PLANT IDENTIFICATION USING MODEL REFERENCE TECHNIQUES

BY

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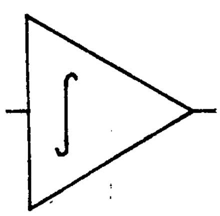
NOMENCLATURE

MRAS	Model Reference Adaptive System.
$\underline{X}$	Underlined letters denote vectors.
$\underline{X}^T$	the transpose of a vector.
$\|\underline{X}\|$	the norm of a vector X.
$\dot{e}$	the derivative of e.
$P \geqslant 0$	a positive definite matrix.
$P > 0$	a positive semidefinite matrix.
I	the identity matrix.



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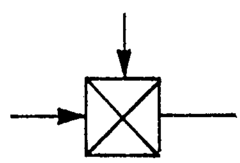

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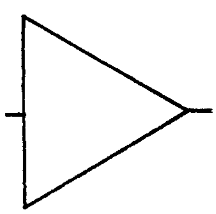

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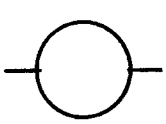
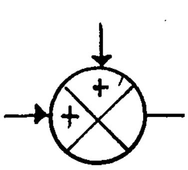


potentiometer.



summing junction.

CHAPTER 1

ADAPTIVE CONTROL - THE MODEL REFERENCE APPROACH

1.1 Introduction

Adaptive control is a branch of control which came to the fore in the early 1950's. It evolved due to the necessity of a systematic technique for the design of optimal control systems when the dynamic characteristics of the plant under consideration are poorly known or when the plant is subjected to large and unpredictable variations.

The basic principal behind adaptive controllers is the continual adaptation of the controller parameters in order to null the effect of drift variations which may occur in the dynamic characteristics of the plant under consideration.

It is this elimination of the effect of drift variations upon the performance of the control system, which makes adaptive control such an attractive scheme as compared to the classical feedback control methods.

A special branch of adaptive systems known in the literature as Model Reference Adaptive Systems (MRAS'S), evolved in the late 1950's.

A MRAS is an adaptive system which operates to null an error between the plant under consideration and a model by adapting the characteristics of the model.

1.2 Application of MRAS's

MRAS's have a "dual" character- they can be used for the control of a process or for the parametric identification and state observation of an unknown plant.

1.2.1 Model Reference techniques in plant control

In this application, the desirable characteristics of the plant are specified in a reference model,and the input signal ("signal synthesis adaptation") or the parameters ("parameter adaptation") of the plant are continually adjusted so that its response duplicates that of the model as closely as possible. The above technique has been extensively used in the fields of aeronautics and nautics.

Early development in the aeronautical field, using Model Reference techniques, included work by Whitaker, Kezer and Yarmon in 1958. They designed adaptive autopilots which compensated for changes in aircraft dynamics due to the wide variations in aircraft speed and altitude - the natural frequency of pitching oscillations of an aircraft changed by a factor of four and the damping ratio changed by a factor of one half from their low speed, zero altitude values. More recent work on the design of adaptive autopilots was conducted by Hoffmann [10].

Nautics is another field which lends itself to Model Reference adaptive techniques - the dynamic characteristics of ships are dependent on the depth of the water in which they are navigating and also on the load which they are transporting. The chief pioneer in this field was J Van Amerongen. He investigated the model reference approach to adaptive steering of ships in the 1970's [7], [8].

Other areas where Model Reference Adaptive Control (MRAC) techniques are becoming popular include Metallurgical processes and Electromechanical systems:

- when dealing with metallurgical processes, the parameters of the dynamic model which characterizes the process vary from batch to batch. Another condition which makes MRAC a useful tool for the control of metallurgical processes is the variation in the working conditions encountered in this field (e.g. the variation of the reactor characteristics).

-in electromechanical systems, large parameter variations are encountered due to their dependence on the moment of inertia and friction of the load.

1.2.2 Model Reference techniques in plant identification

The Model Reference concept is also very extensively used to identify the parameters or to observe the states of an unknown plant.

In the above application, a model of the plant to be identified is specified. The model parameters are then dynamically adjusted so that the model's response duplicates that of the plant as closely as possible. When this condition is reached, the model parameters and states will match the parameters and states of the plant respectively.

It is important to note the duality which exists betweeen model reference adaptive techniques when applied to process control and plant identification. The two applications are theoretically equivalent but the plant and the model play different roles -in model reference control, the plant's response tracks that of the model whilst in plant identification, the model's response tracks that of the plant.

The greatest motivating factor for the use of Model Reference techniques in adaptive control is its high-speed of adaptation. Today due to the higher efficiencies and the superior transient performance of MRAS's, these systems are being used more and more widely in industrial applications as a tool for solving various control, parameter identification and state estimation problems [17].

This text is concerned with investigating the design of plant identifiers using Model Reference techniques.

1.3 MRAS structures

The MRAS structures used in the field of plant identfication are:

i) Parallel Identifier

In this configuration, the adjustable model is placed in parallel with the unknown plant.

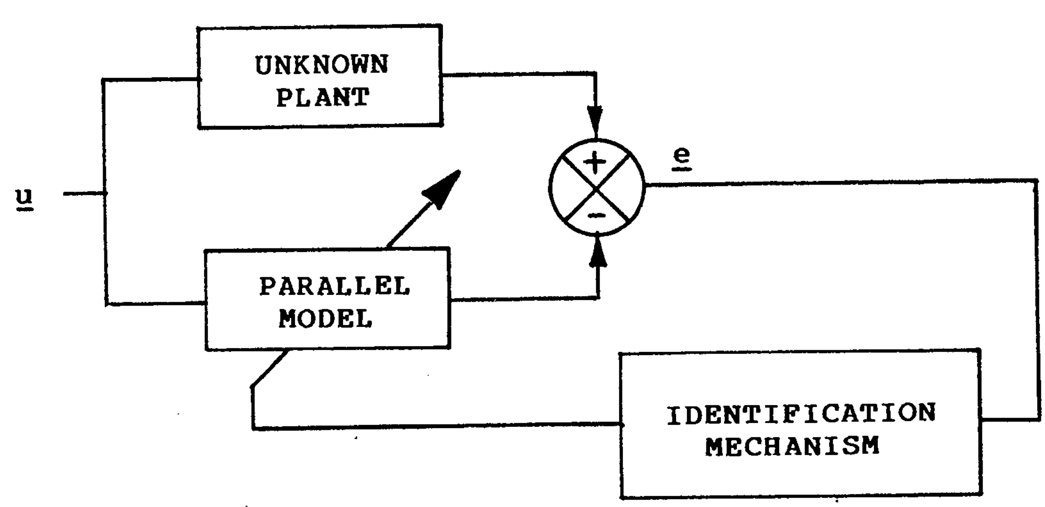


Fig. 1.3-1 Parallel Identifier structure.

ii) Series-Parallel Identifier

In this configuration, the adjustable model is partitioned into two parts - one in parallel and one in series with the unknown plant.

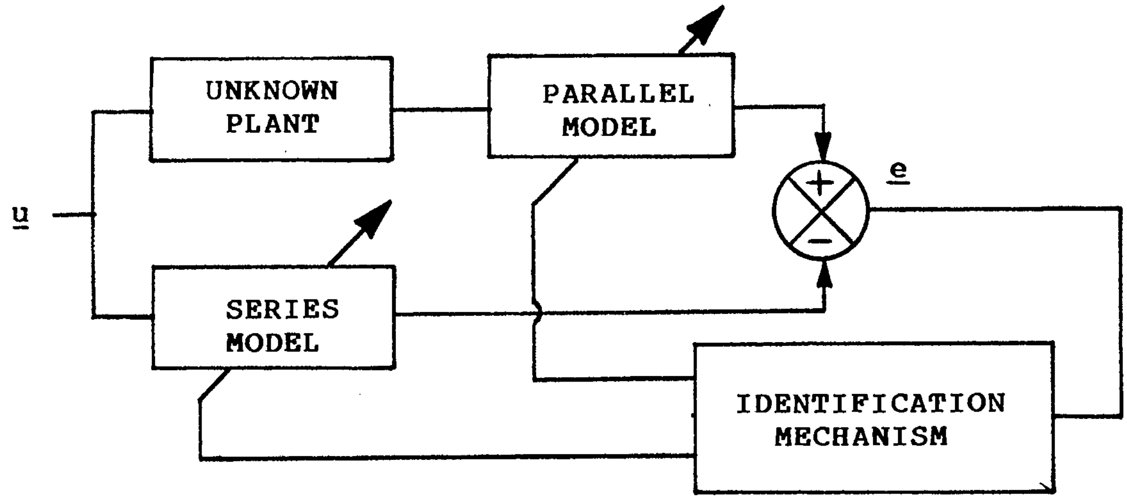


Fig 1.3-2 Series-parallel identifier structure.

iii) Series Identifier

In this configuration, the adjustable reference model is placed in series with the unknown plant.

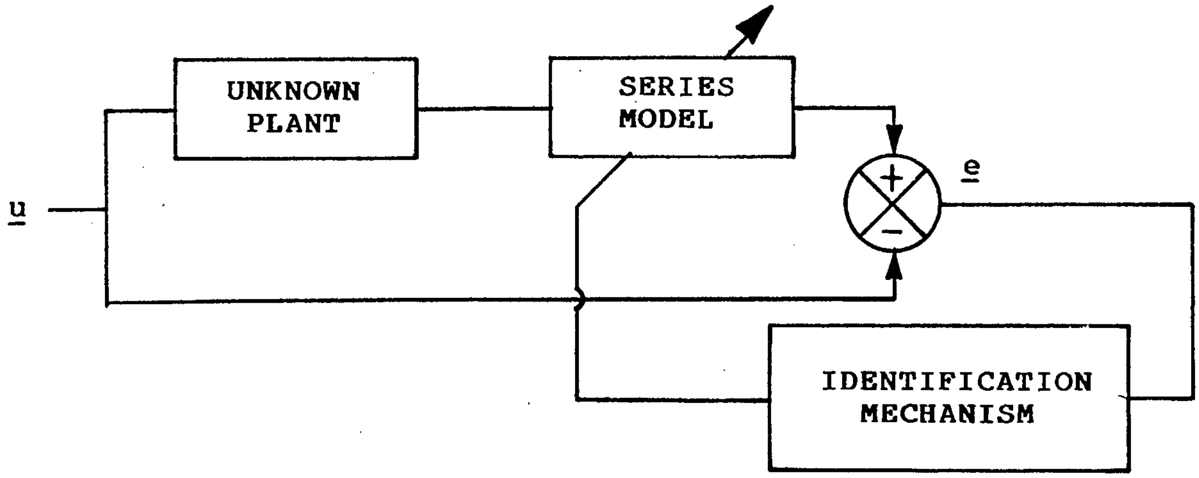


Fig. 1.3-3 Series identifier structure.

The parallel identifier and series parallel identifiers are two most promising and popular approaches for designing plant identifiers using model reference techniques.

1.4 MRAS design methods

The main problem in the development of MRAS is the design of the adaptation mechanism. There exists two general approaches to the design of MRAS. The one is based on the minimization of a performance index whilst the other is based on stability theory.

1.4.1 Design methods based on local parameter optimization theory

These methods were the first to be used for the design of MRAS's.

The basic principal behind these methods is as follows: A quadratic index of performance which expresses the structure and/or state distance between the unknown plant and the adjustable reference model is defined. Surfaces of index of performance equal to a constant are defined in the parameter space in the neighbourhood of the nominal point (i.e. where the index of performance is equal to zero).

Using parametric optimization methods one can design adaptation algorithms for changing the parameters of the adjustable reference model in order to go from one surface of constant index of performance to another surface corresponding to a lower index of performance. When the zero index performance surface is reached, the adjustable reference model should be tracking the unknown plant.

Major developments in the design of MRAS's using minimization of a performance index are stated below:

In the early 1960's Whitaker [13] of the MIT Instrumentation Laboratory proposed the performance index to be the integral square of the state generalised error (i.e. the error between the states of the model and those of the plant). Due to the ease in the practical implementation of this rule, it became a very popular method. This rule is referred to as the MIT design rule.

The disadvantages of the above rule is the slow adaptation speed and the requirement of a large number of sensitivity fiters if multiparameter identification is required.

Donaldson in investigating Model Reference design rules, suggested a more general performance index than that of Whitaker. This rule had the advantage of having a faster speed but required the use of additional sensitivity filters and the measurement of the state vectors.

In 1967 Dressler [14] developed a gradient method which avoided the use of sensitivity filters.

The major drawbacks in local parametric optimization methods can be summarised in point form as:

i) The parametric distance must be small (i.e. the parameters of the reference model must be close to the parameters of the unknown plant).

ii) The speed of adaptation is low.

iii) Once an adaptation law has been developed, there is no guarantee that the resulting adaptive system will be stable. One is thus forced to analyse the stability of the resulting adaptive system. This is not always an easy task to accomplish.

1.4.2 Design methods based on stability theory

Dueto the time varying nonlinear characteristics of MRAS's, stability problems are inherent in the design of such systems. One is thus forced to include a stability analysis in the design of MRAS's.

The classical control problem involves designing a system and then checking whether it is stable or not. When designing MRAS's using stability theory, a reverse philosophy is employed - the conditions for stability are first investigated, and the system is then designed around these stability stipulations. In this manner, system stability will always be assured.

The two stability methods used to design MRAS's are based upon Liapunov's stability theory and Popov's hyperstability and positivity theory.

i) Liapunov theory :

Parks (19) was one of the first to investigate the use of Liapunov theory in the design of MRAS's. He made use of Liapunov's Second Method which provides the sufficient conditions for the stability of nonlinear systems. This method is also sometimes referred to as Liapunov's Direct Method due to the fact that one can directly investigate the system's stability without having to solve the differential equations describing the system.

The basic principle behind Liapunov's Second Method is the definition of a Liapunov function ,V, analagous to the stored energy of the system under consideration. To prove the stability of the system, it is sufficient to show that this V function approaches zero as time approaches infinity.

The major problem in applying this method to nonlinear systems is the degree of difficulty encountered in the construction of the Liapunov V function.

For a general introduction to Liapunov's Second Method the reader is referred to [4]. An example to MRAS design using Liapunov stability is given in [20].

ii) Hyperstability plus positivity theory

The second stability method used in the design of MRAS's is a method developed in the early 1970's using hyperstability and positivity theory.

The concept of hyperstability was introduced by Popov in 1963 as an extension of absolute stability.

The hyperstability approach is being increasingly used as an alternative to Liapunov's method, due to the fact that it provides an easier means of developing more general and flexible adaptation laws.

The basic principle behind the hyperstability concept is as follows:-

The MRAS is recast as a feedback system containing a linear time-invariant operator in the feedforward path and a passive operator in the feedback path. For the system to be hyperstable, one is simply required to make the feedback block satisfy some passivity conditions.

For an extensive treatment on hyperstability and positivity theory, Popov's book [2] is the best source. For a more simplified treatment of Popov's main results, the reader is referred to Appendix A of this text.

Hyperstability and positivity theory is the best available method for the design of MRAS's. The biggest motivating factor for its use is that it translates the MRAS design problem into a general form which can be directly solved by applying a few hyperstability theorems.

It is this ease of application (one does not need to search for a Liapunov function in order to investigate the system stability) which makes it such an attractive scheme.

1.5 Conclusion

This introductory chapter has motivated the use of Model Reference techniques in the design of adaptive control systems. From section 1.4 one concluded that hyperstability and positivity theory presents the best method for the design of MRAS's.

This text will investigate the design of MRAS's in the field of plant identification using hyperstability and positivity theory.

CHAPTER 2

CONTINUOUS-TIME MRAS DESIGN

2.1 Introduction

This chapter will deal with the design of continuous - time MRAS's to be used in the field of plant identification [1]. Two MRAS configurations will be discussed

i) Parallel Identifiers
ii) Series-Parallel Identifiers

Hyperstability theory will be used as the principal design tool. For a brief review of hyperstability theory, the reader is referred to Appendix A, [2].

2.2 Parallel Identifier

This MRAS configuration was briefly introduced in Section 1.3. This section will deal with the design of a parallel identifier using hyperstability theory [11].

2.2.1 Statement of the problem

The parallel identifier will be analysed using state space notation. Consider an unknown plant described by the vector differential equation

$$\dot{\underline{X}}_p = A_p \, \underline{X}_p + B_p \, \underline{u} \qquad (2.2\text{-}1)$$

where : $\underline{X}_p$ is the n-dimensional plant state vector
$\underline{u}$ is the m dimensional input vector
A_p ((nxn)-dim) and B_p ((nxm) dim) are the unknown plant parameters

The model used to track the unknown plant parameters (i.e. the adjustable parallel model) is described by the vector differential equation

$$\dot{\underline{X}}_m = A_m(\underline{v},t)\ \underline{X}_m + B_m(\underline{v},t)\ \underline{u} \qquad (2.2\text{-}2)$$

where : $\underline{X}_m$ is the n-dimensional model state vector
$A_m(\underline{v},t)$ ((nxn) dim) and $B_m(\underline{v},t)$ ((nxm) dim) are the matrices used to asymptotically estimate the corresponding elements of A_p and B_p respectively.
$\underline{v}$ is the output vector of a linear block which processes the state error vector.

The state error vector $\underline{e}$ is defined as the variable vector that represents the difference between the state vector of the model and that of the plant.

$$\underline{e} = \underline{X}_p - \underline{X}_m \qquad (2.2\text{-}3)$$

To attain plant identification, an identification algorithm has to be devised which dynamically adjusts the model parameters in such a way that the following conditions are satisfied.

$$\lim_{t\to\infty} \underline{e}(t) = \underline{0} \qquad (2.2\text{-}4)$$

$$\lim_{t\to\infty} A_m(\underline{v},t) = A_p \qquad (2.2\text{-}5)$$

$$\lim_{t\to\infty} B_m(\underline{v},t) = B_p \qquad (2.2\text{-}6)$$

By definition, the condition expressed by equation (2.2-4) implies that the parallel identifier must be globally asymptotically stable whilst the conditions expressed by equations (2.2-5) and (2.2-6) imply that the model parameters must converge asymptotically towards the unknown parameters of the plant.

Using the classical proportional plus integral control laws, the identification algorithm can be defined by the following equations

$$A_m(\underline{v},t) = \int_0^t f_i(\underline{v},t,\tau)d\tau + f_p(\underline{v},t) + A_m(0) \qquad (0 \leqslant \tau \leqslant t) \qquad (2.2\text{-}7)$$

$$B_m(\underline{v},t) = \int_0^t g_i(\underline{v},t,\tau)d\tau + g_p(\underline{v},t) + B_m(0) \qquad (0 \leqslant \tau \leqslant t) \qquad (2.2\text{-}8)$$

where $A_m(0)$ and $B_m(0)$ represent the initial model parameters and τ is the elapsed time. A schematic representation of the above described parallel identifier structure is shown in Fig. 2.2-1.

The first term in the above two equations is the integral term which ensures that the identification mechanism pocesses memory. This property ensures that the good parameter values, once found, will be memorized.

The second term of these two equations is the proportional term which goes off to zero at equilibrium (i.e. when $\underline{e} = \underline{0}$).

The design objective can now be summarised as follows:

Determine the functions $f_i(\underline{v},t,\tau)$, $f_p(\underline{v},t)$, $g_i(\underline{v},t,\tau)$ and $g_p(\underline{v},t)$ such that equations (2.2-4) to (2.2-6) are satisfied for any initial condition $\underline{e}(0)$, $(A_p-A_m(0))$ and $(B_p-B_m(0))$.

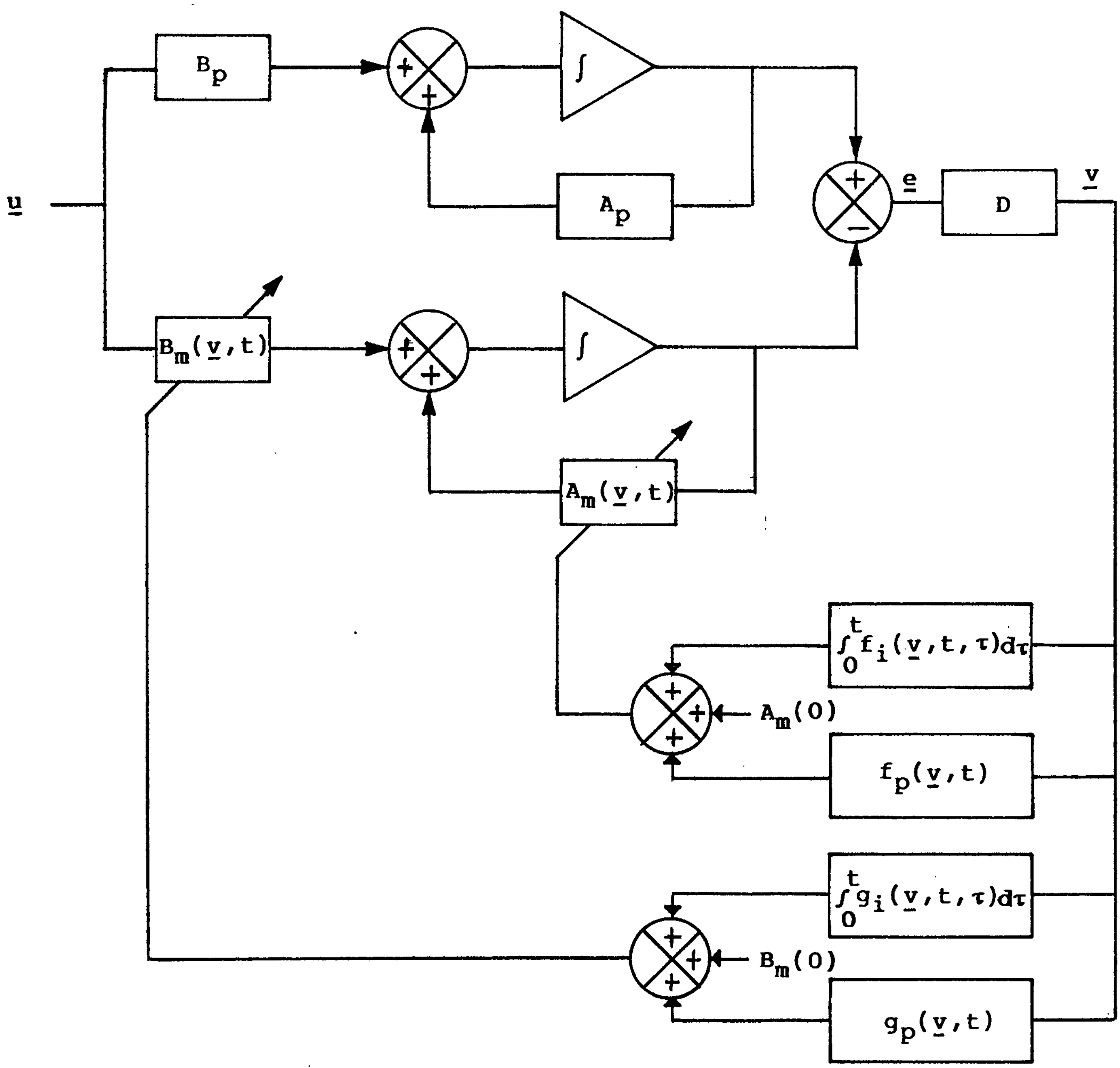


Fig 2.2-1 Parallel identifier structure

2.2.2 Design of the Parallel Identifier

As stated previously, the design of an identifier has to achieve two objectives:

a) The identifier must be globally asymptotically stable (i.e. equation (2.2-4) must be verified).

b) The identifier must ensure perfect asymptotic parameter convergence.

The design of the parallel identifier will thus be divided into two sections defined by the above two objectives.

a) Design for asymptotic stability

The first step in the design of a stable MRAS using Hyperstability theory, is to transform the given MRAS structure into an equivalent feedback system of the type considered by Popov (see Appendix A, [1], [2]).

Due to the fact that the state error vector $\underline{e}$ is the main source of information on the difference between the model and plant, we shall define $\underline{e}$ as the new state vector of the system and then try and obtain a differential equation which characterises the dynamics of $\underline{e}$.

Subtracting equations (2.2-2) from (2.2-1) one obtains

$$\dot{\underline{X}}_p - \dot{\underline{X}}_m = A_p \underline{X}_p + B_p \underline{u} - A_m(\underline{v},t)\, \underline{X}_m - B_m(\underline{v},t)\, \underline{u}$$

therefore

$$\dot{\underline{e}} = A_p \underline{X}_p - A_m(\underline{v},t)\, \underline{X}_m + (B_p - B_m(\underline{v},t))\, \underline{u} \qquad (2.2\text{-}9)$$

Now adding and subtracting the term $A_p.\underline{X}_m$ to the right hand side of equation (2.2-9) one obtains

$$\dot{\underline{e}} = A_p\,\underline{e} + (A_p - A_m(\underline{v},t))\,\underline{X}_m + (B_p - B_m(\underline{v},t))\,\underline{u} \tag{2.2-10}$$

Inserting equation (2.2-7) and (2.2-8) into the above equation

$$\dot{\underline{e}} = A_p\,\underline{e} + (A_p - \int_0^t f_i(\underline{v},t,\tau)d\tau - f_p(\underline{v},t) - A_m(0))\,\underline{X}_m + (B_p - \int_0^t g_i(\underline{v},t,\tau)d\tau - g_p(\underline{v},t) - B_m(0))\,\underline{u} \tag{2.2-11}$$

From equation (2.2-11), Popov's equivalent feedback system can be mathematically represented as follows

i) The nonlinear time-varying feedback block:

Using the notation of Fig A-1, one can represent this block as

$$\underline{w} = (\int_0^t f_i(\underline{v},t,\tau)d\tau + f_p(\underline{v},t) + A_m(0) - A_p)\,\underline{X}_m + (\int_0^t g_i(\underline{v},t,\tau)d\tau + g_p(\underline{v},t) + B_m(0) - B_p)\,\underline{u} \tag{2.2-12}$$

ii) The linear time-invariant feedforward block:

This block is represented by

$$\dot{\underline{e}} = A_p\,\underline{e} - I\,\underline{w} \tag{2.2-13}$$

$$\underline{v} = D\,\underline{e} \tag{2.2-14}$$

The general parallel identifier structure has thus been transformed into an equivalent feedback system of the type considered by Popov.

This equivalent feedback configuration is diagramatically represented below.

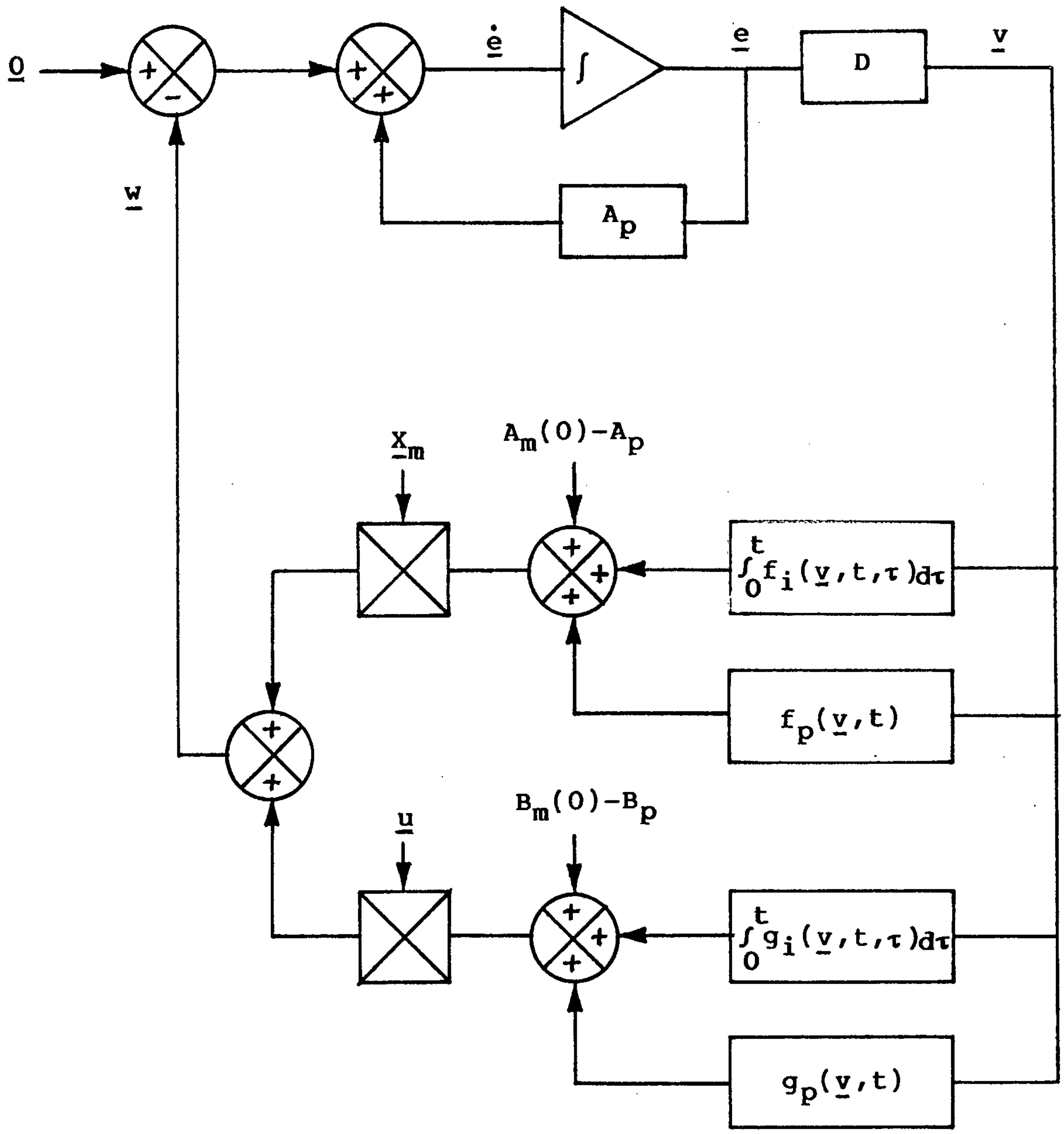


Fig 2.2-2 Equivalent feedback representation of a parallel identifier

The next step in the design of the parallel identifier involves finding solutions for $f_i(\underline{v},t,\tau)$, $f_p(\underline{v},t)$, $g_i(\underline{v},t, \;)$ and $g_p(\underline{v},t)$ such that the nonlinear time-varying equivalent feedback block described by equation (2.2-12) statisfies the Popov integral inegrality

$$\eta(0,t_1) = \int_0^{t_1} \underline{v}^T \underline{w} \, dt \geq -c^2 \qquad (2.2\text{-}15)$$

where c^2 is a finite positive constant.

By inserting equation (2.2-12) into the above equation one obtains

$$\eta(0,t_1) = \int_0^{t_1} \underline{v}^T (\int_0^{t} f_i(\underline{v},t,\tau)d\tau + f_p(\underline{v},t) + A_m(0) - A_p)\underline{X}_m dt$$
$$+ \int_0^{t_1} \underline{v}^T (\int_0^{t} g_i(\underline{v},t,\tau)d\tau + g_p(\underline{v},t) + B_m(0) - B_p)\underline{u} \, dt \geq - c^2 \qquad (2.2\text{-}16)$$

The above type of inequality is solved in Appendix B. The following solutions are obtained [1], [11]

$$f_i(\underline{v},t,\tau) = F_A(t-\tau) \; \underline{v}(\tau) \; (G_A \; \underline{X}_m(\tau))^T \qquad (2.2\text{-}17)$$

$$f_p(\underline{v},t) = F'_A(t) \; \underline{v}(t) \; (G'_A(t) \; \underline{X}_m(t))^T \qquad (2.2\text{-}18)$$

$$g_i(\underline{v},t,\tau) = F_B(t-\tau) \; \underline{v}(\tau) \; (G_B \; \underline{u}(\tau))^T \qquad (2.2\text{-}19)$$

$$g_p(\underline{v},t) = F'_B(t) \; \underline{v}(t) \; (G'_B(t) \; \underline{u}(t))^T \qquad (2.2\text{-}20)$$

where : $F_A(t-\tau)$ and $F_B(t-\tau)$ are positive definite matrix kernels.
G_A and G_B are positive definite constant matrices.
$F'_A(t)$, $F'_B(t)$, $G'_A(t)$ and $G'_B(t)$ are time varying positive definite matrices for all $t \geq 0$.

