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Chapter 1

Introduction

The Black-Scholes model is still the most widely used option-pricing model today, despite its restrictive assumptions and well-known imperfections, due to its inherent simplicity. Numerous attempts have been introduced to circumvent these restrictions and these have ranged from simple modifications to the Black-Scholes model to distinctly new models of the behaviour of security prices.

The Black-Scholes model assumptions are that the stock price follows a continuous path through time, the security's volatility is known and constant, the short-term interest rate is constant, there are no transaction costs for either securities or options, an investor's trades do not affect the taxes he pays, stocks pay no dividends, investors are not allowed to exercise options early, and there are no takeovers or other events that end the life of the option early. The three assumptions that I am going to discuss in this paper are that the stock price follows a continuous path through time, the instantaneous volatility of the stock rate of return is constant, and transaction costs are zero. I present models which relax these assumptions and consider other alternative models as well.

I shall compare how well the different models price options on the Standard and Poor's 500 index (S&P500) sold on 15 August 2001, with expiration lengths of the options ranging from 2 days to 857 days - 20 December 2003. If a market option price \( c \) does not satisfy the necessary lower boundary condition

\[ S_0 e^{-qr} - K e^{-rt} \leq c, \]

(where \( S_0 \) is the index value at close on 15 August 2001, \( q \) is the continuous dividend that \( S(0) \) provides, \( K \) is the strike value of the option, \( t \) is the lifetime of the option, and \( r \) the continuously compounded interest rate applicable over the life of the option), then any model's option price which is not the same as this market option price, will not be considered as incorrect.

In Chapter 2, I discuss the Alternatives to the Black-Scholes model; in Chapter 3, I discuss my method of obtaining my results; in Chapter 4, I discuss the results; and Chapter 5, is the Conclusion.
Chapter 2

Alternatives to the Black-Scholes Model

I present three sets of model which are alternatives to the Black-Scholes model, viz 1) models in which the security price changes are not continuous, 2) models in which the volatility is not a deterministic function, and 3) other models. Theoretical ingenuity has long since outrun definitive empirical knowledge. Most empirical work designed to test these alternatives, has suffered from a number of deficiencies, including failure to provide a separate test of the mathematical structure of option pricing formulae disjoint from problems of input measurement, severe limitations created by use of closing option and stock prices, and limited samples of calendar time or underlying stocks. Despite the theoretical advances, the state of our empirical knowledge is in considerable disarray. The reports of biases from Black-Scholes and other values both from traders and academic studies are conflicting. At present, no consensus exists. [18]

The models compared to the Black-Scholes model, are

1. Pure jump model (Cox and Ross)
2. Mixed diffusion-jump model (Merton)
3. Constant elasticity of variance diffusion model (Cox and Ross)
4. Stochastic volatility model (Hull and White)
5. Compound option model (Geske)
6. Displaced diffusion model (Rubinstein)
7. Non-zero transaction costs model (Leland)
8. GARCH(1,1) model (Hull)
2.1 Discontinuous security price changes

Now, the Black-Scholes solution is not valid, even in the continuous limit, when the stock price dynamics cannot be represented by a stochastic process with a continuous sample path. In essence, the validity of the Black-Scholes formula depends on whether or not the stock price changes satisfy a kind of local Markov property. The Black-Scholes model has many assumptions, but most of these can be relaxed. The one that needs special attention is that security price changes must be continuous.

2.1.1 Pure Jump Model

Since the development of the Black-Scholes formula, much of the research in option pricing theory has focused on the pricing of options written on stocks with jumps.[2] Jump models allow the security to change price continuously, but also allow for a low probability of a sudden large jump in price. The jump process follows a deterministic movement upon which are superimposed discrete jumps.

The Pure Jump model is similar to the Binomial model [2] and works as follows: during an infinitesimal time interval $dt$, the security can jump in price from $S$ to $Su$ with probability $\lambda P_J dt$ and fall to $Se^{-udt}$ with probability $1-\lambda P_J dt$, where $0 \leq \lambda \leq 1$ is a constant, $u$ is some positive real number, $u$ is the up tic and $d = e^{-udt}$ is the down tic. The general jump process formula is derived by analysing the asymptotic behaviour of the binomial formula using the Poisson Limit Theorem (stated below). (This limit theorem is one of the most elementary of the central limit theorems, and by applying it, full advantage can be taken of the simple structure of the binomial return process, in deriving the Pure Jump option pricing formula.)

2.1.1.1 The Binomial Formula

Assume that the following two assumptions are valid.

(A-1) For each $n \in N$, the underlying stock's return process is a stationary binomial process with tic sizes $u_n$ and $d_n$, and the subinterval riskless rates $i_n$ are stationary and nonstochastic such that $d_n < i_n < u_n$, where $u_n, i_n, d_n \in \mathbb{R}$.

(A-2) The underlying stock does not pay any dividends during $[0,T]$.

Then, under assumptions (A-1) and (A-2), the price at time 0 of a discrete-time binomial call option is given by [4]

\[
C_n(0) = S(0) \sum_{j=a^*(n)}^{n} b(j|p_n) - K(i_n)^{-n} \sum_{j=a^*(n)}^{n} b(j|p_n), \quad (2.1)
\]
where
\[ c_n(0) = t = 0 \text{ price of a European call option}, \]
\[ S(0) = \text{ current asset price}, \]
\[ a(n) = \frac{\ln(K/S(0)) - n\ln d_n}{\ln w_n - \ln d_n}, \]
\[ a^*(n) = \text{ smallest integer greater than } a(n), \]
\[ K = \text{ the strike price}, \]
\[ i_n = \text{ interest rate at time } t_n, \]
\[ u_n = \text{ up-tic at time } t_n, \]
\[ d_n = \text{ down-tic at time } t_n, \]
\[ b(j|p_n) = \left[ \frac{n!(n-j)!}{j!} \right] [p'_n]^j [1 - p'_n]^{n-j}, \]
\[ = \text{ the probability of exactly } j \text{ up-tics in the stock's return in } n \text{ tics,} \]
\[ \text{given that the probability of an up-tic on any one tic is } p'_n, \]
\[ p_n = (i_n - d_n)/(u_n - d_n), \]
\[ p'_n = [p_n u_n/i_n]. \]

2.1.1.2 The Limit Result

In this section, the conditions are identified under which the binomial option pricing formula approximates the continuous-time jump process formula. The basic idea is to subdivide the interval \([0, T]\) into smaller and smaller subintervals of the form \([t - 1/2]T/n, tT/n]\) by increasing \(n\), and then to examine the limiting behaviour of the discrete-time formula (2.1). In particular, the Poisson limit theorem \([7]\) is used to identify conditions that guarantee the convergence of (2.1). Here is the Poisson limit theorem \([1]\):

**Theorem 1 (Poisson 1832)** If \( \lim_{n \to \infty} n p_n = \lambda_{p_f} \), where \( \lambda_{p_f} \in [0, +\infty) \), then for each \( j = 1, 2, \ldots \)
\[ \lim_{n \to \infty} b(j|p_n) = \frac{\lambda_{p_f}^j e^{-\lambda_{p_f}}}{j!}. \]

