

**INVESTIGATING RURAL HIGH SCHOOL MATHEMATICS TEACHERS'
TECHNOLOGICAL PEDAGOGICAL CONTENT KNOWLEDGE DEVELOPMENT
USING GEOGEBRA THROUGH A LESSON STUDY APPROACH**

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ABSTRACT

The study aimed to investigate the Technological Pedagogical Content Knowledge (TPACK) development of rural high school mathematics teachers when using GeoGebra and a school-based lesson study (LS) continuous professional development (CPD) approach to teach transformation geometry. The research objectives that guided this study's aim were to:

- Establish the extent of the use of GeoGebra and the LS as a school-based CPD approach in mathematics teaching in high schools in the JTG district prior to the main study.
- Investigate the level of TPACK developed when rural high school mathematics teachers use GeoGebra to teach transformation geometry during a school-based LS CPD approach.
- Investigate the experiences of rural high school mathematics teachers when teaching transformation geometry using GeoGebra.

The primary research question underlying this study is: *What is the TPACK development of mathematics teachers from rural high schools when using GeoGebra and the LS approach as a form of school-based CPD to teach transformation geometry?* The study employed a mixed methods complementarity design, blending quantitative and qualitative approaches, guided primarily by an interpretive paradigm, while acknowledging pragmatic considerations. A total of 135 mathematics teachers from the 31 high schools in the Northern Cape Province's JTG district completed a baseline survey. The baseline survey focused on the extent of technology use (in particular, GeoGebra) in teaching mathematics and the nature of CPDs – a school-based LS CPD approach, in this case to which teachers in the JTG district were exposed. Following the baseline survey, six teachers from two specific schools (A and B) were selected purposively from the 31 high schools in the district for further study. These teachers taught Senior and Further Education and Training (FET) Phase mathematics at two rural high schools selected for research. Each of the two schools formed a LS group of three teachers. These schools' selection criteria included *inter alia* that they were located in low socio-economic areas, and that neither had a computer laboratory, although the teachers had laptops and mobile projectors. The Theory of Didactical Situations (TDS) was used to foreground this study.

The data were collected from multiple sources, including a baseline survey, focus group interviews (FGIs) conducted after the teacher training session on GeoGebra and the LS with the six teachers and the three transformation geometry research lessons (RLs), and observation

guides. The three transformation geometry RLs focused on the concepts of reflection about the x- and y-axis, reflection about the line $y = x$, and translation. The baseline survey findings showed limited usage of GeoGebra and the LS in mathematics teaching within the JTG district's high schools. Factors such as limited mathematical content knowledge and skills, lack of adequate technological resources and training, among others, were noted to be preventing the teachers from integrating the software into their lessons. Additionally, factors including *inter alia* lack of support from school administrators prevented teachers from implementing the LS in their schools. This necessitated training teachers from the two schools on how to utilise GeoGebra to teach transformation geometry, using a school-based LS CPD approach to develop their TPACK.

The data gathered from the FGIs conducted after the teacher training session on the use of GeoGebra and the LS, as well as the three-transformation geometry RLs were analysed using the rubric of Niess et al. (2009). The aim was to track individual teachers' TPACK development levels in each of the four descriptors of the five TPACK levels, as shown by their self-reported scores and the researchers' interrater reliability scores. The statements from individual teachers' reflections on their TPACK development levels from training to RL3 are provided. These statements give an indication of where an individual teacher's TPACK development level was in the five TPACK level descriptors. The two schools' findings revealed that individual teachers transitioned from pre-recognising to the higher TPACK development levels (exploring and advancing) from training to RL3.

Additionally, a quantitative data analysis using a repeated-measure analysis of variance (ANOVA) test was used to establish whether researchers' ratings on teachers' TPACK knowledge in schools A and B differed significantly across the four stages of professional development (training, RL1, RL2, and RL3) for each of the four TPACK dimensions: curriculum and assessment, learning, teaching, and access. This analysis merged data from the two schools (A and B). The findings revealed that the researchers' ratings of teachers' TPACK knowledge in schools A and B increased from the training stage to RL3 across the four TPACK dimensions. Notably, no discernible changes were detected between the training and RL1 phases, suggesting that meaningful improvements manifest later in the intervention.

The study contributes to mathematics teaching when using GeoGebra and the LS as a school-based CPD in teacher education. A larger-scale study with a diverse sample would help to confirm and generalise the findings.

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DEDICATION

Firstly, to my beloved parents (Mr Mugendawala Lawrence and Ms Nakizaye Teddy), in loving memory, I dedicate this thesis as a tribute to your enduring influence on my life and achievements. Secondly, for my daughters (Michelle and Rochelle Ssenyomo), my love and inspiration; may this thesis be a testament to the power of perseverance and dedication.

DECLARATION OF ORIGINALITY

I, Ssenyomo Bernard John, student number (SSNBER001), hereby declare that this thesis titled “Investigating rural secondary school mathematics teachers' technological pedagogical content knowledge development using GeoGebra through a lesson study approach” is entirely my own work and has not been submitted elsewhere for academic purposes. I have properly cited and acknowledged all sources, adhering to the University of Cape Town’s Faculty of Science Research Ethics Committee reference guide.

Signed by candidate

Bernard John Ssenyomo

Date: 01 December 2025

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ACRONYMS USED IN THIS STUDY

AMESA	Association for Mathematics Education of South Africa
ANA	Annual National Assessment
ANOVA	Analysis of Variance
ATP	Annual Teaching Plan
B.Ed.	Bachelor of Education
BSc	Bachelor of Science
CAPS	Curriculum and Assessment Policy Statement
CAS	Computer Algebra Systems
CAT	Computer Application Technology
CPD	Continuous Professional Development
CK	Content Knowledge
XK	ConteXtual Knowledge
DBE	Department of Basic Education
DGS	Dynamic Geometry Software
DHET	Department of Higher Education and Training
DOE	Department of Education
DS	Didactical Situations
F	Test statistic value
FET	Further Education and Training
FGI	Focus Group Interview
FSREC	Faculty of Science Research Ethics Committee
ICT	Information and Communication Technology

IST	In-service Teachers
JTG	John Taolo Gaetsewe District
KO	Knowledgeable Other
LS	Lesson Study
LS-TPACK	Lesson Study Technological Pedagogical Content Knowledge Assessment Model
M	Mean
MKT	Mathematical Knowledge for Teaching
MLS	Microteaching Lesson Study
NCDoe	Northern Cape Department of Education
NCS	National Curriculum Statement
NCTM	National Council of Teachers of Mathematics
NSC	National Senior Certificate
P	Probability Value
PCK	Pedagogical Content Knowledge
PGCE	Postgraduate Certificate in Education
PK	Pedagogical Knowledge
PST	Pre-Service Teachers
RL	Research Lesson
SD	Standard deviation
SMT	School Management Team
SP	Senior Phase
TCK	Technological Content Knowledge
TDS	Theory of Didactical Situations
TIMSS	Trends in International Mathematics and Science Study

TK	Technological Knowledge
TPACK	Technological Pedagogical Content Knowledge
TPK	Technological Pedagogical Knowledge

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CHAPTER ONE

1.1 Introduction

The study aimed to investigate rural high school mathematics teachers' TPACK development when using GeoGebra and the LS approach as a form of school-based CPD to teach transformation geometry. This chapter discusses the study's context and background. Following this is a discussion of the problem statement. The study's aim, objectives and questions are presented next; and then a summary of the methodology, as well as a discussion of the study's significance. Following this, the study's delimitations, limitations and operational definitions are addressed. Finally, the thesis's structure is summarised.

1.2 Context and background of the study

The study worked with mathematics teachers from Senior and FET Phases in rural JTG district high schools, located in South Africa's Northern Cape Province. In the education setting in South Africa, the Senior Phase includes grades 8 and 9, whilst the FET Phase encompasses grades 10 to 12. In the context of this study, "Rural areas" nest learners with low-income parents and/or guardians. Most schools located in rural areas are under-resourced (Mthethwa et al., 2020; Saal et al., 2020); thus, this study worked with under-resourced schools in the JTG district.

In South Africa, the grades R-12 National Curriculum Statement (NCS), as outlined by the Department of Basic Education (Department of Basic Education (DBE), 2012), encourages a learning approach that is learner-centred, with the teacher guiding collaborative and critical thinking. When employing TDS – the roots of which depend on a constructivist approach (Laborde, 2007) – active pedagogy and learning are inseparable (Winsløw et al., 2018). Thus, the South African mathematics curriculum is premised on TDS, which was this study's theoretical home.

Within the mathematics syllabus, there are several justifications for the inclusion of transformation geometry. Among these is that transformation geometry provides learners with opportunities to tackle advanced reasoning tasks using various visual and symbolic representations (Hollebrands, 2003). It is also argued by Mukamba and Makamure (2020) that transformation geometry has practical applications in both mathematics curricula and everyday life. Given the significance of transformation geometry in studying mathematics, the South

African 2014 Annual National Assessment (ANA) diagnostic report for the DBE intimated that questions on geometry topics, particularly transformation geometry, were difficult for learners to answer (DBE, 2014). The 2019 Trends in International Mathematics and Science Study (TIMSS) report also revealed that South African grade 8 learners struggle with geometry (Mullis et al., 2020). These reports indicated the main cause of the problem to be inappropriate pedagogical strategies for teaching transformation geometry concepts, because of limited resources. In a similar vein, Ada and Kurtulus (2010) and Idris (2006) affirmed that inappropriate instructional methods used by teachers and inadequate mathematics teaching facilities are the main causes of learner challenges in understanding transformation geometry concepts.

However, GeoGebra has been found to improve learner transformation geometry performance and learning, when compared to traditional pen and paper methods (Guven, 2012; Küçük & Gün, 2023). GeoGebra was developed by Markus Hohenwarter in 2001, during the course of his master's programme at the university of Salzburg, Australia (Hohenwarter & Preiner, 2007); according to Kepceoğlu and Yavuz (2016, p. 575), GeoGebra is “a dynamic mathematics software for all levels of education that brings together geometry, algebra, spreadsheets, graphing, statistics and calculus in one easy-to-use package”. The software allows users to explore transformation geometry concepts in an environment that is dynamic and interactive (Uymaz, 2024). GeoGebra is a preferred tool in the CPD of the teachers in this study because of some of its features: firstly, the software is an open-source and user-friendly technological tool that requires no licensing and allows learners and teachers to use it at home and in their classes at no cost (Rahman & Puteh, 2017). Secondly, Hohenwarter et al. (2008) asserted, the software is designed specifically for teaching and learning purposes – transformation geometry, in this case.

Despite GeoGebra's pertinence in mathematics instruction, and transformation geometry in particular, the literature reveals that many teachers struggle to integrate the software into the mathematics classroom effectively (Jezdimirović, 2014; Mokotjo & Mokhele, 2021). Jezdimirović (2014) found that GeoGebra use in mathematics instruction in Serbian classrooms is often marked by challenges including (but not limited to) teachers' incompetence in terms of mathematical content, inadequate technological resources, and limited skills for incorporating the software. Mokotjo and Mokhele (2021) concurred that South African mathematics teachers are becoming demotivated concerning GeoGebra integration into their lessons due to insufficient CPD, a lack of support from school management teams (SMT), and limited

resources and GeoGebra skills for making lessons interesting and productive. In addressing the challenges constraining effective GeoGebra integration in the mathematics classroom, CPD for the teachers is recommended (Jezdimirović, 2014; Mokotjo & Mokhele, 2021; Mukamba & Makamure, 2020). The LS is recognised both locally and internationally as an effective collaborative teacher CPD approach to changing classroom practices (Fauskanger et al., 2022; Joubert et al., 2020).

As a CPD approach for teachers of mathematics, in the LS cycle, teachers identify a topic that is challenging to the learners – transformation geometry, in this case – and discuss the objectives of the topic, with guidance from curriculum documents. Using the selected objectives as a guide, teachers design a ‘research lesson’ (RL) to improve learners’ conceptual understanding of transformation geometry concepts. After planning, one teacher volunteers to teach the transformation geometry RL, while the others observe and collect data on learner learning using an observation guide. Following this is an in-depth reflection on the data collected throughout the RL, and then using this evidence to improve the RL for future presentation (Lewis, 2009). This provides rich teacher CPD focusing on classroom practice. In this study, the LS approach, which was implemented in a RL, was used as a school-based CPD approach for developing rural high school mathematics teachers’ TPACK when using GeoGebra to teach reflections and translations. A RL, the heart of LS, is a lesson that is the subject of research and is examined through the stages of a LS framework (Clivaz, 2018). The LS framework that this study uses has four phases: identification of (transformation geometry) objectives, planning, implementing/observing, and reflecting on the entire RL process (Mhakure, 2019) for the purpose of developing teachers’ TPACK through engaging them in a school-based LS CPD approach. This begs the question: Why use a school-based LS CPD approach to develop teachers’ TPACK when teaching transformation geometry using GeoGebra in this study?

In education department practice in South Africa, teachers in rural areas attend generic CPD mathematics workshops outside of their schools with teachers from other schools; the workshops are facilitated by external experts including (but not limited to) subject advisors, provincial coordinators, mathematics education researchers, and teacher educators – a one-size-fits-all scenario. These experts are not familiar with the contexts and teaching cultures of the individual schools; which renders their CPD efforts inadequate, as they barely address the challenges experienced by the teachers within their individual schools (Mhakure, 2019). In

other words, educational contexts in under-resourced schools differ from those in schools that are well-resourced. Under-resourced schools are also different from each other – each school is unique. This is what led to this researcher’s interest in investigating the TPACK development via a school-based LS CPD approach in selected rural high schools; the use of a school-based LS CPD approach in developing rural high school mathematics teachers’ TPACK offers them opportunities that are rarely found in traditional CPD activities, which rely on a ‘top-down’ approach – in this case, where CPD activities are driven predominantly by external experts (Posthuma, 2012; Mhakure, 2019; Woodland & Mazur, 2015). These opportunities include (but are not limited to) the following.

Firstly, school-based CPD activities for the TPACK development of teachers using GeoGebra and the LS allows teachers to use the minimum resources (e.g. laptops and projectors) that they have at their schools to improve transformation geometry teaching and learning (Posthuma, 2012). Secondly, CPD activities for the TPACK development of teachers using GeoGebra and a school-based LS approach can be embedded in teachers’ school and classrooms contexts, giving them the chance to see transformation geometry teaching and learning instantaneously as it occurs at their schools (Mhakure, 2019). Lastly, CPD activities on teachers’ TPACK development using a GeoGebra-based LS as school-based CPD approach “are predicated on the use of teacher-generated, classroom-based, real-time formative and summative assessment data about student (transformation geometry) learning” (Woodland & Mazur, 2015, p. 10). Hence, teachers work with data that is current and meaningful from their own schools.

1.3 Problem statement

The TPACK framework extends Shulman’s (1986) pedagogical content knowledge concept by integrating technological knowledge (GeoGebra in this case) to explain effective (transformation geometry) teaching using GeoGebra (Zelkowski et al., 2013). TPACK highlights the intricate knowledge base necessary for teachers to effectively engage in planning, implementing and reflecting on transformation geometry RLs on reflection, translation, enlargement and reduction in the South African Senior Phase mathematics curriculum, while guiding learners’ learning using GeoGebra (Niess, 2012). The TPACK and the LS frameworks are guided by learner thinking in planning, implementing and analysing transformation geometry RLs using GeoGebra (Niess et al., 2009). These two frameworks (TPACK and LS) are therefore inextricably interwoven and interdependent in this study’s investigation of mathematics teachers’ TPACK development.

Some local and international TPACK studies specific to teachers of mathematics in CPD have incorporated the LS (Huang et al., 2021; Joubert et al., 2020; Kurt & Çakıroğlu, 2018). Results indicate that implementing the LS significantly enhanced teachers' TPACK in mathematics. Huang et al.'s (2021) study aimed to explore how cross-cultural online learning among Chinese and South Australia primary mathematics teachers impacted these teachers' learning when teaching probability using several instructional resources (e.g. online resources, manipulatives and physical devices). Kurt and Çakıroğlu's (2018) study examined the TPACK development of pre-service teachers (PST) in Turkey when using virtual manipulatives to teach statistics through a microteaching LS (MLS). Unlike Huang et al. (2021) and Kurt and Çakıroğlu (2018), Joubert et al. (2022) conducted their study in Africa. Joubert et al.'s (2022) study (administered by the University of Pretoria) worked with 52 teachers across different education levels from all over South Africa and Botswana, and representatives from almost all the subject areas (including mathematics) were among the teachers. These teachers were isolated and could not meet face-to-face on a regular basis. The study explored a blended LS approach (combining face-to-face and online aspects) to enhance teachers' skills in mobile technology incorporation into their lessons. The mobile technologies used included (but were not limited to) Maths quiz grade 7 app, triangle calculator App, and power point slides. While the literature (Huang et al., 2021; Joubert et al., 2020; Kurt & Çakıroğlu, 2018) reveals that LS promotes mathematics teachers' TPACK development, no research has been done in South Africa using GeoGebra and a traditional face-to-face school-based LS CPD to develop rural high school mathematics teachers' TPACK knowledge.

Additionally, prior to the main study, a baseline survey was carried out with 135 mathematics teachers from the JTG district's Senior and FET Phase. The JTG district is located in a rural part of South Africa's Northern Cape Province. The baseline survey aimed to establish the extent of the use of technology (in particular GeoGebra) in mathematics teaching, and the nature of CPDs – LS as a school-based CPD approach, in this case – to which the JTG district's teachers were exposed. The results showed limited usage of GeoGebra and the LS in mathematics teaching in JTG district high schools, thereby strengthening the study's position on the problem statement. The study aimed to bridge a literature void by equipping mathematics teachers at two rural under-resourced high schools in the JTG district with GeoGebra skills for teaching transformation geometry, through a school-based LS CPD approach. Hence, the study investigated the TPACK development among rural high school mathematics teachers when using GeoGebra and the LS approach as a form of school-based

CPD to teach transformation geometry, and was guided by the research aim and by the questions presented below:

1.4 Research aim

The study aimed to investigate the TPACK development of rural high school mathematics teachers when using GeoGebra and the LS approach as a form of school-based CPD to teach transformation geometry. The study's aim was directed by the research objectives, to be discussed next.

1.4.1 Research objectives

The research objectives for this study were to:

- Establish the extent of the use of GeoGebra and the LS as a school-based CPD approach in mathematics teaching in high schools in the JTG district prior to the main study.
- Investigate the level of TPACK developed when rural high school mathematics teachers use GeoGebra to teach transformation geometry during a school-based LS CPD approach.
- Investigate the experiences of rural high school mathematics teachers when teaching transformation geometry using GeoGebra.

1.4.2 Research questions

The primary research question underlying this study is: *What is the TPACK development of mathematics teachers from rural high schools when using GeoGebra and the LS approach as a form of school-based CPD to teach transformation geometry?*

This question branches out into the sub-questions below:

- What is the extent of the use of GeoGebra and the LS as a school-based CPD approach in the teaching of mathematics in JTG district's high schools prior to the main study?
- What is the level of TPACK developed when rural high school mathematics teachers use GeoGebra to teach transformation geometry during a school-based LS CPD approach?
- What are the experiences of rural high school mathematics teachers using GeoGebra to teach transformation geometry?

1.5 Methodology

An interpretive paradigm, whose aim is to gain deeper insight into how teachers interpret and make meaning within their own contexts foregrounded this study (Bertram & Christiansen, 2014). To preserve the phenomena's integrity under study, "efforts are made to get inside the person (teacher) and understand from within" (Cohen et al., 2011, p. 21). For example, immediately after a RL was taught, teachers in the two schools (A and B) were given a reflection guide (see Appendix 8) to rate their competence in using GeoGebra to teach transformation geometry concepts of reflection and translation – which they carried to the FGI for the researcher to make a follow-up entry on their TPACK level ratings. The study employed a mixed methods complementarity design, blending quantitative and qualitative approaches, guided primarily by an interpretive paradigm, while acknowledging pragmatic considerations.

The study used two methods. The first was a baseline survey, which was administered to all teachers in the 31 high schools in the JTG district. The schools had mathematics teachers covering both Senior and FET Phases. The second method was a multiple case study of six teachers from two under-resourced rural high schools (A and B) that participated in a school-based LS CPD approach to develop teachers' TPACK when teaching transformation geometry using GeoGebra. The data were collected using a baseline survey, FGIs conducted after the teacher training sessions on GeoGebra and LS and each of the three transformation geometry RLs, and observation guides. The FGI guide on the four RL stages – identification of (transformation geometry) objectives, planning, implementing/observing, and reflecting – included a reflection guide.

A total of 135 teachers responded to an anonymous baseline survey that was administered to mathematics teachers in all the 31 JTG district high schools. The baseline survey aimed to establish the extent of the use of technology (in particular, GeoGebra) in the teaching of mathematics, and the types of CPDs – LS as a school-based CPD approach, in this case – to which teachers in this district were exposed. After the baseline survey, two high schools (A and B) were purposively selected from the JTG district's 31 high schools, based on criteria that included, *inter alia*, the absence of a computer laboratory at the school, as long as the teachers had laptops and mobile projectors. Each school formed a school-based LS group of three Senior and FET Phase mathematics teachers. The baseline survey was undertaken within the geographical delimitations of the JTG district, with a research sample comprising six teachers from two schools (A and B). The baseline survey was followed by two introductory training

sessions that were conducted per school. In the first session, ISTs were introduced to GeoGebra by the same knowledgeable other (KO), with the aim of empowering them on how to use the software in transformation geometry instruction.

A KO is an individual from outside the LS team but with a deep understanding of the field who “may bring new (transformation geometry) knowledge from research and the curriculum, show the connection between theory and practice and help others learn how to reflect on teaching and learning” (Fauskanger et al., 2022). The study’s KO was a teacher-participant from one of the sample schools, who the researcher had identified before the training as knowledgeable about using GeoGebra and how it works in mathematics teaching, and also experienced in transformation geometry teaching and assessment. The need to conduct this training was informed by the baseline survey’s results regarding the teachers’ knowledge of GeoGebra and its application in mathematics instruction.

The second training session focused on familiarising teachers with the LS approach, as the school-based CPD on TPACK that the teachers had to undergo and was to be carried out within the framework of the LS cycle. Hence, the teachers needed to understand the phases of the LS – identification of (transformation geometry) objectives, planning, implementing/observing, and reflecting – as they were expected to operationalise them using the RL. Like the activities of the GeoGebra training session on transformation geometry teaching, the activities of the LS training session were informed by the data collected during the baseline survey on the nature of CPDs to which mathematics teachers in JTG district high schools were exposed.

In preparation for the data collection phase, the study’s data collection methodology (the three transformation geometry RLs, worksheets, observation, and interview guides including reflection guides) was piloted by the researcher with one school in the JTG district whose demographics were similar to those of the schools used as research samples. The pilots sought to identify areas that needed to be improved, in terms of the instruments’ quality and the methodology itself.

After the training, three transformation geometry RL plans – on the concepts of reflection about the x- and y-axis, reflection about the line $y = x$, and translation – which were created by the researcher were given to the teachers to critique according to their learners’ learning needs and the curriculum. The designing of the three RLs was informed by the South African Senior Phase CAPS document. The teachers in each school and the KO jointly discussed the objectives of each transformation geometry RL.

Using these objectives as a guide, the teachers and the KO planned the transformation geometry RL together using GeoGebra. The teachers in the two schools included some of the comments from RL1 to improve on the planning and implementation of RL2. These teachers also included some of the comments from RL1 and RL2 to improve on the planning and implementation of RL3. In the planning session, the KO expressed his opinions on using GeoGebra for teaching transformation geometry concepts of reflection and translation, while the researcher introduced the teachers to the data collection instruments – observation and FGI guides (including reflection guides) – to be used within the LS cycle. The aim was to acquaint the teachers with the instruments in terms of what they would be observing and reflecting on during and after the actual implementation of each transformation geometry RL respectively. The teacher participants from the two sample schools also identified the three learner categories – lower, middle and upper performers – and the three transformation geometry RLs were designed around their learning needs and the curriculum using GeoGebra.

After planning a particular transformation geometry RL, one of the three teachers facilitated this RL, while the other two teachers and the KO observed and collected data on learner learning using an observation guide. The KO also stepped in to provide clarity in terms of GeoGebra use when the teacher encountered an obstacle during teaching. Immediately after a transformation geometry RL was taught, the teachers were given a reflection guide, which they completed and brought to the FGI for the researcher to record a follow-up note on their TPACK development in each descriptor of the five TPACK levels. Additionally, the teachers and the KO also reflected on the four phases of the transformation geometry RL and suggested ways to use GeoGebra to improve on the next RL. The four phases of each of the three transformation geometry RLs were video- and audio-recorded for analysis purposes.

To analyse the data collected from each research instrument – a baseline survey, FGI conducted after the teacher training session on GeoGebra and LS in transformation geometry teaching and each of the three RLs, and observation guides – the study employed qualitative and/or quantitative methods. The researcher utilised descriptive statistics to analyse qualitative data from the baseline survey. Baseline survey findings were presented utilising tables, graphs, and percentages. Data analysis from teacher training on GeoGebra and the LS was based on the recorded FGI transcriptions, which was conducted after the training session in each school. The recorded FGIs were transcribed verbatim, read several times and analysed using deductive thematic analysis. Niess et al.'s (2009) rubric guided the coding of the FGI transcriptions. The data were organised according to the four descriptors of the five levels of TPACK to track

individual teachers' TPACK development levels, as shown by their self-reported scores and the researchers' interrater reliability scores in the descriptor of the five TPACK levels after training. In addition, a quantitative data analysis using a repeated-measure ANOVA test was used to establish whether researchers' ratings on teachers' TPACK knowledge in schools A and B differed significantly across the four stages of professional development (training, RL1, RL2, and RL3) for each of the four TPACK dimensions: curriculum and assessment, learning, teaching, and access. The analysis combined schools A and B.

As alluded to earlier, the research instruments (the three transformation geometry RLs, observation, and FGI guides including reflection guides) were piloted to check their data collection efficiency. The transcribed data from the three RLs and the LS recordings were analysed according to the RL stages alluded to earlier. Data analysis in the implementation stage was based on observation guides that were used to observe learners' learning of the concepts of reflection and translation. The observation guides were analysed in two steps. While data analysis from the first step arose from teachers' comments on learners' learning when they manually solved questions from a worksheet, data analysis from the second step arose from teachers' comments on learners' learning when the teacher used GeoGebra to teach the same pen and paper activity. Data analysis in the reflection stage was based on the transcripts of FGIs with teachers in the two schools (A and B) after a transformation geometry RL was taught. Since the interviews comprised textual data, Braun and Clarke's (2006) deductive thematic analysis approach was employed in analysing this data. FGI transcripts were reread many times. As the researcher went through this data, he marked important sections and added descriptive names or codes to these sections to identify them for further analysis. These codes were analysed to find similarities, and were then grouped into descriptors of the five TPACK levels. Sentences that were vital were identified to be utilised for further articulation. This data aided the researcher to track individual teachers' TPACK development levels, as shown by their self-reported scores and the researchers' interrater reliability scores in the descriptors of the five TPACK levels after each RL. Additionally, a quantitative data analysis using a repeated-measure ANOVA test was used to establish whether researchers' ratings on teachers' TPACK knowledge in schools A and B differed significantly across the four stages of professional development (training, RL1, RL2, and RL3) for each of the four TPACK dimensions: curriculum and assessment, learning, teaching, and access. The analysis combined schools A and B.

1.6 Significance of the study

Firstly, the study makes a contribution to mathematics teachers' TPACK development in the context of South Africa by testing the significant difference between researchers' interrater reliability scores regarding the teachers' TPACK development levels. Other studies on TPACK and the LS (Joubert et al., 2020; Kurt & Çakıroğlu, 2018; Riales, 2011) did not use the ANOVA test.

Secondly, in addition to what other studies on TPACK and the LS in mathematics teaching did (Joubert et al., 2020; Kurt & Çakıroğlu, 2018; Riales, 2011), this study makes a contribution to mathematics teachers' TPACK development in the South African context by using TDS as a theory to underpin LS in school-based CPDs.

Thirdly, the study looks at a novel method of CPD in South Africa informed by using a GeoGebra-based LS as a form of school-based CPD to teach transformation geometry. South African education department teachers mainly attend generic CPD workshops outside of their schools facilitated by experts (e.g. mathematics subject advisors and teacher educators) who are not familiar with their schools' contexts and teaching cultures (Mhakure, 2019). This renders their CPD activities inadequate, because they seldom address the challenges faced by individual teachers at their schools.

Lastly, from both a theoretical and a practical point of view, this study is an exemplar for how the LS and GeoGebra could be used in teacher education.

1.7 Delimitations and limitations of the study

A baseline survey was carried out in the Northern Cape Province's JTG district, South Africa, with a purposive sample of six teachers from two schools within the study's geographical delimitations. It would therefore be unwise to generalise the baseline survey findings as pertaining to all mathematics teachers in all high schools across the Northern Cape Province.

The study worked with a small sample – only two rural under-resourced high schools. The small number of research sites limits the generalisability of the study's findings.

Teachers from the sample schools (A and B) were lacking in knowledge regarding GeoGebra. These were teachers who had never heard of GeoGebra or who had heard about it but never used it to teach a mathematics topic prior to this study. This limitation of the teachers' lack of the knowledge of GeoGebra was minimised by training the teachers.

The two schools' teachers also lacked LS knowledge. The teachers had never heard about LS or had heard about it but never participated in it prior to this study. The teachers were exposed mostly to workshops organised outside of their schools, conducted by external experts including but not limited to mathematics subject advisors and provincial coordinators. The limitation of the teachers' lack of knowledge regarding LS was also minimised by training the teachers.

1.8 Operational definitions

Technological pedagogical content knowledge (TPACK) is defined as “a way of thinking about the complex relationships between technology [*GeoGebra*], pedagogy, and content in a specific context [*transformation geometry*], which is represented through the carefully considered implementation of technology [*GeoGebra*] in a classroom setting in order to help students [*learners*] better understand a particular topic [*transformation geometry*]” (Cox, 2008, pp. 50-51).

GeoGebra refers to a mathematical software that enables users to explore transformation geometry concepts in an environment that is dynamic and interactive (Uymaz, 2024).

Transformation geometry as a mathematical topic entails changing the object, either in position or shape (DBE, 2012). For example, using GeoGebra's dynamism facility, the object's position and/or shape can be changed by resizing it or dragging the same object to various positions. This study focused on transformation geometry concepts of reflection about the x- and y-axis, reflection about the line $y = x$, and translation.

LS as a CPD form for teachers of mathematics is defined as a strategy:

...in which teachers work together to: formulate (transformation geometry) goals for learner learning and long-term development; collaboratively plan a 'research lesson' (using GeoGebra) designed to bring to life these goals; conduct the lesson in a classroom, with a team member teaching and others gathering evidence on learner learning and development; and discuss the evidence gathered during the lesson, using it to improve the lesson, the unit, and instruction more generally” (Lewis, 2009, p. 95)

This study uses the LS approach as a school-based CPD approach to develop the TPACK of rural high school teachers when utilising GeoGebra to teach transformation geometry concepts of reflection and translation in the Senior Phase of the South African mathematics curriculum.

1.9 Outline of the thesis

The structure of the thesis has five chapters.

Chapter One – Context and background

Chapter one comprises nine sub-headings, namely, Introduction, Context and background, Problem statement, Research aim, Methodology, Significance, Delimitations and limitations, Operational definitions, and Outline of the thesis.

Chapter Two – Literature review and theoretical framing

The chapter begins by discussing technology used in mathematics education, and TDS as a theory that acts as a guide to understanding this study. Next is a discussion of the theoretical frameworks – TPACK and the LS – that the researcher used to inform the collection of data and its analysis. Following this is a discussion about studies on LS and TPACK, and TDS as a theory underpinning the LS. Assessing TPACK development through the LS is also discussed, before the section concludes with a summary.

Chapter Three – Research methodology

The study's research approach and methodology are described in this chapter, which consists of six sub-headings: Research aim, Research orientation, Research design and rationale, Methodology, Ethical considerations, and Conclusion.

Chapter Four – Findings of the study

This chapter's analysis is informed by data from the baseline survey and the FGI conducted after the teacher training session on GeoGebra and LS use in transformation geometry teaching, the three recorded and transcribed transformation geometry RLs, observation guides, and FGIs (including reflection guides) conducted with teachers after a RL was taught. Baseline survey data is analysed first in this chapter. Following this is an analysis of data gathered from the transcribed FGI on the teacher training session on GeoGebra and LS use in transformation geometry teaching. An analysis of data from the three transformation geometry RLs is also discussed, before summarising the findings of each research instrument.

Chapter Five: Discussion and conclusions

This chapter presents the study results, and outlines how the research contributes to theory – TPACK development levels – and practice on the use of GeoGebra and the LS in transformation geometry teaching. Following this is a presentation of the study limitations and implications, and recommendations for further study. Finally, the researcher presents a summary of this chapter.

1.10 Summary

The context and background for this study were discussed first in this chapter. The problem statement was presented next. Following this was a presentation of the research aim, objectives and questions. Thereafter, the study's methodology and significance were summarised. Additionally, the study's delimitations, limitations and operational definitions used were presented. A concise thesis structure outline concludes the chapter. The next chapter reviews the literature relevant to this study.

CHAPTER TWO: LITERATURE REVIEW AND THEORETICAL FRAMING

2.1 Introduction

The study aimed to investigate rural high school mathematics teachers' TPACK development when using GeoGebra and the LS approach as a form of school-based CPD to teach transformation geometry. The chapter first discusses the technology used in mathematics education. In addition, TDS as a theory that acts as a guide to understand this study is discussed. A discussion of the theoretical frameworks – TPACK and the LS – that the researcher used to inform this study's data collection and analysis is presented. Following this is a discussion about studies on the LS and TPACK. TDS as a theory underpinning the LS in developing teachers' TPACK is discussed next. Finally, assessing TPACK development through the LS is also discussed, before the section concludes with a summary.

2.2 Technology used in mathematics education

Technology integration in mathematics instruction is discussed here. According to Chizwina and Mhakure (2018), technology refers to engaging with software tools – GeoGebra, in this case – that can be utilised in designing various illustrative transformation geometry examples to facilitate learners' conceptual understanding. Quality education is considered a crucial factor in improving a country's economic development (Schwab, 2017). As a result, many countries all over the world are making large-scale investments regarding the incorporation of education technology into their school curricula to enhance educational standards in schools. For example, the United States of America's Pennsylvania state spent heavily in introducing the 1:1 technology initiative in its secondary schools, by providing each student with a laptop (Graham et al., 2021). With the goal of fostering classroom technology integration, Rwanda is also investing heavily in smart classrooms, not only equipping her schools with computers but also with full-time internet access (Karuhanga, 2018). Locally, in an effort to refine teaching and learning quality, the 'e-Education White Paper' is a policy introduced by the South African government to utilise educational technology in schools (DBE, 2004). In response, different educational technology projects, known as 'paperless' or 'smart' classrooms, have been launched by the Gauteng and Western Cape Education Departments, and primarily implemented in under-resourced schools in low socio-economic areas in these provinces (Saal et al., 2020). The educational technology that the selected schools were equipped with included, *inter alia*, projectors, smart boards, tablets, and laptops. Not only in Gauteng and Western Cape (two of South Africa's nine provinces) but nationally, the South African government "has

initiatives in place to equip schools with devices, connectivity, digital content, and information and communication technology (ICT) integration training for teachers” (Graham et al., 2021, p. 4).

Despite the large sums of money spent by governments worldwide to equip schools with educational technologies, most mathematics teachers still struggle with effective technology integration into their lessons (Saal et al., 2020; Trujillo-Torres et al., 2020). In a study which investigated 73 Spanish mathematics high school ISTs’ perceptions on introducing technology, Trujillo-Torres et al. (2020) found that teachers considered training to be key to effectively integrating technology into their mathematics lessons; similar to Saal et al. (2020), who investigated the factors facilitating and hindering technology-enhanced mathematics instruction in depressed socio-economic areas in South Africa. Saal et al. (2020) recommended policy makers provide CPD to mathematics teachers on effective technology integration in their (transformation geometry) lessons, tailored to the pace of the annual teaching plan. Overall, the literature demonstrates that CPD for mathematics teachers is essential to promote educational technology integration into their lessons (Saal et al., 2020; Trujillo-Torres et al., 2020). Hence this study, which investigated rural high school mathematics teachers’ TPACK development in South Africa’s JTG district, used the LS as a form of school-based CPD and GeoGebra dynamic geometry software (DGS) to teach transformation geometry.

2.2.1 DGS

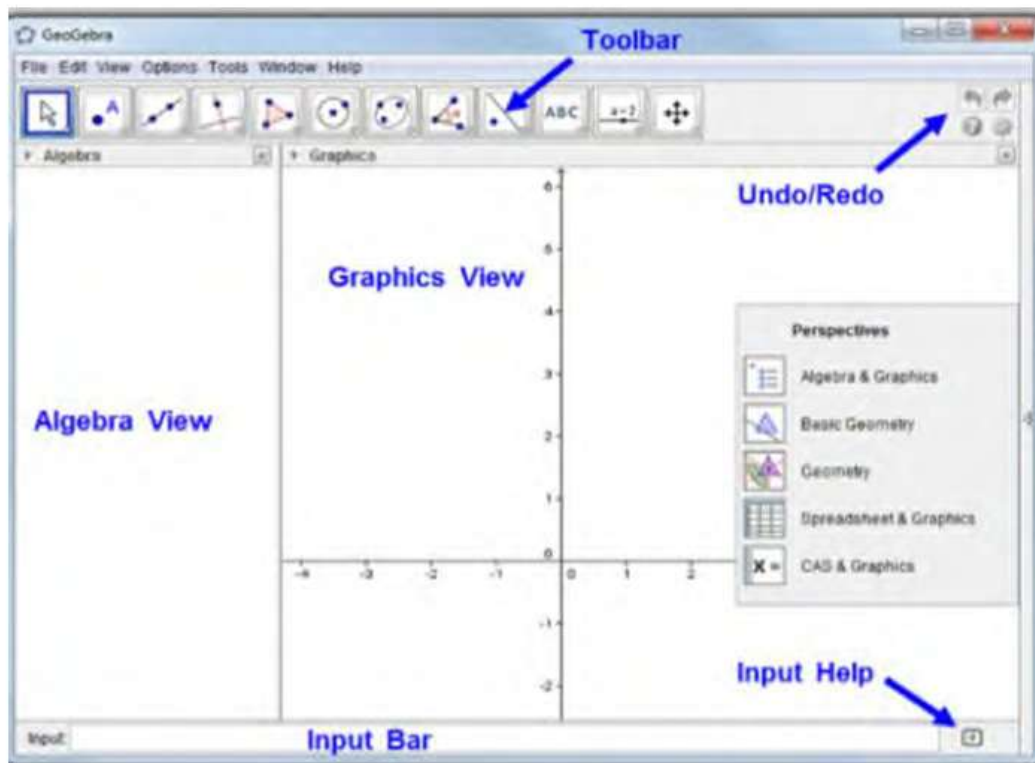
The term ‘dynamic’ can be defined as describing a process of action or motion on a computer screen (Denbel, 2015). DGS packages – including Geometer’s Sketchpad, Cabri Geometry, and GeoGebra – provide robust technological support for transformation geometry teaching and learning, leveraging motion and change on the computer screen (Dikovic, 2009). These software packages focus on the geometric relations (e.g. coordinates, size and distance) that exist between an object and its image, which can be enhanced by the dynamism capability of the software programmes (Kepceoğlu & Yavuz, 2016). This provides learners with opportunities for exploring the geometric relations that exist between an object and its image. The ‘dragging’ feature is a distinctive aspect of DGS compared to other geometry software (Güven, 2012). Additionally, “the dynamic environments where beyond the geometry, other mathematical domains like algebra or analysis can be studied are called dynamic mathematical software” (Kabaca et al., 2010, as cited in Kepceoğlu & Yavuz, 2016). GeoGebr is one of the

most preferred mathematical software suites that has such a property (Kepceoğlu & Yavuz, 2016), and this software is discussed in the section below.

2.2.2 GeoGebra as a DGS in mathematics classes

GeoGebra is a dynamic mathematical software package, created by Markus Hohenwarter at the university of Salzburg in Austria 2002 as part of his master's thesis (Hohenwarter & Preiner, 2007), incorporating geometry, algebra and calculus in a dynamic and interactive way, allowing its users to explore transformation geometry concepts (Uymaz, 2024). GeoGebra is a preferred tool in the CPD of mathematics ISTs in this study, because of its unique features. Firstly, the software is an open-source and user-friendly technological tool that requires no licensing and allows learners and teachers to utilise it in class and at home, at no cost (Rahman & Puteh, 2017). Secondly, the software does not require the user to be expertly computer literate; it is suitable for anyone, and designed to be used at all levels of mathematics education (Preiner, 2008). Thirdly, GeoGebra is tailor-made to facilitate teaching and learning (Hohenwarter et al., 2008) and supports dynamic scenes (Wassie & Zergaw, 2019). Lastly, GeoGebra affords its users an opportunity to see two representations of a transformation geometry object on the same screen (Kepceoğlu & Yavuz, 2016). In other words, the software's layout features multiple sections for diverse representations of transformation geometry objects. Figure 2.1 below shows the GeoGebra window with different sections.

Figure 2.1 Different sections of the GeoGebra window (Source: www.geogebra.org)



Initially developed to incorporate geometry, algebra and calculus in a dynamic and interactive way (Uymaz, 2024), GeoGebra has now been fully developed to incorporate other mathematics disciplines as well, including statistics, measurement, probability, trigonometry and finance. The software has the capability of revolutionising mathematics teaching and learning (Majerek, 2014), in particular transformation geometry.

Several studies have revealed that GeoGebra plays multiple crucial roles in mathematics instruction and learning (Mensah et al., 2023; Mokotjo, 2020; Mthethwa et al., 2020; Zulnaidi & Zamri, 2017). Mensah et al.'s (2023) study investigated GeoGebra used as a tool to enhance the teaching and learning of mathematics by teachers in Ghana. The findings were that the software improved the teaching and the learning of mathematics in the classroom. Zulnaidi and Zamri's (2017) study conducted in Malaysia ascertained that GeoGebra has the potential to create teaching and learning environments that are favourable to learners exploring and making meaning of transformation geometry concepts, through multiple representations and dynamic observations that are not possible with pen and paper. Similar results have been generated by studies conducted in South Africa. Mthethwa et al. (2020) carried out a study in KwaZulu-Natal, South Africa that examined how GeoGebra impacted teacher beliefs and learners

learning. The findings showed that learners who learnt circle geometry using GeoGebra outperformed their peers (who were exposed to this topic in a traditional lecture-based way) in solving problems and justifying their statements. Additionally, in a parallel and complementary investigation, the mathematics teacher's attitudes towards utilising GeoGebra as an instructional tool were qualitatively considered (Mthethwa et al., 2020). The findings were that even in depressed socio-economic areas where technological resources are inadequate, GeoGebra use positively impacted teacher beliefs about mathematics teaching and learning. Mokotjo's (2020) study also conducted in South Africa found GeoGebra helpful in fostering a learning environment where learners can explore, conjecture, and visualise abstract concepts in mathematics.

Despite GeoGebra's pertinence, the literature reveals that many teachers still struggle with integrating this software in mathematics lessons effectively (Jezdimirović, 2014; Mokotjo & Mokhele, 2021; Stols et al., 2015; Wassie & Zergaw, 2019). Wassie and Zergaw (2019) referred to inadequate technical resources and pedagogical knowledge for integrating the software, inadequate belief and technology fluency from teachers, and congested classrooms as some of the obstacles of delivering effective GeoGebra mathematics lessons in Ethiopia. Jezdimirović (2014) confirmed that GeoGebra use in mathematics instruction in Serbia is typically characterised by numerous challenges, including (but not limited to) inadequate technical resources and skills required for incorporating the software into the teaching process. Stols et al. (2015) reported that limited content knowledge and technical skills prohibited South African mathematics teachers from using technology, in particular GeoGebra, in their instruction. Mokotjo and Mokhele (2021) concurred that South African mathematics ISTs become demotivated concerning GeoGebra integration into their lessons, due to inadequate CPD and support from their school managers, and limited resources and skills, which hinder their efforts to effectively integrate the software into mathematics teaching. The factors identified in the above studies, such as inadequate technical resources, congested classrooms, and lack of CPD and support from school management, are among the organisational and situational challenges that teachers experience at their individual schools in their attempts to integrate GeoGebra (Mishra, 2019). These studies (Jezdimirović, 2014; Mokotjo & Mokhele, 2021; Stols et al., 2015; Wassie & Zergaw, 2019) recommended professional training to effectively incorporate GeoGebra in mathematics lessons.

Although GeoGebra is crucial in mathematics teaching and learning, the reviewed literature (Jezdimirović, 2014; Mokotjo & Mokhele, 2021; Stols et al., 2015; Wassie & Zergaw, 2019) made it clear that limited GeoGebra skills due to inadequate or no training was a fundamental challenge to teachers' integration of the software into their lessons. This study therefore sought to engage teachers from two rural high schools in the JTG district in a school-based LS CPD approach, empowering them to use GeoGebra to teach transformation geometry to develop their TPACK.

2.2.3 Teaching transformation geometry using GeoGebra

Transformation geometry is a mathematics topic that involves altering an object's position or shape (DBE, 2012; Mukamba & Makamure, 2020). These alterations comprise movements that include translations, enlargements, reductions, reflections about the x- and y-axis, and reflections about the line $y = x$ of geometric shapes and figures (DBE, 2012). For example, using GeoGebra's dynamic facility, an object can be dragged to different positions on the cartesian plane. Hence, the object's position has been altered. The transformation geometry topic is part of the Senior Phase (grades 8 and 9) mathematics curriculum in South Africa, and is applied in mathematics topics including but not limited to algebraic functions, trigonometry functions and analytical geometry in the FET Phase mathematics curriculum (grades 10 to 12). Transformation geometry plays a crucial role in studying mathematics. According to Hollebrands (2003), transformation geometry provides learners with opportunities to think about crucial mathematical concepts including functions; a context for learners to view mathematics as an interconnected discipline; and opportunities for learners to engage in activities that require higher-level reasoning using various representations. Additionally, Mukamba and Makamure (2020) asserted that transformation geometry has a practical application in the mathematics curricula and everyday life.

Despite transformation geometry's pertinence in studying mathematics, South Africa's 2014 Annual National Assessment (ANA) diagnostic report for the DBE indicated that geometry questions, particularly transformation geometry, were challenging for learners to engage with (DBE, 2014). The TIMSS of 2019 also reported that South African grade 8 learners have poor achievement in geometry tests (Mullis et al., 2020). These reports indicated that learners are not profiting from the transformation geometry teaching approaches, due to resource constraints. This perspective is corroborated by other studies (Ada & Kurtulus, 2010; Idris, 2006), which affirmed that inappropriate instructional methods and inadequate teaching

facilities are the main causes of learner challenges in understanding transformation geometry. GeoGebra has been found to improve learner transformation geometry performance and learning, when compared to the traditional pen and paper methods, by Guven (2012), Küçük and Gün (2023), and Mukamba and Makamure (2020), who all recommended training mathematics teachers to effectively utilise the software for transformation geometry teaching. Hence, this study sought to develop the TPACK of rural high school mathematics teachers by training them to utilise GeoGebra to teach transformation geometry using a school-based LS CPD approach.

In the process of developing rural high school mathematics' TPACK when teaching transformation geometry using GeoGebra and the LS school-based CPD approach, a combination of teaching and learning processes take place (Ngwabe & Schäfer, 2023). Thus, there was a need for this study to be guided by a theory that informs active teaching and learning. When employing TDS, active pedagogy and learning are inseparable (Winsløw et al., 2018). Thus, TDS became this study's theoretical home.

2.3 Theory

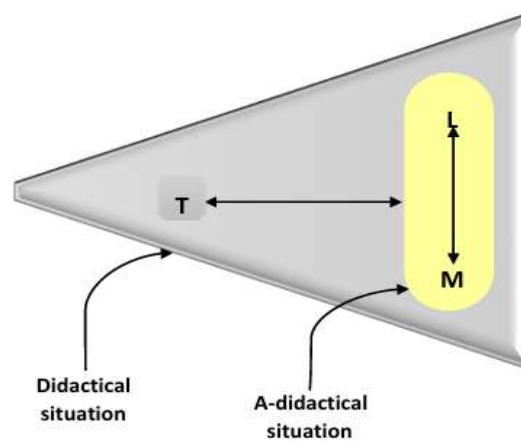
The section begins by discussing TDS as the main theory where this study was foregrounded, before discussing how the theory has been used in pre-service or in-service studies internationally and nationally.

2.3.1 TDS

This section discusses TDS – the overarching theory for this study – and its constructs, as used in developing the TPACK of rural high school mathematics ISTs when teaching transformation geometry using GeoGebra and a school-based LS approach as a CPD.

The study was undergirded by Brousseau's (1997) TDS. TDS, whose roots depend on a constructivist approach (Laborde, 2007), models learners' learning, teachers' transformation geometry knowledge and the teaching thereof using GeoGebra "in terms of didactic situations (DS), which is a dynamic triad unfolding in time consisting of an interaction between teachers, students (learners) and a didactic milieu" (Winsløw et al., 2018, p. 129) – see Figure 2.2. According to Mangiante-Orsola et al. (2018), TDS underscores the teacher's role in facilitating learners' learning of transformation geometry content using GeoGebra, thus representing a DS.

Figure 2.2 DS as interplay between a teacher (T), Learners (L), and milieu (M) – adapted from Winsløw et al. (2018, p. 129)



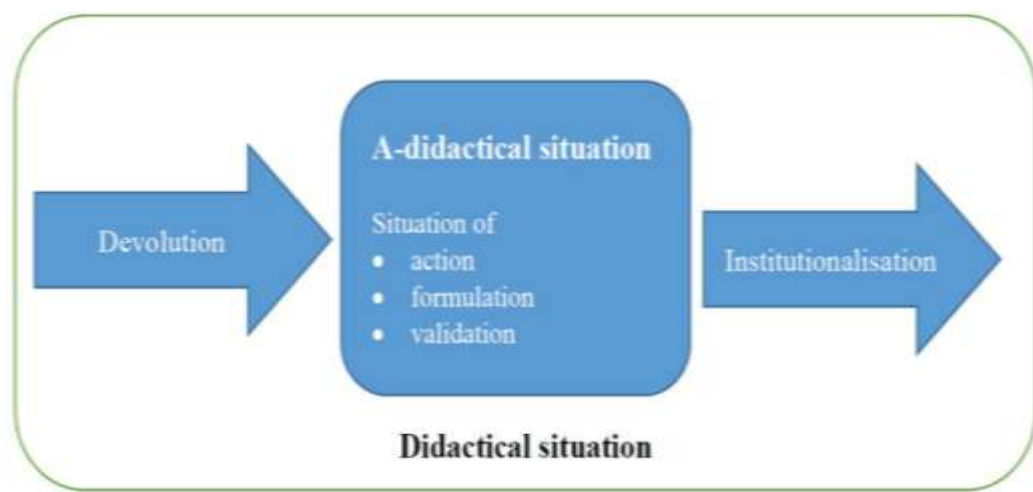
TDS puts forward a fairly complex set of theoretical constructs to articulate what occurs in the lesson and to enable the teacher to better manage learners' transformation geometry learning when using GeoGebra. The paragraphs below briefly describe some of TDS's key concepts used in this study.

The 'milieu', according to Jessen et al. (2023), is defined as "the environment, including problem situations and artifacts to use and manipulate, with which the student (learner) interacts to obtain new (transformation geometry) knowledge" (p. 54). In this study, besides all learners receiving a copy of the transformation geometry worksheet in their groups, the milieu further comprised, *inter alia*, GeoGebra, learners' interactions with fellow learners and the teacher, learners' prior knowledge, classroom arrangement, rules for operating in the situation and measurement tools such as rulers. 'Learning in TDS' is viewed as learners adapting their knowledge to solving the transformation geometry problem (a crucial part of the milieu) using GeoGebra, with the teacher initiating and guiding this interaction (Winsløw et al., 2018). The milieu's function is thus situation-dependant and influenced by its traits.

A situation is defined as the conditions in which learners need to learn transformation geometry concepts (Warfield, 2014) using GeoGebra, and can either be didactical or a-didactical – see Figure 2.2. As per TDS, learners' learning of a particular transformation geometry concept that

the teacher is intending to teach using GeoGebra is modelled according to the construct of DS, where interactions take place between three elements: learners, teacher, and milieu (Clivaz, 2018). For example, when using GeoGebra to consolidate a pen and paper activity, after translating and resizing the object, the teacher poses questions that require learners to establish patterns; hence the interaction between the teacher, learners and the transformation geometry problem on translations – the milieu. Each DS comprises three key stages: devolution, an a-didactical situation and institutionalisation (Novotná & Hošpesová, 2022) – see Figure 2.3.

Figure 2.3 Schema of a DS – adopted from Novotná and Hošpesová (2022, p. 305)



Devolution, a crucial part of a DS, entails putting learners into a relationship with a transformation geometry problem (a crucial part of the milieu), allowing the teacher to partially exclude themselves (Brousseau & Warfield, 2014). For instance, during the implementation phase of the RL, the teacher began by explaining the transformation geometry activity on the worksheet for learners to understand what they were supposed to do, but did not help them on how to do it.

A special kind of DS is the a-didactical. According to Brousseau (2002), in a-didactical situations learners leverage their learning through independent engagement with the proposed transformation geometry problem – a crucial part of the milieu. The learner is directly answering to the transformation geometry problem, and not to the teacher (Clivaz, 2015) – see Figure 2.2. During devolution, the responsibility for discovering and learning transformation geometry concepts is handed over to the learner by the teacher. A-didactical situations in the learning environment basically come in three different phases classified according to their structure as seen in Figure 2.3. These are explained in the paragraphs below.

Within a situation of *action*, learners are interacting with the transformation geometry problem (their milieu) and are implicitly developing a number of approaches (Warfield, 2014).

In the situation of *formulation*, learners are expressing explicitly the implicit ideas that they developed during the action phase (Warfield, 2014). As learners share their hypotheses, they get immediate feedback from the groupmates who understand or do not understand their explanation.

A situation of *validation* is the phase in which learners convince other learners about the correctness of their transformation geometry ideas, previously developed during formulation (Clivaz, 2018; Gök & Ertem-Akbaş, 2019). In the process of validation, the teacher plays the role of a neutral chair in classroom debate, only intervening to organise the debate, drawing attention to possible variations, and encouraging learners to come up with convincing justifications for their proposed approaches (Brousseau, 1997; Genc & Ergen, 2022).

Institutionalisation follows an a-didactical situation – see Figure 2.3. In the institutionalisation stage, the teacher collects transformation geometry knowledge that learners developed in the action, formulation and/or validation phases, and revises, shapes and transfers it into the official mathematics curriculum knowledge (Novotná & Hošpesová, 2022). In other words, the teacher aims to aid learners using GeoGebra's dynamism facility to recognise the transformation geometry knowledge they gained in the context of their a-didactical work and transforming it into knowledge usable in solving other problems. In this study's context, using GeoGebra's dynamism capability, the teacher: asked additional transformation geometry questions with reference to statements made during the validation phase, with the goal of enriching learner's knowledge systems; assigned transformation geometry problems to learners so that they could have a chance to apply the knowledge that they had acquired while solving these problems (Novotná & Hošpesová, 2022); and asked learners to solve a similar transformation geometry problem.

The didactical contract is also among the key constructs of TDS. The contract regulates the interaction between the learners and the teacher in the situation, as well as specifying their respective responsibilities regarding the transformation geometry knowledge they are dealing with (Genc & Ergen, 2022). For example, the teacher may use GeoGebra to apply dynamism to the object, and learners are expected to establish the geometric relationships that exist between the object and its image. The didactical contract is what most affects the transformation geometry knowledge development process (Brousseau, 2004). Therefore,

throughout the transformation geometry teaching and learning process, the teacher can adjust the didactical contract (it is not always stable) through the devolution of the milieu and institutionalisation of the knowledge that is acquired or established (Miyakawa & Winslow, 2009).

2.3.2 Research on TDS and mathematics teaching

In the previous section, the researcher discussed TDS, the theory that acted as a guide in understanding this study – investigating rural high school mathematics teachers' TPACK development when teaching transformation geometry using GeoGebra through a school-based LS CPD approach. This section discusses some evidence of how TDS has been used in research in mathematics education, and how it is used differently or similarly in this study. What distinguishes this study from those in the literature is that no research has ever been done in South Africa using a GeoGebra-based LS as a school-based CPD approach to investigate rural high school mathematics teachers' TPACK development with TDS as a guide.

Firstly, Novotná and Hošpesová's (2022) study investigated how combining concepts from TDS in mathematics with the learner's perspective study data could enrich classroom data analysis – the institutionalisation case. The results revealed that the combination of video recordings, together with interviews with the teacher and the learner pair, the teacher-prepared material, and the productions by learners allowed the researchers to gain a deeper insight of institutionalisation's role in mathematics education. What distinguishes Novotná and Hošpesová's (2022) study from the present study is firstly that Novotná and Hošpesová's (2022) study focused on events in the Czech Republic mathematics classrooms, while teaching linear equations without using technology through the lens of TDS, i.e. about institutionalisation. The current study aimed to investigate rural high school mathematics teachers' TPACK development when using GeoGebra and the LS approach as a form of school-based CPD to teach transformation geometry. Secondly, while TDS was used as a theoretical framework in Novotná and Hošpesová's (2022) study, TDS was used as a guide to understand the present study.

Secondly, Gök and Ertem-Akbaş (2019) examined changes in pre-service elementary teachers' (PST) attitudes towards mathematics while teaching in an environment that had been designed in accordance with a constructivist approach, which differed from the way that they were taught mathematics during their years of elementary and high school. The results of Gök and Ertem-Akbaş's (2019) study showed that the training programme given to PST, which was in line with

TDS, had a positive influence on changing and developing their attitudes towards the courses of mathematics education. Gök and Ertem-Akbaş's (2019) study and the present study differed in the following ways: Firstly, Gök and Ertem-Akbaş's (2019) study aimed to investigate whether the training programme given to PST within the TDS framework in a mathematics education course could change their attitudes towards this course in a teaching process lasting 11 weeks. The present study aimed to investigate rural high school mathematics teachers' TPACK development when using GeoGebra and the LS approach as a form of school-based CPD to teach transformation geometry. Secondly, while Gök and Ertem-Akbaş's (2019) study used TDS as a theoretical framework, this study used TDS as its main theory.

Lastly, Mosimege and Winnaar (2021) carried out a study in South Africa, which analysed the instructional approaches used by teachers in mathematics classrooms, based on TIMSS 2015 questionnaire data. The findings revealed that two instructional approaches – teacher-guided problem solving and teacher-learner interactions – have a significant link to learner performance across algebra, numbers, geometry, and data and chance topics. Although both studies worked with ISTs, what distinguishes the current study from Mosimege and Winnaar's (2021) study is that TDS was used as a theory in the former study and a theoretical framework in the latter.

The next section discusses the theoretical frameworks this study used.

2.4 Theoretical frameworks

The section establishes the theoretical frameworks – TPACK and the LS – that were used to inform data collection and analysis in this study, which investigated rural high school mathematics teachers' TPACK development when using GeoGebra and a school-based LS CPD approach to teach transformation geometry.

