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The Arbitrage Pricing Theory : An assessment of the robustness of empirical techniques employed under conditions of thin trading and in the presence of non-normalities

by
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It has now been well-established that Bayes' rule is not an apt characterization of how individuals actually respond to new data (DeBondt and Thaler, 1985:793).

In making this statement the authors sought to suggest that individuals are poor Bayesian decision makers because they tend to overreact to recent information at the expense of prior earlier information. I would like to thank all those colleagues and friends who have provided the exception to the views of DeBondt and Thaler. Not because of a tendency on their part to correctly react to recent information, but because, by steadfastly ignoring all the accumulated evidence and continuing to support my efforts and hold to the belief that I would successfully complete this thesis, they showed themselves to be non-Bayesian thinkers. In particular I wish to mention the following:

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Abstract

Ever since its original development by Ross in 1976, and in spite of the objections of Shanken (1982), proponents of the Arbitrage Pricing Theory have claimed that the theory offers a testable alternative to the Capital Asset Pricing Model. Over the last decade however, empirical research into the APT has not proved to be universally successful, particularly when consideration is given to the fact that the majority of tests of the validity of the theory have used significantly overlapping NYSE datasets. The number of priced factors *discovered* commonly ranges from one (Tryzinka, 1986) to five or six (Cho, 1984). Research into the identification of the APT factors has also proved to be difficult. While the economic identification of the factors is not a necessary condition for tests of the APT itself, the importance of this aspect as far as future practical application of the theory is concerned cannot be overrated.

This research seeks to add to the literature on the testability of the APT by showing that, theoretical objections aside, the current empirical procedures employed in the determination and pricing of the factors lack power. On the basis of an extensive simulation study the statistical procedures of principal components analysis, principal factor analysis and generalised least squares regression are shown to produce *results* that appear independent of the true number of underlying factors generating risky asset returns. When market microstructure effects and significant levels of thin trading are simulated into the data the power of the procedures is further reduced. Interestingly, in contrast to the suggestions of some prior researchers (Pari and Chen, 1984), when non-normalities of the order observed on the JSE are simulated into the idiocyncratic risk component of returns they have no noticable effect on the *results*. This finding also bears testimony to the low power of the procedures.

A comparison of the use of APT and CAPM methodologies is presented in the final section of the research and, in spite of the power problems, the APT is found to be preferable. In an analysis of both unit trust performance and price/earnings and size anomalies the APT methodology is found to provide results that are either as good as, or improve on, those obtained using a traditional CAPM approach. This finding applies in spite of the uncertainty as to the appropriate number of factor mimicking portfolios to use.

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Introduction and Background

What kind of information is used and how it is put together to frame an estimate of future conditions is important to understand because the character of dynamic processes is typically very sensitive to the way expectations are influenced by the actual course of events. Furthermore, it is often necessary to make sensible predictions about the way expectations would change when either the amount of available information or the structure of the system is changed (Muth, 1961:315).

1.1 Introduction

The Capital Asset Pricing Model (CAPM) and the Arbitrage Pricing Theory (APT) are two leading models that have been proposed for analysing cross-sectional variations in risky asset expected returns. The Capital Asset Pricing Model is developed under the assumption of equilibrium in capital markets and is based on the concept of mean-variance analysis, first introduced by Markowitz (1952). Its full development is attributed to Sharpe (1964), Lintner (1965), Mossin (1966), and Black (1972). The model proposes that the expected return for any asset is linearly related to its covariance with the market portfolio and is given by;

$$E(\tilde{r}_i) = E(\tilde{r}_z) + \beta_i(E(\tilde{r}_m) - E(\tilde{r}_z)) \tag{1.1}$$

where; $\beta_i = \sigma_{im}/\sigma_m^2$ is the beta of the asset and measures the quantity of risk; σ_{im} and σ_m^2 measure the covariance between the asset and the market and the market variance respectively; $E(\tilde{r}_z)$ is the expected return on the Black (1972) zero-beta portfolio; and, $E(\tilde{r}_m)$ is the expected return on the market portfolio.

Aside from the restrictive assumptions concerning the distribution of security returns and/or the form of investor utility curves, the principal critique of the CAPM relates to its empirical testability. Roll has shown that joint nature of the required test implies empirical research into the theory cannot be unambiguously undertaken (Roll, 1977:129-130).

Unlike the CAPM, the Arbitrage Pricing Theory was originally developed by Ross (1976) on the basis of an arbitrage argument only. As such, in its original form, it did not require the constraints imposed by capital market equilibrium and Ross suggested it gave an appropriate pricing relationship *in all but the most profound cases of disequilibria* (1976:343). Although the APT equation only holds approximately in Ross, when additional constraints are imposed such as those by Dybvig (1983) and Grinblatt and Titman (1983) they result in the model becoming exact for all assets. The model proposes that the expected return for any asset is given by (Roll and Ross, 1980:1078);

$$E(\tilde{r}_i) = \lambda_0 + \sum_{i=1}^{k} (\lambda_j \beta_{ij}) \qquad (1.2)$$

where; β_{ij} is the j[th] factor loading for the i[th] asset; λ_0 is the expected return on all zero-beta portfolios; and, λ_j is the expected excess return on the j[th] factor.

The development and increased popularity of the APT is attributable to two primary factors. Firstly, as a multi-factor model, it is consistent with the intuition behind the multi-beta extensions of the CAPM proposed by numerous researchers such as Brennan (1970) and Mayers (1972). Secondly, it is perceived to offer a testable alternative to the CAPM. Unlike the CAPM, it does not require either the identification of the market portfolio or that it be established as mean-variance efficient (Roll and Ross, 1980:1074; Dybvig and Ross, 1985:1184).

1.2 Justification and objectives of the research

The justification and objectives for this research relate to the ongoing investigations into, and development of, capital market theory. The Arbitrage Pricing Theory appears to potentially offer considerable advantages in its explanation of the pricing of risky assets. Although current research into the topic is extensive, the conclusions drawn by different authors are somewhat contradictory.

1.2.1 Justification

The test of any new theory must surely be whether or not it can explain certain anomalies that existing theories cannot. It is the pursuit of this objective that leads to increased understanding of the environment and, as highlighted by Muth in the opening quotation, enables sensible expectations to formed. After over a decade of theoretical development and empirical research, the debate as to whether the Arbitrage Pricing Theory achieves this goal still continues.

Numerous theoretical papers have been produced in an effort to make the APT more empirically tractable. These include the examination of the theory in finite as opposed to limit economies (Dybvig, 1983; Chen and Ingersoll, 1983; Grinblatt and Titman, 1983), an assessment of the theory as a full equilibrium model rather than purely basing the development on an arbitrage argument (Grinblatt and Titman, 1983), and investigations into the relationship between the APT and mean-variance efficiency (Chamberlain and Rothschild, 1983; Grinblatt and Titman, 1987; Tiemann 1988).

The initial empirical research into the APT focused on the determination of the number of priced factors[1], or risk premia, necessary to explain the systematic risk (covariance) common to all risky assets. This focus followed the work of Roll and Ross who suggested that factor identification itself was not central to tests of the validity of the theory. They concluded that, in testing the APT, it is no more appropriate for one to examine this issue than it would be for tests of the CAPM to examine what, if anything, causes returns to be multivariate normal (1980:1077). To date the research in this area has yielded conflicting results, with the number of priced factors ranging from a low of one (Trzcinka, 1986) to a high of six (Cho, 1984). These results are further exacerbated by the fact that much of the research has been undertaken using significantly overlapping New York Stock Exchange data and by the fact that researchers such as Dhrymes, Friend and Gultekin (1984) have criticized the use of factor analysis by Roll and Ross, and others, on the grounds that empirical evidence suggests that the number of significant factors increases with sample size. The counter argument to this view however is that it is not the number of significant factors from the factor analysis procedure that is important in tests of the APT, but the number that are priced. Roll and Ross contend that the number of priced factors does not increase with sample size (1984:349).

Shanken in his critique of the testability of the APT has even suggested that the theory cannot be unambiguously tested on subsets of data using the usual empirical formulation of the theory, since inconsistent conclusions could be reached for a given set of securities (Shanken, 1982:1135-1137).

[1]A factor is "priced" if any investor can expect to be compensated for carrying the risk inherent in the factor. In terms of equation 1.2 therefore, a priced factor is defined as one for which λ_j is greater than zero.

Given the congruency problem of factor structure that occurs between different sample portfolios[2], and the debate as to the true number of factors, the identification of the factors has become an increasingly important issue. Although the formulation of the APT does not make any statement about the economic characteristics of the factors, it has long been acknowledged that identification is an important area of research. Clearly, if consistent economic meaning can be given to the risk premia, identified through factor analysis and cross-sectional regression procedures, the congruency problem becomes far easier to address. Once identified, the risk premia themselves can be compared across different sample portfolios rather that just the intercept (risk-free rate or zero-beta portfolio) in the linear pricing relationship. The initial research into factor identification was undertaken by Oldfield and Rogalski (1981) and Fogler, John and Tipton (1981). Oldfield and Rogalski sought to show that the same set of factors influence both treasury bills and common stock returns, while Fogler et al. found evidence of an overall market wide factor, a factor related to the three-month government bill returns, and a final factor dependent on the long-term Aa-utility bond return. Subsequent research by Roll and Ross (1984), and Chen, Roll and Ross (1986) has identified the term structure of interest rates, expected and unexpected inflation, unanticipated changes in industrial production, and the spread in risk premia between high and low grade bonds as being sources of significant risk premia underlie common stock returns. Barr (1990:25) has identified the gold price, interest rates, foreign share markets and a local property effect as being the major economic forces determining security price movement on the Johannesburg Stock Exchange.

Scope for further work into all aspects of the Arbitrage Pricing Theory clearly exists before any sense of finality can be reached as to the magnitude of the contribution made by the theory. This research hopes, through successfully achieving the objectives outlined below, to add to the existing research into the APT's ability to extend our understanding of portfolio theory and to provide improved economic explanations of security pricing.

1.2.2 Research Objectives

There are three major objectives of this research, all of which relate to the empirical validity of the Arbitrage Pricing Theory.

The first objective is to examine the appropriateness of employing the multivariate

[2]The congruency problem occurs when empirical research is carried out using numerous subsets of data and the consistency of the factors across the subsets is called into question.

statistical techniques commonly used in investigations into the APT. Factor analysis, principal components analysis, and multivariate linear regression are the principal techniques used to test for the number and pricing of the APT factors. These techniques rely on certain multivariate distributional assumptions which may be violated by security returns data. If, as suggested by Mandelbrot (1963) and Fama (1965), returns are distributed as non-normal stable Paretian this would imply the population variance is undefined and statistical techniques using higher than first order powers are inappropriate. In order to achieve the first objective therefore, an extensive investigation into the nature of the distribution of security returns is undertaken. The null hypothesis that returns are distributed as non-normal stable Paretian is, in particular, comprehensively tested since a possible explanation for the many contradictory prior empirical results may be the use of inappropriate techniques on unstable sample variance and covariance data.

If, contrary to the above proposition, security returns do have finite population variance but are still significantly non-normal, statistical procedures assuming normality can still lead to erratic sample dependent conclusions. The second objective of the research therefore, is to examine the power of the multivariate techniques used in tests of the APT. This objective is met by conducting an extensive simulation study. Returns data, simulated as multivariate normal with mean and variance characteristics of listed securities on the Johannesburg Stock Exchange and with known factor structures are used as an initial database. Thereafter, non-normally distributed data are generated and the effects of various levels of thin trading are also examined. It has been long recognized that thin trading exacerbates the non-normality condition (Dimson, 1979; Barr and Bradfield, 1989).

The third, and final, objective is to investigate the performance of the Arbitrage Pricing Theory in allied investigations in financial theory. Three areas of research that have received considerable attention over the last two decades are; the measurement of portfolio performance; the testing of market efficiency using event studies; and, the investigation of numerous anomalies found in tests of capital market theory such as the size and earnings-to-price ratio effects (Banz, 1981; Basu, 1983; Blume and Stambaugh, 1983; Fama and French, 1992).

Since performance can only be assessed relative to a particular benchmark, much of the academic research into these areas is based on the Capital Asset Pricing Model which, until the development of the APT, provided the most appropriate theoretical benchmark. As such the true value of the empirical formulation of the APT may lie in its ability to explain some of the phenomena unexplained by the more traditional empirical approaches. In this phase of the research therefore, the comparative performance of APT and CAPM based benchmarks is assessed in two ways. Firstly, the relationship between excess returns, as defined by the different benchmarks, and firm size and earnings for securities traded on the Johannesburg Stock Exchange is investigated. Secondly, the issue of whether professional fund managers in South Africa have been able to outperform the market on a risk adjusted basis is examined. The methodologies employed for the comparative assessment are developed from the approaches adopted by Brown and Warner (1980), Chang and Lewellen (1985), Connor and Korajcyzk (1986) and Jaffe, Keim and Westerfield (1989).

1.3 Contribution

While economic identification of the APT factors does not form part of this research it does seek to make a contribution in several areas. Through the simulation study examining the robustness of the techniques used in empirical research into the Arbitrage Pricing Theory new insight into the reasons for the contradictory results achieved in prior international research into the APT will be provided. As one of the first comprehensive investigations into the theory in South Africa, the research also sets out to confirm; (a) that security returns on the Johannesburg Stock Exchange are best described by a multiple factor arbitrage pricing model, and (b) that, even without adequate identification of the economic variables underlying security returns, an APT approach can have significant advantages when conducting empirical research into security price behaviour. The value of the theory as an instrument for investigations into the efficient market hypothesis and for the measurement of portfolio performance will hopefully be convincingly illustrated, in spite of the conclusion drawn by Kryzanowski and To that *unambiguous identification of the factors is necessary before the APT can be used by practitioners* (1983:33).

1.4 Thesis organization

As outlined above, three principal objectives are investigated, and the empirical analysis is therefore broken down into three sections. The first section, consisting of chapters three and four, addresses the issue of the distribution of security returns and the implications of non-normalities and thin trading on covariance estimation. Chapter five, the second section, deals with the power of the multivariate statistical techniques used in empirical research into the Arbitrage Pricing Theory, while the third and final section, consisting of chapter six, examines the implications of using the APT in tests of the efficient market hypothesis and in the measurement of portfolio performance. Each of the chapters contains a review of the prior empirical research as a precursor to the presentation of methodology and analysis contained in the chapter.

Chapter two provides the theoretical justification for the research. A critical review of relevant aspects of capital market theory is presented. It includes both a critique of rationality as a model of human behaviour and the development of utility theory. An analysis of relevant aspects of the CAPM as the principal asset pricing model preceding the development of the APT is also provided. Finally the APT is developed and reviewed in the context of capital market theory.

Chapter three investigates the distribution of security returns and the resultant implications for the methodologies used in the estimation of the number of factors underlying these returns. Two hundred and forty-four Johannesburg Stock Exchange securities are used and the tests undertaken address both the infinite variance stable Paretian hypothesis as well as the finite variance compound events alternatives proposed in the literature. The investigation is carried using both weekly and monthly (four-weekly) returns data.

In chapter four, the returns distribution parameters estimated in the previous chapter are used to simulate data for the investigation of the impact of non-normalities and thin trading on, in particular, the estimation of covariance. The degree of thin trading and serial correlation (resulting from other market microstructure effects) simulated into the data is based on three years of daily trading data for the two hundred and forty-four securities selected. A comparative analysis is also carried out to assess the ability of various estimation techniques to reduce any biases in covariance estimation and to establish the impact of such procedures on the efficiency of the resultant estimates.

The robustness and power of factor analysis, principal components analysis, and generalized least squares regression used to identify the number and pricing of factors in empirical research into the APT are investigated in chapter five. A series of simulations are undertaken using data generated in a fashion similar to that employed for the analysis of chapter four except for the fact that the data is generated to conform to particular priced factor structures. In the first stage of the simulations principal components analysis and principal factor analysis, together with the appropriate statistics, are employed to assess the ability of the procedures to correctly identify the number of factors. In the second stage generalized least squares regression is used to establish the power of the procedure in its ability to correctly determine the number of factors that are priced.

Chapter six uses the methodologies developed in chapters four and five as the basis for comparison of the Arbitrage Pricing Theory and other capital asset pricing models. The implications of model selection on tests of the efficient market hypothesis are addressed in the first part of the chapter through an empirical analysis of size and earnings-to-price (E/P) ratio anomalies. A standard portfolio approach is used where the securities are formed into portfolios on the basis of both size and E/P ratio, and the comparative performance of the models is assessed using regression procedures based on the differing abnormal returns estimates produced by the models. The second part of the chapter extends the analysis through the development of an appropriate measurement instrument for the comparison of portfolio performance. The measurement instrument developed is contrasted to the traditional performance measures of Sharpe (1966), Treynor (1965), and Jensen (1968) using South African Unit Trust data.

Finally, chapter seven contains a summary of the investigation and highlights the key conclusions. Additional areas for further research are outlined.

1.5 Data and data manipulation

The database used in this investigation consists of two distinct components. The first component consists of the security price data used in the study. Prices were extracted from a database, originally provided by the Johannesburg Stock Exchange to the University of Cape Town Graduate School of Business, that is updated and maintained by the Statistical Sciences department of the University. This data was supplemented with additional stock split, and capitalization issue information obtained from the monthly bulletins published by the Johannesburg Stock Exchange. The second component of the database consists of the set of earnings, dividends, and preliminary announcement date data used for the market efficiency tests and for the size and E/P ratio anomaly comparative tests. These data were extracted from both the Ivor Jones Roy & Company I-Net database and from the Johannesburg Stock Exchange monthly bulletins.

The dataprocessing needed for the empirical work was carried out on both the University of Cape Town VAX 6000-330 mainframe and on a Miad AT micro-computer with a 40 megabyte hard disk drive. The software packages used were the programming languages Fortran F77 and Microsoft Fortran 5, the NAG scientific subroutines, the statistical packages SPSS (Statistical Package for the Social Sciences) and TSP (Time Series Package), and the spreadsheet package Lotus 1-2-3.

2

Theoretical overview

In the social sciences we are suffering from a curious mental derangement. We have become aware that the orthodox doctrines of economics, politics and law rest upon a tacit assumption that man's behavior is dominated by rational calculation. We have learned further that this is an assumption contrary to fact. But we find it hard to avoid the old mistake, not to speak of using the new knowledge. In our prefaces and introductory chapters some of us repudiate hedonism and profess volitional psychology or behaviorism. Others among us assert that economics at least can have no legitimate relations with psychology in any of its warring forms. In the body of our books, however, we relapse into reasonings about behavior that apply only to creatures essentially reasonable. (Wesley C. Mitchell, 1918:161)

2.1 Introduction

The development of modern portfolio theory is based upon the premise that man is a rational being, and it is this key concept of rationality which forms the basis of all utility theory. In developing the Capital Asset Pricing Model (CAPM), Mossin stated;

> *(i)f one or more individuals do not behave rationally, the whole foundation of the analysis is destroyed, and the concept of equilibrium, and hence also of the market line becomes meaningless (1966:779).*

Additionally, in much of the theoretical work it is further assumed that utility can be quantified beyond the point of ordinal ranking of preferences, and many of the developments rely upon the validity of the concept of cardinal utility. As illustrated by the quotation from Mitchell, the concept of rationality has been debated extensively for well over a century and cannot, in any sense, be considered a universally accepted principal within the social sciences. Even within the field of economics where the concept of economic rationality is generally accepted, the translation to utility theory is not without controversy.

This chapter begins by outlining the debate into utility theory in order to highlight the premises of Von Neumann and Morgenstern. Their work provides the base for much of the research into portfolio theory. Thereafter the theoretical work that preceded the seminal paper by Ross (1976) on the Arbitrage Pricing Theory is critically appraised. In particular, some of the theoretical and empirical limitations of the CAPM are presented. Finally the development of the APT is presented, and the theoretical and empirical advantages of the model over its predecessor, and principal competitor, the CAPM of Sharpe (1964), Lintner (1965) and Mossin (1966), discussed.

2.2 Key developments in Utility Theory

Although much of the finance literature refers back to the concept of cardinal utility as described by Von Neumann and Morgenstern (1947:15-31), an attempt to quantify utility was initially made during the early part of the nineteenth century when Bentham developed *felicific calculus*. He suggested that the value of *pleasure or pain* could be estimated by an individual on the basis of seven criteria, namely; intensity, duration, certainty, propinquity, fecundity, purity, and extent. Through his work Bentham sought to become the *Newton of the moral world* and, while recognising the problems presented by diminishing utility of wealth, he suggested that money should be the instrument of measuring pleasure or pain (Mitchell, 1918:164-165).

Stigler suggests that the economists of Bentham's time did not pursue the direction he took with his theories on utility because of the influence of Ricardo who, in accepting Adam Smith's distinction between value in use and value in exchange concluded that value cannot be measured. Stigler states that;

> ..... *utility theory finally began to win a place in generally accepted economics in the 1870s, under the triple auspices of Jevons, Menger, and Walras* (1950:315).

With respect to the measurement issue however, Stigler felt that, while *Mengler glossed over the problem of measurability of utility*, he found *Jevons' attack on the problem of measurability was characteristically frank and confused as (w)ith gallant inconsistency, he proceeded to devise a way to measure utility* (1950:317).

Majumdar (1958:xi) analysed the meaning and economic consequences of measurable utility as proposed by several notable economists and suggested their views could be categorized according to two principal dimensions. The primary dimension he related to the contending hypotheses of ordinalism and cardinalism. Samuelson, Hicks and Allen were the major economists he saw as following the ordinalist perspective, while Marshall, Von Neumann and Morgenstern, and Freidman and Savage were perceived as supporting the cardinalist perspective. Majumdar defined the ordinalists as those economists who believed that utility could be measured up to the point of ranking by preference. This implies individuals are able to consistently choose a more preferred item but are unable to quantify the extent of their preference. In terms of this definition therefore, utility is only measurable up to monotonic transformation. The cardinalists were defined to include all economists who accepted utility could be measured with more precision than ranking. Cardinal utility therefore included the strict Marshallian idea of addible utility as well as the neo-cardinalist view that utility is merely a term used to describe how an individual makes choice between items, and how strongly the individual feels about the choice. It is the strength of feeling aspect that extends the neo-cardinalist view on measurement beyond ranking since it is this aspect that reflects the concept of marginal utility and allows for measurement of utility up to a linear transformation (1958:35-37).

As a second dimension, Majumdar proposed that the contending views on utility measurement could be further categorized depending upon *whether the definition of utility had been made in behaviourist or introspectionist terms* (1958:xiii). A behaviourist perceives the act of comparison as the act of choice and consequently defines utility in terms of choice, while an introspectionist views the act of comparison as being behind the act of choice and therefore defines utility in terms of preference. While the subtlety of this distinction is not central to the current study, Samuelson, Von Neumann and Morgenstern, and Friedman and Savage developed their work from a behaviourist perspective while Marshall, Hicks and Allen followed an introspective approach.

In comparing the four variants of utility theory that he identified, namely introspective-cardinalist, introspective-ordinalist, behaviourist-cardinalist, and behaviourist-ordinalist, Majumdar states that it is only the behaviourist-cardinalist variant which does not make the tacit assumption that the choice facing an individual contains no element of uncertainty.

> *It is easy enough to see that any type of utility theory must be futile in the presence of pure (complete) uncertainty. So long as uncertainty can be ruled out, it is axiomatic both in ordinalist and in cardinalist utility theory that a rational man necessarily prefers a larger amount of a given commodity to a*

smaller amount. ..... This is the basic axiom of rational choice without which it seems impossible to build up a workable utility theory. And it is this axiom which breaks down completely with the introduction of pure uncertainty: it can no longer be regarded as obviously rational, for example, that two birds (in the bush) should be preferred to one (in hand).

What is true of pure uncertainty is also true of measurable risk at least so far as the ordinalist theory is concerned. There can be no ordinalist ground for retaining the rationality axiom in the form, say, 'a more than 50 per cent chance of catching two birds in the bush is worth more than having one in the hand' (Majumdar, 1958:96-97).

It is this ability to cope with uncertainty, as much as the assumption that utility can be treated as a numerical measurable quantity, which can be considered the major attraction of the utility theory of Von Neumann and Morgenstern to academic research in the field of portfolio theory.

2.2.1 Von Neumann and Morgenstern

Von Neumann and Morgenstern suggested that the assumption of measurable utility was not as radical a departure from the nonnumerical approaches of indifference curve analysis as assumed in the literature of the time. They believed that *under the conditions on which indifference curve analysis is based very little extra effort is needed to reach numerical utility* (1947:17). Their theory was developed by extending the concept of an individual being able to compare any two certain events to allow for comparison between combinations of events. Von Neumann and Morgenstern believed that if it was possible for an individual to indicate a preference between two certain events then the same individual could indicate a preference between a certain event and a combination of two mutually exclusive events with outcome probabilities α and $1-\alpha$. On the basis of this belief, and through the establishment of the five axioms of utility, namely completeness, consistency, strong independence, measurability, and ranking (Copeland and Weston, 1988:79), they provided the necessary framework for the construction of cardinal utility curves that could meet specific criteria. They also note that the presence of complete information is an important additional assumption (1947:30).

Von Neumann and Morgenstern provided a simple illustration of their hypothesis using three certain mutually exclusive events A, B and C. If an individual prefers A to B and C to A, then there exists a unique probability α such that the individual is indifferent between event A and a combination of events B and C with probability of B equal to α

and probability of C equal to $1-\alpha$, and the probability can therefore be used *as a numerical estimate for the ratio of the (individual's) preference of A over B to that of C over B* (1947:18-19).

2.2.2 Utility curves used in finance

A key assumption made in the literature of finance and portfolio theory, additional to the rational investor assumption of positive marginal utility of wealth, is that of risk aversion. This implies that any investor will always prefer the actuarial value of a gamble to the gamble itself. Any mathematical form chosen to represent a utility curve must therefore be such that the utility of the gamble is always less than the utility of the expected outcome of the gamble. Written mathematically;

$$U[G(a,b:\alpha)] < U[E\{G(a,b:\alpha)\}] \tag{2.1}$$

where; $U[\ ]$ represents the utility function, $E\{\ \}$ is the expectations operator, and $G(a,b:\alpha)$ represents a gamble with two possible outcomes, a with probability α and b with probability $1-\alpha$[1].

Two measures of risk aversion can be defined. The first, termed absolute risk aversion (ARA), measures the risk aversion of an individual for a given level of wealth. As such it gives an indication of how an individual will behave in the face of risk. The second form of risk aversion, termed relative risk aversion (RRA), measures the 'local' aversion to risk as a proportion of wealth (Copeland and Weston, 1988:89; Pratt, 1964:122). An individual with constant relative risk aversion for instance, has a constant aversion to a proportionate loss of wealth even though the absolute loss increases as does wealth. The two measures are written as;

$$ARA = -U''[W]/U'[W] \tag{2.2}$$

$$RRA = -WU''[W]/U'[W] \tag{2.3}$$

where; W is the individual's level of wealth, and U' and U'' are the first and second derivatives of the utility function respectively.

[1]Clearly, a risk neutral investor is one who is indifferent between taking a gamble and receiving its actuarial (expected) value with certainty, while a risk taker would actually prefer to take the gamble.

Pratt suggests that logical utility functions should exhibit decreasing absolute risk aversion with increasing wealth, although in some instances the individual's relative risk aversion might be expected to first decrease and then begin increasing (1964:122-123). Friend and Blume found empirical evidence to suggest that portfolio investors had decreasing absolute risk aversion and constant relative risk aversion (1975:920).

Although a variety of mathematical functions have been used in the development of financial theory, including both power and quadratic curves, they have not all met the two conditions of decreasing absolute risk aversion and constant (or decreasing) relative risk aversion. The quadratic utility function used by both Markowitz, in his paper on portfolio selection, and in the development of the Capital Asset Pricing Model is particularly deficient in this respect. Pratt (1964:132) states that;

> *... a quadratic utlity cannot be decreasingly risk-averse on any interval whatever. This severely limits the usefulness of quadratic utility, however nice it would be to have expected utility depend only on the mean and variance of the probability distribution.*

He also suggests that convenient utility functions for which (absolute risk aversion) is decreasing are not so very easy to find (1964:122).

From an empirical point of view Lintner has also suggested that a risk aversion parameter too small to account for the degree of risk aversion generally found was necessary to prevent the quadratic function from implied negative marginal utility far too early (1965:18).

2.3 The early development of portfolio theory

From the analysis of section two above it is apparent that any assumption that individuals are rational agents who always endeavour to maximize their utility functions places constraints on subsequent theoretical developments. This position is further compounded by the assumptions made as to the form of the utility function itself.

The origins of portfolio theory are rightfully traced back to the seminal paper on portfolio selection by Markowitz (1952). In it he introduced the concept of mean-variance efficiency and provided the theoretical justification for the portfolio diversification observed in practice. Additionally, the approach showed that the mean-variance hypothesis, by suggesting diversification across a wide spectrum of securities to remove sector specific risk, would lead to the *right kind* of diversification for the *right reasons*

(1952:89).

Sharpe (1964), Lintner (1965), and Mossin (1966) extended the work of Markowitz by combining it with Tobin's developments into liquidity preference theory, and the seperation theorem (1958), to produce the first equilibrium theory of risky asset prices, namely the Capital Asset Pricing Model (Sharpe, 1964:427). All sought to show that, under the assumptions made, the model provided an equilibrium allocation of assets that was Pareto optimal and as such no reallocation was possible which would increase the utility of an individual without reducing the utility of others (Mossin, 1966:773).

Although they followed essentially the same argument in developing the capital market line, the three authors of the model did not make identical assumptions or reach precisely the same conclusion. The areas in which there was some difference related to the probability distribution of risky asset returns, the form of the utility function, and the uniqueness of the equilibrium risky asset portfolio.

With respect to the equilibrium risky asset portfolio, Sharpe incorrectly concluded that, although all efficient portfolios must obviously lie on the capital market line, *many alternative combinations of risky assets are efficient, and thus the theory does not imply that all investors will hold the same combination* (1964:435). Both Lintner and Mossin showed that there can be only a single tangent point on the efficient frontier, and all individuals must hold the same proportion of the total outstanding stock of all risky assets (Lintner, 1965:17; Mossin, 1966:775).

2.3.1 The Capital Asset Pricing model: Assumptions, extensions, and limitations

One of the two conditions which Markowitz stated must be satisified for efficient portfolios to be developed in the fashion he suggested is that investors *must desire to act according to the E-V (mean-variance) maxim* (1952:83). This implies that either security returns must be distributed as multivariate normal or investors must possess quadratic utility functions (Tobin, 1958:82-85).

While Sharpe placed the same constraints on the distribution of returns and form of the utility function as Markowitz, Lintner suggested that this was unnecessary as the same conclusion follows from an earlier theorem of Roy's without dependence on quadratic utilities or normality under the assumption that investors are *safety firsters* who wish to minimize the probability that the realized outcomes of their investments will fall below the risk-free rate (Lintner, 1965:13-19). Mossin explicitly rejects the assumption of a quadratic utility function by assuming individual utility functions have positive first derivatives and negative second derivatives (1966:772), although he does assume individuals base their investment decisions on mean and variance only (1966:769).

Today it is generally accepted that, with respect to the characteristics of investors, the assumption necessary for the development of the model is that they are risk averse utility maximizers and, if risky asset returns are not multivariate normally distributed, their utility functions are quadratic (Ross, 1976:341; Copeland and Weston, 1988:194)

A further crucial assumption required for the development of the model is that investors have identical perceptions about the probability distribution of all risky assets (Sharpe, 1964:433; Lintner, 1965:14; Mossin, 1966:769). Borch has argued that this assumption is not that radical an additional assumption since;

> *(w)hether two rational persons on the basis of the same information can arrive at different evaluations of the probability of a specific event, is a question of semantics. That they may act differently on the same information is well known, but this can usually be explained assuming that the two persons attach different utilities to the event.* (Borsh, 1962:439).

The validity of Borsh's statement hinges on the implicit assumption of frictionless (perfect) markets where all information is costlessly and instantly available to all participants. This assumption is also made in developing the model together with the assumptions that the quantity of assets in the market is fixed, there are no restrictions on short sales, and that there exists a risk-free rate at which all investors can lend and borrow (Sharpe, 1964:432; Lintner, 1965:15; Mossin, 1966:772). Lintner showed that dropping the restriction on borrowing did not effect the optimum portfolio mix or the applicability of Tobin's seperation theorem (1965:16). He did however point out that disallowing short sales would complicate the analysis if the returns on different stocks are correlated (1965:21).

Since its development, considerable research addressing specific assumptions of the model has been undertaken in an effort to extend, and improve, the Capital Asset Pricing Model as a positive micro economic theory dealing with conditions of risk. Included amongst this research is the work by Lintner (1969) into heterogeneous expectations, Fama (1970) into multiperiod decisions, Brennan (1970) into the implications of differential taxation, Mayers (1972) into the existence of nonmarketable assets, Black (1972) into the issue of restricted borrowing, and Blume and Friend (1973) into the assumption of unrestricted short sales.

Probably the most noteable of the studies is the one by Black who, believing the assumption of unlimited risk-free borrowing was the most restrictive, showed that in the absence of a risk-free asset every efficient portfolio can be constructed as a weighted combination of the market portfolio and the minimum variance zero-beta portfolio. When a risk-free asset exists but cannot be held short, the efficient set of portfolios consist of two parts. One a weighted combination of the market portfolio and the minimum variance zero-beta portfolio, and the other a weighted combination of the risk-free asset and a single efficient portfolio of risky assets (1972:452).

Even with the additional theoretical developments mentioned above, empirical research into the model is subject to considerable debate. Roll has gone so far as to state that the model is not testable because the efficiency of the market portfolio and the validity of the model are joint hypotheses that are almost impossible to test because of the difficulty of measuring the true market portfolio (1977:130-131). Notwithstanding Roll's critique however, extensive empirical research has been undertaken and today it is generally accepted that the pure theoretical form of the model does not agree well with reality.

The empirical work shows evidence that the unsystematic risk associated with individual equities can be explained to a statistically significant extent by other "common" variables. For instance, Black, Jensen and Scholes (1972) found evidence of a second factor, independent of the market, that varied through time and was significantly positive in the postwar period (Black, 1972:446). This finding has been supported by numerous subsequent studies some of which are highlighted by Copeland and Weston (1988:215).

> *Factors other than beta are successful in explaining that portion of security returns not captured by beta. Basu (1977) found that low price/earnings portfolios have rates of return higher than could be explained by the CAPM. Banz (1981) and Reinganum (1981) found that the size of a firm is important. Smaller firms tend to have high abnormal rates of return. Litzenberger and Ramaswamy (1979) found that the market requires higher rates of return on equities with high dividend yields. Keim (1983,1985) reports seasonality in stock returns - a January effect.*

2.3.2 A concluding comment

The CAPM, while undoubtedly one of the major theoretical developments in portfolio theory, has been shown above to have numerous limitations in its ability to explain the economic phenomenon of risky asset pricing. From a theoretical viewpoint it is clear that some of the assumptions are not only restrictive, but run contrary to fundamental evidence concerning investor behaviour and the distribution of security returns. Investors do not behave in a fashion consistent with quadratic utility functions and there is an overwhelming body of evidence to indicate security returns deviate significantly from multivariate normality. Empirically, given the caveat of Roll's critique, the model has been found deficient in numerous respects. This is particularly evident with the requirement for additional factors.

The Arbitrage Pricing Theory, developed by Ross (1976) has attained much of its popularity because of its ability to overcome some of the theoretical and empirical difficulties with the CAPM. In this sense it can be considered the the next logical step in the continuing development of capital market theory.

2.4 The Arbitrage Pricing Theory

In this section the original development of the Arbitrage Pricing Theory is presented. Thereafter some subsequent extensions are highlighted including the applicability of the theory in finite economies.

2.4.1 The development of the APT[2]

While recognising certain limitations and weaknesses of the approach, Ross initially presented an heuristic argument for his theory. Beginning with *the neoclassical assumptions of perfectly competitive and frictionless markets*, Ross assumed that the random returns of a set of assets could be described by a simple factor generating model of the form (Roll and Ross, 1980:1076);

$$\tilde{r}_i = E(\tilde{r}_i) + \beta_{i1}\tilde{\delta}_1 + \ldots\ldots + \beta_{ik}\tilde{\delta}_k + \tilde{\varepsilon}_i, \quad i = 1,\ldots\ldots,n \tag{2.1}$$

where; $E(\tilde{r}_i)$ is the expected return on the i[th] asset; $\tilde{\delta}_j$ is the mean zero j[th] factor common to the returns of all assets; β_{ij} represents the sensitivity of the i[th] asset to the j[th] factor; and, $\tilde{\varepsilon}_i$ is the idiocyncratic risk of the i[th] asset and is sufficiently independent to permit the law of large numbers to apply. As suggested by Roll and Ross, *(t)oo strong a dependence in the $\tilde{\varepsilon}_i$'s would be like saying that there are more than simply the k hypothesised factors* (1980:1076).

[2]The development of the theory as presented here closely follows that presented in the articles by Ross (1976) and Roll and Ross (1980). Given their clarity, little can be gained by attempting to extensively modify the approach of these two papers.

The initial step used by Ross (1976) in the development of the theory involved forming a well diversified arbitrage portfolio using all assets in the set. Being well diversified, the absolute amount of wealth invested in each security should be of the order $1/n$, and, as an arbitrage portfolio the net investment must be zero, ie;

$$\mathbf{x} \bullet \mathbf{1} = 0 \tag{2.2}$$

where; $\mathbf{x} = \left[x_1 \; x_2 \; \ldots\ldots \; x_n \right]$ is a row vector giving the proportion of wealth invested in each of the n assets; and, $\mathbf{1}$ is a unit column vector of length n.

The return on the constructed portfolio is therefore given by;

$$\mathbf{x} \bullet \tilde{\mathbf{r}} = \mathbf{x} \bullet \mathbf{E}(\tilde{\mathbf{r}}) + \mathbf{x} \bullet \boldsymbol{\beta} \bullet \tilde{\boldsymbol{\delta}} + \mathbf{x} \bullet \tilde{\boldsymbol{\varepsilon}}$$

where; $\tilde{\mathbf{r}}$ and $\mathbf{E}(\tilde{\mathbf{r}})$ are column vectors containing the random return and expected return for each of the n securities respectively; $\boldsymbol{\beta}$ is an $n \times k$ matrix containing the sensitivities of each security to each factor; $\tilde{\boldsymbol{\delta}}$ is a column vector containing the k mean zero factors common to all n assets; and, $\tilde{\boldsymbol{\varepsilon}}$ is the column vector of idiocyncratic risks for all n assets.

In the second step of his derivation, Ross invoked the law of large numbers to suggest that, given the well diversified nature of the portfolio, the influence of the idiocyncratic risk components would be negligible and the portfolio return could be approximated as;

$$\mathbf{x} \bullet \tilde{\mathbf{r}} \approx \mathbf{x} \bullet \mathbf{E}(\tilde{\mathbf{r}}) + \mathbf{x} \bullet \boldsymbol{\beta} \bullet \tilde{\boldsymbol{\delta}}.$$

If the arbitrage portfolio is also constructed to have no systematic risk then;

$$\begin{aligned} &\mathbf{x} \bullet \boldsymbol{\beta}_j = 0, \forall\, j \in \{1, 2, \ldots\ldots, k\}, \text{ or equivalently;} \\ &\mathbf{x} \bullet \boldsymbol{\beta} = \mathbf{0}, \text{ and;} \\ &\mathbf{x} \bullet \tilde{\mathbf{r}} \approx \mathbf{x} \bullet \mathbf{E}(\tilde{\mathbf{r}}). \end{aligned} \tag{2.3}$$

By constructing the portfolio in this fashion its random return has been *engineered to be equivalent to a certain return.* (Ross, 1976:242). Since no net wealth has been used in constructing the portfolio, to prevent arbitrarily large disequilibria from developing, the certain return must equal zero; ie,

$$\mathbf{x} \bullet \mathbf{E}(\tilde{\mathbf{r}}) = 0. \tag{2.4}$$

Equations (2.2) to (2.4) are merely statements in linear algebra and, given that the restriction of equation (2.4) must apply to all arbitrage portfolios constructed in the manner described, any vector constructed to be orthogonal to both the constant vector and the asset sensitivities matrix must also be orthogonal to the expected returns vector. This implies that the expected returns vector must be a linear combination of the constant vector and the sensitivities matrix; ie,

$$\mathbf{E}(\tilde{\mathbf{r}}) = \rho\mathbf{1} + \boldsymbol{\beta}\boldsymbol{\lambda}$$

where; ρ is a scalar; λ is a column vector of length k; and, for the i^{th} security the equation yields;

$$E(\tilde{r}_i) = \rho + \beta_{i1}\lambda_1 + \beta_{i2}\lambda_2 + \dots + \beta_{ik}\lambda_k. \tag{2.5}$$

Clearly ρ must be the rate of return on any riskless asset and, even if no such asset exists, ρ is the rate of return on all zero-beta portfolios (Ross, 1976:343). The natural interpretation of each λ_j is that it represents the excess return, or market premium, on a portfolio with only j^{th} factor systematic risk. This interpretation arises because any portfolio constructed such that $\beta_{Pj} = 1$ and $\beta_{Pi} = 0, \forall\, i \neq j$ will provide an expected return $E(\tilde{r}_P) = \rho + \lambda_j$, giving $\lambda_j = E(\tilde{r}_P) - \rho$.

Unfortunately the heuristic argument presented above presumes that economic agents do not become increasingly risk averse[3] as the number of assets increases and the law of large numbers acts to ensure that the idiocyncratic risk component becomes negligible. If such agents do exist, the presence of some idiocyncratic risk can continue to influence the pricing relationship. In order to rule out this possibility Ross suggested five assumptions needed to be made relating to the preferences of economic agents (1976:347-351).

(1) At least one asset must exist with limited liability so that there is some bound to the losses for which an agent is liable.

(2) At least one risk averse agent exists, who is not asymptotically negligible and for whom the coefficient of relative risk aversion is uniformly bounded, who believes that returns are generated by a equation (2.1).

(3) All agents are risk averse and have homogeneous expectations.

(4) The aggregate demand for all assets as a function of total wealth is positive. While

[3]As the number of assets increases aggregate wealth will, in general, also increase. Additionally, with increasing wealth it is conceiveable that some economic agents may become more risk averse.

any asset can be in excess supply, the economy as a whole must wish to hold some of each asset.

(5) The expected returns of the sequence of assets must be uniformly bounded.

While equation (2.5) can be interpreted as a multifactor generalisation of the CAPM, the approach used in its development is quite different. The argument provided by Ross suggests that it holds in all but the most profound cases of disequilibria. As such the theory provides an ex-ante pricing equation that does not rely on market equilibrium. Contrary to the CAPM, no central role exists for the market portfolio and its identification would therefore appear to be unnecessary when undertaking empirical research. Additionally, while a necessary assumption is that all economic agents have equal ex-ante expectations, the theory does not require that all agents believe that returns are generated in a fashion described by equation (2.1). Unlike the CAPM, the Arbitrage Pricing Theory *does not require the stringent homogeneity of anticipations of the mean-variance theory* (Ross, 1976:355).

2.4.2 Extensions of the APT

In spite of the simple heuristic employed by Ross, several assumptions were necessary to rigorously derive the Arbitrage Pricing Theory from the return generating equation described by equation (2.1). Since Ross's original development several researchers have provided additional derivations and approximations to account for such issues as the fact that the diversification of the type described by Ross can only apply approximately in an economy containing a finite number of assets. Some of these subsequent derivations have relied on arbitrage alone and others have sought to provide more robust derivations under equilibrium conditions.

While not seeking to extend or alter Ross's conclusions, Huberman (1982) sought to simplify his proof by providing an explicit definition of arbitrage under the conditions of a limit economy[4] and thereby reduce the reliance of the theory on the nature of the preferences of economic agents. Using his definition of arbitrage and a sequence of economies containing increasing number of risky assets, Huberman showed how his asymptotic no arbitrage condition related to the convergence of the Arbitrage Pricing Theory. He concluded by stating (1982:190);

> *... a result of the type "no arbitrage implies a certain behavior of returns," should involve no consideration of the preference structure of the agents*

[4]A limit economy is one defined to contain an infinite number of risky assets.

involved. Our analysis is in this spirit, because it involves no assumptions on utilities.

Chamberlain (1983), also under the conditions of a limit economy, defined a less restrictive factor structure and utilized the distinction between factor variance and idiosyncratic variance to show it was sufficient to imply equation (2.5). In addition, he derived upper and lower bounds on the approximation error which were necessary and sufficient conditions for exact arbitrage pricing, and showed that the assumption that the idiosyncratic risk be uncorrelated across assets was unnecessarily strong and could be relaxed (1983:1306).

In the same issue of Econometrica, Chamberlain and Rothschild (1983) also examined the implications of the absence of arbitrage in a large asset market which does not necessarily have the strict factor structure of Ross. They showed that as long as the covariance matrix of asset returns has a limited number of exploding eigenvalues[5], then an approximate factor structure exists that is both unique and sufficient to derive Ross's result. In addition they established that the eigenvalues corresponding to the exploding eigenvectors asymptotically converge and thereby play the role of factor loadings[6]. In a subsequent study, Grinblatt and Titman (1985) provided an economic intuition for the approximate factor structure of Chamberlain and Rothschild and showed that, while the approach provided important insights, it did not constitute a significantly weaker restriction on the asset return generating process than Ross's exact factor structure.

Two other significant contributions under the assumption of a limit economy are those of Stambaugh (1983), who extended the Arbitrage Pricing Theory by developing a no arbitrage pricing restriction in a setting where economic agents possess information about future asset returns, and Ingersoll (1984), who provided a stronger version of Huberman's pricing theorem by developing sufficient conditions for the Arbitrage Pricing Theory equation to be unique in the presence of correlated residuals.

In contrast to the above researchers who assumed limit economies in their examination of the theory, Dybvig (1983) and Grinblatt and Titman (1983), using equilibrium rather than arbitrage arguments, and Chen and Ingersoll (1983) investigated the robustness of the

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[5] An eigenvalue is considered "exploding" if its value increases as the number of assets analysed increases.

[6] Aside from its theoretical contribution, the article of Chamberlain and Rothschild, with its focus on eigenvalues and eigenvectors, was the first to provide a justification for the use principal components analysis in empirical research into the APT rather than some form of factor analysis as originally suggested by Roll and Ross (1980).

Arbitrage Pricing Theory in economies containing a finite number of assets. Starting with assumptions about preferences and the nature of the factor model, Dybvig established bounds for individual asset deviations from the APT prices that he contended were directly observable or easily estimable (1983:484). On the basis of these he further concluded that *a rough computation has indicated that the deviations from APT prices are negligible in our economy* (1983:495). Grinblatt and Titman reached similar conclusions when they provided a derivation which described the deviation of an asset's mean return from that predicted by the linear pricing equation. Given that their equilibrium model only placed restrictions on assets that were priced, they additionally suggested it was *applicable in an economy where some assets are either not traded or are traded with high transactions costs* (1983:504). Chen and Ingersoll showed that an exact pricing equation can be inferred without resorting to asymptotic mathematics as long as least one economic agent chooses not to bear idiosyncratic risk.

2.4.3 The testability of the APT

While, as highlighted above, numerous researchers have provided what they considered to be necessary and sufficient conditions for the development of the Arbitrage Pricing Theory, empirical verification of the theory has not taken place without considerable controversy. Firstly, Shanken (1982) has disputed the views of Ross (1976) and Roll and Ross (1980) that the APT is inherently more testable than the CAPM and suggested that, since any finite sum of squared deviations is finite, the boundary condition of Ross (1976) cannot be empirically verified. He further suggested that *the usual empirical formulation of the APT rules out the very expected return differentials which the theory attempt to explain* (1982:1140). Secondly, Dybvig, Friend, Gultekin and Gultekin (1985) have questioned the appropriateness of the theory in a multi-period setting by disputing the stability of the number of priced factors over time. While considerably less strong than Shanken's critique, if their contention is valid then the parsimonious aspects of the theory are substantially weakened. Finally, discussions concerning the general robustness and appropriateness of the empirical techniques currently employed have also appeared in the literature (Brown, 1989; Shanken, 1987).

2.5 Conclusions

This chapter has traced the development of capital market theory from its origins in utility theory through to the development of the Arbitrage Pricing Theory. The seminal work of Markowitz (1952) as well as the subsequent development of the Capital Asset Pricing

Model by Sharpe (1964), Lintner (1965) and Mossin (1966) were presented in order to highlight the evolution that has taken place in the field. While the Arbitrage Pricing Theory can be viewed as a multi-factor generalisation of the Capital Asset Pricing Model, it has been derived under less restrictive assumptions and, in spite of the contrary opinion expressed by Shanken, is consequently seen by many as offering a testable alternative to the CAPM. The major empirical advantages of the theory are believed to be firstly, that it can appropriately be tested on a subset of risky assets, and secondly, that the market portfolio does not play a central role and need not therefore be identified.

A full review of empirical research into the APT is left for later chapters where the empirical research conducted in this thesis is also described.

3

The distribution of security returns

3.1 Introduction

Certain distributional assumptions concerning security returns have been made throughout the development of modern portfolio theory. The concept of mean-variance efficiency presupposed that the distribution of returns can be fully described by knowing the mean and variance of the distribution[1]. This is only the case if, as suggested by Bachelier and Osborne, the summation of successive returns approaches the normal distribution as the number summed increases[2]. Kon (1984:147) states that mean-variance efficiency follows from the assumption of multivariate normality with parameters that are stationary over time, together with the additional assumption of risk aversion.

Empirical research by numerous authors has provided conclusive evidence that returns are distributed in a more leptokurtic fashion than suggested by the normal distribution. A family of distributions, known as the stable Paretian, has been used extensively in empirical investigations into the behaviour of security prices. As a class this family includes both symmetric and asymmetric distributions, and contains both the normal and Cauchy as members. Other studies however, suggest the use of the scaled t-distribution, the lognormal distribution, or a complex mixture of normal distributions[3]. In terms of the random walk hypothesis therefore, the findings suggest that it is possible that the stochastic process from which returns are drawn is not of the Wiener type. It is important

[1]As mentioned in chapter two, the statement that multivariate normality is a necessary condition for mean-variance efficiency actually presumes that investor utility functions are not quadratic (Tobin, 1958:82-85).

[2]For further reference see Cootner, 1964; Fama, 1977:17; Roll and Ross, 1980:1076.

[3]Research investigating the stable Paretian includes work by Fama (1965), Mandelbrot (1967), Teichmoeller (1971), Officer (1972), Hsu, Miller and Wichern (1974), Perry (1983) and Kon (1984). The other distributions mentioned have been proposed by such authors as Praetz (1972), Blattberg and Gonedes (1974), Copeland and Weston (1988:209) Press (1967), Clark (1973) and Ball and Torous (1983).

to note however that the stationarity assumption plays a critical role in empirical research using longitudinal data. Its retention is necessary in order that time series data can be used to gain an understanding of the distribution from which returns in any single future period will be drawn since it is this distribution which is of interest in financial theory (Copeland and Weston, 1988:147).

3.1.1 Implications for the Arbitrage Pricing Theory

Although the distribution of returns does not have a direct bearing on the formulation of the Arbitrage Pricing Theory, which does not require the rigorous assumptions of its predecessors in its development, it does have implications for empirical research into the theory. Existing empirical work, particularly with reference to the identification of the number of priced factors, is undertaken using multidimensional linear procedures such as factor analysis and principal component analysis. In order to draw correct inferences about the population parameters from these procedures it is necessary that certain assumptions be valid. With respect to principal component analysis (PCA) Jolliffe (1986:41) states that;

> *It can be argued that PCA should only ever be done for data which is, at least approximately, multivariate normal, for it is only then that "proper" inferences can be made regarding the underlying population PCs.*

Roll and Ross used the maximum likelihood factor analysis procedure because of the availability of a Chi-square statistic of the likelihood ratio for testing the hypothesis of a specific number of factors. This procedure also relies on the assumption that the original variables have a multivariate normal distribution (Harman, 1976:184).

In this chapter the distribution of security returns on the Johannesburg Stock Exchange is examined as an initial step in the determination of the robustness and power of the multivariate procedures used in investigations into the number and pricing of the APT factors influencing security returns. In common with the approach adopted in prior research (Fama, 1965,1977; Teichmoeller, 1971; Clark, 1973; Perry, 1983; Affleck-Graves and McDonald, 1989), a combination of univariate tests rather than direct multivariate testing procedures is utilized. The focus is considered appropriate because of the higher power of the univariate tests. If the hypothesis that security returns are normally distributed is rejected on the basis of univariate tests, the data are by implication not multivariate normal. The extent of any deviation from multivariate normality is however demonstrated in the final section of the chapter by plotting the generalized Mahalanobis distances of each vector of returns against the appropriate χ^2 percentiles.

3.2 Prior empirical research

As stated in the introduction, considerable research has been undertaken into the distribution of security returns. Many of the investigations questioning the Gaussian assumptions of Bachelier and Osborne (Cootner, 1964) have been based on the concept of a subordinated stochastic process, where it is assumed that price changes between transactions are normally distributed, but the number of transactions occuring in any given time interval is itself stochastic. The distributional characteristics of returns when measured over calendar time intervals are consequently determined by the probability distribution describing the number of transactions occuring in the calendar time interval (known as the directing process). By using different distributions to describe the directing process, different authors have derived different models to explain security returns (Mandelbrot, 1973:157). Examples include the stable Paretian distribution suggested by Mandelbrot and Fama, the lognormal-normal distribution suggested by Clark, and the Student t-distribution suggested by Praetz, and Blattberg and Gonedes.

In this section the findings of several of the post Bachelier and Osborne investigations into security returns are presented together with brief descriptions of the methodologies employed.

3.2.1 Mandelbrot (1963, 1967)

One of the first researchers to question the Gaussian distribution assumptions of speculative asset returns was Mandelbrot who proposed that the empirical distributions suggested a *radical new approach to the problem of price variation* (1963: 395). He sought to show that a particular type of distribution could;

> *account for substantial features of price series (of various degrees of volatility) without non-stationarity, without mixture, without master processes, without contamination, and with a choice of increasingly accurate assumptions about interdependence of successive price changes* (1967:397).

Mandelbrot's seminal paper was principally a theoretical exposition of the stable Paretian hypothesis. It drew extensively on the original work of Paul Levy (1925) who developed the characteristic function for the stable Paretian family of distributions. In the paper he suggested that the stable Paretian probability distribution could explain the outliers observed in returns[4] as well as the erratic behaviour of sample higher order moments (1963: 403). The outliers resulting from the peakedness of the underlying distribution relative to the normal, and the erratic nature of the higher order moments resulting from the fact that the distribution only has finite moments of order less than the characteristic exponent, unless it equals two when all moments are finite[5].

Although in his 1963 paper Mandelbrot limited the empirical testing to an examination of speculative returns in cotton using daily, weekly and monthly data over periods ranging from 1816 to 1958, four years thereafter he provided additional empirical evidence in support of his hypothesis by examining wheat over the period 1883 to 1895, railroad securities and rates of interest over the period 1857 to 1936, and rates of exchange over the period 1803 to 1895 (1967:393). The methodology employed in all cases involved the examination of probability plots of returns over different intervals using double logarithmic graph paper[6] and Mandelbrot's final conclusion was that the empirical evidence supported his hypothesis and that all the speculative returns he examined exhibited stability with characteristic exponents less than two (ie. stable non-normal).

3.2.2 Fama (1963, 1965)

In extending some of the work of Mandelbrot, Fama suggested that certain aspects of investor behaviour, difficult to explain under the Gaussian hypothesis, could be partially explained if the market is actually stable Paretian in nature. The inherent riskiness of a stable Paretian market could lead some investors to avoid speculative markets altogether and others to hold more risk-free liquid assets than would seem appropriate under the assumptions of a Gaussian hypothesis (1963:427).

Like Mandelbrot's, Fama's 1963 paper was principally a theoretical exposition of the stable Paretian hypothesis. In it he provided further insights into the hypothesis, particularly with respect to the stability properties of the distribution. He showed that if individual pieces of information, combining to determine returns from transaction to transaction, are asymptotically Paretian with a common characteristic exponent then the returns over longer time intervals will be Paretian with the same characteristic exponent

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[4]Throughout the thesis returns refers to logarithm price relatives.

[5] The normal distribution is a unique member of the stable Paretian family with a characteristic exponent of two and an index of skewness of zero.

[6]The tests of whether the underlying distributions were stable Paretian utilised the stability of the plots, particularly in the tails, for daily, weekly, monthly and annual data. The characteristic exponents of the distributions were then inferred from the shape of the plots.

(1963:426).

In his 1965 paper Fama carried out extensive empirical research into the behaviour of security prices using daily data between January 1956 and September 1962 for the thirty securities making up the Dow-Jones Industrial index. The research was divided into three sections. In the first, frequency distributions and normal probability graphs were used to compare each stock's return distribution against the normal. The results showed the empirical distributions contained a higher frequency near the mean and at the tails than would be the case for the normal distribution (1965:45-55). In the second section, Fama checked the stability of the distributions under addition by examining the normal probability graphs using four day returns. He also used three methods to estimate the characteristic exponent of each stock. The methods involved estimating the slope of the tails of double-logarithm graphs, using a range analysis approximation, and using a sequential variance approach (1965:62-68). The results generally indicated that the returns distributions seemed stable under addition and had characteristic exponents of less than two. Of the three methods of estimating the exponent, Fama found the range analysis approach the most reliable. In the final section of the paper three tests of independence were used as a test of the random walk hypothesis. The methods used involved serial correlation analyses, runs tests, and tests of a mechanical trading rule known as Alexander's filter technique (1965:69-83). None of the three methods showed evidence of significant dependence.

Fama's overall conclusion was that the empirical evidence supported the random walk hypothesis of security prices in that successive returns were independent and conformed to some probability distribution, namely stable Paretian with characteristic exponent less than two. In commenting on the use of only blue-chip securities in his analysis, Fama also stated that it was conservative in terms of the stable Paretian hypothesis since the blue-chip securities were more likely to have been stable with less extreme returns than other securities (1965:46).

3.2.3 Press (1967)

Press proposed an alternative compound events model of returns with a finite second moment that, he believed, could explain observed variances as well as the non-Gaussian stable process suggested by Mandelbrot. The model was perceived to have the advantage that it *might not require all previous portfolio selection theory based upon finite variance distributions and quadratic loss functions be abandoned* (1967:318).

In terms of the model, the logarithm of the price of a security at any given time is composed of a Wiener process, and a sum of stationary and independent increments that follow a compound Poisson process. The increments represent price changes of random size taking place as the result of a random number of events during the interval of time between observations. Press showed analytically that this distribution is generally more peaked in the vicinity of its mean than the normal, is leptokurtic in character, has a higher probability in the tails than for the comparable normal, and is asymmetric[7] (1967:320-321).

Although Press extended his model to a multivariate setting, he restricted his empirical tests to the use of univariate procedures on monthly data from 1926 to 1960 for ten of the component securities of the Dow Jones Industrial Average. The methodology involved sub-dividing the data into three time periods, estimating the four model parameters for each period and for each security, and producing two types of graphical output. The first graph type involved superimposing the estimated model trend lines plus or minus one standard deviation on the actual plots of logarithm of price against time, while for the second graph type the empirical cumulative density functions were plotted together with the theoretical cumulative density functions using the parameter estimates. As the infinite series representation of the cumulative density function for the compound events model results in the maximum likelihood method being unable to yield explicit estimators, Press used a procedure of *cumulant matching* to estimate the parameters of the distribution and his overall conclusion was that the cumulative density function curves were remarkably close considering *that the parameter estimates leave something to be desired* (1967:335). He further suggested that larger sample sizes would show better results.

3.2.4 Teichmoeller (1971)

Under the assumption that returns are symmetrically distributed, Teichmoeller (1971) used the properties of the characteristic exponent of the Paretian distribution in order to test whether returns might not better be described as a simple mixture of normal distributions.

He examined thirty securities quoted on the New York Stock Exchange between July 1962 and June 1967 for which he had 999 daily returns. By examining the trend in the estimated characteristic exponent of each of the securities using the daily data and non-overlapping sums two, five and ten days respectively, he concluded that security returns do not exhibit properties indicating that they come from a simple mixture of normal

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[7]The asymmetric nature of the distribution applies when the mean of the distribution is not zero.

distributions. Given that the estimate of the characteristic exponent did not appear to increase monotonically as the sum size increased, he therefore suggested that returns are distributed as either symmetric stable with a characteristic exponent of less than two or as a more complex mixture of variables with finite second moments.

3.2.5 Praetz (1972)

Praetz, in his empirical research, rejected the stable Paretian hypothesis in favour of the normal distribution of security returns modified to take account of the changing variance of the market. He proposed that the non-normality observed in returns results because the variance of returns does not remain constant through time, since *any market often has long periods of relative activity, followed by long periods of relative inactivity* (1972:50). Given the evidence of changing variance from year to year, Praetz therefore modified the Osborne model to take into account a variance term which is distributed as an inverted gamma distribution. The resultant distribution of returns he suggested should follow a scaled t-distribution.

Using seventeen weekly share price indices quoted on the Sydney Stock Exchange between 1958 and 1966, Praetz conducted chi-squared goodness of fit tests to compare his model against the Osborne normal distribution model, the stable Paretian model, and the compound event model of Press. The results showed that the stable Paretian, with a characteristic exponent ranging from 1.66 to 1.96, gave marginally lower minimum chi-squared values than the normal but significantly higher values than obtained using the scaled t-distribution. Using a one percent level of significance, the scaled t-distribution could not be rejected for any of the indices while the other three models were rejected in all but four cases. In his conclusion Praetz acknowledges that for individual securities his distribution is *not as hopeful due to the discrete nature of the price changes and, in particular, to the large number of zero price changes that seem to occur* (1972:54). He does however suggest that the scaled t-distribution overcomes the difficulties involved in dealing with the stable Paretian, namely, the infinite variance issue, the unknown nature of the distribution function except in a few unique cases, and the problems involved in parameter estimation.

3.2.6 Officer (1972)

Using a variety of procedures, Officer tested the stable Paretian hypothesis by examining the distribution of a sample of one hundred and thirty-six randomly selected CRSP[8]

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[8]Officer (1972) and Singleton and Wingender (1986) refer to the CRSP database as source rather than to

securities covering the period February 1926 to June 1968. The methodology involved partitioning the sample in several different ways in order to assess the stability of the characteristic exponent and scale parameter (1972:808).

As a first step the stability of the distribution parameters across two consecutive periods was examined[9]. While the results suggested different distributions for the two periods, subsequent analysis of the second period indicated apparent stability within that period. This led Officer to conclude that the distributional characteristics are subject to discrete changes due to macro events rather than being subject to continual change.

Both the longitudinal and cross-sectional stability of the characteristic exponent was examined using daily and monthly data. For the longitudinal tests, the results for the monthly data suggested there was no tendency for the exponent to increase as the returns summed approached five months. This was contradicted by the results using daily data where the average characteristic exponent increased by approximately seventeen percent as the number of days summed increased to twenty. The cross-sectional stability of the exponent was tested by divided the database into varying size portfolios and comparing the portfolio characteristic exponent estimates against the averages obtained for the individual components. For these tests the results suggested that the portfolio characteristic exponent estimates were good approximations of the averages of the estimates obtained for the individual component securities when the total returns were examined but not when the market model residuals were examined.

With respect to the scale parameter, the research showed that, as expected for the stable Paretian hypothesis, the estimates suggested invariance for monthly returns summed for up to five months. Officer did however find that the standard deviation was *suprisingly well behaved.* This was confirmed using daily data and he therefore concluded that the distributions *have some properties which true non-normal stable distributions do not have.* In particular he concluded that (1972:811);

> *(i)t may be that a class of fat-tailed distributions with finite second moments will be found to give better approximations of stock returns, but as yet this remains to be clearly demonstrated.*

His overall conclusion was therefore that the returns distributions exhibited a number of

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the primary market from which their data are extracted, ie. NYSE or AMEX. The reference has therefore been retained in this literature review. Refer also to section 3.2.13.

[9]The two periods examined were February 1926 to February 1947 and March 1947 to June 1968.

properties inconsistent with the stable Paretian hypothesis.

3.2.7 Clark (1973)

Clark hypothesised that the distribution of speculative returns measured over constant calendar time intervals is subordinate to the normal distribution (1973:143). He suggested that this occurs because price series evolve at different rates during identical intervals of time due to the varying rate at which information relevant in amending price expectations arrives at the market. When the information arrival is fast, expectations are revised rapidly and the evolution of price change is also fast. When the information arrival is slow the price process also evolves slowly.

Clark showed analytically that when consecutive returns are independent and normally distributed the stochastic nature of the information directing process will result in a distribution that is leptokurtic. He also proved that if the directing process has a finite second moment, contrary to the stable Paretian hypothesis, the subordinated returns process will have finite variance.

Empirical testing of the hypothesis was carried out in two stages using daily data on cotton price futures over the period January 1947 to February 1955. The first stage involved testing to see if trading volume could be used as a measure of the speed of price evolution, and therefore as a surrogate for the directing process. This was done by ranking the returns on the basis of trading volume, dividing the data into equal size groups, and examining the relationship between average group volume and group sample variance. The results showed both a clear curvilinear relationship and that individual group kurtoses were within two standard deviations of what would be expected for normal data[10]. Clark therefore concluded that trading volume could be used as an *imperfect clock* approximation of the directing process. Additionally, he found that the directing process was itself lognormally rather than normally distributed. The second phase of the empirical work involved a direct comparison of the performance of the stable Paretian and the lognormal-normal subordinated process using both Bayesian and Kolmogorov-Smirnov maximum likelihood procedures (1973:149-152). For both procedures Clark found evidence in favour of the subordinated process and the concluded that;

> *(t)he standard Central Limit Theorem holds only when the number of random*

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[10]For the entire sample the the kurtosis was more than one hundred standard deviations from that expected for the normal distribution (Clark, 1973:143-144).

variables being added is constant (in probability limit, at least); in the the case of speculative markets, this restriction is violated, and the limit distribution of price changes is subordinate to the normal distribution (1973:153).

3.2.8 Hsu, Miller and Wichern (1974)

Hsu, Miller and Wichern supported the findings of Praetz and Officer in that they rejected the assumption that security returns conform to a stable non-normal distribution with stationary parameters. Using daily data of four firms over the period January 1963 to December 1970, they carried out chi-square goodness of fit tests under the null hypothesis that the returns were drawn from symmetric stable Paretian distributions. For each security, the expected values for the goodness of fit tests were calculated over a grid of characteristic exponent and scale parameters in the region of the Fama and Roll estimates (Fama and Roll; 1968) and the minimum calculated chi-square statistic used as a conservative estimate in the hypothesis test (1974:109). Their results indicated that symmetric stable distributions are not consistent with the data for daily returns.

Further confirmation of their findings resulted when they examined monthly data of twenty firms over the period January 1926 to December 1960 using holding period returns rather than returns calculated as logarithm price relatives. Consistent with the findings of Officer (1972), Hsu, Miller and Wichern found evidence of parameter nonstationarity from the pre-war to post-war period. They also suggested that parameter changes were more likely to have occured at discrete points in time as a result of economic phenonena, than in a continuous fashion (1974:111). Using the Studentized range statistic they further concluded that monthly post-war rates of return could be reasonably described by the normal probability distribution. As a final test of stability, longitudinal sums of monthly returns were examined using the methodology employed by both Teichmoeller (1971) and Officer, supplemented by an additional test which involved first randomizing the time series data. The randomization procedure was adopted in an effort to ensure the results were not spurious (1974:112). Their final conclusion was to reject the hypothesis that monthly returns follow a stable Paretian distribution. As an alternative Hsu, Miller and Wichern proposed a normal probability model with a nonstationary variance subject to step changes.

3.2.9 Blattberg and Gonedes (1974)

Blattberg and Gonedes examined the Student t-distribution and the symmetric stable Paretian distribution as two alternative models for describing observed returns characteristics. They suggested that both distributions are consistent with observed data and are *fat tailed*, and both contain the Cauchy and Normal as special cases. The major difference between the two being that the Student t-distribution is non-stable. The methodologies they employed involved the use of likelihood ratio tests and stability tests of the degrees-of-freedom parameter and the characteristic exponent (1974:257-259). Given the complexities of the methodologies employed, an extensive Monte Carlo simulation study was undertaken to establish the power of the tests.

Daily returns over the period January 1956 to September 1962 for the thirty securities making up the Dow Jones Industrial Index were used as data for the study and the likelihood ratio for each of the securities was computed by calculating the likelihood function for each of the models using estimates of the underlying parameters[11]. Maximum-likelihood parameter estimates were used for the Student t-distribution and Fama-Roll parameter estimates for the stable Paretian distribution. Although this biases the likelihood ratio in favour of the Student t-distribution (1974:258), Blattberg and Gonedes showed, through their simulation study, that the bias was moderate. The results indicated that the Student model was superior to the stable Paretian for all thirty securities.

The stability tests used exploited the non-stable nature of the Student t-distribution. Given that the Student t-distribution converges to the normal for summed sequences of independent random variables, the degrees-of-freedom estimates are not stable when returns are added longitudinally and the underlying data conforms to the distribution. This is in contrast to the stability of the characteristic exponent for summed sequences when the underlying data is stable Paretian. The results showed that the average degrees-of-freedom estimate increased from 4.79 to 11.22 as the number of days summed increased to five, while the average characteristic exponent increased from 1.65 to 1.72. This result was found to be consistent with the findings of the simulation study when the underlying data conformed to the Student t-distribution and not when the underlying data conformed to the stable Paretian distribution.

The final conclusion of the study was that the Student model had the greater descriptive validity. Blattberg and Gonedes therefore concurred with the findings of Praetz even though they expressed serious doubts as to the validity of his methodology (1974:256).

3.2.10 Ball and Torous (1983)

[11]Blattberg and Gonedes used the same database as Fama (1965).

Ball and Torous developed a simplified model which retained the economic intuition underlying the model proposed by Press (1967), yielded leptokurtic security return distributions, and was more amenable to statistical testing. They utilized the concept that total returns could be decomposed into normal and abnormal components, where the abnormal changes result from information arrivals causing ***more than a marginal change in price*** (1983:53), and suggested that for reasonably small time increments it could be assumed that no more than one abnormal information arrival would occur within each increment. Consequently abnormal returns could be modeled as a Bernoulli jump process without the need to use the full Poisson process. They did note however, that for finer time intervals, the Bernoulli model converges to the Poisson model (1983:56).

Ball and Torous believed that their model had the principal advantages that improved empirical analyses were possible, statistical testing for the presense of a jump component could be undertaken, and maximum likelihood estimation of the relevant parameters was *practically and economically implementable*. This avoided many of the problems associated with parameter estimation using the method of cumulants where negative variance estimates could be obtained (1983:56-58).

Empirical testing of the model was carried out using daily data over the period September 1975 to September 1977 for forty-seven NYSE listed securities. Since it was found that only five securities did not exhibit the presense of jumps at the five percent level, Ball and Torous concluded that there was strong evidence to support the presense of jumps in security returns. Additionally, their comparison of the method of cumulants and maximum likelihood parameter estimation showed that consistency in variance estimation was only possible with the maximum likelihood procedure where no negative estimates were obtained (1983:60).

3.2.11 Perry (1983)

Given the importance of the second moment in much of the development of financial theory, and the extended debate as to whether security returns have a finite variance, Perry proposed a method for testing for the existence or otherwise of finite variance.

Building on the work of Cootner (1964), the methodology involved computing the variance for successively larger samples of data and examining the trend in the magnitude of the successive estimates. If the true population variance is infinite, the trend line should be significantly positively sloped and the null hypothesis of a finite variance can be rejected (1983:213). In order to account for the possibility that the variance could be finite but changing through time, Perry suggested three trend estimation procedures be followed. A forward procedure where the increased sample size develops cronologically with time, a backward procedure where the reverse applies, and a procedure where the data is randomized before successively larger sample size variance estimates are computed.

Before carrying out empirical tests, Perry endeavoured to validate his methodology using a simulation study on data generated to follow a series of stable non-normal distributions and Student t-distributions. The simulations showed that a parametric test of the significance of the mean slope using the t-statistic had low power and often failed to reject the null hypothesis that the variance was finite when it was false (1983:214). To circumvent this weakness, he therefore proposed an additional nonparametric sign test of the number of positive slopes in a set number of trials.

The empirical research was carried out using daily returns, over the period July 1962 to December 1977, for thirty-seven securities quoted on the NYSE. Both total returns and market model residual returns were examined because of possible impact of cross-sectional dependency in the data. The results suggested that there was;

> ..... *very strong evidence with both the parametric test and the nonparametric test that the variance of the security return distribution is finite but changing over time, and that this pattern of change is a complex one* (1983:219).

Perry therefore concluded that the stable non-normal hypothesis of security returns is untenable, that the observed high kurtosis can most likely be explained by a finite but changing variance, and that the parameter shifts possibly occur at more frequent intervals than suggested by Hsu, Miller and Wichern (1974).

3.2.12 Kon (1984)

In providing the rationale for the use of a discrete mixture of normal distributions to explain the skewness and kurtosis observed in security returns, Kon noted that it is the normality assumption that is crucial to models of financial theory, and rejection of the stationary normal distribution model on the basis of empirical evidence could be due to non-stationarity alone. He further suggested that theory actually predicts that changes in investment and financing decisions by firms will result in the underlying distribution parameters changing. Kon therefore proposed that the combination of macro and micro economic variables impacting on securities would result in a mixture of several normal distributions being necessary to account for firm-specific events, cyclical changes, and structural changes (1984:148-152).

Empirical verification of the model was undertaken by estimating the parameters for models consisting of between one and five discrete mixtures of normals and using Chi-square likelihood ratio tests to draw conclusions as to the appropriate mixture. Additionally, Kon compared the results against the symmetric Student t-distribution of Blattberg and Gonedes (1974). The data used for the research consisted of daily returns from July 1962 to December 1980 for the thirty component securities of the Dow Jones Industrial Index plus the Standard and Poors, Equally Weighted and Value Weighted Indices.

The overall results showed strong evidence in favour of the mixture of normals over the Student t-distribution for twenty-three of the securities and for all three indices. The number of normals in the mixture did however appear to vary, ranging from two for twelve securities, three for eleven securities and the indices, and four for seven securities. Finally, Kon noted that, while the Student t-distribution obviously couldn't account for any of the observed skewness in the data, the mixture of normals could explain both the skewness and kurtosis. This was possible because of the significant differences in the mean and variance estimates across the component normals making up the mixture (1984:160-161).

3.2.13 Singleton and Wingender (1986)

Prior to the work of Singleton and Wingender, increasing evidence of the persistence of positive asymmetry in aggregate security returns (Beedles and Simkowitz, 1980) resulted in the development of portfolio approaches extending the mean-variance framework by utilizing the first three moments of security returns distributions. While acknowledging the consistency in the frequency of positively skewed security returns, Singleton and Wingender carried out empirical research into the persistence of skewness for individual securities and portfolios through time. They argued that only if skewness persisted for individual securities and portfolios was there any merit in investment policies that encouraged investors to concentrate on skewed securities (1986:335).

Monthly returns from 1961 to 1980 for five hundred and fifty-one securities, obtained from the CRSP database, were used for the study. The methodology employed to test for the persistence of skewness involved dividing the database into four time periods of five years and computing Pearson skewness coefficients for each time period, and for each security and portfolio. Pairwise comparisons, to check the proportion of securites and portfolios that remained positively skewed, the proportion that changed from positive to negative skewness, and Spearman rank correlations of the actual skewness coefficients, were undertaken over the three sequentially paired periods (1/2, 2/3, 3/4). The portfolios tested consisted of equally weighted random portfolios of five and twenty securities and, portfolios consisting of five securities formed on the basis of the skewness coefficient ranking in the earlier period for each pairwise comparison. The skewness ranked portfolios were examined as a more rigorous procedure to ensure the largest range of portfolio asymmetry possible (1986:339).

The overall results suggested that *positive skewness is not due to a constant set of securities and investors cannot rely on ex post data to predict which stock will offer skew returns* (1986:339). Singleton and Wingender therefore concluded that skewness in equity returns does not persist and three moment equilibrium models might not be appropriate for examining common equity portfolios.

3.2.14 Concluding comments

From the review of research presented above it is apparent that, although there is still considerable doubt as to the true nature of the distribution of security returns, the more recent findings tend to reject the stable Paretian hypothesis in favour of some complex mixture of distributions with finite variance. Given the centrality of the finite variance assumption to the empirical procedures generally employed in research into Capital Market Theory this trend must be considered very encouraging. What still remains inadequately addressed however are the implications of non-normality, even under conditions of finite variance, for the robustness of the univariate and multivariate procedures employed in empirical research[12]. In particular, the implications of non-normalities for the factor analysis procedures used in the identification of the number of APT factors have not as yet been comprehensively addressed. As this is the central objective of the current research, and given that the bulk of the comprehensive analyses undertaken have used well traded American securities, the distribution of security returns on the Johannesburg Stock Exchange is examined below.

