Bootstrapping the OIS Curve in a South African Bank

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Declaration

I declare that this dissertation is my own, unaided work. It is being submitted for the Degree of Master of Philosophy in the University of the Cape Town. It has not been submitted before for any degree or examination in any other University.

Signed by candidate

September 7, 2017

Abstract

The financial crisis in 2007 highlighted the credit and liquidity risk present in interbank (LIBOR) rates, and resulted in changes to the pricing and valuation of financial instruments. The shift to Overnight Indexed Swap (OIS) discounting and multi-curve framework led to changes in the construction of interest rate zero curves, with the OIS curve being central to this methodology. Developed markets, such as the European (EUR), were able to adopt this framework due to the existence of a liquid OIS market. In the case of the South African (ZAR) market, the lack of such tradeable instruments poses the issue of how to construct or infer the OIS curve. Jakarasi *et al.* (2015) proposed a method to infer the OIS curve through the statistical relationship between SAFEX ROD and 3M JIBAR. The extension of the statistical relationship used by Jakarasi *et al.* (2015) to more statistically rigorous models, capable of capturing more information relating to the relationship between the rates, arises from the expected cointegrating relationship exhibited between rates. This dissertation investigates the implementation of such statistical models to infer the OIS curve in the ZAR market.

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Chapter 1

Introduction

Since the financial crisis in 2007, significant attention has been drawn to the discount rates and methods used when valuing financial instruments. Prior to the crisis, the assumption that highly rated banks did not run the risk of default led to the use of LIBOR rates as a proxy for the risk-free rate. The relationship and negligible spread between LIBOR and OIS rates supported this assumption, and allowed for the use of the single-curve framework for the pricing and valuation of financial instruments (i.e. no consideration was given to the different tenors of LIBOR associated with each market instrument, and both forecasting and discounting used a single LIBOR curve).

During the crisis, it was observed that banks and other highly rated financial institutions could default on payments. This led to spreads between the different tenor LIBOR and OIS rates widening, based on credit and liquidity premiums attached to each tenor (Clarke, 2010c). Noting that each tenor LIBOR had unique credit and liquidity risk premiums, a single forecasting curve could not explain such differences, thus requiring unique forecasting curves for each tenor LIBOR. Furthermore, under the collateralisation of transactions, the collateral posted on transactions earned the reference overnight rate, which was forecasted and discounted using the OIS curve. Hence, the OIS curve was adopted as the discounting curve for such transactions. These factors led to the implementation of the multi-curve framework, with the OIS rates used as the discounting rates, and different tenor LIBOR rates as the respective forecasting rates. This multi-curve framework has been implemented in most developed markets, due to the existence of a liquid OIS market.

When considering the case of the ZAR market, it is evident that the market lacks a tradeable overnight rate (Jakarasi *et al.*, 2015). Whilst there exist overnight reference rates, such as SAFEX ROD and SABOR, no instruments trade on these rates. This prevents the construction of the fundamental OIS curve. In addition, the lack of liquid market instruments with different tenor JIBAR hampers the implementation of a complete multi-curve framework. Jakarasi *et al.* (2015) proposed a statistical method to infer the OIS curve, under the assumption that LIBOR (3M JIBAR) and OIS (SAFEX ROD) rates exhibit a cointegrating relationship. This serves as basis for the use of statistical methods to infer relationships between rates, with the extension to models such as the vector-error correction model (VECM) of interest due to the cointegrating relationships.

The focus of this dissertation is thus to identify and implement a method of bootstrapping the OIS curve used to correctly discount collateralised transactions in the ZAR market. The use of statistical methods to accomplish the above is investigated, with implementation in a developed (EUR) market allowing for the robustness and performance of the models to be tested. An outline of the research aims and considerations are highlighted below:

1. Identification of current methods used to imply/bootstrap the OIS curve from market instruments. Develop an understanding of the implementation of such procedures in a developed market (EUR), including the construction of the curve using current bootstrapping methods.

2. Determination of the statistical relationship between LIBOR and OIS rates, using historical reference rate data, in both the EUR and ZAR markets. Identification of cointegrating relationships and estimation of statistical model parameters using the Johansen and Engle-Granger procedures are of interest.

3. Implementation of the statistical model in a bootstrapping procedure in both the EUR and ZAR markets, identifying the need for different methods under single- and multi-curve frameworks. Ascertain the validity of the statistically inferred OIS curves by comparison to the current market method. Investigate the implied OIS curves under multivariate cases in the EUR market, understanding the influence of the number and tenor of rates used in the models.

Chapter 2

Literature Review

2.1 OIS Discounting

Under the post-crisis setting and Credit Support Annex (CSA), the collateralization of transactions was implemented to reduce the risk of counterparty default. Clarke (2010c) and Hull and White (2013) note that the collateral posted earns interest at the overnight reference rate. As the collateral must reflect the current mark to market value of the transaction, Clarke (2010c) highlights that this overnight reference rate must be used to discount future expected cashflows on such collateralized transactions to satisfy no arbitrage.

Given the widespread use of OIS discounting, the need to bootstrap the OIS curve is fundamental to correct valuation of transactions.

2.2 Bootstrapping in Developed Markets

Developed markets, such as EUR, were quick to adopt the multi-curve framework, with the use of OIS discounting. The existence of an actively traded overnight rate was fundamental to the implementation of this methodology.

2.2.1 OIS

The OIS curve is constructed under the single-curve framework, as the curve is used to both forecast and discount cash flows. The standard instrument is that of the OIS, with the simple fair swap rate $R^o(t_0, t_i)$ given by,

$$R^o(t_0, t_n) = \frac{1}{\tau}\left[\prod_{i=1}^{n}(1 + D^o(t_{i-1}, t_i)\tau_i) - 1\right] \tag{2.1}$$

where

- τ is the year fraction from t_0 to t_n

- τ_i is the year fraction from t_{i-1} to t_i
- $D^o(t_{i-1}, t_i)$ is the simple overnight reference rate that applies for the i-th business day

Clarke (2010b) highlights an alternate method of bootstrapping the short-end of the OIS curve. Daily compounding of overnight reference rates is used to construct the curve out over the short-end (under the assumption that the reference rate will remain largely unchanged for certain periods, in line with market dynamics and expectations). Under such a framework, interpolation between dates on the short-end is not viable, as the step-function of the reference rates could lead to significant errors (i.e. when interpolated over a step). The construction of the mid- and long-end then uses the market available OIS rates, with the use of interpolation to determine intermediate rates (as per the standard bootstrapping procedure).

For the purposes of the dissertation, the standard single-curve framework will be used to bootstrap the market OIS curve, with no consideration to the method of Clarke (2010b) for the short-end. This is left as a possible extension to the dissertation. The particular OIS instruments used in the EUR market are given in Appendix A.

2.2.2 LIBOR Curves under OIS Discounting

Ametrano and Bianchetti (2009) propose a method for multi-curve discounting and bootstrapping, a brief overview of which is given for reference. The method involves firstly constructing a single discounting curve (the OIS curve as per the arguments above). Thereafter homogeneous LIBOR market instruments (referencing LIBOR rates with the same underlying tenor) are selected, based on availability and liquidity. Finally, the yield curves for each of the respective tenors can be constructed by forecasting cashflows with these homogeneous LIBOR market rates and discounting these cashflows with the OIS curve.

For the purpose of this dissertation, only cash deposit-, forward rate agreement-, and swap rates will be used in the bootstrapping procedure. Furthermore, these rates associated with these instruments were assumed to be deterministic. The specific market instruments used to bootstrap each of the diferent tenor LIBOR curve can be found in Appendix A.

Cash Deposit

The cash deposit (DEP) rate is the simple rate corresponding to the interest a party will earn when depositing cash for a given maturity. The equivalent NACC zero rate $r^x(t_0, t_n)$ for tenor x is given by,

$$r^x(t_0, t_n) = \frac{1}{\tau} \ln(1 + L^x(t_0, t_n)\tau) \tag{2.2}$$

where

- τ is the year fraction from t_0 to t_n
- $L^x(t_0, t_n)$ is the respective tenor simple market rate

Forward Rate Agreement

A forward rate agreement (FRA) is the interest a party will earn on a deposit starting at some point in the future t_j, for a given maturity t_n. When using these instruments for bootstrapping, it is important to note that the tenor of the FRA (difference between near and far dates) must coincide with the tenor of the zero curve being bootstrapped. The equivalent NACC zero rate $r^x(t_0, t_n)$ for tenor x is given by,

$$r^x(t_0, t_n) = \frac{f^x(t_0; t_j, t_n)\tau_n + r^x(t_0, t_j)\tau_j}{\tau} \tag{2.3}$$

where

- τ is the year fraction from t_0 to t_n
- τ_j is the year fraction from t_0 to t_j
- τ_n is the year fraction from t_j to t_n
- $f^x(t_0; t_j, t_n)$ is the fair NACC forward rate

Interest Rate Swap

Swap (SWP) rates are the fair rate at which a party can exchange floating LIBOR payments over a number of tenor periods, for a given given maturity t_n. The equivalent simple floating LIBOR rate for the final reset period $L^x(t_{n-1}, t_n)$ is given by,

$$L^x(t_{n-1}, t_n) = \frac{K^x \sum_{i=1}^{n} \tau_i Z^o(t_0, t_i) - \sum_{i=1}^{n-1} L^x(t_{i-1}, t_i)\tau_i Z^o(t_0, t_i)}{\tau_n Z^o(t_0, t_n)} \tag{2.4}$$

where

- K^x is the fair swap rate for the respective tenor swap
- τ_i is the year fraction from t_0 to t_i
- $Z^o(t_0, t_i)$ is the discount factor for t_i, determined from the discounting (OIS) curve

This expected tenor LIBOR rate can then be used to determine the equivalent NACC zero rate $r^x(t_0, t_n)$.

2.3 Bootstrapping in the ZAR Market

In the ZAR market, there exists no tradable overnight rate. Whilst there exist overnight reference rates, such as SAFEX ROD and SABOR, no instruments trade on these rates (i.e. no OIS instruments). As a result, there is no readily available OIS curve that can be used as a proxy for the risk-free rate.

Jakarasi *et al.* (2015) propose a method for bootstrapping the ZAR OIS curve through the assumption of a cointegrating relationship between the SAFEX ROD and 3M JIBAR. 3M JIBAR was selected based on it being the most liquid tenor traded in the ZAR market, and the SAFEX ROD rate was used above that of the SABOR as the overnight reference rate, based on the characteristics and construction of each rate. Jakarasi *et al.* (2015) begin by determining the realised floating OIS rate for a given tenor (3M given the liquidity of interbank rates in the ZAR markets). Thereafter, the cointegration relationship between the floating OIS rate and interbank rate of corresponding tenor (3M JIBAR) is calculated, allowing for the simultaneous bootstrapping of the OIS and JIBAR curves.

Jakarasi *et al.* (2015) propose two Engle-Granger cointegration models for the 3M floating OIS rate (3M FL), using the 3M JIBAR rate (3M JIBAR) and 3M JIBAR-SAFEX spread (SPD). The two models are as follows,

$$(\text{3M FL})_t = \beta_1(\text{3M JIBAR})_t + \beta_2(\text{SPD})_t + \alpha, \tag{2.5}$$

$$(\text{3M FL})_t = \beta_1(\text{3M JIBAR})_t + \alpha. \tag{2.6}$$

The first model takes into consideration both the above mentioned instruments/indices, with the second model considering only the 3M JIBAR rate. In addition, when checking the robustness of the models on developed markets, Jakarasi *et al.* (2015) found that the first model performed better on long dated swaps, with the second model performing better on short dated swaps. This led to the proposal of a hybrid model, using the LIBOR rate and SPD model for the short-end and the LIBOR only model for the long-end.

The work of Jakarasi *et al.* (2015) shows that the modeling of the OIS rate using a cointegration based approach produces reasonable results.

2.4 Cointegration

Cointegration is used to describe the relationship between to variables or processes that appear to have a common long run trend. Engle and Granger (1987) show that given two or more integrated/non-stationary I(1) variables, such that some linear combination of the variables is stationary I(0), then the variables can be considered to exhibit a cointegrating relationship.

2.4.1 Stationarity, Unit Root and Dickey-Fuller Tests

An integrated non-stationary I(1) series contains one unit root, such that when differenced the series is stationary I(0). This is shown by considering the process

$$y_t = \rho y_{t-1} + u_t, \tag{2.7}$$

where u_t is a stationary process, such that

$$\Delta y_t = (\rho - 1)y_{t-1} + u_t. \tag{2.8}$$

Banerjee *et al.* (1993) highlight that for Δy_t to be stationary, we require $\rho = 1$ (i.e. the series contains one unit root). In checking a univariate time series for stationarity, the null hypothesis of $H_0 : \rho = \rho_0 = 1$ is tested. Ordinary least squares (OLS) regression is used to estimate the test statistic $\hat{\rho}$, which is then used to construct the test statistic,

$$\frac{\hat{\rho} - \rho_0}{SE(\hat{\rho})} \tag{2.9}$$

Banerjee *et al.* (1993) further note that $\hat{\rho}$ has a non-asymptotic, non-symmetrical distribution. As a result, it is compared to critical values, tabulated by Dickey and Fuller (1979).

