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THE IMPLICATIONS OF FROTH STRUCTURE AND SURFACE APPEARANCE FOR FLOTATION PERFORMANCE

PhD Thesis
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STATEMENT OF ORIGINALITY

This thesis has not been submitted whole or in part to any other institution for another degree

Signed: D. Hatfield
SYNOPSIS

In this work, the flotation step of mineral extraction has been investigated to determine the mechanisms through which input variables impact on flotation separation efficiency and, in particular, the froth zone in flotation. The novel contribution of the work is in terms of demonstrating the importance of solids in governing the froth stability and hence the froth structure. Further, it has been proposed that the two most important factors that determine the impact of the solids on the froth are the degree of particle coverage on the bubble surface and the particle hydrophobicity. The mechanism through which the solids affect the froth stability has been demonstrated through experimental observations of flotation metallurgical performance, measurement of the froth surface appearance using machine vision and froth modelling. The insight obtained through understanding of the influence of solids on the froth zone will yield improved design and operation of industrial flotation circuits.

In order to thoroughly study froth stability and the role it plays in determination of the froth structure, froth stability has been defined as the inverse of the rate of lamellar rupture per number of lamellae within a given froth volume. Current understanding of the froth structure in terms of its geometry, dynamics and the distribution of solids and solution species within the froth has been presented. The relationships between the rate of drainage from the froth, the lamellar robustness and the froth stability have been explored, with specific reference to the effect of these factors on the drainage of the entrained material and the detachment and drainage of the attached material that enters the froth.

In addition to the traditional metallurgical metrics of flotation performance, additional information regarding the froth surface appearance has been used to assist in confirming the mechanisms governing the process of froth formation. An image analysis system, SmartFroth, has been used to provide measurements of appropriate froth surface descriptors. Froth surface bubble size and velocity were developed in parallel with this work and are used in this thesis whereas surface burst rate is a measure of froth stability and forms part of this work. The algorithms used to extract these descriptors are presented.

Further, an illustrative model of the froth has been developed, based on the fundamentals of froth geometry and the proposed definition of froth stability, which demonstrates the
proposed mechanisms in terms of the impact of a given input parameter on the froth surface appearance, and on the metallurgical performance of the flotation system. The inputs to the model are the air addition rate to the flotation cell, the froth depth, the dry mass flow rate through the froth (this is also predicted by the model), the percent solids of the concentrate, the pulp bubble size distribution, the cross-sectional area of the Plateau borders, the cell geometry, the particle size distribution and the solids density. The outputs of the model are the bubble size distribution at all heights in the froth zone, the fraction of air overflowing the weir, the froth surface flow speed, the water content profile through the froth and the mass flow rate of dry solids and water recovered from the froth.

Three modes of stabilisation of the froth were hypothesised:

1. Viscous stabilisation resulting from the time taken for bubbles to come into contact and for the lamellae to thin to the point at which rupture can occur
2. Solution stabilisation owing to the Gibbs-Marangoni and Plateau effects and the disjoining pressure developed within the lamellae, retarding the rupture of lamellae when considering a two-phase air-solution system, owing to the presence of surfactants (primarily frother) dissolved in solution
3. Solids stabilisation owing to the mechanism of bubble 'armouring' and the formation of a layer of closely packed particles on the lamellae that experience strong, mutually attractive hydrophobic bonding forces and whose collective impact on the lamellae is to form a strong physical barrier to thinning and rupture. The magnitude of this effect is governed by the particle concentration on the bubble lamellae, or degree of close packing of the particles, as well as the particle hydrophobicity, shape and surface roughness.

Investigation of the impact of these parameters on the outputs of the model demonstrated the importance of the solid species in stabilisation of froth. It was shown that unless the solids were taken into account, the predictions of the model were inadequate.

Selected chemical and hydrodynamic factors were investigated in terms of their effect on the solution and solid characteristics in the froth and the froth hydrodynamics. Hypotheses regarding the mechanisms through which the frother and depressant addition rates, the froth depth and the air addition rate impact on flotation metallurgical performance and the froth
surface appearance were tested with measurements of the flotation system on both batch and industrial scales. The froth surface bubble size and velocity and the mass rate of recovery of solids predicted by the model were compared with measurements of these parameters in the experimental systems.

Coalescence and bursting were identified as key phenomena with respect to the impact of the input parameters on froth performance. It was shown that highly stable froth resulted in recovery of relatively small bubbles to the concentrate. This condition led to a high rate of recovery of entrained material to the concentrate in the dense, stable Plateau border network. Conversely, it was shown that unstable froth rejected the entrained material from the Plateau border network through coalescence, resulting in a high rate of drainage of entrained material and high grade. However, there was some degree of rejection or drop-back of valuable material in a highly unstable froth, resulting in lower recovery under these conditions. Lastly, comparison of experimental results with outputs from froth modelling showed that the froth stability varies to a significant degree from the bottom to the top of the froth zone in an industrial flotation system.

Altering the froth depth altered only the froth residence time. Thus, reducing the froth depth reduced the time available for coalescence which led to a smaller froth surface bubble size, a reduction in froth drainage and an increase in the rate of recovery of attached and entrained material to the concentrate. These conclusions were supported by laboratory tests of the impact of changing froth depth on the froth appearance and performance.

The rate of depressant addition to an industrial flotation system impacts on the degree to which naturally floatable gangue is rendered non-floatable. Thus, increasing depressant addition reduced the mass (or mass rate) of particles that were amenable to flotation which reduced the froth stability by removing the stabilising effect of floatable gangue. Consequently, increasing depressant addition, while increasing the concentrate grade by suppressing the flotation of gangue, also increased the concentrate grade by reducing the recovery of entrained material. This conclusion was supported by laboratory tests comparing the froth appearance and flotation performance with and without depressant. The froth model was used to demonstrate the effect of varying depressant addition on the froth stability, structure and performance.
Air addition altered a number of factors impacting on the froth stability and the degree of coalescence at the froth surface. Increased air addition to the flotation cell, while increasing the rate of attached solids recovery into the froth due to the increased bubble surface area flux in the pulp zone, also increased the rate of recovery of entrained material into the froth. Increased air addition also reduced the froth residence time, reducing the time available for drainage and coalescence. While these factors have important and well understood metallurgical implications, the most important impact of increasing air addition - from the point of view of the froth stability - was that increasing air addition reduced the load of solids per unit bubble surface area entering the froth. This resulted in a decrease in the degree of froth stabilisation afforded by the solids at a given bubble size in the froth. Thus, increasing the air addition rate increased the rate of coalescence in the froth to the extent that a higher air addition rate and consequent shorter froth residence time led to a larger bubble size on the froth surface and a relatively small increase in the rate of recovery of solids to the concentrate.

The role of frother in terms of its impact on the froth zone was primarily to increase the robustness of the lamellae separating the bubbles, reducing the lamella rate of rupture and increasing the froth stability. Thus, at higher frother addition rate the froth had higher lamella surface area and Plateau border length per unit volume, and hence higher water content per unit volume. The result was that with increasing frother concentration in solution the rate of recovery of entrained material across the froth increased. Further, frother stabilised the lamellae in a completely homogeneous manner. Therefore, increasing frother concentration increased the degree of mono-disperisty of the bubble size on the froth surface. These observations confirmed the action of frother as a froth stabilising agent. The conclusions were demonstrated by the froth model and supported by experimental evidence from an industrial plant.

The froth model demonstrates that under changing input conditions, the proposed mechanistic framework generates outputs for the froth surface appearance and the metallurgical performance which are in line with expectations and measured industrial data.

The work provides insight into the nature and behaviour of the froth and provides a basis for further understanding of the flotation system and the froth in particular. For the first time, a
mechanistic description of the contribution of solids and solution factors to the froth structure and stability has been presented. However, further work is required, primarily in terms of expanding the fundamental basis and capabilities of the froth model and in terms of identifying the impact of variables that were not tested in the course of this work on the flotation performance and froth structure.
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NOMENCLATURE

$A_c$  
horizontal cross sectional area of the cell at the froth surface

$A_{pb}$  
cross sectional area of the Plateau border

$A_T$  
sub-pixel accuracy of the tracking algorithm for the inter-frame motion vector

$C_{pb}$  
viscous drag coefficient in the Plateau borders

$d_b^i$  
mean, equivalent spherical diameter of the bubbles on a volumetric basis in the $i^{th}$ time step

$d_{32}$  
Sauter mean bubble diameter

$\overline{F}_c$  
capillary force vector

$\overline{F}_d$  
viscous dissipation force vector

$\overline{F}_g$  
gravitational force vector

$FRT$  
froth retention time

$g$  
gravitational acceleration

$h$  
height of the froth over the weir

$H_f^{\text{max}}$  
maximum froth height

$k_{\text{viscous}}$  
viscous stabilisation constant

$J_{\text{froth}}$  
superficial velocity of the froth

$J_g$  
superficial gas velocity

$J_{g,\text{surf}}^i$  
superficial gas velocity out of the froth surface in the $i^{th}$ time step

$J_{pb}$  
superficial flow rate of water in a Plateau border

$J_{\text{slurry}}$  
superficial velocity of slurry

$L_w$  
length of the weir

$L_{pb}$  
mean Plateau border length

$\dot{M}_s$  
mass flow rate of solids to the concentrate

$\dot{M}_w$  
mass flow rate of water to the concentrate

$N_b$  
number of bubbles

$N_{\text{burst}}^i$  
number of bubbles burst in the $i^{th}$ time step

$N_{\text{frames}}$  
number of frames in the image sequence
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{N}_{\text{surf}}$</td>
<td>specific number of lamellae on the froth surface per unit cell surface area</td>
</tr>
<tr>
<td>$\bar{N}_{\text{pix}}$</td>
<td>average number of pixels per bubble</td>
</tr>
<tr>
<td>$N_{\text{pix,x}}$</td>
<td>pixel resolution in the x direction</td>
</tr>
<tr>
<td>$N_{\text{pix,y}}$</td>
<td>pixel resolution in the y direction</td>
</tr>
<tr>
<td>$P$</td>
<td>probability of lamellar rupture leading to coalescence</td>
</tr>
<tr>
<td>$P_{\text{burst}}$</td>
<td>probability of lamellar rupture to the atmosphere (bursting)</td>
</tr>
<tr>
<td>$P_c$</td>
<td>perimeter length of the cell lip</td>
</tr>
<tr>
<td>$Q_A$</td>
<td>volumetric rate of gas into the cell</td>
</tr>
<tr>
<td>$Q_{A,\text{surf}}$</td>
<td>volumetric rate of gas flow out of the froth surface</td>
</tr>
<tr>
<td>$Q_{A,\text{weir}}$</td>
<td>volumetric rate of gas overflowing the weir</td>
</tr>
<tr>
<td>$Q_{\text{conc}}$</td>
<td>volumetric flow rate of concentrate overflowing the weir</td>
</tr>
<tr>
<td>$Q_{f,\text{weir}}$</td>
<td>volumetric flow rate of the bulk froth (gas and slurry) over the weir</td>
</tr>
<tr>
<td>$r_b$</td>
<td>Sauter mean bubble radius</td>
</tr>
<tr>
<td>$r_p$</td>
<td>Sauter mean particle radius</td>
</tr>
<tr>
<td>$R_{pb}$</td>
<td>radius of curvature of the Plateau borders</td>
</tr>
<tr>
<td>$S_{\text{af}}$</td>
<td>specific surface area of bubbles in the froth on a volume basis</td>
</tr>
<tr>
<td>$S_b$</td>
<td>bubble surface area flux in the froth</td>
</tr>
<tr>
<td>$S_{\text{bubbles}}$</td>
<td>specific surface area of the bubbles on a volume basis</td>
</tr>
<tr>
<td>$S_{\text{froth}}$</td>
<td>specific surface area of the froth on a volume basis</td>
</tr>
<tr>
<td>$S_{\text{lam}}$</td>
<td>specific surface area of the lamellae on a volume basis</td>
</tr>
<tr>
<td>$S_p$</td>
<td>particle cross sectional surface area flux</td>
</tr>
<tr>
<td>$S_{\text{particles}}$</td>
<td>specific cross sectional surface area of the particles on a mass basis</td>
</tr>
<tr>
<td>$S_{pb}$</td>
<td>specific surface area of the Plateau borders on a volume basis</td>
</tr>
<tr>
<td>$S_i$</td>
<td>equivalent ellipsoidal surface area of the $i^{th}$ bubble</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
</tr>
<tr>
<td>$u$</td>
<td>overall velocity of the liquid relative to an external frame of reference</td>
</tr>
<tr>
<td>$u_{\text{rel}}$</td>
<td>apparent velocity of the fluid relative to the foam structure</td>
</tr>
</tbody>
</table>
\( \nu \) velocity of the froth relative to an external frame of reference
\( \bar{\nu}_f \) height averaged velocity of the froth towards the weir
\( V_i \) equivalent ellipsoidal volume of the \( i \)th bubble
\( \alpha \) fraction of unburst bubbles overflowing the weir
\( \delta_i \) mean lamella thickness
\( \varepsilon \) slurry content of the froth on a volumetric basis
\( \varepsilon_g \) gas hold-up
\( \varepsilon_{p,\text{lam}} \) contribution to the volume fraction of slurry from the lamellae
\( \varepsilon_{p,\text{pb}} \) contribution to the volume fraction of slurry from the Plateau border network
\( \varepsilon_{\text{surf}} \) slurry content of the froth at the froth surface
\( \varphi \) "golden ratio" for the system based on geometric considerations of the Kelvin foam structure
\( \varphi_s \) mass fraction of solids in the slurry
\( \varphi_{p,\text{pb}} \) mass fraction of solids in the Plateau borders
\( \varphi_{\text{lam}} \) mass fraction of solids in the lamellae
\( \gamma \) surface tension at the air-water interface
\( \phi_s \) volume fraction of solids in the concentrate
\( \lambda \) Plateau border length per unit volume of froth
\( \mu \) viscosity of the solution
\( \mu^* \) bulk viscosity of the slurry
\( \Pi \) lamellar robustness
\( \rho_{\text{conc}} \) bulk density of the concentrate
\( \rho_p \) bulk pulp density
\( \rho_s \) mean solids density in the concentrate
\( \rho_w \) density of water
\( \Sigma_f \) dynamic froth stability factor
\( \tau \) bubble lifetime, defined as the time taken for half of the bubbles of a given size to coalesce into larger bubbles
\( \tau_{\text{overall}} \) bubble lifetime with respect to coalescence in the froth zone
\( \tau_{\text{solids}} \) contribution to the bubble lifetime by solids stabilisation of the lamellae

\( \tau_{\text{soln}} \) contribution to the bubble lifetime by solution factors

\( \tau_{\text{surf}} \) bubble lifetime on the froth surface

\( \tau_{\text{viscomt}} \) contribution to the bubble lifetime by the fluid inhibition of bubble contact, dominated by the fluid viscosity
GLOSSARY
Within the context of this thesis, the following meanings and definitions apply.

**Air addition rate**
The volumetric rate at which air or other flotation gases are added to the flotation cell.

**Air-solution interface**
The interface of the air bubble and the liquid or solids in the lamellae and Plateau borders.

**Attachment**
Following collision between a particle and an air bubble, the particle will attach to the bubble if the sliding or contact time between particle and bubble is greater than the induction or dewetting time of the particle. The *hydrophobicity* of the particle influences both the induction time and the strength of the attachment between the particle and the bubble. Attachment occurs in the *pulp zone* of the *flotation cell*.

**Bubble**
A gas filled cavity in a liquid or solidified liquid. In the context of flotation, an air filled cavity in the *pulp*, or surrounded by *slurry* in the froth. Bubbles are spherical in the pulp and deform into polyhedra in the froth, due to surface tension.

**Bubble shell**
The term bubble shell is restricted to describing a layer of agglomerated liquid and solids adjacent to the bubble while in the *pulp zone*. Material in the bubble shells is re-distributed into the *lamellae* and *Plateau borders* above the *froth-pulp interface*.

**Bubble wake**
The bubble wake is a roughly ellipsoidal region extending below the bubble which retains material in eddies formed by the upward motion of the bubble through the pulp. This region is distinct from the spherical bubble shell of attached material.

**Bubble interstices**
In the region of initial froth formation close to the froth-pulp interface, closely packed bubble swarms entrain material in the bubble interstices which are the initial stages of the vertices present in a well-dried froth structure. Since there are effectively no distinct Plateau borders in such a wet structure, the interstices contain all of the material not attached to bubbles.

Chemical control variables

See reagents.

Concentrate

One of the two product streams of the flotation process, the other being the flotation tails. The concentrate stream is the valuable stream which is further processed, typically by smelting and refining. Ideally, the concentrate should contain a high proportion of the valuable minerals, or high grade.

Contact angle

The interfacial angle between a solid surface and a water drop, or contact angle, is measured using a goniometer, with higher contact angles being correlated with more hydrophobic surfaces.

Degree of dispersity

This metric describes the standard deviation of the bubble size from the population mean. In a mono-disperse froth, all bubbles have the same size. In a highly poly-disperse froth, the bubble sizes vary to a large degree.

Detachment

The movement of solid species and solution from the air-solution interface into the Plateau borders of the froth. Detachment typically occurs as a result of lamella rupture.

Drainage

Drainage is the mechanism whereby gangue material and excess solution that is hydraulically entrained into the froth, flows downwards under gravity through the
Plateau border network back into the pulp. Effective drainage is important for rejection of entrained gangue from the froth.

Entrainment

Entrainment is the non-selective recovery of material into and across the froth zone. Entrainment is a mechanism by which gangue is recovered to the concentrate, reducing the concentrate grade. Entrained material resides in the bubble wakes in the flotation pulp zone, in the bubble interstices at the froth-pulp interface and in the Plateau borders and vertices of the froth zone.

Floatability (P)

Floatability denotes the probability of attachment of a particle in the pulp zone of the flotation cell. Particles having high floatability typically consist of highly liberated material, where the mineral making up the particle has a moderate to highly hydrophobic nature (contact angle > 70°). This combination of particle properties is correlated with a high probability of attachment of the particles in the pulp zone. A group of particles in flotation will typically present a distribution of floatabilities, but for convenience they are frequently classed as fast floating with high floatability, slow floating with moderate to low floatability and non-floating with zero floatability.

Flotation cell

A machine within which froth flotation occurs. Typically, flotation cells are cylindrical or rectangular tanks containing an impeller mechanism to maintain solids suspension and disperse the air as fine bubbles within the flotation pulp zone. The flotation cell can contain an addition point for compressed air close to the impeller, or can be self-aerated, drawing atmospheric air into the dispersion zone close to the impeller. The flotation cell has a lip on one or more sides of the cell over which the froth flows into a launder to form the concentrate stream.

Foam

Two-phase matrix of gaseous bubbles in liquid.

Froth
Three-phase substance consisting of gas bubbles separated by liquid and solid species.

**Froth depth**
Distance between the *froth-pulp interface* and the lip of the *flotation cell*.

**Froth-pulp interface**
The upper boundary of the *pulp zone* and lower boundary of the *froth zone*. Defined as the height within the flotation cell at which bubbles become close packed.

**Froth stability**
The negative inverse of the rate of *lamella* rupture, normalised with respect to the total number of lamellae.

**Froth structure**
The physical form of the *froth*; comprised of *lamellae*, *Plateau borders*, *vertices* and gas bubbles.

**Froth surface descriptors**
Outputs of a *machine vision system* that quantifies aspects of the froth surface appearance. Typical froth surface descriptors are the bubble size distribution, froth velocity and a measurement correlated with the froth surface burst rate.

**Froth viscosity**
The shear rate of the froth under a given shear stress.

**Froth zone**
The region above the pulp zone; bounded by the *froth-pulp interface* at the bottom, the interface of the froth with the atmosphere (the froth surface) at the top, and the walls of the flotation cell at the sides.

**Gangue minerals**
The gangue minerals are the mineral types in the ore that contain none of the desired elements. This material should report exclusively to the tailings or underflow from the flotation cell, given perfect liberation and separation.

**Grade**

The mass fraction of a solid sample or the solids of a process stream that is comprised of a chosen *valuable* element. Typically expressed as a percentage when the mass fraction of valuables in the ore is relatively high (as in base metal flotation), or as grams per ton for precious metal flotation.

**Hydrodynamic control variables**

The *froth depth* and *air addition rate*, input variables to the flotation cell which predominantly impact on the hydrodynamic characteristics of the froth, as opposed to the *chemical control variables*.

**Hydophilic**

Having an affinity for water, or readily wettable by water. A hydophilic particle is not repelled by water and, thus, there is a low driving for *attachment* of a hydophilic particle to an air bubble in the flotation *pulp zone*. Hence, hydophilic particles are generally *non-floatable*.

**Hydrophobic**

Lacking an affinity for water, or not readily wettable by water. Hydrophobicity is a surface property that describes the magnitude of the reduction in Gibbs free energy when the surface changes from being in contact with water to being in contact with something other than water. In the context of flotation, the surface of interest is usually that of a solid particle, and the reduction in Gibbs free energy is the driving force for *attachment* of the solid to an air bubble. Thus, hydrophobic particles are generally *floatable*.

**Hydrophobic particle bridging**

The process whereby a coarse, highly *hydrophobic* particle, upon contact with the *air-solution interfaces* on either side of a *lamella* can cause the interfaces to migrate over
the particle surface, resulting in lamella rupture. The particle forms a bridge between
the bubbles on either side of the lamella.

Lamella
A lamella is the planar liquid or liquid-solid layer separating two gas bubbles within
the froth structure or the curved layer separating a bubble from the atmosphere at the
froth surface.

Lamella rupture
Following formation of a hole in the lamella due to hydrophobic particle bridging,
mechanical disturbance or complete drainage of the lamella; surface tension draws the
exposed edges of the lamella back from the initial hole, towards the remaining contact
of the lamella with the froth structure, namely the Plateau border surrounding the
lamella. This process is known as lamella rupture.

Liberation
The degree to which the valuable minerals are separated from the host gangue
material. Completely liberated particles are pure mineral species, while poorly
liberated particles consist of mixtures of the valuable and gangue minerals. Effective
liberation of the valuables from the gangue is the purpose of milling and a pre-
condition for effective flotation.

Machine vision
Machine vision enables digital systems to perform visual recognition tasks, typically
to provide measurements for automatic control systems or to assist humans in
processing large quantities of visual data. Machine vision measures aspects of the
froth surface appearance, generating froth surface descriptors.

Mass pull
The mass fraction of dry solids in the feed that reports to the concentrate.

Mechanism
A causal chain that describes the process through which a change in the inputs to a system result in a particular outcome or outcomes.

**Mobility**

The rate of transport of froth to the concentrate, combining the net effect of the driving force for froth motion and the froth viscosity.

**Mono-disperse**

See Degree of dispersity.

**Monotonic**

Adjective describing a series or function that consistently increases or decreases over the range of interest.

**Naturally floatable gangue**

Gangue that is hydrophobic and hence floatable under typical flotation conditions, in the absence of surface modifying reagents. The talcaceous minerals are examples of naturally floatable gangue.

**Non-floatable**

See floatability.

**Ore**

The output from mining operations that is a combination of host rock and valuable product and is the feed to a mineral processing plant.

**Plateau border**

Intersection of three lamellae.

**Plateau border network**

A three dimensional ‘web’ of Plateau borders that intersect at vertices; the channels through which drainage from the froth occurs.
Slurry

The combined solids and solution in the Plateau borders and lamellae of the froth. Distinct from the pulp in the pulp zone, due to the differences in location and composition of the material. Slurry excludes the air bubbles in the froth.

Tails

The underflow or waste stream from the flotation cell, which ideally contains all of the gangue material and none of the valuable minerals.

Tetradekahedron

Fourteen sided geometric shape, a candidate for the typical shape adopted by bubbles in a regular mono-disperse froth structure.

Valuable minerals

The desired product from the flotation system that should ideally report exclusively to the concentrate. The valuable minerals consist of all mineral types in the ore that contain the desired elements in a payable proportion.

Vertex

The tetrahedral intersection of four Plateau borders.
uncharged, as in the case of guar gum depressants; or charged, as with carboxymethylcellulose (CMC) depressants.

**Collector**
Collector selectively adsorbs on the valuable mineral surface with one end of the collector molecule and presents a *hydrophobic* tail to the solution, inducing *hydrophobicity* on the *valuable mineral* surface and increasing *recovery* of the *valuable mineral*. The most common collectors in sulphide flotation are the short-chain hydrocarbon xanthates.

**Activator**
Activator increases the efficacy of the *collector* adsorption process, by creating or regenerating a reactive surface on the *valuable mineral*. Copper sulphate is typically used as an activator in sulphide *flotation*.

**Recovery**
The mass percentage of a valuable element in the feed that reports to the *concentrate* stream, which is used as a metallurgical indicator of the effectiveness of the separation.

**Sauter mean bubble diameter**
Ratio of the sum of the equivalent ellipsoidal bubble volumes to the sum of the equivalent ellipsoidal surface areas for a group of bubbles.

**Solids coverage**
Defined as the cross sectional surface area of attached solids per unit surface area of bubble in the pulp. Alternatively, defined as the cross sectional surface area of attached solids per unit surface area of lamellae and Plateau borders in the froth, or equivalently as the ratio of the attached particle cross-sectional surface area flux to the bubble surface area flux.

**Solids load**
Mass of solids per unit surface area of bubble or per unit surface area of lamellae and Plateau borders at the solids-solution interface in the froth.
Slurry

The combined solids and solution in the Plateau borders and lamellae of the froth. Distinct from the pulp in the pulp zone, due to the differences in location and composition of the material. Slurry excludes the air bubbles in the froth.

Tails

The underflow or waste stream from the flotation cell, which ideally contains all of the gangue material and none of the valuable minerals.

Tetradekahedron

Fourteen sided geometric shape, a candidate for the typical shape adopted by bubbles in a regular mono-disperse froth structure.

Valuable minerals

The desired product from the flotation system that should ideally report exclusively to the concentrate. The valuable minerals consist of all mineral types in the ore that contain the desired elements in a payable proportion.

Vertex

The tetrahedral intersection of four Plateau borders.
CHAPTER 1: INTRODUCTION

1.1 SCOPE

This thesis addresses the flotation step of mineral extraction and investigates the mechanisms through which selected physical and chemical input parameters impact on flotation process separation efficiency, with a focus on their effect on the froth zone. The froth zone is studied in terms of its structure, mobility and stability. Specific reference is made to the relationship between descriptors of the froth state and the drainage of the entrained material from the froth and the detachment and drainage of the attached material that enters the froth. The crucial role of froth stability in controlling the froth structure and performance is explored in depth and a definition of froth stability is proposed. Further, the roles of solid and solution species as determinants of the froth stability are hypothesised.

In addition, a machine vision system (SmartFroth) is described in terms of its measurement techniques and the significance of its outputs for determining flotation performance. As a part of this thesis a measurement of the froth surface stability has been developed and is described. Further, the machine vision outputs are used to assist in identifying and quantifying the mechanisms through which the inputs impact on the flotation system.

An illustrative mechanistic model of the froth zone is presented, with which the roles of the solid and solution species are demonstrated.

This thesis addresses the following key questions:

1. How can the froth structure best be described and what are the mechanisms governing froth formation within the context of this structure?

2. Mechanistically, how do selected hydrodynamic and reagent variables alter the froth state, froth surface appearance and metallurgical performance?

3. Can these mechanisms be confirmed by appropriate measurements of the froth surface appearance from machine vision algorithms?

4. Can a model be developed to illustrate the effect of the input variables on the froth structure and, thus, on the relevant froth surface descriptors and flotation performance parameters?
Hypotheses are developed and a series of batch flotation tests as well as several industrial campaigns are used to address the Key questions.

Changes in the air flow-rate, pulp level, frother and depressant dosages are used to investigate the mechanisms through which the froth residence time, the solid and solution species impact on metallurgical performance and the froth surface appearance (Key question 2). The machine vision measurements illuminate the mechanisms through which the inputs impact on the froth zone and quantify observations that are used to make flotation control decisions (Key question 3). An illustrative model of the froth zone is used to demonstrate the effect of the above input parameters on flotation performance and relevant froth surface descriptors (Key question 4).
Figure 1-1: Schematic of conceptual relationships between the factors affecting the froth, the froth surface appearance and metallurgical flotation performance.
1.2 ORGANISATION OF THE THESIS

The literature is reviewed in Chapter 2 – Literature review, to explore the pre-existing knowledge base regarding the froth zone and to provide a context for the work. Chapter 2 ends with the proposed definition of froth stability and addresses Key question 1. The froth surface descriptors used as metrics of the froth structure, mobility and stability and the image analysis approach to measuring these descriptors are covered in Chapter 3 – Machine vision. The machine vision measurements were used to confirm the proposed hypotheses.

Chapter 4 – Model of the froth zone, presents the mathematical background to the model used to illustrate the mechanisms that are proposed by the hypotheses. The results are presented in Chapter 5 – Results and Key questions two and three are addressed, with the discussion of the results appearing in Chapter 6 – Illustrative modelling of an industrial system which addresses Key question four. Conclusions and recommendations regarding the outcomes of the work appear in Chapter 7 – Conclusions.

CHAPTER 2: LITERATURE REVIEW

2.1 INTRODUCTION

In this chapter, the literature regarding the nature of the froth is critically reviewed and summarised. By establishing the current level of understanding regarding the froth, areas that are currently undeveloped are identified - setting the scene for the contribution of this thesis. The literature review addresses the first key question, by exploring the manner in which the froth structure can best be described and the known mechanistic action of solid and solution species on the froth. Further, in light of the known froth structure and its behaviour, a novel definition of froth stability is proposed.

