



**OCTET AND DECUPLET BARYON MASSES
AT FINITE TEMPERATURE**

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ABSTRACT

Octet and decuplet baryon masses are studied at finite temperature in the framework of finite energy QCD sum rules. In all cases it is found that the mass stays fairly constant over a range of temperatures, increasing sharply near the critical temperature for deconfinement T_c .

CONTENTS

1.	Introduction	1
2.	Theoretical preliminaries.....	6
3.	Finite temperature spectral functions in the QCD sector.....	13
3.1	Octet	13
3.1.1	Nucleon	13
	(a) Annihilation region - perturbative diagram	13
	(b) Annihilation region - non-perturbative diagram	29
	(c) Scattering region - perturbative diagram	33
	(d) Scattering region - non-perturbative	36
3.1.2	Sigma.....	39
3.1.3	Lambda	42
3.1.4	Cascade.....	43
3.2	Decuplet.....	44
3.1.1	Omega	44
	(a) Annihilation region - perturbative diagram	44
	(b) Annihilation region - non-perturbative	50
	(c) Scattering region - perturbative diagram	52
	(d) Scattering region - non-perturbative diagram	54
3.1.2	Delta	56
3.1.3	Sigma*	56
3.1.4	Cascade*	57
4.	Hadronic contributions to the spectral functions	58
5.	Applying the Finite Energy Sum Rules	62
6.	Results	65

APPENDICES

- A. Wick's theorem
- B. Current commutators
- C. Dirac algebra
- D. Hadronic spectral functions

1. INTRODUCTION

Since the work by Shifman, Vainshtain and Zakharov [1], the QCD sum rule method, based on the operator product expansion of current correlators and on analyticity properties of spectral functions, has been extensively used as a systematic tool to study various resonance properties based on QCD. At present it stands as a powerful and successful framework in which to analyse the hadronic spectrum, coupling constants, form factors, etc. (see [2] for a review). In particular, the masses of octet and decuplet baryons have been well-reproduced in terms of the vacuum condensates [3-6].

In recent years, modification of hadronic properties at finite temperature has been receiving greater attention in connection with the attempted production of a quark-gluon plasma in relativistic heavy ion collisions. Due to the success of the sum rule formalism, it thus seemed worthwhile to try to extend its range of applications to the finite-temperature domain [7-9]. Attempts in this direction include the phase transitions to colour deconfinement and chiral symmetry restoration, as well as the temperature behaviour of the particle spectrum. Although some criticism [10] has been levelled against the extension [7] of the sum rule program to non-zero temperature, convincing evidence in support of the extension can be found from QCD, as well as from the sigma and Schwinger models [11].

Two phase transitions are required in order for the standard model to be consistent with known facts. At low temperatures colour is confined and chiral symmetry is spontaneously broken. At high temperatures, on the other hand, asymptotic freedom predicts that the interaction between quarks and gluons is weak, such that colour is liberated and chiral symmetry is

restored. Thus the restoration of chiral symmetry and the liberation of colour occur in either one or two phase transitions from the low-temperature hadronic phase to the high temperature quark-gluon-plasma phase. The problem is clearly a difficult one, since even at $T = 0$ the confinement phenomenon has not yet been explained.

The crucial step is to define a meaningful order parameter associated with the particular phase transition. In the case of chiral symmetry (say $N_f = 2$), the order parameter is obviously the quark condensate $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle \equiv \langle \bar{q}q \rangle$, or equivalently $f_\pi(T)$, which are expected to become zero for T larger than the critical chiral temperature (T_c). An expression for $f_\pi(T)$ valid over the whole range of temperatures up to T_c has been derived in the finite- T composite operator formalism [12].

For colour deconfinement, authors working with finite-energy QCD sum rules (FESR) [7,9] have suggested a characterisation in terms of the asymptotic freedom threshold $s_0(T)$, which is the effective threshold separating the low energy resonance region from the continuum region (described by perturbative QCD) in the typical hadronic spectral function. For increasing temperature one intuitively expects resonances to become wider, and the more massive ones progressively to become part of the continuum, causing $s_0(T)$ to decrease as the temperature increases. The conjecture is that deconfinement occurs at a temperature T_d such that $s_0(T_d) = 0$. This is illustrated in Figure 1.1 as obtained from the dimension $d = 2$ FESR in the axial-vector channel [9,13], using the composite operator formalism for $f_\pi(T)$ [14]. Here $T/T_c = 1$ is the temperature at which $f_\pi(T) = 0$.

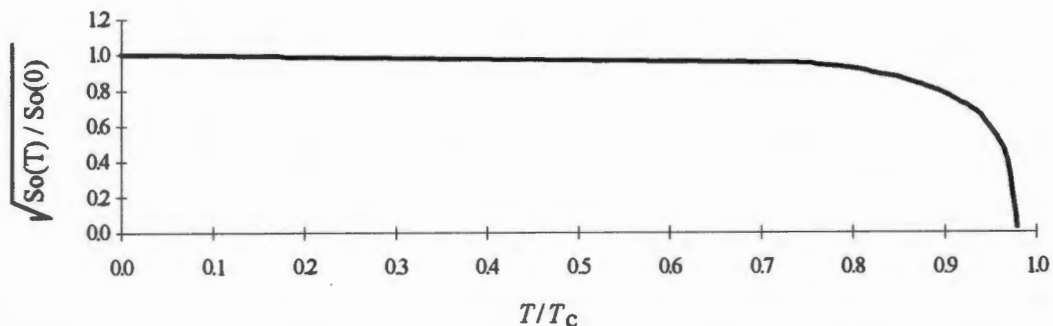


FIGURE 1.1 : Temperature dependence of the asymptotic freedom threshold

Equivalently, one can take as an order parameter the hadronic width of a particle [9], since as the hadrons melt with increasing temperature, their hadronic widths should increase such that at the critical temperature for deconfinement (T_d), the hadronic spectral function becomes totally smooth and is described entirely by the quark degrees of freedom.

So far, however, it has not been possible to conclusively decide on the phase structure, even though it has been shown that the critical temperatures for chiral symmetry restoration and colour deconfinement occur nearly or even exactly at the same temperature [14]. Hence in Figure 1.1, although $s_0(T)$ vanishes at a slightly smaller temperature than T_c , one can safely assume that deconfinement and chiral symmetry restoration take place at a common temperature $T = T_c = T_d$. If however T_c and T_d are not coincident, one would intuitively expect deconfinement to occur before chiral restoration, with an intermediate phase of deconfined quarks and gluons but chirality still broken.

As said above, the other area of application of finite- T QCD sum rules is the temperature behaviour of the particle spectrum, which is also the topic of this thesis. Although the finite- T properties of various mesons have been extensively studied in this framework [7,15,16,17], baryon properties have not been seriously investigated except for the nucleon [18,19,20].

The temperature behaviour of the nucleon propagator has been studied recently by Dominguez and Loewe [20] in the framework of FESR. At $T = 0$ the nucleon mass $M_N \propto \langle \bar{q}q \rangle^{1/3}$ [2], and thus a naive extrapolation of QCD sum rules to $T \neq 0$ would imply a vanishing of $M_N(T)$ at the critical temperature for chiral-symmetry restoration. However, as pointed out in [7], the $T \neq 0$ spectral functions receive additional contributions from scattering of quarks and pions off the currents.

The authors of [20] showed that these contributions (absent at $T = 0$), together with T -dependent annihilation contributions, conspire to prevent the nucleon mass from vanishing. It was found that $M_N(T)$ remains approximately constant in a wide range of temperatures, and then it increases as the critical temperature is approached. Correspondingly, the coupling of nucleons to quarks decreases as the critical temperature is approached.

This confirmed the earlier results obtained by Leutwyler and Smilga [18] in the framework of chiral perturbation theory and the virial expansion. In addition to the above-mentioned behaviour of the mass and coupling, they found that, similarly to what happens with the pion, the nucleon develops a (hadronic) width which grows with increasing temperature. (In referring to a particle's mass and width, one understands the real and imaginary parts of the pole position, respectively, while the coupling to quarks corresponds to the residue of the pole). This lends support to the above-mentioned

suggestion for the hadronic width of a particle as the candidate phenomenological order parameter characterising quark deconfinement.

In this thesis, as an extension to the work of Dominguez and Loewe [20], all the correlators of the octet (N , Σ , Λ , Ξ) and decuplet (Δ , Σ^* , Ξ^* , Ω) baryons are studied at finite temperature in the framework of finite energy QCD sum rules.

2. THEORETICAL PRELIMINARIES

As pointed out by Dominguez and Loewe [13], a naive extension of the QCD sum rules program to $T \neq 0$ faces some problems if carried beyond the lowest dimensional FESR. The reason is that dimension $d = 4$ FESR in the vector and axial-vector channel predict a temperature behaviour of the gluon condensate in strong disagreement with expectations. This invalidates Laplace transform QCD sum rules, which are equivalent to an infinite set of FESR of increasing dimensionality.

However, dimension $d = 2$ FESR are free from problems; they predict a T -dependence of the asymptotic freedom threshold $s_0(T)$ in accordance with expectations. As mentioned in the introduction, this threshold corresponds to the energy squared beyond which resonance peaks in the hadronic spectral function merge into the continuum described by perturbative QCD.

Since it is only retarded (or advanced) Green functions that are analytic at $T \neq 0$ and have spectral representations, the standard quantity for studying the finite- T behaviour of baryons is the retarded two-point correlation function

$$\Pi(\omega, \mathbf{p}, T) = i \int d^4x \exp(i \mathbf{p} \cdot \mathbf{x}) \theta(x_0) \langle\langle \{\eta(\mathbf{x}), \bar{\eta}(0)\} \rangle\rangle \quad \dots (2.1)$$

where η is an external current with the given quantum numbers, and

$$\langle\langle A \cdot B \rangle\rangle = \frac{\sum_n \exp(-E_n/T) \langle n | A \cdot B | n \rangle}{\text{Tr}(\exp(-H/T))} \quad \dots (2.2)$$

denotes the Gibbs average (zero chemical potential, non-zero temperature), and $|n\rangle$ is a complete set of eigenstates of the Hamiltonian H . Also $\rho \equiv (\omega, \rho)$, but due to Lorentz invariance breaking at $T \neq 0$, Π depends on the two variables ω and ρ separately.

The standard choice of currents for the $L = 0$ octet and decuplet baryons that will be investigated, is given by (see [2] p.62) :

$$\begin{aligned}
 \eta^N(x) &= \varepsilon_{abc} (u^a(x) C \gamma^\mu u^b(x)) \gamma_5 \gamma_\mu d^c(x) \\
 \eta^\Sigma(x) &= \varepsilon_{abc} (u^a(x) C \gamma^\mu u^b(x)) \gamma_5 \gamma_\mu s^c(x) \\
 \eta^\Lambda(x) &= \sqrt{\frac{2}{3}} \varepsilon_{abc} \left\{ (u^a(x) C \gamma^\mu s^b(x)) \gamma_5 \gamma_\mu d^c(x) - (d^a(x) C \gamma^\mu s^b(x)) \gamma_5 \gamma_\mu u^c(x) \right\} \\
 \eta^\Xi(x) &= - \varepsilon_{abc} (s^a(x) C \gamma^\mu s^b(x)) \gamma_5 \gamma_\mu u^c(x)
 \end{aligned}$$

... (2.3)

for the $J^P = \frac{1}{2}^+$ octet, and

$$\begin{aligned}
 \eta_\mu^\Delta(x) &= \varepsilon_{abc} (u^a(x) C \gamma^\mu u^b(x)) u^c(x) \\
 \eta_\mu^{\Sigma^*}(x) &= \sqrt{\frac{1}{3}} \varepsilon_{abc} \left\{ 2(u^a(x) C \gamma^\mu s^b(x)) u^c(x) + (u^a(x) C \gamma^\mu u^b(x)) s^c(x) \right\} \\
 \eta_\mu^{\Xi^*}(x) &= \sqrt{\frac{1}{3}} \varepsilon_{abc} \left\{ 2(s^a(x) C \gamma^\mu u^b(x)) s^c(x) + (s^a(x) C \gamma^\mu s^b(x)) u^c(x) \right\} \\
 \eta_\mu^\Omega(x) &= \varepsilon_{abc} (s^a(x) C \gamma^\mu s^b(x)) s^c(x)
 \end{aligned}$$

... (2.4)

for the $J^P = \frac{3}{2}^+$ decuplet.

Here C is the charge conjugation matrix, and (a, b, c) are colour indices.

In the case of the octet, the two-point function (2.1) can be decomposed into

$$\Pi(\omega, \mathbf{p}, T) = \Pi_1(\omega, \mathbf{p}, T) + \not{\mathbf{p}}\Pi_2(\omega, \mathbf{p}, T) \quad \dots (2.5)$$

The spectral functions $\text{Im } \Pi_1$ and $\text{Im } \Pi_2$ can be obtained from Π by using suitable projections :

$$\text{Im } \Pi_1 = \frac{1}{4} \text{Tr} [\text{Im } \Pi] \quad \dots (2.6)$$

$$\text{Im } \Pi_2 = \frac{1}{4\omega} \text{Tr} [\gamma_0 \text{Im } \Pi]$$

Although the decuplet currents have a slightly different form than those of the octet, QCD expressions for the spectral functions can be obtained in the same manner as for the octet currents.

For the decuplet, one extra complication arises because of the tensor nature of the currents, which allows the two-point function $\Pi_{\mu\nu}$ to be decomposed into

$$\begin{aligned} \Pi_{\mu\nu} &= a_1 g_{\mu\nu} \not{\mathbf{p}} + a_2 \gamma_\mu \gamma_\nu \not{\mathbf{p}} + a_3 (\gamma_\mu p_\nu - p_\mu \gamma_\nu) + a_4 \frac{p_\mu p_\nu}{p^2} \not{\mathbf{p}} + a_5 (\gamma_\mu p_\nu + p_\mu \gamma_\nu) \\ &\quad + b_1 g_{\mu\nu} + b_2 \gamma_\mu \gamma_\nu + b_3 \frac{(\gamma_\mu p_\nu - p_\mu \gamma_\nu)}{p^2} \not{\mathbf{p}} + b_4 \frac{p_\mu p_\nu}{p^2} + b_5 \frac{(\gamma_\mu p_\nu + p_\mu \gamma_\nu)}{p^2} \not{\mathbf{p}} \\ &\equiv g_{\mu\nu} \Pi_1(p^2) + g_{\mu\nu} \not{\mathbf{p}} \Pi_2(p^2) + \dots \end{aligned} \quad \dots (2.7)$$

from which it is clear that $\Pi_1 \equiv b_1$ and $\Pi_2 \equiv a_1$. The coefficients a_i and b_i can be obtained by using suitable projections. In the case of the a_i the five projections required are

$$\begin{aligned} \text{Tr} \{ g^{\mu\nu} p \Pi_{\mu\nu} \} &= p^2 (16a_1 + 16a_2 + 4a_4 + 8a_5) \equiv p^2 A \\ \text{Tr} \{ \gamma^\mu \gamma^\nu p \Pi_{\mu\nu} \} &= p^2 (16a_1 + 16a_2 + 24a_3 + 4a_4 + 8a_5) \equiv p^2 B \\ \text{Tr} \{ p^\mu p^\nu p \Pi_{\mu\nu} \} &= p^4 (4a_1 + 4a_2 + 4a_4 + 8a_5) \equiv p^4 C \\ \text{Tr} \{ \gamma^\mu p^\nu \Pi_{\mu\nu} \} &= p^2 (4a_1 + 16a_2 + 12a_3 + 4a_4 + 20a_5) \equiv p^2 D \\ \text{Tr} \{ \gamma^\nu p^\mu \Pi_{\mu\nu} \} &= p^2 (4a_1 - 8a_2 - 12a_3 + 4a_4 + 20a_5) \equiv p^2 E \end{aligned} \quad \dots (2.8)$$

Solving for the a_i 's one finds

$$\begin{aligned} a_1 &= \frac{1}{24} (A + B - 2C - D + E) \\ a_2 &= \frac{1}{24} (A - B + D - E) \\ a_3 &= \frac{1}{24} (B - A) \\ a_4 &= \frac{1}{24} (-2A + 12C - 2D - 2E) \\ a_5 &= \frac{1}{24} (-2C + D + E) \end{aligned} \quad \dots (2.9)$$

Similarly, one can obtain the b_i by projecting

$$\begin{aligned}
 \text{Tr} \{g^{\mu\nu} \Pi_{\mu\nu}\} &= (16b_1 + 16b_2 + 4b_4 + 8b_5) \equiv V \\
 \text{Tr} \{\gamma^\mu \gamma^\nu \Pi_{\mu\nu}\} &= (16b_1 - 32b_2 - 24b_3 + 4b_4 + 8b_5) \equiv W \\
 \text{Tr} \{p^\mu p^\nu \Pi_{\mu\nu}\} &= p^2 (4b_1 + 4b_2 + 4b_4 + 8b_5) \equiv p^2 X \\
 \text{Tr} \{\gamma^\mu p^\nu \not{p} \Pi_{\mu\nu}\} &= p^2 (4b_1 - 8b_2 - 12b_3 + 4b_4 - 4b_5) \equiv p^2 Y \\
 \text{Tr} \{\gamma^\nu p^\mu \not{p} \Pi_{\mu\nu}\} &= p^2 (4b_1 + 16b_2 + 12b_3 + 4b_4 - 4b_5) \equiv p^2 Z \quad \dots (2.10)
 \end{aligned}$$

Solving for the b_i 's one finds

$$\begin{aligned}
 b_1 &= \frac{1}{24}(V + W - 2X - Y + Z) \\
 b_2 &= \frac{1}{24}(V - W + Y - Z) \\
 b_3 &= \frac{1}{24}(-V + W - 2Y + 2Z) \\
 b_4 &= \frac{1}{24}(-2V + 4X + 2Y + 2Z) \\
 b_5 &= \frac{1}{24}(2X - Y - Z) \quad \dots (2.11)
 \end{aligned}$$

In the following sections, each current in the octet and the decuplet will be investigated separately. Of course, since the currents of the octet and of the decuplet are structurally equivalent, calculations concerning the different

currents are also similar to a certain extent. To avoid repetitiveness, detailed calculations are shown for one current from each group only, the calculations of the remaining currents being discussed only insofar they differ from the original ones.

In each case, one starts by considering the leading order contribution to the spectral functions $\text{Im } \Pi_{1,2}$ at the quark level. In Figure 2.1 the relevant diagrams for the time-like ($p^2 = \omega^2 - \mathbf{p}^2 > 0$) conversion processes in the annihilation region is shown. Figure 2.1a corresponds to the purely perturbative diagram where the current is briefly annihilated into its constituent quark components, whereas Figure 2.1b corresponds to the non-perturbative case of a quark-antiquark pair condensing into the vacuum. The processes are first considered at $T = 0$ (mainly in order to verify their correctness), and then the calculations are extended to include their temperature dependence.

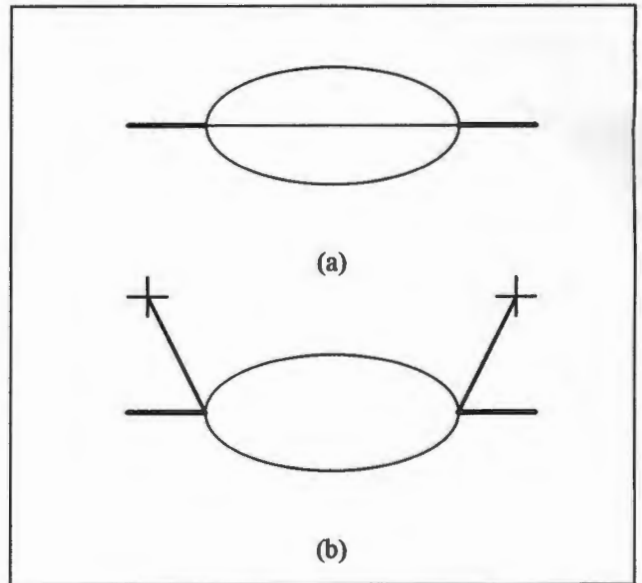


FIGURE 2.1 : Leading order contributions to the QCD spectral functions $\text{Im } \Pi_2$ (a) and $\text{Im } \Pi_1$ (b) in the time-like region $p^2 > 0$.

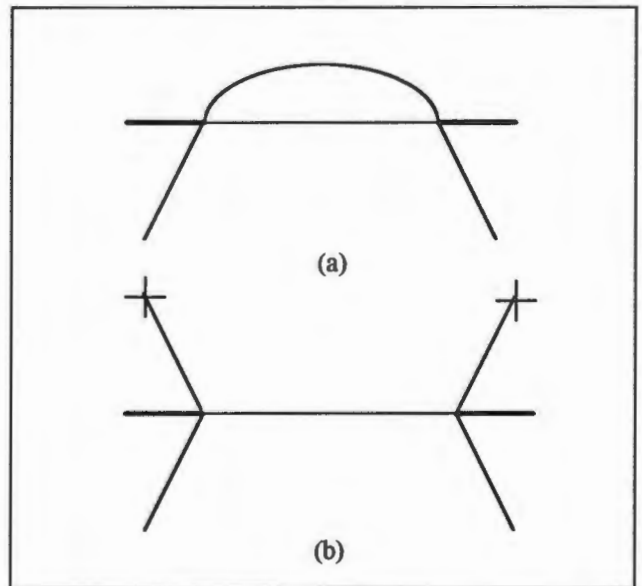


FIGURE 2.2: Leading order contributions to the QCD spectral functions $\text{Im } \Pi_2$ (a) and $\text{Im } \Pi_1$ (b) in the space-like region $p^2 < 0$.

As mentioned in the introduction, there is at $T \neq 0$ the additional possibility of space-like ($p^2 = \omega^2 - \mathbf{p}^2 < 0$) reactions, corresponding to the scattering of the current off (anti-) quarks present in the medium. The leading order diagrams contributing to $\text{Im} \Pi_{1,2}$ are shown in Figure 2.2.

For simplicity, each particle will be considered in its rest frame ($\mathbf{p} = 0$). Also, as a first approximation, the chiral limit ($m_q = 0$) is understood. Corrections of order m_s will be considered afterwards as an improved second approximation.

3. FINITE TEMPERATURE SPECTRAL FUNCTIONS IN THE QCD SECTOR

3.1 OCTET ($J^P = 1/2^+$)

The octet currents are given in (2.3). The nucleon will be discussed first and its spectral function $\text{Im } \Pi^N$ calculated in detail, setting the example of how to obtain similar results for the remaining currents, which will just be quoted.

3.1.1 Nucleon spectral function $\text{Im } \Pi^N$

The nucleon current η^N is given in (2.3) as

$$\begin{aligned}\eta^N &= \varepsilon_{abc} (u^a C \gamma_\mu u^b) \gamma^5 \gamma^\mu d^c \\ &\equiv \varepsilon_{abc} (u^a \Gamma u^b) \Gamma' d^c\end{aligned}\quad \dots (3.1)$$

For the adjoint current one finds

$$\begin{aligned}\bar{\eta}^N &= \varepsilon_{abc} \bar{d}^c \bar{\Gamma}' (\bar{u}^b \bar{\Gamma} \bar{u}^a) \\ &= -\varepsilon_{abc} \bar{d}^c \gamma^\nu \gamma^5 (\bar{u}^b \gamma_\nu C \bar{u}^a)\end{aligned}\quad \dots (3.2)$$

since by definition, for any product of gamma matrices Γ , its adjoint is given by

$$\bar{\Gamma} = \gamma^0 \Gamma^\dagger \gamma^0 \quad \dots (3.3)$$

which leads to

$$\overline{C\gamma_\mu} = \gamma_\mu C \quad \text{and} \quad \overline{\gamma^5\gamma^\mu} = -\gamma^\mu\gamma^5 \quad \dots (3.4)$$

using the standard properties of gamma matrices and the charge conjugation operator C (Appendix C). Keeping in mind (2.1), one thus finds

$$\begin{aligned} \eta^N(x)\bar{\eta}^N(0) &= -\varepsilon_{abc}\varepsilon_{a'b'c'} [(u^a C\gamma_\mu u^b)\gamma_5\gamma^\mu d^c]_x \\ &\quad \cdot [\bar{d}^{c'}\gamma^\nu\gamma_5(\bar{u}^{b'}\gamma_\nu C\bar{u}^{a'})]_0 \\ &= -\varepsilon_{abc}\varepsilon_{a'b'c'} [u_i^a(x)C_{ij}(\gamma_\mu)_{jk} u_k^b(x)(\gamma_5)_{lm}(\gamma^\mu)_{mn} d_n^c(x)] \\ &\quad \cdot [\bar{d}_{n'}^{c'}(0)(\gamma^\nu)_{n'm'}(\gamma_5)_{m'l'}\bar{u}_{i'}^{b'}(0)(\gamma_\nu)_{i'j'}C_{j'k'}\bar{u}_{k'}^{a'}(0)] \end{aligned} \quad \dots (3.5)$$

Consider the $T=0$ case first. Applying Wick's theorem (A17), it follows that only the terms with all fields contracted contributes, since the vacuum expectation value of a normal product vanishes. As mentioned in Appendix A, the non-vanishing contractions are the Feynman propagators

$$\begin{aligned} \langle 0|T(q_m^a(x)\bar{q}_n^b(y))|0\rangle &= \overline{\delta^{ab}q_m(x)\bar{q}_n(y)} \\ &\equiv \delta^{ab}iS_F^q(x-y)_{mn} \end{aligned} \quad \dots (3.6)$$

where $S_F(x-y)$ is the fermion propagator in co-ordinate space.