2.4.2 Enlge-Granger Representation

Engle and Granger (1987) proposed a two-step procedure to identify and determine the cointegrating relationship between variables. Given two time series variables $\{x_t\}, \{y_t\}$, the processes must first be tested for the existence of a unit root such that both can be confirmed to be integrated I(1). This allows for the application of the Dickey-Fuller test discussed above. Having confirmed non-stationarity of the processes, and under the assumption of a cointegrating relationship, the following regression is considered,

$$y_t = \beta x_t + v_t, \tag{2.10}$$

where v_t contains stationary I(0) dynamics. OLS regression can then be used to estimate $\hat{\beta}$, under the omission of v_t,

$$\hat{\beta} = \left(\sum_{t=1}^{T} x_t y_t \right) \left(\sum_{t=1}^{T} x_t^2 \right)^{-1} \tag{2.11}$$

The residual estimates, $\hat{v}_t = y_t - \beta x_t$, are then tested for stationarity, again using the Dickey-Fuller tests. Should the residuals $\hat{v}_t$ prove to be stationarity, the null hypothesis of a cointegrating relationship cannot be rejected and the variables can be assumed to be cointegrated. Under the Engle-Granger approach, the estimates can then be extended to the error correction model. This is achieved through the inclusion of stationary terms, such as autoregressive components (based on the ADF test) and lagged cointegrating relationships. This dissertation will use the Johansen procedure to estimate the VECM model, with the Engle-Granger OLS regression above of interest based on the its application by Jakarasi *et al.* (2015).

Should the residuals prove to be non-stationary, the variables can not be considered to exhibit a cointegrating relationship. This leads to a case of spurious regression, with the VECM and OLSM unable to correctly capture the relationship.

2.4.3 Vector Autoregression (VAR), Vector Error Correction Model (VECM) Representation

A vector autoregressive (VAR) model can be used to describe the interrelationship between two (or more) stationary variables and the previous values of each of the variables. The VECM representation of a VAR process allows for the interrelationship between two (or more) variables that are stationary in the first differences to be determined, again based on the previous values of the first differences of the variables. Furthermore, the VECM representation allows for the determination of a cointegration relationship between the variables, as shown by Johansen and Juselius (1990).

Engle and Granger (1987) derive the error correction model (ECM) for an n-dimensional vector autoregressive process (VAR) of order p. The notation and representation follows that of Johansen and Juselius (1990).

$$\Delta \boldsymbol{X}_t = \boldsymbol{\Pi} \boldsymbol{X}_{t-1} + \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_i \Delta \boldsymbol{X}_{t-i} + \boldsymbol{\Phi} \boldsymbol{D}_t + \boldsymbol{\mu} + \boldsymbol{\epsilon}_t \tag{2.12}$$

where

- $\boldsymbol{\Pi}$ is the long-run multiplier matrix
- $\boldsymbol{\Gamma}_i$ is the i^{th} lag matrix
- $\boldsymbol{\phi}$ is a matrix
- $\boldsymbol{D}_t$ is a vector of deterministic terms
- $\boldsymbol{\epsilon}_t$ is independent identically distributed multivariate, correlated errors

From Johansen and Juselius (1990) and Banerjee *et al.* (1993), the nature of the relationship between the variables is dependent on the rank of the long-run multiplier matrix, $rank(\boldsymbol{\Pi}) = r$. For the case of $r = 0$, no cointegration exists between the variables, and the relationship should be respecified as vector autoregressive in first differences i.e. VAR($p-1$). For the case of $r = n$, the variables are stationary, i.e. VAR(p) model is stable. Finally, for the case of $0 < r < n$, a cointegration relationship exists between the variables. Under such conditions, $\boldsymbol{\Pi}$ can be split into the loading matrix $\boldsymbol{\alpha}$ and cointegrating matrix $\boldsymbol{\beta}$, with both $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ of size $n \times r$, such that $\boldsymbol{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}'$. Furthermore, it is noted that the decomposition of $\boldsymbol{\Pi}$ is not unique.

Johansen Procedure

The Johansen procedure allows for the determination of the model parameters of the VECM. Rearranging Equation 2.12, by Johansen and Juselius (1990) we have the following form of the VECM:

$$\boldsymbol{Z}_{0t} = \boldsymbol{\Pi} \boldsymbol{Z}_{kt} + \boldsymbol{\Gamma} \boldsymbol{Z}_{1t} + \boldsymbol{\epsilon}_t \tag{2.13}$$

where

- $\boldsymbol{Z}_{0t} = \Delta \boldsymbol{X}_t$
- $\boldsymbol{Z}_{kt} = \boldsymbol{X}_{t-k}$

- $\boldsymbol{Z}_{1t}$ denotes the stacked variables $[\Delta\boldsymbol{X}_{t-1},\ldots,\Delta\boldsymbol{X}_{t-k+1},\boldsymbol{D}_t,\mathbf{1}]'$
- $\boldsymbol{\Gamma}$ denotes the parameters $[\boldsymbol{\Gamma}_1,...,\boldsymbol{\Gamma}_{k-1},\boldsymbol{\Phi},\boldsymbol{\mu}]$

Johansen and Juselius (1990) highlights the method of solving for the model parameters, by maximising the log likelihood function. The following derivation of the estimation of cointegrating vectors follows Banerjee *et al.* (1993), with notation consistent with that of (Johansen and Juselius, 1990). Starting with the general form of the VECM in Equation 2.12, excluding deterministic terms $\boldsymbol{D}_t$, the distribution of the erros $\boldsymbol{\epsilon}_t$ are assumed to follow a multivariate normal distribution,

$$\boldsymbol{\epsilon}_t \sim N(\mathbf{0},\boldsymbol{\Omega}). \tag{2.14}$$

From the multivariate normal distribution, the log-likelihood function can be derived,

$$\begin{aligned} L(\boldsymbol{\Gamma}_1,...,\boldsymbol{\Gamma}_{k-1},\boldsymbol{\Pi},\boldsymbol{\Omega}|(\boldsymbol{X}_1,...,\boldsymbol{X}_T)) = &-\frac{T_n}{2}\log(2\pi)-\frac{T}{2}\log|\boldsymbol{\Omega}| \\ &-\frac{1}{2}\sum_{t=1}^{T}\boldsymbol{\epsilon}_t'\boldsymbol{\Omega}^{-1}\boldsymbol{\epsilon}_t. \end{aligned} \tag{2.15}$$

Concentrate $L(\boldsymbol{\Gamma}_1,...,\boldsymbol{\Gamma}_{k-1},\boldsymbol{\Pi},\boldsymbol{\Omega}|(\boldsymbol{X}_1,...,\boldsymbol{X}_T))$ with respect to $\boldsymbol{\Omega}$ followed $(\boldsymbol{\Gamma}_1,...,\boldsymbol{\Gamma}_{k-1})$ in order to reduce the likelihood function to $L^*(\boldsymbol{\Pi})$.
This is achieved by letting $\boldsymbol{Z}_{kt}=\boldsymbol{X}_{t-k}$ and $\boldsymbol{Z}_{t1}=(\Delta\boldsymbol{X}'_{t-1},...,\Delta\boldsymbol{X}'_{t-k+1})'$, and using regression to partial out the effect of $\boldsymbol{Z}_{kt}$ and $\boldsymbol{Z}_{t1}$ on $(\boldsymbol{\Gamma}_1,...,\boldsymbol{\Gamma}_{k-1})$. The residuals $\boldsymbol{R}_{0t},\boldsymbol{R}_{kt}$ are defined as,

$$\boldsymbol{R}_{0t}=\Delta\boldsymbol{X}_t-\sum_{i=1}^{k-1}\hat{\boldsymbol{\Gamma}}_i\Delta\boldsymbol{X}_{t-i} \tag{2.16}$$

where

$$(\hat{\boldsymbol{\Gamma}}_1,...,\hat{\boldsymbol{\Gamma}}_{k-1})=\left(\sum_{t=1}^{T}\Delta\boldsymbol{X}_t\boldsymbol{Z}'_{t1}\right)\left(\sum_{t=1}^{T}\Delta\boldsymbol{Z}_{t1}\boldsymbol{Z}'_{t1}\right)^{-1}, \tag{2.17}$$

and

$$\boldsymbol{R}_{kt}=\boldsymbol{X}_{t-k}-\sum_{i=1}^{k-1}\tilde{\boldsymbol{\Gamma}}_i\Delta\boldsymbol{X}_{t-i} \tag{2.18}$$

where

$$(\tilde{\boldsymbol{\Gamma}}_1,...,\tilde{\boldsymbol{\Gamma}}_{k-1})=\left(\sum_{t=1}^{T}\Delta\boldsymbol{X}_{t-k}\boldsymbol{Z}'_{t1}\right)\left(\sum_{t=1}^{T}\boldsymbol{Z}_{t1}\boldsymbol{Z}'_{t1}\right)^{-1}. \tag{2.19}$$

Thus, the concentrated likelihood function $L^*(\boldsymbol{\Pi})$ is given,

$$L^*(\boldsymbol{\Pi}) = K - \frac{T}{2}\log\left|\sum_{t=1}^{T}(\boldsymbol{R}_{0t} - \boldsymbol{\Pi}\boldsymbol{R}_{kt})(\boldsymbol{R}_{0t} - \boldsymbol{\Pi}\boldsymbol{R}_{kt})'\right|. \tag{2.20}$$

The second moment matrices and cross-products can then be determined from the residuals $\boldsymbol{R}_{0t}, \boldsymbol{R}_{kt}$,

$$\boldsymbol{S}_{ij} = \frac{1}{T}\sum_{t=1}^{T}\boldsymbol{R}_{it}\boldsymbol{R}_{jt}', \quad i,j = 0,k. \tag{2.21}$$

Rewriting Equation 2.20,

$$L^*(\boldsymbol{\Pi}) = K_0 - \frac{T}{2}\log\left|\boldsymbol{S}_{00} - \boldsymbol{\Pi}\boldsymbol{S}_{k0} - \boldsymbol{S}_{0k}\boldsymbol{\Pi}' + \boldsymbol{\Pi}\boldsymbol{S}_{kk}\boldsymbol{\Pi}'\right|. \tag{2.22}$$

The restriction $\boldsymbol{\Pi} = \boldsymbol{\alpha\beta}'$ is now imposed on the system, giving

$$L^*(\boldsymbol{\alpha},\boldsymbol{\beta}) = K_0 - \frac{T}{2}\log\left|\boldsymbol{S}_{00} - \boldsymbol{\alpha\beta}'\boldsymbol{S}_{k0} - \boldsymbol{S}_{0k}\boldsymbol{\beta\alpha}' + \boldsymbol{\alpha\beta}'\boldsymbol{S}_{kk}\boldsymbol{\beta\alpha}'\right|. \tag{2.23}$$

Next, $L^*(\boldsymbol{\alpha},\boldsymbol{\beta})$ is further concentrated with respect to $\boldsymbol{\alpha}$, giving an expression for the MLE of $\boldsymbol{\alpha}$ as a function of $\boldsymbol{\beta}$, and a concentrated likelihood function depending on $\boldsymbol{\beta}$. From Equation 2.23,

$$\frac{\partial L^*(\boldsymbol{\alpha},\boldsymbol{\beta})}{\partial\boldsymbol{\alpha}} = 0, \tag{2.24}$$

giving

$$\hat{\boldsymbol{\alpha}} = \boldsymbol{S}_{0k}\boldsymbol{\beta}(\boldsymbol{\beta}'\boldsymbol{S}_{kk}\boldsymbol{\beta})^{-1}. \tag{2.25}$$

Substituting the above into Equation 2.23,

$$L^{**}(\boldsymbol{\beta}) = K_1 - \frac{T}{2}\log\left|\boldsymbol{S}_{00} - \boldsymbol{S}_{0k}\boldsymbol{\beta}(\boldsymbol{\beta}'\boldsymbol{S}_{kk}\boldsymbol{\beta})^{-1}\boldsymbol{\beta}'\boldsymbol{S}_{k0}\right|. \tag{2.26}$$

Differentiating $L^{**}(\boldsymbol{\beta})$ with respect to $\boldsymbol{\beta}$ is achieved by applying partitioned inversion results,

$$\begin{aligned}\left|\boldsymbol{S}_{00} - \boldsymbol{S}_{0k}\boldsymbol{\beta}(\boldsymbol{\beta}'\boldsymbol{S}_{kk}\boldsymbol{\beta})^{-1}\boldsymbol{\beta}'\boldsymbol{S}_{k0}\right| &= \left|\boldsymbol{\beta}'\boldsymbol{S}_{kk}\boldsymbol{\beta}\right|^{-1}\left|\boldsymbol{S}_{00}\right|\left|\boldsymbol{\beta}'\boldsymbol{S}_{kk}\boldsymbol{\beta} - \boldsymbol{\beta}'\boldsymbol{S}_{k0}\boldsymbol{S}_{00}^{-1}\boldsymbol{S}_{0k}\boldsymbol{\beta}\right| \\ &= \left|\boldsymbol{\beta}'\boldsymbol{S}_{kk}\boldsymbol{\beta}\right|^{-1}\left|\boldsymbol{S}_{00}\right|\left|\boldsymbol{\beta}'(\boldsymbol{S}_{kk} - \boldsymbol{S}_{k0}\boldsymbol{S}_{00}^{-1}\boldsymbol{S}_{0k})\boldsymbol{\beta}\right|.\end{aligned} \tag{2.27}$$

