



This is the post-print of Le Roux, K. 2009. Flu viruses, a lucky community and cosine graphs: the possibilities opened up by the use of a socio-political perspective to study learning in an undergraduate access course in mathematics. *African Journal of Research in Mathematics, Science and Technology Education*. 13(2): 44-63. DOI: <http://dx.doi.org/10.1080/10288457.2009.10740656>.

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Flu viruses, a lucky community and cosine graphs: The possibilities opened up by the use of a socio-political perspective to study learning in an undergraduate access course in mathematics

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Abstract

In this paper I present a perspective of mathematics education and learning, termed a 'socio-political perspective'. Classroom mathematical activity, in which certain ways of acting, behaving and knowing are given value, is located in a wider network of socio-political practices. Learning in mathematics is regarded as coming to participate in the discourse of the community that practises the mathematics. I argue that the use of a socio-political perspective allows the researcher and teacher to view classroom mathematical activity as a product of the network of socio-political practices in which it is located, rather than as a product of individual cognitive ability. I illustrate the use of this perspective by drawing on a study of learning in a first-year university access course in Mathematics at a South African university. Fairclough's method for critical discourse analysis, supplemented with work by Sfard and Morgan in mathematics education, was used to analyse both the text of a 'real world' problem in mathematics and a transcript representing the activity as a group of five students solved the problem. This analysis suggests that, despite containing traces of discourses from outside of mathematics, the problem text constructs the activity as solving a mathematical problem with features of a school mathematical word problem. When solving the problem the students draw on practices associated with school mathematics and their university mathematics course, some of which enable and others constrain their participation. For example, they refer to named functions learned at school, they have difficulty making productive links between the mathematical functions and the 'real world' context, and they have varied opportunities for mathematical talk in the group. The study identifies as key to the students' progress the presence of an authority (in this case a tutor) who can make explicit the ways of thinking, acting, and talking that are valued in the discourse of undergraduate mathematics, and who provides opportunities for mathematical talk.

Keywords: critical discourse analysis, real-world problems, socio-political perspective of learning, undergraduate mathematics, word problems

Introduction

Traditionally, research into the learning of mathematics at undergraduate level has drawn mainly on theoretical perspectives from mathematics and psychology. Literature from the 1980s and 1990s is dominated by studies of the cognitive processes involved when individuals learn mathematical concepts such as function, limit and proof (e.g. Tall, 1991). "Approaches to learning" research, influential in educational research in higher education, has also been taken up at the level of undergraduate mathematics (e.g. Crawford, Gordon, Nicholas, & Prosser, 1994). A review of more recent research publications suggests that a psychological perspective of learning still maintains a key presence at this level (e.g. Habre & Abboud, 2006; Inglis & Simpson,

2008; Przenioslo, 2004; Semadeni, 2008). This, in spite of the “social turn” which emerged in mathematics education research at school level in the late 1980s, a trend which saw a move away from a focus on the acquisition of knowledge by the individual to “theories that see meaning, thinking and reasoning as products of social activity” (Lerman, 2000, p.23).

Some recent studies on the learning of mathematics at undergraduate level can be located in a “social turn”, yet this research confirms the view of John-Steiner and Mahn (1996) that such a term includes diverse interpretations of the “social”. For example, the social context can be taken as a textbook or a mathematical task constructed by a teacher, the classroom context, the departmental environment, or the institutional environment (e.g. Bingolbali & Monaghan, 2008; Berger, 2004; Castela, 2004; Jaworski, 2002; Smith, 2006). Furthermore the location of the individual within the social varies in these studies. The work of Solomon (2006, 2007, 2009) and Wistedt and Brattström (2005) is notable, as it regards learning as coming to participate in or acting according to the rules of particular mathematical communities.

The psychological perspective that has dominated research into mathematics learning at undergraduate level has made a considerable contribution to our understanding of learning, yet this research needs to be challenged from two points of view; in practice certain students continue to fail mathematics, and in terms of research the dominant perspective results in certain types of studies being conducted and not others. In her 1991 presentation of an ontological-psychological perspective on the nature of mathematical concepts, Sfard (1991, p.1) commented that despite the accumulated knowledge on mathematics education, a solution to the considerable problems encountered by students in practice “seems to be as elusive as a cure for the common cold”. Eighteen years later the challenge remains. While acknowledging the complex relationship between educational research and practice (Skovsmose, 2006), from a moral standpoint we cannot ignore the fact that while some students may be succeeding in mathematics, certain students remain marginalized from the discipline and continue to fail. In South Africa, for example, educational experience at school remains inequitable and performance in school mathematics, while generally poor overall by international standards, can be linked to the related factors of race and social class (Reddy, 2006; Simkins, Rule, & Bernstein, 2007). Moreover, while students may gain access to study at tertiary institutions, their success rates at these institutions, particularly in science and engineering where mathematics learning plays a key role, remain very poor (Scott, Yeld, & Hendry, 2007). The empirical research quoted here suggests that mathematics teaching and learning practices, both at school and tertiary level, position participants in different ways in society, providing support for the view that these practices are socio-political (Valero, 2008).

Furthermore, from the perspective of educational research, Valero (2008) argues:

A choice of theoretical and methodological approach in mathematics education research (or in any research in general) is not an accidental act. ... different possibilities are opened and closed by different approaches. (p.56)

The community has gained considerable knowledge by adopting a psychological perspective of the learning of mathematics, yet this perspective has allowed us to ask and to answer some questions about mathematics learning and not others. For example, Sfard (2007, p.569) argues that a focus on the acquisition of concepts by the individual means that the “messy personal interaction” that takes place during learning is not considered, and Lerman (2001) notes that perspectives from mathematics and psychology do not allow us to address the view of schooling as reproduction and the role of culture and power in the mathematics classroom.

