

Estimating Credit Default Swap Spreads from Equity Data

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A dissertation submitted to the Department of Actuarial Science,
Faculty of Commerce, at the University of the Cape Town, in partial
fulfilment of the requirements for the degree of Master of Philosophy.

25th May 2014

*Master of Philosophy specializing in Mathematical Finance,
University of the Cape Town,
Cape Town.*



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Declaration

I declare that this dissertation is my own, unaided work. It is being submitted for the Degree of Master of Philosophy in the University of the Cape Town. It has not been submitted before for any degree or examination in any other University.

25th May 2014

Abstract

Corporate bonds are an attractive form of investment as they provide higher returns than government bonds. This increase in returns is usually associated with an increase in risk. These risks include liquidity, market and credit risk. This dissertation will focus on the modelling of a corporate bond's credit risk by considering how to estimate the credit default swap (CDS) spread of a firm's bond. A structural credit model will be used to do this. In this dissertation, we implement an extension of Merton's model by Hull, Nelken and White (2004), which is based on the use of the implied volatilities of options on the company's stock to estimate model parameters. Such an approach provides an insight into the relationship between credit markets and options markets.

Acknowledgements

I would like to thank Dr Steven Cunanne for providing the topic and his continuous guidance along the way. His help with the review of this dissertation is also acknowledged. In no particular order of importance, I would also like to thank the following people and organisations

- Professor David Taylor, Dr Marchand van Rooyen and Dr Thomas McWalter for providing me with the required background knowledge to tackle the problem presented by Steven.
- Obeid Mahomed for providing guidance and answering any questions I had relating to my dissertation.
- Nishlen Govender for editing this dissertation.
- The Liberty Life Group for sponsoring me throughout my academic career. Without their generous financial support and understanding none of this would have been possible.
- The John Davidson Educational Trust for sponsoring me during my Masters.
- The MPhil class of 2014 for making my Masters's year a memorable one.
- Manchester United Football Club for making me believe that anything is possible.

Finally, I would like to thank my parents and sisters who have supported me through every step of the way. My loving family made me believe that during the difficult times there is always light at the end of the tunnel. They have moulded me into the person I am today.

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Chapter 1

Introduction to credit risk

1.1 A Corporate Bond and Credit Risk

Asset managers, banks and insurers in South Africa usually invest a large percentage of their assets in a variety of bonds. As corporate bonds provide better returns than government bonds, financial institutions have directed much of their long term fixed interest investments towards them. As a result of the increased risk associated with corporate bonds, holders of these instruments need an understanding of the increased credit risk they are exposed to. Another constraint for some financial institutions is that these corporate bonds need to be valued at fair value for accounting purposes. This means that they need to be valued using “market” data.

To calculate the probabilities of default, we implement an extension of Merton’s structural model by Hull, Nelken and White (2004). This model allows one to estimate the volatility of the firm’s assets from the implied volatilities of options written on the firm’s stock. This method is attractive to us as it estimates the probability of default from market data. Once probabilities of default are calculated using this model, we can estimate the Credit Default Swap (CDS) spread for a name and compare the calculated spreads to observed spreads in the market.

The structure of the dissertation will be as follows. In Section 2 we look

at Credit Default Swaps, their history and their features. Thereafter, we introduce the two major types of credit models in Section 3 . We then discuss structural models in detail by conducting a review of the extensions to the Merton and First Passage Time models. Following that, in Section 4 we choose the models we will be working with and explain them in detail. We then perform an empirical investigation in Section 5 where we evaluate the CDS spreads for nine firms. In Section 6 we present our findings from this investigation. Finally, we present our conclusion in Section 7.

Chapter 2

Credit Default Swaps

A Credit Default Swap or CDS transfers the credit risk associated with a bond issued by an entity, usually a firm, from one investor to another. O’Kane and Turnbull (2003) define a CDS as follows: *“In a standard CDS contract one party purchases credit protection from another party, to cover the loss of the face value of an asset following a credit event. This protection lasts until some specified maturity date. The protection is limited to a percentage of the face value. This percentage is referred to as the recovery rate. To pay for this protection, the protection buyer makes a regular stream of payments, known as the premium leg, to the protection seller. These payments are made until a credit event occurs or until maturity, whichever occurs first.”* The CDS spread should not be confused with a credit spread. A credit spread is the difference in yield between a “risk free” bond and a risky bond. To calculate the premium of the CDS, we use the quoted swap spread. This spread would be a reflection of the credit worthiness of the firm.

There are many sources, which differ on the origin of credit default swaps. However, many agree that they were created in the early 1990s by JP Morgan and Deutsche Bank. The very first CDSs were created as a way for financial institutions, mainly banks, to indemnify themselves and their shareholders against their exposure to large corporate bonds. The market has grown rapidly during the last decade from a gross notional in the low hundred billions in

the late 1990s, to a gross notional which is close to £17 trillion today (Wilson, 2011). According to McLean and Nocera (2011), J.P. Morgan constructed its first CDS in 1994. J.P.Morgan had a \$4.8 billion line of credit exposure to Exxon as a result of the Exxon Valdez oil spill. This exposure required J.P. Morgan to tie up millions of dollars in capital, which had to be placed in reserve. Blythe Masters, an employee of JP Morgan, developed the idea of using a CDS to mitigate their exposure. The European Bank for Reconstruction and Development (EBRD) in London agreed to cover any loan default by Exxon in exchange for a steady fee from J.P. Morgan. The agreement provided a stream of cash flows for the EBRD and allowed JP Morgan to remove some risk from their balance sheet. Most European credit default swap agreements use a template called a “master agreement”. This is the standard agreement template written by the International Swaps and Derivatives Association (ISDA). We should note that almost all CDSs are traded on the OTC market and not on an exchange.

A CDS is triggered by a so-called credit event. The technical condition for a default is difficult to determine. To help with this problem, there are five regional “ISDA credit determinations committees”. These committees are comprised of banks and major credit investors. They act as the “ultimate arbiters” on claims relating to possible credit or default events. If there is a possible claim, the holder of the CDS would submit a claims request to the determinations committee. The committee will first decide whether they should accept the query. If they do, the committee will then review the claim to determine whether the CDS should be triggered.

As the CDS market is fairly liquid in the USA and Europe, it can be used to measure the credit quality of a name. In our case, the market CDS spreads will be used to determine the suitability of our model. This will be done by comparing market spreads to our model’s calculated CDS spreads. For the remainder of this dissertation, we will focus on the 5 year CDS.

