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A F A C T O R I A L S T U D Y O F

I N F E R I O R I T Y

BY

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CONTENTS

I N T R O D U C T I O N

The technique of factor analysis is one which is used for determining, by the statistical analysis of a set of intercorrelations among a number of tests or ratings, whether a few elements run as common factors through all the tests; and, if there are such common factors, with what weight each of the tests is loaded with each of the factors.

In order to grasp the full meaning of the above definition, it is necessary to follow it with another equally important definition - that of the term "factors". This definition is clearly set out by Professor Gordon Allport: "A factor is an empirical, a posteriore construct or generalization. It is defined completely by its mode of deviation. Starting always with a battery of tests or ratings, heuristically presumed to cover the function that is the object of study, the factorial description of these tests reduces the scores of all the diverse items to the smallest number of independent variables. The overlap is consolidated, and the number of non-correlating factors discovered is assumed to provide the list of elements basic to the function."[1]

BRIEF HISTORY OF FACTOR ANALYSIS

Charles E. Spearman opened this type of enquiry in England near the beginning of the present century, taking the intercorrelations in combinations of four by the so-called "tetrad difference method", in which he tested the hypothesis that only one factor might be common to all the tests. Webb, who was a student of Spearman, carried out an analysis in which he discovered a temperamental factor which covered a wide range of behaviour and which he called "w" - will.

He...../

1. Gordon W. Allport: "Personality" (Constable & Co., Ltd., London), 1937; pp. 242-3.

He was undecided, however, as to what other qualities were to be found in his material. After Professor Cattell, who was greatly influenced by Spearman's Two Factor Theory, had worked on Webb's factors a great deal of further analysis took place, but this further analysis was made possible only by a new technique. The next step was the evolution of a method which could deal with more than one factor. This new method was developed by two people, Professor Cyril Burt in London, and Professor Thurstone in Chicago.

According to Spearman, a correlation matrix must be a hierarchy if there is only one common factor. The present day test for the hierarchy order in a correlation matrix is to calculate all the tetrad differences and ascertain whether or not they are sufficiently small. We can, of course, not always expect the tetrad differences to be zero.

The Two Factor method of analysis began with the idea that a matrix of correlations would ordinarily show perfect hierarchical order if care was taken to avoid unduly similar tests.

Later group factors had to be admitted within as small a number as possible, though always with the greatest reluctance, from exponents of the school. It had, however, become clear that the Theory of Two Factors had been superceded by a theory of many factors, although the methods of the Two Factor Theory remained as an analytical device for indicating their presence and for isolating them in comparative purity. Under those circumstances some experimenters turned their attention to the possibility of multiple factor analysis by which a matrix of correlations could be analysed direct into its factors. It was Professor Thurstone of Chicago who saw that one solution of this problem could be reached by a generalisation of Spearman's idea of zero -

Tetrad...../

Tetrad Difference. This method, the Centroid Method, is comparatively simple and fairly rapid; it has, however, one disadvantage, for it involves the guessing of the Communalities which have to be placed in the diagonal cells of the matrix. Burt's attitude is that the correlations of the tests with themselves (i.e., ± 1) should be placed in the diagonal cells as communalities. Thurstone disagrees with this on the grounds that tests do not always agree 100% and suggests that the communalities should not be the agreement a test should have with itself but only the amount of agreement that would be due to the common factor.

METHOD OF OBTAINING MATERIAL FOR THE FOLLOWING ANALYSIS.

The material used in the following factoral analysis was obtained from Professor H. Reyburn of the Psychology Department, University of Cape Town.

Professor Reyburn devised the Personality Rating Scale, which consists of 45 question headings.

PERSONALITY RATING SCALE.

June, 1947.

1. ATTITUDE TO THE PROBLEMS OF LIFE. - The capacity or tendency to face up squarely to problems as they arise, to recognise the real issues involved and not to run away from them or dodge them. This does not mean a tendency to seek for trouble, but merely a willingness to look things in the face.

1. Always faces up; 2. nearly always; 3. fairly generally but with some real avoidance; 4. shows definite tendency to avoid many difficult issues and to disguise them from himself; 5. nearly always avoids them.

2. OVERBOLDNESS - tendency to be pretentious, readiness to take on anything.

1. Never shows this tendency; 2. very occasionally; 3. moderately...../

3. moderately often; 4. often; 5. has strong marked ambitious tendency - can be counted on to show it.

3. SYSTEMATIC APPROACH TO LIFE.

1. Pedantic; 2. generally systematic and methodical in his attitudes an behaviour; 3. moderately regular, but with a good deal of variation and noticeable exceptions; 4. very irregular; 5. quite confused.

4. DILATION. Is the subject full and free in emotional reaction and in spontaneity, or restrained and unable to let himself go either in behaviour or thought?

1. Very restricted and inhibited; 2. restricted on the whole, but with occasional real exceptions; 3. fifty-fifty, restraint and freedom both marked - perhaps alternating; 4. pretty free in the warmth and spontaneity of his outlook on life; 5. very unrestrained, brimming over, and full of life.

5. EMOTION. - Is it easily stirred up?

1. Impassive; 2. not easily stirred up, although not without some emotional capacity; 3. emotionally responsive, but with the emotions under reasonable control; 4. more emotional than the average, and somewhat impulsive emotionally; 5. highly emotional.

6. PERSISTENCE OF EMOTIONAL MOODS.

1. Very persistent and enduring; 2. fairly tenacious, a little more so than the average; 3. average; 4. changes in mood rather easily - neither good nor bad temper retained very long; 5. highly volatile emotionally - prediction impossible.

7. RESTRAINT OF EMOTION. - Tendency when emotionally moved to control the emotion and not abandon himself to it.

1. Very guarded and self-controlled; 2. generally guarded, but subject to very occasional outbursts; 3. moderate control, but no feeling of general

inhibition...../

inhibition; 4. expresses emotions pretty freely; 5. subject to frequent emotional storms.

8. RELAXATION. Can the subject release himself from strain, turn away from work, and relax when work is over and there is no need to do anything?

1. Hardly ever relaxes - must always do something; 2. relaxes occasionally, but as a rule not too quickly; 3. relaxes fairly easily, but occasionally carries some strain over from work, etc.; 4. relaxes quickly and naturally; 5. automatically lets go, and throws work and strain aside.

9. RESPONSE TO BRIGHTER AND WARMER ASPECTS OF THINGS.

1. Responds quickly and regularly to the cheerful and stirring aspects of things; 2. responds easily, but with some shadows; 3. generally sees the vivid side, with some lapses and failures to respond emotionally; 4. poor emotional response - poorer than average; 5. emotionally blunted - and unresponsive.

10. TENDENCY TO REDUCE EMOTIONAL TONE. Tendency to respond to emotional situations in a quiet way, and to prefer situations that are not too stimulating.

1. Quite free from this tendency, likes bright colours, and exciting things; 2. exercises a little restraint, but on the whole responds fairly freely and does not avoid strong emotion in any marked way; 3. average - likes excitement and brightness but not too much of it; 4. tends to avoid strong emotions and emotional situations; 5. prefers quiet tones, gentle actions, and non-exciting situations, always tones things down.

11. EASILY ACCESSIBLE TO GLOOMY IDEAS. If a distressing situation is put to subject, is he quickly and easily moved by it?

1. Not easily moved, takes things very calmly and

refuses...../

refuses to get down-hearted; 2. on the whole not easily moved, but is not blind to distress; 3. responds fairly easily to the darker side of things, and can easily be made sad; 4. sees the dark side quite quickly - more easily and more often than the average; 5. is greatly upset by the many and grave distressing situations he finds, seeing all there is of them.

12. EASILY GETS GLOOMY IDEAS OF HIS OWN.

1. Quite free from spontaneous gloom; 2. very occasionally gets sad ideas; 3. has them fairly frequently, but not overwhelmingly so - is not so down in the mouth as he might be; 4. very prone to sad reflections and dismal feelings; 5. almost perpetually melancholic.

13. SUBJECT TO MARKED FITS OF DEPRESSION.

1. Never; 2. rarely; 3. now and again, not very often; 4. frequently; 5. very often indeed.

14. MENTALLY COURAGEOUS IN ATTITUDE TO LIFE.

1. Never shirks a test, and never makes excuses; 2. rarely does so; 3. average - is somewhat apprehensive of testing situations, and tends at times to avoid them or discount them, but on the whole goes through with things; 4. prone to avoid tests, is glad when he has an excuse not to be tested, and is quite ready with reasons for failure; 5. avoids being put to the test and excuses his failure to a marked degree, and can be counted on to do so.

15. SENSE OF INFERIORITY.

1. Quite free from it; 2. almost free from it - it appears only in non-essentials; 3. fairly liable to it, but not to a distressing degree; 4. has noticeable sense of inferiority - more than average; 5. strong and distressing feelings of inferiority.

16. SHY - (Judge by behaviour, but try to get behind the

surface...../

surface.) Does subject feel diffident and embarrassed by strangers and authorities, and is he inhibited in dealing with them?

1. Behaves perfectly naturally and feels at ease; 2. very slightly affected - natural on the whole; 3. not quite at ease, but not really distressed; 4. definitely embarrassed - unwilling to come forward into the limelight; 5. very shy.

17. ASSERTIVE. - Quick and ready to maintain his own point of view.

1. Very slow to assert himself - (may retain his own point of view, but, if so, he keeps it to himself); 2. generally slow to assert himself, but can do so on occasion without excessive stimulus; 3. quite ready to state his own view when need arises; 4. quick and competent in asserting his view - does not let it be passed over in silence; 5. strongly assertive, finds it difficult to let anything go by that does not please him, or with which he does not agree - challenges it almost automatically.

18. SUBMISSIVE.

1. Not at all; 2. submits occasionally, but with difficulty; 3. submits fairly readily; 4. often - follows the lead of others easily, and by preference; 5. always gives in - always docile.

19. HAS HIS OWN WAYS OF LOOKING AT THINGS.

1. Lacks all spontaneity, has no attitudes of his own; 2. spontaneity weak, accepts what comes along rather than dominates it; 3. does not always accept, and is sometimes inclined to give his own colouring to situations, though not excessively so; 4. rather subjective in his estimations, tends to look at the world from his own points of view, or in the light of his own mood or moods;

5. very...../

5. very subjective - finds it difficult to see any other points of view than his own.

20. HAS FIXED WAYS OF LOOKING AT THINGS, OF TAKING THEM IN PRACTICE.

1. Has a very variable outlook - one can't tell what he is going to be like, do, or think; 2. rather variable on the whole; 3. moderate - doesn't swing about too much, but is not confined to one mood or outlook; 4. rather fixed in his attitude - tends generally to look at life and its problems from the same point of view or through the same mood; 5. obsessed by some one outlook.

21. SUGGESTION.

1. Not at all suggestive - bases his views as far as possible on facts and evidence; 2. slightly suggestible, but is objective on the whole; 3. fairly suggestive - accepts ideas easilt in certain moderately common situations; 4. definitely suggestible to many people and in several situations; 5. very suggestible - accepts almost any idea put to him.

22. CONTRA-SUGGESTION.

1. Does not at all resent having ideas put to him, never shows a hostile reaction to them merely because they are not his; 2. generally fair minded, but occasionally dislikes having ideas prescribed to him; 3. lays moderate weight on his independence, and dislikes being dictated to - insists on thinking for himself; 4. tends to reject ideas suggested to him - though not excessively; 5. is generally contrary in his opinions to everyone who seems to him to be authoritative - is strongly contra-suggestible.

23. COMPLIANCY:.

1. Is compliant, ingratiating, eager to be appreciated, is excessively willing to agree to others; 2. is generally compliant, occasionally in a fairly high degree;

3. moderately...../

3. moderately compliant - generally does not like incurring opposition; 4. agrees with and conciliates others only if doing so does not conflict with anything else he wants; 5. is very little interested in doing anything to get the goodwill of others, and tends to regard their views with indifference.

24. DOMINANCE. Does subject tend to dominate those he is with, to be leader and boss them?

1. Not at all; 2. very little; 3. moderately; 4. pretty frequently; 5. yes, generally.

25. SELF-CRITICISM. - Is the subject inclined to take the blame for anything that goes wrong?

1. Not at all; 2. slightly and only when responsibility is clearly brought home; 3. realises his own mistakes readily; 4. apt to blame himself unnecessarily; 5. blames himself incessantly.

26. HIGH IDEAL. - This is not a question of actual level of behaviour reached, but of standard set and arrived at.

1. Has very high and rigid ideal of conduct, which must not be departed from; 2. high ideal, higher than is usually feasible, but not completely rigid; 3. definite standard and clear expectations, but fair amount of allowance made both for self and others; 4. question of standard of conduct and attainment not generally raised, takes things rather realistically; 5. hardly raises question of standard at all - may behave quite decently, but concerns himself very little about goodness and badness of conduct and achievement.

27. AGGRESSION.

1. Never aggressive - not necessarily compliant, but never attacking; 2. occasionally aggressive, though but seldom - occasionally takes a hostile attitude; 3. aggressive now and then, though not excessively, and

generally...../

generally on good grounds, as some hostility is shown on the other side; 4. apt to be aggressive and hostile without real cause; 5. generally takes up a hostile or suspicious attitude and criticism or attacks, even if there is no need and no justification for the attitude - looks for trouble.

28. WITHDRAWAL. - Shows tendency to keep remote from people, not to take them into his confidence, to be stand-offish and not let them really come into intimate contact with him.

1. Altogether free from this tendency; 2. apt to show it on occasion, or with reference to a few people, but as an exception; 3. rather careful about giving himself away, but not excessively so - withdraws only when there is some cause; 4. rather prone to keep apart; 5. very stand-offish - very hard indeed to get in touch with him.

29. SELF-ESTEEM (to be distinguished from conceit).

1. Has unduly low opinion of himself and his abilities - too humble - generally really underestimates himself; 2. takes a lower view of himself than the average, but is not abject; 3. has a fair amount of self-esteem with, however, also a moderate amount of misgiving; 4. takes fairly high view of his own capacities, but without exaggeration; 5. completely free from false modesty, and fully realises his worth.

30. CONCEIT. Has the subject an inflated conception of himself, does he over-value himself and his qualities?

1. Not at all; 2. very little; 3. moderately; 4. a good deal; 5. tremendously.

31. SELF-RELIANCE.

1. Hardly any self-reliance - always shouting for help; 2. has some self-reliance, but is easily discouraged; 3. average - fairly willing and able to do things and

take...../

take responsibility; 4. needs little help, and masters his own forces pretty well; 5. fertile in resource, and can be counted on to do a thing if it is within his power - never asks for help unless it is absolutely necessary.

32. RELIABILITY. If subject has something to do within his capacity, can he be relied upon to finish it faithfully?

1. Very unreliable; 2. shows some reliability but subject to considerable and important lapses; 3. fairly reliable, but by no means completely so; 4. really reliable, with few lapses; 5. completely reliable, with no lapses or failures of what is within his control.

33. PERSEVERANCE. Does the subject persist in what he has undertaken to do and refuse to be put off by difficulties or boredom?