Arising out of the international evidence, the analysis presented below addresses three issues for the South African securities market. Firstly, confirmation is sought that returns for Johannesburg Stock Exchange listed securities exhibit distributional characteristics that are inconsistent with the non-normal stable Paretian family. Secondly, the issue of whether the varying rate that information comes to the market, as reflected in trading

[12]Research into this area includes the paper by Affleck-Graves and McDonald (1989) examining the robustness to departures from normality of the multivariate test proposed by Gibbons, Ross and Shanken (1986), and the research by Brown and Weinstein (1985) into the the use of daily data in event studies.

volumes, results in the distribution of returns being subordinate to the normal is examined. Finally, two non-parametric runs tests are utilized to investigate the stationarity of both the mean and variance of security returns.

3.3 The distribution of security returns on the J.S.E.

3.3.1 Data selection

The data used in the analysis were extracted from the weekly database of Johannesburg Stock Exchange security prices and trading volumes maintained by the Statistical Sciences Department at the University of Cape Town. The database covers a period of approximately twenty years and at the end of July 1992 contained information for six hundred ninety-six companies. The ordinary equity of the two hundred and forty-four companies that traded continuously from February 20, 1973 to March 13, 1992 were selected for analysis[13]. As shown in table 3.1 the selected data represents approximately sixty-six percent of the market capitalization of the stock exchange and is well spread across all sectors.

Consistent with prior research, security returns were computed using first differences of the natural logarithm of security prices after allowing for any market capitalization issues, stock splits, and dividend receipts. Written mathematically;

$$r_t = \ln\left(\frac{p_t + d_t}{p_{t-1}}\right) \tag{3.1}$$

where; r_t is the return or logarithm price relative from t-1 to t; p_t is the security price at time t; and, d_t is the value of any dividend accruing between t-1 and t.

Given the relatively thin trading of the South African stock market, and the problem that this presents with respect to the difference between recorded prices and true underlying prices (Bradfield and Barr, 1989a), the number of weeks during consecutive fifty-two week periods within which trading actually occurred was recorded. Figure 3.1 shows the number of securities that exceeded any particular minimum number of weeks traded over the full nineteen years examined. The tables presented in this chapter are restricted to the forty-five securities that traded for fifty or more weeks per year over all nineteen years.

[13]Appendix 3.1 lists the selected companies together with their JSE code, number of ordinary shares in issue, market capitalization at the end of December 1991, and sector classification.

Given that this method of security selection results in a bias towards the companies that are the larger, more important, and better researched on the Johannesburg Stock Exchange, the results for the full dataset are also presented in appendices. As was stated by Fama (1965:46), focussing on the larger and better traded securities implies the empirical results cannot be considered representative of the entire market, although they can be considered conservative in tests of the null hypothesis that security returns are normally distributed. Rejection of the hypothesis on the basis of the chosen sample can be reasonably extended to less watched securities which are likely to be *less well behaved.*

Figure 3.1 Number of securities that exceed a certain minimum number of weeks trading per annum between February 20, 1973 and March 13, 1992

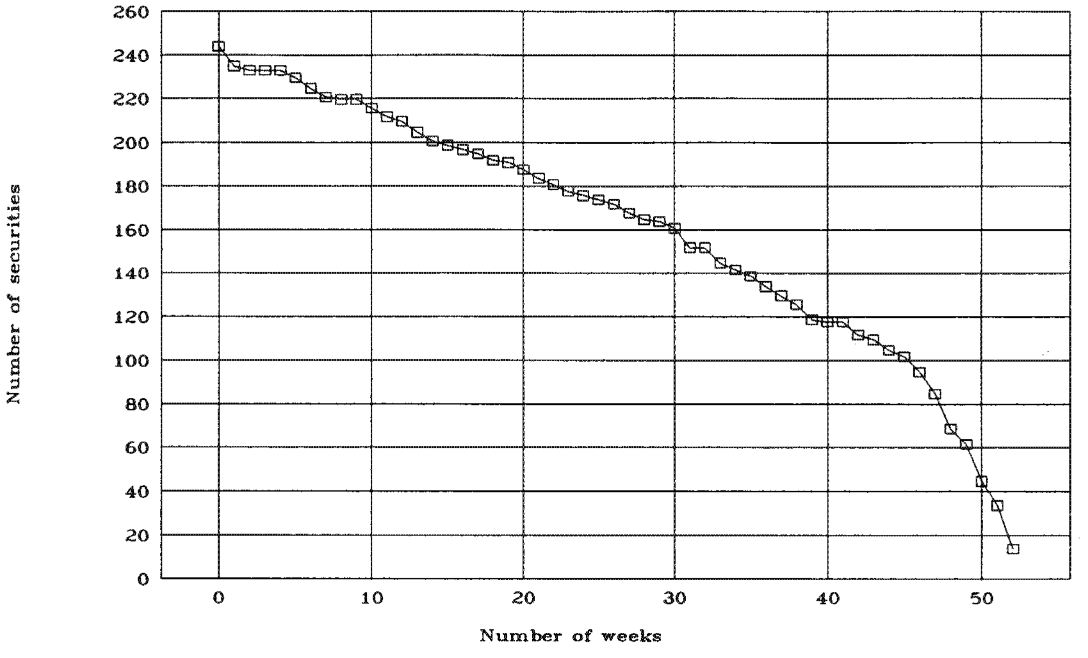


Table 3.1 · Distribution of selected securities across the sectors of the Johannesburg Stock Exchange at December 31, 1991

Sector	Selected securities		Total for sector		
	No	Capitalization (R millions)	No	Capitalization (R millions)	
Banks & Financial Services	4	9519.1	25	23642.8	40.3%
Beverages, Hotels & Leisure	2	15725.7	19	33798.8	46.5%
Building & Construction	14	4663.0	26	5171.8	90.2%
Chemicals & Oils	4	8091.3	9	18910.1	42.8%
Clothing, Footwear & Textile	16	363.3	41	1511.1	24.0%
Coal	6	5540.7	9	5618.0	98.6%
Copper	3	2294.9	4	2302.2	99.7%
Development Capital	0	0.0	26	228.1	0.0%
Diamonds	4	45532.3	7	46490.1	97.9%
Electronics etc.	10	4098.1	44	8849.3	46.3%
Engineering	10	3841.2	36	6102.2	62.9%
Fishing	2	138.8	4	202.6	68.5%
Food	7	11417.3	15	18048.7	63.3%
Furniture & Household	4	668.9	15	1117.4	59.9%
Gold	39	27946.2	63	38092.6	73.4%
Industrial Holdings	27	31469.2	60	71679.1	43.9%
Insurance	7	14433.9	21	23105.9	62.5%
Investment Trusts	2	463.2	12	5588.0	8.3%
Manganese	2	6182.3	2	6182.3	100.0%
Mining Exploration	1	32.2	12	557.4	5.8%
Mining Holdings	15	19777.2	21	20211.0	97.9%
Mining Houses	7	58173.3	11	68024.9	85.5%
Motor	9	2505.5	18	2803.1	89.4%
Other Metals & Minerals	2	22.0	12	524.1	4.2%
Paper & Packaging	9	12195.1	20	13403.9	91.0%
Pharmaceutical & Medical	3	1636.7	14	3168.3	51.7%
Platinum	2	8013.1	9	14266.5	56.2%
Printing & Publishing	2	1563.8	11	1975.0	79.2%
Property	6	515.0	26	1081.9	47.6%
Property Trust	2	208.8	16	4719.1	4.4%
Retailers & Wholesalers	10	6935.4	59	19769.5	35.1%
Steel & Allied	1	1043.0	4	5261.8	19.8%
Sugar	2	1583.6	3	2054.4	77.1%
Tin	2	6.4	2	6.4	100.0%
Tobacco & Match	6	24746.1	6	24746.1	100.0%
Transportation	2	2204.6	9	2752.6	80.1%
Venture Capital	0	0.0	5	23.4	0.0%
Totals	244	333551.4	696	501990.6	66.4%

3.3.2 Experimental design

The empirical research is divided into four sections. In the first section the sample dataset is analysed on a weekly and four-weekly basis over the entire period in order to examine the extent of the deviations from the normal distribution assumption. In the second section the characteristic exponents and trends in variance estimates for increasing sample sizes are examined to establish whether returns show evidence of being distributed as stable Paretian with characteristic exponent less than two. In the third section the sample dataset is broken down on the basis of trading volume to test the *imperfect clock* hypothesis of Clark (1973). Finally, stability tests are conducted of the mean and variance of security returns.

Six test statistics are computed under the null hypothesis that security returns are normally distributed. Consistent with previous research in the field, all tests are undertaken at the one and five percent levels of significance, and, given the increasing evidence of leptokurtic empirical distributions, one tailed tests are undertaken where appropriate. The statistics selected are chosen from the nine empirically tested for comparative sensitivity by Shapiro, Wilk and Chen (1968). They are the Studentized range, the standard third and fourth moments, the Chi-squared statistic, the Kolmogorov-Smirnov statistic, and the Shapiro-Wilk W statistic[14].

Studentized range statistic: This statistic, developed by David, Hartley and Pearson (1954), is computed by dividing the sample range by the standard deviation (Fama, 1977:36-40). Although found to be poor in the case of skewed distributions, the statistic is as robust as the Shapiro-Wilk W for symmetric long-tailed distributions (Shapiro, Wilk and Chen, 1968:1371). The statistic is given as;

$$SR = \frac{r_{\max} - r_{\min}}{s_r} \tag{3.2}$$

Third and Fourth moment statistics: As the names imply, these statistics are based on the standardized third and fourth moments about the sample mean. They are denoted by $\sqrt{b_1}$ and b_2 and given by (Pearson, 1963:95; Keeping, 1962:35);

$$\begin{aligned} \sqrt{b_1} &= \frac{m_3}{m_2^{3/2}} \\ b_2 &= \frac{m_4}{m_2^2} \end{aligned} \tag{3.3}$$

[14]The Shapiro-Wilk statistic was found by Shapiro, Wilk and Chen to provide a generally superior omnibus measure of nonnormality (1968:1366). The large sample sizes used in studies of this nature however, results in the unavailability of tables of normal order statistics (Harter, 1961:158-165; Pearson and Hartley, 1972:205-209) and explains the lack of use of the statistic in early research.

where; $m_r = \sum_{i=1}^{n}(x_i - \bar{x})^r / n$ is the r$^{\text{th}}$ moment about the sample mean computed from n observations.

D'Agostino and Pearson undertook extensive simulations to establish the probability integrals of b_2 for sample sizes of between twenty and two hundred drawn from a normal population. They also presented tables of coefficients for computing the normalized transformation of $\sqrt{b_1}$, from which the probability integral of $\sqrt{b_1}$ can be established using standardized normal probability tables, for sample sizes between eight and a thousand (1973:616-622).

In this study the sample sizes are nine hundred and eighty-eight for the weekly data and two hundred and forty-seven for the four-weekly data. As the graphs of D'Agostino and Pearson for b_2 could not therefore be used for the weekly or four-weekly data, their methodology was replicated to produce probability integrals of b_2 for sample sizes of fifty-two, one hundred, two hundred and forty-seven, and nine hundred and eighty-eight. Fifty thousand simulations of each sample size where generated[15]. Appendix 3.2 presents the results of the simulations study and compares them to the figures from the graphs of D'Agostino and Pearson. The consistency of the results for samples of size fifty-two and one hundred suggests they can be used to establish the critical values for samples consisting of two hundred and forty-seven and nine hundred and eighty-eight observations.

Chi-squared statistic: This goodness-of-fit test is used to determine the likelihood that a sample is drawn from a population that conforms to a specified type of probability distribution (Kohler, 1985:432-434). As with the previous test statistics, the probability of a Type II error is difficult to define since an alternative distribution is not explicitly specified. Large sample sizes therefore have to be relied upon to protect against a Type II error. The statistic is given by;

$$\chi^2_{df} = \sum \frac{(f_{oi} - f_{ei})^2}{f_{ei}} \tag{3.4}$$

where; df = c - d - 1 is the degrees of freedom for c classes and p estimated parameters; f_{oi} is the observed frequency in the i$^{\text{th}}$ cell; and, f_{ei} is the expected frequency in the i$^{\text{th}}$ cell.

[15]The simulations were run on the University of Cape Town VAX 6000-330 with the random numbers generated using NAG subroutines.

Each Chi-squared test in the study is undertaken using ten equiprobable cells (given the null is true) with the parameters μ and σ^2 being estimated from the sample. The number of degrees of freedom is therefore equal to seven in all cases.

Kolmogorov-Smirnov statistic: The Kolmogorov-Smirnov maximum deviation test compares the cumulative relative frequency distribution of the sample and the theoretical distribution pertaining to the null hypothesis (Kohler, 1985:479). The test statistic is given by;

$$D_{KS} = \max\left|F_o - F_e\right| \tag{3.5}$$

where; F_o is the observed cumulative relative frequency; and, F_e is the expected cumulative relative frequency.

Compared to the Chi-squared test, the Kolmogorov-Smirnov test has the principal disadvantage that the population parameters (under the null hypothesis) should not be estimated from the sample but established *a priori* (Kohler, 1985:482). As such the results presented for this test should be viewed as providing additional supportive evidence only.

Shapiro-Wilks W statistic: This omnibus statistic utilizes all the order statistics of the sample and has been found to dominate the Studentized range statistic (Affleck-Graves and McDonald, 1989:893). The statistic is given by;

$$W = \left(\sum_{i=1}^{n} a_{i,n} x_{(i)}\right)^2 \Bigg/ \sum_{i=1}^{n} \left(x_i - \bar{x}\right)^2 \tag{3.6}$$

where; $x_{(i)}$ is the i^{th} ordered observation; and $a_{i,n}$ is the i^{th} normalized coefficient tabulated in Shapiro and Wilk (1965).

Table 3.2 presents a summary of the test statistics and gives the ninety-five and ninety-nine percent confidence limits for the required sample sizes. Note that all the tests are single tailed except for the third moment statistic for which no prior evidence exists to suggest a consistent direction of skewness.

Table 3.2 Test statistics and critical values for testing at the 5% and 1% levels of significance for sample sizes of fifty-two, two hundred and forty-seven, and nine hundred and eighty-eight

Test Statistic	Sample Size	Critical value(s) 5%	1%	Source
Studentized Range	52	5.38	5.80	
(SR)	247	6.47	6.94	Fama (1977:40)
	988	7.32	7.79	
Third Moment	52	0.6363	0.8729	D'Agostino & Pearson (1973:621)
($\sqrt{b_1}$)	247	0.3031	0.4040	Pearson & Hartley (1972:156-159)
	988	0.1525	0.2011	
Fourth Moment	52	3.987	4.821	D'Agostino & Pearson (1973:616)
(b_2)	247	3.523	3.875	Simulated in current study
	988	3.265	3.408	
Chi-squared	52	14.067	18.475	
(χ^2)	247	14.067	18.475	Pearson & Hartley (1972:163)
	988	14.067	18.475	
Kolmogorov-Smirnov	52	0.1664	0.2067	
(D_{KS})	247	0.0865	0.1037	Kohler (1985:T-31)
	988	0.0433	0.0519	
Shapiro-Wilks	52	0.931	0.948	NAG Fortran Library,
(W)	247	0.971	0.975	Mark 15 (1991:G01DDF)
	988	0.981	0.983	

In the second section of the analysis two approaches are used to test the hypothesis that returns are distributed as stable Paretian with characteristic exponent less than two. Following the procedure of Fama (1965), Teichmoeller (1971) and Officer (1972), in the first approach the stability of the characteristic exponent is assessed under addition. If the sample is drawn from a stable Paretian distribution the characteristic exponent should remain constant under addition (Fama, 1963:426). This implies the estimates obtained using weekly data should be consistent with the estimates from four-weekly data. In the second approach the methodology of Perry (1983) is used to test if sample variance increases with sample size. If samples are drawn from a stable Paretian distribution, with characteristic exponent less than two, the true variance is infinite and the estimated variance should increase with increasing sample size (Perry, 1983:213).

The characteristic exponent and scale parameters are estimated from the appropriate fractiles of the sample and are given by (Fama and Roll, 1971:331-333);

$$\begin{aligned} \gamma &= \tfrac{1}{2}0.827\left(x_{0.72} - x_{0.28}\right) \\ \alpha_f &= G\left(f, z_f\right) \end{aligned} \tag{3.7}$$

where; γ is an estimate of the scale parameter; α_f is an estimate of the characteristic exponent; $x_{.f}$ is the $f(N+1)^{st}$ order statistic; $z_f = \left(x_{.f} - x_{1-.f}\right)/2\gamma$; and, G is a function that uniquely maps the fractile z_f and the cumulative probability f into α_f[16].

Appropriate tables for the function G are given in Fama and Roll (1968:822-823), and the suggested fractile to use in estimating the characteristic exponent is 0.97 (Fama and Roll, 1971:337; Blattberg and Gonedes, 1974:261).

For the full sample of two hundred and forty-four securities[17] and for the subgroup of forty-five well traded securities, ordinary least squares regressions are run with the weekly characteristic exponent estimates as the independent variables and the four-weekly estimates as the dependent variables. If the data are consistent with the stable Paretian hypothesis, the true regression lines must pass through the origin and have beta coefficients of one. Beta estimates significantly greater than one indicate that the exponent increases under addition and are inconsistent with the hypothesis. A nonparametric binomial test is also used to test the proportion of the estimates that increase under addition. Under the null hypothesis of no change in the true value, the sign of the difference in the weekly and four-weekly estimates is distributed as a binomial with a probability of being positive equal to fifty percent.

Tests for evidence of a trend in variance estimates, as sample sizes increase, are undertaken using the weekly data. The time series of observations for each security is used to calculate nineteen variance estimates using sub-samples ranging in size from fifty-two observations to the full series in increments of fifty-two. Ordinary least squares regressions of variance estimate against sample size are then carried out and the beta estimate and standard error of the beta estimate computed to test if the true beta is significantly greater than zero. A nonparametric binomial test is again also employed to supplement this test. To avoid drawing incorrect conclusions, the procedure is carried out

[16]Note that the notation here is different to that used for the Shapiro-Wilks W statistic because of the distinct difference in definition. $x_{(i)}$ refer to the i[th] ordered observation, while $x_{.f}$ is the $f(N+1)^{st}$ order statistic.

[17]Securities for which the characteristic exponent could not be estimated because of both the extreme level of thin trading and because the estimates fell outside the range of values provided by Fama and Roll (1968) were excluded from the analysis.

by constructing the increasing size samples using the data in chronological order, in reverse order, and by first randomizing the series. As pointed out by Perry, any trend present as a result of changes in the true variance through time can be isolated in this fashion (1983:217-218). If the data is truly stable Paretian with characteristic exponent less than two, all three of the regressions should have significantly positive slopes.

In contrast to Mandelbrot's proposal that the violation of the central limit theorem is due to returns not having finite variance, the third section of the analysis examines the possibility that the reason for the inapplicability of the theorem is a varying rate of price evolution through time. Following the methodology of Clark (1973:143), each security's time series of returns is first sorted by trading volume and then broken down into nineteen equal sized sub-samples of fifty-two[18]. Tests for normality are undertaken on each sub-sample using the same test statistics as employed in the first section of the analysis. The major assumption in this phase of the analysis is that trading volume can be used as a surrogate measure of the rate of price evolution. This opens the possibility that rejection of the null hypothesis that the sub-samples are normally distributed can be due the selection of an inappropriate surrogate rather than because of a failing of the actual underlying subordinated stochastic model.

The fourth and final section of the analysis examines the stationarity of both the mean and variance of returns over the nineteen year period. In preference to arbitrarily dividing the data series into sub-samples, within which it is presumed that the stationarity assumption is valid, and then comparing across sub-samples, a simple non-parametric procedure is used. The stationarity of the mean for each security is assessed by first computing the deviation of each week's return from the overall nineteen year average. A non-parametric runs test of the sign of the deviation is then undertaken. Evidence of non-stationarity is reflected in unacceptably long positive (periods of higher than average return) and negative (periods of lower than average return) runs. The test of variance stationarity for each security is conducted in a similar fashion except that the weekly absolute deviations are computed and the compared to the median absolute deviation to establish the sign of the "deviation". Although the sampling distribution of the number of runs follows the hypergeometric distribution it can be approximated by the normal distribution (Kohler, 1985:474). If n^+ and n^- are defined as the number of positive and negative signs respectively then, under the assumption that the consecutive signs are random, the

[18]Although most of the analysis conducted in this research utilised nine hundred and eighty-eight weeks of return data, nine hundred and ninety-five weeks of returns were available. This section of the analysis only utilised data for weeks within which trading actually took place. As a result, even with the extra seven observations, not all of the forty-five securities included nineteen sub-samples.

sampling distribution for the number of positive runs can be approximated as normal with an expected value and standard deviation given by[19];

$$\begin{aligned} \mu_{runs} &= \frac{n^+\left(n^- + 1\right)}{n^+ - n^-} \\ \sigma_{runs} &= \sqrt{\frac{n^+ n^-\left(n^+ - 1\right)\left(n^- + 1\right)}{\left(n^+ + n^-\right)^2\left(n^+ + n^- - 1\right)}} \end{aligned} \tag{3.8}$$

3.3 Results of the analysis

Tables 3.3 and 3.4 present the basic descriptive statistics for the weekly and four-weekly data and the results of the tests for normality over the entire period examined. Only one of the securities, namely Messina Limited, had a negative mean return between February 20, 1973 and March 13, 1992. For all forty-five securities the average mean and average standard deviation were 0.32% and 6.01% for the weekly data, and 1.29% and 12.83% for the four-weekly data. The independence of successive returns is confirmed by these results since the four-weekly average standard deviation is, as required, approximately twice (2.136) the weekly average standard deviation.

The high proportion of securities for which the null hypothesis of normality has to be rejected strongly supports the international research into speculative returns. When testing the weekly data at the one percent level of significance, thirty-eight, twenty, forty-four, forty-one, twenty-nine, and thirty-one out of the forty-five securities were found to be significantly nonnormal using the Studentized range, third and fourth moments, Chi-squared, Kolmogorov-Smirnov, and Shapiro-Wilk *W* statistics respectively. The corresponding figures for the four-weekly data are thirty-two, nineteen, thirty-two, eight, two, and nine. The decline in the presence of nonnormality from the weekly to four-weekly datasets is in agreement with the results for the component securities of the Dow Jones Industrial Index and suggests that the stable Paretian hypothesis is untenable since, as stated by Fama, this evidence is *contrary to the implications of the of the hypothesis that ... returns conform roughly to the same type of stable nonnormal distribution* (Fama, 1977:33). There is substantial evidence however, that four-weekly returns are still distributed in a more leptokurtic fashion than the normal distribution with forty-four

[19]The (unlikely) possibility of zero deviations was handled by assuming them to be part of any existing run. For instance, a zero deviation between two negative deviations is registered as a negative run while one at the end of a positive run is assumed to be part of that positive run.

having fourth moments that are higher than the median value obtained using the methodology of D'Agostino and Pearson (1973)[20].

The high proportion of the securities that exhibited negative skewness over the period studied (twenty-eight weekly; thirty-seven four-weekly), and the number for which the direction of skewness changed from the weekly to four-weekly tests (nine) provides some support for the findings of Lau, Wingender and Lau (1989:1142) concerning skewness reliability, and the appropriateness of using a two-tailed test for normality.

[20]The median value obtained on the basis of the fifty thousand simulations for sample sizes of two hundred and forty-seven was 2.95.

Table 3.3 Descriptive statistics and the results of the tests of normality using weekly returns over the period February 20, 1973 to March 13, 1992

	$\bar{r}$	σ_r	γ	α_f	SR	$\sqrt{b_1}$	b_2	χ^2	D_{KS}	W
NEDCOR	0.0025	0.0437	0.016	1.300	11.26**	-0.21**	6.78**	143.8**	0.092**	0.965**
SA-BREWS	0.0042	0.0397	0.015	1.332	10.10**	-0.17*	5.59**	37.4**	0.066**	0.981**
AECI	0.0020	0.0424	0.014	1.263	14.70**	-0.47**	12.92**	217.2**	0.102**	0.948**
AMCOAL	0.0041	0.0426	0.015	1.293	8.19**	-0.15	4.89**	89.0**	0.095**	0.970**
TRANS-NTL	0.0036	0.0480	0.018	1.352	9.50**	0.20*	5.09**	51.1**	0.066**	0.980**
PALAMIN	0.0044	0.0455	0.015	1.313	13.26**	0.58**	8.93**	133.0**	0.093**	0.965**
DEBEERS	0.0031	0.0445	0.016	1.352	12.73**	-0.63**	9.40**	58.2**	0.062**	0.966**
METKOR	0.0028	0.0622	0.015	1.138	12.29**	-0.21**	10.97**	943.7**	0.190**	0.885**
DRIES	0.0037	0.0538	0.024	1.445	6.51	-0.02	3.41*	9.9	0.031	0.985
E-T-CONS	0.0048	0.0769	0.028	1.325	8.05**	0.10	4.44**	286.5**	0.119**	0.972**
ELSBURG	0.0004	0.0733	0.032	1.465	7.72*	0.01	3.93**	45.0**	0.052**	0.986
GROOTVLEI	0.0033	0.0795	0.032	1.398	9.63**	0.30**	5.42**	39.6**	0.043*	0.980**
HARMONY	0.0023	0.0629	0.027	1.427	7.69*	0.08	3.78**	16.6*	0.039	0.987
KINROSS	0.0037	0.0631	0.029	1.469	7.57*	-0.15	3.56**	24.2**	0.040	0.988
KLOOF	0.0035	0.0614	0.024	1.363	8.83**	-0.10	4.27**	26.6**	0.042	0.988
LESLIE	0.0032	0.0709	0.030	1.406	8.58**	-0.02	4.25**	52.2**	0.054**	0.987
LIBANON	0.0013	0.0738	0.031	1.421	8.54**	0.28**	4.69**	33.9**	0.047*	0.982*
LORAINE	0.0012	0.0844	0.037	1.452	13.22**	-0.11	7.75**	39.2**	0.048*	0.984
RANDFONTN	0.0043	0.0589	0.023	1.344	7.49*	0.03	4.16**	32.4**	0.050*	0.978**
RD-LEASE	0.0000	0.1070	0.039	1.365	10.68**	-0.15	6.87**	206.2**	0.095**	0.961**
SOUTHVAAL	0.0035	0.0592	0.025	1.437	8.02**	-0.10	3.80**	13.1	0.031	0.990
VAAL-REEF	0.0037	0.0536	0.023	1.412	9.01**	-0.09	4.17**	25.0**	0.035	0.991
VENTERS	0.0020	0.0875	0.038	1.434	8.04**	0.09	4.59**	56.8**	0.046*	0.979**
VILLAGE	0.0046	0.0980	0.038	1.349	9.41**	0.29**	5.52**	180.6**	0.101**	0.968**
W-R-CONS	0.0027	0.0911	0.033	1.316	8.65**	0.32**	5.37**	111.3**	0.078**	0.963**
WELKOM	0.0031	0.0643	0.028	1.457	8.82**	0.07	4.40**	27.7**	0.045*	0.988
WES-AREAS	0.0007	0.0738	0.031	1.420	8.63**	0.01	4.32**	28.4**	0.041	0.987
ZANDPAN	0.0039	0.0559	0.024	1.392	7.62*	0.11	3.94**	84.8**	0.080**	0.982*
AMIC	0.0031	0.0387	0.013	1.260	10.70**	-0.38**	7.56**	166.6**	0.110**	0.944**
BARLOWS	0.0033	0.0393	0.016	1.410	12.49**	-0.72**	10.25**	26.5**	0.041	0.973**
CGSMITH	0.0040	0.0423	0.012	1.219	18.80**	-0.49**	24.08**	235.8**	0.120**	0.886**
MALBAK	0.0028	0.0541	0.019	1.288	10.05**	-0.17*	5.58**	78.3**	0.077**	0.973**
MESSINA	0.0001	0.0761	0.028	1.380	14.43**	-0.44**	12.67**	97.0**	0.072**	0.947**
PLATE-GL	0.0032	0.0433	0.014	1.216	8.68**	-0.23**	6.01**	206.0**	0.104**	0.948**
AMGOLD	0.0031	0.0463	0.020	1.409	7.97**	-0.03	3.85**	17.8*	0.037	0.988
GENBEL	0.0038	0.0533	0.021	1.384	9.42**	-0.35**	5.96**	116.7**	0.077**	0.969**
MID-WITS	0.0049	0.0644	0.016	1.086	6.93	0.10	3.95**	704.0**	0.170**	0.945**
NEW-WITS	0.0033	0.0617	0.023	1.320	13.24**	0.10	7.63**	101.2**	0.088**	0.979**
R-M-PROPS	0.0030	0.0613	0.020	1.223	9.23**	0.34**	5.72**	203.3**	0.096**	0.957**
ANGLO-AM	0.0036	0.0437	0.017	1.372	10.46**	-0.27**	6.01**	41.4**	0.050*	0.980**
GFSA	0.0036	0.0535	0.022	1.386	10.65**	-0.37**	6.15**	48.1**	0.056**	0.981**
JOHNNIES	0.0063	0.0458	0.015	1.242	12.84**	-0.74**	10.41**	136.6**	0.083**	0.958**
TONGAAT	0.0026	0.0456	0.015	1.262	8.89**	-0.13	5.82**	112.1**	0.078**	0.962**
REMBR-BEH	0.0059	0.0619	0.007	1.000	9.89**	-0.27**	7.44**	1228.3**	0.219**	0.866**
REMGRO	0.0060	0.0531	0.007	1.000	11.49**	-0.05	7.23**	1062.4**	0.191**	0.904**
	0.0032	0.0601	0.022	1.331		-0.09	6.52			

** : significant at the 1% level * : significant at the 5% level

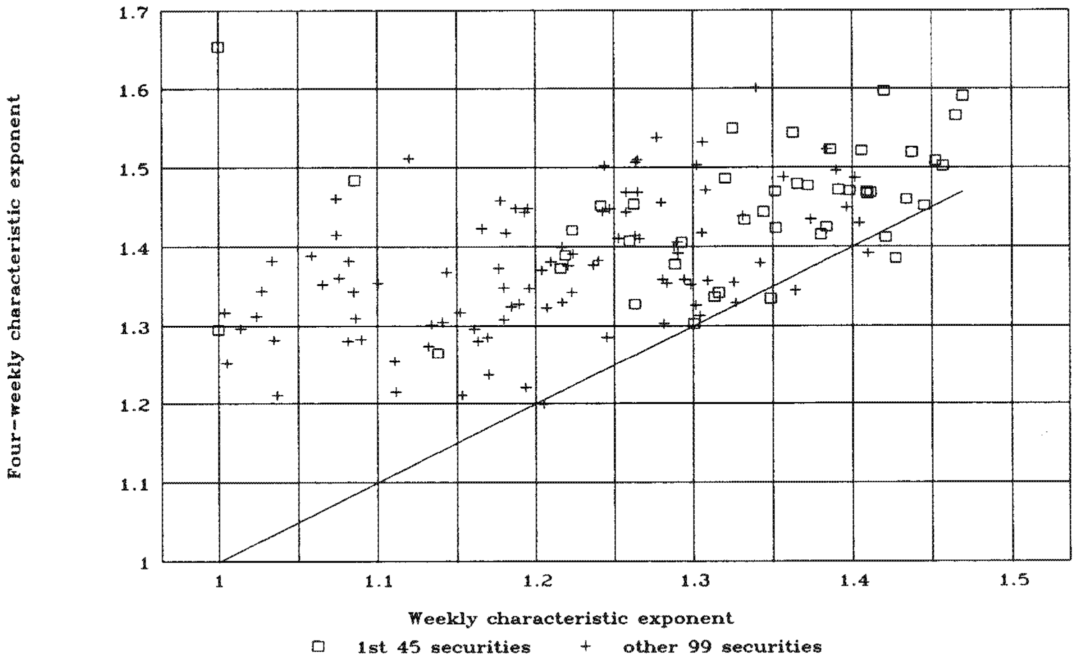
Table 3.4 Descriptive statistics and the results of the tests of normality using four-weekly returns over the period February 20, 1973 to March 13, 1992

	$\bar{r}$	σ_r	γ	α_f	SR	$\sqrt{b_1}$	b_2	χ^2	D_{KS}	W
NEDCOR	0.0100	0.0883	0.032	1.303	7.51**	-0.56**	4.92**	18.2*	0.090*	0.977
SA-BREWS	0.0168	0.0819	0.035	1.434	7.79**	-0.71**	5.48**	8.3	0.054	0.979
AECI	0.0081	0.0854	0.033	1.328	7.54**	-0.08	4.32**	14.0	0.066	0.989
AMCOAL	0.0163	0.0867	0.038	1.406	6.02	-0.35*	3.85*	4.9	0.055	0.976
TRANS-NTL	0.0142	0.0940	0.042	1.424	5.65	-0.04	3.48	17.1*	0.064	0.978
PALAMIN	0.0177	0.0888	0.034	1.337	5.84	-0.10	3.52	8.0	0.063	0.979
DEBEERS	0.0124	0.1015	0.043	1.470	9.30**	-1.50**	12.16**	15.0*	0.070	0.935**
METKOR	0.0111	0.1127	0.040	1.265	7.47**	-0.31*	5.53**	40.6**	0.095*	0.963**
DRIES	0.0148	0.1099	0.047	1.453	6.54*	-0.21	4.27**	5.4	0.045	0.979
E-T-CONS	0.0194	0.1588	0.074	1.550	7.21**	-0.15	4.03**	11.6	0.063	0.991
ELSBURG	0.0017	0.1588	0.081	1.567	6.59*	-0.17	3.48	11.7	0.039	0.989
GROOTVLEI	0.0132	0.1711	0.074	1.471	7.23**	0.26	4.83**	14.3*	0.058	0.978
HARMONY	0.0091	0.1387	0.060	1.386	6.14	0.00	3.41	8.6	0.060	0.985
KINROSS	0.0150	0.1420	0.072	1.591	5.94	-0.03	2.90	6.5	0.035	0.988
KLOOF	0.0142	0.1215	0.057	1.544	7.23**	-0.44**	4.41**	11.5	0.041	0.985
LESLIE	0.0128	0.1504	0.069	1.522	6.80*	-0.03	3.48	9.2	0.057	0.986
LIBANON	0.0052	0.1627	0.070	1.412	5.97	0.08	3.73*	5.4	0.052	0.982
LORAINE	0.0050	0.1931	0.093	1.509	7.03**	-0.13	3.84*	5.2	0.041	0.992
RANDFONTN	0.0173	0.1322	0.057	1.444	7.33**	0.13	4.27**	6.2	0.042	0.988
RD-LEASE	0.0000	0.2183	0.099	1.480	6.89*	-0.01	3.87*	12.1	0.071	0.987
SOUTHVAAL	0.0139	0.1373	0.064	1.520	6.83*	-0.09	3.75*	6.6	0.045	0.990
VAAL-REEF	0.0149	0.1171	0.050	1.469	6.82*	-0.40**	4.85**	11.0	0.049	0.971*
VENTERS	0.0080	0.2041	0.088	1.461	7.30**	-0.14	4.44**	10.2	0.055	0.987
VILLAGE	0.0182	0.1907	0.071	1.335	8.32**	0.43**	6.05**	18.1*	0.069	0.974*
W-R-CONS	0.0108	0.2154	0.083	1.343	8.10**	0.33*	5.07**	22.7**	0.059	0.984
WELKOM	0.0123	0.1430	0.065	1.503	7.21**	0.01	3.69*	2.6	0.034	0.997
WES-AREAS	0.0030	0.1671	0.082	1.597	6.99**	-0.26	4.09**	28.7**	0.070	0.979
ZANDPAN	0.0158	0.1167	0.049	1.472	6.95**	-0.09	4.41**	8.6	0.046	0.980
AMIC	0.0125	0.0825	0.033	1.408	8.03**	-0.99**	6.89**	15.5*	0.082	0.963**
BARLOWS	0.0134	0.0796	0.035	1.467	8.99**	-0.66**	6.75**	12.4	0.045	0.980
CGSMITH	0.0159	0.0937	0.032	1.390	11.34**	-1.36**	14.91**	22.0**	0.083	0.929**
MALBAK	0.0113	0.1110	0.043	1.378	8.28**	-0.60**	6.08**	15.7*	0.057	0.978
MESSINA	0.0002	0.1671	0.064	1.416	11.67**	-1.51**	17.16**	22.0**	0.069	0.927**
PLATE-GL	0.0129	0.0983	0.040	1.374	8.08**	-0.67**	5.35**	9.2	0.077	0.978
AMGOLD	0.0123	0.1030	0.046	1.470	6.76*	-0.22	4.04**	8.3	0.059	0.984
GENBEL	0.0150	0.1218	0.054	1.425	7.80**	-0.58**	5.33**	6.8	0.065	0.977
MID-WITS	0.0196	0.1267	0.059	1.485	6.02	-0.16	3.16	54.2**	0.114**	0.983
NEW-WITS	0.0132	0.1379	0.056	1.487	10.34**	-0.31*	7.99**	17.0*	0.056	0.975
R-M-PROPS	0.0118	0.1356	0.055	1.421	8.64**	0.26	5.72**	16.3*	0.081	0.978
ANGLO-AM	0.0142	0.1013	0.043	1.477	9.22**	-0.73**	7.59**	11.8	0.047	0.976
GFSA	0.0142	0.1261	0.055	1.524	7.79**	-0.77**	6.56**	13.1	0.056	0.964**
JOHNNIES	0.0251	0.1066	0.045	1.452	9.77**	-1.35**	11.18**	12.7	0.066	0.953**
TONGAAT	0.0103	0.0900	0.039	1.454	7.94**	-1.00**	6.80**	11.3	0.080	0.959**
REMBR-BEH	0.0237	0.1022	0.037	1.296	7.26**	-0.48**	4.85**	112.4**	0.139**	0.962**
REMGRO	0.0239	0.1004	0.052	1.654	8.19**	-0.59**	5.82**	54.1**	0.100*	0.974*
	0.0129	0.1283	0.055	1.448		-0.36	5.61			

** : significant at the 1% level * : significant at the 5% level

Besides the evidence of the trend to normality as returns are measured over longer periods, further support for the rejection of the stable Paretian hypothesis is found by examining the characteristic exponent estimates. Forty-two of the forty-five securities have higher estimates for the four-weekly data than for the weekly data. Using the binomial test, this is sufficient evidence to reject the hypothesis that there is no change in the characteristic exponent, with a greater than one percent level of significance. This is illustrated graphically in figure 3.2.

Figure 3.2 Comparison of four-weekly characteristic exponents plotted against weekly characteristic exponents for sample of forty-five well traded securities and an additional ninety-nine securities



The majority of the paired characteristic exponent estimates plot well above the forty-five degree line extending from the origin. An ordinary least squares regression produces a coefficient of determination of 0.100, a beta estimate of 0.234, and a standard error of beta equal to 0.107. This implies the hypothesis that the true beta is equal to one can be rejected with a greater than one percent level of significance. When all the securities for which estimates of the characteristic exponent could be obtained are examined, an identical conclusion is reached[21]. These results are also presented in the figure. In

[21]The characteristic exponent for one hundred and forty-four securities could be estimated using the procedure of Fama and Roll (1971) and of these one hundred and thirty-eight had greater four-weekly exponents. The regression results yielded a coefficient of determination of 0.288, a beta estimate of

addition to the evidence of the change in characteristic exponent, tables 3.3 and 3.4 show that the average scale parameter increases by a multiple of 2.469 and not 4.000 as required if the data is distributed as stable Paretian (Fama, 1965:44).

Table 3.5 gives the results of the regressions of variance estimate against sample size for each of the forty-five securities. The beta estimates and standard errors of beta are presented for regressions run with the sample size increasing by following a forward time sequence, a backward time sequence, and random time sequence. The critical t-statistic for testing at the one percent level of significance, given eight degrees of freedom, is 2.896 and the table shows that five of the forward, twelve of the backward, and thirteen of the random sequence slopes are significantly positive. While this represents a fairly high number of slopes it is important to note that none of the securites exhibit a significantly positive slope across all three regressions, as would be expected if the samples were drawn from stable Paretian distributions with characteristic exponents less than two.

A binomial test of the sign of the slopes also leads to a rejection of the infinite variance hypothesis. The number of positive slopes for all three regression sequences do not exceed the critical numbers of twenty-eight (at the five percent level of significance) and thirty (at the one percent level of significance) out of forty-five required in order to reject the null hypothesis that the probability of a positive slope is 0.5. The regression tests indicate therefore that security returns seem to conform to a distribution with finite variance, but that this variance changes with time.

0.400, and a standard error of beta equal to 0.053.

Table 3.5 Beta estimates and standard errors of beta for regressions against sample size

	Forward sequence		Backward sequence		Random sequence	
	$\hat{\beta}$	$se(\hat{\beta})$	$\hat{\beta}$	$se(\hat{\beta})$	$\hat{\beta}$	$se(\hat{\beta})$
NEDCOR	-0.0605	0.0179	0.0241*	0.0108	-0.0111	0.0038
SA-BREWS	-0.0197	0.0063	-0.0005	0.0075	-0.0081	0.0053
AECI	0.0199*	0.0080	-0.0024	0.0107	0.0209**	0.0057
AMCOAL	-0.0052	0.0071	0.0502**	0.0066	0.0077*	0.0042
TRANS-NTL	-0.0399	0.0136	-0.0063	0.0105	-0.0039	0.0053
PALAMIN	0.0037	0.0082	0.0571**	0.0058	0.0156	0.0125
DEBEERS	-0.0280	0.0122	0.0155	0.0174	-0.0136	0.0077
METKOR	0.1126**	0.0133	-0.0193	0.0277	0.0510*	0.0208
DRIES	-0.0516	0.0065	0.0028	0.0046	-0.0113	0.0032
E-T-CONS	-0.3679	0.0249	-0.0067	0.0359	0.0558**	0.0151
ELSBURG	0.0151	0.0098	-0.1065	0.0182	-0.0123	0.0081
GROOTVLEI	-0.1159	0.0281	-0.3501	0.0585	-0.0506	0.0102
HARMONY	-0.0772	0.0092	-0.0480	0.0208	0.0149**	0.0055
KINROSS	-0.0856	0.0092	0.0037	0.0140	0.0033	0.0096
KLOOF	-0.0643	0.0113	0.0026	0.0086	0.0501**	0.0101
LESLIE	-0.1365	0.0148	-0.0756	0.0165	-0.0589	0.0181
LIBANON	-0.0688	0.0259	-0.3270	0.0681	0.0406*	0.0213
LORAINE	-0.1106	0.0235	-0.2343	0.0668	-0.0133	0.0186
RANDFONTN	-0.1855	0.0256	-0.0677	0.0156	-0.0448	0.0132
RD-LEASE	-0.2888	0.0628	-0.5760	0.1330	-0.2222	0.0240
SOUTHVAAL	-0.1091	0.0110	-0.0445	0.0171	-0.0306	0.0053
VAAL-REEF	-0.0939	0.0136	-0.0436	0.0081	0.0316**	0.0071
VENTERS	-0.0710	0.0237	-0.2090	0.0555	-0.0459	0.0134
VILLAGE	-0.5357	0.0522	0.0807*	0.0333	-0.0468	0.0103
W-R-CONS	-0.3403	0.0535	-0.2149	0.0368	-0.0148	0.0149
WELKOM	-0.1212	0.0141	0.0107	0.0159	0.0346**	0.0061
WES-AREAS	0.0192*	0.0102	-0.1318	0.0165	0.0502**	0.0108
ZANDPAN	-0.1596	0.0162	-0.0130	0.0104	0.0024	0.0120
AMIC	0.0389**	0.0057	0.0150	0.0098	0.0001	0.0050
BARLOWS	-0.0098	0.0069	-0.0077	0.0079	-0.0036	0.0061
CGSMITH	0.0346**	0.0072	0.0450*	0.0200	0.0327**	0.0113
MALBAK	0.0366**	0.0096	0.0457*	0.0244	-0.0155	0.0076
MESSINA	0.1628**	0.0248	-0.0368	0.0726	-0.0322	0.0138
PLATE-GL	0.0033	0.0078	0.0313**	0.0078	0.0155*	0.0070
AMGOLD	-0.0787	0.0110	0.0195**	0.0043	0.0183**	0.0029
GENBEL	-0.0545	0.0126	0.0218**	0.0072	-0.0082	0.0053
MID-WITS	-0.0923	0.0224	0.0706**	0.0125	-0.0287	0.0098
NEW-WITS	-0.1213	0.0278	0.0563**	0.0177	-0.0240	0.0122
R-M-PROPS	-0.1313	0.0249	0.0835**	0.0072	0.0624**	0.0159
ANGLO-AM	-0.0502	0.0116	0.0220*	0.0101	-0.0003	0.0029
GFSA	-0.0361	0.0089	0.0311**	0.0113	-0.0452	0.0171
JOHNNIES	-0.0404	0.0139	-0.0132	0.0127	0.0096**	0.0025
TONGAAT	0.0183*	0.0101	0.0389**	0.0061	-0.0064	0.0045
REMBR-BEH	-0.2833	0.0358	0.0944**	0.0153	0.0474**	0.0097
REMGRO	-0.1315	0.0299	0.0507**	0.0110	0.0446**	0.0134

** : significant at the 1% level   * : significant at the 5% level

The regression equation used is given by $s_i^2 = \hat{\alpha} + \hat{\beta} SIZE_i + e_i$, where; s_i^2 is the i^{th} variance estimate and $SIZE_i$ is the sample size used to estimate s_i^2.

Table 3.6 Summary statistics and the number of times each security exhibits returns significantly different from normal using each statistic when the sample is broken into nineteen groups of fifty-two on the basis of trading volume

	Trading volume deciles (000's)				Number significant at the 5% level						Number significant at the 1% level					
	1st	median	9th	No	SR	$\sqrt{b_1}$	b_2	χ^2	D_{KS}	W	SR	$\sqrt{b_1}$	b_2	χ^2	D_{KS}	W
NEDCOR	25148	73526	240763	18	10	10	15	8	0	9	3	8	7	4	0	9
SA-BREWS	66971	151663	327199	18	4	6	7	4	0	3	2	2	3	1	0	2
AECI	7605	22800	81460	18	9	6	10	9	4	7	6	4	7	7	1	5
AMCOAL	2664	11626	34218	18	6	5	9	7	0	5	6	2	5	4	0	2
TRANS-NTL	16399	52656	159391	18	5	3	6	1	0	3	3	2	5	0	0	1
PALAMIN	3000	11254	46900	18	8	7	8	4	0	8	5	4	6	3	0	5
DEBEERS	90224	219270	694625	19	6	6	8	1	0	7	3	5	5	0	0	4
METKOR	5600	24883	135983	18	15	11	17	18	11	17	10	8	15	17	7	15
DRIES	23162	65684	242800	19	4	1	3	1	0	2	0	1	1	1	0	0
E-T-CONS	36000	146000	556000	18	6	0	5	13	1	3	2	0	3	12	1	1
ELSBURG	32257	122068	336883	19	5	1	5	2	0	0	1	0	0	0	0	0
GROOTVLEI	16350	54200	159633	18	8	1	9	5	0	3	5	0	5	1	0	2
HARMONY	5149	22429	72260	18	4	3	4	1	0	2	2	0	1	0	0	0
KINROSS	3300	13300	48600	18	6	3	6	2	0	2	2	1	0	0	0	0
KLOOF	17400	59200	173112	19	4	3	6	2	0	4	2	3	3	0	0	2
LESLIE	24700	86950	269260	18	6	4	8	1	0	4	4	2	3	0	0	1
LIBANON	4200	13840	80750	19	5	7	6	3	0	3	1	2	2	0	0	1
LORAINE	22100	78800	232520	19	4	5	5	1	1	5	4	5	4	1	0	2
RANDFONTN	31000	94060	260370	19	4	5	5	2	1	2	2	0	2	1	0	0
RD-LEASE	4200	31600	149400	18	8	6	7	8	1	7	6	3	7	6	0	4
SOUTHVAAL	8350	22320	56182	19	2	3	3	2	0	1	2	0	2	0	0	0
VAAL-REEF	2925	12720	42650	19	4	5	6	2	0	5	0	4	2	0	0	2
VENTERS	9200	27750	100000	19	6	3	6	3	0	2	0	2	2	1	0	2
VILLAGE	9500	49700	181300	18	9	6	11	6	0	5	4	5	5	3	0	3
W-R-CONS	4100	14800	60860	18	10	7	9	0	0	5	6	5	6	0	0	5
WELKOM	2620	14600	51176	18	7	3	7	0	0	0	3	0	3	0	0	0
WES-AREAS	13266	48005	153888	19	4	4	6	3	0	2	2	2	2	0	0	2
ZANDPAN	66000	206180	592320	18	5	5	5	2	0	2	4	0	4	2	0	0
AMIC	3543	11548	42942	18	12	9	12	10	1	9	10	5	10	5	0	7
BARLOWS	59552	126371	306750	19	7	4	6	1	0	3	5	2	4	0	0	1
CGSMITH	1700	8347	28748	18	11	9	13	10	2	11	7	6	9	7	1	10
MALBAK	20357	57648	159343	18	5	4	7	5	0	7	4	3	5	2	0	5
MESSINA	5010	21700	85510	18	8	6	10	2	0	4	5	4	6	0	0	3
PLATE-GL	1500	10300	46437	18	7	8	11	8	0	12	5	5	8	6	0	4
AMGOLD	1475	5933	28693	18	4	0	4	0	0	0	2	0	2	0	0	0
GENBEL	48000	185000	534520	19	8	5	7	4	0	5	4	4	5	2	0	3
MID-WITS	97500	379485	1216600	18	6	0	8	17	10	13	3	0	3	16	3	12
NEW-WITS	6699	16174	44500	18	6	7	7	7	0	8	5	5	5	6	0	6
R-M-PROPS	2452	9900	41223	18	9	5	11	8	0	5	3	1	4	3	0	4
ANGLO-AM	28321	79511	219522	19	6	7	9	0	0	8	2	2	6	0	0	3
GFSA	6535	25650	77805	18	4	3	5	1	0	2	4	1	4	0	0	1
JOHNNIES	15100	62000	150410	18	7	4	10	7	1	4	4	2	5	3	0	2
TONGAAT	8850	30073	103335	18	8	2	7	10	0	5	6	0	7	3	0	2
REMBR-BEH	84000	406500	1297200	18	13	9	16	18	14	16	7	6	12	17	11	15
REMGRO	169950	594000	1756500	18	8	7	13	16	11	15	3	2	6	16	9	12
				824	303	218	358	235	58	245	169	118	211	150	33	160

The results of the tests of normality, relating to the subordinated stochastic process with finite variance theory of Clark (1973), are presented in table 3.6. The wide range of weekly trading volumes for each security can be seen by examining the first decile, median and ninth decile of the volume distributions. If, as proposed by Clark, volume can be used as a surrogate for the rate of price evolution, this would suggest that prices do evolve at a considerably changing rate. The last twelve columns of table 3.6 relate to the tests carried out on each of the nineteen sub-samples when the security returns series are broken down into groups of fifty-two on the basis of trading volume[22]. The columns give the number of times the null hypothesis of normally distributed sub-samples had to be rejected at the five percent and one percent levels of significance on the basis of each test statistic. While there is still considerable evidence of nonnormality remaining in the weekly data after accounting for transaction volume, the results show a significant reduction relative to table 3.3. For the Studentized range, third and fourth moments, Chi-squared, Kolmogorov-Smirnov, and Shapiro-Wilks W statistics, the number of securities that have two or less of the nineteen sub-samples showing significant departures from normality when testing at the one percent level of significance is fifteen, twenty-six, eleven, twenty-seven, forty-one, and twenty-five, while the number that have more than five of the sub-samples showing significant departures from normality is only nine, four, fourteen, ten, three, and eight respectively. In general the results indicate some support for the central hypothesis of Clark that the distribution of security returns measured over constant calendar time intervals is subordinate to the normal distribution. However, contrary to the findings of Clark, the data suggests trading volume is not that good a surogate for the rate of price evolution.

The results of the non-parametric tests of stationarity for both the mean and variance of security returns are presented in table 3.7. The evidence supports the contention that security returns exhibit non-stationary behaviour with respect to both mean and variance. Although not directly reported in this study, evidence was found to suggest that trading volume is itself variable through time. Securities appear to have periods of low trading volume, and trading variability, and periods of relatively higher trading volume accompanied by higher trading variability. This increased volatility, if translated into the speed of price evolution, supports the notion of variance non-stationarity. The above argument is also supported by the dominance of negative z-scores for both mean and variance in the table. Negative scores result from longer runs than randomness would

[22]Due to limited thin trading in the selected data, weeks during which no trading occured were present for certain of the securities. The lowest trading volume sub-group for each security was not included in the estimation unless trading occured in all fifty-two weeks.

suggest while positive scores are caused by too frequent an occurrence of sign reversals. Table 3.7 is therefore consistent with the proposal of Hsu, Miller and Wichern (1974) that speculative asset returns are subject to non-stationary variance subject to step changes.

Table 3.7 Non-parametric runs tests for mean and variance stationarity

	$z_{\bar{r}}$	$z_{s_r^2}$		$z_{\bar{r}}$	$z_{s_r^2}$
NEDCOR	-1.6238	-1.9713*	VILLAGE	-0.3986	-2.9883**
SA-BREWS	-1.0759	-1.5899	W-R-CONS	-2.3186*	-2.9890**
AECI	-1.2122	-3.7522**	WELKOM	-2.8415**	-1.7171
AMCOAL	-2.6519**	-4.0066**	WES-AREAS	-2.1067*	-0.1908
TRANS-NTL	-0.4705	-3.1160**	ZANDPAN	0.2380	-1.5892
PALAMIN	-2.0300*	-1.8428	AMIC	-3.1713**	-2.4800*
DEBEERS	-1.4603	-3.6250**	BARLOWS	-0.8215	-1.9715*
METKOR	-3.3486**	-5.5327**	CGSMITH	-3.9387**	-3.1148**
DRIES	-0.5585	-1.5897	MALBAK	0.9584	-3.2434**
E-T-CONS	-2.1735*	-3.3683**	MESSINA	-1.2421	-0.6996
ELSBURG	-2.1868*	1.0811	PLATE-GL	-3.2560**	-2.2252*
GROOTVLEI	-2.6389**	-0.1908	AMGOLD	-0.0366	-3.6250**
HARMONY	-1.2886	-2.3528*	GENBEL	-3.2694**	-1.7171
KINROSS	-3.1949**	-1.7171	MID-WITS	-2.5630*	-2.4803*
KLOOF	0.7399	-2.3531*	NEW-WITS	-3.0130**	-2.0984*
LESLIE	-1.5800	0.1916	R-M-PROPS	-2.7249**	-3.3704**
LIBANON	-2.5772**	-2.3531*	ANGLO-AM	-1.3348	-1.4625
LORAINE	-2.6489**	-0.9539	GFSA	-3.6004**	-4.0064**
RANDFONTN	-2.9867**	-1.4625	JOHNNIES	-5.2031**	-3.6250**
RD-LEASE	-1.1683	-2.6039**	TONGAAT	-0.1924	-3.7499**
SOUTHVAAL	-2.8618**	-1.8443	REMBR-BEH	-3.7985**	-6.6776**
VAAL-REEF	-2.0963*	-0.1908	REMGRO	-3.8285**	-4.3882**
VENTERS	-2.6686**	-2.0984*			

The z-scores for the runs tests of randomness of the deviation signs for each security's returns mean and variance are given by $z_{\bar{r}}$ and $z_{s_r^2}$ respectively.

** : significant at the 1% level   * : significant at the 5% level

3.4 Conclusions

From the evidence presented above it is clear that the pattern of returns for well traded Johannesburg Stock Exchange securities is similar to that found internationally for speculative price series.

While there is strong evidence of returns being distributed in a more leptokurtic fashion than suggested by the normal distribution, there is less evidence of persistent skewness when returns are measured over weekly and four-weekly intervals. Given that four-weekly returns are distributed closer to the normal than weekly returns, the Central Limit

Theorem appears to hold under addition. This finding, together with the consistent increases in the characteristic exponent estimates and the size of the change in the average scale parameter from weekly to four-weekly returns, supports the conclusion of Officer (1972) that the distributions exhibit a number of properties inconsistent with the stable Paretian hypothesis suggested by Mandelbrot. Although imprecise, the impact of volume variability on the distributions for well traded Johannesburg Stock Exchange securities lends credence to the finite variance subordinated process models of Praetz (1972), Clark (1973), Ball and Torous (1983), and Kon (1984). The overall conclusion from the research is therefore that the infinite variance stable Paretian hypothesis is rejected in favour of a finite variance subordinated stochastic process model. As stated in the introduction, this conclusion is important in allowing more rigorous empirical research to be undertaken into risky asset pricing models. In particular it permits the use of numerous multivariate analysis techniques that would be inappropriate if speculative returns had infinite variance.

One caveat to the above conclusion is the evidence of non-stationarity of both the mean and variance of individual security returns which results in the miss-estimation of distribution parameters when using time series data. This problem is of significant concern when undertaking empirical research into the Arbitrage Pricing Theory given the reliance of the procedures on estimated covariance matrices. If, as suggested by Hsu, Miller and Wichern (1974), the non-stationarity is the result of step changes then using shorter time series of returns in the estimation of covariance should be considered[23].

Finally, some statement is necessary concerning the extrapolation of the findings to all securities on the Johannesburg Stock Exchange. As indicated previously, conclusions based on the sample of forty-five securities analysed in this chapter might be considered conservative if rejecting the normal hypothesis can be extended, with a high degree of confidence, to less well traded securities. The appropriateness of such an extrapolation is supported by the evidence presented in appendices 3.3 and 3.4 for the full sample of two hundred and forty-four securities. While this chapter has focused on univariate approaches to normality tests because of their increased power, further evidence of the conservative nature of the conclusions can be made by utilising multivariate normality tests of both the group of forty-five well traded securities and the entire sample of two hundred and forty-four securities. One such test involves a simple graphical procedure where the square of the generalized Mahalanobis distances of the cross-sectional returns

[23]The increasing tendency of researchers to use monthly data over a period of sixty months for the estimation of betas and other security returns paramaters is to a significant extent based on the need to mitigate against non-stationarity problems. See for instance Affleck-Graves and McDonald (1989:894).

vectors are plotted against the appropriate χ^2 percentiles. The square of the generalized Mahalanobis distance is given by;

$$d_t^2 = (\mathbf{r}_t - \bar{\mathbf{r}})' \mathbf{s}^{-1} (\mathbf{r}_t - \bar{\mathbf{r}}) \tag{3.9}$$

where; $\mathbf{r}_t$ is a vector of returns for time period t; $\bar{\mathbf{r}}$ is the vector of sample means; and, $\mathbf{s}$ is the sample covariance matrix.

If the sample is drawn from an n-variate normal distribution d_t^2 is distributed as χ^2 with n degrees of freedom, and a plot of $\chi^2_{(n)}$ percentiles against ordered d_t^2 should lie on a straight line passing through the origin (du Toit, Steyn and Strumf, 1986:50).

Figures 3.3 and 3.4 present the curves using both the weekly and four-weekly data for the forty-five well traded securities and the total sample of two hundred and forty-four respectively. For ease of presentation the squared Mahalanobis distances and $\chi^2_{(n)}$ percentiles have been standardized by dividing by the corresponding maximum values for each of the four curves. This transformation clearly does not affect the linear characteristics of the curves[24]. The presence of multivariate non-normality for the well traded securities is evident from the shape of the two curves displayed in figure 3.3 and is consistent with the univariate analysis undertaken. Additionally, the trend to normality under addition appears supported in the multivariate approach. This is reflected in four-weekly curve fitting closer to a straight line passing through the origin. The substantial increase in non-normality exhibited in figure 3.4 is not suprising and confirms the conservative nature of the conclusions drawn previously with respect to returns computed from quoted price data.

Chapter four extends the analysis of the current chapter by assessing the impact on covariance estimation of the general level of non-normality highlighted above for well traded Johannesburg Stock Exchange listed securities. Additionally, the chapter also examines the exacerbating influence of thin trading. The impact of both effects on covariance estimation constitutes an important initial step in the examination of the robustness of procedures used in empirical research into the Arbitrage Pricing Theory.

[24]For the forty-five well traded securities the maximum values for the squared Mahalanobis distances and chi-square statistics are 250.7 and 82.8, and 124.9 and 77.1 for the weekly and four-weekly returns series respectively. For the full sample of two hundred and forty-four securities the corresponding figures are 696.1 and 245.0, and 323.3 and 312.4.

Figure 3.3 Plot of $\chi^2_{(n)}$ percentiles against ordered squared Mahalanobis distances d_t^2 for forty-five well traded securities

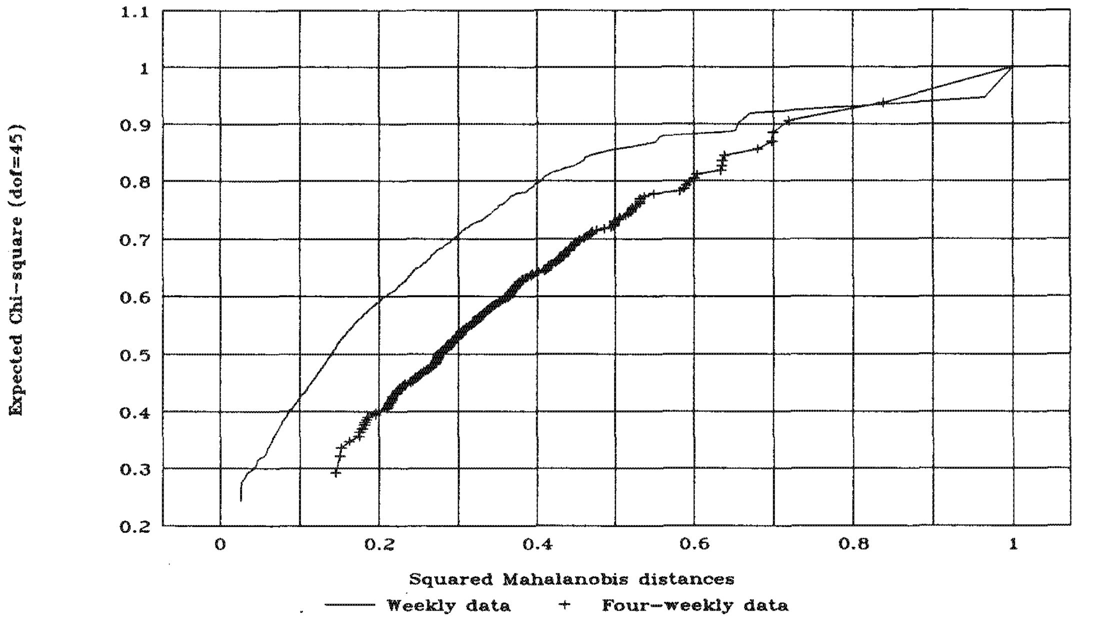
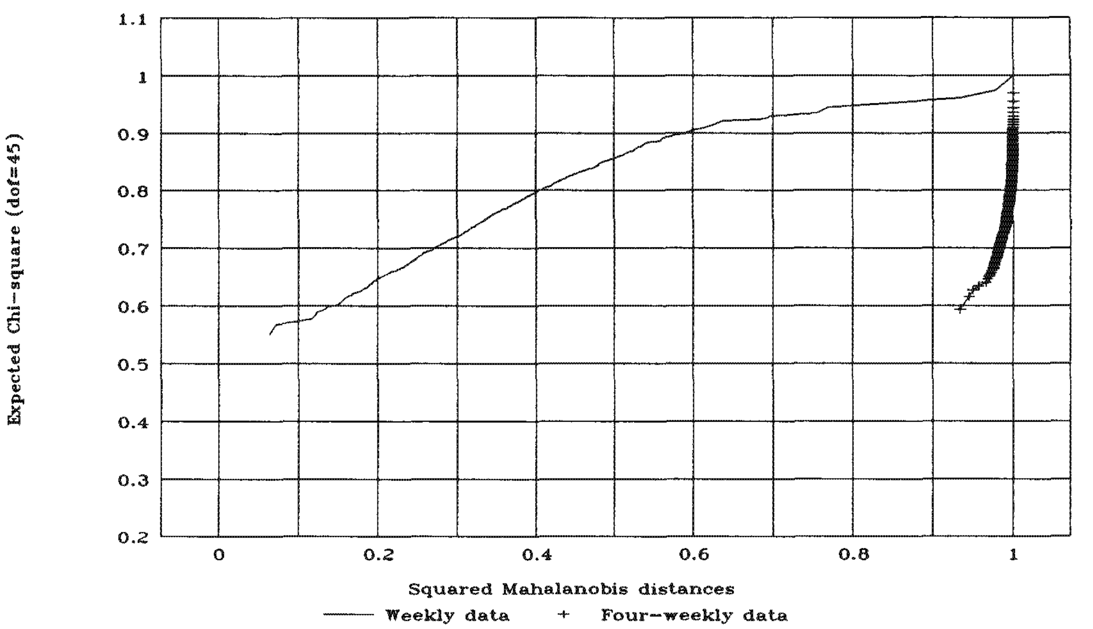


Figure 3.4 Plot of $\chi^2_{(n)}$ percentiles against ordered squared Mahalanobis distances d_t^2 for two hundred and forty-four securities



4

The impact of non-normalities and thin trading on covariance estimation

4.1 Introduction

> *The estimation of covariance matrices may be called the key to multivariate statistics. Robust estimators of these matrices open the door to the robustification of the classical normal-theory multivariate procedures* (Hampel, Ronchetti, Rousseeuw and Stahel, 1986:270).

Chapter three has shown, through an examination of prior international empirical evidence and on the basis of the South African evidence, that security price changes do not represent drawings from multivariate normal distributions. Although the evidence is inconsistent with the infinite variance stable Paretian hypothesis, the distributions do appear to be consistent with some form of subordinated stochastic process that may or may not be a complex mixture of normal distributions. Given the conclusion that the underlying distribution of security price changes is not multivariate normal, the variance of the sample covariance matrix does not conform with the Wishart distribution (Harman, 1976:201). Any hypothesis tests, such as the maximum likelihood χ^2 test, that are contingent upon the Wishart distribution must therefore be questioned.

Additional to the non-normality issue which impacts on the distributional characteristics of the covariance matrix estimate, is the issue of bias and efficiency with respect to point estimates of the pairwise covariances. Considerable theoretical and empirical research has been undertaken in this area with the principal focus being on single factor market model beta estimation and bi-variate covariance estimation in thin traded markets[1]. Much of the initial research involved the development of techniques for obtaining better, less biased beta estimates in thinly traded markets, and for thinly traded securities within relatively

[1]For further reference see Scholes and Williams, 1977; Dimson, 1979; Cohen, Hawawini, Maier, Schwartz and Whitcomb, 1980, 1983a, 1983b; Jog and Riding, 1986; and Heinkel and Kraus, 1988.

well traded markets. The principal adjustment procedures that have been suggested are the Scholes-Williams consecutive time period trading method, a trade-to-trade approach, or a series of methodologies involving the use of lagged, contemporaneous, and leading regression or multiple regression estimates (Dimson, 1979:206).

In this chapter an extensive simulation is carried out to assess the efficiency and reliability of covariance estimation techniques in the presence of thin trading conditions and given different levels of non-normality. This is followed, in chapter five, by a further simulation which extends the covariance analysis through a direct examination of the impact of these two conditions on the identification of both the number and pricing of factors determining security returns in terms of the Arbitrage Pricing Theory. To date there has been little consensus as to the number of priced factors influencing security returns with numbers ranging from one to eight appearing in the literature. One of the reasons proposed for this is the that the statistical procedures lack power (Brown, 1989:1261). The purpose of the simulation procedures employed in the two chapters is to show that biases in the covariance matrix induced by thin trading as well as the degree of non-normality further exacerbate the power of the factor analysis procedures as traditionally employed, but that the power can be improved through the use of robust covariance estimates and the reverse Helmert rotation procedure suggested by Brown (1989:1260).

4.2 Market microstructure and its impact on security returns

Although much of the research in financial economics assumes frictionless trading processes, improved understanding of financial markets requires the assessment of the impact on pricing models of the interaction between market participants, trading mechanisms, and the dynamics of pricing under conditions where trading frictions exist (Cohen et. al., 1980:249). Of particular interest in this study are the biases in parameter estimation that result from security market microstructure under conditions where frictions impede the trading process and cause price-adjustment delays. These delays can be attributed to both the actions of specialists and the periodic trading that results from information, decision and transaction costs (Shanken, 1987:222)[2].

[2]The dynamics of the trading system of the Johannesburg Stock Exchange varies quite significantly from that of other international exchanges and the friction induced microstructure effects cannot be presumed to mirror those of, for instance, the New York Stock Exchange. While, in common with other exchanges, trading takes place on the JSE floor by open outcry with prices being recorded on the price board whenever there is a change, dealing only takes place between brokers. Settlement between brokers occurs at the end of the week and there are no jobbers or specialists. Additionally, commission rates and other client costs are not freely negotiable between client and broker but determined by a schedule. Marketable

While much of the research into thin trading and non-synchronous data issues has been devoted to the estimation of the single factor or market model beta, Shanken (1987) explicitly investigated the implications for the covariance-factor structure of returns. Using the analytical framework proposed by Cohen et. al. (1983b) he found that the average magnitude of the adjusted covariance estimates were approximately twice those of the unadjusted estimates. The data used in the study consisted of a sample of daily returns for three hundred CRSP securities over the period July 3, 1962 to December 31, 1972[3]. Although the research indicated a significant impact, the extent to which bias is reduced through using the adjustment procedures remains unclear since the study did not address the implication of non-conformity of underlying returns to the Cohen et. al. assumptions. Consequently there is an element of joint hypothesis testing involved in the methodology employed. Additionally, Shanken makes no reference to any implications for the standard error of the estimate as additional leads and lags are used in the estimation procedure. These issues are addressed in the analysis presented below through the use of simulation techniques where the underlying prices are generated so as to conform to the proposed pricing model[4].

In 1979 Dimson developed a model to explicitly examine the implication of non-continuous trading on observed prices. The model was based on the assumption that quoted prices recorded at discrete time intervals could be the result of transactions occurring at some time prior to the exact time of observation (1979:197). As such, any observed price p_t^o represents the price at the time of the most recent transaction and it is only at the time of transaction that the underlying price p_t and observed price p_t^o are the same. Dimson's model was extended by Cohen, Hawawini, Maier, Schwartz and Whitcomb (1980:250-252). They developed a broader price adjustment model for the relationship between observed returns and underlying returns by incorporating two additional price adjustment delays to supplement the thin trading error induced by measurements typically being taken at fixed intervals with the most recent transaction prices used as closing prices. The first of the additional adjustments related to inventory issues while the second dealt with frictions attributable to transaction costs.

[3]Cohen et. al. refer to the CRSP database as source rather than to the primary market from which their data are extracted, i.e.. NYSE or AMEX. The reference has therefore been retained in this literature review.

[4]For the purpose of this thesis the term "underlying" prices refers to prices that would result if information was instantaneously and costlessly impounded into the market. In this context therefore, the term refers to simulated prices before allowing for thin trading and other microstructure effects. The terminology is used in preference to the term "true" prices used by Dimson (1979) and Cohen et. al. (1980, 1983) because simulated rather than real market data is being referred to.

Given the centrality of the Dimson and Cohen et. al. models to the simulation analysis carried out in the chapter, both models are discussed in some depth below.

4.2.1 The Dimson analytical framework

In order to develop his aggregated coefficient approach for the measurement of systematic risk and the estimation of beta in the presence of thin trading, Dimson proposed a model of security price evolution using a discrete time framework. By generating simulated data conforming to the model he was then able to compare five approaches for beta estimation. These were the standard market model regression approach, the Scholes-Williams variant of the standard simple regression, the trade-to-trade approach, the full Scholes-Williams approach, and his proposed aggregated coefficients method (1979:206-208).

Dimsons' model is based on the assumption that underlying changes in value are serially uncorrelated and generated by the market model, and that under conditions of continuous trading transaction prices would be equal to underlying prices. He further assumed that at any instant in time t, the probability for the most recent transaction of security j having occurred at time t-i $(i \geq 0)$ is $\theta_{j,t,i}$, and that the distribution of price changes described by $\theta_{j,t,i}$ is stationary and identically distributed over time (1979:201)[5]. Consequently;

$$\theta_{j,t,i} \geq \theta_{j,t,i-\tau}, \ \forall\, \tau > 0; \text{ and,}$$

$$\sum_{i=0}^{\infty} \theta_{j,t,i} = 1$$

and, the expected value of the observed price at time t equals the product of the probability density function and the underlying historical price distribution, which can be written mathematically as;

$$E\left(p^{o}_{j,t}\right) = \sum_{i=0}^{\infty} \theta_{j,t,i}\, p_{j,t-i}$$

Additionally, the continuously compounded return for time period t, and its expected value are given by;

[5]The equations presented are consistent with (3a), (3b) and (6) in Dimson (1979:201) except that no finite time interval is specified within which the security must trade. The summations presented are therefore assessed over the interval $[0,\infty]$ and not [0,n]. While possibly not material, Dimsons' assumption of a finite interval within which securities must trade is inconsistent with his assumption that $\theta_{j,i}$ is stationary and identically distributed over time.

$$r^{o}_{j,t} = \ln\left(p^{o}_{j,t}\right) - \ln\left(p^{o}_{j,t-1}\right); \text{ and,}$$

$$E\left(r^{o}_{j,t}\right) = \sum_{i=0}^{\infty} \theta_{j,t,i}\, r_{j,t-i}$$

where; $r_{j,t}$ is the underlying instantaneous return at time t for security j.

Given that the probability of a security trading in any discrete time period is constant; the $\theta_{j,t,i}$, the average age of quoted prices, and the probability of no trade occurring in any time period can be inferred using the binomial distribution. Following the approach of Dimson and using the unit of time as one day gives the results presented in table 4.1.

Table 4.1 Distribution of trading probabilities assuming a constant and independent probability of trading per day

Probability of trading per day	Probability density of the most recent transaction Number of days since last trade 0	1	2	3	4	5	6	Average Age (days)	Probability of trading per week	Expected number of weeks trading pa
0.90	0.900	0.090	0.009	0.001	0.000	0.000	0.000	0.11	1.000	52.00
0.75	0.750	0.188	0.047	0.012	0.003	0.001	0.000	0.33	0.999	51.95
0.50	0.500	0.250	0.125	0.063	0.031	0.016	0.008	1.00	0.969	50.38
0.25	0.250	0.188	0.141	0.105	0.079	0.059	0.044	3.00	0.763	39.66
0.10	0.100	0.090	0.081	0.073	0.066	0.059	0.053	9.00	0.410	21.29
0.05	0.050	0.048	0.045	0.043	0.041	0.039	0.037	19.00	0.226	11.76

4.2.2 The Cohen, Hawawini, Maier, Schwartz and Whitcomb analytical framework

While the model described above can explain the serial cross-correlations amongst securities, the phenomenon of weak autocorrelation in individual securities cannot be explained by the approach. Although non-zero autocorrelation patterns in security returns occur partially as a result of the direct impact of bid-ask spreads on security prices, other market microstructure effects also result in a delay in the rate at which the underlying return in any period is incorporated in subsequent observed returns.

Cohen et. al. (1980,1983a,1983b) extended the work of prior researchers by developing a procedure for measuring systematic risk that more efficiently addressed the serial cross-correlations, autocorrelations and errors-in-variables type beta bias noted by Scholes-Williams (1977). They distinguished between delays in the price adjustment process as transaction price adjustments lag behind quotation price adjustments (the Fisher effect), and additional frictions resulting from inventory positions and transaction costs (1983b:264).

If it is assumed that quotes of specialists and dealers are influenced by their inventory positions then inventory imbalances accruing in one trading period will be rectified by transacting in subsequent periods. In this way the price adjustments attributable to one period are carried forward into subsequent periods. Additionally, given the significance of transaction costs, individual traders will not trade continuously but accumulate information for finite periods before entering the market. This periodic entry to the market results in price adjustment delays as observed price changes resulting through transaction will be caused by the accumulation of underlying price changes since the previous transaction (1980:251-252).

In the development of their model Cohen et. al. sought to overcome the problems inherent in prior methodologies. While acknowledging that a Fisher effect induced intervalling bias can be overcome by using closing quotation prices or matched trading times, they suggest neither is generally feasible because of data availability[6]. Additionally, they suggest that regression approaches using synchronous and non-synchronous index returns, while being more general and able to handle price adjustment delays over and above the Fisher effect, suffer from efficiency problems.

As a final rationale for their procedure, Cohen et. al. state that;

> ...... *it is trivial to show that the serial cross-covariance decays only gradually as the measurement interval is increased beyond the maximum length of the price-adjustment delays. Thus a correction procedure is needed that is not limited in its ability to encompass an extensive lead lag structure* (1983b:265).

