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1. AN ELECTRO-MECHANICAL CRYSTAL STRUCTURE
FACTOR COMPUTER.
2. CONSTANT CURRENT SOURCE FOR DIRECT CURRENT
ARCS.
3. THE CONTROL AND STABILIZATION OF THE MAGNETIC
FIELD FOR A MASS SPECTROMETER.

by

N. T. VAN DER WALT.

A Thesis submitted to the Department of Electrical Engineering
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~~Doctor of Philosophy.~~

1956.

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The author is also indebted to the Director of the National Physical Laboratory and the President of the Council for Scientific and Industrial Research for permission to use this work as a Ph.D. thesis.

I N T R O D U C T I O N .

The thesis describes three distinct scientific instruments designed and built for the expediency of research physicists and to render the results of their investigations more accurate and reliable. The work was done, as part of the Author's normal duties, in the Electrotechnology Subdivision of the South African National Physical Laboratory, Pretoria.

1. An Electro-mechanical Crystal Structure Factor Computer is presented in the exact form in which it was accepted for publication by The Review of Scientific Instruments (and scheduled to appear in November or December of 1956), except for Appendix II which is added for the sake of completion.
2. Constant Current Source for Direct Current Arcs is presented in the form of a copy from The Review of Scientific Instruments, Volume 27, No. 3, pages 157 - 161, March 1956, except for the Appendix which is added for the sake of completion.
3. The Control and Stabilization of the Magnetic Field for a Mass Spectrometer is presented in the form of a reprint from The Review of Scientific Instruments, Volume 24, No. 6, pages 413 - 416, June 1953.

N. T. VAN DER WALT.

South African National Physical Laboratory, Pretoria.

S U M M A R Y.

An electro-mechanical device is described for computing the two-dimensional structure factor formulae:

$$\sum_{r=1}^N f_r \begin{Bmatrix} \sin \\ \cos \end{Bmatrix} 2\pi hu_r \begin{Bmatrix} \sin \\ \cos \end{Bmatrix} 2\pi kv_r \text{ and } \sum_{r=1}^N f_r \begin{Bmatrix} \sin \\ \cos \end{Bmatrix} 2\pi (hu_r + kv_r).$$

f_r , u_r , v_r are fractional and may be fed in to the third decimal place; h and k may vary from -20 to $+20$. Each term in the series is computed individually to an accuracy of approximately $\frac{1}{2}\%$ and is automatically added to the existing sum whilst the next term is being computed. The speed of computation is generally equal to the speed with which the variables of each term are fed in to the computer. The machine makes use of "magflip" resolvers for computation of the trigonometrical functions.

I. INTRODUCTION.

For reducing to reasonable proportions, the number of tedious man-hours involved in doing structure factor computations, a wide variety of devices have been developed according to the needs of various X-ray crystallographic laboratories. Such devices vary from digital machines such as the well-known Beevers-Lipson strips⁽¹⁾ and its ~~may~~ variations, to electronic digital computers⁽²⁾ and from analogue machines such as simple optical devices⁽³⁾, to the large and most elegant electronic analogue computer (X-RAC) developed by Pepinsky and his colleagues⁽⁴⁾. Summaries of, and comprehensive reference indices to probably all the more promising devices that have been developed are contained in treatises on the subject of X-ray Crystallography such as Booth⁽⁵⁾, Pepinsky⁽⁶⁾, Lipson and Cochran⁽⁷⁾, and Robertson⁽⁸⁾. Some more recent machines are described by Robertson⁽⁹⁾ and Azaroff⁽¹⁰⁾. The latter machine, based on an earlier machine by McLachlan⁽¹¹⁾, is a one-dimensional Fourier synthesizer/.

synthesizer using a large number of resolvers. Another machine using resolvers has also been mentioned by Rogers(12).

In the South African National Physical Laboratory structure factor computations have in the past been done by two skilled scientists using trigonometrical tables and desk calculators. The need for a computing device was felt, the eventual design of which was based on the following considerations:

1. The machine should be able to compute the formula

$$\sum_{r=1}^N f_r \begin{Bmatrix} \sin \\ \cos \end{Bmatrix} 2\pi hu_r \begin{Bmatrix} \sin \\ \cos \end{Bmatrix} 2\pi kv_r$$

which is the form to which any two dimensional structure factor computation can be reduced(13); and where u_r and v_r are the fractional co-ordinates of the atom in the unit crystal cell, and f_r is the atomic scattering factor expressed as a fraction of the largest scattering factor in consideration. However, the very useful formula

$$\sum_{r=1}^N f_r \begin{Bmatrix} \sin \\ \cos \end{Bmatrix} 2\pi (hu_r + kv_r)$$

was included in the design since it appeared that this computation could be done accurately at the extra cost of a resolver and a differential gear assembly.

2. The machine should be able to handle values of h and k up to 20.
3. There should be a large saving in man-hours.
4. A relatively unskilled operator should be able to handle the machine.
5. If possible, the machine should not be less accurate than the desk calculator method, say approximately 1%.
6. The cost of the components for the machine should be in accordance with the amount of computational work to be done.

II. GENERATION OF THE TRIGONOMETRICAL FUNCTIONS.

For generating the sine or cosine functions, the newly developed "mark II" mag-slip resolvers were used. If these devices are used in conjunction with feed back amplifiers such as is shown in Fig. 1., the coupling factor between the applied sinusoidally alternating voltage and the voltage induced in the appropriate rotor winding, is proportional to the cosine of the angle between stator and rotor (as measured from the position of maximum coupling) to a nominal accuracy of 0.1% and with negligible phase shift and very little distortion(14).

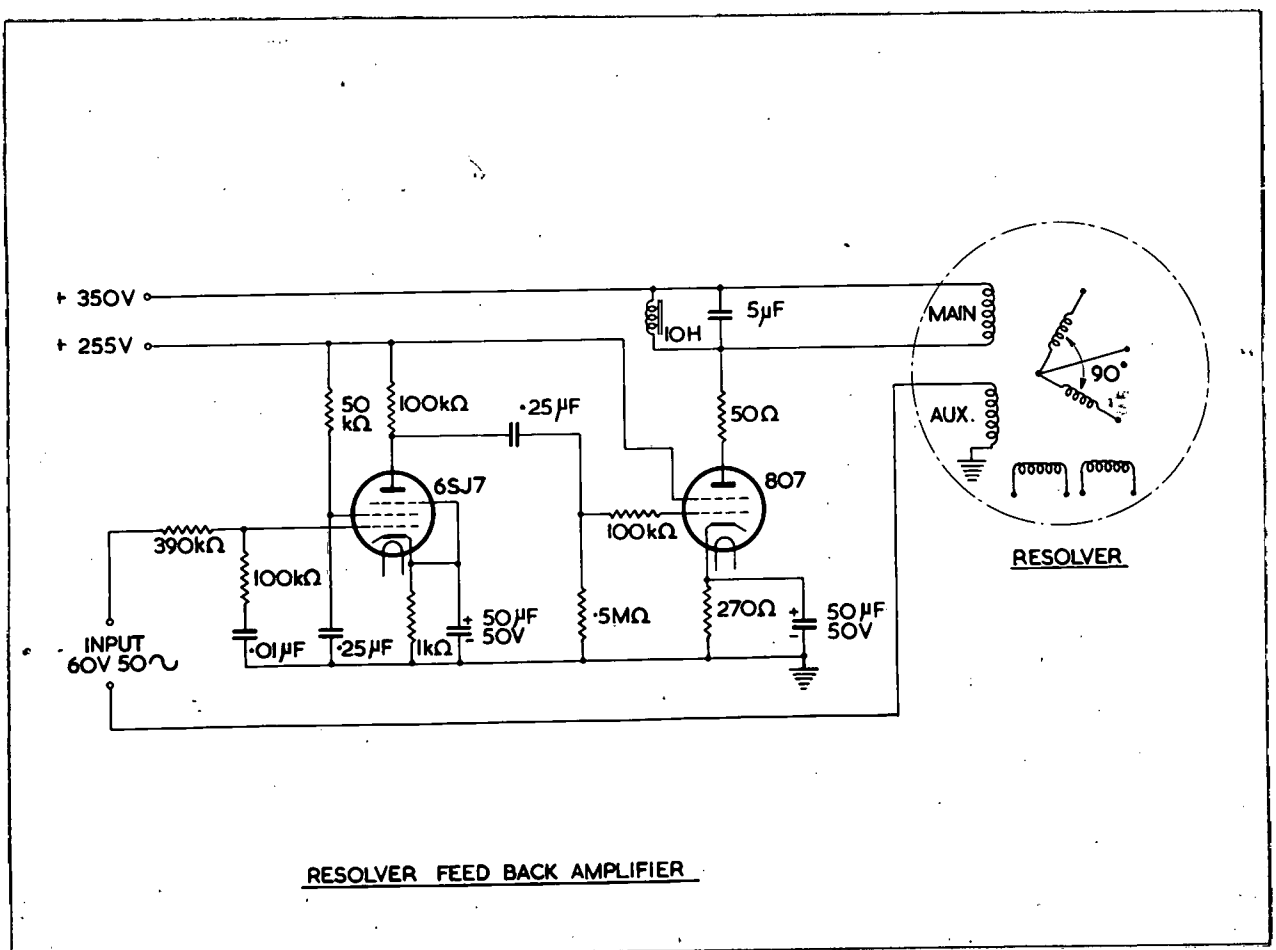


FIGURE 1: Diagram showing the excitation of a resolver stator through a feedback amplifier.

A similar amplifier system is applied to the quadrature stator windings. If there is no input to these stator windings, the input of the amplifier is short circuited, the effect of which is an enhancement of the nominal accuracy.

It is obvious that two or more such resolver systems may be

connected/.

connected in cascade for the generation of a wide variety of sums, products and the sums of products of the sines and cosines of different angles.

The resolvers used in this machine were Muirhead Magslip Resolvers Type E-24-C/1. The maximum applied voltage at 50 cycles per second is 60 volts r.m.s. and the maximum coupling coefficient is .98.

To the rotor shafts of these resolvers must now be applied angular displacements $2\pi hu$ and $2\pi kv$ radians (i.e. hu and kv revolutions) as accurately as possible.

III. GENERATION OF hu AND kv REVOLUTIONS.

After considering various means of multiplying a fraction of a revolution (u or v) by a variable integer (h or k), it became apparent that a gear system held the most promise. A fixed gear ratio for each value of h seemed too complicated and expensive and ratios such as 7, 11, 13, 17 and 19 may be difficult to realize. A gear changing system suffers from the additional and rather serious disadvantage that the zero must be reset or checked whenever the gear ratio is altered.