1. Has little persistence, easily distracted in almost anything he does; 2. weak persistence except on odd occasions in certain directions; 3. average; 4. is persistent in general, though occasionally put off and apt to lose interest; 5. distressed if he cannot finish what he has begun - no matter what it is - sticks at it.

34. CHEERFULNESS (as opposed not to gloom but to absence of feeling tone).

1. Sober and impassive, never rejoices; 2. occasionally bright and happy, but needs a fair amount of provocation; 3. average; 4. brightens up easily, and generally shows a cheerful face, never takes problems seriously unless they merit it; 5. generally bubbling over and is not depressed except by grave distress - full of good humour.

35. IRRITABILITY.

1. Not at all irritable; 2. sometimes irritable, but not often, and generally only for good cause; 3. occasionally irritable when the situation does not really warrant it; 4. easily made cross - has to be approached carefully;

5. very...../

5. very touchy - for ever in a temper about something.

36. EXCITABILITY.

1. Hardly ever excited, very difficult to work up; 2. on the whole very calm and collected, but sometimes gets worked up; 3. average - excitable on occasion, but generally cool enough; 4. rather easily excited - more so than average; 5. very excitable - hypomanic.

37. FLURRY.

1. Keeps very cool and collected in emergency, never loses his head; 2. does not always act up to his best level, but is at a loss only for a short time and considerable pressure; 3. usually fairly cool and efficient, but can not always preserve this under a strain; 4. rather easily rattled and apt to get flurried; 5. nearly always rattled by something and very apt to lose his head.

38. SUSPICION.

1. Not at all suspicious, trusting - easy to tell things to; 2. generally trustful, but apt to raise questions and doubts upon occasion; 3. trustful or suspicious according to the circumstances - not easily taken in but free from needless suspicion; 4. rather apt to distrust people and entertain unnecessary doubts; 5. very suspicious, always wondering what people are up to, never takes anything at its face value - constantly afraid of being taken in.

39. CHARITY - kindly, open-hearted and sympathetic by nature. 1. Callous and indifferent to the needs and sufferings of others; 2. kindly at times, but not very generally or freely; 3. quite open-hearted, unless other motives arise to check sympathy; 4. easily touched, and generally fond of most people; 5. very kind by nature to everyone.

40. SENSITIVE...../

40. SENSITIVE TO THE FEELINGS OF OTHERS.

1. Very unperceptive of what others feel (not necessarily callous - just ignorant); 2. rather dense in perceiving how others are affected; 3. has average sensitivity; 4. more sensitive than normal to the feelings of others; 5. extremely responsive to their feelings are careful of them.

41. SENSITIVE TO SLIGHTS AND INJURIES.

1. Very thick-skinned; 2. not easily hurt, can take offence, but not readily; 3. average; 4. rather easily hurt, even though offence is not intended; 5. very sensitive - fancies slights where none exist.

42. EASILY PLEASED OR SOOTHED.

1. Not easily placated; 2. less ready than average to recover from hurt or to cheer up; 3. average; 4. fairly easily soothed - cheers up fairly quickly; 5. very easily brought to good humour and good spirits - very responsive to others.

43. EXTRA-PUNITIVE.

1. Very slow to blame other people for difficulties - never blames them for the injustice of fate; 2. blames them only occasionally, and usually when there is some colourable ground for doing so, or when irritation is extreme; 3. average; 4. rather prone to blame others for his own mistakes or for misfortunes; 5. nearly always does so.

44. MATURE STABLE AND WELL BALANCED.

1. Is mature and well balanced in every respect, with good sense of proportion in life, adequate emotional response to the world, and control over his own behaviour; 2. generally well-balanced with a very occasional extravagance or loss of control; 3. good general balance, but still capable of considerably greater development and

integration...../

integration; 4. has rather unbalanced and childish attitude to life; 5. has quite undisciplined outlook and behaviour, lacking all reasonable integration.

45. SOCIABILITY.

1. Good mixer; 2. generally good mixer, but not with everyone and under all circumstances; 3. mixes well in the right crowd; 4. does not mix easily except under very favourable circumstances; 5. bad mixer at almost any time.

The set of questions, which were to be marked on a five point scale, were issued to 80 university students, divided into two groups of 40 each. Each student was required to answer the questions with regard to two friends, ringing the scale number which applied to each individual chosen.

As each of the 80 students was required to answer the questions with regard to two friends, 160 cases were obtained. Care was taken that no two students answered the questions with respect to the same two individuals. The names of the rated individuals were submitted to the Professor, who ascertained that in each case different individuals were rated.

On the five point scale of the questionnaire, numbers 1 and 5 are the extremes.

The totals on the various points on the scale were calculated and a table of correlations between each of the 45 variables was obtained by the Rank One method.

METHODS EMPLOYED IN THE ANALYSIS.

As a study was being carried out on Inferiority, the twelve variables which correlated most highly with the inferiority variable (15) were extracted from the Professor's

original...../

original table of correlations and placed in a correlation table. The method employed in extracting the factors was as follows:

(1) The highest quality (irrespective of sign) in each column of the correlation table was placed in the diagonal cell as the communality for that particular column.

(2) Reflection was carried out in order to make the table as positive as possible.

(3) The positive total and the negative total of each column and row was found.

(4) In each column and row the totals (+ minus -) were obtained. To these totals was added the specific communality of that particular row and column.

(5) The grand total of the columns and the rows was found.

(6) The square root of the grand total was calculated and the reciprocal of the square root of the grand total was found.

(7) The reciprocal of the square root of the grand total was multiplied by the sum of each column, the result being the first factor loads.

(8) Each factor load was then multiplied by itself and by each of the other factor loads, resulting in a table of correlations with the first factor.

(9) The total of each row and column was found, the error between these totals and those in the first table being not greater than .003. This is the Table of Cross Multiplication.

(10) The Table of Cross Multiplication was then compared with the original table of correlations. The tables were compared cell by cell, entries in the second table being subtracted from those in the first.

(11) The results of this comparison were placed in the

Table...../

Table of Residuals. The total of each column and the total of each row was found. The errors of each column and of each row in the table were the same as those in the corresponding columns and rows in the Table of Cross Multiplications.

During the analysis, all the necessary checks were carried out.

In order to ascertain the number of factors to extract from the material the Probable Error of the mean correlation of the first correlation table was found.

The Mean Residual of the Table of Residuals was found. When the mean residual is found to be less than or equal to the Probable Error, all factors have been exhausted. In this particular case, the mean residual was found to be equal to the Probable Error after four factors had been extracted.

For the second, third and fourth extraction of factors, the same method was employed; in each case, the highest quality, irrespective of sign in each column was placed in the diagonal cell of that column as the communality.

When the four factors had been extracted in this way, H^2 had to be ascertained. H^2 is the sum of the squares of the factor loadings and should correspond closely to the original "guessed" communalities.

As the ascertained communalities and the "guessed" communalities were found to approximate fairly closely, it was considered unnecessary to repeat the analysis using the ascertained communalities in the diagonal cells of the original correlation table.

GRAPHICAL ROTATION OF THE AXIS.

So far, four "centroid" factors had been extracted from the material. By means of rotating the axis objective

(psychologically...../

(psychologically significant) factors were obtained from the centroid ones. The factor loads were plotted in three series, A, B and C, suitable points were chosen for the swinging of the axis, and accurate measurements were recorded. In order to check the graphical rotation, the Transformation Matrix was obtained after each pair of factors had been plotted. The Transformation Matrices were themselves checked by the process of multiplication.

From the Transformation Matrices the communalities were found.

These communalities were found to check with the communalities obtained by the centroid analysis.

The most significant correlations (those above .200) were determined, and the interpretation of the result was carried out.

P A R T I.

THE EXTRACTION OF THE CENTROID FACTORS.

The twelve variables which correlate with variable 15 (Sense of Inferiority) were found to be:-

Variable.		Correlation Error.
29	Self Esteem	1 in 1000
16	Shyness	" " "
12	Easily Gets Gloomy Ideas of His Own	" " "
17	Assertiveness	" " "
41	Sensitive to Slights and Injuries	" " "
23	Compliancy	" " "
34	Cheerfulness	" " "
44	Nature Stable and Well-Balanced	" " "
1	Attitudes to Problems of Life	" " "
45	Sociability	" " "
21	Suggestion	" " "
13	Subject to Marked Fits of Depression	" " "

The intercorrelations of the 13 variables were extracted from the original correlation table and placed in a table of correlations (Table I, page 19).

Table I...../

TABLE I.

TABLE OF CORRELATIONS.

	15	29	16	12	17	41	23	34	44	1	45	21	13
15													
29	-.577												
16	.468	-.319											
12	.375	-.189	.155										
17	-.375	.530	-.433	-.137									
41	.369	-.250	.242	.318	-.101								
23	-.366	.236	-.011	.152	.427	.026							
34	-.334	.206	-.350	-.418	.428	-.314	-.159						
44	.334	-.155	.177	.204	-.194	.193	.042	.006					
1	.304	-.327	.125	.267	-.282	.139	-.057	-.110	.527				
45	.280	-.256	.557	.324	-.258	.122	.103	-.578	.059	.004			
21	.269	-.303	.164	.085	-.397	-.026	-.352	.108	.315	.515	-.046		
13	.262	-.120	.040	.492	-.114	.316	.048	-.399	.164	.226	.081	.084	

TABLE II.

REFLECTION.

	15	-29	16	12	-17	41	-23	-34	44	1	45	21	13
15	0												
-29	+	0											
16		+	0										
12		+		0									
-17	+	+	+	+	0								
41		+			+	0							
-23	+	+	+	-	+	-	0						
-34	+	+	+	+	+	+	-	0					
44		+			+		-	-	0				
1		+			+		+	+		0			
45		+			+		-	+			0		
21		+			+	-	+	-			-	0	
13		+			+		-	+					0
TOTAL (+)	4	12	4	3	12	3	6	9	2	4	3	3	3
TOTAL (-)	0	0	0	1	0	2	6	3	2	0	2	3	1

The variables 29, 17, 23 and 34 have been reflected.

The reflected qualities were placed in the reflected table (Table III). The communalities were found by placing the highest quality in each column in the diagonal cells of that column.

(1) Square Root of Grand Total (41·935) = 6·47573

(2) Reciprocal = ·15442

(3) Check $\frac{1}{\frac{1}{T}} \times T = T$

(4)	(a)	·15442 x 4·890	=	·75511
	(b)	4·045	=	·62463
	(c)	3·598	=	·55560
	(d)	3·204	=	·49476
	(e)	4·109	=	·63451
	(f)	2·681	=	·41400
	(g)	1·346	=	·20785
	(h)	3·442	=	·53151
	(i)	2·801	=	·43253
	(j)	3·410	=	·52657
	(k)	2·848	=	·43979
	(l)	2·819	=	·43531
	(m)	2·742	=	·42342
				6·47559

These are the first factor loads.

TABLE III.

	15	-29	16	12	-17	41	-25	-34	44	1	45	21	13	TOTAL(+)	TOTAL(-)	TOTAL	C	TOTAL
15	(.577)	.577	.468	.375	.375	.369	.366	.334	.334	.304	.280	.269	.262	4.313		4.313	.577	4.890
-29	.577	(.577)	.319	.189	.530	.250	.236	.206	.155	.327	.256	.303	.120	3.468		3.468	.577	4.045
16	.468	.319	(.557)	.155	.433	.242	.011	.350	.177	.125	.557	.164	.040	3.041		3.041	.557	3.598
12	.375	.189	.155	(.492)	.137	.318	-.152	.418	.204	.267	.224	.085	.492	2.864	.152	2.712	.492	3.204
-17	.375	.530	.433	.134	(.433)	.101	.427	.428	.194	.282	.258	.397	.114	3.676		3.676	.433	4.109
41	.369	.250	.242	.318	.101	(.369)	-.026	.314	.193	.139	.122	-.026	.316	2.364	.052	2.312	.369	2.681
-25	.366	.236	.011	-.152	.427	-.026	(.427)	-.159	-.042	.057	-.103	.352	-.048	1.449	.530	.919	.427	1.346
-34	.334	.206	.350	.418	.428	.314	-.159	(.578)	-.006	.110	.578	-.108	.399	3.137	.273	2.864	.578	3.442
44	.334	.155	.177	.204	.194	.193	-.042	-.006	(.527)	.527	.059	.315	.164	2.322	.048	2.274	.527	2.801
1	.304	.327	.125	.267	.282	.139	.057	.110	.527	(.527)	.004	.515	.226	2.883		2.883	.527	3.410
45	.280	.256	.557	.224	.258	.122	-.103	.578	.059	.004	(.578)	-.046	.081	2.419	.149	2.270	.578	2.848
21	.269	.303	.164	.085	.397	-.026	.352	-.108	.315	.515	-.046	(.515)	.084	2.484	.180	2.304	.515	2.819
13	.262	.120	.040	.492	.114	.316	-.048	.399	.164	.226	.081	.084	(.492)	2.298	.048	2.250	.492	2.742
TOTAL(+)	4.313	3.468	3.041	2.864	3.676	2.364	1.449	3.137	2.322	2.883	2.419	2.484	2.298	36.718				
TOTAL(-)				.152		.052	.530	.273	.048	.117	.149	.180	.048		1.432			
TOTAL	4.313	3.468	3.041	2.712	3.676	2.312	.919	2.864	2.274	2.883	2.270	2.304	2.250			35.286		
C	.577	.577	.557	.492	.433	.369	.427	.578	.527	.527	.578	.515	.492				6.649	
TOTAL	4.890	4.045	3.598	3.204	4.109	2.681	1.346	3.442	2.801	3.410	2.848	2.819	2.742					41.935

TABLE IV.

TABLE OF CROSS-MULTIPLICATION [TABLE OF CORRELATION WITH THE FIRST FACTOR].

	.7551	.6246	.5556	.4948	.6545	.4140	.2079	.5515	.4325	.5266	.4398	.4353	.4234	T	E	T
.7551	(.570)	.472	.420	.374	.479	.313	.157	.401	.327	.398	.332	.329	.320	4.892	-.002	4.890
.6246	.472	(.390)	.347	.309	.396	.259	.130	.332	.270	.329	.275	.272	.265	4.046	-.001	4.045
.5556	.420	.347	(.309)	.275	.353	.230	.116	.295	.240	.293	.244	.242	.235	3.599	-.001	3.598
.4948	.374	.309	.275	(.245)	.314	.205	.103	.262	.214	.261	.218	.215	.210	3.206	-.002	3.204
.6545	.479	.396	.353	.314	(.408)	.262	.132	.337	.274	.334	.279	.276	.269	4.109	.000	4.109
.4140	.313	.259	.230	.206	.265	(.171)	.086	.220	.179	.218	.182	.180	.175	2.681	.000	2.681
.2079	.157	.130	.116	.103	.132	.086	(.043)	.111	.090	.110	.091	.091	.088	1.348	-.002	1.346
.5515	.401	.332	.295	.262	.337	.220	.111	(.283)	.230	.280	.234	.231	.225	3.442	.000	3.442
.4325	.327	.270	.240	.214	.274	.179	.090	.230	(.187)	.228	.190	.188	.183	2.800	.001	2.801
.5266	.398	.329	.293	.261	.334	.218	.110	.280	.228	(.277)	.232	.229	.223	3.412	-.002	3.410
.4398	.332	.275	.244	.218	.279	.182	.091	.234	.190	.232	(.193)	.192	.186	2.848	.000	2.848
.4353	.329	.272	.242	.215	.276	.180	.091	.231	.188	.229	.192	(.189)	.184	2.819	.000	2.819
.4234	.320	.265	.235	.210	.269	.175	.088	.225	.183	.223	.186	.184	(.179)	2.742	.000	2.742
TOTAL	4.892	4.046	3.599	3.206	4.109	2.681	1.348	3.442	2.800	3.412	2.848	2.819	2.742	41.944		
ERROR	-.002	-.001	-.001	-.002	.000	.000	-.002	.000	.001	-.002	.000	.000	.000		-.009	
TOTAL	4.890	4.045	3.598	3.204	4.109	2.681	1.346	3.442	2.801	3.410	2.848	2.819	2.742			41.935

TABLE V.