Noting that maximising $L^{**}(\boldsymbol{\beta})$ with respect to $\boldsymbol{\beta}$ corresponds to minimising the generalised variance ratio, with $|\boldsymbol{S}_{00}|$ constant,

$$\frac{\left|\boldsymbol{\beta}'(\boldsymbol{S}_{kk} - \boldsymbol{S}_{k0}\boldsymbol{S}_{00}^{-1}\boldsymbol{S}_{0k})\boldsymbol{\beta}\right|}{\left|\boldsymbol{\beta}'\boldsymbol{S}_{kk}\boldsymbol{\beta}\right|}. \tag{2.28}$$

Under the normalisation $\beta' \boldsymbol{S}_{kk}\beta = I$, this results in the minimisation of,

$$\left|\beta'(\boldsymbol{S}_{kk} - \boldsymbol{S}_{k0}\boldsymbol{S}_{00}^{-1}\boldsymbol{S}_{0k})\beta\right|. \tag{2.29}$$

This reduces to solving the eigenvalue problem

$$\left(\boldsymbol{\lambda}\boldsymbol{S}_{kk} - \boldsymbol{S}_{k0}\boldsymbol{S}_{00}^{-1}\boldsymbol{S}_{0k}\right)\boldsymbol{\beta} = 0 \tag{2.30}$$

$$\left|\boldsymbol{\lambda}\boldsymbol{S}_{kk} - \boldsymbol{S}_{k0}\boldsymbol{S}_{00}^{-1}\boldsymbol{S}_{0k}\right| = 0 \tag{2.31}$$

for largest eigenvalues $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_r \geq \lambda_n \geq 0$
Giving $\boldsymbol{\beta} = (\boldsymbol{v}_1, \boldsymbol{v}_2, ..., \boldsymbol{v}_r)$ of the corresponding eigenvectors. The remaining parameters are obtained by solving backwards as functions of the MLE of $\boldsymbol{\beta}$.

The results of the parameter estimation are dependent on the correct estimation of the number of cointegrating relationships (i.e. the cointegration rank) Johansen and Juselius (1990); Banerjee *et al.* (1993). Two well-known tests associated with the Johansen procedure are that of the trace test and maximum eigenvalue test. It is proposed that the maximum eigenvalue test will be used, thus the trace test will not be discussed further. For the maximum eigenvalue test, the null hypothesis H_r, is tested against the alternative H_{r+1}, with the likelihood ratio test statistic

$$\zeta_r = -T\ln(1-\lambda_{r+1}), \quad r = 0, 1, ..., n-1. \tag{2.32}$$

The above test statistic is compared against the limiting distribution, as per Johansen (1995), to accept or reject the hypotheses. See Johansen and Juselius (1990) or Banerjee *et al.* (1993) for a detailed discussion of the VECM and above procedure.

Heteroskedasticity and the Autoregressive Conditional Heteroskedastic (ARCH) Test

Heteroskedasticity is the condition whereby the variance of a process is not constant over time. When considering a statistical model, such as that of OLS, changes in the variance of fitted values with observed values is a strong indication of heteroskedasticity. Engle (1982) proposed the ARCH class of models, along with the ARCH test. As the ARCH model is not considered, and heteroskedasticity and the ARCH test do not have a major influence on the aims or results of this dissertation, the concepts are introduced for reference only. See Engle (1982) for a more detailed discussion of heteroskedasticity and the ARCH test.

2.5 Cross Currency Implied OIS Curve

The use of foreign exchange (FX) instruments to bootstrap a domestic (ZAR) OIS curve can be considered on the basis that the collateral could be posted in the foreign (EUR) market. The existence of an OIS curve and different tenor LIBOR curves in the EUR market, along with multiple liquid FX instruments for ZAR-EUR, allows for the ZAR OIS curve to be implied through these FX market instruments.

White (2012) highlights the use of FX instruments to determine the discounting curve in one currency (ZAR), given the existence of such a curve in the other currency (EUR). The construction of the short-end of the OIS curve is based on forward exchange rates, with the long-end based on cross currency basis swaps (floating for floating). Knowing the EUR OIS curve, along with a set of EUR and ZAR tenor LIBOR curves (such as 3M LIBOR/JIBAR), the ZAR OIS curve can be implied. Clarke (2010a) provides further insight and justification for the use of FX instruments, consistent with that of White (2012). As with the above, this requires the existence of a discounting curve in the currency/market (EUR) in which the collateral is posted. Clarke (2010a) shows that some swap, when valued using the unknown (ZAR) discount curve, should price to a fair value V_Z. This can be converted to EUR at the spot exchange rate, giving V_E (the collateral amount). This V_E is then invested at the EUR collateral rate (OIS rate) to the ZAR swap cash flow dates, and converted back to ZAR using forward foreign exchange rates. Thus the ZAR discount curve should present value the cash flows to fair swap value, allowing the ZAR discount curve to be determined given the existence of the required instruments and rates. The use of forward foreign exchange rates is viable up to 1 year, after which cross currency basis swaps should be used.

When considering the use of FX instruments, it is important to note the existence of an associated country risk-premium. This premium results in the corresponding OIS curve being recovered at a premium to the true OIS curve, inducing credit and liquidity risk components that are not unique to the OIS rate. As a result, this method of implying the OIS curve will not be considered in this dissertation, with methods capable of implying a clean OIS curve being preferred.

Chapter 3

Statistical Analysis and Parameter Estimation

Statistical analyses were performed on EUR (EONIA, 1M, 3M, 6M, 12M) and ZAR (SAFEX ROD, 3M JIBAR) reference rates, over the period 01 January 2000 - 31 December 2015. The data set was further broken into to pre- and post-crisis periods, to examine the influence of the different characteristics of each period. The pre-crisis period was 1 January 2000 - 31 July 2007, and the post-crisis period 1 July 2009 - 31 December 2015. All tests were run at the 1% significance level with the number of lags set to 0, where applicable. The main aims of the analyses were to check for the existence of cointegrating relationships, and estimate the parameter values for each of the statistical models.

3.1 Developed Markets - EUR

The historical data for the EUR market is shown in Figure 3.1.

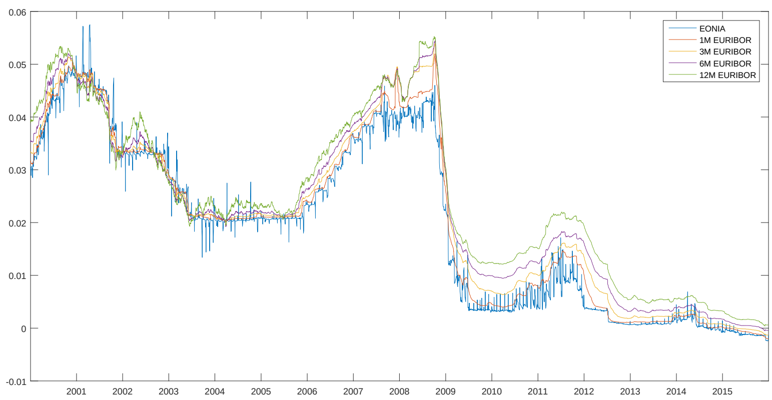


Fig. 3.1: EUR historical reference rates

3.1.1 Cointegrating Relationships

Of the methods and models discussed in Chapter 2, the VECM and Johansen procedure were used to test the historical EUR rates data for cointegrating relationships. The cointegration rank was examined through the use of the maximum eigenvalue test statistics, with these given in Table 3.1 (1% critical test values are given for reference). If the test statistic was found to be greater than the critical, the null hypothesis of cointegration of that rank could be rejected.

It can be seen from Table 3.1 that there exists cointegration between EUR rates. The entire data set exhibits cointegration rank 3, pre-crisis rank 4, and post-crisis rank 3.

Tab. 3.1: EUR maximum eigenvalue test statistics

Data Set	Cointegration Rank				
	0	1	2	3	4
1% Critical Values	39.3693	32.7172	25.8650	18.5200	6.6349
Entire	595.7623	228.1295	54.2363	8.9552	2.3721
Pre-crisis	460.5301	152.8202	76.3340	27.4817	3.9490
Post-crisis	394.1090	192.0820	67.6369	5.2034	2.8228

Multivariate Cases

In order to establish if the results were influenced by the number of variates (rates) included in the analysis, the tests were repeated for different multivariate cases. This was done using the entire data set only. Table 3.2 shows the maximum eigenvalue test statistics against the 1% critical values.

Tab. 3.2: EUR Maximum eigenvalue test statistics - multivariate post-crisis

Data Set	Cointegration Rank (Lowest - Highest)				
1% Critical Values	39.3693	32.7172	25.8650	18.5200	6.6349
EONIA-1M				350.9166	1.6443
EONIA-3M				186.6138	2.1466
EONIA-1M-3M			503.9676	18.1250	1.8647
EONIA-3M-6M			462.2516	35.9075	1.8149
EONIA-1M-3M-6M		535.9252	219.5935	7.5986	2.0001
EONIA-3M-6M-12M		498.4087	89.1419	18.6970	2.0142

In the EUR market, when considering the bivariate cases, it was found that there exists cointegration of rank 1. The tri-variate cases show cointegration of rank 2 in the case of EONIA-3M-6M, yet only rank 1 in the case of EONIA-1M-3M. It was noted that the test statistic lies close to the critical value for the latter case. The quad-variate cases provide further insight, with the case of EONIA-1M-3M-6M and exhibiting rank 2, and EONIA-3M-6M-12M exhibiting cointegration of rank 3. Due to each of the multivariate cases exhibiting strong cointegration relationships, it was found that it may not be necessary to include all rates in the cointegration/statistical model. If one considers the historical data for the post-crisis world, it can be seen that the different tenored rates appear to shifted versions of one another (see Figure 3.1), thus including all tenors may not provide additional information in terms of the cointegrating relationship.

3.1.2 Parameter Estimation

Parameter estimates are presented for both the Engle-Granger regression model (henceforth referred to as OLSM) and VECM. Estimates are given for the case of all LIBOR rates, over the entire, pre-crisis, and post-crisis data sets. Note that in the case of the VECM, the parameter estimates take into consideration the order of cointegration.

OLSM

The Engle-Granger procedure discussed in Chapter 2 was used to estimate the OLSM parameters. The augmented Dickey-Fuller test was performed to check for stationarity (or non-stationarity) of the variables.

ADF test statistics for each of the EUR rates are reported in Table 3.3, with the critical value at the 1% level -2.568 for each case. In order to reject the null hypothesis, a test statistic lower than that of the critical value was required. It was found that in all cases the null hypothesis was unable to be rejected, indicating the existence of a unit root and non-stationarity of the rates. The one exception to the above was that of the post-crisis EONIA, however the increased volatility over parts of this time period could lead to a poor test statistic and result. Based on visual inspection and the non-stationarity of the different LIBOR tenors, it was thus assumed that the post-crisis EONIA data was also non-stationary.

Tab. 3.3: EUR ADF test statistics

	EONIA	1M	3M	6M	12M
Entire	-1.0509	1.1749	1.9046	1.8760	1.5204
Pre	-1.0025	-1.3144	-1.4905	-1.2657	-0.7970
Post	-3.8808	1.3163	3.1816	3.7445	3.2882

Having established non-stationarity of the rate processes, the parameter estimates for the EUR OLSM, of the form $\text{OIS} = \beta_1 1\text{M} + \beta_2 3\text{M} + \beta_3 6\text{M} + \beta_4 12\text{M} + \beta_0$, were estimated by OLS regression. These values are shown in Table 3.4. It was found that the OLSM assigned different weightings to different tenor LIBOR rates when considering each of the data sets. These weightings can be interpreted as holdings in each of the tenor LIBOR rates, such that the net holding replicates a long position of 1 in the OIS rate. Furthermore, it was noted that for each data set significant holdings were required in the 1M and 3M tenor LIBOR rates, with smaller holdings in the 6M and 12M tenor LIBOR rates.

Tab. 3.4: EUR OLSM parameters

	Entire	Pre-crisis	Post-crisis
β_1	1.8405	1.3818	1.4969
β_2	-1.5264	-0.0181	-1.8267
β_3	1.1312	-0.4637	1.0283
β_4	-0.4600	0.1052	0.0237
β_0	0.0003	-0.0004	-0.0007

Lastly, in order to confirm that the rates were correctly modeled as cointegrated, the residual values were checked for stationarity using the ADF test as above. Table 3.5 shows that for each of the data sets the null hypothesis of a unit root was strongly rejected, indicating that the residuals were stationary and the OLS was valid.

Tab. 3.5: EUR residual ADF test statistics

	EONIA
Entire	-21.6562
Pre	-20.4708
Post	-17.6935

VECM

Having performed the Johansen procedure in checking for cointegrating relationships, the parameter estimates were recovered and examined. As the Johansen procedure was run with no lags, only the estimates for the α and β parameters are shown. Comparison of these parameters allows for some insight into the impact of the choice of data set, due to the fact that α and β constitute the long run relationship and this is of key importance when considering the performance of the VECM. Note the parameter estimates shown in Table 3.6 are in matrix form.