A. THE DISCS:

The system used in this computer is shown schematically in Fig. 2. There is a fixed gear ratio of 1:10 between disc A_1 and the rotor shaft of resolver A and a further gear ratio of 2:1 between disc A_1 and disc A_2 . The resolver shaft will, therefore, make 10 complete revolutions for 1 complete revolution of A_1 and 20 complete revolutions for 1 complete revolution of A_2 . Since for $h = 1$ only .1 of a revolution of disc A_1 is required for a full revolution of the resolver rotor, a complete scale for u varying from 0 to 1.000 is engraved on an arc of radius 8 inches and subtending an angle of 36° at the disc centre. The length of the scale is approximately 5 inches, i.e. a scale length of .005 inches corresponds to $\Delta u = .001$. For $h = 2$, another complete scale for u varying from 0 to 1.000 is marked on an arc subtending an angle of

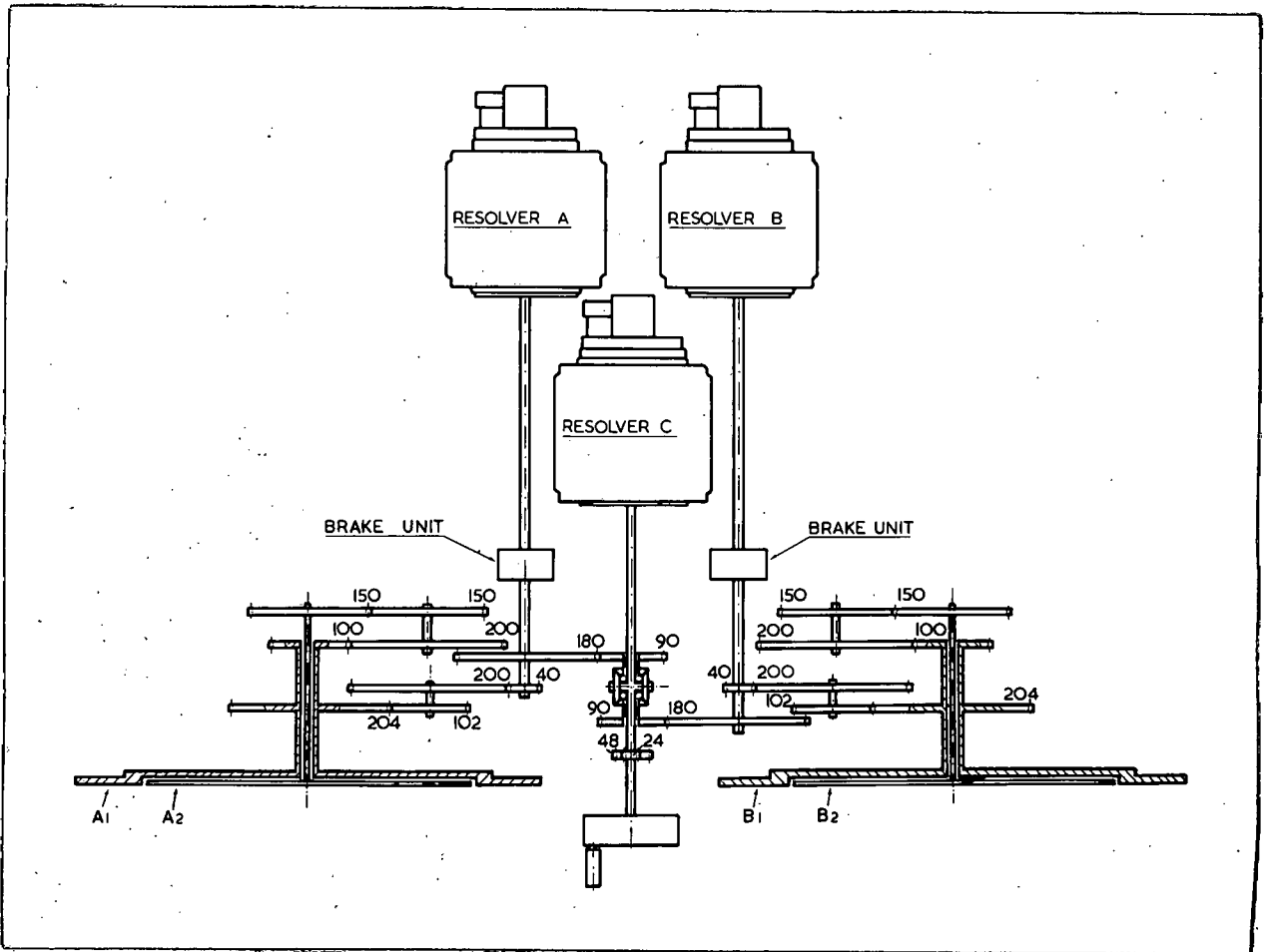


FIGURE 2: Schematic diagram of the disc and gear system.

72° and of a radius .32 inches less than that for $h = 1$. In this manner 10 different scales for u are marked on disc A_1 (as shown in Fig. 3) covering $h = -1$ to 10.

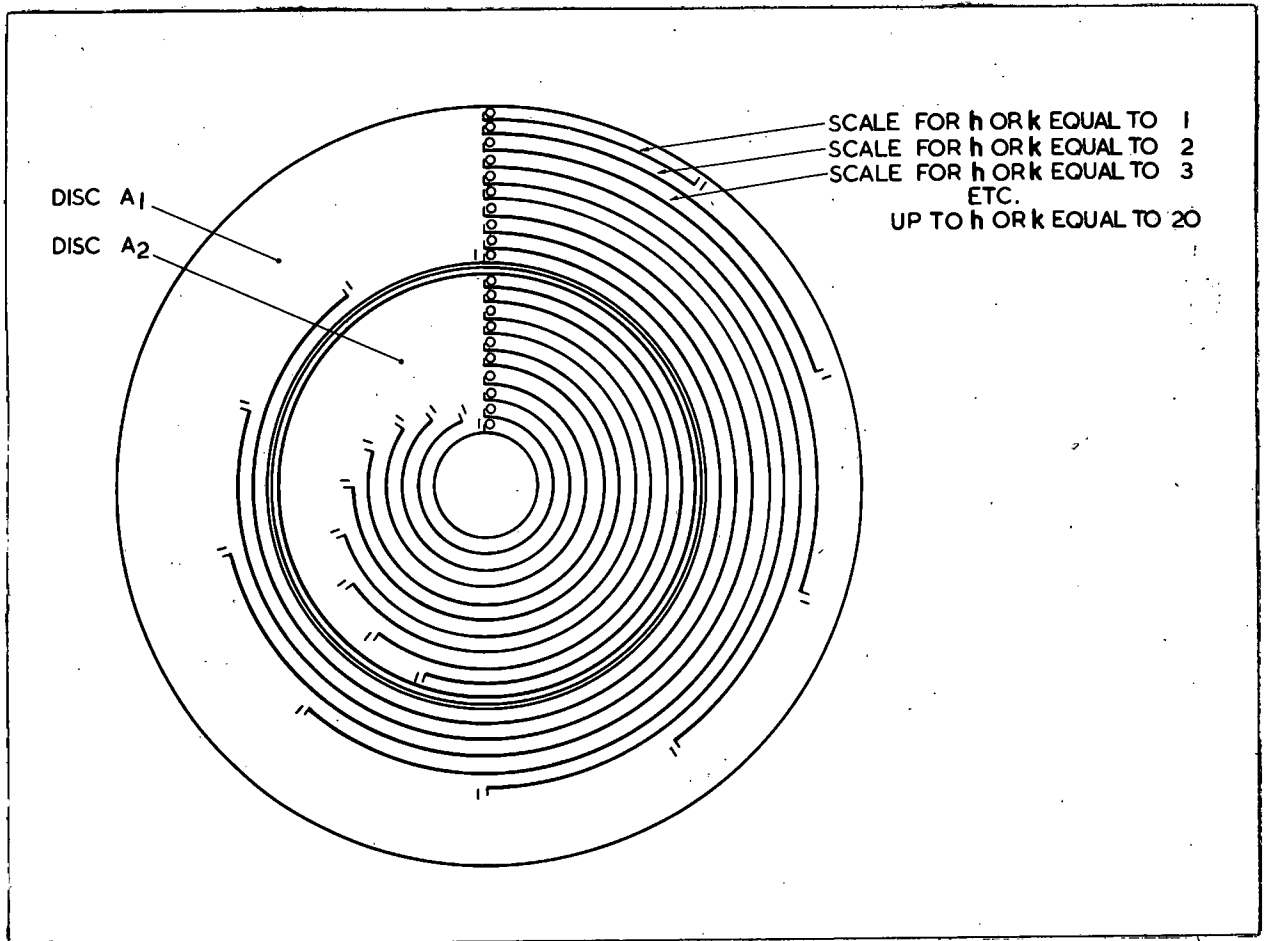


FIGURE 3: Diagram showing the arcs on the discs on which the scales were marked.

For $h = 11$ to 20 similar scales are marked on disc A_2 each complete scale subtending an angle of $18h$ degrees at the centre. Multiplication is, therefore, achieved by two fixed gear ratios, but by selecting that scale for u corresponding to the required value of h .

All the scales are numbered in intervals of .05 and subdivided into 100 equal divisions except the scales for $h = 6$ to 10, which are subdivided into 200 equal divisions. For positive values of hu , the scales are marked in black, in a clockwise direction and above the scale markings. For negative values of hu the scales are numbered in red, in an anti-clockwise direction and below the scale markings.

The scales were subdivided by means of an accurate jig borer and it is estimated that engraving inaccuracies (assuming a perfect gear system) did not exceed .0003 radians for disc A_1 and .0006 radians for disc A_2 in the output shaft.

All the scales had a common zero and the required scale is selected by means of a cursor. The fixed reference line covering all the scales had a width of .005 inches and is situated close to the disc faces to prevent parallax. Assuming that u can be set reasonably quickly to an accuracy equal to the width of the reference line, the error in the output shaft would be $\frac{.05}{r}$ radians for disc A_1 and $\frac{.1}{r}$ radians for disc A_2 (where r is the radius in inches of the particular scale being considered). This corresponds to .006 to .01 radians for disc A_1 and .03 to .09 radians for disc A_2 . (A possible method for reducing these errors at the higher values of h is described in Appendix I). In the preliminary stages of structure factor computations where the emphasis is on speed rather than on accuracy, these errors are not so important, but in the final stage more care should be exercised in setting u and v for higher values of h and k .

A final rather important point concerning the scales on the discs is the following: since the 1.000 position on any scale is an exact number of output revolutions from the zero position, this 1.000 position is in fact another zero position for that particular scale/.

scale so that another scale naturally following on to the first, may be marked on that arc. There are, therefore, 10 scales for $h = 1$ covering the complete 360° of arc, 5 scales for $h = 2$, $3\frac{1}{3}$ for $h = 3$ and so on (Fig. 6.). If the operator has, for example, set $u = .5$ for $h = 10$ and subsequently wishes to set $u = .5$ for $h = 1$, he need not rotate the disc almost half a revolution in order to "find" the scale for $h = 1$ (as is evident from Fig. 3), but may simply take that scale for $h = 1$ which happens to be opposite the reference line. In those cases where a fraction of a scale joins onto the original zero position, a warning mark is added onto the scale to avoid confusion.



FIGURE 6: Photograph showing the discs and also the servo motor assembly.

B. THE GEARS.

In order to reduce angular inaccuracies to a minimum, high quality anti-backlash 48 pitch spur gears, mounted on ball bearings were used. The rest of the mechanical components were manufactured to fine tolerances (15) and most carefully assembled (Fig. 4).

Figure/.

