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THE DESIGN OF HIGH POWER ULTRASONIC TRANSUDCERS.

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Prepared for: The Department of Electrical Engineering at the University of Cape Town.

2 April 2002

Thesis prepared in fulfillment of the requirements for the Degree of MSc. in Electrical Engineering.
Acknowledgements

The author would like to express his most sincere thanks to the following people who provided assistance during the thesis period.

- Dr Jon Tapson who was always helpful and ready to answer any questions asked of him, and for his help in seeing “the big picture.”
- Dr Bruce Mortimer for all his help in the many aspects of this thesis, for always being willing to share his knowledge and time, for allowing me the use of the facilities at the CIR, for his constant encouragement and support.
- Mr Jevon Davies for his willingness to share his broad knowledge of ultrasonics and piezoelectricity, for his endless enthusiasm, positive attitude and cheer.
- Mr Victor Balden for his extensive help with Abaqus, for his dedication to providing help whenever he could, for his constant reassurance and motivation.
- All those at Engineering Acoustics Inc. who were always willing to help in way that they could, for their patience while I wrote up the thesis. Special thanks go to Mr Tom Ensign who patiently and unselfishly shared his vast knowledge and experience with me, on all subjects to do with ultrasound and sonar.
Terms of Reference

This thesis report fulfils the requirements for the degree of MSc in Electrical Engineering. The thesis was supervised and commissioned by Dr. J Tapson at the University of Cape Town on the 25th February 2000. My specific instructions for the project were:

- To investigate the background and development of equivalent circuit design methods.
- To use equivalent circuit methods and Finite Element Method (FEM) to design and optimise three types of high-power ultrasonic transducers.
- To construct working models of the optimised transducers and compare the results with those predicted by the equivalent circuit models and FEM.
- To examine the effect of practical assembly and construction of the transducer on performance.
- To use the transducers to investigate some of the effects of high power ultrasound.
Synopsis

The purpose of this thesis is to investigate the design and construction of high power ultrasound transducers as well as some of the effects of high power ultrasound. The two design approaches that are researched and applied are the equivalent circuit method and Finite Element Method (FEM). These methods are used in conjunction with each other to design three different transducers, which are then built and tested at low and high powers.

Chapter one gives a brief introduction to ultrasound; its background, production, uses and some of the effects associated with high power ultrasound, such as cavitation and acoustic streaming. The motivation for the project is identified as the need for a reliable and effective design method for high power ultrasonic transducers, incorporating FEM (specifically using the package Abaqus). The specific objectives of the thesis are laid out, followed by a brief outline of the thesis.

Chapter two begins by giving a more detailed look at the background, description and development of the piezoelectric sandwich transducer, the governing piezoelectric equations and the equivalent circuit design approach. The next section of Chapter two deals with the derivation of various equivalent circuits; first for a stack of piezoelectric rings, then for composite transducers. The derivation primarily follows work from Mason[19] and Woollett[30]. The outcomes from this derivation are three design methods of varying complexity, namely the frequency equation, the lumped element equivalent circuit and the distributed parameter equivalent circuit. There follows a discussion on velocity transforming sandwich transducers and how they can be incorporated into the various design methods. Finally, a brief description of some useful parameters and their significance for sandwich transducers is given.

Chapter three deals with the finite element method in more detail. Firstly, a general introduction to the method is given. This deals with some of the history behind the method and shows how Abaqus actually performs the calculations using a simple example. A brief description of how to model a piezoelectric disc is then outlined, giving the basic steps required to run a simulation successfully. The simulation results for the disc are then compared to those measured on the Network Analyser with
accuracy of 97% and above being achieved. A full 3-D model of the disc is also tested, with the conclusion that the slight increase in accuracy does not justify the extra computing time and memory required. An attempt to model the disc more exactly (in terms of magnitude and shape of response curves) is then made using various damping and driving methods. Although some improvements are made, it is discovered that the numerical results from Abaqus are purely imaginary due to the lack of losses in the model, thus making it impossible to model the disc exactly.

In Chapter four, the design process for the nodal-mounted transducer is dealt with. A general overview of the design process is given, followed by the design specifications for the transducer. The transducer is then designed using the frequency equation, a lumped element equivalent circuit, a distributed parameter equivalent circuit and finally FEM. Emphasis is placed on achieving the correct resonant frequency with the minimum amount of flexure in the head of the transducer. Both of these objectives are achieved with the appropriate results from each of the methods matching well. Finally, the transducer is modelled radiating into water and shows good coupling as the main piston mode is damped significantly.

The design of the face-mounted transducer begins with a short description of the system, followed by the design specifications for the transducer. The same design method as was used for the nodal-mounted transducer is applied to this transducer. There are however some significant differences that should be noted. The first is the modification of the lumped element circuit to include the compliance of the head and tail due to their extra length. The second is the introduction of asymmetry in the transducer to induce greater movement in the head. Both of these changes improve the results as intended, and the matching between appropriate modelling results is again good. The results for the transducer in water also show a well-coupled system.

Chapter six has to do with the design of the horn transducer. Once the specifications for the transducer have been given, the same design method is again followed. The design process for this horn driver is generally simpler than the other transducers due to the cylindrical shape of the head and tail. The horn adds complexity to the system and is designed using a 1-D equation, distributed parameter analysis and mostly FEM. The objective of increasing displacement of the transducer through the horn is clearly
met. Once again, the comparison between appropriate models shows good agreement. The damping of the piston mode response clearly shows good coupling between transducer and load, and also demonstrates the lack of damping to other significant modes.

The important issue of transducer construction is the topic of Chapter seven. In this chapter, emphasis is placed on the careful checking of the various characteristics of all the parts that make up the transducer and their precise assembly. Pre-stressing of the transducers and some of the theory of the method is discussed in detail. Some results showing the increase in resonant frequency of the transducers with pre-stress are shown. An example of the effects of poor construction is given in the form of results and photographs from the second nodal-mounted transducer. Epoxying of the transducers is discussed and performed on one transducer, with the results showing little difference between the glued and dry transducers. However, these results were only taken at low power. Finally, transducer specific issues to do with construction are noted.

In Chapter 8, a comparison of the predicted and measured results is done. The difference in performance between the first and second attempts of the nodal-mounted transducers is shown. The second attempt out-performs the first by a significant margin, especially shown by the damping of the piston mode in water. All of predicted and measured results are then shown in tabular form, with the final FEM result being approximately 99% accurate. The measured results from the face transducers are then shown, highlighting the differences between the tapered and flat tails and the single and double transducer systems. The effects of standing waves and frequency pulling are briefly discussed before showing the table of results for the face-mounted transducer. An accuracy of above 97% is achieved with the final FEM and measures results. Finally, the measured results for the horn transducer are shown, noting the effect of changing the bolt material from titanium to high-tensile steel. The table of results summarises the findings for the horn transducer, with an accuracy of 98% between final predicted and measured results.

Chapter nine investigates some of the effects of driving the transducers at high power. After describing the set-up and experimental procedure, the results for the tests are
given. They show the expected changes, namely the shift downward in the resonant frequency and the decrease in “Q” and magnitude of admittance of the transducers. These changes are explained in work done by Berlincourt. [64] The hysteresis phenomenon due to the persistence of cavitation bubbles is then demonstrated.

The efficiencies of the nodal and horn transducers are measured calorimetrically in Chapter ten. The experimental set-up and method is described and the results are given for various levels of power. The two nodal transducers operate at efficiencies of about 75% and lower powers, and about 60% at higher powers. The horn transducer was measured twice over the period of a few weeks, with very different results. Initial tests show efficiencies of 50% and 44% for lower and higher powers, and the later results show 35% and 27%. The difference in these results could be due to error or catastrophic degradation of the sub-standard PZT discs from over-exposure to high temperatures. These results agree with Berlincourt’s [64] theory that increasing power increases the losses in the ceramic, lowering the overall electroacoustic efficiency of the transducers.

Chapter eleven first gives a brief summary of what has been done in this project. The design methods are then discussed, looking at the strengths and weaknesses of each. One of the main conclusions is that FEM can be used as a highly effective and accurate design method. This is as long as it is used in conjunction with the appropriate 1-D models, or with enough insight into the design process that the models are no longer required. Also, some of the construction and experimental methods are discussed. Finally, some recommendations for future work are made.
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Chapter 1: Introduction

1.1 Background

Ultrasound transducers have been used to produce ultrasound since 1917, when Paul Langevin \textsuperscript{[1]} developed the first piezoelectric sandwich transducer. Since then, there have been a vast number of developments in the fields of material, mechanical and electrical engineering that have all contributed to the ongoing development of ultrasonics. Thanks to these advances in technology, we now have a multitude of materials and design approaches to choose from. These methods range from using initial mathematical formulae and trial and error, right up to present day, where high-powered computer design packages are used. This project makes use of one of these methods, namely Finite Element Method (FEM), using a package called ABAQUS by Hibbitt, Karlsson & Sorenson, Inc. \textsuperscript{[2]}

Initially, one of the primary applications of ultrasound was in sonar. The ultrasonic industry has grown substantially since then, and applications now include such diverse fields as Non-Destructive Testing (NDT), medical diagnostics, atomisation of liquids, cleaning, welding (metals and plastics) and food and liquid processing. As an example of the growth of the industry, it is interesting to note that by 1979, ultrasonic cleaning and welding were both $50 million industries. \textsuperscript{[3]}

1.2 Production of high power ultrasound

There are two main methods of producing high power ultrasound, namely using magnetostrictive and piezoceramic transducers. Magnetostrictive transducers are based on the effect of magnetostriction: a magnetic field induces strains in electromechanically active substance (such as ferromagnetics or ferrites), resulting in changes in the shape and dimensions of the substance. \textsuperscript{[4]} However, this type of transducer tends to have a relatively low electro acoustic efficiency, \textsuperscript{[4]} hence the increasing popularity of the piezoceramic transducer. This type of transducer employs the piezoelectric effect to produce the necessary change in shape and dimensions. The “direct” piezoelectric effect is produced by inducing a mechanical stress in a piezoelectric substance, resulting in the electrical polarization of the material. \textsuperscript{[4]}
Conversely, the "indirect" effect produces strain in the material due to an induced electrical field. [4]

The emphasis of this thesis will be on piezoelectric transducers. There are basically three types of piezoelectric transducers used in the production of high power ultrasound. These are the tonpilz, horn and sonotrode designs. [5] The type of design depends on the intended application: the tonpilz is used for low intensity, high volume applications; the horn for high intensity, low volume applications; and the sonotrode for high intensity, high volume applications. All three types of transducer normally operate at about 20 - 40kHz, although there are some designs that operate at up to 800kHz. The three types of transducer are shown in Figure 1 below. This thesis will concentrate on the design of tonpilz and horn transducers designed to operate at close to 20kHz.

![Diagram showing the three types of high power transducer](image)

Figure 1: Diagram showing the three types of high power transducer, (a) tonpilz, (b) horn, (c) sonotrode. [5]

1.3 Effects of high power ultrasound

Some of the physical effects produced by high intensity ultrasound include cavitation, acoustic streaming, sonochemical reactions and sonoluminescence. Cavitation can be defined as the formation of voids or cavities in a liquid under the influence of a negative pressure. [6] It is the expansion or rarefaction phase of an acoustic wave that overcomes the tensile strength of the liquid. The pressure at which this process takes
place will depend entirely on the characteristics of the liquid and it’s environment. Cavitation is a nucleated process, so the presence of impurities or small bubbles will dramatically decrease the cavitation threshold. Once the cavities or voids have been created, they normally grow, pulsate then collapse. The behaviour of the cavities is also determined by the properties of the liquid, as well as those of the ultrasonic field. When the cavities collapse, it is speculated that vast amounts of energy are released in the form of very high temperatures and pressures. If the cavities collapse near a solid, they tend to implode asymmetrically. This process is due to the formation of a jet of water through the bubble towards the solid. This jet of water can reach considerable velocities, capable of doing extensive damage to the solid surface. Cavitation is used extensively in the ultrasonic cleaning industry. However, hydrodynamic cavitation causes unwanted damage in ships propellers, pumps and turbines. Other side effects of acoustic cavitation include sonoluminescence and sonochemical reactions.

Another of the effects of high intensity ultrasound is acoustic streaming. This effect creates a flow in the medium (gaseous or liquid) due to the high intensities of energy produced near the transducer, and due to the energy loss in the sound waves. The type of streaming observed depends on the properties of the medium, the driving frequency and on the shape and structure of its environment.

1.4 Motivation

Our research group at the Centre for Instrumentation Research has for several years been involved in the development of high power ultrasonic technology. In particular, applications involving liquid processing have been investigated. Previous and current work involves projects such as food processing, amplifier design and frequency and power control systems.

A need was identified for the development of a transducer design process. In particular, it was felt that the Abaqus FEM package could be adapted for numerical design work. This was completed in part during an undergraduate research project. However, large differences between experimental and FEM results were observed,
especially at power. It was therefore decided to investigate this further as an MSc project.

1.5 Objectives

The specific objectives of this thesis are the following:

- To test Finite Element Modelling as a design method, and prove that it is a reliable and highly effective method of design.
- To use FEM to design three different types of high power ultrasonic transducers, namely the nodal-mounted transducer, the face-mounted transducer and a high-intensity horn transducer.
- To construct the transducers and test them against the predicted results from Abaqus.
- To examine the effect of practical construction and assembly of the transducer on performance.
- To use the transducers to investigate the effects of high intensity ultrasound, and to test various driving circuits.
- To make conclusions and recommendations from the results of the tests, aiding future designers in the optimum design of high-power ultrasonic transducers.

1.6 Outline of thesis

The outline of this thesis will be as follows: in the next chapter, previous work done in the field of piezoelectric sandwich transducer design will be discussed in detail. This is followed in Chapter 3 by a general introduction to Finite Element Analysis, and its application to piezoelectric disc simulation. Chapters 4, 5 and 6 will deal with the actual design of the three different types of transducer, from initial design through to final design drawings. In Chapter 7, the construction of the transducers will be dealt with thoroughly, as this plays an important part in the quality of the final transducer. Experimentation and testing will form the bulk of Chapters 8, 9 and 10 with the results and discussion of those results being included. Finally, conclusions and recommendations for further work will be made in Chapter 11.
Chapter 2: Piezoelectric Sandwich Transducers and Equivalent Circuit Theory

2.1 Background of Piezoelectric Sandwich Transducers

The idea of the sandwich transducer was first used in 1917.\(^1\) It was optimised many years later and continues to form the basis for the majority of high power ultrasound transducer design. Langevin originally used the idea to create a low-frequency resonator out of thin sheets of quartz, the only available active substance at the time. This type of transducer was used mainly in underwater signalling applications, and it was not until the 1960’s, when piezoceramics first came into use, that H. B. Miller\(^9,10\) first used it as a high power emitter type device.\(^11\) The basic design of a pre-stressed sandwich transducer is shown below.

![Diagram of sandwich transducer](image)

**Figure 2:** The basic design of a pre-stressed sandwich transducer as per Neppiras\(^11\)

Figure 1b shows the distribution of stress and velocity along the length of the transducer. This shows that the maximum velocity and hence maximum displacement

---

\(^1\) Basic reference for sandwich transducer.
occurs at the faces of the transducer. This design had numerous advantages compared to its predecessors, namely:

- Only thin discs of piezoceramic are required, as the metal end pieces bring the resonant frequency of the transducer down, thus keeping material costs low.
- As the discs are thin, they have low electrical impedance.
- The pre-stress from the bolt increases the mechanical strength of the structure, as piezoceramics have low tensile strength. It also improves mechanical contact between parts, leading to improved acoustic coupling.
- The metal end pieces act as good heat sinks, thus allowing the transducer to be driven at higher levels without overheating the discs beyond their Curie temperature.
- The design lends itself to the addition of other components, such as boosters or horns, using stubs or bolts to form a reliable joint.\textsuperscript{[11]}

Although the pre-stressed sandwich design was very popular at the time, some designers were not entirely impressed with its performance, and modified the design. Two British patents, the first relating to the use of several symmetrically placed discs\textsuperscript{[12]}, the second to using several peripheral bolts instead of one centre bolt\textsuperscript{[13]}, were filed in 1961 and 1964 respectively. The design using peripheral bolts experienced problems with multiple modes. Also, N. Maropis\textsuperscript{[14]} designed a transducer that used an external threaded shell to pre-stress the system, which he called the “tension shell” transducer. However, it would appear that the original pre-stressed sandwich design is still the most popular and widely used configuration today.

2.2 The Governing Piezoelectric Equations.

Before we get into the design process for the piezoelectric sandwich transducer, we should fully understand the governing piezoelectric equations or “equations of state.”\textsuperscript{[30]} These are a set of linear equations that relate the equations of elasticity to the charge equations of piezoelectricity by means of the piezoelectric constants. The fact that the equations are linear implies that the elastic, piezoelectric and dielectric coefficients are treated as constants independent of the magnitude and frequency of applied mechanical stresses and electric fields.\textsuperscript{[29]} Work by Poincare,\textsuperscript{[73]} Wieg\textsuperscript{[74]} and Voigt\textsuperscript{[33]} led to the development of the governing equations in their current form. The derivation of the equations uses the conservation of energy and thermodynamics.
The most common form of these equations, where electric field and stress are chosen as the independent variables, is the following \[^{[31]}\]:

\[
S = s^T T + d^T E \\
D = d T + e^T E
\]

The symbols used denote the following:

- \( S \) = Strain
- \( T \) = Stress
- \( D \) = Charge Density
- \( E \) = Electric Field
- \( s^T \) = Elastic compliance
- \( e^T \) = Dielectric constant
- \( d_t \) = Piezoelectric constant (\( t = \text{transpose} \))

In the above equations, the superscripts denote which variable was held constant while determining the elastic, piezoelectric or dielectric constants. The piezoelectric constants relate the stress or strain tensors to the charge density or electric field vectors and are therefore third-order tensors. The elastic constants relate two second-order symmetric tensors and are therefore fourth-order tensors. The dielectric constants relate two vectors and are therefore second-order tensors. \[^{[31]}\] The alternative forms of the governing equations and the relationships between the various piezoelectric constants can be seen in Appendix A-1.

2.3 Development of Equivalent Circuit Method

The principal method for the design and analysis of piezoelectric transducers is the equivalent circuit method. Mason did much of the original work in his earlier papers \[^{[34, 35, 36, 37]}\] and later in his book "Electromechanical Transducers and Wave Filters"\[^{[19]}\] and his equivalent circuit model still forms the basis of many of today's models. An earlier model for a mechanically vibrating system valid around a single resonance was developed by Butterworth \[^{[38]}\] in 1914. Work by Cady \[^{[39]}\] and Van Dyke \[^{[40]}\] led to the development of the familiar Butterworth-Van Dyke circuit, which is shown on the following page. Another popular form of equivalent circuit is the "KLM" model, after Krimholtz, Leedom and Matthaei, in which a Transmission Line (TL) schematic is used. Ballato \[^{[41]}\] gives a very good summary of this method and its advantages and disadvantages. We will concentrate primarily on the Mason equivalent model.
Figure 3: BVD one-port resonator equivalent circuit, valid in the vicinity of a single resonance.

The equivalent circuit method is a very powerful design and analysis tool for a variety of reasons. One of the reasons is that the physical interactions of the transducer are quite accurately mirrored in the circuit, which is also laid out in such a way as to correspond to the geometry of the transducer. This makes it easier to visualize the interplay of the waves in the structure simply by inspecting the circuit. In fact, the circuit contains no more information than is supplied by the governing equations, but it puts it across in a succinct and readily interpretable form. We will be using the impedance form of the equivalent circuit. Although the mobility equivalent has a more direct topological correspondence with the mechanical system,\(^\text{[30, 31]}\) we will be doing all our analysis in electrical terms, so the impedance analogy is preferred. The equivalent circuit can be analysed using the powerful and highly developed concept of network analysis to obtain the measures of performance required.\(^\text{[41]}\) Berlincourt et al.,\(^\text{[31]}\) Woollett,\(^\text{[30]}\) and Ballato\(^\text{[41]}\) all give very good accounts of the development of the equivalent circuit theory.

2.4 Derivation of equivalent circuit for stack of piezoelectric rings.

In order to derive the equivalent circuit of a piezoelectric transducer, there are three equations required; the most suitable form of the two governing piezoelectric equations (dependent on the type of transducer and boundary conditions) and the equation of motion for the transducer (specific solution to the wave equation). By applying the relevant boundary conditions at the active (loaded) faces and integrating them across the transducer, we get three coupled equations describing the system.
These equations are in terms of three dependant and three independent variables, representing a network consisting of one pair of electrical terminals and two pairs of mechanical terminals. By introducing an ideal electromechanical transformer, we can achieve an equivalent circuit whose electrical parameters are represented by the coefficients of the terms in the equations. All of the above work deals primarily with a single electromechanical segment. However, many of today’s transducers are composed of multiple identical segments, often arranged in the form of a stack. The derivation of the equivalent circuit for segmented electromechanical systems was originally done by Martin. This derivation follows the same basic method as is outlined above, with some slightly different boundary conditions and assumptions. We will now go through the method outlined above in more detail, basically adapting Woollett’s “Segmented bar, axial field” derivation to suit a stack of piezoelectric rings.

2.4.1 Description of the system

\[ \xi = \text{displacement in } x \text{ direction}, \quad f = \text{force in } x \text{ direction}, \quad V = \frac{\partial \xi}{\partial t} = \text{velocity}. \]

Figure 4: Stack of piezoelectric rings
The primary assumption for this derivation is that \( \frac{\ell}{r} \ll \lambda \) where \( r \) is no. of rings and \( \lambda \) = wavelength. This holds true for our case, as we will be dealing with frequencies around 20 kHz.

### 2.4.2 Equations of State.

For the above system, we will make the assumption that \( \frac{\partial E_3}{\partial x_3} = 0 \). We can justify this statement by noticing that the electrodes allow the charge flow necessary to maintain \( E_2 \) constant. So although there will be slight variation in each segment, we are assuming the volume of ceramic to be equipotential and the voltage in each segment to be equal to the average value of \( E_3 = [E_3]_{\text{ave}} = \int_0^l \frac{\varepsilon}{r} \). As we have made the assumption that \( \frac{\partial E_3}{\partial x_3} = 0 \), we need to choose equations of state in which \( E_3 \) is the independent variable in order to simplify the wave equation:

\[
S_3 = s_{33}^e T_3 + d_{33} E_3
\]

\[
D_3 = d_{33} T_3 + \varepsilon_{33}^T E_3
\]

Noting that \( \varepsilon_{33}^T - \frac{d_{33}^2}{s_{33}^e} = \varepsilon_{33}^s \) and that \( \varepsilon_{33}^s = \varepsilon_{33}^T (1 - k_{33}^2) \), we can convert equations 3 and 4 to:

\[
T_3 = \frac{d_{33}}{s_{33}^e} E_3 + \frac{1}{s_{33}^e} S_3
\]

\[
D_3 = \left( \varepsilon_{33}^T - \frac{d_{33}^2}{s_{33}^e} \right) E_3 + \frac{d_{33}}{s_{33}^e} S_3
\]

### 2.4.3 Wave Equation

In order to derive the wave equation for the piezoceramic, we follow the basic method outlined by Kinsler et al. for the longitudinal wave equation in a bar. For this derivation, only forces in the \( x_3 \) direction will be considered.
Let us consider a small volume element of ceramic, with density \(\rho\), mass \(\rho A_2 dx_3\), and force on the element \(A_2 \frac{\partial T_1}{\partial x_3} dx_3\). Substituting these values into Newton’s Law (\(F = ma\)), we get:

\[
\rho A_2 dx_3 \frac{\partial^2 \xi}{\partial t^2} = A_2 dx_3 \frac{\partial T_1}{\partial x_3}
\]

\[
\Rightarrow \rho \frac{\partial^2 \xi}{\partial t^2} = \frac{\partial T_1}{\partial x_3}
\] \[\text{[7]}\]

Substituting \(T_3[5]\) into [7]:

\[
\rho \frac{\partial^2 \xi}{\partial t^2} = -d_{33} \frac{\partial E_3}{\partial x_3} + \frac{1}{s_{33}} \frac{\partial S_3}{\partial x_3}
\]

Defining strain as \(S_3 = \frac{\partial \xi}{\partial x_3}\):

\[
\Rightarrow \rho \frac{\partial^2 \xi}{\partial t^2} - \frac{1}{s_{33}} \frac{\partial^2 \xi}{\partial x_3^2} + \frac{d_{33}}{s_{33}} \frac{\partial E_3}{\partial x_3} = 0
\]

Initially we assumed that \(\frac{\partial E_3}{\partial x_3} = 0\):

\[
\therefore \frac{\partial^2 \xi}{\partial x_3^2} = \left( e_3^e \right)^2 \frac{\partial^2 \xi}{\partial x_3^2} = 0
\] \[\text{[8]}\]

where \(c_3 = \sqrt{\frac{1}{\rho s_{33}}} = \sqrt{\frac{Y}{\rho}}\) \((Y = \frac{1}{s_{33}} = \text{Young’s Modulus})\). This is the common form of the wave equation.

2.4.4 Travelling wave Solution

Since we will be interested in the response of the stack to harmonic excitation, we should consider travelling waves going in both directions. The solution:

\[
\xi(x_3,t) = \xi_+ e^{(-jkx_3 + j\omega t)} + \xi_- e^{(jkx_3 + j\omega t)}
\] \[\text{[9]}\]

represents one sine wave going in the positive \(x_3\) direction (1st term) and another going in the negative \(x_3\) direction (2nd term). \(k = \frac{\omega}{c_3} = \frac{2\pi}{\lambda}\) is the wave number. By substituting this equation, we can get the other variables in terms of travelling waves. These equations relate the displacement of the piezoceramic (related to time and position) to voltage, stress etc. In equation [10], \(V\) is velocity.
\[ V(x, t) = \frac{\partial \xi}{\partial t} = j\omega [\xi_0 e^{-j\omega t} + \xi_- e^{j\omega t}] e^{j\omega t} \] [10]

\[ S_3(x, t) = \frac{\partial \xi}{\partial x_3} = jk [\xi_0 e^{-j\omega t} + \xi_- e^{j\omega t}] e^{j\omega t} \] [11]

\[ T_3(x, t) = -\frac{d_{33}}{s_{33}} E_3 + \frac{1}{s_{33}} S_3 = -\frac{d_{33}}{s_{33}} E_3 + \frac{jk}{s_{33}} [-\xi_0 e^{-j\omega t} + \xi_- e^{j\omega t}] e^{j\omega t} \] [12]

\[ D_3(x, t) = \varepsilon_{33} E_3 + \frac{d_{33}}{s_{33}} S_3 = \varepsilon_{33} E_3 + \frac{jkd_{33}}{s_{33}} [-\xi_0 e^{-j\omega t} + \xi_- e^{j\omega t}] e^{j\omega t} \] [13]

### 2.4.5 Three port transducer equations

Using transmission line theory, a three-port network as shown below can represent the two pairs of mechanical variables and pair of electrical variables.

![Three port transducer network](image)

**Figure 5: Three port transducer network**

We can now express the magnitude of the travelling waves \( \xi_0 \) and \( \xi_- \) in terms of \( V_0 \) and \( V_t \) at the ends of the stack (\( F \)=force). The following boundary conditions apply:

at \( x = 0 \), \( \nu(0, t) = V_0 e^{j\omega t} \) where \( V_0 = j\omega [\xi_0 + \xi_-] \) [14]

at \( x = \ell \), \( \nu(\ell, t) = V_t e^{j\omega t} \) where \( V_t = j\omega [\xi_0 e^{-j\omega \ell} + \xi_- e^{j\omega \ell}] \) [15]

\[ \xi_0 : V_t - V_0 e^{j\omega t} = j\omega \xi_0 e^{-j\omega \ell} - e^{j\omega t} \]

\[ \Rightarrow \xi_0 = \frac{V_t - V_0 e^{j\omega t}}{2\omega \sin k\ell} \] [16]

\[ \xi_- : V_t - V_0 e^{-j\omega t} = j\omega \xi_- e^{j\omega \ell} - e^{-j\omega t} \]

\[ \Rightarrow \xi_- = \frac{V_t - V_0 e^{-j\omega t}}{2\omega \sin k\ell} \] [17]
The next set of boundary conditions assumes that only the ends of the stack are subjected to loads (compressive) and that there are no shear forces applied. Therefore:

\[
\text{at } x_3 = 0, \quad T_3 = -\frac{f_o}{A_{21}} = -\frac{F_0 e^{j\omega \ell}}{A_{21}}
\]

\[
\text{at } x_3 = \ell, \quad T_3 = -\frac{f_\ell}{A_{21}} = -\frac{F_\ell e^{j\omega \ell}}{A_{21}}
\]

Substituting the expressions for \( \xi \) and \( \zeta \) into the stress equation [12] and applying the above boundary conditions results in the following:

\[
\text{at } x_3 = 0, \quad -\frac{F_0}{A_{21}} = -\frac{d_{33}}{s_{33}} \hat{E}_3 + \frac{jk}{s_{33}} \left[ \frac{-V_\ell - V_0 e^{-j\omega \ell}}{2 \sin k\ell} - \frac{V_0 V_\ell e^{-j\omega \ell}}{2 \sin k\ell} \right]
\]

\[= -\frac{d_{33}}{s_{33}} \hat{E}_3 + j\rho c_3 \left[ \frac{-V_\ell}{\sin k\ell} + \frac{V_0}{\sin k\ell} \right] \quad \text{[18]} \]

\[
\text{at } x_3 = \ell, \quad -\frac{F_\ell}{A_{21}} = -\frac{d_{33}}{s_{33}} \hat{E}_3 + \frac{jk}{s_{33}} \left[ \frac{-V_\ell e^{-j\omega \ell} - V_0}{2 \sin k\ell} - \frac{V_0 e^{-j\omega \ell} - V_\ell}{2 \sin k\ell} \right]
\]

\[= -\frac{d_{33}}{s_{33}} \hat{E}_3 + j\rho c_3 \left[ \frac{V_0}{\sin k\ell} - \frac{V_\ell}{\tan k\ell} \right] \quad \text{[19]} \]

where \( \hat{E}_3 \) is a complex amplitude \( (E_3 = \hat{E}_3 e^{j\omega \ell}) \).

We now need to find an expression relating the electrical current to the other piezo variables. We start by defining charge as:

\[
q = \sum_{i=1}^{n} A_{21} D_3^{(i)}
\]

where \( D_3^{(i)} \) is the charge density in the \( i \)th segment. As \( D_3 \) is independent of \( x_3 \) within each segment but varies from segment to segment, we will take the average of \( D_3 \) over the whole stack, \([D_3]_{\text{ave}}\):

\[
\Rightarrow q = r A_{21} [D_3]_{\text{ave}}
\]

Substituting the expression for \( D_3 \), we get that:

\[
[D_3]_{\text{ave}} = \frac{1}{\ell} \int_0^\ell D_3 dx_3 = e_{33} \frac{1}{\ell} \int_0^\ell E_3 dx_3 + \frac{d_{33}}{s_{33}} \frac{1}{\ell} \int_0^\ell S_3 dx_3 \quad \text{[20]}
\]
But we know that: \[
\frac{1}{r} \int_{r_0}^{r} E_3 \, dx_3 = \frac{e}{r} \quad \text{and} \quad \frac{1}{r} \int_{r_0}^{r} S_3 \, dx_3 = \frac{1}{r} \int_{r_0}^{r} \frac{\partial E}{\partial x_3} \, dx_3 = \frac{1}{r} (\xi_r - \xi_{r_0})
\]

\[\therefore \quad q = \frac{r^2 A_{12}^3 e^{33}}{\xi_0} \, E + \frac{r A_{12}^3 d_{33}}{\xi^3_{33}} (\xi_r - \xi_{r_0})\]

For harmonic excitation, the current:

\[I = \frac{j \omega r^2 A_{12}^3 e^{33}}{\xi_0} \, E + \frac{r A_{12}^3 d_{33}}{\xi^3_{33}} (V_r - V_0)\]  \[\text{[21]}\]

where \( q = Q e^{j \omega t} \) and \( I = j \omega Q \), and \( E = \frac{\ell}{r} \hat{E}_3 \) (E is the electric field).

We finally obtain the transducer equations by using the equations for \( F_0 \), \( F_t \) and \( Q \) and the above relationships:

\[F_0 = \frac{d_{33} A_{12}^3 r}{\xi^3_{33}} \, E - \frac{j \omega r^2 A_{12}^3 e^{33}}{\tan k \ell} V_0 + \frac{j \omega r^2 A_{12}^3 e^{33}}{\sin k \ell} V_r\]  \[\text{[22]}\]

\[F_t = \frac{d_{33} A_{12}^3 r}{\xi^3_{33}} \, E - \frac{j \omega r^2 A_{12}^3 e^{33}}{\sin k \ell} V_0 + \frac{j \omega r^2 A_{12}^3 e^{33}}{\tan k \ell} V_t\]  \[\text{[23]}\]

\[I = \frac{j \omega r^2 A_{12}^3 e^{33}}{\xi_0} \, E - \frac{d_{33} A_{12}^3 r}{\xi^3_{33}} \, V_0 + \frac{d_{33} A_{12}^3 r}{\xi^3_{33}} \, V_t\]  \[\text{[24]}\]

2.4.6 Application to Equivalent Circuit

We now have equations that relate the voltages and currents impressed on the stack to the corresponding forces and velocities at the ends of the stack. These equations can be represented by an equivalent circuit of the following form:  \[\text{[30]}\]

---

Figure 6: Form of equivalent circuit to represent transducer equations.
Circuit analysis of the above equivalent form gives the following network equations:

\[ F_1 = Z_{11}V_1 + Z_{12}V_2 + NE \]  
\[ F_2 = Z_{12}V_1 + Z_{22}V_2 + NE \]  
\[ I = -NV_1 - NV_2 + Y_s E \]

Comparing equations 22, 23 and 24 with 25, 26 and 27 respectively, we observe that:

\[ Z_{11} = \frac{-jZ_0}{\tan k\ell} = jZ_0 \left( \frac{1 - \cos k\ell}{\sin k\ell} - \frac{1}{\sin k\ell} \right) \]

\[ = jZ_0 \tan \frac{k\ell}{2} - \frac{jZ_0}{\sin k\ell} = Z_1 + Z_{12} \]

\[ Z_{22} = \frac{-jZ_0}{\tan k\ell} = Z_{11} = Z_2 + Z_{12} \]

\[ Z_{12} = \frac{jZ_0}{\sin k\ell} \quad Z_1 = Z_2 = jZ_0 \tan \frac{k\ell}{2} \]

where transformation ratio \( N = \frac{d_3 A_{21} r}{s_{33}^r \ell}, Y_b = j\omega s_{33}^r A_{21} r^2 \Rightarrow C_b = \frac{\varepsilon_s s_{33}^r A_{21} r^2}{\ell} \) (\( C_b \) = blocked capacitance), and characteristic acoustic impedance \( Z_0 = \rho c_3 A_{21} \).

We can now substitute the above values into the circuit and get Mason’s standard equivalent circuit:

\[ k = \frac{\omega}{c_s}, \quad c_e = \sqrt{\frac{1}{\rho s_{33}^e}} \] (\( c_e \) = speed of sound in stack).

Figure 7: Equivalent circuit for stack of piezoelectric rings.
2.5 Stack loaded on one end only.

Many transducers are designed to radiate from one face only, i.e. only one face is loaded. We can therefore modify the equivalent circuit as is shown below:

![Equivalent circuit for stack loaded at one end.](image)

**Figure 8: Equivalent circuit for stack loaded at one end.**

This circuit can be simplified using Norton’s equivalent network theory that represents a T-network as a transformer coupled network. This represents a more intuitive view of the physical operation:

![Norton’s equivalent network theory.](image)

**Figure 9: Norton’s equivalent network theory.**

If we apply the above theory to our circuit, the following relationships apply:

\[
Z_b = Z_1 + Z_2 = j2Z_0 \tan \frac{k\ell}{2}
\]

\[
Z_a = \frac{Z_1}{Z_2} (Z_1 + Z_2) = j2Z_0 \tan \frac{k\ell}{2}
\]

\[
\phi = \frac{Z_1 + Z_2}{Z_2} = 2 \quad Z_d = \phi^2 Z_{12} + Z_a
\]

\[
\Rightarrow Z_d = -j4Z_0 \tan \frac{k\ell}{2} = j2Z_0 \left[ -\frac{2}{\sin \frac{k\ell}{2} \cos \frac{k\ell}{2}} + \frac{\sin \frac{k\ell}{2}}{\cos \frac{k\ell}{2}} \right] = -j2Z_0 \cot \frac{k\ell}{2}
\]

So substituting the values for \(Z_b\) and \(Z_d\), we get the final equivalent circuit for the stack loaded at one end. This is shown in Figure 10 on the following page.
2.6 Lumped parameter approximation

Most transducers are designed to operate in a narrow band of frequency around the resonant frequency. This means that we can approximate the transducer by using a lumped parameter circuit that is only valid around resonance.

If we consider the graph of $-\cot \frac{k\ell}{2}$, we see that it passes through zero at the points where $\omega_r = n \frac{\pi c}{\ell}$, mode number $n = 1, 3, 5...$ and $\ell = \frac{n\lambda}{2}$. Therefore, in order to approximate the transducer at resonance, we need a circuit that also has zeros at the $\omega_r$ frequencies. The ideal circuit here is the series resonant (LC) circuit, as can be seen below, where the dotted line represents the response of the LC circuit and the solid line $-\cot \frac{k\ell}{2}$.

Figure 11: Approximation of cotangent function by resonant LC circuit as per Mason\textsuperscript{[19]}
Let us assume that \( Z_1 \) is the series impedance and that \( Z_2 \) is the parallel impedance. Also, \( M_1 \) and \( C_1 \) are the equivalent mechanical components of the series LC circuit, with \( Z'_1 \) as the impedance. Then:

\[
Z_1 = -j2Z_0 \cot \frac{\omega \ell}{2c_3}; \quad \omega = \omega_r, \]

\[
Z'_1 = j\omega M_1 + \frac{1}{j\omega C_1} = jM_1 \left( \omega - \frac{\omega_r^2}{\omega} \right); \quad \omega_r = \sqrt{\frac{1}{M_1 C_1}}.
\]

As a second condition, we match the slope of the reactance at \( \omega_r \):

\[
\frac{dZ_1}{d\omega} = jZ_0^2 \csc^2 \frac{\omega \ell}{2c_3}; \quad \frac{dZ'_1}{d\omega} = jM_1 \left( 1 + \frac{\omega_r^2}{\omega^2} \right)
\]

at \( \omega = \omega_r \) \Rightarrow \[
\frac{dZ}{d\omega} = jZ_0^2 \csc \frac{\omega \ell}{c_3} = \frac{dZ'_1}{d\omega} = j2M_1
\]

From the above equations, we get that:

\[
M_1 = \frac{1}{2} \frac{Z_0^2 \ell}{c_3} = \frac{1}{2} \rho A_{21} \ell \quad \left( = \frac{1}{2} \text{ static mass of stack} \right) \quad [28]
\]

\[
C_1 = \frac{Z_0^2}{M_1 \omega_r^2} = \frac{2\ell}{n^2 \pi^2 \rho c_3^2 A_{21}} = \frac{2}{n^2 \pi^2} \left( \frac{\eta^2 \ell}{A_{21}} \right) \quad \left( = \frac{2}{n^2 \pi^2} \times \text{static compliance} \right) \quad [29]
\]

Using a similar method, we can find the values of the circuit components for \( Z_2 \), except that now we notice that the admittance of \( Z_2 \) has zeros at \( \omega_r \). We therefore use a parallel resonant circuit to approximate the admittance zero. It turns out that:

\[
Y_2 = \frac{1}{Z_2} = -j \frac{\cot \frac{\omega \ell}{2c_3}}{2Z_0} = \frac{Z_1}{4Z_0^2}
\]

\[
\therefore C_2 = \frac{M_1}{4Z_0^2} = \frac{\ell}{8Z_0 c_3} = \frac{\ell}{8 \rho c_3^2 A_{21}} \quad \left( = \frac{1}{8} \text{ static compliance} \right) \quad [30]
\]

\[
M_2 = 4Z_0^2 C_1 = \frac{8\ell Z_0}{n^2 \pi^2 c_3} = \frac{8}{n^2 \pi^2} \rho A_{21} \ell \quad \left( = \frac{8}{n^2 \pi^2} \times \text{static mass of stack} \right) \quad [31]
\]

Normally, we assume the mode number, \( n \), to be one. The final equivalent circuit is shown on the next page.
Figure 12: Lumped parameter approximation of stack loaded at one end.