Tab. 3.6: EUR VECM parameters

	$\alpha \times 10^{-3}$				$\beta \times 10^{3}$			
Entire	0.3256	0.1038	0.0273		−0.5537	−0.2490	−0.0461	
	0.0274	−0.0372	0.0077		0.7640	1.1766	−0.5860	
	0.0333	−0.0267	−0.0043		−0.4440	−1.8533	3.1068	
	0.0352	−0.0277	−0.0052		0.7746	0.7226	−4.3720	
	0.0374	−0.0285	−0.0126		−0.5653	0.2380	1.8804	
Pre	0.4546	0.0524	0.0336	−0.0065	−0.7463	−0.1392	0.0252	−0.0189
	0.0022	−0.0420	0.0054	−0.0031	0.9979	−0.1400	−2.7274	0.4097
	0.0201	−0.0439	−0.0038	−0.0051	0.1700	−0.1677	5.7156	1.5705
	0.0243	−0.0463	0.0116	−0.0095	−0.4458	0.2726	−3.7282	−3.8712
	0.0270	−0.0461	0.0183	−0.0249	0.0287	0.1778	0.7023	1.9109
Post	−0.2878	−0.1723	−0.0279		0.8094	0.5052	0.1072	
	−0.0199	0.0117	0.0066		−0.0806	−2.4485	−0.8544	
	−0.0169	0.0130	0.0012		−0.4552	3.4741	3.1613	
	−0.0133	0.0120	−0.0005		−0.0462	−1.5886	−1.6994	
	−0.0122	0.0114	0.0001		0.0480	0.0482	−0.4109	

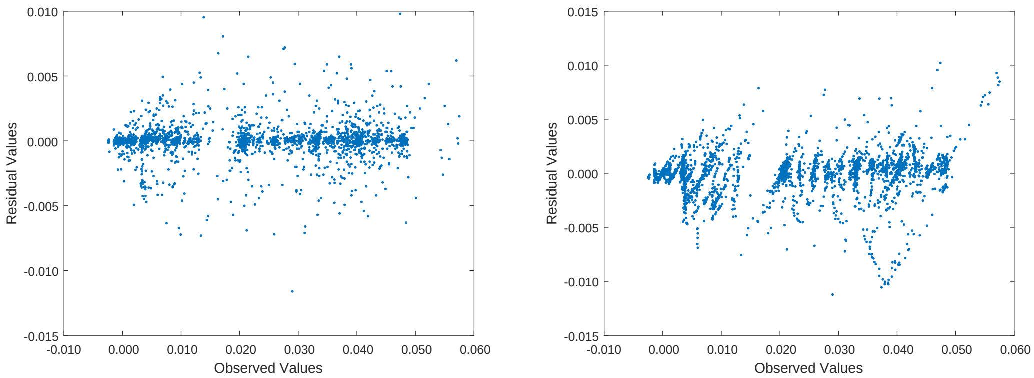
From Table 3.6 it can be seen that the parameters do not vary significantly in terms of the magnitude of the values, however there are differences across the data sets. Differences in the signs of parameters can be explained by the different trends over the respective periods. Any variation in the parameters can be explained by the different nature of the rates over the entire data set, with the rates pre-crisis having a notably different nature to those of the post-crisis. This suggests that the time period over which the parameters are estimated should be considered in order to produce the better estimates.

Real world interpretation of the VECM parameters proves challenging, as these correspond to holdings in the daily rate changes and lagged realisations of the different tenor LIBOR rates.

In particular, the general trend of the rates over a given time period has a significant impact on the parameters. General upward or downward trends give rise to different signs, along with values to which the rates will converge towards in the long-run. The impact of the above will discussed in more detail in following sections, with the application of the VECM to forecasting rates.

Heteroskedasticity

In order to check for heteroskedasticity in each of the models, the residuals were plotted against the observed values, and the ARCH test was performed on all residuals. The ARCH test statistics were compared to the critical value of 6.6349, with values above indicating rejection of the null hypothesis of no ARCH effects.



(a) VECM **(b)** OLS

Fig. 3.2: EUR EONIA residuals vs. observed for entire data set

Tab. 3.7: EUR rates ARCH test

	VECM					OLS
Data Set	EONIA	1M	3M	6M	12M	EONIA
Entire	315.2512	0.7697	10.8252	34.0722	34.9072	2369.8269
Pre-crisis	112.2798	3.3939	0.9393	5.4914	14.8794	431.8585
Post-crisis	286.8176	7.0360	1.3828	0.4520	0.1859	413.0432

Figures 3.2a and 3.2b show no noticeable increase in the variance (greater dispersion of residuals) with observed values, thus it cannot be concluded that the rates exhibit heteroskedasticity.

The ARCH test, Table 3.7, highlighted the existence of heteroskedasticity, with all tests relating to EONIA rejecting the null hypotheses that there exist no ARCH effects. The existence of heteroskedasticity can be in part explained by the difference in volatility between rates and the existence of periods of greater/less volatility, seen in Figure 3.1. Whilst the existence of heteroskedasticity does weaken the cointegrating relationship, it does not invalidate the use of cointegrating relationships and the VECM. With regard to the OLSM, the existence of heteroskedasticity does result in the OLSM estimates no longer being the most efficient, however they remain unbiased. Due to the nature of the rates, it was accepted that heteroskedasticity would always exist to some extent and thus no changes were made to the models or method of parameter estimation.

3.2 Developing Market - ZAR

The historical data for the ZAR market is shown in Figure 3.3.

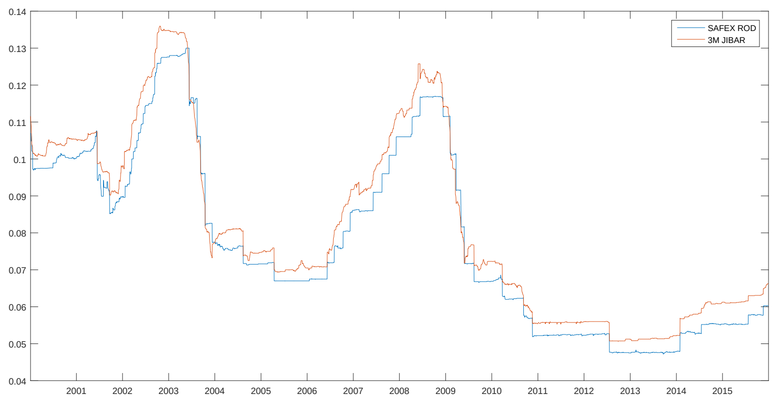


Fig. 3.3: ZAR historical reference rates

3.2.1 Cointegrating Relationship

As was the case with the EUR market, the VECM and Johansen procedure was used to check for a cointegrating relationship between the ZAR rates. The maximum eigenvalue test statistics for the ZAR data are reported in Table 3.8.

It was observed that over all time periods SAFEX ROD and 3M JIBAR are cointegrated with rank 1. This was as expected and allowed for a cointegration based bootstrapping procedure to be used.

Tab. 3.8: ZAR maximum eigenvalue test statistics

Data Set	Cointegration Rank	
	0	1
1% Critical Values	18.5200	6.6349
Entire	316.8702	1.7505
Pre-crisis	144.7263	1.6313
Post-crisis	31.2864	0.0000

3.2.2 Parameter Estimation

The parameter estimates are presented for each model, over the entire, pre-crisis, and post-crisis data sets. Note that in the case of the VECM, the parameter estimates take into consideration the order of cointegration.

OLSM

The parameter estimation for the OLSM followed the procedure used in the EUR market.

ADF test statistics for the ZAR rates are reported in Table 3.9. Given the null hypothesis of the existence of a unit root, it was found that in all cases we were unable to reject the null hypothesis, indicating that the rates are non-stationary.

Tab. 3.9: ZAR ADF test statistics

	SAFEX ROD	3M
Entire	0.8434	1.5281
Pre	0.3195	0.5204
Post	0.9340	0.9157

Having established non-stationarity of the rate processes, the parameters for the ZAR OLSM, of the form $\text{OIS} = \beta_1 3\text{M} + \beta_0$, were estimated using OLS regression. These estimates are shown in Table 3.10. As with the EUR market, the parameter β_1 corresponds to a long holding in 3M JIBAR (noting a fractional holding is required in order to replicate a long position of 1 in the OIS rate). It was found that these parameters agree with those estimated by Jakarasi *et al.* (2015), thus further validating the potential implementation of the model.

Tab. 3.10: ZAR OLSM parameters

	Entire	Pre-crisis	Post-crisis
β_1	0.9729	0.9487	0.9300
β_0	−0.00201175	0.00057655	−0.00003709

Lastly, the residual values were tested for stationarity. Table 3.11 shows the test statistics for each of the data sets. It can be seen that in all cases the null hypothesis of a unit root was rejected, implying stationarity of the residuals and the cointegrating relationship under the OLSM was validated.

Tab. 3.11: ZAR ADF test statistics

	SAFEX ROD
Entire	-6.7714
Pre	-4.5884
Post	-3.0953

VECM

The parameter estimates for the VECM are shown in Table 3.12. Only estimates for $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are shown, with each taking into consideration the order of cointegration. It can be seen that as with the EUR market, the estimates vary with the data set. Note as with the EUR case, parameter estimates are given in matrix form, and real world interpretation of the parameters proves challenging due to the form of the VECM.

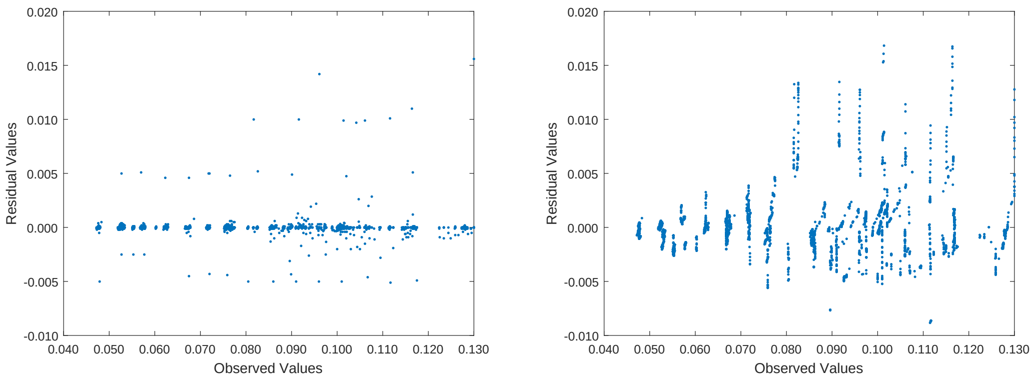
Tab. 3.12: ZAR VECM parameters

	$\boldsymbol{\alpha} \times 10^3$	$\boldsymbol{\beta} \times 10^{-3}$
Entire	0.0522	0.3809
	0.1152	−0.3697
Pre	0.0508	0.3405
	0.1210	−0.3215
Post	0.0134	1.0796
	0.0263	−0.9761

Heteroskedasticity

In order to check for heteroskedasticity, the residuals were plotted against the fitted values, and the ARCH test was performed on all residuals. The ARCH test statistics were compared to the critical value of 6.6349, with values above indicating rejection of the null hypothesis.

Figure 3.4 shows the residual values against the observed values for both the VECM and OLSM. It can be seen from Figure 3.4a that the residuals for the VECM exhibit clustering, however no increase in variance with observed values is visible. The clustering are a result of the the stepped nature of the post-crisis 3M JIBAR and SAFEX ROD as seen in Figure 3.3. Furthermore, the residuals from rate change are clearly noticeable. In contrast, Figure 3.4b clearly shows changes in the variance of residuals with observed values, indicating heteroskedasticity.



(a) VECM **(b)** OLS

Fig. 3.4: ZAR SAFEX ROD residuals vs. observed for entire data set

Tab. 3.13: ZAR rates ARCH test

	VECM		OLS
Data Set	SAFEX ROD	3M	SAFEX ROD
Entire	0.0124	0.0365	3354.1578
Pre-crisis	0.0024	0.0003	1634.6713
Post-crisis	0.0349	0.0088	1414.6205

The ARCH test results, as seen in Table 3.13, confirmed the observations made in above using Figure 3.4, with the VECM residuals exhibiting no heteroskedasticity (unable to reject null hypothesis) and the OLSM residuals exhibiting heteroskedasticity (rejection of the null hypothesis).

3.3 Chapter Summary

The Johansen procedure was used to examine the nature of the relationship between reference rates within markets. It was found that there exists a cointegrating relationship between rates in all markets tested. Furthermore, when considering multivariate cases (of different combinations of rates) it was found that the cointegrating relationships still exist. This supports the possibility of a statistical model with less covariates being used to describe the relationship still producing reasonable results.