	15	-29	16	12	-17	41	-23	-34	44	1	45	21	13
15	(.577)-.570 =.007												
-29	.577-.472 =.105	(.577)-.390 =.187											
16	.468-.420 =.048	.319-.847 =.028	(.557)-.309 =.248										
12	.375-.374 =.001	.189-.309 =.120	.155-.275 =.120	(.492)-.245 =.247									
-17	.375-.479 =.104	.530-.396 =.134	.433-.353 =.080	.137-.314 =.177	(.433)-.403 =.030								
41	.369-.313 =.056	.250-.259 =.009	.242-.230 =.012	.318-.205 =.113	.101-.263 =.162	(.369)-.171 =.198							
-23	.366-.157 =.209	.236-.130 =.106	.011-.116 =.105	-.152-.103 =.256	.427-.132 =.295	-.026-.086 =.112	(.427)-.043 =.384						
-34	.334-.401 =.067	.206-.332 =.126	.350-.295 =.055	.418-.263 =.155	.428-.337 =.091	.314-.220 =.094	-.159-.111 =.270	(.578)-.283 =.295					
44	.334-.327 =.007	.155-.270 =.115	.177-.240 =.063	.204-.214 =.010	.194-.274 =.080	.193-.179 =.014	-.042-.090 =.132	-.006-.230 =.236	(.527)-.187 =.340				
1	.304-.398 =.094	.327-.329 =.002	.125-.293 =.168	.267-.261 =.006	.282-.334 =.052	.139-.218 =.079	.057-.110 =.053	-.110-.280 =.170	.527-.228 =.299	(.527)-.277 =.250			
45	.280-.332 =.052	.256-.275 =.019	.557-.244 =.313	.224-.218 =.006	.258-.279 =.021	.122-.182 =.060	-.103-.091 =.194	-.078-.234 =.344	.059-.190 =.131	.004-.232 =.228	(.578)-.193 =.385		
21	.269-.329 =.060	.303-.272 =.031	.164-.242 =.078	.085-.215 =.130	.397-.276 =.121	-.026-.180 =.206	.352-.091 =.261	-.008-.231 =.359	.315-.188 =.127	.515-.229 =.286	-.046-.192 =.238	(.515)-.190 =.325	
13	.262-.320 =.058	.120-.265 =.145	.040-.235 =.195	.492-.210 =.282	.114-.269 =.155	.316-.175 =.141	-.048-.088 =.136	-.099-.225 =.478	.164-.183 =.019	.226-.223 =.003	.081-.186 =.105	.084-.184 =.100	(.492)-.179 =.313
FIRST FACTOR LOADS	.75511	.62463	.55560	.49476	.63451	.41400	.20785	.53151	.45253	.52657	.43979	.43531	.42342

TABLE VI.

TABLE OF RESIDUALS.

	15	-29	16	12	-17	41	-23	-34	44	1	45	21	13	TOTAL
15	(.007)	.105	.048	.001	.104	.056	.209	.067	.007	.094	.052	.060	.058	-.002
-29	.105	(.187)	.028	.120	.134	.009	.106	.126	.115	.002	.019	.031	.145	-.001
16	.048	.028	(.248)	.120	.080	.012	.105	.058	.065	.168	.313	.078	.196	-.001
12	.001	.120	.120	(.247)	.177	.113	.255	.156	.010	.006	.006	.130	.282	-.002
-17	.104	.134	.080	.177	(.030)	.162	.295	.091	.080	.052	.021	.121	.155	.000
41	.056	.009	.012	.113	.162	(.198)	.112	.094	.014	.079	.060	.206	.141	.000
-23	.209	.106	.105	.255	.295	.112	(.384)	.276	.132	.053	.194	.261	.136	-.002
-34	.067	.126	.055	.155	.091	.094	.270	(.298)	.236	.170	.344	.339	.174	.000
44	.007	.115	.065	.010	.080	.014	.132	.236	(.340)	.299	.131	.127	.019	.001
1	.094	.002	.168	.006	.052	.079	.053	.170	.299	(.250)	.228	.286	.003	-.002
45	.052	.019	.313	.006	.021	.060	.194	.344	.131	.228	(.385)	.238	.105	.000
21	.060	.031	.078	.130	.121	.206	.261	.339	.127	.286	.238	(.325)	.100	.000
13	.058	.145	.195	.282	.155	.141	.136	.174	.019	.003	.105	.100	(.313)	.000
TOTAL	-.002	-.001	-.001	-.002	.000	.000	-.002	.000	.001	-.002	.000	.000	.000	-.009

EXTRACTION OF THE SECOND FACTOR

Pablo VII..../

TABLE VII.

	15	-29	16	12	-17	41	-23	-34	44	1	45	21	13
15													
-29													
16		-											
12		-	-										
-17	-			-									
41		-			-								
-23			-	-		-							
-34	-	-					-						
44		-	-	-	-		-	-					
1	-	-	-		-	-	-	-					
45	-	-			-	-	-		-	-			
21	-		-	-		-		-			-		
13	-	-	-		-		-		-		-	-	

TABLE VIII.

REFLECTION.

	-15	29	16	12	17	41	-23	-34	-44	-1	45	-21	13
-15	0												
29	+	0											
16	-	+	0										
12	-	+	-	0									
17	-	+	-	+	0								
41	-	+			+	0							
23	+	+	+	+	+	+	0						
-34	+	+			-		+	0					
-44	+	-	+	+	-	-	-	+	0				
-1	-	-	+	-	-	+	-	+	+	0			
45	+	+			+	-	+		+	+	0		
-21	-	+	+	+	+	+	+	+	+	+	+	0	
13	+	+	-		+		+		+	-	-	+	0
TOTAL (+)	6	10	5	5	7	5	10	6	8	6	7	11	6
TOTAL (-)	5	2	4	3	5	3	2	1	4	6	2	1	3

TABLE IX.

	-15	29	16	12	17	41	23	-34	-44	-1	45	-21	13	TOTAL(+)	TOTAL(-)	TOTAL	C	TOTAL
-15	(.209)	.105	-.048	-.001	-.104	-.056	.209	.067	.007	-.094	.052	-.060	.058	.498	.363	.135	.209	.344
29	.105	(.145)	.028	.120	.134	.009	.106	.126	-.115	-.002	.019	.051	.145	.823	.117	.706	.145	.851
16	-.048	.028	(.313)	-.120	-.080	.012	.105	.055	.063	.168	.313	.078	-.195	.822	.443	.379	.313	.692
12	-.001	.120	-.120	(.282)	.177	.113	.255	.155	.010	-.006	.006	.130	.282	1.248	.127	1.121	.282	1.403
17	-.104	.134	-.080	.177	(.295)	.162	.295	-.091	-.080	-.052	.021	.121	.155	1.065	.407	.658	.295	.953
41	-.056	.009	.012	.113	.162	(.206)	.112	.094	-.014	.079	.060	.206	.141	.928	.130	.798	.206	1.004
23	.209	.106	.105	.255	.295	.112	(.295)	.270	-.132	-.053	.194	.261	.136	1.943	.185	1.758	.295	2.053
-34	.067	.126	.055	.155	-.091	.094	.270	(.339)	.236	.170	.344	.339	.174	2.030	.091	1.939	.339	2.278
-44	.007	-.115	.063	.010	-.080	-.014	-.132	.236	(.299)	.299	.131	.127	.019	.892	.341	.551	.299	.850
-1	-.094	-.002	.168	-.006	-.052	.079	-.053	.170	.299	(.299)	.228	.286	-.005	1.230	.210	1.020	.299	1.319
45	.052	.019	.313	.006	.021	-.060	.194	.344	.131	.228	(.344)	.238	-.105	1.546	.165	1.381	.344	1.725
-21	-.060	.031	.078	.130	.121	.206	.261	.339	.127	.286	.238	(.339)	.100	1.917	.060	1.857	.339	2.196
13	.058	.145	-.195	.282	.155	.141	.136	.174	.019	-.005	.105	.100	(.282)	1.210	.303	.907	.282	1.189
TOTAL (+)	.498	.823	.822	1.248	1.065	.928	1.943	2.030	.892	1.230	1.546	1.917	1.210	16.152				
TOTAL (-)	.363	.117	.443	.127	.407	.130	.185	.091	.341	.210	.165	.060	.303		2.942			
TOTAL	.135	.706	.379	1.121	.658	.798	1.758	1.939	.551	1.020	1.381	1.857	.907			13.210		
C	.209	.145	.313	.282	.295	.206	.295	.339	.299	.299	.344	.339	.282				3.647	
TOTAL	.344	.851	.692	1.403	.953	1.004	2.053	2.278	.850	1.319	1.725	2.196	1.189					16.857

The variables 15, 29, 17, 23, 44, 1 and 21 have been reflected.

The qualities were placed in the large reflected table, Table IX. The highest quality in each column was placed in the diagonal cell of that particular column.

(1) Square root of grand total (16.857) = 4.10535

(2) Reciprocal = .24356

(3) Check $\frac{1}{\sqrt{T}} \times T = \sqrt{T}$

(4)

.24356	x .344	= .08379
	x .851	= .20727
	x .692	= .16854
	x 1.403	= .34172
	x .953	= .23211
	x 1.004	= .24453
	x 2.053	= .50003
	x 2.278	= .55483
	x .850	= .20703
	x 1.319	= .32126
	x 1.725	= .42014
	x 2.196	= .53486
	x 1.189	= .28959
		4.10570

These are the second factor loads.

Table X...../

TABLE X

TABLE OF CROSS MULTIPLICATION [TABLE OF CORRELATIONS WITH THE SECOND FACTOR]

	.0838	.2073	.1685	.3417	.2321	.2445	.5000	.5548	.2070	.3213	.4201	.5349	.2896	T	E	T
.0838	(.007)	.017	.014	.029	.02o	.021	.042	.047	.017	.027	.035	.045	.024	.345	-.001	.344
.2073	.017	(.043)	.035	.071	.048	.051	.104	.115	.043	.067	.087	.111	.060	.852	-.001	.851
.1685	.014	.035	(.028)	.058	.039	.041	.084	.094	.035	.054	.071	.090	.049	.692	.000	.692
.3417	.029	.071	.058	(.117)	.079	.084	.171	.190	.071	.110	.144	.183	.099	1.406	-.003	1.403
.2321	.020	.048	.039	.079	(.054)	.057	.116	.129	.048	.075	.098	.124	.067	.954	-.001	.953
.2445	.021	.051	.041	.084	.057	(.060)	.122	.136	.051	.079	.103	.131	.071	1.007	-.003	1.004
.5000	.042	.104	.084	.171	.116	.122	(.250)	.277	.104	.161	.210	.268	.145	2.054	-.001	2.053
.5548	.047	.115	.094	.190	.129	.136	.277	(.308)	.115	.178	.233	.297	.161	2.280	-.002	2.278
.2070	.017	.043	.035	.071	.048	.051	.104	.115	(.043)	.067	.087	.111	.060	.852	-.002	.850
.3213	.027	.067	.054	.110	.075	.079	.161	.178	.067	(.103)	.135	.172	.093	1.321	-.002	1.319
.4201	.035	.087	.071	.144	.098	.103	.210	.233	.087	.135	(.177)	.225	.122	1.727	-.002	1.725
.5349	.045	.111	.090	.183	.124	.131	.268	.297	.111	.172	.225	(.286)	.155	2.198	-.002	2.196
.2896	.024	.060	.049	.099	.067	.071	.145	.161	.060	.093	.122	.155	(.084)	1.190	-.001	1.189
TOTAL	.345	.852	.692	1.406	.954	1.007	2.054	2.280	.852	1.321	1.727	2.198	1.190	16.878		
ERROR	-.001	-.001	.000	-.003	-.001	-.003	-.001	-.002	-.002	+.002	-.002	-.002	-.001		-.021	
TOTAL	.344	.851	.692	1.403	.953	1.004	2.053	2.278	.850	1.319	1.725	2.196	1.189			16.857

TABLE XI

	-15	29	16	12	17	41	23	-34	-44	-1	45	-21	13
-15	(.209)-.007 =.208												
29	.106-.017 =.088	(.145)-.043 =.102											
16	-.048-.014 =.062	.028-.036 =.007	(.513)-.028 =.285										
12	-.001-.029 =.030	.120-.071 =.049	-.120-.058 =.178	(.282)-.117 =.165									
17	-.104-.020 =.124	.134-.048 =.086	-.080-.039 =.119	.177-.079 =.098	(.295)-.054 =.241								
41	-.066-.021 =.077	.009-.051 =.042	.012-.041 =.029	.115-.084 =.029	.162-.057 =.105	(.206)-.060 =.146							
23	.209-.042 =.167	.106-.104 =.002	.106-.084 =.021	.255-.171 =.084	.295-.116 =.179	.112-.122 =.010	(.295)-.250 =.045						
-34	.067-.047 =.020	.126-.115 =.011	.055-.084 =.029	.155-.190 =.035	-.091-.129 =.220	.094-.136 =.042	.270-.277 =.007	(.359)-.308 =.051					
-44	.007-.017 =.010	-.115-.043 =.158	.065-.035 =.028	.010-.071 =.061	-.080-.048 =.128	-.014-.051 =.065	-.132-.104 =.236	.236-.115 =.121	(.299)-.043 =.256				
-1	-.094-.027 =.121	-.002-.067 =.065	.168-.054 =.114	-.006-.110 =.116	-.062-.075 =.127	.079-.079 =.000	-.053-.161 =.214	.170-.178 =.008	.299-.067 =.232	(.299)-.103 =.196			
45	.052-.035 =.017	.019-.087 =.068	.313-.071 =.242	.006-.144 =.138	.021-.098 =.077	-.060-.105 =.165	.194-.210 =.015	.364-.253 =.111	.151-.087 =.044	.228-.135 =.093	(.344)-.177 =.167		
-21	-.060-.045 =.105	.031-.111 =.080	.078-.090 =.012	.130-.183 =.053	.121-.124 =.003	.206-.131 =.075	.261-.268 =.007	.339-.297 =.042	.127-.111 =.016	.286-.172 =.114	.238-.225 =.013	(.339)-.286 =.053	
13	.058-.024 =.034	.145-.060 =.085	-.195-.049 =.244	.282-.099 =.183	.155-.067 =.088	.141-.071 =.070	.156-.145 =.009	.174-.161 =.013	.019-.060 =.041	-.005-.093 =.088	-.105-.122 =.227	.100-.155 =.055	(.282)-.084 =.198
SECOND FACTOR LOADS	.08579	.20727	.16854	.34172	.23211	.24453	.50005	.55485	.20703	.32126	.42014	.53486	.28989

TABLE XII.

