The copyright of this thesis vests in the author. No quotation from it or information derived from it is to be published without full acknowledgement of the source. The thesis is to be used for private study or non-commercial research purposes only.

Published by the University of Cape Town (UCT) in terms of the non-exclusive license granted to UCT by the author.
The Risk Premium in Commodity Futures Pricing: from Keynes’ (1930) Theory of Normal Backwardation to Dusak’s (1973) Futures Capital Asset Pricing Model

A literature review and an empirical study of risk premia in precious metals futures
This dissertation is submitted in fulfilment of the requirements for the degree Master of Commerce, in the School of Economics at the University of Cape Town. I confirm that the work is my own, and that it has not been submitted, either in whole or in part, for purposes of any previous degree or examination.

I wish to acknowledge the assistance of my initial supervisor, Dr. S Hugh High, now based in the United States, and my subsequent supervisor, Professor Brian S. Kantor. I also want to thank the Commerce Faculty at the University of Cape Town for granting me permission to complete the dissertation outside South Africa and for purchasing the data set used in the empirical research. Finally, I want to thank Deborah Louw and Loane Sharp for their help in producing the dissertation in English.

Support for this dissertation was received from the National Research Foundation. The conclusions reached do not necessarily reflect those of the sponsor.
# TABLE OF CONTENTS

## INTRODUCTION

Diagram 1: The structure of the literature review and the position of the empirical study

## PART I: LITERATURE REVIEW AND RESEARCH HYPOTHESIS

### Chapter One: Introducing Futures Contracts and Markets and Hedging & Speculation

1. Forward and futures contracts
   - 2. The evolution of futures markets and contracts
      - 2.1 Brief historical overview
      - 2.2 Contract standardisation and design
   - 3. The nature and operation of futures markets
      - 3.1 Marking - to - market
      - 3.2 Zero sum game
      - 3.3 Margin
         - 3.3.1 Can the ‘margin’ constitute an investment?
      - 3.4 The clearinghouse and regulation
   - 3.5 Open interest
      - Graph 1: Daily open interest on ‘December 1997 - palladium’ contract
      - Graph 2: Daily open interest on ‘April 1997 - platinum’ contract
      - Graph 3: Daily open interest on ‘April 1997 - gold’ contract
   - 4. The economic roles of futures markets
   - 5. Hedging and speculation
      - 5.1 The hedging - speculation dichotomy
      - 5.2 Dominant view of hedging - motive (in retrospect)
      - 5.3 Alternative views of hedging - types
   - 5.4 Speculation
   - 5.5 Net hedging and speculation positions (link to futures pricing)

## Chapter Two: Fundamentals of Commodity Futures Prices

1. Notation
   - 3. Commodity markets and commodity futures prices
      - Table 1: Storage vs. expectation mechanism and commodity pricing theories
   - 4. Underlying links between spot and futures prices
      - 4.1 Arbitrage and the law of one price
      - 4.2 Delivery and settlement
      - 4.3 Storage mechanism
      - 4.4 Deriving the cost of carry model
         - Table 2: Deriving the cost of carry model as two arbitrage strategies
         - Graph 4: Daily gold spot prices vs. gold futures prices
         - Graph 5: Daily silver spot prices vs. silver futures prices
         - Graph 6: Daily platinum spot prices vs. platinum futures prices
   - 5. Terminological identification
      - Table 3: Summary of terminology and pricing relationships
   - 6. Basis and arbitrage hedging
      - Table 4: Summary of short and long arbitrage hedging
      - Graph 7: Daily basis for different maturities ‘1997 - gold’ contracts
Chapter Three: Pricing Theories and Keynes' Risk Premium

1. Introduction
2. The theory of storage
   - Diagram 2: Relationship between the basis and level of inventory
   - 2.1 The net marginal cost of storage function
   - 2.2 Digression on risk premium and expected spot price
   - 2.3 Empirical evidence
3. The theory of forecast power and risk premium
   - 3.1 The expectations hypothesis
   - 3.2 The risk premium (Keynes' theory)
     - 3.2.1 Normal Backwardation and Normal Contango
     - 3.2.2 Terminological verification
     - 3.3 Review of empirical studies of Keynes' theory
6. Working vs. Keynes - the link between the two theories

Chapter Four: Applying Modern Portfolio Theory to Futures

1. Introduction
2. The theory of systematic risk in futures markets
3. Modern portfolio theory
   - 3.1 Theoretical review of the Capital Assets Pricing Model
   - 3.2 Brief empirical review
4. The 'Futures-CAPM' Re: derivation and interpretation
   - 4.1 The notion and computation of returns on futures contracts
   - 4.2 Theoretical derivation of the 'Futures-CAPM' equation
     - 4.2.1 Black's approach
   - 4.3 The risk premium hypothesis: CAPM vs. traditional theories

Chapter Five: The Futures-CAPM: Empirical Review and Research Hypotheses

1. Introduction
   - Table 5: A summary of previous theoretical and empirical studies
2. The empirical Futures-CAPM and appropriate market benchmark
3. Stochastic systematic risk
4. Summary and testable hypotheses

PART II: DATA DESCRIPTION AND METHODOLOGY

1. Data
   - Table 6: Precious metals futures contracts and sample period
2. Log relative returns
3. Time series construction
   - 3.1 Fixed-delivery-month method
     - Table 7: Constructing a fixed-delivery-month time series
     - Table 8: Fixed-delivery-month method - constructed time series
   - 3.2 Fixed-interval-to-maturity method
     - Table 9: Fixed-interval-to-maturity method - constructed time series
Table 10: The construction of four fixed-interval-to-maturity time series 103
3.3 Comparing the two methods 103
4. The CAPM assumptions 105
4.1 Normal Distribution 105
4.2 Serial independence 107

PART III: EMPirical FINDINGS 108
1. Tests for serial independence 108
   Table 11: Daily serial correlation coefficients 109
2. Empirical estimation of the futures-CAPM coefficients 110
   2.1 General 110
   2.2 The intercept term ($a_f$ coefficient) 110
       Table 12: Estimated regression parameters 111
   2.3 The risk premium ($b_f$ coefficient) and the CAPM interpretation 112
   2.4 Further interpretation of the regression results 113
3. The CAPM as a test of the traditional theories (2nd hypothesis) 114
4. Further analysis of futures risk and returns 115
   Table 13: Means, standard deviations and kurtosis of futures and S&P500 daily returns 116
4.1 Mean and standard deviation 117
4.2 The Keynesian measure of asset risk 118
4.3 Kurtosis and underlying distribution 119
4.4 The two time series methods – concluding observations 120

CONCLUSION 122

BIBLIOGRAPHY 128

APPENDICES 135

Appendix A: Contracts under study 136
Appendix B: Forward and futures – Contract value and price 138
Appendix C: Statistical measures and tests 145
Appendix D: Time series construction method 147
Appendix E: Gold and Silver – The behaviour of futures prices time series 148
Appendix F: The behaviour of open interest 150
Appendix G: Ordinary least square regression results 157
ILLUSTRATIONS

Diagram 1: The structure of the literature review and the position of the empirical study 8
Diagram 2: The theory of storage - relationship between the basis and level of inventory 56
Diagram 3: The relationships between the futures price and the expected future spot price 64
Diagram 4: Futures price of a storable commodity, risk premium and traders' positions in the context of backwardation, contango, expectations and net hedging hypothesis 67

Graph 1: Daily open interest on 'December 1997 – Palladium’ Contract 21
Graph 2: Daily Open Interest on ‘April 1997 – Platinum’ Contract 21
Graph 3: Daily Open Interest on ‘April 1997 – Gold’ Contract 21
Graph 4: Daily Gold Spot Prices vs. Daily Futures Settlement Prices of Two 1998 Contracts 47
Graph 5: Daily Silver Spot Prices vs. Daily Futures Settlement Prices of Two 1998 Contracts 47
Graph 6: Daily Platinum Spot Prices vs. Daily Futures Settlement Prices of Two 1998 Contracts 47
Graph 7: Daily Basis for 1997’s Gold Contracts 52
Graph 8: Daily Basis for 1998’s Silver Contracts 52
Graph 9: Daily Basis for 1997’s Platinum Contracts 52

Table 1: Storage vs. Expectation Mechanism and Commodities Futures Pricing 39
Table 2: The Derivation of the Cost of Carry Model as Two Competing Arbitrage Strategies 44
Table 3: Summary of Terminology and Pricing Relationships in Full Carry vs. Inverted Market 49
Table 4: Summary of Short and Long Arbitrage Hedging 50
Table 5: A Summary of Previous Theoretical and Empirical Studies Related to the Application of the CAPM to Futures Contracts 87
Table 6: Precious Metals Futures Contracts and Sample Period 97
Table 7: Constructing a Fixed-Delivery-Month Time Series for a ‘March – Silver’ Contract 101
Table 8: Fixed-Deliver-Month Method – Constructed time series 101
Table 9: Fixed-Interval-to-Maturity Method – Constructed time series 102
Table 10: The Construction of the Four Fixed-Interval-to-Maturity Time Series of Platinum 103
Table 11: Daily Serial Correlation Coefficients for Gold, Silver, Platinum and Palladium Contracts 109
Table 12: Estimated Regression Parameters for Gold, Silver, Platinum and Palladium Contracts 111
Table 13: Means, Standard Deviations and Kurtosis of Daily Returns of Futures Contracts and S&P500 daily returns 116
INTRODUCTION

The objectives of this dissertation are twofold. Firstly, the dissertation organises the futures markets literature to provide a simpler, structured framework for future research. As the review of the literature indicates, the futures markets literature is currently highly disorganised, following several disparate strands, each of which reaches a different conclusion about the behaviour of futures prices, with no apparent, unifying research agenda. The first contribution of this dissertation is thus to present several competing frameworks in futures markets research in a single, accessible format.

Secondly, the dissertation evaluates futures pricing in the context of modern portfolio theory, including an empirical study of the pricing of precious metals contracts. The overwhelming body of empirical research in futures markets is inconclusive, based mainly on the competing and sometimes contradictory theoretical and empirical frameworks in use. The second contribution of this dissertation is thus to distinguish, by way of an empirical test, between the traditional Keynesian approach and the modern portfolio approach to the pricing of futures contracts.

The notion of a risk premium in futures contracts is the underlying topic of this dissertation. In terms of the traditional Keynesian approach, participation in futures markets is encouraged by the existence of a risk premium, paid by commodity producers or hedgers, who seek to reduce the risk of commodity price fluctuations, to speculators, who need to be compensated, according to the traditional theory, for doing so. By contrast, in terms of the modern portfolio approach, the risk of futures contracts is diversifiable, in the sense that futures contracts, as part of a well-diversified aggregate portfolio, should not offer compensation of this kind. The essence of the dissertation is thus to distinguish between the two competing approaches.

As a starting point in the dissertation, it is necessary to reach a clear and theoretically consistent interpretation of risk in futures markets. However, the interpretation of risk—and, by implication, the existence of a risk premium—in the pricing of futures contracts, is highly controversial. The controversy reaches back to the early days of futures markets research, and, combined with the generally unstructured and occasionally confusing historical literature, we conducted a thorough and wide-ranging literature review. As two prominent financial economists indicate, in 1971, at a comparatively early point in the modern futures markets literature,

[anyone who undertakes a survey of the literature on futures trading is confronted with an amorphous and rather disjointed list of publications (Gray and Rutledge, 1971: 57)]

The earliest literature on futures markets related exclusively to agricultural commodities. Furthermore, for agricultural products, conventional wisdom dictated that only storable commodities were suitable
for futures trading. However, other storable commodities, including metals, were introduced in the early 1930s, and non-storable commodities, including cattle and hogs, were introduced during the 1960s. Moreover, interest rate contracts were introduced in 1975, followed by stock index futures in 1982. As the range of commodities expanded, the traditional explanation of, for instance, the role of hedgers and speculators, and ultimately the process underlying price- and expectations-formation in futures markets, had to take a new, more extended form, forcing futures markets theory out of its traditional narrow ambit. Futures markets are currently an integral component of the financial system, requiring a general and inclusive theory of the pricing of futures contracts.

In introducing the notion of a risk premium in futures markets, the dissertation consequently encountered two notable difficulties, related both to the disorganised structure of the literature and to the absence of a single, inclusive approach to the underlying forces at work.

The first difficulty has to do with the fact that Keynes' theory of normal backwardation is the point of departure for almost all of the early theoretical work (and for most of the subsequent empirical work) on futures pricing. For instance, most of the academic material that deals directly with our topic—including, for instance, most current textbooks—begins with Keynes' theory. However, as indicated in this dissertation, this point of departure is inadequate since it fails to position the Keynesian approach (and thus the risk premium) within the broader conceptual framework of futures markets and futures pricing. In particular, subsequent work and departures from the theory of normal backwardation have been considerably more important, not only because they reposition the topic in its proper theoretical location, but also because they cast new light on the concept itself as well as on the various issues involved.

This difficulty, in turn, presents a second problem: how, on the one hand, to present and discuss the Keynesian theory itself, given its importance to our topic, and how, on the other hand, to review the various issues and concepts introduced by the theory to the literature, given that each concept takes a different theoretical and empirical form.

It is important to note that, in many ways, Keynes' (1930) theory of normal backwardation and its counterpart Hicks' (1946) theory of normal contango were manifestations of an early approach in which the analysis of futures price formation and behaviour was closely intertwined with the analysis of hedging and speculation. Specifically, Keynes and Hicks—two of the most prominent economists addressing futures markets at that time—viewed speculation as a substitute for missing insurance markets in commodities. In this context, they suggested the existence of a risk premium as a reward for speculators for underwriting the risks of spot price fluctuations faced by the holders of commodities, the hedgers. Based on the assumed positions of hedgers and speculators in the futures markets, Keynes and Hicks then constructed a fairly solid pricing model to explain the behaviour of futures prices in terms of an expected spot price and a risk premium. This pricing model—subsequently known as 'price
insurance theory', 'the hedging imbalance hypothesis', the 'risk premium theory', or, more commonly, 'Keynes' theory of normal backwards'—became a cornerstone in the literature of futures markets. In fact, for several years, the concept of a risk premium in the Keynesian context was so deeply entrenched that many economists adhered to it even when the theory failed to account for observed price and speculator behaviour and as new, alternative lines of analysis were emerging.

At the same time, however, it was the shortcomings of Keynes’ theory, as well as advances in the research on its various components (hedging and speculation, the formation of futures prices, and the notion of an expected spot price), that ignited alternative pricing theories. To a degree, the literature progressed such that the components of Keynes’ dominant theory eventually became larger than the theory itself, with the notion of a risk premium in futures contracts surviving by several decades the gradual theoretical erosion of the theory of normal backwards.

We therefore believe it appropriate to include a comprehensive literature review that progressively introduces the economics of futures markets, reviews concepts and jargon, and distinguishes between the various issues in a manner that properly positions our topic within the broad literature of futures markets. A graphical representation of the structure of the dissertation is given in Diagram 1. In Chapters One and Two, the dissertation investigates the pertinent issues in considerable detail, taking into account original, subsequent and current writings on the subject. Chapter One introduces fundamental aspects of futures markets: the institutional apparatus, contract mechanics, the role of futures markets, and hedging and speculation. Chapter Two introduces and explains important price-related concepts: the difference between the value and the price of a futures contract; the cost of carry, storage, basis, and 'arbitrage hedging'; and the notion of an expected spot price. The first two chapters thus form the basis of the discussion of broader pricing theories in Chapter Three.

Chapter Three presents Keynes' theory of normal backwards, under a broader pricing framework that we have termed the 'theory of forecasting power and the risk premium'. The notion of a risk premium is fully explored as part of the analysis of two theories, the theory of storage and the theory of forecasting power and the risk premium, and ultimately the application of modern portfolio theory to futures markets. Chapter Four, in turn, reviews the literature of modern portfolio theory and the theoretical adaptation of the capital asset pricing model (CAPM) to futures contracts. In particular, Chapter Four examines the theoretical aspects of the futures-CAPM, in particular the mathematical derivation of the futures-CAPM equation to be tested later, coupled with a review of modern portfolio theory and of the Sharpe-Lintner version of CAPM. The empirical hypothesis is introduced in Chapter Five. Following Dusak's (1973) approach, we use the Sharpe-Lintner single-index single-period version of the CAPM, firstly, to generalise from the Keynesian formulation and, secondly, to test whether our set of data on returns from gold, silver, platinum and palladium futures conforms better to the portfolio approach than to the Keynesian model.
Diagram 1: The Structure of the Literature Review and the Position of the Empirical Study

- **Cash Market** → **Forward Market** → **Credit Market** → General equilibrium pricing for any financial assets
  - Trade centralisation & contracts' standardisation
  - Basis
  - Cost of Carry
  - Equilibrium Value & Price

- **Participants** → **Hedging & Speculation**

- **Futures Market**

- **Futures Prices**
  - **Expected Spot Prices**
  - **Risk Premium**

- **1. Theory of Storage**
  - Competing theory
  - Keynes' theory of Normal Backwardation
  - Expectations Hypothesis

- **2. Theory of Forecast Power and Risk Premium**

- **3. Capital Asset Pricing Model**
  - Empirical Relation

- **Empirical Case Study**
  - 'Futures-CAPM'

- **(Chapter 1)** Introducing futures contracts, futures markets and hedging and speculation
- **(Chapter 2)** Fundamentals of commodities futures prices
- **(Chapter 3)** Pricing theories and Keynes' risk premium
- **(Chapter 4 and 5)** Applying modern portfolio theory to futures: Theoretical and empirical review of the literature
Specifically, we test the following empirical hypothesis:

If the pricing of commodity futures contracts conforms to the CAPM approach, and if commodity futures returns over time are independent and normally distributed, then expected returns on holding these contracts should be commensurate with the level of systematic risk. More specifically,

*Either*: If gold, silver, platinum and palladium futures contracts are risky assets, and that risk is undiversifiable, then the contracts will command a risk premium (reflected in a positive slope parameter, or beta) and ex post returns will be positive on average (reflected in a positive intercept parameter, or alpha);

*Or*: If the entire risk of gold, silver, platinum and palladium futures contracts is diversifiable (reflected in a zero beta, implying no systematic risk) then investors will not be rewarded for taking that risk and ex post returns will be zero on average (reflected in a zero alpha).

Following the differentiation proposed by Telser (1958) and, later, by Fama and French (1987, 1988), we conclude that the literature suggests three fundamental analytical frameworks for the modelling of futures prices and returns.

The first framework is the theory of storage, which is rooted in the functional and arbitrage relationships between the futures price and the current spot price of a storable commodity, known as the cost-of-carry model. In particular, the theory of storage explains the difference between contemporaneous futures and spot prices in terms of financing costs, physical storage costs, and a convenience yield on the holding of inventories. In expanding the theory to embrace non-storable commodities as well, the relevant price spread becomes that between the expected spot price and the current spot price, which in turn invites the (debatable) inclusion of a risk premium factor in the model, as hypothesised, in effect, in Keynes' theoretical framework. The theory of storage, which was most notably formulated by Working (1948, 1949b), Brennan (1958) and Telser (1958), is well accepted and relatively uncontroversial.

The second framework is the theory of forecasting performance and risk premium, which is, in turn, divided into two interrelated theories: the expectations theory associated with Telser (1958) and the risk premium theory associated with Keynes (1930). This analytical framework is rooted in the price discovery role of futures markets (that is, in the forecasting ability and information content of futures prices) and in the relationship between futures and expected spot prices. More specifically, the theory splits the futures price into a forecast of the future spot price and an expected risk premium. In contrast to the theory of storage, this framework has long been debated and therefore encompasses a large variety of analytical models. The debate centres largely on the question of whether futures prices represent the anticipation of spot prices on the delivery date and, if so, whether they constitute unbiased
anticipation. The first component of this analytical framework, the expectations theory or 'hypothesis of unbiased futures price' associated with Telser (1958), implies no expected risk premium.

On the other hand, the second component of this analytical framework, the risk premium theory associated with Keynes (1930), Hicks (1946) and, later, Cootner (1960a, 1960b), suggests that, under 'normal' conditions, the futures price is a downward-biased expectation of the future spot price, with the difference or bias equalling a risk premium. Simply stated, if the futures price is below the expected spot price and speculators buy futures, then on maturity of the contract speculators pocket the difference between the futures price and the then-prevailing spot price. In this somewhat indirect way, the risk premium theory suggests that speculators are rewarded for bearing risk.

The third theoretical framework is the application of modern portfolio theory, as represented primarily by the Sharpe-Lintner CAPM, to the pricing of futures contracts. We have termed this body of literature the 'futures-CAPM', and it forms the central element of our empirical investigation, given its solid theoretical basis in modern financial theory. It is important to note that, while this body of literature can be viewed as an empirical extension of the theory of normal backwardation (and is treated as such in most textbooks), it can also be viewed as an alternative interpretation (in its entirety) to the Keynesian theory. Both approaches are addressed and empirically tested in this dissertation.

As noted above, Dusak (1973) was the first to put forward the 'portfolio approach' as a test as well as a competing analysis of returns to speculators to that of Keynes. The first and most apparent link between the two theories is indeed an empirical one. Among Keynes' main propositions is a positive relationship between the risk assumed by speculators for investing in the risky commodity and the return for assuming this risk. This hypothesis of a positive relationship between risk and return is also the main premise of the CAPM. It is therefore appropriate to test the theory of normal backwardation and contango, or more specifically, the existence of a risk premium in futures contracts, using the well-known CAPM apparatus.

However, portfolio theory provides a very specific definition of a risk premium for any asset. In effect, the application of portfolio theory to futures contracts fine-tunes this abstract concept of a risk factor by actually defining the relevant risk, thereby providing an explicit (and empirically feasible) measure for the risk premium.

Firstly, the risk in a portfolio context is the covariance or the joint variability of futures prices and the prices of an (hypothetical) aggregate and well-diversified portfolio of assets, known in the literature as the 'market portfolio'. The portfolio approach further divides that concept of risk into systematic and non-systematic risk. Markowitz's (1952) Portfolio Model and its extension, the CAPM devised by Sharpe (1963, 1964) and Lintner (1965a, 1965b), are the principal models used to analyse and quantify the risk and the return of any asset. In contrast, the risk emphasised in Keynes' theoretical framework is
the price risk incurred by commercial participants due to routine inventory holding, which is managed by transacting with speculators on futures market. That risk is identified solely with the fluctuations of futures prices.

Secondly, as noted above, Keynes' analysis assumes that hedgers must pay the speculators a (implicit) monetary premium as an incentive to trade in futures. By contrast, in a portfolio context (within an integrated set of competitive asset markets), hedgers need not pay any premium to induce other agents to trade. Instead a risk premium, if it indeed exists, depends upon the covariance between the payoffs to the hedge asset and the economy-wide risk faced by all agents. Specifically, if the hedge asset is not correlated with market fluctuations, it will carry no risk premium, since investors should not be rewarded for risk that can be diversified away. The asset may, however, still offer value to a particular hedger owing to its specific income stream or superior contractual characteristics. Only a hedge asset that protects its holder against market risk will carry a risk premium or compensation for systematic risk that cannot be diversified away.

Thus, in contrast to the Keynesian framework (the theories of normal backwardation and contango, and the expectations theory), the CAPM does not stipulate the sign or the size of the risk premium. It hypothesises instead that returns on any risky asset, including futures, are a function of the asset's contribution to the risk of the market portfolio. Only this systematic risk, measured by the asset's beta, will command a risk premium in the market. Our prime research hypothesis, stated above, encapsulates the empirical (statistical) implications of the futures-CAPM as a stand-alone pricing theory.

In addition, the CAPM can be used to accept or reject the three traditional risk premium theories, namely, backwardation, contango and expectations. The empirical results are evaluated relative to the sign and significance of the beta coefficient, as the measure of the risk premium, and relative to the expected returns on the futures contract. This approach depends on the researcher's willingness to reinterpret the implications of the traditional theories within the CAPM framework.

Our second hypothesis, which is effectively jointly tested with the first hypothesis, addresses this approach as encapsulated in the following (ex-ante) expression:

<table>
<thead>
<tr>
<th>Expected Return:</th>
<th>Risk:</th>
<th>As predicted by:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E(\Delta \tilde{F}) &gt; 0 )</td>
<td>( \mathrm{Cov}(\Delta \tilde{F}, \tilde{R}_m) &gt; 0 )</td>
<td>Backwardation (a)</td>
</tr>
<tr>
<td>( = )</td>
<td>( = )</td>
<td>Expectations (b)</td>
</tr>
<tr>
<td>( &lt; )</td>
<td>( &lt; )</td>
<td>Contango (c)</td>
</tr>
</tbody>
</table>

In the above equations, \( E \) is an expectations operator, \( \Delta \) is an \( n \)-period difference operator, and \( \sim \) indicates that the variable concerned is a random variable. In other words, \( \Delta \tilde{F} \) represents the \( n \)-period
change in the futures price (rather than the percentage return, for reasons indicated in Chapter Four), and \( \text{Cov}(\Delta \tilde{F}, \tilde{R}_m) \) represents the joint variability of futures price changes and the value of the market portfolio.

It is important to note, at the outset, that a review of the literature does not indicate an obvious preference for any one of the above theoretical approaches. The advantage of the empirical investigation in this dissertation is that it conducts an empirically feasible test between the alternative theories. However, the empirical application of the Sharpe-Lintner CAPM to futures contracts in general, and to the traditional risk premium theories in particular, involves several conceptual and computational difficulties associated with the special features of futures contracts. These complications have been progressively addressed by the literature and are important to the understanding of our topic and research hypotheses. Conceptually, for instance, futures contracts do not represent net wealth, since no capital is invested in a futures position and the value of all outstanding futures contracts sums to zero. The interpretation of 'return' in the context of futures contracts is thus problematic. Computationally, in addition, futures contracts have relatively short lives (a year, on average, between contract initiation and maturity) and a variable period (since, from the moment of initiation, the time remaining to maturity falls). The construction of a long time series of futures contract prices is consequently, computationally, highly involved.

The empirical test in this dissertation is based on the application of the CAPM to precious metals futures. Much of the historical empirical literature in futures markets has concentrated on agricultural commodities, although more recently Taylor (1986) and Chang, Chen and Chen (1990) have conducted empirical tests for metals futures. The contribution of this dissertation is thus to extend the work of Chang et al (1990) to precious metals not yet tested, and to incorporate empirical modifications, including advances in time series construction, raised in earlier studies of agricultural commodities. For this purpose, a data set was constructed for gold, silver, platinum and palladium futures contracts traded on the New York Mercantile Exchange from 1980 to 1998, representing 368 different contracts and over 90,000 data points. For most of these contracts, nineteen years of daily returns have been computed, using methods that facilitate the construction of long futures price and return time series. The dissertation meticulously investigates the conceptual and computational issues arising from empirical work in futures markets in Chapters Four and Five.

The overall structure of the dissertation is as follows:

- **Part I: Literature Review** encompasses five chapters, structured along the lines highlighted above.

- **Part II: Data Description and Methodology** describes the data collected, adjustments made, and the two time-series methods employed in constructing long samples of log-relative daily
returns on manually structured contracts. Part II further explains the statistical implications and treatment of the CAPM's assumptions of normal distribution and serial independence.

- **Part III: Empirical Findings** examines the results of the tests for serial independence, of the empirical estimate of the futures-CAPM coefficients, and of the CAPM as a test of the traditional theories. Part III also evaluates the Keynesian measures of risk and return relative to the CAPM measures. It further examines the data's kurtosis and underlying distribution and the implications of the two time-series methods.

- **Conclusions** summarises the principal theoretical and empirical findings.
PART I: LITERATURE REVIEW

Chapter One
Introducing Futures Contracts, Futures Markets, and Hedging and Speculation

1. FORWARD AND FUTURES CONTRACTS

“A futures contract is to a forward contract as payment in currency is to payment by cheque”
(Telser and Higinbotham, 1977, p.969).

A forward contract is an agreement between a buyer (the holder of the ‘long position’) and a seller (the holder of a ‘short position’) who agree to exchange a stated amount (the contract size) of a stated asset (the deliverable item, or the underlying asset) at a stated time in the future (the ‘settlement date’, ‘maturity date’ or ‘expiration date’ of the contract). The seller is obliged to deliver on the contract, while the buyer is obliged to pay the specified amount or the forward price for the asset on delivery.

The traditional purpose of such contracts was the protection of both parties from price fluctuations that were expected to occur in the underlying asset during the term of the contract. Forward contracting is an old and very common practice found in almost all commercial markets. In some markets, however, forward contracts were so widely used that market forces gradually led to their standardisation and the institutionalisation of their trade. Contract standardisation and institutional arrangements led, in turn, to the creation of a more efficient mode of transacting, namely, a futures contract. The ability to transact without personal contact and on predetermined terms resulted in lower transaction costs and a higher volume of trade. The limit to such a standardisation process was the establishment of an organised futures exchange. Although a futures contract is less flexible than a forward agreement, it is nevertheless a highly liquid standardised form of a forward contract that has been (i) depersonalised, (ii) guaranteed by an exchange and (iii) traded in public markets.

The futures exchange sets all the terms of the contract except its price and authorises trading. In addition, a clearing-house is interposed between the buyer and the seller. In this role the clearing-house essentially becomes the seller for the actual buyer and the buyer for the actual seller. Its interposition essentially breaks each trade into two separate contracts and eliminates default risk. This mechanism also enables both parties to unwind their position at any time by engaging in “reverse trading” with the clearing-house and thus avoiding delivery.1 The buyer can ‘sell its long position’ (that is, enter a contract to sell the asset) and a seller can ‘cover its short position’ (that is, enter a long position), thereby closing out their positions, with the clearing-house. In this way, rather than taking or making

1 Reverse trading is also referred to as ‘principle of offset’, ‘clearinghouse offset’, or ‘ring settlement method’. 
delivery of the commodity, most traders would enter reversing trades to cancel their original position. In fact, it is estimated that the number of contracts actually resulting in physical delivery range from 1% to 3% of all contracts, depending on the commodity and the activity in the contract (Bodi, Kane and Marcus, 1993, p. 710).

Furthermore, forward and futures contracts generate very different cash flows to their holders. Parties entering futures contracts are required to put up margin and settle their contracts with the exchange on a daily basis. In doing so, profits and losses are realised at the end of each trading day. By contrast, a forward position is carried to maturity, only on which time profits and losses can be realised. A futures contract could therefore be viewed as a forward contract on an instalment plan.

This settlement difference turns out to be very important in the theoretical pricing models of these two contracts as discussed in Chapter Two and further explored and formulated in Appendix B (see Black, 1976; Cox, Ingersoll and Ross, 1981; Richard and Sundaresan, 1981; Jarrow and Oldfield, 1981).

2. THE EVOLUTION OF FUTURES MARKETS AND CONTRACTS

2.1 BRIEF HISTORICAL OVERVIEW

The development of organised futures markets progressed from, at first, traditional cash commodity trading (traced back to the Roman and Greek empires) through the primitive forward arrangements operating in Europe (in the eleventh to the fourteenth centuries) to, finally the emergence of organised forward markets for grains in Chicago in the second half of the nineteenth century. Gray and Rutledge (1971, p. 58) noted that “forward trading of one sort or another is of course virtually as old as commerce itself, but markets organised for the conduct of futures trading have become prominent only during the past century”. Irwin (1954) and Goss and Yamey (1976) claimed that the origin of futures trading can be traced back to the seventeenth century in Amsterdam, in grains, brandy, whale oil and coffee agreements. It was also claimed that the practice of forward trading developed independently and concurrently in Japan in rice contracts (Goss and Yamey, 1976; Peck, 1985). According to Veljanovski (1986, p. 25), the origin of organised futures trading in its modern form can be traced back to the New York Produce Exchanges, which appeared in 1752. It is difficult to judge, from the information available today to what extent the early trading in commodity contracts resembled what is now understood as futures trading, but it was clearly a primitive form of the activity.

Futures trading has grown out of existing merchandising practices. Leuthold, Jumkus and Cordier (1989, p. 22) emphasised that “futures markets are firmly rooted in commerce and serve as an extension of the cash market; they would not exist otherwise”. Gray and Rutledge (1971) noted that the possibility that futures markets were organised by persons outside the trade wishing to speculate in price movement finds no historical support. Instead, business people such as merchants, dealers, farmers and
processors who were dealing in cash and forward trading organised the markets to improve the trading in which they engaged. The documents traded, however, appear to differ from modern futures contracts in that they manifest a lack of standardisation, homogeneity, and formalised trading rules.

The emergence of the modern organised futures markets can be traced with greater accuracy to the nineteenth century, with the opening of the Chicago Board of Trade (CBT) in 1848. The development of the modern exchange corresponds closely with the growth of Chicago as a commercial centre. Initially, wheat corn and other grains contracts were simply forward commitments. Gradually, merchants found third parties that were willing to buy these contracts and assume the price risk, and contracts began changing hands several times before delivery occurred. These early markets can be characterised as delivery markets. Transactions were facilitated by the provision of uniform rules on the grade and the quantity of the commodity to be delivered (Irwin, 1954; Gray and Rutledge, 1971; Veljanovski, 1986). These basic rules were aimed at assuring competition and liquidity. Clearing arrangements guaranteed individual contracts and most deliveries were carried out. Gray and Rutledge (1971, p. 58) also noted that "probably not the role, and certainly not the extent of hedging as it presently emerged were contemplated when markets were organised". Such method of forward contracting continued to evolve until futures contracts, as opposed to forward contracts, were finally traded on the CBT in 1865. It is important to note, however, that futures contracts did not replace forward contracts entirely. It soon became clear that futures contracts were more effective as temporary substitutes for forward contracts, while forward contracts continued to serve the purpose of actual deliveries. Other modern agricultural futures exchanges were independently established in New Orleans, New York, Liverpool, London, Berlin, and Tokyo during the nineteenth century.

From that point onward, the literature identifies several stages in the development of the modern futures exchange that can be summarised as a progressive adaptation of various trading provisions and institutional arrangements. The literature further emphasises the iterative nature of this development as it progresses toward the creation of more efficient means of dealing with price risk and other related trading risks. The various provisions and institutional arrangements, for example, margin, marking-to-market and clearing-house, now provide a clear economic distinction between futures trading and forward trading. (These specific institutional arrangements are discussed below.)

The final element in the creation of modern futures markets, as distinct from the physical (or cash) and the forward markets, was the development of the practice of 'reverse trading', also known as 'clearing house offset'. This practice started in 1883 at the CBT with the formation of the ring settlement procedures, while the first complete and formal clearing-house system was established only in 1891, at

---

2 For more details about the development of futures markets in Chicago and in other American states see Peck (1985) and Leuthold et al (1989).

3 Peck (1985, p. 8-9) provides a brief description of the development of futures market in these locations and in commodities other than grain. Her thorough analysis is based primarily on the work of Irwin (1954).
the Minneapolis Grain Exchange. Separate grain futures markets soon emerged at nearly all majors
terminal markets in the United States and in Europe. Naturally, the lowest transaction-cost markets
flourished, while other less efficient markets inevitably vanished. Peck (1985) noted that the sustained
proliferation of early futures markets was a direct result of their evolutionary character. It took the
merchants more than two decades to recognise the precise value of this evolution as well as the
fundamental usefulness of these markets for hedging and storage requirements. It is therefore not
surprising that it took so much longer to formulate and centralise the actual trading. In addition, many
merchants initially viewed the market as a highly dangerous gambling operation and it took some time
for this perception to change.

Overall, it took some 50 years for organised futures markets to develop into the form recognised today.
For almost 100 years they were dominant by agricultural commodities, where among agricultural
products conventional wisdom dictated that only storable commodities were adaptable to futures
trading. Non-storable commodities, however, were soon also introduced, including Pork Bellies (1961),
Live Cattle (1964) and Hogs (1966). This development added a new dimension to the evolving market
and its lagging economic theory, in particular to prevailing pricing theories.

The first metal contracts traded were copper and tin in 1929 and silver in 1931. Platinum and palladium
began trading in 1956 and 1968 respectively. Gold futures were not introduced until 1973 when a ban
on ownership of gold by United States citizens was lifted. Metals futures were the only exception to the
agricultural bounds until 1972 when currency contracts were introduced. The introduction of interest
rates contracts in 1975 and stock index futures in 1982 finally transformed the industry from being the
almost exclusive province of agricultural interest into an integral component of financial markets
(Silber, 1985). Since then the pace of product innovations by futures exchanges has been phenomenal.4
Gay and Laux (1991, p.1) emphasised that the United States markets, in particular, have been a
remarkable success story in terms of volume growth and new product development.

Nevertheless, US futures exchanges now face many competitive challenges arising from (i) the
proliferation of new futures markets in other countries; (ii) off-exchange instruments such as swaps and
hybrids; and (iii) over-the-counter electronic markets. The US markets traditionally responded to such
challenges by introducing innovative products that offered new risk-shifting opportunities, but, as noted
by Gay and Laux (1991), this is unlikely now to be sufficient “as evident by the mimicry of contracts,
which carry no patent protection, by other futures exchanges.” The focus of futures exchanges seems to
have shifted to the institutional arrangements and trading structure in order to stay ahead of the
competition. This shift should ultimately lead to the more efficient execution of orders, lower
transactions costs, increased liquidity and a better flow of information. In the academic literature, this
area of research is known as ‘market microstructure’, and we discuss it further in Section 3 below.

---

4 Further elaboration of successes, failures, survival rates, and longevity of contracts and competition among
2.2 CONTRACT STANDARDISATION AND DESIGN

In contrast to cash and forward markets, the object traded on futures markets is a futures contract, not the actual underlying commodity. There is typically one standard contract per commodity; and all the terms of each contract, except the price, are predetermined by the rules of the exchange. Therefore the only negotiable term during trading is the buy/sell price of the contract. The standardisation characteristic of futures contracts typically comprises five elements (or contract's terms): quantity, quality, delivery time, delivery location, and anonymous trading. These five elements may be easily identified in Appendix A, where the specification of the four precious metal contracts - gold, silver, platinum and palladium - is presented for later investigation under our research hypothesis.

The standardisation of futures contracts has significant implications for the costs of transacting in this financial instrument and for the efficiency and net benefits of futures markets (Goss and Yamey, 1976; Silber, 1985; Veljanovski, 1986). First, standardisation eliminates the cost of bargaining over non-price terms and of enforcing contract provisions. Secondly, it reduces the monitoring costs that are inherent in markets in which a principal-agent relationship exists. These costs are much higher in spot and forward markets in which the terms of contracts are negotiated and carefully drawn, and which clearly provide the broker with various opportunities to take advantage of the principal.

Thirdly, it makes all contracts of a particular commodity or of a particular maturity month highly substitutable - a feature that is very useful for hedging purposes. This property of 'fungibility' is not found in spot or forward markets. In this context, as noted by Telser (1981, p. 12-13), a futures contract has the same attributes as money: the contract is essentially a temporary abode of purchasing power, the transaction costs of buying and selling are kept at a minimum, and, provided trade volumes are high, there will be highly elastic excess demand for futures contracts. Thus, while money is the most liquid of all assets, a futures contract is the most liquid of all contractual instruments of trade. An organised futures market is the essential device that enables such liquidity. It is this liquidity feature (or the immediacy of transaction at lowest cost) that best distinguishes futures markets from other markets. Other technical definitions will typically fall short in one way or another.

Closely related to standardisation of contracts is the contracts' design, which is considered crucial to the success of futures trading. The design problem centres on the actual specifications of the contracts and the standardisation elements, that is, contracts' delivery grade and location, contracts' size, contracts' delivery months, units for price quotations, daily price limits, margin requirements etc. As noted by Veljanovski (1986) and Silber (1981), in designing a contract the exchange is confronted with the basic marketing problem of translating economic prerequisites for efficient trading while maintaining sufficient appeal to both hedgers and speculators. A poorly designed contract - for instance, one that does not reflect commercial reality or favours a specific group of participants over others - would lack the necessary appeal and would fail to attract sufficient volume of trade necessary for liquidity.
The standardisation and design features of futures contracts are not without drawbacks, which makes the spot and forward markets important substitutes as well as complementary markets to the futures markets and explains their parallel existence. Standardisation gives the seller very limited options in term of grade, quantity and the time and place of delivery, which give rise to what is known as basis risks, which have to be borne by hedgers. In spot and forward markets, on the other hand, contracts’ terms can be tailor-made to suit parties’ requirements. (These so-called ‘seller options’ and the concept of ‘basis risk’ are discussed below.)

Many futures contracts allow the seller to deliver either the specified contract grade or any one of a prescribed list of alternative deliverables. Not all contracts, however, allow such substitution. Metals contracts, for example, are typically single-grade contracts. Since only one delivery grade can govern the futures price, it is important that the contract is based on the grade that will best reflect the price movements of the hedged commodity in order to attract the hedgers (or in order to reduce the ‘grade basis’). Nevertheless, standardisation would necessarily diminish the hedging properties of a contract when there are (even slight) disparities between price movements in the contract grade and the actual deliverable grade held by the hedger (or any other disparities). Similarly, the contracts usually give the seller the choice of delivery location in one of a number of exchanged licensed warehouses or depositories (which thus gives rise to ‘spatial basis’).

Lastly, futures contracts always refer to a particular delivery month. The seller is typically at liberty to make the delivery on any day within that month subject to giving a notice of the intention to deliver (which gives rise to ‘time basis’, also known as ‘the basis’, discussed in length in Chapter Three).

These seller’s options or the resultant basis risks suggest that standardisation also makes futures contracts an inferior and unfavourable delivery instrument. This in turn helps to explain the observed fact that between 1% and 3% of all futures contracts actually settled with physical delivery.\(^5\)

It seems clear that all these drawbacks could have been resolved if the homogeneity benefits of futures contracts standardisation and design were traded off against the heterogeneity benefits of less pre-determined or negotiable specifications. However, the contracts’ heterogeneity would necessarily benefit some groups of hedgers at the expense of others and would increase transaction costs by reducing market liquidity. As noted by Working (quoted in Veljanovski, 1986), “it is understandable that many hedgers should prefer a ‘poor’ hedge that is cheap to a more perfect hedge that is relatively expensive”. In fact, hedgers use the futures contract as a temporary set-off for a cash market position while they closely monitors the unparallel co-movement in cash and spot prices, and thus willingly assuming (cheap) ‘basis risk’.

\(^5\) See Kamara (1982) for a further discussion on the above and other dimensions of the basis.
3. THE NATURE AND OPERATION OF FUTURES MARKETS

As mentioned above, the object traded on futures markets is the futures contract. There is typically one standardised contract per commodity relating to a large quantity and a specified quality, while the contract price (that is, the futures price) is the only negotiable term during trading. When a contract is entered into, the value of both long and short positions is equal to zero. During trading, the price of the contract fluctuates in line with supply and demand and in relation to the spot price, resulting in unrealised gains and losses relative to the original agreed futures price. At the end of every trading day, each party’s net worth will be either larger or smaller (depending on the price movement) than it would be had the position not taken.

Clearly, therefore, the party that was adversely affected by the price change and is faced with (potential) loss has the incentive to dishonour the agreement and disavow the contract. In order to prevent such reneging and to guarantee the integrity of standardised futures contracts, the futures exchanges have built three levels of protection into contracts: margin, marking-to-market and clearing-house. In this context it is important to note that the margin and marking-to-market deposits are not actually ‘down-payments’ or investment toward the purchase of the deliverable item under the futures contract. Instead, they are a means by which the clearing-house can enter into a contract with an unknown party through a public market mechanism and still be reasonably certain that this party will honour the agreement even if the asset’s price moves against that party during the life of the contract. Such operational arrangements as well as other rules and regulations governing the futures markets (discussed below) were introduced progressively in order to enhance the market’s market-microstructure. A good market-microstructure promotes liquidity, pricing efficiency and least-cost mode of transacting, all of which are necessary to attract many participants and lead to active trading.

3.1 MARKING-TO-MARKET

This is the term given to the daily settlement procedure used in all organised exchanges, which requires ‘losers’ to settle up with ‘winners’ in response to daily changes in the futures price. Specifically, at the end of each trading day the clearing-house requires all positions to recognise daily-accrued gains and losses. The contract is then rewritten at the new futures price and its value set to zero. In practice, the price at which contracts are settled and subsequently rewritten is known as the ‘settlement price’ or

---

6 See Chapter Two and Appendix B.
7 The holder of the long position will gain from a futures price increase while the short position will gain from a price decrease. Pricing relationship between spot and futures are discussed in the next chapter.
8 The economic speciality known as ‘market microstructure’ refers to the formal and informal institutional arrangements established for trading securities in individual markets that are important for the achievement of external and internal efficiency in the allocation of resources. As noted by Hasbrouck (1991, p. 7) “[it] provides a scientific basis for preferring one form of market structure over another”.
9 Gay and Laux (1991) provide an in-depth analysis of current issues futures exchanges are facing with respect to market-microstructure.
'closing price' and is calculated as an average of the prices of the last several trades of the day.\textsuperscript{10} These paper-gains or losses are daily credited or debited to the brokers' interest-bearing margin accounts with the clearing-house, and the specific brokerage firm then acts as the clearing-house for its clients. If, for example, the day's settlement price, $X$, is higher than the price at which a contract was carried at or entered into during the day, $Y$, then funds equal to the amount of $(X - Y)$ per contract will be transferred from the short trader's account to that of the long trader. The reverse will occur when the futures price decreases during the day (that is, when $X$ is lower than $Y$). (This procedure is explained further below.)

3.2 ZERO SUM GAME

In order for a futures contract to open, a buyer and a seller must be matched by the exchange. Thus, whenever a futures contract is entered into, one long open position and one short open position are created. Black notes (1976, p.168) that "the total long interest in commodity contracts of any type must equal the total short interest". If all open positions in all futures contracts are added up and all short positions are counted as negative, the sum of all positions is always zero (ignoring commission) and therefore the net supply of futures is zero. Evidently, the futures market is a zero-sum game: the incremental paper gain of one party is exactly matched by an equal incremental loss to another party. Irrespective of what the spot price of the underlying asset turns out to be on settlement day, the payoffs of the long and short positions (of the same contract) exactly offset each other.

In other words, price fluctuation in the spot market which occur over the life of futures contracts produce winners and losers in equal amounts within the futures markets itself. There is thus no net spillover impact on the spot market for the underlying asset due to futures trading. (The relationship between futures and spot prices is discussed in Chapter Two. The payoff to a futures position is illustrated and further explored in Appendix B.)

3.3 MARGIN

While the marking-to-market protects the integrity of the futures contract over its entire life, the margin is a collateral that insures the integrity of the contract at the time it is entered into. This assurance is obtained by the requirement that both parties make a 'margin' or 'good faith' deposit at the time the contract is initiated. Failure to honour the contract leads to forfeiture of the margin. As Bodie and Rosansky (1980, p.38) explain: "A commodity contract is a viable instrument of intent. To legalise the agreement, and to provide some assurance of the financial ability of the buyer and seller to make good their losses, the commodity exchange set a 'performance bond' - commonly called the 'margin'." If the required cash balance in the broker's account (or 'commodity account') only equals the unrealised profit/loss (on open positions) from the marking-to-market obligation, the clearing-house would have

\textsuperscript{10} The settlement price is determined by a formula that uses the range of prices recorded within each day closing period, usually the last minutes of trading. It is determined by the exchange settlement committee and is intended to indicate the fair value of the futures contract at the close of trading (Edwards and Ma, 1992, p.76).
very little protection against a party that dishonoured these procedures, especially if unusually strong price movements occurred. Therefore, on the initiation of a contract both parties are required to deposit an *initial margin*, which is usually between 5% and 10% of the value of the contract. Thereafter, a *maintenance margin* amount must be kept in the account. This margin is typically 75%-80% of the initial margin. If the equity in the account falls below the maintenance margin requirement, a *margin call* will be sent by the broker to the customer, requiring additional collateral funds to be deposited. When daily gains exceed the required margin, the broker may withdraw the excess.

The broker account with the exchange can be summarised as follows (Sharpe, 1978, p.397):

\[
\text{Net equity position} = \frac{\text{Margin}}{\text{Plus unrealised gains on open position}} - \frac{\text{Less unrealised losses on open position}}{}
\]

The margin is thus a "cushion" which insures that the parties to a contract always carry an equity balance that is greater than their paper-losses on open positions. It varies from contract to contract but is typically set at a level likely to cover few days' worth of potential marking to market requirements. "In general, the greater the value of the contract and the variability of its price, the larger will be the required margins" (Sharpe, 1978, p.397).

Consequently, while daily price changes in the underlying commodity determine the marking-to-market cash flows, the size of the margin needed as a cushion to keep the two parties honest depends on the estimated volatility of the asset price rather than on the value of the entire contract itself.

3.3.1 **CAN THE 'MARGIN' CONSTITUTE AN INVESTMENT?**

The margin in futures trading differs significantly from the margin in shares or bonds trading. In the stock market, for instance, 'margin' refers to the amount of money deposited with the broker in order to purchase the security, while the balancing amount (of the total purchasing price) is typically borrowed. It is also possible to pay some of the amount in cash and borrow the margin from the broker. This margin essentially backs both the equity position in the security and the loan, and should therefore comprise a substantial percentage of the value of the asset purchased (usually between 20% and 25%). In this respect a futures contract is *not* an asset, since although a margin is deposited no balance needs to be paid or borrowed. A futures' margin need cover only a potential marking-to-market requirement that is based upon daily price changes in the underlying asset. These daily spreads are usually a small fraction of the total value of the asset traded and consequently futures margins are fairly low (that is, as a percentage of the underlying asset price), which clearly enables an investor to enjoy a considerable leverage. The 'leverage factor' is equal to the required margin percentage.