Under the assumption that underlying returns are generated by the market model, Cohen et. al. modeled the relationship between underlying and observed returns as (1980:251)[7];

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[6]The intervalling-effect bias results from price adjustment delays that cause security returns to exhibit serial cross-correlations with each other and the market index. This results in beta bias that is a function of the trading interval over which the returns are calculated. As the measurement interval increases, the bias in the beta estimate declines.

[7]As for the Dimson equation the representation here does not define a finite time period within which the true return must be incorporated into the observed return pattern.

$$r^o_{j,t} = \sum_{i=0}^{\infty}\left(\gamma_{j,t-i,i}\, r_{j,t-i} + \varphi_{j,t-i,i}\right)$$

where; $\gamma_{j,t-i,i}$ represents the proportion of the underlying return for security j generated in period t-i that is incorporated in the observed return i periods later at t, and the random variable $\varphi_{j,t-i,i}$ reflects the impact of the bid-ask spread on observed returns[8].

By further assuming that;

$\gamma_{j,t,m}$ and $\gamma_{k,\tau,n}$ are independent for all $j \neq k, t, \tau, m, n$;

$E\left(\gamma_{t,j,m}\right) = E\left(\gamma_{j,\tau,n}\right)$ for all j, t, τ, m ; and,

$E\left(\sum_{i=0}^{\infty}\gamma_{j,t,i}\right) = 1$ for all j, t

an equation for the contemporaneous covariance between the underlying returns of two securities j and k can be derived as (Cohen et. al. 1983b:276);

$$\operatorname{cov}\left(r_{j,t};r_{k,t}\right) = \operatorname{cov}\left(r^o_{j,t};r^o_{k,t}\right) + \sum_{i=1}^{\infty}\operatorname{cov}\left(r^o_{j,t};r^o_{k,t-i}\right) + \sum_{i=1}^{\infty}\operatorname{cov}\left(r^o_{j,t-i};r^o_{k,t}\right)$$

The difference between the approaches of Dimson and Cohen et. al. is reflected in the nature of the delay functions. The Dimson $\theta_{j,t,i}$ represents the probability distribution for the most recent transaction and the model assumes the observed price equals the underlying price at the time of transaction. The Cohen et. al. $\gamma_{j,t,m}$ on the other hand relates the underlying return to a series of subsequent observed returns and in this way the model allows for additional frictions in the trading process and does not just address the thin trading issue.

4.2.3 South African evidence and the selection of simulation parameters

Simulation procedures used to assess the impact of security market microstructure on the robustness of standard approaches used to estimate covariance matrices and subsequent factor structures must be based on certain assumptions about the distributions of

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[8]While the φ_j's induce negative autocorrelation for security j, they have no impact on any serial cross-correlation between securities (1980:254). Consequently they are excluded from subsequent analysis.

parameters such as the Dimson $\theta_{j,t,i}$ and the Cohen et. al. $\gamma_{j,t,m}$. In order to ensure that appropriate simulation parameters are employed, this section examines the characteristics of daily returns for two hundred and forty of the securities used in the analysis of chapter three for the period December 28, 1987 to March 20, 1992[9].

An analysis of trading price ages for the sample is shown in table 4.2[10]. The securities were initially ranked on the basis of the average transaction age. Thereafter they were divided into deciles consisting of twenty-four securities each. From the table it can be seen that the most frequently traded securities on the Johannesburg have prices that are often less than one day old and almost always less than two days old, while the lesser traded securities have prices that are over five days old more than thirty percent of the time[11]. Given the limited length of the time series used in the analysis the findings of the table are remarkably similar to those produced by Smithers for the London Stock Exchange over the period January 1955 to December 1974 (Dimson, 1979:211).

Table 4.2 Analysis of trading price ages for two hundred and forty securities trading on the Johannesburg Stock Exchange

trading decile	Distribution of Price Ages (days) 0	1	2	3	4	5	6-10	11-20	21-30	31-40	41-50	50+	Range of mean ages Min	Max
1	0.988	0.011	0.001										0.001	0.044
2	0.918	0.067	0.011	0.003	0.001								0.045	0.169
3	0.794	0.137	0.041	0.016	0.007	0.003	0.003						0.174	0.440
4	0.696	0.180	0.065	0.027	0.014	0.007	0.010	0.001					0.474	0.685
5	0.609	0.201	0.086	0.043	0.023	0.013	0.021	0.003					0.692	1.054
6	0.530	0.206	0.102	0.059	0.035	0.021	0.035	0.009	0.002	0.001			1.079	1.591
7	0.397	0.203	0.121	0.076	0.053	0.036	0.078	0.030	0.005	0.001			1.607	2.953
8	0.326	0.179	0.117	0.081	0.060	0.046	0.115	0.056	0.012	0.004	0.002	0.001	2.975	3.913
9	0.216	0.145	0.107	0.081	0.065	0.053	0.157	0.113	0.036	0.014	0.006	0.006	4.048	8.413
10	0.132	0.081	0.063	0.053	0.045	0.040	0.146	0.163	0.087	0.052	0.035	0.103	8.988	77.629
Total	0.561	0.141	0.071	0.044	0.030	0.022	0.057	0.038	0.014	0.007	0.004	0.011	0.001	77.629

[9]The database of daily Johannesburg Stock Exchange securities available from the Statistical Sciences department at the University of Cape Town only extends back to December 28, 1987. Four of the securities, namely Jade, Norimed, Putprop and Urqhart, were excluded also from the analysis because some of their daily data were missing from the database.

[10]The analysis for this section utilized a daily database that contains only the volume traded, daily high, daily low and closing price for each security. No intra-day information is provided relating to transaction times and as a consequence, for this analysis, it is assumed that any trading occurs at the close. Trading price ages are therefore measured in discrete days and refer to the number of days since the last transaction.

[11]Thirty-three percent and fifty-nine percent of the prices for the ninth and tenth decile respectively are over five days old.

While the use of deciles in the presentation of table 4.2 limits the degree of comparability with the numbers presented in table 4.1, the empirical results are not inconsistent with those one would expect under the assumption of a binomial distribution of trading probabilities. Simulation of trading frequencies using Dimson $\theta_{j,t,i}$'s conforming to drawings from independently distributed daily trading probabilities of between 0.05 and 0.95 therefore appears reasonable.

The existence and implications of autocorrelation in security price changes has been under considerable investigation since the 1960's[12]. Schwartz and Whitcomb focused specifically on market model residuals as it was their contention that these residuals are more nearly independent across securities and that any autocorrelation more correctly reflects the unique components of security returns. They based this assumption on the suggestion in the literature of the time that the autocorrelations result because returns follow a random walk with a reflecting barrier, and because investors have a preference for discrete price changes (Schwartz and Whitcomb, 1977:44).

As discussed previously, in extending this area of research, Cohen et. al. proposed that zero autocorrelation in underlying returns for a security does not exclude the possibility of observed returns exhibiting non-zero autocorrelations. Additionally, while the sign of the autocorrelations and their relationship to thinness of trading is theoretically indeterminate, empirical evidence suggests a predominantly positive relationship for well traded securities and predominantly negative relationship for more thinly trading issues (Cohen et. al., 1980:254).

The autocorrelation structure for the selected sample of Johannesburg Stock Exchange securities was estimated by first computing time series of daily returns, $r^o_{j,t}$, using;

$$r^o_{j,t} = \ln\left(p^o_{j,t}\right) - \ln\left(p^o_{j,t-1}\right),\ \forall \left\{V_{j,t-1} \neq 0 \cup V_{j,t} \neq 0\right\}; \text{otherwise missing}$$

where; $V_{j,t}$ is the volume of trading during day t for security j.

Thereafter the autocorrelations of lag τ were computed as $\hat{\rho}_{j,\tau} = corr\left(r^o_{j,t-\tau}; r^o_{j,t}\right)$, where all non-missing pairs of observations are used in the estimation of the correlation. Significance testing of the autocorrelations were undertaken using the standard procedure for testing the hypothesis that a correlation coefficient is not significantly different from

[12]For further reference see Osborne, 1962; Niederhoffer and Osborne, 1966; Fama, Fisher, Jensen and Roll, 1969.

zero. The students-t statistic based on a standard error of estimate[13] was employed in preference to the procedure normally used to test autocorrelation structures proposed by Bartlett (Pindyck and Rubinfeld, 1981:500). The technique used is considered more appropriate because the non-trade induced missing values result in the set of autocorrelations for a security being estimated using significantly different sample sizes, which do not necessarily even have a tendency to decrease monotonically with increasing lag. For the same reason, the joint hypothesis Q-statistic of Box and Pierce used to test that all of the autocorrelation coefficients are zero also could not be used (Pindyck and Rubinfeld, 1981:549).

Table 4.3 and appendix 4.1 summarize the results for the first ten lags, for each of the two hundred and forty securities[14]. The appendix, which presents the results for each security, gives the number of observations used in the computation of the autocorrelation estimates, and whether each is significantly different from zero at the 5% and 1% level. The results for the more frequently trading deciles confirm prior empirical evidence (Cohen et. al. 1980:254). Security autocorrelations are predominantly positive for at least the first two lags, and more are significant than can be expected by chance. There is also some evidence to suggest that, while fewer are significant, there is an increasing proportion of negative autocorrelations for lags of three and greater. The results for the thinner trading deciles, while less clear, are still consistent with the more frequently trading securities for lags of one and two.

Summary autocorrelations for each decile are presented in table 4.3. They were computed by standardizing the returns series for each security, grouping the securities into their respective trading deciles, and then computing aggregate autocorrelation coefficients for each decile. While it can be questioned whether the assumption of equivalent autocorrelation patterns for securities within narrowly defined trading frequency ranges is valid, this assumption is considered no less valid than the implicit assumption of stationarity of the structure for an individual security whose trading frequency will change across time. Additionally, table 4.3 also shows consistency with prior research by indicating significant positive autocorrelations for lags one and two in spite of some indication of negative autocorrelation for the more frequently trading securities for lags three and above.

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[13]This estimate is given by $\sqrt{\left(1-\hat{\rho}^2\right)/(N-2)}$.

[14]In contract to the other appendices, the ordering of securities in appendix 4.1 is based on average price age with the most frequently trading securities being presented first.

Table 4.3 Summary table of autocorrelations based on aggregated trading decile standardized returns

Trading Decile	Autocorrelations 1	2	3	4	5	6	7	8	9	10
						sample size aggregate correlation average correlation / standard deviation				
1	24450	24194	24160	24138	24112	24083	24062	24037	24004	23980
	0.026**	0.000	-0.028**	0.001	-0.009	-0.008	-0.006	0.014*	0.016*	-0.020**
	0.025/0.053	0.000/0.032	-0.028/0.042	0.002/0.041	-0.009/0.032	-0.008/0.032	-0.007/0.029	0.014/0.028	0.016/0.030	-0.020/0.032
2	20059	18794	18716	18653	18639	18642	18624	18548	18513	18524
	0.049**	0.018*	-0.028**	-0.014	0.002	-0.009	0.020**	0.009	0.002	-0.011
	0.049/0.067	0.018/0.043	-0.027/0.034	-0.013/0.037	0.002/0.039	-0.008/0.028	0.020/0.040	0.010/0.049	0.002/0.037	-0.010/0.042
3	14104	12119	11934	11849	11854	11819	11746	11699	11740	11781
	0.056**	0.009	-0.004	-0.029**	0.012	-0.014	-0.026**	0.015	-0.003	-0.011
	0.055/0.104	0.011/0.087	0.002/0.059	-0.028/0.062	0.014/0.052	-0.016/0.061	-0.028/0.049	0.016/0.043	-0.003/0.042	-0.012/0.051
4	10000	7873	7795	7750	7668	7592	7501	7469	7490	7526
	0.062**	0.012	-0.018	0.001	0.015	0.003	-0.020	0.020	0.008	-0.019
	0.062/0.071	0.014/0.064	-0.010/0.085	0.003/0.072	0.011/0.063	0.007/0.072	-0.015/0.065	0.022/0.056	0.012/0.062	-0.017/0.080
5	7298	5411	5350	5253	5230	5224	5137	5107	5065	5013
	0.017	0.001	-0.004	0.015	-0.019	0.006	-0.005	0.007	-0.010	-0.026
	0.017/0.124	0.017/0.100	0.002/0.079	0.015/0.084	-0.015/0.095	0.003/0.085	-0.010/0.074	-0.002/0.074	-0.011/0.076	-0.031/0.066
6	5418	3809	3710	3621	3576	3525	3502	3480	3457	3416
	0.069**	0.042**	0.022	-0.048**	-0.005	-0.018	0.021	0.002	-0.008	0.008
	0.082/0.121	0.056/0.123	0.050/0.102	-0.019/0.121	-0.001/0.064	-0.009/0.125	0.017/0.086	-0.006/0.084	-0.001/0.098	0.004/0.075
7	2675	1564	1514	1458	1422	1368	1371	1383	1337	1322
	0.038*	0.082**	-0.003	0.009	-0.006	0.045	-0.018	0.036	0.004	0.044
	0.030/0.151	0.082/0.134	0.019/0.139	-0.010/0.096	-0.033/0.182	0.048/0.122	-0.017/0.156	0.054/0.133	0.010/0.130	0.053/0.197
8	1922	1109	1081	1026	1011	978	992	954	921	918
	0.137**	0.131**	0.117**	0.099**	0.043	0.021	0.067*	0.037	-0.008	0.033
	0.102/0.130	0.051/0.123	0.107/0.121	0.088/0.104	0.030/0.120	-0.009/0.113	0.027/0.107	0.019/0.100	-0.039/0.106	0.011/0.128
9	785	383	349	366	364	332	320	299	313	304
	0.043	0.051	0.083	0.065	0.031	-0.001	-0.067	-0.104	-0.016	-0.022
	0.050/0.114	0.015/0.101	-0.001/0.083	0.025/0.087	-0.013/0.090	-0.067/0.078	-0.022/0.071	-0.080/0.059	0.005/0.054	-0.018/0.070
10	779	573	573	573	570	546	539	539	542	539
	-0.016	-0.016	0.024	-0.090*	-0.038	-0.031	-0.017	-0.015	-0.006	0.005
	0.154/0.119	-0.021/0.114	0.095/0.099	-0.103/0.113	-0.032/0.064	0.143/0.092	-0.066/0.088	-0.162/0.088	0.017/0.109	0.116/0.066
Total	87490	75829	75182	74687	74446	74109	73794	73515	73382	73323
	0.045**	0.013**	-0.015**	-0.008*	0.000	-0.006	-0.002	0.012**	0.005	-0.013**

** : significant at the 1% level * : significant at the 5% level

On the basis of the sample evidence, the Cohen et. al. $\gamma_{j,t,m}$, excluding non-trade effects, is presumed to induce autocorrelation for up to three lags. A simple heuristic was employed to obtain values for $\gamma_{j,t,m}$, which assumed that the values for all time periods were drawings from independent normal distributions and conformed to the following:

$$\gamma_{j,t,0} \sim N\left(\delta;\sigma^2\right);$$
$$\gamma_{j,t,1} \sim N\left(\tfrac{1}{2}(1-\delta);\sigma^2\right);$$
$$\gamma_{j,t,2} \sim N\left(\tfrac{1}{2}(1-\delta);\sigma^2\right);\text{ and,}$$
$$\gamma_{j,t,3} = 1-\gamma_{j,t,0}-\gamma_{j,t,1}-\gamma_{j,t,2}$$

The results of using this heuristic to simulate price adjustment delays are shown in table 4.4 for different values of σ and δ. For each σ and δ, the table gives the average, standard deviation, minimum and maximum of the first ten autocorrelations for fifty time series of observed returns generated from independent normally distributed simulated underlying returns series[15] [16].

The table illustrates that, as expected, increasing δ results in reduced average autocorrelations of lag one and two. This occurs because the δ represents the proportion of each underlying return that is expected to be immediately reflected in the observed return. Only the residual is carried forward into subsequent observed returns thereby inducing autocorrelation. Increasing σ, for each of the selected δ's, on average reduces the first two mainly positive autocorrelations and makes the third autocorrelation slightly more negative. On the basis of a comparison between table 4.4 and appendix 4.1, the simulations discussed in the subsequent sections were carried out using $\delta = 0.80$ and $\sigma = 0.10$.

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[15]Normally distributed random numbers generated for this thesis were either computed using the method developed by Box and Muller (1958), which uses logarithmic and trigonometric functions to create two pseudo- standard normal random variates from two independent random variates distributed uniformly on [0;1] (Newman and Odell, 1971:19), or using the NAG Fortran Library routine G005DDF.

[16]In computing the observed returns the true returns were first transformed to nearest integer prices with an initial value of 100c. Each simulated series consisted of two hundred observations.

Table 4.4 Impact of price adjustment delay parameters on the autocorrelation structure for simulated data

δ / σ	Lagged Correlations : giving average, standard deviation, minimum, and maximum									
	1	2	3	4	5	6	7	8	9	10
0.70/0.00	0.221**	0.163**	-0.005	-0.016	0.004	-0.005	-0.019	-0.026	-0.014	-0.014
	0.083	0.070	0.069	0.069	0.075	0.076	0.091	0.077	0.074	0.083
	0.045	-0.004	-0.126	-0.195	-0.164	-0.194	-0.234	-0.205	-0.240	-0.191
	0.422	0.380	0.164	0.127	0.160	0.161	0.135	0.136	0.123	0.186
0.70/0.05	0.217**	0.167**	-0.004	-0.008	-0.010	-0.026	-0.024	-0.029	-0.013	-0.033
	0.077	0.075	0.077	0.074	0.054	0.070	0.073	0.078	0.075	0.077
	0.093	-0.014	-0.128	-0.151	-0.112	-0.152	-0.198	-0.162	-0.159	-0.203
	0.403	0.314	0.157	0.163	0.131	0.119	0.112	0.136	0.182	0.093
0.70/0.10	0.203**	*0.152**	-0.043	-0.033	-0.013	-0.009	-0.011	-0.009	0.002	0.015
	0.072	0.067	0.075	0.079	0.072	0.082	0.062	0.065	0.082	0.072
	-0.002	0.035	-0.212	-0.262	-0.203	-0.269	-0.137	-0.143	-0.182	-0.131
	0.349	0.342	0.108	0.148	0.128	0.164	0.131	0.135	0.230	0.182
0.70/0.15	0.152**	0.125**	-0.037	-0.002	-0.021	-0.006	-0.022	0.007	-0.014	0.009
	0.102	0.066	0.070	0.083	0.077	0.062	0.076	0.074	0.071	0.065
	-0.295	-0.037	-0.178	-0.209	-0.238	-0.190	-0.185	-0.147	-0.146	-0.126
	0.338	0.273	0.138	0.202	0.148	0.152	0.172	0.173	0.152	0.156
0.80/0.00	0.152**	0.123**	0.008	-0.012	-0.010	-0.012	-0.012	-0.022	-0.015	-0.038
	0.064	0.060	0.064	0.072	0.059	0.061	0.070	0.073	0.082	0.071
	-0.003	0.000	-0.118	-0.151	-0.155	-0.224	-0.130	-0.189	-0.297	-0.168
	0.336	0.256	0.141	0.214	0.140	0.091	0.138	0.149	0.158	0.117
0.80/0.05	0.149**	0.122**	-0.002	-0.003	0.006	-0.004	-0.003	-0.009	-0.004	0.004
	0.087	0.081	0.082	0.091	0.062	0.085	0.066	0.068	0.071	0.055
	-0.017	-0.077	-0.174	-0.198	-0.186	-0.137	-0.139	-0.189	-0.166	-0.087
	0.415	0.285	0.150	0.180	0.127	0.218	0.133	0.158	0.141	0.122
0.80/0.10	0.106**	0.086**	-0.011	0.012	0.015	-0.006	0.006	-0.003	0.003	-0.020
	0.086	0.069	0.065	0.059	0.078	0.071	0.071	0.065	0.077	0.071
	-0.051	-0.060	-0.125	-0.174	-0.141	-0.189	-0.134	-0.146	-0.159	-0.218
	0.302	0.245	0.115	0.129	0.236	0.180	0.172	0.138	0.170	0.108
0.80/0.15	0.077**	0.055**	-0.022	0.000	-0.005	-0.031	-0.017	-0.024	-0.001	-0.030
	0.076	0.075	0.074	0.067	0.055	0.067	0.066	0.075	0.074	0.069
	-0.059	-0.117	-0.161	-0.151	-0.103	-0.140	-0.181	-0.212	-0.136	-0.177
	0.250	0.206	0.152	0.164	0.133	0.134	0.100	0.164	0.207	0.122
0.90/0.00	0.041**	0.055**	0.000	-0.009	-0.018	-0.011	-0.006	-0.003	-0.006	-0.009
	0.070	0.070	0.070	0.083	0.075	0.079	0.062	0.087	0.077	0.069
	-0.084	-0.078	-0.142	-0.212	-0.165	-0.160	-0.127	-0.179	-0.153	-0.193
	0.172	0.242	0.135	0.183	0.119	0.116	0.112	0.151	0.163	0.126
0.90/0.05	0.057**	0.039**	0.002	0.013	-0.016	-0.010	0.001	-0.003	0.007	0.001
	0.069	0.075	0.065	0.064	0.074	0.062	0.074	0.084	0.083	0.077
	-0.073	-0.193	-0.212	-0.110	-0.184	-0.163	-0.142	-0.190	-0.188	-0.143
	0.195	0.209	0.115	0.202	0.124	0.110	0.174	0.282	0.156	0.204
0.90/0.10	0.019*	0.014	-0.015	-0.010	-0.015	-0.008	-0.009	-0.011	-0.007	-0.005
	0.063	0.067	0.074	0.068	0.065	0.067	0.071	0.072	0.074	0.072
	-0.160	-0.178	-0.208	-0.178	-0.193	-0.146	-0.142	-0.137	-0.201	-0.195
	0.154	0.136	0.120	0.205	0.120	0.177	0.156	0.143	0.156	0.144
0.90/0.15	0.025*	0.024**	-0.035	-0.005	-0.004	-0.033	0.009	-0.010	-0.011	-0.016
	0.079	0.060	0.074	0.068	0.057	0.065	0.072	0.082	0.061	0.074
	-0.219	-0.147	-0.171	-0.162	-0.152	-0.164	-0.191	-0.194	-0.132	-0.226
	0.146	0.140	0.108	0.167	0.152	0.141	0.170	0.203	0.106	0.180

** : significant at the 1% level   * : significant at the 5% level

4.3 The impact of non-normalities and thin trading on covariance estimation

The value of a simulation is closely tied to the fidelity with which it represents the real-world environment that it attempts to model (Vale and Maurelli, 1983:465).

In spite of the obvious validity of the above statement it is also important that conclusions relating to bias in parameter estimation using simulation methods are based on known "population" parameters. For this reason, rather than adopting the approach of Shanken (1987), the simulations conducted in the following sections use artificially created data in preference to actual security market data.

In this section an extensive simulation study is conducted to assess the impact of non-normalities in underlying returns and the impact of thin trading and market microstructure on covariance estimates based on observed (traded) price data. The impacts of non-normalities, thin trading, and market microstructure are examined both jointly and severally in order to assess the importance of each on the robustness of covariance estimation. The section is divided into two components. The first outlines the simulation methodology employed, and the techniques for computing the covariance estimates. The second presents the results and discusses the implications thereof.

4.3.1 Simulation methodology

The simulation design followed a four step procedure.

(a) Underlying daily returns series spanning a five year period (thirteen hundred days), and conforming to specific population parameters, were generated for each of two securities. The parameters specified included the first four moments of each security as well as the correlation between the two. This phase of the simulation was conducted using the methodologies of Fleishman (1978) and Vale and Maurelli (1983).

Fleishman's method of simulating non-normal variates utilizes a power transformation to produce variates that conform to a distribution with known first four moments. The parameters used in the transformation are determined by solving a set of non-linear equations so as to achieve the required moments. Given the non-linear nature of the equations an optimization routine such as Newton's method, which involves using first order partial derivatives and iterating to a final solution, must be employed (Sokolnikoff and Redheffer, 1966:657-659). Fleishman's transformation is given as (1978:522);

$$\tilde{Y} = a + b\tilde{X} + c\tilde{X}^2 + d\tilde{X}^3$$

where; $\tilde{X}$ is a standardized normal variate, and a, b, c and d are the transformation constants.

Tadikamalla (1980), in criticizing the method, pointed out that one of its limitations is that the exact nature of the resulting distribution is unknown. He did however acknowledge its ease of implementation and speed of execution (Vale and Maurelli, 1983:465).

Generation of bivariate or multivariate non-normal data with a predefined correlation structure is complicated by the fact that the correlations are altered in the process of transforming the intermediate normally distributed multivariate data. By developing an equation for computing a required intermediate correlation between pairs of normally distributed variables, Vale and Maurelli extended Fleishman's methodology, such that, when transformed into non-normal variables, the desired final correlation is obtained (1983:465-467). The equation is given as;

$$\rho_{Y_1;Y_2} = \rho_{X_1;X_2}\left(b_1b_2 + 3b_1d_2 + 3d_1b_2 + 9d_1d_2\right) + \rho^2_{X_1;X_2}\left(2c_1c_2\right) + \rho^3_{X_1;X_2}\left(6d_1d_2\right)$$

where; $\rho_{X_1;X_2}$ is the intermediate correlation; $\rho_{Y_1;Y_2}$ is the final correlation; and, b_1, c_1, d_1 and b_2, c_2, d_2 are the Fleishman transformation constants.

Two sets of simulations were run with each set consisting of three independent simulations. For all the simulations each security was presumed to have a mean daily return of 0.001 and a standard deviation of daily returns of 0.022. These represent values that are consistent with the averages presented in chapter three. Within each set of simulations different standardized third and fourth moments were used. One simulation was conducted assuming these moments were equal to zero for both securities (normally distributed returns assumption). A second assumed both securities had standardized third moments of zero and fourth moments of 7.5 (equivalent to a kurtosis of 4.5). Finally, a third was based on the assumption that both securities had standardized third moments of 0.5 and fourth moments of three (equivalent to a kurtosis of zero). The choice of moments was again based upon the findings of chapter three, and selected to investigate, in particular, the implications of the leptokurtic nature of returns on covariance estimation. The two sets of simulations using different correlations were conducted in order to investigate the sensitivity of the results to the underlying correlation between the two securities. Correlations of 0.25 and 0.50 were selected for the purpose based on sample evidence from prior research (Page, 1986:41)[17].

Once the data had been simulated, descriptive statistics for the "bivariate" sample were computed and used to assess the reliability of the simulation methodology.

(b) The underlying returns were next converted into three sets of observed returns series[18]. The first set involved adjusting the series to reflect the market microstructure frictions, excluding the non-trade aspect, as defined by the Cohen et. al. $\gamma_{j,t,m}$. The second set was adjusted to reflect the degree of thin trading as defined by the Dimson $\theta_{j,t,i}$. The third set incorporated both adjustments and therefore represents the composite, or total simulation. The choice of daily trading probabilities of 0.90, 0.50, and 0.10 was based on the output presented in table 4.1 and table 4.2, and discussed in section 4.2.3. These values are also consistent with trading deciles one, five and nine given in Bradfield and Kroon for the South African market over the period January 1981 to December 1986 (1990:16). It must be noted that, for both this and the previous step in the simulation procedure, the choice of parameters reflects a compromise between the desire to undertake as comprehensive an analysis as possible by using a wide range of parameter selections, and the cost of the significant processing time necessary to undertake each simulation sequence[19].

(c) The five years of daily observed returns were transformed into weekly (five-daily) and monthly (four-weekly) series and the descriptive statistics computed. For each series of returns an additional series was also computed to flag whether or not a trade had taken place on at least one day during the week or month concerned. This was necessary to allow for the possibility of a zero return even when trading had taken place. The significant difference between the procedure adopted here and that used by Bradfield and Kroon is that this research allows for non-synchronous trading within each week. By directly generating weekly data their study did not allow for the situation where trading occurs within the same week but on different days (1990: 7-8).

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[17]Under the assumption that the average of the correlations between individual securities and an overall market index represents a slightly downwardly biased estimate of the communality, reasonable correlation correlation estimates for use in the simulation can be inferred from coefficients of determination obtained when running market model regressions. The results of Page (1986) produced values for these coefficients of the order of 0.05 to 0.30.

[18]The simulation presumes that when trading occurs it does so at the close. Intra-day trading activity is not allowed for in the simulation because of the vast computer resources necessary to simulate such data.

[19]Each of the six simulations ran for a total elapsed time of 2:22:00.0 and used 1:01:00.0 of CPU time when run on the University of Cape Town VAX 6000-330.

(d) Estimation of the correlation between the two securities using the observed monthly, weekly and daily returns was undertaken using a series of approaches. The first of these involved simply using the closing prices (period-to-period observed returns) and reflects the approach of replacing non-trading day prices with the most recent traded price. The second method used is a variant of the Williams-Scholes approach which involves only computing returns for those periods when two consecutive traded prices exist (Scholes and Williams, 1977). The estimated correlation is then based on the limited time series of returns computed when both securities trade in consecutive periods. By implication therefore, this approach, in common with the third method, can only be employed when supplementary data on trading volumes is available. Method three used a trade-to-trade approach where returns were computed over the differing length intervals spanning the periods when both securities traded. As the time series of returns calculated in this fashion are not measured over equal intervals, the means, variances and covariances were computed using the formulae;

$$\overline{X} = \sum_{i=1}^{n} X_i \Big/ \sum_{i=1}^{n} t_i ;$$

$$s_X^2 = \frac{1}{n-1} \sum_{i=1}^{n} t_i \left(\frac{X_i}{t_i} - \overline{X} \right)^2 \text{; and,}$$

$$s_{X;Y} = \frac{1}{n-1} \sum_{i=1}^{n} t_i \left(\frac{X_i}{t_i} - \overline{X} \right) \left(\frac{Y_i}{t_i} - \overline{Y} \right)$$

where; X and Y are the trade-to-trade returns; t_i is the number of periods contained within trade-to-trade interval i; and, n is the number of trade-to-trade returns in the sample.

Finally, the Cohen et. al. procedure outlined in section 4.2.2 was used (1983b:276). This procedure involved summing leading and lagging covariance terms computed using the full series of period-to-period observed returns. On the basis of the findings of Bradfield and Kroon (1990) and Bradfield and Barr (1989a) between zero and three leads and lags were used in the simulations.

Steps **(a)** through **(d)** were repeated one thousand times and summary descriptive statistics computed for each set of estimates. In contrast to the method of Scholes and Williams, box and whisker plots were used to highlight the bias and variability of the sample estimates (Dillon, Madden, Firtle, 1990:474). Scholes and Williams plotted deviations of the estimates from the true parameter values in their comparison of measured and true betas (1977:324).

4.3.2 Results and discussion

The results of the simulations are presented in three sections. The impact of non-normalities, thin trading and market microstructure on the distributional characteristics of individual security returns are discussed first. Thereafter, the effect on the pairwise correlation estimates is assessed. Finally, the implications of using adjustment procedures to reduce any biases in the estimates is examined, as well as the efficiency of these different approaches.

Figures 4.1 to 4.4 present box and whisker plots of the first four moment based statistics over the one thousand iterations for each of the three security distributions examined, namely D(0.001;0.022;0;0), D(0.001;0.022;0;4.5) and D(0.001;0.022;0.5;0)[20]. The plots contained within each figure give the results for daily, weekly (five-daily) and monthly (four-weekly) data. The robustness of the simulation procedure and the random number generator used is confirmed by the box and whisker plots labeled (a) in each of the figures. The distribution of the one thousand first four moment based estimates using the underlying return series is consistent with the means and variances of these estimates one should obtain from repeated unbiased random sampling of the data.

From figure 4.1 it is clear that the effects of thin trading and market microstructure on the estimation of the mean return is negligible. As expected, the range of means for the daily data is consistent with the sample size of 1300 observations and population mean and standard deviation of 0.001 and 0.022. Based on these values the 99.9% confidence interval for the mean should span a theoretical range of 0.0040. Similarly, for the weekly and monthly data where the sample sizes are 260 and sixty-five, the 99.9% confidence limits cover theoretical ranges of 0.0203 and 0.0842 respectively[21].

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[20]The notation $D(\mu;\sigma;\gamma;\kappa)$ refers to a distribution having a mean μ, a standard deviation σ, a skewness γ, and a kurtosis κ.

[21]The independence of the daily returns implies that returns measured over n day intervals will have a mean of $n\mu$ and a variance of $n\sigma^2$, and that the 99.9% confidence interval for the mean should span the theoretical range $2t_{n-1;0.005}\,\sigma/\sqrt{n}$.

Figure 4.1 Simulation box and whisker plots for the average return estimates[22]

Key

a : underlying return series

Cohen et. al. microstructure effects only

b : return series including the microstructure effects

Dimson thin trading effects

c: returns allowing for a trading probability of 0.1

d : returns allowing for a trading probability of 0.5

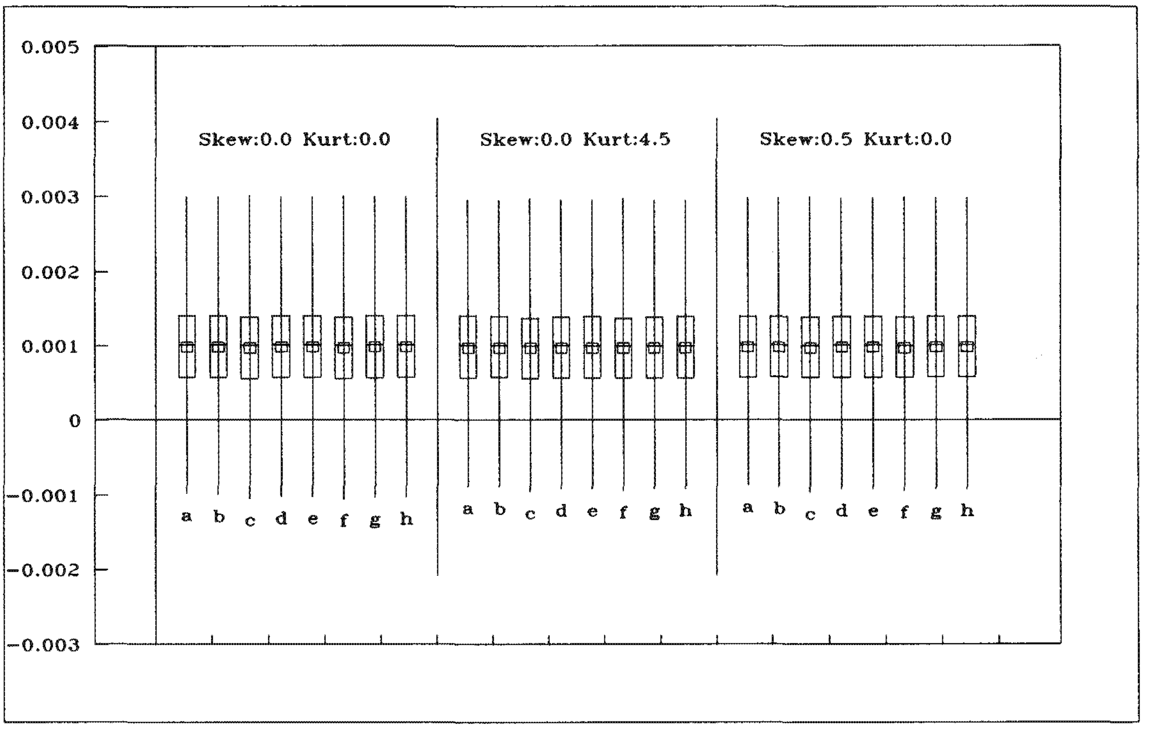
e : returns allowing for a trading probability of 0.9

Cohen et. al. and Dimson effects

f: returns allowing for a trading probability of 0.1

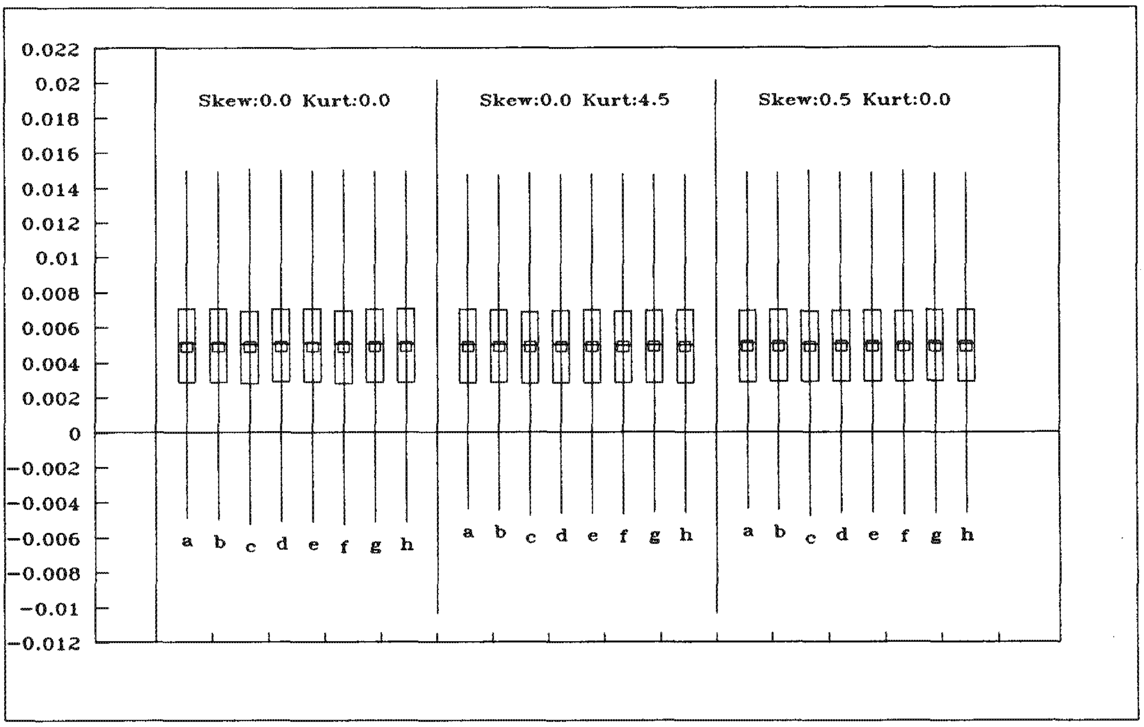
g : returns allowing for a trading probability of 0.5

h : returns allowing for a trading probability of 0.9

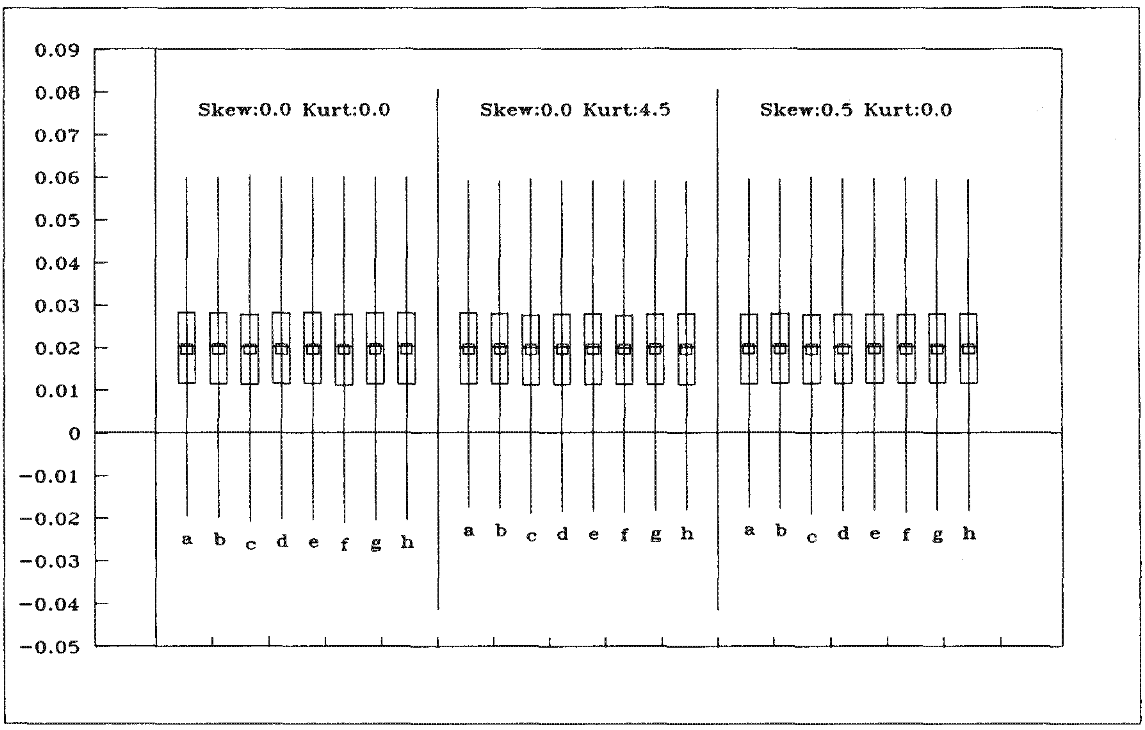


Panel A : Daily data

[22]The box and whisker plots presented in figures 4.1 to 4.5 give the range, first and third quartiles, median and average for each set of simulations.



Panel B : Weekly (five days) data



Panel C : Monthly (twenty days) data

The impact of thin trading on the estimation of security standard deviation is to increase the variability of the estimates. This finding is evident when the range for the underlying returns series (box and whisker plots (a)) is compared to the trend in the series allowing for thin trading levels ranging from 0.1 to 0.9 presented in figure 4.2 (box and whisker plots (c) (d) (e)). Thin trading does not however induce bias in the estimate. All the bias that is evident in the plots is caused by the smoothing effect of the lag in observed price changes due to the market microstructure effects of inventory balancing and transaction costs. Interestingly, the impact of market microstructure is shown to be more significant for well traded securities. For the thinner trading securities, the thin trading itself dominates and increasingly eliminates the limited autocorrelation effects on the variance estimates. Non-normality of underlying returns, while not biasing the estimate, reduces the confidence in point estimates of the population parameter by increasing the range of the estimates.

There is also a reduction in bias from daily through to monthly data. This reduction occurs because, as discussed in section 3.1, the microstructure induced delays in price changes are of short duration. For the monthly data in particular, the thin trading effect dominates and, while there is a reduction in the efficiency of point estimates, most of the bias is eliminated.

The third and fourth moment estimates presented in figure 4.3 and figure 4.4 are upwardly biased for very thinly traded securities and the confidence intervals for the population parameters based on sample estimates become significantly wider. This is particularly the case for the fourth moment where estimates in excess of three hundred were obtained for the thinly traded securities when using daily data. These extremely high values highlight the problem of using closing price data to estimate the distributional characteristics of securities in thinly traded markets. Finally, consistent with what should occur when sampling from non- stable Paretian populations, both the skewness and kurtosis reduce as the returns are measured over longer time intervals.

Figure 4.2 Simulation box and whisker plots for the return standard deviation estimates

Key

a : underlying return series

Cohen et. al. microstructure effects only

b : return series including the microstructure effects

Dimson thin trading effects

c: returns allowing for a trading probability of 0.1

d : returns allowing for a trading probability of 0.5

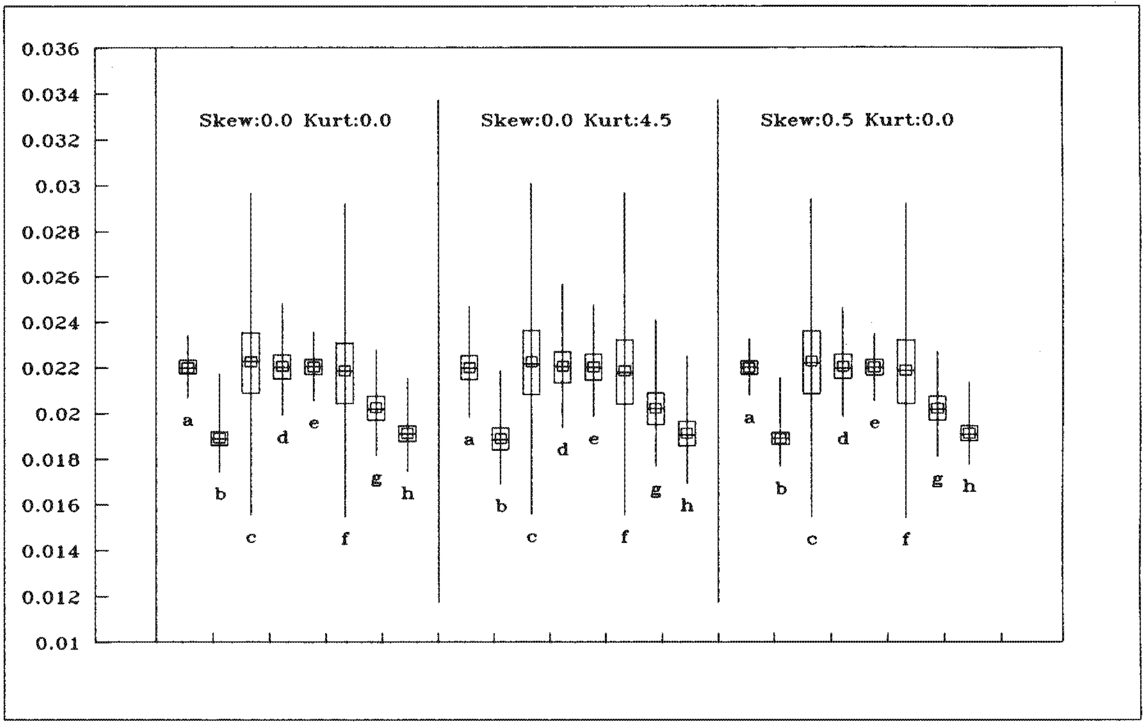
e : returns allowing for a trading probability of 0.9

Cohen et. al. and Dimson effects

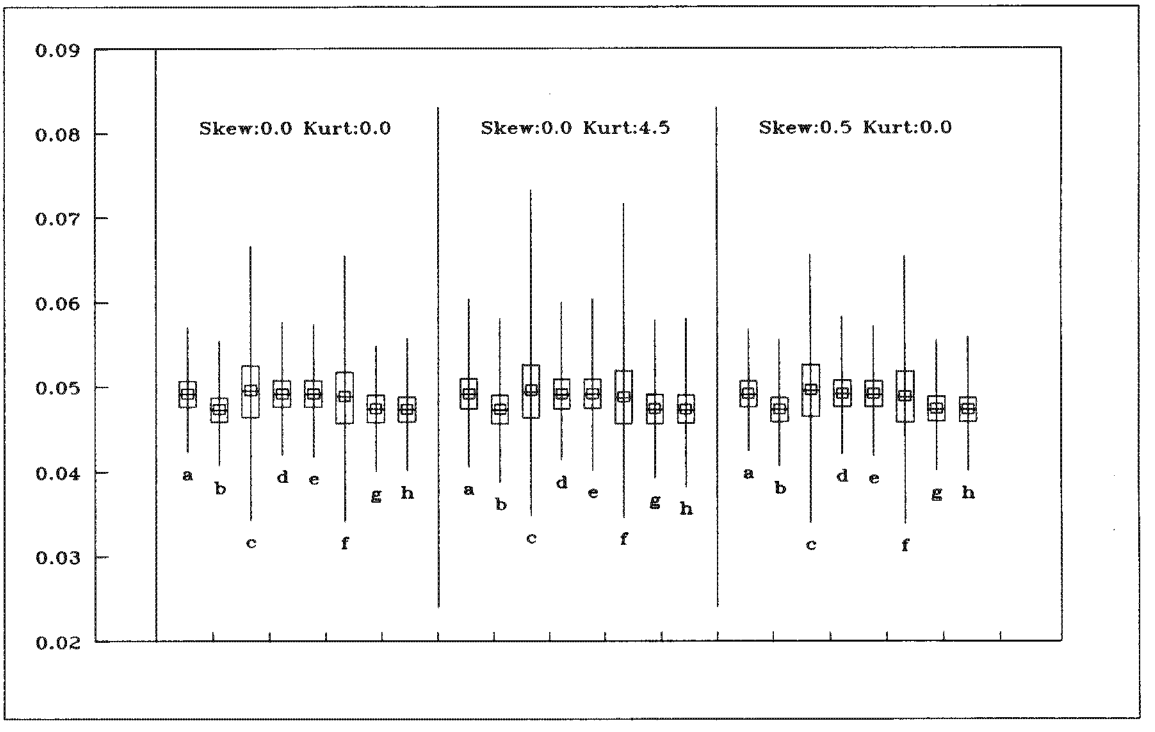
f: returns allowing for a trading probability of 0.1

g : returns allowing for a trading probability of 0.5

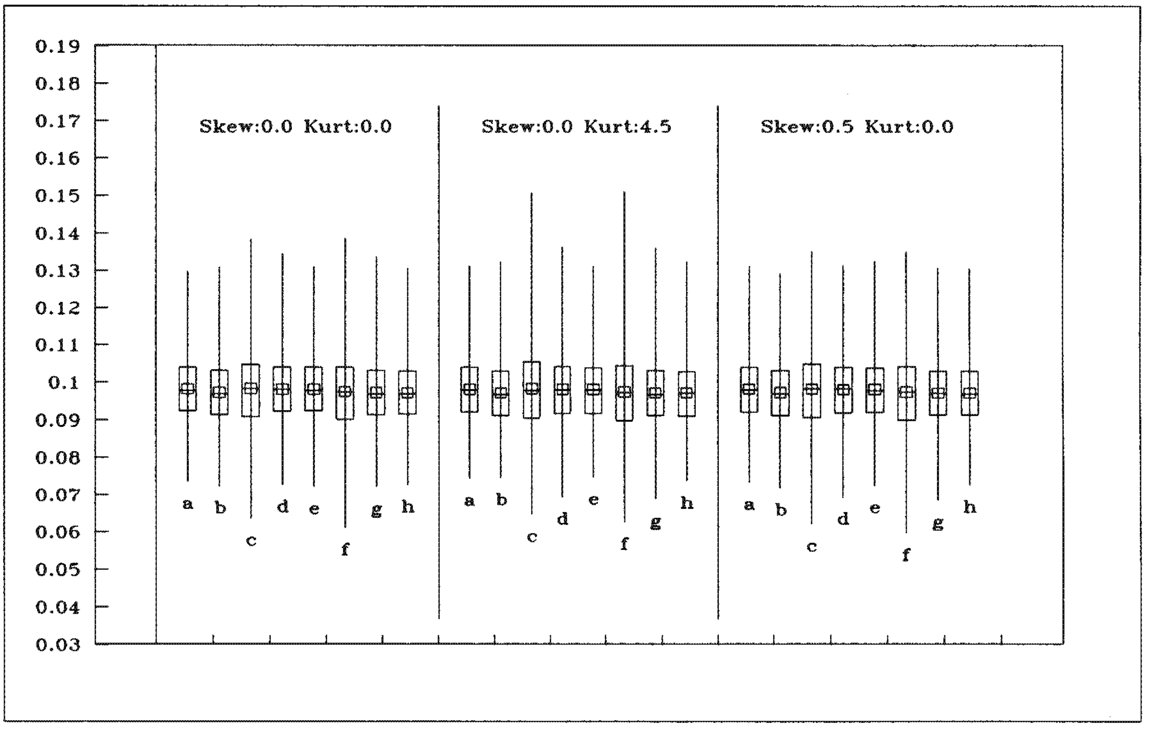
h : returns allowing for a trading probability of 0.9



Panel A : Daily data



Panel B : Weekly (five days) data



Panel C : Monthly (twenty days) data

Figure 4.3 Simulation box and whisker plots for the return third moment estimates

Key

a : underlying return series

Cohen et. al. microstructure effects only

b : return series including the microstructure effects

Dimson thin trading effects

c: returns allowing for a trading probability of 0.1

d : returns allowing for a trading probability of 0.5

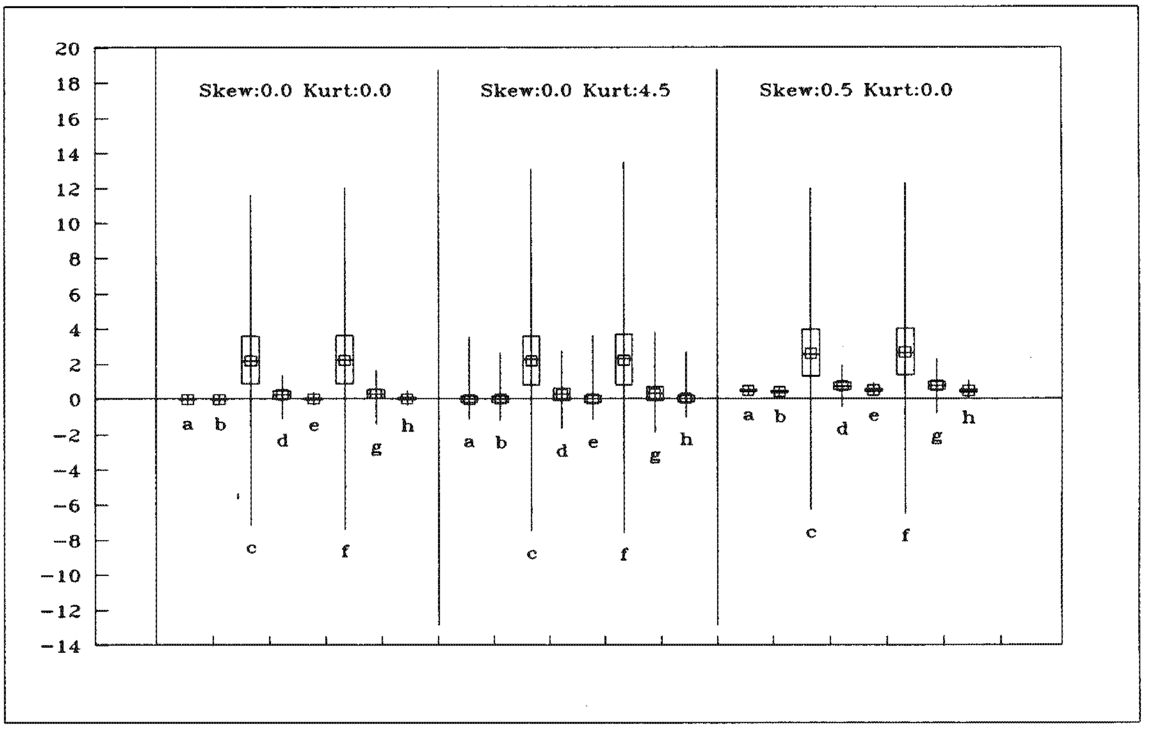
e : returns allowing for a trading probability of 0.9

Cohen et. al. and Dimson effects

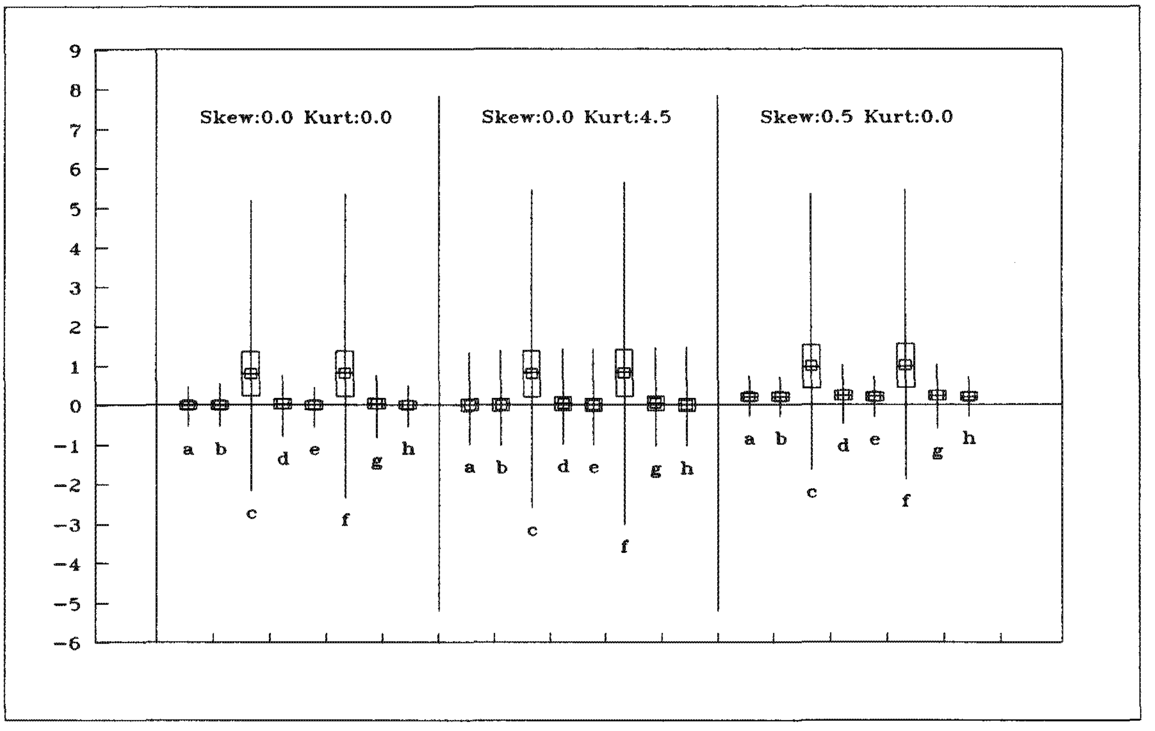
f: returns allowing for a trading probability of 0.1

g : returns allowing for a trading probability of 0.5

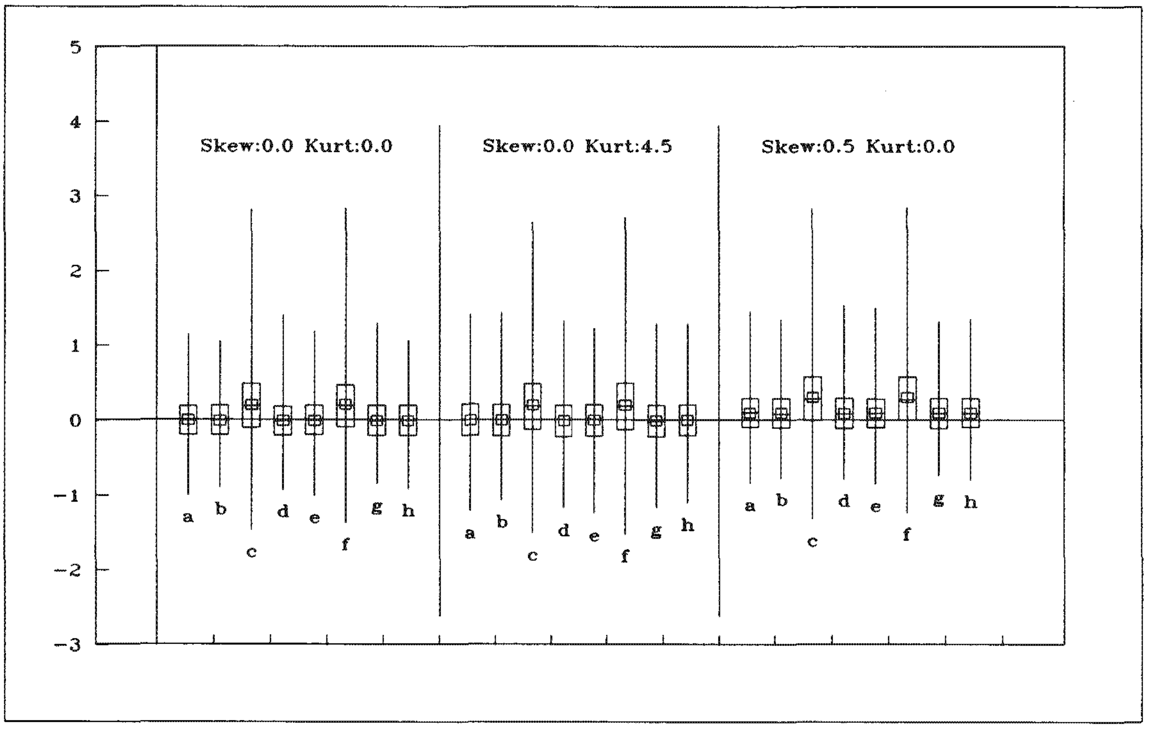
h : returns allowing for a trading probability of 0.9



Panel A : Daily data



Panel B : Weekly (five days) data



Panel C : Monthly (twenty days) data

Figure 4.4 Simulation box and whisker plots for the return fourth moment estimates

Key

a : underlying return series

Cohen et. al. microstructure effects only

b : return series including the microstructure effects

Dimson thin trading effects

c: returns allowing for a trading probability of 0.1

d : returns allowing for a trading probability of 0.5

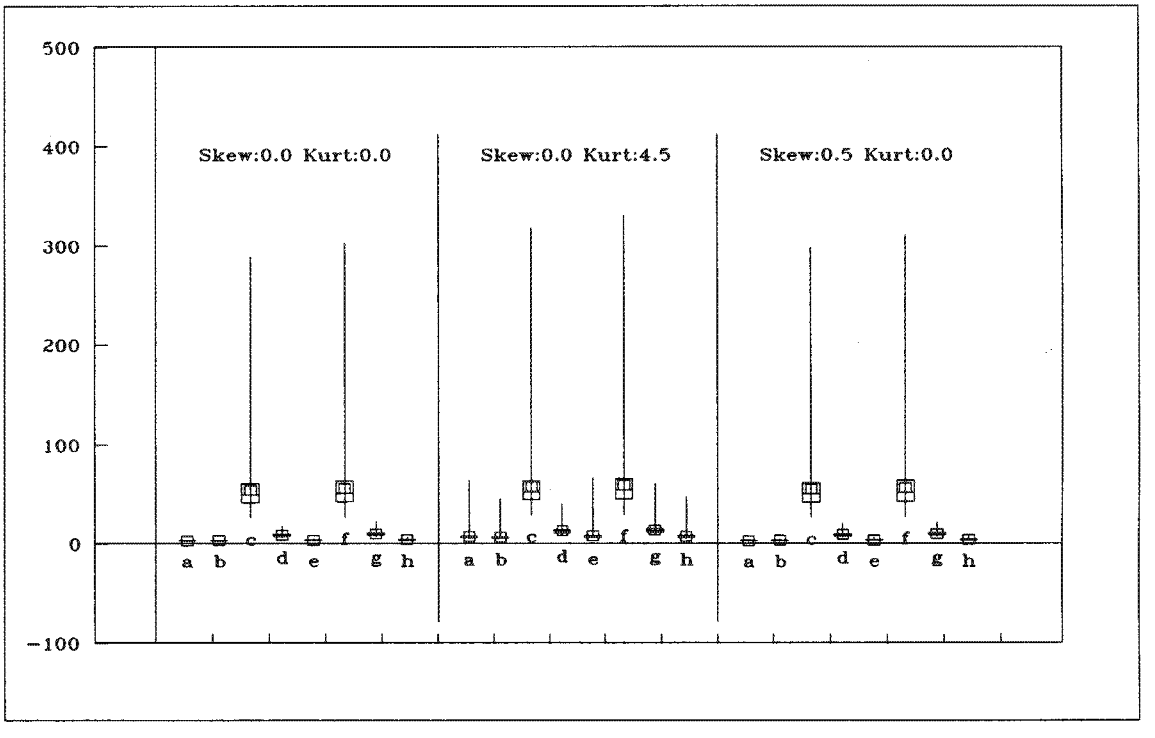
e : returns allowing for a trading probability of 0.9

Cohen et. al. and Dimson effects

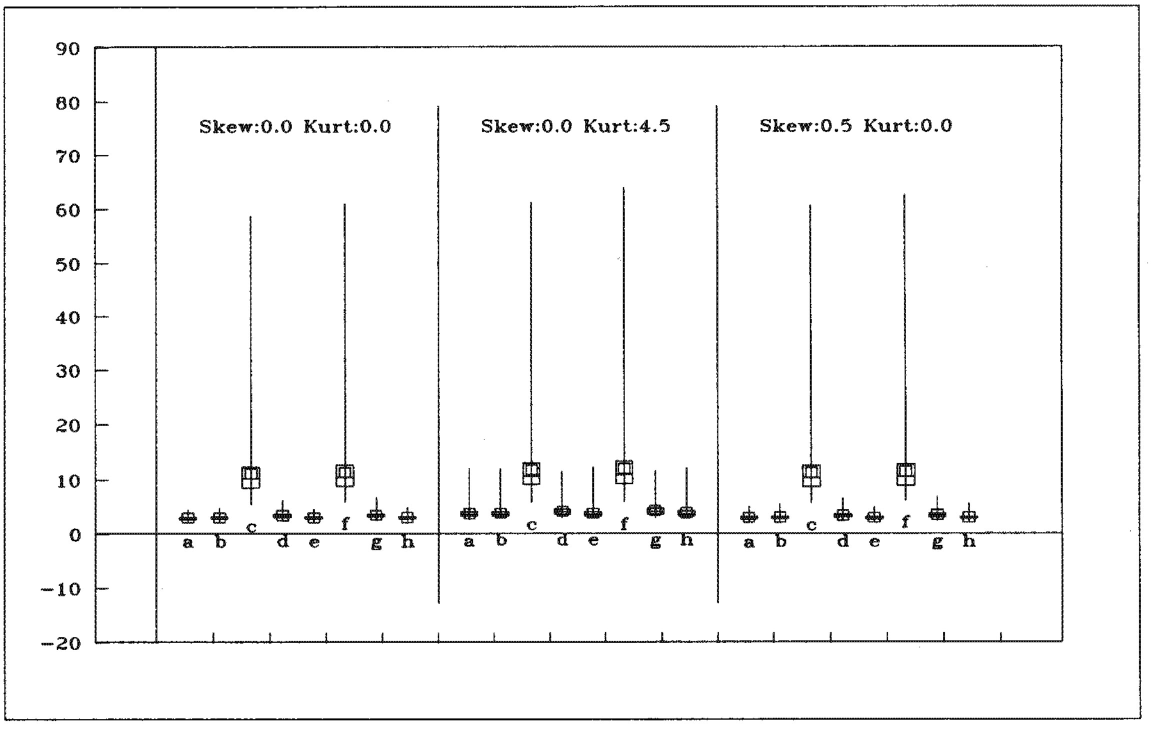
f: returns allowing for a trading probability of 0.1

g : returns allowing for a trading probability of 0.5

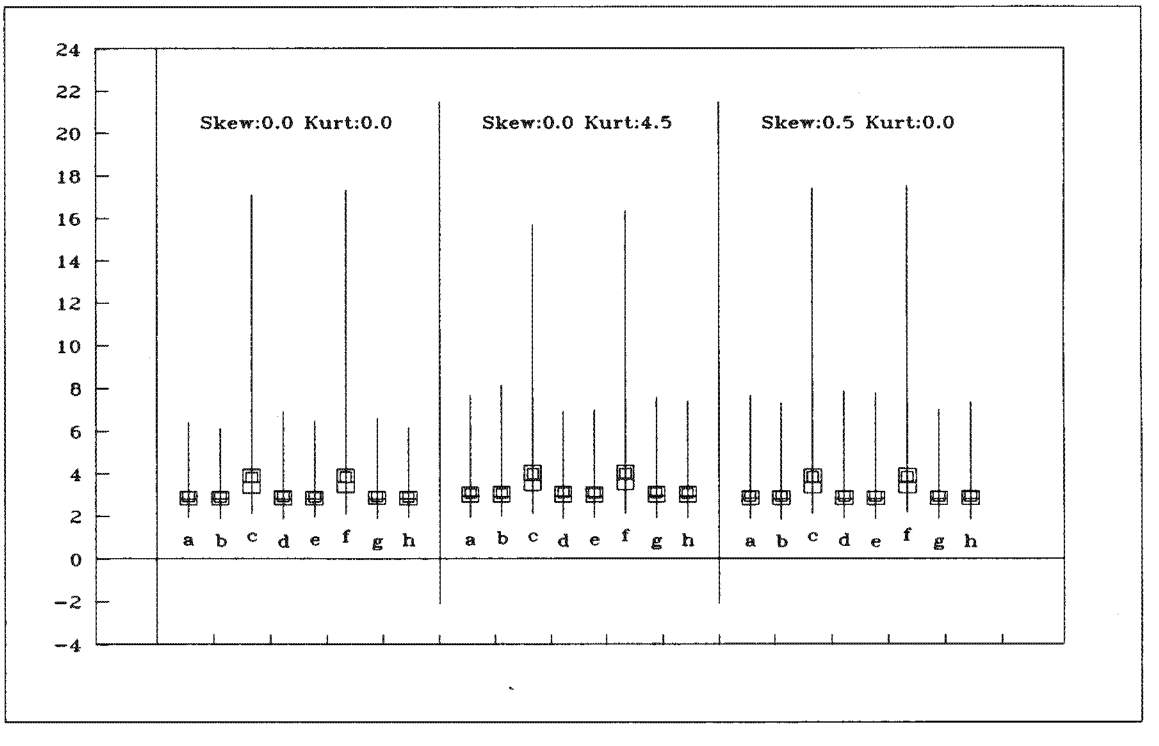
h : returns allowing for a trading probability of 0.9



Panel A : Daily data



Panel B : Weekly (five days) data



Panel C : Monthly (twenty days) data

As shown by the results of the simulations presented in figure 4.5 the impact of thin trading and market microstructure on pairwise correlation estimates can be significant. The results presented in the figure are based on an assumed population correlation of 0.50 and, given that the findings are identical to those for the simulations run using the lower population correlation of 0.25, only the one set of results is discussed in detail.

In contrast with the results for the variance estimation, the major bias in the estimation of correlation is caused by thin trading. As modeled, the autocorrelation effect causes a significantly smaller downward bias in the correlation estimate. In all cases where one of the securities traded thinly the bias is extreme with average estimates ranging from ten percent of the population value for daily data to sixty percent for monthly data. Additionally, as can be seen from the box and whisker plots (plots (c) (d) (e) and (i) (j) (k)), in excess of eighty percent of the one thousand estimates are below the population value for the monthly data, and all the estimates are below the population value for both the daily and weekly data. As the trading frequency of both securities increases the bias is reduced, and for monthly data when both securities have daily trading probabilities of fifty percent and above it is less than ten percent. While this result might suggest that the problem of bias is not that significant for "well" traded securities it must be recognized that a daily trading probability of fifty percent implies that a security is expected to trade during fifty weeks per annum. As shown in figure 3.1, this frequency represents only a limited proportion of securities trading on the Johannesburg Stock Exchange. The importance of bias in correlation and covariance estimation in empirical research using South African and other thinly traded market data should therefore not be underrated.

Figure 4.5 Simulation box and whisker plots of the pairwise Pearson correlation estimates

Key

a : underlying return series

Cohen et. al. microstructure effects only

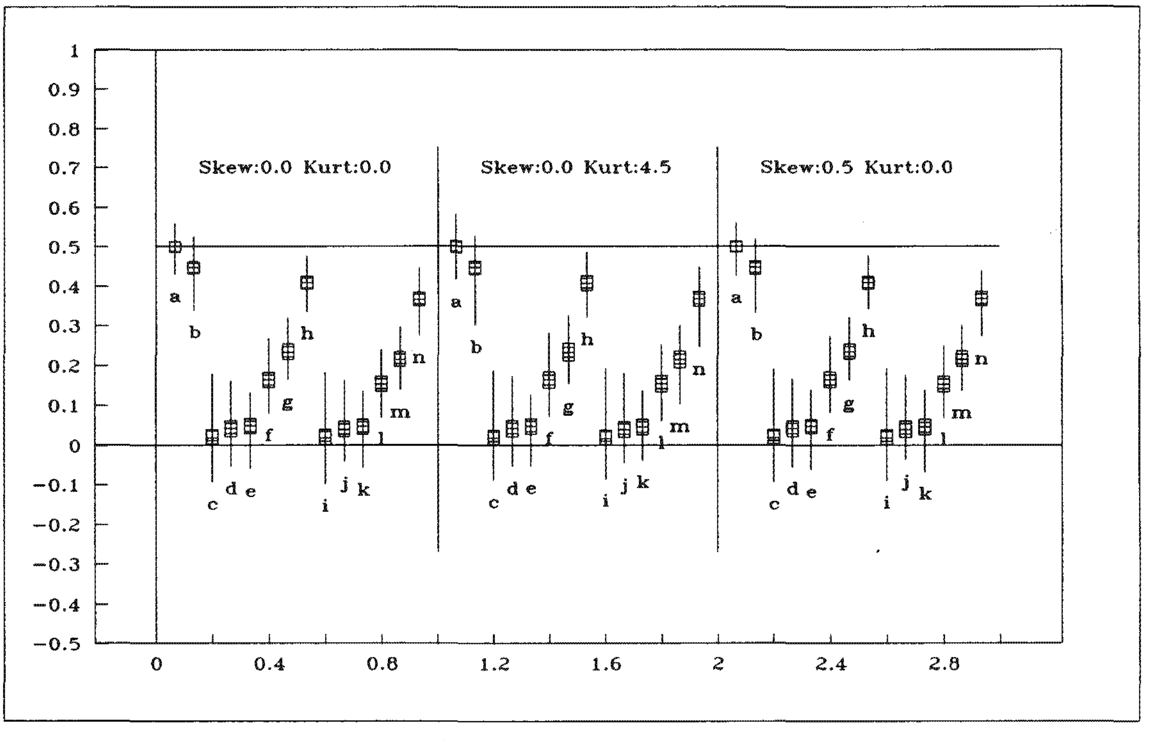
b : return series including the microstructure effects

Dimson thin trading effects

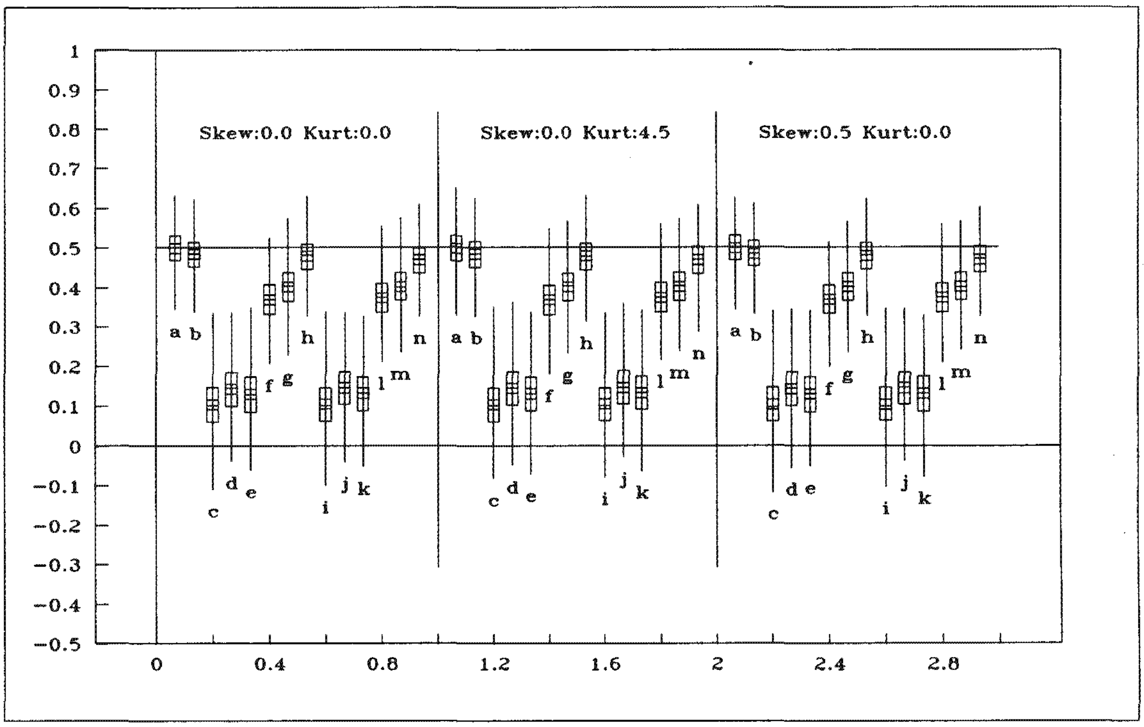
c : security trading probabilities of 0.1 and 0.1
d : security trading probabilities of 0.1 and 0.5
e : security trading probabilities of 0.1 and 0.9
f : security trading probabilities of 0.5 and 0.5
g : security trading probabilities of 0.5 and 0.9
h : security trading probabilities of 0.9 and 0.9

Cohen et. al. and Dimson effects combined

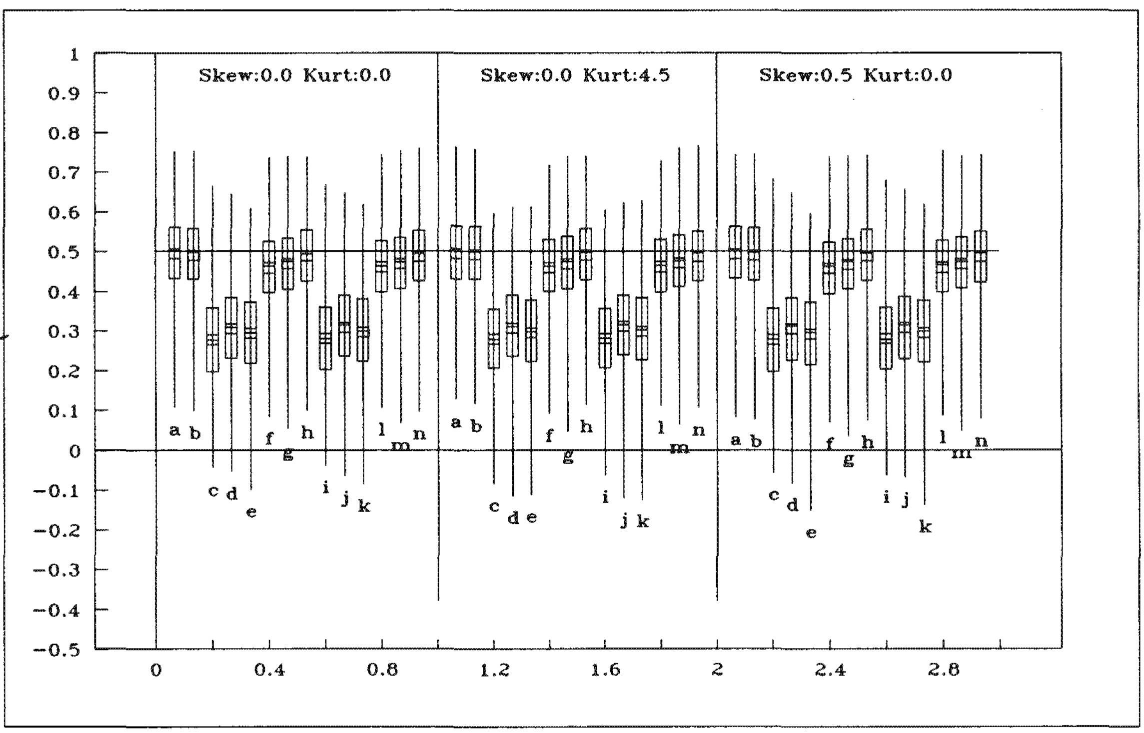
i : security trading probabilities of 0.1 and 0.1
j : security trading probabilities of 0.1 and 0.5
k : security trading probabilities of 0.1 and 0.9
l : security trading probabilities of 0.5 and 0.5
m : security trading probabilities of 0.5 and 0.9
n : security trading probabilities of 0.9 and 0.9



Panel A : Daily data



Panel B : Weekly (five days) data



Panel C : Monthly (twenty days) data

The extent to which the procedures of Cohen et. al., Scholes and Williams, and the trade-to-trade approaches impact on pairwise correlation estimation is shown in tables 4.5 to 4.7. The tables give the mean and standard deviation of the one thousand correlation estimates computed for each of the market imperfection conditions simulated. Additionally, the mean and standard deviation of the correlation estimates using the underlying and observed returns series are also presented.