To establish a conceptual framework for discussion of the factors affecting froth behaviour and to clarify aspects of the froth modelling, a description of the froth structure is presented in Section 2.2; focussing on the role of the lamellae and Plateau borders in forming the froth structure. Certain common assumptions and approximations to the froth structure, found in the literature, are explored and investigated.
In Section 2.3 the distribution of attached, entrained and detached components within the froth is explored, with respect to the independent contributions of liquid and solids in each class to the composition of the Plateau border network and the froth lamellae.

Drainage is a key mechanism in determining the recovery of entrained material and, hence, a major contributor to the froth separation efficiency. Further, drainage of entrained material impacts on, and is affected by, the froth stability. In Section 2.4, drainage of the solution and solid components is explored in detail, highlighting two mechanisms: intra-Plateau border drainage and drainage promoted by structural changes to the froth.

Drainage and lamellar persistence are the dominant factors governing lamellar rupture. Having considered promotion and inhibition of drainage in Section 2.4, the solution and solid parameters impacting on lamellar robustness are considered in Sections 2.5 and 2.6 respectively.

Section 2.7 draws together Sections 2.4, 2.5 and 2.6 by proposing a conceptual framework within which the rates of drainage and the lamellar persistence combine to determine the rate of lamellar rupture and hence froth stability. Section 2.8 investigates the significance of stability as a concept and reviews existing definitions and metrics of froth stability. Section 2.9 summarises and identifies the contradictions reported in the literature.

Section 2.10 summarises the objectives and presents hypotheses to address the Key questions and the mechanistic effects of selected physical and chemical factors on the froth stability and structure, the froth surface appearance and the physical and modelled flotation performance.
2.2 The froth structure

In flotation, the function of the froth zone is to enhance the overall selectivity of the flotation process. The froth achieves this by preferentially rejecting the pulp that separates the bubbles in the pulp phase and reducing the recovery of entrained material to the concentrate stream, while preferentially retaining the attached material. Since the entrained material contains a high proportion of gangue and the attached material contains a high proportion of valuable minerals, the function of the froth is to substantially increase the concentrate grade while resulting in as small a reduction as possible in the recovery of valuables. The relationship between recovery and grade is a trade-off that needs to be managed according to operational constraints. As the final separation phase in a flotation cell, the froth zone is a crucial determinant of the grade and recovery of the flotation process.

Establishing a common framework for discussion of the froth structure allows for discussion of the mechanistic role of the input parameters in changing the nature of the froth. The structure of a static, non-draining three-phase froth or two-phase foam is governed by energy minimisation of the froth under surface tension acting at the gas-solution interfaces. In dynamic, meta-stable flotation froth, drainage due to gravitational and viscous dissipation forces also plays a role in determining the froth structure. In Section 2.2, the fully drained, mono-dispersed case of a froth having uniform bubble size is presented. The role of gravitational and viscous dissipation forces in drainage are explored in Section 2.4.

Figure 2-1 shows an effective conceptual model of froth formation, given the limitations of the two-dimensional page. Figure 2-1 shows attachment in the pulp zone, entrainment into the froth in bubble interstices, as well as the increase in particle coverage on the lamella surfaces with coalescence and a high degree of coverage of solids on the lamellae at higher levels in the froth. Limitations of Figure 2-1 include the range of scales present in a real froth. Bubble diameters in real froth range from 1 mm at the froth-pulp interface to 10 cm at the froth surface. This is an increase of two orders of magnitude and this change is not accurately depicted. Linked to the three dimensional, multi-scalar nature of real froth, Figure 2-1 does not accurately show the froth-pulp interface; where bubbles pack into a randomly distributed, close packed structure and undergo rapid drainage and coalescence. For the purposes of this thesis, it is assumed that the bubble shells cease to exist at the froth-pulp
interface and the material in the shells is re-distributed into the lamellae and Plateau borders of the froth.

The differences between the two and three-dimensional structures are crucial for effective modelling of the froth zone, as demonstrated in Chapter 4. The two-dimensional representation of the froth is expanded to a more realistic and hence, accurate, three-dimensional picture in Sections 2.2.1 through 2.2.4.

Figure 2-1: A two dimensional representation of the froth structure

2.2.1 Close packed polyhedral structures

The study of space-filling shapes within mathematical and physics literature has generated many space-filling arrangements for closely packed spheres. Lord Kelvin proposed a packing of tetrahedra as an optimal three-dimensional space-filling arrangement and posed the challenge of analytically finding whether this was the case. This challenge remains unanswered and current study of foam geometry is based largely on computational methods. Figure 2-2 shows a recently discovered structure, consisting of alternating dodecahedra and a
tetradecahedra (Weaire and Phelan, 1994). This structure is currently thought to be an optimal packing of bubbles of equal volume.

Figure 2.2: Recently proposed candidate for the optimal surface area minimising, mono-disperse space-filling structure from Weaire and Phelan (1994)

2.2.2 Lamellae

The planar layers, consisting of solution and solids, separating the bubbles, are known as lamellae. Lamellae are films stabilised by surfactant molecules and solids present in the lamellae. In the absence of solids, the lamellae are 7 to 100nm thick (Taylor et al., 2004). In the case of lamellae containing solid species Hemmings (1981); Flynn and Woodburn, (1987); Woodburn et al., (1994); Asplin et al. (1998); Sadr-Kazemi and Gilliers (2000) have shown that the lamellar thickness in three-phase froth is approximately equivalent to the d_{50} of the particle size.
Figure 2-3: Schematic of lamella during early stages of froth formation, showing some of the factors impacting on lamella drainage.

An exception to the planar nature of the lamellae is evident at the froth surface, where the lamellae deform from a planar conformation to a shape similar to an ellipsoidal section, pinned at the edges by adjacent Plateau borders.

2.2.3 Plateau borders

Plateau borders, named after Plateau (1871), are channels that occur at the interface of three lamellae (Podval et al., 1996). Since the froth is subject to Gibbs free energy minimisation under the force of surface tension, the lamellae in a mono-dispersed froth intersect at 120° to each other. One can visualise three lamellae as three planes intersecting and stopping along a line. This line is the centre line of the Plateau border. Due to the surface tension at the air-water interface, the junction of any two lamellae has a curved intersection with a variable radius of curvature. The ‘sharpness’ of the intersection or radius of curvature of the Plateau
border ($R_{vo}$) is determined by its liquid content. A smaller radius of curvature corresponds to a drier froth. The radius of curvature is important for the capillary force, discussed in Section 2.4.2. Figure 2-4 shows the intersection of three lamellae at a Plateau border.

![Figure 2-4: Three dimensional structure of a Plateau border, where $R_{vo}$ denotes the radius of curvature of the Plateau border and $A_{vo}$ its cross sectional area](image)

2.2.4 Vertices

The intersection of any four Plateau borders occurs in the three dimensional form of a tetrahedron (Neethling et al., 2000). This is the shape dictated by Gibbs free energy minimisation under the force of surface tension. All of the channels are at roughly $109.5^\circ$ to each other. At the intersection, a vertex is formed that also contains an appreciable quantity of fluid and entrained gangue. As the Plateau borders shrink the vertices lose water and entrained solids, becoming smaller and having a smaller radius of curvature. Figure 2-5 shows the intersection of four Plateau borders at a vertex. For the purpose of visual clarity, the Plateau borders are portrayed as lines.
Figures 2.6 and 2.7 demonstrate the overall structure of the froth; combining the Plateau borders and vertices for a mono-dispersed, regular Kelvin foam and randomly distributed mono-dispersed foam respectively. The analytical and computational difficulty of dealing with multi-scalar poly-disperse foam (highlighted in Section 4.6) renders investigations of real froth structures beyond the scope of current geometric investigations.
2.2.5 Common approximations to the froth structure

Two common approximations for the dimensions of the froth structure appear in the literature.

2.2.5.1 Bubble shells

The first is to consider all of the material, both attached and entrained solids and water in the froth, to reside in a layer of particles and water surrounding the bubbles (Flynn and Woodburn, 1987; Murphy et al., 1998; Malysa, 1998; Neethling and Cilliers, 1999; Schwarz, 2003). When this approximation is used, the term ‘bubble shell’ is used to describe this material. Thus, despite the fact that bubble shells are defined in the glossary as existing only in the pulp phase, in this section bubble shells refer to a lumped lamella and Plateau border term in the froth.

Since the lamellae represent the majority of the surface area surrounding a given bubble; this ‘shell’ is approximately equivalent, in terms of its total area, to the lamellae. Flynn and Woodburn (1987) express this relationship implicitly by defining $S/2$ as the volume of suspension per unit area of bubble surface and then state that, for a froth composed essentially of two-sided films, $\delta$ may be regarded as the mean lamella thickness. A layer thickness is then calculated based on the volumetric recovery of material and froth geometric considerations.
Subsequent to this analysis, Malysa (1998) and Schwarz (2003) present the following relationship for lamella thickness:

$$\delta = \frac{2(1-\varepsilon_g)\frac{\sum (r^2)}{N}}{3\varepsilon_g \frac{\sum (r)}{N}}$$

[2-1]

where

- $\delta$ = thickness of the layer of particles and water
- $\varepsilon_g$ = gas hold-up
- $r$ = average bubble radius
- $N$ = number of bubbles

or equivalently, as used by Neethling and Cilliers, (1999):

$$\delta_i = \frac{\Phi_w}{\alpha \times Q_a \times S_{af}}$$

[2-2]

where

- $\delta_i$ = average lamella thickness
- $\Phi_w$ = rate of water overflowing the weir
- $\alpha$ = fraction of unburst bubbles overflowing the weir
- $Q_a$ = volumetric rate of air addition to the cell
- $S_{af}$ = specific surface area of bubbles in the froth on a volume basis

While useful for comparative purposes within a given set of experiments, Equations 2-1 and 2-2 are somewhat misleading, in the sense that the majority of the non-gaseous material in the froth (by mass or volume) has been shown to reside in the Plateau borders and vertices of the froth structure and not the lamellae (Weaire and Hutzler, 1999; Neethling et al., 2000; Ventura-Medina and Cilliers, 2002). Further, this approach leads to consideration of a mean froth composition, which is misleading in that a higher proportion of the material in the
Plateau borders is entrained compared with the predominantly attached material in the lamellae (Ventura-Medina and Cilliers, 2002).

2.2.5.2 Combined Plateau border and vertex structure

One can consider all material that does not reside on the lamellar surfaces to reside in the Plateau borders, as opposed to separating the Plateau border and vertex contributions to bulk volumetric flow (Neethling et al., 2000). To achieve equivalent capillary force across the froth structure, the radius of curvature (and thus the surface tension force) must be equal in the Plateau borders and vertices. This, in turn, implies that the vertices do not have a significantly larger volume per unit length than the Plateau borders and this approximation is reasonable. In the work that follows, a reference to a loss of Plateau border length (or volume) implies a loss of both Plateau border volume and of the associated vertex volume.
2.3 Distribution of components in the froth

This section investigates the distribution of attached and entrained material within the previously described froth structure, to populate the structure with its components. The sections that follow deal explicitly with the mechanisms by which the material enters, is distributed within and leaves the froth.

Typical flotation froth contains gas bubbles (air or nitrogen), water containing dissolved ions and surfactants in solution and solid particles. Particles and solution in the froth can be classified according to their source. Material can be entrained into the froth in the pulp surrounding the rising air bubbles, or can be carried into the froth attached to air bubbles. Once in the froth, material that is recovered by true flotation either remains attached to the air bubbles or detaches from the air bubbles. Thus, the non-gaseous material in the froth will be considered as belonging to one of three classes, each with two sub-classes:

- Attached
  - Solids
  - Water
- Detached
  - Solids
  - Water
- Entrained
  - Solids
  - Water

It is implicitly understood that the majority of attached material will be valuable species strongly attached to the lamellae, due to hydrophobic forces. The less hydrophobic gangue or less liberated species will tend to enter the froth through entrainment, rather than as attached species on the bubble surfaces.

2.3.1 Attached

2.3.1.1 Solids

During froth formation, the capillary force draws solution and entrained solids (pulp) from the lamellae into the adjacent Plateau borders and vertices. The movement of pulp from the lamellae into the Plateau borders causes rapid lamella thinning in the froth close to the pulp-
froth interface and the removal of the majority of the entrained material from the lamellae. Thus, the material in the lamellae derives primarily from true flotation and the solid species in the lamellae are primarily attached particles (Neethling and Cilliers, 2000; Ventura-Medina, 2002). This implies that the contribution of entrained solids to the lamella composition is negligible.

2.3.1.2 Water (and reagents in solution)

In the pulp, water is attached in a shell surrounding the bubbles as shown by Krzan et al., (2004); Sam (1995) and Morar (2003); with the thickness of the layer being dependent on the concentration and type of surfactant and ionic species at the air-solution interface. The attached water shell travels across the froth-pulp interface with the bubbles.

The attached solution has a higher bulk concentration of frother molecules than the entrained material due to the higher concentration of frother at the air-water interface. The attached water shell is distinct from the entrained material in terms of its origin and, hence, in terms of the mechanism which governs the contribution of the detached water to the entrained material in the Plateau borders of the froth. Specifically, the thickness of the water shells is affected by the frother concentration, while the material entering the froth due to hydraulic entrainment in the bubble wakes and interstices (see Section 2.3.3) is assumed not to be affected by the frother concentration.

Attached water is subject to the same capillary forces as outlined in Section 2.3.1.1 and hence the bubble shells containing the attached material cease to exist at the froth-pulp interface as the solution is drawn into the Plateau border network and lamellae of the froth.

2.3.2 Detached

Detached material derives from the redistribution of the material in the ruptured lamella and adjacent Plateau borders following a coalescence or bursting event (lamellar rupture) in the froth. As a lamella ruptures, the material within can either:

1. Be released as independent particle-water aggregates in the bubble void and fall into adjacent Plateau borders, thus becoming detached material, equivalent in terms of further drainage to the entrained material;
2. Be drawn into the lamellae of the larger bubble and remain attached;
3. Be drawn into the Plateau borders of the new, larger bubble becoming detached to join the entrained material.

2.3.2.1 Particles

Whether re-attachment of detached solids occurs and whether re-attachment is selective with respect to the size or hydrophobicity of particles is the subject of ongoing debate. Despite the likelihood that all of the above mechanisms play a role in the movement of solids following the inherently abrupt and stochastic process of lamellar rupture, it is proposed that redistribution of attached material to adjacent lamellae is the dominant mechanism of the above three. If attached material stays attached as coalescence proceeds, the solids loading per unit lamella surface area increases. The probability of material remaining attached during coalescence is, in all likelihood, dependent on the existing lamella loading, although this has not been confirmed.

2.3.2.2 Water

Detached water is not expected to be subject to preferential re-attachment and is assumed to behave identically to entrained water once detached. Detached water enters the Plateau borders and drains through these channels under the influence of the viscous dissipation, capillary and gravitational forces.

2.3.3 Entained

2.3.3.1 Particles

Entrained material is carried across the froth-pulp interface when the rising bubbles carry material with them in their upward passage. Leaving aside the material in the attached bubble shell (Section 2.3.1), the entrained material is in the bubble wake of individual bubbles and in the interstices between bubbles of a bubble swarm at the froth-pulp interface (Woodburn et al., 1994).

Since such selectivity with respect to particle size and density in the entrained material entering the froth would be additive with the preferential drainage of coarse, entrained material with a rapid settling rate from the froth (discussed in Section 2.4), these effects would be additive in a typical flotation system and an empirical adjustment would effectively adjust for both.
The dominant factor governing the flux of entrained solids in the bubble wakes and in the interstices of the bubble swarm is the bulk flux of bubbles across the froth-pulp interface. Thus, the superficial gas velocity and bubble surface area flux plays an important role in determining the flux of ‘purely entrained’ material into the froth.

2.3.3.2 Water

The entrained material is drawn from the bubble wakes and interstices into the Plateau borders of the froth in the early stages of froth formation, and thus the bubble wakes and interstices cease to exist above the froth-pulp interface. The subsequent flow of the entrained material within the Plateau borders forms the basis of the section that follows.

Again, the dominant factor controlling the flux of entrained water into the froth is the bulk flux of bubbles across the froth-pulp interface and this in turn is controlled by the superficial gas velocity entering the froth.
2.4 DRAINAGE

2.4.1 Factors governing drainage

Drainage denotes the flow of material in the Plateau borders, resulting in the rejection of a portion of the material that is entrained into the froth (increasing grade), along with the loss of a fraction of the valuable detached material (reducing recovery). This leaves the remaining entrained material, the attached material and the remaining detached material to be recovered to the concentrate.

As a result, drainage is the mechanism through which upgrading occurs in the froth, and the determining factor for the froth recovery. Drainage affects and is affected by the froth stability (defined in Section 2.10), and hence is described in detail in this section with respect to the forces that govern the drainage of solution and solid particles through the froth. The movement of solid species relative to the solution is not a focus of this work, having been thoroughly investigated by, among others, Savassi (1998). Consequently, for the purposes of this discussion, the draining material will be considered as a homogenous fluid with given density and viscosity. Lastly, the impact of coalescence and bursting on drainage is considered. The impact of drainage on the froth stability is covered in Sections 2.8 and 4.2.

Drainage is a rate process and as such requires time to occur. The total time for drainage is the froth residence time, determined by the depth of the froth and the rate at which the froth rises, which in turn is controlled by the superficial gas velocity in the froth. Varying rates of drainage within the froth as a function of height complicate this picture in that the degree to which the froth drains within a given height segment in the froth is dependent both on the amount of time the froth remains in that segment, as well as the other factors affecting drainage described below. However, it is readily apparent that an increase in the time for drainage (such as an increase in the froth depth) leads to an increase in the degree of drainage of the froth at the froth surface. This leads to a decrease in the volume fraction of water in the froth at the surface for a deeper froth, independent of any changes in the froth bubble size. The interaction of drainage with changes in the bubble size is discussed in Section 2.4.2.4.

Verbist et al., (1996) derived a model to describe the transient drainage of liquid out of foam by conducting a force balance around a unit section of the Plateau border. Three forces are considered in the model:
1. Gravitational forces,
2. Capillary forces; and
3. Forces due to viscous dissipation.

In this approach, Verbist et al., (1996) like others (Goldshtein et al., 1996; Neethling et al., 2000) ignore inertial forces since the fluid in the Plateau borders is slow moving relative to the Plateau border structure. In the derivation that follows, the drainage forces are approximated as having only a vertical component and ‘upwards’ is taken to be the positive direction. This simplification is justified on the grounds that this discussion aims to highlight the impact of changes in the flotation conditions on drainage in the vertical direction. In accordance with the above list of forces, the mathematical derivation is:

2.4.1.1 Gravitational force

\[ \bar{F}_g = -\rho g \]  \hspace{1cm} [2-3]

where
\[ \bar{F}_g = \text{gravitational force vector} \]
\[ \rho = \text{apparent bulk fluid (pulp) density} \]
\[ g = \text{gravitational acceleration} \]

2.4.1.2 Capillary force

Capillary force arises from the action of surface tension on the decreasing cross sectional area of the Plateau borders and the attendant decrease in the radius of curvature of the Plateau borders with height. This thinning with height causes a pressure gradient, opposing the gravitational force which for the one dimensional case, can be expressed as:

\[ \bar{F}_c = -\frac{dP_i}{dy} \]  \hspace{1cm} [2-4]

where
\[ \bar{F}_c = \text{capillary force vector} \]
\[ P_g - P_l = \frac{\gamma}{R_{pb}} \]  

where

\[ P_g = \text{pressure of the gas phase} \]
\[ P_l = \text{pressure of the liquid in the Plateau border} \]
\[ \gamma = \text{surface tension} \]
\[ R_{pb} = \text{radius of curvature of the Plateau border} \]

Assuming that the gas pressure in the bubbles is very similar on each side of a given Plateau border (which is reasonable as the foam structure will deform to ensure that this is the case), Equations 2-4 and 2-5 can be simplified to:

\[ F_c = \frac{C\gamma}{2} A_{pb}^{-1.5} \frac{dA_{pb}}{dy} \]  

where

\[ A_{pb} = C^2 R_{pb}^2 \]

and

\[ C = \sqrt{\left( \sqrt{3} - \frac{\pi}{2} \right)} \]

where

\[ A_{pb} = \text{cross-sectional area of the Plateau border} \]
\[ C = \text{a constant} \]

2.4.1.3 Viscous dissipation force

Viscous dissipation is the 'drag force' resulting from the flow of the pulp within the Plateau borders relative to the Plateau border walls.

\[ \bar{F}_d = -\frac{k_1 \times k_2 \times \mu \times \bar{u}_{rel}}{A} \]  \[2-9\]

where

- \( \bar{F}_d \) = viscous dissipation force
- \( k_1 \) = shape factor due to morphology of the Plateau borders \( (50) \)
- \( k_2 \) = the number of times further, on average, the liquid has to flow by means of randomly oriented Plateau borders relative to a straight line between two points \( (3) \)
- \( \mu \) = bulk viscosity of the slurry
- \( \bar{u}_{rel} \) = apparent velocity vector of the fluid relative to the froth structure

The liquid velocity can be found from the apparent vertical velocity of the liquid relative to the froth structure and the bulk flow of the entire froth structure by summation:

\[ \bar{u} = \bar{u}_{rel} + \bar{v} \]  \[2-10\]

where

- \( \bar{u} \) = overall velocity of the liquid relative to an external frame of reference
- \( \bar{u}_{rel} \) = apparent velocity of the fluid relative to the foam structure
- \( \bar{v}_{f} \) = velocity of the froth relative to an external frame of reference

The sum of the three forces acting on the unit Plateau border must be zero:

\[ \bar{F}_g + \bar{F}_c + \bar{F}_d = 0 \]  \[2-11\]

and by substitution and solving for the overall liquid velocity, \( \lambda \).
\[ \bar{u} = -k_3 A_{pb} - \frac{k_4}{\sqrt{A_{pb}}} \frac{dA_{pe}}{dy} + v \]  \[ 2-12 \]

where

\[ k_3 = \frac{\rho g}{150 \mu} \]  \[ 2-13 \]

and

\[ k_4 = \frac{C \gamma}{300 \mu} \]  \[ 2-14 \]

Neethling (1999) combined the above with the continuity equation to extend the above derivation for a single Plateau border to the entire froth zone. The continuity equation is simply a volume balance around a volume of froth:

\[ \nabla (\bar{u} A_{pb} \lambda) = 0 \]  \[ 2-15 \]

where

\[ \lambda = \text{Plateau border length per unit volume of froth} \]

Neethling (1999) further developed a relationship between bubble size in the froth and the Plateau border length per volume of froth (\( \lambda \)) based on the geometry of mono-disperse Kelvin foam.

\[ \lambda = \frac{5\sqrt{3}}{\pi \varphi \times r^2} \]  \[ 2-16 \]

where

\[ \varphi = \frac{1 + \sqrt{5}}{2} \]  \[ 2-17 \]
Finally, it is clear that for foam, and assuming the bulk of the liquid to be present in the Plateau borders, the liquid content of the froth is the product of the cross-sectional area of the Plateau borders and the Plateau border length per unit volume.

\[ \varepsilon_L = A_{pb} \times \lambda \]  

[2-18]

where

- \( \varepsilon_L \) = fractional liquid content of the froth
- \( A_{pb} \) = cross sectional area of the Plateau border
- \( \lambda \) = Plateau border length per unit volume of froth

### 2.4.2 Promotion and inhibition of drainage

The rate of drainage and the lamellar robustness determine the froth stability, as discussed in Section 2.8. Leja (1982) reports that when viscosity and hence drainage is mainly responsible for froth stability, a change in the conditions such that either the viscosity is reduced or the drainage is facilitated will often collapse the froth.

Given the preceding analytical presentation of the forces acting on the entrained material in the Plateau borders, it is possible to present a qualitative discussion of the impact of various process parameters on the rate of drainage.

As presented in Section 2.3, the majority of the solution in the froth is contained in the Plateau borders (with the entrained and detached solids) and consequently we expect the water content of the froth to vary with a change in either:

1. the Plateau border cross sectional area, \( A_{pb} \)
2. the bubble size, and hence the length of Plateau border per volume of froth, \( \lambda \)
Sections 2.4.3.1 through 2.4.3.3 deal with the factors that alter the rate of intra-Plateau border drainage, and consequent impact on $A_{fb}$ without considering changes in the bubble size. Section 2.4.3.4 considers the impact of coalescence and changing froth structure on drainage.

2.4.2.1 Gravity

In general, changes in the rate of gravitational acceleration are not expected. Thus, changes in the gravitational force are proportional to the mass of the entrained material. This force can be considered largely independent of the commonly altered input parameters to the froth, with the exception of changes in the bulk density of the entrained material. Given that on the plants studied in this work, addition of dilution water is used to control the pulp density of the feed to the industrial flotation plants, disturbances in this variable were uncommon and the gravitational force was considered to have a consistent impact on the rate of drainage.

2.4.2.2 Viscous dissipation and pulp viscosity

On the other hand, changes in the viscosity of the slurry feed to flotation are expected to occur, due to changes in the particle size of the feed to float, changes in ore mineralogy, and the presence or absence of colloidal ferrous iron hydroxide precipitates (>300nm) or fine ferrous particles (1-5 micron) identified by Smart, (1991). These iron precipitates or fine particles can derive from the ore or from mild steel grinding media and are thought to impact on the viscosity of the slurry (Fornasierio and Ralston, 1992; Lovell, 1976).

Vand (1948) proposed the following relationship between bulk slurry viscosity and the solution and solid characteristics:

$$\mu^* = \mu(1 + 2.5\varphi_s + 4.275\varphi_s^2)$$  \[2-19\]

where

$$\varphi_s = \frac{X_s}{X_s + \frac{\rho_s}{\rho}}$$  \[2-20\]

and

$$\mu^* = \text{bulk viscosity of the slurry}$$
\[ \mu = \text{viscosity of the solution} \]
\[ \phi_s = \text{volumetric fraction of solids in the solution} \]
\[ X_s = \text{solid concentration in the slurry by mass} \]
\[ \rho_s = \text{solid density} \]
\[ \rho = \text{solution density} \]

This equation has proven useful for the determination of order of magnitude estimates for the viscosity of slurry in the Plateau borders of the froth (Ata et al., 2003).

However, it has been proposed that particle characteristics such as size and surface chemistry play a significant role in pulp viscosity in the context of froth flotation. Klassen and Mokrousov, (1963) identified the importance of the relative amounts and particle size distributions of hydrophobic and hydrophilic particles for the mobility of a flotation froth. Gaudin (1957) noted that ore bodies containing a high content of clay type minerals are typically treated as slurries with low solid concentrations. Leja (1982) reported that at high concentration, these particles yield slurries that have high viscosities. Researchers have identified changes in the bulk viscosity of the material in the froth due to changes in the concentration of fine hydrophilic solids in the entrained material as an important factor governing the water recovery from froth (Szatkowski and Freyburger, 1985; Ata et al., 2003; Tao et al., 2000; Lovell, 1976).

Stevenson et al., (2003) studied the effect of varying the viscosity of a water-glycerol solution on the performance of an aqueous froth column by changing the glycerol content from 0 to 0.6 volume fraction. The attendant change in viscosity was from 0.99 to 10.72 mPa.s. It was concluded that increasing the slurry viscosity retards the drainage of entrained solution from the froth and the obvious conclusion is that it does so by increasing the viscous dissipation force.

2.4.2.3 Capillary force and frother addition

Neethling et al., (2000) conclude that the capillary force and its counter-balancing gravitational force are the dominant forces in determining the water content of the froth, when reasonably dry polyhedral froth is present on the froth surface. Since the capillary force derives from surface tension at the gas-liquid interface, decreasing the surface tension through
the addition of frother should reduce the capillary force and accelerate drainage. This is the opposite of what is observed in practice, where frother addition retards drainage and increases the water recovery. The increase of water recovery with increasing frother concentration can be explained through two mechanisms:

1. The size of the bubble shells entering the froth increases (Sam, 1995) and the increased positive disjoining pressure in the lamellae promotes retention of water once in the froth (Derjaguin and Titievskaya, 1953)

2. As proposed by Plateau (1871), the surfactant introduces an interfacial viscous dissipation force term into the force balance, which retards drainage in a similar manner to the bulk viscous dissipation force.

Work by Sam (1995) has indicated a possible method for determining bubble shell thickness in the pulp and studies in this regard are ongoing (Morar, 2003).

The most important implication of the balance between the constant gravitational force and the drainage dependent capillary force is that, for given surface tension (relatively constant chemical composition of the surfactants in solution), the rate of drainage is dependent on the radius of curvature and therefore the thickness of the Plateau borders. This finding implies that after the period of rapid drainage close to the froth-pulp interface (predominantly governed by the viscous and gravitational forces) the Plateau border cross sectional area equilibrates to a fairly constant value, due to an equilibrium being established between the gravitational and capillary forces. This justifies the use of an assumed constant Plateau border cross sectional area in the determination of the water content in Chapter 4.

2.4.2.4 Effect of froth stability on drainage

Thus far, promotion and inhibition of drainage have been investigated in terms of factors that impact primarily on intra-Plateau border drainage, essentially assuming a constant froth structure. An equally important cause of changes in the water recovery (and hence recovery by entrainment) are changes in the froth structure.

On lamellar rupture, entrained material flows from the (now unstable) Plateau borders surrounding the ruptured lamella, into the adjacent Plateau borders of the new bubble and this, in turn, increases the volume of the Plateau borders, while decreasing the specific Plateau border length.
Consequently, the cross sectional area and radius of curvature of the Plateau borders increases and the capillary force holding the entrained and detached material in the froth decreases. Thus, the rate of drainage increases following coalescence and unstable froth is subject to greater rates of drainage than stable froth. This implies that the water recovery for an unstable froth is lower than for a more stable froth.