Chapter 3

Credit Risk Modelling

3.1 Credit Risk Models: Structural Versus Intensity-Based Approach

The very first way in which credit risk was mathematically modelled is now referred to as the structural modelling of credit risk. This first approach goes back to the fundamental papers by Black and Scholes (1973) as well as Merton (1974). This approach uses economic fundamentals, such as the capital structure of a company, in order to model the likelihood of a default event. Along with this, the total value of the firm's assets and the default triggering barrier are the major components of the model. The firm's assets, V_t , are modelled by Geometric Brownian motion with the following dynamics:

$$dV_t = V_t (rdt + \sigma_V dW_t). \quad (3.1)$$

Here, W_t is standard Brownian motion. The firm's equity, E_t , is modelled as a derivative with V_t as the underlying. In this framework, if the asset value, V_T , is insufficient to pay back the debt, the firm is considered to have defaulted. We model this as V_T crossing some default threshold barrier, D . Usually, this default threshold, D , will coincide with the debt level of the firm. This argument only allows for a firm to default at time T . The following equations describe how the firm's equity can be valued:

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$$E_T = \max(V_T - D, 0). \quad (3.2)$$

By the Black-Scholes equation, the initial value of the equity can be expressed as:

$$E_0 = V_t \Phi(d_1) - D e^{-r(T)} \Phi(d_2), \quad (3.3)$$

where d_1 and d_2 are as follows

$$d_1 = \frac{\ln\left(\frac{e^{r(T)} V_t}{D}\right) + \frac{1}{2} \sigma_V^2(T)}{\sigma_V \sqrt{T}} \quad (3.4)$$

$$d_2 = d_1 - \sigma_V \sqrt{T}. \quad (3.5)$$

Finally, the probability of default (PD) can be expressed as follows:

$$PD_T = P[V_T < D] = \Phi(-d_2). \quad (3.6)$$

Heath, Jarrow and Morton (1992) introduced another way of modelling credit risk. This is now known as the reduced form framework. Here, they model the default process and how the process evolves through different stages. The evolution of default is defined as a jump process. In this model, a firm's credit quality may move from states of no-default to an absorbing default state. To extract the default intensity of a firm, these models make use of market prices of the firm's defaultable bonds and CDSs. This hazard rate is usually state dependent and can be used to calculate PDs as well as the prices of other defaultable instruments. Under these models, the capital structure of the firm is not considered at all. Reduced form models make exclusive use of market data, in particular, bond prices, as the only source of information regarding the firms credit structure.

With a structural model, one can model both credit risk and equity. This is done through linking the two through the firm's asset value, V_t . As this

fits in with the problem of modelling the term structure of the probability of default from market data, structural models will be considered exclusively in this dissertation.

3.2 Criticism and Extensions of the Merton Model

The one major advantage (and disadvantage) of Merton's model is that it employs the theory of European option pricing, which was developed by Black and Scholes in 1973. Thus, many assumptions are required to allow the theory to hold. Assumptions were made with regard to the firm's capital structure, the dynamics of its asset value process as well as the interest rate term structure. With any model, there is a trade-off between plausibility and tractability. Merton's model sacrifices plausibility for the sake of tractability. Following Merton's work, many extensions were made to this model. As these models were made more realistic, one of the main goals was for the model to have a closed form solution for the probability of default. These new models saw the introduction of more realistic assumptions. Although some of these models did not produce closed form expressions for the required probabilities, most of them provided expressions for the probabilities which could be evaluated using numerical methods. Some of these extensions to the original model were presented by Merton (1974). These extensions, among others, include stochastic interest rates, accounting for coupon bearing and callable bonds as well as relaxing the assumption that the Modigliani-Miller Theorem holds.

A major drawback of Merton's model is that default can only occur at maturity. By doing this, the model rules out the event of an early default. This means that if the firm's value falls down to levels below its liabilities and recovers to meet its commitments at maturity, the firm will not have

defaulted under the approach of Merton. Another problem with this model is the assumption around a firm's capital structure. If we consider a firm, it would be safe to say that its capital structure is far more complex than a simple zero-coupon bond. Geske (1977) extends the model by considering the relationship between the liability structure of the firm and a coupon bearing bond. Under this model, each coupon payment is treated as a compound option. This model allows for default at each possible coupon payment. If the firm fails to make a coupon payment, the firm is considered to have defaulted. This accounts for the possibility of an early default. Geske also considers other extensions to the model. These include the consideration of features such as safety covenants, sinking funds, debt subordination and pay-out restrictions.

The relaxation of the assumption of flat term structure of interest rates is an extension which we encounter a few times across the literature. Jones *et al.* (1984) suggests that *"there exists evidence that introducing stochastic interest rates, as well as taxes, would improve the model's performance."* With the inclusion of stochastic interest rates, it enables the modeller to account for correlation between assets of a firm and the interest rates. Hsu *et al.* (2003), Longstaff and Schwartz (1995), Ronn and Verma (1986), Briys and De Varenne (1997) and Kim *et al.* (1993) have all considered the above extension.

One popular extension was related to the calibration of the Merton model. Most of the works we reviewed used market data to calibrate the volatility of a firm's assets, σ_V . Jones, Mason and Rosenfeld (1984) use the value of the company's stock price along with its instantaneous volatility to estimate σ_V . The most interesting approach we came across was that of Hull, Nelken and White (2004). Their model calibration extension is based on the use of the implied volatilities of options on the company's equity to estimate model parameters. They test their model by calculating a credit spread and comparing it to market CDS spread data. They note that using CDS spreads are an attract-

ive alternative to the use of a bond's credit spread. Credit spreads calculated from a bond price, usually depend on the bond's liquidity. To determine the credit component of the spread, one would also need an estimate of the true risk-free rate.

Another general extension of the Merton model was the introduction of a jump process in the asset's dynamics. Delianedis and Geske (2001) investigate how the credit spread of a bond can be broken down. In particular, they focus on the proportion of a bond's credit spread that can be attributed to default risk. This is done by using the frameworks provided by Merton and Geske. Delianedis and Geske come to the conclusion that default risk only explains a small amount of a bond's credit spread. They show that the rest of the spread can be explained by market risk factors, liquidity, taxes as well as jumps. To investigate the effect of a jump process, the authors include a jump component in the Merton model. They conclude that: *“while jumps may explain a portion of the residual spread it is unlikely that jumps can explain it entirely.”*

Arora, Bohn and Zhua (2006) empirically compare two structural models, the basic Merton and Vasicek-Kealhofer (VK) as well as one reduced-form model. To investigate the relative merits of each model, they test the ability of each model to predict spreads in the CDS market. Built on the original insights of Black-Scholes-Merton, Vasicek and Kealhofer developed a straightforward variant of the Merton model. Arora *et al.* employ the commercial adaptation of the VK model by Moody's KMV. They find that the VK model outperforms the other two models when it comes to predicting CDS spreads.

Zhang *et al.* (2009) provide a model to determine CDS spreads. Their model is a stylized structural model, which is used to interpret the observed effects of volatility and jump risks on credit spreads. They calibrate four nested models: A Merton model, a jump-diffusion, a stochastic volatility and jump-diffusion stochastic volatility model. They find that the model which incorporates both jump-diffusion and stochastic volatility is the best predictor

of observed CDS spreads. It should be noted that all of these models provided spreads, which are lower than the observed spreads in the market.

3.3 First Passage Time Models

Black and Cox (1976) introduced the original First Passage Model (FPM). This was the result of a natural extension to the Merton model. Black and Cox (1976) extended the model to allow for the case when a particular firm may default at any time and not only at the maturity date of the debt. Under the measure Q we assume the assets follow the same dynamics as described in the previous section:

$$dV_t = V_t(rdt + \sigma_V dW_t). \quad (3.7)$$

We also assume that there exists a lower level of the firm's asset value such that if the firm's assets fall below this level, it defaults. We assume that this default barrier is constant. Conditional on default not having occurred yet ($V_t > D$), the time of default at time $t = 0$ is given by:

$$\tau = \inf [s \geq t | V_s \leq D]. \quad (3.8)$$

Following the reflection principle properties of Brownian motion W_t , it can be shown that the default probability from time t to time T can be expressed as:

$$PD(t) = P[\tau \leq T | \tau \geq t] = \Phi(h_1) + \exp\left[\frac{2}{\sigma_V^2} \left(r - \frac{\sigma_V^2}{2} \ln\left(\frac{D}{V_T}\right)\right)\right] \Phi(h_2), \quad (3.9)$$

where

$$h_1 = \frac{\ln\left(\frac{D}{e^{r(T-t)}V_t}\right) + \frac{1}{2}\sigma_V^2(T-t)}{\sigma_V\sqrt{T-t}} \quad (3.10)$$

and

$$h_2 = h_1 - \sigma_V \sqrt{T - t}. \quad (3.11)$$

3.4 First Passage Time Models: Extensions and Criticisms

There have been many extensions made to the original works of Black and Cox (1976). Elizalde (2006) provides a review of most of these extensions. These extensions include: time-dependent and stochastic default barriers, stochastic interest rates, jumps in the asset value process, allowing for a strategic default, taxes and debt subordination. Naturally, these extensions make the model more realistic. The major downside is that they increase the models analytical complexity. First Passage Time models are known to be analytically complex and this complexity is increased even further if we consider some of the extensions in the list mentioned above. This mathematical complexity associated with First Passage Time models makes it difficult for one to obtain a closed form expression for the value of the default probability. As a result of this drawback, one would usually resort to numerical methods to solve for the default probabilities.