Two factors need to be considered when assessing the distributional characteristics of the sample correlation coefficient. Firstly, it is usually assumed that the two variables are normally distributed. The impact of departures from normality on the traditional t-test and F-test used in hypothesis testing has been investigated by such researchers as Bartlett (1935) and Srivastava (1958). The conclusions suggest that the tests give *quite good results even for considerable departures from normality*, kurtosis in particular only has a small effect on the t-test, and that the impact of non-normality diminishes with sample size (Keeping, 1962:207). Secondly, the Student t-distribution used to test the significance of the correlation coefficient assumes that the parent population is uncorrelated. When this condition does not hold the distribution of the sample correlation becomes *quite complicated* and, when the absolute value of the population correlation is near to one, the distribution of the sample value is *far from normal* (Keeping, 1962:307).

An expression for the distribution of the sample correlation when the population value is not equal to zero has been developed by David (1938). From it, series expressions for the cumulants can be derived which give the following for the expected value and variance of the correlation estimate (Keeping, 1962:307).

$$E\left(\hat{\rho}_{X;Y}\right) = \rho - \frac{\rho\left(1-\rho^2\right)}{2(n-1)}\left[1 - \frac{1-9\rho^2}{4(n-1)} + \ldots\right]; \text{ and,}$$

$$\sigma^2_{\hat{\rho}} = \frac{\left(1-\rho^2\right)^2}{n-1}\left[1 - \frac{11\rho^2}{2(n-1)} + \ldots\right]$$

where; n is the sample size, and ρ is the population correlation[23].

[23]The David approximation is used in this research in preference to Fisher's transformation which involves transforming the correlation estimates into an approximately normally distributed variable using the relation $z = \frac{1}{2}\ln\left[(1+\rho)/(1-\rho)\right]$. This transformed variable has a variance of $1/(n-3)$ (Fisher, 1938).

Table 4.5 Daily data - means and standard deviations of the one thousand correlation estimates using different adjustment procedures

Daily trading probabilities	Underlying returns	Observed returns	Trade-to-trade	Scholes-Williams	Cohen et. al. One lead & lag	Cohen et. al. Two leads & lags	Cohen et. al. Three leads & lags
	Skewness : 0.000 Kurtosis : 0.000 - mean / standard deviation						
0.1/0.1	0.5013/0.021	0.0235/0.030	0.4549/0.272		0.0684/0.050	0.1106/0.064	0.1489/0.072
0.1/0.5		0.0418/0.029	0.4828/0.097	0.4630/0.329	0.1093/0.050	0.1630/0.065	0.2058/0.077
0.1/0.9		0.0464/0.028	0.4805/0.073	0.4200/0.279	0.1051/0.054	0.1539/0.070	0.1945/0.084
0.5/0.5		0.1549/0.028	0.4716/0.042	0.4424/0.094	0.3469/0.048	0.4669/0.063	0.5253/0.078
0.5/0.9		0.2166/0.027	0.4620/0.032	0.4454/0.049	0.4103/0.051	0.5295/0.069	0.5742/0.085
0.9/0.9		0.3681/0.025	0.4509/0.025	0.4473/0.028	0.5450/0.054	0.6490/0.075	0.6559/0.094
	Skewness : 0.000 Kurtosis : 4.500 - mean / standard deviation						
0.1/0.1	0.5014/0.023	0.0230/0.030	0.4588/0.271		0.0681/0.051	0.1107/0.064	0.1494/0.073
0.1/0.5		0.0419/0.031	0.4842/0.095	0.4573/0.364	0.1099/0.052	0.1637/0.067	0.2069/0.078
0.1/0.9		0.0464/0.029	0.4810/0.074	0.4231/0.287	0.1056/0.055	0.1542/0.071	0.1957/0.085
0.5/0.5		0.1552/0.031	0.4718/0.044	0.4456/0.101	0.3474/0.051	0.4670/0.066	0.5259/0.078
0.5/0.9		0.2163/0.030	0.4621/0.034	0.4461/0.053	0.4105/0.054	0.5292/0.071	0.5744/0.086
0.9/0.9		0.3679/0.028	0.4509/0.028	0.4474/0.031	0.5446/0.056	0.6480/0.078	0.6550/0.095
	Skewness : 0.500 Kurtosis : 0.000 - mean / standard deviation						
0.1/0.1	0.5011/0.021	0.0236/0.030	0.4594/0.269		0.0683/0.051	0.1102/0.065	0.1488/0.073
0.1/0.5		0.0417/0.029	0.4825/0.098	0.4588/0.323	0.1090/0.052	0.1631/0.066	0.2061/0.078
0.1/0.9		0.0459/0.029	0.4807/0.073	0.4204/0.277	0.1047/0.055	0.1535/0.071	0.1945/0.084
0.5/0.5		0.1548/0.029	0.4714/0.043	0.4431/0.096	0.3470/0.048	0.4668/0.064	0.5255/0.078
0.5/0.9		0.2171/0.027	0.4617/0.033	0.4454/0.050	0.4107/0.051	0.5299/0.069	0.5749/0.085
0.9/0.9		0.3684/0.025	0.4511/0.026	0.4475/0.028	0.5453/0.054	0.6492/0.076	0.6561/0.094

The mean and standard deviations for the underlying returns correlation estimates presented in the first column of the tables are consistent with the theoretical values derived from the above equations. For a population correlation of 0.5 and daily, weekly and monthly sample sizes of 1300, 260 and sixty-five respectively the equations give values of 0.4999 and 0.0208, 0.4993 and 0.0465, and 0.4971 and 0.0927. The effects of the thin trading and market microstructure illustrated in figure 4.5 are represented by the unadjusted correlation estimation results given in the observed returns columns. The significant downward bias, particularly for the daily data, is again confirmed by the tables. For the thinner trading securities the increased variability of the estimates is also evident. Finally, as suggested above, the implications of skewness and kurtosis in security returns appears small.

Table 4.6 Weekly data - means and standard deviations of the one thousand correlation estimates using different adjustment procedures

Daily trading probabilities	Underlying returns	Observed returns	Trade-to-trade	Scholes-Williams	Cohen et. al. One lead & lag	Cohen et. al. Two leads & lags	Cohen et. al. Three leads & lags
	Skewness : 0.000 Kurtosis : 0.000 - mean / standard deviation						
0.1/0.1	0.5002/0.046	0.1054/0.063	0.4388/0.125	0.3381/0.345	0.2590/0.103	0.3521/0.130	0.4059/0.153
0.1/0.5		0.1460/0.062	0.4062/0.081	0.3465/0.141	0.3022/0.108	0.3810/0.139	0.4291/0.164
0.1/0.9		0.1328/0.063	0.3937/0.083	0.3276/0.142	0.2847/0.110	0.3721/0.138	0.4205/0.162
0.5/0.5		0.3734/0.053	0.3981/0.053	0.3949/0.055	0.5196/0.109	0.5205/0.154	0.5152/0.183
0.5/0.9		0.4020/0.053	0.4165/0.053	0.4151/0.054	0.5222/0.111	0.5217/0.154	0.5155/0.186
0.9/0.9		0.4696/0.049	0.4696/0.049	0.4696/0.049	0.5261/0.112	0.5216/0.157	0.5166/0.188
	Skewness : 0.000 Kurtosis : 4.500 - mean / standard deviation						
0.1/0.1	0.5001/0.047	0.1055/0.063	0.4378/0.126	0.3377/0.348	0.2601/0.103	0.3539/0.128	0.4065/0.150
0.1/0.5		0.1471/0.063	0.4067/0.082	0.3474/0.144	0.3037/0.110	0.3831/0.139	0.4308/0.163
0.1/0.9		0.1338/0.065	0.3950/0.083	0.3285/0.143	0.2869/0.112	0.3741/0.139	0.4222/0.162
0.5/0.5		0.3739/0.055	0.3986/0.055	0.3954/0.057	0.5199/0.110	0.5221/0.153	0.5165/0.182
0.5/0.9		0.4024/0.054	0.4168/0.055	0.4154/0.056	0.5232/0.112	0.5229/0.154	0.5164/0.186
0.9/0.9		0.4697/0.050	0.4697/0.050	0.4697/0.050	0.5265/0.112	0.5226/0.156	0.5173/0.188
	Skewness : 0.500 Kurtosis : 0.000 - mean / standard deviation						
0.1/0.1	0.5003/0.046	0.1050/0.063	0.4399/0.125	0.3428/0.345	0.2582/0.105	0.3514/0.131	0.4056/0.154
0.1/0.5		0.1459/0.062	0.4065/0.082	0.3469/0.140	0.3019/0.109	0.3809/0.140	0.4288/0.167
0.1/0.9		0.1323/0.064	0.3937/0.083	0.3281/0.144	0.2844/0.112	0.3720/0.140	0.4206/0.166
0.5/0.5		0.3737/0.053	0.3983/0.053	0.3950/0.055	0.5202/0.110	0.5205/0.156	0.5153/0.185
0.5/0.9		0.4023/0.053	0.4167/0.053	0.4152/0.054	0.5228/0.113	0.5220/0.156	0.5157/0.187
0.9/0.9		0.4700/0.049	0.4700/0.049	0.4700/0.049	0.5273/0.114	0.5222/0.159	0.5167/0.189

Substantial reductions in the bias induced by thin trading occurs when the different adjustment approaches are used. The efficiency of the various approaches does however differ quite markedly.

A comparison of the trade-to-trade and Scholes-Williams approaches shows the first produces an average correlation that is consistently higher than that produced by the Scholes-Williams method. The efficiency of the estimate is also improved because of the larger sample size generally used. For the trade-to-trade method the price data is used for all periods during which both securities trade while for the Scholes-Williams method only those returns calculated when both securities trade in common consecutive periods are used. The reduced comparative benefit of the trade-to-trade approach as the returns are measured over longer time periods results from the definition of consecutive trading intervals. In the simulation procedure used in this study cognizance was not taken of the exact time within the trading/measurement interval that each security traded. A final disadvantage of the Scholes-Williams approach is that it cannot always be used for thinly traded securities, particularly when the correlation is to be estimated using daily data. For the daily data presented in table 4.5, over the one thousand iterations the correlation could not be estimated when both securities had independent trading probabilities of ten percent and the summary results are therefore not presented in the table[24].

Table 4.7 Monthly data - means and standard deviations of the one thousand correlation estimates using different adjustment procedures

Daily trading probabilities	Underlying returns	Observed returns	Trade-to-trade	Scholes-Williams	Cohen et. al. One lead & lag	Cohen et. al. Two leads & lags	Cohen et. al. Three leads & lags
	Skewness : 0.000 Kurtosis : 0.000 - mean / standard deviation						
0.1/0.1	0.4957/0.095	0.2814/0.114	0.3667/0.123	0.3493/0.144	0.4520/0.199	0.4594/0.278	0.4545/0.329
0.1/0.5		0.3098/0.114	0.3609/0.117	0.3530/0.124	0.4491/0.208	0.4634/0.276	0.4548/0.332
0.1/0.9		0.2972/0.115	0.3482/0.117	0.3396/0.125	0.4479/0.208	0.4612/0.272	0.4524/0.332
0.5/0.5		0.4622/0.099	0.4622/0.099	0.4622/0.099	0.4803/0.216	0.4669/0.285	0.4603/0.345
0.5/0.9		0.4705/0.099	0.4705/0.099	0.4705/0.099	0.4796/0.213	0.4645/0.282	0.4588/0.346
0.9/0.9		0.4878/0.096	0.4878/0.096	0.4878/0.096	0.4804/0.214	0.4646/0.282	0.4595/0.347
	Skewness : 0.000 Kurtosis : 4.500 - mean / standard deviation						
0.1/0.1	0.4968/0.096	0.2819/0.113	0.3675/0.123	0.3509/0.142	0.4530/0.199	0.4605/0.278	0.4562/0.334
0.1/0.5		0.3124/0.116	0.3635/0.120	0.3558/0.127	0.4503/0.209	0.4645/0.278	0.4571/0.339
0.1/0.9		0.3003/0.117	0.3512/0.121	0.3426/0.128	0.4491/0.209	0.4626/0.274	0.4549/0.339
0.5/0.5		0.4632/0.100	0.4632/0.100	0.4632/0.100	0.4825/0.217	0.4699/0.285	0.4628/0.351
0.5/0.9		0.4715/0.101	0.4715/0.101	0.4715/0.101	0.4816/0.214	0.4679/0.283	0.4615/0.353
0.9/0.9		0.4887/0.097	0.4887/0.097	0.4887/0.097	0.4830/0.214	0.4685/0.282	0.4621/0.354
	Skewness : 0.500 Kurtosis : 0.000 - mean / standard deviation						
0.1/0.1	0.4953/0.095	0.2815/0.115	0.3670/0.124	0.3498/0.144	0.4520/0.198	0.4603/0.276	0.4538/0.327
0.1/0.5		0.3093/0.115	0.3602/0.118	0.3524/0.125	0.4508/0.209	0.4639/0.276	0.4532/0.330
0.1/0.9		0.2962/0.117	0.3470/0.118	0.3385/0.127	0.4496/0.208	0.4614/0.273	0.4508/0.331
0.5/0.5		0.4618/0.100	0.4618/0.100	0.4618/0.100	0.4817/0.216	0.4669/0.284	0.4601/0.343
0.5/0.9		0.4702/0.100	0.4702/0.100	0.4702/0.100	0.4809/0.214	0.4645/0.282	0.4584/0.345
0.9/0.9		0.4875/0.097	0.4875/0.097	0.4875/0.097	0.4815/0.215	0.4639/0.283	0.4587/0.346

The Cohen et. al. procedure needs to be used with some degree of caution. It is clear from the tables that while additional leads and lags reduce the bias in the correlation estimate, the efficiency of the estimate is significantly reduced as well. The number of leads and lags that should optimally be employed when using the approach is contingent upon the relative trading frequency of the two securities. Although not explicitly addressed in this analysis, consideration also needs to be given to what leads and lags to include when the two securities have differing trading probabilities. The reduction in efficiency resulting from the incremental addition of non-synchronous coefficients is explained by the fact that if non-synchronous coefficients added to the synchronous estimate are sufficient to include all the underlying correlation effect then the k random

[24]When the Scholes-Williams approach was used for the daily data, and the two securities had simulated trading probabilities of 0.1 and 0.5, the correlation could only be estimated in 10.5% of the simulations. This figure rose to 93.9% when the securities had trading probabilities of 0.1 and 0.9. In the case of weekly data it was only when both securities had trading probabilities of 0.1 that the coefficient could not always be estimated and was only calculated for 71.9% of the simulations.

variables (correlation estimates) can be viewed as a transformation of k independent random variables. Of these k-1 have an expected value of zero and variance of $1/(n-2)$ and one has an expected value and variance as given in the formulae of David. As such the expected value and variance of the sum of is given by the sum of the values for the independent random variables. For the variance this gives;

$$\sigma_{\hat{\rho}}^2 = \frac{\left(1-\rho^2\right)^2}{n-1}\left[1-\frac{11\rho^2}{2(n-1)}+\dots\right]+\frac{k-1}{n-2}.$$

Abstracting from the non-normality issue the theoretical standard deviations of the correlation estimate for the daily, weekly and monthly data are therefore 0.0444, 0.0996 and 0.2009 when one lead and lag is used; 0.0593, 0.1329 and 0.2685 when two leads and lags are used; and, 0.0711, 0.1594 and 0.3222 when three leads and lags are used. These values are again close to those resulting from the simulations. Additionally, the similarity between the normal, skew and high kurtosis data confirms the conclusions about the robustness of traditional tests under conditions of departure from normality.

While the trade-to-trade approach was found to be consistently better than the Scholes-Williams approach, comparison between the trade-to-trade and Cohen et. al. approaches does not lead to as clear a conclusion. For daily data the trade-to-trade approach provides slightly downwardly biased but reasonably robust estimates. For the Cohen et. al. approach significantly more than three leads and lags are required to eliminate bias when the securities are very thinly traded, and for well traded securities between one and two leads and lags should be employed. The high estimate standard deviation for the trade-to-trade approach when both securities have daily trading probabilities of ten percent results because the sample size is greatly reduced by the exclusion of much of the returns data (all the non-synchronous trading periods). For weekly data the Cohen et. al. procedure consistently gives less biased estimates when the appropriate number of leads and lags are used but suffers from reduced efficiency. If both securities have daily trading probabilities of fifty percent or more the Cohen et. al. procedure with a single lead and lag appears to be the preferable compromise between bias and efficiency. When monthly data is used the results suggest that the trade-to-trade approach is best for well traded securities and, given the definition of interval trading used in the simulations, is equivalent to the Scholes-Williams method and the value obtained using observed data. For thinly traded securities the addition of leads and lags beyond the first does not reduce the bias by much but does greatly reduce the efficiency of the estimate.

In conclusion, for the level of thin trading and market microstructure effects simulated in the study, the trade-to-trade approach appears superior to the others when daily data is used to estimate correlation coefficients even though it results in slightly downwardly biased estimates. For weekly and monthly data however, the Cohen et. al. approach should be adopted. If the trading probabilities for the individual securities cannot be estimated then three leads and three lags should be considered when using weekly data and one lead and lag when using monthly data. Estimation can be improved using the Cohen et. al. approach if records of daily trading volumes are used to estimate the trading probability for each security so that varying leads and lags can be used based on the trading frequency of the securities involved. Finally, while the simulation conditions were derived on the basis of the evidence of the Johannesburg Stock Exchange, the conclusions drawn have relevance for all thinly traded stock exchanges and even for thinly traded securities within the better traded markets such as the New York Stock Exchange, the American Stock Exchange, the Tokyo Stock Exchange, and possibly the London Stock Exchange.

5

The impact of non-normalities and thin trading on factor determination

5.1 Introduction

This chapter extends the analysis of the previous chapter by examining the effect of market microstructure, thin trading and non-normality on the methodologies currently used in empirical research into the number and pricing of the Arbitrage Pricing Theory factors. Over the last decade there has been considerable investigation and debate into this aspect of the theory, and the lack of consensus is evidenced by the fact that the number of priced factors "found" commonly range from one (Trzinka, 1986) to five (Roll and Ross, 1980; Cho, Eun and Senbet, 1986). Much of the debate as to the reason for these inconsistent findings has focused on the small sample properties of the factor analytic techniques used, and on such issues as the convergence or otherwise of the Ross strict factor structure and the more general approximate factor structure of Chamberlain and Rothschild (Ross, 1976; Roll and Ross, 1980; Chamberlain and Rothschild, 1983; Brown, 1989).

The implications of thin trading and non-normality examined in this chapter, while alluded to by many researchers, have only been expressly addressed by Shanken (1987). As discussed further below, a principal components methodology is used in preference to the maximum likelihood procedure of Roll and Ross (1980). This approach is consistent with the approximate factor structure approach and can be tested using an approximate χ^2 procedure under the standard assumptions of multivariate normality (Bartlett, 1950:77)[1,2].

[1]Given the conceptual difference between components analysis and factor analysis, comparative analyses are also conducted using principal factor procedures where the squared multiple covariances (correlations) are used as (initial) communality estimates.

[2]Appendix 5.1 outlines the essential difference between principal components analysis and principal factor analysis as it applies to this research debate.

The chapter is divided into three sections. In the first the relevant research into the area is discussed. Reference is made to both the methodologies employed and the consequent findings. In the second section the implications of market microstructure effects, thin trading and non-normalities on the robustness of the test procedures are investigated[3]. This is done by comparing the simulation results against what would theoretically be expected in the absence of market microstructure and other effects, and against benchmark simulations conducted in the absence of such market imperfections. The third and final section assesses the potential implications for empirical research of using robust covariance estimates.

5.2 Prior empirical research

Empirical research into the validity of the Arbitrage Pricing Theory can be broadly classified into three areas. The first relates to the empirical testability of the theory itself, the second focuses on the estimation of the number and pricing of the factors, and the third examines the ability of the model to explain certain CAPM anomalies. While numerous researchers have, to a greater or lesser extent, addressed more than one area, the review presented below is broken down into these three areas rather than being presented in chronological order. The identification of the factors is not addressed below because this aspect has no direct bearing on the validity of the theory. As stated by Roll and Ross (1980:1077);

> *in testing the APT, it is no more appropriate for us to examine this issue than it would be for tests of the CAPM to examine what, if anything, causes returns to be multivariate normal.*

5.2.1 The testability of the APT

In support of his view that Roll's critique remains relevant for empirical tests of the Arbitrage Pricing Theory, Shanken has, on two occasions, raised questions as to whether the theory is more amenable to empirical verification than the Capital Asset Pricing Model. In his first paper dealing with the subject he suggested that the methodology whereby one seeks to test whether the expected returns vector is a linear combination of the unit vector and the factor loadings matrix *rules out the very expected return differentials which the theory seeks to explain* (1982:1134). In 1985, when Dybvig and

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[3]The reader is referred to chapter four for the definition of market microstructure effects as used within the context of this research.

Ross developed an approach that they contended established equilibrium restrictions to factor models that led to testable implications for the theory, Shanken again reiterated his concerns by stating that the restrictions involved testing *joint hypotheses that cannot be verified without observing the entire universe of assets* (1985:1189). Dybvig and Ross reject this conclusion and argue that testing the theory on a subset is valid and that the degenerate cases of the type developed by Shanken for his *empirical APT have little relevance for actual empirical tests* (1985:1184). Finally they stress that any bias when using subsets of data is towards rejection of the theory and, contrary to the assertion of Shanken, that the chance of spurious acceptance is small.

Chamberlain and Rothschild defined an approximate factor structure as an *(a)ppropriate concept for investigating the relationship between factor loadings and asset prices* (1982:1282). Although less restrictive than the strict factor structure employed by Ross (1976) in his development of the Arbitrage Pricing Theory, Chamberlain and Rothschild suggest the approximate structure is still sufficient as a condition for reaching Ross's conclusions. Based on their analysis they also suggest that if security returns can be explained by an approximate k-factor structure then only k eigenvalues are unbounded and principal components analysis can be applied to test the relationship since *factor analysis and principal components analysis are asymptotically equivalent* (1982:1296-1301).

While not rejecting the testability of the theory outright, Kryzanowski & To (1983) suggest several assumptions need to be tested before unambiguous testing of the Arbitrage Pricing Theory can be carried out. The assumptions, which they stress are not prerequisites for the theory, are; (a) the mean return vector and variance-covariance matrix must be intertemporally stationary; (b) security returns must be characterized by an explicit underlying factor structure with one or more general factors impacting on all securities; (c) the underlying factor structure is congruent (across time and across subsets of assets); and (d) the loading coefficients are also intertemporally stationary (1983:32-33). In tests using both Rao and Alpha factor analysis across eleven samples of fifty NYSE securities and three samples of sixty Toronto Stock Exchange securities over the period January 1948 to December 1977 Kryzanowski and To concluded that the first factor *has perfect generalizability* and that the second of their stated assumptions is therefore met (1983:42).

In an extensive critique of the methodologies employed in early empirical tests of the Arbitrage Pricing Theory, Dhrymes, Friend and Gultekin (1984) suggested that they suffer from several basic limitations. Firstly, they contended that it is impermissible to carry out independent tests on whether a single risk-factor is priced using the two stage procedure of factor analysis followed by cross-sectional generalized least squares regressions. Only *unambiguous "F-tests" or asymptotic chi-square tests on the significance of the vector of risk premia* should be carried out (1984:345). Secondly, they suggested that the three-to-five factors of Roll and Ross (1980) are not robust and depend on the size of the group analyzed. In reply, Roll and Ross, disputed these conclusions and stated that not only can t-tests be appropriately employed but that, *if there actually are fewer than thirty "pervasive" factors generating returns, then factor analyzing groups of size thirty or more is equivalent in every way except statistical power and computational cost* (1984:349). They further suggested that the increasing number of common factors is expected but that these will tend to be non-priced industry specific factors and have no associated risk premium.

5.2.2 The number and pricing of the APT factors

In spite of Shanken's position, a vast body of literature has developed over the last decade involving empirical tests of the Arbitrage Pricing Theory. The majority of these have focused on the determination of the number and pricing of the APT factors. In the first major empirical research published in the literature, Roll and Ross (1980) developed a test procedure that they believed was designed to be definitive. The core methodology has been subsequently utilized by many other researchers and forms the basis of the simulation study conducted in this thesis. Using maximum likelihood factor analysis and generalized least squares regression procedures on daily data covering the period July 1962 to December 1972 for twelve hundred and sixty securities, Roll and Ross reached the conclusion that three factors, and possibly a fourth, are definitely priced for New York and American Stock Exchange securities. Although recognizing that the problem of comparing across forty-two groups of thirty securities results in some positive dependence and leads to an overstatement of the significance of the relationship *between explanatory variables and expected returns*, they none-the-less maintained that their Hotelling T^2 test procedure showed no evidence that adjacent group intercept estimates differed (1980:1099-1100). In contrast to the approach of Roll and Ross, Brown and Weinstein used a bilinear paradigm introduced by Kruskal to test for the equivalence of the factor prices. Using the same data as Roll and Ross, but by combining consecutive groups to obtain twenty-one groups of sixty securities, Brown and Weinstein tested for bilinear equivalence across the groups based on three, five and seven factors. They finally concluded in favour of the three factor model since, *while the adequacy of the given factor model (measured by the Chi-square statistics) increases with the number of factors, the proportion of securities for which the factors are the same across groups* *decreases with the number of factors* (1983:728).

In order to resolve the factor comparability problem across groups, Cho (1984) used the technique of inter-battery factor analysis. This procedure examines the inter-group correlation matrix in order to extract the factors common to two groups and is conceptually similar to canonical correlation analysis. The procedure also permits the analysis of larger groups of securities because only a sub-matrix of covariances is considered. Using daily data for approximately twelve hundred securities spanning the period used by Roll and Ross, Cho concluded, firstly that the number of factors ranges from two to nine and is dependent upon industry groups selected, and secondly that there is no tendency for the number of common factors to increase with sample size. When testing at a ten percent level rather than the fifty percent level employed by Roll and Ross, Cho found that *there are five or six inter-group common factors that generate daily returns for two groups and that these inter-group common factors do not depend on the size of groups* (1984:1499). Finally, Cho could not reject the hypothesis that risk-free rate and risk premia were common across groups and different from zero.

With respect to the two stage testing procedure generally followed in empirical research, Chen has suggested the number of factors assumed in the generalized least squares regression test impacts on the result. She suggested that too few results in the underestimation of the Type I error while too many adversely impact on the power of the test because of the high probability of a Type II error (1983:1400). Using larger groups of securities of up to one hundred and eighty, and by following a multi-step procedure designed to estimate *portfolio loading, so as to "balance estimation errors with other desirable properties"*, she did however reject the null hypothesis that the selected five factor loading coefficients were equal to zero (1983:1397).

In contrast to much of the prior debate, Pari and Chen (1984) have suggested that significant residual correlations violate the Arbitrage Pricing Theory development and that tests using the maximum likelihood estimation procedures implicitly recognize this. They also suggest that the use of daily data is inappropriate because of the resultant departure from multivariate normality. Using monthly returns data for equally weighted industry portfolios over the six year period 1975 to 1980, Pari and Chen reached the conclusion that a three factor return-generating process resulted in no statistically significant cross-sectional dependence among the residuals and that all three factors were priced over the period studied while the residual risk was not. This result they suggest supports the APT in preference to the CAPM. As a final component of their research Pari and Chen also used varimax rotation to facilitate the economic interpretation of the systematic factors.

This aspect of their research can however be criticized as an approach to factor identification since, while the real underlying factors must be pervasive macroeconomic forces that span the k-factor space, any assumption that they will be orthogonal is questionable.

In one of the early simulation studies of the robustness of the Arbitrage Pricing Theory, Cho, Elton and Gruber generated returns consistent with the zero beta CAPM as well as with actual historical returns[4]. When running tests of comparison against Roll and Ross's results they found that seven factors were required to obtain results consistent with those of Roll and Ross. Finally, based on the simulated data Cho, Elton and Gruber concluded that the commonly employed two stage procedure had *a slight tendency to overstate the number of factors at work in the market* (1984:8).

Dhrymes, Friend, Gultekin and Gultekin (1985) were amongst the first researchers to examine the instability of the number of factors determined empirically. They found that the number of factors increases as both the number of observations and number of securities increase. When groups of ninety securities were examined as many as thirteen factors were evident from the factor analysis phase of the procedure. Additionally, a joint χ^2 test of the risk premia pricing proved to be exceedingly variable across groups and *provides very little support for the key implication of the APT model.*[5]. Dhrymes et. al. do however acknowledge the influence of the number of factors specified from the factor loading estimation when running the generalized least squares procedure, and that the number of priced factors is much smaller than the number determined in the first stage factor analysis. Subsequent work by Trzcinka (1986) supported the finding that the number of factors identified is a function of the size of group analyzed. On the basis of the proof provided by Chamberlain and Rothschild that the k eigenvalues of the covariance matrix should grow without bound if returns are generated by a linear k factor model, Trzcinka computed the eigenvalues for successively larger groups up to a maximum of eight hundred and sixty-five securities. The study utilized weekly data because of the expense of calculations for daily data as well as the non-normality problem, and because monthly data resulted in too few degrees of freedom for the larger groups. In order to test the (un)boundedness of the eigenvalues Trzcinka ran a series of

[4]In their simulation procedure, and in subsequent testing, Cho, Elton and Gruber (1984) came across several Heywood cases which they had to exclude from the analysis. This condition can be particularly problematic when using standard procedures for variance-covariance matrix estimation in thinly traded markets. The issue is addressed in subsequent sections of the chapter.

[5]Dhrymes et. al. (1984) used a joint χ^2 test of the entire vector because of their criticism of independent t-statistics of the pricing condition.

regressions of eigenvalues against the number of securities using both random and nested sequences of increasing size groups to examine the extent to which the k eigenvalues dominate the trace of the matrix (1986:352). The overriding conclusions of the study were two-fold. Firstly, all eigenvalues grew larger with number of securities and the χ^2 statistic could not accept the hypothesis that a constant k principal components could adequately capture the variation in asset returns (1986:367). Secondly, Trzcinka stated that *(o)ne should not conclude from this study that only one factor is important for security pricing. We have found evidence that there is one large factor and no obvious way to choose more than one* (1986:367).

Burmeister and McElroy carried out empirical research into Arbitrage Pricing Theory by testing of the *nesting* process[6] using three different regression procedures and by investigating both measured macro economic factors and other non-observed factors. The multivariate regression procedures employed were iterated nonlinear weighted least squares regression, iterated nonlinear seemingly unrelated regression, and iterated nonlinear three stage least squares regression, and the conclusion of the study was that while the CAPM restrictions on the APT are rejected, the APT restrictions on the linear factor model are not (1988:732). This result is contrary to the findings of Gultekin and Gultekin (1987) and Cho and Taylor (1987) but, as stated by Brown, *the research provides new measures and interpretations of the factors of the Arbitrage Pricing Theory* and has *developed new and innovative methods of jointly estimating factor sensitivities and risk premia* (1988:734).

Much of the inconsistency of the above findings might be attributed to the low power of the statistical procedures and additional small sample problems with the techniques employed. Shanken (1987) investigated the impact of poor covariance estimates resulting from non-synchronous trading on the factor structure of returns. He used ten groups of thirty securities from the CRSP daily database that had complete daily data from July 1962 to December 1972 and estimated the covariances and factor structure in two ways. Firstly, by using the standard covariance estimator and secondly, by using the Cohen et. al. (1983a, 1983b) covariance adjustment with three lags. Consistent with the evidence of the previous chapter Shanken found that the estimates were significantly downwardly biased when using the standard estimator. Additionally, he found that the first five factors explained almost double the covariance when using the adjusted estimators. As an extension of this area of research, Brown employed an extensive simulation study to

[6]In a universe where returns are generated by a linear factor model, Burmeister and McElroy suggest that the CAPM is *nested* inside the APT which is in turn *nested* in the linear factor model.

show that, in an exact k-factor economy, principal components and eigenvalue analysis of the covariance matrix will *lead an investigator to the false inference that the one important "factor" is the return on an equally weighted market index* (1989:1247).

5.2.3 The APT and CAPM anomalies

Tests of the Arbitrage Pricing Theory through the assessment of the theory's ability to explain previously unexplained anomalies are based on the notion that *a minimum empirical requirement for an alternative model should be that it accounts for empirical anomalies that arise within the CAPM* (Reinganum, 1981:320). Empirical research utilizing this approach includes the work of Roll and Ross (1980), Chen (1983), Dhrymes, Friend, Gultekin and Gultekin (1985), Cho and Taylor (1987) and Gultekin and Gultekin (1987).

Roll and Ross tested for the explanatory power of own variance and concluded that, *except for the spurious influence of skewness*, it was not significant (1980:1075). While this finding is consistent with that of Chen (1983) who utilized daily data over the period 1963 to 1978, it is the opposite of the conclusion reached by Reinganum (1981) and Dhrymes, Friend, Gultekin and Gultekin (1985). It is of interest to note that, although he reached contrary conclusions, Reinganum used data covering the same period as Roll and Ross. Dhrymes et. al. even went so far as to conclude that unique risks as measured by residual standard deviation appear at least as important as common risks measured by factor risk premia.

Reinganum also extended the own variance analysis by seeking to establish if the Arbitrage Pricing Theory could *account for other empirical anomalies that arise within the CAPM* (1981:320). While recognizing the limitations of the joint testing procedure employed, he found that the model could not explain the small firm effect even when risk was measured using five factors. Reinganum's method of estimating the factor loadings utilized an approach which exploits the notion that if one knows the factor loadings for k securities, then the k factor loadings for all securities can be estimated. The approach was considered necessary because of the expense (at that time) of decomposing a five hundred security variance-covariance matrix. Reinganum's conclusion is the opposite of that of Chen (1983) who found she could not reject the null hypothesis that size had no explanatory effect after adjusting for risk using the estimated factors of the APT.

More recent empirical research into the Arbitrage Pricing Theory has involved the examination of seasonality and the January effect. Cho and Taylor used daily data over the period January 1973 to February 1983 and found there was some fluctuation in the covariances across calendar months with January coefficients being higher than average. While they contended that this effect may explain the instability in the estimated number of return-generating factors (1987:1205), they also concluded that *(t)he APT pricing relationship does not seem to be supported by (their) testing methodology* (1987:1210). Gultekin and Gultekin supported the these conclusions by finding that once the January effect had been removed there was no significant relationship between expected returns and *APT predicted risk measures* (1987:1223).

5.2.4 Conclusion

The above review highlights the conflicting views of researchers undertaking empirical analysis of the Arbitrage Pricing Theory. The conclusions range from the very favourable to such comment as that by Dhrymes, Friend, Gultekin and Gultekin who, when commenting on the initial findings of Roll and Ross (1980) suggested, *(i)t is difficult to imagine a more complete rejection of the crucial implications of such APT models, using the flawed methodology of splitting the universe of assets into 30 security groups* (1985:674). The research outlined below seeks to add further insight to the debate through the use of an extensive simulation combined with several of the techniques used in prior empirical work. By simulating data with known returns characteristics clearer conclusions can be reached relating to the inherent empirical validity of the theory versus the power of the techniques currently employed.

5.3 The impact of non-normality and thin trading on factor estimation

5.3.1 Methodology

The analysis conducted in this section is based on returns data for securities drawn from several k-factor simulated economics. In order to directly assess the impact of the number of factors on the statistical techniques employed, single, three and five factor economics are simulated. The choice of a single factor economy is based on the continuing prominence of the single factor market model developed by Sharpe et. al., while the three and five factor economies are representative of the number of priced factors generally found in empirical research into the Arbitrage Pricing Theory.

Written mathematically, the returns for the i^{th} security drawn from a k-factor economy is given by:

$$\tilde{r}_i = \lambda_0 + \sum_{j=1}^{k}\left(\lambda_j + \tilde{\delta}_j\right)b_{i,j} + \tilde{\varepsilon}_i \tag{5.1}$$

where $\tilde{\delta}_j$ is the j[th] mean zero factor common to all assets; λ_j can be interpreted as the risk premium corresponding to the j[th] factor; $b_{i,j}$ is the sensitivity of the i[th] asset's return to the j[th] factor; and, $\tilde{\varepsilon}_i$ is the risk component idiosyncratic to the i[th] asset with $E\left\{\tilde{\varepsilon}_i \middle| \tilde{\delta}_j\right\}$ equal to zero for all j.

5.3.1.1 Simulation design

The above equation was used in the development of the three stage simulation design for the production of weekly underlying and observed returns outlined below[7]. The choice of weekly data in preference to daily or monthly data is based, in part, on the evidence of chapter four which showed the implications of market microstructure and thin trading to be more profound for weekly data than monthly data[8]. Numerous researchers including Roll and Ross (1980), Trzcinka (1986) and Perry (1982) have also found it preferable to use weekly data (or *every other observation* when using daily data) because the problems of non-synchronous trading, non-normalities, unstable variance and autocorrelations induced by bid-ask spread have been shown to have a more profound impact when using daily data. Additionally, most empirical research in South Africa needs to be undertaken using weekly or monthly returns series because of the paucity of data[9]. Carrying out the simulation based on similar interval data therefore provides a point of reference for subsequent empirical research.

(a) One thousand three hundred daily returns for the k priced factors were first generated under the assumption that the factors are orthogonal, normally distributed, and contribute equally to the variance of returns for the economy as a whole[10]. Since it is possible to

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[7]As was the case for chapter four, the term "underlying" is used to represent the return that would occur if prices adjusted continuously in the absence of microstructure effects and were measured as such.

[8]The use of monthly data for multivariate tests of the type carried in this research is also problematic because it restricts the number of securities that can be examined simultaneously. Difficulties arise when the correlation/covariance matrix becomes singular as the number of securities examined tends to the length of the time series of returns. Counteracting this tendency through the use of extended time series is itself of questionable value because of the stationarity assumptions that must be made.

[9]The Johannesburg Stock Exchange only maintains three years of daily data on their computer system as opposed to approximately twenty years of weekly data. Additionally, companies that delist have their time series of prices removed from the database.

[10]Again, as was the case for the simulations of chapter four, trading is assumed to occur once per day at

rotate any set of m correlated factors into an equivalent set of k orthogonal factors ($k \leq m$) the orthogonality assumption imposes no constraints on the generalizability of the results. The assumption of equal contribution to average variance is the same as the approach used by Brown and, as he has shown, is not inconsistent with the empirical evidence suggesting one dominant market index having the major impact on security returns (1989:1247). For each of the twelve simulated economies the factors were simulated as equivalent normally distributed independent random variables having the parameters presented in table 5.1. As outlined in (b) below, the mean and variance of the factors were selected so that, once the idiosyncratic risk components had been added, the individual securities would have daily returns distributions similar to those presented in chapter three for actual Johannesburg Stock Exchange listed securities.

Table 5.1 Parameters used for the twelve simulated economies

Simulated economy	Number of factors (k)	μ_k	σ_k^2	σ_ε^2	Average proportion of variance explained
1 and 4	1	0.0010000	0.0001080	0.0003760	22.3%
2 and 5	3	0.0003333	0.0000360	0.0003760	22.3%
3 and 6	5	0.0002000	0.0000216	0.0003760	22.3%
7 and 10	1	0.0010000	0.0002430	0.0002410	50.2%
8 and 11	3	0.0003333	0.0000810	0.0002410	50.2%
9 and 12	5	0.0002000	0.0000486	0.0002410	50.2%

For each of the simulated economies μ_k is the mean and σ_k^2 the variance of each of the independent factors while σ_ε^2 is the variance of the simulated idiosyncratic component.

(b) For each of the m securities, the factor loadings for each security (the $b_{i,j}$'s) and the idiosyncratic risk components were then simulated. The $m \times k$ factor loadings were obtained as random drawings from a normal distribution with unit mean and a variance of 0.04, while the idiosyncratic risk components were simulated from both normal and non-normal distributions[11]. The generation of non-normal idiosyncratic risk components enables the implications of non-normalities in underlying returns to be directly investigated as opposed to merely relying on the non-normalities induced through thin trading. For all securities the idiosyncratic risk components were assumed to be

the close.

[11]The pairs of simulated economies presented in the table, namely 1/4, 2/5, 3/6, 7/10 ,8/11 and 9/12, differ only to the extent that the securities for first of each pair of economies were simulated to have normal idiosyncratic risk components while the securities for the second were simulated to have fourth moments of 7.5 (kurtosis of 4.5).

identically distributed with zero mean and skewness, and with variance given in table 5.1. Daily underlying returns for each security were thereby finally produced as;

$$r_{i,t} = \sum_{j=1}^{k} b_{i,j} f_{j,t} + \varepsilon_{i,t}$$

where; for each of the k factors $f_{j,t} \sim N(\mu_k;\sigma_k)$ and $\varepsilon_{i,t} \sim N(0;\sigma_\varepsilon)$ or $D(0;\sigma_\varepsilon;0;4.5)$.

Abstracting from the bias induced by the randomly distributed factor loadings, the orthogonal nature of the factors as well as their zero correlation with the idiosyncratic risk components implies that, on average, the daily security returns will be distributed with a mean of 0.001 and variance of 0.000484[12]. These numbers are consistent with the empirical findings presented in chapter three. In addition, as shown in the table, the proportion of variance explained by the factor(s) will average 22.3% for factor economies 1 to 6 and 50.2% for factor economies 7 to 12. The choice of two sets of simulations with differing security proportion of variance explained allows for additional tests of the sensitivity of the results[13].

(c) As a final stage of the data simulation procedure, the daily underlying data were transformed into both underlying and observed weekly data series using the methodology described in chapter four, and by assuming the securities were equally likely to have daily trading probabilities of ten percent, fifty percent or ninety percent.

5.3.1.2 Test procedure

For each of the simulated economies, and for each iteration, portfolios of securities varying in size from twenty through to sixty were analyzed to investigate the number and pricing of the factors[14]. The analyses were carried out on both the underlying and observed weekly data. As an initial step principal components analysis of the variance-covariance matrix was carried out. This approach is conceptually simpler than factor analysis in that the resultant approximate factor structure based on the first k eigenvalues is unique and does not suffer from the rotational problems commonly associated with

[12] $\mu_s = k\mu_k = 0.001$ and $\sigma_s^2 = k\sigma_k^2 + \sigma_\varepsilon^2 = 0.000484$

[13] As outlined in chapter four the proportion of variance was selected on the basis of prior sample evidence from the Johannesburg Stock Exchange (Page, 1986:41).

[14] The approach adopted in this study is based on the procedure adopted originally by Roll and Ross (1980). It has subsequently been employed by the majority of researchers into the APT and can be contrasted with the multivariate linear regression test developed by Jobson (1982).

factor analysis (Brown, 1989:1248). In addition, while conventional factor analysis remains the dominant approach in tests of the APT its appropriateness has been called into question (Raveh, 1985). The approximate factor structure also allows for weak correlation between the residuals and does not make the implicit assumption that residual returns unexplained by the priced factors be uncorrelated across securities[15]. Relaxing this assumption explicitly allows for the existence of non-priced factors which have been mentioned by numerous researchers including Roll and Ross (1980:1075). It is the existence of such common factors that has led to conclusions being reached which suggest that, while increasing the number of securities analyzed may result in more factors being found, the number of priced factors should not increase[16]. Lee and Comrey do point out however that component analysis does not make the assumption that each variable (security) consists of a common and unique part and that the factor dimensions using component analysis mix common, specific and random error variance together (1979:302). Finally, through an analysis of twenty years of data, Shukla and Trzcinka (1990) concluded that the one-vector principal components approach resulted in an empirical pricing error smaller than or equal to the five factor maximum likelihood approach. They do however suggest that the *superiority of any technique must be decided on a case-by-case basis* even though principal components and eigenvectors should not be *ruled out a priori* (1990:1563).

The significance of the principal component eigenvalues were tested using an approximate χ^2 likelihood statistic developed by Bartlett (1950:77-82). For a variance-covariance matrix of order m, the statistic for testing the equivalence of the remaining eigenvalues after the first k have been removed is given by:

$$\chi^2 = -\left\{n - \tfrac{1}{6}(2m + 11) - \tfrac{2}{3}k\right\}\ln\left(V_{m-k}\right) \tag{5.2}$$

where; n is the length of the series of data used in the estimation of the matrix; $\mathbf{V}$ is the estimated variance-covariance matrix; $\frac{1}{2}(m - k - 1)(m - k + 2)$ is the number of degrees of freedom; and, V_{m-k} is given by;

$$V_{m-k} = |\mathbf{V}| \Big/ \left\{\lambda_1\lambda_2\ldots.\lambda_k\left[\frac{m - \lambda_1 - \lambda_2\ldots. - \lambda_k}{m - k}\right]^{m-k}\right\} \tag{5.3}$$

[15]See Appendix 5.1.

[16]Brown and Weinstein make explicit mention of the impact of non-priced factors on mean zero stochastic error terms even though they assume the matrix is diagonal in their testing of the APT using the bilinear paradigm (1983:717).

Given the dominance of factor analytic procedure in the literature, comparative principal factor analyses were also carried out for selected portfolio sizes and for a subset of the simulated economies. As opposed to principal components analysis, principal factor analysis uses an adjusted covariance matrix, with the main diagonal replaced with communality estimates, as input. While several approaches can be adopted for estimating the communality, the squared multiple covariance (SMC) has the advantage of providing a lower bound estimate of the communality and, *(l)argely because of this property, Guttman (1956) recommends the SMC as the "best possible" estimate* (Harman, 1976:87)[17]. For each security the squared multiple covariance is computed as[18];

$$SMC_i = \sigma_i^2 - \frac{1}{v^{ii}}$$

where, v^{ii} is the diagonal element of $\mathbf{V}^{-1}$ corresponding to the i[th] security.

In addition to replacing the diagonals with communality estimates the decision must be made as to whether one should iterate to an optimal solution through continuous re-estimation and substitution of the communality estimates (Nie, Hull, Jenkins, Steinbrenner and Bent, 1975:479-480). Analyses were carried out using both procedures to assess the implications of using poorer communality estimates. Statistical testing for the number of factors when using a principal factor procedure relies on a χ^2 procedure developed by Ripp (1953) which is *independent of the type of factor solution* (Harman, 1976:184). The specific statistic to test for the significance of k factors is given by;

$$U_k = \left(n - \tfrac{1}{6}(2m + 11 - 4k)\right)\left\{\ln\left|\mathbf{BB}' - \mathbf{U}^2\right| - \ln|\mathbf{V}| + tr\left[\mathbf{V}\left(\mathbf{BB}' - \mathbf{U}^2\right)^{-1}\right] - m\right\}$$

where; $\mathbf{B}$ is the $m \times k$ loading array; U^2 is the corresponding diagonal array of security communalities; and, U_k is asymptotically distributed as χ^2 with $\frac{1}{2}\left[(m-k)^2 + m - k\right]$

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[17]Thurston (1947) has suggested that the final factor loadings depend directly on the sizes of the diagonals (communalities) that are placed in the correlation (covariance) matrix.

[18]It must be noted that the SMC's in this research are computed from the variance-covariance matrix and are analogous to, but not equal to, the squared multiple correlations given in Harman (1976:87). The approach used here reflects the fact that the analyses are undertaken using the variance-covariance matrix directly rather than the correlation matrix.

degrees of freedom[19].

Finally, it must be noted that the above formulae relate to statistical significance and;

> *it does not follow that all the factors which reach statistical significance in a large sample necessarily remove a very large fraction of variance; and hence some of them may be comparatively unimportant in practice. Again, even if they are numerically important, this has no necessary implications of psychological or other reality of the factors* (Bartlett, 1950:82).

Just as not all statistically significant common factors remove a large fraction of the (co)variance, not all can be presumed to be priced. Roll and Ross (1980) therefore proposed a method of cross-sectional generalized least squares regressions to identify the number of priced factors[20]. The power of cross-sectional regression procedures to identify pricing is assessed in the simulation by using the methodology outlined by Roll and Ross (1980:1090-1091) and Cho, Elton and Gruber (1984:6-7), together with the reverse Helmet rotation proposed by Brown (1989:1253). The importance of using generalized least squares procedures in preference to ordinary least squares is well documented (Brown, 1989:1252; Pindyck and Rubinfeld, 1981:164-168; Roll and Ross, 1980:1090). It is normally used in a time series context to correct for bias and inefficiency in ordinary least squares caused by autocorrelation and heteroscedasticity in the residuals. While the cross-sectional pricing regressions do not use time series data, heteroscedasticity is present in the general factor model because the common factors do not necessarily explain the requisite proportion of each security's variance to ensure equal residual (idiosyncratic) variance across all securities. The equivalent of autocorrelation also exists because of the possible existence of numerous non-priced factors.

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[19]The use of the variance-covariance matrix in preference to the correlation matrix changes the number of degrees of freedom. When using the correlation matrix an additional m degrees of freedom are lost and the resultant number of degrees of freedom becomes $\frac{1}{2}\left[(m-k)^2 - m - k\right]$ (Harman, 1976:205).

[20]Generalized least squares regression is based on the assumption that $E(\varepsilon\varepsilon')=\sigma^2\Omega$ as opposed to the ordinary least squares regression assumption that $E(\varepsilon\varepsilon')=\sigma^2 I$. Under the condition that Ω is positive definite, a non-singular matrix **H** can be derived such that $\mathbf{H}\Omega\mathbf{H}'=\mathbf{I}$ and the original model can be transformed by computing where $\mathbf{Y}=\mathbf{HR}$ and $\mathbf{X}=\mathbf{HB}$ where **R** is the cross-sectional security returns vector and **B** is the factor loading matrix. Testing of the regression is then carried out using ordinary least squares regression procedures with **Y** as the dependent variable and **X** as the independent variable. The computation of **H** involves parallel manipulation of the positive definite matrix and an identity matrix of the same rank until the original matrix has been transformed to have all elements in the strict lower diagonal equal to zero. In the process the identity matrix is transformed into the required transformation matrix. The interested reader is referred to Shayle R. Seatle, "Matrix Algebra useful for statistics", John Wiley & Sons, New York, 1982, pp. 199-200.

The generalized least squares procedure used in this study differs from that employed by Roll and Ross (1980) and Cho, Elton and Gruber (1984) in that it is appropriately adjusted to allow for loadings based on the variance-covariance matrix in the case of the principal components analysis rather than the correlation matrix used in the traditional principal factor procedure. While recognizing that some econometric problems do exist, Roll and Ross suggest that a *natural generalised least squares cross-sectional regression for each day t is*;

$$\hat{\lambda}_t = \left(\hat{\mathbf{B}}'\hat{\mathbf{V}}^{-1}\mathbf{B}\right)^{-1}\hat{\mathbf{B}}'\hat{\mathbf{V}}^{-1}\mathbf{r}_t = \Gamma\mathbf{r}_t$$

where, $\hat{\mathbf{B}}$ is the estimated factor loading matrix; $\hat{\mathbf{V}} = \hat{\mathbf{B}}\hat{\mathbf{B}}' + \mathbf{D}$ is the estimated covariance matrix; $\mathbf{r}_t$ the vector of returns at time period t; and $\hat{\lambda}_t$ is the vector of estimated premia for time period t.

Using the above, together with the fact that the principal component and principal factor procedures constrain the covariance matrix of $\hat{\lambda}_t$ to be diagonal, allows one to conduct mutually independent t-tests for the significance of the risk premia. The approach is based on the arithmetic mean sample returns with the premia estimates being given by;

$$\bar{\hat{\lambda}} = \Gamma\bar{\mathbf{r}},$$

and having a diagonal covariance matrix of[21,22];

$$\tfrac{1}{T}\left(\hat{\mathbf{B}}'\hat{\mathbf{V}}\hat{\mathbf{B}}\right)^{-1}$$

As the simulation procedure did not involve the addition of a zero-beta or risk-free coefficient it is assumed equal to zero for the generalized least squares procedure and the difficulties associated with using an augmented matrix of loadings, namely $\left[\mathbf{1}:\hat{\mathbf{B}}\right]$, is thereby avoided.

It has been suggested by Brown that the procedure outlined above is biased towards

---

[21]As pointed out by Roll and Ross (1980), the estimation errors in $\hat{\mathbf{B}}$ result in the significance tests for $\hat{\lambda}$ being only asymptotically correct.

[22]The factor loadings and uniquenesses are re-adjusted where appropriate so that the t-statistics are not under-estimated (Cho, Elton and Gruber, 1984:6).

finding a single priced factor and several non-priced factors since;

> ....*suppose that a generalised least squares regression of mean returns against the underlying factor loading revealed that each of the original k factors were priced (with t-values equally significant and greater than two). Then it is possible to show that the first principal factor, the equally weighted market index, will appear to be a priced factor (t-value greater than* $2\sqrt{k}$*) and that the remaining factors implied by the principal factor solution will not be priced (t-value equal to zero)* ...... (Brown, 1989:1252).

As a solution to the problem, he suggests that one consider a rotation which, although not affecting the Arbitrage Pricing Theory relationship, will *retrieve the k "equally important" factors.* The rotation proposed is the inverse of the Helmet matrix, and according to Brown it lends to more powerful tests of the asset pricing relationships to the extent that each of the factor prices is measured with equal statistical precision (1989:1253)[23,24]. The Helmet matrix is given as;

$$\mathbf{T} = \begin{pmatrix} 1/\sqrt{k} & 1/\sqrt{2} & 1/\sqrt{2\times 3} & \cdot & \cdot & 1/\sqrt{(k-1)k} \\ 1/\sqrt{k} & 1/\sqrt{2} & 1/\sqrt{2\times 3} & \cdot & \cdot & 1/\sqrt{(k-1)k} \\ \cdot & 0 & -2/\sqrt{2\times 3} & \cdot & \cdot & 1/\sqrt{(k-1)k} \\ \cdot & 0 & 0 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1/\sqrt{k} & 0 & 0 & 0 & 0 & -(k-1)/\sqrt{(k-1)k} \end{pmatrix} \tag{5.3}$$

The rotation approach was utilized in a second generalized least squares analysis and compared to the pricing relationship found based on the unrotated loading matrix[25].

For each of the twelve economics examined one hundred and fifty iterations were carried out and summary statistics computed for the eigenvalues, χ^2 significances, and the cross-sectional generalized least squares rotated and unrotated pricing regressions.

---

[23]Of course the problems highlighted by Chen (1983:1400) still remain. If too few factors are extracted the Type I error is underestimated (even after rotation) while if too many are extracted the power of the test is weak and the chance of a Type II error large.

[24]This approach is different to the varimax rotation procedure often chosen as the default option in many factor analysis procedures. The problems of using varimax rotation over too many factors and causing excessive dispersion of variance is also not applicable here (Lee and Comrey, 1979:320).

[25]For k=2 the rotation procedure produces, as expected, a matrix $\mathbf{T}^{-1} = \mathbf{T}'$ that rotates the factor loading matrix through $\pi/4$ radians.

5.3.2 Results and Discussion

The results of the simulation are presented in two sections. In the first the results of the principal component analyses for each of the twelve economies are discussed. In the second section the evidence of the generalized least squares pricing regression is presented.

Figures 5.1 to 5.3 plot the first five eigenvalues as a function of portfolio size for the one, three and five factor normally distributed security idiosyncratic risk component economies where the percentage of security returns explained by the factors averages 22.3%[26]. Appendices 5.2 to 5.4 present the same plots but for the simulations where the percentage of security returns explained by the factors averages 50.2%. Each figure presents two graphs with differently scaled vertical axes so as to more clearly display the dispersion over the second through fifth eigenvalue curves. Numerous aspects of the figures merit discussion, particularly given the high degree of similarity between the average eigenvalue curves. Theoretical eigenvalue curves are also presented in each figure using formulae derived by Brown (1989:1250) for a k-factor economy. The formulae are based on *large* portfolios analyzed using principal components analysis and give[27];

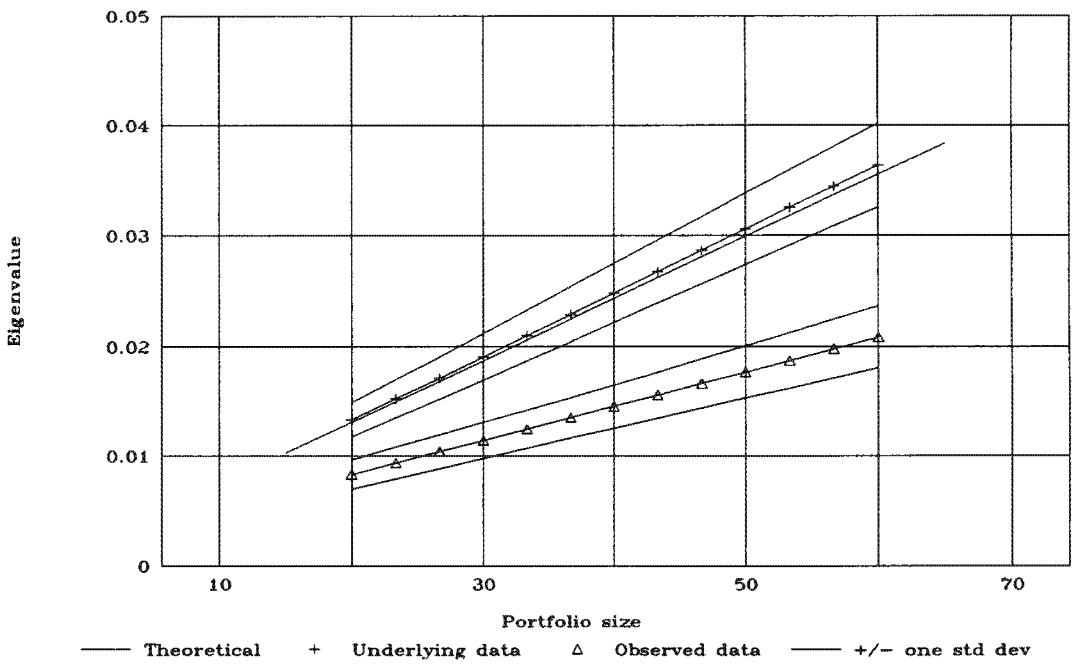
$$
\begin{aligned}
\lambda_1 &= \sigma_\varepsilon^2\left[\frac{R^2}{\left(1-R^2\right)k}\left[(m-1)\sigma_b^2+km\right]+1\right] \\
\lambda_{2,3,\ldots,k} &= \sigma_\varepsilon^2\left[\frac{R^2}{\left(1-R^2\right)k}(m-1)\sigma_b^2+1\right] \qquad (5.4) \\
\lambda_{k+1,\ldots,m} &= \sigma_\varepsilon^2
\end{aligned}
$$

where; m equals the number of securities being analyzed; σ_f^2 is the variance of the k independently and identically distributed factors; σ_b^2 is the variance of the k independently and identically distributed factor loadings; σ_ε^2 is the variance of the idiosyncratic risk component for each security; and, $R^2 = k\sigma_f^2/\left(k\sigma_f^2+\sigma_\varepsilon^2\right)$

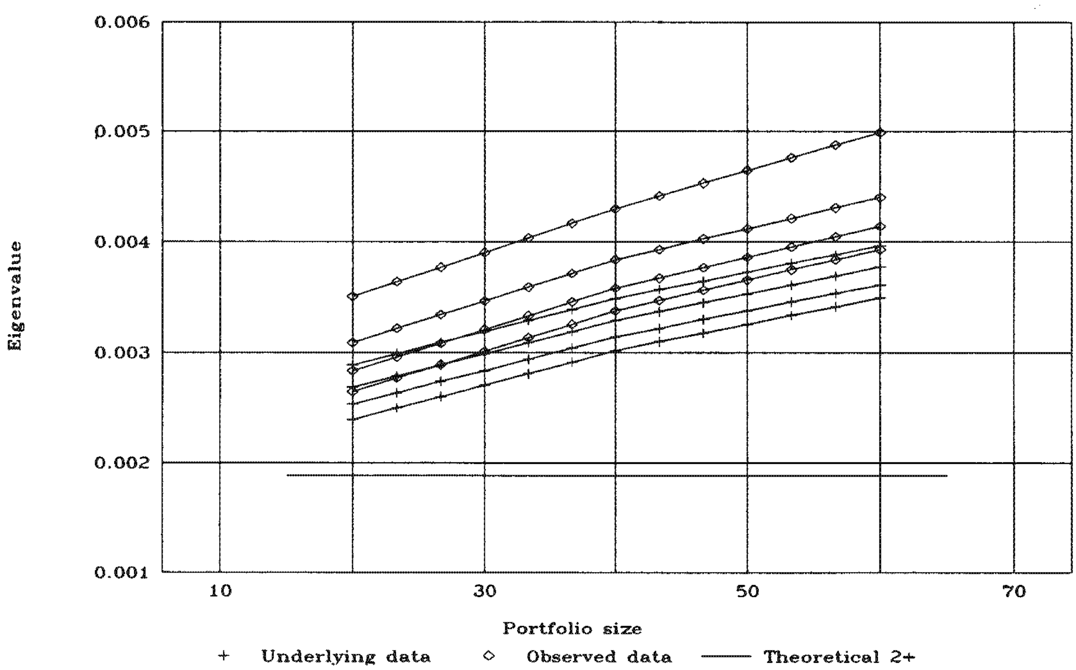
[26]The graphs for the non-normally distributed idiosyncratic risk condition were found to be almost identical and are therefore not presented.

[27]*Large* in this context refers to the number of securities in the portfolio and not the number of observations used in the computation of the variance-covariance matrix.

Figure 5.1 Eigenvalues as a function of portfolio size:
Single factor economy - Average variance explained of 22.3%

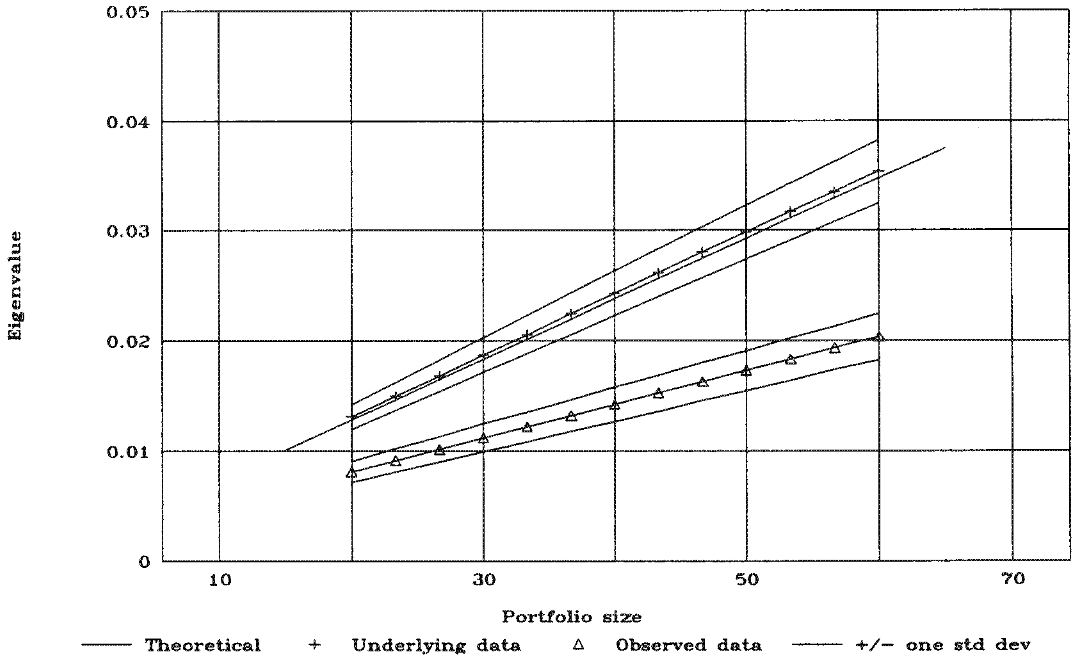


(a) Largest eigenvalue

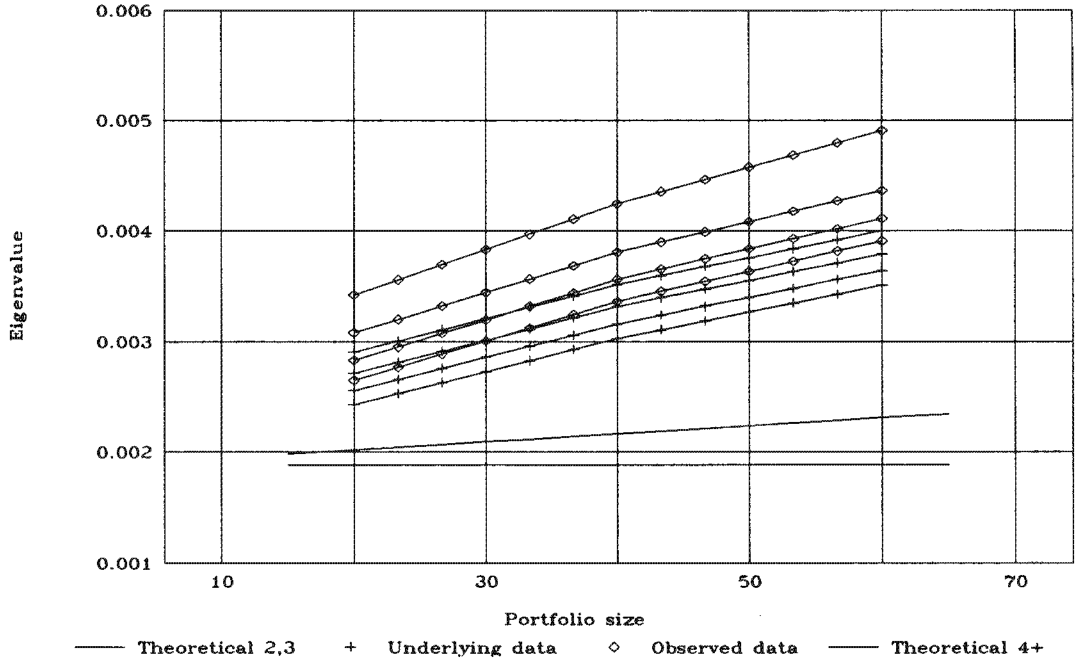


(b) Eigenvalues two to five

Figure 5.2 Eigenvalues as a function of portfolio size:
Three factor economy - Average variance explained of 22.3%

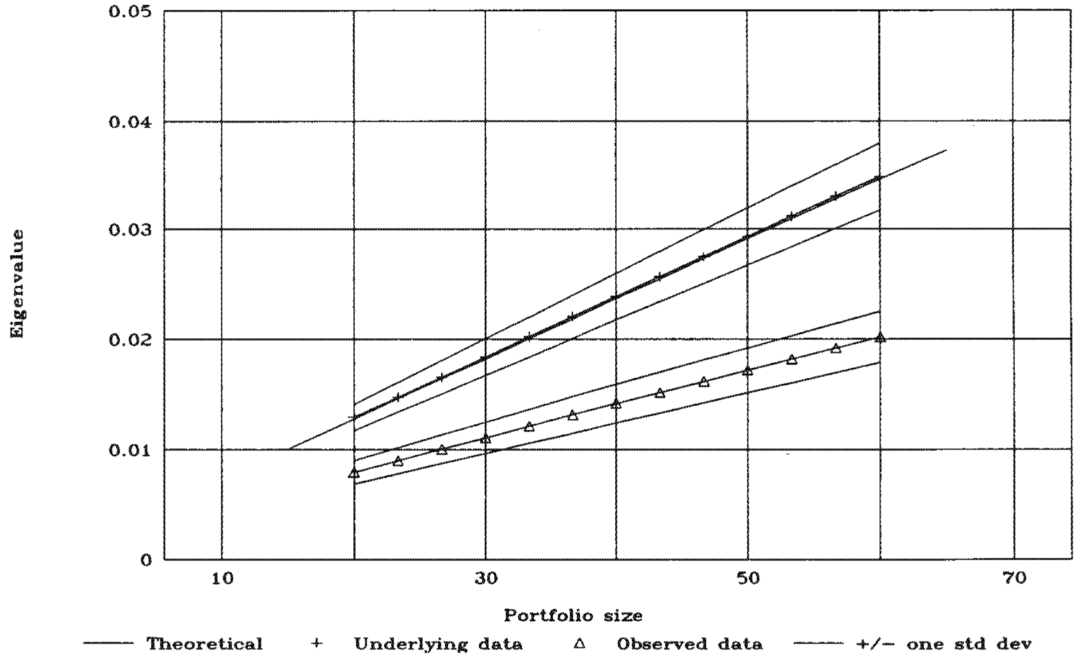


(a) Largest eigenvalue

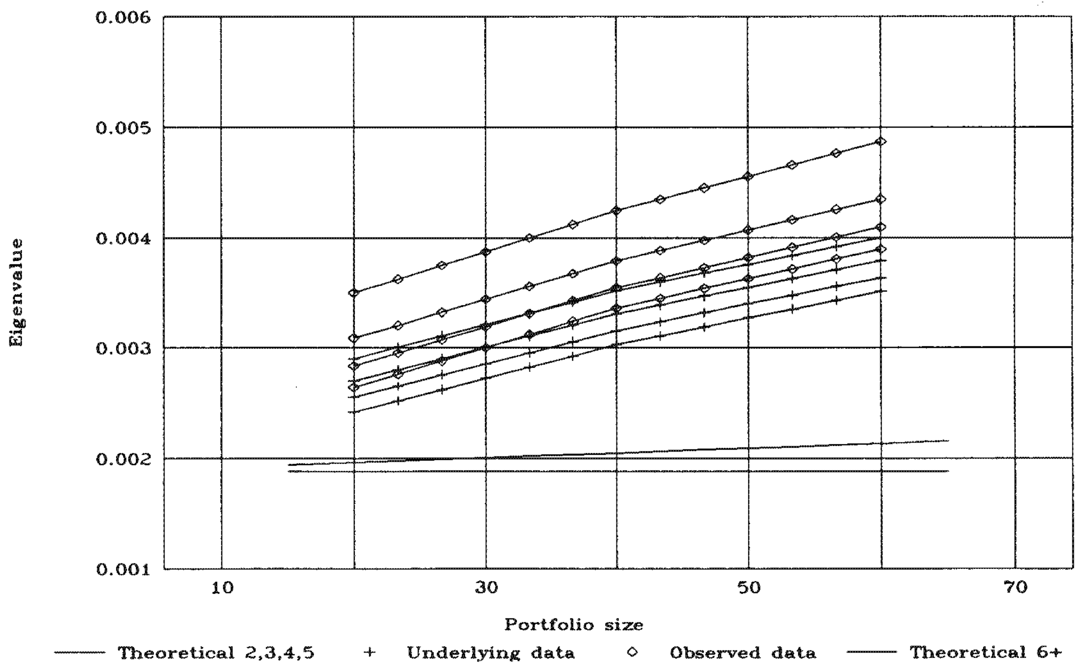


(b) Eigenvalues two to five

Figure 5.3 Eigenvalues as a function of portfolio size:
Five factor economy - Average variance explained of 22.3%



(a) Largest eigenvalue



(b) Eigenvalues two to five

For the simulations based on the 22.3% average variance explained, the weekly simulation parameters of $\sigma_\varepsilon^2 = 0.00188$, $\sigma_b^2 = 0.04$ and $k\sigma_f^2 = 0.00054$ result in identical R^2 values and the difference between the eigenvalues across the one, three and five factor economies represented in figures 5.1 to 5.3 is therefore determined exclusively by k[28,29].

Table 5.2 gives the theoretical eigenvalues for the six economies containing securities with normally distributed idiosyncratic risk components for portfolios ranging in size from twenty through to sixty.

Table 5.2 Theoretical eigenvalues against the number of securities in the portfolio analyzed for the six simulated economies containing securities with normally distributed idiosyncratic risk components

Economy	k	m	σ_f^2	λ_1	$\lambda_{2\ldots k}$	$\lambda_{k+1,\ldots}$	$\partial\lambda_1/\partial m$	$\partial\lambda_{2\ldots k}/\partial m$
Economies simulated to have (a) normal idiosyncratic risk & (b) average communality of 22.3%								
1	1	20	0.000540	0.013090		0.001880	0.000562	
		40	0.000540	0.024322		0.001880	0.000562	
		60	0.000540	0.035554		0.001880	0.000562	
2	3	20	0.000180	0.012817	0.002017	0.001880	0.000547	0.000007
		40	0.000180	0.023761	0.002161	0.001880	0.000547	0.000007
		60	0.000180	0.034705	0.002305	0.001880	0.000547	0.000007
3	5	20	0.000108	0.012762	0.001962	0.001880	0.000544	0.000004
		40	0.000108	0.023648	0.002048	0.001880	0.000544	0.000004
		60	0.000108	0.034535	0.002135	0.001880	0.000544	0.000004
Economies simulated to have (a) normal idiosyncratic risk & (b) average communality of 50.2%								
7	1	20	0.001215	0.026428		0.001205	0.001264	
		40	0.001215	0.051700		0.001205	0.001264	
		60	0.001215	0.076972		0.001205	0.001264	
8	3	20	0.000405	0.025813	0.001513	0.001205	0.001231	0.000016
		40	0.000405	0.050437	0.001837	0.001205	0.001231	0.000016
		60	0.000405	0.075061	0.002161	0.001205	0.001231	0.000016
9	5	20	0.000243	0.025690	0.001390	0.001205	0.001225	0.000010
		40	0.000243	0.050184	0.001584	0.001205	0.001225	0.000010
		60	0.000243	0.074678	0.001778	0.001205	0.001225	0.000010
R^2=0.223140 for economies 1,2 & 3, and R^2=0.502066 for economies 7,8 & 9. $\partial\lambda_{k+1,\ldots}/\partial m = 0.000$.								

From the table it can be seen that for the second economy, simulated under the assumption of three orthogonal factors explaining an average of 22.3% of individual

[28]Under the assumption of independence of successive returns, the weekly variances are equal to five times the daily parameters.