In addition, a number of researchers have argued that studies conducted from a psychological perspective can be accompanied by a deficit model of students that locates success or failure in mathematics in the individual student (e.g. Zevenbergen & Flavel, 2007). Valero (2004a) claims that particular research perspectives build constructs which guide our activity both as researchers and as teachers. By constructing certain students as in deficit, the psychological perspective exerts power, and as the brief literature review in this paper suggests, this power is being reproduced in mathematics education research at undergraduate level. This argument provides support for the view that mathematics education research as a practice is socio-political (Valero, 2004b).

The aim of this paper is to present a socio-political perspective of mathematics learning. This perspective draws on work from two fields, namely critical linguistics and mathematics education research at school level. My intention in presenting this perspective is not to discount research conducted from other perspectives. Rather, in the light of Valero's (2008, p.56) comment, I prefer to consider what possibilities are "opened up" by my particular choice of perspective that may be "closed" by other approaches. In considering these possibilities, I draw on an ongoing study in which I am investigating the use of problems with "real-world" contexts in an undergraduate access course in mathematics (which I will refer to as the Course from this point).¹ The broad aim of the study is to identify what enables and constrains the students' learning of mathematics when they solve these problems, and to explain this in terms of the socio-political practices of the classroom and of the wider context in which the classroom is located.

I begin the paper by presenting a broad theoretical notion of mathematics education as a network of socio-political practices, with a focus on student learning as a socio-political practice. I then provide a description of the study and the methodology adopted. I present two aspects of the study in this paper. Firstly, I present the analysis of the text of a real-world problem, which I call the Flu Virus Problem. I discuss what meanings are valued by the text and what identities are made available to the student by the text. I attempt to explain this in the wider socio-political context in which the problem is embedded. Secondly, I present an analysis of the interaction between five students as they solve the Flu Virus Problem. This analysis focuses on the features of the interaction that appear to enable or constrain the development of the valued mathematical discourse. I explain this interaction in terms of the socio-political practices of the classroom and the wider socio-political practices in which the classroom activity is located. I use the results to argue that a socio-political perspective can be used to represent undergraduate classroom activity as a product of the network of socio-political practices rather than as a product of individual student ability.

Theoretical framework

In challenging the psychological perspective of mathematics learning in the introduction to this paper I have argued that the practices of mathematics teaching and learning and the practice of mathematics education research are socio-political. In this section I expand on this argument by presenting the broad theoretical notion of mathematics education as a network of socio-political

1 I am taking "real-world" to mean "everything that has to do with nature, society or culture, including everyday life as well as school and university subjects or scientific or scholarly disciplines different from Mathematics" (The International Commission for Mathematics Instruction, 2002, p.230). The term "real-world problem" is a broad term that can be applied to a variety of mathematical problems. Developing a description of the "real-world problems" used in the Course is one aspect of the larger study (descriptions of two such problems in the Course are presented in Le Roux (2008a, 2008b)).

practices. I then elaborate on the particular aspect of this notion that is the focus of this paper, that is, a socio-political perspective of learning mathematics.

Mathematics education as a network of socio-political practices

In this paper I adopt Valero's (2007, p.226) use of the term "network of mathematics education practices" to refer to the many practices where the notion of mathematics teaching and learning is given meaning, for example, mathematics classroom activity, policy making, teacher education and research. These practices are "social practices", in the sense used by Fairclough (2003, p.205), as they are relatively stable forms of social activity, each with certain activities, participants, social relations, objects, position in time and space, values and discourse.² Fairclough (2003, p.23) argues that these practices are "social" in two respects. Firstly, they are embedded in a wider network of practices in which their meaning is constructed and reconstructed. For example, the activity of a teacher and students in an undergraduate mathematics classroom is socially determined through its relationship to other practices. Secondly, this mathematics classroom activity has social effects with regard to reproducing or changing practices and social relationships.

So if we regard the practices of mathematics education as social, in what sense are they also political? Social practices are also political since certain activities, behaviours, knowledge and values are given status in these practices. For example, in an undergraduate mathematics classroom, particular ways of approaching a problem, writing a solution or relating to one's peers may be valued. Valero (2007) draws on the work of Foucault to argue that power is distributed when people participate in social practices. The term "power" here does not refer to a fixed characteristic of institutions or of individuals which is imposed on others, but refers to the capacity of people to position themselves in relation to what is valued in the practices. This theoretical view of mathematics education is complemented by a growing body of empirical research located in school mathematics arguing that the practices of schools position mathematics students differently (e.g. Cooper & Dunne, 2000; Cotton & Hardy, 2004; Dowling, 1996).

Mathematics learning as a socio-political practice

Having established the notion that the varied practices of mathematics education are both social and political, I now turn to what this perspective means for student learning in mathematics. Firstly, I draw on a socio-cultural perspective of learning which argues that since the practices of mathematics are social practices, learning mathematics involves coming to participate in the discourse of the community that practises the mathematics (e.g. Lerman, 2001; Sfard, 2001b).³ Of course there are a number of communities that practise mathematics. Both Morgan (1998) and Sfard (2000a) agree that there is something distinctive about the various social practices that we would label as being "mathematical", yet certain aspects of these practices may differ considerably, for example, what is considered an appropriate argument in school mathematics and in academic mathematics. Furthermore, since mathematics education is inherently political,

2 The term "discourse" here is used for written and spoken language as well as other forms of semiosis such as body language and visual images (Fairclough, 2003, p.205).

3 The term "discourse" here is used in the abstract sense to refer to the particular ways of using written and spoken language, as well as other forms of semiosis such as symbolic tools and body language, that are considered meaningful in a particular practice or community. Fairclough (1989, p.90) refers to this as the "conventions, norms, codes of practice underlying actual discourse".

becoming a participant in the discourse of a community not only involves grappling with the content and skills of the community and using the tools of the community appropriately, but also determining what is valued in the community and negotiating one's identity and position in that community.