According to Kivunja (2018, p. 46), a theoretical framework “is a structure or a data mining lens that uses knowledge from research done to date in your field (mathematics education in this case), to make sense of the data in your own research study.” A theoretical framework allows the researcher to develop an informed lens to:

- collect, examine and analyse data
- interpret and discuss the findings in the research data
- make recommendations and conclusions to the study

- add academic rigour and skills – through cross-referencing between your own research findings and existing findings from other studies (Kivunja, 2018).

It has also been asserted that theoretical “frameworks play a pertinent role by guiding the kinds of research questions researchers can ask, the nature of evidence that is to be collected, the methodologies that are appropriate for collecting this evidence, the strategies available for analysing the data and finally [the] interpretations we make from this analysis” (Mishra & Koehler, 2006, p. 1039).

According to Upadhyay (2015), the researcher can synthesise more than one theoretical perspective or concepts resident within more than one theory, to develop a theoretical background that guides a study. In this study, the operationalisation of the constructs under investigation were guided by the synthesis of two main theoretical perspectives: the LS and Niess et al.’s (2009) TPACK development model. These theoretical frameworks – TPACK and the LS – include a focus on learner thinking to plan, implement, and analyse GeoGebra transformation geometry RLs. Hence, the two theoretical frameworks were inextricably interwoven and interdependent when used to investigate the TPACK development of rural high school mathematics teachers when using GeoGebra and the LS approach as a form of school-based CPD to teach transformation geometry.

2.4.1 TPACK Framework

This section begins with a discussion of TPACK, a teacher knowledge framework that outlines the expertise required for effective teaching using GeoGebra. Following this is a discussion of how some studies have assessed mathematics teachers’ TPACK and the researcher’s intended direction for structuring the assessment and/or measurement of TPACK in this study. Lastly, the TPACK development model is discussed.

While there are alternative theories related to the use of technology in education, the study chose TPACK. The higher TPACK levels (exploring and advancing) prioritise learner transformation geometry thinking. These levels require planning, implementing/observing and reflecting on GeoGebra-based transformation geometry RLs with the aim of promoting learners’ thinking and conceptual understanding of this content (Niess et al., 2009). These practices are integral to the LS CPD approach, which aims to enhance transformation geometry instruction with GeoGebra in this study.

2.4.1.1 TPACK

The focus of this section is on a teacher knowledge framework called TPACK – one of the theoretical frameworks the researcher chose to use in this study – that presents a structure for thinking about and understanding the knowledge needed by teachers to teach transformation geometry with GeoGebra (Niess, 2013).

Technology integration into teaching has become a critical expectation as teachers develop their technological knowledge with respect to specific content (transformation geometry, in this case) and for use in the teaching process (Aldemir Engin et al., 2023; Niess, 2011). This type of knowledge, derived from integrating technological knowledge into the other types of knowledge that teachers need for teaching in the digital age, has been identified as TPACK (Angeli & Valanides, 2009). TPACK is a dynamic framework that describes the knowledge that teachers must rely on in designing, implementing and evaluating transformation geometry instruction at the Senior Phase in the South African mathematics curriculum, while guiding learners' learning with GeoGebra (Niess, 2012).

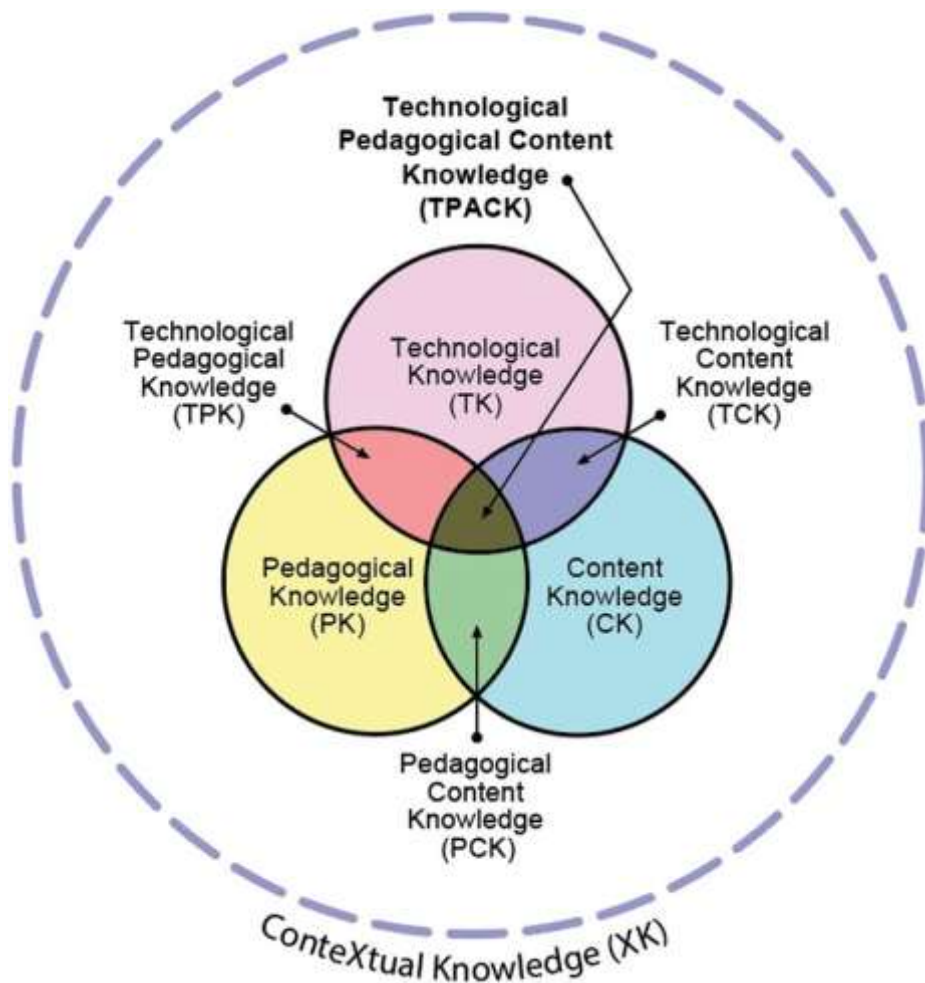
The TPACK framework according to Tan et al. (2019) is grounded in the idea that effective GeoGebra integration in transformation geometry teaching requires a thoughtful blend of transformation geometry content, instructional strategies, and GeoGebra. This argument was further supported by M. Koehler et al. (2013), who emphasise that for teachers to efficiently incorporate GeoGebra into their transformation geometry lessons, their TPACK must be developed. TPACK builds on pedagogical content knowledge (Shulman, 1987) with the addition of technological knowledge to explain effective transformation geometry teaching using GeoGebra (Zelkowski et al., 2013). However, Niess (2012) warned that meaningful integration of GeoGebra requires teachers not only to be skilful and knowledgeable with the software but also to know how to use it to teach this content. Other studies affirm that for effective transformation geometry instruction and engaging learners with diverse needs, teachers must know ways of teaching this content with GeoGebra (Alemdag et al., 2020), hence the importance of understanding the way the three knowledge domains – transformation geometry content, instructional strategies and GeoGebra – are woven together. TPACK is therefore a total package required for integrating the knowledge of GeoGebra, instructional strategies and transformation geometry content to design instructions for thinking and learning this content with this software (Niess et al., 2009). However, the challenge is that knowledge of GeoGebra is often seen as separate from knowledge of instructional strategies and

transformation geometry content, a treatment similar to the problem identified in the 1980s by Shulman (Mishra & Koehler, 2006). Similarly, Jiménez Sierra et al. (2023) contended that most technology-related CPD to date is techno-centric – the design and features of technology are (over) emphasised, rather than the learning that it can support.

However, Hughes (2005) pointed out that GeoGebra knowledge is context-dependent, and that good (transformation geometry) teaching involves understanding the software's relationship with instructional approaches and transformation geometry content. The benefit of incorporating a new educational technology (GeoGebra, in this case) for (transformation geometry) teaching is that it drives teachers to confront educational issues that are basic, because GeoGebra forces them to deal with all three factors – GeoGebra, instructional strategies, and transformation geometry content – and their relationships (Mishra & Koehler, 2006). Within this study's context, GeoGebra provided a learning milieu in which transformation geometry content and instructional strategies were inextricably linked in the transformation geometry RL stages of identification of objectives, planning, implementing/observing, and reflecting (see Section 3.5.3.3).

M. J. Koehler and Mishra (2009) stated that effective integration of technology in teaching hinges on teachers' mastery of three key knowledge areas: (transformation geometry) content, pedagogy (instructional strategies), and technology (GeoGebra), plus the four relationships between them – see Figure 2.4. This is the knowledge which teachers utilise in planning, implementing, and reflecting on transformation geometry content and teaching, as they guide learners' thinking and learning with GeoGebra (Niess, 2013).

Figure 2.4 Revised version of the TPACK image – adopted from Mishra (2019, p. 17)



Content knowledge (CK) is the knowledge of a teacher’s understanding of transformation geometry content (Mishra & Koehler, 2006).

Pedagogical knowledge (PK) is defined by M. Koehler et al. (2013, p. 15) as “teachers’ deep knowledge about the processes and practices or methods of (transformation geometry) teaching and learning”. In this study, PK includes RL plan development, teachers’ instruction methods, the assessment of learner transformation geometry learning, and the skills of classroom management.

Technology knowledge (TK) according to Schmidt et al. (2009, p, 125) is the “the knowledge about various technologies, ranging from low-tech technologies such as pencil and paper to digital technologies such as the internet, digital videos, interactive whiteboards, and software programs”. In the current study, TK encompasses teachers’ GeoGebra use in transformation geometry teaching and learning.

Pedagogical Content Knowledge (PCK) refers to how transformation geometry content can be represented and structured to facilitate learner understanding (Shulman, 1986; 1987). In other words, it is an interface between PK and transformation geometry content (CK). In this study, PCK relates to how teachers use their PK to teach transformation geometry content to their learners; indeed, it refers to participating teachers' representation of transformation geometry concepts to their learners using GeoGebra.

Technological Content Knowledge (TCK) is the knowledge of a teacher's understanding of how to use GeoGebra to teach transformation geometry (Mishra & Koehler, 2006). More specifically, in this study TCK encompasses teachers' understanding of how GeoGebra use influences learners' understanding and practice of transformation geometry.

Technological Pedagogical Knowledge (TPK) is defined as the "knowledge of the existence, components and capabilities of various technologies (GeoGebra, in this case) as they are used in teaching and learning settings, and conversely, knowing how teaching might change as the result of using particular technologies" (Mishra & Koehler, 2006, p. 1028). TPK in this study involves a teacher's skill in applying GeoGebra in transformation geometry activities prepared during the planning session, and their ability to implement their strategies (designed during planning) of using the software for transformation geometry teaching.

Technological pedagogical content knowledge (TPACK) is "a way of thinking about the complex relationships between technology (GeoGebra), pedagogy (instructional strategies), and content in a specific context (transformation geometry), which is represented through the carefully considered implementation of technology in a classroom setting in order to help students (learners) better understand a particular topic (transformation geometry)" (Cox, 2008, pp. 50-51).

Contextual Knowledge (XK) refers to the teachers' understanding of their contexts, and this was formerly labelled as 'Contexts' (Mishra, 2019). XK's inclusion in the diagram in Figure 2.4 emphasises the organisational and situational challenges that teachers experience at their individual schools (Mishra, 2019) in their attempts at technology (GeoGebra) integration. Mishra (2019) further asserted that "the success of their (teachers') efforts depends not as much on their knowledge of T, P, C and its overlaps, but rather on their knowledge of the context" (p. 77). Thus, this study employed the LS approach as a form of school-based CPD to afford teachers an opportunity of seeing transformation geometry teaching and learning, as it occurs

instantaneously in their schools and classroom contexts (Mhakure, 2019) to develop their TPACK.

According to Schmidt et al. (2009), the TPACK framework aims to design and evaluate teachers' knowledge, which is focused on effective learner learning in different mathematics topics, particularly transformation geometry. This framework therefore provides a valuable lens to help in understanding the required knowledge for successful GeoGebra incorporation into the transformation geometry lessons.

In the section that follows, the researcher presents the different assessment instruments that researchers have used in assessing PST' TPACK. The advantage of using instruments that have been tested already is that they increase the validity of the findings.

2.4.1.2 Measuring and assessing TPACK

The preceding section discussed TPACK, a model for understanding teacher knowledge needed to effectively teach transformation geometry using GeoGebra. This section starts with a discussion of how some studies in mathematics education have assessed mathematics teachers' TPACK. The section ends with a discussion of the direction in which the researcher intended to structure the assessment and/or measurement of TPACK in this study.

Various researchers have reviewed the literature on measuring TPACK, and acknowledge that a variety of assessment tools have been developed by researchers to evaluate teachers' TPACK (Abbitt, 2011; M. J. Koehler et al., 2012). M. J. Koehler et al. (2012) claimed that "little effort has been made to provide a comprehensive account of TPACK measures in a systematic manner" (p. 17), categorising measures of teachers' TPACK "into five types (i.e. self-report measures, open-end questionnaires, performance assessments, interviews, and observations)" (p. 18). Researchers predominantly use self-reports to measure TPACK. Self-reported data are often collected using surveys because they are easy and budget-friendly to administer, and reach large numbers of teachers (Lyublinskaya & Tournaki, 2014).

Tournaki and Lyublinskaya's (2014) study investigated PST' TPACK development via a course utilising Schmidt et al.'s (2009) survey of PST' knowledge of teaching and technology. The results revealed that upon completion of the course requirements, teachers had significant improvements in their TPACK from the pre- to post-test. Besides that study, M. J. Koehler and Mishra (2005) designed an online survey tool to assess student and faculty participants' TPACK perceptions after they worked together to design online courses. The results showed that there

was significant improvement in the participants' TPACK, both as individuals and as a group. Furthermore, Zelkowski et al. (2013) reported findings from over 300 surveys completed by secondary mathematics PST at 15 institutions. The findings suggested that TPACK, TK, CK, and PK constructs were valid and reliable, while PCK, TCK and TPK constructs remained challenging for PST to separate and self-report.

Apart from surveys, researchers have also used teaching artifacts to collect data for measuring teachers' TPACK including but not limited to transcripts of planning and implementation sessions (Lyublinskaya & Tournaki, 2014). M. Koehler et al. (2007) utilised techniques of discourse analysis in coding group discussion excerpts in a design-based seminar where faculty and graduate students collaborated on developing online courses. Quantitative findings indicated a shift towards a more holistic understanding of TPACK in both design teams. Qualitative analyses revealed similarities and differences between both design teams at various levels, indicating that TPACK development is a multi-layered process. M. Koehler et al. (2007) acknowledged limitations, such as coding bias and subjective analysis. Harris et al. (2010) designed a valid and reliable rubric to assist teacher educators in assessing teachers' TPACK through evaluating their teaching artifacts, including but not limited to RL plans. This valid instrument measured TCK, TPK and TPACK.

The TPACK assessment literature discussed above revealed that most researchers have structured their assessment of TPACK using self-assessment measures of TPACK through surveys. However, Lawless and Pellegrino (2007) warned that when relying on self-reports in assessing a construct, there is a potential disconnect between the teacher's self-reported perceptions and their actual teaching methods. Similarly, Saralar et al. (2017) asserted that while a survey gives a broader overview of teachers' TPACK, it fails to provide detailed insight into measuring TPACK in a specific topic (e.g. utilising GeoGebra to teach transformation geometry).

Due to these weaknesses of self-reported data, Lyublinskaya and Tournaki (2010) asserted that external evaluation of those practices and related artifacts, corroborated by teacher self-reports, could provide deeper insights into the essence of teachers' TPACK. Thus, in this study, transcripts of videos of training sessions, video-recorded RL plan meetings, classroom observations and recordings of implemented RLs provided data related to practice that minimised self-reporting weaknesses from observation, reflection and interview guides.

Given the lack of assessments tailored for ISTs, a specialised model was required to evaluate their TPACK development. Niess et al. (2009) specifically designed a mathematics teachers' TPACK development framework, which they employed to assess TPACK progression among PST in teaching mathematics with Excel spreadsheets. Mishra and Koehler (2009) noted that knowing teachers' TPACK levels is vital to evaluating technology-enhanced teaching effectiveness. Hence, this study adapted the TPACK rubric of Niess et al. (2009) to design FGI guides (that included reflection guides), and used the instrument to measure ISTs' TPACK levels by coding teachers' reflections on the four RL stages: identification of transformation geometry objectives, planning, implementing/observing, and reflecting.

2.4.1.3 TPACK development model

The previous section started by discussing how some studies in mathematics education have assessed mathematics teachers' TPACK, before discussing the researcher's intended direction for structuring the TPACK assessment in this study. This section demystifies the adapted TPACK assessment model (Niess et al., 2009) used to investigate how GeoGebra and the LS approach as a form of school-based CPD impacted on rural mathematics teachers' TPACK in transformation geometry.

Niess (2005) pointed out that there are four themes – 'curriculum and assessment', 'learning', 'teaching' and 'access' – that aid a researcher to assess teachers' TPACK. According to Niess (2005), all the TPACK subcomponents (seven in total – CK, PK, TK, PCK, TCK, TPK and TPACK) are collated in these four themes. In other words, the four themes are a combination of all the seven TPACK components discussed in Section 2.4.1.1. An explanation of the four themes follows, illustrating how CK, TK and PK culminate in comprehensive TPACK levels.

Curriculum and Assessment is defined as the knowledge of transformation geometry concepts and assessment of these concepts that needs to be known by the teacher to effectively incorporate GeoGebra into their teaching. This theme also includes teachers' GeoGebra knowledge required to assess learners' conceptual grasp of transformation geometry, beyond just curriculum content – transformation geometry in this case. According to Niess (2008), the combination of TPACK components of CK and TCK creates a curriculum and assessment theme.

Learning refers to a teacher's awareness of learners' understanding, thinking and common misconceptions when using GeoGebra to learn transformation geometry. Säljö described learning as "the increase in (transformation geometry) knowledge (through assessing); memorising; the acquisition of facts, procedures etc., which can be retained and/or utilised in practice; the abstraction of meaning; and an interpretative process aimed at the understanding of reality" (1979, cited in Purdie & Hattie, 2002, p. 17). Apart from encompassing a teacher's conception and understanding of learners' transformation geometry learning and how they learn and understand this content, the theme also includes the representations of transformation geometry concepts with GeoGebra. The learning theme according to Niess (2008) encompasses TPACK components of PK, PCK, and TPK.

Teaching encompasses a deep understanding of how to facilitate learner learning in transformation geometry using GeoGebra. In other words, it refers to knowledge of transformation geometry teaching using GeoGebra to make this content easy for learners to learn. The theme encompasses a teacher's understanding of transformation geometry content, key learning outcomes, and GeoGebra's role in supporting learner learning (Niess, 2007). In addition, the teaching theme informs instructional choices, prioritising effective GeoGebra integration to enhance learner access to transformation geometry concepts. The TPACK components of CK, PK, PCK and TPK are integrated into the teaching dimension (Niess, 2008).

Access refers to knowing effective "instructional strategies and representations for teaching" (Saralar, 2016, p. 8) transformation geometry using GeoGebra. The theme encompasses the teacher's understanding of how to provide learners with access to GeoGebra (Niess, 2005). The teaching theme, according to Niess (2008), consists of TK, TPK and TCK of TPACK. Table 2.1 summarises the themes – curriculum and assessment, learning, teaching and access – as well as their descriptions.

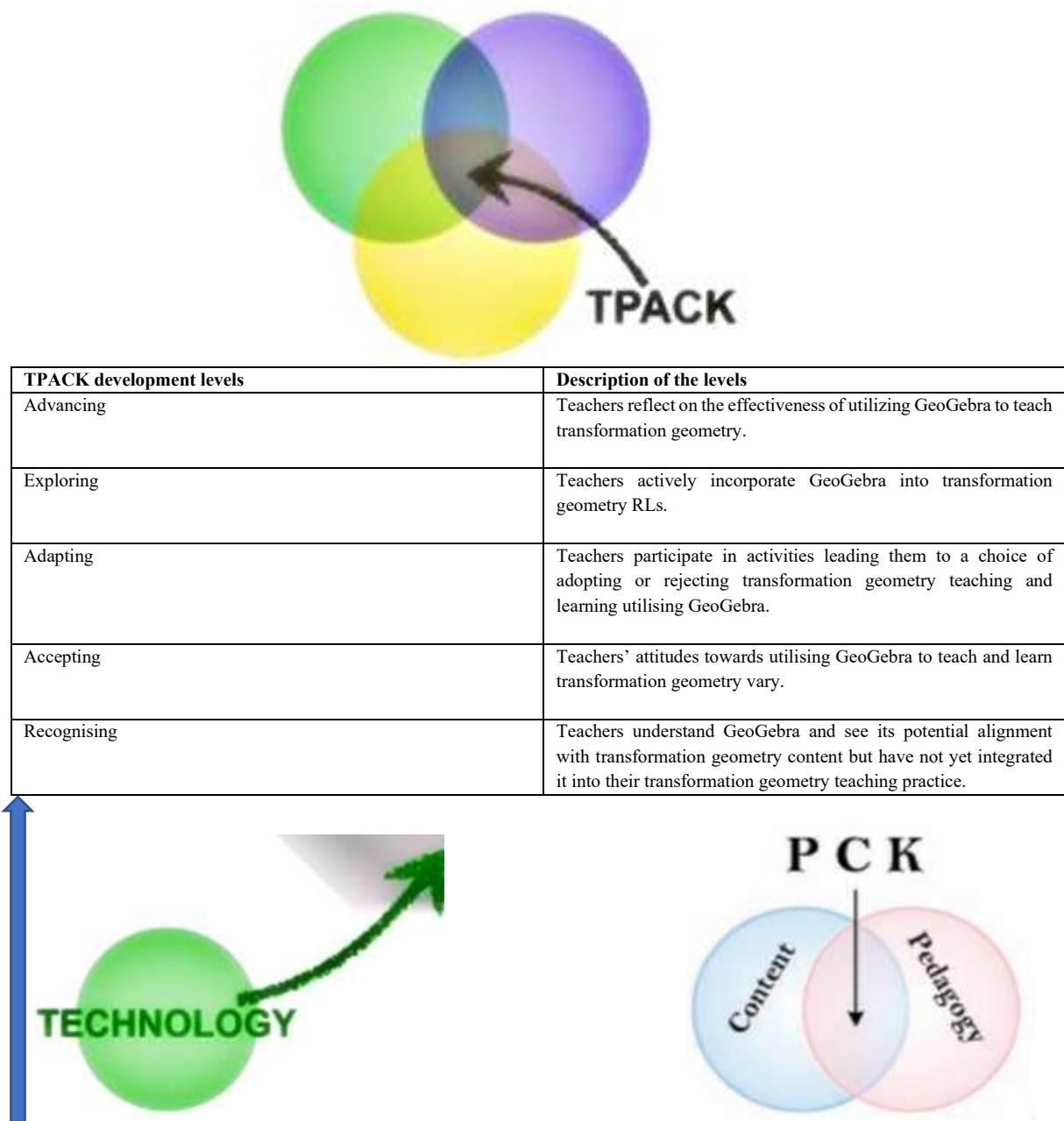
Table 2.1 Descriptors for Major Themes in the Mathematics Teacher TPACK Development Model (Niess, 2008, p. 11)

Theme	Descriptor
Curriculum and Assessment	- Curriculum – the treatment of transformation geometry content on reflection and translation. - Assessment – assessing learners’ transformation geometry understandings using GeoGebra.
Learning	- Focus on subject matter – i.e. learning of transformation geometry. - Demonstration of conceptions of how learners learn transformation geometry – i.e. development of learners’ transformation geometry thinking skills.
Teaching	- Focus on subject matter – i.e. learning of transformation geometry topic. - Instructional approaches. - Classroom environment. - Professional development.
Access	- Usage – whether learners are allowed to use GeoGebra. - Barriers – how teachers address the factors hindering GeoGebra integration. - Availability – how GeoGebra makes higher levels and more transformation geometry concepts available for investigation for greater numbers of more diverse learners.

Subsequently, Niess et al. (2009) presented a framework describing mathematics teachers’ progression in TPACK. In other words, this model illustrates mathematics teachers’ progression in TPACK in integrating GeoGebra to teach transformation geometry. Niess et al.’s (2009) TPACK framework was framed around four major themes in Table 2.1. According to the TPACK development model (Figure 2.5), TPACK develops as teachers progress through phases (recognising, accepting, adapting, exploring and advancing) for each theme (see Table 2.1), building on their existing PCK, when integrating new technology (Riales, 2011) – GeoGebra, in this case, in teaching transformation geometry. As teachers progress through the TPACK developmental model, their GeoGebra knowledge (TK) integrates with instructional strategies used (PK) and transformation geometry content (CK), forming a comprehensive TPACK framework (Niess et al., 2009) – a knowledge base where, in this study’s context, teachers actively engage in guiding learner transformation geometry learning with GeoGebra.

Teachers’ TPACK development for teaching transformation geometry using GeoGebra involves a cognitive growth process (Niess, 2011). Figure 2.5 illustrates the levels of teacher knowledge development, integrating multiple knowledge bases: GeoGebra, transformation geometry content, and instructional strategies (Niess et al., 2009). The following paragraphs elaborate on each of the five TPACK development levels.

Figure 2.5 A contextualised description of the TPACK development model – adapted from Niess et al. (2009, p. 10)



Recognising level: At this level, teachers know about GeoGebra. However, this software is not integrated into their transformation geometry lessons because they believe that the software may hinder learners' transformation geometry learning, thinking or reasoning skills (Aldemir Engin et al., 2023).

Accepting (persuasion) level. Teachers are aware of GeoGebra and practise with it; but the software's transformation geometry teaching potential is not yet a consistent consideration (Lyublinskaya & Tournaki, 2014). In other words, teachers find difficulty in transiting from traditional to GeoGebra-integrated approaches in transformation geometry teaching. Additionally, accepting teachers worry that GeoGebra use might hinder the development of learners' transformation geometry thinking skills. So, in this study, learners worked on the paper and pen activity first for the teachers to identify their strengths and weaknesses; then the software was used to consolidate the same activity. Teachers may also have a concern that learners' attention may be diverted from transformation geometry content to a focus on GeoGebra. As a result, they may limit GeoGebra usage, particularly when introducing and developing key transformation geometry concepts. At the accepting level, classroom management concerns also arise with the use of GeoGebra, which forces use of teacher-centred and teacher-led instruction (Aldemir Engin et al., 2023).

Adapting (decision) level. At this level, teachers begin to demonstrate an understanding of the GeoGebra's potential for enhancing transformation geometry teaching and learning. They start to engage in exploring and experimenting with GeoGebra integration as a learning tool, weighing adoption or rejection (Saralar et al., 2017). While considering GeoGebra useful, teachers who are adapting worry about learners' transformation geometry skills. They try ideas for integrating GeoGebra into transformation geometry teaching; but in teaching learners, at best they tell learners to use drills and practices of the ideas with the software (Lyublinskaya & Tournaki, 2014). Therefore, GeoGebra supplements teacher-led transformation geometry instruction.

Exploring level. Teachers at this level actively incorporate GeoGebra into transformation geometry lessons (Niess et al., 2009). They use the software to foster critical thinking through engaging learners in interactive transformation geometry explorations. Guided by learner thinking, exploring teachers "plan, implement and reflect on (transformation geometry) teaching and learning" (Niess et al., 2009, p.12) using GeoGebra. Furthermore, these teachers seek and demonstrate new approaches to transformation geometry instruction using GeoGebra

(Niess, 2012). They are also more willing to let learners explore transformation geometry concepts using GeoGebra via learner-led learning, while acting as guides.

Advancing level. Teachers at this level willingly use GeoGebra in various ways to teach transformation geometry, and in doing so, they demonstrate an integrated transformation of their transformation geometry content, pedagogy, and GeoGebra knowledge (Niess, 2013). These teachers use GeoGebra-enhanced instruction to engage learners in higher-order thinking and self-directed transformation geometry learning, enabling them to achieve sophisticated understanding (Aldemir Engin et al., 2023). Additionally, advancing teachers reflect on GeoGebra's effectiveness for teaching transformation geometry. They are also committed to thoughtful planning, teaching and reflection to enhance learners' transformation geometry thinking and understanding using GeoGebra (Niess, 2013).

Niess et al.'s (2009) TPACK rubric offers a comprehensive approach to evaluating mathematics teachers' TPACK, focusing on four distinct themes (see Table 2.1) when using GeoGebra to teach transformation geometry. Each of these themes comprises a number of descriptors (see Appendix 1). The TPACK levels can be utilised in evaluating teachers' expertise in integrating GeoGebra, pedagogy, and transformation geometry content for each theme's descriptors. Each theme's descriptors are elaborated across five TPACK levels, with specific mathematics examples illustrating each level in the TPACK model (see Appendix 1). Appendix 1 presents the TPACK levels rubric from Niess et al. (2009) used in this research. Using a validated instrument increases the results' validity and reliability. Niess et al.'s (2009) rubric was slightly modified to suit this research's context. An example of this adaptation is changing the term 'mathematics' to 'transformation geometry' and the term 'technology' to 'GeoGebra' in the sentence that follows: "An IST engages learners in high-level thinking activities (such as project-based and problem solving and decision-making activities) for learning transformation geometry using GeoGebra as a learning tool." (Niess et al., 2009, pp. 22-23). This adapted rubric provides detailed actions that ISTs can take when integrating GeoGebra to support transformation geometry instruction and learning. Using the themes of Niess et al. (2009) as a base, this study investigated rural high school mathematics teachers' TPACK development when using GeoGebra and the LS approach as a form of school-based CPD to teach transformation geometry.

The TPACK framework illustrates a teacher's progression from recognising GeoGebra's potential for achieving transformation geometry objectives to adopting the software's use,

modifying transformation geometry RLs to leveraging GeoGebra, actively incorporating GeoGebra into transformation geometry teaching, and advancing the teaching and learning of this content using the software. The higher TPACK levels (exploring and advancing) prioritise learner transformation geometry thinking. These levels require planning, implementing/observing and reflecting on GeoGebra-based transformation geometry RLs with the aim of promoting learners' thinking and conceptual understanding of this content. These practices are integral to the LS CPD approach, which aims to enhance transformation geometry instruction with GeoGebra.

2.4.2 The LS framework

The previous section discussed the TPACK development model, highlighting that its more advanced stages – exploring and advancing – require teachers' planning, implementing/observing, and reflecting on GeoGebra-based transformation geometry RLs; hence the phases of the LS. This section firstly unpacks the construct of the LS – what is the LS? Secondly, it discusses how the LS has influenced mathematics education, before discussing its relevance to this study.

“The graveyards of US [*South African*] educational reform are littered with once-promising innovations that were poorly understood, superficially implemented, and consequently pronounced ineffective. If lesson study (LS) is to be any different, it will require a deep understanding of what it is and why it has been useful to Japanese teachers, and how it can be adapted to the very different setting of the US [*South Africa*].” (Lewis, 2004, p. 134)

2.4.2.1 What is the LS?

This section unpacks the construct of LS, which is used as both a CPD approach and a theoretical framework in the main study. The LS is the second of the two theoretical frameworks adopted for this study, which investigated rural high school mathematics teachers' TPACK development when engaged in a CPD that used GeoGebra to teach transformation geometry.

Lesson study, a CPD approach, gained popularity in Japan during the 1960s, but its roots actually stretch back to the 19th century in Japan (Ono & Ferreira, 2010). LS has been used in Japanese schools for more than 40 years, but it only gained international recognition in the 1990s through the TIMSS results and video evidence (Stiegler & Hiebert, 1999). LS represents

a new approach in teacher development within South Africa, mirroring global trends. Ono and Ferreira (2010) documented the events that led to LS's inception in South Africa. The growing interest in LS in South Africa can be linked to the period between 1999 and 2006, driven by initiatives from Japan International Co-operation Agency (JICA). The implementation of LS in South African schools experienced a period of stagnation, likely unforeseen, due to obstacles like curriculum reform processes (Ono & Ferreira, 2010). The revival and implementation of LS in numerous public schools across four South African provinces, including two with large learner and teacher populations, was evident from 2011 onwards, resulting from a partnership between the DBE and JICA (Sekao & Engelbrecht, 2022). The outcome led to a more organised method of utilising LS as a model for mathematics teachers' development.

What is lesson study? LS as a CPD form for mathematics teachers is defined as a method

“... in which teachers work together to: formulate goals for learner learning and long-term development; collaboratively plan a ‘research lesson’ designed to bring to life these goals; conduct the lesson in a classroom, with a team member teaching and others gathering evidence on learner learning and development; and discuss the evidence gathered during the lesson, using it to improve the lesson, the unit, and instruction more generally.” (Lewis, 2009, p. 95)

This study used the LS as a form of school-based CPD to develop the TPACK of rural high school teachers when utilising GeoGebra to teach the transformation geometry concepts of reflection and translation at the Senior Phase in the South African mathematics curriculum. School-based CPD using the LS through the construct of a RL, is a possible solution for developing TPACK in rural mathematics ISTs. A RL, central to the LS process, is a lesson that is a subject of research and is examined through the stages of the LS framework (Clivaz, 2018). In other words, a RL is a jointly planned, taught or observed and reflected-on lesson by the LS group of teachers. Therefore, the LS framework that this study used when teaching transformation geometry using GeoGebra was implemented on a RL. The LS framework that this study used has four phases: identification of (transformation geometry) objectives, planning, implementing/observing, and reflection (Mhakure, 2019), for the purpose of developing ISTs' TPACK by engaging them in a school-based LS CPD approach using GeoGebra to teach transformation geometry. Figure 2.6 portrays these phases, including the activities that rural mathematics teachers enter into in each phase and how these activities contribute to their TPACK development. The RL plans utilised in this study were created by the researcher, and critiqued by teachers according to their learners' learning needs and the

curriculum in their respective schools, and used in the teachers' TPACK development when using GeoGebra to teach transformation geometry.

Figure 2.6 Contextualised description of the Phases of a RL – adapted from Mhakure (2019, p.

6)

Phases of a RL	Teachers' and the knowledgeable other's (KO) co-learning activities from the transformation geometry RL	Outcomes
1. Identification of objectives	The LS group of teachers discuss objectives of the transformation geometry RL within the context of learners' learning needs and the curriculum; the team identifies a research question that they want to investigate to deepen their knowledge about learners' learning on the transformation geometry concept under discussion.	Teachers' school-based CPD using GeoGebra on transformation geometry leads to improvement in: teaching and assessing learners' conceptual understanding of transformation geometry concepts; facilitation of the development of learners' conceptual knowledge of transformation geometry thinking skills; creation of new instructional approaches and a classroom environment that promotes the learning of transformation geometry concepts; addressing factors hindering GeoGebra integration, and making higher levels of transformation geometry concepts available for investigation to diverse learners.
	Between phases 1 & 2: Teachers and the KO brainstorm and research transformation geometry concepts, and think about the concepts that are likely to present conceptual challenges to learners.	
2. Planning	Developing a plan for the transformation geometry RL using GeoGebra; deciding on the approach to facilitate this RL – using a worksheet first, then GeoGebra, to solve the same transformation geometry problem – and its justification; teachers learn how to utilise GeoGebra to: teach and assess learners' understanding of transformation geometry concepts, facilitate the development of learners' conceptual knowledge of transformation geometry thinking skills, create new instructional approaches and a classroom environment that promotes the learning of transformation geometry concepts, address factors preventing GeoGebra integration in mathematics instruction, and make complex transformation geometry concepts explorable for diverse learners; deciding on the data collection strategies (observation, reflection and interview guides); anticipating and responding to learners' transformation geometry conceptual challenges using case learners when using pen and paper and when using GeoGebra.	
	Between phases 2 & 3: Performing trial runs of the data collection methodology – the three transformation geometry RLs, observation, reflection, and interview guides; acclimatizing learners to the various data collection methods – worksheets and video-recording; familiarising teachers with the data collection instruments – observation, reflection, and interview guides.	
3. Implementing	One teacher facilitates the transformation geometry RL using GeoGebra; the other teachers and the KO observe and collect data on learner transformation geometry learning using a pre-determined research instrument – an observational guide, in this case; data is also gathered using audio and video-recordings; and samples of learners' work are collected.	
	Between phases 3 & 4: Teachers complete a reflection guide to rate their GeoGebra competence in teaching transformation geometry.	
4. Reflection	The LS group of teachers reconvene in a reflection session for the researcher to follow up on their scorings from the reflection guide; the teachers draw from the four RL stages – identification of objectives, planning, implementing/observation and reflection – as they reflect on their scorings. More information is acquired about any unclear answers on teachers' ratings, and the relevant TPACK levels are adjusted; the KO reflects on the four phases of each RL; discussions relevant to the transformation geometry RL itself take place; and the LS group also discusses the three case learners.	
	Between phases 1 & 4: Evidence collected is used to inform decisions on whether or not to re-teach the transformation geometry RL; whether revisions to the RL are needed to improve learners' accessibility of the next RL; and whether innovative approaches to deepen transformation geometry learning should be executed.	

Firstly, in the *identification of objectives* phase, rural high school mathematics teachers at schools A and B meet formally in their respective schools to discuss their learners' learning needs and the curriculum (Jiménez Sierra et al., 2023) – the Senior Phase transformation geometry topic, in this case. Table 2.1 portrays the objectives for the three transformation geometry RLs, which focused on the concepts of reflection about the x- and y-axis, the $y = x$ line and translation.

Table 2.2 A summary of the transformation geometry objectives for the three RLs

RL	Transformation geometry objectives
One	- Identify which coordinate changed in sign when reflection is done about either the x- or the y-axis - Show that the size of the object and its image are congruent. - Show that the corresponding vertices of the object and its image are equidistant from the line of reflection.
Two	- Identify how the x- and y-coordinates changed after a reflection about the line $y = x$. - Show that the object and its image are congruent. - Show that the object and its image are equidistant from the line of reflection.
Three	- Show that a translation produces congruent objects (identical in size and shape). - Show that the distances between the coordinates of the corresponding vertices of the object and its image are equal.

With guidance from the Senior Phase CAPS document, mathematics teachers concurred with the transformation geometry objectives of the three RLs (see Table 2.2). Additionally, in this phase, the teachers identified a research question they wanted to investigate to learn and deepen their knowledge about the learners' learning on the transformation geometry concepts (Fujii, 2014; Lewis & Hurd, 2011). The LS in this study started from the teachers' experience that their Senior Phase learners tended to have difficulties with the geometric relationships that exist between an object and its image in transformation geometry (DBE, 2014; Mullis et al., 2020; Rollick, 2009); hence, each RL's research question addressed the different geometric relationships that can be observed between the object and its image in transformation geometry. Table 2.3 lists the research questions for each of the three transformation geometry RLs.

Table 2.3 Research questions for each of the three RLs

RL	Research question
One	What are the different geometric relationships that can be observed between the object and its image reflecting on the x- and y-axis on the Cartesian plane?
Two	What are the different geometric relationships that can be observed between the object and its image reflecting on the line $y = x$ on the Cartesian plane?
Three	What are the different geometric relationships that can be observed between the object and its image when carrying out translations on the Cartesian plane?

Between the identification of objectives and planning phases, the LS group of teachers (including the KO) participated in brainstorming activities focusing on key transformation geometry concepts and possible learner conceptual learning hurdles.

Secondly, in the *planning phase*, using the selected goals as a guide, teachers (with the aid of GeoGebra) developed a transformation geometry RL plan justifying the chosen instructional approach that was informed by the need to address learners' learning needs. In this study, the teaching approach agreed on in each RL plan involved using a worksheet first, followed by using GeoGebra to solve the same transformation geometry problem. The argument was that since transformation geometry is part of the Grade 8 curriculum, which was covered in the previous year, the worksheets were to be used to ascertain learners' prior learning, and GeoGebra was then to be implemented to build from and make connections to that knowledge (Kilpatrick et al., 2001). During the planning stage, teachers learned how to utilise GeoGebra to: teach and assess learners' conceptual understanding of transformation geometry concepts of reflection and translation in the South African Senior Phase mathematics curriculum; foster the development of learners' conceptual knowledge of transformation geometry thinking skills; create new instructional approaches and a classroom environment, which fosters and facilitates the learning of transformation geometry concepts; address factors hindering its integration; and accommodate more learners in exploring complex and additional transformation geometry concepts.

Given the evidence-based nature of RLs, teachers and researchers decide the data collection methods (Jiménez Sierra et al., 2023). On this basis, plans of what was to be observed and reflected on during and after teaching the transformation geometry RL with GeoGebra were incorporated into the RL plan by the researcher, and the observation was linked to the RL's research question (Fujii, 2014). This study's observation guides were used to collect data on learners' transformation geometry learning, first using a worksheet and then GeoGebra. The interview guides (including reflection guides) were used to collect data on mathematics ISTs' TPACK levels in the five TPACK level descriptors throughout the LS phases alluded to earlier. In addition, part of this phase's discussions focused on anticipating and responding to learners' transformation geometry conceptual challenges, using 'case learners' first with a worksheet, then with GeoGebra.

Drawing from Mhakure (2019), and chosen based on this study's context, 'case learners' are three learners known to the LS group of teachers, around whom the transformation geometry RL is developed using GeoGebra. That is to say, the planning phase focuses on specific identified learners in a transformation geometry RL, one in each of the three categories – low, medium and top achievers – and a RL is created around their learning needs and the curriculum utilising GeoGebra. The idea behind this is that, as the LS group of teachers design the transformation geometry RL, they must predict each of the case learners' interactions – and by extension, those of other learners in the classroom – during each RL stage (Cajkler et al., 2014). Dudley (2015) warns that teachers frequently misidentify case learners.

Between the RL planning and implementation phases, the researcher finalised his data collection strategies by piloting the data collection methodology (the three RLs, observation and interview guides including reflection guides – see Section 3.5.3) to improve the quality of the instruments, and the methodology itself.

Thirdly, following the planning stage is *implementation*. In this stage, a volunteer teacher implemented the collaboratively developed transformation geometry RL, while other teachers and the KO observed, collecting data on learner learning using a pre-determined observation guide (Perry & Lewis, 2009). Learner observation is key to the LS, as it sheds light on how the transformation geometry RL impacts learners' thinking and cognition (Dudley, 2015). The transformation geometry RL activities were recorded using audio and video. This data was used to track the overall TPACK development levels of ISTs within their respective school-based LS groups in each descriptor of the five TPACK levels after a transformation geometry RL. Samples of learners' work were also garnered to provide insight into their ways of transformation geometry thinking. The KO and the observer teachers closely monitored the case learners' interactions, and their performance during the transformation geometry RL was noted.

Between the implementation and reflection phases, teachers and the KO completed a reflection guide to rate their competence in GeoGebra use, to teach the transformation geometry concepts of reflection and translation.

Fourthly, the implementation stage is followed by the *reflection phase*. During this phase, the researcher and the teachers held a FGI for the researcher to follow up on their ratings on the reflection guide. This allows more information to be acquired about any unclear answers on teachers' ratings, and then the relevant TPACK levels are adjusted. As the teachers reflect on their ratings of the items on the reflection guide, they draw from the four transformation geometry RL stages – identification of objectives, planning, implementing/observing, and reflection. The KO also reflects on the four phases of the transformation geometry RL and suggests recommendations on using GeoGebra to enhance the next RL. Additionally, the phase includes discussions relevant to the transformation geometry RL itself, such as: classroom discourses, learner learning moments and/or strategies, and achieving the objectives. The LS group of teachers analysed whether the predictions about the three case learners' interactions were met.

Between the identification of objectives and reflection phases, the data from the previous four phases informed the LS group of teachers' decisions on: whether to re-teach or refine the transformation geometry RL to improve on the next one; whether to pursue innovative approaches for improving deep transformation geometry learning; and how learner-teacher interactions and engagement in transformation geometry RL classroom practices could be improved (Mhakure, 2019).

2.4.2.2 Research on the LS and mathematics teaching

In the previous section, the overall construct of LS was discussed. This section discusses some evidence regarding how the LS has been used in mathematics education research, particularly focusing on ISTs' CPD. What distinguishes this study from existing research is that no research has ever been done in South Africa using LS as a school-based CPD approach in investigating the TPACK development of mathematics teachers in rural under-resourced schools via GeoGebra as a visual mediating tool.

Various studies have been conducted locally and internationally on the LS and mathematics teaching, enhancing teachers' professional growth (Fauskanger et al., 2022; Letloenyane & Jita, 2015; Ni Shuilleabhain & Seery, 2017; Posthuma, 2012). Firstly, Fauskanger et al. (2022) investigated the development of Malawian primary mathematics teachers' classroom practices using the LS. Their findings were that before this CPD, the teachers reported traditional views of mathematics teaching and learning. Post-LS cycle participation, the teachers highlighted the need for learner-centred approaches in mathematics lessons and the value of learner autonomy

in discovering mathematics concepts. In other words, adoption of the LS method promoted a transition from teacher-centred views towards a greater focus on learner participation in the mathematics lesson. What distinguishes Fauskanger et al.'s (2022) study from the present study? Firstly, Fauskanger et al. (2022) explored how LS impacted teachers' perspectives on teaching number sense without using technology in rural Malawian primary schools, while the present study aimed to investigate rural high school mathematics teachers' TPACK development when using GeoGebra and a school-based LS CPD approach to teach transformation geometry. Secondly, each LS group in Fauskanger et al.'s (2022) study chose a LS topic on counting and received feedback on their RL plan drafts from professional development instructors (teacher educators). In this study, transformation geometry RL plans were created by the researcher and given to the teachers in their respective LS groups – which comprised the same KO – to critique according to their learners' learning needs and the curriculum goals. Lastly, while Fauskanger et al.'s (2022) study collected data using teachers' written reflections from workshops and RL plans, the present study used data from FGIs conducted with each school after the teacher training session on GeoGebra and LS, and the three-transformation geometry RLs and observation guides.

Secondly, Ni Shuilleabhain and Seery's (2017) qualitative case study investigated LS's impact on teachers' shift to constructivist teaching practices. The findings showed that participating in successive LS cycles supported teachers in implementing new teaching approaches in their lessons. The constructivist approaches involved a teacher acting as a guide in supporting learners' mathematical explorations and thinking. The traditional practices use a blend of explanation and practice in which learners are lectured to by the teacher, in turn they listen, give answers to closed questions when called upon, copy examples, and when instructed, work on an exercise similar to the examples they were dealing with. In both this study and Ni Shuilleabhain and Seery's (2017) study, the LS approach was a new school-based CPD approach to the teachers. This CPD approach required the teachers, within their respective LS groups, to collaboratively plan, conduct or observe and reflect on RLs over successive LS cycles. However, what distinguishes the approach used to train teachers in this study from that reported in Ni Shuilleabhain and Seery's (2017) study is that the current study used the LS approach, via GeoGebra, to investigate the TPACK development of ISTs from two rural high schools on the topic of transformation geometry. Ni Shuilleabhain and Seery's (2017) study, conducted in one urban secondary school in the Republic of Ireland and without using technology, investigated changes in teachers' teaching practices and beliefs in the context of

curriculum reform, following participation in successive LS cycles. Notably, most of the teachers (three out of five) who participated in Ni Shuilleabhain and Seery's (2017) study were not qualified mathematics teachers, while all the participants in the present study were qualified mathematics teachers.

Thirdly, Letloenyane and Jita (2015) carried out a study that assessed the impact of a school–university partnership using a Japanese LS approach as a vehicle for an intervention. The results showed that the CPD partnership positively impacted ISTs' perceived curriculum decisions and instructional practices. However, what distinguishes Letloenyane and Jita's (2015) study from this study is that firstly, Letloenyane and Jita's (2015) study, carried out in South Africa's Free State province, worked with 110 teachers from primary and secondary school using the LS approach to foster mathematics instruction without using technology. The present study, conducted in two rural under-resourced high schools in the Northern Cape province's JTG district, used a LS approach via GeoGebra to investigate the TPACK development of six ISTs when teaching transformation geometry. Secondly, the methodology used for data collection in Letloenyane and Jita's (2015) study was the retrospective pre-testing approach using self-reports. Teachers were requested to complete a questionnaire six months after the workshop, reflecting on changes in their teaching practices and curriculum decisions influenced by the intervention. The current study, which was administered by the researcher, collected data using FGIs conducted immediately after the teacher training session on GeoGebra and the LS, the three transformation geometry RLs, observations of these RLs and reflection on each RL, which was done in real time.

Lastly, another South African study in the Free State province involved five rural secondary school mathematics teachers participating in an adapted LS approach (Posthuma, 2012). The results revealed that the LS adaptation fostered teacher reflection, self-knowledge, and effective mathematics teaching. While this study used a traditional LS approach to investigate ISTs' TPACK development when using GeoGebra to teach transformation geometry, Posthuma's (2012) study, which used an adapted LS without technology, investigated ISTs' reflective practice as they planned and reflected on the teaching of linear equations as a group.

Despite LS's benefits, research indicates that there are challenges hindering its effective and long-term implementation (Darling-Hammond, 2013; Jhang, 2019; Ogegbo et al., 2019; Sekao & Engelbrecht, 2022). Darling-Hammond (2013) asserted that school-based CPD activities are not provided with sufficiently adequate resources for them to function optimally, thus

discouraging teachers from implementing the LS in their schools. In Taiwan, Jhang (2019) referred to lack of familiarity with the LS process, fearing criticism and being labelled as incompetent, and having limited time and support from schools, as some of the challenges to participating in the LS. Research in South Africa has yielded comparable findings. Sekao and Engelbrecht (2022) identified lack of support from the SMT, large class sizes, the “free-rider effect”, where some participants do most of the work while others ride along as passengers, and teachers’ discomfort with classroom observations, as the challenges experienced by teachers when participating in the LS. Ogegbo et al.’s (2019) study also conducted in South Africa identified similar challenges to implement the LS such as time constraints, absence of institutional support, fear of being criticised, and lack of confidence due to inadequate content knowledge and teaching skills.

The following precautions were taken in this study to mitigate some of the identified challenges hindering LS implementation. Firstly, the researcher obtained a commitment from the SMT to provide support, including arranging a projector, venue, and releasing teachers from after school duties (e.g., meetings) on our meeting day. Secondly, teachers were offered training on the LS process to reduce fear of criticism and increase confidence. Thirdly, during planning, every teacher was given an opportunity to respond to a posed question, thus minimising the “free-rider effect”. Fourthly, in enhancing teachers’ content knowledge and teaching skills, during planning, teachers were trained how to teach transformation geometry content using GeoGebra. Fifthly, the researcher made it clear to the teachers that their lesson observations were not an evaluation on their performance, thus encouraging a culture where the teachers support each other. Lastly, the challenge of large class sizes was addressed by working with 25 learners per school.

While the LS is becoming popular in South Africa, no research exists that uses the LS (Posthuma, 2012) as a school-based CPD form in developing rural high school mathematics teachers’ TPACK via GeoGebra. This study therefore attempted to address this void by investigating rural high school mathematics teachers’ TPACK development when using GeoGebra and the LS approach as a form of school-based CPD to teach transformation geometry.

2.4.2.3 Why use the LS as a school-based CPD in this study?

The preceding section reviewed studies on the LS both locally and internationally and in what ways they differed from or were similar to the current study. This section provides an explanation of what makes the LS a form of CPD approach suitable for developing TPACK in mathematics ISTs in this study.

In Education Departments in South Africa, CPD activities often occur outside schools, led by external experts (Mhakure, 2019). These experts include (but are not limited to) mathematics subject advisors, provincial coordinators, education researchers and teacher educators from higher learning institutions. Teachers in rural schools attend generic CPD workshops outside of their schools, with teachers from other schools, that are facilitated by external experts – a one-size-fits-all scenario. These experts are not familiar with these schools' contexts and their teaching cultures, thus rendering their CPD efforts inadequate because they seldom address the institutional challenges experienced by teachers within their individual schools (Mhakure, 2019). In the current study, school-based CPD activities on teachers' TPACK development are at the heart of their professional learning in selected disadvantaged schools located in the Northern Cape Province's JTG district. The question is: Why would a school-based LS CPD approach be suitable for this research?

As alluded to earlier, the school-based CPD on the development of TPACK that the teachers underwent was carried out within the framework of the LS cycle. Hence, the teachers needed to understand the phases of the LS, as they were expected to operationalise them using the transformation geometry RL. This study used GeoGebra and a school-based LS CPD approach to develop mathematics teachers' TPACK in selected rural under-resourced schools in the Northern Cape Province's JTG district. The educational contexts in under-resourced schools differ from those in schools that are well-resourced. Under-resourced schools are also different from one another, making each school unique. This led to the researcher's interest in investigating the TPACK development via a school-based LS CPD approach in selected rural high schools, as it would offer their teachers opportunities that are rarely found in traditional CPD activities. These opportunities include but are not limited to the following:

Firstly, school-based CPD activities on teachers' TPACK development using GeoGebra and the LS offers teachers the possibility of using the minimum of resources (e.g. a laptop and

mobile projector) that they have at their schools to improve transformation geometry teaching and learning (Posthuma, 2012).

Secondly, CPD activities on teachers' TPACK development using GeoGebra and a school-based LS approach are embedded within the contexts of teachers' schools and classrooms, affording them the chance to see transformation geometry teaching and learning as it occurs, instantaneously, at their schools (Mhakure, 2019). Therefore, according to Chong and Kong (2012) and Erickson et al. (2005), there is need for school administrators to avail the necessary resources to support teachers' school-based CPD activities, and do away with structural obstacles which hamper the processes of change.

Lastly, CPD activities on teachers' TPACK development using a GeoGebra school-based LS CPD approach "are predicated on the use of teacher-generated, classroom-based, real-time formative and summative assessment data about student (transformation geometry) learning" (Woodland & Mazur, 2015, p. 10). Hence, teachers are able to work with data that is current and meaningful from their schools.

This study uses the LS as a school-based CPD approach via integrating GeoGebra to develop rural high school mathematics teachers' TPACK in teaching transformation geometry. As alluded to earlier, the higher TPACK levels (exploring and advancing) place emphasis on learner thinking in planning, implementing and analysing GeoGebra transformation geometry RLs (see Section 2.4.1.3). Hence, participating in the LS could enhance these TPACK development levels.

2.5 TPACK and LS

In the previous section, the researcher explained the theoretical frameworks (TPACK and the LS) that informed data collection and analysis in the present study, which investigated rural high school mathematics teachers' TPACK development when using GeoGebra and a school-based LS CPD approach to teach transformation geometry. This section discusses some TPACK and LS studies which have been carried out in mathematics education, highlighting what distinguishes the current study from the studies reported in the literature. What differentiates the present study from the reviewed studies is that no research has ever been done in South Africa using a GeoGebra-based LS as a school-based CPD approach to investigate rural high school mathematics teachers' TPACK development. This study could therefore be said to shed new light on TPACK development for rural mathematics teachers in South Africa.

The literature indicates that TPACK development strategies often involve teachers working together to address challenging mathematics topics (M. J. Koehler & Mishra, 2005) – transformation geometry, in this case. The LS fits into this perspective, as it entails collaborative planning, implementation and reflection on GeoGebra-based transformation geometry RLs (Fernández, 2002).

Some local and international TPACK studies specific to teachers of mathematics in CPD have incorporated the LS (Huang et al., 2021; Joubert et al., 2020; Kurt & Çakıroğlu, 2018; Riales, 2011). Firstly, Huang et al.'s (2021) study called for mathematics ISTs' TPACK development through an online intercultural LS. The findings showed enhancement in teachers' TPACK and mathematical knowledge for teaching (MKT), the development of expertise in instructional task creation, and effectiveness in addressing online teaching challenges. Some differences were noted between the present study and that of Huang et al. (2021). Firstly, the present study, which focused on the Senior Phase topic of transformation geometry, used a GeoGebra-based LS as a school-based CPD approach to develop rural high school mathematics teachers' TPACK in the Northern Cape Province's JTG district, South Africa. Huang et al.'s (2021) study aimed to explore how cross-cultural online learning among Chinese and South Australia primary mathematics teachers impacted these teachers' learning when teaching probability using several instructional resources (e.g. online resources, manipulatives and physical devices). Secondly, unlike the current study, which collected data on learner learning using an observation guide, learner data in Huang et al.'s (2021) study was not collected, due to online platform teaching limitations such as inadequate interactions between learners and teachers.

Secondly, Joubert et al.'s (2020) study investigated blended LS to support teachers having limited face-to-face collaboration opportunities. The findings of Joubert et al.'s (2020) study showed that blended LS supports isolated teachers in effectively incorporating technology into their lessons. Notable differences between Joubert et al.'s (2020) study and the present study include (but are not limited to): Firstly, Joubert et al.'s (2020) study explored a blended LS approach to enhance teachers' skills in mobile technology incorporation into their lessons. The current study utilised GeoGebra and a school-based traditional LS CPD approach to develop rural high school mathematics teachers' TPACK when teaching transformation geometry. Secondly, Joubert et al.'s (2020) study (administered by the University of Pretoria) worked with 52 teachers across different education levels from all over South Africa and Botswana, and representatives from all subject areas (including mathematics) were among the teachers. These schools' geographical distances apart contributed to the isolation of teachers, hence the adaption of the LS into a blended format online. The current study, administered by the researcher, worked with only six mathematics teachers from two school-based LS groups from rural under-resourced high schools in the Northern Cape Province's JTG district.

Thirdly, Kurt and Çakıroğlu's (2018) study examined the TPACK development of PST when using virtual manipulatives to teach statistics through a microteaching LS (MLS). The findings revealed that MLS enhanced PST' TPACK as they showed notable growth in specific TPACK domains. What distinguishes the present study from Kurt and Çakıroğlu's (2018) study is that firstly, the present study used the LS as a form of school-based CPD approach via integrating GeoGebra to develop rural high school mathematics teachers' TPACK on the Senior Phase topic of transformation geometry. Kurt and Çakıroğlu (2018) used the MLS via virtual manipulatives to develop PST' TPACK in statistics teaching at Grade 8. Secondly, the theoretical frameworks in the present study were predicated on the TPACK development model (Niess et al., 2009) and the LS, while Kurt and Çakıroğlu's (2018) study used M. J. Koehler and Mishra's (2009) TPACK framework.

Lastly, Riales (2011) conducted a qualitative study in the USA which examined secondary school mathematics ISTs' TPACK when utilising technology in teaching mathematics through the LS approach. Riales's (2011) study employed the TPACK framework (Niess et al., 2009) in examining group interactions and individual cases (four out of the six teachers) in LS meetings. The results showed that a technology-based LS design enabled participants to practise higher-level TPACK skills: exploring and advancing. Individual TPACK development patterns differed, with some participants (50% of the teachers) performing actions at the higher

TPACK development levels based on post-study observations. Riales (2011) recommended further research with larger samples and ongoing LS, as well as extensive classroom observations, to validate and expand these results. The similarities between Riales's (2011) study and the present study include (but are not limited) to: Firstly, both studies utilised the TPACK development model of Niess et al. (2009) in guiding data collection and analysis. Secondly, Riales's (2011) study and the present study were carried out with secondary school mathematics teachers from rural settings. Riales (2011) differs from the present study in the following ways: Firstly, Riales's (2011) study described secondary school mathematics ISTs' TPACK progression within a technology-based LS context that used TI-84 graphing calculators to teach algebra. The present study utilised GeoGebra and the LS approach to develop rural high school mathematics teachers' TPACK when teaching transformation geometry. Secondly, most participants in Riales's (2011) study had previously used the TI-Navigator system. These participants' classrooms had a "set of TI-84 plus graphing calculators, a TI-Navigator system, and an interactive whiteboard" (Riales, 2011). The participants of the current study had no prior experience with GeoGebra. Thirdly, the scope of Riales (2011) focused on a single LS cycle with teachers from one school, and their learners had access to a graphing calculator. The current study investigated the effect of participation in three LS cycles with teachers from two under-resourced rural high schools, with each school forming a LS group of mathematics teachers; and learners had no access to the computer.