If we consider the above circuit in the frequency range in which it is valid, we will notice that the impedance of the parallel circuit is very much larger than any practical mechanical loading. [31] It is for this reason that one will often see the lumped parameter equivalent circuit for a transducer showing only the series impedance.

In order to work out the values of the actual circuit components for this type of circuit, for example to use in a simulation, one needs to use a transformation factor. This essentially transforms all of the mechanical impedance elements through the electromechanical coupler so that we end up with a purely electrical equivalent circuit. Details of this method can be found in Hueter and Bolt. [24] This method was tested [18] and found to be accurate.

2.7 Equivalent circuit theory for composite transducers.

Although all of the above theory applies to the stack of ceramic rings, it can quite easily be adapted to suit other systems such as a single disc, bars or almost any other piezoceramic configuration. However, most transducers today are composite, such as the one in Figure 2. The equivalent circuit for the composite uses the same theory as above, but with the non-active parts (head, tail and bolt) being represented by passive T-circuits. [19] In order to put together the distributed parameter circuit for a composite transducer, one simply cascades the various T-sections together. Any analysis or simplification required can then be carried out. An example of a composite transducer equivalent circuit is shown on the next page, Figure 13. The subscripts c, t, h and b stand for ceramic, tail, head and bolt respectively and the dotted lines indicate the various sections of the transducer.
Figure 13: Distributed parameter circuit for composite transducer\textsuperscript{[444]}

In many cases, the lumped parameter circuit is perfectly adequate for design purposes. Although the distributed parameter circuit is an exact representation of the transducer, the increase in the accuracy of the results does not warrant the added computation time required. The lumped parameter equivalent circuit for the composite transducer is shown below.\textsuperscript{[444]}

Figure 14: Lumped-parameter circuit for composite transducer.

For “simple” composite transducers, Neppiras \textsuperscript{[111]} uses a very useful design equation, the frequency equation, first developed by Langevin.\textsuperscript{[52]} By “simple” here, we mean transducers that are usually symmetrical in terms of dimensions, the end masses are non-tapered cylindrical elements, are made of the same materials and have the same surface area. The basic pre-stressed sandwich transducer is designed to be a symmetric $\lambda/2$-extensional-mode system. This means that each $\lambda/4$ section can be
dealt with separately. In order to establish the exact dimensions of the transducer, the frequency-determining equations must be found. These equations relate the resonant frequency (ω) to the impedances, sound velocities and lengths of the elements.\textsuperscript{[14]} The equations are derived from the equivalent circuit for ceramic loaded at one end, including only the series impedance term and using transmission line theory. The derivation is shown in Appendix A-2. The equation comes out as follows:

\[
\tan \theta_c \tan \theta_h = \frac{Z_c}{Z_h} \tag{32}
\]

where \( \theta_c = k_c \ell_c \); \( \theta_h = k_h \ell_h \); \( Z_c = \rho_c c_c A_c \); \( Z_h = \rho_h c_h A_h \), and \( c \) is the extensional mode velocity of sound in the material. Equation [32] can now be used to design a transducer. Once the material properties, disc thickness and area and desired frequency are known, one can determine the lengths of the required end pieces. This is also a flexible design method, as many different shape and material combinations will give the same resonant frequency.\textsuperscript{[11]} This method of design has been very popular since its discovery, and probably still forms the basis of many transducer designs.\textsuperscript{[11,14,15,16,17]}

2.8 Velocity transforming sandwich transducers

It is often advantageous to make the end sections out of materials with widely different \( \rho c \)'s as this results in a velocity transformation. A common combination is steel and titanium, where the titanium would form the head and have a greater velocity than the steel due to its low acoustic impedance. Titanium also has the advantage of having very high strength and resistance to cavitation. The same design equations can be used for this type of transducer, as long as it is still symmetric about the ceramic. This practice is regularly used in liquid-load applications.

Another method of achieving velocity transformation is to increase the surface area of the work-face. This is usually done by making the head a truncated cone. The design equations for this type of transducer now become more complex, because of the velocity transformation in the cone. Increasing the surface area of the cone decreases the velocity at the face of the transducer. This is the typical form of the tonpilz (German for mushroom) transducer.
According to Stansfield,\textsuperscript{[22]} the acoustic impedance (viewed from $A_1$ face) of a truncated cone can be calculated using the following equation:

$$Z_{in} = R_{11} \left( \frac{R_{12} \sin k \ell + j R_r \frac{\sin k(\ell - \vartheta_2)}{\sin k \vartheta_2}}{\frac{R_r}{\sin k(\ell + \vartheta_1 - \vartheta_2)} - j \frac{R_{12}}{\sin k(\ell + \vartheta_1)} \frac{\sin k \vartheta_1}{\sin k \vartheta_2}} \right)$$ \textsuperscript{[33]}

where $R_{11} = \rho_i c_i A_1$ and $R_{12} = \rho_i c_i A_2$.

$A_1$, $A_2$ = surface areas of rear and front ends of cone.

$\rho_i c_i$ refer to material of the head.

$R_r = \rho c A_2$, the radiation resistance at the radiating surface.

$k \vartheta_1 = \tan^{-1} k \ell_1$

$k \vartheta_2 = \tan^{-1} k \ell_2$.

An alternative method is suggested by Woollett,\textsuperscript{[30]} in which he gives us the impedances of the T-section in the equivalent circuit. Woollett also gives some useful references on the subject as Morse,\textsuperscript{[46]} Mohammed\textsuperscript{[47]} and Eisner.\textsuperscript{[48]} So the T-section for the equivalent circuit of a non-piezoelectric bar\textsuperscript{[49]} is shown on the next page with the relevant equations.
Figure 16: Equivalent T-section for non-piezoelectric bar.

The following equations apply to the above circuit:

\[ F_1 = Z_{11}V_1 + Z_{12}V_2 \]
\[ F_2 = Z_{12}V_1 + Z_{22}V_2 \]

The equations relevant to a truncated cone are as follows:

\[ Z_{11} = -jZ_0 \sqrt{\frac{A_1}{A_2} \left( \cot k\ell + \frac{1}{kx_0} \right)} \]  \[ \text{[34]} \]

\[ Z_{22} = -jZ_0 \sqrt{\frac{A_2}{A_1} \left( \cot k\ell - \frac{1}{k(x_0 + \ell)} \right)} \]  \[ \text{[35]} \]

\[ Z_{12} = Z_{21} = -\frac{jZ_0}{\sin k\ell} \]  \[ \text{[36]} \]

Another common application of velocity transformation is to increase the velocity of the face. This is achieved by using a stepped or Mason horn as is shown below. \[ \text{[50]} \]

Figure 17: Mason or stepped horn.

For this type of horn, the dimensions \( \ell_1 \) and \( \ell_2 \) are usually one-quarter wavelength long as this leads to the maximum amplification. \[ \text{[51]} \] The impedance transformation
ratio is given by \( \left( \frac{A_2}{A_1} \right)^2 \) and the amplitude transformation ratio is given by \( \left( \frac{A_2}{A_1} \right) \). \[5.0\]

Increasing the ratio will obviously increase the displacement at the radiating face. However a compromise has to be reached, as increasing the ratio will also increase the level of stress at the transition section of the horn.

Other types of sandwich transducer can also be designed by using the frequency determining equations. One such type is the asymmetric transducer, which can be useful if the transducer has to be mounted on one of the end pieces with the minimum amount of damping. A diagram of two different asymmetric transducers and the relevant design equation is shown on the next page. This equation is derived in the same way as for a symmetrical transducer.

![Diagram of asymmetric sandwich transducers](image)

**Figure 18:** Asymmetric sandwich transducers: (a) transducer with mounting flange, (b) general form of simple asymmetric sandwich as per Neppiras. \[11\]
Another form of transducer that can be designed using the frequency-determining equations is the multi-element sandwich. This type of transducer can use any even number of PZT discs within reason. Some of the advantages include being able to achieve a range of electrical impedances, and the fact that the power-handling capacity of the transducer is increased. There are some drawbacks, however: including more discs increases the number of joints, thereby increasing losses, and overheating may become a problem. The general equation for this type of transducer is a summation of all the elements, taken two at a time:

$$\sum \frac{Z_r}{Z_s} \tan \theta_r \theta_s = 1$$ \[37\]

In this project, sandwich transducers with both two and four discs will be designed and built.

2.9 Some useful parameters for sandwich transducers.

There are a number of useful parameters relating to piezoelectric sandwich transducers that are often used to give an indication of the performance of a transducer. Most of the terms in the following equations pertain to the one-port lumped element approximation circuit for the transducer radiating into water.

\[
I_p = \frac{M + M_r}{N^2}; \quad C_p = N^2 C^E_M; \quad R_p = \frac{R_M + R_r}{N^2}; \quad N = \frac{d_{33} A_{33}}{s^{33} t} (t = \text{thickness of disc}).
\]

**Figure 19:** One-port lumped element approximation circuit for the transducer radiating into water (including losses). \[45\]

In figure 19, the following relationships apply:
Mechanical impedance = \( Z_M^e = R_M + j\omega M + \frac{1}{j\omega C_M^e} \)

Radiation impedance = \( Z_r = R_r + j\omega M_r \).

The following parameters are most useful in characterising the performance of the transducer: \(^{[45]}\)

Series resonance in water: \( f_w = \frac{1}{2\pi \sqrt{L_r C_r}} \)

Mechanical storage factor: \( Q_M = \frac{\omega_w L_r}{R_y} \)

\( Q_M \) gives an indication of the sharpness of the transducer response at resonance. For high power applications, \( Q_M \) should be high.

Electromechanical coupling factor: \( k = \sqrt{\frac{C_r}{C_b + C_r}} \)

The electromechanical coupling coefficient is probably the most important parameter, as it gives an indication of the performance potential of the transducer. \( k \) lies between 0 and 1, and for a good transducer is anywhere from 0.5 and upwards. Mason defines the coupling factor \( k^2 \) as the ratio of energy stored in mechanical form to total input energy. \(^{[19]}\)

Electric dissipation factor: \( \tan \delta = \frac{G_b}{B_b} = \frac{G_b}{\omega C_b} \)

It is assumed here that electric dissipation is due to dielectric hysteresis loss, so \( \tan \delta \) is independent of frequency while \( G_b \) is proportional to frequency.

Electrical \( Q \) at resonance: \( Q_e = \frac{B_{in}}{G_{in}} = \frac{1}{\tan \delta + \frac{k^2}{1 - k^2} Q_M} \)

This gives an inverse indication of the power factor at the electrical input and so should be low.

Electro acoustical efficiency: \( \eta_{ea} = \eta_{em} \eta_{ma} \)

The electro acoustical efficiency gives an overall indication of the ability of the transducer to convert electrical to acoustic energy. Obviously, this should be as high as possible. The two factors that make up \( \eta_{ea} \) are shown below; \( \eta_{ma} \) varies little with frequency, while \( \eta_{em} \) is strongly frequency dependant. \(^{[45]}\)
Mechanoacoustical efficiency: \[ \eta_{ma} = \frac{R_r}{R_m + R_r} \]

Electromechanical efficiency (at resonance): \[ \eta_{em} = \frac{1}{R_r} \left( \frac{1}{G_n} + \frac{1}{R_r} \right) = \frac{1}{k^2 Q_M} \left( 1 + \frac{\tan \delta}{1 - k^2} \right) \]

Woollett \cite{30} covers most of these parameters in more detail. Another useful tool when considering transducer parameters is the admittance plot. One can glean much important information merely by looking over the admittance plot. An example of such a plot is shown in Appendix A-3.

2.10 Conclusion

In this chapter, the following points have been discussed: the background and origin of the piezoelectric sandwich transducer and equivalent circuit theory, the derivation of some pertinent equivalent circuits, the design of some velocity transforming transducers and some useful parameters for sandwich transducers. Much of the material mentioned in this chapter will be applied in Chapters 4, 5 and 6 in the design of the high power ultrasonic transducers.

In the next chapter, the Finite Element method will be considered. Firstly, the background of the method will discussed, then its application to the modelling of a piezoelectric disc will be examined.
Chapter 3: The Finite Element Method

3.1 Introduction to Finite Element Method

The Finite Element Method has proven to be a powerful tool for modelling, analysing and solving complex problems. It is currently used in a wide range of engineering fields ranging from Civil and Mechanical to Biomedical and Nuclear. The theory of FEM is based on the concept of solving a very complicated problem by replacing it with a simpler one. The solution to the simpler problem will obviously not be the exact solution to the more complex problem, but rather an approximation. However, this is often the only method of getting an approximate solution, which can later be improved by refining the mesh and increasing the number of iterations. One of the earliest and simplest examples of this method was its use by ancient mathematicians to find the circumference of a circle by approximating it with the perimeter of a polygon. By using two polygons, one inscribing and one circumscribing the circle, the upper and lower limits of the circumference could be found. Also, by increasing the number of sides (elements) on the polygon, the approximate solution approaches the exact one. These characteristics are common to many finite element applications today. A diagram of the circle example is shown below:

![Diagram of circle with upper and lower bound polygons]

Figure 13: Circle showing upper and lower bound polygons.

The basis of this method is the element. Before one can perform any sort of analysis, one first has to create the mesh, which is made up of elements interconnected at joints called nodes or nodal points. Each element is defined by a certain number of nodes, depending on which type of element is chosen.
The nodes and elements are then numbered in a logical manner according to the global coordinate system. In Figure 3, the blue numbers refer to the element numbers, while the red ones refer to the associated node numbers. Since the variation of the field variable (stress, voltage, temperature, etc.) is not known, it is approximated using an interpolation or shape function. The interpolation functions are defined in terms of the values of the field variables at the nodes, and are normally in the form of polynomials that can be of any order necessary to generate approximations of the required accuracy. Once the field equations (the defining equations) for the whole system have been written, the new unknowns will be the nodal values of the field variable. The field equations, which are normally in the form of matrices, are then solved for the nodal values using the appropriate boundary conditions. Using these new nodal values, the interpolation functions then define the field variable throughout the system. This process is summarised into six steps by Rao [23] using static structural problems as a general case.

In order to gain some insight into how Abaqus actually solves problems, we will consider a simple mechanical model, namely the mass, spring and damper system. This is also useful since the 1-D (one dimensional) equivalent model of a piezoelectric sandwich transducer is made up of the same components.
The governing (field) equation of this system is:

$$\begin{align*}
[m] \frac{d^2 x}{dt^2} + [c] \frac{dx}{dt} + [k] x &= F
\end{align*}$$

[38]

where $m$ is the mass, $c$ is the damping and $k$ the spring constant. In a 1-D system, these are all scalar values. If it were a more complex system then $m$, $c$ and $k$ would become matrices. An interpolation or shape function in the form of the particular solution for this equation is:

$$x = X e^{\omega t}$$

[39]

When this is substituted into the governing equation, we get

$$- \omega^2 [m] X + [k] X = 0$$

[40]

Therefore,

$$\begin{align*}
[m] \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} &= \lambda \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\
[k] \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} &= \lambda \begin{bmatrix} 1 \\ 1 \end{bmatrix}
\end{align*}$$

[41]

We can express this using eigen-values and eigen-vectors as follows.

$$A \nu = \nu \lambda$$

[42]

Abaqus actually solves this equation for $\nu$ and $\lambda$ (the field variables), where $\nu$ are the mode shapes of the displaced system, and $\lambda$ are the frequencies at which the modes occur (since $\lambda = \frac{1}{\omega^2}$ and therefore $\omega = \sqrt{\frac{1}{\lambda}}$). These modes of vibration are called “eigenmodes”, and the displaced shapes can be viewed using Abaqus Post. For a 1-D system, these equations are all relatively simple, but when complex 3-D models are used, the equations become much larger and more difficult to solve. This is especially true for our case, since we are using piezoelectric discs, which involve coupled governing equations relating mechanical, electro-mechanical and electrical parameters.

Possibly the main advantage of using the finite element method to design piezoelectric transducers is that there are many characteristics of the transducer than can be judged from the FEM simulation results. This allows most, if not all, the refinement and optimization to be done before the transducer is actually built. In
previous years, the cut-and-try method was used which was not only time-consuming and wasteful, but also unreliable.

3.2 Finite Element Simulation of PZT disc

Obviously, one of the most important components of any sandwich transducer is the piezoelectric disc. The quality, alignment and poling of the discs will all play a vital role in determining the performance of the transducer. Therefore, one must ensure that the discs are modeled as accurately as possible to have any hope of modeling the whole transducer correctly.

The first step of the process is to generate the mesh for the disc. It is often a good idea to make a basic plan of the proposed mesh on paper before entering it into the code. This will later act as a good reference for node/element numbers and sets. Once the nodes have been defined and generated, the elements are then generated between the nodes. The elements now need to have their material properties defined.

The discs for this project came from a company called Ferroperm Piezoceramics in Denmark. In order to input the material properties, we required the data sheet for the discs from Ferroperm. This can be seen in Appendix B-1 (PZ26 discs were used). Abaqus requires the following three sets of material constants in order to simulate a piezoelectric disc: dielectric, piezoelectric and elastic constants. These can all be found on the data sheet. It should be remembered that the relative dielectric constants are given, so they still have to be multiplied by the permittivity of free space $\varepsilon_r$.

Using the Abaqus Standard Users Manual and looking up the keywords dielectric, piezoelectric and elastic, one can transfer the parameters from the table into the matrix format required by Abaqus. The resultant matrices are shown in Appendix B-2.

The final stage in the input deck is to put in the analysis steps and appropriate boundary conditions. The usual steps for piezoelectric transducers are the following: closed circuit modal analysis, open circuit modal analysis and frequency analysis. The closed and open circuit analysis gives the series and parallel resonances respectively, while the frequency analysis gives the response of the transducer to a 1 Volt
excitation at the various frequencies in the specified range. The results required from
the frequency sweep are also specified in the final step. The input deck for an
axisymmetric model of a piezoelectric disc is shown in Appendix B-3. As the disc is
symmetric about the z-axis, we only have to model one small section that can then be
swept 360° to produce a 3-D model. However, the actual analysis is only carried out
on the small section, we therefore expect all the results to be symmetrical as well.
This type of model requires much less computing time than a full 3-D model. Out of
interest, a full 3-D model was generated and the input deck can be seen in Appendix
B-4.

Once the simulation has been run, the results can be seen using Abaqus Post. This is a
post-processing program that allows one to view an image of the transducer, the
displaced shaped of the transducer at its various resonant frequencies and graphs of
the transducer's response to the 1 Volt excitation.

3.3 Results of PZT disc simulation

![Figure 15: Axisymmetric model of piezoelectric disc](image)
Figure 16: 3-D model of disc, generated from axisymmetric model.

Figure 17: Graph showing predicted admittance of transducer from 1 Volt excitation.
3.4 Network Analyzer

In order to test whether this design method is in fact effective and reliable, we have to have some method of testing the transducers once they have been constructed. For this thesis, we will be using the HP Network Analyzer to test the transducers. The results from it are considered to be reliable and accurate. To make the testing of the transducers as quick and easy as possible, a program was written in HPVee to control the Network Analyzer and download the information directly onto a computer. A diagram of the HPVee program can be seen in Appendix B-5.

3.5 Comparison of results

<table>
<thead>
<tr>
<th>Modes</th>
<th>Measured (NetAn)</th>
<th>Axisym model</th>
<th>3-D model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Breathing (kHz)</td>
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<td>43.08</td>
</tr>
<tr>
<td>Wall-thickness (kHz)</td>
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<td>130.3</td>
<td>130.8</td>
</tr>
<tr>
<td>Thickness (kHz)</td>
<td>323.0</td>
<td>320.9</td>
<td>320.3</td>
</tr>
</tbody>
</table>

Table 1: Predicted and measured values of resonant frequencies for PZT disc.

As can be seen from the above table, the predicted results from Abaqus are very close to the measured ones. This implies that the material properties used are the correct ones and they are in the correct format. Although the results from the 3-D model are slightly more accurate in the first two cases, the extra computing time required for the 3-D model does not justify the marginal increase in accuracy.

3.6 Exact simulation of PZT disc

As can be seen from the graph in figure 17 and the results, the Abaqus simulation of the disc is relatively accurate for predicting the resonant frequency. However, the results do not give us a reasonable indication of the magnitude or "Q" (sharpness of curve) of the response. An attempt was made to obtain more realistic results from Abaqus, so that we would be able to accurately predict the both the magnitude and the "Q" of the transducer. This would in turn allow us to quantitatively ascertain some of the important characteristics of the transducers.
Various methods were tried to get a more realistic simulation result. The first attempt involved changing the way the disc was being driven in Abaqus. Initially, this was done by assigning a value of 1V to the voltage variable, i.e. the disc was being driven by a voltage. As an alternative, the variable “cecharge” (current equivalent) was used to drive the disc. This had the effect of changing the variables that were available as outputs, but did not have any useful effect on the final results. The next change was to try and include some damping in the model. Abaqus has two kinds of damping: mass proportional (alpha) and stiffness damping (beta). The first type will apply at any point in the model where there is mass in motion and cause damping proportional to the figure given for alpha (using the mass matrix). Stiffness damping on the other hand is associated with the material itself and is actually calculated as a percentage of the stiffness matrix. Both methods of damping were tried, and the stiffness method, after some iteration, gave more accurate results. The results are shown below.

![Graph showing actual and simulated values of G & B using stiffness damping.](image)
As can be seen from the graph above, the results appear to match up almost exactly. Changing the stiffness damping allowed us to control the magnitude of the response, and the frequency had to be shifted up slightly by decreasing the density of the disc. The damping did not seem to affect the "Q" of the response at all.

From the results, it looks like we are able to model the disc exactly. However, the above simulation is for a single disc. As soon as we try to model a more complex system or transducer, the results stray from the actual values again by a significant amount. This implies that the damping is model specific, i.e., we would have to change it each for any addition or significant change to the model. It may be possible that there exists some optimum combination of both mass and stiffness damping that would give us reasonable results with all models, but this would require much more investigation.

Some enquiries were then made to H.K.S as to why we were not able to exactly model a PZT disc. H.K.S replied saying that there were in fact no losses included in the PZT capabilities of the program. This implies that the numerical results obtained from Abaqus are purely imaginary. So the lack of any form of damping leads to unrealistically high "Q" values and the lack of losses gives us only the imaginary part of the results. This means that important parameters that require the real part of the results, such as efficiency, cannot be found using this program. We can however still predict the resonant frequencies and their mode shapes with considerable accuracy.

3.7 Conclusion
Now that we are confident that we can model a piezoelectric disc accurately, we can now attempt to design the sandwich transducers. In the following chapters, all three types of transducer will be designed using both the equivalent circuit and FEM methods.
Chapter 4: Nodal-Mounted Tonpilz Transducer Design using Equivalent Circuit Theory and FEM

4.1 Introduction

In order to design the three types of high power ultrasonic transducers, we will now integrate the design methods that have been discussed in the previous two chapters. We will try to keep the design process as concise as well as accurate as possible. The first step in the design process for all three transducers will be the frequency equation (equation 32 in Chapter 2). It is accepted that this is a very simplistic design method, but we are only using it as it gives us some basic dimensions to work with. These basic dimensions will then be used in a lumped element analysis of each of the transducers. The lumped element equivalent circuit that we will use is shown in the Brush Electronics manual "Piezoelectric Technical Data." [53] The derivation of this circuit can be seen in Appendix C-1. This will allow us to refine the transducers to some extent and even get some initial measures of performance of the transducers. The final type of equivalent circuit we will use to approximate the transducers is the distributed parameter circuit shown in Woollett. [45] This is the closest approximation we can make to the actual transducer using equivalent circuit theory. This method does involve more complex calculations than the other methods, but with packages such as Mathcad, [54] computational time is greatly reduced. One of the main advantages of the method is that it can handle the more complex head shapes such as the cone, as we shall see later in the chapter. The design of the horn and the final refinements and optimisation of the transducers will be done using Abaqus. Where possible, the transducers will also be modelled with water loading using Abaqus. Also, knowledge from a previous model [18] will be used as a guideline in the design process

4.2 Design of the nodal-mounted tonpilz transducer

The nodal-mounted transducers will basically be suspended by a nodal plate, with their faces slightly submerged in the liquid. They will be expected to produce acoustic intensity sufficient to cause cavitation at and around the face.
4.2.1 Design specifications

This transducer will be designed to meet the following specifications:

- It will be made using the piezoceramic discs supplied by Ferroperm. The disc dimensions are $38.5 \times 12.7 \times 6.4$ mm, so the transducer will be designed around these.

- It will be held at a central nodal plate.

- The transducer will be designed to operate in its longitudinal "piston" mode around 22 kHz, with the maximum displacement occurring at the head. In this mode, there must be as little "flapping" (flexure) as possible in the head.

- The head of the transducer will be in direct contact with and induce cavitation in liquids, therefore the head material should be resistant to corrosion from both rust and cavitation.

- The costs of material and manufacture should be kept at a minimum.

4.2.2 Design method

In order to get a first approximation of the dimensions of the transducer, we will use the frequency equation method. Once we have the basic dimensions of the head and tail, we will then use the lumped element circuit to get a better approximation and develop the transducer as far as we can with this method. Then we will include the effect of the tapered head using the distributed parameter analysis. Finally, we will use an iterative method in Abaqus to compare the results with the 1-D models, then gradually build up and "test" an exact model of the proposed transducer. The transducer will then be refined in Abaqus for optimum performance.

4.2.2.1 Frequency Equation Design

The frequency equation as per Neppiras\textsuperscript{111} is as follows:

$$\tan \vartheta_a \tan \vartheta_t = \frac{Z_c}{Z_s}$$

where $\vartheta_a = \frac{\omega t}{c_s}$, $\vartheta_t = \frac{\omega t}{c_L}$, $Z_c = \rho c_s A_s$, $Z_s = \rho c_L A_L$.

$$\tan \frac{2\pi f t}{c_s} \tan \frac{2\pi f t}{c_L} = \frac{\rho c_s A_s}{\rho c_L A_L}$$

[43]
In the above expression, the subscript c refers to the ceramic and subscript L refers to either the head or the tail of the transducer. In this expression, our only unknown is the length of the head or tail (all parts are assumed to be straight cylinders). We used Mathcad \cite{3} to manipulate the figures as the program facilitates the easy change of variables.

In order to meet the design specifications for the nodal-mounted tonpilz, it was decided that stainless steel should be used for the head, and brass for tail. Their widely different \( \rho v \)'s will ensure that the displacement occurs predominantly at the head, and stainless steel is rust resistant and will also resist cavitation corrosion, as the intensity of cavitation at the face is reduced by the conical shape of the head. It was decided that four piezoelectric discs would be used to increase the power handling capacity and output of the transducer. This means that \( n=2 \) in equation \( [43] \). The diameter of the passive materials (head and tail) was chosen to be 40mm, as this is slightly bigger than the discs. The design frequency used in the equation is 22kHz. The constants and variables used and the calculations for the nodal-mounted tonpilz can be seen in Appendix C-2. For this initial calculation, some of the finer details of the transducer, such as the bolt, nodal plate and head shape are not considered, as this is the first approximation. After substituting all the relevant values in, we get that the head should be 17mm long and the tail 15mm long.

4.2.2.2 Lumped Element Approximation

The lumped element circuit for the nodal mounted tonpilz is shown below. This circuit could actually be used for practically any sandwich transducer. In the circuit below, \( M \) stands for the static mass of the components, \( C_{mE} \) is the compliance of the ceramic stack and \( C_b \) is the blocked capacitance of the stack. For the lumped element circuit, the series resonance frequency is calculated quite simply using the formula \( f_r = \frac{1}{\sqrt{MC}} \), where \( M \) and \( C \) are the equivalent mass and compliance of the circuit.
Instead of using the simple formula above for calculating the resonant frequency, we calculated the overall impedance of the circuit, and then used this to give both the series and parallel resonances of the circuit. From these values we were able to calculate the effective coupling factor, a useful measure of the transducer’s performance. Mathcad was again used to facilitate the ease of calculation. Once we input the lengths of the head and tail from the tan approximation into the lumped element circuit, we get following results: \( f_s = 22.78\text{kHz} \), \( f_r = 31.17\text{kHz} \) and \( k_{eff} = 0.683 \). We expect the coupling factor to be about 0.5 for a good transducer, so this value is obviously very high. This is because there are no losses included in the model. The full calculation can be seen in Appendix C-3.

The next step in the design process is to include the pre-stress bolt in the approximation. For the purposes of this simplistic calculation, it should be noted that this is not a whole bolt being modelled, it is only a rod of steel basically joining the front and back masses. The rod is easily included by putting the compliance of the rod in series with the compliance of the ceramic (the mass of the rod is neglected). We expect the resonance frequency of the system to increase with the inclusion of the bolt, as the system is now stiffer. The results come out as follows with an 8mm diameter rod:

\[ f_s = 24.76\text{kHz}, \quad f_r = 32.66\text{kHz} \text{ and } k_{eff} = 0.652. \]
The final refinement we can perform at this stage of the design is to make the bolt as close to the actual bolt as possible. So as a rough approximation, we will make a hole in the tail for the bolt to go through, increase the length of the bolt and include some extra mass for the head of the bolt. We will assume that an 8mm bolt is used with a head diameter of 22mm and thickness of 6 mm. This will add about 15g to the mass of the tail if the bolt is made of steel. The result on the resonant frequencies and effective coupling factor are the following: \( f_1 = 23.96\text{kHz} \), \( f_2 = 32\text{kHz} \) and \( k_{ep} = 0.663 \). We would expect the frequency to drop slightly due to the extra compliance in the longer bolt, and the extra mass of the bolt head.

Although this design method is relatively simplistic, it is useful because it allows us to get a fair amount of detail into the model at this early stage of design. Its simplicity is also one of its advantages. There are other 1-D models that are perhaps more accurate than the lumped element method, such as Kagawa and Yamabuchi's \(^{35}\) finite-element equivalent circuit which is valid over a wider range of frequencies. Their circuit is shown in Appendix C-4. However, we must keep in mind that we are only approximating the transducer, so it would seem prudent to keep the initial models as simple but as accurate as possible.

### 4.2.2.3 Distributed parameter approximation

For the distributed parameter approximation, we will initially model the earlier approximations to see how they compare with this method. Then, the transducer will be optimised as far as possible using the distributed parameter analysis. When it comes to adding the bolt into the analysis, we will be using the simplified version of a rod "tying" the head and tail together as shown below in figure 20 (a). The reason for this simplified version is that if we try to model the actual bolt going through the tail, this type of analysis becomes a lot more complex. The main difference is that the stress bolt can no longer be modelled using a T-circuit; it now needs an H-circuit to be accurately represented. It would be difficult to justify this added complication, as the simpler method appears to deliver perfectly adequate results, as we shall see later. The diagrams of the rod and the actual bolt approximation are shown below.
Shown below is the distributed parameter equivalent circuit of the transducer using the rod approximation for the bolt.

\[ Z_{t1} = j \rho \epsilon \sigma A, \tan \frac{\alpha}{2}, \text{ and} \]
\[ Z_{t2} = -j \frac{P_c A}{\sin(kf)} \]

with the same functions applying to the other components, replacing the relevant parameters. For this type of analysis, a tool like Mathcad is necessary, as doing these calculations manually would be an arduous task.
We will start by modelling the transducer with an untapered head, 4 PZT discs and the tail. The equivalent circuit for this model would be exactly the same as figure 20, just without the bolt components. The method used to analyse the circuit is as follows: simplify the circuit as far as possible, translate the mechanical impedances across the transformer to yield the equivalent electrical terms, derive the impedance matrix for the circuit, solve for the currents flowing in each mesh then plot the frequency versus electrical admittance. When we do this for the transducer with a 17mm stainless steel head and 15mm brass tail, we get the following results: $f_n = 22.04\text{kHz}$, $f_p = 28.95\text{kHz}$ and $k_{eff} = 0.648$. The complete calculation can be seen in Appendix C-5. Once again the coupling coefficient is unrealistically high due to the lack of losses.

Including the rod in the approximation increases the resonance frequency as expected. However, the increase predicted by the distributed parameter method is not as substantial as that predicted by the lumped element method. The results are as follows: $f_n = 23.70\text{kHz}$, $f_p = 30.0\text{kHz}$ and $k_{eff} = 0.613$. The changes in the calculation and relevant results are shown in Appendix C-5.

The next stage of the design is to taper the head. There are quite a few factors involved in the selection of the head diameter. The first factor to be considered is the intended application, as this will determine some vital properties of the transducer. We require a high power, narrowband transducer with optimum matching to the liquid load, as opposed to the high power, broadband transducer that is often used in sonar applications. According to Yao, increasing the diameter of the head will improve the radiation impedance of the transducer, but will also lower the flexural ("flapping") mode of the head. This mode is considered wasteful as the piston mode of the transducer is now coupling with the flexural mode of the head, thereby reducing the power output from the transducer. Also, Stansfield recommends that the flexural mode is at least one and a half times the main resonant frequency. Therefore a compromise must be reached. We can make a rough approximation of the first flexural mode of the head by using the
formula for a free circular plate, fixed at the centre, of thickness \( t_h \) and radius \( a \) from Leissa,\(^{[57]}\)

\[
\tilde{f}_j = \frac{0.172 l_h}{a^2} \sqrt{\frac{Y}{\rho(1-\sigma^2)}}
\]

where \( Y \) = Young's modulus, \( \rho \) = density and \( \sigma \) = Poisson's ratio. This will be a very rough approximation, but will give us an idea of where the flapping mode will be. We can assume that the actual flapping mode of the head will be substantially higher than predicted by the formula, as we will be using a cone instead of a flat plate. If we try with a head diameter of 70mm, keeping the length at 17mm, the predicted flapping frequency comes out at 12.527kHz. Even considering the increase in frequency due to the cone shape, this is too low. Also, the series resonance drops to 20.87kHz, which is unacceptable. If we reduce the diameter to 55mm, the flexural mode moves up to 20.292kHz. Considering the cone shape, this may be acceptable. The other results are \( f_1 = 22.13\text{kHz} \), \( f_a = 28.01\text{kHz} \) and \( k_{eff} = 0.613 \). Once again, the changes in the calculations and full results can be seen in Appendix C-5.

This method appears to be very useful in a number of ways. One of the main advantages of this method is its versatility; after the calculations have been done once, it takes little effort to change the material properties or include a new component such as a bolt. Up to now, this method also appears to be quite accurate, as it compares well to the previous methods. The transducer will now be checked and optimised using FEM.

**4.2.2.4 FEM Design of the nodal-mounted transducer**

Initially for the FEM model, we will use the dimensions used in the previous design methods to see how the results compare. A similar progression of gradually making the model more accurate will again be followed. The transducer will then be refined and optimised as far as possible. Once we have reached what we believe is a good design, it will then be simulated radiating into water, as this is a vital test of its performance. All of the models for the transducers will be axisymmetric, as this saves a huge amount of computer memory and time compared to 3-D models.
We start off modelling the head and tail of the transducer separately, using the dimensions from the frequency approximation. As the frequency equation assumes that the transducer is clamped at the nodal plate, the simulated version is also clamped along the face of the central piezoelectric disc. Shown below is a 180-degree view of the head and two discs, and its piston mode.

**Figure 22: Pictures from Abaqus of the head and two discs and its piston mode**

In the picture of the piston mode, the red mesh depicts the undeformed transducer and the blue the deformed. These pictures of the deformed and undeformed mesh are highly exaggerated, as the real transducer only produces microns of displacement, but they are useful for showing the different mode shapes. The relevant frequencies for the head and tail sections are as follows:

- **Head:** $f_1 = 24.506\text{kHz}$, $f_2 = 27.966\text{kHz}$ and $k_{\text{eff}} = 0.482$.
- **Tail:** $f_1 = 23.450\text{kHz}$, $f_2 = 26.497\text{kHz}$ and $k_{\text{eff}} = 0.466$.

These results are somewhat higher than the 22kHz predicted by the frequency equation, but we shall leave the dimensions unchanged for now to see how the FEM compares to the other models.

The next step in the design process is to couple the head and tail sections together. In the actual Abaqus deck, all of the parts of the transducer (discs, head and tail) are held together by multi-point constraints or mpc's. These ensure that all degrees of freedom
(e.g. displacement, voltage) are “tied” together. This implies that there will be perfect coupling between the pieces, which is obviously not possible in the actual transducer, but it is a close enough approximation. The coupled head and tail are shown below, with the piston mode next to it.

![Coupled head and tail and piston mode](image)

**Figure 23:** Coupled head and tail and piston mode.

The results for this combination are as follows: $f_h = 21.417\,\text{kHz}$, $f_d = 25.112\,\text{kHz}$ and $k_{eff} = 0.522$

As with the previous design methods, we now include the 8mm rod in the model.

![Head and tail with rod](image)

**Figure 24:** Head and tail with rod
As with the other models, the bolt has the effect of increasing the resonant frequency $f_r = 23.146 \text{kHz}$, $f_n = 26.666 \text{kHz}$ and $k_{\text{eff}} = 0.497$.

We now taper the head out to 70mm and see what kind of mode shape is given.

Figure 25: (a) Transducer with 70mm tapered head, (b) very “flappy” piston mode, (c) flexural mode of head
As can be seen from the above pictures, the piston mode is strongly coupled to the flexural mode, which is unacceptable. The flexural mode ($f_f$) of the head is also low in frequency $f_f = 20.403\text{kHz}$, $f_s = 23.175\text{kHz}$, $f_t = 32.757\text{kHz}$ and $k_{ef} = 0.474$. So we decrease the diameter of the head to 55mm, which gives us a vastly improved piston mode and significantly higher flexural mode.

Figure 26: (a) Transducer with 55mm head, (b) piston mode, (c) flexural mode
As one can see from the above pictures, the piston mode is no longer coupled to the flexural mode, which itself has coupled with the flexural mode of the tail at a much higher frequency: $f_1 = 21.863\, \text{kHz}$, $f_2 = 25.176\, \text{kHz}$, $f_3 = 41.686\, \text{kHz}$ and $k_{\text{eff}} = 0.496$

We now start to refine and optimise the transducer. The first step is to modify the head slightly by including the small, untapered sections at the base and face of the head. The modified transducer is shown below.

![Transducer with modified head](image)

**Figure 27: Transducer with modified head.**

Including these untapered sections appears to increase the magnitude of admittance of the transducer quite drastically, considering the magnitude of the changes. It is not known why the performance of the transducer is enhanced by the new head shape; perhaps it is due to the fact that the untapered sections allow uniform propagation of the sound waves through the structure. A comparison of the graphs for the transducers is shown in Appendix C-6.

The next step in refining the transducer is to include a small taper at the base of the tail. It is thought that this taper in the tail improves the distribution of pre-stress when the bolt is torqued up to the required value. This will in turn prolong transducer life.
The final step in the design is to include details such as the actual bolt, nodal plate and electrodes. A steel nodal plate in between the two central discs, with a 70mm diameter and 1mm thickness, was used to make the transducer easier to grasp without interfering with the wiring and piezoelectric discs. The edges of the nodal plate were then constrained, as they would be in reality. Four beryllium-copper electrodes were used to provide the voltage across the discs, although they actually make very little difference in the Abaqus simulation. An 8mm high tensile steel bolt, with a 20mm head was also included in the simulation. The final transducer is shown in Figure 29 below.

Figure 28: Nodal-mounted transducer with modified head and tail

Figure 29: Final design of nodal-mounted transducer, and its piston mode.
Figure 30: Other modes of the transducer: nodal plate mode (left), and flexural mode of head (right)

The results for the final transducer are as follows: $f_r = 21.784$ kHz, $f_n = 24.443$ kHz, $f_p = 37.980$ kHz and $k_{ad} = 0.453$.

4.2.2.5 Water testing in FEM

The next important part of the design process is to simulate the transducer with a water load. This is one method that can be used to test the efficacy of the transducer. If one considers the output admittance characteristics of the transducer in water, they will clearly be damped by the loading effect of the water. It is the magnitude of this damping that one can use to gauge the how well the transducer couples to its liquid load; the greater the damping of the admittance, the better the coupling between transducer and load.

For the actual deck in Abaqus, there are a few points to take note of that are critical to the validity of the results. The first point is that we have to simulate the transducer radiating into a large body of water, or one with a non-reflective boundary wall. This is to prevent interference from reflected energy and the creation of standing waves. As we try to keep the models as small as possible in Abaqus, it is obviously not an option to model a large body of water, so we instead model a smaller hemispheric of water with a non-reflective
boundary. The minimum distance to the non-reflective boundary is decided by the maximum frequency of interest, which will define the wavelength in the liquid. The minimum distance to the boundary is then \( x = \frac{\lambda}{2D} \), where \( \lambda = \text{wavelength} \) and \( D = \text{diameter of sound source (head)} \). For this case, we will say the maximum frequency of interest will be 30kHz. This means that we require a hemisphere with a radius of approximately one meter, which creates a very large model. Although this model takes a long time to run, it is still plausible to do. In the deck, one must not forget to define the non-reflective boundary elements as well as the interface elements between the head and the water. The full deck for the simulation of the nodal-mounted transducer radiating into the hemisphere of water can be seen in Appendix C-7.