**Theorem 2** Suppose (A-1) to (A-6) hold, where
(A-3) \( \lim_{n \to \infty} u_n = u, \lim_{n \to \infty} d_n = d, \lim_{n \to \infty} i_n = i \), where \( u, d, \) and \( i \) are positive real numbers,
(A-4) \( \lim_{n \to \infty} n p_n = \lambda_{p_f} \), where \( \lambda_{p_f} \in [0, \infty) \), and where \( p_n = (i_n - d_n)/(u_n - d_n) \),
(A-5) \( \lim_{n \to \infty} n \ln d_n = l \), where \( l \in (-\infty, \infty) \), and
(A-6) \( \lim_{n \to \infty} [i_n]^n = r \in (0, \infty) \).
Then the continuous-time jump process formula is

$$\lim_{n \to \infty} c_n(0) = S(0) \sum_{j=0}^{\infty} \frac{[u \lambda_{P,j}]i^j e^{-(u \lambda_{P,j})i}}{j!} - K - 1 \sum_{j=a^*}^{\infty} \frac{\lambda_{P,j} e^{-\lambda_{P,j}}}{j!}, \quad (2.2)$$

where

$$a = \lim_{n \to \infty} a(n) = \lim_{n \to \infty} \frac{\ln[K/S(0)] - n \ln d_n}{\ln u_n - \ln d_n}$$

$$a^* = \text{the smallest integer greater than } a.$$

**Proof** Suppose that $a$ is not an integer. Firstly, since $\lim_{n \to \infty} a(n) = a$ and $a$ is not an integer, there exists an $n^*$ such that $a^* > a(n) > a^* - 1$ for all $n > n^*$. Thus, for all $n > n^*$, $a^*$ is the smallest integer greater than $a(n)$, so that for all $n > n^*$, $a^* = a^*(n)$. Secondly, for all $n$,

$$a^*-1 \sum_{j=0}^{a^*-1} b(j)p'_n + \sum_{j=a}^{n} b(j)p'_n$$

$$= \sum_{j=0}^{a^*-1} \frac{[u \lambda_{P,j}]i^j e^{-(u \lambda_{P,j})i}}{j!} + \sum_{j=a}^{\infty} \frac{[u \lambda_{P,j}]i^j e^{-(u \lambda_{P,j})i}}{j!}, \quad (2.3)$$

since the left-hand side of equation (2.3) is simply the sum of the binomial probabilities over all possible $j$ (which is equal to 1), and the right-hand side is simply the sum of the Poisson probabilities over all possible $j$ (which is also equal to 1). Rearranging equation (2.3) gives, for all $n$,

$$\sum_{j=a^*}^{n} b(j)p'_n - \sum_{j=a^*}^{\infty} \frac{[u \lambda_{P,j}]i^j e^{-(u \lambda_{P,j})i}}{j!} = \sum_{j=0}^{a^*-1} \left[ \frac{[u \lambda_{P,j}]i^j e^{-(u \lambda_{P,j})i}}{j!} - b(j)p'_n \right]. \quad (2.4)$$

Given (A-3) and (A-4), it follows from the Poisson Limit Theorem that the right-hand side of equation (2.4) converges to zero as $n$ tends to infinity. Thus,

$$\lim_{n \to \infty} \sum_{j=a^*}^{n} b(j)p'_n = \sum_{j=a^*}^{\infty} \frac{[u \lambda_{P,j}]i^j e^{-(u \lambda_{P,j})i}}{j!}.$$

Also, from the Poisson Limit Theorem, it follows that

$$\lim_{n \to \infty} \sum_{j=a^*}^{n} b(j)p_n = \sum_{j=a^*}^{\infty} \frac{\lambda_{P,j} e^{-\lambda_{P,j}}}{j!}.$$
(Note that $\sum_{j=a^*_n}^{\infty} \frac{\lambda_j^{-1} e^{-\lambda_j z_j}}{j!}$ is defined, since
\[
\left| \frac{T_{j+1}}{T_j} \right| = \left| \frac{\lambda_j}{j+1} \right| = \frac{\lambda}{j+1} < 1 \text{ for large } j,
\]
where $T_j := \frac{\lambda_j e^{-\lambda_j}}{j!}$.

Now, if we take limits in equation (2.1), then we obtain that
\[
\lim_{n \to \infty} c_n(0) = S(0) \lim_{n \to \infty} \sum_{j=a^*(n)}^{\infty} b(j|p_n) - K \lim_{n \to \infty} (i_n)^{-n} \lim_{n \to \infty} \sum_{j=a^*(n)}^{\infty} b(j|p_n),
\]
\[
(2.5)
\]
since $\lim_{n \to \infty} (i_n)^{-n} = r^{-1}$ exists (by assumption), and we've shown above that $\lim_{n \to \infty} \sum_{j=a^*(n)}^{\infty} b(j|p_n)$ also exists. Thus, equation (2.5) becomes
\[
\lim_{n \to \infty} c_n(0) = S(0) \sum_{j=a^*}^{\infty} \left[ \frac{(u \lambda_{p_j})^j}{j!} e^{-(u \lambda_{p_j})j} \right] - K r^{-1} \sum_{j=a^*}^{\infty} \frac{\lambda_{p_j} e^{-\lambda_{p_j}}}{j!},
\]
since $a^*(n) = a^*$ for all $n \geq n^*$.

Next, suppose that $a$ is an integer. The problem here is that, if the sequence $(a(n))_{n \in N}$ oscillates about $a$, then $a^*(n) = a$ for infinitely many $n$ (i.e., for all $n$ where $a(n) < a$), and $a^*(n) = a^* = a + 1$ for infinitely many $n$ (i.e., for all $n$ where $a + 1 > a(n) \geq a$). So, equation (2.1) becomes
\[
c_n(0) = \text{common} + \left\{ \begin{array}{ll}
0 & \text{for } a(n) \geq a \\
S(0)b(a^* - 1)p_n - K(i_n)^{-n}b(a^* - 1)p_n & \text{for } a(n) < a
\end{array} \right.,
\]
where $\text{common} := S(0) \sum_{j=a^*}^{\infty} b(j|p_n) - K(i_n)^{-n} \sum_{j=a^*}^{\infty} b(j|p_n)$. So,
\[
\lim_{n \to \infty} c_n(0) = \text{formula (2.2)} + \lim_{n \to \infty} \left\{ \begin{array}{ll}
0 & \text{for } a(n) \geq a \\
S(0)b(a|p_n) - K(i_n)^{-n}b(a|p_n) & \text{for } a(n) < a
\end{array} \right.,
\]
since $a + 1 = a^*$. So, we need to show that
\[
\lim_{n \to \infty} [S(0)b(a|p_n) - K(i_n)^{-n}b(a|p_n)] = 0.
\]

Now, note that
\[
b(j|p_n) = \left[ \frac{n!(n-j)!}{j!} \right] \left[ \frac{p_n u_n}{i_n} \right]^j \left[ \frac{1 - p_n u_n}{i_n} \right]^{n-j}
\]
\[
= b(j|p_n) \left( \frac{u_n}{i_n} \right)^j \left[ \frac{1 - e^{u_n}}{[1 - p_n]^{n-j}} \right]^{n-j}
\]
\[ b(j|p_n) \left( \frac{u_n}{i_n} \right)^j \left[ 1 - \frac{u_n - d_n}{u_n + d_n} \right]^{n-j} \left[ 1 - \frac{i_n - d_n}{u_n + d_n} \right]^{n-j} \]
\[ = b(j|p_n)\left(\frac{d_n}{i_n}\right)^{n-j} \]

So, now we need to show that

\[ \lim_{n \to \infty} b(a|p_n) \left( \frac{d_n}{i_n} \right)^{n-a} S(0) - \frac{a(n)}{d_n} \frac{d_n^{n-a(n)} S(0)}{a(n)} S(0) = 0, \]

since, for all \( n \),

\[ a(n) = \frac{\ln[K/S(0)] - n \ln d_n}{\ln u_n - \ln d_n} \Leftrightarrow \frac{u_n^{a(n)} d_n^{n-a(n)} S(0)}{a(n)} = K. \]

Now, \( \lim_{n \to \infty} [u_n^{a(n)} d_n^{n-a(n)} S(0) - \frac{a(n)}{d_n} \frac{d_n^{n-a(n)} S(0)}{a(n)} S(0)] = 0 \), since \( \lim_{n \to \infty} a(n) = a \).

Thus,

\[ \lim_{n \to \infty} \left[ S(0)b(a|p_n) - K(i_n)^{-n} b(a|p_n) \right] = 0, \]

since \( \lim_{n \to \infty} b(a|p_n) \) and \( \lim_{n \to \infty} (i_n)^{-n} \) both exist. Thus, the theorem is proved.

Here are two special cases \([1]\) of formula (2.2).

1) Let \( u_n = u \) and \( d_n = 1 \) for all \( n \), and let \( i_n = e^{rT/n} \), where \( u > \exp[rT/n] > 1 \).