Although many empirical tests have been carried out on structural models and FPM models, in general, they have not yielded much success. Eom, Helwege and Huang (2004) carried out a review of five models (Collin-Dufresne and Goldstein, Merton, Leland and Toft, Longstaff and Schwartz, and Geske). The authors perform an empirical analysis on these models. Eom, Helwege and Huang (2004) find that these five structural bond pricing models produce prices, which are not consistent with corporate bond prices in the market. The authors also indicate that difficulties are not limited to the under-prediction of spreads. In particular, the authors note the dispersion of predicted spreads

by the various models.

Another major criticism of these structural models we discussed above is the so-called predictability of defaults. Elizalde (2006) states that the process which drives the asset value of a firm under the FPM model does not allow for a sudden default. He argues that sudden defaults are sometimes observed in the market. He also states that the model has almost a zero probability of a sudden default. A major reason why these FPM models produce small short-term credit spreads is due to the problems associated with “predictability of default” described above. These short-term credit spreads produced by FPM models are thus inconsistent with the spreads observed in the market. In the market, we usually observe that short-term credit spreads are far higher than those calculated by a FPM model. This can be explained by the fact that the market incorporates the possibility of unexpected default or a sudden deterioration in the firm’s credit quality. It should be noted that the same properties of these structural models that make default predictable also make recovery predictable.

In most structural models, when a firm defaults, it is assumed that bondholders or creditors get the remaining value (besides those that consider strategic default). This “wind-up” value coincides with the default threshold at default. If we consider a situation with complete information about the asset value and the default threshold, then the recovery rate is also a predictable quantity. In the literature we reviewed, we came across two ways to deal with the problem of these predictability effects. The predictability of default is a result of the assumption of perfect knowledge or information about a company’s assets and debt. If we consider the real world, it is impossible to deduce the value of the firm’s assets, V_t , its debt level, its asset’s volatility, σ_V , or the default threshold simply by looking at the capital structure of the firm. In the first approach, we assume that we have only incomplete information about the firm’s asset process and its default level. This results in a model whereby the

market can only determine a distribution function for these processes. This makes default impossible to predict under these models. This approach is considered, among others, in Duffie and Lando (2001) and Jarrow and Protter (2004).

The other way of dealing with this problem is to incorporate jumps in the dynamics of the firm value. The result of this is that it allows for the asset value of the firm to suddenly drop. These jumps (drops) can reduce the distance to default (the gap between asset value and default threshold), or even cause a default if the drop is big enough. The net result is that default is no longer a predictable event. It can be observed that under these models, short-term PDs do not tend to zero. This results in short-term credit spreads which are higher than those which we observe under traditional FPM models. Most of the literature agrees that these models also overestimate the credit spread. Zhou (2001) and Hilberink and Rogers (2002) investigate the effects of incorporating a jump process or a jump component into the asset value of a firm. Zhou takes the method of Longstaff and Schwartz (1995) and extends it by adding a lognormally distributed jump component. Hilberink and Rogers (2002) researched the use of certain Levy processes to model the firm's asset value. They only considered processes that could only have downward jumps. They implement this by extending the model of Leland (1994). The models by Zhou (2001) and Hilberink and Rogers (2002) circumvent the problem of the predictability of default.

Incorporating jumps in a model has an interesting consequence. It converts the recovery payment at default into a random variable. This comes about as the assets of the firm can jump or drop suddenly below the default threshold. If the firm's assets followed a diffusion process without the inclusion of any jump processes, the value of the firm at default would be the debt value. Fouque *et al.* (2006) investigates the use of introducing stochastic volatility in FPM. The authors find that this extension results in increased short-term spreads.

The last set of extensions we came across investigate liquidity and its effect on credit spreads. Davydenko (2005) is critical of the traditional literature on structural models as most authors do not consider liquidity reasons as a major determinant of default. He indicates that firms with high funding costs are more likely to default. The author uses a sample of American bond issuers from 1996 to 2003 to show that liquidity shortages are likely to cause default for firm, especially if the firm's cost of external financing is high. Another important part of Davydenko's work is that he presents an argument and evidence against the traditional structural model. He argues against the idea that default is caused when the asset value crosses a particular debt threshold.

3.5 Information Content of Market Data for CDS Spreads

Cao, Yu and Zhong (2010) investigate the relationship between CDS spreads and option-implied volatility. To do this, the authors analyse a sizeable sample of firms with both CDS and options data. Cao, Yu and Zhong (2010) observe that a firm's put option-implied volatility is better at explaining the variation of Credit Default swap spreads over time than historical volatility. Cao, Yu and Zhong (2010) argue that a CDS and an out-of-the-money put option are similar, as they both offer protection against downside risk. Cremers *et al.* (2008) further support this claim. Their paper introduces measures of volatility and jump risk that are based on individual stock options to explain credit spreads on corporate bonds. They also show that implied volatilities of individual options contain useful information for credit spreads.

Zhang *et al.* (2009) also investigate the relationship between CDS spreads and volatility. In addition to this, they consider a firm's stock price jump risk. The authors then calibrate a Merton-type structural model that incorporates

stochastic volatility and jumps. They find that their jump-diffusion stochastic volatility model outperforms the alternative models in fitting the observed credit spreads.

Chapter 4

Model Choices

After considering the extensions in Section 3, we now choose the models to be implemented. The models selected should allow accurate estimation of market CDS spreads using only equity data. The approach of Hull, Nelken and White (2004) is most suitable. We extend their analysis one step further by calculating CDS spreads instead of the usual credit spread. The section below motivates the choice of this model. We also investigate an extension to the First Passage Time (FPT) model of Black and Cox (1976) using the approach of Hull, Nelken and White (2004).

4.1 Hull-Nelken-White Model

If we consider Merton's model, it can be shown that a vanilla option on the equity of a firm can be represented as a compound option on the company's assets. Geske (1977) provided a valuation formula for compound options. In the paper by Hull, Nelken and White (2004), the authors use the work of Geske to extend Merton's model one step further. They demonstrate how the credit spread in Merton's model can be estimated from the implied volatilities of equity options. We will now explain how they implement this.

As we have shown previously in equation 3.3, the equity of a firm can be expressed as a call option on its assets if its dynamics can be described

by equation 3.1. Following this, an option on the equity of a firm can be expressed as a compound option. If you consider a put option and a CDS, they both provide the holder with a form of downside protection. A put option provides you with protection if the equity price drops sharply. A CDS protects a bondholder in the event of a default. If the market anticipates a default, we would expect stock prices to drop. The major reason for choosing this model is that it links the options market to the credit market. Consider the following payoff of a put option on a firm's equity with maturity \bar{T} :

$$\max(K - E_{\bar{T}}, 0), \quad (4.1)$$

where $E_{\bar{T}}$ is a call option on the assets at \bar{T} .