In momentum space,

$$\langle 0|T(q_m^a(x)\bar{q}_n^b(y))|0\rangle = \delta^{ab} \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} iS_F^q(p)_{mn} \quad \dots (3.7)$$

where the momentum-space fermion propagator $S_F^q(p)$ is given by

$$\begin{aligned}
 S_F^q(p) &\equiv \frac{1}{\not{p} - m_q + i\varepsilon} = \frac{\not{p} + m_q}{p^2 - m_q^2 + i\varepsilon} \\
 &= \frac{\not{P}}{p^2} \quad (\text{chiral limit})
 \end{aligned}
 \tag{3.8}$$

(a) Annihilation region - perturbative diagram - Fig. 2.1(a)

In this case, all the quark fields have to be contracted, in all possible ways. Using (3.7), and ignoring the phase space factors for the time being, one possible contraction for (3.5) is given by

$$\begin{aligned}
 \langle 0 | T(\eta^N(x) \bar{\eta}^N(0)) | 0 \rangle_1 &= \dots (-1) (-\varepsilon_{abc} \varepsilon_{a'b'c'}) \delta^{aa'} \delta^{bb'} \delta^{cc'} \\
 &\quad \cdot i S_F^u(k_1)_{ki'} (\gamma_\nu)_{i'j'} C_{j'k'} i (-) S_F^{uT}(k_2)_{k'i} C_{ij} (\gamma_\mu)_{jk} \\
 &\quad \cdot (\gamma_5)_{lm} (\gamma^\mu)_{mn} i S_F^d(k_3)_{nn'} (\gamma^\nu)_{n'm'} (\gamma_5)_{m'l'} \\
 &= \dots -i^3 \varepsilon_{abc} \varepsilon^{abc} \text{Tr} \left[S_F^u(k_1) \gamma_\nu C S_F^{uT}(k_2) C \gamma_\mu \right] \\
 &\quad \cdot \gamma_5 \gamma^\mu S_F^d(k_3) \gamma^\nu \gamma_5
 \end{aligned}
 \tag{3.9}$$

where the (-1) in front arises from the definition of the generalised normal product of a number of fermion fields (A15) :

$$\overbrace{(u_1 u_2 d)(\bar{d} \bar{u}_2 \bar{u}_1)} \rightarrow (-1)^3 \overbrace{(u_2 \bar{u}_2)} \overbrace{(\bar{u}_1 u_1)} \overbrace{(d \bar{d})} \tag{3.10}$$

The power 3 arises from the number of fermion interchanges required to change the order from $(u_1 u_2 d)$ to $(u_2 u_1 d)$, and from $(\bar{d} \bar{u}_2 \bar{u}_1)$ to $(\bar{u}_2 \bar{u}_1 \bar{d})$.

The minus sign in front of $S_F^{u^T}(k_2)_{k_4}$ comes from the identity $\langle 0 | T(\bar{u}_k^a(0) u_i^a(x)) | 0 \rangle = i S_F^{u^T}(0-x) = -i S_F^{u^T}(x)$.

Now $\varepsilon_{abc} \varepsilon^{abc} = 6$, and using the chiral limit expression (3.8) for the propagator, it follows that

$$\begin{aligned}
 \langle 0 | T(\eta^N(x) \bar{\eta}^N(0)) | 0 \rangle_1 &= \dots -6i^3 \frac{k_1^\alpha}{k_1^2} \frac{k_2^\beta}{k_2^2} \frac{k_3^\delta}{k_3^2} \cdot \text{Tr} \left[\gamma_\alpha \gamma_\nu C \gamma_\beta^T C \gamma_\mu \right] \\
 &\quad \cdot \gamma_\delta \gamma^\mu \gamma_\nu \gamma^\nu \gamma_\delta \\
 &= \dots 6i^3 \frac{k_1^\alpha}{k_1^2} \frac{k_2^\beta}{k_2^2} \frac{k_3^\delta}{k_3^2} \cdot \text{Tr} \left[\gamma_\alpha \gamma_\nu \gamma_\beta \gamma_\mu \right] \\
 &\quad \cdot \gamma^\mu \gamma_\delta \gamma^\nu \\
 &\dots (3.11)
 \end{aligned}$$

using results from Appendix C.

Furthermore

$$\begin{aligned}
 &k_1^\alpha k_2^\beta k_3^\delta \cdot \text{Tr} \left[\gamma_\alpha \gamma_\nu \gamma_\beta \gamma_\mu \right] \cdot \gamma^\mu \gamma_\delta \gamma^\nu \\
 &= k_1^\alpha k_2^\beta k_3^\delta \cdot 4 \left[g_{\alpha\nu} g_{\beta\mu} - g_{\alpha\beta} g_{\nu\mu} + g_{\alpha\mu} g_{\nu\beta} \right] \cdot \gamma^\mu \gamma_\delta \gamma^\nu \\
 &\dots (3.12) \\
 &= 4 \left[k_{1\nu} k_{2\mu} - (k_1 \cdot k_2) g_{\nu\mu} + k_{1\mu} k_{2\nu} \right] \cdot \gamma^\mu \gamma_\delta \gamma^\nu \\
 &= 4 \left[k_2 k_3 k_1 - (k_1 \cdot k_2) \gamma^\mu k_3 \gamma_\mu + k_1 k_3 k_2 \right]
 \end{aligned}$$

To simplify this further, one notes that

$$\begin{aligned}
\mathbb{k}_1 \mathbb{k}_3 \mathbb{k}_2 + \mathbb{k}_2 \mathbb{k}_3 \mathbb{k}_1 &= k_1^\mu k_2^\alpha k_3^\nu (\gamma_\mu \gamma_\nu \gamma_\alpha + \gamma_\alpha \gamma_\nu \gamma_\mu) \\
&= k_1^\mu k_2^\alpha k_3^\nu [\gamma_\mu (2g_{\nu\alpha} - \gamma_\alpha \gamma_\nu) + \gamma_\alpha (2g_{\nu\mu} - \gamma_\mu \gamma_\nu)] \\
&= k_1^\mu k_2^\alpha k_3^\nu [2\gamma_\mu g_{\nu\alpha} - (2g_{\mu\alpha} - \gamma_\alpha \gamma_\mu) \gamma_\nu + 2\gamma_\alpha g_{\nu\mu} - \gamma_\alpha \gamma_\mu \gamma_\nu] \\
&= 2k_1^\mu k_2^\alpha k_3^\nu [g_{\nu\alpha} \gamma_\mu - g_{\mu\alpha} \gamma_\nu + g_{\nu\mu} \gamma_\alpha] \\
&= 2\mathbb{k}_1(k_2 \cdot k_3) - 2\mathbb{k}_3(k_1 \cdot k_2) + 2\mathbb{k}_2(k_1 \cdot k_3)
\end{aligned}
\tag{3.13}$$

and thus, using (C5) to simplify the second term in (3.12), one obtains

$$\begin{aligned}
&k_1^\alpha k_2^\beta k_3^\delta \cdot \text{Tr}[\gamma_\alpha \gamma_\nu \gamma_\beta \gamma_\mu] \cdot \gamma^\mu \gamma_\delta \gamma^\nu \\
&= 8[\mathbb{k}_1(k_2 \cdot k_3) + \mathbb{k}_2(k_1 \cdot k_3)]
\end{aligned}
\tag{3.14}$$

Thus the final result for the one possible way of contracting (3.5), now including the phase space factors, is given by

$$\begin{aligned}
\langle 0 | T(\eta^N(x) \bar{\eta}^N(0)) | 0 \rangle_1 &= 48i^3 \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \frac{d^4 k_3}{(2\pi)^4} e^{-i(k_1+k_2+k_3) \cdot x} \\
&\quad \cdot \frac{1}{k_1^2} \frac{1}{k_2^2} \frac{1}{k_3^2} \cdot [\mathbb{k}_1(k_2 \cdot k_3) + \mathbb{k}_2(k_1 \cdot k_3)]
\end{aligned}
\tag{3.15}$$

The only other possible way to contract the quark fields in (3.5) is

$$\begin{aligned}
\langle 0 | T(\eta^N(x) \bar{\eta}^N(0)) | 0 \rangle_2 &= \dots (+1) (-\varepsilon_{abc} \varepsilon_{a'b'c'}) \delta^{ba'} \delta^{ab'} \delta^{cc'} \\
&\quad \cdot i S_F^u(k_1)_{kk'} \cdot i S_F^u(k_2)_{ii'} (\gamma_\nu)_{ij'} C_{j'k'} C_{ij} (\gamma_\mu)_{jk} \\
&\quad \cdot (\gamma_5)_{lm} (\gamma^\mu)_{mn} i S_F^d(k_3)_{nn'} (\gamma^\nu)_{n'm'} (\gamma_5)_{m'l'} \\
&= \dots -i^3 \varepsilon_{abc} \varepsilon^{bac} S_F^u(k_1)_{kk'} C_{kj'}^T (\gamma_\nu)_{j'i'}^T S_F^u(k_2)_{ii'} C_{ij} (\gamma_\mu)_{jk} \\
&\quad \cdot (\gamma_5)_{lm} (\gamma^\mu)_{mn} S_F^d(k_3)_{nn'} (\gamma^\nu)_{n'm'} (\gamma_5)_{m'l'} \\
&= \dots +i^3 \varepsilon_{abc} \varepsilon^{abc} \text{Tr} [S_F^u(k_1) C^T \gamma_\nu^T S_F^u(k_2) C \gamma_\mu] \\
&\quad \cdot \gamma_5 \gamma^\mu S_F^d(k_3) \gamma^\nu \gamma_5 \\
&\dots (3.16)
\end{aligned}$$

where now by definition of a generalised normal product a factor of (+1) appears :

$$\overbrace{(u_1 u_2 d)} \overbrace{(d \bar{u}_2 \bar{u}_1)} \rightarrow (-1)^2 \overbrace{(u_1 \bar{u}_2)} \overbrace{(u_2 \bar{u}_1)} \overbrace{(d \bar{d})} \dots (3.17)$$

Also, in the third step, use has been made of the anti-symmetric nature of ε^{abc} . Proceeding in the same manner as before, it follows that

$$\begin{aligned}
\langle 0 | T(\eta^N(x) \bar{\eta}^N(0)) | 0 \rangle_2 &= \dots -6i^3 \frac{k_1^\alpha}{k_1^2} \frac{k_2^\beta}{k_2^2} \frac{k_3^\delta}{k_3^2} \cdot \text{Tr} [\gamma_\alpha C^T \gamma_\nu^T \gamma_\beta^T C \gamma_\mu] \\
&\quad \cdot \gamma_5 \gamma^\mu \gamma_\delta \gamma^\nu \gamma_5 \\
&\dots (3.18) \\
&= \dots 6i^3 \frac{k_1^\alpha}{k_1^2} \frac{k_2^\beta}{k_2^2} \frac{k_3^\delta}{k_3^2} \cdot \text{Tr} [\gamma_\alpha \gamma_\nu \gamma_\beta \gamma_\mu] \\
&\quad \cdot \gamma^\mu \gamma_\delta \gamma^\nu
\end{aligned}$$

where the last step follows from

$$\begin{aligned}
C^T \gamma_\nu^T \gamma_\beta^T C &= -C \gamma_\nu^T (-CC) \gamma_\beta^T C \\
&= C \gamma_\nu^T CC \gamma_\beta^T C \\
&= \gamma_\nu \gamma_\beta
\end{aligned}
\tag{3.19}$$

by (C17) and (C18).

The result in (3.18) is the same as that in (3.11), and hence using (3.15),

$$\begin{aligned}
\langle 0 | T(\eta^N(x) \bar{\eta}^N(0)) | 0 \rangle &= \langle 0 | T(\eta^N(x) \bar{\eta}^N(0)) | 0 \rangle_1 + \langle 0 | T(\eta^N(x) \bar{\eta}^N(0)) | 0 \rangle_2 \\
&= 96i^3 \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \frac{d^4 k_3}{(2\pi)^4} e^{-i(k_1+k_2+k_3) \cdot x} \\
&\quad \cdot \frac{1}{k_1^2} \frac{1}{k_2^2} \frac{1}{k_3^2} \cdot [k_1 (k_2 \cdot k_3) + k_2 (k_1 \cdot k_3)]
\end{aligned}
\tag{3.20}$$

At $T = 0$, the two-point correlation function in (2.1) thus becomes

$$\begin{aligned}
\Pi(\omega, p, T = 0) &= i \int d^4 x e^{ip \cdot x} \theta(x_0) \langle 0 | T(\eta^N(x) \bar{\eta}^N(0)) | 0 \rangle \\
&= 96i^4 \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \frac{d^4 k_3}{(2\pi)^4} d^4 x e^{i(p-k_1-k_2-k_3) \cdot x} \theta(x_0) \\
&\quad \cdot \frac{1}{k_1^2} \frac{1}{k_2^2} \frac{1}{k_3^2} \cdot [k_1 (k_2 \cdot k_3) + k_2 (k_1 \cdot k_3)]
\end{aligned}
\tag{3.21}$$

Integrating over x , one finds

$$\begin{aligned} \Pi(\omega, p, T=0) &= 96i^4 \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \frac{d^4 k_3}{(2\pi)^4} \theta(k_1^0) \theta(k_2^0) \theta(k_3^0) \\ &\quad \cdot \frac{1}{k_1^2} \frac{1}{k_2^2} \frac{1}{k_3^2} \cdot [k_1(k_2 \cdot k_3) + k_2(k_1 \cdot k_3)] \\ &\quad \cdot (2\pi)^4 \delta^3(p - k_1 - k_2 - k_3) \delta(\omega - k_1^0 - k_2^0 - k_3^0) \end{aligned} \quad \dots (3.22)$$

In order to find the spectral function $\text{Im } \Pi$, note that

$$\text{Im} \left[\left(\frac{1}{p_1^2 - m_1^2} \right) \left(\frac{1}{p_2^2 - m_2^2} \right) \cdots \left(\frac{1}{p_n^2 - m_n^2} \right) \right] = \frac{1}{2i} \prod_{a=1}^n (-2\pi i \delta(p_a^2 - m_a^2)) \quad \dots (3.23)$$

which means that

$$\begin{aligned} \text{Im} \Pi(\omega, p, T=0) &= 48i^4 \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \frac{d^4 k_3}{(2\pi)^4} \theta(k_1^0) \theta(k_2^0) \theta(k_3^0) \\ &\quad \cdot (2\pi)^3 \delta(k_1^2) \delta(k_2^2) \delta(k_3^2) \cdot [k_1(k_2 \cdot k_3) + k_2(k_1 \cdot k_3)] \\ &\quad \cdot (2\pi)^4 \delta^3(p - k_1 - k_2 - k_3) \delta(\omega - k_1^0 - k_2^0 - k_3^0) \end{aligned} \quad \dots (3.24)$$

Using the identity

$$\int d^4 k \theta(k^0) \delta(k^2 - m^2) = \int \frac{d^3 k}{2k^0} \quad \dots (3.25)$$

(which holds in the chiral limit as well) one finds

$$\begin{aligned}
\text{Im}\Pi(\omega, p, T=0) &= 48i^4 \int \frac{d^3k_1}{(2\pi)^4} \frac{d^3k_2}{(2\pi)^4} \frac{d^3k_3}{(2\pi)^4} \frac{1}{2k_1^0} \frac{1}{2k_2^0} \frac{1}{2k_3^0} \\
&\quad \cdot (2\pi)^3 \cdot [\mathbf{k}_1(k_2 \cdot k_3) + \mathbf{k}_2(k_1 \cdot k_3)] \quad \dots (3.26) \\
&\quad \cdot (2\pi)^4 \delta^3(p - k_1 - k_2 - k_3) \delta(\omega - k_1^0 - k_2^0 - k_3^0)
\end{aligned}$$

In the chiral limit, $k^2 = k^{\mu^2} - k^2 = 0$, implying that $k^0 = |k|$. Thus

$$\begin{aligned}
\text{Im}\Pi(\omega, p, T=0) &= 48i^4 \int \frac{d^3k_1}{(2\pi)^4} \frac{d^3k_2}{(2\pi)^4} \frac{d^3k_3}{(2\pi)^4} \frac{1}{2|k_1|} \frac{1}{2|k_2|} \frac{1}{2|k_3|} \\
&\quad \cdot (2\pi)^3 \cdot [\mathbf{k}_1(k_2 \cdot k_3) + \mathbf{k}_2(k_1 \cdot k_3)] \quad \dots (3.27) \\
&\quad \cdot (2\pi)^4 \delta^3(p - k_1 - k_2 - k_3) \delta(\omega - |k_1| - |k_2| - |k_3|)
\end{aligned}$$

Since the above function is clearly symmetric in k_1 , k_2 , and k_3 , the square brackets can be simplified to $2\mathbf{k}_1(k_2 \cdot k_3)$.

Bringing all the factors to the front, one finds

$$\begin{aligned}
\text{Im}\Pi(\omega, p, T=0) &= \frac{12}{(2\pi)^5} \int \frac{d^3k_1}{|k_1|} \frac{d^3k_2}{|k_2|} \frac{d^3k_3}{|k_3|} \cdot [\mathbf{k}_1(k_2 \cdot k_3)] \\
&\quad \cdot \delta^3(p - k_1 - k_2 - k_3) \delta(\omega - |k_1| - |k_2| - |k_3|) \quad \dots (3.28)
\end{aligned}$$

Comparing with (2.5) and (2.6), it is clear that in this case, for the perturbative diagram, there is no contribution to Π_1 .

In the rest frame ($\mathbf{p} = 0$) the contribution to Π_2 is given by

$$\begin{aligned}
 \text{Im} \Pi_2(\omega, \mathbf{p} = 0, T = 0) &= \frac{3}{\omega (2\pi)^5} \int \frac{d^3 k_1}{|k_1|} \frac{d^3 k_2}{|k_2|} \frac{d^3 k_3}{|k_3|} \text{Tr}[\gamma_0 \not{k}_1] (k_2 \cdot k_3) \\
 &\quad \cdot \delta^3(-k_1 - k_2 - k_3) \delta(\omega - |k_1| - |k_2| - |k_3|) \\
 &\quad \dots (3.29) \\
 &= \frac{12}{\omega (2\pi)^5} \int \frac{d^3 k_1}{|k_1|} \frac{d^3 k_2}{|k_2|} \frac{d^3 k_3}{|k_3|} |k_1| (k_2 \cdot k_3) \\
 &\quad \cdot \delta^3(-k_1 - k_2 - k_3) \delta(\omega - |k_1| - |k_2| - |k_3|)
 \end{aligned}$$

since $\text{Tr}[\gamma_0 \not{k}_1] = 4k_1^0 = 4|k_1|$. The first delta function gives $k_3 = -(k_1 + k_2)$ and hence

$$\begin{aligned}
 |k_3| &= \sqrt{[-(k_1 + k_2)] \cdot [-(k_1 + k_2)]} \\
 &= \sqrt{|k_1|^2 + |k_2|^2 + 2|k_1||k_2|\cos\theta_{12}} \\
 &\quad \dots (3.30) \\
 &= \omega - |k_1| - |k_2|
 \end{aligned}$$

where $\cos \theta_{12}$ is the angle between k_1 and k_2 , and the last step follows from the second delta function.

Letting $\cos \theta_{12} \equiv z$, and using the identity

$$\delta(f(z)) = \sum_n \frac{\delta(z - z_n)}{|f'(z_n)|} \quad \text{where } f(z_n) = 0 \quad \dots (3.31)$$

the second delta function becomes

$$\delta(\omega - |k_1| - |k_2| - |k_3|) = \frac{\omega - |k_1| - |k_2|}{|k_1||k_2|} \delta(z - z_0) \quad \dots (3.32)$$

$$\text{where } z_0 = 1 + \frac{\omega}{2|k_1||k_2|} (\omega - 2|k_1| - 2|k_2|) \quad \dots (3.33)$$

Furthermore

$$\begin{aligned} k_2 \cdot k_3 &= |k_2||k_3| - k_2 \cdot k_3 \\ &= |k_2|(\omega - |k_1| - |k_2|) + k_2 \cdot (k_1 + k_2) \quad \dots (3.34) \\ &= |k_2|[\omega - |k_1|(1-z)] \end{aligned}$$

Substituting into (3.29), one finds

$$\begin{aligned} \text{Im} \Pi_2(\omega, p=0, T=0) &= \frac{12}{\omega (2\pi)^5} \int \frac{d^3 k_1}{|k_1|} \frac{d^3 k_2}{|k_2|} \frac{|k_1||k_2|}{\omega - |k_1| - |k_2|} \\ &\quad \cdot [\omega - |k_1|(1-z)] \frac{\omega - |k_1| - |k_2|}{|k_1||k_2|} \delta(z - z_0) \quad \dots (3.35) \\ &= \frac{12}{\omega (2\pi)^5} \int \frac{d^3 k_1}{|k_1|} \frac{d^3 k_2}{|k_2|} [\omega - |k_1|(1-z)] \delta(z - z_0) \end{aligned}$$

Next, note that

$$\begin{aligned} d^3 k_1 &= |k_1|^2 d|k_1| d(\cos\theta_1) d\phi_1 \\ d^3 k_2 &= |k_2|^2 d|k_2| dz d\phi_2 \end{aligned} \quad \dots (3.36)$$

where in the case of k_2 , the z-axis is chosen parallel to k_1 , which means that $\cos\theta_1 = \cos\theta_{12} = z$. Then (3.35) becomes

$$\begin{aligned} \text{Im}\Pi_2(\omega, p=0, T=0) &= \frac{24}{\omega (2\pi)^3} \int d|k_1| d|k_2| dz |k_1| |k_2| [\omega - |k_1|(1-z)] \delta(z-z_0) \\ &= \frac{24}{\omega (2\pi)^3} \int d|k_1| d|k_2| |k_1| |k_2| [\omega - |k_1|(1-z_0)] \\ &= \frac{12}{(2\pi)^3} \int d|k_1| d|k_2| |k_1| [\omega - 2|k_1|] \end{aligned} \quad \dots (3.37)$$

where the last step follows after substituting (3.33).

In order to proceed, one needs to find the limits of integration for $|k_1|$ and $|k_2|$. From (3.33),

$$\begin{aligned} z_0 \leq 1 &\Rightarrow \frac{\omega}{2} \leq |k_1| + |k_2| \quad \dots(i) \\ z_0 \geq -1 &\Rightarrow \begin{cases} |k_1| \leq \frac{\omega}{2} \Rightarrow |k_2| \leq \frac{\omega}{2} \\ |k_1| \geq \frac{\omega}{2} \Rightarrow |k_2| \geq \frac{\omega}{2} \end{cases} \quad \dots(ii) \end{aligned} \quad \dots (3.38)$$

From the second δ -function in (3.29) one can obtain the further condition

$$\omega - |k_1| - |k_2| = |k_3| = |k_1 + k_2| \geq ||k_1| - |k_2|| \quad \dots (3.39)$$

$$\Rightarrow \begin{cases} |k_1| \geq |k_2| \Rightarrow |k_1| \leq \frac{\omega}{2} & \dots(i) \\ |k_2| \geq |k_1| \Rightarrow |k_2| \leq \frac{\omega}{2} & \dots(ii) \end{cases}$$

Figure 3.1 shows the domains of $|k_1|$ and $|k_2|$, which leads to integration limits of $[0, \omega/2]$ for $|k_1|$, and $[\omega/2 - |k_1|, \omega/2]$ for $|k_2|$.