Moreover, suppose that the binomial return process is given by

\[ r_{nt} = \begin{cases} u & \text{with probability } q \\ 1 & \text{with probability } (1 - q) \end{cases}, \]

where \( q \) is fixed for all \( n \). Then from Theorem 2

\[ e_1(0) = \lim_{n \to \infty} \left[ S(0) \sum_{j=a^*(n)}^{n} b(j|p_n) - K(i_n)^{-n} \sum_{j=a^*(n)}^{n} b(j|p_n) \right] \]
\[ = S(0) \sum_{j=a^*}^{\infty} [urT/(u - 1)]^j \exp[-urT/(u - 1)] \]
\[ - Ke^{-rT} \sum_{j=a^*}^{\infty} [rT/(u - 1)]^j \exp[-rT/(u - 1)], \]

where \( a = \ln \left( \frac{K}{S(0)} \right) \ln u. \)
2) Suppose example (1) is modified so that a limit result can be obtained under risk neutrality. In particular, let \( u_n = u \) and \( d_n = 1 \) for all \( n \), and more importantly, let the binomial return process be given by

\[
 r_{nt} = \begin{cases} 
 u & \text{with probability } q_n = \frac{\lambda_p T}{n} \\
 1 & \text{with probability } (1 - q_n) 
\end{cases}
\]

so that the probability of an up-tick decreases as the number of subintervals increases. Then from Theorem 2

\[
 c_2(0) = \lim_{n \to \infty} \left[ S(0) \sum_{j=a^*}^{n} b(j)p_n) - K(i_n)^{-n} \sum_{j=a^*}^{n} b(j)p_n) \right]
\]

\[
 = S(0) \sum_{j=a^*}^{\infty} [u \lambda_p T] j \exp(-u \lambda_p T) \cdot \frac{1}{j!} 
\]

\[
 - Ke^{-\lambda_p T (u-1)} \sum_{j=a^*}^{\infty} [\lambda_p T]^j \exp(-\lambda_p T) \cdot \frac{1}{j!}
\]

where \( a = \ln \left( \frac{K}{S(0)} \right) \ln u \). (We shall use this option price expression in the calculation of the model call option values for the Pure Jump model.)

2.1.2 The Mixed Jump-Diffusion model

2.1.2.1 The stock price and option price dynamics

In this model, the total change in stock price is posited to be comprised of:

i) normal random movements in price, for example, due to a temporary imbalance between supply and demand, or new information that causes marginal changes in the value of the security; this component can be modelled by a standard Brownian motion with constant variance; ii) the abnormal random movements in price due to the arrival of important new information. These would have to occur at discrete times or else one would not be able to distinguish them from component (i); also they will occur randomly.

Just as once the natural prototype process for the continuous component of the stock price change is a Wiener process, so the prototype for the jump component is a Poisson process.

**Definition 1** A random variable \( X \) has a Poisson distribution \([8]\) if its possible values can only be nonnegative integers \( (n \in N) \) and

\[
 P(\text{Probability } (X = n) = \frac{(\lambda_{MJD})^n e^{-\lambda_{MJD}}}{n!}
\]

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where $\lambda_{MJD} \in (0, \infty]$ satisfies the identity
\[ \sum_{n=0}^{\infty} n \times \text{Probability } (X = n) = \lambda_{MJD}, \]
(which shows that $\lambda_{MJD}$ is the mean of the Poisson process.)

**Definition 2** Suppose we have the triple $(\Omega, F, P)$, where $\Omega$ is the set of 'elementary outcomes', $F$ is a σ-algebra of a subsets of $\Omega$, and $P$ is a probability measure on $F$. A Poisson process is random, countable subset $\Pi$ of $R$ such that

1) for any disjoint, measurable sets in $F$, $A_1, A_2, ..., A_n$, the random variables $N(A_1), N(A_2), ..., N(A_n)$ are independent (where $N(A_i) : \Omega \to \{0, 1, 2, ..., \infty\}$ is the counting function), and

2) $N(A)$ has a Poisson distribution with mean $\lambda_{MJD} \in (0, \infty]$. We can extend this definition of Poisson processes [8] to include time as follows:

\[ P(n \text{ elementary events occurring in a time interval } [0, t]) = \frac{(\lambda_{MJD} t)^n e^{-\lambda_{MJD} t}}{n!} \]

So, in a small time interval of length $dt$,

\[ P(n \text{ elementary events occurring in a time interval } dt) = \frac{(\lambda_{MJD} dt)^n e^{-\lambda_{MJD} dt}}{n!} \]

Let $P_n(t)$ denote the probability that $n$ elementary events will occur in the time interval $[0, t]$. Then we have that

\[ P_0(dt) = e^{-\lambda_{MJD} dt}, \text{ and } P_1(dt) = (\lambda_{MJD} dt) e^{-\lambda_{MJD} dt}. \]

So,

\[ P_{>1}(dt) = 1 - P[0 \text{ or } 1 \text{ events in } dt] = 1 - P_0(dt) - P_1(dt) \]

\[ = 1 - [1 - (\lambda_{MJD} dt) + \frac{(\lambda_{MJD} dt)^2}{2!} - O((dt)^3)] - (\lambda_{MJD} dt)[1 - (\lambda_{MJD} dt) + \frac{(\lambda_{MJD} dt)^2}{2!} - O((dt)^3)] \]

\[ = \frac{(\lambda_{MJD} dt)^2}{2!} - O((dt)^3) \]

\[ = 0 \quad [\text{up to } O(dt)]. \]
\[
P_{\geq 1}(dt) = P_1(dt) = 1 - P_0(dt) = \lambda_{MJD} dt + O((dt)^2), \quad \text{and} \\
P_0(dt) = 1 - P_1(dt) = 1 - \lambda_{MJD} dt + O((dt)^2).
\]

Let \( q(t) \) be a Poisson process with mean \( \lambda_{MJD} \), and let an elementary event of \( q \) be a jump of size \( k \in \mathbb{R} \). Then, from above, we have that
\[
dq = \begin{cases} 
0 & \text{with probability } 1 - \lambda_{MJD} dt \\
k & \text{with probability } \lambda_{MJD} dt
\end{cases}, \quad (2.6)
\]
where \( dq \) denotes the infinitesimal change in \( q \) over \( dt \).

(Back to the Mixed Jump-Diffusion Model.) Given that the Poisson event occurs (i.e. some important information on the stock arrives), there is a 'drawing' from a distribution to determine the impact of this information on the stock price. In other words, if \( S(t) \) is the stock price at time \( t \) and \( Y \) is the random variable description of this drawing, then, neglecting the continuous part, \( S(t + dt) \) will be the random variable \( S(t + dt) = S(t)Y \), given that one such arrival occurs between \( t \) and \( t + dt \). The occurrence of such an event (such as a jump event) will increase the security price by a proportional amount \( k \). This implies that the jumps lead to an average growth rate of \( k\lambda_{MJD} \), and thus that the drift rate due to the geometric Brownian motion must be \( \alpha - k\lambda_{MJD} \), where \( \alpha \) is the instantaneous expected return on the security. Thus we have the following model
\[
\frac{dS}{S} = (\alpha - k\lambda_{MJD}) dt + \sigma dz + dq, \quad (2.7)
\]
where \( \alpha \) is the instantaneous expected return on the stock, \( \sigma^2 \) is the instantaneous variance rate of the return, \( dz \) is a standard Gauss-Wiener process, \( q(t) \) is an independent Poisson process with mean \( \lambda_{MJD} \), \( k := E[Y - 1] \) (where \( Y - 1 \) is the random variable percentage change in the stock price, neglecting the continuous part, if the Poisson event occurs\(^1\)), \( E \) is the expectation operator over the random variable \( Y \), and \( dz \) and \( dq \) are independent processes. The derivation of an equation for the price of a derivative contingent upon a security \( S \) described by such a process is complicated by several factors. First of all, in addition to Ito's lemma, one must apply a corresponding (and considerably more complicated) lemma pertaining to Poisson processes.