[29]A similar situation applies for the three economies represented in appendices 5.1 to 5.3 where $\sigma_\varepsilon^2 = 0.001205$, $\sigma_b^2 = 0.04$ and $k\sigma_f^2 = 0.001215$.

security returns, a principal components analysis of a portfolio of size forty should theoretically yield the following results: A first eigenvalue of 0.023761, second and third eigenvalues of 0.002161, and all subsequent eigenvalues of 0.001880. In addition the first eigenvalue should increase (decrease) by 0.000547 as the portfolio size analyzed increases (decreases) by one, the second and third should increase (decrease) by 0.000007, while all subsequent eigenvalues should remain unchanged.

For all economies and for the underlying return data, the average of the simulated first eigenvalues for the different portfolio sizes are consistent with the theory. The level of thin trading and the microstructure effects simulated into the data results in an almost fifty percent reduction in the first eigenvalue across all portfolio sizes. This decrease is consistent across all the simulation iterations as reflected in the fact that the standard deviation does not increase.

The curves of the second through fifth eigenvalue averages are almost identical for the one, three and five factor economies and also similar to those for the second through tenth presented by Brown for his four factor simulated economy (1989:1256). In all cases the plots for the second through to k^{th} eigenvalues highlight that the values are greater than theory would suggest and also that the rate of change in eigenvalue as a function of portfolio size is greater than given by the partial derivative of equation 5.4b, namely;

$$\frac{\partial \lambda_{2,3,\dots,k}}{\partial m} = \frac{\partial}{\partial m}\left(\sigma_e^2\left[\frac{R^2}{\left(1-R^2\right)k}(m-1)\sigma_b^2+1\right]\right)$$
$$= \sigma_e^2\frac{R^2}{\left(1-R^2\right)k}\sigma_b^2$$

In addition, the $k+1^{th}$ and higher eigenvalues exhibit patterns similar to the 2^{nd} through to k^{th} eigenvalues and their slopes clearly deviate from the expected value of zero[30]. This result is supported by Trzcinka's analysis of 865 securities extracted from the CRSP data tapes. He found that all the eigenvalues, including the smallest, increased with the size of the group analyzed (1986:358).

[30]The results of taking the partial derivatives with respect to portfolio size for all eigenvalues are presented in table 5.2.

Contrary to what was found to be the case with the first eigenvalue, the impact of thin trading and market microstructure on the subsequent eigenvalues is to increase them. The combination of these two effects is to significantly alter the eigenvalue plot and its use as a criteria for selecting the appropriate number of factors is of questionable value when principal components analysis is used in the presence of thin trading[31,32]. Given the similarity between the eigenvalue plots across the one, three and five factor economies, together with the simulated eigenvalue patterns for the second through to k^{th}, and $k+1^{th}$ and higher eigenvalue groups, the technique is also unlikely to provide a reliable cut-off even for well traded portfolios. This finding has profound implications for international empirical research into the APT, particularly given the relatively thin trading position of many international stock markets.

In common with the findings of Brown (1989:1258), the simulations conducted in this study suggest that the small sample properties of eigenvalues may lead to the conclusion of one dominant factor and a *multiplicity of smaller pervasive factors responsible for security returns*[33]. The χ^2 tests conducted using principal components analysis for portfolios of size twenty and sixty securities are reported in tables 5.3 and 5.4 for the six economies simulated to have an average communality of 22.3% and in appendices 5.5 and 5.6 for the six economies simulated to have an average communality of 50.2%[34]. The interpretation of the tables is best described by an example. Consider the 4.7 in the third row of table 5.3 under the column headed 0.5. This number implies that 4.7% of the one hundred and fifty simulations of portfolios of size twenty (for the first economy) resulted in p-values of between 0.5 and 0.6 that, after the extraction of the first three, the remaining eigenvalues were not significantly different. Additionally, 91.3% of the simulations had p-values greater than 0.5 that, after the extraction of the first three, the remaining eigenvalues were not significantly different[35].

---

[31]Cattel (1966) has suggested that the trend in a plot of all eigenvalues will exhibit a "scree" pattern and that the scree invariably begins at the k^{th} eigenvalue when k is the true number of factors (Harman, 1976:163).

[32]Kryzanowski and To (1983:49) produce scree diagrams for samples of Canadian securities.

[33]It must be noted however that Brown (1989) used an inappropriate number of degrees of freedom in computing the theoretical χ^2 distribution as he failed to distinguish between principal component analyses based on the full covariance matrix versus the correlation matrix.

[34]The format used in the tables and appendices is the same as that employed by Roll and Ross (1980) and as displayed in their table II.

[35]The 91.3% is obtained by adding the 49.3%, 15.3%, 12.7%, 9.3% and 4.7%.

Table 5.3 Cross-sectional distribution of the χ^2 statistic when using principal components analysis on returns simulated to have an average communality of 22.3% : Underlying returns data

Economy	m	n	Probability that the remaining eigenvalues are not significantly different after the removal of the first "n"									
			0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0
Securities simulated to have normally distributed idiosyncratic risk components												
1	20	1	8.0	9.3	9.3	17.3	10.0	11.3	12.0	7.3	7.3	8.0
	60		8.0	8.0	11.3	6.7	8.0	12.7	8.7	11.3	14.7	10.7
	20	3	49.3	15.3	12.7	9.3	4.7	2.7	2.7	2.0	0.7	0.7
	60		36.0	16.7	11.3	9.3	8.7	9.3	4.7	2.0	2.0	0.0
	20	5	73.3	10.7	9.3	4.0	0.0	1.3	0.7	0.7	0.0	0.0
	60		66.0	16.0	9.3	5.3	2.7	0.7	0.0	0.0	0.0	0.0
2	20	1	10.7	8.7	14.0	9.3	6.0	6.7	7.3	12.7	14.0	10.7
	60		10.7	9.3	6.7	8.0	10.0	13.3	8.0	14.7	7.3	12.0
	20	3	43.3	12.7	10.0	8.7	10.0	4.7	2.7	4.7	2.0	1.3
	60		38.0	19.3	15.3	7.3	6.0	2.7	2.7	3.3	3.3	2.0
	20	5	62.7	15.3	10.7	2.0	2.0	2.7	2.7	1.3	0.7	0.0
	60		72.7	12.7	3.3	3.3	3.3	0.7	1.3	2.0	0.7	0.0
3	20	1	8.7	11.3	11.3	7.3	16.0	8.0	8.0	13.3	8.7	7.3
	60		8.0	8.0	8.7	8.7	8.0	12.7	9.3	10.7	12.0	14.0
	20	3	40.0	21.3	13.3	9.3	4.0	4.0	4.7	0.0	2.0	1.3
	60		33.3	20.0	12.7	12.0	4.0	6.7	1.3	6.0	4.0	0.0
	20	5	66.7	14.0	7.3	4.7	3.3	2.0	0.7	0.7	0.0	0.7
	60		68.0	14.7	6.7	2.7	3.3	3.3	1.3	0.0	0.0	0.0
Securities simulated to have non-normally distributed idiosyncratic risk components												
4	20	1	5.3	5.3	7.3	6.7	10.0	10.0	12.0	10.0	11.3	22.0
	60		4.0	4.7	5.3	6.7	10.7	10.7	7.3	14.0	14.7	22.0
	20	3	25.3	19.3	14.0	10.7	6.0	5.3	4.7	6.7	6.7	1.3
	60		24.7	17.3	12.7	6.0	14.0	6.0	6.7	3.3	5.3	4.0
	20	5	49.3	18.7	9.3	6.0	5.3	3.3	2.7	4.0	1.3	0.0
	60		56.7	14.0	10.7	6.7	2.7	3.3	2.7	1.3	1.3	0.7
5	20	1	2.7	8.7	6.7	5.3	8.7	11.3	10.7	12.0	14.7	19.3
	60		2.7	7.3	4.0	5.3	8.7	10.7	11.3	13.3	13.3	23.3
	20	3	28.0	18.0	10.7	8.7	11.3	6.7	7.3	2.0	4.7	2.7
	60		26.7	12.7	18.0	9.3	7.3	8.0	7.3	3.3	3.3	4.0
	20	5	49.3	19.3	8.0	9.3	5.3	2.0	4.0	1.3	0.7	0.7
	60		62.7	14.7	6.7	6.7	3.3	0.0	3.3	2.0	0.0	0.7
6	20	1	8.7	6.7	6.0	8.0	8.7	13.3	12.0	12.0	10.0	14.7
	60		4.7	4.0	6.7	6.0	10.7	9.3	13.3	13.3	8.0	24.0
	20	3	29.3	25.3	13.3	10.0	6.0	6.0	3.3	4.0	2.7	0.0
	60		27.3	12.7	17.3	10.0	5.3	7.3	5.3	8.7	4.0	2.0
	20	5	60.7	17.3	9.3	6.0	2.0	2.0	2.0	0.7	0.0	0.0
	60		56.7	16.7	8.7	7.3	4.7	3.3	1.3	0.0	1.3	0.0

The results presented in the top half of table 5.3 show a remarkable degree of similarity between the single, three and five factors simulated economies when the underlying returns data are used and the idiosyncratic risk components are normal. For all three economies represented there is no consistent evidence to suggest that the results are influenced by portfolio size. For all three economies approximately half of the simulated portfolio groups had at least an even chance that the eigenvalues after the first were equal, about eighty-five percent had at least an even chance that the eigenvalues after the third were equal, and about ninety-five percent had at least an even chance that the eigenvalues after the fifth were equal[36].

The impact of non-normalities in the idiosyncratic risk component is highlighted in the lower half of the table and shows some consistency with the views of Kryzanowski and To that the χ^2 *test statistic is quite sensitive to departures from normality* (1983:42). A lower percentage of the portfolios simulated have at least an even chance that the remaining eigenvalues are equal after the removal of the first k, for all values of k. The percentages that had at least an even chance that the eigenvalues after the first, third and fifth were equal averaged thirty-five, seventy-five and ninety-two respectively across the three economies.

The impact of market microstructure and thin trading on factor structure estimation, highlighted by the comparative eigenvalue curves shown in figures 5.1 and 5.2, is clearly evident when tables 5.3 and 5.4 are compared. The existence of market microstructure, and thin trading in particular, results in more common factors being required before the remaining show statistical evidence of being equivalent. In the case of the single factor economy, when analyzing the size twenty portfolios, less than five percent of the simulated portfolio groups had at least an even chance that the eigenvalues after the first were equal, less than thirty percent had at least an even chance that the eigenvalues after the third were equal, and less than seventy percent had at least an even chance that the eigenvalues after the fifth were equal. A similar pattern of results is evident for the three and five factor (normal and non-normal idiosyncratic risk) economies. This trend is consistent with Shanken's finding that the first five factors account for double the co-variation in daily returns when using an adjusted covariance matrix in preference to the standard estimator of Pearson. In contrast to the underlying returns results, there appears to be little difference between the results for normal and non-normal idiosyncratic risk economies when based on the observed returns data. This is consistent with the findings

[36]These values are computed by summing the contents of columns 0.5 to 0.9 for each row of the tables.

of Brown and Weinstein who, while noting that the bilinear testing procedure they adopted requires that the idiosyncratic risk components of returns be normal, also found their results were not materially affected by the non-normal probability distribution of daily returns (1983:721). The contrast with the results presented in table 5.4 results because the thin trading and other microstructure induced biases dominate the moderate levels of non-normality simulated into the underlying returns for economies four to six and ten to twelve.

In addition to the substantial increase in the number of factors required when using principal components analysis and χ^2 test statistics in the presence of thin trading, the trend of an increasing number of factors being necessary as portfolio size increases is also apparent. For all three economies the percentage of portfolios that had at least an even chance that the eigenvalues after the k were equivalent declines markedly as portfolio size increases. When analyzing the size sixty portfolios for the single factor economy, less than one percent of the simulated portfolio groups had at least an even chance that the eigenvalues after the first were equal, less than ten percent had at least an even chance that the eigenvalues after the third were equal, and approximately thirty-five percent had at least an even chance that the eigenvalues after the fifth were equal. This result contrasts sharply with the evidence of table 5.2 but is consistent with much of the empirical evidence[37].

---

[37]See for example Dhrymes, Friend and Gultekin (1983,1984); Upton (1985); Trzcinka (1986).

Table 5.4 Cross-sectional distribution of the χ^2 statistic when using principal components analysis on returns simulated to have an average communality of 22.3% : Observed returns data

Economy	m	n	Probability that the remaining eigenvalues are not significantly different after the removal of the first "n"									
			0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0
Securities simulated to have normally distributed idiosyncratic risk components												
1	20	1	0.7	0.0	0.0	1.3	0.0	1.3	1.3	2.0	6.7	86.7
	60		0.0	0.0	0.0	0.0	0.7	0.0	0.0	1.3	2.0	96.0
	20	3	6.0	4.7	2.7	8.0	7.3	7.3	7.3	9.3	15.3	32.0
	60		0.7	0.7	2.0	1.3	2.7	6.0	3.3	6.0	14.0	63.3
	20	5	20.7	15.3	11.3	10.7	10.7	8.0	7.3	5.3	6.0	4.7
	60		5.3	10.0	5.3	8.0	6.7	9.3	6.0	12.0	17.3	20.0
2	20	1	0.0	1.3	0.7	0.7	0.7	1.3	4.7	4.7	6.7	79.3
	60		0.0	0.0	0.0	0.0	0.0	0.0	0.7	0.0	4.0	95.3
	20	3	7.3	6.0	3.3	4.7	8.7	5.3	10.7	14.7	10.7	28.7
	60		0.0	0.0	2.7	4.0	2.0	4.0	4.7	14.0	13.3	55.3
	20	5	22.0	13.3	18.7	7.3	8.0	12.7	6.0	4.7	4.7	2.7
	60		6.7	8.7	6.7	8.7	7.3	6.7	12.0	12.0	14.7	16.7
3	20	1	0.0	0.7	0.7	1.3	2.0	0.7	3.3	4.7	4.7	82.0
	60		0.0	0.0	0.0	0.0	0.7	0.0	0.0	1.3	7.3	90.7
	20	3	9.3	6.7	8.7	6.0	6.0	6.7	4.0	12.0	17.3	23.3
	60		1.3	2.0	4.7	5.3	1.3	6.0	6.7	11.3	11.3	50.0
	20	5	31.3	10.0	10.0	10.7	10.7	8.0	6.7	6.7	2.7	3.3
	60		13.3	6.7	8.7	13.3	6.0	6.7	7.3	10.7	12.7	14.7
Securities simulated to have non-normally distributed idiosyncratic risk components												
4	20	1	0.0	0.0	0.0	0.0	1.3	0.7	1.3	1.3	2.0	93.3
	60		0.0	0.0	0.0	0.0	0.7	0.0	0.7	0.0	0.7	98.0
	20	3	4.0	3.3	4.7	4.7	5.3	8.7	8.0	12.0	17.3	32.0
	60		0.7	0.7	0.7	0.0	1.3	2.7	4.7	5.3	10.0	74.0
	20	5	18.0	12.0	12.0	8.7	11.3	9.3	8.0	8.7	4.7	7.3
	60		3.3	4.7	4.7	5.3	6.7	8.0	9.3	10.7	12.7	34.7
5	20	1	0.0	0.7	1.3	0.7	1.3	0.7	3.3	4.0	8.0	80.0
	60		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.0	98.0
	20	3	5.3	6.0	6.7	6.0	5.3	9.3	11.3	9.3	8.0	32.7
	60		0.0	1.3	1.3	2.7	4.0	2.7	5.3	3.3	13.3	66.0
	20	5	25.3	10.7	11.3	8.7	4.7	10.7	5.3	6.0	7.3	10.0
	60		7.3	5.3	7.3	3.3	6.7	9.3	8.7	14.0	11.3	26.7
6	20	1	0.0	0.0	0.0	0.7	0.7	2.0	1.3	4.7	6.0	84.7
	60		0.0	0.0	0.0	0.0	0.0	0.7	0.0	0.7	3.3	95.3
	20	3	4.7	5.3	7.3	4.7	7.3	7.3	7.3	10.0	16.7	29.3
	60		0.7	1.3	2.0	2.7	0.7	2.7	6.7	7.3	13.3	62.7
	20	5	22.7	15.3	8.7	12.7	9.3	6.0	5.3	6.0	6.0	8.0
	60		6.0	6.0	6.7	8.0	5.3	10.7	6.0	15.3	11.3	24.7

The overall results of the principal components analysis highlights two issues. Firstly, the technique has low power. There is little to distinguish between the simulation results when the data were generated by a single factor economy from those when the data were generated by three or five factor economies. This result applies both when using the less formal scree diagram approach and when using the χ^2 likelihood ratio statistic suggested by Bartlett (1950). Secondly, the significant bias introduced into the variance-covariance matrix as a result of thin trading and market microstructure leads to empirical conclusions that there is a higher number of significant common factors and that the number increases as the size of the portfolio analyzed increases[38]. This finding is consistent with that of both Kryzanowski and To (1983), who found for United States data that as sample size increases from ten to fifty Rao and Alpha factor analyses produce minimum numbers of factors (at the one percent level of significance) ranging from two to eight, and Trzcinka (1986), who showed that for k greater than one there is no obvious way to choose the number of factors and that the number *found* tends to increase as the group size increases[39]. However, as suggested by Roll and Ross (1980) when they first conducted empirical research into the Arbitrage Pricing Theory, this aspect is not necessarily critical in tests of the theory. As long as more factors are suggested by the factor analytic technique employed than are truly pervasive, the cross-sectional pricing regression procedure should result in a stable number of priced factors as the size of the portfolio being analyzed increases.

Although the simulation analysis has highlighted some deficiencies in the principal components approach suggested by Brown (1989), it cannot be automatically inferred that these apply when the factor approaches used by other researchers are employed[40]. Tables 5.5 and 5.6 present the simulation results when principal factor analyses (with and

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[38]This bias was shown through the simulation study conducted in chapter four.

[39]Dhrymes, Friend and Gultekin tentatively suggest that the methodology employed in empirical research "discovers" factors equal to ten percent of the sample/group analyzed (1984:346). In reply, Roll and Ross (1983) contend however that the increased factors relate to non-priced industry specific factors. Upton (1985) suggests that the effects of group size and length of time period analyzed on the number of significant factors using factor analysis is influenced by the differing effect these two parameters have on the χ^2 statistic and its corresponding degrees of freedom. He suggests that for large time series the number of significant factors will most certainly be overstated while the positive relationship between number of factors and group size observed empirically is strengthened in that an opposite effect is suggested based on the partial derivatives of the χ^2 statistic and its degrees of freedom as group size increases.

[40]See for example Roll and Ross (1980); Reinganum (1981); Kryzanowski and To (1983); Cho, Elton and Gruber (1984); Cho (1984); Shanken (1987).

without iteration to the optimal solution) are used in preference to components analyses. Given the extensive computer processing time involved, only the three economies containing securities having normal idiosyncratic risk and with average communality of 22.3% were simulated. In addition, only seventy-five iterations were undertaken.

A comparison of tables 5.5 and 5.6 suggests that the use of squared multiple covariances (SMCs) as communality estimates is sufficiently robust in that there appears to be no marked and consistent change in the distribution of the χ^2 statistic when iterations are carried out to obtain improved communality estimates. The use of principal factor analysis over principal components analysis does however result in an increased likelihood that fewer factors are necessary. As the table highlights, there was a greater than even chance of concluding that one factor is sufficient in over eighty percent of the groups, across all three economies. Additionally, the technique provides a substantial improvement in the consistency of the results for the observed data relative to the underlying data. This finding suggests that factor analytic techniques may be preferable to the approach suggested by Brown (1989) in spite of his comments concerning the uniqueness of the solution when employing principal components and its consequent advantage over factor analysis.

The improved results found for the principal factor technique in the presence of this trading can be explained directly by the impact of thin trading on the variance-covariance matrix[41]. The analysis in chapter four showed that thin trading had a significant effect on pairwise covariance estimation by both biasing it downwards and by reducing the efficiency of the estimate. A uniform level of thin trading across a portfolio of securities results in the off-diagonal elements being downwardly biased relative to the main diagonal (the variance estimates are less affected by thin trading). For principal factor analysis (as well as other factor analysis techniques) the main diagonal is replaced with estimates of the communalities. The squared multiple correlations are just a lower bound of the estimates (Harman, 1976:87). As alternative estimates include the highest off-diagonal *covariance*, the adjusted matrix when employing factor analysis merely becomes a less efficient estimate of the variance-covariance matrix (multiplied by a scalar) when there is a uniform level of thin trading. In contrast to this, the principal components approach, which does not involve replacement of the main diagonal with communality estimates, uses a variance-covariance matrix that has reduced efficiency and a higher level of bias.

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[41] As shown in chapter four the thin trading effect dominates the other market microstructure effects in its impact on covariance estimation.

Table 5.5 Cross-sectional distribution of the χ^2 statistic when using principal factor analysis on returns simulated to have an average communality of 22.3% : Underlying returns data

Economy	m	n	Probability that "n" factors are sufficient to explain the communality									
			0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0
SMCs as communality estimates and no iteration												
1	20	1	40.0	22.7	9.3	6.7	8.0	2.7	2.7	4.0	2.7	1.3
	60		34.7	14.7	13.3	10.7	8.0	5.3	5.3	4.0	2.7	1.3
	20	3	86.7	4.0	6.7	1.3	0.0	1.3	0.0	0.0	0.0	0.0
	60		74.7	12.0	8.0	4.0	0.0	1.3	0.0	0.0	0.0	0.0
	20	5	93.3	5.3	0.0	1.3	0.0	0.0	0.0	0.0	0.0	0.0
	60		94.7	5.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	20	1	37.3	25.3	9.3	6.7	6.7	4.0	2.7	1.3	5.3	1.3
	60		34.7	14.7	13.3	9.3	10.7	2.7	4.0	5.3	1.3	4.0
	20	3	81.3	8.0	4.0	2.7	1.3	2.7	0.0	0.0	0.0	0.0
	60		76.0	9.3	6.7	2.7	1.3	1.3	1.3	1.3	0.0	0.0
	20	5	92.0	2.7	2.7	2.7	0.0	0.0	0.0	0.0	0.0	0.0
	60		92.0	4.0	1.3	2.7	0.0	0.0	0.0	0.0	0.0	0.0
3	20	1	37.3	20.0	10.7	8.0	9.3	8.0	2.7	1.3	1.3	1.3
	60		26.7	18.7	9.3	6.7	6.7	12.0	2.7	6.7	8.0	2.7
	20	3	84.0	9.3	4.0	0.0	0.0	1.3	0.0	0.0	0.0	1.3
	60		66.7	13.3	4.0	6.7	5.3	2.7	1.3	0.0	0.0	0.0
	20	5	92.0	6.7	0.0	0.0	0.0	0.0	0.0	1.3	0.0	0.0
	60		86.7	10.7	1.3	1.3	0.0	0.0	0.0	0.0	0.0	0.0
SMCs as initial communality estimates with iteration to final solution												
1	20	1	44.0	17.3	6.7	8.0	13.3	2.7	5.3	1.3	0.0	1.3
	60		34.7	14.7	14.7	5.3	9.3	8.0	2.7	6.7	1.3	2.7
	20	3	84.0	8.0	4.0	1.3	2.7	0.0	0.0	0.0	0.0	0.0
	60		78.7	6.7	6.7	4.0	2.7	0.0	1.3	0.0	0.0	0.0
	20	5	94.7	5.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	60		96.0	2.7	1.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	20	1	29.3	25.3	8.0	12.0	8.0	5.3	2.7	4.0	2.7	2.7
	60		18.7	20.0	5.3	13.3	10.7	8.0	4.0	12.0	5.3	2.7
	20	3	81.3	9.3	2.7	2.7	1.3	2.7	0.0	0.0	0.0	0.0
	60		65.3	10.7	12.0	1.3	6.7	1.3	1.3	1.3	0.0	0.0
	20	5	93.3	5.3	0.0	1.3	0.0	0.0	0.0	0.0	0.0	0.0
	60		86.7	8.0	2.7	1.3	1.3	0.0	0.0	0.0	0.0	0.0
3	20	1	37.3	20.0	10.7	8.0	9.3	8.0	2.7	1.3	1.3	1.3
	60		26.7	18.7	10.7	6.7	6.7	12.0	1.3	6.7	8.0	2.7
	20	3	86.7	6.7	4.0	0.0	1.3	0.0	0.0	0.0	0.0	1.3
	60		66.7	14.7	4.0	5.3	6.7	1.3	1.3	0.0	0.0	0.0
	20	5	96.0	2.7	0.0	0.0	0.0	0.0	1.3	0.0	0.0	0.0
	60		86.7	12.0	0.0	1.3	0.0	0.0	0.0	0.0	0.0	0.0

Table 5.6 Cross-sectional distribution of the χ^2 statistic when using principal factor analysis on returns simulated to have an average communality of 22.3% : Observed returns data

Economy	m	n	Probability that "n" factors are sufficient to explain the communality 0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0
SMCs as communality estimates and no iteration												
1	20	1	30.7	17.3	8.0	6.7	8.0	9.3	5.3	6.7	6.7	1.3
	60		10.7	12.0	9.3	8.0	9.3	14.7	9.3	9.3	8.0	9.3
	20	3	77.3	6.7	6.7	4.0	2.7	1.3	1.3	0.0	0.0	0.0
	60		53.3	17.3	9.3	8.0	6.7	1.3	2.7	0.0	1.3	0.0
	20	5	89.3	4.0	2.7	1.3	2.7	0.0	0.0	0.0	0.0	0.0
	60		85.3	9.3	2.7	1.3	0.0	1.3	0.0	0.0	0.0	0.0
2	20	1	32.0	12.0	10.7	13.3	5.3	5.3	6.7	8.0	4.0	2.7
	60		14.7	20.0	4.0	12.0	6.7	10.7	10.7	4.0	8.0	9.3
	20	3	73.3	12.0	4.0	5.3	2.7	1.3	1.3	0.0	0.0	0.0
	60		56.0	24.0	8.0	1.3	4.0	2.7	2.7	0.0	0.0	1.3
	20	5	88.0	8.0	4.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	60		89.3	4.0	4.0	0.0	0.0	2.7	0.0	0.0	0.0	0.0
3	20	1	17.3	20.0	16.0	8.0	13.3	2.7	9.3	2.7	6.7	4.0
	60		10.7	13.3	12.0	8.0	8.0	12.0	6.7	10.7	10.7	8.0
	20	3	77.3	13.3	2.7	4.0	2.7	0.0	0.0	0.0	0.0	0.0
	60		54.7	17.3	9.3	10.7	1.3	1.3	2.7	2.7	0.0	0.0
	20	5	92.0	4.0	1.3	2.7	0.0	0.0	0.0	0.0	0.0	0.0
	60		89.3	5.3	1.3	1.3	2.7	0.0	0.0	0.0	0.0	0.0
SMCs as initial communality estimates with iteration to final solution												
1	20	1	18.7	12.0	17.3	14.7	8.0	6.7	6.7	4.0	6.7	5.3
	60		9.3	12.0	9.3	10.7	5.3	6.7	8.0	10.7	8.0	20.0
	20	3	72.0	12.0	10.7	4.0	0.0	0.0	0.0	0.0	1.3	0.0
	60		46.7	16.0	9.3	6.7	4.0	4.0	5.3	2.7	2.7	2.7
	20	5	89.3	9.3	0.0	0.0	1.3	0.0	0.0	0.0	0.0	0.0
	60		74.7	8.0	8.0	4.0	2.7	0.0	1.3	1.3	0.0	0.0
2	20	1	30.7	17.3	9.3	8.0	6.7	6.7	6.7	4.0	2.7	8.0
	60		6.7	8.0	9.3	16.0	8.0	10.7	10.7	9.3	8.0	13.3
	20	3	70.7	14.7	5.3	2.7	2.7	1.3	2.7	0.0	0.0	0.0
	60		48.0	20.0	10.7	6.7	4.0	1.3	6.7	0.0	2.7	0.0
	20	5	88.0	8.0	2.7	0.0	1.3	0.0	0.0	0.0	0.0	0.0
	60		84.0	4.0	6.7	2.7	1.3	1.3	0.0	0.0	0.0	0.0
3	20	1	17.3	20.0	16.0	8.0	13.3	4.0	8.0	2.7	6.7	4.0
	60		10.7	13.3	12.0	8.0	8.0	12.0	6.7	10.7	10.7	8.0
	20	3	80.0	12.0	2.7	4.0	1.3	0.0	0.0	0.0	0.0	0.0
	60		56.0	17.3	13.3	6.7	0.0	1.3	2.7	2.7	0.0	0.0
	20	5	96.0	1.3	2.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	60		89.3	5.3	1.3	2.7	1.3	0.0	0.0	0.0	0.0	0.0

In common with the preceding analysis, the results of the second stage testing of the Arbitrage Pricing Theory highlight the low power of the methodology. The generalized least squares regressions used to investigate the *effect of factors on equilibrium returns* (Cho, Elton and Gruber, 1984:6) do not produce clearly discernible differences for the one, three and five factor simulated economies. As stated in the methodology the zero beta coefficient λ_0 needs to be estimated and, for the current analysis, an external estimate of $\lambda_0 = 0$ was used in preference to augmenting the factor loading matrix. As the study utilizes simulated returns that the *true* λ_0 is known[42]. Additionally, Roll and Ross have suggested that by not augmenting the loading matrix the estimates of the factor premia remain statistically independent and testing for the number of priced factors is thereby reduced to a simple t-test (1980:1091).

Table 5.7 presents the percentage of portfolios with at least a specified number of risk premia being significant when testing at the five percent level of significance. The results are presented for portfolios of size twenty, forty and sixty. Additionally, the table is based on generalized least square regressions used the first five unrotated component loadings[43,44]. The low power of the procedure is clearly evident from the table. Little difference is evident between the results for the one, three and five factor simulated data. For the underlying returns data only a single factor appears to be priced. When testing at the five percent level one would expect to find two or more factors significant five percent of the time if only one priced factor described the returns generating process. When the observed data is analyzed however, the results for all three simulated economies suggest two, or possible three, priced factors are necessary. As was found to be the case with the χ^2 test of the principal component analysis itself, the impact of thin trading leads to the conclusion that more priced factors are necessary than is suggested when analyzing the underlying data. Additionally, there is some evidence to suggest that the number of priced factors found using the generalized least squares procedure may be influenced by the size of the portfolio analyzed. The results for the underlying data exhibit a tendency for the percentage of groups having at least two factors significant to decrease as the size of the portfolio increases. A similar pattern occurs at the third factor in the case of the observed

[42]This approach is consistent with that used by Cho, Elton and Gruber (1984).

[43]These loadings parallel the factor loadings produced when factor analysis is used but result from the principal components analysis.

[44]Only the results for the three economies consisting of securities having normally distributed idiosyncratic risk components are presented. The results for the non-normal condition exhibited an almost identical pattern.

data.

Finally, it is clear from table 5.7 that the percentage of groups having at least k factors significant decreases as the average communality increases. This result is somewhat surprising and can possibly be explained by that fact that the generalized least squares regressions were constrained to have a λ_0 equal to zero.

The reverse Helmert rotation suggested by Brown as a means of measuring the factor pricing with *equal statistical precision* (1989:1253) results in an increased number of factors being found to be significant. These results are presented in table 5.8. The lower power of the generalized least squares procedure remains however in that the results for the one, three and five factor economics remain remarkable similar. Using the rotation procedure results in an over-estimation of the number of priced factors for the single factor economies and an under-estimation for the five factor economies. This problem is also exacerbated in the presence of thin trading.

Table 5.7 Cross-sectional generalized least squares regressions of mean returns on component loadings: Percentages of groups with at least k factors significant at the five percent level: Unrotated loading matrix

		k				
Economy	m	1	2	3	4	5
Underlying data : Average security communality of 22.3%						
1	20	87.3	2.7	0.0	0.0	0.0
	40	87.3	1.3	0.0	0.0	0.0
	60	88.7	0.0	0.0	0.0	0.0
2	20	84.7	5.3	0.0	0.0	0.0
	40	88.7	2.7	0.0	0.0	0.0
	60	89.3	2.0	0.0	0.0	0.0
3	20	88.0	1.3	0.0	0.0	0.0
	40	91.3	0.7	0.0	0.0	0.0
	60	90.7	0.0	0.0	0.0	0.0
Underlying data : Average security communality of 50.2%						
7	20	54.0	2.0	0.0	0.0	0.0
	40	54.7	1.3	0.0	0.0	0.0
	60	56.0	0.0	0.0	0.0	0.0
8	20	56.7	4.7	0.0	0.0	0.0
	40	56.7	2.0	0.0	0.0	0.0
	60	58.7	1.3	0.0	0.0	0.0
9	20	65.3	0.7	0.0	0.0	0.0
	40	65.3	0.7	0.0	0.0	0.0
	60	64.7	1.3	0.0	0.0	0.0
Observed data : Average security communality of 22.3%						
1	20	91.3	30.7	8.0	0.7	0.0
	40	94.7	42.0	3.3	0.7	0.0
	60	94.0	45.3	4.0	0.7	0.0
2	20	90.0	26.7	6.0	0.0	0.0
	40	94.0	27.3	2.0	0.7	0.0
	60	94.7	36.7	2.7	0.7	0.0
3	20	91.3	20.7	2.7	0.0	0.0
	40	94.0	32.7	4.7	0.0	0.0
	60	94.7	42.0	2.7	0.0	0.0
Observed data : Average security communality of 50.2%						
7	20	72.0	33.3	6.0	0.7	0.0
	40	72.7	45.3	3.3	0.7	0.0
	60	73.3	52.7	1.3	0.0	0.0
8	20	74.7	34.7	5.3	0.0	0.0
	40	75.3	47.3	2.0	0.7	0.0
	60	75.3	53.3	0.7	0.0	0.0
9	20	74.0	33.3	5.3	0.0	0.0
	40	77.3	47.3	0.7	0.0	0.0
	60	77.3	55.3	0.0	0.0	0.0

Table 5.8 Cross-sectional generalized least squares regressions of mean returns on component loadings: Percentages of groups with at least k factors significant at the five percent level: <u>Rotated loading matrix</u>

		k				
Economy	m	1	2	3	4	5
Underlying data : Average security communality of 22.3%						
1	20	51.3	22.0	6.0	1.3	0.0
	40	56.7	15.3	5.3	2.7	0.7
	60	48.7	18.7	4.0	2.0	0.0
2	20	53.3	18.7	4.7	0.0	0.0
	40	52.0	16.0	1.3	0.0	0.0
	60	53.3	20.0	4.7	0.7	0.0
3	20	62.0	17.3	4.7	0.7	0.0
	40	60.0	22.0	4.7	1.3	0.0
	60	54.0	24.7	6.7	2.7	0.7
Underlying data : Average security communality of 50.2%						
7	20	34.0	6.0	0.7	0.0	0.0
	40	22.0	5.3	2.0	0.0	0.0
	60	22.7	3.3	2.0	0.7	0.0
8	20	28.0	10.0	2.7	0.0	0.0
	40	26.7	4.7	0.0	0.0	0.0
	60	27.3	7.3	0.7	0.0	0.0
9	20	35.3	11.3	0.7	0.0	0.0
	40	30.7	3.3	1.3	0.0	0.0
	60	32.7	4.0	2.7	0.0	0.0
Observed data : Average security communality of 22.3%						
1	20	86.7	52.0	16.0	6.7	1.3
	40	85.3	56.0	22.0	6.7	0.0
	60	86.7	59.3	26.7	7.3	0.0
2	20	80.7	46.0	18.7	0.7	0.0
	40	80.0	55.3	22.7	2.0	0.0
	60	84.0	53.3	23.3	2.7	0.0
3	20	86.7	56.7	21.3	2.7	0.7
	40	88.0	57.3	25.3	3.3	0.0
	60	89.3	56.0	25.3	4.0	0.0
Observed data : Average security communality of 50.2%						
7	20	68.0	28.0	10.7	2.0	0.0
	40	69.3	30.0	8.7	2.0	0.0
	60	69.3	29.3	10.7	2.7	0.0
8	20	66.0	32.0	7.3	2.7	0.0
	40	64.7	32.0	6.0	0.7	0.0
	60	71.3	27.3	5.3	0.7	0.0
9	20	67.3	33.3	8.7	0.7	0.0
	40	67.3	32.7	6.7	1.3	0.0
	60	72.0	32.0	9.3	2.7	0.0

5.4 Conclusion

Dybvig and Ross have argued that tests of the Arbitrage Pricing Theory on subsets of data are *typically valid* and that, in cases for which the testability becomes biased, any bias is towards rejection. They consequently suggest *there is little danger of spurious acceptance of the APT* (Dybvig and Ross, 1985:1195).

The simulations conducted in this chapter provide evidence that contradicts the statement of Dybvig and Ross. Under conditions of thin trading, the low power of the empirical procedures commonly employed is likely to lead to the conclusion that security returns are more consistent with the Arbitrage Pricing Theory than the Capital Asset Pricing Model. The finding results because of the significant biases induced in the variance-covariance matrix by thin trading and is consistent with the results provided by Kryzanowski and To (1983:50). They found, in a comparative analysis of NYSE and Toronto Stock Exchange securities, that the Canadian factor structures produced a first factor of lower relative importance and generalizability and a larger number of relevant factors[45].

Eigenvalue plots were shown to be remarkably similar for both one, three and five factor economies with the presence of significant levels of thin trading resulting in the first eigenvalue being biased upwards. Additionally, for both the well traded and thinly traded conditions, the eigenvalues did not conform to what one would theoretically expect. The $k+1^{th}$ and higher eigenvalues were found to increase as the size of the portfolio analyzed increased rather than remaining constant. Principal components analysis was shown to be a poor substitute for factor analysis when thin trading is present. The χ^2 test procedure is more biased towards a multiplicity of factors when based on the full variance- covariance matrix than when squared multiple covariances are used as communality estimates.

Although the above suggests that empirical evidence supporting the Arbitrage Pricing Theory may be spurious, rejection of the theory based on the evidence that the number of factors increases as the size of the portfolio being analyzed increases is inappropriate. This study has shown that this *evidence* can result from biases in covariance estimation due to market microstructure and thin trading effects. When data are generated by a k-factor economy downward bias in covariance estimation results in more factors being found as sample size increases. This trend does not however occur when the underlying

[45]In their context generalizability referred to the factor being common across all securities.

variance- covariance matrix is used in the analysis.

As with the χ^2 analysis, the generalized least squares regression procedure was shown to lack power. The results of the pricing regressions for the one, three and five factor economies show no discernible differences and in both cases the underlying data using the unrotated loading matrix lead to the conclusion of one priced factor with slight evidence that there may be two. This result is consistent with Brown's contention that regression based on the unrotated matrix will be biased towards finding a single priced factor. For the thin-traded observed data two priced factors were found. The reverse Helmet rotation results in more factors being priced but the pattern remains as for the unrotated generalized least squares regression analysis. The power of the procedure remains low in its ability to differentiate between the one, three and five factor economies. More priced factors are also found when the loading matrix is estimated using a thin trading induced biased variance-covariance matrix.

The impact of non-normalities in the idiosyncratic risk component of security returns was found to be negligible. While this aspect of the research was limited to the consideration of a moderate amount of kurtosis with no simulations conducted using skewed data, the finding is none-the-less relevant given the characteristics of the empirical distributions presented in chapter three. The evidence of the Johannesburg Stock Exchange is that while daily returns (logarithm price relatives) exhibit leptokurtic characteristics there is no evidence of consistent skewness.

The overall conclusion of the simulation study is therefore three-fold. Firstly, the diversity in the results of the vast body of empirical research into Arbitrate Pricing Theory as an alternative to the Capital Asset Pricing Model can to a large extent be explained by the low power of the procedures currently employed. Both the principal components/principal factor analysis and the subsequent generalized least squares regressions are unable to clearly distinguish between the one, three and five factor economies. A researcher's predilection to one or the other is therefore likely to influence the conclusions drawn. Secondly, thin trading, with its impact on covariance estimation, biases the results towards a multifactor conclusion being reached. It does not bias towards rejection of the APT as suggested by Dybvig and Ross. Finally, the existence of moderate non-normalities in the returns distribution does not significantly impact on the results. The simulation study therefore supports the findings of Brown who has stated:

> *... mechanical application of purely statistical approaches to determining the number of pervasive factors in equity returns may lead to false inferences.*

> *Unfortunately, it seems that economic analysis and intuition are essential ingredients to the process* (1989:1261).

The power problem highlighted by this research has also been alluded to by Brown and Weinstein and they have suggested the adoption of a Bayesian approach which allows *degrees of belief in respective models to be modified by the data* since with very many observations it is possible to *reject any hypothesis at one's favourite level of statistical significance* (1983:733-735).

Robust estimation of the variance-covariance matrix is a necessary step in any testing of the Arbitrage Pricing Theory. While bivariate approaches exists for covariance estimation in the presence of thin trading problems result when using the approach in a multivariate context. Covariance matrices estimated using a full dataset are, by construction, positive semi-definite and as such have eigenvalues greater than or equal to zero. Recognizing the rank/matrix inversion necessity of having a gramian or positive definite matrix, these matrices can nonetheless be factor analyzed. Pairwise estimation of covariance using Dimson (1979) and Cohen, Hawawini, Maier, Schwartz and Whitcomb (1980, 1983a, 1983b) procedures on the other hand, can result in an estimated variance-covariance matrix that is not positive semi-definite. As a consequence the eigenvalues can be negative and the factor loading matrix is undefined[46]. Additionally χ^2 test procedures cannot be used. While re-estimation of the covariance matrix based on the positive eigenvalue eigenvectors might be considered, this technique is inherently biased towards fewer factors[47]. It also does not recognize the relative efficiency of the pairwise covariance estimates. Chapter Four has shown that for higher levels of thin trading increasing the leads and lags reduces the estimation bias but also makes for less efficient estimates.

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[46]This condition is equivalent to the Heywood case whereby the estimated residual variance for a security is found to be negative. The condition has been referred to by prior researchers who have tended to deal with the problem by exclusion (Shanken, 1987:225).

[47]The rank of the matrix becomes equal to the number of positive eigenvalues.

6

CAPM and APT based benchmarks: An empirical comparison

6.1 Introduction

Over the last two decades extensive empirical research has been undertaken in order to investigate the level of efficiency of risky asset markets with the accumulated evidence tending to support the notion that markets are efficient in the weak and semi-strong forms but not in the strong form[1]. While most of this research is based on the theoretical underpinnings of the asset-pricing model of Sharpe (1964), Lintner (1965) and Black (1972), two other empirical models have also been frequently used[2]. In spite of being severely criticized by Roll (1977), the CAPM based methodology has retained its popularity because of its theoretical appeal. As discussed in chapter two, Roll's critique is based on the premise that the true market portfolio cannot be measured and as a consequence tests of the efficiency of the market portfolio and the validity of the CAPM are joint. Additionally, he suggests that ex-post portfolio performance measures cannot be based on a cross-sectional security market line.

Since the market portfolio does not play the central role in the Arbitrage Pricing Theory that it does in the CAPM, Roll and Ross (1980) suggest that, because of its reliance on the concept of large asset markets, the APT can be tested on a subset of the universe of risky assets. If this view is accepted then more robust and valid tests of market efficiency become possible when the research is based on the theoretical underpinnings of the APT. Shanken has refuted this claim by suggesting that the empirical formulation for testing the

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[1]The terms weak, semi-strong and strong form of efficiency were developed by Fama (1970b,1977) and are in common use as operational definitions. They can be contrasted with the far stricter definitions proposed by Rubinstein (1975) and Latham (1985).

[2]The first is the *market model* which argues that security returns are linearly related to the returns on the "market" portfolio with parameters that remain constant over the time period examined. The second is the *empirical market line* which argues that cross-sectional security returns are linearly related to security betas with parameters that are not necessarily constant through time (Copeland and Weston, 1988:362).

APT precludes the returns differentials the theory is trying to explain. He does acknowledge however that;

> *(t)he rapidly growing volume of empirical analysis purporting to test the theory indicates that this view has achieved a significant level of acceptance in the finance research community* (Shanken, 1982:1137).

In spite of the body of evidence in favour of market efficiency in the semi-strong form, several enigmatic findings persist. These include the anomalies of firm size, earnings-to-price ratio and book-to-market value of equity (Banz, 1981; Basu, 1977; Fama and French, 1992); the apparent existence of long-term stock market over-reaction (De Bondt and Thaler, 1985, 1987); and the persistent discount on newly issued stock (Stigler, 1964; Logue, 1973; Ibbotson, 1975). In addition, while assessment of the performance of managed funds in the United States (Jensen, 1968; Kim, 1978; Grinblatt and Titman, 1986) has led to the conclusion that such funds do not yield returns that are superior to naive investment strategies[3], different conclusions have been reached for some smaller and thinner traded markets (Knight and Firer, 1989).

The acceptance of the testability of the APT has resulted in increased interest in the theory as the basic framework for tests of market efficiency (Brown and Weinstein, 1985; Chang and Lewellen, 1985; Chen, Copeland and Mayers, 1987), with the principal advantage being that the return-generating equation is itself the basis for the ex-ante return relationship. This advantage is however counterbalanced by the disadvantage that the number of factors must be determined empirically, and, as shown in chapter five, the power of current statistical techniques used in determination of the number of priced factors is low.

This chapter uses South African data to assess the comparative performance of the single factor CAPM based framework and the multi-factor APT based framework in the investigation of market efficiency. The performance of Black's (1972) two parameter form of the CAPM is compared to the performance of one factor, three factor and five factor forms of Ross's Arbitrage Pricing Theory[4]. The chapter is divided into four sections. In the first, the techniques used for measuring *normal* performance when

[3]Although the managed funds appeared to outperform with respect to gross rates of return, their net performance after subtracting management fees and transaction costs was found not to be superior.

[4]The selection of the one, three and five factor forms is based on the findings of Page (1986) and Gilbertson and Goldberg (1981) who concluded that security returns on the Johannesburg Stock Exchange are explained by *at least a two factor model*, as well as in recognition of the low power of the methodology as discussed in chapter five.

applying the two frameworks are outlined. In the second section these techniques are applied to an investigation of size and earnings-to-price ratio (E/P) anomalies and in the third they are applied in the evaluation of unit trust performance. Conclusions are presented in the final section.

6.2 The measurement of "normal" performance

Any assessment of abnormal performance must, of necessity, be measured against some particular benchmark which defines *normal* performance. In an ex-post analysis of security and portfolio returns data, the abnormal returns are then described as the difference between the actual returns and those predicted by the benchmark model. Brown and Warner (1980) examined the robustness of three general CAPM based representations of the process generating ex-ante returns, namely the mean adjusted, market adjusted, and market and risk adjusted returns models. Of the three, the market and risk adjusted model involves the least restriction on the general formulation of the CAPM (Brown and Warner, 1980:207-208; Brown and Warner, 1985:6-7). A derived general factor model was added by Brown and Weinstein in 1985. It can be viewed as a general factor extension of the market model and as such is consistent with the formulation of the Arbitrage Pricing Theory (1985:491-495). The approach was also adopted by Chang and Lewellen as a benchmark in assessing mutual fund performance.

In an efficient market a security or portfolio cannot systematically earn returns that differ from those which are predicted and, while there will be times when the realized return differs from that predicted, the expected value of the abnormal return cannot be significantly different from zero. Written mathematically, if the return realized on security or portfolio i in time period t is given by;

$$r_{it} = r_{it}^{\exp} + \varepsilon_{it} \tag{6.1}$$

where; $r_{it}^{\exp}$ is the ex-ante expected return on the security or portfolio; and, ε_{it} is the abnormal, or unexpected, component of return, then in an efficient market;

$$\varepsilon_{it}^{\exp} = 0. \tag{6.2}$$

Recasting the ex-ante models in terms of ex-post values yields, for the CAPM and APT based models respectively;

$$r_{it} - \rho_t = \beta_i\left(r_{mt} - \rho_t\right) + \varepsilon_{it} \text{ ; and,} \tag{6.3}$$

$$r_{it} - \rho_t = \sum_{j=1}^{k} \beta_{ij}\left(\delta_{jt} - \rho_t\right) + \varepsilon_{it}. \tag{6.4}$$

where; r_{it} is the realized return on security or portfolio i in time period t; ρ_t is the return on the zero-beta portfolio (or risk-free rate if such exists)[5]; r_{mt} is the return on the market portfolio in time period t; δ_{jt} is the return on a well diversified portfolio with unit systematic risk on factor j and no systematic risk on the other factors; and, β_i and β_{ij} measure the sensitivity of the security or portfolio to the market portfolio and j^{th} factor respectively.

Clearly, the first step in the comparative tests of market efficiency involves the estimation of the market portfolio, the returns on the factor mimicking portfolios, and the establishment of the appropriate risk-free rate, or the estimation of the zero-beta portfolio[6]. For this purpose, the database of two hundred and forty-four Johannesburg Stock Exchange listed securities used in the distribution analysis conducted in chapter three was selected. As discussed previously, these securities traded continuously over the period February 20, 1973 to March 13, 1992 and the time series of returns for each was computed using weekly logarithm price relatives after adjusting for such issues as sub-divisions and capitalizations[7].

6.2.1 Estimation of the market and zero-beta portfolio

The market index was constructed as an equally weighted index of the two hundred and forty-four securities rather than as a market value or capitalization weighted index. This approach was originally justified by Brown and Warner (1980:243) who, in an extensive simulation study, suggested that using an equally weighted index *is no less likely, and in fact slightly more likely, to pick up abnormal performance* than a value weighted index. Banz has also suggested that an equally weighted index is preferable to one that is value weighted because it reduces the problem of beta over- and underestimation (1981:8).

[5]Throughout the thesis the notation ρ_t rather than r_{ft} is used to distinguish the zero-beta portfolio from an actual risk-free (default free) rate such as a treasury bill rate.

[6]We temporarily abstract from Roll's critique in assuming that the market portfolio, or a proxy for it, can be estimated.

[7]The database did not contain dividend information and these data were collected directly from the Johannesburg Stock Exchange monthly bulletin for inclusion in the assessment of the unit trust performance. The exclusion of dividends would render any performance comparison meaningless given the differing objectives of fund managers with respect to such issues as income and capital growth.

Empirical research which has reported results using both types of index further confirms the acceptability of using equally weighted indices and that they can be beneficial in thin traded markets (Fama and French, 1992:431; Dimson, 1979:213; Chan and Chen, 1988:314; Basu, 1983). For the South African market Barr and Bradfield go so far as to suggest that;

> *(a)lthough the equally weighted index itself does not escape the problem of thin trading, it does appear to yield more intuitively appealing estimates of beta* ... (1989b:363)

The beta for each security was estimated by regressing the security's weekly rates of return against the return on the market index. Total returns rather than excess returns were used because the estimation of the betas is a necessary first step in estimating the zero-beta portfolio[8]. A Black zero-beta portfolio approach was used in preference to using a Treasury bill risk-free rate because there is no guarantee that the figures available for the debt market in South Africa are synchronous with the database of security prices. The estimation involves establishing a weighted portfolio of the two hundred and forty-four securities that is uncorrelated with the equally weighted market index. While a large number of such portfolios exist, only the efficient, or minimum variance, portfolio is the theoretically appropriate alternative to the risk-free security within the context of the CAPM (Black, 1972). This portfolio is found by minimizing the quadratic function describing its variance in terms of the variances and covariances of the securities making up the portfolio, subject to the constraint that the resultant portfolio has a beta of zero and subject to the wealth constraint. Formally, the objective is to find the vector of security loadings that solves the minimization problem;

$$\text{minimize } \mathbf{XVX}' \; ; \tag{6.5}$$

subject to the constraints that

$$\mathbf{X}\boldsymbol{\beta}' = 0$$
$$\mathbf{Xe}' = 1.$$

where; $\mathbf{X}$ is the required row loading vector; $\mathbf{V}$ is the calculated covariance matrix; β is a row vector of the security beta estimates; and, $\mathbf{e}$ is a unit row vector.

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[8]While the regression constant is influenced by the choice of excess or total returns in the estimation process, beta is not systematically affected.

Finding the quadratic solution based on the individual securities would have required the determination of weightings for all two hundred and forty-four securities. As this procedure is expensive in computer time a method similar to that of Chang and Lewellen (1985) was therefore adopted. The individual securities were first ranked alphabetically and aggregated into sixty-one "clusters" of four[9]. These equally weighted intermediate portfolios were then used in the quadratic optimization procedure.

6.2.2 Estimation of the factor mimicking portfolios

Benchmark models conforming to the APT framework were constructed under the assumption that security returns behaviour can be explained by single, three and five factor economies. For each of these defined economies both principal components and principal factor techniques were used in the estimation. Estimation of the priced factors was carried out using a two step procedure. In the first step a principal components analysis of the full covariance matrix of the two hundred and forty-four securities used in the construction of the equally weighted index was undertaken. The procedure was used to extract the first five principal component loadings for each security. In addition, the first five principal factor loadings were also extracted by replacing the main diagonal of the covariance matrix with communality estimates calculated as the squared multiple covariances[10]. As outlined by Roll and Ross (1980), the loadings are the APT equivalent of the CAPM security betas for each factor.

The second step of the procedure is similar to that adopted in the estimation of the Black zero-beta portfolio outlined above. However, for the APT based benchmarks, not only must the zero-beta portfolio be estimated but efficient factor mimicking portfolios also need to be constructed for each of the factors. Since the econometric variables influencing security returns are not defined *a priori*, they must be estimated using factor mimicking portfolios. For each of the factors a quadratic optimization procedure was utilized to construct a minimum variance portfolio with unit loading on the factor and zero loading on all the others. Written mathematically, the objective is to;

$$\text{minimize } \mathbf{XVX}' \; ; \tag{6.6}$$

[9]Ranking by beta was not undertaken because it is believed that the same "clusters" should be used for estimating the APT factors so as to not bias the results towards any particular approach.

[10]Although the non-iterative factor analysis procedure used in this analysis reduces the efficiency of the estimates, the approach was shown in chapter five not to significantly impact on the results. The method also has the advantage of requiring considerably less computer processing resources and does not require different analyses be undertaken for each of the one, three, and five factor economies in estimating the factor loadings.

subject to the constraints that

$$\mathbf{X}\beta_i' = 0 \text{ for all } i \neq j$$
$$\mathbf{X}\beta_j' = 1$$
$$\mathbf{X}\mathbf{e}' = 1.$$

where; β_j is the row vector of loadings for the j^{th} factor; and, all the other variables are as defined before.

In the estimation of the factor mimicking portfolios the security weightings were not constrained to be greater than zero. While this constraint is often considered appropriate in estimating efficient indices (the market portfolio) due to the necessity for all assets to be in positive aggregate demand, the factor mimicking portfolios are merely surrogates for the underlying and unobserved true macroeconomic factors that systematically influence the pricing of risky assets[11].

The methodology of factor estimation outlined above is based on the approach of Chang and Lewellen (1985) although several distinct differences exist. Because of a size constraint imposed by the factor analysis programs they had available, Chang and Lewellen adopted the grouping procedure used in previous research (Roll and Ross, 1980; Gehr, 1975; Reinganum, 1981b). In addition, as a result of grouping their data into thirty-three groups of thirty securities, a rotation technique developed by Berges (1982) had to be utilized to *reduce the different structures to a common space* and obtain more efficient pooled estimates (Chang and Lewellen, 1985:20). The specification of the number of factors was also based on χ^2 tests using maximum likelihood factor analysis. Estimation of the factor loadings, or factor betas, was then obtained by regressing the returns for each security against the pooled standardized factor scores of each of the hypothesized factors. In this research the programs developed for the analysis were able to cope with the entire two hundred and forty-four securities in the estimation of the component and factor loadings. The analysis was also undertaken using the covariance matrix directly rather than being based on the correlation matrix. Given that only one large group of securities was analyzed no rotation needed to be employed and the loadings could be used directly in the subsequent analysis without the need to run regressions of securities against standardized factor scores. Chang and Lewellen also discuss in some depth their rationale for employing cluster analysis as an intermediate

[11]The zero-beta portfolio for each of the assumed economies is estimated by constraining all the portfolio factor loadings to be equal to zero.

step in estimating the final mimicking portfolios. The use of such a procedure to ensure groupings of securities that are as internally homogeneous and as externally heterogeneous as possible with respect to the loadings array has great merit when constraints exist on the size of the quadratic minimization analysis that can be handled[12]. In this research the procedure of Oldfield and Rogalski (1981) was used in that naive intermediate portfolios were formed. Sixty-one equally weighted portfolios of four securities were constructed from the database of two hundred and forty-four securities with the grouping done alphabetically. While less robust than the procedure of Chang and Lewellen, the procedure was adopted for three main reasons[13]. Firstly, although naive clustering does tend to result in similar factor risks across portfolios, this tendency is unlikely to cause problems when the portfolios consist of only four securities. Secondly, the cluster procedure can result in clusters of significantly varying sizes and consequently the clustered portfolios have differing levels of idiosyncratic risk. Finally, for comparative purposes it is beneficial to use the same "clusters" for all the estimations (for the Black zero-beta portfolio and for the various assumed economy factor mimicking portfolios).

6.3 The relationship between abnormal returns, firm size and earnings[14]

Possibly the most significant anomalies in empirical research into market efficiency relate to the size effect of Banz (1981) and Reinganum (1981a, 1981b, 1983), and to the E/P ratio effect (Basu, 1977, 1983). In a recent paper, market-to-book value of equity has also been found to explain cross-sectional variations in security returns (Fama and French, 1992). As much of the research is based on the theoretical framework of the CAPM, the debate concerning these anomalies has prompted researchers to conclude that either the CAPM is misspecified or the market is inefficient (Reinganum, 1981a:20; Banz, 1981; Basu, 1983; Fama and French, 1992:459). Not all have been convinced

[12]The principle merit being that the cluster procedure ensures that the intermediate portfolios do not have similar risk levels which could result in a singular Hessian matrix. The Hessian matrix is the second order partial derivative of the function to be optimised and is nothing other than the covariance matrix in the context of this research.

[13]It must also be noted that the cluster procedure that was available could not directly cluster the two hundred and forty-four individual securities and naive clusters of at least two needed to be formed prior to using the procedure. This process was tested and rejected in favour of that described above.

[14]A proportion of the theoretical overview discussed here is drawn from a paper published by Palmer and myself (Page and Palmer, 1991). As this work evolved from primary research undertaken by Francis Palmer for his University of Cape Town MBA research report thesis (proposed and supervised by myself) I wish to acknowledge his contribution.

however and other researchers have concluded that the empirical results are due to data biases (Roll, 1981:882; Banz and Breen, 1986:791).

This study focuses on the size and E/P ratio anomalies. It adds to the debate about whether the anomalies result from model misspecification or are consistent with market inefficiency by replicating some of the prior research using both a CAPM based framework and several versions of a comparable APT based framework. Comparison across the frameworks is undertaken by establishing whether the prior results and conclusions are materially altered by using an alternative, and possibly more appropriate, multi-factor framework.

6.3.1 Prior research

In much of the research into size and earnings effects the debate has revolved not around the existence of the anomalies as such but rather around which, if either, of the effects dominates. Reinganum (1981a, 1981b) found that risk adjusted returns from small firms outperformed those from larger firms by approximately twenty percent and concluded that the E/P ratio is subsumed by the size effect. The study was based on both NYSE and AMEX securities and used daily data over the period 1962 to 1975. These results were supported by Banz (1981) using a sample spanning forty years, who concluded that a size effect that appears to persist over forty years is more likely the result of model misspecification than market inefficiency.

In contrast to the above, Basu (1983:150) suggested that the E/P ratio effect was dominant and subsumed the size effect. He suggested that Reinganum's methodology did not adequately adjust for risk in that security abnormal returns were computed as *the difference between a given portfolio's realized return and that earned by an equally weighted NYSE-AMEX index* (Basu, 1983:130). The conclusion that the inadequate risk adjustment increased the apparent size effect was confirmed using a database of nine hundred NYSE securities over the period 1962 to 1978.

In an attempt to resolve the issue of which effect dominated, Cook and Rozeff (1984), and Jaffe, Keim and Westerfield (1989) also controlled for the January effect. In both papers the authors concluded that it was the January effect that dominated, and that once it had been controlled for, size and E/P ratio effects were independently significant. Fama and French have recently extended the analysis by adding the associated variables of book-to-market value of equity and leverage. On the basis of a dataset covering the period 1963 to 1990 they reached the conclusion that size and book-to-market value of equity were the major variables explaining cross-sectional variations in security returns (1992:450).

Roll (1981) and Blume and Stambaugh (1983) investigated the effect of data bias due to the use of daily data and resulting from such issues as bid-ask spread. The problem of serial correlation, and its impact on beta estimation, was found to exaggerate the size effect[15]. When allowing for these microstructure effects, Blume and Stambaugh showed that the size effect reported by Reinganum was halved.

The South African evidence, to a certain extent, contradicts the United States findings. In an early study into the small firm effect on the Johannesburg Stock Exchange, de Villiers, Lowings, Pettit and Affleck-Graves (1986) concluded that larger firms actually outperformed smaller during the 1973 to 1982 period. A more recent study by Page and Palmer (1991) using data spanning the period 1978 to 1988 found significant evidence of an E/P effect only[16]. Although the size effect was not found to be significant by Page and Palmer, the signs of the coefficients related to size were positive and therefore consistent, at least in orientation, with the findings of de Villiers, Lowings, Pettit and Affleck-Graves.

Finally, as stated above, it should be noted that most of the analyses into stock market anomalies have utilized the Sharpe, Lintner and Black CAPM in establishing the benchmark for measuring abnormal performance. There have, none the less, been numerous methodological issues raised in the research literature. These include the data problems of ex-post selection bias and look-ahead bias (Banz and Breen, 1986), and discussions as to the most appropriate regression techniques to employ in the analysis[17].

6.3.2 Research Methodology

One hundred and forty-five companies for which both security price data and financial data were available were selected for the analysis. Their names are listed in appendix 6.1.

[15]This contention was supported by Reinganum's (1983) subsequent analysis using Dimson's leads and lags methodology of beta estimation.

[16]The study used the both the traditional approach in the estimation of beta and the abnormal returns, and the Dimson thin trading adjustment to obtain improved beta estimates. The conclusions relating to significance of the size and E/P ratio effects were unchanged by the choice of beta estimation procedure (Page and Palmer, 1991:70).

[17]Both three-stage least squares and seemingly unrelated regression techniques have been proposed (Jaffe, Keim and Westerfield, 1989). Additionally different studies have employed both cross-sectional and longitudinal beta estimation approaches (de Villiers et. al., 1986; Fama and French, 1992).

The financial statement data was obtained from the Ivor Jones Roy and Company I-Net database and consisted of the preliminary announcement date, the E/P ratio, and the market value of equity figures. These data were extracted over the period January 1978 to September 1991 and resulted in a sample database consisting of 1525 company years of records. Security price data were extracted from the database maintained at the University of Cape Town. The prices were extracted for the same period as used for the estimation of the zero-beta portfolios, equally weighted index and factor mimicking portfolios, namely February 20, 1973 to March 13, 1992. Announcement date data rather than year-end data was utilized to avoid the problem of look-ahead bias. This bias is introduced when financial year-end data is used as a basis for portfolio selection prior to the information being publicly available. Banz and Breen (1986:791) suggest that look-ahead bias has the effect of introducing an E/P ratio effect. The issue of ex-post selection bias (survivor bias) was not addressed in the research because the South African databases used did not contain information on delisted companies. While this is a deficiency of the study, two points need to be noted. Firstly, ex-post selection bias results from the exclusion of companies that have underperformed and either failed or delisted and from the exclusion of those companies that have been acquired in mergers and takeovers. As failed or delisted companies tend to be the smaller companies in the South African context, their exclusion is likely to bias the results in favour of a United States oriented size effect. This effect is however counteracted to a certain extent by the takeover of smaller companies by larger companies. Secondly, as stated by Jaffe et. al. (1989:137);

> *"Banz and Breen find that the estimation of E/P effect is not very sensitive to ex-post selection bias but is quite sensitive to look-ahead bias. Their analysis .... suggests that much of the measured E/P effect is due to the failure to account for look-ahead bias.*

For each company announcement date, estimates of the regression parameters were obtained by regressing the previous two hundred weeks of returns against the equally weighted market index, the zero-beta portfolios and the factor mimicking portfolios. In this way beta estimates and factor loading estimates were obtained for each of the seven benchmark models[18], for each of the 1525 announcement dates. The abnormal returns over the twenty-six weeks post preliminary announcement date were then computed for each benchmark model using the estimated parameters and equations (6.3) and (6.4). These were subsequently summed over the twenty-six weeks to produce seven estimated

[18]The seven models used being the Black (1972) CAPM based model; the one, three and five factor principal component APT based models; and, the one, three and five factor principal factor APT based models.

cumulative abnormal returns for each company year. Twenty-six weeks were selected for the analysis in preference to a full year so as to reduce the impact, if any, of the publication of interim financial statements and their subsequent impact on price.

The next step in the procedure involved constructing portfolios of company years on the basis of size and E/P ratio. A within groups method was employed in order to obtain portfolios that contain an approximately equal number of company years. This method is widely used to reduce sampling error (Jaffe, Keim and Westerfield, 1989:137; Basu, 1983:133)[19]. The relatively small sample of 1525 announcements spanning eight years and nine months required that each company year be treated independently and it was therefore possible to have the same company represented more than once in any given portfolio. In order to ensure the data were consistent across time periods, the market value of each firm at announcement date was adjusted to reflect January 1988 prices. As the E/P ratio is non-dimension no correction was deemed necessary. The 1525 company years of data were initially ranked by size and divided into seven groups of one hundred and ninety-one and one group (containing the largest size data) of one hundred and eighty-eight company years. Secondary ranking was then undertaken on each of the eight size based groups by placing all the company year data with negative earnings into one portfolio and dividing the rest on the basis of ranked E/P ratio into seven equal size groups (any slight rebalancing necessary was accommodated in the largest E/P ratio portfolio). Using this approach fifty-six portfolios of approximately equal size were created together with eight smaller portfolios consisting of the company years for which the reported earnings were negative. The positive earnings portfolios are consequently likely to be equally diversified and more so than the negative earnings portfolios. Each of the sixty-four portfolios was constructed as an equally weighted portfolio of its individual components, and the average size, E/P ratio and seven different model based cumulative abnormal returns calculated. The equally weighted approach avoids biasing the portfolios towards the larger companies thereby reducing the price volatility and portfolio beta (Banz, 1981:10)[20].

Finally, the existence of size and earnings effects were tested for by using the average data of the sixty-four portfolios to run multiple regressions of portfolio abnormal return against portfolio E/P ratio and natural logarithm of size. The natural logarithm of size was

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[19]Cook and Rozeff (1984:542) describe the various portfolio construction procedures that can be employed.

[20]The analysis was also conducted by first ranking by E/P ratio and then by size and the outcome of using this alternative procedure is reported in the results section.

selected as it has been shown to be linearly related to abnormal returns (Brown, Kleidon and Marsh, 1983). Written mathematically the regression equation is;

$$\tilde{\varepsilon}_p^m = \alpha_0^m + \alpha_1^m \ln\left(Size_p\right) + \alpha_2^m (E/P)_p + \tilde{\xi}_p^m \qquad (6.7)$$

where; $\tilde{\varepsilon}_p^m$ is the twenty-six week cumulative abnormal return for portfolio p established using benchmark model m; and, α_0^m, α_1^m and α_2^m are the parameters for benchmark model m that require estimation.

The adequacy of the model, and the theoretical framework upon which it is based, is assessed by considering the significance of the three estimated regression parameters. If the market is efficient, and the model correctly specified, the portfolio abnormal returns should not be systematically different from zero, and size and E/P ratio should not have any predictive ability on portfolio returns adjusted to adequately reflect portfolio systematic risk. If any of the regression parameters are found to be significant then either the model is misspecified or the market is inefficient. Independent examination of the regression parameters, should they prove to be significant, enables conclusions to be drawn as to the relative importance of size and E/P ratio.

6.3.3 Results

Tables 6.1 and 6.2 present the descriptive statistics for the constructed portfolios using the presentation format of Jaffe, Keim and Westerfield (1989:139) and Page and Palmer (1991:68-69). The five panels of table 6.1 give the statistics for the portfolios sorted first by size and then by E/P ratio while the panels of table 6.2 give the statistics for the portfolios sorted first by E/P ratio and then by size. In each of the tables, panels A to D present the average twenty-six week total return, market model beta, size, and E/P ratio for the company years comprising each portfolio. The standard deviations of the values are also presented. Panel E gives the number of company years making up each portfolio.

The statistics presented in panel A of both tables show that, while most of the portfolios achieved a positive total return over the twenty-six weeks after the announcement date for the period studied, those portfolios that had their values decline had negative or comparatively low E/P ratios. This finding suggests that the market interprets the total earnings number in the weeks following announcement and not just the unexpected component of earnings. There is no cogent reason why low E/P ratios should be viewed by the market as generally being surprisingly low or high E/P ratios surprisingly high. The result is however appealing because it suggests that the impact of ex-post selection, or survival, bias is likely to have been minimal. Ex-post selection bias would tend to artificially improve the performance of the low E/P ratio portfolios due to the exclusion of failed companies. There is no discernible trend in total return as portfolio size increases in either of the tables.

Although there is no trend relating E/P ratio and beta, the row averages in panel B of table 6.1 and the column averages in panel B of table 6.2 show a strong relationship between firm size and market model beta estimate. This trend must however be viewed with caution. Stoll and Whaley (1983:64) and Roll (1981:884) found that thin trading induces a higher degree of autocorrelation between returns and results in the underestimation of beta. Given the inverse relationship between firm size and level of trading, it is therefore not surprising to find the beta estimates appearing to be smaller for the smaller firms. The issue of concern is rather the impact of downward bias on the abnormal returns estimation for the post announcement twenty-six week period. Finally, the relatively low betas in the table result from a peculiarity in the South African capital market where the industrial sector is overall less risky (in a beta sense) than the mining sector. Given that this study examines the size and E/P ratio effect for industrial companies, and that the equally weighted market index was computed from securities spanning the whole JSE, the average betas will therefore appear low.

While the row averages in panel C and D of table 6.1 suggest a weak negative relationship between size and E/P ratio, this conclusion is not supported by the column totals of the same panels in table 6.2. In contrast to the conclusions drawn by Jaffe, Keim and Westerfield (1989:140) therefore, there does not appear to be any consistent correlation between size and the E/P ratio for the South African data.