TABLE OF RESIDUALS.

	-15	29	16	12	17	41	23	-34	-44	-1	45	-21	13	TOTAL
-15	(.202)	.088	.062	.030	.124	.077	.167	.020	.010	.121	.017	.105	.034	-.001
29	.088	(.102)	.007	.049	.086	.042	.002	.011	.158	.069	.068	.080	.095	-.001
16	.062	.007	(.285)	.178	.119	.029	.021	.039	.028	.114	.242	.012	.244	.000
12	.030	.049	.178	(.165)	.098	.029	.084	.035	.061	.116	.138	.053	.183	-.003
17	.124	.086	.119	.098	(.241)	.105	.179	.220	.128	.127	.077	.003	.088	-.001
41	.077	.042	.029	.029	.105	(.146)	.010	.042	.065	.000	.163	.075	.070	-.003
23	.167	.002	.021	.084	.179	.010	(.045)	.007	.256	.214	.018	.007	.009	-.001
-34	.020	.011	.039	.035	.220	.042	.007	(.031)	.121	.008	.111	.042	.013	-.002
-44	.010	.158	.028	.061	.128	.065	.256	.121	(.256)	.232	.044	.016	.041	-.002
-1	.121	.069	.114	.116	.127	.000	.214	.008	.232	(.196)	.093	.114	.096	-.002
45	.017	.068	.242	.138	.077	.163	.018	.111	.044	.093	(.167)	.013	.227	-.002
-21	.105	.080	.012	.053	.003	.075	.007	.042	.016	.114	.013	(.053)	.055	-.002
13	.034	.095	.244	.183	.088	.070	.009	.013	.041	.096	.227	.055	(.198)	-.001
	-.001	-.001	.000	-.003	-.001	-.003	-.001	-.002	-.002	-.002	-.002	-.002	-.001	-.021

EXTRACTION OF THE THIRD FACTOR

Table XIII...../

TABLE XIII

	-15	29	16	12	17	41	23	-34	-44	-1	45	-21	13
-15													
29													
16	-	-											
12	-		-										
17	-		-										
41	-	-	-										
23						-							
-34			-	-	-	-	-						
-44	-	-		-	-	-	-						
-1	-	-		-	-		-	-					
45		-		-	-	-	-						
-21	-	-	-	-	-		-						
13			-				-		-	-	-	-	

TABLE XIV

REFLECTION.

	15	-29	16	12	-17	-41	-23	-34	-44	-1	45	-21	-13
15	0												
-29	+	0											
16	+	+	0										
-12	-	+	+	0									
-17	-	+	+	+	0								
-41	-	-	+	+	+	0							
-23	+	+	-	+	+	-	0						
-34	-	-	-	+	+	+	+	0					
-44	+	+	+	+	+	+	+		0				
-1	+	+	+	+	+	-	+	-		0			
45	-	+	+	+	+	+	+				0		
-21	+	+	-	+	+	-	+					0	
-13	+	+	+	+	+	+	-	-	+	+	+	+	0
TOTAL (+)	7	10	9	11	11	7	9	4	8	7	7	6	10
TOTAL (-)	5	2	3	1	1	5	3	5	0	2	1	2	2

TABLE XV

	16	-29	16	12	-17	-41	-23	-34	-44	-1	45	-21	-15	TOTAL(+)	TOTAL(-)	TOTAL	C	TOTAL
16	(.167)	.088	.062	-.030	-.124	-.077	.167	-.020	.010	.121	-.017	.105	.034	.587	.268	.319	.167	.486
-29	.088	(.158)	.007	.049	.086	-.042	.002	-.011	.158	.069	.068	.080	.085	.692	.053	.639	.158	.797
16	.062	.007	(.244)	.178	.119	.029	-.021	-.039	.028	.114	.242	.012	.244	1.023	.072	.951	.244	1.195
12	-.030	.049	.178	(.185)	.098	.029	.084	.035	.061	.116	.138	.055	.185	1.024	.030	.994	.185	1.177
-17	-.124	.086	.119	.098	(.220)	.105	.179	.220	.128	.127	.077	.005	.088	1.230	.124	1.106	.220	1.326
-41	-.077	-.042	.029	.029	.105	(.165)	-.010	.042	.065	-.000	.165	.075	.070	.503	.204	.299	.165	.462
-23	.167	.002	-.021	.084	.179	-.010	(.236)	.007	.236	.214	.016	.007	-.009	.912	.040	.872	.236	1.108
-34	-.020	-.011	-.039	.035	.220	.042	.007	(.220)	.121	-.008	.111	.042	-.015	.578	.091	.487	.220	.707
-44	.010	.158	.028	.061	.128	.065	.236	.121	(.236)	.232	.044	.016	.041	1.140		1.140	.236	1.376
-1	.121	.069	.114	.116	.127	-.000	.214	-.008	.232	(.232)	.095	.114	.096	1.296	.008	1.288	.232	1.520
45	-.017	.068	.242	.138	.077	.165	.016	.111	.044	.095	(.242)	.015	.227	1.192	.017	1.175	.242	1.417
-21	.105	.080	-.012	.055	.005	-.075	.007	.042	.016	.114	.015	.114)	.055	.488	.087	.401	.114	.515
-15	.034	.085	.244	.185	.088	.070	-.009	-.015	.041	.096	.227	.055	(.244)	1.123	.022	1.101	.244	1.345
TOTAL(+)	.587	.692	1.023	1.024	1.230	.503	.912	.578	1.140	1.296	1.192	.488	1.123	11.788				
TOTAL(-)	.268	.053	.072	.030	.124	.204	.040	.091		.008	.017	.087	.022		1.016			
TOTAL	.319	.639	.951	.994	1.106	.299	.872	.487	1.140	1.288	1.175	.401	1.101			10.772		
C	.167	.158	.244	.185	.220	.165	.236	.220	.236	.232	.242	.114	.244				2.659	
TOTAL	.486	.797	1.195	1.177	1.326	.462	1.108	.707	1.376	1.520	1.417	.515	1.345					15.431

TABLE XV

	15	-29	16	12	-17	-41	-23	-34	-44	-1	45	-21	-13	TOTAL(+)	TOTAL(-)	TOTAL	C	TOTAL
15	(.167)	.088	.062	-.030	-.124	-.077	.167	-.020	.010	.121	-.017	.105	.054	.587	.268	.319	.167	.486
-29	.088	(.158)	.007	.049	.086	-.042	.002	-.011	.158	.069	.068	.080	.085	.692	.053	.639	.158	.797
16	.062	.007	(.244)	.178	.119	.029	-.021	-.039	.028	.114	.242	.012	.244	1.023	.072	.951	.244	1.195
12	-.030	.049	.178	(.185)	.098	.029	.084	.035	.061	.116	.136	.053	.183	1.024	.030	.994	.183	1.177
-17	-.124	.086	.119	.098	(.220)	.105	.179	.220	.128	.127	.077	.003	.088	1.230	.124	1.106	.220	1.326
-41	-.077	-.042	.029	.029	.105	(.163)	-.010	.042	.065	-.000	.163	.075	.070	.503	.204	.299	.163	.462
-23	.167	.002	-.021	.084	.179	-.010	(.236)	.007	.236	.214	.016	.007	-.009	.912	.040	.872	.236	1.108
-34	-.020	-.011	-.039	.035	.220	.042	.007	(.220)	.121	-.008	.111	.042	-.015	.578	.091	.487	.220	.707
-44	.010	.158	.028	.061	.128	.065	.236	.121	(.236)	.232	.044	.016	.041	1.140		1.140	.236	1.376
-1	.121	.069	.114	.116	.127	-.000	.214	-.008	.232	(.232)	.095	.114	.096	1.296	.008	1.288	.232	1.520
45	-.017	.068	.242	.136	.077	.163	.016	.111	.044	.095	(.242)	.015	.227	1.192	.017	1.175	.242	1.417
-21	.105	.080	-.012	.055	.003	-.075	.007	.042	.016	.114	.015	.114)	.055	.486	.087	.401	.114	.515
-13	.054	.085	.244	.183	.088	.070	-.009	-.013	.041	.096	.227	.055	(.244)	1.125	.022	1.101	.244	1.345
TOTAL(+)	.587	.692	1.023	1.024	1.230	.503	.912	.578	1.140	1.296	1.192	.486	1.125	11.788				
TOTAL(-)	.268	.053	.072	.030	.124	.204	.040	.091		.008	.017	.087	.022		1.018			
TOTAL	.319	.639	.951	.994	1.106	.299	.872	.487	1.140	1.288	1.175	.401	1.101			10.772		
C	.167	.158	.244	.183	.220	.163	.236	.220	.236	.232	.242	.114	.244				2.659	
TOTAL	.486	.797	1.195	1.177	1.326	.462	1.108	.707	1.376	1.520	1.417	.515	1.345					13.431

The variables 15, 29, 17, 41, 23 and 13 have been reflected.

(1) Square root of grand total (13.431) = 3.66483;

(2) Reciprocal = .27286;

(3) Check: $\frac{1}{\frac{1}{\sqrt{T}}} = \sqrt{T}$

(4)

.27286	x	.486	= .13261
	x	.797	= .21747
	x	1.195	= .32607
	x	1.177	= .32116
	x	1.326	= .36181
	x	.462	= .12606
	x	1.108	= .30233
	x	.707	= .19291
	x	1.376	= .37546
	x	1.520	= .41475
	x	1.417	= .38664
	x	.515	= .14052
	x	1.345	= .36700
			3.66479

These are the third factor loads.

TABLE XVI.

TABLE OF CROSS-MULTIPLICATION [A TABLE OF CORRELATION WITH THE THIRD FACTOR].

	.1326	.2175	.3261	.3212	.3618	.1261	.3023	.1929	.3756	.4148	.3866	.1405	.3670	T	E	T
.1326	(.018)	.029	.043	.043	.048	.017	.040	.026	.050	.055	.051	.018	.049	.487	-.001	.486
.2175	.029	(.047)	.071	.070	.079	.027	.066	.042	.082	.090	.084	.031	.080	.798	-.001	.797
.3261	.043	.071	(.106)	.105	.118	.041	.099	.063	.123	.135	.126	.046	.120	1.196	-.001	1.195
.3212	.043	.070	.105	(.103)	.116	.041	.097	.062	.121	.133	.124	.045	.118	1.178	-.001	1.177
.3618	.048	.079	.118	.116	(.151)	.046	.109	.070	.136	.150	.140	.051	.133	1.327	-.001	1.326
.1261	.017	.027	.041	.041	.046	(.016)	.038	.024	.047	.052	.049	.018	.046	.462	.000	.462
.3023	.040	.066	.099	.097	.109	.038	(.091)	.058	.114	.125	.117	.043	.111	1.108	.000	1.108
.1929	.026	.042	.063	.062	.070	.024	.058	(.037)	.072	.080	.075	.027	.071	.707	.000	.707
.3756	.050	.082	.123	.121	.136	.047	.114	.072	(.141)	.156	.145	.053	.138	1.378	-.002	1.376
.4148	.055	.090	.135	.133	.150	.052	.125	.080	.156	(.172)	.160	.058	.152	1.518	.002	1.520
.3866	.051	.084	.126	.124	.140	.049	.117	.075	.145	.160	(.150)	.054	.142	1.417	.000	1.417
.1405	.018	.031	.046	.045	.051	.018	.043	.027	.053	.058	.054	(.020)	.052	.516	-.001	.515
.3670	.049	.080	.120	.118	.133	.046	.111	.071	.138	.152	.142	.052	(.135)	1.347	-.002	1.345
TOTAL	.487	.798	1.196	1.178	1.327	.462	1.108	.707	1.378	1.518	1.417	.516	1.347	13.439		
ERROR	-.001	-.001	-.001	-.001	-.001	.000	.000	.000	-.002	.002	.000	-.001	-.002		-.008	
TOTAL	.486	.797	1.195	1.177	1.326	.462	1.108	.707	1.376	1.520	1.417	.515	1.345			13.451

TABLE XVII.

	15	-29	16	12	-17	-41	-25	-34	-44	-1	45	-21	-13
15	(.167)-.018 =.149												
-29	.068-.029 =.069	(.188)-.047 =.111											
16	.062-.045 =.019	.007-.071 =.064	(.244)-.108 =.188										
12	-.030-.045 =.075	.048-.070 =.021	.178-.105 =.073	(.153)-.105 =.080									
-17	-.124-.048 =.172	.066-.079 =.007	.118-.116 =.001	.098-.116 =.018	(.220)-.131 =.089								
-41	-.077-.017 =.094	-.048-.027 =.069	.029-.041 =.012	.029-.041 =.012	.105-.046 =.059	(.165)-.016 =.147							
-25	.167-.048 =.127	.002-.066 =.064	-.021-.099 =.120	.084-.097 =.013	.179-.109 =.070	-.010-.058 =.048	(.236)-.091 =.145						
-34	-.020-.026 =.046	-.011-.062 =.050	-.059-.065 =.106	.055-.082 =.027	.220-.070 =.150	.042-.024 =.018	.007-.058 =.151	(.220)-.037 =.183					
-44	.010-.060 =.049	.158-.082 =.076	.038-.123 =.085	.061-.121 =.060	.128-.136 =.008	.065-.047 =.018	.256-.114 =.122	.121-.072 =.049	(.236)-.141 =.095				
-1	.121-.055 =.066	.069-.090 =.021	.114-.185 =.071	.116-.133 =.017	.127-.150 =.023	-.000-.082 =.082	.214-.125 =.089	-.098-.080 =.018	.252-.156 =.096	(.252)-.172 =.080			
45	-.017-.051 =.068	.068-.084 =.016	.242-.126 =.116	.188-.134 =.014	.077-.140 =.063	.165-.049 =.114	.018-.117 =.101	.121-.075 =.046	.044-.145 =.101	.093-.180 =.087	(.242)-.150 =.092		
-21	.105-.018 =.087	.080-.031 =.049	-.012-.046 =.058	.083-.045 =.038	.008-.081 =.048	-.075-.018 =.056	.007-.042 =.035	.042-.027 =.015	.016-.053 =.037	.114-.058 =.056	.018-.084 =.041	(.114)-.020 =.094	
-13	.084-.049 =.015	.085-.080 =.006	.244-.120 =.124	.183-.118 =.065	.088-.153 =.065	.070-.046 =.024	-.009-.111 =.130	-.065-.071 =.006	.041-.138 =.097	.096-.152 =.056	.287-.142 =.085	.055-.062 =.008	(.244)-.136 =.109

THIRD FACTOR LOADS	.15261	.21747	.32607	.32116	.36181	.12606	.30235	.12891	.37846	.41475	.58664	.14062	.36700

TABLE XVIII

TABLE OF RESIDUALS.