Let \( F(S, t) \) be the time \( t \) price of some derivative with underlying \( S \), and assume that \( F \) is twice continuously differentiable in \( S \) and \( t \). Then, by Ito's formula, we have that

\(^1\)Now, \( \Delta S = SY - S = S(Y - 1) \), so, \( \frac{\Delta S}{S} = Y - 1. \)
\[ dF = F_t dt + F_S [(α - kλ_{MJD}) S dt + σ S dz + S dq] + \frac{1}{2} S^2 F_{SS} [σ^2 dt + 2(α - kλ_{MJD}) + 2σ dq dz + (dq)^2]. \]  

(2.8)

From equation (2.6),

\[ E[dq] = 0(1 - λ_{MJD} dt) + k(λ_{MJD} dt) = λ_{MJD} dt. \]

Thus, \( dq \) is \( O(dt) \) and so \((dq)^2\) is \( O((dt)^2)\). So, equation (2.8) now becomes,

\[ dF = [F_t + (α - kλ_{MJD}) SF_S + \frac{1}{2} σ^2 S^2 F_{SS}] dt + SF_S dq + σ SF_S dz. \]  

(2.9)

If we take expectations in (2.9), then we obtain that

\[ E[dF] = [F_t + (α - kλ_{MJD}) SF_S + \frac{1}{2} σ^2 S^2 F_{SS}] dt + E[SF_S dq]. \]  

(2.10)

Now, \( E[SF_S dq] = E[dF \frac{dS}{dS}] \) for very small \( dS \). From equation (2.7), if we neglect the \( dt \) and \( dz \) terms, then \( dS = Sdq \). And so

\[ E[SF_S dq] = E[F(S(t + dt)) - F(S(t))] \]

\[ = \frac{1}{2} \sigma^2 S^2 \frac{dS}{dS} [(S(t + dt)) - F(S(t))] \]

\[ = (1 - λ_{MJD} dt) E[F(S) - F(S)] + (λ_{MJD} dt) EY[F(SY) - F(S)]. \]

Thus, equation (2.10) now becomes

\[ E[dF] = [F_t + (α - kλ_{MJD}) SF_S + \frac{1}{2} σ^2 S^2 F_{SS}] dt + (λ_{MJD} dt) EY[F(SY) - F(S)]. \]

Now, the expected change in \( F \) has no uncertainty in it (i.e. no \( dz \) term), thus, we expect \( F \) to grow at the risk-free rate \( r \) over \( dt \). So,

\[ [F_t + (α - kλ_{MJD}) SF_S + \frac{1}{2} σ^2 S^2 F_{SS}] dt + λ_{MJD} EY[F(SY) - F(S)] dt = r F dt, \]

which gives us the partial differential equation

\[ 0 = \frac{1}{2} σ^2 S^2 F_{SS} + (α - kλ_{MJD}) SF_S + F_t - r F + λ_{MJD} EY\{F(SY, t) - F(S, t)\}, \]

(2.11)

subject to the boundary conditions \( F(0, t) = 0 \) and \( F(S, 0) = \max(S - K, 0) \).

While a complete closed-form solution to (2.11) cannot be written down without a further specification of the distribution for \( Y \), a partial solution which is in a reasonable form for computation can be.

Define \( W(S, r, K, r, σ^2) \) to be the Black-Scholes option pricing formula for the no-jump case. Define the random variable \( X_n \) to have the same distribution
as the product of \( n \) independently and identically distributed random variables, each identically distributed to the random variable \( Y \), where it is understood that \( X_0 = 1 \). Define \( E_n \) to be the expectation operator over the distribution of \( X_n \). Then the solution to equation (2.11) for the option price when the current stock price is \( S \) can be written as [2]

\[
F(S, \tau) = \sum_{n=0}^{\infty} e^{-\tau \lambda_{MJD} (\tau \lambda_{MJD})^n} E_n[W(SX_n e^{-k \tau \lambda_{MJD}}, \tau, K, \sigma^2, r)].
\]  

(2.12)

There are two special cases where (2.12) can be vastly simplified. The one we consider, is that which is described by Samuelson [12] where there is a positive probability of immediate ruin (i.e. if the Poisson event occurs, then the stock price goes to zero). In our notation, this case corresponds to \( Y = 0 \) with probability one. Clearly, then \( X_n = 0 \) for \( n \neq 0 \), and \( k = -1 \). So, in this case, equation (2.12) can be written as

\[
F(S, \tau) = W(S, \tau; K, \sigma^2, r + \lambda_{MJD}).
\]  

(2.13)

(We shall use this option price expression in the calculation of the model call option values for the Mixed Jump-diffusion model.) Formula (2.13) is identical to the standard Black-Scholes solution, but with a larger 'interest rate' \( r' = r + \lambda_{MJD} \), substituted in the formula.

### 2.2 Non-constant volatility

#### 2.2.1 Constant Elasticity of Variance Diffusion

Both casual empiricism and economic rationale tend to support the inverse relationship between a stock price and its volatility. If a firm's stock price falls, then the market value of its equity tends to fall more rapidly than the market value of its debt, causing the debt-equity ratio to rise, hence the riskiness of the stock increases. A similar effect could be observed even if the firm has almost no debt. Since every firm faces fixed costs which have to be met irrespective of its income, a decrease in income will decrease the value of the firm and at the same time increase its riskiness.

The constant elasticity of variance diffusion class of stock price distributions establishes a theoretical framework within which this inverse relationship can be empirically tested. The instantaneous standard deviation of the percentage price change for this class is given as \( \sigma_S t^{(\alpha-2)/2} \) where \( 0 \leq \alpha \leq 2 \). Since \( \alpha = 2 \) implies the diffusion process is lognormal, the Black-Scholes model is one special case of these constant elasticity of variance models. More formally, the model for the stock price is

\[
dS = \mu S dt + \sigma S \theta dB_t.
\]
The standard deviation of the return distribution fluctuates inversely with the level of the stock price.

Cox [3] derived an option pricing formula which holds if the stock price follows a constant elasticity of variance diffusion. Assuming risk-neutrality, the value of an option is merely the discounted expected value of the option payoff. The solution to the option pricing problem then depends on finding the distribution of the stock price at expiration. Cox found that, if the stock price follows the constant elasticity of variance diffusion, then the continuous part of the density of \( S_T \), conditional on \( S_t(t < T) \), is

\[
f(S_T, T; S_t, t) = (2 - \alpha) \sqrt{k^{1-2\alpha} (xy^{1-2\alpha})^{1/2-\alpha}} e^{-rT} f_1/2-\alpha(2\sqrt{xy}), \quad (2.14)
\]

where

- \( \tau = T - t \),
- \( k = \frac{2\mu}{\sigma^2(2 - \alpha)(e^{\mu(2-\alpha)r} - 1)} \),
- \( x = kS_t^{2-\alpha} e^{\mu(2-\alpha)r} \),
- \( y = kS_T^{2-\alpha} \), and
- \( f_1 = \text{modified Bessel function of the first kind of order } q \).

The probability that \( S_T = 0 \) is given by \( G \left( \frac{\alpha}{2-\alpha}, r \right) \), where \( G(m, n) \) is the complimentary gamma distribution and \( r \) is the risk-free rate. Given that the probability distribution of \( S_T \) is known, Cox obtains the following option price:

\[
c(S, \tau) = S_t \sum_{n=0}^{\infty} g(n+1, x) G(n + 1 + \frac{1}{2 - \alpha}, kK^{2-\alpha}) - K e^{-r\tau} \sum_{n=0}^{\infty} g(n + 1 + 1, x) G(n + 1, kK^{2-\alpha}),
\]

where

\[
g(m, n) = \frac{e^{-nm}n^{m-1}}{\Gamma(m)}
\]

is the gamma density function.

- \( k = \frac{2r}{\sigma^2(2 - \alpha)(e^{(2-\alpha)r} - 1)} \),
- \( x = kS_t^{2-\alpha} e^{(2-\alpha)r} \), and
- \( K \) = strike price.