Table 6.1 Average weekly returns, market model betas, market capitalization, E/P ratios, and portfolio sizes for sixty-four portfolios constructed using market capitalization as the primary ranking

Panel A : Average weekly returns									
Size	**E/P Ratio**								
	<0	**2**	**3**	**4**	**5**	**6**	**7**	**High**	**Avg**
Small	-0.04	0.00	-0.01	0.05	0.14	0.08	0.21	0.25	0.09
	0.39	*0.37*	*0.38*	*0.24*	*0.28*	*0.34*	*0.37*	*0.39*	
2	0.29	-0.01	-0.01	0.09	0.17	0.13	0.17	0.22	0.13
	0.63	*0.27*	*0.30*	*0.34*	*0.35*	*0.28*	*0.26*	*0.29*	
3	0.04	0.00	0.02	0.12	0.07	0.02	0.09	0.17	0.07
	0.39	*0.30*	*0.34*	*0.29*	*0.21*	*0.34*	*0.27*	*0.23*	
4	-0.20	-0.09	0.08	0.02	0.10	0.16	0.13	0.18	0.05
	0.43	*0.38*	*0.24*	*0.30*	*0.28*	*0.21*	*0.19*	*0.21*	
5	0.07	0.05	0.08	0.07	0.01	0.11	0.18	0.17	0.09
	0.32	*0.30*	*0.27*	*0.31*	*0.19*	*0.20*	*0.20*	*0.24*	
6	-0.03	0.05	0.14	0.09	0.04	0.18	0.11	0.05	0.08
	0.43	*0.29*	*0.26*	*0.26*	*0.21*	*0.19*	*0.30*	*0.30*	
7		-0.04	0.08	0.06	0.10	0.15	0.19	0.07	0.08
		0.34	*0.23*	*0.20*	*0.24*	*0.19*	*0.22*	*0.22*	
Large	0.12	0.10	0.05	0.12	0.04	0.09	0.06	0.11	0.09
	0.23	*0.28*	*0.26*	*0.23*	*0.19*	*0.18*	*0.22*	*0.26*	
Avg	0.03	0.01	0.05	0.08	0.08	0.12	0.14	0.15	0.08
Panel B : Average market model betas									
Size	**E/P Ratio**								
	<0	**2**	**3**	**4**	**5**	**6**	**7**	**High**	**Avg**
Small	0.58	0.57	0.30	0.36	0.36	0.41	0.39	0.73	0.46
	0.54	*0.54*	*0.46*	*0.30*	*0.47*	*0.43*	*0.41*	*0.49*	
2	0.45	0.51	0.65	0.51	0.75	0.43	0.68	0.51	0.56
	0.44	*0.56*	*0.51*	*0.34*	*0.41*	*0.43*	*0.52*	*0.36*	
3	0.42	0.47	0.54	0.65	0.63	0.76	0.59	0.45	0.56
	0.63	*0.40*	*0.64*	*0.36*	*0.31*	*0.41*	*0.41*	*0.39*	
4	0.58	0.64	0.50	0.57	0.64	0.69	0.75	0.91	0.66
	0.57	*0.33*	*0.37*	*0.43*	*0.56*	*0.42*	*0.36*	*0.46*	
5	0.63	0.70	0.60	0.68	0.74	0.66	0.75	0.78	0.69
	0.65	*0.51*	*0.46*	*0.39*	*0.42*	*0.68*	*0.46*	*0.44*	
6	0.50	0.73	0.65	0.90	0.76	0.62	0.83	0.67	0.71
	0.37	*0.45*	*0.53*	*0.31*	*0.51*	*0.36*	*0.48*	*0.50*	
7		0.77	0.89	0.92	0.85	0.79	0.88	0.71	0.72
		0.50	*0.35*	*0.25*	*0.40*	*0.27*	*0.26*	*0.54*	
Large	0.93	0.95	1.17	1.00	0.97	1.09	1.03	1.16	1.04
	0.26	*0.35*	*0.30*	*0.18*	*0.27*	*0.27*	*0.24*	*0.32*	
Avg	0.51	0.67	0.66	0.70	0.71	0.68	0.74	0.74	0.68

Panel C : Average market capitalization (millions)

Size	E/P Ratio <0	2	3	4	5	6	7	High	Avg
Small	10	15	14	14	13	14	14	9	13
	8	*7*	*7*	*8*	*7*	*8*	*6*	*6*	
2	37	38	38	38	40	40	39	42	39
	8	*11*	*8*	*9*	*10*	*10*	*9*	*10*	
3	78	81	80	83	75	76	76	80	79
	20	*15*	*14*	*14*	*12*	*15*	*15*	*14*	
4	115	141	139	133	135	137	136	135	134
	17	*20*	*20*	*20*	*18*	*19*	*19*	*23*	
5	205	240	231	223	231	231	226	229	227
	25	*36*	*30*	*38*	*34*	*36*	*35*	*40*	
6	365	384	380	396	391	394	396	407	389
	58	*57*	*67*	*71*	*65*	*67*	*69*	*68*	
7		838	917	894	806	808	795	806	733
		189	*217*	*240*	*225*	*223*	*188*	*202*	
Large	4752	5190	5254	2961	3522	2950	4891	5000	4315
	2209	*4095*	*4713*	*1721*	*2599*	*1535*	*4983*	*5571*	
Avg	695	866	882	593	652	581	822	838	741

Panel D : Average E/P Ratios

Size	E/P Ratio <0	2	3	4	5	6	7	High	Avg
Small	-0.43	0.06	0.14	0.18	0.21	0.27	0.32	0.47	0.15
	0.49	*0.03*	*0.02*	*0.01*	*0.01*	*0.02*	*0.02*	*0.14*	
2	-0.43	0.07	0.13	0.16	0.19	0.22	0.28	0.38	0.13
	0.41	*0.03*	*0.01*	*0.01*	*0.01*	*0.01*	*0.02*	*0.07*	
3	-0.44	0.07	0.11	0.15	0.18	0.21	0.25	0.35	0.11
	0.84	*0.02*	*0.01*	*0.01*	*0.01*	*0.01*	*0.02*	*0.05*	
4	-0.21	0.07	0.10	0.13	0.15	0.17	0.22	0.35	0.12
	0.27	*0.02*	*0.01*	*0.01*	*0.00*	*0.01*	*0.02*	*0.10*	
5	-0.12	0.05	0.08	0.10	0.13	0.15	0.19	0.29	0.11
	0.12	*0.02*	*0.01*	*0.01*	*0.01*	*0.01*	*0.02*	*0.07*	
6	-0.05	0.05	0.09	0.11	0.13	0.15	0.19	0.28	0.12
	0.07	*0.03*	*0.01*	*0.01*	*0.01*	*0.01*	*0.02*	*0.05*	
7		0.05	0.07	0.09	0.11	0.13	0.16	0.23	0.10
		0.02	*0.00*	*0.00*	*0.01*	*0.01*	*0.01*	*0.06*	
Large	-0.04	0.03	0.06	0.08	0.09	0.11	0.13	0.17	0.08
	0.02	*0.02*	*0.01*	*0.00*	*0.00*	*0.00*	*0.01*	*0.03*	
Avg	-0.21	0.05	0.10	0.12	0.15	0.18	0.22	0.32	0.12

Panel E : Portfolio Size

Size	E/P Ratio <0	2	3	4	5	6	7	High
Small	26	24	24	24	24	24	24	21
2	12	26	26	26	26	26	26	23
3	5	27	27	27	27	27	27	24
4	5	27	27	27	27	27	27	24
5	9	26	26	26	26	26	26	26
6	6	27	27	27	27	27	27	23
7	0	27	27	27	27	27	27	29
Large	12	25	25	25	25	25	25	26

Numbers in italics give the standard deviations for the sample of securities making up each portfolio

Table 6.2 Average weekly returns, market model betas, market capitalization, E/P ratios, and portfolio sizes for sixty-four portfolios constructed using E/P ratio as the primary ranking

Panel A : Average weekly returns

E/P	Size								
Ratio	**Small**	**2**	**3**	**4**	**5**	**6**	**7**	**Large**	**Avg**
<0	-0.24	0.02	0.31	0.22	-0.13	0.01	0.09	0.09	0.05
	0.32	*0.38*	*0.35*	*0.70*	*0.35*	*0.33*	*0.36*	*0.23*	
2	0.06	-0.06	-0.02	0.04	0.01	0.01	0.08	0.08	0.03
	0.40	*0.29*	*0.41*	*0.29*	*0.35*	*0.27*	*0.24*	*0.30*	
3	0.00	0.03	0.10	0.07	0.12	0.12	0.08	0.06	0.07
	0.22	*0.25*	*0.24*	*0.23*	*0.27*	*0.24*	*0.21*	*0.21*	
4	-0.01	0.00	0.08	0.07	0.07	0.09	0.10	0.07	0.06
	0.28	*0.37*	*0.22*	*0.30*	*0.25*	*0.22*	*0.21*	*0.15*	
5	-0.01	0.13	0.06	0.02	0.11	0.11	0.18	0.10	0.09
	0.31	*0.32*	*0.32*	*0.26*	*0.22*	*0.24*	*0.26*	*0.22*	
6	0.07	0.05	0.14	0.12	0.07	0.15	0.12	0.11	0.10
	0.33	*0.30*	*0.28*	*0.28*	*0.14*	*0.14*	*0.24*	*0.27*	
7	0.16	0.20	0.09	-0.04	0.08	0.19	0.13	0.05	0.11
	0.33	*0.28*	*0.31*	*0.32*	*0.26*	*0.26*	*0.24*	*0.28*	
High	0.16	0.20	0.13	0.19	0.26	0.16	0.15	0.12	0.17
	0.41	*0.36*	*0.27*	*0.28*	*0.18*	*0.22*	*0.21*	*0.26*	
Avg	0.03	0.07	0.11	0.09	0.08	0.10	0.12	0.09	0.08

Panel B : Average market model betas

E/P	Size								
Ratio	**Small**	**2**	**3**	**4**	**5**	**6**	**7**	**Large**	**Avg**
<0	0.54	0.47	0.59	0.52	0.59	0.63	0.57	0.89	0.60
	0.36	*0.67*	*0.56*	*0.52*	*0.51*	*0.65*	*0.45*	*0.29*	
2	0.54	0.55	0.66	0.79	0.63	0.93	1.02	1.09	0.78
	0.61	*0.44*	*0.47*	*0.48*	*0.44*	*0.38*	*0.31*	*0.37*	
3	0.49	0.52	0.73	0.54	0.83	0.94	0.91	1.06	0.75
	0.41	*0.38*	*0.45*	*0.51*	*0.37*	*0.31*	*0.23*	*0.19*	
4	0.37	0.63	0.53	0.69	0.82	0.80	0.92	1.13	0.74
	0.57	*0.66*	*0.39*	*0.34*	*0.38*	*0.35*	*0.27*	*0.27*	
5	0.54	0.63	0.56	0.67	0.91	0.66	0.77	1.04	0.72
	0.35	*0.45*	*0.38*	*0.50*	*0.69*	*0.43*	*0.35*	*0.32*	
6	0.42	0.54	0.61	0.62	0.72	0.78	0.73	1.03	0.68
	0.36	*0.43*	*0.34*	*0.33*	*0.49*	*0.43*	*0.45*	*0.38*	
7	0.36	0.49	0.53	0.81	0.63	0.71	0.71	0.87	0.64
	0.45	*0.44*	*0.40*	*0.44*	*0.43*	*0.39*	*0.45*	*0.46*	
High	0.65	0.50	0.43	0.67	0.52	0.65	0.79	0.68	0.61
	0.49	*0.46*	*0.37*	*0.54*	*0.37*	*0.46*	*0.42*	*0.60*	
Avg	0.49	0.54	0.58	0.66	0.71	0.76	0.80	0.98	0.69

Panel C : Average market capitalization (millions)									
E/P	Size								
Ratio	Small	2	3	4	5	6	7	Large	Avg
<0	3	11	24	45	114	219	1045	5573	879
	1	*4*	*5*	*10*	*31*	*39*	*943*	*1915*	
2	21	79	201	317	570	998	2399	8101	1586
	8	*29*	*38*	*42*	*125*	*152*	*661*	*4616*	
3	54	134	209	314	531	976	1721	4535	1059
	32	*21*	*28*	*35*	*106*	*143*	*355*	*2443*	
4	32	89	155	253	416	709	1533	5244	1054
	16	*16*	*27*	*34*	*64*	*154*	*401*	*4629*	
5	26	74	116	158	248	373	614	3160	596
	11	*17*	*9*	*16*	*37*	*40*	*115*	*2348*	
6	12	33	63	113	178	293	608	3261	570
	7	*6*	*10*	*19*	*24*	*48*	*125*	*5463*	
7	12	32	52	68	103	171	334	1270	255
	6	*6*	*6*	*4*	*14*	*30*	*64*	*2614*	
High	5	14	25	43	64	100	170	871	162
	3	*3*	*4*	*5*	*9*	*11*	*32*	*2089*	
Avg	21	58	106	164	278	480	1053	4002	770

Panel D : Average E/P Ratios									
E/P	Size								
Ratio	Small	2	3	4	5	6	7	Large	Avg
<0	-0.51	-0.37	-0.45	-0.31	-0.34	-0.12	-0.05	-0.03	-0.27
	0.61	*0.44*	*0.37*	*0.39*	*0.63*	*0.12*	*0.06*	*0.01*	
2	0.04	0.05	0.05	0.05	0.05	0.06	0.04	0.04	0.05
	0.02	*0.02*	*0.02*	*0.02*	*0.02*	*0.01*	*0.02*	*0.02*	
3	0.09	0.09	0.09	0.09	0.09	0.08	0.09	0.09	0.09
	0.01	*0.01*	*0.01*	*0.01*	*0.01*	*0.01*	*0.01*	*0.01*	
4	0.11	0.12	0.11	0.11	0.11	0.11	0.12	0.11	0.11
	0.01	*0.01*	*0.01*	*0.01*	*0.01*	*0.01*	*0.01*	*0.01*	
5	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14
	0.01	*0.01*	*0.01*	*0.01*	*0.01*	*0.01*	*0.01*	*0.01*	
6	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17
	0.01	*0.01*	*0.01*	*0.01*	*0.01*	*0.01*	*0.01*	*0.01*	
7	0.22	0.22	0.22	0.21	0.22	0.22	0.22	0.22	0.22
	0.02	*0.02*	*0.02*	*0.02*	*0.02*	*0.02*	*0.02*	*0.02*	
High	0.40	0.34	0.32	0.32	0.34	0.34	0.33	0.31	0.34
	0.15	*0.08*	*0.06*	*0.05*	*0.07*	*0.09*	*0.08*	*0.05*	
Avg	0.08	0.09	0.08	0.10	0.10	0.13	0.13	0.13	0.11

Panel E : Portfolio Size								
E/P	Size							
Ratio	Small	2	3	4	5	6	7	Large
<0	10	10	10	10	9	9	9	9
2	26	26	26	26	26	26	26	25
3	26	26	26	26	26	26	26	25
4	26	26	26	26	26	26	26	25
5	26	26	26	26	26	26	26	25
6	26	26	26	26	26	26	26	25
7	26	26	26	26	26	26	26	25
High	26	26	26	26	26	26	26	25

Numbers in italics give the standard deviations for the sample of securities making up each portfolio

The regression results of abnormal return against the natural logarithm of portfolio size and E/P are reported in table 6.3. The table gives the results for portfolio abnormal returns based on each of the seven benchmark models and based on the alternative approaches to portfolio construction. The results are remarkably consistent across all seven models with respect to both the E/P effect and size effect. For the portfolios constructed using size as the primary method of ranking, all the models produce regressions for which the size effect is not significant but for which the E/P effect is significant at the 1% level. Additionally, the constant of the regression is negative for all models, although only significantly so for the multi-factor principal component and principal factor approaches. The results for the regressions where the portfolios were constructed using E/P ratio as the primary ranking method are very similar except in that the coefficients of determination and significances of the E/P ratio coefficients are slightly lower. Although not significant, the t-values of the size coefficients bear further comment. As discussed above, and shown by Dimson (1979), Roll (1981), Cohen, Hawawini, Maier, Schwartz and Whitcomb (1983), Stoll and Whaley (1983), and Barr and Bradfield (1989b), thin trading results in the downward bias of beta (and similarly factor loadings). The inverse relationship between trading volume and size therefore creates a larger downward beta bias and consequent greater upward abnormal return bias for the smaller firm portfolios and should induce a negative relationship between size and abnormal return. The positive coefficients and reasonably high t-values observed in the table contradict this tendency and lend support to the findings of de Villiers, Lowings, Pettit and Affleck-Graves (1986). Further research is clearly warranted before a final conclusion can be reached concerning the size effect.

6.3.4 Discussion

The results presented above are not very encouraging for researchers seeking to promote the cause of an Arbitrage Pricing Theory based approach to investigations into anomalies and market efficiency. While one should obviously be cautious about generalizing from just one study, the evidence is that none of the APT based models was able to explain the size and earnings anomalies any better than the CAPM based model. The addition of up to five factors using both the principal components and principal factor methodologies in no way diminished the size and earnings anomaly. If anything, the coefficients of determination presented in table 6.3 suggest that reduced efficiency of the estimates as more factors were added actually increased the apparent existence of the anomalies[21]. Additionally, for the three and five factor models the constants of the regression were also found to be significant and, in absolute magnitude, greater than the constants for the CAPM based and single factor models.

Table 6.3 Regressions of various benchmark model portfolio abnormal returns against E/P ratio and natural logarithm of portfolio size

Portfolio Construction using market capitalization as the primary ranking							
	CAPM Approach	APT Single Factor (PC)	APT Three Factor (PC)	APT Five Factor (PC)	APT Single Factor (PF)	APT Three Factor (PF)	APT Five Factor (PF)
α_0	-0.0412	-0.0456	-0.0719	-0.0710	-0.0414	-0.0636	-0.0639
SE(α_0)	0.0281	0.0277	0.0270	0.0276	0.0281	0.0266	0.0267
t-value	-1.466	-1.645	-2.666**	-2.570*	-1.474	-2.394*	-2.397*
α_1	0.0056	0.0067	0.0086	0.0084	0.0058	0.0070	0.0062
SE(α_1)	0.0049	0.0048	0.0047	0.0048	0.0049	0.0046	0.0047
t-value	1.140	1.376	1.831	1.747	1.182	1.504	1.329
α_2	0.1521	0.1527	0.1801	0.1787	0.1519	0.1779	0.1862
SE(α_2)	0.0505	0.0498	0.0484	0.0496	0.0505	0.0477	0.0479
t-value	3.014**	3.066**	3.717**	3.601**	3.007**	3.728**	3.888**
SER	0.0661	0.0653	0.0635	0.0650	0.0662	0.0625	0.0627
adj R^2	0.1408	0.1502	0.2110	0.1998	0.1412	0.2027	0.2112
dof	60	60	60	60	60	60	60
Portfolio Construction using E/P Ratio as the primary ranking							
	CAPM Approach	APT Single Factor (PC)	APT Three Factor (PC)	APT Five Factor (PC)	APT Single Factor (PF)	APT Three Factor (PF)	APT Five Factor (PF)
α_0	-0.0374	-0.0410	-0.0672	-0.0650	-0.0376	-0.0612	-0.0556
SE(α_0)	0.0284	0.0281	0.0262	0.0260	0.0284	0.0262	0.0248
t-value	-1.318	-1.462	-2.561*	-2.497*	-1.322	-2.336*	-2.242*
α_1	0.0060	0.0069	0.0085	0.0082	0.0062	0.0072	0.0056
SE(α_1)	0.0051	0.0051	0.0047	0.0047	0.0051	0.0047	0.0045
t-value	1.1738	1.3640	1.7986	1.7498	1.2132	1.5199	1.2596
α_2	0.1225	0.1249	0.1640	0.1593	0.1220	0.1662	0.1632
SE(α_2)	0.0515	0.0510	0.0477	0.0472	0.0516	0.0476	0.0450
t-value	2.379*	2.451*	3.441**	3.372**	2.363**	3.492**	3.624**
SER	0.0726	0.0718	0.0672	0.0666	0.0728	0.0671	0.0635
adj R^2	0.1093	0.1211	0.2086	0.2014	0.1097	0.2012	0.2023
dof	61	61	61	61	61	61	61

** : significant at the 1% level   * : significant at the 5% level

The regression used is given by $\bar{\varepsilon}_p^m = \alpha_0^m + \alpha_1^m \ln(Size_p) + \alpha_2^m (E/P)_p + \bar{\xi}_p^m$ where; $\bar{\varepsilon}_p^m$ is the twenty-six week cumulative abnormal return for portfolio p established using benchmark model m; and, α_0^m, α_1^m and α_2^m are the parameters for benchmark model m that require estimation., where;

[21] The reduced efficiency results because of the additional constraints on the quadratic optimisation routine used to estimate the minimum variance factor mimicking portfolios. Each additional factor adds one additional constraint.

A discussion of whether the results are consistent with model misspecification is deferred until after the results of the investigation into the performance of South African professional fund managers are presented.

6.4 The performance of professionally managed funds

Over the last two decades a considerable body of research has been undertaken in an attempt to assess the performance of professionally managed funds. The first comprehensive studies of mutual fund performance were undertaken by Sharpe (1966) and Jensen (1968). While, as with the anomaly studies mentioned above, most of the research has been conducted using risk-adjusted CAPM based benchmarks, techniques involving the use of both risk-class and weighted index benchmarks have also been proposed (Kim, 1978:388-391; Rennie and Cowhey, 1990). The dominance of the risk-adjusted approach is reflected in the extensive use of four well known *yardsticks*, all of which emanate, at least in part, from the CAPM framework. With Sharpe's measure, the average excess return of a portfolio[22] is adjusted for risk by dividing by the standard deviation of the portfolio's returns. This measure is then compared to the same measure for the market portfolio[23]. Treynor's measure involves dividing the mean excess return of a portfolio by its beta coefficient, and then comparing the result to the average excess return on the market portfolio. Jensen's measure focuses on the intercept term of the regression of excess rates of return of the portfolio against the excess returns of the market portfolio. The Treynor-Black appraisal ratio involves dividing Jensen's intercept term by the idiosyncratic standard deviation (Lehmann and Modest, 1987:250).

Clearly two of the above measures are inappropriate when one wishes to compare the relative merits of a CAPM based approach to portfolio performance assessment with multi-factor APT based approaches. Sharpe's measure, by dividing by standard deviation of return is not influenced by the choice of framework, and Treynor's measure can only be employed for a single risk factor model. In this study therefore, Jensen's measure and the Treynor-Black appraisal ratio are used to assess whether professional fund managers outperform the market[24]. It is hypothesized that the apparent superior performance of

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[22]The excess return for a portfolio or security is defined as the total return less the risk-free rate of return.

[23]The assumption in this comparison is that either portfolio is itself fully diversified or the investor holds no other risky asset portfolio and is therefore interested in total portfolio risk.

[24]Reporting the ordinary least squares t-statistic for the intercept term is equivalent to reporting the Treynor-Black appraisal ratio since the statistic is directly proportional to the appraisal ratio (Lehmann and Modest, 1987).

South African fund managers reported by Knight and Firer (1989) results from the misspecification of what constitutes normal performance.

6.4.1 Prior Research into managed fund performance

Professional fund managers[25] claim to provide two services for the public. Firstly, because of the large investment base accumulated, they may minimize the diversifiable risk the investor has to bear. Secondly, because of asymmetric information issues and the cost of obtaining such information, the professional manager may obtain superior performance through better security selection and better timing of trades. While the first claim may have some merit, the second is contrary to the concept of semi-strong form efficiency (unless the professional managers have access to inside or non-public information). Empirical research into mutual fund performance therefore provides a test for semi-strong form efficiency.

Jensen (1968) examined the performance of one hundred and fifteen mutual funds over the period 1955 to 1964 and concluded that, on average, the funds did not outperform passive buy-and-hold strategies over the period studied. In addition, when allowance was made for commissions and other expenses the funds tended to underperform the passive strategies. Mains (1977) in criticizing the work of Jensen suggested that his use of annual data with the attendant assumption that dividends are re-invested at year-end and expenses paid at year-end biased the results against the professionally managed funds. In a re-analysis using monthly data, Mains concluded that the gross return of mutual funds exceeded those of randomly selected portfolios but that once costs had been deducted their performance was equal to those of naive strategies. Using quarterly data for one hundred and thirty-eight funds over the seven year period 1969 to 1975 Kim reached conclusions that were consistent with those of Jensen rather than Mains. He concluded that;

> *.... most mutual funds, especially those funds with high risk objectives, were engaged in losing games. .... In view of the fact that most of the sample funds were outperformed by the benchmark portfolios, the investment experience of the seven-year period is consistent with the efficient market hypothesis, as many previous studies of mutual funds were* (Kim, 1978:25).

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[25]In this context professional fund management relates to the management of mutual funds or unit trusts. As the term unit trust is in common use in South Africa both terms will be used interchangeably in the analysis.

Chang and Lewellen (1985) were amongst the first to employ an APT framework in the assessment of mutual fund performance. They used a variant of Jensen's measure[26] in their assessment of sixty-seven mutual funds over the period January 1971 to December 1979. Two conclusions were reached. Firstly, Chang and Lewellen found that for those funds for which the Jensen's type measure was significant, it tended to be negative rather than positive and therefore signaled under- rather than superior performance. Secondly, they found the APT framework produced more powerful, but consistent, results and that the analysis suggested that;

> *a multi-factor model provides at least a somewhat better description of the securities-return-generating process that was at work in the equity marketplace during the time period investigated than does the traditional CAPM-based portrayal of that process* (Chang and Lewellen, 1985:27).

In a second application of the APT framework, Lehmann and Modest (1987) focused the attention of their research on whether different measurement procedures lead to different conclusions and whether the number of factors assumed to underlie the economy impacts on the findings. They utilized variants of both Jensen's measure and the Treynor-Black appraisal ratio and concluded that the measures are *quite sensitive to the method used to construct the APT benchmark* and that the choice of what constitutes normal performance is therefore important in the evaluation of portfolios (1987:263). With the respect to the performance of the mutual funds, Modest and Lehmann found persistent large and negative Jensen measures which they believed could not be conclusively ascribed to bias in their constructed factor portfolios.

Although the overwhelming evidence of the performance of mutual funds in the United States supports the contention that professionally managed portfolios do not achieve superior performance and that the market is therefore semi-strong form efficient, the problem of stationarity remains inadequately addressed. Active management of a mutual fund results in the composition and objectives of the managed fund changing through time. This dynamic process is likely to change the risk profile of the fund and therefore impacts on any time series derived risk-adjusting performance measure. In order to address this problem Rennie and Cowhey (1990) have proposed the use of benchmark portfolios that reflect the style of an investment manager while Lee and Rahman (1990) employed a regression technique in an attempt to separate stock selection ability from

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[26]The measure is termed a variant of Jensen's performance measure because, while it remains the estimate of the regression constant, the theoretical justification for regression itself is based on the APT and not the CAPM.

timing ability and explore market timing and selectivity performance of a sample of mutual funds.

Recent additional empirical research of the United Sates market include the investigation of biases resulting from the fact that only surviving funds are usually tested for performance (Grinblatt and Titman, 1989); further tests of relationship between expenses and charges affiliated with mutual funds and their superior results (Ippolito, 1989); and a study by Okunev (1990) using a stochastic dominance approach.

Although unit trusts were first launched in South Africa in the 1960s, it is only recently that there has been a significant increase in the number of such funds available. In January 1987 there were only fourteen funds in existence but by July 1992 this number had grown to forty-six[27]. Gilbertson and Vermaak (1982) analyzed eleven unit trusts over the period 1974 to 1981 and concluded that generally these funds outperformed benchmark portfolios on a risk adjusted basis. In a recent article, Knight and Firer (1989) reported the results of their study of the performance of ten South African unit trusts during the 1977 to 1986 period. They investigated both beta parameter stability and the stationarity of the unit trusts, and examined the performance of the unit trusts both on a risk adjusted and non-risk adjusted basis. In stating several reservations concerning the interpretations of their results, Knight and Firer make specific reference to the choice of the market portfolio as well as Roll's critique (1977). For the full period examined, they found that half of the unit trusts outperformed that market and none performed significantly worse than the market. They also suggest that their results *show better risk adjusted performance from unit trusts than was reported in previous South African studies* but acknowledge that the analysis did not take transaction costs into account (1989:62-66).

6.4.2 Research Methodology

Twenty-five unit trusts spanning the four year period February 28, 1988 to March 20, 1992 formed the database for the study. Weekly closing prices were obtained from the University of Cape Town securities database and supplemented with cash dividend information obtained from the Johannesburg Stock Exchange Quarterly Handbook. Appendix 6.2 lists the unit trusts selected for the investigation. Weekly rates of return were again computed using dividend adjusted logarithm price relatives.

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[27]The breakdown by type of unit trust was twenty-two general equity funds, seventeen specialist equity funds, and seven high income and gilt funds.

For each of the unit trusts seven Jensen type performance measures were estimated. The measures were obtained by regressing the unit trust weekly excess returns against the excess returns on the market portfolio and factor mimicking portfolios for each of the seven benchmark models[28] discussed in sections 6.2 and 6.3. Written mathematically, for the CAPM and APT based approaches respectively, the regression equations are given by;

$$r_{it} - \rho_t = \alpha_{0i} + \alpha_{1i}\left(r_{mt} - \rho_t\right) + \varepsilon_{it} \text{ ; and,} \tag{6.8}$$

$$r_{it} - \rho_t = \alpha_{0i}^m + \sum_{j=1}^{k} \alpha_{ji}^m\left(\delta_{jt} - \rho_t\right) + \varepsilon_{it}. \tag{6.9}$$

where; r_{it} is the return of the security or portfolio i in time period t; ρ_t is the return on the zero-beta portfolio (or risk-free rate if such exists); r_{mt} is the return on the market portfolio in time period t; δ_{jt} is the return on a well diversified portfolio with unit systematic risk on factor j and no systematic risk on the other factors; α_{0i} and α_{1i} are the regression constant and slope coefficient to be estimated when regressing against the excess return on the equally weighted market index; and α_{0i}^m and α_{ji}^m are the constant and slope coefficient for the j^{th} factor to be estimated when regressing against excess factor returns using the m^{th} APT based framework.

The degree of diversification of each of the unit trusts in terms of the proposed frameworks is then assessed using the adjusted coefficient of determination, while the significance of the Jensen type risk adjusted over- or under performance measure requires the examination of the significance level of the constant of the regression.

While the use of only four years of data may initially be viewed as a disadvantage when compared to the ten plus years employed in most international studies, several reasons exist that justify the choice of time period. The use of weekly data in preference to monthly, quarterly or annual data provided more observations over a four year period than most other studies had over their ten year plus periods of analysis. The shorter time span also has a theoretical advantage in that parameter stationarity assumptions of managed funds are more tenable over a shorter time period. Finally, in the context of the

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[28]For the unit trust analysis the zero-beta portfolios and factor mimicking portfolios were estimated as described in section 6.2 but based on weekly data spanning the period February 28, 1988 to March 20, 1992. As mentioned in an earlier footnote, security returns were also computed inclusive of dividends.

South African market, longer time periods result in restrictions on the number of unit trusts that can be analyzed.

6.4.3 Results

Table 6.4 presents the estimates of the intercept terms, together with the significance levels, for each of the twenty-five unit trusts for the period under investigation and for each of the seven alternative benchmark models.

Several observations are immediately evident from the results. Firstly, the coefficients (Jensen type measures) which serve as measures of each unit trust's actual risk adjusted under- or over performance, are significantly positive only when the unit trusts were evaluated against the CAPM and single factor APT based approaches. In common with the results of Knight and Firer (1989), several unit trusts apparently managed to outperform the market portfolio over the period studied. Eight of the unit trusts produced performance coefficients which were significantly positive at the one percent level and two produced significantly positive coefficients at the five percent level. More than half of the unit trusts did not, however, manage to significantly beat the market on a risk adjusted basis. As is evident from the table, the single factor APT benchmarks provided results similar to the CAPM approach and the majority of the apparently overperforming unit trusts were the general equity funds. The similarity in the estimated alpha coefficients of the unit trusts as measured by the three alternative single index benchmarks is not surprising since the single factor APT approaches can be considered as alternative representations of the *market portfolio*. On average, when employing the traditional CAPM based Jensen's performance measure, or an equivalent single factor APT based measure, the unit trusts seem to have provided unit holders with a risk adjusted annualized excess return of between nine and twelve percent. If this can be considered a true reflection of superior performance by professional fund managers then it is surely sufficient to justify the fees which the unit trusts are charging their unit holders[29].

[29]Over and above Marketable Securities Tax, brokerage and management fees up to a maximum of 5% are also charged. While the various components of the charges are governed by legislation, the spread between the buying and selling price for equity funds is approximately 6.5% and for income funds approximately 1% (there is no Marketable Securities Tax on debt instruments). The initial charge can also drop to between 0.5% and 3% when switching within the same "family" of Unit Trusts.

Table 6.4 Comparative Jensen measures across the various benchmark models for twenty-five Unit Trusts

	CAPM Approach	APT Single Factor (PC)	APT Three Factor (PC)	APT Five Factor (PC)	APT Single Factor (PF)	APT Three Factor (PF)	APT Five Factor (PF)
GRD	0.0033**	0.0038**	-0.0006	-0.0001	0.0036**	-0.0006	0.0003
MTF	0.0022*	0.0028*	-0.0015	-0.0012	0.0027*	-0.0014	-0.0007
MOM	0.0021*	0.0026*	-0.0008	-0.0001	0.0024*	-0.0006	0.0000
OMI	0.0033**	0.0041**	-0.0007	-0.0003	0.0039**	-0.0009	-0.0001
SGE	0.0030**	0.0036**	-0.0008	-0.0003	0.0034**	-0.0008	-0.0001
SNDX	0.0033**	0.0040**	-0.0009	-0.0008	0.0038**	-0.0012	-0.0006
SNTR	0.0027**	0.0033**	-0.0012	-0.0010	0.0031**	-0.0013	-0.0007
STD	0.0027	0.0029*	-0.0008	-0.0010	0.0029	-0.0010	-0.0005
SYG	0.0037**	0.0042**	0.0004	0.0010	0.0041**	0.0006	0.0012
UAL	0.0033**	0.0038**	-0.0011	-0.0008	0.0037**	-0.0012	0.0000
GRDR	0.0024	0.0033	-0.0008	-0.0008	0.0031	-0.0011	-0.0001
OMMF	0.0006	0.0018	-0.0017	-0.0017	0.0016	-0.0022	-0.0018
SAGR	0.0015	0.0025	-0.0014	-0.0010	0.0023	-0.0014	-0.0002
SNDV	0.0020	0.0025	-0.0012	-0.0010	0.0023	-0.0016	-0.0002
SNID	0.0015	0.0023	-0.0015	-0.0018	0.0021	-0.0022	-0.0018
SNMN	-0.0003	0.0017	0.0002	-0.0004	0.0013	-0.0005	-0.0007
STDG	0.0028**	0.0028*	-0.0015	-0.0009	0.0028*	-0.0013	-0.0009
UALM	0.0019	0.0029*	-0.0016	-0.0016	0.0027*	-0.0018	-0.0010
UALSO	0.0015	0.0015	-0.0017	-0.0011	0.0015	-0.0014	-0.0010
GRDI	0.0003	0.0001	0.0002	0.0001	0.0001	-0.0001	-0.0004
CRB	0.0002	0.0002	-0.0007	-0.0005	0.0002	-0.0006	-0.0008
SENG	0.0005	0.0006	-0.0007	-0.0001	0.0006	-0.0005	-0.0007
SENY	0.0004	0.0006	-0.0009	-0.0005	0.0006	-0.0008	-0.0009
STDI	0.0002	0.0001	-0.0006	-0.0007	0.0001	-0.0007	-0.0009
UALGT	0.0003	0.0004	0.0001	0.0000	0.0003	0.0000	-0.0001
Avg	0.0018**	0.0023**	-0.0009**	-0.0007**	0.0022**	-0.0010**	-0.0005**
Std	0.0012	0.0013	0.0006	0.0006	0.0013	0.0007	0.0006

** : significant at the 1% level * : significant at the 5% level

The conclusion about the superior performance of the unit trusts changes completely however when the performance measures are based on any of the multi-factor APT based benchmarks. These benchmarks represent a more complex environment and are obviously more appropriate when security returns are systematically influenced by several macro-economic variables. In the context of the three and of the five factor models, for both the components approach to factor estimation and the factor analysis approach, none of the twenty-five unit trusts exhibits any significant over performance. Rather, almost all of the unit trusts produced negative risk adjusted performance measures. These results are consistent with the findings of Lehmann and Modest (1987) for United States mutual funds. Additionally, while none of the individual coefficients was found to be statistically significant, the average under performance of all of the unit trusts is both negative, and significantly so at the one percent level for all four of the multi-factor benchmarks. In contrast to the single factor approaches therefore, average negative risk adjusted excess rates of return of between two and five percent per annum result when using the multi-factor approaches. Clearly, the choice of the model for the assessment of unit trust performance in South Africa is of great importance. Again, as mentioned in the review of prior empirical research, this finding is similar that of Lehmann and Modest (1987:263).

Table 6.5 displays the adjusted coefficients of determination of the regressions for each of the unit trusts and for each of the benchmark models. The statistics provided by the table measure the extent to which the variability of the rates of return on each of the unit trusts is captured or *explained* by the benchmark used in the regression. The coefficients of determination are adjusted to reflect the loss of degrees of freedom resulting because of the different number of independent variables in the regressions. As evident from the table, the two single factor APT based benchmarks provided relatively poor explanations of the variability of unit trust excess returns. While most of the regressions were significant at the one percent level, the average percentages explained were only of the order of twenty to twenty-five percent. These are particularly low when one recognizes that one of the objectives of unit trust investment is to diversify away idiosyncratic risk. The CAPM based benchmark, while producing similar Jensen performance measures, indicated that the degree of diversification of the unit trusts is somewhat higher.

The explanatory power of the variability of the rates of return of the unit trusts improved considerably when the multi-factor benchmarks were employed with the average adjusted coefficient of determination rising by around twenty-five percent from its level for the CAPM based approach, and more than doubling relative to the single factor APT benchmarks[30]. The increase in the adjusted coefficients of determination when moving from the three to five factor benchmarks however is slight. This lends support to the contention that, in the context of the South African capital market, a three-factor model is sufficient in providing explanations of market related phenomena. Such a model might therefore be appropriate for event studies, portfolio performance studies and other analyses of features of the Johannesburg Stock Exchange securities market.

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[30]Clearly, the coefficient of determination is influenced by the index against which a unit trust is regressed. If it is a sector specific or focussed unit trust, and not just a general equity fund, regressing it's returns against the specific sector index should produce a substantially higher coefficient of determination. In spite of the however, in terms of the CAPM and portfolio diversification, the overall market index is the appropriate independent variable.

The sufficiency of the three factor approach is further evidenced by examining those regressions for which the adjusted coefficients of determination were relatively low. All resulted from the application of the single index benchmarks to the high income and gilt funds. The significant increase in adjusted coefficients of determination when using the three factor APT based benchmarks, followed by the only marginal further improvement found when using the five factor benchmarks supports the contention that three APT factors may be sufficient.

Table 6.5 Regression Adjusted R^2 across the various benchmark models for twenty-five Unit Trusts

	CAPM Approach	APT Single Factor (PC)	APT Three Factor (PC)	APT Five Factor (PC)	APT Single Factor (PF)	APT Three Factor (PF)	APT Five Factor (PF)
GRD	0.5836**	0.2775**	0.6765**	0.6879**	0.3023**	0.6970**	0.7099**
MTF	0.5408**	0.2738**	0.6186**	0.6220**	0.3010**	0.6418**	0.6473**
MOM	0.5570**	0.2656**	0.6162**	0.6412**	0.2865**	0.6507**	0.6545**
OMI	0.5675**	0.3191**	0.6256**	0.6308**	0.3429**	0.6430**	0.6487**
SGE	0.5869**	0.2825**	0.6726**	0.6789**	0.3025**	0.6945**	0.7021**
SNDX	0.5201**	0.2633**	0.6175**	0.6169**	0.2850**	0.6386**	0.6411**
SNTR	0.5805**	0.3010**	0.6813**	0.6818**	0.3263**	0.6964**	0.7007**
STD	0.2764**	0.0953**	0.4007**	0.3991**	0.1147**	0.4108**	0.4091**
SYG	0.5570**	0.2606**	0.6324**	0.6504**	0.2819**	0.6669**	0.6707**
UAL	0.6056**	0.2833**	0.7164**	0.7212**	0.3080**	0.7267**	0.7508**
GRDR	0.4470**	0.2711**	0.4802**	0.4770**	0.2872**	0.4824**	0.4897**
OMMF	0.5421**	0.4030**	0.5685**	0.5664**	0.4228**	0.5738**	0.5722**
SAGR	0.6580**	0.4324**	0.6498**	0.6566**	0.4573**	0.6728**	0.6900**
SNDV	0.1560**	0.0639**	0.1822**	0.1787**	0.0665**	0.1872**	0.2015**
SNID	0.3539**	0.2103**	0.4009**	0.3994**	0.2158**	0.4076**	0.4034**
SNMN	0.6193**	0.6454**	0.6790**	0.6809**	0.6615**	0.6915**	0.6900**
STDG	0.3769**	0.0798**	0.6195**	0.6382**	0.0901**	0.6616**	0.6619**
UALM	0.6641**	0.4392**	0.7047**	0.7033**	0.4691**	0.7158**	0.7241**
UALSO	0.3211**	0.0683**	0.4547**	0.4734**	0.0826**	0.4833**	0.4854**
GRDI	-0.0005	0.0000	0.0504**	0.0425*	-0.0011	0.0568**	0.0594**
CRB	0.1011**	0.0126	0.1694**	0.1788**	0.0161*	0.1926**	0.1921**
SENG	0.1374**	0.0350**	0.1982**	0.2197**	0.0393**	0.2164**	0.2172**
SENY	0.1361**	0.0407**	0.1956**	0.2038**	0.0457**	0.2076**	0.2078**
STDI	0.0353**	-0.0046	0.1540**	0.1675**	-0.0038	0.1613**	0.1677**
UALGT	0.2886**	0.0717**	0.3840**	0.3848**	0.0820**	0.4025**	0.4037**
Avg	0.4085	0.2156	0.4860	0.4920	0.2313	0.5032	0.5080
Std	0.2119	0.1657	0.2119	0.2127	0.1727	0.2158	0.2178

** : significant at the 1% level * : significant at the 5% level

The extent to which the choice of the benchmark affects ones judgment about the performance of the unit trusts can further be inferred by ranking the different unit trust according to the estimated constant of the regression. While the focus of this research is not on the determination of the extent to which unit trusts maintain their position and rank from one period to another, the consistency of unit trust ranking by the different benchmark models is an important aspect of the research. Table 6.6 provides information about the rank order of the unit trusts according to the seven different benchmark models.

Table 6.6 Ranking across the various benchmark models for twenty-five Unit Trusts

	CAPM Approach	APT Single Factor (PC)	APT Three Factor (PC)	APT Five Factor (PC)	APT Single Factor (PF)	APT Three Factor (PF)	APT Five Factor (PF)
GRD	5	5	5	6	5	8	2
MTF	11	12	20	22	12	21	17
MOM	12	13	12	4	13	6	3
OMI	4	2	7	7	2	12	5
SGE	6	6	10	8	6	11	8
SNDX	2	3	15	15	3	15	13
SNTR	8	8	17	18	7	18	14
STD	9	9	11	20	9	13	12
SYG	1	1	1	1	1	1	1
UAL	3	4	16	14	4	16	4
GRDR	10	7	13	13	8	14	6
OMMF	18	17	25	24	17	25	25
SAGR	15	14	19	17	15	19	10
SNDV	13	15	18	19	14	22	9
SNID	16	16	21	25	16	24	24
SNMN	25	18	2	9	19	4	15
STDG	7	11	22	16	10	17	19
UALM	14	10	23	23	11	23	23
UALSO	17	19	24	21	18	20	22
GRDI	21	24	3	2	24	3	11
CRB	24	23	9	11	23	7	18
SENG	19	20	8	5	20	5	16
SENY	20	21	14	10	21	10	20
STDI	23	25	6	12	25	9	21
UALGT	22	22	4	3	22	2	7

Two aspects of the table stand out. Firstly, while the rank order of the unit trusts are very similar in all of the single index benchmarks, they are quite different when using the multi-factor APT benchmarks. Secondly, despite the fact that the Jensen type measure was not significantly positive in any of the multi-factor benchmarks, one unit trust, Syfrets Growth Fund (SYG), was ranked first across all seven performance measures. As an extension of the analysis of the consistency across benchmarks, Spearman's pairwise rank order correlation coefficients were computed and are presented in table 6.7. The rank correlations between the CAPM based approach and the single factor APT based approaches proved to be extremely high and in all three cases exceeded 0.95. The rank correlations across the two three factor approaches is also an impressive 0.93, and far exceeds the rank correlation between the two five factor benchmarks. It must be noticed however that the pairwise correlation coefficients between the rank order of the unit trusts using either of the three-factor approaches and the rank order estimated using the CAPM based approach and the single factor APT based approaches are all negative, albeit not statistically significant. In the same vein, the correlation coefficients between the rank order of the unit trusts measured by the five-factor principal component approach and those measured by all three single index approaches are negative although not statistically significant. These results again confirm that performance measurement is highly sensitive to the *yardstick* used in the estimation of risk adjusted performance, and consequently that one should be somewhat cautious in the declaration of *winners* and *losers* when assessing professional fund managers.

Table 6.7 Spearman rank correlations across the various benchmark models for twenty-five Unit Trusts

	APT Single Factor (PC)	APT Three Factor (PC)	APT Five Factor (PC)	APT Single Factor (PF)	APT Three Factor (PF)	APT Five Factor (PF)
CAPM Approach	0.9523**	-0.0985	-0.0069	0.9669**	-0.1731	0.5415**
APT Single Factor (PC)	1.0000	-0.0477	-0.0192	0.9969**	-0.1615	0.5477**
APT Three Factor (PC)		1.0000	0.8685**	-0.0662	0.9269**	0.5592**
APT Five Factor (PC)			1.0000	-0.0285	0.9362**	0.6446**
APT Single Factor (PF)				1.0000	-0.1746	0.5400**
APT Three Factor (PF)					1.0000	0.4992*

** : significant at the 1% level * : significant at the 5% level

Finally, it is interesting to note that the rank order of the unit trusts using the five factor principal factor benchmark was significantly positively correlated with the rank order by all other benchmarks at, at least, the five percent level. This result is possibly surprising and can best be explained by recognizing that the principal factor analysis was undertaken without iterating to a final optimal solution. As more factors are extracted the absence of sufficient iterations gives rise to relatively poorer factor estimates and the estimated regression constants in the five factor principal factor approach should therefore be viewed with caution. Note however that the correlations presented in the table for this approach, while all significant, are in magnitude somewhere between the extreme values found and reported in the rest of the table.

6.4.4 Discussion

The issue of unit trust performance in the South African capital market is by no means simple. Different conclusions as to whether professionally managed unit trusts outperform naive buy-and-hold investment strategies[31] are obtained depending upon which benchmark model is employed. What is at issue therefore is whether the return generating process in the South African capital market is best described by the Capital Asset Pricing Model or by a multi-factor Arbitrage Pricing Model. Despite all of its theoretical elegance, more and more evidence is being reported which is in contradiction to the basic hypotheses which emanate from the CAPM[32].

In this analysis an extended sample of unit trusts was employed relative to that of Knight and Firer (1989). In the empirical investigation an improved version of the CAPM was also employed by forming an equally weighted index as a more robust representation of the *market portfolio* and by deriving the minimum variance zero-beta portfolio as a theoretically preferred zero systematic risk portfolio. The results were found to be similar to those reported earlier by Knight and Firer, in that the Jensen measures for all but one of the unit trusts were positive, although not highly significant. Conceivably, the improvement in the estimation procedure of the market index surrogate as well as the use of a zero-beta portfolio acted to mitigate their conclusions that unit trusts do indeed outperform naive buy-and-hold investment strategies.

The principal appeal of the multi-factor APT based benchmarks is that their use leads to the conclusion that the unit trusts did not outperform naive buy-and-hold strategies over the period examined. This finding is theoretically appealing because it suggests that the market is semi-strong from efficient, although the aggregate under performance observed in both this study and in the study by Lehmann and Modest (1987) requires explanation. The under performance can possibly be explained, at least in the South African context, by recognizing that the implications of transaction costs incurred through the active trading of securities and management of unit trust portfolios is reflected in the price of the units. The naive buy-and-hold strategies assumed in the estimation of the factor

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[31]A naive buy-and-hold investment strategy refers to the process whereby a random portfolio of securities is chosen and subsequently held for a set time horizon without any trading activity taking place.

[32]Recent criticism of the empirical validity of the CAPM is provided by Fama and French (1992).

mimicking portfolios did not take account of any of the microstructure implications[33]. Clearly more research into this aspect is required before definitive conclusions can be reached about the real economic significance of the apparent two to five percent per annum under performance.

The similarity between the results using the three and five factor benchmarks, together with the substantial increase in percentage explained relative to all the single index approaches, suggests that a three factor APT based benchmark is adequate for use in investigations into managed fund performance in South Africa. The ability of the three factor approach to explain a significant, albeit a small proportion, of the variability of returns for the high income and gilt funds is particularly encouraging in this context.

6.5 Conclusions

The major objective of the research discussed in this chapter was to compare the performance of a traditional CAPM based approach to empirical investigations of capital market theory with several multi-factor APT based approaches. Overall the evidence lends support to the APT based approaches since, for the investigations carried out, the results show that they perform at least as well as, and at times better than, the CAPM based approach. In spite of the low power of the statistical techniques employed in identifying the number of priced factors, it appears that returns for South African Capital Market securities are better described by a multi-factor pricing equation. In the context of the unit trust study in particular, it must be noted that the improved explanation when using the multi-factor approaches is reflected in the adjusted coefficients of determination and the conclusion cannot therefore be drawn that this improvement results merely from having more independent variables in the regression. Additionally, support for a multi-factor APT model must be based on its ability to account for more of the communalities between individual securities rather than on its ability to explain total security variance.

[33]The individual wishing to construct a fully diversified portfolio would require considerable wealth when cognisance is taken of the necessity to purchase securities in finite lot sizes. Additionally, for similar reasons, the reinvestment of dividend income while maintaining diversification is difficult.

Table 6.8 Correlation amongst estimated zero-beta portfolios and amongst the equally weighted index and the first factor mimicking portfolios

	APT Single Factor (PC)	APT Three Factor (PC)	APT Five Factor (PC)	APT Single Factor (PF)	APT Three Factor (PF)	APT Five Factor (PF)
Estimated Zero-beta Portfolios : February 20, 1973 to March 13, 1992						
CAPM Approach	0.9583	0.8700	0.8677	0.9587	0.8660	0.8415
APT Single Factor (PC)	1.0000	0.7719	0.7694	0.9998	0.7667	0.7438
APT Three Factor (PC)		1.0000	0.9967	0.7706	0.9925	0.9596
APT Five Factor (PC)			1.0000	0.7681	0.9836	0.9525
APT Single Factor (PF)				1.0000	0.7652	0.7428
APT Three Factor (PF)					1.0000	0.9708
Estimated Zero-beta Portfolios : February 28, 1988 to March 13, 1992						
CAPM Approach	0.9356	0.8691	0.8711	0.9417	0.8620	0.8582
APT Single Factor (PC)	1.0000	0.8042	0.7960	0.9995	0.7840	0.7829
APT Three Factor (PC)		1.0000	0.9898	0.8089	0.9766	0.9756
APT Five Factor (PC)			1.0000	0.7999	0.9876	0.9842
APT Single Factor (PF)				1.0000	0.7877	0.7867
APT Three Factor (PF)					1.0000	0.9987
Equally Weighted Index and First Factor mimicking portfolios : February 20, 1973 to March 13, 1992						
CAPM Approach	0.7459	0.7892	0.7890	0.7304	0.7751	0.7718
APT Single Factor (PC)	1.0000	0.9572	0.9566	0.9989	0.9503	0.9446
APT Three Factor (PC)		1.0000	0.9993	0.9548	0.9971	0.9906
APT Five Factor (PC)			1.0000	0.9543	0.9951	0.9886
APT Single Factor (PF)				1.0000	0.9493	0.9437
APT Three Factor (PF)					1.0000	0.9942
Equally Weighted Index and First Factor mimicking portfolios : February 28, 1988 to March 13, 1992						
CAPM Approach	0.7698	0.8149	0.8170	0.7742	0.8180	0.8159
APT Single Factor (PC)	1.0000	0.9845	0.9844	0.9940	0.9749	0.9745
APT Three Factor (PC)		1.0000	0.9999	0.9851	0.9948	0.9944
APT Five Factor (PC)			1.0000	0.9845	0.9948	0.9944
APT Single Factor (PF)				1.0000	0.9851	0.9848
APT Three Factor (PF)					1.0000	0.9997
All correlations are significant at the 1% level						

Estimation of the factor mimicking portfolios, although more complex than the calculation of a value weighted or equally weighted index as a market surrogate, is by no means overly difficult. Any concern with using such an approach in empirical analysis must therefore relate to the question of the robustness of the estimation process itself. Table 6.8 addresses this concern, at least partially, by presenting the correlations between the seven estimated zero-beta portfolios, and the correlations between the equally weighted index and the first factor mimicking portfolios for each of the six APT based approaches[34] .Clearly, if the estimation procedures are robust, the zero-systematic risk portfolios should be highly correlated as should the first estimated factors and the equally weighted index. The consistently high correlations presented in the table suggest therefore that the procedures are sufficiently robust to warrant the use of multi-factor approaches in empirical research. It is also interesting to note that the magnitudes of the correlations were unaffected by whether the estimated zero-beta and factor mimicking portfolios were based on the full database from February 20, 1973 to March 13, 1992 or the shorter period database from February 28, 1988 to March 13, 1992.

Given its potential advantages, an APT based approach to research into capital market anomalies, event studies, and performance assessments should be seriously considered. When employing such an approach however, despite the correlations presented above, it is recommended that a principal components analysis or an iterative factor analysis procedure be used in the estimation of the factor mimicking portfolios. The results presented in this chapter suggest that the use of a non-iterative procedure with squared multiple covariances as communalities provides increasingly poorer estimates as the number of assumed factors increases.

Aside from any general merits of adopting an APT approach, several specifics about the two empirical studies conducted in this research warrant highlighting. As mentioned in section 6.3.4, the results of the size and E/P ratio anomaly study are not very favourable for the APT. The E/P ratio dominated the size effect across all seven benchmarks and the inclusion of up to five APT factors did not, in any way, affect its significance. Whether these findings are an indication of market inefficiency or model misspecification cannot be categorically stated. It is more probable however, that the results support the notion of model misspecification since, as suggested by Banz (1981), the persistence over an extended period of time of an anomaly that represents a real *arbitrage* opportunity is unlikely. One caveat to this conclusion is the recognition that the limited size database necessitated the use of a longitudinal approach to parameter estimation as well as the aggregation of data across time in the portfolio construction. The more advanced and rigorous procedures that are less dependent upon the assumption of stationarity were not adopted[35]. The factor mimicking portfolios were constructed under the assumption that

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[34]Both the components analysis and factor analysis utilise ranked eigenvalues and eigenvectors in the determination of the component and factor loadings respectively. As such the first *factor* explains the most variance or covariance amongst the securities used in the estimation and surrogates for the market index.

[35]These procedures include the use of seemingly unrelated regressions, three stage least squares procedures and the cross-sectional approaches to parameter estimation.

the pairwise covariances between the two hundred and forty-four securities used in their estimation were stationary over the period February 20, 1973 to March 13, 1992. This assumption can obviously be questioned and, given the consequent methodological limitations of the study, the results may merely reflect low power in the procedures adopted rather than model misspecification or market inefficiency.

Unlike the anomaly study, the assessment of unit trust performance using multi-factor APT benchmarks resulted in significantly different conclusions to those reached when relying upon a single factor or CAPM based approach. The results of the multi-factor benchmark analyses show that the professionally managed funds studied did not achieve superior performance over the period under investigation. This finding is consistent with efficiency in the semi-strong form and suggests earlier conclusions drawn for the South African market may have been the result of model misspecification resulting from the use of CAPM based frameworks. The use of a shorter time period for the unit trust analysis could explain the improved performance of the multi-factor approaches relative to the size and E/P anomaly analysis. The assumption of stationarity over a four year period is clearly more tenable.

Although a Fama-MacBeth (1973) type of procedure was adopted for the size and E/P anomaly study[36], the equally weighted index, factor mimicking portfolios, and zero-beta portfolios were themselves only estimated once using the entire twenty years of data for the selected two-hundred and forty-four securities. While re-estimation of the factor mimicking portfolios and zero-beta portfolios for each announcement, using a shorter period of data ending immediately prior to each announcement, would better address possible non-stationarity in underlying security sensitivities to the pervasive economic factors, the process is extremely resource intensive. For the current study, for instance, the process would require one hundred and forty-five estimations[37].

Although the research conducted in this chapter did not have the objective of identifying the number of priced APT factors underlying the South African securities market, the similarity between the results for the three and five factor approaches for both empirical

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[36]For each preliminary earnings announcement, the CAPM beta and the factor "betas" for the six APT approaches were estimated using the two hundred weeks of returns ending just prior to the announcement.

[37]Each estimation would, in turn, require the estimation of a two hundred and forty-four security covariance matrix, undertaking a principal components and principal factor analysis, and running twenty-five quadratic optimization routines to estimate the required zero-beta portfolios and factor mimicking portfolios.

analyses indicate that three factors are sufficient. No significant gain in explanatory power was achieved through the addition of the fourth and fifth factors.

Roll's (1977) critique of both the empirical testability of the CAPM itself as well as the validity of its use in ex-post performance measurement provides the further justification for adopting an APT based approach to studies of the type discussed above. Despite Shanken's (1982) opinion, it is generally accepted that the APT approach can be legitimately employed in empirical research using a subset of data without the need to identify the universe of risky assets.

7

Conclusions

In spite of the objections of Shanken (1982), proponents of the Arbitrage Pricing Theory continue to claim that the theory offers a testable alternative to the Capital Asset Pricing Model. Over the last decade however, empirical research into the APT has not proved to be universally successful. The number of priced factors *discovered* by different researchers commonly ranges from one to five or six (Tryzinka, 1986; Cho, 1984), even though the majority of tests have utilized significantly overlapping New York and American Stock Exchange datasets. In addition, empirical identification of the APT factors has also proved to be elusive and, while the economic identification of the factors cannot be considered a necessary condition for tests of the theory itself, the importance of this aspect as far as future practical application of the theory is concerned cannot be overrated.

This research adds to the literature on the Arbitrage Pricing Theory by showing that, theoretical objections aside, the multivariate procedures commonly employed in tests of the number and pricing of the factors lack power. Consequently, the conclusions drawn by different researchers are to a substantial extent influenced by their predispositions to single or multi-factor outcomes.

The χ^2 tests developed for both principal components analysis and principal factor analysis are based on the assumption that the data being analyzed is distributed as multivariate normal. In this research an empirical analysis of the distribution of security returns was therefore undertaken as a first step in the assessment of the power the multivariate procedures since any significant level of non-normality must, of necessity, impact on the test statistics. The distribution of returns (logarithm price relatives) of Johannesburg Stock Exchange listed securities was investigated in chapter three and the conclusions confirm the international evidence that security returns are distributed in a more leptokurtic fashion than suggested by the normal distribution. Contrary to the propositions of Mandelbrot (1963, 1967) and Fama (1963, 1965) however, the returns exhibit various characteristics inconsistent with the infinite variance stable Paretian hypothesis[1] and appear to conform more to some finite variance subordinated stochastic process model of the type suggested by Praetz (1972), Clark (1973), Ball and Torous (1983) and Kon (1984). This finding is important because of the fundamental role that the variance-covariance matrix plays in modern portfolio theory generally, and because of the particular role that matrix plays in the multivariate methodologies employed in tests of the APT.

In addition to the underlying distributional characteristics of securities, prior research has also shown that the estimation of returns parameters is influenced by both general market microstructure effects[2] and by the level of trading activity. Thin trading has, in particular, been shown to exacerbate the non-normal characteristics of observed returns distributions (Dimson, 1979; Cohen, Hawawini, Maier, Schwartz and Whitcomb, 1980). The extent of the impact of both effects was assessed in chapter four using an extensive simulation analysis. This technique was considered appropriate because it allows one to draw sample data from a known population and unambiguously assess the joint and several impact of microstructure effects and thin trading. Daily *true* returns for pairs of securities with a predefined correlation were first generated with distributional characteristics of the order observed for the Johannesburg Stock Exchange. These returns were then transformed into traded prices and returns by simulating a microstructure induced lag in the price evolution and by imposing various levels of thin trading. The results showed that, while the levels of non-normality observed on the Johannesburg Stock Exchange have no discernible impact on either the bias or efficiency of point estimates of pairwise covariance, thin trading does induce a significant downward bias in the estimate. The use of trade-to-trade, Dimson, and Cohen et. al. leading and lagging adjustments was found to significantly reduce both the bias and the efficiency of the estimate. Additionally, it was evident from the simulation that no single adjustment procedure provides the best solution in all cases. The trade-to-trade approach, while still resulting in a slight downward bias, proved to be superior to other methods when using daily data, and the Cohen et. al. approach provided better estimates when using weekly and monthly data. It must be noted however, that the number of leads and lags employed when using the Cohen et. al. adjustment procedure should be selected on the basis of the degree of thin trading of the pair of securities under examination. Blind adoption of a certain number of leads and

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[1]The sample distributions tend to the normal under addition and the sample variance remains stable as sample size increases.

[2]Relating to such issues as inventory positions, discrete pricing and settlement conventions.

lags in order to achieve an unbiased estimate can needlessly reduce the efficiency of the estimate without any significant reduction of bias if the securities are relatively well traded in the first place. The development of a heuristic to relate degree of thin trading to the choice of number of leads and lags when using daily, weekly or monthly data is clearly an area that warrants additional research.

In chapter five the simulation analysis was extended into a multivariate context in order to assess the power of the methodologies commonly used in tests of the Arbitrage Pricing Theory. While the methodologies examined have their origin in the empirical research of Roll and Ross (1980), the dominance of principal components rather than principal factor methods in the analysis is more consistent with the approximate factor structure development of the theory credited to Chamberlain and Rothschild (1983). The findings of the simulations proved to be discouraging as far as empirical tests of the APT are concerned. Two factors support this conclusion.

Firstly, even when, for benchmark purposes, the returns were simulated as well traded multivariate normal, the techniques proved unable to distinguish between single, three and five factor economies. Eigenvalue plots and χ^2 tests based on both components analysis and principal factor analysis provided results that where indistinguishable across the three simulated economies consisting of securities having an average communality of 22.3%, and across the three simulated economies consisting of securities having an average communality of 50.2%. For all six economies the exploding eigenvalues[3] did not stop at the k^{th} (where k is the number of factors) but continued for all of the eigenvectors extracted. The generalized least squares procedure, both with and without the reverse Helmert adjustment suggested by Brown (1989), also proved to lack power. When the regressions were based on the unrotated loading matrix only one factor appeared priced across all economies and when they were based on the rotated loading matrix two factors appeared to be priced. The reverse Helmert rotation, while leading to the conclusion that more factors are priced, cannot be considered an improvement since the technique had no impact on the power of the test. The appropriate number of priced factors was still not identified.

Secondly, the problems highlighted above proved to be significantly magnified in the presence of thin trading because of the bias introduced into the covariance matrix. When employing principal components analysis this bias was found to significantly decrease the size first eigenvalue and increase the relative sizes of all subsequent

[3]As discussed in section 2.4.2, an eigenvalue is considered "exploding" if its value increases without bound as the number of assets analysed increases.

eigenvalues. Contrary to the suggestion of Dybvig and Ross (1985), more rather than less factors are identified as a result. Additionally, the impact on the eigenvalue structure makes the use of a scree diagram as a means of determining the number of factors highly suspect. The simulation also showed that the generalized least squares procedure produces more priced factors in the presence of thin trading. The inability to distinguish across economies does however remain unaffected. The results of the simulations conducted for this research clearly contradict the suggestion of Roll and Ross (1980) that, even if too many factors are extracted in the first step of the test procedure, the cross-sectional regression technique should correctly find k to be significantly different from zero. *Evidence* provided from real stock market data apparently supporting the contentions of Roll and Ross are more likely to be the result of the inherently low power of the procedures and the tendency for the techniques to identify more factors in the presence of thin trading and market microstructure effects than they would in the absence of such effects.

The use of trade-to-trade and Cohen et. al. procedures to reduce the bias in the covariance matrix induced by thin trading proved to be problematic when attempting to employ multivariate procedures. Under conditions of thin trading of the order observed for the Johannesburg Stock Exchange, pairwise adjustments to covariance estimates produce an adjusted matrix that has a high probability of being non-gramian and, as a consequence, numerous Heywood cases appear to exist. Any subsequent adjustment to the matrix to force it to be semi-definite positive should be viewed with caution unless recognition and account is taken of the relative trading frequencies of the different securities making up the portfolio or group. As stated above, the degree of bias and efficiency is determined almost exclusively by this factor.

The overall conclusion of the simulation study is that the diversity in the results of the vast body of empirical research into the Arbitrage Pricing Theory can to a great extent be explained by the power problems of the empirical procedures employed. Conclusions drawn as to the validity or otherwise of the theory will remain open to debate until more powerful techniques become available.

In spite of the significant estimation problems, the Arbitrage Pricing Theory has been used by several researchers in market efficiency tests and as a benchmark in the investigation of market anomalies. Abstracting from the power problems addressed in earlier chapters, the comparative performance of CAPM and APT benchmarks for assessing market anomalies and portfolio performance was examined in chapter six. The difficulty of determining the correct number of APT factors was addressed by constructing three sets of factor mimicking portfolios - one under the assumption that only a single factor is priced, one assuming three factors are priced, and one assuming five factors are priced[4]. In contrast to the negative methodological results pertaining to tests of the APT itself, the use of the model in allied tests of capital market theory holds some promise. While the two studies conducted in chapter six did not provide evidence to suggest the APT is consistently superior to the CAPM, it would appear that the use of a multi-factor APT based approach provides a benchmark that will perform as well as a CAPM based benchmark and, in some cases may provide one that is superior. Although the E/P ratio and size effect anomalies found on the Johannesburg Stock Exchange (Page and Palmer, 1991) remained unchanged when use was made of up to a five factor APT based benchmark, the conclusions reached about the performance of South African unit trust managers were significantly influenced by the choice of benchmark.

If examined on its own, the finding of E/P ratio and size anomaly study would suggest that no purpose can be gained through the adoption of the more complex procedures necessary to construct factor mimicking portfolios, since, as stated by Reinganum (1981:320), ***a minimum empirical requirement for an alternative model should be that it accounts for empirical anomalies that arise within the CAPM.*** This conclusion must however be tempered by results of the unit trust study. Contrary to the conclusions drawn when employing a CAPM based benchmark, the application of multi-factor APT risk adjustment procedures produced the more intuitively appealing result that, consistent with semi-strong form efficiency, no professionally managed fund achieved a significantly superior performance over the period studied.

While numerous research avenues remain open in investigations into the Arbitrage Pricing Theory, two specific areas not addressed in this study deserve mention. Firstly, in the assessment of the comparative performance of the APT and CAPM further work of the type undertaken by Brown and Warner (1980) could prove fruitful. They adopted a "bootstrapping" procedure to test the power of alternative CAPM based benchmarks by repeatedly simulating "hypothetical" events into real security data. Clearly, there is considerable scope to extend this analysis to include APT based benchmarks. In particular, the methodology allows for the problem of self selection bias where securities with higher (lower) risk on one specific factor have a greater probability of

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[4]Three sets are refered to here because the six approaches mentioned in chapter six deal with the methodological issue of whether to the use of principal components analysis or principal factor analysis in determining the factor mimicking portfolios as well as the assumption as to the true factor structure underlying security returns on the Johannesburg Stock Exchange.

experiencing the "event". Secondly, the identification of the APT factors is an area that requires further investigation since, in spite of the comments by Roll and Ross (1980) that the issue is not relevant to tests of the APT itself, without the identification of the pervasive factors the value of the theory in normative analysis is greatly diminished. While considerable investigation has been undertaken internationally, the only published work on factor identification in South Africa is by Barr (1990). His methodology involved using a covariance biplot procedure on the factor scores for the first two factors obtained when factor analyzing twenty-six non-gold Johannesburg Stock Exchange sector indices. Given the low power of the factor analysis procedure identified in this research, alternative approaches to the identification issue warrant further investigation, particularly given the problems of the paucity of data and of obtaining synchronous econometric and security price data for the South African market.

Appendices

Appendix 3.1 Securities selected for the investigation of the distributional characteristics of security returns and for the estimation of the equally weighted index, factor mimicking portfolios, and the zero-beta portfolios

No.	JSE Code	Company Name	No. of Shares (000s)	Price at 31/12/91 (c)	Market Capital. (R m)
		Banks & Financial			
1	BANKORP	Bank Holdings Corp. of SA	419451	275	1153.5
2	BOLAND	Boland Bank	13451	1050	141.2
3	NEDCOR	Nedcor	185962	1560	2901.0
4	SBIC	Standard Bank Inv Corp	102373	5200	5323.4
		Beverages, Hotels & Leisure			
5	SA-BREWS	South African Breweries	268135	5450	14613.4
6	SUNCRUSH	Suncrush	2713	41000	1112.3
		Building & Construction			
7	ANG-ALPHA	Anglo Alpha	30076	3925	1180.5
8	BLUE-CIRC	Blue Circle	27547	3100	854.0
9	BOUMAT	Boumat	25941	420	109.0
10	CONCOR	Concor	22816	410	93.5
11	EVERITE	Everite	88927	250	222.3
12	GOLDSTEIN	S.M. Goldstein	9995	200	20.0
13	GRINAKER	Grinaker Holdings	34949	675	235.9
14	GYPSUM	Gypsum Industries	8183	1000	81.8
15	L-T-A	LTA	26388	360	95.0
16	MASONITE	Masonite (Africa)	6666	950	63.3
17	OTIS	Otis Elevator Company	17000	200	34.0
18	PORTHLD	Portland Holdings	42998	45	19.3
19	PPC	Pretoria Portland Cement	40406	4050	1636.4
20	YORKCOR	York Timber Organisation	9447	190	17.9
		Chemicals & Oils			
21	AECI	AECI	154667	900	1392.0
22	CHEMSERVE	Chemical Services	2001	4000	80.0
23	ENGEN	Engen	153931	3850	5926.3
24	SENTRCHEM	Sentrachem	115480	600	692.9
		Clothing & Textiles			
25	ADONIS	Adonis Knitwear	3545	190	6.7
26	AF-&-OVER	African & Overseas	1250	845	10.6
27	AF-OVER-A	African & Overseas -A-	1250	845	10.6
28	ALLWEAR	Allwear	14004	75	10.5
29	BOLTONS	Bolton Industrial Holdings	5174	410	21.2
30	BURLINGTN	Burlington Industries	600	307	1.8
31	FENIX	Fenix Industries	11000	290	31.9
32	GUBINGS	Gubb and Inggs	2031	710	14.4
33	JADE	Jaffe-Delswa Investments	3199	120	3.8
34	NINIAN	Ninian and Lester Holdings	3219	600	19.3
35	PROGRESS	Progress Industries	2804	160	4.5
36	REX-TRUE	Rex Trueform Clothing Co	2730	930	25.4
37	ROMATEX	Romatex	26515	450	119.3
38	SEARDEL	Seardel Investment Corp	23423	340	79.6
39	T-E-J	Towles, Edgar Jacobs	2508	70	1.8
40	TEX-MILLS	Textile Mills (1947) Hldgs	1991	90	1.8
		Coal Mines			
41	AMCOAL	Anglo American Coal Corp	25147	12400	3118.2
42	G-F-COAL	Goldfields Coal	16863	810	136.6
43	RANDCOL	Randfontein Collieries	102415	1250	1280.2
44	TRANS-NTL	Trans Natal Coal Corporation	79669	1225	975.9

Appendix 3.1 (continued) Securities selected for the investigation of the distributional characteristics of security returns and for the estimation of the equally weighted index, factor mimicking portfolios, and the zero-beta portfolios

No.	JSE Code	Company Name	No. of Shares (000s)	Price at 31/12/91 (c)	Market Capital. (R m)
45	VIERFONTN	Vierfontein Colliery	4000	15	0.6
46	WANKIE	Wankie Colliery Co. (Zim)	25333	115	29.1
		Copper Mining			
47	BOTREST	Botswana RST	17979	45	8.1
48	PALAMIN	Palabora Mining Company	28316	7600	2152.0
49	Z-C-I	Zambia Copper Investments	122560	110	134.8
		Diamond Mining			
50	ANAMINT	Anglo American Inv Trust	100000	11000	11000.0
51	BROADACRE	Broadacres Investments	3409	45	1.5
52	DEBEERS	De Beers Consolidated Mines	380141	9000	34212.7
53	I-G-I	Incorporated General Ins	13830	2300	318.1
		Electronics etc.			
54	ABACUS	Abacus	56737	15	8.5
55	ABERDARE	Aberdare Cables Africa	14577	2900	422.7
56	AF-CABLE	African Cables	24327	570	138.7
57	ALTECH	Allied Technologies	10474	8900	932.2
58	ALTRON	Allied Electronics Corp	18984	4900	930.2
59	CAFCA	Central African Cables	30600	165	50.5
60	DELTA	Delta Electrical Industries	41095	880	361.6
61	REUNERT	Reunert	31482	1800	566.7
62	TEDELEX	Television & Electrical Hldgs	60521	200	121.0
63	VENTRON	Ventron Corporation	26952	2100	566.0
		Engineering			
64	AFROX	African Oxygen	29957	7500	2246.8
65	BERZACK	Berzack Brothers Holdings	26544	1075	285.3
66	BIVEC	Berzack Illman Inv Corp.	19214	700	134.5
67	BUFFCOR	Buffalo Corporation	6400	165	10.6
68	CEMENCO	Cementation Company (Africa)	8952	250	22.4
69	DORBYL	Dorbyl	31823	1900	604.6
70	ED-LBATE	Edward L. Bateman	27363	690	188.8
71	FRALEX	Fralex Holdings	16110	590	95.0
72	G-I-C	Goldfields Industrial Corp	3951	500	19.8
73	METKOR	Metkor Investments	108544	215	233.4
		Fishing			
74	NAMFISH	Namibia Fishing	3150	425	13.4
75	OCFISH	Oceana Fishing Group	9392	1335	125.4
		Food			
76	CADSWEP	Cadbury Schweppes SA	35118	3150	1106.2
77	FEDFOOD	Federale Voedsel	29750	1575	468.6
78	I-&-J	Irwin and Johnson	28633	4000	1145.3
79	ICH	Industrial and Com Hldgs Group	37218	1325	493.1
80	KANHYM	Kanhym Investments	55000	500	275.0
81	PREM-GRP	Premier Group	78863	3250	2563.0
82	TIGR-OATS	Tiger Oats	139378	3850	5366.1
		Furniture and Household Goods			
83	AFCOL	Associated Furniture Company	23714	1200	284.6
84	ELLERINE	Ellerine Holdings	6900	5000	345.0
85	MATH-ASH	Mathieson and Ashley Holdings	11460	162	18.6
86	PICAPLI	Picardi Appliances	25933	80	20.7

Appendix 3.1 (continued) Securities selected for the investigation of the distributional characteristics of security returns and for the estimation of the equally weighted index, factor mimicking portfolios, and the zero-beta portfolios

No.	JSE Code	Company Name	No. of Shares (000s)	Price at 31/12/91 (c)	Market Capital. (R m)
		Gold Mines			
87	AFR-LEASE	Afrikaner Lease	6728	70	4.7
88	BLYVOOR	Blyvooruitzicht Gold Mining Co	24000	600	144.0
89	BRACKEN	Bracken Mines	14000	140	19.6
90	BUFFELS	Buffelsfontein Gold Mining Co	11000	3450	379.5
91	DBN-DEEP	Durban Roodepoort Deep	2325	2100	48.8
92	DRIES	Driefontein Consolidated	204000	3875	7905.0
93	E-R-P-M	East Rand Proprietary Mines	16632	900	149.7
94	E-T-CONS	Eastern Transvaal Cons Mines	86334	310	267.6
95	ELSBURG	Elsburg Gold Mining Co	30203	305	92.1
96	FALCON	Falcon Mines	19972	141	28.2
97	GROOTVLEI	Grootvlei Proprietary Mines	11439	475	54.3
98	HARMONY	Harmony Gold Mining Co	26885	1950	524.3
99	HARTIES	Hartebeesfontein Gold Mining Co	112000	1500	1680.0
100	KINROSS	Kinross Mines	18000	4450	801.0
101	KLOOF	Kloof Gold Mining Co	121100	3200	3875.2
102	LESLIE	Leslie Gold Mines	16000	300	48.0
103	LIBANON	Libanon Gold Mining Co	40000	300	120.0
104	LORAINE	Loraine Gold Mines	16367	330	54.0
105	MODDER	Consolidated Modderfontein	16100	60	9.7
106	RANDFONTN	Randfontein Estates Gold Mining	61136	1650	1008.7
107	RD-LEASE	Rand Leases Gold Mining Co	118505	43	51.0
108	SALLIES	SA Land & Exploration Company	9314	140	13.0
109	SIMMERS	Simmer and Jack Mines	19913	230	45.8
110	SOUTHVAAL	Southvaal Holdings	26000	6400	1664.0
111	ST-HELENA	St Helena Gold Mining Company	9625	2300	221.4
112	STH-RODPT	South Roodepoort Main Reef	17455	35	6.1
113	STILFTN	Stilfontein Gold Mining Company	13063	325	42.5
114	VAAL-REEF	Vaal Reefs Exploration & Mining	19114	20100	3841.9
115	VENTERS	Venterspost Gold Mining Co	20200	225	45.5
116	VILLAGE	Village Main Reef Gold Mining	6068	180	10.9
117	VLAKS	Vlakfontein Gold Mining Co	6800	100	6.8
118	W-R-CONS	West Rand Consolidated Mines	4250	500	21.3
119	WAVERLEY	Waverley Gold Mines	8500	210	17.9
120	WELKOM	Welkom Gold Mining Company	35351	1665	588.6
121	WES-AREAS	Western Areas Gold Mining Co	40307	510	205.6
122	WINKELS	Winkelhaak Mines	12180	3500	426.3
123	WIT-NIGEL	Witwatersrand Nigel	29366	21	6.2
124	WSTN-DEEP	Western Deep Levels	27712	11400	3159.2
125	ZANDPAN	Zandpan Gold Mining Company	130203	275	358.1
		Industrial Holdings			
126	A-V-I	Anglovaal Industries	28585	11800	3373.0
127	AMIC	Anglo American Ind Corp	54774	7300	3998.5
128	BARLOWS	Barlow Rand	190629	4925	9388.5
129	BIDVEST	Bidvest	6566	2700	177.3
130	BTRDUN	BTR Dunlop	23512	2100	493.8
131	CGSMITH	CG Smith	47012	10900	5124.3
132	CULLINAN	Cullinan Holdings	14453	435	62.9
133	EUREKA	Eureka Industrial	60000	18	10.8

Appendix 3.1 (continued) Securities selected for the investigation of the distributional characteristics of security returns and for the estimation of the equally weighted index, factor mimicking portfolios, and the zero-beta portfolios

No.	JSE Code	Company Name	No. of Shares (000s)	Price at 31/12/91 (c)	Market Capital. (R m)
134	F-S-I	Form-Scaff Industries	54168	400	216.7
135	FARM-AG	Farm-Ag	14348	215	30.8
136	HLH	Hunt, Leuchars & Hepburn Hldgs	149087	1600	2385.4
137	MALBAK	Malbak	145886	1250	1823.6
138	MALHOLD	Malhold	41224	3225	1329.5
139	MCPHAIL	MacPhail Holdings	14203	175	24.9
140	MESSINA	Messina	12965	525	68.1
141	METJE-&-Z	Metje and Ziegler	3467	325	11.3
142	MICOR	Micor Holdings	23506	65	15.3
143	NICTUS	Nictus Finansiele Instelling	7125	45	3.2
144	OCEANA	Oceana Development Trust	20382	1900	387.3
145	PICBEL	Picardi Beleggings	4444	360	16.0
146	PICHOLD	Picardi Holdings	6050	325	19.7
147	PLACOR	Placor Holdings	22179	2700	598.8
148	PLATE-GL	Plate Glass & Shatterprufe	16472	5650	930.7
149	RENTBEL	Rentmeesterbeleggings	3625	700	25.4
150	SA-BIAS	SA Bias Holdings	19249	450	86.6
151	TGH	Tollgate Holdings	74720	550	411.0
152	W-&-A	W & A Investment Corporation	109928	415	456.2
		Insurance			
153	GARDIAN	Guardian National Insurance	10048	2250	226.1
154	I-C-S	Imperial Cold Storage	15127	550	83.2
155	LIB-HOLD	Liberty Holdings	45685	9500	4340.1
156	LIBERTY	Liberty Life Association of SA	227756	3650	8313.1
157	M-&-F	Mutual & Federal Insurance Co	46841	2500	1171.0
158	PROSURE	Protea Assurance Group	7865	1300	102.2
159	SAGE-LTD	Sage Holdings	24028	825	198.2
		Investment Trusts			
160	COM-FUND	Common Fund Inv Society	7321	6000	439.3
161	INTRUST	Investec Investment Trust	1226	1950	23.9
		Manganese			
162	ASS-MANG	Assoc Manganese Mines of SA	3548	32750	1162.0
163	SAMANCOR	Samancor	167345	3000	5020.4
		Mining Exploration			
164	FREDDEV	Free State Dev & Inv Corp	22227	145	32.2
		Mining Holdings			
165	AMGOLD	Anglo American Gold Inv Co	24147	22000	5312.3
166	ASSORE	Associated Ore & Metal Corp	1400	19000	266.0
167	COR-SYND	Coronation Syndicate	6000	80	4.8
168	DUIKERS	Duiker Exploration	14390	600	86.3
169	EGOLI	Egoli Mining Co.	42348	45	19.1
170	GENBEL	Gencor Investment Corp	432231	670	2895.9
171	LONFIN	London Finance & Inv Group	24967	70	17.5
172	MID-WITS	Middle Wits. (Western Areas)	321642	825	2653.5
173	MINORCO	Minerals & Resources Corp	170312	4625	7876.9
174	NEW-CENT	New Central Wits Areas	1766	3600	63.6
175	NEW-WITS	New Wits. Gold Exploration	30635	1050	321.7
176	R-M-PROPS	Rand Mines Properties	12403	1225	151.9
177	RAND-LON	Rand London Corporation	137306	8	11.0
178	TWEEFONTN	Tweefontein United Collieries	1735	1060	18.4

Appendix 3.1 (continued) Securities selected for the investigation of the distributional characteristics of security returns and for the estimation of the equally weighted index, factor mimicking portfolios, and the zero-beta portfolios