From a structural perspective, a froth that has undergone a high degree of coalescence, and thus has large bubbles, has a low Plateau border length and lamella area per unit volume. Assuming a constant Plateau border cross sectional area in the froth, which is reasonable given the constant balance of forces in the Plateau borders, the slurry content of the froth by volume is low under these conditions and, hence, the rate of water and solids recovery is low.
2.5 Effect of Surfactants in Solution on Lamellar Robustness

Surfactants in solution adsorb at the air-water interface, reducing the interfacial surface tension and increasing the froth stability. Frother is added to the flotation system to achieve the required level of lamellar robustness and froth fluidity for effective recovery of the flotation concentrate. Three theories have gained wide recognition as explanations of the role of surfactants in lamella stabilisation for two-phase systems (Leja, 1982). They are presented below, in chronological order.

2.5.1 Marangoni

A film is subject to constant mechanical disturbance from sources ranging from air currents and eddies to the vibration generated by the mechanism of an industrial flotation cell. Marangoni (1871) proposed that foam stability derived from the formation of regions of higher surface tension in expanding areas of the film and lower surface tension in compressed regions. Marangoni concludes that this phenomenon dampens disturbances acting on the films. This has been confirmed in experiments on dilute solutions of pure surfactants, which do not impart significant surface viscosities. In recent studies of flotation frothers, the reduction in the surface tension at either a static or a dynamic air-solution interface has been correlated with the froth stabilising action of the frother (Comley et al., 2002; Sweet et al., 1997). This method of determining the stabilising action of the frother molecules relies on the Marangoni effect.

2.5.2 Plateau

Plateau (1873), who conducted extensive work in the area of foams - particularly with respect to their structure, proposed that the rate of drainage of liquid from the inter-bubble lamellae was governed by the viscosity of the adsorbed surfactant film.

This theory is borne out by experiment for some systems, but not all. This supports the distinction between drainage dominated phenomena and lamella robustness in polyhedral foam as causes of froth stability. Oldroyd (1950) has shown that a complete description of the rheological properties of an interface requires two viscosity coefficients and two elasticity (dilational) coefficients. The experimental difficulty in directly determining these coefficients is significant and, hence, the relationship between surface viscosity and froth stability has not been fully tested.
2.5.3 Gibbs

Gibbs proposed a very similar explanation to that of Marangoni. However, he explicitly invoked the movement of surfactant molecules from thin regions (depletion of surfactant) to contracted regions (increased concentration) during disturbances. He then proposed that a local region of high surface tension arises in the thin region and the molecules move to restore the equilibrium surface concentration, thus reducing disturbances.

The net effect of the Gibbs and Marangoni explanations (Gibbs-Marangoni effect) of the impact of surfactants on the air-solution interface is the same, as they both relate to changes in the surface elasticity by different mechanisms. Certain limitations apply to this theory, particularly the fact that the magnitude of the surface tension gradients is difficult to measure, as is the surfactant concentration on the films.
2.6 Effect of solid species on lamellar robustness

A number of authors have observed that solid species in the flotation froth can either inhibit or enhance rupture of the lamellae, thus altering the froth stability (Gaudin, 1957; Johansson and Pugh, 1992; Dippenaar and Harris, 1982). Subrahmanyan and Forssberg (1988) identify the particle hydrophobicity distribution, solid concentration, particle shape or aspect ratio, state of aggregation and particle size distribution as factors affecting the impact of solid species on lamellar robustness. From among these variables, experimental studies have shown that the particle hydrophobicity and degree of coverage of the particles on the lamellae have a highly significant effect.

Further, experimental evidence suggests that hydrophobicity and lamellar coverage interact such that particles of a given hydrophobicity may have different effects on the froth stability at varying coverage on the bubble surfaces (Dippenaar and Harris, 1982; Aveyard et al., 1994). To decouple the effect of the two parameters, the impact of particles on lamellar robustness is presented for the case of particles 'in isolation' in Section 2.6.1, while the formation of particle layers is treated in Section 2.6.2.

2.6.1 Impact of the hydrophobicity of particles in isolation on lamellar robustness

This sub-section addresses the effect of particles at low coverage on the lamellae. Under these conditions the particles effectively act in isolation, since there is insufficient solid material for particle layers to form.

On the basis of hydrophobicity, particles can be divided into the following categories:

- Hydrophilic
- Slightly hydrophobic
- Moderately (or critically) hydrophobic with mixed hydrophobic as a special case
- Highly hydrophobic

For the discussion that follows, hydrophobicity will be considered as synonymous with contact angle, with more hydrophobic particles having higher contact angles. The relationship between contact angle, hydrophobicity and floatability is complex, particularly when one considers the difficulties of obtaining reproducible contact angle measurements.
(Harris, 2002). However, the relationship between these three variables is expected to be isotonic, that is, constantly positively correlated.

2.6.1.1 Hydrophilic species

Hydrophilic species do not attach to air bubbles due to the repulsive hydration forces and are therefore not present as attached species in the bubble lamellae. Entrained species may alter the viscosity, and hence drainage, from the froth thus inhibiting lamellar rupture. Once the entrained species are removed from the lamellae into the Plateau borders by capillary suction (Section 2.3) they are not expected to play a further role in altering the lamellar robustness.

2.6.1.2 Slightly hydrophobic

As predicted by Garrett (1979) and confirmed by measurements using cinematography (Dippenaar, 1982a), when the contact angle of the particle is relatively low (<50°); film thinning occurs in the solution region some distance away from the particle. At this point, the film could either equilibrate to the solution stabilised thickness, or rupture. The measurements made by Dippenaar (1982a) indicate that film rupture occurred at a distance of approximately 1 particle radius away from the particle. Thus, as with the hydrophilic species, slightly hydrophobic particles have limited impact on the lamellar robustness.

2.6.1.3 Moderately (critically) hydrophobic species

The role of a single particle of intermediate contact angle on retarding drainage from a film during the dynamics of film thinning is presented in Figure 2-8. The dynamics are represented by successive drainage steps from condition one through two, to equilibrium at condition three.

At the point at which the particle comes into contact with the air-solution interfaces on both sides, the film has a thickness twice that of the particle radius (Case 1). If the three-phase line of contact can move easily over the particle surface (in the case that the surface is smooth), the film thins to the point at which the three phase line of contact and the film thickness are in equilibrium with respect to the contact angle on the particle (Case 2). If the film drains to a thickness below the equilibrium thickness with respect to the particle (Case 3), the real contact angle on the particle is smaller than the equilibrium contact angle and the particle exerts a counteracting force on the adjacent solution, retarding drainage in the film adjacent to
the particle. Thus, moderately hydrophobic particles in isolation retain solution in the lamellae and reduce the rate of rupture of the lamellae.

If surface roughness inhibits the movement of the three phase line of contact, a thicker film will be stabilised, resulting in a greater stabilising action by the particle.

A special mineralogical case of the moderately hydrophobic species are particles presenting mixed hydrophobicity such as talcaceous minerals (talc) which are anisotropic and have highly hydrophobic crystal faces and edges of significantly lower hydrophobicity (Fuerstenau and Huang, 2003). These minerals are known to have a strong froth stabilising effect (Roberston et al., 2003).

![Diagram](image)

Figure 2-8: Moderately hydrophobic particle inhibiting film drainage and rupture from Garrett (1979)

2.6.1.4 Highly hydrophobic species

In the drainage process presented above, if the particle is highly hydrophobic, the high contact angle causes the three-phase boundaries to move over the particle surface in an attempt to reach equilibrium; until the two boundaries encounter each other and capillary suction induces
rupture from this initial hole. The dynamics of the movement of the three phase boundaries over the particle surface for the case of high contact angles is presented in Figure 2-9.

![Diagram](image)

Figure 2-9: Highly hydrophobic spherical particle inducing film rupture from Garrett (1979)

Evidence for this mechanism is presented by Garrett (1979) and is shown in Figure 2-10, where particles in a column increase the rate of froth breakdown, decreasing froth stability for increasing particle hydrophobicity. The feed solids concentration is very low (0.08 wt%) and this implies that the observed effects are the effect of particles in isolation inducing rupture in the lamellae.
Figure 2-10: Correlation between volume of froth destroyed per second and contact angle of PTFE powder at 0.08 wt% in the feed. (A) Solution only; (B) 0.01 M SDS in 0.1 M NaCl; (C) 0.01 M SDS; (D) 2 x 10^{-3} M DDPS from Garrett (1979)

These results are qualitatively similar to those obtained by Aveyard et al., (1994) for individual hydrophobic rods inserted into lamellae, in that no stabilised regime is observed and only froth destabilisation is observed at increasing hydrophobicity. However, the contact angle ranges reported by these two authors are notably different, possibly due to the overestimation of contact angle by Aveyard et al., (1994).

Dippenaar (1982a), confirmed the mechanism of lamellar rupture for individual highly hydrophobic galena particles in films, with additional notes regarding the effect of the orientation of the cubic galena particles with respect to the film.

Studies (Johansson and Pugh, 1992; Schwarz and Grano, 2002; Ata et al., 2003) report the hydrophobicity ranges that lead to either froth stabilising or froth breaking effects. Johansson and Pugh (1992) report that, for a sparsely mineralised (2 wt% in the feed) quartz system:

- Hydrophilic particles $\theta < 40^\circ$ had no effect on the froth stability
- Moderately / critically hydrophobic particles ($\theta \approx 65^\circ$) stabilised the froth
- Highly hydrophobic particles ($\theta > 90^\circ$) destabilised the froth

Schwarz and Grano (2002) give the following contact angle ranges for similar effects at a similar feed concentration of hydrophobic species:
• Slightly hydrophobic particles ($\theta < 53^\circ$) had a limited effect on froth stability
• Critically hydrophobic particles ($\theta \approx 63^\circ$) stabilised the froth
• 'Highly hydrophobic' particles ($\theta \approx 69^\circ$) de-stabilised the froth

The above results are in excellent agreement regarding the trends that are observed on increasing hydrophobicity. For particles at low concentration in the froth, increasing hydrophobicity first increases the froth stability up to a critical contact angle and decreases the froth stability thereafter.

In a second set of results Aveyard et al, (1994) presented shake tests, using an unspecified concentration of particles in solution. As discussed above, the single rod tests showed rapid destabilisation of the films at contact angles above $85^\circ$, with no stabilising effect at intermediate hydrophobicity. However, the tests for the particles showed significant stabilisation of the lamellae at intermediate contact angles between $80^\circ$ and $90^\circ$. This suggests that particle concentration interacts with the particle hydrophobicity to govern the overall impact of solids on the stability of froth lamellae.

Ata et al., (2003) investigated the impact of glass beads with three levels of hydrophobicity on the stability of the froth in a froth column. They also investigated the effect of adding fine, hydrophilic silica material to the feed. The concentration of glass beads in the feed was 7.5 wt%, and the same concentration of fine (5\,\mu m) silica was added to the feed for two of the three sets of tests. The experiments with entrained silica at three levels of hydrophobicity were repeated with two air spargers. This work indicated that:

• Addition of fine silica retarded coalescence, in agreement with Section 2.4.2.2
• Slightly hydrophobic particles ($\theta \approx 50^\circ$) had no effect on the froth stability
• Moderately hydrophobic particles ($\theta \approx 66^\circ$) stabilised the froth
• Highly hydrophobic particles ($\theta \approx 82^\circ$) had an intermediate effect on froth stability, imparting some stability to the froth

Ata et al. (2003), concluded that their results did not verify the findings of Dippenaar, (1982a); Johansson and Pugh (1992) and Aveyard et al (1994), in that highly hydrophobic solids were not found to destroy the froth and, instead, partially stabilised the froth. The work was done at higher particle concentration in the feed. This leads to consideration of the
noted that stabilising action by the particles relative to the solution case was observed during the course of the test work. This implied that the surface area of the particles had a particular significance for their stabilising or destabilising effect and that the highly hydrophobic sulphides used by Dippnaar (1982a, 1982b) caused froth stabilisation for specific size ranges and concentrations. Thus, a particle layer may have a different effect on froth stability to that which would be expected, given the effect of individual particles on lamellae.

Figure 2-12: Diagram of particles stabilizing a lamella

Figure 2-12 represents a layer of closely packed particles within a stabilised froth lamella. In flotation practice, the range of particle diameters present is typically approximately an order of magnitude smaller than the film thickness and the particle size range in the lamellae is far greater than is depicted in the figure. Hence, the particles pack more effectively than shown, increasing the surface area of adjacent particles in contact.

Hydrophobic inter-particle attraction forces (Derjaguin, 1985; Yoon and Aksoy, 1999; Nguyen et al., 2003; Hu and Dai, 2003) increase the mutual attraction of the particles in the lamellae and, hence, contribute to the rigidity and robustness of the lamellae in the froth. Thus from the literature and current understanding particle concentration as well as particle hydrophobicity are critical for the stabilising effect of particles on lamellae.

It is proposed that the most logical terms in which to consider particle surface coverage is the particle cross sectional surface area flux ($S_p$) per unit bubble surface area flux ($S_b$) and this concept is developed and formalised in Chapter 4.
2.7 The roles of the rate of drainage and lamellar rupture in determining stability

The impact of selected input parameters on the rate of drainage and the lamellar robustness has been presented in the preceding sections. Below is an overview of the manner in which drainage and persistence of the lamellae determine the probability of rupture of lamellae or froth stability.

2.7.1 Combining lamellar robustness and drainage

For a lamella to rupture, it must first thin (drain) to the point at which rupture can occur. The time taken for the films to reach this point of potential rupture is governed by the rate of drainage out of the lamellae and the initial froth water content. The time taken for a film to thin to the point at which rupture can occur can be considered as the lower bound on the time (or height for given superficial gas velocity) for which a froth zone must remain stable. Dippenaar, (1982a) concludes that in the system of individual highly hydrophobic particles in lamellae, the time to drain is the rate determining step for lamella rupture, with rupture following rapidly after sufficient drainage has occurred; such that the galena particle comes into contact with both of the air-solution interfaces.

Changes in the lamellar robustness manifest under well-drained conditions, when the froth adopts a polyhedral structure. Once the films are fully drained, the Gibbs/Marangoni effect, disjoining pressure and particulate effects on lamella robustness determine the probability of rupture of the thin films. Thus, the stability of polyhedral froth is dominated by the lamellar robustness; as opposed to the rate of drainage. The froth can be subdivided into two zones: the region close to the froth-pulp interface, dominated by the froth drainage characteristics and the region closer to the froth surface in which lamellar robustness determines the froth stability.

The rate of drainage impacts on the time (height) after which coalescence can occur and the lamellar robustness governs the rate of rupture after (above) that point. Both influence the ultimate degree to which the froth has coalesced by the time it reaches the concentrate. Stabilising factors, whether related to the rate of drainage or the lamellar persistence, extend the froth life-time; effectively increasing the terminal froth height at a given superficial gas velocity and increasing the water recovery at a given height (Leja, 1982).
possible mechanism by which higher concentrations of solids may contribute to film stability and a discussion of the formation of particle layers in Section 2.6.2.

2.6.2 Lamella coverage and formation of particle layers

This section explores the formation of particle layers, which have been mentioned by numerous authors as significant for the froth behaviour, particularly the froth stability (Johansson and Pugh, 1992; Aveyard et al., 1994 and Gaudin 1957). At the froth-pulp interface, the attached particles form a cluster on the underside of the bubbles to which they are attached, as shown in Figure 2-11. They cover a small proportion of the bubble surface, since aggregates containing a large mass of particles have negative buoyancy (Bradshaw and O'Connor, 1996).

![Figure 2-11: Formation of bubble-particle aggregates in the pulp phase from Plate (1986)](image)

As coalescence proceeds in the froth, the surface area available for attached particles decreases. As per the discussion of Section 2.3.3.1, the reduction in available surface area causes an increase in the attached particle surface area coverage per unit lamella surface area. Thus as the froth coalesces, the particles become more closely packed on the lamella surfaces and form a particle layer. At this point, the froth has reached maximum carrying capacity.

Dippenaar and Harris (1982) note that correcting the mass of particles for the particle radius squared, such that a constant surface area of particles was added to the feed of the system, induced a constant level of instability in the froth. Further, Dippenaar and Harris (1982)
CHAPTER 3: MACHINE VISION FOR FLOTATION

3.1 INTRODUCTION

Recent developments in digital imaging capacity have promoted the development of machine vision systems, which can approximate human perception of aspects of a scene presented to a computer via a camera, communications link and digitising hardware. The analysis is performed in software and if well designed and implemented, the measurements generated by the system capture appropriate aspects of human understanding of what is observed in a quantitative, robust, accurate and reproducible manner.

A number of commercial machine vision systems for flotation have recently been developed including VisioFroth (CISA Vision Systems, 2003), FrothMaster™ (Outokumpu, 2004) and JK Frothcam (JKTech, 2001). The interested reader is referred to the review of these systems, and other work in the field of froth image analysis in Francis (2001). The rising popularity of these commercial systems and the high level of industrial interest in image analysis of flotation indicates that flotation plants recognise the need for additional, reliable measurements of the flotation process, and recognise that machine vision systems are well suited to providing these measurements.

SmartFroth has been developed to conduct fundamental research into the relationships between machine vision measurements and flotation performance and the algorithms developed and implemented in SmartFroth perform analysis of the froth surface appearance in order to generate outputs that are termed froth surface descriptors. The algorithms are presented, with notes regarding the application of these measurements to industrial systems. A comprehensive description of the algorithms can be found in Wright (1999), Francis (2001), Forbes (2004) and in patents SmartFroth 2, 3 & 4 (2004).

Prior to initiation of this work in 2000, a machine vision system had been developed that performed fast watershed bubble segmentation (Wright, 1999). During the time over which the work described by this thesis was performed, the SmartFroth software has been optimised and coded within a general machine vision architecture, and three independent froth velocity algorithms, a colour analysis module, a froth texture algorithm and a froth surface stability detection algorithm have been included in the software. While all image analysis
concentration of solids and solution per unit volume of froth, and the rate of recovery of concentrate is less than would be expected from the increase in the flux into the froth.

2.10.5 The effect of the rate of frother addition on froth performance and appearance

It is hypothesised that increased frother addition increases the robustness of the lamellae in the froth and thus increases the froth stability and reduces the rate of coalescence in the froth zone. The reduced rate of coalescence reduces the bubble size at any given height in the froth, and this reduces the rate of drainage from the froth, as presented in Section 2.4.2.

Further, the increased strength of association between the froth-solution interface and the solution, mediated by the frother molecules, increases the thickness of the lamellae in the froth. Thus, both reduced coalescence and increased lamella thickness contribute to the increase in the rate of water recovery at higher frother addition rate and this is associated with an increase in the rate of recovery of entrained material to the concentrate. Increased stability and reduced coalescence results in reduced froth surface bubble size and increased froth surface stability.

When frother is added, the rate of detachment of valuable particles from lamellae decreases due to the reduced rate of coalescence, and thus the rate of recovery of attached species is increased by the reduced rate of drop-back of valuable solids to the pulp.
and slightly increase the froth surface burst rate. Alternatively, the nearly instantaneous bursting of bubbles on the froth surface can lead to a smaller observed surface bubble size and a very high froth surface burst rate. Hence, there is a non-linear relationship between the froth surface bubble size and the froth stability when increasing depressant addition. The effect of increasing depressant dosage on the froth surface bubble size depends on the flotation system, the solids loading and the type of solids in the froth before and after depressant addition.

An additional subtlety in terms of the impact of solids of different hydrophobicity on the froth surface appearance and performance is seen when the impact of depressant addition on the froth surface stability is monitored over time and hence as the solids are removed from the system. When depressant is not added the floatable gangue stabilises the froth and the froth stability decreases as the solids are removed. When depressant is added and the floatable gangue does not enter the froth, the removal of solids from the system increases the froth stability. This shows that, in agreement with Section 2.6.1, the moderately hydrophobic floatable gangue species stabilise the froth while the highly hydrophobic sulphide species destabilise the froth in the absence of the stabilisation afforded by the floatable gangue.

2.10.4 The effect the rate of air addition on froth performance and appearance

It is hypothesised that increasing the rate of air addition increases the rate of recovery of solids and solution to the concentrate. The increase is significantly lower than could be expected given theoretical consideration of the increased flux into the froth and the increased superficial velocity through the froth, and hence shorter froth residence time.

An important factor limiting the rate of recovery of solids and solution under conditions of high air addition is the interaction of the air rate with the froth stability, mediated by the role of solids in stabilising the froth. Increased air addition and increasing bubble surface area flux decreases the solids attachment per bubble in the pulp and hence decreases the particle coverage per unit bubble surface area of the bubbles entering the froth.

The reduced solids coverage on the bubbles entering the froth reduces the froth stabilising action of the solids on the lamellae and hence increases the rate of coalescence in the froth. Thus, despite the expected decrease in bubble size resulting from the reduced FRT, the froth surface bubble size increases with increasing air addition. Hence, there is a lower
Attached solids in the froth are not subject to drainage, whereas entrained and detached solids in the Plateau borders and water in the Plateau borders and the lamellae are subject to drainage to a greater or lesser extent. The resulting imbalance between the rates of drainage of solids and solution causes an increase in the solids concentration of the slurry in the froth as drainage proceeds. Thus, an increase in the froth residence causes an increase in the percent solids of the concentrate.

Since unattached (entrained and detached) particles in the Plateau borders are subject to a greater rate of drainage than the particles in the lamellae of the froth, and the unattached solids are of a lower grade than the attached solids, an increase in the froth residence time leads to an increase in the concentrate grade.

A reduction in the recovery of solids from the froth with increasing froth depth can be due to both increased drainage of entrained material and possibly due to detachment and drainage of valuable material from the froth.

Changes in the froth surface bubble size and froth surface stability when changing the froth depth can be measured using a machine vision instrument.

2.10.3 The effect of gangue depression on froth performance and appearance

It is hypothesised that solid species play a key role in stabilisation of the froth structure. The presence of floatable gangue species (including talc) stabilises the froth. The removal of the stabilising gangue species when depressant is added leads to a less stable froth and an increase in the rate of coalescence and bursting.

Unstable froth is subject to a higher rate of coalescence and bursting than stable froth, and a high rate of coalescence results in an increase in the rate of drainage from the froth, as outlined in Section 2.4. Thus, in addition to the reduction in the recovery of attached gangue to the concentrate the recovery of water and entrained gangue decreases when depressant is added to the flotation system.

Increased depressant dosage which removes froth stabilising floatable gangue solids causes increased coalescence and bursting (less stable froth) and this can lead to one of two results. The increased rate of coalescence within the froth can increase the froth surface bubble size
2.10 OBJECTIVES AND HYPOTHESES

2.10.1 Research Objectives

The objectives of this thesis are to define froth stability, develop a model to test the effect of input parameters on the froth stability and experimentally test the proposed mechanisms with metallurgical performance indicators and measurements of froth surface appearance using a newly developed froth machine vision system.

In order to extend the current understanding of froth stability the following key questions will be addressed:

1. How can the froth structure best be described and what are the mechanisms governing froth formation within the context of this structure?
2. Mechanistically, how do selected hydrodynamic and reagent variables alter the froth state, froth surface appearance and metallurgical performance?
3. Can these mechanisms be confirmed by appropriate measurements of the froth surface appearance from machine vision algorithms?
4. Can a model be developed to illustrate the effect of the input variables on the froth structure and, thus, on the relevant froth surface descriptors and flotation performance parameters?

Hypotheses have been developed to test the key questions as they relate to the selected input variables. The frother dosage, depressant dosage, air addition rate and froth depth are chosen for investigation as inputs that are typically altered in platinum flotation to vary the performance of the process. In particular, how do the selected variables alter the froth structure, froth surface appearance and metallurgical performance and can these mechanisms be confirmed by appropriate measurements of the froth surface appearance from machine vision algorithms?

2.10.2 The effect of changing froth depth on froth performance and appearance

It is hypothesised that changing froth depth changes the froth retention time and that this, in turn, alters the time for coalescence and drainage in the froth. It is further hypothesised that the rate of coalescence in the froth decreases as time in the froth zone increases due to stabilisation of the lamellae by solid species.
The theoretical basis for the impact of the solution species on the lamella robustness has been discussed in terms of the theories of Plateau, Marangoni and Gibbs in Section 2.5 and this is identified as key factor affecting the froth stability.

In Section 2.6, the solid species have been shown to have an impact on the lamellar robustness, with the effect of the particles depending on numerous factors, key among which are both the particle hydrophobicity and the particle coverage on the lamellae. Contradictory observations of the effect of highly hydrophobic particles on lamella robustness have been noted. Johanssen and Pugh (1992), Schwarz and Grano (2002) and Aveyard et al. (1994), show destabilisation of lamellae by highly hydrophobic particles at low solids concentration in the froth. Dippenaar (1982), Aveyard et al. (1994), and Ata et al. (2003), note stabilisation of lamellae by relatively hydrophobic particles but these observations were at higher particle concentrations in the froth. This shows a need to consolidate these findings regarding the impact of particle properties including concentration and hydrophobicity on film stability.

The rate of drainage of entrained material from the froth, as well as the intrinsic lamellar robustness contributes to the froth stability. The exploration of the relationship between these variables and the froth stability in Section 2.7 draws on and extends current theory. These findings demonstrate that drainage both determines, and is determined by, the froth stability and highlights the importance of the stability of the froth as a determinant of the metallurgical performance in flotation.

Coalescence and bursting, both recognised as important mechanistic phenomena in the froth, are associated with the rate of lamellar rupture in the froth and the rate of lamellar rupture is associated with the froth stability. This association, combined with consideration of previously proposed definitions of froth stability in Section 2.8, shows the need for a new definition of froth stability.
2.9 LITERATURE SUMMARY

In Section 2.1, an appropriate description of the froth structure for the purposes of determining the mechanistic action of input variables on the froth structure, surface appearance and flotation performance has been presented.

In Section 2.2, the froth structure has been described in terms of the lamellae separating the bubbles and the Plateau border network at the lamellar interfaces. The limitations of existing descriptions of the froth structure have been noted and the approach of Neethling et al., 2000 which considers a loss of Plateau border length to be equivalent to the loss of Plateau border length and associated vertex volume is proposed for use in further development.

The distribution of components within the froth structure is crucial for determination of the mechanism through which an input parameter impacts on the froth. A change in a given input may alter either the solid or solution species, and may alter either the character of the material in the lamellae or Plateau borders. In Section 2.3 the distribution of the solid and solution components in a well-dried froth has been discussed, highlighting the presence of predominantly attached species in the lamellae, with entrained and detached material resident in the Plateau border network, and thus subject to drainage. The impact of lamellar rupture on the re-distribution of material between the lamellae and Plateau border network has been presented.

Drainage has been discussed in Section 2.4 in terms of the factors that accelerate and inhibit this phenomenon; which are changes in the balance of the gravitational, capillary and viscous forces. Intra-Plateau border drainage and drainage due to coalescence causing structural changes in the froth have been identified as two major contributors to the overall rate of drainage. Of particular relevance to the rate of intra-Plateau border drainage is the impact of fine hydrophilic species on the recovery of entrained material in that fine particles at high concentration can increase the viscosity of the draining material and thus increase the water content of the froth. It has been noted that the effect of the rate of coalescence on drainage is insufficiently explored and given that coalescence results in a loss of stable lamella and Plateau border volume it is proposed that this mechanism is an important contributing factor to the rate of drainage.
\[ \alpha = \frac{Q_w}{Q_A} \]  

where

\[ \alpha \quad = \quad \text{fraction of air overflowing the weir} \]

\[ Q_A \quad = \quad \text{volumetric rate of air into the cell (m}^3/\text{s}) \]

\[ Q_w \quad = \quad \text{volumetric rate of air overflowing the weir (m}^3/\text{s}) \]

And \( Q_w \) is determined by a volume calculation

\[ Q_w = v_F \times H_w \times P_c \]  

where

\[ v_F \quad = \quad \text{velocity of the froth surface towards the weir} \]

\[ H_w \quad = \quad \text{height of the froth over the weir} \]

\[ P_c \quad = \quad \text{perimeter length of the weir} \]

The measured variables \( \alpha \) and \( \Sigma \) are related to lamellar rupture through the bubble size, as detailed in the Froth modelling section.
2.8.2 The dynamic froth stability factor

A metric of froth stability was first formalised by Bikerman (1938). The measurement involved observing the rate of breakdown of froth from the equilibrium height attained in a sparged column. Bikerman defined the Dynamic Froth Stability Factor as:

\[ \Sigma = \frac{\Delta H}{\Delta t} \]  

where:

- \( \Sigma \) = dynamic froth stability factor
- \( H \) = height
- \( t \) = time

Although intended as a quantitative measure of froth stability, tests conducted using different experimental setups have yielded different results and, although this test (and the equivalent ‘shake test’) remains popular as a qualitative measure of stability (Dippenaar, 1982b), it can be used only as a measure of relative stability using a given experimental rig. Further, a single operator is required in the case of the shake test to achieve reasonable reproducibility of the results (Aveyard et al., 1994).

Since the measurement is equivalent to a measure of the rate of loss of volume of the froth per unit column cross sectional area, one can consider it as the superficial gas velocity out of the top of the froth. In a continuous flotation system where air is added to the bottom of the froth and a fraction of the air overflows the weir in unbroken bubbles, this measure is conceptually equivalent to the alpha parameter described below; except that they are inverse measures or, in other words, high \( \Sigma \) is measured for an unstable froth, while high alpha is measured for a stable froth.