Moreover, the exchange permits either cash or cash equivalents (that is, short-term interest-bearing securities) to be posted as margin. If only cash is allowed, the required margin can to some extent be
considered an investment that supports open futures positions. If, however, interest-bearing securities are deposited instead of cash, essentially no investment is made and open positions can be regarded as bets that have yet to be settled. As further noted by Sharpe (1978, p.397), "the individual's funds are invested but in Treasury bills". Clearly, a rational investor would choose to post cash-equivalent securities rather than cash, since he could earn interest on them. It is also important to note that ownership of securities is not transferred to the exchange except on default, the securities are deposited as 'initial margin' and 'margin calls' are typically settled in cash. In both circumstances the margin, as explained earlier, will be very small relative to the total exposure of an open position and even relative to the possible gains and losses. "Thus, the buyer of the futures contract does not actually 'pay' for it, and, of course, the seller really receives no money for it" (Chance, 1991, p.328). This situation had led many commentators to argue that futures are 'highly levered' and very risky investment. If one takes the margin as an 'investment base', futures do seem very risky. This, however, is not really the true position (as will become evident in Chapter Four).

3.4 THE CLEARINGHOUSE AND REGULATION

Futures markets are highly organised, self-regulated markets. The exchange governs the conduct of trading; it decides on each contract's specifications and design; and it sets the rules of exchange. The exchange and its independent 'operating arm' - the clearing-house - ensure that all contracts are settled according to their terms. It also has the power to discipline members who breach the rules.

The clearing-house is unique to futures markets and critical to their success. It matches and reconciles all transaction and money flows; it assures the financial integrity of each contract; and it provides the delivery mechanism if delivery actually occurs. Its financial resources are also important in assuring the integrity of contracts. Its existence as a counter-party to all contracts means that if a member fails to honour its obligation, the clearing-house is the entity that suffers the loss, rather than the other party to the contract. This feature clearly provides more liquidity to futures markets, since potential traders need look only to the creditworthiness of the clearing-house for assurance that contracts will be honoured.

The exchange members' obligations are to the clearing-house and not to each other.\textsuperscript{11} The governance supplied by the exchange removes the risk of default in the settlement of futures transactions by holding members of the exchange liable to the clearing-house. It also protects the client of a member broker from loss due to the broker's insolvency or dishonesty. Together with the margin practice - which caters for the broker's risk of client default - this governance structure effectively eliminates default and reliability risk from the contracts. In addition, the exchange rules, especially in relation to its members, severely limit their ability to behave opportunistically.

\textsuperscript{11} Only members of the exchange, and in fact only a small number of them, can become clearing-house members because the financial requirements and responsibilities are substantial. Exchange members who are not clearing members must clear their transactions through a clearing member for a fee.
The sum of contract standardisation, contract design, centralisation of trading and exchange governance have led to the development of futures markets as the most convenient, secured, liquid and least-cost method of dealing with price risk and transferring the bundle of right attached to the futures contract. Veljanovski (1986, p.35) noted that “although the hectic and competitive nature of futures trading has led many to refer to it as the ‘last frontier of capitalism’, it is nonetheless highly regulated capitalism, both in the form of self-regulations, … or government regulation.”

US futures trading is regulated through a system of self-regulatory organisation and a federal regulator operating at three levels: the Commodity Futures Trading Commission (CFCT), the National Futures Association (NFA) and the designated futures exchange (for example, CBOT and NYME). The CFCT is an independent federal agency based in Washington DC that adopts and enforces regulations under the Commodity Exchange Act and monitors industry self-regulatory organisations. The NFA, whose principle office is in Chicago, is an industry-wide, self-regulatory organisation whose programmes include registration of industry professionals, auditing of certain registrants and arbitration. Each US futures exchange also has self-regulatory obligations in relation to its members and its markets. (Although the intervention and regulation of futures markets is a vast and important topic, which has been receiving much attention in the academic literature, it is beyond the scope of this paper.)

3.5 OPEN INTEREST

‘Open interest’ refers to the total number of outstanding contracts for each maturity month at the end of each trading day or, alternatively, the number of futures contracts for which delivery is outstanding. Each futures transaction creates two contractual positions with the exchange, a long one and a short one, but only one contract of open interest (since there is only one contract in existence for which delivery is obliged). The calculation of open interest is therefore simply the number of all short (or long) open positions. When a new contract is first sold, the open interest advances from zero to one. During the early months of the trading life of a specific contract, for instance ‘March 1999 - Gold’, increasing numbers of contracts progressively open, and the open interest could rise to thousands of contracts within a few months. Later, as the contract’s delivery or maturity date nears, more and more positions are closed out (or rolled over) with reverse trading to avoid delivery, and open interest falls substantially. On the delivery date the (few) parties with remaining open interest must fulfil their obligation by making or taking delivery and open interest reduces to zero. Open interest will thus be greatest for nearby maturity contracts (typically two to four months to ‘delivery’) and relatively low for succeeding distant maturity. Commercial users place most of their trades in the nearby maturities, because more information about these months’ futures and spot price is already in the market.

---


13 Futures contracts are typically referred to by their delivery month and year. See fn.26 and methodology section.
Graph 1 to 3 below plot the cumulative frequency of open interest on three metal contracts: 'December 1997 - Palladium', 'April 1997 - Platinum' and 'April 1997 - Gold'. It should be noted that while the behaviour of open interest during the life of each contract is similar, the size of open positions in gold contracts is far larger than that in platinum and palladium contracts. Moreover, gold contracts typically trade for long periods; and, as with silver contracts, they can sometimes 'go public' up to 60 months prior to delivery.\textsuperscript{14}

\textit{Graph 1: Daily Open Interest on 'December 1997 - Palladium' Contract}

\textit{Graph 2: Daily Open Interest on 'April 1997 - Platinum' Contract}

\textit{Graph 3: Daily Open Interest on 'April 1997 - Gold' Contract}

\textit{Source: Futures Industry Institution Data Centre, United States}

\textsuperscript{14} See Table 1 and 2 in Appendix A for more details on these metal contracts' design and trading characteristics.
4. THE ECONOMIC ROLES OF FUTURES MARKETS

The two most frequently cited economic contributions of futures markets are *hedging* and *price discovery*. They seem to constitute the standard explanations for the existence and benefits of futures markets and have thus attracted significant attention from the literature (Hardy, 1940; Irwin, 1954; Johnson, 1960; Working, 1962; Stoll, 1979; Yamey, 1986; Hirshleifer, 1988). Other functions typically referred to in the literature are speculation, greater market efficiency, risk management, transactional superiority and institutional advantages (Kaldor, 1939; Houthakker 1957; Gray and Rutledge, 1971; Peck, 1985; Goss, 1981 and 1986; Canarella and Pollard, 1985; Veljanovski, 1986; Copeland and Weston 1988).

Futures contracts represent a hedging tool, insofar as they are used by risk-averse commercial users in order to reduce or eliminate an underlying business risk associated with producing and marketing the commodity or providing a service. The terms ‘hedging role/function’ and ‘risk-transfer role/function’ are essentially synonymous and the analogy with ‘insurance’ is commonly made. More generally, the hedging role of futures markets is closely related to firms’ risk management. A large number of commercial and non-commercial firms use futures markets to manage the price risk inherent in their underlying business. Because futures prices are related to the underlying asset’s spot prices, trading in futures contracts can be used to cover or artificially create an exposure to the actual asset. Futures contracts further enable those wishing to reduce risk to transfer it in the marketplace to those willing to assume it. Because this market is so effective at reallocating risk, participants need not assume an uncomfortable level of risk. Investors are consequently willing to supply more funds to financial markets, which in turn enable firms to raise more capital and keep the cost of capital as low as possible.

Similarly, futures contracts can be used as an investment tool, in that they can be used to modify the “risk and return” characteristic of an investment portfolio, enable convenient and low-cost exposure to commodities’ prices and also expand the range of investment opportunities. In this section, however, the term “hedging” is restricted to instances in which futures are used to offset more fundamental business risk, rather than to tailor the risk characteristics of an investment portfolio.

The second contribution, price discovery, is also referred to as the ‘informational role of futures markets’. It is based on the idea that futures trading motivate the collection, assimilation, and dissemination of relevant information that is embodied in highly visible futures prices. Through futures trading, the market’s expectations regarding current and future supply and demand conditions as well as current and expected spot prices are instantaneously revealed.

---

15 For further discussion see Keynes (1930), Hardy (1940), Hicks (1946), Working (1953a and 1953b), Johnson (1960), Peck (1977), Scholes (1981), Telser (1981), Yamey (1986). Also most undergraduate textbooks explain the role of futures markets and pricing theories in this insurance context, which is too simplistic.
Information about expected spot prices is socially valuable for two reasons. First, in relation to storable commodities, it is an important determinant of commercial firms' storage decisions and of smooth supply conditions. Secondly, and relatedly, a knowledge of expected future prices affects both production and consumption decisions in a way that smooths and improves the allocation of commodities over time (Peck, 1985; Edwards and Ma, 1992).

Several economists have argued that if futures markets are to perform their price discovery role efficiently, futures prices should be good predictors of expected spot prices and therefore of actual future spot prices (Telser, 1958; Stein, 1961; Samuelson, 1965; Danthine, 1977; Goss, 1981; French, 1986). This argument form the basis of numerous lines of research regarding the behaviour and formation of futures prices, since it naturally lends itself to different interpretations and empirical modelling. The theory of forecast power and performance (explored in more details in Chapter Three) is one such example.

The ability of futures markets to provide information about current and future spot prices is the central rationale for their existence. Speculators' participation in futures markets plays an important part in carrying out this function. The beneficial effects of speculation on the price-discovery process are several. First, the participation of speculators lowers the aggregate risk aversion in the market. This may have the results that hedgers respond more strongly to price signals, which improves the price formation process (Goss and Yamey 1976; Peck, 1985). Secondly, new speculators bring additional information into the market, improving the informational content of prices. Thirdly, since speculators' returns depend primarily on their trading expertise, the incentive to gather information is greater than that of commercial firms who use the market to hedge. Lastly, as suggested by Peck (1985, p.70), "in processing information, speculators must take into account the responses of all participants to the prices implied by any single piece of information, thus improving the rationality of market prices".

Silber (1985, p.85) noted that both hedging and price discovery functions are listed in Guideline No. 1 of the Commodity Futures Trading Commission (CFTC). The guideline offers simple definitions of each of these important functions of futures markets. It states that the price-discovery function of futures markets will be satisfied if "prices involved in transactions for futures delivery in the contract are generally quoted and disseminated as a basis for determining prices to producers, merchants, or consumers of such commodity". The hedging use of a futures market is indicated when "transactions are utilised by producers, merchants, or consumers engaged in handling such commodity as a means of hedging themselves against possible loss through fluctuations in price". These definitions point to the conclusion that the price discovery is an information-based contribution of futures markets whereas hedging implies a transactional role for futures contracts. Thus, the main contribution of futures markets appears to lie in the establishment of prices for a potential future delivery of a commodity and for providing a forum for transacting at such prices.
In two studies relating to the use of information in futures markets, Goss (1981 and 1986) has provided a more exhaustive list of the functions performed by these markets. In his 1986 paper, he noted that “[t]he literature contains no comprehensive framework for analysing the performance of futures markets; nevertheless, there is probably a reasonable degree of agreement on the functions of these markets” (p.2). The function he has listed may be summarised (with some adaptation) as follows:

1. Futures markets provide facilities for risk management because they provide opportunities for hedging.
2. For storable commodities (in particular, seasonal commodities), futures markets facilitate stockholding in private hands. This is because the forward premium acts as a guide to inventory control and may be interpreted as a return on hedged stock (at least in situations where the hedge is held to maturity).\(^{16}\)
3. Futures markets perform a forward pricing function and thus facilitate intertemporal allocation of resources.
4. Futures markets act as a mechanism for the collection and dissemination of information.

While a further discussion of each of these functions is beyond the scope of this review, it is nevertheless clear from this list that they are all intimately intertwined with hedging and speculation and with futures price formation and behaviour. Some of these issues will therefore be discussed or referred to in relevant contexts in the following sections and chapters.

5. **HEDGING AND SPECULATION**

As mentioned above, organised futures markets facilitate two main kinds of activities: hedging and speculation. The *New Palgrave Dictionary* defines hedging and speculation as follows: “Speculation is the purchase (or temporary sale) of goods for later resale (repurchase), rather than use, in the hope of profiting from the intervening price change”.\(^{17}\) In principle, any durable good could be the subject of speculative purchase, but if carrying costs are high or the good is illiquid, than the margin between the buying and selling price will be large and speculation in that good will normally be unattractive.\(^{18}\)

*Hedging* on the other hand typically refers to a transaction on a futures market undertaken to reduce the risks arising from some other risky activity, producing the commodity, storing it, or processing it for

---

\(^{16}\) In the next two chapters it will be shown that Working (1953b) has interpreted the forward-premium as the price of storage and that Brennan (1958) and Telser (1958) explained the holding of inventory in times of a forward-discount in term of convenience yield.

\(^{17}\) The *New Palgrave Dictionary of Money and Finance*, s.v. “Futures market, hedging and speculation”.

\(^{18}\) According to the dictionary (*ibid.*) liquidity in this context means that there is a perfect or near perfect market in which the underlying asset can be sold immediately for a well-defined price. This liquidity requirement severely limits the range of assets available for large-scale speculation. The properties of the main two types of asset traded in organized futures markets - commodities and financial assets - make them particularly attractive to speculation.
final sale. The *New Palgrave Dictionary* also noted that the concepts of hedging and speculation have always fascinated academics and practitioners alike, perhaps because of their inconsistent definitions, occasional misunderstanding, and genuine economic importance.

A review of the vast literature on hedging and speculation indicates a disorderly development in the understanding of the terms themselves and the role played by hedgers and speculators on futures markets and in the formation of futures prices. The result is a proliferation of definitions and interpretations of the two terms. The definition of hedging, in particular, has long been a matter of dispute and has produced numerous publications. As noted by Paul (1976, p.4): “The more the concept of hedging is examined, the more it becomes evident that the term is used with many different shades of meaning. Although hedging can be sharply defined for specific purposes, no single definition meets everyone’s needs. The only feature that is common to all hedging in commodity futures is that such transactions or positions are somehow related to business needs.” Indeed, almost all firms engaging in any production, processing, distribution and marketing of agricultural and metal products would consider the use; in fact, most do use futures markets extensively in their commercial transactions and decisions. They are all typically referred to as hedgers. As noted by Peck (1985), the diversity of firms that find futures markets useful in the course of their normal business operations indicates that a single definition of hedging would never encompass all legitimate potential business uses of these markets.

Peck added that it is nevertheless clear that hedgers are *commercial users* and their activities relate to risk management. With futures markets, commercial firms can use futures positions as a temporary substitute for intended purchases or sale of the commodity, thus separating physical ownership from price change speculation. In the absence of a futures market, such speculation on commodity prices will require a large investment in storage and transport facilities to acquire and maintain ownership. Conceptually, futures markets could exist if trading were restricted to commercial firms or hedgers only. This structure would, however, require coincidence in the participating firms’ buying and selling decision and presumably also in the timing of the production and consumption of commodities, which would clearly result in a highly illiquid and inefficient futures markets (Breman, 1958; Telser and Higinbotham, 1977; Peck, 1985; Hirshleifer, 1988; Copeland and Weston 1988). In this context, speculators - defined as the *non-commercial participants* in futures markets - play an important role in absorbing the frequently unbalanced demands and supply of the commercial buyers and sellers. Imbalances could result from uneven numbers of total buyers and sellers, a discrepancy between the timing of buying and selling within a day or inequality in term of maturity requirements.

### 5.1 THE HEDGING – SPECULATION DICHOTOMY

It is interesting to note that, in the past, futures markets were regarded as speculative markets (Kaldor, 1939; Hardy, 1940; Working 1953a and 1962). As noted by Holbrock Working (1962, p. 435) “at Chicago … dealing in futures was initially regarded in the grain trade itself as a disreputable
speculative business; for more than a decade the CBOT refused to allow such transactions in its quarters”. This perception was soon changed, however, and a review of the literature points to a dramatic shift in the opinion of practitioners and academic alike toward viewing futures markets as a hedging market, a view most strongly advocated by Working. He (1962) suggested that the concept that futures markets depend on speculation for their existence should be replaced with the new concept that these markets depend primarily on hedging.

This change in attitude was preceded by a persistent focus on hedging activities and the related economic roles of futures markets, and to some extent to a neglect of speculation. It was motivated by research suggesting that, at least in commodity futures, strong involvement by hedgers is critical to the success of futures markets. Among the first evidence that futures markets were primarily commercial and not speculative was the observation that the level of trading activity showed a pronounced seasonal pattern, which mirrored the accumulation and decumulation of inventories of grains. Trading positions were consistent with storage needs in agricultural markets or with the so-called ‘arbitrage hedging’ explained below (Working, 1953a and 1953b; Irwin, 1954; Johnson, 1960; Stoll, 1979; Peck, 1985; Yamey, 1986).

Although economists recognised that a futures market composed only or predominantly of hedgers is not plausible, the prevailing view was nevertheless that substantial trading activities among commercial users (that is, trading among themselves) had first to be present before speculators could be attracted. The focus had shifted to what can be termed the hedging-speculation dichotomy, in which commercial users were seen as risk-averse traders who were willing to offer a premium to other traders - the speculators - in order to sell futures contracts as a mean of hedging against spot price fluctuations at harvest time (Johnson, 1960; Sharpe, 1978; Copeland and Weston 1988). In this context, speculation was essentially a passive by-product of hedging. Speculators were seen to participate in the market in order to gain the premium offered by hedgers for sharing the risk.

The hedging-speculation dichotomy was expressed in a vast body of literature during the first half of the twentieth century, the most prominent of which contained the theories of ‘normal backwardation and contango’ and ‘risk premium and bias in futures prices’, associated with Keynes (1930), Hick (1946), Telser (1958, 1960) and Cootner (1960a, 1960b). In addressing the issue, Keynes (1930) and Hicks (1946) - to whom the terms “backwardation” and “contango” are attributed - had in essence brought this body of literature into mainstream economics. (These theories are discussed in Chapter Three.) Subsequently, as research on futures markets and prices progressed, a less passive and more significant role was assigned to speculators, who are now finally viewed as professional traders, absorbing the imbalances of positions and specialising and devoting resources to forecast prices. As noted by Yamey (1986, p.34), it became clear that a futures market with no or only weak speculation activity “would have difficulty in ensuring continuity in trading and a high degree of liquidity for contracts; and the costs of using such a market for hedging could be unacceptably high".
5.2 DOMINENT VIEW OF HEDGING – MOTIVE (IN RETROSPECT)

In a literature review regarding the evolution of hedging definitions and motives for hedging, Yamey (1986, p.77) had pointed out the following:

"Once upon a time hedging by means of transactions in futures contracts was almost universally regarded as a practice intended by the hedger to avoid, reduce or eliminate risk of price changes by shifting that risk on to others willing to bear it [speculators]. Hedging was almost always defined, described or discussed in these terms, both in material addressed to those engaged in business and also in academic publications."

Gray and Rutledge (1971, p.79), in an attempt to trace the development of various concepts of hedging, emphasised the convenience to "demarcate four classes of hedging theories; each distinguished from the others on the basis of the assumption made about hedgers' attitude toward risk and hedgers' motivation to profit from futures operations". The first two categories relate to price risk: (i) hedging could be carried out to eliminate risks associated with price fluctuations; or (ii) it could be carried out to reduce risks of price changes. The risk-elimination view of hedging, the most naive view of all, was most commonly expounded before the Second World War. The archetypal hedger was seen as a risk-averse commercial firm, who wished to "cover" itself in anticipation of an adverse commodity price movement so that it was not later forced to make a disadvantageous purchase or sale. This view led to numerous publications by prominent economists such as Alfred Marshall,19 Paul Samuelson,20 Keynes (1930), Kaldor (1939), Hicks (1946) and many others. The importance of the risk-elimination view was that it permitted the analogy between hedging-speculation and insurance. In this context, Keynes and Hicks viewed speculation as a substitute for missing insurance markets in commodities, in that gains from futures trading were linked to differences in traders' willingness to take on risk and consequently to the taking of opposite positions. Telser (1981) has named that body of literature as "the price insurance theory".

Post-war developments in the literature represent a significant break from this traditional risk avoidance/reduction approach. Economists, predominantly Holbrook Working, have made a great effort to avoid the association of hedging with such motives. It is now widely agreed that 'pure risk avoidance' hedging is virtually non-existent in modern markets (Working, 1962; Kamara, 1982) since it can deny the objective of maximising shareholders' value. Hedging as a tool to shed some price risk is, however, widely practised. As noted by Yamey (1986 p.79), "[this] change in attitude cannot be accounted for as having been inspired by changes in the practice of hedging". Hedging of price risk through the whole or partial set-off of transactions in futures markets is still an important justification for hedgers' participation and for the establishment and operation of futures market. In an early paper, Yamey (1984) had already claimed that this extensive post-war literature in no way questioned or

weakened this justification. Although he acknowledged Working's exceptional and dominant contribution to the understanding of hedging activities, he nevertheless concluded that "it is unfortunate that [Working's] definition gives no hint of the motives or reasons for hedging, and no indication that active futures markets and hedging in futures contracts are not to be found except where there is considerable volatility in price". The empirical findings of Higinbotham and Telser (1977) support Yamey's conclusion that the hedging use of futures market is largely motivated by the desire to reduce the risk of adverse price changes.  

Hedgers' motives notwithstanding, it seems that the common tendency to associate hedging with price risk has had the unfortunate effect of obscuring the presence of other types of actual hedging operations, which had less to do with risk avoidance or reduction. Again, it is due to Working (in a series of papers beginning 1948)  

that a more complete and balanced view of hedging has developed and now prevails (see the brief outline below). In retrospect, we can safely say that Working and his followers explored the diversity of situations or ways in which hedging can be practised by those who produce, handle or process commodities. Their focus was on the opportunities presented to hedgers in the futures market, and they did not directly address motives. The shift in attitude represents a refinement - a better understanding of the operation of futures markets and their use by various participants, including hedgers, who were found to constitute a large and heterogeneous group of users.

In addition, as noted earlier, academic attention shifted from the uses to which the institution (the futures market/exchange) was put towards the properties of the institution. This shift occurred chiefly because the practice of hedging could not account for many of the characteristics of futures contracts or for the nature of trading, and existing theories lacked predictive power. Nevertheless, the view that hedging is primarily concerned with the risk of price change is still dominant in public discussion, in much of the literature and also in information issues by futures exchanges and brokers (see Goss and Yamey 1976; Silber, 1985). Risk reduction remains the best motivational explanation for hedging and therefore also (amongst other things) for the existence of futures market.

5.3 ALTERNATIVE VIEWS OF HEDGING - TYPES

Below is a brief description of four main types of hedging that illustrates Working's elaboration of the concept. It is important to note that the distinction between the various categories hinges primarily on the behaviour of the 'basis'. As noted by Peck (1977, p.152), "the continuing concern with the basis and its expected changes is at the centre of each form of hedging Working delineates".

---

21 Ironically, Higinbotham and Telser's paper is in fact an attack on this traditional hedging explanation; as they pointed out "an organised futures market is not necessary in order to obtain the advantage of hedging".

22 Yamey (1986, p.79) commented that "Working's contribution to the study of the economics of futures trading overshadow those of any of his predecessors, contemporaries or successors".
Routine Hedging
This term refers to the hedging of price risks, which involves the routine ‘covering’ of the trader’s position in spot by the making of an equivalent off-setting transaction in futures. Such a strategy is attractive to commercial users whose main business requires the holding of inventories but who find it inexpedient or costly to study the market and form own judgement about prospective price movement (Goss and Yamey, 1976). This category is similar to Working’s (1953b and 1962) ‘operational hedging’ discussed in Peck (1985) and Kamara (1982), except that the hedge is held for a very short time, for example while the product is being processed or transported. Goss and Yamey (1976, p.20) add that “routine hedging is encouraged where banks require or prefer that a borrower’s stock be hedged so that his insolvency is not impaired by a drastic fall in market price”.

Selective or Discretionary Hedging
This term refers to the hedging of commodity inventories according to price expectations. In contrast to routine hedging, the trader exercises judgment about price changes in situations where the decision of when and how much inventories to hedge (over-hedged or under-hedged) depends on price expectations and other business considerations, such as the extent of spot commitment, the ability to withstand losses and the cost of hedging. A selective short hedge, for example, hedges when he expects the price to fall but, at the same time, finds it necessary to carry some inventories for business purposes; selling the futures contract would substitute for selling the commodity. The objective of such hedge is to reduce the risk of making a loss. Selective hedging introduces a speculative element to hedging.

Carrying Charge or Arbitrage Hedging
This is probably the most notable contribution of Working to the understanding of hedging and, more generally, to the economics of futures markets. (See next Chapter for a detailed discussion of arbitrage and the carrying charge theory.) In the current context, it is important to note that this type of hedging is associated with the storage of a commodity and expectation of making direct pecuniary profits. The trader will purchase and store the commodity and simultaneously sell futures for direct profit from expected favourable changes in price relationship - the spot-futures differential (basis) - rather than risk changes in the price level. More specifically, the hedger speculates that a favourable change in the basis will exceed his costs of carrying inventories over the period and other transaction costs.

Goss and Yamey (1976) noted that the term carrying charge emphasises the remuneration aspect or motive of such hedging; while the term arbitrage is derived from Working’s remark that this type of hedging ‘is a sort of arbitrage’ between spot and futures market and serves to keep prices in the two markets in line. This type of hedging is the most common and, according to Gray and Rutledge (1971, p.81), who call it “hedging carried out to profit from movement in the basis”, it integrates the hedging decision into the overall management strategy. The perceived opportunity to hedge stocks profitably and the decision to accumulate them are, in this context, interdependent. The similarity between carrying-charge hedging and selective hedging is apparent. Yet, according to Working (1962, p.438),
 unlike selective hedging, with carrying-charge hedging the decision “is not primarily to hedge or not, but whether to store or not” and the hedger exercises his judgement based on his expectation of changes in the basis and not on expectation about changes in prices. Furthermore, since such description of selective hedging is closely related to the risk-avoidance approach, an association Working wished to avoid, he added that “the purpose of [selective hedging] is not risk avoidance, in the strict sense, but avoidance of loss” (1962, p.440). Clearly, Yamey’s criticism remains intact.23

Anticipatory Hedging
This term refers to hedging guided by price expectations. Here the sale or purchase of futures is made before the spot market commitment is entered into. As theorised by Working (ibid. p.441), “the anticipatory hedge serves as a temporary substitute for a merchandising contract that will be made later”. The hedge typically involves either the purchases of futures contracts against anticipated raw material requirements or the sale of contracts by producers in advance of the completion of production. Since there is no offsetting cash position, risk exposure can be higher or lower depending on movements in the price level. Although this strategy might prove profitable, it is essentially speculation rather than hedging. Perhaps Working classified it as hedging because it is carried out by commercial users and provides an example of the diversity of uses of futures markets.

In summary, it seems clear that all the categories of hedging described above, except perhaps arbitrage hedging, operate as a response to some uncertainty about spot price fluctuations. Furthermore, they all contain some speculative element. As to arbitrage hedging, Goss and Yamey (1976, p.20) pointed out that “[w]hilst futures market may be said to serve the needs of hedgers when they engage in routine or selective hedging, these markets may be said to provide opportunities for hedging when [arbitrage] hedging is involved”. The hedger’s futures position was seen to be motivated partially by the desire to stabilise income and reduce price uncertainty and partly by the desire to increase expected profits. Thus, all hedging positions are a mixture of hedging and speculation. As concluded by Kamara (1982, p.263), “hedging and speculation are not the opposite of each other but similar activities with different prices serving as the basis of speculation: hedgers speculate on both the basis and price levels while speculators speculate only on price level”.

23 In fact, Yamey (1986, p.81) also criticised that view, stating “there is no indication here or elsewhere how confident the hedger must be about his price expectation for his resulting hedging decision to be classified as one stemming from the desire to avoid loss rather than from the desire to avoid risk”. 
5.4 SPECULATION

We have already noted the tendency to emphasise the hedging function of futures markets and the general neglect of speculators and their roles. This neglect can be attributed to three factors. First, it is partly due to the fact that it was harder to demonstrate the price discovery function, in which speculators play an important role, than that of the hedging function, and the role played by speculators was therefore generally ignored. Secondly, Keynes’s traditional explanation of hedging as an insurance strategy assigned a very specific and limited role to the speculators, which was very different from that assumed in the context of the price discovery function. Lastly, the popular portrayal of speculators as gamblers causing wide price fluctuations from which they profit was deep-rooted.

Nevertheless, as the following analysis will show, it now seems clear that the minimal roles of speculators are (i) to help to make the market continuous, (ii) to increase its liquidity, and in this way (iii) to reduce the costs of using the market for hedging purposes, as well as for speculation itself. In contrast to the disputes and subtleties associated with the classification of hedging, the literature seems to concur that speculation can be functionally classified into three main categories as detailed below (Gray and Rutledge, 1971; Goss and Yamey, 1976; Kamara 1982; and Peck, 1985).

Position Trading or Price level trading
This is the most frequently identified form of speculation. Speculators are assumed to absorb the imbalances between the aggregate positions of commercial buyers and sellers of contracts on any given day. This form of trading is based on information regarding current and future supply and demand, and operates when trading is carried out in expectation of profiting from price changes over very short or longer period. Position traders use both a fundamental analysis and technical analysis to help them judge whether the current price level is higher or lower than or equal to the level warranted. Peck (1985, p.26) noted that “their ranks include professional traders, most of the so-called amateur traders, professionally managed account and commodity mutual funds”.

Spread Trading
These speculators are seen to absorb imbalances in the degree of futurity required by hedgers; that is, they match nearby maturity need with more distant futures need. Spreaders seek to profit from predicting changes in relative prices rather than prices per se. They will hold simultaneous positions to buy and sell different maturity futures contracts within one market (‘intramarket spread’) or similar maturity contracts between two or more markets (‘intermarket spread’). The most common intramarket spread is what is known as the “old crop-new crop spread”; for example, May-July wheat price or December-March silver spread. An intermarket spread, by contrast, would involve, for example, a December gold contract traded both in Chicago and in New York (Peck, 1985; and Kolb, 1988).
Market Making or Scalping

These speculators trade in large volumes during the daily trading session but rarely carry open positions overnight. Kamara (1982) noted that such trading is concerned primarily with buying on price dips and selling on price bulges. These dips and bulges are small and short-lived (that is, they last for only a few minutes or - very rarely - for only a few days). They typically arise from speculative buying or selling and from hedging carried out through market orders for immediate execution. The essential feature of scalpers is that they trade on transitory price changes and profit from skilfully accommodating the flow of orders as they come to market. Peck emphasised the scalpers' absorption of temporary imbalances in the timing of buy/sell orders within a trading day. She did note, however, that the direct association with imbalances in commercial users' demands breaks down somewhat, since scalpers do not distinguish among orders arriving on the floor, and will necessarily trade with other speculators as well.

Kamara (1982, p.264) noted that “few if any speculators confine themselves to only one form of speculation and most make some attempt to make use of dips and bulges”. Further, most of the floor trading is scalping, which Kamara holds as being largely responsible for the observed reversal pattern of consecutive price change.

In summary, it seems clear that speculation on commodity (and other assets) prices would occur with or without futures markets. Futures markets do, however, contribute significantly toward speculative activities by making it possible for individuals to engage in speculation without having to engage or invest in facilities to produce, handle, store, or otherwise use the commodity. The analysis in Chapter One would suggest that the standardisation of futures contracts facilitates such specialisation by speculators. Standardisation also enables speculators to take up either long or short positions easily, according to their expectation of price movement. Moreover, the organisation of futures markets enables the specialist speculator to economise in his operations, since transaction costs and capital commitments are minimised. The ease and speed of transacting according to one's judgment of current market conditions is not matched in markets without an accompanied futures market. Yet, such efficiency in speculative activities also gives rise to much criticism of futures trading. Trading by both professional and amateur speculators who 'climb aboard the bandwagon' can influence the price level and price movement of commodities equally readily in either direction, with serious effects on the fortunes of commercial users and consumers. Much attention has been paid by the literature to the stabilising and destabilising effects of speculation on spot price and trading in general, as well as to the role speculators and their contribution to market efficiency and price formation. None the less, it is widely agreed that futures markets facilitate the generally beneficent activity of speculators, which in turn improves the formation of prices so that they better reflect available market information and efficient intertemporal resource allocation.

24 Peck (1985, p.29) noted that “unlike specialists on the stock market [i.e. equity market-makers], scalpers are not assigned to any one market, nor do they hold an ‘inventory’ of public orders. Their income is derived solely from their trading”. See Silber (1984) for further details on market-makers.
5.5 NET HEDGING AND SPECULATION POSITIONS (LINK TO FUTURES PRICING)

This final section links the topics of hedging and speculation directly to our research hypothesis. As noted above, hedging, in its usual meaning in the context of futures trading, involves concomitant and opposite transactions in the related cash and futures markets for a commodity. It is also a characteristic of most futures markets that at most times short hedging exceeds long hedging in volume. That is, hedgers are most often net sellers of futures contracts or net-short on aggregate (Keynes, 1930; Houthakker, 1957; Goss and Yamey, 1976; Stoll, 1979; Kamara, 1982; Chang, 1985). Various attempts have been made to explain this phenomenon of net-short hedging, the findings of which are summarised in Goss and Yamey (1976) and Kamara (1982). The fact that short hedging is not typically matched in volume by long hedging implies that speculators (position traders) are typically net-long in futures, since buying and selling must match. Goss and Yamey (1976, p.34) noted that “[I]t would be surprising, however, to find that aggregate open interest of speculators with long positions exactly equalled the net short open positions of hedgers. One would not expect such a degree of ‘balance’ because of discontinuities in trading by both hedgers and speculators ... and because of differences in market expectations among speculators which would cause some to be net long and others net short at any given time.”

As far as this research is concerned, the important point to note is the assumption of a net long position of speculators. It is this feature that provided a foundation for the theory of normal backwardation. As noted in the Introduction, several economists of the time, most notably Keynes and Hicks, have formulated their futures pricing theories based on this assumption, suggesting a certain price relationship between expected spot and futures prices of commodities and positive returns for speculators who absorbed hedging imbalances. The New Palgrave Dictionary, in defining hedging and referring to this body of literature, noted that “an economically interesting question is whether agents ‘pay a premium’ to hedge”, while Peck (1985, p.26-27) noted that:

“Although it is convenient to think of position traders as absorbing the imbalances in commercial positions in a futures market, it is not an easy matter to link the profits of these traders to returns to speculators for assuming this net position. Nevertheless, a number of economists have proposed such links. Keynes was the first to propose a direct link.”

The critical point is that the Keynesian framework anticipated, in effect, a positive relationship between ‘risk and return’ (to speculators in futures markets), which is also the main premise of the Capital Asset Pricing Model. It is exactly this connection that must be explored in order to appreciate fully the literature on the subject-matter of this dissertation and its research hypotheses. We do so in Chapters Three, Four and Five.
Chapter Two
Fundamentals of Commodity Futures Prices

1. NOTATION

At the outset, it is important to define the following terms clearly in the manner in which they will be referred to in the following sections and chapters:

\[ T = \] the time to contract’s expiration or the time when the contract is settled (that is, commodity delivery or cash settlement).

\[ t = \] any time prior to contract expiration, where \( t = 0 \) is the time the contract is entered into, \( t = t \) is usually the current time and \( 0 > t > T \). Occasionally, for generalisation purposes we will use \( t \) as the time a contract is signed.

\[ S_t = \] spot or cash price at time \( t \) prior to expiration, or the price at which a commodity can be bought or sold for immediate delivery for cash at any time \( t \).

\[ S_T = \] spot price at contract’s expiration.

\[ f_{t,T} = \] the forward price, or a forward contract’s exercise price, is the price at time \( t \) for a transaction that will occur at time \( T \). Specifically: \( f_{0,T} = \) today’s forward price, \( f_{t,T} = \) forward price at \( t \) and \( f_{T,T} = \) forward price at expiration, all for delivery at \( T \). Note that the forward contract’s exercise price stays fixed over the life of a specific contract, and is privately negotiated.

\[ F_{t,T} = \] the futures price, or exercise price, is conceptually and for modelling purposes the same as the forward price except that this price is publicly established in organised futures markets and is continuously changing during trading hours.\(^{26} \) Thus: \( F_{0,T} = \) today’s futures price (open of trade and previous day close), \( F_{t,T} = \) futures price at \( t \) (during the day) and \( F_{T,T} = \) futures price at expiration (end of the day).

\[ E(S_{t,T}) = \] the expected spot price is the price that investors are expecting at time \( t \) to be the spot price in the future time, i.e. at \( T \), when the corresponding futures contract is due for settlement.

\(^{26}\) A number of different futures contracts will be traded simultaneously on the same commodity, each calling for (potential) delivery at a different time in the future. This results in the establishment of many different futures prices for the same commodity at any given time. For example, Silver futures contracts traded on NYMEX call for delivery on the following six months: January, March, May, July, September, and December; moreover, prices are quoted for each delivery month for the next two years. July and December contracts are quoted for a 5-year trading period. Thus at any time there may be as many as 18 futures contracts traded and thus 18 different silver futures prices.
2. FORWARD AND FUTURES – CONTRACTS VALUE AND PRICE

Futures and forward contracts are not assets in the traditional sense but are rather contractual instruments. As such the term value and price in the context of future transactions take on a different form than that used for traditional assets such as shares. As emphasised by Black (1976) and Jarrow and Oldfield (1981), the price of a forward or a futures contract is distinct from the value of such contracts; while the futures price is an observable number, its value is less obvious. Black (1976) and French (1983) noted that futures and forward prices are very much like the odds on a sport bet. Since the underlying commodity can be purchased just before delivery and re-sold immediately afterwards, the contracts are bets about the maturity price of the commodity. The payoff on such bets is equal to the difference between the futures (or forward) price and the maturity spot price. The contract price is the equilibrating factor in futures and forward markets; it adjusts to balance the demand and the supply to buy or sell the commodity in the future. Through the exchange or privately, buyers and sellers negotiate the contract’s exercise price in an attempt to find a value that will make entering the contract today economically worthwhile.\textsuperscript{27} The two parties will reach an agreement but no money will change hands at the outset.

Because on initiation of the contract neither party pays or invests anything of monetary value and neither receives anything of monetary value, the exercise price is set so that the contract’s value is zero (Sharpe, 1978; Black, 1976). A market value of zero is an equilibrating factor that yields an equal amount of short and long positions with payoffs that exactly offset each other.\textsuperscript{28} With an initial value of zero, provided the exercise price of the contract does not change, neither party can execute an offsetting trade that will generate profit. During their lives, however, the value of a futures and a forward contract is not necessarily equal to zero and as the exercise price changes the parties involved will pay or receive money. Valuation models of forward and futures contracts therefore attempt to find these exercise prices that are consistent with a fixed initially market value of zero.

As noted in Chapter One, forward and futures contracts differ in a fundamental way as they promise fundamentally different payoffs. A futures contract makes interim payments during its life while a forward contract does not. But this different form of payoff payment does not mean that a futures contract is simply a forward contract rewritten each day, which is a common misconception in the literature and was clarified by Black (1976), Jarrow and Oldfield (1981) and French (1976). These economists as well as Cox, Ingersoll and Ross (1981) have all (independently) discovered that this difference in payment schedule implies different exercise prices under certain conditions. The studies by Black (1976), Cox et al (1981), Jarrow and Oldfield (1981), Richard and Sundaresan (1981), French

\textsuperscript{27} Sharpe (1978, p.396) noted that if the contract’s price was set arbitrarily – like the exercise price of an option contract – some sort of payment between the two parties would undoubtedly be required to obtain an agreement, which is the case with option contracts where the premium is negotiated and paid at the outset.

\textsuperscript{28} See equations (2) and (7) and Diagram 1 in Appendix B.
(1983), and Park (1985) have all developed general equilibrium pricing equations of futures and forward contracts. They also examined the theoretical implication of the differences between forward and futures prices and the relationship of these prices to expected spot prices.

It is interesting to note the common threads of these studies. First, unlike traditional futures pricing models, the discussion and propositions are independent of (i) production and storage decisions; (ii) ‘hedging and speculation’; and (iii) ‘storable/non-storable’ frameworks. Secondly, these studies recognise that futures contracts are fundamentally distinguishable from forward contracts and therefore focus on the marking-to-market process as the prime differentiating factor between them. Thirdly, most studies are based on arbitrage arguments and are thus relatively general. The formulation is based on the ‘law’ of one price (defined later) and consistently makes use of portfolio strategies (concurrent investments positions), which replicate the payoffs from a forward or futures contract. By using a default-free bond to represent the credit or fixed-income market together with a spot position in the actual asset, these studies show that equilibrium forward and futures prices are equal to the values of other financial assets. Nevertheless, although a position in a derivative contract could be synthetically replicated with both credit and cash instruments, it does not follow that such contracts are redundant securities (see Silber, 1985). Fourthly, all the studies emphasise the role of interest rates in the pricing of such securities. Lastly, the models assume no taxes or transaction costs (in other words, perfect frictionless markets) and assume that traders can borrow and lend at the same nominal interest rate.

(See Appendix B for a review of the mathematical derivation of futures and forward contracts value and exercise price equations using arbitrage-based models as formulated in these studies. It is important to note that the cost-of-carry model, discussed below, is an arbitrage-based pricing model.)

Despite these differences between futures and forward contracts, most commodity traders and many writers treat futures contracts as though they were forward contracts. French (1983, p.312) noted that “many economists seem to agree that the daily settling-up has a negligible effect on futures prices”. For example, Dusak (1976), Stoll (1976), Grauer and Litzenberger (1979), Chang (1985) and others, in developing their models, abstracted from the marking-to-market provisions completely. Other authors, most notably Black (1976), explicitly recognised the daily settling up, but still concluded that futures and forward prices would be the same. Chance (1991) pointed to other studies that held that the difference between forward and futures prices is not economically significant.

29 As noted by Cox et al (1981, p.323) “[t]his allows one to apply any framework for valuing assets to the determination of forward and futures prices, even though they are not themselves asset prices”.
30 French (1983, p.313) noted that “the models also assume that investors will not default on any contracts, which implies that there is a finite upper bound on the daily price changes and on the daily interest rates”.
31 Where the models differ is in their assumptions of: (i) discrete versus continuous time equilibrium models, (ii) multi-good versus single-good economies and (iii) utility-based versus arbitrage-based models.
On the other hand, French (1981) did find significant differences between these prices but emphasised the difficulty with, first, forward prices data, which is generally unavailable for many commodities and, secondly, the failure of theoretical valuation models in discriminating among price differences. Chance (1991) concluded that although some unexplained differentials do exist, they are not consistent.

In line with these findings, it seems reasonable to accept the prevailing approach and assume that forward and futures prices are essentially equal and to refer from now on to futures prices only. Therefore, in the analysis below we present several models relating to futures prices, spot prices and storage costs that implicitly model futures contracts as forward contracts. We are no longer concerned with the relationship between futures and forward prices. In the context of pricing analysis, the focus here is on the relationship between futures prices and current and expected spot prices, and also between successive futures prices.

3. COMMODITY MARKETS AND COMMODITY FUTURES PRICES

Commodity market theory is the theory of intertemporal price formation on speculative markets (Adams and Behrman, 1978; Ghosh, Gilbert and Hughes Hallett, 1987; Kamara 1982). Commodity markets differ from markets for industrial goods in a number of respects. First, commodities are readily storable; secondly, they have a high degree of homogeneity; thirdly, commodity markets are highly competitive and pricing power is almost zero. There are, of course major exceptions to each of these principles but, as noted by Ghosh et al (1987, p.24), “a paradigmatic primary commodity may be thought of as exhibiting all three of these characteristics”.

The storability feature of commodities focuses attention on the intertemporal relationship between various prices and markets for identical goods, in particular between current and expected spot prices and futures prices. The term ‘basis’, for instance, refers to the intertemporal difference between spot prices and futures prices and is typically analysed in the context of hedging strategies. The theory of storage and the theory of forecasting power and the risk premium are the two dominant analytical frameworks that explain the relationship between futures prices and expected spot prices, and explain the returns to the holders of futures contracts. The characteristics of competitiveness and homogeneity translate into a lack of pricing power by producers and imply a particular form for these pricing relationships, for example, arbitrage pricing. They also explain the existence of organised futures markets alongside cash and forward markets, as discussed in Chapter One.

In the context of futures price formation and for purposes of analysis most commodities’ contracts fall into one of two categories: (i) storable and (ii) non-storable for contract delivery. Storable commodities are further divided according to their characteristics of seasonal and non-seasonal production or supply. Metals, like petroleum and financial instruments, are typically referred to as storable and non-seasonal (or ‘constantly renewable’) commodities. The assumptions of ‘indefinite storability’ and ‘constant
renewability' commodities are somewhat exaggerated, since most commodities do not exhibit such behaviour, yet in relative terms and as a theoretical framework such classification is appropriate. Precious metals are 'indefinitely storable' in a way that perishable commodities such as foodstuffs are not, and are 'constantly renewable' in a way in which grains, for example wheat, are not. The earthly supply of precious metals is finite and extraction is an increasingly costly exercise. Relative to many agricultural commodities, however, the extraction and the supply of metals is a continuous process, while the production of grains is a discrete process and supply is highly seasonal.

More importantly, the categories are appropriate in terms of the price structure in the commodity market. Discontinuous production processes and supply conditions, coupled with steady consumption and demand, lead to seasonal behaviour in the inventories level and prices of these commodities. Strong seasonal patterns are exhibited in most agricultural prices, owing to natural constraints, and inventories are typically carried forwards to smooth out supply. In contrast to agricultural commodities, the production of metals is relatively smooth (at least on a world-wide basis) and prices do not exhibit distinct seasonal patterns. The prices of metals are affected by general business conditions, by economic agents' expectations of future supply and demand and, importantly, by inflation expectations; prices and inventories are also highly sensitive to supply and demands shocks (Barro, 1986; Fama and French, 1988). It is important to note that, as with agricultural commodities, most metal inventories are held primarily for their consumption value. But precious metals, particularly gold and silver, are also held for their investment value - a factor that plays a role in the pricing of such futures contract.

The storability feature of commodity futures forms the basis of a well-defined functional relationship between spot prices and futures prices, as well as between nearby and distant futures prices. This relationship is formally expressed in the cost-of-carry model, also referred to as the spot-futures-parity theorem, which is the dominant model for the formation of storable commodities' futures prices. Non-storable or perishable commodities, of which live cattle and hogs are the principle 'archetypes', are characterised by varying supply conditions and fluid and heterogeneous inventories. They are thus characterised by the inability to carry inventories into the futures and redeliver them on a later maturity contract. They therefore do not follow the theoretical expected carrying-cost pattern between cash and futures prices. Instead, as is suggested by the literature, the futures price reflects traders' expectations of future supply and demand and consequently implicitly reflects the expected future spot price. Some writers link this proposition to the price discovery role of futures markets, while the carry model is occasionally associated with the forward pricing role of futures markets (Goss 1981; Peck, 1985; Leuthold et al, 1989). The link between these two alternative views will be discussed in the next chapter. While the notion that futures prices should equal the expected spot price is the dominant theory for the pricing of non-storable commodities, it clearly applies to storable commodities as well.

This analysis indicates that both storage and expectation mechanisms serve to link futures prices and spot prices. Similarly, the formation of futures prices is open to two alternative interpretations. The
first, which is based on the storage mechanism, sees futures prices as a cost-of-carry concept. This is in fact a functional explanation for the formation of prices, suggesting that the futures price is an accounting estimate of the cost of carrying the commodity into the future. The second, which is rooted in the expectation mechanism, sees the futures price as a predictor of the subsequent future spot price.

It is important to note, however, that expected spot prices themselves would also reflect carrying charges, just like futures prices would. Clearly, in the absence of future markets, expected spot prices would be rationally formed according to the estimate made by an investor of the cost of carrying the commodity into the future. In other words, both futures prices and expected spot prices for storable commodities would reflect carrying charges. Thus, the storage or cost-of-carry mechanism serves as a link between both (i) the current spot price - \( S_t \) and the futures price - \( F_{t,T} \) and (ii) the expected spot price - \( E(S_{t,T}) \) and the future price \( F_{t,T} \). The expectation or forecasting mechanism, on the other hand, is a theoretical relationship based on the price discovery role of futures markets and operates as a link only between expected spot prices and futures prices.

It should further be noted that these two mechanisms do not compete with each other but rather are alternative views of futures price formation, and it is the concept of ‘expected spot price’ that serves to link the two views. (See the review in Chapter Three of the two analytical apparatuses of modelling futures prices and returns, namely, the theory of storage and the theory of forecasting power and the risk premium.)

In the following sections, we focus on four fundamental links between the spot and the futures markets and prices, an analysis that is appropriate for the sake of conceptual clarity. The table below summarises the points already made and their relationship to the pricing theories discussed below as well as in the next chapter.

| Table 1: Storage vs. Expectation Mechanism and Commodity Futures Pricing Theories |
|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| **Commodity**                  | **Futures price formation**     | **Intertemporal pricing relationship (Basis)** | **Telser and Fama & French Pricing Theory** |
| Storable                        | Cost of carry \( F_{t,T} = S_t e^{R[(T-t)/365]} \) (Chapter Two) | \( F \) and \( S \) | Theory of Storage \( F_{t,T} = S_t e^{E[(T-t)/365]} + W'_{t,T} - \gamma'_{t,T} \) (Chapter Three) |
| Non-storable                    | Predictor of subsequent spot prices \( F_{t,T} = E_d(S_T) \) (Chapter Three) | \( F \) and \( E(S) \) | Forecast power and risk premium \( F_{t,T} - S_t = E_d(S_T) - S_t + E(RP_{t,T}) \) (Chapter Three) |
4. UNDERLYING LINKS BETWEEN SPOT AND FUTURES PRICES

Chapter One examined the complementary functional relationship between futures markets and spot markets. In term of pricing and performance the two markets are inextricably linked. As already noted, spot and futures markets for storable commodities combine to form carrying-charge markets; that is, there is a charge for holding the physical commodity, but not for holding futures contracts. The prices prevailing in the cash and futures markets at any time are therefore closely related. There are basically four important links and mechanisms that explain this close relationship: arbitrage and the law of one price; delivery and settlement; storage; and of course the cost-of-carry model itself.