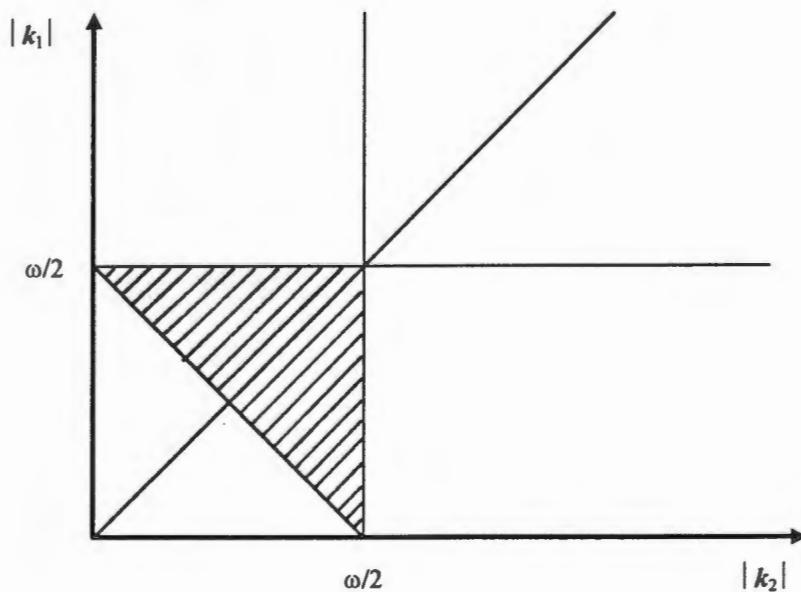


FIGURE 3.1 : Domains of variation of $|k_1|$ and $|k_2|$

Hence integrating (3.37), the final expression for the contribution of the annihilation diagram in Figure 2.1(a) to the imaginary part of the spectral function at zero temperature is given by

$$\text{Im}\Pi_2(\omega, \mathbf{p} = 0, T = 0) = \frac{\omega^4}{64\pi^3} \quad \dots (3.40)$$

This result is in agreement with [3] and [20].

In order to extend this result to the finite- T region, note that for $T > 0$ the propagator in (3.8) is modified and becomes (with one thermal insertion)

$$\begin{aligned} S_F^q(p) &\equiv (\not{p} + m_q) \left(\frac{1}{p^2 - m_q^2 + i\epsilon} + 2\pi i \delta(p^2 - m_q^2) n_F\{|p^0|/T\} \right) \\ &= \not{p} \left(\frac{1}{p^2} + 2\pi i \delta(p^2) n_F\{|p^0|/T\} \right) \quad (\text{chiral limit}) \end{aligned} \quad \dots (3.41)$$

where

$$n_F(z) = \frac{1}{e^z + 1} \quad \dots (3.42)$$

is the Fermi thermal factor.

Denoting a thermal insertion by a bar, the finite- T version of the diagram in Figure 2.1(a) is shown in Figure 3.2 below.

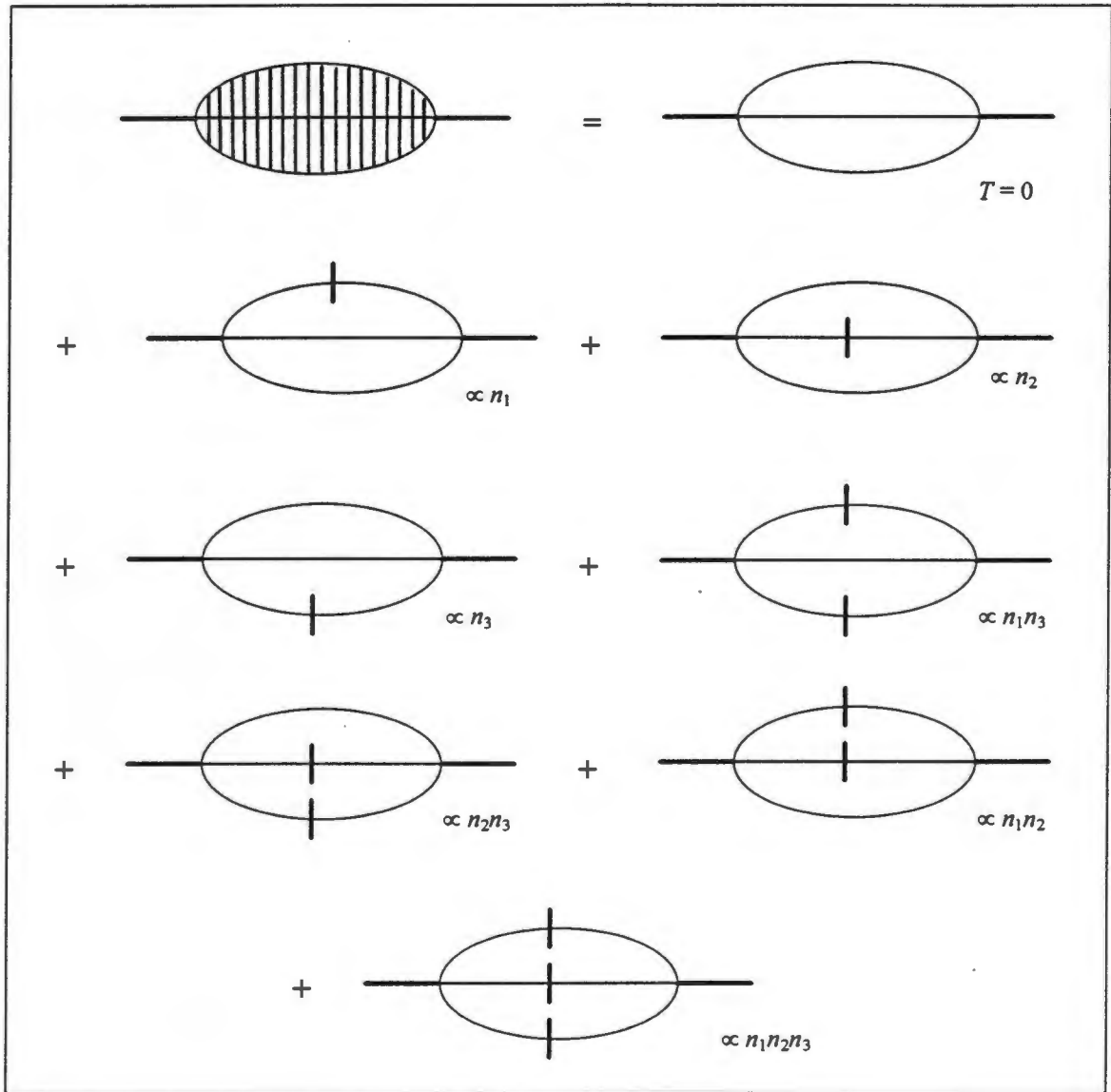


FIGURE 3.2 : Leading order contributions to the QCD spectral function $\text{Im } \Pi_2$ in the time like region, at $T \neq 0$.

From (3.23) it follows that in order to obtain the expression for $\text{Im } \Pi$ with n thermal insertions, one simply has to multiply the $T = 0$ expression by a factor of

$$(-1)^n n_F \{ |p_1^0|/T \} n_F \{ |p_2^0|/T \} \dots n_F \{ |p_n^0|/T \}.$$

One has to be careful, though, since for three thermal insertions, (3.22) shows that

$$\begin{aligned}
\Pi(\omega, \mathbf{p}, T) &= 96i^4 \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \frac{d^4 k_3}{(2\pi)^4} \theta(k_{1_0}) \theta(k_{2_0}) \theta(k_{3_0}) \\
&\quad \cdot n_F \left\{ \frac{k_1^0}{T} \right\} n_F \left\{ \frac{k_2^0}{T} \right\} n_F \left\{ \frac{k_3^0}{T} \right\} \cdot [k_1 (k_2 \cdot k_3) + k_2 (k_1 \cdot k_3)] \\
&\quad \cdot (2\pi)^4 \delta^3(\mathbf{p} - \mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3) \delta(\omega - k_1^0 - k_2^0 - k_3^0)
\end{aligned} \tag{3.43}$$

which is real. Hence the overall thermal factor is given by

$$F(T) = 1 - n_F^1 - n_F^2 - n_F^3 + n_F^1 n_F^2 + n_F^1 n_F^3 + n_F^2 n_F^3 \tag{3.44}$$

where $n_F^1 \equiv n_F(p_1^0/T)$ etc.

Using this together with (3.37), (3.40) and Figure 3.1, the contribution of the annihilation diagram (Figure 2.1(a)) to the imaginary part of the spectral function at non-zero temperature is given by

$$\begin{aligned}
\text{Im}\Pi_2(\omega, \mathbf{p} = 0, T) &= \frac{\omega^4}{64\pi^3} + \frac{3}{2\pi^3} \int_0^{\omega/2} dx \int_0^{\omega/2} dy x(\omega - 2x) \\
&\quad \left\{ -n_F(x/T) - n_F(y/T) + n_F(x/T)n_F(y/T) \right. \\
&\quad \left. + n_F[(\omega - x - y)/T] [n_F(x/T) + n_F(y/T) - 1] \right\} \tag{3.45} \\
&\equiv \frac{\omega^4}{64\pi^3} + \frac{3}{2\pi^3} \frac{F^\circ(\omega, T)}{\omega}
\end{aligned}$$

where the 'o' in the function $F^o(\omega, T)$ stands for octet, in anticipation that this same function will appear in the spectral functions of the other octet currents.

The result (3.45) differs slightly from the one obtained in [20], but after having redone the calculation the authors of [20] now agree with the result in (3.45).

**(b) Annihilation region - non-perturbative (condensate) diagram -
Fig. 2.1 (b)**

In the case where a quark-antiquark pair condenses into the vacuum, their contraction no longer yields the standard propagator of (3.7), but rather the properly normalized quark condensate (to first order in m , and neglecting m_u and m_d since $m_{u,d} \ll m_s$)

$$\langle 0 | q_m^a(x) \bar{q}_n^b(0) | 0 \rangle \mapsto \frac{1}{12} \langle \bar{q}q \rangle \delta^{ab} \left[\delta_{mn} + \frac{i}{4} m_q x_\mu (\gamma^\mu)_{mn} + \dots \right] \quad \dots (3.46)$$

Since the contribution from quark condensates will only be considered to first order, there are only 3 possible ways of contracting the quark fields in (3.5). From (3.9) and (3.16), the two possibilities involving the $\langle \bar{u}u \rangle$ condensate lead to

$$\dots \text{Tr} \left[\gamma_\nu C S_F^{uT}(k_2) C \gamma_\mu \right] = \dots \text{Tr} \left[\gamma_\nu S_F^u(k_2) \gamma_\mu \right]$$

and

... (3.47)

$$\dots \text{Tr} \left[S_F^u(k_1) \gamma_\nu C C \gamma_\mu \right] = \dots \text{Tr} \left[S_F^u(k_2) \gamma_\nu \gamma_\mu \right]$$

both of which are zero because of the odd number of gamma matrices.

In the case of $\langle \bar{d}d \rangle$, one finds

$$\langle 0 | T(\eta^N(x) \bar{\eta}^N(0)) | 0 \rangle_{1,2} = \dots \frac{1}{2} \langle \bar{d}d \rangle i^2 \frac{k_1^\alpha}{k_1^2} \frac{k_2^\beta}{k_2^2} \text{Tr}[\gamma_\alpha \gamma_\nu \gamma_\beta \gamma_\mu] \gamma^\mu \gamma^\nu \quad \dots (3.48)$$

where the sign can be reconciled with (3.11) if one notes that now there is no minus sign due to the interchange $\gamma_\delta \gamma_5 = -\gamma_5 \gamma_\delta$, and also that Wick's theorem now gives one minus sign less.

Since

$$k_1^\alpha k_2^\beta \text{Tr}[\gamma_\alpha \gamma_\nu \gamma_\beta \gamma_\mu] \gamma^\mu \gamma^\nu = 4 [2 k_2 \cdot k_1 - 4 k_1 \cdot k_2] \quad \dots (3.49)$$

it follows that (in the chiral limit)

$$\begin{aligned} \langle 0 | T(\eta^N(x) \bar{\eta}^N(0)) | 0 \rangle &= -8i^2 \langle \bar{d}d \rangle \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} e^{-i(k_1+k_2) \cdot x} \\ &\quad \cdot \frac{1}{k_1^2} \frac{1}{k_2^2} [2(k_1 \cdot k_2) - k_1 \cdot k_2] \end{aligned} \quad \dots (3.50)$$

Now the correlation function at $T = 0$ becomes

$$\begin{aligned} \Pi(\omega, p, T = 0) &= -8i^3 \langle \bar{d}d \rangle \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \theta(k_1^0) \theta(k_2^0) \\ &\quad \cdot \frac{1}{k_1^2} \frac{1}{k_2^2} [2(k_1 \cdot k_2) - k_1 \cdot k_2] \\ &\quad \cdot (2\pi)^4 \delta^3(p - k_1 - k_2) \delta(\omega - k_1^0 - k_2^0) \end{aligned} \quad \dots (3.51)$$

which, using (3.23) and (3.25), leads to

$$\begin{aligned} \text{Im}\Pi(\omega, \mathbf{p}, T=0) &= \frac{-1}{(2\pi)^2} \langle \bar{d}d \rangle \int \frac{d^3 k_1}{|k_1|} \frac{d^3 k_2}{|k_2|} \cdot [2(\mathbf{k}_1 \cdot \mathbf{k}_2) - \mathbf{k}_1 \mathbf{k}_2] \\ &\dots (3.52) \\ &\cdot \delta^3(\mathbf{p} - \mathbf{k}_1 - \mathbf{k}_2) \delta(\omega - |\mathbf{k}_1| - |\mathbf{k}_2|) \end{aligned}$$

Comparing with (2.5), it is clear that for the non-perturbative diagram there is no contribution to Π_2 . In the rest frame ($\mathbf{p} = 0$) the contribution to Π_1 is given by

$$\begin{aligned} \text{Im}\Pi_1(\omega, \mathbf{p} = 0, T=0) &= \frac{-1}{4(2\pi)^2} \langle \bar{d}d \rangle \int \frac{d^3 k_1}{|k_1|} \frac{d^3 k_2}{|k_2|} \text{Tr} [2(\mathbf{k}_1 \cdot \mathbf{k}_2) - \mathbf{k}_1 \mathbf{k}_2] \\ &\cdot \delta^3(-\mathbf{k}_1 - \mathbf{k}_2) \delta(\omega - |\mathbf{k}_1| - |\mathbf{k}_2|) \\ &= \frac{-1}{(2\pi)^2} \langle \bar{d}d \rangle \int \frac{d^3 k_1}{|k_1|} \frac{d^3 k_2}{|k_2|} (\mathbf{k}_1 \cdot \mathbf{k}_2) \\ &\dots (3.53) \\ &\cdot \delta^3(-\mathbf{k}_1 - \mathbf{k}_2) \delta(\omega - |\mathbf{k}_1| - |\mathbf{k}_2|) \end{aligned}$$

Integrating over k_2 , the first δ -function gives $\mathbf{k}_1 = -\mathbf{k}_2$, and $|\mathbf{k}_1| = |\mathbf{k}_2|$, forcing $\mathbf{k}_1 \cdot \mathbf{k}_2 = 2|\mathbf{k}_1|^2$. The second δ -function becomes $\delta(\omega - 2|\mathbf{k}_1|) = \frac{1}{2} \delta(|\mathbf{k}_1| - \omega/2)$. Integration over k_1 then yields

$$\begin{aligned} \text{Im}\Pi_1(\omega, \mathbf{p} = 0, T=0) &= -\frac{1}{\pi} \langle \bar{d}d \rangle \int d|\mathbf{k}_1| |\mathbf{k}_1|^2 \delta(|\mathbf{k}_1| - \omega/2) \\ &\dots (3.54) \\ &= -\frac{\omega^2}{4\pi} \langle \bar{d}d \rangle \end{aligned}$$

in agreement with [2].

In order to extend this result to finite temperature, one has to consider the diagrams in Figure 3.3 below.

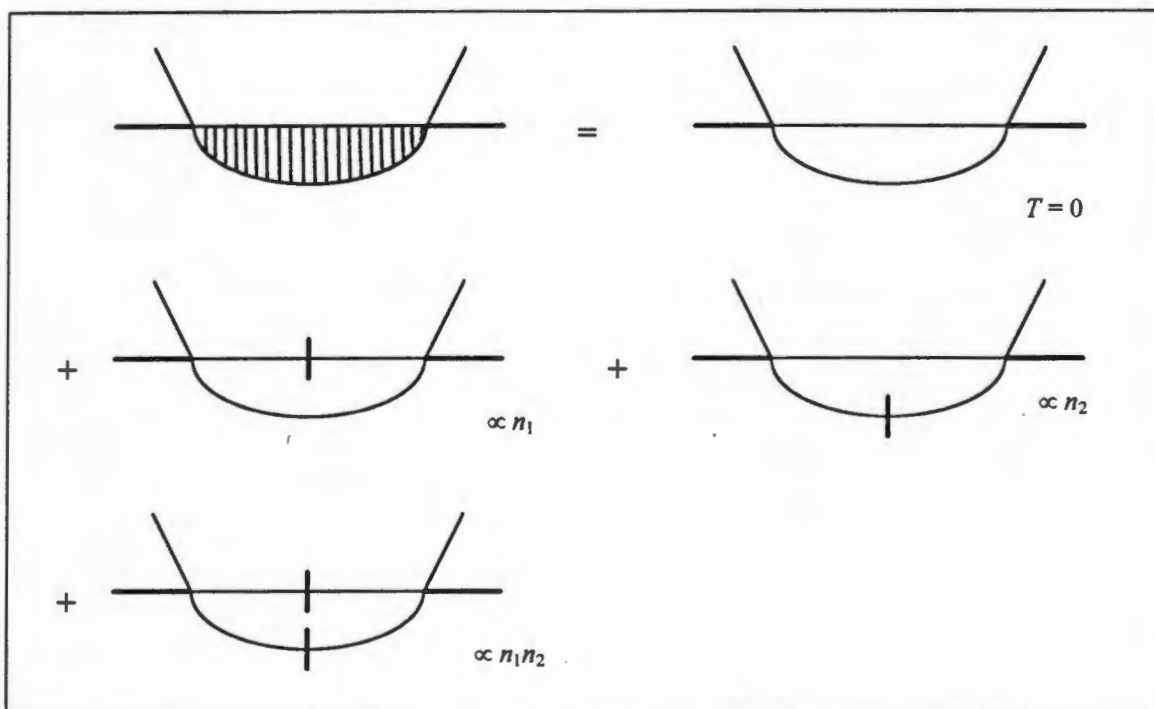


FIGURE 3.3 : Leading order contributions to the QCD spectral function $\text{Im } \Pi_1$ in the time like region, at $T \neq 0$.

As in the perturbative case, the correlation function corresponding to the diagram with 2 thermal insertions is real, and hence the thermal factor is given by

$$F(T) = 1 - n_F \{ |k_1|/T \} - n_F \{ |k_2|/T \} = 1 - 2n_F \{ |k_1|/T \} \quad \dots (3.55)$$

since $|k_1| = |k_2|$.

The final expression for the contribution to the spectral function is thus given by

$$\begin{aligned}
 \text{Im}\Pi_1(\omega, \mathbf{p} = 0, T) &= -\frac{1}{\pi} \langle \bar{d}d \rangle_T \int d|k_1| |k_1|^2 \delta(|k_1| - \omega/2) [1 - 2n_F(|k_1|/T)] \\
 &= -\frac{\omega^2}{4\pi} \langle \bar{d}d \rangle_T [1 - 2n_F(\omega/2T)]
 \end{aligned}$$

... (3.56)

which agrees with the result in [20].

(c) Scattering region - perturbative diagram - Fig. 2.2(a)

In the scattering region ($p^2 = \omega^2 - \mathbf{p}^2 < 0$) the nucleonic current scatters off antiquarks present in the medium. Mathematically this is equivalent to the annihilation process, except that now $(E_1, \mathbf{k}_1) \rightarrow (-E_1, -\mathbf{k}_1)$ for the antiquark.

From (3.29) it follows that the contribution of diagram 2.2(a) to the spectral function is

$$\begin{aligned}
 \text{Im}\Pi_2(\omega, \mathbf{p} = 0, T) &= \frac{12}{\omega (2\pi)^5} \int \frac{d^3k_1}{|k_1|} \frac{d^3k_2}{|k_2|} \frac{d^3k_3}{|k_3|} |k_1| (k_2 \cdot k_3) F(T) \\
 &\quad \cdot \delta^3(\mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3) \delta(\omega + |k_1| - |k_2| - |k_3|)
 \end{aligned}$$

... (3.57)

where $F(T)$ is the thermal factor (to be considered later). The first δ -function gives $\mathbf{k}_3 = \mathbf{k}_1 - \mathbf{k}_2$ and hence the second δ -function becomes

$$\delta(\omega + |k_1| - |k_2| - |k_3|) = \frac{\omega + |k_1| - |k_2|}{|k_1||k_2|} \delta(z - z_0) \quad \dots (3.58)$$

$$\text{where } z_0 = 1 - \frac{\omega}{2|k_1||k_2|} (\omega + 2|k_1| - 2|k_2|) \quad \dots (3.59)$$

$$\text{Also } k_2 \cdot k_3 = |k_2| [\omega + |k_1|(1-z)] \quad \dots (3.60)$$

Substituting into (3.57), one finds

$$\text{Im} \Pi_2(\omega, p=0, T) = \frac{12}{(2\pi)^3} \int d|k_1| d|k_2| |k_1| (\omega + 2|k_1|) F(T) \quad \dots (3.61)$$

The limits of integration for $|k_1|$ and $|k_2|$ can be found using the same method as before (cf. the perturbative diagram in the annihilation region), and they turn out to be $[0, \infty)$ for $|k_1|$, and $[\omega/2, |k_1| + \omega/2]$ for $|k_2|$.

To obtain the thermal factor $F(T)$, note that, by definition (3.42) of n_F ,

$$n_F(-x) = 1 - n_F(x) \quad \dots (3.62)$$

Using this with (3.44) it follows that

$$\begin{aligned} F(T) &= 1 - n_F^{1(-)} - n_F^2 - n_F^3 + n_F^{1(-)} n_F^2 + n_F^{1(-)} n_F^3 + n_F^2 n_F^3 \\ &= n_F^1 (1 - n_F^2 - n_F^3) + n_F^2 n_F^3 \end{aligned} \quad \dots (3.63)$$

Thus the perturbative scattering contribution to the spectral function is

$$\begin{aligned} \text{Im } \Pi_2(\omega, \mathbf{p} = 0, T) &= \frac{3}{2\pi^3} \int_0^\infty d|k_1| \int_{\omega/2}^{|k_1|+\omega/2} d|k_2| |k_1| (\omega + 2|k_1|) \\ &\quad \left\{ n_F(|k_1|/T) \left[1 - n_F(|k_2|/T) - n_F((\omega + |k_1| - |k_2|)/T) \right] \right. \\ &\quad \left. + n_F(|k_2|/T) n_F((\omega + |k_1| - |k_2|)/T) \right\} \quad \dots (3.64) \end{aligned}$$

which, again slightly differs from the result in [20] (apart from the fact that the authors of [20] assigned the momentum k_3 to the incoming anti-quark; which, as said before, makes no difference). In the meantime the authors have redone the calculation and now agree with the result in (3.64).

In the scattering region, $\omega^2 - p^2 \leq 0$, or $\omega < |p|$. Thus the limit $p \rightarrow 0$, forces $\omega \rightarrow 0$. In this case (3.64) becomes

$$\begin{aligned} \text{Im } \Pi_2(\omega = 0, \mathbf{p} = 0, T) &= \frac{3}{\pi^3} \int_0^\infty dx \int_0^x dy 2x^2 \\ &\quad \left\{ n_F(x/T) \left[1 - n_F(y/T) - n_F((x-y)/T) \right] \right. \\ &\quad \left. + n_F(y/T) n_F((x-y)/T) \right\} \quad \dots (3.65) \end{aligned}$$

Figure 3.4 shows the integral evaluated with various finite x -upper limits. The results are plotted against these upper limits, for various values of T between 0 and T_c .

Clearly the integral (3.65) vanishes, so there are no scattering contributions to $\text{Im } \Pi_2$ in the case of the nucleon.