The identification laws that ensure that the nonlinear feedback block satisfies Popov's Integral Inequality are

$$A_m(\underline{v},t) = \int_0^t F_A(t-\tau)\ \underline{v}(\tau)\ (G_A\ \underline{X}_m(\tau))^T\ d\tau$$

$$+ F'_A(t)\ \underline{v}(t)\ (G'_A\ \underline{X}_m(t))^T + A_m(0) \qquad (2.2\text{-}21)$$

$$B_m(\underline{v},t) = \int_0^t F_B(t-\tau)\ \underline{v}(\tau)\ (G_B\ \underline{u}(\tau))^T\ d\tau$$

$$+ F'_B(t)\ \underline{v}(t)\ (G'_B(t)\ \underline{u}(t))^T + B_m(0) \qquad (2.2\text{-}22)$$

The next stage in the design of the identifier involves finding the conditions to be satisfied by the equivalent feedforward block, in order that the feedback system will be asymptotically hyperstable.

From Eqs. (2.2-13) and (2.2-14)one can derive the transfer matrix of the equivalent feedforward block to be

$$H(s) = D\ (sI - A_p)^{-1} \qquad (2.2\text{-}23)$$

where I is the identity matrix.

Applying Theorem A-1, one finds that for the equivalent feedback system to be asymptotically hyperstable (for the type of feedback block satisfying Popov's Integral Inequality), the equivalent feedforward block described by equations (2.2-13) and (2.2-14) must be a strictly positive real transfer matrix.

The reason for having to introduce the linear transformation

$$\underline{v} = D\ \underline{e}$$

into the design, was so as to ensure that the necessary conditions for the feedforward block to be a hyperstable block are met. The matrix gain D, known as a linear compensator, is freely chosen so as to ensure that the feedforward block is a strictly positive real transfer matrix. This matrix can be computed by solving the Liapunov equation (see Appendix A)

$$A^T{}_p \, D + D \, A_p = - Q \qquad (2.2\text{-}24)$$

where Q is an arbitrary positive definite matrix.

It is important to note that the positivity condition that the feedforward block has to meet is not a necessary condition but merely a sufficient condition. This implies that if H(s) is not strictly positive real then it does not necessarily mean that the identifier will be unstable.

The final structure of the parallel theoretically asymptotically stable identifier is shown in Fig 2.2-3.

b) Design for perfect asymptotic parameter convergence

The structure represented by Fig 2.2-3 will ensure that the MRAS will be asymptotically stable. However for the purpose of plant identification one also requires that the model parameters converge towards those of the unknown plant.

The conditions for perfect asymptotic parameter convergence are analysed in Appendix C, [1], [20]. These conditions are stated below:

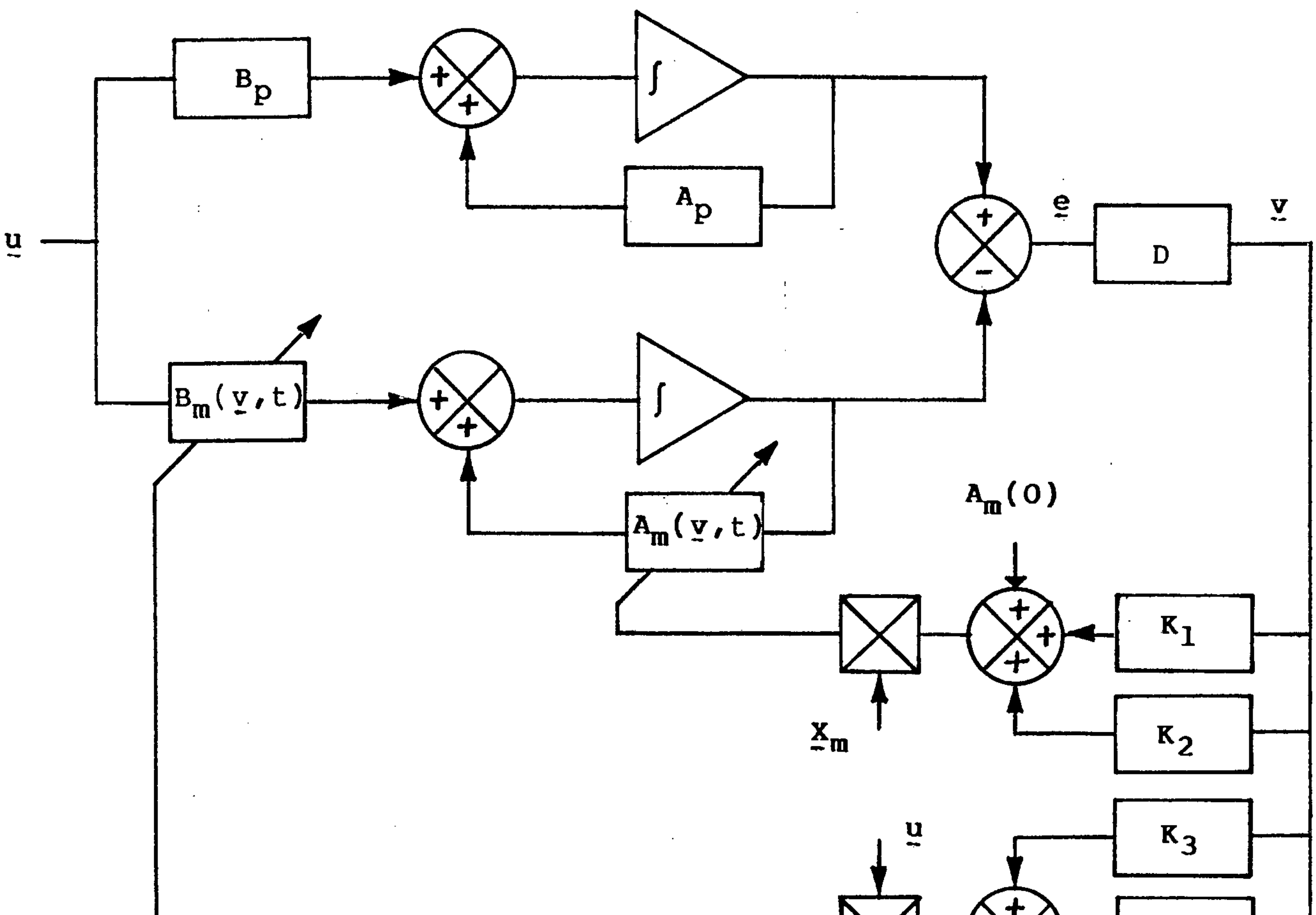
A globally asymptotically stable parallel identifier will achieve perfect parameter identification if the input vector $\underline{u}$ and the plant state vector $\underline{X}_p$ are linearly independent. The conditions on $\underline{u}$ to assure independence of $\underline{u}$ and $\underline{X}_p$ are

i) the plant to be identified must be completely controllable

ii) the components of the vector $\underline{u}$ must be linearly independent

iii) each component of the vector $\underline{u}$ must contain at least $(n+1)/2$ distinct frequencies, where n denotes the model and plant state vector dimension.

One can thus conclude that the accuracy of a parallel identifier will not be dependent on the identification algorithm only but on the nature of the input vector $\underline{u}$.



$K_1 = \int_0^t F_A(t-\tau)d\tau$ $K_3 = \int_0^t F_B(t-\tau)d\tau$

$K_2 = F'_A(t)G'_A$ $K_4 = F'_B(t)G'_B$

Fig 2.2-3 Asymptotically stable parallel identifier structure.

2.2.3 The effect of noise on the precision of identification

Since noise is an inevitable characteristic of any physical system, one has to investigate its effects when designing a control system.

A brief analysis on the effect of noise on the accuracy of the parallel identifier designed in Section 2.2.2 is presented below.

To simplify the analysis of the noise effect we will consider as an example, a first order unknown plant described by

$$\dot{X}_p = a_p\ X_p + b_p\ u \tag{2.2-25}$$

where we will assume that b_p is known and a_p is the unknown plant parameter. The parallel model is described by

$$\dot{X}_m = a_m(v,t)\ X_m + b_m\ u \tag{2.2-26}$$

Using the integral part only of the identification law specified by equation (2.2-21) and selecting $F_A(t-).G_A = f$ one obtains

$$a_m(v,t) = \int_0^t f\ v(\tau)\ X_m(\tau)\ d\tau + a_m(0) \tag{2.2-27}$$

If the plant output is corrupted by a zero mean noise of finite variance such that v becomes v+n, the identification law will become

$$a_m(v,t) = \int_0^t fv(\tau)X_m(\tau)d\tau + \quad fn(\tau)X_m(\tau)d\tau + a_m(0) \tag{2.2-28}$$

If the identification gain f is low then the second term on the right hand side of equation (2.2-28) will be small and can thus be considered negligible. The estimated parameter will therefore be very accurate.

If the identification gain f is high then the second term on the right hand side of equation (2.2-28) can no longer be neglected and will lead to the following phenomena

i) the mean value of the estimated parameter will become biased.

ii) The estimated parameter will oscillate around the biased mean value.

A parallel identifier will therefore achieve accurate identification in the presence of noise if the identification gain is low. A low identification gain however has the disadvantage of leading to large identification times.

2.3 Series-Parallel Identifier

This MRAS configuration was briefly introduced in Section 1.2.2. This section will deal with the design of a series-parallel identifier using hyperstability theory [1], [20].

The procedure for the design of the series-parallel identifier will follow the same procedure as the design of the parallel identifier in Section 2.2.

2.3.1 Statement of the problem

Consider an unknown plant to be identified described by the vector differential equation

$$\dot{\underline{X}}_p = A_p \underline{X}_p + B_p \underline{u} \qquad (2.3\text{-}1)$$

where : $\underline{X}_p$ is the n-dimensional plant state vector
$\underline{u}$ is the m-dimensional input vector
A_p ((nxn)-dim) and B_p ((nxm)-dim) are the unknown plant parameters.

The series-parallel estimation model is described by the vector differential equation

$$\dot{\underline{X}}_m = A_m(\underline{v},t)\, \underline{X}_p + B_m(\underline{v},t)\, \underline{u} - K\, \underline{e} \qquad (2.3\text{-}2)$$

where : $\underline{X}_m$ is the n-dimensional model state vector
$\underline{u}$ is the m-dimensional input vector
$A_m(\underline{v},t)$ ((nxn)-dim) and $B_m(\underline{v},t)$ ((nxm)-dim) are used to asymptotically estimate the corresponding elements of A_p and B_p respectively.

$\underline{e}$ is the state error vector.

K is a stable matrix.

The state error vector is defined by the expression

$$\underline{e} = \underline{X}_p - \underline{X}_m \tag{2.3-3}$$

To attain plant identification, an identification algorithm has to be devised which dynamically adjusts the model parameters in such a way that the following conditions are satisfied

$$\lim_{t\to\infty} \underline{e}(t) = \underline{0} \tag{2.3-4}$$

$$\lim_{t\to\infty} A_m(\underline{v},t) = A_p \tag{2.3-5}$$

$$\lim_{t\to\infty} B_m(\underline{v},t) = B_p \tag{2.3-6}$$

By definition the condition expressed by equation (2.3-4) implies that the series-parallel identifier must be globally asymptotically stable whilst the conditions expressed by equations (2.3-5) and (2.3-6) imply that the model parameters must converge asymptotically towards the unknown plant parameters.

Using the classical proportional plus integral control laws, the indentification alogorithm can be defined by the following equations

$$A_m(\underline{v},t) = \int_0^t f_i(\underline{v},t,\tau)\, d\tau + f_p(\underline{v},t) + A_m(0) \tag{2.3-7}$$

$$B_m(\underline{v},t) = \int_0^t g_i(\underline{v},t,\tau)\, d\tau + g_p(\underline{v},t) + B_m(0) \tag{2.3-8}$$

where $A_m(0)$ and $B_m(0)$ represent the initial model parameters and τ is the elapsed time. A schematic representation of the above described series-parallel identifier structure is shown in Fig. 2.3-1.

The design objective can thus be summarised as follows:

Determine the functions $f_i(\underline{v},t,\tau)$, $f_p(\underline{v},t)$, $g_i(\underline{v},t,\tau)$ and $g_p(\underline{v},t)$ such that equations (2.3-4) to (2.3-6) are satisfied for any initial condition $\underline{e}(0)$, $(A_p - A_m(0))$ and $(B_p - B_m(0))$.

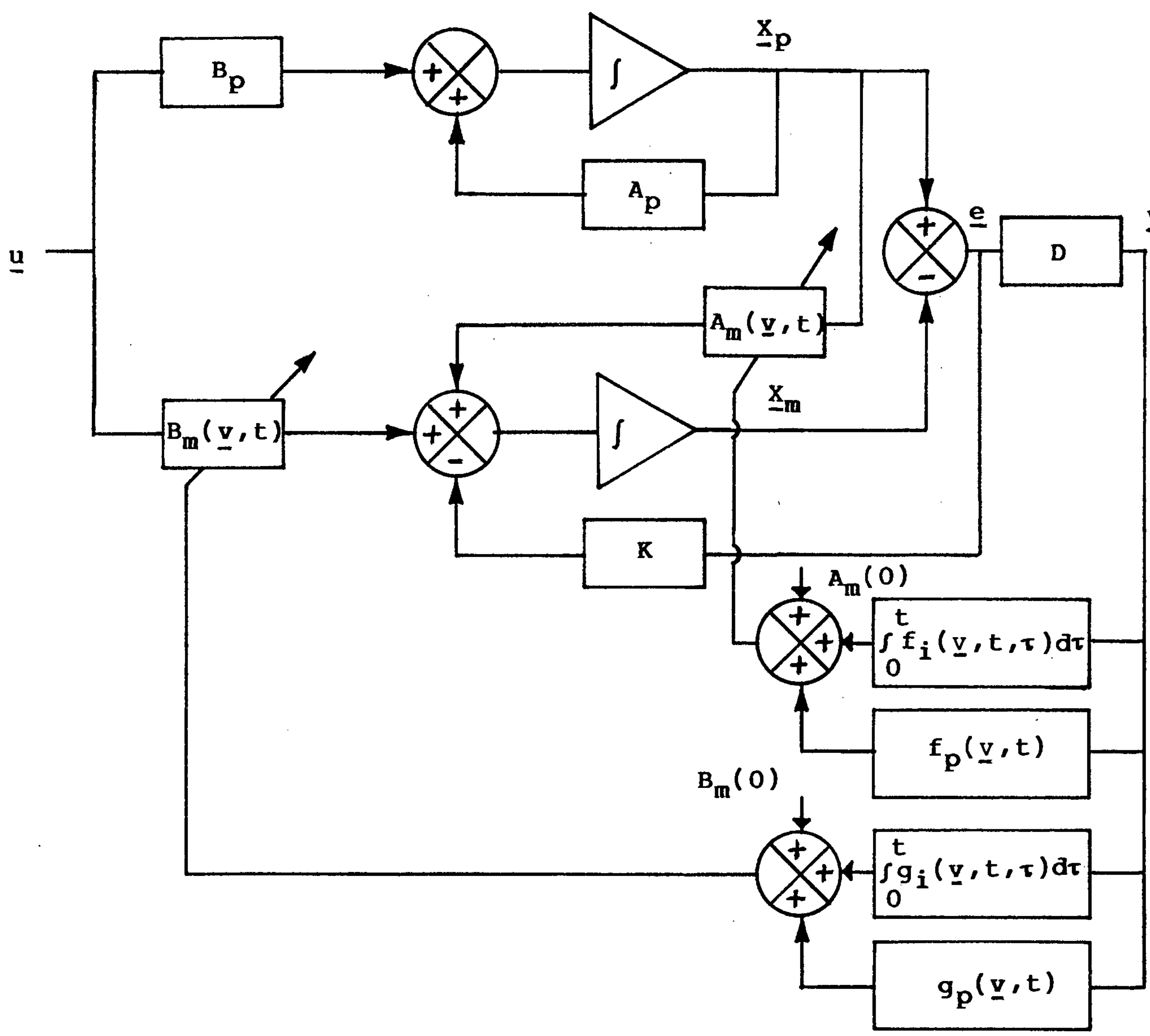


Fig 2.3-1 Series-parallel Identifier structure

2.3.2 Design of the Series-parallel identifier

As stated previously, the design of the series-parallel identifier must achieve two objectives

a) The identifier must be globally asymptotically stable (i.e. equation (2.3-4) has to be verified).

b) The identifier must achieve perfect asymptotic parameter convergence (i.e. verification of equations (2.3-5) and (2.3-6)).

These two objectives are dealt with below [20].

a) Design for asymptotic stability

The first stage in the design of the series-parallel identifier involves transforming the given MRAS structure into the equivalent feedback system of the type considered by Popov (see Fig A-1).

Subtracting equation (2.3-2) from (2.3-1) one obtains

$$\dot{\underline{X}}_p - \dot{\underline{X}}_m = A_p \underline{X}_p + B_p \underline{u} - (A_m(\underline{v},t) \underline{X}_p + B_m(\underline{v},t) \underline{u} - K\underline{e}) \qquad (2.3\text{-}9)$$

$$\dot{\underline{e}} = K\underline{e} + (A_p - A_m(\underline{v},t)) \underline{X}_p + (B_p - B_m(\underline{v},t)) \underline{u} \qquad (2.3\text{-}10)$$

Inserting equation (2.3-7) and (2.3-8) into the above equation

$$\dot{\underline{e}} = K\underline{e} + (A_p - \int_0^t f_i(\underline{v},t,\tau)d\tau - f_p(\underline{v},t) - A_m(0)) \underline{X}_p$$
$$+ (B_p - \int_0^t g_i(\underline{v},t,\tau)d\tau - g_p(\underline{v},t) - B_m(0)) \underline{u} \qquad (2.3\text{-}11)$$

One can thus represent Popov's equivalent feedback system as follows :

i) The nonlinear time-varying feedback block: using the notation of Fig A-1 one can represent this block by

$$\underline{w} = \left(\int_0^t f_i(\underline{v},t,\tau)d\tau \;\; + f_p(\underline{v},t) + A_m(0) \quad A_p \right) \underline{X}_p$$
$$+ \left(\int_0^t g_i(\underline{v},t,\tau)d\tau \;\; + g_p(\underline{v},t) + B_m(0) \quad B_p \right) \underline{u} \qquad (2.3\text{-}12)$$

where $\underline{w}$ and $\underline{v}$ are the output and input vectors respectively.

ii) The linear time invariant feedforward block:

This block is represented by

$$\dot{\underline{e}} = K \, \underline{e} \quad I \, \underline{w} \qquad (2.3\text{-}13)$$

$$\underline{v} = D \, \underline{e} \qquad (2.3\text{-}14)$$

where w and v are the input and output vectors of this block respectively.

This equivalent feedback configuration is diagramatically represented by Fig 2.3-2.

The next step in the design involves finding solutions for $f_i(\underline{v},t,\tau)$, $f_p(\underline{v},t)$, $g_i(\underline{v},t,\tau)$ and $g_p(\underline{v},t)$ such that the nonlinear time-varying equivalent feedback block described by equation (2.3-12) satisfies the Popov integral inequality

$$\eta(0,t_1) = \int_0^{t_1} \underline{v}^T \, \underline{w} \, dt \quad \geq -c^2 \qquad (2.3\text{-}15)$$

where c^2 is a finite positive constant.

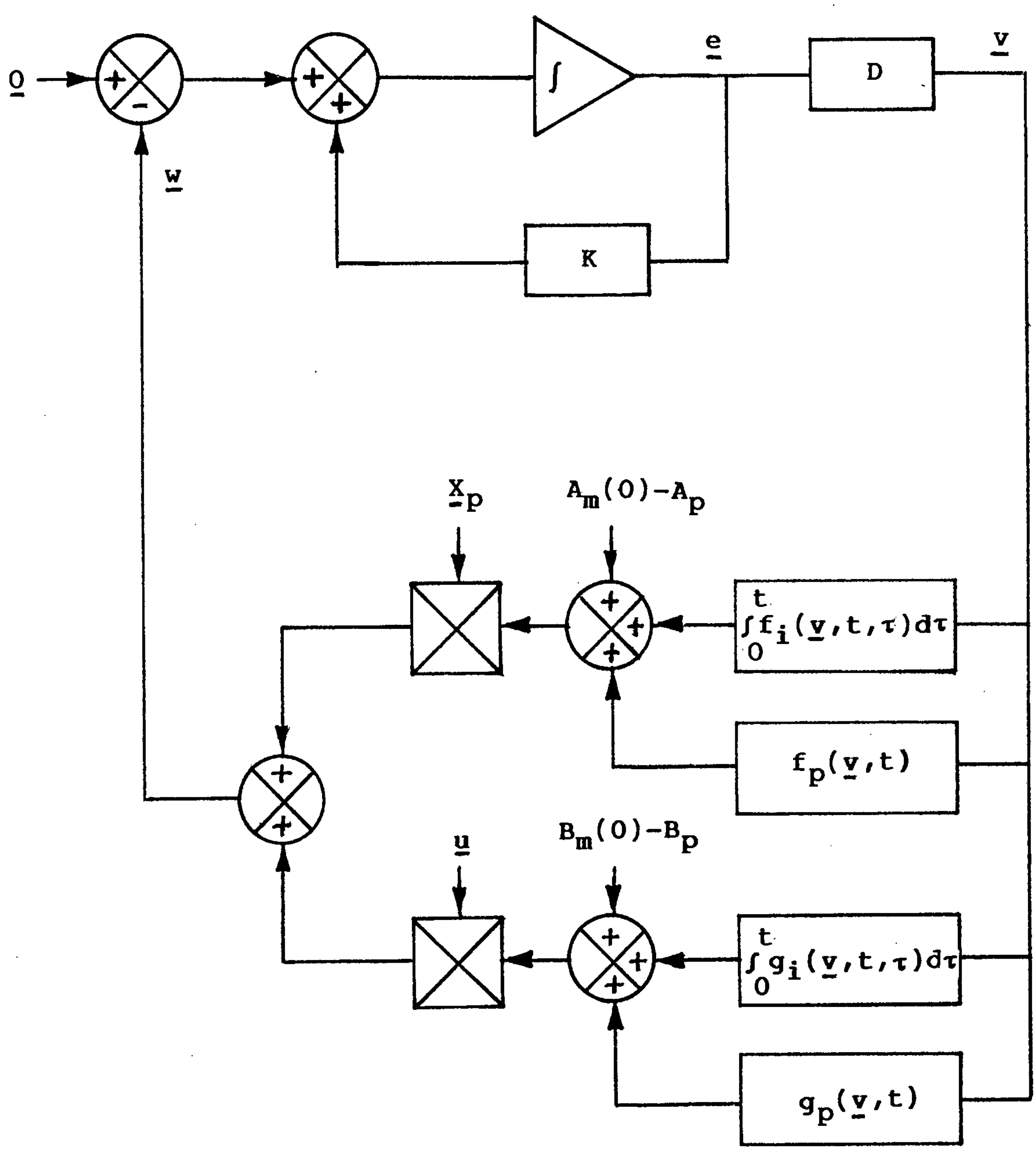


Fig 2.3-2 Equivalent feedback representation of a series-parallel identifier.

Inserting equation (2.3-12) into the above equation

$$\eta(0,t_1) = \int_0^{t_1} \underline{v}^T(\int_0^t f_i(\underline{v},t,\tau)d\tau + f_p(\underline{v},t) + A_m(0) - A_p)\underline{X}_p \, dt$$
$$+ \int_0^{t_1} \underline{v}^T(\int_0^t g_i(\underline{v},t,\tau)d\tau + g_p(\underline{v},t) + B_m(0) - B_p) \, \underline{u} \, dt \geq - c^2 \qquad (2.3\text{-}16)$$

The above type of inequality is solved in Appendix B, [1], [20]. Using the results obtained in Appendix B, the following solutions are obtained

$$f_i(\underline{v},t,\tau) = F_A(t-\tau) \, \underline{v}(\tau) \, (G_A \, \underline{X}_p(\tau))^T \qquad (2.3\text{-}17)$$

$$f_p(\underline{v},t) = F'_A(t) \, \underline{v}(t) \, (G'_A(t) \, \underline{X}_p(t))^T \qquad (2.3\text{-}18)$$

$$g_i(\underline{v},t,\tau) = F_B(t-\tau) \, \underline{v}(\tau) \, (G_B \, \underline{u}(\tau))^T \qquad (2.3\text{-}19)$$

$$g_p(\underline{v},t) = F'_B(t) \, \underline{v}(t) \, (G'_B(t) \, \underline{u}(t))^T \qquad (2.3\text{-}20)$$

where : $F_A(t-\tau)$ and $F_B(t-\tau)$ are positive definite matrix kernels.
G_A and G_B are positive definite constant matrices.
$F'_A(t)$, $F'_B(t)$, $G'_A(t)$ and $G'_B(t)$ are time-varying positive definite matrices for all $t>0$.

The identification laws that ensure that the nonlinear feedback block satisfies Popov's integral inequality are

$$A_m(\underline{v},t) = \int_0^t F_A(t-\tau) \, \underline{v}(\tau) \, (G_A \, \underline{X}_p(\tau))^T d\tau$$
$$+ F'_A(t) \, \underline{v}(t) \, (G'_A(t) \, \underline{X}_p(t))^T + A_m(0) \qquad (2.3\text{-}21)$$

$$B_m(\underline{v},t) = \int_0^t F_B(t-\tau) \, \underline{v}(\tau) \, (G_B \, \underline{u}(\tau))^T d\tau$$
$$+ F'_B(t) \, \underline{v}(t) \, (G'_B(t) \, \underline{u}(t))^T + B_m(0) \qquad (2.3\text{-}22)$$

The final step in the design of the identifier involves finding the conditions to be satisfied by the equivalent feedforward block, described by Eqs. 2.3-13 and 2.3-14, in order that the feedback system will be asymptotically hyperstable.

The transfer matrix of the equivalent feedforward block is

$$H(s) = D\,(sI - K)^{-1} \qquad (2.3\text{-}23)$$

Applying Theorem A-1, one finds that for the equivalent feedback system to be asymptotically hyperstable (for the class of feedback block satisfying Popov's integral inequality), the equivalent feedforward block described by equation (2.3-13) and (2.3-14) must be a strictly positive real transfer matrix.

The matrix D which satisfies the above condition can be computed by solving the Liapunov equation (see Appendix A).

$$K^T D + D K = - Q \qquad (2.3\text{-}24)$$

where Q is an arbitrary positive definite matrix.

Unlike the case of the parallel identifier where computation of D becomes impossible when A_p is unknown, the series-parallel identifier has the advantage that D can be directly calculated using equation (2.3-24).

The final structure of the asymptotically stable series-parallel identifier is shown in Fig 2.3-3.

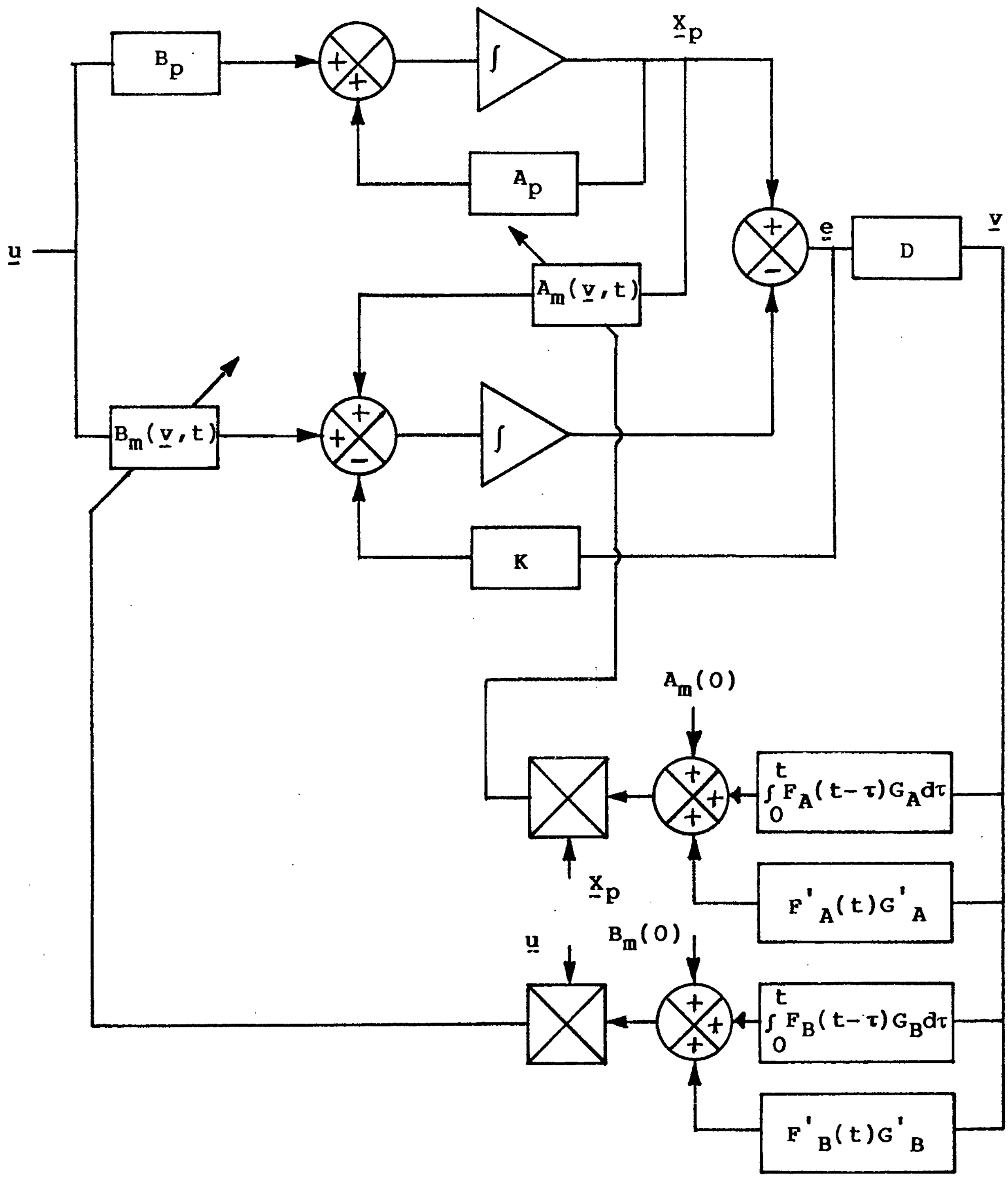


Fig 2.3-3 Series-parallel identifier.

b) Design for perfect asymptotic parameter convergence

The structure represented by Fig 2.3-3 will ensure that the MRAS will be asymptotically stable. However for the purpose of plant identification one also requires that the model parameters converge towards those of the unknown plant.

The conditions for perfect asymptotic parameter convergence are mathematically analysed in Appendix C, [1], [20] and are identical to the ones derived for the parallel identifier in Section 2.2.2. These conditions are stated below:

A globally asymptotically stable series-parallel identifier will achieve perfect parameter identification if the input vector $\underline{u}$ and the plant state vector $\underline{X}_p$ are linearly independent. The conditions on u to assure the independence of $\underline{u}$ and $\underline{X}_p$ are

i) the plant to be identified must be completely controllable.

ii) the components of the vector $\underline{u}$ must be linearly independent.

iii) each component of the vector $\underline{u}$ must contain at least (n+1)/2 distinct frequencies, where n denotes the model and plant state vector dimension.

2.3.3 The effect of noise on the precision of identification

A brief analysis on the effect of noise on the accuracy of the series-parallel identifier designed in Section 2.3.2 is presented below.