The study

The research study that I draw on in this paper is located in a first-year university access Course in mathematics at a South African university. This Course forms part of an extended curriculum programme which is specifically designed to provide students, disadvantaged by the schooling system, with access to tertiary studies in science. After six weeks of the academic year, this group of students is joined in the Course by those students who are performing poorly on the mainstream first-year mathematics course. In the year that the data was collected, I played the role of researcher, lecturer on the Course, convenor of the Course, and student advisor to the students in the extended programme.

In this paper I focus on a sub-question of one of the six real-world problems used in the study. The group of five students represented in this paper were videoed as they worked in a group on this sub-question in the regular weekly afternoon workshop (the part of the problem to be discussed in this paper is given in Figure 1). The video was filmed by a professional from the university television services and, although I was present in the workshop venue during the filming, I did not interact with the students in any way. The students had access to a tutor and resources such as course notes. The tutor, whom I refer to as the Tutor in this paper, had pursued postgraduate studies in mathematics and in education and was selected on the basis of his excellent record as a tutor on access courses. Ethics agreements were negotiated with the student participants and the Tutor prior to the study.

6. A flu virus has hit a community of 10 000 people. Once a person has had the flu he or she becomes immune to the disease and does not get it again. Sooner or later everybody in the community catches the flu. Let $P(t)$ denote the number of people who have, or have had the disease t days after the first case of flu was recorded.
- (a) Draw a rough sketch of the graph of P as a function of t , clearly showing the maximum number of people who get infected, **and do not continue until you have had your graph checked by a tutor.**

Figure 1: The Flu Virus Problem, Question 6a, Workshop 10, 2007 Resource Book, page 54

The solution to this sub-question which is valued in the course, and is provided to students in the written solutions a few days after the workshop, is given in Figure 2⁴.

4 It should be noted that this "solution" is problematic as a model for the spread of the disease. However, a discussion of the problematic features of the graph is not included in this paper, since the students had not seen this solution when the data used in this study was collected. The Tutor, who had been exposed to this problematic solution when preparing for the workshop, provided an alternative solution for the students.

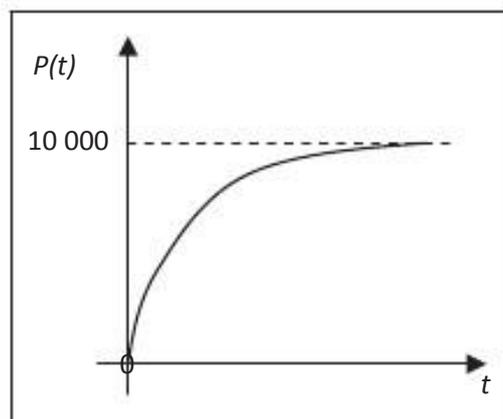


Figure 2: Worked Solution for Question 6a, Workshop 10

The video-footage made was selective as it could not include all aspects of the action. This footage was transcribed by the researcher to represent both the verbal and non-verbal action of the students. The resulting transcript is a “re-presentation” of that video-footage, as the transcription process is selective and is shaped by the theoretical perspective of the researcher (Setati, 2003, p.294).

Methodology

The analytic tools used in the study are based on the work of Fairclough (1989, 1992, 2003, 2006) in the field of critical linguistics, but supplemented with the work of Sfard (2000a, 2000b, 2001a, 2001b, 2007) and Morgan (1998, 2006) within mathematics education. In my presentation of the methodology below I discuss the analytic tools with particular reference to my study of classroom mathematical activity and mathematical texts.

Fairclough’s method for critical discourse analysis

Fairclough (2003, p.205) regards text, such as the written text of a mathematics problem or the written transcript representing student action, as a “social event” which is shaped by socio-political practices and social agents.⁵ Such a text is a “mode of representation” in the sense that it reflects wider socio-political practices, but it is also a “mode of action” as it gives meaning to these practices by constituting them (Fairclough, 1992, p.64). He identifies three ways in which meaning is constructed by text (Fairclough, 2003, p.26-27):

1. “Representation” refers to how text represents aspects of the physical, social and material world, for example, how mathematics or the everyday world is represented.
2. “Action” refers to how text acts and interacts by enacting relations between participants.
3. “Identification” refers to how text identifies people and their values.

According to Fairclough (1989) analysis of a social event involves focusing on the discursive processes of text production and interpretation. These processes are socially constrained, firstly, by the “members’ resources”, that is, the internalized social structures and conventions that

5 Fairclough’s use of the term “social practice” is inclusive and includes political practice. Given my specific theoretical orientation I have chosen to refer to “socio-political practice” in this paper.

individuals bring to the setting. Secondly, they are socially constrained by the specific socio-political practice of which the members are part (Fairclough, 1989, p.25).

Fairclough (1992) argues that we cannot reconstruct the processes of production and interpretation, but we can look for traces of this in the text. Using critical discourse analysis as a methodology to analyse the text of a mathematics problem and a transcript representing classroom activity thus involves two related activities:

1. Looking at the text, line by line, in terms of the three types of meaning and how these are “realized” in various features of the text (p.72). Fairclough (1989, 1992, 2003) provides a list of textual features that can be used to identify these meanings in the text, and I have supplemented this list by drawing on the work of Janks (2005), McCormick (2005), Morgan (1998), and Pimm (1987).
2. Making a connection between the concrete social event and more abstract socio-political practices. Fairclough (2003, p.28) states that “discourse”, “genre” and “style” are relatively stable ways of “representing”, “acting”, and “identifying” respectively. Hence we can make a link to the wider socio-political practices by asking which discourses, genres and styles are articulated in the text.

