## APPENDIX: MATHEMATICAL DERIVATION OF THE MODEL

This model assumes that the cigarette market consists of only one price segment. In practical terms this means that the model is applicable to countries where there is not a large spread in cigarette prices.

The model consists of (1) an initial equilibrium, (2) a subsequent excise tax change, which changes the average price, overall consumption and the market distribution, and (3) a new equilibrium. The model considers the growth in a number of variables between the two equilibrium states. Subscript *1* refers to the baseline equilibrium, and subscript *2* refers to the new equilibrium.

The model focuses on short-term (one year) impacts of one-off changes in the excise tax. To keep the model tractable, it is assumed that cigarette excise tax and price changes do not increase illicit trade. Also, the model does not consider the impact of inflation, population or income growth, since these are typically longer-term effects.

The retail price (*P*) is broken into three components: (1) the excise tax (*ET*), (2) general sales tax (*ST*) (e.g. Value-added Tax), and (3) the industry price (*IP*). The industry price is a catch-all category that represents the revenue distributed among all players along the tobacco value chain, i.e. primary producers, manufacturers, importers, logistical companies, wholesalers and retailers.[1] At the outset

$$P_1 = IP_1 + ET_1 + ST_1 \qquad (1)$$

At the outset the existing excise tax burden (defined as $ET_1/P_1$) is assumed known. A sales tax of (100 $\tau$)% is levied on $(IP_1 + ET_1)$. Thus follows

$$P_1 = (IP_1 + ET_1)(1 + \tau) \qquad (2)$$

The industry price is obtained as

$$IP_1 = P_1/(1 + \tau) - ET_1 \qquad (3)$$

Total cigarette consumption at the outset is $Q_1$. Aggregate values are obtained as follows:

total expenditure by consumers: $P_1 \times Q_1$

total excise tax revenue: $ET_1 \times Q_1$ (4)

total industry revenue: $IP_1 \times Q_1$

If the excise tax (*ET*) is levied as a *specific* tax (i.e. a specific amount per cigarette), *ET* is determined independently of the industry price (*IP*). However, if *ET* is *ad valorem*, the tax is typically levied as a percentage of a component of the industry price (*IP*). The

[1] The industry price also includes import duties and other duties on imported raw product, but these are ignored for the sake of simplicity.

relative size of that component differs from country to country.[2] The model calculates an initial "pseudo" ad valorem tax rate as $ET_1/IP_1$.

Once the initial model has been set up, the excise tax is allowed to increase. For a (100 $\psi$) % increase in the specific tax, the new tax is calculated as

$$ET_2 = ET_1 (1 + \psi) \tag{5}$$

For an ad valorem excise tax, the percentage increases from $ET_1/IP_1$ to $(ET_1/IP_1) (1 + \psi)$.

In many countries tobacco companies have some control over the industry price (*IP*), given the highly concentrated nature of the industry. The public announcement of an excise tax increase is often the ideal time to change *IP*. Thus we assume that the industry changes *IP* by (100 $\lambda$)% at the same time as the excise tax change. Thus

$$IP_2 = IP_1 (1 + \lambda) \tag{6}$$

If the excise tax is levied as a specific tax, the new retail price ($P_2$) is calculated as

$$P_2 = \{IP_1 (1 + \lambda) + ET_1 (1 + \psi)\} \times (1 + \tau) \tag{7}$$

If the excise tax is levied ad valorem, the new retail price is calculated as

$$P_2 = IP_1 (1 + \lambda) \{1 + (ET_1/IP_1) (1 + \psi)\} \times (1 + \tau) \tag{8}$$

Once the new average price ($P_2$) has been calculated, one can use the arc formulation of the price elasticity ($\varepsilon_P = \frac{Q_2 - Q_1}{P_2 - P_1} \times \frac{P_1 + P_2}{Q_1 + Q_2}$), to solve for $Q_2$ as follows:

$$Q_2 = Q_1\left[1 + \varepsilon_P\left(\frac{P_2 - P_1}{P_1 + P_2}\right)\right] / \left[1 - \varepsilon_P\left(\frac{P_2 - P_1}{P_1 + P_2}\right)\right] \tag{9}$$

One can then easily calculate the following aggregates:

total expenditure by consumers: $P_2 \times Q_2$
total excise tax revenue: $ET_2 \times Q_2$ (10)
total industry revenue: $IP_2 \times Q_2$

In the final step, the model calculates the growth rates in the following variables: (1) average retail price, (2) consumption, (3) total expenditure, (4) total excise tax revenue, and (5) total industry revenue.