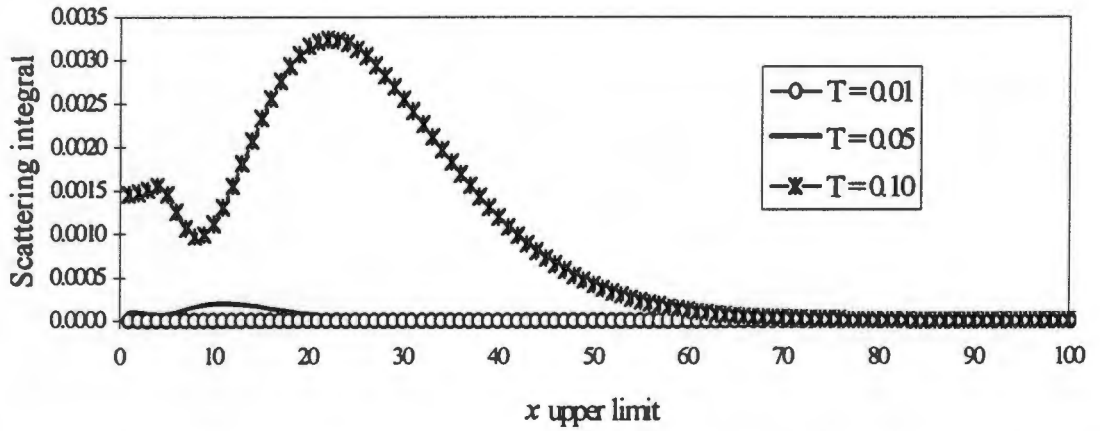


FIGURE 3.4 : Nucleonic scattering contributions to $\text{Im } \Pi_2$ for various T (MeV).

(d) Scattering region - non-perturbative (condensate) diagram - Fig. 2.2(b)

From (3.53), the non-perturbative scattering contribution to the spectral function is (keeping p non-zero for the time-being)

$$\begin{aligned} \text{Im } \Pi_1(\omega, p, T) = & \frac{-1}{(2\pi)^2} \langle \bar{d}d \rangle_T \int \frac{d^3 k_1}{-|k_1|} \frac{d^3 k_2}{|k_2|} (k_1 \cdot k_2) F(T) \\ & \cdot \delta^3(p + k_1 - k_2) \delta(\omega + |k_1| - |k_2|) \end{aligned} \quad \dots (3.66)$$

The first δ -function gives $k_2 = p + k_1$, leading to

$$\delta(\omega + |k_1| - |k_2|) = \frac{\omega + |k_1|}{|k_1||p|} \delta(z - z_0) \quad \dots (3.67)$$

$$\text{where } z_0 = \frac{\omega^2 - p^2 + 2\omega|k_1|}{2|k_1||p|} \quad \dots (3.68)$$

Now $k_1 \cdot k_2 = \frac{1}{2}(\omega^2 - p^2)$ and then

$$\text{Im} \Pi_1(\omega, \mathbf{p}, T) = \frac{1}{2\pi} \langle \bar{d}d \rangle_T \frac{\omega^2 - p^2}{|p|} \int d|k_1| F(T) \quad \dots (3.69)$$

The limits of integration can again be obtained by using the same method as before. One finds $|k_1| \in [(|p| - \omega)/2, \infty)$.

From (3.55) and (3.62), the thermal factor becomes

$$\begin{aligned} F(T) &= 1 - n_F[-|k_1|/T] - n_F[|k_2|/T] \\ &= n_F[|k_1|/T] - n_F[|k_2|/T] \quad \dots (3.70) \\ &= n_F[|k_1|/T] - n_F[(\omega + |k_1|)/T] \end{aligned}$$

where the last step follows from the δ -function in (3.66). Performing the integral, one finds

$$\text{Im} \Pi_1(\omega, \mathbf{p}, T) = \frac{1}{2\pi} \langle \bar{d}d \rangle_T \frac{\omega^2 - p^2}{|p|} \left[\omega + T \ln \left(\frac{n_F\left(\frac{|p| - \omega}{2T}\right)}{n_F\left(\frac{|p| + \omega}{2T}\right)} \right) \right] \quad \dots (3.71)$$

Taking the limit $\omega \rightarrow 0$ first, the round brackets becomes unity and hence the square brackets $\rightarrow 0$. The same holds for the term in front of the brackets, and thus $\text{Im } \Pi_1$ vanishes under the limits $(\omega^2 - p^2) \leq 0, p \rightarrow 0$.

The same result is obtained if p is put equal to zero from the start, in (3.66). In that case, $k_1 = k_2$, so that $k_1 \cdot k_2 = 0$ in the chiral limit. Thus there are no QCD scattering contributions to the spectral functions at all.

3.1.2 Sigma spectral function $\text{Im } \Pi^\Sigma$

Using η^Σ as given in (2.3), and following the procedure as for the nucleon, one obtains

$$\text{Im } \Pi_1^\Sigma(\omega, p=0, T) = -\frac{\omega^2}{4\pi} \langle \bar{s}s \rangle_T [1 - 2n_F(\omega/2T)] \quad \dots (3.72)$$

$$\text{Im } \Pi_2^\Sigma(\omega, p=0, T) = \frac{\omega^4}{64\pi^3} + \frac{3}{2\pi^3} \frac{F^0(\omega, T)}{\omega} \quad \dots (3.73)$$

where $F^0(\omega, T)$ is as given for the nucleon in (3.45).

However, since m_s is of the same order of magnitude as the condensates, one has to include terms $\propto m_s$. Firstly, this means that one must replace the chiral limit expression $S_F = \not{k}/k^2$ for the strange quark propagator by the normal expression $S_F = (\not{k} + m_s)/(k^2 - m_s^2)$. The only effect of the m^2 in the denominator will be an extra factor of m^2 in the expression for z_0 (cf. (3.33) in the case of the nucleon):

$$z_0 = 1 + \frac{1}{2|k_1||k_2|} (\omega^2 - 2\omega|k_1| - 2\omega|k_2| - m_s^2) \quad \dots (3.74)$$

Restricting ourselves to terms up to first order in the strange quark mass m_s , it is clear that one can ignore the m^2 in the denominator of the propagator.

Calculating the new term in the standard fashion gives an extra factor (in the rest frame)

$$\text{Im}\Pi_1^\Sigma(\omega, \mathbf{p} = 0, T) = \frac{m_s \omega^4}{32 \pi^3} + \frac{3m_s}{4 \pi^3} \frac{F_m^\Sigma(\omega, T)}{\omega} \quad \dots (3.75)$$

where

$$F_m^\Sigma(\omega, T) = \int_0^{\omega/2} dx \int_{\omega/2-x}^{\omega/2} dy \omega^2 (2x + 2y - \omega) F^*(T) \quad \dots (3.76)$$

and

$$F^*(T) = -n_F \{x/T\} - n_F \{y/T\} + n_F \{x/T\} n_F \{y/T\} + n_F \{(\omega - x - y)/T\} [n_F \{x/T\} + n_F \{y/T\} - 1] \quad \dots (3.77)$$

Secondly, one should now consider the next to leading order term in the expression for the condensate (3.46). Using the fact that

$$x_\lambda e^{i(p-k_1-k_2)x} = i \frac{\partial}{\partial k_\lambda} [e^{i(p-k_1-k_2)x}] \quad \dots (3.78)$$

and

$$\frac{\partial S_F(k)}{\partial k^\lambda} = \frac{\partial}{\partial k^\lambda} \left[\frac{1}{k^\lambda \gamma_\lambda - m} \right] = \frac{-\gamma_\lambda}{(k^\lambda \gamma_\lambda - m)^2} = -S_F(k) \gamma_\lambda S_F(k) \quad \dots (3.79)$$

this contribution is calculated as

$$\text{Im}\Pi_2^\Sigma(\omega, \mathbf{p} = 0, T) = \frac{m_s \langle \bar{s}s \rangle_T}{4\pi} [1 - 2n_F(\omega/2T)] \quad \dots (3.80)$$

Thus finally

$$\begin{aligned} \text{Im}\Pi_1^\Sigma(\omega, \mathbf{p} = 0, T) &= -\frac{\omega^2}{4\pi} \langle \bar{s}s \rangle_T [1 - 2n_F(\omega/2T)] \\ &+ \frac{m_s \omega^4}{32\pi^3} + \frac{3m_s}{4\pi^3} \frac{F_m^\Sigma(\omega, T)}{\omega} \end{aligned} \quad \dots (3.81)$$

and

$$\begin{aligned} \text{Im}\Pi_2^\Sigma(\omega, \mathbf{p} = 0, T) &= \frac{\omega^4}{64\pi^3} + \frac{3}{2\pi^3} \frac{F^\circ(\omega, T)}{\omega} \\ &+ \frac{m_s}{4\pi} \langle \bar{s}s \rangle_T [1 - 2n_F(\omega/2T)] \end{aligned} \quad \dots (3.82)$$

As in the case of the nucleon, as in fact for all octet currents, scattering contributions vanish in the rest frame and in the chiral limit.

3.1.3 Lambda spectral function $\text{Im } \Pi^\Lambda$

Although η^Λ is slightly more complicated than the other octet currents, results can be obtained straightforwardly by using the by now standard approach. One finds

$$\begin{aligned} \text{Im } \Pi_1^\Lambda(\omega, \mathbf{p} = 0, T) &= -\frac{\omega^2}{12\pi} \left(4\langle \bar{q}q \rangle_T - \langle \bar{s}s \rangle_T \right) \left[1 - 2n_F(\omega/2T) \right] \\ &\quad - \frac{m_s \omega^4}{96\pi^3} - \frac{m_s}{4\pi^3} \frac{F_m^\Lambda(\omega, T)}{\omega} \end{aligned} \quad \dots (3.83)$$

$$\begin{aligned} \text{Im } \Pi_2^\Lambda(\omega, \mathbf{p} = 0, T) &= \frac{\omega^4}{64\pi^3} + \frac{3}{2\pi^3} \frac{F^\circ(\omega, T)}{\omega} \\ &\quad - \frac{m_s}{12\pi} \left(4\langle \bar{q}q \rangle_T - 3\langle \bar{s}s \rangle_T \right) \left[1 - 2n_F(\omega/2T) \right] \end{aligned} \quad \dots (3.84)$$

where $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle \equiv \langle \bar{q}q \rangle$ as said in the introduction, and

$$F_m^\Lambda(\omega, T) = \int_0^{\omega/2} dx \int_{\omega/2-x}^{\omega/2} dy \omega^2 (2x + 2y - \omega) F^*(T) \quad \dots (3.85)$$

as for the Σ particle in (3.76).

3.1.4 Cascade spectral function $\text{Im } \Pi^{\Xi}$

As expected, the results are the same as for the nucleon :

$$\text{Im } \Pi_1^{\Xi}(\omega, \mathbf{p} = 0, T) = -\frac{\omega^2}{4\pi} \langle \bar{q}q \rangle_T [1 - 2n_F(\omega/2T)] \quad \dots (3.86)$$

$$\text{Im } \Pi_2^{\Xi}(\omega, \mathbf{p} = 0, T) = \frac{\omega^4}{64\pi^3} + \frac{3}{2\pi^3} \frac{F_a(\omega, T)}{\omega} \quad \dots (3.87)$$

3.2 DECUPLET ($J^P = 3/2^+$)

3.2.1 Omega spectral function $\text{Im } \Pi^\Omega$

(a) Annihilation region - perturbative diagram - Fig. 2.1 (a)

In the perturbative case, there are six ways in which the quark fields of η^Ω can be contracted :

$$\begin{aligned}
 & T[(s_1 s_2 s_3)_x (\bar{s}_3 \bar{s}_2 \bar{s}_1)_0]_{\text{no e.t.c.}} \\
 & \quad \Downarrow \\
 & \begin{array}{ll}
 (i) \overbrace{s_1 \bar{s}_3} \overbrace{s_2 \bar{s}_2} \overbrace{s_3 \bar{s}_1} & (iv) \overbrace{s_1 \bar{s}_3} \overbrace{s_2 \bar{s}_1} \overbrace{s_3 \bar{s}_2} \\
 (ii) \overbrace{s_1 \bar{s}_1} \overbrace{s_2 \bar{s}_3} \overbrace{s_3 \bar{s}_2} & (v) \overbrace{s_1 \bar{s}_1} \overbrace{s_2 \bar{s}_2} \overbrace{s_3 \bar{s}_3} \\
 (iii) \overbrace{s_1 \bar{s}_2} \overbrace{s_2 \bar{s}_1} \overbrace{s_3 \bar{s}_3} & (vi) \overbrace{s_1 \bar{s}_2} \overbrace{s_2 \bar{s}_3} \overbrace{s_3 \bar{s}_1}
 \end{array}
 \end{aligned} \tag{3.88}$$

Groups (i) - (iii) involve an even number of quark field interchanges and hence has an associated factor of (+1) from Wick's Theorem (A15). Likewise groups (iv) and (vi) involve an odd number of interchanges, and hence has an associated factor of (-1).

Starting with case (i) (labelled by 1), and proceeding in the same way as for the nucleon (in the chiral limit), one finds

$$\begin{aligned}
 \text{Im } \Pi_1^{\mu\nu}(\omega, p, T=0) &= \frac{3}{8(2\pi)^5} \int \frac{d^3 k_1}{|k_1|} \frac{d^3 k_2}{|k_2|} \frac{d^3 k_3}{|k_3|} \not{k}_1 \gamma^\nu \not{k}_2 \gamma^\mu \not{k}_3 \\
 &\quad \cdot \delta^3(p - k_1 - k_2 - k_3) \delta(\omega - |k_1| - |k_2| - |k_3|)
 \end{aligned} \tag{3.89}$$

With the limits of integration of $|k_1|$ and $|k_2|$ turning out to be the same as in the case of the nucleon (3.38, 3.39), applying the projections as defined in (2.8) in the rest frame ($p = 0$), one finds that only the projections involving the α_i 's (i.e. Π_2) survive. It turns out that

$$A = B/4 = 10C = D = E = \frac{-\omega^4}{256\pi^3} \quad \dots (3.90)$$

so that, from (2.9),
$$\alpha_1^I = \frac{-\omega^4}{5 \cdot 2^8 \pi^3} \quad \dots (3.91)$$

Cases (ii), (iv) and (vi) lead to the same result, but cases (iii) and (v) (labelled by III) give

$$\begin{aligned} \text{Im} \Pi_{\text{III}}^{\mu\nu}(\omega, p, T=0) &= \frac{3}{2(2\pi)^5} \int \frac{d^3 k_1}{|k_1|} \frac{d^3 k_2}{|k_2|} \frac{d^3 k_3}{|k_3|} \\ &\cdot [k_1^\mu k_2^\nu - (k_1 \cdot k_2) g^{\mu\nu} + k_1^\nu k_2^\mu] k_3 \\ &\cdot \delta^3(p - k_1 - k_2 - k_3) \delta(\omega - |k_1| - |k_2| - |k_3|) \end{aligned} \quad \dots (3.92)$$

Applying the projections one obtains

$$A = B = -5C = -2D = -2E = \frac{-\omega^4}{64\pi^3} \quad \dots (3.93)$$

so that
$$\alpha_1^{\text{III}} = \frac{-\omega^4}{5 \cdot 2^7 \pi^3} \quad \dots (3.94)$$

Hence from (3.91) and (3.94) the final expression for the spectral function in the chiral limit at $T = 0$ is

$$\text{Im}\Pi_2(\omega, \mathbf{p} = 0, T = 0) = 4a_1^I + 2a_1^{\text{III}} = \frac{-\omega^4}{160\pi^3} \quad \dots (3.95)$$

In the finite temperature case, the thermal factor is (as given in (3.44) for the nucleon and other octet currents) for three propagators :

$$F(T) = 1 - n_F(|k_1|/T) - n_F(|k_2|/T) + n_F(|k_1|/T)n_F(|k_2|/T) + n_F(|k_3|/T)[n_F(|k_1|/T) + n_F(|k_2|/T) - 1] \quad \dots (3.96)$$

From the calculations in the $T = 0$ case, and now adding the thermal factor $F(T)$, one finds for each of the cases (i), (ii), (iv) and (vi) that

$$\text{Im}\Pi_2^I(\omega, \mathbf{p} = 0, T) = -\frac{\omega^4}{5 \cdot 2^8 \pi^3} - \frac{1}{64\pi^3} \int d|k_1|d|k_2| F^*(T) \cdot \left\{ (\omega - |k_1| - |k_2|) \left(8 \frac{|k_1||k_2|}{\omega} - 2\omega + 5|k_1| + 3|k_2| \right) + |k_2|^2 - |k_1|^2 \right\} \quad \dots (3.97)$$

where $F^*(T) = F(T; k_3 = \omega - k_1 - k_2) - 1$ as in the case of the octet (3.77).

For cases (iii) and (v) one finds

$$\begin{aligned} \text{Im} \Pi_2^{\text{III}}(\omega, p=0, T) &= -\frac{\omega^4}{5 \cdot 2^7 \pi^3} + \frac{1}{16\pi^3} \int d|k_1| d|k_2| F^*(T) \\ &\cdot \left\{ (\omega - |k_1| - |k_2|) \left(\omega - 2|k_1| - 2|k_2| - 4 \frac{|k_1||k_2|}{\omega} \right) \right\} \end{aligned} \quad \dots (3.98)$$

Thus the total perturbative contribution to the spectral function in the chiral limit at finite T is

$$\text{Im} \Pi_2(\omega, p=0, T) = \frac{-\omega^4}{160\pi^3} + \frac{1}{16\pi^3} \frac{F^d(\omega, T)}{\omega} \quad \dots (3.99)$$

where

$$\begin{aligned} F^d(\omega, T) &= \int_0^{\omega/2} dx \int_{\omega/2-x}^{\omega/2} dy F^*(T) \cdot \left\{ \omega(\omega - 2x - 2y)(4\omega - 5x - 3y) \right. \\ &\quad \left. - 16xy(\omega - x - y) \right\} \end{aligned} \quad \dots (3.100)$$

Considering terms $\propto m_s$, cases (i), (ii), (iv) and (vi) yield

$$\begin{aligned} \text{Im} \Pi_1^{\mu\nu}(\omega, p, T=0) &= \frac{3m_s}{8(2\pi)^5} \int \frac{d^3k_1}{|k_1|} \frac{d^3k_2}{|k_2|} \frac{d^3k_3}{|k_3|} \\ &\cdot (\gamma^\nu k_2 \gamma^\mu k_3 - k_1 \gamma^\nu \gamma^\mu k_3 + k_1 \gamma^\nu k_2 \gamma^\mu) \\ &\cdot \delta^3(p - k_1 - k_2 - k_3) \delta(\omega - |k_1| - |k_2| - |k_3|) \end{aligned} \quad \dots (3.101)$$

leading to

$$V = \frac{2W}{5} = 16X = 16Y = 16Z = \frac{-m_s \omega^4}{16\pi^3} \quad \dots (3.102)$$

so that
$$b_1^I = \frac{-9m_s \omega^4}{2^{10} \pi^3} \quad \dots (3.103)$$

Cases (iii) and (v) yields

$$\begin{aligned} \text{Im} \Pi_{\text{III}}^{\mu\nu}(\omega, p, T=0) &= \frac{3m_s}{2(2\pi)^5} \int \frac{d^3 k_1}{|k_1|} \frac{d^3 k_2}{|k_2|} \frac{d^3 k_3}{|k_3|} \\ &\cdot [k_1^\mu k_2^\nu - (k_1 \cdot k_2) g^{\mu\nu} + k_1^\nu k_2^\mu] \quad \dots (3.104) \\ &\cdot \delta^3(p - k_1 - k_2 - k_3) \delta(\omega - |k_1| - |k_2| - |k_3|) \end{aligned}$$

so that

$$V = W = -8X = -8Y = -8Z = \frac{-m_s \omega^4}{16\pi^3} \quad \dots (3.105)$$

and thus
$$b_1^{\text{III}} = \frac{-3m_s \omega^4}{2^9 \pi^3} \quad \dots (3.106)$$

Thus the overall result at $T = 0$ is

$$\text{Im} \Pi_1(\omega, p = 0, T = 0) = 4b_1^I + 2b_1^{\text{III}} = \frac{-3m_s \omega^4}{64\pi^3} \quad \dots (3.107)$$

The negative sign for the spectral function might seem a bit surprising, but the corresponding hadronic spectral function (5.4) turn out to be negative too, thus preventing the mass (5.7) from becoming imaginary.

At $T \neq 0$, one finds

$$\begin{aligned} \text{Im} \Pi_1^I(\omega, \mathbf{p} = 0, T) = & -\frac{9m_s \omega^4}{2^{10} \pi^3} - \frac{m_s}{32\pi^3} \int d|k_1| d|k_2| F^*(T) \\ & \cdot \left\{ \omega^2 + 4|k_1||k_2| + 4(|k_1| + |k_2|)(\omega - |k_1| - |k_2|) \right\} \end{aligned} \quad \dots (3.108)$$

and

$$\begin{aligned} \text{Im} \Pi_1^{\text{III}}(\omega, \mathbf{p} = 0, T) = & -\frac{3m_s \omega^4}{2^9 \pi^3} + \frac{m_s}{16\pi^3} \int d|k_1| d|k_2| F^*(T) \\ & \cdot \left\{ \omega (\omega - 2|k_1| - 2|k_2|) - 4|k_1||k_2| \right\} \end{aligned} \quad \dots (3.109)$$

Thus the contribution of terms $\propto m_s$ to the spectral function at finite T is

$$\text{Im} \Pi_1(\omega, \mathbf{p} = 0, T) = \frac{-3m_s \omega^4}{64 \pi^3} + \frac{m_s}{4 \pi^3} \frac{F_m^\Omega(\omega, T)}{\omega} \quad \dots (3.110)$$

where

$$F_m^\Omega(\omega, T) = \int_0^{\omega/2} dx \int_{\omega/2-x}^{\omega/2} dy F^*(T) \cdot \omega \left\{ 2x^2 + 2y^2 - 3\omega(x+y) \right\} \quad \dots (3.111)$$

(b) Annihilation region - non-perturbative (condensate) diagram
- Fig. 2.1(b)

Starting with cases (iii) and (v) of (3.88), only the last of the three pairs of quark fields can be turned into condensates (i.e. their momenta $\rightarrow 0$), which, in the chiral limit, at $T = 0$, leads to

$$\text{Im}\Pi_{\text{III}}^{\mu\nu}(\omega, p, T=0) = -\frac{1}{4} \frac{\langle \bar{s}s \rangle}{(2\pi)^2} \int \frac{d^3 k_1}{|k_1|} \frac{d^3 k_2}{|k_2|} (k_1^\mu k_2^\nu - g^{\mu\nu} k_1 \cdot k_2 + k_1^\nu k_2^\mu) \cdot \delta^3(p - k_1 - k_2) \delta(\omega - |k_1| - |k_2|) \quad \dots (3.112)$$

Applying the projections one finds that only those involving b_i 's (i.e. Π_1) survives :

$$V = W = \frac{\langle \bar{s}s \rangle \omega^2}{2\pi} \quad (X = Y = Z = 0) \quad \dots (3.113)$$

Hence

$$b_1^{\text{III}} = \frac{\langle \bar{s}s \rangle \omega^2}{24\pi} \quad \dots (3.114)$$

The reason why only the last of the three pairs of quark fields in cases (iii) and (v) can be turned into condensates, is that in these two cases, where each quark in the omega current is contracted with its own conjugate, writing out the time-ordered product as in (3.9) leads to

$$\text{Tr}[S_F^z(k_1) \gamma_\mu S_F^z(k_2) \gamma^\mu] S_F^z(k_3) \quad \dots (3.115)$$

When k_1 or $k_2 \rightarrow 0$, the above trace becomes zero (trace of an odd number of gamma functions).