2.8.3 Fraction of air overflowing the weir

The fraction of air overflowing the weir has been proposed as a measurement of froth stability (Woodburn et al., 1994; Neethling and Cilliers, 1998). This fraction is determined from the ratio
2.8 Froth Stability

Numerous authors report froth stability as a key parameter in flotation performance (Gaudin, 1957; Subrahmanyan and Forssberg, 1988). Despite the emphasis on this parameter, there is little consensus regarding the definition of froth stability. There are a range of definitions with respect to the meaning of stability as used in the English language and an appropriately wide range of parameters have been used to measure and describe film or froth stability in the flotation literature (Bikerman, 1938; Ventura-Medina and Cilliers, 2004; Woodburn et al., 1994; Aveyard et al., 1994). Each measurement implicitly defines froth stability in terms of the measured parameter, but a mathematical definition is required that is independent of a given stability measure and that derives directly from the fundamental mechanism governing froth stability: lamellar rupture.

A selection of conceptual definitions of stability and the measures proposed to date is presented and an attempt is made to define stability as it relates to froth performance. A few of the mathematical consequences of such a definition are presented, from the point of view of froth modelling.

2.8.1 Conceptual froth stability

Presented below are a few dictionary definitions of stability:

- Resistance to change, deterioration, or displacement
- Fixedness; as opposed to fluidity
- The quality of being free from change or variation
- The property of a body that causes it, when disturbed from a condition of equilibrium or steady motion, to develop forces or moments that restore the original condition
- Resistance to chemical change or to physical disintegration

The concepts of displacement and fluidity refer more accurately to the mobility and viscosity of the froth. Further, the concept of equilibrium is inappropriate in the context of flotation, since coalescence is irreversible. Conceptually, froth stability is most closely linked to the resistance of the froth to change or physical disintegration.
2.7.2.4 Kugelschaum

Kugelschaum essentially consists of a bubble suspension in solution. The defining characteristic of Kugelschaum is that the bubble films rupture rapidly once they encounter each other (Kitchener, 1964). The bottom layers of flotation froth have been likened to Kugelschaum and, thus, a range of drainage rates is expected, potentially leading to partial stabilisation by a reduction in the rate of drainage.

2.7.2.5 Solid foam

One can argue that lamellar rupture and coalescence would occur for glass foam should drainage occur. The zero or near-zero drainage rates prevent this coalescence, thus accounting for the persistence of the foam. A similar argument applies to expanded polystyrene or polyurethane foam. These foams are labelled ‘solid foam’ in Figure 2-13 and this is a descriptive rather than technically accepted term.

2.7.2.6 Meta-stable froth

It is possible to place flotation froths within this framework. Flotation froth has been termed ‘meta-stable’. Froth with a persistence as low as Kugelschaum would not have sufficient residence or robustness to withstand transport to the weir of a flotation cell, although many bench scale floats exhibit this froth type. Flotation froth should not be as persistent as polyhedral detergent based foam, since downstream processing requires the froth to break down in the flotation cell launders after recovery to the concentrate.

Hydrophobic inter-particle bonding between close-packed particles in the lamellae may generate froth that approximates the stability and low drainage rates of solid foam, as the solution is prevented from draining by its association with the solid species. This froth type may remain stable in the absence of significant mechanical stress, but typically breaks down when not constantly regenerated by the addition of air. This polyhedral, well drained froth type minimises recovery by entrainment and results in a high grade concentrate.
2.7.2 Foam types

2.7.2.1 Fine foam

Whipped cream, an example of highly stable foam, has been labelled ‘fine foam’. Whipped cream drains relatively slowly (relative to flotation froth), due to the high viscosity of cream. However, this is largely irrelevant, due to the very high persistence of the lamellar films that are formed in the foam, due to the presence of high concentrations of highly surface-active proteins in the cream. A similar argument applies to shaving foam. ‘Fine foam’ is not a recognised class of foam and is intended to be descriptive of the examples provided here.

2.7.2.2 Persistent foam

Persistent polyhedral foams (Kitchener, 1964) drain quickly, assuming that they consist of a relatively inviscid aqueous solution. The persistence of the foam derives from the Marangoni (or disjoining pressure) stabilisation of the lamellar films, that form once the froth has drained. The stabilising agents are highly surface-active molecules in solution. The most common example of this foam is an agitated solution of dishwashing liquid. It should be noted that, depending on the mode of agitation, fine foam structures may result from a persistent foam solution. However they can be expected to have a somewhat lower overall stability due to differences in the nature of the relevant surfactants.

2.7.2.3 Transient foam

Transient foam (Kitchener, 1964) may result from the action of relatively weakly surface-active molecules in solution on a bubble generation system, or by the action of highly surface-active molecules at low concentration. An example of transient foam, of great interest for the study of froth flotation, is a frother solution in water; in the case of a two-phase study of frothers or froth structure.

Transient foam shares many characteristics with flotation froth, although it may drain faster than flotation froth, due to the absence of solids in the foam that increase the viscosity of the interstitial phase. The impact of the solid species on the persistence of the lamella (relative to transient foam) is of particular relevance for froth flotation. Further, transient foam differs from flotation froth in terms of the thickness of the lamella in the two cases.
development was collaborative, the development of the froth surface stability algorithm in particular is an original contribution stemming from this work.

The following froth visual parameters have been identified as most significant for quantifying the froth surface appearance within the context of Platinum group metal (PGM) flotation and are used in this thesis (Botha, 1999):

1. Bubble size distribution described by the Sauter mean bubble diameter and bubble size homogeneity, using the watershed segmentation, as existed prior to this thesis.
2. Froth surface velocity as detected using three independent metrics, developed in conjunction with this thesis.
3. Stability index or rate of bubble bursting on the froth surface developed in this thesis.
3.2 Watershed segmentation and measurement of bubble size

Watershed segmentation is used to determine the boundaries between bubbles allowing for determination of the froth surface bubble size distribution, Sauter mean diameter and bubble size homogeneity. Wright (1999) applied the Fast Watershed algorithm of Vincent and Soille (1991) to segmentation of the froth into bubbles. The code was updated and optimised to improve the accuracy (Francis, 2001) and speed (Botha, 1999; Old, 2001) of the segmentation.

The description of the algorithm relies on a morphological analogy to the underlying two dimensional matrix represented by pixels in an image. The grey levels of the image are taken to be equivalent to the elevation of a region in the image. The use of a single, bright illuminant for the image capture ensures a single bright highlight on each of the bubble surfaces. The highlights are the ‘peaks’ of the image in the morphological context. Figures 3-1 to 3-3 illustrate the transform from the brightness of the pixels of the image to the elevation of a ‘landscape’.

![Image](image-url)

Figure 3-1: The original image before pre-processing
Figure 3-2: The contrast-stretched image as a contour map

Figure 3-3: The image represented as a topological surface, showing the highlights of the bubbles as peaks

Figure 3-4: The inverted image as a topological surface showing the 'valleys' and the ridge-lines or watersheds of the 'terrain'
The image of the bubbles, once inverted, consists of a field of hollows or valleys (at the image highlights), separated by ridges at the boundaries between the bubbles (see Figure 3-4). The algorithm metaphorically ‘pierces’ the terrain at the bottom of the valleys using the marker image, and fills the valleys until adjacent regions intersect at the ridges. In geographic terms, these ridges would be the ‘watersheds’ of the terrain, hence the name of the algorithm. In terms of image analysis of flotation, they are the boundaries between the bubbles.

The watershed algorithm consists of the following steps:

3.2.1 Contrast enhancement

This consists of setting a configurable number of pixels (typically 5%) to the minimum and maximum values (0 and 255 respectively), and linearly scaling the grey-scale histogram of the image to map from the original minimum and maximum values to the new values. This insures the reliability of thresholds used in the marker extraction.

3.2.2 Low pass filtering

Low-pass filtering removes noise and artefacts of the image capture process from the image to be processed. Low-pass filtering consists of a configurable number of passes over the image (convolutions) with a square structuring element of configurable dimensions. This operation is termed a ‘box blur’ operation. The values for the parameters are two passes with a 3x3 or 5x5 structuring element. The number of passes defines the ‘smoothness’ of the result with three passes yielding a reasonable approximation to a Gaussian smoothing operation. Large numbers of passes have a high computational cost. The size of the structuring element defines the degree of smoothing, or cut-off spatial frequency of the low-pass filter.

Increasing the size of the filter reduces the likelihood of over-segmentation of the large bubbles, and a blurring of the small bubbles to the point where regions of bubbles on the froth surface having the same dimensions as the pulp bubble size are under-segmented into large regions representing spurious bubbles. Conversely, a small structuring element reduces the chances of under-segmentation of the small bubbles, while increasing the risk of over-segmentation of the large bubbles.
3.2.3 Extraction of maxima

3.2.3.1 Subtraction and grey-scale reconstruction

The marker extraction consists of subtraction of a configurable value from the input image, followed by grey-scale reconstruction of the subtracted image under the original image. The grey-scale reconstruction "spreads" the image across constant grey-scale values until it touches the original image. Using the morphological analogy the spreading is a horizontal expansion under the original image until the maxima are isolated peaks between the original and grey-scale reconstructed images.

3.2.3.2 Subtraction and thresholding

The reconstructed image is subtracted from the original image to yield only the 'peaks', or maxima, and the image is thresholded at a value of 1 to yield a binary image consisting of the peaks in white on a black background. This image is the 'marker image', in that it marks the positions of the maxima (or minima on the original image).

3.2.4 Homotopic modification of input image with marker image

Using the morphological analogy, the process known as homotopic modification is used to 'smooth the sides' of the valleys in the original image (using the information in the marker image) so that the subsequent segmentation does not get 'stuck' on rough edges on the walls of the valleys.

3.2.5 Vincent's fast watershed algorithm

This procedure 'fills' the valleys at increasing grey levels, until adjacent regions intersect and define the bubble boundaries. The output of the above algorithm on the sample image region is shown in Figure 3-5.
3.2.6 Determination of the Sauter mean bubble diameter

The bubble boundaries having been identified, the bubble edges are fitted with ellipses using standard square error minimization techniques. The volume and surface area of the bubbles is approximated by rotation of the ellipse around the major axis, and

\[ d_{30} = \frac{\sum V_i}{\sum S_i} \quad [3-1] \]

where

- \( d_{30} \) = Sauter mean bubble diameter
- \( V_i \) = equivalent volume of the \( i^{th} \) bubble
- \( S_i \) = equivalent surface area of the \( i^{th} \) bubble

3.2.7 Bubble Shape

The ellipse fitting, coded by Wright (1999) with additional measures and modifications by the author and Morar (2002) is described in the patent SmartFroth 2 (2004). In general, spherical bubbles on the froth surface have not dried substantially before reaching the froth surface. This implies high entrainment and low grade with high recovery. Polygonal bubbles are the proposed ideal shape where significant froth drainage and removal of gangue has occurred, but the bubbles are not excessively deformed. Highly elongated or 'kidney shaped' bubbles
are indicative of a highly mineralised, 'mature froth' which has had time to be exposed to
deformation in the froth zone.

The ellipse fitting procedure described above can be used to determine a shape factor, which
is the ratio of the lengths of the major to minor axes of the ellipse. Alternatively, the length of
the perimeter of a circle that encloses the measured area of the bubble divided by the
measured length of the bubble perimeter, is an alternative metric of the bubble circularity.
3.3 Measurement of Froth Surface Velocity

The froth surface velocity is a froth transport parameter. The froth velocity directly relates to the rate of recovery of concentrate over the cell lip (see Chapter 4 for details). Thus, the froth velocity relates directly to the froth recovery. Highly mobile froths are usually linked to low grade, high recovery conditions. Slower froths are dry, mineralised and sticky and are linked to high grade and low recovery. Very slow moving or stationary froth implies that little or no concentrate is being recovered.

The importance of this parameter, and extensive testing of the validity of the parameter under a wide range of froth flow speed conditions led to the development, and implementation in SmartFroth, of three distinct measures of the froth surface flow speed namely bubble tracking velocity, block matching velocity and burst rate velocity.

3.3.1 Bubble tracking velocity

The bubble tracking velocity algorithm was developed and tested by Rapacz (2002). In the bubble tracking velocity algorithm the software captures a sequence of images (of configurable length, but typically 5) and performs watershed segmentation on each of the images using the algorithm described in Section 3.2. The bubbles in each frame are then matched to the bubbles in the next frame using the relative area of the bubbles and the bubble proximity in successive frames as the closeness of match criteria. It is the bubble proximity criterion that ensures accurate matches between frames, but renders the algorithm inaccurate at high flow speeds. The matching process is formally known as ‘annealing’ that ensures one-to-one matching of the elements and achieves overall optimisation of the matches. The annealing algorithm consists of an inner loop of iterative row and column normalisation to convergence, followed by increasing the errors by replacing each error value with its exponential value in an outer loop, and iteration in the outer loop until convergence is reached and the matches are found.

StoneThree Digital Signal Processing incorporated the algorithm in SmartFroth according to specification (SmartFroth 2, 2004). The bubble tracking velocity has very high accuracy, and is appropriate for the measurement of sub-pixel flow of the froth. The measurement accuracy can be calculated by:
where:

- $A_r$ = sub-pixel accuracy of the tracking algorithm for the inter-frame motion vector
- $N_b$ = number of bubbles in the froth image
- $\tilde{N}_{pix}$ = average number of pixels per bubble
- $N_{pix,x}$ = pixel resolution in the x direction
- $N_{pix,y}$ = pixel resolution in the y direction
- $N_{frames} - 1$ = number of frame pairs in the image sequence

Using the standard image size in SmartFroth, which has a horizontal resolution of 320 pixels and a vertical resolution of 240 pixels and 5 frames, the theoretical error is $3.3 \times 10^{-6}$ pixels/frame. However, the algorithm begins to break down and under-estimate the flow velocity at a velocity above 3 pixels per frame, owing to erroneous bubble matches.

Thus, the algorithm is appropriate for slow-moving froth where small changes in the flow speed have important consequences for the flotation performance. This is the case for platinum froth where the low concentration of floatable solids (notably in the roughers) leads to very slow moving froth, and where control is targeted at maintaining this flow speed. A typical set-point value for the rougher flow speed is 1 pixel/frame, highlighting the importance of sub-pixel accuracy.
Figure 3-6 shows the positions of the bubble centroids and watershed lines for a typical image sequence. In this example the weir is situated just out of the field of view at the top of the image, hence the dominant flow in that direction. Figure 3-6 demonstrates that the tracking algorithm is dependent on the consistency of the watershed segmentation, and the reason for the high algorithm accuracy. Each bubble centroid is determined by the average of its pixels, and the overall motion vector is determined from the vector sum of the differences in the centroid positions between the last and first frames. This yields a true 4 pair average of the motion vector. Thus, the resultant motion vector is an average of an average of averages, with excellent sub-pixel accuracy. Bubbles for which tracking was 'lost' and a match was not found between one of the pairs of image frames are shown without centroids in the above image. Loss of tracking on all of the bubbles can result in failure of the algorithm to accurately detect fast flow.

Lastly, the Figure demonstrates that some deformation of the froth does occur across the visible field of view. This is evident from the different angles of the motion vectors in different parts of the image, with the vectors in the bottom right of the image being more...
tangential to the weir relative to the motion in the top left corner, which is almost perpendicular to the weir. Deformation such as that observed in this image sequence is typical of froth motion as the froth 'swirls', 'surges' and consequently warps under the influence of turbulence at the pulp-froth interface.

3.3.2 Block matching

This algorithm is commonly used for motion detection in digital image processing, and exists in a multitude of variants, most of which have been highly optimised to minimise processor load. A variant of block matching is used in the real-time MPEG motion compression algorithm typically used for satellite TV video compression. This algorithm was implemented in SmartFroth by StoneThree Digital Signal Processing and following further testing by the author, a correlation algorithm based on a 40x40 pixel block size and a comprehensive search of the overlap search space was found to yield robust results. A grid of configurable density, but typically 10x10, yields motion vectors over a central 280x200 sub-region in the centre of the image. Each block detects the integral pixel change between the first and second images of each of four pairs (for a five frame sequence).

The advantage of a block match algorithm is the relatively low processor load it requires with the result that real-time motion detection is possible using modest processing resources and this opens up the possibility for multiple cameras per computer machine vision systems. However, the maximum accuracy of the algorithm is dictated by the integral motion matches of the blocks. If substantial froth deformation does not occur (as in highly stable platinum froth) the maximum accuracy of the algorithm for an average over 5 frames (4 image frame pairs) is 1/4 of a pixel per frame. Since this is not the difference between the initial and final positions of the blocks, the average is not a 'true' average in the sense that the tracking velocity is a true average. For example, if the motion of the froth is 0.4 pixels per frame in a given direction, the motion match for the image pairs will report a zero value for the motion for each pair, and the average of these will be zero.

\[ v_f = \Sigma \text{int}(v_f) \]  

On the other hand a 'true' average would yield 1.6 for the entire motion, rounded to 2, yielding 0.5 pixels/frame.
Comparing Equation 3-3 and 3-4, it is evident that the block match algorithm performs less accurately than an equivalent algorithm that determines an average of the motion over the relevant number of image pairs. Under conditions where substantial deformation of the froth is expected, the 100 individual blocks will report somewhat different results for the motion vectors, and the accuracy is satisfactory. For example, in froth types where the froth stability is low, and subject to rapid, constant deformation the block match algorithm yields satisfactory accuracy.

Further, since the output of the matches is integral, the relative error \( \frac{1}{v_f} \) is reduced with increasing flow speed. An error of one pixel/frame relative to a true flow speed of 0.5 pixels/frame is 2, but the relative error at a true flow speed of 6.5 pixels/frame is 0.15. Set-points for copper froth motion are substantially higher than for platinum froth owing to the relative richness of the feed and high concentrate flow rates. In practice, the block-matching algorithm has proven unsuitable for analysis of process conditions on rougher flotation cells in a platinum flotation plant, but well suited to copper flotation conditions.

At very high froth flow speed (approximately > 8 pixels/frame) the block match algorithm finds spurious matches between blocks, and since the algorithm inherently 'forces' each block to report a result, the accuracy of the algorithm declines, ultimately resulting in a zero reported velocity when the matches are random. At typical zoom settings, froth flow speed above 8 pixels/frame is abnormal, and usually indicative of a major process deviation from optimal conditions.

### 3.3.3 Burst rate velocity

The burst rate velocity is derived from the burst rate algorithm, essentially consisting of a single, large block match over a 128x256 block region. The accuracy of the algorithm is very poor, equivalent to the block match under conditions of no deformation and the results of inter-frame matching are integers. On the other hand, no maximum for the detectable froth motion has been encountered when using the burst rate velocity, and this algorithm is known to be applicable to froth motion as high as 14 pixels/frame.

\[
v_f = \text{int}(\Sigma(v_f'))
\]
This is the highest flow speed thus far encountered in any image sample set, and it is assumed that at this high flow speed the accuracy of the measure is not of prime importance, since the process is almost certainly operating in a non-optimal regime of excessively fast flow. Further, at this flow speed the degree of blurring of the froth image due to motion is unacceptably high, and froth images should not be captured at a zoom level such that blurring is evident.
3.4 Measurement of Froth Stability

The burst rate algorithm was developed as a novel contribution to the image analysis for this thesis.

The rate of bursting of bubbles on the froth surface can be used to determine the froth surface lamellar robustness. Factors affecting the lamellar robustness are covered in detail in Chapters 2 and 5. The burst rate is correlated with, but distinct from, the rate of coalescence and works in conjunction with coalescence to determine the observed froth surface bubble size distribution. Thus, the froth surface burst rate provides intuitively meaningful information regarding the froth state and performance that is independent of the information gathered by the other measures.

The algorithm for determination of the froth surface burst rate relies on the premise that regions of the froth surface that undergo a large change in the grey level value correspond to regions where a bubble burst or coalescence event has occurred. Since the tops of bubbles are highlighted and the edges between bubbles are dark, this is a reasonable assumption. Image registration (or matching the images spatially) is a prerequisite for accurate detection of the areas where a significant change has occurred, in order to prevent image motion being erroneously considered as surface instability. Thus, the first part of the surface burst detection algorithm consists of determining the correct integral motion vector for displacement of the second image prior to the image comparison (subtraction).

3.4.1 Image pre-processing

The image is contrast enhanced and low-pass filtered according to the algorithms described in Section 3.2. This improves the robustness of the subsequent steps by reducing the sensitivity of the algorithm to noise in the images. A central region is extracted from both images, consisting of a 128x256 block. The sub-image dimensions are dictated by the fact that the Fast Fourier Transform (FFT) operates fastest on matrices having dimensions that are a power of two.
Figure 3-7 shows the sub-image pair for the following demonstration of the surface burst determination algorithm.

3.4.2 Determining the motion vector

The algorithm now consists of taking the real component of the inverse two-dimensional FFT of the dot product of the FFT of the first image and the complex conjugate of the FFT of the second image to yield the correlation matrix shown in Figure 3-8.
The \((x,y)\) position of the matrix maximum relative to the centre of the matrix gives the motion vector for the image pair. For this example, the motion is 8 pixels right and 8 pixels up.

3.4.3 Subtraction

The appropriate (shifted) sub-image is selected from the second image in the process known as image registration, and the images are subtracted from one another, to yield the difference image.

The metrics derived from this analysis are sum of the absolute pixel gray level difference between the original image and the registered image, normalised with respect to the number of pixels and the maximum grey level change, and the maximum height of the correlation peak, normalised with respect to the mean of the autocorrelation of the two sub-images.

3.5 Concluding Remarks

The froth surface stability measurement as developed in this thesis and froth surface Sauter mean bubble diameter were very useful in determining the impact of depressant addition on the froth structure and stability in a laboratory flotation cell, providing insight into the role of floatable gangue species and highly hydrophobic sulphide species in terms of their impact on the froth stability. The froth surface stability is correlated with, but not equivalent to bubble bursting on the froth surface. As a measurement of the image analysis system which has no direct physical equivalent (unlike bubble size, for example), it is not possible to include it as a model input or output, although it is correlated with the burst rate on the froth surface (which can be modelled, but not directly quantitatively measured). This implies that one must compare trends of bursting rates, rather than absolute values.

Secondly, the froth surface flow speed was used to model the rate of recovery of wet concentrate from an industrial flotation cell under widely varying process conditions, and in separate test work the Sauter mean bubble diameter and froth surface flow speed were used to model the weight fraction of solids in the concentrate and the rate of water recovery from an industrial flotation cell under conditions of changing air addition and froth depth, demonstrating the mechanisms through which changing froth retention time and air addition rate impact on the recovery of floatable particles, recovery by entrainment, froth structure and appearance.
The froth surface flow speed was controlled during tests of the effect of changing the frother addition rate on the froth structure and stability. This was done in order to maintain the rate of concentrate recovery at a constant level. This approach was highly successful, maintaining a constant rate of recovery of attached species. Further, the Sauter mean bubble diameter and bubble size homogeneity correlated well with the frother addition rate and percent solids in the concentrate, confirming the proposed mechanistic role of frother in altering the froth structure and flotation performance.

The froth surface descriptors assisted in testing the proposed hypotheses by providing information regarding the froth structure, stability (as developed in this thesis) and mobility of the froth under changing input conditions.
CHAPTER 4: DEFINING AND MODELLING FROTH STABILITY

4.1 INTRODUCTION

This Chapter proposes a novel definition of froth stability and incorporates this definition within a novel model framework to illustrate the mechanistic effect of the hydrodynamic and reagent variables on the froth structure, surface appearance and metallurgical performance.

The model proposes to illustrate the dependence of the froth surface appearance and flotation performance on the stabilising and de-stabilising factors acting in the froth zone. Rather than a fully comprehensive predictive model, certain assumptions regarding the froth and inputs are made which would be inappropriate for a fully predictive model.

Measured parameters of the flotation metallurgical performance such as the mass recovery of solids and the solids concentration in the concentrate stream are used in conjunction with the proposed mechanisms to generate estimates of the independently measured froth surface bubble size distribution, froth surface flow speed and the mass rate of recovery of solids in order to obtain estimates of the froth stabilising parameters and illustrate the action of the mechanisms on the froth structure, appearance and metallurgical performance.
4.2 PROPOSED DEFINITION OF FROTH STABILITY

As discussed in Section 2.8 a need has been identified for a clearly bounded novel definition of froth stability to advance understanding of the interaction of flotation conditions with this crucial parameter.

As recognised in Section 2.8, coalescence and bursting are the two most important aspects of froth stability. The fundamental unit process for both coalescence and bursting is lamellar rupture, the definition of froth stability is posed in terms of the normalised rate of lamellar rupture. To make stability positively correlated with 'conceptual' stability and thus to present a measure of stability (as opposed to instability) the normalised rate of rupture is inverted.

4.2.1 Definition of froth stability

The froth stability index is the inverse of the rate of lamella rupture per second, per number of lamellae, within a given froth volume.

This definition is subdivided for the two cases of bursting and coalescence, where bursting is a special case of coalescence, involving rupture of a lamella that separates a bubble from the atmosphere.

Thus, using the notation for stability proposed by Bikerman (1938), and inserting the subscript F to avoid confusion with the summation operator Σ,

\[
Σ_F = -\frac{N_t}{dN_L/dt}
\]

[4-1]

where

- \( Σ_F \) = froth stability (s)
- \( N_L \) = number of lamellae
- \( \frac{dN_L}{dt} \) = rate of change of number of lamellae (1/s)
This definition implicitly normalises both the rate of change of lamellae per unit time and the total number of lamellae by the froth volume under consideration.

### 4.2.2 Mathematical consequences of the definition

The probability of rupture of a lamella at any given instant is defined as:

$$ P = - \frac{dN_l}{N_l} = \frac{1}{\Sigma} $$  \[4-2\]

Then the bubble lifetime is assumed to be the time after which half of the lamellae within a given froth volume have ruptured, and thus

$$ \int_0^\infty P dt = 0.5 $$  \[4-3\]

where

- $\tau_o$ = mean bubble lifetime
- $P$ = probability of rupture of a lamella

which, assuming a constant probability of lamellar rupture becomes

$$ \tau = \frac{0.5}{P} $$  \[4-4\]

Lamellar robustness is defined as positively correlated with the lamella (or bubble lifetime) in that a highly robust lamella has a lower probability of rupture at a given instant, and on average has a longer lifetime before rupture. Thus, for a well drained lamella,

$$ \Pi = 1 - P $$  \[4-5\]

where

- $\Pi$ = lamellar robustness
- $P$ = probability of rupture of a lamella
The results and discussion sections expand on the hypothesised action of solid and solution species as determinants of the froth stability, and both the model development and discussion of the model justify the above definitions, in that they lead to a conceptually consistent model, and results that agree with observed experimental results.

Further, the rate of change of specific surface area over the specific surface area of the froth is equal to the negative of the probability of lamella rupture, or

\[
\frac{1}{S_{\text{froth}}} \frac{dS_{\text{froth}}}{dt} = -P
\]  

where

\[S_{\text{froth}}\]  

specific surface area of the froth

Thus, highly unstable froth and particularly froth with small bubbles (and hence high specific surface area) close to the froth-pulp interface, undergoes a high rate of loss of surface area as coalescence proceeds, leading to a rapid increase in the surface area coverage of the lamellae by particles, as presented in Section 2.6.

4.2.3 Contributors to bubble lifetime

The extent to which a given factor contributes to retarding or accelerating bubble coalescence is expressed in terms of the contribution of that factor to the overall bubble lifetime. The bubble lifetime is used due to the fact that is an intuitive concept, rendering the relationships described below mechanistically meaningful.

Three modes of stabilisation of the lamellae are proposed:

1. Viscous stabilisation resulting from the time taken for bubbles to come into contact and for the lamellae to thin to the point at which rupture can occur.
2. Solution stabilisation owing to the Gibbs-Marangoni and Plateau effects and disjoining pressures developed within the lamellae, retarding the rupture of lamellae when considering a two-phase air-solution system, owing to the presence of frother dissolved in solution.
3. Solids stabilisation owing to the mechanism of bubble 'armouring' and the formation of a layer of closely packed particles on the lamellae that experience strong, mutually
attractive hydrophobic bonding forces and whose collective impact on the lamellae is to form a strong physical barrier to thinning and rupture. The magnitude of this complex effect is governed by the particle concentration on the bubble lamellae, or degree of close packing of the particles, as well as the particle hydrophobicity, shape and surface roughness.

Further, smooth, highly hydrophobic solid particles have been shown, both theoretically and experimentally, to rupture lamellae very rapidly when acting "in isolation" by accelerating movement of the three phase lines of contact over the solid surface, rapidly resulting in lamellar rupture.

Lastly, the lamellae on the froth surface are subject to different stresses and stabilising factors than the bulk solution, and are treated as a special case of the general lamella stabilisation. Due to the presence of windows on the bubble surface lamellae, it is proposed that the solution and viscous characteristics of the froth dominated the rate of bursting on the froth surface.

Through the equations of Section 4.3.2, the bubble lifetime is closely related to the probability of rupture of the lamellae, and hence the lamella robustness. It is proposed that the bubble lifetime (and hence lamella stability) in the froth zone can be expressed by the relationship:

\[ \tau_{overall} = \tau_{viscous} + \tau_{sol} + \tau_{solid} \]  

where

- \( \tau_{overall} \) = bubble lifetime with respect to coalescence in the froth zone
- \( \tau_{viscous} \) = contribution to the bubble lifetime by the fluid inhibition of bubble contact, dominated by the fluid viscosity
- \( \tau_{sol} \) = contribution to the bubble lifetime by solution factors
- \( \tau_{solid} \) = contribution to the bubble lifetime by solids stabilisation of the lamellae

It is stipulated that the relationship between the degree of froth stabilisation and the physical parameters, while mechanistically justifiable is an estimate of the form of the relationship
between each of these parameters and the probability of rupture of a given lamella. As such, they should be subject to review as further information regarding the complex interactions of these factors with the froth structure becomes available.