Parameters were recovered using the Johansen procedure and OLS regression, and the estimates over different data sets were compared. It was found that the parameters do change when considering different periods, due to differences in the general trend of the rates over the given period. In particular, general upward or downward trends alter the signs of the parameter estimates, the impact of which was found to be significant. This highlights the need for consideration into the data used to estimate model parameters when implementing the statistical models. Real world interpretation of the parameters is reasonable in the case of the OLSM, with the parameters representing long and short positions in each of the tenor LIBOR rates (noting fractional holdings are required). In the case of the VECM, interpretation proves challenging as the parameters represent holdings in the day to day differences and lagged realisations of the tenor LIBOR rates, potentially unattainable in a market setting.

In order to further understand the relationship, the residuals were tested for heteroskedasticity. This was done by examining residual plots and performing an ARCH test on the residuals. Under the VECM, it was found that heteroskedasticity exists to some extent in the EUR market but not in the ZAR market. Under the OLSM, it was found that heteroskedasticity exists in both the EUR and ZAR market. Due to the nature of rates, with OIS exhibiting greater variance and changes in variance over time, these effects are hard to mitigate or remove and are thus only noted for reference.

Chapter 4

Statistical Bootstrap Implementation and Results

The implementation of the statistical relationships determined in the Chapter 3 allowed for the OIS and different tenor LIBOR zero curves to be bootstrapped from the respective LIBOR market instruments. All bootstrapping took place assuming ZAR business days (for both EUR and ZAR market cases), using raw interpolation where required, and for 16 July 2015.

4.1 Developed Market

The developed market presented a combination of challenges when attempting to bootstrap the OIS curve. The underlying cause of which was the multicurve framework, which results in market prices being derived under OIS discounting and the respective tenor LIBOR forecasting. However these same LIBOR curves were required to infer the OIS curve using the statistical relationships determined in Chapter 3. These challenges resulted in the need to simultaneously bootstrap the OIS and different tenor LIBOR curves. As a baseline result, the OIS and LIBOR curves were bootstrapped under the current market method, with the zero curves shown in Figure 4.1.

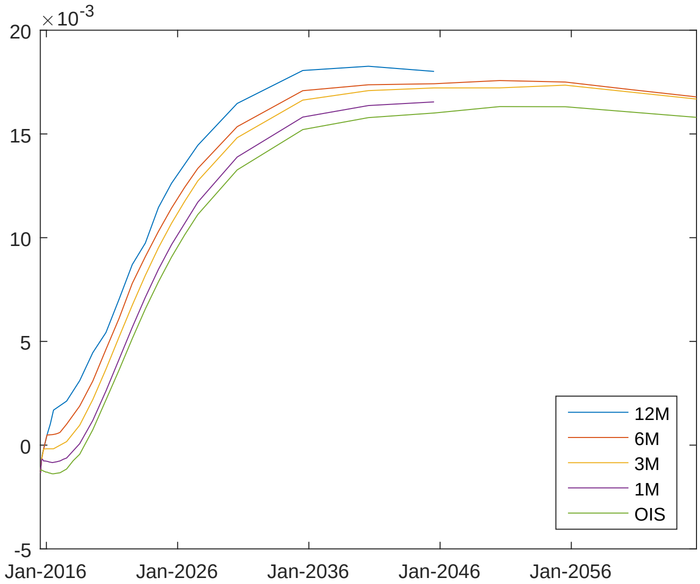


Fig. 4.1: EUR market implied zero curves

4.1.1 Method

It was noted that even under the multi-curve framework, the DEP and FRA zero rates could be bootstrapped independently of the OIS curve. This allowed for the standard bootstrapping procedure to be applied to these instruments, requiring only the SWP rates to be simultaneously bootstrapped. The simultaneous bootstrapping procedure developed is detailed below.

1. Identify the market instruments and rates to be used in the bootstrapping procedure for each tenor LIBOR, along with the initial visible OIS reference rate

2. Bootstrap the DEP and FRA zero rates for each of the different tenor LIBOR curves, and set up 'dummy' rates for each of the SWP zero rates such that an initial guess of each of the tenor LIBOR curves is obtained

3. (VECM only) Using the market visible and previously realised OIS and tenor LIBOR reference rates, initialise the VECM forecasting model

4. For the following business day, imply each of the different tenor LIBOR forward rates, using the current guess for each of the tenor LIBOR zero curves

5. Apply the statistical model (VECM/OLSM) to forecast the OIS rate for the

given business day, using the above forward LIBOR rates as the expected value of the tenor LIBOR reference rates

6. Stepping through each of the successive business days, out till the end of the zero curve, imply each of the successive forward LIBOR rates and infer the corresponding OIS rates

7. From the successive daily forecasts of the OIS rate, recover the OIS zero curve (see Equation 2.1)

8. Using this inferred OIS curve and estimated LIBOR curves, recover each of the SWP zero rates under the multi-curve pricing framework

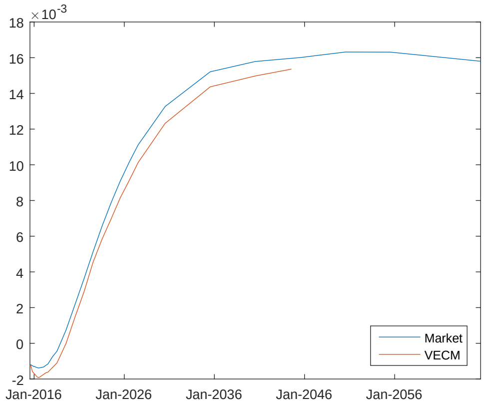
9. Repeat until convergence is obtained for the each of the SWP zero rates

The simultaneous multi-curve bootstrapping procedure required that each of the different tenor LIBOR curves be estimated at each convergence iteration, in order to allow for the OIS curve to be inferred. Thereafter, all of the required SWP zero rates (corresponding to all swaps of all LIBOR tenors) were repriced. This ensured convergence of the system as a whole for a given set of market rates, inferred OIS curve, and tenor LIBOR curves.

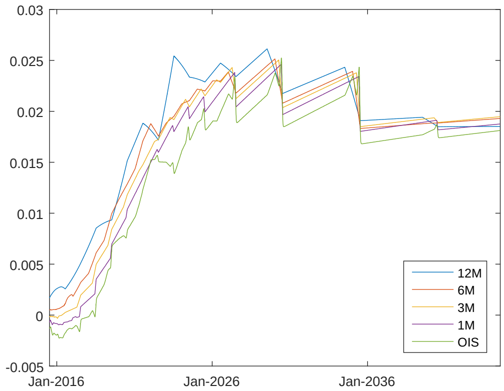
Furthermore, it was noted that the OIS curve obtained from the above procedure was limited by the shortest maturity LIBOR curve, minus the longest tenor of LIBOR rate used. For example, if the 1M curve extended out to 30 years, and 12M was the longest tenor LIBOR rate used, then the OIS curve would extend out to 29 years. This was due to the fact that in order to estimate the OIS rate at a given pillar date, each tenor forward LIBOR rate was required at that pillar date, thus requiring a LIBOR zero curve that extended the respective tenor beyond the pillar date. Furthermore, at each convergence iteration this required the last SWP rate to be estimated, such that the full tenor LIBOR curve could be recovered. As the OIS curve would not extend to this last pillar date, it was not possible to recover the rate under OIS discounting. The proposed solution was to bootstrap each the last tenor SWP rates under the single-curve framework, as simply using the market visible fair swap rate as an estimate for the zero rate was highly inaccurate.

4.1.2 VECM Inferred

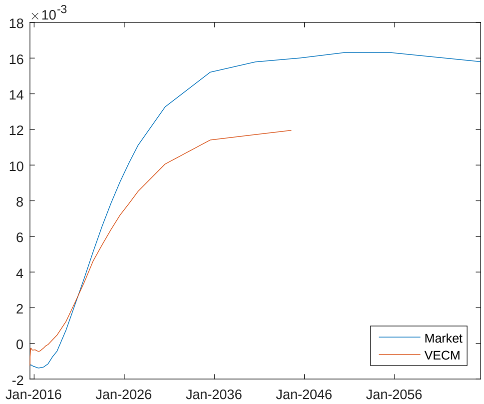
The following results are those of the VECM inferred OIS curve in the EUR market. In order to investigate the influence of the data set, and corresponding parameters, the procedure was implemented using both the entire and post-crisis data set.



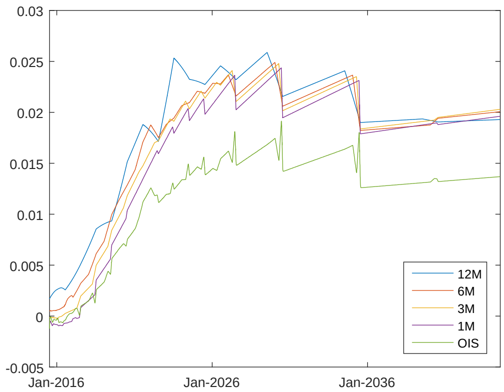
(a) OIS curves: entire data set



(b) Daily rate forecasts: entire data set



(c) OIS curves: post-crisis data set



(d) Daily rate forecasts: post-crisis data set

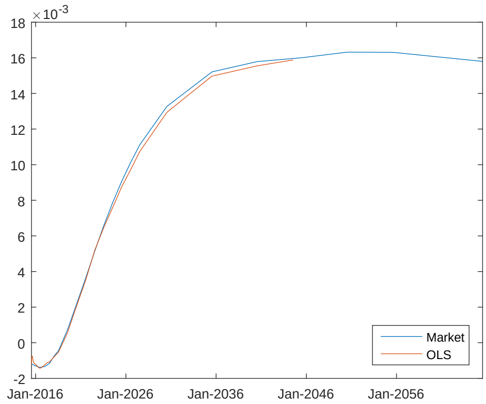
Fig. 4.2: EUR VECM inferred OIS curves and daily rate forecasts

Figure 4.2 shows the VECM inferred OIS curves and daily rate forecasts for both the entire and post-crisis data set, with the market implied OIS curve given for reference in each case. Figures 4.2a and 4.2c show the impact of the data set and parameters on the results of the inferred OIS curves. Under the entire data set, the inferred OIS curve was able to capture the correct term structure, and appeared to be a shifted version of the market implied OIS curve. Under the post-crisis data set, the VECM relationship was unable to capture the correct term structure, failing to infer any reasonable OIS curve. The shifted OIS curve visible in Figure 4.2a was a result of the autoregressive component of the VECM. The dependence of OIS forecasts on previous forecasts reduces the ability of the VECM to capture the term structure, as this term structure was visible only in the forward LIBOR rates. The contribution of this autoregressive component, which was compounded by the number of successive daily forecasts, was enough to offset the OIS curve from that of the market implied OIS curve.

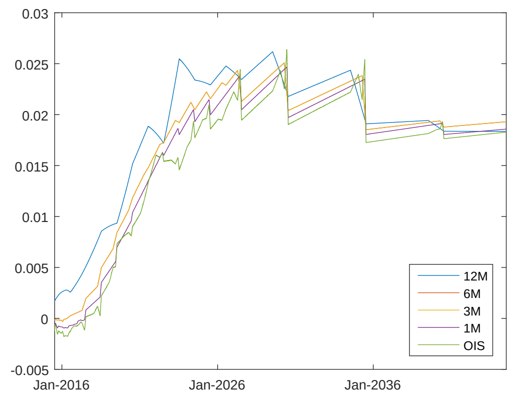
The VECM parameters, specifically the long run relationship, had a significant impact on the daily forecasts of the OIS rate. In Figure 4.2d, the under estimation of the daily OIS rates is visible. As in the post-crisis world there exists noticeable spread between OIS and different tenor LIBOR rates, when this spread starts to decrease, and forward LIBOR rates converge, the relationship produces poor estimates for the OIS rate. When considering the entire data set, the existence of the pre-crisis period with little or no spread between different tenor LIBOR rates allows for the OIS forecasts to be less effected by the decreasing spread visible in forward LIBOR rates. The decreasing spread visible in forward LIBOR rates indicates the possibility that future LIBOR rates will no longer maintain the post-crisis spread levels, returning to a state similar to that of the pre-crisis period. When implementing the VECM, the parameters are designed to capture long-term trends, and thus specific data sets with unique characteristics (such as the spread observed in the post-crisis period) should not be used.

4.1.3 OLSM Inferred

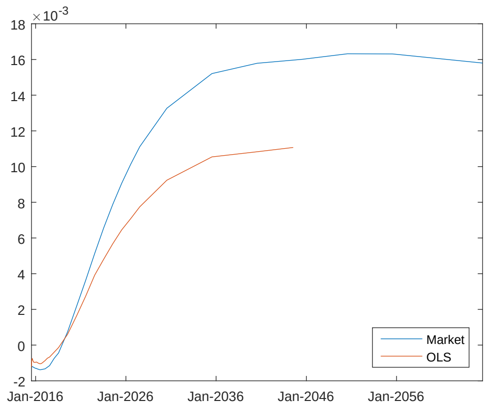
The following of results are those of the OLSM implied OIS curve in the EUR market. As was the case with the VECM, the influence of the data set was investigated.