To simplify the analysis of the noise effect we will consider as an example, a first order unknown plant described by

$$\dot{X}_p = a_p X_p + b_p u \qquad (2.3\text{-}25)$$

where we will assume that b_p is known and a_p is the unknown parameter. The series-parallel model is described by

$$\dot{X}_m = a_m(v,t) X_p + b_m u - K e \qquad (2.3\text{-}26)$$

Using the integral part only of the identification law specified by equation (2.3-21) and selecting $F_A(t-).G_A = f$ one obtains

$$a_m(v,t) = \int_0^t f\, v(\tau)\, X_p(\tau)\, d\tau + a_m(0) \qquad (2.3\text{-}27)$$

If the plant output is corrupted by a zero mean noise of finite variance, such that e becomes e+n and thus v becomes equal to (v+nD), the identification law will become

$$a_m(v,t) = \int_0^t f(v(\tau) + n(\tau)D)(X_p(\tau) + n(\tau))d\tau + a_m(0) \qquad (2.3\text{-}28)$$

The above equation will produce a n^2 term. Even though the identification gain f is low, this term will always cause a bias on the identified parameter.

Series-parallel identifiers thus have the disadvantage of introducing a bias on the identified parameter in the presence of measurement noise, even if the identification gains are small.

2.4 Conclusion

This chapter dealt with the design of continuous-time parallel and series-parallel identifiers. A brief summary of the structure and main features of the two identifiers is given below:

Parallel identifier:

- The structure of the parallel identifier designed is depicted diagramatically in Fig. 2.2-3.

- In order to ensure the asymptotic stability of this MRAS structure, one must ensure that the transfer matrix of the feedforward block be a strictly positive real transfer matrix.

- Parallel identifiers offer accurate parameter identification in the presence of a zero mean random measurement noise if the identification gains are low. Low identification gains will however lead to very slow identification times.

Series-parallel identifier:

- The structure of the series-parallelidentifier designed is depicted diagramatically in Fig 2.3-3.

- In order to ensure the asymptotic stability of this MRAS structure, one must ensure that the transfer matrix of the feedforward block be a strictly positive real transfer matrix.

- Series-parallel identifiers have the disadvantage of offering inaccurate plant identification in the presence of noise.

In the next chapter the two identifiers designed above will be tested.

CHAPTER 3

CONTINUOUS-TIME IDENTIFICATION OF A SECOND ORDER PLANT

3.1 Introduction

In Chapter 2, the design of continuous-time identifiers using model reference techniques was discussed. This chapter will deal with the practical implementation of the identifiers discussed in Chapter 2 in order to identify second order plants.

Two schemes will be used to implement the identifiers, namely
i) analog computer simulation
ii) digital computer simulation

3.2 Implementation of the parallel identifier

The plant to be identified is characterised by the following differential equation

$$\frac{d^2y_p}{dt^2} + a_{p1}\frac{dy_p}{dt} + a_{p0}y_p = b_p u \qquad (3.2\text{-}1)$$

where b_p , a_{p0} and a_{p1} are the unknown plant parameters, u is the plant input and y_p is the plant output.

Expressing Equation (3.2-1) in state space notation, one obtains

State equation $$\dot{\underline{x}}_p = A_p\,\underline{x}_p + B_p\,\underline{u} \qquad (3.2\text{-}2)$$

Output equation $$y_p = x_{p1} \qquad (3.2\text{-}3)$$

where: $A_p = \begin{bmatrix} 0 & 1 \\ -a_{p0} & -a_{p1} \end{bmatrix}$ and $B_p = \begin{bmatrix} 0 \\ b_p \end{bmatrix}$

$\underline{x}^T{}_p = (x_{p1}\ ,\ x_{p2}\ )$ is the state vector of the plant.
y_p is the plant output.

The parallel estimation model is of the same form and order as that of the plant

$$\frac{d^2Y_m}{dt^2} + a_{m1}(\underline{v},t)\frac{dY_m}{dt} + a_{m0}(\underline{v},t)Y_m = b_m(\underline{v},t)\underline{u} \qquad (3.2\text{-}4)$$

where $b_m(\underline{v},t)$, $a_{m0}(\underline{v},t)$ and $a_{m1}(\underline{v},t)$ are the adjustable model parameters, $\underline{u}$ is the model input and y_m is the model output.

The model can be expressed in state space notation as

State equation $\qquad \underline{X}_m = A_m\ \underline{X}_m + B_m\ \underline{u} \qquad (3.2\text{-}5)$

Output equation $\qquad y_m = X_{m1} \qquad (3.2\text{-}6)$

where: $A_m = \begin{bmatrix} 0 & 1 \\ -a_{m0} & -a_{m1} \end{bmatrix}$ and $B_m = \begin{bmatrix} 0 \\ b_m \end{bmatrix}$

$\underline{x}^T{}_m = (x_{m1},\ x_{m2}\ )$ is the state vector of the model
y_m is the model output

In Chapter 2 it was derived that the parallel identifier described above will achieve perfect asymptotic parameter identification if the following identification mechanism is employed

$$\underline{v} = D\ \underline{e} \qquad (3.2\text{-}7)$$

where D can be computed by solving the Liapunov equation

$$A^T{}_p\ D + D\ A_p = -Q \qquad (3.2\text{-}8)$$

where Q is an arbitrary positive definite matrix

and

$$A_m(\underline{v},t) = \int_0^t F_A(t-\tau)\ \underline{v}(\tau)\ (G_A\ \underline{X}_m(\tau))^T\ d\tau + F'_A(t)\ \underline{v}(t)\ (G'_A\ \underline{X}_m(t))^T + A_m(0) \qquad (3.2\text{-}9)$$

$$B_m(\underline{v},t) = \int_0^t F_B(t-\tau)\ \underline{v}(\tau)\ (G_B\ \underline{u}(\tau))^T\ d\tau + F'_B(t)\ \underline{v}(t)\ (G'_B(t)\ \underline{u}(t))^T + B_m(0) \qquad (3.2\text{-}10)$$

where $F_A(t-\tau)$ and $F_B(t-\tau)$ are positive definite matrix kernels.
G_A and G_B are positive definite constant matrices.
$F'_A(t)$, $F'_B(t)$, $G'_A(t)$ and $G'_B(t)$ are positive definite matrices for all $t>0$.

Due to the interactive and nonlinear nature of the identifier, it is impossible to theoretically calculate the variables in the identification mechanism which result in optimal plant identification. The common technique of tuning had to therefore be employed whereby the identifier underwent a large amount of practical testing to determine what format the identification mechanism was to take in order to achieve optimal plant identification.

The following identification algorithm was generally found to produce the best results

$$A_m(\underline{v},t) = \int_0^t g_i\ I\ \underline{v}(\tau)\ (\underline{X}_m(\tau))^T\ d\tau + g_p\ I\ \underline{v}(t)\ (\underline{X}_m(t))^T + A_m(0) \qquad (3.2\text{-}11)$$

$$B_m(\underline{v},t) = \int_0^t f_i\ I\ \underline{v}(\tau)\ (\underline{u}(\tau))^T\ d\tau + f_p\ I\ \underline{v}(t)\ (\underline{u}(t))^T + B_m(0) \qquad (3.2\text{-}12)$$

where I is the identity matrix
g_i and f_i are the integral identification gains
g_p and f_p are the proportional identification gains

Using the previous two equations, the expressions for the three parameters to be identified thus become

$$a_{m0}(\underline{v},t) = -\int_0^t g_i \; v_2(\tau) \; X_{m1}(\tau) \; d\tau \quad - \quad g_p \; v_2(t) \; X_{m1}(t) + a_{m0}(0) \tag{3.2-13}$$

$$a_{m1}(\underline{v},t) = -\int_0^t g_i \; v_2(\tau) \; X_{m2}(\tau) \; d\tau \quad - \quad g_p \; v_2(t) \; X_{m2}(t) + a_{m1}(0) \tag{3.2-14}$$

$$b_m(\underline{v},t) = +\int_0^t f_i \; v_2(\tau) \; u_{\underline{2}}(\tau) \; d\tau \quad + \quad f_p v_2(t) \; u_2(t) + b_m(0) \tag{3.2-15}$$

where $a_{m0}(0)$, $a_{m1}(0)$, $b_m(0)$ are the initial model parameters
$\underline{v}(t)^T = (v_1(t), \; v_2(t))$
$\underline{u}(t)^T = (u_1(t), \; u_2(t))$

3.2.1 Analog computer simulation

The plant and model were simulated on a VIDAC 169 analog computer whilst the identification mechanism was implemented on a large AEI-TR48 analog computer.

A complete analog computer patch diagram of the parallel identifier is shown in Fig (3.2-1).

A general analysis of the performance of the parallel identifier will now be investigated by means of various identification examples.

The first three examples that we will look at assume that two of the three plant parameters are known. One therefore only needs to identify one unknown parameter.

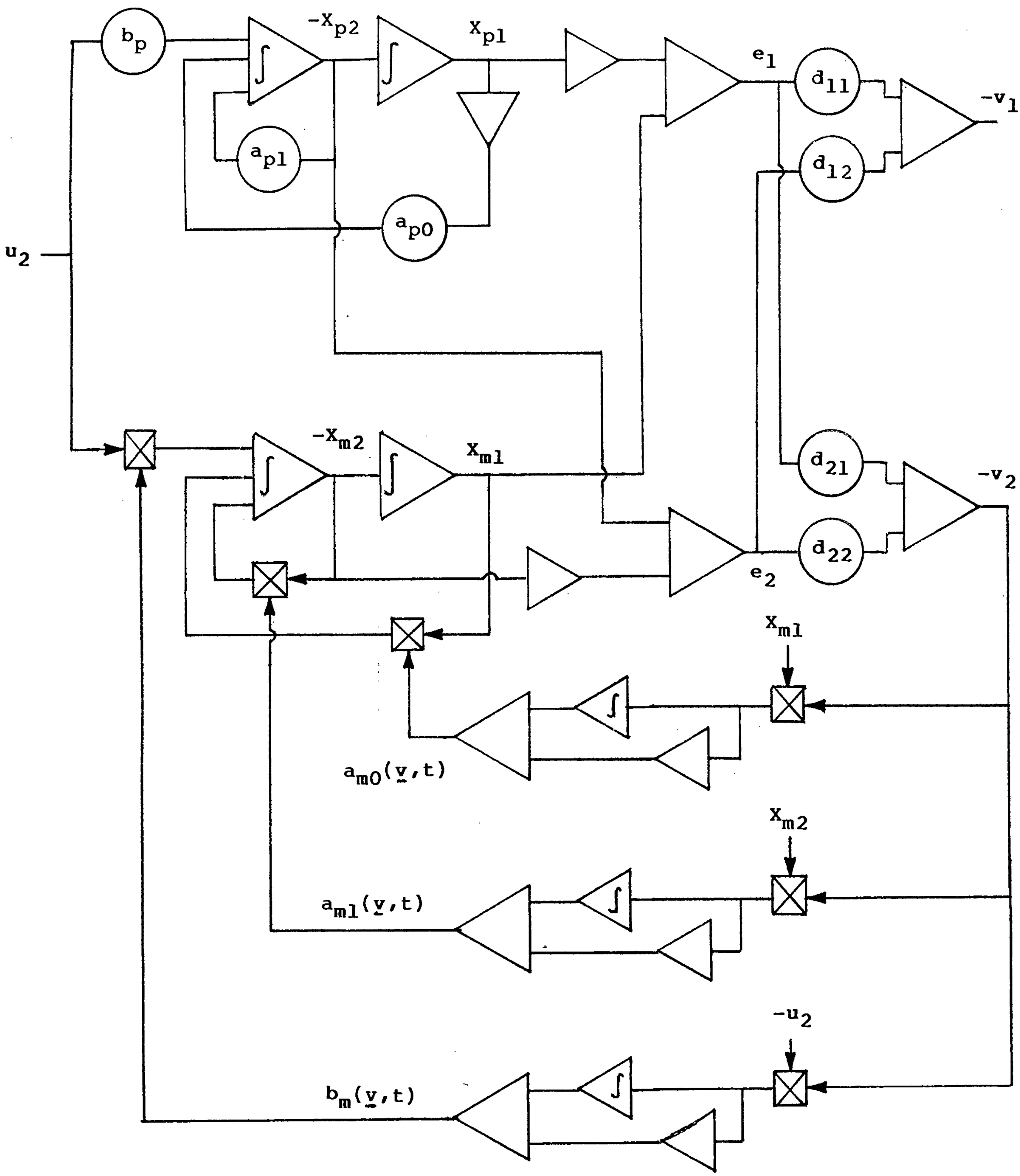


Fig. 3.2-1 Analog computer patch diagram of the parallel identifier

EXAMPLE 3.2-1

In this example the plant parameters a_{p0} and a_{p1} are known and b_p is to be identified.

The plant parameters were chosen to be

$$b_p = 0.3$$
$$a_{p0} = 0.5$$
$$a_{p1} = 0.6$$

The following identification mechanism was used :

$$\begin{aligned} b_m(\underline{v},t) &= \int_0^t v_2(\tau)\; u_2(\tau)d\tau \;+ v_2(t)\; u_2(t) \\ a_{m0}(\underline{v},t) &= a_{p0} \\ a_{m1}(\underline{v},t) &= a_{p1} \end{aligned}$$

The plant/model input was chosen to be 0.2 Hz square waveform and the linear compensator matrix D is

$$D = \begin{bmatrix} 0.37 & 0.2 \\ 0.2 & 0.5 \end{bmatrix}$$

The results for the above example are shown in Fig (3.2-2).

EXAMPLE 3.2-2

In this example, the plant parameters b_p and a_{p1} are known whilst a_{p0} is to be identified.

The plant parameters chosen were identical to those of the previous example and the identification mechanism is

$$\begin{aligned} b_m(\underline{v},t) &= b_p \\ a_{m0}(\underline{v},t) &= -\int_0^t v_2(\tau)\; X_{m1}(\tau)\; d\tau \;- v_2(t)\; X_{m1}(t) \\ a_{m1}(\underline{v},t) &= a_{p1} \end{aligned}$$

The same plant/model input and linear compensator matrix used in the previous example were again used.

The results obtained are illustrated in Fig (3.2-3).

<u>EXAMPLE 3.2-3</u>

In this example the plant parameters b_p and a_{p0} are known whilst the plant parameter a_{p1} is to be identified.

The plant parameters, plant/model input and the linear compensator matrix used in this example is identical to those used in the previous two examples.

The identification mechanism was chosen to be

$$\begin{aligned} b_m(\underline{v},t) &= b_p \\ a_{m0}(\underline{v},t) &= a_{p0} \\ a_{m1}(\underline{v},t) &= - \int_0^t v_2(\tau)\, X_{m2}(\tau)\, d\tau \quad - \quad v_2(t)\, X_{m2}(t) \end{aligned}$$

The results obtained for the above example are shown in Fig (3.2-4).

The above three examples illustrate that if the parameters are identified independently, very fast and accurate identification is easily achieved.

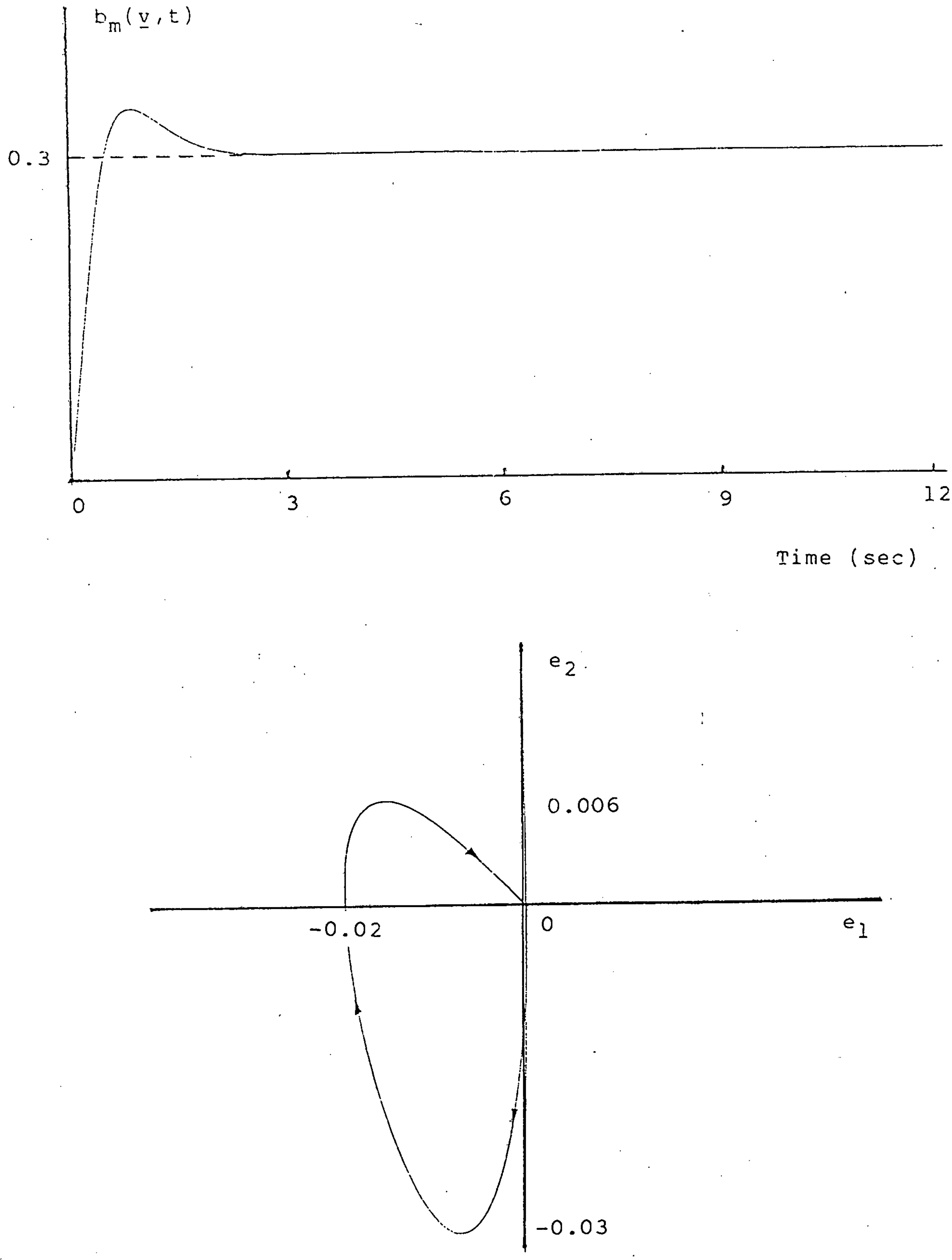


Fig. 3.2-2 Results of Example 3.2-1.

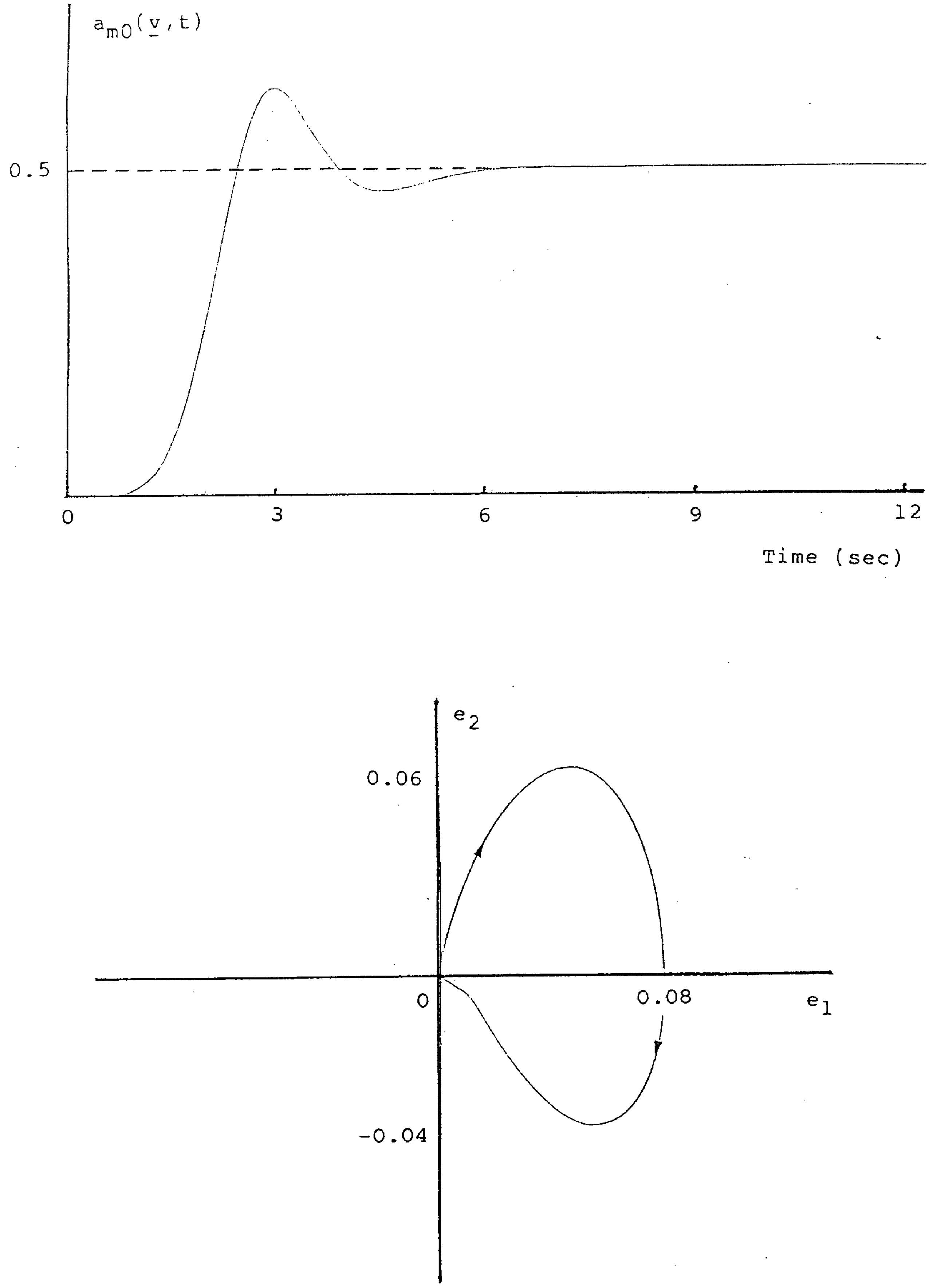


Fig. 3.2-3 Results of Example 3.2-2.

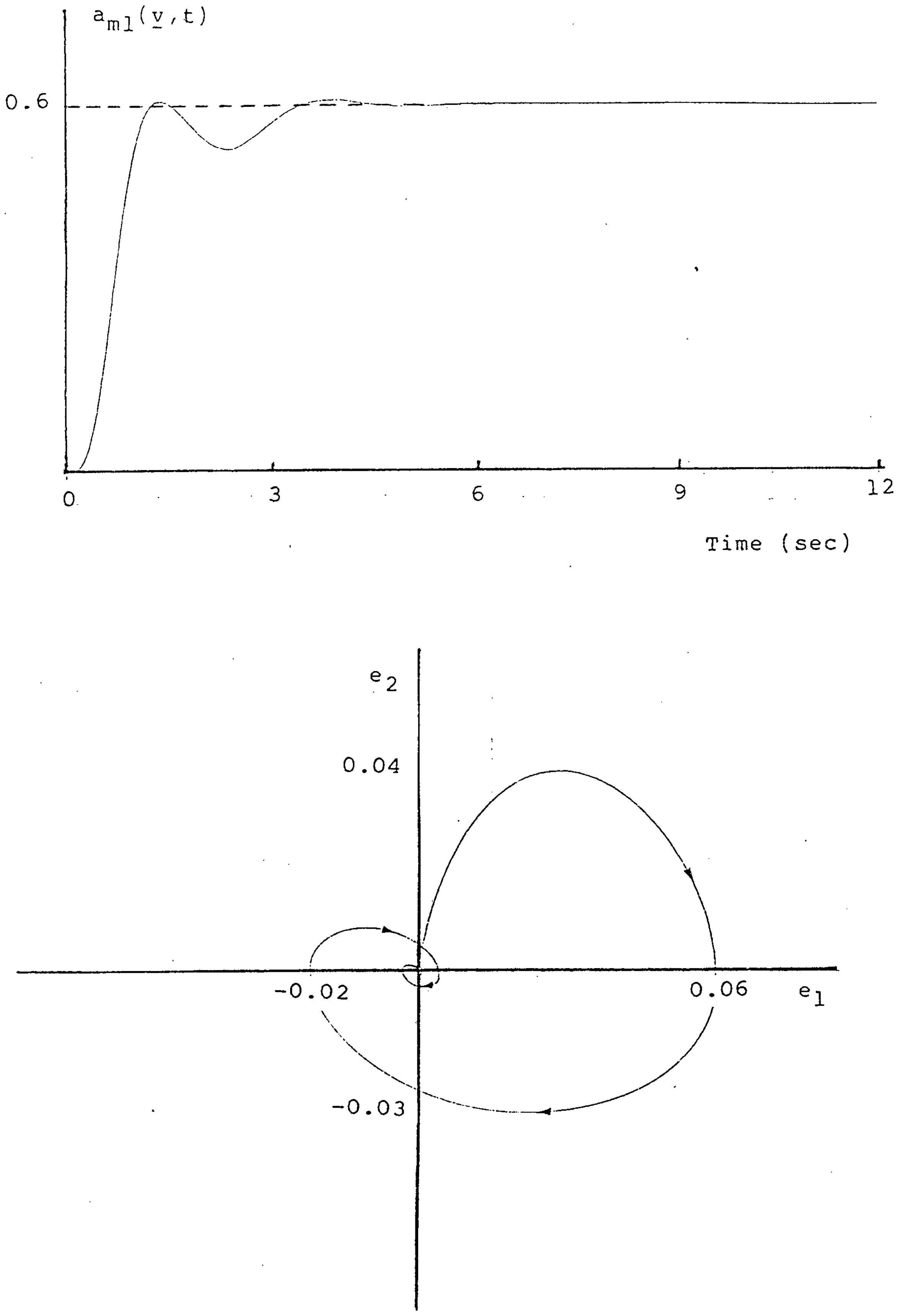


Fig. 3.2-4 Results of Example 3.2-3.

The simultaneous identification of all three plant parameters will now be investigated.

EXAMPLE 3.2-4

The plant parameters were chosen to be

$$b_p = 0.4$$
$$a_{p0} = 0.7$$
$$a_{p1} = 0.5$$

The parameter identification laws were chosen as

$$b_m(\underline{v},t) = \int_0^t v_2(\tau)\ u_2(\tau)d\tau + v_2(t)\ u_2(t)$$

$$a_{m0}(\underline{v},t) = -\int_0^t v_2(\tau)\ x_{m1}(\tau)d\tau - v_2(t)\ x_{m1}(t)$$

$$a_{m1}(\underline{v},t) = -\int_0^t v_2(\tau)\ x_{m2}(\tau)d\tau - v_2(t)\ x_{m2}(t)$$

The linear compensator matrix is

$$D = \begin{bmatrix} 0.37 & 0.2 \\ 0.2 & 0.5 \end{bmatrix}$$

The plant and model input is chosen to be a 0.1 Hz square waveform.

The results from this example are shown in Fig 3.2-5. From these results one immediately notices that the identification is not very accurate. From a large number of tests performed on the identifier it was found that the inclusion of proportional control in the identification laws results in a definite degradation in the accuracy of identification. For accurate identification it was found that the identification mechanism should contain integral control only. This is shown by the next example.

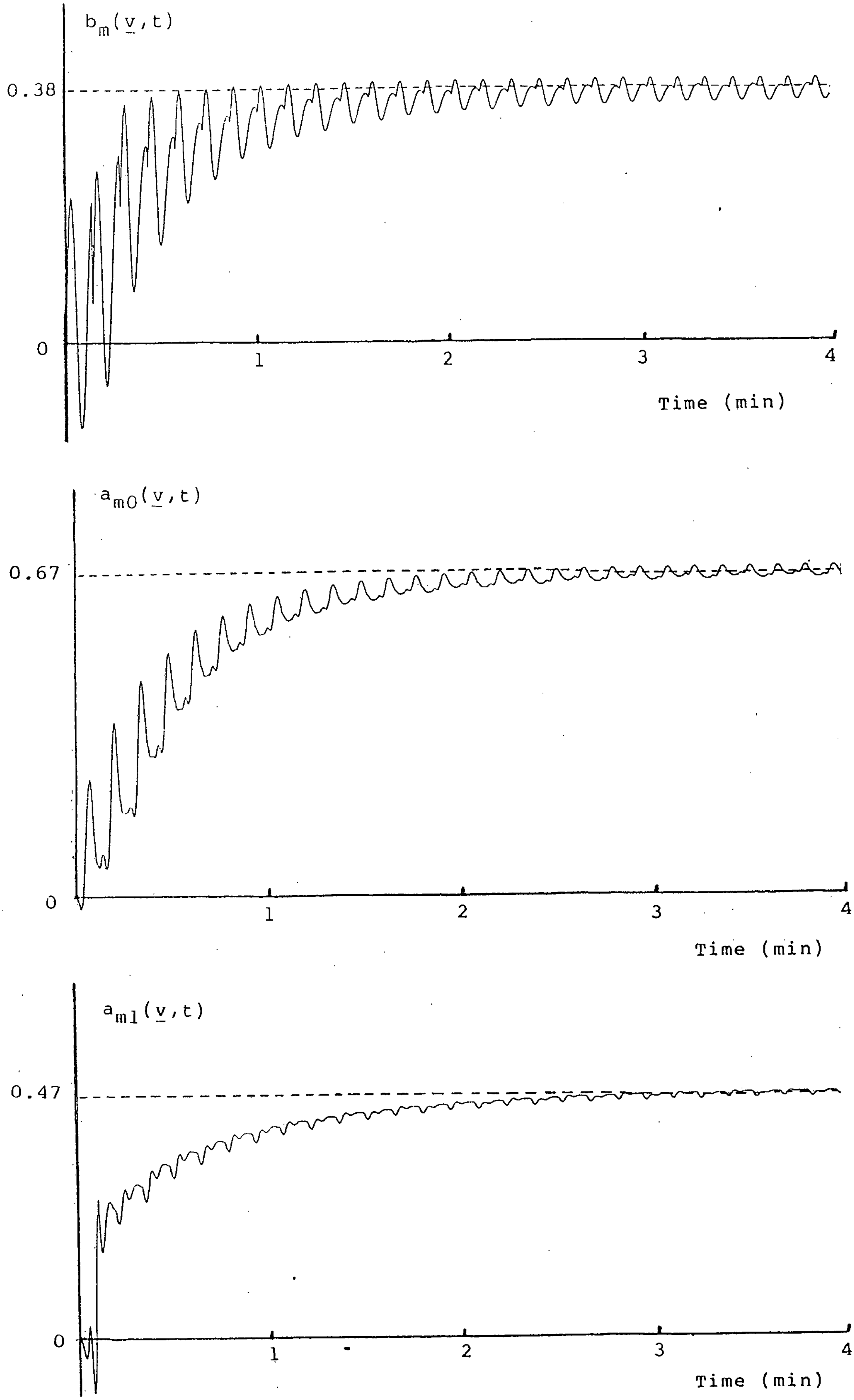


Fig. 3.2-5 Results of Example 3.2-4.

EXAMPLE 3.2-5

This example is exactly identical to the previous example except that the identification laws now become

$$b_m(\underline{v},t) = \int_0^t v_2(\tau)\ u_2(\tau)d\tau$$

$$a_{m0}(\underline{v},t) = -\int_0^t v_2(\tau)\ x_{m1}(\tau)d\tau$$

$$a_{m1}(\underline{v},t) = -\int_0^t v_2(\tau)\ x_{m2}(\tau)d\tau$$

The results of this example are shown in Fig 3.2-6. From these results it is evident that accurate plant identification is achieved, when identifying the three parameters simultaneously, if the identification mechanism makes use of integral control only.

In section 2.2.2 (b), it was stated that the accuracy of the parallel identifier is dependent on the nature of the plant and model input vector $\underline{u}$. In the previous example it was shown that perfect asymptotic parameter convergence is attained if the input waveform is a square wave. The next example will investigate the effect of the plant/model input waveform on the accuracy of identification.

EXAMPLE 3.2-6

This example is exactly identical to the previous example except that the input waveform $\underline{u}$ is a 0.02 Hz sinusoidal waveform.