4.1 ARBITRAGE AND THE LAW OF ONE PRICE

Although futures contracts are traded in an entirely different marketplace from that of actual commodities, the ‘law of one price’ and the assumption of perfectly competitive markets ensures that two identical goods will not sell for different prices. Otherwise, any misalignment between the spot and the futures price will create a risk-free arbitrage opportunity. More generally, the law of one price holds that \textit{investment strategies that have the same payoffs must have the same current value} (Cox et al, 1981, p.323). By taking opposite positions (and thus inverse payoffs) in the two markets, arbitrageurs bridge pricing gaps between them. This action prevents the futures price from drifting too far away from its theoretical value, which is measured relative to the spot price of the underlying commodity. The theoretical value of the futures contract is governed by the cost-of-carry model explained below. More specifically, if the futures contract become too cheap relative to the spot price plus carrying cost, arbitrageurs buy the futures contract and sell or short the spot commodity. The demand for a long futures position forces the futures price to rise while the spot price is driven down by excess selling, thus re-establishing the proper relationship between them (namely, the carry model) as well as the law of one price. If the futures price is lower than the spot price, the reverse will take place.

The following points should, however, be noted: first, it is not always possible to sell a commodity short (see Kolb, 1997); secondly, such activities are not unique to ‘arbitrageurs’. As noted in Chapter One, hedgers and speculators may equally be involved in arbitrage transactions and any distinction is therefore made primarily for analysis purposes. Thirdly, participants can take advantage of price misalignment between futures and spot prices without risk only if the futures price exceeds the spot by more than the \textit{carry charge} - the cost of holding inventories until the maturity of the futures contract.

4.2 DELIVERY AND SETTLEMENT

When a futures contract is near expiration it calls for an immediate delivery. All holders of remaining short open interests have to deliver what they have sold while holders of remaining long positions have to accept and pay for what they have bought. Clearly the sellers will not deliver anything that could be sold at a higher price in the spot market, nor will the buyers take delivery of anything that is cheaper
elsewhere. As explained above, an asset available from two competing sources must be priced identically, for otherwise an arbitrage opportunity opens up. Therefore an expiring futures contract is theoretically equivalent to a spot transaction. Almost no futures traders holds his position until the contract expires; instead traders use the market’s liquidity to enter into offsetting transactions or reverse trading. The fact that delivery is viable, however, and will occur on those open positions at expiration makes it an important consideration in the pricing of all derivatives instruments.

With futures, the delivery mechanism dictates an unambiguous contract’s value at expiration, which is determined by the spot price. As the contract’s maturity date nears, the futures price will progressively converge toward the spot price. This will continue until the contract’s expiration date (T), at which time the futures contract’s price and value will equal the spot price of the deliverable commodity. This ultimate equality is known as the convergence property. Being so widely anticipated by market participants, this property will also influence the pricing relationship between the futures and the spot price during the life of each contract. This equality can be expressed as follows:

\[ F_{t,T} = S_T \]  
(Generalising: \( F_{t,t} = S_t \) for all \( t \))  

(1)

4.3 STORAGE MECHANISM

The holding of inventories is a form of storage, and storage is a form of investment in which an investor defers selling the commodity in the hope of (i) obtaining a higher price at a later date; (ii) meeting higher expected and unexpected demand in the future; and (iii) benefiting from pure speculation on prices. The demand for storage is derived from the demand for stable consumption but varying production conditions. The supply of storage refers to the relationship between the expected increase in a commodity’s price and the amount of inventory that people are willing to store; where the causal relationship runs from expected price changes to the amount stored. The literature regarding intertemporal supply and demand schedules for commodity stocks or consumption and production decision is beyond the scope of this research. The focus here is on the holders of stock, primarily hedgers, their willingness to carry inventories, the role of futures markets in their storage decisions and, primarily, on price formation.

The terms ‘carrying charges’ (or cost of carry), ‘basis’, ‘spread’ and ‘theory of storage’ are interrelated. The first three concepts are important in defining the functional relationship between the spot price, futures price and successive futures prices, particularly in a hedging context (typically in textbooks) in order to explain hedgers’ behaviour. When supplies are ample, the difference between two simultaneously quoted prices for successive delivery dates for a storable commodity would clearly

32 The price of an expiring future contract and the related spot price can still differ by a small amount (see Garbade and Silber, 1983; Siegel and Siegel, 1990; Kolb, 1997).
33 See Appendix B for more details.
34 Sharpe (1973) pointed out that for predictive purposes the order could be reversed.
relate to the costs of storing that commodity between the two dates. The difference between the cash and futures price at the time a storable commodity is bought and stored is variously called the basis, the spot premium or discount and the carrying charge or, if negative, the inverse carrying charge (Peck, 1985, p.14). The difference between prices of near and distant futures contracts is termed the ‘spread’. The predictive power of the basis and of the spread regarding the level of inventory and expected returns to storage form the basis of the theory of storage (see Chapter Three). The basis and its interpretation in the context of arbitrage or carrying charge hedging is analysed later on.

The fact that the futures market provides simultaneous quotations of value for a commodity deliverable at successively distant dates clearly points to the important economic effects attributed to this market, derived - in this context - from its role in facilitating storage decisions and the role of forward pricing. To the extent that futures prices affect storage decisions, spot prices of the commodity over time will also be affected. In the absence of futures markets, the price difference that affects storage decisions is the difference between today’s cash price and an expected cash price. Simply stated, if prices are expected to increase, merchants are willing to carry higher level of inventory than otherwise (see Kaldor, 1939; Brennan, 1958). The important factor for merchants is the expected change in spot prices relative to the carrying costs or the magnitude of projected price changes and returns for storage. In a world of certainty, spot prices will increase by an amount equal to the carrying charges. In a world of uncertainty today’s spot price will be equal to the expected future spot price minus the cost of carry (and ignoring risk premium for our present purposes).

Therefore in the absence of a futures market the storage return is speculative and depends entirely on events occurring after the decision is made whether or not to store (Kaldor, 1939; Leuthold et al, 1992). With a futures market, storage returns can largely be determined when the decision to store is made because the futures price is, amongst other things, a carrying cost concept (Working, 1953b), which implies that it is a rationally formed, ‘cost accounting’ type of price for holding stock. The futures price for a storable asset is theoretically bounded by the (net) cost of carry for a predetermined holding period, and it is the market forces of arbitrage that can be relied on to ensure equilibrium pricing.

In summary, storing inventories entails risk arising from spot price fluctuations and uncertain storage and other carrying costs. Futures contracts may be used to reduce this risk by providing a means through which future selling prices for commodities are established today. Furthermore, if the risk can be removed completely, an overall investment of futures plus spot will offer the risk-free rate only.

---

35 Pack (1985, p.14) also noted that the terms ‘carrying charge’ and ‘inverse carrying charge’ are usually reserved for differences between futures prices and not between cash and futures prices. ‘Carrying charge’ is the term commonly used in the trade, while ‘cost of carry’ is the term usually found in textbooks. It was Holbrook Working who, in a series of papers starting in 1948, introduced the term ‘carrying charge’ to the academic literature.
4.4 DERIVING THE COST-OF-CARRY MODEL

The underlying principles of the cost-of-carry model are twofold: (i) the holding of concurrent spot and futures position in the form of covered riskless arbitrage; and (ii) accounting for the various carrying cost in the arbitrage position. Carrying costs or charges encompass four categories: storage costs (warehousing and spoilage), transportation costs, insurance costs and financing costs; they do not include the value of the commodity itself. The first three categories are loosely referred to as total physical costs or simply storage costs and for most storable agricultural and (consumption) metal commodities they can be substantial. Physical costs are generally constant for commodities with stable, ample supplies. For other assets such as stocks and bonds, the cost of storage are insignificant. Some assets even offer a return from storage, or positive yield, in the form of dividend, coupon interest or fees on lending gold or silver owned. Financing costs refer to the rate paid on funds borrowed to buy the asset or the interest forgone while funds are tied in inventory holding. Finance costs for metals are not constant; in fact most of the variation in precious metals futures prices is explained by the variation in short-term interest rates (Peck, 1985; Monro and Cohn, 1986; Neal, 1988; Fama and French, 1988).

For seasonal commodities, a third carrying cost element is the so-called ‘convenience yield’, a concept that was first introduced by Kaldor (1939). It refers to the advantages derived from carrying additional units of stock above immediate requirements (rather than holding the equivalent amount in cash and buying the stock later) either to meet unexpected demand from clients or in times of shortage. In periods of ample supply the convenience derived from actual ownership is zero, while with falling supplies convenience rises, representing the premium earned by those who hold inventories of a commodity that is in short supply (Brennan, 1958; Telser, 1958). The convenience yield implies a positive return from storage and is thus deducted from the total carrying cost.

The following notations are used for the various costs:

\[ W_{LT} = \text{physical cost of carrying the commodity: warehouse, transport, insurance etc.} \]
\[ Y_{LT} = \text{convenience yield} \]
\[ R_{LT} = \text{riskless rate of interest or opportunity cost of funds tied in inventory. In practice, the relevant financing rate is the ‘repo’ (repurchase agreement) rate, which is typically slightly higher than the rate on equivalent maturity government paper.} \]
\[ C_{LT} = \text{constant total cost of carry. For non-seasonal consumption commodities } C_{LT} = R_{LT} + W_{LT}; \text{ for seasonal commodities } C_{LT} = R_{LT} + W_{LT} - Y_{LT}. \text{ For metals that are held solely for investment purposes carrying costs are only financing costs (see below).} \]

(In the following section, all costs are expressed as annualised percentage and, in line with advanced valuation formulas for derivative securities, we use continuously compounded interest and other costs.)
The derivation of the ‘cost-of-carry model’ for a storable commodity involves two synthetically created competing strategies. Either strategy (or position) is initiated at time $t = 0$ and facilitates the purchase of one unit of the physical asset at time $T$. Table 2 provides a summary of the following literal derivation. The first strategy is the purchase of $e^{(C - R)T/365}$ units of the commodity at a cost of $S_0$ per unit. This spot position is held until $T$. The difference $(C - R)$ is the component of carrying cost that reflects out-of-pocket storage costs and is theoretically the rate at which inventory would have to be sold off to cover the storage costs and leave one unit of the asset at time $T$ (Stoll, 1979).

The second strategy is a long position in a futures contract (no initial investment) and the purchase of $F_{0,T} e^{R*T/365}$ default free bonds. At time $T$, the second position yields two payoffs: (i) $S_T - F_{0,T}$ the value of the long futures position, and (ii) $F_{0,T}$ the proceeds from the bond. The total payoff $(S_T - F_{0,T} + F_{0,T})$ therefore equals $S_T$, which is the amount required to buy one unit of the commodity. This strategy provides the same amount of inventory at $T$ but for a cost of $F_{0,T} e^{R*T/365}$ dollars. Since both strategies have the same terminal values, their initial costs or current value must be equal (law of one price). Thus:

$$S_0 e^{(C - R)T/365} = F_{0,T} e^{-R*T/365}$$  \hspace{1cm} (2)

$$F_{0,T} = S_0 e^{C*T/365}$$  \hspace{1cm} (3)

<table>
<thead>
<tr>
<th>Strategy 1</th>
<th>Market</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy $e^{C - R*T/365}$ at $S_0$ and carry until time $T$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Strategy 2</th>
<th>Market</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy $F_{0,T} e^{T/365}$ bonds</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Buy one unit of asset at $F_{0,T}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CF at $t$</th>
<th>Value at $T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>- $S_0 e^{(C - R)T/365}$</td>
<td></td>
</tr>
<tr>
<td>$S_T$</td>
<td></td>
</tr>
<tr>
<td>- $F_{0,T} e^{-R*T/365}$</td>
<td></td>
</tr>
<tr>
<td>$0$</td>
<td></td>
</tr>
<tr>
<td>$(S_T - F_{0,T})$</td>
<td></td>
</tr>
<tr>
<td>$S_T$</td>
<td></td>
</tr>
</tbody>
</table>

Equation (3) is the most common expression of the ‘cost-of-carry model’, or the ‘spot-futures parity theorem’ in the literature. In this general form it holds, essentially, that as long as the underlying commodity is in ample supply, so that spot holding (of inventories) can be carried forward, the futures price equals the spot price plus carrying costs, where those costs are primarily the net interest costs of holding the cash commodity from the current date until the settlement date of the futures (Silber 1985).

Similarly, the carry model will define the relationship between two futures prices. If $F_{t,T1}$ is the price of nearby maturity contract, and $F_{t,T2}$ the price of more distant maturity, we find:

$$F_{t,T2} = F_{t,T1} e^{C*T/365}, \hspace{1cm} (T2 > T1)$$  \hspace{1cm} (4)

Importantly, the components of the carry cost $C_{t,T}$ and their dependency on the asset’s price are each a function of the nature of the underlying commodity, and thus (3) and (4) could take various forms:-
For financial assets and investment commodities, such as gold and silver, the only carrying cost is interest. Generalising the time period to $T-t$ (i.e. from any $t$ to $T$), with $T-t$ being the holding period during a year, equations (3) and (4) could therefore be rewritten as:

$$F_{t,T} = S_t e^{R(T-t)/365}$$

$$F_{t,T_2} = F_{t,T_1} e^{R(T-T_2)/365}, \quad (T_2 > T_1)$$

Any deviation from equalities (5) and (6) would lead to an arbitrage opportunity. If $F > S_t e^{R(T-t)/365}$ or if $F_{t,T_2} > F_{t,T_1} e^{R(T-T_2)/365}$ the so-called 'cash and carry arbitrage' strategy will take place: borrow and buy the spot commodity and simultaneously sell futures; while if $F < S_t e^{C^{**}/365}$ or if $F_{t,T_2} < F_{t,T_1} e^{C^{**}/365}$ a 'reverse cash and carry arbitrage strategy' will be exploited: short sell the commodity, invest the proceeds and purchase (long) futures, driving the prices back into equilibrium.

For commodities that are held for consumption or convenience purposes and are in ample supply (most metals), carrying costs include interest costs plus storage costs. In addition to incorporating storage costs, however, equations (5) and (6) need a further modification in the form of partial equality. Assuming that storage costs per unit are a constant proportion of the spot price (and, like interest costs are regarded as negative yield), the (modified) carry model becomes:

$$F_{t,T} = S_t e^{(R + W)(T-t)/365}$$

$$F_{t,T_2} = F_{t,T_1} e^{(R + W)(T-T_2)/365}, \quad (T_2 > T_1)$$

Like financial assets, arbitrage cash and carry ensures an upper limit to the extent to which the futures price can exceed the spot price plus carrying costs; that is, any disequilibrium ($F_{t,T} > S_t e^{(R + W)(T-t)/365}$) will be short-lived or arbitraged away. It is important to note that the operation of this 'arbitrage constraint' on the carry model is independent of (i) the size of inventories, (ii) their ownership and (iii) of market expectations when the temporary disequilibrium occurs. But, in contrast to financial assets, reverse cash and carry arbitrage cannot be relied on to provide the (theoretical) lower limit of the futures price, and in fact there is no process of riskless and profitable dealing that could prevent $F_{t,T}$ from being less than $S_t e^{(R + W)(T-t)/365}$.

At any time the maximum volume of reverse arbitrage trading is dependent upon the volume of physical inventories held by certain categories of traders who alone can take advantage of the prevailing intermarket price relationship. These traders include firms that are holding stock for consumption purposes and against their own forward sale commitments in the cash market.

36 It is also possible that the yield derived from a financial asset, for example, the coupon from holding a bond, dividends from shares or a foreign interest payment will offset the interest cost, which can result in $C < R$.

37 See Kolb (1988) for a detailed summary of such strategies and underlying assumptions and Section 5 below.

38 Some scholars, however, take storage to be independent of the price, in which case equation (5) becomes: $F_{t,T} = S_t e^{R(T-t)/365} + W_{t,T}$ (e.g. Sharpe, 1978; Fama and French, 1988).
Such firms are faced with two alternatives. First, they can sell their stock now and replace it with the purchase of suitable quantity and maturity futures contracts, then accept delivery on the futures and honour their forward sell commitment at the prevailing spot price. This course of action, at the postulated relative spot and futures prices, enables them to reduce their carrying costs and increase profits without involving them in additional risks (other than ‘basis risk’, as discussed in Section 5 below).

Secondly, if such firms had hedged their stock by sales of futures contracts, they could take advantage of the spot premium \( (F_{S,T} < S_t e^{(R + W)(T - \theta)/365}) \) by selling their stock and closing out their futures position by buying futures. The crucial point is that at any time only a given volume of potential arbitrage is possible since the stock held in the appropriate trading context is limited and short selling facilities are generally non-existent for such commodities. This volume of arbitrage may be insufficient to bring about increases in futures price and reductions in the spot price, which explains the inequality signs (Goss and Yamey, 1976; Peck, 1985; Chance, 1991; and Hull, 1997).

The equality sign in equations (5), (6), (7) and (8) denotes the relationship between the futures price and the underlying asset price in the so-called ‘full carry’ or ‘normal market’. In this market, futures prices fully compensate a trader for the cost of purchasing (or holding) a commodity today, storing it and delivering it in the future. In a normal market, assuming ample supplies, the cost of carry \( e^{(R + W)(T - \theta)} \) is positive (it is therefore typically referred to as a ‘positive carry charge market’). This in turn implies that at any time before the expiration of the futures contract the futures price is greater than the spot price and, similarly, the price of a more distant contract is higher than the nearby contract by the full cost of carry between the two periods.

Alternatively, the market can be in a ‘less than full carry’ structure but still normal with futures prices above spot prices (and distant prices above nearby future prices) but by less than full carrying costs, as equations (7) and (8) indicate. That is:

\[
\begin{align*}
F_{t,T} &\leq S_t e^{(R + W)(T - \theta)/365} & \text{and} & \quad F_{t,T} > S_t & (R + W > 0) \\
F_{t,T2} &\leq F_{t,T1} e^{(R + W)(T2 - T1)/365} & \text{and} & \quad F_{t,T2} > F_{t,T1} & (R + W > 0)
\end{align*}
\]

It is important to note that a ‘less than full carry market’ may also indicate expectations of forthcoming shortage and holders of consumption stock may therefore be reluctant to sell the commodity. This situation typically arises with seasonally produced agricultural commodities, as explained below. While equations (7) and (8) place limits on the relative values of various futures prices and are almost never violated, it does not imply that they must move in lockstep with each other. Nevertheless, the links are

---

39 Sometimes, however, practitioners use the concept of ‘carry’ to mean coupon and/or interest earned minus financing cost, rather than ‘cost of carry’. They would thus refer to positive cost of carry (the usual situation) as ‘negative carry’ since a net-cash-outflow is results. We will use the concept of the cost of carry.
very close, as shown in the accompanying Graphs 4-6, which plot the daily prices of two different futures contracts and the concurrent spot price over a two-year period for gold, silver and platinum. A normal market structure and the convergence property are clearly evident for the given periods.

Lastly, for seasonal agricultural commodities the carrying cost includes interest costs plus storage costs less convenience yield. When a commodity is subject to a discontinuing production process or when current supplies are relatively low (for example, before harvest) the owners of the remaining stock place progressively more value on the flexibility provided by their ownership. As emphasised by Peck (1985, p.42), when scarcity increases "ownership yields convenience ... and the degree of benefit [such as continued production or accommodating orders] increases with the decreases in available stocks".

In this situation, the convenience yield could exceed all other costs resulting in negative costs of storage, which are also called 'inverse carrying charges', and in a so-called 'inverted market' structure. In this type of market not only is arbitrage unable to prevent the futures price from falling to less than full carry cost (equations 7a and 8a), but futures prices could fall to a discount-to-spot price and nearby futures prices to levels below distant prices (Brennan, 1958; Telser, 1958). Thus, the carry model is:

\[ F_{t,T} \leq S_t e^{(R + W - Y)/(T - t)/365} \]
\[ F_{t,T2} \leq F_{t,T1} e^{(R + W - Y)/(T2 - T1)/365}, \quad (T2 > T1) \] and
\[ F_{t,T2} < F_{t,T1} \quad (R + W - Y < 0) \] (9)

While there is an upper limit on how much the futures price can exceed the spot price, there is theoretically no limit on how far the spot price may be in a premium to futures, since there is no upper limit to the potential value of ownership. The assumption of constant C is unlikely to be satisfied in an inverted market where changing expectations of the coming harvest cause frequent unanticipated changes in the relation between spot and futures prices and thus in the value of C. Accordingly, the price of agricultural commodities changes with the seasonal production cycle: prices are frequently inverted before a new harvest, reflecting seasonal temporary shortage of grain, are at full carry immediately afterwards (to encourage storage), and are progressively less than full carry until the next harvest (Stoll, 1979; Brennan, 1958; Peck, 1985; Leuthold et al, 1989).

Fama and French (1988) examined the hypothesis of convenience yields in metals suggesting that there are some parallels between the price behaviour of metals and of agricultural commodities (see Chapter Three, Section 2.3).
Graph 4: Daily Gold Spot Prices vs. Daily Gold Futures Settlement Prices of Two 1998 Contracts (from the first trading day to maturity).

Graph 5: Daily Silver Spot Prices vs. Daily Silver Futures Settlement Prices of Two 1998 Contracts (from the first trading day to maturity).

Graph 6: Daily Platinum Spot Prices vs. Daily Platinum Futures Settlement Prices of Two 1997 Contracts (from the first trading day to maturity).

Source: Futures Industry Institution Data Centre, United States
5. TERMINOLOGICAL IDENTIFICATION

In part, the seemingly disordered state of the literature has arisen out of academia’s tendency to assign a new vocabulary to an existing concept (or theory or model) once it is elaborated in a new or different context. In addition, the evolution of futures markets economics was led primarily by futures industry experts, who employed a great deal of industry jargon and agricultural language. We have therefore constructed the following table, which aims to classify, place in context and simplify all the terminology typically encountered in the relevant literature and therefore used in this research. Some of the terminology in the table is only introduced in Chapter Three.

| Table 3: Summary of Terminology and Pricing Relationships in Full Carry vs. Inverted Market |
|-----------------------------------------------|------------------|------------------|
| Pricing relationship                        | Normal Market    | Inverted market  |
|                                             | (Full carry or Less than full carry) |                  |
| Arbitrage strategy                         | Long asset, short futures | Short asset, long futures |
| Basis (defined as $F - S$)                 | Positive         | Negative         |
| Cost of carry (Literature, see fn. 35)     | Positive         | Negative         |
|                                             | (i.e. carrying charge) | (i.e. inverse carrying charge) |
| Carry (Industry practice, see fn. 39)      | Negative         | Positive         |
|                                             | (i.e. financing costs) |                  |
| Spot price                                 | Spot discount    | Spot Premium     |
| Forward price                              | Forward premium  | Forward discount |
| Storage Theory (assets with no futures)    | $E(S_{t+T}) > S_t$ | $E(S_{t+T}) < S_t$ |
| Industry description                       | Contango         | Backwardation    |
| Keynes Hicks Approach                      | Normal Backwardation | Normal Contango |
| (Relates to expectation theory and bias)    | $F < \text{Exp.}(S_T)$ | $F > \text{Exp.}(S_T)$ |

6. BASIS AND ARBITRAGE HEDGING

For a hedger with a position in the actual commodity and an opposite position - of equal magnitude - in a futures contract the difference between the futures price, which is determined by the carry model, and the price of the commodity for immediate delivery is significant. The usefulness of a futures contract as a hedging tool depends on the degree to which the price behaviour of the futures contract mimics that of the underlying commitment. If the association between the two prices - spot and futures - is not perfect, ‘basis risk’ exists. As mentioned in Chapter One, the basis has three dimensions: time, grade (or quality) and spatial (or location). The most important dimension to active markets participants is the ‘time’ or ‘intertemporal’ basis, which as noted above represents the cost of carrying the underlying commodity over time. This basis plays an important role in the storage decisions of commercial users as captured in Working’s theory of storage, and hedging strategies in general.

The literature refers to three types of intertemporal basis:
Initial Basis: \( b_0 = F_{0,T} - S_0 \)
The basis: \( b_t = F_{t,T} - S_t \) \( 0 < t < T \)
Maturity Basis: \( b_T = F_{T,T} - S_T \)

(where \( b_t \) would typically referred to as ‘opening basis’, and \( b_{t+k} \) as the ‘closing basis’)

As noted earlier, Working (1953a, 1953b) placed great emphasis on hedging entered into with the intention and expectation of making direct pecuniary profits, which he termed carrying charge or arbitrage hedging. As proposed by Yamey (1986, p.80), such hedging “is said to be motivated by the prospects, indeed sometimes the near certainty of pecuniary profits resulting from changes in the basis relative to the cost of carrying inventory over time”. It seems reasonable to assume that commercial firms will hedge whenever they expect hedging to increase income: pecuniary and non-pecuniary. For the common short hedger, direct pecuniary income would be derived from positive changes in the basis, while indirect pecuniary income would be that derived from convenience yield. Non-pecuniary income, owing to reduced exposure to price changes, would be earned by risk-averse hedgers.

The nature of short and long arbitrage hedging and their respective outcomes are illustrated in Table 4.

For simplicity, we assume that two opposite positions of equal magnitude are taken in the two markets - cash and futures - in order to avoid the discussion of the appropriate hedge ratio. Transaction costs and carrying costs are also ignored. With short hedging we postulate that a trader is holding inventories of a commodity at time \( t \), which he intends to sell at time \( t+k \), and simultaneously hedges this spot position by selling futures contracts maturing (or reversed) at \( t+k \). With long hedging we suppose that at time \( t \) a trader is expecting the purchase of a commodity at time \( t+k \), or alternatively, a trader has sold the commodity short for delivery at time \( t+k \), and simultaneously hedges the spot position by buying futures contracts maturing at \( t+k \).

**Table 4: Summary of Short and Long Arbitrage Hedging**

<table>
<thead>
<tr>
<th>Market Structure</th>
<th>Short Arbitrage Hedging (Typical) (Cash and Carry Arbitrage)</th>
<th>Long Arbitrage Hedging (Reverse Cash and Carry Arbitrage)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basis</td>
<td>Full carrying charge</td>
<td>Inverted Market</td>
</tr>
<tr>
<td>Termination</td>
<td>= Long the basis</td>
<td>= Short the basis</td>
</tr>
<tr>
<td>Opening Position: ( t )</td>
<td>Long position in actual commodity, short position in futures contract</td>
<td>Short position in actual commodity, long position in futures contract</td>
</tr>
<tr>
<td>Closing Position: ( t+k )</td>
<td>Sell the commodity and close out future, (or deliver on the contract)</td>
<td>Buy the commodity with futures contract and deliver on the spot forward commitment</td>
</tr>
<tr>
<td>Total Payoff ( (= \text{change in } b) )</td>
<td>( = -S_t + S_{t+k} - (F_{t+k,T} - F_{t,T}) ) ( = (F_{t,T} - S_t) - (F_{t+k,T} - S_{t+k}) ) ( = b_t - b_{t+k} )</td>
<td>( = S_t - S_{t+k} + (F_{t+k,T} - F_{t,T}) ) ( = -(F_{t,T} - S_t) + (F_{t+k,T} - S_{t+k}) ) ( = -b_t + b_{t+k} )</td>
</tr>
<tr>
<td>Profit</td>
<td>if basis narrows (i.e. spot price rises by more than futures price)</td>
<td>if basis widens (i.e. spot price rises by less than futures price)</td>
</tr>
<tr>
<td>Losses</td>
<td>If basis widens</td>
<td>if basis narrows</td>
</tr>
<tr>
<td>Payoff If hedge held to maturity</td>
<td>( = -S_t + S_T - (F_{T,T} - F_{t,T}) ) ( = (F_{t,T} - S_t) - (F_{T,T} - S_T) ) ( = b_t - 0 )</td>
<td>( = S_t - S_T + (F_{T,T} - F_{t,T}) ) ( = -(F_{t,T} - S_t) + (F_{T,T} - S_T) ) ( = -b_t + 0 )</td>
</tr>
</tbody>
</table>

50
As seen in the table, theoretically the long arbitrage hedge is simply the reverse of short. In practice, however, the hedging outcomes are not identical. In fact, it is suggested in the literature that arbitrage hedging is far less significant for the long hedger than it is for the short (Yamey and Goss, 1976). Arbitrage hedging is carried out to profit from reliably predictable difference in prices in the spot and futures markets. Evidently, the profit from the hedge is simply the change in the basis, and the uncertainty how the basis will change is the basis risk. A hedge substitutes the change in the basis for the change in the spot prices. As noted by Chance (1991, p. 358), “the basis change is far less variable than the spot price change; hence, the hedged position is far less risky than the unhedged position”. A trader who ‘completely’ hedged a commodity position has, in affect, exchanged price risk for basis risk; the only remaining uncertainty is the difference between the value of the commodity and that of the futures on closure. The trader can therefore be said to ‘speculate the basis’, which is the point advocated so strongly by Working (1953), regarding spot commodity holders not as passive short hedgers but as professional speculators on the movement of the basis over time.

Sharpe (1978, p.402) noted that many dealers speculate the basis intentionally “for success requires knowledge of local conditions that affect price differences, not global conditions that affect price levels”. Thus arbitrage hedging is a speculative activity but one producing a risk level that is much lower than that of an unhedged position.

Due to the convergence property, a hedged position held to maturity theoretically has a closing basis of zero and its opening basis is analogous to the value of a forward contract at expiration. However, although it is convenient in theoretical and some empirical studies to treat the price of a matured futures contract as if it were the corresponded spot price, the maturity basis is typically negative. As can be seen in Graphs 7-9, the basis approaches zero during the life of a given contract but is highly variable over that period and negative at maturity. The variation is primarily due to differences in deliverable grade and location. Goss and Yamey (1976) pointed out that it is the discount-adjusted price of the cheapest grade that governs the price of a matured contract rather than the deliverable or basis grade and consequently some of the variations in the maturity basis are reasonably predictable.\textsuperscript{40}

Other possible reasons for non-zero basis relates to transaction costs and other carrying costs and to the fact that commodity contracts specify a delivery month rather than a delivery day. That is, contracts can be exercised at any time during that month at the short investor’s discretion and the futures prices would vary significantly during that month.

\textsuperscript{40} See Garbade and Silber, 1983; and Goss and Yamey, (1976) for more explanations.
Graph 7: Daily Basis ($F_{t,T} - S_t$) for 1997’s Gold Contracts (from the first trading day to maturity)

Graph 8: Daily Basis ($F_{t,T} - S_t$) for 1998’s Silver Contracts (from the first trading day to maturity)

Graph 9: Daily Basis ($F_{t,T} - S_t$) for 1997’s Platinum Contracts (from the first trading day to maturity)
As noted above, because of the existence of inventories, all futures prices for a commodity relate to each other. Furthermore, since the costs of carrying inventories are more or less constant in the short term or within a storage season, cash and futures prices are functionally related by these costs, as formulated in the carry model, and the set of prices move together as a constellation (see Graphs 4-6). Similarly, as seen in the Graphs 7-9 above (which were generated from the same data set), all the bases of a specific commodity with different maturities contracts also move closely together, with the basis of the more distant futures contract consistently higher than the nearby contract by the cost of carry between the two periods.41

Nevertheless, it can also be seen that the basis takes values within a wide range and varies significantly between different commodities. The literature suggests that such variations could be the result of various factors, including (i) the preferences of inventory holders for the security provided by hedging relative to the prevailing cost of hedging; (ii) expectation of changes in the basis and in storage costs which may affect hedging decisions and in turn may affect the supply of futures; and (iii) expectations of speculators and commercial participants regarding futures supply and demand conditions, which in turn affect the demand for and supply of futures (Goss and Yamey, 1976; Peck, 1985; Kolb, 1988; Kamara. 1988).

In summary, the essence of arbitrage hedging is the predictability of the basis and changes in the basis over time by market participants. For many futures contracts the basis follows a predictable pattern and traders must recognise deviations from that pattern. Peck (1985) emphasised that in contrast to agricultural commodities, in which arbitrage requires storage space and is therefore carried out largely by commercial elevator firms, almost anyone can engage in arbitrage hedging in precious metals, since specialised storage facilities are not required. It is therefore mainly in agricultural commodities, to the extent that the basis is both more stable and predictable than absolute price levels over the relevant storage periods, that arbitrage hedging reduces the business risk inherent in storage. This in turn implies that futures markets can be used to reduce storage margins. In metals, it is price level risk rather than storage risk that is shed, and arbitrage hedging is therefore more of a speculative nature.

41 In relation to the discussion of the cost-of-carry model above, theoretically, there is an upper limit but no lower bound for the value of the basis, or for how much the futures price can exceed the spot price. The maximum positive value of the basis is the marginal carrying cost of holding a unit of the commodity from any time \( t \) until the maturity of the futures contract \( T \); or based on equation (9) and using simple interest that is:

\[ F_{t,T} - S_t = \frac{S_t (R + W - Y)^{\nu / 365}}{(1 + R)^{T - t}} \]

(Also known as maximum contango or full carry as shown in Table 3).

Clearly, any basis larger than this marginal cost offers arbitrageurs or storers the opportunity for riskless profit by buying the spot commodity and simultaneously selling futures contracts (short arbitrage hedging in Table 4). Thus, arbitrage-hedging activities prevent the basis from exceeding the full marginal cost of carry. The basis can, however, be lower than this maximum and even negative, as is the case in an inverted market and backwardation, although this is not common in metals. The marginal carrying cost function is discussed further in Chapter Three in the context of theory of storage.
Chapter Three
Pricing Theories and Keynes’ Risk Premium

1. INTRODUCTION

The lack of consensus on the underlying process, as well as the determinants of futures prices, has led researchers to take a variety of approaches in developing models of futures contracts. Nevertheless, it was in effect Telser, in his important early paper in 1958, followed much later by Fama and French (1987), in a more explicit manner, who suggested two basic analytical approaches to the modelling of futures prices and returns. Although the literature differs on this subject, we regard the distinction drawn by these economists as according most closely with our understanding of the literature and with the precepts underlying this review.

The first theoretical framework centres around the theory of storage. The second involves the theory of forecasting performance and risk premium. While this second framework is deeply rooted in Keynes’ theory of normal backwardation, it has attracted so much research that it has developed into a voluminous body of literature, encompassing much more than the Keynesian theory. None the less, our focus is primarily on the theory of normal backwardation, since it is this theory that prompted the application of Modern Portfolio Theory to futures markets.

While the theory of storage is well accepted and uncontroversial, the question of whether futures prices reveal expectations about future spot prices (and whether futures prices command a risk premium, as do the spot prices of other assets) has long been debated in the literature. This question is now part of a broader debate regarding the efficiency and price discovery role of futures markets.

In reviewing the literature, we found a wide diversity of theoretical and empirical lines of research that relate either directly or indirectly to the price discovery role. The most dominant strands of empirical studies are the following:

(i) Studies related to the so-called forward pricing efficiency of futures markets. According to this hypothesis, if spot and futures prices fully reflect all available information, they can each be regarded as an unbiased predictor of future spot prices. Alternatively, there should be no predictable bias in futures prices’ forecasts and the futures price should equal the expected spot price (see Samuelson, 1965; Goss, 1981; French, 1986; Canarella and Pollard 1985 and 1986; Neal, 1988).

42 It is interesting to note that many such studies include the word ‘efficiency’ in their heading although they each deal with very different topics. We found that ‘efficiency’ in futures markets could take various interpretations and if loosely used could be very misleading.
(ii) A closely related line of research includes studies that attempt to determine whether futures prices contain a risk premium. If significant unpredictable risk premium exists, futures prices will not be good predictors of expected spot prices. These studies directly relate to our research topic.

(iii) Studies that have examined the efficient market hypothesis (sometimes referred to as the forward pricing performance of futures markets) and can take various forms. Some economists have used mechanical filters or trading rules to examine whether speculators in futures markets consistently earn profits. If futures prices are rationally formed and incorporate all available information, profitable trading strategies cannot exist and expected excess returns to speculators trading on the basis of publicly available information should be zero (see Houtakker, 1961; Cargill and Rausser, 1975; Hartzmark, 1987). Related to these are studies that have employed the traditional random walk model and the Martingale hypothesis to test for serially independent expected futures price changes (see Danthine, 1977; Stevenson and Bear, 1970; Aggarwal and Sundararaghavan, 1987).  

(iv) Lastly, the Palgrave Dictionary points to a line of research in which economists have examined whether there are models that can make 'out-of-sample' forecasts of future spot prices which are superior to those made using futures prices only. If such models exist, the futures market cannot be viewed as providing important new information.

The first and second lines of research identified here - forward pricing or the unbiased predictor hypothesis, and the risk premium - are closely related to the two pricing frameworks identified above and in fact serve as a theoretical link between them. They are dealt with in more detail below, but the other 'price discovery issues' are beyond the scope of this review.

This chapter will focus on a comparison of various theories and models of futures prices and returns with the aim, first, of postulating the notion of a risk premium and then of applying the Capital Asset Pricing Model to test for that risk premium (in Chapter Four).

2. THE THEORY OF STORAGE

The supply of storage theory was pioneered by Kaldor (1939), was most notably formulated by Working (1948, 1949b) within the realm of futures markets and hedging, and was then formally

---

43 Bilson (1981) noted that the forward pricing efficiency or the unbiased hypothesis (UPH) is distinguishable from the efficient market hypothesis (EMH). On the theoretical level the UPH has come to be associated with the EMH and the rational expectation hypothesis (REH). Bilson, however, convincingly argued that UPH is not a necessary condition for either EMH or REH to hold and showed that it is possible to construct models in which markets are efficient in the sense described above and expectations are rational but UPH is violated. In order to distinguish the forward pricing efficiency from EMH and REH, he redefined the UPH as the 'speculative efficiency hypothesis'.

44 The New Palgrave Dictionary of Money and Finance, s.v. “informational role of futures markets.”
modelled and tested by Brennan (1958) and Telser (1958) for agricultural commodities and by Fama and French (1986, 1987) for metals. As noted in the previous chapter, the term 'supply of storage' refers to the relationship between the expected increase in a commodity's price and the amount of inventories firms are willing to store.

More specifically, as explain by Kamara (1981, p.265) in his review of this literature: "[T]he price spread \[ E(S_T) - S \] is an increasing function of the amount of stocks carried from \([r \text{ to } T]\). The firm will hold that amount of stocks which will equate the marginal net cost of storage per period with the expected price change during the period." It is important to note, however, that the theory of storage and the concept of declining marginal convenience yield on inventory were formulated by Working to explain observed seasonal behaviour of spot and futures prices for agricultural commodities; in particular, futures prices that are below spot prices before harvests (that is, inverse carrying charge or inverted market), when inventories are low and the marginal convenience yield on inventory is high (Working, 1948 and 1949b; Fama and French, 1988). As such, the theory in the context of futures market - where the relevant intertemporal price difference is the basis or futures' spreads - preceded the more general rule pertaining to all commodities (with or without futures markets and hedging), where the price spread is that between the current and the expected spot prices.45

The main theoretical foundations of the theory were outlined in Chapter Two, namely, the storage mechanism that links cash and futures markets, the cost-of-carry model, basis and arbitrage hedging. Therefore in the current context we focus only on link to inventory levels, the notion of a risk premium and expected spot prices. The relationship between the expected price change and the amount carried in inventory, or similarly between the basis and levels of inventory, as predicted by the theory of storage, is shown in Diagram 2.

**Diagram 2: The Theory of Storage - Relationship between the Basis and Levels of Inventory**

45 This issue is discussed further later in the chapter.
The supply of storage curve slopes upwards, reflecting the fact that firms or professional speculators will store more of a commodity when they anticipate significant price increases. The curve is horizontal over its mid-range when available storage space is rented out or used for other purposes without putting pressure on the renting costs. This situation represents normal market conditions, in which the expected price change - as reflected in the basis - equals the typical carrying costs for storage (see below). The right hand side of the curve, however, is steeper, reflecting the situation in which large carrying costs, and therefore larger expected price changes are necessary to induce larger or more storage.

To the left of $X_0$, where smaller quantities of inventory are in storage and lower-cost storage facilities become available, the curve gradually slopes down to $X_1$, reflecting a positive but declining return to storage. Then to the left of $X_1$, the curve drops off steeply, indicating an inverted market in which inventories are held at time of negative expected price changes. The minimum level of inventories that is held ($X_2$) regardless of ‘losses’ on the carrying charges is explained by the convenience yield.

2.1 THE NET MARGINAL COST OF STORAGE FUNCTION

The following discussion draws on the work of Kaldor (1939), Working (1948, 1949b), Brennan (1958), Telser (1958) and French (1986) and focuses on the derivation of ‘equilibrium condition’ for the supply of storage in a two-period model by a firm that maximises storage returns. The net marginal cost of storage function (MCS) is derived from the total cost function incurred by holders of inventories, which is dependent on the amount of inventories carried out of period $t$, denoted $X_t$. The total cost function, $TC_t(X_t)$, can be expressed as follows:

\[ TC_t(X_t) = R_{t,T}(X_t) + W_{t,T}(X_t) + RP_{t,T}(X_t) - Y_{t,T}(X_t) = TC \] (11)

In line with the notation in Chapter Two: $R_{t,T}(X_t)$ is total financing costs, reflecting the opportunity cost of the capital invested in inventory (where $R_{t,T}$ is the interest rates prevailing at $t$, for the period $T - t$), $W_{t,T}(X_t)$ is the total physical costs and $Y_{t,T}(X_t)$ is the total convenience yield, all of which are incurred in carrying $X_t$ units of the commodity from $t$ to $T$. $RP_{t,T}(X_t)$ represent the total risk premium or, as Brennan termed it, the ‘total risk aversion factor’. This risk premium, if it exists, arises out of the uncertainty regarding the realised selling price $S_T$ and is expected to be an increasing function of inventory, of the degree of risk aversion of the trading firm and of price volatility (Leuthold et al, 1989). Based on the expected first and second derivatives of each of the cost components, the net marginal cost function of storage may be written as:

\[ TC'_{t,T}(X_t) = R'_{t,T}(X_t) + W'_{t,T}(X_t) + RP'_{t,T}(X_t) - Y'_{t,T}(X_t) = MCS \] (12)

(Where: $R'_{t,T} > 0$, $R''_{t,T} \geq 0$; $W'_{t,T} > 0$ $W''_{t,T} \geq 0$; $RP'_{t,T} > 0$, $RP''_{t,T} \geq 0$; and $Y'_{t,T} \geq 0$ $Y''_{t,T} \leq 0$)

---

46 A complete analysis of the theory of storage further incorporates commodity production and consumption equilibrium conditions and the demand for inventories or storage, which fall beyond the scope of our research. For further discussion see Weymar (1966); Kamara (1981); Peck (1985); French (1986); and Leuthold et al (1989).
In (12) financing costs, storage costs and the risk factor are all increasing functions of \( X \), with their marginal costs either increasing or a constant function of inventory levels. As noted by Goss and Yamey (1976, p.14) and empirically confirmed by Brennan (1958) and Fama and French (1988), at any given time financing and storage costs per unit of the commodity are likely to remain at a constant level for a wide range of total inventories of the commodity. This is particularly true for precious metals. For very high levels of inventory, however, marginal carrying costs are likely to rise progressively due to more expensive storage facilities, obsolescence or wastage and tougher credit arrangements.

The convenience yield is also an increasing function of inventories but the marginal convenience declines and can reach zero at large levels of inventory. Although such a yield is not directly observable, it will be reflected in an observed (inverse) price spread or spot premium, and its existence is widely accepted. The inclusion of a risk premium factor is disputable and we shall discuss it below. The net marginal cost function has a similar algebraic format and signs to the total cost function, and can be positive or negative depending on current supplies conditions in the market. As explained in Chapter Two (Section 4.4), normally when inventory levels are high \( Y'_{t,T} \) is small or even zero and MCS is positive, but when inventories are relatively low \( Y'_{t,T} \) may be very large and may outweigh the other costs so that MCS is negative.

For a firm that carries inventory from time \( t \) to time \( T \), total expected net income will equal total revenue less total costs or: \( TR'_{t,T}(X_t) - TC'_{t,T}(X_t) \). The quantity of inventory that maximises net income in a competitive market is calculated by differentiating expected net income with respect to \( X_t \) and setting the derivative equal to zero, which gives \( TR'_{t,T}(X_t) = TC'_{t,T}(X_t) = MCS \). In other words, a firm will maximise profit when marginal cost is equal to the net marginal revenue. In equilibrium and (with or without futures markets), for any given level of stock, firms will store the commodity until marginal revenue or the expected capital gain \( E(R_{S_F}) - S_t \) is equal to the MCS over the time inventories are held. That is:

\[
TR'_{t,T}(X_t) = E(R_{S_F}) - S_t = MCS = R'_{t,T}(X_t) + W'_{t,T}(X_t) + RP'_{t,T}(X_t) - Y'_{t,T}(X_t)
\]

(13)

In equilibrium firms will store commodities or carry inventories until the expected capital gain equals the sum of the interest cost, net physical storage cost (less convenience) and the cost of bearing the commodity price risk (French, 1986). Accordingly, the market is said to exhibit a ‘full carrying charge’ when the price differential approximates the marginal cost of interest and physical storage. In a shortage, the marginal convenience yield may be larger than the sum of the other components and the market is then said to exhibit an ‘inverse carrying charge’, which is simply a negative price of storage. Clearly the availability of futures markets with cost of carry formed futures prices permits the storage decisions of firms and traders to be hedged, thus providing a reliable, predictable return to storage, which explains Working’s interpretation of the basis as ‘return to storage’.
2.2 DIGRESSION ON RISK PREMIUM AND EXPECTED SPOT PRICE

Brennan’s assumption about the supply of storage was that it reflects actual storage costs plus an appropriate expected return on the investment involved. The appropriate return clearly depends on the relevant risk of the position and thus the inclusion of a risk premium factor in the model. However, the inclusion of \( R^p_{t, T} \) as a factor in the net marginal storage costs and the existence of such a risk premium is a highly controversial, much-debated subject. In fact, in many studies of the theory of storage it does not appear as a factor at all. As noted in the Introduction, some economists, predominantly Working (1953b), Telser (1958) and Fama and French (1987), have separated the risk premium apparatus from the theory of storage entirely. A review of the literature suggests that this approach is highly appropriate, although such a conclusion can only be appreciated after an analysis of the second theoretical framework (see below).

For the present we note that, in general, for commodities trading on futures markets the risk premium is related to traders’ expectations of spot price changes, their risk aversion, their hedging/speculation decisions and to the forward pricing role of futures markets. However, its existence can be hypothesised only if one can establish a definitive relation between the expected price and the futures price. Consequently, central to the concept of a risk premium is the notion of an expected spot price (Telser, 1958; Edwards and Ma, 1992). The second theoretical framework of forecasting performance and risk premium posits a hypothesis of precisely such a link. Accordingly, after the discussion of this framework we shall return to Working’s theory of storage to illustrate the link between the two (see Section 4). It is nevertheless important to note the following at this point.

According to Brennan, Telser and French, the relevant price spread is that between the expected spot price and current spot price or, as hypothesised by Working, between successive futures prices. For commodities traded on futures markets, we could use the basis as the relevant spread, implicitly assuming the futures price to be an unbiased estimate of the expected spot price (via the second pricing framework), which implies no risk premium.

The inclusion of the risk premium as part of the price of storage, as done by Brennan (1958), French (1986), Chance (1991) and Leuthold et al (1989), could, however, be left out. Gray (1961, p.253) noted that “Brennan [made] what appears … to be a fatal error when he finally [verified] his finding of the risk premium by substituting futures prices for expected spot prices”. Working, Telser and Fama and French (1987 and 1988), on the other hand, did not incorporate the premium in their formulation and interpretation of the theory.

---

47 As we shall see in Chapter Four, the application of portfolio theory to futures markets fine-tunes this abstract concept of a risk factor by actually defining the relevant risk of the position and thus providing an explicit measure for the risk premium.

48 Gray (1961, p.253) pointed out that “[t]he same ‘risk premium’ is revealed after the substitution as before, which, contrary to Brennan’s reasoning, should not be the case, because the risk premium must be the difference between futures prices and expected spot prices”.
It is essential to understand that the ‘expected price’ differs significantly from the ‘futures price’ for the same time horizon. They differ because of the way in which they are formed. The actual price in the future is unknown; it can only be forecasted. The expected price for a future date represents only the most likely (rationally formed) market expectations at that moment in time and its formation necessarily depends on (among other things) on investors’ expected reward and attitude toward risk.

On the other hand, as explained in Chapter Two, the futures price for storable commodities is a cost-of-carry concept and its relation to the spot price can be determined by observing the trading opportunities open to arbitrageurs. Under such a theoretical framework, we need not know anything about investors’ preferences or attitudes toward risk; therefore the risk premium should not be a factor in estimating the futures price or in determining the price of storage (Siegel and Siegel 1990). For non-storable commodities, the futures price is not bounded by the cost of carry (that is, inventories and storage costs do not exist to bound prices together) and thus the differences between the expected future spot price and the futures price diminish.

2.3 EMPIRICAL EVIDENCE

The theory of storage is the dominant model of commodity futures prices. It is widely accepted and well documented. While most studies of the theory have used inventory data to test the hypothesis of a negative relationship between the marginal convenience yield and inventory levels, others have tested the implication of the theory as reflected in basis variability, which is in turn a function of the variability in carrying costs and, in particular, financing costs. Carrying costs studies are essentially tests of the theory of storage as it manifests in the basis and in the carry model. Numerous studies have empirically tested the theory for all agricultural commodities and are reviewed in Kamara (1982).

Fama and French (1988) have tested the theory for six industrial metals and for gold, silver and palatinum, working on the hypothesis that the marginal convenience yield declines at higher inventory levels but at a decreasing rate. The tests were carried out indirectly, however, based on the implication in the hypothesis that futures prices vary less than spot prices and that the variation of futures prices is a decreasing function of maturity. They noted that “the success of the theory in describing metals prices over the business cycle in our sample [1972 – 1983] is interesting because the predicted price behaviour is generated by general business conditions rather than by the harvest seasonals that motivated the development of the theory” (p. 1076).