It should be noted that the call option on the assets or equity has an expiry, T such that $T > \bar{T}$. It can be shown, using the work by Geske (1977) together with the Black-Scholes framework, that the price of such an option can be expressed by the following equation:

$$p = De^{-rT} \text{BVN} \left[-a_2, d_2; \sqrt{\frac{\bar{T}}{T}} \right] - V_0 \text{BVN} \left[-a_1, d_1; -\sqrt{\frac{\bar{T}}{T}} \right] \quad (4.2)$$

$$+ Ke^{-rT} \Phi(-a_2), \quad (4.3)$$

where D is the debt level and

$$a_1 = \frac{\ln \left(\frac{V_0}{V_{\bar{T}}^* e^{-r\bar{T}}} \right) + \frac{1}{2} \sigma_V^2(\bar{T})}{\sigma_V \sqrt{\bar{T}}} \quad (4.4)$$

$$a_2 = a_1 - \sigma_V \sqrt{\bar{T}}. \quad (4.5)$$

In the above equation, BVN is the cumulative bivariate normal distribution function. $V_{\bar{T}}^*$ is the critical asset value at time \bar{T} such that the equity value (i.e. the call option on the firms's assets) equals K . That is, $V_{\bar{T}}^*$ is the asset value

below which the put option on the equity will be exercised. The expressions for d_1 and d_2 are listed in equations 3.4 and 3.5 respectively. We also express:

$$K = \kappa E_0 e^{r\bar{T}} \quad (4.6)$$

$$V_{\bar{T}}^* = \alpha V_0 e^{r\bar{T}}. \quad (4.7)$$

It can be seen that α is the ratio of the critical asset price, $V_{\bar{T}}^*$ to the forward asset price. Hull, Nelken and White (2004) refer to it as the implied strike level. The parameter κ is the ratio of the option strike price to the forward equity price and the authors refer to it as the option's moneyness.

If vanilla option trades on the market, the market convention is to quote the implied volatility of the option. The following equations for a put option illustrate this:

$$\begin{aligned} P_0 &= K e^{-r\bar{T}} \Phi(-d_2^*) - E_0 \Phi(-d_1^*) \\ &= \kappa E_0 \Phi(-d_2^*) - E_0 \Phi(-d_1^*), \end{aligned} \quad (4.8)$$

where

$$\begin{aligned} d_1^* &= \frac{\ln\left(\frac{e^{r\bar{T}} E_0}{K}\right) + \frac{1}{2} v^2(\bar{T})}{v\sqrt{\bar{T}}} \\ &= \frac{\ln(\kappa) + \frac{1}{2} v^2(\bar{T})}{v\sqrt{\bar{T}}} \end{aligned} \quad (4.9)$$

$$d_2^* = d_1^* - v\sqrt{\bar{T}}. \quad (4.10)$$

If we have a market price, P_0 , we can solve for v . We denote the above put option's implied volatility by v .

We can now express a put option on a firm's equity in two different ways. If we set these two equations equal to one another and solve for the $V_{\bar{T}}^*$, we

can determine the volatility of the assets. As we can express $V_{\bar{T}}^*$ as $\alpha V_0 e^{r\bar{T}}$, we can solve for α instead. The volatility of the assets, σ_V , together with implied strike level, α , can be determined by solving the following set of equations:

$$\begin{aligned} \kappa E_0 \Phi(-d_2^*) - E_0 \Phi(-d_1^*) &= De^{-rT} BVN \left[-a_2, d_2; \sqrt{\frac{\bar{T}}{T}} \right] \\ &- V_0 BVN \left[-a_1, d_1; -\sqrt{\frac{\bar{T}}{T}} \right] \\ &+ Ke^{-rT} \Phi(-a_2) \end{aligned} \quad (4.11)$$

and

$$\begin{aligned} \kappa E_0 e^{r\bar{T}} &= V_{\bar{T}}^* \Phi(d_{2,\bar{T}}) - De^{-r(T-\bar{T})} \Phi(d_{1,\bar{T}}) \\ &= V_{\bar{T}}^* \left[\Phi(d_{2,\bar{T}}) - \frac{L}{\alpha} \Phi(d_{1,\bar{T}}) \right], \end{aligned} \quad (4.12)$$

where $L = \frac{De^{rT}}{V_0}$ (the leverage ratio) and

$$d_{1,\bar{T}} = \frac{-\ln\left(\frac{L}{\alpha}\right) + \frac{1}{2}\sigma_V^2(T-\bar{T})}{\sigma_V \sqrt{T-\bar{T}}} \quad (4.13)$$

$$d_{2,\bar{T}} = d_{1,\bar{T}} - \sigma_V \sqrt{T-\bar{T}}. \quad (4.14)$$

Equation 4.11 is used to equate the two methods for pricing a put option on the equity of the firm. Equation 4.12 is used to find α . Given σ_V , solving this equation will give us a value for α such that the call option on the assets (the equity) of the firm is equal to K . In other words, we are solving for the critical value of the assets, $V_{\bar{T}}^*$, such that a call option on the assets equals K . Equation 4.12 is derived from the usual Black-Scholes equation for a vanilla option. Equations 4.11 and 4.12 can be solved simultaneously to get values

for α and σ_V . These two equations allow us to get a market consistent σ_V . With the above pricing formula for a put option on a firm's equity, we can now relate the asset's volatility σ_V to an implied volatility on the firm's stock. After solving for σ_V , we use equation 3.6 to determine the probability of default.

We chose this model as past research has shown there is a strong link between equity option and the credit derivatives markets. This relationship may allow one to get market consistent probabilities of default and CDS spreads.

4.2 First Passage Time

We shall make use of the standard first passage time model by Black and Cox (1976) with the calibration extension described in Section 4.1. This allows us to determine two sets of probabilities and CDS spreads. This extension is simple in nature. It involves calibrating σ_V to an implied volatility of a put option on the name's stock in the same way as we have described in section 4.1. We then use this estimate of σ_V to calculate the probability of default using equation 3.9.

Chapter 5

Empirical Investigation

To evaluate the suitability of our model, we conducted an empirical investigation. We used the models to calculate the probabilities of default for a set of firms. These PDs were used together with the appropriate discount curves to calculate CDS spreads. In the section below, the methodology is explained in detail. The results are presented at the end of the section.

5.1 Data

All the data needed for the calculations was collected from Bloomberg. Data was collected for selected American, European and South African Stocks. The credit worthiness for the firms listed in Table 5.1 was investigated:

Firm's Name	Ticker	S&P Rating
The Goldman Sachs Group Inc	GS	A-
JPMorgan Chase & Co	JPM	A
AK Steel Holding Corp	AKS	B
Telefonica SA	TEF	BBB
Target Corp	TGT	A+
British Petroleum	BP	A
FirstRand Ltd	FSR	BBB
BHP Billiton PLC	BHP	A+
Standard Bank Group Ltd	SBK	BBB

Tab. 5.1: List of firms together with their S&P credit rating as at November 2013.

The period of investigation was from 1 January 2013 to 31 October 2013. For each European and American stock, we downloaded CDS spreads for each business day in the period. We used five-year CDS quotes in our analysis. For each firm, implied volatility surfaces were also downloaded for the first business day of each month. For the South African firms, volatility surfaces were only available from April to September. Balance sheets were retrieved for each firm on the list and were used to calculate the debt outstanding and the asset value per share. As per the ISDA CDS valuation model, ISDA CDS fixing swap curves were downloaded for both the American and European markets for the concerned period. South African swap curves were downloaded to value the South African stocks' CDS spreads.