Investigating mathematical discourse

Fairclough’s tools have been used for studying texts in various social contexts, but for my purposes they required development for the study of mathematical discourse. Acknowledging that discourse varies across “mathematical” practices, both Morgan (1998) and Sfard (2000a, 2001a, 2001b) point to certain features of discourse that are related to these practices, namely, particular ways of using language and symbolism, characteristics processes, certain types of objects, particular rules of argument, and certain rules of engagement between participants.

Commenting on the wider field of discourse analysis, Sfard (2000b, p.298) notes that, while this method has been used to study the “rules and norms constituting mathematical practices”, little attention has been given to using the method for the study of mathematical content and in particular mathematical objects. Sfard (2001a, p.34) proposes the use of “focal analysis” for studying the mathematical content of communication. Two parts of the “discursive focus” can be identified in the semiotic activity of student interaction, that is, the “pronounced focus” which refers to the words the student uses when identifying “the object of her or his attention” and the “attended focus” which is what the student is “looking at, listening to” (Sfard, 2000b, p.304).

Results: The Flu Virus Problem

I begin by presenting an analysis of the text of the Flu Virus Problem (see Figure 1). Using the analytic tools described in the previous section, I describe what the text represents and identify what relationships and identities it constructs for the student. I provide the textual evidence for these descriptions, although space restraints prevent me from providing all the detail. I also explain the nature of the text with reference to the wider socio-political context in which the problem is located. This analysis forms the background for the analysis of the interaction between five students as they solve the Flu Virus Problem which follows.

Analysis of the Flu Virus Problem

The first three sentences of the problem represent the problem text as a problem about a “flu virus”. This representation suggests links to both the study of epidemiology and a reform mathematics discourse which promotes making the study of mathematics relevant through the use of real-world contexts. ⁶However, certain textual features in these first three sentences suggest that the epidemiology discourse is not the focus of the text, and that the task context can be linked to an everyday discourse. The text represents the study of disease more in everyday, rather than the scientific terms, for example, this “flu virus” is not named and is used interchangeably with other everyday terms such as “the flu” and “the disease”. In sentence 3 the term “immune” is explained in everyday language as “does not get it again”, thus positioning the student as someone who needs the specialist terms of epidemiology explained. The community infected by the virus and the “people” in this community are represented in general terms, with no names or location in space and time (other than the vague everyday term “sooner or later” in sentence 3). These “people” are passive in relation to the disease, they simply “catch” or are “hit” by the flu, and their identification as either male or female suggests another link to a reform mathematics discourse which promotes inclusivity in terms of gender.

The presentation of the flu virus, the community and the people in the community as not real in time or in space links the problem to the discourse of school mathematical word problems. Gerofsky (1996, p.40) argues that a school mathematical word problem “pretends that” a particular situation exists. The tense of the Flu Virus Problem presents the situation as ongoing; Gerofsky (1996, p.40) notes that tense is not important in school mathematical word problems. The statements in sentences 1 to 3 are presented in the declarative mood, with the author (whose voice is absent) instructing the student. The information about the task context is presented with certainty, and the student is positioned as someone who accepts the information as true, without supporting evidence.

Sentence 4 provides a clue to the focus of the text by giving meaning to the first three sentences as sentences in a mathematics problem. It does this by linking the flu virus with a mathematical object, that is, a function that can be represented using the symbols P and t . The instruction in sentence 4 is given as an imperative “Let...”, a feature that is common in school mathematics texts, and hides the human voice of the author (Morgan, 1998, p.16). Thus sentence 4 can be likened to the “set-up” aspect of a school mathematical word problem (Gerofsky, 1996, p.37). Sentence 4 is followed by an instruction to the student to draw “the graph of P as a function of t ”, thus reinforcing the representation as a mathematical problem. This instruction is numbered, representing a question addressed to the student and following on from the “information” and “set-up” aspects of the school mathematical word problem (Gerofsky, 1996, p.37). The use of the definite article “the” for the graph suggests that there is only one correct graph. The presentation of one possible graph in the written solutions to the problem further reinforces this suggestion (see Figure 2).

Three features of question (a) serve as reminders to the student and position him/her as someone needing help with the problem. Firstly, the adjective “rough” is used to describe the required sketch graph, although the term “sketch” has the notion of “rough” implicit in it. Secondly,

6 The process of naming discourses has the effect of fixing these discourses in time and space and of setting up boundaries. Yet, by definition, discourses are overlapping, changing, and may vary across communities (Moschkovich, 2007). Furthermore, my classification of discourses is influenced by my understanding of the setting of the study, and unavoidably reflects some value judgments about this setting.

the student is provided with an additional instruction to “clearly” show “the maximum number of people who get infected”. Thirdly, the student is provided with an additional instruction (presented in bold as an additional reminder) to have his/her graph “checked by a tutor”. In this instruction the student is addressed directly as “you”, a deviation from what one would expect to see in an undergraduate mathematics textbook. The nature of the Flu Virus Problem as a problem to be solved in an interactive workshop session certainly makes such a deviation possible, yet this feature also represents the activity of solving the problem as different from the activity of solving problems presented a textbook. Although the possessive pronoun “your” gives the student ownership of his/her graph, the fact that the tutor is given the power to evaluate the graph suggests that a student’s graph may not be the correct one.

Results: The student interaction when solving the Flu Virus Problem

Using the previous analysis of the Flu Virus Problem as a basis, I now present an analysis of a transcript representing the interaction of five students (Bongani, Mpumelelo, Siyabulela, Vuyani and Lungiswa) as they solve this Problem. ⁷ Since this analysis does not make use of the full set of data used in the wider study, it is unavoidable that certain students feature more prominently than others in the presentation. This should not be taken to be a reflection on what took place in other interactions, and which will be documented in the wider study.