Overall, the studies showed that LS implementation had a consistently positive impact on mathematics teachers' TPACK development. However, it is apparent from the literature that no research has ever been done in South Africa using GeoGebra and the LS as a school-based CPD to develop the TPACK knowledge of mathematics teachers in rural schools via integrating GeoGebra as a visual mediating tool. This study is vital because it helps to fill a void in the literature on using LS and GeoGebra to develop rural high school mathematics teachers' TPACK in South Africa.

2.6 TDS as a theory underpinning the LS in promoting teachers' TPACK

Literature related to TPACK and the LS in mathematics education has been discussed in the previous section, highlighting what distinguishes the current study from the reviewed studies. This section discusses how TDS promotes rural high school mathematics teachers' TPACK development within the LS framework when teaching transformation geometry via GeoGebra in the South African mathematics curriculum.

A teaching event (a transformation geometry RL, in this case) typically encompasses various teaching situations – didactical and a-didactical (Bahn & Winsløw, 2019) – see Section 2.3.1. The transformation geometry RL is a DS, and the milieu for teachers' learning (TPACK development) in the LS, in which teachers plan, implement and evaluate specific hypotheses about learner learning in the DS (Winsløw et al., 2018) using GeoGebra – see Figure 2.7. Therefore, a transformation geometry RL is at the core of the LS process.

Figure 2.7 The learning situation of lesson studies: the interplay between the teachers (T) and their milieu (M), the DS (based on Clivaz, 2018, p. 176)

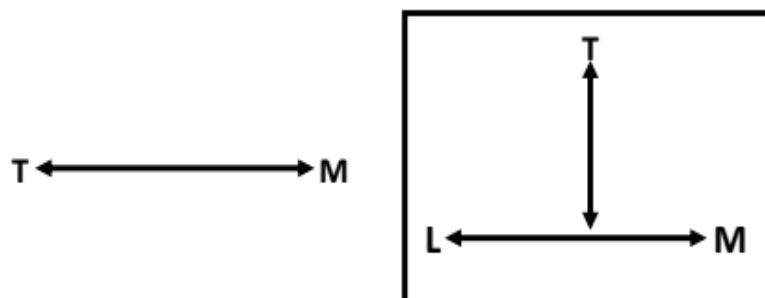


Figure 2.7 shows that the DS (transformation geometry RL) is the milieu for teachers' TPACK development. TDS supports LS's central elements (Lendínez Muñoz et al., 2024; Miyakawa & Winsløw, 2009). In other words, TDS supports teachers' planning, teaching/observing and reflecting on how their design choices affect learners' transformation geometry learning in a RL using GeoGebra. This enables an in-depth analysis of teachers' TPACK development throughout LS stages (Bahn, 2018).

In this study, the researcher employed 'TDS-LS' to describe a school-based LS process grounded in TDS principles. The aim was to develop rural high school mathematics teachers' TPACK knowledge during planning and teaching transformation geometry RLs using GeoGebra guided by TDS principles. In TDS-LS, teachers possibly develop TPACK knowledge through the transformation geometry RL phases of identification of objectives,

planning, implementing/observing and debriefing. In the context of this study, the identification of objectives phase was embedded within the planning phase. During the planning phase, teachers learned (using GeoGebra) to:

- Design a learning environment that enables self-directed learner exploration (a-didactical situation).
- Modify the transformation geometry problem (milieu) to prompt learners to evolve their approaches and/or conjectures. For example, after reflecting the object (drawn in quadrant 1) about the y-axis, the teacher can resize it or change its position to various quadrants for learners to re-examine their conjectures regarding how the coordinates of the object and image would change.
- Deepen their understanding of learner-milieu interactions.
- Structure the devolution phase – the kind of instructions that they need to give to learners to put them into a relationship with a transformation geometry problem.
- Create new instructional approaches and a classroom environment that promotes the learning of transformation geometry.

During implementation/observation and reflection on the GeoGebra transformation geometry RL, teachers had opportunity to refine their prior knowledge, while acquiring new learning linked to:

- The actual handover of a transformation geometry problem free from clues about the knowledge they want the learners to learn.
- The analysis of approaches which learners utilise in an a-didactical situation and connecting them with specific traits of the transformation geometry problem.
- Fine-tuning the interactions of learners in the situation through dynamic adjustments to didactical variables in response to learner transformation geometry learning needs or desired learning outcomes. For example, with the situation of institutionalisation, the teacher can change the position of the object to various quadrants after reflecting it about the x-axis, to assist learners with visualisation of how the coordinates change.
- How to summarise from learner strategies from the pen and paper activity. Once learners connect with the mathematical idea (the geometric relationships that exist between the object and its image), teacher guidance refines their understanding. With the aid of the dynamism capability of GeoGebra, the teacher can make explicit the transformation geometry knowledge that learners gain in the context of their a-

didactical work, and add it to the learners' system of mathematical knowledge (institutionalisation phase) (Lendínez Muñoz et al., 2024; Novotná & Hošpesová, 2022).

With TPACK gaining prominence in teacher education, effective assessment tools are needed to measure teachers' mastery of this critical knowledge area (Groth et al., 2009), throughout the LS cycle within the RL itself. This assessment model is discussed in the section below.

2.7 Assessing TPACK development through the LS

The previous section discussed how TDS promotes mathematics teachers' TPACK development within the LS framework when utilising GeoGebra to teach transformation geometry in the South African Senior Phase mathematics curriculum. The "lesson study technological pedagogical content knowledge (LS-TPACK) assessment model" of Groth et al. (2009) is discussed here. This model offers away to evaluate TPACK development exhibited by individual teachers within different LS groups, integrating transformation geometry, GeoGebra and pedagogical knowledge: instruction approaches, classroom management skills, RL plan development, and understanding and evaluating of learner learning strategies.

Groth et al. (2009) designed a LS-TPACK model to evaluate the TPACK development of teachers through LS cycles. Their study showcased the LS-TPACK model's application, underscoring the assessment framework rather than teachers' TPACK development (Groth et al., 2009). In the paragraphs that follow, the procedures for implementing the assessment model of LS-TPACK are explained.

In the LS-TPACK process, teachers start by analysing the study plan to identify a problematic topic (transformation geometry, in this case) and establish the learning goal(s). Once the learning goals have been identified, the teachers draft four-column written RL plans that incorporate (GeoGebra) technology. To enhance RL quality, a university faculty reviews RL plans and worksheets, providing feedback on pedagogy and content. Teachers then select which feedback to incorporate to improve the RL plans before implementation. Teachers can request assistance from a university faculty in planning the RL (Groth et al., 2009). In maintaining consistency and alignment to this study's research questions and objectives of the three transformation geometry RL plans, the researcher created the four-column written RL plans with the aid of GeoGebra. After drafting these RL plans, the researcher sent them to the teachers in their school-based LS groups in the two schools (A and B). With the KO taking centre stage,

the teachers collaboratively critiqued the transformation geometry RL plans according to the learning needs of their learners and the curriculum.

Once the RL plans have been refined, a volunteer teacher from the LS group teaches the RL, while peers observe. The RL is audio- and video-recorded. The video is later viewed by the entire LS group and a university faculty member. While ideally all LS team members would observe the RL live (Lewis, 2002), video recording allows them to share and discuss it remotely, accommodating scheduling conflicts. The LS group of teachers and faculty members view the RL video together during the debriefing session. While viewing this video, participants note the strengths and areas of improvement. After watching the RL video, participants discuss the RL's strengths and weaknesses, starting with the teacher that implemented it and ending with the faculty member's insights. After a debriefing participant has shared their views, a more unstructured conversation may occur in which the discourse may shift to the objectives of the subsequent related RL (Groth et al., 2009). In this study, while a teacher from the LS group volunteered to teach the transformation geometry RL, the researcher video-recorded this RL, and other teachers (including the KO) observed and collected data on learner learning using an observation guide. Each LS group held a debriefing session, analysing data and notes from the RL phases – identification of the objectives, planning, implementing/observing and reflection – with discussion led by the teacher who taught the RL to the KO. There was no need for the team to view the transformation geometry RL video during the debriefing because this session was conducted immediately after the RL was taught, and all LS group participants had been present when the RL was implemented.

The researcher collects qualitative data through the LS-TPACK process and compiles it into a case study database (Yin, 2003), the data being transcripts of the identification of transformation geometry objectives session, transcripts of the planning session, the original and revised RL plans, transcripts and videos of implemented RLs, observation guides, reflection guides and transcripts of the reflection sessions about the four RL stages. The researcher(s) draw conclusions about teachers' TPACK level (based on the five TPACK level descriptors) from the above data in the data-base throughout the cycle of LS within the RL itself (Groth et al., 2009). In this study, inferences about individual teachers' TPACK levels in the five TPACK level descriptors were drawn by the researchers from the above data in the data base.

Beyond describing the LS-TPACK assessment framework and a pilot example, Groth et al. (2009) discussed its advantages and disadvantages. Among the strengths were the following. Firstly, the model's strength lies in its nuanced capture of TPACK development as a fluid, contextual, and collaborative process. Secondly, the LS-TPACK model intertwines assessment of TPACK with CPD in real time. Thirdly, the teachers and KO critiquing the transformation geometry RL plans given to them by the researcher provides CPD (learning opportunities) to the teachers. Fourthly, the LS-TPACK model allows for the simultaneous study of transformation geometry concepts of reflection and translation; instruction approaches, classroom management skills, RL plan development, and the assessment of learner learning; and of GeoGebra use in transformation geometry instruction. Lastly, the model leverages the expertise of the KO and the teachers, combining their distinct perspectives in transformation geometry objective setting, RL planning and review, and debriefing. A limitation of the LS-TPACK model is its focus on group-level TPACK development, without providing a way to assess individual teachers' TPACK (Groth et al., 2009).

2.8 Summary

The higher TPACK levels (exploring and advancing) prioritise learner transformation geometry thinking. These levels require planning, implementing/observing and reflecting on GeoGebra-based transformation geometry RLs, with the aim of promoting learners' thinking and conceptual understanding of this content. The practices (planning, implementing/observing and reflecting on the RL) are the core elements that underpin the LS, a CPD approach which aims to enhance transformation geometry instruction with GeoGebra. Some local and international TPACK studies specific to teachers of mathematics in CPD have incorporated the LS. Results from the literature indicate that implementing the LS significantly enhances teachers' TPACK in mathematics (Huang et al., 2021; Joubert et al., 2020; Kurt & Çakıroğlu, 2018; Riales, 2011). Despite the LS promoting mathematics teachers' TPACK development, no research has been done in South Africa using GeoGebra and the LS as a school-based CPD to develop rural high school mathematics teachers' TPACK. This research is significant as it addresses a knowledge gap on using LS and GeoGebra to develop rural high school mathematics teachers' TPACK in South Africa. The next chapter discusses the research methodology used in investigating mathematics teachers' TPACK development when utilising a GeoGebra-based LS as a form of school-based CPD to teach transformation geometry in rural under-resourced high schools.

CHAPTER THREE: RESEARCH METHODOLOGY

3.1 Introduction

The method of inquiry used in this study is described in detail in this chapter. There are six sections: research aim, research orientation, research design and rationale, methodology, ethical considerations and conclusion.

3.2 Research aim

The aim of this study was to investigate the TPACK development of rural high school mathematics teachers when using GeoGebra and the LS approach as a form of school-based CPD to teach transformation geometry. This aim was guided by the following objectives.

3.2.1 Objectives of the study

The specific objectives were to:

- Establish the extent of the use of GeoGebra and the LS as a school-based CPD approach in mathematics teaching in high schools in the JTG district prior to the main study.
- Investigate the level of TPACK developed when rural high school mathematics teachers use GeoGebra to teach transformation geometry during a school-based LS CPD approach.
- Investigate the experiences of rural high school mathematics teachers when teaching transformation geometry using GeoGebra.

3.2.2 Main research question

The primary research question underlying this study is: *What is the TPACK development of mathematics teachers from rural high schools when using GeoGebra and the LS approach as a form of school-based CPD to teach transformation geometry?*

The above question may be divided into the following sub-questions:

- What is the extent of the use of GeoGebra and the LS as a school-based CPD approach in the teaching of mathematics in JTG district's high schools prior to the main study?
- What is the level of TPACK developed when rural high school mathematics teachers use GeoGebra to teach transformation geometry during a school-based LS CPD approach?

- What are the experiences of rural high school mathematics teachers using GeoGebra to teach transformation geometry?

3.3 Philosophical Paradigm.

A research paradigm is “a conceptual model of a person’s worldview, complete with the assumptions that are associated with that view” (Clark & Creswell, 2008, p. 73). The research paradigm framework is supported by four pillars:

- Ontology focuses on what counts as reality (What is real?).
- Epistemology deals with how we can know this reality (How do we generate knowledge?).
- Axiology deals with the integration of values into a researcher’s work.
- Methodology focuses on how we can investigate this reality/ How we can go about studying it (What methods should we use?) (Maree, 2016).

This study adopted an interpretive paradigm, seeking to understand and interpret the personal values, meanings, and experiences that participants (teachers) associate with their own (classroom) experiences (Bertram & Christiansen, 2014). According to interpretivists, reality is subjective and can only be partially understood since it is created within individual minds (Pretorius, 2024). In other words, the researcher cannot fully grasp another teacher’s experience objectively, but can attempt to interpret and derive significance from it. Interpretivism is centred on the idea that teachers are best suited to understand their own classroom experiences (regarding their TPACK levels in this case). Therefore, interpretivists seek to explore and understand the meanings teachers assign to these experiences (Cohen et al., 2018). The interpretive paradigm prioritises individual perspectives and emotions that influence how teachers perceive their surroundings, highlighting the significance of context in forming these viewpoints (Pretorius, 2024). As a result, researchers following this approach encourage participants to introspect about their personal experiences and explain their interpretations. For example, in this study, immediately after each RL was taught, teachers were given a reflection guide (see Appendices 12a, 12b, 12c) to rate their competence in using GeoGebra to teach a particular transformation geometry concept, which they also brought to the FGI so that the researcher could follow up on their TPACK level ratings.

Additionally, B. S. Merriam (2009) asserted that in interpretive research, multiple interpretations of reality exist, and there is no one objective truth that can be generalised. This

was significant to this study, as the researcher captured and portrayed all the facets of teachers' experiences (regarding their TPACK levels) as they used GeoGebra to teach the transformation geometry RLs on the concepts of reflection and translation using a GeoGebra school-based LS CPD approach.

The study employed a mixed methods complementarity design, blending quantitative and qualitative approaches, guided primarily by an interpretive paradigm, while acknowledging pragmatic considerations. According to Greene et al. (1989), a complementarity mixed methods approach is a design which employs both qualitative and quantitative methods to enhance interpretability of assessments of a single phenomenon (individual teachers' TPACK levels) – for example via different levels of analysis. Pragmatism also allows for a mix of quantitative and qualitative techniques to provide a more comprehensive understanding of complex issues (individual teachers' TPACK levels), allowing for a richer understanding by leveraging the strengths of both approaches (Pretorius, 2024). For example, in this study, the data gathered from teacher training sessions on GeoGebra and the LS, as well as the three-transformation geometry RLs in each of the two schools (A and B) was analysed using deductive thematic analysis. In addition, a small component of quantitative data analysis using the ANOVA test was employed. The intent was to establish whether researchers' ratings on teachers' TPACK knowledge in schools A and B differed significantly across the four stages of professional development (training, RL1, RL2, and RL3) for each of the four TPACK dimensions: curriculum and assessment, learning, teaching, and access – see Sections 3.5.4.2 and 3.5.4.3.2. This flexibility allows the researcher to leverage the strengths of both methods, leading to a more thorough understanding of teachers' TPACK levels (Almalki, 2016; Pretorius, 2024).

3.4 Research design and rationale

A research design serves as a road map for a study, detailing the methods for data collection, analysis, and interpretation (Creswell & Plano Clarke, 2011) in answer to the research question(s). The study primarily employed two methods. First, a baseline survey was administered to teachers in all 31 high schools in JTG district, a remote district of the Northern Cape Province in South Africa. This study included mathematics ISTs from both Senior and FET Phases. The second method was a multiple case study of two under-resourced rural high schools (A and B) that participated in a school-based LS CPD approach to develop teachers' TPACK when teaching transformation geometry using GeoGebra.

3.4.1 Baseline survey

A baseline survey is an instrument that is employed in collecting initial data on the current state of affairs (Cohen et al., 2011). Hence, establishing a starting point for future comparisons. In this study, the baseline survey aimed to establish the extent of the use of technology (in particular, GeoGebra) in mathematics instruction, and the CPD types – LS as a school-based CPD approach, in this case – to which teachers in this district had been exposed.

A total of 135 teachers responded to an anonymous baseline survey that was administered to both Senior and FET Phases mathematics teachers in the 31 high schools in JTG district, Northern Cape Province. For analysis purposes, the teachers were coded as Tr1, Tr2, ..., Tr135. The data from the baseline survey responses was collected in the first term of the 2023 academic year. The baseline survey consisted of five parts: A, B, C, D and E (see Appendix 8). Section A covered general information about the teacher's profile. Sections B and C focused on technology and GeoGebra use in the mathematics classroom. Section D comprised questions on the types of CPDs of mathematics teachers currently taking place in schools, and section E consisted of questions on LS as a form of school-based CPD.

3.4.2 Case study

Since this study worked with six teachers split into two LS groups (A and B) across their schools, the research used a qualitative methodology to study multiple cases as the second method. A multiple case study allows researchers to investigate many cases (the six teachers split into two school-based LS groups, in this case), highlighting commonalities and distinctions in terms of the teachers' TPACK development levels within their respective schools (Baxter & Jack, 2008). Additionally, through a case study, researchers can thoroughly explore a topic in its natural setting (J. Creswell, 2012) using diverse data sources, a strategy that enhances the credibility of the data (Yin, 2003). This enables a richer understanding of the phenomenon (teachers' TPACK development, in this case), revealing its various aspects. The study collected data through a baseline survey, FGIs with teachers on GeoGebra and the LS training, observation guides and FGIs on the three transformation geometry RLs.

The main study focused on a case of six selected Senior and FET Phase mathematics ISTs from two LS groups within their respective schools (A and B), who received training on using

GeoGebra and the LS through the RL to teach transformation geometry. Therefore, the unit of analysis was analysed in two distinct ways: firstly, to identify the level of TPACK developed when selected rural high school mathematics teachers use GeoGebra and the LS as a form of school-based CPD approach to teach transformation geometry; and secondly, to identify the same teachers' experiences in teaching transformation geometry using GeoGebra.

3.5 Methodology

The section presents the LS as a research methodology and a CPD approach for rural high school mathematics teachers' TPACK when using GeoGebra to teach transformation geometry concepts of reflection and translation. This section is divided into five parts: research population, research sample, data collection tools, analysis procedures, and validity measures to ensure research quality.

3.5.1 Research population

This section describes the population from which the research sample emanated. The research took place in the JTG district, a rural region in South Africa's Northern Cape Province. The research population was made up of all the 31 district high schools and approximately 160 mathematics teachers. According to Anthanasou et al. (2012), a population refers to the entire group of individuals (teachers) and institutions (schools) that a researcher aims to study, understand, or describe. It is noteworthy that a researcher cannot include every school of the target population in their study. Therefore, a sample of schools from the population was chosen for this research.

3.5.2 Research Sample

This section describes the schools in which the study was conducted, as well as the sampling procedure used in their selection. Purposive sampling was employed to collect in-depth and valid data for the study. As per Fraenkel and Wallen (2006), purposive sampling involves selecting a specific group of participants (rural mathematics ISTs, in this case), based on a study's requirements. Additionally, Mertens (2005) noted that in an interpretive paradigm, researchers choose samples that provide rich, detailed insights for in-depth examination. The study's implementation occurred in two rural under-resourced high schools. The schools included mathematics teachers from both the Senior and FET Phases. In South Africa's education system, the Senior Phase covers grades 7 to 9, but more precisely in this study grades 8 and 9, and the FET Phase covers grades 10 to 12. The schools were selected using purposive

sampling, with the following criteria: the schools had to be located in a low socio-economic area; the teachers had to be willing to be part of the study; though there was no computer laboratory at the school, the teachers had to have laptops and mobile projectors; and convenient access – the schools had to be close to the town where the researcher resides. The pseudonyms ‘A’ and ‘B’ were used for the two schools, to protect their identities.

These schools are located in South Africa’s Northern Cape Province, in the JTG district. As it is a remote district, schools located in the JTG district are largely under-resourced. The sample, therefore, came from under-resourced schools in a disadvantaged district of South Africa’s Northern Cape Province. Neither school had a computer laboratory, but the teachers had laptops and mobile projectors, though until this study they were not used for mathematics teaching. In this study’s context, ‘disadvantaged areas’ are viewed as areas nesting learners from economically challenged homes. Two rural under-resourced high schools participated in this study, each forming a LS group consisting of in-service Senior and FET Phase mathematics teachers. For this study, a school-based LS group encompasses mathematics teachers within a school department aiming to boost learner learning through jointly identifying problematic areas in mathematics (transformation geometry, in this case), planning transformation geometry RLs, implementing them (one teacher volunteers to teach, while others observe), and finally reflecting on the RLs. The next section profiles the two groups of teachers in school-based LS groups, A and B.

3.5.2.1 Senior and FET Phase school-based mathematics LS groups of teachers

The section discusses the LS groups of teachers who provided the data for this study. Bearing in mind the importance of the context with varied resource availability in schools (Mishra, 2019), two school-based LS groups of mathematics teachers were used in this study. The groups were taken from the research sample (see Section 3.5.2), with each school forming its own LS group of teachers. LS groups A and B were the pseudonyms used for the two groups. The groups comprised both Senior and FET Phase mathematics teachers in each school. For discussion purposes, the teachers in each LS group were assigned pseudonyms. In LS group A, the names used were Mike, Tom and Bernard, while in LS group B, the names used were Sarah, Bosco and Frank. The selection of these teachers was purposive, based on their willingness to be part of the main study after the teacher training workshops explained in section 3.5.2.2. Table 3.1 summarises participating teachers’ profiles; including that of the KO, whose pseudonym was Lawrence.

Table 3.1 Teacher profiles for the two participating schools

Pseudonym	Gender	School	Area of specialization	Mathematics teaching experience	Qualification
Mike	Male	A	Mathematics and Geography	15	Bachelor of Education in mathematics education
Tom	Male	A	Mathematics and Physics	11	Bachelor of Science and Postgraduate Certificate in Education (PGCE)
Bernard	Male	A	Mathematics and Life Skills	12	Diploma in Education
Sarah	Female	B	Mathematics and Physics	6	Bachelor of Science and PGCE
Bosco	Male	B	Mathematics, Computer Application Technology (CAT) and Mathematics Literacy	10	Bachelor of Education in mathematics and computer science
Frank	Male	B	Mathematics and Physics	2	Bachelor of Science Honours in science education; PGCE in mathematics
Lawrence	Male	B	Mathematics and CAT	25	Master of Education in mathematics education

3.5.2.2 Teacher training on GeoGebra and the LS

This section presents the two introductory training sessions that were conducted per school, each lasting two hours. The first session introduced mathematics ISTs to GeoGebra, focusing on its application in teaching transformation geometry. The second training session introduced teachers to LS, as the school-based CPD on TPACK that the teachers had to undergo which was carried out within the framework of the LS cycle. Hence the teachers needed to understand the phases of the LS, as they were expected to operationalise them using the RL.

3.5.2.2.1 Introducing GeoGebra to participating mathematics teachers

In this session, an introductory GeoGebra training workshop for each school-based LS group of teachers was conducted. The training was done by the same KO (see Section 3.5.2.3), who the researcher had identified before the workshop. This was a teacher in one of the participating schools who was knowledgeable about GeoGebra and how it works in mathematics instruction. The need to conduct this training was informed by the baseline survey results, which assessed teachers' familiarity with GeoGebra and its use in mathematics teaching. The training started off by showing teachers what constitutes the software and its application in mathematics instruction in general. This was followed by showing teachers how to teach the transformation geometry concepts of reflection, translation and enlargement using GeoGebra. Since one of the

teachers had to volunteer to teach in the actual RL, during the training, there was a need to give teachers a hands-on activity, using GeoGebra to teach translation in transformation geometry. The aim was to get the teachers acquainted with how the software operates in teaching transformation geometry concepts, thus providing them with an opportunity to strengthen their TPACK.

When learning how to use GeoGebra to teach transformation geometry, teachers faced time constraints, hindering their ability to explore its features and causing unease with using the software for teaching. The teachers also struggled with the software due to little or no prior experience, leading to uncertainty when using it to teach transformation geometry. To address these challenges, the knowledgeable other (KO) advised teachers to allocate more time for practicing GeoGebra, emphasising that mastering its tools and menus requires ongoing practice and hands-on experience. The KO also encouraged teachers to watch You-Tube videos to learn more about using the software to teach transformation geometry.

3.5.2.2.2 Introducing the LS to the participating teachers

The second workshop introduced teachers to the construct of LS as a school-based CPD approach. Like the activities of the GeoGebra training session, the activities of the LS training session were informed by the data collected during the baseline survey on the nature of CPDs to which the teachers in the JTG District were exposed. The reason participating teachers were introduced to the LS was that the school-based CPD on TPACK that the teachers had to undergo was to be carried out within the framework of the LS cycle. Hence the teachers needed to understand the LS phases, as they were expected to operationalise them using the RL.

In addition, the researcher introduced the teachers to the data collection instruments – observation and the interview guides (including reflection guides) – to be used within the LS cycle. The aim was to get the teachers acquainted with the instruments in terms of what they would be observing and reflecting on, both during and after the actual implementation of the RL. In the case of the observation guide, the researcher made it clear to the teachers that during implementation, when one teacher is teaching, the other teachers would use the observation guide to observe learner transformation geometry learning. For the reflection instrument, the researcher explained to teachers that after a RL was observed and concluded, they had to complete a reflection guide to rate their competence in using GeoGebra to teach a particular transformation geometry concept. The teachers then had to bring the reflection guides to the FGI for the researcher to further probe their responses and follow up on their TPACK

development in each descriptor of the five TPACK levels after each RL. The researcher made it clear to the teachers that these data collection instruments were not an evaluation of their performance, thus keeping them calm. Lastly, the teachers willing to participate in this research were given consent forms – see Appendix 4.

3.5.2.3 KO

A ‘Knowledgeable Other’ or KO, according to Takahashi and McDougal (2016), is an expert in the field who can share research and mathematics curriculum-based insights, bridge theory and practice, and guide others in (transformation geometry) teaching and learning that is reflective. A KO anticipates future learner transformation geometry learning paths and broadens teachers’ thinking beyond the current RL (Takahashi, 2014). This study expected the KO to possess knowledge of GeoGebra. The KO used in this study was an experienced mathematics teacher from one of the selected schools, who worked with teachers in the identification of transformation geometry objectives, planning, implementing/observing learner learning and evaluating transformation geometry RLs within the LS framework. The two schools (A and B) worked with the same KO; this was a mathematics teacher with expertise in using GeoGebra and experienced in teaching and assessing transformation geometry.

3.5.2.4 Roles of the researcher and KO

In this section, the different roles performed by the researcher and the KO in this study are presented. The roles played by the researcher are presented first, followed by those of the KO.

Roles of the researcher

S. Merriam (1998) underscores the importance of clarifying the researcher’s role in qualitative studies. The researcher took on different roles in this research. Firstly, the researcher was responsible for organising a meeting with the KO to talk to him about his role prior to the main study. Secondly, the researcher introduced the teachers and the KO to the LS construct as a school-based CPD approach. Thirdly, the researcher created the three transformation geometry RL plans for teachers in the two schools (A and B) to critique according to the learning needs of their learners and the curriculum. Lastly, the researcher was responsible for video recording the introductory training workshops, LS group meetings and RL implementations, and conducting FGIs with the teachers of both schools.

Roles of the KO

The KO trained teachers during the introductory training workshop on how GeoGebra works in teaching mathematics, particularly transformation geometry. Being part of the two school-based LS groups of teachers in schools A and B, the KO was present throughout all the LS phases: identification of transformation geometry objectives, planning the RL with a research question, implementing/observing learner learning and reflecting on the entire RL. Therefore, the KO shared his views in the four RL stages.

During the identification of objectives phase on the transformation geometry concepts of reflection and translation, the KO shared his views on each transformation geometry concept and why it is important. When planning, the KO offered guidance on using GeoGebra to teach transformation geometry. Apart from stepping in to provide clarity in terms of GeoGebra use when the teacher was challenged during teaching, the KO also observed learner transformation geometry learning during teaching. In addition, the KO reflected on the four phases of the RL and suggested recommendations for how to improve the next RL using GeoGebra. The views of the KO are therefore seen through all the RL stages.

3.5.3 Data collection instruments

The section discusses the data collection tools utilised in this study. These instruments are summarised in Table 3.2. Data was collected during the second half of the 2023 academic year from high school mathematics teachers through a baseline survey, FGI conducted after the teacher training session on GeoGebra and the LS, the three transformation geometry RLs, observation and interview guides (including reflection guides) on the four RL stages with the teachers.

Table 3.2 An overview of data collection instruments

Type of instrument	Data generated	Intent	Research question addressed
Baseline survey	Qualitative and descriptive statistics	To give an understanding of the extent of the use of technology, in particular GeoGebra, in mathematics teaching in the JTG district and the types of CPDs in schools – LS as a school-based CPD approach – to which the district’s teachers were exposed.	1
FGIs – after teacher training sessions on GeoGebra and the LS, and each of the three transformation geometry RLs.	Qualitative and quantitative data	To track the teachers’ TPACK development levels as shown by their self-reported scores and the researchers’ interrater reliability scores before and after the training, and after each of the three RLs on the transformation geometry concepts of reflection about the x- and y-axis, reflection about the $y = x$ line and translation.	2
Research lessons – these lessons were piloted.	Qualitative and quantitative data	To track the overall TPACK development level of individual teachers in their respective LS groups in each descriptor of the five TPACK levels after each of the three RLs on the transformation geometry concepts of reflection about the x- and y-axis, reflection about the $y = x$ line and translation.	2 & 3
Observation guide	Qualitative data	To observe learner learning in each of the three transformation geometry RLs on the concepts of reflection about the x- and y-axis, reflection about the $y = x$ line and translation.	3

3.5.3.1 Baseline survey

This section describes the baseline survey, which was used to gather data about the extent of technology integration (in particular, GeoGebra) in mathematics teaching, and the nature of CPDs – LS as a school-based CPD approach, in this case – to which the JTG district’s teachers were exposed.

Before the commencement of this study, 135 Senior and FET Phase mathematics teachers completed an anonymous baseline survey administered to them by the researcher. The baseline survey aimed to establish the extent of the use of technology (in particular GeoGebra) in mathematics teaching and the CPD types – a school-based LS CPD approach, in this case – to which teachers in this district were exposed. Prior to administering the baseline survey to the research population at workshop sites, it was piloted with mathematics teachers in one school. Pilot testing with teachers ensured the questionnaire was clear and yielded relevant data. Teachers informed the researcher that questions 13 and 17 were unclear, due to unfamiliar terminology. Teachers also indicated that question 8 needed refinement to accommodate teachers who use or have used more than one mathematics-related software package in their lessons. The pilot study’s feedback allowed the researcher to improve and fine-tune the baseline survey instrument, making adjustments such as allowing multiple responses for question 8 and adding definitions for LS as a school-based CPD approach and KO to enhance clarity.

Following the pilot, a final baseline survey was disseminated to all relevant mathematics teachers in the JTG district through workshops led by mathematics subject advisors. The baseline survey enabled the researcher to gather various types of data. Data collected included ...teacher demographics, technology integration in mathematics teaching, and teachers' familiarity with GeoGebra and the LS. For example, in the category of teachers' familiarity with the LS, the researcher sought details on teachers' participation in various CPD activities within their school's mathematics department. The baseline survey data was used by the researcher to gauge the extent of technology use, particularly GeoGebra use, in mathematics teaching and the nature of CPD programmes that teachers were exposed to in the district. While the findings of the baseline survey provide data to answer the study's initial research question – the extent of the use of GeoGebra and the LS as a school-based CPD approach in mathematics teaching in JTG district's high schools prior to the main study – the same findings strengthened the study's position on the problem statement (see Section 4.2).

3.5.3.2 Teacher training on GeoGebra and the LS – interview guide

This section discusses the FGI that probed the teachers in two ways: firstly, to ascertain their understanding of GeoGebra use in teaching mathematics topics prior to the training workshop; and secondly, to ascertain what GeoGebra knowledge the teachers gained through the training to teach the Senior Phase transformation geometry topic in the South African mathematics curriculum. This comparison assisted the researcher to track and compare individual teachers' TPACK development levels' progression in each descriptor of the five TPACK levels, based on their experiences with GeoGebra before and after the training.

A FGI according to Kitzinger (1995) capitalises on participant discussions to gather insights. Kitzinger (1995) further asserted that the focus group approach enables participants to explore and refine their opinions through group interaction, yielding richer insights than individual interviews. In this part of the study, a FGI lasting 30 to 40 minutes was conducted with the teachers after the training, to identify their individual TPACK levels in each descriptor of the five TPACK levels within their school-based LS groups. Immediately after the training, teachers had to reflect on the training session by completing a reflection guide, to rate their competence on GeoGebra use in teaching transformation geometry concepts of reflection, translation and enlargement (Appendix 9). A FGI was then conducted for teachers to elaborate on their scoring on each item of the reflection guide in using GeoGebra to teach these transformation geometry concepts. This process allowed the researcher to assess teachers'

knowledge growth in GeoGebra from pre-training to post-training. The data gathered from the interview aided the researcher in comparing teachers' progression through the TPACK development model. These FGIs were audio-recorded and transcribed later for analysis.

3.5.3.3 The three RLs in the two-school based LS groups using GeoGebra

The section discusses the three RLs on the transformation geometry concepts of reflection along the x- and y-axis, reflection along the $y = x$ line and translation, which provided data to track individual teachers' progress in the TPACK development model after each RL.

The three transformation geometry RLs alluded to earlier were piloted first, before their actual implementation. Forgasz and Kaur (1997) asserted that “no matter how well conceived, some aspects of a research study may not run entirely to plan” (p. 196). For this reason, piloting or trialling research methods can increase the project's overall quality (Henk, 1987). A research pilot is “a study conducted prior to the major research study that in some way is a small-scale model of the major study: conducted for the purpose of gaining additional information by which the major study can be improved – for example, an exploratory use of the measurement instrument with a small group for the purpose of refining the instrument” (Wiersma, 1991, p. 427). The benefits of research pilots thus include substantial methodological improvements (Forgasz & Kaur, 1997). The main study's procedures were piloted to gain experience and assess potential shifts in TPACK, when teachers used GeoGebra and the LS as a school-based CPD to teach transformation geometry. Before the main study, the researcher piloted the data collection methodology (the three RLs, worksheets, observation, and interview guides including reflection guides) with one school in the JTG district whose demographics were similar to those of the schools used as research samples. The pilots aimed to identify areas for improvement, refine the research instruments, and enhance the methodology. The three transformation geometry RLs (RL1, RL2, and RL3) are presented below.

RL1 – reflection about the x- and y-axis

RL1 (see Appendix 10a) was the first of the series of three, and the mathematical topic addressed was that of reflection about the x- and y-axis. This RL collected qualitative and quantitative data that tracked the TPACK development levels for individual teachers in each descriptor of the five TPACK levels throughout the four RL stages of identification of objectives, planning, implementing/observing and reflecting. The researcher created the RL plan for RL1 for teachers in LS groups A and B to critique according to their learners' learning needs and the curriculum. LS groups A and B comprised teachers from schools A and B respectively. During the identification of objectives phase, the KO and each school's teachers jointly discussed the objectives of RL1, with guidance from the South African Senior Phase Curriculum and Assessment Policy Statement (CAPS). The RL focus was to ensure that learners can: show that the object ($\triangle EFG$) and its image ($\triangle E'F'G'$) are congruent, show that the corresponding vertices of $\triangle EFG$ and $\triangle E'F'G'$ are of equal distance from the reflection line, and identify which coordinate changes in sign when reflection is done about either the x- or the y-axis.

Using the selected objectives as a guide, the teachers of LS groups A and B and the KO planned RL1 jointly, using GeoGebra, within their respective schools. During planning, the KO shared his insights on teaching the transformation geometry concept of reflection about the x- and y-axis using GeoGebra, while the researcher introduced the teachers to the data collection instruments: observation and interview guides (including reflection guides) to be used within the LS cycle. The aim was to acquaint the teachers with the instruments in terms of what they would observe and reflect on both during and after the actual implementation of RL1. The planning stage also looked at specific identified learners from each school, one from each of three achievement levels (lower, middle and top achievers), and RL1 was tailored to their learning needs using GeoGebra. The mathematics teacher(s) in each school selected the three learners. While planning this RL, the teachers anticipated how the lower, middle and top achievers would engage with transformation geometry when reflecting $\triangle EFG$ about the x- and y-axis, using a worksheet first and then GeoGebra. Teachers in the two schools – A and B – reviewed and critiqued the RL plan provided by the researcher to ensure it met their learners' learning needs: In LS group A, when using a worksheet, the prediction was that the top achiever would successfully carry out learning activities (a) to (d), the middle achiever would struggle with learning activities (c) and (e), and the low achiever would only successfully carry out learning activity (a). When the learners were made to use GeoGebra, the prediction was that

all the categories of learners would successfully carry out the learning activities except the low achiever who would still struggle with activity (e). In LS group B, when using a worksheet, the prediction was that the top achiever would successfully carry out all the learning activities (a) to (e), the middle achiever would struggle with learning activity (e), and the low achiever would struggle with learning activities (b) and (e). When GeoGebra was to be employed, the prediction was that all the categories of learners in school B would successfully carry out all the learning activities except for the low achiever, who would still struggle with activity (e).

After planning, one of the three teachers facilitated RL1 while the other two teachers and the KO observed and collected data on learner learning of the transformation geometry concepts of reflection about the x- and y-axis, using an observation guide (see Appendix 11a) to determine how they performed in this RL. The teacher began RL1 by explaining the transformation geometry activity of reflecting $\triangle EFG$ about the x- and y-axis so that learners would understand what they were to do, but not helping the learners regarding how to do it – devolution. The activity required learners to reflect $\triangle EFG$ about the x- and y-axis using paper and pen. The learners began by trying to understand the transformation geometry problem autonomously within their groups (action situation), after which they had to formulate and share hypotheses about the problem (formulation situation) with their groupmates. This was followed by the learners convincing their classmates of the validity of the transformation geometry ideas on reflecting $\triangle EFG$ about the x- and y-axis that they had developed: a validation situation. With GeoGebra, the teacher helped the learners to recognise the transformation geometry knowledge gained in the context of their a-didactical work (reflecting $\triangle EFG$ about the x- and y-axis), and transform it into knowledge usable for solving other problems: an institutionalisation situation. The teacher used GeoGebra to consolidate the same pen and paper activity. The learners interacted with the teacher during the implementation of this activity using GeoGebra, while the other teachers and the KO were observing learner learning. When teaching RL1, the KO stepped in to provide clarity when the teacher got stuck while using GeoGebra.

Immediately after the RL was taught, the teachers were given a reflection guide (see Appendix 12a), which they completed and brought to the FGI for the researcher to follow up on their overall TPACK development. The teachers and the KO reflected on the four phases of RL1 – identification of objectives, planning, implementing/observing and reflecting – suggesting

recommendations for improving RL2 using GeoGebra. All four phases of the RL were captured on video and audio for later analysis by the researcher.

RL2 – reflection about the $y = x$ line

RL2 (see Appendix 10b) was the second of the series of three, and the mathematics topic addressed was that of reflection about the line $y = x$. As with RL1, RL2 also garnered qualitative and quantitative data that tracked the TPACK development levels for individual teachers in each descriptor of the five TPACK levels.

Just like in RL1, the researcher created the RL plan for RL2 for teachers in the two schools – A and B – to critique according to their learners’ learning needs and the curriculum. The KO and the teachers in each school’s LS group discussed the objectives of RL2 jointly, with the aid of the Senior Phase CAPS document. The RL focus was to ensure that learners could: identify how the x - and y -coordinates changed after reflecting rectangle ABCD about the line $y = x$, show that the object (rectangle ABCD) and its image (rectangle A'B'C'D') were congruent, and show that the corresponding vertices of rectangles ABCD and A'B'C'D' were equidistant from the line $y = x$.

Using these objectives as a guide, the teachers and the KO planned RL2 together using GeoGebra. When planning RL2, the teachers included some of the recommendations from RL1, to improve on the planning and implementation of RL2. During the planning, the KO shared his insights on leveraging GeoGebra to teach the transformation geometry concepts of reflection about the line $y = x$, while the researcher introduced the teachers to the data collection instruments: observation and interview guides (including reflection guides) to be used within the LS cycle. The aim was to get the teachers accustomed to the instruments in terms of what they would be observing and reflecting on both during and after the actual implementation of RL2. Similarly to the events of RL1, as the teachers planned RL2 they predicted how the lower, middle and top achievers would engage with transformation geometry when reflecting rectangle ABCD about the line $y = x$, first using a worksheet then GeoGebra. In both schools A and B the teachers critiqued the RL plan 2 presented to them by the researcher (see Appendix 10b) to meet their learners’ learning needs, as follows. In school A, when using a worksheet, the prediction was that the top achiever would successfully carry out learning activities from (a) to (e), the middle achiever would struggle with learning activity (c), and the low achiever would struggle with all the activities. When using GeoGebra in school A, the

prediction was that all the categories of learners would successfully carry out the learning activities except for the low achiever, who would still struggle with activity (e). In school B, when using a worksheet, the prediction was that the top achiever would successfully carry out all the learning activities from (a) to (e), the middle achiever would struggle with learning activity (e), and the low achiever would successfully carry out only learning activity (c). When using GeoGebra, the prediction was that all three categories of learners – low, middle and top achievers – in school B would successfully carry out all the learning activities except for the low achiever, who would still struggle with activity (e).

After planning, one of the three teachers facilitated RL2 while the other two teachers and the KO observed and collected data on the learner learning of the transformation geometry concepts of reflection about the line $y = x$, using an observation guide (see Appendix 11b). The teacher began RL2 by explaining the transformation geometry activity of reflecting rectangle ABCD about the line $y = x$ so that learners understood what they were to do, without giving them hints on how to do it. The activity required the learners to reflect rectangle ABCD about the line $y = x$ using the worksheet. The learners began by trying to understand this transformation geometry problem individually within their groups, after which they had to formulate and share their hypotheses with the other group members. Following this, the learners had to convince their classmates of the validity of the transformation geometry ideas that they had developed regarding reflecting rectangle ABCD about the $y = x$ line. The teacher then aided the learners to recognise the transformation geometry knowledge they had gained in the context of their a-didactical work (reflecting rectangle ABCD along the line $y = x$) and transform it into knowledge usable for solving other problems. After the pen and paper activity had been completed, the teacher utilised GeoGebra to illustrate and engage learners in reflecting rectangle ABCD along the line $y = x$. The learners interacted with the teacher during GeoGebra implementation for this activity, while the other teachers and the KO observed learner learning. During the implementation of RL2, the KO stepped in to provide clarity to the teacher in terms of GeoGebra use when they were challenged.

Immediately after RL2 was implemented, the teachers were given a reflection guide (see Appendix 12b), which they completed and brought to the FGI for the researcher to follow up on their overall TPACK development. The teachers and the KO reflected on the four phases of RL2 – identification of objectives, planning, implementing/observing and reflecting –

suggesting recommendations for improving the next RL using GeoGebra. As for RL1, the researcher video- and audio-recorded the four phases of RL2 for analysis purposes.

RL3 – translation

This RL (see Appendix 10c) was the last of the series of three, and the mathematical topic addressed was that of translation. As with RL1 and RL2, RL3 also collected qualitative and quantitative data, which tracked the TPACK development levels for individual teachers in each descriptor of the five TPACK levels.

Just like for RL1 and RL2, the researcher created the RL plan for RL3 for teachers in the two schools (A and B) to critique according to their learners' learning needs and the curriculum. The KO and the teachers of school A and B jointly discussed the objectives of RL3. The RL focus was to ensure that learners could show that: a translation produces congruent objects, and the distances between the coordinates of the corresponding vertices of the object (square ABCD, in this case) and its image (square A'B'C'D') are equal.

Using the geometric translation objectives as a guide, the teachers and the KO planned RL3 together using GeoGebra. LS groups A and B teachers and the KO included some of the comments from RL1 and RL2 to improve on the planning and implementation of RL3. During the planning, the KO shared insights on utilising GeoGebra to teach the transformation geometry concept of translation, while the researcher introduced the teachers to the data collection instruments – observation and interview guides to be used within RL3. The aim was to acquaint the teachers with the instruments in terms of what they would be observing and reflecting on during and after the actual implementation of the RL. Similarly to the events of RL1 and RL2, as the teachers planned RL3 they predicted how the lower, middle and top achievers would engage with transformation geometry when translating square ABCD 3 units to the left and 4 units upwards, first using a worksheet and then GeoGebra. In both schools A and B the teachers critiqued the RL plan 3 presented to them by the researcher (see Appendix 10c) to meet their learners' learning needs, as follows. In school A, when using a worksheet, the prediction was that the top and middle achievers would all successfully carry out learning activities (a) to (d), and the low achiever would struggle with learning activity (d). In school B, the prediction was that the top and middle achievers would successfully carry out all the learning activities from (a) to (d) on the worksheet, and the low achievers would only struggle with learning activity (d). When using GeoGebra, the prediction was that for both schools, all

the learner categories – low, middle and top achievers – would successfully carry out all the learning activities on the worksheet.

After planning RL3, one of the three teachers facilitated the RL while the other two teachers and the KO observed and collected data on learner geometric translation learning using an observation guide (see Appendix 11c). The teacher began by explaining the transformation geometry activity on translation to the learners so that they understood what they were supposed to do, but without helping them regarding how to do it. The activity required learners to translate square ABCD 3 units to the left and 4 units upwards, using paper and pen. Each learner began by trying to understand this transformation geometry problem autonomously within their group, after which they had to formulate and share their hypotheses with the other group members. This was followed by the learners convincing their classmates of the validity of the transformation geometry ideas on translating square ABCD 3 units to the left and 4 units upwards that they had developed. The teacher then helped the learners to recognise the geometric translation knowledge gained in the context of their a-didactical work (translating square ABCD 3 units to the left and 4 units upwards), and transforming it into knowledge usable for solving other geometric translation problems. After the pen and paper activity had been completed, the teacher utilised GeoGebra to consolidate this activity. The learners interacted with the teacher during the implementation of this activity using GeoGebra, while the other teachers and the KO were observing learner geometric translation learning. When teaching RL3 using GeoGebra, the KO stepped in to provide clarity to the teacher when he got stuck.

Immediately after the RL was taught, the teachers were given a reflection guide (see Appendix 12c), which they completed and brought to the FGI for the researcher to follow up on their overall TPACK development. The teachers and the KO reflected on the four phases of RL3 and suggested recommendations of improving the next RL using GeoGebra. Like RL1 and RL2, the researcher video and audio-recorded the four Phases of RL3 for analysis purposes.

3.5.3.4 Observation guides

The observation guides (see Appendix 11a, 11b, 11c) that collected data on learners' transformation geometry learning in the three transformation geometry RLs – which focused on the concepts of reflection about the x- and y-axis, reflection about the $y = x$ line and translation – are discussed in this section.

Observation according to Anthanasou et al. (2018, p. 91) is “a systematic process of collecting data that relies on a researcher's ability to gather data through their sense without questioning or communicating with participants.” Observation is a key data gathering technique in qualitative research, because it “offers the investigator (researcher) the opportunity to gather live data from naturally occurring social settings, allowing the researcher to look directly at what is taking place in situ rather than relying on second-hand accounts” (Cohen et al., 2011, p. 456). As observation was an integral part of the LS, for consistency in data collection when implementing the three transformation geometry RLs, the teachers and the KO were given a pre-determined observation guide in each RL to observe pre-defined aspects of the RL plans (McKechnie, 2008). The observation guides included questions related to the success criteria of each of the three transformation geometry RLs, and provided space for the teachers to comment on their chosen opinion. While a teacher in each LS group voluntarily taught a RL, the other two teachers and the KO observed and collected data on learner learning of transformation geometry concepts of reflection and translation, using a worksheet first and then GeoGebra, which helped in answering research question three: What are rural high school mathematics ISTs' experiences in using GeoGebra to teach transformation geometry?

3.5.3.5 Interview (FGI) guides for each of the three RLs

This section describes FGIs which garnered data that tracked individual teachers' TPACK development levels in each descriptor of the five TPACK levels, as shown by their self-reported scores and the researchers' interrater reliability scores after each RL.

The TPACK framework shaped the reflection guides used in the interview guides (see Appendix 12a, 12b, 12c). Immediately after a RL was taught, the researcher gave the teachers a reflection guide which required them to tick an appropriate box to rate their competence in using GeoGebra to teach a particular transformation geometry concept. The teachers carried the reflection guide with them to the FGI for the researcher to follow up on their ratings. This allowed for clarification of any unclear answers on the teachers' ratings so that the relevant

TPACK levels could be adjusted. As the teachers reflected on their ratings of the items on the reflection guide, they drew from the four stages of the transformation geometry RL – identification of objectives, planning, implementing/observing and reflecting. The KO also reflected on the four phases of each RL, and suggested recommendations for improving the next RL using GeoGebra.

Following Groth et al.'s (2009) LS-TPACK design (see Section 2.7), the researcher asked teachers to share their perceptions of the transformation geometry RL during the reflection meeting, beginning with the teacher who taught it and concluding with the KO. These interviews, which lasted approximately 30 to 40 minutes each, aided the researcher in tracking the TPACK development levels of individual teachers as shown by their self-reported scores and researchers' interrater reliability scores, thus addressing the study's second question: What is the level of TPACK developed when rural high school mathematics teachers use GeoGebra to teach transformation geometry during a school-based LS CPD approach?

3.5.4 Data analysis procedures

The preceding section described the data collection instruments. This section details how the data from each research tool were analysed.

According to S. Merriam (1998), analysis involves systematically dissecting data to extract insights and meaning. The study used qualitative and/or quantitative methods to analyse data from each research instrument – the baseline survey, the FGI guides on teacher training on GeoGebra and LS, the three recorded and transcribed transformation geometry RLs, the observation guides, and FGIs conducted after each of the three RLs. Firstly, the researcher discussed how the baseline survey data were analysed. Secondly, the researcher discussed how the qualitative and quantitative data were analysed that had been gathered from FGI transcriptions of the teacher training session on using GeoGebra and the LS. Thirdly, the data analysis from the three recorded and transcribed RLs and the LS was discussed. Before the main study, the RLs and associated data collection tools (worksheets, observation guides, reflection and interview guides – including reflection guides) underwent piloting to validate their data collection efficacy.

3.5.4.1 Data analysis from the baseline survey

The baseline survey data were qualitatively analysed using descriptive statistics. Tables, graphs and percentages were used to present the results from the baseline survey. The baseline survey aimed to give an understanding of the use of technology in particular, GeoGebra – in mathematics teaching in the JTG district, and the CPD types in schools (a school-based LS CPD approach, in this case) that the district's teachers were exposed to thus, addressing the initial research question – What is the extent of the use of GeoGebra and the LS as a school-based CPD approach in the teaching of mathematics in JTG district's high schools prior to the main study? The base line survey results (see Section 4.2) also reinforced the study's position on the problem statement: no research has ever been done in South Africa using GeoGebra and the LS as a school-based CPD to develop the TPACK knowledge of mathematics teachers in rural high schools.

3.5.4.2 Data analysis from teacher training on GeoGebra and the LS

Data analysis from teacher training on GeoGebra and the LS was based on the recorded FGI transcriptions, which FGI was conducted immediately after the training session with each school's LS group. A deductive approach was used for data analysis at this stage. Deductive thematic analysis applies existing theoretical frameworks to identify and analyse themes in data. Deductive thematic analysis starts with previous research and theories, aiming to validate or extend theoretical knowledge (Braun & Clarke, 2006). Recorded FGIs were transcribed verbatim, read several times, and analysed using deductive thematic analysis. The coding of FGI transcripts was guided by the TPACK rubric of Niess et al. (2009) (see Appendix A). The data were organised based on the descriptors of the five TPACK levels. This data helped the researcher to track teachers' TPACK development levels as shown by their self-reported scores and the researchers' interrater reliability scores in each descriptor of the five TPACK levels after training.

Additionally, a quantitative data analysis using a repeated-measure ANOVA test was used to establish whether researchers' ratings on teachers' TPACK knowledge in schools A and B differed significantly across the four stages of professional development (training, RL1, RL2, and RL3) for each of the four TPACK dimensions: curriculum and assessment, learning, teaching, and access. The analysis combined schools A and B. In the quantitative data analysis using the ANOVA test, 'M' stands for the mean, 'F' for the test statistic value, and 'P' for the

probability value. The teachers and the researchers' self-rated scores are per training session and per RL conducted in each school. The mean of the researchers' interrater reliability scores for the two schools' training sessions (one session each) and RLs (three RLs per school) was obtained by adding the researchers' interrater reliability scores of the six teachers across four descriptors of the five TPACK development levels and dividing by 6, the total number of the researchers' interrater reliability scores.

At this stage, the researcher was looking for data on teachers' understanding of GeoGebra use before the training and what GeoGebra knowledge they had gained throughout the training to teach transformation geometry. The data aided the researcher to track teachers' advancement through the TPACK model, informed by their experience with GeoGebra before and after the training, thus answering the second research question – the level of TPACK developed when rural high school mathematics teachers use GeoGebra to teach transformation geometry during a school-based LS CPD approach.

3.5.4.3 Data analyses from three recorded and transcribed RLs and the LS

Data analysis from the three recorded and transcribed transformation geometry RLs (on the concepts of reflection about the x- and y-axis, reflection about the line $y = x$ and translation) and the LS is presented here.

Prior to the actual data collection, these three transformation geometry RLs (comprising RL plans, observation and interview guides) were piloted at a school with similar demographics to those of the schools used as research samples. Following the research pilots was the actual data collection from the two schools (A and B). Each school conducted three transformation geometry RLs (on the concepts of reflection along the x- and y-axis, reflection along the line $y = x$ and translation) which were recorded and transcribed for analysis purposes. The piloted instruments' findings and actual data analysis are discussed through the RL stages articulated in the sections below.

3.5.4.3.1 Implementation stage

The section discusses the findings from the pilots of the observation guides, before discussing how the actual data collected by these instruments were analysed.

In the implementation stage, observation guides (see Appendixes 11a, 11b, 11c) were used to observe learners' transformation geometry learning. When piloting each of the three-transformation geometry RLs, each RL's observation guide was also piloted. The intention was to test the instrument's efficiency in collecting data before proceeding with the main study. Data from the observation guide was used to brief the next RL teacher to ensure each of the three case learners (low, medium and top) would have an opportunity to respond to a posed question, making it easier for the observer teachers to comment on the learning of each of these three learner categories on the observation guide.

The actual data from the observation guides were analysed in two steps. Data analysis from the first step arose from teachers' comments on learners' learning when they solved questions on the worksheet first, which they worked on manually. Data analysis from the second step arose from teachers' comments on learners' learning when the teacher utilised GeoGebra to consolidate the pen and paper activity. This data helped address the third research question – the experiences of rural high school mathematics teachers teaching transformation geometry using GeoGebra.

3.5.4.3.2 Reflection stage

In this section, the findings from the pilots of the interview guides are discussed first, followed by an explanation of the data analysis process for the main study.

In the reflection stage, the researcher used an interview guide including a reflection guide for teachers to reflect on the RL stages of identification of objectives, planning, implementing/observing and reflecting (see Appendixes 12a, 12b and 12c). Before the main data collection, the interview guides were tested during trial runs of the three transformation geometry RLs. The data from the pilot FGIs conducted after each RL pilot informed improvements to the instrument's quality, by including a question (question 2) that required teachers to reflect on the four RL phases.

Given the textual nature of interview data, a deductive thematic analysis approach was applied to the FGIs conducted after each transformation geometry RL. According to Braun and Clarke

(2006, p. 79), thematic analysis is “a method for identifying, analysing and reporting patterns (themes) within data.” Interviews were transcribed and reread many times. As the researcher went through this data, he marked important sections and added descriptive names or codes to these sections to identify them for further analysis. These codes were analysed to find similarities and then grouped into descriptors of the five TPACK levels. Sentences that were vital were identified to be utilised for further articulation. This data aided the researcher to track teachers’ TPACK development levels, as shown by their self-reported scores and the researchers’ interrater reliability scores in the five TPACK level descriptors.

In addition, a quantitative data analysis using a repeated-measure ANOVA test was used to establish whether researchers’ ratings on teachers’ TPACK knowledge in schools A and B differed significantly across the four stages of professional development (training, RL1, RL2, and RL3) for each of the four TPACK dimensions: curriculum and assessment, learning, teaching, and access. As with the training, the teachers’ self-rated scores and researchers’ interrater reliability scores are per RL conducted in each school. In the quantitative data analysis using the ANOVA test, ‘M’ stands for the mean, ‘F’ for the test statistic value, and ‘P’ for the probability value. The teachers and the researchers’ self-rated scores are per training session and per RL conducted in each school. The mean of the researchers’ interrater reliability scores for the two schools’ training sessions (one session each) and RLs (three RLs per school) was obtained by adding the researchers’ interrater reliability scores of the six teachers across four descriptors of the five TPACK development levels and dividing by 6, the total number of the researchers’ interrater reliability scores.

The interview guides helped the researcher to track teachers’ TPACK development levels as shown by their self-reported scores and researchers’ interrater reliability scores in each descriptor of the five TPACK levels, thus answering the second research question – the level of TPACK developed when rural high school mathematics teachers use GeoGebra to teach transformation geometry during a school-based LS CPD approach.

3.5.5 Trustworthiness of the study

The preceding section discussed how the data collected by each research instrument were analysed. In this section, the researcher discusses the trustworthiness strategies used to demonstrate the accuracy of the research findings.

The study employed a mixed methods complementarity design, blending quantitative and qualitative approaches, guided primarily by an interpretive paradigm, while acknowledging pragmatic considerations. The researcher considered qualitative trustworthiness criteria (credibility, transferability, confirmability, and dependability) to ensure these findings were rigorous. For quantitative findings, he focused on reliability and construct validity to ensure they were trustworthy. The study's strategies for establishing these trustworthiness criteria are explained in detail below.

Credibility refers to how much confidence one can have in the truth of research results (Macnee & McCabe, 2008). Credibility determines if research findings accurately reflect the information from participants' original data and correctly interpret their views (Graneheim & Lundman, 2004). There are multiple ways in which credibility can be enhanced when collecting data and when analysing the data. In this study, to ensure credibility, mechanical means to record the data, triangulation, and member checks techniques were employed.