The specifics of the equations for determining the minimum lifetime, solution and solids contributions to the bubble lifetime are presented in Section 4.5.1 through 4.5.3.

4.2.3.1 Viscous stabilisation

As discussed in the literature review in Section 2.7.1, the lamellae must first drain to the point at which rupture can occur, after which the lamellar robustness dominates the rupture mechanics. Inhibition of drainage by fine, clay-like particles at high concentration in suspension or viscous fluids such as glycerol is thought to be responsible for the stabilisation of some froth types by rendering the interstitial fluid highly viscous.

As an approximation to this coalescence inhibiting factor, the contribution of viscous forces to overall bubble lifetime was taken to be a constant, multiplied by the slurry content of the froth and the slurry viscosity.

\[ \tau_{\text{viscous}} = k_{\text{viscous}} \varepsilon \mu \]  

[4-8]

where

\[ \tau_{\text{viscous}} \] = contribution to the bubble lifetime from inhibition of bubble contact dominated by the fluid viscosity

\[ k_{\text{viscous}} \] = viscous stabilisation constant

\[ \varepsilon \] = slurry content of the froth on a volumetric basis

\[ \mu \] = bulk slurry viscosity

However, for the system under investigation, it was found that the froth dynamics required no contribution to the froth stability from the viscous term to achieve acceptable modelling of the froth behaviour. Further, since the solution stabilisation and drainage contributions to the bubble lifetime are dominant only in the lower reaches of the froth (see below) the two contributions to stability are additive and largely indistinguishable.
Lastly, the viscosity of slurry in the system investigated is not particularly high, and this further confirms that the drainage term can be considered to be insignificant. In order to minimise the number of degrees of freedom in the model parameters, the drainage constant was therefore taken to be zero. The literature suggests that this is not the case for all mineralogical systems, and that drainage may play a crucial role in froth stabilisation in other systems, particularly those subject to ultra-line grinding and/or having clay-like gangue constituents.

4.2.3.2 Solution stabilisation

The presence of frother in solution at the air-solution interfaces stabilises the lamellae through the Gibbs-Marangoni and disjoining pressure effects. Due to the homogeneous distribution of frother in solution, and the equilibrium adsorption levels of frother at the air-solution interfaces, the contribution to froth stability from this constituent of the solution is evenly spatially distributed and there is assumed to be no variation of the solution contribution to bubble lifetime through the froth zone. Thus, the solution contribution to the bubble lifetime can be expressed as a constant

\[ \tau_{\text{solution}} = k_{\text{solution}} \]

where

- \( \tau_{\text{solution}} \) = solution contribution to the bubble lifetime
- \( k_{\text{solution}} \) = solution stabilisation constant

4.2.3.3 Solids stabilisation

Arguably the most complex, and most important, contribution to the froth stability is made by the particles attached to the air-solution interfaces in the lamellae of the froth. As argued in Section 2.6.2, these particles can form a strongly bonded layer on the lamellae and thus mechanically increase the lamella robustness, to the point that under certain highly stabilised conditions froth can be observed to dry out, yet retain its structure.

Since the degree of stabilisation of the lamellae by the particles is arguably related to the degree to which the particles form a continuous 'layer' on the lamellae, a surface coverage function was derived to describe this aspect of the effectiveness of froth stabilisation by solid particles.
Solids coverage is defined as the cross sectional surface area of attached solids per unit surface area of bubble in the pulp or per unit surface area of lamellae and Plateau borders at the solids-solution interface in the froth, or equivalently as the ratio of the attached particle cross-sectional surface area flux to the bubble surface area flux.

Assuming that the majority of the solids in the concentrate (by mass) arrive there by true flotation, as opposed by entrainment, the mass flow rate of attached material in the concentrate can be approximated by the mass flow rate of bulk solids to the concentrate. If this is not the case for a given system, then the flotation performance is notably sub-optimal. Then the ratio of interest can be expressed as:

\[
\frac{S_p}{S_b} = \frac{\bar{S}_{\text{particles}} M_{c,\text{solids}}}{\bar{S}_{\text{bubble}} Q_A}
\]

where

- \(S_p\) = particle cross sectional surface area flux
- \(S_b\) = bubble surface area flux at a given height in the froth
- \(\bar{S}_{\text{particles}}\) = specific cross sectional surface area of the particles on a mass basis
- \(\dot{M}_{c,\text{solids}}\) = mass flow rate of dry solids to the concentrate
- \(\bar{S}_{\text{bubble}}\) = specific surface area of the bubbles on a volume basis
- \(Q_A\) = volumetric flow rate of gas into the cell

then, assuming that the particles are spherical, the cross sectional surface area per mass

\[
\bar{S}_{\text{particles}} = \frac{4\pi r_{p}^2}{3 \rho_{s}} = \frac{3}{4\pi r_{p}\rho_{s}}
\]

where

- \(r_{p}\) = Sauter mean particle radius
- \(\rho_{s}\) = mean solids density in the concentrate
and the surface area per volume for the bubbles is

$$\bar{S}_{\text{bubbles}} = \frac{4\pi r_b^2}{3}\frac{3}{r_b}$$  \[[4-12]\]

where

$$r_b = \text{Sauter mean bubble radius at the appropriate height in the froth}$$

Substituting Equations 4-11 and 4-12 into Equation 4-10 yields the relatively simple relationship

$$\frac{S_p}{S_b} = \frac{M_{\text{solid}}}{4Q_g r_p \rho_s}$$  \[[4-13]\]

where:

- $S_p$ = particle cross sectional surface area flux
- $S_b$ = bubble surface area flux in the froth
- $M_{\text{solid}}$ = mass flow rate of dry solids to the concentrate
- $Q_g$ = volumetric flow rate of gas into the cell
- $r_p$ = Sauter mean particle radius
- $r_b$ = Sauter mean bubble radius
- $\rho_s$ = mean solids density in the concentrate

This relation bears a few attractive features. Firstly, all of the parameters in the relationship are relatively easy to measure experimentally. Secondly, these parameters, or aspects of the experimental system can be used as proxies for the above parameters in much of the experimental work reported in the literature. Lastly, the relationship quantitatively predicts the role of changing particulate properties and rate of recovery into the froth on the degree of solid stabilisation.
This ratio is not meant as a literal determination of the degree of surface coverage of the bubbles by the particles. Rather, it is linearly correlated with the coverage and should be adjusted by shape and packing factors of unknown magnitude in order to explicitly predict the bubble coverage.

![Graph](image)

**Figure 4-1:** Proposed form of relationship between degree of bubble coverage by particles and contribution to bubble lifetime by solids

An exponential form is proposed for the relationship between the bubble coverage and the solids contribution to bubble lifetime (shown in Figure 4-1) due to the experimentally observed rapidly changing region of the froth, followed by a relatively quiescent region over which the froth structure is remarkably constant. That is, the changes in flotation performance with changes in froth height are remarkably small over a wide range of heights, above a zone in which rapid change is observed with increasing height.

Considering coalescence as a rate phenomenon, a remarkable degree of stabilisation is required at increasing froth heights in order to predict the observed trends with respect to the
bubble size and flotation performance presented below. This topic is further discussed in Chapter 6.

Then assuming that the degree of bubble coverage subsumes the other factors affecting the solid stabilisation of the froth, such as surface roughness, particle shape and solids hydrophobicity, the following relationship is proposed for the role of solids in froth stabilisation

\[ \tau_{\text{solids}} = \exp \left( k_{1,\text{solids}} \left( \frac{S_p}{S_b} - k_{2,\text{solids}} \right) \right) - 1 \]

where

- \( \tau_{\text{solids}} \) = solution contribution to the bubble lifetime
- \( k_{1,\text{solids}} \) = solids stabilisation constant
- \( k_{2,\text{solids}} \) = solution stabilisation constant
- \( \frac{S_p}{S_b} \) = ratio of the particle surface area flux to the bubble surface area flux

\( k_{1,\text{solids}} \) is a fitted constant, describing the degree to which the solids contribute to the froth stability. \( k_{2,\text{solids}} \) is a second constant which shifts the x-axis of the exponential such that a significant degree of stabilisation occurs at the appropriate surface coverage. The experimental work presented below, as well as that by Ventura-Medina and Cilliers (2002), Honaker and Oszever (2003) and Bradshaw and O’Connor (1996) indicates that over a wide range of industrial systems, the froth is typical recovered at a surface area coverage \( \left( \frac{S_p}{S_b} \right) \) of 30-40%. Thus,

\[ k_{2,\text{solids}} = 0.4 \]  

This function is consistent with the proposed mechanistic role of solids in stabilisation of the lamellae in the froth, and as such can be considered semi-empirical. This relation could be subject to significant improvements to incorporate the role of particle roughness, shape and...
hydrophobicity on the degree of solid stabilisation of the lamellae. However, for the system investigated, the relationship is adequate for predicting the degree of stabilisation of the froth by solid species with varying height in the froth zone.

The destabilising role of highly hydrophobic solids at low concentrations in the froth is dealt with in section 2.6, and disregarded in the above equation, due to certain properties common to the majority of industrial flotation systems. Firstly, since extremely hydrophobic solids (θ>100°) have a highly destabilising effect on the froth regardless of the particulate concentration on the bubble surfaces (Aveyard et al., 1994), levels of collector under which this degree of hydrophobicity is achieved are avoided in industrial flotation practice.

Secondly, owing to the excellent pulp attachment probability for highly hydrophobic 'fast floating' species, the rate of recovery of these species to the froth is likely to be high whenever such particles are present in the pulp zone. Thus, when highly hydrophobic particles are present in the pulp they occur in the froth at high degrees of froth coverage. This phenomenon is evident in PGM flotation, which arguably has the lowest concentration of highly floatable material of the common industrial flotation systems. None the less, early rougher cells have the appearance of a densely packed 'sulphide' type float, albeit diluted with floatable gangue and particulate PGMs. Thus, the destabilising impact of the highly hydrophobic species is mitigated by their high concentration in the froth, and hence the formation of lamella stabilising layers.

Lamellar and Plateau border 'overloading' with solids could lead to the mechanical collapse of the froth under the weight of the solids, but this condition is undesirable in industrial flotation practice, and due to the low concentration of floatable material in the pulp, is not frequently observed in PGM flotation operations and is thus omitted from the impact of solids on froth stability. This phenomenon may of greater financial significance for flotation systems where the froth contains a heavy load (per unit surface area) of coarse material, as in some base metal flotation operations, and the relationship for the froth stabilising action of the solids could require a destabilising component at very high solids coverage.
4.3 Parameters defining the form of the model

The inputs to the model are the rate of air addition to the flotation cell, the froth depth, the bubble size distribution in the pulp, the fiftieth and ninetieth percentiles of the particle size distribution, the solids and liquid densities, the mass flow-rate of solids from the cell (which can be validated as an output), the estimated constants for solids, solution and viscous stabilisation of the froth, the cell geometry, the solids concentration of the concentrate stream and the cross-sectional area of the Plateau borders at the froth surface.

Among the most important outputs of the model are the froth surface bubble size distribution, the froth surface flow speed towards the weir, the fraction of air overflowing the weir ($\alpha$), and the rate of recovery of pulp from the cell.

At this stage, no distinction is made between solids and solution in the froth and the pulp in the froth lamellae and Plateau borders was treated as a non-settling, homogeneous suspension with a viscosity and density calculated from the specified concentrate percent solids. Similarly, no distinction is made between attached and entrained material, at this stage, and the model makes no predictions regarding the concentrate grade, although the model is well suited to this potential extension.

This froth model is primarily concerned with the process of coalescence and the interplay between the factors affecting coalescence (and thus change in the froth structure) and the probability of coalescence. Since coalescence close to the froth-pulp interface is known to be very rapid, it was a requirement of the model that it be able to approximately model the froth conditions from the point of entry of bubbles into the froth to the froth surface.

This imposes certain restrictions on the form of the model, in that the computational load of fundamentally modelling the froth was prohibitive, given that 1 million bubbles of 1mm diameter are required to generate one bubble of 100mm diameter. Thus, a model form was chosen that classed bubbles into 'generations' on the basis of the number of times that they had coalesced and approximated the froth state at each level in the froth zone based on the bubble size distribution at that level.
The froth is defined as beginning at the ‘froth-pulp interface’ and this is taken to be the point at which the bubbles achieve randomly spatially distributed spherical close packing. This simplifies the problem of froth modelling by leaving the region just below the defined interface (in which significant detachment occurs, and the bubbles interact with the material draining out of the froth) in the pulp region.

On the other hand, this choice of lower boundary for the froth zone violates the assumptions of reasonably well-dried, polyhedral structure required for the use of the model proposed by Neethling et al., (2003) in the lower regions of the froth. This region is explicitly included in the model since bubbles of a size close to that of the pulp bubble size are commonly seen on the froth surface, and this implies that the contribution of these lower layers to overall performance is non-trivial and cannot safely be ignored.

In Section 4.3 the relationships between the elements of the model are presented diagrammatically and then the five sections of the model are explained in detail in Sections 4.4 through 4.8. Section 4.9 presents results which confirm the validity of the froth structure equations.
4.4 Diagrams of the Model Relationships

Figure 4-2: Diagram of relationships between the variables and constants in the illustrative froth model showing the first four elements of the model structure.
Figure 4.3: Diagram of relationships between the variables and constants in the illustrative froth model showing the determination of the froth slurry content, the fifth element of the froth model.
4.5 Calculation Procedure

As shown in Figures 4-2 and 4-3, the model consists of five parts. First, the bubble size distribution is determined (or specified for the first step), and then in part two the factors affecting the froth stability are calculated to determine the solids, solution and drainage contributions to the lifetime of bubbles within the froth, and at the froth surface.

In the third part the rate of lamellar rupture at the surface and consequent rate of gas flux out of the top surface is calculated, and by difference with the rate of gas entering the froth, the rate of gas overflowing the weir is calculated. This has been labelled "Surface bursting" in Figure 4-2.

The gas is contained in the slurry, the quantity being determined by the slurry content \( \varepsilon \) of the froth. In the fourth part of the model the total volumetric rate of overflow of material is used to determine the froth surface flow speed and the rate of overflow of slurry is used to calculate the mass and water recovery from the cell. Part four of the model has been labelled "Flotation performance" in Figure 4-2.

The slurry content of the froth at all heights has a critical impact on the flotation performance in dictating the influence of the drainage component on the bubble lifetime, and on the rate of recovery of slurry to the concentrate. Thus, a froth structure model is used to estimate the slurry content of the froth including contributions from the Plateau borders \( \varepsilon_{\text{pg}} \) and from the lamellae \( \varepsilon_{\text{lam}} \). This, the fifth part of the model, is shown in Figure 4-3.

Figures 4-2 and 4-3 show arrows into a box where a value is derived from later in the equation set, setting up an iterative loop. Values that are used elsewhere in the calculations, either as iterative components or further down in the calculation, but where space prohibits drawing direct lines to the relevant box, are shown with arrows out to the right.

Thus, for example, the fifth part of the model (calculation of the slurry content) requires the calculated Plateau border length per unit volume from part 2 and the slurry content is then used in calculation of the metallurgical performance in part four of the model. An exception to this rule, the overall bubble lifetime \( \tau_{\text{ont}} \), is used in the next iteration step. It was found that under a wide range of conditions the model was stable and converged to a reproducible
result. The exception to this finding was upon closing the loop between the mass flow rate of slurry into and out of the froth, and hence these parameters are decoupled, as stated in Section 4.1.
4.6 Determination of the Bubble Size Distribution

Upon coalescence, two bubbles join to form a new bubble with a volume that is the sum of the volumes of the initial bubbles. One can consider the bubbles as belonging to a particular 'generation' depending on the number of times they have undergone coalescence with adjacent bubbles, with bubbles at the pulp-froth interface being in generation one.

In the model the bubbles in each generation were assumed to derive from two smaller bubbles of identical size. This assumption was tested, and it was found that the correction for the impact of coalescence between non-consecutive generations was small, while adding significantly to the model complexity.

Figure 4-4: Representation of the number and size of bubbles in generations one through seven

Figure 4-5: Three dimensional representation of the number and size of bubbles in generations one through five

Figures 4-4 and 4-5 show the dramatic change in the number of bubbles and fairly gradual (albeit by a power law relationship) change in the diameter of the bubbles upon coalescence.
These Figures are clearly not literal representations of the froth structure, being more closely visually related to coalescence in the pulp, but are intended to convey the dramatic impact of coalescence on the nature of the froth. In both images, the volume in each level is the same, with the exception of Figure 4-5, where generation 5 should have 1.5 bubbles, but is shown with one. The change in the total surface area of the bubbles on coalescence is also readily appreciated, and this has important implications for the impact of solid species on the froth stability as detailed in Section 4.5.3. Figures 4-4 and 4-5 show coalescence to the seventh and fifth generations respectively, but given that for 1.2mm bubbles entering the froth, a bubble with a diameter of \(~100\)mm is in the \(20^{th}\) generation, the extremely dynamic nature of the froth is evident.

Beginning at a given mean pulp bubble diameter, at a froth time (or height) of zero, the froth model progresses through \(i = 1,2,\ldots,n\) steps from \(t = 0\) to the froth retention time (FRT) with the FRT determined by the relationship

\[
FRT = \frac{H_f^{\text{max}}}{J_s}
\]  

[4-16]

where

- \(FRT\) = froth retention time
- \(H_f^{\text{max}}\) = maximum froth height (a given)

and the superficial gas velocity

\[
J_s = \frac{Q_A}{A_{cell}}
\]  

[4-17]

where

- \(Q_A\) = air flow rate into the cell (a given)
- \(A_{cell}\) = horizontal cross sectional area of the cell at the relevant height in the froth
The rate of coalescence of bubbles is dictated by the bubble lifetime. The determination of the bubble lifetime is explained in Section 4.5. In this section, the overall bubble lifetime from the preceding time step is assumed to be known and \( \tau_{\text{overall}} \) is abbreviated as \( \tau \).

Considering the first generation bubbles, by definition half of the bubbles coalesce within the bubble lifetime and the number of bubbles in the first generation can be expressed by the equation

\[
P(t) = 2^{\left(\frac{-t}{\tau}\right)}
\]  

[4-18]

where

\( \tau = \) overall bubble lifetime

Then the number of bubbles in each successive generation can be calculated by a volume balance over the bubble generation. Expressed in terms of the number of bubbles, in discrete form:

\[
N^g(t) = N^g(t - \Delta t)P(\Delta t) + \frac{1}{2}N^{g-1}(t - \Delta t)(1 - P(\Delta t))
\]  

[4-19]

where

\( N^g(t) = \) number of bubbles in generation \( g \) at time \( t \)
\( N^1(0) = \) number of bubbles in the 1st generation at \( t = 0 \)
\( P(t) = \) proportion of bubbles that have not coalesced at time \( t \)
\( \Delta t = \) time step

The factor 2 in the denominator of the second term derives from the volume balance, in that for each bubble coalescence event in the previous generation, half of the number of bubbles report to the current generation.

Calculated over the height of the froth zone this yields height profiles for the number (or volume) of bubbles in each class, and a bubble size distribution at each time step. A typical
set of height profiles for the volume fraction of bubbles in a selection of generations is shown in Figure 4-5.

Figure 4-5: The volume fraction of gas in a selection of bubble generations as height increases through the froth

Figure 4-5 shows the depletion of volume in earlier generations and the formation of larger (later generation) bubbles as height in the froth increases.
To extract the data for Figure 4-6 the froth surface video sequence was drawn from the second cell of a rougher bank (primary extraction stage) on a plant processing Merensky reef ore. Watershed segmentation, as per Section 3.2, was used to determine the froth surface bubble size distribution. During the time that the sequence was taken, the plant was operating normally. See Section 5.4.3 for further details.

It is noteworthy that the model predicts a log-normal bubble size distribution, in excellent agreement with the sample distribution determined experimentally under the same conditions as those used in the modelling. Thus, the bubble size distribution derived from the model is consistent with the observed appearance of the flotation froth for the system under investigation.
4.7 BURSTING AT THE FROTH SURFACE

Bursting at the froth surface is a special case of coalescence involving coalescence between a bubble and the atmosphere. It is proposed that the probability of bursting is affected by a different mechanism from the mechanism of coalescence within the froth due to the different stabilising factors and stresses acting on the surface lamellae.

The presence of windows or breaks in the solids layer on bubble surfaces and their expansion prior to bursting suggests that solids have less of an effect on bursting than on coalescence. Consequently, it is proposed that the solution and drainage characteristics of the froth dominate the probability of bursting. Thus the equation for the bubble lifetime on the surface is expected to have the following form:

\[ \tau_{\text{surf}} = \tau_{\text{viscous}} + \tau_{\text{soln}} \]  \[4-20\]

where

- \( \tau_{\text{surf}} \) = bubble lifetime on the froth surface
- \( \tau_{\text{viscous}} \) = contribution to the bubble lifetime by the fluid inhibition of bubble contact, dominated by the fluid viscosity
- \( \tau_{\text{soln}} \) = contribution to the bubble lifetime by solution factors

The number of lamellae per unit area on the froth surface is given by:

\[ \tilde{N}_{t,\text{surf}} = \frac{A_c}{k_{\text{geom, surf}} \sqrt{\pi} \rho} \]  \[4-21\]

where

- \( \tilde{N}_{t,\text{surf}} \) = number of lamellae on the froth surface per unit cell surface area
- \( A_c \) = horizontal cross sectional area of the cell at the froth surface = 1
\[ k_{\text{geom,corr}} = \text{correction for the approximately hexagonal shape of the bubbles, relative to the ideal circular shape} \left( \frac{3}{\pi} \right) \]

\[ r_b = \text{Sauter mean bubble radius} \]

and the probability of bursting is related to the surface bubble lifetime by:

\[ P_{\text{burst}} = \frac{0.5}{\tau_{\text{surf}}} \]

where

\[ P_{\text{burst}} = \text{probability of lamellar rupture to the atmosphere (bursting)} \]

The mean lamellae lifetime on the froth surface in Equation 4-23 is determined from Equation 4-21.

The equation for the rate of lamellar rupture is:

\[ \frac{d}{dt} \tilde{N}_{L,\text{surf}} = \frac{\Delta \tilde{N}_{L,\text{surf}} P_{\text{burst}}}{\Delta t} \]

where

\[ \Delta \tilde{N}_{L,\text{surf}} = \text{number of lamellae burst between} \ t - \Delta t \ \text{and} \ t \ \text{per unit cell cross sectional area} \]

\[ \tilde{N}_{L,\text{surf}} = \text{number of lamellae on the froth surface at time} \ t - \Delta t \ \text{per unit cell surface area} \]

\[ P_{\text{burst}} = \text{probability of lamellar rupture to the atmosphere (bursting)} \]

and Equations 4-22 and 4-23 are substituted into Equation 4-24 to determine the rate of change of the number of lamellae on the froth surface.

Then the superficial gas velocity out of the top surface of the froth is given by:
where

\[ J_{g,\text{surf}} = \frac{d}{dt} \Delta N_{\text{surf}} V_b \]  \hspace{1cm} [4-24]

and the volumetric rate of gas flow out of the top of the froth is given by:

\[ Q_{\text{surf}} = J_{g,\text{surf}} A_{\text{cell}} \]  \hspace{1cm} [4-25]

Then the volumetric flow rate of gas over the weir is the difference between the gas flow rate into the cell and the gas flow rate out of the top surface of the froth:

\[ Q_{\text{weir}} = Q_4 - Q_{\text{surf}} \]  \hspace{1cm} [4-26]

The flow rate of air over the weir is determined by substitution of Equation 4-24 into Equation 4-25, Equation 4-25 into Equation 4-26 and Equation 4-26 into 4-27.

This system of equations can also be used to calculate the fraction of air overflowing the weir as defined by Neethling and Cilliers (2000), and used by Ventura-Medina and Cilliers (2002, 2004).
\[ \alpha = \frac{Q_{\text{air,weir}}}{Q_{d}} \]  

where

\[ \alpha \]  
fraction of air overflowing the weir

\[ Q_{\text{air,weir}} \]  
volumetric gas flow rate over the weir

\[ Q_{d} \]  
volumetric gas flow rate into the cell

This output provides an additional check on the validity of the model outputs.
4.8 Flocculation Performance

4.8.1 Mass flow rate of solids and water

The froth surface velocity towards the weir and the rate of solids and water recovery to the concentrate are relatively easy to calculate given the water content of the froth overflowing the weir, the rate of gas recovery over the weir and given cell and material parameters.

The total volumetric rate of overflow of slurry and gas over the weir and therefore to the concentrate:

\[ Q_{f,\text{weir}} = \frac{Q_{f,\text{weir}}}{(1 - \kappa)} \]  

where

- \( Q_{f,\text{weir}} \) = volumetric flow rate of the bulk froth (gas and slurry) over the weir
- \( \kappa \) = liquid (slurry) content of the froth

and the volumetric rate of recovery of the concentrate is given by:

\[ Q_{\text{conc}} = Q_{\text{com, weir}} \kappa \]  

where

- \( Q_{\text{conc}} \) = volumetric flow rate of concentrate over the weir

The relation used to determine the volume fraction of solids is:

\[ \phi_r = \frac{\rho_s \phi_{\text{conc}}}{\rho_r} \]  

where

- \( \phi_r \) = volume fraction of solids in the concentrate
- \( \rho_s \) = mass fraction of solids in the concentrate
\[ \rho_{\text{conc}} = \text{density of the concentrate} \]
\[ \rho_s = \text{density of the solids} \]

Then the mass flow rate of solids and water to the concentrate can be calculated by:

\[ \dot{M}_s = \phi_s Q_{\text{conc}} \rho_s \]

and

\[ \dot{M}_w = (1 - \phi_s) Q_{\text{conc}} \rho_w \]

where
\[ \dot{M}_s = \text{mass flow rate of solids to the concentrate} \]
\[ \dot{M}_w = \text{mass flow rate of water to the concentrate} \]
\[ \phi_s = \text{volume fraction of solids in the concentrate} \]
\[ \rho_s = \text{density of the solids} \]
\[ \rho_w = \text{density of water} \]

The mass flow rate of water and solids to the concentrate were measured on a number of industrial campaigns, and these experimental measurements are compared with the model outputs in Chapter 6.

4.8.2 Froth velocity

The froth surface velocity can be detected by the machine vision system. The definition of the froth velocity is the magnitude of the component of the motion vector on the froth surface in the direction of the cell lip measured at the cell lip, averaged over the height of the froth over the lip. Thus for a co-ordinate system having the line from the centre of the cell to the cell lip as the positive y axis then strictly,

\[ v_f = \bar{v}_{f,y} \]

where
\( v_f \) = froth velocity towards the cell lip

The terms froth flow speed and froth velocity can be used interchangeably. Due to the no-flow boundary condition at the point at which the froth contacts with the lip or weir, there is a flow profile over the lip itself with a typical parabolic shape for laminar flow over a plate. However, the froth velocity is averaged over the froth area from the lip backwards towards the centre of the cell, and in this region, (for a reasonably deep froth) the froth can be assumed to experience a constant flow speed towards the lip for a depth into the froth at least equal to the height of the froth over the lip.

Thus, the area averaged froth velocity is assumed to be the same as the height averaged velocity over the cell lip. Given the above assumption, one would expect acceleration of the froth surface over the lip itself, and this is indeed observed, with the froth surface experiencing tensile stress that accelerates froth breakdown prior to discharge into the concentrate launder.

In agreement with Ventura-Medina and Cilliers (2002) the froth surface flow speed can be calculated from the bulk volumetric flow of froth over the cell lip, the height of the froth over the weir and the perimeter length of the cell lip:

\[
v_f = \frac{Q_{f,\text{net}}}{P_c h}
\]  

where

\( P_c \) = perimeter length of the cell lip (given)

\( h \) = height of the froth over the lip (given)

Modelling of the height of the froth over the weir is complex and would require the development of a sophisticated froth transport model that takes into account the bulk froth viscosity, probably as a function of the solution and solids content of the froth and the froth bubble size distribution, the bubble size distribution itself, and the air rate overflowing the weir.
As a result, the froth height over the weir was not modelled and the need to specify the height of the froth over the cell lip is a significant weakness of this section of the modelling. However, this parameter introduces a uniform bias to the results, and was set to the average value observed on the industrial plant campaigns of 11 cm. Thus, the observed trends remain valid, despite the possible inclusion of a bias from the specified froth height parameter.

The froth velocity has been measured under a range of input conditions on an industrial system, and these measurements are compared with the model prediction of the froth surface flow speed in Chapter 6.
4.9 SLURRY CONTENT

The slurry content of the froth is a key parameter in the illustrative model of the froth, primarily due to the importance of this variable in determining the rate of recovery of the concentrate. Development of models of the froth water content is currently an area of active research and existing models did not cover the full range of conditions encountered in this model.

This model was derived based on the froth structure and performs well in the prediction of the slurry content over a wide range of bubble sizes. However, due to assumptions inherent in certain of the froth structural relationships, the model does not perform well close to the froth-pulp interface at heights below ~5% of the total froth height for the system under investigation. None the less, the model is most accurate for prediction of the water content close to the froth surface, and hence adequately predicts the important metallurgical performance parameters presented in Section 4.8.