However in the remaining four cases (*i*, *ii*, *iv* and *vi*) no trace part appears; the time-ordered product gives

$$[S_F^s(k_1)\gamma^\mu S_F^s(k_2)\gamma^\nu S_F^s(k_3)]_{nn'} \quad (\text{no trace}) \quad \dots (3.116)$$

which means that there are three possible ways of contracting these fields, corresponding to whether k_1 , k_2 or k_3 is zero. One finds (after relabelling of k 's)

$$\begin{aligned} \text{Im}\Pi_1^{\mu\nu}(\omega, p, T=0) &= -\frac{1}{16} \frac{\langle \bar{s}s \rangle}{(2\pi)^2} \int \frac{d^3k_1}{|k_1|} \frac{d^3k_2}{|k_2|} \\ &\quad [k_1\gamma^\nu k_2\gamma^\mu - k_1\gamma^\nu\gamma^\mu k_2 + \gamma^\nu k_1\gamma^\mu k_2] \quad \dots (3.117) \\ &\quad \delta^3(p - k_1 - k_2) \delta(\omega - |k_1| - |k_2|) \end{aligned}$$

which gives

$$V = \frac{2W}{5} = 8X = 4Y = \frac{\langle \bar{s}s \rangle \omega^2}{2\pi} \quad (Z=0) \quad \dots (3.118)$$

and thus

$$b_1^I = \frac{\langle \bar{s}s \rangle \omega^2}{16\pi} \quad \dots (3.119)$$

Thus, in the annihilation region, the final non-perturbative contribution to $\text{Im } \Pi_1$, in the chiral limit, at zero temperature and in the rest frame, is

$$\text{Im } \Pi_1(\omega, \mathbf{p} = 0, T = 0) = 4b_1^{\text{I}} + 2b_1^{\text{III}} = \frac{\langle \bar{s}s \rangle \omega^2}{3\pi} \quad \dots (3.120)$$

At non-zero T , the thermal factor is the same as for the nucleon (3.55)

$$F(T) = 1 - n_F(|\mathbf{k}_2|) - n_F(|\mathbf{k}_1|) = 1 - 2n_F(|\mathbf{k}_2|) \quad \dots (3.121)$$

Hence, still in the chiral limit,

$$\text{Im } \Pi_1(\omega, \mathbf{p} = 0, T) = \frac{\langle \bar{s}s \rangle \omega^2}{3\pi} \left[1 - 2n_F(\omega/2T) \right] \quad \dots (3.122)$$

For non-perturbative terms $\propto m_s$, the same method used in the case of the octet currents (3.78, 3.79) yields

$$\text{Im } \Pi_2(\omega, \mathbf{p} = 0, T) = \frac{3m_s \langle \bar{s}s \rangle_T}{4\pi} \left[1 - 2n_F\left(\frac{\omega}{2T}\right) \right] \quad \dots (3.123)$$

(c) Scattering region - perturbative diagram - Fig. 2.2(a)

Making the replacement $(E_1, \mathbf{k}_1) \rightarrow (-E_1, -\mathbf{k}_1)$ in (3.98), and finding that the integration limits and the thermal factor are the same as in the case of the nucleon, it follows that

$$\begin{aligned} \text{Im}\Pi_2(\omega, p=0, T) = & \frac{-1}{16\pi^3} \int_0^\infty dx \int_{\omega/2}^{\omega/2+x} dy \left\{ \omega(\omega+2x-2y)(4\omega+5x-3y) \right. \\ & \left. + 16xy(\omega+x-y) \right\} \\ & \cdot \left\{ n_F(x/T) \left[1 - n_F(y/T) - n_F((\omega+x-y)/T) \right] \right. \\ & \left. + n_F(y/T) n_F((\omega+x-y)/T) \right\} \end{aligned} \quad \dots (3.124)$$

Since in the scattering region $\omega \rightarrow 0$ as $p \rightarrow 0$, one strictly has to put $\omega = 0$ in the above expression :

$$\begin{aligned} \text{Im}\Pi_2(\omega=0, p=0, T) = & \frac{1}{\pi^3} \int_0^\infty dx \int_0^x dy xy(y-x) \\ & \cdot \left\{ n_F(x/T) \left[1 - n_F(y/T) - n_F((x-y)/T) \right] \right. \\ & \left. + n_F(y/T) n_F((x-y)/T) \right\} \end{aligned} \quad \dots (3.125)$$

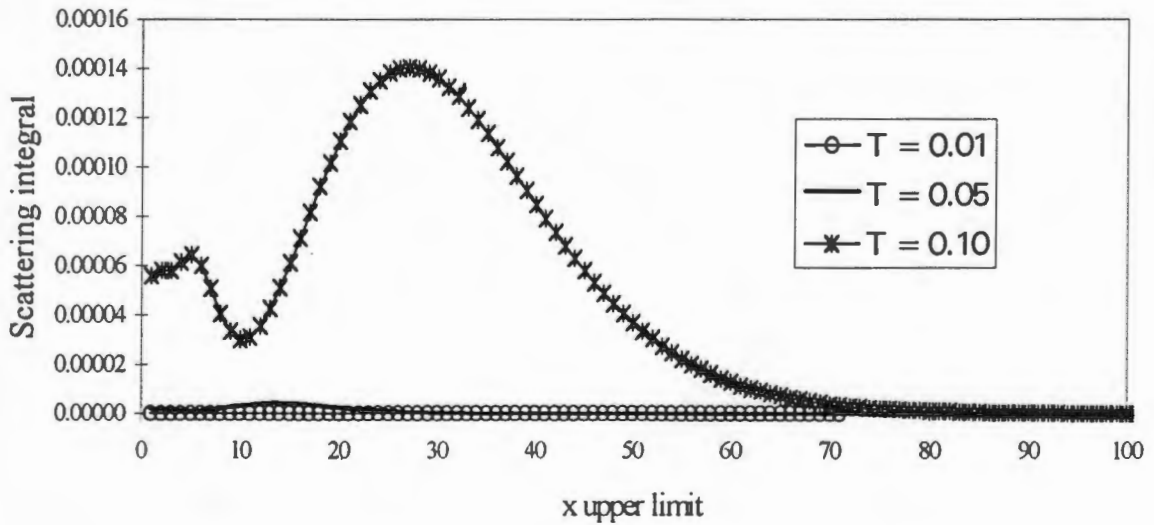


FIGURE 3.5 : Omega scattering contributions to $\text{Im}\Pi_2$ for various T .

Figure 3.5 shows the numerically integrated value of the integrand in (3.125), versus the upper limit of x integration, for various values of T . Clearly as the upper limit approaches infinity, the integral goes to zero, for all values of T below $T_c = 0.102$ MeV. Thus there is no scattering contribution to the spectral function in the perturbative case.

(d) Scattering region - non-perturbative (condensate) diagram - Fig. 2.2(b)

Making the replacement $(E_1, \mathbf{k}_1) \rightarrow (-E_1, -\mathbf{k}_1)$ in (3.112) and (3.117), keeping \mathbf{p} non-zero, and again noting that the integration limits and thermal factor turn out to be the same as in the case of the nucleon (3.70), one finds

$$\text{Im}\Pi_1(\omega, \mathbf{p}, T) = -\frac{1}{\pi} \langle \bar{s}s \rangle_T \frac{\omega^2 - \mathbf{p}^2}{|\mathbf{p}|} \left[\omega + T \ln \left(\frac{n_F\left(\frac{|\mathbf{p}|-\omega}{2T}\right)}{n_F\left(\frac{|\mathbf{p}|+\omega}{2T}\right)} \right) \right] \quad \dots (3.126)$$

which is of exactly the same form as the corresponding result for the nucleon (3.71). Thus it also vanishes in the limit $p \rightarrow 0$ and $\omega \rightarrow 0$, so that there is no non-perturbative scattering contribution to the spectral function either.

In summary, the final results for the Ω spectral functions are

$$\begin{aligned} \text{Im}\Pi_1^\Omega(\omega, \mathbf{p} = 0, T) &= \frac{\omega^2}{3\pi} \langle \bar{s}s \rangle_T \left[1 - 2n_F(\omega/2T) \right] \\ &\quad - \frac{3m_s \omega^4}{64\pi^3} + \frac{m_s}{4\pi^3} \frac{F_m^\Omega(\omega, T)}{\omega} \end{aligned} \quad \dots (3.127)$$

$$\begin{aligned} \text{Im} \Pi_2^\Omega(\omega, p=0, T) &= \frac{-\omega^4}{160\pi^3} + \frac{1}{16\pi^3} \frac{F^d(\omega, T)}{\omega} \\ &+ \frac{3m_s}{2\pi} \langle \bar{s}s \rangle_T [1 - 2n_F(\omega/2T)] \end{aligned} \quad \dots (3.128)$$

where $F_m^\Omega(\omega, T)$ and $F^d(\omega, T)$ are given in (3.111) and (3.100) respectively.

3.2.2 Delta spectral function $\text{Im } \Pi^\Delta$

Using η^Δ as given in (2.4), and following the same procedure as for the omega, one obtains

$$\text{Im } \Pi_1^\Delta(\omega, p = 0, T) = \frac{\omega^2}{3\pi} \langle \bar{q}q \rangle_T [1 - 2n_F(\omega/2T)] \quad \dots (3.129)$$

$$\text{Im } \Pi_2^\Delta(\omega, p = 0, T) = \frac{-\omega^4}{160\pi^3} + \frac{1}{16\pi^3} \frac{F^d(\omega, T)}{\omega} \quad \dots (3.130)$$

where $F^d(\omega, T)$ is as given for the omega in (3.100). As in the case of the omega, and in fact for all decuplet currents, scattering contributions vanish in the rest frame and in the chiral limit.

3.2.3 Sigma* spectral function $\text{Im } \Pi^{\Sigma^*}$

Although η^{Σ^*} (and η^{Ξ^*}) are slightly more complicated than the other decuplet currents, results follow straightforwardly by using our previous methods. One finds

$$\begin{aligned} \text{Im } \Pi_1^{\Sigma^*}(\omega, p = 0, T) = & + \frac{\omega^2}{9\pi} (2\langle \bar{q}q \rangle_T + \langle \bar{s}s \rangle_T) [1 - 2n_F(\omega/2T)] \\ & - \frac{m_s \omega^4}{64\pi^3} + \frac{m_s}{12\pi^3} \frac{F_m^{\Sigma^*}(\omega, T)}{\omega} \end{aligned} \quad \dots (3.131)$$

$$\begin{aligned} \text{Im}\Pi_2^{\Sigma^*}(\omega, p=0, T) &= \frac{-\omega^4}{160\pi^3} + \frac{1}{16\pi^3} \frac{F^d(\omega, T)}{\omega} \\ &+ \frac{m_s}{12\pi} (4\langle \bar{q}q \rangle_T - \langle \bar{s}s \rangle_T) [1 - 2n_F(\omega/2T)] \end{aligned} \quad \dots (3.132)$$

where $F_m^{\Sigma^*}(\omega, T)$ and $F^d(\omega, T)$ are given by (3.111) and (3.100) respectively, as for the omega.

3.2.3 Cascade* spectral function $\text{Im}\Pi^{\Xi^*}$

In this case one finds

$$\begin{aligned} \text{Im}\Pi_1^{\Xi^*}(\omega, p=0, T) &= \frac{\omega^2}{9\pi} (\langle \bar{q}q \rangle_T + 2\langle \bar{s}s \rangle_T) [1 - 2n_F(\omega/2T)] \\ &- \frac{m_s \omega^4}{32\pi^3} + \frac{m_s}{6\pi^3} \frac{F_m^{\Xi^*}(\omega, T)}{\omega} \end{aligned} \quad \dots (3.133)$$

$$\begin{aligned} \text{Im}\Pi_2^{\Xi^*}(\omega, p=0, T) &= \frac{-\omega^4}{160\pi^3} + \frac{1}{16\pi^3} \frac{F^d(\omega, T)}{\omega} \\ &+ \frac{m_s}{6\pi} (2\langle \bar{q}q \rangle_T + \langle \bar{s}s \rangle_T) [1 - 2n_F(\omega/2T)] \end{aligned} \quad \dots (3.134)$$

where $F_m^{\Xi^*}(\omega, T)$ and $F^d(\omega, T)$ are given by (3.111) and (3.100) respectively, as for the omega.

4. HADRONIC CONTRIBUTIONS TO THE SPECTRAL FUNCTIONS

At low temperatures, the hadronic contribution to the spectral function is due only to thermal pions, since the effects of massive particles are exponentially (Boltzmann-) suppressed. For this reason it suffices to consider a gas of massless pions. The relevant lowest order (in T) diagrams corresponding to the interaction of the current with the thermal pion gas are shown in Figure 4.1 below. Diagram (a) corresponds to production of a fermion by the current in the presence of the thermal pion gas, while in diagram (b) the same process is accompanied by the induced production of a pion. Clearly these

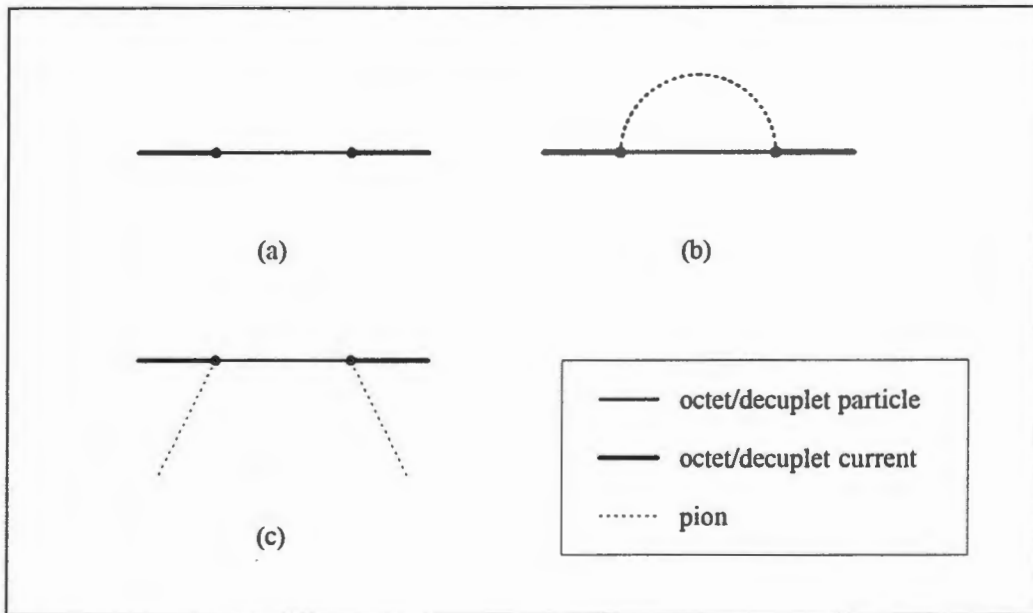


FIGURE 4.1 : Lowest order (in T) contributions to the hadronic spectral functions.

two diagrams contribute to the hadronic spectral functions in the annihilation region ($\omega^2 - p^2 > M^2$). In diagram (c) a thermal pion from the heat bath is scattered off the current, with the associated production of a fermion, a

process which contributes to the hadronic spectral functions in the scattering region ($\omega^2 - p^2 < M^2$). However, being governed by the strong interaction, conservation of isospin applies, and hence this process may not be allowed in all cases. Particularly, in the case of the Ω and the Λ , both of which have zero isospin ($I = 0$), the processes $\eta(\Lambda) \rightarrow \Lambda + \pi$ and $\eta(\Omega) \rightarrow \Omega + \pi$ are forbidden.

Diagram 4(a) will be treated in the narrow-width approximation. Despite the results of [18], showing that the nucleon develops a hadronic width $\Gamma_N(T)$ which increases with temperature, the above approximation is justifiable since it is well known that QCD sum rules are in general not very sensitive to the presence of a width in the spectral function, and even less so in the case of mass predictions using ratios of FESR. Moreover, since two sum rules are needed to obtain the quark coupling to the current, $\lambda^2(T)$, as well as the relevant mass $M(T)$, and there are only two FESR to leading order in dimensionality, attempts to predict $\Gamma_N(T)$ as well would entail considering FESR of higher dimensionality. However, due to a lack of knowledge of higher dimensional vacuum condensates, and problems faced by the FESR formalism at next to leading order dimensionality (as mentioned in §2), this is not feasible.

Considering diagram 4(b), it is evident that at $T = 0$ it becomes a loop correction to the fermion mass. Because of the way in which the FESR are normalized at $T = 0$ (cf. §5), this contribution will be absorbed into the continuum and hence is not relevant here.

In order to calculate diagram 4(c) one needs to know the matrix elements $\langle 0 | \eta_N | N \pi \rangle$ for the nucleon, $\langle 0 | \eta_\Sigma | \Sigma \pi \rangle$ for the sigma, etc. They can be

estimated using PCAC and are expressed in terms of the commutators of the current η with the axial charge Q_5^a . For example, in the case of the nucleon

$$\langle 0 | \eta_N | N \pi^a \rangle = \frac{i}{f_\pi} \langle 0 | [\eta_N, Q_5^a] | N \rangle \quad \dots (4.1)$$

and

$$\langle 0 | \eta_N | N \rangle = \lambda_N u_N \quad \dots (4.2)$$

where u_N is the nucleon spinor, and λ_N is the nucleon pole residue (representing the quark coupling to the nucleonic current). Using the results in (B13) and (B18), the required matrix elements can now be calculated. One obtains

$$\text{Im} \Pi(\omega, p=0, T) = \pm \frac{3\lambda^2}{32\pi f_\pi^2} \left[M - \gamma_0 \frac{(\omega^2 + M^2)}{2\omega} \right] \cdot \frac{\frac{M^2 - \omega^2}{2\omega^2}}{\exp\left[\frac{M^2 - \omega^2}{2|\omega|T}\right] - 1} \quad \dots (4.3)$$

for the N and Δ , and

$$\text{Im} \Pi(\omega, p=0, T) = \pm \frac{\lambda^2}{6\pi f_\pi^2} \left[M - \gamma_0 \frac{(\omega^2 + M^2)}{2\omega} \right] \cdot \frac{\frac{M^2 - \omega^2}{2\omega^2}}{\exp\left[\frac{M^2 - \omega^2}{2|\omega|T}\right] - 1} \quad \dots (4.4)$$

for the Σ , Ξ , Σ^* , and Ξ^* , where the \pm sign corresponds to whether $0 < \omega < M$ or $-M < \omega < 0$ respectively.

The 2 different cases (they differ by a factor of 16/9) arise because of the fact that the nucleon and the delta consist of u and d quarks only, and hence SU(2) symmetry applies, whereas for the rest of the currents SU(3) applies. In the case of the nucleon, the result is $\frac{1}{2}$ times the result obtained in [19] and [20]. (The authors of [19] and [20] have redone the calculations and now agree with (4.3).) Also, as stated in the two references, the two equations show that the scattering contribution for $T \ll M$ is exponentially suppressed except for a narrow region in the vicinity of $\omega = \pm M$, where $\text{Im } \Pi_{1,2}$ is of order $O(T)$.

Due to the \pm sign in (4.3) and (4.4), pion current scattering contributes only to the FESR involving $\text{Im } \Pi_1$:

$$\begin{aligned} \text{Im } \Pi_1(\omega, p=0, T) &= \pm \frac{3\lambda^2}{64\pi f_\pi^2 \omega^2} \cdot \frac{M(M^2 - \omega^2)}{\exp\left[\frac{M^2 - \omega^2}{2|\omega|T}\right] - 1} \\ &\equiv \pm \frac{3\lambda^2 M}{64\pi f_\pi^2 \omega} H(\omega, T) \end{aligned} \quad \dots (4.5)$$

for N and Δ , and

$$\begin{aligned} \text{Im } \Pi_1(\omega, p=0, T) &= \pm \frac{\lambda^2}{12\pi f_\pi^2 \omega^2} \cdot \frac{M(M^2 - \omega^2)}{\exp\left[\frac{M^2 - \omega^2}{2|\omega|T}\right] - 1} \\ &\equiv \pm \frac{\lambda^2 M}{12\pi f_\pi^2 \omega} H(\omega, T) \end{aligned} \quad \dots (4.6)$$

for Σ , Ξ , Σ^* , and Ξ^* , where

$$H(\omega, T) \equiv \frac{M^2 - \omega^2}{\omega} \cdot \frac{1}{\exp\left[\frac{M^2 - \omega^2}{2|\omega|T}\right] - 1} \quad \dots (4.7)$$

5. APPLYING THE FINITE ENERGY SUM RULES

The lowest dimension FESR corresponding to the two-point function in (2.1) are

$$\frac{1}{\pi} \int_0^{s_0(T)} ds \text{Im} \Pi_{1,2}(s, T) \Big|_{\text{QCD}} = \frac{1}{\pi} \int_0^{s_0(T)} ds \text{Im} \Pi_{1,2}(s, T) \Big|_{\text{HAD}} \quad \dots (5.1)$$

where $s_0(T)$ is the asymptotic freedom threshold. In general, the results for $\text{Im} \Pi_{1,2}$ are

$$\begin{aligned} \text{Im} \Pi_1 \Big|_{\text{QCD}} &= A \langle \bar{q}q \rangle_T \omega^2 [1 - 2n_F(\omega/2T)] + B\omega^4 + C \frac{F_m(\omega, T)}{\omega} \\ &\dots (5.2) \\ \text{Im} \Pi_2 \Big|_{\text{QCD}} &= X \langle \bar{q}q \rangle_T \omega^2 [1 - 2n_F(\omega/2T)] + Y\omega^4 + Z \frac{F^{o/d}(\omega, T)}{\omega} \end{aligned}$$

where it is assumed that $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = \langle \bar{s}s \rangle \equiv \langle \bar{q}q \rangle$, which for our purposes, where one is interested only in the relative magnitude $M(T)/M(0)$, is a reasonable assumption. The function $F^{o/d}(\omega, T)$ corresponds to either $F^o(\omega, T)$ or $F^d(\omega, T)$ and comes from the zeroth order (in mass) terms in the spectral function, while $F_m(\omega, T)$ arises from the first order mass terms.

In the hadronic case, the spectral function is saturated with the particular pole (e.g. nucleonic), so that higher $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ resonances are absorbed in the continuum modelled by perturbative QCD. In Appendix D, where the details of the calculation are contained, it is shown that for the octet

$$\text{Im}\Pi(p)|_{\text{HAD}} = \pi\lambda^2 (p+M) \delta(p^2 - M^2) \quad \dots (5.3)$$

and for the decuplet

$$\text{Im}\Pi_{\mu\nu}(p)|_{\text{HAD}} = -\pi\lambda^2 \delta(p^2 - M^2) \{g_{\mu\nu}p + M g_{\mu\nu} + \dots\} \quad \dots (5.4)$$

From this, and §4 equations (4.5) and (4.6), it follows that

$$\begin{aligned} \frac{1}{\pi} \text{Im}\Pi_1(s, T)|_{\text{HAD}} &= \lambda^2 M(T) \delta(s - M^2(T)) + \theta(s - s_0) \frac{1}{\pi} \text{Im}\Pi_1(s, T)|_{\text{QCD}} \\ &\quad + \frac{1}{\pi} \frac{GM(T)}{2\omega} H(\omega, T) \end{aligned} \quad \dots (5.5)$$

where $H(\omega, T)$ is given by (4.7), and G is fixed by (4.5) and (4.6). Also

$$\frac{1}{\pi} \text{Im}\Pi_2(s, T)|_{\text{HAD}} = \lambda^2 \delta(s - M^2(T)) + \theta(s - s_0) \frac{1}{\pi} \text{Im}\Pi_2(s, T)|_{\text{QCD}} \quad \dots (5.6)$$

Before applying the FESR, inspection of the spectral functions in (5.2) at the quark level shows that the thermal corrections are of order $O(T)$ in the case of $\text{Im}\Pi_2$ and order $O(T^4)$ in the case of $\text{Im}\Pi_1$. Hence the latter can be neglected in the whole range $0 < T < T_c$.

Substituting (5.2,5.6) into (5.1), and solving for $M(T)$, one finds

$$M(T) = \frac{\frac{3A \langle \bar{q}q \rangle_T}{2Y s_0(T)} + \frac{B}{Y} + \frac{6C}{Y s_0^3(T)} \int_0^{\sqrt{s_0(T)}} F_m(\omega, T) d\omega}{\left[1 + \frac{G}{\pi\lambda^2} \int_0^{\sqrt{s_0(T)}} H(\omega, T) d\omega \right] \left[1 + \frac{6Z}{Y s_0^3(T)} \int_0^{\sqrt{s_0(T)}} F^{o/d}(\omega, T) d\omega + \frac{3X \langle \bar{q}q \rangle_T}{2Y s_0(T)} \right]} \quad \dots (5.7)$$

At $T = 0$, it follows from (5.2) and (4.5-6) that $G = C = Z = 0$ and so

$$M(0) = \frac{\frac{3A \langle \bar{q}q \rangle_0}{2Y s_0(0)} + \frac{B}{Y}}{1 + \frac{3X \langle \bar{q}q \rangle_0}{2Y s_0(0)}} \quad \dots (5.8)$$

Thus one can now obtain $M(T)/M(0)$, which can be further simplified by using the result from [14]

$$\frac{\langle \bar{q}q \rangle_T}{\langle \bar{q}q \rangle_0} \approx \frac{s_0(T)}{s_0(0)} \quad \dots (5.9)$$

The only unknown is $s_0(0)$, but from (5.8) it follows that

$$s_0(0) = - \langle \bar{q}q \rangle_0 \frac{3(A - XM(0))}{2(B - YM(0))} \quad \dots (5.10)$$

For example, for the nucleon $s_0(0) = -24\pi^2 \frac{\langle \bar{q}q \rangle_0}{M_N(0)} \quad \dots (5.11)$

Since we are not interested here in the accurate prediction of M itself, but rather in its temperature behaviour, the above expression can be used to normalize s_0 .