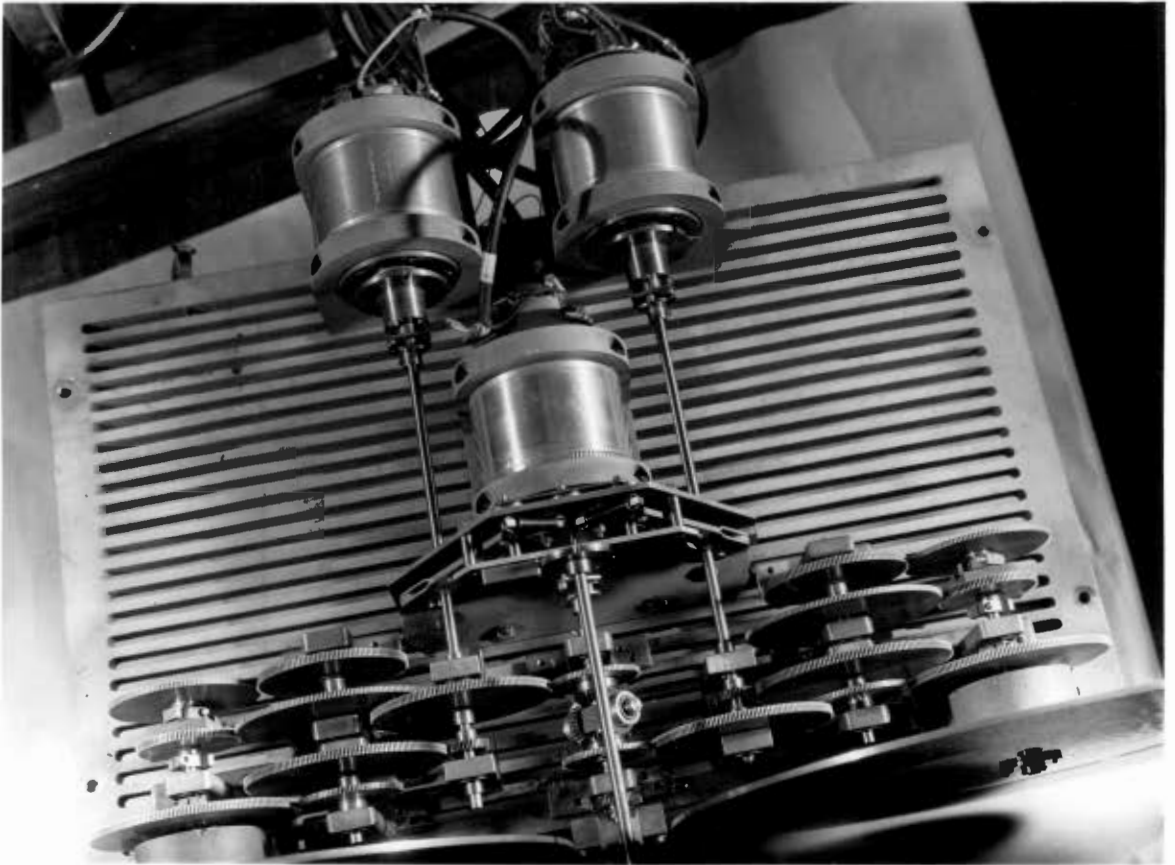


FIGURE 4: Photograph showing the complete gear system on a slotted mounting plate.

The generation of k_v is the same as that for h_u . Furthermore, the rotor shafts of resolvers A and B are coupled by means of a high quality differential gear assembly (backlash practically zero) for the generation of $(h_u + k_v)$ which is applied to the rotor of a third resolver C.

To the differential gear assembly is also suitably coupled a hand crank. For setting h_u the rotor shaft of resolver B is mechanically braked so that discs A_1 and A_2 only respond to the hand crank. For subsequently setting k_v , rotor shaft B is released while A is braked. The two brakes are controlled by a single knob.

IV. THE METHOD OF COMPUTATION.

Means have still to be provided for bringing f suitably into the computation and for recording and successively adding each individual computation. This is best explained by means of an example:

Figure 5 shows the computer diagrammatically set up for computing/.

ting

$$\sum_{r=1}^N f_r \cos 2\pi hu_r \cos 2\pi kv_r.$$

From a mains transformer 60 volts at 50 cycles per second is applied to a stator system of resolver A. It is obvious that if hu and kv revolutions have been applied to the rotors of resolvers A and B respectively, the ratio of the voltage selected from resolver B to the input voltage is $K_a K_b \cos 2\pi hu \cos 2\pi kv$ (where K_a and K_b are the coupling factors of the resolvers when $hu = kv = 0$ and are equal to .98). For multiplying this function by f , a 10 K. wire-wound potentiometer is connected across the appropriate rotor winding of resolver B by means of a selector switch.

This potentiometer has a useful angle of approximately 358° for which the makers specify a resistance linearity of 0.1% and minimum incremental value of 0.04%. Fitted to the potentiometer is a six inch dial, (dividing the useful angle into exactly 100 divisions), with vernier on which f can be set to the third decimal place.

Fed from the same mains transformer is a second potentiometer (P_2) consisting of a 10 turn 10 K. wirewound Helipot (for which the makers specify a resistance linearity of 0.1% and minimum incremental value of .015%) on which a voltage equal and opposite in phase to that on P_1 can be obtained. Equality is obtained by connecting two high quality 1 megohm resistors between the sliders of P_1 and P_2 and adjusting for zero voltage between the common point of these resistors and the common earth point of the potentiometers. This is done automatically as follows:

The detector of the network described above is an amplifier which supplies one phase of a 2-phase servo motor mechanically coupled to the slider of P_2 . The direction of rotation is such as to make the voltage on the slider of P_2 equal to that on P_1 . The total voltage across P_2 is so selected and finally adjusted by means of series resistor R_1 that exactly 4 turns of P_2 corresponds

to/.

to the voltage on P_1 when $f_r \cos 2\pi hu_r \cos 2\pi kv_r = 1.000$. The centre position of P_2 corresponds to zero and the opposite direction of rotation to negative values of the function being computed.

Also mechanically connected to the motor via a magnetic clutch and a suitable gear ratio (Fig. 6), are two counters rotating in opposite directions relative to each other. The gear ratio is so chosen that the counters will make exactly 100 revolutions for 4 revolutions of P_2 i.e. the number 1000 will appear on one counter for $f_r \cos 2\pi hu_r \cos 2\pi kv_r = 1.000$. The other counter records the complement of the first counter, i.e. it will show a "positive" number for negative values of the computation. Provided the magnetic clutch is engaged, a number will therefore appear on the appropriate counter equal to $f_r \cos 2\pi hu_r \cos 2\pi kv_r$.

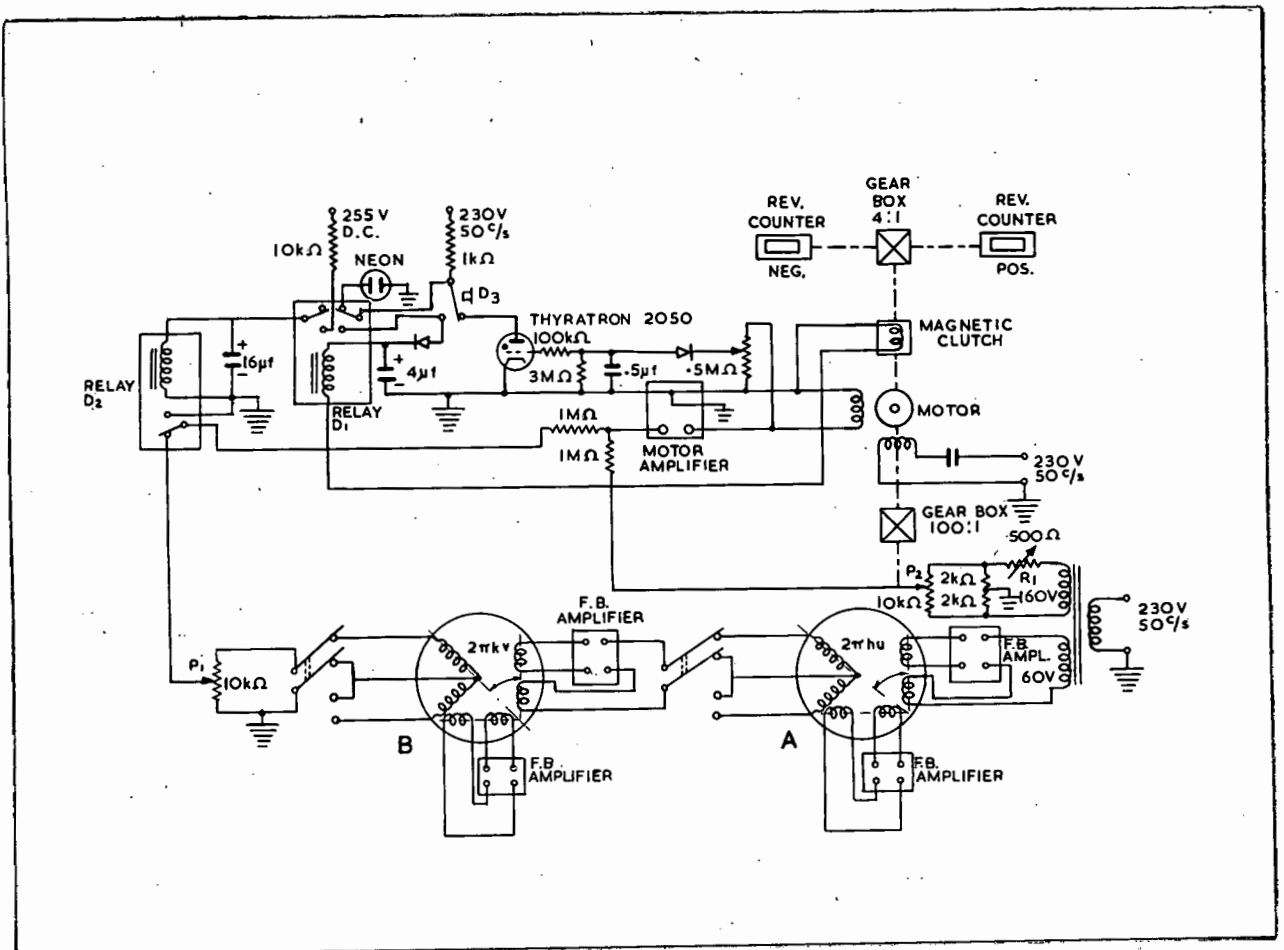


FIGURE 5: The computer schematically set up to compute

$$\sum_{r=1}^N f_r \cos 2\pi hu_r \cos 2\pi kv_r.$$

The/..

The motor amplifier is conventional in design and the overall gain is so adjusted by means of negative feedback that the motor system just does not oscillate. A few millivolts input to the amplifier causes full voltage to be applied to the motor and under these circumstances the system will successively repeat exactly on the counters any fixed setting of the variables on the computer and at a speed of approximately 100 digits on the counters per second. Furthermore, since potentiometer P_2 and resolver A are fed from the same mains transformer, the system is independent of mains voltage or frequency variations.

V. AUTOMATIC SUMMATION OF SUCCESSIVE COMPUTATIONS.

With the system as shown in Fig. 5, the slider of P_2 will automatically follow the value of $f_1 \cdot \cos 2\pi hu_1 \cdot \cos 2\pi kv_1$ whilst f_1 , u_1 and v_1 are being adjusted and usually does so fast enough to attain its final position a second or two after the variables have been set. During this time the magnetic clutch has been disengaged, i.e. the counters have remained at zero. When spring biased switch D_3 is momentarily pressed, relay D_1 and the magnetic clutch are energized and held in this state by D_1 itself. Relay D_2 is subsequently energized by D_1 causing one input to the motor amplifier to be disconnected from P_1 and earthed. The motor then returns the slider of P_2 to the zero position and a number equal to $f_1 \cdot \cos 2\pi hu_1 \cdot \cos 2\pi kv_1$ appears on the counters.

Whilst the motor is rotating, an appropriate proportion of the voltage across it is rectified and applied to the grid of the thyatron in order to cut it off. As soon as the motor stops rotating, this negative bias disappears and the thyatron conducts, thus short circuiting relay D_1 and the magnetic clutch which both release. D_2 subsequently releases and the motor amplifier is automatically reconnected to P_1 . The slider of P_2 then proceeds to take up its new position corresponding to the values f_2 , u_2 and v_2 which in the meantime may have been set on the computer.

When/.

When D_3 is again pressed the second computation is automatically added to the first on the counters.

It is to be noted that as soon as D_3 has been pressed, the operator may proceed to feed in the new variables to the computer without affecting the "recording" of the previous computation. Except for computations which approach unity, the previous computation will have been recorded and the slider of P_2 have taken up its new position simultaneously with the final adjustment of the new variables. The speed of computation is generally, therefore, equal to the speed with which the variables are fed into the computer.

VI. COMPUTATION OF $\begin{Bmatrix} \sin \\ \cos \end{Bmatrix} 2\pi(hu + kv)$.

For this computation there are two possibilities:

The first method consists of expanding $\sin 2\pi(hu + kv)$, say, into $\sin 2\pi hu \cdot \cos 2\pi kv + \sin 2\pi kv \cdot \cos 2\pi hu$ and applying the outputs from both rotor windings of resolver A to both stator windings of resolver B. The nominal accuracy of each resolver is reduced to 0.2% and switching is not always straightforward. For example, since the two rotor windings of resolver A have a common connection, a complete set of stator terminals of resolver B must be reversed in order to effect the positive sign in the above expansion of $\sin 2\pi(hu + kv)$.