In their 1987 study they also tested the theory of storage for storable and non-storable agricultural commodities and precious metals indirectly, according to two of its assumptions. The first was “that inventory seasonals generate seasonals in the marginal convenience yield and in the basis” (p.56), and accordingly they tested for seasonals in the basis as reflected in the basis’ standard deviations during different months and over various sub-periods. The second assumption was that, “controlling for
variation in the marginal storage cost and the marginal convenience yield, the $T - t$ period basis for any stored commodity should vary one-for-one with the $T - t$ period interest rate". Both methods have provided support for the theory of storage in precious metals. According to the first approach the authors found that metals had low basis standard deviations since they are not high-storage-cost commodities and are therefore not subject to high seasonals in supply and demand. Using the second method they found that "metals prices are consistent with the hypothesis that the basis tracks nominal interest rates" (p.61). Following their second empirical methodology, Page (1990) tested the theory of storage in short-term gold futures contracts over the period 1983-1988, the findings of which have also supported the theory of storage.

3. THE THEORY OF FORECASTING POWER AND THE RISK PREMIUM

This theory splits the futures price $F_{t,T}$ into a forecast of the future spot price $E(S_T)$ formed at time $t$, and an expected risk premium $E(RP_{t,T})$:

$$F_{t,T} = E_t(S_T) + E(RP_{t,T})$$

(14)

In contrast to the theory of storage, this view has long been debated. The debate may be divided into two interrelated theoretical questions. The first has to do with the forecasting ability of futures prices. The second concerns whether the expected premium is non-zero: that is, does a risk premium exist? We focus our attention primarily on the debate surrounding the risk premium and to a lesser extent on the forecasting performance of futures markets.

For many years, economists have been arguing whether futures prices represent the anticipation of delivery-date spot prices and, if so, whether they constitute unbiased anticipation. The expectation theory and the theory of normal backwardation support the anticipation or forecasting-ability hypothesis but differ in relation to the bias hypothesis. The expectations theory supports the unbiased hypothesis, while Keynes (1930) argued that under 'normal' conditions and if agents' expectations were correct, the futures price would be a downward-biased anticipation of the expected spot price, with the difference or bias equalling the marginal risk premium.

As noted earlier, the relationship between futures prices and expected spot prices is closely linked to the theory of storage. Therefore at the end of this section we draw the link between the two frameworks.

3.1 THE EXPECTATIONS HYPOTHESIS

The expectations hypothesis, also known as the 'hypothesis of unbiased futures price' or the 'no-risk-premium hypothesis', can be written symbolically as follows:

$$F_{t,T} = E_t(S_T)$$

(15)
The hypothesis was first postulated by Telser (1958, 1960, 1967\textsuperscript{49}) and Gray (1961), who contended that on average today’s futures price for delivery at \( T \) will accurately forecast the expected spot price at contract maturity. This theory corresponds with the price discovery role of futures markets, under which futures prices are viewed as forecasts, although technically they are much more than that.\textsuperscript{50} Evidently, one of the benefits of futures markets is the creation of uniform and publicly known prices regarding the future delivery of a commodity. Thus for many participants futures prices provide unambiguous information about the expected but unobservable future spot prices. It is widely held that if all available information, including the (rational) expectations of economic agents, is fully accounted for in the price formation process, current futures prices may be regarded as the market’s unbiased consensual expectation of subsequent spot prices. Equation (15) should therefore hold at all times.

This view corresponds to an equilibrium approach, which assumes risk neutrality, no uncertainty, homogenous expectations and thus perfect pricing efficiency. In such a world, hedgers will transact with other hedgers who hold contrary obligations, while speculators will be absent since futures prices conform with their expectation (that is, \( F_{t,T} = E_t(S_T) \)) and on average they will expect neither profit nor loss (zero return for speculation). Moreover, by entering the market the speculators will take on superfluous risk, since their expectation might prove incorrect, with no compensating return.

Empirical studies of this hypothesis investigate, in essence, the informational or forward pricing efficiency of futures markets and typically ask how well information is reflected in futures prices relative to current spot prices, including, for instance, storage and other carrying costs.\textsuperscript{51} Similarly, such studies would also examine the informational content of the basis. In general, empirical findings regarding the expectations hypothesis are contradictory. Yet according to many of the theorists, the overall empirical evidence shows that for most commodities the futures price is not an unbiased estimate of the spot price at maturity date. See Goss (1986); Kamara (1982); French (1986); Fama and French (1987); and Page (1990) for further details.\textsuperscript{52}


\textsuperscript{50} Futures prices are market determined prices which traders can buy and sell at and respond to in a production sense; they also reflect expectations and represent instant interpretation of information. Moreover they functionally relate to cash prices. Leuthold et al (1989, p. 107), noted that “as guides for allocating inventory, futures prices as forecasts may be self-fulfilling but in guiding long-run production they may be self-defeating”.

\textsuperscript{51} Detecting superior spot price forecasts in futures prices vs. current spot prices is one of the testable hypotheses in forward pricing studies. If all available information is fully reflected in the process of current spot price formation, then the best possible anticipation of the price relating to a later date is the current spot price and thus there may be nothing for the futures market to forecast. Hence, both are potentially valid forecasts. See also Stein (1961), Telser (1958), Goss (1981), French (1986) and Neal (1988).

\textsuperscript{52} Although direct tests of the expectations hypothesis lie beyond the scope of our research, it is important to note that the hypothesis is possibly jointly tested with the theory of normal backwardation. That is, an acceptance (rejection) of the backwardation and/or contango hypothesis could imply either directly or indirectly a rejection (acceptance) of the unbiasedness hypothesis and vice versa, depending on the specific hypothesis and test.
3.2 THE RISK PREMIUM (KEYNES' THEORY)

The risk premium was the king's raiment; if the king be espied naked, one observed that the king was wearing no clothes, and thereby reaffirmed the assumption that kings wear clothes. Read Kaldor (1961), who was clearly in doubt regarding the substance of the King's raiment, but who was not about to question the postulate of garmented kings. Better to anticipate the "see-through" garment than to acknowledge nudity (Gray and Rutledge, 1971, p. 68).

In contemporary economics of futures markets, the prevailing explanation for the rejection of the expectations hypothesis is the existence of a risk premium. However, it is important to note that chronologically the risk premium theory preceded the expectations theory as well as most other pricing theories. In fact, the risk premium theory was never a rejoinder to any theoretical apparatus but rather a catalyst that, primarily due to its shortcoming, ignited alternative models. The expectations theory, for example, was formulated as an (empirical) objection to the notion of bias. Similarly, Working's (1948) theory of inverse carrying charges was stimulated by the failure of the prevailing risk premium apparatus to explain observed futures price versus spot price behaviour like full carry and inverted market structures. As the above quotation demonstrates, the concept of a risk premium was so deeply entrenched that economists adhered to it (with the exception of Working) even when it failed to account for observed price and market participants' behaviour.

As noted before, the notion of a risk premium originated from Keynes' theory of normal backwardation (1930). This theory and its counterpart, the theory of normal contango formulated by Hicks (1946), are closely linked to the insurance view of futures markets as well as to the hedging-speculation dichotomy, which dominated early literature and largely overshadowed the price discovery contribution. This body of literature constitutes the traditional explanation for the differences between futures prices and expected spot prices. It is sometimes referred to as the hedging pressure explanation (Siegel and Siegal 1990), the hedging imbalance hypothesis (Edwards and Ma, 1992), or simply the theory of the risk premium.

3.2.1 NORMAL BACKWARDATION AND NORMAL CONTANGO

The theory of normal backwardation is a cornerstone in the literature of futures markets economics. Because its development occurred early in the evolution of the literature and its theoretical and empirical implications were so pervasive, it is hard to find an academic paper that did not - directly or indirectly - formalise, analyse or refer to it.

In previous sections we noted that the theory supplied the dominant theoretical links between (i) risk reduction or transfer role of futures markets, (ii) the assumed behaviour and market positions of hedgers and speculators, and (iii) futures prices formation and behaviour. More specifically, it postulated a

---

53 See Goss (1986) for other economic grounds sought to account for the rejection of the expectation hypothesis.
connection between net hedging, biased futures prices and speculative profits. Using these concepts and links, Keynes (1930) Hicks (1946) and Cootner (1960a, 1960b), to whom the overall theory is largely attributed, constructed a solid pricing model to explain the behaviour of futures prices in terms of an expected spot price and a risk premium. Since this model supports the notion of biased futures prices, it clearly competes with or provides alternative explanation to Telser and Gray’s expectations hypothesis.

Diagram 3 below summarises the relationship between the theories of normal backwardation, contango, and expectations or unbiased futures prices. It also shows the hypothesised relationship between futures prices, expected spot prices and the risk premium coupled with the assumed positions of hedgers and speculators - all of which are discussed next.

**Diagram 3: Relationships between the Futures Price and the Expected Future Spot Price**

```
<table>
<thead>
<tr>
<th>Hedgers</th>
<th>Futures Price</th>
<th>Hedgers</th>
<th>Futures Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Inclined to go short in</td>
<td>F&lt;sub&gt;T&lt;/sub&gt; = E&lt;sub&gt;T&lt;/sub&gt;(S&lt;sub&gt;T&lt;/sub&gt;)</td>
<td>(Inclined to go long in</td>
<td>F&lt;sub&gt;T&lt;/sub&gt; &gt; E&lt;sub&gt;T&lt;/sub&gt;(S&lt;sub&gt;T&lt;/sub&gt;)</td>
</tr>
<tr>
<td>the commodity. Hedge</td>
<td></td>
<td>the commodity. Hedge</td>
<td></td>
</tr>
<tr>
<td>against price decline)</td>
<td></td>
<td>against price increase)</td>
<td></td>
</tr>
<tr>
<td>e.g. Gold mine sells</td>
<td></td>
<td>e.g. Jeweler buys futures</td>
<td></td>
</tr>
<tr>
<td>Futures</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F&lt;sub&gt;T&lt;/sub&gt; &lt; E&lt;sub&gt;T&lt;/sub&gt;(S&lt;sub&gt;T&lt;/sub&gt;)</td>
<td></td>
<td>F&lt;sub&gt;T&lt;/sub&gt; &gt; E&lt;sub&gt;T&lt;/sub&gt;(S&lt;sub&gt;T&lt;/sub&gt;) - F&lt;sub&gt;T&lt;/sub&gt; &lt; 0</td>
<td>Negative risk premium (Normal Contango)</td>
</tr>
<tr>
<td>E&lt;sub&gt;T&lt;/sub&gt;(S&lt;sub&gt;T&lt;/sub&gt;) - F&lt;sub&gt;T&lt;/sub&gt; &gt; 0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Positive risk premium (Normal Backwardation)
```

**NORMAL BACKWARDATION**

Central to the theory of normal backwardation are the assumptions that (i) commercial firms are faced with financial risk arising from positive stockholding over time against changes in spot prices, which they wish to hedge; and (ii) that speculators have no forecasting ability. Under these assumptions, futures markets could be used as an insurance scheme in which risk-averse hedgers found speculators who would agree to underwrite the risks of spot price fluctuations in return for an insurance premium. Both Keynes and Hicks argued that in normal conditions hedgers would usually be long in the physical commodity and would thus hedge themselves against a price decline with offsetting short futures positions. This hedging activity would drive the speculators to take the opposite, long positions in the contract and thus on aggregate speculators would hold a net long position in futures contracts. Moreover, in order to entice the speculators to take the net long positions perennially, the hedgers would agree to sell the commodity forward (or hedge it by selling futures) at a price lower than the price expected to prevail in the spot market on contract maturity.

---

54 The diagram draws on Figure 1 in Page (1990, p.2)
Speculators would, in turn, be handsomely rewarded for their willingness to bear some risk, because they would be able to buy the commodity ‘low’ (paying the contract’s price $F_{t,T}$) and immediately sell it ‘high’ in the cash market collecting $E_d(S_T)$. Under normal backwardation the expected ‘built-in’ difference between $F_{t,T}$ and $E_d(S_T)$ yields a positive return for speculation or a risk premium, which Keynes estimated would be modest at 10% per annum (Keynes, 1930, p. 143). By implication, the theory also suggests that, since hedgers are normally net short, it will also be normal for $F_{t,T}$ to be less than $E_d(S_T)$ by the amount hedgers are willing to sacrifice to avoid risk, and speculators are rewarded for assuming it, or by the amount Keynes termed ‘backwardation’. As explained by Keynes, and symbolically represented in equation (16):

\[ [T]he \ quoted \ forward \ (futures) \ price, \ though \ above \ the \ present \ spot \ price, \ must \ fall \ below \ the \ anticipated \ future \ spot \ price \ by \ at \ least \ the \ amount \ of \ the \ normal \ backwardation; \ and \ the \ present \ spot \ price \ since \ it \ is \ lower \ than \ the \ quoted \ forward \ price, \ must \ be \ much \ lower \ than \ the \ anticipated \ future \ spot \ price \ (1930, \ p.144). \]

Backwardation $\Rightarrow$ \quad $F_{t,T} < E_d(S_T)$ and $E_d(S_T) = F_{t,T} + E(RP_{t,T})$ \hspace{1cm} (16)

Subsequent supporters of the theory contended that the futures market is (implicitly) a semi-efficient mechanism in which futures prices are downward-biased estimates of future spot prices due to the existence of a positive risk premium (Chang, 1985; Park, 1985). It is therefore ‘normal’ for futures prices to be unreliable estimates of spot prices at contract maturity.

Furthermore, since futures contracts are sold at a discount to the expected spot price, they should yield a rate of return higher than the risk-free rate (Copeland and Weston, 1988). The contract price will therefore rise (on average) during the life of the contract until it converges with the spot price prevailing at contract maturity (see Diagram 4).

NORMAL CONTANGO

The contango hypothesis is the inverse of normal backwardation:

\[ Contango \Rightarrow \quad F_{t,T} > E_d(S_T) \quad \text{and} \quad E_d(S_T) = F_{t,T} - E(RP_{t,T}) \] \hspace{1cm} (17)

In this scenario, the natural hedger is the purchaser of a commodity (for example, millers or jewellers), rather than the seller or supplier. The hedger, who is theoretically short in the physical commodity, would hedge himself by taking a long futures position, and thus on aggregate speculators would be net short. For the latter to take the short side of the contract it follows that $F_{t,T} > E_d(S_T)$. The long hedger agrees to pay the higher futures price in order to eliminate price uncertainty and for deferring delivery while the upward bias is the return for the speculator.

\[ 55 \text{ Keynes also detailed three other costs, namely deterioration, warehouse and insurance, but elaborated on the risk premium or risk cost (i.e. backwardation) in particular.} \]
It is crucial to note that both backwardation and contango hypotheses make no reference to the differential forecasting skills of traders. Risk premium is simply the average reward for the willingness to assume a risky position in a risk-averse world. Therefore the rewards for speculators should not be conditional on any superior judgment or inside information, which further implies that the risk premium must be net of any changes in the spot price level (Chang 1985). As noted by Gray and Rutledge (1971, p. 74), “if spot prices rise, carrying futures prices with them, a speculator might have forecasted the rise, but its gain is fortuitous from a risk premium standpoint”.

3.2.2 TERMINOLOGICAL VERIFICATION

An unfortunate but innocent contribution of Keynes and Hicks to the literature of futures markets arose from their terminology, which resulted in persistent subsequent confusion. In Table 3 in Chapter Two, we distinguished between ‘industry terminology’ and ‘Keynes-Hicks terminology’ without clarifying the evident inconsistency between the two. It was Gray and Rutledge (1971) who provided a clear explanation for this terminological issue. They noted that at the London Stock Exchange the term ‘backwardation’ was (and still is) used to describe the premium paid by the seller to the buyer for allowing him to defer delivery of the stock certificate, while ‘contango’ is the premium paid by the buyer to the seller for allowing him to defer acceptance of the certificate. They also noted that in commodity markets the British refer to a forward discount (or spot premium) as backwardation, while in America it is referred to as ‘inverse carrying charge’. Similarly, a forward premium is called contango in the United Kingdom and ‘carrying charge’ in the United States. They consequently concluded:

Keynes might better have referred to a risk premium component of a contango, rather than a backwardation component which entailed a contradiction in terms. He could then have said that markets sometimes reflect contango and sometimes backwardation, but that in either case a risk premium is a normal component of the difference between spot and forward prices (1971, p.65).

It seems that Keynes considered the risk premium (backwardation) to be a ‘normal’ part of the difference between futures prices and expected spot prices in both kinds of markets, carrying charge market or inverse carrying charge market. In other words, for Keynes the risk premium was an integral part of futures markets, irrespective of the difference between futures prices and the current spot prices. As also noted by Gray and Rutledge (1971, p.65) it was subsequently due to Kaldor that the term ‘normal backwardation’ was replaced with the concept of a marginal risk premium, thus “affording an escape from awkward and ambiguous terminology, which has unfortunately not been uniformly traversed in subsequent writings”.
3.3 REVIEW OF EMPIRICAL STUDIES OF KEYNES’ THEORY

Direct empirical tests of the normal backwardation hypothesis are somewhat limited, since ex ante expected spot prices of commodities are unobservable. Consequently, various indirect lines of empirical research have been taken to test the theory. The application of portfolio theory to futures contracts, for example, was originally suggested as one such test. Because of its nature, however, portfolio theory provides a very specific definition for the notion of a risk premium, which can then be subjected to direct empirical testing. In contrast, early empirical studies analysed futures prices in isolation from other asset markets and the existence of a risk premium was only indirectly inferred in terms of the assumptions of Keynes’ theory and their implications.

Three assumptions regarding speculators’ behaviour were originally made in order to motivate the risk premium and to contend that the profit flow between hedgers and speculators is analogous to the flow of insurance premium or a reward for risk bearing and not for forecasting skills. The three assumptions were that speculators: (i) had to be risk averse; (ii) held net long positions (that is, there was no variation in the level of hedging); and (iii) were unable to forecast futures prices (Rockwell, 1967, Gray and Rutledge, 1971; Chang, 1985).

By implication, from an empirical testing standpoint, all three assumptions will be met if \( F_{LT} \) is a downward biased estimate of \( E(S_t) \). Accordingly, the futures price would tend to rise, on average, over the life of the contract (under normal conditions or no shortage) until it converges with the spot price at expiration. With normal contango (under inverted market conditions), where \( F_{LT} > E(S_t) \), at contract’s maturity the futures price should, on average, drift downwards.

The hypothesised behaviour of the futures price relative to the expected spot price, assuming the expected spot price remains unchanged during the life of a hypothetical future contract, is summarised in Diagram 3. It also demonstrates the assumed positions of speculators in response to hedging imbalances (the second assumption) as discussed below.

**Diagram 4: The Futures Price of a Storable Commodity, the Risk Premium and Traders’ Positions in the Context of Normal Backwardation, Contango, Expectations and Net Hedging Hypothesis**
A second implication of all three assumptions is that, by merely holding a long position in futures over a sufficiently lengthy period (as a response to continuing short selling of contracts by hedgers against their inventory positions), speculators would earn significant profits. Similarly, with long hedging (contango), continuous short speculation would be profitable: speculators would theoretically reap a risk premium by being consistently on the opposite side of the market from hedgers.

In two related studies, accepting all three assumptions, Telser (1958, 1967 in fn. 49) tested the existence of an upward trend in cotton and wheat futures prices as they approach maturity as well as the returns to speculators who maintained long positions in wheat corn and soybeans contracts. He found no such trend and no systematic positive return to long speculation. This finding led him to reject the normal backwardation theory and to formulate the unbiased futures prices hypothesis. It is important to note, however, that he did agree with the ‘short hedging - long speculation - risk transfer’ paradigm but contended that the number of potential speculators would be sufficiently large so that any bias would disappear. In other words, he did not formulate his hypothesis in the context of the forward pricing role or speculators’ forecasting ability.

Cootner (1960a, 1960b) disagreed with both Telser’s finding and his conclusion. Looking at wheat futures over the same sample period, he did find evidence for an upward trend in prices. He further criticised Telser’s adoption of the second assumption, instead recognising that short hedging does not predominate at all times for all commodities and that speculators can still obtain a risk premium even if prices do not rise on the average (see net hedging in Diagram 3). According to Chang (1985), Cootner showed it to be possible for speculators to profit merely by being long after the peak of net short hedging and by being short after the peak of net long hedging. Kamara (1982) concluded that while there is evidence of trends in futures prices that can be associated with hedging patterns, these trends do not by themselves prove that a risk premium exists. We tend to agree with this observation, as the following evidence suggests.

Houthakker (1957) and Rockwell (1967) recognised that both assumptions two and three should be relaxed simultaneously. They examined the actual profit flow of various categories of traders, for instance, large reporting speculators and large hedgers compared with small non-reporting traders. Houthakker concluded that his sample of speculators did earn profit, and even ingeniously estimated the share of profits attributable to risk premium versus that relating to forecasting skill (Chang, 1985). Nevertheless, Houthakker’s results were now challenged by Telser (1958) for not deducting transaction costs from speculators’ income and for using a sample of unstable prices to confirm a bias.

Rockwell, using Houthakker’s methodology, further emphasised the need to distinguish between rewards for taking risk, which he tested by assuming a hypothetical speculator with a naïve strategy of simply holding the contrary position to (net) hedgers, and rewards for forecasting skills, which he confirmed by the failure of the naïve strategy and by analysis of the profit flow for large speculators
relative to the time period of their position.\textsuperscript{56} He found no support for normal backwardation (as represented by the naive strategy), but did find evidence supporting speculators' forecasting skills. Further studies regarding the flow of profit to traders are summarised in Kamara (1982). Gray (1961) challenged the findings of both Houthakker and Cootner and presented evidence of price trends that varied between major agricultural futures markets but bore no relation to the level of risk as measured by the variability (or standard deviation) of futures prices themselves. He therefore suggested that bias need not be interpreted as the transfer of a risk premium but could be explained in term of the structural characteristics of a particular market or what he termed a \textit{characteristic bias}.\textsuperscript{57}

Gray therefore rejected the view of Keynes, Cootner, and Houthakker of a price bias as a manifestation of the risk premium and instead supported Telser's hypothesis of 'unbiasedness'. He also emphasised the concept of 'market balance', which refers to a market with sufficient volume of speculation to respond to any hedging imbalances. A considerable amount of empirical work was subsequently devoted to the detection of a positive or a negative risk premium in various futures markets and to other explanations of bias. The work was carried out primarily in agricultural commodities. The results are inconclusive at best. Kamara (1982, p.273) has concluded as follows:

\begin{quote}
While it is widely accepted that futures markets are used by risk-averse hedgers, the evidence suggests that they have been able to purchase the insurance very cheaply, so that on average futures prices do not contain a significant risk premium. Analysis of the returns to speculators showed that speculation should be regarded as a skilled occupation and not merely as risk bearing, in which the returns vary greatly with the ability and knowledge of the speculators.
\end{quote}

A more recent study by Park (1985) re-examined the theory of normal backwardation in metal contracts, including gold, silver, platinum and copper. In sharp contrast to the studies detailed above, Park based his tests on the general equilibrium valuation models of forward and futures prices discussed in Chapter Two and Appendix B. His testable hypothesis was formulated in the context of utility-based models for the formation of futures and forward prices and the relationship between forward, futures and expected spot prices. While these models are beyond our scope, we nevertheless note that Park's results were consistent with the theory of normal backwardation (as implied by: $E_t(S_T) > F_{t,T} > f_{t,T}$) for all metals contracts. Park also referred to a 1982 study by O'Brian and Schwarz that supported the normal backwardation hypothesis for gold by comparing implicit expected spot prices in gold options with gold futures prices.

\textsuperscript{56} See Rockwell, p.156 and Houthakker, p.144.
\textsuperscript{57} See Gray and Rutledge (1971, p.75-76) and Kamara (1982, p.273-274) for other causes of bias and further analysis.

69
4. WORKING vs. KEYNES – THE LINK BETWEEN THE TWO THEORIES

Most of the studies reviewed above merely examined the statistics as an empirical test of the Keynes theory. None of them has actually provided an alternative theory that can accommodate observed contradictory facts. Clearly, conflicting empirical evidence does not render a theory invalid, unless a better theory that also accommodates statistical evidence can be put forward.

In this sense the approach of Working (1948, 1949a, 1949b) to the problem and his more general interpretation of spot-futures pricing relationships may be regarded as superior to that of his contemporaries. First, his framework was capable of incorporating other pricing elements in addition to the risk premium as the facts might warrant - for instance, convenience yield. It was thus also capable of explaining observed price behaviour (including the entire range from full carrying charges to steep inversion) that was not explained by the Keynesian approach. In contrast to the proponents of both normal backwardation and expectations theory, Working disliked the interpretation of futures prices as predictors of subsequent spot prices. As noted by Gray and Rutledge (1971, p.70), he rejected as inadequate those explanations that viewed prices at two different times as being separately determined. He favoured the view that intertemporal price spreads (positive or negative), or the basis, reflect prices of storage between the relevant dates, while new information would be expected to affect the whole spectrum of prices.

Nevertheless, as noted by Goss (1986, p.5), Working did believe that all prices are forecasts in the sense that they all embody information including expectations, and therefore both the futures prices and expected spot prices reflect carrying charges. This is partly the reason why the literature of futures markets has also interpreted the forward pricing function of futures markets as an extension of his 'price of storage' concept (see Goss, 1981). It also explains the frequent substitution of futures prices for expected spot prices and the interpretation of the basis as the price of storage.

In this context, Fama and French (1987), in comparing and testing the theory of storage and the theory of forecast power risk premium, have formulated the following two equations which express the basis:

\[ F_{t,T} - S_t = S_t R_{t,T} + W_{t,T} - Y_{t,T} \]  \hspace{1cm} (18)

\[ F_{t,T} - S_t = E_t[(S_T) - S_t] + E(RP_{t,T}) \]  \hspace{1cm} (19)

They noted that expression (19) and the theory of storage in (18) are alternative but not competing views of the basis; “variation in the expected premium or the expected changes in the spot price [in (19)] translate into variation in the carrying costs” (p.62). For example, a positive basis for metals (where inventories are typically high) can be explained in terms of interest and storage costs that outweigh marginal convenience yields, but a positive basis can equally be explained in terms of an expected increase in the spot price necessary to induce storage.
Chapter Four
Applying Modern Portfolio Theory to Futures

1. INTRODUCTION

The futures pricing models presented in Chapter Three evaluated the price of a futures contract in relation to the spot price, expected spot prices, returns to storage and hedging and speculation uses. The risk of the futures contract was identified by looking at the fluctuations in futures prices in an isolated futures market framework. As the theoretical and empirical review indicated, in an examination of futures markets on a stand-alone basis the definitions and measurements of a risk premium are implicit, at best, in the various pricing models and very imprecise.

Furthermore, the theories regarding the roles and motives of market participants ignored the risk-shifting opportunities available in capital markets as a whole. Clearly, from the viewpoint of a portfolio manager, not only does risk have to share the stage with returns, but it is also more appropriate to evaluate the risk and expected returns of futures contracts relative to the expected returns on other assets in a portfolio. Such an alternative ‘portfolio approach’ to futures pricing would evaluate the futures contracts in an equilibrium situation within the larger capital market framework. In such a wider market the futures market is only one of many candidates competing for a place in a portfolio.

The inclusion of futures contracts in a portfolio will be governed by several factors: (i) the risk-return profile of the underlying spot commodity and, closely linked to this, the risk-return profile of the contract; (ii) the correlation of futures contracts with other asset classes (shares, bonds and cash), with the market portfolio and with inflation; and (iii) their unique trading characteristics, such as transaction costs, liquidity and short selling, amongst others. If futures contracts offer superior risk-adjusted returns and diversification benefits which can reduce the overall risk of the portfolio, an investor should consider including them in his portfolio.

2. THE THEORY OF SYSTEMATIC RISK IN FUTURES MARKETS

The risk emphasised so far is the price risk incurred by commercial participants due to (routine) inventory holding over time, which is managed by transacting with speculators on futures markets. The two traditional hypotheses - expectations and normal backwardation - assume the existence of a mass of speculators who are willing to enter either side of the futures market if it compensates them sufficiently for the price risk they incur.

Modern portfolio theory (MPT) fine-tunes this approach by refining the notion of risk used in the determination of risk premium. The risk in a portfolio context concerns the variability of the asset’s
expected returns, while MPT expands the concept of ‘risk’ (as measured by the standard deviation of the asset’s returns) by dividing it into two component: systematic risk and unsystematic risk. Markowitz’s Portfolio Model and its extension, the Capital Asset Pricing Model (hereafter CAPM) devised by Sharpe (1963, 1964) and Lintner (1965a, 1965b), are the prime models used to analyse the risk and return profiles of individual assets and classes of assets.

Katherine Dusak from Chicago University was the first to test the implication of the theory of normal backwardation in the context of the CAPM setting. She introduced the subject matter as follows:

_This paper offers another and quite different interpretation of the returns to speculators in futures markets. It is argued that futures markets are no different in principle from the markets for any other risky portfolio assets. Futures markets are perhaps more colourful than many other sub-segments of the capital market, and the terminology of futures markets is perhaps more arcane, but these differences in form should not obscure the fundamental properties that futures market assets share with other investment instruments: in particular, they are all candidates for inclusion in the investor’s portfolio (1973, p.1388)._

Keynes’ analysis assumed that hedgers must pay other agents - the speculators - an (implicit) monetary premium as an incentive to trade futures. By contrast, within an integrated set of competitive asset markets, when the standard ‘risk-return decision process’ applies, hedgers need not pay any premium to induce other agents to trade. Instead the existence of a risk premium, if it indeed exists, depends upon the covariance between the payoffs to the hedge asset and the economy-wide risk faced by all agents. If the hedge asset is not correlated with market fluctuations, it will carry no risk premium, although it may still offer value to a particular hedger owing to its specific income stream or superior contractual characteristics. Only a hedge asset that protects the hedger against market risk will carry a risk premium - or compensation for systematic risk that cannot be diversified way.

In contrast to normal backwardation and contango the CAPM does not, by itself, stipulate the sign or the size of the risk premium. It hypothesises that returns on any risky asset, including futures, are a function of the asset’s contribution to the risk of an aggregate and well-diversified portfolio of assets or the so-called ‘market portfolio’. Only this systematic risk, measured by the asset beta (defined later), will command a risk premium in the market, since investors should not be rewarded for risks that can be diversified away.
3. MODERN PORTFOLIO THEORY

It seems clear that investors in futures markets bear risk. Investors with long or short positions are exposed to profits and losses arising from changes in the futures prices. Furthermore, the behaviour of futures prices is clearly highly correlated with the underlying commodity spot price. Like commodities' spot prices, futures prices are highly sensitive to changes in the business cycle, inflation expectations and changes in interest rates. If we assume this risk to be systematic, the well-known general equilibrium CAPM equation can be used to measure it. Before applying this equation to futures contracts, we briefly review the development of the CAPM, its theoretical implications and its empirical application.

3.1 THEORETICAL REVIEW OF THE CAPITAL ASSETS PRICING MODEL

The 'mean-variance model of investor behaviour' developed by Markowitz (1952) laid the foundation for the CAPM. His was the first attempt to model investor's portfolio choices rigorously in terms of expected (or mean) returns and the variance of returns (that is, risk) of alternative combinations of assets in a 'single-period' world. His contention was that investors would optimally hold a mean-variance efficient portfolio, defined as a portfolio with the highest expected return for a given level of variance (or with lowest variance for a given level of expected return).

Markowitz's most notable contribution to finance and economic theory was the conceptualisation of the notion of risk. For the first time a sound definition and vigorous statistical measurement of the risk associated with individual assets and with portfolios comprising different combinations of such assets was put forward. For individual assets, risk represents the likely dispersion of actual returns about their mean value and it can thus be quantitatively measured by the variance (or standard deviation) of returns as derived from probability theory. For a portfolio of two or more assets, risk is measured by the weighted sum of the variances of each asset adjusted for the co-variability of each asset's returns with the returns of all other assets included in the portfolio (see Copeland and Weston, 1988, p.153-161).

Sharpe (1963, 1964) and thenLintner (1965a, 1965b) significantly extended Markowitz's framework and overcame some of its shortcomings. The combined work of these two distinguished economists came to be known as the CAPM, which became the premier theoretical equilibrium model of asset prices. Despite the development of theoretically superior and more general intertemporal asset pricing models, the single-period CAPM is still widely used by investment practitioners and academic alike.

58 The discussion is based on Markowitz (1952); Sharpe (1963 and 1964); Copeland and Weston (1988); Blake (1990), and Campbell, Lo and Mackinlay (1997).
59 The Markowitz model was based on several assumptions, the most noteworthy being the normal distribution of the assets' returns. This is a necessary condition for the variance to be a sufficient measure of risk (see also Data Description and Methodology, Section 4.1).
60 The statistical measures of risk and return for a single asset are given in Appendix C.
Initially, Sharpe's 'Single Factor Model of Securities Returns' greatly simplified the computational complexity and consequently the practical implementations of Markowitz's portfolio selection techniques. Sharpe's key assumption was that assets' returns are related only through a common relationship with some 'basic underlying factor'. This assumption not only eliminated the laborious calculation of the Variance-Covariance Matrix between all assets in a portfolio, but also enabled Sharpe to derive the following model, which expresses the \textit{ex post} assets' returns as being determined solely by random factors and this single underlying factor,

\begin{equation}
R_i = \alpha_i + \beta_i R_m + \epsilon_i
\end{equation}

where \( R_i \) is the return on any asset \( i \), \( R_m \) is the return on some broad market index over a specified period of time and \( \epsilon_i \) is a random variable or the error term with a mean of zero and a constant variance for all \( R_i \) values. (That is, the classical linear regression assumptions hold.) The equation's parameters describe the underlying relationship between the asset return and the return on the index; specifically, \( \alpha_i \) represents the asset's return when the return on the index is zero and \( \beta_i \) represents the change in \( R_i \) for a unit change in the index. Equation (20) is sometimes called the \textit{market model}, or the \textit{characteristic line} of the \( i \)th asset, and has all the characteristics of a regression equation.

This simple model of assets' \textit{ex post} returns laid the groundwork for new theoretical and empirical concepts and ultimately to the CAPM equation. A few issues deserve a brief closer look. First, through its focus on the nature and source of an asset's returns, the model demonstrated that these returns can be broken down into two separate components: (i) the so-called specific or unsystematic component, as represented by \( \alpha_i + \epsilon_i \), which is uncorrelated with any exogenous (common) factor and is solely a function of the asset's unique characteristics; and (ii) the so-called systematic component, as represented by \( \beta_i R_m \), which depends on the correlation with the market index. This exposition of an asset's 'return characteristics' provided a rigorous framework for quantifying the asset-specific (or diversifiable) risk and its market-related (undiversifiable) risk, derived through a similar and interrelated analysis of the variance of \( R_i \). It also provided the theoretical rationale for the benefits of diversification.

Secondly, by implication, the single factor model required that an index be incorporated as a separate variable in the formulation of the problem of portfolio selection, which therefore required the restatement of the problem of portfolio analysis. This (unobserved) single factor, which was to represent what came to be known as the 'market portfolio', was regarded by Sharpe (1963, p.281) as the most important single influence on expected returns from all asset. Although this market portfolio was not strictly specified and therefore opened the door to empirical adaptability, it was clear that it had

\footnotesize{\textsuperscript{62} See explanation following equation (26) \textsuperscript{63} See Copeland and Weston (1988) for further details.}
to represent some measure of total accumulated wealth: that is, a portfolio containing all existing assets (broadly defined) weighted according to their outstanding proportions in a portfolio of total wealth.  

Thirdly, while the model could be used to obtain values for the parameters from historical data, it also lent itself to adaptation as an expectational model, which was not only necessary but also proved to be highly useful in Markowitz's portfolio selection context. While the Markowitz model could only identify the 'frontier' of an efficient risky portfolio in an expected return-variance space, it could not select a unique optimal portfolio.

With the introduction of a risk-free asset, and assuming that investors can borrow and lend at a 'pure' risk-free interest rate, Sharpe and Linnter provided an ingenious solution to this selection problem. They showed that if investors have homogeneous expectations and optimally hold mean-variance efficient portfolios, in the absence of market frictions, the portfolio of all invested wealth - the market portfolio - would itself be a Markowitz mean-variance efficient portfolio.

Moreover, the (linear) combination of this (and only this) portfolio with the riskless asset would yield the Capital Market Line (CML), along which every investor's optimal (mixed) portfolio would be formed in accordance with the expected return-variance rule. All other risky portfolios, when combined with the riskless asset, would fall below the CML and result in sub-optimal returns. The precise combination (or weighting) of the market portfolio and the risk-free asset will depend on the investor's degree of risk aversion and is thus a 'choice' variable which will determine the expected return-risk on such a hybrid portfolio.

Although this examination of the CML is incomplete, we can safely note that it is essentially a display of capital market equilibrium, portfolio optimality and the pricing of portfolios.

The Sharpe-Lintner version of the CAPM equation, stated below, is a model for the pricing of individual risky assets and implies the totality of the CML and the characteristic line. In the context of portfolio and asset pricing - as prescribed by Markowitz and depicted in the CML - not only do the

---

64 Black (1976, p.172) noted that the market portfolio includes corporate securities, personal assets such as real estate, and assets held by non-corporate businesses. He stressed that to the extent that corporations hold stocks of commodities, commodities are implicitly included in the market portfolio, whereas to the extent that they are held by individual and non-corporate or private businesses they are explicitly included in that portfolio. We will refer to this important observation in Section 4 below and Chapter Five.

65 The concept of a 'risk-free' asset or investment is, in a sense, notional as all investments are subject to some risk, for instance default risk or liquidity risk, for which extra compensation will be required by investors over and above the time value of money or the real rate of return. Furthermore, the assumption of equal borrowing and lending rates at the riskless rate casts a shadow over the CAPM's usefulness as a representative model.

66 For further details on the CML see Blake (1990), Campbell et al (1997), and Copeland and Weston (1988).

67 A complete discussion of capital market equilibrium and a thorough derivation of the CAPM should further include a review of (i) the dynamics of capital market equilibrium, (ii) the implication of the model's assumptions and (iii) the intervening theoretical and empirical contribution of many eminent economists - the sum of which resulted in modern portfolio and investment analysis.
risk-return co-ordinates for individual assets fall below the CML, because of their sub-optimality, but also no consistent (linear) relationship between the asset's expected return and risk is observable.

For equilibrium pricing to prevail, however, this inadequacy of the CML cannot suggest that the pricing of individual asset should not be a function of risk and should not conform to the expected returns-risk trade-off. It is the CAPM that provides the necessary extension of equilibrium risk-return pricing to individual assets, by further incorporating the theoretical implications of the characteristic market line. As suggested above, the only relevant risk to individual assets is that which is related to the single external factor - the market index - and is therefore undiversified. Therefore, in the context of the CAPM extension it is the relationship between the asset's expected return and its relative risk - as measured by the beta coefficient in the single index model - that is important to risk-averse, rational investors and determines efficient asset pricing. This relationship can be captured in a discernible and consistent linear relationship termed the security market line (SML), which corresponds directly with the CAPM.

There are two important implications of this analysis. First, in equilibrium all assets would be priced so that they lie on the SML, while all efficient portfolios would be priced so that they lie on the CML. Inefficient portfolios would not lie along the CML but would still be priced to reflect only the undiversifiable risk that they contained. Secondly, given the existence of a risk-free asset, the market or the equilibrium price of risk is the excess return - or the risk premium - relative to the market, over and above the risk-free rate.

The well-known Sharpe-Lintner version of the CAPM (the 'usual' equation of assets pricing), which captures the essence of all these 'intermediate-models' above, holds that in equilibrium the expected return on any asset should conform to the following relationship:

\[
E(\tilde{R}_i) = R_f + \beta_i [E(\tilde{R}_m) - R_f]
\]

\[
\beta_i = \frac{\text{Cov}(\tilde{R}_i, \tilde{R}_m)}{\sigma^2(\tilde{R}_m)}
\]

where \( \tilde{R}_i \) is the random rate of returns on asset \( i \), expressed as a percentage change (or typically the change in the log value) of the initial investment, \( R_f \) is the risk-free rate of interest, usually estimated as the returns on short-term interest-bearing security, and \( \tilde{R}_m \) is the random rate of returns on the market portfolio or on total wealth, typically approximated by a broad stock market index or some measure of total consumption or gross domestic product. \( E(\tilde{R}_i) \) and \( E(\tilde{R}_m) \) are the respective expected returns. \( \beta_i \) represent the marginal contribution of asset \( i \) to the variability of returns of the market portfolio.

Equation (21) holds that in equilibrium the expected return on any asset \( i \) will be equal to the risk-free rate of interest plus a risk premium proportional to the contribution of the asset to the variability of
returns on total wealth. It further implies the following general conclusions about the required (risk-adjusted) rate of returns on an asset if it is to be held in equilibrium:

If \( \beta = 0 \) asset \( i \) is not a risky asset and \( E(R_i) = R_f \)

(An asset whose returns have a zero covariance with the return on the market is not risky, consequently its expected return \( E(R_i) \) is only equal to that of a risk-free asset. Changing its proportion in the market portfolio would not affect the variability of returns on the portfolio).

If \( \beta > 0 \) asset \( i \) is a risky asset and \( E(R_i) > R_f \) (known as positive-beta asset).

If \( \beta < 0 \) asset \( i \) can reduce the portfolio risk and \( E(R_i) < R_f \) (known as negative-beta asset).

If \( \beta > 1 \) asset \( i \) is more volatile than the market and \( E(R_i) > E(R_m) \) (known as aggressive asset).

(An asset whose returns have a high covariance with the return on the market is risky, consequently it has a positive expected return \( E(R_i) \) since increasing its proportion in the market portfolio increases the variability of returns on the market portfolio).

If \( \beta < 1 \) asset \( i \) is less volatile than the market and \( E(R_i) < E(R_m) \) (known as defensive asset).

If \( \beta = 1 \) asset \( i \) has the same risk as the risk on the market and \( E(R_i) = E(R_m) \).

The single-period CAPM has been the most widely tested general model of asset pricing under the condition of uncertainty. It makes the very strong prediction that in equilibrium the expected excess returns across assets is proportional to their betas relative to the market portfolio.

### 3.2 BRIEF EMPIRICAL REVIEW

The Sharpe-Lintner version of the CAPM can be concisely expressed in terms of the \( i \)th asset's returns in excess of the risk-free rate or simply in terms of excess return. In other words, if \( \tilde{Z}_i = E(\tilde{R}_i) - R_f \), the result is:

\[
E(\tilde{Z}_i) = \beta_i E(\tilde{Z}_m)
\]

\[
\beta_i = \text{Cov}(\tilde{Z}_i, \tilde{Z}_m) / \sigma^2(\tilde{Z}_m) \tag{23}
\]

Because, theoretically the risk-free rate is treated as being non-stochastic, equations (22) and (24) are equivalent. In empirical research, proxies for the risk-free rate are stochastic and thus the betas can differ. The impact of the use of proxies will result in a higher risk-free rate and smaller risk premium than the model suggests. Most of the empirical studies related to the Sharpe-Lintner CAPM model employ excess return and thus use (23) and (24). Such tests focus, amongst other things, on two main implications of (23): that the intercept is zero, and that the market risk premium is positive.\(^68\)

\(^68\) Another important implication of the CAPM is that the beta completely explains the cross-sectional variation of expected excess returns. This implication is rooted in the ‘SML view’ of the CAPM, or the so-called ‘cross-sectional regression approach’ developed by Fama and MacBeth (1973) which assumes a linear relation between expected returns and market betas as the only explanatory variable. Since these betas are unobservable, scholars have relied on Sharpe’s market model in (20) to estimate the beta on the \( i \)th asset and then run a regression for each cross-section of \( N \) assets. See Copeland and Weston (1988), and Campbell et al (1997) for more details.
The CAPM is a single-period model and hence (23) (and 21) do not have a time dimension. For an econometric analysis of the model, it is necessary to add an assumption about the time-series behaviour of returns over time in order to estimate the model over time. The assumption adopted for that purpose is that returns are independently and identically distributed through time and are jointly multivariate normal. Campbell, Lo and MacKinlay (1997, p.183) noted that while the assumption is strong, and some departure from it has been observed, "it has the benefit of being theoretically consistent with the CAPM holding period by period" and was adopted mainly for empirical purposes.

To test the validity of the CAPM model in equation (23) empirically, it is necessary to transform it from an ex ante (or expectational) form into an ex post version using observed data. As noted by Copeland and Weston (1988, p.212) this could be done by assuming that the rate of return on any asset is a fair game. That is, on average, realised return is equal to the expected rate. The ex post equivalent of (23) is:

\[ Z_t = \beta_t Z_m + \varepsilon_{it} \]  

(25)

Its empirical counterpart, which can be estimated using normal linear regression model, is

\[ Z_{it} = \alpha_i + \beta_i Z_{mt} + \varepsilon_{it} \]  

(26)

where \( Z_t \) and \( Z_m \) are the realised rate of returns on the \( i \)th asset and the market respectively. \( \alpha_i \) and \( \beta_i \) are the regression parameters both expressed in a risk premium form. \( \alpha_i \) is the intercept term assumed to be equal to zero and \( \beta_i \) measures the systematic risk, or the excess return on the asset relative to that on the market, and is loosely referred to as 'the risk premium co-efficient'. It could be negative, positive or equal to zero, the implications of which were discussed above. \( \varepsilon_{it} \) is a random error term assumed to be normally distributed with \( E(\varepsilon_{it}) = 0 \) for all \( t \); \( \text{VAR}(\varepsilon_{it}) = \sigma^2 \) a constant for all \( t \); \( E(\varepsilon_{it}, \varepsilon_{jt}) = 0 \) for all \( t \) and \( E(R_{mt} \varepsilon_{it}) = 0 \). These so-called 'classical assumptions', which form part of the Gauss-Markov Theorem, are required in order to derive the 'best linear unbiased estimators' of the true (or population) regression parameters in (26) when using ordinary-least-squares estimation techniques.\(^{69}\)

Voluminous research regarding tests of the CAPM, primarily with respect to equity, has been carried out to date. Almost every assumption used in the derivation of the original model has been challenged, reformulated and empirically examined. Our concern, however, is with those studies that have applied and tested the Sharpe-Lintner CAPM version in futures contracts only.

\(^{69}\) We note, however, that Roll (1977) criticised the use of equation (26) for measuring performance, arguing that the CAPM could never be satisfactorily tested since the true market portfolio is unobservable. According to Roll, any empirical test of the CAPM using ex post returns would only be a test of the chosen market portfolio proxy. However, as noted by Campbell et al (1997, p.214), later studies considered a number of broader proxies (broader, for instance, than the widely used S&P500 Index) and showed that inferences are similar whether one uses stock-based proxy, a stock- and a bond-based proxy, or a stock- bond- and real-estate-based proxy. Campbell et al concluded that "[t]his suggests that inferences are not sensitive to the error in the proxy when viewed as a measure of the market portfolio and thus Roll's concern is not an empirical problem". We will refer to the portfolio proxy issue in Chapter Five, Section 2.
4. THE ‘FUTURES-CAPM’ RE: DERIVATION AND INTERPRETATION

The theoretical and empirical application of Sharpe-Lintner CAPM equation to futures contracts in general, and to the hypothesis of normal backwardation in particular, involves several conceptual and computational complications. These complications have been progressively addressed by the literature. We noted earlier that the market model was originally applied by Dusak to test the Keynesian view of a price bias as a manifestation of a risk premium. The empirical progression of what we termed the futures-CAPM can be characterised by (i) the extent to which the assumptions regarding speculators’ behaviour were initially adopted and subsequently relaxed and (ii) criticism of the specifications of Dusak’s original model’s and suggested modifications. The model also depends on the assumptions underlying the CAPM itself. The principal tenets of the futures-CAPM can be best examined within the following framework, which focuses on the theoretical and empirical interpretation and application of the CAPM to futures and systematically lead to our research hypothesis:

1. The notion and computation of returns on futures contracts
2. The theoretical derivation of the futures-CAPM equation
3. The risk premium hypothesis: CAPM vs. traditional theories
4. The model’s specifications: empirical review of the futures-CAPM
   (a) The appropriate market benchmark
   (b) Changing speculative position - Stochastic systematic risk

The model’s specifications, combined with a review of previous empirical studies and the methodology used, are discussed in Chapter Five.

4.1 THE NOTION AND COMPUTATION OF RETURNS ON FUTURES CONTRACT

The first problem encountered by scholars in applying the CAPM to futures was that of defining the appropriate ‘capital asset’ and its expected rate of returns. First, as argued by Black (1976), unlike stocks of commodities, commodities futures are not included in the market portfolio, since commodities contracts are like pure bets: that is, for every short position there is a matched long position and if all contracts are taken together, they net out to zero. Baxter Conin and Tamarkin (1985, p.124) also noted that “given that futures contracts should not be included in the market portfolio ...[the question arises] whether futures contracts can legitimately be regressed against the market portfolio”.

Secondly, as noted in Chapter Two and further explored in Appendix B, unlike most assets, the price of a futures contract is not analogues to its value. This poses a ‘problem’ for the calculation of rate of return under the CAPM paradigm, since the initial value of the contract (and the value at the end of each day) is zero. Moreover, since holders of futures contracts do not invest any capital in the contract, the percentage change in the futures price cannot be interpreted as ‘rate of return’.

---

70 See also footnote 64.
In addition, as noted by Dusak (1973) and Bodie and Rosansky (1980), the margin posted by the investor cannot constitute the initial capital investment required in the calculation of rate of return (that is, to calculate \( E(R) \) as the ratio of the net profit at close-out to the initial margin). The reason for ruling out this appealing computation is that the margin on futures, as explained in Chapter One, is merely a good faith deposit or a 'performance bond'. It is not transferred from buyers to sellers but simply kept by the broker until the positions are closed, and it is therefore not a meaningful number for the CAPM and need not even exist if other guarantees were in place.

None the less, zero net investment does not imply a riskless position in futures markets. The variability in futures prices implies that investors with positions in futures markets do bear risk even though the value of their positions at the end of each day is zero. Pricing in a CAPM context should therefore still be appropriate but subject to some modification in the CAPM equation, owing to the uniqueness of futures contracts.