The slurry content of the froth can be broken down into a Plateau border contribution and a lamella contribution:

\[ \varepsilon = \varepsilon_{pb} + \varepsilon_{lm} \]  

\[ \varepsilon_{pb} \quad \text{slurry content contribution of the Plateau border network} \]

\[ \varepsilon_{lm} \quad \text{slurry content contribution of the lamellae} \]

The volume fraction in the Plateau borders is the length per unit volume of the Plateau borders multiplied by the cross sectional area of the Plateau borders:

\[ \varepsilon_{pb} = \lambda A_{pb} \]  

where

\[ \varepsilon_{pb} \quad \text{slurry content contribution of the Plateau borders} \]

\[ \lambda \quad \text{length of Plateau border per unit volume} \]

\[ A_{pb} \quad \text{effective cross-sectional area of the Plateau borders} \]
and the contribution of the lamellae to the slurry content is given by

\[ e_{\text{lam}} = \tilde{S}_{\text{lam}} \delta_l \]  \[\text{[4-37]}\]

where

- \( e_{\text{lam}} \) = contribution to the volume fraction of slurry from the lamellae
- \( \tilde{S}_{\text{lam}} \) = specific surface area of the lamellae on a volume basis
- \( \delta_l \) = mean lamellae thickness

The determination of the Plateau border length per unit volume, the effective cross-sectional area of the Plateau borders, the specific surface area of the lamellae and the lamella thickness are treated separately in Sections 4.9.1 through 4.9.4 and the iteration procedure is presented in Section 4.9.5.

### 4.9.1 Plateau border length

The Plateau border length per unit volume, as derived by Neethling et al., (2000) can be determined from froth geometry for a reasonably well-dried, polyhedral froth using

\[ \lambda = \frac{10\sqrt{3}}{\pi \left[1, \sqrt{\frac{d_b}{2}}\right]^2} \]  \[\text{[4-38]}\]

where

- \( \lambda \) = the length of Plateau border per unit volume of froth
- \( d_b \) = mean diameter of the circumscribed sphere around the bubble

This equation imposes the lower limit on the height in the froth zone that can be adequately modelled using this slurry content model, since the froth close to the froth-pulp interface is not well-dried and polyhedral. Under these conditions the above equation over-predicts the length of Plateau border in the froth, and hence over-predicts the contribution to the slurry content from this element of the froth structure. This introduces the requirement for a maximum in the predicted slurry content which is discussed in Section 4.9.5.
4.9.2 Plateau border cross sectional area

As presented in Section 4.9, the model requires that this parameter be specified. However, as the froth height increases, or the assumed Plateau border cross sectional area decreases the sensitivity of the model to this parameter decreases. Thus, for typical conditions investigated in this work, increasing the effective diameter of the Plateau borders by a factor of ten from 0.1mm to 1mm and hence changing the cross sectional area of the Plateau borders by a factor of 100 increases the water content at the froth surface by 19%.

However, a Plateau border cross sectional area of 0.1mm results in an estimated 99.6% of the material in the froth being present in the lamellae at the surface, and due to the observed rate of recovery of entrained material, this estimate is unfeasible. A Plateau border effective diameter of 1mm yields an estimate of 64% of the material being present in the lamellae at the froth surface, and this result is reasonable in terms of the expected recovery of entrained material to the concentrate, and this value was used in this work.

4.9.3 Surface area of the lamellae

The surface area of the lamellae per unit volume of froth can be approximated from the difference between the surface area of a close packed dodecahedral structure and the surface area of the Plateau border network, adjusted for the packing of bubbles in the froth.

\[ S_{\text{lam}} = S_{\text{froth}} - S_{\text{pb}} \]  \[ 4-39 \]

where

- \( S_{\text{lam}} \) = specific surface area of the lamellae (surface area per unit volume of froth)
- \( S_{\text{froth}} \) = specific surface area of the froth
- \( S_{\text{pb}} \) = specific surface area of the Plateau borders

Considering an individual dodecahedron, the surface area of the shape is somewhat smaller than the surface area of the circumscribing sphere and the ratio is approximated as being somewhat higher than the ratio of the length of the sides of a hexagon to the circumference of
its circumscribed circle, which is \(3/\pi \approx 0.955\). Thus \(k_{34} \approx 0.97\). Possible inaccuracy in this approximation is mitigated by the fact that the correction is, in any case, small.

Then, considering that each surface in the bulk froth is shared between two bubbles,

\[
\tilde{S}_g = \sum_{s=1}^{g} \frac{\tilde{N}_b s g^s k_{s g}}{2}
\]  

[4-40]

where

- \(s_b^g\) = surface area of an individual bubble in the bubble generation \(g\)
- \(\tilde{N}_b^g\) = number of bubbles in the bubble generation \(g\)

By geometry, the surface area of the Plateau borders in the froth with each Plateau border shared between three bubbles is given by:

\[
\tilde{S}_{ph} = \lambda \sqrt{3} \tilde{N}_b \frac{r_{ph}}{3(1-e)\cos(\pi/6)}
\]  

[4-41]

The above equation requires an estimate of the radius of curvature of the Plateau borders that can be obtained from the relation

\[
A_{ph} = \frac{r_{ph}}{\lambda}
\]  

[4-42]

and after Neethling and Cilliers (2003)

\[
r_{ph} = \sqrt{\frac{A_{ph}}{C^2}}
\]  

[4-43]

where

- \(C = 0.402\)

By substituting Equation 4-43 into Equation 4-42 and substitution of Equations 4-42 and 4-41 into Equation 4-40 one obtains an estimate of the specific surface area of the lamellae.
4.9.4 Lamella thickness

Hemmings (1981) and Sadr-Kazemi and Cilliers (2000) found that the lamellae thickness is close to the 90th percentile of the particle size distribution \(d_{p,90}\). Since the particle size distribution is specified, the lamella thickness can be directly estimated from

\[ \delta_l = d_{p,90} \]  \[4-44\]

4.9.5 Iteration procedure

The iteration procedure presented below is most easily understood with reference to Figure 4-2. The Plateau border length per unit volume and the specific surface area of the froth can be calculated directly from the bubble size distribution. Given the cross-sectional area of the Plateau borders and using the known Plateau border length per unit volume, the Plateau border contribution to the slurry content can be calculated using Equation 4-46.

\[ \varepsilon_{Pb} = \lambda A_{Pb} \]  \[4-45\]

With an initial estimate of the slurry content and the Plateau border contribution to the slurry content, the specific surface area of the lamellae can be calculated by the procedure presented in Section 4.8.3. Given the 90th percentile of the particle diameter, the lamella thickness can be estimated, and the lamella contribution to the slurry content can be calculated using

\[ \varepsilon_{lam} = \tilde{S}_{lam} \delta_l \]  \[4-46\]

The total slurry content of the froth per unit volume is determined by summation of the contributions from the Plateau borders and the lamellae. However, the limitations of the estimate of the Plateau border length dictate that it is necessary to cap the slurry content at 0.36 using:

If \( \varepsilon > 0.36 \) then \( \varepsilon = 0.36 \)

The value of 0.36 is the slurry content of randomly distributed close packed spherical bubbles and slurry content above this level is only present in the pulp, given the definition of the position of the pulp-froth interface. Despite the loss of predictive capability with respect to the slurry content in the lower regions of the froth zone this procedure maintains the validity.
of the model structure and does not introduce a significant bias to the model results for the experimental system under consideration. However, for systems that operate with shallow froth depth, it is necessary that a correction for the Plateau border length calculation be introduced, in order to allow for accurate modelling of the froth slurry content close to the froth-pulp interface.

The slurry content contribution of the Plateau borders is then re-calculated from the capped total slurry content value, and this estimate is returned to the calculation of the lamella surface area, along with the capped slurry content estimate itself. This model structure is highly robust and converges rapidly to an estimate of the slurry content of the froth as a function of the height in the froth. Figure 4-7 presents a typical slurry content profile through the froth zone.

![Slurry content profile](image)

**Figure 4-7:** Slurry content profile as a function of height through the froth zone showing invalid region close to the froth-pulp interface and rapid drainage thereafter to a stable slurry content in higher regions of the froth.
4.10 Concluding remarks

Froth stability has been defined and an illustrative model of the froth has been developed that predicts the froth surface appearance in the form of the froth surface flow speed and the froth surface bubble size distribution. In terms of metallurgical indicators, although the model does not predict the concentrate grade, it predicts the mass and volumetric rate of recovery of slurry from the cell.

The model illustrates the hypothesised mechanisms of froth stabilisation that determine the rate of bubble coalescence and bursting. Further, the model is structurally consistent with current understanding of the froth structure, and it has been shown that the model generates results with respect to the froth structure that are consistent with the expected structure of the froth, validating the model implementation. The froth surface bubble size distribution generated under typical operating conditions has been compared with an experimentally determined bubble size distribution and excellent agreement was found between the predicted and experimental results, with both sets of data indicating that the bubble size distribution on the froth surface is log-normal.

The predicted outputs from the model are compared with image analysis and metallurgical results on an industrial system over a range of input conditions in Chapter 6.
CHAPTER 5: RESULTS AND DISCUSSION

5.1 INTRODUCTION

5.1.1 Key questions

This chapter presents results that address key questions 2 and 3, as presented in Chapter 1:

- Mechanistically, how do select hydrodynamic and reagent variables alter the froth state, froth surface appearance and metallurgical performance?
- Can these mechanisms be confirmed by appropriate measurements of the froth surface appearance from machine vision algorithms?

A brief description of relevant characteristics of the platinum flotation system are presented in Section 5.1.2, the research methodology is presented in Section 5.1.3, the results and discussion appear in Sections 5.2 through 5.5 and Section 5.6 presents an overview of key findings and a few concluding remarks. Sections 5.2 through 5.6 contain the relevant hypothesis to be tested with the data, a comparison of two levels of the input conditions and a discussion of the findings from the experimental work.

5.1.2 Platinum flotation system

Both the laboratory and industrial tests were conducted using Merensky ore. Merensky ore contains 1 - 1.5% sulphide mineral species including chalcopyrite, pyrite and pentlandite. Since the Platinum Group Elements (PGEs) are closely associated with the sulphides, the performance data with respect to these species were taken to be correlated with the flotation performance of the PGEs for which the ore is mined. Consequently, sulphur analysis was used in this work as an indicator of the flotation performance of the system.

The primary gangue species are pyroxene and feldspar. Of particular importance for the flotation performance of the ore, is the fact that it contains naturally floatable altered magnesium silicates, including the talcaceous minerals (talc) at a concentration of up to 5% by weight. These minerals, being naturally floatable, contribute significantly to the recovery of floatable gangue species in the absence of depressant. 50 grams of non-floatable pure manganese was added to the batch flotation system as an indicator of the behaviour of the entrained species. Table 5-1 presents a summary of the experimental conditions.
5.1.3 Methodology

The approach to answering the above questions involved test work on both industrial systems and laboratory apparatus, with the selection of the test system made on the basis of feasibility of the test on an industrial scale and the accuracy required of the measurements to achieve statistical confidence in the results. The tests to determine the impact of air addition rate and frother addition rate on flotation performance and froth appearance were performed on industrial mineral processing plants, while testing of the impact of the depressant addition rate and the froth depth was performed on a batch flotation scale.

To identify the impact of a given input on the measured outputs, the inputs were set to ‘low’ and ‘high’ conditions and statistical t-tests were performed to establish the difference in the mean of the measured variables and the significance of the estimated difference. The relative difference was determined by normalisation of the difference, with respect to the initial value and conversion to a percentage. In subsidiary tests, it was established that all of the relationships were monotonic, thus avoiding the danger of straddling a reversal of trend in the relationship between the input and the output.
5.2 THE EFFECT OF FROTHER DEPTH ON FROTHER PERFORMANCE AND APPEARANCE

5.2.1 Introduction

From the literature, as presented in Sections 2.3, 2.4 and Equation 4-16, one can determine that increasing the froth depth in flotation proportionally increases the froth residence time (FRT) without altering the flow of entrained or attached solids or water into the froth. Thus, when changing the froth depth, changes in the flotation performance can be directly attributed to changes in the froth performance, as opposed to a combination of pulp and froth factors.

Further, the literature shows that changes in the froth depth do not have an impact on the solids and solution factors governing the lamella robustness. Thus, when changing the froth depth, changes in the degree to which the froth has coalesced and drained when it reaches the froth surface are dependent on the change in the froth residence time.

5.2.2 Hypothesis

It is hypothesised that changing froth depth changes the froth retention time (FRT) and that this, in turn, alters the time for coalescence and drainage in the froth. It is further hypothesised that the rate of coalescence in the froth decreases as time in the froth zone increases due to stabilisation of the lamellae by solid species.

Attached solids in the froth are not subject to drainage, whereas entrained and detached solids in the Plateau borders and water in the Plateau borders and the lamellae are subject to drainage to a greater or lesser extent. The resulting imbalance between the rates of drainage of solids and solution causes an increase in the solids concentration of the slurry in the froth as drainage proceeds. Thus, an increase in the froth residence causes an increase in the percent solids of the concentrate.

Since unattached (entrained and detached) particles in the Plateau borders are subject to a greater rate of drainage than the particles in the lamellae of the froth, and the unattached solids are of a lower grade than the attached solids, an increase in the froth residence time leads to an increase in the concentrate grade.
A reduction in the recovery of solids from the froth with increasing froth depth can be due to both increased drainage of entrained material and possibly due to detachment and drainage of valuable material from the froth.

Changes in the froth surface bubble size and froth surface stability when changing the froth depth can be measured using a machine vision instrument.

5.2.3 Experimental Method

Tests of the impact of the froth depth on the flotation performance were performed in a batch flotation cell. The experimental method presented in this section applies to both the batch tests of this section and to those of Section 5.3. The experimental design tested the impact of three independent variables on the flotation performance and froth surface appearance: the impact of depressant addition, froth depth and dispersant addition (Robertson, 2003). Two levels of depressant addition, two levels of froth depth and three levels of dispersant addition resulted in twelve conditions and tests were conducted in triplicate for each of the twelve conditions, for a total of 36 (3x3x2x2) tests. The experimental method is presented in detail in Appendix C. Detailed results of these tests can be found in Appendix A.

The effect of froth depth on the froth performance and appearance was tested in the absence of depressant. Thus, 9 individual tests were used for each of the froth height conditions and paired t-tests were used to eliminate the impact of the dispersant addition on the statistical validity of identifying a difference in the mean of the populations. The means themselves and, hence, the relative differences observed, represent the average of the three dispersant conditions. This ensures that the results are robust with respect to the level of dispersant in the system.
Table 5-1: Summary of experimental conditions

<table>
<thead>
<tr>
<th>Constants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cell type</td>
</tr>
<tr>
<td>Cell volume</td>
</tr>
<tr>
<td>Impeller speed</td>
</tr>
<tr>
<td>Air flow-rate</td>
</tr>
<tr>
<td>Ore</td>
</tr>
<tr>
<td>Solid concentration</td>
</tr>
<tr>
<td>Feed size</td>
</tr>
<tr>
<td>Collector (type and dosage)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Frother (type and dosage)</td>
</tr>
<tr>
<td>Concentrates collected</td>
</tr>
<tr>
<td>Scrapping interval</td>
</tr>
<tr>
<td>Tracer for entrainment</td>
</tr>
<tr>
<td>Dispersant addition</td>
</tr>
<tr>
<td>Depressant</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Froth depth</td>
</tr>
</tbody>
</table>

The concentrate and tails samples were analysed for sulphur and manganese. The total entrained mass recovered was calculated from the entrained manganese recovery and the total mass of material in the cell. All masses of floatable material recovered were corrected for entrainment, to determine the mass recovered by true flotation. The mass of floatable gangue recovered was calculated by the difference between the total floated mass and the sum of the entrained mass and sulphide mass. Consequently, the floatable gangue mass contains the contribution of both the independently floatable gangue particles and the gangue associated with floatable sulphide particles.
5.2.4 Results and discussion

Figure 5.1 shows that the changes in entrained gangue recovery, water recovery, percent solids and the froth surface bubble size and stability were highly statistically significant, with the probability of a real difference in the mean values greater than 99.9%, while the changes in sulphide recovery and the mass recovery of floatable gangue were not as highly significant. There was no significant change in the total mass of solids recovered.

As hypothesised, the increased froth residence time caused by the increased froth depth resulted in a substantial (~28%) reduction in the recovery of entrained gangue and water across the froth. This supports the mechanistic understanding of the impact of drainage on the entrained material initially present in the Plateau borders.
Further, in line with the mechanism of drainage of entrained slurry from the Plateau borders and water from the lamellae, the solids concentration of the froth increased with increasing froth residence time. The observed 14% increase in sulphide grade resulted from preferential drainage of the entrained solids in the Plateau borders of the froth, relative to the attached solids in the froth lamellae.

In all cases, the relative change in the measured output was at least a factor of three smaller than the magnitude of the change in the input variable. The froth depth was increased by 100% and the relative insensitivity of the metallurgical and froth surface parameters to this change indicated that the mechanisms of drainage and coalescence were non-linear.

In agreement with the hypothesis, the froth surface bubble size was successfully measured using the machine vision system and the increased time for coalescence provided by the deeper froth resulted in an increase in the froth surface bubble size. Assuming a bubble size of 1.2mm in the pulp of the flotation cell, the number of coalescence events required to reach 3.00mm bubble diameter was 4.00, then a further 0.59 coalescence events were required to reach 3.44mm diameter in the case of the deeper froth. Thus, the rate of coalescence in the second half of the froth depth was, on average, 15% of the rate in the first half for the deeper froth.

Thus, it was concluded that the coalescence rate in the froth was not constant and that froth stability increased as the froth depth increased. As coalescence occurred in the froth, the specific surface area of the froth (surface area per unit volume) decreased, while the load of solids on the lamellae decreased more slowly, if at all, due to detachment. Thus, as coalescence occurred, the solids coverage on the bubbles, defined in Section 4.5.3 as the attached solids cross sectional surface area per unit bubble surface area, increased. This led to increasing stabilisation of the lamellae by solids as the froth coalesced, as well as a lower rate of coalescence in the second half of the froth height than in the first. As hypothesised, the solid species increased the lamella robustness and stabilised the froth.

It is recommended that these tests be conducted on an industrial flotation plant to confirm the findings of this section, particularly with respect to the impact of increasing froth depth on the recovery of attached species.
5.2.5 Summary

Froth depth was found to impact primarily on the degree to which the froth had coalesced and drained by the time the froth was recovered to the concentrate, as hypothesised. Increasing the froth depth and, hence the froth retention time, reduced the recovery of water and entrained solids to the concentrate, increased the concentrate percent solids and increased the froth surface bubble size and surface stability. The difference between the rate of coalescence in the shallow froth and the inferred rate of coalescence in the upper half of the deep froth demonstrated that the solids retarded coalescence to a greater degree as the bubbles became larger. This resulted in a smaller than expected increase in the froth surface bubble size when the froth depth was increased. Further, the deeper froth had a more stable froth surface; again supporting the hypothesis that the rate of coalescence decreased as height in the froth increased.

Additional evidence that increased coverage by attached solid species was responsible for the decreased rate of coalescence in the upper region of the froth, as opposed to another surface area or time dependent mechanism, is presented in Section 5.3 which investigates the effect of depressant and in Section 5.4 which demonstrates the effect of the rate of air addition. Section 5.5 investigates the impact of frother on the froth stability.
5.3 THE EFFECT OF GANGUE DEPRESSION ON FROTH PERFORMANCE AND APPEARANCE

5.3.1 Introduction
Depressant is added to reduce the floatability of the potentially or naturally floatable gangue species, such as the talcaceous minerals. This reduces the flow of attached floatable gangue from the pulp into the froth. However, if the dosage is managed correctly, depressant addition has an insignificant impact on the recovery of the sulphide species.

5.3.2 Hypothesis
It is hypothesised that solid species play a key role in stabilisation of the froth structure. The presence of floatable gangue species (including talc) stabilises the froth. The removal of the stabilising gangue species when depressant is added leads to a less stable froth and an increase in the rate of coalescence and bursting.

Unstable froth is subject to a higher rate of coalescence and bursting than stable froth, and a high rate of coalescence results in an increase in the rate of drainage from the froth, as outlined in Section 2.4. Thus, in addition to the reduction in the recovery of attached gangue to the concentrate the decreased froth stability causes a reduction in the rate of recovery of water and entrained gangue when depressant is added to the flotation system.

Increased depressant dosage which removes froth stabilising floatable gangue solids causes increased coalescence and bursting (less stable froth) and this can lead to one of two results. The increased rate of coalescence within the froth can increase the froth surface bubble size and slightly increase the froth surface burst rate. Alternatively, the nearly instantaneous bursting of bubbles on the froth surface can lead to a smaller observed surface bubble size and a very high froth surface burst rate. Hence, there is a non-linear relationship between the froth surface bubble size and the froth stability when increasing depressant addition. The effect of increasing depressant dosage on the froth surface bubble size depends on the flotation system, the solids loading and the type of solids in the froth before and after depressant addition.

An additional subtlety in terms of the impact of solids of different hydrophobicity on the froth surface appearance and performance is seen when the impact of depressant addition on the froth surface stability is monitored over time and hence as the solids are removed from the
system. When depressant is not added the floatable gangue stabilises the froth and the froth stability decreases as the solids are removed. When depressant is added and the floatable gangue does not enter the froth, the removal of solids from the system increases the froth stability. This shows that, in agreement with Section 2.6.1, the moderately hydrophobic floatable gangue species stabilise the froth while the highly hydrophobic sulphide species destabilise the froth in the absence of the stabilisation afforded by the floatable gangue.

5.3.3 Experimental Method

The experiments used are those described in Section 5.2.4, the test work having been conducted as one multivariate set of tests.

This section highlights the impact of depressant addition on the froth surface appearance and flotation performance. The tests presented in Section 5.3.4 were a comparison between depressant addition and no depressant addition at 2cm froth depth. The choice of these tests was made on the basis that the deeper froth depth allowed the froth more time to respond to the different input conditions and, hence, was more sensitive to the input changes. Therefore the deeper froth tests were more representative of industrial conditions, where froth depth is commonly an order of magnitude greater than in a batch flotation test. That said, the results for the 1cm froth depth were, in terms of the trends observed, identical to the 2cm depth tests.

Due to the testing of the impact of dispersant on the froth, each of the depressant conditions and froth depths were replicated at three dispersant conditions and each condition was tested in triplicate. This resulted in 9 individual tests per depressant and level condition and paired t-testing was used to eliminate the impact of the varying dispersant conditions on the statistical confidence in a difference between the means of the conditions. The means themselves represent the mean of the nine tests and, hence, the mean of the results for the three dispersant addition rates.

Table 5-1 (see Section 5.2.3) shows the conditions of the tests in this Section, where a froth depth of 2cm was used and the depressant addition rate was changed from 0 to 100 g/ton of APX4M.
5.3.4 Results and discussion

Figure 5-2: The impact of depressant addition on the metallurgical performance and surface appearance of a batch flotation test

Figure 5-2 shows that, as expected, the addition of depressant significantly reduced the recovery of floatable gangue, without significantly altering the recovery of attached sulphides. The changes in recovery had the expected impact on the concentrate grade, increasing the grade by \(\approx 48\%\).

Note that the recovery of entrained material also decreased, albeit by \(\approx 17\%\), relative to the \(\approx 40\%\) reduction in the recovery of floatable gangue. Since entrained material was non-selectively recovered into the froth, the change in the recovery of entrained material can be attributed to the reduction in froth stability resulting from the absence of froth stabilising talc.

The reduction in the recovery of entrained solids and water can be attributed to a reduction in the froth stability when depressant was added. Since the reduction in recovery of naturally floatable gangue from the pulp into the froth was the only variable altered in terms of the pulp
conditions, these findings support the hypothesis that depressant addition and a reduction in the recovery of floatable gangue into the froth reduced the froth stability. This further supports the hypothesis that solid species play a crucial role in the stabilisation of the froth structure.

Figure 5-2 shows that when depressant was added the froth surface stability decreased by $\approx 40\%$. In addition, the froth surface bubble size decreased. This represents the second hypothesised result with respect to the impact of depressant on the froth surface bubble size. When depressant was added, the flotation conditions resulted in nearly instantaneous bursting of the bubbles on the froth surface. The rapid bursting prevented the formation of large bubbles on the froth surface and the observed froth surface bubble size decreased by $\approx 24\%$ on average.

The large and highly significant reduction in the froth surface stability on depressant addition is in agreement with conclusions regarding the impact of solids on the froth stability based on the reduced recovery of entrained gangue and water.

Figures 5-3 and 5-4 show the change in the froth surface stability index and the froth surface bubble diameter, over the course of the batch flotation tests. The discrete data points represent the average of the froth surface descriptors over the course of the collection interval, with the original data (shown as lines on Figures 5-3 and 5-4) having been sampled at two second intervals.
Figure 5-3: Froth surface stability index as a function of time over the course of the batch flotation tests, showing the decrease in froth stability as the solids were removed when depressant was not added and the increase in froth stability as the solids were removed when depressant was added.
Figure 5-4: Froth surface bubble size as a function of time over the course of the batch flotation tests, showing the decrease in bubble size as the test progressed for both conditions and the larger initial bubble size for the condition with no depressant.

Figure 5-3 shows that in the case when depressant was not added, the froth surface stability decreased as the test progressed, in agreement with the hypothesis that the floatable gangue species stabilised the froth. As the floatable gangue was recovered, the froth stability decreased. When depressant was added, the froth stability increased as the test progressed and the solids were removed. The opposite direction of these trends showed that the solid particles had opposite impacts on the froth stability, depending on whether depressant was added or not. This was due to the nature of the floatable gangue, including talc, which was rendered hydrophilic by the depressant.

Figure 5-4 indicates that the froth surface bubble size remained constant at or near the lower detection limit of the machine vision system when depressant was added, accompanied by highly unstable froth. When no depressant was added the froth surface bubble size was initially large and decreased over the course of the test, as the floatable species were removed from the flotation system. In agreement with prior findings, this implies that the observed
froth surface bubble size in a batch flotation test is dominated by bursting on the froth surface, rather than by coalescence within the froth zone and shows the effect of changing solids concentration in the froth stability and bubble size.

When depressant was added the species in the froth were predominantly the highly hydrophobic sulphides since the recovery of valuables was not affected by depressant addition, while the floatable gangue was rendered hydrophilic by depressant addition. This implied that the highly hydrophobic sulphide species, when acting on the froth in isolation from the floatable gangue, destabilised the froth. This finding is in agreement with the literature of Section 2.6.1.

5.3.5 The effect of hydrophobicity and bubble coverage on froth stability

The impact of particle hydrophobicity and lamella coverage on the froth stability can be further explored by supplementing the results presented in this Section with results from the literature.

The naturally floatable gangue species in the ore used are moderately hydrophobic. Moderately hydrophobic particles, particularly those with 'critical' hydrophobicity (θ = 63° to 65°), act as effective froth stabilising agents, whether in closely packed layers or as individual particles in the lamella. The sulphide particles in the ore are highly hydrophobic and have been shown to act as effective froth destabilising agents at low particle concentration in the froth. However, it is hypothesised that highly hydrophobic particles (θ = 68° to 80°) may also increase the lamellar robustness, if present in sufficient concentration in the froth to aggregate in the lamellae. This phenomenon is demonstrated by data from coal flotation systems, where a high concentration of highly hydrophobic solids strongly stabilised the froth, as observed by Horaker and Oszever (2003).

Figure 5-5 shows the qualitative relationship between the froth stability, the particle surface coverage and the particle hydrophobicity. The froth stability is plotted on an arbitrarily selected scale of ±15. The stability is plotted as a function of the coverage metric $S_p/S_b$ and the particle hydrophobicity. The data is drawn from the combined work of this thesis; Alta et al., (2003); Ventura-Medina et al., (2004) and Johansson and Pugh, (1992). In cases where quantitative data regarding the froth stability was not available and qualitative descriptions
were given (such as highly unstable, etc.), these were mapped onto the stability scale of ±15 according to the author's judgement.

The plotted surface is empirically fitted to the data, and derives from a radial basis neural network trained on the data with a low number of neurons to ensure generality of the results. The surface is meant as a visual aid for determining the trend in the data. The vertical lines above and below the data points depict the deviation of the data from the surface. Figure 5-5 has been put together as a conceptual aid and should not be considered as complete or necessarily highly accurate on a quantitative basis. However, it does show the relationship between the surface coverage of particles on the froth lamellae, the particle hydrophobicity and the froth stability.

![Figure 5-5: Relationship between particle surface coverage, particle hydrophobicity, and froth stability; incorporating data from this thesis, Aveyard et al., (1994), Ata et al., (2003) and Johannsen and Pugh, (1992)](image)

Figure 5-5 combines the results of this thesis; Aveyard et al., (1994), Ata et al., (2003) and Johannsen and Pugh, (1992) in terms of the results of these authors, with respect to the stabilising or destabilising conditions observed under various conditions of the particle
coverage and hydrophobicity. The hydrophobicity is closely related to the particle contact angle, and can be understood as such for the purposes of Figure 5-5.

The data points are coded as follows:

- **Blue:** Results from the batch tests of this thesis
- **Black:** Arceyard et al., (1994)
- **Green:** Johansson and Pugh (1992)

Figure 5-5 demonstrates key points regarding the relationship between particle hydrophobicity and lamella coverage on the one hand, and the froth stability on the other. The impact on the froth of species with contact angles below 40° is minimal, particularly since these species are not, in general, recovered in the pulp and are therefore not present as attached species in the froth. Conversely, highly hydrophobic species with contact angle above 90° destabilise the froth and this is consistent with the reduction in recovery of sulphides in base metal flotation when the collector addition is too high. In the intermediate hydrophobicity range, the particles stabilise the froth, regardless of particle concentration: with the degree of stabilisation increasing with increasing particle coverage on the lamellae. However, increasing particle coverage increases the hydrophobicity at which particles have a stabilising impact on the froth. If this trend is monotonic, this suggests that highly hydrophobic species can stabilise the froth at sufficiently high concentration on the bubble surfaces.