The second method consists of adding hu and kv with a differential gear assembly and applying $(hu + kv)$ revolutions resolver to C. An extra resolver and a high quality differential gear is needed, but this method was preferred because of the nominal accuracy of 0.1% obtainable. In order to economize on feedback amplifiers a complete pair of feedback amplifiers was transferred from resolver B to resolver C by means of a multipole two position switch. Also by means of this switch potentiometer P_1 was transferred to the selector switch of the rotor windings of C and another resistor R_1 (preadjusted to match K_C) was substituted for R_1 .

VII. ACCURACY.

To test the accuracy of the instrument, the following procedure was carried through:

Selecting/.

Selecting the function $(\cos 2\pi hu. \cos 2\pi kv)$ and adjusting it firstly to equal + 1.000 and subsequently -1.000, the readings on the counters were noted for f varying from 0 to 1.000. The errors remained within the limits $\pm .002$ except for values of f above .97 where the error was $\pm .003$. These errors are a combination of the errors of potentiometers P_1 and P_2 .

The additional errors due to the resolvers are $\pm .002$ for $\left\{\frac{\sin}{\cos}\right\} 2\pi hu$ $\left\{\frac{\sin}{\cos}\right\} 2\pi kv$ and $\pm .001$ for $\left\{\frac{\sin}{\cos}\right\} 2\pi (hu + kv)$.

Apart from the fact that the angular errors due to the discs and gears are not readily determined, the effect of such errors depends on the computation itself. For example, the error in $\cos 2\pi hu. \cos 2\pi kv$ due to angular errors $(\Delta 2\pi hu)$ and $(\Delta 2\pi kv)$ radians is $-\sin 2\pi hu. \cos 2\pi kv. (\Delta 2\pi hu) + \sin 2\pi kv. \cos 2\pi hu. (\Delta 2\pi kv)$.

However, in order to get some idea of the errors to be expected, $f. \cos 2\pi kv$ was set equal to unity and $\cos 2\pi hu$ then computed for values of hu varying from 0 to 20 in steps of .05 (i.e. 400 computations in all and corresponding to 1 complete revolution of disc A_2). The deviation from the correct values varied between $-.006$ and $+.006$ except at those positions where $\cos 2\pi kv$ should equal zero where the deviation was up to $\pm .010$.

The computation for

$$\sum_{n=1}^{n=20} \frac{1}{n} \sin 2\pi(.25n); \cos 2\pi(xn)$$

(square wave) was carried through for $x = 0$ to 1.0 in steps of .05 and the deviations from the calculated values were within $\pm .010$.

VIII. SPEED OF COMPUTATION.

In the above computations reasonable care was taken with the setting of the variables f_r , u_r and v_r . In actual structure factor computations, accuracy, speed of computation and the experience of/.

of the operator are closely related. Furthermore, button D_3 is so situated that it may be operated by the palm of the right hand which also operates the hand crank (Fig. 7), whilst the mechanical brakes are operated by the left hand.



FIGURE 7: Photograph showing the front panel of the computer.

It may perhaps be mentioned that the first time a particular operator (relatively unskilled in the use of scientific machines) used the computer, he did 10 structure factor computations in an hour, and except for the first computation, the others agreed exactly with the structure factors as computed by the desk calculator method (two skilled scientists) at a rate of 7 to 10 minutes each.

ACKNOWLEDGEMENTS.

The author wishes to thank Dr. J. N. van Niekerk of the X-ray Crystallographical Section of the South African National Physical Laboratory, for presenting the problem, Mr. E. W. Spencer of the C.S.I.R. Workshop for his very accurate mechanical work on the computer and the South African Council for Scientific and Industrial Research for permission to publish this paper.

A P P E N D I X.

I.

Figure 8 shows an alternative method for the generation of h_u or kv revolutions. Four six inch diameter hollow drums E_1 to E_4 are used instead of discs A_1 and A_2 . The scales for $h = 1$ to 5 are marked on drum E_1 , each scale subtending an angle of $72 h$ degrees at the axis of the drum; the scales for $h = 6$ to 10 are marked on E_2 , each scale subtending $36 h$ degrees, the scales for $h = 11$ to 15 on E_3 , each scale subtending $24 h$ degrees and the scales for $h = 16$ to 20 on E_4 , each scale subtending $18 h$ degrees. The angular error which the width of a .005 inch wide reference line would cause in the output shaft is constant at approximately .008 radians for drum E_1 , .017 radians for E_2 , .025 radians for E_3 and .033 radians for E_4 .

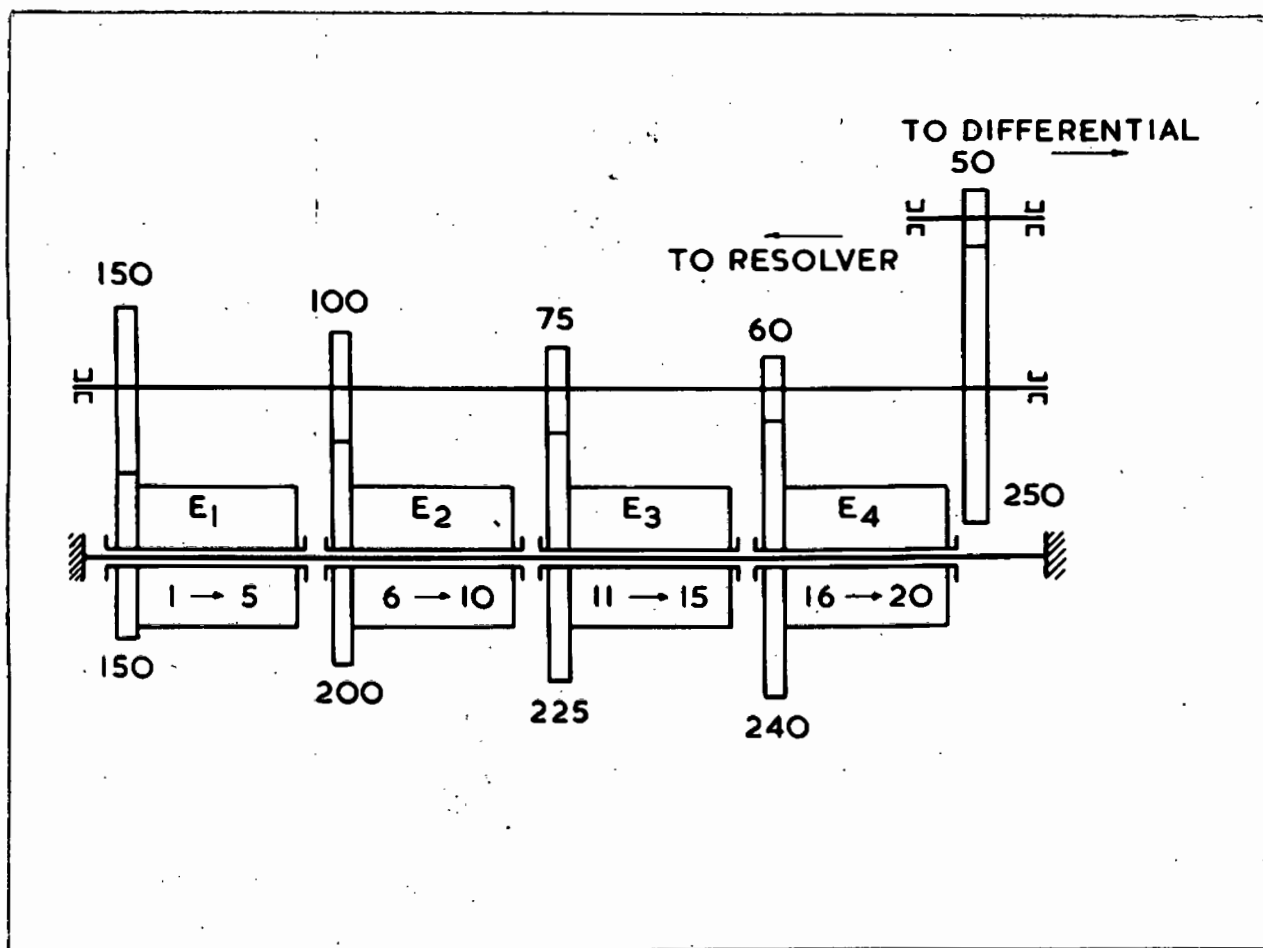


FIGURE 8: Diagram of an alternate method for the generation of h_u or kv revolutions.

A P P E N D I X.

II.

The circuit diagram of the servo motor amplifier is shown in Fig. 9 and the circuit diagram of the power supplies of the computer in Fig. 10.

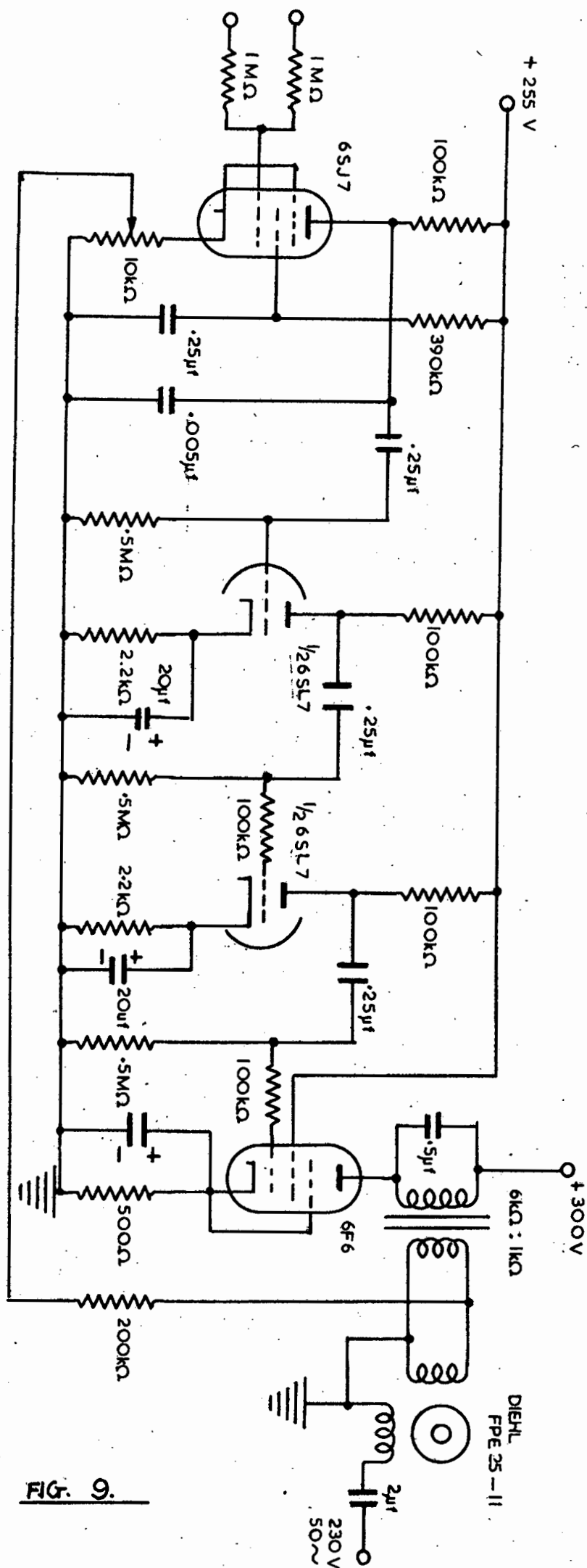


FIG. 9.