The first strategy used to ensure credibility was use of mechanical means to record the data. The researcher utilised an audio-recording device to capture interview verbatim, allowing the transcripts to be more precise compared to if the researcher had only taken notes during the interview. The second strategy used in the study was member checking. Member checking entails sharing data and interpretations with participants of the study for feedback (J. W. Creswell & Miller, 2000). Teachers were given reflection guides to complete and self-rate, which were then discussed in the FGI. The researcher returned the findings and transcribed FGIs to the teachers to verify their accuracy and confirm they reflected their views. This allowed the researcher to correct errors of fact from the reflection guides. Lastly, triangulation techniques were used to increase data credibility. Triangulation involves utilising multiple and different approaches to cross-validate and confirm findings (Bertram & Christiansen, 2014). Triangulation assumes that single-method approaches are inherently biased and limited, so multiple methods are employed to get a more comprehensive understanding of a phenomenon under study (individual teachers' TPACK levels, in this case) (Greene et al., 1989). Triangulation therefore aids the researchers to cross-check findings from multiple instruments

for consistency (Migiro & Magangi, 2011). This study utilised multiple data-generating methods to ensure credibility. Data were gathered via a baseline survey, FGIs on teacher training on GeoGebra and the LS, and the three recorded and transcribed transformation geometry RLs using GeoGebra, and observation guides. Using different data collection methods aided the researcher to verify each individual teacher's TPACK levels in each descriptor of the five TPACK levels. For example, findings from reflection guides were cross-checked with FGI data for verification. During FGIs, the researcher asked follow-up questions on the teachers' ratings on the reflection guide. This helped to check whether the teacher's TPACK level (on a particular descriptor of the five TPACK levels) on the reflection guide confirmed or contradicted that from the FGI. During teaching, the two teachers and the KO in each school (A and B) simultaneously observed learner transformation geometry learning using a pre-determined observation guide. This aided the researcher to verify if data from one teacher's observation guide aligned with or diverged from data from other teachers' guides. The teachers' responses on the observation guides were also validated using information from the video-recorded RLs to ascertain whether what they jotted down was what had transpired during the implementation of the RL. Additionally, FGI data were cross-validated with recorded RL planning meetings and RL implementation data to see if the data from each source (in relation to the descriptors of the five TPACK levels) contradicted or supported the others. Furthermore, the study employed methodological triangulation through using different research methods (Lincoln & Guba, 1985). The study primarily used two methods. First, a baseline survey was administered to all teachers in the 31 high schools in JTG district, a remote district of the Northern Cape Province in South Africa. The baseline survey aimed to establish the extent of the use of technology (in particular, GeoGebra) in mathematics instruction, and the CPD types – LS as a school-based CPD approach, in this case – to which teachers in this district had been exposed. The second method was a multiple case study of two under-resourced rural high schools (A and B) that participated in a school-based LS CPD approach to develop their teachers' TPACK when teaching transformation geometry using GeoGebra. The two schools were selected from the 31 high schools in the district.

Transferability means how well the findings of a qualitative study can be applied or carried over to different contexts or with other groups of people (Tobin & Begley, 2004). A qualitative researcher increases the credibility of their study by using transferability tactics like providing detailed (thick) descriptions and applying purposive sampling (Bitsch, 2005). Each strategy is discussed below.

The first strategy used for transferability was providing thick descriptions. Thick description “enables judgments about how well the research context fits other contexts, thick descriptive data, i.e. a rich and extensive set of details concerning methodology and context, should be included in the research report” (Li, 2004, p. 305). In ensuring transferability of this study, the researcher detailed every step of the research – from the study’s context and data gathering to creation of the final report. This will assist other researchers in deciding if the results can be applied to different contexts. The second strategy used to ensure transferability was purposive sampling. Purposive sampling yields more detailed insights compared to other probability sampling techniques (Cohen et al., 2011). In this study, following the baseline survey, two rural schools were selected purposively from the 31 high schools in the JTG district for further study.

Confirmability is “concerned with establishing that data and interpretations of the findings are not figments of the inquirer’s imagination, but are clearly derived from the data” (Tobin & Begley, 2004, p. 392). It involves addressing the question: How do we know if the findings come solely from participants and the investigation was not influenced by the bias, motivations or interests of the researchers? (Guba, 1981). Confirmability is crucial in qualitative research because it boosts objectivity and reduces researcher bias (Trochim, 2006). In this study, the researcher’s role was described thoroughly to reduce the impact of research bias.

Dependability refers to “the stability of findings over time” (Bitsch, 2005, p. 86). It involves addressing the question: How can researchers be sure that the results would be consistently reproduced with the same (similar) participants in an identical setting? (Guba, 1981). Dependability in this study was established using triangulation. Additionally, the transformation geometry RL stages (of identification of objectives, planning, implementing/observing, and reflecting) were captured on video and audio, and these recordings were examined repeatedly to objectively assess individual teachers’ TPACK levels. The researcher transcribed these recordings manually.

Construct validity is the extent to which the data collection methods capture what the researcher is after (Bertram & Christiansen, 2014). To ensure validity of the research instrument, the researcher-developed instrument underwent rigorous validation procedures. Firstly, another researcher (not involved in the study, but familiar with TPACK) reviewed the data collection tools (baseline survey, the three transformation geometry RLs, worksheets, observation and interview guides – including reflection guides). Secondly, before rolling out the baseline survey district-wide, the researcher tested it with mathematics teachers in a single

school, and their feedback was used to fine tune the baseline survey. This helped identify questions that were unclear, and ensured the baseline survey generated data relevant to the study. Lastly, in preparation for the data collection phase, the study's data collection methodology was pilot tested by the researcher with one school in the JTG district whose demographics were similar to those of the schools used as research samples. The pilots sought to identify areas that needed to be improved, in terms of the instruments' quality and the methodology itself.

Reliability concerns the consistency of findings when other researchers follow the same procedures (Campbell & Ahrens, 1998). To enhance result reliability of quantitative findings, inter-rater reliability was employed. The consistency of the findings or dependability of the data was promoted by the researcher working with two other postgraduate students (a master's and a PhD student) who were familiar with the TPACK development model. These students assisted in independently rating the overall TPACK level of an individual teacher in each descriptor of the five TPACK levels. Where there was no variation in the ratings of the three raters, the rating was given, and where there were variations in the ratings, the researcher and assistants discussed the ratings together to come up with a common rating.

3.6 Ethical considerations

The section discusses the ethical principles that the researcher followed to ensure responsible and credible findings in this study.

Ethics plays a crucial role in research, especially when humans or animals are involved, and concerns moral principles (Bertram & Christiansen, 2014). The researcher followed the University of Cape Town's Faculty of Science Research Ethics Committee's (FSREC) principles, upholding respect and dignity, transparency and honesty, accountability and responsibility, and integrity and academic professionalism. More information on these principles can be found in the following sections.

3.6.1 Respect and dignity

Before obtaining consent, the researcher fully informed teachers about the study's objectives, and process, and their expected involvement. Teachers were assured they could leave the study whenever they wanted. Participants were also guaranteed confidentiality, with data only accessible to the researcher and supervisor, unless consent was given (see Appendix 4). Additionally, the researcher obtained the consent of both the learners (see Appendix 6) and

their parents or guardians (see Appendix 7) by sending a letter to each one of them with an attached reply slip. The letter articulated the study's nature, assuring parents or guardians of learners' confidentiality and anonymity, and of the restricted access to recordings – the recordings would only be accessible to the research team.

To protect identities, fictitious names were assigned to teachers and research sites in the final thesis. The names used for the research sites were 'school A' and 'school B'. 'Bosco', 'Sarah', and 'Frank' were the pseudonyms used for the teachers at school A, while 'Bernard', 'Tom', and 'Mike' were the fictive names used for school B's teachers. 'Lawrence' was the pseudonym chosen for the KO.

On completion of the study, the findings will be shared with the teachers in their respective LS groups, either through direct communication or in a feedback workshop. The researcher will also make the findings from this study available to the Northern Cape Department of Education (NCDoE) and the principals of the participating schools. Feedback will be shared with teachers and other stakeholders in this study in a clear and understandable manner (e.g. a suitable summary rather than a copy of the dissertation).

3.6.2 Transparency and honesty

Prior to the study, the researcher secured ethical clearance from the FSREC at the University of Cape Town (see Appendix 2). This was followed by the researcher seeking permission from the head of the NCDoE to conduct research in the selected schools in JTG district. Lastly, permission was obtained from the principals of the schools where the research was conducted (see Appendix 5). Transcripts of the FGIs were returned to teachers for validation and comment.

3.6.3 Accountability and responsibility

The study adhered to the ethical principles and guidelines stipulated by the University of Cape Town's FSREC. During the research journey, the researcher was responsible for creating a conducive environment. Additionally, the researcher was responsible for keeping all the collected data safe. The data was stored securely on the researcher's password-protected laptop throughout the study. The researcher also contacted the supervisor for advice regarding safely keeping this data.

3.6.4 Integrity and academic professionalism

The thesis is the researcher's original work, with proper citations and references to others' ideas, following the University of Cape Town's referencing guidelines. The study's findings are presented authentically, without any data manipulation.

3.7 Summary

The study's research methodology and approach were presented in this chapter. The researcher began by outlining the research aim. Additionally, there was discussion of an interpretive research paradigm in which the study was foregrounded. The study's research design was then articulated, incorporating a baseline survey and case study approach. Thereafter, the research population and research sample were discussed. Following this, a discussion was presented regarding the data collection instruments – baseline survey, FGIs conducted after the teacher training session on GeoGebra and the LS, and the three recorded and transcribed transformation geometry RLs, and observation guides. These research instruments (a baseline survey, the three RLs, observation and interview guides – including reflection guides) were piloted to check their efficiency in terms of collecting data. The data analysis approach for each of the instruments was further elaborated. The study's trustworthiness and ethical aspects were discussed in the chapter's final section. The following chapter describes the results from the data collection instruments.

CHAPTER FOUR: FINDINGS OF THE STUDY

4.1 Introduction

The study aimed to investigate rural high school mathematics teachers' TPACK development when using GeoGebra and the LS approach as a form of school-based CPD to teach transformation geometry. This chapter's analysis is informed by data from the baseline survey, the FGI conducted after the teacher training session on GeoGebra and the LS, the three recorded and transcribed transformation geometry RLs, observation guides, and FGIs (including reflection guides) conducted with teachers after a RL was taught. This chapter first analyses the baseline survey data. Following this is an analysis of data gathered from the transcribed FGI on the teacher training session on GeoGebra and the LS. An analysis of the data from the three transformation geometry RLs is also discussed, before a summary of each research instrument's findings.

The teachers' real work and words from the data collection instruments (see Section 3.5.3) are used to support the data presentation, with teachers identified by code names as described in section 3.5.2.1.

4.2 The baseline survey

The section reports on the findings of the baseline survey (see Appendix 8), which was carried out with the Northern Cape province of South Africa's JTG district's Senior and FET Phase mathematics teachers prior to the main study. The baseline survey aimed to establish the extent of the use of technology (in particular, GeoGebra) in mathematics teaching, and the CPD types (LS as a school-based CPD approach, in this case) to which teachers in this district were exposed. Data collection occurred during term one of the 2023 academic year, through a baseline survey with 135 Senior and FET Phase mathematics ISTs. The baseline survey was conducted in the Northern Cape Province of South Africa's JTG district, with a purposive sample of six teachers from the 135 teachers at two high schools (A and B) within the study's geographical delimitations. The baseline survey data were qualitatively analysed using descriptive statistics. The findings of the baseline survey were presented utilising tables, graphs, and percentages. To maintain confidentiality, each teacher's baseline survey was coded with a number from Tr1 to Tr135 (see Section 3.4.1). The analysis was structured around three key areas: teacher demographics, technology use (specifically GeoGebra) in mathematics teaching, and familiarity with LS.

4.2.1 Demographic information of teachers in the baseline survey

In this section, the demographic characteristics of the surveyed teachers are discussed. This information includes the teachers' gender, the quintile ranking of their schools, their teaching qualifications and their teaching experience.

In the baseline survey, there were 82 males and 53 female teachers. Of these 135 surveyed teachers, 81% and 16% respectively indicated that they were teaching in government schools where learners are exempted or not exempted from paying school fees, while 3% were teaching in private schools. In the basic education context of South Africa, public schools are grouped into five quintiles, primarily for allocation of funds (Van Dyk & White, 2019). Schools in the quintile 1-3 category do not collect school fees from learners, as these serve low-income socio-economic communities, while learners attending schools categorised in quintile 4 and 5 pay fees. The quintile ranking ranges from 1 (poorest schools) to 5 (wealthiest schools). In quintile 1 schools, which are poorly resourced and in low-income socio-economic communities, educational technology integration, such as GeoGebra, is scarce (Mthethwa et al., 2020).

Additionally, the researcher noted that all participating mathematics teachers were qualified for their specific teaching roles. The baseline survey showed nearly three-quarters of the respondents (72%) were in possession of at least a mathematics teaching degree, while only 28% held either a diploma or a certificate. These results differ from those of Stols et al. (2015), who asserted that South Africa struggles with a number of socio-economic problems – including but not limited to poorly- or under-qualified mathematics and science teachers – that are deeply entrenched in the social fibre of the country. The researcher also noted that most (70%) of the teachers who completed the baseline survey have taught mathematics for five years or more.

4.2.2 The extent of technology use in mathematics teaching

The preceding section discussed the teacher demographics from the baseline survey. This section discusses findings from technology use in mathematics teaching. Following that is a discussion of the teachers' experiences with technology in mathematics teaching.

4.2.2.1 Technology use in mathematics teaching

In the baseline survey, 68 (50%) of all the teachers surveyed indicated they did not use technology in teaching mathematics, while 62 (46%) indicated that they did. The latter specified the mathematics-related software that they use or have used in mathematics teaching (see Table 4.1).

Table 4.1 Mathematics software used by teachers in their lessons

Mathematical software	SageMath	FreeMat	GeoGebra	None	Other(s)	Spoilt
Percentage (%)	5	2	34	50	7	2

While a number of software packages are utilised by the JTG district's teachers (see Table 4.1), GeoGebra is utilised the most. Forty-six (34%) of the teachers surveyed indicated using GeoGebra, while 87 (64%) of the respondents indicated that they did not. It is therefore clear that GeoGebra is underutilised by JTG district's teachers. Teachers recounted their experiences using GeoGebra in mathematics classes.

4.2.2.2 Teachers' experiences of integrating GeoGebra into the mathematics classroom

This part of the baseline survey discusses the benefits and challenges of using GeoGebra in mathematics teaching. The legends (D, N, A and SA) used in the figures below stand for: D = disagree, N = neither agree nor disagree, A = agree, and SA = strongly agree.

4.2.2.2.1 Advantages of using GeoGebra in the teaching of mathematics

This section presents the advantages of using GeoGebra in mathematics teaching according to the teachers who indicated using the software in Table 4.1. Since GeoGebra is used alongside other software in mathematics classes, it has the same advantages as the general use of technology; there was general agreement among the teachers surveyed about technology's benefits in mathematics instruction. However, GeoGebra's specific benefits – this study's focus – are summarised in Figure 4.1.

Figure 4.1 Advantages of GeoGebra in transformation geometry teaching

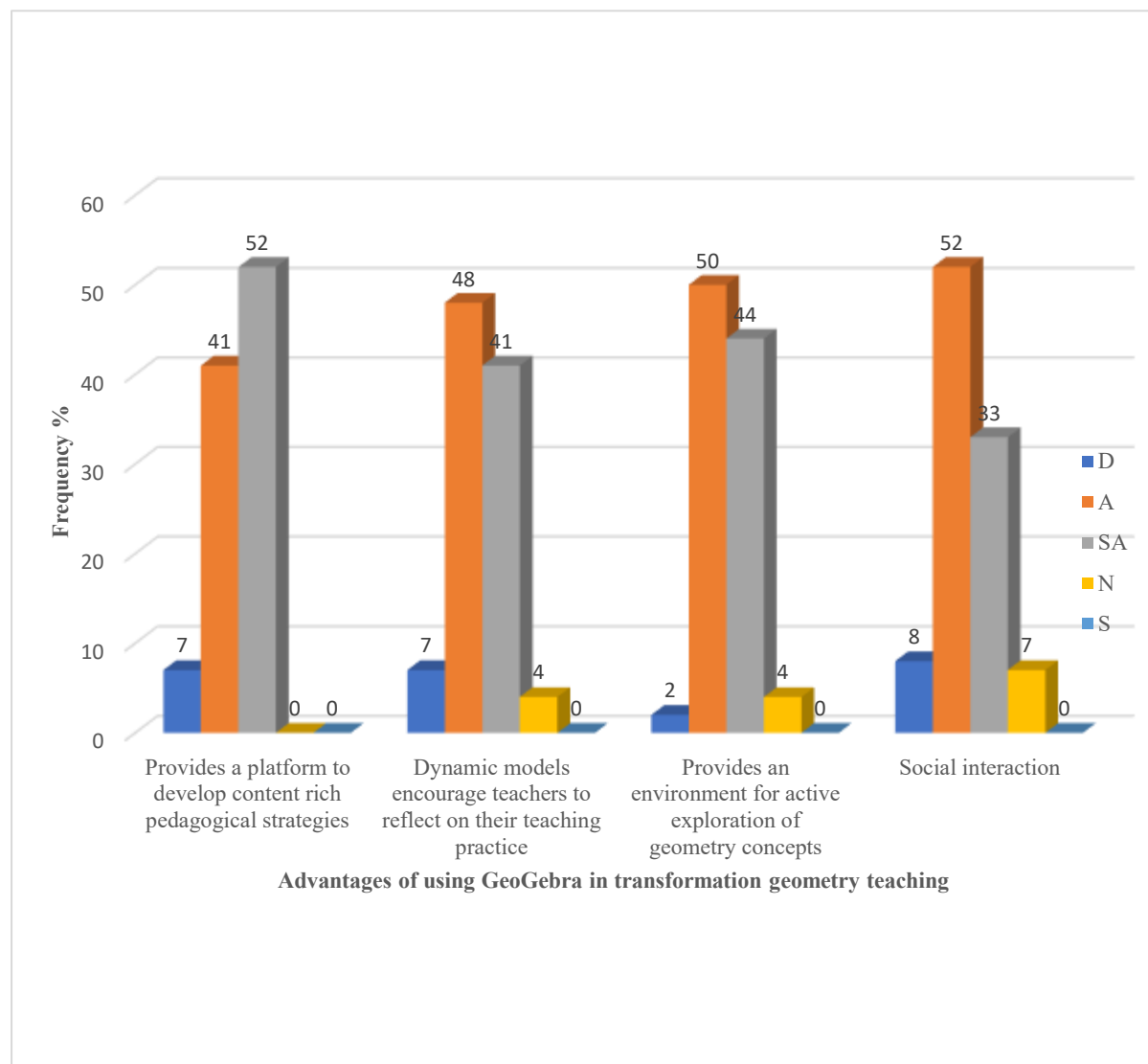


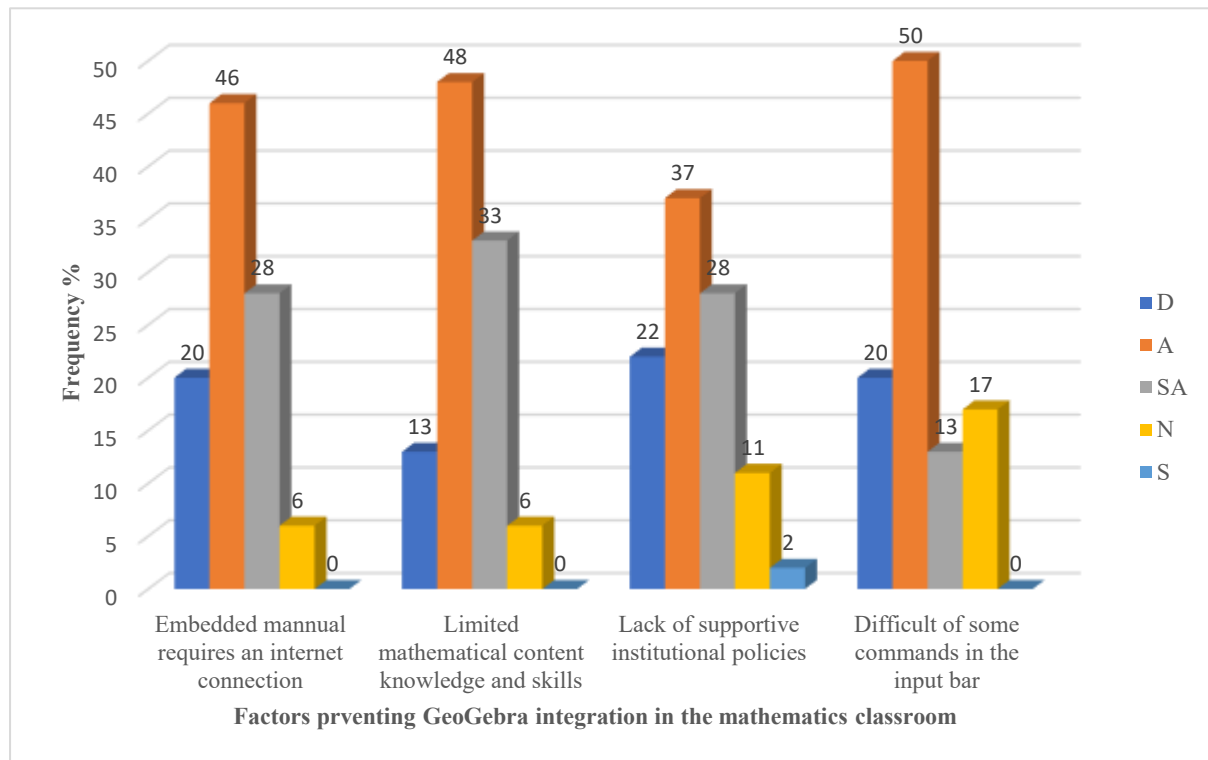
Figure 4.1 reveals there was broad consensus among the surveyed teachers on the benefits of GeoGebra in transformation geometry teaching. However, what is most prominent here is that GeoGebra enables active exploration of transformation geometry concepts as 94% of the teachers agreed or strongly agreed with this advantage. This finding is consistent with what Zulnaidi and Zamri (2017) found in Malaysia: that GeoGebra has the capacity to create a conducive teaching and learning environment, in which learners can explore and make meaning of transformation geometry concepts through multiple representations and dynamic observations that are not possible with pen and paper, thus increasing their achievement. In studies from South Africa, Mokotjo (2020) and Mthethwa et al. (2020) attested that GeoGebra facilitates discovery, exploration and visualisation of abstract mathematics concepts, thus improving learners' academic performance.

4.2.2.2.2 Factors that prevent teachers from integrating GeoGebra into the classroom

However, despite recognising GeoGebra's benefits in mathematics instruction, participating teachers reported significant challenges in its use. This section presents the factors that led to teachers' inability to integrate GeoGebra in mathematics lessons, as reported by the teachers who indicated using this software in Table 4.1.

Since GeoGebra is often used alongside other software, the factors that prevent GeoGebra integration into mathematics instruction mirror the broad challenges of technology adoption in teaching. From the baseline survey responses, there was general agreement among the teachers surveyed about the factors leading to their inability to incorporate technology into mathematics instruction. The factors that prevent participating mathematics teachers from incorporating GeoGebra specifically into their lessons – the study's focus – are summarised in Figure 4.2.

Figure 4.2 Factors that prevent teachers from integrating GeoGebra into the classroom



Of the 46 teachers (34%) out of all the respondents who indicated having taught with GeoGebra (see Table 4.1), most agreed that several factors prevented them from incorporating the software into their mathematics lessons (see Figure 4.2). However, 81% of the respondents agreed or strongly agreed that limited mathematical content knowledge and skills prevented them from integrating GeoGebra into their lessons.

Other factors preventing GeoGebra integration in mathematics lessons were also identified by the same teachers in answer to the open-ended question (see Appendix 7, question 16). These included loadshedding (power cuts), device limitations, short periods, inadequate training, classroom management challenges and work overload. Lack of training was the factor identified by most of the respondents – 24 out of the 46 teachers who indicated using GeoGebra in Table 4.1. For instance, one teacher (Tr114) pointed out: “*I lack the knowledge of how to use GeoGebra to teach mathematics concepts.*” Tr5’s baseline survey response aligned with the challenge highlighted by Tr114: “*...a lack of support from the department. When we attend workshops, the officials encourage us to use technology but they do not support us on how to integrate it in the classroom.*”

While other factors that prevent GeoGebra integration in the mathematics classroom are important, the major ones which are most linked to this study and cited by most of the

respondents were a dearth of training, a shortage of devices, and limited skills. Jezdimirović (2014) and Wassie and Zergaw (2019) highlighted challenges in incorporating GeoGebra in Serbian and Ethiopian mathematics classrooms respectively, including teachers' limited content knowledge, inadequate technological resources, limited pedagogical knowledge, inadequate CPD and skills. These challenges were echoed by Mokotjo and Mokhele (2021) in South Africa.

It is evident from the baseline survey that the few teachers who indicated using GeoGebra (see Table 4.1) generally agreed on the factors that prevented them from integrating the software into the mathematics classroom. The factors pertinent to this study included, *inter alia*, inadequate training, shortage of resources and limited GeoGebra skills. This necessitated engaging the research sample of teachers from two schools (A and B) in a LS as a school-based CPD to develop their TPACK in using GeoGebra for teaching transformation geometry.

4.2.3 The extent to which teachers are familiar with the LS

The previous section presented findings on technology integration in mathematics teaching, before discussing the findings regarding teachers' experiences with technology in their classes. This section discusses the findings relating to the types of CPDs that teachers have participated in in their school mathematics departments. The advantages of being part of the LS are then discussed, followed by a discussion of the findings regarding the challenges in implementing the LS.

4.2.3.1 Types of CPDs teachers have participated in in their school mathematics department

Table 4.2 summarises the findings from the types of CPDs that high school mathematics teachers in the JTG district have participated in in their school mathematics department.

Table 4.2 Types of CPDs taking place in schools

Types of CPDs taking place in schools	Workshops outside school	LS as a school-based CPD	Seminars at school	Seminars outside school	Other(s)	Spoilt
Percentage (%)	52	26	6	9	2	5

While teachers in the JTG district had been exposed to different types of CPDs, CPDs outside school were the most attended. Of the teachers surveyed, 43 (32%) indicated participating in CPDs at school, whilst 85 (63%) indicated attending CPDs outside school (see Table 4.2). This finding corroborates Mhakure's (2019) assertion that in Education Departments in South

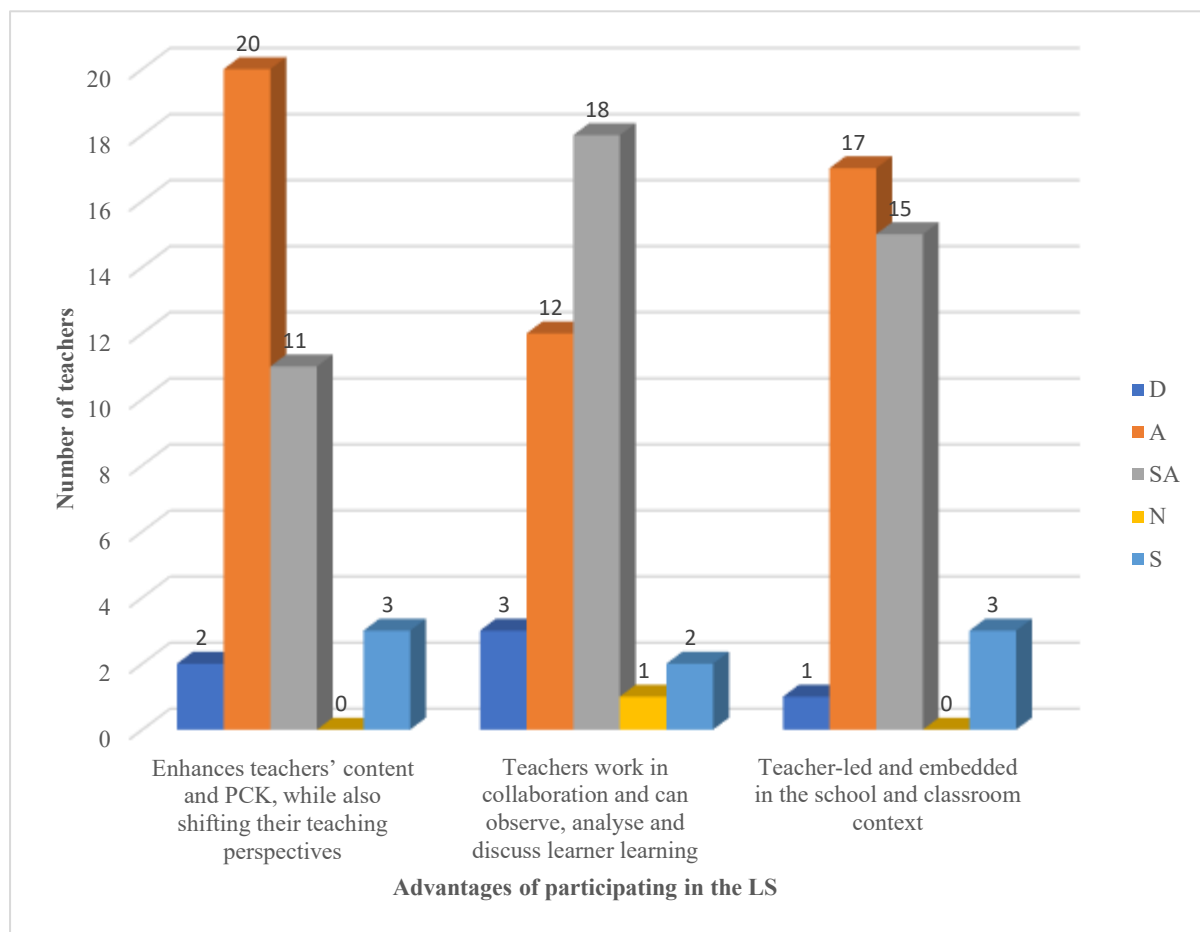
Africa, CPD activities often occur outside schools, led by external experts. The experts include, *inter alia*, mathematics subject advisors and teacher educators from higher learning institutions. Teachers in rural schools attend generic CPD workshops outside of their schools with teachers from other schools that are facilitated by external experts – a one-size-fits-all scenario. These experts are not familiar with these schools’ contexts and teaching cultures, thus rendering their CPD efforts inadequate because they barely address the institutional challenges experienced by the teachers within their individual schools (Mhakure, 2019).

Additionally, it is evident from Table 4.2 that most of the surveyed teachers in the JTG district were not familiar with the LS – out of all the respondents, only 26% indicated participating in the LS, while 93 (69%) of the respondents indicated they had not. Despite its potential for meaningful teacher CPD, LS is not commonly used as a CPD approach in South Africa (Letloenyane & Jita, 2015; Posthuma, 2012). There was therefore a need to introduce teachers to the LS as a form of school-based CPD to develop their TPACK.

4.2.4.3 Advantages of being part of the LS

This section discusses the advantages of participating in the LS reported by teachers who indicated familiarity with the LS in Table 4.2. Figure 4.3 summarises these results.

Figure 4.3 Advantages of participating in the LS



The data in Figure 4.3 reveals strong agreement between respondents on the advantages of LS participation. However, the most prominent benefit of the LS highlighted by the teachers is that the LS is teacher-led and embedded in the school and classroom context – 32 out of the 36 teachers (see Table 4.2 and Figure 4.3) who indicated having participated in LS before agreed or strongly agreed with this advantage. CPD activities using the LS are embedded within the contexts of teachers' schools and classrooms, affording them a chance to see transformation geometry teaching and learning, as it occurs instantaneously at their schools (Mhakure, 2019).

4.2.4.3 Challenges in implementing the LS

The challenges in implementing the LS faced by teachers who indicated participating in the LS in Table 4.2 are presented in this section. Figure 4.4 summarises the results.

Figure 4.4 Challenges in implementing the LS

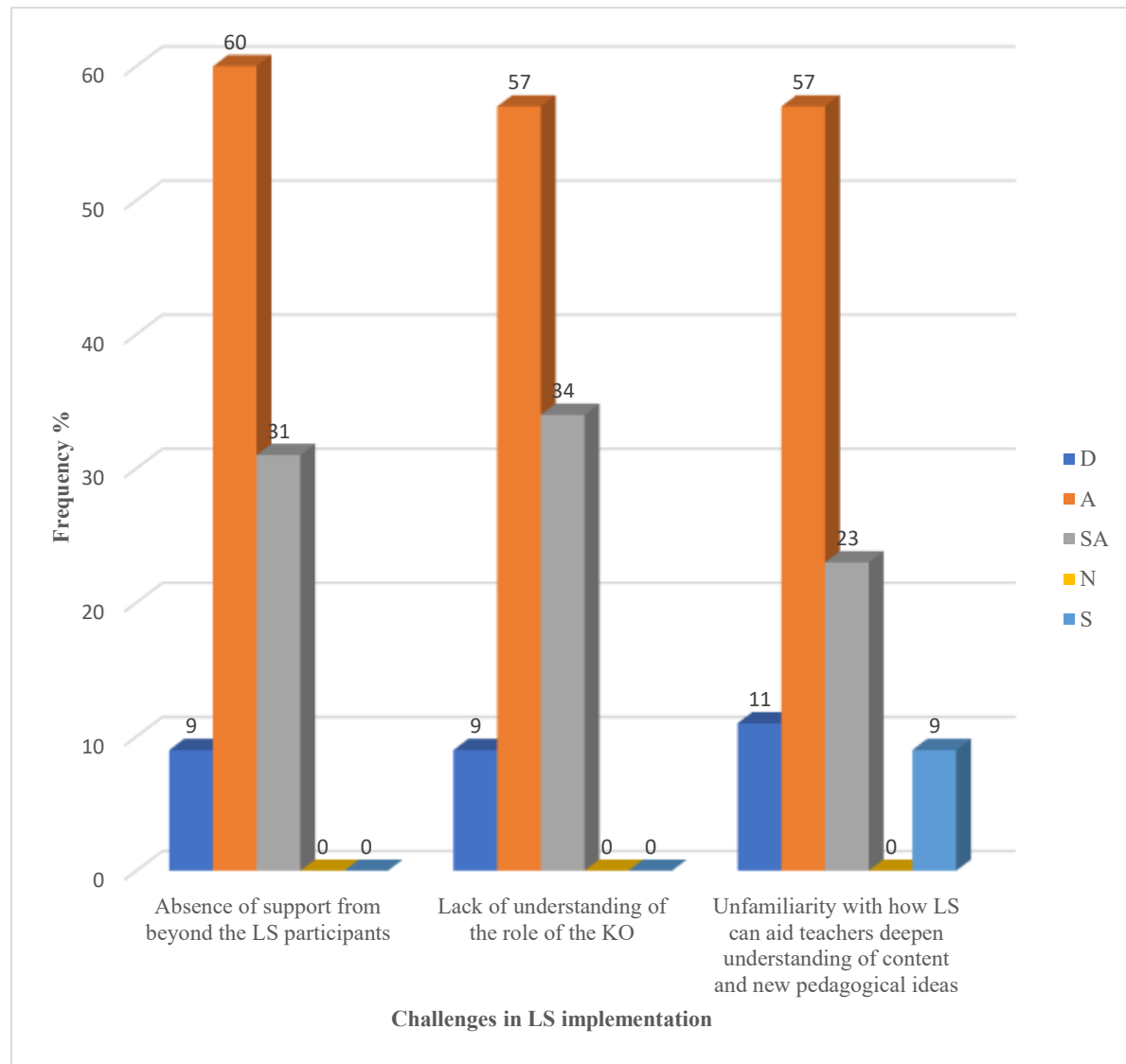


Figure 4.4 shows that most teachers who reported prior experience with the LS (see Table 4.2) agreed that there were factors that prohibited them from implementing it in their school mathematics departments. Absence of support from outside the LS participants was cited by most of the respondents, with 91% agreeing or strongly agreeing that this factor prevented them from implementing the LS. This result supports Darling-Hammond's (2013) assertion that school-based CPD activities are under-resourced, which hinders their optimal functioning. School administrators therefore need to make the necessary resources available to support

teachers' school-based CPD activities and address structural factors that impede change (Chong & Kong, 2012; Erickson et al., 2005).

The findings from the FGIs with teachers from the two schools (A and B) on teacher training on GeoGebra and the LS are presented in the sections below.

4.3 TPACK development levels – findings from FGIs of the two LS groups on teacher training on GeoGebra and the LS

This section first reports the findings of the FGIs (on the transformation geometry concepts of reflection, translation and enlargement), which were conducted after the teacher training sessions on GeoGebra and the LS with the research sample of teachers from the two schools (A and B). Following this is a presentation of the KO's reflections on teachers' levels of TPACK development before and after the training in each school.

As per the methodology, each of the two schools (A and B) formed a LS group of three teachers from the same mathematics department; their training was school based, with teachers from the Senior and FET Phases. Each LS group of teachers had two introductory training sessions – one on GeoGebra use and the other on the LS – and sessions lasted approximately two hours each. The training on GeoGebra use focused on teaching the Senior Phase transformation geometry concepts of translation, reflection and enlargement in South Africa's mathematics curriculum. Although enlargements were covered during the training, they were not part of the RLs that teachers designed, implemented and reflected on. The FGI aimed to track the teachers' TPACK development levels as shown by their self-reported scores and the researchers' interrater reliability scores before and after the training (see Sections 4.3 and 4.4).

A deductive thematic approach was employed to qualitatively analyse the data gathered from each school on the teacher training session on GeoGebra and the LS. Additionally, a quantitative data analysis using a repeated-measure ANOVA test was used to establish whether researchers' ratings on teachers' TPACK knowledge in schools A and B differed significantly across the four stages of professional development (training, RL1, RL2, and RL3) for each of the four TPACK dimensions: curriculum and assessment, learning, teaching, and access. The analysis combined schools A and B. In the quantitative data analysis using the ANOVA test, 'M' stands for the mean, 'F' for the test statistic value, and 'P' for the probability value. The teachers and the researchers' self-rated scores are per training session and per RL conducted in each school. The mean of the researchers' interrater reliability scores for the two schools'

training sessions (one session each) and RLs (three RLs per school) was obtained by adding the researchers' interrater reliability scores of the six teachers across four descriptors of the five TPACK development levels and dividing by 6, the total number of the researchers' interrater reliability scores.

4.3.1 TPACK development levels – findings from the FGI on teacher training on GeoGebra and the LS with teachers from school A

This section presents the findings from the FGI conducted with school A teachers after the GeoGebra and LS training sessions. The individual teachers' TPACK development levels in school A before and after training are summarised in Table 4.3 below, pseudonyms for the school A teachers (Tom, Bernard and Mike) in the first column and TPACK theme descriptors in the third column. Teachers' initial TPACK scores are represented in the second column. The teachers' self-reported scores and the researchers' interrater reliability scores on the TPACK development levels after training based on the five TPACK development level descriptors are indicated using numbers ranging from 0 to 5, in the fourth and fifth column respectively, with 0 meaning Pre-recognising level, 1 = Recognising level, 2 = Accepting level, 3 = Adapting level, 4 = Exploring level, and 5 = Advancing level. Teachers' reflection statements on their TPACK development level before and after training are shown in the last column. The statements were interpreted to indicate the individual teachers' TPACK levels after training in terms of the five TPACK level descriptors.

Table 4.3 TPACK development levels of school A's teachers before and after training

LS group A's teachers	Initial TPACK before training	Themes and their descriptors	Teachers' self-reported scores	Researchers' interrater reliability Scores	Teachers' reflections on their TPACK development level
Tom	0	Curriculum and assessment – Curriculum and assessment descriptor	4	3	I had never used GeoGebra before this training. After participating in the training, in teaching the transformation geometry concept of translation per se, after drawing and translating the object, I can ask learners to establish and apply the geometric relationships that exist between the object and its image.
	0	Learning - Conception of learner thinking descriptor	3	3	I had never used GeoGebra prior to this training, but after engaging in the training, I can now draw the object, reflect it about the line $y = x$, then pose questions to evaluate learners' thinking and investigate points misunderstood regarding the relationships that exist between the object and its image.
	0	Teaching – Instructional and environment descriptor	4	3	GeoGebra was new to me, but after taking part in the training, I can now construct the object, ask learners to reflect it about the x- axis ... then use GeoGebra to reflect it. If there are any misconceptions, GeoGebra is used to correct them by asking learners to establish the patterns that exist between the object and its image.
	0	Access – Barriers and availability descriptor	2	2	GeoGebra was new to me. Inasmuch as GeoGebra allows me to draw and reflect various objects when teaching the concepts of reflection about the x- and y-axis for learners to establish patterns between them after training, I still lack the necessary skills to teach these concepts using this software. Hence, I fear to frustrate myself and the learners during the RL.
Bernard	0	Curriculum and assessment descriptor	3	3	The training was my first encounter with GeoGebra. After training, when teaching the concept of enlargement, for example, I can now enlarge the object with a certain scale factor for learners to see and identify the geometric relationships between the object and its image.
	0	Learning – Conception of learner thinking descriptor	4	3	Before participating in the training, I did not know about GeoGebra. After taking part in the training, I can enlarge the object in GeoGebra then ask questions to the learners to comprehend their thinking, misconceptions and understanding in terms of the geometric relationships between the object and its image.
	0	Teaching – Instructional and environment descriptor	3	2	I did not use GeoGebra before this training. After participating in the training, when teaching geometric enlargements, after explaining to the learners that the coordinates of the image are obtained by multiplying the coordinates of the corresponding vertices of the object by the scale factor, I then use GeoGebra to reinforce this concept.
	0	Access – Barriers and availability descriptor	3	2	While GeoGebra allows me to translate different objects for learners to develop relationships that exist between the object and its image, being my first time to engage with the software, I am still not comfortable in using it. I can end up getting learners confused and/or unsettled as I demonstrate this transformation geometry concept.

Mike	0	Curriculum and assessment descriptor	4	3	I did not know about GeoGebra before the training. After participating in this training, I can now draw and reflect the object along the y-axis, then probe learners with questions addressing the objectives of the transformation geometry concepts of reflection about the y-axis. GeoGebra supports the visualisation of these objectives, thus honing learner understanding.
	0	Learning – Conception of learner thinking descriptor	3	3	GeoGebra was new to me prior to this training. It is because of taking part in the training that I can now construct the object in a particular quadrant, reflect it about the x- or y-axis, then ask learners questions aligned to the objectives of this concept to gauge their understanding.
	0	Teaching – Instructional and environment descriptor	2	2	This training was my first encounter with GeoGebra. Like what Bernard said, after engaging in the training, when teaching the transformation geometry concepts of reflection about the line $y = x$, I can start by explaining how the coordinates of the object and its image relate. After that, I can utilise GeoGebra in drawing and reflecting the object to show learners how this relationship comes about.
	0	Access – Barriers and availability descriptor	3	2	GeoGebra was new to me in this training. When teaching enlargement now, for example, after enlarging the object, I can ask learners to establish how the areas of the object and its image relate. However, I am not well versed with the running of this software, hence I may end up frustrating myself in front of the learners as I teach this concept.

It is quite conspicuous from Table 4.3 that prior to the training, the scoring of teachers from school A in each descriptor of the five TPACK levels was the same: zero. They were at the pre-recognising stage. A teacher at this stage in the context of this study is one who has never heard about GeoGebra or one who has heard about the software but never used it in teaching mathematics topics, especially transformation geometry.

In the curriculum and assessment descriptor of the ‘curriculum and assessment’ theme, school A’s teachers uniformly displayed adapting-level TPACK capabilities post-training. The teachers in school A indicated that they could demonstrate transformation geometry concepts with GeoGebra, then ask learners questions that required them to establish and apply the relationships that existed between the object and its image.

Regarding the learning theme descriptor ‘conception of learner thinking’: after training, all school A teachers exhibited adapting TPACK level traits in beginning to explore GeoGebra integration as a tool for learning to teach transformation geometry.

Guided by the teaching theme’s descriptors, specifically ‘instructional and environment’, an analysis of Bernard and Mike’s excerpts suggested that they were receptive to using GeoGebra. Bernard and Mike’s descriptions of introducing transformation geometry concepts algorithmically and using GeoGebra to reinforce these concepts demonstrated accepting-level

TPACK thinking. Tom's TPACK development was characterised as adapting level, as evidenced by his approach to the teaching theme, instructional and environment aspects. Tom described incorporating GeoGebra-based transformation geometry activities that were tailored to learners' learning needs, as well as using teacher-directed instructional strategies to control the RL flow.

Analysis of school A's teachers' excerpts, using the 'barriers and availability' descriptor of the access theme, suggested that these teachers operated at the accepting TPACK development level. Apart from the teachers in school A describing using GeoGebra for learners to explore various transformation geometry examples, they also raised concerns such as "access and management issues" (Niess et al., 2009, p. 21) when incorporating GeoGebra into transformation geometry teaching.

Additionally, the teachers at school A identified factors, including (but not limited to) vandalism, that would frustrate the integration of GeoGebra into their mathematics lessons. For example, Mike noted that, *"Our school being in a low socio-economic status community, the frequent break-ins have left the school vulnerable in terms of technology integration in the classroom, particularly GeoGebra. Due to vandalism, the sockets in most classrooms are not working and all the computers, and tablets went missing."*

It can be concluded from School A's teachers' reflections that there was a slight improvement in individual teachers' competencies in using GeoGebra to teach the transformation geometry concepts of reflection, translation and enlargement after training, when compared to where they had started before training. The teachers transitioned from the pre-recognising level before training to (primarily) the adapting level (a lower level of the TPACK development) after training.

Reflections by the KO on the training session on GeoGebra and the LS with teachers from school A

In this section, the researcher reports on the KO's reflections on school A's teachers' TPACK development, pre- and post-GeoGebra and LS training, across the five TPACK level descriptors. Lawrence was the KO for the two schools, A and B. During the teachers' reflections on the training session, Lawrence also reflected on their TPACK development levels before and after the training and his reflections were aligned to the four key descriptors – 'curriculum and assessment', 'conception of learner thinking', 'instructional and environment', and 'barriers and availability'. At the beginning of the training, Lawrence noted that school A's teachers were encountering GeoGebra for the first time. After the training, Lawrence indicated that the teachers' competencies in using this software to teach transformation geometry concepts had improved slightly for each descriptor of the five TPACK levels.

In terms of the curriculum and the assessment theme descriptor 'curriculum and assessment', Lawrence observed that teachers in school A were beginning to develop an understanding of how to leverage GeoGebra for teaching and assessing learners' conceptual understanding of the transformation geometry concepts of reflection, translation and enlargement. For example, Lawrence remarked: *"Teachers could use GeoGebra to demonstrate the transformation geometry concepts and pose questions that require learners to establish patterns between the object and its image, which they were unable to establish prior to the training."*

With the learning theme descriptor 'conception of learner thinking', Lawrence noted that teachers in school A were starting to see the potential of GeoGebra in fostering learners' transformation geometry thinking and conceptual understanding: *"The teachers could draw and reflect the objects in GeoGebra, then ask questions to comprehend learners' thinking about the geometric relationships that existed between the object and its image."*

Through the teaching theme descriptor 'instructional and environment', Lawrence's observation was that most of the teachers in school A could now use GeoGebra to validate the transformation geometry concepts that they introduced algorithmically. For example, Lawrence noted: *"GeoGebra use by most teachers in school A was non-exploratory. These teachers underscored using the software to reinforce the transformation geometry relationships that existed between the object and its image, which they had shown to the learners on the chalkboard."*

Lastly, regarding the access theme descriptor ‘barriers and availability’, Lawrence noted that although teachers in school A acknowledged GeoGebra’s role in teaching the transformation geometry concepts of reflection, translation and enlargement, they noted that there were challenges to teaching these concepts using the software. Lawrence pointed out: *“Inasmuch as teachers reported using GeoGebra to draw and reflect different objects for learners to establish patterns that existed between them, the same teachers expressed discomfort when using the software to teach these concepts.”* Lawrence encouraged teachers to create time for practising with the software: *“Teachers need more time to go through all what we have done in the training, because GeoGebra tools and menus need continuous practice for one to [be] perfect in using the software. Teachers also need to watch You-Tube videos to learn more about this software.”*

It is evident from Lawrence’s reflections that school A’s individual teachers’ competencies in using GeoGebra to teach the transformation geometry concepts of reflection, translation and enlargement improved from the pre-recognising stage before training, to (predominantly) the adapting stage (a lower level of TPACK development) after training.

4.3.2 TPACK development levels - findings from the FGI of school B on teacher training on GeoGebra and the LS

The previous section discussed the TPACK development levels of school A’s individual teachers, highlighting changes from pre-training to post-training. Discussed here are the TPACK development levels of individual teachers from school B, comparing their skills before and after training, based on the five TPACK level descriptors. The analysis of Group B follows, similar to that of Group A, and the information is summarised in Table 4.4 in the same way as in Table 4.3. The pseudonyms, Bosco, Sarah and Frank are used for teachers in LS group B.

Table 4.4 TPACK development levels of school B's teachers before and after training

LS group B's teachers	Initial TPACK before training	Themes and their descriptors	Teachers' self-reported scores	Researchers' interrater reliability scores	Teachers' reflections on their TPACK development level
Bosco	0	Curriculum and assessment – Curriculum and assessment descriptor	3	2	I was not aware of GeoGebra before the training; but after taking part in this training, I now understand the transformation geometry concepts better, although I am still not conversant with how to use the software to teach and assess these concepts.
		Learning – Conception of learner thinking descriptor	4	3	I was not using GeoGebra before the training; but after engaging in this training, I can draw and reflect on the object, then ask learners to come up with the geometric relationships between the object and its image on their own.
	0	Teaching – Instructional and environment descriptor	2	2	Before the training, I was unaware of GeoGebra. When teaching the transformation geometry concepts of reflection about the x- or y-axis per se, I begin by explaining this concept, while relating it to real life contexts. This is followed by giving learners an activity to work on after which I use GeoGebra to consolidate.
	0	Access – Barriers and availability descriptor	2	2	I was not using GeoGebra before this training. But now I acknowledge that with the software, several objects can be quickly drawn, reflected and investigated. However, I lack confidence in using GeoGebra to teach transformation geometry concepts, particularly enlargement, and this can end up causing the learners to become disconcerted during the RL.
Sarah	0	Curriculum and assessment descriptor	2	2	Like what Bosco said, this training was my first encounter with GeoGebra. After participating in the training, although I can draw and reflect the object, I am still not comfortable with teaching and assessing transformation geometry concepts of reflection, translation and enlargement using GeoGebra.
	0	Learning – Conception of learner thinking descriptor	3	3	As I have alluded to earlier, GeoGebra was new to me prior to this training, but after taking part in the training, I can now reflect the object about the x- or y-axis. This is followed by checking learners' understanding and application of the geometric relationships that exist between the object and its image through asking them questions.
	0	Teaching – Instructional and environment descriptor	3	2	I came across GeoGebra for the first time during this training. In teaching translations with GeoGebra, for example, I can start off by translating the object on the whiteboard as the learners observe, then use GeoGebra to verify what I have been doing.
	0	Access – Barriers and availability descriptor	2	2	GeoGebra was new to me during the training. After participating in this training, since I can now draw and reflect the object, then ask learners to comment on the geometric relationships that exist between the object and its image, I am still struggling to, for instance, measure the distance between corresponding vertices of the object and its image.

Frank	0	Curriculum and assessment descriptor	4	3	The training was my first time engaging with GeoGebra. After participating in the training, I can now use the software to draw and reflect the object about the y-axis, then ask learners to establish the geometric relationships that exist between the object and its image. It is learning by visualisation.
	0	Learning - Conception of learner thinking descriptor	4	3	Like what I indicated before, I had never come across GeoGebra before this training. But when using GeoGebra now, learners can see how the transformation of the object occurs. This gives them an opportunity to understand the properties of the transformation geometry concepts of reflection, translation ... through visualisation.
	0	Teaching – Instructional and environment descriptor	3	2	GeoGebra was new to me during training. After training, after explaining to the learners how the coordinates of the object and its image relate when the object is reflected either about the y-axis, or the line $y = x$, I can now use the software to validate this relationship just as Sarah indicated.
	0	Access – Barriers and availability descriptor	4	2	The training was my first encounter with GeoGebra. After participating in this training, I can now easily draw and translate different objects, then ask learners to establish the geometric relations that exist between them. However, I fear that it may be difficult for me to implement what I have learnt because we only have one project at school, hence fading in memory.

As with the teachers in school A (see Table 4.3), prior to the training, the scoring of teachers in school B in each of the four descriptors of the five TPACK levels was also zero (see Table 4.4). After participating in the training, school B's teachers' competencies in using GeoGebra to teach the transformation geometry concepts of reflection, translation and enlargement had improved slightly.

Regarding the curriculum and assessment descriptor of the 'curriculum and assessment' theme, Bosco and Sarah's excerpts represented the accepting TPACK level. Bosco and Sarah expressed a desire to replace (but demonstrated difficulty in replacing) traditional approaches to teaching with approaches that integrate GeoGebra effectively in the teaching of transformation geometry (Lyublinskaya & Tournaki, 2014), and had a limited view of its utilisation in assessment. Frank illustrated the adapting TPACK level by indicating demonstrating transformation geometry concepts using GeoGebra, and posing different questions to establish the learners' understanding of these concepts.

The comments of the teachers from school B, as seen in the learning theme descriptor 'conception of learner thinking', suggested an adapting TPACK level. School B's teachers described constructing and reflecting the objects in GeoGebra, while posing questions to uncover the learners' thinking, identify misconceptions, and assess their understanding.

The teachers in school B demonstrated an accepting TPACK level, as evidenced by their excerpts of the teaching theme – instructional and environment descriptor – by describing how they introduced transformation geometry concepts algorithmically and used GeoGebra to tightly manage the verification of these concepts.

With the access theme, school B teachers demonstrated an accepting TPACK level, as seen in their perspectives on ‘barriers and availability’ descriptor. These teachers intimated that although they could use GeoGebra to create various transformation geometry examples to be explored by learners, they were still worried about accessing the software to teach the transformation geometry concepts of reflection, translation and enlargement.

Furthermore, the teachers in school B identified overcrowded classrooms, inadequate resources and time as the major hinderances to integrating GeoGebra into the South African mathematics classroom, particularly in rural schools. For example, Frank said: *“GeoGebra requires technological devices, and a lot of time for preparation and practice. However, in our school, we do not have a computer laboratory, the teachers’ workload is too much, the annual teaching plan (ATP), and the classrooms are congested. This makes it difficult for us to integrate the software into the mathematics classroom.”*

It is quite clear from the reflections of school B’s teachers that there was a slight improvement in individual teachers’ competencies in using GeoGebra to teach the transformation geometry concepts of reflection, translation and enlargement in each descriptor of the five TPACK levels. The teachers transitioned from the pre-recognising level before training to primarily the accepting level (a lower level of TPACK development) after training.

Reflections by the KO on the training sessions on GeoGebra and the LS with teachers from school B

The previous section discussed the individual school B teachers' TPACK development levels pre- and post-training, based on the five TPACK level descriptors. This section presents the KO's comments on school B's teachers' TPACK development levels before and after training. As he did for the teachers in school A, Lawrence (the KO) noted that at the beginning of the training, the teachers from school B were also novice users of GeoGebra, receiving their initial exposure during the training session; and post-training, these teachers' competencies in using the software to teach transformation geometry concepts had improved slightly.

In terms of the curriculum and assessment descriptor of the 'curriculum and the assessment' theme, Lawrence noted that after the training, the teachers in school B had varied experiences regarding how to teach and assess transformation geometry concepts using GeoGebra. For example, Lawrence pointed out: *"While Frank could use GeoGebra to draw, reflect ... the object to show (or ask) learners the geometric relationships that existed between the object and its image, Bosco and Sarah expressed a desire to use GeoGebra but could not think about how the software might support transformation geometry teaching and assessment."*

Regarding the learning theme's descriptor 'conception of learner thinking', Lawrence noted that school B's teachers were starting to explore incorporating GeoGebra as a learning tool. Lawrence pointed out: *"The teachers were beginning to develop learners' appropriate transformation geometry thinking skills by drawing and reflecting the object, then allow[ing] them to discover the geometric relationships that exist between the object and its image."*

Within the teaching theme, focusing on 'instructional and environmental' descriptor, Lawrence observed that instruction with GeoGebra by teachers in school B was teacher-centred and teacher-led. For example, he noted: *"Teachers had concerns that learners do not develop appropriate transformation geometry thinking skills when GeoGebra is used. So, they preferred to use the software in a non-exploratory way, in validating transformation geometry concepts that were introduced to the learners procedurally."*

In relation to the access theme descriptor 'barriers and availability', Lawrence noted that although teachers in school B acknowledged that GeoGebra allowed various transformation geometry examples to be explored by learners, teachers raised access and management issue concerns about incorporating the software into the classroom. This is what Lawrence said: *"With enlargement per se, teachers still had difficulties in creating the slider. Hence, they feared*

to embarrass themselves in front of the learners.” In mediating this challenge, Lawrence encouraged the teachers to practise more with the software. “I will share You-Tube videos with you, which you can watch and practise in your free time how the slider is created and used in teaching transformation geometry concepts.”

In conclusion, the KO’s observations revealed that school B’s individual teachers’ competencies in using GeoGebra to teach the transformation geometry concepts of reflection, translation and enlargement improved from the pre-recognising level before training to predominantly the accepting level (a lower level of TPACK development) after training.

The next section discusses the TPACK level findings from the three RLs, which focused on the transformation geometry concepts in the South African mathematics curriculum of reflection about the x- and y-axis, reflection about the line $y = x$ and translation.

4.4 TPACK development levels – findings from the three transformation geometry RLs

The preceding section discussed individual teachers’ TPACK development levels in the two schools (A and B) before and after training. The TPACK development level findings from the implementation/observation of each of the three transformation geometry RLs on the concepts of reflection about the x- and y-axis, reflection about the line $y = x$ and translation in each of the two schools (A and B), are reported in this section. Following this is a presentation of the TPACK development level findings from the FGIs on each of the three transformation geometry RLs with teachers in each school. The section ends by highlighting the importance of the findings in relation to the study’s research questions.

In accordance with the methodology, three transformation geometry RLs – on the concepts of reflection about the x- and y-axis, reflection about the line $y = x$ and translation – were taught by each of the two schools, A and B. The RLs were created by the researcher and given to teachers to critique according to their learners’ learning needs and the curriculum. The design of these RLs was informed by the South African Senior Phase CAPS document. The data from each transformation geometry RL was qualitatively analysed through a deductive thematic analysis approach. Additionally, a quantitative data analysis using a repeated-measure ANOVA test was used to establish whether researchers’ ratings on teachers’ TPACK knowledge in schools A and B differed significantly across the four stages of professional development (training, RL1, RL2, and RL3) for each of the four TPACK dimensions: curriculum and assessment, learning, teaching, and access. The analysis combined schools A and B.

4.4.1 TPACK development levels – findings from RL1 from the two schools

This section reports on the TPACK development level findings from RL1, which was the first of the series of three, and the mathematical topic addressed in this RL was that of reflection about the x- and y-axis. RL1 collected qualitative and quantitative data that tracked the TPACK development levels for each teacher in each of the five TPACK level descriptors.