The results are shown in Fig 3.2-7. From these results it is evident that a sinusoidal input is not a suitable input to achieve perfect identification. This is due to the fact that a sinusoid has only one frequency component, thus violating the necessary conditions stated in section 2.2.2 (b) for the achievement of perfect asymptotic parameter identification.

EXAMPLE 3.2-7

This example is identical to the previous example except that the plant/model input waveform is a step waveform.

The results for this example are illustrated in Fig 3.2-8.

Once again one can note that very poor identification is achieved. The above two examples demonstrate that for perfect parameter identification, the plant/model input waveform must satisfy the conditions expressed in section 2.2.2 (b) i.e. the input waveform must contain a large frequency content. It is for this reason that a square waveform is such a suitable plant/model input.

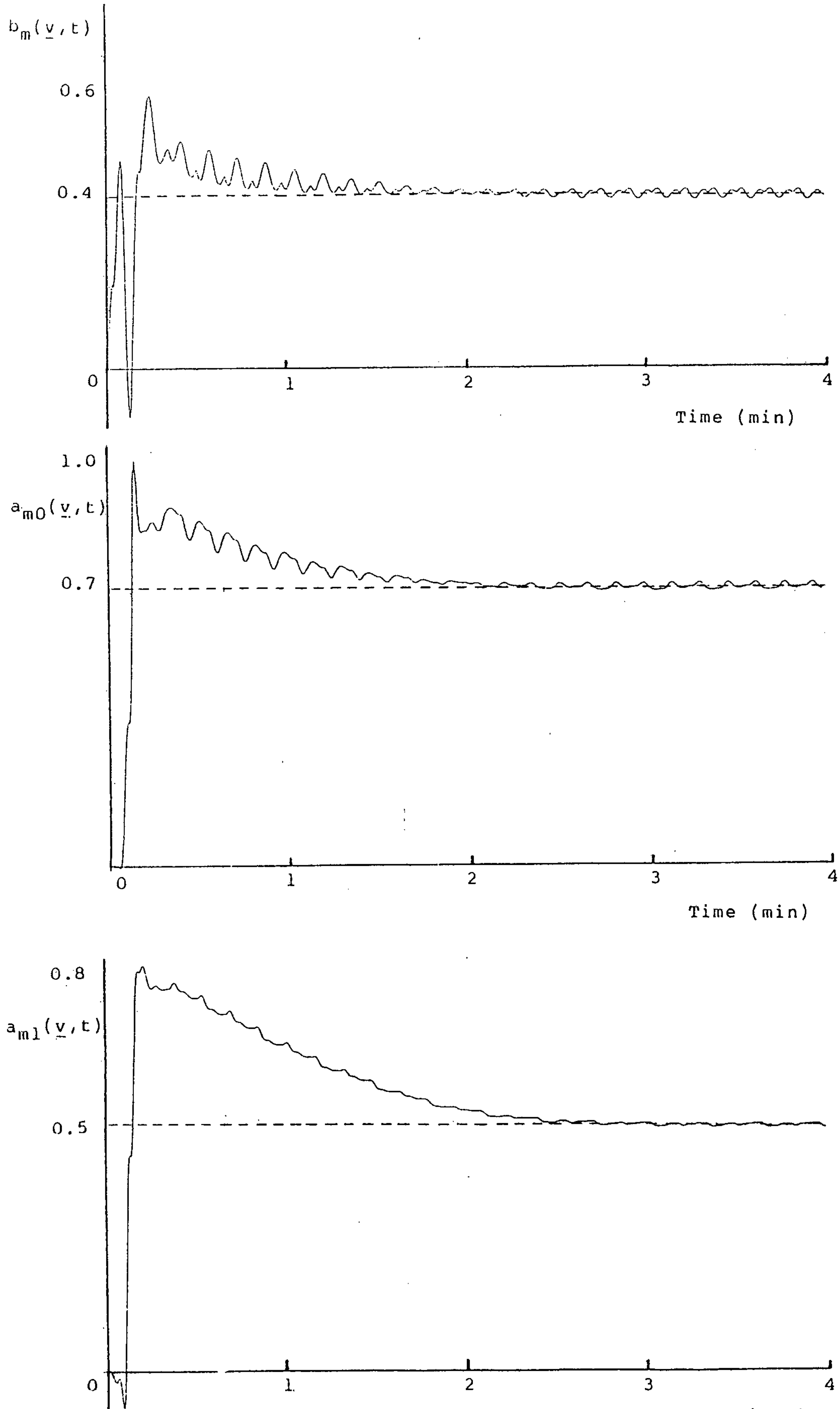


Fig. 3.2-6 Results of Example 3.2-5.

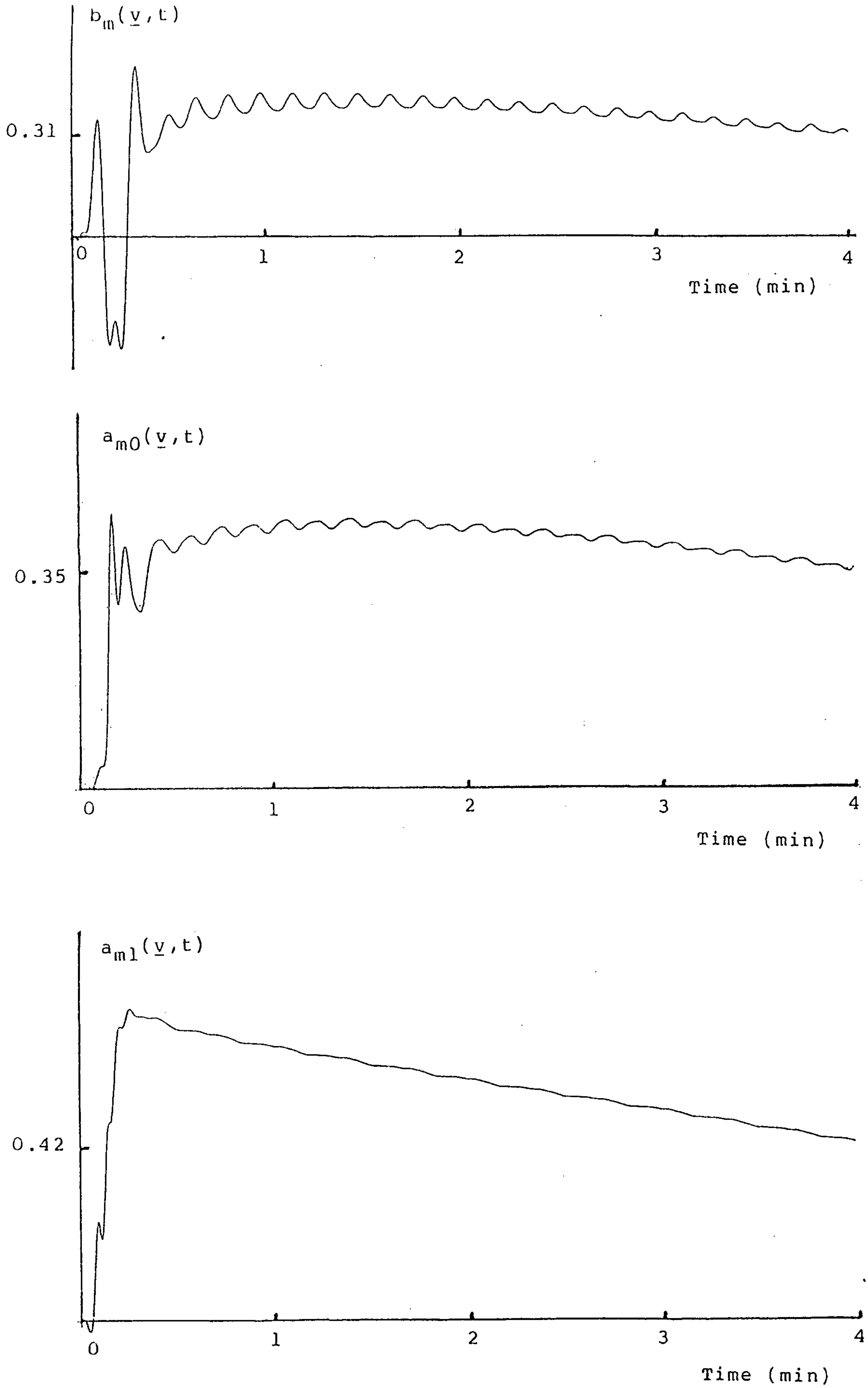


Fig. 3.2-7 Results of Example 3.2-6.

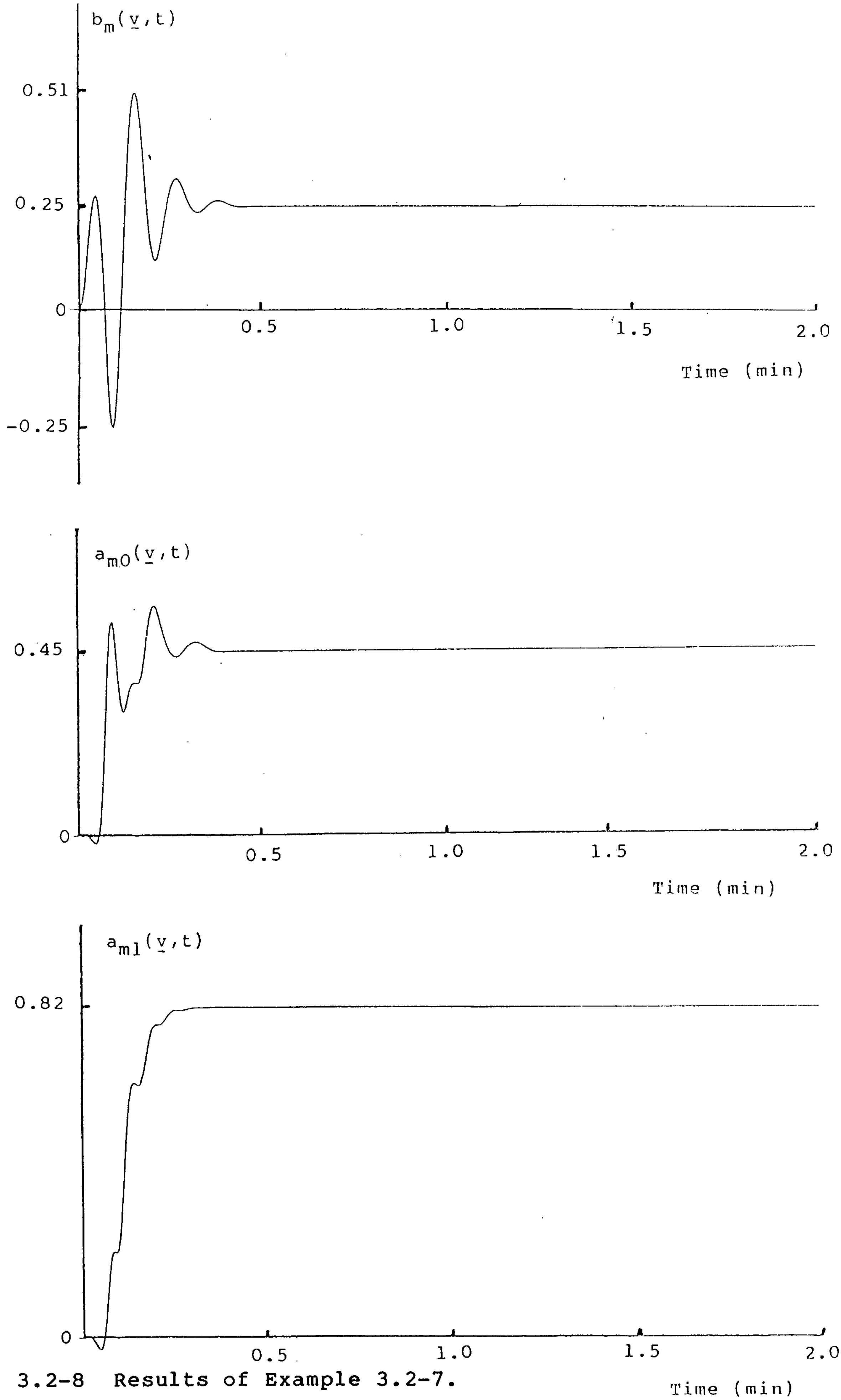


Fig. 3.2-8 Results of Example 3.2-7.

The final example demonstrates the effect of changing the integral identification gain.

<u>**EXAMPLE 3.2-8**</u>

This example is identical to Example 3.2-5, except that the integral identification gains are reduced from 1 to a value of 0.2.

The identification laws thus become

$$b_m(\underline{v},t) = 0.2 \int_0^t v_2(\tau)\ u_2(\tau)d\tau$$

$$a_{m0}(\underline{v},t) = -0.2 \int_0^t v_2(\tau)\ X_{m1}(\tau)d\tau$$

$$a_{m1}(\underline{v},t) = -0.2 \int_0^t v_2(\tau)\ X_{m2}(\tau)d\tau$$

The results for this example are shown on Fig 3.2-9.

From these results it can immediately be seen that a decrease in the identification gains causes a greater degree of accuracy in the parametric identification. In Example 3.2-5 it was noted that the model parameters oscillated about the plant parameters due to the higher gain. This gain was however not large enough to bias the model parameters as theoretically predicted in Section 2.2.3. If one tries to further increase the identification gain, the analog computer begins to saturate.

One can thus conclude that by decreasing the identification gain, more accurate identification is achieved. A decrease in the identification gain however leads to longer identification times.

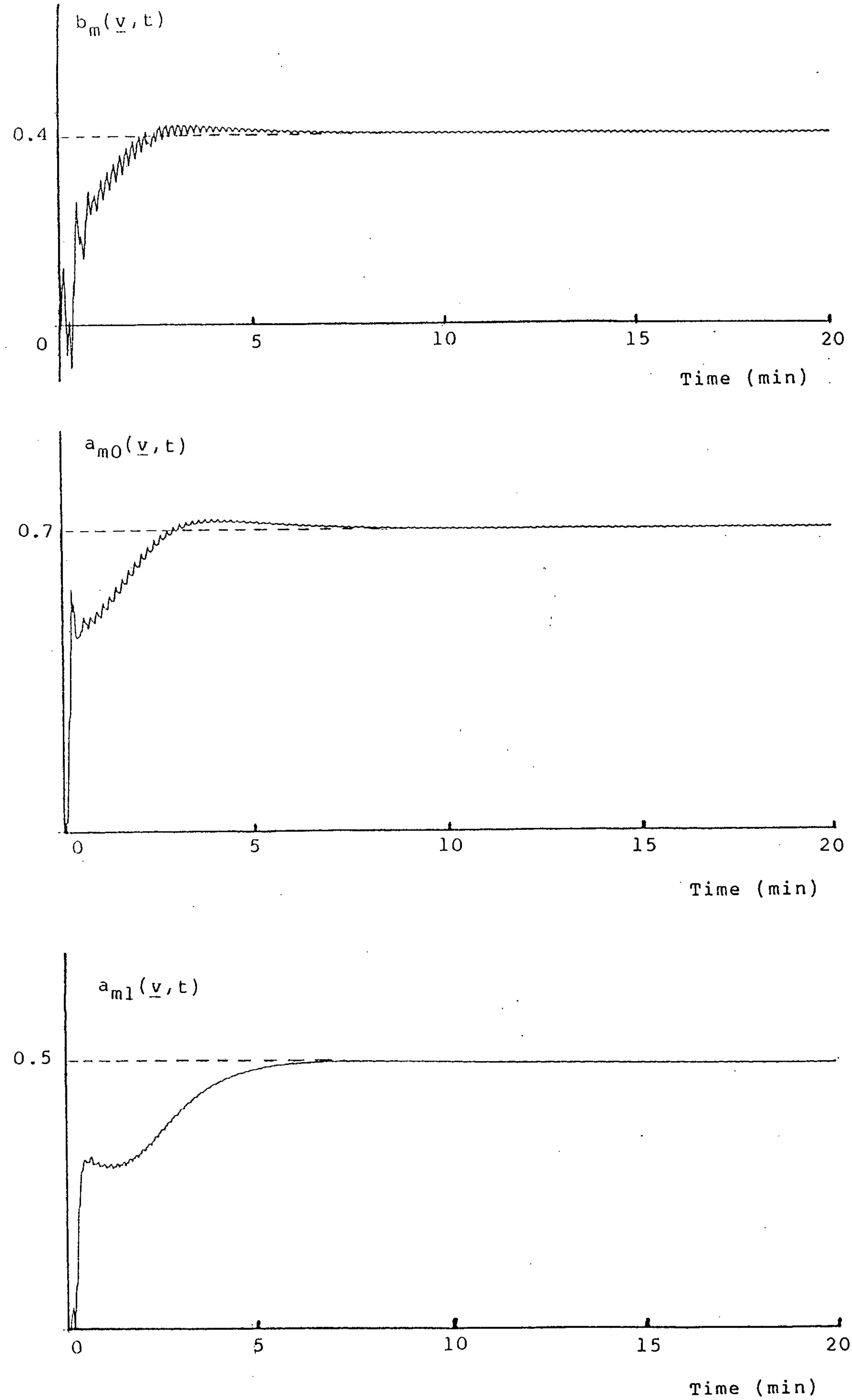


Fig. 3.2-9 Results of Example 3.2-8.

3.2.2 Digital computer simulation

The digital computer has become the most popular tool for simulating modern control systems. This Section will deal with the simulation of the parallel identifier designed in Section 2.2 on the HP 85.

The differential equations characterising the MRAS are solved by means of the fourth order Runge-kutta numerical method. For a brief review of this method, the reader is referred to Appendix D.

The digital computer program simulating the identifier can be seen in Appendix E.

In order to investigate the performance of the parallel identifier, the following example was implemented on the HP 85.

EXAMPLE 3.2-9

The plant was selected to have the following parameters

$b_p = 1$

$a_{p0} = 2$

$a_{p1} = 2$

The input waveform is a 2 Hz square waveform.

The linear compensator matrix was selected to be

$$D = \begin{bmatrix} 2.5 & 0.5 \\ 0.5 & 0.8 \end{bmatrix}$$

The identification gain was chosen to be 100.

Results obtained for the above example can be seen in the figure below.

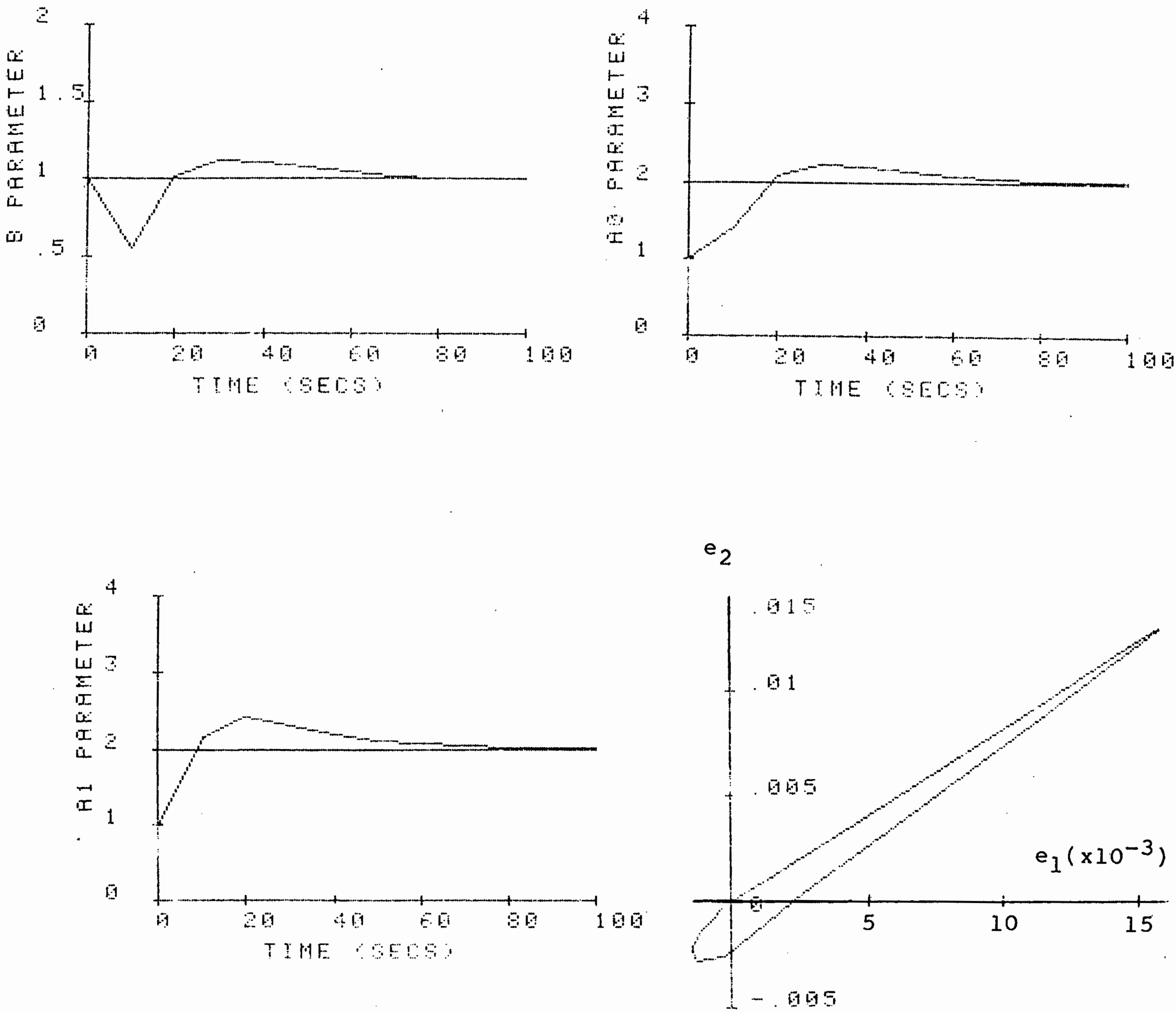


Fig. 3.2-10 Results of Example 3.2-9.

From these results and many other results performed on the digital simulator, one can see that the digital simulator offers perfect asymptotic parameter convergence. Results obtained using the digital simulator verified the conditions stated in section 3.2.1. -i.e.

- very accurate plant identification is achieved if the identification algorithm makes use of integral control laws only.

- the accuracy of plant identification is dependent on the nature of the input waveform.

- the identification gain determines the speed of identification.

3.3 Implementation of the series-parallel identifier

The second order plant to be identified is characterised by the following differential equation

$$\frac{d^2 y_p}{dt^2} + a_{p1}\frac{dy_p}{dt} + a_{p0} y_p = b_p u \qquad (3.3\text{-}1)$$

where b_p, a_{p0} and a_{p1} are the unkown plant parameters, u is the plant input and y_p is the plant output.

Expressing Eq. (3.3-1) in state space notation, one obtains

State equation $\qquad \dot{\underline{x}}_p = A_p \underline{x}_p + B_p \underline{u} \qquad (3.3\text{-}2)$

Output equation $\qquad y_p = x_{p1} \qquad (3.3\text{-}3)$

where : $A_p = \begin{bmatrix} 0 & 1 \\ -a_{p0} & -a_{p1} \end{bmatrix}$ and $B_p = \begin{bmatrix} 0 \\ b_p \end{bmatrix}$

$\underline{x}^T{}_p = (x_{p1}, x_{p2})$ is the state vector of the plant.
y_p is the plant output.

The series-parallel estimation model is defined in state space notation as

State equation $\qquad \dot{\underline{x}}_m = A_m(\underline{v},t)\, \underline{x}_p + B_m(\underline{v},t)\, \underline{u} - K\, \underline{e} \qquad (3.3\text{-}4)$

Output equation $\qquad y_m = x_{m1} \qquad (3.3\text{-}5)$

where : $A_m = \begin{bmatrix} 0 & 1 \\ -a_{m0} & -a_{m1} \end{bmatrix}$ and $B_m = \begin{bmatrix} 0 \\ b_m \end{bmatrix}$

y_m is the output of the series-parallel model.

In Chapter 2 it was derived that the series-parallel identifier will achieve perfect asymptotic parameter identification if the following identification algorithm is employed :

- i) $$\underline{v} = D\,\underline{e} \qquad (3.3\text{-}6)$$

where D can be computed by solving the Liapunov matrix equation

$$K^T D + D K = -Q \qquad (3.3\text{-}7)$$

where Q is an arbitrary positive definite matrix.

- ii) $$A_m(\underline{v},t) = \int_0^t F_A(t-\tau)\,\underline{v}(\tau)\,(G_A\,\underline{X}_p(\tau))^T\,d\tau + F'_A(t)\,\underline{v}(t)\,(G'_A\,\underline{X}_p(t))^T + A_m(0) \qquad (3.3\text{-}8)$$

$$B_m(\underline{v},t) = \int_0^t F_B(t-\tau)\,\underline{v}(\tau)\,(G_B\,\underline{u}(\tau))^T\,d\tau + F'_B(t)\,\underline{v}(t)\,(G'_B(t)\,\underline{u}(t))^T + B_m(0) \qquad (3.3\text{-}9)$$

where : $F_A(t-\tau)$ and $F_B(t-\tau)$ are positive definite matrix kernels
G_A and G_B are positive definite constant matrices.
$F'_A(t)$, $F'_B(t)$, $G'_A(t)$ and $G'_B(t)$ are positive definite matrices for all $t>0$.

As in the case of the parallel identifier, the technique of tuning had to be employed to determine which identification mechanism consistently produced the best results.

The following identification algorithm was generally found to produce the best results:

$$A_m(\underline{v},t) = \int_0^t g_i\,I\,\underline{v}(\tau)\,(\underline{X}_p(\tau))^T\,d\tau + g_p\,I\,\underline{v}(t)\,(\underline{X}_p(t))^T + A_m(0) \qquad (3.3\text{-}10)$$

$$B_m(\underline{v},t) = \int_0^t f_i\,I\,\underline{v}(\tau)\,(\underline{u}(\tau))^T\,d\tau + f_p\,I\,\underline{v}(t)\,(\underline{u}(t))^T + B_m(0) \qquad (3.3\text{-}11)$$

where : I is the identity matrix (2x2 dimensional).
g_i and f_i are the integral identification gains.
g_p and f_p are the proportional identification gains.

The expressions for the three parameters to be identified can thus be expressed as follows:

$$a_{m0}(\underline{v},t) = - \int_0^t g_i \; v_2(\tau) \; X_{p1}(\tau) \; d\tau - g_p \; v_2(t) \; X_{p1}(t) + a_{m0}(0) \tag{3.3-12}$$

$$a_{m1}(\underline{v},t) = - \int_0^t g_i \; v_2(\tau) \; X_{p2}(\tau) \; d\tau - g_p \; v_2(t) \; X_{p2}(t) + a_{m1}(0) \tag{3.3-13}$$

$$b_m(\underline{v},t) = \int_0^t f_i \; v_2(\tau) \; u(\tau) \; d\tau + f_p \; v_2(t) \; u_2(t) + b_m(0) \tag{3.3-14}$$

where : $a_{m0}(0)$, $a_{m1}(0)$ and $b_m(0)$ are the initial model parameters
$\underline{v}(t)^T = (v_1(t), v_2(t))$
$\underline{u}(t)^T = (u_1(t), u_2(t))$

The series-parallel model and the plant were implemented on a VIDAC 169 analog computer, whilst the identification algorithm was implemented on an AEI-TR48 analog computer.

A complete analog patch diagram of the series-parallel identifier is given in Fig. (3.3-1).

A general analysis of the performance of the series-parallel identifier will now be investigated by looking at various identification examples.

The first three examples that we will look at assume that two of the three plant parameters are known.

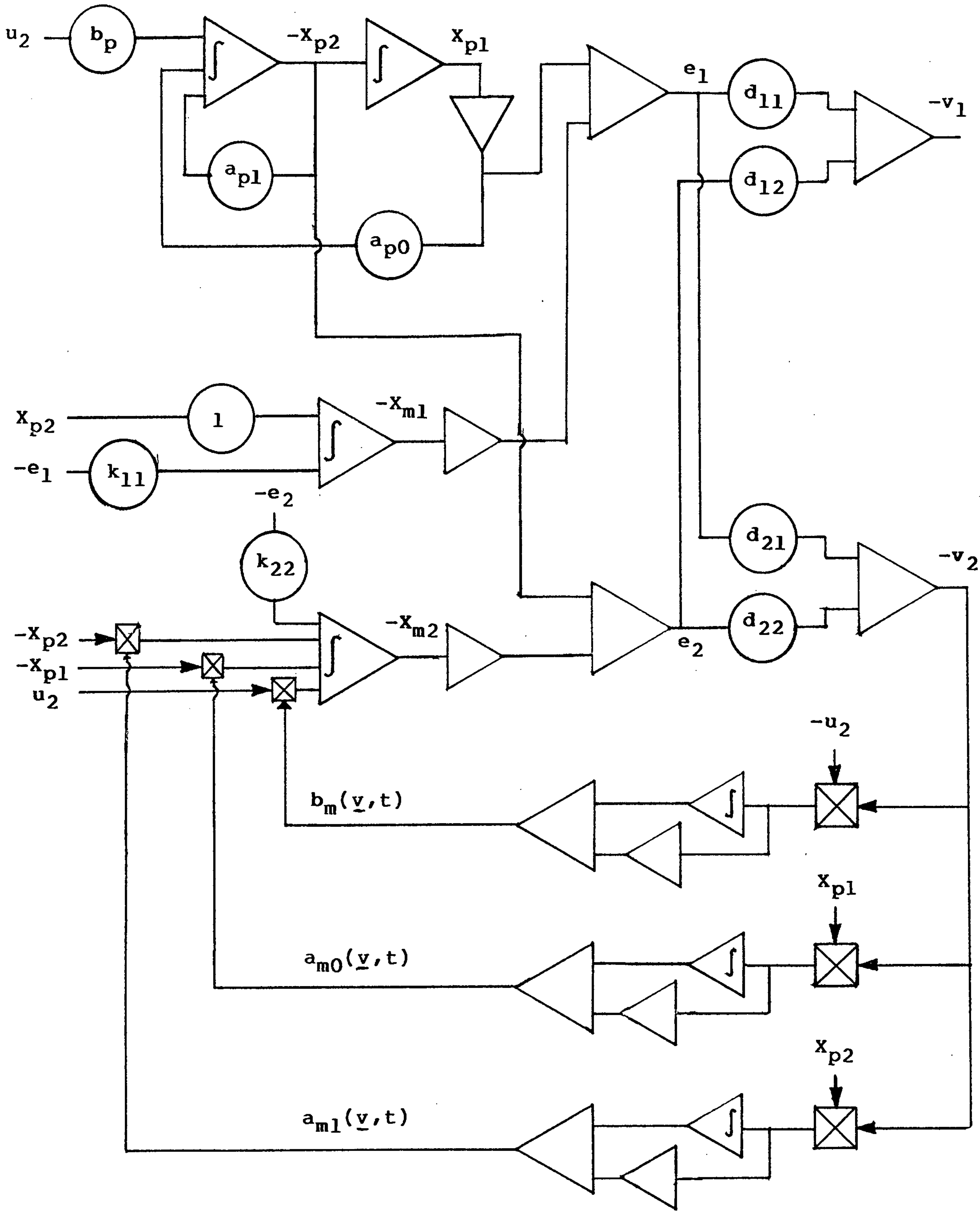


Fig. 3.3-1 Analog computer patch diagram of the series-parallel identifier.

EXAMPLE 3.3-1

In this example, the plant parameters a_{p0} and a_{p1} are known and b_p is to be identified.

The plant parameters were chosen to be

$$b_p = 0.3$$
$$a_{p0} = 0.6$$
$$a_{p1} = 0.5$$

The following identification algorithm was used :

$$b_m(\underline{v},t) = \int_0^t v_2(\tau)\ u_2(\tau)\ d\tau + v_2(t)\ u_2(t)$$
$$a_{m0}(\underline{v},t) = a_{p0}$$
$$a_{m1}(\underline{v},t) = a_{p1}$$

The plant/model input was chosen to be a 0.2 Hz square waveform.

To find the value of the linear compensator matrix, D, that ensures perfect asymptotic stability, the Liapunov matrix equation (Eq. (3.3-7)) has to be solved. If one selects the positive definite matrix Q = 10.I and K = -I, then the positive definite matrix D obtained is

$$D = 10.I$$

The results for the above example are shown in Fig. 3.3-2

EXAMPLE 3.3-2

In this example, the plant parameters b_p and a_{p1} are known whilst a_{p0} is to be identified.