CRYSTAL STRUCTURE FACTOR COMPUTER
— MOTOR AMPLIFIER —

HTEK. EN DATUM
SIG. & DATE

[Signature]
3/11/55

NASIONALE FISIESTE LABORATORIUM.
NATIONAL PHYSICAL LABORATORY.
PRETORIA

W.N.N.R.
C.S.I.R.

TEK. No. DRG. No.

E 295

Constant Current Source for Direct Current Arcs

N. T. VAN DER WALT

National Physical Laboratory, Council for Scientific and Industrial Research, Pretoria, South Africa

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A method is described by means of which the direct current of a spectrochemical arc is kept constant at any value between $\frac{1}{2}$ and $20\frac{1}{2}$ amp in steps of $\frac{1}{2}$ amp, independent of arc gap or the material to be excited. The principle involved is phase control on the grids of thyratrons and the design features which affect this particular application are discussed.

INTRODUCTION

REPRODUCTIONS of typical patterns of arc current using a 240 v 300 amp-hr battery are shown in Fig. 1(a) and 2(a). Figure 1 shows the arc current for a conducting material (Pfund arc⁷) and Fig. 2 for a nonconducting material. For the excitation of nonconducting materials the arc current is preset on the "short circuit" value by means of a series ballast resistor, while the Pfund arc is usually regulated to 4 or 6 amp while the arc is operating.

Reviewing the literature the usefulness of a constant current arc source became apparent.¹⁻⁴ The specifications and desirable features set by the Spectrochemical Subdivision are summarized below.

DESIGN CONSIDERATIONS

1. The regulation should be such that the current variations are eliminated or considerably reduced.
2. It should be possible to select any value of constant current up to 20 amp and any such selected value should be accurately repeatable at any time and should be independent of the material to be excited.
3. The arc source should be supplied directly off 50 cps three phase or single phase mains and the constancy of arc current should be independent of mains frequency and voltage variations.
4. The maximum dc output voltage should be approximately 200.

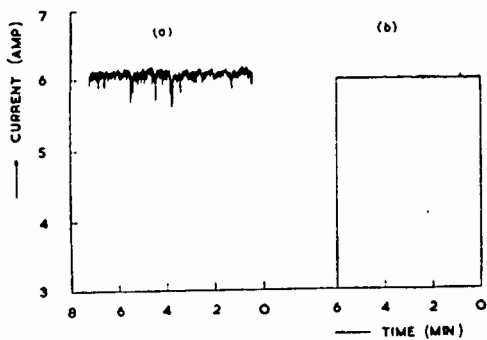


FIG. 1. Records of arc current for a conducting material using (a) batteries, and (b) the constant current source.

¹ Green and Kuper, *Rev. Sci. Instr.* **11**, 250 (1940).

² Potter and Scott, *Rev. Sci. Instr.* **18**, 722 (1947).

³ Fetterley and Hazel, *J. Opt. Soc. Am.* **40**, 76 (1950).

⁴ Smith and Wiggins, *Spectrochim. Acta* **3**, 327 (1948).

5. The percentage "ripple" should not exceed the value generally obtained in some commercially available rectifier arc source systems.

6. The unit should be transportable between rooms.

7. A relatively unskilled operator should be able to handle the unit for routine analyses.

8. The unit should be suitably protected against overcurrent and short circuits and should not be dangerous to the operator.

DESCRIPTION OF THE CIRCUIT

Considering the 4-kw output power required it was considered desirable to use the available 400 v 50 cps, three phase mains supply as input.

The basic circuit diagram is shown in Fig. 3.

From the three-phase mains is obtained, by means of two single phase "Scott"-connected transformers an isolated 250 v rms, 1.8 kva per phase, four phase system. (V_a in Fig. 3).

From this system are obtained:

1. The anode and filament supplies (through appropriate filament transformers not shown in Fig. 3) for four FG172 General Electric thyratrons, each having a continuous rating of 6.4 amp. (25.6 amp total).

2. The filament and 300 v high tension supplies for the dc amplifier and its regulated power supply (also not shown in Fig. 3).

3. Two 75 v supplies, tapped off appropriate phases for the ac bridge systems which supply variable phase grid voltages to the thyratrons.

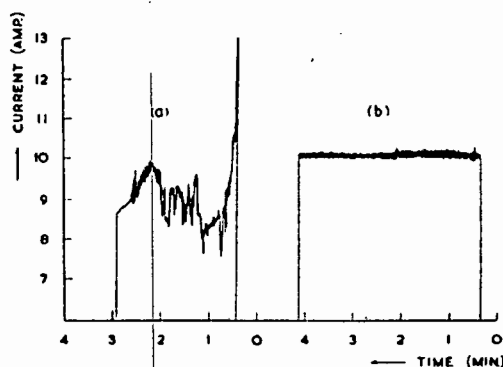


FIG. 2. Records of arc current for a nonconducting material using (a) batteries, and (b) the constant current source.

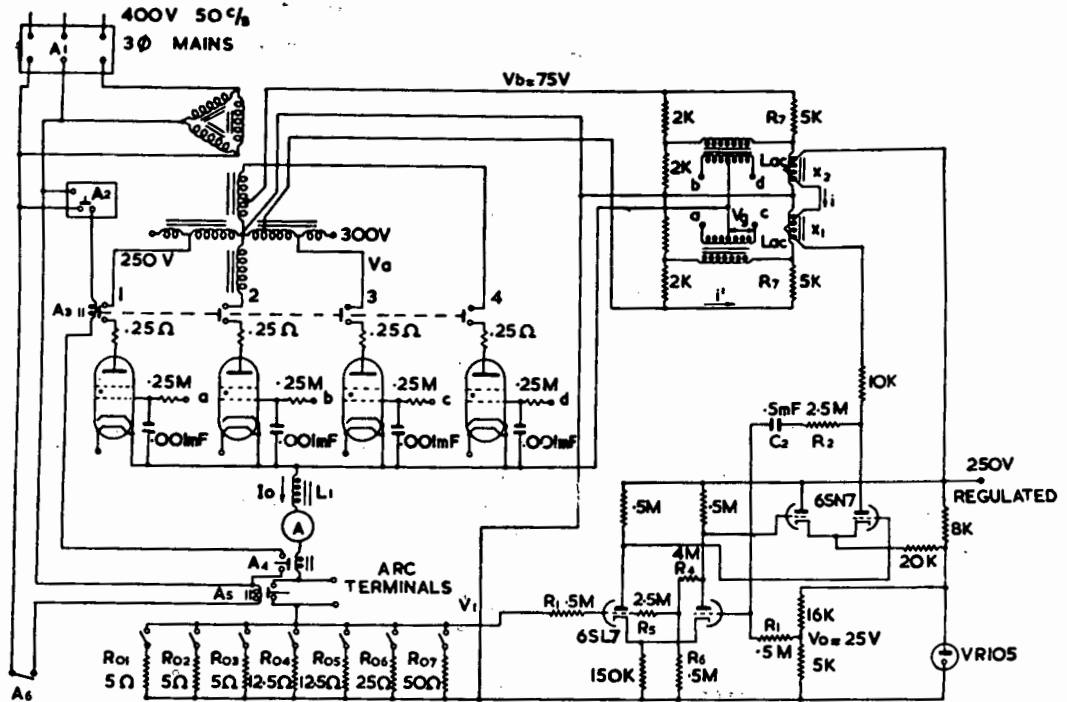


FIG. 3. Basic circuit diagram of the unit.

Series connected in the common cathode circuit are a current smoothing choke L_1 , an ammeter, the coil of an over-current relay A_4 , the arc, and a system of "standard" resistors R_{01} to R_{07} which may be switched in any parallel combination.

The difference in voltage between V_1 , the voltage across R_0 , and V_0 a "standard" voltage obtained from the regulated power supply is amplified in a dc amplifier and appears as a variation in i , the dc control current of saturable reactors X_1 and X_2 . A variation in i from 0 to 0.8 ma changes the alternating inductance L_{ac} of the reactors from approximately 60 henries to practically zero.

By means of positive feedback through resistors R_4 , R_5 , and R_6 the forward gain K_1 of the dc amplifier is increased to almost "infinity" over its operating

range.⁶ The negative transient feed-back through C_2 and R_2 serves not only to stabilize the system as a whole and to improve the transient response, but also to smooth out the ripple in V_1 , which may otherwise have overloaded the amplifier.

The output voltages from the bridge circuits of which X_1 and X_2 form part, are fed to the grids of the thyratrons by means of transformers and resistance-capacitance networks.

A vector diagram of a bridge circuit showing the phase relationship between the anode and grid voltages of thyatron No. 3 is shown in Fig. 4, and the wave-shapes are shown in Fig. 5. It can be seen that as i increases, L_{ac} decreases and α increases, making available to the arc current an average voltage V_t proportional to the shaded areas in Fig. 5. Neglecting the volt drop across the thyratrons a curve showing the relationship between V_t and i is shown in Fig. 6.

A fairly large inductance L_1 in series with the arc is necessary for reducing ripple and for preventing the arc current from being discontinuous (especially for $\alpha < 90^\circ$). This choke consists of two series connected windings of 150 turns # 10 SWG wire each, wound on opposite legs of a 10-in. square laminated iron core of 3-in. sq cross section. The choke is used without an air-gap and has a direct current resistance of 0.4 ohm.

For V_0 a value of 25 v was chosen and any change

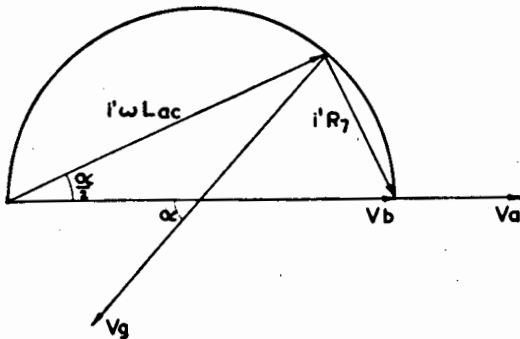


FIG. 4. Vector diagram of the phase shifting bridge circuits.

⁶ Valley and Wallman, *Vacuum Tube Amplifiers*, Radiation Laboratory Series, Vol. 18, p. 475. (McGraw-Hill Book Company Inc., New York, 1948).

in its value due to voltage or resistance changes was observed to be negligible.

Since $1/R_0 = 1/R_{01} + 1/R_{02} + \dots + 1/R_{07}$ each admittance unit contributes a fixed value of current to I_0 . The total admittance unit consists of three 5 amp units, two 2 amp units, one 1 amp unit and a $\frac{1}{2}$ amp unit so that any value of I_0 between $\frac{1}{2}$ and $20\frac{1}{2}$ amp may be selected in steps of $\frac{1}{2}$ amp. The units were made of Eureka wire and were large enough for any resistance change due to self-heating to be negligible.

For improving the accuracy of spectrochemical analyses especially when a large number of successive exposures are to be made at the same current, it has been found expedient to extinguish the arc by short circuiting it. This is done by a 40 amp solenoid operated switch (A_6) remotely controlled by a switch (A_6) usually attached to the arc cabinet door. By this method any cooling down of the components of the source unit between exposures is prevented, thus keeping the conditions at which successive exposures are made more nearly the same.

The apparatus was built into a steel cabinet $2 \times 2 \times 4\frac{1}{2}$ ft high and was mounted on wheels.

PROTECTION AND SWITCHING

Referring to Fig. 3, A_1 is a 7.5 kva three phase starting switch, with over-current relays, and switches on the four phase transformer, and consequently the thyatron filaments, the dc amplifier and its regulated power supply and time delay switch A_2 .

Switch A_2 is of the self-resetting type and is set to have a time delay of six minutes for the protection of the thyatrons. It controls the solenoid of switch A_3 which is similar to A_1 but adapted for four lines.

In series with the solenoid of A_3 is also a switch A_4 which is controlled by a 25 amp over-current coil in the cathode circuit of the thyatrons. A_4 can also be switched by hand.