No.	JSE Code	Company Name	No. of Shares (000s)	Price at 31/12/91 (c)	Market Capital. (R m)
179	VOGELS	Vogelstruisbult Metal Holdings	18394	425	78.2
		Mining Houses			
180	ANGLO-AM	Anglo American Corporation	231950	12400	28761.8
181	ANGLOVAAL	Anglovaal	17832	7800	1390.9
182	CHARTER	Charter Consolidated	105468	3050	3216.8
183	GENCOR	General Mining Union Corp	706645	1220	8621.1
184	GFSA	Gold Fields of South Africa	96179	7750	7453.9
185	JOHNNIES	Johannesburg Cons. Inv Co	147497	5175	7633.0
186	RANDMIN	Rand Mines	14910	7350	1095.9
		Motor			
187	GENTYRE-A	General Tyre & Rubber Co	10165	2350	238.9
188	MCCARTHY	McCarthy Group	86622	480	415.8
189	METAIR	Metair Investments	5633	1275	71.8
190	PORT	Brian Porter Holdings	2830	400	11.3
191	SAFICON	Saficon Investments	31155	615	191.6
192	SAKERS	Sakers Finance & Inv Corp	9574	935	89.5
193	TOYOTA	Toyota (SA)	40668	2400	976.0
194	URQHART	Urqhart Industries	12883	115	14.8
195	WESCO	Wesco Investments	8402	5900	495.7
		Other Metals & Minerals			
196	CON-MURCH	Consolidated Murchison	6240	120	7.5
197	MSAULI	Msauli Asbestos	6451	225	14.5
		Paper & Packaging			
198	CARLCOR	Carlto Paper Corporation	15762	4400	693.5
199	COATES	Coates Brothers	3400	2200	74.8
200	CONSOL	Consol	64196	3635	2333.5
201	CTP	CTP Holdings	22683	692	157.0
202	HARWILL	Harwill Investments	9782	200	19.6
203	HOLDAIN	Holdains	24285	4050	983.5
204	HORTORS	Hortors	51563	100	51.6
205	NAMPAK	Nampak	47703	6700	3196.1
206	SAPPI	Sappi	125785	3725	4685.5
		Pharmaceuticals & Medical			
207	ADCOCK	Adcock Ingram	27384	5850	1602.0
208	NORIMED	Norimed	3028	410	12.4
209	UNI-COLD	Union Cold Storage of SA	1300	1720	22.4
		Platinum Mining			
210	LYD-PLAT	Lydenburg Platinum	14400	4300	619.2
211	RUSPLAT	Rustenburg Platinum Hldgs	125320	5900	7393.9
		Printing & Publishing			
212	ARGUS	Argus Holdings	41888	2850	1193.8
213	T-M-L	Times Media	22091	1675	370.0
		Property			
214	AMAPROP	Anglo American Properties	44979	725	326.1
215	BESTER	Bester Beleggings	12000	50	6.0
216	GF-PROPS	Gold Fields Properties	10224	725	74.1
217	N-KLEINS	New Kleinfontein Properties	2010	800	16.1
218	PUTPROP	Putco Properties	26425	75	19.8
219	SABLE	Sable Holdings	7288	1000	72.9

Appendix 3.1 (continued) Securities selected for the investigation of the distributional characteristics of security returns and for the estimation of the equally weighted index, factor mimicking portfolios, and the zero-beta portfolios

No.	JSE Code	Company Name	No. of Shares (000s)	Price at 31/12/91 (c)	Market Capital. (R m)
		Property Trust			
220	PRIMA	Prima Property Trust	88740	60	53.2
221	SANLAND	Sanland Property Trust	194400	80	155.5
		Retailers & Wholesalers			
222	BOYMANS	Boymans	10751	130	14.0
223	CNAGALO	CNA Gallo	33163	2575	853.9
224	EDGARS	Edgars Stores	50818	5250	2667.9
225	EUROPA	Europa	3764	50	1.9
226	FOSCHINI	Foschini	10735	3625	389.1
227	GRESHAM	Gresham Industries	52459	63	33.0
228	O-K	OK Bazaars (1929)	11921	980	116.8
229	PEPKOR	Pepkor	15053	1050	158.1
230	PICKNPAY	Pick 'n Pay Stores	78262	2225	1741.3
231	WOOLTRU	Wooltru	13902	6900	959.2
		Steel & Allied Industries			
232	HIVELD	Hiveld Steel & Vanadium Corp	70954	1470	1043.0
		Sugar			
233	CROOKES	Crookes Brothers	12000	567	68.0
234	TONGAAT	Tongaat-Hulett Group	74844	2025	1515.6
		Tin Mines			
235	ROOIBERG	Rooiberg Tin	2075	275	5.7
236	UNION-TIN	Union Tin Mines	2400	30	0.7
		Tobacco and Matches			
237	LIONMATCH	Lion Match Company	45378	385	174.7
238	REMBR-BEH	Rembrandt Beherende Belegg	360000	1850	6660.0
239	REMGRO	Rembrandt Group	522000	2450	12789.0
240	TEGKOR	Tegniese Beleggingskorporasie	166441	1525	2538.2
241	TIB	Tegniese & Industriele Belegg	132000	1700	2244.0
242	UTICO	Utico Holdings	6075	5600	340.2
		Transportation			
243	MOBILE	Mobile Industries	28363	2575	730.3
244	TRENCOR	Trencor	14313	10300	1474.2

Appendix 3.2 Empirical probability curves of the sample standardised fourth moment in samples of size fifty-two, one hundred, two hundred and forty-seven, and nine hundred and eighty-eight estimated over sets of fifty thousand simulations

Cum. Probability	52		100		247	988
	Simulation	D'A&P	Simulation	D'A&P	Simulation	Simulation
0.0010	1.817	1.82	2.046	2.04	2.289	2.594
0.0025	1.874	1.88	2.096	2.10	2.343	2.626
0.0050	1.924	1.93	2.133	2.14	2.381	2.651
0.0100	1.984	2.00	2.188	2.19	2.426	2.680
0.0250	2.077	2.09	2.271	2.27	2.490	2.721
0.0500	2.159	2.17	2.351	2.35	2.549	2.759
0.1000	2.271	2.27	2.443	2.45	2.626	2.805
0.9000	3.635	3.62	3.525	3.52	3.368	3.196
0.9500	3.987	3.98	3.779	3.78	3.523	3.265
0.9750	4.349	4.34	4.035	4.02	3.678	3.331
0.9900	4.821	4.84	4.409	4.37	3.875	3.408
0.9950	5.196	5.24	4.678	4.65	4.025	3.467
0.9975	5.634	5.68	4.969	4.94	4.169	3.532
0.9990	6.241	6.08	5.410	5.36	4.412	3.613
D'A&P	Figures from D'Agostino and Pearson (1973:615-616)					

Appendix 3.3 Descriptive statistics and the results of the tests of normality using weekly returns over the period February 20, 1973 to March 13, 1992

	$\bar{r}$	σ_r	γ	α_f	SR	$\sqrt{b_1}$	b_2	χ^2	D_{KS}	W
BANKORP	0.0013	0.0464	0.017	1.306	9.80**	-0.02	5.58**	380.5**	0.129**	0.968**
BOLAND	0.0028	0.0363	0.005	n/a	10.05**	-0.10	7.90**	1065.3**	0.210**	0.901**
NEDCOR	0.0025	0.0437	0.016	1.300	11.26**	-0.21**	6.78**	143.8**	0.092**	0.965**
SBIC	0.0032	0.0383	0.013	1.266	11.26**	0.11	6.69**	205.2**	0.115**	0.960**
SA-BREWS	0.0042	0.0397	0.015	1.332	10.10**	-0.17*	5.59**	37.4**	0.066**	0.981**
SUNCRUSH	0.0058	0.0437	0.006	n/a	17.09**	-1.38**	27.84**	1687.3**	0.251**	0.820**
ANG-ALPHA	0.0046	0.0384	0.010	1.120	10.22**	-0.07	6.79**	602.4**	0.178**	0.933**
BLUE-CIRC	0.0043	0.0520	0.016	1.217	14.59**	-0.98**	14.30**	368.4**	0.154**	0.932**
BOUMAT	0.0026	0.0443	0.010	1.058	9.83**	0.28**	6.58**	898.0**	0.187**	0.921**
CONCOR	0.0015	0.0755	0.007	n/a	11.91**	-0.11	8.70**	1353.9**	0.226**	0.898**
EVERITE	0.0027	0.0412	n/a	n/a	23.09**	-3.03**	63.13**	1708.3**	0.246**	0.779**
GOLDSTEIN	0.0020	0.0763	0.023	1.193	9.58**	0.27**	6.38**	728.3**	0.175**	0.931**
GRINAKER	0.0035	0.0530	0.014	1.170	15.54**	-0.36**	14.69**	522.8**	0.168**	0.904**
GYPSUM	0.0039	0.0375	n/a	n/a	21.47**	-2.65**	59.64**	4668.2**	0.377**	0.559**
L-T-A	0.0013	0.0554	0.019	1.243	9.34**	0.12	5.38**	478.2**	0.156**	0.954**
MASONITE	0.0027	0.0631	0.005	n/a	13.34**	0.96**	15.83**	1492.5**	0.231**	0.812**
OTIS	0.0037	0.0513	0.015	1.205	12.31**	-0.01	7.63**	534.9**	0.152**	0.947**
PORTHLD	0.0005	0.0779	n/a	n/a	16.31**	-0.43**	16.88**	3164.9**	0.322**	0.757**
PPC	0.0043	0.0354	0.007	1.014	16.14**	-0.07	13.52**	703.7**	0.211**	0.912**
YORKCOR	0.0042	0.0587	n/a	n/a	18.93**	-0.79**	49.48**	5663.4**	0.415**	0.434**
AECI	0.0020	0.0424	0.014	1.263	14.70**	-0.47**	12.92**	217.2**	0.102**	0.948**
CHEMSERVE	0.0036	0.0400	0.004	n/a	13.00**	0.13	11.33**	1522.1**	0.245**	0.842**
ENGEN	0.0050	0.0529	0.016	1.245	11.02**	-0.02	7.46**	408.5**	0.141**	0.943**
SENTRCHEM	0.0026	0.0504	0.018	1.305	10.36**	-0.09	5.86**	197.1**	0.111**	0.969**
ADONIS	0.0025	0.0718	n/a	n/a	12.50**	0.04	11.08**	2031.9**	0.265**	0.839**
AF-&-OVER	0.0020	0.0425	n/a	n/a	16.17**	-0.96**	24.06**	3969.7**	0.361**	0.637**
AF-OVER-A	0.0021	0.0405	n/a	n/a	18.11**	-1.48**	30.53**	3805.3**	0.355**	0.650**
ALLWEAR	-0.0009	0.0789	0.024	1.223	12.95**	0.30**	8.97**	687.9**	0.175**	0.939**
BOLTONS	0.0024	0.0476	n/a	n/a	16.01**	2.17**	27.04**	2667.5**	0.302**	0.759**
BURLINGTN	0.0016	0.0405	n/a	n/a	23.97**	0.80**	74.17**	7701.3**	0.479**	0.259**
FENIX	0.0015	0.0706	0.022	1.196	10.31**	0.14	6.71**	959.4**	0.194**	0.928**
GUBINGS	0.0034	0.0509	n/a	n/a	13.61**	1.21**	18.93**	3848.4**	0.344**	0.635**
JADE	0.0017	0.0567	n/a	n/a	14.54**	-0.83**	21.71**	4860.6**	0.390**	0.575**
NINIAN	0.0029	0.0441	n/a	n/a	13.74**	-1.47**	18.55**	3877.2**	0.363**	0.668**
PROGRESS	0.0019	0.0696	0.008	n/a	10.24**	0.10	7.32**	1294.2**	0.226**	0.899**
REX-TRUE	0.0019	0.0346	n/a	n/a	14.26**	-0.75**	16.20**	4298.2**	0.382**	0.652**
ROMATEX	0.0022	0.0493	0.010	1.037	13.42**	-0.08	10.07**	856.4**	0.181**	0.904**
SEARDEL	0.0040	0.0634	0.014	1.065	11.30**	-0.21**	9.06**	854.0**	0.179**	0.902**
T-E-J	-0.0008	0.0819	n/a	n/a	11.64**	0.65**	11.38**	2620.0**	0.299**	0.791**
TEX-MILLS	0.0035	0.0491	n/a	n/a	26.20**	0.81**	96.65**	8057.9**	0.486**	0.229**
AMCOAL	0.0041	0.0426	0.015	1.293	8.19**	-0.15	4.89**	89.0**	0.095**	0.970**
G-F-COAL	0.0047	0.0498	0.018	1.304	10.84**	0.02	5.57**	202.7**	0.109**	0.978**
RANDCOL	0.0053	0.0450	0.009	1.005	9.70**	-0.05	7.42**	679.2**	0.178**	0.902**
TRANS-NTL	0.0036	0.0480	0.018	1.352	9.50**	0.20*	5.09**	51.1**	0.066**	0.980**
VIERFONTN	0.0028	0.0709	0.024	1.288	13.04**	-0.40**	10.14**	556.6**	0.152**	0.927**
WANKIE	0.0009	0.0658	n/a	n/a	11.18**	0.50**	10.89**	1944.5**	0.256**	0.830**
BOTREST	-0.0028	0.0941	0.021	1.074	10.90**	0.43**	8.53**	891.8**	0.191**	0.902**
PALAMIN	0.0044	0.0455	0.015	1.313	13.26**	0.58**	8.93**	133.0**	0.093**	0.965**
Z-C-I	-0.0001	0.0880	0.010	n/a	10.18**	1.11**	11.04**	1282.6**	0.230**	0.844**
ANAMINT	0.0046	0.0463	0.012	1.204	19.85**	-1.80**	31.40**	390.2**	0.143**	0.887**

Appendix 3.3 (continued) Descriptive statistics and the results of the tests of normality using weekly returns over the period February 20, 1973 to March 13, 1992

	$\bar{r}$	σ_r	γ	α_f	SR	$\sqrt{b_1}$	b_2	χ^2	D_{KS}	W
BROADACRE	0.0011	0.0896	n/a	n/a	13.99**	0.58**	11.68**	2148.8**	0.276**	0.843**
DEBEERS	0.0031	0.0445	0.016	1.352	12.73**	-0.63**	9.40**	58.2**	0.062**	0.966**
ICH	0.0051	0.0457	n/a	n/a	15.25**	0.24**	21.63**	4440.5**	0.371**	0.600**
ABACUS	0.0024	0.0993	n/a	n/a	13.96**	0.87**	15.50**	3774.6**	0.366**	0.680**
ABERDARE	0.0042	0.0421	n/a	n/a	19.12**	-0.55**	29.70**	2580.1**	0.301**	0.710**
AF-CABLE	0.0021	0.0483	n/a	n/a	17.99**	0.67**	25.05**	3402.5**	0.326**	0.682**
ALTECH	0.0050	0.0511	0.006	n/a	15.08**	0.10	16.11**	1017.8**	0.198**	0.830**
ALTRON	0.0024	0.0431	n/a	n/a	20.32**	-1.91**	43.42**	2564.2**	0.295**	0.714**
CAFCA	0.0016	0.0553	n/a	n/a	17.46**	-0.57**	38.37**	5973.1**	0.420**	0.433**
DELTA	0.0035	0.0534	0.008	n/a	12.66**	0.34**	10.64**	1245.1**	0.218**	0.879**
REUNERT	0.0050	0.0479	0.004	n/a	17.03**	1.65**	26.56**	1593.8**	0.248**	0.787**
TEDELEX	0.0020	0.0641	0.022	1.294	11.18**	-0.22**	6.75**	280.6**	0.107**	0.962**
VENTRON	0.0027	0.0739	0.017	1.141	14.78**	-1.06**	20.22**	754.7**	0.174**	0.843**
AFROX	0.0049	0.0406	0.011	1.170	10.80**	-0.22**	7.74**	370.6**	0.169**	0.930**
BERZACK	0.0088	0.0562	n/a	n/a	18.69**	2.67**	40.77**	4817.0**	0.389**	0.552**
BIVEC	0.0039	0.0607	n/a	n/a	15.91**	-0.74**	18.60**	2607.8**	0.291**	0.775**
BUFFCOR	0.0031	0.0920	n/a	n/a	13.93**	0.42**	17.65**	2636.6**	0.288**	0.736**
CEMENCO	0.0017	0.0516	0.005	n/a	13.48**	-0.97**	14.18**	1335.3**	0.221**	0.868**
DORBYL	0.0031	0.0389	0.011	1.153	13.34**	-0.32**	10.74**	503.7**	0.147**	0.908**
ED-LBATE	0.0052	0.0471	0.006	n/a	14.61**	0.20*	12.83**	1588.5**	0.240**	0.853**
FRALEX	0.0031	0.0909	0.019	1.082	14.83**	-0.33**	16.64**	999.2**	0.185**	0.831**
G-I-C	0.0036	0.0644	0.004	n/a	10.81**	-0.18*	8.69**	1459.9**	0.227**	0.864**
METKOR	0.0028	0.0622	0.015	1.138	12.29**	-0.21**	10.97**	943.7**	0.190**	0.885**
NAMFISH	0.0025	0.0556	0.012	1.035	13.28**	-0.93**	12.72**	994.9**	0.191**	0.862**
OCFISH	0.0037	0.0487	0.007	n/a	13.07**	-0.49**	10.94**	1170.0**	0.215**	0.893**
CADSWEP	0.0047	0.0494	0.006	n/a	17.02**	-0.41**	19.61**	1187.0**	0.218**	0.835**
FEDFOOD	0.0029	0.0461	0.013	1.177	11.38**	0.21**	9.57**	426.6**	0.148**	0.916**
I-&-J	0.0048	0.0509	0.016	1.224	10.83**	-0.45**	7.37**	325.9**	0.151**	0.948**
ICS	0.0027	0.0492	0.014	1.185	14.00**	0.23**	12.64**	491.0**	0.163**	0.909**
KANHYM	0.0022	0.0621	0.017	1.152	9.82**	0.30**	6.75**	467.8**	0.139**	0.931**
PREM-GRP	0.0036	0.0420	0.012	1.210	13.44**	-0.19*	11.22**	239.5**	0.107**	0.925**
TIGR-OATS	0.0045	0.0399	0.013	1.290	14.68**	-0.15	11.16**	145.6**	0.108**	0.940**
AFCOL	0.0021	0.0509	0.012	1.090	10.79**	-0.04	8.34**	794.1**	0.177**	0.903**
ELLERINE	0.0044	0.0455	0.006	n/a	9.99**	-0.17*	8.42**	1150.0**	0.222**	0.868**
MATH-ASH	0.0049	0.0582	n/a	n/a	15.90**	-0.19*	22.40**	4455.2**	0.380**	0.601**
PICAPLI	0.0019	0.0937	n/a	n/a	13.49**	-0.37**	15.79**	2752.7**	0.301**	0.729**
AFR-LEASE	0.0025	0.0907	0.029	1.237	11.77**	-0.15	8.22**	412.6**	0.126**	0.943**
BLYVOOR	0.0012	0.0624	0.024	1.364	11.88**	0.49**	8.26**	91.8**	0.066**	0.971**
BRACKEN	0.0024	0.0705	0.030	1.410	7.98**	-0.12	4.40**	98.1**	0.070**	0.980**
BUFFELS	0.0028	0.0607	0.025	1.374	7.59*	-0.12	3.76**	42.1**	0.050*	0.986
DBN-DEEP	0.0017	0.0903	0.032	1.302	12.76**	0.31**	8.23**	115.7**	0.065**	0.968**
DRIES	0.0037	0.0538	0.024	1.445	6.51	-0.02	3.41*	9.9	0.031	0.985
E-R-P-M	0.0011	0.0797	0.028	1.298	10.84**	0.41**	6.25**	160.0**	0.079**	0.968**
E-T-CONS	0.0048	0.0769	0.028	1.325	8.05**	0.10	4.44**	286.5**	0.119**	0.972**
ELSBURG	0.0004	0.0733	0.032	1.465	7.72*	0.01	3.93**	45.0**	0.052**	0.986
FALCON	0.0043	0.0669	n/a	n/a	14.57**	-0.49**	18.13**	3023.3**	0.314**	0.714**
GROOTVLEI	0.0033	0.0795	0.032	1.398	9.63**	0.30**	5.42**	39.6**	0.043*	0.980**
HARMONY	0.0023	0.0629	0.027	1.427	7.69*	0.08	3.78**	16.6*	0.039	0.987
HARTIES	0.0039	0.0627	0.025	1.397	8.49**	-0.33**	5.28**	30.4**	0.042	0.973**
KINROSS	0.0037	0.0631	0.029	1.469	7.57*	-0.15	3.56**	24.2**	0.040	0.988

Appendix 3.3 (continued) Descriptive statistics and the results of the tests of normality using weekly returns over the period February 20, 1973 to March 13, 1992

	$\bar{r}$	σ_r	γ	α_f	SR	$\sqrt{b_1}$	b_2	χ^2	D_{KS}	W
KLOOF	0.0035	0.0614	0.024	1.363	8.83**	-0.10	4.27**	26.6**	0.042	0.988
LESLIE	0.0032	0.0709	0.030	1.406	8.58**	-0.02	4.25**	52.2**	0.054**	0.987
LIBANON	0.0013	0.0738	0.031	1.421	8.54**	0.28**	4.69**	33.9**	0.047*	0.982*
LORAINE	0.0012	0.0844	0.037	1.452	13.22**	-0.11	7.75**	39.2**	0.048*	0.984
MODDER	0.0011	0.0870	0.021	1.111	10.88**	0.83**	8.44**	704.6**	0.166**	0.918**
RANDFONTN	0.0043	0.0589	0.023	1.344	7.49*	0.03	4.16**	32.4**	0.050*	0.978**
RD-LEASE	0.0000	0.1070	0.039	1.365	10.68**	-0.15	6.87**	206.2**	0.095**	0.961**
SALLIES	0.0027	0.0879	0.033	1.342	9.68**	0.02	5.91**	99.8**	0.057**	0.969**
SIMMERS	0.0014	0.0856	0.031	1.302	8.58**	0.29**	5.16**	251.0**	0.114**	0.965**
SOUTHVAAL	0.0035	0.0592	0.025	1.437	8.02**	-0.10	3.80**	13.1	0.031	0.990
ST-HELENA	0.0020	0.0601	0.023	1.331	8.95**	-0.40**	4.91**	114.3**	0.075**	0.976**
STH-RODPT	-0.0025	0.1034	0.036	1.258	10.62**	-0.21**	6.08**	176.1**	0.089**	0.971**
STILFTN	0.0021	0.0671	0.022	1.290	11.91**	-0.18*	9.22**	163.3**	0.070**	0.944**
VAAL-REEF	0.0037	0.0536	0.023	1.412	9.01**	-0.09	4.17**	25.0**	0.035	0.991
VENTERS	0.0020	0.0875	0.038	1.434	8.04**	0.09	4.59**	56.8**	0.046*	0.979**
VILLAGE	0.0046	0.0980	0.038	1.349	9.41**	0.29**	5.52**	180.6**	0.101**	0.968**
VLAKS	0.0002	0.0818	0.034	1.404	9.36**	0.13	4.96**	81.7**	0.064**	0.983*
W-R-CONS	0.0027	0.0911	0.033	1.316	8.65**	0.32**	5.37**	111.3**	0.078**	0.963**
WAVERLEY	0.0017	0.1039	0.032	1.221	11.00**	0.69**	7.09**	495.3**	0.163**	0.942**
WELKOM	0.0031	0.0643	0.028	1.457	8.82**	0.07	4.40**	27.7**	0.045*	0.988
WES-AREAS	0.0007	0.0738	0.031	1.420	8.63**	0.01	4.32**	28.4**	0.041	0.987
WINKELS	0.0026	0.0606	0.025	1.402	8.37**	-0.13	4.35**	40.9**	0.042	0.983
WIT-NIGEL	0.0004	0.1023	0.036	1.326	11.46**	0.22**	8.16**	288.1**	0.112**	0.944**
WSTN-DEEP	0.0034	0.0572	0.023	1.357	7.27	0.10	3.69**	39.6**	0.058**	0.983
ZANDPAN	0.0039	0.0559	0.024	1.392	7.62*	0.11	3.94**	84.8**	0.080**	0.982*
A-V-I	0.0046	0.0424	0.013	1.189	9.14**	-0.17*	5.88**	262.8**	0.141**	0.951**
AMIC	0.0031	0.0387	0.013	1.260	10.70**	-0.38**	7.56**	166.6**	0.110**	0.944**
BARLOWS	0.0033	0.0393	0.016	1.410	12.49**	-0.72**	10.25**	26.5**	0.041	0.973**
BIDVEST	0.0025	0.0466	n/a	n/a	15.99**	-0.40**	20.12**	2138.1**	0.278**	0.765**
BTRDUN	0.0043	0.0491	0.016	1.264	10.93**	-0.33**	7.38**	333.2**	0.138**	0.946**
CGSMITH	0.0040	0.0423	0.012	1.219	18.80**	-0.49**	24.08**	235.8**	0.120**	0.886**
CULLINAN	0.0022	0.0491	0.010	1.034	13.12**	0.19*	9.13**	1053.2**	0.203**	0.915**
EUREKA	0.0038	0.0772	n/a	n/a	17.95**	1.11**	30.72**	4081.9**	0.369**	0.644**
F-S-I	-0.0002	0.0640	n/a	n/a	15.48**	-1.21**	19.13**	2116.0**	0.264**	0.787**
FARM-AG	0.0021	0.0696	0.017	1.112	11.97**	0.19*	11.44**	1018.3**	0.190**	0.867**
HLH	0.0031	0.0492	0.011	1.085	17.30**	-2.11**	32.20**	756.9**	0.182**	0.862**
MALBAK	0.0028	0.0541	0.019	1.288	10.05**	-0.17*	5.58**	78.3**	0.077**	0.973**
MALHOLD	0.0039	0.0468	0.013	1.161	9.59**	0.00	6.85**	613.8**	0.174**	0.926**
MCPHAIL	0.0023	0.0751	n/a	n/a	10.76**	0.38**	9.47**	1917.4**	0.264**	0.842**
MESSINA	0.0001	0.0761	0.028	1.380	14.43**	-0.44**	12.67**	97.0**	0.072**	0.947**
METJE-&-Z	0.0013	0.0562	n/a	n/a	17.11**	0.17*	29.18**	5343.7**	0.402**	0.524**
MICOR	0.0028	0.0525	n/a	n/a	12.83**	-0.10	12.26**	2357.5**	0.299**	0.799**
NICTUS	-0.0002	0.0795	n/a	n/a	14.64**	-0.34**	14.50**	3235.2**	0.320**	0.769**
OCEANA	0.0030	0.0597	n/a	n/a	22.08**	0.03	56.02**	5412.3**	0.405**	0.434**
PICBEL	0.0031	0.0741	0.016	1.027	13.43**	0.27**	9.07**	907.3**	0.182**	0.920**
PICHOLD	0.0026	0.0610	0.013	1.086	13.48**	0.69**	12.90**	860.6**	0.169**	0.885**
PLACOR	0.0035	0.0558	0.017	1.188	8.69**	-0.07	5.48**	348.6**	0.140**	0.948**
PLATE-GL	0.0032	0.0433	0.014	1.216	8.68**	-0.23**	6.01**	206.0**	0.104**	0.948**
RENTBEL	0.0021	0.0565	n/a	n/a	11.78**	0.44**	10.35**	2055.0**	0.264**	0.832**
SA-BIAS	0.0042	0.0472	n/a	n/a	17.95**	2.77**	36.10**	3692.4**	0.351**	0.629**

Appendix 3.3 (continued) Descriptive statistics and the results of the tests of normality using weekly returns over the period February 20, 1973 to March 13, 1992

	$\bar{r}$	σ_r	γ	α_f	SR	$\sqrt{b_1}$	b_2	χ^2	D_{KS}	W
TGH	0.0027	0.0719	n/a	n/a	16.05**	-0.15	21.48**	3358.5**	0.334**	0.675**
W-&-A	0.0045	0.0619	0.015	1.100	8.62**	0.17*	4.96**	749.2**	0.179**	0.944**
GARDIAN	0.0038	0.0506	n/a	n/a	16.40**	0.77**	23.70**	3787.1**	0.347**	0.643**
I-G-I	0.0034	0.0597	n/a	n/a	14.30**	0.37**	14.02**	1938.1**	0.265**	0.824**
LIB-HOLD	0.0048	0.0442	0.016	1.265	8.82**	0.10	5.18**	249.0**	0.129**	0.963**
LIBERTY	0.0044	0.0392	0.013	1.258	11.23**	0.56**	7.71**	278.1**	0.127**	0.945**
M-&-F	0.0047	0.0395	0.004	n/a	16.04**	0.16*	17.45**	1997.1**	0.278**	0.814**
PROSURE	0.0023	0.0482	n/a	n/a	14.09**	-1.09**	16.47**	2746.2**	0.313**	0.757**
SAGE-LTD	0.0026	0.0481	0.013	1.134	11.24**	-0.55**	7.96**	856.7**	0.196**	0.920**
COM-FUND	0.0041	0.0331	n/a	n/a	17.16**	1.28**	24.64**	3491.7**	0.334**	0.659**
INTRUST	0.0033	0.0563	n/a	n/a	18.70**	-1.39**	31.79**	3508.7**	0.326**	0.641**
ASS-MANG	0.0044	0.0304	n/a	n/a	24.88**	2.52**	81.27**	4740.4**	0.387**	0.458**
SAMANCOR	0.0049	0.0509	0.017	1.253	11.80**	0.30**	7.68**	159.7**	0.095**	0.957**
FREDDEV	0.0015	0.0756	0.027	1.308	9.81**	-0.12	6.22**	201.6**	0.101**	0.963**
AMGOLD	0.0031	0.0463	0.020	1.409	7.97**	-0.03	3.85**	17.8*	0.037	0.988
ASSORE	0.0039	0.0439	n/a	n/a	15.94**	0.84**	26.17**	4233.5**	0.362**	0.577**
COR-SYND	0.0022	0.0831	0.021	1.074	9.76**	0.02	6.89**	824.2**	0.176**	0.911**
DUIKERS	0.0020	0.0613	0.017	1.180	16.45**	-0.93**	22.17**	599.1**	0.163**	0.898**
EGOLI	0.0022	0.1007	0.028	1.180	10.86**	0.96**	8.73**	620.9**	0.164**	0.913**
GENBEL	0.0038	0.0533	0.021	1.384	9.42**	-0.35**	5.96**	116.7**	0.077**	0.969**
LONFIN	0.0008	0.0710	0.018	1.144	11.33**	-0.03	7.50**	807.6**	0.185**	0.930**
MID-WITS	0.0049	0.0644	0.016	1.086	6.93	0.10	3.95**	704.0**	0.170**	0.945**
MINORCO	0.0029	0.0537	0.015	1.166	14.64**	0.23**	10.90**	377.1**	0.136**	0.927**
NEW-CENT	0.0034	0.0544	0.014	1.132	15.48**	-1.49**	22.82**	546.1**	0.158**	0.900**
NEW-WITS	0.0033	0.0617	0.023	1.320	13.24**	0.10	7.63**	101.2**	0.088**	0.979**
R-M-PROPS	0.0030	0.0613	0.020	1.223	9.23**	0.34**	5.72**	203.3**	0.096**	0.957**
RAND-LON	-0.0009	0.0966	0.029	1.207	12.00**	0.42**	7.75**	383.4**	0.138**	0.944**
TWEEFONTN	0.0023	0.0572	0.008	n/a	19.98**	-0.55**	28.08**	1340.7**	0.217**	0.838**
VOGELS	0.0034	0.0616	0.023	1.327	8.83**	0.24**	5.20**	192.9**	0.099**	0.968**
ANGLO-AM	0.0036	0.0437	0.017	1.372	10.46**	-0.27**	6.01**	41.4**	0.050*	0.980**
ANGLOVAAL	0.0057	0.0424	0.006	n/a	13.13**	-0.35**	10.75**	907.7**	0.206**	0.883**
CHARTER	0.0022	0.0581	0.017	1.194	14.30**	0.11	11.02**	474.9**	0.133**	0.917**
GENCOR	0.0042	0.0459	0.018	1.390	8.64**	-0.09	4.65**	99.2**	0.086**	0.980**
GFSA	0.0036	0.0535	0.022	1.386	10.65**	-0.37**	6.15**	48.1**	0.056**	0.981**
JOHNNIES	0.0063	0.0458	0.015	1.242	12.84**	-0.74**	10.41**	136.6**	0.083**	0.958**
RANDMIN	0.0030	0.0492	0.014	1.182	9.52**	0.04	6.91**	279.3**	0.107**	0.933**
GENTYRE-A	0.0049	0.0514	n/a	n/a	18.11**	-1.36**	31.55**	2334.3**	0.277**	0.730**
MCCARTHY	0.0042	0.0558	0.018	1.217	8.94**	-0.12	5.45**	237.4**	0.118**	0.960**
METAIR	0.0042	0.0621	n/a	n/a	14.76**	0.58**	15.20**	2877.4**	0.309**	0.741**
PORT	0.0022	0.0589	n/a	n/a	14.50**	-1.30**	19.60**	3477.0**	0.342**	0.702**
SAFICON	0.0024	0.0663	0.011	1.004	16.58**	-1.50**	22.00**	1000.0**	0.210**	0.837**
SAKERS	0.0031	0.0616	0.005	n/a	17.07**	-1.72**	26.64**	1602.2**	0.250**	0.810**
TOYOTA	0.0048	0.0624	0.006	n/a	14.79**	-0.31**	14.01**	1250.9**	0.208**	0.870**
URQHART	-0.0012	0.0853	0.005	n/a	12.88**	-1.29**	14.16**	1425.3**	0.228**	0.830**
WESCO	0.0045	0.0641	0.003	n/a	9.45**	0.39**	7.16**	1582.5**	0.240**	0.859**
CON-MURCH	-0.0020	0.0695	0.023	1.265	12.08**	-0.83**	12.18**	136.2**	0.088**	0.928**
MSAULI	0.0024	0.0809	0.028	1.263	8.93**	0.39**	5.43**	209.7**	0.101**	0.957**
CARLCOR	0.0042	0.0429	n/a	n/a	12.70**	0.30**	10.98**	1927.8**	0.261**	0.835**
COATES	0.0042	0.0497	n/a	n/a	16.33**	1.00**	25.03**	4044.6**	0.355**	0.638**
CONSOL	0.0069	0.0320	n/a	n/a	20.42**	-0.65**	31.27**	2856.9**	0.306**	0.732**

Appendix 3.3 (continued) Descriptive statistics and the results of the tests of normality using weekly returns over the period February 20, 1973 to March 13, 1992

	$\bar{r}$	σ_r	γ	α_f	SR	$\sqrt{b_1}$	b_2	χ^2	D_{KS}	W
CTP	0.0023	0.0632	0.006	n/a	12.84**	0.29**	10.61**	1338.8**	0.222**	0.874**
HARWILL	0.0019	0.1048	n/a	n/a	13.23**	0.96**	16.38**	4544.4**	0.377**	0.626**
HOLDAIN	0.0041	0.0424	0.010	1.081	10.83**	-0.07	8.40**	694.4**	0.185**	0.907**
HORTORS	0.0012	0.0679	n/a	n/a	11.90**	-0.27**	8.81**	1511.1**	0.232**	0.884**
NAMPAK	0.0041	0.0365	0.012	1.280	13.29**	-0.55**	9.90**	224.2**	0.145**	0.944**
SAPPI	0.0041	0.0482	0.017	1.309	11.01**	-0.57**	7.73**	117.2**	0.089**	0.966**
ADCOCK	0.0048	0.0335	n/a	n/a	17.41**	0.63**	22.66**	3722.8**	0.355**	0.671**
NORIMED	0.0012	0.0677	n/a	n/a	15.90**	-0.60**	24.67**	3865.2**	0.346**	0.639**
UNI-COLD	0.0026	0.0304	n/a	n/a	25.13**	9.14**	156.47**	6871.8**	0.469**	0.316**
LYD-PLAT	0.0037	0.0638	0.024	1.340	9.17**	-0.21**	5.42**	71.8**	0.063**	0.973**
RUSPLAT	0.0039	0.0610	0.025	1.384	10.44**	-0.21**	5.86**	48.1**	0.047*	0.979**
ARGUS	0.0031	0.0374	n/a	n/a	14.69**	0.20*	13.46**	2466.2**	0.297**	0.784**
T-M-L	0.0050	0.0564	n/a	n/a	17.27**	-0.65**	22.61**	2473.9**	0.290**	0.726**
AMAPROP	0.0018	0.0601	0.018	1.195	11.92**	0.28**	8.14**	501.9**	0.149**	0.938**
BESTER	-0.0008	0.0694	0.019	1.178	11.19**	-0.18*	8.33**	736.1**	0.180**	0.920**
GF-PROPS	0.0029	0.0671	0.022	1.247	10.43**	0.55**	7.25**	292.6**	0.128**	0.943**
N-KLEINS	0.0038	0.0667	n/a	n/a	15.50**	0.06	20.21**	2746.4**	0.293**	0.724**
PUTPROP	-0.0007	0.0943	0.032	1.282	10.91**	-0.18*	8.62**	626.3**	0.158**	0.921**
SABLE	0.0036	0.0638	n/a	n/a	16.33**	-1.03**	19.59**	1821.2**	0.253**	0.825**
PRIMA	0.0024	0.0401	0.013	1.240	9.05**	-0.13	5.37**	762.3**	0.175**	0.947**
SANLAND	0.0031	0.0429	0.006	n/a	11.02**	-0.29**	7.50**	1223.4**	0.220**	0.914**
BOYMANS	0.0018	0.0660	n/a	n/a	19.10**	0.61**	30.11**	4323.5**	0.373**	0.622**
CNAGALO	0.0042	0.0536	0.012	1.076	13.54**	-0.07	11.37**	699.0**	0.181**	0.897**
EDGARS	0.0041	0.0377	0.003	n/a	14.19**	0.29**	13.63**	1685.8**	0.255**	0.813**
EUROPA	-0.0003	0.0695	0.007	n/a	14.52**	-0.06	13.66**	1307.2**	0.219**	0.864**
FOSCHINI	0.0066	0.0410	n/a	n/a	17.85**	2.51**	33.61**	2941.9**	0.317**	0.657**
GRESHAM	0.0013	0.0682	0.012	n/a	10.66**	0.16*	9.23**	1291.3**	0.211**	0.879**
O-K	0.0015	0.0390	0.014	1.277	9.82**	-0.20*	5.51**	220.1**	0.103**	0.968**
PEPKOR	0.0036	0.0471	0.013	1.163	10.63**	-0.13	8.38**	705.4**	0.176**	0.908**
PICKNPAY	0.0050	0.0461	0.015	1.280	12.17**	-0.61**	8.77**	243.5**	0.136**	0.946**
WOOLTRU	0.0038	0.0423	0.015	1.283	10.73**	0.15	6.49**	146.7**	0.102**	0.963**
HIVELD	0.0034	0.0495	0.016	1.244	10.48**	0.14	6.97**	188.7**	0.104**	0.948**
CROOKES	0.0029	0.0396	n/a	n/a	14.88**	0.03	20.81**	4463.2**	0.371**	0.587**
TONGAAT	0.0026	0.0456	0.015	1.262	8.89**	-0.13	5.82**	112.1**	0.078**	0.962**
ROOIBERG	-0.0006	0.0533	0.004	n/a	11.71**	-0.15*	9.95**	1386.4**	0.230**	0.850**
UNION-TIN	0.0041	0.0760	n/a	n/a	15.17**	1.26**	16.43**	1610.7**	0.247**	0.826**
LIONMATCH	0.0030	0.0391	n/a	n/a	12.87**	0.07	12.12**	2057.9**	0.281**	0.816**
REMBR-BEH	0.0059	0.0619	0.007	1.000	9.89**	-0.27**	7.44**	1228.3**	0.219**	0.866**
REMGRO	0.0060	0.0531	0.007	1.000	11.49**	-0.05	7.23**	1062.4**	0.191**	0.904**
TEGKOR	0.0063	0.0516	n/a	n/a	11.16**	0.04	15.30**	2992.5**	0.319**	0.675**
TIB	0.0063	0.0517	n/a	n/a	11.14**	0.13	12.39**	2996.3**	0.310**	0.712**
UTICO	0.0036	0.0501	0.010	1.024	11.95**	0.36**	9.40**	1025.2**	0.198**	0.893**
MOBILE	0.0058	0.0455	n/a	n/a	13.47**	-0.12	13.53**	3664.7**	0.341**	0.705**
TRENCOR	0.0057	0.0413	n/a	n/a	22.51**	-3.41**	81.68**	4023.5**	0.355**	0.559**

** : significant att the 1% level   * : significant at the 5% level

Appendix 3.4 Descriptive statistics and the results of the tests of normality using four-weekly returns over the period February 20, 1973 to March 13, 1992

	$\bar{r}$	σ_r	γ	α_f	SR	$\sqrt{b_1}$	b_2	χ^2	D_{KS}	W
BANKORP	0.0053	0.0891	0.042	1.533	7.13**	0.04	3.97**	17.9*	0.098*	0.990
BOLAND	0.0111	0.0716	0.023	1.267	9.03**	-0.71**	8.53**	35.5**	0.092*	0.945**
NEDCOR	0.0100	0.0883	0.032	1.303	7.51**	-0.56**	4.92**	18.2*	0.090*	0.977
SBIC	0.0129	0.0848	0.036	1.411	7.23**	0.20	4.17**	8.6	0.059	0.989
SA-BREWS	0.0168	0.0819	0.035	1.434	7.79**	-0.71**	5.48**	8.3	0.054	0.979
SUNCRUSH	0.0231	0.0953	0.028	1.193	8.41**	-0.70**	7.93**	83.1**	0.128**	0.941**
ANG-ALPHA	0.0185	0.0844	0.039	1.512	6.33	-0.06	3.65*	7.7	0.045	0.986
BLUE-CIRC	0.0171	0.1051	0.039	1.331	9.34**	-0.63**	7.39**	20.9**	0.092*	0.974*
BOUMAT	0.0104	0.0944	0.038	1.389	5.98	-0.02	3.31	9.1	0.053	0.986
CONCOR	0.0059	0.1334	0.052	1.334	5.77	-0.07	3.53*	73.9**	0.121**	0.975
EVERITE	0.0106	0.0889	0.028	1.298	10.69**	-0.95**	13.81**	47.2**	0.105**	0.924**
GOLDSTEIN	0.0081	0.1481	0.062	1.444	8.00**	-0.19	4.67**	22.0**	0.065	0.992
GRINAKER	0.0138	0.1209	0.042	1.286	9.94**	-1.46**	14.04**	23.3**	0.082	0.925**
GYPSUM	0.0158	0.0768	0.013	n/a	10.89**	-0.52**	12.02**	232.5**	0.213**	0.865**
L-T-A	0.0052	0.1038	0.047	1.445	6.22	-0.16	3.45	7.3	0.093*	0.985
MASONITE	0.0107	0.1309	0.048	1.261	8.39**	0.70**	6.87**	42.8**	0.098*	0.955**
OTIS	0.0150	0.1122	0.034	1.200	7.51**	-0.38*	6.11**	50.2**	0.101*	0.949**
PORTHLD	0.0021	0.1345	0.039	1.167	7.39**	-0.21	5.27**	164.1**	0.164**	0.947**
PPC	0.0173	0.0828	0.032	1.297	7.57**	-0.19	4.90**	9.7	0.067	0.975
YORKCOR	0.0167	0.1254	n/a	n/a	9.81**	0.24	13.00**	702.1**	0.314**	0.712**
AECI	0.0081	0.0854	0.033	1.328	7.54**	-0.08	4.32**	14.0	0.066	0.989
CHEMSERVE	0.0146	0.0913	0.028	1.240	7.73**	-0.38*	6.29**	39.1**	0.093*	0.952**
ENGEN	0.0202	0.0980	0.037	1.286	6.74*	0.34*	4.23**	12.0	0.064	0.980
SENTRCHEM	0.0105	0.1029	0.042	1.418	7.20**	-0.02	4.46**	8.6	0.052	0.985
ADONIS	0.0101	0.1274	0.045	1.274	7.37**	0.32*	4.93**	47.9**	0.111**	0.973*
AF-&-OVER	0.0080	0.0903	0.019	1.024	7.61**	-0.05	6.19**	198.2**	0.173**	0.914**
AF-OVER-A	0.0086	0.0915	0.020	1.076	9.53**	-0.65**	10.15**	149.2**	0.162**	0.892**
ALLWEAR	-0.0035	0.1364	0.050	1.343	6.65*	0.37*	4.48**	19.8**	0.088*	0.974*
BOLTONS	0.0095	0.1012	0.033	1.232	6.85*	0.47**	4.42**	31.0**	0.107**	0.970**
BURLINGTN	0.0064	0.0880	n/a	n/a	14.28**	2.50**	38.39**	1308.0**	0.405**	0.517**
FENIX	0.0061	0.1270	0.048	1.348	7.03**	0.33*	4.28**	62.4**	0.134**	0.976
GUBINGS	0.0135	0.1032	0.025	1.070	8.01**	0.58**	7.02**	138.0**	0.166**	0.901**
JADE	0.0067	0.1195	0.020	n/a	8.19**	0.02	6.88**	276.5**	0.209**	0.897**
NINIAN	0.0117	0.0895	0.016	n/a	6.78*	-0.60**	5.21**	203.7**	0.199**	0.923**
PROGRESS	0.0076	0.1361	0.048	1.307	8.67**	0.66**	6.62**	58.3**	0.110**	0.970**
REX-TRUE	0.0076	0.0745	0.019	1.109	7.89**	-0.28	5.97**	156.0**	0.201**	0.930**
ROMATEX	0.0087	0.0991	0.034	1.211	6.05	-0.02	3.83*	43.7**	0.076	0.971*
SEARDEL	0.0158	0.1326	0.050	1.353	7.97**	0.11	4.91**	28.5**	0.086	0.981
T-E-J	-0.0031	0.1605	0.051	1.234	7.69**	0.74**	5.65**	109.6**	0.168**	0.947**
TEX-MILLS	0.0139	0.1055	n/a	n/a	13.72**	1.52**	28.10**	1583.0**	0.449**	0.463**
AMCOAL	0.0163	0.0867	0.038	1.406	6.02	-0.35*	3.85*	4.9	0.055	0.976
G-F-COAL	0.0189	0.1009	0.039	1.313	6.87*	-0.44**	4.73**	18.0*	0.089*	0.970**
RANDCOL	0.0211	0.0959	0.033	1.252	6.56*	-0.16	4.34**	19.3**	0.072	0.972*
TRANS-NTL	0.0142	0.0940	0.042	1.424	5.65	-0.04	3.48	17.1*	0.064	0.978
VIERFONTN	0.0114	0.1263	0.047	1.401	8.64**	0.56**	7.68**	36.9**	0.090*	0.949**
WANKIE	0.0035	0.1241	0.040	1.208	6.50*	0.41**	4.70**	87.5**	0.134**	0.952**
BOTREST	-0.0112	0.1804	0.072	1.416	6.85*	0.03	4.85**	27.4**	0.099*	0.970**
PALAMIN	0.0177	0.0888	0.034	1.337	5.84	-0.10	3.52	8.0	0.063	0.979
Z-C-I	-0.0004	0.1655	0.053	1.288	8.73**	1.34**	9.39**	75.1**	0.134**	0.932**
ANAMINT	0.0184	0.0956	0.037	1.371	9.17**	-1.29**	10.26**	20.5**	0.077	0.930**

Appendix 3.4 (continued) Descriptive statistics and the results of the tests of normality using four-weekly returns over the period February 20, 1973 to March 13, 1992

	$\bar{r}$	σ_r	γ	α_f	SR	$\sqrt{b_1}$	b_2	χ^2	D_{KS}	W
BROADACRE	0.0045	0.1753	0.052	1.230	9.69**	1.45**	12.47**	105.2**	0.164**	0.898**
DEBEERS	0.0124	0.1015	0.043	1.470	9.30**	-1.50**	12.16**	15.0*	0.070	0.935**
ICH	0.0206	0.0969	0.017	n/a	9.02**	1.04**	11.36**	244.9**	0.209**	0.823**
ABACUS	0.0096	0.1900	n/a	n/a	8.48**	0.40*	7.05**	571.9**	0.296**	0.844**
ABERDARE	0.0168	0.0836	0.024	1.175	7.76**	-0.37*	6.51**	57.2**	0.108**	0.936**
AF-CABLE	0.0084	0.1022	0.021	1.016	10.12**	-0.28	10.22**	199.3**	0.152**	0.888**
ALTECH	0.0201	0.1141	0.039	1.277	9.94**	-0.76**	9.31**	26.2**	0.081	0.955**
ALTRON	0.0096	0.0998	0.008	n/a	9.85**	-1.46**	14.61**	345.9**	0.203**	0.816**
CAFCA	0.0064	0.1264	n/a	n/a	10.76**	0.26	14.15**	647.1**	0.278**	0.734**
DELTA	0.0139	0.1074	0.046	1.421	6.77*	-0.16	4.32**	7.5	0.068	0.979
REUNERT	0.0201	0.1107	0.030	1.212	10.35**	0.04	9.07**	53.1**	0.121**	0.942**
TEDELEX	0.0078	0.1249	0.046	1.360	7.26**	-0.57**	5.35**	12.1	0.070	0.967**
VENTRON	0.0107	0.1480	0.048	1.305	11.20**	-0.67**	11.09**	26.8**	0.091*	0.945**
AFROX	0.0195	0.0867	0.028	1.238	8.82**	0.08	6.68**	33.5**	0.111**	0.963**
BERZACK	0.0352	0.1172	0.008	n/a	8.96**	1.30**	9.54**	422.4**	0.242**	0.827**
BIVEC	0.0156	0.1291	0.033	1.136	9.27**	-0.03	7.11**	120.1**	0.135**	0.946**
BUFFCOR	0.0124	0.1761	0.046	1.114	7.96**	0.31*	7.22**	167.7**	0.171**	0.909**
CEMENCO	0.0069	0.0974	0.038	1.349	7.64**	-0.40*	5.81**	44.7**	0.099*	0.963**
DORBYL	0.0123	0.0898	0.030	1.211	7.64**	-0.30	5.89**	43.8**	0.090*	0.952**
ED-LBATE	0.0208	0.0989	0.038	1.372	7.31**	-0.11	4.75**	28.3**	0.103*	0.980
FRALEX	0.0123	0.1681	0.062	1.382	7.47**	-0.67**	6.91**	29.2**	0.080	0.938**
G-I-C	0.0145	0.1353	0.050	1.343	7.70**	-0.42**	5.41**	34.8**	0.102*	0.967**
METKOR	0.0111	0.1127	0.040	1.265	7.47**	-0.31*	5.53**	40.6**	0.095*	0.963**
NAMFISH	0.0099	0.1202	0.042	1.283	7.52**	-0.18	5.06**	36.0**	0.092*	0.970**
OCFISH	0.0149	0.1000	0.042	1.461	8.16**	-0.58**	5.58**	7.0	0.065	0.984
CADSWEP	0.0187	0.1090	0.035	1.222	8.67**	-0.18	7.26**	39.1**	0.107**	0.944**
FEDFOOD	0.0118	0.1023	0.041	1.373	8.41**	-0.31*	5.93**	8.3	0.057	0.981
I-&-J	0.0194	0.0964	0.040	1.392	7.79**	-0.70**	5.75**	24.8**	0.081	0.973*
ICS	0.0109	0.1022	0.040	1.326	7.64**	0.10	4.70**	14.3*	0.075	0.982
KANHYM	0.0089	0.1201	0.044	1.318	6.71*	-0.41**	4.49**	26.4**	0.081	0.973*
PREM-GRP	0.0145	0.0896	0.039	1.382	6.95**	-0.27	4.59**	13.0	0.049	0.978
TIGR-OATS	0.0181	0.0799	0.033	1.406	6.97**	-0.37*	4.55**	11.0	0.049	0.981
AFCOL	0.0085	0.1014	0.036	1.283	7.46**	-0.32*	4.71**	16.6*	0.092*	0.982
ELLERINE	0.0176	0.0976	0.035	1.277	6.44	-0.60**	4.34**	23.8**	0.112**	0.965**
MATH-ASH	0.0196	0.1360	0.015	n/a	9.14**	0.72**	9.59**	338.5**	0.230**	0.843**
PICAPLI	0.0077	0.1952	0.049	1.091	7.35**	0.64**	5.68**	175.7**	0.186**	0.923**
AFR-LEASE	0.0100	0.1763	0.070	1.378	6.48*	0.45**	4.18**	23.7**	0.075	0.974*
BLYVOOR	0.0049	0.1235	0.048	1.345	6.55*	0.08	3.68*	19.5**	0.068	0.985
BRACKEN	0.0097	0.1523	0.067	1.392	5.32	0.19	3.25	3.1	0.056	0.978
BUFFELS	0.0112	0.1287	0.058	1.434	6.87*	0.20	3.47	6.6	0.042	0.992
DBN-DEEP	0.0069	0.2153	0.082	1.327	6.84*	0.10	4.03**	20.0**	0.066	0.985
DRIES	0.0148	0.1099	0.047	1.453	6.54*	-0.21	4.27**	5.4	0.045	0.979
E-R-P-M	0.0043	0.1804	0.071	1.353	6.80*	0.63**	4.88**	24.9**	0.084	0.956**
E-T-CONS	0.0194	0.1588	0.074	1.550	7.21**	-0.15	4.03**	11.6	0.063	0.991
ELSBURG	0.0017	0.1588	0.081	1.567	6.59*	-0.17	3.48	11.7	0.039	0.989
FALCON	0.0172	0.1367	0.040	1.210	8.34**	0.78**	8.14**	93.9**	0.145**	0.929**
GROOTVLEI	0.0132	0.1711	0.074	1.471	7.23**	0.26	4.83**	14.3*	0.058	0.978
HARMONY	0.0091	0.1387	0.060	1.386	6.14	0.00	3.41	8.6	0.060	0.985
HARTIES	0.0156	0.1307	0.057	1.450	6.77*	-0.35*	4.50**	5.3	0.039	0.979
KINROSS	0.0150	0.1420	0.072	1.591	5.94	-0.03	2.90	6.5	0.035	0.988

Appendix 3.4 (continued) Descriptive statistics and the results of the tests of normality using four-weekly returns over the period February 20, 1973 to March 13, 1992

	$\bar{r}$	σ_r	γ	α_f	SR	$\sqrt{b_1}$	b_2	χ^2	D_{KS}	W
KLOOF	0.0142	0.1215	0.057	1.544	7.23**	-0.44**	4.41**	11.5	0.041	0.985
LESLIE	0.0128	0.1504	0.069	1.522	6.80*	-0.03	3.48	9.2	0.057	0.986
LIBANON	0.0052	0.1627	0.070	1.412	5.97	0.08	3.73*	5.4	0.052	0.982
LORAINE	0.0050	0.1931	0.093	1.509	7.03**	-0.13	3.84*	5.2	0.041	0.992
MODDER	0.0046	0.1774	0.059	1.255	7.18**	0.42**	4.69**	109.1**	0.146**	0.964**
RANDFONTN	0.0173	0.1322	0.057	1.444	7.33**	0.13	4.27**	6.2	0.042	0.988
RD-LEASE	0.0000	0.2183	0.099	1.480	6.89*	-0.01	3.87*	12.1	0.071	0.987
SALLIES	0.0108	0.1918	0.072	1.380	8.37**	0.44**	5.55**	18.5**	0.070	0.973*
SIMMERS	0.0056	0.1609	0.070	1.504	8.08**	0.40*	4.42**	12.6	0.077	0.987
SOUTHVAAL	0.0139	0.1373	0.064	1.520	6.83*	-0.09	3.75*	6.6	0.045	0.990
ST-HELENA	0.0081	0.1249	0.054	1.439	6.86*	-0.09	3.82*	11.7	0.038	0.991
STH-RODPT	-0.0098	0.2219	0.092	1.469	7.63**	0.00	4.37**	5.6	0.065	0.990
STILFTN	0.0082	0.1450	0.059	1.392	6.36	-0.13	3.88**	10.8	0.048	0.983
VAAL-REEF	0.0149	0.1171	0.050	1.469	6.82*	-0.40**	4.85**	11.0	0.049	0.971*
VENTERS	0.0080	0.2041	0.088	1.461	7.30**	-0.14	4.44**	10.2	0.055	0.987
VILLAGE	0.0182	0.1907	0.071	1.335	8.32**	0.43**	6.05**	18.1*	0.069	0.974*
VLAKS	0.0006	0.1696	0.075	1.431	6.31	-0.03	3.74*	5.8	0.050	0.983
W-R-CONS	0.0108	0.2154	0.083	1.343	8.10**	0.33*	5.07**	22.7**	0.059	0.984
WAVERLEY	0.0066	0.1984	0.080	1.377	6.45	0.53**	4.12**	21.7**	0.085	0.976
WELKOM	0.0123	0.1430	0.065	1.503	7.21**	0.01	3.69*	2.6	0.034	0.997
WES-AREAS	0.0030	0.1671	0.082	1.597	6.99**	-0.26	4.09**	28.7**	0.070	0.979
WINKELS	0.0103	0.1365	0.060	1.488	6.85*	-0.09	3.57*	5.8	0.038	0.992
WIT-NIGEL	0.0017	0.2157	0.083	1.356	6.55*	-0.16	4.13**	27.8**	0.064	0.978
WSTN-DEEP	0.0135	0.1244	0.059	1.489	6.88*	-0.33*	4.16**	2.7	0.034	0.986
ZANDPAN	0.0158	0.1167	0.049	1.472	6.95**	-0.09	4.41**	8.6	0.046	0.980
A-V-I	0.0182	0.0921	0.034	1.329	7.34**	-0.68**	5.81**	20.7**	0.080	0.960**
AMIC	0.0125	0.0825	0.033	1.408	8.03**	-0.99**	6.89**	15.5*	0.082	0.963**
BARLOWS	0.0134	0.0796	0.035	1.467	8.99**	-0.66**	6.75**	12.4	0.045	0.980
BIDVEST	0.0101	0.1059	0.029	1.136	9.58**	-0.41**	9.07**	78.1**	0.120**	0.915**
BTRDUN	0.0174	0.0981	0.043	1.507	7.21**	-0.74**	5.76**	8.0	0.065	0.963**
CGSMITH	0.0159	0.0937	0.032	1.390	11.34**	-1.36**	14.91**	22.0**	0.083	0.929**
CULLINAN	0.0088	0.1019	0.042	1.383	6.66*	-0.20	3.99**	12.7	0.071	0.985
EUREKA	0.0151	0.1491	n/a	n/a	9.76**	0.70**	12.41**	616.0**	0.301**	0.775**
F-S-I	-0.0006	0.1394	0.034	1.110	8.04**	-0.60**	6.62**	200.0**	0.163**	0.931**
FARM-AG	0.0083	0.1415	0.046	1.216	6.75*	0.13	4.41**	29.8**	0.082	0.972*
HLH	0.0124	0.1027	0.036	1.344	9.08**	-1.07**	10.23**	21.5**	0.072	0.937**
MALBAK	0.0113	0.1110	0.043	1.378	8.28**	-0.60**	6.08**	15.7*	0.057	0.978
MALHOLD	0.0156	0.1065	0.036	1.296	8.17**	-0.17	5.19**	24.1**	0.077	0.981
MCPHAIL	0.0093	0.1441	0.044	1.264	9.92**	-0.85**	10.87**	131.0**	0.148**	0.937**
MESSINA	0.0002	0.1671	0.064	1.416	11.67**	-1.51**	17.16**	22.0**	0.069	0.927**
METJE-&-Z	0.0052	0.1180	n/a	n/a	8.15**	0.19	7.05**	408.1**	0.239**	0.867**
MICOR	0.0113	0.1061	0.034	1.223	8.11**	0.11	5.75**	42.1**	0.116**	0.966**
NICTUS	-0.0008	0.1338	0.028	1.040	8.26**	-0.27	5.96**	293.0**	0.212**	0.928**
OCEANA	0.0121	0.1381	n/a	n/a	13.45**	2.04**	28.70**	940.7**	0.353**	0.637**
PICBEL	0.0124	0.1494	0.057	1.345	7.45**	0.64**	5.34**	21.8**	0.069	0.970**
PICHOLD	0.0105	0.1293	0.045	1.310	7.84**	0.43**	5.97**	41.2**	0.087*	0.961**
PLACOR	0.0141	0.1018	0.046	1.449	5.68	-0.04	3.15	11.3	0.069	0.982
PLATE-GL	0.0129	0.0983	0.040	1.374	8.08**	-0.67**	5.35**	9.2	0.077	0.978
RENTBEL	0.0083	0.1075	0.037	1.273	8.22**	0.67**	6.60**	62.9**	0.121**	0.954**
SA-BIAS	0.0168	0.1054	0.007	n/a	11.23**	0.84**	15.62**	439.8**	0.248**	0.769**

Appendix 3.4 (continued) Descriptive statistics and the results of the tests of normality using four-weekly returns over the period February 20, 1973 to March 13, 1992

	$\bar{r}$	σ_r	γ	α_f	SR	$\sqrt{b_1}$	b_2	χ^2	D_{KS}	W
TGH	0.0108	0.1601	0.028	n/a	10.24**	1.19**	11.75**	227.3**	0.194**	0.858**
W-&-A	0.0181	0.1250	0.049	1.355	7.52**	-0.23	4.59**	42.1**	0.098*	0.983
GARDIAN	0.0152	0.1134	0.023	1.024	8.23**	0.69**	7.71**	181.4**	0.166**	0.897**
I-G-I	0.0137	0.1211	0.034	1.235	10.70**	1.80**	16.57**	74.7**	0.128**	0.883**
LIB-HOLD	0.0193	0.0906	0.042	1.510	7.54**	-0.38*	4.72**	7.9	0.065	0.988
LIBERTY	0.0174	0.0855	0.035	1.444	6.92*	-0.42**	5.07**	8.4	0.049	0.973*
M-&-F	0.0187	0.0804	0.029	1.290	8.15**	0.11	6.28**	34.9**	0.099*	0.961**
PROSURE	0.0093	0.0952	0.026	1.145	7.13**	-0.46**	4.93**	126.1**	0.162**	0.947**
SAGE-LTD	0.0104	0.0959	0.034	1.302	6.47	0.02	3.99**	48.5**	0.102*	0.973*
COM-FUND	0.0164	0.0737	0.014	n/a	7.93**	0.48**	6.56**	147.9**	0.164**	0.911**
INTRUST	0.0132	0.1184	0.027	1.068	9.65**	-0.30*	8.92**	169.9**	0.141**	0.901**
ASS-MANG	0.0174	0.0687	0.010	n/a	12.00**	1.10**	19.26**	436.6**	0.261**	0.732**
SAMANCOR	0.0197	0.1118	0.042	1.411	9.28**	-0.85**	8.49**	17.1*	0.086	0.946**
FREDDEV	0.0062	0.1622	0.070	1.472	8.37**	-0.68**	7.07**	13.4	0.052	0.971*
AMGOLD	0.0123	0.1030	0.046	1.470	6.76*	-0.22	4.04**	8.3	0.059	0.984
ASSORE	0.0157	0.0939	0.011	n/a	8.72**	0.13	8.70**	273.4**	0.206**	0.839**
COR-SYND	0.0089	0.1706	0.074	1.461	8.08**	0.03	5.00**	12.9	0.048	0.988
DUIKERS	0.0080	0.1290	0.051	1.349	9.69**	-0.24	8.12**	40.9**	0.093*	0.958**
EGOLI	0.0088	0.1984	0.079	1.309	6.77*	0.61**	5.04**	44.3**	0.096*	0.961**
GENBEL	0.0150	0.1218	0.054	1.425	7.80**	-0.58**	5.33**	6.8	0.065	0.977
LONFIN	0.0030	0.1381	0.052	1.368	8.93**	-0.22	6.41**	40.8**	0.095*	0.976
MID-WITS	0.0196	0.1267	0.059	1.485	6.02	-0.16	3.16	54.2**	0.114**	0.983
MINORCO	0.0115	0.1098	0.044	1.423	8.14**	0.19	4.98**	8.6	0.060	0.987
NEW-CENT	0.0135	0.1216	0.041	1.275	8.34**	-0.34*	6.33**	23.2**	0.076	0.971*
NEW-WITS	0.0132	0.1379	0.056	1.487	10.34**	-0.31*	7.99**	17.0*	0.056	0.975
R-M-PROPS	0.0118	0.1356	0.055	1.421	8.64**	0.26	5.72**	16.3*	0.081	0.978
RAND-LON	-0.0035	0.1741	0.067	1.324	7.23**	0.34*	4.07**	32.6**	0.069	0.983
TWEEFONTN	0.0093	0.1221	0.035	1.192	9.87**	-0.10	7.93**	96.6**	0.121**	0.951**
VOGELS	0.0138	0.1215	0.045	1.330	6.31	0.12	3.93**	17.8*	0.077	0.978
ANGLO-AM	0.0142	0.1013	0.043	1.477	9.22**	-0.73**	7.59**	11.8	0.047	0.976
ANGLOVAAL	0.0227	0.0974	0.036	1.366	7.75**	-0.44**	6.06**	20.0**	0.082	0.958**
CHARTER	0.0088	0.1163	0.039	1.221	7.02**	0.30	4.48**	23.3**	0.101*	0.972*
GENCOR	0.0169	0.0998	0.043	1.497	8.08**	-0.56**	5.29**	12.6	0.053	0.984
GFSA	0.0142	0.1261	0.055	1.524	7.79**	-0.77**	6.56**	13.1	0.056	0.964**
JOHNNIES	0.0251	0.1066	0.045	1.452	9.77**	-1.35**	11.18**	12.7	0.066	0.953**
RANDMIN	0.0122	0.1135	0.047	1.418	6.87*	-0.08	4.16**	7.0	0.052	0.986
GENTYRE-A	0.0195	0.1094	0.029	1.187	10.11**	-0.63**	10.74**	68.4**	0.128**	0.915**
MCCARTHY	0.0169	0.1116	0.046	1.400	6.72*	-0.35*	4.32**	11.3	0.067	0.978
METAIR	0.0169	0.1359	0.038	1.182	9.72**	0.57**	8.21**	98.7**	0.140**	0.930**
PORT	0.0088	0.1146	0.031	1.117	7.08**	-0.55**	6.05**	109.2**	0.167**	0.926**
SAFICON	0.0097	0.1356	0.047	1.318	7.81**	-1.44**	9.47**	33.9**	0.096*	0.910**
SAKERS	0.0125	0.1244	0.039	1.266	8.33**	-1.41**	9.85**	30.6**	0.116**	0.912**
TOYOTA	0.0192	0.1357	0.052	1.385	7.60**	-0.09	5.22**	24.5**	0.086	0.973*
URQHART	-0.0047	0.1676	0.042	1.087	8.98**	-1.92**	13.78**	150.3**	0.167**	0.860**
WESCO	0.0182	0.1274	0.038	1.176	6.73*	0.58**	4.57**	68.1**	0.127**	0.954**
CON-MURCH	-0.0081	0.1364	0.059	1.469	8.29**	-0.39*	5.90**	23.8**	0.098*	0.976
MSAULI	0.0097	0.1554	0.061	1.414	8.18**	-0.22	5.45**	19.8**	0.078	0.977
CARLCOR	0.0168	0.0898	0.031	1.251	6.91*	0.46**	4.41**	49.4**	0.105**	0.969**
COATES	0.0168	0.1043	0.015	n/a	7.78**	0.26	6.42**	215.6**	0.188**	0.907**
CONSOL	0.0276	0.0669	0.018	1.168	7.98**	-0.33*	6.63**	89.8**	0.160**	0.937**

Appendix 3.4 (continued) Descriptive statistics and the results of the tests of normality using four-weekly returns over the period February 20, 1973 to March 13, 1992

	$\bar{r}$	σ_r	γ	α_f	SR	$\sqrt{b_1}$	b_2	χ^2	D_{KS}	W
CTP	0.0093	0.1168	0.036	1.197	7.29**	0.29	5.23**	56.0**	0.114**	0.953**
HARWILL	0.0075	0.1783	n/a	n/a	7.78**	0.57**	7.15**	552.9**	0.268**	0.828**
HOLDAIN	0.0164	0.0940	0.035	1.281	6.55*	-0.17	3.97**	14.9*	0.074	0.978
HORTORS	0.0049	0.1258	0.059	1.482	7.39**	-0.11	4.33**	26.2**	0.090*	0.988
NAMPAK	0.0165	0.0784	0.030	1.360	7.08**	-0.60**	4.96**	13.8	0.070	0.974*
SAPPI	0.0165	0.0970	0.038	1.358	7.50**	-0.67**	5.39**	6.7	0.064	0.976
ADCOCK	0.0192	0.0744	0.021	1.168	9.04**	0.32*	7.36**	129.9**	0.137**	0.933**
NORIMED	0.0047	0.1266	0.024	n/a	10.14**	0.31*	8.88**	255.7**	0.202**	0.907**
UNI-COLD	0.0105	0.0640	n/a	n/a	12.38**	4.74**	39.20**	890.6**	0.373**	0.569**
LYD-PLAT	0.0150	0.1384	0.066	1.601	8.45**	-0.96**	7.90**	3.2	0.031	0.963**
RUSPLAT	0.0156	0.1306	0.058	1.523	8.85**	-0.89**	7.58**	4.8	0.048	0.973*
ARGUS	0.0126	0.0877	0.026	1.163	8.05**	0.07	5.86**	47.9**	0.123**	0.957**
T-M-L	0.0201	0.1229	0.033	1.157	9.21**	-0.39*	8.39**	80.3**	0.140**	0.918**
AMAPROP	0.0073	0.1118	0.047	1.449	6.95**	0.58**	4.82**	5.9	0.055	0.976
BESTER	-0.0031	0.1326	0.056	1.459	6.93*	0.42**	4.22**	16.8*	0.084	0.984
GF-PROPS	0.0118	0.1234	0.057	1.448	5.72	0.15	3.28	12.6	0.051	0.982
N-KLEINS	0.0153	0.1360	0.032	1.075	8.82**	0.57**	7.45**	137.7**	0.159**	0.920**
PUTPROP	-0.0028	0.1623	0.059	1.303	6.82*	0.14	4.50**	31.3**	0.099*	0.972*
SABLE	0.0142	0.1308	0.041	1.260	9.42**	0.27	7.15**	49.6**	0.111**	0.959**
PRIMA	0.0097	0.0694	0.029	1.383	7.70**	-0.04	4.97**	36.8**	0.083	0.981
SANLAND	0.0125	0.0710	0.025	1.284	6.82*	-0.20	4.08**	61.0**	0.103*	0.975
BOYMANS	0.0071	0.1266	0.013	n/a	9.30**	0.61**	8.97**	330.0**	0.244**	0.860**
CNAGALO	0.0168	0.1159	0.046	1.361	7.53**	-0.27	4.66**	11.6	0.069	0.986
EDGARS	0.0164	0.0888	0.030	1.292	8.04**	-0.27	5.81**	37.9**	0.085	0.970**
EUROPA	-0.0011	0.1321	0.040	1.180	7.27**	-0.23	5.48**	98.8**	0.132**	0.949**
FOSCHINI	0.0265	0.0873	0.017	1.035	11.61**	0.62**	14.06**	149.2**	0.158**	0.861**
GRESHAM	0.0051	0.1310	0.048	1.244	6.66*	0.24	4.52**	58.4**	0.079	0.968**
O-K	0.0061	0.0850	0.041	1.539	6.25	-0.01	3.18	5.4	0.047	0.991
PEPKOR	0.0146	0.1067	0.038	1.281	8.46**	-0.40**	7.25**	31.0**	0.104**	0.942**
PICKNPAY	0.0200	0.0921	0.042	1.456	7.07**	-0.64**	4.76**	12.1	0.056	0.976
WOOLTRU	0.0151	0.0839	0.033	1.355	6.53*	-0.28	4.38**	26.1**	0.077	0.972*
HIVELD	0.0135	0.0948	0.044	1.503	7.76**	0.10	4.26**	11.2	0.075	0.989
CROOKES	0.0117	0.0838	0.012	n/a	8.06**	-0.56**	7.62**	214.0**	0.202**	0.875**
TONGAAT	0.0103	0.0900	0.039	1.454	7.94**	-1.00**	6.80**	11.3	0.080	0.959**
ROOIBERG	-0.0024	0.1189	0.042	1.272	6.82*	-0.04	4.00**	25.8**	0.078	0.983
UNION-TIN	0.0164	0.1401	0.046	1.307	9.83**	1.44**	11.07**	71.5**	0.134**	0.910**
LIONMATCH	0.0119	0.0765	0.028	1.318	7.57**	0.07	5.37**	27.9**	0.098*	0.966**
REMBR-BEH	0.0237	0.1022	0.037	1.296	7.26**	-0.48**	4.85**	112.4**	0.139**	0.962**
REMGRO	0.0239	0.1004	0.052	1.654	8.19**	-0.59**	5.82**	54.1**	0.100*	0.974*
TEGKOR	0.0251	0.0924	0.025	1.092	6.36	-0.25	5.23**	302.6**	0.188**	0.903**
TIB	0.0253	0.0968	0.027	1.093	5.94	-0.19	4.14**	276.4**	0.215**	0.924**
UTICO	0.0145	0.1081	0.043	1.313	7.56**	0.07	4.72**	13.7	0.066	0.978
MOBILE	0.0233	0.0955	0.026	1.152	8.05**	-0.17	5.71**	260.0**	0.197**	0.939**
TRENCOR	0.0228	0.0892	0.018	1.089	10.91**	-1.70**	18.29**	250.1**	0.189**	0.844**

** : significant att the 1% level * : significant at the 5% level

Appendix 4.1 Autocorrelation structure of daily security returns over the period December 28, 1987 to March 20, 1992

Name	Lag 1 Lag 6	Lag 2 Lag 7	Lag 3 Lag 8	Lag 4 Lag 9	Lag 5 Lag 10
	Trading Decile Number 1		(size) : autocorrelation : significance		
DRIES	(1050) 0.024	(1048) 0.002	(1047) 0.015	(1046) -0.027	(1045) -0.014
	(1044) -0.006	(1043) -0.006	(1042) -0.051	(1041) 0.037	(1040) -0.066*
DEBEERS	(1050) 0.131**	(1048) 0.045	(1047) -0.001	(1046) 0.009	(1045) -0.033
	(1044) 0.042	(1043) 0.021	(1042) 0.025	(1041) -0.021	(1040) 0.067*
BARLOWS	(1050) 0.123**	(1048) 0.013	(1047) -0.033	(1046) -0.049	(1045) -0.011
	(1044) -0.061*	(1043) -0.031	(1042) 0.011	(1041) -0.040	(1040) -0.016
ANGLO-AM	(1050) 0.070*	(1048) 0.031	(1047) 0.046	(1046) -0.027	(1045) -0.024
	(1044) 0.001	(1043) 0.024	(1042) 0.024	(1041) -0.008	(1040) -0.020
RUSPLAT	(1047) 0.104**	(1044) 0.029	(1043) 0.028	(1042) 0.024	(1041) -0.004
	(1040) 0.010	(1039) -0.006	(1038) -0.053	(1037) 0.005	(1036) 0.011
KLOOF	(1047) 0.036	(1044) 0.012	(1043) -0.011	(1042) -0.025	(1041) -0.005
	(1040) -0.011	(1039) 0.008	(1038) -0.032	(1037) 0.004	(1036) -0.052
ELSBURG	(1044) -0.048	(1040) -0.016	(1039) -0.069*	(1038) 0.046	(1037) -0.002
	(1036) 0.007	(1035) 0.028	(1034) 0.036	(1033) 0.006	(1032) -0.007
GENCOR	(1046) 0.014	(1044) -0.018	(1044) -0.017	(1043) -0.050	(1042) -0.004
	(1041) -0.056	(1040) -0.011	(1039) 0.032	(1038) -0.001	(1037) -0.021
SA-BREWS	(1038) 0.077*	(1032) 0.026	(1031) -0.082**	(1031) -0.059	(1031) -0.009
	(1030) -0.013	(1029) 0.008	(1028) 0.002	(1027) 0.024	(1026) -0.028
VAAL-REEF	(1038) 0.035	(1032) 0.004	(1031) -0.017	(1030) 0.005	(1029) 0.022
	(1028) -0.019	(1027) 0.027	(1026) 0.025	(1025) 0.009	(1024) -0.037
ZANDPAN	(1042) -0.083**	(1039) -0.053	(1038) -0.121**	(1037) 0.036	(1035) -0.019
	(1033) -0.007	(1032) -0.021	(1031) 0.035	(1030) 0.015	(1029) -0.045
HARTIES	(1040) 0.024	(1035) 0.010	(1033) -0.060	(1032) -0.003	(1031) 0.017
	(1030) 0.029	(1029) 0.009	(1029) -0.008	(1029) 0.027	(1028) -0.053
REMGRO	(1034) 0.004	(1027) 0.002	(1025) -0.060	(1024) -0.038	(1023) -0.046
	(1022) 0.004	(1021) -0.032	(1020) -0.008	(1019) 0.024	(1018) -0.003
GENBEL	(1034) 0.010	(1027) -0.024	(1025) -0.066*	(1025) 0.014	(1025) 0.044
	(1024) -0.048	(1023) -0.042	(1022) 0.027	(1022) 0.046	(1022) 0.018
SOUTHVAAL	(1017) -0.017	(1005) 0.011	(1006) 0.030	(1005) 0.008	(1003) 0.000
	(1002) -0.004	(1001) 0.032	(1000) 0.034	(999) 0.081*	(999) -0.049
REMBR-BEH	(1027) 0.050	(1018) -0.088**	(1015) -0.069*	(1014) -0.023	(1013) -0.045
	(1012) -0.036	(1011) 0.021	(1010) 0.001	(1009) -0.001	(1008) 0.003
LORAINE	(1018) 0.016	(1006) -0.008	(1004) -0.011	(1005) 0.047	(1006) 0.008
	(1005) -0.011	(1002) -0.014	(999) 0.006	(997) 0.056	(996) -0.025
LIBANON	(990) -0.027	(969) 0.003	(967) 0.034	(967) 0.050	(966) -0.002
	(963) -0.063	(961) 0.030	(961) 0.029	(962) 0.047	(962) -0.051
WSTN-DEEP	(975) -0.013	(949) 0.061	(946) -0.018	(946) 0.034	(947) 0.005
	(948) 0.019	(949) -0.019	(945) 0.039	(940) -0.003	(939) -0.034
BANKORP	(983) 0.086**	(963) -0.016	(960) -0.060	(959) -0.081*	(956) -0.079*
	(952) -0.021	(951) -0.042	(950) 0.020	(949) -0.031	(949) -0.034
WES-AREAS	(960) 0.017	(933) -0.019	(932) -0.018	(932) 0.049	(931) -0.033
	(929) 0.068*	(930) -0.024	(930) 0.051	(926) 0.015	(924) -0.005
LESLIE	(961) -0.032	(936) -0.041	(933) -0.005	(934) 0.061	(934) 0.022
	(931) -0.030	(930) 0.000	(932) 0.014	(929) 0.048	(924) -0.059
RANDFONTN	(970) 0.022	(952) 0.026	(953) -0.061	(947) 0.051	(943) 0.064
	(945) 0.022	(946) -0.065*	(943) 0.051	(939) -0.012	(940) 0.014
MALBAK	(939) -0.015	(907) 0.011	(904) -0.048	(901) -0.015	(898) -0.068*
	(896) -0.005	(895) -0.058	(894) 0.037	(893) 0.062	(891) 0.023