Below are the results of the simulations, the first into air, and the second into the hemisphere of water.

![Admittance vs Frequency (Nodal-mounted in air)](image)

**Figure 31: Results of nodal-mounted transducer radiating into air**
Figure 32: Results of nodal-mounted transducer radiating into hemisphere of water.

As one can see from the above graphs, the resonance frequency drops and the magnitude of the admittance is damped substantially when the transducer is loaded with the water. From the results so far, this transducer appears to meet all the specifications. The design drawings for the transducer can be seen in Appendix C-8.

4.3 Conclusion

In this chapter, the nodal-mounted transducer has been designed using four methods of increasing complexity: the frequency equation, lumped element analysis, distributed parameter analysis and finally FEM. Most of the results seem to agree very well, although those from Abaqus tend to be slightly high. This will be discussed in a later chapter. If we consider the fact that all of the design methods are in fact approximations of varying simplicity, it is quite surprising how accurate the results are. The face-mounted transducer will now be designed using the same method.
Chapter 5: Face-Mounted Tonpilz Transducer Design using Equivalent Circuit Theory and FEM

5.1 Introduction

These transducers will be suspended from the bottom of a specially designed bucket. The base of the bucket will be a stainless steel plate, about 200mm in diameter and 1mm thick. The transducer will be bonded to the plate, and will be expected to sonify the liquid contents of the bucket.

5.2 Design Specifications

The face-mounted tonpilz transducer should meet the following design specifications:

- The piezoelectric discs will again be provided by Ferroperm [23] and have the same dimensions as those used for the nodal-mounted transducer.
- The transducer will be mounted on the base of a specially designed bucket using Araldite epoxy. The base will be a stainless steel plate, 1mm x 198mm diameter.
- The head of the transducer will not be in contact with any liquids.
- The transducer will induce the maximum amount of cavitation in any liquid within the bucket.
- The operating frequency of the transducer will be around 26kHz, and this transducer will also operate in its longitudinal or piston mode.
- The costs of material and manufacture should be kept at a minimum.

5.3 Design Method

For the face-mounted transducer, we will basically use the same method that we did for the nodal mounted transducer. It should be kept in mind, however, that it will be mounted on its face, so there is no nodal point at the centre of the PZT discs. We will still design the transducer as a symmetrical sandwich, as the frequency equation makes that assumption, and then modify it to suit its particular application.
5.3.1 Frequency Equation Design

For this design, the head and tail were chosen to be made from aluminium and brass respectively. These two materials have very different $\rho c$'s which will ensure that the bulk of the movement occurs at the head of the transducer. Also, the head will not be in contact with the liquid, so corrosion is not an issue. The diameter of the parts was again chosen to be 40mm. It was decided that two PZT discs would be sufficient for this application.

Using the properties for the aluminium, brass and ceramic, the results show that the head should be 38 mm long and the tail 19mm long. It must be remembered that this calculation assumes that the end pieces are straight cylinders. The calculation can be seen in Appendix D-1.

5.3.2 Lumped Element Approximation

If we use the circuit given in figure 19 as an approximation of the face mounted transducer, the results appear to be way off from the expected 26 kHz. The results we get are the following: $f_1 = 34.29$kHz, $f_n = 46.91$kHz and $k_{eff} = 0.682$. In the equivalent circuit in figure 19, we notice that the compliance of the head and tail are ignored, as their effect on the resonance frequency would be very small.

![Figure 22: Revised lumped element circuit to include compliance of head and tail.](image-url)
However, because of the increased length of the head and tail for the face-mounted transducer, the compliance of the end pieces becomes significant. So we must revise the lumped element circuit as shown on the previous page. If we use the circuit in figure 22, we get the following results: \( f_1 = 25.07 \text{kHz}, f_2 = 29.19 \text{kHz} \) and \( k_{\text{eff}} = 0.512 \). These are much closer to the expected values.

Following the same design method as for the nodal-mounted transducer, we then include a rod between the tail and head as an approximation of the bolt. This produces the result that: \( f_1 = 26.27 \text{kHz}, f_2 = 29.71 \text{kHz} \) and \( k_{\text{eff}} = 0.467 \).

As a final approximation, we can try to model the actual bolt more closely. Again using the same method as previously, we make a hole in the tail, increase the length of the bolt and add some mass for the bolt head. For this approximation, the final results come out as: \( f_1 = 26.74 \text{kHz}, f_2 = 30.54 \text{kHz} \) and \( k_{\text{eff}} = 0.483 \).

The above results appear to be consistent with what we would expect and can all be seen in Appendix D-2. The transducer will now be modelled using the distributed parameter analysis.

### 5.3.3 Distributed parameter analysis

For the face-mounted transducer, we will be using the same distributed parameter circuit (figure 21) as we did for the nodal-mounted transducer, obviously taking into account the different materials and number of discs. Using this method with the basic dimensions from the previous section and not including the bolt, we get the following results: \( f_1 = 26.05 \text{kHz}, f_2 = 29.80 \text{kHz} \) and \( k_{\text{eff}} = 0.485 \). These and the rest of the results for the distributed parameter approximation can all be found in Appendix D-3.

Using the same progression as previously, we then include the rod between the head and tail, giving us: \( f_1 = 27.15 \text{kHz}, f_2 = 30.25 \text{kHz} \) and \( k_{\text{eff}} = 0.441 \).
For the tapered head of the transducer, we first try an 80mm diameter head, then a 60mm diameter, which, according to Leissa, will have flexural frequencies at 21.65 and 38.49kHz respectively. Obviously, we prefer the higher the flexural mode. Using the 80mm diameter head in the distributed parameter approximation lowers the resonant frequency quite substantially $f_1 = 22.50\text{kHz}$, $f_2 = 25.24\text{kHz}$ and $k_{eff} = 0.453$. Decreasing the diameter of the head to 60mm increases the frequencies to more acceptable levels: $f_1 = 24.23\text{kHz}$, $f_2 = 27.03\text{kHz}$ and $k_{eff} = 0.444$.

The transducer that we now have is still basically a symmetrical one, that is, about the nodal point between the two discs. However, this transducer is being mounted on its face, so there is no need for it to be symmetrical. This allows us to refine this transducer so that we get the maximum amount of movement from the face. The easiest method of doing this is by increasing the mass of the tail while decreasing the mass of the head. Since the materials have very different $\rho c^2$'s, this is very easily done by changing the relevant lengths of the head and tail. In this way, the mass of the whole transducer is heavily biased towards the tail, thereby allowing increased movement in the head. Reducing the head to 20mm and increasing the tail length to 26mm gave the following results: $f_1 = 28.40\text{kHz}$, $f_2 = 32.22\text{kHz}$ and $k_{eff} = 0.472$. If one considers the graph of the results (Appendix D-3), we also see that the magnitude of admittance (relative) increases, as does the coupling coefficient. Although the resonant frequency is higher than the design specification, we can easily correct this by refining the lengths of the components. This will be done in Abaqus.

5.3.4 FEM design of the face-mounted transducer

Once again, we will go through the iterative process of building up the transducer from the basic components, comparing the results with the 1-D models as we go. Some refinements will then be made, before the final drawings are made for production.
We start by modelling the head and tail of the transducer, each with one PZT disc. The dimensions of the head and tail are from the frequency equation calculation. Below, the head is shown, with its piston mode.

![Figure 33: Head assembly and its piston mode](image)

Once again, we are treating the transducer as if it were a symmetrical transducer, so the top of the PZT disc is constrained. The results for the frequencies of the head and tail were as follows: Head: \( f_r = 27.181 \text{kHz} \), \( f_a = 28.248 \text{kHz} \) and \( k_{ae} = 0.272 \).

Tail: \( f_r = 28.189 \text{kHz} \), \( f_a = 30.084 \text{kHz} \) and \( k_{ae} = 0.349 \).

Once again, the FEM results come out substantially higher than those predicted by the frequency equation.

Now we couple the head and tail together, and then add the pre-stress bolt. The results for these combinations are as follows: Without rod: \( f_r = 24.990 \text{kHz} \), \( f_a = 27.127 \text{kHz} \) and \( k_{ae} = 0.389 \); with rod: \( f_r = 26.208 \text{kHz} \), \( f_a = 28.129 \text{kHz} \) and \( k_{ae} = 0.363 \). The pictures of the transducers can be seen on the next page in figure 34.
We then taper the head, first out to 80mm, then 60mm diameter. The results are shown below.

Figure 35: Transducer with 80mm diameter head, piston mode, flexural head mode

For the above transducer, the results are low as expected: $f_p = 21.386\,\text{kHz}$, $f_o = 22.682\,\text{kHz}$, $f_f = 35.291\,\text{kHz}$ and $k_{ef} = 0.333$. One can also see that that piston mode is again quite strongly coupled to the flexural head mode, and the flexural mode itself is too low. So we try with the head diameter of 60mm.
Reducing the head diameter has the effect of increasing the relevant frequencies, both the piston and flexural modes. $f_p = 23.576\text{kHz}$, $f_s = 25.207\text{kHz}$, $f_f = 44.096\text{kHz}$ (tail flexural mode) and $k_{df} = 0.354$. With the 60mm head, we notice that the tail flexural mode is lower than that of the head, but the frequency is high enough not to worry about the flexural effect. The next stage is to modify the transducer so that more of the displacement occurs in the head. This is done by biasing the mass towards the tail as is shown below.

![Modified transducer with 20mm long head and 26mm tail](image)
One can see that the relative displacement of the head (to the tail) is greater than previously. Also, the resonant frequency is closer to the specifications: \( f_r = 27.272\text{kHz} \), \( f_o = 29.325\text{kHz} \), \( f_t = 43.891\text{kHz} \) and \( k_{\text{eff}} = 0.367 \).

We now refine the transducer to include the details such as the electrodes, the actual bolt, and the plate to which the face is glued. The plate will obviously make a significant difference to the transducer, as some of the plate modes will couple with those of the transducer. The whole system is shown below.

![Figure 38: Face-mounted transducer on plate](image)

The edges of the plate are constrained in Abaqus, as they would be by the actual bucket. In order to connect the face of the transducer to the plate in the Abaqus deck, we again use multi-point constraints. This will assume perfect coupling between the head and the plate, whereas we will probably incur small losses in the glue joint between the two pieces. It is possible to model the glue joint if one has the necessary properties of the cured epoxy, but it appears that the mpc's give a close enough approximation. The results for the above system are as follows: \( f_r = 25.704\text{kHz} \), \( f_o = 27.199\text{kHz} \), \( f_t = 33.320\text{kHz} \) and \( k_{\text{eff}} = 0.327 \). There is another mode at around 23kHz that appears to be a mode of the plate, as there is little movement in the transducer. This and the other modes of interest are shown on the following page.
Figure 39: Modes of the face-mounted transducer: (a) Plate mode at 23.060kHz, (b) Piston mode coupled with plate mode at 25.704kHz, (c) Flexural mode of head coupled with plate mode at 33.320kHz.
The admittance graph for the transducer mounted on the bucket is shown below.

Figure 40: Graph showing admittance vs. frequency for face-mounted transducer

Figure 41: Admittance vs. frequency for modified face-mounted transducer
The final refinement that we can make to the transducer is to taper the tail slightly to improve the distribution of pre-stress. Once again, we include a three millimetre taper at the base of the tail. This small change appears to have quite a drastic influence on the output of the transducer as shown on the previous page in figure 41. We do not however expect the actual transducer to improve by the same margin, as this would be improbable, but we will manufacture both types of tail and then test the final transducers to see if it does make some difference.

5.3.5 Water testing in FEM

Once again, we will "test" the transducer radiating into water, to try and gauge the performance of the transducer. This transducer was simulated differently in water compared to the nodal-mounted transducer. The main reason for this is that because this transducer radiates upwards into its load, allowing us to define an exact amount of water and the boundary conditions on the surface of the liquid. So we modelled the transducer radiating into the bucket filled with about 56mm of water. This depth is significant as it is close to the wavelength of a standing wave in the water. When the depth of the water is equal to the wavelength of a standing wave, the impedance of the load drops significantly and the admittance correspondingly increases.

So the transducer was modelled radiating into 56mm of water, with the pressure on the surface of the water being approximated to zero. Once again, interface elements between the plate and the water are very important for meaningful results. The graph of the output is shown on the following page. One notices this time that the resonance frequency has actually increased. This is due to the frequency-pulling effect of the water mode (standing wave) on the transducer mode. This will be discussed further in a later chapter.
5.4 Conclusion

Once again, the different design methods appear to agree well, but not quite as well as the nodal-mounted transducer. The main reason for the slightly larger discrepancies is that the face-mounted transducer is a more complex system. This means that we have to make more assumptions about aspects such as the mounting of the transducer and the face. These assumptions could lead to a slightly increased margin of error. A full comparison and discussion of the results will be given in a later chapter when we can compare them to measured results. We will now use the same method to design the horn transducer.
Chapter 6: Horn Transducer Design using Equivalent Circuit Theory and FEM

6.1 Introduction

The design of the horn driver will be the simplest of all the transducers, as it is a straightforward symmetrical sandwich transducer with no tapered parts. The horn itself will be a challenge to design due to its complex shape. This transducer is very different from the previous two in that the horn is designed to intensify the sound field produced by the driver, unlike the tonpilz transducers that decrease the sound intensity. This adds some new dimension to the design process.

6.2 Design Specifications

The high intensity horn transducer will be designed to meet the following specifications:

- The Ferroperm [23] discs will again be used for this transducer.
- The driver will be designed as a separate part from the horn, as they are both \( \frac{\lambda}{2} \) sections. This also means that the driver can be used for different output sections.
- Maximum displacement will occur at the head of the transducer, which will then be coupled to the horn, so that velocity amplification can take place in the horn.
- Due to the intense nature of the cavitation expected, and due to the high stresses and strains involved, the head, bolt and horn of the transducer will have to be made from an extremely tough and corrosion resistant material. The material will also have to be relatively light to achieve widely different \( \rho c \)’s.
- The transducer should operate at around 22kHz.
- The driver will be constrained on the edge of a nodal plate in the middle of the piezoelectric discs.

6.3 Design Method

This transducer will be designed using the same method as the previous two designs i.e. the frequency equation, the lumped element approximation, the distributed parameter
approximation and finally FEM. The horn itself will be designed as a separate piece, and then added to the models where appropriate.

6.3.1 Frequency Equation Design

Due to the high stresses and strains and the intense nature of the cavitation to be produced by this transducer, it was decided that the bolt, head and horn would be made from Titanium 5 (Grade 5 – 6% Al, 4% V). This material, as well as being extremely strong, is also very light in comparison with others of similar tensile strength. This will make it ideal for the bolt, as it is has a high tensile strength while still being reasonably compliant. It is also a very tough material and so should be able to withstand the intense cavitation at the tip of the horn. Unfortunately titanium is an expensive material, but it is perfectly suited to this application and therefore will be used. Stainless steel was chosen as the material for the tail, as it has a ρc different enough from titanium and is readily available at relatively low cost. It was again decided that 4 PZT discs would be used for this application.

Using the frequency equation with the relevant parameters for the transducer, we get that the head should be 27mm long and the tail 17mm. Both parts have an outer diameter of 40mm. The full calculation can be seen in Appendix E-1.

6.3.2 Lumped element approximation

For this transducer, we can still use the simpler lumped element circuit shown in figure 19. Using the dimensions calculated in the frequency equation, the lumped element approximation predicts the results for the transducer without the rod as follows: \( f_r = 22.94 \text{kHz}, \ f_a = 31.39 \text{kHz} \) and \( k_{ef} = 0.683 \). All of the full calculations for the lumped element approximation can be seen in Appendix E-2.

Once again, we then include the rod between the head and the tail, and then approximate the actual bolt going through the tail and into the head with the following results:
With rod: \( f_r = 24.16 \text{kHz}, \quad f_o = 32.30 \text{kHz} \) and \( k_{\text{eff}} = 0.664 \).

With approximated bolt: \( f_r = 23.61 \text{kHz}, \quad f_o = 31.86 \text{kHz} \) and \( k_{\text{eff}} = 0.672 \).

### 6.3.3 Distributed Parameter Analysis

For the distributed parameter analysis of the horn transducer, we will again be using the equivalent circuit shown in figure 21. This analysis is fairly simple as there is no tapered head to include, and we will follow the same progression as previously, without and with the bolt, using the dimensions given by the frequency equation:

Without rod: \( f_r = 21.95 \text{kHz}, \quad f_o = 28.52 \text{kHz} \) and \( k_{\text{eff}} = 0.638 \).

With rod: \( f_r = 22.90 \text{kHz}, \quad f_o = 29.07 \text{kHz} \) and \( k_{\text{eff}} = 0.616 \).

The full calculations can be seen in Appendix E-3.

### 6.3.4 Horn design

The first choice one has to make with regards to the horn, is what type of horn to choose i.e. what profile do we want the horn to have. The five most common types are stepped, conical, exponential, catenoidal, and Fourier. Each type of horn has its own advantages depending on the type of intended application, as this will define the velocity-magnification required, maximum allowable stress and material choice. For our application, we require velocity amplification, and the maximum allowable strain will be quite high, as we will be using titanium as the horn material. Ideal for this application would be the Fourier horn, as this allows high amplification with relatively low strain. However, our main limitation in this case will be machining capability. The Fourier profile would be very difficult to machine, especially from a tough material such as titanium. One of the simpler designs, namely the stepped horn, will be the easiest of the choices to machine. A diagram of the basic stepped horn is shown in Chapter 2, figure 17.

As mentioned earlier, one of the main issues with horns is the maximum allowable strain. One has to ensure that that the strain is maintained as low as possible throughout the length of the horn so that the material will not fail due to fatigue. For the stepped horn,
the region of maximum strain is the region of smallest diameter, \[^{58}\] i.e. between the sections of large and small cylinder diameters. Say that we needed an amplification factor of two; this would imply that the tip of the horn would have a 20mm diameter. A 20mm tip is still quite solid, so would not flex too much, and is still large enough to process a reasonable amount of liquid. So the filet between the two sections would have a 10mm radius. This is a relatively small radius and will cause high concentrations of stresses and strains in the region, perhaps leading to fatigue failure. As we do not wish to decrease the diameter of the horn tip any further, but want to decrease the strain in the filet region, we have to increase the radius of the filet. This implies increasing the diameter of the first section of the horn, which does not have any major implications on its performance apart from a slight increase in material losses, as the overall amplification factor is still the same. So if we taper the diameter of the first section of the horn out to 60mm, this means that the filet radius is now 20mm. An initial design of the horn and the horn driver is shown below.

![Diagram of horn design](image)

**Figure 43: Initial design of horn shown with horn driver.**

In the drawing, the filet section is also shown as a taper, as this is the way we will model that section using distributed parameter analysis. The letters above the horn indicate the sections of the horn as they will be modelled.

In order to calculate the lengths of the sections of the horn, we will use the method given in "Ultrasonic Engineering," \[^{58}\] pages 90 and 91. We will treat sections w and x as one, and sections y and z as one, to simplify the calculation. The calculation uses the wavelength in the horn material at the desired resonant frequency, the radii of the two sections and the design curves given on page 90. If we put in all the relevant information
for our horn design, we get that the large diameter section should be 55.1mm and the
smaller section 58.8mm. We now have all the relevant dimensions for the horn, so we can
model it with the horn driver using distributed parameter analysis.

In order to model the horn as accurately as possible, each of the sections of the horn has
to be modelled using its own T-piece in the equivalent circuit. The T-pieces are then
cascaded together and joined to the equivalent circuit for the driver. A basic schematic is
shown below.

![Schematic](image)

**Figure 44: Basic schematic of equivalent distributed parameter circuit for horn
driver with horn**

Once again, this type of analysis would be very difficult and time consuming to do
manually, so we will use Mathcad. Using the dimensions from the earlier calculation, we
get the following results: \( f_1 = 22.59 \text{kHz}, \ f_a = 24.83 \text{kHz} \) and \( k_{ef} = 0.415 \). The full
calculation can be seen in Appendix E-4.

We could try to refine and optimise this model more using distributed parameter analysis,
as this method seems quite accurate. However, FEM is a much more powerful tool for
doing this kind of optimisation, so we will now go on to the FEM modelling of the horn
transducer.
6.3.5 FEM design of the horn transducer

As mentioned earlier, the horn driver is the simplest of the three transducers as it is a symmetrical sandwich transducer. We will again follow the same design steps as we did for the 1-D modelling to see how the results compare. We will not however include all the pictures of the steps, as they are very similar to those from the first steps of the previous designs and do not teach us anything new.

The head and tail of the transducer are first modelled on their own with two PZT discs and the results are the following.

**Head:** 
\[ f_c = 23.597 \text{kHz}, \quad f_a = 26.443 \text{kHz} \text{ and } k_{ac} = 0.431. \]

**Tail:** 
\[ f_c = 24.506 \text{kHz}, \quad f_a = 27.966 \text{kHz} \text{ and } k_{ac} = 0.482. \]

Again, the results from Abaqus are somewhat higher than the 1-D results. We next model the head and tail assemblies together, without and then with the rod.

**Without rod:** 
\[ f_c = 21.612 \text{kHz}, \quad f_a = 25.222 \text{kHz} \text{ and } k_{ac} = 0.516. \]

**With rod:** 
\[ f_c = 22.676 \text{kHz}, \quad f_a = 26.133 \text{kHz} \text{ and } k_{ac} = 0.497. \]

The next step is to taper the tail of the transducer slightly. This again improves the magnitude of admittance. The transducer, its piston mode, and the graphs of the output are shown below.

![Figure 45: Horn driver and its piston mode](image-url)
The results for the transducer are as follows:

\[ f_s = 24.457 \text{kHz}, \quad f_a = 26.724 \text{kHz} \text{ and } k_{eff} = 0.403 \]

The final stage for the horn driver is to include all the details such as the nodal plate, electrodes, and the actual bolt. The actual bolt goes all the way through the head and extends beyond the head, as the horn will be screwed on to this section. The final transducer and its piston mode are shown on the next page. The results for the final transducer are as follows: \( f_s = 20.963 \text{kHz}, \quad f_a = 23.801 \text{kHz} \text{ and } k_{eff} = 0.474 \).
We will now model the horn for the transducer. Firstly, we model the horn and transducer from the 1-D design to compare the results. We will then refine and optimise the horn as far as possible in FEM. Using the dimensions obtained from the calculation method in "Ultrasonic Engineering," the results for the simulation of the basic horn system are as follows: $f_1 = 24.667\text{kHz}$, $f_2 = 25.135\text{kHz}$ and $k_{\text{eff}} = 0.192$.

The horn is then modelled as accurately as possible and optimised to produce the maximum displacement at the tip of the horn. A small nodal ridge is included close to the nodal point of the horn to provide another option for mounting or supporting the transducer. The filet between the large and small cylinders is also included. The final design of the horn system and some of its significant modes are shown below.

**Figure 47: Final horn driver and its piston mode**
The output of the transducer radiating into air is shown below.
Figure 51: Output of horn transducer radiating into air

In the above graph, the other modes are the bending mode of the nodal plate and the mode shown in figure 52. The results for the final transducer with the horn are as follows: \( f_s = 20.971 \text{kHz}, f_a = 21.275 \text{kHz} \) and \( k_{eff} = 0.168 \). It seems that including the horn in the model decreases the effective coupling factor significantly. It is not known what causes this effect, but it may have something to do with the large increase in the “Q” of the device when the horn is added.

6.3.6 Water testing

In order to test the performance of the transducer, we again test it in water to see how the piston mode is damped. We radiate once again into a large hemisphere of water to ensure that there is no interference from reflected energy. The output of the transducer in water in shown on the next page in figure 54. Notice how only the piston mode is damped significantly, while the other modes remain largely unaffected.
From the above results, we conclude that the design of the transducer is satisfactory. The Abaqus code to simulate the transducer radiating into water can be seen in Appendix E-5. The dimensions for the transducer were taken from the Abaqus simulation, and the design drawings can be seen in Appendix E-6.

6.4 Conclusion
Agreement between the various design methods is again good. Due to the complex shape of the horn, the lumped element method was not used in the design of the horn. The fact that the distributed element method can handle this type of complex system shows how powerful this method can be. The FEM results clearly show the damping phenomenon of the piston mode when it is loaded in water, while the other modes are essentially unchanged. The next chapter will discuss the very important topic; the manufacture and assembly of all of the transducers.
Chapter 7: Transducer Construction

7.1 Introduction

The next step in the process is the manufacture and assembly of the transducers. This is an extremely important step as poor machining and assembly can ruin a perfectly well designed transducer. There are many important factors to be considered here such as the PZT discs themselves, surface finish and pre-stress, all of which will now be discussed in detail. The general factors that apply to all the transducers will be discussed first, followed by those that only apply to the specific transducers.

7.2 PZT discs

7.2.1 Electrical Characteristics

The first two characteristics that should be checked are the electrical capacitance and the dissipation factor. Both of these can easily be measured using a standard LCR bridge set on low frequency. The expected capacitance of the discs can be found using the standard formula \[ C = \frac{\varepsilon_r A}{t} \] \( (\varepsilon_r = \text{dielectric constant}, A = \text{cross-sectional area}, t = \text{thickness}) \). For high power, high frequency applications the dissipation factor should be as low as possible, normally below about 0.004, as higher values will lead to excessive heating in the discs during operation. If either the capacitance or the dissipation factor stray from their expected values by a significant amount, the discs are more than likely substandard and should not be used.

Another electrical characteristic of the PZT discs that should be tested is the poling direction. Most manufacturers indicate the poling direction on the disc somehow, but this is not always marked very clearly. Obviously having a disc the wrong way around in an assembly would cause it to fail, so it is worth performing this simple test. All it basically requires is a simple op-amp circuit \[^{18} \] that allows you to squeeze the disc and observe to see if you get the expected output.
7.2.2 Mechanical Characteristics

Most ceramic manufacturers will ship a specification sheet with their products or the customer will define their required specifications. Either way, all of the physical dimensions of the discs should be thoroughly checked to see that they are within the required tolerance. Not only should the diameters and thickness of the discs be checked, but also the flatness / curvature. Obviously, this should be kept to an absolute minimum to prevent the discs from cracking when being pre-stressed. It is also a good idea to weigh the discs to check that the density of the piezo-ceramic is correct.

Another physical aspect to check is the surface finish / condition of the discs. There should be no visible cracks and chips must be relatively small for them not to have a significant effect. The surface finish of the discs should be smooth enough to ensure maximum contact within the assembly. After manufacture, the ceramic can sometimes have a somewhat powdery finish to it due to the manufacturing process. It is therefore advisable to specify a surface finish to the manufacturer if this is possible. The final surface characteristic to check is the electrode. One should make certain that both sides of the disc have evenly coated surfaces to ensure proper electrical contact. If connections are going to be soldered onto the electrodes, one should verify that this can be done with relative ease and repeatability.

7.2.3 Resonance Characteristics

The final criteria that must be checked before the discs are used are the resonance characteristics. These are best viewed on a network analyser which can give you real-time data on all the necessary admittance / impedance characteristics. The first test to perform is a broadband sweep of the admittance curve to check for any spurious resonances. The presence of unexpected peaks usually means one of two things; either the disc is cracked or it has an uneven distribution of piezoelectric coefficient. Both of these faults could lead complete failure of the system and so should be avoided at all costs. It should also be checked that the fundamental resonances of the disc occur in the expected frequency ranges. Below is an example showing the thickness resonance of a
good disc as opposed to a disk that is either cracked or has an uneven distribution of
piezoelectric coefficient.

Figure 53: Graph showing thickness resonance of a good disc.

Figure 54: Graph showing thickness resonance of a faulty disc.
7.3 Machined parts

By machined parts here we obviously mean the rest of the parts of the transducer; the head, tail, flange, bolt, electrodes and isolation ring (iso ring). One should check to ensure that all of the pieces are within the specified tolerances. Of critical importance are the perpendicularity and surface finish tolerances. The perpendicularity ensures that there will be no spurious modes induced by bending modes, and the surface finish is vital to ensure good coupling, especially if glue is not being used. In fact, the head, tail, nodal plate, and electrodes (if possible) should all be lapped in order to ensure maximum contact between all of the interacting surfaces. Another important point not to forget is the alignment/concentricity of all the parts. If any of the pieces are off-centre, they could cause spurious modes, uneven wear and possibly failure of the transducer. This is why we include the iso ring, short for isolation ring, as it also isolates the bolt from the high voltages on the PZT discs. Finally, all of the parts, including the discs, should be thoroughly cleaned before being assembled. This means cleaning with ordinary soap as well as with a good degreasing agent such as M. E. K (methyl ethyl ketone).

7.4 Pre-Stressing the transducers

This is a very important part of the transducer construction as it not only protects the discs from possible fracture but also plays a vital role in the coupling of the transducer. It is a well-known fact that PZT discs have a much higher compressive than tensile strength, so sufficient pre-stress will ensure that the discs never go into tension. If the transducer is not being glued together, pre-stress will ensure that there is maximum contact between parts and enhance the coupling factor.

7.4.1 Theory

To make sure that the maximum compressive stress for the piezoceramic discs is not exceeded during the process, the relevant values should be calculated beforehand. According to Berlincourt et al.,\(^3\) the maximum allowable compressive stress for PZT 4 piezoceramics is approximately 55Mpa at 25°C and 28Mpa at 100°C. PZT 4 is the same type of ceramic as PZ26 so these values apply to the discs we are using. These maximum values however are not set in stone, as many authors give different values, as noted by
Butler et al. [60] As our values of torque and pressure are substantially lower than most of the values given, we are not overly concerned with the absolute maximum possible values. In order to work out what pressure is applied to the discs for a given torque, we have to use the formula below given in Shigley. [61]

\[ T = F_r \left[ \frac{d_m}{2} \left( \frac{\tan \lambda + \mu \sec \alpha}{1 - \mu \tan \lambda \sec \alpha} \right) + \frac{\mu_c d_c}{2} \right] \]

The symbols in the above equation stand for the following:
- \( T \) = torque,
- \( F_r \) = force,
- \( d_m \) = mean diameter,
- \( \lambda \) = lead angle,
- \( \mu \) = coefficient of friction,
- \( 2\alpha \) = thread angle,
- \( \mu_c \) = coefficient of collar friction,
- \( d_c \) = mean collar diameter.

There is a simpler equation given in Shigley, but this only applies to standard bolts, not machined ones, as ours is. Care should be taken to use the correct formula as they give quite different results. We will use the formula to get the force obtained from certain torques, then covert the force into a pressure by dividing by the surface area of the discs.

For our 8mm bolt, the following results are obtained:

<table>
<thead>
<tr>
<th>Torque (Nm)</th>
<th>Force (kN)</th>
<th>Pressure (Mpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>7.351</td>
<td>7.085</td>
</tr>
<tr>
<td>40</td>
<td>9.802</td>
<td>9.448</td>
</tr>
<tr>
<td>50</td>
<td>12.252</td>
<td>11.809</td>
</tr>
<tr>
<td>60</td>
<td>14.702</td>
<td>14.171</td>
</tr>
<tr>
<td>65</td>
<td>15.927</td>
<td>15.351</td>
</tr>
</tbody>
</table>

Table 1: Table showing expected pressure for given torque

So we see that we can quite safely go up to 65 Nm torque without damaging the discs at all. However, using a torque wrench alone is not an accurate enough measurement of torque applied due to friction effects in the threads and at the collar. A more accurate method of measuring the torque / pressure applied is to use the pre-stressing method outlined in Bodholt’s [62] paper. This method measures the potential difference generated across the piezoceramic stack during pre-stressing and relates it back to pressure and hence the torque applied. The following circuit was used to measure the voltage generated.
Figure 55: Circuit for measuring voltage generated during pre-stressing.

Although the op amp in the above circuit helps to reduce the leakage current while the reading is being taken, it is still difficult to get a stable and reliable measurement. However, the readings seem accurate enough, as the correlation between the predicted and results is quite good. The equation given by Bodholt [62] is the following:

\[ T_3 = \frac{UC}{g_{33} \varepsilon_{33} nA} \]

where \( T_3 \) = stress (pressure), \( U \) = voltage, \( C \) = capacitance of capacitor C1, \( g_{33} \) = piezoelectric constant, \( \varepsilon_{33} \) = dielectric constant, \( n \) = number of discs and \( A \) = surface area of discs. Using the above formula, we can calculate the expected voltage for a given torque.

7.4.2 Results and some effects of pre-stress

The table below shows the results for torquing the horn driver, comparing the measured and predicted voltages.

<table>
<thead>
<tr>
<th>Torque (Nm)</th>
<th>V predicted (V)</th>
<th>V measured (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>4.40</td>
<td>3.51</td>
</tr>
<tr>
<td>40</td>
<td>5.87</td>
<td>4.62</td>
</tr>
<tr>
<td>50</td>
<td>7.34</td>
<td>5.85</td>
</tr>
<tr>
<td>60</td>
<td>8.81</td>
<td>6.78</td>
</tr>
<tr>
<td>65</td>
<td>9.54</td>
<td>7.71</td>
</tr>
</tbody>
</table>

Table 2: Table showing predicted and measured voltages for given torque.
As one can see from the above table, the measured voltages are somewhat lower than the predicted ones. This is however expected, as some of the torque is "lost" to friction in the threads and nut collar. A small amount of lubricating oil should be placed on the threads and the collar to try and minimise the friction between parts. All of the transducers were pre-stressed using this method, and all gave similar results. One should note that each measurement in the above table was not done sequentially i.e. the torque was taken from zero to the relevant value each time, making sure that there was no residual charge left on the transducer from the previous measurement.

One of the effects of pre-stress is that it increases the resonant frequency of the transducer quite substantially. Results from a nodal-mounted tonpilz and the horn driver are shown below.

![Admittance vs Frequency Nodal 1 (Various torque levels)](image)

**Figure 56:** Graph for nodal-mounted transducer showing increase in resonant frequency with torque increase.
Figure 57: Graph for Horn driver of increasing resonant frequency for increasing torque.

One can clearly see from the above graphs that resonant frequency does increase quite substantially with increased pre-stress. Intuitively it would seem that that this increase in resonant frequency is due to the fact that the bolt is being “stretched.” If we consider the mechanical model of the transducer, stretching the “spring” would increase the overall resonant frequency of the transducer. However, if one tries to model this, it does not work, as the actual Young’s modulus of the bolt does not change as the pre-stress is increased. Arnold and M+hlen [63] conducted experiments that show that this increase in resonant frequency is due to improved acoustical contact between the transducer pieces and not due to any alteration in the physical parameters of the ceramics or metallic pieces. This appears to be true if one considers the graph of the nodal-mounted transducer. At 30Nm there is not enough pre-stress to create sufficient coupling of the pieces. Only after 40Nm do you get enough pressure to induce sufficient contact of the pieces. As you increase the pre-stress further, it appears that the admittance no longer changes, only frequency. This might suggest that coupling only increases up to a certain point with increasing pre-stress.
It is interesting to observe what happens to the characteristics of the transducer after pre-stressing. Below is a graph showing the progression of the horn driver after being torqued up to 65Nm.

![Admittance vs Frequency (Horn Driver after torquing)](image)

**Figure 58: Graph showing progression of horn driver after torquing.**

Looking at the above graph, we can see that there is quite a drastic change immediately after torquing the transducer. This effect is thought to be due to the transducer “settling,” or allowing the stresses to distribute themselves evenly throughout the transducer. Driving the transducer also had a significant effect on the characteristics of the admittance curve. Not only does the resonant frequency increase, but the magnitude of the admittance and the “Q” of the transducer decrease substantially. It could be argued that the increase in frequency is again due to improved acoustic coupling after the mechanical agitation of being run for an hour. This theory does not however account for the drop in the “Q” of the transducer. There is another quite drastic change after the transducer has been run, probably again due to “settling.” Then, over the next eight months, the resonant frequency changes very little.
After the two nodal-mounted transducers had been assembled, they were tested and used in some experiments. It was noticed that the 2\textsuperscript{nd} transducer was performing poorly compared to the first. This seemed strange, as the two transducers are identical. It was decided they should be disassembled and checked for faults. Having done so, it was discovered that the 2\textsuperscript{nd} transducer had a cracked disc. Some photographs of the various discs from both transducers are shown below.

![Photographs of discs](image_url)

Figure 59: Photographs of some of the discs used in the nodal mounted transducers.

*top row from 2\textsuperscript{nd} transducer, bottom row from 1\textsuperscript{st} transducer.*

From the wear shown on the discs from the 2\textsuperscript{nd} transducer, it appears that the fracture occurred in the disc at an early stage. It is apparent that it was causing some asymmetric modes that made the discs wear unevenly. One can see that the discs from the 1\textsuperscript{st} transducer wore relatively evenly. The crack in disc 22 could have been caused by a number of factors. One of the factors may be that the disc wasn’t perfectly flat and cracked as soon as the transducer was torqued. The flatness of the discs could not be tested at the time due to the lack of the proper equipment. Another possibility is that the transducer was over-torqued, as the transducers were not accurately pre-stressed the first time. This is quite unlikely though, as the discs can withstand very high levels of compressive stress. The poor standard of the electrodes is another possibility for the
cause of the fracture. The electrodes were hand-cut out of a sheet of copper and were therefore not flat, and the diameter of the hole through the electrodes was too big, allowing them to sit off-centre in the assembly. Both of these factors could have caused asymmetrical modes, which may have in turn caused the fracture of the disc. The final possibility to consider is that the transducer was over-driven when it was first used, but this is again unlikely as it takes very high voltages to physically crack a disc.

The construction of the two nodal-mounted transducers was generally poor the first time. All of the factors, from the sub-standard electrodes to inaccurate torquing contributed to poor transducers. This point illustrates the importance of proper assembly methods and practices, as the second time the transducers had no performance problems after being correctly assembled.

7.5 Gluing of the transducers

7.5.1 Preparation

If the transducers are going to be glued, most of the preparation and assembly techniques are the same, but there are a few critical differences. The two most important factors when preparing to glue the assembly together are the cleanliness and condition of the interacting surfaces. There is no need to lapp the surfaces as before, in fact they need to be abraded to ensure a good glue bond. For the PZT discs, this entails scrubbing them lightly with a scouring pad and powder, such as “Comet” or “Vim.” Care should be taken here to not scrub too hard, as the electrode is usually a very thin layer that could easily be damaged or rubbed off completely in places. For the rest of the components, all of the contacting surfaces should be sanded with light / medium weight sand paper, just enough to “rough up” the surfaces. This improves by the glue bond in two ways: it removes any oxide / dirt layer that may be on the surface of the material and it increases the bondable surface area. Following this, all the pieces should be thoroughly cleaned with an effective degreasing agent such as M.E.K. to ensure that there is absolutely no grease / oil on the surfaces. The presence of any oily substance will compromise the integrity of the bond, thus possibly compromising the performance of the transducer.
7.5.2 The epoxy adhesive process

For the gluing of the nodal-mounted transducer, we require a high temperature, high strength epoxy. The choice of epoxy used here cannot be disclosed as it is proprietary information, but it was chosen for its high bond strength and ability to operate at temperatures up to 230°C continuous, or 280°C short-term operation. Its high-temperature ability means that it has a high "glass transition" temperature. This term is used to describe the temperature when the epoxy begins the transition between solid and liquid. We obviously need this limit to be as high as possible, as surpassing it would lead to substantial losses in the transducer.

Once all of the parts have been thoroughly cleaned, the epoxy should be mixed according to the manufacturer's instructions. A very thin layer should then be applied to both surfaces of each interface before joining them. It should be noted that the epoxy itself should not form a layer on its own; it is merely the bonding medium between the parts. A thin layer of epoxy will also ensure good electrical contact, as any excess will easily be squeezed out. Once the pieces of the transducer are all joined, the pre-stress bolt should initially be very lightly torqued, increasing the torque gradually over the next few hours. As the epoxy has a long working life (more than 12 hours), this allows the excess to steadily be squeezed out. Once there are no longer visible signs of any excess, the torque should be taken down to zero before torquing up to the full value, using the pre-stress circuit as before to ensure accurate torquing. After checking that there is full electrical contact in the transducer, it should be put in an oven to cure as per the manufacturer's guidelines. The easiest method of checking for electrical connection is to measure the capacitance of the stack and make sure that it comes to the expected value.

7.5.3 Results

After the transducer had been cured and cooled down, it was tested on the network analyser. It was then tested again that same day in the evening, to see if any change had occurred. All of the results are shown below.
Figure 60: Graph showing progression of Rex1 with gluing.