Drawing from the methodology, the researcher created the RL plan for RL1 for teachers in the two schools (A and B) to critique according to their learners' learning needs. Each school's teachers and the KO jointly discussed the objectives of RL1. The objectives included learners being able to: show that the object and its image were congruent; show that the object and its image's corresponding vertices were equidistant from the x- and y-axis; and identify which coordinate changes in sign when reflection is done about either the x- or the y-axis.

Using these objectives as a guide, the teachers and the KO planned RL1 together using GeoGebra. During the planning of RL1, the KO shared his views on teaching the transformation geometry concept of reflection about the x- or the y-axis utilising GeoGebra, while the researcher introduced the teachers to the data collection instruments: observation and interview guides (including reflection guides) to be used within this RL. The aim was to get the teachers acquainted with the instruments in terms of what they would be observing and reflecting on during and also after the actual implementation of RL1. School A and B teachers also identified the three learner categories – lower, middle and upper performer – for which RL1 was developed using GeoGebra.

After planning RL1, one of the three teachers in each school facilitated this RL, while the other two teachers and the KO observed and collected data on learner learning of the transformation geometry concept of reflection about the x- and y-axis, using an observation guide. The KO also stepped in to provide clarity regarding GeoGebra use when the teacher encountered obstacles while teaching. Immediately after RL1 was taught, the teachers were given a reflection guide, which they completed and brought to the FGI for the researcher to follow up on their TPACK development in each descriptor of the five TPACK levels in the four themes. The teachers and the KO also reflected on the four phases of RL1 – identification of objectives, planning, implementing/observing and reflecting – and suggested recommendations for improving RL2 with GeoGebra. The researcher video- and audio-recorded these phases of RL1 for later analysis.

4.4.1.1 TPACK development levels – findings from implementation/observation of RL1 in School A

In this section, data is discussed from the implementation/observation of RL1 by teachers from school A. Qualitative data was collected using a predetermined observation guide (see Appendix 11a) to observe learner learning when teaching the transformation geometry concept of reflection about the x- and y-axis. This data shed light on the third research question – the experiences of rural high school mathematics teachers teaching transformation geometry using GeoGebra.

Mike was the teacher who taught RL1, while Tom, Bernard and Lawrence (the KO) observed learner learning when the transformation geometry concept of reflection about the x- and y-axis were taught using a worksheet, and with GeoGebra. Learners first solved questions from a worksheet, which they worked on manually; after which the teacher used GeoGebra to teach the same activity. The teacher instructed learners to work independently within their groups initially, followed by formulating and sharing hypotheses about the transformation geometry problem of reflecting $\triangle EFG$ about the x- and y-axis with their groupmates. This was followed by the learners convincing their classmates of the validity of the transformation geometry ideas on reflecting $\triangle EFG$ about the x- and y-axis that they had developed. Using GeoGebra, the teacher then assisted the learners to recognise the transformation geometry knowledge gained in the context of their a-didactical work (reflecting $\triangle EFG$ about the x- and y-axis) and transforming it into knowledge they could use to solve various problems. The findings from teaching RL1 using a worksheet and then GeoGebra are presented and analysed in the sections below.

Teachers' experiences on the use of the worksheet to teach the transformation geometry concepts of reflection about the x- and y-axis.

This section presents the experiences of teachers from school A when learners manually solved the pen and paper activity on the transformation geometry concepts of reflection about the x- and y-axis.

When using a worksheet, based on the position of $\triangle EFG$ (the object), which was drawn in quadrant I, the teachers in school A acknowledged that all learners were able to identify which coordinate changed in sign when $\triangle EFG$ was reflected about the x- and y-axis. Bernard said: *"All learners concluded that the signs of the x- and y-coordinates changed to negative when $\triangle EFG$ was reflected about the y- and x-axis respectively."* Furthermore, the teachers noted that some learners were unable to discern that the sizes of $\triangle EFG$ and its images were congruent. For example, as Tom mentioned: *"Except for the low and middle achievers, top achievers were able to show congruency of $\triangle EFG$ and its images using the formula of the area of a right-angled triangle."* The teachers also observed that some learners were not able to show that the distances between the corresponding vertices of $\triangle EFG$ and its image from the x- or the y-axis were equidistant. Lawrence had this to say: *"Through counting the blocks, all the learners except the low achievers showed that the corresponding vertices of $\triangle EFG$ and its images were equidistant from the x- or the y-axis ..."*

During the planning of RL1, the prediction was that the low achievers would have challenges with all activities on the worksheet except activity (a) (drawing the positions of the images after reflecting $\triangle EFG$ about the x- and y-axis); the middle achievers would struggle with activities (c) (comparing the sizes of $\triangle EFG$ and its images) and (e) (discussing their solutions showing the relationship between $\triangle EFG$, the x- or the y-axis, and its images); and the top achievers would have difficulties with activity (e) – see Appendix 10a. However, when observing RL1, the teachers noted that each of these three categories of learners performed better than expected. For example, Bernard reported: *"The low achievers did not struggle with activity (b) (How are the signs of the coordinates of the vertices of $\triangle EFG$ affected after reflecting it about the x- and y-axis?) as was predicted, but only struggled with activities (c), (d) and (e). The middle achievers only struggled with activity (c), but not (e). The top achievers did not struggle with activity (e) as had been anticipated."*

Although the three categories of learners performed better than expected, it can be concluded from school A's teachers' observations that not all success criteria were achieved – the low achievers struggled with almost every activity on the worksheet.

Observations on the use of GeoGebra to teach the transformation geometry concept of reflection about the x- and y-axis.

The previous section described the experiences of teachers from school A when learners manually solved the pen and paper activity on the transformation geometry concepts of reflection about the x- and y-axis. This section presents school A's teachers' experiences of the use of GeoGebra to teach the same pen and paper activity.

When teaching the same activity using GeoGebra, there was an agreement among the teachers at school A that GeoGebra did not assist the learners with the visualisation of how the coordinates of the points of the vertices of $\triangle EFG$ changed in sign when reflection was done about the x- and y-axis. For example, Lawrence pointed out: *“Failure by Mike to change the position of $\triangle EFG$ (that was drawn in quadrant 1) to different quadrants when teaching the objective of coordinates left the learners with a misconception from the worksheet – the signs of the x- and y-coordinates always change to negative when reflection is done about the y- and x-axis respectively.”* The teachers also agreed that GeoGebra assisted the learners with visualising how the areas and distances between the corresponding vertices of $\triangle EFG$ and its images from the x- or the y-axis related. Mike said: *“After calculating the areas and the distances between the corresponding vertices of $\triangle EFG$ and its images from the x- or the y-axis, I changed the positions of $\triangle EFG$ but within quadrant 1. Even the low and middle achievers who struggled with these questions on the worksheet could now see that the triangles remained congruent in size, and their corresponding vertices were always equidistant from either the x- or [the] y-axis.”*

During the planning of RL1, the prediction was that when GeoGebra was used to consolidate the pen and paper activity, all three categories of learners (low, middle and top) would successfully carry out the learning activities except the low achiever, who would still struggle with activity (e). However, when observing RL1, the teachers in school A noted that all three learner categories struggled with activity (a), contrary to the prediction made during planning. For example, Bernard pointed out: *“ $\triangle EFG$ was drawn in quadrant 1, and Mike [the teacher who taught RL1] did not change its position to various quadrants but only in the first quadrant.*

Hence learners walked away with a false conjecture from the pen and paper activity – the signs of the x and y coordinates always change to negative when reflection is done about the y- and x-axis respectively.”

In conclusion, it is conspicuous that the school A teachers acknowledged that GeoGebra assisted the learners with the visualisation of the geometric relationships that existed between the object and its image except for the objective of coordinates. The teachers noted that failure by Mike to change the positions of $\triangle EFG$ to different quadrants when teaching the objective of coordinates left learners with a misconception about how the signs of the coordinates of the object were affected after reflecting it about the x- or the y-axis.

4.4.1.2 TPACK development levels – findings from the reflections on RL1 in school A

In the previous section, the researcher discussed school A's teachers' experiences in using a worksheet to teach the transformation geometry concepts of reflection about the y- and x- axis, before discussing the same teachers' experiences in using GeoGebra to consolidate the same pen and paper activity. This section tracks school A's teachers' TPACK development levels, as shown by their self-reported scores and the researchers' interrater reliability scores in Table 4.5. The pseudonyms Tom, Bernard and Mike are used for the teachers in school A, and the descriptors of the TPACK themes are presented in the first and second column respectively. The teachers' self-reported scores and the researchers' interrater reliability scores on the TPACK development levels after RL1 based on the descriptors of the five TPACK levels are indicated by numbers ranging from 0 to 5, in the third and fourth column respectively: 0 means Pre-recognising level, 1 = Recognising level, 2 = Accepting level, 3 = Adapting level, 4 = Exploring level, and 5 = Advancing level. The teachers' reflections on their TPACK development levels after RL1 are presented in the last column. These statements give an indication of where an individual teacher's TPACK level is in the five TPACK level descriptors.

Table 4.5 TPACK development levels of individual teachers in school A after RL1

LS group A's teachers	Themes and descriptors	Teachers' self-reported scores	Researchers' interrater reliability scores	Teachers' reflections on their TPACK development level
Mike	Curriculum and assessment – Curriculum and assessment descriptor	4	3	When I was teaching RL1, I observed that some learners could not identify that ΔEFG (the object) and its images were of the same size after reflecting the object about the x- and y-axis during the pen and paper activity. After reflecting on the object about the x- and y-axis using GeoGebra, I changed its position; but within quadrant 1. Even the low achievers could now conjecture that the object and its images were congruent in size.
	Learning – Conception of learner thinking descriptor	4	3	As alluded to earlier, when I was teaching RL1, I noticed that some learners struggled to figure out that ΔEFG and its images were congruent in size when working on the pen and paper activity. I measured the areas of ΔEFG and its image in GeoGebra and changed the positions of ΔEFG within quadrant 1. On asking learners for their observations, they all concluded that their areas remained the same.
	Teaching – Instructional and environment descriptor	5	3	When I was teaching RL1, I noted that some learners could not identify that the distances between the vertices of ΔEFG and its image's corresponding vertices were equidistant from the x- or y-axis during the worksheet activity. After reflecting and measuring these distances, I changed the positions of ΔEFG , but within quadrant 1. On asking learners to comment on how the distances were related, they all conjectured that they were equal.
	Access – Barriers and availability descriptor	3	2	When I was teaching, I was able to draw and reflect ΔEFG about the x- and y-axis for learners to establish the geometric relationships that existed between these objects, hence making abstract concepts visible. However, I am still not confident in using GeoGebra to teach the transformation geometry concepts of reflection about the x- and y-axis to enhance learner understanding. For example, I still struggle with removing labels, and this overcrowds the whiteboard.
Tom	Curriculum and assessment descriptor	5	3	When planning RL1, measuring the sizes of the object and its image in GeoGebra was not clear to me. This only became clear as I was observing this RL. I can now use the area tool in GeoGebra to calculate the areas of the object and its image, then ask learners to make deductions regarding their sizes – it is discovery learning.
	Learning – Conception of learner thinking descriptor	3	3	During the planning of RL1, measuring the distances between the vertices of ΔEFG and its image's corresponding vertices from either the x- or y-axis was theoretically discussed in GeoGebra, and this was not clear to me. This only became clear as I was observing the RL. I can now use GeoGebra to measure these distances, then pose questions to comprehend learner thinking and misconceptions.
	Teaching – Instructional and environment descriptor	4	3	Drawing from today's RL observation, the teacher's use of GeoGebra gave me a practical shift in my teaching approaches, which were largely theoretical during planning. I can now use GeoGebra to draw and reflect ΔEFG about the x- axis ... then ask learners for their deductions rather than me having to tell them the geometric relationships that exist between the object and its images.
	Access – Barriers and availability descriptor	2	2	When planning RL1, it was not clear to me on how to measure the distances between the corresponding vertices of the object and its image from the x- or y-axis using GeoGebra. Although this became clear as I observed this RL, I still have challenges in using the dynamism capability of the software to enable learners to investigate the patterns that exist between the object and its image.

Bernard	Curriculum and assessment descriptor	4	3	During planning, it was not clear to me on how to use GeoGebra to demonstrate the distances between the corresponding vertices of the object and its image from the x-axis ... This became clear as the teacher rectified the misconception from the worksheet. I can now use the software to measure these distances, then ask learners for their observations.
	Learning – Conception of learner thinking descriptor	3	3	As a result of observing RL1, I can now utilise GeoGebra to measure the areas of $\triangle EFG$ and its images, then ask learners to comment on how these areas are related. This was not clear to me during planning as it was not practically discussed.
	Teaching – Instructional and environment descriptor	4	3	When observing RL1, I noted that Mike's use of GeoGebra reduced teacher talk and teaching by telling, which was not that clear during planning. When teaching the transformation geometry concepts of reflection about the x- and y-axis, I can now draw and reflect $\triangle EFG$ about the x- or y-axis, then let the learners study the objects for patterns.
	Access – Barriers and availability descriptor	2	2	Like what I saw during the planning of RL1, when observing this RL, I have learnt to use GeoGebra to create and reflect different objects, then engage learners in exploring the geometric relationships that exist between the object and its image. However, I am worried about frustrating learners because I am not yet good at using this software to teach the transformation geometry concepts of reflection about the x- and y-axis. Hence, this can lead to disengagement.

It is evident from Table 4.5 that with the curriculum and assessment descriptor of the 'curriculum and assessment' theme, all school A's teachers demonstrated adapting-level TPACK. These teachers described using GeoGebra to demonstrate the geometric relationships that existed between $\triangle EFG$ and its image, and posed questions for learners to make conjectures about these relationships.

Regarding the learning theme's descriptor 'conception of learner thinking', school A teachers' considerations in drawing and reflecting $\triangle EFG$ about the x- and y-axis and asking learners questions to comprehend their thinking and misconceptions from the pen and paper activity showcased that their performance was at an adapting level.

With the teaching theme, focusing on instructional and environment descriptor, the school A teachers' excerpts represent adapting TPACK level features. The school A teachers described utilising GeoGebra to draw and reflect $\triangle EFG$ about the x- and y-axis, and then give learners the opportunity to explore the geometric relationships that existed between $\triangle EFG$ and its images. The teachers therefore controlled GeoGebra to provide effective classroom management.

School A's teachers' thinking, as reflected in their excerpts, was aligned with the accepting-level TPACK capabilities in the 'barriers and availability' descriptor of the access theme. Inasmuch as teachers in school A described using GeoGebra to engage learners in exploring

different examples on the transformation geometry concepts of reflection about the x- and y-axis, they also raised concerns about access and management when using GeoGebra to teach these concepts.

In addition, in South Africa, there are other issues: loadshedding, and a shortage of facilities such as computer laboratories in schools, particularly in lower-level socio-economic communities – teachers mentioned these as some of the factors that prohibit them from using GeoGebra in their mathematics lessons. Mike said: *“Sometimes you set the apparatus in a class, then loadshedding takes place. Furthermore, in our school, there’s no computer laboratory, making the integration of GeoGebra difficult in the classroom. The school therefore needs to establish a computer laboratory to enable the integration of GeoGebra in the mathematics classrooms.”*

When reflecting on RL1, the teachers in school A felt that Mike’s teaching approach to the objective of coordinates using GeoGebra needed improvement. Bernard said: *“Mike did not change the position of $\triangle EFG$ (drawn in quadrant 1) to other quadrants, leaving learners with a misconception that the signs of the x- and y-coordinates always change to negative when reflection is done about the y- and x-axis respectively. This could have emanated from the planning session, because dynamism was theoretically emphasised after reflecting the object.”* Going forward to RL2, school A teachers expressed the need to do things differently. For instance, Tom pointed out: *“We need to make use of the dynamism capability of GeoGebra by changing the positions of the object to different quadrants, or to overlap quadrants, for learners to investigate the geometric relationships that exist between the object and its image. This will assist them in overcoming their thinking errors and/or misconception.”*

Additionally, the teachers in school A indicated that they were looking forward to RL2 – they wanted to pay attention to learner learning. For example, Tom mentioned: *“Observing learner learning needed to be supplemented by us moving around during the pen and paper activity, to see what the low, middle and top learners had written down on their worksheets prior to the whole group and class discussions.”*

It can be concluded from school A’s teachers’ reflections that after RL1, individual teachers demonstrated mainly the adapting level (a lower level of TPACK development). This meant that teachers in school A were beginning to have an understanding of how to utilise GeoGebra in the teaching transformation geometry concept of reflection about the x- and y-axis.

Reflections by the KO on RL1 with teachers from school A

In this section, the researcher reports on the KO's reflections on school A's teachers' competencies in using GeoGebra to teach the transformation geometry concept of reflection about the x- and y-axis in each of the four descriptors – 'curriculum and assessment', 'conception of learner thinking', 'instructional and environment', and 'barrier and availability' – of the five TPACK levels.

Regarding the curriculum and assessment descriptor of the 'curriculum and assessment' theme, Lawrence (the KO) acknowledged that after RL1, the teachers in school A were beginning to have an understanding of how to teach and assess learners' conceptual understanding of reflections about the x- and y-axis with GeoGebra. For example, Lawrence noted: *"Teachers in school A could now draw and reflect $\triangle EFG$ about the x- and y-axis, measure the areas and distances between the corresponding vertices of $\triangle EFG$ and its image from either the x- or the y-axis, and probe learners to comment on these geometric relationships."*

For the learning theme descriptor 'conception of learner thinking', Lawrence observed that after RL1, the teachers in school A were beginning to develop learners' appropriate transformation geometry thinking skills with GeoGebra as a learning tool. Lawrence had this to say: *"Teachers could now draw and reflect $\triangle EFG$ in GeoGebra and get the learners to establish the geometric relationships that existed between $\triangle EFG$ and its image on their own without having to tell them what is going on."*

Focusing on the teaching theme – instructional and environment descriptor – Lawrence noted that teachers in school A were beginning to adapt instructional strategies, which afforded learners the chance to explore using GeoGebra, as well as enabling teachers not to lose control of the classroom. For example, Lawrence reported: *"Teachers could use GeoGebra to draw and reflect $\triangle EFG$ about the x- and y-axis, and ask learners to establish the geometric relationships that existed between $\triangle EFG$ and its image."*

With the access theme descriptor 'barriers and availability', Lawrence's observation was that the teachers in school A were not yet competent in using GeoGebra to teach the transformation geometry concepts of reflection about the x- and y-axis. Lawrence said: *"Although the teachers in school A could draw and reflect $\triangle EFG$ about the x- and y-axis for learners to explore patterns that existed between the object and its image, these teachers struggled with harnessing the dynamism capability of GeoGebra. Hence, this hindered their effective integration of this*

software in teaching the transformation geometry concept of reflection about the x- and y-axis, leaving learners with a misconception regarding the objective of coordinates.” Going forward to RL2, Lawrence encouraged the teachers to practise moving the object to different quadrants to rectify this misconception. For example, Lawrence mentioned: *“Teachers need to make use of the dynamism capability of GeoGebra, as this will help learners in overcoming their thinking errors and/or misconceptions from the pen and paper activity regarding the objective of coordinates.”*

While reflecting on RL1, Lawrence emphasised that he was not happy with the way in which the planning of this RL with GeoGebra took place. For example, Lawrence said: *“Since I had already trained the teachers before, when planning RL1, I just drew the object, reflected it, and we started theoretically discussing how to use GeoGebra to teach and facilitate the development of learners’ conceptual knowledge of geometric reflection thinking skills. Some teachers did not bring their laptops along.”* Lawrence recommended that in RL2, they do things differently. He said: *“Inasmuch as teachers were trained before, in moving to RL2, we needed to practically go through the entire process of reflecting the object about the line $y = x$ during planning, with teachers being in possession of their laptops to get acquainted to the software. This will help us to not repeat the same mistake that took place in today’s RL of Mike not changing the position of the object when teaching the objective of coordinates.”*

Lawrence’s comments imply that after RL1, the teachers in school A were beginning to have an understanding of how to utilise the software in teaching transformation geometry concepts of reflection about the x- and y-axis, and thus in the main demonstrating the adapting TPACK level for most descriptors.

4.4.1.3 TPACK development levels – findings from the implementation/observation of RL1 in School B

The section presents the findings from the implementation/observation of RL1 by teachers from school B. The analysis of the implementation/observation of the teachers of school B's RL1 follows a similar analysis to that performed for school A. Frank is the teacher who taught RL1, while Bosco, Sarah and Lawrence (the KO) were observing learner learning when the transformation geometry concepts of reflection about the x- and y-axis were taught using a worksheet and using GeoGebra. Learners solved questions from a worksheet first, which they worked on manually, after which Frank used the software to consolidate the same activity. The findings from teaching RL1 using a worksheet first and then GeoGebra by teachers in school B are presented and analysed in the sections below.

Teachers' experiences on the use of the worksheet to teach the transformation geometry concept of reflection about the x- and y-axis.

In this section, the researcher presents school B's teachers' experiences when learners manually solved the pen and paper activity on the transformation geometry concept of reflection about the x- and y- axis.

With reference to the worksheet, school B's teachers agree that most learners could identify which coordinate changed in sign when reflection was done about either the x- or the y-axis. For example, Bosco highlighted the following: *"All learners except the low achievers conjectured that the sign of the x- coordinate ... changes from positive to negative after reflecting $\triangle EFG$ (which was drawn in quadrant 1) about the y-axis ..."* However, some teachers raised concerns that the learners' conjecture could lead them into believing that their conclusion would always be correct, no matter in which quadrant the object is drawn. Sarah cautioned: *"This conclusion applies only when $\triangle EFG$ is drawn in quadrant 1 and 4."*

Additionally, school B's teachers observed that their learners were able to show that the sizes of $\triangle EFG$ and its images were the same. Frank said: *"I calculated the area of $\triangle EFG$ (the object), then asked learners to calculate the areas of $\triangle E'F'G'$ and $\triangle E''F''G''$ (the images of $\triangle EFG$ after reflecting it about the y- and x-axis respectively), and they could see that they were the same."* However, some teachers in school B felt that Frank should not have shown learners how to calculate the area of $\triangle EFG$. Lawrence pointed out: *"Giving learners hints on how to calculate the area of $\triangle EFG$ limited their thinking ..."* School B's teachers also agreed that learners could

identify that the distances from either the x- or the y-axis between the vertices of $\triangle EFG$ and the corresponding vertices of its images were the same. Bosco, for example, stated: *“Learners counted the unit blocks from each line of reflection to the corresponding vertices of $\triangle EFG$ and its image, and could see that they are equal.”*

When planning RL1, the prediction was that the top achievers would successfully carry out all the learning activities (a) to (e), the middle achievers would struggle with learning activity (e), and low achievers would struggle with learning activities (b) and (e) – see Appendix 10a. When observing RL1, school B’s teachers indicated that these three categories of learners performed as predicted. For example, Mike said: *“The low, middle and top achievers performed as predicted during planning.”*

It is evident from school B’s teachers’ observations that all success criteria except the objective of coordinates were achieved when learners worked on the pen and paper activity.

Observations on GeoGebra use to teach the transformation geometry concept of reflection about the x- and y-axis.

The previous section presented school B’s teachers’ experiences when learners manually solved the pen and paper activity on the transformation geometry concept of reflection about the x- and y-axis. This section presents school B’s teachers’ observations when GeoGebra was used to teach the same activity regarding the transformation geometry concept of reflection about the x- and y-axis.

When using GeoGebra to teach the same pen and paper activity, school B’s teachers acknowledged that the software did not assist the learners with visualisation of how the coordinates of $\triangle EFG$ ’s vertices changed in sign when reflection was done about either the x- or the y-axis. For example, Sarah noted: *“After Frank reflected $\triangle EFG$ (the object), which was drawn in quadrant 1, he did not change the position of the object. Hence, learners walked away with a misconception from the pen and paper activity that the signs of the x- and y-coordinates always become negative when reflection is done about either the y- or the x-axis respectively.”*

Additionally, the teachers jointly agreed that GeoGebra assisted the learners with visualisation of how the sizes of $\triangle EFG$ and its images related. Frank pointed: *“Even when I changed the sizes and/or positions of $\triangle EFG$ within quadrant 1, learners could see that the sizes of $\triangle EFG$ and its image(s) remained the same, which dynamism was not possible with the worksheet activity.”*

The teachers also observed that GeoGebra assisted the learners with the visualisation of how the distances between the vertices of $\triangle EFG$ and the corresponding vertices of its images from either the x- or the y-axis were related. Bosco stated: *“Although Frank did not apply dynamism to $\triangle EFG$, he measured the distances of the corresponding vertices of $\triangle EFG$ and its images from either the x- or the y-axis and learners could see that the answers were the same as those from their pen and paper activity.”*

During the planning of RL1 by the teachers in school B, the prediction was that when GeoGebra was used to consolidate the pen and paper activity, all three categories of learners – low, middle and top – would successfully carry out the learning activities, except that the low achievers would still struggle with activity (e). However, when observing RL1, school A’s teachers noted that all three categories of learners answered activity (a) incorrectly. Lawrence pointed out: *“When teaching the objective of coordinates, Frank did not apply dynamism to $\triangle EFG$. Hence, learners were made to believe that the signs of the y- and x-coordinates always change to negative when reflection is done about the x- and y-axis respectively, which is not the case always.”*

As with the teachers in school A, it can be concluded from the observations of RL1 by the teachers in school B that all the success criteria except the objective of coordinates were achieved when GeoGebra was used to consolidate the pen and paper activity.

4.4.1.4 TPACK development levels – findings from the reflections on RL1 in school B

The observations of school B’s teachers regarding using worksheets and GeoGebra in teaching reflections about the x- and y-axis were presented in the preceding section. This section tracks school B’s individual teachers’ TPACK development levels, as shown by their self-reported scores and the researchers’ interrater reliability scores. Like Table 4.5 for school A, Table 4.6 summarises RL1’s findings for school B.

Table 4.6 TPACK development levels of individual teachers in school B after RL1

LS group B's teachers	Themes	Self-ratings by the teachers	Interrater reliability	Teachers' reflections on their TPACK development level
Frank	Curriculum and assessment – Curriculum and assessment descriptor	3	2	When I was teaching RL1, after reflecting ΔEFG (drawn in quadrant 1) about the y-axis ... all learners except the low achievers concluded that the sign of the x- coordinate ... changed to negative. When using GeoGebra, I reflected and examined the coordinates of ΔEFG and its image, then asked learners whether they agree that the sign of the x- coordinate ... changed to negative after reflecting ΔEFG about the y-axis ... and they agreed.
	Learning - Conception of learner thinking descriptor	4	3	When I was teaching RL1, I noticed that the low achievers could not identify how the coordinates of ΔEFG and its image were related after reflecting ΔEFG about the x- axis ... during the pen and paper activity. In developing this thinking skill with GeoGebra, I drew ΔEFG in quadrant 1 and reflected it about the x- axis ... The low achievers could now see and conjecture that the sign of the y- coordinate changed to negative ...
	Teaching – Instructional and environment descriptor	4	2	As alluded to earlier, when teaching RL1, I noticed that some learners could not identify which coordinate changed in sign after reflecting ΔEFG about the y-axis ... After drawing and reflecting ΔEFG about the y-axis ... in GeoGebra, I examined the signs of the coordinates of one of the corresponding vertices of ΔEFG and its image, then asked learners to examine the other vertices. Even the low achievers could now conclude that the x- coordinate ... changed to negative after reflecting ΔEFG about the y-axis ...
	Access – Barrier and availability descriptor	2	2	When I was teaching RL1, after reflecting ΔEFG about the y-axis per se, I measured its area and that of its image ($\Delta E'F'G'$) in GeoGebra. I then tried to change the position of $\Delta E'F'G'$, but it was not moving. This got me frustrated, and the learners were beginning to be unsettled. The KO stepped in and told me to move ΔEFG instead of $\Delta E'F'G'$, and this enabled us to explore different examples.
Sarah	Curriculum and assessment descriptor	3	2	The objectives of RL1 were well discussed, and guided the planning of this RL. However, calculating the area of the object and its image in GeoGebra during planning was not clear to me. This became clear as I was observing the RL. I am now able to utilise GeoGebra to calculate the area of ΔEFG and its image(s) to show learners that they are congruent in size.
	Learning - Conception of learner thinking descriptor	2	3	Using GeoGebra in facilitating the development of learners' thinking skills of reflection about the x- and y-axis was not clear to me during planning, as the discussion was mainly theoretical. When observing RL1, I have learnt that in mitigating the misconception regarding the objective of coordinates, one needs to draw and reflect an object that overlaps quadrants, then ask learners to establish how the coordinates are related.
	Teaching – Instructional and environment descriptor	3	2	During the planning of RL1, the teaching approaches to use with GeoGebra were just ideas. When observing today's RL I have learnt that after explaining to learners the geometric relationships that exist between ΔEFG and its image, I can use GeoGebra to strengthen these geometric relationships.
	Access – Barrier and availability descriptor	2	2	During the planning of this RL, we only worked with integer coordinates. When observing this RL, I realized that the coordinates of point E in ΔEFG were decimals. The teacher did not know how this came about. He tried applying dynamism to change the coordinates to integers, but he failed – the teacher was frustrated. This happened towards the end of the RL, and he decided to end it. I also have challenges in this regard.

Bosco	Curriculum and assessment descriptor	4	3	Like what I saw during the planning of RL1, I can draw and reflect ΔEFG about the x- and y-axis, then ask learners for the geometric relationships that exist between ΔEFG and its images. GeoGebra aids learners to see concepts that may appear abstract.
	Learning – Conception of learner thinking descriptor	3	3	During the planning of RL1, the object was only restricted to quadrant 1. Like what Sarah said, when observing this RL, I have learnt that it is vital to draw and reflect an object (ΔEFG) that shares quadrants, then ask learners to comment on how the signs of the coordinates were affected. This aids learners in overcoming their misconceptions or thinking errors from the pen and paper.
	Teaching – Instructional and environment descriptor	2	2	Like what Sarah said, after seeing the planning and implementation of RL1, in teaching reflections about the x- and y-axis, I can now start by explaining the geometric relationships that exist between the object and its image, while relating them to real life. This is followed by using GeoGebra to draw and reflect the object about the x- and y-axis to reinforce these geometric relationships that exist between the object and its image.
	Access – Barrier and availability descriptor	3	2	After participating in planning and observing RL1, I can now use GeoGebra to draw and reflect ΔEFG about the x- axis ... then engage learners in investigating patterns that exist between them. However, I still have some challenges in using the software to teach the transformation geometry concept of reflection about the x- axis ... For example, I still need support on how to apply dynamism to the object and make the object bigger so that the whole class can see.

It can be concluded from Table 4.6 that Frank and Sarah exhibited accepting TPACK level characteristics, as evidenced by their curriculum and assessment descriptor excerpts. Frank and Sarah expressed a desire to use GeoGebra but struggled to move beyond traditional methods and had limited ideas for using GeoGebra when assessing the transformation geometry concept of reflection about the x- and y-axis. Regarding ‘curriculum and assessment’ descriptor of the curriculum and assessment theme, Bosco demonstrated the adapting-level TPACK. Bosco reported using GeoGebra to demonstrate the transformation geometry concept of reflection about the x- and y-axis, after which he posed questions to gauge the learners’ understanding.

With the learning theme, teachers typified adapting TPACK level features regarding the ‘conception of learner thinking’ descriptor. They described using GeoGebra to draw and reflect an object that overlaps quadrants and asking questions to comprehend learners’ thinking and misconceptions from the pen and paper activity.

It is evident from school B’s teachers’ excerpts that their thinking implied the accepting of GeoGebra. The teachers’ desire to use GeoGebra to verify the activities of the transformation geometry concept of reflection about the x- and y-axis, which was introduced procedurally, while tightly managing instruction, indicates that they were thinking on the accepting TPACK level with regard to the ‘instructional and environment’ descriptor of the teaching theme.

The school B teachers illustrated the traits of an accepting TPACK level, as evidenced by their ‘barriers and availability’ descriptor excerpts in the access theme. School B teachers intimated

that they could draw and reflect ΔEFG about the x- or the y-axis for learners to investigate the patterns that exist between ΔEFG and its image. However, the teachers raised concerns regarding access and management when utilising GeoGebra to teach the transformation geometry concepts of reflection about the x- and y-axis.

The teachers in school B also acknowledged inadequate time as a prohibiting factor regarding the effective use of GeoGebra in teaching reflections about the x- and y-axis. Sarah said: *“GeoGebra requires a lot of practice, and some of us are failing to create time to practise with this software due to the work overload and the congested ATP. As a result, we end up being frustrated – and frustrating learners, too – during the RL, because we lack the confidence in using the software. The SMT should therefore set aside time on the time-table to enable school-based teacher CPD activities.”* Additionally, in South Africa’s education context, a limited number of schools have mathematics centres, and the teachers in school B highlighted the lack of teacher-permanent classrooms where learners come to them as a hindering factor to effective GeoGebra integration. For example, Bosco said: *“A teacher has to move from one class to the other, while connecting the gadgets. This discourages us from integrating the software in our classrooms, as it is time-consuming for one to keep on connecting the gadgets.”*

According to their observations, school B’s teachers jointly agreed that they were not impressed with the way the objective of coordinates was taught using GeoGebra. Sarah said: *“Like what transpired during the planning of RL1, when teaching the objective of coordinates, Frank did not change ΔEFG ’s position to different quadrants after reflecting it about the x- and y-axis. Hence, learners were made to believe that the signs of the x- and y-coordinates always change to negative after reflecting the object about the y- and x-axis respectively.”* Moving forward to RL2, school B’s teachers stressed the need to do things differently. For example, Bosco mentioned: *“During the planning of RL2, dynamism needed to be applied to the object so that the teachers get acquainted to how the software works to enrich learners’ conceptual understanding of the geometric relationships that exist between the object and its image during implementation.”*

The teachers in school B were also not happy with the planning when a question was posed. They recommended that in the next RL, each teacher should have something to say during planning. Frank said: *“When a question is posed during the planning session, every teacher should be given an opportunity to respond.”*

It can be concluded from school B's teachers' reflections that after RL1, individual teachers primarily demonstrated the accepting level (a lower TPACK development level).

Reflections by the KO on RL1 with teachers from school B

The section presents the KO's comments on school B's teachers' competencies in using GeoGebra to teach the transformation geometry concepts of reflection about the x- and y-axis in the four descriptors – 'curriculum and assessment', 'conception of learner thinking', 'instructional and environment', and 'barriers and availability'.

In the curriculum and assessment descriptor of the 'curriculum and assessment' theme', Lawrence noted that teachers in school A had varied competencies in teaching and assessing learners' conceptual understanding of the concept of reflection about the x- and y-axis using GeoGebra. For example, Lawrence said: *"While Frank and Sarah could use GeoGebra to draw and reflect the object about the x- and y-axis to show learners the geometric relationships that existed between the object and its image, Bosco could use the software to reflect the object about the x-axis ... then ask learners for the patterns that existed between the object and its image."*

Regarding the learning theme descriptor 'conception of learner thinking', Lawrence observed that teachers in school B were beginning to get the learners to explore the geometric relationships that existed between the object and its image without their guidance. For example, Lawrence reported: *"In developing learners' thinking skills with the aid of GeoGebra, teachers could now draw and reflect the object about the x- and y-axis, then ask learners to establish how the coordinates of the object and its image related after this transformation."*

With a focus on the teaching theme's descriptors, particularly 'instructional and environment', Lawrence noted that the teachers in school B tightly managed instruction with GeoGebra in a non-exploratory way, while validating the activity on the worksheet on reflection about the x- and y-axis. Lawrence noted: *"The teachers used GeoGebra to draw and reflect $\triangle EFG$ (the object) about the x- and y-axis, to verify the geometric relationships that existed between the object and its image."*

Regarding the access theme's descriptor 'barriers and availability', Lawrence observed that although teachers in school A could now use GeoGebra to develop various examples of the transformation geometry concept of reflection about the x- and y-axis for learners to explore,

they expressed concerns regarding access and management when integrating GeoGebra in their lessons. For instance, Lawrence pointed out: *“Inasmuch as teachers could draw and reflect different objects for learners to investigate patterns, these teachers still had difficulties in using GeoGebra, which included but [were] not limited to resizing and/or changing the position of the object.”* Moving on to RL2, Lawrence encouraged teachers to capitalise on the dynamism capability of GeoGebra to foster learners’ understanding of the geometric relationships that exist between the object and its image. Lawrence said: *“Teachers needed to practise moving the object to different positions to enable learners in investigating the patterns that were not possible with the pen and paper activity, and also to enrich their understanding of the geometric relationships that exist between the object and its image.”*

It is evident from Lawrence’s observations that after RL1, school B’s individual teachers’ competencies in using GeoGebra to teach the transformation geometry concept of reflection about the x- and y-axis were predominantly at the accepting level – a lower level of TPACK development.

4.4.2 TPACK development levels – findings from RL2 with the two schools

The preceding section discussed the findings of the TPACK development levels from RL1 with the teachers from the two schools, A and B. This section discusses the TPACK development level results from RL2, which was the second of the series of three, and the mathematical topic addressed in this RL was that of reflection about the line $y = x$. RL2 collected qualitative and quantitative data that aided in tracking the progression of individual teachers’ TPACK levels based on their GeoGebra experiences when implementing RL2.

As per the methodology, RL2 was created by the researcher and given to teachers, within their respective LS groups, to critique according to their learners’ learning needs and the curriculum. The KO and the teachers in the two schools jointly discussed the objectives of RL2 in their school’s LS group. The objectives included learners being able to: identify how the x- and y-coordinates change after reflecting rectangle ABCD about the line $y = x$; show that rectangle ABCD and its image are congruent; and show that rectangle ABCD and its image’s corresponding vertices are equidistant from the line $y = x$. During the identification of objectives phase, the KO shared his views on the transformation geometry concept of reflection about the line $y = x$, and why it is important.

When planning RL2, the teachers of schools A and B included some recommendations from RL1 to improve on the planning and implementation of this RL. During the planning of RL2, the KO shared his views on teaching reflections about the line $y = x$ using GeoGebra, while the researcher introduced the teachers to the data collection instruments – observation and interview guides (including reflection guides) to be used in RL2. The aim was to get the teachers acquainted with these instruments, in terms of what they would be observing during and reflecting on after the actual implementation of RL2. Both schools' teachers also identified the three learner categories – lower, middle and upper performers – and RL2 was designed, with GeoGebra, around their learning needs and the curriculum.

After the planning session for RL2, one of the three teachers facilitated this RL while the other two teachers and the KO observed and collected data on learner learning, using an observation guide. The KO also stepped in to provide clarity in terms of GeoGebra use when the teacher faced challenges during teaching. Immediately after RL2 was taught, the teachers in each school were given a reflection guide, which they completed and brought to the FGI for the researcher to follow up on their overall TPACK development after this RL. The teachers and the KO also reflected on the four phases of RL2 (identification of objectives, planning, implementation or observation, and reflection) and suggested recommendations on how to use GeoGebra to improve the next RL: RL3. The four phases of RL2 were video- and audio-recorded by the researcher for analysis purposes.

4.4.2.1 TPACK development levels – findings from implementation/observation of RL2 in School A

The section discusses the findings from the implementation/observation of RL2 by teachers from school A. Qualitative data was collected using an observation guide that was used to observe learner learning when teaching the transformation geometry concept of reflection about the $y = x$ line. This data assisted in answering the third research sub-question: the experiences of rural high school mathematics teachers teaching transformation geometry using GeoGebra.

RL2 was taught by Bernard, while Tom, Mike and Lawrence (the KO) observed learner learning when the transformation geometry concept of reflection about the $y = x$ line was taught using a worksheet and GeoGebra. Learners began by manually working on a pen and paper activity, after which Bernard used GeoGebra to consolidate the same activity. The

findings from the teaching of RL2 using a worksheet first, then GeoGebra, are presented and analysed in the sections below.

Teachers' experiences on the use of the worksheet to teach the transformation geometry concepts of reflection about the line $y = x$

The section presents the experiences of teachers from school A when learners manually solved the pen and paper activity on the transformation geometry concept of reflection about the $y = x$ line.

The general consensus from the school A teachers was that when using a worksheet, most learners managed to identify how the x- and y-coordinates changed when reflecting rectangle ABCD (the object) about the line $y = x$. For example, Mike observed: *“Except for the low achievers, all the learners were able to see that the y- and x-coordinates interchanged.”* The teachers also agreed that all learners could recognise that the sizes and distances of the corresponding vertices from the line $y = x$ of rectangles ABCD and A'B'C'D' (the image) were the same. Tom said: *“Learners were able to show that the sizes of rectangles ABCD and A'B'C'D' were congruent. Some learners counted the unit squares inside the rectangles and compared them, while others measured their lengths and breadths and used the formula for the area of a rectangle to calculate the areas. With regard to the distance, some learners counted the blocks and ascertained that the corresponding vertices of rectangles ABCD and A'B'C'D' were equidistant from the line $y = x$, while others used rulers to measure.”*

When planning RL2, the prediction was that the low achievers would be challenged by all the activities on the worksheet; the middle achievers would struggle with activity (d) (compare the distances between the corresponding vertices of rectangles ABCD and A'B'C'D' from the line $y = x$); and the top achiever would have no difficulty in carrying out this pen and paper activity – see Appendix 10b. However, while observing RL2, the teachers in school A noted that the middle and lower achievers performed better than predicted, while the top achievers performed as predicted. For example, Mike noted: *“The low achievers did not struggle with activities (c) (compare the sizes of rectangles ABCD and A'B'C'D') and (d), but struggled with activities (a), (b) and (e). The middle achiever did not have any challenges with activity (d), as was predicted, and the top achiever got every question in this activity correct.”*

It is evident from school A's teachers' observations that all the success criteria except the objective of coordinates were achieved during the pen and paper activity that focused on the transformation geometry concept of reflection about the line $y = x$.

Observations on GeoGebra use to teach the transformation geometry concept of reflection about the line $y = x$.

The preceding section presented the experiences of school A's teachers when learners manually solved the pen and paper activity on the transformation geometry concept of reflection about the $y = x$ line. The experiences of school A's teachers when using GeoGebra to teach the same transformation geometry activity on reflection about the line $y = x$ are presented in this section.

When using GeoGebra to teach the same pen and paper activity, school A's teachers agreed that the software assisted all the learners with the visualisation of how the x- and y-coordinates changed when reflection was done about the line $y = x$. For example, Bernard stated: *"I changed the position of rectangle ABCD to various quadrants and all learners could now correctly conjecture that the x- and y-coordinates interchanged."* Finally, the teacher institutionalised the relationship between the coordinates of the object and its image after reflecting the object about the line $y = x$, as the coordinates swap. Furthermore, the teachers acknowledged that GeoGebra had assisted learners with visualisation of how the sizes of rectangles ABCD and A'B'C'D' related after reflection about the line $y = x$. Lawrence said: *"After calculating the areas of rectangles ABCD and A'B'C'D' in GeoGebra, the teacher changed the positions and sizes of rectangle ABCD, and learners concluded that the sizes remained the same."* The teachers also agreed that GeoGebra had assisted learners with visualisation of how the distances from the $y = x$ line between the corresponding vertices of rectangles ABCD and A'B'C'D' related. Mike stated: *"Changing the positions of rectangle ABCD to various quadrants assisted the learners in appreciating that the distances between the corresponding vertices of rectangles ABCD and A'B'C'D' from the line $y = x$ always remained the same, thus enhancing learners' understanding. This dynamism that was applied to the object was not evident with the pen and paper activity."*

During the planning of RL2, the prediction was that when GeoGebra was used to consolidate the pen and paper activity, the three learner categories – low, middle and top – would successfully carry out the learning activities on the worksheet, except that the low achiever would still struggle with activity (e). When observing RL2, the lower, middle and top achievers

performed as predicted during the planning. For example, Tom said: *“Apart from the lower achievers struggling with activity (e), the other questions on the worksheet were answered correctly by these three categories of learners. Inasmuch as the lower achievers understood the geometric relationships that existed between rectangles ABCD and A'B'C'D', they had difficulties in expressing themselves, because of language barriers.”*

It is quite clear from the observations of the teachers from school A that GeoGebra use assisted learners in visualising the geometric relationships that existed between the object and its image, when compared to the pen and paper activity.

4.4.2.2 TPACK development levels – findings from the reflections on RL2 in school A

The preceding section presented the observations of school A’s teachers on teaching the transformation geometry concept of reflections about the $y = x$ line with a worksheet and GeoGebra. This section tracks individual teachers’ TPACK development levels as shown by their self-reported scores and the researchers’ interrater reliability scores in each descriptor of the five TPACK levels. Similarly to the tables shown for RL1, the RL2 findings are captured in Table 4.7.

Table 4.7 TPACK development levels of individual teachers in school A after RL2

LS group A's teachers	Themes and their descriptors	Teachers' self-reported scores	Researchers' interrater reliability scores	Teachers' reflections on their TPACK development level
Bernard	Curriculum and assessment – Curriculum and assessment descriptor	4	4	When I was teaching RL2, I observed that the low achievers could not identify how the coordinates related after reflecting rectangle ABCD about the $y = x$ line during the pen and paper activity. Changing the positions of rectangle ABCD (the object) to various quadrants and probing learners to establish how the coordinates related enabled the low achievers to also see this pattern.
	Learning – Conception of learner thinking descriptor	5	4	As alluded to earlier, when I was teaching RL2, I noted that some learners could not establish how the coordinates changed on the worksheet activity. I changed the positions of rectangle ABCD to overlap quadrants in GeoGebra, and all learners could now conjecture that the x- and y-coordinates swapped.
	Teaching – Instructional and environment descriptor	4	4	When I was teaching, I observed that some learners struggled with the objective of coordinates on the worksheet. After reflecting rectangle ABCD, I changed its position to different quadrants to create different situations. I then posed questions for learners to think about the geometric relationships between rectangle ABCD and its image. All learners could now conjecture that the coordinates interchanged.
	Access – Barrier and availability descriptor	3	3	When teaching RL2 I changed the position of the object, but it was only moving along the x-axis. I had to start afresh to move the object to whatever position I wanted it to be for learners to investigate the traits that existed between the object and its image – especially those that challenged them during the pen and paper activity, such as one on how the x- and y-coordinates changed after a reflection about the $y = x$ line.
Tom	Curriculum and assessment descriptor	5	4	When we were planning RL2, we only changed the position of the object, but not its size and shape. When observing this RL, I have learnt that after drawing and reflecting rectangle ABCD about the line $y = x$, it is also important to resize it to form different shapes, then ask learners to comment on the geometric relationships using different objects.
	Learning – Conception of learner thinking descriptor	4	4	After seeing the planning and implementation of RL2, I can now draw and reflect the object (rectangle ABCD) about the line $y = x$, change its position to overlap the line $y = x$, then probe learners to establish the patterns that exist between the object and its image.
	Teaching – Instructional and environment descriptor	4	4	Like what I saw during planning, when observing RL2, I have learnt that it is vital to engage learners in the exploration of the geometric relationships that exist between the object (rectangle ABCD) and its image by changing its position to various quadrants, and asking them to comment on the geometric relationships that exist between them. Even the low achievers could now conjecture that the coordinates swap.
	Access – Barrier and availability descriptor	4	3	When observing RL2, I have learnt that changing the positions of rectangle ABCD after reflecting it about the line $y = x$ aided the low achievers to see that the x- and y-coordinates interchange, which pattern they could not establish during the pen and paper activity.
	Curriculum and	4	4	In planning RL2, when measuring the distances between the corresponding vertices of the object and its image, lines were

Mike	assessment descriptor			used and no dynamism was applied. When observing RL2, I noted that Bernard did the same thing. In changing the positions of the object, the lines were static. After the KO intervened, I learnt that it is vital to use line segments in joining the corresponding vertices of the object and its image, change the object's position, and ask learners to comment on the distances.
	Learning – Conception of learner thinking descriptor	5	4	Like what I saw during the planning of RL2, when observing this RL, I have learnt that after drawing and reflecting rectangle ABDC (object) about the $y = x$ line, I need to vary its position for learners to develop conjectures about the geometric relationships that exist between the object and its image – particularly those that they struggled with during the pen and paper activity, such as the objective of coordinates.
	Teaching – Instructional and environment descriptor	4	4	When planning, dynamism was not applied when measuring the distances between the corresponding vertices of the object and its image from the line $y = x$, which were joined using lines. In observing RL2, I have learnt that it is key to join these vertices using line segments instead of lines (the lines remained stationary with dynamism), change the position of the object, and ask learners to comment on these distances.
	Access – Barrier and availability descriptor	3	3	When planning RL2, in measuring distance, the lines (instead of line segments) were used to connect the corresponding vertices of the object and its image from the line $y = x$, and the object's position was not changed. When observing this RL, I have learnt that one needs to use line segments because when Bernard applied dynamism to the object, the lines did not move along with the vertices, making it difficult to explore the trait of distance.

It can be concluded from Table 4.7 that all teachers demonstrated the exploring-level TPACK traits in the curriculum and assessment descriptor of the ‘curriculum and assessment’ theme. School A teachers reported using GeoGebra for learners to establish the geometric relationships that existed between rectangle ABCD and its image when rectangle ABCD was reflected about the line $y = x$ by changing its position to various quadrants, and asking learners to formulate conjectures about what they saw.

The learning theme descriptor ‘conception of learner thinking’, revealed that the teachers demonstrated the exploring TPACK level. School A’s teachers reflected on teaching the transformation geometry concept of reflection about the $y = x$ line, focusing on supporting learners’ understanding of its properties.

That the teachers demonstrated exploring-level TPACK features was also evidenced by their excerpts regarding the teaching theme ‘instructional and environment’ descriptor. The teachers reported changing the positions of rectangle ABCD to various quadrants after reflecting it about the line $y = x$, and posing questions that engaged learners in thinking about the geometric relationships that existed between rectangle ABCD and its image.

With the access theme descriptor ‘barriers and availability’, school A’s teachers’ thinking, as reflected in their excerpts, suggested an adapting TPACK level. These teachers reported using GeoGebra as a tool to provide learners with a new approach to accessing the geometric relationships that existed between rectangle ABCD and its image that were formerly out of reach, particularly when looking at the objective of coordinates during the pen and paper activity.

The teachers in school A observed that Bernard struggled with measuring the distances between the corresponding vertices of the object (rectangle ABCD) and its image from the line $y = x$ when teaching the objective of distance. For example, Mike noted: *“When measuring the distances between the corresponding vertices of the object and its image from the $y = x$ line during planning, lines were used to join these vertices, and the object’s position was not changed; but dynamism was emphasised by ‘word of mouth’. When implementing RL2, Bernard followed the same procedure when measuring the distance; and on changing the position of rectangle ABCD, the lines remained static, making it difficult to explore this objective. It was after the KO intervened that this challenge was rectified.”* The teachers recommended that in the next RL, the teacher who was going to teach RL3 should rehearse what they had planned before teaching this RL. Tom, for instance, pointed out: *“If Bernard had rehearsed prior to teaching RL2, the challenge that arose in measuring the distance between the corresponding vertices of rectangle ABCD and its image from the $y = x$ line using lines instead of line segments could have been picked up and rectified before implementing this RL. Therefore, there is [a] need for the teacher who is going to teach to rehearse before the RL, to guard against the challenges that may arise during this RL.”*

Additionally, school A’s teachers suggested that the teacher who was going to teach the next RL with a focus on the transformation geometry concept of translation should be the one demonstrating how to use GeoGebra to teach this concept during planning. This is what Bernard had to say: *“Since the KO trained us already, let the teacher who is going to teach RL3 be the one demonstrating how to use GeoGebra to teach this concept, while the KO serves as a guide. This will help the teacher(s) to get more acquainted with how to use the software in teaching the transformation geometry concept of translation, hence minimising the challenges that may arise during the RL.”*

It is evident from school A’s teachers’ reflections that after RL2, individual teachers practised actions primarily at the exploring level – a higher TPACK development level.

Reflections by the KO on RL2 with teachers from school A

This section reports on the KO's reflections on school A's teachers' competencies in using GeoGebra to teach the transformation geometry concepts of reflection about the line $y = x$ in the four descriptors – 'curriculum and assessment', 'conception of learner thinking', 'instructional and environment', and 'barriers and availability'.

With regard to the curriculum and assessment descriptor of the 'curriculum and assessment' theme, Lawrence observed that after RL2, the teachers in school A could actively integrate the software into teaching and assessing learners' conceptual understanding of the transformation geometry concept of reflection about the line $y = x$. For example, Lawrence remarked: *"The teachers in school A could use GeoGebra to engage learners in exploration by resizing and/or changing the position of rectangle ABCD (the object) in different quadrants, then probe them for the geometric relationships that they observed between the object and its image."*

Under the learning theme descriptor 'conception of learner thinking', Lawrence noted that when teaching the concept of reflection about the line $y = x$, school A's teachers enabled learners to explore the relationships that existed between the object and its image, while taking on a facilitative role, guiding learners' exploration without dictating the process. For example, Lawrence said: *"In teaching the transformation geometry concept of reflection about the $y = x$ line, teachers in school A took the role of a facilitator through engaging learners in exploration and posing probing questions to guide them in understanding."*

Within the descriptors of the teaching theme, in particular 'instructional and environment', Lawrence observed that the teachers in school A engaged learners in exploring the geometric relationships that existed between rectangle ABCD (the object) and its image on the transformation geometry concept of reflection about the line $y = x$, while guiding rather than directing the exploration. Lawrence reported: *"Teachers could use GeoGebra to change the positions of the object, then pose questions that engaged learners in exploring the patterns between the object and its image, while acting as guides."*

Regarding the access theme descriptor 'barriers and availability', Lawrence noted that teachers in school A used GeoGebra to provide learners with a novel approach to understanding geometric relationships between the object and its image, particularly those which they could not establish during the pen and paper activity. Lawrence said this: *"Teachers in school A used GeoGebra to vary the positions of the object to assist learners in establishing the patterns that*

existed between the object and its image, especially those that they could not during the worksheet activity – for example, the success criteria that focused on how the coordinates changed after reflecting the object about the $y = x$ line.”

When observing RL2, Lawrence noted that Bernard experienced a challenge when using GeoGebra to teach the objective of distance, which emanated from the planning phase. Lawrence said: *“During planning, in measuring the distances between corresponding vertices of the object and its image from the line $y = x$, we used lines to join these vertices instead of line segments, and dynamism was not practically, but theoretically applied to the object. Bernard followed the same process in measuring this distance when implementing RL2, and on changing the position of rectangle ABCD, the lines were not moving.”* Moving to RL3, Lawrence recommended harnessing the dynamism capability of GeoGebra not only during implementation, but also in planning the RL. This is how Lawrence put it: *“Inasmuch as I intervened when Bernard got stuck in measuring the distances between the corresponding vertices of rectangle ABCD and its image from the $y = x$ line during RL2, I realised that there was a need to practically apply dynamism to the object during planning when measuring these distances. This could have helped to identify this challenge, which arose during the RL.”*

It can be concluded from Lawrence’s observations that after RL2, the individual school A teachers’ competencies in using GeoGebra to teach the transformation geometry concept of reflection about the line $y = x$ were predominantly at the exploring level – a higher TPACK development level.

4.4.2.3 TPACK development levels – findings from implementation/observation of RL2 in School B

This section discusses the results from the implementation/observation of RL2 by teachers from school B. The analysis of the implementation/observation of school B’s teachers’ RL2 is similar to that of school A’s teachers. Bosco is the teacher who taught RL2, while Sarah, Frank and Lawrence observed learner learning when the transformation geometry concept of reflection about the $y = x$ line was taught, using a worksheet and using GeoGebra. Learners solved questions from a worksheet first, which they were working on manually, after which Frank used the software to teach the same activity. The findings from the teaching of RL2 using a worksheet first, then GeoGebra, are presented and analysed next.

Teachers' experiences on the use of the worksheet to teach the transformation geometry concept of reflection about the line $y = x$.

This section presents the experiences of teachers from school B when learners manually solved the pen and paper activity on the transformation geometry concept of reflection about the line $y = x$.

There was acknowledgement by school B's teachers that when using a worksheet, most learners could identify how the x- and y-coordinates changed when rectangle ABCD (the object) was reflected about the line $y = x$. For example, Sarah said: *"All [the] learners, excluding the low achievers, were able to identify that the x- and y-coordinates swapped after rectangle ABCD was reflected about the $y = x$ line."* Additionally, the teachers observed that the majority of the learners could not correctly identify that rectangle ABCD and A'B'C'D' (the image) were congruent in size. This is what Bosco observed: *"Very few learners were able to determine that rectangle ABCD and its image were congruent in size. Most learners used the wrong formula – area of a right-angled triangle – instead of the formula for the area of a rectangle."* The teachers also noted that most learners could identify that the corresponding vertices of rectangles ABCD and A'B'C'D' were equidistant from the line $y = x$. Frank pointed out: *"Except for the low achievers, all [the] learners were able to identify that the corresponding vertices of rectangles ABCD and A'B'C'D' were equidistant from the $y = x$ line."*

During the planning of RL2, the prediction was that the low achievers would be challenged by all the activities on the worksheet, the middle achievers would struggle with activity (d) (compare the distances from the line $y = x$ of the corresponding vertices of rectangles ABCD and A'B'C'D') and (e) (discuss your solutions, showing the relationship between rectangle ABCD, the line $y = x$, and rectangle A'B'C'D'), and the top achievers would answer all the questions on the pen and paper activity correctly – see Appendix 10b. However, while observing RL2, school B's teachers noted that unlike the top achiever, who did not perform as well as expected, the low and middle achievers performed better than predicted. For example, Frank noted: *"The low achievers only struggled with activities (b) (establishing how the x- and y-coordinates changed after reflecting rectangle ABCD about the line $y = x$), (c) (comparing the sizes of rectangles ABCD and A'B'C'D'), (d) and (e), as was predicted, but not (a) (drawing the image of rectangle ABCD and label it A'B'C'D'). The middle achievers did not struggle with activity (d) as predicted, but only struggled with activity (e); and the top achievers only had difficulty with activity (c) – yet the prediction was that they would get it correct."*

It can therefore be concluded from school B's teachers' experiences from the pen and paper activity that most of the success criteria were not achieved.

Observations on GeoGebra use to teach the transformation geometry concept of reflection about the line $y = x$.

The previous section presented the experiences of teachers from school B when learners manually solved the pen and paper activity on the transformation geometry concept of reflection about the line $y = x$. This section presents school B's teachers' experiences of using GeoGebra to teach the transformation geometry concept of reflection about the line $y = x$.

After utilising GeoGebra to teach the same pen and paper activity, the teachers from school B agreed that the software had assisted learners with visualisation of how the x- and y-coordinates changed when rectangle ABCD was reflected about the line $y = x$. For example, Lawrence said: *"The lower achievers could now conjecture that the coordinates swapped when the positions and/or sizes of rectangle ABCD were changed to different quadrants."* Additionally, school B's teachers noted that GeoGebra assisted learners with visualising how the sizes of rectangles ABCD and A'B'C'D' were related after reflecting rectangle ABCD about the line $y = x$. Bosco pointed out: *"I resized rectangle ABCD several times, and all learners could now see and conjecture that its size and that of its image remained the same."* The teachers also observed that the software assisted learners with visualising how the distances from the $y = x$ line of the corresponding vertices of rectangles ABCD and A'B'C'D' were related. Sarah pointed out: *"The low achievers could now see that the distances from the $y = x$ line of the corresponding vertices of rectangles ABCD and A'B'C'D' remained the same, even when the teacher changed the position of rectangle ABCD."*

When planning RL2, the prediction by teachers in school B was that when GeoGebra was used to teach the same pen and paper activity, the low, middle and top achievers would successfully carry out the learning activities on the worksheet, except that the low achievers would have difficulties with activity (e). When RL2 was observed, the middle and top achievers performed as predicted, while the lower achievers performed better than predicted during planning. For example, Lawrence said: *"The lower achievers rightly answered activity (e), yet the prediction was that 'they will struggle'. The top and middle achievers performed as predicted – they correctly answered all the activities on the worksheet with the software."*

It is evident from the teachers' observations that GeoGebra assisted the learners with the visualisation of the geometric relationships that existed between the object and its image, particularly those that they struggled with on the pen and paper activity when reflecting rectangle ABCD about the $y = x$ line.