I present the analysis in the form of six “discursive actions” (Sfard, 2001b, p.6) which appear to determine the direction of the discourse and to be key in enabling or constraining the students’ participation in the discourse of mathematics. I also attempt to explain these actions in the light of the socio-political practices of the classroom and by referring to the discourses, genres and styles of the wider space. Although I present the six actions separately, they are inter-related and interact to produce the discourse represented in the transcript.

Analysis of the student interaction

1. The students identify the required population graph with known, named graphs.

Once the students have read through the question, Mpumelelo is the first student to make a claim public in the group. When he asks, “It won’t it be like a cos graph?” he identifies the population graph with a particular type of trigonometric graph, the cosine graph. He also presents a standard visual image of the graph by physically tracing the shape in the air with his pen (see Figure 3). Later in the interaction the Tutor persuades the students that the graph is increasing, and Mpumelelo asks whether the graph will go “up and down or will it be just a straight line?”. His use of the words “or” and “just” here suggest that he has only two choices. When the Tutor has intervened to explain that the graph cannot be a straight line, Siyabulela and Mpumelelo co-construct an alternative description, this time using the more general word “curves” to describe the graph.

⁷ The names of the students have been changed. Ismail (2008, p.5) refers to the “politics of representation” when noting that the research writing process constructs identities for the research participants. The names Bongani (thanks), Mpumelelo (success), Siyabulela (thanks), Vuyani (happy) and Lungiswa (perfect) have been selected to acknowledge my admiration for these five students and my gratitude for their willingness to take part in the study.

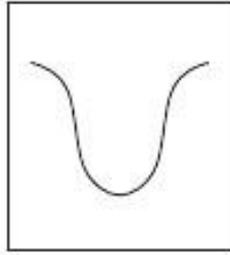


Figure 3

Mpumelelo consistently identifies himself as a student who is prepared to make his ideas public either verbally or as visual images. His claims are always tentative, as suggested by his presentation of his idea as a question and his use of the modalized negative “won’t” in “It won’t it be like a cos graph?”. Yet the graphs he names and the accompanying visual images he presents become the pronounced and the attended focus in the discussion between the five students, to the exclusion of other possible graphs. In contrast, Bongani draws possible solutions in his book, one of which is the same as the solution valued in the Course (see Figure 2), but he does not make his ideas public in the same way that Mpumelelo does.

In identifying the required population graph with known, named graphs, Mpumelelo seems to be drawing on and getting his confidence from his experience of the school mathematics discourse, in which the classification of functions is a key activity. Furthermore, the standard version of the cosine graph he presents is typical of the presentation of the cosine graph in school textbooks, and this is reproduced in the materials for the undergraduate access course, that is, in the Course discourse. Mpumelelo’s choice may also be a result of the complex interplay between the school mathematics discourse, the Course discourse and the everyday discourse represented by the task context. Explaining his choice of the cosine graph in line 30 of Transcript 1 below, Mpumelelo’s pronounced focus is “this maximum” and he seems to attend to the word “maximum” or the number “10 000” in the problem text. It is possible that his focus on the maximum in the everyday discourse of the text promotes his choice of graph, a graph that starts and ends at its maximum value.

Transcript 1: The Flu Virus Problem, lines 30 to 33⁸

line	student	what they say	what they do
30	Mpumelelo:	So it’s like we have that ... this maximum now right?	He traces a circle in the problem text
31	Lungiswa:	Uh hum ↑	
32	Bongani:	10 000	
33a	Mpumelelo:	Ja it’s 10 000	
33b	Mpumelelo:	But then when I get there the ... number of who get it ... that would be minimum	He uses his hand to trace the decreasing part of the cosine graph, starting at the maximum value

8 In this transcript an up arrow (↑) indicates a rising intonation and three dots (...) are used for a short pause.

Alternatively, it seems that Mpumelelo's choice of a standard cosine graph may be influencing his interpretation of the task context. In line 33b of Transcript 1 he uses his hand to trace the decreasing part of the cosine graph (see Figure 4). When his hand gets to the minimum value of the graph, he introduces into the task context the "minimum" number of people.



Figure 4

2. *The students focus on certain properties of the graph and not others, and the Tutor plays a role in alerting them to particular features of the required graph.*

Lines 30 to 33b of Transcript 1 indicate that the students focus on the maximum and minimum values of the cosine graph. They also discuss whether the cosine graph will increase after the initial decrease, mainly using the everyday terms of the graph going "down" and "up" for decreasing and increasing, a description that is often used in the school mathematics discourse. Siyabulela later introduces a discussion about whether the function values can be negative. However, although some of the students provide axes for their graphs in their drawings, the position of the graph on the axes is not in focus. Moreover, the students do not focus on the concavity of the graph.

How can this observation be explained? Firstly, as I have argued, the presence of the word "maximum" in the text of the Flu Virus Problem may determine what becomes the mathematical focus. Secondly, it is possible that the presentation of the standard version of the cosine graph, together with the students' prior experiences of this graph at school and in the Course, influences which properties become the focus. For example, the maximum and minimum values of the cosine graph and the increasing/decreasing nature of the graph are features of the school mathematics discourse and the Course discourse. The concavity of the graph on the other hand would not have been an explicit aspect of study in either of these discourses (although the concept had been dealt with informally in certain real-world problems in the Course).

The concavity of the graph only becomes a focus when the tutor attempts to persuade the students that the graph will not be a straight line and talks about the "rate of new infections". This seems to encourage the students to draw graphs in which the concavity varies, for example, while the Tutor is talking about the rate Bongani traces the graph in Figure 5 in the air and then draws it in his answer book. The rest of the interaction is dominated by students presenting the graphs in Figures 5 to 8 and debating which to select as the answer.