The plant parameters chosen were identical to those of the previous example and the identification mechanism is

$$b_m(\underline{v},t) = b_p$$

$$a_{m0}(\underline{v},t) = -\int_0^t v_2(\tau)\ x_{p1}(\tau)\ d\tau - v_2(t)\ x_{p1}$$
$$a_{m1}(\underline{v},t) = a_{p1}$$

The same plant/model input and linear compensator matrix used in the previous example were again used.

The results obtained are shown in Fig. 3.3-3

EXAMPLE 3.3-3

In this example, the plant parameters b_p and a_{p0} are known whilst the parameter a_{p1} is to be identified.

The plant/model input, plant parameters and linear compensator matrix used in this example are identical to those used in the previous two examples.

The identification algorithm was chosen to be

$$b_m(\underline{v},t) = b_p$$
$$a_{m0}(\underline{v},t) = a_{p0}$$
$$a_{m1}(\underline{v},t) = -\int_0^t v_2(\tau)\ x_{p2}(\tau)\ d\tau - v_2(t)\ x_{p2}(t)$$

The results obtained for the above example are given in Fig. 3.3-4

The above three examples illustrate that very fast and accurate plant identification can be achieved when identifying the plant parameters independently. These three examples basically serve as a proof that the identification algorithm for the series-parallel identifier does work. The real test is to however try and identify all three plant parameters simultaneously. This will be looked at in the next examples.

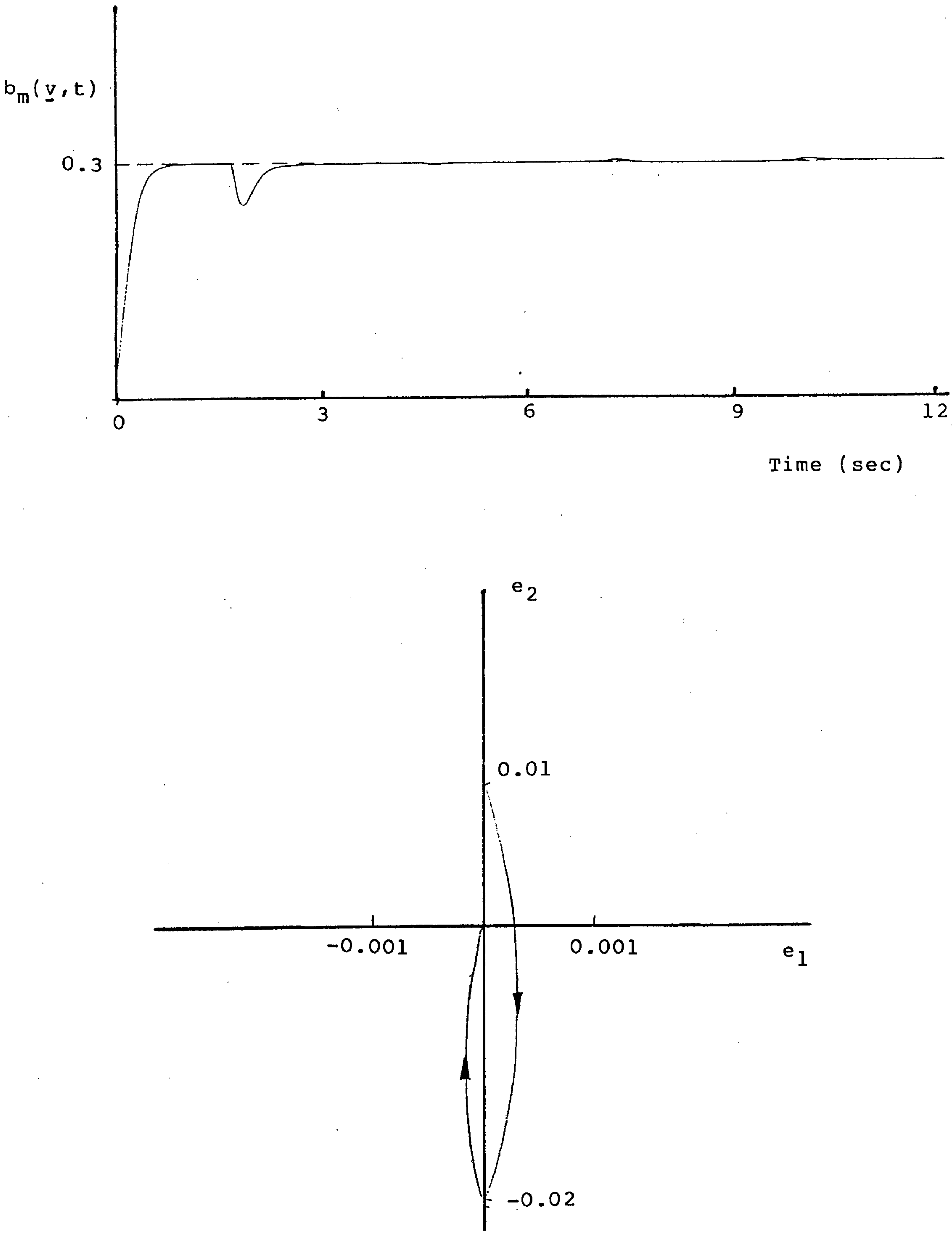


Fig. 3.3-2 Results of Example 3.3-1.

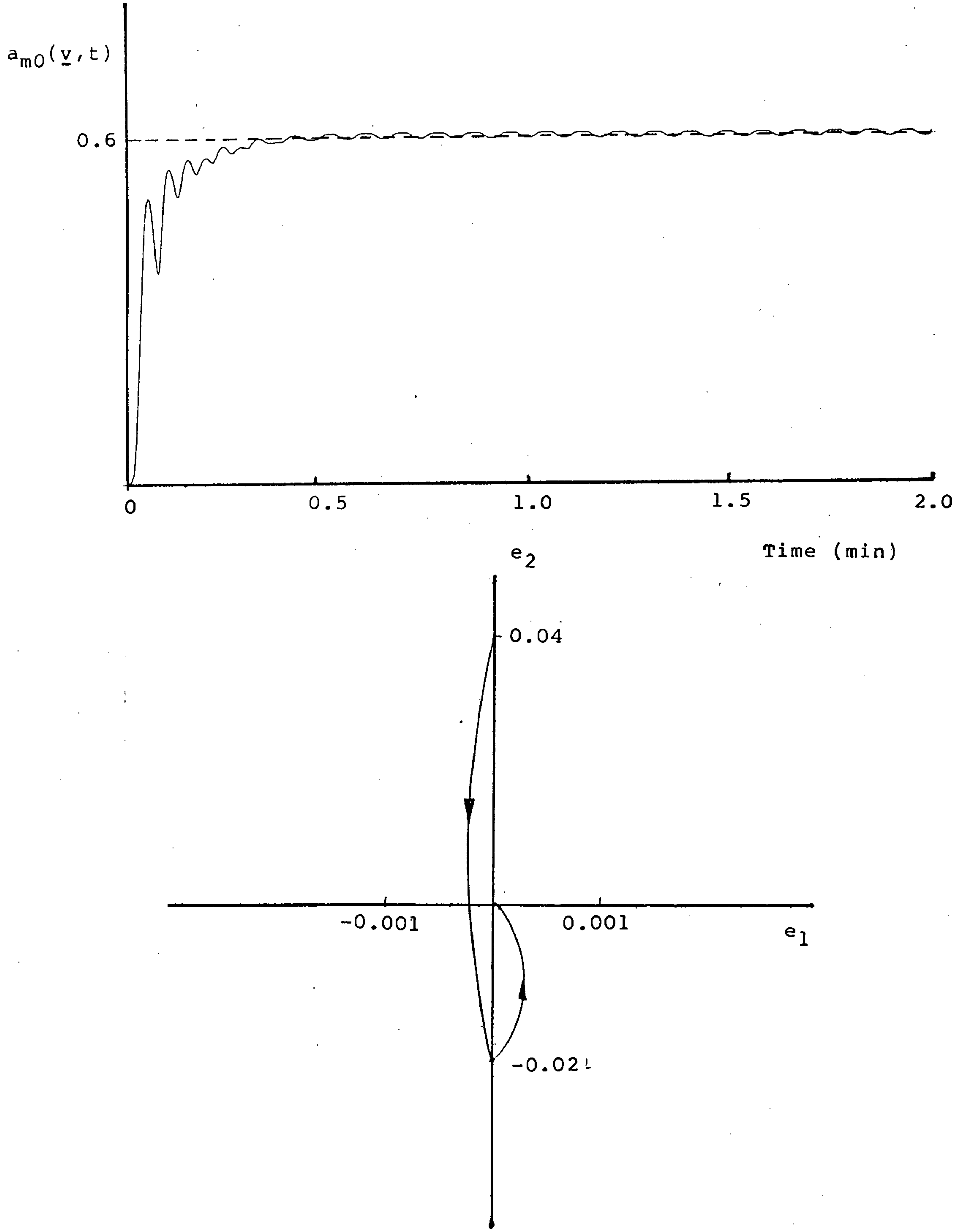


Fig. 3.3-3 Results of Example 3.3-2.

Method B

A similar method to that described by Silberman and Fisher [30] was used for the dissolution of fly ash and coal ash samples.

About 0.5 g aliquots of the samples were transferred to 250 ml polypropylene volumetric flasks and 10 ml of water added to wet the samples. 10 ml 48% HF were added from plastic tipped pipettes; the flasks were sealed and shaken overnight (about 15 hours) at low speed, with the flasks half immersed in a water bath kept at 70 $\pm$ 2°C. After cooling, 70 ml of saturated (at room temperature) aqueous boric acid solution as added, followed by about 4 more hours of shaking in the water bath. After cooling, the solutions were transferred to 200 ml volumetric flasks and diluted to the mark with water. Further dilutions were required for the determination of Fe, Al and Si. Black residues were obtained and these were filtered off. All solutions were stored in plastic bottles.

The instrumental parameters are shown in table 5.2. It was found more convenient to use 6 analytical programmes combining elements as follows:

1	Ca, Mg, Ti	4	Fe
2	Al, Si	5	Na
3	Mn, Ba	6	P

Table 5.2 Method B, Instrumental Parameters

Elements	Spectral line (nm)	Torch Power	Nebulizer pressure (psi)	Obs height (mm)	Sample flow rate (ml/min)
Ca	315.89	4	30	16	1.2
Mg	285.21	4	30	18	1.2
Ti	334.94	4	30	18	1.2
Al	396.15	3	25	14	1.0
	308.22	3	25	14	1.0
Si	288.16	3	25	14	1.0
	251.61	3	35	16	1.0
Mn	257.61	4	30	18	1.2
Ba	493.41	4	30	16	1.2
Fe	259.94	4	30	16	1.2
Na	589.00	0	40	6	1.5
P	213.62	5	25	14	1.5

Calibration of the spectrometer was done using mixed standards containing exactly the same final amounts of acids as the samples. For Si and Al two spectral lines for each element were used for comparison; there was no noticeable difference between the two sets of data. Results for Si and Al determination by this method in the NBS-SRM 1633a is a mean of the two sets of results obtained using both lines. Each determination is a mean value of six separate 5 seconds integration time readings. Three separate analyses (sample dissolutions and standard preparations) were done for the NBS-SRM 1633a sample and two for the other samples.

Estimated detection limits for the elements determined by this method are listed in table 5.3.

The samples analysed by this method are NBS-SRM 1633a, PFA 4, FA2, CA1.

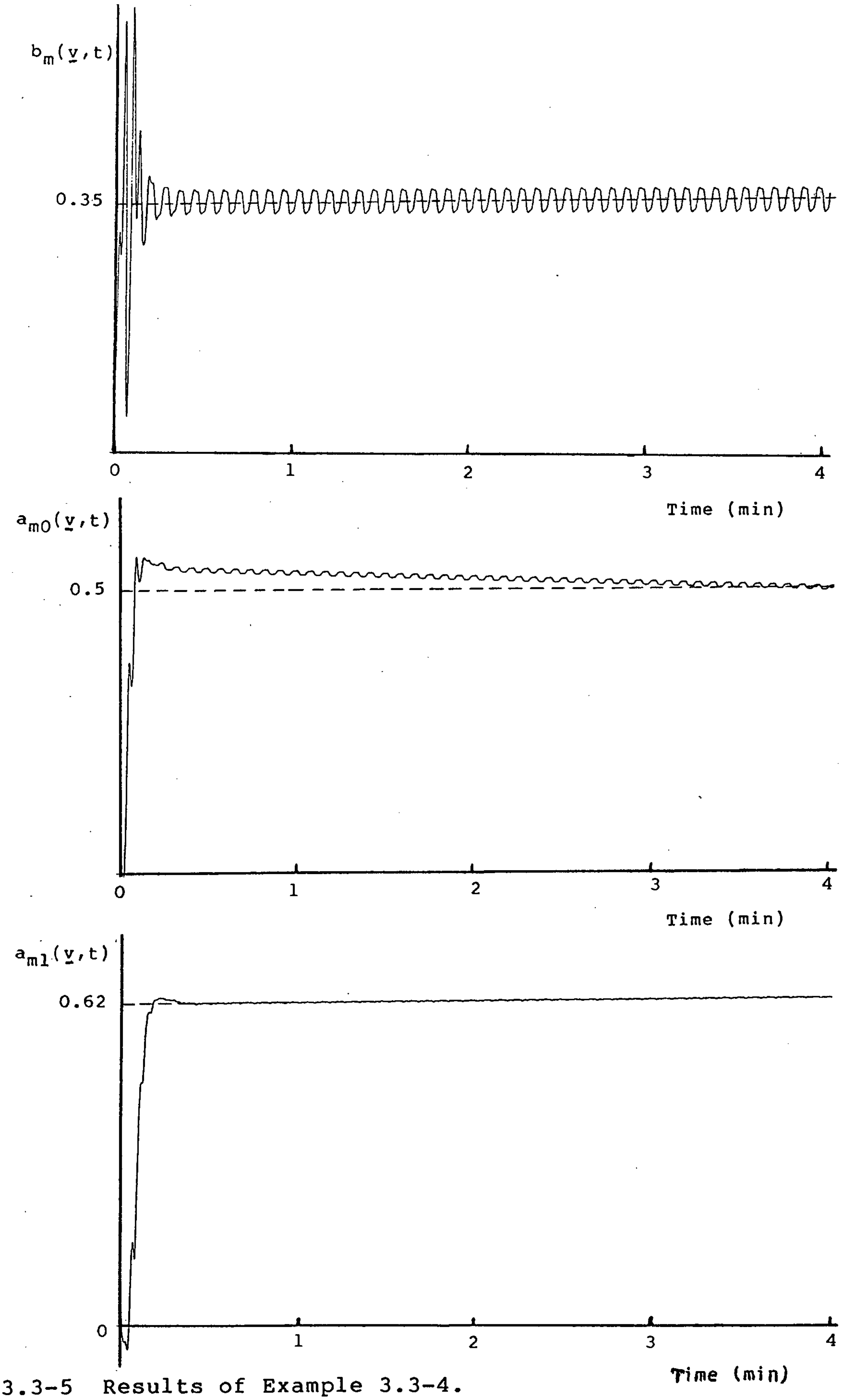


Fig 3.3-5 Results of Example 3.3-4.

EXAMPLE 3.3-5

This example is identical to the previous one except that the proportional term in the identification mechanism is set to zero. The identification laws thus become

$$b_m(\underline{v},t) = \int_0^t v_2(\tau)\, u_2(\tau)\, d\tau$$

$$a_{m0}(\underline{v},t) = -\int_0^t v_2(\tau)\, x_{p1}(\tau)\, d\tau$$

$$a_{m1}(\underline{v},t) = -\int_0^t v_2(\tau)\, x_{p2}(\tau)\, d\tau$$

The results from this example are shown in Fig. 3.3-6. From these results it is evident that although the proportional term in the identification mechanism does not detrimentally affect the stability of the system, it degrades the accuracy of plant identification.

Another point which will be investigated is the affect of different plant/model input waveforms on the accuracy of identification. In Section 2.3.2 (b), it was stated that asymptotic parameter convergence will occur if the input waveform contained a large frequency content. A basic Fourier analysis of the square waveform used in all of the previous examples shows that the square waveform does contain a large frequency content and thus accurate identification is achieved. The next example will look at the use of an input waveform that does not satisfy the above condition.

EXAMPLE 3.3-6

This example is set out in the same way as the previous example except that now the input waveform is a 0.2 Hz sinusoid.

The results from this example are shown in Fig. 3.3-7.

From these results one notices that when the input waveform is changed from a square to a sinusoidal waveform, the accuracy of identification decreases. This is obviously due to the fact that a sinusoid only contains one distinct frequency component, thus violating the necessary condition for asymptotic parameter convergence that was expressed in Section 2.3.2 (b).

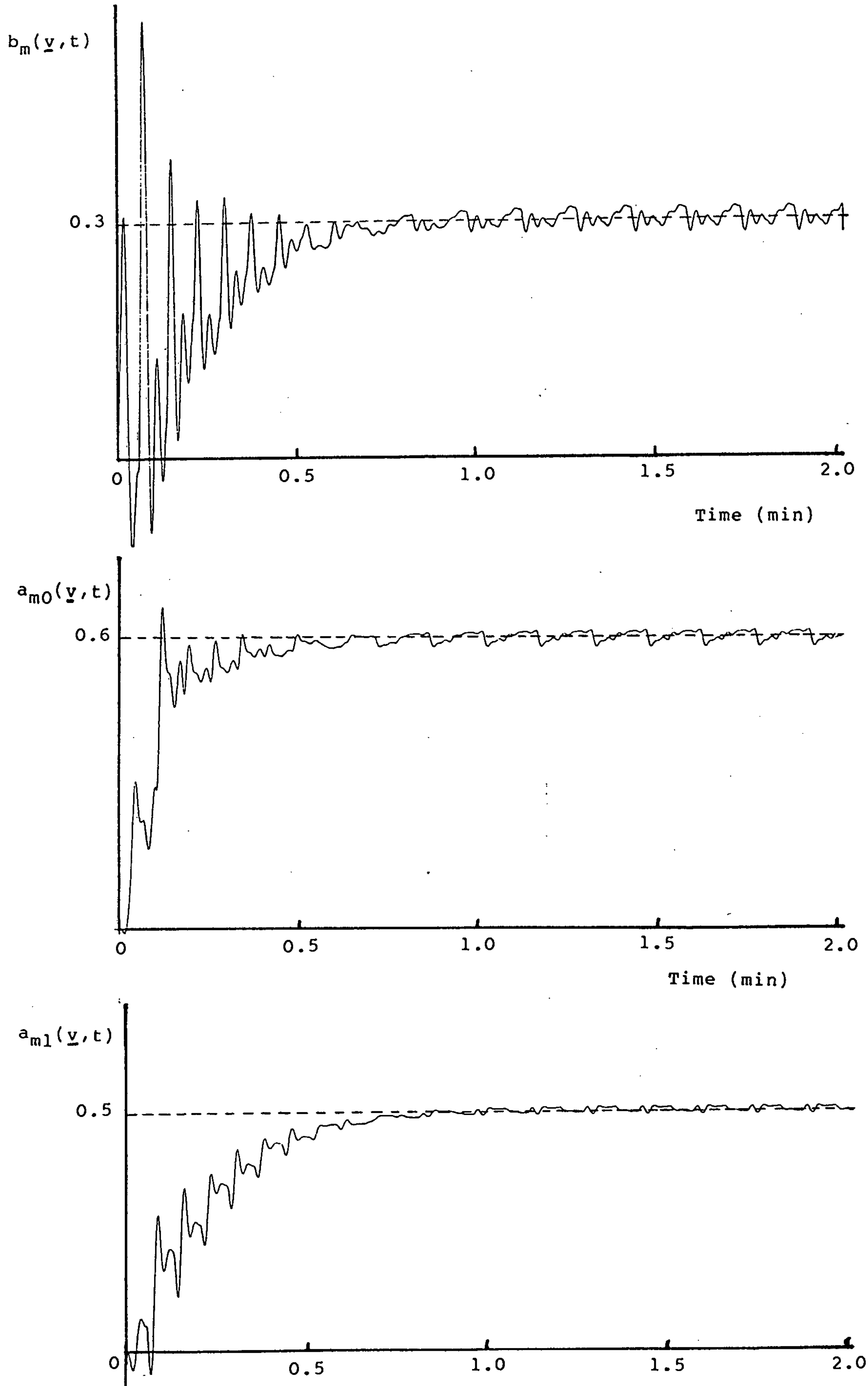


Fig. 3.3-6 Results of Example 3.3-5

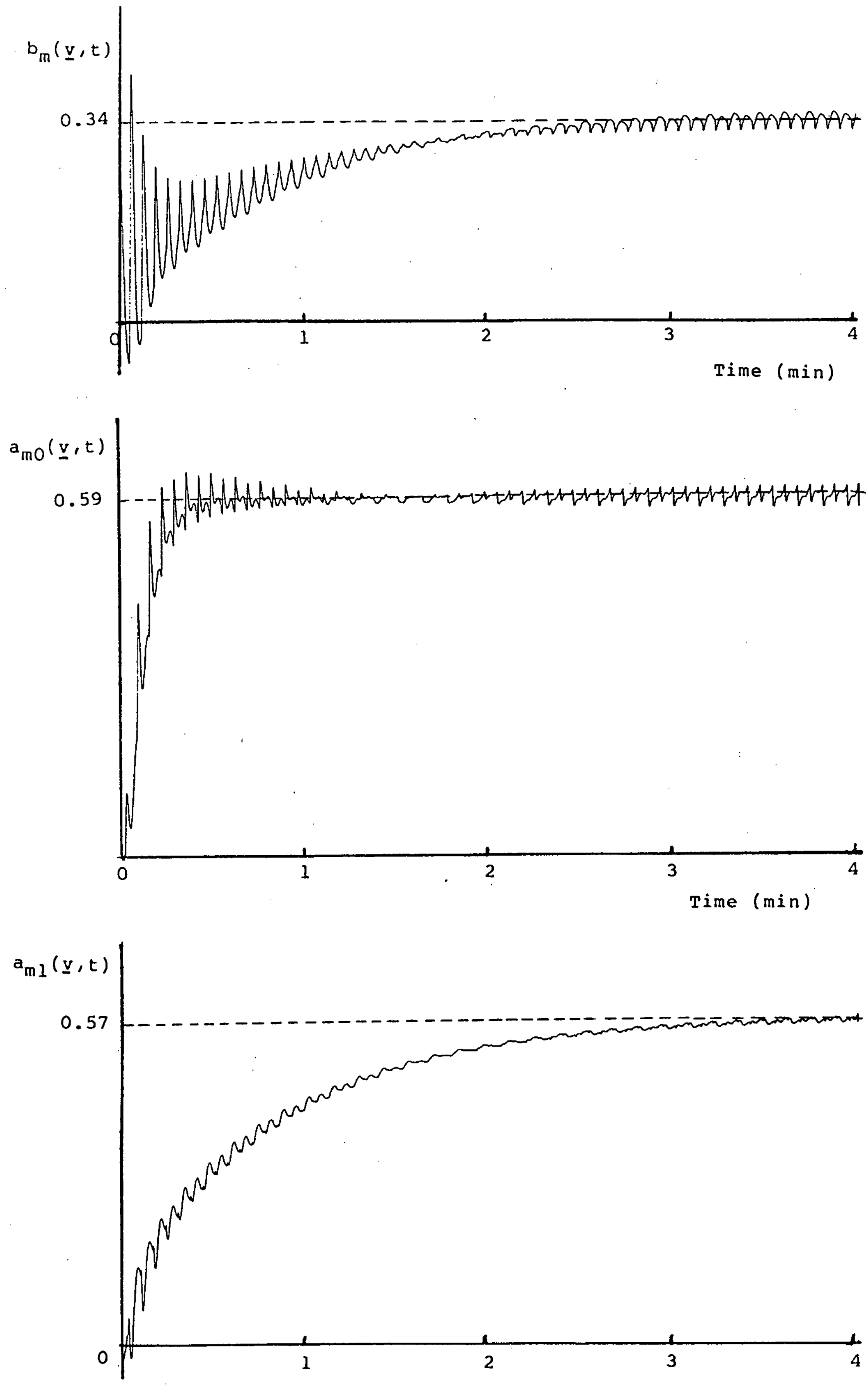


Fig. 3.3-7 Results of Example 3.3-6.

3.4 Conclusion

A brief summary of the main results arrived at in this chapter follows.

- Parallel Identifiers:

i) To achieve accurate plant identification, it was found that the identification algorithm had to contain integral control laws only. This ensured that the identification mechanism contained memory - the ideal parameter values, once found, would be memorised by the algorithm.

ii) The best identification results were achieved when the plant/model input waveform could be decomposed into a large number of sinusoidal components. It was impossible to identify any plant when the plant/model input waveform was a sinusoid.

iii) A decrease in the identification gain caused a decrease in the speed of identification. It however increased the accuracy of identification. In Example 3.2-5, it was noted that the model parameters oscillated about the unknown plant parameters. In Example 3.2-8, where the identification gain was decreased, it was noted that the model parameters approached the unknown plant parameters more accurately. One can thus conclude that in the case of the parallel identifier, if the identification gain is kept low then the identification will be more precise. This point was theoretically observed in Section 2.2.3.

iv) Another important point that was observed in this chapter was that the linear compensator is merely a sufficient stability condition and not a necessary stability condition. This can be seen by the fact that in many of the examples looked at, the system remained stable irrespective of the linear compensator being used.

- Series-parallel identifier:

  i) For accurate parameter identification it was observed that, as in the case of the parallel identifier, the identification algorithm had to contain integral control laws only.

  ii) The precision of identification was dependent on the nature of the plant/model input signal. This signal had to be rich in sinusoidal content. A square waveform input was thus found to achieve good parameter identification.

  iii) One of the advantages of series-parallel identifiers over parallel identifiers is the fact that the computation of the linear compensator matrix does not involve having to have any prior knowledge of the plant parameters. One can thus compute the linear compensator matrix directly by solving the Liapunov equation. This is however not such a major advantage due to the fact that the stability condition imposed by the linear compensator matrix is a sufficient and not a necessary stability condition.

On comparing the overall results obtained for the parallel and series-parallel identifier one observes that the parallel identifier offered

- more accurate identification (especially at low identification gains).

- less oscillatory parameter responses.

One can thus conclude that for the purposes of parameter identification, the parallel identifier is slightly superior to the series-parallel identifier.

The other aspect that this chapter looked at was the performance of a digital simulator as opposed to the normal analog computer approach. From various examples tried out on the digital simulator, it was found that it performed much better than its analog computer counterpart.

This is due to the following imperfections of the analog computer:

- To achieve faster identification, the identification gain had to be increased. This could be easily achieved on the digital simulator but caused opamp saturation problems when implemented on the analog computer.

- Some of the opamps on the analog computer contained drift and input offset voltage problems. Although these effects were minor and did not effect to a great degree the precision of identification, they affected the speed of identification.

- Multiplier inaccuracies also caused definite degradation of results, especially when the multipliers had to handle signals containing high frequency components.

The digital simulator can implement every analog computer device ideally. This results in a far superior and easier to implement system.

CHAPTER 4

DISCRETE-TIME MRAS DESIGN

4.1 Introduction

The implementation of continuous-time MRAS's by means of analog devices is often hindered by the difficulties encountered in simulating the identification algorithm by means of analog devices.

The digital computer provides a flexible and easy means of implementing MRAS's. It is this ease of implementation coupled with the evolution of the digital computer over the last few years that has transformed discrete-time MRAS's into one of the most popular techniques for the design of adaptive systems.

Due to the fact that MRAS's are time-varying nonlinear systems and have an inherent one sample delay in the identification loop, the design of a discrete-time MRAS does not simply involve the discretization of the corresponding continuous-time MRAS. One is therefore forced to design specific identification algorithms for discrete-time MRAS's.

One of the most attractive features which the digital computer offers in the field of MRAS design is the ability to design identification algorithm using time-varying identification gains.

This chapter will deal with the design of a discrete-time parallel identifier using time-varying identification gains.

4.2 Design of a discrete-time parallel identifier

4.2.1 Statement of the problem

The discrete-time parallel identifier design will be described by means of difference equations [1], [44], [45].

The plant to be identified is described by the difference equation

$$X_p(k) = \sum_{i=1}^{n} a_i \; X_p(k-i) + \sum_{i=0}^{m} b_i \; u(k-i) \qquad (4.2\text{-}1)$$

where : $X_p(k)$ is the plant output at sample k.
$u(k)$ is the plant input at sample k.
$a_i(k)$ and $b_i(k)$ are the plant parameters at sample k.

Now defining

$$\underline{p}^T = (a_1, \; \ldots \; , a_n, b_0, \; \ldots \; , b_m) \qquad (4.2\text{-}2)$$

and

$$\underline{X}^T{}_{k-1} = (X_p(k-1), \; \ldots \; , X_p(k-n), \; u(k), \; \ldots \; , u(k-m)) \qquad (4.2\text{-}3)$$

Eq. (4.2-1) can be rewritten as

$$X_p(k) = \underline{p}^T \; \underline{X}_{k-1} \qquad (4.2\text{-}4)$$

The parallel estimation model is described by

$$X_m(k) = \sum_{i=1}^{n} a_i(k) \; X_m(k-i) + \sum_{i=0}^{m} b_i(k) \; u(k-i) \qquad (4.2\text{-}5)$$

Now defining

$$\underline{p}^T(k) = (a_1(k), \; ... \; , a_n(k), b_0(k), \; ... \; , b_m(k)) \tag{4.2-6}$$

and

$$\underline{Y}^T{}_{k-1} = (X_m(k-1), \; ... \; ,X_m(k-n), u(k), \; ... \; ,u(k-m)) \tag{4.2-7}$$

Eq. (4.2-5) can be rewritten as

$$X_m(k) = \underline{p}^T(k) \, \underline{Y}_{k-1} \tag{4.2-8}$$

The above equation describes the model output calculated using the values of the adjustable parameters at sample k (i.e. after the identification has taken place). This is known as the a posteriori output.

The model output calculated using the value of the adjustable parameters at sample (k-1) is known as the a priori output. The equation for the a prior model output is shown below

$$X^o{}_m(k) = \underline{p}^T(k-1) \, \underline{Y}_{k-1} \tag{4.2-9}$$

The previous two expressions show the one sample delay which is inherent in discrete-time MRAS's.

The generalised error is defined as

a priori: $$e^o(k) = X_p(k) - X^o{}_m(k) \tag{4.2-10}$$

a posterior: $$e(k) = X_p(k) - X_m(k) \tag{4.2-11}$$

By analogy to the continuous case, the identification algorithm also has a linear compensator generating a signal v(k).

a priori: $$v^o(k) = e^o(k) + \sum_{i=1}^{n} d_i\, e(k-i) \qquad (4.2\text{-}12)$$

a posteriori: $$v(k) = e(k) + \sum_{i=1}^{n} d_i\, e(k-i) \qquad (4.2\text{-}13)$$

In Chapter 3 it was noted that proportional control leads to poor identification. The identification algorithm for this discrete identifier will therefore only make use of integral control. This leads to the following identification law

$$\underline{p}(k) = \underline{p}(k-1) + \underline{I}(v(k)) = \underline{p}(-1) + \sum_{j=0}^{k} \underline{I}(v(j)) \qquad (4.2\text{-}14)$$

The design objective can thus be summarised as follows:

- find the expression for $\underline{I}(v(k))$ such that the equivalent feedback block satisfies Popov's integral inequality.
- find the value of the linear compensator such that the equivalent system is asymptotically stable.

4.2-2 An asymptotically stable parallel identifier

The identification algorithm for the parallel identifier described in Section 4.2-1 is given in theorem form below. This is followed by the proof showing that this algorithm assures that the resulting MRAS structure is globally asymptotically stable.