Two such modifications have been suggested. The first, recommended by Bodie and Rosansky (1980), is to compute returns under the assumption that an investor uses Treasury bills to post a 100% margin. The second, recommended by Dusak (1973), Black (1976), Scholes (1981) and So (1987), is to rewrite the CAPM equation using the spot commodity prices or in absolute futures price changes rather than percentage returns as outlined below (see also Methodology, Section 2). As noted by So (1987, p.313), "even though the investor's investment in the position is zero, the position may have a positive or negative expected dollar return". We will use the second method, since it is more realistic and consistent with the majority of empirical tests written to date.

4.2 THEORETICAL DERIVATION OF THE 'FUTURES-CAPM' EQUATION\(^\text{71}\)

Underlying the following derivation of a modified CAPM equation, which is applicable to futures contracts prices, is the observation that while the expected return on asset \( i \) [i.e. the \( E(\widetilde{R}) \) in the \textit{ex ante} CAPM equation] cannot be interpreted as the percentage change in the futures price, it can easily be interpreted as the return on the (unhedged) spot commodity underlying the futures contract and which price is closely correlated with the futures price.

Assuming no transaction costs and no basis risk, we can write the one-period (net of storage) expected rate of return for an investor who hold an unhedged position in the (risky) commodity \( i \) as follows:

\[
E(\widetilde{R}_t) = E[(\widetilde{S}_{t+1} - \bar{S}_t) / \bar{S}_t]
\]  

(27)

where \( \bar{S}_t \) is the beginning price of commodity \( i \) (or the current price) and \( \widetilde{S}_{t+1} \) is the (unknown) end of the period price. Substituting (27) into the CAPM equation in (21) we find:

\(^{71}\) The following discussion draws on the work of Dusak (1973), Black (1976), Baxter et al (1985), So (1987), Copeland and Weston (1988) and Hull (1997)
\[ E[(\tilde{S}_{i,t+1} - S_{i,t})/S_{i,t}] = R_f + \beta_i'[E(\tilde{R}_m) - R_f] \]

where:
\[ \beta_i' = \text{cov}((\tilde{S}_{i,t+1} - S_{i,t})/S_{i,t} , \tilde{R}_m) / \sigma^2(\tilde{R}_m) \]

As with any risky asset, \( E(\tilde{R}_i) \) should compensate the investor for the interest on the capital invested in the commodity plus a return, positive or negative, over and above the risk-free rate due to unanticipated changes in the spot price. With futures markets, however, an investor has the choice of hedging his holding, in which event he could renounce his risky position in favour of a risk-free position, to earn only the risk-free rate (owing to arbitrage). In order to introduce the futures price into this equation we note that the buyer of the futures contract, who assumes the risk from the hedger, invests no capital in this position and thus should not earn the 'pure' time return or the risk-free rate. By implication, and in equilibrium, the investor (or speculator) should receive the expected return on the unhedged commodity less the risk-free rate (i.e. \( E(\tilde{R}_i) - R_f \)). Alternatively, and as noted by Chang, Chen and Chen (1990, p.30), because futures are traded on margin, traders do not require as much compensation for deferred consumption. This analysis will clearly apply to both long and short positions.

In addition, as noted in Chapter Two and elaborated in Appendix B, this expected return should equal the value of the futures contract at expiration (or just prior to the cash payout, before the contract is rewritten and its value set to zero), which equals the change in the futures price during the holding period. As expressed in equation (9) in the Appendix and restated below, adjusting the subscripts and assuming a long position:

\[ V_{F,(t,T)} = F_{t+1,T} - F_{t,T} = E(\Delta \tilde{F}) = E(\tilde{R}_i) - R_f \]

Based on the equalities in equation (27) and (29) we can thus rewrite (28) as follows:

\[ E[(\tilde{S}_{i,t+1} - S_{i,t})/S_{i,t}] - R_f = E(\tilde{R}_i) - R_f = E(\Delta \tilde{F}) = \beta_i'' [E(\tilde{R}_m) - R_f] \]

where:
\[ \beta_i'' = \text{cov}(\Delta \tilde{F}, \tilde{R}_m) / \sigma^2(\tilde{R}_m) \]

Equation (30) represents one way of calculating the risk premium on futures contracts in the context of the CAPM framework and is similar to that derived by Dusak (1973, p.1393). From the equalities in the equation we can see first, that the expected change in the futures price is equivalent to the risk premium on the spot commodity (i.e. \( E[(\tilde{S}_{i,t+1} - S_{i,t})/S_{i,t}] - R_f = E(\Delta \tilde{F}) \)) and, secondly, that the interpretation of returns on futures contracts is as a risk premium; that is - as a return over and above the risk-free rate (i.e. \( E(\Delta \tilde{F}) = E(\tilde{R}_i) - R_f \)).

### 4.2.1 BLACK'S APPROACH

An alternative way to derive the futures-CAPM equation is the one developed by Fischer Black (1976), who took the approach of using futures prices instead of spot prices in equation (28). That is:
\[
E((\tilde{F}_{t+1} - F_{t,t})/F_{t,t}) = R_f + \beta'_i[E(\tilde{R}_m) - R_f] \\
\text{where:} \\
\beta'_i = \frac{\text{cov}((\tilde{F}_{t+1} - F_{t,t})/F_{t,t}, \tilde{R}_m)}{\sigma^2(\tilde{R}_m)}
\]

(31)

In doing so Black (1976, p.172) argued that since equation (21) “cannot be applied directly to a futures contract, because the initial value of the contract is zero ... [we] will rewrite the equation so that it applies to [the expected] dollar returns rather than [expected] percentage return.” Baxter et al (1985) and So (1987) have followed this practice. Multiplying (31) by \(F_{t,t}\) and expressing \(\beta''\) in full, we get Black’s CAPM expression for the expected money returns on the commodity:

\[
E(\tilde{F}_{t+1} - F_{t,t}) - R_f F_{t,t} = \frac{\text{cov}((\tilde{F}_{t+1} - F_{t,t}), \tilde{R}_m)}{\sigma^2(\tilde{R}_m)} [E(\tilde{R}_m) - R_f]
\]

(32)

Black noted that since the beginning of the period value of a futures contract is zero, one could set \(F_{t,t}\) equal to zero. The value of the contract at the end of the period, before the position is marked-to-market and its value reset to zero, is the change in the futures price over the period or \(\Delta \tilde{F}\) (usually during the day).\(^72\) Thus we can set \(\tilde{F}_{t+1} = \Delta \tilde{F}\) to get the modified, empirical futures-CAPM equation:

\[
E(\Delta \tilde{F}) = \frac{\text{cov}(\Delta \tilde{F}, \tilde{R}_m)}{\sigma^2(\tilde{R}_m)} [E(\tilde{R}_m) - R_f]
\]

or

\[
E(\Delta \tilde{F}) = \beta''_i [E(\tilde{R}_m) - R_f]
\]

(33)

Equation (33) holds that the expected change in the futures price is proportional to what Black (p.173) termed the ‘dollar beta’ of the futures price, as in (30). For the simplicity of exposition and further explanation, we rewrite (30) and (33) using the conventional CAPM notation, that is:

\[
E(\tilde{R}_t) = \beta_i [E(\tilde{R}_m) - R_f]
\]

(34)

Equation (34) shows that when the CAPM is applied to futures \(R_f\) is not included as the intercept term, or, similarly, excess returns are replaced with ‘raw’ returns that reflect the risk premium only.

An examination of the differences between a share transaction and futures trading provides further insight into the rationale behind this equation. A share is purchased with an immediate cash payment, while a futures contract is only an agreement for future delivery and payment. Because the payment for a share is made upfront the expected return should also reflect the time value of money (\(R_f\)); that is, a risky asset should yield a return over and above the risk-free rate of return. The return on a futures position should not include \(R_f\) since, essentially, no money is put up to buy the futures contract and interest can even be earned on money put up as a margin. This situation implies that the return for a share, with comparable riskiness to that of a commodity as represented by a futures contract, should be \(R_f\)-greater than the return for the futures contract (Bodie and Rosansky, 1980; Elem and Vaught, 1988).

\(^72\) As noted by Black (1976, p.173), in practice, the commodity exchange set daily limits which constrain the reported change in the futures price and the daily gains and losses of traders. These price limits are traditionally aimed at reducing extreme price fluctuations, yet this regulation seems dubious, as protection against fluctuation in equilibrium prices cannot realistically be granted. Following Black, we will assume such limits do not exist.
4.3 THE RISK PREMIUM HYPOTHESIS: CAPM vs. TRADITIONAL THEORIES

An important insight into the implications of the Sharpe-Lintner CAPM for futures contracts may be gleaned by an analysis of the relationship between spot prices, futures prices, expected spot prices and expected returns, both in the context of the CAPM framework and in relation to the theories of normal backwardation and contango. The aim of the following section is therefore to establish a definitive relationship between (i) the expected rate of returns on the ith asset (a commodity underlined by a futures contract) and its beta as a manifestation of risk premium under the CAPM apparatus and (ii) the relationship between \( F_{iT} \) and \( E(S_T) \) as a manifestation of the risk premium and bias as prescribed by the theories of normal backwardation, contango and expectations.

As noted above, spot commodities are assets that form part of an investor's portfolio, either directly or indirectly. When the returns on commodities are defined net of all storage costs and convenience yield, commodities, like any other asset, can be priced according to the Sharpe-Lintner CAPM expressed in equations (21) and (23). The expected return on a spot commodity was also expressed in (27).

For ease of comparison with the discussion in Chapter Two and Three, we ignore the tilde signs and refer to time \((t+1)\) as time \( T \) to represent the end of the investment or contract period. This alteration does not affect the formulation of the equations or their theoretical content. Equating (21) with (27), both of which express expected return, and solving for the current spot commodity price, the equilibrium risk-return relation for commodity \( i \) can be expressed as:

\[
S_{i,t} = \frac{E(S_{i,T}) - S_{i,t} \beta_i [E(R_m) - R_f]}{1 + R_f}
\]  

Equation (35) says that the current price of any commodity \( i \) is the discounted value (at the risk-free rate with simple compounding) of its expected future spot price at time \( T \), adjusted downward for risk by the factor \( S_{i,t} \beta_i [E(R_m) - R_f] \), which equals the risk premium based on the commodity's systematic risk and proportional to its current spot price. (Dusak, 1973)

To link equation (35) with futures prices, one should refer to the discussion in Chapter Two regarding the functional relationship between futures and spot prices due to arbitrage hedging, as formulated in the cost-of-carry equations. The futures contract quotes a price for the purchase of the spot commodity but with deferred delivery; for this delayed payment benefit, the buyer must pay one period credit or borrowing charge, as expressed in the cost-of-carry model. It is important to note that we are assuming that the only cost of carry is a financing cost, which is assumed to be the risk-free rate under capital market equilibrium.\(^73\) Thus the futures price under normal conditions (using simple compounding) is:

\(^73\) As discussed in Chapter Two, in practice the financing cost in futures markets is the Repo rate. We also do not pay much attention here to other carrying charges, since as we are dealing with precious metals that are almost always in ample supply and are typically held as investment asset. Also as suggested by Dusak (1973, p.1392) we can assume that the differences between the returns on spot and futures market assets, from sources such as transaction costs, basis risk, the business risk of storage or limitations on borrowing, are so small and so unsystematic relative to the variations in returns due to price fluctuations that they can safely be ignored.
\[ F_{t,T} = S_{t,t} (1 + R_f) \]  
(36)

where \( F_{t,T} \) is the current futures price on commodity \( i \) to be delivered at \( T \). Given (36), if we now multiply \( S_t \) in equation (35) by \( (1 + R_f) \), we have the following expression for the futures price:

\[ F_{t,T} = S_t (1 + R_f) = E(S_{t,T}) - S_{t,t} \beta_t [E(R_m) - R_f] \]  
(37)

or, rearranging (37), to separate between absolute prices and the risk premium we get:

\[ [E(S_{t,T}) - F_{t,T}] / S_t = \beta_t [E(R_m) - R_f] \]  
(38)

An alternative useful expression for the futures price, based on the analysis in Chapter Two is:\(^74\)

\[ F_{t,T} = E_t(S_T)e^{[R_f - E(R_i)](T-t)365} \quad \text{or} \quad F_{t,T} = E_t(S_T) [(1 + R_f)(1 + E(R_i))]^{T-t}365 \]  
(39)

Equation (39), in either form, implies that the price of a futures contract relative to the expected future spot price of the underlying commodity is a function of the discount rate applied to the expected future spot price of the commodity \([1 + E(R_i)]\), relative to the cost of carry or the risk-free rate \([1 + (R_f)]\) over the holding period of the spot commodity and the futures position. Looking at both equations (38) and (39), we can now finally express the theory of forecasting power and the risk premium and its implications in the context of the CAPM model. Concisely stated: -

If \( \beta = 0 \) \( \Rightarrow \) \( F_{t,T} = E_t(S_T) \) and \( E(R_i) = R_f \) \( (\therefore \text{zero-beta asset} \iff \text{"Expectation"}) \)

If the underlying commodity has zero systematic risk, the futures price will be an unbiased estimate of the ultimate spot price and the expected return on the spot asset and on the futures position will be the risk-free rate. Both long and short futures positions carry no systematic risk and on average expect neither profit nor loss. Furthermore, if a commodity futures contract has no systematic risk, its price will not tend to rise or to fall as it matures, while fluctuation in the futures price would reflect fluctuations in interest rates only (that is, in line with the cost of carry).

If \( \beta > 0 \) \( \Rightarrow \) \( F_{t,T} < E_t(S_T) \) and \( E(R_i) > R_f \) \( (\therefore \text{positive-beta asset} \iff \text{"Backwardation"}) \)

If the ultimate realisation of the commodity spot price involves positive systematic risk, the profit to the contract buyer (or the long position) also involves such a risk. Market participants with well-diversified portfolios will be willing to buy futures only if they expect positive profits as compensation for bearing that risk. Their expected profit is positive only if \( E_t(S_T) \), at which they can sell, is greater than \( F_{t,T} \) at which they can buy, implying that investors expect a return on a long futures position that is greater than the risk-free return. For the short position, positive beta futures imply expected loss (the reverse of the long).

\(^74\) Using continuous compounded interest rate as in Chapter Two, equation (27) is analytically equivalent to:

\[ S_t = E(S_T)e^{[R_f - E(R_i)](T-t)365} \], while (36) is equivalent to \( F_{t,T} = S_t e^{(R_f - E(R_i))(T-t)365} \). Solving the later for \( S_t \) and substituting the result into the former we get: \( F_{t,T} = E_t(S_T)e^{[R_f - E(R_i)](T-t)365} \), or \( F_{t,T} = E_t(S_T) [(1 + R_f)(1 + E(R_i))]^{(T-t)365} \) with simple interest, where the time period can be ignored without significantly changing the result.
If $\beta < 0 \implies F_{t,R} > E(S_T)$ and $E(R_t) < R_f$ \qquad (: negative-beta asset $\iff$ "Contango")

Holding a long futures position involves bearing negative systematic risk and yields an expected loss, which is perfectly consistent with the CAPM, since an asset with negative beta will on average earn less than the risk-free rate. Diversified investors taking long positions in futures contracts will be willing to suffer expected losses of $F_{t,R} - E(S_T)$ in order to lower the portfolio risk or in return for the 'insurance' provided by long, negative beta positions. For the short position, this is a positive beta asset.

Essentially, if the CAPM and all its assumptions hold at all time futures pricing will be consistent with the expectation theory or zero beta futures, implying that the risk of investing in futures contract is independent of the risk of changes in the value of all assets taken together. Investors therefore do not have to be paid for that risk (Black, 1976; Dusak, 1973; Carter, Rausser and Schmitz 1983). Market forces of supply and demand for futures contracts will drive the prices of positive beta futures up (bringing them back into line with expected spot prices) and expected returns down so that futures contracts yield only the risk-free rate. The reverse will take place with negative beta futures. Mispriced futures contracts according to the CAPM will earn excess profit\textsuperscript{75}.

\textsuperscript{75} This conclusion abstracts from the possibility of changing speculative positions and assumes net long hedging positions, an issue discussed in the next chapter.
Chapter Five
The Futures-CAPM: Empirical Review and Research Hypotheses

1. INTRODUCTION

The purpose of this chapter is to review previous empirical work that has tested and in some instances restated Dusak's original futures-CAPM model. As noted earlier, our review of the literature suggests that the empirical progression of the futures-CAPM can be characterised by two observations. The first is the extent to which Keynes' original assumptions regarding speculators' behaviour were initially adopted and subsequently relaxed. The second is the restatement of Dusak's model in order to improve on its identified shortcomings. The importance of the review in this chapter is that it enables us to arrive at our research hypothesis and at the methodology used to test it in a coherent way that also highlights its contribution to the literature.

Essentially, our research methodology builds on the various empirical studies that re-examined Dusak's model and are listed in Table 5. Most of these studies have tested for the existence of a risk premium in agricultural commodities, typically the same commodities tested by Dusak, with the exception of the work of Taylor (1986) and Chang et al (1990), which tested metals contracts. Our study extends the work of Chang et al (1990) to other precious metals not yet tested, and we further addresses additional empirical modifications and computational issues that were raised in the agricultural studies.

Table 5 summarises the results of an extensive literature survey of all the studies we found that are either directly or indirectly related to the application of modern portfolio theory to commodities futures contracts. The table lists, in chronological order, each of these studies in terms of the following categories: (i) the commodities tested; (ii) the data sample and the method of computing the returns on futures contracts; and (iii) the nature of the studies, for instance a theoretical analysis of the application of the CAPM to futures contracts or an empirical study testing the futures-CAPM.

The last two columns describe the methodological approach taken by scholars in addressing two of the specifications of Dusak's original model: first, the approach taken with respect to the appropriate proxy for the market index, and secondly, the assumption made regarding the stationarity of the model's parameters - in particular the beta parameter, which represents systematic risk. It is around these two issues that controversy arose and modifications were suggested. They serve to distinguish between all the empirical work that proceeded from Dusak's original work. We discuss these issues in details below followed by a concluding summary and the research hypotheses.
Table 5: A Summary of Previous Theoretical and Empirical Studies Related to the Application of the CAPM to Futures Contracts

<table>
<thead>
<tr>
<th>Year</th>
<th>Scholar(s)</th>
<th>Commodities</th>
<th>Data sample and Returns</th>
<th>Nature of the Study</th>
<th>Market Index</th>
<th>Systematic Risk ((\beta))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1976</td>
<td>Black</td>
<td></td>
<td></td>
<td>Theoretical CAPM Model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1979</td>
<td>Stoll</td>
<td></td>
<td></td>
<td>Theoretical Hedging Model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1984</td>
<td>Jagannathan</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1984</td>
<td>Hazuka</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1985</td>
<td>Chang</td>
<td>Wheat, Corn &amp; Soybeans</td>
<td>1951 – 1980</td>
<td>Non parametric test</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>1986</td>
<td>Taylor</td>
<td>Corn, Cocoa, Coffee, Sugar, Wool</td>
<td>Corn (1963-76); Cocoa &amp; coffee (1971-80); sugar (1961-81); wool (1966-78) daily log relative</td>
<td>Empirical CAPM</td>
<td>FT30</td>
<td>Stationary</td>
</tr>
<tr>
<td>1990</td>
<td>Chang, Chen &amp; Chen</td>
<td>Copper, Platinum &amp; Silver</td>
<td>1964 – 1983 monthly log-relative</td>
<td>Empirical CAPM</td>
<td>90% S&amp;P500 10% DJ-CCI**</td>
<td>Stationary</td>
</tr>
</tbody>
</table>

* Unless otherwise stated, log relative returns refer to the first difference of the natural logarithms of futures prices over the stated interval, that is \(R_{F,t} = \ln(F_{F,t} / F_{F,t-1})\). (see Data Description and Methodology)

** DJ - CFI = Dow Jones Commodity Futures Index, and DJ - CCI = Dow Jones Commodity Cash Index.
2. **THE EMPIRICAL FUTURES-CAPM AND APPROPRIATE MARKET PROXY**

The empirical ex post futures-CAPM equation used by Dusak (1973) to test for the existence of risk premium in wheat, corn and soybeans futures during 1952-1967 was the following:

\[ R_{F,t} = \alpha_F + \beta_F Z_{m,t} + \varepsilon_{F,t} \]  

Equation (40) is similar to (26) except that \( R_{F,t} \) is the one-period (realised) rate of return on the \( F \)th commodity futures in period \( t \) and \( \alpha_F \) and \( \beta_F \) are the normalised regression parameters, estimated by regressing the futures contract historical returns against the excess return on a proxy of total wealth, where all the classical assumptions regarding the error term, \( \varepsilon_{F,t} \), are assumed to hold (as in (26)).

Dusak implicitly accepted the three assumptions made by Keynes regarding the existence of a risk premium, that is: (i) speculators are risk averse; (ii) speculators hold net long positions; and (iii) speculators are unable to forecast future prices. She found no evidence in support of normal backwardation and risk premium, since the commodities' betas and alphas were not significantly different from zero. As a proxy for the return on total wealth Dusak selected the returns on the Standard and Poor’s 500 Index (S&P500), supporting this choice by noting that: “Common stock, after all, represent an important fraction of total wealth, so that even in a more comprehensive index they would be heavily weighted” (1973, p.1401). She added that the S&P500 Index has been the standard approach followed in most assets pricing studies.

Carter, Rausser and Schmitz (1983) (hereafter CRS) were the first to re-specify the futures-CAPM as formulated by Dusak. Their first criticism of Dusak’s model (or ‘principal deficiency’, to use their words) was the assumption that “the well-diversified portfolio of speculators contains only common stocks” (1983, p.320). CRS argued that taken alone, the S&P500 does not account for the price instability of the country’s stock of commodities, nor does it recognise the importance of commodity assets to the economy, as measured by total value traded relative to stocks. This inadequacy, they argued, “results in biased estimates of the degree of systematic risk one should expect for futures contracts if the CAPM is properly interpreted” (p.323).

To correct for Dusak’s omission of commodities from the proxy of the market portfolio, CSR constructed an index that gives equal weight to stocks, as represented by the S&P500, and to commodities, as represented by the Dow Jones Commodity Futures Index (DJ-CFI). Using this proxy, they found the required risk premium on wheat corn and soybeans during 1966-1976 to be positive and of economically significant magnitude, thus rejecting Dusak’s findings and providing support for the

---

76 CRS have justified their equal-weightings proxy in two ways. First, they compared the value of commodities represented by traded futures contracts in 1977 ($1,230bn) with the value of listed stocks in the same year ($950bn). Secondly, they cited the value of total farm assets ($655bn).
normal backwardation hypothesis. While CRS' rationale and respecification of the model were generally followed in subsequent research, a controversy has risen over the issue of the weightings assigned to commodities in the new market portfolio's proxy.

Marcus (1984) argued that CRS' contrary findings were due to the inappropriateness of an equal-weightings index, which in effect assigns far too great a weight to commodities and thereby biases the estimated betas upward. He noted that essentially commodities futures prices were regressed against an index dominated by those very prices, which makes the beta of a typical commodity futures price an increasing function of the weight assigned to the commodity index. Consequently, with a zero weight, as in Dusak's model, the betas would average to zero, and as the weight approached one the betas would average to one.

Marcus's argument was verified by Elam and Vaught (1988, p.85), who showed that the sample beta coefficient "indeed increases as the weight given to commodities in the market portfolio increases". Instead, Marcus (1984, p.163) recommended a value-weighted index of all assets in the economy. After calculating an 11.2% share of agricultural farm assets in the households' portfolio of total wealth, he suggested "roughly" a 90-10 weighting of stocks v. commodities. Following Marcus' recommendation both So (1986\(^7\)), using 90-10 weighting, and Baxter, Conine and Tamarkin (1985) (hereafter BCT), assigning 93.7-6.3 weighting, found no evidence of a risk premium in wheat, corn and soybeans contracts, thus supporting Dusak's results.

BCT, however, replaced the DJ Commodity Futures Index with the DJ Cash Commodity Index (DJ-CCI). They did so as a critique to CRS procedures, arguing that they "do not believe that the value of futures can be reasonably compared with the value of stocks in constructing a market portfolio .... [T]he net supply of futures contracts is zero; therefore, they should not be included in the market portfolio. It is theoretically correct, however, to include the value of cash commodities in the market portfolio" (1985, p.124). Following BCT, Elam and Vaught (1988) and Chang et al (1990) extended the analysis to other commodities using a combination of 90% S&P500 and 10% DJ-CCI. However, in contrast to BCT, both studies did find evidence of significant systematic risk.

While we found the value-weighted approach for the market benchmark intuitively appealing, we nevertheless believe that the use of a single market index, and in particular the use of the S&P500 Index as its proxy, is appropriate for three reasons.

First, it is reasonable to assume that, at the margin, the S&P500 does capture, to a sufficient extent, the holdings of commodity stock in a representative portfolio of total wealth. This is clearly explained by Black (1976, p.172) who noted that "the market portfolio ... includes corporate securities, personal assets such as real estate, and assets held by non-corporate businesses. To the extent that corporations

hold stocks of commodities, commodities are implicitly included in the market portfolio, whereas to the extent that they are held by individual and non-corporate or private businesses they are explicitly included in that portfolio”.

Indeed a breakdown of the S&P500 Index total market capitalisation during the years 1997-1999, for instance, clearly supports Black’s argument. The basic material and energy sectors, which encompass companies that produce and hold commodities and therefore have a direct exposure to commodities prices, constitute about 8% of the index total market capitalisation during that period. Furthermore, the non-cyclical sector, which includes sub-sectors such as food, beverages, tobacco, pharmaceuticals, all of which holds commodities as raw materials, constitutes about 18% of the index market capitalisation.

Similarly, the industrial sector, which is very much exposed to metals prices, constitutes about 10% of the index. Thus, in its broad exposure to all major industries and as a function of changes in investors' preferences to these industries, the S&P500 provides a fair representation of the distribution of total wealth - directly and indirectly - amongst available assets, including all commodities.

Secondly, we dislike the idea of weighting two indices which are themselves weighted averages of many other assets prices.

Thirdly, although the S&P500 might not be the best representative of the true portfolio of total wealth, it is nevertheless the most dominant proxy used by practitioners and academics alike. In this context, we refer to the comment of Campbell et al (1997, p.214) regarding Roll’s critique of the unobservability of the market portfolio, stated in footnote 69. It seems reasonable that Campbell’s analysis, which showed that empirical inference is not sensitive to the use of a proxy in place of the market portfolio and thereby concluding that Roll’s concern is not an empirical problem, could equally apply to CRS’s critique of Dusak’s proxy. It would therefore be methodologically consistent with standard CAPM studies in other assets classes to use the S&P500 Index, which would thus render the empirical results and conclusions more useful and comparable to such other studies.

3. STOCHASTIC SYSTEMATIC RISK

The second criticism raised by CRS (1983) of the Dusak model was its failure to account for changing speculative positions on futures markets. As noted above, an implicit assumption in Dusak’s model was that speculators hold net long positions. Relaxing this assumption allows for changing speculative positions, which in a CAPM context translates to relaxing the assumption that the β parameter is stable over time (CRS, 1985; So, 1987; Elam and Vaught, 1988; Chang et al, 1990). CRS argued that in the

78 The analysis was conducted using data extracted from Bloomberg Financial Markets.
79 See Campbell et al (1997, p.213-215). However, this view should clearly be subjected to further empirical testing.
security market literature it is no longer traditionally assumed that the covariance between returns on a security and returns on the market (the \( \beta \) value) are stable or stationary over time.

They also noted that there has been almost no discussion in the literature of the nature of the systematic risk in futures contracts. Following CSR, So (1987) also noted that many studies have assumed that the systematic risk is stable and used the ordinary least square method to estimate the coefficients of equation (40). Both CRS and So quoted Sharpe (1978) to support their criticism and suggested modification, saying that the systematic risk of futures positions might change during seasons or owing to supply and demand shocks, as the positions of speculators and hedgers change.

Changing speculative positions may be caused by various factors, systematic and non-systematic. Fabozzi and Francis (1977 and 1978) suggested that changes in the stock market or the industry in which a firm operates could alter the firm's single index market model statistics - the \( \alpha \) and \( \beta \) parameters. In futures markets, for example, seasonal changes prompting changes in the supply and demand for an agricultural commodity will cause hedgers and speculators to adjust the riskiness of their positions. In addition, periods of price support and other intervening regulations lead to different price patterns for commodity futures. One would expect, for example, that in periods of price stabilisation, the risk premium, if present, would be stable.

The non-stationarity of the parameters may also arise from non-systematic factors, such as changing technology within an industry and variable management decisions or accounting practices (CSR, 1983). On a macroeconomic level, changes in overall economic conditions - for instance, the growth or recession stages of the business cycle, changes in the money supply and interest rates and other structural changes in the economy - may all affect the intertemporal instability of the systematic risk (Francis and Fabozzi, 1979; Anderson, 1985; Fama and French 1988).

The non-stationarity in agricultural futures was explained by Gray (1962)\(^{80} \) and Peck (1985). Gray suggested that it may be due to hedging costs instead of a risk premium; Peck, on the other hand, proposed that the level of long or short activities on futures markets reflects commercial need, while total market use of futures reflects the seasonality in commercial stock.

According to So and CRS, since there is no \textit{a priori} reason to adapt Dusak's assumption that the level of non-systematic returns and the level of systematic risk - the \( \alpha \) and \( \beta \) parameters in equation (40) respectively - for a futures contract are stationary, this assumption should be altered. Instead, an alternative statistical technique should be employed to cater for random or stochastic parameter(s) and therefore for net speculative positions. CRS assumed that both the \( \alpha \) and the \( \beta \) parameters are stochastic and included in their model a variable for the positions of large speculators that also catered for net

---


On the other hand, So assumed non-stationarity of the $\beta$ parameter only and followed the random coefficient method of Theil (1971) and Hildreth and Houck (1968).\footnote{Hildreth, C. and Houck, J.P. (1968). Some Estimators for a Linear Model with Random Coefficients. \textit{Journal of the American Statistical Association}. 63: 584-595.} As noted above, CRS did find statistical evidence confirming the existence of systematic risk in wheat corn and soybeans futures contracts, while So did not.

While we acknowledge the importance of accounting for changing speculative positions, following BCT (1985), Taylor (1986), Elam and Vaught (1988) and Chang et al (1990) we assume stationarity in the model parameters in accordance with the Sharpe-Lintner single-index CAPM as originally applied by Dusak. As was insightfully argued by BCT (1985, p.123, fn.4), the specification error identified and accounted for by CRS in their reformulation of Dusak's model “brings up a theoretical issue of whether the empirical methodology employed by CRS is consistent with the theoretical CAPM … [The] CAPM as formulated by Dusak to include the risk-free rate is an equilibrium model in which short positions do not exist. Thus, Dusak should not be faulted for the exclusion of short positions; rather, CRS may be questioned for using an empirical methodology which is contrary to the underlying theoretical model. The question is the robustness of the CAPM to include short positions.”

The problems of accounting for speculative imbalances and for changing speculative positions (and related to those difficulties, the controversy surrounding the inadmissibility of short positions under the CAPM framework) as well as the models developed to address the non-stationarity of systematic risk all fall beyond the scope of our research. Following Dusak (and the other, similar studies listed above) we use the CAPM framework to generalise the Keynesian formulation and to test whether data on precious metals futures returns during our sample period conform better to the capital markets model than to the Keynesian model.

In addition, in line with Working’s approach to futures markets and in contrast to Keynes, Hicks, Cootner and Telser, we do not rest our hypothesis on differences in taste or attitude toward risk among hedgers and speculators. In our context, we simply look at an investor in futures market and its expected returns from holding a long position in the market, in like manner to looking at returns to the holder of an equity or bond investment. Thus within the CAPM framework we would take the returns to short speculators to be the inverse of the long hedgers or speculators.
4. SUMMARY AND TESTABLE HYPOTHESES

The analysis in Chapters Four and Five has four important implications. First, it suggests that the risk underlying a futures contract is the risk attributed to the behaviour of the spot commodity price and in particular that associated with changes in expected spot prices.\(^3\)

Secondly, fundamental to the CAPM application and specification to futures contracts is the interpretation of futures returns as a risk premium, that is, as a return over and above the risk-free rate.

Thirdly, it demonstrates that the difference between the futures price and the expected spot price is analogous to the difference between the risk-free rate (or the expected return on a default-free bond or on a Repo agreement) and the expected return on a commodity (or a pure asset) with the same systematic price risk as the futures contract.\(^4\)

Lastly, referring to equations (28) and (30), there are two ways of applying the CAPM to futures or, similarly, of calculating the risk premium. One is to calculate the expected return on the spot commodity over the holding period or any required interval, less the risk-free rate. The second is to calculate the change in the futures price over the required interval. And thus despite the uniqueness of futures contracts, as was concluded by Dusak (1973, p.1393), “futures contracts, properly interpreted, poses no problem for capital market theory”. (See Data Description and Methodology, Section 2.)

The notion that futures contracts are priced like any other asset means that investors who own futures are able to diversify away that part of risk that is diversifiable. In turn, if agents can share risk through futures markets, expected returns from futures contracts holdings should be consistent with the Sharpe-Lintner single-index single-period CAPM. Relative to the traditional theories of risk and return in futures markets - backwardation and contango, and expectations - the CAPM approach can therefore be seen from two perspectives:

(i) As an alternative interpretation (in its entirety) to the Keynes and Hicks ‘insurance’ interpretation.

(ii) As explained in Chapter Four, the CAPM may also be employed to empirically test for the existence of a risk premium as hypothesised by the three traditional theories.

---

\(^3\) As noted in the context of the theory of storage, the risk of storing or holding a commodity between two periods, arising because the spot price at the end of the holding period is an unknown in the first period, is identical to the risk associated with holding a long futures contract (French, 1986).

\(^4\) Relating this to the pricing fundamentals of Chapter Two, we can also say that this relationship between the futures price and the expected spot price of the underlying commodity must always hold, since we can always (synthetically) create the spot commodity position by holding a default-free bond and a long futures position (depending on the arbitrage objectives).
The approach in point (i) is the one taken by Dusak and is, in fact, her contribution to the literature of futures markets. She was the first to put forward (and contrast) the portfolio approach as a competing analysis of returns to speculators to that of Keynes, concluding - based on her empirical tests - that futures returns conform better to the portfolio point of view than to the Keynesian interpretation. According to Dusak, in contrast to the Keynesian approach, which hypothesises positive returns to speculators (due to contracts' riskiness as reflected in their price variability), the portfolio approach, by itself, makes no presumption as to the sign or the magnitude of these returns. Instead it says that "returns on any risky capital asset, including futures market assets are governed by that asset contribution, positive, negative, or zero, to the risk of a large and well-diversified portfolio of assets" (Dusak, 1973, p.1388).

Another related difference between the two approaches is the measure and interpretation of risk. While risk in the Keynesian framework is identified solely with fluctuation in futures prices (or the standard deviation of futures price change), within the portfolio framework risk is the covariance or the joint variability of futures prices and market portfolio prices. Thus, by broadening the number of financial assets analysed to include essentially the entire capital market, the CAPM abstracts from the characteristics peculiar to a specific futures market. Futures markets are subsumed under a more general market for financial assets.

In this context, the first hypothesis to be empirically tested is the pricing of futures contracts within the framework of the CAPM, as was done by Dusak, that is:

*If futures contracts pricing is to conform to the CAPM approach, and if futures returns over time are independent and normally distributed, then expected returns on holding futures contracts should be commensurate with the level of systematic risk.*

More specifically, this hypothesis can take two different forms (or testable outcomes) – either of which is analogous to it and will support it:

*If futures contracts are risky assets, and that risk is undiversifiable, then futures contracts will command a risk premium (reflected in a positive beta) and ex post returns will be positive on average (reflected in a positive intercept-term or alpha).*

*If, however, futures contracts' entire risk is diversifiable (reflected in a zero beta and implying no systematic risk), which seems highly plausible given the existence of spot commodity markets where opposite positions can be taken, then investors will not be rewarded for taking that risk and ex post returns will be zero on average (reflected in a zero alpha).*

The approach in point (ii) is basically equivalent to the question: what is the outcome if the CAPM is employed as an empirical test of the three traditional theories of the risk premium? This seems to be the approach taken by studies that followed Dusak's methodology and used the CAPM to accept or reject
the Keynesian theory of normal backwardation. They have done so based on the sign and the significance of the beta coefficient as a measure of the risk premium, assuming of course that the question is essentially an empirical one. Clearly, however, as noted by Chang et al (1990, p.36), the question whether the results of a statistical futures-CAPM test can be viewed as rejecting or accepting the Keynesian theory will depend on the researcher’s willingness to reinterpret the implications of the theory in the CAPM framework.

In line with our analysis so far, which was based on the approach taken by Dusak as well as by BCT (1985) and Elam and Vaught (1988), the Keynesian theory of normal backwardation - in the CAPM framework - predicts that all price risks of futures contracts should be rewarded by the market. Contracts’ expected returns should in turn compensate for this entire risk. This can be translated to the following (ex-ante) expression, which is in fact three jointly-tested hypotheses, suggested by BCT, which expresses the implication of the CAPM [as applied to futures and expressed in equations (30) or (33)], for the traditional theories of futures price determination:

<table>
<thead>
<tr>
<th><strong>Expected Return:</strong></th>
<th><strong>Risk:</strong></th>
<th><strong>As predicted by:</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>If $E(\Delta \bar{F}) &gt; 0$ when $\text{Cov}(\Delta \bar{F}, \bar{R}_m) &gt; 0$ then</td>
<td>Backwardation ( (a) )</td>
<td></td>
</tr>
<tr>
<td>= &lt; &lt; &lt;</td>
<td>Expectations ( (b) )</td>
<td></td>
</tr>
<tr>
<td>&lt;</td>
<td>Contango ( (c) )</td>
<td></td>
</tr>
</tbody>
</table>

As in the first hypothesis, an empirical ex post futures-CAPM equation such as equation (40) is employed to test this theoretical expression, where the alpha and beta parameters (estimated using ordinary least square regression) represent the mean returns and the systematic risk respectively. (See also equation (41) in Data Description and Methodology.) The theoretical rationale for this expression was explored and explained in Section 4.3 in Chapter Four.

It is interesting to note that Chang et al (1990) had in fact used a different interpretation for the Keynesian theory in the CAPM framework. They argued that “one may also reinterpret the [Keynesian] theory in the sense that it only requires a reward in proportion to the amount of the systematic risk borne” (p.36, fn.14). While this reasoning sound perfectly in line with the CAPM, it was used to suggest that the following set of results to support the theory of normal backwardation:

$E(\Delta \bar{F}) = 0$ when $\text{Cov}(\Delta \bar{F}, \bar{R}_m) > 0$

(That is, empirically, their alpha coefficients were statistically indistinguishable from zero while their betas were all positive and significantly different from zero.)

We find these results to be inconsistent with the above expression (particularly with (a)) and with Dusak’s approach. Perhaps what Chang et al meant is that backwardation is evident in their data set
because the 'low' reward for speculation, even of zero (as reflected in a zero alpha), is in proportion to the small size of their beta coefficient (that is, of the risk premium). We, however, take the above expression as the proper interpretation of the three traditional hypotheses within the CAPM framework, and as the second testable hypothesis in the research (jointly tested with the first hypothesis above).

Lastly, another implication derived from the literature review regarding the futures-CAPM is that futures positions are settled on a daily basis, and therefore the most appropriate holding period for the calculation of futures returns in our sample data is one trading day. This observation was also made by BCT (1985), although they have used semi-monthly returns in their study in order to replicate Dusak's study more closely. Taylor (1986) also suggested the use of daily returns. No other CAPM-related study reviewed here has tested daily returns on futures contracts.

While the literature on the subject of futures contracts and modern portfolio theory is very rich, in particular the studies related to appropriate measures of the systematic risk, our concern is with studies falling within the scope of the Sharpe-Lintner CAPM framework as formulated by Dusak. Within this framework the contribution of the empirical part of this research to the literature is its coverage of metals that were not tested before, such as gold and palladium; its sample period, which is significantly longer than all other studies covered in this paper; and the testing of daily returns which had not yet been tested under the re-specified futures-CAPM.

85 Many other studies have considered the risk premium in the context of the so-called 'intertemporal CAPM', which derives equilibrium expected returns on assets for more general economies than that assumed by the Sharpe-Lintner CAPM. While these models are perhaps more realistic (given the austere assumptions of the Sharpe-Lintner model) they are highly mathematically complex and fall beyond the scope of the framework created so far. It is also important to note that there are several problems with such a straightforward application of the CAPM to futures. In a multi-period, multi-good framework, the CAPM holds under the assumptions of stationary investment and consumption opportunity set. These assumptions imply constant relative prices and constant interest rates. Under these conditions a commodity futures contract would be a riskless asset.
PART II: DATA DESCRIPTION AND METHODOLOGY

1. DATA

The securities under investigation are gold, silver, platinum and palladium futures contracts traded on the New York Mercantile Exchange (NYMEX). Table 1 and 2 in Appendix A provide a detailed description of each commodity contract in terms of the contract specification and design. There are six different futures contracts traded (simultaneously) during a year in gold and silver, and four contracts in platinum and palladium. Futures contracts are identified according to their delivery month and year, which typically constitutes the contract's name. For instance, 'March 1999 - Gold' refers to a gold contract for delivery on March 1999. Table 6 below presents the delivery months for the four precious metals traded on the NYMEX, the sample period of data collected, and the number of contracts analysed. In total, this research dealt with 368 different contracts, involving over 90,000 data points.

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Delivery Month</th>
<th>Sample Period</th>
<th>Number of Contracts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold</td>
<td>February, April, June, August, October, December</td>
<td>January 1980 - December 1998</td>
<td>114 (19yrs X 6 contracts)</td>
</tr>
<tr>
<td>Silver</td>
<td>January, March, May, July, September, December</td>
<td>January 1980 to December 1998</td>
<td>114 (19yrs X 6 contracts)</td>
</tr>
<tr>
<td>Platinum</td>
<td>January, April, July, October</td>
<td>January 1980 to December 1998</td>
<td>76 (19yrs X 4 contracts)</td>
</tr>
<tr>
<td>Palladium</td>
<td>March, June, September, December</td>
<td>January 1983 to December 1998</td>
<td>64 (16yrs X 4 contracts)</td>
</tr>
</tbody>
</table>

The systematic risk for gold, silver, platinum and palladium was estimated using the following empirical ex post futures-CAPM equation, using a normal linear regression model:

$$R_{F,t} = a_F + b_F (R_{mt} - R_F) + e_{Ft}, \quad t = 1, 2, \ldots, T$$  \hspace{1cm} (41)

where $R_{F,t}$ is the first difference in the natural logarithms of daily settlement prices for the $F$th commodity futures contract in period $t$ being a day. The use of the logarithm transformation is explained below with $R_{F,t}$ specified in equation (42). A daily database, from which contracts’ settlement prices were extracted, was purchased from the “Futures Industry Institution Data Centre” in the United States. For most contracts nineteen years of daily returns were computed, where $T$ equals the number of observations in each sample. $R_F$ is the risk-free rate in period $t$, proxied by the computed one-day Treasury Bill rate, collected from St. Louis Fed’s FRED® Database, and $R_{mt}$ is the one-day log relative return on the market portfolio proxied by the S&P500 Index collected from Bloomberg Financial Markets Database. $a_F$ and $b_F$, are the normalised regression coefficients, estimated by regressing each contract’s historical returns against the excess returns on the S&P500 Index, and $e_{Ft}$ is the residual.

---

86 The database also includes daily data of opening, high, low and cash prices, volume and open interest for each commodity futures contract.
87 URL: http://www.stls.frb.org/fred/
disturbance term. More specifically, $b_F$ is the slope of the regression line, representing systematic risk, and is an estimate of the (true) $\beta$ parameter of the futures contract. $a_F$ is the intercept-term, representing the unsystematic risk of futures returns (or the mean ex post returns earned by each contract) expressed in a risk premium form.

2. LOG RELATIVE RETURNS

As indicated earlier, in applying the CAPM to futures the random variable rate of returns is the dollar return or the absolute change in futures prices rather than the percentage returns. However, a review of previous studies shows that none had actually used the theoretical dollar-return specification. Furthermore, with the exception of BCT (1985), none had actually acknowledged this apparent anomaly. In all tests, returns were computed as the first differences in the natural logarithms of futures prices over the selected time interval, typically semi-monthly or monthly. That is, assuming a one-day time interval, the random variable return $R_{t,t}$ on contract $F$ was generated by taking the natural logarithm of the settlement price for a given day $F_t$ discounted by the settlement price of the previous day $F_{t-1}$; or:

$$R_{F,t} = \ln(F_t / F_{t-1}) \quad t = 1, 2, \ldots$$

(42)

with $F_0$ being the futures price in the beginning date (or the first day) of a contract series.

$F_t / F_{t-1}$ is the ‘relative’ price change (effectively a percentage change) rather than the absolute price change, which has the effect of discounting the magnitude of the underlying price. However, since $\ln(F_t / F_{t-1}) = \ln(F_t) - \ln(F_{t-1})$, the transformation via the natural logarithms converts the series of relative changes into a series whose terms combine to give the cumulative relative changes over any sequence of days in a manner analogous to that of the absolute price changes (Cornwell, Town and Crowson, 1984; So, 1987). Thus, as noted by So (1987, p.317), "[the] use of log price changes is consistent with the price change argument proposed by Dusak (1973) and can avoid the price-level effect due to the positive relationship between variability and the price levels".

In addition, the use of log-price changes rather than simple price changes is a convention used in all econometric studies of financial time series, including futures. It is used because the first two moments of a sample distribution - the mean and the standard deviation - have no meaningful interpretation in a time series model except if the sample data is stationary, and stationarity can be achieved via the log transformation.\(^{88}\)

\(^{88}\) As noted by Taylor (1986, p.17), a stochastic process whose first and second order moments do not change with time is said to be second order stationary (typically abbreviated to stationary). He notes (p.241) that stationary stochastic processes often appear to be acceptable for daily returns. Like many others, he argued that the price process responsible for generating daily prices indexed by the time variable $t$ is non-stationary, while the returns process may well be considered stationary when it is being defined by the first difference in the log price.
Measuring returns using simple price changes may be heteroskedastic over the life of a typical contract approaching maturity, or whenever there are dramatic changes in the volatility of prices. Furthermore, due to thin trading of distant contract, daily price changes are often positively serially correlated. The use of the first difference in log price can mitigate these potential statistical problems. Taking the natural logs of prices induces linearity in the series as natural logs are base independent; while taking the natural logs and then the first difference in the logs prices induces stationarity in the resultant returns time series and reflects the natural, continuous growth rates.

3. TIME SERIES CONSTRUCTION

In order to construct a long time series of futures prices, which is subsequently used to calculate a long time series of ex post returns on futures contracts, numerous contracts have to be used. Two methods of constructing such price time series were suggested in the literature. The first, which is very much a universal practice, is based on the aggregation of futures prices over time for contracts with a specific delivery month. We termed this the fixed-delivery-month method. The second is based on the aggregation of futures prices over time for artificially created contracts with a specific maturity, which we termed fixed-interval-to-maturity method.

It is important to note that under both methods delivery-month prices (and thus returns) were intentionally kept out of all the constructed time series. The reason for the omission is that metal futures contracts do not have a specific maturity date but rather there is a delivery-period of three to four weeks from the beginning of the delivery/maturity month. During this period trading volume drops sharply as positions are rolled over to later maturity contracts. As a result, settlement prices during delivery months represent thin and somewhat erratic trading.

Also excluded were (i) observations in days when trading volume was zero; (ii) outliers, which are defined as observations lying outside three standard deviations of the returns distribution; and (iii) observations that are too far distant from maturity (longer than a year; see below). These sample-screening procedures were suggested in Fabozzi, Ma and Briley (1994), who also dealt with daily futures returns over long sample periods. In total we dealt with over 170,000 observations.

3.1 FIXED-DELIVERY-MONTH METHOD

Under this approach daily returns were calculated from price series for various metal contracts defined by their maturity month, for example, daily returns for a ‘February-Gold’ contract, or for a ‘January-Platinum’ contract. All price series begin at January 1980 except for palladium, which starts at January 1983 for reasons of data availability. For most contracts, it was possible to construct a sample that ended at December 1998 except for the palladium contracts and the July and October platinum contracts that have a shorter sample due to constraints embodied in the method itself.
The fixed-delivery-month price time series were created by rolling over a specific delivery month contract from one year to the next. In this process (known as “rolling” a futures position), in order to avoid delivery and continuously maintain a long position in a specific contract (say, a ‘June-Gold’ contract), a contract is “sold” each year just prior to its maturity. Straight away, a new contract that has not yet reached delivery (that is, the next year ‘June-Gold’) is “purchased”. Thus, in constructing the multi-year series, every contract contributes a block of data that starts at the first trading day of the delivery month one year prior to that month and ends a year later (that is, on the last day of the month preceding the delivery month). Table 7 illustrates the construction of such time series for a ‘March - silver’ contract.