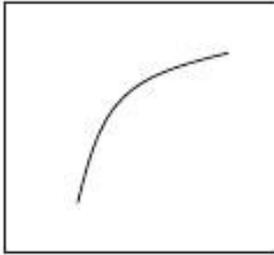


Figure 5

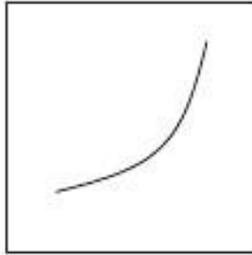


Figure 6

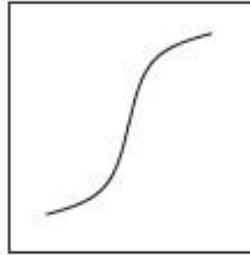


Figure 7

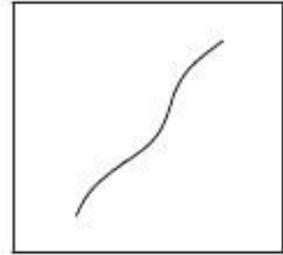


Figure 8

In debating the alternatives the students reproduce the view presented in the Flu Virus Problem that there is only one possible solution to the question. At one stage Lungiswa seems to challenge this view when she presents the graph in Figure 8 as an alternative to Vuyani's choice of Figure 6 and says, "But you can also draw it like this". However the other students want to settle on one possible answer. For example, Vuyani says (laughing), "Now now we have to find which way are we going to ... draw this thing. So like this or like this?". When the Tutor returns and asks whether they have "an agreement on a graph", Mpumelelo says, "I think guys ... we must take this one", beginning a process in which all the students (including Lungiswa) each select one of the graphs they have been discussing. Although the Tutor portrays the expectation that they present one possible answer, he then presents two possible graphs as solutions; the graph in Figure 5 is presented as the "basic" graph, and the graph in Figure 7 is presented as the "sophisticated" answer.

3. *The students have difficulty making links between the everyday discourse of the task context and the mathematical concepts. The tutor plays a key role in helping them to make these links.*

The students frequently use the task context as the grounds to explain their claims regarding the shape of the graph. For example, Siyabulela correctly explains that the function values cannot be negative, since "You can't have a negative number of the population of people", and Mpumelelo gives a similar explanation for this property of the graph. However, at times the link between the mathematical and task context is inadequate. Proposing a cosine graph that is decreasing from the maximum value of 10 000, Mpumelelo uses the task context to argue that "first it get ... the few of them". This apparent absence of a useful link could be related to the students' experience of school mathematical word problems, for in these problems they only need to "pretend that" the situation described in the task context exists (Gerofsky, 1996, p.40). This argument that the students do not take the task context seriously is reinforced by the observation that Siyabulela actually makes a joke about it; having clarified with the Tutor that eventually the whole community will be infected, he jokes, "So that is one lucky community", and the other students laugh with him. Mpumelelo is also not challenged by the other students about this lack of coherence, and possible reasons for this are discussed below.

In contrast to the students' difficulties working across the contexts, the Tutor consistently engages the students in relational processes that identify the mathematical concepts with their meaning in the task context. For example, in his first interaction with the students he asks them to identify the labels on their axes. The students respond, talking about the "number of people" and the "time". First, he responds positively with the word "okay", and then rewords their responses, adding information that makes a link between the task context and the variables P and t , for example "Okay and t is the time in days as you've got". In this

example he gives further positive feedback “as you’ve got”. He also develops the students’ description of P: “it’s not just the number of people ... it’s the number of people who have the flu or have had the flu”, slowing down his speech towards the end for emphasis. He uses this elaboration of the meaning of P to help students to focus on the errors in their current graphs.

4. *When the tutor is not present, the students evaluate one another’s responses. This is often in the form of positive feedback and tends not to address the content of the claims. When the tutor is present, he is the sole evaluator and the students do not have opportunities to respond to one another.*

The students identify with the role of evaluators of one another’s ideas as they give one another positive feedback in statements like “Ja”, and by nodding their heads. This feedback does not refer to the properties of the graph or to the task context, the type of feedback that might help the students to challenge Mpumelelo’s explanation for his choice of a cosine graph.

The students also prompt one another to continue speaking, and these prompts often contain implicit positive feedback, for example, “Uh ... hum”. The use of positive feedback and prompts are a consistent feature of Lungiswa’s speech in particular. By asking Mpumelelo to explain his initial claim about the “cos graph”, she sets up the expectation that students explain their reasoning, yet at the same time she also creates enabling social relations for this to occur by listening and responding.

Negative feedback features mainly when the students draw different graphs, such as those presented in Figures 5 to 8. This may be a product of what is represented in the text of the Flu Virus problem; since the students expect only one correct solution, they set about finding fault in the alternatives. Moreover, most of this negative feedback involves presenting alternative visual representations by tracing and drawing graphs and by pointing to the visual representations. For example, Vuyani stretches across to Lungiswa’s book and draws the graph in Figure 6 next to her version of the graph in Figure 8, and says, “No like ... like like this”. Siyabulela, on the other hand, can be identified as making statements that refer to the content. For example, he agrees with Vuyani’s unexplained claim that there must be a difference between the graphs in Figure 6 and 7, and argues that “It has to do like with the with the rate of how much they get affected by time or something”.

The nature of the students’ interactions and particularly of their feedback as described here may be a result of their experiences as school mathematics students working in groups and in the Course. Adler (1997) notes that a participatory-inquiry approach to teaching and learning mathematics, in which students’ knowledge and contributions are valued and students often work with their peers, can inadvertently constrain rather than enable mathematical activity. The nature of this feedback could also be related to issues of academic literacy; the students are speaking English as a second language, they may not have developed the necessary academic literacy related to the mathematical discourse, and their school experience may have involved learning by rote rather than thinking critically and debating (M. Paxton, personal communication, July 17, 2008).