5.2 Methodology

After all the data for the investigation was collected the probabilities of default were calculated for each firm. For each firm, we calculated the asset per share, V_0 , and debt per share, D . This was done by dividing the total assets and total liabilities, respectively, by the number of shares outstanding for each firm. The risk free rate, r , was chosen from the appropriate swap curve so that it had the same tenor as the option. To calculate the asset volatility we used the approach of Hull, Nelken and White (2004). To do this, we used 80 percent out-the-money put options. These options were chosen as they were the furthest out-the-money put options available with data for all the stocks. To solve equations 4.11 and 4.12, we used Matlab's `fsolve`. This function allows one to solve two non-linear simultaneous equations. As inputs, it takes the set of equations as well as the initial estimates for the two parameters being solved for. We performed this analysis for various tenors of the 80 per cent out-the-money put options. This was consistent with what was done in previous research. As we show later in the findings section, the implied volatility of these options and the CDS spreads for a particular firm are highly correlated.

It is important to note that the asset volatility, σ_V , is then calculated for different tenors to create a volatility skew. This is done by solving equations 4.11 and 4.12 for different values of T . We then proceeded to calculate the probability of default. Two sets of probabilities were calculated. To calculate the Merton probabilities we used equation 3.6 and to calculate the first passage time (FPT) probabilities we used equation 3.9. PDs were calculated for each year from 1 to 5 for each method i.e. $PD_T = P[\text{Time of default} < T]$ for $T \in [1, 2, 3, 4, 5]$. It is important to note that these are cumulative probabilities of default. Below is an example for Goldman Sachs:

T	PD_t
1	0.000655
2	0.043568
3	0.067241
4	0.101715
5	0.111123

Tab. 5.2: An example of PDs calculated using the Merton model for Goldman Sachs for May 2013 using a 3 month option.

Thereafter, we proceed to calculate probabilities of default for each quarter. We first create a hazard rate term structure, by finding out the hazard rate for each of the 5 years. The hazard rate λ_t is given by:

$$\lambda_t = \frac{\ln(S_t)}{-t}, \quad (5.1)$$

where S_t the survival probability is given by $S_t = 1 - PD_t$.

t	λ_t
1	0.000655
2	0.022273
3	0.023203
4	0.026817
5	0.023559

Tab. 5.3: May 2013 Goldman Sachs annual Merton hazard rates.

We assumed the hazard rates remained constant over each year and then calculated the probabilities of default for each quarter using the following equation:

$$1 - PD_t = \exp(\lambda_t t). \quad (5.2)$$

Table 5.4 below illustrates this.

With these PDs, we then proceeded to calculate the CDS spread for each name. To do this we need to “strip” the required swap curves. This was done by coding up a curve stripper for US, European and South African swap curves. As per the ISDA model for US and European CDSs, the ISDA CDS fixing swap curves were used. The following equation was used to calculate the CDS spread for a name:

$$\text{spread} = \frac{\sum S_{t-1}(PD_t - PD_{t-1}) B(0, t) (1 - \text{REC})}{\sum \tau S_t B(0, t)}, \quad (5.3)$$

where REC is the recovery rate set at the outset of the CDS, $B(0, t)$ is the discount factor from time t to time zero and τ is the fraction of the year between reset dates. The formula is derived by equating the cash flows for the premium leg of the CDS to the cash flows of the protection leg. The numerator of the formula represents the protection leg. The term $S_{t-1}(PD_t - PD_{t-1})$ represents the probability of surviving to the start of the period and then defaulting in that period. The denominator represents the protection leg of the CDS. This spread is calculated for both sets of probabilities (Merton and FPT) and then compared to the observed spreads in the market.

In summary, we solve equations 4.11 and 4.12 for different implied volatilities and tenors. For each of the 6 US and European firms and each time point (first business day of each month) we calculate:

- An asset volatility skew with the 3 month implied volatility as an input;
- An asset volatility skew with the 6 month implied volatility as an input;
- For each model PDs out to 5 years for both the 3 month and 6 month implied volatilities; and
- CDS spreads for each model.

t	λ_t	PD_t
0.25	0.0007	0.000164
0.5	0.0007	0.000327
0.75	0.0007	0.000491
1	0.0007	0.000655
1.25	0.0223	0.027457
1.5	0.0223	0.032857
1.75	0.0223	0.038227
2	0.0223	0.043568
2.25	0.0232	0.050867
2.5	0.0232	0.056356
2.75	0.0232	0.061814
3	0.0232	0.067241
3.25	0.0268	0.083465
3.5	0.0268	0.089589
3.75	0.0268	0.095673
4	0.0268	0.101715
4.25	0.0236	0.095277
4.5	0.0236	0.10059
4.75	0.0236	0.105872
5	0.0236	0.111123

Tab. 5.4: May 2013 Quarterly Merton PDs and hazard rates for Goldman Sachs.

5.3 Results

In this section, we present the results observed after performing the calculation for the US and European stocks. The results for the South African stocks will be presented in a later section as there were no observable CDS quotes for these stocks.

5.3.1 Relationship between Implied Volatility and CDS spreads

The correlation between CDS spreads and implied volatility was observed to be 0.974. This indicates that there is a strong relationship between CDS spreads and implied volatilities. Below is a scatter plot of all the US and European stocks CDS spreads versus their implied volatilities.

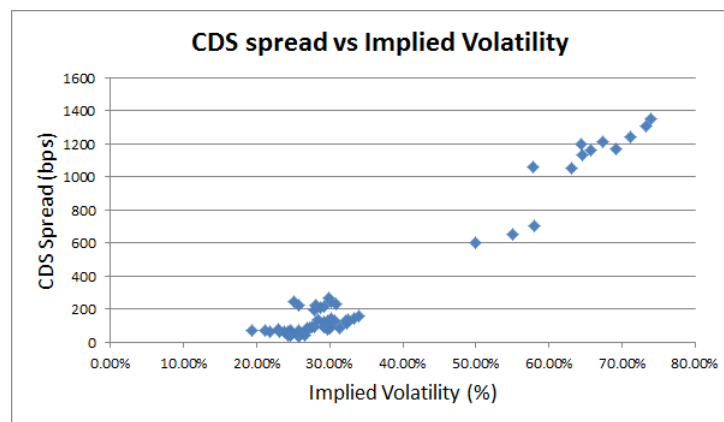


Fig. 5.1: Illustration of the correlation between CDS spreads and implied volatilities (3 and 6 month).

For names with a high CDS spread, we observe high implied volatility for their options. This indicates that put options on this name, as well as CDS spreads, will both be expensive which may be as a result of the increased risk of default. As both credit default swaps and put options provide downside

protection, this is as expected.

5.3.2 Asset Volatility

Before we calculate the probabilities of default, we need to determine the asset volatility (σ_V) structure. This is done by solving equations 4.11 and 4.12 using Matlab. Figure 5.2 shows how the volatility changes over time for BP. We can see the short term volatility is higher than the longer term volatility. We also observe that the asset volatility is generally higher when using the 3 month volatility as an input for solving equations 4.11 and 4.12. This is a consequence of implied volatilities generally decreasing as the tenor of the out-the-money put option decreases. This decreasing asset volatility has an effect on the probabilities of default.