The results obtained are discussed in the next section.

6. RESULTS AND CONCLUSIONS

Numerically, the ω -integrals in equation (5.7) requires the most effort. The ones involving $F_m(\omega, T)$ and $F^{o/d}(\omega, T)$ are 3-dimensional, with integration variables x, y and ω , where x and y correspond to the momenta k_1 and k_2 of two quarks. These integrals, as well as the (scattering) integral over $H(\omega, T)$, have been numerically evaluated using the 8-point Gauss-Legendre integration formula. Figure 6.1 shows the temperature behaviour of $|I_a|$, I_s and $|I_m|$ in the case of the Σ , where (cf. 5.7)

$$I_a = \frac{6Z}{Y_{S_0^3}(T)} \int_0^{\sqrt{s_0(T)}} F(\omega, T) d\omega \quad \dots (6.1)$$

$$I_s = \frac{G}{\pi\lambda^2} \int_0^{\sqrt{s_0(T)}} H(\omega, T) d\omega \quad \dots (6.2)$$

$$I_m = \frac{6C}{Y_{S_0^3}(T)} \int_0^{\sqrt{s_0(T)}} F_m(\omega, T) d\omega \quad \dots (6.3)$$

and where 'a', 's' and 'm' refer to annihilation, scattering and mass (of strange quark) respectively. Taking into account (5.9), it is clear that the above three terms are the only temperature dependent terms in the expression for $M(T)$. The values used are $\langle \bar{q}q \rangle_0 \approx -0.01 \text{ GeV}^3$ and $f_\pi(T=0) = 87 \text{ MeV}$.

From Figure 6.1 it is seen that $|I_a|$ and $|I_m|$ are monotonically increasing functions of T . However in the case of the hadronic scattering contribution I_s , an extra complication arises, because the restraint $\omega^2 < M^2(T)$ (see §4) results in the upper limit of integration for the integral $\int_0^{\sqrt{s_0(T)}} H(\omega, T) d\omega$ effectively being $M(T)$ as long as $s_0(T) > M^2(T)$. One expects this to be the case at small T , since in general (5.10) shows that $s_0(0) > M(0)$. Though, as T increases, $s_0(T)$ decreases (see Figure 1.1) and thus at some stage it becomes less than $M(T)$ so that the upper limit becomes $s_0(T)$. As a result of this, I_s initially increases with temperature, and then after reaching a maximum, decreases again, vanishing at $T=T_c$. In the case of the Σ , this maximum clearly occurs at $T/T_c = 0.93$.

The above comments and analysis hold in general for the other octet and decuplet particles.

The problem of having the unknown $M(T)$ as an upper limit of integration still remains, but it can be overcome by using the secant root finding method to obtain a final value for $M(T)$ at each temperature. Using equations (5.7), (5.8) and (5.9) together with the expressions for the imaginary parts of the octet and decuplet spectral functions, a numerical value for the ratio $M(T)/M(0)$ can be obtained for each of the octet and decuplet particles over the temperature range $0 < T < T_c$. The results are shown in Figures 6.2 - 6.9. The masses of the particles as well as the normalizing factors $s_0(0)$ are given in each case.

In general the ratio $M(T)/M(0)$ is found to be approximately equal to one up to a point, after which it increases sharply. This sharp increase is partly due to the (hadronic) scattering contribution, as discussed above (but not only to

it, since the same happens in the case of the Λ and the Ω where the scattering contribution is absent). This behaviour is also found to be relatively insensitive to the errors of uncertainty in the constants used (f_π , the vacuum condensate and the masses of the particles at $T = 0$). It does of course depend crucially on the various terms (and their signs) in the imaginary parts of the spectral functions.

The temperature evolution of $M(T)$ for the octet and decuplet particles agrees with previous results for the nucleon [20], and does not support sporadic claims made that hadron masses should vanish at the critical temperature.

