADD,P US.
        EXCESS RESERVES
        @ADD,P EXR.
        TRG EXOGENOUS PROXY
        @ADD,P TRG.
        TOTAL GOVT. LENDING
        @ADD,P TGL.
        MONEY BASE
        @ADD,P MB.
        ENDA
        TREND    21
        COSNA     1    7    1
        COSNA    14   20    1
        LAG       1    8
        LAG       2    9
        LAG       3   10
        LAG       4   11
        LAG.      5   12
        LAG       6   13
        LAG       7   29
        LAG      14   22

        LAG      15   23
        LAG      17   25
        LAG      16   24
        LAG      19   27
        LAG      20   28
        REDUC     2    1   44
        FINAL
        FINISH
END DATA.
@FIN
```

(b) RESIMUL

```
94567 7c1 3456787c1234567890123456789012345678901234567890123456789012345678901234567890123456789012345

IN HRS,A0227-R008/ANAZ,WYHER,S20,20

G RUN.

STA,L RUN.
A T7 RL70-5 07/25-18:38:45
          C......................C......................CV......................C............
          START,1                B.O.P. (2)
              -1     7    13    44    22    22     2     1     0     3     0     1     0
               5     2
          F   14    1P    -1    7F     2   13P    -2   14F    22    8F    11    5F    15   12F   18    20
          F    3    2P     3    9F     4    1F     5    8F    +7   19F   -5   14F   -19   20
          C    1    3F    -8   13F     9    4F   -10    5P     4   11P    10   12P   40   15F   -20   20
          F   13    4F    16    5F   -17   13P     5   11F     1   12F   -21   20
          C    1    5P    -7    6P     6   16
          C    1    6P    -8   17P    -9   12F    -6    5
          C    1    7F   -12    5P   -16   12P   -17   18P   -22   20
          F1=C1P15 $ F2=P1P13 $ F3=C1+C2P3 $ F4=C2P3P12 $ F5=P3P12 $ F6=C2P9  $ F7=P3P11 $
           F8=  P4P13 $ F9=C2P4 $ F10=C2P10 $ F11=C2P1P14$F12=C2P16$F13=C1+C2P5
          $ F14=C1+C2P1+C2P2 $ F15=P1P14 $ F16=C2P15   $ F17=P5P13 $
          F18 = P1P19 $ F19= P3P20 $ F20= P4P21 $ F21= P5P21$ F22=P1+P2 $ S
            0.5
          1.        0.5
             1   29   44    1    3    0    2
             1    2    3    4    5    6    7   21   22   23   24   25   26   27   28   29   30   31   32   33
             34   35   36   37   38   39   40   41   42
          B.O.P. (2)
          X1=X1-X21
          X2=X2-X22
          X3=X3-X23
          X4=X4-X24
          X5=X5-X25
          X6=0.5(X6+X26)
          X7=0.5(X7+X42)
          X26=0.5(X27+X35)
          X27=0.5(X28+X36)
          X28=0.5(X29+X37)
          X29=0.5(X30+X38)
          X30=0.5(X31+X39)
          X31=0.5(X33+X41)
          X32=0.785(X34-22.5)
          X32=X32/99.483
          X33=0.785
          FINAL
             0    0    1    4    4
          FINISH
D DATA.