RIPPLE

The 200 cps ripple in the arc current has the effect of alternating current superimposed on the direct current and since alternating current may cause stroboscopic effects when rotating sectors are used in spectrochemical analysis, the percentage ripple should not be excessive.

The larger the effective time constant T_1 of the arc current circuit, the smaller is the percentage ripple. It can be shown that T_1 is almost independent of arc current but decreases as the average value of arc voltage increases. On the other hand for larger values of arc voltage, the voltage pulses (Fig. 4) made available by the thyatrons are larger and the amount of smoothing required on the arc current is smaller.

The percentage ripple, measured at 50 v across the arc, remained very nearly constant at 21% over the whole current range except between 4 and 6 amp where

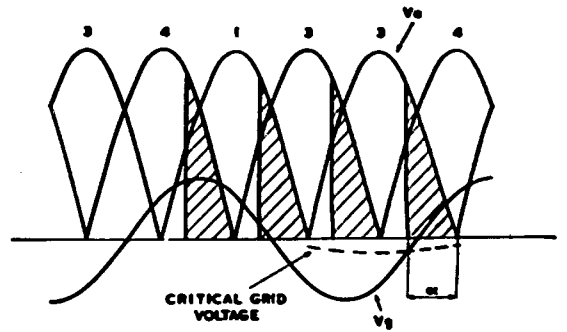


FIG. 5. Diagram showing the relative wave shapes of the anode and grid voltages of a thyatron.

the ripple went up to 24%. At 100 v across the arc, the ripple decreased to 14%; at 150 v to 10%.

Although the percentage ripple is generally larger than the 18% which was measured in a commercially available rectifier arc source, no stroboscopic effects were noticed when a 2 to 1 sector rotating at 600 rpm was used. A six phase system would of course yield less ripple and so would a larger choke; however, another perhaps more important effect of too large a choke is that the ability of the system to "recover" from a disturbance would be adversely affected.

AUTOMATIC REGULATION AND TRANSFER FUNCTIONS

Considering the system as a whole it can be seen that if I_0 drops from the desired value (because of an increase in the equivalent arc resistance R_a say), V_1 becomes less than the reference voltage V_0 , i increases, L_{ac} decreases, α increases and eventually V_i increases to compensate for the increase in R_a and to bring I_0 again to the desired value.

A block diagram with the relevant circuit elements is shown in Fig. 7.

Assuming that $K_1 \rightarrow \infty$ (i.e., the dc amplifier behaves as a perfect integrator by virtue of the negative feedback through C_2), it can be shown that the Laplace transform of the closed loop transfer function of the system is

$$I_0(s) = \frac{V_0 K \cdot G(s)}{R_0 (1 + K \cdot G(s))} \tag{1}$$

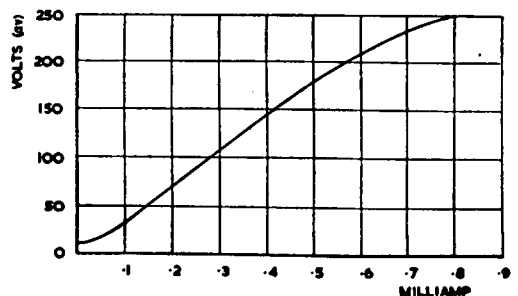


FIG. 6. Curve showing the average voltage on the cathodes of the thyatrons versus the control current of the saturable reactors.

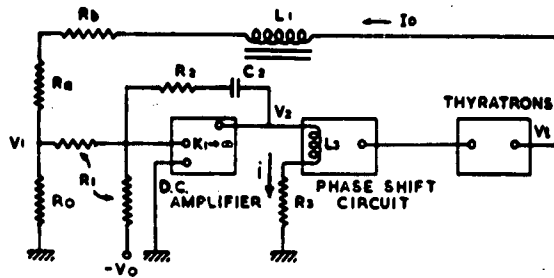


FIG. 7. Equivalent block diagram of the control system.

where

$$K = \frac{R_0}{R_0 + R_a(s) + R_b} \cdot \frac{1}{C_2} \cdot \frac{1}{R_1 \cdot R_3} \cdot Z_a \quad (2)$$

and

$$G(s) = \frac{1}{s} \cdot \frac{1 + T_2 s}{(1 + T_1 s)(1 + T_3 s)} \quad (3)$$

$R_0 + R_a + R_b$ is the total effective resistance of the arc current circuit. $T_2 = C_2 R_2$ and $T_3 = L_3 / R_3$ where L_3 is a function of i and R_3 is the total effective resistance in the circuit of i . Since both i and V_t are functions of α we write $V_t(s) = Z_a i(s)$ where Z_a as a function of i can be determined empirically (see Fig. 6).

From the above equations it is evident that within the operating range of the instrument ($\alpha < 135^\circ$) and in the ideal case when K_1 approaches "infinity" the value of I_0 in the steady state ($s \rightarrow 0$) is equal to V_0 / R_0 and is independent of the other parameters in the circuit. For K_1 finite it can be shown that the fractional error in the steady state is equal to $1 / K \cdot K_1 \cdot R_1 C_2$. K is nonlinear and depends on arc voltage but by making K_1 sufficiently large the steady state error can be reduced to the proportions of drift effects in the amplifier over most of the operating range of the unit.

TRANSIENT RESPONSE

Variations in I_0 are caused by variations in equivalent arc resistance. A rigorous analysis of $I_0(s)$ in terms of $\Delta R_a(s)$ is not readily made but some relevant remarks concerning the "recovery" of the system from such a

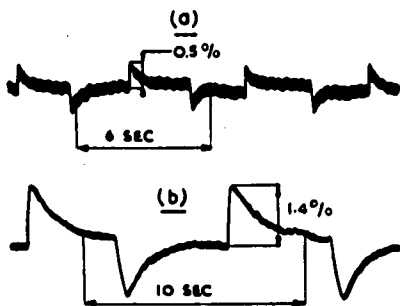


FIG. 8. Records of the change in arc current due to a sudden change of 10% in arc resistance at (a) 5 amp arc current and 50 v across the arc, and (b) 10 amp arc current and 150 v across the arc.

disturbance can be made by inspection of the parameters in the open loop transfer function $K \cdot G(s)$.

Provided the various time constants are such that the system is stable and adequately damped, the effect of R_a on I_0 will be smaller for a high value of K and the recovery will be faster.

1. Since $R_0 / (R_0 + R_a + R_b) = V_0 / I_0 \cdot I_0 / V_t$, K is independent of I_0 but is inversely proportional to V_t . Furthermore Z_a decreases as V_t increases so that the response time will be slower at higher arc voltages but will be almost unaffected by arc current.

2. A relatively small value of C_2 should be chosen when making up the required value of T_2 .

3. It is desirable that the dc amplifier should have a high input impedance so that R_1 should be relatively large.

4. R_3 is composed of the dc resistance of X_1 and X_2 , the dynamic plate resistance of the 6SN7 and a series 10K resistor. Whereas it seems desirable that R_3 should be small a relatively large value reduces T_3 to negligible proportions and the 10K resistor increases the amount of voltage available for feed-back through $R_2 C_2$.

5. Since T_3 is small it seems from $G(s)$ that the response time of the system will be fastest when T_2 is equal to the incremental value of T_1 when the disturbance occurs. However, this incremental value of T_1 is almost independent of I_0 but decreases as V_t increases so that T_2 was experimentally so chosen that the recovery of the system was fastest at arc voltages which are most likely to occur (i.e., between 40 and 70 v).

To test the response of the system over its whole range of operation the arc was replaced by two parallel resistors of appropriate values. The one resistor had a value ten times that of the other and by switching it in and out an effective step function change of 10% was

TABLE I. Percentage instantaneous change in arc current due to a sudden 10% change in equivalent arc resistance and the equivalent time constant with which such a change in current decreases to zero taken at different arc currents and arc voltages.

Current amp	Volts across arc	ΔI_0 max	Equivalent time constant
2.5	50	0.6	0.7 sec
	100	0.9	0.8
	150	1.4	0.9
5	50	0.6	0.6
	100	0.8	1.0
	150	1.2	1.0
10	50	0.6	0.8
	100	0.8	1.1
	150	1.4	1.6
15	50	0.7	0.5
	100	0.9	1.2
	150	1.9	2.1
20	50	0.7	0.6
	100	1.0	1.5
	150	2.0	2.3

TABLE II. Relative deviations (R) of spectral line intensity ratios.

	Sr4078 Cr4254	Ba4554 Cr4254	Mn4030 Cr4254	Mg3336 Sn3262	Na3302 Sn3262	Ca3274 Sn3262	Ca3179 Sn3262	Al3082 Sn3262	Fe3021 Sn3262	Mn2802 Sn2839
Constant current source	16%	21%	12%	7%	12%	10%	14%	11%	11%	10%
Batteries	19%	19%	13%	10%	21%	13%	25%	17%	11%	21%

achieved in R_a . A dc amplifier was connected between R_0 and an external stable supply of 25 v. A fast pen recorder with a flat frequency response up to 100 cps was connected to the output of the amplifier in order to record ΔI_0 . The amplifier input circuit was given a time constant of 0.2 sec for smoothing out excessive ripple in V_1 which would otherwise mask the effect of ΔI_0 .

Figure 8(a) shows a record of ΔI_0 taken at 5 amp and 50 v across the arc and 8(b) a similar record taken at 10 amp and 150 v across the arc. The almost instantaneous change ΔI_0 max which occurs when the step function is applied decreases to zero almost exponentially. In Table I is shown the percentage values of ΔI_0 max due to an instantaneous change of 10% in R_a and also the equivalent time constants with which ΔI_0 max decreases to zero.

EXPERIMENTAL RESULTS

Shown in Fig. 1(b) and 2(b) are records of arc current taken with similar electrodes and the same materials as the curves on the left hand sides but with the above apparatus as current source instead of batteries. It is clear that the apparatus is capable of compensating almost entirely for slow variation in arc resistance. For rapid variations in arc resistance such as would appear in the excitation of nonconducting materials the variation in arc current is approximately 1 to 2% although the average is much more constant.

In order to determine what improvements, if any, are obtained when the constant current source is used to excite the analytic samples for spectrochemical analysis, the standard and relative deviations of the intensity ratios of certain line pairs were calculated. The sample used was a synthetic plant material containing lithium carbonate as buffer. Ten exposures were made using the constant current source and ten using batteries. A four step rotating sector was used in both cases. The intensity of the lines was measured on a modified Zeiss "Spektrollinien Photometer" and the intensity ratios and their standard (S) and relative (R) deviations calculated. The intensity ratios were calculated in a modified A.R.L. calculating board fitted with Seidel scales and the deviations according to the formulas:

$$S = \left(\frac{d^2}{n-1} \right)^{1/2} \text{ and } R = \frac{S}{x} \times 100\%$$

where d =difference of each value from the mean; n =number of values; x =arithmetic mean.

The relative deviations can be seen in Table II from which it appears that in general the deviations when using the constant current source are never much greater and often much smaller than when using batteries.

As a further comparison of the two types of current sources forty spectra were recorded at 4 amp and forty at 6 amp, on an Ilford Ordinary plate from a Pfund iron arc, a standard light source that is often used for plate calibration and absorption photometry.⁶ At each value of current twenty spectra were obtained from the constant current source and twenty from the batteries. The intensities of two of the iron lines Fe 3163.9 and Fe 3172.1 used by Hasler⁷ for plate calibration were measured and the relative deviations of the intensities of the two lines were calculated. Table III gives the relative deviations obtained from the constant current source showing a very definite improvement as far as the reproducibility of the intensity of the line from exposure to exposure is concerned.

TABLE III. Relative deviation (R) of spectral line intensity from Pfund iron arc.

	4 amp		6 amp	
	3163.9 A	3172.1 A	3163.9 A	3172.1 A
Constant current source	7%	7%	4%	5%
Batteries	10%	10%	10%	10%

A further paper by Strasheim and Keddy in connection with the use of the constant current source in spectrochemical analysis will appear shortly in *Spectrochimica Acta*.