	15	-29	16	12	-17	-41	-23	-34	-44	-1	45	-21	-13	TOTAL
15	(.149)	.059	.019	.073	.172	.094	.127	.046	.040	.066	.068	.087	.015	-.001
-29	.059	(.111)	.064	.021	.007	.069	.064	.053	.076	.021	.016	.049	.005	-.001
16	.019	.064	(.138)	.073	.001	.012	.120	.102	.095	.021	.116	.058	.124	-.001
12	.073	.021	.073	(.080)	.018	.012	.013	.027	.060	.017	.014	.008	.065	-.001
-17	.172	.007	.001	.018	(.089)	.059	.070	.150	.008	.023	.065	.048	.045	-.001
-41	.094	.069	.012	.012	.059	(.147)	.048	.018	.018	.052	.114	.093	.024	.000
-23	.127	.064	.120	.013	.070	.048	(.145)	.051	.122	.089	.101	.036	.120	.000
-34	.046	.053	.102	.027	.150	.018	.051	(.183)	.049	.088	.036	.015	.084	.000
-44	.040	.076	.095	.060	.008	.018	.122	.049	(.095)	.076	.101	.037	.097	-.002
-1	.066	.021	.021	.017	.023	.052	.089	.088	.076	(.060)	.067	.056	.056	.002
45	.068	.016	.116	.014	.065	.114	.101	.036	.101	.067	(.092)	.041	.085	.000
-21	.087	.049	.058	.008	.048	.093	.036	.015	.037	.056	.041	(.094)	.003	-.001
-13	.015	.005	.124	.065	.045	.024	.120	.084	.097	.056	.085	.003	(.109)	-.002
TOTAL	-.001	-.001	-.001	-.001	-.001	.000	.000	.000	-.002	.002	.000	-.001	-.002	-.008

EXTRACTION OF THE FOURTH FACTOR

TABLE XIX.

	15	-29	16	12	-17	-41	-23	-34	-44	-1	45	-21	-13
15													
-29													
16		-											
12	-	-											
-17	-			-									
-41	-	-	-	-									
-23		-	-	-		-							
-34	-	-	-	-			-						
-44	-		-	-	-								
-1		-	-	-	-	-		-					
45	-	-			-		-		-	-			
-21			-		-	-	-		-		-		
-13	-				-		-	-	-	-			

TABLE XX.

REFLECTION.

	15	-29	-16	-12	-17	41	-23	-34	-44	-1	-45	-21	13
15	0												
-29		0											
-16	-	+	0										
-12	+	+	+	0									
-17	-		-	+	0								
41	+	+	-	-	-	0							
-23		-	+	+		+	0						
-34	-	-	+	+		-	-	0					
-44	-		+	+	-	-			0				
-1		-	+	+	-	+		-		0			
-45	+	+	+	+	+	+	+	-	+	+	0		
-21			+	-	-	+	-		-		+	0	
13	+	-	+	+	+	+	+	+	+	+	+	-	0
TOTAL (+)	4	4	9	10	3	7	5	3	4	5	11	3	10
TOTAL (-)	4	4	3	2	6	5	3	6	4	3	1	5	1

TABLE XXI

	15	-29	-16	-12	-17	41	-23	-34	-44	-1	-45	-21	13	TOTAL(+)	TOTAL (-)	TOTAL	C	TOTAL
15	(.172)	.059	-.019	.073	-.172	.094	.127	-.046	-.040	.066	.068	.087	.015	.589	.277	.312	.172	.484
-29	.059	(.076)	.064	.021	.007	.069	-.064	-.053	.076	-.021	.016	.049	-.005	.361	.143	.218	.076	.294
-16	-.019	.064	(.124)	.073	-.001	-.012	.120	.102	.095	.021	.116	.058	.124	.773	.032	.741	.124	.865
-12	.073	.021	.073	(.073)	.018	-.012	.013	.027	.060	.017	.014	-.008	.065	.381	.020	.361	.073	.434
-17	-.172	.007	-.001	.018	(.172)	-.059	.070	.150	-.008	-.023	.063	-.048	.045	.353	.311	.042	.172	.214
41	.094	.069	-.012	-.012	-.059	(.114)	.048	-.018	-.018	.052	.114	.093	.024	.494	.119	.375	.114	.489
-23	.127	-.064	.120	.013	.070	.048	(.127)	-.051	.122	.089	.101	-.036	.120	.810	.151	.659	.127	.786
-34	-.046	-.053	.102	.027	.150	-.018	-.051	(.150)	.049	-.088	-.036	.015	.084	.427	.292	.135	.150	.285
-44	-.040	.076	.095	.060	-.008	-.018	.122	.049	(.122)	.076	.101	-.037	.097	.676	.103	.573	.122	.695
-1	.066	-.021	.021	.017	-.023	.052	.089	-.088	.076	(.089)	.067	.056	.056	.500	.132	.368	.089	.457
-45	.068	.016	.116	.014	.063	.114	.101	-.036	.101	.067	(.116)	.041	.085	.786	.036	.750	.116	.866
-21	.087	.049	.058	-.008	-.048	.093	-.036	.015	-.037	.056	.041	(.087)	-.003	.399	.132	.267	.087	.354
13	.015	-.005	.124	.065	.045	.024	.120	.084	.097	.056	.085	-.003	(.124)	.715	.008	.707	.124	.831
TOTAL(+)	.589	.361	.773	.381	.353	.494	.810	.427	.676	.500	.786	.399	.715	7.264				
TOTAL(-)	.277	.143	.032	.020	.311	.119	.151	.292	.103	.132	.036	.132	.008		1.756			
TOTAL	.312	.218	.741	.361	.042	.375	.659	.135	.573	.368	.750	.267	.707			5.508		
C	.172	.076	.124	.073	.172	.114	.127	.150	.122	.089	.116	.087	.124				1.546	
TOTAL	.484	.294	.865	.434	.214	.489	.786	.285	.695	.457	.866	.354	.831					7.054

The variables 16, 12, 41, 45, and 13 have been reflected.

(1) Square root of Grand total (7.054) = 2.65651

(2) Reciprocal = .37643

(3) Check $\frac{1}{\sqrt{T}} \times T = \sqrt{T}$

(4)

.37643	x .484	=	.18219
	x .294	=	.11067
	x .865	=	.32561
	x .434	=	.16337
	x .214	=	.08056
	x .489	=	.18407
	x .786	=	.29587
	x .285	=	.10728
	x .695	=	.26162
	x .457	=	.17203
	x .866	=	.32600
	x .354	=	.13326
	x .831	=	.31281
			2.65534

These are the fourth factor loads.

TABLE XXII.

TABLE OF CROSS MULTIPLICATION [A TABLE OF CORRELATIONS WITH THE FOURTH FACTOR]

	.1822	.1107	.3256	.1634	.0806	.1841	.2959	.1073	.2616	.1720	.3260	.1333	.3128	TOTAL	ERROR	TOTAL
.1822	(.033)	.020	.059	.030	.015	.034	.054	.020	.048	.031	.059	.024	.057	.484	.000	.484
.1107	.020	(.012)	.036	.018	.009	.020	.033	.012	.029	.019	.036	.015	.035	.294	.000	.294
.3256	.059	.036	(.106)	.053	.026	.060	.096	.035	.085	.056	.106	.043	.102	.863	.002	.865
.1634	.030	.018	.053	(.027)	.013	.030	.048	.018	.043	.028	.053	.022	.051	.434	.000	.434
.0806	.015	.009	.026	.013	(.007)	.015	.024	.009	.021	.014	.026	.011	.025	.215	-.001	.214
.1841	.034	.020	.060	.030	.015	(.034)	.055	.020	.048	.032	.060	.025	.058	.491	-.002	.489
.2959	.054	.033	.096	.048	.024	.055	(.088)	.032	.077	.051	.097	.039	.093	.787	-.001	.786
.1073	.020	.012	.035	.018	.009	.020	.032	(.012)	.028	.019	.035	.014	.034	.288	-.003	.285
.2616	.048	.029	.085	.043	.021	.048	.077	.028	(.068)	.045	.085	.035	.082	.694	.001	.695
.1720	.031	.019	.056	.028	.014	.032	.051	.019	.045	(.030)	.056	.023	.054	.458	-.001	.457
.3260	.059	.036	.106	.053	.026	.060	.097	.035	.085	.056	(.106)	.044	.102	.865	.001	.866
.1333	.024	.015	.043	.022	.011	.025	.039	.014	.035	.023	.044	(.018)	.042	.355	-.001	.354
.3128	.057	.035	.102	.051	.025	.058	.093	.034	.082	.054	.102	.042	(.098)	.833	-.002	.831
TOTAL	.484	.294	.863	.434	.215	.491	.787	.288	.694	.458	.865	.355	.833	7.061		
ERROR	.000	.000	.002	.000	-.001	-.002	-.001	-.003	.001	-.001	.001	-.001	-.002		-.007	
TOTAL	.484	.294	.865	.434	.214	.489	.786	.285	.695	.457	.866	.354	.831			7.054

TABLE XXIII.

	15	-29	-16	-12	-17	41	-23	-34	-44	-1	-45	-21	13
15	(.172)-.033 =.139												
-29	.059-.028 =.039	(.076)-.012 =.064											
-16	-.019-.059 =.078	.064-.036 =.028	(.124)-.106 =.018										
-12	.073-.030 =.043	.021-.018 =.003	.073-.053 =.020	(.073)-.027 =.046									
-17	-.172-.015 =.187	.007-.009 =.002	-.001-.026 =.027	.018-.013 =.005	(.172)-.007 =.165								
41	.094-.034 =.060	.069-.020 =.049	-.012-.060 =.072	-.012-.030 =.042	-.059-.015 =.074	(.114)-.034 =.080							
-23	.127-.054 =.073	-.064-.033 =.097	.120-.096 =.024	.013-.048 =.035	.070-.024 =.046	.048-.055 =.007	(.127)-.088 =.039						
-34	-.046-.020 =.066	-.053-.012 =.065	.102-.035 =.067	.027-.018 =.009	.150-.009 =.141	-.018-.020 =.038	-.051-.032 =.083	(.150)-.012 =.138					
-44	-.040-.048 =.088	.076-.029 =.047	.095-.085 =.010	.060-.043 =.017	-.008-.021 =.029	-.018-.048 =.066	.122-.077 =.045	.049-.028 =.021	(.122)-.068 =.054				
-1	.066-.031 =.035	-.021-.019 =.040	.021-.056 =.035	.017-.028 =.011	-.023-.014 =.037	.052-.032 =.020	.089-.051 =.038	-.088-.019 =.107	.076-.045 =.031	(.089)-.030 =.059			
-45	.068-.059 =.009	.016-.036 =.020	.116-.106 =.010	.014-.053 =.039	.063-.026 =.037	.114-.060 =.054	.101-.097 =.004	-.036-.035 =.071	.101-.085 =.016	.067-.056 =.011	(.116)-.106 =.010		
-21	.087-.024 =.063	.049-.015 =.034	.058-.043 =.015	-.008-.022 =.030	-.048-.011 =.059	.093-.025 =.068	-.036-.039 =.075	.015-.014 =.001	-.037-.035 =.072	.056-.023 =.033	.041-.044 =.003	(.087)-.018 =.069	
13	.015-.057 =.042	-.005-.035 =.040	.124-.102 =.022	.065-.051 =.014	.045-.025 =.020	.024-.058 =.034	.120-.093 =.027	.084-.034 =.050	.097-.082 =.015	.056-.054 =.002	.085-.102 =.017	-.003-.042 =.045	(.124)-.098 =.026
FOURTH FACTOR LOADS	.18219	.11067	.32561	.16337	.08056	.18403	.29580	.10726	.26156	.17199	.32591	.13322	.31281

TABLE XXIV

TABLE OF RESIDUALS

	15	-29	-16	-12	-17	41	-23	-34	-44	-1	-45	-21	13	TOTAL
15	(.139)	.039	.078	.043	.187	.060	.073	.066	.088	.035	.009	.063	.042	.000
-29	.039	(.064)	.028	.003	.002	.049	.097	.065	.047	.040	.020	.034	.040	.000
-16	.078	.028	(.018)	.020	.027	.072	.024	.067	.010	.035	.010	.015	.022	.002
-12	.043	.003	.020	(.046)	.005	.042	.035	.009	.017	.011	.039	.030	.014	.000
-17	.187	.002	.027	.005	(.165)	.074	.046	.141	.029	.037	.037	.059	.020	-.001
41	.060	.049	.072	.042	.074	(.080)	.007	.038	.066	.020	.054	.068	.034	-.002
-23	.073	.097	.024	.035	.046	.007	(.039)	.083	.045	.038	.004	.075	.027	-.001
-34	.066	.065	.067	.009	.141	.038	.083	(.138)	.021	.107	.071	.001	.050	-.003
-44	.088	.047	.010	.017	.029	.066	.045	.021	(.054)	.031	.016	.072	.015	.001
-1	.035	.040	.035	.011	.037	.020	.038	.107	.031	(.059)	.011	.033	.002	-.001
-45	.009	.020	.010	.039	.037	.054	.004	.071	.016	.011	(.010)	.003	.017	.001
-21	.063	.034	.015	.030	.059	.068	.075	.001	.072	.033	.003	(.069)	.045	-.001
13	.042	.040	.022	.014	.020	.034	.027	.050	.015	.002	.017	.045	(.026)	-.002
TOTAL	.000	.000	.002	.000	-.001	-.002	-.001	-.003	.001	-.001	.001	-.001	-.002	-.007

To find the Probable Error of the mean correlation of the first correlation table (Table II):

$$.6745 \times \frac{(1 - r^2)}{\sqrt{n}} ,$$

where n = number of cases, and

r = correlation coefficient.

Total = 19.075

Mean $= \frac{19.075}{66} = .28901$.

Whence, $.6745 \times \frac{(1 - r^2)}{\sqrt{n}} = .6745 \times \frac{1 - .28901^2}{\sqrt{160}}$

$= .6745 \times \frac{1 - .03353}{12.64911}$

$= .6745 \times .07245$

$= .04888$

The Probable Error of the mean correlation of the first correlation table = .049.

The mean Residual of the Table of Residuals (Table XVIII), (third factor), was found.

Total = 4.510

Mean $= \frac{4.510}{66}$

Therefore, Mean Residual = .068.

The mean Residual of the Table of Residuals (Table XXIV) was found.

Total = 3.209

Mean $= \frac{3.209}{66} = .04862$

Therefore, Mean Residual = .049.

As the mean Residual of the Table of Residuals, (Table XXIV - fourth factor) was found to be not greater than the Probable Error of the mean correlation of the first correlation table, the analysis had been completed, four factors having been extracted from the material.

ASCERTAINING H^2.

H^2 had then to be ascertained. H^2 is the sum of the squares of the factor loadings - the communalities which should be placed in the diagonal cells of the original correlation matrix. The value of H^2 acts as a check to the guessed communalities in the original table of correlations.

FACTORS (Former Communalities in Brackets).