The "no glue" curve was measured just before the transducer was disassembled, the "wet glue" curve just after torquing the transducer up to full torque, the "glue 9am" after cooling from the oven the next day and the "glue 6pm" measurement was taken at 6 pm that evening. As can be seen from the graph, the only significant change is that the resonant frequency has increased slightly. This suggests that there is only a slight improvement in the coupling of the transducer when epoxy is used. It must be noted though, that this measurement is at low signal.

In order to check if the coupling into water improved at all with the transducer being glued, measurements were also taken before and after the gluing process. It should be noted in the following graph that the resonance we are interested in is the first one. The second mode in the graph is a water mode and is a function of how far the face of the transducer is from the bottom of the measurement vessel.
Figure 61: Graph showing the transducer results in water.
In the above graph, the transducer mode is the one that is marked at 21.672 kHz. This would appear to be correct as we expect the resonant frequency to decrease and the magnitude of the admittance to be damped substantially.

7.6 Transducer specific factors
The factors that must be taken into consideration for each transducer will now be discussed.

7.6.1 Nodal-mounted transducer
If the construction and assembly guidelines discussed earlier are followed for this transducer, there should be no significant problems. One problem that was noticed after the transducer had been assembled was vibration in the nodal plate. This would seem strange as the name nodal plate implies that the plate is placed at a velocity node in the assembly. If one looks at figure 29 in Chapter 4, one can see a small amount of movement in the nodal plate in the piston mode, so perhaps more care should be taken to minimise the problem during this stage of design. However, in practice, this is very
difficult to do. So to reduce the amount of vibration, we simply reduced the diameter of the nodal plate to 50mm. This achieved the desired result.

Another factor that has to be taken into account for this transducer is how to tighten the assembly. As the head of the transducer would be damaged if put directly into a vice, a vice grip was designed that fitted snugly around the head. The drawing for the vice grip can be seen in Appendix C-8.

7.6.2 Face-mounted transducer

Initially, the face-mounted transducers were designed to be used on a one per bucket basis. However, due to limited power / cavitation capabilities of the system, it was decided that the two transducers with the tapered tails should both be mounted on a single plate. This would increase the power capabilities of the system, hence allowing a greater volume of liquid to be processed, or smaller quantities to be processed at higher intensities. This was done successfully, as shown in the photographs later in the chapter.

Once again, we have to design a special vice grip for the head of the transducer so that the assembly can be torqued without damaging any parts. The vice grip for this transducer can be seen in Appendix D-5 with the other design drawings.

For the face-mounted transducer, great care should be taken when gluing the transducers onto the buckets. Both surfaces should be sanded to roughen them slightly, then thoroughly cleaned with soap and water followed by a good degreasing agent such as M.E.K. Once the epoxy has been mixed and the parts joined, a large pressure should be applied to the parts to ensure that all of the excess epoxy is squeezed out, as we want as thin a joint as possible. Of course, care must be taken so that the parts do not shift at all during the curing so that the transducer stays centred on the plate. Finally, as the bucket is joined to the plate with bolts, silicon grease or some other suitable sealing agent should be used in between the plate and the bucket to prevent leaks from occurring.
7.6.3 Horn transducer

Once again, we require vice grips for the head, as well as the horn for torquing the assembly, the first to torque the horn driver, the second to torque the horn onto the head. They are however a little simpler this time, as there is no taper to take into account. The vice grips can be seen in Appendix E-6.

During the machining of the parts for the horn transducer, a problem was encountered with the head and bolt. As per the design from Abaqus, both the head and the bolt were to be made from grade 5 Titanium. The problem occurred when the machinist tried to test the threads on the bolt by screwing it into the head. During this process, the threads galled and it became impossible to remove the bolt from the head. Galling is a fairly common occurrence, especially when the bolt and hole are made from the same material. It is possible to avoid galling by taking great care throughout the machining and testing process, or by having the threads rolled instead of cut. There are times however when it is unavoidable. The outcome of this problem was that we had to use a high tensile steel bolt instead, as the Titanium had been imported from the USA at great cost. So the replacement bolt was first simulated in Abaqus, and appeared to only have the effect of reducing the magnitude of the admittance and decreasing the resonant frequency slightly. Both of these effects were expected as the high tensile bolt is not as compliant as the Titanium bolt, but substantially more dense, giving it a lot more mass.

Photographs of all three assembled transducers are shown on the next page.
7.7 Photographs of assembled transducers

Figure 62: Nodal-mounted transducer

Figure 63: Face-mounted transducers, tapered tails on left, flat tail on right

Figure 64: Horn transducer
7.8 Conclusion

It is clear from this chapter that the manufacture and assembly of the parts that make up these transducers play a vital role. It is essential that the utmost care be taken throughout the whole process to ensure the most efficient and effective transducers. Probably the most difficult process to perform accurately is the torquing of the transducers. This is because of the friction involved in the method. An alternative method should be looked into for this process. The rest of the processes are fairly straightforward and should result in good transducers if the steps are followed correctly. Unfortunately, due to time constraints, the glued transducer was not tested at high power. It is vital that the effects of gluing the transducer together be investigated at high power, as it is likely that the efficiency of the transducer would be increased dramatically.

In the following chapter, the transducers will be used in some tests and experiments, and we will compare the results from the design chapters to the actual measured results from the transducers.
Chapter 8: Comparison between predicted and measured results.

8.1 Introduction

The first set of tests will essentially be comparison testing between the predicted results (from the 1-D and FEM modelling) and the measured results of the actual transducers. All of these measurements will be performed on the Network Analyser at low signal.

8.2 Nodal-mounted transducers

The nodal-mounted transducers went through various stages before we were satisfied that that they had been assembled and were performing satisfactorily. To prove a point of how important assembly and construction is, the first few results will be from the first attempt at assembling the transducers. The results below are from the 2nd transducer; the results for the 1st one are similar, though not quite as bad. For reference, the predicted results from Abaqus are shown again below.

![Admittance vs Frequency (Nodal-mounted in air)](image)

Figure 65: Predicted results for nodal-mounted transducer
The graph below shows the results from the 2\textsuperscript{nd} transducer after it had been poorly assembled. It was not torqued accurately, the electrodes had been badly made and not enough care was taken during the assembly process.

![Admittance vs Freq (Nodal-mounted #2 in air)](image)

**Figure 66: Nodal-mounted #2 after poor assembly**

One notices in figure 65 that the resonant frequency is very low, as is the magnitude of admittance. Both of these factors suggest that there is very poor coupling between the parts.

![Admittance vs Freq (Nodal-mounted #2 in water)](image)

**Figure 67: Nodal-mounted #2 in water**
We then tested the transducers in water; the results for the 2nd transducer are shown on the previous page. One can see from the above graph that the magnitude of the main resonance is barely damped at all. This implies that there is hardly any coupling into the water, another sign of a poor transducer. The slightly higher mode is probably a standing wave in the water.

![Admittance vs Frequency](image)

**Figure 68: Nodal-mounted transducers after settling and being driven**

So the transducers were taken apart and re-assembled with new electrodes and better-matched discs, with the rest of the parts obviously being the same as before. They were then torqued up carefully using a torque wrench and the charge circuit, and then re-tested at the various stages. It turned out that the 1st transducer had to be torqued up to 65Nm to match the 2nd in frequency (torqued to 50Nm). This may have been due to a number of reasons such as a slight difference in the characteristics of the discs or a small difference in surface finish of the parts, which lead to greater friction in the one transducer. Directly after torquing, the two transducers were matched to within 10 Hz of each other at 21840 and 21830Hz. After settling and being driven briefly, the resonant frequencies had increased slightly as shown below. Now the resonant frequencies are much closer to the predicted value and the magnitude of the admittance has increased substantially. The
difference in magnitudes is again probably due to variation in the discs. When the transducers were tested in water, the results were vastly improved as shown in figure 68. We can see that the damping of the piston mode is much greater than previously, which implies that the transducers are performing much better.

All of the above results just go to show the importance and absolute necessity of careful construction and assembly of the transducers.

![Admittance vs. Freq (Nodal 1 & 2 in water)](image)

**Figure 69: The improved transducers radiating into water**

Table 3 is given as a summary of the significant predicted and measured results.
Looking through the figures, we see that the predicted values match up with the measured ones remarkably well. This is especially true of the series resonance frequency. Even the effective coupling factors are reasonably accurate, especially given the simplicity of the 1-D approximations.

From these results, we conclude that the design method works very well for the nodal-mounted transducers.

### 8.3 Face-mounted transducers

The first set of results we will show is the comparison of the transducers with and without the tapered tail. Abaqus predicted that there would be an increase in the magnitude of admittance and frequency with the tapered tail. This is shown to be true in the graph below.

<table>
<thead>
<tr>
<th></th>
<th>Series Resonance (kHz)</th>
<th>Parallel Resonance (kHz)</th>
<th>Effective Coupling Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lumped (No taper)</td>
<td>23.960</td>
<td>32.0</td>
<td>0.663</td>
</tr>
<tr>
<td>Distributed (No taper)</td>
<td>23.70</td>
<td>30.0</td>
<td>0.613</td>
</tr>
<tr>
<td>FEM (No taper)</td>
<td>23.146</td>
<td>26.666</td>
<td>0.497</td>
</tr>
<tr>
<td>Distributed (Tapered)</td>
<td>22.13</td>
<td>28.01</td>
<td>0.613</td>
</tr>
<tr>
<td>FEM (Tapered)</td>
<td>21.863</td>
<td>25.176</td>
<td>0.496</td>
</tr>
<tr>
<td>FEM (Final)</td>
<td>21.784</td>
<td>24.443</td>
<td>0.453</td>
</tr>
<tr>
<td>Actual # 1</td>
<td>21.950</td>
<td>25.870</td>
<td>0.529</td>
</tr>
<tr>
<td>Actual # 2</td>
<td>21.960</td>
<td>26.140</td>
<td>0.542</td>
</tr>
</tbody>
</table>

Table 3: Table comparing predicted and actual results for nodal transducers
Figure 70: Comparison of transducers with and without taper in tail

We now compare the results for the transducers mounted on the bucket. The Abaqus results are shown again below for reference.

Figure 71: Abaqus results for face-mounted transducer
Figure 72: Transducer with untapered tail, mounted on plate

Figure 73: Transducers with tapered tails, mounted on same plate, measured separately
From figure 72, we see that the resonant frequency of the single transducer with the untapered tail is slightly higher than that predicted by Abaqus, but still quite close. For the two transducers with tapered tails on the same bucket, we did not simulate this combination in Abaqus as it would have required a large 3-D model that would have required too much computing time and memory. However, we see that the resonant frequency of each of the transducers is only slightly lower than the single transducer. It is difficult to predict what to expect of the resonant frequency with the double transducer system, as it is now a fairly complex system with factors to consider such as the mass loading effect of one transducer on the other, the increased stiffness of the plate and more complex and numerous modes due to the extra transducer. Hence, the rather “messy” output in figure 73. Obviously, when we measure the two transducers together, the outputs simply add as shown in figure 74 below.

![Admittance vs Frequency (Tapered tail transducers measured together)](image)

**Figure 74: Double transducer system, transducers measured in parallel**

When we put water into the buckets (single and double systems), we get the following results.
Looking at figures 75 and 76, one would believe that the results were wrong, as the resonant frequency has increased, as has the magnitude of admittance in both cases.
However, these seemingly anomalous results are due to the effect of standing waves in the water. Consider the formula \( f_n = \frac{c}{\lambda} \) where \( f = \) frequency, \( n = \) mode number, \( c = \) speed of sound in water and \( \lambda = \) wavelength (depth of water). The speed of sound is constant (1428m/s) and the resonant frequency of the transducer is around 26kHz. Therefore, if \( n=1 \), we expect a standing wave to occur when the depth of water is approximately 55mm, and at 110mm when \( n=2 \) and so on. This effect can actually be observed on the Network Analyser when water is slowly added to the bucket, the standing wave can be seen moving down in frequency, then it couples with the resonant mode of the transducer and then continues to decrease in frequency as the bucket is filled. So what we are seeing in the graphs above is actually the transducer mode coupled with the standing wave. The slight increase in frequency is due to the frequency-pulling effect that occurs when the resonant frequencies are close together.

Unfortunately, because we designed the face-mounted transducer as a symmetrical sandwich and then modified it, we cannot compare some of the 1-D results with the measured ones. So the results from the symmetrical approximation will be compared with each other, and then the results for the actual transducer and its approximations will be compared in table 4 on the next page. In the table, sym. stands for symmetric.
<table>
<thead>
<tr>
<th></th>
<th>Series Resonance (kHz)</th>
<th>Parallel Resonance (kHz)</th>
<th>Effective Coupling Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lumped (No taper, sym.)</td>
<td>26.74</td>
<td>30.54</td>
<td>0.483</td>
</tr>
<tr>
<td>Distributed (No taper, sym.)</td>
<td>27.15</td>
<td>30.25</td>
<td>0.441</td>
</tr>
<tr>
<td>FEM (No taper, sym.)</td>
<td>26.208</td>
<td>28.129</td>
<td>0.363</td>
</tr>
<tr>
<td>Distributed (Tapered, sym.)</td>
<td>24.23</td>
<td>27.03</td>
<td>0.444</td>
</tr>
<tr>
<td>FEM (Tapered, sym.)</td>
<td>23.576</td>
<td>25.207</td>
<td>0.354</td>
</tr>
<tr>
<td>Distributed (Tapered, asym.)</td>
<td>28.40</td>
<td>32.22</td>
<td>0.472</td>
</tr>
<tr>
<td>FEM (Tapered, asym.)</td>
<td>27.272</td>
<td>29.325</td>
<td>0.367</td>
</tr>
<tr>
<td>FEM (final system)</td>
<td>25.704</td>
<td>27.199</td>
<td>0.327</td>
</tr>
<tr>
<td>Actual</td>
<td>26.330</td>
<td>28.820</td>
<td>0.407</td>
</tr>
</tbody>
</table>

Table 4: Comparison of results for face-mounted transducer

Once again, it appears that the relevant figures match quite well. The matching for this transducer is not as good as it was for the nodal-mounted, but then one would expect this as the face-mounted system is more complex and therefore more difficult to model accurately in both 1-D and FEM. However, the above results are quite satisfactory.

8.4 Horn transducer

As mentioned in Chapter 7, the titanium bolt had to be replaced by a high tensile steel one. This means that most of the modelling was done for the titanium bolt. So we will still compare the modelled results for the titanium bolt, and then only compare the last few Abaqus simulations with the actual transducer. The bolt only makes a small difference to the output, as is shown below in the Abaqus results.
As one can see, the change is not that significant, so the high tensile bolt was used. The horn driver was then assembled with great care, and torqued up to 65Nm as was shown in Chapter 7. Once the transducer had been driven for a while and allowed to settle, it was then measured on the network analyser. The results are shown below.

Figure 77: FEM comparison of the horn driver with different bolts
Figure 78: Results of the horn driver, measured on Network Analyser. One can see that the resonant frequency is very closely matched to that predicted by Abaqus. The horn is then coupled with the driver, with the results shown below.

Figure 79: Horn transducer in air
The transducer is then tested in water, with the results shown below.

**Figure 80: Horn transducer radiating into water**

If we consider the results from Abaqus (figures 51 and 52 in Chapter 6), we notice that all three modes of the transducer are quite accurately predicted. Even the results of the transducer in water are well predicted. This gives us confidence that these types of transducers can be modelled very accurately using Abaqus.

Table 5 on the next page is given as a summary of the various results from the design process of the horn transducer.
<table>
<thead>
<tr>
<th></th>
<th>Series Resonance (kHz)</th>
<th>Parallel Resonance (kHz)</th>
<th>Effective Coupling Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lumped (Driver)</td>
<td>23.61</td>
<td>31.86</td>
<td>0.672</td>
</tr>
<tr>
<td>Distributed (Driver)</td>
<td>22.90</td>
<td>29.07</td>
<td>0.616</td>
</tr>
<tr>
<td>FEM (Driver)</td>
<td>24.457</td>
<td>26.724</td>
<td>0.403</td>
</tr>
<tr>
<td>FEM (final driver Ti5 bolt)</td>
<td>20.963</td>
<td>23.801</td>
<td>0.474</td>
</tr>
<tr>
<td>FEM (final driver SS bolt)</td>
<td>20.626</td>
<td>21.893</td>
<td>0.335</td>
</tr>
<tr>
<td>Distributed (Horn)</td>
<td>22.59</td>
<td>24.83</td>
<td>0.415</td>
</tr>
<tr>
<td>FEM (Horn)</td>
<td>24.667</td>
<td>25.135</td>
<td>0.192</td>
</tr>
<tr>
<td>FEM (final horn Ti5 bolt)</td>
<td>20.963</td>
<td>21.271</td>
<td>0.169</td>
</tr>
<tr>
<td>FEM (final horn SS bolt)</td>
<td>20.971</td>
<td>21.275</td>
<td>0.168</td>
</tr>
<tr>
<td>Actual with SS bolt</td>
<td>21.320</td>
<td>21.620</td>
<td>0.166</td>
</tr>
</tbody>
</table>

Table 5: Comparison of results from horn transducer

Once again, apart from a few slight deviations, the results seem to match up well.

8.5 Conclusions

Looking at the various results from all three transducers, we can see that the transducers have been accurately modelled. Most of the 1-D results are surprisingly accurate as well, especially when one considers the simplicity of some of the methods. From the tables, we can see that the largest deviation between predicted and measured results is only a few percent. The increased performance of the 2nd nodal-mounted transducer again shows how important careful assembly is.

Chapter 9 deals with the investigation of some of the effects of high-power ultrasound.
Chapter 9: Transducer tests at high power

9.1 Introduction

Ultrasonic transducers have a wide range of applications, from sonar and medical diagnostics to industrial cleaning. The transducer and its characteristics will obviously depend on the specific application, determining the transducer power and frequency requirements.

The design of most, if not all, transducers and the accompanying matching/locking circuitry is done at small signal levels. This design method is adequate if the transducer is to be used at low power, but introduces problems when trying to design a high-power transducer. It is well known that the characteristics of ultrasonic transducers change at high power levels. [21, 64-69]

The three most notable changes are a shift in the resonant driving frequency, a decrease in both the ‘Q’ of the transducer and the admittance at resonance. It has also been noted that there is a hysteretic frequency effect when increasing or decreasing power. It is believed that these effects are primarily due to the non-linearity of the piezoelectric material when driven at high power as well as non-linear effects in the water. In the following tests, we will try to show experimental proof of these effects, propose theories as to what may cause them and show a method of characterising high-power ultrasonic transducers.

9.2 Experimental set-up and method

We will now give a basic description of the system used for testing the transducers at high power. An HP digital function generator was used to provide the input waveforms in the experiment. This allowed easy and accurate variation of frequency or input voltage when necessary, which helped keep the tests short and prevented the PZT discs from heating up excessively. A Crest commercial audio amplifier was used in order to get the necessary power at the optimum frequency to the transducer. A matching circuit was required for optimum driving of the transducer. This consisted of an inductor in parallel
with the transducer and an impedance matching transformer. The inductor cancels out the effect of the static capacitance of the discs, and the transformer matches the impedance of the amplifier to that of the transducer. The value of the inductor was calculated using the following formula:

\[ L = \frac{1}{(2\pi f_m)^2 C_0} \]

where \( f_m \) is the resonant frequency and \( C_0 \) is the static capacitance. The nodal-mounted transducers were used in all the power tests, primarily Nodal #1 as it appeared to give more reliable test data. The voltage across the transducer and the current through it were measured using an HP digital scope. The current measurement was taken from a current transformer on the ground lead of the transducer.

**Figure 81: Experimental set-up for power tests**

The transducers were suspended by their nodal plates during testing in a rectangular stainless steel bath, with approximately 2mm of the heads covered with water. The bath is approximately 60mm deep, 100mm wide and 300mm long. The frequency was then swept through the required range, while ensuring that the input voltage remained constant, with the relevant readings being taken. The readings were the voltage across the
transducer, the current through the transducer and the phase difference between the two waveforms. From these values, we could then calculate the impedance and hence admittance of the transducer at various frequencies and power levels. Care was taken to perform the tests as quickly as possible, so as not to let the temperature of the discs surpass their Curie temperature, and to minimise the effect of varying temperature on the results.

9.3 Results from power tests

![Graph showing effects of driving Nodal #1 at high power](image)

*Figure 82: Graph showing effects of driving Nodal #1 at high power*

In order to convince ourselves that these results were valid, the same test was performed on Nodal #2. The results are shown on the next page.
Figure 83: Graph showing effects of driving Nodal #2 at high power

The graphs in Figures 82 and 83 clearly show the effects of driving the transducers at higher powers: namely the shift in resonant frequency of the transducers and the decrease in “Q” and magnitude of the admittance. The cause of these effects is clearly explained by Berlincourt. As power is increased, the dielectric loss also increases due to nonlinearity in the dielectric displacement - electric field relationship (hysteresis). This hysteresis is caused mainly by electrical domain reorientation. With the increase in dielectric loss comes an associated increase in heat generated due to the losses and a subsequent decrease in electroacoustic efficiency. Hence the decrease in the magnitude of admittance. Also, the change of permittivity with the increased electric field will upset the inductive tuning of the blocked capacitance in the transducer. Hence, the change in resonant frequency of the transducer. Although the hysteresis effect was observed during these experiments, it cannot be seen clearly on the graphs in Figures 82 and 83. A separate set of tests was carried out in order to show this effect.
9.4 Hysteresis tests

The experimental set up for this set of measurements was very similar to that for the power tests, the exceptions being the exclusion of the function generator and the inclusion of the phase lock loop. The function of the phase lock loop is to establish the optimum driving frequency of the transducer, whatever the input power, and drive the transducer at that frequency. It is therefore unnecessary to measure the phase between voltage and current, as it is assumed to be approximately zero at all times. The diagram of the experimental set-up is shown on the next page.

![Diagram of experimental set-up for hysteresis tests]

**Figure 84: Experimental set-up for hysteresis tests**

Initially, the tests were performed in the stainless steel bath used in the power measurements. The power was adjusted to the minimum possible while ensuring that the phase lock loop was locked on the correct frequency; approximately 22kHz.
Measurements of current and voltage were then taken at increasing power levels, until saturation began to occur. The same measurements were then taken while decreasing the power to approximately the same level as the starting point. The admittance and input power could then be worked out from the voltage and current.

Due to some unexpected results that were produced from these tests, further testing was done to determine the origin of the anomalies. It was noted that there was a sharp increase in admittance around the 100V level while decreasing the power. Therefore, the tests were run again, but this time, another sweep of power up and down was done immediately after the first sweep. This was in order to see if the effect persisted in partially degassed water while the transducer was still hot. To eliminate the effect of the partially degassed water, new water was used for the second sweep, again as soon as possible after the first sweep, while the transducer was still hot. Finally, to investigate whether the effect was a function of wavelength, the tests were conducted in a jug, approximately 200mm deep.

9.5 Results

![Admittance vs Input Voltage (Nodal-mounted if 1 in new water at start)](image)

**Figure 85:** Graph showing hysteresis effect due to onset of cavitation
In Figure 85, one can see the gradual decrease in the magnitude of admittance, albeit not linearly, due to the increasing power level. This is expected from the previous set of results. Around 250V, one can see a sharp increase in the magnitude of admittance. This is brought about by the sudden onset of cavitation. The point at which the onset of cavitation will occur is dependant on many factors including temperature, ambient pressure, concentration of cavitation nuclei (such as gas, impurities), medium, input power etc. So at this point, the conditions are clearly optimum for cavitation to occur. With the onset of cavitation, a bubble or cavitation cloud is formed at the front face of the transducer. As the load in front of the transducer is now a “mixed-phase” load, the transducer will “see” a lesser impedance, therefore the admittance increases drastically.

Once the magnitude of the admittance has reached a maximum, it begins to decrease, even though the input power is being increased. This phenomenon is known as “saturation”, and is due to the “unloading” effect. What this means is that the transducer cannot transfer any more energy into the water. One of the main factors in this effect is the bubble cloud, as it forms a kind of barrier around the transducer head, so that energy cannot be transferred to the rest of the load.

Once the power begins to decrease, the unloading effect continues until the point of onset of cavitation is reached. At this point, the admittance increases slightly and then remains at the higher magnitude well beyond the point at which cavitation began. This is due to the fact that the bubble cloud persists at the front face of the transducer and is known as the hysteresis effect. The reason why the bubble cloud persists is that once it has been formed, it requires less energy to maintain it, as the bubbles themselves form cavitation nuclei and are “seen” as a lesser impedance. As the power is decreased further, a point is reached where there is no longer sufficient energy to maintain the cavitation cloud, and the cavitation is “quenched.” This leads to a dramatic decrease in the magnitude of the admittance. The admittance then increases steadily as the power is decreased, which is expected from the power tests. The unexpected result is the peak in admittance that occurs at slightly less than 100V; so further testing was carried out to determine the source of the anomaly.
Figure 86 on the following page shows the results from sweeping the power up and down twice, without changing the water in between sweeps. From this graph, one can see the onset of cavitation again, although it is somewhat earlier than in previous tests. Saturation then occurs as expected, and the cavitation is then quenched before hysteresis can occur. The reason for this is probably because of the unexpected early onset of cavitation. Then, as with the previous results, the admittance gradually increases to a peak at about 75V. During the second sweep, one can see that the peak does not occur around 80V, so it is clearly a function of decreasing power only. Also, the onset of cavitation is much later in the second sweep. This is due to the fact that the water has been partially degassed during the first sweep, so a higher power level is required to produce cavitation. The peak then occurs again on the downward sweep.

![Graph showing results of double power sweep](image)

Figure 86: Graph showing results of double power sweep

There are some deductions that can be made from this graph. The first is that the peak occurs in both fresh water and partially degassed water. Therefore, one can assume that water is not a factor in the formation of the peak. However, the transducer does heat up quite substantially during the testing, so temperature may well be the cause of the peak. To test this hypothesis, the transducer will be tested in fresh water, thereby heating up the transducer. The water will then be changed for fresh water, and re-tested while the transducer is still hot from the previous sweep. The results are shown below.
Figure 87: Graph showing the effect of changing the water for the second sweep
Although the peak cannot be seen in the first downward sweep, it is highly accentuated in
the second. This would tend to suggest that the elevated temperature of the transducer
plays a major role in producing the 100V peak. To eliminate the possibility that the depth
of the test vessel may play a role in the creation of this peak, the transducer was tested in
a jug approximately 200mm deep.

Figure 88: Results of testing in a deep jug of water, using the same water for the
second sweep
Figure 89: Results of testing in the jug, changing the water between sweeps
It is difficult to conclude anything concrete from the above results. In Figure 88, the only peak around 100V occurs on the first upward sweep, whereas in Figure 89, it occurs on the second upward sweep. The saturation and cavitation hysteresis can still be clearly seen, but it is unclear what effect the depth of the test vessel has on the 100V peak.

9.6 Photographs

Figure 90: Steel test bath for nodal-mounted transducers
9.7 Conclusions

In these tests, the effects of driving ultrasonic transducers at high drive levels have been clearly illustrated. From the results of the power tests, it has been shown that when ultrasonic transducers are driven at high powers, the following effects will occur: the resonant driving frequency \( f_r \) will decrease, the magnitude of the admittance will decrease and the water-loaded mechanical "Q" of the transducer will decrease. These
facts emphasise the importance of using the high power characteristics of the transducer to design both the matching and control circuits for driving the transducer.

The saturation and hysteresis effects due to the persistence of cavitation bubbles has also been shown.

The origin of the peak in admittance around 100V is still not clear. The deductions that can be made from the testing are that the admittance peak seems more prevalent in the steel bath, and at elevated temperatures. No other trends are clearly apparent from the results. Further testing is clearly necessary to establish the source of this peak.
Chapter 10: Calorimetric tests to determine electro-acoustic efficiency of the transducers

10.1 Introduction

In some applications of ultrasound, particularly in the biochemical and sonochemical fields, it is vital to know the exact amount of energy being delivered to the load. In many cases, the “dose” of ultrasound is critical to the results. One such example is given by J. Van Zyl in his thesis. The application is the processing of wine in order to improve the taste characteristics in a very short period of time. In this case, the over or under-exposure to ultrasound by as little as 15 seconds could make a significant difference. Hence the need to accurately measure the amount of ultrasonic energy going directly into the load.

10.2 Experimental set-up and method

The basic concept of the experiment is that the input electrical power to the transducer is measured and compared to the amount of ultrasonic energy absorbed by the liquid. The ultrasonic energy heats up the water, so the heating of the water will be proportional to the amount of energy absorbed. The temperature rise in the water will be measured using the calorimetric method outlined below.

The experimental set-up was basically the same as for the transducer tests at power, except that we now also had a PM100 power analyser from Voltech that made the measurement of input electrical power much easier. So the amplifier drove the transducer via the matching network, and the corresponding temperature increase was measured by the calorimeter. This consisted of an RTD (resistive temperature detector) in the liquid (contained in thermos flask) and a data-logging device connected to the serial port of a computer. A PT100 RTD was used that gave us a 0.1°C resolution. A block diagram of the data-logger is shown below in figure 93.
A program was written in Visual Basic 6 to read the data in from the data logging system and the results could then be manipulated using Microsoft Excel.

The first step of the experiment was to calibrate the calorimeter. This was done using an immersion heater, a variable ac supply, a magnetic stirrer, the temperature probe and the data logging system. The Voltech power analyser measured the various power levels going into the heating element, which was immersed in 1 litre of tap water. The time taken for a specific temperature rise was measured for different power settings and entered into a calibration curve. This is shown below.

\[
y = 0.0122x - 0.0082 \\
R^2 = 0.9986
\]
The transducers were then driven into 1 L of fresh tap water, and the results compared to the calibration curve to get the efficiencies. The temperature readings were taken at 1-second intervals for increases on temperature of at least 3°C.

10.3 Results

![Graph](image)

Figure 95: Output from calorimeter system for Nodal #1

Above is an example of one of the graphs for the Nodal #1 transducer. From this graph, we take the gradient and enter it into the spreadsheet with the relevant calculations. The results for the two nodal transducers and the horn transducer are shown on the following page in Table 6.
### Table 6: Results of efficiency tests on transducers

<table>
<thead>
<tr>
<th>Nodal #1</th>
<th>Electrical Power (W)</th>
<th>Gradient</th>
<th>Acoustic Power (W)</th>
<th>Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>37</td>
<td>0.0093</td>
<td>27.41</td>
<td>74.09</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.0119</td>
<td>37.66</td>
<td>75.91</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.0177</td>
<td>61.47</td>
<td>61.47</td>
</tr>
<tr>
<td>Nodal #2</td>
<td>37</td>
<td>0.0094</td>
<td>27.82</td>
<td>75.19</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.0122</td>
<td>39.17</td>
<td>78.34</td>
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<td></td>
<td>100</td>
<td>0.0188</td>
<td>65.93</td>
<td>65.93</td>
</tr>
<tr>
<td>Horn 1st</td>
<td>60</td>
<td>0.0101</td>
<td>30.66</td>
<td>51.10</td>
</tr>
<tr>
<td></td>
<td>97</td>
<td>0.0145</td>
<td>48.50</td>
<td>50.00</td>
</tr>
<tr>
<td></td>
<td>120</td>
<td>0.0157</td>
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<td>44.47</td>
</tr>
<tr>
<td>Horn 2nd</td>
<td>60</td>
<td>0.0077</td>
<td>20.93</td>
<td>34.88</td>
</tr>
<tr>
<td></td>
<td>125</td>
<td>0.0111</td>
<td>34.71</td>
<td>27.77</td>
</tr>
</tbody>
</table>

10.4 Photographs

Figure 96: Photo showing horn transducer in calorimeter and probe with data logging system on right
10.5 Conclusions

From the results, we see that the nodal-mounted transducers are 75% efficient at lower powers, dropping to about 60% at higher power due to the increase in dielectric losses. This is quite high for this type of transducer. However, there is a significant source of error in that the transducer itself heats up due to internal losses in the piezoceramic. Some of this heat is conducted into the head and tail of the transducer. Obviously, the temperature rise in the head will then be transferred into the water. This implies that the temperature increase in the water is due both to the ultrasonic energy and the losses in the piezoceramic. Unfortunately, the PZT discs that we use tend to have relatively high loss factors, and thus heat up quite quickly. This means that the error margin for this particular experiment could be substantial.

The two sets of horn transducer readings were taken a few weeks apart, the second set being taken on behalf of the author. The large discrepancy between the two sets of readings is very strange. It implies that either there was a serious error in one of the sets of readings or that there had been a significant change in the transducer. Unfortunately, when the horn transducer was assembled, the only discs left were sub-standard. All the remaining discs had an uneven distribution of piezoelectric coefficient, which lead to a double peak at the thickness resonance.

![Graph: Admittance curves for discs used in Horn driver](image)

Figure 97: Admittance curves for discs used in Horn driver
The admittance graphs shown above are for the 4 discs used, but they are very closely matched so that only three can be seen. The possible change in the transducer may have been a side effect of the sub-standard PZT discs. As the discs have an uneven distribution of piezoelectric coefficient, it is likely that the heating in the discs will be localized to certain areas. In the event of excessive heating, which is quite likely in this case, the localised areas of high temperature may well have caused partial or full depolarisation of the piezoceramic. Obviously, this would cause the efficiency of the transducer to drop off dramatically.

The sub-standard discs are probably also the cause of the poor efficiency of the horn transducer. If the discs have an uneven distribution of piezoelectric coefficient, it is very likely that the discs will not perform as well as they should.

A point that should be noted here is that the effect of the losses in the PZT discs contributing to the heating of the water can basically be neglected here. This is because of the large amount of metal between the discs and the water; it would take a very large temperature increase in the discs to induce a significant rise in temperature of the tip of the horn. This means that the results for the horn are quite accurate.

Another source of error in this experiment is the straying from resonance of the transducer. At the time of the experiment, the phase lock loop was unavailable, so the transducer was kept at resonance by ensuring that the phase angle between voltage and current was maintained as close to zero as possible. This was not a completely satisfactory method of driving the transducer. Also, there were some intermediate power levels at which the transducer was even more difficult to drive as it was on the point of cavitating the water, but did not have the necessary power to go into hard cavitation. This made the resonance frequency change frequently, which made it difficult to maintain a constant power level into the transducer.
Chapter 11: Conclusions and Recommendations

11.1 Summary

In this project, we have done the following:

- Given a brief background to development and theory of piezoelectric sandwich transducers,
- Explored the origin and development of some of the design and analysis methods such as the various equivalent circuit models,
- Given a brief introduction to Finite Element Method, and its uses in modelling PZT discs,
- Designed three different types of transducers using 1-D, equivalent circuit and FEM design methods,
- Manufactured and assembled the transducers using “state of the art” techniques,
- Tested the transducers and compared the results with those predicted by the three design methods,
- Used the transducers in various tests.

11.2 Discussion and Conclusions

If we look back at the comparison between predicted and measured results for all three transducers, we see that they match very well. All of the design methods appear to be able to predict the resonant frequencies of a transducer satisfactorily, as long as the limitations of the method are kept in mind.

The frequency equation method, the simplest of the three, is quite basic; therefore we expect its capabilities to be the same. However, as a starting point for the design of a transducer, this method works very well.

A similar rationale applies to the other two equivalent circuit methods, the more complex the method, the greater the capabilities. How far one goes with the equivalent circuit method will obviously also depend on whether one has access to a better design method.
such as FEM. It is possible, for example, to include water loading in the lumped and distributed parameter analysis of a transducer. This was not done in this project due to time constraints and also due to the fact that the author could use FEM to predict the effects of water loading. One of the pitfalls of the equivalent circuit methods though, is that they require very careful mathematical manipulation, whether a computer program is used or not. As was discovered by the author, it is very easy to enter the wrong variable, or leave a variable out of an equation, and this can lead to completely anomalous results. So great care has to be taken when developing the models for the transducers.

As mentioned at the beginning of this project, FEM is a very powerful design and analysis tool. In this project, it has proved to be very useful and, for the most part, completely accurate in its ability to predict resonant frequencies and the mode shapes at those frequencies. It too has the downfall that great care has to be taken at the outset of the project when the model is being developed. Once again, small errors or emissions can lead to strange or completely incorrect results. Another point that should be noticed is that without the proper insight and experience with the model that one is dealing with, FEM can be quite uninformative and even misleading. For example, one has to know what sort of results to expect from the simulations and how to interpret them, or they will be of no use or significance. This is why using the 1-D and equivalent circuit models is very useful, as one develops a sense of what to expect and the implications of various changes. One of the downfalls of Abaqus, rather than FEM itself, is that the program has not yet been developed far enough to fully simulate piezoceramics accurately. The fact that we are unable to predict the expected magnitude of admittance is quite a serious downfall. Being able to do so would allow us a much broader scope of possibilities, such as being able to predict efficiencies of the transducers. There are other FEM packages that have been developed specifically for the simulation of piezoelectric transducers, such as PZFlex, \cite{72} which have all of the necessary capabilities.

We have seen the effect that poor manufacture and assembly can have on the piezoelectric transducers. This point cannot be stressed enough. Even the methods described in this project are somewhat crude and could be vastly improved. One such
example is the method that we used to torque the transducers. During the torquing operation, a lot of the actual torque goes to overcoming friction between the threads of the head and the bolt and the collar and bolt head. One can never really be certain of exactly how much torque is being lost in this process, although it can be estimated. Also, the whole assembly or various parts of it sometimes tend to rotate during the process, which could cause extra friction and possibly misalignment. A much better way to pre-stress the transducers would be to have a fixture that would allow the assembly to be pre-stressed without the bolt. The bolt could then be screwed in while the transducer was still under compression, then the compression slowly released so that the bolt could take up the tension. This would eliminate both the friction and rotation problems.

The test data from the transducers measured on the Network Analyser can be considered to be accurate. A suggestion that would help us measure the transducers in this case would be a large test bath so that the results would not have any interference from standing waves and reflected energy.

The high power testing of the transducers gave some interesting results. There was also the possibility of errors in these measurements though. One of the main sources of error would be the measurement of phase by the oscilloscope. This is not a very accurate method of measuring the phase, and a better method should certainly be used in future. The results however are accurate enough and still agree with theory.

The efficiency tests conducted with the transducers were very informative. This seems to be a good method for testing the efficiency of transducers, but care must be taken to eliminate the errors mentioned in Chapter 10.

11.3 Recommendations

There are a number of recommendations that can be made for future work based on topics covered in this project. On the subject of 1-D modelling, it would be very interesting to see how the models performed with the inclusion of water loading. This is not a very difficult step, and would be very useful in adding to our design capabilities.
With regards the FEM design of the transducers, it would be most interesting to have access to a design package such as PZFlex. A program with all the functionality of PZFlex, designed specifically for piezoelectric transducers, would allow us to fully develop and explore additional design possibilities in the field of piezoelectric transducers.

Due to time constraints, we were not able to test the high power performance of a glued transducer versus one that had been assembled dry. I believe that this is a critical issue, and one that needs to be explored in great depth in order to adequately ascertain the effects of bonding the transducer together. It is very possible that finding the correct epoxy or gluing method may vastly improve the efficiency and efficacy of the transducers.

There is ongoing research into the electronics and matching circuit technology for driving piezoelectric transducers. This is another very important field that plays a critical role in the use of high power ultrasonic transducers.
APPENDIX A-1

Below are the alternative governing piezoelectric equations. The form of equation used will depend on which variables are chosen to be independent. The choice of the right set of independent variables often simplifies later derivations.

\( T \) and \( D \) independent variables:
\[
S = s^D T + g, D \\
E = -g T + \beta^T D
\]

\( S \) and \( E \) independent variables:
\[
T = e^S S - e, E \\
D = eS + e^S E
\]

\( S \) and \( D \) independent variables:
\[
T = e^D S - h, D \\
E = -h S + \beta^S D
\]

In the above equations, \( g, e, \) and \( h \) represent the piezoelectric constants, and \( c^e \) is elastic stiffness, and \( \beta^s \) is dielectric impermeability. Elastic stiffness and compliance, and dielectric constant and impermeability are related by the following equations \(^{[31]}\):

\[
c_{ij} = \frac{(-1)^{i+j} \Delta_{ij}}{\Delta}
\]

where \( \Delta \) is the determinant and \( \Delta_{ij} \) is the minor of the \( s_{ij} \) matrix.

\[
\beta_{mk} = \frac{(-1)^{m+k} \Delta_{mk}}{\Delta}
\]

where \( \Delta \) is the determinant and \( \Delta_{mk} \) is the minor of the \( \varepsilon_{mk} \) matrix.
The four piezoelectric constants, \( d, g, e \) and \( h \), are interrelated as follows \[^{[31]}\):

\[
\begin{align*}
d_{m} &= \epsilon_{m}^{T}g_{m} = \epsilon_{m}^{s}s_{ji}^{e} \\
g_{m} &= \beta_{m}^{T}d_{m} = h_{m}^{s}s_{ji}^{e} \\
e_{m} &= \epsilon_{m}^{s}h_{m} = d_{m}^{e}s_{ji}^{e} \\
h_{m} &= \beta_{m}^{s}e_{m} = g_{m}^{e}s_{ji}^{e}
\end{align*}
\]

where \( m, n = 1 \ldots 3 \), and \( j, i = 1 \ldots 6 \).