ACKNOWLEDGMENTS

The author is indebted to Dr. A. Strasheim and Mr. R. Keddy of this laboratory for the spectrochemical results and to the South African Council for Scientific and Industrial Research for permission to publish this paper.

⁶ M. Slavin, Ind. and Eng. Chem. Anal. Ed. 12, 131 (1940).
⁷ M. F. Hasler, J. Opt. Soc. Am. 31, 140 (1941).

A P P E N D I X,

The circuit diagram of the "Scott"-connected transformers for obtaining a four-phase system, the filament transformers for the thyristors and the 250 volt, 60 milliamp d.c. regulated power supply, is shown in Fig. 9.

The specifications of the "Scott"- connected transformers are:-

(a) MAIN TRANSFORMER.

Primary: 400 volts centre tapped, 10.39 amps 4.16 K.V.A.

Secondary: 500 volts centre tapped (with a tap also at 75 volts from the centre tap) 7.2 amps, 3.6 K.V.A.

Extra turns capable of carrying 0.2 amps are provided at the end of 500 volt winding in order to obtain a 600 volt centre tapped supply for the d.c. power supply.

(b) TEASER TRANSFORMER.

Primary: 346.4 volts, 10.39 amps, 3.6 K.V.A.

Secondary: 500 volts centre tapped (with a tap also at 75 volts from the centre tap) 7.2 amps, 3.6 K.V.A.

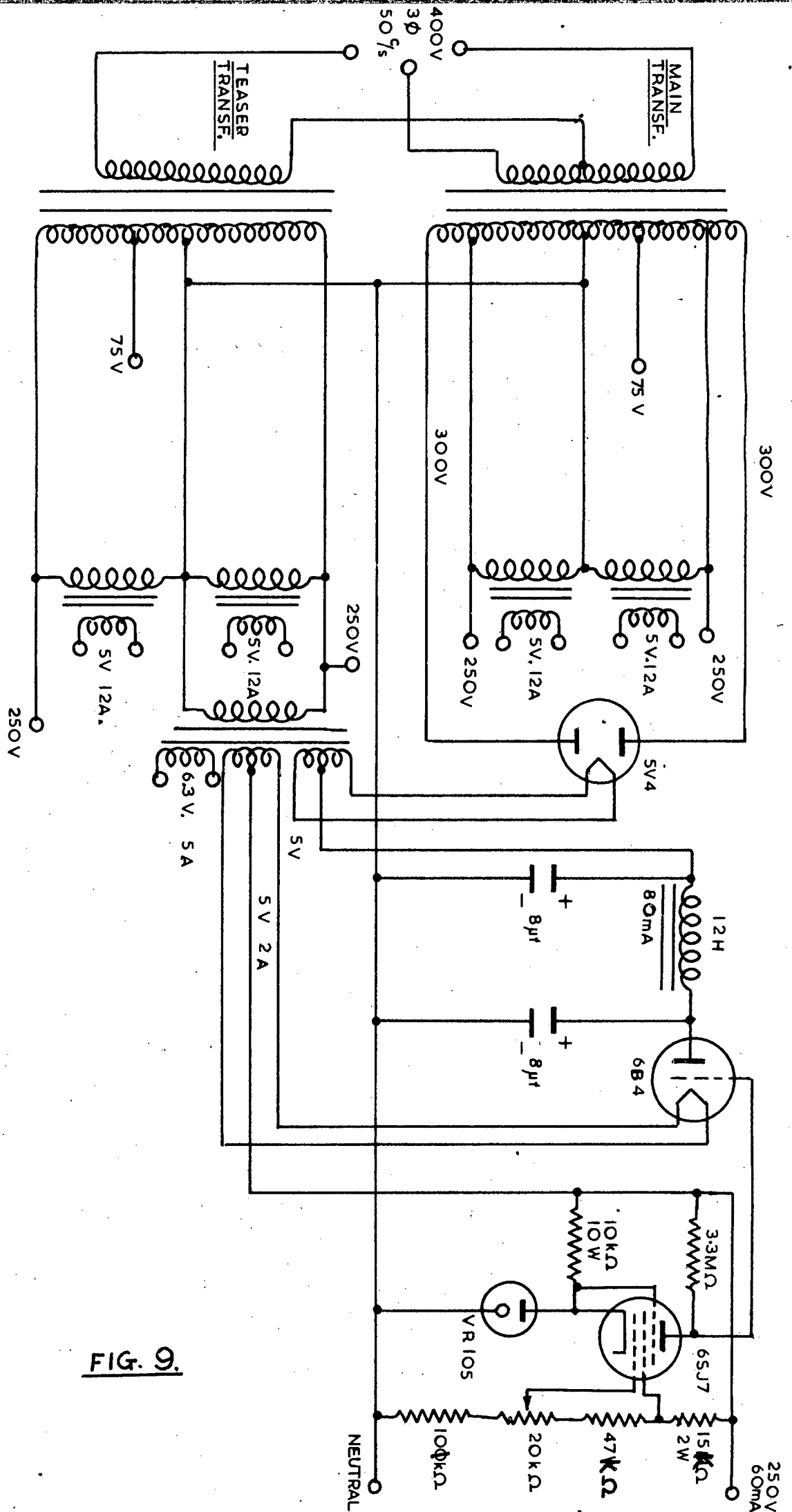


FIG. 9.

CONSTANT CURRENT ARC SOURCE FOR DIRECT CURRENT ARCS
 — FOUR PHASE TRANSFORMER, FILAMENT SUPPLIES
 AND REGULATED D.C. POWER SUPPLY —

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The Control and Stabilization of the Magnetic Field for a Mass Spectrometer

N. T. VAN DER WALT

National Physical Laboratory, Council for Scientific and Industrial Research, Pretoria, South Africa

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A method is described by means of which the magnetic field for a mass spectrometer is stabilized to within one part in 25 000 over a period of 30 minutes at any value of flux density within its variable range of 580 gauss to 9.3 kilogauss.

INTRODUCTION

IN the magnetic deflection type of mass spectrometer all ions of the same m/e ratio are focused onto a collector by a particular magnetic field of flux density B_1 (say). B_1 must be sufficiently stable to allow the resolution of 1 mass unit from ion beams containing masses up to 450. Although a stability of 1 part in 5000 seemed to be sufficient,¹ the aim was a stability of 1 part in 50 000. Furthermore, it must be possible to vary B_1 according to the particular value of m/e , and in order to study the shape of the output peak on the collector, B_1 must be varied smoothly and linearly over the relevant range.

An electromagnet is generally used to produce B_1 . The control and stability of B_1 is usually achieved by controlling and stabilizing the magnetizing current.² A stable relation between current and flux density is relied upon.

Rotating fluxmeters are frequently used for measuring B_1 . In some cases the output of a search coil S_1 rotating at a constant speed in B_1 is measured,² while in other cases the output of S_1 is compared against the output of a similar search coil S_2 rotating synchronously in time and space with S_1 in a magnetic field (B_2) of known or calculable flux density.^{3,4} In the system described below (Fig. 1) a permanent magnet is used for a "reference" field (B_2) in a manner similar to that described by Caldecourt.⁵

The outputs of S_1 and S_2 are rectified mechanically

(M_1 and M_2) and compared in a dc amplifier which controls the magnetizing current of the spectrometer magnet so as to equalize the outputs of S_1 and S_2 . In this manner the stability of B_1 is referred to B_2 , and the control is referred to the fraction of the output of S_2 tapped off by the slider of a helically wound potentiometer. Instabilities in amplifier gain, relation between B_1 and magnetizing current, etc., are reduced to second-order effects.

In order not to influence the operation of the spectrometer, it was necessary to mount the driving motor and "reference" magnet some distance from the electrometer tube and magnet.

THE CONTROL CIRCUIT

A circuit diagram of the dc amplifier is shown in Fig. 1.

e_1 , the rectified output of S_1 and e_2 , a fraction of the rectified output from S_2 , tapped off by means of a precision "Helipot" potentiometer R_3 , are fed to the grids of differential amplifier V_1 . Whatever the respective values of e_1 and e_2 , only the amplified difference between e_2 and e_1 appears across the smoothing condenser C_p . This difference is further differentially amplified by V_2 and brought to a single output terminal before it is fed via a level changing battery to the grids of four current amplifier tubes in parallel. The sense of the current change in the electromagnet is so as to reduce the absolute value of $(e_2 - e_1)$. It should be noted here that the action of the circuit is to control e_1 relative to e_2 . The control of B_1 then depends on its relation to e_2 and e_1 .

V_1 and V_2 were specially selected to have identical μ 's and g_m 's in each section. This feature, together with "constant current devices" used as common cathode

¹ A. O. Nier, Rev. Sci. Instr. 18, 398 (1947).

² Hipple, Grove, and Hickam, Rev. Sci. Instr. 16, 69 (1945).

³ L. M. Langer and F. R. Scott, Rev. Sci. Instr. 21, 522 (1950).

⁴ Hedgran, Siegbahn, and Svartholm, Proc. Phys. Soc. (London) 63, 960 (1950); further details by A. Hedgran in Manne Siegbahn Anniversary Volume, pages 268-279, November, 1951.

⁵ V. J. Caldecourt, Rev. Sci. Instr. 21, 772 (1950).

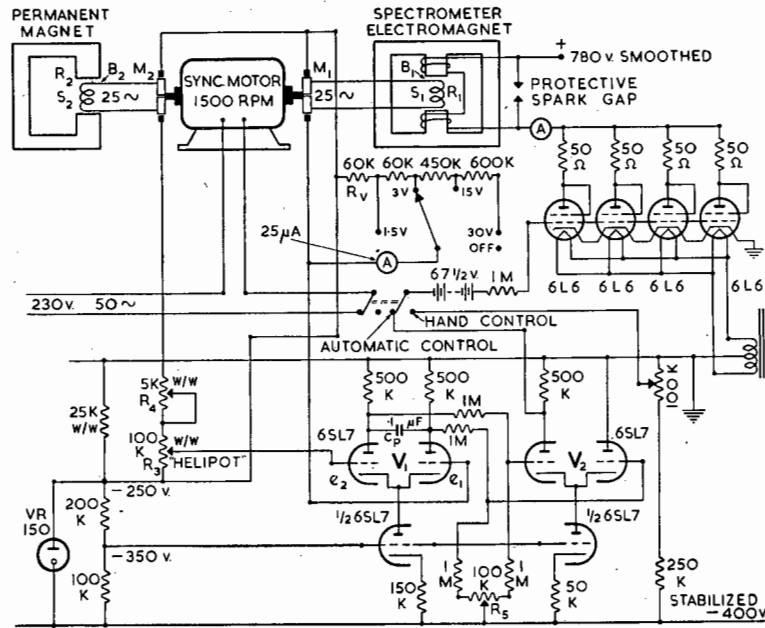


Fig. 1. Circuit diagram.

loads, makes the differential amplifiers independent, to a large extent, of common mode variations and drift resulting from heater voltage variations. Their outputs are also more nearly linear because of mutual cancellation of plate resistance variations. All these effects are conducive to a highly stable gain and freedom from drift.

Nichrome wire wound resistors and high precision Welwyn carbon resistors of sufficiently high power ratings to prevent any self-heating were used throughout. The -400 -volt power supply had a stability of 1 in 3000 for a 5 percent change in mains voltage. No current was drawn from the $67\frac{1}{2}$ -volt battery, but it was replaced regularly together with other batteries used in the spectrometer.

The high resistance voltmeter connected across M_1 measures B_1 to the accuracy of the driving motor speed.

Series resistance R_4 was so adjusted that the 780 -volt power supply is not overloaded when the slider of R_3 is turned to its maximum value.