THEOREM 4.2-1

The parallel identifier described in Section 4.2-1 is globally asymptotically stable if the following identification algorithm is employed [1], [44].

$$\underline{p}(k) = \underline{p}(k-1) + \frac{G(k-1)\, \underline{Y}_{k-1}\, v^o(k)}{1 + \underline{Y}^T_{k-1}\, G(k-1)\, \underline{Y}_{k-1}} \qquad (4.2\text{-}15)$$

$$G(k) = G(k-1) - \frac{G(k-1)\ \underline{Y}_{k-1}\ \underline{Y}^T_{k-1}\ G(k-1)}{L + \underline{Y}^T_{k-1}\ G(k-1)\ \underline{Y}_{k-1}} \qquad (4.2\text{-}16)$$

$$v^o(k) = X_p(k) - (\underline{p}(k-1))^T\ \underline{Y}_{k-1} + \sum_{i=1}^{n} d_i\ e(k-i) \qquad (4.2\text{-}17)$$

where: $G(0)$ is an arbitrary positive definite matrix

L is a constant which allows one to weight the rate of the decrease of the identification gain matrix.

$d_1, \ldots, d_n$ are chosen such that the discrete transfer function

$$H(z) = \frac{1 + \sum_{i=1}^{n} d_i\ Z^{-i}}{1 - \sum_{i=1}^{n} a_i\ Z^{-i}} - \frac{1}{2L} \qquad (4.2\text{-}18)$$

is strictly positive real.

The above identification algorithm makes use of the popular least squares identification method.

PROOF OF THEOREM 4.2-1:

The first step in the proof involves transforming the parallel identifier into the type of equivalent feedback system considered by Popov (see Appendix A - Fig A-1).

Subtracting Eq. (4.2-5) from Eq. (4.2-1) one obtains

$$X_p(k) - X_m(k) = \sum_{i=1}^{n} (a_i X_p(k-i) - a_i(k) X_m(k-i)) + \sum_{i=0}^{m} (b_i - b_i(k)) u(k-i) \qquad (4.2\text{-}19)$$

Now adding and subtracting the term $a_i\ X_m(k-i)$ on the right hand side of Eq. (4.2-19) and using Eq. (4.2-11), one obtains

$$e(k) = \sum_{i=1}^{n} a_i\ e(k-i) + \sum_{i=1}^{n} (a_i - a_i((k)) X_m(k-i) + \sum_{i=0}^{m} (b_i - b_i(k)) u(k-i) \qquad (4.2\text{-}20)$$

This equation can be expressed at instant (k+1) as

$$e(k+1)=\sum_{i=1}^{n} a_i\, e(k+1-i) + \sum_{i=1}^{n} (a_i-a_i(k+1))X_m(k+1-i)$$

$$+\sum_{i=0}^{m} (b_i-b_i(k+1))u(k+1-i) \qquad (4.2\text{-}21)$$

Defining

$$\underline{a}^T = (a_1, \ldots, a_n) \qquad (4.2\text{-}22)$$

$$\underline{e}^T = (e(k), \ldots, e(k+1-n)) \qquad (4.2\text{-}23)$$

$$\underline{d}^T = (d_1, \ldots, d_n) \qquad (4.2\text{-}24)$$

Eq. (4.2-21) can now be rewritten as

$$e(k+1) = \underline{a}^T \underline{e} + (\underline{p}-\underline{p}(k+1))^T \underline{Y}_k \qquad (4.2\text{-}25)$$

and Eq. (4.2-13) can be expressed at instant (k+1) as

$$v(k+1) = e(k+1) + \sum_{i=1}^{n} d_i\, e(k+1-i)$$

$$= e(k+1) + \underline{d}^T \underline{e} \qquad (4.2\text{-}26)$$

In a similar manner, Eqs. (4.2-10) and (4.2-12) can be expressed at instant (k+1) as

$$e^o(k+1) = \underline{a}^T \underline{e}(k) + (\underline{p}-\underline{p}(k))^T \underline{Y}_k \qquad (4.2\text{-}27)$$

and

$$v^o(k+1) = e^o(k+1) + \underline{d}^T \underline{e} \qquad (4.2\text{-}28)$$

Subtracting Eq. (4.2-28) from Eq. (4.2-26) one obtains

$$v(k+1) - v^o(k+1) = e(k+1) - e^o(k+1) \qquad (4.2\text{-}29)$$

Now substituting Eqs. (4.2-25) and (4.2-27) into the above equation one obtains

$$v(k+1) - v^{o}(k+1) = (\underline{p}(k)-\underline{p}(k+1))^{T}\ \underline{Y}_{k} \qquad (4.2\text{-}30)$$

Inserting the identification law described by Eq. (4.2-15) into Eq. (4.2-30) one obtains

$$v(k+1) - v^{o}(k+1) = -\frac{\underline{Y}^{T}_{k}\ G(k)\ \underline{Y}_{k}\ v^{o}_{k+1}}{1 + \underline{Y}^{T}_{k}\ G(k)\ \underline{Y}_{k}} \qquad (4.2\text{-}31)$$

Solving the above equation for v(k+1) one obtains

$$v(k+1) = \frac{v^{o}_{k+1}}{1 + \underline{Y}^{T}_{k}\ G(k)\ \underline{Y}_{k}} \qquad (4.2\text{-}32)$$

Substituting the above equation into Eq. (4.2-15) at instant (k+1) one obtains

$$\underline{p}(k+1) = \underline{p}(k) + G(k)\ \underline{Y}_{k}\ v(k+1) \qquad (4.2\text{-}33)$$

If one now defines an auxiliary variable

$$\underline{q}(k) = \underline{p}(k) - \underline{p} \qquad (4.2\text{-}34)$$

Therefore

$$\underline{q}(k+1) = \underline{q}(k) + G(k)\ \underline{Y}_{k}\ v(k+1) \qquad (4.2\text{-}35)$$

Defining

$$\begin{aligned} w(k+1) &= (\underline{p}(k+1) - \underline{p})^{T}\ \underline{Y}_{k} \\ &= \underline{Y}^{T}_{k}\ (\underline{p}(k+1) - \underline{p}) \qquad (4.2\text{-}36) \end{aligned}$$

and inserting Eqs. (4.2-24) and (4.2-34) into the above equation one obtains

$$w(k+1) = \underline{Y}^T{}_k\ \underline{q}(k) + \underline{Y}^T{}_k\ G(k)\ \underline{Y}_k\ v(k+1) \qquad (4.2\text{-}37)$$

Using Eqs. (4.2-25) and (4.2-26), one can define the equivalent feedback system as follows

- Linear time-invariant feedforward block:

$$e(k+1) = \underline{a}^T\ \underline{e} - w(k+1) \qquad (4.2\text{-}38)$$

$$v(k+1) = e(k+1) + \underline{d}^T\ \underline{e} \qquad (4.2\text{-}39)$$

- Nonlinear time-varying feedback block:

$$\underline{q}(k+1) = \underline{q}(k) + G(k)\ \underline{Y}_k\ v(k+1) \qquad (4.2\text{-}40)$$

$$w(k+1) = \underline{Y}^T{}_k\ \underline{q}(k) + \underline{Y}^T{}_k\ G(k)\ \underline{Y}_k\ v(k+1) \qquad (4.2\text{-}41)$$

If we now define a new variable $v^*(k+1)$ as

$$v^*(k+1) = v(k+1) + \frac{w(k+1)}{2L} \qquad (4.2\text{-}42)$$

The equivalent feedback system is thus now represented as follows

- Linear time-invariant feedback block:

$$e(k+1) = \underline{a}^T\ \underline{e} - w(k+1) \qquad (4.2\text{-}43)$$

$$v^*(k+1) = e(k+1) + \underline{d}^T\ \underline{e} + \frac{w(k+1)}{2L} \qquad (4.2\text{-}44)$$

- Nonlinear time-varying feedback block:

$$\underline{q}(k+1) = \underline{q}(k) + G(k)\ \underline{Y}_k(v^*(k+1) - \frac{w(k+1)}{2L}) \qquad (4.2\text{-}45)$$

$$w(k+1) = \underline{Y}^T{}_k\ \underline{q}(k) + \underline{Y}^T{}_k\ G(k)\ \underline{Y}_k(v^*(k+1) - \frac{w(k+1)}{2L}) \qquad (4.2\text{-}46)$$

The next step of the proof involves showing that the equivalent feedback block is such that it satisfies Popov's integral inequality:

$$\sum_{k=0}^{k_1} w(k+1)\ v^*(k+1) \geqslant -c^2 \qquad (4.2\text{-}47)$$

where c^2 is an arbitrary positive finite constant and k_1 is greater than zero.

In order to show that the equivalent feedback block (i.e. Eqs. (4.2-45) and (4.2-46)) satisfies Popov's integral inequality, one decomposes this feedback block into two blocks connected by feedback and then evaluates the sum of the input/output products for each of these blocks.

Due to the fact that the sum of the input/output products is independent of the input/output sequence, one can define a new input/output sequence:

$$u_k = v^*(k+1) \qquad (4.2\text{-}48)$$

$$o_k = w(k+1) \qquad (4.2\text{-}49)$$

The feedback block described by Eqs. (4.2-45) and (4.2-46) can now be expressed as follows

$$\underline{q}(k+1) = \underline{q}(k) + G(k)\ \underline{Y}_k\ (u_k - \frac{o_k}{2L}) \qquad (4.2\text{-}50)$$

and

$$o_k = \underline{Y}^T{}_k\ \underline{q}(k) + \underline{Y}^T{}_k\ G(k)\ \underline{Y}_k\ (u_k - \frac{o_k}{2L}) \qquad (4.2\text{-}51)$$

This feedback block is now decomposed into a feedforward block and a feedback block:

- The feedforward block is described as follows

$$\underline{q}(k+1) = \underline{q}(k) + G(k)\, \underline{Y}_k\, u^1{}_k \qquad (4.2\text{-}52)$$

$$o^1{}_k = o_k = \underline{Y}^T{}_k\, \underline{q}(k) + \underline{Y}^T{}_k\, G(k)\, \underline{Y}_k\, u^1{}_k \qquad (4.2\text{-}53)$$

where

$$u^1{}_k = u_k - \frac{o^1{}_k}{2L} \qquad (4.2\text{-}54)$$

and $u^1{}_k$ and $o^1{}_k$ are the input and output respectively of the feedforward block.

- The feedback block is described as follows

$$o^2{}_k = \frac{o^1{}_k}{2L} = \frac{u^2{}_k}{2L} \qquad (4.2\text{-}55)$$

where $u^2{}_k$ and $o^2{}_k$ are the input and output respectively of this second block.

The two blocks are schematically represented by Fig. (4.2-1).

Using the above relations, Popov's integral inequality becomes

$$\begin{aligned} \sum_{k=0}^{k_1} w(k+1)\, v^*(k+1) &= \sum_{k=0}^{k_1} o_k\, u_k \\ &= \sum_{k=0}^{k_1} o_k\, (u^1{}_k + u^2{}_k) \\ &= \sum_{k=0}^{k_1} o^1{}_k u^1{}_k + \sum_{k=0}^{k_1} o^2{}_k u^2{}_k \geq -c^2 \qquad (4.2\text{-}56) \end{aligned}$$

The left hand side of the above inequality has thus been decomposed into two sums of input/output products, each sum corresponding to one of the blocks in which the equivalent feedback system has been decomposed.

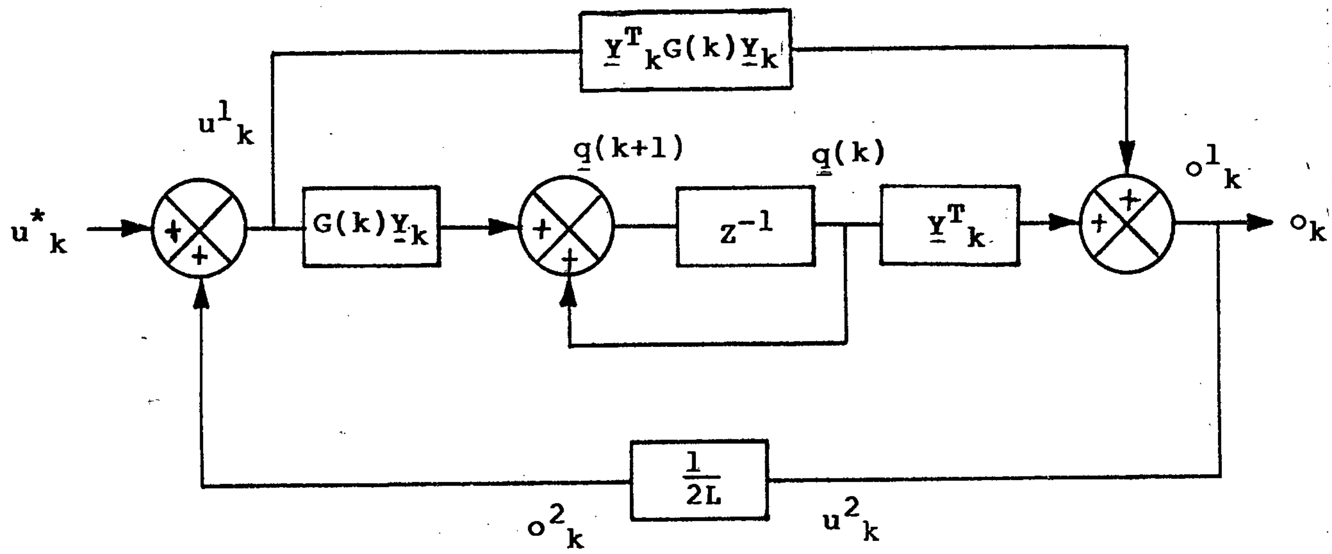


Fig. 4.2-1: Representation of the two blocks formed from the decomposition of the equivqlent feedback block given by Eqs. (4.2-50) and (4.2-51).

The two sum terms found in Eq. (4.2-56) will now be evaluated separately.

- First term:

To evaluate this term, Lemma (A-5) will be used.

The correspondence between the notation used in the Lemma and the notation used in this proof is given below:

$$\underline{X}_k = \underline{q}(k) \quad ; \quad A_k = I \quad ; \quad B_k = G(k)\ \underline{Y}_k$$

$$\underline{C}_k = \underline{Y}^T_k \quad ; \quad D_k = \underline{Y}^T_k\ G(k)\ \underline{Y}_k \qquad (4.2\text{-}57)$$

To evaluate the sum term using Eq. (A-28), one must first find the matrices P_k, Q_k, R_k and S_k such that the equations in Lemma (A-4) are satisfied.

$$S^T_k = \underline{Y}^T_k - G(k)\underline{Y}^T_k(G^{-1}(k) + \frac{\underline{Y}_k\underline{Y}^T_k)}{L}$$

$$= \frac{(\underline{Y}^T_k G(k)\underline{Y}_k)\underline{Y}^T_k}{L} \qquad (4.2\text{-}67)$$

$$R_k = 2\underline{Y}^T_k G(k)\underline{Y}_k - G(k)\underline{Y}^T_k(G^{-1}(k) + \tfrac{1}{L}\underline{Y}_k\underline{Y}^T_k)G(k)\underline{Y}_k$$

$$= 2\underline{Y}^T_k G(k)\underline{Y}_k - G(k)\underline{Y}^T_k(G^{-1}(k+1) - \tfrac{1}{L}\underline{Y}_k\underline{Y}^T_k + \tfrac{1}{L}\underline{Y}_k\underline{Y}^T_k)G(k)\underline{Y}_k$$

$$= 2\underline{Y}^T_k G(k)\underline{Y}_k - \underline{Y}^T_k(I + \tfrac{1}{L}G(k)\underline{Y}_k\underline{Y}^T_k)G(k)\underline{Y}_k$$

$$= \underline{Y}^T_k G(k)\underline{Y}_k - \tfrac{1}{L}(\underline{Y}^T_k G(k)\underline{Y}_k)^2 \qquad (4.2\text{-}68)$$

Using Eq. (A-28) of Lemma A-5 one can express the first term to be

$$\sum_{k=0} o^1_k\, u^1_k = \tfrac{1}{2}\underline{q}^T(k_1+1)G^{-1}(k_1+1)\underline{q}(k_1+1) - \tfrac{1}{2}\underline{q}^T(0)G^{-1}(0)\underline{q}(0)$$

$$-\frac{1}{2L}\sum_{k=0}^{k_1}(\underline{q}^T(k)\underline{Y}_k\underline{Y}^T_k\underline{q}(k) + 2u^1_k\underline{Y}^T_k G(k)\underline{Y}_k\underline{Y}^T_k\underline{q}(k)$$

$$+ u^1_k(\underline{Y}^T_k G(k)\underline{Y}_k)^2 u^1_k) + \tfrac{1}{2}\sum_{k=0}^{k_1} u^1_k\underline{Y}^T_k G(k)\underline{Y}_k u^1_k \qquad (4.2\text{-}69)$$

The second term will now be evaluated

- Second term:

$$\sum_{k=0}^{k_1} o^2_k\, u^2_k = \sum_{k=0}^{k_1} u^2_k\, u^2_k = \sum_{k=0}^{k_1}(o_k)^2 \qquad (4.2\text{-}70)$$

Inserting Eqs. (4.2-51) and (4.2-54) into the above equation one obtains

$$\sum_{k=0}^{k_1} o^2_k\, u^2_k = \sum_{k=0}^{k_1}(\underline{Y}^T_k\underline{q}(k) + \underline{Y}^T_k G(k)\underline{Y}_k u^1_k)^2/2L$$

$$= \sum_{k=0}^{k_1}(\underline{q}^T(k)\underline{Y}_k\underline{Y}^T_k\underline{q}(k) + 2u^1_k\underline{Y}^T_k G(k)\underline{Y}_k Y^T_k\underline{q}(k)$$

$$+ u^1_k(\underline{Y}^T_k G(k)\underline{Y}_k)^2 u^1_k)/2L \qquad (4.2\text{-}71)$$

Now adding the first and second sum terms

$$\sum_{k=0}^{k_1} o_k\ u_k = \sum_{k=0}^{k_1} o^1{}_k\ u^1{}_k + \sum_{k=0}^{k_1} o^2{}_k\ u^2{}_k$$

$$= \tfrac{1}{2}\underline{q}^T(k_1+1)G^{-1}(k_1+1)\underline{q}(k_1+1) - \tfrac{1}{2}\underline{q}^T(0)G^{-1}(0)\underline{q}(0)$$

$$+ \tfrac{1}{2}\sum_{k=0}^{k_1} u^1{}_k \underline{Y}^T{}_k G(k)\underline{Y}_k u^1{}_k \qquad (4.2\text{-}72)$$

Since $G^{-1}(k_1+1)$, $G^{-1}(0)$ and $G(k)$ are positive definite matrices, the above equation can be expressed as follows

$$\sum_{k=0}^{k_1} o_k\ u_k > -\tfrac{1}{2}\underline{q}^T(0)G^{-1}(0)\underline{q}(0) = -c^2 \qquad (4.2\text{-}73)$$

One thus concludes that the equivalent feedback block described by Eqs. (4.2-45) and (4.2-46) and satisfying the conditions imposed by Theorem 4.2-1, satisfies Popov's integral inequality.

The final step in the proof of Theorem 4.2-1 is to find the conditions that assure that the feedforward block is asymptotically hyperstable. From Theorem A-2 one finds that for the above to be satisfied, the transfer matrix which characterises the feedforward block of the equivalent feedback system given by Eqs. (4.2-43) to (4.2-46) must be a strictly positive discrete transfer matrix.

This transfer matrix, which can be derived from Eqs. (4.2-43) and (4.2-44) is expressed below in the z-domain

$$H(z) = \frac{1 + \sum_{i=1}^{n} d_i\ z^{-i}}{1 - \sum_{i=1}^{n} a_i\ z^{-i}} - \frac{1}{2L} \qquad (4.2\text{-}74)$$

where a_i $(i=1,2, \ldots ,n)$ are the coefficients of the denominator of the discrete transfer function characterising the unknown plant and d_i $(i=1,2, \ldots ,n)$ are the coefficients of the linear compensator.

The linear compensator coefficients can be suitably chosen so as to ensure that H(z) is strictly positive real. The computation of these coefficients will be discussed in Section 4.2-3.

Since the above positivity condition is stated in Theorem 4.2-1, one can thus conclude that the identification algorithm stated by Theorem 4.2-1 assures the global asymptotic stability of the MRAS.

The block diagram for the discrete parallel identifier using time-varying identification gains is illustrated below

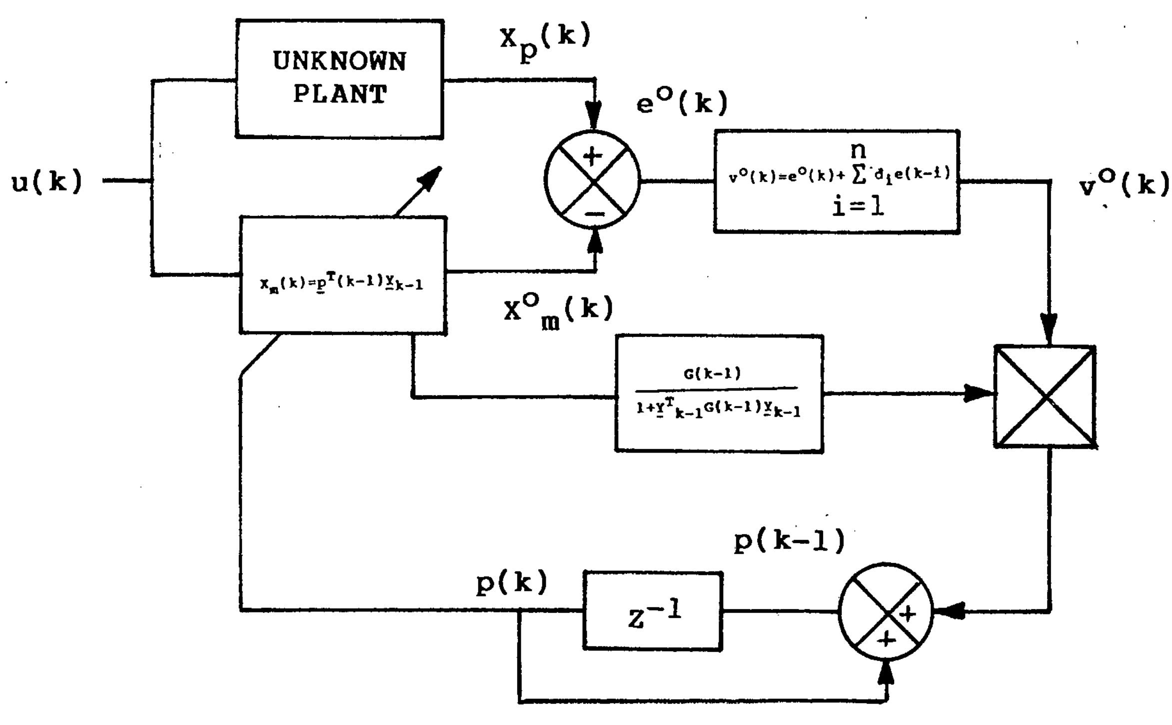


Fig. 4.2-2: Discrete-time parallel identifier using time-varying identification gains.

4.2-3 Computation of linear compensator coefficients

Theorem 4.2-1 requires one to select suitable values of the coefficients of the linear compensator, d_i, such that the discrete transfer function given by Eq. (4.2-74) be strictly positive real.

In order to compute the coefficients d_i, one must describe H(z) by state space notation.

The feedforward block is described by Eqs. (4.2-43) and (4.2-44). These equations are expressed in state space notation as

$$e(k+1) = A\underline{e} - w(k+1) \qquad (4.2\text{-}75)$$

$$v^*(k+1) = (\underline{d} + \underline{a})^T\underline{e} - (1 - \frac{1}{2L})w(k+1) \qquad (4.2\text{-}76)$$

where

$$A = \begin{bmatrix} 0 & & \left[\quad I \quad \right] & \\ 0 & & & \\ a_n & a_{n-1} & \cdots\cdots & a_1 \end{bmatrix}$$

In order to ensure that the transfer function is strictly positive real, one uses Lemma A-2.

Selecting $P_{n,n} < 2 - \frac{1}{L}$ where $L > 0.5$, the system characterised by Eqs. (4.2-75) and (4.2-76) can be decomposed into two parallel systems. The first system is defined as

$$e(k+1) = A\underline{e} - w(k+1) \qquad (4.2\text{-}77)$$

$$v^1(k+1) = (\underline{d} + \underline{a})^T\underline{e} - \tfrac{1}{2}P_{n,n}w(k+1) \qquad (4.2\text{-}78)$$

The second system is defined as

$$v^2(k+1) = (\tfrac{1}{2}P_{n,n} + \frac{1}{2L} - 1)w(k+1) \qquad (4.2\text{-}79)$$

The system described by Eqs. (4.2-77) and (4.2-78) will be characterised by a strictly positive real transfer matrix if the following two equations are satisfied

$$A^T PA - P = -Q \qquad Q > 0 \qquad (4.2\text{-}80)$$

$$P_{n,n}\underline{a}^T + (0,\ P_{n,1},\ P_{n,2},\ \dots\ ,\ P_{n,n-1}) = (\underline{a} + \underline{d})^T \qquad (4.2\text{-}81)$$

where $P = [P_{i,j}]$

The above two equations are obtained by evaluating Eqs. (A-19) and (A-20).

From Eqs. (4.2-80) and (4.2-81) one obtains

$$d_i = (P_{n,n} - 1)a_i + P_{n,n-i} \qquad i=1,2,\dots,n-1 \qquad (4.2\text{-}82)$$

$$d_n = (P_{n,n} - 1)a_n \qquad (4.2\text{-}83)$$

where P satisfies Eq. (4.2-80).

The above two equations are used to find the coefficients of the linear compensator that ensure that the discrete feedforward transfer matrix of the equivalent feedback system is strictly positive real.

As in the continuous-time case, it is apparent that one cannot directly compute the linear compensator coefficients using the above two equations due to the fact that the plant parameters are unknown. This limitation is however not as severe as it might seem due to the fact that the linear compensatormerely introduces a sufficient stability condition and not a necessary stability condition.

4.2-4 Effects of noise

The effects of noise on the precision of discrete-time parallel identifiers is summarised in Theorem form below [37], [38].

THEOREM 4.2-2

The parameters of the estimation model of a parallel identifier using time-decreasing identification gains will, in the presence of measurement noise, converge to those of the unknown plant with unity probability if in the absence of noise the parameters of the estimation model converge to those of the unknown plant.
The proof of the above theorem can be found in [37].

4.3 Conclusion

This chapter dealt with the design of a discrete-time parallel identifier using time-decreasing identification gains.

The above feature derives the following advantages over algorithms using constant identification gains

- The performannce of the MRAS will not be dependent on the values of the identification gains.
- Very accurate identification can be achieved rapidly even in the presence of noise.

CHAPTER 5

PRACTICAL IMPLEMENTATION OF A DISCRETE-TIME IDENTIFIER

5.1 Introduction

This chapter will deal with the practical implementation and testing of the discrete-time parallel identifier designed in Chapter 4.

The identifier will be used to identify a second order plant. This unknown plant is described in the z-domain as

$$H_p(z) = \frac{X_p(z)}{u(z)} = \frac{b_{p1}z^{-1} + b_{p2}z^{-2}}{1 - a_{p1}z^{-1} - a_{p2}z^{-2}} \qquad (5.1\text{-}1)$$

where X_p and u are the plant output and input respectively and b_{p1}, b_{p2}, a_{p1} and a_{p2} are the z-domain plant parameters.

The parallel estimation model is chosen to have the same structure as the plant. The transfer function of the model can therefore be represented as

$$H_m(z) = \frac{X_m(z)}{u(z)} = \frac{b_{m1}z^{-1} + b_{m2}z^{-2}}{1 - a_{m1}z^{-1} - a_{m2}z^{-2}} \qquad (5.1\text{-}2)$$

where $X_m(z)$ and u(z) are the model output and input respectively and b_{m1}, b_{m2}, a_{m1} and a_{m2} are the adjustable model parameters used to asymptotically identify the corresponding plant parameters.

A computer program simulating the discrete-time parallel identifier designed in Chapter 4 was written on the HP 85. A program listing can be found in Appendix E.

Due to the fact that the plant parameters are unknown, one would be unable to use the above method to calculate the linear compensator coefficients. The linear compensator assures the stability of the system. As has been mentioned before, the linear compensator does not impose a necessary stability condition but merely a sufficient condition for stability. A large amount of practical testing showed that setting the linear compensator coefficients to zero ensured the stability of the system for all the examples considered. It is important to note that by setting the linear compensator to zero, from Eq. (4.2-13), $v(k)=e(k)$. This is not to be confused with the continuous-time case where setting the linear compensator to zero forces $v(t)=0$.

The initial estimation model parameters were all set to zero.

Results for the above example are shown in Fig. 5.2-1.

From these plots one can see that the parametric identification is fast and very accurate. This is due to the use of time-decreasing identification gains, which are initially very high. This initial high gain rapidly adjusts the model parameters until they tend towards the plant parameters. Once they are close to the plant parameters, the identification gain becomes very low thus ensuring that model parameters converge accurately towards the plant parameter.

EXAMPLE 5.2-2

This example is identical to Example 5.2-1, except that a zero mean noise (max ampilitude =1) was added to the plant output.

The result obtained are illustrated in Fig 5.2-2.

It can be deduced form these results that even in the presence of noise, very accurate identification is achieved.

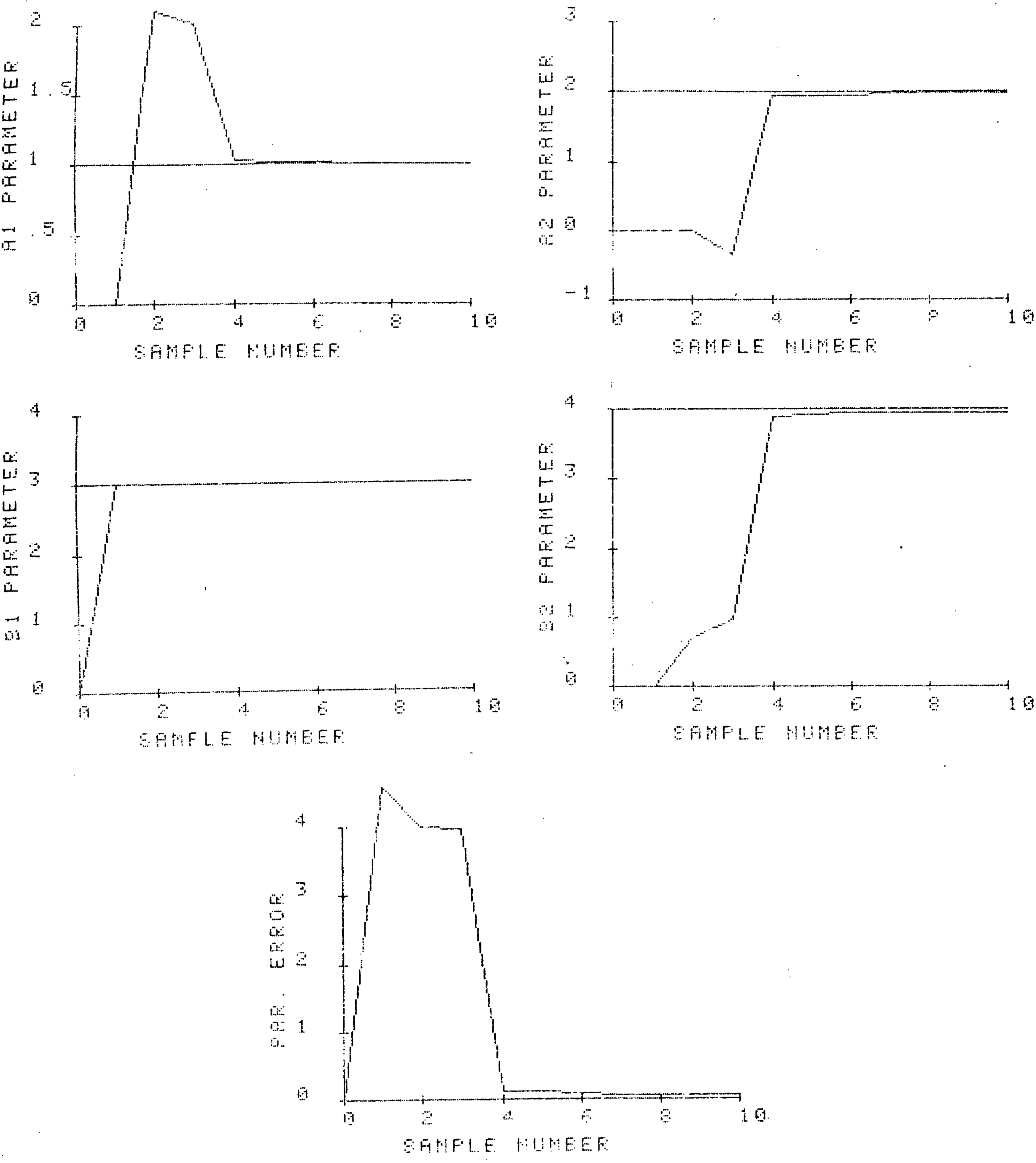


Fig. 5.2-1 Results obtained for Example 5.2-1.