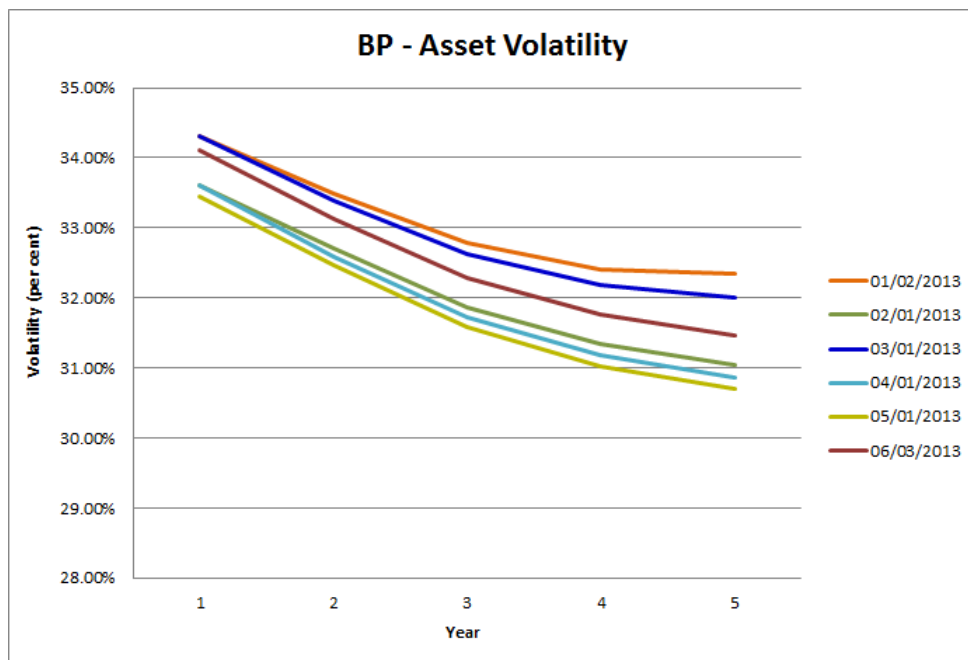


Fig. 5.2: Asset volatility over time for BP.

5.3.3 Probabilities of Default

To calculate the above CDS spreads for both models, we need to calculate the probabilities of default. This is done by using equation 3.6 and 3.9. We observe that the probabilities calculated using the Merton Model are generally lower than the probabilities calculated using the FPT model. As the volatility decreases over time, this creates a problem as it allows for cumulative probabilities of default that are not strictly increasing over time. This problem is overcome by making the assumption of a constant hazard rate over each year and restricting $PD_t - PD_{t-1}$ to positive values. An example of the probabilities of default is presented in the Section 5.2; we will not present any here.

5.3.4 Merton CDS spreads

Here we present the results relating to the CDS spreads calculated using the Merton model. We will refer to CDS spreads calculated using probabilities from the Merton model as “Merton CDS spreads”. The same applies for the FPT model. It can be seen below that the Merton CDS spreads are highly correlated with the market CDS spread. The correlation between Market CDS spreads and all the Merton CDS spreads was 0.850. The 3 month implied volatility based spreads have a higher correlation with the market rates than the 6 month implied volatility based spreads.

	Correlation
3 month CDS	0.890
6 month CDS	0.811
3 and 6 month CDS	0.850

Tab. 5.5: The correlation between the calculated Merton spreads and the market spreads.

It was noted that although the Merton model CDS spreads were accur-

ate for most firms, it did overestimate the spread for BP and TGT. This is indicated by the scatter on the lower right hand side of Figure 5.3.

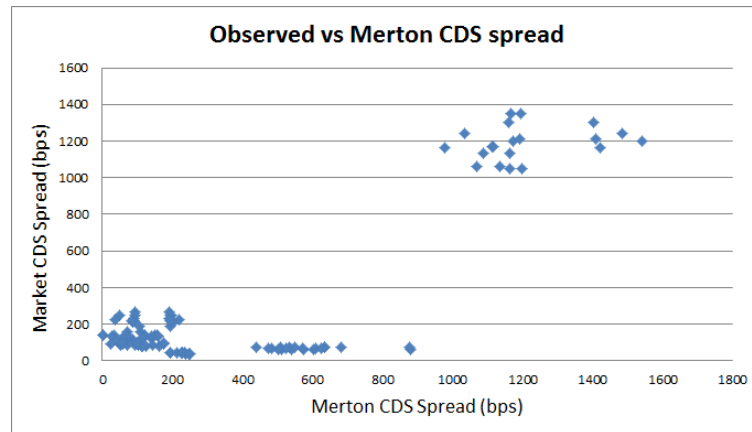


Fig. 5.3: A scatter plot of the Merton CDS spreads vs the market CDS spreads for all firms.

If we take a look at some of the individual firm's results, we see that there is a difference between using the 3 month and 6 month option implied volatility when calculating the CDS spreads. This effect varies by firms and this is illustrated in the two figures below. The 6 month implied volatility spreads are higher than the market spreads for AKS while they are lower than the market spreads for GS.

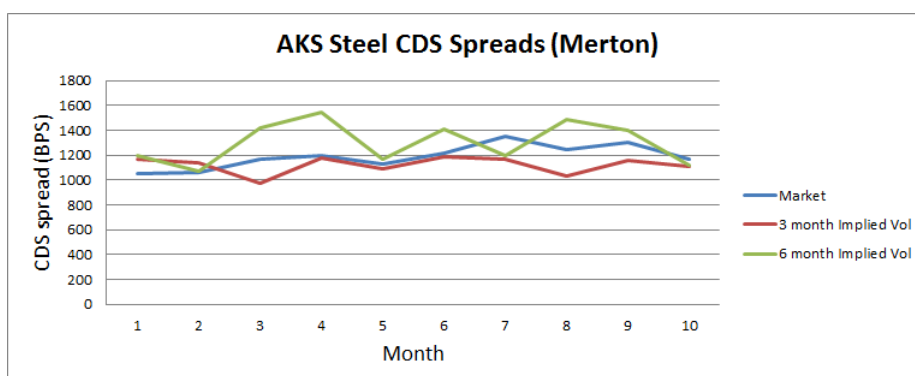


Fig. 5.4: The Merton CDS spreads using the 3 and 6 month volatility for AK Steel over the investigation period.

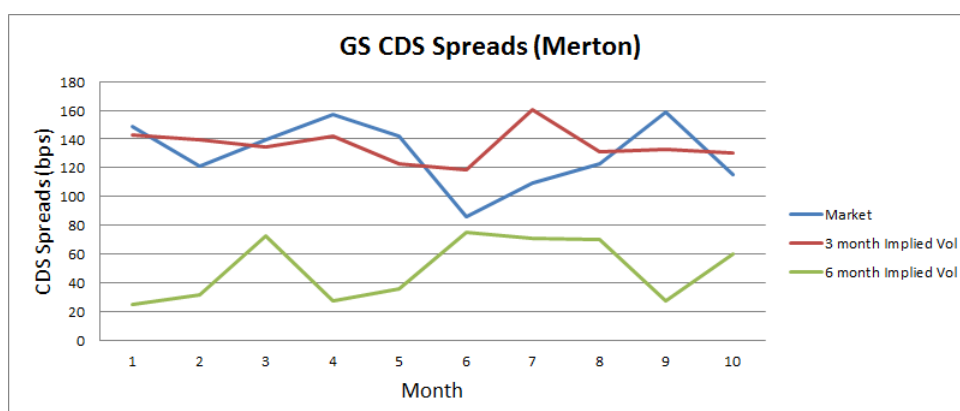


Fig. 5.5: The Merton CDS spreads using the 3 and 6 month volatility for Goldman Sachs over the investigation period.