4.4.2.4 TPACK development levels – findings from the reflections on RL2 in school B

The preceding section discussed the results from the implementation/observation of RL2 by teachers from school B, when a worksheet and then GeoGebra were utilised to teach the transformation geometry concept of reflection about the $y = x$ line. This section tracks school B individual teachers' TPACK development levels as shown by their self-reported scores and the researchers' interrater reliability scores on each descriptor of the five TPACK levels. In the same way as the previous tables, Table 4.8 summarises the findings of RL2.

Table 4.8 TPACK development levels of individual teachers in school B after RL2

LS group B's teachers	Themes and their descriptors	Teachers' self-reported scores	Researchers' interrater reliability scores	Teachers' reflections on their TPACK development level
Bosco	Curriculum and assessment – Curriculum and assessment descriptor	5	4	When I was teaching, I observed that most learners used the formula of a right-angled triangle in calculating areas of rectangles ABCD and A'B'C'D' on the worksheet. I calculated the areas in GeoGebra, changed the sizes of rectangle ABCD. On asking the learners for their observations, all learners could now see that the correct formula was that of the area of a rectangle, [which had been] presented by one of the top achievers during the worksheet activity.
	Learning – Conception of learner thinking descriptor	4	4	When I was teaching RL2, I noted that some learners had difficulty in identifying how the coordinates changed when rectangle ABCD was reflected about the line $y = x$ during the pen and paper activity. When using GeoGebra, after reflecting rectangle ABCD, I changed its positions and asked learners to comment on the coordinates of the corresponding vertices. All learners conjectured that the coordinates swap.
	Teaching – Instructional and environment descriptor	5	4	When I was teaching RL2, I noticed that some learners struggled to identify that the corresponding vertices of rectangle ABCD and its image were equidistant from the $y = x$ line. After drawing and reflecting rectangle ABCD, I measured these distances, changed its position and asked the learners to comment on these distances. The exploration and questioning methods were interactive and increased learner participation.
	Access – Barrier and availability descriptor	3	3	As alluded to earlier, when I was teaching RL2, some learners struggled with establishing the relationship between the distances of the corresponding vertices of the object and its image from the line $y = x$. I measured these distances several times, but they were different. After the KO stepped in to mitigate this challenge, I changed the position of the object, and all learners could now conclude that the distances were equal.
Sarah	Curriculum and assessment descriptor	4	3	When we planned RL2, teaching and assessing learners' conceptual grasp of the concept of reflection about the line $y = x$ using GeoGebra was not clear to me – the discussion was more in theory rather than in practice. This became clear as I was observing this RL. I can now draw and reflect rectangle ABCD, then pose questions that require learners to establish and apply the patterns that exist between the object and its image.
	Learning – Conception of learner thinking descriptor	3	3	During the planning of RL2, it was not clear to me how to foster learners' thinking skills development on the transformation geometry concept of reflection about the line $y = x$ using GeoGebra. As a result of observing RL2, I can now draw and reflect rectangle ABCD, then probe learners to comprehend their understanding and thinking errors from the pen and paper activity.
	Teaching – Instructional and environment descriptor	4	3	When we were planning RL2, creating teaching approaches and a classroom environment that promotes the learning of the transformation geometry concept of reflection about the line $y = x$ using GeoGebra was not clear to me. After observing RL2, I can now draw and reflect rectangle ABCD about the line $y = x$ – then, for instance, ask learners to establish the relationship that exists between the coordinates of rectangle ABCD and its image.
	Access – Barrier and availability descriptor	2	3	When planning RL2, I did not understand how to measure the distances between the corresponding vertices of the object and its image from the line $y = x$. This became clear when observing RL2. I can now measure these distances in GeoGebra, then probe learners to comment on them, hence rectifying their thinking errors and misconceptions from the worksheet activity.

Frank	Curriculum and assessment descriptor	4	3	Like what Sarah said, when planning RL2, the discussion on how to teach and assess learners' conceptual understanding of reflections about the line $y = x$ was predominantly theoretical. Observing this RL has given me a practical shift on how to utilise GeoGebra in teaching and assessing learners' understanding of this concept. I can now draw and reflect the object, then ask learners to establish the patterns between the objects.
	Learning – Conception of learner thinking descriptor	3	3	When observing RL2, I have learnt that after reflecting rectangle ABCD (the object), it is crucial to ask learners questions that allow them to investigate the patterns that exist between the object and its image, particularly those which they could not establish during the pen and paper activity; this was not clear to me during planning.
	Teaching – Instructional and environment descriptor	4	3	Like Sarah indicated, during the planning of RL2 the instructional approaches to use with GeoGebra were not clear to me – the planning was mainly theoretical. When observing this RL, I have learnt that after reflecting rectangle ABCD, I need to engage learners by posing questions that require them to establish the geometric relationships that exist between the object and its image.
	Access – Barrier and availability descriptor	3	2	Like what I saw during planning, when observing RL2, I have learnt that various situations can quickly be created with GeoGebra. However, in changing the positions of rectangle ABCD, coordinates with decimal numbers appeared and Bosco was unaware about how this happened. The same thing happened to me in RL1. I still need support in this regard. I am also not confident in teaching the objective of distance with GeoGebra.

Table 4.8 shows that Bosco illustrated exploring TPACK level traits, as showcased by his excerpt of the curriculum and assessment descriptor of the 'curriculum and assessment' theme. Bosco utilised exploration to develop learners' transformation geometry knowledge on the concept of reflection about the line $y = x$ by posing questions for learners to come up with conjectures about the ideas that they were investigating. Sarah and Frank showcased characteristics of the adapting TPACK development level. Both reported demonstrating the transformation geometry concept of reflection about the line $y = x$ with GeoGebra, and asking learners questions that required them to establish and apply the patterns that existed between the object and its image. Some teachers also demonstrated an understanding of some advantages of teaching and learning the concept of reflection about the line $y = x$ when using GeoGebra. For example, Sarah mentioned: *"When presenting their pen and paper solutions, some learners calculated the area of a rectangle using the formula, $A = l \times b$, while the other learners used the formula for the area of a square, $A = (\text{side})^2$. The use of the software (GeoGebra) helped the learners to decide on the correct formula for calculating the area of a rectangle."*

The thinking revealed through Bosco's excerpt implies the exploring TPACK level, as evidenced by Bosco's excerpt from the learning theme descriptor 'conception of learner thinking'. Bosco pointed out how GeoGebra could be used in fostering the teaching and learning of the objective of coordinates when reflecting rectangle ABCD about line the $y = x$,

while focusing on guiding learners' understanding. Sarah and Frank exemplified the characteristics of the adapting TPACK level in beginning to develop appropriate thinking skills regarding reflection about the line $y = x$ when utilising GeoGebra as a learning tool.

Regarding the teaching theme descriptor 'instructional and environment', Bosco demonstrated exploring TPACK level characteristics. Bosco reported using exploration and question-and-answer methods with GeoGebra in engaging learners in thinking about the characteristics of the concept of reflection about the line $y = x$. Sarah and Frank exhibited the adapting TPACK development level by incorporating a GeoGebra-based RL on the concept of reflection about the line $y = x$ that was tailored to learners' learning needs, as well as beginning to adapt teaching methods that would enable learners to explore with this software.

With the access theme descriptor 'barriers and availability', Bosco and Sarah illustrated the adapting TPACK development level. Bosco and Sarah reported using GeoGebra to provide access to the objective of distance, which was formerly out of reach to some learners during the pen and paper activity. Frank exemplified the accepting level of TPACK development. Although Frank acknowledged that the software allowed a greater number of examples of the concept of reflection about the line $y = x$ to be explored, he worried about access in terms of integrating GeoGebra into the classroom.

Additionally, some teachers in school B acknowledged GeoGebra's role in teaching higher-order concepts of reflection about the line $y = x$. For example, Bosco said: *"The object can occupy at least two quadrants or one of its vertices can be drawn on the line $y = x$ for learners to apply some thinking in drawing the image and figuring out the geometric relationships that exist between the object and its image."*

When reflecting on RL2, school B's teachers indicated that in the next RL they would need to make sure that the teacher who was going to teach the RL was comfortable with using GeoGebra. For example, Frank stated: *"The focus of the next RL on translation should be on teaching and learning of this concept, and not on learning how to use GeoGebra to teach this concept in front of learners. My suggestion therefore is that when planning RL3, the KO should take us through again on how to use the software to teach the transformation geometry concept of translation."* The teachers also recommended that in the next RL on the transformation geometry concept of translation, the teacher should reduce teacher talk while presenting the RL. As Sarah pointed out: *"While investigating the patterns that exist between the object and*

its image with GeoGebra, the learners should do more talking and exploration of these geometric relationships than the teacher.”

It is evident from school B’s teachers’ reflections that after RL2, most of the teachers practised actions predominantly at the adapting level – a lower TPACK development level.

Reflections by the KO on RL2 with teachers from school B

This section reports the KO’s reflections on school B’s teachers’ competencies in using GeoGebra to teach the transformation geometry concept of reflection about the $y = x$ line in each of the four descriptors: ‘curriculum and assessment’, ‘conception of learner thinking’, ‘instructional and environment’, and ‘barrier and availability’.

For the curriculum and assessment descriptor of the ‘curriculum and assessment’ theme, Lawrence noted that school B’s teachers had differing competencies in teaching and assessing learners’ conceptual understanding of the concept of reflection about the line $y = x$ with GeoGebra. For example, Lawrence noted: *“While Bosco could draw, reflect, drag the object, and ask learners for the geometric relationships that existed between the object and its image, Sarah and Frank could only draw and reflect the object, then ask learners for the patterns that existed between them, without applying dynamism to the object.”*

With the learning theme descriptor, ‘conception of learner thinking’, Lawrence observed that school B’s teachers’ competencies differed in terms of using GeoGebra to facilitate the development of the learners’ conceptual knowledge of geometric reflection thinking skills about the line $y = x$. Lawrence pointed out: *“When teaching RL2, Bosco guided learners in understanding the geometric relationships that existed between the object and its image through exploration, while Frank and Sarah were at the stage of beginning to explore integrating GeoGebra as a tool for learning the transformation geometry concept of reflection about the line $y = x$.”*

Concerning the ‘instructional and environment’ descriptor of the teaching theme, Lawrence noted that Frank and Sarah were beginning to adapt instructional approaches with GeoGebra that allowed learners the opportunity to explore the geometric relationships that existed between the object and its image. Bosco engaged learners in the exploration of the patterns that existed between the object and its image, while serving as a guide. This is what Lawrence had to say: *“While serving as a guide, Bosco used GeoGebra to engage learners in the exploration*

of the distances between the corresponding vertices of rectangle ABCD and its image from the line $y = x$. Sarah and Frank used instructional strategies with GeoGebra which were teacher-directed (such as reflecting rectangle ABCD about the line $y = x$, and asking learners to comment on the patterns that they saw) to maintain control of how the activity progressed.”

Regarding the access theme ‘barriers and availability’ descriptor, Lawrence observed that Bosco and Sarah used GeoGebra to provide learners with access to the traits of the concept of reflection about the line $y = x$ that were formerly out of reach during the worksheet activity, such as how the coordinates changed after reflecting the object about the line $y = x$. In Frank’s case, Lawrence noted that the teacher used the software to give varied examples for learners to investigate the patterns that existed between the object and its image, though he expressed concerns regarding access with respect to using the software. As examples, Lawrence offered this: *“Bosco and Sarah used GeoGebra to mitigate learners’ misconceptions and thinking errors from the worksheet activity, while Frank lacked the confidence in using the software to teach the transformation geometry concept of reflection about the line $y = x$ – although he acknowledged that with this software, various examples could quickly be created for learners to explore.”*

Lawrence observed that Bosco struggled when teaching the objective of distance. For example, Lawrence noted: *“In measuring the distances between the corresponding vertices of the rectangle ABCD (object) and its image from the line $y = x$, the distances were not equal. I then intervened by drawing the object again, reflected it, and measured the distances from one pair of the corresponding vertices of the object and [of] its image from the line $y = x$. Bosco then measured the distances between the other corresponding vertices from the line $y = x$, applied dynamism to the object, and all learners were now able to establish that these distances were equal.”* Regarding RL3, Lawrence recommended that the teacher who was going to teach RL3 would need to practise with the software before the RL. Lawrence had this to say: *“The reason ... the challenge in measuring the distances between the corresponding vertices of the object and its image arose during RL2 was because Bosco did not practise with the software prior to the RL. Hence, the teacher who was going to teach RL3 would need to rehearse prior to teaching this RL.”*

Lawrence also noted that the focus of RL2 was on only the learners chosen as examples of the three categories of learners – the low, middle and top achievers. Lawrence explains: *“Bosco forgot about the rest of the learners, and only focused on these three learner categories. As a*

result, some learners – particularly the lower achiever [one of the three case learners] began to disengage because he was struggling, while the other learners in the classroom were mimicking the teacher on whom he was going to select next. So, in the next RL, the teacher needs to strike a balance by accommodating all learners in the classroom.”

It is quite clear from Lawrence’s observations that after RL2, school B’s individual teachers’ competencies in teaching the transformation geometry concept of reflection about the line $y = x$ with GeoGebra were mainly at the adapting stage – a lower TPACK development level.

4.4.3 TPACK development levels – findings from RL3 with the two schools

The findings of the TPACK development levels from RL2 with teachers from the two schools (A and B) were discussed earlier. This section discusses the TPACK development level findings from RL3, which was the last of the series of three, and the mathematical topic addressed in this RL was translation. Like RL1 and RL2, RL3 collected qualitative and quantitative data that tracked individual teachers’ TPACK levels in each descriptor of the five TPACK levels in the four themes.

With reference to the methodology, the researcher created the RL plan on the transformation geometry concept of translation for teachers in the two schools (A and B) to critique according to their learners’ learning needs and the curriculum. The teachers in each school and the KO jointly discussed the objectives of RL3. The objectives included learners being able to show that a translation produces congruent objects, and that the distances between the vertices of square ABCD and the corresponding vertices of its image were equal. During the identification of objectives phase, the KO shared his views on the transformation geometry concept of translation, and why it was pertinent.

The two LS groups of teachers in schools A and B included some comments from RL1 and RL2 to improve on the planning and implementation of RL3. During the planning of RL3, the KO shared his views on teaching the transformation geometry concept of translation with GeoGebra, while the researcher introduced the teachers to the data collection instruments – the observation and interview guides (including reflection guides) to be used in RL3. The aim was to get the teachers acquainted with the instruments in terms of what they would be observing during and reflecting on after the actual implementation of RL3. The teachers in both schools (A and B) also identified the three categories of learners – lower, middle and upper performers – in their respective schools, around whom RL3 was developed using GeoGebra.

After planning RL3, one of the three teachers facilitated this RL, while the other two teachers, and the KO observed and collected data on learner learning of the transformation geometry concept of translation, using a pre-determined observation guide. The KO also stepped in to provide clarity in terms of GeoGebra use when the teacher got stranded during their teaching. Immediately after RL3 was taught the teachers were given a reflection guide, which they completed and brought to the FGI for the researcher to follow up on their overall TPACK development. The teachers and the KO also reflected on the four phases of RL3 – identification of objectives, planning, implementing or observing, and reflecting – and suggested recommendations for how to use the software to improve the next RL. The four phases of RL3 were video- and audio-recorded by the researcher for analysis purposes.

4.4.3.1 TPACK development levels – findings from the implementation/observation of RL3 in School A

The section discusses the findings from the implementation/observation of RL3 by teachers from school A. Qualitative data was collected using an observation guide that was used to observe learner learning when teaching the transformation geometry concept of translation. This data assisted in answering the third research question – the experiences of rural high school mathematics teachers teaching transformation geometry using GeoGebra.

Tom taught RL3 and Mike, Bernard and Lawrence observed learner learning when the transformation geometry concept of translation was taught, using a worksheet and then using GeoGebra. Learners solved questions from a worksheet first, which they worked on manually; after which Tom used GeoGebra to consolidate the same activity. The findings from teaching RL3 using a worksheet first, then GeoGebra, are presented and analysed here.

Teachers' experiences on the use of the worksheet to teach the transformation geometry concept of translation

In this section, the researcher presents school A's teachers' experiences when learners manually solved the pen and paper activity on the transformation geometry concept of translation.

When using a worksheet, school A's teachers agreed that all learners could identify that square ABCD and its image were of the same size. For example, Tom said: *“All learners managed to calculate the areas of square ABCD and its image, and concluded that they were identical in size. Some learners used the formula $A = (\text{side})^2$, while the other learners counted the unit*

boxes in the objects and found out that the objects were congruent in size.” School A’s teachers also observed that most of their learners could not identify that the distances between the vertices of square ABCD and its image’s corresponding vertices were all equal in length. This is what Bernard noted: *“Instead of measuring the distances between the corresponding vertices of square ABCD and its image, some learners measured the distances between these vertices from the x-axis and concluded that they were not equal.”*

When RL3 was planned, the prediction was that the low achievers would only face challenges with activity (d) (discussing their solutions showing the relationships between square ABCD and its image), and the middle and top achievers would correctly answer the questions on the pen and paper activity – see Appendix 10c. However, while observing RL3, the teachers in school A noted that unlike the top achievers, who performed as predicted, the low and middle achievers did not perform as well as predicted. For example, Bernard observed: *“The low achievers did not only struggle with activity (d) as had been predicted, but also struggled with activity (c) (measuring and comparing the distances between the corresponding vertices of square ABCD and its image). The middle achievers struggled with activity (c), yet the prediction was that they would get everything correct on the worksheet; and the top achievers performed as had been anticipated.”*

It can be concluded from school A’s teachers’ observations that all the success criteria except the objective of distance were achieved when learners solved the questions on the pen and paper activity.

Observations on GeoGebra use to teach the transformation geometry concept of translation

The previous section presented school A’s teachers’ experiences when learners manually solved the pen and paper activity on the transformation geometry concept of translation. This section presents school A’s teachers’ experiences when teaching the transformation geometry concept of translation using GeoGebra.

When utilising GeoGebra in teaching the same pen and paper activity, there was an acknowledgement by the teachers in school A that the software assisted the learners with visualising how the size of square ABCD and that of its image related. For example, Mike mentioned: *“Tom calculated the areas using GeoGebra, changed the position and/or size of square ABCD (the object), and learners could see and conjecture that their areas remained the same. This was not possible with the pen and paper activity, as the object was static, and in*

quadrant 1. Tom also hid the object and asked learners to draw it given the image and the translation vector. On using GeoGebra to verify learners' answers, Tom also calculated their areas, changed the position and size of the object, and learners conjectured that they were congruent in size." School A's teachers also concurred that the software assisted the learners with visualising how the distances between the corresponding vertices of square ABCD and its image related. Lawrence said: *"With the aid of GeoGebra, all learners understood how to measure the distances between the corresponding vertices of square ABCD and its image; unlike on the worksheet, where most of the learners measured these distances from the x-axis. With GeoGebra, learners could see that the distances between the corresponding vertices of the object and its image are always equal, even when the teacher changed the position of the object using the translation vector."*

During the planning of RL3, the prediction was that when GeoGebra is used to consolidate the pen and paper activity, the three case learners in the three learner categories – low, middle and top – would successfully carry out the same pen and paper activity. When observing RL3, these three case learners performed as predicted. For example, Tom pointed out: *"All the questions on the worksheet were answered correctly by these three categories of learners with GeoGebra."*

It can be argued from school A's teachers' observations that GeoGebra assisted the learners with the visualisation of the geometric relationships that existed between square ABCD and its image.

4.4.3.2 TPACK development levels – findings from the reflections on RL3 in school A

The preceding section presented the findings from the implementation/observation of RL3 by teachers from school A when a worksheet and GeoGebra were used to teach the same transformation geometry activity on the concept of translation. This section tracks school A's individual teachers' TPACK development levels, as shown by their self-reported scores and the researchers' interrater reliability scores, in each descriptor of the five TPACK levels. Like the tables in RL1 and RL2, Table 4.9 summarises the findings of RL3.

Table 4.9 TPACK development levels of individual teachers in school A after RL3

LS group A's teachers	Themes and their descriptors	Teachers' self-reported scores	Researchers' interrater reliability scores	Teachers' reflections on their TPACK development level
Tom	Curriculum and assessment – Curriculum and assessment descriptor	5	5	When I was teaching RL3, I observed that some learners had difficulty with establishing the relationship between the distances of the corresponding vertices of the square ABCD (object) and its image on the worksheet. When using GeoGebra, I measured these distances, changed the positions of the object, and all [the] learners conjectured that they were all equal. I then hid the translation vector, and asked [the] learners to establish the rule.
	Learning – Conception of learner thinking descriptor	4	5	When I was teaching RL3, after measuring the distances between the corresponding vertices of square ABCD and its image, I changed the positions of this object using the translation vector; and all learners could now conjecture that these distances were equal. I also hid the object, gave learners the image and translation vector, then asked them to draw and justify the position of the object, thus enhancing their understanding.
	Teaching – Instructional and environment descriptor	4	5	When I was teaching RL3, in presenting their worksheet solutions, I observed that some learners were saying that the y-coordinate changes to positive. After translating square ABCD, I changed its position to different quadrants, hence countering learners' thinking error from the worksheet. I also designed higher-order questions with GeoGebra that required learners to draw the object given its image and the translation vector.
	Access – Barrier and availability descriptor	4	4	When I was teaching, I noted from the worksheet activity that some learners were conjecturing that the y-coordinate becomes negative after translating the object. After translating the object in GeoGebra, I realized that some learners still conjectured that the y-coordinate changed to negative. I rectified this thinking error by changing the translation vector to various positions, thus aiding learners to overcome their thinking error.
Mike	Curriculum and assessment descriptor	4	5	When planning RL3, the object, image and translation vector were not hidden. This became clear when observing this RL. Beyond measuring the areas ... of the object and its image in GeoGebra and resizing the object for learners to establish conjectures, I can now construct the image and translation vector, then ask learners to draw the object, which has been hidden.
	Learning – Conception of learner thinking descriptor	5	5	During the planning of RL3, the object, image and translation vector were not hidden. When observing this RL, I learnt that beyond drawing, reflecting and resizing square ABCD (the object) for learners to establish patterns that exist between the object and its image, I can also draw and translate the object, then hide the translation vector and ask learners to establish the translation rule using the given information.
	Teaching – Instructional and environment descriptor	4	5	When I was observing RL3, I learnt to construct and reconstruct various polygons in GeoGebra, then pose questions that require higher-level thinking such as asking learners to draw the object given its image and the translation vector. I did not see the latter during planning.
	Access – Barrier and availability descriptor	4	4	During the planning of RL3, the image, translation vector ... were not hidden. When observing this RL, I learnt that beyond changing the positions of the object and asking learners to establish the patterns that exist between the object and its image, I can now ask learners to draw the object given the image and translation vector, hence advancing their thinking skills.
	Curriculum and assessment descriptor	4	4	Like what I saw during planning, when observing RL3, I learnt the importance of changing the positions of square ABCD (object) and asking learners to comment on how the ... distances between the corresponding vertices of the object and its image relate. Even the learners who could not

Bernard				establish this relationship during the pen and paper activity could now see and correctly conjecture it with GeoGebra.
	Learning – Conception of learner thinking descriptor	5	4	Like what I saw during planning, when observing RL3, I noted the importance of adjusting the positions of square ABCD using the translation vector, and posing questions for learners to establish the geometric relationships that exist between the object and its image, particularly those that they struggled with during the pen and paper activity – such as the distances between vertices of the object and its image’s corresponding vertices.
	Teaching – Instructional and environment descriptor	4	4	Like what I learnt during planning, observing RL3 has taught me to put learners at the centre by changing the positions of the object using the translation vector, and asking them to comment on the areas and distances between the corresponding vertices of the object and its image.
	Access – Barrier and availability descriptor	4	4	Like what Tom indicated, during the pen and paper activity and when GeoGebra was used, some learners still conjectured that the y-coordinate sign changed from negative to positive. When observing RL3, I learnt the pertinence of changing the translation vector’s position to different quadrants, then asking learners to comment on the coordinates. This process helped learners to overcome their thinking errors.

Table 4.9 shows that with the curriculum and assessment descriptor of the ‘curriculum and assessment’ theme, Tom and Mike exemplified features of the advancing TPACK development level. Tom and Mike reported using GeoGebra to engage learners in exploring the transformation geometry concept of translation, as well as creating innovative assessments to gauge learners’ conceptual understanding of this concept’s traits embedded in the software. Bernard illustrated exploring-level TPACK capabilities. Bernard reported using GeoGebra to engage learners in exploring the transformation geometry concept of translation by dragging the object and asking them to establish conjectures about the ideas they were investigating.

Regarding the learning theme’s descriptor ‘conception of learner thinking’, Tom and Mike exemplified advancing TPACK level elements in using GeoGebra to design assessments that promoted advanced levels of translation understanding for learners. Bernard’s excerpt implied the exploring TPACK development level, as he leveraged GeoGebra to promote learner engagement and self-direction in learning the transformation geometry concept of translation.

In terms of the teaching theme – instructional and environment descriptor – Tom and Mike exemplified characteristics of the advancing TPACK development level. Tom and Mike reported maintaining learners’ engagement through managing GeoGebra-enhanced geometric translation activities, by engaging them in exploration and asking high-order questions to prompt learners to think about the geometric translation ideas that they were investigating. Bernard typified an exploring TPACK development level, by reporting using GeoGebra to engage learners in exploration of the transformation geometry concept of translation, while playing a guiding role.

Analysis of School A's teachers' excerpts, guided by the access theme descriptor 'barriers and availability' suggested that these teachers epitomised the exploring TPACK development level. The school A teachers pointed out that in addition to simultaneously illustrating multiple situations of a geometric translation task using symbolic, graphic and numeric representations, they could explore ideas to minimise the impact of challenges, which may arise when teaching this concept with GeoGebra.

The teachers at school A observed that when presenting RL3, Tom went an extra mile in fostering learners' understanding of the transformation geometry concept of translation by developing higher-order questions that involved hiding either the object or the translation vector, which they had not discussed during planning. This is what Bernard had to say: *"It was interesting to see Tom enriching learners' geometric translation conceptual understanding by developing innovative assessments; such as hiding the object and giving learners the image and translation vector, then asking them to draw the object. We did not talk about this when planning RL3."* Moving forward to the next RL, the teachers in school A recommended exhausting various options when planning to enrich learners' conceptual understanding. For example, Mike suggested: *"We need to focus more on designing innovative assessments that capture learners' understanding of geometric enlargements with GeoGebra in the next RL."*

Additionally, the teachers in school A observed that when Tom was presenting RL3, the whiteboard was crowded with a lot of information, which shifted the learners' attention from a particular geometric translation objective that the teacher wanted them to acquire. For example, Bernard indicated: *"When Tom was measuring the distances between the corresponding vertices of the object and its image, small letters that represented these distances showed up and the areas of the objects were also not removed when measuring distance, hence crowding learners with a lot of information on the whiteboard."* Going forward to the next RL, the teachers in school A indicated that they needed to do things differently by removing irrelevant information that could shift learners' attention from a particular geometric relationship that the teacher wanted them to learn. Mike had this to say: *"When planning and teaching the next RL, we need to focus more on removing information that has been used in examining a particular geometric relationship before moving on to the next one, and also hiding irrelevant information that shows up when measuring area and distance."*

It is quite clear from the reflections of school A's teachers that after RL3, individual teachers demonstrated higher TPACK levels of – exploring and advancing.

Reflections by the KO on RL3 with teachers from school A

In this section, the researcher reports on the KO's reflections on school A's teachers' competencies in using GeoGebra to teach the transformation geometry concept of translation in each of the four descriptors – 'curriculum and assessment', 'conception of learner thinking', 'instructional and environment', and 'barrier and availability'.

In the curriculum and assessment descriptor of the 'curriculum and assessment' theme, Lawrence's observation was that school A's teachers had varied competencies in teaching and assessing learners' translation conceptual understanding with GeoGebra. For example, Lawrence stated: *"Beyond changing the position of square ABCD (the object) and asking learners to establish the geometric relationships that existed between the object and its image, Tom and Mike also showed competence in hiding one of the three factors – the object, image and translation vector – and posing questions that required higher-order thinking. The questions included, inter alia, asking learners to draw the object given the image and the translation vector. Bernard reported constructing the object in GeoGebra, changing its position and asking learners for the geometric relationships that existed between this object and its image."*

With the learning theme descriptor 'conception of learner thinking', Lawrence noted that school A's teachers' competencies with the software in fostering the development of learner's conceptual knowledge of the transformation geometry concept of translation differed. Lawrence observed: *"While Bernard used GeoGebra to guide learners in understanding geometric translation traits by changing the positions of the object and posing questions to comprehend their understanding, Tom and Mike could use the software as a learning tool to engage learners in high-level thinking activities, such as asking them to draw the object given its image and the translation vector."*

Concerning the teaching theme's descriptors, particularly 'instructional and environment', while Lawrence noted the two teachers' effective management of the GeoGebra-enhanced activities in maintaining learner engagement and autonomy in learning translations, the other teacher engaged learners in the exploration of geometric translation traits, while serving as a guide. Lawrence observed: *"While Bernard used GeoGebra to change the position of the object to guide learners in establishing the geometric relationships that existed between the object and its image, Tom and Mike drew and translated the objects, then posed higher-order*

questions that required the learners to draw the object given the translation vector and the image.”

With the access theme descriptor, ‘barriers and availability’, Lawrence observed that the teachers in school A recognised the challenges for teaching the transformation geometry concept of translation with GeoGebra, and proactively sought solutions to overcome them. Lawrence reported *“The teachers engaged [the] learners in exploring the transformation geometry concept of translation, and also assessed this concept using multiple representations of the concept that included but [were] not limited to graphic and numeric representations.”*

When observing RL3, Lawrence noted that Tom did not recap on the previous RL when, introducing RL3. Lawrence observed: *“Tom did not reflect of the previous RL when introducing RL3. Learners confused translation for reflection, particularly when measuring the distances between the corresponding vertices of square ABCD and its image during the pen and paper activity. Most learners were measuring these distances from the x- axis, referring to this axis as a line of reflection.”* In the next RL, Lawrence indicated that when introducing the RL, the teacher should start by summarising the previous RL. For example, Lawrence said: *“When introducing the new RL, the teacher needs to reflect on the previous RL for learners to see how the two RLs relate.”*

It is evident from Lawrence’s observations that after RL3, school A’s individual teachers’ competencies in using GeoGebra to teach the transformation geometry concept of translation were at the higher TPACK development level– exploring and/or advancing.

4.4.3.3 TPACK development levels – findings from implementation/observation of RL3 in School B

The section discusses the results from the implementation/observation of RL3 by teachers from school B. The analysis of the implementation/observation of school B’s RL3 follows a similar analysis to that for school A’s RL3. Sarah is the teacher who taught RL3, while Bosco, Frank and Lawrence observed learner learning when the transformation geometry concept of translation was taught using a worksheet and with GeoGebra. Learners solved questions from a worksheet first, which they were working on manually; after which Frank used the software to consolidate the same activity. The findings from teaching RL3 using a worksheet first, then GeoGebra, are presented and analysed here.

Teachers' experiences on the use of the worksheet to teach the transformation geometry concept of translation

The section presents the experiences of teachers from school B when learners manually solved the pen and paper activity on the transformation geometry concept of translation.

School B teachers agreed that all learners could identify that square ABCD and its image were of the same size when using a worksheet. For example, Frank said: *“All learners were able to conclude that square ABCD and its image were equal in size. Some counted and compared the blocks in each square, while the others used the formula of the area of a square.”* School B teachers also noted that some learners could not identify that the distances between the vertices of square ABCD and the corresponding vertices of its image were all equal. Bosco pointed out: *“Some learners referred to the x-axis as the mirror line, and measured the distances between the corresponding vertices of square ABCD and its image from the x-axis and concluded that they were not equal.”*

When planning RL3, the prediction was that the low achievers would successfully carry out all the learning activities on the worksheet except for activity (d) (discussing their solutions showing the relationships that exist between square ABCD, and its image), and the middle and top achievers would correctly answer every question on the worksheet – see Appendix 10c. However, when observing RL3 the teachers in school A observed that while the top achievers performed as expected, the low and middle achievers performed less well than predicted. For example, Bosco noted: *“The lower achiever did not struggle with activity (d) as had been predicted, but struggled with activity (c) (measuring and comparing the distances between the corresponding vertices of square ABCD and its image). The middle achievers struggled with activity (c), yet the prediction was that ‘they will answer every question on the worksheet correctly’; and the top achiever performed as had been anticipated – they got every activity correct.”*

It can be concluded from school B's teachers' observations that all the success criteria were not achieved when learners manually solved questions from the worksheet. The learners struggled with establishing how the distances between the vertices of square ABCD and those between its image's corresponding vertices were related.

Observations on GeoGebra use to teach transformations using translation

The previous section discussed the experiences of teachers from school B when learners manually solved a pen and paper activity on the transformation geometry concept of translation. This section presents the experiences of school B's teachers in using GeoGebra to teach the transformation geometry concept of translation.

When teaching the same pen and paper activity using GeoGebra, school B's teachers concurred that GeoGebra assisted learners with the visualisation of the relationship between the size of square ABCD and its image. For example, Sarah said: *"I calculated the area of square ABCD and its image using GeoGebra, and learners concluded that their areas remained the same, even when the object was resized. This was not possible with the pen and paper activity, thus enhancing learners' understanding of this objective."* School B's teachers also acknowledged that GeoGebra assisted learners with visualising how the distances between the vertices of square ABCD and the corresponding vertices of its image were related. Lawrence noted: *"In mitigating learners' misconception when they measured the distances between the corresponding vertices of square ABCD and its image from the x-axis on the worksheet, Sarah measured these distances in GeoGebra and changed the positions of square ABCD. Even learners who struggled with this objective could see how these distances were measured, and correctly conjectured that they were equal."*

It is evident from the observations of the teachers in school B that GeoGebra assisted the learners with visualising the geometric translation relationships that existed between square ABCD and its images.

4.4.3.4 TPACK development levels – findings from the reflections on RL3 in school B

The preceding section discussed the findings from the implementation/observation of RL3 by teachers from school B when a worksheet and GeoGebra were used to teach the same geometric translation activity. This section tracks the TPACK development levels of individual teachers from school B, as shown by their self-reported scores and the researchers' interrater reliability scores, in each of the descriptors of the five TPACK levels. In a similar way as for RL1 and RL2, Table 4.10 summarises the findings of RL3.

Table 4.10 TPACK development levels of individual teachers in school B after RL3

LS group B's teachers	Themes and their descriptors	Teachers' self-reported scores	Researchers' interrater reliability scores	Teachers' reflections on their TPACK development level
Sarah	Curriculum and assessment – Curriculum and assessment descriptor	4	4	When teaching RL3, I noted that during the worksheet activity, some learners were measuring the distances between the corresponding vertices of square ABCD and its image from the x-axis. In mitigating this thinking error, I measured these distances, moved the object using the translation vector, and asked learners to comment on how they related. All learners could now see how these distances were measured, and conjectured that they were of length equal to the translation vector.
	Learning – Conception of learner thinking descriptor	4	4	When I was teaching RL3, I observed that some learners could not establish that the distances between the vertices of square ABCD and its image's corresponding vertices were equal. I measured these distances in GeoGebra, changed the translation rule (using the translation vector) and the sizes of the object. Learners could now see where their misconception in measuring these distances was on the worksheet.
	Teaching – Instructional and environment descriptor	5	4	When I was teaching RL3, I observed that some learners were conjecturing that the y-coordinates changed to negative after translating square ABCD (the object) on the worksheet. After drawing and translating the object, I changed the position of the object using the translation vector, while asking learners to comment on the coordinates. GeoGebra aided learners to overcome their mistakes from the worksheet.
	Access – Barrier and availability descriptor	3	3	When I was teaching RL3, during the pen and paper activity, I noted that some learners had difficulty in measuring the distances between the vertices of square ABCD and its image's corresponding vertices. When using GeoGebra, I measured these distances, changed the position and size of the object, and learners could now establish that these distances were equal.
Bosco	Curriculum and assessment descriptor	5	4	Like what I saw during planning, when observing RL3, I also noted that after constructing and translating square ABCD (object), it is crucial to calculate the areas and distances between the corresponding vertices of the object and its image, change the positions of the translation vector and the sizes of the object, then ask learners to comment on the geometric relationships that exist between the object and its image.
	Learning – Conception of learner thinking descriptor	4	4	Like what I saw during the planning of RL3, when observing this RL, I noted the importance of changing the position and size of square ABCD, the object. The learners correctly conjectured that the areas of square ABCD and its image were always the same, thus enriching their understanding. On the pen and paper activity, the object was static.
	Teaching – Instructional and environment descriptor	5	4	Like what I learnt during the planning of RL3, when observing this RL, I noted that it is important to change the positions and sizes of square ABCD, while posing questions that engage learners in thinking about the geometrical translation relationships that they are investigating.
	Access – Barrier and availability descriptor	4	3	When we were planning RL3, we did not anticipate that learners could measure the distances between the vertices of the object and its image's corresponding vertices from the x-axis. Observing this RL empowered me with how to mitigate this thinking error by measuring these distances, changing the positions of the object for learners to see how they are measured and also to establish a conjecture about them.
	Curriculum and assessment descriptor	4	4	Like what I saw during planning, when observing RL3, I learnt that it is vital to change the position of the translation vector. This enhanced learners' investigation of the geometric translation ideas such as the distances between the vertices of the object and its image's corresponding vertices, which most of

Frank				them struggled to measure on the worksheet – this became clear to all learners with GeoGebra, hence mitigating their thinking error.
	Learning – Conception of learner thinking descriptor	3	4	Like what I saw during the planning of RL3, when observing this RL, I have learnt that after constructing square ABCD [object] and the translation vector, I can use this vector to change the position of the object and pose questions that target the misconceptions that learners experienced on the worksheet – such as how to measure the distances between the vertices of the object and its image’s corresponding vertices.
	Teaching – Instructional and environment descriptor	4	4	During the planning of RL3, the teaching approaches for using GeoGebra to foster the learning of transformation geometry translations were primarily theoretical. Observing this RL has enabled me to experience these approaches practically; I can now change the position and size of square ABCD, then probe learners for the geometric relationships that exist between square ABCD and its image.
	Access – Barrier and availability descriptor	4	3	When presenting their worksheet solutions, some learners conjectured that after translating square ABCD, the y-coordinate always changes to positive. We did not foresee this when planning this RL. When observing RL3, I noted the importance of moving the translation vector to different quadrants, as this helped the learners in overcoming this misconception by examining the coordinates of the images.

It is evident from Table 4.10 that in terms of the curriculum and assessment descriptor of the ‘curriculum and assessment’ theme, all school B teachers epitomised exploring TPACK level traits. The teachers reported constructing and dragging the object in GeoGebra, and asking learners questions that required them to show their understanding of the geometrical translation ideas with which they were dealing.

In the learning theme descriptors, with a focus on ‘conception of learner thinking’, school B’s teachers illustrated features of the exploring TPACK level. The school B teachers reported facilitating the instruction of the concept of translation with the goal of fostering learners’ understanding.

In the teaching theme – instructional and environment descriptor – school B’s teachers’ excerpts showcased the exploring TPACK level. The teachers in this school reported using the software to engage the learners in exploring geometric translation concepts, accompanied by asking the learners questions that engaged them in thinking about these concepts.

The teachers in school B portrayed an adapting TPACK development level, as evidenced by their excerpts in the access theme descriptor – ‘barriers and availability’. These teachers reported using GeoGebra to provide learners with access to geometric translation concepts that had been out of reach during the pen and paper activity.

When observing RL3, the teachers in school B noted that Sarah was not confident enough in using GeoGebra to teach the transformation geometry concept of translation. For example, Bosco said: *“In some instances, Sarah knew what she wanted to do, but was not confident.*

Sarah took some time trying to figure out what was supposed to be done, though she eventually figured it out.” The teachers in school B recommended the need for more practice with the software, particularly for the teacher who was going to teach the next RL. This is what Frank had to say: *“In the next RL, after planning this RL, the teacher who is going to teach needs to rehearse to acquaint themselves more with how to use the tools in GeoGebra, such as those that require measuring the area, resizing, and zooming the object ... so that they do not practise ‘trial and error’ in front of the learners.”*

Furthermore, school B’s teachers observed that during the pen and paper activity, some learners conjectured that the sign of the y-coordinate always changed to positive. Sarah pointed out: *“During planning, we did not anticipate that learners would think like this. Although I harnessed the dynamism capability of GeoGebra to mitigate this misconception – by changing the position of the object, with the aid of the translation vector – in the next RL, we must exhaust various ways of learner thinking during planning.”*

The teachers in school B also indicated that they were looking forward to higher-order questions in the next RL, to see how learners would respond to these questions. Bosco observed: *“Sarah tried to create a higher-order question; she provided learners with the object and its image, and asked them to establish the translation rule – but forgot to hide the translation vector. When planning the next RL, we need to incorporate higher-order questions to enrich [the] learners’ conceptual understanding during the RL.”*

It can be concluded from the reflections of the teachers in school B that after RL3, the teachers practised actions predominantly at the exploring stage – a higher TPACK development level.

Reflections by the KO on RL3 with teachers from school B

The section reports on Lawrence the KO’s comments on school B’s teachers’ competencies in teaching the transformation geometry concept of translation with GeoGebra for each of the four descriptors – ‘curriculum and assessment’, ‘conception of learner thinking’, ‘instructional and environment’, and ‘barriers and availability’.

Regarding the curriculum and assessment descriptor of the ‘curriculum and assessment’ theme, Lawrence noted that teachers from school B had the same competencies in teaching and assessing learners’ conceptual understanding of translation when the software is used. For example, Lawrence observed: *“All teachers in school B could use GeoGebra to draw and drag the object using the translation vector, then probe learners with questions that were aligned to*

the objectives, particularly those that they struggled with on the worksheet – measuring the distances between the corresponding vertices of the object and its image.”

With the learning theme descriptor ‘conception of learner thinking’, Lawrence observed that the competencies of school B’s teachers in facilitating the development of learners’ conceptual knowledge of the transformation geometry concept of translation with GeoGebra were the same. Lawrence noted: *“School B’s teachers engaged learners in the exploration of the geometric translation relationships that existed between the object and its image by changing the positions of the object using the translation vector and posing questions to guide learners in understanding.”*

Within the teaching theme, focusing on the ‘instructional and environmental’ descriptor, Lawrence noted that school B’s teachers used different instruction strategies – exploring and question/answer – to engage learners in investigating the geometric translation relationships that existed between the object and its image, while serving as guides. This is what Lawrence had to say: *“The teachers drew and translated the object, changed its position and size, then posed questions that required learners to establish the geometric relationships that existed between the object and its image.”*

In relation to the access theme, focusing on ‘barriers and availability’ descriptor, Lawrence observed that school B’s teachers used GeoGebra for learners to examine the geometric translation relationships that existed between the object and its image, particularly those that they struggled with on pen and paper. For example, Lawrence said: *“The teachers in school B used the software (GeoGebra) to provide learners with access to geometric translation relationships that existed between the object and its image, which were formerly out of reach when solving the questions on the worksheet, such as the distance between the corresponding vertices of the object and its image.”*

Lawrence observed that teachers in school B needed to create more time to practise with GeoGebra so that they could get more accustomed to the tools offered by this software. For example, Lawrence reported *“... Sarah took more time in thinking about her actions with GeoGebra. In the next RL, the teacher [who is] going to teach this RL needs to rehearse prior to the RL, so that their actions with the software are automatic.”*

In conclusion, Lawrence's observations revealed that after RL3, school B's individual teachers' competencies in teaching the transformation geometry concept of translation were mainly at the exploring level – a higher TPACK development level.

4.5 TPACK development levels – findings from researchers' reflections on the training session and the three RLs in two schools

This section presents the quantitative findings of researchers' ratings of teachers' TPACK knowledge in the two schools (A and B), covering training to RL3 and looking at all four TPACK dimensions: curriculum and assessment, learning, teaching, and access. A repeated-measures ANOVA test was conducted to examine whether researchers' ratings on teachers' TPACK knowledge in schools A and B differed significantly across four stages of professional development (Training, RL1, RL2, RL3) for each of four TPACK dimensions. The analysis combined schools A and B. Researchers' ratings of teachers' TPACK knowledge from training up to RL3 in both schools are summarised in Table 4.11. The first column lists the TPACK dimensions, and the second column shows the mean scores for each of the four TPACK dimensions, comparing training to RL3. Columns three and four display the ANOVA test results and the trend in the mean scores of the four TPACK dimensions as the training progresses from RL1 to RL2 and then to RL3. The rightmost column provides a summary of what the ANOVA test results and trend analyses actually indicate for each of the four TPACK dimensions after training to RL3.

Table 4.11 Summary of the ANOVA test results and trends across four TPACK Dimensions

TPACK Dimension	Means (Training → RL1 → RL2 → RL3)	ANOVA Test Results	Trend	Interpretation
Curriculum and assessment	2.67, 2.67, 3.67, 4.33	$F(3,15) = 30.00, p < .001$, Partial $\eta^2 = .86$	Strong linear increase ($F \approx 90, p < .001$); slight cubic	Significant improvement, especially after RL1.
Learning – Conception of learner thinking	3.00, 3.00, 3.67, 4.33	$F(3,15) = 22.00, p < .001$, Partial $\eta^2 = .81$	Linear increase; quadratic significant ($p \approx .025$)	Significant improvement; mostly linear with curvature.
Teaching – Instructional and environment	2.50, 2.50, 3.67, 4.33	$F(3,15) = 63.57, p < .001$, Partial $\eta^2 = .93$	Very strong linear effect; quadratic & cubic significant	Largest effect size; major improvement after RL1.
Access – Barriers and availability	2.00, 2.00, 2.83, 3.50	$F(3,15) = 31.67, p < .001$, Partial $\eta^2 = .86$	Strong linear effect; quadratic ($p \approx .025$)	Improvement but from lowest baseline.

Note. RL = Research Lesson; η^2 = Eta squared.

Looking at the descriptive statistics from the researchers' ratings of the six teachers TPACK knowledge in schools A and B showed that ratings increased from the training stage to RL3 across all the four TPACK dimensions, with no missing data or outliers. Mauchly's test of sphericity could not be computed due to singular matrices, but Greenhouse–Geisser corrections were applied where necessary.

For the curriculum and assessment descriptor, the ANOVA test was statistically significant, $F(3,15) = 30.00, p < .001$, partial $\eta^2 = .86$, indicating a large effect of stage on researchers' ratings on teachers' TPACK knowledge. Post-hoc contrasts revealed a strong linear trend, $F(1,5) = 90.00, p < .001$, suggesting researchers' ratings on teachers' TPACK knowledge improved steadily from Training ($M = 2.67$) to RL3 ($M = 4.33$). However, there was no difference in means within training and RL1.

Regarding the learning theme – conception of learner thinking descriptor, the effect of stage was also significant, $F(3,15) = 22.00, p < .001$, partial $\eta^2 = .81$. Researchers' ratings on teachers' TPACK knowledge increased from training ($M = 3.00$) to RL3 ($M = 4.33$), with a significant linear trend, $F(1,5) = 37.69, p = .002$, and a smaller quadratic component, $p = .025$. However, no difference was observed between training and RL1 means.

With the teaching theme – instructional and environment descriptor, the ANOVA test yielded $F(3,15) = 63.57, p < .001$, partial $\eta^2 = .93$, the largest effect size among all the four TPACK themes. Researchers' ratings on teachers' TPACK knowledge rose from training ($M = 2.50$) to RL3 ($M = 4.33$), with significant linear ($p < .001$) and quadratic trends. However, there was no difference in means within training and RL1.

Finally, for the access theme – barriers and availability descriptor, the effect of stage was significant, $F(3,15) = 31.67, p < .001$, partial $\eta^2 = .86$. Researchers' ratings on teachers' TPACK knowledge improved from training ($M = 2.00$) to RL3 ($M = 3.50$), with a strong linear trend ($p < .001$) and a smaller quadratic effect ($p = .025$). However, no difference was observed between training and RL1 means.

4.6 Summary

This section ends with a summary of the findings from the research instruments – a baseline survey, FGIs conducted after the teacher training session on GeoGebra and LS, the three transformation geometry RLs, observation guides, and FGIs after each RL – in relation to the study's research questions.

Summary of findings from the baseline survey

The section presents a summary of the key findings from the baseline survey in relation to addressing the first research question:

What was the extent of the use of GeoGebra and LS as a school-based CPD approach in mathematics teaching in high schools in the JTG district prior to the main study?

The baseline survey findings showed that GeoGebra and LS use in the JTG district was scant. Factors such as limited mathematical content knowledge and skills, a lack of adequate technological resources and lack of training, among other factors, were noted as preventing the teachers from integrating the software into the classroom. Additionally, other factors that prevented the teachers from implementing the LS in their schools included, *inter alia*, the absence of support from beyond the LS teachers and unfamiliarity with how LS can aid teachers to deepen understanding of mathematical (transformation geometry) content and new pedagogical ideas.

In addition to the baseline survey findings providing data to address the first research question, these findings strengthened the study's position on the problem statement – no research has ever been done in South Africa using GeoGebra and the LS as a school-based CPD to develop rural high school mathematics teachers' TPACK knowledge.

Summary of findings from the FGIs on teacher training on GeoGebra and the LS and the three transformation geometry RLs

This section presents a summary of the findings from the FGIs on teacher training on GeoGebra and the LS, as well as the three transformation geometry RLs, on the concepts of reflection about the x- and y-axis, reflection about the $y = x$ line and translation – in relation to the study's second research question:

What is the level of TPACK developed when rural high school mathematics teachers use GeoGebra to teach transformation geometry during a school-based LS CPD approach?

No research has been carried out previously in South Africa using GeoGebra and LS as a school-based CPD to develop rural high school mathematics teachers' TPACK knowledge. This study aimed to fill the void in the identified literature by training rural high school mathematics teachers at two under-resourced schools in South Africa's Northern Cape Province, in the JTG district, on how to utilise GeoGebra to teach transformation geometry using a school-based LS CPD approach. Hence, the study investigated the TPACK development of rural high school mathematics teachers using GeoGebra through a LS approach.

The two schools' qualitative findings revealed that from training through to RL3, individual teachers transitioned from pre-recognising to the higher TPACK development levels (exploring and advancing). At the start of the training, the teachers' TPACK development levels in both schools were at the pre-recognising stage. After training and RL1, school A and school B teachers primarily demonstrated lower TPACK development levels (adapting and accepting respectively). After RL2, the teachers showcased mainly the exploring and adapting levels, in school A and school B respectively. After RL3, school A teachers exemplified the higher TPACK development levels (exploring and advancing levels), and school B teachers primarily epitomised the exploring level.

The findings from the quantitative analysis – merging data from schools A and B – revealed that the researchers' ratings of teachers' TPACK knowledge increased from the training stage to RL3 across the four TPACK dimensions: curriculum and assessment, learning, teaching, and access. However, there was no difference in means within training and RL1 across the four TPACK dimensions.

Summary of findings from mathematics ISTs' experiences in teaching transformation geometry using GeoGebra

The section summarises the findings from the observation guides as they relate to addressing the study's third question:

What are the experiences of rural high school mathematics teachers using GeoGebra to teach transformation geometry?

Observation guides collected data to address this question. The observation guides collected data on learner learning, first when using a worksheet, then when using GeoGebra to teach the same pen and paper transformation geometry activity. During the implementation and observation of the three transformation geometry RLs, the teachers in schools A and B showed appreciation for what the software could achieve that the worksheet could not. The teachers noted that GeoGebra helped the 'teaching' teacher to resize and change the position of the object to various quadrants, and so assisting learners to overcome their pen and paper misconceptions and/or thinking errors regarding the geometric relationships that existed between the object and its image. The software was also used as a learning tool, to facilitate high-order thinking in learners – for example, by hiding the object and asking them to draw it when given its image and the translation vector.

CHAPTER FIVE: DISCUSSION AND CONCLUSIONS

5.1 Introduction

The study aimed to investigate the TPACK development of rural high school mathematics teachers when using GeoGebra and the LS approach as a form of school-based CPD to teach transformation geometry. In this chapter, which concludes the study, the researcher discusses the findings and outlines how the study contributes to the theory (in terms of TPACK development levels) and practice of GeoGebra and LS use in teaching transformation geometry. Following this, the limitations of the study are presented, as well as its implications and some recommendations for further research. Finally, there is a summary of the chapter.

5.2 Discussion of the findings

This section discusses the findings from the data collection instruments – a baseline survey, FGIs conducted after the teacher training session on GeoGebra and LS and the three RLs and observation guides – as they related to answering the three research questions.

Research question one

What was the extent of the use of GeoGebra and the LS as a school-based CPD approach in mathematics teaching in high schools in the JTG district prior to the main study?

To answer this first research question, a baseline survey was conducted. The data from the baseline survey was used to establish the extent of the use of technology (in particular GeoGebra) in mathematics instruction, and the CPD types – LS as a school-based CPD approach, in this case – to which JTG district's high school mathematics teachers were exposed. The results showed limited usage of GeoGebra and the LS in mathematics teaching in the JTG district's high schools. The teachers reported factors that prohibited them from integrating the software into their classrooms, including limited mathematical content knowledge and skills, loadshedding (power cuts), inadequate devices, and lack of training. Jezdimirović (2014) and Wassie and Zergaw (2019) reported challenges in incorporating GeoGebra into Serbian and Ethiopian mathematics classrooms respectively, including teachers with limited content knowledge, inadequate technological resources, limited pedagogical knowledge, and inadequate CPD and skills; in a South African study, Mokotjo and Mokhele (2021) concurred with their assessments.

Additional, factors that prevented teachers from implementing the LS in their schools included (but were not limited to) absence of support from beyond the LS participants. This result confirms that of Darling-Hammond (2013), who asserted that school-based CPD activities are not provided with sufficiently adequate resources for them to function optimally. While the findings of the baseline survey provided the data to answer the study's first question, they also reinforced the position of the study on the problem statement – no research has ever been done in South Africa on the use of GeoGebra and the LS as a school-based CPD to develop rural high school mathematics teachers' TPACK knowledge. This necessitated training the teachers from the two schools (A and B) that took part in the study on how to utilise GeoGebra to teach the transformation geometry concepts of reflection, translation and enlargement using a school-based LS CPD approach to develop their TPACK. Enlargements were only part of the training, not the main study topic.

Research question two

What is the level of TPACK developed when rural high school mathematics teachers use GeoGebra to teach transformation geometry during a school-based LS CPD approach?

To address this research question, FGIs that included reflection guides were used. The FGIs required teachers in the two schools (A and B) to reflect on the school-based teacher training sessions on GeoGebra and LS, as well as the four stages of each of the three transformation geometry RLs – identification of objectives, planning, implementing/observing and reflecting. The data gathered tracked individual teachers' TPACK development levels in the five TPACK level descriptors, as shown by their self-reported scores and the researchers' interrater reliability scores.

The qualitative findings from the two schools revealed that individual teachers transitioned from pre-recognising to the higher TPACK development levels (exploring and advancing) from training to RL 3. At the beginning of the training, in both schools, the teachers' levels of TPACK development were at the pre-recognising stage. After training and RL1, the teachers demonstrated mainly the adapting (school A) and accepting (school B) levels – the lower TPACK levels. After RL2, the teachers showed primarily the exploring (school A) and adapting (school B) levels. After RL3, school A teachers exemplified higher TPACK levels (exploring and advancing), and school B teachers demonstrated mostly the exploring stage.

For the quantitative analysis – merging data from schools A and B – a repeated-measures ANOVA test was conducted to examine whether researchers’ ratings on teachers’ TPACK knowledge in schools A and B differed significantly across four stages of professional development (Training, RL1, RL2, RL3) for each of four TPACK dimensions: curriculum and assessment, learning, teaching, and access. The analysis combined schools A and B.

An ANOVA test revealed a statistically significant effect of stage on researchers’ ratings of teachers’ TPACK knowledge for the curriculum and assessment descriptor, $F(3,15) = 30.00$, $p < .001$, partial $\eta^2 = .86$, indicating a large effect. Post-hoc contrasts demonstrated a strong linear trend, $F(1,5) = 90.00$, $p < .001$, with mean ratings increasing from the training stage ($M = 2.67$) to RL3 ($M = 4.33$). No significant difference was found between the training and RL1 means.

The effect of stage on teachers’ TPACK knowledge for the learning theme descriptors, with a focus on “conception of learner thinking” was also significant, $F(3,15) = 22.00$, $p < .001$, partial $\eta^2 = .81$. Researchers’ ratings rose from training ($M = 3.00$) to RL3 ($M = 4.33$), displaying a significant linear trend, $F(1,5) = 37.69$, $p = .002$, and a smaller quadratic component ($p = .025$). Means with training and RL1 did not differ significantly.

ANOVA test results for the teaching theme – instructional and environment descriptor showed the largest effect size among the four TPACK themes ($F(3,15) = 63.57$, $p < .001$, partial $\eta^2 = .93$). Researchers’ ratings on teachers’ TPACK knowledge increased from training ($M = 2.50$) to RL3 ($M = 4.33$) with significant linear ($p < .001$) and quadratic trends. No difference was observed between training and RL1 means.

For the access theme – barriers and availability descriptor, the effect of stage was significant, $F(3,15) = 31.67$, $p < .001$, partial $\eta^2 = .86$. Researchers’ ratings on teachers’ TPACK knowledge improved from training ($M = 2.00$) to RL3 ($M = 3.50$), exhibiting a strong linear trend ($p < .001$) and a smaller quadratic effect ($p = .025$). Means for training and RL1 were statistically equivalent.

The quantitative findings indicate a consistent pattern across all four TPACK dimensions: researchers’ ratings of teachers’ TPACK knowledge increased significantly from the training stage to RL3, predominantly following a linear trajectory with some quadratic influences. Notably, no discernible changes were detected between the training and RL1 phases, suggesting that meaningful improvements manifest later in the intervention.

It can therefore be concluded from the above qualitative and quantitative findings from the two schools (A and B) that teachers' participation in a school-based LS CPD approach using GeoGebra to teach transformation geometry aided them in developing their TPACK. These findings support earlier research results that implementing the LS significantly enhanced teachers' TPACK in mathematics (Huang et al., 2021; Joubert et al., 2020; Kurt & Çakıroğlu, 2018; Riales, 2011).