IN


RUNID: HRS         ACCT: A0227-R008        PROJECT: WYHER

TIME:      TOTAL: 00:00:02.851        CBSUPS:   000136960

           CPU:   00:00:00.076        I/O:   00:00:00.209

           CC/ER: 00:00:02.565        WAIT:  00:00:00.000
```

COMPUTER PROGRAMS OF C.R. WYMER

A set of programs has been developed for handling linear systems of equations of the form

(1) $B(\theta)y_t + C(\theta)z_t = u_t$

and non-linear systems where each equation has the form

(2) $\phi_i(y_t, z_t, \theta) = u_{it}$, ϕ being any function.

y_t is a vector of endogenous variables, z_t a vector of predetermined variables, θ a vector of parameters and u_t a vector of disturbances, generally assumed to be independently normally distributed $N(0,\Omega)$. Some equations may be identities. Input and output of the programs is standardised such that information can be stored on magnetic tape or disc and used in other programs. Special features of these programs allow the exact or approximate discrete models equivalent to any mixed order differential equation system to be estimated and their properties analysed easily. A brief note on each program, identified by their code name, follows.

SIMUL

Calculates two stage least squares, three stage least squares, and full information maximum likelihood estimates of model (1) where all restrictions are linear. The reduced form is found if required. Chi-square values for the likelihood ratio test are calculated. Uses a modified Newton-Raphson procedure to maximise the likelihood function.

RESIMUL

Calculates full information maximum likelihood estimates of model (1) where the coefficients of the system are any functions, algebraic or transcendental, of the set of parameters. This allows non-linear restrictions within and across equations to be imposed as well as certain types of inequality constraints. The program evaluates analytical first derivatives of the functions of parameters and uses a Newton-Raphson procedure, beginning with arbitrary initial values, to maximise the likelihood function.

RESIMULA

Modified version of RESIMUL which calculates full information maximum likelihood estimates of model (1) with a moving average disturbance of the form

$$u_t = \{ I - R(L,\theta)\} \varepsilon_t$$

where R is a matrix polynomial of any order in the lag operator L and ε_t is a vector of independent normally distributed disturbances. The elements of B,C and R are any functions of the parameters θ. This allows the moving average coefficients to be restricted, if necessary to some function of the parameters in the deterministic part of the model, as well as allowing non-linear restrictions within and across equations. The program estimates the model by multiplying the system by the inverse of the coefficient matrix of ε_t, expanding as a Taylor's series and truncating the resulting infinite series after a given number of terms. The system may be re-estimated automatically taking more terms in the expansion in order to determine whether the truncation point is satisfactory; the corresponding estimates of R may be output to disc for calculating the eigenvalues using CONTINEST.

DISCON

Calculates full information maximum likelihood estimates of the exact discrete model equivalent to a mixed order linear differential equation system written as the first order system

$$(3) \quad dy(t) = A(\theta)y(t)dt + C(\theta)z(t)dt + d\zeta(t)$$

where the coefficients are any functions of the set of parameters. This allows non-linear restrictions within and across equations to be imposed. Uses a Newton-Raphson procedure to minimise the determinant of the residual variance matrix.

CONTINEST

Calculates the eigenvalues and eigenvectors of a matrix as well as the corresponding variance matrices. This program is used in particular with mixed order linear difference or differential equation systems estimated by SIMUL, RESIMUL or DISCON. Given initial conditions the program will calculate the coefficients of the complementary function. Uses modified QR procedure.

PREDIC

Forecasting or simulation program for the general linear model (1) or the exact discrete model (3). Point prediction estimates with standard errors of forecasts and prediction intervals at a given confidence level are calculated for as many periods ahead as desired. Simulation may be either "ex post" using observed values of the exogenous variables, or "ex ante" using given values of exogenous variables which may be changed as desired. In each simulation random normal disturbances may be included such that these disturbances have an expected variance matrix which is given, generally being the estimated variance matrix of residuals in the structural model. For simulation special features allow either the parameters or the variables in the model to be altered according to some specified control system. Thetic and anti-thetic samples may be generated for Monte-Carlo studies.

ASIMUL

Calculates pseudo full information maximum likelihood estimates of the general system (2), non-linear in the variables and subject to non-linear within and across equation restrictions on the parameters. The program automatically evaluates analytical first derivatives of the variable and parameter functions with respect to the parameters, and calculates the derivatives of the variable functions with respect to the endogenous variables. Incorporate a method of solving systems of non-linear equations. Uses Newton-Raphson procedure to maximize the likelihood of the structural model but also evaluates the likelihood of the implicit reduced form.

APREDIC

Forecasting or simulation program using the general non-linear model (2). This is a non-linear version of PREDIC which may be used with models estimated by ASIMUL. Facilities similar to those of PREDIC are incorporated within the program except that standard errors of forecasts and prediction intervals are not found.

Other programs which have been developed are:

TRANSF

Data processing program which reads from cards, tape or disc and writes to tape or disc. Graph plotting facilities are included. This program incorporates a simple form of compiler which can read and execute Fortran type arithmetic statements. The other programs listed here incorporate a modified version of these transformations.

ORLESQ

Calculates least square estimates and usual statistics, as well as the F statistic required to test the hypothesis that a subset of coefficients are zero.

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