Lastly, the above plot shows a scale of 0-3% on the scale of lamella coverage and modelling of the froth suggests that, at the froth surface, the particle coverage in industrial systems in the primary separation role is typically closer to 30-40%. Thus, a range of experimental systems generate froth that is not representative of industrial systems, with respect to the stabilising impact of the solids on the lamella surfaces. This finding implies that reports in the literature frequently under-estimate the role of solids in stabilisation of the froth.

The findings of this section, particularly with respect to the impact of depressant addition on the froth surface bubble size, should be confirmed by experiments conducted on an industrial flotation system.
5.3.6 Summary

Comparing tests containing no depressant to tests with depressant addition confirmed that, in addition to the depressant fulfilling the traditionally accepted role of reducing the floatability and recovery of naturally floatable gangue, depressant addition simultaneously reduced the stability of the froth. This was identified by the reduction in water recovery and recovery of entrained material, and by a decrease in the froth surface stability when depressant was added. These results identified the solid species as the agent responsible for the reduction in the froth stability, as opposed to another time or surface area dependent phenomenon. Further, it was found that when depressant was added, the highly hydrophobic species present in the froth at the beginning of the test destabilised the froth, in agreement with a number of other researchers who have identified destabilisation of the froth by highly hydrophobic solid species. However, when depressant was not added, the removal of the moderately hydrophobic solids decreased the froth stability and this again confirmed the importance of solid particles in stabilisation of the froth.
5.4 THE EFFECT OF THE RATE OF AIR ADDITION ON FROTH PERFORMANCE AND APPEARANCE

5.4.1 Introduction

As presented in the literature, increasing the air addition rate to the flotation cell increases the flow of attached and entrained material into the froth. On the basis of the results in the literature, the flux of entrained solids and solution and the flux of attached water into the froth should increase by the same proportion as the increase in the rate of air addition. However, the increase in the rate of recovery of attached solids into the froth is lower than the increase in the air addition rate. Hence, as the air rate increases, the solids loading per unit bubble surface area of the bubbles entering the froth decreases.

As illustrated by Equation 4-16, an increase in the rate of air addition to the froth leads to an equal decrease in the froth residence time and, due to the reduction in the time for coalescence and drainage in the froth, this would be expected to increase the recovery of solids and solution across the froth, again arguably by the same proportion as the increase in the rate of air addition. Thus, the compound impact of the increase in the flow of material into the froth and the decrease in the froth residence time would be expected to lead to a large increase in the rate of recovery of material to the concentrate.

5.4.2 Hypothesis

It is hypothesised that increasing the rate of air addition increases the rate of recovery of solids and solution to the concentrate. The increase is significantly lower than could be expected given theoretical consideration of the increased flux into the froth and the increased superficial velocity through the froth, and hence shorter froth residence time.

An important factor limiting the rate of recovery of solids and solution under conditions of high air addition is the interaction of the air rate with the froth stability, mediated by the role of solids in stabilising the froth. Increased air addition and increasing bubble surface area flux decreases the solids attachment per bubble in the pulp and hence decreases the particle coverage per unit bubble surface area of the bubbles entering the froth.

The reduced solids coverage on the bubbles entering the froth reduces the froth stabilising action of the solids on the lamellae and hence increases the rate of coalescence in the froth. Thus, despite the expected decrease in bubble size resulting from the reduced FRT, the froth...
surface bubble size increases with increasing air addition. Hence, there is a lower concentration of solids and solution per unit volume of froth, and the rate of recovery of concentrate is less than would be expected from the increase in the flux into the froth.

5.4.3 Experimental method

Experiments were conducted on an industrial system with low and high air addition, within typical industrial ranges. Subsidiary tests were conducted to check for non-isotonic behaviour between the limits. To maximise statistical significance, air was increased from 2 m$^3$/min to 5 m$^3$/min, to cover the maximum range of input levels typically used on the industrial flotation plant. This represented a 150% increase. Full results for these tests can be found in Appendix A.

The cells used for the test work were continuous rougher cells performing the initial flotation step within the flotation circuit. There was no recycle of cleaner tails material to the roughers, and hence the feed to the cells was composed entirely of fresh feed from the primary milling stage.

All other process conditions were governed by the operating personnel, to effectively simulate typical plant operating conditions. Table 5-2 lists the plant operating conditions during the time of the test work.
Table 5-2: Summary of experimental conditions

<table>
<thead>
<tr>
<th>Constants</th>
<th></th>
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</thead>
<tbody>
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</tr>
<tr>
<td>Cell volume</td>
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</tr>
<tr>
<td>Froth surface area</td>
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<tr>
<td>Ore type</td>
<td>Merensky</td>
</tr>
<tr>
<td>Solid concentration</td>
<td>27.5 wt%</td>
</tr>
<tr>
<td>Feed size</td>
<td>60% passing 75μm</td>
</tr>
<tr>
<td>Collector (type and dosage)</td>
<td>SIBX (20g/ton)</td>
</tr>
<tr>
<td></td>
<td>DTP, Senkol 5 (10g/ton)</td>
</tr>
<tr>
<td>Frother (type and dosage)</td>
<td>Dow 250 (10g/ton)</td>
</tr>
<tr>
<td>Depressant</td>
<td>Guar type (15g/ton ore)</td>
</tr>
<tr>
<td>Froth depth</td>
<td>30cm</td>
</tr>
</tbody>
</table>

| Variable                  |             |
| Air flow-rate             | 2 - 5m³/min |
5.4.4 Results and discussion

![Figure 5-6: The impact of increasing air addition on the metallurgical performance and surface appearance of an industrial flotation cell](image)

Figure 5-6 shows that, in agreement with the hypothesised role of air addition in altering the froth performance, increasing the rate of air addition increased the froth surface bubble size and the froth surface flow speed by ≈35% and 37% respectively. This had the predicted impact on the rate of recovery of solids, since the increased volumetric rate of recovery of froth was counter-balanced to some extent by the increased bubble size and, therefore, the relative sparseness of the froth in terms of its slurry content per unit volume.

The water recovery increased by more than the solids recovery under conditions of high air addition and this was as expected. As outlined previously, the model does not take into account the different rates of drainage of solution and solids. In the case of increasing air addition, it was observed that the net rate of drainage of solution was lower than the rate of drainage of solids. Future work is required to identify the nature of the time-dependent drainage relationship independently for solids and solution.
It could be argued that the increase in the froth surface bubble size was induced by an increase in the pulp bubble size. However, the empirical relationship derived by Gorain (1999) indicated that, for an increase in the air addition rate, the increase of the pulp bubble size is the fourth root of the increase in the air addition rate. Using this relationship, the increase in the pulp bubble size was 26%. Thus, even if one assumed a constant coalescence rate and the surface bubble size increased by 26% as a result of the increase in the pulp bubble size, the change was insufficiently large to cause the observed increase in the froth surface bubble size, relative to that achieved by changing the FRT only.

Further, the variable, non-linear dependence of the coalescence rate on the height in the froth implies that the froth surface bubble size is less sensitive to changes in the pulp bubble size than a constant coalescence rate would imply. Modelling of the froth in Chapter 6 indicates that a 100% increase in the pulp bubble diameter causes a 20% increase in the froth surface bubble size. Thus, an increase in the pulp bubble size was not responsible for the change from an expected decrease in the surface bubble size to an observed increase in the surface bubble size, when the air addition rate was increased.

The increase in froth surface bubble size on increasing air addition, despite the expected decrease due to the reduced FRT, is a crucial result that strongly supports the theory that solids (or at least some surface area dependent factor) had a critical and arguably dominant role to play in the formation of the froth structure. This effect on the froth would be due to its impact on the rate of coalescence in the froth. The results strongly support the hypothesised importance of the solids species in stabilisation of the froth.

5.4.5 Summary

As expected, increasing the air addition in an industrial flotation system caused an increase in the recovery of solids and water from the flotation cell, a decrease in the percent solids of the slurry in the froth and an increase in the froth surface flow speed. However, an increase in the air addition rate caused an increase in the froth surface bubble size, despite the decrease in the bubble size that would be expected from the associated reduction in the froth residence time. It was concluded that the increase in the froth surface bubble size was due to a reduction in the load of solids per bubble entering the froth. This, in turn, decreased the degree of solids stabilisation of the lamellae in the froth and increased the rate of coalescence such that the
degree of coalescence (or bubble size) increased at the froth surface despite the shorter residence time. This result implies that in an industrial system, the solids may have an overriding effect on the froth stability and, hence, a significant impact on the flotation performance.

This is further discussed in Chapter 6 where results of illustrative modelling of the froth are presented.
5.5.1 Introduction

In flotation practice, frother is added to stabilise the bubbles in the pulp and in the froth, and to increase the recovery of froth to the concentrate. As presented in the literature, frother has been shown to reduce the surface tension at the air-water interface. The reduction in surface tension brought about by alignment of the frother molecules at the air-water interface, reduces the Gibbs free energy change upon lamella rupture and, thus, increases the resistance of the lamella to rupture in the presence of mechanical disturbances.

Further, the literature has shown that higher concentrations of frother molecules at the air-solution interfaces in the froth increase the thickness of the lamellae, thus retarding drainage and increasing the water content of the froth.

To isolate the effect of frother addition on the structure from the impact of frother on the froth mobility, the volumetric rate of recovery of froth (air, solids and solution) was kept as constant as possible during the course of the experiments. This was achieved by automatically maintaining a controlled froth surface flow speed to the concentrate, by changing the air addition rate to the flotation cell.

5.5.2 Hypothesis

It is hypothesised that increased frother addition increases the robustness of the lamellae in the froth and thus increases the froth stability and reduces the rate of coalescence in the froth zone. The reduced rate of coalescence reduces the bubble size at any given height in the froth, and this reduces the rate of drainage from the froth, as presented in Section 2.4.2.

Further, the increased strength of association between the froth-solution interface and the solution, mediated by the frother molecules, increases the thickness of the lamellae in the froth. Thus, both reduced coalescence and increased lamella thickness contribute to the increase in the rate of water recovery at higher frother addition rate and this is associated with an increase in the rate of recovery of entrained material to the concentrate. Increased stability and reduced coalescence results in reduced froth surface bubble size and increased froth surface stability.
When frother is added, the rate of detachment of valuable particles from lamellae decreases due to the reduced rate of coalescence, and thus the rate of recovery of attached species is increased by the reduced rate of drop-back of valuable solids to the pulp.

### 5.5.3 Experimental method

Frother addition rates were set as mass addition rates relative to the feed rate of dry solids to the system, as is typical in industrial flotation practice. The addition of 15 g/t of frother (Senkol 75) was alternated with the addition of no frother to the system.

The experiments of this section were conducted on the same industrial flotation plant as those of Section 5.4. The froth flow speed was controlled during the tests, by changing the air addition rate to the flotation cells using a standard Proportional-Integral-Derivative (PID) controller. Froth depth was unchanged during the changes to the frother addition rate.

![Bar chart](image-url)

**Figure 5-7:** The impact of control of the air addition rate on the froth velocity with changing frother addition.
Figure 5-7 indicates that control of the air addition rate was successful in maintaining a constant froth velocity on the surface of the flotation cell, and the froth depth was constant during the tests. The air addition rate was reduced by 10% at higher frother conditions, but this result is not highly statistically significant.

The results were gathered over the course of five consecutive days, with an average of one and a half hours between samples, to allow the system to reach steady-state at the new frother addition rate.

No significant coalescence of bubbles in the pulp zone was expected or experienced, given the low frother concentration required to fulfill the coalescence inhibiting function of frother, and the presence of surface active reagents such as collectors in the flotation pulp.

The prevailing plant operating conditions for this set of experiments were the same as those presented in Table 5-2 with the exception that the air addition rate was adjusted as outlined above, to maintain a constant froth surface flow speed, and the frother addition rate was changed between no addition of frother and 15g/ton of frother addition.
5.5.4 Results

Figure 5-8: The impact of increased frother addition on the froth performance and flotation appearance of an industrial flotation cell

Figure 5-8 shows that control of the froth surface flow speed maintained a constant rate of recovery of sulphides at constant grade, with a reduction in drop-back compensating for any reduction in pulp attachment that may have occurred as a result of reduced air addition with higher frother addition rates. The insignificant change in the rate of recovery of dry solids indicated that control of the froth surface flow speed had the desired effect and maintained a constant rate of recovery of froth to the concentrate. This highlighted the impact of increased frother addition on the froth structure and water content, as hypothesised.

The reduced rate of air addition may have reduced the rate of attachment of solids in the pulp (although the 10% difference in the air addition rate is not highly significant). It is therefore proposed that, to achieve the most reliable results, the froth depth should be selected as the manipulated variable for control of the froth surface velocity in future testing of the impact of frother addition on the froth structure and performance.
However, the insignificant change in the rate of solids recovery and valuable sulphides implies that neither the rate of attachment in the pulp, nor the rate of drop-back of valuable particles from the froth, was a highly significant mechanism in altering the froth performance.

The non-floatable gangue mineral chromite tends to occur as fully liberated particles in the size fractions that are subject to entrainment and is thus suitable for use as a tracer for entrainment in this experimental system. However, the low concentrations of entrained chromite in the pulp and particularly the concentrate, leads to high variability in the assay for chromite. Thus, the 24.8% increase in the recovery of chromite is only significant at the 89% confidence level. None the less, it is believed that the use of chromite as a tracer, in this context, is justified. The increase in chromite recovery indicates that increased frother addition increased the recovery of water and entrained material to the same degree, with an attendant reduction in the solids concentration of the froth.

As hypothesised, a machine vision system was successfully used to detect a reduction in the froth surface bubble size and an increase in the froth surface stability index, when frother was added. The change in the froth structure and surface burst rate occurred due to solution stabilisation of the lamellae by the frother molecules. The increased lamella robustness led to a decrease in the rate of coalescence in the froth and, hence, a smaller froth surface bubble size and a decrease in the rate of lamella rupture on the froth surface.

5.5.5 Discussion of the effect of frother in solution on lamellar robustness

The hypotheses regarding the role of frother within the froth zone and the mechanisms of gangue entrainment and lamella stabilisation, which were expected on the basis of the literature review, were confirmed by the observed results. The results regarding the impact of frother addition on the froth mobility, recovery of water shells across the froth-pulp interface and lamellar robustness, both within the froth and at the froth surface, indicates two phenomena. Firstly, that increased frother addition increased the size of the water shells containing entrained material as they entered the froth and, secondly, that frother stabilised the lamellae through the Gibbs-Marangoni effect.

It was shown that the frother molecules reduced the rate of lamellar rupture in the froth and, thus increased the bubble lifetime throughout the froth zone, leading to smaller bubbles on the
froth surface. Increasing frother addition also reduced the froth surface stability index, indicating that frother plays a significant role in the stabilisation of the froth surface lamellae.

The above mechanisms would also apply to the action of surfactants on two-phase foams, without considering the role of solid species in flotation froth.

5.5.6 Summary

As hypothesised, it was found that increasing the rate of frother addition increased the recovery of water and entrained material across the froth, due to the increased thickness of the lamellae and the stronger association of the air-solution interfaces in the froth with the solution when frother was added. Stabilisation of the lamellae by the frother molecules in solution reduced the rate of coalescence in the froth, reduced the froth surface bubble size and increased the froth surface stability.
5.6 CONCLUDING REMARKS

This chapter sought to address the following key questions:

- Mechanistically, how do select hydrodynamic and reagent variables alter the froth state, froth surface appearance and metallurgical performance?
- Can these mechanisms be confirmed by appropriate measurements of the froth surface appearance from machine vision algorithms?

These key questions have been addressed through analysis of experimental work from a comprehensive set of batch flotation tests and two industrial sampling campaigns. The impact of two hydrodynamic variables: the air addition rate and the froth depth, as well as two reagent variables: frother and depressant addition rate; has been studied in terms of their impact on the froth structure and mobility and, hence, on the froth performance and surface appearance.

The impact of the froth depth was studied in a batch flotation cell and, as expected, it was found to impact primarily on the degree to which the froth had coalesced and drained by the time the froth was recovered to the concentrate. Increasing the froth depth and, hence the froth retention time, reduced the recovery of water and entrained solids to the concentrate increased the concentrate percent solids and increased the froth surface bubble size and surface stability. The impact of froth depth on the performance and appearance of the froth, demonstrated the usefulness of an accurate structural understanding of the froth, when interpreting metallurgical results of flotation tests. The difference between the rate of coalescence in the shallow froth and the inferred rate of coalescence in the upper half of the deep froth, demonstrated the impact of solids in stabilising the froth. This resulted in a smaller than expected increase in the froth surface bubble size when the froth depth was increased. Further, the deeper froth had a more stable froth surface; again supporting the hypothesis that solids play an important role in stabilisation of the froth structure.

Comparing tests containing no depressant to tests with depressant addition confirmed that, in addition to the depressant fulfilling the traditionally accepted role of reducing the floatability and recovery of naturally floatable talcaceous gangue, depressant addition simultaneously reduced the stability of the froth. This was identified by the reduction in water recovery and recovery of entrained material, and by a decrease in the froth surface stability when
depressant was added. These results identified the solid species as the agent responsible for the reduction in the froth stability, as opposed to another time or surface area dependent phenomenon. Further, it was found that when depressant was added, the highly hydrophobic species present in the froth at the beginning of the test destabilised the froth, in agreement with a number of other researchers who have identified destabilisation of the froth by highly hydrophobic solid species. When depressant was not added, the removal of the moderately hydrophobic solids decreased the froth stability. Thus, when depressant was not added, the highly hydrophobic sulphides did not destabilise the froth to the same extent as had been observed when depressant was added. The observations regarding the impact of depressant on the froth stability confirmed the important role of solid particles in stabilisation of the froth.

As expected, increasing the air addition in an industrial flotation system caused an increase in the recovery of solids and water from the flotation cell, a decrease in the percent solids of the slurry in the froth and an increase in the froth surface flow speed. However, an anomalous result was observed, in that an increase in the air addition rate caused an increase in the froth surface bubble size, despite the decrease in the bubble size that would be expected from a 60% reduction in the froth residence time. An increase in the pulp bubble size as the mechanistic cause for the increase in the froth surface bubble size was eliminated. It was concluded that the increase in the froth surface bubble size was due to a reduction in the load of solids per bubble entering the froth. This, in turn, decreased the degree of solids stabilisation of the lamellae in the froth and this increased the rate of coalescence to such a degree that the bubbles increased to a larger size at the surface in the case of the high air addition, despite there being only 40% of the time available in which coalescence could take place. This result implies that in an industrial system, the solids may have an overriding effect on the froth stability and, hence, a significant impact on the flotation performance.

By controlling the volumetric rate of recovery of froth to the concentrate, it was possible to study the impact of frother addition on the froth structure, in isolation from the impact of frother addition on the froth mobility. As expected, it was found that increasing the rate of frother addition increased the recovery of water and entrained material froth, due to the increased thickness of the lamellae and the stronger association of the air-solution interfaces in the froth with the solution when frother was added. Again, as expected, the stabilisation of
the lamellae by the frother molecules in solution reduced the rate of coalescence in the froth, reduced the froth surface bubble size and increased the froth surface stability.

These findings addressed the first of the above key questions, by identifying the mechanisms through which the input variables impacted on the froth. The key role played by the froth surface descriptors: the froth surface bubble size, stability and velocity, confirms that, in answer to the second key question, measurements of the froth surface appearance can be used to confirm the hypothesised mechanisms, by providing a quantitative measurement of the froth structure, stability and mobility.

A comparison of the above results with predicted results from modelling of the froth in an illustrative mechanistic model provides confirmation of the validity of the mechanisms on which the model is based. This comparison is presented in Chapter 6.
CHAPTER 6: MODELLING OF THE EFFECT OF AIR AND LEVEL ON THE FROTH

6.1 INTRODUCTION

In this chapter, the illustrative model developed in Chapter 4 is used to compare the outputs of the model; bubble size, velocity and rate of mass recovery with the experimental results, thus evaluating the model validity. In addition, the model is used to illustrate the response of the modelled industrial froth to input conditions that were not directly tested. The impact of changes in air addition rate and froth depth on the froth is directly compared with the experiment results and the effect of changing solution and solid stabilisation on the froth due to changes in frother and depressant addition rates is explored.

The results regarding the impact of air and level on the froth are then extrapolated to a two dimensional grid of results, simulating a multivariate experimental design. This is done to demonstrate the overall characteristics of the response surfaces for the Sauter mean bubble diameter, the froth velocity and the rate of solids recovery; given certain assumptions regarding the factors affecting the froth stability.

The Sauter mean bubble diameter on the froth surface was selected as a key indicator of the froth state, since it relates directly to the froth structure and the degree to which bubbles have coalesced by the time they reach the froth surface. The Sauter mean bubble diameter was determined from the watershed segmentation of the froth surface as described in Chapter 3. Further, the Sauter mean bubble size was used (as opposed to the median or mean bubble size), due to the importance of the surface area per volume of the froth for the flotation performance, as detailed in Chapter 4.

The froth surface velocity is the rate of flow of the froth surface towards the weir of the cell, as measured by the machine vision system; again as described in Chapter 3. As demonstrated in Chapter 4, the froth surface flow speed is an important indicator of the froth mobility and the rate of recovery of concentrate from the cell. The tracking velocity was selected as the appropriate measure of the froth surface velocity, by reason of the sub-pixel accuracy of this measurement which is appropriate to the slow flow conditions observed in the experiments.
At present, the model treats all the material in the froth as slurry of intermediate composition; focussing exclusively on the effect of input parameters of the froth stability, structure and mobility. This limits the model to an illustrative role, in that it goes no further than demonstrating the proposed mechanisms in a consistent mathematical framework. Therefore, the different rates of drainage of solution and solids are not modelled at present and water recovery from the model is calculated based on the experimentally determined solids concentration in the recovered material and the rate of dry solids recovery. This represents an area requiring further attention to separately identify the factors impacting on solids and water drainage under various input conditions. Further, the model does not separately model attached and entrained solid species. Thus, the link between the model outputs and the concentrate grade can only be inferred and is not presented.

The rate of solids recovery is a key indicator of the metallurgical performance of the flotation system, since it is linked to the recovery of the flotation system, as well as being indirectly linked with the grade of the flotation concentrate through an inverse (though non-linear) relationship. Further, the rate of dry solids recovery is linked with the rate of recovery of water to the concentrate. The limitations of the model with regard to the concentrate grade and water recovery present scope for future improvement of the model, as does prediction of the surface burst rate.

However, the froth structure itself is now adequately modelled, as is the rate of recovery of solids from the concentrate. In addition, as demonstrated by Chapter 5, it is advantageous to model the water recovery and the rate of recovery of attached and entrained species within such a structurally sound framework, particularly by taking into account the interdependent, but also largely independent contributions of Plateau borders and lamellae to the concentrate composition.
6.2 Solution stabilisation of the froth

As presented in Section 4.2.3, the overall bubble lifetime is determined by three contributing components. These consist of a contribution to the bubble lifetime from the viscosity of the fluid, a solution contribution and a contribution from solids stabilisation of the bubble. It was proposed that the solution contribution derives primarily from the effect of surfactants (e.g. frother) at the air solution interface, while the solids contribution derives from bubbles forming a layer within the lamella of the froth effectively 'armouring' the bubbles against coalescence.

\[ \tau_{\text{overall}} = \tau_{\text{nu}} + \tau_{\text{sol}} + \tau_{\text{solid}} \]  

where

\( \tau_{\text{overall}} \) = bubble lifetime with respect to coalescence in the froth zone

\( \tau_{\text{nu}} \) = contribution to the bubble lifetime by the fluid inhibition of bubble contact, dominated by the fluid viscosity

\( \tau_{\text{sol}} \) = contribution to the bubble lifetime by solution factors

\( \tau_{\text{solid}} \) = contribution to the bubble lifetime by solids stabilisation of the lamellae

This Section briefly highlights the effect of changing the solution contribution to the bubble lifetime, in terms of the effect of this variable on the modelled flotation response.

Section 6.3 presents the effect of changes in the strength of the solids contribution to froth stability on the modelled flotation response.
Figure 6.1: Response of the froth surface Sauter mean bubble diameter, froth velocity and mass recovery to increasing solution stabilisation with base case solid stabilisation.

Figure 6.2: Response of the fractional water content to increasing solution stabilisation with base case solid stabilisation.
Figure 6-1 shows the model response to increasing solution stabilisation of the froth at intermediate solid stabilisation. As would be expected for increasing lamellar robustness induced by increasing frother concentration; without any stabilisation, no froth is recovered and the froth surface velocity is zero. As the frother concentration increases, the froth velocity rapidly increases and then stabilises. The froth surface bubble size decreases with increasing frother concentration, due to inhibition of coalescence in the froth zone. The rate of dry mass recovery is the product of the volumetric rate of overflow of the froth, which is linearly related to the froth velocity and the volume fraction of slurry in the overflowing froth, shown in Figure 6-2. Thus, the initial increase in the froth velocity causes the dry mass recovery to increase rapidly with increasing solution stabilisation of the froth at low levels. At higher solution stabilisation, the exponential shape of the slurry content in the froth becomes dominant and the mass recovery again increases rapidly.
6.3 Solid Stabilisation of the Froth

Figure 6-3: Response of the froth surface Sauter mean bubble diameter, froth velocity and mass recovery to increasing solids stabilisation, with base case solution stabilisation
Figures 6-3 and 6-4 show the response of the metallurgical and image analysis parameters to increasing solid stabilisation of the froth. The froth velocity is relatively insensitive to changes in the degree of solid stabilisation of the froth and, hence, the dry mass recovery is dominated by the approximately linear increase in the slurry content of the froth, with increasing solid stabilisation. As expected, the bubble size decreases with increasing solid stabilisation, due to solids on the lamellae reducing the probability of lamellar rupture and coalescence in the froth.
6.4 Comparison of model and experimental results with respect to the effect of increased froth depth on froth performance and appearance

Figure 6-5: The impact of doubling the froth depth on selected indicators of the flotation performance and froth surface appearance, comparing the experimental results presented in Section 5.2 with the results of the froth model described in Chapter 4.

Due to the froth depth experiments having been conducted on a batch flotation system, while the model was designed for an industrial system, the experimental results in terms of the absolute solids recovery and bubble size were estimated from the performance of the industrial system. Thus, the performance of the industrial system under typical operating conditions, as derived from the experimental work on the impact of air addition and frother addition were used as a 'base case' condition. The relative changes in the measured parameters were then used to determine the change in the industrial parameters under equivalent conditions. Since there is no froth overflow in a batch system, there was no basis on which to calculate an equivalent change in the industrial froth surface velocity. However, the model results with respect to the froth surface velocity are shown in Figure 6-5.
From the perspective of the mechanistic determinants of froth behaviour, the sole impact of increasing the froth depth was to increase the froth residence time. This, in turn, allowed more time for the performance and structure determining processes of coalescence, detachment and drainage of detached and entrained solids and solution. The results presented in Figure 6.5 compare the results of the batch flotation tests presented in Section 5.2 with the results of modelling of the froth for a doubling of the froth depth.

In the batch system, the 100 percent change in the froth depth physically resulted in a negligible (and not significant) 1% reduction in the mass recovery of the system. This result implies that the solids in the froth were not subject to appreciable detachment, possibly due to the shallow froth of the batch flotation system and the forced recovery of concentrate by scraping. By contrast, the model predicts an appreciable 11.4% reduction in the solids recovered from the system. This is more realistically representative of the reduction in solids recovery expected with increasing froth height in an industrial system.

The negligible change in the recovery of solids from the froth is also, to some extent, due to the relatively small change of 15.7% in the froth surface bubble size, which indicates that a low rate of coalescence was observed in the second half of the deeper froth. The lower than expected change in the froth surface bubble size indicates that the coalescence rate in the froth is not constant and this observation is duplicated in the froth modelling by the inclusion of the solids stabilising component. The correspondence between the experimental and modelled results is achieved by specifying a significant contribution of the solids stabilising factor to the modelled results. The model predicts a 9.7% increase in the bubble size for the same change in the froth depth. This indicates that the model is duplicating the performance of the experimental system to within a reasonable margin of error. To duplicate the experimental results, the solids contribution at the froth surface in the model is 251 times that of the solution contribution to the bubble lifetime.

The duplication of the experimental results with an illustrative model of the froth provides confirmation of the validity of the mechanisms on which the model is based and, in particular, highlights the importance of solid stabilisation for the froth appearance and performance.
6.5 Comparison of model and experimental results with respect to the effect of increased air addition on froth performance and appearance

Figure 6-6: The impact of increasing the air addition rate from 2m³/min to 5m³/min on selected indicators of the flotation performance and froth surface appearance, comparing the experimental results presented in Section 5.4 with the results of the froth model described in Chapter 4.

As discussed in Section 5.4, increasing the rate of air addition into the cell proportionally increased the flow of solids and solution into the froth and reduced the froth retention time. However, the reduction in the load of solids on the bubbles due to depletion of floatable solids in the pulp reduced the contribution of the solids to the lamella robustness in the froth. Therefore, the rate of coalescence in the froth increased. The increase in the rate of coalescence more than compensated for the decrease in the time available for coalescence (the froth retention time), leading to a larger bubble size on the froth surface when the air addition rate was high, than when the air addition rate was low.
In Chapter 4, a model was described that was designed to illustrate the physical implications of certain hypothesised mechanisms on the froth structure and appearance. Despite an over-prediction of the change in the velocity of the froth on increasing air addition and an under-prediction of the absolute bubble size, modelling of the froth with conditions of changing air addition generated indicators of the froth performance and surface appearance that were both quantitatively and qualitatively in line with those determined on the experimental system as shown in Figure 6-6. Given that the model is designed to be illustrative of the mechanisms acting in the froth, as opposed to an accurate predictive model, the differences between the experimental results and the predicted results in this context are negligible.