Table 7: Constructing a Fixed-Delivery-Month Time Series for a ‘March – Silver’ Contract

<table>
<thead>
<tr>
<th>Futures contract used in time series construction</th>
<th>Block of data used to construct the time series (by dates)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>Contract life (trading dates)</td>
</tr>
</tbody>
</table>

Since the length of time covered by a particular contract’s block of data is 12 months, contracts that were traded for less than a year (for instance, ‘September 1992 - Palladium’ which traded between 17/12/91 - 25/9/92) have had to be excluded in order to avoid the typical compromise of series discontinuity. Thus for some contracts the testable sample is shorter. The options of allowing series discontinuity or averaging or substituting data from other contracts, as in other studies, did not seem appropriate, since it could bias the results according to subjective data choice. It is also an inconsistent approach. As shown in Table 8, this method resulted in six different time series for gold, five for silver and four for platinum and palladium.
Table 8: Fixed-Deliver-Month Method – Constructed Time Series

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Delivery Month</th>
<th>Sample</th>
<th>Number of returns observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold</td>
<td>February</td>
<td>January 1980 – December 1998</td>
<td>4783</td>
</tr>
<tr>
<td></td>
<td>April</td>
<td>January 1980 – December 1998</td>
<td>4783</td>
</tr>
<tr>
<td></td>
<td>June</td>
<td>January 1980 – December 1998</td>
<td>4783</td>
</tr>
<tr>
<td></td>
<td>August</td>
<td>January 1980 – December 1998</td>
<td>4783</td>
</tr>
<tr>
<td></td>
<td>October</td>
<td>January 1980 – December 1998</td>
<td>4783</td>
</tr>
<tr>
<td></td>
<td>December</td>
<td>January 1980 – December 1998</td>
<td>4783</td>
</tr>
<tr>
<td>Silver</td>
<td>March</td>
<td>January 1980 – December 1998</td>
<td>4783</td>
</tr>
<tr>
<td></td>
<td>May</td>
<td>January 1980 – December 1998</td>
<td>4783</td>
</tr>
<tr>
<td></td>
<td>July</td>
<td>January 1980 – December 1998</td>
<td>4783</td>
</tr>
<tr>
<td></td>
<td>September</td>
<td>January 1980 – December 1998</td>
<td>4783</td>
</tr>
<tr>
<td></td>
<td>December</td>
<td>January 1980 – December 1998</td>
<td>4783</td>
</tr>
<tr>
<td></td>
<td>April</td>
<td>January 1980 – December 1998</td>
<td>4771</td>
</tr>
<tr>
<td></td>
<td>July</td>
<td>January 1980 – June 1997</td>
<td>4394</td>
</tr>
<tr>
<td></td>
<td>October</td>
<td>January 1980 – September 1997</td>
<td>4459</td>
</tr>
<tr>
<td>Palladium</td>
<td>March</td>
<td>January 1983 – February 1994</td>
<td>2802</td>
</tr>
<tr>
<td></td>
<td>June</td>
<td>January 1983 – May 1994</td>
<td>2865</td>
</tr>
<tr>
<td></td>
<td>September</td>
<td>January 1983 – August 1991</td>
<td>2176</td>
</tr>
<tr>
<td></td>
<td>December</td>
<td>January 1983 – November 1997</td>
<td>3744</td>
</tr>
</tbody>
</table>

3.2 FIXED-INTERVAL-TO-MATURITY METHOD

Following Elam and Vaught (1988) and Chang et al (1990), realised returns were calculated for artificially created futures contracts in which the time to maturity of contracts was kept constant. Prices for each metal were divided into 4, 5 or 6 groups based on a fixed interval of time to maturity (or delivery). The number of price groups (and consequently of return time series) was in effect a technical function of: (i) the number of different contracts traded during a year; (ii) the monthly distribution of contracts throughout the year (for example, following a bi-monthly cycle or quarterly cycle); and (iii) data availability. Table 9 lists all the constructed time series for the four metals according to their different maturity groups. As can be seen, this method resulted in six different time series (or contracts) for gold, five for silver and four for platinum and palladium.

With platinum and palladium, for example, since trading is conducted in a quarterly cycle (January, April, July, October for platinum, and March, June, September, December for palladium), the price data were easily divided into four interval-to-maturity groups: 1-3 months, 4-6 months, 7-9 months and 10-12 months to maturity. The results were four return times series for each commodity. As in the fixed-delivery-month method, however, since some palladium contracts were traded for less than a year, it was not possible to construct time series extending to December 1998 without discontinuities, in particular for the 7-9 months and 11-12 months to maturity groups.
### Table 9: Fixed-Interval-to-Maturity Method – Constructed Time Series

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Time to maturity</th>
<th>Sample</th>
<th>Number of observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold</td>
<td>1-2 months</td>
<td>January 1980 – December 1998</td>
<td>4783</td>
</tr>
<tr>
<td></td>
<td>3-4 months</td>
<td>January 1980 – December 1998</td>
<td>4783</td>
</tr>
<tr>
<td></td>
<td>5-6 months</td>
<td>January 1980 – December 1998</td>
<td>4783</td>
</tr>
<tr>
<td></td>
<td>7-8 months</td>
<td>January 1980 – December 1998</td>
<td>4783</td>
</tr>
<tr>
<td></td>
<td>9-10 months</td>
<td>January 1980 – December 1998</td>
<td>4783</td>
</tr>
<tr>
<td></td>
<td>11-12 months</td>
<td>January 1980 – December 1998</td>
<td>4783</td>
</tr>
<tr>
<td>Silver</td>
<td>1-3 months</td>
<td>January 1980 – December 1998</td>
<td>4783</td>
</tr>
<tr>
<td></td>
<td>3-5 months</td>
<td>January 1980 – December 1998</td>
<td>4783</td>
</tr>
<tr>
<td></td>
<td>5-7 months</td>
<td>January 1980 – December 1998</td>
<td>4783</td>
</tr>
<tr>
<td></td>
<td>7-9 months</td>
<td>January 1980 – December 1998</td>
<td>4783</td>
</tr>
<tr>
<td></td>
<td>9-11 months</td>
<td>January 1980 – December 1998</td>
<td>4783</td>
</tr>
<tr>
<td>Platinum</td>
<td>1-3 months</td>
<td>January 1980 – December 1998</td>
<td>4771</td>
</tr>
<tr>
<td></td>
<td>4-6 months</td>
<td>January 1980 – December 1998</td>
<td>4771</td>
</tr>
<tr>
<td></td>
<td>7-9 months</td>
<td>January 1980 – December 1998</td>
<td>4771</td>
</tr>
<tr>
<td></td>
<td>10-12 months</td>
<td>January 1980 – December 1998</td>
<td>4771</td>
</tr>
<tr>
<td>Palladium</td>
<td>1-3 months</td>
<td>January 1983 – December 1998</td>
<td>4017</td>
</tr>
<tr>
<td></td>
<td>4-6 months</td>
<td>January 1983 – December 1998</td>
<td>4017</td>
</tr>
<tr>
<td></td>
<td>7-9 months</td>
<td>January 1983 – November 1991</td>
<td>2239</td>
</tr>
<tr>
<td></td>
<td>10-12 months</td>
<td>January 1983 – August 1991</td>
<td>2176</td>
</tr>
</tbody>
</table>

The actual construction of a fixed-interval-to-maturity price time series is shown in Table 10, which illustrates the construction of the four time series groups for platinum, from January 1980 to March 1981. ‘Standing’ in the month of January 1980 (in terms of the time series dates), for instance, the nearby contract is January-1980, the next is April-1980, then July-1980 and October-1980. Although during January 1980 the nearby futures is January-1980, January is the delivery month of that contract and was therefore excluded. Instead, prices from April-1980 contract were placed in group 1, which includes prices of contracts maturing in 1-3 months. Similarly (and still ‘standing’ in January 1980), prices from the next available contract - the July-1980 contract - were placed in group 2, which captures prices of contracts maturing in 4-6 months. The October-1980 contract contributed the prices for group 3 (7-9 months to maturity), and the January-1981 contract contributed the data for group 4 since (standing in January) this contract has 12 months to maturity.

A similar process of data allocation was carried out standing in the month of February 1980, then March, April, and so on to December 1980 and was then repeated in a similar manner for all the years from 1981 to 1998, as shown in Table 10. The number of months in brackets is the number of months to maturity of the specified contract, excluding the delivery month.
Table 10: The Construction of the Four Fixed-Interval-to-Maturity Time Series of Platinum

<table>
<thead>
<tr>
<th>Platinum Time series dates</th>
<th>Futures contracts contributing the price data and the time to maturity (in brackets) of these data points relative to the time-series dates (in column 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-31/1/1980</td>
<td>April80 (3 months) 1-3month to maturity</td>
</tr>
<tr>
<td>1-28/2/1980</td>
<td>April80 (2 months) 1-3month to maturity</td>
</tr>
<tr>
<td>1-31/3/1980</td>
<td>April80 (1 month) 1-3month to maturity</td>
</tr>
<tr>
<td>1-30/4/1980</td>
<td>July80 (3 months) 1-3month to maturity</td>
</tr>
<tr>
<td>1-31/5/1980</td>
<td>July80 (2 months) 1-3month to maturity</td>
</tr>
<tr>
<td>1-30/6/1980</td>
<td>July80 (1 month) 1-3month to maturity</td>
</tr>
<tr>
<td>1-31/7/1980</td>
<td>October80 (3 months) 1-3month to maturity</td>
</tr>
<tr>
<td>1-31/8/1980</td>
<td>October80 (2 months) 1-3month to maturity</td>
</tr>
<tr>
<td>1-30/9/1980</td>
<td>October80 (1 month) 1-3month to maturity</td>
</tr>
<tr>
<td>1-31/10/1980</td>
<td>January81 (3 months) 1-3month to maturity</td>
</tr>
<tr>
<td>1-30/11/1980</td>
<td>January81 (2 months) 1-3month to maturity</td>
</tr>
<tr>
<td>1-31/12/1980</td>
<td>January81 (1 month) 1-3month to maturity</td>
</tr>
<tr>
<td>1-31/1/1981</td>
<td>April81 (3 months) 1-3month to maturity</td>
</tr>
<tr>
<td>1-28/2/1981</td>
<td>April81 (2 months) 1-3month to maturity</td>
</tr>
<tr>
<td>1-31/3/1981</td>
<td>April81 (1 month) 1-3month to maturity</td>
</tr>
<tr>
<td>etc...</td>
<td>etc...</td>
</tr>
</tbody>
</table>

For silver, since contracts are not distributed evenly throughout the year (that is, there are both January and December contracts and no consistent quarterly or bi-monthly cycle), all January contracts were excluded in order to avoid two price observations in a group for a given month.\(^9\) The January contracts also happened to be highly illiquid. This method has therefore resulted in only five time series with overlapping intervals to maturity. The construction of each is shown in Appendix D.

### 3.3 COMPARING THE TWO METHODS

With the exception of Elam and Vaught (1988) and Chang et al (1990) all the empirical studies described in Table 5 used the fixed-delivery-month method, as originally adopted by Dusak (1973). Dusak raised the option of constructing futures time series using the fixed-interval-to-maturity method, based on the proposition made by Samuelson (1965) that variation in futures prices is a decreasing function of maturity, but suggested that the theoretical and empirical justification for looking at returns in this way is weaker.\(^9\)

---

\(^9\) This treatment was suggested by Elam and Vaught (1988, p.83), in their study of cattle and hog futures, as a way to improve the serial correlation test in the regression residual.

\(^9\) Results of tests of the Samuelson hypothesis are mixed. A detailed review of this literature is given in Kamara (1982). We note, however, that Taylor (1986), in a comprehensive futures study, concluded that the standard deviation does not increase systematically during the final six trading months. These results confirm an alternative hypothesis that suggests there is no necessity for the variance of futures prices to increase as the maturity date approaches; rather it depends on the distribution of the underlying variables (see Richard and Sundaresan, 1981; Anderson, 1985; Kamara, 1982). We also note that Dusak did not explain her suggestion further but expressed her intention to do so in a later paper, which unfortunately we could not trace.
In justifying their use of the fixed-time-to-maturity rather than the fixed-delivery-month method, Chang et al (1990, p.32) noted that: "[A]s a result of the fixed delivery time the remaining life of each contract changes over time. Thus, each return time series usually contains both realised returns on a contract with a remaining life of up to one year and as short as one month". They further referred to studies (though in the bond market) that have shown that the systematic risk of an asset with limited life is a function of the remaining life of the asset.\footnote{Chang et al argued that only a small effort is needed to generalise the results from the bond market to futures contracts, with no further explanation, which for us seemed necessary since futures and bonds are two very different assets.} They concluded that "conceivably, failure to fix the maturity of the return time series may reduce the statistical significance of the beta estimate" (p.32).

In order to fix a specific time to maturity (i.e. 3-, 5-, 7-, 9-month) Chang et al used a weighted average of two futures price quotations in months in which the maturity of available contracts did not match the required maturity of the specified time series. Elam and Vaught (1988), on the other hand, fixed an interval to maturity (the approach taken in this paper as well). They did not, however, explain their compilation choice, which as noted above was different from all previous studies. Fixing the maturity period over an interval rather than one period offers the advantage of leaving the futures price data 'as is', with minimum subjective computation (for example, weighted averages of different contracts).

From the review of the empirical literature it seems clear that there is no obvious preference for any one method, nor is there a strong argument in favour of one over the other. The first method is the one typically followed, almost by default, being a universal practice of futures-exchanges and data-providers. We therefore find it appropriate, given the comprehensive database collected for the purpose of this research and the resultant long samples of daily returns, to test the CAPM in futures contracts using both time series methods. It is important to note that a preliminary, graphical examination of the resultant price and return time series under both methods indicates that while the behaviour of the price time series - as reflected in the behaviour of 'open interest' and actual prices - appears different under the two methods, the behaviour of the return time series is very similar. (See Appendix E and F for a comparison between the resultant price time series under the two methods.)

In Appendix E we display the behaviour of the actual price series under the two methods for silver and gold. In Appendix F we display the behaviour of the corresponding daily open-interest for all four metals. The graphs plot the compiled open-interest time series (38 in total) under both methods, over each metal entire sample as an indication of trading activity.

The graphical analysis of actual futures prices and of open-interest presented in these appendices was conducted for two reasons. The first was to ascertain whether the two different "rolling" processes worked successfully and the multi-year daily series behaved as expected. That is, a successful compilation of all the time series would be reflected, for example, in a consistent pattern in the
behaviour of open-interest under the ‘fixed-delivery-month’ method, or in a clear full-carrying-charge structure in futures prices under the ‘fixed-interval-to-maturity’ method. Secondly, and relatedly, this analysis enabled us to check the integrity of the data subsequently used to generate the log-returns series and to examine each contract’s tradability (or liquidity), which would in turn give us an indication of the informational efficiency of the process generating the underlying futures prices.

As can be seen in Appendices 5 and 6, although the behaviour patterns of open interest between the two methods are very different (see explanation in Appendix F), the behaviour of the futures prices is very similar. Under both methods the futures prices in all the time series follow the behaviour of the spot prices fairly closely, although it may be said that the fixed-interval-to-maturity method is more consistent with the theory of storage. However, when it comes to daily returns the two methods seem unlikely to differ significantly in term of systematic risk.

The behaviour and distribution parameters of the resultant returns time series are discussed in the Empirical Findings section. Indeed, the results indicate that, at least for daily returns, the two methods do not significantly differ from each other. It is not the intention of this dissertation, however, to examine the behaviour and statistical properties of the actual price time series.

4. THA CAPM ASSUMPTIONS

As noted in the empirical review, since the Sharpe-Lintner CAPM is a single-period model it is necessary, for statistical analysis, to add the assumptions that returns are: (i) independently and identically distributed through time, and (ii) jointly multivariate normal - in order to estimate the model over time. The independently and identically distributed (IID) assumption is of more significance than the normality assumption; in fact IID is an important property of the family of stable Pareto distributions, of which the normal distribution is a special case (Fama, 1963 and 1965; Mann and Heifner, 1976; Campbell et al, 1997).

While we have tested for serial independence in our sample data (as explained below and reported in the Empirical Findings section), we provide only a brief empirical review of the normality assumption in futures and the important findings. A complete analysis and statistical tests of the actual distribution of our sample and its parameters would be a lengthy exercise, falling beyond our scope.

4.1 NORMAL DISTRIBUTION

The normal (Gaussian) distribution assumption is important for the derivation of the finite-sample properties of CAPM tests. In reviewing the literature on the distribution of futures prices, Kamara (1982, p.278) has pointed out that “the empirical evidence strongly suggests that the distribution of daily price changes (or in logarithms transformation) does not conform to the normal curve but is
highly leptokurtic". He noted that several explanations for the phenomenon of leptokurtosis have been suggested, the most notable of which is that of Benoit Mandelbrot, in a study that was brought to the forefront in this area by Fama (1965) in studying the behaviour of stock market prices.

As explained in Kamara (p.279), Mandelbrot, examining commodity (spot) prices, argued that "the individual effects composing a price change do not have a finite variance. Hence, the Central Limit Theory does not apply and the limiting distribution could be any member of [what he labelled as] the stable Paretoian [or non-Gaussian] family, which includes the normal distribution as a special case". He added further that "these stable distributions have an unbounded kurtosis and usually give high values for any measured sample kurtosis, thereby making them good candidates for the distribution of futures price changes".

In Table 13 in the Empirical Findings section, we present kurtosis values for our data, which clearly confirm the high leptokurtosis in the distribution of futures log-price changes under both methods of computing the time series. Fitting a stable Paretoian distribution, rather than a normal distribution, allows the random variable to have an infinite variance (Fama, 1965, p.44).

To the best of our knowledge, the most comprehensive empirical study regarding the distribution of daily log-changes in futures prices, testing 18 different futures contracts including gold and silver, is the one conducted by Cornew et al (1984). The data selected had time periods ranging from three to nine years, and the time series were constructed using a method similar to the fixed-interval-to-maturity method with the objective of choosing price data from periods containing the greatest trading activity. Since they had only one time series per commodity, we assume their series resemble our 1-2 or 1-3 months to maturity time series.

Like Dusak (1973), who examined the normality assumption in her sample of semi-monthly log-returns on wheat, corn, and soybeans, their results also confirmed that the distribution of futures returns corresponded more closely to the family of stable distributions than to the narrower class of normal distribution. (See also Mann and Heifner (1976) in support of the stable Paretoian hypothesis in commodity futures.)

Statistical problems are posed by the fact that the underlying distribution conforms better to the stable non-Gaussian than to the normal distribution, in particular the computation of sample variance and standard deviations (to measure risk), which are always finite. Nevertheless, it can be shown that the ordinary least squares coefficients are consistent, although not necessarily efficient, estimators of the

---

92 See Appendix C for the definition and statistical measure of kurtosis.
93 Mandelbrot, B. (1963). The Variation of Certain Speculative Price. Journal of Business. 36: 394-419. In Fama (1965) and Kamara (1982). As noted by Fama (1965, p.42), the Gaussian hypothesis was not seriously questioned and academic research has too readily neglected the implications of the leptokurtosis usually observed in empirical distribution of price changes until the work of Benoit Mandelbrot began to appear.
corresponding population parameters and that the loss of efficiency is not likely to be significance for large samples (Fama, 1965; Dusak, 1973; Taylor, 1986). The size of all our 38 data samples is always greater than 2000 observations.

4.2 SERIAL INDEPENDENCE

Testing for serial independence of daily returns is important on two grounds. First, from a statistical point of view, it is necessary to establish serial independence, since the procedures for computing subsequent statistical measures, particularly ordinary least square estimation, assume it. Secondly, from an investor’s viewpoint - as noted by Fama (1965, p.69) in the context of stock prices but equally applicable to futures - dependencies in the return time series can be used by investors to increase expected profits.

The serial correlation coefficient, $r_t$, which measure the relationship between the value of a random variable in time $t$ - in our case daily futures returns $\tilde{R}_{F,t}$ - and its value $\tau$ days earlier, is defined as:

$$r_t = \frac{\text{Cov}(\tilde{R}_{F,t}, \tilde{R}_{F,t-\tau})}{\text{Var}(\tilde{R}_{F,t})}$$  \hspace{1cm} (43)

The standard error of $r_t$ - defined for a large sample and under the assumption that the distribution of $\tilde{R}_{F,t}$ has a finite variance - is: $\sigma(r_t) = [1/(T - \tau)]^{\frac{1}{2}}$, with $T$ being the sample size.

As noted above, however, the literature indicates that the distribution of futures returns is a stable non-Gaussian one, for which finite variance does not exist; thus the assumption of a finite variance is inappropriate and the use of equation (43) is questionable. Nevertheless, as noted by Fama (1965, p.69), it has been shown that $r_t$ is a consistent and unbiased estimate of the true population’s serial correlation for a stable Pareto distribution (of the nature observed by Cornwell et al and Dusak94), particularly since the sample size is very large.

In addition, as noted by Dusak (1973, p.1395), it was also shown that $r_t$ is an adequate descriptive measure of the serial correlation in the population, in the sense that it behaves the same as its counterpart from a normally distributed sample of observations.

---

94 That is, more precisely, a stable Pareto distribution with a characteristic exponent $\alpha$ greater than 1. The average $\alpha$ of gold and silver in the study of Cornwell et al (1984, p.541) were 1.54 and 1.71 respectively.
PART III: EMPIRICAL FINDINGS

1. TESTS FOR SERIAL INDEPENDENCE

The first reported results are those of the serial independence tests of the hypothesis that daily returns on metal contracts are independently and identically distributed through time.

Table 11 presents the sample correlation coefficient computed for each daily log-return time series for lag $\tau = 1, 2, \ldots, 10$ days. Also shown is the standard error of $r$, as defined above. It can be seen that the serial correlation coefficients for all the samples are very small in absolute terms and fluctuate above and below zero. The largest value is only 0.098. There is a roughly equal number of positive and negative coefficients, generally with no consistent pattern in the signs with respect both to a particular commodity and to a construction method.

Out of 380 coefficients, all of which are below 0.1, only 114 are more than twice their computed standard errors and are equally found in the two construction methods. This, however, should not be regarded as an important factor for two reasons: first, because for our large samples the standard errors are very small - serial correlation coefficients as small as 0.03, for instance for gold, silver and some platinum series, are twice their computed standard errors. Secondly, as noted by Fama (1965, p.70), the formulae for computing the standard error underestimate the true variability of the coefficients.

Thus, overall, it seems clear that such a small order of magnitude of serial dependency is, for all intended statistical and investment purposes, insignificant. We can therefore accept the assumption underlying the CAPM that the daily returns are serially independent and proceed with the tests of the ordinary least square regression coefficients.
### Table 11: Daily Serial Correlation Coefficients for Gold, Silver, Platinum and Palladium ($t = 1, \ldots, 10$)

<p>| | | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T = 4783$ for all contracts</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sigma(\hat{\rho}_t)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Gold</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-2</td>
<td>-0.015</td>
<td>0.051</td>
<td>0.009</td>
<td>-0.098</td>
<td>0.010</td>
<td>-0.042</td>
<td>-0.004</td>
<td>-0.002</td>
<td>-0.017</td>
<td>0.005</td>
</tr>
<tr>
<td>3-4</td>
<td>0.001</td>
<td>0.033</td>
<td>0.011</td>
<td>-0.092</td>
<td>0.002</td>
<td>-0.043</td>
<td>-0.008</td>
<td>-0.008</td>
<td>-0.015</td>
<td>0.004</td>
</tr>
<tr>
<td>5-6</td>
<td>0.007</td>
<td>0.054</td>
<td>0.011</td>
<td>-0.089</td>
<td>0.000</td>
<td>-0.042</td>
<td>-0.008</td>
<td>-0.009</td>
<td>-0.014</td>
<td>0.004</td>
</tr>
<tr>
<td>7-8</td>
<td>0.013</td>
<td>0.055</td>
<td>0.010</td>
<td>-0.086</td>
<td>0.001</td>
<td>-0.043</td>
<td>-0.009</td>
<td>-0.009</td>
<td>-0.013</td>
<td>0.003</td>
</tr>
<tr>
<td>9-10</td>
<td>0.019</td>
<td>0.037</td>
<td>0.010</td>
<td>-0.083</td>
<td>0.000</td>
<td>-0.038</td>
<td>-0.010</td>
<td>-0.010</td>
<td>-0.014</td>
<td>0.002</td>
</tr>
<tr>
<td>11-12</td>
<td>0.024</td>
<td>0.037</td>
<td>0.011</td>
<td>-0.080</td>
<td>0.003</td>
<td>-0.037</td>
<td>-0.012</td>
<td>-0.010</td>
<td>-0.014</td>
<td>0.001</td>
</tr>
<tr>
<td>Ave. $\hat{\rho}$</td>
<td>0.008</td>
<td>0.055</td>
<td>0.014</td>
<td>0.010</td>
<td>-0.088</td>
<td>0.001</td>
<td>-0.041</td>
<td>-0.009</td>
<td>-0.008</td>
<td>-0.015</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Silver</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-3</td>
<td>0.043</td>
<td>0.051</td>
<td>0.037</td>
<td>0.025</td>
<td>0.200</td>
<td>0.000</td>
<td>0.028</td>
<td>-0.020</td>
<td>-0.015</td>
<td>-0.007</td>
</tr>
<tr>
<td>3-5</td>
<td>0.047</td>
<td>0.052</td>
<td>0.039</td>
<td>0.039</td>
<td>0.034</td>
<td>0.000</td>
<td>0.029</td>
<td>-0.016</td>
<td>-0.021</td>
<td>-0.003</td>
</tr>
<tr>
<td>5-7</td>
<td>0.048</td>
<td>0.054</td>
<td>0.036</td>
<td>0.037</td>
<td>0.033</td>
<td>0.000</td>
<td>0.026</td>
<td>-0.016</td>
<td>-0.019</td>
<td>0.002</td>
</tr>
<tr>
<td>7-9</td>
<td>0.052</td>
<td>0.057</td>
<td>0.042</td>
<td>0.033</td>
<td>0.031</td>
<td>0.000</td>
<td>0.026</td>
<td>-0.016</td>
<td>-0.025</td>
<td>0.001</td>
</tr>
<tr>
<td>9-11</td>
<td>0.052</td>
<td>0.060</td>
<td>0.042</td>
<td>0.031</td>
<td>0.033</td>
<td>0.000</td>
<td>0.026</td>
<td>-0.016</td>
<td>-0.022</td>
<td>0.003</td>
</tr>
<tr>
<td>Ave. $\hat{\rho}$</td>
<td>0.048</td>
<td>0.055</td>
<td>0.039</td>
<td>0.033</td>
<td>0.030</td>
<td>0.000</td>
<td>0.027</td>
<td>-0.017</td>
<td>-0.020</td>
<td>-0.001</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Platinum</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-3</td>
<td>0.017</td>
<td>0.050</td>
<td>0.014</td>
<td>-0.061</td>
<td>-0.002</td>
<td>-0.028</td>
<td>-0.046</td>
<td>0.001</td>
<td>-0.028</td>
<td>0.012</td>
</tr>
<tr>
<td>3-6</td>
<td>0.018</td>
<td>0.052</td>
<td>0.015</td>
<td>-0.057</td>
<td>-0.007</td>
<td>-0.027</td>
<td>-0.051</td>
<td>-0.004</td>
<td>-0.027</td>
<td>0.016</td>
</tr>
<tr>
<td>7-9</td>
<td>0.023</td>
<td>0.048</td>
<td>0.013</td>
<td>-0.054</td>
<td>-0.006</td>
<td>-0.025</td>
<td>-0.047</td>
<td>-0.005</td>
<td>-0.025</td>
<td>0.010</td>
</tr>
<tr>
<td>10-12</td>
<td>0.026</td>
<td>0.047</td>
<td>0.013</td>
<td>-0.053</td>
<td>-0.005</td>
<td>-0.022</td>
<td>-0.046</td>
<td>-0.007</td>
<td>-0.025</td>
<td>0.008</td>
</tr>
<tr>
<td>Ave. $\hat{\rho}$</td>
<td>0.021</td>
<td>0.049</td>
<td>0.014</td>
<td>-0.056</td>
<td>-0.005</td>
<td>-0.026</td>
<td>-0.048</td>
<td>-0.004</td>
<td>-0.026</td>
<td>0.012</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Palladium</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-3</td>
<td>0.056</td>
<td>-0.045</td>
<td>-0.042</td>
<td>0.003</td>
<td>-0.017</td>
<td>-0.017</td>
<td>0.019</td>
<td>-0.012</td>
<td>0.003</td>
<td>-0.029</td>
</tr>
<tr>
<td>3-6</td>
<td>0.048</td>
<td>-0.033</td>
<td>-0.036</td>
<td>-0.023</td>
<td>0.003</td>
<td>-0.023</td>
<td>0.012</td>
<td>0.000</td>
<td>-0.006</td>
<td>-0.008</td>
</tr>
<tr>
<td>7-9</td>
<td>0.027</td>
<td>-0.041</td>
<td>-0.038</td>
<td>-0.023</td>
<td>0.010</td>
<td>-0.026</td>
<td>0.002</td>
<td>0.009</td>
<td>0.006</td>
<td>0.033</td>
</tr>
<tr>
<td>10-12</td>
<td>0.026</td>
<td>-0.039</td>
<td>-0.039</td>
<td>-0.027</td>
<td>0.010</td>
<td>-0.029</td>
<td>0.002</td>
<td>0.011</td>
<td>0.008</td>
<td>0.034</td>
</tr>
<tr>
<td>Ave. $\hat{\rho}$</td>
<td>0.039</td>
<td>-0.045</td>
<td>-0.039</td>
<td>-0.018</td>
<td>0.002</td>
<td>-0.024</td>
<td>0.009</td>
<td>0.002</td>
<td>0.003</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Since the sample size is very large the SE, with three decimal places, does not vary between the different lags.
2. EMPIRICAL ESTIMATION OF THE FUTURES-CAPM COEFFICIENTS

Table 12 summarises the results of the ordinary least square regression model, as expressed in equation (41). The estimated regression coefficients, $a_F$ and $b_F$, their corresponding standard errors (SE) and $t$-values, the coefficients of determination ($r^2$) and the Durbin-Watson statistic (DW) for the sample period 1980 - 1998 (with some exceptions, outlined in the table) under both construction methods are displayed in the Table for each of the 38 commodity contracts. 95 (The actual regression output for each contract is given in Appendix G.)

2.1 GENERAL

The first outstanding feature of Table 12 is the very small size of the regression coefficients for all contracts under both methods. In particular, all the estimated $a_F$ coefficients (the intercept term) are, with no exception, zero, while all estimated $b_F$ coefficients, except two, are less than 0.1 (in absolute terms) with gold, silver and platinum contracts all showing positive betas and palladium contracts negative betas. In fact, with the exception of the platinum contracts, all $b_F$'s are less than 0.05.

The low $r^2$'s are consistent with the small $b_F$ coefficients since, as shown in Appendix C, $\beta_F$ is equals to $r$ times the ratio of $\sigma(\hat{R}_F)/\sigma(\hat{R}_w)$. A $r^2$ of approximately zero implies a beta of approximately zero and suggests no correlation and no explanatory power in the model. The DW statistics do not reject the null hypothesis of random residuals for any regression set at the 5% level of significance, implying non-significant autocorrelation. The very low serial correlation of the residuals suggests that the assumption of independence in the residuals (that is, the $E(e_{it}e_{it-1}) = 0$ for all $t$, as discussed in Chapter Four) upon which the calculation of the SE of the coefficients is predicted is a tenable one.

2.2 THE INTERCEPT TERM ($a_F$ COEFFICIENT)

Not only are all the intercept terms zero, under both compilation methods, but the SE are also zero (when rounded up to three decimal places96). The similar size of the $a_F$ coefficients and their corresponding SE is reflected, with few exceptions, in a $t$-statistics of around one for most contracts. It can be seen in the Table that the sizes and the signs of the $t$-values for gold and platinum are consistent between the two methods while the sizes of the $t$-values, but not their signs, are consistent in the silver contracts. There is no consistent pattern in the $t$-values of the palladium contracts under the two methods. Nevertheless (irrespective of these slight differences), the overall results for all contracts of all four metals are that the intercept term is not significantly different from zero, since none of the $t$-values rejects the null hypothesis that $a_F = 0$ at the 5% level of significance. These results are similar to those obtained by Dusak (1973), BCT (1985) and Elam and Vaught (1988).

95 The $r^2$, which measure the strength of the linear relationship between returns on the futures and returns on the market; and the DW statistic, which test for first-order autocorrelation in the residuals are defined in Appendix C
96 The alpha estimates were all less than 0.001. They ranged between (±) 0.0001 and 0.0004.
<table>
<thead>
<tr>
<th></th>
<th>Fixed interval to maturity</th>
<th>Fixed delivery month</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a_F$ ($t_0$)</td>
<td>$SE(a_F)$ ($t_0$)</td>
</tr>
<tr>
<td><strong>Gold</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-2</td>
<td>0.000 (1.831)</td>
<td>0.000 (1.696)</td>
</tr>
<tr>
<td>3-4</td>
<td>0.000 (0.862)</td>
<td>0.000 (1.470)</td>
</tr>
<tr>
<td>5-6</td>
<td>0.000 (0.886)</td>
<td>0.000 (1.239)</td>
</tr>
<tr>
<td>7-8</td>
<td>0.000 (0.907)</td>
<td>0.000 (1.020)</td>
</tr>
<tr>
<td>9-10</td>
<td>0.000 (0.927)</td>
<td>0.000 (0.795)</td>
</tr>
<tr>
<td>11-12</td>
<td>0.000 (0.947)</td>
<td>0.000 (0.592)</td>
</tr>
<tr>
<td><strong>Silver</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-3</td>
<td>0.000 (1.394)</td>
<td>0.000 (1.334)</td>
</tr>
<tr>
<td>3-5</td>
<td>0.000 (1.432)</td>
<td>0.000 (1.349)</td>
</tr>
<tr>
<td>5-7</td>
<td>0.000 (1.438)</td>
<td>0.000 (1.157)</td>
</tr>
<tr>
<td>7-9</td>
<td>0.000 (1.457)</td>
<td>0.000 (1.021)</td>
</tr>
<tr>
<td>9-11</td>
<td>0.000 (1.470)</td>
<td>0.000 (0.791)</td>
</tr>
<tr>
<td><strong>Platinum</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-3</td>
<td>0.000 (-0.720)</td>
<td>0.000 (4.466)</td>
</tr>
<tr>
<td>4-6</td>
<td>0.000 (-0.721)</td>
<td>0.000 (3.996)</td>
</tr>
<tr>
<td>7-9</td>
<td>0.000 (-0.728)</td>
<td>0.000 (3.853)</td>
</tr>
<tr>
<td>10-12</td>
<td>0.000 (-0.727)</td>
<td>0.000 (3.646)</td>
</tr>
<tr>
<td><strong>Palladium</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-3 &amp; 4-6 months:</td>
<td>0.000 (1.066)</td>
<td>0.000 (-0.772)</td>
</tr>
<tr>
<td>7-9 months:</td>
<td>0.000 (1.112)</td>
<td>0.000 (-1.465)</td>
</tr>
<tr>
<td>10-12 months:</td>
<td>0.000 (0.247)</td>
<td>0.000 (-0.906)</td>
</tr>
</tbody>
</table>

**Table 12: Estimated Regression Parameters for Gold, Silver, Platinum, and Palladium Contracts**

$T =$ numbers of observations, $SE =$ standard error of coefficients, $t =$ t-stat., critical value at 5% & 1% levels of significance are 1.96 & 2.58 respectively.
Theoretically, and as suggested by Jensen (1968), the intercept term is a performance measure of the asset or contract, since it measures the abnormal returns or excess returns earned by holding the futures contract. The finding of insignificant intercept terms, or zero mean excess returns, supports the argument that speculators are unable to earn extra returns from being long or short during the duration of all contracts, whether the strategy is based on a particular delivery month or on a particular maturity.

This finding is also consistent with the theoretical futures-CAPM equation expressed in (34) in which the risk-free rate is not included as the intercept term and the return on futures contracts is interpreted as a risk premium. Zero or very small negative mean returns are also reported in Table 13, which is discussed later on.

2.3 THE RISK PREMIUM (b_F COEFFICIENT) AND THE CAPM INTERPRETATION

Beside the uniformity in size across all contracts under both methods, the other striking and more important observation regarding the b_F’s coefficients is that, with the exception of the platinum contracts, none is significantly different from zero. On aggregate the SE of the b_F’s are approximately the same size as, if not somewhat larger than, the regression coefficients themselves. It should also be remembered that the SE presented in the Table may be understated because, as noted above, ordinary least squares coefficients are not necessarily efficient ones if the distribution of the underlying returns is non-Gaussian. And thus the t-values for all the gold, silver and palladium contracts are very small and do not reject the null hypothesis that b_F = 0 at the 5% level of significance. Again, these results are similar to those of Dusak (1973), BCT (1985) and Elam and Vaught (1988).

The theoretical implication of insignificant b_F estimates is that the systematic risk for gold, silver, and palladium is almost zero. Within the CAPM framework, this suggests that gold silver and palladium contracts are not, on average, risky assets, since the returns on holding them - irrespective of the method used to construct the time series - have a zero covariance with the returns on the market.97 Coupled with mean returns of zero (or zero a_F’s) this set of results confirms our first hypothesis with respect to gold silver and palladium futures contracts. That is, if futures contracts pricing is to conform to the CAPM approach, expected returns on holding futures contracts should be commensurate with the level of systematic risk. The results indicate that the entire risk of these contracts is readily diversifiable and investors should not be rewarded for taking such a risk.

97 It is important, however, to highlight the comment made by Dusak (1973, fn.25, p.1403) regarding zero-beta assets: "[A] zero beta asset has only zero covariance with other assets on average. With some assets its return will be positively correlated, and with others, negatively correlated. In fact, since the zero β assets will themselves be part of total wealth, they must be negatively correlated, on balance, with all other assets in the market portfolio. Because of this covariance with other assets, the zero β asset does make a sufficient contribution to the diversification and hence the risk reduction of the total portfolio to justify its inclusion even at a mean return (over and above interest) of zero".
Furthermore, the low systematic risk of these contracts, or similarly their low correlation with stock returns, implies that that gold, silver and palladium futures should be useful in diversifying, or reducing the return variability of, a stock portfolio.

It is interesting to note that, in contrast to gold and silver contracts, the $b_F$'s of all the palladium contracts are consistently negative, which suggests that palladium contracts are negative beta assets and can thus reduce the overall risk of a portfolio. However, the small size of these betas and their statistical insignificance renders such a conclusion a very tentative one (which could be subjected to further empirical testing).

As to the **platinum** futures, all the contracts under both methods have significant beta both at the 5% and 1% level of significance. The average beta for the fixed-interval-to-maturity contracts is 0.097, relative to 0.020 for gold, 0.031 for silver and -0.032 for palladium. Under the fixed-delivery-month method the contribution of the average platinum contract to the variability of returns on the S&P500 market portfolio is 0.101, relative to 0.020 for gold, 0.024 for silver and -0.037 for palladium. Having significant, positive systematic risk, the platinum contracts can therefore be viewed as risky financial assets when held as part of a large portfolio of assets, although the relatively small size of their $b_F$'s indicates that there is relatively low systematic risk in owning them.

It is not clear, however, that our samples of platinum futures are priced to conform to the CAPM interpretation of risk and return. The positive and significant levels of systematic risk found in these contracts imply that the return on platinum futures should be, on average, positive although low, given the low levels of that systematic risk. However, as noted above, the mean excess returns on all the platinum contracts are not significantly different from zero. Moreover, as reported in Table 13, the means of the daily rate of returns for all eight platinum contracts over the entire sample (1980-1998) and over two sub-periods (1980-1989 and 1990-1998) are actually negative.\(^9\) We therefore conclude that this set of results rejects the first hypothesis that platinum contracts are priced according to the CAPM, since the levels of systematic risk borne are not commensurate with the levels of realized returns.

### 2.4 FURTHER INTERPRETATION OF THE REGRESSION RESULTS

Notwithstanding their low size and statistical insignificance, some further observations regarding the estimated betas and their corresponding standard errors (SE) should be made. First, there seems to be a negative relationship between the estimated betas and contract maturity. It can be seen in Table 12 that the longer the interval to maturity, the lower the betas for all contracts. This pattern is not found in

---

contracts with a fixed delivery month where the size of the betas displays no obvious relationship with a particular month.

Furthermore, lower betas in contracts with longer intervals to maturity are accompanied by lower SE. However, the decline in the SE (which implies an increase in the statistical reliability of the coefficient estimates) is not as large as that in the betas, effectively resulting in lower $t$-values for contracts with longer intervals to maturity and thus an even lower statistical significance for their systematic risk.

Thus, when comparing the two compilation methods of futures returns it seems that under the fixed-interval-to-maturity holding strategy, daily returns on longer maturity contracts (for all metals) have somewhat lower covariance with the returns on the market's proxy - reflected in lower and less significant betas. On the other hand, contracts that differ in terms of their delivery month are essentially indistinguishable in terms of the behaviour of the systematic risk and its SE. Nevertheless, in terms of the other regression statistics (that is, $a_F$, DW, and $R^2$) the two methods are indistinguishable. (The two time series methods are further discussed in Section 4.4.)

3. THE CAPM AS A TEST OF THE TRADITIONAL THEORIES (2nd Hypothesis)

Our second research hypothesis deals with the use of the CAPM as a means of accepting or rejecting the three traditional theories of the risk premium (backwardation, contango and expectations) based on the sign and the significance of the beta and the alpha coefficients. Below we restate, in an empirical format, the three jointly tested hypotheses [the theoretical expression of which was stated in Chapter Five as expressions (a), (b) and (c)].

\[
\begin{align*}
\text{If } b_F &= 0 \quad \text{and} \quad a_F = 0 \\
\text{risk premium} &< 0 \\
\text{mean excess returns} &< 0 \\
\end{align*}
\]

Backwardation (i)  
Expectations (ii)  
Contango (iii)

The findings of Table 12, of $b_F$ estimates that are not significantly different from zero for gold, silver and palladium contracts or of zero risk premium, coupled with zero $a_F$ estimates for these contracts, under both methods clearly support the expectation theory as expressed in (ii) above. As noted in Chapter Four, according to the expectation hypothesis, when the underlying commodity has zero systematic risk, the futures price will be an unbiased estimate of the ultimate spot price and expected return on the spot asset and on the futures position will be the risk-free rate. Both long and short futures positions carry no systematic risk and on average expect neither profit nor loss. Indeed, the findings of zero $a_F$'s – the intercept term, expressed in a risk premium format (i.e. returns over and above the risk-free rate) – support the proposition that the return to holders is the risk-free rate of return.
With respect to the platinum contracts, "strictly speaking" the findings in Table 12 of positive and significant systematic risk (i.e. $b_P > 0$) and zero intercept term (i.e. $a_P = 0$) do not support any of the traditional theories, since they are inconsistent with all of the expressions (i), (ii) and (iii) above.

We do note, however, that similar results to those we obtained for all eight platinum contracts were obtained by Chang et al (1990) for all of their copper, platinum and silver contracts over the period 1964-1983 (for copper and platinum) and 1969-1983 (for silver). They interpreted these results as providing support for the theory of normal backwardation. As discussed in Chapter Five, unlike our approach, Chang et al adopted the approach that the Keynesian theory only requires a reward in proportion to the amount of systematic risk borne. And thus mean excess returns of zero coupled with a risk premium of a low magnitude provide sufficient support for backwardation.

4. FURTHER ANALYSIS OF FUTURES RISK & RETURN

Further insight into the application of portfolio theory to futures contracts relative to the traditional Keynesian interpretation of risk and return may be gained from the statistics presented in Table 13. This table presents the mean, the standard deviations, and the kurtosis measure of the daily rate of return for each futures contract and for the S&P500 Index in order to facilitate comparison between futures and equity investment. These descriptive statistics (or estimates of the distribution parameters) were computed for the gold, silver and platinum contracts under both methods over the entire sample period (January 1980 to December 1998) and over two ten year sub-periods (January 1980 to December 1989 and January 1990 to December 1998; see also fn. 98).

The sample period and first sub-period of the palladium contracts commenced in January 1983. Owing to the short sample size of many of the contracts (see Table 8), the statistics were only computed when the sample was comparably long enough. Accordingly, the palladium contracts are largely excluded from the analysis.

---

99 Chang et al tested five, log-relative returns time series with a fixed maturity of 3, 5, 7, 9 and 12 months for copper and silver futures and three time series with a fixed maturity of 4, 7 and 10 months for platinum.

100 See Appendix C for statistical formulas.
<table>
<thead>
<tr>
<th></th>
<th>Mean (%)</th>
<th>SD (%)</th>
<th>Kurtosis</th>
<th>Mean (%)</th>
<th>SD (%)</th>
<th>Kurtosis</th>
<th>Mean (%)</th>
<th>SD (%)</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gold</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-2</td>
<td>-0.0140</td>
<td>1.250</td>
<td>11.55</td>
<td>-0.0133</td>
<td>1.569</td>
<td>8.07</td>
<td>-0.0149</td>
<td>0.750</td>
<td>16.41</td>
</tr>
<tr>
<td>3-4</td>
<td>-0.0145</td>
<td>1.232</td>
<td>10.09</td>
<td>-0.0139</td>
<td>1.541</td>
<td>6.99</td>
<td>-0.0152</td>
<td>0.750</td>
<td>16.42</td>
</tr>
<tr>
<td>5-6</td>
<td>-0.0150</td>
<td>1.223</td>
<td>9.79</td>
<td>-0.0146</td>
<td>1.529</td>
<td>6.77</td>
<td>-0.0154</td>
<td>0.749</td>
<td>16.49</td>
</tr>
<tr>
<td>7-8</td>
<td>-0.0154</td>
<td>1.215</td>
<td>9.52</td>
<td>-0.0151</td>
<td>1.516</td>
<td>6.57</td>
<td>-0.0156</td>
<td>0.748</td>
<td>16.38</td>
</tr>
<tr>
<td>9-10</td>
<td>-0.0157</td>
<td>1.207</td>
<td>9.26</td>
<td>-0.0156</td>
<td>1.505</td>
<td>6.39</td>
<td>-0.0159</td>
<td>0.747</td>
<td>16.25</td>
</tr>
<tr>
<td>11-12</td>
<td>-0.0161</td>
<td>1.200</td>
<td>9.05</td>
<td>-0.0161</td>
<td>1.495</td>
<td>6.23</td>
<td>-0.0161</td>
<td>0.745</td>
<td>16.18</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td>-0.0151</td>
<td>1.221</td>
<td></td>
<td>-0.0148</td>
<td>1.526</td>
<td></td>
<td>-0.0155</td>
<td>0.748</td>
<td></td>
</tr>
<tr>
<td><strong>February</strong></td>
<td>-0.0172</td>
<td>1.270</td>
<td>13.52</td>
<td>-0.0192</td>
<td>1.575</td>
<td>9.49</td>
<td>-0.0149</td>
<td>0.806</td>
<td>22.40</td>
</tr>
<tr>
<td><strong>April</strong></td>
<td>-0.0145</td>
<td>1.328</td>
<td>22.06</td>
<td>-0.0139</td>
<td>1.666</td>
<td>16.01</td>
<td>-0.0152</td>
<td>0.800</td>
<td>19.25</td>
</tr>
<tr>
<td><strong>June</strong></td>
<td>-0.0150</td>
<td>1.304</td>
<td>18.46</td>
<td>-0.0146</td>
<td>1.633</td>
<td>13.35</td>
<td>-0.0154</td>
<td>0.791</td>
<td>18.19</td>
</tr>
<tr>
<td><strong>August</strong></td>
<td>-0.0154</td>
<td>1.302</td>
<td>17.59</td>
<td>-0.0151</td>
<td>1.626</td>
<td>12.70</td>
<td>-0.0156</td>
<td>0.798</td>
<td>19.55</td>
</tr>
<tr>
<td><strong>October</strong></td>
<td>-0.0157</td>
<td>1.305</td>
<td>19.94</td>
<td>-0.0156</td>
<td>1.640</td>
<td>14.41</td>
<td>-0.0159</td>
<td>0.777</td>
<td>16.83</td>
</tr>
<tr>
<td><strong>December</strong></td>
<td>-0.0161</td>
<td>1.286</td>
<td>17.75</td>
<td>-0.0161</td>
<td>1.611</td>
<td>12.86</td>
<td>-0.0161</td>
<td>0.778</td>
<td>16.69</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td>-0.0157</td>
<td>1.299</td>
<td></td>
<td>-0.0158</td>
<td>1.625</td>
<td></td>
<td>-0.0155</td>
<td>0.792</td>
<td></td>
</tr>
<tr>
<td><strong>Silver</strong></td>
<td>-0.0379</td>
<td>1.868</td>
<td></td>
<td>-0.0695</td>
<td>2.149</td>
<td></td>
<td>-0.0026</td>
<td>1.495</td>
<td></td>
</tr>
<tr>
<td><strong>March</strong></td>
<td>-0.0374</td>
<td>1.914</td>
<td>7.67</td>
<td>-0.0691</td>
<td>2.210</td>
<td>6.78</td>
<td>-0.0022</td>
<td>1.517</td>
<td>6.97</td>
</tr>
<tr>
<td><strong>May</strong></td>
<td>-0.0377</td>
<td>1.878</td>
<td>5.70</td>
<td>-0.0696</td>
<td>2.160</td>
<td>4.70</td>
<td>-0.0022</td>
<td>1.502</td>
<td>7.02</td>
</tr>
<tr>
<td><strong>July</strong></td>
<td>-0.0378</td>
<td>1.868</td>
<td>5.79</td>
<td>-0.0696</td>
<td>2.149</td>
<td>4.81</td>
<td>-0.0025</td>
<td>1.494</td>
<td>6.98</td>
</tr>
<tr>
<td><strong>September</strong></td>
<td>-0.0380</td>
<td>1.846</td>
<td>5.78</td>
<td>-0.0695</td>
<td>2.121</td>
<td>4.79</td>
<td>-0.0029</td>
<td>1.481</td>
<td>7.03</td>
</tr>
<tr>
<td><strong>December</strong></td>
<td>-0.0383</td>
<td>1.835</td>
<td>5.83</td>
<td>-0.0697</td>
<td>2.103</td>
<td>4.85</td>
<td>-0.0034</td>
<td>1.479</td>
<td>7.05</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td>-0.0379</td>
<td>1.868</td>
<td></td>
<td>-0.0695</td>
<td>2.149</td>
<td></td>
<td>-0.0026</td>
<td>1.495</td>
<td></td>
</tr>
<tr>
<td><strong>Platinum</strong></td>
<td>-0.0372</td>
<td>1.930</td>
<td>7.58</td>
<td>-0.0690</td>
<td>2.215</td>
<td>6.59</td>
<td>-0.0041</td>
<td>1.526</td>
<td>7.63</td>
</tr>
<tr>
<td><strong>March</strong></td>
<td>-0.0374</td>
<td>1.940</td>
<td>9.75</td>
<td>-0.0691</td>
<td>2.249</td>
<td>8.81</td>
<td>-0.0022</td>
<td>1.521</td>
<td>7.67</td>
</tr>
<tr>
<td><strong>April</strong></td>
<td>-0.0376</td>
<td>1.936</td>
<td>8.21</td>
<td>-0.0690</td>
<td>2.234</td>
<td>7.11</td>
<td>-0.0025</td>
<td>1.537</td>
<td>8.37</td>
</tr>
<tr>
<td><strong>July</strong></td>
<td>-0.0378</td>
<td>1.933</td>
<td>7.55</td>
<td>-0.0690</td>
<td>2.227</td>
<td>6.48</td>
<td>-0.0029</td>
<td>1.540</td>
<td>8.04</td>
</tr>
<tr>
<td><strong>September</strong></td>
<td>-0.0379</td>
<td>1.920</td>
<td>7.97</td>
<td>-0.0690</td>
<td>2.216</td>
<td>7.02</td>
<td>-0.0034</td>
<td>1.522</td>
<td>7.57</td>
</tr>
<tr>
<td><strong>December</strong></td>
<td>-0.0383</td>
<td>1.920</td>
<td>7.58</td>
<td>-0.0689</td>
<td>2.215</td>
<td>6.59</td>
<td>-0.0041</td>
<td>1.526</td>
<td>7.63</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td>-0.0378</td>
<td>1.930</td>
<td></td>
<td>-0.0690</td>
<td>2.226</td>
<td></td>
<td>-0.0030</td>
<td>1.530</td>
<td></td>
</tr>
<tr>
<td><strong>Palladium</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>January</strong></td>
<td>-0.0147</td>
<td>1.698</td>
<td>8.54</td>
<td>-0.0165</td>
<td>2.105</td>
<td>6.53</td>
<td>-0.0128</td>
<td>1.093</td>
<td>5.99</td>
</tr>
<tr>
<td><strong>April</strong></td>
<td>-0.0142</td>
<td>1.726</td>
<td>10.08</td>
<td>-0.0150</td>
<td>2.144</td>
<td>7.63</td>
<td>-0.0133</td>
<td>1.085</td>
<td>5.94</td>
</tr>
<tr>
<td><strong>July</strong></td>
<td>-0.0127</td>
<td>1.731</td>
<td>7.87</td>
<td>-0.0148</td>
<td>2.106</td>
<td>5.95</td>
<td>-0.0100</td>
<td>1.033</td>
<td>7.54</td>
</tr>
<tr>
<td><strong>October</strong></td>
<td>-0.0119</td>
<td>1.755</td>
<td>9.92</td>
<td>-0.0147</td>
<td>2.153</td>
<td>7.49</td>
<td>-0.0083</td>
<td>1.033</td>
<td>6.81</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td>-0.0134</td>
<td>1.727</td>
<td></td>
<td>-0.0152</td>
<td>2.127</td>
<td></td>
<td>-0.0111</td>
<td>1.057</td>
<td></td>
</tr>
<tr>
<td><strong>S&amp;P500</strong></td>
<td>0.0513</td>
<td>0.996</td>
<td>67.34</td>
<td>0.0479</td>
<td>1.104</td>
<td>81.67</td>
<td>0.0551</td>
<td>0.861</td>
<td>9.54</td>
</tr>
</tbody>
</table>
4.1 MEAN AND STANDARD DEVIATION

The first notable finding of Table 13 is that the mean daily returns of all the gold silver and platinum contracts over the entire sample period and over the two sub-periods, under both time series methods, are consistently negative. That is, on average the holders of long positions in these contracts incurred losses. The mean returns of the palladium contracts, on the other hand, are all positive. In the case of gold, all 12 contracts’ means are roughly of the same magnitude, averaging -0.0150 under both methods and over both sub-periods as well as the entire sample period. This, however, is not the position with the silver and the platinum contracts. While their mean returns are of the same magnitude within each construction method, their size is different in the two sub-periods.