Additionally, it is worth highlighting that in Riales (2011) – the scope of which focused on a single LS cycle, with teachers from one school – individual TPACK development patterns differed, with some participants (50% of the teachers) performing actions at higher TPACK levels according to post-lesson observation feedback. The TI-84 graphing calculator was the technology used to teach algebra, in this case. Most participants in Riales's (2011) study had previously used the TI-Navigator system. These participants had classrooms equipped with a “set of TI-84 plus graphing calculators, a TI-Navigator system, and an interactive whiteboard” (Riales, 2011, p. 133).

However, the current study investigated the effect of participation in three GeoGebra-based LS cycles, by teachers from two under-resourced rural high schools, with regard to their teachers' TPACK development. In contrast to Riales's (2011) findings, after training and RL1, schools A and B's teachers illustrated mainly lower TPACK development levels (adapting and accepting, respectively). The current study's participants had no prior experience with GeoGebra. Schools A and B both lacked adequate technological equipment: neither school had a computer laboratory, but their teachers had laptops and mobile projectors that at the time were not used for mathematics teaching. While most of the teachers in Riales's (2011) study demonstrated higher TPACK development levels (exploring and advancing) in one LS cycle, the current study's teachers reached the higher TPACK development levels only after RL2 and/or RL3.

As alluded to earlier, teachers' participation in this study's LS as a school-based CPD approach, while using GeoGebra to teach transformation geometry assisted them in developing their TPACK. Mhakure (2019) also acknowledged that effective teacher learning occurs when CPD is contextualised within the teacher's own school and classroom settings. However, although the teachers acknowledged that participation in a school-based LS CPD approach led to an improvement in their individual TPACK development levels, they also noted that practising the LS approach required a lot of resources, including but not limited to time and technological

gadgets (extension cables, laptops and projectors). Hence, the teachers called upon the SMTs in their respective schools to intervene by providing the necessary support. This call is similar to those of Chong and Kong (2012), and Erickson et al. (2005), who stated that school administrators must make available the resources necessary to support teachers' school-based CPD activities, and do away with structural obstacles that hinder the processes of change.

Research question three

What are the experiences of rural high school mathematics teachers using GeoGebra to teach transformation geometry?

The researcher used data collected from the implementation/observation of the three transformation geometry RLs – on the concepts of reflection about the x- and y-axis, reflection about the $y = x$ line and translation – to address the study's third research question. One teacher in each of the two schools (A and B) taught one RL, while the other two teachers and the KO observed learners' transformation geometry learning using an observation guide when a worksheet and GeoGebra were used. Learners solved questions from a worksheet first, which they worked on manually, after which the teacher used GeoGebra to consolidate the same pen and paper activity. During the implementation and observation of the three transformation geometry RLs, the teachers at schools A and B showed knowledge of what the software could achieve that the worksheet could not.

In teaching RL1, teachers at the two schools (A and B) agreed that GeoGebra assisted learners with visualising how the areas and the distances from the x-or the y-axis between the corresponding vertices of $\triangle EFG$ and its images were related. The teachers changed the position of $\triangle EFG$ (but within quadrant 1), thus helping learners to overcome their misconceptions and/or thinking errors from the worksheet activity. However, when teaching the objective of coordinates, the teachers in both schools agreed that GeoGebra did not assist the learners with visualising how the coordinates of the vertices of $\triangle EFG$ changed in sign when reflection was done about the x- and y-axis. The 'teaching' teachers failed to change the position of $\triangle EFG$ (which was drawn in quadrant 1) to different quadrants, leaving the learners with a misconception from the worksheet – that the x- and y-coordinate signs always change to negative when reflection is done about the y- and x-axis respectively.

In RL2 and RL3, the teachers in schools A and B acknowledged that GeoGebra assisted learners with visualising the geometric relationships that existed between the objects and their images, when compared to the pen and paper activity. When using the software, after drawing and reflecting or translating the object, the teachers resized the object or changed its position to different quadrants, and this aided the learners in overcoming their misconceptions and/or thinking errors about the geometric relationships that existed between the object and its image from the pen and paper activity. Additionally, during RL3 the teachers in school A used the software as a learning tool to facilitate advanced thinking skills in learners; for example, by hiding the object and asking them to draw it when given its image and the translation vector. These findings are in line with those of Guven (2012), Küçük and Gün (2023) and Mukamba and Makamure (2020), who determined that GeoGebra improves learner transformation geometry performance and learning compared to traditional pen and paper methods.

5.3 Contributions of the study

This section presents the main contributions of this research to the theory (TPACK development levels) and practise of GeoGebra and LS use in teaching transformation geometry.

Firstly, the study makes a contribution to mathematics teachers' TPACK development in the South African context, by testing the significance of the difference between the researchers' interrater reliability scores, in terms of TPACK development levels. Other studies on TPACK and the LS (Huang et al., 2021; Joubert et al., 2020; Kurt & Çakıroğlu, 2018; Riales, 2011) did not use the ANOVA test.

Secondly, in addition to what other studies on TPACK and the LS in mathematics teaching have contributed (Huang et al., 2021; Joubert et al., 2020; Kurt & Çakıroğlu, 2018; Riales, 2011), this study makes a contribution to mathematics teachers' TPACK development in the context of South Africa, by using TDS as a theory to underpin LS in school-based CPDs.

Thirdly, the study looks at a novel way of achieving CPD in South Africa, informed using a GeoGebra-based LS as a form of school-based CPD to teach transformation geometry. In the South African Department of Education, teachers mainly attend generic CPD workshops outside of their schools, facilitated by experts (e.g. subject advisors, provincial coordinators and teacher educators) who are not familiar with their schools' contexts and teaching cultures (Mhakure, 2019). This renders their CPD activities inadequate, because they barely address the challenges faced by individual teachers within their schools.

Lastly, from both a theoretical and a practical point of view, this study could be an exemplar for how the LS and GeoGebra could be used in teacher education.

5.4 Limitations of the study

The limitations of this study are discussed in this section.

The study worked with a small sample – only two rural under-resourced high schools. The limited number of research sites does not allow the study's findings to be generalised. However, while the study's findings are context-specific, they provide valuable insights into teachers' experiences, informing future TPACK research and addressing a gap in the literature.

The teachers in the two schools lacked knowledge about GeoGebra. These were teachers who had either never heard about GeoGebra or who had heard about it prior to this study but had never used it before to teach a mathematics topic. This limitation of teachers' lack of GeoGebra knowledge was minimised by training the teachers.

The teachers in the two schools also lacked knowledge about the lesson study (LS). They had either never heard about the LS, or had heard about it but had never participated in one prior to this study. For the most part the teachers had been exposed to workshops organised outside of their schools and conducted by external experts, including but not limited to mathematics subject advisors and provincial coordinators. The limitation of teachers' lack of LS knowledge was also minimised by training the teachers.

The inability to compute Mauchly's test of sphericity due to singular matrices and reliance on Greenhouse-Geisser corrections constrained the quantitative analysis. Future research should consider larger samples or alternative analytical approaches.

5.5 Implications and recommendations for future research

This section discusses the current study's implications for teacher educators, policy-makers and scholars of mathematics education, as well as suggesting recommendations for further research.

Firstly, the study's findings should be disseminated into the broader South African mathematics education context – for example, through district and provincial workshops, and the national Association for Mathematics Education of South Africa (AMESA) platforms. Among other things, this could help to inform teacher CPD practices, influence policy or programme development, and support improved mathematics education outcomes in South Africa.

Secondly, teacher training programmes should equip trainees with the skills to integrate technology such as GeoGebra, and the LS approach, into mathematics instruction.

Thirdly, the findings from the GeoGebra school-based LS CPD approach underscore the importance of sustained professional development opportunities that extend beyond initial training to foster continued growth in teachers' TPACK competencies.

Fourthly, with the sample used in this study being small (two rural under-resourced high schools), a future study could use the same study approach but focus on a bigger and more diverse sample for the results to be generalisable to a wider context.

Fifthly, the study investigated the TPACK levels developed when rural high school mathematics teachers use GeoGebra and the LS as a school-based CPD approach to teach transformation geometry concepts. Additionally, the study investigated rural high school mathematics teachers' experiences when using GeoGebra to teach transformation geometry. With the focus of this study being only on teachers, future research could be conducted that focuses on learners' knowledge and experiences with the use of a GeoGebra-based LS to learn the same concepts.

Lastly, the current study investigated the TPACK development of rural high school mathematics teachers when using GeoGebra and the LS as a school-based CPD approach to teach transformation geometry. Future research could investigate the effect of using a GeoGebra-based LS on other areas of mathematics.

5.6 Summary

The aim of the study was to investigate the TPACK development of rural high school mathematics teachers when using GeoGebra and the LS approach as a form of school-based CPD to teach transformation geometry. This research was conducted in the JTG district of the Northern Cape Province, a remote region in South Africa. As the area is remote, most of the high schools located in the JTG district are under-resourced. The study therefore worked with two under-resourced rural high schools from this district. While the teachers were from under-resourced schools and were also encountering GeoGebra and the LS for the first time, they managed to appreciate that participating in a GeoGebra-based LS to teach transformation geometry empowered them to use the limited technology resources at their schools to improve the teaching of this topic, in line with the findings of Posthuma (2012). Additionally, the teachers intimated at the beginning of the study (during training and RL1) that they were not comfortable with using GeoGebra to teach transformation geometry. However, as the study progressed (during RL2 and RL3), the teachers' beliefs regarding GeoGebra integration in the mathematics classroom became more positive. Other studies have shown similar results (Huang et al., 2021; Joubert et al., 2020; Riales, 2011). The study makes a contribution towards the teaching of mathematics using GeoGebra and the LS in teacher education. Future research should focus on using a bigger and more diverse sample of the same study to generalise the findings to a larger population.

References

- Abbitt, J. T. (2011). Measuring technological pedagogical content knowledge in preservice teacher education: A review of current methods and instruments. *Journal of Research on Technology in Education*, 43(4), 281–300.
<https://doi.org/10.1080/15391523.2011.10782573>
- Ada, T., & Kurtulus, A. (2010). Students' misconceptions and errors in transformation geometry. *International Journal of Science and Technology*, 41(7), 901–909.
<https://doi.org/10.1080/0020739x.2010.486451>
- Aldemir Engin, R., Karakuş, D., & Niess, M. L. (2023). TPACK development model for pre-service mathematics teachers. *Education and Information Technologies*, 28(4), 4769–4794.
- Alemdag, E., Cevikbas, S. G., & Baran, E. (2020). The design, implementation and evaluation of a professional development programme to support teachers' technology integration in a public education centre. *Studies in Continuing Education*, 42(2), 213–239. <https://doi.org/10.1080/0158037x.2019.1566119>
- Almalki, S. (2016). Integrating quantitative and qualitative data in mixed methods research challenges and benefits. *Journal of Education and Learning*, 5(3), 288–296.
- Angeli, C., & Valanides, N. (2009). Epistemological and methodological issues for the conceptualization, development, and assessment of ICT–TPCK: Advances in technological pedagogical content knowledge (TPCK). *Computers & Education*, 52(1), 154–168. <https://doi.org/10.1016/j.compedu.2008.07.006>
- Anthanasou, J. A., Di Fabio, A., Elias, M. J., Ferreira, R., Gitchel, W. D., Jansen, J. D., Malindi, M. J., McMahon, M., Morgan, B., Mpofu, E., Nieuwenhuis, J., Perry, J. C., Panulla, S., Pretorius, G., Seabi, J., Sklar, R. H., Theron, L. C., & Watson, M. (2012). *Complete your thesis or dissertation successfully: Practical guidelines*. Edited by JG (Kobus) Maree. Juta.
- Bahn, J. (2018). *Inquiry based mathematics education and lesson study* [Unpublished doctoral dissertation]. University of Copenhagen, Denmark.
- Bahn, J., & Winsløw, C. (2019). Doing and investigating lesson study with the theory of didactic situations. In R. Huang, A. Takahashi, & J.-P. Da Ponte (Eds.), *Theory and Practice of Lesson Study in Mathematics Education*, (pp. 83–102), Springer Nature, New York.

- Baxter, P., & Jack, S. (2008). Qualitative case study methodology: Study design and implementation for novice researchers. *The Qualitative Report*, 13(4), 544–559.
- Bertram, C., & Christiansen, I. (2014). *Understanding Research*. Hatfield, Pretoria: Van Schaik Publishers.
- Bitsch, V. (2005). Qualitative research: A grounded theory example and evaluation criteria. *Journal of Agribusiness*, 23(1), 75-91.
- Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology*, 3(2), 77–110. <https://doi.org/10.1191/1478088706qp063oa>
- Brousseau, G. (1997). *Theory of Didactical Situations in Mathematics* (Trans. by Balacheff, N., Cooper, M., Sutherland, R. and Warfield, V.). Dordrecht: Kluwer.
- Brousseau, G. (2002). *Theory of Didactical Situations in Mathematics*. Dordrecht, The Netherlands: Springer.
- Brousseau, G. (2004). *Théorie des Situations Didactiques* (2nd ed.). La Pensée sauvage.
- Brousseau, G., & Warfield, V. (2014). Didactic situations in mathematics education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education*. Springer.
https://doi.org/10.1007/978-94-007-4978-8_47
- Cajkler, W., Wood, P., Norton, J., & Pedder, D. (2014). Lesson study as a vehicle for collaborative teacher learning in a secondary school. *Professional Development in Education*, 40(4), 511–529. <https://doi.org/10.1080/19415257.2013.866975>
- Campbell, R., & Ahrens, C. E. (1998). Innovative community services for rape victims: An application of multiple case study methodology. *American Journal of Community Psychology*, 26, 537–571.
- Chizwina, S., & Mhakure, D. (2018). Exploring how the attributes of technology affect adoption in teaching mathematics in a higher education institution in South Africa. *African Journal of Research in Mathematics, Science, and Technology Education*, 22(3), 276–286. doi: 10.1080/18117295.2018.1499459
- Chong, W. H., & Kong, C. A. (2012). Teacher collaborative learning and teacher self-efficacy: The case of lesson study. *The Journal of Experimental Education*, 80(3), 263–283.
- Clivaz, S. (2015). French didactique des mathématiques and lesson study: A profitable dialogue? *International Journal for Lesson and Learning Studies*, 4(3), 245–260. <https://doi.org/10.1108/IJLLS-12-2014-0046>.

- Clivaz, S. (2018). Lesson study as a fundamental situation for the knowledge of teaching. *International Journal for Lesson and Learning Studies* 7(3), 172–183.
<https://doi.org/10.1108/IJLLS-03-2018-0015>.
- Cohen, L., Manion, L., & Morrison, K. (2011). *Research Methods in Education* (7th ed.) London: Routledge.
- Cohen, L., Manion, L., & Morrison, K. (2018). *Research Methods in Education* (8th ed.) London: Routledge.
- Cox, S. (2008). *A conceptual analysis of technological pedagogical content knowledge* [Doctoral dissertation]. Brigham Young University. 1482.
<https://scholarsarchive.byu.edu/etd/1482>
- Creswell, J. (2012). *Research Design: Qualitative and Quantitative Approaches*. Thousand Oaks.
- Creswell, J. W., & Miller, D. L. (2000). Determining validity in qualitative inquiry. *Theory Into Practice*, 39(3), 124–130. https://doi.org/10.1207/s15430421_tip3903_2
- Creswell, J. W., & Plano Clark, V. L. (2011). *Designing and Conducting Mixed Methods Research* (2nd ed.) Thousand Oaks, CA: Sage.
- Darling-Hammond, L. (2013). *Getting Teacher Evaluation Right: What really matters for effectiveness and improvement*. New York, NY: Teachers College Press.
- Denbel, D. G. (2015). Students' learning experiences when using a dynamic geometry software tool in a geometry lesson at secondary school in Ethiopia. *Journal of Education and Practice*, 6(1), 24–38.
- Department of Basic Education (DBE). (2004). *White Paper on e-Education*. Government Gazette (No. 236734). Pretoria: Government Printing Works.
- Department of Basic Education (DBE). (2012). *Curriculum and Assessment Policy Statement [CAPS], Mathematics Grades 7–9*. Pretoria: Government Printing Works.
- Department of Basic Education (DBE). (2014). *Annual National Assessments: 2014 Diagnostic Report*. Pretoria: Government Printers.
- Dikovic, L. (2009). Implementing dynamic mathematics resources with GeoGebra at the college level. *International Journal of Emerging Technologies in Learning*, 4(3), 51–54. <https://doi.org/10.3991/ijet.v4i3.784>.
- Dudley, P. (2015). *Lesson Study: Professional learning for our time*. London and New York: Routledge.

- Erickson, G., Brandes, G. M., Mitchell, I., & Mitchell, J. (2005). Collaborative teacher learning: Findings from two professional development projects. *Teaching and Teacher Education*, 21, 787–798.
- Fauskanger, J., Helgevold, N., Kazima, M., & Jakobsen, A. (2022). Challenging Malawian primary teachers' views on mathematics teaching and learning through lesson study. *International Journal for Lesson & and Learning Studies*, 11(1), 26–39.
<https://doi.org/10.1108/ijlls-10-2021-0087>
- Fernández, C. (2002). Learning from Japanese approaches to professional development: The case of lesson study. *Journal of Teacher Education*, 53(5), 393–405.
- Forgasz, H., & Kaur, B. (1997). The role of the pilot study in mathematics education research. *The Mathematics Educator*, 2(2), 187–196.
- Fraenkel, J. R., & Wallen, N. E. (2006). *How to Design and Evaluate Research in Education*. Boston: McGraw-Hill.
- Fujii, T. (2014). Implementing Japanese lesson study in foreign countries: Misconceptions revealed. *Mathematics Teacher Education and Development*, 16(1), 65–83.
- Genc, M., & Ergan, S. (2022). Teaching geometry through didactical situations: The case of the triangle inequality. *Acta Didactica Napocensia*, 15(2), 123–141.
<https://doi.org/10.24193/adn.15.2.8>
- Gök, M., & Ertem-Akbaş, E. (2019). Examining the attitude change of pre-service elementary school teachers towards a course of mathematics education within the framework of Theory of Didactical Situations. *International Online Journal of Education and Teaching*, 6(4), 879–904.
<http://iojet.org/index.php/IOJET/article/view/665>
- Graham, M. A., Stols, G. H., & Kapp, R. (2021). Integrating Classroom Technology: South African mathematics teachers. *Computers in the Schools*, 38(3), 189–213.
<https://doi.org/10.1080/07380569.2021.1953951>
- Graneheim, U. H., & Lundman, B. (2004). Qualitative content analysis in nursing research: Concepts, procedures and measures to achieve trustworthiness. *Nurse Education Today*, 24(2), 105–112.
- Greene, J. C., Caracelli, V. J., & Graham, W. F. (1989). Toward a conceptual framework for mixed method evaluation designs. *Educational Evaluation and Policy Analysis* 11(3), 255–274. <http://dx.doi.org/10.3102/01623737011003255>
- Groth, R., Spickler, D., Bergner, J., & Bardzell, M. (2009). A qualitative approach to assessing technological pedagogical content knowledge. *Contemporary Issues in*

- Technology and Teacher Education*, 9(4), 392–411. Retrieved from <http://www.citejournal.org/vol9/iss4/mathematics/article1.cfm>
- Guba, E. G. (1981). Criteria for assessing the trustworthiness of naturalistic inquiries. *Educational Communication and Technology Journal*, 29(2), 75-91.
- Güven, B. (2012). Using dynamic geometry software to improve eight grade students' understanding of transformation geometry. *Australasian Journal of Educational Technology*, 28(2), 364–382. <https://doi.org/10.14742/ajet.878>
- Harris, J., Grandgenett, N., & Hofer, M. (2010). Testing a TPACK-based technology integration assessment rubric. In D. Gibson & B. Dodge (Eds.), *Proceedings of Society for Information Technology & Teacher Education International Conference 2010* (pp. 3833-3840). Chesapeake, VA: AACE.
- Henk, W. A. (1987). Methodological concerns in experimental reading research: All that glitters... *Reading Psychology: An International Quarterly*, 8(2), 61–71.
- Hohenwarter, M., Kreis, Y., & Lavicza, Z. (2008). Technology professional development and research collaboration: Towards an international GeoGebra institute. *Beiträge zum Mathematikunterricht*, pp. 307-310.
- Hohenwarter, M., & Preiner, J. (2007). Dynamic mathematics with GeoGebra. *Journal of On-line Mathematics and its Applications*, 7.
- Hollebrands, K. F. (2003). High school students' understanding of transformation geometry in the context of a technological environment. *Journal of Mathematical Behaviour*, 22, 55–72. [http://dx.doi.org/10.1016/S0732-3123\(03\)00004-X](http://dx.doi.org/10.1016/S0732-3123(03)00004-X)
- Huang, X., Lai, M. Y., & Huang, R. (2021). Teachers' learning through an online lesson study: An analysis from the expansive learning perspective. *International Journal of Lesson and Learning Studies*, 10, 202–216.
- Hughes, J. (2005). The role of teacher knowledge and learning experiences in forming technology-integrated pedagogy. *Journal of Technology and Teacher Education*, 13(2), 277–302.
- Idris, N. (2006). Exploring the effects of T1-84 plus on achievement and anxiety in mathematics. *Eurasia Journal of Mathematics, Science, and Technology Education*, 2(3), 66–78.
- Jessen, B., Bos, R., Doorman, M., & Winsløw, C. (2023). Lesson study in mathematics with the Theory of Didactic Situations and Realistic Mathematics Education as theoretical support: Two cases from the European TIME project. *International Journal for*

Lesson and Learning Studies, 12(1), 52–64.

<https://doi.org/10.1108/IJLLS-01-2022-0009>

- Jezdimirović, J. (2014). Computer based support for mathematics education in Serbia. *International Journal of Technology and Inclusive Education*, 3(1), 277–286.
- Jhang, F. H. (2019). Teachers' attitudes towards lesson study, perceived competence, and involvement in lesson study: evidence from junior high school teachers. *Professional Development in Education*, 46(1), 82–96.
- Jiménez Sierra, Á. A., Ortega Iglesias, J. M., Cabro-Almenara, J., & Palacios-Rodríguez, A. (2023). Development of the teacher's technological pedagogical content knowledge (TPACK) from the lesson study: A systematic review. *Frontiers in Education*, 8, 1078913. <https://doi.org/10.3389/feduc.2023.1078913>
- Joubert, J., Callaghan, R., & Engelbrecht, J. (2020). Lesson study in a blended approach to support isolated teachers in teaching with technology. *ZDM Mathematics Education*, 52, 907–925.
- Karuhanga, J. (2018, September 18). Smart classroom project gathers momentum. *The New Times*. <https://www.newtimes.co.rw/news/smart-classroom-project-gathers-momentum>
- Kepceoğlu, I., & Yavuz, I. (2016). Teaching a concept with GeoGebra: Periodicity of trigonometric functions. *Educational Research and Reviews*, 11(8), 573–581.
- Kilpatrick, J., Swafford, J., & Findel, B. (2001). *Adding It Up: Helping children learn mathematics*. Washington, DC: National Academy Press.
- Kitzinger, J. (1995). Qualitative research: Introducing focus group interviews. *British Medical Journal*, 311, 299–302.
- Kivunja, C. (2018). Distinguishing between theory, theoretical framework, and conceptual framework: A systematic review of lessons from the field. *International Journal of Higher Education*, 7(6), 44–53.
- Koehler, M., Mishra, P., & Cain, W. (2013). What is technological pedagogical content knowledge (TPACK)? *Journal of Education*, 193(3), 13–19. <https://doi.org/10.1177/002205741319300303>
- Koehler, M., Mishra, P., & Yahya, K. (2007). Tracing the development of teacher knowledge in a design seminar: Integrating content, pedagogy and technology. *Computers & Education*, 49, 740–762.

- Koehler, M. J., & Mishra, P. (2005). What happens when teachers design educational technology? The development of technological pedagogical content knowledge. *Journal of Educational Computing Research*, 32(2), 131–152.
- Koehler, M. J., & Mishra, P. (2009). What is technological pedagogical content knowledge (TPACK)? *Contemporary Issues in Technology and Teacher Education*, 9(1), 60–70.
- Koehler, M. J., Shin, T. S., & Mishra, P. (2012). How do we measure TPACK? Let me count the ways. In R. N. Ronau, C. R. Rakes, & M. L. Niess (Eds.), *Educational Technology, Teacher Knowledge, and Classroom Impact: A research handbook on frameworks and approaches* (pp.16–31). Hershey, PA: Information Science Reference.
- Küçük, K., & Gün, Ö. (2023). The effects of GeoGebra-assisted transformation geometry instruction on student achievement, attitudes, and beliefs. *Journal of Computer and Education Research*, 11(22), 671–690.
- Kurt, G., & Çakıroğlu, E. (2018). Preservice mathematics teachers' TPACK development in statistics teaching: A microteaching lesson study. In M. A. Sorto, A. White, & L. Guyot (Eds.), *Looking Back, Looking Forward. Proceedings of the Tenth International Conference on Teaching Statistics*. International Statistical Institute.
- Laborde, C. (2007). Towards theoretical foundations of mathematics education. *ZDM Mathematics Education*, 39, 137–144.
- Lawless, K. A., & Pellegrino, J. W. (2007). Professional development in integrating technology into teaching and learning: Knowns, unknowns, and ways to pursue better questions and answers. *Review of Educational Research*, 77(4), 575–614.
- Lendínez Muñoz, E. M., García García, F. J., Lerma Fernández, A. M., & Abril Gallego, A. M. (2024). Increase in self-efficacy in prospective teachers through theory-based lesson study. *Journal of Mathematics Teacher Education*, 27(4), 717–742.
<https://doi.org/10.1007/s10857-023-09597-0>
- Letloenyane, M. D., & Jita, L. C. (2015). School-University Partnerships for Professional Development of Teachers: A case of lesson study intervention in mathematics. *Journal of Higher Education in Africa*, 13, 147–167.
- Lewis, C. (2002). *Lesson Study: A handbook of teacher-led instruction changes*. Philadelphia, PA, USA: Research for Better schools Inc.
- Lewis, C. (2004). Does lesson study have a future in the United States? *Journal of Social Science Studies*, 3(1), 113–137.

- Lewis, C. (2009). What is the nature of knowledge development in lesson study? *Educational Action Research*, 17(1), 95–110.
- Lewis, C., & Hurd, J. (2011). *Lesson Study Step by Step: How teacher learning communities improve instruction*. Portsmouth, NH: Heinemann.
- Li, D. (2004). Trustworthiness of think-aloud protocols in the study of translation processes. *International Journal of Applied Linguistics*, 14(3), 301–313.
- Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. Newbury Park, CA: Sage.
- Lyublinskaya, I., & Tournaki, E. (2014). A study of special education teachers' TPACK development in mathematics and science through assessment of lesson plans. *Technology and Teacher Education*, 22(4), 449–470.
- Lyublinskaya, I., & Tournaki, N. (2010). Integrating TI-Nspire technology into algebra classrooms: Selected factors that relate to quality of instruction. In D. Gibson & B. Dodge (Eds.), *Proceedings of Society for Information Technology & Teacher Education International Conference 2010* (pp. 1513–1520). Chesapeake, VA: AACE.
- Macnee, L. C., & McCabe, S. (2008). *Understanding nursing research: Using research evidence-based practice*. Philadelphia, PA: Lippincott Williams & Wilkins.
- Majerek, D. (2014). Application of GeoGebra for Teaching Mathematics. *Advances in science and technology. Research Journal*, 8(24), 51–54.
- Mangiante-Orsola, C., Perrin-Glorian, M. J., & Strømskag, H. (2018). Theory of didactical situations as a tool to understand and develop mathematics teaching practices. *Annales de Didactique et de Sciences Cognitives*, 145–174.
- McKechnie, A. E. (2008). Phenotypic flexibility in basal metabolic rate and the changing view of avian physiological diversity: A review. *Journal of Comparative Physiology B*, 178, 235–247.
- Mensah, P. A. A., Ansul, B., Karadaar, I. B., & Junior, J. G. (2023). GeoGebra as a tool for improving mathematics instruction and learning by Ghanaian teachers. *American Journal of Education and Technology*, 2(2), 69–75.
- Merriam, B. S. (2009). *Qualitative Research: A guide to design and implementation*. San Francisco CA: Jossey-Bass.
- Merriam, S. (1998). *Qualitative Research and Case Study Applications in Education*. San Francisco: Jossey-Bass.
- Mertens, D. M. (2005). *Research Methods in Education and Psychology: Integrating diversity with quantitative and qualitative approaches* (2nd ed.) Thousand Oaks: Sage.

- Mhakure, D. (2019). School-based mathematics teacher professional learning: A theoretical position on the lesson study approach. *South African Journal of Education*, 39, 1-8.
- Migiro, S. O., & Magangi, B. A. (2011). Mixed methods: A review of literature and the future of the new research paradigm. *African Journal of Business Management*, 5(10), 3757–3764.
- Mishra, P. (2019). Considering contextual knowledge: The TPACK diagram gets an upgrade. *Journal of Digital Learning in Teacher Education*, 35(2), 76–78.
<https://doi.org/10.1080/21532974.2019.1588611>
- Mishra, P., & Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record*, 108(6), 1017.
<https://doi.org/10.1111/j.1467-9620.2006.00684.x>
- Mishra, P., & Koehler, M. J. (2009). What is technological pedagogical content knowledge? *Teachers College Record*, 9, 1017–1054.
- Miyakawa, T., & Winsløw, C. (2009). Didactical designs for students' proportional reasoning: An open approach lesson and a fundamental situation. *Educational Studies in Mathematics*, 72(2), 199–218.
- Mokotjo, L. G. (2020). *The integration of GeoGebra in the teaching of mathematics in South African high schools* [Unpublished doctoral dissertation]. University of the Free State, Bloemfontein.
- Mokotjo, L. G., & Mokhele, M. L. (2021). Challenges of integrating GeoGebra in the teaching of mathematics in South African high schools. *Universal Journal of Educational Research*, 9(5), 963–973.
- Mosimege, M., & Winnaar, L. (2021). Teachers' instructional strategies and their impact on learner performance in Grade 9 mathematics: Findings from TIMSS 2015 in South Africa. *Perspectives in Education*, 39(2), 324–338.
- Mthethwa, M., Bayaga, A., Bosse, M. J., & Williams, D. (2020). GeoGebra for learning and teaching: A parallel investigation. *South African Journal of Higher Education*, 40(2), 1–12. <https://doi.org/10.15700/saje.v40n2a1669>
- Mukamba, E., & Makamure, C. (2020). Integration of GeoGebra in teaching and learning transformation geometry at the ordinary level in Zimbabwe. *Contemporary Mathematics and Science Education*, 1(1), 1–8.

- Mullis, I. V. S., Martin, M. O., Foy, P., Kelly, D. L., & Fishbein, B. (2020). *TIMSS 2019 International Results in Mathematics and Science*. Chestnut Hill, MA: TIMSS & PIRLS International Study Center.
- Ngwabe, A. H., & Schäfer, M. (2023). *Exploring the potential of how GeoGebra can be used as a visual mediator between preservice teachers' mathematical pedagogical content knowledge and teaching proficiency*. Paper presented at the 31st Annual Conference of the Southern African Association for Research in Mathematics, Science and Technology Education, University of the Free State.
- Niess, M. L. (2005). Preparing teachers to teach science and mathematics with technology: Developing a technology pedagogical content knowledge. *Teaching and Teacher Education*, 21(5), 509–523.
- Niess, M. L. (2007). Developing teacher's TPACK for teaching mathematics with spreadsheets. *Technology and Teacher Education Annual*, 18(4), 2238–2245.
- Niess, M. L. (2008). Guiding preservice teachers in developing TPACK. In AACTE Committee on Innovation and Technology (Eds.), *Handbook of Technological Pedagogical Content Knowledge (TPCK) for Educators* (pp. 223-250). New York, NY: Routledge.
- Niess, M. L. (2011). Investigating TPACK: Knowledge growth in teaching with technology, *Journal of Educational Computing Research*, 44(3), 299–317.
- Niess, M. L. (2012). Teacher Knowledge for Teaching with Technology: A TPACK lens. In R. Ronau, C. Rakes, & M. Niess (Eds.), *Educational Technology, Teacher Knowledge, and Classroom Impact: A research handbook on frameworks and approaches* (pp. 1-15). Hershey: IGI Global.
- Niess, M. L. (2013). Central component descriptors for levels of technological pedagogical content knowledge. *Journal of Educational Computing Research*, 48(2), 173–198.
- Niess, M. L., Ronau, R. N., Shafer, K. G., Driskell, S. O., Harper, S. R., Johnston, C., Browning, C., Ozgun-Koca, S. A., & Kersaint, G. (2009). Mathematics teacher TPACK standards and development model. *Contemporary Issues in Technology and Teacher Education*, 9(1), 4–24.
- Ni Shuilleabhain, A., & Seery, A. (2017). Enacting curriculum reform through lesson study: A case study of mathematics teacher learning. *Professional Development in Education*, 44, 1–15.

- Novotná, J., & Hošpesová, A. (2022). Theory of Didactical Situations in mathematics as one of the theoretical approaches to the LPS analysis of classroom data: The case of institutionalization. *ZDM Mathematics Education*, 54(2).
- Ogegbo, A. A., Gaigher, E., & Salagaram, T. (2019). Benefits and challenges of lesson study: A case of teaching Physical Sciences in South Africa. *South African Journal of Education*, 39(1), 1-9.
- Ono, Y., & Ferreira, J. G. (2010). A case study of continuing teacher professional development through lesson study in South Africa. *South African Journal of Education*, 30(1), 59-74.
- Perry, R. R., & Lewis, C. C. (2009). What is successful adaptation of lesson study in the US? *Journal of Education Change*, 10, 365–391.
- Posthuma, A. B. (2012). Mathematics teachers' reflective practice within the context of adapted lesson study. *Pythagoras*, 33(3), 1–9.
- Preiner, J. (2008). *Introducing dynamic mathematics software to mathematics teachers: The case of GeoGebra* [Unpublished doctoral dissertation]. University of Salzburg.
- Pretorius, L. (2024). Demystifying research paradigms: Navigating ontology, epistemology, and axiology in research. *The Qualitative Report*, 29(10), 2698-2715.
- Purdie, N., & Hattie, J. (2002). Assessing students' conception of learning. *Australian Journal of Educational and Developmental Psychology*, 2, 17–32.
- Rahman, M. H. A., & Puteh, M. (2017). Learning Trigonometry Using GeoGebra Learning Module: Are under achiever pupils motivated? *Sains Humanika*, 1(2), 39–42.
- Riales, J. W. (2011). *An examination of secondary mathematics teachers' TPACK development through participation in a technology-based lesson study* [Unpublished doctoral dissertation]. University of Mississippi.
<http://search.proquest.com/docview/879634838?accountid=13380>
- Rollick, M. B. (2009). Toward a definition of reflection. *Mathematics Teaching in the Middle School*, 14(7), 396–398.
- Saal, P. E., Graham, M. A., & Van Ryneveld, L. (2020). Integrating educational technology in mathematics education in economically disadvantaged areas in South Africa. *Computers in the Schools*, 37(4), 253–268.
<https://doi.org/10.1080/07380569.2020.1830254>
- Saralar, I. (2016). *A pre-service mathematics teacher's technological pedagogical content knowledge regarding different views of 3-D figures in geometry* [Unpublished master's thesis]. Middle East Technical University, Ankara.

- Saralar, I., İşıksal-Bostan, M., & Akyüz, D. (2017). The evaluation of a pre-service mathematics teacher's TPACK: A case of 3D shapes with GeoGebra. *International Journal for Technology in Mathematics Education*, 25(2), 3–21.
https://doi.org/10.1564/tme_v25.2.01
- Schmidt, D. A., Baran, E., Thompson, A. D., Mishra, P., Koehler, M. J., & Shin, T. S. (2009). Technological pedagogical content knowledge (TPACK): The development and validation of an assessment instrument for preservice teachers. *Journal of Research on Technology in Education*, 42(2), 123–149.
- Schwab, K. (2017). The global competitiveness report 2017–2018. Retrieved from <https://www.weforum.org/reports/the-global-competitiveness-report-2017-2018>
- Sekao, D., & Engelbrecht, J. (2021). South African primary mathematics teachers' experiences and perspectives about lesson study. *International Journal of Science and Mathematics Education*, 20(7), 1431–1453.
- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15, 4–14. <https://doi.org/10.2307/1175860>
- Shulman, L. S. (1987). Knowledge in teaching: Foundation of the new reform. *Harvard Educational Review*, 57(1), 1–23.
<https://doi.org/10.17763/haer.57.1.j463w79r56455411>
- Stigler, J., & Hiebert, J. (1999). The teaching gap. New York: Free Press.
- Stols, G., Ferreira, R., Pelser, A., Olivier, W. A., Van der Merwe, A., De Villiers, C., & Venter, S. (2015). Perceptions and needs of South African mathematics teachers concerning their use of technology for instruction. *South African Journal of Education*, 35(4), 1209–1221. <https://doi.org/10.15700/saje.v35n4a1209>
- Takahashi, A. (2014). The role of the knowledgeable other in lesson study: Examining the final comments of experienced lesson study practitioners. *Mathematics Teacher Education and Development*, 16(1), 4–12.
- Takahashi, A., & McDougal, T. (2016). Collaborative lesson research: maximizing the impact of lesson study. *ZDM Mathematics Education*, 48(4). doi:10.1007/s11858-015-0752-x
- Tan, L., Chai, C., Deng, F., Zheng, C., & Drajati, N. (2019). Examining pre-service teachers' knowledge of teaching multimodal literacies: A validation of the TPACK survey. *Educational Media International*, 56(4), 285–299.
- Tobin, G. A., & Begley, C. M. (2004). Methodological rigour within a qualitative framework. *Journal of Advanced Nursing*, 48(4), 388–396.

- Tournaki, N., & Lyublinskaya, I. (2014). Preparing special education teachers for teaching mathematics and science with technology by integrating the TPACK framework into the curriculum: A study of teachers' perceptions. *Journal of Technology & Teacher Education*, 22(2), 243–259.
- Trochim, W. M. (2006). *The Research Methods Knowledge Base* (2nd ed.).
- Trujillo-Torres, J. M., Hossein-Mohand, H., Gómez-García, M., Hossein-Mohand, H., & Cáceres-Reche, M. P. (2020). Mathematics teachers' perceptions of the introduction of ICT: The relationship between motivation and use in the teaching function. *Mathematics*, 8(12), 2158.
- Upadhyay, P. (2015). Designing theoretical and conceptual frameworks. *Research Journal of Sociology/Anthropology*, 1, 27–51.
- Uymaz, A. S. (2024). The development of an ICT-generated guideline for high school 3D geometry lessons based on the perceptions of pre-service teachers and their instructors [Unpublished master's thesis]. İhsan Doğramacı Bilkent University, Ankara.
- Van Dyk, H., & White, C. J. (2019). Theory and practice of the quintile ranking of schools in South Africa: A financial management perspective. *South African Journal of Education*, 39(1), 1–9.
- Warfield, V. (2014). *Invitation to Didactique*. New York, NY: Springer.
- Wassie, Y. A., & Zergaw, G. A. (2019). Some of the potential affordances, challenges and limitations of using GeoGebra in mathematics education. *Eurasia Journal of Mathematics, Science and Technology Education*, 15(8).
- Wiersma, W. (1991). *Research Methods in Education: An introduction* (5th ed.). Sydney: Allyn and Bacon.
- Winsløw, C., Bahn, J., & Rasmussen, K. (2018). Theorizing lesson study: Two related frameworks and two Danish case-studies. In M. Quaresma, C. Winsløw, S. Clivaz, J. P. Da Ponte, A. N. Shúilleabháin, A. Takahashi, & T. Fujii (Eds.), *Mathematics Lesson Study Around the World: Theoretical and methodological issues* (pp. 123–142). Chambersburg, PA: Springer.
- Woodland, R. H., & Mazur, R. (2015). Beyond hammers versus hugs: Leveraging educator evaluation and professional learning communities into job-embedded professional development. *NASSP Bulletin*, 99(1), 5–25.
- Yin, R. K. (2003). *Case study research: Design and methods* (3rd ed.) Thousand Oaks, CA: Sage.

- Zelkowski, J., Gleason, J., Cox, D. C., & Bismark, S. (2013). Developing and validating a reliable TPACK instrument for secondary mathematics preservice teachers. *Journal of Research on Technology in Education*, 46(2), 173–206.
- Zulnaidi, H., & Zamri, S. (2017). The effectiveness of GeoGebra: The intermediary role of procedural knowledge on students' conceptual knowledge and their achievement in mathematics. *Eurasia Journal of Mathematics Science and Technology Education*, 13(6), 2155–2180.

APPENDICES

Appendix 1: Mathematics Teacher TPACK Development rubric, adapted from Niess et al. (2009)

CURRICULUM & ASSESSMENT

C: Curriculum descriptor A: Assessment descriptor Ex: Mathematics Example

Recognising

C: Acknowledges that mathematical ideas displayed with the technologies can be useful for making sense of topics addressed in the curriculum.

Ex: Creates graphs of multiple linear functions using graphing calculators to provide a visual representation for varying slopes. Considers these visuals as making sense of the idea of slope but is unsure of how this might help students learn the basic concept.

A: Resists idea of technology use in assessment indicating that technology interferes with determining students' understanding of mathematics.

Ex: Does not allow calculator use when assessing students' understanding of solving linear equations.

Accepting

C: Expresses desire but demonstrates difficulty in identifying topics in own curriculum for including technology as a tool for learning.

Ex: Attends and participates in mathematics dynamic geometry system workshop to identify curricular ideas for incorporating the technologies as learning tools. Mimics the incorporation of a dynamic geometry system idea from the workshop to display measuring the sum of the angles of a triangle that upon multiple changes of the triangle suggests that the sum of the angles of any triangle is 180 degrees.

A: Acknowledges that it might be appropriate to allow technology use as part of assessment but has a limited view of its use (e.g. use of technology on a section of an exam).

Ex: Attends and participates in mathematics assessment professional development

to consider ideas for assessing students' understanding of solving systems of linear functions using the calculator as a tool. Mimics the assessment idea to explain the use of the calculator for solving systems of linear functions by using the trace function to identify the intersection. Often retests technology questions with paper and pencil questions to be sure that the concept was learned the right way.

Adapting

C: Understands some benefits of incorporating appropriate technologies as tools for teaching and learning the mathematics curriculum.

Ex: Targets key topics students investigate with technology. Develops lessons to demonstrate mathematics concepts with technology and activities for students to use technology to verify or reinforce those concepts. After students have learned to create graphs of specific linear functions, students are challenged to use the spreadsheet to verify the graphical representation of the ordered pairs.

A: Understands that if technology is allowed during assessment, then different questions/items must be posed (i.e. conceptual vs. procedural understandings).

Ex: Allows use of calculator in an assessment but designs the assessment to focus on gathering students' conceptual understanding of solving systems of linear functions in addition to their procedural understanding.

Exploring

C: Investigates the use of topics in own curriculum for including technology as a tool for learning; seeks ideas and strategies for implementing technology in a more integral role for the development of the mathematics that students are learning.

Ex: Adapts own previous mathematics lesson to include technology.

Ex: Develops own ideas about using technology to enhance current curriculum; thus, begins altering pre-existing activities or creating new activities for current curriculum.

A: Actively investigates use of different types of technology-based assessment Items and questions (e.g. technology active, inactive, neutral or passive).

Ex: Designs assessments where students are expected to show their understanding of mathematical ideas using an appropriate technology that extends

beyond paper and pencil type questions.

Advancing

C: Understands that sustained innovation in modifying own curriculum to efficiently and effectively incorporate technology as a teaching and learning tool is essential.

Ex: Develops innovative ways to use technology to develop mathematical thinking in students such as using virtual algebra tiles to extend ideas of handheld manipulatives to focus on variables in algebraic expressions.

Ex: Modifies and advances curriculum to take advantage of technology as a tool for teaching and learning such as using CAS to explore more complex algebraic expressions.

A: Reflects on and adapts assessment practices that examine students' conceptual understandings of the subject matter in ways that demand full use of technology.

Ex: Develops innovative assessments to capture students' understandings of the mathematics embedded in the particular technology.

LEARNING

M: Mathematics learning descriptor C: Conception of student thinking descriptor Ex: Mathematics example

Recognising

M: Views mathematics as being learned in specific ways and that technology often gets in the way of learning.

Ex: Mathematical exploration with technology rarely seen.

C: More apt to accept the technology as a teaching tool rather than a learning tool.

Ex: Technology is used only outside of normal classroom activities, such as checking homework, calculating large numbers, etc.

Accepting

M: Has concerns about students' attention being diverted from learning of appropriate

mathematics to a focus on the technology in the activities.

Ex: Limits student technology use, particularly during the introduction and

development of key topics.

C: Is concerned that students do not develop appropriate mathematical thinking skills when the technology is used as a verification tool for exploring the mathematics.

Ex: Activities that use technology are almost always redone without technology to be certain students really learned the particular concept.

Adapting

M: Begins to explore, experiment and practise integrating technologies as mathematics learning tools.

Ex: Students explore some mathematics topics using technology.

C: Begins developing appropriate mathematical thinking skills when technology is used as a tool for learning.

Ex: Although students use technology for most topics, assessing student thinking remains mostly technology free.

Exploring

M: Uses technologies as tools to facilitate the learning of specific topics in the mathematics curriculum.

Ex: Students explore numerous topics using technology, sometimes ranging outside the topic at hand.

C: Plans, implements, and reflects on teaching and learning with concern for guiding students in understanding.

Ex: Technology activities are implemented and evaluated with respect to Student learning of mathematics and student attitudes towards mathematics.

Ex: Manages technology-enhanced activities towards directing student engagement and self-direction in learning mathematics.

Advancing

M: Plans, implements and reflects on teaching and learning with concern and Personal conviction for student thinking and understanding of the mathematics to be enhanced through integration of the various technologies.

Ex: Students explore mathematics topics, integrating various technologies in attempts to better understand mathematical concepts.

C: Technology-integration is integral (rather than in addition) to development of the mathematics students are learning.

Ex: Engages students in high-level thinking activities (such as project-based

and problem solving and decision-making activities) for learning mathematics using the technology as a learning tool.

Ex: Technology is used to develop advanced levels of understanding of mathematical concepts.

TEACHING

M: Mathematics learning descriptor I: Instructional descriptor E: Environment descriptor PD: Professional development descriptor Ex: Mathematics example

Recognising

M: Concerned that the need to teach about the technology will take away time from teaching mathematics.

Ex: Students use technology on their own and little or no instruction with technology is present.

I: Does not use technology to develop mathematical concepts.

Ex: Technology, if used in class, is used for menial or rote activities.

E: Uses technology to reinforce concepts taught without technology.

Ex: Focus on linear functions where students practice creating graphs by hand to explore different functions. After students have demonstrated competence with linear functions, summarize the knowledge, with a spreadsheet example or a graphing calculator example.

PD: Considers attending local professional development to learn more about technologies.

Ex: Attends local workshops that focus on gaining skills with the technology; context of the learning activities is mathematics.

Accepting

M: Uses technology activities at the end of units, for ‘days off’, or for activities peripheral to classroom instruction.

Ex: Technology-enhanced activities are not used for topics that require more advanced technology skills.

I: Merely mimics the simplest professional development mathematics curricular Ideas for incorporating the technologies.

Ex: Introduces the Pythagorean Theorem algorithmically; teacher use of dynamic geometry to verify the Pythagorean Theorem; students find solutions to example problems using paper and pencil.

E: Tightly manages and orchestrates instruction using technology.

Ex: Technology is directed, in a tightly sequenced, step-by-step process. Skill-based, non-exploratory technology use.

PD: Recognises the need to participate in technology-related PD.

Ex: Seeks out technology-related professional development workshops that are directed at developing the technology in the learning of mathematics.

Adapting

M: Uses technology to enhance or reinforce mathematics ideas that students have learned previously.

Ex: Students use technology to reinforce previously teacher-taught concepts.

I: Mimics the simplest professional development activities with the technologies but attempts to adapt lessons for his/her mathematics classes.

Ex: Technology-based lessons are incorporated that are tailored to students' needs.

E: Instructional strategies with technologies are primarily deductive, teacher-Directed in order to maintain control of the how the activity progresses.

Ex: Begins to adapt instructional approaches that allow students opportunities to explore with technology for part of lessons.

PD: Continues to learn and explore ideas for teaching and learning mathematics Using only one type of technology (such as spreadsheets).

Ex: Shares ideas from professional development with other mathematics teachers in the building.

Exploring

M: Engages students in high-level thinking activities (such as project-based, problem-solving and decision-making activities) for learning mathematics using the technology as a learning tool.

Ex: Teachers share classroom-tested, technology-based lessons, ideas and successes with peers.

I: Engages students in explorations of mathematics with technology where the teacher is in role of guide rather than director of the exploration.

Ex: Students use technology to explore new concepts as the teacher serves mostly as a guide.

E: Explores various instructional strategies (including both deductive and inductive strategies) with technologies to engage students in thinking about the

mathematics.

Ex: The teacher incorporates a variety of technologies for numerous topics.

PD: Seeks out and works with others who are engaged in incorporating technology in mathematics.

Ex: Organises teachers of similar mathematics and grade level in investigating the mathematics curriculum to integrate appropriate technologies.

Advancing

M: Active, consistent acceptance of technologies as tools for learning and teaching mathematics in ways that accurately translate mathematical concepts and processes into forms understandable by students.

Ex: Teacher is seen as a resource as novel ideas for helping students learn mathematics with technology.

I: Adapts from a breadth of instructional strategies (including both deductive and inductive strategies) with technologies to engage students in thinking about the mathematics.

Ex: The teacher helps students move fluently from one tool to another while demonstrating a focus on and a joy of deeply understanding mathematical topics.

E: Manages technology-enhanced activities in ways that maintain student engagement and self-direction in learning the mathematics.

Ex: The teacher forms and reforms learning groups where individual and group learning is valued and encouraged.

PD: Seeks ongoing PD to continue to learn to incorporate emerging technologies.

Continues to learn and explore ideas for teaching and learning mathematics with multiple technologies to enhance access to mathematics.

Ex: Engages teachers in the district in evaluating and revising the mathematics curriculum to more seamlessly integrate technology throughout the grades, adjusting the curriculum for a 21st-century mathematics curriculum with appropriate technologies.

ACCESS

U: Usage descriptor B: Barrier descriptor A: Availability descriptor Ex: Mathematics example

Recognising

- U: Permits students to use technology only after mastering certain concepts.
Ex: Mathematical exploration with technology tools is challenged by beliefs about how students need to learn mathematics.
- B: Resists consideration of changes in content taught although it becomes accessible to more students through technology.
Ex: Student access to technology is limited to after they have learned the given concepts using paper and pencil procedures and only for rote activities.
- A: Notices that authentic problems are more likely to involve unfriendly numbers and may be more easily solved if students had calculators.
Ex: Assigns some mathematics problems using school and community data but saves then for ‘extra credit’ work if students have calculators.

Accepting

- U: Students use technology in limited ways during regular instructional periods.
Ex: Student activities with technology are limited to brief tightly controlled situations.
- B: Worries about access and management issues with respect to incorporating technology in the classroom.
Ex: Students can only use technology in isolated situations or non-important learning situations.
- A: Calculators permit greater number of examples to be explored by students.
Ex: Student use calculators to investigate patterns and functions.

Adapting

- U: Permits students to use technology in specifically designed units.
Ex: Access to and use of technology is available for exploration of new topics, usually with the teacher’s demonstration.
- B: Uses technology as a tool to enhance mathematics lessons in order to provide students a new way to approach mathematics.
Ex: Concepts learned with technology are not assessed with technology.
- A: Concepts are taught differently since technology provides access to connections formerly out of reach.

Ex: Students use dynamic geometry software to investigate and make connections between trigonometry functions.

Exploring

U: Permits students to use technology for exploring specific mathematical topics.

Ex: Access to and use of technology is available and encouraged for mathematics exploration during most class times.

B: Recognises challenges for teaching mathematics with technologies, but explores strategies and ideas to minimise the impact of those challenges.

Ex: Technology is used extensively in assessments. Seeks out ways to obtain technology for classroom use and begins creating methods for technology management issues.

A: Through the use of technology, key topics are explored, applied and assessed incorporating multiple representations of the concepts and their connections.

Ex: Simultaneous equations are developed from an authentic situation, solved, and interpreted using graphs, tables, symbols and data.

Advancing

U: Permit students to use technology in every aspect of mathematics class.

Ex: Technology is seen as an opportunity to challenge notions of what Mathematics students can master.

B: Recognises challenges in teaching with technology and resolves the challenges through extended planning and preparation for maximizing the use of available resources and tools.

Ex: Technology is used to expand the mathematics concepts that can be accessed by students.

A: Students are taught and permitted to explore more complex mathematics topics or mathematical connections as part of their normal learning experience.

Ex: Using the Internet to find interesting mathematical problems, students investigate the role that technologies can play in finding solutions to the

problems.

Appendix 2: Ethical clearance



Department of Oceanography

Professor Isabelle Ansong
FSREC Chair 2025-2027

University of Cape Town Private Bag Rondebosch 7701
South Africa

22nd April 2025

Dear Mr Ssenyomo

Investigating the mediating role of GeoGebra among in-service mathematics teachers' technological pedagogical content knowledge (TPACK) and teaching proficiency using the Lesson Study.

Application - FSREC/ 086-2023

I am pleased to inform you that as the current Chair of the FSREC I have approved the above-named application for research ethics clearance, on the basis that you have fulfilled the conditions outlined in the original approval letter dated 9th October 2023.

Conditions outlined in the original letter

- Researcher should get approval from the NCDiE and the two participating schools and send this approval to sri-rec@uct.ac.za
- Research may not commence in a given school until permission has been given by the NCDiE and the school.

I have received documentation as well as confirmation from your supervisor Prof David Erwin that approval has been granted by both the targeted schools and the NCDiE and therefore confirm that you have met the approval for full ethical clearance.

Please contact me if you need advice on ethical issues that arise.

Expiration date: 31 December 2025

I wish you great success in your research.

Kind Regards,

A handwritten signature in blue ink, appearing to read 'Isabelle'.

Isabelle Ansong
FSREC Chair

Appendix 3: Head of the Northern Cape Department of Education



DEPARTMENT OF EDUCATION

Enquiries: J N Horne
 Contact No: (053) 839 6536
 Reference: L2.10.2.4.3
 Date: 30 November 2022

90 Cillierslaan Vle 2
 Kuruman
 8400

Dear Mr. B Ssennyomo

APPROVAL TO CONDUCT RESEARCH

The aforesaid matter with Ref: L2.10.2.4.3, bears reference

This letter serves to indicate that approval is granted to conduct research for the thesis titled: "Investigating the mediating role of GeoGebra among in-service mathematics teachers' technological pedagogical content knowledge (TPACK) and teaching proficiency using the lesson study"

The onus rests with you as the researcher to arrange appropriate and relevant time schedules with the sampled teachers in order to conduct the research. A copy of this approval letter must be presented to the District Director and the school (Principal and SGB) as proof that permission for the research has been granted.

The following conditions must be strictly applied during the conduct of your research in the Northern Cape Department of Education (NCDoE). Approval may be withdrawn should any of the conditions listed below be flouted.

Criteria for approval	Comment
Value of the proposed research	The research topic covers a policy position of the Basic Education Sector and thus has value. The research study aims to understand how selected in-service Grade 9 Mathematics teachers develop and use GeoGebra applets to teach Geometry.
Policy and Strategic Alignment	The research is aligned to policy and to the Medium Term Strategic Framework (MTSF). The outcome of the research serves to assist in teacher development which falls within the ambit of the priorities of the Basic Education Sector.



Approval to Conduct Research: Mr. B Ssennyomo

Page 1 of 2

Appendix 4: Informed consent from participating teachers

Sisetyomo Bernard John
0734221461
sisetyomo@gmail.com
20/01/2023

Department of Mathematics and Applied Mathematics
University of Cape Town
Private Bag X3
Rondebosch 7701
South Africa

Project Title

Investigating the Technology Pedagogical Content Knowledge development of rural secondary school mathematics teachers when using GeoGebra software to teach geometric transformations at the Senior Phase through Lesson Study as a form school-based continuous professional development.

Dear sir/madam,

Re: **Informed voluntary consent to participate in research study**

You are invited to participate in a research study conducted with in-service Senior, and FET Phase mathematics teachers at selected schools in the John Taolo Gaetsewe District. The study aims to investigate the Technology Pedagogical Content Knowledge (TPACK) development of rural secondary school mathematics teachers when using GeoGebra software to teach geometric transformations at the Senior Phase through Lesson Study (LS) as a form school-based continuous professional development (CPD).

Your participation in this study will be beneficial to you (including me) in the sense that the use of GeoGebra software in teaching will provide us with many opportunities for enhancing our TPACK and mathematics teaching proficiency through the use of LS.

This study will be in form of an intervention program. During the study, you will be asked to participate in an intervention program that will consist of two workshops: the first session will deal with training teachers on how to use GeoGebra software to teach geometric transformation concepts of reflection, translation and enlargement. The second workshop will introduce teachers to the LS as a school-based CPD. I will take photographs, audio recordings or videos as part of the study. If you object to this, please indicate this below.

Are you comfortable with the data collection tools (photographs, audio and video recording) to be used in this study?
A. Yes ☒ B. No

There are no potentially harmful risks related to your participation in this study. In addition, your participation is completely voluntary; you may refuse to participate, and you may withdraw at any time without having to state a reason and without any prejudice or penalty against you. Should you choose to withdraw, the researcher commits not to use any of the information you have provided without your signed consent. Note that the researcher may also withdraw you from the study at any time. Furthermore, all information collected in this study will be kept private in that you will not be identified by name or by affiliation to an institution. Confidentiality and anonymity will be maintained as pseudonyms will be used.

By signing this consent form, you agree to participate in this research study. The aim, procedures to be used, as well as the benefits of your participation have been explained verbally to you in detail, using this form. Refusal to participate in or withdrawal from this study at any time will have no effect on you in any way. You are free to ask me, to ask questions or request further information, at any time during this research.

I agree to participate in this research (tick one box)

Yes ☒ No

(signature)

I agree to be photographed/audio-recorded/video-recorded (strike through as applicable) (optional)

Yes ☒ No

(signature)

I agree to participate in focus and/or individual interviews.

Yes ☒ No

(signature)

I agree to the use of properly anonymized photographs/audio recordings/videos in webinars/forums for research purposes (strike through as applicable)

Yes ☒ No

(signature)

Name of Participant: _____ Signature of Participant: _____

Date: 25/01/2023

Date: 25/01/2023

Simeonina Bernard John
0734221861
simeon@me.com
20/01/2023

Department of Mathematics and Applied Mathematics
University of Cape Town
Private Bag X3
Rondebosch 7703
South Africa

Project Title

Investigating the Technology Pedagogical Content Knowledge development of rural secondary school mathematics teachers when using GeoGebra software to teach geometric transformations at the Senior Phase through Lesson Study as a form school-based continuous professional development

Dear sir/madam

Re: Informed voluntary consent to participate in research study

You are invited to participate in a research study conducted with in-service Senior, and FET Phase mathematics teachers at selected schools in the John Taolo Gaetsewe District. The study aims to investigate the Technology Pedagogical Content Knowledge (TPACK) development of rural secondary school mathematics teachers when using GeoGebra software to teach geometric transformations at the Senior Phase through Lesson Study (LS) as a form school-based continuous professional development (CPD).

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Are you comfortable with the data collection tools (photographs, audio and video recording) to be used in this study?