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[2] A common practice is to base the ad valorem tax on an Ex Works (EXW) value, i.e. the value of the cigarettes as they leave the factory, or in the case of imported cigarettes, on the Free on Board (FOB) value. The EXW and FOB values are less than the *IP*, as used in this paper, because the *IP* includes logistical and storage costs, as well as wholesale and retail margins. The EXW/*IP* and FOB/*IP* ratios are likely to differ significantly among countries.

While the main focus is on the fiscal and consumption effects of a change in the excise tax and/or industry price, the model also estimates the following: (1) changes in smoking prevalence and smoking intensity, and (2) the number of lives potentially saved because of the increase in the price of cigarettes.

A decrease in cigarette consumption $Q$ is achieved in a combination of two ways: a decrease in the percentage of people smoking (i.e. smoking prevalence) and/or a decrease in the average number of cigarettes smoked by smokers (i.e. smoking intensity). The user sets the percentage of the decrease in consumption attributed to a decrease in smoking prevalence at $100\rho\%$. If the initial smoking prevalence is $SP_1$, the new smoking prevalence $SP_2$ is calculated as

$$SP_2 = SP_1\left[1+\left\{\frac{Q_2 - Q_1}{(Q_1 + Q_2)/2}\right\}\right]\rho \qquad (11)$$

An index of smoking intensity is initially calculated as

$$SI_1 = Q_1/SP_1 \qquad (12)$$

At the new equilibrium it is calculated as $SI_2 = Q_2/SP_2$.

In order to estimate the number of lives saved from tobacco-related deaths due to the excise tax increase, the model requires the following inputs: adult population size ($N$), smoking prevalence at the outset ($SP_1$), and an assumption about the smoking-related mortality that could be averted if a smoker quits smoking ($\omega$).[3] Based on the findings that smoking duration is far more important than the number of cigarettes smoked per day (Flanders, et al, 2003) and the assumption that low-price cigarettes are equally dangerous as high-price cigarettes, tobacco-related mortality is determined solely by smoking prevalence and not at all by smoking intensity.[4] The number of lives saved is calculated as

$$N \times (SP_1 - SP_2) \times \omega \qquad (13)$$

***Bibliography***

[3] This fraction should not be confused with the oft-quoted statistic that regular smokers have a 50% probability of dying from a smoking related disease. A smoker who quits at a relatively advanced age still faces a substantial tobacco-related mortality risk. In fact, a recent study (Ranson et al, 2002, quoted in Barber et al, 2008) suggests that a smoker who quits at age 60 or older would avert only 10% of the tobacco-related mortality risk, compared to a 95% risk reduction is a smoker quits before age 29. The implication is that $\omega$ is substantially smaller than 0.50; in a recent study on Indonesia, $\omega$ is estimated at 0.35 (Barber et al., 2008).

[4] Thus a person who smoked ten cigarettes per day for 20 years has a much higher mortality risk than a person who smoked 20 cigarettes per day for ten years, even though the number of cigarettes smoked is the same. However, to the extent that a decrease in smoking intensity reduces smoking-related mortality, the model presents a conservative estimate of the number of lives saved through this intervention.

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Flanders, WD, Lally, CA, Zhu, B, Henly, SJ and Thun, MJ, 2003. Lung Cancer Mortality in Relation to Age, Duration of Smoking, and Daily Cigarette Consumption. *Cancer Research* 63: 6556-6562.