TRANSIENT RESPONSE

It is often necessary to "scan" the spectrum by varying B_1 linearly from one value to another. This may be done by driving the slider of R_3 at a constant rate over the relevant range. Assuming an ideal relation between e_1 and B_1 , the speed of response of e_1 to a sudden change in e_2 may conveniently be tested by suddenly shorting R_4 (of appropriate value for this particular purpose).

Noting that in the transient state e_1 and e_2 are functions of time, the Laplace transform of the system

differential equation is given by

$$e_1(s) = \frac{K_1 K_2 K_3 K_4 K_5}{(1+sT_p)(1+sT_f)} [e_2(s) - e_1(s)], \quad (1)$$

where K_1 = voltage gain of $V_1 = 60$, K_2 = voltage gain of $V_2 = 30$, K_3 = gain of current amplifier = 2.6 mA/v , K_4 = sensitivity of the electromagnet = 36 gauss/mA , K_5 = sensitivity of $S_1 = 2.9 \text{ v/kilogauss}$, T_p = smoothing time constant = $C_p R_p = 0.025 \text{ sec}$, R_p = output impedance of $V_1 = 250 \text{ K}$, T_f = time constant of the electromagnet = 1.5 sec , and $K = K_1 K_2 K_3 K_4 K_5 = 500$ (approx).

From Eq. (1) it can be seen that in the steady state the "error"

$$(e_2 - e_1) = e_1/K \quad (2)$$

increases linearly with e_1 . However, by means of potentiometer R_5 , $(e_2 - e_1)$ may be set to vary from a negative to a positive value. Furthermore, at high magnetizing currents K_4 is smaller as a result of magnetic leakage, and at low currents K_3 is smaller because of the current amplifier "cutting off."

For a step function input of e_2 (i.e., $e_2(s) = 1/s$) the response of e_1 is given by

$$e_1(s) = \frac{K}{s[(1+sT_p)(1+sT_f) + K]} \quad (3)$$

As a result of almost instantaneous overloading of V_2 , Eq. (3) is linear only over a range of $(e_2 - e_1)$ of less than 0.1 volt. However, when V_2 overloads, the current amplifier forces a current through the electromagnet which tends, with a time constant T_f , to a value corresponding to the voltage on its grids. This state of affairs persists until e_1 is of such a value as to bring the

system within its linear range. It is, therefore, to be expected that the response will vary depending on the initial value of e_1 , but that the response will be faster than the response of the magnetizing current to a step voltage input directly to the electromagnet. Transient oscillations are sensitive to T_p/T_f and therefore limit the value of C_p . Any ripple in the magnetizing current is further almost entirely eliminated by the high time constant T_f .

The response of e_1 to a step function change in e_2 from 20 to 10 volts is shown in Fig. 2. Because of this relatively fast time of response, a constant rate of scan of B_1 up to 200 gauss per second may be achieved by driving the slider of R_3 at a constant speed.

By using integrating amplifiers well known in servo-mechanism practice ($e_2 - e_1$) may be practically reduced to zero and the time of response slightly increased. However, it has been found that stability is adversely affected and an extra amplifying stage to rectify matters was hardly considered worth while.

STABILITY

Assuming that M_1 and M_2 are mechanically sound and that e_1 and e_2 feed into "infinite" impedances,

$$e_1 = 2f \left[1 - \frac{R_1'}{R_v'} \right] (A_1 N_1) B_1 \times 10^{-8} \text{ volts (av),} \quad (4)$$

and

$$e_2 = 2fk \left[1 - \frac{R_2''}{R_v''} \right] (A_2 N_2) B_2 \times 10^{-8} \text{ volts (av).} \quad (5)$$

for

$$\frac{R_1'}{R_v'} \quad \text{and} \quad \frac{R_2''}{R_v''} \ll 1,$$

where R_v' = resistance of the voltmeter across $M_1 = 60K$ at the 1.5-volt multiplier setting; R_1' = resistance of S_1 plus $M_1 = 750$ ohms; $R_v'' = R_3 + R_4 = 103K$ (R_3 and R_4 are considered as a single resistance because they have identical electrical properties). R_2'' = resistance of S_2 plus $M_2 = 1500$ ohms; $A_1 N_1$ = area turns of S_1 ; $A_2 N_2$ = area turns of S_2 ; f = frequency of the driving motor = $\frac{1}{2}$ mains frequency; k = fraction of $R_3 + R_4$ tapped off by the slider of R_3 ; and $e_2/k = 30$ volts (average).

From Eqs. (2), (4), and (5), and if $1/K \ll 1$,

$$B_1 = k \left[1 - \frac{1}{K} \right] \left[1 - \frac{R_2''}{R_v''} \right] \left[1 + \frac{R_1'}{R_v'} \right] \frac{A_2 N_2}{A_1 N_1} B_2. \quad (6)$$

By inspection of Eq. (6) the following remarks can be made concerning the stability of B_1 :

(1) Since B_1 is directly proportional to k , the dial of potentiometer R_3 may be calibrated in terms of B_1 .

(2) Since $K = 500$ a 1 percent instability in K would cause an instability of 2 parts in 100 000 in B_1 .

(3) Contact resistance changes in M_2 being negligible compared with R_v'' , temperature changes are the main

cause for variations in R_2''/R_v'' . R_2'' (copper) having a temperature coefficient of $+0.4$ percent/ $^{\circ}C$ and R_v'' (nichrome) $+0.02$ percent/ $^{\circ}C$, a temperature variation of $1^{\circ}C$ would cause a variation in B_1 of 6 parts in 100 000.

(4) Similarly R_1' having a temperature coefficient of $+0.4$ percent/ $^{\circ}C$ and R_v' (Welwyn resistor) a temperature coefficient of -0.02 percent/ $^{\circ}C$, a temperature variation of $1^{\circ}C$ would cause a variation of 5 parts in 100 000 in B_1 at the 1.5-volt multiplier setting. This variation in B_1 is inversely proportional to the multiplier setting.

(5) Effects (3) and (4) oppose each other.

(6) Except if S_1 and S_2 are loosely wound or badly constructed, variations in $A_2 N_2/A_1 N_1$ can only occur if A_1 and A_2 have different coefficients of expansion.

(7) B_1 is directly dependent on B_2 . For a good quality permanent magnet with a low temperature coefficient, instabilities in B_2 over a period of about an hour are very small, provided that the room temperature remains reasonably constant. It is also essential that, while the spectrometer is in use, no ferromagnetic material is moved near the magnet so as to influence B_2 .

(8) The only other effect that may influence B_1 is zero drift in the amplifier. This effect is difficult to estimate and depends to a large extent on a choice of components. However, every reasonable precaution has been taken to eliminate zero drift.

Whereas it is difficult to measure the absolute stability of B_1 directly, the fractional stability $\Delta B_1/B_1$ may be measured to a fair degree of accuracy by measuring $\Delta(e_2 - e_1)/e_1$, as is evident from the following remarks:

(1) Since $\Delta_1 B_1/B_1 = (1/K)(\Delta K/K)$ (Eq. 6) and $\Delta K/K = -\Delta_1(e_2 - e_1)/e_1$ (Eq. 2), $+\Delta_1 B_1/B_1$ caused by a variation ΔK is equal to $-(1/K)[\Delta_1(e_2 - e_1)/e_1]$.

(2) $+\Delta_2 B_1/B_1$ resulting from variations $+\Delta(R_1'/R_v')$, and $-\Delta(A_1 N_1)$ (Eq. 6) is equal to $-\Delta_2 e_1/e_1$ because of the same variations (Eq. 4).

(3) Similarly, $+\Delta_3 B_1/B_1 = +\Delta_3 e_2/e_2$ resulting from $-\Delta(R_2''/R_v'')$, $+\Delta(A_2 N_2)$ and $+\Delta B_2$ [Eqs. (6) and (5)].

(4) A variation $+\Delta f$ in f causes no variation in B_1 but would cause $+\Delta e_1/e_1 = +\Delta e_2/e_2$ (Eqs. 4 and 5). For

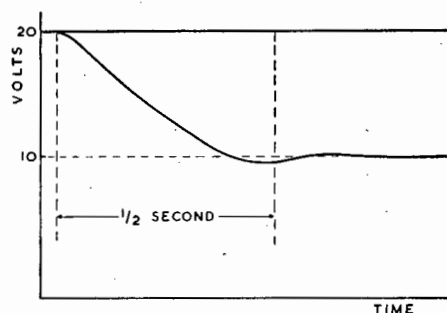


FIG. 2. Response of e_1 to a step function change in e_2 .

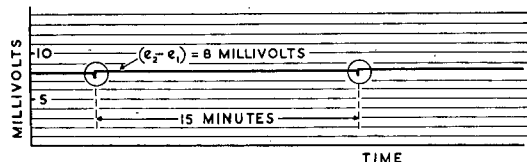


FIG. 3. Record of $(e_2 - e_1)$ over a period of approximately 30 minutes.

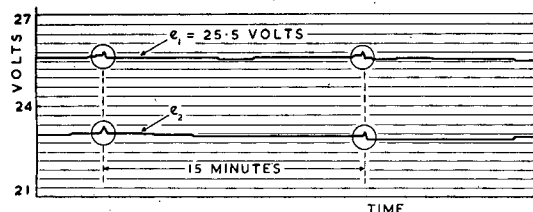


FIG. 4. Records of e_1 and e_2 over periods of approximately 30 minutes each.

e_1 very nearly equal to e_2 $\Delta(e_2 - e_1)/e_1$, a result of Δf , is zero.

(5) Since zero drift directly causes $\Delta_4(e_2 - e_1)$,

$$\frac{\Delta_4 B_1}{B_1} = \frac{\Delta_4(e_2 - e_1)}{e_1}$$

Summing the above we have that total

$$\frac{\Delta B_1}{B_1} = \frac{1}{K} \frac{\Delta_1(e_2 - e_1)}{e_1} + \frac{\Delta_2 e_1}{e_1} + \frac{\Delta_3 e_2}{e_2} + \frac{\Delta_4(e_2 - e_1)}{e_1}$$

For $e_1 = e_2$ (nearly)

$$\frac{\Delta B_1}{B_1} = \frac{\Delta^1(e_2 - e_1)}{e_1} + \frac{1}{K} \frac{\Delta_1(e_2 - e_1)}{e_1}, \quad (7)$$

where Δ^1 denotes a variation in $(e_2 - e_1)$ caused by all the above effects except ΔK . Total $\Delta(e_2 - e_1) = \Delta^1(e_2 - e_1)$

$+ \Delta_1(e_2 - e_1)$ may be measured by connecting a Brown recording potentiometer directly across the grids of V_1 . At balance, the effective impedance of the potentiometer is infinite. This potentiometer automatically standardizes itself against an internal standard cell every 15 minutes (the encircled points on Figs. 3 and 4).

A typical record of $(e_2 - e_1)$ taken over a period of about 30 minutes is shown on Fig. 3 and a record of e_1 taken immediately afterwards and without disturbing the system, on Fig. 4.

By inspection of Eq. 7 it can be seen that unless $\Delta_1(e_2 - e_1)$ and $\Delta^1(e_2 - e_1)$ are such that they oppose each other almost perfectly over the whole period of the record on Fig. 3 (which is highly improbable), $\Delta B_1/B_1$ is nearly equal to $\Delta(e_2 - e_1)/e_1$.

The variation $\Delta(e_2 - e_1)$ at a level of e_1 of about 25 volts is considerably less than 1 millivolt. This yields a fractional stability $\Delta B_1/B_1$ of better than one part in 25 000 over a period of half an hour. At no time, after a reasonable warming-up period, has the stability been found to be worse.

A record of e_2 was also taken on Fig. 4, but at a different setting of B_1 so that the two traces did not coincide. These two records show that the variations in e_1 and e_2 are of the same order. These variations are typical of the local mains frequency (which is hand controlled at the power station).

Finally it could be added that the system lived up to all expectations when used on the mass spectrometer as a whole. It could, of course, also be used in other problems where a controlled magnetic field is required.

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