The above figures show that when using the Merton model with implied volatility to calibrate the model, it produces spreads which are comparable to market CDS spreads. Overall, the results are encouraging. Although the model generally overestimates the spread, it sometimes falls below the market spread. To conclude our analysis of this model we regress all the Merton CDS spreads (both the 3 and 6 month implied volatility based CDS spreads) against the market CDS spreads. We also set the intercept term to 0. The β coefficient

for this model is 0.801. This indicates that the model provides a fairly good estimate, although it overestimates the spread by around 25 per cent. The ideal β coefficient would be 1. Also the R^2 is fairly high, indicating that most of the variability in the market CDS spreads is explained by the Merton model.

β Coefficient	0.801
R^2	0.812
t stat	22.475
p-value	4.07341E-43

Tab. 5.6: Summary of the regression of the Merton CDS spreads against the market CDS spreads.

5.3.5 FPT CDS spreads

As in the previous section we present the results relating to the calculated CDS spreads using the FPT model. Although the FPT CDS spreads are correlated with the market CDS spreads, the relationship is not as strong at the relationship with the Merton model. The correlation between market spreads and all the FPT spreads was 0.695. In this case the 3 month implied volatility based spreads have a lower correlation with the market rates than the 6 month implied volatility based spreads.

	Correlation
3 month CDS	0.681
6 month CDS	0.711
3 and 6 month CDS	0.695

Tab. 5.7: The correlation between the calculated FPT spreads and the market spreads.

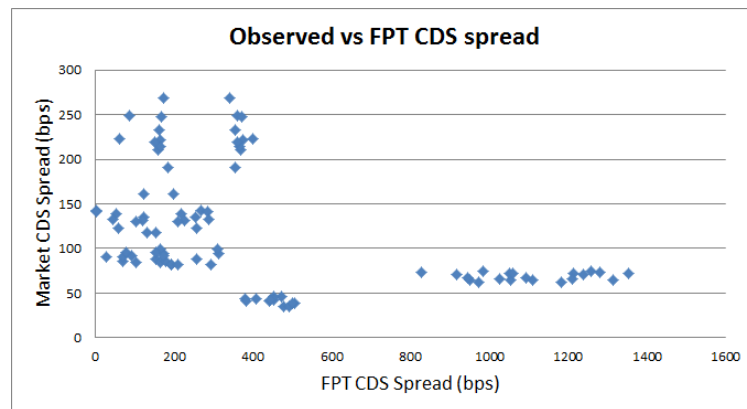


Fig. 5.6: A scatter plot of the FPT CDS spreads vs the market CDS spreads for all firms besides AK Steel.

The FPT model tends to overestimate the CDS spread and this can be seen in the above figure. The reason AK Steel's results were omitted from the diagram is that the FPT model produced very large CDS spreads. Including them in the diagram would distort the scatter plot.

As the FPT model allows default to occur at any point in time and not just at maturity, we would expect to observe larger spread for this model. This is since the probabilities of default would be higher for the FPT model than the Merton model. Taking a look at the results for Telefonica and Goldman Sachs, we can see this effect of the higher probabilities of default.

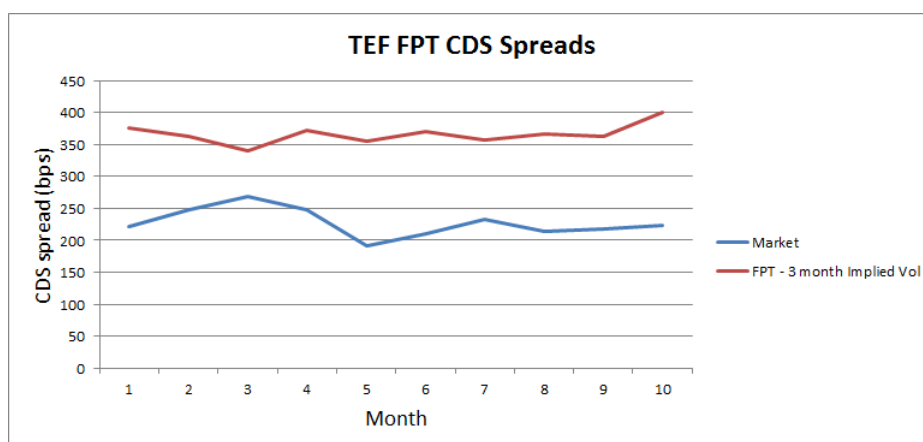


Fig. 5.7: The FPT CDS spreads using the 3 month volatility for Telefonica over the investigation period.

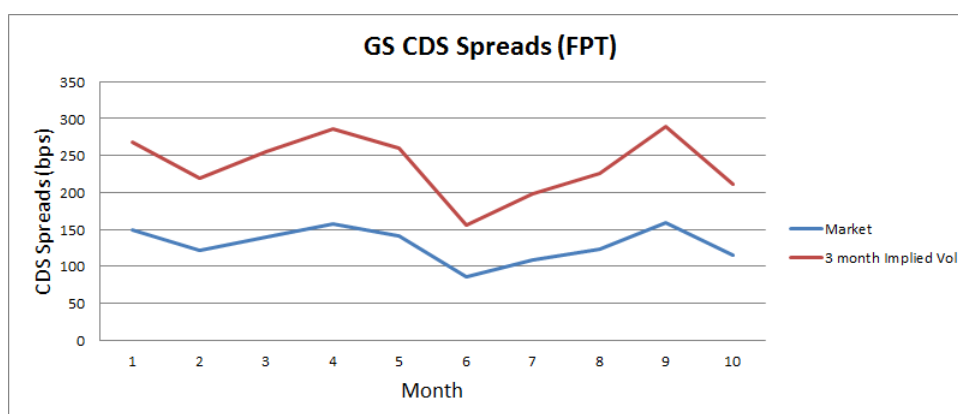


Fig. 5.8: The FPT CDS spreads using the 3 month volatility for Goldman Sachs over the investigation period.

Another observation was the volatility of the spreads for some stocks. This can be observed in the figure below for JP Morgan. The results for both implied volatilities are presented.

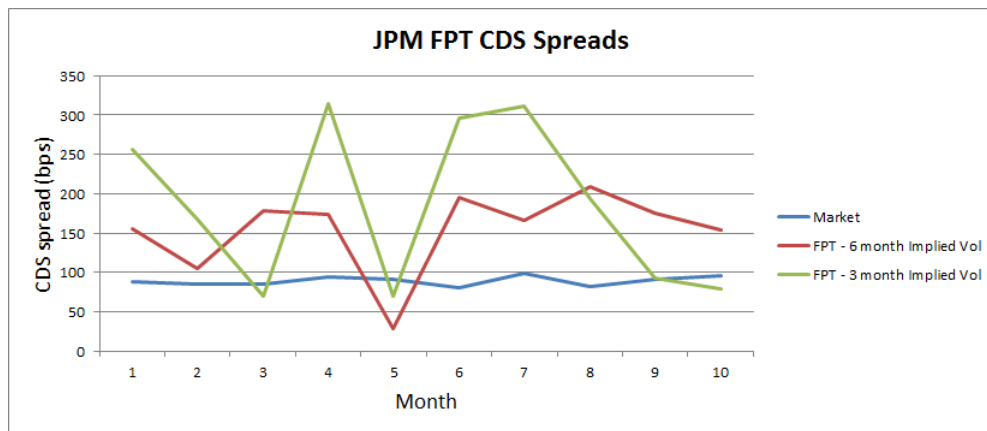


Fig. 5.9: The FPT CDS spreads using the 3 and 6 month volatility for JP Morgan over the investigation period.