Mason \[^{[32]}\] has a useful diagrammatic method of summarising the relationships between the electrical and mechanical variables involved piezoelectricity.

![Diagram](image)

**Figure A-1:** Mason's representation of the relationships between mechanical and electrical variables. \[^{[32]}\] The numbers indicate the order of tensor representation.
APPENDIX A-2

To derive the frequency-determining equation, we use equivalent circuit for the ceramic loaded at one end, including the mass of the head as a load. The resultant circuit with the load is shown below.

Figure A-2: Equivalent circuit for ceramic loaded at one end with head mass.

According to Bolef and Miller, the input impedance at \( y = 0 \) for a resonator of length \( \ell \), terminated at \( y = \ell \) in some arbitrary impedance \( Z(\ell) \) is given by

\[
Z_{in} = Z_0 \frac{Z(\ell) + Z_0 \tanh \vartheta_\ell}{Z_0 + Z(\ell) \tanh \vartheta_\ell} \tag{20}
\]

where \( \vartheta = \alpha + ik \), \( \alpha \) is the attenuation coefficient, and \( k = \frac{\omega}{c} \) is the ultrasonic propagation constant. Here, we will assume that \( \alpha = 0 \). It should also be noted that \( j \tanh \vartheta_c = \tan j \vartheta_c \).

Firstly, looking in at \( X \), the only section we have is \( Z_L \), so

\[
Z_{inX} = Z_h \tanh \vartheta_h = jZ_h \tan \vartheta_h \tag{21}
\]

since \( Z(\ell) = 0 \), as the mass is radiating into air.

Now, looking in at \( Y \), we have the ceramic and the load, therefore

\[
Z_{inY} = Z_c \frac{jZ_h \tan \vartheta_h - jZ_c \cot \vartheta_c}{Z_c - jZ_h \tan \vartheta_h \cot \vartheta_c} \tag{22}
\]
For the condition of resonance, the input impedance tends to zero; therefore we equate the numerator of (22) to zero:

\[ Z_c jZ_h \tan \vartheta_h - j Z_c^2 \cot \vartheta_c = 0 \]  

(23)

\[ \Rightarrow Z_h \tan \vartheta_h = \frac{Z_c}{\tan \vartheta_c} \]

\[ \Rightarrow \tan \vartheta_c \tan \vartheta_h = \frac{Z_c}{Z_h} \]

APPENDIX A-3

![Diagram](image)

Figure A-3: Admittance plot of piezoelectric resonator.
An explanation of the symbols used in the diagram on the previous page is given below:

\[ f_s = \text{motional (series) resonance frequency} \]
\[ f_p = \text{parallel resonance frequency} \]
\[ f_r = \text{Resonance frequency (B = 0)} \]
\[ f_a = \text{antiresonance frequency (B = 0)} \]
\[ f_m = \text{frequency at maximum admittance} \]
\[ f_n = \text{frequency at minimum admittance} \]
\[ f_1, f_2 = \text{quadrantal frequencies (or frequencies at half max. conductance, half power points)} \]
\[ Y_{m/n} = \text{maximum and minimum admittance.} \]

If we consider the one-port equivalent circuit diagram for a transducer (Chapter 2, figure 19), we see that the transducer has two branches: a motional branch \((L_M, C_M, R_M)\) and a blocked branch \((C_b)\). If we only considered the motional branch, the admittance plot would be symmetrical about the x-axis. It is the blocked branch that gives the plot the offset: the bigger \(C_b\) is the greater the offset. If \(C_b\) is large, the power factor of the transducer becomes low and the efficiency of the driving amplifier suffers. This problem can be corrected by adding a series or parallel tuning reactance.

When the transducer is perfectly tuned, the plot will have no offset. In this case, \(f_m = f_s = f_r\) and \(f_n = f_a = f_p\). Figure A-3 assumes that the transducer is lossless. If the transducer has losses, the plot is offset by an angle \(\delta\) and the circle becomes a cissoidal loop.\(^{45}\)

From the admittance plot, we can find the following parameters:

\[ Q_M = \frac{f_s}{f_2 - f_1}, \quad Q_e = \frac{B_s}{G_s}, \quad k^2 = \frac{1}{1 + \frac{B_b Q_M}{G_s}} = 1 - \frac{f_s^2}{f_p^2} = \frac{C_M}{C_s + C_M}, \]
\[ L_T = \frac{R_T Q_M}{\omega_s}, \quad C_T = \frac{1}{L_T \omega_s^2}, \quad G_b = B_b \tan \delta, \quad R_T = \frac{1}{G_s}, \]
\[ f_s = \frac{1}{2\pi \sqrt{L_M C_M}}, \quad f_p = \frac{1}{2\pi \sqrt{L_M C_b + C_M}}. \]
In the above expressions, $C_b$ is the blocked susceptance evaluated at $f_s$ and $G_s$ is the conductance at $f_s$.

The efficiency of the transducer can also be worked out using the values for the maximum conductance in air and water:

$$\eta_{ma} = \frac{G_{air} - G_{water}}{G_{air}}, \quad \eta_{mu} = \frac{G_{water}}{G_{air}}.$$
APPENDIX B-1

The table below shows the material properties of the discs supplied by Ferroperm. The properties relevant to the simulations are highlighted. The symbols stand for the following: \( \varepsilon_r \) = relative dielectric constants
e = piezoelectric constants
c = elastic constants.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Pz21</th>
<th>Pz23</th>
<th>Pz24</th>
<th>Pz26</th>
<th>Pz27</th>
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<td>( \varepsilon_1 ), ( \varepsilon_3 )</td>
<td>C/N</td>
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<td>8.10E+02</td>
<td>1.10E+03</td>
<td>1.80E+03</td>
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<td>9.14E+02</td>
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<td>5.97E+10</td>
<td>9.38E+10</td>
<td>1.40E+11</td>
<td>9.33E+10</td>
<td>8.75E+10</td>
</tr>
<tr>
<td>$C_{33}^D$</td>
<td>N/m$^2$</td>
<td>1.42E+11</td>
<td>1.54E+11</td>
<td>1.75E+11</td>
<td>1.58E+11</td>
<td>1.44E+11</td>
</tr>
<tr>
<td>$C_{44}^D = C_{55}^D$</td>
<td>N/m$^2$</td>
<td>4.10E+10</td>
<td>4.87E+10</td>
<td>4.34E+10</td>
<td>3.66E+10</td>
<td></td>
</tr>
</tbody>
</table>

Table B-1: Material properties of PZT discs supplied by Ferroperm.
APPENDIX B-2

The following matrices represent the material properties of the PZ26 discs in the Abaqus format.

Elastic = \( c = \begin{bmatrix} 168 & 99.9 & 110 & 0 & 0 & 0 \\ 0 & 123 & 99.9 & 0 & 0 & 0 \\ 0 & 0 & 168 & 0 & 0 & 0 \\ 0 & 0 & 0 & 30.1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 30.1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 30.1 \end{bmatrix} \text{ GPa} \)

Piezoelectric = \( e = \begin{bmatrix} 0 & -1.98 & 0 \\ 0 & 14.7 & 0 \\ 0 & -1.98 & 0 \\ 9.86 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 9.86 \end{bmatrix} \text{ C/m}^2 \)

Dielectric = \( \varepsilon_r = \begin{bmatrix} 11.771 & 0 & 0 \\ 0 & 10.531 & 0 \\ 0 & 0 & 11.771 \end{bmatrix} \text{ } 10^{-9} \text{ F/m} \)
APPENDIX B-3

The Abaqus code to simulate a piezoelectric disc is shown below.

*heading
  Ferroperm Disc 38.5 x 12.7 x 6.4
  * support conditions free-free
  *preprint, model=yes
**-------------------------------------------------------------------
** pzt definition
**-------------------------------------------------------------------
*node
  1, 0.00635, 0.
  21, 0.01925, 0.
  **
  401, 0.00635, 0.0064
  421, 0.01925, 0.0064
  **
*ngen, nset=layerB
  1, 21, 1
*ngen, nset=layerT
  401, 421, 1
  **
*nfill
  layerB, layerT, 8, 50
  **
*element, type=cax8e, elset=pzt_pos
  1, 1,3,103,101, 2,53,102,51
*elgen, elset=pzt_pos
  1, 10,2,1, 4,100,100
  **
*solid section, elset=pzt_pos, material=pzt4_p2, orientation=2_pos
**-------------------------------------------------------------------
** electrode electrical detail
**-------------------------------------------------------------------
*node,nset=ns_ref
  100000,,
*equation
  2
  layerT,9,1.0, 100000,9,-1.0
** material definitions

*orientation, name=2_pos
1.,0.,0., 0.,1.,0.
1,0.0
*orientation, name=2_neg
-1.,0.,0., 0.,-1.,0.
1,0.0
**
** poled in 2 direction!!
**
*MATERIAL,NAME=PZT4_p2
*elastic, type=ortho
16.8E10,9.99E10,12.3E10,11.0E10,9.99E10,16.8E10,3.01E10,3.01E10,
2.88E10
*piezoelectric, type=s
0.,0.,0.,9.86,0.,0.,-1.98,14.7
-1.98,0.,0.,0.,0.,0.,0.,0.
0.,9.86
*dielectric, type=aniso
11.771E-9,0.,10.531e-09,0.,0.,11.771E-9
*density
7700.
**

*step,perturbation
  closed circuit
  * frequency
  50,500.e3,1.
  **
  *restart, write
  **
*boundary
  layerB, 9,9, 0.0
  100000, 9, 9, 0.0
  **
  *node print,f=0
  *el print, f=0
  *end step
  **
*step,perturbation

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open circuit
*frequency
50, 500.e3, 1.
**
*restart, write
**
*boundary
layerB, 9, 9, 0.0
**
*node print, f=0
*el print, f=0
*end step
**===================================================================
*step, perturbation
frequency sweep
*steady state dynamic, direct
10.e3, 500.e3, 9800, 1, 1
**
*restart, write, f=980
**
*boundary
layerB, 9, 9, 0.0
100000, 9, 9, 1.0
**
*node file, f=1, nset=ns_ref
RCHG, PHCHG, EPOT, PHPOT
**
*nset, nset=ns_ref2
1, 21, 401, 421
*node file, f=1, nset=ns_ref2
U, V, A, PU
**
*node print, f=0
*el print, f=0
*monitor, node=1, dof=2
*end step

APPENDIX B-4

Abaqus input deck to simulate a full 3-D model of the PZT disc.
*heading
3D Ferroperm Disc 38.5 x 12.7 x 6.4
* support conditions free-free
*preprint, model=yes

**--------------------------------
** pzt definition
**--------------------------------

*nnode
1, 0.00635, 0.
21, 0.01925, 0.
**
401, 0.00635, 0.0064
421, 0.01925, 0.0064

**-------------------------
*nngen, nset=layerBl
1, 21, 1

**-------------------------
*nngen, nset=layerT1
401, 421, 1

**-------------------------
*nfill, nset=face1
layerBl, layerT1, 8, 50
**

**
*nncopy, change number=5000, old set=face1, shift, multiple=36
0., 0., 0.
0., 0., 0., 0., 1., 0., 10.
**

**node set of actual nodes used on face1 ie not mid face nodes!!
*nset, nset=face1a, generate
  1, 21, 1
  51, 71, 2
  101, 121, 1
  151, 171, 2
  201, 221, 1
  251, 271, 2
  301, 321, 1
  351, 371, 2
  401, 421, 1
**node set of actual nodes used on face2 ie not mid face nodes!!
*nset, nset=face2, generate
180001, 180021, 1
180051, 180071, 2
180101, 180121, 1
180151, 180171, 2
180201, 180221, 1
180251, 180271, 2
180301, 180321, 1
180351, 180371, 2
180401, 180421, 1

**

**mpc's tying first and last face together(all d.o.f, includes dof 9)
**

*mpc
tie, face2, facela
**

** face2 node set is redundant!
**

** electrode electrical detail
**

*nset, nset=n_s_ref
100000,,
*equation
2
top,9,1.0, 100000,9,-1.0

** elements stuff
**

*element, type=c3d20e, elset=pzt_pos
1, 1,3,10003,10001, 101,103,10103,10101, 2,5003,10002,5001,
102,5103,10102,5101, 51,53,10053,10051
*elgen, elset=pzt_pos
1, 10,2,1, 4,100,100, 18,10000,500
**

*solid section, elset=pzt_pos, material=pzt4_p2, orientation=2_pos
**

** material definitions
*orientation, name=2_pos
1.0,0.0,0.1.0,0.
1,0.0
*orientation, name=2_neg
-1.0,0.0,0.0,-1.0,0.
1,0.0
**
** poled in 2 direction!!
**
*MATERIAL, NAME=PZT
*elastic, type=ortho
16.8E10,9.99E10,12.3E10,11.0E10,9.99E10,16.8E10,3.0E10,3.0E10,2.88E10
*piezoelectric, type=s
0.0,0.0,0.9.86.0,0.0.-1.98,14.7
-1.98,0.0,0.0,0.0,0.0,0.0,0.0,0.9.86
*dielectric, type=aniso
11.771E-9,0.0,10.531e-9,0.0,0.0,11.771E-9
*density
7700.
**
**===================================================================
**frequency sweep from 321kHz, top voltage not constrained, bot=0V
**===================================================================
*step, perturbation
open circuit
*frequency
50,340.e3,103.041e9
**
*restart, write
**
*boundary
bot, 9,9,0.0
100000, 9,9,0.0
**
*node print, f=0
*el print, f=0
*end step
**===================================================================
**frequency sweep from 15kHz, top voltage not constrained, bot=0V**

*step, perturbation
  open circuit
*frequency
  50,100.e3,225.e6
**
*restart, write
**
*boundary
  bot, 9,9, 0.0
  100000, 9,9, 0.0
**
*node print, f=0
*el print, f=0
*end step
**
**frequency sweep from 130kHz, top voltage not constrained, bot=0V**

*step, perturbation
  open circuit
*frequency
  50,160.e3,16.9e9
**
*restart, write
**
*boundary
  bot, 9,9, 0.0
  100000, 9,9, 0.0
**
*node print, f=0
*el print, f=0
*end step
**
**top and bot voltage constrained to zero**

*step, perturbation
  closed circuit
*frequency
  50,500.e3,1.
**
*restart, write  
**  
*boundary  
bot,  9,9, 0.0  
100000, 9,9, 0.0  
**  
*node print,f=0  
*el print, f=0  
*end step  
**===================================================================
**frequency sweep like NetAn  
**===================================================================
*step, perturbation  
no preload  
*steady state dynamic, direct  
10.e3, 500.e3, 9800, 1,1  
**60.e3, 500.e3, 9800, 1,1  
**  
*restart, write,f=980  
**  
*boundary  
bot, 9,9, 0.0  
100000, 9,9, 1.0  
**  
*node file, f=1, nset=ns_ref  
RCHG, PHCHG, EPOT, PHPOT  
**  
nset, nset=ns_ref2  
1,21,401,421  
*node file, f=1, nset=ns_ref2  
U,V,A,PU  
**  
*node print,f=0  
*el print, f=0  
*monitor, node=1, dof=2  
*end step  
**===================================================================  
nset, nset=bot, generate  
1,21,1  
5001, 5021, 2  
10001, 10021, 1
15001, 15021, 2
20001, 20021, 1
25001, 25021, 2
30001, 30021, 1
35001, 35021, 2
40001, 40021, 1
45001, 45021, 2
50001, 50021, 1
55001, 55021, 2
60001, 60021, 1
65001, 65021, 2
70001, 70021, 1
75001, 75021, 2
80001, 80021, 1
85001, 85021, 2
90001, 90021, 1
95001, 95021, 2
100001, 100021, 1
105001, 105021, 2
110001, 110021, 1
115001, 115021, 2
120001, 120021, 1
125001, 125021, 2
130001, 130021, 1
135001, 135021, 2
140001, 140021, 1
145001, 145021, 2
150001, 150021, 1
155001, 155021, 2
160001, 160021, 1
165001, 165021, 2
170001, 170021, 1
175001, 175021, 2
**180001, 180021, 1
*nset, nset=top, generate
  401,  421,  1
  5401,  5421,  2
 10401, 10421,  1
 15401, 15421,  2
 20401, 20421,  1
 25401, 25421,  2
30401, 30421, 1
35401, 35421, 2
40401, 40421, 1
45401, 45421, 2
50401, 50421, 1
55401, 55421, 2
60401, 60421, 1
65401, 65421, 2
70401, 70421, 1
75401, 75421, 2
80401, 80421, 1
85401, 85421, 2
90401, 90421, 1
95401, 95421, 2
100401, 100421, 1
105401, 105421, 2
110401, 110421, 1
115401, 115421, 2
120401, 120421, 1
125401, 125421, 2
130401, 130421, 1
135401, 135421, 2
140401, 140421, 1
145401, 145421, 2
150401, 150421, 1
155401, 155421, 2
160401, 160421, 1
165401, 165421, 2
170401, 170421, 1
175401, 175421, 2
**180401, 180421, 1
**end
**===================================================================
APPENDIX B-5

Below is the HPVee program written to control the Network Analyzer. The user inputs the start frequency, stop frequency and resolution and the program then calculates the step size and number of files to generate. The Network Analyzer steps through the specified frequencies, recording the admittance at each frequency, then transmits it to the computer where it is stored in the designated file. The data can then be manipulated using the appropriate graphic program.[20]

Figure B-1: HPVee program to control the Network Analyzer.
APPENDIX C-1

The derivation of the lumped element equivalent circuit approximation that we will use is shown below. The circuit can be used for any of the components of the transducer; if it is used to represent the ceramic, it will obviously include the active section (the blocked capacitance of the ceramic across a 1:N transformer in series with $C_m E$).

![Equivalent lumped element circuit diagram]

Figure C-1: Equivalent lumped element circuit

In the above circuit, the symbols stand for the following: $m = \rho A = $ static mass of component, $C = $ mechanical compliance $= \frac{I}{YA}$ where $Y = $ Young's modulus,

$$c_p = \sqrt{\frac{Y}{\rho}} = \text{speed of sound in material}.$$  

The first boundary condition that we will assume is that both ends of the component are free, i.e. both ends of the circuit are short-circuited. It then follows that:

$$l = \frac{\lambda}{2} = \frac{c_p}{2l} \Rightarrow f = \frac{c_p}{2l} \Rightarrow \omega = \frac{\pi c_p}{l} = \frac{\pi}{l} \sqrt{\frac{Y}{\rho}}.$$  

Also, $\omega = \frac{1}{\sqrt{MC}} = \frac{1}{\sqrt{\left(\frac{\alpha}{2} + \beta\right)MC}} = \frac{\pi}{l} \sqrt{\frac{Y}{\rho}}$ and $mC = \frac{\rho l^2}{Y}$.

Therefore: $\omega = \frac{1}{\left(\frac{\alpha}{2} + \beta\right)l} \sqrt{\frac{\rho}{\pi}} \Rightarrow \frac{\alpha}{2} + \beta = \frac{1}{\pi^2}.$

The second boundary condition that we will assume is that one end is clamped (i.e. open-circuited) and the other end is free:
\[ l = \frac{\lambda}{4} \Rightarrow \omega = \frac{\pi \sqrt{Y/\rho}}{2l} \]. Also \[ \omega = \frac{1}{\sqrt{mC}} = \frac{1}{\sqrt{(\alpha + \beta)mC}} = \frac{\pi \sqrt{Y/\rho}}{2l} \].

So \[ \omega = \frac{1}{\sqrt{(\alpha + \beta)l \sqrt{\rho/Y}}} = \frac{\pi}{2l \sqrt{\rho/Y}} \Rightarrow \alpha + \beta = \frac{4}{\pi^2} \].

From the two equations for \( \alpha \) and \( \beta \), we conclude that:

\( \alpha = \frac{6}{\pi^2} \approx 0.6 \) and \( \beta = -\frac{2}{\pi^2} \approx -0.2 \).
APPENDIX C-2

Tan approx for transducers-Nodal Mounted Tonpilz

Head Specs (Stainless Steel)

\( \text{lh} \) \hspace{10mm} \text{Length}  
\( \text{rh} := 0.02 \) \hspace{10mm} \text{Radius}  
\( \text{ph} := 7800 \) \hspace{10mm} \text{Density}  
\( \text{Ah} := \pi \cdot \text{rh}^2 \) \hspace{10mm} \text{Ah} = 1.257 \times 10^{-3} \quad \text{Section Area}  
\( \text{ch} := 4936 \) \hspace{10mm} \text{Speed of sound in material}  

Ceramic Specs (Half Stack)

\( \text{Ac} := \frac{\pi \cdot (0.0385^2 - 0.0127^2)}{4} \) \hspace{10mm} \text{Section area of PZT discs}  
\( \text{tc} := 0.064 \) \hspace{10mm} \text{Thickness of discs}  
\( \text{nc} := 2 \) \hspace{10mm} \text{Number of discs}  
\( \text{pc} := 7700 \) \hspace{10mm} \text{Density of discs}  
\( \text{s33E} := 1.96 \times 10^{-11} \) \hspace{10mm} \text{Elastic constant E-field}  
\( \text{cc} := \sqrt{\frac{1}{\text{pc} \cdot \text{s33E}}} \) \hspace{10mm} \text{Sound speed for constant E-field or shorted terminals}  

Tail Specs (Brass)

\( \text{lt} \) \hspace{10mm} \text{Length}  
\( \text{rt} := 0.02 \) \hspace{10mm} \text{Radius}  
\( \text{pt} := 8250 \) \hspace{10mm} \text{Density}  
\( \text{At} := \pi \cdot \text{rt}^2 \) \hspace{10mm} \text{At} = 1.257 \times 10^{-3} \quad \text{Section Area}  
\( \text{ct} := 3303 \) \hspace{10mm} \text{Speed of sound in material}  

Frequency

\( \text{fr} := 22 \cdot 10^3 \) \hspace{10mm} \text{Desired resonant frequency}  

Tan Approx

\[ \text{lh} = \text{ch} \cdot \left( \frac{\text{ph} \cdot \text{ch} \cdot \text{Ah}}{\tan \left( \frac{2 \cdot \pi \cdot \text{fr} \cdot \text{nc} \cdot \text{tc}}{\text{cc}} \right)} \right) \]

\[ \text{lh} = 0.017 \quad \text{Head Length}  

\[ \text{lt} = \text{ct} \cdot \left( \frac{\text{pt} \cdot \text{ct} \cdot \text{At}}{\tan \left( \frac{2 \cdot \pi \cdot \text{fr} \cdot \text{nc} \cdot \text{tc}}{\text{cc}} \right)} \right) \]

\[ \text{lt} = 0.015 \quad \text{Tail Length}  

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APPENDIX C.3

Nodal Mounted Tonpilz: Lumped Element Approx

Head Specs (Stainless Steel)

\[ \text{lh} := 0.017 \quad \text{Length} \]
\[ \text{rh} := 0.02 \quad \text{Radius} \]
\[ \rho h := 7800 \quad \text{Density} \]
\[ \text{Ah} := \pi \cdot \text{rh}^2 \quad \text{Ah} = 1.257 \times 10^{-3} \quad \text{Section Area} \]
\[ \text{Mh} := \text{lh} \cdot \rho h \cdot \text{Ah} \quad \text{Mh} = 0.167 \quad \text{Static mass of head} \]

Ceramic Specs

\[ \text{Ac} := \frac{\pi \left(0.0385^2 - 0.0127^2\right)}{4} \quad \text{Section area of PZT discs} \]
\[ \text{tc} := 0.0064 \quad \text{Thickness of discs} \]
\[ \text{nc} := 4 \quad \text{Number of discs} \]
\[ \rho c := 7700 \quad \text{Density of discs} \]
\[ \varepsilon c := 13308.854 \times 10^{-12} \quad \text{Dielectric constant of piezoelectric ceramic material} \]
\[ s_{33} E := 1.96 \times 10^{-11} \quad \text{Elastic constant E-field} \]
\[ d_{33} := 328 \times 10^{-12} \quad \text{Longitudinal strain per unit E-field} \]
\[ k_{33} := \frac{d_{33}}{\varepsilon c \cdot s_{33} E} \quad \text{Coupling factor} \]
\[ \text{CmE} := \frac{n_c \cdot t_c \cdot s_{33} E}{\text{Ac}} \quad \text{CmE} = 4.836 \times 10^{-10} \quad \text{Compliance of stack} \]
\[ \text{Mc} := n_c \cdot t_c \cdot n_c \cdot \text{Ac} \quad \text{Mc} = 0.205 \quad \text{Static mass} \]
\[ \varepsilon c \cdot \text{Ac} \cdot n_c \quad \text{Ce} := \frac{\text{t_c}}{\text{Ce}(1 - k_{33}^2)} \quad \text{Ce} = 7.636 \times 10^{-9} \quad \text{Free capacitance of stack} \]
\[ \text{Cb} := \varepsilon c \cdot (1 - k_{33}^2) \quad \text{Cb} = 4.077 \times 10^{-9} \quad \text{Blocked capacitance of stack} \]
\[ N := \frac{d_{33} \cdot \text{Ac}}{s_{33} E \cdot t_c} \quad \text{N} = 2.713 \quad \text{Blocked force per volt} \]

Tail Specs (Brass)

\[ \text{lt} := 0.015 \quad \text{Length} \]
\[ \text{rt} := 0.02 \quad \text{Radius} \]
\[ \rho t := 8250 \quad \text{Density} \]
\[ \text{At} := \pi \cdot \text{rt}^2 \quad \text{At} = 1.257 \times 10^{-3} \quad \text{Section Area} \]
\[ \text{Mt} := \text{lt} \cdot \rho t \cdot \text{At} \quad \text{Mt} = 0.156 \quad \text{Static mass of tail} \]
Compressive stress bias bolt

\[ ts = \pi e_1 \ell_c \]

\[ ts = 0.02 \ell_c \] \hspace{1cm} Length of bolt

\[ As = \pi e_0 \ell_c \]

\[ As = 0.004 \] \hspace{1cm} Section Area

\[ \rho s = 7800 \]

\[ \rho s = 19.3 \times 10^6 \] \hspace{1cm} Density of bolt material

\[ Y_S = 19.3 \times 10^6 \]

\[ Y_S = 19.3 \times 10^6 \] \hspace{1cm} Young's Modulus for bolt material

\[ C_S = \frac{ts}{Y_S As} \]

\[ C_S = 2.639 \times 10^{11} \] \hspace{1cm} Compliance of bolt

\[ MS = \rho S As ts \]

\[ MS = 0.01 \] \hspace{1cm} Static mass of bolt

Swept frequency increments and range

\[ nf = 1012 \]

\[ \omega_j = \omega_1 \]

\[ \omega_j = 2\pi f_j \]

Lumped circuit calculation without bolt t

\[ Z_1 = \frac{1}{j \omega_1 C_{m1}} - \frac{2 j \omega_1 M_c}{j \omega_1 C_{m1}} + \frac{j \omega_1 (C_0 (6M_c + M_0) + (6M_c + M_l))}{j \omega_1 C_{m1}} \]

\[ \left| \frac{N_j}{Z_1} \right| \]

Plot magnitude of electrical drive admittance
Determine mechanical resonance, anti-resonance frequency and effective coupling coefficient

\[ f_1 = f_1 \left( Y_1 = \max(Y) \right) \quad \text{fr} = \max(fr) \quad \bar{fr} = 2.278 \times 10^4 \]
\[ f_a = f_1 \left( Y_1 = \min(Y) \right) \quad fa = \max(fa) \quad \bar{fa} = 3.117 \times 10^4 \]
\[ k_{eff} = \sqrt{1 - \left(\frac{f_1}{\bar{fr}}\right)^2} \quad k_{eff} = 0.683 \]

**Lumped Circuit calculation with rod**

\[ Z_1 = \frac{1}{j \omega_1 C_{ri}} + \frac{1}{j \omega_1 C_b} - 2j \omega_1 \eta_1 \bar{C} \eta_1 \left( 6 \bar{M}c + \bar{M}l \right) \left( 6 \bar{M}c + \bar{M}l \right) \\
\left( 6 \bar{M}c + \bar{M}l \right) + \left( 6 \bar{M}c + \bar{M}l \right) \]
\[ Y_1 = \left( j \omega_1 C_b \right) + \frac{R_1}{Z_1} \]

**Plot magnitude of electrical drive admittance**

Determine mechanical resonance, anti-resonance frequency and effective coupling coefficient

\[ fr_1 = f_1 \left( Y_1 = \max(Y) \right) \quad \text{fr} = \max(fr) \quad \bar{fr} = 2.476 \times 10^4 \]
\[ fa_1 = f_1 \left( Y_1 = \min(Y) \right) \quad fa = \max(fa) \quad \bar{fa} = 3.266 \times 10^4 \]
\[ k_{eff} = \sqrt{1 - \left(\frac{fr}{fa} \right)^2} \quad k_{eff} = 0.652 \]
Lumped Circuit calculation with bolt

Tail Specs with bolt (Brass)

\[ \begin{align*}
\text{Length} \quad l_1 & = 0.015 \\
\text{Radius} \quad r_1 & = 0.02 \quad r_2 = 0.0055 \\
\text{Section Area} \quad A_1 & = \pi \left( r_1^2 - r_2^2 \right) = 1.162 \times 10^{-3} \\
\text{Static mass of tail (inc bolt head)} \quad M_1 & = l_1 \cdot r_1 \cdot A_1 + 0.015 \quad M_1 = 0.159
\end{align*} \]

Compressive stress bias bolt

\[ \begin{align*}
\text{Length of bolt} \quad l & = 0.044 \\
\text{Compliance of bolt} \quad C_S & = \frac{1}{Y_S A_S} = 4.185 \times 10^{-9} \\
\text{Static mass of bolt without bolt head} \quad M_S & = \rho_3 A_S l \quad M_S = 0.016
\end{align*} \]

\[ Z_i = \frac{1}{j \omega_1 C_i E} + \frac{1}{j \omega_1 C_i S} = \frac{2 \omega_1 M_C}{\omega_1 (\delta M + M_t + \delta M_t + M_h)} \\
Y_i = \left( j \omega_1 C_i b \right) + \frac{N^2}{Z_i} \\
\]

Plot magnitude of electrical drive admittance

Determine mechanical resonance, anti-resonance frequency and effective coupling coefficient

\[ \begin{align*}
fr & = f_1 \left( Y_1 = \max(Y) \right) \\
fa & = f_1 \left( Y_1 = \min(Y) \right) \\
\text{keff} & = \sqrt{1 - \left( \frac{fr}{fa} \right)^2} \\
fr & = \max(fr) \quad fr = 2.396 \times 10^6 \\
fa & = \max(\text{fa}) \quad fa = 3.2 \times 10^4 \\
\text{keff} & = 0.063
\end{align*} \]
Figure C-2: Kagawa and Yamabuchi’s FE circuit.

The above circuit could represent any of the components of the transducer. One would obviously need to include the active section when modelling the ceramic. The main advantage of this circuit is that it is valid over a much larger frequency range than most other lumped element circuits.
APPENDIX C-5

Nodal tonpilz - distributed parameter, no bolt, untapered

**Head cylinder - Stainless steel**

- \( \bar{b}_h = 0.17 \)
- \( A_h = \frac{\pi \bar{b}_h^2}{4} \)
- \( \rho_h = 7800 \)
- \( Y_h = 193 \times 10^9 \)
- \( c_h = \frac{Y_h}{\sqrt{\rho_h}} \quad c_h = 4.974 \times 10^3 \)
- \( M_h = \rho_h A_h \bar{b}_h \quad M_h = 0.167 \)
- \( C_h = \frac{\bar{b}_h}{Y_h A_h} \quad C_h = 7.009 \times 10^{-11} \)

**Ceramic Assembly**

- \( A_c = \frac{\pi \left(0.085^2 - 0.027^2 \right)}{4} \)
- \( t_c = 0.0064 \)
- \( n_c = 4 \)
- \( \varepsilon_c = 13300.85 \times 10^{-12} \)
- \( \rho_c = 7700 \)
- \( s_{33E} = 1.96 \times 10^{-11} \)
- \( c_c = \frac{1}{\sqrt{\rho_c s_{33E}}} \quad c_c = 2.574 \times 10^3 \)
- \( d_{33} = 328 \times 10^{-12} \)
- \( k_{33} = \frac{d_{33}}{\sqrt{\varepsilon_c s_{33E}}} \)
- \( C_e = \frac{\varepsilon_c A_c}{A_c} \)
- \( C_b = C_e \left(1 - k_{33}^2 \right) \)
- \( N = \frac{d_{33}}{s_{33E}/t_c} \)
- \( C_mE = \frac{n_e. \varepsilon_c. s_{33E}}{A_c} \)
- \( M_c = \rho_c A_c t_c C_c \)

**Length of head**

**Section area head**

**Density of head material**

**Youngs modulus**

**Speed of sound in head**

**Static mass of head**

**Section area of ceramic rings**

**Thickness of ceramic rings**

**Number of ceramic elements**

**Dielectric constant of piezoelectric ceramic material**

**Density of ceramic rings**

**Elastic constant E-field**

**Sound speed for constant E-field or shorted terminals**

**Longitudinal strain per unit E-field**

- \( k_{33} = 0.683 \)
- \( C_b = 7.636 \times 10^{-6} \)
- \( C_m = 2.713 \)
- \( C_mE = 4.836 \times 10^{-10} \)

**Coupling factor**

**Free capacitance of stack**

**Blocked capacitance of stack**

**Blocked force per volt**

**Compliance of stack**

**Static mass**
Tail mass - Brass

\[ A_l = \frac{\pi 0.44 T^2}{4} \]

\[ \sigma = 0.15 \]

\[ \rho_l = 8250 \]

\[ \gamma_l = 90 \times 10^5 \]

\[ c_l = \sqrt{\frac{\gamma_l}{\rho_l}} \]

\[ c_l = 3.303 \times 10^3 \]

\[ M_l := \rho_l \cdot A_l \cdot t \]

\[ \zeta_l = \frac{\pi}{2} \]

\[ C_l = 1.326 \times 10^{-10} \]

Swept frequency increments and range

\[ f_l = 1012 \]

\[ f_l = 10000 \]

\[ f_2 = 40000 \]

\[ j = \sqrt{-1} \]

\[ \omega_1 := 2 \pi f \]

Equivalent circuit elements assuming mechanical impedance analogy and a plane wave traveling on length axis of the resonator (single degree of freedom).

**Forward section**

\[ Z_{h1} := j \cdot \rho_l \cdot c_h \cdot A_l \cdot t \cdot \frac{\omega_1 \cdot t_h}{2 \cdot \chi_h} \]

\[ Z_{h2} := \frac{\rho_l \cdot c_h \cdot A_l}{j \cdot \sin \left( \frac{\omega_1 \cdot t_h}{\chi_h} \right)} \]

**Ceramic circuit elements**

\[ Z_{c1} := j \cdot \rho_c \cdot c_c \cdot A_c \cdot \tan \left( \frac{\omega_1 \cdot t_c}{2 \cdot c_c} \right) \]

\[ Z_{c2} := \frac{\rho_c \cdot c_c \cdot A_c}{j \cdot \sin \left( \frac{\omega_1 \cdot t_c}{c_c} \right)} \]

**Tail section**

\[ Z_{t1} := j \cdot \rho_t \cdot c_t \cdot A_t \cdot \tan \left( \frac{\omega_1 \cdot t_t}{2 \cdot c_t} \right) \]

\[ Z_{t2} := \frac{\rho_t \cdot c_t \cdot A_t}{j \cdot \sin \left( \frac{\omega_1 \cdot t_t}{c_t} \right)} \]

Translate mechanical impedances through the ideal transformer to yield an equivalent electrical network.

\[ Z_1 = \frac{1}{j \omega_1 \cdot C_h} \quad \text{Blocked capacitance term} \]

\[ Z_2 = \frac{Z_{c2}}{N^2} \quad \text{Compliance related terms for ceramic} \]

\[ Z_3 = \frac{Z_{c1} + Z_{t1} + \frac{2Z_1 \cdot Z_{t2}}{Z_{t1} + Z_{t2}}}{N^2} \quad \text{Aft section of resonator} \]
\[
\frac{Z_{e1} + Z_{h1} + \frac{Z_{h1}Z_{h2}}{Z_{h1} + Z_{h2}}}{N^2}
\]

Forward section of resonator

Impedance matrix for equivalent circuit:
\[
\begin{pmatrix}
Z_1 & -Z_1 & 0 \\
-Z_1 & Z_1 + Z_2 + Z_3 & -Z_3 \\
0 & -Z_3 & Z_3 + Z_4
\end{pmatrix}
\]

Drive resonator with 1 volt at electrical terminals and compute currents flowing in each mesh
\[
\begin{pmatrix}
v_1 \\
v_2
\end{pmatrix}
= Z(0)^{-1}
\begin{pmatrix}
v_t \\
0
\end{pmatrix}
\]

Plot magnitude of electrical drive admittance
\[
Y_{mag} = \frac{10^6}{|V_t|}
\]

Determine mechanical resonance, anti-resonance frequency and effective coupling coefficient
\[
\begin{align*}
f_r &= f_t \left( Y_{mag} = \max(Y_{mag}) \right) \\
fr &= \max(f_t) \\
f_i &= \min(Y_{mag}) \\
f_a &= \min(f_t)
\end{align*}
\]

\[
k eff^2 = \frac{(f_r - f_i)^2}{f_r} \\
k eff = 0.648
\]
Nodal tonpilz - rod approximation

Compressive stress bias bolt

\[
\begin{align*}
\tau_s &= \rho \omega^2 \sigma \\
A_s &= \pi \cdot 0.004^2 \\
\rho s &= 7800 \\
Y_s &= 19.5 \times 10^{10} \\
\sigma_s &= \sqrt{\frac{Y_s}{\rho s}} \\
C_s &= \frac{\tau_s}{Y_s A_s} \\
\frac{C_s}{C_m E} &= 5.456 \\
M_s &= \rho s A_s \cdot \tau_s \\
M_s &= 0.01
\end{align*}
\]

Stress bolt section

\[
Z_{s1} = \frac{\rho s A_s}{\tan \left( \frac{\omega_1}{2 \cdot \sigma_s} \right)} \\
Z_{s2} = \frac{\rho s A_s}{\sin \left( \frac{\omega_1}{2 \cdot \sigma_s} \right)}
\]

Translate mechanical impedances through the ideal transformer to yield an equivalent electrical network:

\[
Z_{1} = \frac{1}{j \omega_1 C_b} \\
Z_{2} = \frac{C_{s2}}{N^2} \\
Z_{3} = \frac{Z_{s1} + Z_{h1} + \left( Z_{h1} + Z_{h2} \right)}{N^2} \\
Z_{4} = \frac{Z_{s2}}{N^2} \\
Z_{5} = \frac{Z_{s1} + Z_{h1} + \left( Z_{h1} + Z_{h2} \right) + Z_{s1}}{N^2}
\]

Impedance matrix for equivalent circuit:

\[
Z(0) = \begin{pmatrix}
Z_{1} & -Z_{1} & 0 \\
-Z_{1} & Z_{1} + Z_{2} + Z_{3} + Z_{4} & -Z_{3} \\
0 & -Z_{3} & Z_{3} + Z_{5}
\end{pmatrix}
\]
Drive resonator with 1 volt at electrical terminals and compute currents flowing in each mesh.

\[
V_1 = \frac{1}{Z} \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} = Z \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}
\]

Plot magnitude of electrical drive admittance

\[ Y_{\text{mag}} := \left| \frac{V}{I} \right| \]

Determine mechanical resonance, anti-resonance, frequency and effective coupling coefficient:

\[
\begin{align*}
fr_1 &= f_{r1} \left( Y_{\text{mag}} = \max(Y_{\text{mag}}) \right) \\
mr &= \max(fr) \\
f_{r1} &= 2.37 \times 10^4 \\
fa &= f_{a1} \left( Y_{\text{mag}} = \max(Y_{\text{mag}}) \right) \\
m a &= \max(fa) \\
f_{a1} &= 3 \times 10^4 \\
k_{\text{eff}} &= \sqrt{1 - \left( \frac{fr}{fa} \right)^2} \\
k_{\text{eff}} &= 0.613
\end{align*}
\]
Nodal Mounted Tonpilz - tapered head

Head cone - stainless steel

- \( \text{th} = 0.017 \) Length of head
- \( r_1 = 0.02 \) \( r_2 = 0.0275 \) Radii of two faces of cone
- \( \theta = \tan\left(\frac{\text{th}}{r_2 - r_1}\right) \) \( \alpha_{\text{hm}} = \tan(\theta) \)
- \( \text{thm} = 0.062 \) Length of complete cone
- \( l_r = \text{thm} - \text{th} \) \( l_r = 0.045 \) Difference between head and total cone length
- \( A_{1h} = \pi r_1^2 \) \( A_{2h} = \pi r_2^2 \) Areas of faces
- \( \rho_h = 7800 \) Density of head material
- \( Y_h = 193 \times 10^9 \) Youngs modulus
- \( c_h = \frac{Y_h}{\rho_h} \) \( c_h = 4.974 \times 10^7 \) Speed of sound in head
- \( M_h = \rho_h \cdot \text{th} \cdot \sqrt{A_{2h} \cdot A_{1h}} \) \( M_h = 0.22 \) Static mass of head
- \( C_h = \frac{\text{th}}{Y_h \cdot \sqrt{A_{2h} \cdot A_{1h}}} \) \( C_h = 3.098 \times 10^{-11} \) Compliance of head

Components of head section

- \( Z_{11} = -j \frac{\rho_h \cdot c_h \cdot \sqrt{A_{2h} \cdot A_{1h}}}{A_{2h}} \left( \frac{1}{\tan(\frac{\alpha_{\text{th}} \cdot \text{th}}{c_h})} + \frac{c_h}{\alpha_{\text{th}} \cdot l_r} \right) \)
- \( Z_{12} = \frac{\rho_h \cdot c_h \cdot \sqrt{A_{2h} \cdot A_{1h}}}{j \cdot \sin(\frac{\alpha_{\text{th}} \cdot \text{th}}{c_h})} \)
- \( Z_{21} = j \frac{\rho_h \cdot c_h \cdot \sqrt{A_{2h} \cdot A_{1h}}}{A_{2h}} \left( \frac{1}{\tan(\frac{\alpha_{\text{th}} \cdot \text{th}}{c_h})} - \frac{c_h}{\alpha_{\text{th}} \cdot l_r} \right) \)
- \( Z_{22} = j \frac{\rho_h \cdot c_h \cdot \sqrt{A_{2h} \cdot A_{1h}}}{j \cdot \sin(\frac{\alpha_{\text{th}} \cdot \text{th}}{c_h})} \)
- \( Z_{11} = Z_{11} - Z_{12} \)
- \( Z_{12} = Z_{21} - Z_{12} \)
- \( Z_{21} = Z_{12} \)
- \( Z_{22} = Z_{22} \)

- \( Z_{11} = Z_{11} + Z_{21} + Z_{12} + Z_{22} \)
- \( Z_{12} = Z_{12} \)

- \( Z_{11} = \frac{Z_{11} + Z_{21} + Z_{12} + Z_{22}}{N^2} \) Equivalent impedance of forward section of resonator

Impedance matrix for equivalent circuit

- \( Z(\omega) = \begin{pmatrix} Z_{11} & -Z_{11} & 0 \\ -Z_{11} & Z_{11} + Z_{21} + Z_{12} + Z_{22} & Z_{12} + Z_{22} \\ 0 & -Z_{12} & Z_{11} + Z_{21} \end{pmatrix} \)
Drive resonator with 1 volt at electrical terminals and compute currents flowing in each mesh

\[
\begin{bmatrix}
10^{-1} \\
1 \\
10^{-2}
\end{bmatrix}
= Z(0)^{-1}
\begin{bmatrix}
V_t \\
0 \\
0
\end{bmatrix}
\]

Plot magnitude of electrical drive admittance

\[
Y_{\text{mag}} := \frac{|10^{-1}|}{|V_t|}
\]

Determine mechanical resonance, anti-resonance frequency and effective coupling coefficient

\[
\begin{align*}
fr &= \frac{1}{f} \left( Y_{\text{mag}} = \max(Y_{\text{mag}}) \right) \\
fr &= \max(fr) \\
fr &= 2.213 \times 10^7 \\
f_a &= \frac{1}{f} \left( Y_{\text{mag}} = \min(Y_{\text{mag}}) \right) \\
f_a &= \max(f_a) \\
f_a &= 2.801 \times 10^4 \\
\text{keff} &= \sqrt{1 - \left( \frac{fr}{f_a} \right)^2} \\
\text{keff} &= 0.613
\end{align*}
\]
Figure: Graph showing improved output of nodal-mounted transducer with modified head.
APPENDIX C-7

Below is the full deck to simulate the nodal-mounted transducer radiating into the hemisphere of water.