Appendix 4.1 (continued) Autocorrelation structure of daily security returns over the period December 28, 1987 to March 20, 1992

Name	Lag 1 Lag 6	Lag 2 Lag 7	Lag 3 Lag 8	Lag 4 Lag 9	Lag 5 Lag 10
	Trading Decile Number 2		(size) : autocorrelation : significance		
NEDCOR	(950) -0.068*	(921) 0.016	(921) -0.030	(917) -0.049	(912) -0.059
	(907) -0.041	(906) 0.023	(907) -0.004	(908) -0.007	(907) 0.008
SAPPI	(945) 0.058	(914) 0.051	(910) 0.027	(906) -0.054	(907) -0.001
	(910) -0.034	(909) -0.039	(906) -0.055	(903) 0.009	(902) -0.044
GFSA	(917) 0.134**	(879) 0.033	(879) -0.017	(876) -0.016	(873) 0.020
	(877) 0.032	(878) 0.009	(874) 0.006	(870) 0.099**	(870) -0.047
VENTERS	(923) 0.026	(887) -0.028	(887) -0.009	(889) -0.006	(891) -0.015
	(886) 0.005	(884) 0.066*	(889) 0.045	(889) -0.009	(885) -0.031
MINORCO	(968) 0.131**	(948) 0.000	(945) -0.086**	(945) -0.028	(946) 0.029
	(946) 0.015	(945) 0.013	(945) -0.031	(946) -0.029	(945) -0.030
AMGOLD	(910) 0.025	(875) -0.008	(876) -0.036	(876) -0.012	(878) -0.003
	(873) -0.029	(871) 0.052	(869) 0.062	(865) -0.001	(861) -0.026
MODDER	(887) 0.055	(843) 0.098**	(843) -0.039	(836) -0.064	(835) 0.022
	(842) -0.048	(842) 0.032	(833) 0.032	(827) -0.009	(830) -0.003
GROOTVLEI	(884) 0.022	(838) -0.006	(834) -0.044	(832) -0.038	(828) 0.056
	(824) -0.012	(823) 0.028	(826) 0.055	(827) 0.079*	(827) -0.073*
NEW-WITS	(841) 0.100**	(781) -0.015	(778) 0.001	(777) 0.039	(777) 0.008
	(781) 0.022	(786) 0.069	(778) -0.018	(776) -0.007	(781) -0.019
MID-WITS	(875) 0.028	(833) -0.003	(831) -0.071*	(829) 0.032	(826) -0.045
	(824) -0.015	(821) 0.049	(817) 0.029	(819) -0.067	(818) 0.003
JOHNNIES	(836) 0.043	(788) -0.011	(789) -0.056	(783) -0.033	(778) 0.011
	(779) 0.009	(778) 0.045	(775) -0.003	(775) -0.004	(773) -0.011
TONGAAT	(842) 0.005	(792) 0.109**	(786) -0.092**	(781) 0.008	(784) -0.073*
	(778) 0.002	(775) 0.019	(778) 0.012	(774) -0.030	(775) 0.046
LIBERTY	(833) 0.116**	(781) -0.014	(776) -0.066	(777) -0.046	(776) 0.023
	(776) -0.022	(772) -0.006	(764) -0.083*	(758) 0.026	(755) 0.014
RD-LEASE	(834) -0.104**	(784) 0.030	(783) -0.013	(783) -0.042	(780) 0.050
	(778) -0.049	(781) -0.064	(782) -0.001	(784) 0.027	(785) -0.002
LIB-HOLD	(781) 0.056	(715) 0.055	(706) 0.006	(703) 0.066	(701) -0.029
	(701) 0.006	(701) -0.012	(692) 0.001	(691) -0.015	(692) 0.088*
PICKNPAY	(750) 0.047	(675) 0.092*	(672) 0.001	(668) -0.053	(658) 0.003
	(654) 0.014	(663) 0.035	(666) -0.009	(665) -0.033	(672) 0.001
AECI	(755) 0.021	(682) -0.018	(677) -0.023	(677) -0.050	(683) -0.039
	(679) 0.017	(675) 0.035	(675) 0.089*	(670) -0.043	(667) 0.031
TRANS-NTL	(750) 0.077*	(674) 0.019	(668) 0.019	(666) -0.049	(665) -0.021
	(672) -0.020	(674) -0.072	(664) 0.074	(662) -0.004	(664) -0.067
SBIC	(745) 0.177**	(667) 0.023	(658) -0.071	(654) 0.015	(663) 0.055
	(673) 0.041	(667) 0.022	(656) -0.062	(652) 0.032	(655) 0.009
TIGR-OATS	(784) 0.000	(719) 0.023	(711) -0.011	(709) -0.016	(708) -0.020
	(713) -0.041	(709) -0.026	(704) 0.007	(707) 0.029	(704) -0.014
SAMANCOR	(824) 0.124**	(775) -0.005	(770) 0.012	(765) 0.020	(764) -0.006
	(761) -0.032	(765) 0.010	(763) 0.070	(765) 0.017	(769) 0.041
KINROSS	(769) 0.088*	(703) 0.067	(700) 0.007	(696) 0.054	(697) 0.091*
	(698) 0.023	(692) 0.081*	(686) 0.071	(683) 0.015	(684) -0.062
PRIMA	(719) -0.068	(646) -0.007	(648) -0.014	(647) -0.009	(646) -0.033
	(643) -0.047	(638) 0.065	(639) -0.079*	(642) 0.013	(639) 0.039
WELKOM	(737) 0.080*	(674) -0.068	(668) -0.044	(661) 0.021	(663) 0.024
	(667) 0.013	(669) 0.043	(660) 0.020	(655) -0.044	(664) -0.085*

Appendix 4.1 (continued) Autocorrelation structure of daily security returns over the period December 28, 1987 to March 20, 1992

Name	Lag 1 Lag 6	Lag 2 Lag 7	Lag 3 Lag 8	Lag 4 Lag 9	Lag 5 Lag 10
	Trading Decile Number 3		(size) : autocorrelation : significance		
AMIC	(734) 0.153**	(672) -0.014	(665) -0.108**	(658) -0.125**	(656) 0.035
	(648) -0.013	(641) -0.007	(635) 0.036	(628) -0.010	(626) -0.041
FREDDEV	(713) 0.084*	(635) -0.036	(628) -0.069	(633) -0.065	(632) -0.002
	(629) 0.011	(628) 0.054	(622) 0.016	(615) -0.050	(613) 0.002
PREM-GRP	(637) 0.078*	(537) 0.076	(521) 0.059	(525) 0.041	(540) 0.053
	(550) 0.040	(549) -0.025	(541) -0.006	(539) -0.064	(545) 0.038
MESSINA	(702) 0.070	(637) -0.030	(625) -0.057	(625) -0.045	(628) 0.028
	(621) 0.013	(608) 0.011	(608) -0.009	(621) -0.007	(629) -0.042
HARMONY	(682) 0.071	(616) -0.015	(605) 0.022	(605) -0.034	(607) -0.052
	(608) 0.000	(603) 0.013	(595) 0.011	(594) -0.018	(597) -0.018
METKOR	(673) -0.073	(594) 0.094*	(587) 0.001	(596) -0.048	(595) 0.009
	(587) -0.072	(586) -0.073	(588) 0.005	(586) 0.026	(582) 0.024
SENTRCHEM	(678) 0.002	(604) 0.089*	(601) -0.051	(600) 0.011	(604) 0.013
	(605) 0.085*	(605) -0.061	(600) 0.029	(597) 0.051	(599) -0.035
PALAMIN	(589) 0.087*	(498) 0.007	(479) -0.003	(467) -0.051	(466) -0.048
	(472) -0.025	(467) 0.043	(462) -0.029	(469) -0.002	(472) 0.007
CGSMITH	(526) 0.089*	(422) -0.100*	(414) 0.011	(413) 0.008	(411) 0.030
	(400) 0.018	(399) -0.041	(399) 0.076	(403) 0.032	(404) 0.134**
WOOLTRU	(495) 0.161**	(387) 0.025	(380) 0.029	(376) 0.030	(376) 0.081
	(372) -0.035	(384) -0.091	(384) -0.014	(382) -0.047	(377) -0.010
A-V-I	(510) 0.014	(411) -0.033	(404) -0.005	(398) -0.120*	(388) 0.027
	(380) -0.023	(369) -0.011	(374) 0.061	(388) -0.001	(394) -0.036
R-M-PROPS	(426) -0.103*	(326) -0.056	(328) 0.014	(318) -0.031	(324) 0.069
	(324) -0.073	(307) -0.093	(304) 0.073	(300) 0.014	(305) -0.144*
E-T-CONS	(595) 0.018	(520) -0.045	(514) -0.016	(519) 0.054	(517) 0.007
	(510) -0.079	(514) -0.045	(517) 0.041	(508) -0.051	(503) -0.102*
ENGEN	(616) 0.228**	(547) 0.013	(532) -0.044	(521) -0.115**	(528) -0.041
	(532) -0.023	(535) -0.117**	(539) 0.074	(543) 0.127**	(538) 0.003
KANHYM	(533) 0.065	(446) -0.099*	(443) -0.031	(428) -0.047	(431) -0.057
	(439) 0.034	(437) -0.094*	(443) -0.037	(454) -0.030	(447) 0.021
MALHOLD	(508) -0.070	(407) 0.232**	(398) 0.130**	(392) -0.111*	(397) 0.123*
	(400) -0.176**	(390) -0.064	(382) 0.012	(384) 0.010	(383) -0.039
W-R-CONS	(599) 0.036	(524) -0.023	(510) -0.012	(511) 0.123**	(503) 0.104*
	(497) 0.041	(495) -0.057	(496) 0.028	(502) 0.030	(501) 0.028
MCPHAIL	(540) -0.127**	(456) -0.058	(458) -0.060	(458) -0.075	(465) -0.057
	(471) -0.063	(474) -0.006	(466) 0.082	(448) -0.028	(453) -0.030
PLATE-GL	(542) 0.154**	(462) 0.132**	(457) 0.070	(447) 0.056	(437) 0.000
	(434) 0.030	(429) 0.010	(418) 0.070	(426) -0.002	(426) -0.025
HIVELD	(619) 0.183**	(540) 0.102*	(537) 0.009	(538) -0.031	(541) -0.064
	(542) -0.079	(537) -0.067	(533) -0.013	(537) 0.027	(551) -0.031
NAMPAK	(453) 0.283**	(362) 0.147**	(342) 0.144**	(330) -0.037	(330) 0.052
	(324) 0.104	(324) 0.051	(324) 0.039	(332) 0.012	(336) 0.008
VILLAGE	(583) -0.029	(500) -0.033	(493) -0.036	(489) -0.081	(484) 0.052
	(485) -0.031	(482) 0.017	(477) -0.066	(475) 0.009	(483) -0.002
STH-RODPT	(632) -0.019	(577) -0.132**	(584) 0.034	(585) -0.003	(583) 0.014
	(578) 0.012	(577) -0.005	(580) -0.023	(586) -0.056	(591) 0.002
BRACKEN	(519) -0.033	(439) 0.020	(429) 0.017	(417) 0.011	(411) -0.033
	(411) -0.069	(406) -0.006	(412) -0.061	(423) -0.034	(426) 0.001

Appendix 4.1 (continued) Autocorrelation structure of daily security returns over the period December 28, 1987 to March 20, 1992

Name	Lag 1 Lag 6	Lag 2 Lag 7	Lag 3 Lag 8	Lag 4 Lag 9	Lag 5 Lag 10
	Trading Decile Number 4		(size) : autocorrelation : significance		
AMCOAL	(376) -0.020	(262) -0.013	(266) 0.042	(276) 0.081	(271) 0.012
	(269) -0.139*	(277) -0.168**	(279) 0.148*	(278) -0.021	(275) -0.108
HLH	(551) 0.103*	(469) 0.018	(467) 0.027	(464) 0.010	(453) 0.057
	(447) 0.015	(451) -0.050	(454) 0.027	(449) -0.032	(450) -0.033
W-&-A	(558) 0.098*	(480) 0.066	(469) -0.057	(455) 0.008	(445) -0.020
	(446) 0.036	(436) -0.050	(422) 0.058	(423) 0.006	(435) -0.067
WAVERLEY	(482) 0.005	(401) -0.078	(401) -0.067	(408) -0.017	(408) 0.007
	(393) -0.085	(378) 0.025	(386) 0.107*	(397) 0.062	(399) -0.214**
LYD-PLAT	(436) 0.113*	(340) 0.009	(332) -0.152**	(325) -0.013	(323) 0.070
	(322) 0.119*	(312) -0.059	(309) -0.050	(308) -0.060	(316) 0.084
BOLAND	(377) 0.038	(277) 0.040	(273) 0.053	(271) 0.006	(263) 0.002
	(258) -0.017	(261) -0.036	(257) 0.018	(259) -0.018	(266) 0.059
AMAPROP	(377) -0.085	(285) 0.048	(271) 0.055	(265) -0.051	(254) -0.051
	(255) -0.060	(260) -0.148*	(265) -0.057	(266) 0.056	(271) -0.007
VLAKS	(616) 0.074	(550) -0.049	(549) -0.096*	(541) 0.050	(536) 0.063
	(530) -0.069	(533) -0.014	(536) -0.035	(528) -0.059	(529) 0.023
SIMMERS	(395) 0.074	(302) -0.068	(302) -0.125*	(308) -0.017	(298) -0.075
	(292) -0.120*	(297) 0.019	(303) 0.093	(306) 0.069	(302) -0.064
DORBYL	(386) -0.022	(293) 0.039	(300) 0.074	(306) -0.126*	(296) 0.099
	(294) 0.093	(302) 0.033	(305) 0.063	(294) -0.015	(278) -0.088
ABACUS	(577) 0.034	(517) -0.040	(512) -0.080	(508) -0.076	(503) 0.010
	(504) -0.017	(501) -0.102*	(499) -0.091*	(492) 0.004	(493) 0.019
WINKELS	(400) 0.122*	(297) -0.003	(285) 0.000	(286) -0.001	(290) 0.093
	(287) 0.081	(280) 0.081	(275) 0.008	(277) -0.044	(282) -0.045
I-&-J	(392) -0.110*	(297) -0.003	(298) 0.016	(304) 0.017	(304) 0.031
	(299) 0.011	(278) 0.072	(271) 0.008	(286) -0.053	(282) 0.098
G-F-COAL	(375) -0.002	(284) -0.027	(289) -0.079	(280) -0.085	(266) -0.031
	(267) 0.120*	(266) -0.009	(262) 0.046	(266) -0.031	(269) -0.082
VOGELS	(389) 0.112*	(309) 0.032	(316) 0.134*	(316) -0.038	(308) 0.096
	(311) -0.014	(310) -0.017	(309) 0.034	(305) -0.031	(299) -0.142*
MCCARTHY	(368) 0.107*	(273) 0.002	(269) -0.118	(258) -0.003	(260) 0.081
	(259) 0.081	(256) 0.006	(254) 0.030	(256) -0.024	(256) -0.041
BTRDUN	(327) 0.137*	(231) 0.102	(223) 0.000	(226) 0.028	(234) -0.064
	(231) 0.053	(210) 0.002	(199) -0.033	(198) 0.051	(204) 0.021
PLACOR	(371) 0.125*	(287) 0.106	(270) 0.044	(266) 0.007	(264) -0.055
	(266) -0.031	(265) 0.075	(262) 0.023	(252) 0.095	(252) -0.087
RANDMIN	(284) 0.130*	(187) 0.046	(196) 0.117	(205) 0.137*	(206) -0.044
	(198) -0.013	(185) 0.043	(175) 0.009	(184) 0.112	(201) 0.087
OTIS	(400) 0.073	(314) -0.026	(312) 0.043	(294) 0.058	(290) 0.053
	(293) -0.031	(295) -0.017	(298) -0.044	(293) 0.182**	(291) 0.035
MSAULI	(446) 0.039	(376) 0.048	(378) -0.092	(380) -0.053	(383) -0.038
	(388) -0.021	(376) -0.012	(375) 0.054	(378) 0.036	(382) 0.083
FEDFOOD	(360) 0.135*	(273) 0.036	(258) 0.121	(255) 0.091	(266) 0.101
	(257) 0.090	(250) 0.060	(247) 0.043	(248) 0.070	(248) 0.049
TEGKOR	(325) 0.060	(229) -0.116	(227) -0.108	(231) -0.108	(238) -0.114
	(224) 0.042	(213) -0.024	(214) 0.003	(226) -0.026	(224) -0.007
SAGE-LTD	(432) 0.159**	(340) 0.172**	(332) 0.012	(322) 0.172**	(309) -0.023
	(302) 0.042	(309) -0.081	(313) 0.065	(321) -0.028	(322) 0.024

Appendix 4.1 (continued) Autocorrelation structure of daily security returns over the period December 28, 1987 to March 20, 1992

Name	Lag 1 Lag 6	Lag 2 Lag 7	Lag 3 Lag 8	Lag 4 Lag 9	Lag 5 Lag 10
	Trading Decile Number 5		(size) : autocorrelation : significance		
L-T-A	(268) 0.059	(179) 0.033	(181) -0.030	(186) -0.027	(196) 0.008
	(185) 0.009	(178) 0.045	(177) -0.117	(178) 0.076	(181) -0.175*
SALLIES	(399) 0.133**	(322) 0.028	(327) 0.059	(321) 0.005	(315) -0.044
	(315) 0.085	(320) -0.062	(315) -0.013	(309) 0.000	(309) -0.077
ANAMINT	(364) 0.070	(285) 0.027	(266) -0.040	(255) 0.038	(246) -0.157*
	(242) 0.133*	(245) 0.062	(257) -0.022	(254) -0.036	(245) 0.039
TIB	(270) 0.062	(193) -0.065	(189) -0.017	(181) -0.016	(174) -0.096
	(171) 0.080	(187) -0.042	(192) 0.016	(188) -0.009	(190) 0.019
AFROX	(259) 0.170**	(165) -0.028	(161) -0.035	(164) 0.055	(152) -0.012
	(157) -0.195*	(157) -0.041	(152) 0.032	(165) 0.000	(167) -0.087
CNAGALO	(325) 0.134*	(241) 0.032	(227) 0.071	(218) -0.057	(224) 0.150*
	(230) 0.049	(219) 0.125	(222) -0.007	(219) -0.013	(206) 0.032
SA-BIAS	(331) -0.143**	(244) 0.062	(236) -0.154*	(230) 0.120	(232) -0.130*
	(232) -0.070	(231) 0.000	(228) -0.063	(219) 0.101	(225) 0.008
FARM-AG	(374) 0.018	(301) -0.096	(296) 0.009	(279) -0.002	(276) -0.048
	(288) 0.073	(290) 0.064	(283) 0.036	(275) 0.017	(269) -0.077
GRESHAM	(364) 0.049	(291) -0.101	(292) -0.023	(285) -0.044	(282) -0.116
	(283) 0.044	(283) 0.044	(284) -0.026	(283) 0.018	(275) -0.052
BUFFELS	(272) -0.038	(190) -0.178*	(187) 0.017	(184) -0.016	(179) -0.043
	(171) 0.095	(162) -0.039	(169) 0.092	(177) 0.002	(187) -0.054
GOLDSTEIN	(292) -0.064	(209) 0.004	(208) -0.015	(198) 0.166*	(188) -0.252**
	(192) -0.057	(199) -0.129	(206) 0.009	(208) -0.009	(201) 0.056
GF-PROPS	(240) -0.129*	(158) 0.003	(170) 0.084	(169) -0.043	(174) 0.023
	(162) -0.003	(153) -0.121	(153) -0.146	(153) 0.048	(145) 0.014
SEARDEL	(253) -0.050	(171) 0.076	(167) 0.040	(166) 0.030	(167) -0.007
	(171) -0.020	(159) 0.035	(157) 0.018	(159) -0.042	(156) -0.037
CHARTER	(536) 0.046	(472) -0.075	(463) -0.024	(461) 0.040	(468) 0.037
	(472) 0.025	(464) -0.023	(456) 0.012	(459) 0.028	(466) 0.032
CONCOR	(249) -0.142*	(165) -0.056	(152) -0.027	(150) -0.206*	(141) 0.084
	(142) -0.046	(142) -0.192*	(145) -0.016	(140) 0.034	(133) -0.065
HOLDAIN	(231) 0.213**	(148) 0.166*	(154) 0.084	(158) 0.137	(159) -0.082
	(162) 0.096	(159) 0.020	(146) -0.026	(129) -0.022	(132) -0.095
F-S-I	(380) 0.141**	(302) 0.125*	(291) 0.039	(280) 0.070	(272) 0.010
	(271) -0.078	(260) 0.011	(257) 0.033	(254) 0.037	(249) -0.028
SANLAND	(248) -0.226**	(166) -0.013	(161) 0.024	(162) 0.031	(168) 0.035
	(157) -0.082	(159) 0.004	(152) -0.035	(138) 0.091	(147) -0.157
PPC	(163) 0.031	(86) 0.242*	(101) 0.219*	(105) -0.048	(93) 0.077
	(100) 0.045	(91) -0.060	(86) 0.123	(89) 0.027	(99) 0.075
I-G-I	(281) 0.138*	(209) 0.132	(206) -0.142*	(196) -0.027	(196) -0.047
	(193) -0.078	(185) -0.026	(179) -0.006	(178) -0.163*	(166) 0.040
ALLWEAR	(326) -0.203**	(241) -0.016	(231) -0.115	(233) -0.071	(245) 0.028
	(244) -0.119	(232) 0.089	(233) 0.042	(247) -0.038	(241) 0.005
O-K	(236) 0.045	(167) -0.038	(166) 0.064	(165) 0.163*	(172) 0.101
	(169) -0.059	(156) -0.039	(156) -0.049	(153) -0.238**	(148) -0.057
ALTRON	(187) 0.164*	(123) 0.190*	(135) -0.045	(128) 0.089	(135) 0.137
	(138) 0.130	(123) 0.071	(113) -0.118	(113) -0.089	(107) -0.094
EUREKA	(450) -0.072	(383) -0.053	(383) 0.015	(379) -0.020	(376) -0.003
	(377) 0.013	(383) -0.025	(389) 0.190**	(378) -0.091	(369) -0.008

Appendix 4.1 (continued) Autocorrelation structure of daily security returns over the period December 28, 1987 to March 20, 1992

Name	Lag 1 / Lag 6	Lag 2 / Lag 7	Lag 3 / Lag 8	Lag 4 / Lag 9	Lag 5 / Lag 10
	Trading Decile Number 6		(size) : autocorrelation : significance		
ICS	(267) 0.084	(184) 0.146*	(180) 0.027	(189) 0.050	(184) 0.039
	(174) 0.075	(162) 0.080	(158) 0.051	(145) -0.044	(150) -0.069
EGOLI	(724) 0.041	(675) 0.009	(670) -0.049	(659) -0.070	(659) -0.014
	(660) -0.065	(665) 0.001	(660) 0.026	(655) -0.043	(652) 0.020
BOUMAT	(189) 0.044	(121) 0.085	(110) 0.001	(108) 0.035	(108) 0.048
	(97) 0.046	(105) -0.090	(113) -0.018	(107) -0.255**	(102) 0.096
REUNERT	(171) 0.262**	(95) -0.042	(93) -0.021	(95) 0.200*	(88) 0.122
	(85) -0.204	(90) -0.226*	(89) -0.026	(91) 0.080	(90) 0.089
PEPKOR	(214) -0.088	(146) 0.133	(133) -0.082	(128) -0.002	(126) -0.038
	(125) -0.053	(129) 0.030	(140) -0.020	(133) -0.059	(126) -0.004
ANG-ALPHA	(228) 0.087	(147) 0.108	(136) 0.060	(129) -0.167	(130) 0.091
	(121) 0.023	(123) 0.046	(127) 0.045	(128) -0.002	(132) 0.103
TEDELEX	(235) 0.053	(161) 0.051	(158) 0.055	(149) -0.065	(134) -0.111
	(138) 0.024	(149) 0.025	(149) 0.027	(142) 0.053	(140) -0.066
BLYVOOR	(226) 0.081	(162) 0.067	(153) -0.009	(149) -0.078	(147) 0.006
	(151) -0.199*	(155) 0.165*	(150) -0.010	(144) 0.089	(140) 0.001
CULLINAN	(167) 0.170*	(106) 0.120	(109) 0.192*	(98) 0.118	(99) -0.097
	(93) 0.112	(90) -0.002	(93) 0.060	(97) -0.048	(98) 0.054
ALTECH	(244) -0.053	(164) -0.030	(163) -0.089	(153) 0.017	(152) -0.005
	(159) 0.084	(164) 0.098	(160) -0.010	(163) -0.001	(168) 0.102
ST-HELENA	(216) 0.189**	(144) 0.044	(139) 0.031	(144) -0.058	(149) -0.005
	(152) -0.015	(146) 0.105	(136) 0.123	(136) 0.030	(127) -0.070
LONFIN	(307) 0.030	(249) -0.056	(234) 0.056	(227) -0.233**	(221) 0.041
	(218) -0.057	(216) 0.047	(213) 0.038	(215) 0.064	(214) 0.055
SAFICON	(154) 0.247**	(98) 0.106	(89) 0.132	(75) 0.034	(75) 0.003
	(82) 0.063	(87) 0.106	(79) -0.088	(69) 0.094	(60) 0.096
DELTA	(153) -0.082	(89) -0.039	(90) 0.145	(91) 0.045	(99) 0.002
	(82) 0.034	(73) 0.166	(76) 0.134	(86) -0.150	(88) 0.078
STILFTN	(152) -0.076	(89) -0.358**	(90) 0.268**	(88) -0.179	(88) -0.043
	(87) -0.038	(79) 0.029	(70) 0.033	(86) -0.001	(84) -0.024
FENIX	(374) 0.134**	(297) -0.052	(306) -0.066	(300) -0.138*	(291) -0.055
	(290) 0.042	(288) 0.013	(277) 0.051	(264) -0.040	(257) 0.012
PICHOLD	(178) 0.031	(105) 0.022	(98) 0.009	(91) 0.010	(99) 0.028
	(102) 0.099	(92) -0.012	(98) 0.068	(97) 0.049	(96) -0.006
ANGLOVAAL	(116) 0.126	(66) 0.200	(66) -0.115	(71) 0.109	(62) 0.027
	(52) 0.143	(56) -0.054	(66) -0.141	(63) 0.261*	(53) -0.090
TGH	(305) -0.144*	(225) 0.091	(214) 0.001	(207) -0.171*	(200) -0.043
	(201) 0.015	(206) -0.022	(207) -0.214**	(205) 0.054	(205) -0.088
ROMATEX	(171) 0.084	(97) 0.024	(90) 0.165	(94) -0.103	(104) 0.093
	(106) -0.042	(96) -0.111	(97) 0.019	(100) 0.021	(104) 0.000
VENTRON	(163) 0.263**	(96) 0.146	(89) -0.001	(87) -0.099	(85) -0.109
	(88) 0.155	(78) -0.011	(76) -0.167	(76) -0.085	(78) -0.015
CADSWEP	(153) 0.239**	(100) 0.110	(98) 0.136	(92) 0.261*	(87) 0.078
	(85) 0.048	(76) 0.023	(73) 0.005	(70) -0.054	(72) -0.033
BLUE-CIRC	(146) 0.000	(89) 0.177	(90) 0.187	(86) 0.013	(77) -0.091
	(67) -0.111	(70) -0.028	(73) -0.080	(74) 0.047	(73) -0.188
CON-MURCH	(165) 0.247**	(104) 0.284**	(112) 0.160	(111) 0.018	(112) 0.001
	(110) -0.403**	(107) 0.030	(100) -0.041	(111) -0.072	(107) 0.053

Appendix 4.1 (continued) Autocorrelation structure of daily security returns over the period December 28, 1987 to March 20, 1992

Name	Lag 1 Lag 6	Lag 2 Lag 7	Lag 3 Lag 8	Lag 4 Lag 9	Lag 5 Lag 10
	Trading Decile Number 7		(size) : autocorrelation : significance		
RANDCOL	(112) 0.058	(64) 0.072	(56) -0.148	(49) -0.097	(52) 0.191
	(51) 0.024	(60) -0.209	(59) -0.146	(55) 0.268*	(48) -0.042
CONSOL	(134) 0.123	(79) -0.097	(71) 0.041	(67) -0.012	(68) 0.077
	(67) 0.123	(63) -0.017	(61) 0.002	(63) 0.001	(65) 0.009
GRINAKER	(265) 0.127*	(187) 0.116	(183) 0.057	(185) 0.091	(182) 0.109
	(167) 0.047	(167) 0.154*	(174) 0.016	(183) 0.123	(178) 0.017
VIERFONTN	(185) 0.057	(136) 0.150	(141) 0.015	(146) 0.026	(132) -0.082
	(128) 0.126	(127) -0.128	(122) -0.063	(114) -0.024	(111) -0.076
MOBILE	(102) 0.208*	(55) 0.185	(66) -0.008	(62) 0.134	(55) 0.117
	(52) 0.153	(51) 0.014	(45) -0.073	(43) 0.044	(48) -0.100
FRALEX	(117) 0.001	(69) 0.448**	(64) -0.028	(57) 0.095	(63) 0.079
	(60) -0.145	(59) 0.084	(59) 0.087	(52) -0.175	(52) 0.014
CTP	(155) 0.141	(98) -0.120	(104) 0.021	(100) -0.054	(102) -0.086
	(94) 0.126	(98) -0.048	(102) -0.106	(101) -0.043	(102) -0.086
EDGARS	(96) 0.160	(53) 0.143	(40) -0.035	(38) 0.017	(35) -0.009
	(28) 0.276	(31) -0.002	(37) 0.358*	(34) 0.170	(26) -0.118
PICBEL	(139) 0.061	(76) -0.063	(68) 0.051	(72) 0.121	(69) -0.124
	(64) -0.071	(60) -0.024	(68) 0.085	(71) -0.136	(70) -0.100
E-R-P-M	(182) 0.007	(123) 0.171	(118) -0.003	(117) -0.077	(115) -0.082
	(117) -0.053	(106) -0.060	(105) 0.099	(110) -0.231*	(110) 0.171
LIONMATCH	(94) -0.091	(47) 0.002	(39) -0.044	(36) -0.030	(38) -0.321*
	(46) 0.039	(44) 0.078	(38) -0.130	(41) -0.021	(45) 0.040
M-&-F	(60) 0.040	(23) 0.211	(21) 0.444*	(17) 0.140	(23) 0.096
	(20) 0.029	(15) -0.021	(19) 0.005	(16) 0.108	(18) -0.196
TRENCOR	(65) -0.004	(32) 0.035	(29) 0.008	(27) 0.059	(28) 0.005
	(29) -0.088	(32) 0.369*	(29) 0.109	(28) -0.020	(25) 0.098
UTICO	(85) 0.003	(39) 0.263	(37) -0.047	(34) -0.063	(30) -0.080
	(29) -0.042	(40) -0.297	(45) 0.141	(34) 0.217	(34) 0.042
NAMFISH	(141) -0.302**	(93) -0.096	(93) -0.310**	(88) 0.103	(81) 0.204
	(75) -0.034	(70) -0.012	(79) 0.082	(77) -0.002	(73) 0.702**
PROGRESS	(76) -0.298**	(40) -0.001	(31) -0.081	(26) -0.067	(28) -0.512**
	(29) 0.179	(30) 0.020	(24) 0.074	(23) 0.033	(29) -0.002
CHEMSERVE	(90) -0.014	(53) 0.016	(50) -0.080	(47) -0.145	(43) -0.031
	(40) -0.066	(36) -0.049	(30) 0.279	(23) -0.041	(26) 0.383
AFCOL	(114) 0.092	(69) 0.218	(68) 0.041	(70) 0.033	(72) 0.198
	(71) -0.057	(72) -0.003	(66) 0.018	(63) -0.092	(60) -0.077
SUNCRUSH	(43) 0.065	(15) -0.039	(17) 0.254	(14) -0.126	(16) 0.025
	(19) -0.032	(21) -0.081	(26) -0.040	(22) -0.126	(17) 0.365
DUIKERS	(95) 0.303**	(47) 0.118	(50) 0.048	(45) -0.119	(43) -0.246
	(48) 0.350*	(43) -0.125	(41) 0.221	(41) 0.132	(40) -0.074
FOSCHINI	(66) 0.015	(30) 0.002	(31) 0.079	(30) -0.021	(36) -0.327*
	(34) 0.179	(30) 0.031	(26) -0.066	(24) 0.020	(26) 0.026
NEW-CENT	(125) 0.065	(71) 0.042	(70) -0.031	(65) -0.030	(53) -0.035
	(50) 0.063	(57) -0.003	(59) 0.211	(57) 0.173	(58) 0.033
ED-LBATE	(59) 0.183	(24) 0.004	(26) 0.185	(26) -0.216	(21) -0.135
	(18) -0.042	(21) 0.277	(28) 0.201	(21) -0.179	(24) 0.191
NICTUS	(75) -0.283*	(41) 0.175	(41) 0.034	(40) 0.010	(37) 0.176
	(32) 0.072	(38) -0.365*	(41) -0.071	(41) 0.032	(37) 0.042

Appendix 4.1 (continued) Autocorrelation structure of daily security returns over the period December 28, 1987 to March 20, 1992

Name	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
	Lag 6	Lag 7	Lag 8	Lag 9	Lag 10
	Trading Decile Number 8		(size) : autocorrelation : significance		
ARGUS	(44) 0.190	(17) 0.306	(18) 0.497*	(27) -0.252	(24) 0.077
	(16) 0.267	(15) 0.052	(17) 0.095	(16) 0.288	(19) 0.222
BUFFCOR	(94) 0.097	(57) 0.019	(60) 0.078	(55) 0.069	(50) -0.199
	(44) 0.005	(37) -0.002	(33) -0.028	(30) -0.042	(32) -0.022
SABLE	(24) 0.525**	(6) -0.814*	(10) 0.035	(8) 0.493	(14) -0.078
	(10) -0.003	(15) 0.332	(10) -0.007	(6) -0.443	(6) -0.015
ELLERINE	(94) -0.138	(45) 0.261	(48) -0.239	(47) 0.304*	(45) 0.016
	(48) 0.000	(47) 0.012	(44) -0.026	(34) -0.059	(35) -0.126
BESTER	(81) -0.257*	(49) 0.062	(50) 0.336*	(45) 0.129	(46) -0.024
	(44) -0.151	(46) 0.179	(40) 0.103	(35) -0.274	(35) 0.441**
AFR-LEASE	(83) -0.167	(43) 0.018	(39) -0.128	(44) -0.045	(43) -0.129
	(38) -0.065	(35) 0.113	(31) -0.206	(33) -0.074	(34) 0.060
T-M-L	(62) 0.162	(30) -0.113	(29) 0.146	(24) 0.398	(23) 0.187
	(21) -0.140	(12) -0.160	(11) 0.155	(15) 0.090	(19) 0.093
MASONITE	(67) 0.251*	(29) -0.069	(29) -0.235	(33) -0.116	(30) -0.317
	(31) -0.135	(31) 0.461**	(27) 0.327	(21) -0.012	(25) 0.389
OCFISH	(49) -0.081	(27) 0.017	(22) 0.233	(21) -0.218	(12) 0.011
	(16) 0.247	(16) -0.015	(11) 0.021	(12) -0.003	(10) -0.002
CARLCOR	(51) 0.163	(19) 0.228	(15) 0.139	(13) -0.054	(16) -0.101
	(16) -0.120	(18) 0.052	(21) -0.081	(19) -0.073	(15) 0.188
BOLTONS	(75) 0.070	(37) 0.279	(36) 0.011	(31) 0.004	(31) -0.043
	(28) 0.086	(27) -0.475*	(29) 0.148	(35) 0.112	(31) -0.106
PICAPLI	(49) -0.004	(24) -0.032	(23) -0.005	(21) -0.033	(18) 0.074
	(16) 0.001	(13) -0.297	(12) -0.051	(16) -0.229	(11) 0.018
EVERITE	(58) 0.094	(28) 0.010	(27) 0.038	(22) -0.079	(20) -0.171
	(24) -0.207	(28) -0.084	(23) 0.066	(28) 0.008	(26) -0.339
TOYOTA	(79) 0.268*	(42) 0.125	(39) 0.875**	(34) 0.482**	(36) 0.350*
	(37) 0.112	(38) 0.407**	(38) 0.169	(35) -0.177	(37) 0.059
WIT-NIGEL	(167) 0.024	(117) -0.022	(111) -0.022	(105) -0.231*	(106) -0.080
	(108) -0.016	(105) -0.040	(100) 0.026	(99) 0.085	(92) 0.061
DBN-DEEP	(393) 0.305**	(321) 0.291**	(312) 0.169**	(294) 0.240**	(285) 0.125*
	(282) 0.117*	(284) 0.178**	(278) 0.088	(281) 0.013	(288) 0.011
GENTYRE-A	(72) 0.146	(38) -0.290	(33) -0.117	(23) 0.217	(26) 0.255
	(29) -0.219	(37) -0.159	(36) 0.032	(24) -0.125	(21) 0.023
ABERDARE	(37) 0.173	(12) -0.002	(12) -0.037	(13) 0.767**	(20) 0.406
	(21) -0.038	(21) -0.043	(14) -0.305	(12) -0.074	(13) -0.046
AF-CABLE	(49) -0.109	(17) 0.011	(16) 0.286	(20) -0.403	(22) 0.031
	(15) -0.146	(22) -0.181	(30) -0.036	(22) -0.013	(21) -0.683**
METAIR	(20) -0.713**		(7) -0.067		
		(10) 0.300			(6) -0.465
BERZACK	(91) 0.345**	(55) 0.415**	(54) 0.278*	(52) 0.154	(52) 0.112
	(45) 0.266	(48) -0.018	(52) -0.024	(49) 0.081	(47) 0.231
COR-SYND	(38) 0.318*	(16) 0.204	(19) 0.010	(19) -0.146	(18) -0.164
	(14) 0.000	(19) -0.002	(22) 0.122	(21) 0.181	(20) 0.189
BIDVEST	(77) 0.266*	(45) 0.057	(45) 0.072	(43) 0.171	(40) 0.056
	(39) -0.091	(40) 0.020	(43) -0.135	(45) -0.145	(44) 0.018
ROOIBERG	(68) 0.511**	(30) 0.213	(27) 0.224	(27) 0.160	(32) 0.291
	(31) 0.024	(28) 0.024	(28) -0.027	(29) -0.009	(31) 0.070

Appendix 4.1 (continued) Autocorrelation structure of daily security returns over the period December 28, 1987 to March 20, 1992

Name	Lag 1 Lag 6	Lag 2 Lag 7	Lag 3 Lag 8	Lag 4 Lag 9	Lag 5 Lag 10
	Trading Decile Number 9		(size) : autocorrelation : significance		
BROADACRE	(70) 0.003	(37) -0.042	(36) 0.120	(37) -0.070	(34) -0.184
	(38) 0.073	(38) -0.246	(33) -0.101	(28) -0.056	(29) -0.094
COM-FUND	(22) -0.009	(10) 0.100			(6) 0.026
ADONIS	(17) -0.135			(6) -0.006	
BIVEC	(92) 0.369**	(58) 0.263*	(54) 0.316*	(55) 0.390**	(56) 0.320*
	(58) 0.386**	(54) 0.029	(57) -0.170	(57) -0.096	(50) -0.103
HARWILL	(102) -0.242*	(63) -0.013	(61) 0.051	(65) 0.228	(66) -0.085
	(56) 0.041	(49) 0.215	(51) -0.314*	(59) -0.008	(53) -0.022
UNION	(108) 0.094	(66) 0.173	(63) 0.020	(66) -0.050	(62) 0.042
	(57) -0.190	(58) -0.504**	(57) -0.047	(64) -0.026	(64) 0.038
Z-C-I	(50) 0.009	(27) 0.328	(23) 0.050	(24) -0.089	(20) -0.304
	(18) 0.308	(17) 0.165	(12) -0.551	(14) 0.043	(15) 0.060
CROOKES	(15) 0.115		(7) 0.060	(6) -0.041	
N-KLEINS	(15) 0.305	(6) 0.004			
MICOR	(18) 0.695**	(6) 0.267	(9) -0.059	(10) 0.167	(8) -0.459
	(7) -0.141	(6) 0.410			
GARDIAN					
GYPSUM	(7) -0.180		(6) -0.085	(6) -0.156	
G-I-C	(34) 0.017	(11) -0.049	(6) -0.037	(7) 0.013	(10) 0.020
	(6) 0.014	(10) -0.007	(9) 0.023	(9) 0.006	(9) 0.041
ADCOCK	(25) 0.144	(9) -0.004		(9) 0.001	(11) 0.006
	(9) -0.001	(9) 0.003	(10) -0.003	(9) -0.005	(11) -0.003
RENTBEL	(32) -0.110	(13) -0.951**	(12) -0.853**	(11) 0.209	(16) 0.117
	(11) -0.799**	(10) 0.376	(8) -0.037	(8) -0.129	
GUBINGS	(18) 0.243	(7) -0.050			
CEMENCO	(21) 0.344				
	(6) 0.065	(6) -0.692			
WESCO	(23) 0.021	(10) -0.061	(8) 0.038	(6) 0.023	(7) -0.008
	(7) -0.102	(8) 0.129	(6) 0.137	(7) 0.239	(8) -0.088
SAKERS	(15) -0.364	(6) -0.003			
	(6) -0.101	(6) -0.051			
WANKIE	(6) -0.028				
PORTHLD	(13) 0.427				
T-E-J	(14) 0.379			(9) -0.052	(10) -0.045
OCEANA	(49) -0.109	(27) -0.165	(27) 0.373	(30) -0.198	(31) 0.391*
	(29) -0.422*	(25) -0.108	(24) 0.260	(24) 0.082	(25) 0.010
HORTORS	(15) -0.844**	(6) 0.665			

Appendix 4.1 (continued) Autocorrelation structure of daily security returns over the period December 28, 1987 to March 20, 1992

Name	Lag 1 Lag 6	Lag 2 Lag 7	Lag 3 Lag 8	Lag 4 Lag 9	Lag 5 Lag 10
	Trading Decile Number 10		(size) : autocorrelation : significance		
PROSURE	(9) 0.320				
MATH-ASH	(21) -0.292	(7) -0.098	(7) -0.232		(6) -0.997**
	(10) -0.062	(8) -0.044			
PORT					
REX-TRUE					
NINIAN	(14) 0.328	(6) 0.234			
AF-OVER-A	(7) 0.000				
BOTREST	(43) -0.126	(13) -0.436	(21) 0.544*	(25) -0.132	(19) -0.208
	(14) 0.437	(16) -0.221	(18) -0.742**	(17) 0.159	(15) 0.323
AF-&-OVER					
COATES					
TWEEFONTN					
ASSORE					
ICH	(145) 0.007	(108) -0.043	(104) 0.000	(107) -0.211*	(105) -0.019
	(93) -0.093	(89) -0.177	(91) 0.012	(98) -0.070	(98) 0.041
ASS-MANG					
YORKCOR	(16) 0.960**	(7) -0.262			
BOYMANS	(13) 0.362	(6) 0.354	(9) 0.249	(11) 0.092	(8) 0.146
FALCON					
METJE-&-Z	(6) 0.165				
INTRUST					
CAFCA					
TEX-MILLS					
UNI-COLD					
RAND-LON	(466) -0.068	(404) -0.001	(407) 0.001	(406) -0.078	(412) -0.053
	(408) -0.041	(404) 0.014	(408) 0.010	(408) 0.000	(410) -0.017
EUROPA	(19) 0.037	(10) 0.086	(10) 0.010	(7) -0.184	(9) 0.940**
	(8) 0.472	(8) 0.096	(7) 0.072	(6) -0.019	
BURLINGTN					

** : significant att the 1% level * : significant at the 5% level

Appendix 5.1 An overview of Principal Components Analysis and Principal Factor Analysis and their relevance to the Arbitrage Pricing Theory[1]

Principal components analysis was initially developed by Hotelling (1933) as an extension of the empirical methodology proposed by Pearson (1901) for the reduction of data so as to extract *a maximum amount of variance* (Harman, 1976:14). Given a set of n observed variables, the methodology involves determining a new set of n uncorrelated variables (components) that, in the order in which they are extracted, account for the maximum amount of variance of the observed variables. Written mathematically, if the set of observed variables are given as $\{\tilde{r}_1;\tilde{r}_2;\tilde{r}_3;.....\tilde{r}_n\}$, then;

$$\tilde{r}_i = \alpha_{i,1}\tilde{C}_1 + \alpha_{i,2}\tilde{C}_2 + \alpha_{i,3}\tilde{C}_3 + ...\alpha_{i,n}\tilde{C}_n \quad (i = 1,2,3,....n)$$

where $\tilde{C}_j$ is the j[th] principal component, uncorrelated with all the other principal components, that explains the maximum amount of residual variance in the n observed variables after the first j-1 components have been extracted.

Clearly, given that each successive components account for less of the variance in the observed variables, if they explain a large enough proportion of the total variance, only a few components may be retained. *However, all the components are required to reproduce the correlations among the variables* (Harman, 1976:15).

In contrast to the principal components procedure, principal factor analysis is designed to meet the objective of finding a set of uncorrelated variables (factors) that best reproduce the correlations between the observed variables. Written mathematically, for the set of observed variables $\{\tilde{r}_1;\tilde{r}_2;\tilde{r}_3;.....\tilde{r}_n\}$;

$$\tilde{r}_i = \alpha_{i,1}\tilde{F}_1 + \alpha_{i,2}\tilde{F}_2 + \alpha_{i,3}\tilde{F}_3 + ...\alpha_{i,k}\tilde{F}_k + \beta_i\tilde{U}_i \quad (i = 1,2,3,....n)$$

where; $\tilde{F}_j$ is the j[th] principal factor, uncorrelated with all the other factors, that explains the maximum amount of residual correlation in the n observed variables after the first j-1 factors have been extracted; and, $\tilde{U}_i$ is a unique factor that accounts for the residual

[1] While the appendix refers exclusively to principal factor analysis, the comments as presented here apply equally to all forms of factor analysis. The subtle distinctions between the different types of factor analytic procedures are not discussed. As such, if not for the desire to remain consistent with the body of the thesis, the term "traditional (classical) factor analysis" would be more appropriate in this discussion.

variance of the observed i^{th} variable uncorrelated with all the other variables. Usually k is found to be substantially less than n because the technique is appropriately employed when there is reason to believe that the correlations between a large set of variables are due to a significantly smaller set of underlying constructs.

For both principal components analysis and principal factor analysis the $\alpha_{i,j}$'s are referred to as component or factor loadings.

In terms of the Arbitrage Pricing Theory, principal components analysis allows for the possibility that, after the removal of the common priced factors, there may be other non-priced factors common to subsets of securities. For instance, a group of securities from the same sector of the economy may be influenced by a common sector specific economic risk. However, if through intelligent diversification across sectors it is possible to diversify away this risk, investors cannot expect to earn any additional return for bearing the risk. Written mathematically therefore, if the factor generating model is of the form;

$$\tilde{r}_i = E(\tilde{r}_i) + \sum_{j=1}^{k+m} \beta_{i,j}\,\tilde{\delta}_j + \tilde{\varepsilon}_i \tag{1}$$

where only the first k factors are priced, then it can be shown that;

$$E(\tilde{r}_i) = \lambda_0 + \sum_{j=1}^{k} \beta_{i,j}\,\lambda_j. \tag{2}$$

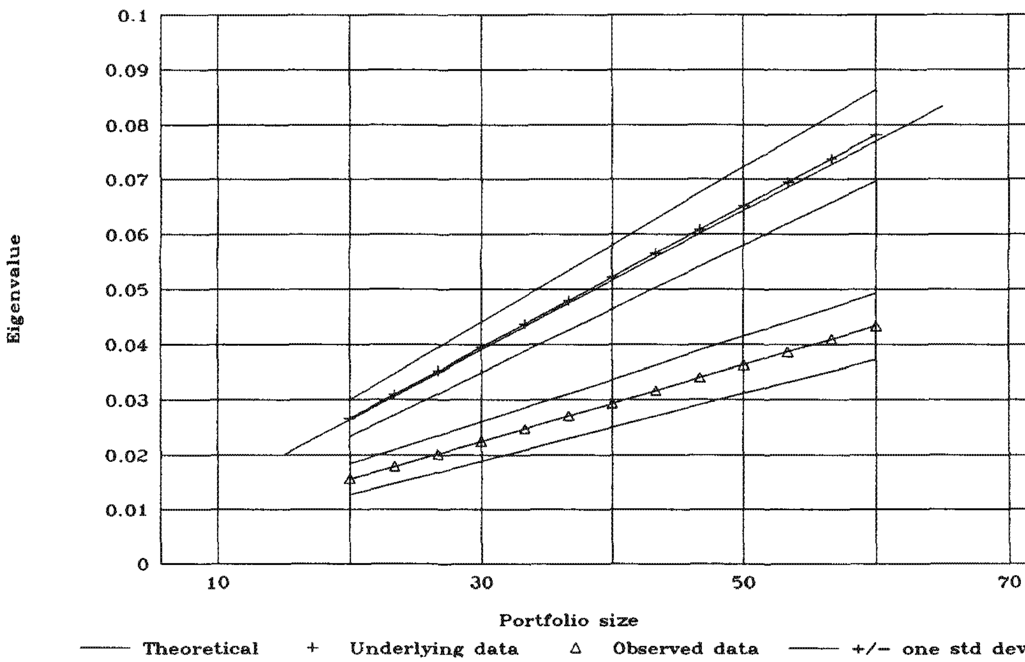
Substituting equation (2) into equation (1) yields;

$$\tilde{r}_i = \lambda_0 + \sum_{j=1}^{k} \beta_{i,j}\left(\lambda_j + \tilde{\delta}_j\right) + \sum_{j=k+1}^{k+m} \beta_{i,j}\,\tilde{\delta}_j + \tilde{\varepsilon}_i \tag{3}$$

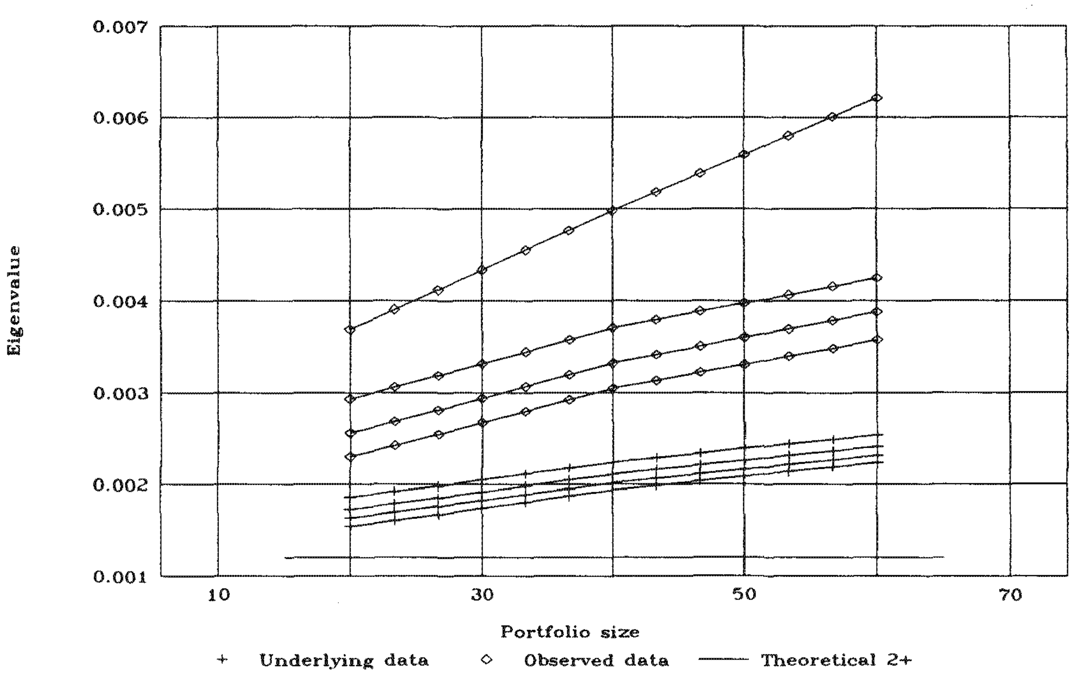
where; λ_0 is the risk-free rate of return, or return on a zero-beta portfolio; λ_j is the risk premium attached to the j^{th} priced factor given $j \in \{1,2,3,...k\}$; and, all the subsequent common factors are unpriced, ie. $\lambda_j = 0\ \forall\ j \in \{k+1, k+2, ...k+m\}$.

Principal factor analysis on the other hand, assumes that the common (priced) factors account for all the correlation between securities. Once the they have been extracted and the correlation between securities explained, the residual risk is presumed to be unique to each of the securities. This procedure is therefore more consistent with the Arbitrage Pricing Theory as developed by Ross (1976) and as described and tested by Roll and Ross (1980).

Appendix 5.2 Eigenvalues as a function of portfolio size:
Single factor economy - Average variance explained of 50.2%

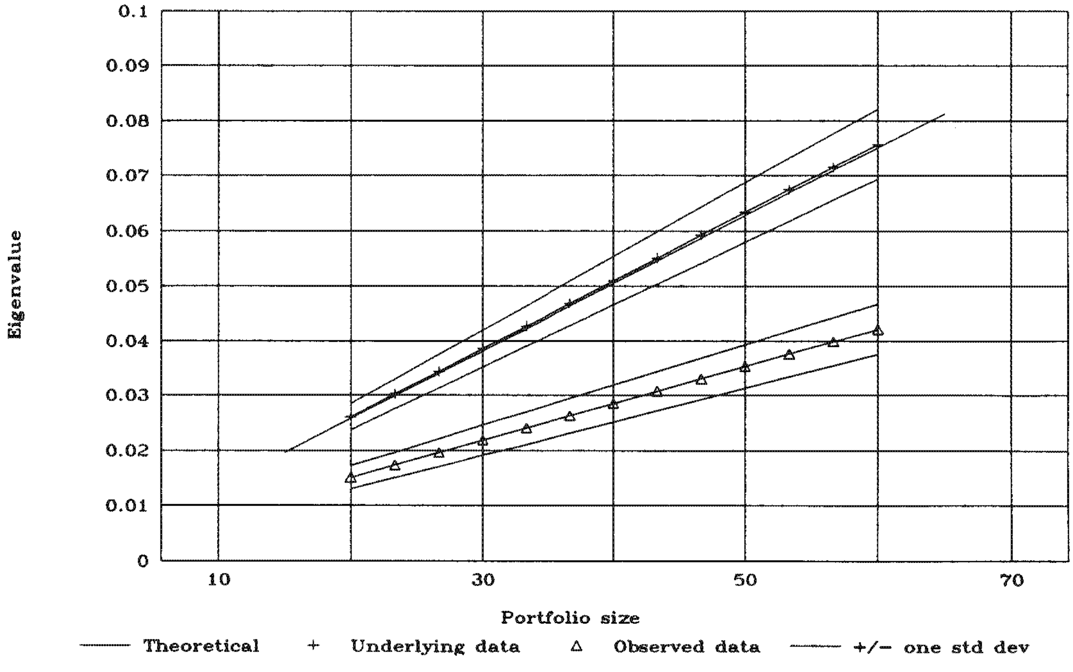


(a) Largest eigenvalue

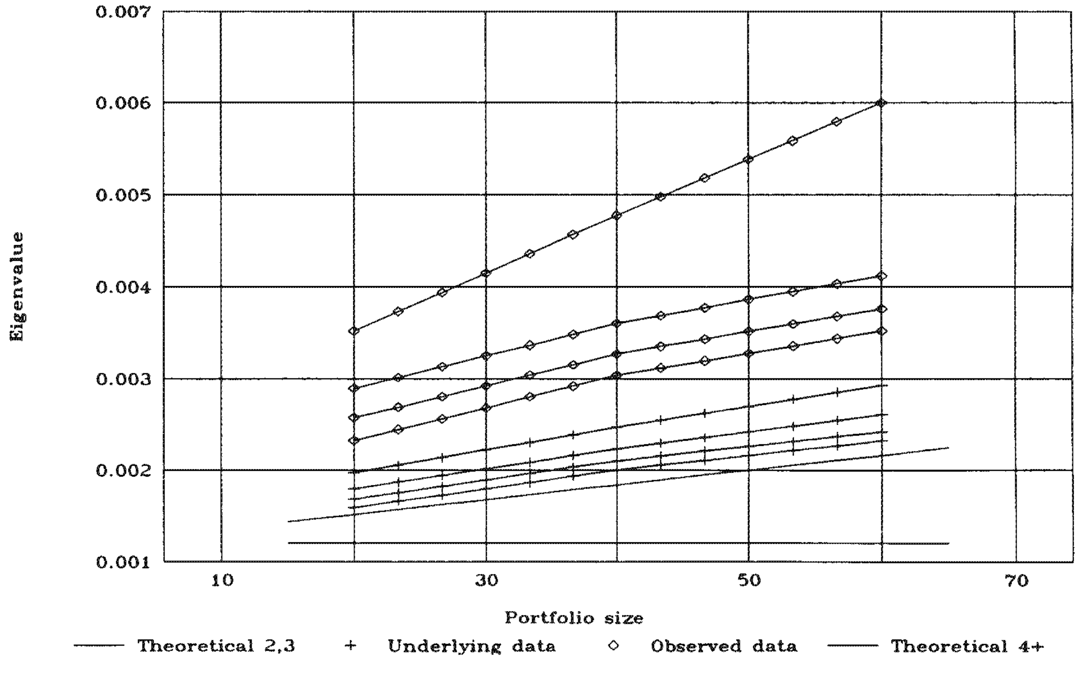


(b) Eigenvalues two to five

Appendix 5.3 Eigenvalues as a function of portfolio size:
Three factor economy - Average variance explained of 50.2%

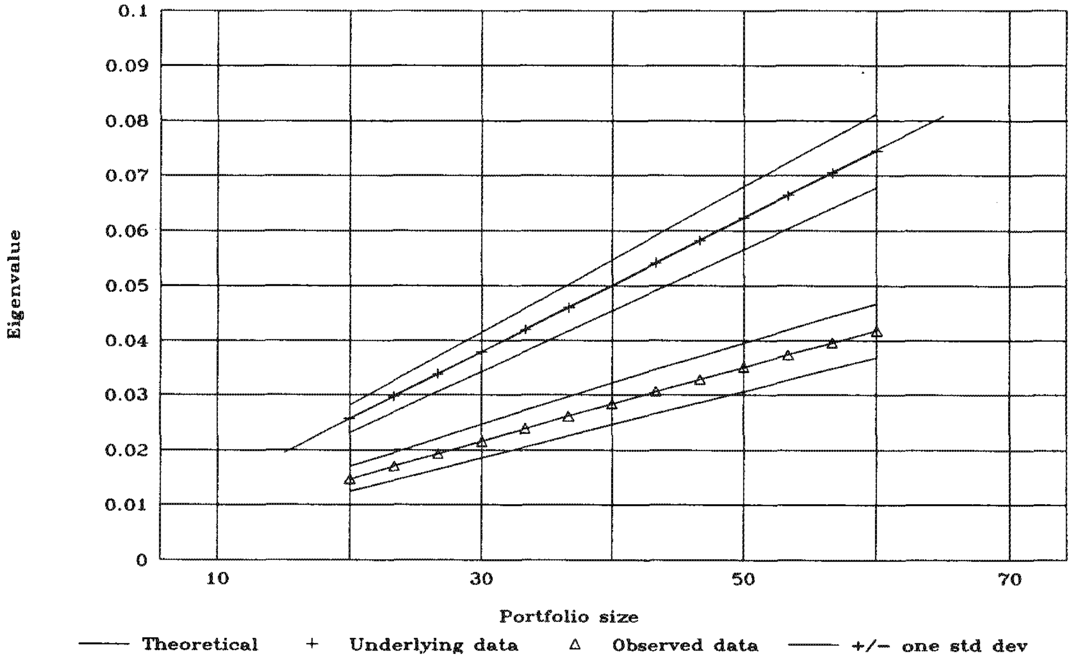


(a) Largest eigenvalue

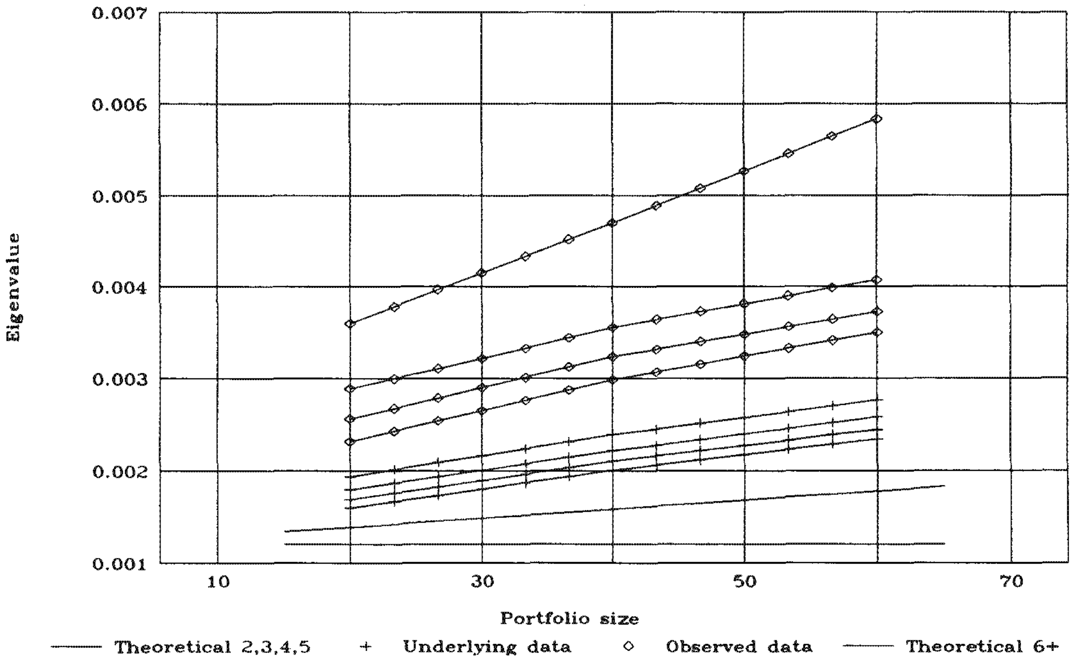


(b) Eigenvalues two to five

Appendix 5.4 Eigenvalues as a function of portfolio size: Five factor economy - Average variance explained of 50.2%



(a) Largest eigenvalue



(b) Eigenvalues two to five

Appendix 5.5 Cross-sectional distribution of the χ^2 statistic when using principal components analysis on returns simulated to have an average communality of 50.2% : Underlying returns data

Economy	m	n	Probability that the remaining eigenvalues are not significantly different after the removal of the first "n" 0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0
Securities simulated to have normally distributed idiocyncratic risk components												
7	20	1	8.0	9.3	8.7	18.0	10.7	10.0	10.7	8.7	8.0	8.0
	60		8.0	8.0	10.7	7.3	8.0	12.7	8.7	10.7	14.7	11.3
	20	3	49.3	15.3	12.7	9.3	4.7	2.7	2.7	2.0	0.7	0.7
	60		36.0	16.7	12.0	10.0	6.0	10.7	5.3	1.3	2.0	0.0
	20	5	73.3	10.0	10.0	3.3	0.7	1.3	0.7	0.7	0.0	0.0
	60		66.0	16.0	8.7	6.0	2.7	0.7	0.0	0.0	0.0	0.0
8	20	1	0.7	4.7	5.3	9.3	8.0	9.3	7.3	6.0	15.3	34.0
	60		0.7	1.3	0.7	2.0	4.0	3.3	5.3	8.0	17.3	57.3
	20	3	28.7	18.7	10.0	7.3	9.3	5.3	7.3	8.0	3.3	2.0
	60		21.3	10.0	13.3	10.0	11.3	12.0	5.3	6.0	3.3	7.3
	20	5	53.3	17.3	13.3	3.3	5.3	2.7	2.7	0.7	1.3	0.0
	60		54.7	16.7	12.0	3.3	3.3	3.3	1.3	3.3	1.3	0.7
9	20	1	4.7	6.7	5.3	8.0	6.7	10.7	11.3	14.0	14.0	18.7
	60		0.7	1.3	2.7	3.3	2.7	5.3	6.0	8.7	14.0	55.3
	20	3	29.3	21.3	10.7	12.0	5.3	6.7	5.3	4.7	3.3	1.3
	60		14.7	8.7	10.7	11.3	11.3	10.0	6.7	9.3	9.3	8.0
	20	5	60.7	14.0	9.3	2.7	6.7	2.0	2.7	0.7	0.0	1.3
	60		44.7	17.3	13.3	5.3	7.3	3.3	1.3	5.3	2.0	0.0
Securities simulated to have non-normally distributed idiocyncratic risk components												
10	20	1	5.3	5.3	6.7	7.3	10.0	11.3	10.7	10.0	11.3	22.0
	60		4.0	4.7	5.3	6.7	10.7	10.7	7.3	14.0	14.7	22.0
	20	3	24.7	18.7	14.7	11.3	4.7	6.7	4.7	7.3	6.0	1.3
	60		24.0	18.0	13.3	4.7	14.7	6.7	6.0	3.3	5.3	4.0
	20	5	50.0	16.7	10.7	5.3	6.7	2.0	3.3	4.7	0.7	0.0
	60		56.0	14.7	10.7	6.7	2.0	4.0	2.7	1.3	1.3	0.7
11	20	1	2.7	2.7	3.3	4.0	6.0	6.0	5.3	12.7	17.3	40.0
	60		0.0	0.0	0.7	0.7	2.0	4.0	2.7	5.3	14.0	70.7
	20	3	22.7	11.3	11.3	9.3	8.7	11.3	6.7	8.0	4.7	6.0
	60		11.3	8.0	12.7	8.0	12.0	9.3	10.7	7.3	10.7	10.0
	20	5	42.7	14.0	12.0	10.7	6.7	5.3	4.7	1.3	2.0	0.7
	60		40.0	20.7	10.0	9.3	4.7	7.3	2.7	1.3	2.7	1.3
12	20	1	3.3	4.7	2.7	5.3	8.0	8.7	9.3	17.3	14.0	26.7
	60		0.0	1.3	0.7	2.0	2.0	2.0	6.7	8.0	14.0	63.3
	20	3	20.7	18.0	18.0	11.3	8.0	6.0	5.3	6.7	3.3	2.7
	60		8.7	8.7	10.7	6.7	12.0	9.3	12.0	8.0	11.3	12.7
	20	5	52.7	16.0	11.3	8.7	2.7	4.0	2.0	1.3	0.7	0.7
	60		36.7	16.7	8.0	12.0	8.0	8.0	1.3	6.0	1.3	2.0

Appendix 5.6 Cross-sectional distribution of the χ^2 statistic when using principal components analysis on returns simulated to have an average communality of 50.2% : Observed returns data

Economy	m	n	Probability that the remaining eigenvalues are not significantly different after the removal of the first "n"									
			0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0
Securities simulated to have normally distributed idiocyncratic risk components												
7	20	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
	60		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
	20	3	0.0	0.0	0.7	0.0	0.0	1.3	0.0	3.3	1.3	93.3
	60		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
	20	5	2.7	1.3	2.7	2.0	2.7	2.7	5.3	8.7	10.0	62.0
	60		0.0	0.0	0.0	0.0	0.0	0.0	0.7	0.7	0.0	98.7
8	20	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
	60		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
	20	3	0.0	0.0	0.0	0.0	1.3	0.0	1.3	0.7	0.7	96.0
	60		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
	20	5	0.0	2.7	1.3	2.0	1.3	1.3	4.0	10.7	11.3	65.3
	60		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.7	99.3
9	20	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
	60		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
	20	3	0.0	0.0	0.0	0.0.	1.3	0.0	0.0	0.7	1.3	96.7
	60		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
	20	5	0.0	0.7	2.0	2.7	4.0	4.7	8.0	8.0	7.3	62.7
	60		0.0	0.0	0.0	0.0	0.0	0.0	0.7	0.0	0.0	99.3
Securities simulated to have non-normally distributed idiocyncratic risk components												
10	20	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
	60		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
	20	3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.3	0.7	98.0
	60		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
	20	5	0.7	0.0	1.3	3.3	2.0	3.3	7.3	5.3	10.7	66.0
	60		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.7	99.3
11	20	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
	60		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
	20	3	0.0	0.0	0.7	1.3	0.7	0.7	0.7	1.3	1.3	93.3
	60		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
	20	5	2.7	1.3	1.3	1.3	6.0	3.3	6.0	7.3	7.3	63.3
	60		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
12	20	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
	60		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
	20	3	0.0	0.0	0.0	0.0	0.0	0.7	0.7	2.0	0.7	96.0
	60		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0
	20	5	0.7	2.0	2.7	1.3	5.3	0.7	5.3	7.3	9.3	65.3
	60		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	100.0

Appendix 6.1 Securities selected for the investigation of the relationship between abnormal returns, firm size, and E/P ratio

No	Name	No	Name
1	Abercom Group	55	Farm-Ag
2	Aberdare Cables Africa	56	Federale Voedsel
3	Adcock Ingram	57	Federale Volksbeleggings
4	Adonis Knitwear	58	Form-Scaff Industries
5	AECI	59	Foschini
6	African Cables	60	Frasers
7	African Oxygen	61	General Tyre and Rubber Company
8	Alexander Lipworth	62	Goldfields Coal
9	Allied Electronics Corporation	63	Goldfields Industrial Corporation
10	Allied Technologies	64	Gresham Industries
11	Amalgamated Retail	65	Grinaker Holdings
12	Anglo American Coal Corporation	66	Group Five Engineering
13	Anglo American Industrial Corp.	67	Gubb and Inggs
14	Anglo American Investment Trust	68	Gypsum Industries
15	Anglovaal Industries	69	Haggie
16	Argus Holdings	70	Harwill Investments
17	Associated Engineering	71	Hiveld Steel and Vanadium Corp.
18	Associated Furniture Company	72	Hortors
19	Barlow Rand	73	Hunt, Leuchars and Hepburn Hld.
20	Berzack Brothers Holdings	74	Imperial Cold Storage
21	Bolton Industrial Holdings	75	Industrial and Commercial Holdings
22	Boymans	76	Irwin and Johnson
23	Brian Porter Holdings	77	Kanhym Investments
24	Broadacres Investments	78	Lion Match Company
25	BTR Dunlop	79	LTA
26	Buffalo Corporation	80	Lucem Holdings
27	Burlington Industries	81	M & S Spitz Footwear Holdings
28	Cadbury Schweppes SA	82	MacPhail Holdings
29	Carlto Paper Corporation	83	Malbak
30	Cementation Company (Africa)	84	Malhold
31	CG Smith	85	Masonite (Africa)
32	CG Smith Foods	86	Mathieson and Ashley Holdings
33	Chemical Services	87	McCarthy Group
34	Chubb Holdings	88	Messina
35	Claude Neon Lights (SA)	89	Metair Investments
36	Clicks Stores	90	Metal Closures Group (SA)
37	CNA Gallo	91	Metje and Ziegler
38	Coates Brothers	92	Metkor Investments
39	Concor	93	Micor Holdings
40	Consol	94	Namibia Fishing
41	Crookes Brothers	95	Nampak
42	CTP Holdings	96	National Bolts
43	Cullinan Holdings	97	Nictus Finansiele Instelling
44	Darling and Hodgson	98	Ninian and Lester Holdings
45	De Beers Consolidated Mines	99	Northern Engineering Ind. Africa
46	Delta Electrical Industries	100	Oceana Fishing Group
47	Dorbyl	101	OK Bazaars (1929)
48	Edgars Stores	102	Otis Elevator Company
49	Edward L. Bateman	103	Pepkor
50	Ellerine Holdings	104	Picardi Appliances
51	Engen	105	Picardi Beleggings
52	Ensign Clothing	106	Picardi Holdings
53	Eureka Industrial	107	Pick 'n Pay Stores
54	Everite	108	Plate Glass & Shatterprufe Ind.

Appendix 6.1 (continued) Securities selected for the investigation of the relationship between abnormal returns, firm size, and E/P ratio

No	Name	No	Name
109	Power Technologies	128	Television & Electrical Holdings
110	Premier Group	129	Tiger Oats
111	Pretoria Portland Cement	130	Times Media
112	Progress Industries	131	Tollgate Holdings
113	Rentmeesterbeleggings	132	Tongaat-Hulett Group
114	Rembrandt Group	133	Towles, Edgar Jacobs
115	Reunert	134	Toyota (SA)
116	Rex Trueform Clothing Company	135	Trans Natal Coal Corporation
117	Romatex	136	Trencor
118	S.M. Goldstein	137	Union Cold Storage of SA
119	Sappi	138	Union Wine
120	Sasol	139	Utico Holdings
121	Seardel Investment Corporation	140	Vierfontein Colliery
122	Sentrachem	141	W & A Investment Corporation
123	Sinclair Holdings	142	Waltons Stationary Company
124	South African Breweries	143	Witbank Colliery
125	Southern Sun Hotel Holdings	144	Wooltru
126	Sterns Diamond Organisation	145	York Timber Organisation
127	Suncrush		

Appendix 6.2 Unit Trusts selected for portfolio performance assessment

No	Type of Fund	Code	Unit Trust Name
1	General Equity Funds	GRD	Guardbank Growth Fund
2		MTF	Metboard Mutual Fund
3		MOM	Momentum Unit Trust Scheme
4		OMI	Old Mutual Investors Fund
5		SGE	Sage Fund
6		SNDX	Sanlam Index Trust
7		SNTR	Sanlam Trust
8		STD	Standard Bank Mutual Fund
9		SYG	Syfrets Growth Fund
10		UAL	UAL Unit Trust
11	Specialist Equity Funds	GRDR	Guardbank Resources Fund
12		OMMF	Old Mutual Mining Fund
13		SAGR	Sage Resources Fund
14		SNDV	Sanlam Dividend Trust
15		SNID	Sanlam Industrial Trust
16		SNMN	Sanlam Mining Trust
17		STDG	Standard Bank Gold Fund
18		UALM	UAL Mining and Resources Unit Trust
19		UALSO	UAL Selected Opportunities Unit Trust
20	High Income and Gilt Funds	GRDI	Guardbank Income Fund
21		CRB	Corbank Gilt Fund
22		SENG	Senbank Gilt Fund
23		SENY	Senbank High Yield Fund
24		STDI	Standard Bank Extra Income Fund
25		UALGT	UAL Gilt Fund

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