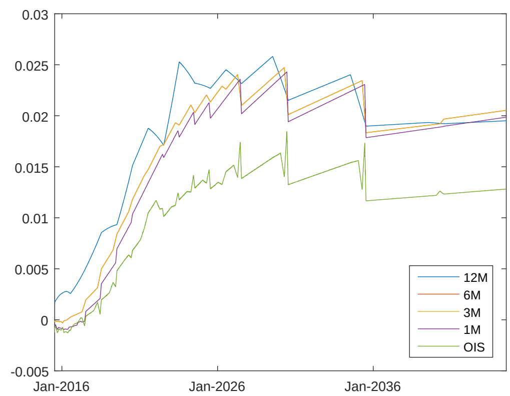
(a) OIS curves: entire data set



(b) Daily rate forecasts: entire data set



(c) OIS curves: post-crisis data set



(d) Daily rate forecasts: post-crisis data set

Fig. 4.3: EUR OLSM inferred OIS curves and daily rate forecasts

Figure 4.3 shows the OLSM inferred OIS curves and daily rate forecasts for both the entire and post-crisis data set, with the market implied OIS curve given for reference in each case. Under the entire data set, Figure 4.3a shows that the OLS inferred OIS curve appeared to coincide with that of the market implied OIS curve. Under the post-crisis data set however, Figure 4.3c shows the OLSM implied OIS curve was unable to correctly capture the required term structure. This failure of the post-crisis OLSM can be attributed to the same shortcomings of the post-crisis VECM discussed above, with Figure 4.3d clearly showing the under estimation of the daily OIS rates.

Tab. 4.1: EUR OLSM inferred OIS against market implied

Date	Market implied (%)	OLSM (%)	Spread (bps)
17-Jul-2015	-0.1220	-0.1240	-0.2000
23-Jul-2015	-0.1210	-0.0872	3.3823
17-Aug-2015	-0.1200	-0.0746	4.5431
16-Sep-2015	-0.1220	-0.0973	2.4741
16-Oct-2015	-0.1255	-0.1118	1.3726
16-Nov-2015	-0.1270	-0.1160	1.0965
17-Dec-2015	-0.1300	-0.1212	0.8825
18-Jan-2016	-0.1300	-0.1239	0.6110
16-Feb-2016	-0.1320	-0.1278	0.4223
16-Mar-2016	-0.1340	-0.1333	0.0727
18-Apr-2016	-0.1350	-0.1377	-0.2693
16-May-2016	-0.1370	-0.1410	-0.4025
17-Jun-2016	-0.1380	-0.1427	-0.4722
18-Jul-2016	-0.1380	-0.1419	-0.3883
16-Aug-2016	-0.1372	-0.1399	-0.2652
16-Sep-2016	-0.1364	-0.1362	0.0132
17-Oct-2016	-0.1355	-0.1326	0.2885
16-Nov-2016	-0.1347	-0.1290	0.5676
19-Dec-2016	-0.1338	-0.1257	0.8094
16-Jan-2017	-0.1330	-0.1229	1.0131
18-Apr-2017	-0.1239	-0.1121	1.1804
17-Jul-2017	-0.1151	-0.1071	0.7956
16-Jul-2018	-0.0440	-0.0521	-0.8053
16-Jul-2019	0.0741	0.0571	-1.6999
16-Jul-2020	0.2178	0.2065	-1.1278
16-Jul-2021	0.3621	0.3504	-1.1649
18-Jul-2022	0.5132	0.5184	0.5169
17-Jul-2023	0.6565	0.6458	-1.0691
16-Jul-2024	0.7873	0.7600	-2.7318
16-Jul-2025	0.9067	0.8761	-3.0602
16-Jul-2026	1.0134	0.9728	-4.0619
16-Jul-2027	1.1114	1.0737	-3.7730
16-Jul-2030	1.3266	1.2937	-3.2913
16-Jul-2035	1.5210	1.4971	-2.3872
16-Jul-2040	1.5787	1.5549	-2.3773
18-Jul-2044	1.5964	1.5882	-0.8285

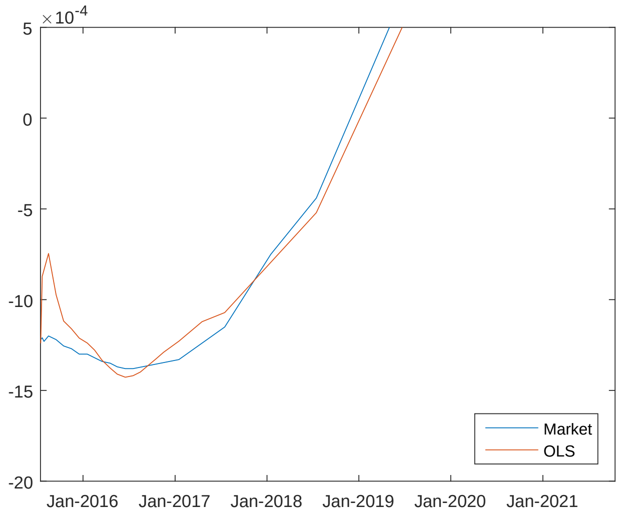
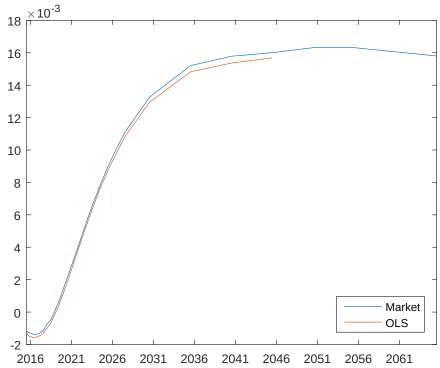


Fig. 4.4: EUR OLS inferred OIS curve at the short end

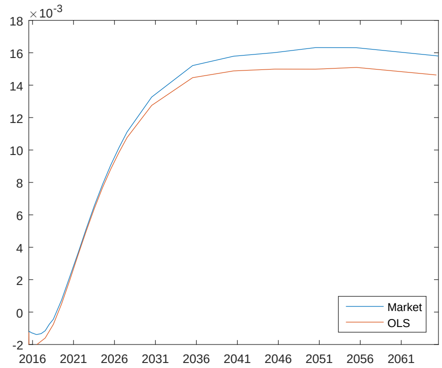
Table 4.1 shows the OLSM inferred and market implied NACC rates for a given set of pillar dates, along with the basis point spread between the two. The minimal spread between rates confirmed the performance of the OLSM, with no spread greater than 5 bps in both the short- and long-end. The reason for the improved performance over the VECM was that the OLSM was able to imply the OIS rate and term structure directly from the forward LIBOR rates, with no further influence from long run relationships or autoregressive components. Minor divergence between the OLSM and market implied OIS curves occurs only in the extreme short- and long-end. The reason for the long-end divergence was that the last maturity SWP rate was bootstrapped under the single-curve framework, with the need to do so previously discussed. Divergence in the short-end, as seen in Figure 4.4, was a result of the forecasting under the statistical relationship, with the short end of different tenor LIBOR curves having to be estimated using the same set of non-homogeneous rates. These non-homogeneous rates do not show the expected spread between different tenor LIBOR, and thus would not fit the statistical model parameters as estimated.

4.1.4 Multivariate Cases

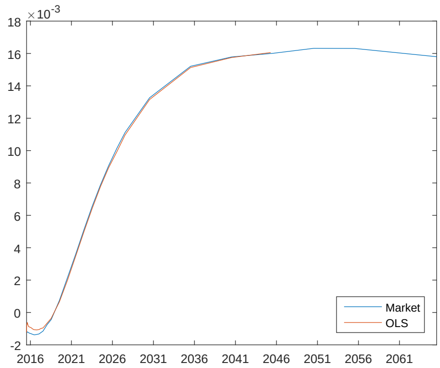
Having established that there exist cointegrating relationships even when considering fewer tenor LIBOR rates, and that the OLSM was capable of recovering the market implied OIS curve, the impact of the implementation of these different multivariate cases was investigated. It was reasoned that shorter tenor LIBOR rates, such as 1M and 3M, would be more closely related to the OIS rate, as a result of less credit and liquidity risk. Given the improved performance of the OLSM under the entire data set, multivariate cases were considered only under this model.



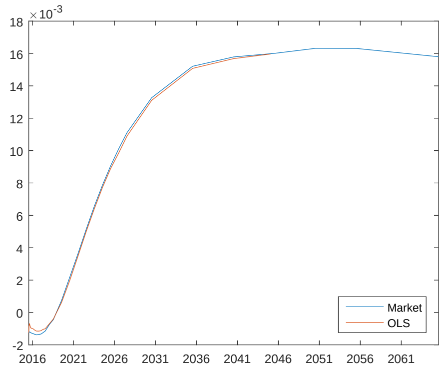
(a) EONIA-1M



(b) EONIA-3M



(c) EONIA-1M-3M



(d) EONIA-1M-3M-6M

Fig. 4.5: EUR multivariate OLSM inferred OIS curves

From Figure 4.5 it can be seen that the number of LIBOR rates included in the statistical model had a noticeable impact. The bivariate cases seen in Figures 4.5a and 4.5b perform worse than that of the tri- and quad variate cases, seen in Figures 4.5c and 4.5d respectively. Furthermore, the case of OIS-3M performs worse than that of OIS-1M, whilst the tri- and quad variate cases appear to perform better than that of the full multivariate case, suggesting that certain rates are more closely related to the OIS rate. This was confirmed when considering Tables 4.2 and 4.3, with the basis spread being relatively low for each of the selected curve dates. Furthermore, it was found that the case of EONIA-1M performed best in the short-end of the curve, with the the tri- and quad-variate cases performing noticeably better in the long-end.

Tab. 4.2: EUR multivariate OLSM inferred OIS rates against market implied

Date	Market implied (%)	1M (%)	3M (%)	1M-3M (%)	1M-3M-6M (%)
17-Jul-2015	-0.1220	-0.1240	-0.1240	-0.1240	-0.1240
23-Jul-2015	-0.1210	-0.1380	-0.1969	-0.0798	-0.0862
17-Aug-2015	-0.1200	-0.1305	-0.2016	-0.0619	-0.0689
16-Sep-2015	-0.1220	-0.1409	-0.2030	-0.0779	-0.0857
16-Oct-2015	-0.1255	-0.1471	-0.2044	-0.0878	-0.0959
16-Nov-2015	-0.1270	-0.1483	-0.2053	-0.0892	-0.0972
17-Dec-2015	-0.1300	-0.1500	-0.2060	-0.0915	-0.0994
18-Jan-2016	-0.1300	-0.1515	-0.2075	-0.0929	-0.1006
16-Feb-2016	-0.1320	-0.1533	-0.2081	-0.0957	-0.1032
16-Mar-2016	-0.1340	-0.1550	-0.2065	-0.0992	-0.1069
18-Apr-2016	-0.1350	-0.1562	-0.2046	-0.1022	-0.1100
16-May-2016	-0.1370	-0.1573	-0.2030	-0.1050	-0.1129
17-Jun-2016	-0.1380	-0.1573	-0.2012	-0.1067	-0.1147
18-Jul-2016	-0.1380	-0.1564	-0.1995	-0.1069	-0.1149
16-Aug-2016	-0.1372	-0.1554	-0.1964	-0.1072	-0.1152
16-Sep-2016	-0.1364	-0.1543	-0.1930	-0.1074	-0.1152
17-Oct-2016	-0.1355	-0.1533	-0.1897	-0.1076	-0.1149
16-Nov-2016	-0.1347	-0.1518	-0.1864	-0.1070	-0.1143
19-Dec-2016	-0.1338	-0.1501	-0.1828	-0.1063	-0.1136
16-Jan-2017	-0.1330	-0.1487	-0.1798	-0.1058	-0.1120
18-Apr-2017	-0.1239	-0.1411	-0.1698	-0.0988	-0.1041
17-Jul-2017	-0.1151	-0.1333	-0.1601	-0.0956	-0.1006
16-Jul-2018	-0.0440	-0.0630	-0.0756	-0.0361	-0.0403
16-Jul-2019	0.0741	0.0493	0.0502	0.0656	0.0613
16-Jul-2020	0.2178	0.1899	0.1963	0.2011	0.1975
16-Jul-2021	0.3621	0.3416	0.3500	0.3499	0.3468
18-Jul-2022	0.5132	0.4922	0.4988	0.5005	0.4990
17-Jul-2023	0.6565	0.6345	0.6390	0.6432	0.6404
16-Jul-2024	0.7873	0.7666	0.7678	0.7765	0.7728
16-Jul-2025	0.9067	0.8833	0.8817	0.8940	0.8900
16-Jul-2026	1.0134	0.9834	0.9841	0.9923	0.9876
16-Jul-2027	1.1114	1.0835	1.0771	1.0957	1.0905
16-Jul-2030	1.3266	1.2954	1.2751	1.3157	1.3105
16-Jul-2035	1.5210	1.4828	1.4463	1.5132	1.5079
16-Jul-2040	1.5787	1.5367	1.4881	1.5750	1.5682
18-Jul-2044	1.5964	1.5628	1.4969	1.6009	1.5936