*heading

Nodal-mounted Tonpilz transducer, SS head, Brass tail, HT steel bolt, 55mm diameter head, 65.6mm long, 4 ReCu electrodes, head mpc'd to water surface

***

*preprint, model=yes

***

**node definition

***

*nodel

1, 0.0, 0.0
5, 0.004, 0.0
27, 0.0275, 0.0

**

101, 0.0, 0.001
105, 0.004, 0.001
127, 0.0275, 0.001

**

301, 0.0, 0.005
305, 0.004, 0.005
327, 0.0255, 0.005

**

801, 0.0, 0.016
805, 0.004, 0.016
809, 0.00635, 0.016
815, 0.01, 0.016
825, 0.01925, 0.016
827, 0.02, 0.016

**

1001, 0.0, 0.018
1005, 0.004, 0.018
1009, 0.00635, 0.018
1015, 0.01, 0.018
1025, 0.01925, 0.018
1027, 0.02, 0.018

**
4201, 0.0, 0.0456
4205, 0.004, 0.0456
4207, 0.0055, 0.0456
4209, 0.00635, 0.0456
4215, 0.01, 0.0456
4225, 0.01925, 0.0456
4227, 0.02, 0.0456
**
5001, 0.0, 0.0606
5005, 0.004, 0.0606
5007, 0.0055, 0.0606
5011, 0.00635, 0.0606
5015, 0.01, 0.0606
5027, 0.02, 0.0576
**
5201, 0.0, 0.0656
5205, 0.004, 0.0656
5211, 0.00635, 0.0656
5215, 0.01, 0.0656
**-----------------------------------------
**head definition
**-----------------------------------------
*ngen, nset=temp1
 1, 5, 1
 5, 27, 1
*ngen, nset=temp2
101, 105, 1
105, 127, 1
*ngen, nset=temp20
805, 809, 1
809, 825, 1
825, 827, 1
**
*ngen, nset=temp3a
301, 305, 1
*ngen, nset=temp21
305, 327, 1
**
*ngen, nset=temp3b
1005, 1009, 1
1009, 1025, 1
1025, 1027, 1
* nset, nset=temp3
  temp3a, temp21
  **
* nfill
  temp1, temp2, 2, 50
  temp2, temp3, 4, 50
  temp21, temp20, 10, 50
  temp20, temp3b, 4, 50
  **
* element, type=cax8, elset=head
  1, 1, 103, 101, 2, 53, 102, 51
* elgen, elset=head
  1, 13, 2, 1, 1, 100, 100
  **
* solid section, elset=head, material=ss
  **
* element, type=cax8, elset=head
  101, 101, 103, 203, 201, 102, 153, 202, 151
* elgen, elset=head
  101, 13, 2, 1, 2, 100, 100
  **
* element, type=cax8, elset=head
* elgen, elset=head
  305, 11, 2, 1, 5, 100, 100
  **
* element, type=cax8, elset=head
  805, 805, 807, 907, 905, 806, 857, 906, 855
* elgen, elset=head
  805, 11, 2, 1, 2, 100, 100
  **
---------------------------------------------------------------
** bolt definition
**---------------------------------------------------------------
* ngen, nset=temp5a
  801, 805, 1
* ngen, nset=temp5b
  1001, 1005, 1
* ngen, nset=temp5
  5001, 5005, 1
  **
*nfill
temp3a, temp5a, 10, 50
*nfill
temp5a, temp5b, 4, 50
*nfill
temp5b, temp5, 80, 50
**
*element, type=cax8, elset=bolt
301, 301,303,403,401, 302,353,402,351
*elgen, elset=bolt
301, 2,2,1, 5,100,100
**
*solid section, elset=bolt, material=ss
**
*element, type=cax8, elset=bolt
801, 801,803,903,901, 802,853,902,851
*elgen, elset=bolt
801, 2,2,1, 2,100,100
**
*element, type=cax8, elset=bolt
1001, 1001,1003,1103,1101, 1002,1053,1102,1051
*elgen, elset=bolt
1001, 2,2,1, 40,100,100
**-----------------------------------------------
*ngen, nset=temp6
4207, 4209, 1
4209, 4225, 1
4225, 4227, 1
*ngen, nset=temp7a
5007, 5015, 1
*ngen, nset=temp7b
5015, 5027, 1
*nset, nset=temp7
temp7a,temp7b
**
*nfill
temp6, temp7, 16, 50
**
*element, type=cax8, elset=tail
4207, 4207,4209,4309,4307, 4208,4259,4308,4257
*elgen, elset=tail
4207, 10,2,1, 8,100,100
**
*solid section, elset=tail, material=brass
**nut definition
**nset, nset=temp8a
temp5, temp8a, temp7a
**nfill
temp8, temp9, 4, 50
**
*element, type=cax8, elset=nut
5001, 5001,5003,5103,5101, 5002,5053,5102,5051
*elgen, elset=nut
5001, 7,2,1, 2,100,100
**
*solid section, elset=nut, material=ss
**nut definition
**node
1109, 0.00635, 0.018
1125, 0.01925, 0.018
**
1509, 0.00635, 0.0244
1525, 0.01925, 0.0244
**
*nset, nset=a1
1109, 1125, 1
*nset, nset=b1
1509, 1525, 1
**
*nfill
a1, b1, 8, 50
**
*element, type=cax8e, elset=pztpos
1109, 1109, 1111, 1211, 1209, 1110, 1161, 1210, 1159
*elgen, elset=pztpos
1109, 8, 2, 1, 4, 100, 100
**
*solid section, elset=pztpos, material=pz26, orientation=2pos
**electrode definition
**node
1609, 0.00635, 0.0244
1625, 0.01925, 0.0244
1627, 0.02, 0.0244
**
1709, 0.00635, 0.02465
1725, 0.01925, 0.02465
1727, 0.02, 0.02465
**
*ngen, nset=a2
1609, 1625, 1
1625, 1627, 1
*ngen, nset=b2
1709, 1725, 1
1725, 1727, 1
**
*nfill
a2, b2, 2, 50
**
*element, type=cax8e, elset=elec
1609, 1609, 1611, 1711, 1709, 1610, 1661, 1710, 1659
*elgen, elset=elec
1609, 9, 2, 1, 1, 100, 100
**
*solid section, elset=elec, material=BeCu
**pzt2 definition
**node
1809, 0.00635, 0.02465
1825, 0.01925, 0.02465
**
2209, 0.00635, 0.03105
2225, 0.01925, 0.03105
**
*ngen, nset=a3
1809, 1825, 1
*ngen, nset=b3
2209, 2225, 1
**
*nfill
a3, b3, 8, 50
**
*element, type=cax8e, elset=pztneg
1809, 1809, 1811, 1911, 1909, 1810, 1861, 1910, 1859
*elgen, elset=pztneg
1809, 8, 2, 1, 4, 100, 100
**
*solid section, elset=pztneg, material=pz26, orientation=2neg
**
*nodal definition
**
*node
2309, 0.00635, 0.03105
2325, 0.01925, 0.03105
2327, 0.02, 0.03105
2339, 0.035, 0.03105
**
2439, 0.035, 0.03155
**
2509, 0.00635, 0.03205
2525, 0.01925, 0.03205
2527, 0.02, 0.03205
2539, 0.035, 0.03205
**
*ngen, nset=Na
2309, 2325, 1
2325, 2327, 1
2327, 2339, 1
*ngen, nset= Nb
2509, 2525, 1
2525, 2527, 1
2527, 2539, 1
**
*nfill
Na, Nb, 4, 50
**
*element, type=cax8, elset=nodal
2309, 2309, 2311, 2411, 2409, 2310, 2361, 2410, 2359
*elgen, elset=nodal
2309, 15, 2, 1, 2, 100, 100
**
*solid section, elset=nodal, material=ss
**
**
**
**
**
**
**
**
**
*nset=a4
2609, 2625, 1
2625, 2627, 1
**
*nset=b4
2709, 2725, 1
2725, 2727, 1
**
*nfill
a4, b4, 2, 50
**
*element, type=cax8, elset=elec
2609, 2609, 2611, 2711, 2709, 2610, 2661, 2710, 2659
*elgen, elset=elec
2609, 9, 2, 1, 1, 100, 100
**
**
**pzt3 definition
*node
2809, 0.00635, 0.0323
2825, 0.01925, 0.0323
**
3209, 0.00635, 0.0387
3225, 0.01925, 0.0387
**
*ngen, nset=a5
2809, 2825, 1
*ngen, nset=b5
3209, 3225, 1
**
*nfill
a5, b5, 8, 50
**
*element, type=cax8e, elset=pztpos
2809, 2809, 2811, 2911, 2909, 2810, 2861, 2910, 2859
*elgen, elset=pztpos
2809, 8, 2, 1, 4, 100, 100
**
**electrode3 definition
**
*node
3309, 0.00635, 0.0387
3325, 0.01925, 0.0387
3327, 0.02, 0.0387
**
3409, 0.00635, 0.03895
3425, 0.01925, 0.03895
3427, 0.02, 0.03895
**
*ngen, nset=a6
3309, 3325, 1
3325, 3327, 1
*ngen, nset=b6
3409, 3425, 1
3425, 3427, 1
**
*nfill
a6, b6, 2, 50
**
*element, type=cax8, elset=elec
3309, 3309, 3311, 3411, 3409, 3310, 3361, 3410, 3359
*elgen, elset=elec
3309, 9, 2, 1, 1, 100, 100
**
**pzt4 definition
**

*nnode
3525, 0.01925, 0.03895
3525, 0.01925, 0.03895
**
3909, 0.00635, 0.04535
3925, 0.01925, 0.04535
**
*ngen, nset=a7
3509, 3525, 1
*ngen, nset=b7
3909, 3925, 1
**
*nfill
a7, b7, 8, 50
**
*element, type=cax8e, elset=pztneg
3509, 3509, 3511, 3611, 3609, 3510, 3561, 3610, 3559
*elgen, elset=pztneg
3509, 8, 2, 1, 4, 100, 100
**
**electrode4 definition
**

*nnode
4009, 0.00635, 0.04535
4025, 0.01925, 0.04535
4027, 0.02, 0.04535
**
4109, 0.00635, 0.0456
4125, 0.01925, 0.0456
4127, 0.02, 0.0456
**
*ngen, nset=a8
4009, 4025, 1
4025, 4027, 1
*ngen, nset=b8
4109, 4125, 1
4125, 4127, 1
**
*nfill
a8, b8, 2, 50
**
*element, type=cax8, elset=elec
4009, 4009,4011,4111,4109, 4010,4061,4110,4059
*elgen, elset=elec
4009, 9,2,1, 1,100,100
**-----------------------------------------------------------
*pzt1 to head mechanical vertical coupling
**-----------------------------------------------------------
*nset, nset=tem10, generate
1009, 1025, 1
*mpc
tie, tem10, a1
**-----------------------------------------------------------
*pzt1 to elecl mechanical vertical coupling
**-----------------------------------------------------------
*nset, nset=tem11, generate
1609, 1625, 1
*mpc
tie, tem11, b1
**-----------------------------------------------------------
*elecl to pzt2 mechanical vertical coupling
**-----------------------------------------------------------
*nset, nset=tem12, generate
1709, 1725, 1
*mpc
tie, tem12, a3
**-----------------------------------------------------------
*pzt2 to Nodal mechanical vertical coupling
**-----------------------------------------------------------
*nset, nset=Noda, generate
2309, 2325, 1
*mpc
tie, Noda, b3
**-----------------------------------------------------------
**Nodal to elec2 mechanical vertical coupling**

* nset, nset=Nodb, generate
  2509, 2525, 1
* nset, nset=temp13, generate
  2609, 2625, 1
* mpc
tie, Nodb, temp13

**elec2 to pzt3 mechanical vertical coupling**

* nset, nset=temp14, generate
  2709, 2725, 1
* mpc
tie, temp14, a5

**pzt3 to elec3 mechanical vertical coupling**

* nset, nset=temp15, generate
  3309, 3325, 1
* mpc
tie, temp15, b5

**elec3 to pzt4 mechanical vertical coupling**

* nset, nset=temp16, generate
  3409, 3425, 1
* mpc
tie, temp16, a7

**pzt4 to elec4 mechanical vertical coupling**

* nset, nset=temp17, generate
  4009, 4025, 1
* mpc
tie, temp17, b7

**elec4 to tail mechanical vertical coupling**

* nset, nset=temp18, generate
  4209, 4225, 1
*nset, nset=b8', generate
4109, 4125, 1
*mpc
tie, b8', templ8
**---------------------------------------------------
**electrode electrical detail
**---------------------------------------------------
*node, nset=ns_ref
100013, 0.036, 0.0
100014, 0.037, 0.0
100015, 0.038, 0.0
*equation
2
b1,9,1.0, a3,9,-1.0
*equation
2
a3,9,1.0, 100013,9,-1.0
*equation
2
b3,9,1.0, a5,9,-1.0
*equation
2
a5,9,1.0, 100014,9,-1.0
*equation
2
b5,9,1.0, a7,9,-1.0
*equation
2
a7,9,1.0, 100015,9,-1.0
** tying +ve elecs together
*equation
2
100013,9,1.0, 100015,9,-1.0
** tying -ve elecs together
*equation
2
a1,9,1.0, b7,9,-1.0
*equation
2
b7,9,1.0, 100014,9,-1.0
**---------------------------------------------------
**material definitions

*orientation, name=2pos
1.0, 0.0, 0.0, 0.0, 1.0, 0.0
1.0

*orientation, name=2neg
-1.0, 0.0, 0.0, 0.0, -1.0, 0.0
1.0

*material, name=pz26
*elastic, type=ortho
16.8E10, 9.99E10, 12.3E10, 11.0E10, 9.99E10, 16.8E10, 3.01E10, 3.01E10, 2.88E10
*piezoelectric, type=s
0.0, 0.0, 0.986, 0.0, -1.98, 14.7
-1.98, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.986
*dielectric, type=aniso
11.771E-9, 0.0, 10.531E-9, 0.0, 0.0, 11.771E-9
*density
7700.

*material, name=ss
*elastic
200.e9, 0.3
*density
7800.

*material, name=brass
*elastic
90.e9, 0.3
*density
9410.

*material, name=BeCu
*elastic
128.e9, 0.3
*density
8250.
*material, name=water
*density
1000.0
*acoustic medium, bulk modulus
2.18e9

**water definition & water / head interface

*node
10001, 0.0, 0.0
10002, 0.001, 0.0
10005, 0.004, 0.0
10027, 0.0275, 0.0
10101, 0.3, 0.0
10199, 1.0, 0.0

**
52002, 0.0, -0.001
52005, 0.0, -0.004
52027, 0.0, -0.0275
52101, 0.0, -0.3
52199, 0.0, -1.0

**
*ngen, line=c, nset=cls1
10002, 52002, 200, 10001
*ngen, line=c, nset=cls2
10005, 52005, 200, 10001
*ngen, line=c, nset=cls3
10027, 52027, 200, 10001
*ngen, line=c, nset=mid
10101, 52101, 200, 10001
*ngen, line=c, nset=far
10199, 52199, 200, 10001

**
*nset, nset=surface, generate
10027, 10199, 1

**
*nfill
cls1, cls2, 3, 1
*nfill
cls2, cls3, 22, 1
*nfill, bias=0.98
cls3, mid, 74, 1
*nfill
**element definition (wedge elements first)**

*element, type=acax8, elset=water

10403, 10403, 10003, 10001, 10001, 10203, 10002, 10001, 10402
10803, 10803, 10403, 10001, 10001, 10603, 10402, 10001, 10802
11203, 11203, 10803, 10001, 10001, 11003, 10802, 10001, 11202
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19203, 19203, 18803, 10001, 10001, 19003, 18802, 10001, 19202
19603, 19603, 19203, 10001, 10001, 19403, 19202, 10001, 19602
20003, 20003, 19603, 10001, 10001, 19803, 19602, 10001, 20002
20403, 20403, 20003, 10001, 10001, 20203, 20002, 10001, 20402
20803, 20803, 20403, 10001, 10001, 20603, 20402, 10001, 20802
21203, 21203, 20803, 10001, 10001, 21003, 20802, 10001, 21202
21603, 21603, 21203, 10001, 10001, 21403, 21202, 10001, 21602
22003, 22003, 21603, 10001, 10001, 21803, 21602, 10001, 22002
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52003, 52003, 51603, 10001, 10001, 51803, 51602, 10001, 52002
**
*element, type=acax8, elset=water
10405, 10405, 10005, 10003, 10403, 10205, 10004, 10203, 10404
10429, 10429, 10029, 10027, 10427, 10229, 10028, 10227, 10428
10503, 10503, 10103, 10101, 10501, 10303, 10102, 10301, 10502
*elgen, elset=water
10405, 105, 400, 400, 12, 2, 2
10429, 105, 400, 400, 37, 2, 2
10503, 105, 400, 400, 49, 2, 2
*solid section, elset=water, material=water
**
**Interface elements
**----------------------------------
*element, type=asi3a, elset=inter
9001, 1,2,3
*elgen, elset=inter
9001, 13,2,1
*interface, elset=inter
**----------------------------------
**Non-reflective boundary definition
**-------------------------------------
*elset, elset=waterbound, generate
10599, 52199, 400
*surface impedance
waterbound, I1,
**-------------------------------------
**mpc's tying head to water
**-------------------------------------
*nset, nset=headwater, generate
10001, 10027, 1
**
*mpc
tie, templ, headwater
**-------------------------------------
*step, perturbation
1V excitation, nodal plate support simulation, water modes inc.
*steady state dynamic, direct
20.0e3, 25.0e3, 100, 1,1
**
*restart, write, f=0
**
*boundary
2439, 1,2, 0.0
100014, 9,9, 0.0
surface, 8,8, 0.0
**
*cecharge
100015, , 1.0
**
*node file, f=1, nset=ns_ref
RCHG, PHCHG, EPOT, PHPOT
**
**nset, nset=ns_ref2
**1,27,5201,5215
**node file, f=0, nset=ns_ref2
**U,V,A,PU
**
*node print, f=0
*el print, f=0
*end step
**----------------------------------------------------------
------
**----------------------------------------------------------
------

APPENDIX C-8

The design drawings for the nodal mounted transducer can be seen on the following pages.
Title: Flange For Nadal Tapiz

<table>
<thead>
<tr>
<th>PART NO.</th>
<th>DESCRIPTION</th>
<th>MATERIAL</th>
<th>NUM.</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Flange</td>
<td>Stainless steel</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

UNIVERSITY OF CAPE TOWN
DEPARTMENT OF MECHANICAL ENGINEERING

Dimensions in millimeters (mm),tolerance unless otherwise stated: ±0.05

Scale: 1:1
Date: 14/6/00
Sheet: 1 of 1

Drawn by: A Green

Drawing Number:
APPENDIX D-1

Tan approx for transducers-Face Mounted Tonpilz

Head Specs (Aluminium)

<table>
<thead>
<tr>
<th>lh</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>rh := 0.02</td>
<td>Radius</td>
</tr>
<tr>
<td>ρh := 2700</td>
<td>Density</td>
</tr>
<tr>
<td>Ah := π · rh^2</td>
<td>Ah = 1.257 × 10^{-3}</td>
</tr>
<tr>
<td>ch := 5055</td>
<td>Speed of sound in material</td>
</tr>
</tbody>
</table>

Ceramic Specs (Half Stack)

<table>
<thead>
<tr>
<th>Ac</th>
<th>Section area of PZT discs</th>
</tr>
</thead>
<tbody>
<tr>
<td>tc := 0.0264</td>
<td>Thickness of discs</td>
</tr>
<tr>
<td>nc := 1</td>
<td>Number of discs</td>
</tr>
<tr>
<td>ρc := 7700</td>
<td>Density of discs</td>
</tr>
<tr>
<td>s33E := 1.96 × 10^{-11}</td>
<td>Elastic constant E-field</td>
</tr>
<tr>
<td>cc := \sqrt{\frac{1}{\rho c \cdot s33E}}</td>
<td>cc = 2.574 × 10^{3}</td>
</tr>
</tbody>
</table>

Tail Specs (Brass)

<table>
<thead>
<tr>
<th>lt</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>rt := 0.02</td>
<td>Radius</td>
</tr>
<tr>
<td>ρt := 8250</td>
<td>Density</td>
</tr>
<tr>
<td>At := π · rt^2</td>
<td>At = 1.257 × 10^{-3}</td>
</tr>
<tr>
<td>ct := 3303</td>
<td>Speed of sound in material</td>
</tr>
</tbody>
</table>

Frequency

| fr := 26 × 10^{-3} | Desired resonant frequency |

Tan Approx

| lh := \frac{\left(\rho c \cdot cc \cdot Ac\right)}{\sqrt{\frac{2 \cdot π \cdot fr \cdot nc \cdot tc}{cc}}} | lh = 0.038 | Head Length |
|----|--------|
| lt := \frac{\left(\rho c \cdot cc \cdot Ac\right)}{\sqrt{\frac{2 \cdot π \cdot fr \cdot nc \cdot tc}{cc}}} | lt = 0.019 | Tail Length |
APPENDIX D-2

Face Mounted Tonpilz: Lumped Element Approx

Head Specs (Aluminium)

\[ l_h := 0.038 \] Length
\[ r_h := 0.02 \] Radius
\[ \rho_h := 2700 \] Density
\[ A_h := \pi r_h^2 \] Section Area
\[ M_h := l_h \rho_h A_h \] Static mass of head
\[ Y_h := 69 \times 10^9 \] Youngs modulus
\[ C_h := \frac{l_h}{Y_h A_h} \] Compliance of head

Ceramic Specs

\[ A_c := \pi \left( \frac{0.0385^2 - 0.012^2}{4} \right) \] Section area of PZT discs
\[ t_c := 0.0064 \] Thickness of discs
\[ N_c := 2 \] Number of discs
\[ \rho_c := 7700 \] Density of discs
\[ \varepsilon_c := 13308.854 \times 10^{-12} \] Dielectric constant of piezoelectric ceramic material
\[ s_{33} := 1.96 \times 10^{-11} \] Elastic constant E-field
\[ d_{33} := 328 \times 10^{-12} \] Longitudinal strain per unit E-field
\[ k_{33} := 0.683 \] Compliance of stack

\[ C_{mE} := \frac{N_c t_c \varepsilon_c s_{33}}{A_c} \] Static mass
\[ M_c := \rho_c t_c N_c A_c \] Free capacitance of stack
\[ C_b := C_c \left( 1 - k_{33}^2 \right) \] Blocked capacitance of stack
\[ N := \frac{d_{33} A_c}{s_{33} E t_c} \] Blocked force per volt

Tail Specs (Brass)

\[ l_t := 0.019 \] Length
\[ r_t := 0.02 \] Radius
\[ \rho_t := 8250 \] Density
\[ A_t := \pi r_t^2 \] Section Area
Mt := l_t·π·A_t \quad Mt = 0.197 \quad \text{Static mass of tail}

Y_t := 90 \cdot 10^9 \quad \text{Youngs modulus}

C_t := \frac{r}{Y_t A_t} \quad C_t = 1.68 \times 10^{-10} \quad \text{Compliance of tail}

\textbf{Compressive stress bias bolt}

\text{ts := nc·tc} \quad \text{ts = 0.013} \quad \text{Length of bolt}

\text{As := \pi \cdot 0.04f^2} \quad \text{Section Area}

\rho_s := 7800 \quad \text{Density of bolt material}

Y_s := 19.3 \cdot 10^{10} \quad \text{Youngs Modulus for bolt material}

C_s := \frac{ts}{Y_s A_s} \quad C_s = 1.319 \times 10^{-9} \quad \text{Compliance of bolt}

M_s := \rho_s A_s \cdot ts \quad M_s = 5.019 \times 10^{-3} \quad \text{Static mass of bolt}

\textbf{Swept frequency increments and range}

\text{nf := 1012} \quad i := 0..nf \quad f_1 := 10000 \quad f_2 := 40000 \quad f_i := f_1 \left( \frac{f_2}{f_1} \right)^{i/nf}

j := \sqrt{-1} \quad \omega_i := 2\pi f_i

\textbf{Lumped circuit calculation without bolt t}

\begin{align*}
Z_{1i} := & \frac{j \omega_i \cdot 6 \cdot M_t \cdot \left(-j \omega_i \cdot 2 \cdot M_t + \frac{1}{j \omega_i \cdot C_t}\right)}{j \omega_i \cdot 4 \cdot M_t + \frac{1}{j \omega_i \cdot C_t}} + j \omega_i \cdot 6 \cdot M_t + j \omega_i \cdot 6 \cdot M_c \quad \text{Tail and ceramic impedance} \\
Z_{2i} := & \frac{j \omega_i \cdot 6 \cdot M_h \cdot \left(-j \omega_i \cdot 2 \cdot M_h + \frac{1}{j \omega_i \cdot C_h}\right)}{j \omega_i \cdot 4 \cdot M_h + \frac{1}{j \omega_i \cdot C_h}} + j \omega_i \cdot 6 \cdot M_h + j \omega_i \cdot 6 \cdot M_c \quad \text{Head and ceramic impedance} \\
Z_{3i} := & \frac{1}{j \omega_i \cdot C_m} - j \omega_i \cdot 2 \cdot M_c \quad \text{Ceramic impedance}
\end{align*}

Z_i := \frac{Z_{1i} \cdot Z_{2i}}{Z_{1i} + Z_{2i}} + Z_{3i}

Y_i := \left| j \omega_i \cdot C_b + \frac{N^2}{Z_i} \right| \quad \text{Overall admittance}
Determine mechanical resonance, anti-resonance frequency and effective coupling coefficient

\[ f_r := \text{fr}(Y = \max(Y)) \quad f_r = \max(fr) \quad f_r = 2.507 \times 10^4 \]
\[ f_a := \text{fa}(Y = \min(Y)) \quad f_a = \max(fa) \quad f_a = 2.919 \times 10^4 \]

\[ \text{keff} := \sqrt{1 - \left(\frac{f_r}{f_a}\right)^2} \quad \text{keff} = 0.312 \]

**Lumped Circuit calculation with rod**

\[ Z_1 := \frac{j \omega \phi \delta M \left(-j \omega \phi \cdot 2 \cdot Mh + \frac{1}{j \omega \phi \cdot Cl}\right)}{j \omega \phi \cdot 4 \cdot Mh + \frac{1}{j \omega \phi \cdot Cl}} + j \omega \phi \cdot 6 \cdot Mh + j \omega \phi \cdot 6 \cdot Mc \]

\[ Z_2 := \frac{j \omega \phi \cdot 6 \cdot Mh \left(-j \omega \phi \cdot 2 \cdot Mh + \frac{1}{j \omega \phi \cdot Ch}\right)}{j \omega \phi \cdot 4 \cdot Mh + \frac{1}{j \omega \phi \cdot Ch}} + j \omega \phi \cdot 6 \cdot Mh + j \omega \phi \cdot 6 \cdot Mc \]

\[ Z_3 := \frac{1}{j \omega \phi \cdot C_{me}} + \frac{1}{j \omega \phi \cdot C_s} - j \omega \phi \cdot 2 \cdot Mc \]
\[ Z_i = \frac{Z_{i1} Z_{i2}}{Z_{i1} + Z_{i2}} + Z_{i3} \]

\[ Y_i = \left( \frac{1}{\omega_i C_i} + \frac{N^2}{Z_i} \right) \]

Plot magnitude of electrical drive admittance

Determine mechanical resonance, anti-resonance frequency and effective coupling coefficient

\[ f_{r} = \frac{1}{\pi \sqrt{\frac{1}{\omega_i C_i} + \frac{N^2}{Z_i}}} \]

\[ f_{a} = \frac{1}{\pi \sqrt{\frac{1}{\omega_i C_i} + \frac{N^2}{Z_i}}} \]

\[ k_{eff} = \sqrt{\frac{1}{\pi \sqrt{\frac{1}{\omega_i C_i} + \frac{N^2}{Z_i}}}} \]

Lumped Circuit calculation with bolt

Tail Specs with bolt (Stainless steel)

<table>
<thead>
<tr>
<th>( L = 0.015 )</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>11 = 0.02 ( r )</td>
<td>Radius</td>
</tr>
<tr>
<td>( At = \pi \left( r_1^2 - r_2^2 \right) )</td>
<td>Section Area</td>
</tr>
<tr>
<td>( Mt = tr \pi At + 0.015 )</td>
<td>Static mass of tail (inc bolt head)</td>
</tr>
</tbody>
</table>

| 190 |
Compressive stress bias bolt

\[ t_s := 0.028 \quad \text{Length of bolt} \]

\[ C_S := \frac{t_s}{Y_s A_S} \quad C_S = 2.866 \times 10^{-9} \quad \text{Compliance of bolt} \]

\[ M_S := \rho S A_S t_s \quad M_S = 0.01 \quad \text{Static mass of bolt without bolt head} \]

\[
Z_1 := \frac{j \omega_i \cdot 6 M_l \left(-j \omega_i \cdot 2 M_l + \frac{1}{j \omega_i \cdot C_l}\right)}{1 + j \omega_i \cdot 4 M_l + \frac{1}{j \omega_i \cdot C_l}} + j \omega_i \cdot 6 M_l + j \omega_i \cdot 6 M_c
\]

\[
Z_2 := \frac{j \omega_i \cdot 6 M_h \left(-j \omega_i \cdot 2 M_h + \frac{1}{j \omega_i \cdot C_h}\right)}{1 + j \omega_i \cdot 4 M_h + \frac{1}{j \omega_i \cdot C_h}} + j \omega_i \cdot 6 M_h + j \omega_i \cdot 6 M_c
\]

\[
Z_3 := \frac{1}{j \omega_i \cdot C_m \omega_i} + \frac{1}{j \omega_i \cdot C_s} - j \omega_i \cdot 2 M_c
\]

\[
Z_i := \frac{Z_1 Z_2}{Z_1 + Z_2} + Z_3
\]

\[
Y_i := \left(1 + \frac{N^2}{Z_i^2}\right)
\]

Plot magnitude of electrical drive admittance

Determine mechanical resonance, anti-resonance frequency and effective coupling coefficient

\[ f_r = \frac{1}{} \left(\frac{Y_i = \text{max}(Y)}{N}ight) \quad f_r = \max(f_r) \quad f_r = 2.674 \times 10^3 \]

\[ f_a = \frac{1}{} \left(\frac{Y_i = \text{min}(Y)}{N}ight) \quad f_a = \max(f_a) \quad f_a = 3.054 \times 10^3 \]

\[ k_{eff} = \frac{N}{1 - \left(\frac{f_r}{f_a}\right)^2} \quad k_{eff} = 0.483 \]
**APPENDIX D-3**

**Face Tonpilz - no bolt, untapered**

**Head cylinder - Al**

- \( \text{th} = 0.038 \)
- \( \text{Ah} = \frac{\pi \times 0.040^2}{4} \)
- \( \rho_h = 2700 \)
- \( Y_h = 69 \times 10^9 \)
- \( c_h = \frac{Y_h}{\sqrt{\rho_h}} \), \( c_h = 5.053 \times 10^3 \)
- \( M_h = \rho_h \cdot Ah \cdot th \), \( M_h = 0.129 \)
- \( C_h = \frac{th}{Y_h/Ah} \), \( C_h = 4.383 \times 10^{-11} \)

**Ceramic Assembly**

- \( A_B = \frac{x \left( 0.0385^2 - 0.0127^2 \right)}{4} \)
- \( \theta_c = 0.064 \)
- \( \eta_c = 2 \)
- \( \eta_c = 13308.854 \times 10^2 \)
- \( \rho_c = 7700 \)
- \( s_{33E} = 1.9610^{-11} \)
- \( \eta_c = \frac{1}{\rho_c \cdot s_{33E}} \), \( \eta_c = 2.574 \times 10^3 \)
- \( d_{33} = 528 \times 10^{-12} \)
- \( k_{33} = \frac{d_{33}}{\sqrt{\eta_c \cdot s_{33E}}} \)
- \( C_e = \frac{\rho_c \cdot A_c \cdot \eta_c}{s_{33E}} \)
- \( C_h = C_e \left( 1 - k_{33}^2 \right) \)
- \( N = \frac{d_{33} \cdot A_e \cdot \eta_c}{s_{33E} \cdot A_c} \)
- \( C_{nE} = \frac{n_c \cdot t_c \cdot s_{33E}}{A_c} \)
- \( M_e = \rho_c \cdot t_c \cdot n_c \cdot A_c \)

- Length of head
- Section area head
- Density of head material
- Youngs modulus
- Speed of sound in head
- Static mass of head
- Compliance of head

**Section area of ceramic rings**

- Thickness of ceramic rings
- Number of ceramic elements
- Dielectric constant of piezoelectric ceramic material
- Density of ceramic rings
- Elastic constant E-field
- Sound speed for constant E-field or shorted terminals

**Longitudinal strain per unit E-field**

- \( k_{33} = 0.083 \)
- Coupling factor
- \( C_e = 3.818 \times 10^{-9} \)
- Free capacitance of stack
- \( C_b = 2.038 \times 10^{-9} \)
- Blocked capacitance of stack
- \( N = 2713 \)
- Blocked force per volt
- \( C_{nE} = 2.418 \times 10^{-11} \)
- Compliance of stack
- \( M_e = 0.102 \)
- Static mass
Tail mass - Br

\[ At = \frac{\pi 0.045^2}{4} \]

\[ a = 0.019 \]

\[ \rho c = 8250 \]

\[ Y_1 = 90 \times 10^9 \]

\[ c_t = \frac{Y_1}{\rho c} \]

\[ c_t = 3.103 \times 10^3 \]

\[ M_t = \rho c A_t \]

\[ C_t = 1.68 \times 10^{-10} \]

Swept frequency increments and range

\[ n' = 1012 \quad i = 0 \ldots m' \quad \Omega = 10000 \quad \Omega_2 = 40000 \quad f_i = \Omega \left( \frac{n'}{n} \right) \]

Equivalent circuit elements assuming mechanical impedance analogy and a plane wave travelling on length axis of the resonator (single degree of freedom):

Forward section

\[ ZH_1 = j \rho c \cdot A_t \cdot \tan \left( \frac{\omega_i \cdot th}{2 \cdot ch} \right) \]

\[ ZH_2 = \frac{j \rho c \cdot A_t}{1 - \sin \left( \frac{\omega_i \cdot th}{ch} \right)} \]

Ceramic circuit elements

\[ Z_{c1} = j \rho e \cdot c \cdot c_e \cdot A \cdot \tan \left( \frac{\omega_i \cdot th}{2 \cdot c_e} \right) \]

\[ Z_{c2} = \frac{j \rho e \cdot c \cdot c_e}{1 - \sin \left( \frac{\omega_i \cdot th}{c_e} \right)} \]

Tail section

\[ ZH_1 = j \rho c \cdot c_t \cdot A \cdot \tan \left( \frac{\omega_i \cdot ti}{2 \cdot ct} \right) \]

\[ ZH_2 = \frac{j \rho c \cdot c_t \cdot A}{1 - \sin \left( \frac{\omega_i \cdot ti}{ct} \right)} \]

Translate mechanical impedances through the ideal transformer to yield an equivalent electrical network

\[ Z_1 = -j \omega_i \cdot C_h \]

\[ Z_2 = \frac{Z_{c1}}{N^2} \]

\[ \text{Compliance related terms for ceramic} \]

\[ \frac{Z_{c1} \cdot ZH_1 + ZH_1 \cdot ZH_2}{ZH_1 + ZH_2} \]

\[ Z_3 = \frac{Z_{c1} + ZH_1 + \frac{ZH_1 \cdot ZH_2}{ZH_1 + ZH_2}}{N^2} \]

\[ \text{Forward section of resonator} \]

\[ \text{Blocked capacitance term} \]
Impedance matrix for equivalent circuit

\[ Z(0) := \begin{pmatrix} Z_1 & -Z_1 & 0 \\ -Z_1 & Z_1 + Z_2 + Z_3 & -Z_3 \\ 0 & -Z_3 & Z_3 + Z_4 \end{pmatrix} \]

Drive resonator with 1 volt at electrical terminals and compute currents flowing in each mesh

\[ \begin{pmatrix} V_1 \\ I_1 \\ I_2 \end{pmatrix} = Z(0)^{-1} \begin{pmatrix} V_1 \\ \varphi \\ o \end{pmatrix} \]