When the Tutor joins the group the students listen to him, answer his questions and ask him for clarification. They give the Tutor feedback on whether they are following his explanations, either by responding with “Ja”, nodding their heads, or co-constructing text

with the Tutor. Yet, the Tutor is identified as the sole evaluator of their responses during this time, thus reproducing the view presented in the Flu Virus Problem that they must have their work “checked by a tutor”. When evaluating the students, however, the Tutor presents his responses in an encouraging way. For example, commenting on Lungiswa’s decreasing graph he personalizes his response by saying “...it looks like to me...” and says her graph would be “okay” if the task were phrased differently, that is, if it were graphing the number of people who have not had the disease.

5. *There are few instances where the students use mathematical discourse in their speech.*

In the discussion so far I have presented a number of instances in the transcript where students trace the shape of graphs in the air, make drawings of graphs in their answer books and point to these visual representations. Such activity is a consistent feature of the interaction of the transcript. What is absent from much of the interaction is verbal descriptions of the graph using mathematical terminology, and this may have implications for the students’ participation in the mathematics community.

Siyabulela identifies himself as a student who participates in some way in the discourse of mathematics. When the Tutor introduces the concept of “rate” into the discussion, Siyabulela draws on his text, using words like “gradient”, “steepness” and “rate”, and sketching small tangent lines on his graphs. This developing chain of texts appears to be key in helping Lungiswa to select the graph in Figure 8 as a possible solution. She picks up on Siyabulela’s use of the word “rate” and asks, “Is the rate increasing?” This question seems to remind her of a population graph in earlier Course material which students used to study increasing and decreasing rates, and she asks Bongani whether he remembers it and draws the graph in Figure 7.

After Mpumelelo’s initial presentation of the required graph as a “cos graph”, the students consistently talk about the graph as “it”. For example, discussing whether the functions values can be negative, Siyabulela asks “It will never go to the negative side will it?” The students also use the pronoun “it” to reference other objects, for example the disease, yet there seems to be a common understanding of what is referenced each time. Yet the fact that graph represented as “it” becomes the pronounced focus, rather than “the cos graph”, may prevent them from interrogating the initial choice of graph as a cosine graph.

The analysis also suggests that different students have varied opportunities to talk, with implications for the students’ participation in the mathematics community. Mpumelelo, Lungiswa and Siyabulela consistently identify themselves as students who are prepared to make their claims public, they engage in longer arguments than the other students, and their arguments are often maintained through the use of prompts and positive feedback as discussed. In contrast, Bongani and Vuyani’s participation in the interaction mainly involves the presentation of individual ideas, rather than extended arguments. For example, in response to Lungiswa’s question “Is the rate increasing?”, Bongani responds, “They increasing and decreasing.” At times both he and Vuyani try to enter the conversation, but without success, for example, soon after Mpumelelo has suggested the cosine graph, Vuyani tries unsuccessfully to make a contribution to the discussion when he starts, “The thing is ...”. These varied opportunities to talk may be explained with reference to how the students identify themselves and one another (as discussed in point 6 below), or may be related to the students prior experiences of talking mathematics.

During the Tutor's interaction with the students he provides varied opportunities for them to speak mathematically. When Mpumelelo asks whether the graph will be oscillating or a straight line he responds, "Uhm ... well you can't just ask me ... I mean try and like okay say something and explain why it would look like that ...". He thus creates the expectation that the students talk about their answers. Yet, at other times the Tutor only provides opportunities for the students to provide one word answers, without requiring an explanation. For example, he asks the students how many people will have the disease on the first day. Before they have an opportunity to respond, he provides two possible solutions for them to choose from, that is, "a lot" or "a little". By providing possible answers, the Tutor shuts down opportunities for the students to talk. These varied practices of the Tutor may be related to his attempts to balance the provision of opportunities to engage in conceptual talk, with making sure that the students progress through the Course material. This concern is illustrated by a comment he makes after an explanation to the students, "Now try and try and move on ...". The Tutor was also responsible for tutoring five other groups in the room, and thus could not use all his time to engage Bongani, Mpumelelo, Siyabulela, Vuyani and Lungiswa in an extended conversation.

6. *The students appear to position themselves and to position one another in particular ways and this impacts on who speaks and what is said.*

In the discussion I have referred to the different ways that students participate in the interaction. For example, I note that Mpumelelo, Lungiswa and Siyabulela are prepared to talk about their tentative claims, and the nature of these public claims seems to determine what is discussed in the group. I have noted that Bongani presents a graph that would have been regarded as correct in the context (see Figure 5), long before the Tutor presented two possible answers to the group. Yet he did not make this graph public to the group. Furthermore, Vuyani seems to struggle to establish a voice at first, yet the analysis of the instances where his short comments are valued, suggests that these comments are useful in taking the discourse forward. In trying to explain this it may be appropriate to note that Vuyani had only been part of this group of students for a month as he had recently transferred to the access Course from the mainstream first-year mathematics course.

Discussion of the Flu Virus Problem and the student interaction

In the analysis of the Flu Virus Problem, I have described how the text represents both the activity and the task context of the problem and sets up subject positions for the participants. The theme of the Flu Virus Problem is presented as being the study of disease, suggesting a link to the discourse of epidemiology, and the use of a "real-world" context points to traces of a reform mathematics discourse in the text. Yet the identification of the flu virus with a mathematical object, a function, identifies the problem as a mathematical task in a pedagogic text. In particular, the structure of the problem, the presentation of the task context and the positioning of the participants links the problem to the discourse of school mathematical word problems (Gerofsky, 1996, p.37). The student is constructed as someone needing to have the task context explained in everyday terms and requiring additional reminders and feedback from an authority in order to proceed as required in the mathematics classroom.