The average mean return of the six fixed-interval-to-maturity silver contracts over the period 1980-1989 is -0.0695, which is the same as the average mean for the six fixed-delivery-month contracts over the same period. The average mean return of the same group of contracts over the sample 1990-1998 is -0.0026, which is similar to the -0.0030 average mean of the six fixed-delivery-month contracts over the same period. Over the entire sample period the average mean return of the silver contracts is -0.0379 (fixed interval to maturity) and -0.0378 (fixed delivery month), clearly some average of the two sub-periods. A similar picture emerges with respect to the platinum contracts, with some exceptions due to shorter sample periods of some contracts (see Table).

It is interesting to note that over the entire sample period platinum contracts suffered the lowest average daily losses and silver contracts the highest, irrespective of whether the investment strategy was based on a particular delivery month or on a particular maturity. Clearly, palladium contracts earned the highest average daily returns. Moreover, the silver contracts were more volatile than both platinum and gold, in particular silver contracts with a fixed delivery month. The average standard deviation of the five fixed-delivery-month silver contracts was 1.930 relative to 1.727 for the four platinum contracts and 1.299 for the six gold contracts. Under the fixed-interval-to-maturity method the average standard deviation was 1.868 for silver, 1.671 for platinum and 1.221 for gold. This exact pattern (or order) of volatility between the silver, platinum and gold futures coupled with the same differences in volatility levels between the two time series methods was also found in the two sub-periods, with the first sub-period (1980-1989) consistently displaying higher volatility levels than the second sub-period (1990-1998). To judge how large these standard deviations are, however, we will compare them later to the corresponding standard deviation of the more familiar S&P500 index.

In fact, looking at the means and the standard deviations of the gold silver and platinum contracts, one can discern apparent shifts in the distribution of returns for all three commodities over the two sub-periods. Both the average mean of daily returns, which are in fact daily ‘losses’, and the average standard deviations of the silver and platinum futures over the period 1990-1998 were significantly lower than those for the period 1980-1989. This observation hold with respect to each and every silver
and platinum contract examined (see Table 13). In the case of gold, only average standard deviations (not average losses on contracts) under both time series methods showed major changes, being significantly lower over the period 1990-1998. Several economic factors such as inflation and interest rate levels prevailing during the 1980s compared with those in the 1990s could explain the changes in the variability of precious metals prices and returns. However, this research does not speculate on the causes of the shifts in returns distribution, which is a subject worthy of future research.

4.2 THE KEYNESIAN MEASURE OF ASSET RISK

The standard deviations (SD) in Table 13 are also taken to represent the Keynesian measure of risk relative to the CAPM measures of risk (and return) presented in Table 12. As noted earlier, within the traditional Keynesian framework risk was identified solely in relation to simple variability of prices. Interpreting the standard deviation (or the variance) of returns as a measure of Keynesian risk was the approach taken by Dusak, who noted that “there [was] nothing in Keynes’s essentially heuristic discussion of futures market risk to suggest the use of any one measure of simple variability over another.” 101 The differences in the proposed measures of risk between the portfolio approach and the Keynesian approach facilitate further comparison between the portfolio and the Keynesian interpretation of investing in futures markets.

Within the above context, the most notable finding of Table 13 is that the variability of each and every futures contract studied, with the exception of gold contracts over the sub-period 1990-1998, is greater than the variability of a diversified portfolio of equities. The variability of these 12 gold contracts is approximately the same as that of the S&P500. In other words, based on the standard deviations of returns (over the entire sample), all four precious metal futures are consistently riskier than a diversified portfolio of equities. Incidentally, these findings suggest that the conventional belief that traders in commodity futures bear above-average risk is supported by the data. 102

Moreover, since the variability of futures returns is greater than that of the S&P500 index, one would expect that, if Keynes was correct in identifying asset risk with simple variability, the mean return over and above the risk-free rate would be greater for the futures asset so as to compensate for that higher risk. This is clearly not the position. As can be seen in Table 13, the mean return on each futures contract is much lower, and in fact for most is actually negative, than the mean return on the S&P500

101 Dusak (1976, p.1396) added, however, that the use of sample variance to measure risk is open to objection if the distribution of returns is stable non-Gaussian, which, as noted in the Methodology section, is likely to be the case for futures contracts returns. Nevertheless, the analysis of financial assets’ variance and standard deviation is widely accepted and a common exercise in financial economics studies.

102 The results of Chang et al (1990) are similar to ours with respect to the high standard deviations of their copper, platinum and silver contracts relative to a portfolio of stocks. However, the mean returns were all positive. Dusak, on the other hand, found that the variability of wheat, corn and soybeans futures was similar to that of the S&P500, concluding that futures contracts were not as risky as conventional belief dictated. Still, as we found, the mean returns in her work were negative or very close to zero and substantially lower than the S&P500.
Index. The mean return on the index would have been even higher had it included dividends in the calculation of total returns.

These results seriously undermine the theory of normal backwardation since they show – according to the Keynesian measure of asset risk – that investors who invested in gold, silver, platinum and palladium futures during 1980-1998 incurred risk for which they ‘received’, on average, no compensation at all. Worse, their returns were negative or, in the case of palladium, very close to zero. Clearly these investors would have been better off investing in a diversified portfolio of equity which, for lower risk, would have received on average substantially higher returns.

These findings very clearly suggest that our data on precious metal futures during the entire sample period and over two sub-periods do not conform well to the Keynesian theory of normal backwardation. This result gives further support to our first hypothesis and more generally to Dusak’s entire approach. The low or negative mean returns are clearly more consistent with the CAPM’s measure of risk presented in Table 12 (except in relation to the platinum contracts).

4.3 KURTOSIS AND UNDERLYING DISTRIBUTION

Table 13 also reflects the kurtosis values for all contracts under scrutiny, which as noted earlier confirms the high leptokurtosis in the distribution of futures returns under both methods of computing the time series. All the sample estimates of the kurtosis are far greater than 3, implying the distribution is peaked (leptokurtic) relative to the normal (which have kurtosis of 3). Lowest kurtosis measures of around 4 are found for the platinum and palladium contracts during the sub-periods 1980-1989 and 1983-1989 respectively. However, this too is a high level of kurtosis relative to the normal distribution, given, as noted in Taylor (1986, p.44), that the standard error of an estimate is \( \sqrt{\frac{24}{n}} \) for Gaussian white noise and these estimates exceeds 3 by far more than four SE. It is very clear that the return-generating process of the data in this study is not even approximately Gaussian. Rather, as explained earlier, it suggests that the returns distribution of our metal futures corresponds better to the family of stable Paretian distributions than to the narrower class of normal distribution – subject, however, to further, advanced empirical testing.

In summary, we conclude with an observation made by Dusak (1973, p.1400), which is seemingly supported by the findings of Table 13 (but subject to further testing as indicated above), that the similarities in the estimated distribution parameters suggest that within any commodity group the distribution of returns has the same parameters; the slight differences that do exist represent mere sampling fluctuation.
4.4 THE TWO TIME SERIES METHODS – CONCLUDING OBSERVATIONS

In regard to the two methods, Table 13 demonstrates noticeable patterns between contract interval to maturity and volatility of contract returns. For all four metal futures and for both the whole period and for the two sub-periods, the volatility of the returns declines monotonically with the contract interval to maturity. That is, the longer the maturity of the contract, the lower is the standard deviation. This pattern is in fact perfectly consistent with the conjecture of Samuelson (1965), noted in the Methodology section, that variation in futures (and spot) prices is a decreasing function of maturity. These results are very similar to those obtained by Chang et al (1990), the only study that examined the means and standard deviations of contracts with a fixed time to maturity. On the other hand, there is no noticeable uniform pattern across all four metals and for all sample periods between contract delivery month and the volatility of contract returns.

It is interesting to note at this stage that these findings are consistent with those of Table 12. There, too, we found a negative relationship between contract interval to maturity and the systematic risk (or estimated beta) of the contract, while no noticeable pattern relating to contract delivery month and the estimated beta was identified. Thus the findings suggest that contracts whose delivery months differ are essentially indistinguishable in terms of volatility of returns and systematic risk, while contracts that differ in their maturity are distinguishable. That is, longer maturity contracts offer long speculators the advantages of reducing the systematic risk of their positions as well as lower volatility of returns.

This pattern in volatility (assessed in isolation from the pattern of average returns, that is, irrespective of whether that risk is commensurate with an appropriate level of return) seems to justify the efforts involved in fixing the maturity of a futures position. Similarly, it appears to be advantageous to invest in futures markets on the basis of a particular interval to maturity rather than on the basis of a particular delivery month.

Table 13 also reveals a pattern between the interval to maturity and mean returns but one that is less noticeable and not uniform across all four metal futures. In the case of gold and silver contracts it may be seen that, both for the whole period and for the two sub-periods, the longer the interval to maturity the more negative the mean returns or, in effect, the higher the losses incurred. Similarly, it may be said that the average loss on gold and silver contracts is (nearly) an increasing function of maturity, the opposite of the pattern identified with respect to volatility. This pattern is not found, in a consistent manner, in platinum contracts or palladium contracts. Likewise, we found no noticeable or consistent pattern between a contract’s delivery month and its mean returns.

It is important to note, however, that the differences in the mean returns of contracts within each metal group (or more precisely the increases in mean losses) are of a very small magnitude. This observation, coupled with the fact that the mean returns on gold and silver contracts are all negative, in effect
renders the relationship between contract maturity and mean returns meaningless, which neither justifies nor rejects the argument for fixing contract maturity.

Furthermore, even if we assumed this relationship between contract maturity and mean returns to be significant, the resultant relationship between maturity and both volatility and average returns – or the resultant 'return to volatility' trade-off – is one that makes no investment sense. That is, neither the trade-off of "greater losses to lower volatility for longer maturity" or that of "lower losses to higher volatility for shorter maturity" is attractive enough to entice speculators to take long positions in gold and silver futures.

In summary, while the overall analysis above has identified some differences between the two time series methods, it is clear that, more often than not, the similarities between the two are far greater than the disparities.
CONCLUSION

The analytical and empirical challenge in financial markets research is to identify the underlying mechanism that governs the price behaviour of financial instruments. The essential difficulty arises from the fact that investors’ expectations of the future are not directly observable; expectations may only be inferred from the prices themselves. In these circumstances, a theory of fluctuations in financial market prices is, necessarily, a theory of the process of expectations formation. In particular, in the context of futures markets, the relationship between the futures price, $F_{tT}$, and the expected spot price, $E(S_T)$, is of particular relevance. Naturally, at the heart of the alternative pricing theories examined in this dissertation is a fundamental disagreement about the expectations formation process.

The Keynesian approach to pricing in futures markets, incorporating the theories of backwardation and contango, has been shown to be a somewhat partial theory of the process of expectations formation. Firstly, the Keynesian approach does not conform to the observed movements of futures market prices, predicting incorrectly, for instance, that the futures price under normal conditions gradually rises during the life of a futures contract. Secondly, the Keynesian approach is not based on the rational, optimising decisions of financial market participants, assigning, for instance, a limited and passive role to speculators and thus discounting the price discovery role of futures markets. Thirdly, most importantly, the Keynesian notion of the risk of an asset is identified solely with the price fluctuations of the asset itself, rather than with the asset’s contribution to the risk of a properly diversified portfolio.

It would be unfair, of course, to criticise Keynes (1930) for his early contribution to futures markets research in the context of subsequent analytical advances. However, as indicated in our literature review, it took several years before the innovations proposed by modern portfolio theory—for instance, the early work of Markowitz (1952) and subsequently of Sharpe (1963) and Lintner (1965)—were incorporated into futures markets research, notably by Dusak (1973) and Black (1976). Certainly, futures markets were, at that stage, comparatively new and partially understood features of the financial system. Moreover, early theoretical advances that incorporated a more realistic version of the expectations formation process—notably Telser (1958) and Gray (1961)—made seriously unrealistic corollary assumptions about the investor behaviour, including risk neutrality, perfect foresight, homogeneous expectations, and perfect pricing efficiency. On the whole, following our review of the literature, it would be fair to say that the early research agenda in futures markets was diverted, largely unintentionally, by the influence of Keynes thinking in general, and the enthusiastic and longstanding adoption of Keynes’ theory by researchers involved in futures markets.

In this context, a notable exception is the research of Working (1948, 1949, 1953, 1962), who resisted the Keynesian approach and formulated the theory of storage as a workable, empirically supported alternative. Two important conclusions emerge from comparing Working’s theory with the Keynesian
approach to intertemporal price differences. Firstly, while both approaches supported the hedging role of futures markets, Working provided a radical revision of the hedging theory. The Keynesian theory of hedging essentially imposed the risk premium as the sole explanation of intertemporal price relations and thus corresponded solely with the insurance view of hedging. Working's theory of storage, on the other hand, was capable of incorporating other pricing elements in addition to the risk premium, such as the convenience yield. It was thus capable of explaining the entire range of (observed) price behaviour, from full carrying charges to steep inversion, which could not be explained by the Keynesian theory. In doing so, Working seriously challenged the prevailing Keynesian, risk-avoidance view of hedging, promoting instead the arbitrage- or carrying-charge hedging. Secondly, in the context of Working's theory of storage, which favoured the view that intertemporal price spreads (the basis) reflect prices of storage between the relevant dates, it was clear that there was no possibility that the risk premium could account for any more than a small fraction of intertemporal price differences—if at all. It is particularly interesting that our empirical findings are broadly consistent with Working's hypothesis whereas they contradict Keynes' hypothesis. We concur with Yamey's (1986) assessment that "Working's contribution to the study of the economics of futures trading overshadows those of any of his predecessors, contemporaries or successors" (1986, p.79).

The other serious alternative to the Keynesian approach is offered by modern portfolio theory. As indicated in this dissertation, these approaches differ substantially in their interpretations of risk. For instance, the Keynesian notion of the risk of an asset is identified exclusively with its price behaviour, abstracting from the more realistic case where investors combine different assets, with hopefully uncorrelated returns into a portfolio. In particular, the CAPM suggests that risk is best defined by beta, or the relationship between the expected rate of return on an asset and the expected rate of return on the market portfolio. However, the modern portfolio approach is not without limitations of its own, related mainly to the practical difficulties of testing the theory empirically. As with all empirical tests in financial markets, ex ante or expected returns are not observable, directly or indirectly. In addition, the market portfolio is essentially a hypothetical construct, in the sense that it is observable only in principle. As a result, the conclusions reached in this dissertation are necessarily contingent on the resolution of these fundamental questions in financial economics. Nonetheless, we argue in this dissertation, following the lead of current research, that distinguishing between alternative pricing theories in futures markets remains an essentially empirical issue.

In choosing to test the existence of a risk premium in precious metals, we opted simply to examine an investor in commodity futures and his expected returns from holding a long position in such contracts, similar to the usual approach for equity or bond investments. In the dissertation, we suggested that precious metals, in particular gold and silver, are held for their investment value as well their consumption value, as is the case with other financial assets. Being storable and non-seasonal commodities, precious metals prices do not exhibit distinct seasonal patterns, in contrast with agricultural commodities, where questions of storage and inventory levels exert a significant influence
on price behaviour. Instead, and similar to financial assets, the prices of precious metals are affected by general business conditions, by economic agents' expectations of future supply and demand, and by inflation expectations. As such it is price level risk rather than storage risk that traders aim to shed in precious metals futures markets, implying that arbitrage hedging in such contracts is more of a speculative nature.

All these factors rendered precious metals a suitable candidate for inclusion in an investor's diversified portfolio of assets. In addition, we concluded that while the relevant risk from a general equilibrium standpoint is the risk inherent in the ownership of the spot commodity itself, the variability in futures prices implies that investors with positions in futures markets do bear risk, even though the value of their positions at the end of each day is zero. Assuming this risk to be systematic, the well-known general equilibrium CAPM equation is broadly appropriate, subject to some modification in the equation owing to the special features of futures contracts.

Our empirical work was based on Dusak's methodology and subsequent scholarly work. In reviewing the empirical literature, we highlighted the controversy surrounding some of Dusak's original methodology, in particular her choice of the S&P500 as the appropriate proxy for the market index and the assumptions she made regarding the stationarity of the model's parameters, in particular the beta parameter. Our review of the modifications to Dusak's model suggested by Carter et al. (1983), Baxter, et al. (1985), So, (1987), and Elam and Vaught (1988) led us to the conclusion that Dusak's original assumptions and specifications were broadly appropriate for purposes of our research. Specifically, we found that the use of a single market index (proxied by the S&P500) and the assumption of stationarity in the model parameters (as estimated by alpha and beta) is broadly consistent with the underlying theoretical model, the Sharpe-Lintner single-index single-period CAPM.

Apart from extending Dusak's analysis of agricultural commodities to precious metals, we refined and expanded Dusak's hypothesis and methodology in three ways. Firstly, we split her original hypothesis into two separate components: to treat the CAPM's application to futures markets as an alternative theory (in line with the modern portfolio approach) and to treat the CAPM's application simply as an empirical test (of the theory of forecasting power and the risk premium). Secondly, we constructed the returns time series in two ways, one based on the delivery month, as in Dusak's model, and the other based on the time to contract maturity. Thirdly, we tested daily rather than monthly data, with a significantly longer sample period, ranging from 1980 to 1998. The use of daily data is suggested by the daily settlement basis of futures positions. No other study has used daily returns, focussing instead on monthly or semi-monthly returns, which we believe are less appropriate, given institutional arrangements in futures markets.

The first hypothesis investigated in this dissertation was that the pricing of gold, silver, platinum and palladium futures conformed to the CAPM approach, in that ex post returns on the holding of these
contracts were commensurate with the level of their systematic risk—that is, conformity between the estimated regression coefficients $a_F$ and $b_F$. Specifically, this hypothesis permitted two possible outcomes. The first possible outcome was that the futures contracts considered in the investigation were risky assets and therefore commanded a risk premium (reflected in a positive beta) commensurate with positive \textit{ex post} returns (reflected in a positive alpha). The second was that the entire risk of these contracts was diversifiable (reflected in a zero beta and implying no systematic risk) and thus \textit{ex post} returns would be zero on average (reflected in a zero alpha) because investors would not be rewarded for diversifiable risk. \textit{Ex post} returns were measured both by the alpha coefficient (Table 12) in accordance with Jensen (1968) and by the mean of the log-relative daily returns (Table 13).

Our statistical tests for gold, silver and palladium futures contracts showed that mean excess returns and systematic risk (given by the estimated regression coefficients $a_F$ and $b_F$, respectively) were not significantly different from zero. For gold, silver and palladium futures, under both time series construction methods, we therefore accept the hypothesis that the risk of precious metals contracts is commensurate with the level of \textit{ex post} returns, as predicted by the \textit{ex post} CAPM approach.

By contrast, our statistical tests for platinum futures contracts showed that, while mean excess returns (given by the estimated regression coefficient $a_F$) were not significantly different from zero, the systematic risk (given by $b_F$) was positive and significantly different from zero. Although the beta estimates for platinum futures were generally small, and certainly well below unity—ranging from 0.09 to 0.11—we could not accept the hypothesis that the risk of precious metals futures contracts is commensurate with the level of \textit{ex post} returns. This finding implies that, although platinum contracts command a risk premium, investors who bought platinum futures were not rewarded for taking such risk—an outcome that contradicts one of the premises of the portfolio approach.

The second hypothesis investigated in this dissertation involved the use of the CAPM to test for the existence of a risk premium as hypothesised by the backwardation, contango and expectations theories. Using the same set of results obtained for the first hypothesis, we inferred from the sign and size of the risk premium (as manifested in the beta coefficient) relative to the size and the sign of realized excess returns (as manifested in the alpha coefficient) which of the three theories were supported by the data. In the case of gold, silver and palladium futures, under both time series construction methods, our findings clearly supported the expectations theory, which implies, given the finding of zero systematic risk, that the futures price of these contracts are unbiased estimates of spot prices at expiration, while the expected return on the futures position is the risk free rate of return.

In the case of platinum contracts, our findings were inconsistent with the traditional theories. Still, as noted in an earlier section, some scholars have argued that such results support the backwardation theory. We, however, would argue that, given the low magnitude of the beta coefficients, it is also possible that they support the expectations theory, even though the platinum contracts’ systematic risk
is significantly different from zero. In any case, the overall findings in regard to the platinum contracts point to another possible conclusion. While the evidence is not consistent with the chosen portfolio approach, the pricing of platinum contracts seems to offer an opportunity to risk-averse investors when becoming “short speculators” in these contracts. Selling short platinum futures that have a positive systematic risk and a zero or negative mean in effect creates an investment that is negatively correlated with the rest of the investor’s portfolio, thereby reducing its risk, and does not reduce the average rate of return on the portfolio as a whole (Dusak, 1973; Bodie and Rosansky, 1980).

Regarding the two methods of constructing the returns time series, which represented for us two alternative strategies of investing in futures. The results indicated that under the fixed-interval-to-maturity method, daily returns on longer maturity contracts (for all metals) had somewhat lower covariance with the returns on the market’s proxy (as reflected in lower and less significant betas) as well as lower volatility of returns (as reflected in lower standard deviations). On the other hand, contracts whose delivery months differ were essentially indistinguishable in terms of the behaviour of the systematic risk (and its standard error) and in terms of the standard deviation of the returns. Nevertheless, in terms of the other regression statistics ($a_f$, DW, and $R^2$), the two time series construction methods were indistinguishable.

While it seems reasonable to suggests that the volatility pattern identified in the fixed-interval-to-maturity contracts justifies the efforts involved in this method, when this volatility pattern is combined with the pattern between contract maturity and mean returns, the resultant trade-off between returns and volatility is one that makes no investment sense. We therefore concluded that while the overall analysis has identified some differences between the two time-series methods, more often than not the similarities between the two are clearly far greater than the disparities. This finding implies that it makes no difference which method the investor has chosen to hold (and roll over) his futures position. It is, of course, easier and cheaper to implement the fixed-delivery-month method, without significantly influencing the empirical results.

An interesting additional test is the application of Keynes’ (1930) method, which relates realised returns to the (own) variability of realised returns, to precious metals markets. Generally, our findings suggested that the Keynesian hypothesis that commodities futures are a high-risk investment with a high rate of return was rejected for our sample of precious metals contracts. Our findings showed that gold, silver, platinum and palladium futures are relatively risky—as reflected by the standard deviation of the daily returns compared to the standard deviation in the returns on the S&P500 index for the entire sample period—but that these futures contracts have not offered high realised returns. That is, our findings indicate that investors in precious metals contracts have not received, on average, any compensation for bearing the risk associated with futures contracts, as reflected in the negative or zero rate of returns on all contracts. Thus, our results contradict Keynes’ theory, using even the limited (own-asset) notion of risk associated with Keynes’ research.
These results thus cast a shadow over the theories of normal backwardation and contango and give further support to our first hypothesis and, more generally, to Dusak's approach of looking at the systematic risk rather than at simple variability of prices and returns. As noted above, the precious metals futures tested in this study are not risky assets in the CAPM framework, with only platinum being a borderline case in the light of the very low size of its beta. The relatively high variability of returns (reflected in high standard deviations) must therefore constitute diversifiable risk rather than systematic risk.

We should, however, acknowledge the fact that Keynes and his contemporaries studied only agricultural commodities. In addition, economic and monetary conditions prevailing during our sample period were significantly different from those prevailing during the Keynesian period. These factors raise the distinct possibility of widely different variability in spot and futures prices and in the resultant risk and return characteristics. To some extent, these observations prevent us from determining a direct conflict between the portfolio and the Keynesian views of risk and return in metal futures. We believe that such a conclusion warrants further research into the behavior of precious metal prices relative to that of the commodities studied by Keynes under a very different economic setting.

It is consequently difficult to reconcile the Keynesian and modern portfolio approaches to futures pricing, on account of advances in the structure and functioning of futures markets—and, indeed, the financial system as a whole—since Keynes' original research. Nonetheless, in light of modern developments in financial theory, and our empirical analysis of recent futures market data, it appears that the Keynesian approach has neither strong analytical nor empirical support. As our empirical results indicate, Keynes' predictions do not pass the test of modern portfolio theory, nor even the more limited measures of risk and return commonly available during the peak of the Keynesian approach to futures pricing. The longevity of the Keynesian approach, for instance, its frequent use as a building block in current texts on futures markets, is consequently surprising, in light of both the historical empirical literature and the empirical results presented in this dissertation.

In summary, the empirical results in this dissertation are generally consistent with our theoretical conclusion that, if the Sharp-Lintner single-index single-period CAPM and its assumptions hold at all times, futures pricing will be consistent with the expectation theory and with zero beta futures. The implication of this observation is that the risk of investing in futures contracts is independent of the risk of changes in the value of all assets taken together and is diversifiable. Investors therefore do not have to be paid for assuming that risk. Market forces of supply and demand for futures contracts will drive the prices of positive beta futures up (bringing them back into line with expected spot prices) and expected returns down so that futures contracts yield only the risk-free rate. The reverse will take place with negative beta futures. Mispriced futures contracts could, according to the CAPM, earn excess profit, on average, or reduce the risk of a portfolio without reducing the overall return on that portfolio.
BIBLIOGRAPHY


131

URL:
http://www.cbot.com/ (Chicago Board of Trade)
Appendices
## Appendix A

### CONTRACTS UNDER STUDY

<table>
<thead>
<tr>
<th>TABLE – 1</th>
<th>GOLD</th>
<th>SILVER</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Market</td>
<td>COMEX Division, New York Mercantile Exchange</td>
<td>COMEX Division, New York Mercantile Exchange</td>
</tr>
<tr>
<td>Trading Hours</td>
<td>Floor hours: 8:20 a.m. - 2:30 p.m. (Eastern). NYMEX ACCESS® hours: Sunday: 7:00 p.m. - 8:00 a.m. (Eastern). Mon. to Thur.: 4:00 p.m. - 8:00 a.m. (Eastern)</td>
<td>Floor hours: 8:25 a.m. - 2:25 p.m. (Eastern) NYMEX ACCESS® hours: Sunday: 7:00 p.m. - 8:00 a.m. (Eastern). Mon. to Thur.: 4:00 p.m. - 8:00 a.m. (Eastern)</td>
</tr>
<tr>
<td>Contract Unit (Delivery quantity &amp; quality/grade)</td>
<td>100 troy ounces (5 percent more or less) of refined gold, assaying not less than .995 fineness, cast either in one bar or in three one-kilogram bars and bearing a serial number and identifying stamp of a refiner approved and listed by the COMEX Division.</td>
<td>5,000 troy ounces (6 percent more or less) of refined silver, assaying not less than .999 fineness, cast bars weighing 1,000 or 1,100 troy ounces each and bearing a serial number and identifying stamp of a refiner approved and listed by the Exchange.</td>
</tr>
<tr>
<td>Delivery Months</td>
<td>Current and next two calendar months and any February, April, August, and October within a 23-month period and any June and December within a 60-month trading period beginning with the current month.</td>
<td>Current and next two calendar months and any January, March, May and September, within a 23-month period and any July and December within a 60-month trading period beginning with the current month.</td>
</tr>
<tr>
<td>Ticker Symbol</td>
<td>GC</td>
<td>SI</td>
</tr>
<tr>
<td>Price Quotation</td>
<td>U.S. dollars per troy ounce.</td>
<td>U.S. cents per troy ounce.</td>
</tr>
<tr>
<td>Minimum Price Fluctuation</td>
<td>Ten cents ($0.10) per troy ounce, or $10 per contract.</td>
<td>One-half of one cent ($0.005) per troy ounce, or $25 per contract for outright transactions. Spread transactions are traded at one-tenth of one cent ($0.001) per troy ounce intervals, or $5 per contract.</td>
</tr>
<tr>
<td>Max. Daily Price Fluctuation</td>
<td>$75 per ounce. Trading is halted for 15 minutes when the price limit is reached, after which the market is reopened at 100% of the initial limit.</td>
<td>$1.50. Trading is halted for 15 minutes when the price limit is reached, after which the market is reopened at 100% of the initial limit.</td>
</tr>
<tr>
<td>Speculative Position Limits</td>
<td>Spot month limit of 3,000 contracts. Overall positions are subject to position accountability standards of 7,500 contracts on a net futures equivalent basis.</td>
<td>Spot month limit of 1,500 contracts. Overall positions are subject to position accountability standards of 7,500 contracts on a net futures equivalent basis.</td>
</tr>
<tr>
<td>Last Trading Day</td>
<td>Third last business day of the delivery month.</td>
<td>Third last business day of the delivery month.</td>
</tr>
<tr>
<td>Delivery Points</td>
<td>By warehouse receipt at Exchange – approved depositories.</td>
<td>By warehouse receipt at Exchange – approved depositories.</td>
</tr>
<tr>
<td>First Notice Day</td>
<td>Last business day of the month prior to the delivery month.</td>
<td>Last business day of the month prior to the delivery month.</td>
</tr>
<tr>
<td>Last Notice Day</td>
<td>Second last business day of the delivery month (up to 12 noon).</td>
<td>Second last business day of the delivery month (up to 12 noon).</td>
</tr>
<tr>
<td>TABLE – 2</td>
<td>PLATINUM</td>
<td>PALLADIUM</td>
</tr>
<tr>
<td>-----------</td>
<td>----------</td>
<td>-----------</td>
</tr>
<tr>
<td>The Market</td>
<td>NYMEX Division, New York Mercantile Exchange</td>
<td>NYMEX Division, New York Mercantile Exchange</td>
</tr>
<tr>
<td>Year Contract Began Trading</td>
<td>December 1956</td>
<td>January 1968</td>
</tr>
<tr>
<td>Trading Hours</td>
<td>Floor hours: 8:20 a.m. - 2:30 p.m. (Eastern) NYMEX ACCESSSm hours: Sunday: 7:00 p.m. - 8:00 a.m. (Eastern) Mon. to Thur.: 4:00 p.m. - 8:00 a.m. (Eastern)</td>
<td>Floor hours: 8:10 a.m. - 2:20 p.m. (Eastern) NYMEX ACCESSSm: Sunday: 7:00 p.m. - 8:00 a.m. (Eastern) Mon. to Thur.: 4:00 p.m. - 8:00 a.m. (Eastern)</td>
</tr>
<tr>
<td>Contract Unit (Delivery Qty)</td>
<td>50 troy ounces. Weight tolerance permitted: 5% plus or minus. Multiple bar delivery with no bar to weigh less than 10 ounces.</td>
<td>100 troy ounces. Weight tolerance permitted: 5% plus or minus. Multiple bar delivery with no bar to weigh less than 10 troy ounces.</td>
</tr>
<tr>
<td>Delivery Months</td>
<td>Trading is conducted over 15 months beginning with the current month and next two consecutive months before moving into the quarterly cycle of January, April, July, and October.</td>
<td>Trading is conducted over 15 months beginning with the current month and next two consecutive months before moving into the quarterly cycle of March, June, September, and December.</td>
</tr>
<tr>
<td>Ticker Symbol</td>
<td>PL</td>
<td>PA</td>
</tr>
<tr>
<td>Price Quotation</td>
<td>US dollars and cents per troy ounce.</td>
<td>US dollars and cents per troy ounce</td>
</tr>
<tr>
<td>Min. Price Fluctuation</td>
<td>$0.10 per troy ounce ($5.00 per contract)</td>
<td>$0.05 per troy ounce ($5.00 per contract)</td>
</tr>
<tr>
<td>Max. Daily Price Fluctuation</td>
<td>$25 per ounce ($1,250 per contract). None during the current delivery month and three business days preceding it. Expanded limits of $37.50 and $50 (maximum) take effect in two steps.</td>
<td>$6 per troy ounce ($600 per contract). None during the current delivery month and the three business days preceding it. Expanded limits of $9 and $12 take effect in two steps.</td>
</tr>
<tr>
<td>Position Limits</td>
<td>1,500 contracts for all months combined, but not to exceed 700 contracts in the delivery month.</td>
<td>625 contracts for all months combined, but not to exceed 225 contracts in the delivery month.</td>
</tr>
<tr>
<td>Delivery Notices</td>
<td>By 5:00 p.m. on the last business day of the month preceding delivery month, or any subsequent business day, but not later than the third business day prior to the end of the month.</td>
<td>By 5:00 p.m. on the last business day of the month preceding the delivery month, or any subsequent business day, but no later than the third business day prior to the end of the delivery month.</td>
</tr>
<tr>
<td>Last Trading Day</td>
<td>Fourth business day prior to the end of the delivery month.</td>
<td>Fourth business day prior to the end of the delivery month.</td>
</tr>
<tr>
<td>Deliverable Grade</td>
<td>Minimum platinum content of 99.95%.</td>
<td>Minimum palladium content of 99.95%.</td>
</tr>
<tr>
<td>Delivery Points</td>
<td>By depository receipt in vaults approved by the Exchange.</td>
<td>By depository receipt in vaults approved by the Exchange.</td>
</tr>
<tr>
<td>First Notice Day</td>
<td>Last business day preceding the delivery month.</td>
<td>First business day preceding the delivery month.</td>
</tr>
<tr>
<td>Last Notice Day</td>
<td>Third business day prior to the end of the delivery month.</td>
<td>Third business day prior to the end of the delivery month.</td>
</tr>
<tr>
<td>First Delivery Day</td>
<td>First business day following first notice day</td>
<td>First business day following first notice day</td>
</tr>
</tbody>
</table>
Appendix B
FORWARD AND FUTURES – CONTRACT VALUE AND PRICE

1. THE VALUE OF A FORWARD CONTRACT

The following discussion is based on the work of Black, 1976; Cox et al, 1981; Richard and Sundaresan, 1981; Jarrow and Oldfield, 1981; and French, 1983. The notations used are the same as those defined in the literature review.

The fundamentals of a futures contract valuation, rests on the value of a forward contract at three specific time periods: at initiation, at expiration and prior to expiration. In many respects the two contracts are identical, while the behaviour of interest rates and the marking-to-market procedures are the differentiating features. Diagram 1 below illustrates the value of both a long and a short position in a forward contract as a function of the contract’s exercise price. The diagram equally applies to a futures contract that is held to maturity under the assumption of fixed interest rates and ignoring marking-to-market.

Diagram 1: The Value and Price of a Forward Contract (entered at \( t = 0 \), maturing at \( T \)).

![Diagram of Forward Contract Value](image)

The current market value of a forward contract for delivery at \( T \) (i.e. \( V_{f(t,T)} \)) is a function of the forward price at current time \( t \) for delivery at \( T \) (i.e. \( f_{t,T} \)), of the exercise price which is the price at the date the contract was written (i.e. \( f_{0,T} \)) and of the delivery date. That is:

\[
V_{f(t,T)} = f_{t,T} f_{0,T} T
\]  

(1)

---

If \( V_f \) is the dollar value of a long position, \(-V_f\) is the value of the short position and total market's value is zero.
AT INITIATION (\(t=0\))

As discussed in Chapter One, when written at time \(t=0\), the exercise price gives the forward contract a value of zero (for both the short and long position). This can be expressed as follows:

\[
At \ t=0 \quad V_{f,\{0, T\}} = f(f_{0,T}, T) = 0
\]  

(2)

where at \(t = 0\), the exercise price \(f_{0,T}\) is equal to the forward price \(f_{t,T}\) in (1). That is: \(f_{t,T} = f_{0,T}\).

After time \(t = 0\) the exercise price remains fixed for the specific contract. However, the exercise price for new contracts may be different from the previous exercise price. By definition, \(f_{t,T}\) is the exercise price for a new forward contract written at \(t\), but with the same delivery date \(T\). And thus the exercise price \(f_{t,T}\) need not equal \(f_{0,T}\) for \(t > 0\).

AT EXPIRATION (\(t=T\))

Due to the delivery obligation and arbitrage forces, an expiring forward contract is theoretically equivalent to a spot transaction. Thus, at contract expiration the forward price equals the underlying asset's spot price (ignoring delivery costs). The value of the forward contract at maturity must therefore equal to the spot price at time \(T\) (or \(S_T\)) less the forward contract's exercise price \(f_{0,T}\). That is:

\[
At \ t= T \quad V_{f,\{T, T\}} = f(S_{0,T}, f_{T,T}, T) = S_T - f_{0,T}
\]  

(3)

In other words, the value of a forward contract at maturity, for the long position, is the value of buying the commodity at the exercise price and immediately reselling it at the spot price. The value could be positive or negative, depending on the spot price movement, since the contract must be exercised.

PRIOR TO EXPIRATION (at \(t=t\))

During the life of a forward contract, its value can be positive or negative, while its exercise price stays fixed until settlement. The value of this existing (or 'old') contract will depend on the exercise prices of newly written contracts. Our aim is therefore to find the value of the original/existing contract at any point \(t\), during it life (i.e. finding: \(V_{f,\{0, T\}}\)).

The analytical method used by the studies listed above in addressing this problem is, first, assuming a strategy of duel short and long positions in a contract and, secondly, assuming the existence of a risk free default bonds that pays $1 on day \(T\). Starting with a long position in the original contract written at \(t = 0\) with exercise price equal to \(f_{0,T}\), these studies assume that some time later at \(t = t\) a short position is written in a new contract with same maturity time and with exercise price of \(f_{t,T}\). We note that since the forward contract is illiquid it cannot be sold, but it does have a value because a new contract can be sold to offset the initial position. At expiration date, \(T\), the contract which is long has a value which is equal to: \((S_T - f_{0,T})\), whereas the short contract yields: \(-(S_T - f_{t,T})\) or \((f_{t,T} - S_T)\). The combined value of this strategy (or overall position) at \(T\), is therefore the sum of the long and short position: \([S_T - f_{0,T}] + (f_{t,T} - S_T)\), or a total payoff of \((f_{t,T} - f_{0,T})\). That is, we bought the commodity at \(f_{0,T}\) and sold it at \(f_{t,T}\).
This payoff is certain at time \( t \) since it is independent of the spot price \( S_T \) or the forward price \( f_{T,T} \) at expiration, which as noted above are equal (i.e. \( f_{T,T} = S_T \)).

Furthermore, the value of such position at time \( t \) - during the life of the original contract and at initiation of a newly written contract - is the value of the original contract, say \( V_{ft} \), less (because we short) the value of the new contract, say \( V_{f_{0}(new)} \). It also equal the guaranteed payoff of: \( f_{t,T} - f_{0,T} \) due at expiration, since at time \( t \) (and relative to the original forward contract) we are long in the commodity at \( f_{0,T} \) and can sell it at \( f_{t,T} \) (which equal \( f_{0,T} \) with respect to the new contract). Thus:

\[
\text{Overall position's value at } t = V_{f_{t}(T)} - V_{f_{0}(T)} = \text{PV (payoff at } T) = \text{PV } (f_{t,T} - f_{0,T})
\]  

(4)

In other words, the difference between two forward contracts' values is the present value of the difference between the two forward prices. However, because at \( t \) the second or 'new' forward contract has just been written and thus has a value of zero (as in (1)), the value of a forward contract prior to expiration is simply the value of the original forward contract, \( V_{f_{t}(T)} \) (our unknown), which is equal to the present value of the final payoff discounted at \( R_{t,T} \) (the risk-free rate or the discount rate on a default-free Treasury Bill). If we define \( B_{t,T} \) as the value at time \( t \) of such default-free government bond that pays one dollar at time \( T \), and assuming continuous compounding, we get:

\[
B_{t,T} = e^{-R_{t,T}T}
\]  

(5)

And thus the value of the forward contract prior to expiration, which preclude profitable arbitrage is:

\[
\text{At } t = t \quad V_{f_{t}(0,T)} = V_{f_{0,T}} = B_{t,T} \times (f_{t,T} - f_{0,T}) = (f_{t,T} - f_{0,T})* e^{-R_{t,T}T}
\]  

(6)

In summary, the value of a forward contract at any time \( t \) is the risk-free discounted value of the forward prices difference, which can be positive \( (f_{t,T} > f_{0,T}) \) or negative \( (f_{t,T} < f_{0,T}) \). French (1983, p. 313) emphasised the intertemporal exchange opportunities provided by forward contracts as follows: A trader can make a delay purchase by initiating one forward contract and investing \( f_{t,T} \times B_{t,T} \) dollars in riskless bonds. When the contract mature, the trader receives \( f_{t,T} \) dollars from the bonds and exchange this for one unit of the commodity. In effect, an investment of \( f_{t,T} \times B_{t,T} \) today, yields one unit of the asset on day \( T \).

2 Alternatively, the trader could reverse the strategy and obtain \( f_{t,T} \times B_{t,T} \) today in exchange for the asset at time \( T \).

3
written contract (with the same maturity month), \( F_{t,T} \) is the futures price that is quoted on the futures exchange. Clearly \( F_{t,T} \) need not equal \( F_{0,T} \), but at expiration date \( F_{T,T} \) necessarily equal to the spot price. Thus:

\[
\begin{align*}
\text{At } t = 0 & \\
V_{F,(0,T)} = f(F_{0,T}, F_{t,T}, T) = 0
\end{align*}
\]

(Where \( F_{0,T} = F_{t,T} \))

However, during the life of a futures contract (i.e. during successive days), its value is influence by the daily marking-to-market distribution of payoffs, which implies that a futures contract has a different cash flow stream from a forward contract. As the futures contract matures, an investor must make or receive the daily marking-to-market payoff payments toward the eventual purchase of the commodity\(^4\).

Richard and Sundaresan (1981, p. 348) noted that: "a futures contract is like a forward contract paid for on a unique instalment plan. ... What makes the futures contract unique among instalment plans is that the daily instalments are not specified in advance in the contract but are determined by the daily change in the futures price." If the futures price rises (falls), the long trader receives (makes) a payment from (to) the short trader and the payment equals the rise (fall) in the futures price from the previous day. As noted by Jarrow and Oldfield (1981, p. 377) "while the forward contract's provision specify its value at two particular times \([t = 0 \text{ and } t = T]\), a futures contract's provisions specify its value at time \([t = 0]\) and its cash flow at the end of every trading day."

We let \( t^* \) represent the end of each trading day (say, the first day). If a contract was written at \( t = 0 \) and was held until the end of the day (to \( t = t^* \)), the effect of the marking to market is basically to rewrite the contract at the closing futures price (which is now the 'new' futures price) while the difference in prices is added (or deducted) from the trader account. The change in a futures contract's value during a day is therefore:

\[
\begin{align*}
\text{At } 0 \leq t \leq t^* & \\
V_{F,(0,T)} - V_{F,(0,T)} = F_{t,T} - F_{0,T}
\end{align*}
\]

(8)

And since \( V_{F,(0,T)} = 0 \), equation (8) becomes:

\[
\begin{align*}
\text{At } 0 < t < t^* & \\
V_{F,(0,T)} = F_{t,T} - F_{0,T}
\end{align*}
\]

(9)

Equation (9) is the futures contract's value at time \( t \) after it is written and before the cash payout is distributed at time \( t^* \). It explicitly provides that the cash flow only occurs at the end of each trading day, at which time the value of outstanding contract, \( V_{F,(0*,T)} = f(F_{t*,T}, F_{t,T}, T) \) return to zero with the marking-to-market procedure at the closing exercise price \( F_{t*,T} \), which is also the settlement price. Like forward contracts, futures can have positive or negative values, depending on whether new exercise

\[\text{We note that the total daily installment will be equal to the difference between the maturity exercise price (which equals the spot price) and the futures price that was specified when the contract was initiated.}\]
prices rise or fall during the day relative to the previous day’s settlement price, but the value of the contract after the daily settlement will always be zero, since the value of newly written contract is zero.

3. **FORWARD VS. FUTURES PRICES**

Forward and futures prices, or exercise prices, are often taken to be synonymous, even though they are defined in two very different contracts. In this section we examine the conditions under which the absence of profitable arbitrage compels forward and futures prices to be equal and the conditions that make them differ, assuming discrete time and single good economy. As noted above, at contracts’ expiration the futures price always equal the forward price since both equal the spot price (that is: \( F_{T,T} = S_T \)). The main condition that drives forward and futures prices to be equal at any time before \( T \) (i.e. \( f_{t,T} = F_{t,T} \)) is that risk-free interest rates (\( R \)) are deterministic. The literature uses portfolio strategies and a series of arbitrage trading, starting one trading day before \( T \) (at \( T-1 \)), then two days prior to expiration (at \( T-2 \)), repeating the argument successively to demonstrate that \( f_{t,T} = F_{t,T} \) for all \( t \).

The key factor on which such arbitrage portfolio is based, is that a risk-free perfect hedge can be constructed by traders who can always know with certainty the proper hedge ratio, or the exact number of units of a commodity to be delivered, which under the assumption of non-stochastic \( R \) is equal to \((1/B_{t,T}) \) or \((e^{R(t-T)}) \). Two trading day strategies are sufficient to prove the pricing equality:

**Strategy at \( (T - 1) \):** A trader can buy a forward contract at the exercise price \( f_{T-1,T} \) and simultaneously sell a futures contract of equal size at the market futures price \( F_{T-1,T} \). Both contracts expire in one day and the transaction costs no money. At \( T \), the payoff for the overall position is the following difference:

\[
(f_{T,T} - f_{T-1,T}) - (F_{T,T} - F_{T-1,T}) = F_{T-1,T} - f_{T-1,T} \quad \text{(since } f_{T,T} = F_{T,T}) \tag{10a}
\]

Unless \( F_{T-1,T} = f_{T-1,T} \) a certain risk-free profit can be obtained at \((T-1)\): if \( F_{T-1,T} > f_{T-1,T} \) the profit for the ‘long forward short futures’ position/strategy is positive at no cost. Excess demand for forward contracts and the selling of futures will derive the forward price up and the futures price down until the two prices are equalised. The opposite strategy of ‘long futures short forward’ will be profitable when \( F_{T-1,T} < f_{T-1,T} \) but will also be arbitraged away thus ensuring the equality between the two prices.