Plot magnitude of electrical drive admittance

\[ Y_{\text{mag}} = \left| \frac{1}{V_1} \right| \]

Determine mechanical resonance, anti-resonance frequency and effective coupling coefficient

- \( f_r = f_r \left( Y_{\text{mag}} = \max(Y_{\text{mag}}) \right) \)
- \( f_a = \max(f_a) \)
- \( f_a = 2.605 \times 10^4 \)
- \( f_a = 2.98 \times 10^4 \)

\[ k_{\text{eff}} = \sqrt{1 - \left( \frac{f_r}{f_a} \right)^2} \]

\[ k_{\text{eff}} = 0.485 \]
Face Tonpilz - rod, untapered

Compressive stress bias bolt (Steel)

\[ t_s = \text{nom} \]
\[ A_s = \pi \cdot d_0^2 \]
\[ \rho_s = 7800 \]
\[ Y_s = 193 \times 10^6 \]
\[ c_s = \sqrt{\frac{Y_s}{\rho_s}} \]
\[ C_s = \frac{2 \cdot \pi \cdot d_0}{Y_s A_s} \]
\[ \frac{C_s}{C_{mln}} = 5.456 \]
\[ M_s = \frac{c_s}{A_s \cdot c_s} \]
\[ M_s = \frac{5.019}{10^{-3}} \]

Stress bolt section

\[ Z_{st} = j \cdot \rho_s \cdot c_s \cdot A_s \cdot \frac{1}{2 \cdot c_s} \]
\[ Z_{st} = j \cdot \frac{\rho_s \cdot c_s \cdot A_s}{\tan \left( \frac{\theta_s \cdot A_s}{c_s} \right)} \]

Translate mechanical impedances through the ideal transformer to yield an equivalent electrical network

\[ Z_1 = \frac{1}{j \omega_1 C_b} \quad \text{Blocked capacitance term} \]

\[ Z_2 = \frac{Z_{st}}{N^2} \quad \text{Compliance related terms for ceramic and bolt} \]

\[ Z_3 = \frac{Z_{st} + Z_{b1} + Z_{b2} + Z_{st}}{N^2} \quad \text{Forward section of resonator} \]

\[ Z_4 = \frac{Z_{st}}{N^2} \quad \text{Bolt} \]

\[ Z_5 = \frac{Z_{st} + Z_{b1} + Z_{b2} + Z_{st}}{N^2} \quad \text{Aft section of resonator} \]

Impedance matrix for equivalent circuit

\[
Z(\omega) = \begin{pmatrix}
Z_1 & -Z_1 & 0 \\
-Z_1 & Z_1 + Z_2 + Z_3 + Z_4 & \sqrt{Z_3} \\
0 & -Z_3 & Z_3 + Z_5
\end{pmatrix}
\]
Drive resonator with 1 volt at electrical terminals and compute currents flowing in each mesh

\[ \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = Z(0)^{-1} \begin{bmatrix} V_L \\ 0 \end{bmatrix} \]

Plot magnitude of electrical drive admittance

\[ Y_{\text{mag}} := \left| \frac{I_1}{V_L} \right| \]

Determine mechanical resonance, anti-resonance frequency and effective coupling coefficient:

\[ f_r := f_i \left( Y_{\text{mag}} = \max(Y_{\text{mag}}) \right) \]
\[ f_a := f_i \left( Y_{\text{mag}} = \min(Y_{\text{mag}}) \right) \]
\[ k_{\text{eff}} = \sqrt{1 - \left( \frac{f_a}{f_i} \right)^2} \quad \text{keff} = 0.441 \]

\[ f_i = 2.715 \times 10^4 \]
\[ f_a = 3.025 \times 10^4 \]
Face Mounted Tonpilz - rod, tapered head

Head cone - aluminium

\[ \theta = 0.038 \]  
\[ r_2 = 0.02 \]  
\[ r_2 = 0.03 \]  

Length of head

Radii of two faces of cone

\[ \theta = \tan \left( \frac{\theta}{2} - r_2 \right) \]  
\[ \theta = 1.313 \]  
\[ \theta_{hm} = \tan (\theta) \]  
\[ \theta_{hm} = 0.114 \]  

Length of complete cone

\[ h_r = \theta_{hm} - \theta \]  
\[ h_r = 0.076 \]  

Difference between head and total cone length

\[ A_{1h} = \pi r_1^2 \]  
\[ A_{2h} = \pi r_2^2 \]  

Areas of faces

\[ \rho_{hm} = 2.700 \]  

Density of head material

\[ Y_{hm} = 69 \times 10^9 \]  

Youngs modulus

\[ c_h = \sqrt{\frac{Y_{hm}}{\rho_{hm}}} \]  
\[ c_h = 3.053 \times 10^3 \]  

Speed of sound in head

\[ M_{hm} = \rho_{hm} \sqrt{A_{2h} - A_{1h}} \]  
\[ M_{hm} = 0.193 \]  

Static mass of head

\[ C_h = \frac{\theta_{hm}}{Y_{hm} \sqrt{A_{2h} - A_{1h}}} \]  
\[ C_h = 2.922 \times 10^{-10} \]  

Compliance of head

Forward section

\[ Z_{11} = - \frac{\rho_{hm} \sqrt{A_{2h} - A_{1h}} \left( \frac{1}{Y_{hm}} \left( \frac{\alpha_{1h}}{\alpha_{1h} - \frac{\theta_{hm}}{\alpha_{1h}}} \right) + \frac{\theta_{hm}}{c_h \tan (\theta)} \right)}{\pi \theta_{hm}} \]  
\[ Z_{12} = \frac{\rho_{hm} \sqrt{A_{2h} - A_{1h}} \left( \frac{1}{Y_{hm}} \left( \frac{\alpha_{1h}}{\alpha_{1h} - \frac{\theta_{hm}}{\alpha_{1h}}} \right) - \frac{\theta_{hm}}{c_h \tan (\theta)} \right)}{\pi \theta_{hm}} \]  

\[ Z_{21} = Z_{21} - Z_{12} \]  
\[ Z_{22} = Z_{22} - Z_{12} \]  
\[ Z_{13} = Z_{13} - Z_{12} \]  
\[ Z_{23} = Z_{23} - Z_{12} \]

\[ Z_{11} + Z_{12} + Z_{21} + Z_{22} + Z_{31} + Z_{32} + Z_{33} = 0 \]  

Forward section of resonator
Impedance matrix for equivalent circuit

\[
Z(\omega) = \begin{pmatrix}
Z_1 & -Z_1 & 0 \\
-Z_1 & Z_1 + Z_2 + Z_3 + Z_4 & -Z_3 \\
0 & -Z_3 & Z_3 + Z_5
\end{pmatrix}
\]

Drive resonator with 1 volt at electrical terminals and compute currents flowing in each mesh

\[
\begin{pmatrix}
V_1 \\
V_2
\end{pmatrix} = Z(\omega)^{-1} \begin{pmatrix}
V_1 \\
0
\end{pmatrix}
\]

Plot magnitude of electrical drive admittance

\[
Y_{mag} := \left| \frac{V_1}{V_1} \right|
\]

Determine mechanical resonance, anti-resonance frequency and effective coupling coefficient

\[
\begin{align*}
fr_1 &= f \left( Y_{mag} = \max(Y_{mag}) \right) \\
fr &= \max(fr) \\
fr &= 2.423 \times 10^6 \\
fa_1 &= f \left( Y_{mag} = \max(Y_{mag}) \right) \\
fa &= \max(fa) \\
fa &= 2.703 \times 10^4 \\
k_{eff} &= \sqrt{1 - \left( \frac{fr}{fa} \right)^2} \\
k_{eff} &= 0.444
\end{align*}
\]
Plot magnitude of electrical drive admittance for asymmetrical transducer

\[ Y_{mag} = \frac{10^4}{V_i} \]

Determine mechanical resonance, anti-resonance frequency and effective coupling coefficient:

- \( f_r = \frac{f_r}{f_i} (Y_{mag} = \text{max}(Y_{mag})) \), \( f_r = \text{max}(f_r) \), \( f_r = 2.84 \times 10^6 \)
- \( f_a = \frac{f_a}{f_i} (Y_{mag} = \text{min}(Y_{mag})) \), \( f_a = \text{max}(f_a) \), \( f_a = 3.22 \times 10^4 \)
- \( k_{eff} = \sqrt{1 - \left(\frac{f_r}{f_a}\right)^2} \), \( k_{eff} = 0.472 \)
APPENDIX D-4

Below is the code used to simulate the face-mounted transducer radiating into water:

*heading
   Face Mounted tonpilz transducer, Al head, Brass tail, HT steel bolt,
   72.3mm long, 60mm diameter head, 2 discs, 2 electrodes, 1mm x 100mm
   disc ss face, 55.7mm water on plate.

*preprint, model=yes

*node definition

*node
  1, 0.0, 0.007
  5, 0.004, 0.007
  7, 0.0055, 0.007
  9, 0.00635, 0.007
  15, 0.01, 0.007
  **
  101, 0.0, 0.01
  105, 0.004, 0.01
  107, 0.0055, 0.01
  109, 0.00635, 0.01
  115, 0.01, 0.01
  **
  201, 0.0, 0.013
  205, 0.004, 0.013
  207, 0.0055, 0.013
  209, 0.00635, 0.013
  215, 0.01, 0.013
  227, 0.02, 0.013
  **
  1101, 0.0, 0.039
  1105, 0.004, 0.039
  1107, 0.0055, 0.039
  1109, 0.00635, 0.039
  1115, 0.01, 0.039
  1125, 0.01925, 0.039
  1127, 0.02, 0.039
  **
2601, 0.0, 0.0523
2605, 0.004, 0.0523
2609, 0.00635, 0.0523
2615, 0.01, 0.0523
2625, 0.01925, 0.0523
2627, 0.02, 0.0523

**
3401, 0.0, 0.0643
3405, 0.004, 0.0643
3427, 0.026, 0.0643

**
3801, 0.0, 0.0723
3805, 0.004, 0.0723
3827, 0.03, 0.0723
3845, 0.088, 0.0723
3849, 0.099, 0.0723

**
3945, 0.088, 0.0728
3949, 0.099, 0.0728

**
4001, 0.0, 0.0733
4005, 0.004, 0.0733
4027, 0.03, 0.0733
4045, 0.088, 0.0733
4049, 0.099, 0.0733

**
**
**

** nut definition
**

*ngen, nset=tempi
1, 5, 1
5, 15, 1

**ngen, nset=temps
**101, 105, 1

**ngen, nset=temps2
**105, 107, 1

**107, 111, 1

*ngen, nset=temps
201, 205, 1
205, 207, 1
207, 209, 1
209, 215, 1
*ngen, nset=tmpl5
201, 205, 1
**
*nfill
**tmpl23, temp5, 2, 50
tmpl1, temp2, 4, 50
**tmpl24, temp2, 2, 50
**
*element, type=cax8, elset=nut
1, 1, 3, 103, 101, 2, 53, 102, 51
*elgen, elset=nut
1, 7, 2, 1, 2, 100, 100
**element, type=cax8, elset=nut
**5, 5, 7, 107, 105, 6, 57, 106, 55
**elgen, elset=nut
**5, 3, 2, 1, 2, 100, 100
**element, type=cax8, elset=nut
**101, 101, 103, 203, 201, 102, 153, 202, 151
**elgen, elset=nut
**101, 2, 2, 1, 1, 100, 100
**
*solid section, elset=nut, material=ss
**
**tail definition
**
*ngen, nset=tmpl3a
207, 215, 1
*ngen, nset=tmpl3b
215, 227, 2
*nset, nset=tmpl3
tmpl3a, tmpl3b
*ngen, nset=tmpl4
1107, 1109, 1
1109, 1125, 1
1125, 1127, 1
**
*nfill
tmpl3, temp4, 18, 50
**
*element, type=cax8, elset=tail
207, 207, 209, 309, 307, 208, 259, 398, 257
*eigen, elset=tail
207, 10, 2, 1, 9, 100, 100
**
*solid section, elset=tail, material=brass
**
**bolt definition
**
nugen, nset=temp6
2601, 2605, 1
**nugen, nset=temp7
**2801, 2805, 1
*nugen, nset=temp8
3401, 3405, 1
**
*nfill
temp5, temp6, 48, 50
*nfill
temp6, temp8, 16, 50
**nfill
**temp7, temp8, 12, 50
**
element, type=cax8, elset=bolt
201, 201, 203, 303, 301, 202, 253, 302, 251
*eigen, elset=bolt
201, 2, 2, 1, 24, 100, 100
**
*solid section, elset=bolt, material=ss
**
element, type=cax8, elset=bolt
2601, 2601, 2603, 2703, 2701, 2602, 2653, 2702, 2651
*eigen, elset=bolt
2601, 2, 2, 1, 6, 100, 100
**
**element, type=cax8, elset=bolt
**2001, 2801, 2803, 2903, 2901, 2802, 2853, 2902, 2851
**eigen, elset=bolt
**2801, 2, 2, 1, 6, 100, 100
**
**head definition
**
**ngen, nset=temp9
2605, 2609, 1
2609, 2625, 1
2625, 2627, 1
**ngen, nset=temp10a
3405, 3427, 1
**ngen, nset=temp10
temp8, temp10a
**ngen, nset=temp11
3801, 3805, 1
3805, 3827, 1
**nfill
temp2, temp10a, 16, 50
temp10, temp11, 8, 50
**element, type=cax8, elset=head
2605, 2605, 2607, 2707, 2705, 2606, 2657, 2706, 2656
**elgen, elset=head
2605, 11, 2, 1, 0, 100, 100
**solid section, elset=head, material=al
**element, type=cax8, elset=head
3401, 3405, 3403, 3503, 3501, 3402, 3453, 3502, 3451
**elgen, elset=head
3401, 13, 2, 1, 4, 100, 100
**face definition
**ngen, nset=temp19
3827, 3845, 1
3845, 3849, 1
**ngen, nset=temp20
temp11, temp19
**ngen, nset=temp21
4001, 4005, 1
4005, 4027, 1
4027, 4045, 1
4045, 4049, 1
*nset, nset=comp22, generate
3945, 3949, 1
**
*nfill

temp20, temp21, 4, 50
**
*element, type=con8, elset=face
3801, 3803, 3903, 3901, 3807, 3853, 3902, 3851
*elgen, elset=face
3801, 24, 2, 1, 2, 100, 100
**
*solid section, elset=face, material=ss
**
**electrode1 definition
**
*node
1209, 0.00635, 0.039
1215, 0.01, 0.039
1225, 0.01925, 0.039
1227, 0.02, 0.039
**
1209, 0.00635, 0.03925
1215, 0.01, 0.03925
1225, 0.01925, 0.03925
1227, 0.02, 0.03925
**
*ngen, nset=s1
1209, 1225, 1
1225, 1227, 1
*ngen, nset=h1
1209, 1325, 1
1325, 1327, 1
**
*nfill
s1, h1, 2, 50
**
*element, type=con8, elset=elec
1209, 1209, 1211, 1311, 1309, 1210, 1261, 1310, 1259
*elgen, elset=elec
1209, 9, 2, 1, 1, 100, 100
**
solid section, elset=elec, material=BeCu

**

**pzt1 definition

**

*node
1409, 0.00635, 0.03925
1415, 0.01, 0.03925
1425, 0.01925, 0.03925
**
1809, 0.00635, 0.04565
1815, 0.01, 0.04565
1825, 0.01925, 0.04565
**
*ngen, nset=a2
1409, 1425, 1
*ngen, nset=b2
1809, 1825, 1
**
*nfill
a2, b2, 8, 50
**
*element, type=cs68e, elset=pztpos
1409, 1409,1411,1511,1509, 1410,1461,1510,1459
*elgen, elset=pztpos
1409, 8,2,1, 4,100,100
**
*solid section, elset=pztpos, material=pz26, orientation=2pos
**
**electrode2 definition
**

*node
1909, 0.00635, 0.04565
1915, 0.01, 0.04565
1925, 0.01925, 0.04565
1927, 0.02, 0.04565
**
2009, 0.00635, 0.0459
2015, 0.01, 0.0459
2025, 0.01925, 0.0459
2027, 0.02, 0.0459
**
*nset=a3
1909, 1925, 1
1925, 1927, 1
*nset=b3
2009, 2025, 1
2025, 2027, 1
**
*nfill
a3, b3, 2, 50
**
*element, type=cax8, elset=elec
*elgen, elset=elec
1909, 9, 2, 1, 1, 100, 100
**
**pzt2 definition
**
*node
2109, 0.00635, 0.0459
2115, 0.01, 0.0459
2125, 0.01925, 0.0459
**
2509, 0.00635, 0.0523
2515, 0.01, 0.0523
2525, 0.01925, 0.0523
**
*nset=a4
2109, 2125, 1
*nset=b4
2509, 2525, 1
**
*nfill
a4, b4, 8, 50
**
*element, type=cax8e, elset=pztneg
2109, 2109, 2111, 2211, 2211, 2209, 2110, 2110, 2110, 2115
*elgen, elset=pztneg
2109, 6, 2, 1, 4, 100, 100
**
*solid section, elset=pztneg, material=pz26, orientation=2neg
**
**Call to elec1 mechanical vertical coupling**

```
*nsel, nset-templ2, generate
1109, 1125, 1
*nsel, nset-templ3, generate
1209, 1225, 1
*mpc
tie, temp13, temp12
```

**elec1 to pzt1 mechanical vertical coupling**

```
*nsel, nset-templ4, generate
1309, 1325, 1
*mpc
tie, temp14, a2
```

**pzt1 to elec2 mechanical vertical coupling**

```
*nsel, nset-templ5, generate
1909, 1925, 1
*mpc
tie, temp15, b2
```

**elec2 to pzt2 mechanical vertical coupling**

```
*nsel, nset-templ6, generate
2009, 2025, 1
*mpc
tie, temp16, a4
```

**pzt2 to head mechanical vertical coupling**

```
*nsel, nset-templ7, generate
2609, 2625, 1
*mpc
tie, temp17, b4
```

**electrode electrical detail**

```
*nnode, nset=ns_ref
1000000, 0.1, 0.0
```
*equation
2
b2, 9, 1.0, a4, 9, -1.0
*equation
2
a4, 9, 1.0, 100000, 5, -1.0

**

**material definitions**

**

*orientation, name=p2oa
1.0, 0.0, 0.0, 0.0, 1.0, 0.0
1.0, 0.0
*orientation, name=2neg
-1.0, 0.0, 0.0, 0.0, -1.0, 0.0
1.0, 0.0
**

*material, name=pz26
*elastic, type=ortho
16.8E10, 9.99E10, 12.3E10, 11.0E10, 9.99E10, 16.8E10, 3.0E10, 3.0E10,
2.0E10
*piezoelectric, type=iso
0., 0., 0., 3.86, 0., 0., -1.98, 14.7
-1.98, 0., 0., 0., 0., 0., 0., 0., 3.86
*dielectric, type=aniso
11.771E-9, 0., 10.531E-9, 0., 0., 11.771E-9
*density
7700.
**

*material, name=sa
*elastic
209, e9, 0.3
*density
7800.
*material, name=brass
*elastic
80.29, e9, 0.3
*density
8410.
*material, name=alu
*elastic
* density
2700.
* material, name=BeCu
* elastic
123.8, 0.3
* density
8050.
* material, name=water
* density
1000 0.
* acoustic medium, bulk modulus
2.18e9
**
** Water definition & interface
**
* node
6701, 0.0, 0.129
6705, 0.004, 0.129
6727, 0.03, 0.129
6745, 0.088, 0.129
**
* nset, nset=water1, generate
4001, 4005, 1
4005, 4027, 1
4027, 4045, 1
* ngen, nset=water2
6701, 6705, 1
6705, 6727, 1
6727, 6745, 1
* nfill
water1, water2, 54, 50
**
* element, type=acax8, elset=water
4001, 4001, 4003, 4103, 4101, 4002, 4053, 4102, 4051
* elgen, elset=water
4001, 22, 2, 1, 27, 100, 100
* solid section, elset=water, material=water
**
* element, type=nds13a, elset=inter
9001, 4001, 4002, 4003
The design drawings for the face-mounted transducer can be seen on the following pages.

APPENDIX D-5
Tan approx for transducers-Horn driver
Head Specs (Titanium)

\[ \text{lh} \quad \text{Length} \]
\[ \text{rh} := 0.02 \quad \text{Radius} \]
\[ \text{ph} := 4420 \quad \text{Density} \]
\[ \text{Ah} := \pi \cdot \text{rh}^2 \quad \text{Ah} = 1.257 \times 10^{-3} \quad \text{Section Area} \]
\[ \text{ch} := 5079 \quad \text{Speed of sound in material} \]

Ceramic Specs (Half Stack)

\[ \text{Ac} := \frac{\pi \cdot \left(0.0385^2 - 0.02127^2\right)}{4} \quad \text{Section area of PZT discs} \]
\[ \text{tc} := 0.0064 \quad \text{Thickness of discs} \]
\[ \text{nc} := 2 \quad \text{Number of discs} \]
\[ \text{pc} := 7700 \quad \text{Density of discs} \]
\[ s33E := 1.96 \cdot 10^{-11} \quad \text{Elastic constant E-field} \]
\[ \text{cc} := \sqrt{\frac{1}{\text{pc} \cdot s33E}} \quad \text{cc} = 2.574 \times 10^3 \quad \text{Sound speed for constant E-field or shorted terminals} \]

Tail Specs (Stainless Steel)

\[ \text{lt} \quad \text{Length} \]
\[ \text{rt} := 0.02 \quad \text{Radius} \]
\[ \text{pt} := 7800 \quad \text{Density} \]
\[ \text{At} := \pi \cdot \text{rt}^2 \quad \text{At} = 1.257 \times 10^{-3} \quad \text{Section Area} \]
\[ \text{ct} := 4936 \quad \text{Speed of sound in material} \]

Frequency

\[ \text{fr} := 22 \cdot 10^3 \quad \text{Desired resonant frequency} \]

Tan Approx

\[ \text{lh} := \text{ch} \cdot \frac{\left(\frac{\text{pc} \cdot \text{cc} \cdot \text{Ac}}{\text{ph} \cdot \text{ch} \cdot \text{Ah}}\right)}{\tan\left(\frac{2 \cdot \pi \cdot \text{fr} \cdot \text{nc} \cdot \text{tc}}{\text{cc}}\right)} \]
\[ \text{lh} = 0.027 \quad \text{Head Length} \]

\[ \text{lt} := \text{ct} \cdot \frac{\left(\frac{\text{pc} \cdot \text{cc} \cdot \text{Ac}}{\text{pt} \cdot \text{ct} \cdot \text{At}}\right)}{\tan\left(\frac{2 \cdot \pi \cdot \text{fr} \cdot \text{nc} \cdot \text{tc}}{\text{cc}}\right)} \]
\[ \text{lt} = 0.017 \quad \text{Tail Length} \]
APPENDIX E-2

Horn Driver: Lumped Element Approx

**Head Specs (Titanium)**

- **lh** := 0.027 Length
- **rh** := 0.02 Radius
- **ρh** := 4420 Density
- **Ah** := \( π \times rh^2 \) Section Area
- **Mh** := lh \( \cdot \) ph \( \cdot \) Ah Static mass of head

**Ceramic Specs**

\[
Ac := \frac{π \left(0.0385^2 - 0.0127^2\right)}{4}
\]

- **tc** := .0064 Thickness of discs
- **nc** := 4 Number of discs
- **ρc** := 7700 Density of discs
- **εc** := 13308.854 \( 10^{-12} \) Dielectric constant of piezoelectric ceramic material
- **s33E** := 1.96 \( 10^{-11} \) Elastic constant E-field
- **d33** := 328 \( 10^{-12} \) Longitudinal strain per unit E-field
- **k33** := 0.683

\[
CmE := \frac{nc \cdot tc \cdot s33E}{Ac}
\]

- **Mc** := pc \( \cdot \) tc \( \cdot \) nc \( \cdot \) Ac Static mass
- **Ce** := \( εc \cdot Ac \cdot nc \)

\[
Cb := Ce \left(1 - k33^2\right)
\]

- **Cb** := 4.077 \( 10^{-9} \) Blocked capacitance of stack
- **N** := \( \frac{d33 \cdot Ac}{s33E \cdot tc} \)

**Tail Specs (Stainless Steel)**

- **lt** := 0.017 Length
- **rt** := 0.02 Radius
- **ρt** := 7800 Density
- **At** := \( π \times rt^2 \) Section Area
- **Mt** := lt \( \cdot \) pt \( \cdot \) At Static mass of tail

\[
CmE = 4.836 \times 10^{-10}
\]

- **Ce = 7.636 \times 10^{-9}** Free capacitance of stack
- **Cb = 4.077 \times 10^{-9}** Blocked capacitance of stack
- **N = 2.713** Blocked force per volt
Compressive stress bias bolt

\[ \text{Length of bolt} \quad ts := nc \cdot tc \quad \text{ts} = 0.026 \]

\[ \text{Section Area} \quad As := \pi \cdot .004^2 \]

\[ \text{Density of bolt material} \quad \rho s := 4420 \]

\[ \text{Young's Modulus for bolt material} \quad Ys := 11.4 \times 10^{10} \]

\[ \text{Compliance of bolt} \quad Cs := \frac{ts}{Ys \cdot As} \quad Cs = 4.468 \times 10^{-9} \]

\[ \text{Static mass of bolt} \quad Ms := \rho s \cdot As \cdot ts \quad Ms = 5.688 \times 10^{-3} \]

Swept frequency increments and range

\[ nf := 1012 \quad i := 0..nf \quad f1 := 10000 \quad f2 := 40000 \quad f_i := f1 \left( \frac{f2}{f1} \right)^{i/nf} \]

\[ j := \sqrt{-1} \quad \omega_i := 2 \pi f_i \]

Lumped circuit calculation without bolt

\[ Z_i := \frac{1}{j \omega_i \cdot C_m E} - .2j \omega_i \cdot Mc + \frac{j \omega_i \cdot (.6Mc + Mt) (.6Mc + Mh)}{(.6Mc + Mt) + (.6Mc + Mh)} \]

\[ Y_i := \left( j \omega_i \cdot Cb \right) + \frac{N^2}{Z_i} \]

Plot magnitude of electrical drive admittance
Determine mechanical resonance, anti-resonance frequency and effective coupling coefficient

\[ f_r = \frac{1}{2\pi} \left( Y_i = \max(Y) \right) \]
\[ f_a = \frac{1}{2\pi} \left( Y_i = \min(Y) \right) \]
\[ f_r = \text{max}(fr) \]
\[ f_a = \text{max}(fa) \]
\[ f_r = 2.294 \times 10^4 \]
\[ f_a = 3.139 \times 10^4 \]

\[ k_{\text{eff}} = \sqrt{1 - \left( \frac{f_r}{f_a} \right)^2} \]
\[ k_{\text{eff}} = 0.683 \]

Lumped Circuit calculation with rod

\[ Z_r = \frac{1}{\omega_1 C_r J} + \frac{1}{\omega_1 C_s J} - 2j\omega_1 M_c J + \frac{j\omega_1 (0.6M_e + M_l)(0.6M_e + M_l)}{(0.6M_e + M_l)(0.6M_e + M_l)} \]

\[ Y_{i1} = \left[ \left( \omega_1 C_r J \right) + \frac{N^2}{Z_r} \right] \]

Plot magnitude of electrical drive admittance

Determine mechanical resonance, anti-resonance frequency and effective coupling coefficient

\[ f_r = \frac{1}{2\pi} \left( Y_i = \max(Y) \right) \]
\[ f_a = \frac{1}{2\pi} \left( Y_i = \min(Y) \right) \]
\[ f_r = \text{max}(fr) \]
\[ f_a = \text{max}(fa) \]
\[ f_r = 2.416 \times 10^4 \]
\[ f_a = 3.23 \times 10^4 \]

\[ k_{\text{eff}} = \sqrt{1 - \left( \frac{f_r}{f_a} \right)^2} \]
\[ k_{\text{eff}} = 0.664 \]
Lumped Circuit calculation with bolt

Tail Specs with bolt (Stainless steel)

<table>
<thead>
<tr>
<th>Length</th>
<th>Radius</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_1 = 0.017$</td>
<td>$r_1 = 0.02$</td>
</tr>
<tr>
<td>$l_2 = 0.0053$</td>
<td>$r_2 = 0.015$</td>
</tr>
</tbody>
</table>

Section Area

$A_t = \pi \left( r_1^2 - r_2^2 \right)$

Static mass of tail (inc bolt head)

$M_t = \rho \pi A_t + 0.015 \ M_h = 0.169$

Compressive stress bias bolt

$\sigma_s = \frac{F_s}{A_s + b_s}$

$\sigma_s = 0.043$

$C_s = \frac{F_s}{\rho_s A_s}$

$C_s = 7.43 \times 10^{-9}$

$M_s = \rho_s A_s b_s$

$M_s = 9.465 \times 10^{-3}$

$Z_1 = \frac{1}{j \omega_1 C_{b_1}} + \frac{1}{j \omega_1 C_s} = \frac{1}{j \omega_1 (M_c + M_t) + j \omega_1 (0.6M_c + M_h) + j \omega_1 (0.6M_c + M_t) + j \omega_1 (0.6M_c + M_h)}$

$Y_1 = \left| \frac{1}{j \omega_1 C_b} + \frac{\omega_1^2}{Z_1} \right|

Plot magnitude of electrical drive admittance

Determine mechanical resonance, anti-resonance frequency and effective coupling coefficient

$fr_1 = \frac{1}{2 \pi} \left( \frac{v_1}{\rho_1} \right)$

$fr_c = \frac{v_1}{\rho_1}$

$fr = 2.36 \times 10^3$

$fa = \frac{v_1}{\rho_1}$

$fa = 3.186 \times 10^4$

$k_c = \frac{v_1}{\rho_1} \frac{1}{\rho_1}$

$k_c = 0.672$
APPENDIX E-3

Horn Driver - Distributed parameter analysis, no bolt

Head cylinder - titanium

\[ \text{Length of head} \]

\[ \text{Section area head} \]

\[ \text{Density of head material} \]

\[ \text{Youngs modulus} \]

\[ \text{Speed of sound in head} \]

\[ \text{Static mass of head} \]

\[ \text{Compliance of head} \]

Ceramic Assembly

\[ \text{Section area of ceramic rings} \]

\[ \text{Thickness of ceramic rings} \]

\[ \text{Number of ceramic elements} \]

\[ \text{Dielectric constant of piezoelectric ceramic material} \]

\[ \text{Density of ceramic rings} \]

\[ \text{Elastic constant E-field} \]

\[ \text{Sound speed for constant E-field or shorted terminals} \]

\[ \text{Longitudinal strain per unit E-field} \]

\[ \text{Coupling factor} \]

\[ \text{Free capacitance of stack} \]

\[ \text{ Blocked capacitance of stack} \]

\[ \text{Blocked force per volt} \]

\[ \text{Compliance of stack} \]

\[ \text{Static mass} \]
Tail mass - stainless steel

\[ A_t = \frac{\pi 0.040^2}{4} \]

\[ \omega_c = 0.17 \]

\[ q_t = 7800 \]

\[ Y_t = 19.3 \cdot 10^9 \]

\[ c_t = \sqrt{\frac{Y_t}{\rho t}} \]

\[ c_t = 4.974 \cdot 10^3 \]

\[ M_t = \rho t A_t \quad M_t = 0.167 \]

\[ C_t = \frac{\omega_t}{Y_t A_t} \quad C_t = 7.00 \text{N/m} \cdot \text{rad}^{-1} \]

Swept frequency increments and range

\[ n_f = 1012 \quad \omega = \omega_c \quad \Omega = 10000 \quad \Omega = 40000 \quad f_4 = 1\left(\frac{\Omega}{\Omega}\right)^{n_f} \]

\[ j = \sqrt{-1} \quad \omega_1 = 2\pi f_1 \]

Equivalent circuit elements assuming mechanical impedance analogy and a plane wave traveling on length axis of the resonator (single degree of freedom)

Forward section

\[ Z_{h1} = j\rho \text{ch} \cdot \text{Ah} \cdot \tan \left(\frac{\omega_1 \text{th}}{2 \text{ch}}\right) \quad Z_{h2} = \frac{j \rho \text{ch} \cdot \text{Ah}}{\sin \left(\frac{\omega_1 \text{th}}{2 \text{ch}}\right)} \]

Ceramic circuit elements

\[ Z_{c1} = j \rho \text{cc} \cdot \text{Ac} \cdot \tan \left(\frac{\omega_1 \text{tc} \cdot \text{ac}}{2 \text{cc}}\right) \quad Z_{c2} = \frac{j \rho \text{cc} \cdot \text{Ac}}{\sin \left(\frac{\omega_1 \text{tc} \cdot \text{ac}}{2 \text{cc}}\right)} \]

Tail section

\[ Z_{t1} = j \rho \text{ct} \cdot \text{Ah} \cdot \tan \left(\frac{\omega_1 \text{th}}{2 \text{ct}}\right) \quad Z_{t2} = \frac{j \rho \text{ct} \cdot \text{Ah}}{\sin \left(\frac{\omega_1 \text{th}}{2 \text{ct}}\right)} \]

Translate mechanical impedances through the ideal transformer to yield an equivalent electrical network

\[ Z_{21} = \frac{1}{j \omega_1 \text{Ch}} \quad \text{Blocked capacitance term} \]

\[ Z_{22} = \frac{z_{c2}^2}{N^2} \quad \text{Compliance related term for ceramic} \]

\[ Z_{21} + Z_{t1} + \frac{z_{t1} \cdot z_{t2}}{Z_{t1} + z_{t2}} \quad \text{Aft section of resonator} \]
Impedance matrix for equivalent circuit

\[
Z(t) = \begin{pmatrix}
Z_{l1} & -Z_{l1} & 0 \\
-Z_{l1} & Z_{l1} + Z_{2} + Z_{3} & -Z_{3} \\
0 & -Z_{3} & Z_{l1} + Z_{4}
\end{pmatrix}
\]

Drive resonator with 1 volt at electrical terminals and compute currents flowing in each mesh:

\[
\begin{pmatrix}
V_1 \\
I_1 \\
I_2
\end{pmatrix} = Z(t) \begin{pmatrix}
V_1 \\
V_2 \\
0
\end{pmatrix}
\]

Plot magnitude of electrical drive admittance

\[
|Y_{mag}| = \frac{|10_t|}{V_1}
\]

Determine mechanical resonance, anti-resonance frequency and effective coupling coefficient

\[
\begin{align*}
fr &= \frac{1}{2\pi} \max(Y_{mag}) \\
fa &= \frac{1}{2\pi} \min(Y_{mag}) \\
\text{keff} &= \sqrt{1 - \left(\frac{fr}{fa}\right)^2}
\end{align*}
\]

\[
\begin{align*}
fr &= 2.195 \times 10^4 \\
fa &= 2.852 \times 10^4 \\
\text{keff} &= 0.638
\end{align*}
\]
Horn Driver including rod

Compressive stress bias bolt (Titanium)

\[ \sigma_s := \frac{\sigma}{\sigma_y} \quad \text{s} = 0.026 \]
\[ A_s := \pi \cdot 0.004^2 \]
\[ p_s := 4420 \]
\[ Y_s := 11.4 \cdot 10^1 \]
\[ \sigma_s := \frac{Y_s}{\sqrt{p_s}} \quad \sigma_s = 5 \cdot 10^3 \]
\[ C_s := \frac{\sigma_s}{Y_s A_s} \quad C_s = 4.468 \cdot 10^{-5} \]
\[ \frac{C_s}{\text{CmE}} = 9.237 \]
\[ M_s := p_s A_s / \sigma_s \quad M_s = 5.688 \cdot 10^{-3} \]

Plot magnitude of electrical drive admittance

\[ \gamma_{\text{mag}} := \frac{|\gamma|}{|V|} \]

Determine mechanical resonance, anti-resonance frequency and effective coupling coefficient

\[ f_r := f_r \left( \gamma_{\text{mag}} = \max( \gamma_{\text{mag}} ) \right) \quad f_r := \max( f_r ) \quad f_r = 2.29 \times 10^4 \]
\[ f_a := f_a \left( \gamma_{\text{mag}} = \min( \gamma_{\text{mag}} ) \right) \quad f_a := \max( f_a ) \quad f_a = 2.907 \times 10^3 \]
\[ \text{keff} := \sqrt{1 - \left( \frac{f_r}{f_a} \right)^2} \quad \text{keff} = 0.616 \]
APPENDIX E-4

Horn Driver with horn - distributed parameter analysis

First taper of Horn

Length of taper

\[ l_w = 0.010 \]

Radii of two faces of cone

\[ r_{w1} = 0.02 \quad r_{w2} = 0.03 \]

Length of complete cone

\[ l_{tw} = \tan^{-1}\left(\frac{r_w}{l_{tw2} - l_{tw1}}\right) \quad l_{tw1} = 0.785 \quad l_{tw2} = \tan(l_{tw1}) r_{w2} \quad l_{tw} = 0.03 \]

Difference between head and total cone length

\[ l_{tw} = l_{tw2} - l_w \quad l_{tw} = 0.02 \]

Areas of faces

\[ A_{tw} = \pi r_w^2 \quad A_{tw2} = \pi r_{w2}^2 \]

Density of material

\[ \rho_w = 4420 \]

Young's modulus

\[ Y_w = 114.1 \times 10^9 \]

Speed of sound

\[ c_w = \sqrt{\frac{Y_w}{\rho_w}} \quad c_w = 5.079 \times 10^2 \]

Static mass

\[ M_w = \rho_w l_w \sqrt{A_{w2} - A_{w1}} \quad M_w = 0.083 \]

Compliance

\[ C_w = \frac{l_w}{Y_w \sqrt{A_{w2} - A_{w1}}} \quad C_w = 4.654 \times 10^{-11} \]

60mm cylinder section of horn

Length of cylinder section

\[ l_y = 0.45 \]

Section area

\[ A_y = \frac{\pi 0.06d^2}{4} \]

Density of material

\[ \rho_y = 4420 \]

Young's modulus

\[ Y_y = 114.1 \times 10^9 \]

Speed of sound

\[ c_y = \sqrt{\frac{Y_y}{\rho_y}} \quad c_y = 5.079 \times 10^2 \]

Static mass

\[ M_y = \rho_y A_y l_y \quad M_y = 0.562 \]

Compliance of section

\[ C_y = \frac{l_y}{Y_y A_y} \quad C_y = 1.306 \times 10^{-10} \]

Second taper of horn

Length of taper

\[ l_y = 0.020 \]

Radii of two faces of cone

\[ r_{y1} = 0.03 \quad r_{y2} = 0.01 \]

Length of complete cone

\[ l_{yw} = \tan^{-1}\left(\frac{r_y}{l_{yw2} - l_{yw1}}\right) \quad l_{yw1} = 0.785 \quad l_{yw2} = \tan(l_{yw1}) r_{y2} \quad l_{yw} = 0.03 \]

Difference between taper and total cone length

\[ l_{yw} = l_{yw2} - l_y \quad l_{yw} = 0.01 \]
\[ A_{1y} = \pi y^2 \quad A_{2y} = \pi y^2 \]  
Areas of faces

\[ \rho y = 4420 \]  
Density of material

\[ Y_{1y} = 114 \times 10^3 \]  
Youngs modulus

\[ c_1 = \frac{\sqrt{Y_{1y}}}{\rho y} \quad c_2 = 5.079 \times 10^3 \]  
Speed of sound

\[ M_x = \rho y y \sqrt{A_{2y} - A_{1y}} \quad M_s = 0.083 \]  
Static mass of section

\[ C_y = \frac{\rho y}{Y_{1y} \sqrt{A_{2y} - A_{1y}}} \quad C_y = 1.861 \times 10^{-10} \]  
Compliance of section

**20mm cylinder of horn**

\[ l_z = 0.39 \]  
Length of cylinder

\[ A_z := \frac{\pi 0.026^2}{4} \]  
Section area

\[ \rho z = 4420 \]  
Density of material

\[ Y_{2z} = 114 \times 10^3 \]  
Youngs modulus

\[ c_2 = 5.079 \times 10^3 \]  
Speed of sound

\[ M_z := \rho z A_{2z} - l_z \quad M_z = 0.054 \]  
Static mass of section

\[ C_z := \frac{l_z}{Y_{2z} A_z} \quad C_z = 1.089 \times 10^{-9} \]  
Compliance of section