A. Yes ☒

B. No

There are no potentially harmful risks related to your participation in this study. In addition, your participation is completely voluntary; you may refuse to participate, and you may withdraw at any time without having to state a reason and without any prejudice or penalty against you. Should you choose to withdraw, the researcher commits not to use any of the information you have provided without your signed consent. Note that the researcher may also withdraw you from the study at any time. Furthermore, all information collected in this study will be kept private in that you will not be identified by name or by affiliation to an institution. Confidentiality and anonymity will be maintained as pseudonyms will be used.

By signing this consent form, you agree to participate in this research study. The aim, procedures to be used, as well as the benefits of your participation have been explained verbally to you in detail, using this form. Refusal to participate in or withdrawal from this study at any time will have no effect on you in any way. You are free to contact me, to ask questions or request further information, at any time during this research.

I agree to participate in this research (tick one box)

Yes ☒ No

(Initials)

I agree to be photographed/audio-recorded/video-recorded (strike through as applicable)

Yes ☒ No

(Initials)

I agree to participate in focus and/or individual interviews

Yes ☒ No

(Initials)

I agree to the use of properly anonymized photographs/audio recordings/videos in websites/institutions for research purposes (strike through as applicable)

Yes ☒ No

(Initials)

Name of Participant: _____

Signature of Participant: _____

Witnessed (for random): _____

Signature of Institution: _____

Date: 25/01/2023

Date: 25/01/2023

Appendix 5: Approval letter from school principals

Ssennyomo Bernard John
0734221861
ssennyomo@gmail.com
20/11/2023

To The Principal
[REDACTED] High School

Dear Sir

Re: Permission to conduct research at your school

My name is Ssennyomo Bernard John, a PhD student at the University of Cape Town. I intend to conduct a study in mathematics, focusing on geometry teaching through the use of GeoGebra software at the Senior and FET Phase. The title of my study is "Investigating the mediating role of GeoGebra among in-service mathematics teachers' technological pedagogical content knowledge (TPACK) and teaching proficiency using lesson study (LS) as a form of school-based continuous professional development approach (CPD)." Research has shown that the concept of geometry has proved to be problematic at the primary, secondary and tertiary levels. This prompted me to embark upon this study for my PhD as its findings could be used to mitigate the pedagogical challenges encountered by teachers in teaching geometry in schools.

The overarching goal of this study is to investigate the mediating role of GeoGebra between in-service mathematics teachers' TPACK and their teaching proficiency using LS as a form of school-based continuous professional development (CPD).

The purpose of this letter is to request your permission to conduct this study at your school with the mathematics department teachers as my participants. The Head of Northern Cape Department of Education has been informed about the intended study and permission has already been granted from her office for me to go ahead with this study. I intend to uphold all the acceptable research ethics such as respect and human dignity, transparency and honesty, integrity and academic professionalism during the data collection process. Issues of confidentiality and anonymity will be addressed by using pseudonyms rather than the real names of schools and teachers involved in this study. Moreover, data collected will be used exclusively for the purpose of this study and it will not be published without the consent of the participants. I undertake to make available to you the findings and recommendations from this study.

I look forward to hearing from you soon.

Sincerely,

Ssennyomo Bernard John
UCT PhD student researcher

If [REDACTED] to conduct research at [REDACTED] High School has been approved/declined.

Signed: Principal [REDACTED] High School



Ssennyomo Bernard John
0734221861
ssennyomo@gmail.com
20/11/2023

To The Principal
[REDACTED] High School

Dear Sir

Re: Permission to conduct research at your school

My name is Ssennyomo Bernard John, a PhD student at the University of Cape Town. I intend to conduct a study in mathematics, focusing on geometry teaching through the use of GeoGebra software at the Senior and FET Phase. The title of my study is "Investigating the mediating role of GeoGebra among in-service mathematics teachers' technological pedagogical content knowledge (TPACK) and teaching proficiency using lesson study (LS) as a form of school-based continuous professional development approach (CPD)." Research has shown that the concept of geometry has proved to be problematic at the primary, secondary and tertiary levels. This prompted me to embark upon this study for my PhD as its findings could be used to mitigate the pedagogical challenges encountered by teachers in teaching geometry in schools.

The overarching goal of this study is to investigate the mediating role of GeoGebra between in-service mathematics teachers' TPACK and their teaching proficiency using LS a form of school-based continuous professional development (CPD).

The purpose of this letter is to request your permission to conduct this study at your school with the mathematics department teachers as my participants. The Head of Northern Cape Department of Education has been informed about the intended study and permission has already been granted from her office for me to go ahead with this study. I intend to uphold all the acceptable research ethics such as respect and human dignity, transparency and honesty, integrity and academic professionalism during the data collection process. Issues of confidentiality and anonymity will be addressed by using pseudonyms rather than the real names of schools and teachers involved in this study. Moreover, data collected will be used exclusively for the purpose of this study and it will not be published without the consent of the participants. I undertake to make available to you the findings and recommendations from this study.
I look forward to hearing from you soon.

[REDACTED] brought at [REDACTED] High School has been approved/declined.

[REDACTED] High School



Sesanyono Bernard John
0754251861
sesanyono@gmail.com
23/11/2023

To The Principal
[REDACTED] High School

Dear Sir

Re: Permission to conduct research at your school

My name is Sesanyono Bernard John, a PhD student at the University of Cape Town. I intend to conduct a study in mathematics, focusing on geometry teaching through the use of GeoGebra software at the Senior and FET Phase. The title of my study is "Investigating the mediating role of GeoGebra among in-service mathematics teachers' technological pedagogical content knowledge (TPACK) and teaching proficiency using lesson study (LS) as a form of school-based continuous professional development approach (CPE)." Research has shown that the concept of geometry has proved to be problematic at the primary, secondary and tertiary levels. This prompted me to embark upon this study for my PhD as its findings could be used to mitigate the pedagogical challenges encountered by teachers in teaching geometry in schools.

The overarching goal of this study is to investigate the mediating role of GeoGebra between in-service mathematics teachers' TPACK and their teaching proficiency using LS a form of school-based continuous professional development (CPD).

The purpose of this letter is to request your permission to conduct this study at your school with the mathematics department teachers as my participants. The Head of Northern Cape Department of Education has been informed about the intended study and permission has already been granted from her office for me to go ahead with this study. I intend to uphold all the acceptable research ethics such as respect and human dignity, transparency and honesty, integrity and academic professionalism during the data collection process. Issues of confidentiality and anonymity will be addressed by using pseudonyms rather than the real names of schools and teachers involved in the study. Moreover, data collected will be used exclusively for the purpose of this study and it will not be published without the consent of the participants. I undertake to make available to you the findings and recommendations from this study.

Yours,

Our research at [REDACTED] High School has been approved/accepted.

[REDACTED] High School

[REDACTED]

PD 30/11/2023

2023-12-15

[REDACTED]
NORTHERN CAPE PROVINCE

Appendix 6: Informed consent of learners' parents or guardians

Ssennyomo Bernard John
0734221861
ssennyomo@gmail.com
17 October 2023

Dear Parents or Guardians

The mathematics teacher of your child will be participating in a research project to enhance transformation geometry instruction using GeoGebra. GeoGebra is a dynamic tool that provides for active, interactive, and energetic mathematics (transformation geometry) instruction. Given the challenges learners face with transformation geometry, using GeoGebra in class could help your child grasp these concepts better.

I am seeking your permission for your child to be present in the classroom, while we record videos of their mathematics teacher. I will be recording three lessons, with the camera focused on the teacher to capture their teaching style and approaches. I will ensure that the camera angles and recording methods avoid capturing the faces of any learners in the class. In cases where a child's face is visible, we will take steps to digitally obscure it, ensuring anonymity. If obscuring the learner's face is not feasible, please know that any visuals or videos featuring your child will be used solely for collecting and analysing data in this project. The recorded visuals will be securely stored and not included in any publications during the study.

If you have any concerns or questions regarding the information provided, please feel free to do discuss them with me before signing.

I....., as the parent or legal guardian of, hereby consent to Mr Ssennyomo Bernard John video recording the class in which my child is a participant.

Signed:

Date:

Ssennyomo Bernard John

Name of Researcher

Signature of Researcher

Ssenyomo Bernard John
 0734221861
ssennyomo@gmail.com
 17 October 2023

Dear learner

Re: Informed voluntary assent to participate in research study

You are invited to participate in a study focused on transformation geometry teaching, involving senior and FET phase teachers and grade 9 learners at your school. By participating in this study, you will benefit from using GeoGebra to explore transformation geometry concepts, which will deepen your understanding of reflections, translations, enlargements, and reductions. Your identity and name will confidentially be kept, and all study data will be securely stored and protected from unauthorised access. The research requires you to give permission to use the following data – *without revealing your name and identity*:

- Audio-recording your participation in lessons
- Video recording your participation in lessons

This data will be used to gauge if there is any difference between the research lessons from the way the teachers presented them. Please be assured that any visuals or videos of you will only be used for the purpose of this study's analysis. The visuals will neither be utilized in any publications nor posted on social media. I greatly appreciate your involvement in this study. Your participation or declining to participate in this study will not have any influence on your mathematics marks. You have the right to withdraw from the study at any time if you decide to do so.

For any queries or concerns regarding this research project, you can contact me at the details provided below:

Ssenyomo Bernard John: ssennyomo@gmail.com (0734221861)

Having read the above paragraphs, please complete the following:

Item	Assent (✓ tick the appropriate response)	
a) Would you like to take part in a recorded focus group interview?	Yes	No
b) Would you be willing to join a video recorded focus group interview?	Yes	No
e) Do you give permission for the researchers to audio record your participation in lessons?	Yes	No
f) Do you give permission for the researchers to video record your participation in lessons?	Yes	No

_____	_____	_____
Name of learner	Date	Signature
Ssenyomo Bernard John	_____	_____
Name of Researcher	Date	Signature

Appendix 8: Baseline survey on John Taolo Gaetsewe District high school mathematics teachers' knowledge of teaching, technology, and school-based continuous professional development

The baseline survey focuses on the extent of technology use (in particular, GeoGebra) in teaching mathematics and the nature of CPD programmes – a school-based LS CPD approach, in this case, to which teachers in the JTG district are exposed.

Rest assured, participation is optional, and your confidentiality throughout the study will be ensured. Please provide your most accurate response to each question. Your honest feedback will be greatly appreciated.

PART A: BACKGROUND INFORMATION

For each question, select an answer by circling the corresponding option.

Gender

A. Female

B. Male

C. I prefer not to say

2. Which quintile does your school fall into?

In the basic education context of South Africa, government schools are categorised into five quintiles primarily for the allocation of funding (Van Dyk & White, 2019). No fees are charged in Quintile 1-3 schools, which serve low socio-economic communities, whereas Quintile 4-5 schools are fee-charging. The quintile ranking ranges from Quintile 1 indicating the most impoverished schools, to Quintile 5, indicating the most affluent.

A. Quintile 1

B. Quintile 2

C. Quintile 3

D. Quintile 4

E. Quintile 5

F. Other: ...

3. What is your level of teaching experience in this subject area?

A. 0—5 years

B. 6—10 years

C. 11—15 years

D. 16—20 years

E. 21+

4. What is/are your teaching qualification(s) in the subject?

A. ACE certificate

B. Diploma

C. B Ed

D. B Ed (Hons)

E. Masters

G. Other:

PART B: THE USE OF TECHNOLOGY IN THE TEACHING OF MATHEMATICS

The questions here aim to investigate technology use in mathematics instruction in JTG district's high schools.

Technology refers to engaging software tools – GeoGebra in this case – that can be utilised in designing various illustrative mathematics (transformation geometry) examples to facilitate learners' conceptual understanding (Chizwina & Mhakure, 2018).

5. Do you incorporate technology into your mathematics lessons?

A. Yes

B. No

6. What are the advantages of incorporating technology into a mathematics class?

Complete all questions; if you are uncertain, “neither agree nor disagree” is a valid response.

Circle one option for each statement below

Disagree = D, Neither Agree/Disagree = N, Agree = A, Strongly Agree = SA

- With technology, mathematics teachers can create an interactive learning environment that supports the integration of content and pedagogy. D A SA N
 - Technology enables teachers to incorporate mathematics concepts in different ways, promoting flexible learning. D A SA N
 - Effective utilisation of technology enhances mathematics teachers' development of subject matter content and enables teaching proficiency. D A SA N
 - Integrating technology is often time saving. D A SA N
-

7. What are the constraining factors to effective technology implementation in the mathematics classroom?

Circle one option for each statement below

Disagree = D, Neither Agree/Disagree = N, Agree = A, Strongly Agree = SA

- | | | | | |
|--|---|---|----|---|
| ☐ Lack of in-service training | D | A | SA | N |
| ☐ Restricted curriculum. | D | A | SA | N |
| ☐ Fears related to technological difficulties or learners surpassing teachers in technology knowledge. | D | A | SA | N |
| ☐ Demands from high-stakes examinations – e.g. NSC examinations | D | A | SA | N |
-

8. Which of the following mathematical software have you used before and/or are still using in your lessons?

You can choose more than one option in this question.

- ☐ SageMath
- ☐ FreeMat
- ☐ GeoGebra
- ☐ None
- ☐ Other(s): -----

PART C: GEOGEBRA USE IN THE TEACHING OF GEOMETRY

The section intends to gather data on the extent to which JTG district high school mathematics teachers used GeoGebra in teaching geometry.

9. Do you use or have you used GeoGebra during your teaching?

A. Yes

B. No

Questions 10-12 should be answered by teachers who use or have used GeoGebra before or during instruction.

10. What mathematics teaching benefits exist in using GeoGebra in geometry teaching?

Circle one option in each of the statements

Disagree = D, Neither Agree/Disagree = N, Agree = A, Strongly Agree = SA

- It provides an environment for active exploration of geometry concepts through multiple representations thus enhancing understanding. D A SA N
 - It offers a robust platform for mathematics teachers to create effective, content-rich teaching strategies for proficient mathematics teaching. D A SA N
 - GeoGebra's dynamic models inspire mathematics teachers to reflect on and innovate their teaching methods. D A SA N
 - There is more direct social interaction. D A SA N
-

11. What factors prevent you from integrating GeoGebra into the mathematics classroom, if any?

Circle one option for each statement below

Disagree = D, Neither Agree/Disagree = N, Agree = A, Strongly Agree = SA

- The embedded manual requires an internet connection. D A SA N
 - Lack of supportive institutional policies. D A SA N
 - Limited understanding of mathematical concepts and ability to implement them in the classroom. D A SA N
 - Challenges with certain input bar functions. D A SA N
-

12. Apart from the above factors, state any other factors that prevent you from integrating this software into the mathematics classroom.

.....

PART D: TYPES OF PD TAKING PLACE IN SCHOOLS

This section intends to gather data on the types of PD of mathematics teachers currently taking place in JTG District's secondary schools.

13. Which of the following types of PDs have you participated in your school mathematics department?

While school-based PD activities are teacher-led and occur at their schools' classroom contexts, workshops and seminars conducted outside school premises are traditional PD projects taking place outside schools, and are led by third-party specialists that are not familiar with these schools' contexts and their teaching cultures (Mhakure, 2019). These experts include but are not limited to mathematics subject advisors, provincial coordinators, education researchers, and teacher educators from higher learning institutions.

- A. School based PD (lesson study/workshops at school) B. Workshops outside school
C. Seminars at school D. Seminars outside school
E. Other(s):

PART E: Lesson study as a school-based PD

This section intends to gather data on school-based PD of teachers with specific reference to lesson study.

14. Have you participated in lesson study as a school-based PD before?

- A. Yes B. No

Questions 15-17 applies to teachers with prior lesson study participation

15. What are the benefits of being part of lesson study?

Circle one option for each statement below

Disagree = D, Neither Agree/Disagree = N, Agree = A, Strongly Agree = SA

- ☐ It is teacher-led and embedded in the school and classroom context.

D A SA N

- It fosters a collaborative environment where teachers observe, analyse, and discuss teaching practices and learner thinking. D A SA N
- It effectively enhances teachers' subject matter and pedagogical knowledge, while also shifting their perspectives on teaching and learning. D A SA N

16. What additional advantages can be gained from participating in lesson study?

17. What challenges are associated with implementing the lesson study?

A Knowledgeable Other (KO), according to Takahashi and McDougal (2016), is an expert in the field who can share research and mathematics curriculum-based insights, bridge theory and practice, and guide others in reflective teaching and learning.

Circle one option in each of the following statements

Disagree = D, Neither Agree/Disagree = N, Agree = A, Strongly Agree = SA

- Absence of support from beyond lesson study participants. D A SA N
- Uncertainty about the KO's role in the lesson study process. D A SA N
- A lack of understanding of how the school-based PD is supposed to aid teachers deepen their understanding of content and of new pedagogical ideas. D A SA N

Thank you for sharing your insights and taking the time to complete this survey!

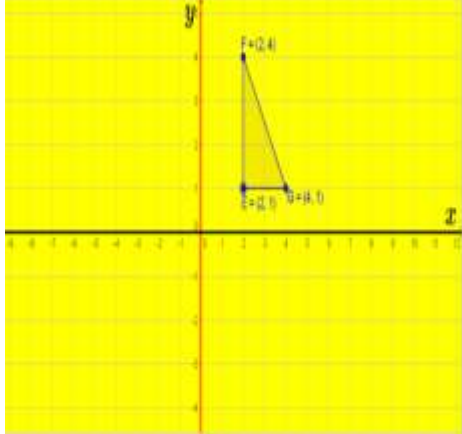
Appendix 9: Reflection guide and FGI after the training

Part A: Reflection guide							
Reflect on the training session by ticking an appropriate box to rate your competence in using GeoGebra for transformation geometry instruction (0 – I still do not understand how to use GeoGebra to teach transformation geometry; 1 – I am now aware of the software, but not able to incorporate it in teaching; 2 – I would like to incorporate GeoGebra into teaching, but I am having difficulty with transformation geometry; 3 – I am beginning to understand how to use the software to teach transformation geometry; 4 – I can actively integrate GeoGebra in transformation geometry instruction; and 5 – I am at an advanced stage in using GeoGebra for transformation geometry instruction.							
Theme	Descriptors in this study	Levels of GeoGebra use					
		0	1	2	3	4	5
Curriculum and Assessment	I acknowledge and understand the curriculum objectives, and how to teach and assess learners' conceptual understanding of these concepts (on reflection, translation and enlargement) using GeoGebra.						
Learning	I can now utilise GeoGebra to facilitate the development of learners' conceptual knowledge of transformation geometry thinking skills.						
Teaching	I can use GeoGebra to create new instructional approaches and a classroom environment that promotes transformation geometry learning.						
Access	I acknowledge GeoGebra's role in the teaching of the high-order transformation geometry concepts and I can identify factors that are barriers to the use of the software to teach these concepts.						
Part B: FGI after training							
As you reflect on your ratings of the items in the reflection guide above, I want you to comment from where you started from regarding the use of GeoGebra and your participation in the training.							
Elaborate on your scoring on each item of the reflection guide with regard to the use of GeoGebra to teach transformation geometry concepts.							

Appendix 10a: RL plan 1 on the transformation geometry concepts of reflection about the x- and y-axis designed by the researcher

Grade	9	Date	
Topic	Transformation geometry	Time	2 hours
Subtopic	Reflection about the x- and y-axis	LSG	
Concepts to be taught	Equidistant objects from the reflection line; change in sign of the y- and x-coordinate after reflection about the x- and y-axis respectively; and congruent shapes.		
Research question	What are the different geometric relationships that can be observed between the object and its image during reflections about the x- and y-axis on the Cartesian plane?		
Success criteria	Upon completing the lesson, learners are expected to: - Identify which coordinate changes sign when reflection is done about either the x- or the y-axis. - Show that the size of the object and its image are congruent. - Show that the corresponding vertices of the object and its image are equidistant from the line of reflection		

Teachers' activities	Learners' activities	Prediction of learners' strategies in reflections	Observations of learning
<u>Introduction</u> <u>Example 1/question 1</u> <u>Solution developmental steps</u> <u>Step 1 – using a worksheet.</u> The teacher presents learners with a problem on a worksheet. The teacher asks learners to: a) draw the position of $\Delta E'F'G'$, the image of ΔEFG after reflection about the x-and y-axis. b) describe what they notice about the sign of the x- and y-coordinate of each point of $\Delta E'F'G'$ after reflecting ΔEFG about the x- and y-axis. c) compare the size of ΔEFG (object) and $\Delta E'F'G'$ (image). d) calculate the distances of the corresponding vertices of ΔEFG and $\Delta E'F'G'$ from each line of reflection and state the relationship. e) discuss their solutions showing the relationship between ΔEFG, mirror, and $\Delta E'F'G'$.	Learners: (a) draw the possible positions of the triangle's images. b) describe which coordinate of ΔEFG's vertices changes in sign after reflection along the x-and y-axis. c) measure the dimensions of the triangle and its image, and compare their sizes (areas). Also use symmetry to relate the size of the triangle to its image. d) measure the distances of the corresponding vertices of the triangle, mirror, and mirror-image and draw conclusions. Also count the units of the corresponding vertices of ΔEFG and $\Delta E'F'G'$ from the line of reflection (e) compare and discuss solutions showing the relationship between the triangle, mirror, and image within their group, other groups, and the whole class.	We predict that the top achiever (learner) will successfully carry out the learning activities in (a), (b), (c), and (d). We predict that the middle achiever (learner) will accurately carry out the learning activities in (a), (b), (d). In activity (b), the medium learner will swap the x-and y-coordinates after reflecting the object about the x- or y-axis. They will also not change the sign of the x- or y-coordinate after reflection about the x- or y-axis. We predict that the low achiever (learner) will struggle to carry out the learning activities in (b), (c), (d) and (e).	The teacher observing the lesson in the classroom should be able to observe whether learners were able to: a) draw the possible positions of the triangle's image. b) describe the coordinate of the points of the vertices of a triangle that changes in sign after reflection about the x- and y-axis. c) describe and compare the size of ΔEFG and $\Delta E'F'G'$. d) measure the distances of the vertices of ΔEFG – mirror, and mirror – the vertices of $\Delta E'F'G'$ and establish a relationship between them. e) discuss their solutions showing the relationship between ΔEFG, mirror and $\Delta E'F'G'$.

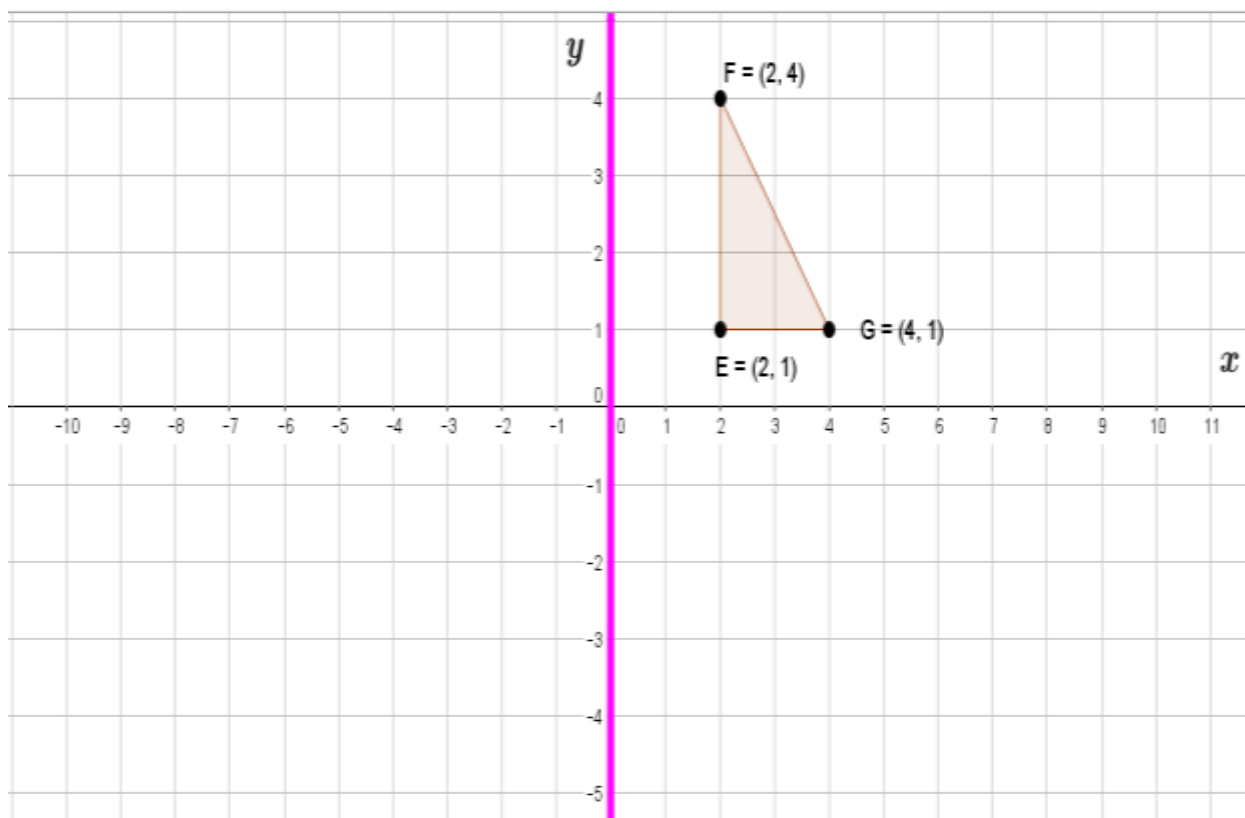
	Teachers and learners interact during GeoGebra implementation on reflections about the x- and y-axis.		
Developmental step Step 2 – Using GeoGebra. Teacher uses GeoGebra to: - illustrate and engage the learners in the above examples. - rectify anticipated and unanticipated misconceptions and thinking errors that emerged from learners' work in step 1 of the lesson's developmental stage.		Top achiever (learner) is not expected to struggle in carrying out and understanding the reflection of $\triangle EFG$ across the x-and y-axis when using GeoGebra. Middle achiever (learner) is also not expected to struggle with the carrying out and understanding the reflection of $\triangle EFG$ across the x- and y-axis using GeoGebra – though s/he might need more time. The low achiever (learner) might not see the direct relationship between the paper presentations and the use of GeoGebra at first but will get by with more interactions. However, the low achievers will struggle with activity (e).	The teacher observing the lesson in the classroom should be able to observe whether learners are able to: a) overcome their misconceptions and thinking errors about how the sizes of $\triangle EFG$ and $\triangle E'F'G'$ and the distances of their corresponding vertices from the mirror are related when GeoGebra is used. b) not swap the x-and y-coordinates after reflection across the x- or y-axis when GeoGebra is used. c) change the sign of the x- or y-coordinates after reflection across the x- or y-axis when GeoGebra is used.
Conclusion 1. Does the size of the object change after reflection? 2. How are the distances of the corresponding vertices of $\triangle EFG$ and $\triangle E'F'G'$ from the line of reflection related? 3. Comment on which coordinate changes in sign when reflection is done along the i) x-axis ii) y-axis.	Learners respond to the teacher's questions	Top achiever is not expected to struggle with answering these questions. Middle achiever is also not expected to struggle with answering these questions. Low achiever might struggle in answering question 3.	Learners being able to respond that: 1. $\triangle EFG$ and $\triangle E'F'G'$ are congruent after reflection. 2. $\triangle EFG$ and $\triangle E'F'G'$ are equidistant from the line of reflection. 3i) the y-coordinate changes in sign after reflection about the x-axis. 3ii) the x-coordinate changes in sign after reflection about the y-axis.

RL – Work sheet 1

QUESTION 1

In the figure below, $\triangle EFG$ with coordinates $E(2; 1)$, $F(2; 4)$ and $G(4; 1)$ is drawn. Reflect $\triangle EFG$ in the

x-and y-axis.

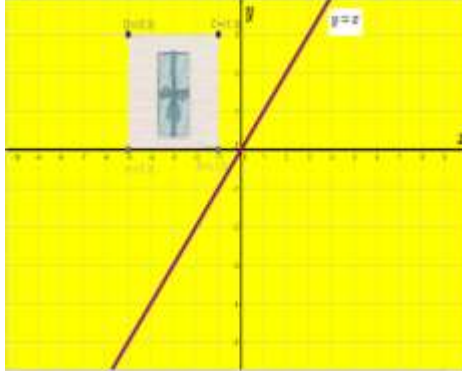


- Draw the position of $\triangle EFG$'s image after reflection about the
 - x-axis ($\triangle E'F'G'$).
 - y-axis ($\triangle E''F''G''$).
- Write down the coordinates of the vertices of each image of $\triangle EFG$ after reflection about the x- and y-axis. Which coordinate of the vertices of $\triangle EFG$ changed in sign after reflection about the
 - x-axis
 - y-axis.
- Compare the size (area) of $\triangle EFG$ (object) and that of its image ($\triangle E'F'G'$)?
- Compare the distance of the corresponding vertices of $\triangle EFG$ and its image ($\triangle E'F'G'$) from each line of reflection.
- Discuss your solutions showing the relationship between the triangle, mirror, and image.

Appendix 10b: RL plan 2 on the transformation geometry concepts of reflection about the line $y = x$ drafted by the researcher

Grade	9	Date	
Topic	Transformation geometry	Time	2 hours
Subtopic	Reflection along the line $y = x$	LSG	
Concepts to be taught	Interchange of y- and x-coordinates after reflection about the line $y = x$, equidistant objects from line $y = x$, and congruent shapes.		
Research questions	What are the different geometric relationships that can be observed between the object and its image during reflections about the line $y = x$ on the Cartesian plane?		
Success criteria	Upon completing the lesson, learners are expected to: - Identify how the x-and y-coordinates are affected by the reflection along the line $y = x$ - Show that the object and its image are of the same size. - Show that the object and its image are equidistant from the line of reflection		

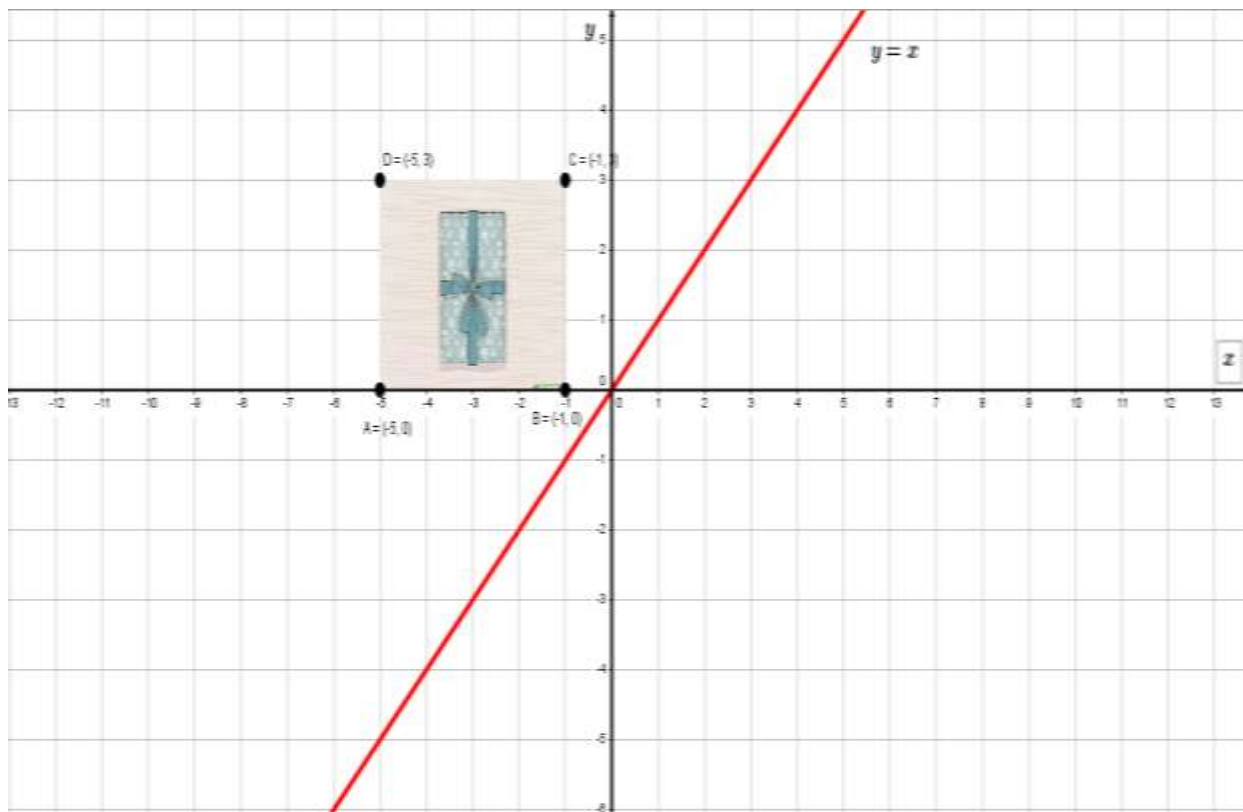
Teachers' activities	Learners' activities	Prediction of learners' strategies in reflections	Observations of learning
<u>Introduction</u> <u>Example 1/question 1</u> <u>Solution developmental steps</u> <u>Step 1 – using a worksheet.</u> 1. The teacher presents learners with a problem on a worksheet showing a picture of Rethabile, a grade 9C learner, who places her birthday gift in front of a mirror. The teacher asks learners to think about the position of the learner's gift (object), its reflection (image) and the mirror. The teacher asks learners to work in groups of two or three to: a) draw the position of the gift's image (rectangle A'B'C' D'). b) describe how the x- and y- coordinates relate to each other after reflecting Rethabile's gift (rectangle ABCD) about the $y = x$. c) compare the size of Rethabile's gift (rectangle ABCD) and its image (rectangle A'B'C' D'). d) calculate the distances of Rethabile's gift (rectangle ABCD) and its image (rectangle A'B'C' D') from the mirror. e) discuss their solutions showing the relationship between Rethabile's gift, mirror, and image.	Learners: (a) draw the possible positions of the gift's image given the gift (object). (b) describe how the x-and y-coordinate relate to each other after a reflection about the line $y = x$. (c) measure the dimensions of the gift and its image then use them to find their respective areas and compare them. Also use symmetry to relate the size of the gift to its image. (d) measure the distances of the vertices of the object – mirror, mirror – image and draw conclusions on the two distances. (e) compare and discuss their solutions showing the relationship between object, mirror, and image within their group, with other groups, and with the whole class.	We predict that the top achiever (learner) will accurately carry out the learning activities in (a)-(e). We predict that the middle achiever (learner) will accurately carry out the learning activities in (a), (b), (d), and (e). We predict that the low achiever (learner) will have conceptual challenges in relation to carrying out the learning activities in (a)-(e).	The teacher observing the lesson in the classroom should be able to observe whether learners were able to: a) draw the possible positions of the gift's image given the gift's picture (object). (b) describe how the x- and y-coordinate relate to each other after a reflection about the line $y = x$. c) compare the sizes of the gift and its image. d) measure the distances of the vertices of the gift – mirror, and mirror – image and draw conclusions on the two distances. e) discuss their solutions showing the relationship between Rethabile's gift, mirror, and image

	Teacher and learners interact during GeoGebra implementation when reflecting the learner's gift (object) across the mirror.		
<u>Developmental step</u> <u>Step 2 – Using GeoGebra.</u> Teacher uses GeoGebra to: • illustrate and engage the learners in the above examples. • rectify anticipated and unanticipated misconceptions and thinking errors that emerged from learners' work in step 1 of the lesson's developmental stage. Hence using the software in teaching reflection along the line $y = x$.		The top learner is not expected to struggle in carrying out and understanding reflection of the learner's gift about the line $y = x$ when GeoGebra is used. The middle achiever is also not expected to struggle with carrying out and understanding reflection of the learner's gift using GeoGebra – though he might need more time. The low achiever might not see the direct relationship between the paper presentations and the use of GeoGebra at first, but will get by with more interaction. However, the low achievers will still struggle with activity (e).	Whether learners were able to overcome their conceptual challenges (misconceptions and thinking errors) about how the x- and y-coordinate relate to each other after a reflection about the line $y = x$, and how the size of the gift (object) and its image and their distance from the mirror are related when GeoGebra was used.
<u>Conclusion</u> 1. Does the size of the object change after reflection? 2. How are the distances of the image and object from the line of reflection related? 3. Comment on how the x- and y-coordinates are affected after reflection about the line $y = x$.	Learners respond to the teacher's questions	The top achiever is not expected to struggle in answering these questions. The middle achiever is also not expected to struggle with answering these questions. The low achiever might struggle to answer question 3.	Learners being able to respond: 1. that the object and its image are congruent after reflection. 2. that the object and its image are equidistant from the line of reflection. 3. the x and y-co-ordinate interchange after reflection along the line $y = x$.

RL 2 – Work sheet 2

QUESTION 1

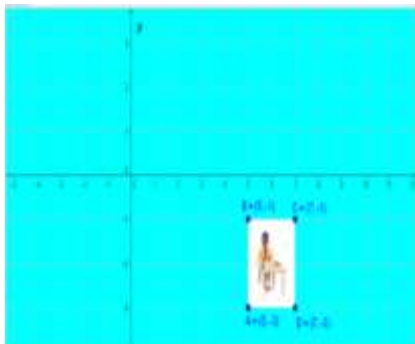
The figure below shows a picture of grade 9 learner Rethabile's birthday gift placed in front of a mirror.



- Draw the position of the image of Rethabile's gift and label its vertices A'B'C'D'.
- Write down the coordinates of the vertices, A'B'C'D', of the image of Rethabile's gift. How are the x-and y-coordinates affected after reflecting Rethabile's gift about the line $y = x$?
- Compare the size (area) of Rethabile's gift (rectangle ABCD) with that of its image (rectangle A'B'C'D')?
- Compare the distances between the corresponding vertices of Rethabile's gift (rectangle ABCD) and those of its image (rectangle A'B'C'D') from the $y = x$.
- Discuss your solutions showing the relationship between Rethabile's gift (rectangle ABCD), mirror (the line $y = x$) and image (rectangle A'B'C'D').

Appendix 10c: RL plan 3 on the transformation geometry concept of translation drafted by the researcher

Grade	9	Date	
Topic	Transformation geometry	Time	2 hours
Subtopic	Translation	LSG	
Concepts to be taught	Equidistant points, and congruent objects		
	What are the different geometric relationships that can be observed between the object and its image when carrying out translations on the Cartesian plane?		
Success criteria	Upon completing the lesson, learners are expected to: - show that a translation produces congruent objects (identical in size and shape) - show that the distances between the coordinates of the corresponding vertices of the object and its image are equal.		

Teachers' activities	Learners' activities	Prediction of learners' strategies in translations	Observations of learning
<u>Introduction</u> <u>Example 1/question 1</u> <u>Solution developmental steps</u> <u>Step 1 – using a worksheet.</u> The teacher presents learners with a problem on a worksheet showing the position of a grade 9 learner who needs to come to the board to present her solution to the classmates. The teacher asks learners to work in groups of two or three to think about the new position of the learner. The teacher asks learners to: - draw the position of the learner's image after sliding the original figure 3 units to the left and 4 units upwards. - compare the size and shape of the learner's picture (the object) and her image. - measure the distances between the corresponding vertices of the object and its image and compare them. - discuss their solutions showing the relationship between the learner's picture and her image. 	Learners: - draw the position of the learner's image after sliding the original figure 3 units to the left and 4 units upwards. - measure the dimensions of the learner's picture and its image then use them to calculate their respective areas to compare their sizes. Also compare the shape of the learner's picture to her image. - measure the distances between the corresponding vertices of the learner's picture and her image and compare them. - compare and discuss their solutions showing the relationship between the learner's picture and her image within their group, other groups, and the whole class.	We predict that the top learner will accurately carry out the learning activities in (a)-(d). We predict that the middle achiever (learner) will also carry out the learning activities in (a) to (d) – though will need more time as compared to the top achiever (learner). We predict that the low achiever (learner) will have challenges in carrying out the learning activity (d).	The teacher observing the lesson in the classroom should be able to observe whether learners were able to: - draw the possible position of the learner's image after the object had been slid horizontally to the left by 3 units, and upwards by 4 units. - compare the sizes and shape of the object and its image. - show that the corresponding vertices of the object and its image are equidistant - discuss their solutions showing the relationship between the learner's picture, and image

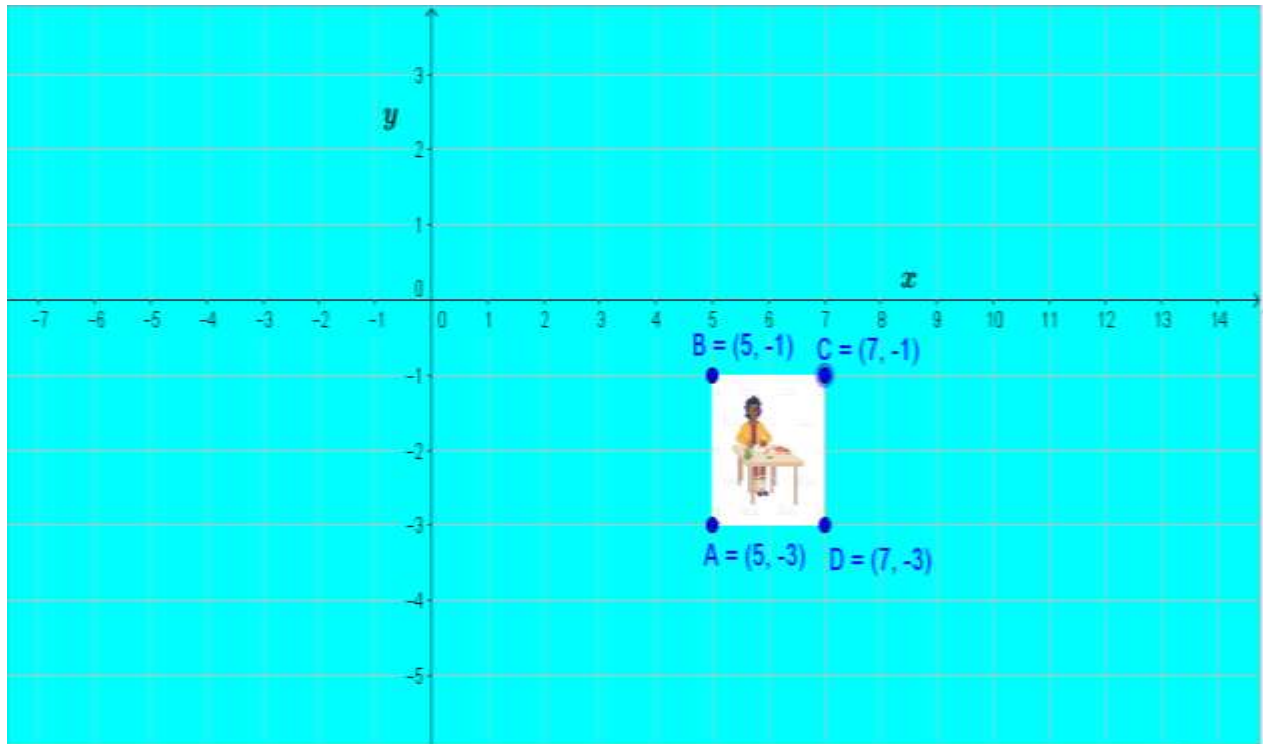
<u>Developmental step</u> <u>Step 2 – Using GeoGebra.</u> Teacher uses GeoGebra to: • illustrate and engage the learners in the above examples. • rectify anticipated and unanticipated misconceptions and thinking errors that emerged from learners’ work in step 1 of the lesson’s developmental stage.	Teachers and learners interact during GeoGebra implementation on translation of the learner’s picture.	Top achiever (learner) is not expected to struggle in carrying out and understanding the translation of the learner’s picture when using GeoGebra. Middle achiever (learner) is also not expected to struggle with carrying out and understanding the translation of the learner’s picture in GeoGebra – though they might need more time. Low achiever (learner) might not see the direct relationship between the paper presentations and the use of GeoGebra at first, but will get by with more interaction.	Whether learners were able to overcome their conceptual challenges regarding the relationship between the size and shape of the learner’s picture and her image, and the lengths of the corresponding vertices of the learner’s picture and her image.
<u>Conclusion</u> 1. Does the size and shape of the object change after translation? 2. Comment on the distances between the coordinates of the corresponding vertices of the object and its image.	Learners respond to the questions posed by the teacher	Top achiever (learner) is not expected to struggle in answering these questions. Middle achiever (learner) is also not expected to struggle with answering these questions. Low achiever (learner) might have difficulties in responding to the second question.	Learners being able to respond: 1. that the object and its image are congruent after translation. 2. the distances between the coordinates of the corresponding vertices of the object and its image are congruent.

□

RL3- Worksheet 3

QUESTION 1

The figure below shows the position of a grade 9 learner who needs to come to the board to present her solution to her classmates. Use it to answer the following questions.



- Draw the position of the learner's image (square A'B'C'D') after sliding the original Figure (square ABCD) 3 units to the left and 4 units upwards.
- Compare the size and shape of the learner's picture (square ABCD) and her image (square A'B'C'D').
- Measure and compare the distances between the corresponding vertices of the object (square ABCD) and its image (square A'B'C'D').
- Discuss your solution showing the relationship between the learner's picture and her image.

Appendix 11a: Observation guide for RL1

Date		Research lesson	1
LSG		Grade	9
Topic	Transformation geometry	Subtopic	Reflection along x-axis, y-axis

For each statement, select Yes (Y) or No (N) to describe your observation of learners' learning of the transformation geometry concepts of reflection about the x-and y-axis. Use the space to elaborate on your choices. You may ask for additional paper if needed.

Lesson stage	Step	Success criteria			
		Upon completing the lesson, learners are expected to: - Identify which coordinate changes sign when reflection is done about either the x- or the y-axis. - Show that the size of the object and its image are congruent. - Show that the object and its image are equidistant from the line of reflection			
<u>Introduction</u> <u>Example1/</u> <u>question 1</u>	<u>One</u> Using a worksheet	1	Were the learners able to identify which coordinate changes sign when reflection is done about either the x- or the y-axis?	Y	N
		Comment on each line of reflection			
		2	Were the learners able to identify that $\triangle EFG$ and its images are of the same size after reflection about the x-and y-axis?	Y	N
			Comment		
		3	Were the learners able to correctly identify that $\triangle EFG$ and its images are equidistant from the x-and y-axis respectively?	Y	N
		Comment on your response?			
		4	Did the low, medium and top achievers (learners) perform as expected?	Y	N
Comment on each category of learner.					

<u>Development</u> <u>Example1/</u> <u>question 1</u>	<u>Two</u> <u>Using</u> <u>GeoGebra</u>	5 Did GeoGebra assist the learners with the visualisation of how the coordinates of the points of the vertices of $\triangle EFG$ change in sign when reflection is done about either the x-or the y-axis? Y N
		You can comment about the learners' misconceptions and/or errors from the exercise - if any?
		6 Did GeoGebra assist the learners with visualising how the size of $\triangle EFG$ and its image are related? Y N
		You can comment about the learners' misconceptions and/or errors from the exercise, if any.
		7 Did GeoGebra assist the learners with visualising how the distance of $\triangle EFG$ and its image from either the x-or the y-axis are related? Y N
		You can comment about the learners' misconceptions and/or errors from the exercise, if any.
		8 Did the low, medium, and top achievers (learners) perform as expected? Y N
		Comment on each category of learner.

Appendix 11b: Observation guide RL2

Date		Research lesson	1
LSG		Grade	9
Topic	Transformation geometry	Subtopic	Reflection along the line $y = x$

For each statement, select Yes (Y) or No (N) to describe your observation of learners' learning of the transformation geometry concepts of reflection about the line $y = x$. Use the space to elaborate on your choices. You may ask for additional paper if needed.

Lesson stage	Step	Success criteria			
		Upon completing the lesson, learners are expected to: - Identify how the x-and y-coordinates are affected by the reflection along the line $y = x$. - Show that the object and its image are of the same size. - Show that the corresponding vertices of the object and its image are equidistant from the line of reflection			
Introduction Example1/ question 1	One Using a worksheet	1	Were the learners able to identify how the x-and y-coordinates are affected when reflection is done about the line $y = x$?	Y	N
		Comment			
		2	Were the learners able to identify that rectangle ABCD and its image, rectangle A'B'C'D', are of the same size (area) after reflection about the line $y = x$?	Y	N
			Comment		
		3	Were the learners able to correctly identify that rectangle ABCD and its image, rectangle A'B'C'D', are equidistant from the line $y = x$?	Y	N
		Comment on your response?			
		4	Did the low, medium and top achievers (learners) perform as expected?	Y	N

		Comment on each category of learner.			
<u>Development</u> <u>Example1/</u> <u>question 1</u>	<u>Two</u> <u>Using</u> <u>GeoGebra</u>	5	Did GeoGebra assist the learners with the visualisation of how the x-and y-coordinates are affected when reflection is done about the line $y = x$?	Y	N
		You can comment about the learners' misconceptions and/or errors from the exercise, if any.			
		6	Did GeoGebra assist the learners with visualising how the size (area) of rectangle ABCD and its image, rectangle A'B'C' D', are related?	Y	N
		You can comment about the learners' misconceptions and/or errors from the exercise, if any.			
		7	Did GeoGebra assist the learners with visualising how the distance of rectangle ABCD and its image, rectangle A'B'C' D', from the line $y = x$ are related?	Y	N
		You can comment about the learners' misconceptions and/or errors from the exercise, if any.			
		8	Did the low, medium, and top achievers (learners) perform as expected?	Y	N
		Comment on each category of learner.			

Appendix 11c: Observation guide for RL3

Date		Research lesson	1
LSG		Grade	9
Topic	Transformation geometry	Subtopic	Translation

For each statement, select Yes (Y) or No (N) to describe your observation of learners' learning of the transformation geometry concept of translation. Use the space to elaborate on your choices. You may ask for additional paper if needed.

Lesson stage	Step	Success criteria			
		On completing the lesson, learners are expected to: - show that a translation produces congruent objects (identical in size and shape) - show that the distances between the coordinates of the corresponding vertices of the object and its image are equal.			
<u>Introduction</u> <u>Example1/</u> <u>question 1</u>	<u>One</u> Using a Worksheet	1	Were the learners able to identify that square ABCD and its image (square A'B'C'D') are of the same size and shape after carrying out the translation?	Y	N
		Comment			
		2	Were the learners able to correctly identify that the distances between the coordinates of the corresponding vertices of the object and its image are equal.	Y	N
		Comment on your response?			
<u>Development</u> <u>Example1/</u>	<u>Two</u> Using GeoGebra	3	Did the low, medium and top achievers (learners) perform as expected?	Y	N
		Comment on each category of learner.			
		4	Did GeoGebra assist the learners with visualising how the size of square ABCD and its image (square A'B'C'D') are related?	Y	N
		You can comment about the learners' misconceptions and/or errors from the exercise - if any?			

<u>question 1</u>				
	5	Did GeoGebra assist the learners with visualising how the distances between the coordinates of the corresponding vertices of the object (square ABCD) and its image (square A'B'C'D') are related?	Y	N
	You can comment about the learners' misconceptions and/or errors from the exercise, if any.			
	6	Did the low, medium, and top achievers (learners) perform as expected?	Y	N
	Comment on each category of learner.			

Appendix 12a: Reflection and interview guides for RL1

When answering these questions, please refer to the RL – identification of objectives, planning, and implementation/observation – in which you participated.

Part A: Reflection guide							
Reflect on the RL by ticking an appropriate box to rate your competence in teaching transformation geometry concepts of reflection about the x- and y-axis using GeoGebra (0 – I still do not understand how to use GeoGebra to teach the transformation geometry concepts of reflection about the x- and y-axis; 1 – I am now aware of GeoGebra, but not able to incorporate it in teaching the transformation geometry concepts of reflection about the x- and y-axis; 2 – I would like to incorporate GeoGebra into teaching, but I am having difficulty with the transformation geometry concepts of reflection about the x- and y-axis; 3 – I am beginning to understand how to use GeoGebra to teach the transformation geometry concepts of reflection about the x- and y-axis; 4 – I can actively integrate GeoGebra in teaching the transformation geometry concept of reflection about the x- and y-axis; and 5 – I am at an advanced stage in using GeoGebra to teach the transformation geometry concept of reflection about the x- and y-axis.							
Formative assessment is a type of assessment which takes place during teaching.							
Theme	Descriptors in this study	levels of GeoGebra use					
		0	1	2	3	4	5
Curriculum and assessment	I acknowledge and understand the curriculum objectives and how to teach and assess learners' conceptual understanding of the transformation geometry concepts of reflection about the x- and y-axis using GeoGebra.						
Learning	I can now use GeoGebra to facilitate the development of learners' conceptual knowledge of transformation geometry thinking skills, particularly reflection about the x- and y- axis.						
Teaching	I can use GeoGebra to create new instructional approaches and a classroom environment that promotes the learning of the transformation geometry concept of reflection about the x- and y-axis.						
Access	I acknowledge GeoGebra's role in the teaching of the high-order transformation geometry concepts of reflection about the x- and y-axis and I can identify factors that are barriers to the use of the software to teach these concepts.						
Part B: Focus group interview guide							
As you reflect on your ratings of the items in the reflection guide above, I want you to draw from the stages of the RL: (a) identification of objectives, (b) planning, (c) implementation (observation) that you carried out.							
1. Elaborate on your scoring on each item of the reflection guide.							
2. Reflect on the RL that focused on the transformation geometry concepts of reflection about the x- and y-axis using GeoGebra from (i) identification of objective, (ii) planning, and (iii) implementation/observation: (a) What would you do differently (if anything) in the next lesson to improve on the discussion of objectives? (b) What would you do differently (if anything) in the next lesson to improve on the planning? (c) What would you do differently (if anything) in the next lesson to improve on the implementation? (d) Reflecting on your observations about learner learning (about achieving the success criteria) – do you think this lesson should be re-taught?							
Thanks for contributing to today's activity.							

Appendix 12b: Reflection and interview guide for RL2

When answering these questions, please refer to the RL – identification of objectives, planning, and implementation/observation – which you participated in.

Part A: Reflection guide							
Reflect on the research lesson by ticking an appropriate box to rate your competence in teaching the transformation geometry concept of reflection about the line $y = x$ using GeoGebra (0 – I still do not understand how to use GeoGebra to teach the transformation geometry concept of reflection about the line $y = x$; 1 – I am now aware of GeoGebra, but not able to incorporate it in teaching the transformation geometry concept of reflection about the line $y = x$; 2 – I would like to incorporate GeoGebra into teaching, but I have challenges of using it to teach the transformation geometry concept of reflection about the line $y = x$; 3 – I am beginning to understand how to use GeoGebra to teach the transformation geometry concept of reflection about the line $y = x$; 4 – I can actively integrate GeoGebra in teaching the transformation geometry concept of reflection about the line $y = x$; and 5 – I am at an advanced stage in using GeoGebra to teach the transformation geometry concept of reflection about the line $y = x$).							
Formative assessment is a type of assessment which takes place during teaching.							
Theme	Descriptors in this study	levels of GeoGebra use					
		0	1	2	3	4	5
Curriculum and assessment	I acknowledge and understand the curriculum objectives and how to teach and assess learners' conceptual understanding of the transformation geometry concept of reflection about the line $y = x$ using GeoGebra.						
Learning	I can now use GeoGebra to facilitate the development of learners' conceptual knowledge of transformation geometry thinking skills, particularly reflections about the line $y = x$.						
Teaching	I can use GeoGebra to create new instructional approaches and a classroom environment that promotes the learning of the transformation geometry concept of reflection about the line $y = x$.						
Access	I acknowledge GeoGebra's role in the teaching of the high-order transformation geometry concepts of reflection about the line $y = x$ and I can identify factors that are barriers to the use of the software to teach these concepts.						
Part B: Focus group interview guide							
As you reflect on your ratings of the items in the reflection guide above, I want you to draw from the stages of the research lesson: (a) identification of objectives, (b) planning, (c) implementation (observation) that you carried out.							
1. Elaborate on your scoring on each item of the reflection guide.							
2. Reflect on the RL that focused on the transformation geometry concepts of reflection about the line $y = x$ using GeoGebra from (i) identification of objective, (ii) planning, and (iii) implementation/observation: (a) What would you do differently (if anything) in the next lesson to improve on the discussion of objectives? (b) What would you do differently (if anything) in the next lesson to improve on the planning? (c) What would you do differently (if anything) in the next lesson to improve on the implementation? (d) Reflecting on your observations about learner learning (about achieving the success criteria) – do you think this lesson should be re-taught?							
Thanks for contributing to today's activity.							

Appendix 12c: Reflection and interview guide for RL3

When answering these questions, please refer to the RL – identification of objectives, planning, and implementation/observation – in which you participated.

Part A: Reflection guide							
Reflect on the research lesson by ticking an appropriate box to rate your competence in teaching the transformation geometry concept of translation using GeoGebra (0 – I still do not understand how to use GeoGebra to teach transformation geometry concept of translation; 1 – I am now aware of GeoGebra, but not able to incorporate it in teaching the transformation geometry concept of translation; 2 – I would like to incorporate GeoGebra into teaching, but I have challenges of using it to teach the transformation geometry concept of translation; 3 – I am beginning to understand how to use GeoGebra to teach transformation geometry concept of translation; 4 – I can actively integrate GeoGebra in teaching transformation geometry concept of translation; and 5 – I am at an advanced stage in using GeoGebra to teach the transformation geometry concept of translation.							
Formative assessment is a type of assessment which takes place during teaching.							
Theme	Descriptors in this study	levels of GeoGebra use					
		0	1	2	3	4	5
Curriculum and Assessment	I acknowledge and understand the curriculum objectives and how to teach and assess learners' conceptual understanding of the transformation geometry concept of translation using GeoGebra.						
Learning	I can now use GeoGebra to facilitate the development of learners' conceptual knowledge of the transformation geometry concept of translation.						
Teaching	I can use GeoGebra to create new instructional approaches and a classroom environment that promotes the learning of the transformation geometry concept of translation.						
Access	I acknowledge GeoGebra's role in the teaching of the high-order geometric translation concepts and I can identify factors that are barriers to the use of the software to teach these concepts.						
Part B: Focus group interview guide							
As you reflect on your ratings of the items in the reflection guide above, I want you to draw from the stages of the research lesson: (a) identification of objectives, (b) planning, (c) implementation (observation) that you carried out.							
1. Elaborate on your scoring on each item of the reflection guide.							
2. Reflect on the RL that focused on the transformation geometry concept of translation using GeoGebra from (i) identification of objective, (ii) planning, and (iii) implementation/observation: (b) What would you do differently (if anything) in the next lesson to improve on the planning? (c) What would you do differently (if anything) in the next lesson to improve on the implementation? (d) Reflecting on your observations about learner learning (about achieving the success criteria) –do you think this lesson should be re-taught?							
Thanks for contributing to today's activity.							

Appendix 12: Certificate of editing

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24 July 2025

To whom it may concern:

EDITING/PROOFREADING OF THESIS

I declare that during July 2025 I language-edited a thesis by Ssenyomo Bernard John titled 'Investigating rural high school mathematics teachers' technological pedagogical content knowledge development using GeoGebra software through a lesson study approach', for submission to the University of Cape Town Department of Mathematics and Applied Mathematics in fulfilment of the requirements for the degree of Doctor of Philosophy.

Yours faithfully