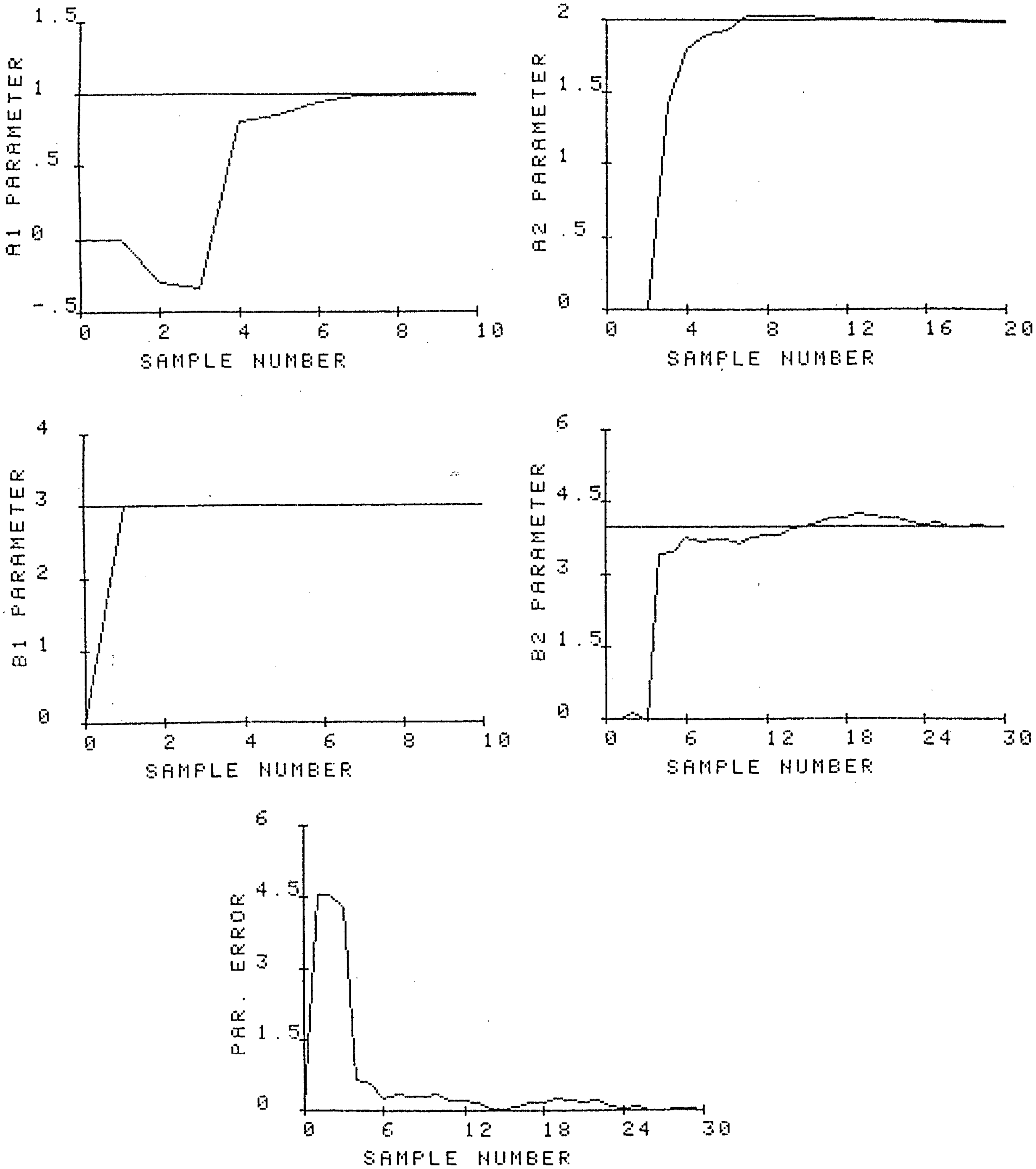


Fig. 5.2-2 Results obtained for Example 5.2-2.

EXAMPLE 5.2-3

This example is set out to show that the precision of identification of the above discrete-time identifier is dependent on the input signal u.

In this example we shall set up the following conditions

$$a_{p1} = 1$$
$$a_{p2} = 1$$
$$b_{p1} = 1$$
$$b_{p2} = 1$$

The input waveform was chosen to be a step input. All the other conditions were set identical to those in the previous example.

The results of this example are shown in Fig 5.2-3.

From these results it can be seen that the identification is rather poor. This is due to the fact that we are using a step as the plant/model input waveform. From practical testing of the identifier, it was found that the nature of the input signal was not as critical to the precision of identification as was the case in the continuous-time identifier.

EXAMPLE 5.2-4

This example is set out to illustrate the effect of the identification gain on the nature of identification. The identification gain was reduced to 100 I.

The results of this example are shown in Fig. 5.2-4.

From these results it can be seen that by decreasing the identification gain, the speed of identification also decreases. A decrease in the identification gain, has however no detrimental effect on the precision of identification.

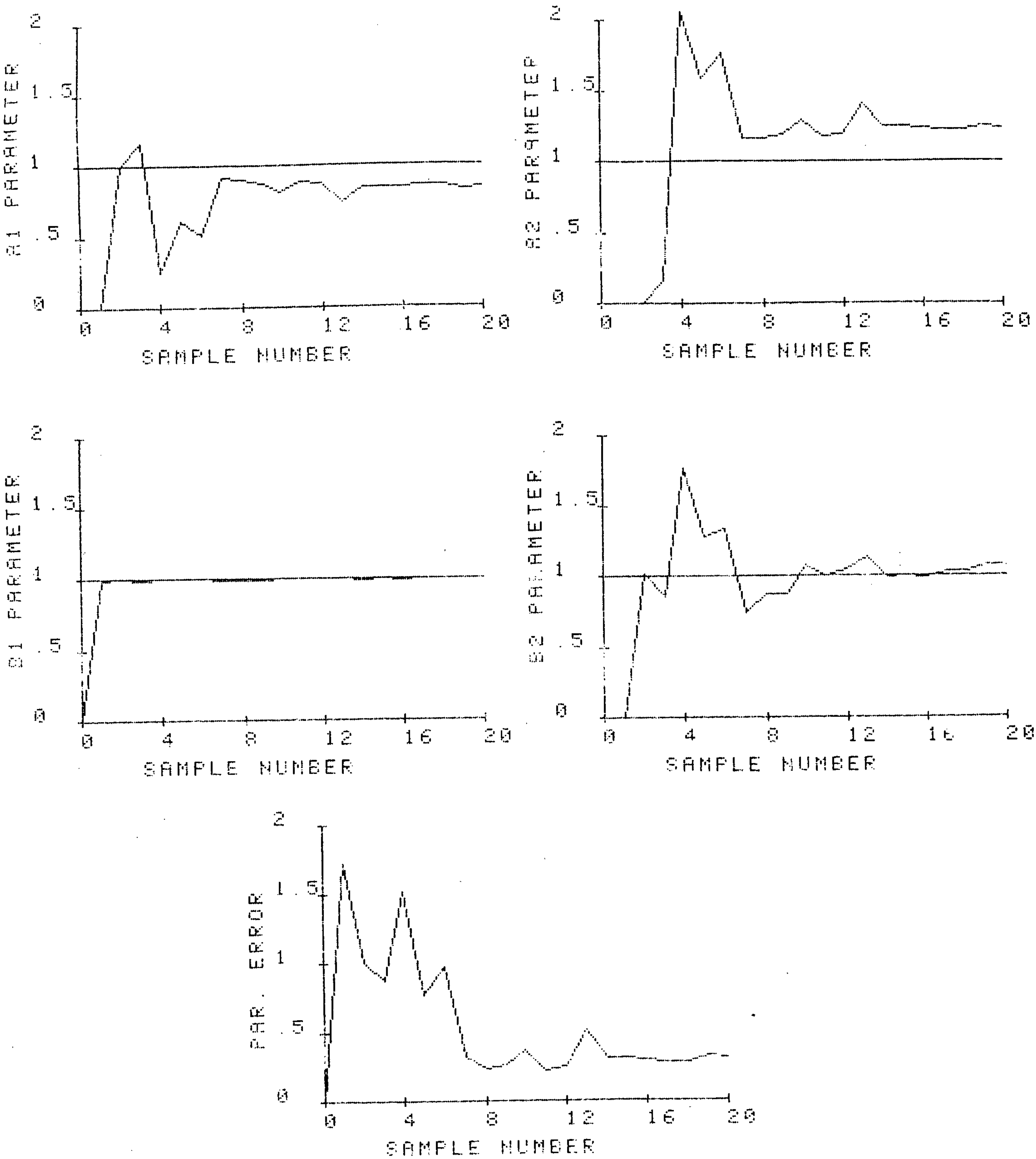


Fig. 5.2-3 Results obtained for Example 5.2-3.

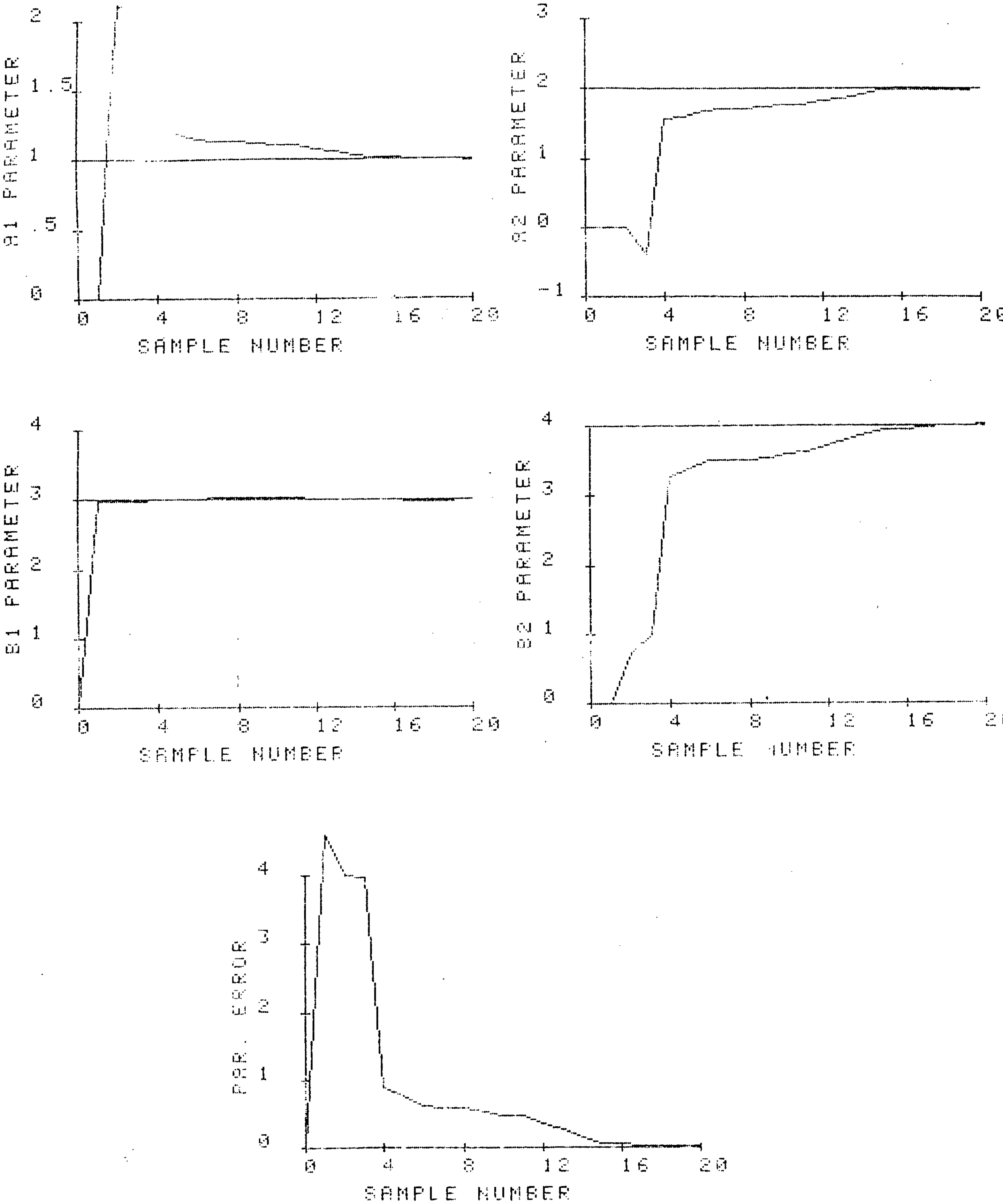


Fig 5.2-4 Results obtained for Example 5.2-4.

5.3 Conclusion

The discrete-time parallel identifier using time-decreasing identification gains was found to perform very well in the presence of noise.

The major advantages which it offers are

i) Very fast parametric identification.
ii) Very accurate parametric identification.
iii) Very easy to implement on a digital computer.

In section 2.2.3 it was stated that, in the presence of noise, parallel identifiers will achieve accurate parameter identification if the identification gain is low. A low identification gain will however lead to slow identification times. When using time-decreasing identification gains one does not need to compromise between speed and accuracy of identification due to the fact that

- initially the identification gain is high thus leading to fast identification times.

- as the model parameters tend towards the plant parameters, the identification gain tends towards zero. In section 2.2.3, it was stated that parallel identifiers will achieve accurate identification in the presence of noise if the identification gain is kept low.

A parallel identifier using time-decreasing gains will thus offer very fast and accurate identification.

Another property of the discrete-time identifier that was analysed in this chapter is the fact that perfect asymptotic parameter identification is dependent on the nature of the plant/model input waveform. In Example 5.2-3, a step was used as the plant/model input. This was found to produce rather poor identification. When the same example was performed with the plant/model input as a square waveform, perfect identification was achieved. It was found that the square waveform input always yielded perfect identification.

From analysing a wide range of examples, it was found that by setting the value of the linear compensator to zero, the system stability was always maintained. The reason for the value of the linear compensator not being so critical to the stability of the system is due to the fact that the linear compensator merely imposes a sufficient stability condition and not a necessary one.

If one however needed to calculate the theoretical value of the linear compensator that ensures stability, one could use the method stated in section 4.2.3. In this method one needs to know the values of the plant parameters in order to compute the linear compensator. Due to the fact that the plant parameters are unknown, one cannot use this method to directly compute the value of the linear compensator. If one really desires to use the theoretical linear compensator values then one can get around the above problem by initially using a different identification method to get approximate plant parameter values (e.g. least squares or series-parallel identifiers). These approximate parameter values can then be used to compute the theoretical linear compensator value using the method described in section 4.2.3. The parallel identifier using the theoretically computed value of the linear compensator can then be used as a final step to achieve very accurate identification of the plant parameters.

From the results obtained in this chapter and the testing of the identifier that took place during the course of this project, it can be deduced that the discrete-time parallel identifier designed in Chapter 4 is an excellent plant identifier, even in the presence of noise.

CHAPTER 6

CONCLUSION

This text investigated the performance of model reference adaptive systems for the purpose of identifying unknown plant parameters.

Two types of identifiers were analysed

- Parallel identifier.
- Series-parallel identifier.

The parallel identifier offered the advantage that accurate plant identification could always be achieved in the presence of measurement noise, provided that the identification gain was kept low. In the case of the series-parallel identifier, noise caused the identified parameters to contain a small bias irrespective of the value of the identification gain.

The series-parallel identifier offered easy theoretical computation of the linear compensator matrix to ensure overall system stability. In the case of the parallel identifier, theoretical computation of this matrix is impossible due to the fact that one does not know the values of the plant parameters. This is however not such a serious disadvantage due to the fact that the linear compensator merely imposes a sufficient and not a necessary stability condition.

It was found that for perfect asymptotic parameter convergence, the plant/model input had to be rich in sinusoidal frequency content. A square waveform always yielded good identification results.

Another aspect investigated was the effect of identification gain on the accuracy of identification. It has already been stated above that in the presence of noise, low identification gains will cause parallel identifiers to achieve more precise parameter convergence. It was however also noted that low identification gains will cause a decrease in the identification times of both identifiers.

For the purpose of plant parameter identification, it can be concluded that due to its better performance in the presence of noise, the parallel identifier achieves more accurate identification than the series-parallel identifier.

Another important aspect that this text investigated was the performance of continuous-time and discrete-time identifiers.

The performance of the continuous-time identifiers designed was degraded by the various imperfections of the analog computer.

The ease in implentation of the discrete-time identifier lead to the incorporation of more versatile and efficient features into the identification algorithm structure. One such feature that was looked at was the use of a discrete-time parallel identifier using time-decreasing identification gains. In the continuous-time case one had to compromise between accuracy and speed of identification. The use of time-decreasing identification gains allow one to obtain both accurate and fast parameter identification.

Discrete-time identifiers are far superior to their continuous-time counterparts due to the fact that they offer

- increased speed of identification.

- increased accuracy of identification.
- easy practical implementation.

The purpose of this text was to analyse the performance of model reference adaptive systems as parameter identifiers.It was found that the one major advantage that model reference systems offer over many other identification methods is its performance in the presence of noise. This is especially true for the case of the parallel identifier which identifies accurately provided the identification gain is low.

From the results obtained in this text and during the course of the project, it can be concluded that model reference adaptive systems are a very usefull and successfull parameter identification tool.

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APPENDIX A

HYPERSTABILITY THEORY

A.1 Introduction [2], [21]

Hyperstability can be defined as the property of a system which requires that the state vector of the system remains bounded if the inputs $\underline{u}$ are restricted to belonging to a subset of the set of all possible inputs.

Consider a closed loop system described by the state space equations

$$\dot{\underline{X}} = A\,\underline{X} + B\,\underline{u} \qquad \text{(A-1)}$$

$$\underline{v} = C\,\underline{X} \qquad \text{(A-2)}$$

and whose feedback block is described by

$$\underline{w} = f(\underline{v},t,\tau) \qquad \tau < t \qquad \text{(A-3)}$$

where : $\underline{X}$ is the state vector of the feedforward block.
$\underline{u}$ and $\underline{v}$ are the input and output respectively of the feedforward block.
$\underline{w}$ is the output of the feedback block.
The matrix pair (A,B) is completely controllable and the matrix pair (A,C) is completely observable.
$\underline{f}$ denotes a nonlinear vectorial dependence between $\underline{w}$ and $\underline{v}$ in the interval $\tau < t$.

Hyperstable system :

The above system is said to be hyperstable if for an input $\underline{u}$, the following inequality holds for some positive constants K and c and all time t

$$\|\underline{x}(t)\| < K(\|\underline{x}(0)\| + c) \qquad (A-4)$$

Asymptotically hyperstable system :

The above system is said to be asymptotically hyperstable if for an input u, the following expression holds

$$\lim \underline{x}(t) = \underline{0} \qquad (A-5)$$

A.2 The application of Popov's results to continuous time systems

In his studies on hyperstability, Popov investigated the global asymptotic stability of systems of the form represented by Fig. A-1 for the class of feedback blocks satisfying the Popov integral inequality

$$n(t_0,t_1) = \int_{t_0}^{t_1} \underline{v}^T \underline{w} \, dt > - c^2 \quad \text{for all } t_1 > t_0 \qquad (A-6)$$

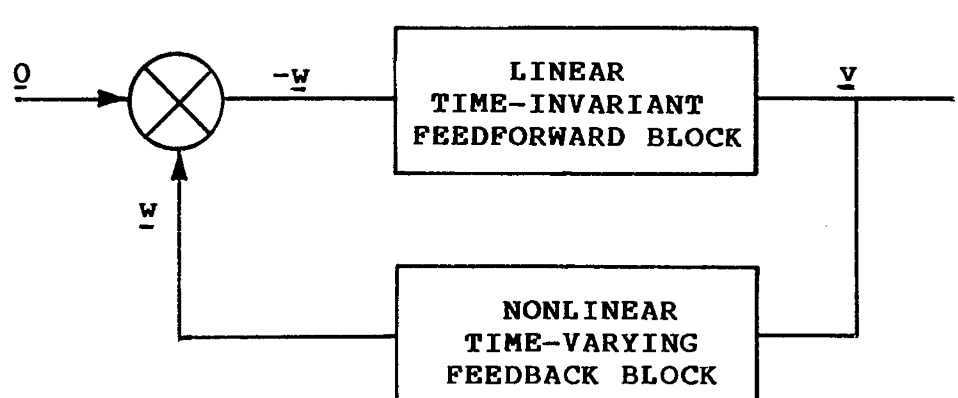

Fig. A-1 Standard multivariable nonlinear time-varying feedback system condidered by Popov.

Due to the fact that the feedback block is completely defined by Eq. A-3, the hyperstability properties of the feedback system will depend only on the nature of the feedforward block. A feedforward block that assures the hyperstabilty of a closed loop system is called a hyperstable block.

The main result which Popov arrived at in his studies on hyperstability is summarised in theorem form below

THEOREM A-1 :

The necessary and sufficient condition for the feedback system described by Eqs. (A-1), (A-2), (A-3) and (A-6) to be (asymptotically) hyperstable is that the transfer matrix which characterises the feedforward block

$$H(s) = C\,(sI-A)^{-1}\,B \qquad (A\text{-}7)$$

be a (strictly) positive real transfer matrix.

The conditions that ensure the positivity of the feedforward block are given in the lemma below

LEMMA A-1

The transfer matrix

$$H(s) = C\,(sI-A)^{-1}\,B$$

is a strictly positive real transfer matrix if there exists a symmetric positive definite matrix P and a symmetric positive definite matrix Q such that the following set of equations are verified

$$A^T\,P + P\,A = -Q \qquad (A\text{-}8)$$

$$B^T P = C \tag{A-9}$$

Eq. (A-8) is known as the Liapunov matrix equation and the above lemma is referred to in the literature as the "positive real lemma" or as the "Kalman-Yakubovitch-Popov lemma".

A direct method for solving the Liapunov matrix equation for a second order system is given below :

The Liapunov matrix equation can be written as

$$\begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix} \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} + \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = - \begin{bmatrix} q_{11} & q_{12} \\ q_{12} & q_{22} \end{bmatrix} \tag{A-10}$$

One is required to find a symmetric matrix P which satisfies the above equation if Q is an arbitrary symmetric positive definite matrix.

Eq. (A-10) reduces to

$$2\, a_{11}\, p_{11} + 2\, a_{21}\, p_{12} = -q_{11} \tag{A-11}$$

$$a_{12}\, p_{11} + (a_{11} + a_{22}) p_{12} + a_{21}\, p_{22} = -q_{12} \tag{A-12}$$

$$2\, a_{12}\, p_{12} + 2\, a_{22}\, p_{22} = -q_{22} \tag{A-13}$$

If matrices A and Q are known, then matrix P can be directly computed by solving the above three equations.

For a study on the various methods of solving the Liapunov matrix equation, the reader is referred to [5].

A.3 The application of Popov's results to discrete-time systems

The discrete-time form of Popov's integral inequality is given below

$$\eta(k_0,k_1) = \sum_{k=k_0}^{k_1} \underline{w}^T{}_k \, \underline{v}_k \geq -c^2 \qquad \text{for all } k_1 \geq k_0 \qquad \text{(A-14)}$$

where : c^2 is an arbitrary positive finite constant.

$\underline{w}_k$ is the output vector of the feedback block at instant k.

$\underline{v}_k$ is the input vector of the feedback block at instant k.

Consider a discrete-time feedback system whose feedforward block is linear and time invariant. A state space realization of this system yields the following :

- The linear time-invariant feedforward block is described by

$$\underline{X}_{k+1} = A\,\underline{X}_k + B\,\underline{u}_k \qquad \text{(A-15)}$$

$$\underline{v}_k = C\,\underline{X}_k + D\,\underline{u}_k \qquad \text{(A-16)}$$

where : $\underline{X}_k$ is the system state vector at instant k.

$\underline{u}_k$ and $\underline{v}_k$ are the input and output vectors respectively of the feedforward block.

The matrix pair (A,B) is completely controllable and the matrix pair (A,C) is completely observable.

- The nonlinear time-varying feedback block is described by

$$\underline{w}_k = f(\underline{v},k,l) \qquad l \leq k \qquad \text{(A-17)}$$

where : $\underline{w}_k$ is the output vector of the feedback block at instant k.

$\underline{f}$ denotes a nonlinear vectorial dependence between $\underline{w}$ and $\underline{v}$.

Popov's main result concerning systems of the above type is given in theorem form below

THEOREM A-2

The necessary and sufficient condition for the feedback system described by equations (A-14) to (A-17) to be (asymptotically) hyperstable is that the transfer matrix which characterises the feedforward block of the equivalent feedback system

$$H(z) = D + C\,(zI - A)^{-1}\,B \tag{A-18}$$

be a (strictly) positive discrete transfer matrix.

For a discrete transfer matrix to be strictly positive real, it is sufficient that it satisfy the conditions specified by one of the following positivity lemma:

LEMMA A-2

The discrete transfer matrix H(z) given by Eq. (A-18) is strictly positive real if there exists a symmetric positive matrix P, a symmetric positive matrix Q and matrices S and R such that

$$A^T PA - P = -Q \tag{A-19}$$

$$B^T PA + S^T = C \tag{A-20}$$

$$D + D^T - B^T PB = R \tag{A-21}$$

LEMMA A-3

The system described by Eqs. (A-15) and (A-16) is said to be strictly positive real if every solution $\underline{X}_k(\underline{X}_0, \underline{u}_k)$ of the above equations satisfy the following equality

$$\sum_{k=0}^{k_1} \underline{v}^T{}_k \underline{u}_k = \tfrac{1}{2}\underline{X}^T{}_{k_1+1} P \underline{X}_{k_1+1} - \tfrac{1}{2}\underline{X}^T{}_0 P \underline{X}_0 + \tfrac{1}{2}\sum_{k=0}^{k_1} (\underline{X}^T{}_k Q \underline{X}_k + 2\underline{u}^T{}_k S^T \underline{X}_k + \underline{u}^T{}_k R \underline{u}_k) \tag{A-22}$$

where : P is a positive definite matrix and matrices P, Q, S and R satisfy Eqs. (A-19) to (A-21).

The above two lemma are for the case when the feedforward block of the equivalent feedback system is time-invariant. The corresponding lemma for the case when the feedforward block of the equivalent feedback system is time-varying will be stated below.

Consider the following feedforward time-varying block

$$\underline{X}_{k+1} = A_k \underline{X}_k + B_k \underline{u}_k \tag{A-23}$$

$$\underline{v}_k = C_k \underline{X}_k + D_k \underline{u}_k \tag{A-24}$$

Lemma (A-2) and (A-3) can now be restated as follows:

LEMMA A-4

The system defined by Eqs. (A-23) and (A-24) is strictly positive real if there exists a symmetric time-varying positive definite matrix P_k, a symmetric time-varying positive semidefinite matrix Q_k and time-varing matrices S_k and R_k such that

$$A^T{}_k P_{k+1} A_k - P_k = -Q_k \tag{A-25}$$

$$B^T{}_k P_{k+1} A_k + S^T{}_k = C_k \tag{A-26}$$

$$D_k + D^T{}_k - B^T{}_k P_{k+1} B_k = R_k \tag{A-27}$$

LEMMA A-5

The system described by Eqs. (A-23) and (A-24) is said to be strictly positive real if every solution $X_k(X_0, u_k)$ satisfies the following equality

$$\sum_{k=0}^{k_1} \underline{v}^T{}_k \, \underline{u}_k = \tfrac{1}{2} \, \underline{X}^T{}_{k_1+1} P_{k_1+1} \underline{X}_{k+1} - \tfrac{1}{2} \, \underline{X}^T{}_0 \, P_0 \, \underline{X}_0$$
$$+ \sum_{k=0}^{k_1} (\underline{X}^k{}_k \, Q_k \, \underline{X}_k + 2 \, \underline{u}^T{}_k \, S^T{}_k \, \underline{X}_k + \underline{u}^T{}_k \, R_k \, \underline{u}_k) \qquad \text{(A-28)}$$

where the matrices P_k, Q_k, S_k and R_k satisfy Eqs. (A-25) to (A-27).

APPENDIX B

SOLUTION FOR POPOV'S INTEGRAL INEQUALITY

B.1 Introduction

One of the mathematical problems encountered in the design of MRAS's using hyperstablity and positivity theory is the need to find a set of functions which solve Popov's integral inequality. This appendix will deal with the derivation of these functions for continuous-time MRAS's. [1]

B.2 Statement of the problem

Consider a block with the input equal to $\underline{v}$ and the output equal to $\underline{w}$, where $\underline{v}$ and $\underline{w}$ are n-dimensional vectors.

The output of the block is defined by

$$\underline{w}(t) = \left(\int_0^t I(\underline{v},t,\tau)d\tau + P(\underline{v},t) + A \right) \underline{x}(t) \tag{B-1}$$

where : $I(\underline{v},t,\tau)$ ((nxm) dimensional) and $P(\underline{v},t)$ ((nxm)-dimensional) are matrix functional s of $\underline{v}(t)$.
A ((nxm)-dimensional) is a constant matrix.
$\underline{x}(t)$ (m-dimensional) is a continuous vector function.

Popov's integral inequality is defined as

$$\eta(0,t_1) = \int_0^{t_1} \underline{v}^T \underline{w} \, dt \geq - c^2 \tag{B-2}$$

for all $t_1 \geq 0$

where c^2 is a positive arbitrary finite constant.

Inserting Eq. (B-1) into Popov's integral inequality, one obtains

$$\eta(0,t_1) = \int_0^{t_1} \underline{v}^T\left(\int_0^{t} I(\underline{v},t,\tau)d\tau + P(\underline{v},t) + A\right) \underline{x}\, dt \geq -c^2 \qquad \text{(B-3)}$$

The objective is to find a general solution for $I(\underline{v},t,\tau)$ and $P(\underline{v},t)$ such that the inequality of Eq.(B-3) holds.

For the inequality of Eq.(B-3) to hold, it is sufficient that the following two inequalities hold

Inequality 1:

$$\eta(0,t_1) = \int_0^{t_1} \underline{v}^T\left(\int_0^{t} I(\underline{v},t,\tau)\, d\tau + A\right) \underline{x}\, dt \geq -c^2 \qquad \text{(B-4)}$$

for all $t_1 > 0$

where c^2 is an arbitrary finite constant.

Inequality 2:

$$\eta(0,t_1) = \int_0^{t_1} \underline{v}^T P(\underline{v},t)\, \underline{x}\, dt \geq -c^2 \qquad \text{(B-5)}$$

for all $t_1 > 0$

where c^2 is an arbitrary finite constant.

B.3 Solution of the problem

The expression for $I(\underline{v},t,\tau)$ and $P(\underline{v},t)$ which satisfy Popov's integral inequality is given in the following two lemma :

Lemma B-1:

The inequality of Eq. (B-4) holds if

$$I(\underline{v},t,\tau) = F(t-\tau)\, \underline{v}(\tau)\, (G\, \underline{x}(\tau))^T \qquad \text{(B-6)}$$

where $F(t-\tau)$ is a positive definite square matrix kernel and G is a positive definite matrix.

Lemma B-2:

The inequality of Eq. (B-5) holds if

$$P(\underline{v},t) = F'(t)\ \underline{v}(t)\ (G'(t)\ \underline{x}(t))^T \qquad \text{(B-7)}$$

where $F'(t)$ and $G'(t)$ are time-varying positive definite matrices for all $t > 0$.