Figure 6.1 : BEHAVIOUR OF TEMPERATURE DEPENDENT INTEGRALS

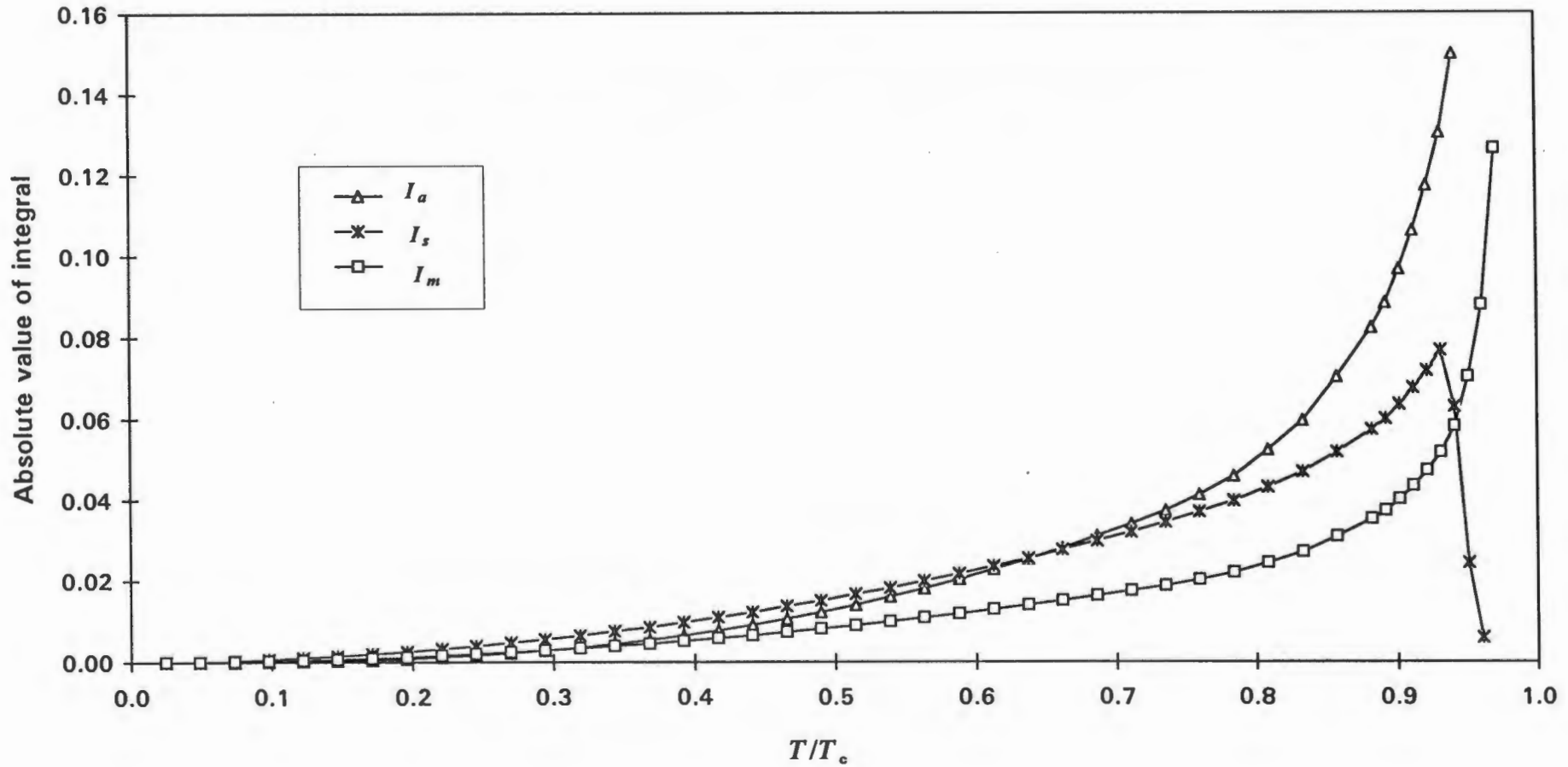


Figure 6.2 : NUCLEON MASS AT FINITE TEMPERATURE

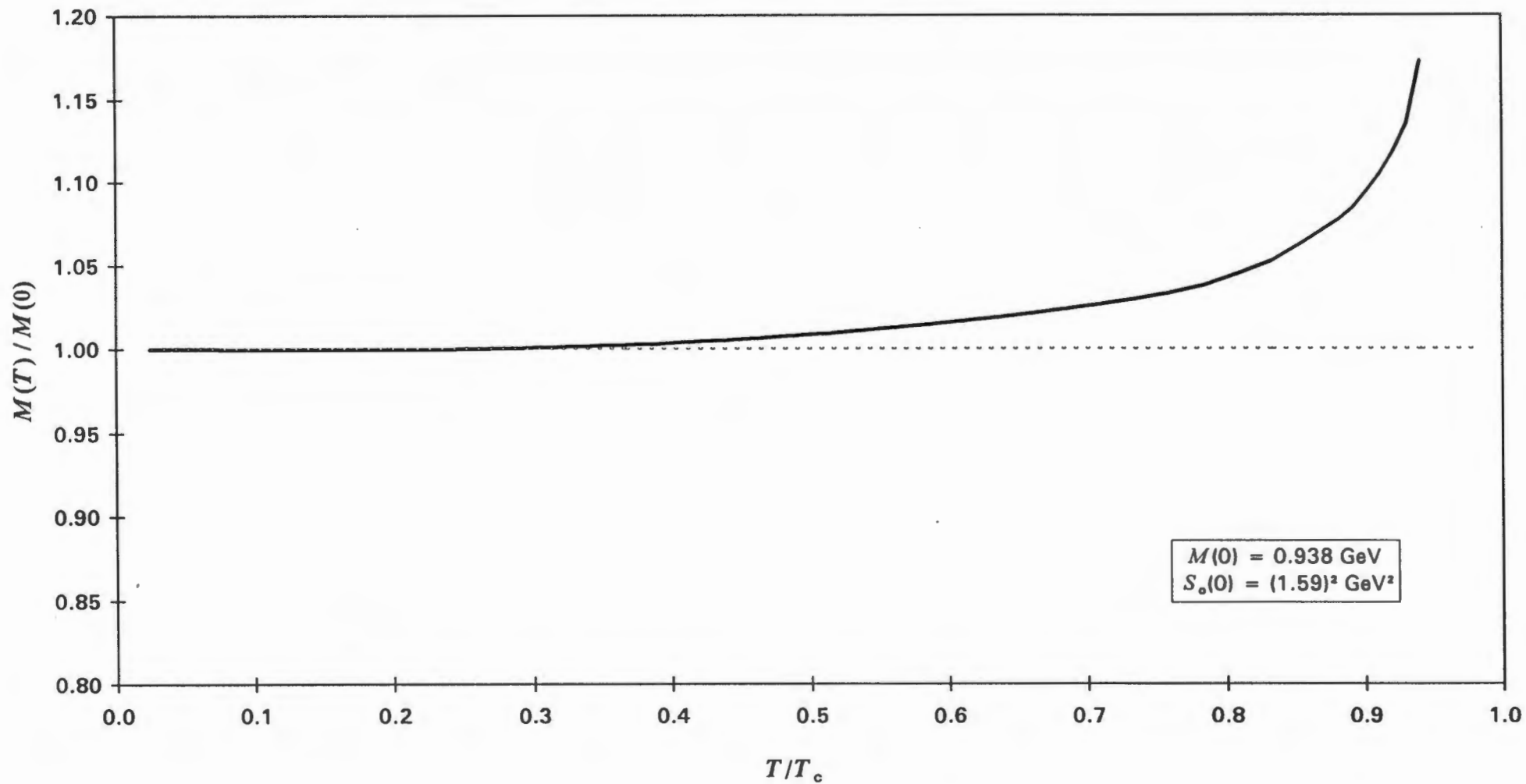


Figure 6.3 : LAMBDA MASS AT FINITE TEMPERATURE

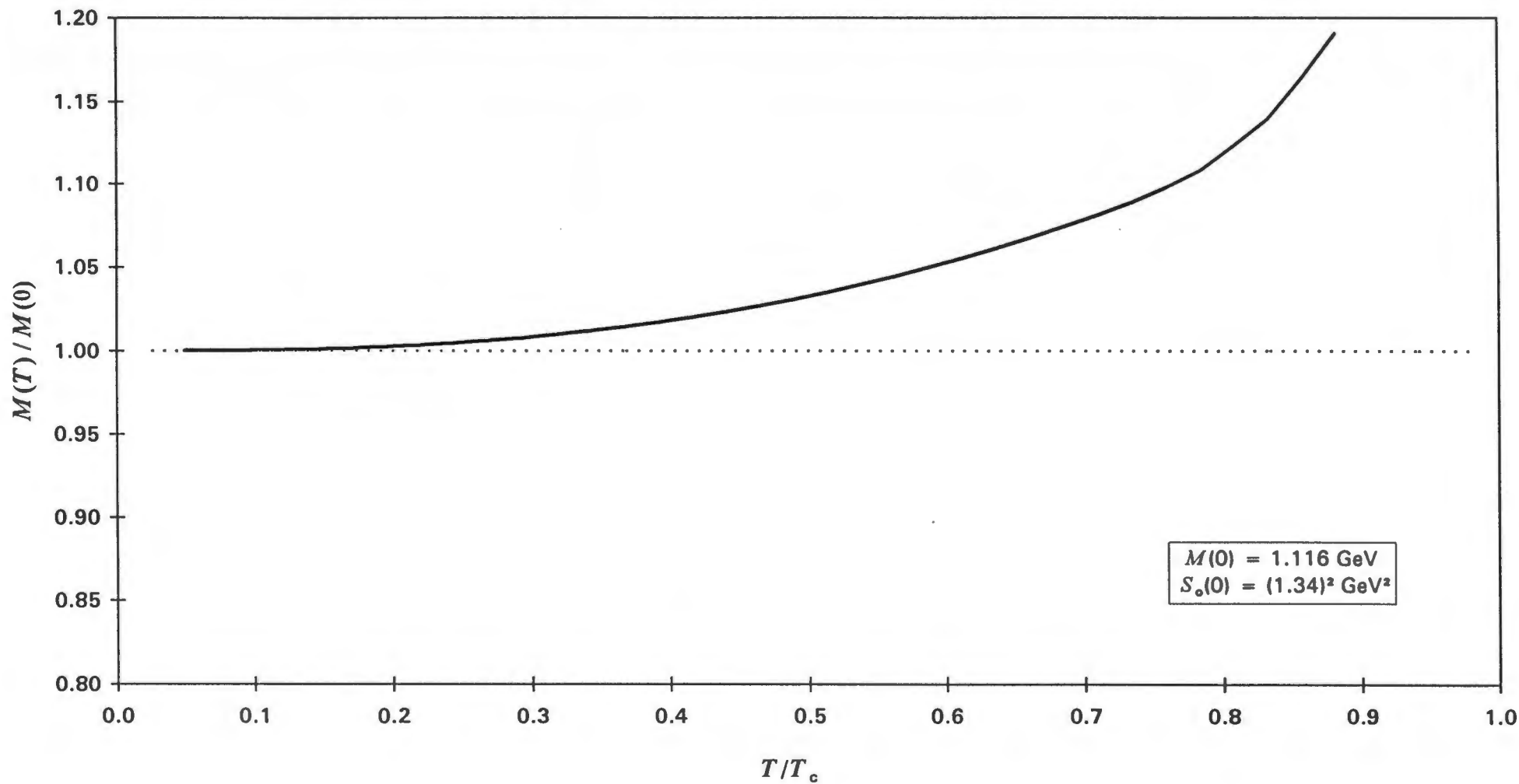


Figure 6.4 : CASCADE MASS AT FINITE TEMPERATURE

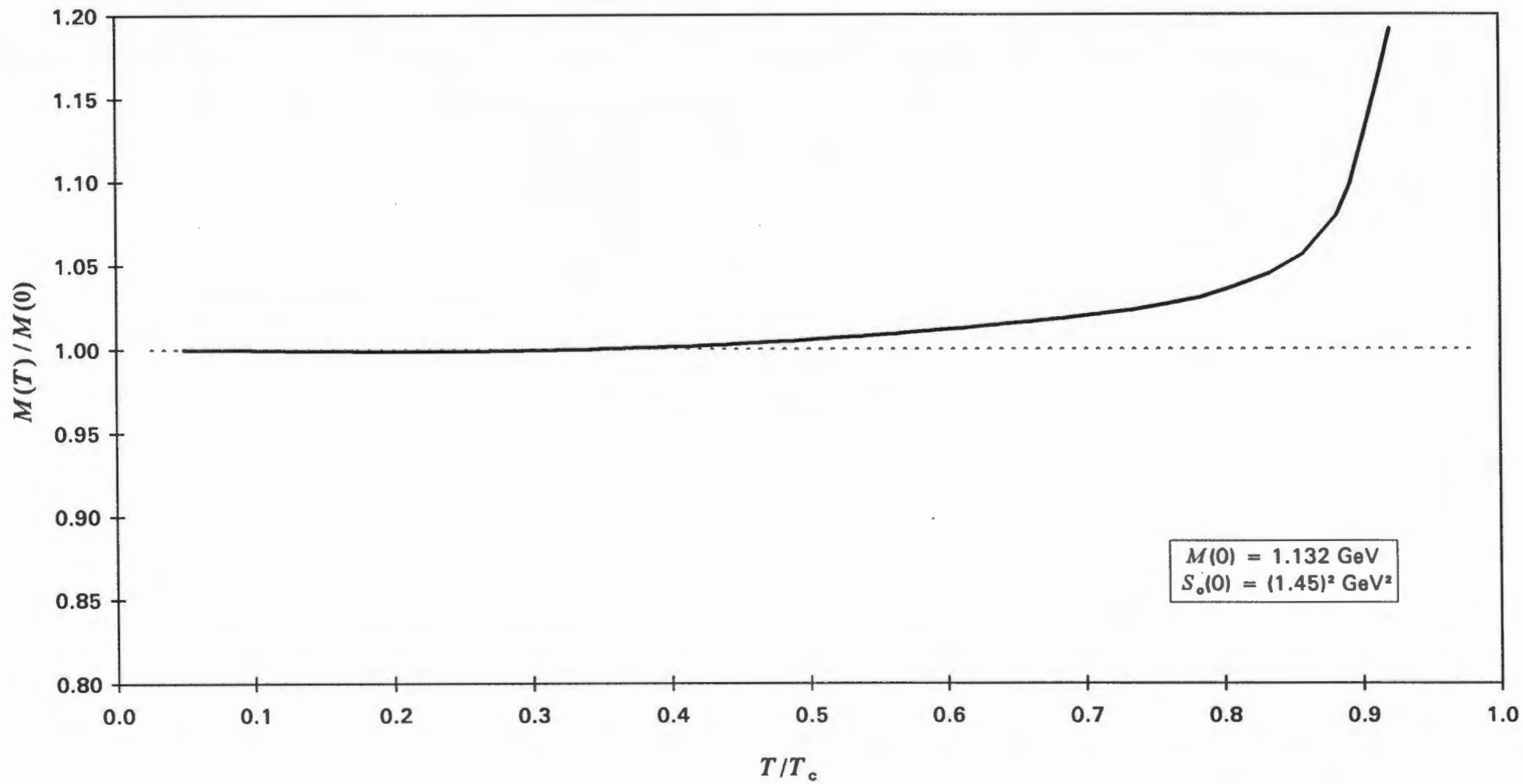


Figure 6.5 : SIGMA MASS AT FINITE TEMPERATURE

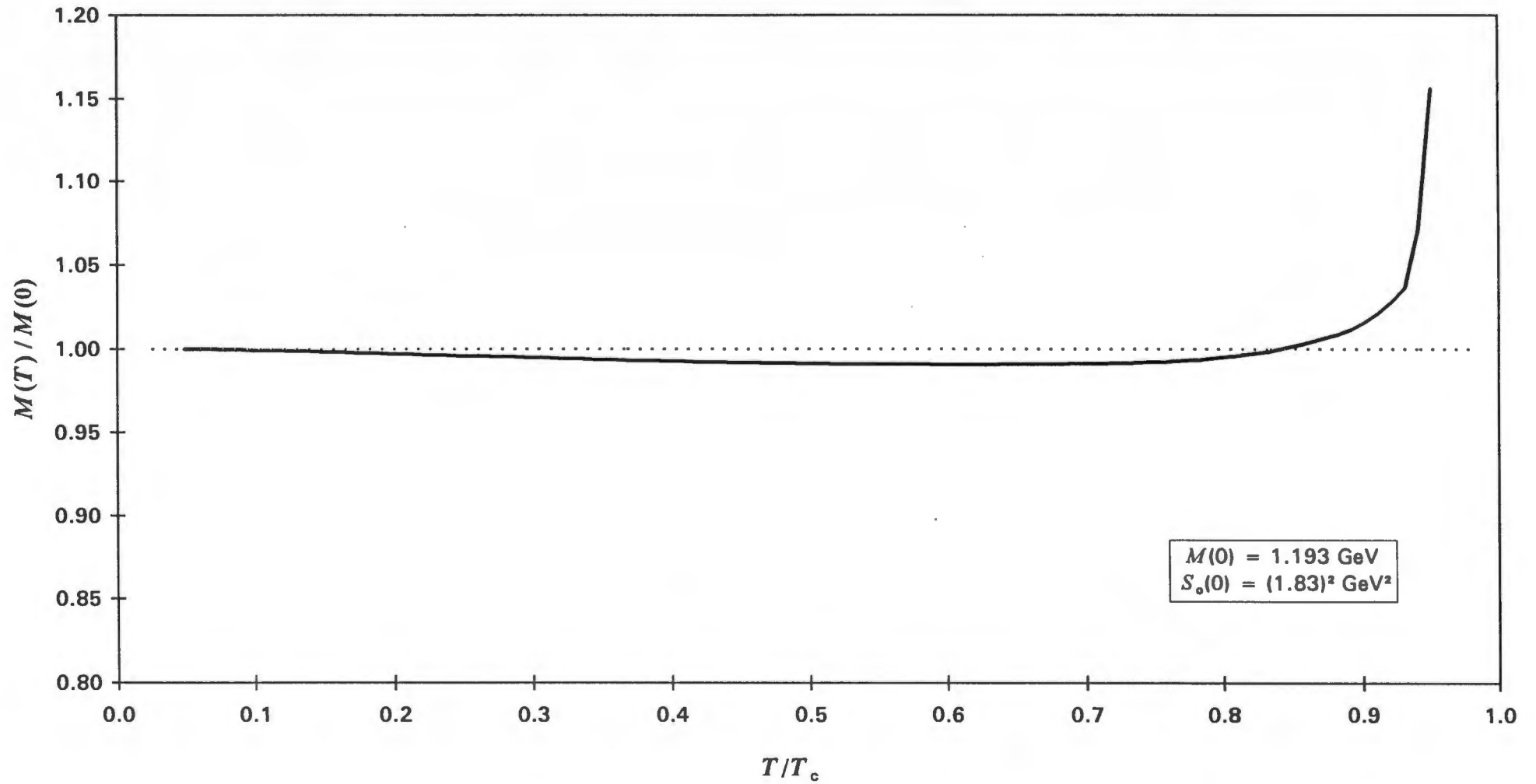


Figure 6.6 : DELTA MASS AT FINITE TEMPERATURE

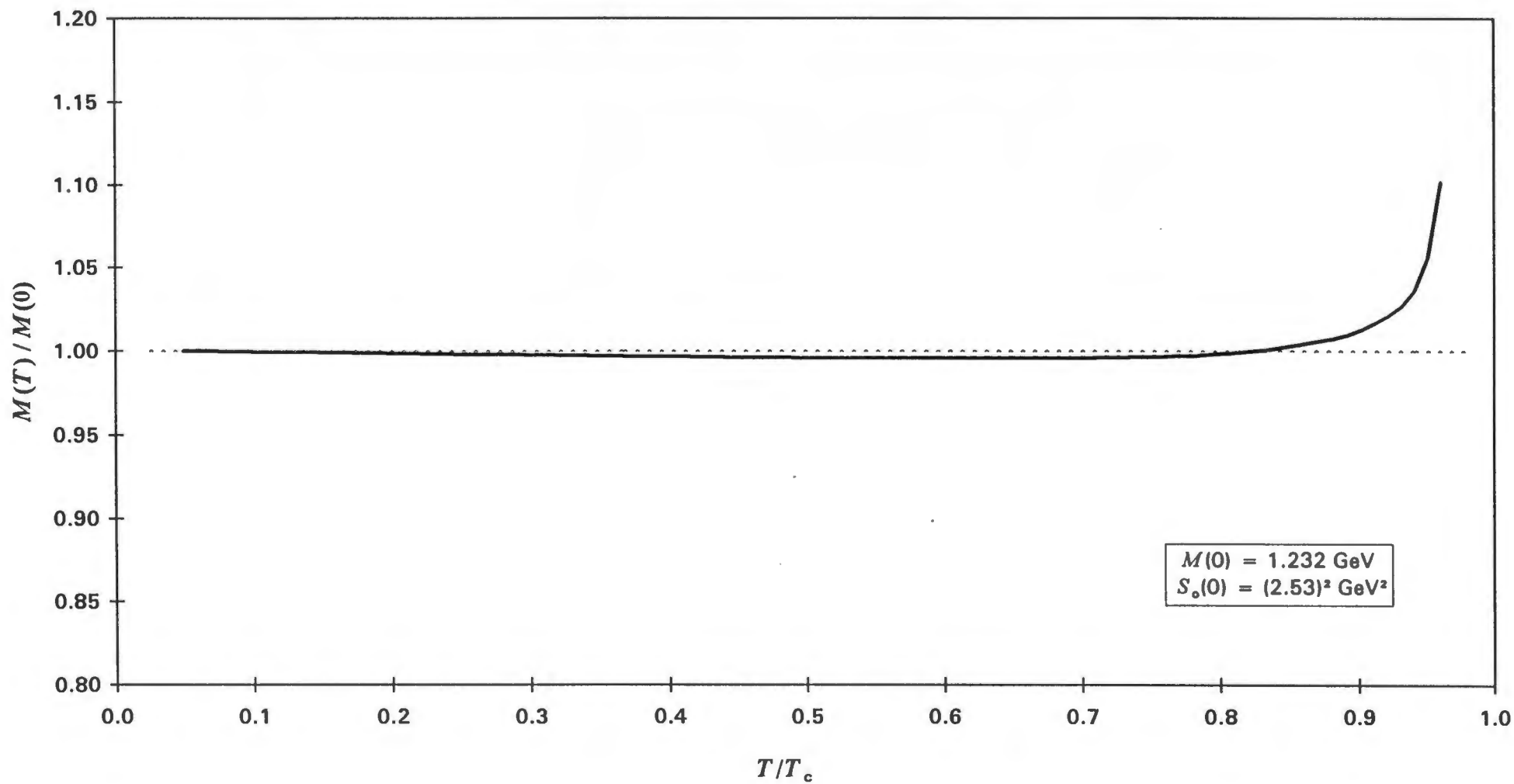


Figure 6.7 : SIGMA* MASS AT FINITE TEMPERATURE

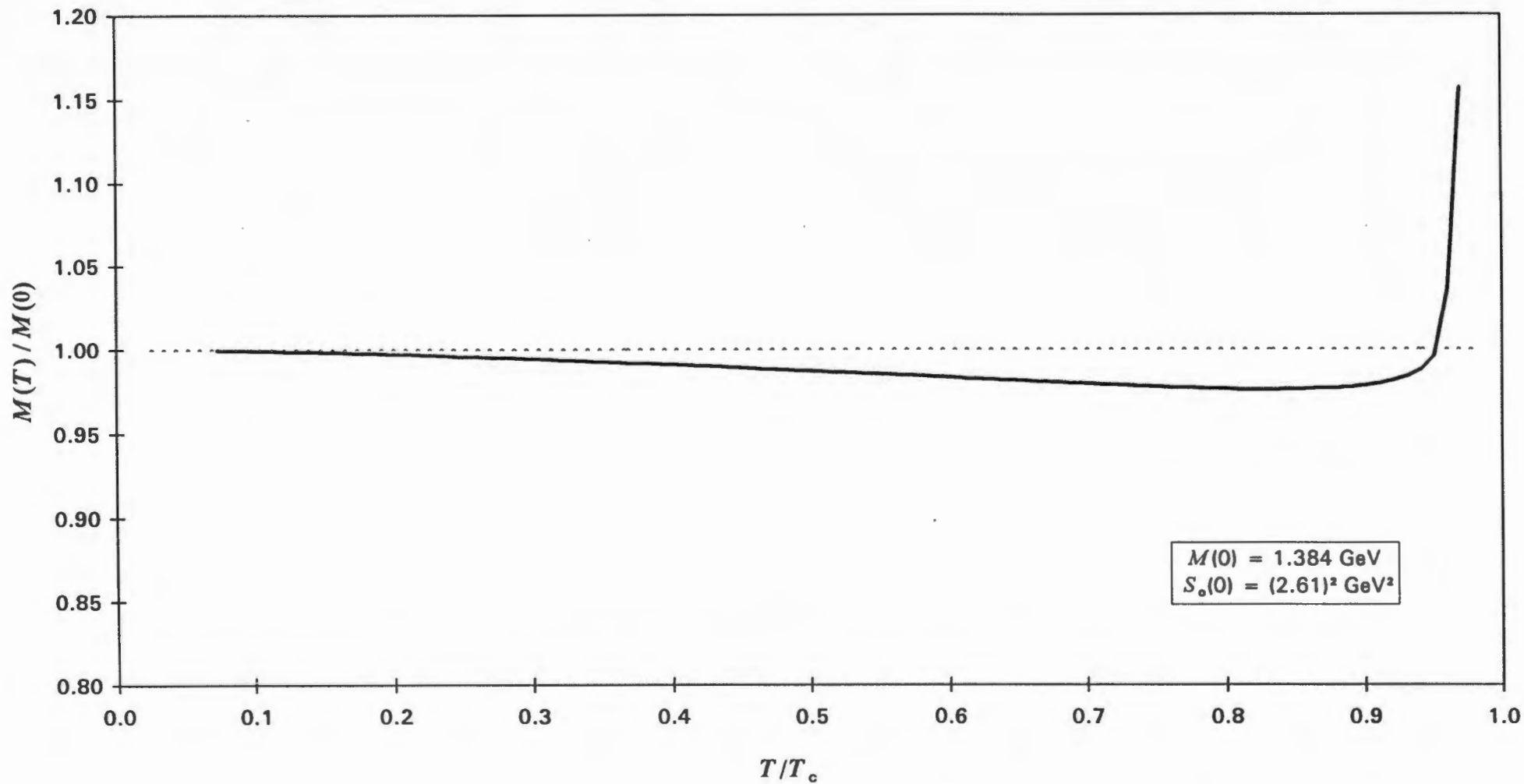


Figure 6.8 : CASCADE* MASS AT FINITE TEMPERATURE

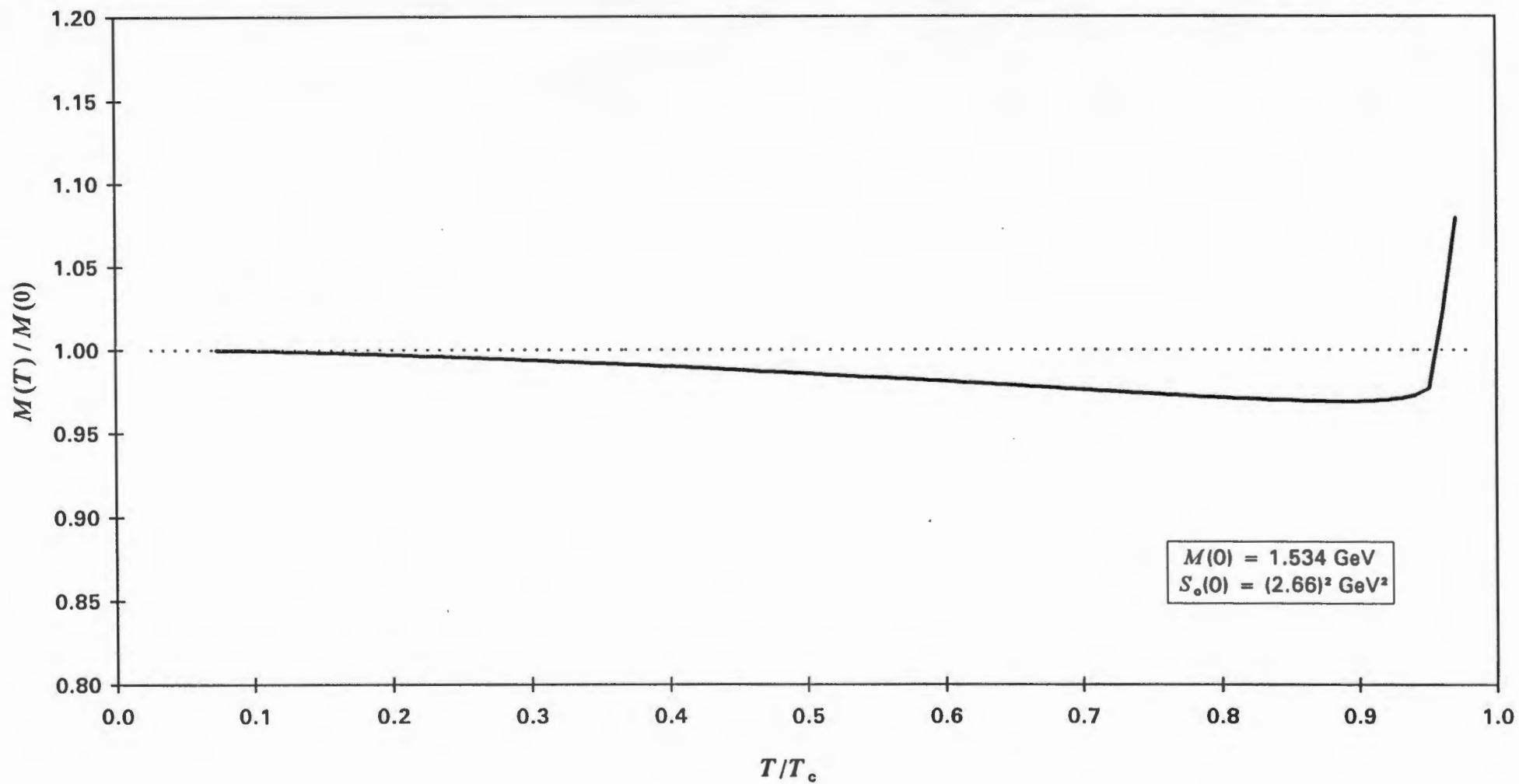
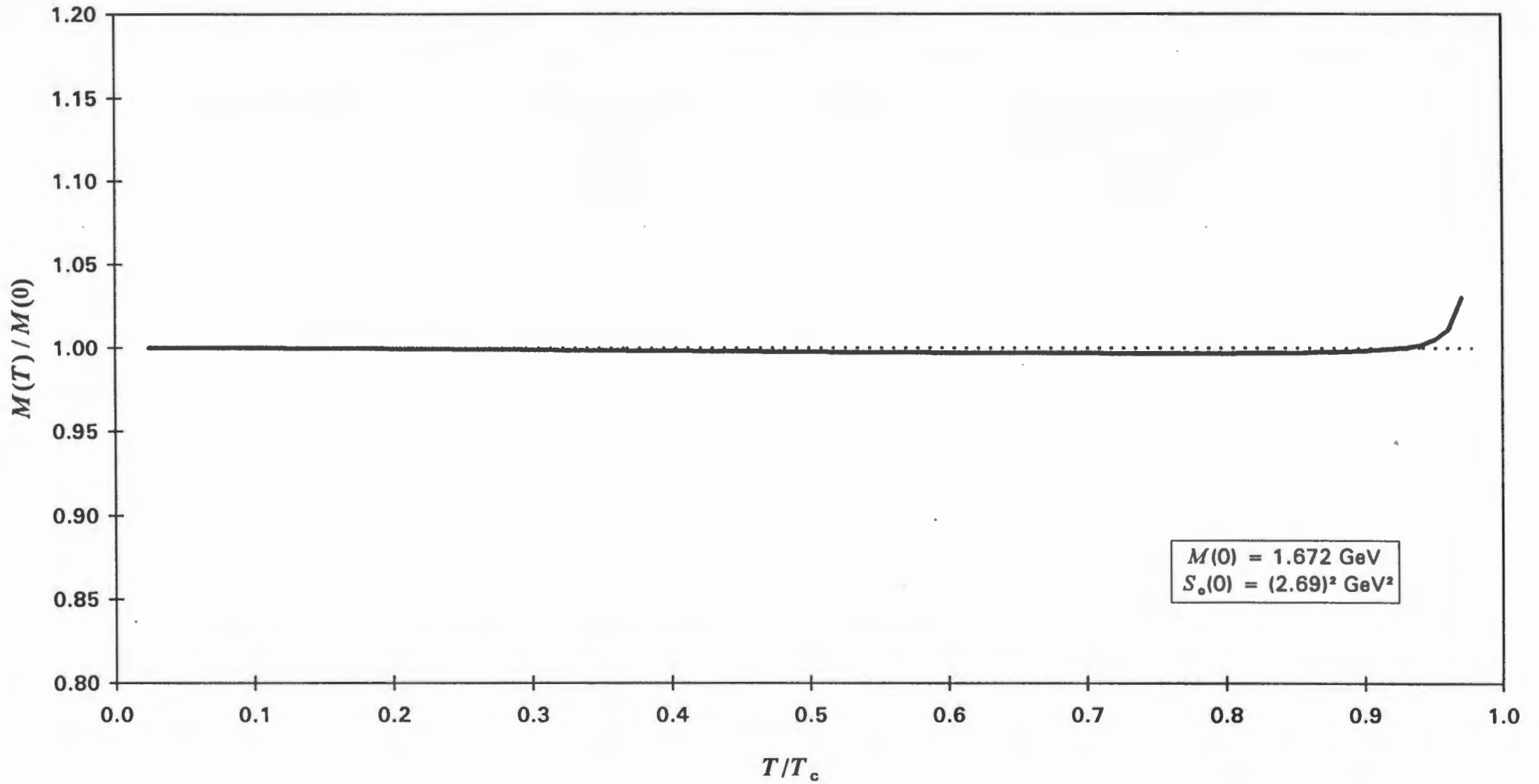


Figure 6.9 : OMEGA MASS AT FINITE TEMPERATURE



APPENDIX A : ORDERING AND WICK'S THEOREM

1. Normal ordering

By definition, **normal ordering** implies that all creation operators stand to the left of all annihilation operators. For example,

$$(a b^\dagger c d^\dagger) \xrightarrow[\text{ordered}]{\text{normally}} (b^\dagger d^\dagger a c) \quad \dots \text{(A1)}$$

Note that normal ordering does not fix the order of creation or annihilation operators among themselves. This follows since they commute among themselves and hence can be written in any order.

The concept of normal ordering is then used to define the **normal product** of a set of standard operators, where standard means linear in either creation or annihilation operators. Let Q, R, \dots, W be a set of such operators. Their normal product, usually denoted by $N(\dots)$ or a pair of colons $(: \dots :)$, is then given by

$$\text{fermions : } N(QR\dots W) = (-1)^P (Q'R' \dots W') \quad \dots \text{(A2)}$$

$$\text{bosons : } N(QR\dots W) = (Q'R' \dots W') \quad \dots \text{(A3)}$$

where Q', R', \dots, W' are the operators Q, R, \dots, W reordered so that all annihilation operators stand to the right of all creation operators. The exponent P is the number of interchanges of neighbouring fermion operators required to change the order $(QR\dots W)$ to $(Q'R' \dots W')$. Hence in arranging a product of boson operators in normal order, one simply treats them as

though all commutators vanish. Likewise fermion operators are treated as though all anti-commutators vanish. Note too that

$$N(QR) = \pm N(RQ) \quad \dots (A4)$$

where the minus sign refers to the case of two fermions, and the plus sign in all other cases.

In general it is also required that the normal product be distributive :

$$N(RS\dots+VW\dots) = N(RS\dots)+N(VW\dots) \quad \dots (A5)$$

One advantage of using the normal product lies in the fact that it avoids the introduction of virtual intermediate particles. Since in a normal product, all annihilation operators stand to the right of all creation operators, such an operator first absorbs a number of particles, and then emits some particles. It does not cause emission and reabsorption of intermediate particles, which greatly simplifies calculations.

An additional property of a normal product is that the vacuum expectation value (vev) of any such product vanishes. This follows since the vacuum state $|0\rangle$, in which no particles are present, is characterised by

$$a|0\rangle = 0 \quad \dots (A6)$$

for any annihilation operator a .

Thus normal ordering of the operators comprising an observable physically means normalisation of that observable with respect to the vacuum.

2. Time ordering

Whereas normal ordering involves ordering operators according to their functions (creation or annihilation), time ordering involves ordering operators chronologically.

In general, for any number of boson and fermion operators, the time-ordered product (or T-product) orders them such that later times stand to the left of earlier times. As in the case of normal ordering, boson (fermion) fields are treated as though their commutators (anti-commutators) vanish :

fermions :

$$T[\phi(x_1)\phi(x_2)\dots\phi(x_n)] = (-1)^P\phi(x_{1'})\phi(x_2)\dots\phi(x_{n'}) \quad \dots (A7)$$

bosons :

$$T[\phi(x_1)\phi(x_2)\dots\phi(x_n)] = \phi(x_{1'})\phi(x_2)\dots\phi(x_{n'}) \quad \dots (A8)$$

where $t_{1'} > t_2 > \dots > t_{n'}$ and again the exponent P is the number of interchanges of neighbouring fermion operators required to change the order $(x_1 x_2 \dots x_n)$ to $(x_{1'} x_2 \dots x_{n'})$.

3. Wick's Theorem

On quantization, any scalar field ϕ becomes a hermitian operator, satisfying the usual equal time commutation relations. To establish contact with

particles, $\varphi(x)$ is expanded in a complete set of positive and negative frequency parts

$$\varphi(x) = \varphi^+(x) + \varphi^-(x) \quad \dots (A9)$$

where φ^+ and φ^- involves only annihilation and creation operators respectively. From (A2) and (A3) it follows that

$$\begin{aligned} N[\varphi(x)\varphi(y)] &= \varphi^+(x)\varphi^+(y) + \varphi^-(x)\varphi^-(y) + \varphi^-(x)\varphi^+(y) + \varphi^-(y)\varphi^+(x) \\ &= \varphi(x)\varphi(y) - [\varphi^+(x), \varphi^-(y)] \quad \dots (A10) \end{aligned}$$

$$\begin{aligned} N[\psi(x)\psi(y)] &= \psi^+(x)\psi^+(y) + \psi^-(x)\psi^-(y) + \psi^-(x)\psi^+(y) + \psi^-(y)\psi^+(x) \\ &= \psi(x)\psi(y) - \{\psi^+(x), \psi^-(y)\} \quad \dots (A11) \end{aligned}$$

for bosons and fermions respectively, where $[,]$ and $\{ , \}$ refer to the commutator and anti-commutator. For two fermion (boson) fields, the anti-commutator (commutator) is a c-number, i.e. it does not involve creation or annihilation operators. Hence for any two operators A and B

$$AB = N(AB) + \langle 0|AB|0\rangle \quad \dots (A12)$$

from which it follows that for $x_1^0 \neq x_2^0$

$$T[A(x_1)B(x_2)] = N[A(x_1)B(x_2)] + \langle 0|T[A(x_1)B(x_2)]|0\rangle \quad \dots (A13)$$

The second term on the right of (A13) is called the *contraction* of $A(x_1)$ and $B(x_2)$ and is denoted by

$$\langle 0 | T[A(x_1)B(x_2)] | 0 \rangle \equiv \underbrace{A(x_1)B(x_2)} \quad \dots (A14)$$

Being a vacuum expectation value, it will vanish unless one of the operators A or B creates particles which the other absorbs. The non-vanishing contractions are the Feynman propagators.

In order to generalise (A13) one needs to define the *generalised normal product* :

$$N(\underbrace{ABCDE\dots MNOP\dots}) = (-1)^p \underbrace{A} \underbrace{N} \underbrace{BC} \underbrace{EP} \dots N(CDF\dots MO\dots) \quad \dots (A15)$$

Using this, Wick has proved the following generalisation of (A13) for unequal times ($x_a^0 \neq x_b^0$ for $a \neq b$) :

$$\begin{aligned} & T(ABCD\dots WXYZ) \\ &= N(ABCD\dots WXYZ) \\ &+ N(\underbrace{ABC\dots YZ}) + N(\underbrace{ABC\dots YZ}) + \dots + N(\underbrace{ABC\dots YZ}) \\ &+ N(\underbrace{ABCD\dots YZ}) + \dots + N(\underbrace{AB\dots WXYZ}) \\ &+ \dots \end{aligned} \quad \dots (A16)$$

The right-hand side contains the sum of all possible generalised normal products that can be formed from $(ABC\dots XYZ)$, preserving the original order throughout.

Wick extended the theorem further to include mixed T-products :

$$T[N(AB\dots)_{x_1} \dots N(AB\dots)_{x_n}] = T[(AB\dots)_{x_1} \dots (AB\dots)_{x_n}]_{\text{no e.t.c.}} \quad \dots (A17)$$

where "e.t.c." stands for "equal-time contraction". Hence when taking the T-product of a group of normally ordered operator sets, one uses the standard rule (A16), but without contracting any equal-time operators.

APPENDIX B : CURRENT COMMUTATORS

1. Aim

The aim of this appendix is to find $[Q_5^a, \eta]$, i.e. the commutator of the axial charge Q_5^a and some current η . The axial charge is given by [19]

$$Q_5^a = \frac{1}{2} \int q_\ell^\dagger(x) \gamma_0 \gamma_5 \lambda_{\ell m}^a q_m(x) d^3x \quad \dots (B1)$$

where q_i denote the quark fields, with flavour indices running from 1 to 3, since only currents consisting of up, down and strange quarks are considered (i.e. SU(3) symmetry). The λ^a ($a = 1 \dots 8$) are the standard Gell-Mann matrices of SU(3). As usual, repeated indices implies summation.

2. Poisson brackets

In order to achieve this aim, one needs some results regarding Poisson brackets. For a discrete system, the *Poisson bracket* of two functions u and v with respect to canonical variables (q, p) is defined as

$$\begin{aligned} [u, v] &= \sum_{i=1}^n \left(\frac{\partial u}{\partial q_i} \frac{\partial v}{\partial p_i} - \frac{\partial u}{\partial p_i} \frac{\partial v}{\partial q_i} \right) \\ &= u_q v_p - u_p v_q \quad (n=1) \end{aligned} \quad \dots (B2)$$

This formulation can be extended to include products of two or more functions :

$$\begin{aligned}
[uv, w] &= (uv)_q w_p - (uv)_p w_q \\
&= uv_q w_p + u_q w_p v - uv_p w_q - u_p w_q v \\
&= u(v_q w_p - v_p w_q) + (u_q w_p - u_p w_q) v \\
&= u[v, w] + [u, w] v \qquad \dots (B3)
\end{aligned}$$

and

$$\begin{aligned}
[uvw, z] &= uv[w, z] + [uv, z]w \\
&= uv[w, z] + u[v, z]w + [u, z]vw \qquad \dots (B4)
\end{aligned}$$

Clearly the same holds for a continuous system, where the Poisson bracket is defined in terms of a pair of densities.

3. Current-quark commutators

The next step is to find an expression for $[Q_5^a, q_k]$. From (B1)

$$\begin{aligned}
[Q_5^a, q_k(y)] &= \frac{i}{2} \int d^3x [q_m^+(x) \gamma_5 \lambda_{mn}^a q_n(x), q_k(y)] \\
&= \frac{i}{2} \gamma_5 \lambda_{mn}^a \int d^3x [q_m^+(x) q_n(x), q_k(y)] \qquad \dots (B5)
\end{aligned}$$

Using (B4), as well as the usual equal-time anti-commutation relations for fermion fields,

$$\{q_m^+(x), q_n^+(y)\} = \{q_m(x), q_n(y)\} = 0 \quad \dots (B6)$$

$$\{q_m^+(x), q_n(y)\} = \{q_m(x), q_n^+(y)\} = \delta_{mn} \delta^3(x-y)$$

it follows that

$$\begin{aligned} [q_m^+(x)q_n(x), q_k(y)] &= q_m^+(x) [q_n(x), q_k(y)] + [q_m^+(x), q_k(y)]q_n(x) \\ &= q_m^+(x) (\{q_n(x), q_k(y)\} - 2q_k(y)q_n(x)) \\ &\quad + (\{q_m^+(x), q_k(y)\} - 2q_k(y)q_m^+(x))q_n(x) \\ &= -2q_m^+(x)q_k(y)q_n(x) - 2q_k(y)q_m^+(x)q_n(x) \\ &\quad + \delta_{mk} \delta^3(x-y)q_n(x) \\ &= -2 \{q_m^+(x), q_k(y)\}q_n(x) + \delta_{mk} \delta^3(x-y)q_n(x) \\ &= -\delta_{mk} \delta^3(x-y)q_n(x) \end{aligned} \quad \dots (B7)$$

Hence

$$\begin{aligned} [Q_5^a, q_k(y)] &= -\frac{i}{2} \gamma_5 \lambda_{mn}^a \delta_{km} \int d^3x \delta^3(x-y) q_n(x) \\ &= -\frac{i}{2} \gamma_5 \lambda_{kn}^a q_n(y) \end{aligned} \quad \dots (B8)$$

The same result can be obtained in a more rigorous manner by using Lorentz indices. The procedure is analogous to the above.