**Head cylinder - titanium**

\[ t_h := 0.27 \]  
Length of head

\[ A_h := \frac{\pi 0.041^2}{4} \]  
Section area head

\[ \rho h = 4420 \]  
Density of head material

\[ Y_{1h} = 114 \times 10^3 \]  
Youngs modulus

\[ c_h := \frac{\sqrt{Y_{1h}}}{\rho h} \quad c_h = 5.079 \times 10^3 \]  
Speed of sound in head

\[ M_h := \rho h A_h - t_h \quad M_h = 0.15 \]  
Static mass of head

\[ C_h := \frac{t_h}{Y_{1h} A_h} \quad C_h = 1.885 \times 10^{-10} \]  
Compliance of head

**Ceramic Assembly**

\[ A_c := \frac{\pi \left(0.085^2 - 0.025^2\right)}{4} \]  
Section area of ceramic rings

\[ t_c := 0.064 \]  
Thickness of ceramic rings
\[ n_e = 4 \]
\[ \alpha = 1330.8854 \times 10^{-12} \]
\[ \rho_e = 7700 \]
\[ s_{33} = 1.96 \times 10^{-11} \]
\[ \alpha = \frac{1}{\sqrt{\rho_e s_{33}}} \]
\[ d_{33} = 3.28 \times 10^{-12} \]
\[ k_{33} = \frac{d_{33}}{\sqrt{\varepsilon_e s_{33}}} \]
\[ C_e = \frac{\varepsilon_e A_c n_c}{l_e} \]
\[ C_b = C_e \left( 1 - k_{33}^2 \right) \]
\[ N = \frac{d_{33} A_c}{s_{33} l_e} \]
\[ C_{mE} = \frac{n_e l_c s_{33} E}{A_c} \]
\[ M_e = \rho_e n_e A_c \]

Tail mass - stainless steel
\[ M_t = \frac{0.0407 \times 10^4}{4} \]
\[ \rho_t = 0.017 \]
\[ Y_t = 19.3 \times 10^6 \]
\[ c_t = \frac{Y_t}{\rho_t} \]
\[ c_t = 4.974 \times 10^9 \]
\[ M_t = \rho_t A_t \times 10^4 \]
\[ C_t = \frac{1}{Y_t A_t} \]
\[ C_t = 7.009 \times 10^{-11} \]

Compressive stress bias bolt (Titanium)
\[ t_s = n_e \times 10 \]
\[ b_s = 0.026 \]
\[ A_b = \pi \times 0.04^2 \]
\[ \rho_b = 44200 \]
\[ Y_s = 11.4 \times 10^9 \]

Number of ceramic elements
Dielectric constant of piezoelectric ceramic material
Density of ceramic rings
Elastic constant E-field
Sound speed for constant E-field or shorted terminals
Longitudinal strain per unit E-field

\[ k_{33} = 0.683 \]
\[ C_e = 7.636 \times 10^{-9} \]
\[ C_t = 4.077 \times 10^{-9} \]
\[ N = 2.713 \]
\[ C_{mE} = 4.883 \times 10^{-10} \]
\[ M_e = 0.208 \]

Free capacitance of stack
Blocked capacitance of stack
Blocked force per volt
Compliance of stack
Static mass
\[ \omega_s = \sqrt{\frac{Y_s}{\rho_s}} \quad \omega_s = 5.079 \times 10^3 \]
\[ C_s = \frac{\varepsilon_s}{Y_s \lambda_{sv}} \]
\[ C_s = \frac{4.68 \times 10^{-9}}{9.237} \]
\[ M_s = \frac{\rho_s}{\lambda_{sv} \lambda_{sv}} \]
\[ M_s = 5.68 \times 10^{-5} \]

Swept frequency increments and range
\[ n^* = 10.12 \quad \omega_1 = 0.1 \quad \Omega = 30 \quad f_2 = 4 \times 10^4 \quad f_{\text{nf}} = f_1 \left( \frac{12}{n} \right) \]

Equivalent circuit elements assuming mechanical impedance analogy and a plane wave traveling on length axis of the resonator (single degree of freedom):

First taper
\[ Z_{11w} = -j \rho w c_\omega \sqrt{A_{2w} A_{1w}} \left( \frac{1}{\tan \left( \frac{\omega_1 t_{sw}}{c_\omega} \right)} + \frac{cw}{\omega_1 t_{rw}} \right) \]
\[ Z_{12w} = \frac{\rho w c_\omega \sqrt{A_{2w} A_{1w}}}{j \sin \left( \frac{\omega_1 t_{rw}}{c_\omega} \right)} \]
\[ Z_{22w} = -j \rho w c_\omega \sqrt{A_{2w} A_{1w}} \left( \frac{1}{\tan \left( \frac{\omega_1 t_{rw}}{cw} \right)} - \frac{cw}{\omega_1 t_{lw}} \right) \]
\[ Z_{w1} := Z_{11w} - Z_{12w} \]
\[ Z_{w2} := Z_{22w} - Z_{12w} \]
\[ Z_{w3} := Z_{12w} \]

60 mm cylinder
\[ Z_{y1} = j \rho s c_\omega A_{xy} \tan \left( \frac{\omega_1 t_{xy}}{2 c_\omega} \right) \]
\[ Z_{y2} = \frac{\rho v c_\omega A_{xy}}{j \sin \left( \frac{\omega_1 t_{xy}}{c_\omega} \right)} \]

Second taper
\[ Z_{11y} = -j \rho v c_\omega \sqrt{A_{2y} A_{1y}} \left( \frac{1}{\tan \left( \frac{\omega_1 t_{vy}}{c_\omega} \right)} + \frac{c_y}{\omega_1 t_{vy}} \right) \]
\[ Z_{12y} = \frac{\rho v c_\omega \sqrt{A_{2y} A_{1y}}}{j \sin \left( \frac{\omega_1 t_{vy}}{c_\omega} \right)} \]
\[ Z_{22y} = -j \rho v c_\omega \sqrt{A_{2y} A_{1y}} \left( \frac{1}{\tan \left( \frac{\omega_1 t_{vy}}{c_y} \right)} - \frac{c_y}{\omega_1 t_{vy}} \right) \]
\[ Z_{y1} = -Z_{11y} - Z_{12y} \]
\[ Z_{y2} = -Z_{22y} - Z_{12y} \]
\[ Z_{y3} = Z_{12y} \]
20 mm cylinder
\[ Z_{21} = j \rho \frac{1}{c} \sin \left( \frac{\omega_1}{2c} \right) \]
\[ Z_{22} = \frac{\rho \frac{1}{c} \sin \left( \frac{\omega_2}{2c} \right)}{1} \]

Head section
\[ Z_{11} = j \rho \frac{1}{c} \sin \left( \frac{\omega_1}{2c} \right) \]
\[ Z_{12} = \frac{\rho \frac{1}{c} \sin \left( \frac{\omega_2}{2c} \right)}{1} \]

Ceramic circuit elements
\[ Z_{11} = j \rho \frac{1}{c} \sin \left( \frac{\omega_1}{2c} \right) \]
\[ Z_{22} = \frac{\rho \frac{1}{c} \sin \left( \frac{\omega_2}{2c} \right)}{1} \]

Tail section
\[ Z_{11} = j \rho \frac{1}{c} \sin \left( \frac{\omega_1}{2c} \right) \]
\[ Z_{12} = \frac{\rho \frac{1}{c} \sin \left( \frac{\omega_2}{2c} \right)}{1} \]

Stress bolt section
\[ Z_{11} = j \rho \frac{1}{c} \sin \left( \frac{\omega_1}{2c} \right) \]
\[ Z_{12} = \frac{\rho \frac{1}{c} \sin \left( \frac{\omega_2}{2c} \right)}{1} \]

Translate mechanical impedances through the ideal transformer to yield an equivalent electric network
\[ Z_{11} = Z_{11} + \frac{Z_{12} Z_{22}}{Z_{12} + Z_{22}} \]
\[ Z_{21} = Z_{21} \]
\[ Z_{12} = \frac{Z_{12} Z_{21}}{Z_{21} + Z_{12}} \]

Horn impedance
\[ Z_h = Z \left( \frac{Z_{11} + Z_{12}}{Z_{12} + Z_{22}} \right) \]

Blocked capacitance term
\[ Z_{11} = \frac{1}{j \omega_1 C_1} \]

Compliance related terms for ceramic
\[ Z_{11} = Z_{11} + \frac{Z_{12} Z_{22}}{Z_{12} + Z_{22}} \]

Forward section of resonator
\[ Z_{11} = \frac{Z_{12} Z_{22}}{Z_{12} + Z_{22}} \]

Bolt
\[ Z_{11} = \frac{1}{N^2} \]
\[ Z_S = \frac{Z_{S_1} + Z_{S_2} + \frac{Z_{S_1}Z_{S_2}}{Z_{S_1} + Z_{S_2}}}{N^2} \]

All section of resonator

Impedance matrix for equivalent circuit

\[
Z_0 = \begin{pmatrix}
Z_{1_1} & -Z_{1_1} & 0 \\
-Z_{1_1} & Z_{1_1} + Z_{2_1} + Z_{1_2} + Z_{4_1} - Z_{3_1} & 0 \\
0 & -Z_{1_1} & Z_{1_1} + Z_{1_2}
\end{pmatrix}
\]

Drive resonator with 1 volt at electrical terminals and compute currents flowing in each mesh

\[
\begin{pmatrix}
V_1 \\
V_2 \\
V_3
\end{pmatrix} = Z_0^{-1} \begin{pmatrix}
10 \\
11 \\
12
\end{pmatrix}
\]

Plot magnitude of electrical drive admittance

\[ Y_{mag} = \frac{1}{V_1} \]

Determine mechanical resonance, anti-resonance frequency and effective coupling coefficient

\[
fr_1 = f_r \left( Y_{mag} = \max(Y_{mag}) \right) \quad fr = \max(fr) \quad fr = 2.259 \times 10^4
\]

\[
f_a = f_r \left( Y_{mag} = \max(Y_{mag}) \right) \quad fa = \max(fa) \quad fa = 2.483 \times 10^4
\]

\[
K_{eff} = \sqrt{1 - \left( \frac{fr}{fa} \right)^2} \quad K_{eff} = 0.415
\]
APPENDIX E-5

The Abaqus code to simulate the horn transducer with the horn, radiating into water is shown below.

*heading
Horn transducer, with horn radiating into large hemisphere of water

**

*preprint, model=eye

**

**node definition

**

*node
1, 0.0, 0.013
2, 0.004, 0.013
3, 0.005, 0.013
4, 0.00635, 0.013
5, 0.01, 0.013

**

201, 0.0, 0.020
205, 0.004, 0.020
207, 0.0055, 0.020
209, 0.00635, 0.020
215, 0.01, 0.020
227, 0.02, 0.025

**

1001, 0.0, 0.0435
1005, 0.004, 0.0435
1007, 0.0055, 0.0435
1009, 0.00635, 0.0435
1015, 0.01, 0.0435
1025, 0.0125, 0.0435
1027, 0.02, 0.0435

**

4201, 0.0, 0.0711
4205, 0.004, 0.0711
4207, 0.0055, 0.0711
4209, 0.00635, 0.0711
4215, 0.01, 0.0711
4225, 0.01925, 0.0711
4227, 0.02, 0.0711

4601, 0.0, 0.0841
4605, 0.004, 0.0841
4607, 0.0055, 0.0841
4609, 0.00635, 0.0841
4613, 0.01, 0.0841
4625, 0.01925, 0.0841
4627, 0.02, 0.0841

4801, 0.0, 0.0889
4805, 0.004, 0.0889
4807, 0.0055, 0.0889
4809, 0.00635, 0.0889
4815, 0.01, 0.0889
4825, 0.01925, 0.0889
4827, 0.02, 0.0889

5201, 0.0, 0.103
5205, 0.004, 0.103
5207, 0.0055, 0.103
5209, 0.00635, 0.103
5215, 0.01, 0.103
5225, 0.01925, 0.103
5227, 0.02, 0.103

*ngen, nset=tempi
1, 5, 1
5, 7, 1
7, 3, 1
9, 15, 1

*ngen, nset=temp2
201, 205, 1
205, 207, 1
.207, 208, 1
.209, 215, 1
**
*ndefill
.temp1, temp2, 4, 50
**
*element, type=cax8, elset=nut
1, 1, 3, 103, 101, 2, 53, 102, 51
*elgen, elset=nut
1, 7, 2, 1, 2, 100, 100
**
*solid section, elset=nut, material=ti5
**
**tail definition
**
*nset, nset=temp3a
207, 209, 1
209, 215, 1
*nset, nset=temp3b
215, 227, 1
*nset, nset=temp3
.temp3a, temp3b
*nset, nset=temp4
1007, 1009, 1
1009, 1025, 1
1025, 1027, 1
**
*ndefill
.temp3, temp4, 16, 50
**
*element, type=cax8, elset=tail
207, 207, 209, 309, 307, 208, 259, 308, 259
*elgen, elset=tail
207, 10, 2, 1, 8, 100, 100
**
*solid section, elset=tail, material=as
**
**bolt definition
**--------------------------------------------------
*ngen, nset=temp5
201, 205, 1
*ngen, nset=temp6
4201, 4205, 1
*ngen, nset=temp7
4601, 4605, 1
*
*nfill
temp5, temp6, 80, 50
*nfill
temp6, temp7, 8, 50
*
*element, type=cax8, elset=bolt
201, 201, 203, 303, 301, 202, 253, 302, 251
*elgen, elset=bolt
201, 2, 2, 1, 40, 100, 100
*
*element, type=cax8, elset=bolt
4201, 4201, 4203, 4303, 4301, 4202, 4253, 4302, 4251
*elgen, elset=bolt
4201, 2, 2, 1, 4, 100, 100
*
*solid section, elset=bolt, material=ti5
*
**--------------------------------------------------
**head definition
**--------------------------------------------------
*ngen, nset=temp8
4205, 4209, 1
4209, 4225, 1
4225, 4227, 1
*ngen, nset=temp9
4605, 4609, 1
4609, 4625, 1
4625, 4627, 1
*nset, nset=temp10
temp7, temp9
*ngen, nset=temp11
4801, 4805, 1
*ngen, nset=temp12
4805, 4809, 1
4809, 4825, 1
4825, 4827, 1
*nset, nset=temp13
temp11, temp12
*ngen, nset=temp14
5205, 5209, 1
5209, 5225, 1
5225, 5227, 1

*nfill
temp8, temp9, 8, 50
temp10, temp13, 4, 50
temp12, temp14, 8, 50

*element, type=cax8, elset=head
4205, 4207, 4307, 4305, 4206, 4257, 4306, 4255
*elgen, elset=head
4205, 11, 2, 1, 4, 100, 100

*element, type=cax8, elset=head
4601, 4603, 4703, 4701, 4602, 4653, 4702, 4651
*elgen, elset=head
4601, 13, 2, 1, 2, 100, 100

*element, type=cax8, elset=head
4805, 4807, 4907, 4905, 4806, 4857, 4906, 4855
*elgen, elset=head
4805, 11, 2, 1, 4, 100, 100

*solid section, elset=head, material=ti5

**

**bolt2 definition
*ngen, nset=temp25
5201, 5205, 1
**
*nfill
temp11, temp25, 8, 50
**
*element, type=cax8, elset=bolt2
4801, 4803, 4901, 4903, 4902, 4851
*elgen, elset=bolt2
4801, 2, 2, 1, 4, 100, 100
**
*solid section, elset=bolt2, material=ti5
**
**-----------------------------------------
**electrode1 definition
**-----------------------------------------
*node
1109, 0.00635, 0.0435
1115, 0.01, 0.0435
1125, 0.01925, 0.0435
1127, 0.02, 0.0435
**
1209, 0.00635, 0.04375
1215, 0.01, 0.04375
1225, 0.01925, 0.04375
1227, 0.02, 0.04375
**
*ngen, nset=al
1109, 1125, 1
1125, 1127, 1
*ngen, nset=bl
1209, 1225, 1
1225, 1227, 1
**
*nfill
al, bl, 2, 50
**
*element, type=cax8, elset=elec
elset=elec
**
*solid section, elset=elec, material=BeCu
**pzt1 definition
*node
definition
*node
1309, 0.00635, 0.04375
1315, 0.01, 0.04375
1325, 0.01925, 0.04375
**
1709, 0.00635, 0.05015
1715, 0.01, 0.05015
1725, 0.01925, 0.05015
**
*nset, nset=a2
1309, 1325, 1
*nset, nset=b2
1709, 1725, 1
**
*nfill
a2, b2, 8, 50
**
*element, type=cax8, elset=pztpos
definition
*element, type=cax8, elset=pztpos
1309, 1309, 1311, 1411, 1409, 1310, 1361, 1410, 1359
*elgen, elset=pztpos
definition
*elgen, elset=pztpos
1309, 8, 2, 1, 4, 100, 100
**
*solid section, elset=pztpos, material=pz26, orientation=2pos
**electrode2 definition
**
*node
definition
*node
1809, 0.00635, 0.05015
1815, 0.01, 0.05015
1825, 0.01925, 0.05015
1827, 0.02, 0.05015
**
1909, 0.00635, 0.0504
1915, 0.01, 0.0504
1925, 0.01925, 0.0504
1927, 0.02, 0.0504
**
*ngen, nset=a3
1809, 1825, 1
1825, 1827, 1
*ngen, nset=b3
1909, 1925, 1
1925, 1927, 1
**
*nfill
a3, b3, 2, 50
**
*element, type=cax8, elset=elec 2
1809, 1809, 1811, 1911, 1909, 1810, 1861, 1910, 1859
*elgen, elset=elec
1809, 9, 2, 1, 1, 100, 100
**
**pzt2 definition
**
*node
2009, 0.00635, 0.0504
2015, 0.01, 0.0504
2025, 0.01925, 0.0504
**
2409, 0.00635, 0.0568
2415, 0.01, 0.0568
2425, 0.01925, 0.0568
**
*ngen, nset=a4
2009, 2025, 1
*ngen, nset=b4
2409, 2425, 1
**
*nfill
a4, b4, 8, 50
**
*element, type=cax8e, elset=pztneg
*elgen, elset=pztneg
2009, 8, 2, 1, 4, 100, 100
**
*solid section, elset=pztneg, material=pz26, orientation=2neg
**-------------------------------------------------------------
**elec3300 definition
**-------------------------------------------------------------
*node
2509, 0.00635, 0.0568
2515, 0.01, 0.0568
2525, 0.01925, 0.0568
2527, 0.02, 0.0568
**
2609, 0.00635, 0.05705
2615, 0.01, 0.05705
2625, 0.01925, 0.05705
2627, 0.02, 0.05705
**
*nset, nset=a5
2509, 2525, 1
2525, 2527, 1
*nset, nset=b5
2609, 2625, 1
2625, 2627, 1
**
*nfill
a5, b5, 2, 50
**
*element, type=cax8e, elset=elec
2509, 2509, 2511, 2611, 2609, 2510, 2561, 2610, 2559
*elgen, elset=elec
2509, 9, 2, 1, 1, 100, 100
**-------------------------------------------------------------
**nodal definition

*node
2709, 0.00635, 0.05705
2715, 0.01, 0.05705
2725, 0.01925, 0.05705
2727, 0.02, 0.05705
2735, 0.025, 0.05705
**
2835, 0.025, 0.058
**
2909, 0.00635, 0.05805
2915, 0.01, 0.05805
2925, 0.01925, 0.05805
2927, 0.02, 0.05805
2935, 0.025, 0.05805
**
*ngen, nset=Na
2709, 2725, 1
2725, 2727, 1
2727, 2735, 1
*ngen, nset=Nb
2909, 2925, 1
2925, 2927, 1
2927, 2935, 1
**
*nfill
Na, Nb, 4, 50
**
*element, type=cax8, elset=nodal
2709, 2709, 2711, 2811, 2809, 2710, 2761, 2810, 2759
*elgen, elset=nodal
2709, 13, 2, 1, 2, 100, 100
**
*solid section, elset=nodal, material=ss
**
pzt3 definition
**
*node
3009, 0.00635, 0.05805
3015, 0.01, 0.05805
3025, 0.01925, 0.05805
**
3409, 0.00635, 0.06445
3415, 0.01, 0.06445
3425, 0.01925, 0.06445
**
*nset, nset=a6
3009, 3025, 1
*nset, nset=b6
3409, 3425, 1
**
*nfill
a6, b6, 8, 50
**
*element, type=cax8e, elset=pztpos
3009, 3009, 3011, 3111, 3109, 3010, 3061, 3110, 3059
*elgen, elset=pztpos
3009, 8, 2, 1, 4, 100, 100
**
**electrode4 definition
**
*node
3509, 0.00635, 0.06445
3515, 0.01, 0.06445
3525, 0.01925, 0.06445
3527, 0.02, 0.06445
**
3609, 0.00635, 0.0647
3615, 0.01, 0.0647
3625, 0.01925, 0.0647
3627, 0.02, 0.0647
**
*nset, nset=a7
3509, 3525, 1
3525, 3527, 1
*ngen, nset=b7
3609, 3625, 1
3625, 3627, 1
**
*nfill
a7, b7, 2, 50
**
*element, type=cax8, elset=elec
3509, 3509, 3511, 3611, 3609, 3510, 3561, 3610, 3559
*elgen, elset=elec
3509, 9, 2, 1, 1, 100, 100
**--------------------------------------------------
**pzt4 definition
**--------------------------------------------------
*node
3709, 0.00635, 0.0647
3715, 0.01, 0.0647
3725, 0.01925, 0.0647
**
4109, 0.00635, 0.0711
4115, 0.01, 0.0711
4125, 0.01925, 0.0711
**
*ngen, nset=a8
3709, 3725, 1
*ngen, nset=b8
4109, 4125, 1
**
*nfill
a8, b8, 8, 50
**
*element, type=cax8e, elset=pzt neg
3709, 3709, 3711, 3811, 3809, 3710, 3761, 3810, 3759
*elgen, elset=pzt neg
3709, 8, 2, 1, 4, 100, 100
**-----------------------------------------------------------
**tail to elecli mechanical vertical coupling
**-----------------------------------------------------------
*nset, nset=temp15, generate
1009, 1025, 1
*nset, nset=temp16, generate
1109, 1125, 1
*mpc
tie, temp15, temp16

**elec1 to pzt1 mechanical vertical coupling

*nset, nset=temp17, generate
1209, 1225, 1
*mpc
tie, temp17, a2

**pzt1 to elec2 mechanical vertical coupling

*nset, nset=temp18, generate
1809, 1825, 1
*mpc
tie, temp18, b2

**elec2 to pzt2 mechanical vertical coupling

*nset, nset=temp19, generate
1909, 1925, 1
*mpc
tie, temp19, a4

**pzt2 to elec3 mechanical vertical coupling

*nset, nset=temp20, generate
2509, 2525, 1
*mpc
tie, temp20, b4

**elec3 to nodal mechanical vertical coupling

*nset, nset=temp21, generate
2609, 2625, 1
*nset, nset=Noda, generate
2709, 2725, 1
*mpc
tie, temp21, Noda

**nodal to pzt3 mechanical vertical coupling

*nset, nset=Nodb, generate
2909, 2925, 1
*mpc
tie, Nodb, a6

**pzt3 to elec4 mechanical vertical coupling

*nset, nset=temp22, generate
3509, 3525, 1
*mpc
tie, temp22, b6

**elec4 to pzt4 mechanical vertical coupling

*nset, nset=temp23, generate
3609, 3625, 1
*mpc
tie, temp23, a8

**pzt4 to head mechanical vertical coupling

*nset, nset=temp24, generate
4209, 4225, 1
*mpc
tie, temp24, b8

**electrode electrical detail

*node, nset=ns_ref
100000, 0.035, 0.0
100001, 0.036, 0.0
100002, 0.037, 0.0
* equation
2
b2, 9, 1.0, a4, 9, -1.0
* equation
2
a4, 9, 1.0, 100000, 9, -1.0
* equation
2
b4, 9, 1.0, a6, 9, -1.0
* equation
2
a6, 9, 1.0, 100001, 9, -1.0
* equation
2
b6, 9, 1.0, a8, 9, -1.0
* equation
2
a8, 9, 1.0, 100002, 9, -1.0
**------------------------------------------------------
** material definitions
**------------------------------------------------------
* orientation, name=2pos
1.0, 0.0, 0.0, 0.0, 1.0, 0.0
1, 0.0
* orientation, name=2neg
-1.0, 0.0, 0.0, 0.0, -1.0, 0.0
1, 0.0
**
* material, name=pz26
* elastic, type=ortho
16.8E10, 9.99E10, 12.3E10, 11.0E10, 9.99E10, 16.8E10, 3.01E10, 3.01E10,
2.88E10
* piezoelectric, type=s
0., 0., 0., 9.86, 0., 0., -1.98, 14.7
-1.98, 0., 0., 0., 0., 0., 0., 0.
0., 9.86
*dielectric, type=aniso
11.771E-9, 0., 10.531E-9, 0., 0., 11.771E-9
*density
7700.
**-------------------
*materail, name=ss
*elastic
200.e9, 0.3
*density
7800.
*materail, name=ti5
*elastic
114.e9, 0.33
*density
4420.
*materail, name=BeCu
*elastic
128.e9, 0.3
*density
8250.
*materail, name=Al
*elastic
69.e9, 0.33
*density
2700.
**--------------------------------------------------------------------
**horn definition
**--------------------------------------------------------------------
*node
5301, 0.0, 0.103
5305, 0.004, 0.103
5309, 0.00635, 0.103
5315, 0.0095, 0.103
5325, 0.01925, 0.103
5327, 0.02, 0.103
**
5901, 0.0, 0.11585
5905, 0.004, 0.11585
**--------------------------
** base horn definition
**--------------------------

*ngeo, nset=base1a
5301, 5305, 1
*ngeo, nset=base1
5305, 5309, 1
5309, 5325, 1
5325, 5327, 1
*ngeo, nset=base2a
5901, 5905, 1
*ngeo, nset=base2
5905, 5927, 1
*ngeo, nset=base3a
6301, 6305, 1
*ngeo, nset=base3b
6305, 6327, 1
*nset, nset=base3
base3a, base3b
*nset, nset=base4
8001, 8027, 1
**
*nfill
base1, base2, 12, 50
base2, base3b, 8, 50
base3, base4, 34, 50
**
*element, type=cax8, elset=horn
5305, 5305, 5307, 5407, 5405, 5306, 5357, 5406, 5355
6301, 6301, 6303, 6403, 6401, 6302, 6353, 6402, 6351
*elgen, elset=horn
5305, 11, 2, 1, 10, 100, 100
6301, 13, 2, 1, 17, 100, 100
*solid section, elset=horn, material=ti5
**
**---------------------------------------------------------------------**
**bolt2 definition**
**---------------------------------------------------------------------**
*nfill
base1a, base2a, 12, 50
*nfill
base2a, base3a, 8, 50
**
*element, type=cax8, elset=bolt2
5301, 5301, 5303, 5403, 5401, 5302, 5353, 5402, 5351
*elgen, elset=bolt2
5301, 2, 2, 1, 10, 100, 100
**
*solid section, elset=bolt2, material=ti5
**
**---------------------------------------------------------------------**
**mid horn definition**
**---------------------------------------------------------------------**
*node
20001, 0.02825, 0.1796
**
*ngen, nset=mid1
8101, 8127, 1
*ngen ,nset=mid2
8127, 8155, 1
*nset, nset=mid3
mid1, mid2
*ngen, line=c, nset=mid4
10101, 10155, 1, 20001
**
*nfill
mid3, mid4, 20, 100
**
*element, type=cax8, elset=horn
8101, 8101,8301,8303,8103, 8201,8302,8203,8102
*elgen, elset=horn
8101, 27,2,1, 10,200,200
**
**---------------------------------------------------------------------
**mpc's tying base and mid together
**---------------------------------------------------------------------
*mpc
tie, 8001, 8127
tie, 8002, 8126
tie, 8003, 8125
tie, 8004, 8124
tie, 8005, 8123
tie, 8006, 8122
tie, 8007, 8121
tie, 8008, 8120
tie, 8009, 8119
tie, 8010, 8118
tie, 8011, 8117
tie, 8012, 8116
tie, 8013, 8115
tie, 8014, 8114
tie, 8015, 8113
tie, 8016, 8112
tie, 8017, 8111
tie, 8018, 8110
tie, 8019, 8109
tie, 8020, 8108
tie, 8021, 8107
tie, 8022, 8106
tie, 8023, 8105
tie, 8024, 8104
tie, 8025, 8103
tie, 8026, 8102
tie, 8027, 8101
**---------------------------------------------------------------------
**top horn
**---------------------------------------------------------------------
*node
8199, 0.0, 0.2262
10199, 0.0095, 0.2262
**
*ngen, nset=top1
8155, 10155, 100
*ngen, nset=top2
8199, 10199, 100
**
*nfill
top1, top2, 44, 1
**
*element, type=cax8, elset=horn
8155, 8155, 8355, 8357, 8157, 8255, 8356, 8257, 8156
*elgen, elset=horn
8155, 10, 200, 200, 22, 2, 1
**
**---------------------------------------------------------------------
**flange definition
**---------------------------------------------------------------------
*node
8301, 0.02825, 0.15185
8901, 0.02825, 0.15485
**
10201, 0.02825, 0.15185
10205, 0.03025, 0.15185
**
10801, 0.02825, 0.15485
10805, 0.03025, 0.15485
**
*nset, nset=flange1
10201, 10205, 1
*nset, nset=flange2
10801, 10805, 1
**
*nfill
flange1, flange2, 6, 100
**
*element, type=cax8, elset=horn
10201, 10201, 10203, 10403, 10401, 10202, 10303, 10402, 10301
*elgen, elset=horn
10201, 2, 1, 1, 1, 3, 1, 200, 200
**
**
*nset, nset=flange3, generate
10201, 10801, 100
*nset, nset=flange4
8301, 8901, 100
**
*mpc
tie, flange3, flange4
**
*nset, nset=flange5, generate
10205, 10805, 100
**
*------------------------------------------------------------------------------------------------------------------------
**mpc's tying head to base of horn
*------------------------------------------------------------------------------------------------------------------------
*nset, nset=head1, generate
5201, 5227, 1
*nset, nset=horn1, generate
5301, 5327, 1
**
*mpc
  tie, head1, horn1
**
**
*water definition
**
*node
  20001, 0.0, 0.2262
  20002, 0.000475, 0.2262
  20021, 0.0095, 0.2262
  20101, 0.1, 0.2262
  20199, 1.0, 0.2262
**
  62002, 0.0, 0.226675
  62021, 0.0, 0.2357
  62101, 0.0, 0.3262
  62199, 0.0, 1.2262
**
*nset, nset=surface, generate
  20021, 20199, 1
**
*nfill
  cls1, cls2, 19, 1
  *nfill, bias=0.98
  cls2, mid, 80, 1
  *nfill, bias=0.98
  mid, far, 98, 1
**
**element definition (wedge elements first)**

*element, type=acax8, elset=water

20403, 20403, 20001, 20001, 20003, 20402, 20001, 20002, 20203
20803, 20803, 20001, 20001, 20403, 20802, 20001, 20402, 20603
21203, 21203, 20001, 20001, 20803, 21202, 20001, 20802, 21003
21603, 21603, 20001, 20001, 21203, 21602, 20001, 21202, 21403
22003, 22003, 20001, 20001, 21603, 22002, 20001, 21602, 21803
22403, 22403, 20001, 20001, 22003, 22402, 20001, 22002, 22203
22803, 22803, 20001, 20001, 22403, 22802, 20001, 22402, 22603
23203, 23203, 20001, 20001, 22803, 23202, 20001, 22802, 23003
23603, 23603, 20001, 20001, 23203, 23602, 20001, 23202, 23403
24003, 24003, 20001, 20001, 23603, 24002, 20001, 23602, 23803
24403, 24403, 20001, 20001, 24003, 24402, 20001, 24002, 24203
24803, 24803, 20001, 20001, 24403, 24802, 20001, 24402, 24603
25203, 25203, 20001, 20001, 24803, 25202, 20001, 24802, 25003
25603, 25603, 20001, 20001, 25203, 25602, 20001, 25202, 25403
26003, 26003, 20001, 20001, 25603, 26002, 20001, 25602, 25803
26403, 26403, 20001, 20001, 26003, 26402, 20001, 26002, 26203
26803, 26803, 20001, 20001, 26403, 26802, 20001, 26402, 26603
27203, 27203, 20001, 20001, 26803, 27202, 20001, 26802, 27003
27603, 27603, 20001, 20001, 27203, 27602, 20001, 27202, 27403
28003, 28003, 20001, 20001, 27603, 28002, 20001, 27602, 27803
28403, 28403, 20001, 20001, 28003, 28402, 20001, 28002, 28203
28803, 28803, 20001, 20001, 28403, 28802, 20001, 28402, 28603
29203, 29203, 20001, 20001, 28803, 29202, 20001, 28802, 29003
29603, 29603, 20001, 20001, 29203, 29602, 20001, 29202, 29403
30003, 30003, 20001, 20001, 29603, 30002, 20001, 29602, 29803
30403, 30403, 20001, 20001, 30003, 30402, 20001, 30002, 30203
30803, 30803, 20001, 20001, 30403, 30802, 20001, 30402, 30603
31203, 31203, 20001, 20001, 30803, 31202, 20001, 30802, 31003
31603, 31603, 20001, 20001, 31203, 31602, 20001, 31202, 31403
32003, 32003, 20001, 20001, 31603, 32002, 20001, 31602, 31803
32403, 32403, 20001, 20001, 32003, 32402, 20001, 32002, 32203
32803, 32803, 20001, 20001, 32403, 32802, 20001, 32402, 32603
33203, 33203, 20001, 20001, 32803, 33202, 20001, 32802, 33003
33603, 33603, 20001, 20001, 33203, 33602, 20001, 33202, 33403
22403, 22403, 20001, 20001, 22003, 22402, 20001, 22002, 22203
22803, 22803, 20001, 20001, 22403, 22802, 20001, 22402, 22603
23203, 23203, 20001, 20001, 22803, 23202, 20001, 22802, 23003
23603, 23603, 20001, 20001, 23203, 23602, 20001, 23202, 23403
24003, 24003, 20001, 20001, 23603, 24002, 20001, 23602, 23803
24403, 24403, 20001, 20001, 24003, 24402, 20001, 24002, 24203
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25603, 25603, 20001, 20001, 25203, 25602, 20001, 25202, 25403
26003, 26003, 20001, 20001, 25603, 26002, 20001, 25602, 25803
26403, 26403, 20001, 20001, 26003, 26402, 20001, 26002, 26203
26803, 26803, 20001, 20001, 26403, 26802, 20001, 26402, 26603
27203, 27203, 20001, 20001, 26803, 27202, 20001, 26802, 27003
27603, 27603, 20001, 20001, 27203, 27602, 20001, 27202, 27403
28003, 28003, 20001, 20001, 27603, 28002, 20001, 27602, 27803
28403, 28403, 20001, 20001, 28003, 28402, 20001, 28002, 28203
28803, 28803, 20001, 20001, 28403, 28802, 20001, 28402, 28603
29203, 29203, 20001, 20001, 28803, 29202, 20001, 28802, 29003
29603, 29603, 20001, 20001, 29203, 29602, 20001, 29202, 29403
30003, 30003, 20001, 20001, 29603, 30002, 20001, 29602, 29803
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30803, 30803, 20001, 20001, 30403, 30802, 20001, 30402, 30603
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32003, 32003, 20001, 20001, 31603, 32002, 20001, 31602, 31803
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32803, 32803, 20001, 20001, 32403, 32802, 20001, 32402, 32603
33203, 33203, 20001, 20001, 32803, 33202, 20001, 32802, 33003
33603, 33603, 20001, 20001, 33203, 33602, 20001, 33202, 33403
34003, 34003, 20001, 20001, 33603, 34002, 20001, 33602, 33803
34403, 34403, 20001, 20001, 34003, 34402, 20001, 34002, 34203
34803, 34803, 20001, 20001, 34403, 34802, 20001, 34402, 34603
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36003, 36003, 20001, 20001, 35603, 36002, 20001, 35602, 35803
36403, 36403, 20001, 20001, 36003, 36402, 20001, 36002, 36203
36803, 36803, 20001, 20001, 36403, 36802, 20001, 36402, 36603
37203, 37203, 20001, 20001, 36803, 37202, 20001, 36802, 37003

256
52803, 52803, 20001, 20001, 52403, 52802, 20001, 52402, 52603
53203, 53203, 20001, 20001, 52803, 53202, 20001, 52802, 53003
53603, 53603, 20001, 20001, 53203, 53602, 20001, 53202, 53403
54003, 54003, 20001, 20001, 53603, 54002, 20001, 53602, 53803
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55203, 55203, 20001, 20001, 54803, 55202, 20001, 54802, 55003
55603, 55603, 20001, 20001, 55203, 55602, 20001, 55202, 55403
56003, 56003, 20001, 20001, 55603, 56002, 20001, 55602, 55803
56403, 56403, 20001, 20001, 56003, 56402, 20001, 56002, 56203
56803, 56803, 20001, 20001, 56403, 56802, 20001, 56402, 56603
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57603, 57603, 20001, 20001, 57203, 57602, 20001, 57202, 57403
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59203, 59203, 20001, 20001, 58803, 59202, 20001, 58802, 59003
59603, 59603, 20001, 20001, 59203, 59602, 20001, 59202, 59403
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60403, 60403, 20001, 20001, 60003, 60402, 20001, 60002, 60203
60803, 60803, 20001, 20001, 60403, 60802, 20001, 60402, 60603
61203, 61203, 20001, 20001, 60803, 61202, 20001, 60802, 61003
61603, 61603, 20001, 20001, 61203, 61602, 20001, 61202, 61403
62003, 62003, 20001, 20001, 61603, 62002, 20001, 61602, 61803
**
*element, type=acax8, elset=water
20405, 20405, 20403, 20003, 20005, 20404, 20203, 20004, 20205
20429, 20429, 20427, 20027, 20029, 20428, 20227, 20028, 20229
20447, 20447, 20445, 20045, 20047, 20446, 20245, 20046, 20247
20451, 20451, 20449, 20049, 20051, 20450, 20249, 20050, 20251
20503, 20503, 20501, 20101, 20103, 20502, 20301, 20102, 20303
*elgen, elset=water
20405, 105,400,400, 12,2,2
20429, 105,400,400, 9,2,2
20447, 105,400,400, 2,2,2
20451, 105,400,400, 26,2,2
20503, 105,400,400, 49,2,2
*solid section, elset=water, material=water
**Interface elements**

*element, type=asi3a, elset=inter
9001, 20001, 20002, 20003

*elgen, elset=inter
9001, 10, 2, 1

*interface, elset=inter

**Non-reflective boundary definition**

*elset, elset=waterbound, generate
20599, 62199, 400

*surface impedance
waterbound, I4,

**mpc's tying head to water**

*nset, nset=headwater, generate
20001, 20021, 1

**

*mpc
tie, top2, headwater

*material, name=water
*density
1000.0

*acoustic medium, bulk modulus
2.18e9

**step, perturbation
closed circuit -- nodal plate support simulation

*frequency
10, 1.e+03

**

*restart, write, f=0

**

*boundary
2835,  1,2, 0.0
a2,  9,9, 0.0
b8,  9,9, 0.0
100000,  9,9, 0.0
100001,  9,9, 0.0
100002,  9,9, 0.0
surface, 8,8, 0.0
**
*node print,f=0
*el print, f=0
*end step
**
*step,perturbation
open circuit -- nodal plate support simulation
*frequency
10,,1.e+03
**
*restart, write, f=0
**
*boundary
2835,  1,2, 0.0
surface, 8,8, 0.0
**
*node print,f=0
*el print, f=0
*end step
**
*step,perturbation
1V excitation
*steady state dynamic, direct
15.e3,  20.5e3, 500, 1,1
20.5e3,  21.5e3, 2000, 1,1
21.5e3,  25.e3, 500, 1,1
**
*restart, write,f=0
**
*boundary
2835,  1,2, 0.0
a2, 9,9, 0.0
b8, 9,9, 0.0
100000, 9,9, 1.0
100001, 9,9, 0.0
100002, 9,9, 1.0
surface, 8,8, 0.0
**
*node file, f=1, nset=ns_ref
RCHG, PHCHG, EPOT, PHPOT
**
*nset, nset=ns_ref2
1,2809,8199,10501
*node file, f=1, nset=ns_ref2
U,V,A,PU
**
*node print,f=0
*el print, f=0
**monitor, node=5, dof=2
*end step
**---------------------------------------------------------------
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APPENDIX E-6

The design drawings for the Horn transducer can be seen on the following pages.
Isoring for Horn Transducer
5mm cap screw

Section A-A

Vice grip for Horn Transducer

UNIVERSITY OF CAPE TOWN
DEPARTMENT OF MECHANICAL ENGINEERING

DRAWN BY: A Green
DRAWING NUMBER: Horn/8

DIMENSIONS IN MILLIMETERS (mm)
TOLERANCES UNLESS OTHERWISE STATED
0.05
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