	I^2	+	II^2	+	III^2	+	IV^2	=	H^2
(a)	.57019	+	.00702	+	.01759	+	.03319	=	.62799 (.577)
(b)	.39016	+	.04305	+	.04729	+	.01225	=	.49275 (.577)
(c)	.30869	+	.02841	+	.10632	+	.10602	=	.54944 (.557)
(d)	.24479	+	.11677	+	.10314	+	.02669	=	.49139 (.492)
(e)	.40260	+	.05388	+	.13091	+	.00649	=	.59288 (.433)
(f)	.17140	+	.05980	+	.01589	+	.03388	=	.28097 (.369)
(g)	.04320	+	.25003	+	.09140	+	.08754	=	.47217 (.427)
(h)	.28250	+	.30784	+	.03721	+	.01151	=	.63906 (.578)
(i)	.18708	+	.04286	+	.14097	+	.06845	=	.43936 (.527)
(j)	.27728	+	.10321	+	.17202	+	.02960	=	.58211 (.527)
(k)	.19342	+	.17652	+	.14949	+	.10628	=	.62571 (.578)
(l)	.18950	+	.28608	+	.01975	+	.01776	=	.51309 (.515)
(m)	.17929	+	.03386	+	.13469	+	.09785	=	.49569 (.492)

As the difference between the ascertained communalities and the guessed communalities was not very large, it was considered unnecessary to repeat the analysis. Graphical rotation of the axis was then performed.

PART II...../

P A R T I I

GRAPHICAL ROTATION OF THE AXIS

Four centroid factors have so far been extracted from the material. These factors are "centroid" factors having no psychological significance.

Rotation of the axis is the process by which objective factors are obtained from the centroid factors. As. G. H. Thompson states, "it cannot be too emphatically pointed out that the first factors which emerge from the 'centroid' process need not have any psychological significance as unitary primary traits. It is only after rotation to a suitable position that this can be expected."[1]

In the first instance, Factors I and II were plotted together. The rotated axis was passed through variable 15 (feelings of inferiority). The choice of this plotted point for passing the axis is due to the fact that this thesis is devoted to the study of feelings of inferiority by the centroid method of factorial analysis and therefore should form the basis of the graphical rotation of the axis.

The factors were plotted in the following manner:

A. FACTORS I_0 and II_0 = I_1 and II_1

I_1 and III_0 = I_2 and III_1

I_2 and IV_0 = I_3 and IV_1

Broken lines represent the rotated axis. In order to check the numerous steps in the rotation the Transformation Matrix was obtained.

Transformation Matrix

$\cos\theta$	$\sin\theta$
$-\sin\theta$	$\cos\theta$

1. G.H.Thompson: "The Factorial Analysis of Human Ability" p. 247.

A. FIGURE I. I_o and II_o Plotted.

V	I_o	II_o	I_1	II_1	I_o	II_o
15	.76	-.08	.76	.00	.7551	-.0838
29	-.63	.21	-.65	.13	-.6246	.2073
16	.56	.17	.54	.23	.5556	.1685
12	.50	.34	.46	.40	.4948	.3417
17	-.64	.23	-.66	.15	-.6345	.2321
41	.41	.25	.38	.30	.4140	.2445
23	-.21	.50	-.26	.47	-.2079	.5000
34	-.53	-.56	-.46	-.61	-.5315	-.5548
44	.43	-.21	.45	-.16	.4325	-.2070
1	.53	-.32	.56	-.26	.5266	-.3213
45	.44	.42	.39	.47	.4398	.4201
21	.44	-.54	.49	-.49	.4353	-.5349
13	.42	.29	.39	.34	.4234	.2896

$$T = \begin{Vmatrix} .9968 & .0859 \\ -.0859 & .9968 \end{Vmatrix}$$

θ = angle of rotation

$\theta = 7^\circ$

F_1 Column

	I_1	II_1
15	.7598	.0000
29	-.6453	.1297
16	.5311	.2350
12	.4494	.3995
17	-.6581	.1531
41	.3812	.2933
23	-.2673	.4711
34	-.4600	-.6155
44	.4545	-.1528
1	.5620	-.2547
45	.3854	.4706
21	.4973	-.4778
13	.3850	.3391

This checked by multiplication.

FIGURE II: I_1 and III_o Plotted.

V	I_1	III_o	I_2	III_1	I_1	III_o
15	.76	.13	.77	.00	.7598	.1326
29	-.65	-.22	-.67	-.11	-.6463	-.2175
16	.54	.33	.59	.22	.5311	.3261
12	.46	.32	.51	.23	.4494	.3212
17	-.66	-.36	-.71	-.22	-.6581	-.3618
41	.38	-.13	.35	-.20	.3812	-.1261
23	-.26	-.30	-.30	-.25	-.2673	-.3023
34	-.46	- .19	-.49	-.10	-.4600	-.1929
44	.45	-.38	.38	-.46	.4545	-.3755
1	.56	-.42	.48	-.52	.5620	-.4148
45	.39	.39	.45	.31	.3854	.3866
21	.49	-.14	.46	-.22	.4973	-.1405
13	.39	-.37	.32	-.44	.3850	-.3670

$$\begin{Vmatrix} .9934 & -.2397 \\ .2397 & .9934 \end{Vmatrix}^T$$

$\theta = 10^o$

F_2 Column

	I_2	III_1
15	.7714	-.0000
29	-.6732	-.1022
16	.5797	.2289
12	.4984	.2383
17	-.7110	-.2420
41	.3535	-.1904
23	-.3157	-.2513
34	-.4866	-.1101
44	.3825	-.4487
1	.4813	-.5061
45	.4467	.3139
21	.4655	-.2248
13	.3155	-.4284

This checked by multiplication.

FIGURE III: I_2 and IV_o Plotted

V	I_2	IV_o	I_3	IV_1	I_2	IV_o
15	.77	.18	.79	.00	.7714	.18212
29	-.67	-.11	-.68	-.05	-.6732	-.1107
16	.59	-.33	.50	-.46	.5797	-.3256
12	.51	-.16	.46	-.27	.4984	-.1634
17	-.71	-.08	-.71	.09	-.7110	-.0806
41	.35	.18	.39	.09	.3535	.1840
23	-.30	-.30	-.37	-.22	-.3157	-.2958
34	-.49	-.11	-.50	.00	-.4866	-.1073
44	.38	-.26	.32	-.34	.3825	-.2616
1	.48	-.17	.43	-.28	.4813	-.1720
45	.45	-.33	.36	-.42	.4467	-.3260
21	.46	-.13	.42	-.24	.4655	-.1333
13	.32	.31	.39	.23	.3155	.3128

$$\begin{Vmatrix} T \\ .9869 & -.3837 \\ .3837 & .9869 \end{Vmatrix}$$

$\theta = 14^\circ$

F_3 Column

	I_3	IV_1
15	.7926	.0000
29	-.6799	.0553
16	.4836	-.4563
12	.4442	-.2792
17	-.7095	.0938
41	.3875	.0930
23	-.3780	-.2107
34	-.4981	.0136
44	.3079	-.3464
1	.4254	-.2833
45	.3545	-.4244
21	.4194	-.2420
13	.3818	.2272

This checked by multiplication.

B. FACTORS II_1 and III_1 = II_2 and III_2

II_2 and IV_1 = II_3 and IV_2

FIGURE IV: II_1 and III_1 Plotted

V	II_1	III_1	II_2	III_2	II_1	III_1
15	.00	.00	.00	-.00	.0088	-.0013
29	.13	-.11	.05	-.17	.1297	-.1022
16	.23	.22	.31	.06	.2350	.2289
12	.40	.23	.46	-.02	.3995	.2383
17	.15	-.22	-.00	-.28	.1531	-.2420
41	.30	-.20	.14	-.33	.2933	-.1904
23	.47	-.25	.25	-.47	.4711	-.2513
34	-.61	-.10	-.57	.25	-.6155	-.1101
44	-.16	-.46	-.38	-.29	-.1528	-.4487
1	-.26	-.52	-.50	-.29	-.2547	-.5061
45	.47	.31	.57	.000	.4706	.3139
21	-.49	-.22	-.52	.09	-.4778	-.2248
13	.34	-.44	.04	-.55	.3391	-.4284

The rotated axis was passed through the centroid point of the plotted points 45, 12 and 16 (Sociability, Easily Gets Gloomy Ideas of His Own, and Shyness).

F_4 Column...../

$\theta = 33^{\circ}$

$$T = \begin{Vmatrix} .9236 & -.7361 \\ .7361 & .9236 \end{Vmatrix}$$

F_4 Column

	II_2	III_2
15	.0000	-.0000
29	.0530	-.1564
16	.3218	.0640
12	.4649	-.0177
17	-.0034	-.2864
41	.1423	-.3193
23	.2582	-.4673
34	-.5763	.2430
44	-.3725	-.2951
1	-.4893	-.2858
45	.5656	.0069
21	-.5230	.0717
13	.0511	-.5439

This checked by multiplication.

FIGURE V...../

FIGURE V: II_2 and IV_1 Plotted.

V	II_2	IV_1	II_3	IV_2	II_2	IV_1
15	.00	.00	.00	-.00	.0067	.0098
29	.05	.05	.01	.07	.0530	.0553
16	.31	-.46	.53	-.17	.3218	-.4563
12	.46	-.27	.53	.07	.4649	-.2792
17	-.00	.09	-.06	.08	-.0034	.0938
41	.14	.09	.06	.16	.1423	.0930
23	.25	-.22	.33	-.01	.2582	-.2107
34	-.57	.00	-.44	-.36	-.5763	.0136
44	-.38	-.34	-.07	-.50	-.3725	-.3464
1	-.50	-.28	-.22	-.53	-.4893	-.2853
45	.57	-.42	.71	.04	.5656	-.4244
21	-.52	-.24	-.26	-.51	-.5250	-.2420
13	.04	.23	-.11	.21	.0511	.2272

T

$$\begin{Vmatrix} .8935 & .7941 \\ -.7941 & .8935 \end{Vmatrix}$$

$\theta = 38.6^\circ$

F_5 Column

	II_3	IV_2
15	.0000	-.0000
29	.0071	.0763
16	.5360	-.1569
12	.5376	.0709
17	-.0610	.0712
41	.0535	.1614
23	.3331	-.0041
34	-.4594	-.3481
44	-.0757	- .5029
1	-.2065	-.5261
45	.7067	.0200
21	-.2587	-.5149
13	-.1015	.2096

This checked by multiplication.

C. III_2 and IV_2 = III_3 and IV_3

FIGURE VI

The rotated axis was passed through the centroid point of the plotted points 1 and 44 (Attitude to Problems of Life, and Mature, Stable and Well Balanced).

V	III_2	IV_2	III_3	IV_3	III_2	IV_2
15	-.00	-.00	-.00	-.000	-.0059	-.0035
29	-.17	.07	-.02	-.18	-.1564	.0763
16	.06	-.17	.12	-.13	.0640	-.1569
12	-.02	.07	.05	-.05	-.0177	.0709
17	-.28	.08	-.09	-.28	-.2864	.0712
41	-.33	.16	-.02	-.37	-.3193	.1614
23	-.47	-.01	-.24	-.40	-.4673	-.0041
34	.25	-.36	-.19	.40	.2430	-.3481
44	-.29	-.50	-.58	.01	-.2931	-.5029
1	-.29	-.53	-.60	.01	-.2858	-.5261
45	.00	.04	.04	-.02	.0069	.0200
21	.09	-.51	-.40	.33	.0717	-.5149
13	-.55	.21	-.10	-.58	-.5439	.2096

F 6 Column...../

$\theta = 30^{\circ}$

T

.9375	.6990
-.6990	.9375

F_6 Column

	III_3	IV_3
15	-.0000	-.0000
29	-.0271	-.1719
16	.1339	-.1038
12	.0479	-.0552
17	-.1060	-.2754
41	-.0509	-.3542
23	-.2714	-.3804
34	-.1458	.3987
44	-.5801	.0011
1	-.5949	.0140
45	.0204	-.0158
21	-.3808	.3541
13	-.1403	-.5657

This checked by multiplication.