For the \( \alpha = 1 \) case, Cox and Ross (1976) obtained the following option price formula:

\[
c(S, \tau) = (S - Ke^{-r\tau}) N(y_1) + (S + Ke^{-r\tau}) N(y_2) + v(n(y_1) - n(y_2)),
\]
where

\[ N(\cdot) = \text{cumulative unit normal distribution function}, \]
\[ n(y) = \frac{1}{\sqrt{2\pi}} e^{-y^2/2}, \]
\[ v = \sigma \left( \frac{1 - e^{-2\tau}}{2\tau} \right)^{1/2}, \]
\[ y_1 = \frac{S - K e^{-\tau}}{v}, \text{ and} \]
\[ y_2 = \frac{-S - K e^{-\tau}}{v}. \]

Similarly, Cox (1975) developed an approximation formula for the value of the option for the absolute root model (\(\alpha = 0\)). Let

\[ y = \frac{4rS}{\sigma^2(1 - e^{-\tau})} \text{ and } z = \frac{4rK}{\sigma^2(e^{-\tau} - 1)}. \]

Let \( w \) be a parameter which takes on the values 0 or 4. Let

\[ h(w) = 1 - \frac{2}{3}(w + y)(w + 3y)(w + 2y)^{-2}, \]
\[ q_1 = \frac{w + 2y}{(w + y)^2}, \]
\[ q_2 = (h - 1)(1 - 3h), \text{ and} \]
\[ q(w) = \frac{1 + h(h - 1)q_1 - \frac{h(2 - h)q_2^2}{2} - \left( \frac{-z}{w+y} \right)^h}{q_1 \{2h^2(1 + q_2)\}^{1/2}}. \]

Then we have the option price

\[ e(S, \tau) = SN(q(4)) - Ke^{-\tau} N(q(0)), \]

where \( N(\cdot) \) is the cumulative unit normal distribution function. (We shall use this option price expression in the calculation of model call option values for the constant elasticity of variance diffusion model.)

### 2.2.2 GARCH(1,1)

Another way of coping with the constant volatility assumption of the Black-Scholes model is not to correct it at all. Instead, if one could accurately predict the future volatility over the remaining lifetime of a derivative, then the Black-Scholes model could be employed to compute its price. Such predictions are usually carried out by modelling volatility as an autoregressive conditional heteroskedastic (ARCH) process first proposed by Engle. [5] These processes restrict
the volatility to be constant while allowing the conditional volatility (conditional upon past data) to vary as a function of prior prediction errors. There are a plethora of variations on the original ARCH process and several are used in options pricing.

2.2.2.1 Estimating volatility from Historical Data

A variable has a lognormal distribution if the natural logarithm of the variable is normally distributed. For a security $S$,

$$\ln \frac{S_T}{S_0} \sim \phi \left[ \left( \mu - \frac{\sigma^2}{2} \right) T, \sigma \sqrt{T} \right],$$

(2.15)

where $S_T$, $S_0$, $\mu$ and $\sigma$ are the security prices at times $T$ and 0, the expected rate of return, and the volatility, respectively, of the security over the period [0,T].

To estimate the volatility of a security price empirically, the security price is usually observed at fixed intervals of time (e.g. every trading day, week, or month).

Define

- $n + 1$ as Number of observations
- $S_i$ as Security price at end of $i$th interval ($i = 0, 1, ..., n$)
- $\tau$ as Length of time interval in years,

and let

$$u_i = \ln \left( \frac{S_i}{S_{i-1}} \right)$$

for $i = 1, 2, ..., n$. Because $S_i = S_{i-1}e^{u_i}$, $u_i$ is the continuously compounded return in the $i$th interval. ($i$ is in trading days, so no concessions are made for weekends, for example.)

Choosing an appropriate value for $n$ is not easy. Generally, more data leads to more accuracy, however, $\sigma$ does change over time and data that are too old may not be relevant for predicting the future. An often used rule of thumb is to set the time period over which the volatility is to be measured equal to the time period over which it is to be applied.

(An important issue is whether time should be measured in calender days (365) or trading days (252) when volatility parameters are to be estimated. Empirical research indicates that days in which the exchange is closed, should be ignored for the purposes of volatility calculation. Also, as tax factors play a part in determining returns around an ex-dividend date, it is probably best to discard altogether data for intervals that include an ex-dividend date.)
2.2.2.2 Estimating volatility

Define $\sigma_n$ as the volatility of a market variable on day $n$, as estimated at the end of day $n - 1$. Recall that the standard approach to estimating $\sigma_n$ from historical data is:

Suppose that the value of a market variable at the end of day $i$ is $S_i$ ($i = 1, \ldots, n$). Then the continuously compounded return during day $i$ (between the end of day $i - 1$ and the end of day $i$) is

$$u_i = \ln \left( \frac{S_i}{S_{i-1}} \right),$$

where $u_n$ is the latest observation. An unbiased estimate of the variance rate per day $\sigma_n^2$ using the most recent $m$ observations on the $u_i$ is

$$\sigma_n^2 = \frac{262}{m-1} \sum_{i=1}^{m} (u_{n-i} - \bar{u})^2,$$

where $\bar{u}$ is the mean of the $u_i$'s:

$$\bar{u} = \frac{1}{m-1} \sum_{i=1}^{m} u_{n-i}.$$

2.2.2.3 ARCH Processes

Given that the objective is to monitor the current level of volatility, it is appropriate to give more weight to recent data. A model that does this is

$$\sigma_n^2 = \sum_{i=1}^{m} \alpha_i u_{n-i}^2,$$  \hfill (2.17)

The variable $\alpha_i$ ($> 0$) is the amount of weight given to observation $n - i$. We wish to give less weight to older observations, so, $\alpha_i < \alpha_j$ when $i > j$. The weights must of course sum to unity.

An extension of equation (2.17) is to assume that there is a long term average volatility $\gamma V$ and that this should be given some weight:

$$\sigma_n^2 = \gamma V + \sum_{i=1}^{m} \alpha_i u_{n-i}^2,$$  \hfill (2.18)

where $\gamma$ is the weight assigned to $V$. The weights must sum to unity, so,

$$\gamma + \sum_{i=1}^{m} \alpha_i = 1.$$

This is known as an ARCH($m$) model.
2.2.2.4 Exponentially Weighted Moving Average Model (EWMA)

The EWMA model is a particular case of the model in equation (2.17) where the weights $\alpha_t$ decrease exponentially as we move back through time (i.e. $\alpha_{t+1} = \lambda \alpha_t$, where $0 < \lambda < 1$). Equation (2.17) then becomes

$$\sigma^2_n = \lambda \sigma^2_{n-1} + (1 - \lambda)u^2_{n-1}.$$  \hspace{1cm} (2.19)

If we substitute for $\sigma^2_{n-1}$ and then for $\sigma^2_{n-2}$ and so on, then we obtain

$$\sigma^2_n = (1 - \lambda) \sum_{j=1}^{m} \lambda^{j-1} u^2_{n-j} + \lambda^m \sigma^2_0.$$  \hspace{1cm} (2.20)

For large $m$, $\lambda^m \sigma^2_0$ is sufficiently small so that equation (2.20) is the same as equation (2.17) with $\alpha_t = (1 - \lambda)\lambda^{t-1}$. Here we see that the weights decline at a rate $\lambda$ as we move back in time.

The EWMA approach has the advantage that relatively little data need to be stored. It is designed to track changes in the volatility, for suppose that there is a big move in the market variable on day $n - 1$ so that $u^2_{n-1}$ is large. From equation (2.20) this causes $\sigma_n$, our estimate of daily volatility for day $n$, to move upward. Now, the value of $\lambda$ governs how responsive the estimate of daily volatility is to the most recent observations on the $u^2_t$'s. A low value of $\lambda$ leads to a great deal of weight being given to $u^2_{n-1}$ when $\sigma_n$ is calculated. In this case, the estimates produced for the volatility on successive days are themselves highly volatile. A high value of $\lambda$ (i.e. close to 1) produces estimates of daily volatility that respond relatively slowly to new information provided by the $u^2_t$.