**Strategy at \( (T - 2) \):** A trader can buy \((1/B_{T-1,T})\) forward contracts at \( f_{T-2,T} \) and simultaneously establish a short position in futures contracts at \( F_{T-2,T} \). We note that since interest rates are deterministic, the next period’s default-free bond price \( B_{T-1,T} \), is known at \((T-2)\). On the following day, at \((T-1)\), the value of the forward position can be established using equation (6) between times \((T-1)\) and \((T-2)\) and multiply it by the numbers of contracts held. That is:

[\[
(B_{t,T}(f_{T-1,T} - f_{T-2,T}))(1/B_{T-1,T}) = f_{T-1,T} - f_{T-2,T} \tag{11}
\]

Or:

\[
[e^{R(T-1)}(f_{T-1,T} - f_{T-2,T})][e^{R(T-1)}(T-2)] = [e^{R365}(f_{T-1,T} - f_{T-2,T})][e^{R365} = f_{T-1,T} - f_{T-2,T}
\]

142
From equation (9) we know that the value of the position in the futures is \( -(F_{T-1,T} - F_{T-2,T}) \), since the trader is short, or \( (F_{T-2,T} - F_{T-1,T}) \). The combined payoff to the overall strategy at \((T-1)\) is therefore:

\[
(f_{T-1,T} - f_{T-2,T}) - (F_{T-1,T} - F_{T-2,T}) = F_{T-2,T} - f_{T-2,T} \quad \text{(since } f_{T-1,T} = F_{T-1,T} \text{)} \tag{10b}
\]

Again, such positive payoff will be known with certainty at \((T-2)\), so that \(F_{T-2,T}\) must equal \(f_{T-2,T}\) to eliminate risk-free profit. Similar strategies will be taken if \(F_{T-2,T} < 0 > f_{T-2,T}\). It is important to note that, in effect, the number of forward contracts purchased (or the quantity of commodity delivered) is the return earned from continuous reinvestment in a default-free bond from any time \(t\) to \((T-1)\).

Clearly, however, if interest rates are stochastic, the hedge factor is a random variable and the above arbitrage hedge cannot be constructed without some risk, and thus \(f_{t,T}\) need not equal \(F_{t,T}\) for all \(t\) prior to \((T-1)\). Cox et al. (1981) and French (1983), in deriving arbitrage models of futures and forward exercise prices under the assumption of “discrete time single good economy”, provided several summarised equations that also incorporate spot prices. These equilibrium equations, two of which are reviewed below, show that futures and forward prices are linked to the future spot price and the difference between these prices is related to the difference between holding a long-term bond and rolling over a series of one-day bonds.

**EXERCISE PRICE EQUATIONS**

French (1983) based his pricing models on the intertemporal exchange that is available through futures and forward contracts discussed above, noting that with such exchange “investors must be marginally indifferent between \(f_{t,T} \cdot B_{t,T}\) today and one unit of the commodity at \(T\)” (p. 314). This indifference may be used to derive equations, expressed in purely monetary terms, for the forward price and, with some qualification, for the futures price as well.

The equation for the forward price basically demonstrates that using a forward contract to purchase a commodity at time \(T\) is equivalent to purchasing the commodity today, except that the forward contracts allows deferred payment. That is:

\[
f_{t,T} \cdot B_{t,T} = PV_{t,T}(\widetilde{S}_T) \tag{12}
\]

\[
\therefore \quad f_{t,T} = (1/B_{t,T}) \cdot PV_{t,T}(\widetilde{S}_T) = e^{R(t-T)} \cdot PV_{t,T}(\widetilde{S}_T) \tag{13}
\]

where \(\widetilde{S}_T\) is the unknown spot price of the commodity at \(T\) and \(PV_{t,T}(\widetilde{S}_T)\) is the present value at \(t\) of a payment to be received at \(T\). Expression (12) shows that \(f_{t,T} \cdot B_{t,T}\) must be the value on day \(t\) of \(\widetilde{S}_T\) dollars on day \(T\). Equivalently (13) shows that the forward price must equal the present value of the maturity spot price times the gross return from a bond held for \((T-t)\). Clearly the present value of the

\[\text{Richard et al. (1981), Cox, et al (1981) and French (1983) had all also developed similar valuation equations for a continuous time continuous state economy in which forward and futures prices can be different.}\]
maturity spot price is not an observable number, but for assets that pay no dividends or other payment the current spot price must equal the present value of the future spot price. Thus the forward price is:
\[ f_{t,T} = e^{R^*(T-t)} * (S_t) \] (14)

Similar expression for the futures price, originally developed by Cox et al (1981), is as follows:
\[ F_{t,T} = PV_{t,T} \{ \exp \left[ \sum_{r=1}^{T-1} \tilde{R}(t^*, t^*+1) \right] * \tilde{S} \} \] (15)

That is, the futures price must equal the present value of the product of the maturity spot price and the gross return \([.]\) expected from rolling over one day bond. \(\tilde{R}(t^*, t^*+1)\) is the (unknown) continuously compounded interest rate on the daily bond (i.e. from day \(t^*\) to day \(t^*+1\)) and gross return is summed from \((t^* = t)\) to \((T-1)\). In contrast to a forward contract the futures contract cannot be shown to be equivalent to a portfolio composed of spot commodity and default free discount bonds alone. French's (1983, p. 315) clear-sighted conclusion regarding contracts and interest rate is most telling:

*The only cash flow that is relevant to the forward trader is agreed on today and paid on the maturity date. Therefore, the relevant interest rate in determining the forward price is the known yield on a multi period bond. On the other hand, while the futures trader knows the total payments he will have to make, the timing of these cash flows is only determined as the contract matures. Because of this uncertainty, the futures price is a function of the unknown one-day interest rates that are expected to arise over the life of a contract.*

Cox et al further developed several propositions regarding the relation between forward and futures prices. We do not discuss these models in detail but make the following general observations. If interest rates and futures prices are positively correlated, the futures contract will offer an advantage over the forward contract and the futures price would be expected to be above the forward price. Positive correlation with interest rates is advantageous for futures contracts because, assuming a long position if futures prices are rising, positive daily cash flows (from the marking-to-market procedures) could be reinvesting at a similarly rising short rates. If futures prices are falling, negative cash flows can accumulate but will be covered by additional borrowing at falling rates. Clearly a negative correlation would be disadvantageous to futures contracts and one would expect forward prizes to be higher than futures prices. In practice, we would expect positive covariance between most commodity futures prices and interest rates (which is analogue to negative covariance with bond price) due to their relation with inflation expectations (French, 1983; Telser 1986; Barro, 1986). We would therefore expect a positive futures-forward price differential.

---

6 More complete models are beyond our scope.
7 Jarrow et al (1981) used this argument to demonstrate that a futures contract is not a redundant security.
8 For financial assets, the covariance between futures and interest rates is almost always negative, as expected, and the forward price should be above the futures price.
Appendix C

STATISTICAL MEASURES AND TESTS

The distribution of a single asset’s observed returns \( R_1, R_2, \ldots, R_N \) can be summarised by their average value or the mean \( (\bar{R}) \), standard deviation \( (s) \) and kurtosis \( (k) \). These statistics estimate the respective parameters of the process generating returns, if the process is strictly stationary. These statistics are defined and calculated as follows:

Sample Mean:
The sum of actual (historical) return observations \( (\bar{R}) \) divided by the number of observations \( (N) \).

\[
\bar{R} = \frac{1}{N} \sum_{i=1}^{N} R_i
\]

Sample Standard Deviation \( (s\ or\ SD) \):
The most commonly used measure of risk is the standard deviation. The standard deviation is a measure of the variability of the series of returns about its mean return value. It is equal to the positive square root of the sum of the squared differences between each return observation \( (\bar{R}) \) and the mean return \( (\bar{R}) \) divided by the number of observations.

\[
s = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (R_i - \bar{R})^2}
\]

Sample Kurtosis \( (k) \):
Measurement of the peakedness or flatness of the distribution of the returns time series.
Normal distributions have kurtosis \( (k) \) of 3. If \( k \) exceeds 3, the distribution is peaked (leptokurtic) relative to the normal distribution; if it is less than 3, the distribution is flat (platykurtic) relative to the normal. High values of \( k \) are caused by more observations several standard deviations away from the mean than predicted by normal distributions (Taylor, 1986, p.44).

\[
k = \frac{1}{N-1} \sum_{i=1}^{N} (R_i - \bar{R})^4 / s^4
\]
Durbin-Watson (DW) Test (Statistic):
A statistical procedure used to test for first-order serial correlation (or autocorrelation) in the
disturbance or error term \( u_t \) or \( e_t \). More formally, the DW statistic measures the linear association
between adjacent residuals from a regression model. It is a test of the hypothesis that \( \rho \) (the first-order
serial correlation coefficient) is equal to zero in the specification:

\[ u_t = \rho u_{t-1} + e_t \]

(where \( e_t \) is random)

Generally speaking, when \( \rho = 0 \) (reflecting no first-order autocorrelation), the DW statistic will be
around 2.0. If there is negative autocorrelation the DW statistic will lie somewhere between 2 and 4,
while a DW statistic of less than 2.0 suggests positive autocorrelation, which is the most commonly
observed form of dependence. More specifically, in testing for positive autocorrelation, a DW of less
than 1.55 rejects the hypothesis of no (positive) autocorrelation, whereas DW greater than 1.65 accepts
it. However, a DW between 1.55 and 1.65 is inconclusive.

Strong autocorrelation in the error terms could imply either that a significant explanatory variable has
been omitted from the model’s specification and its influences are being reflected in the behaviour of
the error term or that the model has been mis-specified.

Coefficient of Determination \((r^2)\)
A common measure of the “goodness of fit” in regression analysis used to assess the degree of
causation between two variables. The coefficient of determination is equivalent to the square of the
‘correlation coefficient’ \((r)\) and reflects the percent of variation in the dependent variable that is
explained by the variations in the independent variable.

Within the context of this dissertation the dependent variable is the random rate of returns on a risky
asset \( i (\widetilde{R}_i) \), where the risky asset being a futures contract, and the independent variable is the random
rate of returns on the market portfolio \( (\widetilde{R}_m) \). The correlation coefficient between these two variable
\( (r_{i,m}) \) is equal to the covariance between the returns on the risky asset and the returns on the market,
divided by the standard deviation of each, as stated below.

The value of the coefficient of determination varies between 0 (0%) and 1 (100%) with higher values
representing better explanatory power of a model in explaining the trends in historical data.

The relation between \( r^2 \) and the \( \beta \) coefficient, noted in the Empirical Findings Section, is as follows:

\[ r_{i,m} = \text{Cov}(\widetilde{R}_i, \widetilde{R}_m)/\sigma(\widetilde{R}_i)*\sigma(\widetilde{R}_m) \]

\[ \text{Cov}(\widetilde{R}_i, \widetilde{R}_m) = r_{i,m} \sigma(\widetilde{R}_i) \times \sigma(\widetilde{R}_m) \]

\[ \beta_i = \text{Cov}(\widetilde{R}_i, \widetilde{R}_m)/\sigma(\widetilde{R}_m) \]

\[ \beta_i = r_{i,m} \sigma(\widetilde{R}_i)/\sigma(\widetilde{R}_m) \]

\[ \beta_i = r_{i,m} \sigma(\widetilde{R}_i)/\sigma(\widetilde{R}_m) \]
### Appendix D

**TIME SERIES CONSTRUCTION METHOD**

#### FIXED INTERVAL TO MATURITY - SILVER

<table>
<thead>
<tr>
<th>Silver</th>
<th>Time series dates</th>
<th>Futures contracts contributing the data (and time to maturity in brackets)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-31/1/1980</td>
<td>1-3mth to maturity</td>
<td><strong>Series 1</strong> 3-5mth to maturity <strong>Series 2</strong> 5-7mth to maturity <strong>Series 3</strong> 7-9mth to maturity <strong>Series 4</strong> 9-11mth to maturity</td>
</tr>
<tr>
<td>1-31/1/1980</td>
<td>March80(2mth) May80(4mth) July80(6mth) Sept80(8mth) Dec80(10mth)</td>
<td></td>
</tr>
<tr>
<td>1-31/1/1980</td>
<td>March80(1mth) May80(3mth) July80(5mth) Sept80(7mth) Dec80(9mth)</td>
<td></td>
</tr>
<tr>
<td>1-31/5/1980</td>
<td>July80(2mth) Sept80(4mth) Dec80(7mth) March81(9mth) May81(10mth)</td>
<td></td>
</tr>
<tr>
<td>1-30/6/1980</td>
<td>July80(1mth) Sept80(3mth) Dec80(6mth) March81(8mth) May81(9mth)</td>
<td></td>
</tr>
<tr>
<td>1-31/7/1980</td>
<td>Sept80(2mth) Dec80(5mth) March81(7mth) May81(8mth) June81(10mth)</td>
<td></td>
</tr>
<tr>
<td>1-31/10/1980</td>
<td>Dec80(2mth) March81(5mth) March81(5mth) May81(7mth) July81(9mth)</td>
<td></td>
</tr>
<tr>
<td>1-31/12/1980</td>
<td>March81(3mth) March81(3mth) May81(5mth) July81(7mth) Sept81(9mth)</td>
<td></td>
</tr>
<tr>
<td>1-28/2/1981</td>
<td>March81(1mth) May81(3mth) July81(5mth) Sept81(7mth) Dec81(9mth)</td>
<td></td>
</tr>
<tr>
<td>1-30/6/81</td>
<td>July81(1mth) Sept81(3mth) Dec81(6mth) March82(9mth) March82(9mth)</td>
<td></td>
</tr>
<tr>
<td>1-31/7/1981</td>
<td>Sept81(2mth) Dec81(5mth) March82(8mth) May82(10mth)</td>
<td></td>
</tr>
<tr>
<td>1-31/8/1981</td>
<td>Sept81(1mth) Dec81(4mth) March82(7mth) May82(9mth) July82(10mth)</td>
<td></td>
</tr>
<tr>
<td>1-30/9/1981</td>
<td>Dec81(3mth) Dec81(3mth) March82(6mth) May82(8mth) July82(9mth)</td>
<td></td>
</tr>
<tr>
<td>1-31/10/1981</td>
<td>Dec81(2mth) March82(5mth) May82(7mth) July82(9mth)</td>
<td></td>
</tr>
<tr>
<td>1-31/11/1981</td>
<td>Dec81(1mth) March82(4mth) May82(6mth) July82(8mth) Sept82(10mth)</td>
<td></td>
</tr>
<tr>
<td>1-31/12/1981</td>
<td>March82(3mth) March82(3mth) May82(5mth) July82(7mth) Sept82(9mth)</td>
<td></td>
</tr>
<tr>
<td>1-31/1/1982</td>
<td>March82(2mth) May82(4mth) July82(6mth) Sept82(8mth) Dec82(11mth)</td>
<td></td>
</tr>
<tr>
<td>1-28/2/1982</td>
<td>March82(1mth) May82(3mth) July82(5mth) Sept82(7mth) Dec82(9mth)</td>
<td></td>
</tr>
<tr>
<td>1-31/3/1982</td>
<td>May82(2mth) July82(4mth) Sept82(6mth) Dec82(9mth) Dec82(9mth)</td>
<td></td>
</tr>
<tr>
<td>1-30/4/1982</td>
<td>May82(1mth) July82(3mth) Sept82(5mth) Dec82(8mth) March83(11mth)</td>
<td></td>
</tr>
<tr>
<td>etc...</td>
<td>etc...</td>
<td>etc...</td>
</tr>
</tbody>
</table>
Appendix E

GOLD AND SILVER—THE BEHAVIOUR OF FUTURES PRICES TIME SERIES

Gold price behavior under the fixed-interval-to-maturity method (Graph 1) and fixed-delivery-month method (Graph 3) for the entire sample period: 1980-1998 and for the sub-period: 1992-1996 (Graphs 2&4). USS/ounce.
Silver price behavior under the fixed-interval-to-maturity method (Graph 1) and fixed-delivery-month method (Graph 3) for the sample period: 1981-1998 and for the sub-period: 1994-1996 (Graphs 2 & 4). US cents per ounce.
Appendix F
THE BEHAVIOUR OF OPEN INTEREST

Open interest is closely related to volume\(^6\) and provides an indication of tradability and general interest (or demand and supply) in futures markets. As can be seen on the right-hand side of the graphs below the fixed-delivery-month method has indeed yielded a consistent 12-month wave-pattern of the open interest in all contracts' time series. Furthermore, that pattern is of an equal magnitude and displays similar variability within each metal group. The exception is the 'October-gold' contract in which open interest is unusually low, presumably since this contract does not attract much trading interest.

In addition, the behaviour of gold's time series is very similar to silver's time series. The open interest pattern revealed by the fixed-delivery-months method is one in which, at the beginning of each 12-month cycle, contracts are progressively opened and open interest rises continuously and steeply. (For example, there could be up to 120,000-140,000 contracts within a few months in gold and up to 5,000-6,000 contracts in palladium.) Later, as each contract's delivery date nears, more positions are progressively closed out and open interest falls substantially. Since we excluded the actual delivery month, in none of the contracts does open interest decline to zero. An almost identical 12-month pattern repeats itself throughout the sample period, with similar levels of open interest. Thus under this method the liquidity of contracts remains relatively constant within each metal group and over the sample years.

The difference in the tradability and liquidity of gold and silver relative to platinum and in particular to palladium is striking.

The behaviour of open interest under the fixed-interval-to-maturity method (the left-hand side of the graphs below) is very different for each metal's maturity contracts but is consistent when each maturity group is compared across the different metals and overall is very much as expected. Essentially this second time series construction method illustrates the fact that open interest and liquidity are greatest for nearby maturity contracts (1-2 months and 3-4 months to delivery), and relatively low for succeeding distant maturity time series groups.

\(^6\) Open-interest stock figures are widely watched market statistics published daily for every commodity contract. They are positively related, albeit not directly, to volume. See Telsor and Hughbotham (1977) for more details. In plotting volume levels for all constructed price time series (38 in total) we obtained a picture very similar to that illustrated by the plots of open interest levels for all contracts. Importantly, however, open interest represents a stock variable and volume represents a flow variable, a distinction that explains the difference between the behaviour of the two.
Gold (cont.)

- Open interest 0-10 months
  - 140,000
  - 120,000
  - 100,000
  - 80,000
  - 60,000
  - 40,000
  - 20,000

- Open interest October
  - 140,000
  - 120,000
  - 100,000
  - 80,000
  - 60,000
  - 40,000
  - 20,000

- Open interest November
  - 140,000
  - 120,000
  - 100,000
  - 80,000
  - 60,000
  - 40,000
  - 20,000

- Open interest December
  - 140,000
  - 120,000
  - 100,000
  - 80,000
  - 60,000
  - 40,000
  - 20,000
Silver contracts

Fixed-interval-to-maturity

Fixed-delivery-month

Open interest 1-3 months

Open interest March

Open interest 3-6 months

Open interest May

Open interest 5-7 months

Open interest June

Open interest 7-9 months

Open interest September
Silver (cont.)

Open Interest 5-11 months

Open Interest December
Platinum contracts

Fixed-interval-to-maturity

Fixed-delivery-

Open interest 1-3 months

Open interest January

Open interest 4-6 months

Open interest April

Open interest 7-8 months

Open interest July (Jan. 1980 - June 1987)

Open interest 10-12 months

Open interest October (Jan. 1987 - Sept. 1987)
 Palladium contracts

Fixed-interval-to-maturity

Open interest 1-3 months (Jan. 1983 - Dec. 1994)

Fixed-delivery-month

Open interest March (Jan. 1983 - Feb. 1994)

Open interest June (Jan. 1983 - May 1994)


Open interest December (Jan. 1983 - Nov. 1994)
## Appendix G

**ORDINARY LEAST SQUARE REGRESSION RESULTS\(^\text{10}\)**

### Gold: Fixed-Interval-to-Maturity Method

**Dependent Variable:** GOLD1-2  
**Method:** Least Squares  
**Sample:** 1/03/1990 12/31/1996  
**Included observations:** 4783

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.000159</td>
<td>0.000181</td>
<td>-0.831200</td>
<td>0.4059</td>
</tr>
<tr>
<td>PREM</td>
<td>0.030759</td>
<td>0.018139</td>
<td>1.695668</td>
<td>0.0900</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.000601</td>
<td></td>
<td>-0.000140</td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.000392</td>
<td></td>
<td>0.012500</td>
<td></td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.012498</td>
<td></td>
<td>-5.926138</td>
<td></td>
</tr>
<tr>
<td>Sum squared resid</td>
<td>0.746747</td>
<td></td>
<td>-5.923432</td>
<td></td>
</tr>
<tr>
<td>Log likelihood</td>
<td>14174.36</td>
<td></td>
<td>2.875291</td>
<td></td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>2.028957</td>
<td></td>
<td>0.090014</td>
<td></td>
</tr>
</tbody>
</table>

**Dependent Variable:** GOLD3-4  
**Method:** Least Squares  
**Sample:** 1/03/1990 12/31/1996  
**Included observations:** 4783

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.000154</td>
<td>0.000178</td>
<td>-0.861716</td>
<td>0.3889</td>
</tr>
<tr>
<td>PREM</td>
<td>0.026265</td>
<td>0.017873</td>
<td>1.469510</td>
<td>0.1418</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.000451</td>
<td></td>
<td>-0.000145</td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.000242</td>
<td></td>
<td>0.012316</td>
<td></td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.012314</td>
<td></td>
<td>-5.956735</td>
<td></td>
</tr>
<tr>
<td>Sum squared resid</td>
<td>0.724970</td>
<td></td>
<td>-5.953028</td>
<td></td>
</tr>
<tr>
<td>Log likelihood</td>
<td>14245.14</td>
<td></td>
<td>2.159495</td>
<td></td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>1.986004</td>
<td></td>
<td>0.141760</td>
<td></td>
</tr>
</tbody>
</table>

**Dependent Variable:** GOLD5-6  
**Method:** Least Squares  
**Sample:** 1/03/1990 12/31/1996  
**Included observations:** 4783

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.000157</td>
<td>0.000177</td>
<td>-0.866586</td>
<td>0.3759</td>
</tr>
<tr>
<td>PREM</td>
<td>0.021993</td>
<td>0.017755</td>
<td>1.238671</td>
<td>0.2155</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.000321</td>
<td></td>
<td>-0.000150</td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.000112</td>
<td></td>
<td>0.012233</td>
<td></td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.012233</td>
<td></td>
<td>-5.969862</td>
<td></td>
</tr>
<tr>
<td>Sum squared resid</td>
<td>0.715430</td>
<td></td>
<td>-5.966275</td>
<td></td>
</tr>
<tr>
<td>Log likelihood</td>
<td>14276.82</td>
<td></td>
<td>1.534305</td>
<td></td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>1.985046</td>
<td></td>
<td>0.215529</td>
<td></td>
</tr>
</tbody>
</table>

**Dependent Variable:** GOLD7-8  
**Method:** Least Squares  
**Sample:** 1/03/1990 12/31/1996  
**Included observations:** 4783

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.000159</td>
<td>0.000176</td>
<td>-0.906991</td>
<td>0.3845</td>
</tr>
<tr>
<td>PREM</td>
<td>0.017995</td>
<td>0.017635</td>
<td>1.020379</td>
<td>0.3076</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.000218</td>
<td></td>
<td>-0.000154</td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.000009</td>
<td></td>
<td>0.012150</td>
<td></td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.012150</td>
<td></td>
<td>-5.862531</td>
<td></td>
</tr>
<tr>
<td>Sum squared resid</td>
<td>0.705802</td>
<td></td>
<td>-5.979924</td>
<td></td>
</tr>
<tr>
<td>Log likelihood</td>
<td>14308.22</td>
<td></td>
<td>1.041173</td>
<td></td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>1.972575</td>
<td></td>
<td>0.307600</td>
<td></td>
</tr>
</tbody>
</table>

\(^{10}\) Generated with EViews software.
### Gold: Fixed-Delivery-Month Method

**Dependent Variable: FEBRUARY**

**Method: Least Squares**

**Sample: 1/03/1980 12/31/1998**

**Included observations: 4783**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.000173</td>
<td>0.000184</td>
<td>-0.939561</td>
<td>0.3475</td>
</tr>
<tr>
<td>PREM</td>
<td>0.002923</td>
<td>0.018440</td>
<td>0.158534</td>
<td>0.8740</td>
</tr>
</tbody>
</table>

**R-squared**

**Adjusted R-squared**

**S.E. of regression**

**Sum squared resid**

**Log likelihood**

**Durbin-Watson stat**

### Gold: Fixed-Delivery-Month Method

**Dependent Variable: APRIL**

**Method: Least Squares**

**Sample: 1/03/1980 12/31/1998**

**Included observations: 4783**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.000152</td>
<td>0.000192</td>
<td>-0.792496</td>
<td>0.4281</td>
</tr>
<tr>
<td>PREM</td>
<td>0.022509</td>
<td>0.019281</td>
<td>1.167419</td>
<td>0.2431</td>
</tr>
</tbody>
</table>

**R-squared**

**Adjusted R-squared**

**S.E. of regression**

**Sum squared resid**

**Log likelihood**

**Durbin-Watson stat**
### Dependent Variable: JUNE

**Method:** Least Squares  
**Sample:** 1/03/1980 12/31/1996  
**Included observations:** 4783

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.000157</td>
<td>0.000189</td>
<td>-0.834233</td>
<td>0.4042</td>
</tr>
<tr>
<td>PREM</td>
<td>0.024055</td>
<td>0.018928</td>
<td>1.270898</td>
<td>0.2038</td>
</tr>
<tr>
<td><strong>R-squared</strong></td>
<td>0.000338</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.000129</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.013041</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sum squared resid</td>
<td>0.813061</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log likelihood</td>
<td>13970.89</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>1.982067</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Dependent Variable: AUGUST

**Method:** Least Squares  
**Sample:** 1/03/1980 12/31/1996  
**Included observations:** 4783

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.000161</td>
<td>0.000189</td>
<td>-0.857482</td>
<td>0.3912</td>
</tr>
<tr>
<td>PREM</td>
<td>0.024298</td>
<td>0.018991</td>
<td>1.286283</td>
<td>0.1964</td>
</tr>
<tr>
<td><strong>R-squared</strong></td>
<td>0.000346</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.000137</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.013015</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sum squared resid</td>
<td>0.809874</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log likelihood</td>
<td>13960.29</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>1.985397</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Dependent Variable: OCTOBER

**Method:** Least Squares  
**Sample:** 1/03/1980 12/31/1996  
**Included observations:** 4783

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.000168</td>
<td>0.000189</td>
<td>-0.890720</td>
<td>0.3731</td>
</tr>
<tr>
<td>PREM</td>
<td>0.033150</td>
<td>0.018931</td>
<td>1.751041</td>
<td>0.0800</td>
</tr>
<tr>
<td><strong>R-squared</strong></td>
<td>0.000641</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.000432</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.013043</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sum squared resid</td>
<td>0.813377</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log likelihood</td>
<td>13969.96</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>1.984606</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Dependent Variable: DECEMBER

**Method:** Least Squares  
**Sample:** 1/03/1980 12/31/1996  
**Included observations:** 4783

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.000166</td>
<td>0.000186</td>
<td>-0.890001</td>
<td>0.3735</td>
</tr>
<tr>
<td>PREM</td>
<td>0.013683</td>
<td>0.018662</td>
<td>0.733215</td>
<td>0.4635</td>
</tr>
<tr>
<td><strong>R-squared</strong></td>
<td>0.000112</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>-0.000097</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.012857</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sum squared resid</td>
<td>0.793071</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log likelihood</td>
<td>14034.58</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>1.970658</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Silver: Fixed-Interval-to-Maturity Method

Dependent Variable: SILVER1-3
Method: Least Squares
Sample: 1/03/1980 12/31/1996
Included observations: 4783

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.000386</td>
<td>0.000277</td>
<td>-1.39415</td>
<td>0.1633</td>
</tr>
<tr>
<td>PREM</td>
<td>0.037089</td>
<td>0.027778</td>
<td>1.334452</td>
<td>0.1821</td>
</tr>
</tbody>
</table>

R-squared: 0.000372
Adjusted R-squared: 0.000163
S.E. of regression: 0.019138
Sum squared resid: 1.751169
Log likelihood: 12136.06
Durbin-Watson stat: 1.917573

Dependent Variable: SILVER3-5
Method: Least Squares
Sample: 1/03/1980 12/31/1996
Included observations: 4783

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.000389</td>
<td>0.000272</td>
<td>-1.431574</td>
<td>0.1523</td>
</tr>
<tr>
<td>PREM</td>
<td>0.036764</td>
<td>0.027256</td>
<td>1.348832</td>
<td>0.1775</td>
</tr>
</tbody>
</table>

R-squared: 0.000380
Adjusted R-squared: 0.000171
S.E. of regression: 0.018779
Sum squared resid: 1.686014
Log likelihood: 12226.73
Durbin-Watson stat: 1.909830

Dependent Variable: SILVER5-7
Method: Least Squares
Sample: 1/03/1980 12/31/1996
Included observations: 4783

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.000389</td>
<td>0.000270</td>
<td>-1.438190</td>
<td>0.1504</td>
</tr>
<tr>
<td>PREM</td>
<td>0.031373</td>
<td>0.027106</td>
<td>1.157382</td>
<td>0.2472</td>
</tr>
</tbody>
</table>

R-squared: 0.000280
Adjusted R-squared: 0.000071
S.E. of regression: 0.018676
Sum squared resid: 1.887515
Log likelihood: 12253.12
Durbin-Watson stat: 1.907005

Dependent Variable: SILVER7-9
Method: Least Squares
Sample: 1/03/1980 12/31/1996
Included observations: 4783

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.000389</td>
<td>0.000267</td>
<td>-1.438694</td>
<td>0.1483</td>
</tr>
<tr>
<td>PREM</td>
<td>0.027371</td>
<td>0.026797</td>
<td>1.021386</td>
<td>0.3071</td>
</tr>
</tbody>
</table>

R-squared: 0.000216
Adjusted R-squared: 0.000009
S.E. of regression: 0.018463
Sum squared resid: 1.629717
Log likelihood: 12307.95
Durbin-Watson stat: 1.899574

Dependent Variable: FUT9-11
Method: Least Squares
Sample: 1/03/1980 12/31/1996
Included observations: 4783

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.000390</td>
<td>0.000265</td>
<td>-1.470025</td>
<td>0.1416</td>
</tr>
<tr>
<td>PREM</td>
<td>0.021052</td>
<td>0.026630</td>
<td>0.790534</td>
<td>0.4293</td>
</tr>
</tbody>
</table>

R-squared: 0.000131
Adjusted R-squared: 0.000078
S.E. of regression: 0.018347
Sum squared resid: 1.809399
Log likelihood: 12337.95
Durbin-Watson stat: 1.897210

---

160
Silver: Fixed-Delivery-Month Method

Dependent Variable: MARCH
Method: Least Squares
Included observations: 4783

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.000386</td>
<td>0.000281</td>
<td>-1.374820</td>
<td>0.1822</td>
</tr>
<tr>
<td>PREM</td>
<td>0.036475</td>
<td>0.028156</td>
<td>1.295471</td>
<td>0.1952</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.000351</td>
<td>Mean dep. var</td>
<td>-0.000374</td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.000142</td>
<td>S.D. dep. var</td>
<td>0.019400</td>
<td></td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.019399</td>
<td>Akaike info criterion</td>
<td>-5.046901</td>
<td></td>
</tr>
<tr>
<td>Sum squared resid</td>
<td>1.799141</td>
<td>Schwarz criterion</td>
<td>-5.044094</td>
<td></td>
</tr>
<tr>
<td>Log likelihood</td>
<td>12071.42</td>
<td>F-statistic</td>
<td>1.678245</td>
<td></td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>1.935655</td>
<td>Prob(F-statistic)</td>
<td>0.195220</td>
<td></td>
</tr>
</tbody>
</table>

Dependent Variable: MAY
Method: Least Squares
Included observations: 4783

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.000382</td>
<td>0.000280</td>
<td>-1.363988</td>
<td>0.1726</td>
</tr>
<tr>
<td>PREM</td>
<td>0.019869</td>
<td>0.028103</td>
<td>0.705985</td>
<td>0.4796</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.000105</td>
<td>Mean dep. var</td>
<td>-0.000376</td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>-0.000105</td>
<td>S.D. dep. var</td>
<td>0.019361</td>
<td></td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.019362</td>
<td>Akaike info criterion</td>
<td>-5.050541</td>
<td></td>
</tr>
<tr>
<td>Sum squared resid</td>
<td>1.792425</td>
<td>Schwarz criterion</td>
<td>-5.047834</td>
<td></td>
</tr>
<tr>
<td>Log likelihood</td>
<td>12080.37</td>
<td>F-statistic</td>
<td>0.499827</td>
<td></td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>1.907547</td>
<td>Prob(F-statistic)</td>
<td>0.479611</td>
<td></td>
</tr>
</tbody>
</table>

Dependent Variable: JULY
Method: Least Squares
Included observations: 4783

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.000385</td>
<td>0.000280</td>
<td>-1.376976</td>
<td>0.1687</td>
</tr>
<tr>
<td>PREM</td>
<td>0.023093</td>
<td>0.028054</td>
<td>0.823185</td>
<td>0.4105</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.000142</td>
<td>Mean dep. var</td>
<td>-0.000376</td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>-0.000067</td>
<td>S.D. dep. var</td>
<td>0.019328</td>
<td></td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.019329</td>
<td>Akaike info criterion</td>
<td>-5.054031</td>
<td></td>
</tr>
<tr>
<td>Sum squared resid</td>
<td>1.786190</td>
<td>Schwarz criterion</td>
<td>-5.051324</td>
<td></td>
</tr>
<tr>
<td>Log likelihood</td>
<td>12068.71</td>
<td>F-statistic</td>
<td>0.677500</td>
<td></td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>1.906325</td>
<td>Prob(F-statistic)</td>
<td>0.410455</td>
<td></td>
</tr>
</tbody>
</table>

Dependent Variable: SEPTEMBER
Method: Least Squares
Included observations: 4783

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.000385</td>
<td>0.000278</td>
<td>-1.396839</td>
<td>0.1656</td>
</tr>
<tr>
<td>PREM</td>
<td>0.018250</td>
<td>0.027868</td>
<td>0.654895</td>
<td>0.5128</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.000090</td>
<td>Mean dep. var</td>
<td>-0.000379</td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>-0.000119</td>
<td>S.D. dep. var</td>
<td>0.019199</td>
<td></td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.019200</td>
<td>Akaike info criterion</td>
<td>-5.067384</td>
<td></td>
</tr>
<tr>
<td>Sum squared resid</td>
<td>1.762488</td>
<td>Schwarz criterion</td>
<td>-5.064677</td>
<td></td>
</tr>
<tr>
<td>Log likelihood</td>
<td>12120.65</td>
<td>F-statistic</td>
<td>0.428874</td>
<td></td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>1.892242</td>
<td>Prob(F-statistic)</td>
<td>0.512574</td>
<td></td>
</tr>
</tbody>
</table>

Dependent Variable: DECEMBER
Method: Least Squares
Included observations: 4783

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.000389</td>
<td>0.000278</td>
<td>-1.401059</td>
<td>0.1613</td>
</tr>
<tr>
<td>PREM</td>
<td>0.020681</td>
<td>0.027874</td>
<td>0.741946</td>
<td>0.4582</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.000115</td>
<td>Mean dep. var</td>
<td>-0.000383</td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>-0.000094</td>
<td>S.D. dep. var</td>
<td>0.019203</td>
<td></td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.019204</td>
<td>Akaike info criterion</td>
<td>-5.069960</td>
<td></td>
</tr>
<tr>
<td>Sum squared resid</td>
<td>1.763235</td>
<td>Schwarz criterion</td>
<td>-5.064253</td>
<td></td>
</tr>
<tr>
<td>Log likelihood</td>
<td>12119.83</td>
<td>F-statistic</td>
<td>0.550483</td>
<td></td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>1.910923</td>
<td>Prob(F-statistic)</td>
<td>0.458157</td>
<td></td>
</tr>
</tbody>
</table>
Platinum: Fixed-Interval-to-Maturity Method

Dependent Variable: PLAT1-3
Method: Least Squares
Included observations: 4771

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.000179</td>
<td>0.000249</td>
<td>-0.720353</td>
<td>0.4713</td>
</tr>
<tr>
<td>PREM</td>
<td>0.111336</td>
<td>0.024929</td>
<td>4.469199</td>
<td>0.0000</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.004165</td>
<td>Mean depend var</td>
<td>-0.000143</td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.003956</td>
<td>S.D. depend var</td>
<td>0.017227</td>
<td></td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.017193</td>
<td>Akaike info criterion</td>
<td>-5.288226</td>
<td></td>
</tr>
<tr>
<td>Sum squared resid</td>
<td>1.409690</td>
<td>Schwarz criterion</td>
<td>-5.265514</td>
<td></td>
</tr>
<tr>
<td>Log likelihood</td>
<td>12617.06</td>
<td>F-statistic</td>
<td>19.94893</td>
<td></td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>1.974253</td>
<td>Prob(f-statistic)</td>
<td>0.000008</td>
<td></td>
</tr>
</tbody>
</table>

Dependent Variable: PLAT4-6
Method: Least Squares
Included observations: 4771

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.000175</td>
<td>0.000242</td>
<td>-0.721021</td>
<td>0.4709</td>
</tr>
<tr>
<td>PREM</td>
<td>0.096846</td>
<td>0.024238</td>
<td>3.995749</td>
<td>0.0001</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.003337</td>
<td>Mean depend var</td>
<td>-0.000143</td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.003128</td>
<td>S.D. depend var</td>
<td>0.016743</td>
<td></td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.016716</td>
<td>Akaike info criterion</td>
<td>-5.344435</td>
<td></td>
</tr>
<tr>
<td>Sum squared resid</td>
<td>1.332638</td>
<td>Schwarz criterion</td>
<td>-5.341723</td>
<td></td>
</tr>
<tr>
<td>Log likelihood</td>
<td>12751.15</td>
<td>F-statistic</td>
<td>15.96601</td>
<td></td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>1.970532</td>
<td>Prob(f-statistic)</td>
<td>0.000065</td>
<td></td>
</tr>
</tbody>
</table>

Dependent Variable: PLAT7-9
Method: Least Squares
Included observations: 4771 after adjusting endpoints

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.000174</td>
<td>0.000239</td>
<td>-0.727589</td>
<td>0.4669</td>
</tr>
<tr>
<td>PREM</td>
<td>0.092216</td>
<td>0.023396</td>
<td>3.582559</td>
<td>0.0001</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.003103</td>
<td>Mean depend var</td>
<td>-0.000144</td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.002894</td>
<td>S.D. depend var</td>
<td>0.018533</td>
<td></td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.016509</td>
<td>Akaike info criterion</td>
<td>-5.369455</td>
<td></td>
</tr>
<tr>
<td>Sum squared resid</td>
<td>1.299709</td>
<td>Schwarz criterion</td>
<td>-5.366743</td>
<td></td>
</tr>
<tr>
<td>Log likelihood</td>
<td>12810.84</td>
<td>F-statistic</td>
<td>14.84221</td>
<td></td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>1.960834</td>
<td>Prob(f-statistic)</td>
<td>0.000118</td>
<td></td>
</tr>
</tbody>
</table>

Dependent Variable: PLAT10-12
Method: Least Squares
Included observations: 4771

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.000172</td>
<td>0.000237</td>
<td>-0.727423</td>
<td>0.4670</td>
</tr>
<tr>
<td>PREM</td>
<td>0.096367</td>
<td>0.023686</td>
<td>3.662275</td>
<td>0.0003</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.002780</td>
<td>Mean depend var</td>
<td>-0.000144</td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.002571</td>
<td>S.D. depend var</td>
<td>0.018357</td>
<td></td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.016336</td>
<td>Akaike info criterion</td>
<td>-5.390460</td>
<td></td>
</tr>
<tr>
<td>Sum squared resid</td>
<td>1.272694</td>
<td>Schwarz criterion</td>
<td>-5.387748</td>
<td></td>
</tr>
<tr>
<td>Log likelihood</td>
<td>12860.94</td>
<td>F-statistic</td>
<td>13.29532</td>
<td></td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>1.955208</td>
<td>Prob(f-statistic)</td>
<td>0.000269</td>
<td></td>
</tr>
</tbody>
</table>
Platinum: Fixed-Delivery-Month Method

Dependent Variable: JANUARY
Method: Least Squares
Sample: 1/03/1980 12/31/1996
Included observations: 4771

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.000178</td>
<td>0.000246</td>
<td>-0.725235</td>
<td>0.4683</td>
</tr>
<tr>
<td>PREM</td>
<td>0.094978</td>
<td>0.024591</td>
<td>3.862390</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

R-squared: 0.000311
Adjusted R-squared: 0.002909
S.E. of regression: 0.016969
Sum squared resid: 1.371723
Log likelihood: 12682.19
Durbin-Watson stat: 1.964078

Dependent Variable: APRIL
Method: Least Squares
Sample: 1/03/1980 12/31/1996
Included observations: 4771

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.000174</td>
<td>0.000250</td>
<td>-0.695833</td>
<td>0.4866</td>
</tr>
<tr>
<td>PREM</td>
<td>0.098384</td>
<td>0.024683</td>
<td>3.938092</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

R-squared: 0.000324
Adjusted R-squared: 0.003032
S.E. of regression: 0.017230
Sum squared resid: 1.415618
Log likelihood: 12606.72
Durbin-Watson stat: 1.938004

Dependent Variable: JULY
Method: Least Squares
Sample: 1/03/1980 6/30/1997
Included observations: 4393

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.000155</td>
<td>0.000261</td>
<td>-0.593490</td>
<td>0.5529</td>
</tr>
<tr>
<td>PREM</td>
<td>0.095562</td>
<td>0.026895</td>
<td>3.553870</td>
<td>0.0004</td>
</tr>
</tbody>
</table>

R-squared: 0.000286
Adjusted R-squared: 0.002641
S.E. of regression: 0.017284
Sum squared resid: 1.311713
Log likelihood: 11594.35
Durbin-Watson stat: 1.987626

Dependent Variable: OCTOBER
Method: Least Squares
Sample: 1/03/1980 9/30/1997
Included observations: 4457

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.000152</td>
<td>0.000262</td>
<td>-0.580057</td>
<td>0.5619</td>
</tr>
<tr>
<td>PREM</td>
<td>0.113197</td>
<td>0.027028</td>
<td>4.188163</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

R-squared: 0.000392
Adjusted R-squared: 0.003698
S.E. of regression: 0.017516
Sum squared resid: 1.369837
Log likelihood: 11703.76
Durbin-Watson stat: 1.986833

163
### Palladium: Fixed-Interval-to-Maturity Method

**Dependent Variable: PALLAD1-3**  
Method: Least Squares  
Included observations: 4017

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.000302</td>
<td>0.000283</td>
<td>1.066301</td>
<td>0.2864</td>
</tr>
<tr>
<td>PREM</td>
<td>-0.022019</td>
<td>0.028536</td>
<td>-0.771828</td>
<td>0.4404</td>
</tr>
</tbody>
</table>

| R-squared                  | 0.000148    | Mean dependent var | 0.000294 |
| S.E. of regression         | 0.017937    | Akaike info criterion | -5.203394 |
| Sum squared resid          | 1.291783    | Schwarz criterion | -5.200259 |
| Log likelihood             | 10453.02    | F-statistic       | 0.595410 |
| Durbin-Watson stat         | 1.884245    | Prob(F-statistic) | 0.440380 |

**Dependent Variable: PALLAD4-8**  
Method: Least Squares  
Included observations: 4017

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.000302</td>
<td>0.000272</td>
<td>1.112136</td>
<td>0.2861</td>
</tr>
<tr>
<td>PREM</td>
<td>-0.040155</td>
<td>0.027406</td>
<td>-1.455195</td>
<td>0.1423</td>
</tr>
</tbody>
</table>

| R-squared                  | 0.000534    | Mean dependent var | 0.000287 |
| Adjusted R-squared         | 0.000285    | S.D. dependent var | 0.017229 |
| S.E. of regression         | 0.017226    | Akaike info criterion | -5.284243 |
| Sum squared resid          | 1.191454    | Schwarz criterion | -5.281107 |
| Log likelihood             | 10615.40    | F-statistic       | 2.146796 |
| Durbin-Watson stat         | 1.898361    | Prob(F-statistic) | 0.142946 |

**Dependent Variable: PALLAD7-9**  
Method: Least Squares  
Included observations: 2239

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-8.17E-05</td>
<td>0.000369</td>
<td>-0.221282</td>
<td>0.8249</td>
</tr>
<tr>
<td>PREM</td>
<td>-0.0335736</td>
<td>0.033511</td>
<td>-1.086390</td>
<td>0.2864</td>
</tr>
</tbody>
</table>

| R-squared                  | 0.000508    | Mean dependent var | -9.05E-05 |
| Adjusted R-squared         | 0.000061    | S.D. dependent var | 0.017466 |
| S.E. of regression         | 0.017486    | Akaike info criterion | -5.256279 |
| Sum squared resid          | 0.689387    | Schwarz criterion | -5.251175 |
| Log likelihood             | 5985.405    | F-statistic       | 1.137167 |
| Durbin-Watson stat         | 1.939646    | Prob(F-statistic) | 0.286363 |

**Dependent Variable: PALLAD10-12**  
Method: Least Squares  
Sample: 1/04/1983 8/30/1991  
Included observations: 2176

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-9.35E-05</td>
<td>0.000379</td>
<td>-0.248557</td>
<td>0.6053</td>
</tr>
<tr>
<td>PREM</td>
<td>-0.030951</td>
<td>0.034153</td>
<td>-0.906252</td>
<td>0.3649</td>
</tr>
</tbody>
</table>

| R-squared                  | 0.000378    | Mean dependent var | -0.000102 |
| Adjusted R-squared         | -0.000082   | S.D. dependent var | 0.017676 |
| S.E. of regression         | 0.017677    | Akaike info criterion | -5.232229 |
| Sum squared resid          | 0.679295    | Schwarz criterion | -5.227003 |
| Log likelihood             | 5994.665    | F-statistic       | 0.821293 |
| Durbin-Watson stat         | 1.940496    | Prob(F-statistic) | 0.364903 |
## Palladium: Fixed-Delivery-Month Method

**Dependent Variable: MARCH**  
Method: Least Squares  
Included observations: 2802

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.000112</td>
<td>0.000329</td>
<td>0.339347</td>
<td>0.7344</td>
</tr>
<tr>
<td>PREM</td>
<td>-0.035401</td>
<td>0.032226</td>
<td>-1.098524</td>
<td>0.2721</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.000431</td>
<td>Mean dependent var</td>
<td>0.000103</td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.000074</td>
<td>S.D. dependent var</td>
<td>0.017415</td>
<td></td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.017414</td>
<td>Akaike info criterion</td>
<td>-5.262373</td>
<td></td>
</tr>
<tr>
<td>Sum squared resid</td>
<td>0.849091</td>
<td>Schwarz criterion</td>
<td>-5.258135</td>
<td></td>
</tr>
<tr>
<td>Log likelihood</td>
<td>7374.585</td>
<td>F-statistic</td>
<td>1.206756</td>
<td></td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>1.941926</td>
<td>Prob(F-statistic)</td>
<td>0.272070</td>
<td></td>
</tr>
</tbody>
</table>

**Dependent Variable: JUNE**  
Method: Least Squares  
Included observations: 2865

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.000110</td>
<td>0.000317</td>
<td>0.347475</td>
<td>0.7283</td>
</tr>
<tr>
<td>PREM</td>
<td>-0.048888</td>
<td>0.031239</td>
<td>-1.564996</td>
<td>0.1177</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.000855</td>
<td>Mean dependent var</td>
<td>9.84E-05</td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.000506</td>
<td>S.D. dependent var</td>
<td>0.016973</td>
<td></td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.016986</td>
<td>Akaike info criterion</td>
<td>-5.314244</td>
<td></td>
</tr>
<tr>
<td>Sum squared resid</td>
<td>0.824323</td>
<td>Schwarz criterion</td>
<td>-5.310083</td>
<td></td>
</tr>
<tr>
<td>Log likelihood</td>
<td>7614.654</td>
<td>F-statistic</td>
<td>2.449214</td>
<td></td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>1.950684</td>
<td>Prob(F-statistic)</td>
<td>0.117694</td>
<td></td>
</tr>
</tbody>
</table>

**Dependent Variable: SEPTEMBER**  
Method: Least Squares  
Sample: 1/04/1983 8/30/1991  
Included observations: 2176

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.000113</td>
<td>0.000382</td>
<td>-0.295440</td>
<td>0.7677</td>
</tr>
<tr>
<td>PREM</td>
<td>-0.026173</td>
<td>0.034424</td>
<td>-0.780303</td>
<td>0.4472</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.000266</td>
<td>Mean dependent var</td>
<td>-0.000120</td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>-0.000194</td>
<td>S.D. dependent var</td>
<td>0.017815</td>
<td></td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.017817</td>
<td>Akaike info criterion</td>
<td>-5.216389</td>
<td></td>
</tr>
<tr>
<td>Sum squared resid</td>
<td>0.690140</td>
<td>Schwarz criterion</td>
<td>-5.211183</td>
<td></td>
</tr>
<tr>
<td>Log likelihood</td>
<td>5677.431</td>
<td>F-statistic</td>
<td>0.570861</td>
<td></td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>1.937654</td>
<td>Prob(F-statistic)</td>
<td>0.447156</td>
<td></td>
</tr>
</tbody>
</table>

**Dependent Variable: DECEMBER**  
Method: Least Squares  
Included observations: 3744

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.000198</td>
<td>0.000289</td>
<td>0.737139</td>
<td>0.4611</td>
</tr>
<tr>
<td>PREM</td>
<td>-0.038164</td>
<td>0.027717</td>
<td>-1.376931</td>
<td>0.1686</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.000506</td>
<td>Mean dependent var</td>
<td>0.000185</td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.000239</td>
<td>S.D. dependent var</td>
<td>0.016438</td>
<td></td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.016436</td>
<td>Akaike info criterion</td>
<td>-5.378138</td>
<td></td>
</tr>
<tr>
<td>Sum squared resid</td>
<td>1.010886</td>
<td>Schwarz criterion</td>
<td>-5.378111</td>
<td></td>
</tr>
<tr>
<td>Log likelihood</td>
<td>10066.87</td>
<td>F-statistic</td>
<td>1.896930</td>
<td></td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>1.930266</td>
<td>Prob(F-statistic)</td>
<td>0.168616</td>
<td></td>
</tr>
</tbody>
</table>