PART III...../

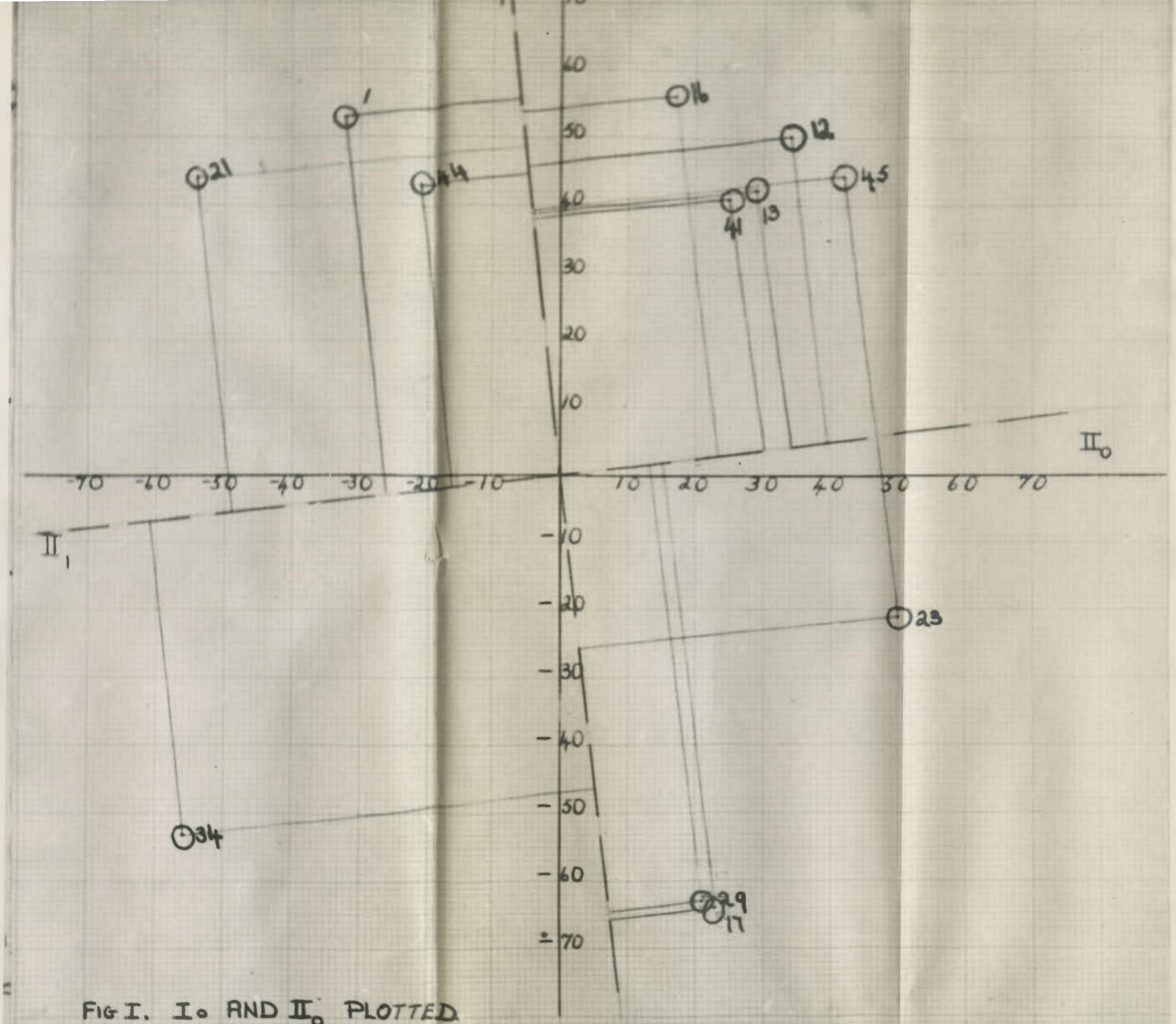


FIG I. I_0 AND II_0 PLOTTED

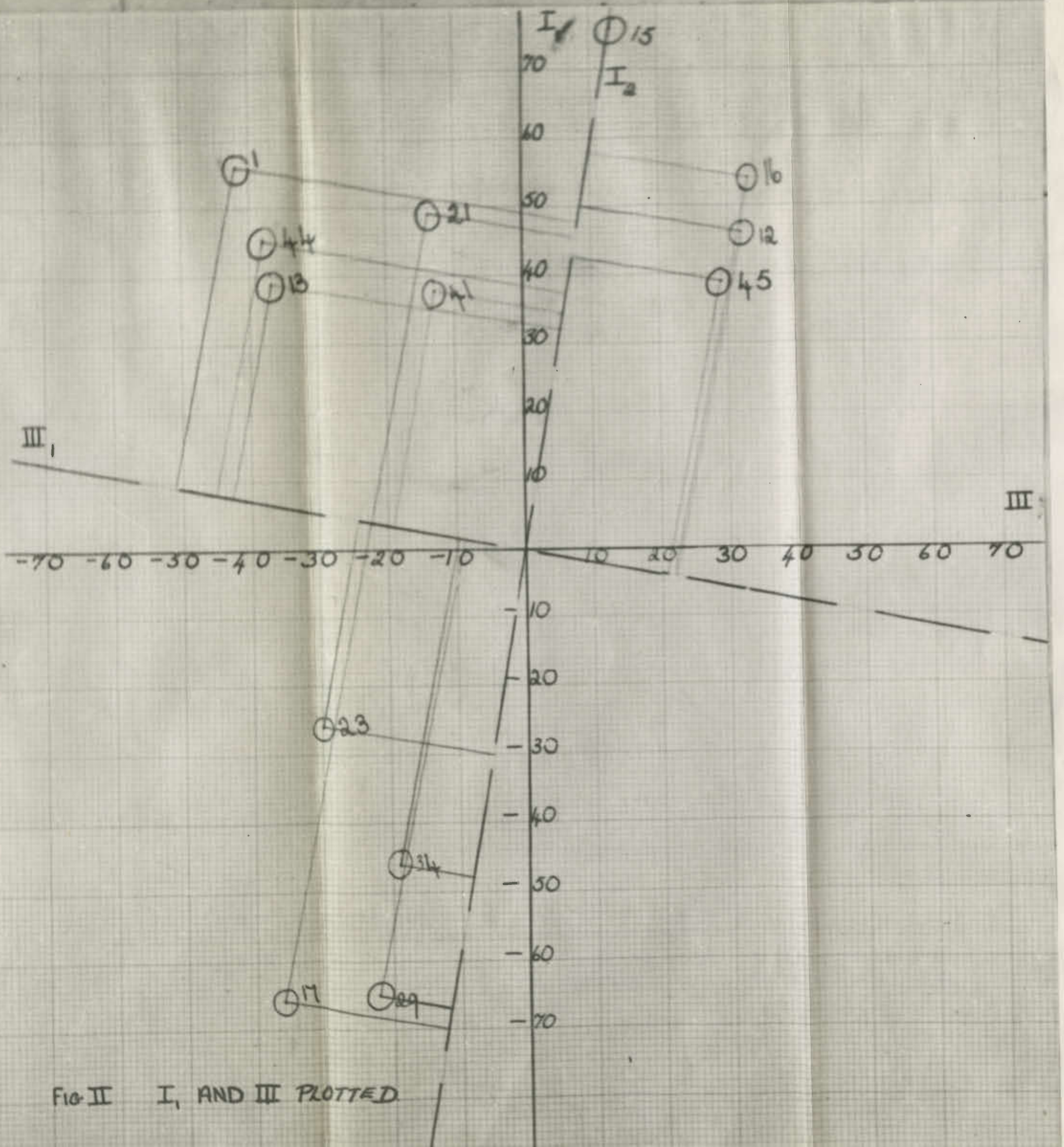


FIG II I, AND III PLOTTED

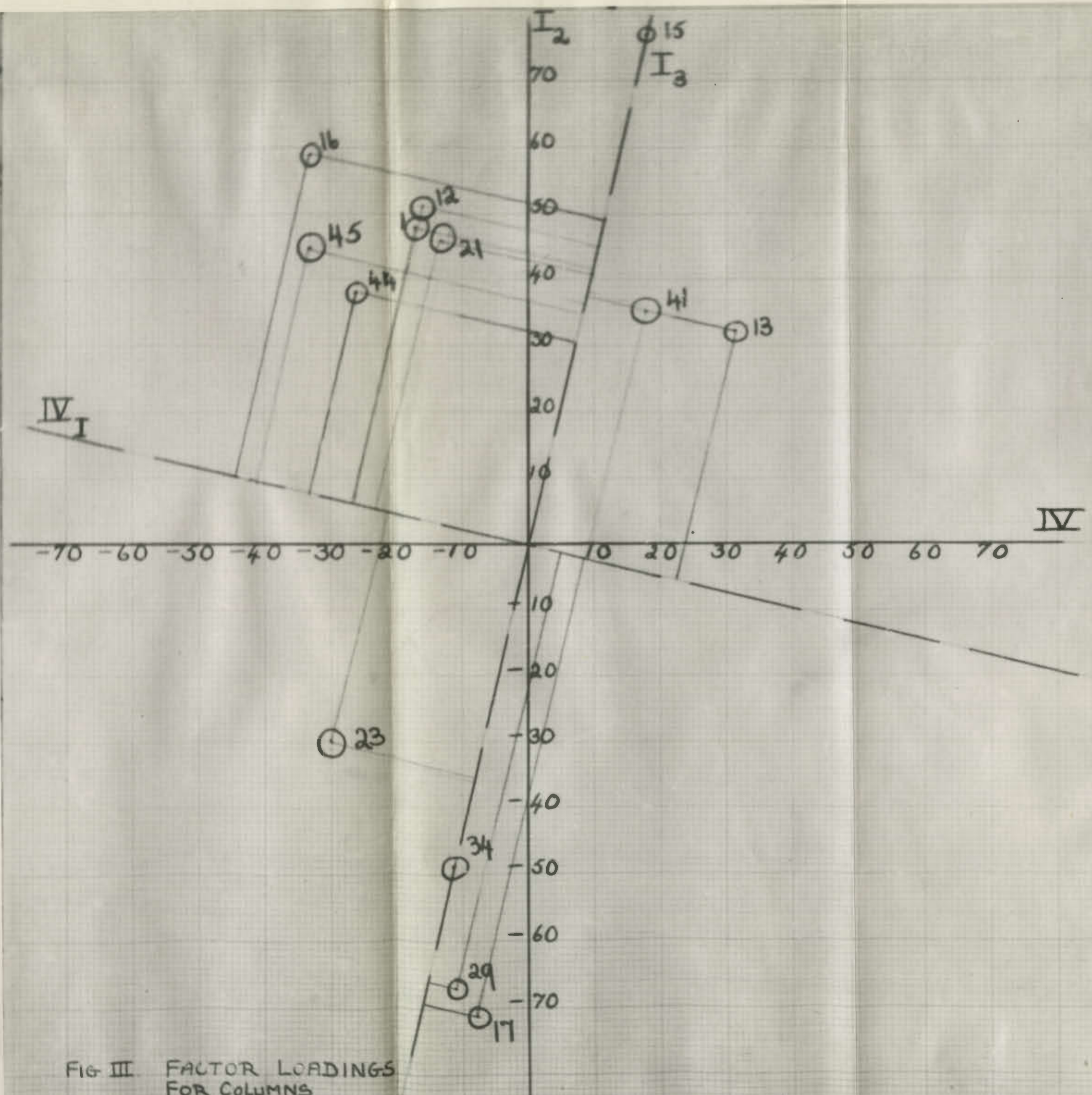


FIG III FACTOR LOADINGS FOR COLUMNS

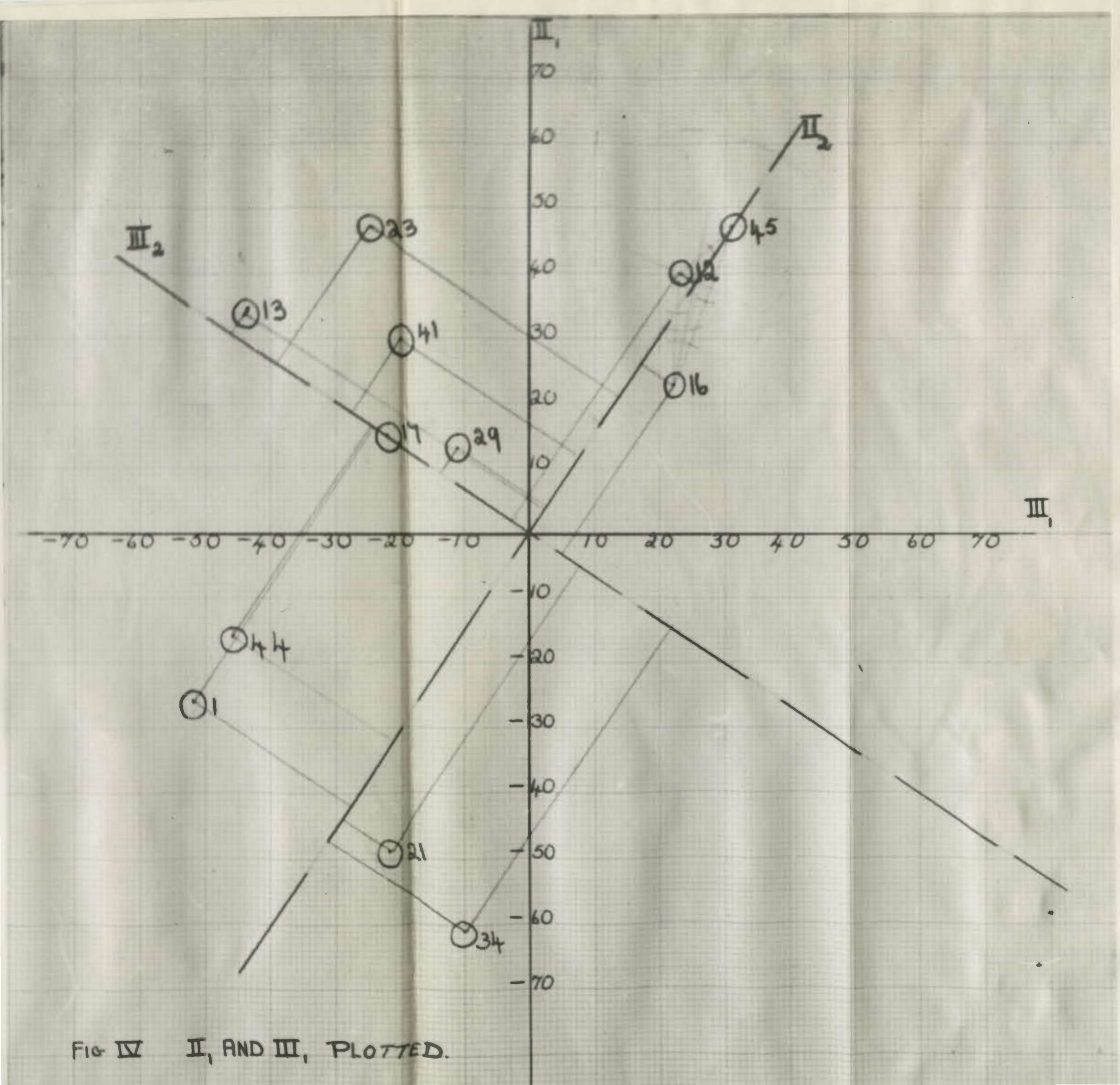


FIG IV II_1 AND III_1 PLOTTED.