2.2.2.5 The Generalised Autoregressive Conditional Heteroskedastic (GARCH) Model

GARCH(1,1) differs from EWMA in that $\sigma^2_t$ is calculated from a long-run average variance rate $\nu$, as well as using $\sigma_{t-1}$ and $u^2_{n-1}$. The equation for GARCH(1,1) is

$$\sigma^2_n = \kappa + \beta \sigma^2_{n-1} + \alpha u^2_{n-1},$$  \hspace{1cm} (2.21)

where $\kappa := \gamma \nu$ and $\gamma, \beta, \alpha$ are constant weights which sum to unity. (When $\kappa$ is zero, then the GARCH(1,1) model reduces to the EWMA model.)

The "(1,1)" indicates that $\sigma_n$ is based on the most recent observation of $u^2$ and the most recent estimate of the variance rate. The more general GARCH(p,q) model calculates $\sigma^2_t$ from the most recent $p$ observations on $u^2$ and the most recent $q$ estimates of the variance rate.

20
2.2.3 Stochastic Volatility

Consider a derivative asset \( f \) with a price that depends upon some security price \( S \) and its instantaneous variance \( V(= \sigma^2) \) which are assumed to obey the following stochastic processes:

\[
dS = \phi S \, dt + \sigma S \, dw \\
dV = \mu V \, dt + \xi V \, dz.
\]

(2.22)

(2.23)

The variable \( \phi \) is a parameter that may depend on \( S, \sigma \) and \( t \). The variables \( \mu \) and \( \xi \) depend on \( \sigma \) and \( t \), but it is assumed that they do not depend on \( S \). The Wiener processes \( dz \) and \( dw \) have correlation \( \rho \). The actual process that a stochastic variance follows is probably fairly complex. It cannot take on negative values, so, the instantaneous standard deviation must approach zero as \( \sigma^2 \) approaches zero.

There is no asset that is clearly instantaneously correlated with the state variable \( \sigma^2 \), thus it does not seem possible to form a hedge portfolio that eliminates all the risk. However, as was shown by Garman [6], a derivative \( f \) with a price that depends on state variables \( \theta \) must satisfy the differential equation

\[
f_t + \frac{1}{2} \sum_{i,j} \rho_{ij} \sigma_i \sigma_j \theta_i \theta_j \frac{\partial^2 f}{\partial \theta_i \partial \theta_j} - rf = \sum_i \theta_i \frac{\partial f}{\partial \theta_i} [-\mu_i + \beta_i (\mu^* - r)],
\]

(2.24)

where \( \sigma_i \) is the instantaneous standard deviation of \( \theta_i \), \( \rho_{ij} \) is the instantaneous correlation between \( \theta_i \) and \( \theta_j \), \( \mu_i \) is the drift rate of \( \theta_i \), \( \beta_i \) is the vector of multiple-regression betas for the regression of the state-variable "returns" \((d\theta/\theta)\) on the market portfolio and the portfolios most closely correlated with the state variables, \( \mu^* \) is the vector of instantaneous expected returns on the market portfolio and the portfolios most closely correlated with the state variables, and \( r \) is the vector with elements that are the risk-free rate \( r \). When variable \( i \) is traded, it satisfies the \((N + 1)\)-factor CAPM, and the \( i \)th element of the right-hand side of (2.24) is \(-r\theta_i \frac{\theta_i}{\theta_i} \).

In the problem under consideration, there are two state variables, \( S \) and \( V \), of which \( S \) is traded. The differential equation (2.24) thus becomes

\[
f_t + \frac{1}{2} \left[ \sigma^2 S^2 f_{SS} + 2 \rho \sigma^3 \xi S f_S V + \xi^2 V^2 f_{VV} \right] - rf
\]

\[
= -r S f_S - [\mu - \beta_V (\mu^* - r)] \sigma^2 f_V,
\]

(2.25)

where \( \rho \) is the instantaneous correlation between \( S \) and \( V \). The variable \( \beta_V \) is the vector of multiple-regression betas for the regression of the variance "returns" \((dV/V)\) on the market portfolio and the portfolios most closely correlated with the state variables, and \( \mu^* \) is defined as above. We shall assume that \( \beta_V (\mu^* - r) \) is zero or that the volatility is uncorrelated with aggregate consumption.
This is not an unreasonable assumption and means that the volatility has zero systematic risk. The derivative asset must then satisfy

\[ f_t + \frac{1}{2} \left[ \sigma^2 S^2 f_{SS} + 2\rho \sigma^3 \xi S f_{SV} + \xi^2 V^2 f_{VV} \right] - rf = -rSf_S - \mu \sigma^2 f_V. \]  

(2.26)

It will also be assumed that \( \rho = 0 \), i.e. that the volatility is uncorrelated with the stock price.

An analytic solution to (2.26) for a European call option may be derived by using the risk-neutral valuation procedure. Since neither (2.26) nor the option boundary conditions depend upon risk preferences, we may assume in calculating the option value that risk neutrality prevails. Thus, \( f(S, \sigma^2, t) \) must be the present value of the expected terminal value of \( f \) discounted at the risk-free rate. The price of the option is therefore

\[ f(S_t, \sigma_t^2, t) = e^{-r(T-t)} \int f(S_T, \sigma_T^2, T)p(S_T|S_t, \sigma_t^2)dS_T, \]  

(2.27)

where

- \( T \) = time at which the option matures,
- \( S_t \) = security price at time \( t \),
- \( \sigma_t \) = instantaneous standard deviation at time \( t \),
- \( p(S_T|S_t, \sigma_t^2) \) = the conditional distribution of \( S_T \) given the security price and variance at time \( t \),
- \( E(S_T|S_t) \) = \( S_t e^{r(T-t)} \), and
- \( f(S_T, \sigma_T^2, T) \) = \( \max[S - X, 0] \).

The condition imposed on \( E(S_T|S_t) \) is given to make it clear that, in a risk-neutral world, the expected rate of return on \( S \) is the risk-free rate.

The conditional distribution of \( S_T \) depends on both the process driving \( S \) and the process driving \( \sigma^2 \). Making use of the fact that, for any three related random variables \( x, y, z \), the conditional density functions are related by

\[ p(x|y) = \int g(x|z)h(z|y) \, dz, \]

equation (2.27) may be greatly simplified.

Define

\[ \bar{V} = \frac{1}{T-t} \int_t^T \sigma_t^2 \, d\tau \]
as the mean variance over the life of the derivative. Then, using this, the distribution of $S_T$ may be written as

$$p(S_T | \sigma^2_t) = \int g(S_T | \tilde{V}) h(\tilde{V} | \sigma^2_t) \, d\tilde{V},$$

where the dependence on $S_t$ is suppressed to simplify the notation. Substituting this into (2.27) yields

$$f(S_t, \sigma^2_t, t) = e^{-r(T-t)} \int \int f(S_T) g(S_T | \tilde{V}) h(\tilde{V} | \sigma^2_t) dS_T d\tilde{V},$$

which can be written as

$$f(S_t, \sigma^2_t, t) = \int \left[ e^{-r(T-t)} \int f(S_T) g(S_T | \tilde{V}) dS_T \right] h(\tilde{V} | \sigma^2_t) d\tilde{V}. \tag{2.28}$$

Under the prevailing assumptions ($\rho = 0$; $\mu$ and $\xi$ independent of $S$) and since $\log(S_T/S_0)$ conditional on $\tilde{V}$ is normally distributed with variance $\tilde{V}$ when $S$ and $\tilde{V}$ are instantaneously uncorrelated, the inner term in (2.28) is the Black-Scholes price for a call option on a security with a mean variance $\tilde{V}$, which is

$$c(\tilde{V}) = S_t N(d_1) - K e^{-r(T-t)} N(d_2),$$

where

$$d_1 = \frac{\log(S_t/K) + (r + \tilde{V}/2)(T-t)}{\sqrt{\tilde{V}(T-t)}},$$

$$d_2 = d_1 - \sqrt{\tilde{V}(T-t)}.$$

Thus, the option value is given by

$$f(S_t, \sigma^2_t) = \int c(\tilde{V}) h(\tilde{V} | \sigma^2_t) d\tilde{V}. \tag{2.29}$$