Tab. 4.3: EUR multivariate spread between OLSM inferred and market implied

Date	1M (bps)	3M (bps)	1M-3M (bps)	1M-3M-6M (bps)
17-Jul-2015	-0.2000	-0.2000	-0.2000	-0.2000
23-Jul-2015	-1.7046	-7.5935	4.1227	3.4781
17-Aug-2015	-1.0526	-8.1625	5.8140	5.1140
16-Sep-2015	-1.8868	-8.0990	4.4059	3.6310
16-Oct-2015	-2.1591	-7.8854	3.7709	2.9655
16-Nov-2015	-2.1340	-7.8251	3.7765	2.9811
17-Dec-2015	-1.9980	-7.5956	3.8501	3.0594
18-Jan-2016	-2.1475	-7.7475	3.7057	2.9425
16-Feb-2016	-2.1330	-7.6077	3.6337	2.8816
16-Mar-2016	-2.0947	-7.2452	3.4834	2.7149
18-Apr-2016	-2.1207	-6.9602	3.2764	2.4977
16-May-2016	-2.0256	-6.6034	3.1975	2.4096
17-Jun-2016	-1.9266	-6.3240	3.1291	2.3308
18-Jul-2016	-1.8361	-6.1503	3.1082	2.3072
16-Aug-2016	-1.8180	-5.9158	3.0057	2.2052
16-Sep-2016	-1.7987	-5.6651	2.8962	2.1156
17-Oct-2016	-1.7793	-5.4144	2.7866	2.0629
16-Nov-2016	-1.7101	-5.1719	2.7664	2.0403
19-Dec-2016	-1.6340	-4.9051	2.7441	2.0135
16-Jan-2017	-1.5694	-4.6787	2.7252	2.0989
18-Apr-2017	-1.7145	-4.5891	2.5140	1.9810
17-Jul-2017	-1.8172	-4.5014	1.9457	1.4457
16-Jul-2018	-1.8930	-3.1558	0.7947	0.3734
16-Jul-2019	-2.4831	-2.3958	-0.8557	-1.2880
16-Jul-2020	-2.7860	-2.1452	-1.6748	-2.0254
16-Jul-2021	-2.0534	-1.2127	-1.2160	-1.5248
18-Jul-2022	-2.1068	-1.4402	-1.2763	-1.4239
17-Jul-2023	-2.1945	-1.7517	-1.3286	-1.6093
16-Jul-2024	-2.0655	-1.9524	-1.0801	-1.4452
16-Jul-2025	-2.3419	-2.4949	-1.2693	-1.6694
16-Jul-2026	-2.9995	-2.9302	-2.1082	-2.5782
16-Jul-2027	-2.7959	-3.4363	-1.5677	-2.0912
16-Jul-2030	-3.1249	-5.1543	-1.0892	-1.6077
16-Jul-2035	-3.8154	-7.4693	-0.7811	-1.3102
16-Jul-2040	-4.1952	-9.0619	-0.3717	-1.0494
18-Jul-2044	-3.3622	-9.9527	0.4434	-0.2835

To further measure the performance of the OLSM, the sum of squared errors (SSE) was calculated for each curve. This compared each of the inferred OIS zero rates to the market implied zero rates, for each business day included in the zero curve, providing a more complete view of the performance of each of the EUR OLSM cases. Note that for all multivariate cases, only points up to the maturity coinciding with that of the full EUR OLSM were considered. These errors confirmed the performance of the different multivariate cases, with the EONIA-1M-3M having the lowest SSE, followed closely by the EONIA-1M-3M-6M case. The validity of the OIS curves recovered under such cases highlight that not all LIBOR rates were required to infer the OIS curve.

Tab. 4.4: EUR SSE $\times 10^4$

EONIA-1M	EONIA-3M	EONIA-1M-3M	EONIA-1M-3M-6M	EONIA-1M-3M-6M-12M
7.4361	29.9843	1.2908	1.7485	4.1927

4.2 ZAR Market

The ZAR market proved comparatively simple to bootstrap, as there was no need to implement a simultaneous multi-curve bootstrap. The 3M JIBAR curve could be bootstrapped under the single-curve framework, and thereafter the cointegrating relationship applied to infer the OIS curve. As there exists no OIS curve in the ZAR market, both the statistical models were implemented across all data sets to investigate the inferred OIS curves.

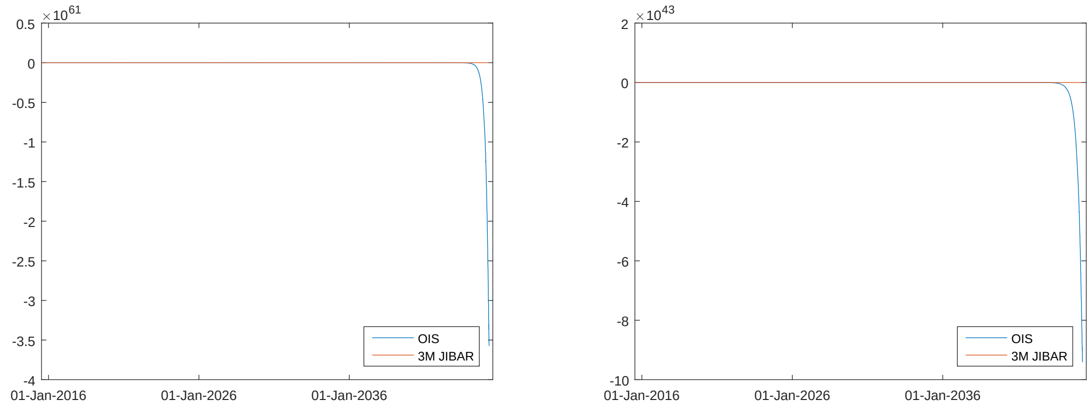
4.2.1 Method

1. Identify the market instruments and rates to be used in the bootstrapping procedure for 3M JIBAR
2. Bootstrap the 3M JIBAR curve under single-curve framework, obtaining convergence in all zero rates
3. (VECM only) Using the market visible and previously realised OIS and 3M JIBAR reference rates, initialise the VECM forecasting model
4. For the following business day, imply the 3M JIBAR forward rates, using the known 3M JIBAR zero curve

5. Apply statistical model (VECM/OLSM) to forecast the OIS rate for the given business day, using the above forward 3M JIBAR rate as the expected value of the 3M JIBAR reference rate
6. Stepping through each of the successive business days, out till the end of the zero curve, imply each of the successive forward 3M JIBAR rates and infer the corresponding OIS rates
7. From the successive daily forecasts of the OIS rate, recover the OIS zero curve (see Equation 2.1)

4.2.2 VECM Inferred

The following results are those of the VECM implied OIS curve in the ZAR market. As with the EUR market, the VECM was applied to both the entire and post-crisis data set.



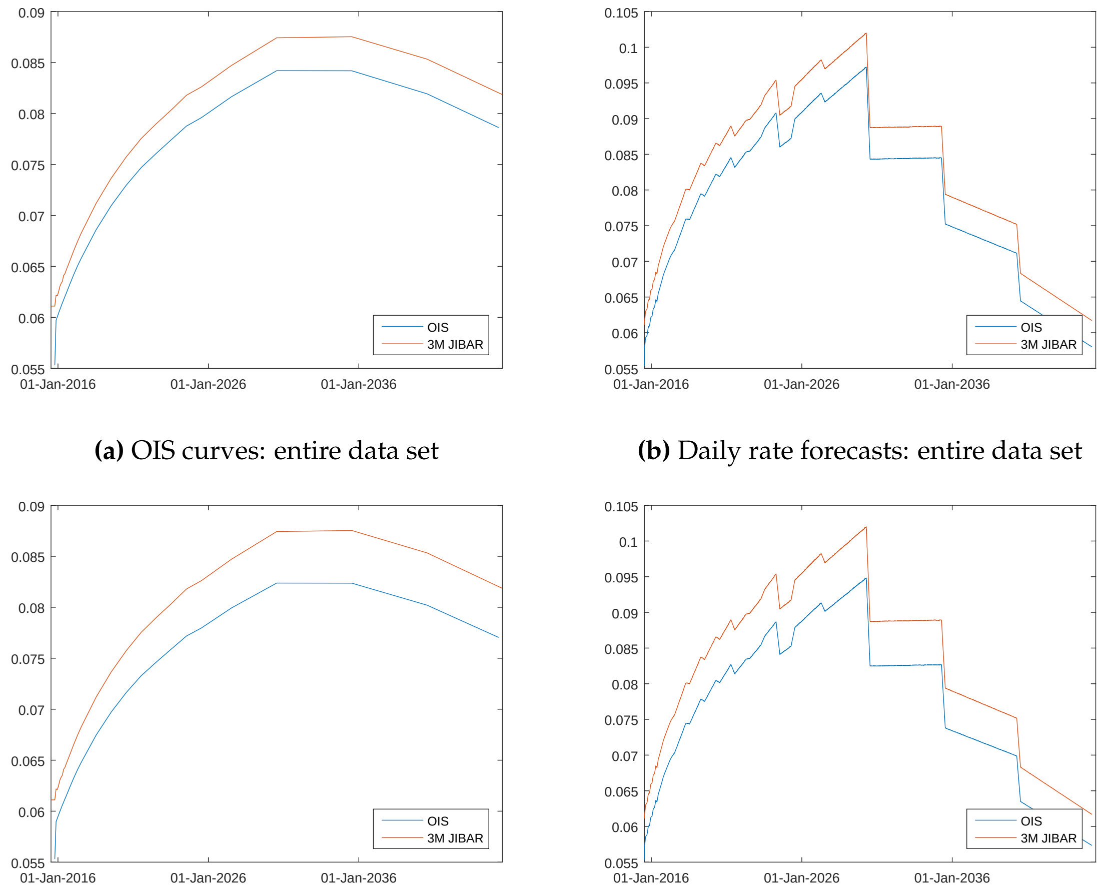
(a) Entire data set **(b)** Post-crisis data set

Fig. 4.6: ZAR VECM daily rate forecasts

Figure 4.6 shows the results of the VECM daily rate forecasts. It is evident that the VECM failed to imply any reasonable OIS forecasts, and no OIS curve could be recovered. In implementing the VECM in the ZAR market, a further shortcoming of the model was discovered. As discussed above, the influence of the data set and corresponding parameters was of key importance. When considering a significant downward trend in historical reference rates, as was the case for the ZAR market, the parameters reflect this long term trend. As a result, when attempting to forecast the daily OIS rates, each successive forecast was forced to follow this downward trend. This was compounded by the autoregressive component of the VECM, with the increasingly negative forecasts being included in the following forecasts. Thus the significant downward trend led to exponential decay in the daily forecasts, and no reasonable OIS curve could be recovered. Given the nature of the VECM, a significantly upward trend could lead to exponential growth in the daily forecasts.

4.2.3 OLSM Inferred

The following results are those of the OLSM inferred OIS curve in the ZAR market. With the failure of the VECM in the ZAR market, and having shown in the EUR market that the OLSM was better able to recover the OIS curve, the OLSM inferred curve was of particular interest.



(a) OIS curves: entire data set

(b) Daily rate forecasts: entire data set

(c) OIS curves: post-crisis data set

(d) Daily rate forecasts: post-crisis data set

Fig. 4.7: ZAR OLSM inferred OIS curves and daily rate forecasts

It can be seen from Figure 4.7 that the OLSM inferred what appeared to be a reasonable OIS curve. Due to the simple regression structure, in which the 3M JIBAR was essentially treated as an exogenous variable, the OLSM did not suffer from the impact of the long term trend and autoregressive component that resulted in the failure of the VECM. As a result, the OLSM was able to infer the OIS curve under significant upward or downward trends in historical reference rate data.

When comparing the performance of the entire and post-crisis data sets, the spread between the 3M JIBAR and inferred OIS curves was considered. It can be seen that the spread between the curves was notably greater in the post-crisis data, which was found to be the case in the EUR market. Thus, considering the EUR market results, it was deduced that the OLSM inferred OIS curve under the entire data set was the better estimate for the ZAR market.

Table 4.5 shows the zero curve rates for both 3M JIBAR and the inferred OIS, as well as the basis point spread between the rates. It can be seen that the spread between the rates lies around 20 bps int he short-end, widening to 30 bps and over in the long-end. The initial spread of around 50 bps quickly to 20bps, before widening back to 30 bps in the long-end. This change in spread level could be attributed to the compounding of the inferred daily OIS rates used to construct the OIS curve.

Tab. 4.5: ZAR OLSM inferred OIS against 3M JIBAR

Date	3M JIBAR (%)	OIS Rate (%)	Spread (bps)
16-Oct-2015	6.1107	5.5300	-58.0698
16-Nov-2015	6.2182	5.9709	-24.7308
17-Dec-2015	6.2119	6.0067	-20.5127
18-Jan-2016	6.2554	6.0433	-21.2131
16-Feb-2016	6.3036	6.0754	-22.8150
16-Mar-2016	6.3318	6.1075	-22.4299
18-Apr-2016	6.3542	6.1430	-21.1232
16-May-2016	6.4102	6.1712	-23.9008
17-Jun-2016	6.4287	6.2040	-22.4670
18-Jul-2016	6.4625	6.2339	-22.8604
17-Oct-2016	6.5571	6.3276	-22.9476
16-Jan-2017	6.6529	6.4182	-23.4669
18-Apr-2017	6.7432	6.5033	-23.9814
17-Jul-2017	6.8254	6.5788	-24.6564
16-Jul-2018	7.1194	6.8601	-25.9295
16-Jul-2019	7.3653	7.0958	-26.9482
16-Jul-2020	7.5743	7.2968	-27.7408
16-Jul-2021	7.7557	7.4704	-28.5222
18-Jul-2022	7.9008	7.6102	-29.0524
17-Jul-2023	8.0362	7.7437	-29.2472
16-Jul-2024	8.1788	7.8764	-30.2418
16-Jul-2025	8.2616	7.9597	-30.1981
16-Jul-2027	8.4718	8.1649	-30.6941
16-Jul-2030	8.7427	8.4199	-32.2751
16-Jul-2035	8.7533	8.4191	-33.4215
16-Jul-2040	8.5334	8.1931	-34.0285
18-Apr-2045	8.2027	7.8623	-34.0459

4.3 Further Discussion and Extensions

Based on the results of the implementation of the statistical models, further points on the application and extension of such models were considered.