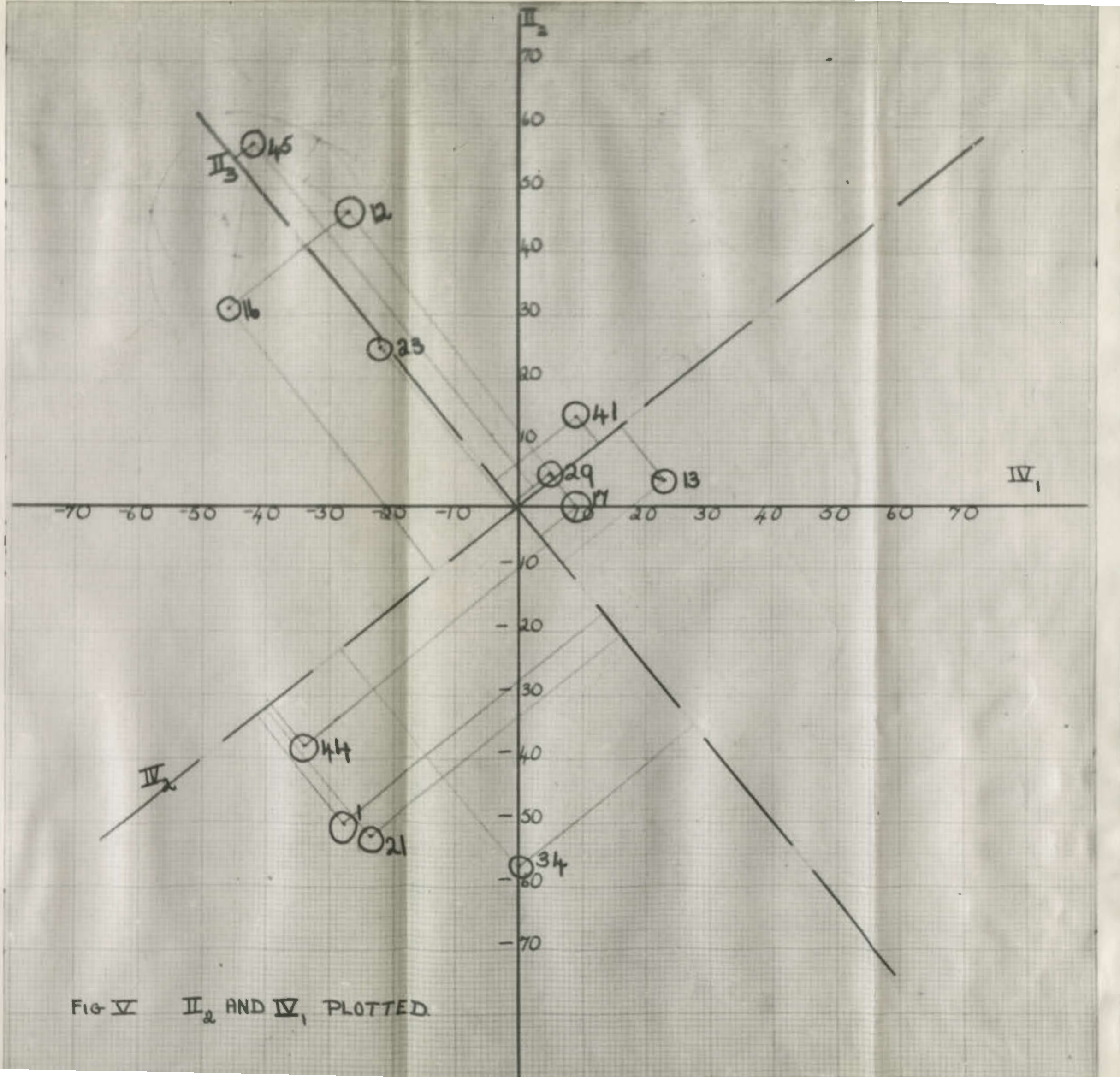
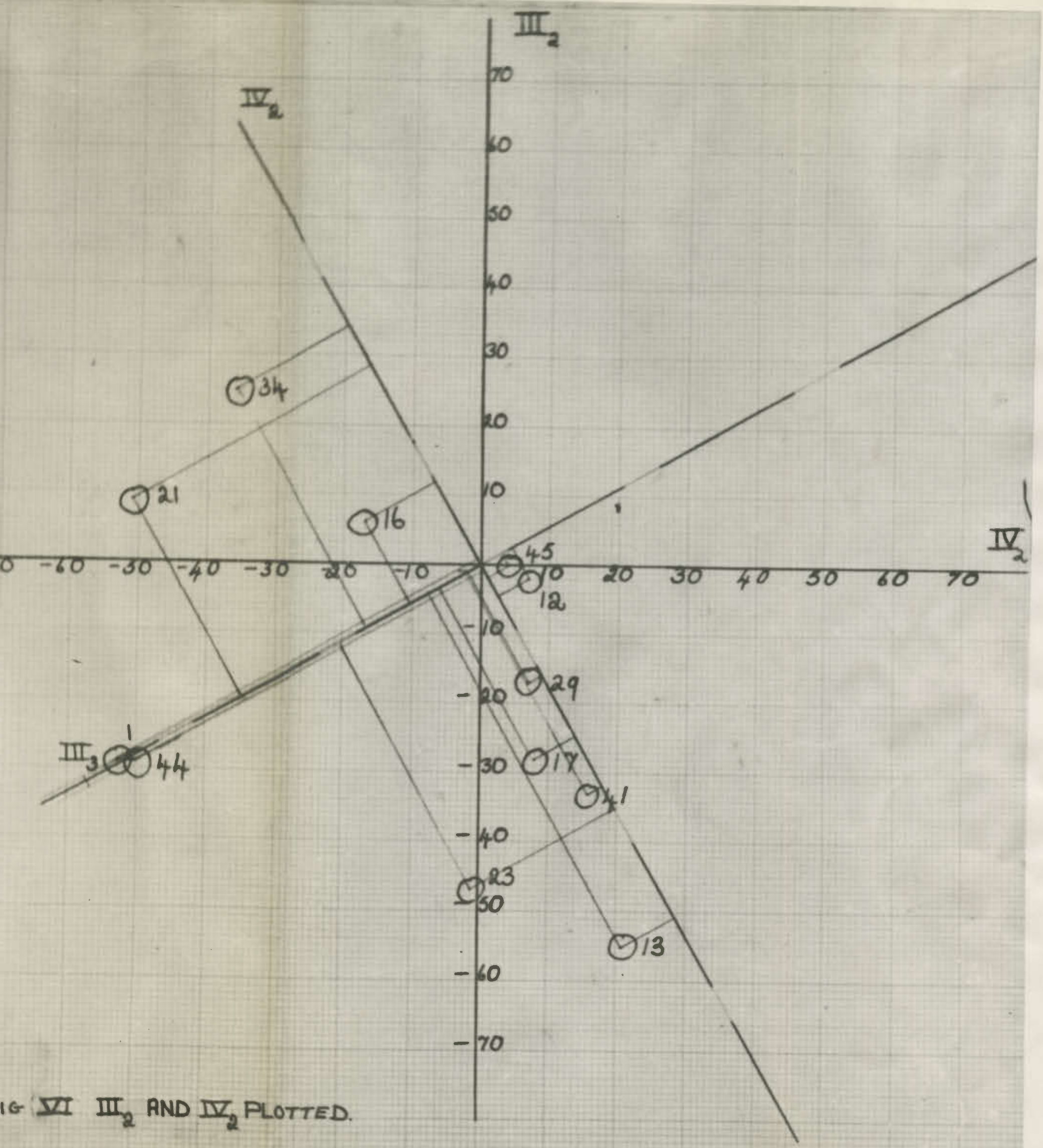
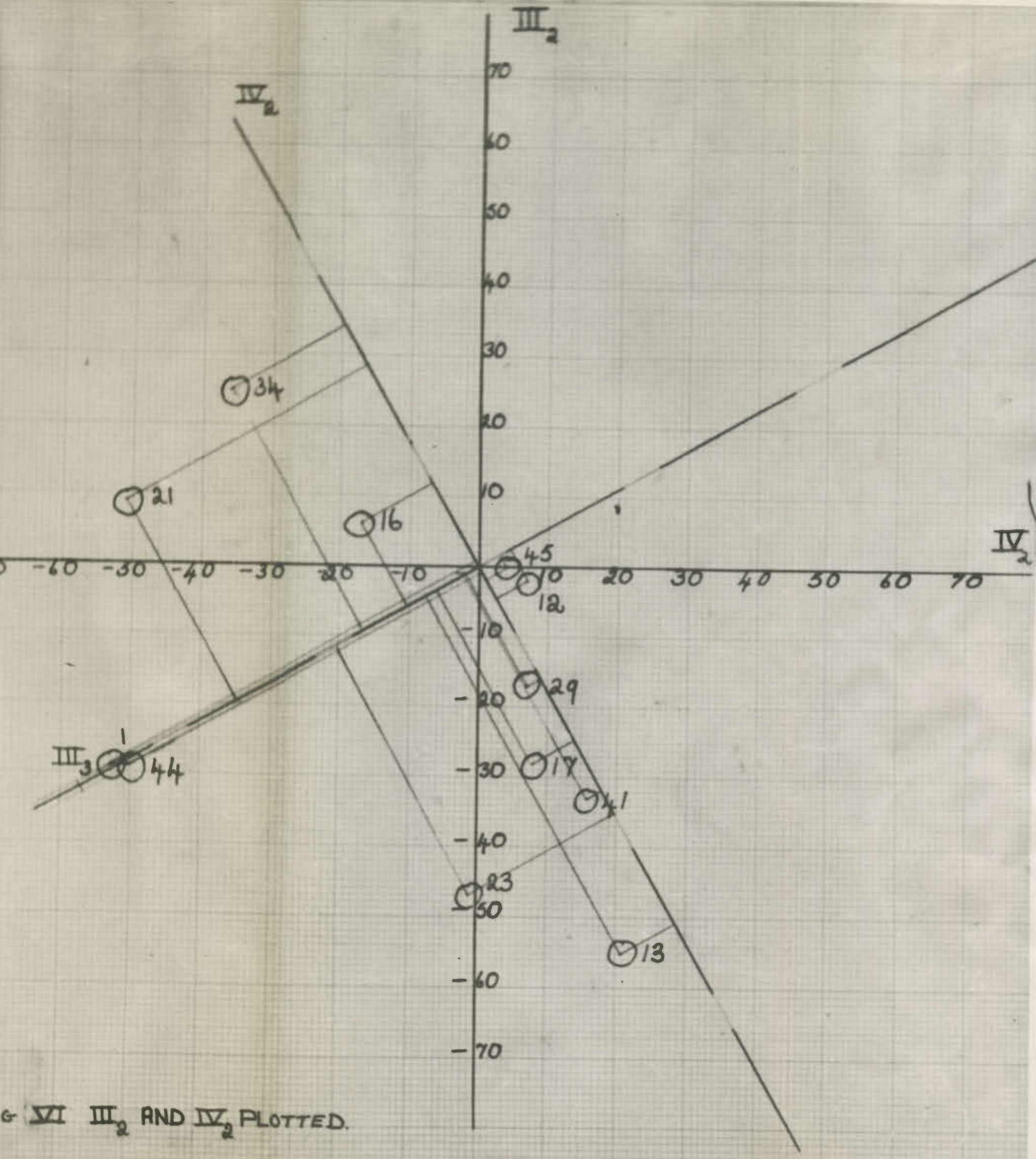


FIG V II_2 AND IV_1 PLOTTED



IG VI III_2 AND IV_2 PLOTTED.



G VI III_2 AND IV_2 PLOTTED.

PART III

INTERPRETATION.

From the Transformation Matrices, the communalities (H_2) were found by squaring the factor loads obtained from the specific Transformation Matrices.

These communalities were compared with those obtained by the centroid method. The difference between each pair of communalities (H_2 and H_1) did not exceed .001.

In determining the significant factor loads, all those below .200 (Table XXV) were neglected as being insignificant, as those factor loads below .200 account for only 4% of the variance of the analysis.

Table XXVI contains those factor loads which were considered to be of significance to the interpretation.

Table XXV...../

TABLE XXV.

		F_1	F_2	F_3	F_4	H_2	$H_2(1)$
15	Sense of Inferiority	.793	.011	-.003	-.006	.629	.628
29	Self Esteem	-.680	.007	-.172	-.027	.493	.493
16	Shy	.484	.536	.134	-.104	.560	.549
12	Easily Gets Gloomy Ideas of His Own	.444	.538	-.055	.048	.492	.491
17	Assertive	-.710	-.061	-.275	-.106	.595	.594
41	Sensitive to Slights and Injuries	.388	.054	-.354	-.051	.281	.281
23	Compliancy	-.378	.333	-.380	-.271	.473	.472
34	Cheerfulness	-.498	-.459	.399	-.146	.639	.639
44	Mature, Stable and Well Balanced	.308	-.076	.048	-.580	.439	.439
1	Attitude to the Problems of Life	.425	-.207	.068	-.595	.582	.582
45	Sociability	.355	.707	-.016	.020	.627	.626
21	Suggestion	.419	-.259	.354	-.381	.513	.513
13	Subject to Marked Fits of Depression	.382	.102	-.566	-.140	.496	.496

TABLE XXVI.

SIGNIFICANT FACTOR LOADS [PERSONALITY RATING TEST MEASURED ON SCALE 5].

		F_1	F_2	F_3	F_4
15	Sense of Inferiority	.793			
29	Self Esteem	-.680			
16	Shyness	.484	.536		
12	Easily Gets Gloomy Ideas of His Own	.444	.538		
17	Assertive	-.710		-.275	
41	Sensitive to Slights and Injuries	.388		-.354	
23	Compliancy	-.378	.333	-.380	-.271
34	Cheerfulness	-.498	-.459	.399	
44	Mature, Stable and Well Balanced	.308			-.580
1	Attitudes to the Problems of Life	.425	-.207		-.595
45	Sociability	.355	.707		
21	Suggestion	.419	-.259	.354	-.381
13	Subject to Marked Fits of Depression	.382		-.566	

FACTOR I.

	+		-
15.	.793	29.	-.680
16.	.484	17.	-.710
12.	.444	23.	-.378
41.	.388	34.	-.498
44.	.308		
1.	.425		
45.	.355		
21.	.419		
13	.382		

Factor I is characterised by a high positive factor load for Variable 15 (Sense of Inferiority);

a high negative factor load for variable 29, indicating a lack of self-esteem;

a high positive factor load for variable 16, indicating shyness;

a high positive factor load for variable 12, indicating the easy manifestation of gloomy ideas;

a high negative factor load for variable 17, indicating a lack of assertiveness;

a fairly high positive factor load for variable 41, indicating sensitivity to slights and injuries;

a high negative factor load for variable 23, indicating compliance;

a high negative factor load for variable 34, indicating lack of cheerfulness;

a fairly high positive factor load for variable 44, immaturity and instability;

a high positive factor load for variable 1, indicating a tendency to avoid problems;

a fairly high positive factor load for variable 45, indicating unsociability;

a high...../

a high positive factor load for variable 21, indicating suggestibility; and

a fairly high positive factor load for variable 13, indicating the manifestation of marked fits of depression.

FACTOR II.

+		-	
16.	.536	1.	-.207
12.	.538	21.	-.259
23.	.333	34.	-.459
45.	.707		

Factor II is characterised by a high positive factor load for variable 16, indicating shyness;

a high positive factor load for variable 12, indicating the easy manifestation of gloomy ideas;

a fairly high positive factor load for variable 23, indicating NON compliance;

a high negative factor load for variable 34, indicating lack of cheerfulness;

a low negative correlation for variable 1, indicating a tendency to face up squarely to the problems of life;

a high positive factor load for variable 45, indicating unsociability; and

a low negative factor load for variable 21, indicating non-suggestibility.

FACTOR III.

+		-	
34.	.399	17.	-.275
21.	.354	41.	-.354
		23.	-.380
		13.	-.566

Factor III is characterised by a low negative factor load for variable 17, indicating non-assertiveness;

a fairly...../

a fairly high negative factor load for variable 41, indicating non-sensitiveness to slights and injuries;

a fairly high negative factor load for variable 23, indicating compliance;

a fairly high positive factor load for variable 34, indicating cheerfulness;

a fairly high positive factor load for variable 21, indicating suggestibility; and

a high negative factor load for variable 13, indicating the absence of marked fits of depression.

FACTOR IV.

+	-
23.	-.271
44.	-.580
1.	-.595
21.	-.381

Factor IV is characterised by a low negative factor load for variable 23, indicating compliance;

a high negative factor load for variable 44, indicating maturity and stability;

a high negative factor load for variable 1, indicating a tendency to face up squarely to problems as they arise; and

a fairly high negative factor load for variable 21, indicating non-suggestibility.

THE INFERIOR INDIVIDUAL.

On the basis of this analysis and with special attention being paid to the results obtained from Factor I, I should like to offer the following description of the individual who suffers from feelings of inferiority.

The...../

The inferior individual who is aware of his inferiority possibly lacks self esteem and consequently has a low opinion of himself and his abilities. Because of this lack of self esteem, the individual easily gets gloomy ideas of his own, is subject to marked fits of depression, and lacks cheerful emotion. He is unbalanced and easily distressed, and this makes him sensitive to slights and injuries. In defence, he tends to be compliant, suggestible, unsociable, and to avoid problems which put him to the test.

An analysis of this kind may be expected to throw some light on possible neurotic trends, but one should not expect too much from it as there was no attempt made to include neurotic subjects and any which did come into the analysis did so by chance, and so the results merely show the relation of qualities and tendencies to each other.

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