It does not seem to be possible to obtain an analytic representation for the distribution of $\tilde{V}$ for any reasonable set of assumptions about the process driving $\tilde{V}$. It is, however, possible to calculate all the moments of $\tilde{V}$ when $\mu$ and $\xi$ are constant.\footnote{For the proofs of the following results, see "Mean Dependent Options." Working Paper, Accounting Group, University of Waterloo, 1985.} For example, when $\mu \neq 0$,

$$E(\tilde{V}) = \frac{e^{\mu T} - 1}{\mu T} V_0,$$

$$E(\tilde{V}^2) = \left[ \frac{2 e^{(2\mu + \xi^2)T}}{(\mu + \xi^2)(2\mu + \xi^2)T^2} + \frac{2}{\mu T^2} \left( \frac{1}{2\mu + \xi^2} - \frac{e^{\mu T}}{\mu + \xi^2} \right) \right] V_0^2.$$
and when $\mu = 0$,

$$
E(\tilde{V}) = V_0,
$$

$$
E(\tilde{V}^2) = \frac{2(\xi^2 T - \xi^2 T - 1)}{\xi^4 T^2} V_0^2,
$$

$$
E(\tilde{V}^3) = \frac{e^{3\xi^2 T} - 9e^{3\xi^2 T} + 6\xi^2 T + 8}{3\xi^6 T^3} V_0^3.
$$

Expanding $c(\tilde{V})$ in a Taylor series about its expected value $c(\tilde{V})$ yields

$$
f(S_t, \sigma^2_t) = c(\tilde{V}) + \frac{1}{2}c_{\tilde{V}\tilde{V}} \int (\tilde{V} - \tilde{V})^2 h(\tilde{V}) d\tilde{V} + ...$$

$$
= c(\tilde{V}) + \frac{1}{2}c_{\tilde{V}\tilde{V}} \left[ \text{Var}(\tilde{V}) + \frac{1}{6}c_{\tilde{V}\tilde{V}\tilde{V}} \text{Skew}(\tilde{V}) + ... \right],
$$

where $\text{Var}(\tilde{V})$ and $\text{Skew}(\tilde{V})$ are the second and third central moments of $\tilde{V}$. For sufficiently small values of $\xi^2(T - t)$, this series converges very quickly. Using the moments for the distribution of $\tilde{V}$ given above, this series becomes (when $\mu = 0$)

$$
f(S, \sigma^2) = c(\sigma^2)$$

$$
+ \frac{1}{2} \left[ \frac{S \sqrt{T - t} N'(d_1) (d_1 d_2 - 1)}{4\sigma^3} \times \left[ \frac{2\sigma^4 (e^k - k - 1)}{k^2} - \sigma^4 \right] \right.
$$

$$
+ \left. \frac{S \sqrt{T - t} N'(d_1) [(d_1 d_2 - 3)(d_1 d_2 - 1) - (d_1^2 + d_2^2)]}{48\sigma^5} \times \sigma^6 \left[ \frac{e^{3k} - (9 + 18k)e^k + (8 + 24k + 18k^2 + 6k^3)}{3k^3} \right] \right] + ..., 
$$

(2.30)

where $k := \xi^2(T - t)$ and the $t$ subscript has been dropped to simplify notation. (We shall use this option price expression in the calculation of the model call option prices for the Stochastic volatility model.)
2.3 Other models

2.3.1 Compound Option Model

The compound option model is actually more of a unique and interesting way of viewing an option than a practical tool for valuing derivatives. The assumptions of this model regarding corporate debt (though some can be relaxed) are rather restrictive - it postulates that \( \sigma_V \) (the volatility of the value \( V \) of the firm) is constant - and is not much less limiting than the Black-Scholes assumption of constant stock price volatility. The compound option solution contains an additional term compared to the Black-Scholes solution, which reflects the debt position of the firm. The derived formula has the desirable attributes of the Black-Scholes model in that it does not depend on knowledge of the expected return on either the stock or the assets of the firm. A formula for the value of a call option \( c \) as a compound option can be derived as a function of \( V \). The following setting describes this perspective.

Consider a corporation that has common stock and bonds outstanding. Suppose the bonds are pure discount bonds, giving the bondholder the right to the face value \( M \) (if the corporation can pay \( M \)), with a maturity of \( T \) years. Suppose that the indenture of the bond stipulates that the firm cannot issue any new senior or equivalent rank claims on the firm, nor pay cash dividends or repurchase shares prior to the maturity of the bonds. Finally, suppose the firm plans to liquidate in \( T \) years, pay off the bonds, if possible, and pay any remaining value to the stockholders as a liquidating dividend. Here the bondholders own the firm's assets and have given the stockholders the option to buy the assets back when the bonds mature.

Now, a call option on the stock of the firm is an option on an option or a compound option. This situation can be represented functionally as \( c = f(S, t) = f(g(V, t), t) \), where \( t \) is current time. Therefore, changes in the value of the call can be expressed as a function of changes in the value of the firm and changes in time. If the value of the firm follows a continuous sample path, and if investors can continuously adjust their positions, then a riskless hedge can be formed by choosing an appropriate mixture of the stock and call options on the stock.

Merton has shown that an American call option will not be exercised early if the underlying asset has no payouts. Thus, the stock, depicted as an option on the value of the firm, will not be exercised early because the firm by assumption makes no dividend or coupon payments. Since the proof by Merton does not rely on any distributional assumptions, the compound call option on the stock will not be prematurely exercised either.

\(^3\)A comparison of the Black-Scholes model shows it to be a special case of the compound option model.

\(^4\)Most of these restrictions can be relaxed. In particular, the firm does not have to liquidate, but could pay off the bonds and refinance.
To derive the compound option formula for a call in continuous time, assume that security markets are perfect and competitive, unrestricted short sales of all assets with full use of proceeds is allowed, the risk-free rate of interest is known and constant over time, trading takes place continuously in time, and changes in the value of the firm follow a random walk in continuous time with a variance rate proportional to the square of the value of the firm. Thus, the return on the firm follows a diffusion described by the following stochastic differential equation formalised by Ito:

\[
\frac{dV}{V} = \alpha_{V} dt + \sigma_{V} dZ_{V},
\]

where \(\alpha_{V}\) is the instantaneous expected rate of return on the firm per unit time, \(\sigma_{V}^{2}\) is the instantaneous variance of the return on the firm per unit time, and \(dZ_{V}\) is a mean zero, normal random variable with variance \(dt\) (i.e. a standard Gauss-Weiner process). Since the call option \(c(V,t)\) is a function of the value of the firm and time, its return also follows a diffusion process that can be described by a related stochastic differential equation,

\[
\frac{dc}{c} = \alpha_{c} dt + \sigma_{c} dZ_{c},
\]

where \(\alpha_{c}\) is the instantaneous expected rate of return on the call per unit time, \(\sigma_{c}^{2}\) is the instantaneous variance of the return on the call per unit time, and \(dZ_{c}\) is a standard Gauss-Weiner process. We now derive an equation for \(c\).