B.4 Proof of the problem

Before stating the proof of the above lemma, we will first define an inequality which will be used in the proof.

Definition B-1:

The square matrix kernel $K(t,\tau)$ is said to be positive definite if for each interval (t_0, t_1) and all the vector functions $\underline{y}(t)$ that are piecewise continuous in (t_0, t_1), the following inequality holds

$$\eta(t_0, t_1) = \int_{t_0}^{t_1} \underline{y}^T(t) \left| \int_0^t K(t,\tau)\ \underline{y}(\tau)\ d\tau \right| dt > 0 \qquad \text{(B-8)}$$

With the aid of the above definition we will now prove the above two lemma independently.

Proof of Lemma B-1 :

Inserting Eq. (B-6) into the inequality of Eq. (B-4) one obtains

$$\eta(0,t_1) = \int_0^{t_1} \underline{x}(t)\ \underline{v}^T(\int_0^t F(t-\tau)\underline{v}(\tau)\underline{x}(\tau)d\tau \ + A)dt > -c^2 \qquad \text{(B-9)}$$

$$\text{for all } t_1 > 0$$

Replacing x(t).v(t) by y(t) the above inequality becomes

$$\int_{0^-}^{t_1} \underline{y}^T \left(\int_0^t F(t-\tau)\, \underline{y}(\tau)\, d\tau \; + A \right) dt > -c^2 \qquad \text{(B-10)}$$

for all t_1 0

Taking the case when the constant matrix A is equal to zero, Eq. (B-10) becomes

$$\int_{0^-}^{t_1} \underline{y}^T \left(\int_0^t F(t-\tau)\, \underline{y}(\tau)\, d\tau \right) dt > -c^2 \qquad \text{(B-11)}$$

Due to the fact that F(t-τ) is a positive definite matrix kernel, from definition B-1, it is evident that Eq. (B-11) must be greater than or equal to zero.

For the case where A is not equal to zero one can consider A to be the output of the system characterised by the matrix kernel F(t-τ) at time t = 0.

i.e. $$A = \int_{-t_0}^{0} F(t-\tau)\; y(-t_0)\; d\tau \qquad \text{(B-12)}$$

The inequality of Eq. (B-10) thus becomes

$$\int_{-t_0}^{t_1} \underline{y}'(t)^T \left(\int_{-t_0}^{t} F(t-\tau)\underline{y}'(\tau) d\tau \right) dt - \int_{-t_0}^{0} \underline{y}(-t_0)^T \left(\int_{-t_0}^{t} F(t-\tau) y(-t_0) d\tau \right) dt > -c^2 \qquad \text{(B-13)}$$

where $$y'(t) = \begin{cases} \underline{y}(t) \text{ for } t > 0 \\ \underline{y}(-t_0) \text{ for } -t_0 < t < 0 \end{cases}$$

Due to the fact that F(t-τ) is a positive definite matrix kernel, all the terms on the left hand side of equation (B-13) are greater than or equal to zero - (see definition B-1)

We can therefore conclude that

$$I(\underline{v},t,\tau) = F(t-\tau)\; \underline{v}(\tau)\; (G\; \underline{x}(\tau))^T$$

satisfies Inequality 1.

Proof of Lemma B-2 :

Inserting Eq. (B-7) into Eq. (B-5) one obtains

$$\eta(0,t_1) = \int_0^{t_1} \underline{v}^T(t)\ (F'(t)\ \underline{v}(t)(G'(t)\ \underline{x}(t))^T\underline{x}(t)\ dt \geq -c^2 \qquad \text{(B-14)}$$

Eq. (B-14) will be greater than or equal to zero for all time greater than zero because the products $\underline{v}^T(t).F'(t).\underline{v}(t)$ and $\underline{x}^T(t).G'(t).\underline{x}(t)$ will be greater than or equal to zero for any time greater than zero.

We can therefore conclude that

$$P(\underline{v},t) = F'(t)\ \underline{v}(t)\ (G'(t)\ \underline{x}(t))^T$$

satisfies Inequality 2.

B.5 Conclusion

Solutions for $I(\underline{v},t,\tau)$ and $P(\underline{v},t)$ that satisfy the inequality given by Eq. (B-3) are

$$I(\underline{v},t,\tau) = F(t-\tau)\ \underline{v}(\tau)\ (G\ \underline{x}(\tau))^T$$

$$P(\underline{v},t) = F'(t)\ \underline{v}(t)\ (G'(t)\ \underline{x}(t))^T$$

where : $F(t-\tau)$ is a positive definite square matrix kernel.
G is a positive definite matrix.
$F'(t)$ and $G'(t)$ are time-varying positive definite matrices for all $t \geq 0$.

APPENDIX C

CONDITIONS FOR PARAMETER CONVERGENCE

C.1 Introduction

When designing MRAS's to be used in the field of plant identification, one is not only interested in ensuring that the final structure is asymptotically stable but also in ensuring that the unknown parameters are perfectly asymptotically identified.

This appendix will state the necessary conditions for the continuous-time identifiers discussed in Chapter 2 to achieve perfect asymptotic parameter identification. [1],[20]

C.2 Parallel identifier parameter convergence conditions

It was shown in Section 2.2.2 that the differential equation that characterises the dynamics of the state vector $\underline{e}$ is

$$\dot{\underline{e}} = A_p \underline{e} + (A_p - A_m(\underline{v},t))\underline{X}_m + (B_p - B_m(\underline{v},t))\underline{u} \qquad \text{(C-1)}$$

Taking the limit of the above equation as time tends towards infinity, one obtains

$$(A_p - \lim_{t\to\infty} A_m(\underline{v},t))\underline{X}_p + (B_p - \lim B_m(\underline{v},t))\underline{u} = 0 \qquad \text{(C-2)}$$

From the definition of linear independence, if u and X_p are linearly independent then the only way that the above expression can equal zero is if the following two equations are satisfied

$$A_p - \lim_{t\to\infty} A_m(\underline{v},t) = 0 \qquad \text{(C-3)}$$

$$B_p - \lim_{t \to \infty} B_m(\underline{v}, t) = 0 \qquad (C-4)$$

The above two equations imply perfect asymptotic parameter identification.

One is therefore interested in finding the conditions that $\underline{u}$ must satisfy in order to ensure that $\underline{u}$ and $\underline{X}_p$ are linearly independent.

These conditions are stated in the theorem below

THEOREM C-1:

Given an asymptotically stable parallel identifier, perfect asymptotic parameter convergence is achieved if

- the plant to the identified is completely controllable

- $\underline{u}$ can be decomposed into linearly independent sinusoidal components

- Each component of the vector $\underline{u}$ contains at least $(n+1)/2$ distinct frequencies, where n is the dimension of the plant or model state vector.

Proof of the above theorem can be found in [20].

C.3 Series-parallel identifier parameter convergence conditions

When dealing with parameter convergence, the series-parallel problem is identical to that of the parallel identifier. The same conditions as stated in theorem C-1 apply in order to ensure the asymptotic parameter convergence of the series-parallel identifier.

C.4 Conclusion

One can therefore deduce that the accuracy of the identifier is not dependent on the identification algorithm (the identification algorithm is simply designed to achieve perfect asymptotic stability) but on the nature of the input signal $\underline{u}$.

APPENDIX D

ANALYSIS OF NUMERICAL METHODS USED FOR CONTINUOUS-TIME MRAS SIMULATION

When simulating a MRAS on a digital computer, one is required to solve differential equations. The method used in the digital simulator tested in chapter 3, was the popular fourth order Runge-kutta method. [49], [50], [52]

Assume that the equation to be solved is a first order equation defined by

$$Y = f(X,Y) \tag{D-1}$$

with the condition (X_n, Y_n) present at step n. Then the solution to (D-1) at step n+1 is found by means of the following algorithm

$$A_n = h\ f(X_n,\ Y_n) \tag{D-2}$$

$$B_n = h\ f(X_n + \tfrac{1}{2}h,\ Y_n + \tfrac{1}{2}A_n) \tag{D-3}$$

$$C_n = h\ f(X_n + \tfrac{1}{2}h,\ Y_n + \tfrac{1}{2}B_n) \tag{D-4}$$

$$D_n = h\ f(X_{n+1},\ Y_n + C_n) \tag{D-5}$$

$$Y_{n+1} = Y_n + (A_n + 2B_n + 2C_n + D_n)/6 \tag{D-6}$$

where h is the time between each step.

To apply the runge-kutta method to solve higher order differential equations, one transforms the higher order differential into first order differential equations by means of state-space notation and then apply the runge-kutta to solve each of these first order differential equations separately.

The Runge-kutta has the advantage of being a very accurate numerical method - it pocesses a truncation error per step of the order h^5.

APPENDIX E

PROGRAM LISTINGS

E.1 Program listing of the digitally simulated parallel identifier

```
10  ! SIMULATION OF A MRAC SYSTE
    M!
20  !                          !
21  !                          !
35  ! DIMENSIONING ARRAYS !
40  !                          !
50  DIM C1(4),C2(3)
59  DIM D(4)
60  DIM B(300),A0(300),A1(300)
65  DIM E1(300),E2(300)
67  DIM T(300),S(300)
70  !                          !
140 ! ...P-K COEFFICIENTS...!
145 !                         !
160 FOR J=1 TO 4
170 READ C1(J)
180 NEXT J
190 DATA 1,2,2,1
200 FOR J=1 TO 3
210 READ C2(J)
220 NEXT J
230 DATA .5,.5,1
231 !                          !
232 ! LINEAR COMPENSATOR       !
233 !                          !
234 FOR J=1 TO 4
235 READ D(J)
236 NEXT J
237 DATA 3,1,1,1
239 !                          !
240 ! SYSTEM INFO  !
241 !                          !
242 DISP "ENTER B,A0,A1 PARAMETE
    RS"
244 INPUT B,A0,A1
246 DISP "ENTER INIT MODEL B,A0,
    A1"
247 INPUT C0,C1,C2
252 DISP "ENTER B,A0,A1 GAINS"
254 INPUT G,G0,G1
256 DISP "ENTER START,STOP TIME"
260 INPUT T0,T1
262 DISP "ENTER STEP SIZE "
265 INPUT H
267 DISP "ENTER INPUT PERIOD"
268 INPUT P
270 DISP "ENTER INITIAL OUTPUT
    P1(0),P2(0)   "
275 INPUT P1,P2
276 M1=P1 @ M2=P2 @ E1=0 @ E2=0
    @ V1=0 @ V2=0
278 B(0)=C0 @ A0(0)=C1 @ A1(0)=C
    2 @ E1(0)=0 @ E2(0)=0
280 I1=0 @ I2=0 @ I3=0
281 ! DEFINE THE SYSTEM INPUT !
283 U=1
300 ! VARIABLE INIT          !
302 Z=10 ! DIM OF PLOT ARRAY!
303 E=IP((T1-T0)/H/Z+.5)
304 IF E=0 THEN E=1
305 N=0
306 C=0
307 D=0
308 !                          !
310 !  SIMULATION LOOP         !
311 !                          !
318 FOR T=T0+H TO T1 STEP H
320 N=N+1
321 C=C+H @ IF C>P OR C=P THEN U
    =-U @ C=0
323 ! CALCULATE PLANT O/P  !
325 GOSUB 592
329 ! CALCULATE MODEL O/P    !
330 GOSUB 641
335 ! CALCULATE ERROR        !
340 GOSUB 672
341 ! ADAPTATION MECHANISM   !
343 GOSUB 2500
344 ! STORE VALUES TO PLOT !
345 IF N<>E THEN GOTO 440
346 D=D+1 @ N=0
347 B(D)=C0 @ A0(D)=C1 @ A1(D)=C
    2
348 E1(D)=E1 @ E2(D)=E2
440 NEXT T
442 ! SELECT WHICH PLOT      !
445 GOSUB 4000
550 END
561 !                          !
562 ! PRINT GRAPH AND TABLE !
564 !                          !
565 DISP "GRAPH PRINTOUT (Y/N)"
566 INPUT C$
569 IF C$="Y" THEN GRAPH @ COPY
570 DISP "PRINTOUT TABLE (Y/N)"
572 INPUT A$
574 IF A$="Y" THEN GOSUB 2000
576 STOP
577 !                          !
580 ! ...SUBROUTINES..........!
585 !                          !
586 !                          !
590 ! ...RUNGE-KUTTA.......!
591 !                          !
592 ! PLANT DIFFERENTIAL EQU   !
595 !                         !
600 FOR K=1 TO 4
605 IF K=1 THEN S1=0 @ S2=0 @ X1
    =P1 @ X2=P2
610 ! ...DX=F(X,U) EQN....!
614 D1=X2
616 D2=B*U-A1*X2-A0*X1
620 S1=S1+C1(K)*D1
621 S2=S2+C1(K)*D2
622 IF K<4 THEN X1=P1+C2(K)*H*D1
     @ X2=P2+C2(K)*H*D2
625 IF K=4 THEN P1=P1+S1*H/6 @ P
    2=P2+S2*H/6
630 NEXT K
635 RETURN
```

```
640  !                              !
641  ! MODEL DIFFERENTIAL EQU!
642  !                              !
645  FOR K=1 TO 4
650  IF K=1 THEN S1=0 @ S2=0 @ Q1
     =M1 @ Q2=M2
653  !     DX=F(X,U) EQN            !
655  D1=Q2
657  D2=C0*U-C2*Q2-C1*Q1
660  S1=S1+C1(K)*D1
663  S2=S2+C1(K)*D2
665  IF K<4 THEN Q1=M1+C2(K)*H*D1
      @ Q2=M2+C2(K)*H*D2
668  IF K=4 THEN M1=M1+S1*H/6 @ M
     2=M2+S2*H/6
670  NEXT K
671  RETURN
672  ! ERROR ROUTINE                 !
673  !                               !
674  E1=P1-M1 @ E2=P2-M2
675  V1=D(1)*E1+D(2)*E2
676  V2=D(3)*E1+D(4)*E2
677  RETURN
679  !                               !
680  ! PLOT PARAMETER AXIS          !
685  !                                !
686  DISP "ENTER Y0,Y1"
687  INPUT Y0,Y1
690  T=T1-T0
700  GCLEAR
705  L=ABS(Y0-Y1)
710  SCALE -(.2*T),T1+.1*T,Y0-.2*
     L,Y1+.2*L
712  IF Y0>=0 AND Y1<=0 THEN I=0
     @ GOTO 720
715  IF Y0<Y1 THEN I=Y0 ELSE I=Y1
720  XAXIS I,T/5,T0,T1
730  YAXIS T0,L/4,Y0,Y1
740  MOVE 0,Y0-.2*L
750  LABEL "        TIME (SECS)"
760  LDIR 90
770  MOVE -(.15*T),Y0+.3*L
780  LABEL P$
790  LDIR 0
800  FOR X9=T0 TO T1 STEP T/5
810  MOVE X9,Y0-.1*L
820  LABEL VAL$(X9)
830  NEXT X9
840  FOR Y9=Y0 TO Y1 STEP L/4
850  MOVE -(.12*T),Y9
860  LABEL VAL$(Y9)
870  NEXT Y9
1010  ! PLOT PARAMATERS           !
1012  FOR N=0 TO Z
1020  IF N=0 THEN MOVE T0,Y @ DRA
      W T1,Y @ MOVE T0,T(0) @ GOT
      O 1045
1040  DRAW N*E*H,T(N)
1045  NEXT N
1080  RETURN
1090  !                              !
1190  !                               !
1200  ! PLOT PHASE-PLANE AXIS !
1205  !                               !
1210  DISP "ENTER Y0,Y1"
1220  INPUT Y0,Y1
1230  DISP "ENTER X0,X1"
1240  INPUT X0,X1
1250  GCLEAR
1260  L=ABS(Y0-Y1)
1270  T=ABS(X0-X1)
1280  SCALE X0,X1,Y0,Y1
1290  XAXIS 0,T/4,X0,X1
1300  YAXIS 0,L/4,Y0,Y1
1310  MOVE .8*X1,.1*Y1
1320  LABEL " E1"
1340  MOVE .2*X0,.8*Y1
1350  LABEL " E2"
1360  FOR X9=X0 TO X1 STEP T/4
1370  MOVE X9,.1*Y0
1375  IF X9=0 THEN MOVE .05*X1,.
      *Y0
1377  IF X9=X1 THEN MOVE .95*X1,
      1*X0
1380  LABEL VAL$(X9)
1390  NEXT X9
1400  FOR Y9=Y0 TO Y1 STEP L/4
1410  MOVE .05*X1,Y9
1415  IF Y9=0 THEN GOTO 1430
1417  IF Y9=Y1 THEN MOVE .05*X1,
      9*Y1
1420  LABEL VAL$(Y9)
1430  NEXT Y9
1440  ! PLOT PHASE-PLANE          !
1460  FOR N=0 TO Z
1470  IF N=0 THEN MOVE E1(0),E2(
      ) @ GOTO 1490
1480  DRAW E1(N),E2(N)
1490  NEXT N
1500  RETURN
2000  ! PRINTOUT SUBROUTINE !
2020  !                              !
2025  PRINT @ PRINT @ PRINT
2030  PRINT "TIME";TAB(10);"   V2
      T) ";TAB(20);"    B(T) "
2040  FOR I=0 TO T1/H
2050  PRINT T(I);TAB(10);INT(V2(
      )*10000+.5)/10000;TAB(20);
      NT(B(I)*10000+.5)/10000
2060  NEXT I
2070  RETURN
2500  !                               !
2510  !    ADAPTATION MECH           !
2520  !                               !
2522  N1=C0 @ N2=C1 @ N3=C2
2525  I4=I1 @ I5=I2 @ I6=I3
2530  I1=G*U*V2 @ I2=G0*V2*M1 @
      3=V2*M2*G1
2535  C0=C0+H*((I1-I4)/2+I4)
2537  C1=C1-H*((I2-I5)/2+I5)
```

```
2540 C2=C2-H*((I3-I6)/2+I6)
2550 DISP "B=";C0
2555 DISP "A0=";C1
2556 DISP "A1=";C2
2558 DISP "E1= ";E1
2559 DISP "E2= ";E2
2570 RETURN
4000 ! PLOT RESULTS          !
4010 DISP "PLOT B,A0,A1,E"
4020 INPUT A$
4030 IF A$="A0" THEN GOTO 4100
4040 IF A$="A1" THEN GOTO 4200
4050 IF A$="B" THEN GOTO 4300
4060 IF A$="E" THEN GOTO 4410
4070 GOTO 4620
4100 P$="A0 PARAMETER" @ Y=A0
4110 FOR N=0 TO Z
4120 T(N)=A0(N)
4130 NEXT N
4135 GOSUB 680
4140 GOTO 4600
4200 P$="A1 PARAMETER" @ Y=A1
4210 FOR N=0 TO Z
4220 T(N)=A1(N)
4230 NEXT N
4235 GOSUB 680
4240 GOTO 4600
4300 P$="B PARAMETER" @ Y=B
4310 FOR N=0 TO Z
4320 T(N)=B(N)
4330 NEXT N
4335 GOSUB 680
4340 GOTO 4600
4410 FOR N=0 TO Z
4420 T(N)=E1(N) @ S(N)=E2(N)
4425 GOSUB 1200
4426 GOTO 4600
4430 NEXT N
4600 GRAPH
4620 DISP "EXIT Y/N"
4630 INPUT A$
4640 IF A$="Y" THEN RETURN
4650 GOTO 4000
```

E.2 Program listing of the discrete-time parallel identifier

```
10 ! DISCRETE ID OF A 2nd ORDER
    PLANT PRECEDED BY A ZOH    !

15 !                            !
20 ! DIMENSIONING OF ARRAYS     !
25 ! *********************      !
30 DIM P(50),M(50)
40 DIM E(50),V0(50)
45 DIM U(50)
50 DIM A1(50),A2(50),B1(50),B2(
   50)
60 DIM F1(50),F2(50),F3(50),F4(
   50)
65 DIM F5(50),F6(50),F7(50),F8(
   50)
70 DIM F9(50),G0(50),G1(50),G2(
   50)
75 DIM G3(50),G4(50),G5(50),G6(
   50)
77 DIM D(50)
78 DIM T(50)
80 !                            !
90 ! VARIABLE INITIALIZATION    !
95 ! **********************     !
100 DISP "ENTER A1,A2,B1,B2"
110 INPUT A1,A2,B1,B2
120 DISP "ENTER AM1,AM2,BM1,BM2"
130 INPUT A1(0),A2(0),B1(0),B2(0
    )
140 DISP "ENTER INITIAL POS"
150 INPUT P(0)
155 M(0)=P(0)
156 DISP "ENTER PLANT DRIVE U"
157 INPUT U(1)
160 DISP "ENTER INPUT PERIOD "
170 INPUT P
180 DISP "ENTER VAL OF GAIN DIAG
    "
190 INPUT F
200 F1(0)=F @ F2(0)=0 @ F3(0)=0
    @ F4(0)=0
202 F5(0)=0 @ F6(0)=F @ F7(0)=0
    @ F8(0)=0
204 F9(0)=0 @ G0(0)=0 @ G1(0)=F
    @ G2(0)=0
206 G3(0)=0 @ G4(0)=0 @ G5(0)=0
    @ G6(0)=F
208 DISP "ENTER NOISE GAIN"
209 INPUT F0
210 DISP "ENTER D1,D2"
220 INPUT D1,D2
225 DISP "ENTER NUMBER OF SAMPLE
    S"
226 INPUT T
230 L=0
240 !                            !
250 ! START OF MAIN PROGRAM      !
251 ! ********************       !
252 N=0 @ Z=0
255 E(0)=0 @ D(0)=0
256 GOSUB 3000
260 GOSUB 500
265 GOSUB 3000
270 FOR N=2 TO T
280 Z=Z+1
290 IF Z=P THEN U(N)=-U(N-1) @ Z
    =0 ELSE U(N)=U(N-1)
295 U=U(N)
300 GOSUB 1000
310 GOSUB 1200
330 GOSUB 2000
340 GOSUB 1100
350 E(N)=P(N)-M(N)
355 GOSUB 1500
360 GOSUB 3000
400 NEXT N
410 GOSUB 4000
420 STOP
480 !                            !
500 ! CALC N=1 PARAMETERS        !
502 ! ******************         !
505 N=1
506 U=U(1)
510 P(1)=A1*P(0)+B1*U
540 V0(1)=P(1)-(A1(0)*M(0)+B1(0)
    *U)+D1*E(0)
550 A=F1(0)*M(0)+F3(0)*U
560 B=F5(0)*M(0)+F7(0)*U
570 C=F9(0)*M(0)+G1(0)*U
580 D=G3(0)*M(0)+G5(0)*U
590 E=F1(0)*M(0)+F9(0)*U
600 F=F2(0)*M(0)+G0(0)*U
610 G=F3(0)*M(0)+G1(0)*U
620 H=F4(0)*M(0)+G2(0)*U
630 I=E*M(0)+G*U
640 K=V0(1)/(1+I)
650 A1(1)=A1(0)+A*K
660 A2(1)=A2(0)+B*K
670 B1(1)=B1(0)+C*K
680 B2(1)=B2(0)+D*K
685 D(N)=((A1(N)-A1)^2+(A2(N)-A2
    )^2+(B1(N)-B1)^2+(B2(N)-B2)^
    2)^.5
690 M(1)=A1(1)*M(0)+B1(1)*U
700 E(1)=P(1)-M(1)
710 F1(1)=F1(0)-A*E/(L+I)
715 F2(1)=F2(0)-A*F/(L+I)
720 F3(1)=F3(0)-A*G/(L+I)
725 F4(1)=F4(0)-A*H/(L+I)
730 F5(1)=F5(0)-B*E/(L+I)
735 F6(1)=F6(0)-B*F/(L+I)
740 F7(1)=F7(0)-B*G/(L+I)
745 F8(1)=F8(0)-B*H/(L+I)
750 F9(1)=F9(0)-C*E/(L+I)
755 G0(1)=G0(0)-C*F/(L+I)
760 G1(1)=G1(0)-C*G/(L+I)
765 G2(1)=G2(0)-C*H/(L+I)
770 G3(1)=G3(0)-D*E/(L+I)
775 G4(1)=G4(0)-D*F/(L+I)
780 G5(1)=G5(0)-D*G/(L+I)
```

```
785 G6(1)=G6(0)-D*H/(L+I)
790 RETURN
980 !                              !
1000 ! CALCULATE PLANT OUTPUT !
1005 ! ********************** !
1010 P(N)=A1*P(N-1)+A2*P(N-2)+B1
     *U+B2*U
1015 P(N)=P(N)+F0*(RND-.5)
1020 RETURN
1050 !                              !
1100 ! CALCULATE MODEL OUTPUT !
1105 ! ********************** !

1110 M(N)=A1(N)*M(N-1)+A2(N)*M(N
     -2)+B1(N)*U+B2(N)*U
1120 RETURN
1180 !                              !
1200 ! CALCULATE V0(N)              !
1205 ! **************               !

1210 M=A1(N-1)*M(N-1)+A2(N-1)*M(
     N-2)+B1(N-1)*U+B2(N-1)*U
1220 V0(N)=P(N)-M+D1*E(N-1)+D2*E
     (N-2)
1230 RETURN
1250 !                              !
1500 ! UPDATE INTEG. GAINS          !
1550 ! *******************          !
1600 K=L+I
1610 Q=E/K
1620 R=F/K
1630 S=G/K
1640 Y=H/K
1650 F1(N)=F1(N-1)-A*Q
1660 F2(N)=F2(N-1)-A*R
1670 F3(N)=F3(N-1)-A*S
1680 F4(N)=F4(N-1)-A*Y
1690 F5(N)=F5(N-1)-B*Q
1700 F6(N)=F6(N-1)-B*R
1710 F7(N)=F7(N-1)-B*S
1720 F8(N)=F8(N-1)-B*Y
1730 F9(N)=F9(N-1)-C*Q
1740 G0(N)=G0(N-1)-C*R
1750 G1(N)=G1(N-1)-C*S
1760 G2(N)=G2(N-1)-C*Y
1770 G3(N)=G3(N-1)-D*Q
1780 G4(N)=G4(N-1)-D*R
1790 G5(N)=G5(N-1)-D*S
1800 G6(N)=G6(N-1)-D*Y
1810 RETURN
1850 !                              !
2000 ! UPDATE PARAMETERS            !
2001 ! ****************             !

2002 A=F1(N-1)*M(N-1)+F2(N-1)*M(
     N-2)+F3(N-1)*U+F4(N-1)*U
2004 B=F5(N-1)*M(N-1)+F6(N-1)*M(
     N-2)+F7(N-1)*U+F8(N-1)*U
2006 C=F9(N-1)*M(N-1)+G0(N-1)*M(
     N-2)+G1(N-1)*U+G2(N-1)*U
2008 D=G3(N-1)*M(N-1)+G4(N-1)*M(
     N-2)+G5(N-1)*U+G6(N-1)*U
2010 E=F1(N-1)*M(N-1)+F5(N-1)*M(
     N-2)+F9(N-1)*U+G3(N-1)*U
2012 F=F2(N-1)*M(N-1)+F6(N-1)*M(
     N-2)+G0(N-1)*U+G4(N-1)*U
2014 G=F3(N-1)*M(N-1)+F7(N-1)*M(
     N-2)+G1(N-1)*U+G5(N-1)*U
2015 A1(N)=A1(N-1)+A*V0(N)/K
2016 H=F4(N-1)*M(N-1)+F8(N-1)*M(
     N-2)+G2(N-1)*U+G6(N-1)*U
2020 I=E*M(N-1)+F*M(N-2)+G*U+H*U
2030 K=1+I
2040 A1(N)=A1(N-1)+A*V0(N)/K
2050 A2(N)=A2(N-1)+B*V0(N)/K
2060 B1(N)=B1(N-1)+C*V0(N)/K
2070 B2(N)=B2(N-1)+D*V0(N)/K
2075 D(N)=((A1(N)-A1)^2+(A2(N)-A
     2)^2+(B1(N)-B1)^2+(B2(N)-B2
     )^2)^.5
2080 RETURN
2500 !                              !
3000 ! PRINT RESULTS ON SCREEN!
3005 ! ***********************!

3010 DISP "A1(N)=";INT(A1(N)*100
     00+.5)/10000
3020 DISP "A2(N)=";INT(A2(N)*100
     00+.5)/10000
3030 DISP "B1(N)=";INT(B1(N)*100
     00+.5)/10000
3040 DISP "B2(N)=";INT(B2(N)*100
     00+.5)/10000
3050 DISP "E(N)= ";INT(E(N)*1000
     0+.5)/10000
3055 DISP "D(N)=";INT(D(N)*10000
     +.5)/10000
3060 DISP
3070 RETURN
3080 !                         !
4000 ! PLOT RESULTS            !
4005 ! ***********             !

4010 DISP "PLOT A1,A2,B1,B2,E D
     ?"
4020 INPUT A$
4030 IF A$="A1" THEN GOTO 4100
4040 IF A$="A2" THEN GOTO 4200
4050 IF A$="B1" THEN GOTO 4300
4060 IF A$="B2" THEN GOTO 4400
4070 IF A$="E" THEN GOTO 4500
4080 IF A$="D" THEN GOTO 4550
4100 P$="A1 PARAMETER" @ Y=A1
4110 FOR N=0 TO T
4120 T(N)=A1(N)
4130 NEXT N
4140 GOTO 4600
4200 P$="A2 PARAMETER" @ Y=A2
4210 FOR N=0 TO T
4220 T(N)=A2(N)
```

```
4230 NEXT N
4240 GOTO 4600
4300 P$="B1 PARAMETER" @ Y=B1
4310 FOR N=0 TO T
4320 T(N)=B1(N)
4330 NEXT N
4340 GOTO 4600
4400 P$="B2 PARAMETER" @ Y=B2
4410 FOR N=0 TO T
4420 T(N)=B2(N)
4430 NEXT N
4440 GOTO 4600
4500 P$="ERROR" @ Y=E
4510 FOR N=0 TO T
4520 T(N)=E(N)
4530 NEXT N
4540 GOTO 4600
4550 P$="PAR. ERROR" @ Y=0
4555 FOR N=0 TO T
4560 T(N)=D(N)
4570 NEXT N
4580 GOTO 4600
4590 !                          !
4600 !         PLOT AXES        !
4610 !         *********        !
4620 DISP "ENTER Y0,Y1"
4630 INPUT Y0,Y1
4632 DISP "ENTER MAX SAMPLE"
4633 INPUT T
4635 GCLEAR
4640 L=ABS(Y0-Y1)
4650 SCALE -(.2*T),T+.1*T,Y0-.2*
     L,Y1+.2*L
4660 IF Y0<Y1 THEN I=Y0 ELSE I=Y
     1
4670 XAXIS I,T/5,0,T
4680 YAXIS 0,L/4,Y0,Y1
4690 MOVE .15*T,Y0-.2*L
4700 LABEL "SAMPLE NUMBER"
4710 LDIR 90
4720 MOVE -(.15*T),Y0+.2*L
4730 LABEL P$
4740 LDIR 0
4750 FOR X9=0 TO T STEP T/5
4760 MOVE X9,Y0-.1*L
4770 LABEL VAL$(X9)
4780 NEXT X9
4790 FOR Y9=Y0 TO Y1 STEP L/4
4800 MOVE -(.12*T),Y9
4810 LABEL VAL$(Y9)
4812 NEXT Y9
4815 ! INFO PRINT               !
4816 MOVE .4*T,Y1+.15*L
4817 LABEL "D1="&VAL$(D1)
4818 MOVE .4*T,Y1+.05*L
4819 LABEL "D2="&VAL$(D2)
4820 MOVE 2/3*T,Y1+.15*L
4821 LABEL "GAIN="&VAL$(F1(0))
4830 ! PLOT VARIABLES           !
4832 MOVE 0,Y
4834 DRAW T,Y
4840 MOVE 0,T(0)
4850 FOR N=1 TO T
4860 DRAW N,T(N)
4870 NEXT N
4875 PAUSE
4893 DISP "EXIT (Y/N)"
4895 INPUT A$
4900 IF A$="Y" THEN STOP
5000 GOTO 4000
5500 RETURN
6000 END
```