4. Current-current commutators

For the baryon octet, the currents are of the form (2.3)

$$\eta(x) = \varepsilon_{abc} (q_i^a(x) C \gamma_\mu q_j^b(x)) \gamma^\mu \gamma^5 q_k^c(x) \quad \dots \text{(B9)}$$

or, denoting Lorentz indices explicitly and keeping the variable implicit,

$$\begin{aligned} \eta_\sigma &= \varepsilon_{abc} \left\{ (q_i^a)_\alpha C_{\alpha\beta} (\gamma_\mu)_{\beta\delta} (q_j^b)_\delta \right\} (\gamma^\mu)_{\sigma\tau} (\gamma^5)_{\tau\rho} (q_k^c)_\rho \\ &\equiv \varepsilon_{abc} \left\{ (q_i^a)_\alpha \Gamma_{\alpha\delta} (q_j^b)_\delta \right\} \Gamma'_{\sigma\rho} (q_k^c)_\rho \end{aligned} \quad \dots \text{(B10)}$$

Here $C\gamma^0$ is the charge conjugation operator, and (a,b,c) are colour indices.

Using (B8) one finds

$$\begin{aligned} [Q_5^a, \eta_\alpha(q_x q_y q_z)] &= \frac{1}{2} (\lambda^a)_{zi} (\gamma^5)_{\alpha\beta} \eta_\beta^1(q_x q_y q_i) \\ &\quad - \frac{1}{2} (\lambda^a)_{yi} \eta_\alpha^2(q_x q_i q_z) \\ &\quad - \frac{1}{2} (\lambda^a)_{xi} \eta_\alpha^3(q_i q_y q_z) \end{aligned} \quad \dots \text{(B11)}$$

where

$$\begin{aligned} \eta_\beta^1(q_x q_y q_z) &\equiv \varepsilon_{abc} \left\{ (q_x^a)_\alpha \Gamma_{\alpha\delta} (q_y^b)_\delta \right\} \Gamma'_{\beta\rho} (q_z^c)_\rho \\ \eta_\beta^2(q_x q_y q_z) &\equiv \varepsilon_{abc} \left\{ (q_x^a)_\alpha \Gamma_{\alpha\delta} (\gamma_5)_{\delta\sigma} (q_y^b)_\sigma \right\} \Gamma'_{\beta\rho} (q_z^c)_\rho \\ \eta_\beta^3(q_x q_y q_z) &\equiv \varepsilon_{abc} \left\{ (q_x^a)_\alpha (\gamma_5^T)_{\alpha\delta} \Gamma_{\delta\sigma} (q_y^b)_\sigma \right\} \Gamma'_{\beta\rho} (q_z^c)_\rho \end{aligned} \quad \dots \text{(B12)}$$

Comparing with (B10), it is clear that currents η^2 and η^3 do not correspond to octet currents, and hence are not relevant. The same applies to some of the η^1 currents, e.g. $\eta^1(q_1q_1q_1)$ or $\eta^1(uuu)$. Thus one finds for the octet currents listed in (2.3)

$$\begin{aligned}
[Q_s^a, \eta_\alpha^N] &= \frac{1}{2}(\tau^a)_{22}(\gamma_s)_{\alpha\beta} \eta_\beta^N \\
[Q_s^a, \eta_\alpha^\Sigma] &= \frac{1}{2}(\lambda^a)_{32}(\gamma_s)_{\alpha\beta} \eta_\beta^N + \frac{1}{2}(\lambda^a)_{33}(\gamma_s)_{\alpha\beta} \eta_\beta^\Sigma \\
[Q_s^a, \eta_\alpha^\Lambda] &= \sqrt{\frac{2}{3}} \frac{1}{2}(\gamma_s)_{\alpha\beta} [(\lambda^a)_{22} \eta_\beta^{\Lambda^1} - (\lambda^a)_{11} \eta_\beta^{\Lambda^2}] \\
[Q_s^a, \eta_\alpha^{\Xi}] &= -\frac{1}{2}(\lambda^a)_{11}(\gamma_s)_{\alpha\beta} \eta_\beta^{\Xi}
\end{aligned} \tag{B13}$$

Note that for the nucleon current, which consists of u and d quarks only, the eight Gell-Mann matrices λ^a of SU(3) are replaced by the three Pauli matrices τ^a of SU(2).

Results for the baryon decuplet follow in a similar manner. The currents are of the form (2.4)

$$\eta(x) = \varepsilon_{abc} \{q_i^a(x) C \gamma_\mu q_j^b(x)\} q_k^c(x) \tag{B14}$$

or, denoting Lorentz indices explicitly and keeping the variable implicit,

$$\begin{aligned}
\eta_\sigma &= \varepsilon_{abc} \{(q_i^a)_\alpha C_{\alpha\beta}(\gamma_\mu)_{\beta\delta} (q_j^b)_\delta\} (q_k^c)_\rho \\
&\equiv \varepsilon_{abc} \{(q_i^a)_\alpha \Gamma_{\alpha\delta} (q_j^b)_\delta\} (q_k^c)_\rho
\end{aligned} \tag{B15}$$

Using (B8) one finds, as in (B11),

$$\begin{aligned} [Q_5^a, \eta_\alpha(q_x q_y q_z)] &= \frac{1}{2}(\lambda^a)_{xi} (\gamma^5)_{\alpha\beta} \eta_\beta^1(q_x q_y q_i) - \frac{1}{2}(\lambda^a)_{yi} \eta_\alpha^2(q_x q_i q_z) \\ &\quad - \frac{1}{2}(\lambda^a)_{xi} \eta_\alpha^3(q_i q_y q_z) \end{aligned} \quad \dots (B16)$$

where

$$\begin{aligned} \eta_\beta^1(q_x q_y q_z) &\equiv \varepsilon_{abc} \left\{ (q_x^a)_\alpha \Gamma_{\alpha\delta} (q_y^b)_\delta \right\} (q_z^c)_\beta \\ \eta_\beta^2(q_x q_y q_z) &\equiv \varepsilon_{abc} \left\{ (q_x^a)_\alpha \Gamma_{\alpha\delta} (\gamma_5)_{\delta\sigma} (q_y^b)_\sigma \right\} (q_z^c)_\beta \\ \eta_\beta^3(q_x q_y q_z) &\equiv \varepsilon_{abc} \left\{ (q_x^a)_\alpha (\gamma_5^T)_{\alpha\delta} \Gamma_{\delta\sigma} (q_y^b)_\sigma \right\} (q_z^c)_\beta \end{aligned} \quad \dots (B17)$$

Once again it is clear, comparing with (B15), that currents η^2 and η^3 do not correspond to decuplet currents, and hence are not relevant. Similarly some of the η^1 currents, e.g. $\eta^1(q_2 q_2 q_2)$ or $\eta^1(ddd)$, do not appear in the decuplet. Thus for the decuplet currents listed in (2.4) one finds

$$\begin{aligned} [Q_5^a, \eta_\alpha^\Delta] &= -\frac{1}{2}(\lambda^a)_{11} (\gamma_5)_{\alpha\beta} \eta_\beta^\Delta \\ [Q_5^a, \eta_\alpha^{\Sigma^+}] &= -\frac{1}{2}(\gamma_5)_{\alpha\beta} \sqrt{\frac{1}{3}} \left[2(\lambda^a)_{11} \eta_\beta^{\Sigma^+} + (\lambda^a)_{33} \eta_\beta^{\Sigma^2} + (\lambda^a)_{31} \eta_\beta^\Delta \right] \\ [Q_5^a, \eta_\alpha^{\Xi^-}] &= -\frac{1}{2}(\gamma_5)_{\alpha\beta} \sqrt{\frac{1}{3}} \left[2(\lambda^a)_{33} \eta_\beta^{\Xi^-} + (\lambda^a)_{11} \eta_\beta^{\Xi^2} + (\lambda^a)_{13} \eta_\beta^\Omega \right] \\ [Q_5^a, \eta_\alpha^\Omega] &= -\frac{1}{2}(\lambda^a)_{33} (\gamma_5)_{\alpha\beta} \eta_\beta^\Omega \end{aligned} \quad \dots (B18)$$

APPENDIX C : DIRAC ALGEBRA

This appendix provides a brief summary of results concerning Dirac algebra relevant to the foregoing sections. The properties of the charge conjugation matrix C will also be quoted.

1. Dirac matrices : definitions and standard properties

The Dirac γ -matrices are hermitian 4×4 matrices γ^μ ($\mu = 0, \dots, 3$) satisfying the anticommutation relations

$$\{\gamma^\mu, \gamma^\nu\} = \gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu} \quad \dots (C1)$$

and the Hermiticity conditions

$$\gamma^{\mu\dagger} = \gamma^0 \gamma^\mu \gamma^0 \quad \dots (C2)$$

Now that the γ -matrices have been properly defined, their further properties can be investigated. Note that what follows are all consequences of equations (C1) and (C2), and hence do not depend on a particular representation of the γ -matrices.

A fifth anticommuting γ -matrix is defined by

$$\gamma^5 \equiv i \gamma^0 \gamma^1 \gamma^2 \gamma^3 \quad \dots (C3)$$

which has the properties

$$\begin{aligned}
\{\gamma^\mu, \gamma^5\} &= 0 \\
(\gamma^5)^2 &= 1 \\
\gamma^{5\dagger} &= \gamma^5
\end{aligned}
\quad \dots \text{(C4)}$$

2. Contraction identities

The manipulation of expressions involving γ -matrices is often greatly facilitated by the use of the following algebraic identities, which follow straightforwardly from the anticommutation relations (C1) :

$$\begin{aligned}
\gamma^\mu \gamma_\mu &= 4 & \gamma^\mu \gamma^\alpha \gamma_\mu &= -2\gamma^\alpha \\
\gamma^\mu \gamma^\alpha \gamma^\beta \gamma_\mu &= 4g^{\alpha\beta} & \gamma^\mu \gamma^\alpha \gamma^\beta \gamma^\sigma \gamma_\mu &= -2\gamma^\sigma \gamma^\beta \gamma^\alpha \quad \dots \text{(C5)} \\
\gamma^\mu \gamma^\alpha \gamma^\beta \gamma^\sigma \gamma^\delta \gamma_\mu &= 2(\gamma^\delta \gamma^\alpha \gamma^\beta \gamma^\sigma + \gamma^\sigma \gamma^\beta \gamma^\alpha \gamma^\delta)
\end{aligned}$$

If A, B, \dots denote four-vectors, and we define $A \equiv \gamma_\alpha A^\alpha$, the above identities yield the following contraction identities :

$$\begin{aligned}
\gamma_\mu A \gamma^\mu &= -2A \\
\gamma_\mu AB \gamma^\mu &= 4AB \\
\gamma_\mu ABC \gamma^\mu &= -2CBA \\
\gamma_\mu ABCD \gamma^\mu &= 2(DABC + CBAD)
\end{aligned}
\quad \dots \text{(C6)}$$

One can also derive the identity

$$AB = 2AB - BA \quad \dots (C7)$$

which shows that, in particular,

$$\begin{aligned} AA &= A^2 \\ AB &= -BA \quad (\text{if } AB = 0) \end{aligned} \quad \dots (C8)$$

3. Traces

When evaluating the trace of a product of γ -matrices, there again exist relations that are useful in simplifying the calculations. Some of them will be quoted below (without derivation).

(i) For any odd number of γ -matrices ($\gamma^\alpha \gamma^\beta \dots \gamma^\sigma \gamma^\delta$)

$$\text{Tr} (\gamma^\alpha \gamma^\beta \dots \gamma^\sigma \gamma^\delta) = 0 \quad \dots (C9)$$

(ii) For a product of an even number of γ -matrices,

$$\text{Tr} (\gamma^\alpha \gamma^\beta) = 4g^{\alpha\beta} \quad \dots (C10)$$

$$\text{Tr} (\gamma^\alpha \gamma^\beta \gamma^\sigma \gamma^\delta) = 4(g^{\alpha\beta} g^{\sigma\delta} - g^{\alpha\sigma} g^{\beta\delta} + g^{\alpha\delta} g^{\beta\sigma}) \quad \dots (C11)$$

and hence

$$\text{Tr}(AB) = 4AB \quad \dots \text{(C12)}$$

$$\text{Tr}(ABCD) = 4\{(AB)(CD) - (AC)(BD) + (AD)(BC)\}$$

(iii) For any product of γ -matrices

$$\text{Tr}(\gamma^\alpha \gamma^\beta \dots \gamma^\sigma \gamma^\delta) = \text{Tr}(\gamma^\delta \gamma^\sigma \dots \gamma^\beta \gamma^\alpha) \quad \dots \text{(C13)}$$

and

$$\text{Tr}(A_1 A_2 \dots A_{2n}) = \text{Tr}(A_{2n} \dots A_2 A_1) \quad \dots \text{(C14)}$$

(iv) For the γ^5 matrix, one can show that

$$\text{Tr} \gamma^5 = \text{Tr}(\gamma^5 \gamma^\alpha) = \text{Tr}(\gamma^5 \gamma^\alpha \gamma^\beta) = \text{Tr}(\gamma^5 \gamma^\alpha \gamma^\beta \gamma^\sigma) = 0 \quad \dots \text{(C15)}$$

4. Charge conjugation matrix C

Introducing the transposed matrices $\gamma^{\mu T}$, it follows from (C2) and (C3) that the matrices $(-\gamma^{\mu T})$ also satisfy the anticommutation relations (C2) and the Hermiticity conditions (C3). Then Pauli's fundamental theorem states that there exists a unitary matrix C such that

$$C \gamma^\mu C^{-1} = -\gamma^{\mu T} \quad \dots \text{(C16)}$$

C is called the charge-conjugation matrix and can be shown to satisfy

$$C = -C^{-1} = -C^T = -C^\dagger \quad \text{and} \quad C^2 = -1 \quad \dots \text{(C17)}$$

Thus one can also write

$$C \gamma^{\mu T} C = \gamma^\mu \quad \dots \text{(C18)}$$

APPENDIX D : HADRONIC SPECTRAL FUNCTIONS

In equation (2.1) the two-point function $\Pi(p)$ is defined as

$$\Pi(q) = i \int d^4x e^{iqx} \langle 0 | T(\eta(x) \bar{\eta}(0)) | 0 \rangle \quad \dots (D1)$$

To evaluate this expression in the hadronic sector, one starts off by inserting a complete set of states, summing over spins and integrating over phase space. This leads to

$$\langle 0 | T(\eta(x) \bar{\eta}(0)) | 0 \rangle \mapsto \sum_s \int \frac{d^3p}{2p_0 (2\pi)^3} e^{-ipx} \langle 0 | \eta(0) | N(p, s) \rangle \langle N(p, s) | \bar{\eta}(0) | 0 \rangle \quad \dots (D2)$$

where the exponential e^{-ipx} comes from shifting $\eta(x) \rightarrow \eta(0)$. In the case of the nucleon, for example, assuming that $N(p, s)$ represents the nucleon, and neglecting all other states,

$$\langle 0 | \eta(0) | N(p, s) \rangle = \lambda_N u_N(p, s) \quad \dots (D3)$$

$$\langle N(p, s) | \bar{\eta}(0) | 0 \rangle = \lambda_N \bar{u}_N(p, s)$$

Using

$$\sum_s u_N(p, s) \bar{u}_N(p, s) = \not{p} + M_N \quad \dots (D4)$$

it follows that

$$\langle 0 | T(\eta(x) \bar{\eta}(0)) | 0 \rangle = \lambda_N^2 \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2p_0} e^{-i p \cdot x} (\not{p} + M_N) \quad \dots (D5)$$

At the end of this Appendix it is shown that

$$S_F(x) = -i \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2p_0} e^{-i p \cdot x} (\not{p} + M) \quad \dots (D6)$$

and thus the (octet) hadronic spectral function turns out to be

$$\begin{aligned} \Pi(q) \Big|_{\text{HAD}}^{\text{OCT}} &= i \int d^4 x e^{i q \cdot x} \lambda_N^2 i S_F(x) \\ &= -\lambda_N^2 S_F(q) \quad \dots (D7) \\ &= -\lambda_N^2 \frac{q + M_N}{q^2 - M_N^2} \end{aligned}$$

with imaginary part given by

$$\begin{aligned} \text{Im} \Pi(q) \Big|_{\text{HAD}}^{\text{OCT}} &= \frac{1}{2i} \left\{ \Pi(q^2 + i\varepsilon) - \Pi(q^2 - i\varepsilon) \right\} \\ &= \pi \lambda_N^2 (q + M_N) \delta(q^2 - M_N^2) \quad \dots (D8) \end{aligned}$$

In the case of the decuplet, the $S=3/2$ objects are obtained by multiplying $(S=1) \otimes (S=\frac{1}{2})$, so that they have the properties of both spin- $\frac{1}{2}$ and vector fields. Expressions for such objects will be obtained using the Rarita-Schwinger formalism.

The Rarita-Schwinger equation is

$$(i\partial - M)\psi_\mu - \frac{i}{3}(\gamma_\mu\partial_\nu + \gamma_\nu\partial_\mu)\psi_\nu + \frac{1}{3}\gamma_\mu(i\partial + M)\gamma_\nu\psi_\nu = 0 \quad \dots (D9)$$

while the appropriate Lagrangian is

$$\mathcal{L}_{R.S.} = -\left\{ \bar{\psi}_\mu(i\partial - M)\psi_\mu - \frac{i}{3}\bar{\psi}_\mu(\gamma_\mu\partial_\nu + \gamma_\nu\partial_\mu)\psi_\nu + \frac{1}{3}\bar{\psi}_\mu\gamma_\mu(i\partial + M)\gamma_\nu\psi_\nu \right\} \dots (D10)$$

from which the propagator is found to be ($i\partial \rightarrow p$)

$$\Delta_{\mu\nu}(p) = \frac{-1}{p^2 - M^2} \left\{ g_{\mu\nu}p - \frac{1}{3}\gamma_\mu\gamma_\nu p + \frac{1}{3}(\gamma_\mu p_\nu - \gamma_\nu p_\mu) - \frac{2}{3} \frac{p_\mu p_\nu p}{M^2} \right. \\ \left. + M \left[g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu + \frac{1}{3}(\gamma_\mu p_\nu - \gamma_\nu p_\mu) \frac{p}{M^2} - \frac{2}{3} \frac{p_\mu p_\nu}{M^2} \right] \right\} \quad \dots (D11)$$

The decuplet Green function (for the delta particle, for example) is given by

$$\Pi_{\mu\nu}(q)|_{\text{HAD}} = i \int d^4x e^{iqx} \sum_s \int \frac{d^4p}{(2\pi)^4} e^{-ipx} \langle 0 | \eta_\mu(0) | \Delta(p, s) \rangle \langle \Delta(p, s) | \bar{\eta}_\nu(0) | 0 \rangle \dots (D12)$$

where

$$\langle 0 | \eta_\mu(0) | \Delta(p, s) \rangle = \lambda_\Delta \psi_\mu(p, s) \quad \dots (D13)$$

and

$$(p - M)\psi_\mu = 0$$

$$\gamma^\mu \psi_\mu = 0 \quad \dots (D14)$$

$$q^\mu \psi_\mu = 0$$

As in the case of the octet, the decuplet propagator in co-ordinate space can be written as

$$\Delta_{\mu\nu}(x) = -i \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2p_0} e^{-i p x} \sum_s \psi_\mu(p, s) \psi_\nu(p, s) \quad \dots (D15)$$

which leads to the (decuplet) hadronic spectral function

$$\Pi_{\mu\nu}(q)|_{\text{HAD}}^{\text{DEC}} = \frac{\lambda_\Delta^2}{q^2 - M_\Delta^2} \{g_{\mu\nu} q + M_\Delta g_{\mu\nu} + \dots\} \quad \dots (D16)$$

with imaginary part

$$\text{Im} \Pi_{\mu\nu}(q)|_{\text{HAD}}^{\text{DEC}} = -\pi \lambda_\Delta^2 \delta(q^2 - M_\Delta^2) \{g_{\mu\nu} q + M_\Delta g_{\mu\nu} + \dots\} \quad \dots (D17)$$

EXPRESSION FOR $S_F(x)$

By definition

$$S_F(x) \equiv \int \frac{d^4 p}{(2\pi)^4} S_F(p) \exp(-i p x) \quad \dots (D18)$$

where

$$S_F(p) = \frac{p + M}{p^2 - M^2} \quad \dots (D19)$$

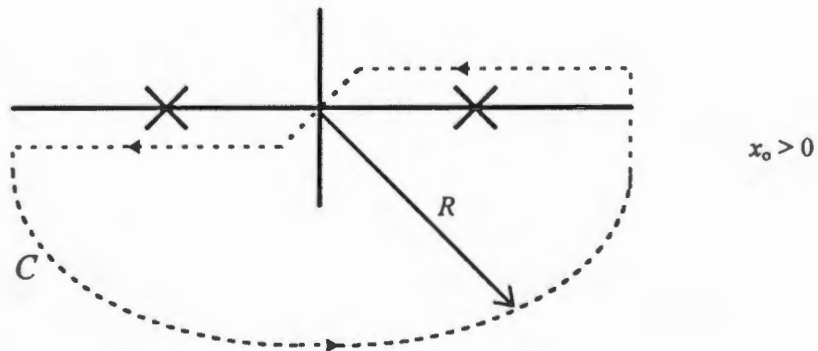
Separating the energy and momentum parts of the integral, and substituting (D19), equation (D18) can be rewritten as

$$S_F(x) = \int \frac{d^3 p}{(2\pi)^3} e^{i p x} \int_{-\infty}^{\infty} \frac{d p_0}{2\pi} e^{-i p_0 x_0} \frac{p + M}{p^2 - M^2} \quad \dots (D20)$$

Writing

$$\begin{aligned} p^2 - M^2 &= p_0^2 - |\mathbf{p}|^2 - M^2 \\ &= \left\{ p_0 - \sqrt{|\mathbf{p}|^2 + M^2} \right\} \left\{ p_0 + \sqrt{|\mathbf{p}|^2 + M^2} \right\} \end{aligned} \quad \dots (D21)$$

it is clear that there are two poles in the complex p_0 -plane. Using the contour C indicated below, the p_0 -integral can be evaluated using Cauchy's Residue Theorem.



Along the contour C ,

$$\oint_C (\dots) = \int_{-\infty}^{\infty} (\dots) + \int_{C(R)} (\dots) = \int_{-\infty}^{\infty} (\dots) = 2\pi i \sum_a \text{Res}(f; a) \quad \dots \text{ (D22)}$$

where the integral along the circular part of contour C vanishes for R very large. In general, for a pole of order m , the residue of a function f at a point a can be obtained from the formula

$$\text{Res}(f; a) = \lim_{z \rightarrow a} \frac{1}{(m-1)!} \frac{d^{m-1}}{dz^{m-1}} [(z-a)^m f(z)] \quad \dots \text{ (D23)}$$

In our case there is only one singularity bounded by contour C (pole of order $m=1$). From (D21), (D22) and (D23) the residue at this singularity is given by

$$\begin{aligned} \text{Res}\left(f; \sqrt{|p|^2 + M^2}\right) &= \lim_{p_0 \rightarrow \sqrt{|p|^2 + M^2}} \left(p_0 - \sqrt{|p|^2 + M^2} \right) \frac{e^{-i p_0 x_0}}{2\pi} \\ &= \frac{e^{-i p_0 x_0}}{2\pi} \frac{p + M}{\left(p_0 - \sqrt{|p|^2 + M^2} \right) \left(p_0 + \sqrt{|p|^2 + M^2} \right)} \quad \dots \text{ (D24)} \\ &= \frac{e^{-i p_0 x_0}}{2\pi} \frac{p + M}{2p_0} \Big|_{p_0 = \sqrt{|p|^2 + M^2}} \end{aligned}$$

Thus from (D20) and (D22)

$$S_F(x) = -i \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2p_0} e^{-i p \cdot x} (p + M) \quad \dots \text{ (D25)}$$

where the negative sign comes from inversion of the limits, and the mass shell relation $p_0 = \sqrt{|p|^2 + M^2}$ is understood.

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