These results lend further credibility to the hypothesised mechanistic framework and re-iterate the importance of the solids in terms of their impact on the froth stability.
6.6 Predictive Modelling of the Response of the Froth to Changing Inputs

The froth was modelled for input conditions covering a range of air addition rates and froth depths in a two dimensional grid, representing a multivariate experimental design. This predictive modelling exercise fulfils two objectives. First, the results can be taken as predictive of the general response of an industrial system to changing input conditions for further testing of the proposed mechanisms and, second, the results clearly demonstrate the manner in which a solids stabilised model deviates from a 'solution only' model. In essence, a solid stabilised model results from assuming a changing rate of coalescence in the froth, while a solution only model demonstrates the response of the froth if a constant coalescence rate is assumed. The model parameters were set such that they matched this experimental system and the model was used to predict the bubble size, froth velocity and dry mass recovery over the same range of air and level input conditions as was used during testing.

In Figures 6-7 through 6-9 the model output is presented for the two cases outlined above. The first case is for a froth stabilised with the 'solution' parameter alone and, thus, the bubble lifetime is constant throughout the froth zone for this case. The second model has the solid contribution to the bubble lifetime set at a high value, while the solution factor is set to a low value. This model corresponds to a highly variable bubble lifetime through the froth, the bubble lifetime being strongly dependent on the solids coverage on the lamellae.
6.6.1 Froth surface Sauter mean bubble diameter

![Figure 6-7: Modelled froth surface Sauter mean bubble diameter under changing air addition and froth depth for a solution only stabilised model and a solids stabilised model](image)

As presented in Sections 6.2 and 6.3, the Sauter mean bubble diameter increased with increasing air addition and froth depth. Figure 6-7 shows that the model that has constant coalescence rate shows an exponential relationship between the bubble size and the froth residence time, regardless of whether the increase in the residence time is caused by an increase in the froth depth or a reduction in the air addition rate. However, Figure 6-7 also shows that the model that incorporates solid stabilisation of the froth as a key mechanism duplicates the observed increasing bubble size with increasing air addition.

Consider coalescence as a stochastic rate process. If the rate of coalescence is constant (as in the solution stabilised case); the average bubble in the bubble population doubles in volume over the course of the bubble lifetime. Thus, the bubble volume increases exponentially with time and changes in the froth residence time would be expected to dominate the froth characteristics. This behaviour is demonstrated in the solution stabilised case in Figure 6-7, where low air addition (and hence low superficial gas velocity) leads to a long residence time, as does a deep froth condition. The increase in bubble volume (and hence diameter) is exponential until the larger bubbles reach the model upper limit in terms of bubble size (307mm diameter). This trend is not observed in practice, in that a doubling of the froth residence time was observed to cause a 15.7% increase in the bubble size.
Similarly, the froth is relatively insensitive to changes in the air addition rate. As the air addition rate increases, the superficial froth velocity increases, the residence time decreases and one would expect the bubbles to coalesce less before reaching the surface. The opposite of this trend is observed, with the bubbles increasing in size with increasing air addition.

The seemingly anomalous industrial results regarding the impact of froth depth and air addition rate on the surface bubble size are explained and successfully modelled, when solid stabilisation of the froth is considered as a fundamental (and possibly dominant) cause of froth stability. The solids stabilisation dominated models both present results that are similar to the observed results in terms of the magnitude of the observed changes and the observed trend in bubble size on changing air addition.

Mechanistically, this derives from the stabilising action of the solids. At the froth-pulp interface, the solids coverage on the lamellae is low. As the bubbles coalesce, the froth loses surface area per unit froth volume on which particles can remain attached and the load of solids per unit surface area increases. The surface area coverage of the bubbles with solid particles increases and the lamellae are stabilised by the solids, retarding coalescence.

Thus, a change in the froth residence time by a factor of four does not impact dramatically on the froth surface bubble size because, although the bubbles continue to coalesce as time progresses, they do so at a slower and slower rate, with the rate depending on the degree to which they have coalesced up to that point.

In addition, the mechanism of solid stabilisation explains the potential for an increase in froth surface bubble size on increasing air addition, despite the fact that increasing air addition reduces the froth residence time. As the air addition rate increases, the coverage of particles per unit lamella surface area entering the froth decreases and solids stabilisation of the froth is reduced. The rate of coalescence is thus higher when the air addition rate is high and the froth surface bubble size can, under some conditions, increase when the air addition rate increases.

The impact of air addition rate on the froth surface bubble size is thus dependent on the balance of the froth residence time and increased coalescence, as a result of reduced solids coverage on the lamellae. This balance of phenomena explains the surprising constancy of
the froth surface bubble size on changing air addition rate and explains why, although the bubble size may increase or decrease with increasing air addition, the resultant change is small relative to the change that would be expected from the equivalent change in the froth residence time. That said, the change is not insignificant and is within the range that can be detected experimentally, as shown in Section 5.4.

6.6.2 Froth surface velocity

Figure 6-8: The fitted and modelled froth surface velocity towards the weir under changing air addition and froth depth

Figure 6-8 indicates that without taking into consideration the role of solids stabilisation of the froth, the range of modelled froth surface flow speed is too great. For long residence time the froth is effectively stationary, while for short residence time a flow speed is predicted that is too high by a factor of three.

By contrast, with solids stabilisation, the results are as expected: with high air addition and shallow froth depth yielding a moderately higher froth surface flow speed.
6.6.3 Rate of dry solids mass recovery

The remarkable feature of the experimental results is that the rate of solids recovery is so constant despite substantial changes in the air addition and froth depth. With reference to Figure 6-9, the solution stabilised model over-predicts the rate of solids recovery by an order of magnitude. The constancy of the experimental results is duplicated for the highly solids stabilised model in order of magnitude terms and the model shows the expected slight increase in the rate of solids recovery for shallow froth depth and high air addition.

The solids stabilised model shows the best overall performance, in terms of its ability to predict the froth surface appearance and flotation performance. This implies that the early rougher cells in PGM flotation are subject to a high degree of stabilisation by the solid species. This result may be general to many flotation systems, in the sense that solids may play an important role in stabilising the froth in many industrial flotation systems.
CHAPTER 7: CONCLUSIONS

7.1 LITERATURE REVIEW AND RESEARCH OBJECTIVES

The froth structure has been described in terms of the lamellae, Plateau borders and vertices forming the three dimensional froth matrix. The distribution and behaviour of attached, detached and entrained solution and solid species within the froth zone and their effect on promotion and inhibition of drainage has been investigated. The role of coalescence in the dynamic formation of flotation froth was explored, and solution and solid species were identified as the key factors impacting on the lamella robustness.

Given the contradictions present within the literature regarding the definition, role and causes of froth stability, the objectives of this thesis were to define froth stability, develop a model to illustrate the effect of input parameters on the froth stability and to experimentally test the proposed mechanisms and hence the model validity with metallurgical performance indicators and measurements of froth surface appearance using a newly developed froth machine vision system.

Specifically, this was to be achieved by addressing the following key questions:

1. How can the froth structure best be described and what are the mechanisms governing froth formation within the context of this structure?
2. Mechanistically, how do selected hydrodynamic and reagent variables alter the froth state, froth surface appearance and metallurgical performance?
3. Can these mechanisms be confirmed by appropriate measurements of the froth surface appearance from machine vision algorithms?
4. Can a model be developed to illustrate the effect of the input variables on the froth structure and, thus, on the relevant froth surface descriptors and flotation performance parameters?

7.2 FROTH STABILITY

In order to further investigate the implications of the rate of lamella rupture for froth performance and appearance, froth stability has been defined as the inverse of the rate of lamella rupture per second, per number of lamellae, within a given froth volume. The structural description drawn from the literature and the definition of froth stability provided a
framework within which to discuss the mechanistic impact of changes in flotation operating conditions on the froth.

The following mechanisms of stabilisation of the froth have been identified, and were used for interpretation of the experimental results and for modelling of the froth:

1. Viscous stabilisation resulting from the time taken for bubbles to come into contact and for the lamellae to thin to the point at which rupture can occur.

2. Solution stabilisation owing to the Gibbs-Marangoni and Plateau effects and disjoining pressures developed within the lamellae, retarding the rupture of lamellae when considering a two-phase air-solution system, owing to the presence of frother dissolved in solution.

3. Solids stabilisation owing to the mechanism of bubble ‘armouring’ and the formation of a layer of closely packed particles on the lamellae that experience strong, mutually attractive hydrophobic bonding forces and whose collective impact on the lamellae is to form a strong physical barrier to thinning and rupture. The magnitude of this complex effect is governed by the particle concentration on the bubble lamellae, or degree of close packing of the particles, as well as the particle hydrophobicity, shape and surface roughness.

The following equation was proposed to describe the combined effect of the viscous, solution and solids characteristics on the froth stability where the froth stability is governed by the bubble lifetime.

\[ \tau_{\text{overall}} = \tau_{\text{viscous}} + \tau_{\text{solut}} + \tau_{\text{solid}} \]  

where

- \( \tau_{\text{overall}} \) = bubble lifetime with respect to coalescence in the froth zone
- \( \tau_{\text{viscous}} \) = contribution to the bubble lifetime by the fluid inhibition of bubble contact, dominated by the fluid viscosity
- \( \tau_{\text{solut}} \) = contribution to the bubble lifetime by solution factors
- \( \tau_{\text{solid}} \) = contribution to the bubble lifetime by solids stabilisation of the lamellae
7.3 MECHANISTIC ACTION OF INPUTS

The experimental work consisted of a comprehensive set of batch flotation tests and two industrial sampling campaigns. The impact of two hydrodynamic variables: the air addition rate and the froth depth, and two reagent variables: frother and depressant addition rate, were studied in terms of their impact on the froth structure and mobility and, hence, on the froth performance and surface appearance. Hypotheses were tested regarding the mechanistic action of each of the inputs within the flotation system.

7.3.1 The effect of changing froth depth on froth performance and appearance

It was shown that changing the froth depth changed the froth retention time and that this altered the time for coalescence and drainage in the froth. It was further shown that the rate of coalescence in the froth decreased as time in the froth zone increased due to stabilisation of the lamellae by solid species. This was demonstrated by the relatively small increase in the froth surface bubble size and by the increase in the froth surface stability when the froth residence time was doubled.

An increase in the froth residence time (FRT) was shown to increase the percent solids in the concentrate due to differences in the rates of drainage of attached and entrained species. Furthermore, an increase in the FRT caused an increase in the concentrate grade due to increased drainage of the low grade entrained species relative to the higher grade attached species.

The changes in the froth surface bubble size and froth surface stability when changing the froth depth were successfully measured using the SmartFroth machine vision instrument.

7.3.2 The effect of gangue depression on froth performance and appearance

It was shown that solid species play a key role in stabilisation of the froth structure. The presence of floatable gangue species (including talc) stabilised the froth and the removal of the stabilising gangue species when depressant was added led to a less stable froth and an increase in the rate of coalescence and bursting. The high rate of coalescence resulted in an increase in the rate of drainage from the froth and thus, in addition to the reduction in the recovery of attached gangue to the concentrate the recovery of water and entrained gangue decreased when depressant was added to the flotation system.
The removal of the froth stabilising gangue species and resulting instability could have led to one of two results. The increased rate of coalescence within the froth could have increased the froth surface bubble size and slightly increased the froth surface burst rate or, nearly instantaneous bursting of bubbles on the froth surface could have led to a smaller observed surface bubble size and a very high froth surface burst rate. In these experiments, the second case was observed as indicated by a very high surface burst rate and small bubbles when depressant was added. It is concluded that there is a non-linear relationship between the froth surface bubble size and the froth stability when increasing depressant addition and that the observed result was due to the low solids loading and highly hydrophobic nature of the solids within the froth when depressant was added.

The impact of solids of different hydrophobicity and concentration on the froth stability was seen when the impact of depressant addition on the froth surface stability was monitored over time. When depressant was not added the floatable gangue stabilised the froth and the froth stability decreased as the solids were removed. When depressant was added and the floatable gangue did not enter the froth and the removal of solids from the system increased the froth stability. Therefore the moderately hydrophobic floatable gangue species stabilised the froth while the highly hydrophobic sulphide species destabilised the froth in the absence of the stabilisation afforded by the floatable gangue.

7.3.3 The effect the rate of air addition on froth performance and appearance

It was shown that increasing the rate of air addition increased the rate of recovery of solids and solution to the concentrate and that this increase was significantly lower than could be expected given theoretical consideration of the increased flux into the froth and the shorter froth residence time.

It was found that an important factor limiting the rate of recovery of solids and solution under conditions of high air addition was the interaction of the air addition rate with the froth stability, mediated by the role of solids in stabilising the froth. Increased air addition reduced the particle coverage per unit bubble surface area.

The reduced solids coverage on the bubbles entering the froth reduced the froth stabilising action of the solids on the lamellae and hence increased the rate of coalescence in the froth. Thus, despite the expected decrease in bubble size resulting from the reduced FRT, the froth
surface bubble size increased with increasing air addition, resulting in lower bulk density of the froth. This indicates that the increase in the rate of coalescence overcame the reduction in the froth residence time as the key determinant of the bubble size and that the solids have a strong impact on the froth stability.

7.3.4 The effect of the rate of frother addition on froth performance and appearance

It was shown that increased frother addition increased the robustness of the lamellae in the froth and thus increased the froth stability and reduced the rate of coalescence in the froth zone. The reduced rate of coalescence reduced the froth surface bubble size and this reduced the rate of drainage from the froth.

Further, the increased strength of association between the froth-solution interface and the solution, mediated by the frother molecules, increased the thickness of the lamellae in the froth and further increased the rate of water recovery at higher frother addition rate. The higher rate of water recovery was associated with an increase in the rate of recovery of entrained material to the concentrate. The increased stability and reduced coalescence also resulted in reduced froth surface bubble size and increased froth surface stability.

When frother was added, the rate of detachment of valuable particles from the lamellae decreased due to the reduced rate of coalescence, and thus the rate of recovery of attached species was increased due to the reduced rate of drop-back of valuable solids to the pulp.

7.4 Froth appearance

The machine vision system SmartFroth has been developed in parallel with this work. Further, the froth surface burst rate, an indicator of the froth surface stability was developed as part of this work. The machine vision algorithms have been described and the changes in the froth surface appearance resulting from changes in the input parameters were successfully detected using SmartFroth. The key role played by the froth surface descriptors: the froth surface bubble size, stability and velocity, in confirmation of the hypothesised mechanistic effect of inputs on the froth confirms that measurements of the froth surface appearance can be used to provide quantitative measurement of the froth structure, stability and mobility.
The froth surface bubble size and froth surface burst rate measurements have proven invaluable as indicators of the rate of coalescence and bursting in the froth, thus indicating the impact of an input on the froth stability.

7.5 FROTH MODELLING

The mechanisms that have been proposed have been formalised in an illustrative froth model, which demonstrates aspects of the flotation performance and froth surface appearance that are qualitatively and quantitatively in line with expectations as well as the measured data for the industrial flotation system. Unification of this froth model with a pulp model could potentially yield a model of the flotation system that predicts the metallurgical performance, froth structure and surface appearance for given input conditions and ore mineralogy. There exists significant scope for refinement and extension of the model and this would be necessary to achieve the above goals.

This work has importance for effective control of the flotation system in an on-line industrial context and particularly for the control of reagent addition. However, it is stipulated that not all of the results were observed on all days of sampling, with disturbance variables playing a key role in the froth appearance on some days. Thus, one can conclude that significant work is required in identifying the impact of disturbance variables on the froth appearance and separation efficiency.

Real-time measurement of the froth surface parameters at a sampling frequency that is appropriate for effective flotation control has been shown to be possible and highly robust. This renders fundamentally informed control attainable with the existing SmartFroth software, when applied in the context of appropriate checks and balances.
7.6 FURTHER WORK

Continuation and expansion of this work could include further development and validation of the froth model, as well as investigation of the mechanisms through which a broader range of control and disturbance variables impact on the flotation performance and froth surface appearance. In terms of model validation, varying particle hydrophobicity by altering the collector dosage would yield insights into the effect of hydrophobicity on the coalescence rate and froth structure in industrial flotation operations. Altering the froth depth in an industrial system would contribute to validation of the predictions of the froth model. The current research was restricted to rougher banks within the flotation operation and further research could extend the current findings to scavenger, cleaner and recleaner applications.

Further development of the froth model could yield a predictive froth model that incorporates input parameters such as the pulp solids concentration, particle size and mineralogy and predicts the grade of the concentrate and water recovery from the froth. The model considers all solution, entrained and attached material in the froth as homogenous slurry. Extension of the model should include separating the contributions of attached and entrained material to the solids in the froth and should optionally consider the grade or strength of attachment of the solid particles. An equation describing the change in Plateau border cross sectional area through the froth would form a crucial contributor to the model of water recovery. This work could yield benefits in terms of understanding the froth behaviour, offline simulation of flotation circuits and, when used in conjunction with image analysis of the froth surface, should yield online model predictive control of the flotation system.

The input variables tested in the course of this work included the frother and depressant addition rates, the froth depth and the air addition rate to the flotation system. A potential unmeasured disturbance variable is the impact of changing air addition or frother concentration on the bubble size in the pulp. The impact of this variable should be measured in future test work. Further, it was found that disturbances to the flotation system could override the impact of changes in input variables on the froth surface appearance. Thus, the impact of the relevant disturbance variables should be studied to understand the impact of disturbance variables on the froth structure and appearance and the flotation performance. In particular, changes in the pulp density and particle size distribution appear to have a dominant effect on the froth performance and surface appearance and therefore require further study.
Additionally, the impact of collector, promoter and activator addition rate on the froth behaviour is also of interest for comprehensive control of the flotation process.

The model and its mechanistic underpinnings require further testing and validation across a range of industrial conditions which will lead to improved understanding of the mechanisms through which inputs and disturbance variables impact on the froth stability and hence the froth structure, performance and appearance. Elements of the froth model, especially the non-linear solids contribution to bubble lifetime are strictly empirical at this stage and require further in depth investigation to determine a physically significant form of the relationship and measure or derive the appropriate parameters. The interaction of the surface active components with the froth viscosity and rate of drainage is also important to achieve a comprehensive understanding of the interaction of reagents with the froth dynamics. However, care should be taken to incorporate the effect of the solids species on such studies due to the potentially dominant effect of the solids on the froth stability.

It is also recommended that ongoing work investigating the fundamental inter-particle hydrophobic forces and the impact of solution species on lamella robustness continue in order to provide a more rigorous basis for the empirical approach employing in this work towards modelling of the impact of solid and solution species on the froth stability. In particular, the assumption of a constant coalescence rate in the absence of solid stabilising species requires probing.
REFERENCES


49. Old C., 2001, Personal communication.


54. Robertson, C., Bradshaw, D.J. and Harris, P.J., 2003, Development of a methodology to de-couple the effects of dispersion and depression in batch flotation, In proceedings of IMPC XXII, Cape Town.


60. SmartFroth1, Patent number ZA200007079, 20001130.

61. SmartFroth2, Patent number ZA200403592, 20040511.

62. SmartFroth3, Patent number ZA200401499, 20040224.

63. SmartFroth4, Patent number ZA200406157, 20040802.


APPENDIX C: FLOTATION TEST PROCEDURE

1. Depressant

APX 4M - a modified guar gum - obtained from Agricultural Products Exchange was used in this investigation. Properties of this reagent are shown in Table C-1. In this investigation, depressant was typically dosed at 100g/t. Tests were also carried out in the absence of depressant.

Table C-1: Properties of the specific sample of APX4M depressant:

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Molecular Weight</td>
<td>252,000</td>
</tr>
<tr>
<td>Viscosity (of 1.5% solution)</td>
<td>7 centipoise</td>
</tr>
<tr>
<td>Purity</td>
<td>75-85%</td>
</tr>
<tr>
<td>%Insolubles</td>
<td>10</td>
</tr>
<tr>
<td>%Moisture</td>
<td>1</td>
</tr>
<tr>
<td>pH (of a 1% solution)</td>
<td>4.84</td>
</tr>
</tbody>
</table>
2. Salts for use in synthetic plant water

Synthetic plant water was used in all the work done in this investigation. Based on chemical analysis of the water quality at the plant, various salts were added to distilled water to simulate the water quality. This is a standard practice in the DRF. The composition of the water is shown in Table C-2.

Table C-2: Salts added to generate synthetic plant water

<table>
<thead>
<tr>
<th>Salt</th>
<th>Formula</th>
<th>Mass added to 1 litre (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calcium chloride</td>
<td>CaCl2</td>
<td>0.111</td>
</tr>
<tr>
<td>Calcium nitrate</td>
<td>Ca(NO3)2.4H2O</td>
<td>0.236</td>
</tr>
<tr>
<td>Magnesium nitrate</td>
<td>Mg(NO3)2.6H2O</td>
<td>0.107</td>
</tr>
<tr>
<td>Magnesium sulphate</td>
<td>MgSO4.7H2O</td>
<td>0.615</td>
</tr>
<tr>
<td>Sodium carbonate</td>
<td>Na2CO3</td>
<td>0.058</td>
</tr>
<tr>
<td>Sodium chloride</td>
<td>NaCl</td>
<td>0.356</td>
</tr>
</tbody>
</table>

3. Collectors:
The collectors Sodium isobutyl xanthate (SIBX) and Senkol 5 (SK5) were supplied by Senmin. The active content of SIBX was 90% and SK5 was 50%. The reagents were added as-is at the dosage specified.

4. Ore Sample
Ore was used was obtained from the Merensky reef at Amandelbuit was described by Viljoen et al. (1986) as being a "pegmetoidal feldspathic pyroxenite". The ore contains 1.5% sulphides, (predominantly chalcopyrite, pentlandite and pyrrhotite) and more than 90% of gangue was made up of feldspar and pyroxene in approximately equal proportions with up to 5% talc and the rest made up of olivine, chlorite and other minor constituents. 1kg of ore was milled to 60% passing 75µm in a rod-mill prior to use.

5. Flotation cell description

A bottom-driven modified Leeds cell was used in all of the tests reported in this thesis. The flotation cell is shown below. The cell was modified slightly to allow for flotation tests to be
A bottom-driven modified Leeds cell was used in all of the tests reported in this thesis. The flotation cell is shown below. The cell was modified slightly to allow for flotation tests to be carried out at different froth heights without changing the pulp volume. This was achieved using a movable front face. The cell used in this investigation has radial baffles in the bottom of the cell (around the impeller). Pyramid shaped baffles are placed on top of these radial baffles to prevent cornering effects and to contain a highly turbulent zone at the bottom of the cell.

![Image of movable front face in flotation cell](image1)

**Figure C-1:** The movable front face in the flotation cell

![Image of baffles in flotation cell](image2)

**Figure C-2:** Position of the baffles in the flotation cell (plan)
The radial baffles in this cell are expected to increase the turbulence in the zone below the pyramidal baffles. Removal of the pyramidal baffles results in rapid overflow of material from the cell as a result of this intense turbulence.

In the absence of air this cell was clearly not perfectly mixed, and one can observe particle settling in the quiescent zone above the pyramidal baffles. Figure 3-7 shows that after the addition of air the distribution of particles in the cell (on a size basis) was identical, and this suggests that the cell was indeed perfectly mixed.

![Graph of Particle Size Distribution](image)

**Figure C-4**: Effect of sampling depth on the particle size distribution in the cell after air addition.
Specific details of the flotation cell are given in Table C-3. In this test-work, impeller speed and air flow-rate are kept constant throughout. Hence the typical values of 1200 rpm and 7 l/min (as used by the DRF for Merensky ore) were used in this work.

Solids content in the cell was maintained as a consistent 30% by milling 1 kg of ore and always filling the cell to the same level.

Table C-3: Details of the flotation cell

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimension (L x B x H)</td>
<td>15 x 15 x 16 cm</td>
</tr>
<tr>
<td>Material</td>
<td>Perspex</td>
</tr>
<tr>
<td>Air-flow rate</td>
<td>7 litres/minute</td>
</tr>
<tr>
<td>Impeller Diameter</td>
<td>6 cm</td>
</tr>
<tr>
<td>Impeller Speed</td>
<td>1200 revolutions per minute</td>
</tr>
<tr>
<td>Solids Content</td>
<td>30%</td>
</tr>
</tbody>
</table>

Flotation tests were carried out in a manner identical to the general procedure followed in the MPRU. This is described briefly below. The addition of MnO2 as a tracer requires one variation to the standard procedure in that the feed sample was taken after MnO2 and depressant addition to allow a mass balance of MnO2 to be carried out.
8. Flotation Test Procedure

- Upon arrival at UCT, ore was crushed to -2mm and split into 1kg sample
- 1kg of ore was milled to 60% passing 75μm in a rod-mill
- (Milling was carried out at 66.6% solids in synthetic plant water, and collectors are added prior to milling)
- Mill product was transferred to a flotation cell and the cell was filled up to the standard level using plant water. (the impeller was turned on during this process)
- Depressant was added to the cell
- 1 minute after depressant addition 50g of dry MnO2 was added to the cell.
- 30 seconds thereafter (after allowing for good mixing of the MnO2) a feed sample (~25 ml) was taken with a syringe.
- 30 seconds thereafter frother was added
- 1 minute thereafter (3 minutes after depressant addition), the air was turned on and flotation began.
- Concentrates are collected by scraping every 15 seconds
- (a blade of a specific length was used to collect the full volume of froth above the layer of the lip of the cell in one motion)
- Concentrates are collected in four separate trays at different stages in the test

Concentrate 1: 0-2 minutes after the addition of air
Concentrate 2: 2-6 minutes “ “
Concentrate 3: 6-12 minutes “ “
Concentrate 4: 12-20 minutes “ “

- Synthetic plant water was used throughout the test to maintain the constant initial pulp level. This was done manually and gauged via markings on the side of the cell.
- Additional wash water was used to rinse the scraper blade and cell lip. It was assumed that all of this water was collected in each subsequent concentrate tray. A separate bottle was used for each concentrate, and the difference in the mass of this bottle before and after the test was used to calculate the mass of wash water added to each concentrate.
• After 20 minutes of flotation, the air was turned off, and two tails samples (~25ml each) are taken with syringes.
• Remaining pulp was rinsed into a bucket
• Concentrate trays are weighed to allow for later determination of the water recovered from the cell (less the wash water).
• All samples are filtered under vacuum and dried overnight in an oven at 700C.

9. Chemical analysis of flotation products

• Dried samples were cooled and weighed
• Samples are "rubbed off" the filter papers using a milling rod
• (This stage was omitted if further screening was required)
• Known masses of each sample are digested over a hotplate with an HF/HCL mixture, HNO3 and HClO4 in turn.
• Concentration of Copper, Nickel and Manganese in the subsequent solution was determined by comparison to standard values using atomic absorption.
• Sulphur grades (%) are obtained from dry samples using a Leco sulphur analyser.
Platinum group elements (PGEs)

The elements Platinum, Palladium, Osmium, Rhodium, Ruthenium and Iridium. Another common abbreviation for the valuables in the platinum industry is 4PGE&Au, referring to the elements that are most financially significant - Platinum, Palladium, Rhodium, Ruthenium and Gold.

Poly-disperse

See Degree of dispersity.

Pulp

The pulp is a relatively homogeneous mixture (relative to the froth) of fine (<200μm) particles and small (~1.5 mm) gas bubbles, in a solution comprising water with dissolved flotation reagents.

Pulp zone

The pulp zone is the region below the froth-pulp interface in a flotation cell which extends to the bottom of the cell and in which bubble-particle collision and attachment occur, prior to transport by buoyancy into the froth.

Reagents

Frother

Frother is added to inhibit coalescence in the pulp zone and to achieve a required level of froth stability, by reducing the Gibbs free energy of the air-solution interface and thus inhibiting lamella rupture following a mechanical disturbance. Frother also enhances froth mobility through reduced froth viscosity and accelerates the kinetics of bubble-particle attachment in the pulp zone. Typically, frother molecules are short-chain hydrocarbons with a hydrophobic head and hydrophilic tail.

Depressant

Depressant is added to reduce the hydrophobicity of naturally floatable gangue species; thus inhibiting their recovery to the concentrate and increasing the concentrate grade. Depressants have a range of chemical compositions, among which are long chain hydrophilic hydrocarbons which can be either
uncharged, as in the case of guar gum depressants; or charged, as with carboxymethylcellulose (CMC) depressants.

**Collector**

Collector selectively adsorbs on the valuable mineral surface with one end of the collector molecule and presents a *hydrophobic* tail to the solution, inducing *hydrophobicity* on the valuable mineral surface and increasing *recovery* of the valuable mineral. The most common collectors in sulphide flotation are the short-chain hydrocarbon xanthates.

**Activator**

Activator increases the efficacy of the collector adsorption process, by creating or regenerating a reactive surface on the valuable mineral. Copper sulphate is typically used as an activator in sulphide flotation.

**Recovery**

The mass percentage of a valuable element in the feed that reports to the *concentrate* stream, which is used as a metallurgical indicator of the effectiveness of the separation.

**Sauter mean bubble diameter**

Ratio of the sum of the equivalent ellipsoidal bubble volumes to the sum of the equivalent ellipsoidal surface areas for a group of bubbles.

**Solids coverage**

Defined as the cross sectional surface area of attached solids per unit surface area of bubble in the pulp. Alternatively, defined as the cross sectional surface area of attached solids per unit surface area of lamellae and Plateau borders in the froth, or equivalently as the ratio of the attached particle cross-sectional surface area flux to the bubble surface area flux.

**Solids load**

Mass of solids per unit surface area of bubble or per unit surface area of lamellae and Plateau borders at the solids-solution interface in the froth.