It is productive to use Fairclough's concept of "order of discourse" as a tool to view this collection of discourses and associated subject positions. Fairclough (1989, p.29-30; 2006, p.31) suggests that in particular social institutions, for example in education, various discourses (ways

of representing the world), genres (ways of acting and interacting) and styles (ways of being) are ordered in particular ways in relation to one another. A particular structuring of these elements is what he calls an “order of discourse”, and the elements may be complementary to one another or may conflict with or be alternative to one another. In the latter case, certain discourses, genres and styles may be dominant over others and be “officially recognized” (Fairclough, 2006, p.31). I have identified traces of different discourses in the Flu Virus Problem, and these are structured in such a way that the dominant element is the pedagogic text in mathematics (that shares features with school mathematical word problems), the focus of which is the production of a mathematical representation, a graph. Whilst this specific ordering might be the one that is recognized by the lecturers on the Course, the question arises as to whether the student recognizes it in this way.

Fairclough (1989, p.29) argues further that the practices and discourses we use are enabled and constrained by the social conventions associated with a particular order of discourse. The Flu Virus Problem is a pedagogic text in mathematics that is used in an undergraduate access course in mathematics; the text and its setting are associated with particular conventions which will both enable and constrain how the student engages with the text. In the analysis of the student interaction presented here I have identified certain discursive actions that appear to enable the students’ solving of the problem; they draw on their knowledge of the properties of graphs from school mathematics, they relate to the task context as if the problem were a school mathematical word problem, their group skills reflect what is valued at school and in the Course, and the Tutor draws on his training as a tutor for the Course by valuing students’ responses and, at times, requiring students to discuss and to explain their answers.

Yet the text and its setting also constrain the extent to which students are able to participate in the discourse of undergraduate mathematics. For example, the classification and naming of graphs is valued in school mathematics, yet this practice appears to constrain their progress in solving the Flu Virus Problem. The students’ apparent lack of concern about the truth value of the task context, the uneven distribution of opportunities for talk, and the nature of this talk constrain meaningful engagement with mathematical discourse. In addition, the Tutor’s opportunities to address these constraining practices are limited by his positioning (by the text) as an authority, and by the competing demands of his role, such as encouraging the students to proceed through all the tasks set for the afternoon workshop and his responsibility to other students in the classroom.

The use of Fairclough’s concept of “order of discourse” suggests, therefore, that the student action (whether enabling or constraining) is not the result of the cognitive ability of the individual students. Rather, the students are working on a pedagogic text in a mathematics classroom, and it is the social conventions associated with this particular practice and the related socio-political practices (such as school mathematics) that both enable and constrain their action.

Conclusion

In the introduction to this paper I signaled my intention to consider what possibilities a socio-political perspective of mathematics learning might “open up” (Valero, 2008, p.56) when used in a research study located in undergraduate mathematics. This theoretical perspective sees learning as coming to participate in the discourse and socio-political practices that are valued by a particular community that practises mathematics. These socio-political practices form part of a network of wider socio-political practices both within mathematics education and in the wider space.

In considering these possibilities, I have presented one part of an ongoing study of the use of real-world problems in an undergraduate access course in mathematics. I use this analysis to argue that the socio-political perspective and the associated analytic tools have a particular “expressive potential” (Gee & Green, 1998, p.121). Firstly, on the micro-level I am able to provide a detailed description of the text of the Flu Virus Problem as well as of the students’ “discursive actions” (Sfard, 2001b, p.6) as they solve it. Secondly, the description of the student interaction enables me to identify those actions that may be enabling or constraining the students’ in their production of a solution, and hence their participation in the discourse of their undergraduate mathematics classroom. Thirdly, I can start to explain this participation with reference (a) to the socio-political practices in the classroom, that is, what is given value during the classroom activity, and (b) to the wider discourses and related socio-political practices on which the students draw. Drawing on the work of Fairclough (1989, p.28) I argue that the specific structuring and ordering of these socio-political practices, that is, the order of discourse, both enables and constrains the extent to which students are able to participate in the discourse. This perspective thus allows me to view the interaction of students as a product of socio-political practices, rather than as a product of cognitive ability of the individual students.

This study, which is still ongoing, presents a number of challenges related to my dual role as researcher and lecturer on the Course. Yet the perspective adopted and the associated tools employed in the research so far have certainly “opened up” (Valero, 2008, p.56) for me as a lecturer what is going on in my classroom. I thus end this paper with some implications and recommendations, based on my personal experience of how the knowledge gained in this study has affected my daily practice as a lecturer. For if I view learning as coming to participate in the discourse and practices of a particular community, in this case the undergraduate mathematics community, then I must consider to what extent the curriculum material, the student interaction, and the related socio-political practices enable or constrain the students’ participation. There is a need to consider to what extent what is valued in the Course is made explicit to students, for example, how productive links can be made between the task context and the mathematical concepts. In addition, how do the valued ways of acting, talking and knowing relate to other discourses and practices that students may draw on? Given the important role of the Tutor in making the valued practices explicit in the workshop setting, attention has to be paid to assisting tutors to play the demanding and sometimes conflicting roles associated with this identity. A related issue is the need to create spaces for students to engage in extended mathematical talk, that is, the ways of talking valued in the discourse of undergraduate mathematics. Lastly, and of particular importance to lecturers engaged in development work with students, what are the implications of a classroom discourse that positions the student in an extended curriculum programme as different to a mainstream student?

Acknowledgements

I would like to express thanks to a number of researchers who have given feedback on drafts of this paper; Bette Davidowitz (University of Cape Town), participants at the 2008 Centre for Higher Education Development (CHED) Writers’ Workshop at the University of Cape Town, and the Mathematics and Science Education PhD group at the University of Witwatersrand. In particular, I extend my thanks to Jill Adler, Moragh Paxton and Bruce Tobias for their detailed feedback at various stages of this research.

This paper is based on work financially supported by the National Research Foundation under Grant number TTK2006040500009. Any opinion, findings and conclusions or recommendations expressed in this material are those of the author and therefore the NRF does not accept any liability in regard thereto.

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