Given the form of the OLSM, it is possible to create a synthetic OIS rate by taking the respective holdings in each of the different tenor LIBOR rates, as per the model parameters. Such a synthetic OIS rate could be used to effect a statistical arbitrage, by comparing the synthetic OIS rate to the realised OIS rate. Should there exist a difference between the two, it could indicate the deviation of the OIS rate from a long run equilibrium relative to the different tenor LIBOR rates.

The synthetic OIS rate could be used as a statistical hedge for the OIS, as taking positions in the longer tenored LIBOR rates would lock in the synthetic OIS rate for a given period of time (dependent on the number of tenor LIBOR rates used in the hedge). Furthermore, the synthetic OIS would be less volatile than the realised OIS rate, as the synthetic volatility is a function of the the less volatile tenor LIBOR rates. It is worth noting that implementing such a statistical hedge could incur high costs, as significant long and short positions would be required in each of the different tenor LIBOR rates.

The implementation of the short-end bootstrapping procedure detailed in Clarke (2010b) is a possible extension of the implementation of the statistical models. The assumption of flat OIS and tenor LIBOR rates out till future meeting dates could be well suited to the application of the OLSM. As it would be expected that the OIS and tenor LIBOR rates remain at similar spreads, the use of the OLSM to infer the level of the OIS based on the different tenor LIBOR rates should allow for the OIS level to be reasonably well inferred. This could then be extended to the short-end bootstrapping procedure.

4.4 Chapter Summary

The implementation of statistical models and bootstrapping procedures showed the potential application of such methods to infer the OIS curve. Key considerations include that of the data set used to estimate the parameters of the model, the existence of autoregressive and trend components, and the number and tenor of rates used in the statistical model. It was found the OLSM significantly outperformed the VECM model in both the EUR and ZAR market. Furthermore, it was shown that use of the entire data set over that of the post-crisis data set produced better OIS curve estimates. In the case of the EUR market, better estimates could be obtained by excluding the 12M tenor LIBOR rate, highlighting that not all tenor

LIBOR rates were required to recover the OIS curve.

Chapter 5

Conclusion

Research of the current market methods allowed for the OIS and different tenor LIBOR curves to be bootstrapped under such procedures. This was done in both the EUR market, under a multi-curve framework, and the ZAR market, under a single-curve framework. The market implied curves in the EUR market would serve as a baseline, allowing for the performance of the statistical models to be measured.

Historical reference rate data was used to test for the existence of cointegrating relationships between OIS and LIBOR rates, in both the EUR and ZAR markets. The Johansen procedure and Engle-Granger approach were used to determine the nature of the cointegrating relationship and estimate parameters for the VECM and OLSM. It was found in general that the different tenor LIBOR and OIS rates exhibited strong cointegrating relationships within each of their respective markets. In the case of the EUR market, cointegrating relationships were still found to exist when considering the different multivariate cases. These cointegrating relationships validated the use of the OLSM and VECM to model the relationship between tenor LIBOR and OIS rates. The parameter estimates for each of the models were found to vary when considering data sets spanning different time periods, as a result of the different nature and trend of reference rates over the period. In particular, the general upward or downward trend and spread between rates were found to have greatest impact on parameter estimates. The VECM and OLSM residuals were tested for heteroskedasticity, with all but the ZAR VECM exhibiting such effects. The existence of heteroskedasticity could be attributed to the nature and volatility of the OIS rates. As these effects do not invalidate the use of the statistical models, no consideration was give to alternative models.

Bootstrapping procedures were developed for both the EUR and ZAR markets, allowing the implementation of the statistical models in both a single- and multi-curve framework. The bootstrapping procedures inferred daily OIS rate forecasts from forward tenor LIBOR rates, allowing for both the OIS and tenor LIBOR curves to be bootstrapped whilst recovering market rates. Under implementation in the simultaneous bootstrapping procedure (EUR market), notable differences in the performance of the VECM and OLSM was observed. These differences could be attributed to the autoregressive and long run trend components of the VECM, and led to the failure of the VECM to capture the correct term structure of the OIS and different tenor LIBOR curves. In the ZAR market, these components resulted in the exponential decay of daily OIS rate forecasts, invalidating the use of the VECM. The treatment of different tenor LIBOR rates as exogenous variables under the OLSM allowed for the correct term structure to be captured, with autoregressive and the long run trend components no longer influencing results. Under the implementation of the procedures, the importance of the data set was highlighted. Use of the entire data set parameters provided reasonable results, whilst the post-crisis data set parameters failed to infer a reasonable OIS curve. This was due to differences in the spread between reference rates across the data sets. When considering multivariate cases in the EUR market, it was found that including too few LIBOR rates produced poor results, however inclusion of all rates was not necessary to recover the OIS curve effectively. The exclusion of the 12M LIBOR rate produced improved results over the full OLSM, highlighting that the shorter tenored LIBOR rates may be closer related to the OIS rate. Based on the results of the implementation of the statistical models in the bootstrapping procedures, the preferred model was found to be that of the OLSM, with parameters estimated over the entire data set. Thus it is recommended that this model be used for the bootstrapping of the OIS curve in the ZAR market.

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Appendix A

Boostrapping Instruments

Tab. A.1: EUR OIS Market Instruments

Instrument	Tenor		Type
EONIA	1	day	DEP
EUSWE1Z	7	day	SWP
EUSWE2Z	14	day	SWP
EUSWEA	1	month	SWP
EUSWEB	2	month	SWP
EUSWEC	3	month	SWP
EUSWED	4	month	SWP
EUSWEE	5	month	SWP
EUSWEF	6	month	SWP
EUSWEG	7	month	SWP
EUSWEH	8	month	SWP
EUSWEI	9	month	SWP
EUSWEJ	10	month	SWP
EUSWEK	11	month	SWP
EUSWE1	12	month	SWP
EUSWE1F	18	month	SWP
EUSWE2	2	year	SWP
EUSWE2F	30	month	SWP
EUSWE3	3	year	SWP
EUSWE4	4	year	SWP
EUSWE5	5	year	SWP
EUSWE6	6	year	SWP
EUSWE7	7	year	SWP
EUSWE8	8	year	SWP
EUSWE9	9	year	SWP
EUSWE10	10	year	SWP
EUSWE11	11	year	SWP
EUSWE12	12	year	SWP
EUSWE15	15	year	SWP
EUSWE20	20	year	SWP
EUSWE25	25	year	SWP
EUSWE30	30	year	SWP
EUSWE35	35	year	SWP
EUSWE40	40	year	SWP
EUSWE50	50	year	SWP

Tab. A.2: EUR 1M Market Instruments

Instrument	Tenor		Type
EONIA	1	day	DEP
EUR001W	7	day	DEP
EUR001M	1	month	DEP
EUSWBV1	2	month	SWP
EUSWCV1	3	month	SWP
EUSWDV1	4	month	SWP
EUSWEV1	5	month	SWP
EUSWFV1	6	month	SWP
EUSWGV1	7	month	SWP
EUSWHV1	8	month	SWP
EUSWIV1	9	month	SWP
EUSWJV1	10	month	SWP
EUSWKV1	11	month	SWP
EUSW1V1	1	year	SWP
EUSW1CV1	15	month	SWP
EUSW1FV1	18	month	SWP
EUSW1IV1	21	month	SWP
EUSW2V1	2	year	SWP
EUSW3V1	3	year	SWP
EUSW4V1	4	year	SWP
EUSW5V1	5	year	SWP
EUSW6V1	6	year	SWP
EUSW7V1	7	year	SWP
EUSW8V1	8	year	SWP
EUSW9V1	9	year	SWP
EUSW10V1	10	year	SWP
EUSW12V1	12	year	SWP
EUSW15V1	15	year	SWP
EUSW20V1	20	year	SWP
EUSW25V1	25	year	SWP
EUSW30V1	30	year	SWP

Tab. A.3: EUR 3M Market Instruments

Instrument	Tenor		Type
EONIA	1	day	DEP
EUR001W	7	day	DEP
EUR001M	1	month	DEP
EUR003M	3	month	DEP
EUFR0AD	4	month	FRA
EUFR0BE	5	month	FRA
EUFR0CF	6	month	FRA
EUFR0DG	7	month	FRA
EUSW1VC	1	year	SWP
EUSW2V3	2	year	SWP
EUSW3V3	3	year	SWP
EUSW4V3	4	year	SWP
EUSW5V3	5	year	SWP
EUSW6V3	6	year	SWP
EUSW7V3	7	year	SWP
EUSW8V3	8	year	SWP
EUSW9V3	9	year	SWP
EUSW10V3	10	year	SWP
EUSW11V3	11	year	SWP
EUSW12V3	12	year	SWP
EUSW15V3	15	year	SWP
EUSW20V3	20	year	SWP
EUSW25V3	25	year	SWP
EUSW30V3	30	year	SWP
EUSW35V3	35	year	SWP
EUSW40V3	40	year	SWP
EUSW50V3	50	year	SWP

Tab. A.4: EUR 6M Market Instruments

Inst	Tenor		Type
EONIA	1	day	DEP
EUR001W	7	day	DEP
EUR001M	1	month	DEP
EUR003M	3	month	DEP
EUR006M	6	month	DEP
EUFR0AG	7	month	FRA
EUFR0BH	8	month	FRA
EUFR0CI	9	month	FRA
EUFR0DJ	10	month	FRA
EUFR0EK	11	month	FRA
EUFR0F1	12	month	FRA
EUFR0G1A	13	month	FRA
EUFR0H1B	14	month	FRA
EUFR0I1C	15	month	FRA
EUFR0J1D	16	month	FRA
EUFR0K1E	17	month	FRA
EUFR011F	18	month	FRA
EUSA2	2	year	SWP
EUSA3	3	year	SWP
EUSA4	4	year	SWP
EUSA5	5	year	SWP
EUSA6	6	year	SWP
EUSA7	7	year	SWP
EUSA8	8	year	SWP
EUSA9	9	year	SWP
EUSA10	10	year	SWP
EUSA11	11	year	SWP
EUSA12	12	year	SWP
EUSA15	15	year	SWP
EUSA20	20	year	SWP
EUSA25	25	year	SWP
EUSA30	30	year	SWP
EUSA35	35	year	SWP
EUSA40	40	year	SWP
EUSA45	45	year	SWP
EUSA50	50	year	SWP

Tab. A.5: EUR 12M Market Instruments

Inst	Tenor		Type	Spread
EONIA	1	day	DEP	
EUR001W	7	day	DEP	
EUR001M	1	month	DEP	
EUR002M	2	month	DEP	
EUR003M	3	month	DEP	
EUR006M	6	month	DEP	
EUR009M	9	month	DEP	
EUR012M	12	month	DEP	
EUSA2	2	year	SWP	EUBSS2
EUSA3	3	year	SWP	EUBSS3
EUSA4	4	year	SWP	EUBSS4
EUSA5	5	year	SWP	EUBSS5
EUSA6	6	year	SWP	EUBSS6
EUSA7	7	year	SWP	EUBSS7
EUSA8	8	year	SWP	EUBSS8
EUSA9	9	year	SWP	EUBSS9
EUSA10	10	year	SWP	EUBSS10
EUSA12	12	year	SWP	EUBSS12
EUSA15	15	year	SWP	EUBSS15
EUSA20	20	year	SWP	EUBSS20
EUSA25	25	year	SWP	EUBSS25
EUSA30	30	year	SWP	EUBSS30

Tab. A.6: ZAR 3M Market Instruments

Instrument	Tenor		Type
JIBA3M	3	month	DEP
SAFR0AD	4	month	FRA
SAFR0BE	5	month	FRA
SAFR0CF	6	month	FRA
SAFR0DG	7	month	FRA
SAFR0EH	8	month	FRA
SAFR0FI	9	month	FRA
SAFR0GJ	10	month	FRA
SAFR0HK	11	month	FRA
SAFR0I1	12	month	FRA
SAFR011C	15	month	FRA
SAFR1C1F	18	month	FRA
SAFR1F1I	21	month	FRA
SAFR1I2	24	month	FRA
SASW3	3	year	SWP
SASW4	4	year	SWP
SASW5	5	year	SWP
SASW6	6	year	SWP
SASW7	7	year	SWP
SASW8	8	year	SWP
SASW9	9	year	SWP
SASW10	10	year	SWP
SASW12	12	year	SWP
SASW15	15	year	SWP
SASW20	20	year	SWP
SASW25	25	year	SWP
SASW30	30	year	SWP