Finally as with the previous model we regress all the FPT CDS spreads (both the 3 and 6 month implied volatility based CDS spreads) against the market CDS spreads. Note that β is far lower for this model, which indicates that the model overestimates the spread. This may indicate that the relationship between the two data sets is not as strong as that relationship observed for the Merton model. Also, the R^2 is lower, indicating that lower amount of the variability in the market CDS spread is explained by the FPT model.

β Coefficient	0.121
R^2	0.714
t stat	16.659
p-value	5.68139E-32

Tab. 5.8: Summary of the regression of the FPT CDS spreads against the market CDS spreads.

5.3.6 South African Firms

To conclude our empirical investigation, we apply the same methodology described in Section 5.2 to three South African firms, First Rand (FSR), Standard Bank (SBK) and BHP Billiton PLC (BIL). The major difference is that, in South Africa, we could not obtain CDS prices for the firms. This is because there is no developed CDS market in South Africa. In Figure 5.10 we graph the spreads obtained for the two South Africa banks we investigated.

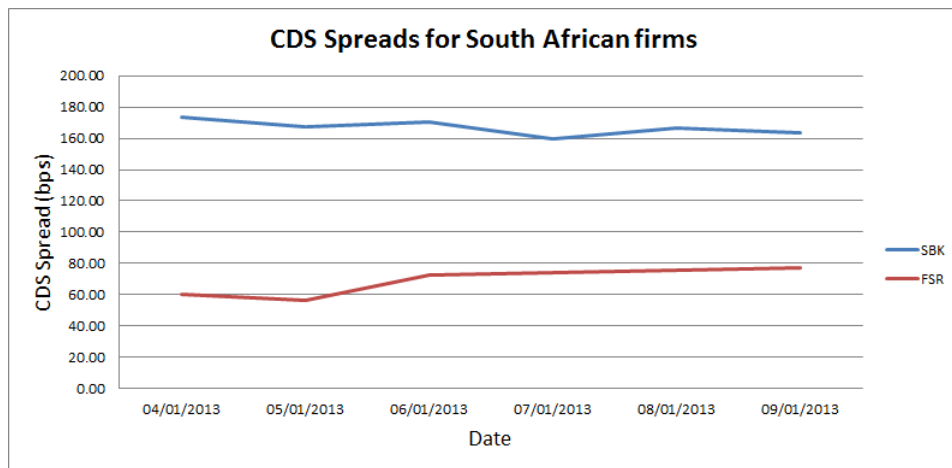


Fig. 5.10: The 3 month implied volatility based Merton CDS spreads for the South African firms.

Again we noted that the FPT model produced spreads that were higher than those produced by the Merton Model. In the table below, we present the result for both models when using the 3 month implied volatility in the calibration process.

	Merton			FPT		
	BIL	SBK	FSR	BIL	SBK	FSR
04/01/2013	699.14	173.21	60.28	1080.03	262.17	99.64
05/01/2013	702.87	167.27	56.63	1089.08	250.98	93.82
06/03/2013	670.73	170.67	72.93	1032.77	256.72	123.00
07/01/2013	705.96	159.81	73.99	1094.91	240.23	123.96
08/01/2013	676.02	166.60	75.48	1042.06	251.30	126.48
09/03/2013	667.17	163.81	77.48	1026.38	247.72	129.63

Tab. 5.9: The CDS spreads for the South African stocks.

Chapter 6

Findings

After performing the calculations above, we can now analyse the performance of each model. First, it is important to note the relationship between option implied volatilities and CDS spreads. We can see that when CDS spreads are high we can expect to find high option implied volatilities for put options on the firms stock. This is because high implied volatilities correspond to high prices for their put options. Both put options and CDS provide downside protection, so we expect there to be some correlation between the two. This is encouraging as we calibrated our models using this option implied volatility. This calibration choice was made to try to ensure a set of probabilities that were market consistent.

We also observed that the asset volatilities we calculated decreased as the tenor of the compound option (i.e. the debt) increased. As the asset volatility was generally higher at the short term, we sometimes observed cumulative PDs that were not strictly increasing over time. On the odd occasion, when solving for the asset volatility using Matlab, we did not find a solution for the equations. This could have distorted some of the results. However, for stocks such as AKS, GS, JPM, SBK, FSR, BIL, all the equations had solutions.

The calculated probabilities of default could not be compared to any market observable data. When comparing the calculated probabilities to the S&P probabilities we observe that they are higher than the historical default rates.

We also note that the PDs calculated using the FPT model are higher than those calculated by the Merton model. This was expected as the Merton model only allows for default at the maturity of the debt.

It is clear that the probabilities calculated using the Merton model result in CDS spreads that are more consistent with market CDS spreads. The CDS spreads calculated using the 3 month implied volatility provide the best estimates. The Merton model produces relatively stable results, although, on average, it slightly overestimates the spread. For all the firms besides BP and Target Corporation the model produced spreads in line with the market. A possible reason for BP's Merton CDS spreads being higher than the market may be a result of the company understating its assets (oil reserves) on its balance sheet.

The FPT model, on the other hand, did not perform as well. The results of the regression analysis support this claim. Although it did provide spreads that were comparable to the market spreads, the model overestimated the CDS spreads. We also note that the model produced CDS spreads for some stocks that were volatile.

When we apply the model to the South African stocks, we find that the spreads for the banks were consistent with those of the American Banks. We again note that the FPT model produces higher spreads than the Merton model.

Chapter 7

Discussion and Conclusions

The aim of this dissertation was to provide a method to calculate market consistent CDS spreads using market or equity data. The idea for this dissertation stemmed from the need to estimate probabilities of default in the market where there was no data available relating to defaults and default rates. We thus conducted a review of credit models to find a method whereby one could estimate probabilities of default from equity data. The obvious connection between the equity market and the bond market was that of put option and credit default swaps. A put option provides the holder of the option with some downside protection by guaranteeing a minimum price at which one could sell a specific stock in the future. A CDS provides a guarantee that one will receive at least the recovery value (set at the outset of the CDS) of the debt in the event of default. In the event of a decrease in credit quality, we would expect equity prices to drop sharply. If we follow this theory, we should expect CDS spreads and option implied volatility to be correlated. This theory was supported by the Section 5.3.1. The implied volatilities may also allow us to capture some of the premiums in the CDS spread which are not attributable to default risk, such as liquidity and market risk. A possible reason for the correlation between put options and CDS spreads is that banks are hedging credit derivatives with equity derivatives.

The calibration extension proposed by Hull, Nelken and White (2004) made

use of this relationship. We applied this extension to the standard Merton and FPT models. After performing an investigation of these models, we noted that the Merton model outperforms the FPT model. The main reason for this is that the FPT model overestimates the probability of default compared to the market. This is because the FPT model allows for default to occur before the maturity of the debt. On the other hand, The probabilities generated by the Merton model resulted in spreads that were similar to those which we observed in the market. It is important to note that these results were calculated in a post 2008 credit crisis environment where CDS spreads have risen. In South Africa, where the CDS market is not as developed as Europe and America such a method would be a useful. Although, we should also note that the options market in South Africa lacks liquidity. Overall, we can conclude that the implied volatility skews are potentially useful as predictors of probabilities of default.

There are a few possible extensions that can be made. Firstly, the moneyness of the options together with its tenor could be varied to determine its effect on predicting the CDS spread of a name. As the time period we considered was limited this investigation could be extended over a larger period of time. In particular, we could perform this investigation over the credit crisis period between 2008 and 2009. It could also be useful to investigate the possibility of a lag effect of information reaching the bond and stock markets at different times. Lastly, the other obvious major extension would be to consider more stocks.

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