By Ito's lemma,

\[
dc = \frac{\partial c}{\partial t} dt + \frac{\partial c}{\partial V} dV + \frac{1}{2} \frac{\partial^{2} c}{\partial V^{2}} V^{2} \sigma_{V}^{2} dt.
\]

Since the firm is now regarded as an underlying security, we can set up a portfolio \(\Pi = c - \Delta V\), where \(\Delta \in \mathbb{R}\). Then we have that

\[
d\Pi = \left(\frac{\partial c}{\partial t} dt + \frac{1}{2} \frac{\partial^{2} c}{\partial V^{2}} V^{2} \sigma_{V}^{2} dt\right) + \left(\frac{\partial c}{\partial V} - \Delta\right) dV.
\]

If we set \(\Delta = \frac{\partial c}{\partial V}\), then we have eliminated the risky part of the return, hence

\[
d\Pi = r \Pi dt,
\]

and so the equation for \(c\) is

\[
\frac{\partial c}{\partial t} = rc - rV \frac{\partial c}{\partial V} + \sigma_{V}^{2} V^{2} \frac{1}{2} \frac{\partial^{2} c}{\partial V^{2}} V^{2}.
\]

Since the stock price is an option on the value of the firm, it follows a related diffusion and, by Ito's lemma, we have

\[
dS = \frac{\partial S}{\partial t} dt + \frac{\partial S}{\partial V} dV + \frac{1}{2} \frac{\partial^{2} S}{\partial V^{2}} V^{2} \sigma_{V}^{2} dt.
\]
By constructing a similar hedging strategy between the stock, the firm and a riskless security, the stock’s equilibrium path can be described by

\[ \frac{\partial S}{\partial t} dt = rS - rV \frac{\partial S}{\partial V} + \sigma^2 \frac{1}{2} \frac{\partial^2 S}{\partial V^2} V^2. \]  

(2.32)

With the boundary condition \( S_T = \max \{ V_T - M, 0 \} \), the solution to the above equation, is the well-known Black-Scholes result

\[ S = V N_1(k + \sigma \sqrt{T}) - M e^{-r T} N_1(k), \]  

(2.33)

where

- \( S \) = current market value of the stock,
- \( V \) = current market value of the firm,
- \( M \) = face value of the debt,
- \( r \) = risk-free interest rate,
- \( \sigma^2 \) = the instantaneous variance of the return on the assets of the firm,
- \( t \) = current time,
- \( T \) = maturity date of the debt,
- \( \tau \) = \( T - t \), and
- \( N_1(\cdot) \) = univariate cumulative normal distribution function.

In a risk-neutral world where all assets earn the same expected rate of return - the riskless rate - the current value of an option is the following riskless discounted expected value of the option payoff at expiration \( t^* \):

\[ c = e^{-r(t^*-t)}E_V \{ \max \{ S_{t^*}(V) - K, 0 \} \}. \]  

(2.34)

Given the assumed relation between the stock and the firm, if the conditional distribution \( F(V_t | V) \) for the value of the firm at the option’s expiration date is known, then substituting from equation (2.33) for \( S(V, t^*) \), equation (2.34) becomes

\[ c = e^{-rT} \left\{ \int_K^\infty V N_1(k + \sigma \sqrt{T_1 - \tau_2}) F(V_t | V) dV - \int_K^\infty K F(V_t | V) dV 
- \int_K^\infty M e^{-r(T_1 - \tau_2)} N_1(k) F(V_t | V) dV \right\}. \]  

(2.35)

Evaluating these integrals yields the compound option formula

\[ c = V N_2(h + \sigma \sqrt{\tau_1}, k + \sigma \sqrt{\tau_2}, \sqrt{\tau_1/\tau_2}) - M e^{-r \tau_2} N_2(h, k; \sqrt{\tau_1/\tau_2}) 
- Ke^{-r \tau_1} N_1(h), \]  

(2.36)

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where

\[ h = \frac{\ln(V/V) + (r - \frac{1}{2} \sigma^2 \tau_1)}{\sigma \sqrt{\tau_1}}, \]

\[ k = \frac{\ln(V/M) + (r - \frac{1}{2} \sigma^2 \tau_2)}{\sigma \sqrt{\tau_2}}, \]

\[ \dot{V} = \text{that value of } V \text{ such that } S_t - K = VN_1(k + \sigma \sqrt{\tau}) - Me^{-r \tau} N_1(k) - K = 0, \]

where \( r = T - t^* \), and,

\[ N_2(\cdot) = \text{bivariate cumulative normal distribution function with } h \text{ and } k \]

as upper integral limits and \( \sqrt{\tau_1/\tau_2} \) as the correlation coefficient,

where \( \tau_1 = t^* - t \) and \( \tau_2 = T - t \).

(We shall use this option price expression in the calculation of the model call option prices for the Compound option model.)

One advantage of the Black-Scholes option pricing model is that only five input variables are required to calculate option prices: \( \sigma_{BS} = f(S, \sigma, r, t^*, K) \). All of these variables are either known or directly observable except \( \sigma \), the instantaneous variance of the stock return. The compound option formula requires seven input variables: \( \sigma_{CO} = g(V, \sigma, r, t^*, T, K, M) \). The two extra variables necessary to capture the leverage effects are \( M \), the face value of the debt, and \( T \), the maturity date of the debt. Three of these variables, \( r, t^* \), and \( K \) are directly observable and the other four can be computed. \( V \) and \( \sigma \) can be found either by defining \( V = S + B \) ( \( B \) is the market value of the firm’s debt) and using empirical data, or by solving for \( V \) and \( \sigma \) from past stock price data, using \( S(V, \sigma) \) and \( \sigma_{S}(V, \sigma) \). One can input the current stock price \( S \) and the current instantaneous volatility \( \sigma_{S} \) of the stock rate of return, and then solve numerically equation (2.33)

\[ S = VN_1(k + \sigma \sqrt{\tau_2}) - Me^{-r \tau} N_1(k), \]

and

\[ \sigma = \frac{\partial S}{\partial V} \sigma_{S} V/S = N_1(k + \sigma \sqrt{\tau_2}) \sigma_{S} V/S. \] (2.37)

For empirically realistic parameter values, a multivariate Newton-Raphson search converges quickly to the solution.

### 2.3.2 Displaced Diffusion

Much like the compound option model, the displaced diffusion model attempts to price equity options by taking into account the structure of the underlying firm. However, it leads to positive correlations between stock price and volatility.
Firms are assumed to consist of risky assets which have a constant volatility and riskless assets which are used to pay off debt. Following Rubinstein [16], the total value of the firm is represented by \( V \) and \( a \) denotes the fraction of \( V \) that is comprised of risky assets. The value of the risky assets \( aV \) can be described by geometric Brownian motion, while the remaining assets grow at the risk-free annual rate which is denoted by \( r \). After \( t \) years the value of the firm will grow to

\[
[ae^y + (1 - a)(1 + r)^t]V
\]

where \( y \) is a normally distributed random variable with instantaneous volatility \( \sqrt{\sigma_V(t)} \). If we let \( \beta \) denote the debt-to-equity ratio, then the value of the firm’s stock in \( t \) years will be given by

\[
S_{\text{future}} = [ae^y + (1 - a)(1 + r)^t](1 + \beta)S - S\beta(1 + r)^t,
\]

where \( S \) is the current market price of the stock.

A key parameter \( a \) in the model is defined as

\[
a = a(1 + \beta).
\]

If \( a > 1 \), then the amount of debt in the displaced diffusion model exceeds the riskless assets. It was found that, netting the riskless assets off against the debt, the model becomes very similar to the compound option model. Unlike the compound option model, the displaced diffusion model does not take into account the possibility of default on the debt.

If \( a < 1 \), then the amount of debt is less than the amount of riskless assets. The model then has properties that are markedly different from that of the compound option model. Netting off the debt against the riskless assets, we can write

\[
S = S_A + S_B
\]

where \( S \) is the stock price, \( S_A \) is the value of the risky assets and \( S_B \) is the value of net riskless assets. When \( S_A \) increases (decreases) quickly, \( S \) increases (decreases) and the volatility of \( S \) also increases (decreases). This is because risky assets have become a proportionately large (small) part of \( S \). It follows that the volatility and stock price are positively correlated. The model therefore leads to an implied distribution with a fat right tail and thin left tail - the reverse of that observed in practice. The formula for pricing a European call option under the displaced diffusion model is given below [16]

\[
c = aS_0N(d_1) - (K - bS_0)e^{-rT}N(d_2),
\]

(2.40)
where

\[ a = \alpha(1 + \beta), \]
\[ b = (1 - a)e^{T}, \]
\[ \sigma_R = \text{is the volatility of the risky assets}, \]
\[ d_1 = \frac{\ln[aS_0/(K - bS_0)] + (r + \frac{1}{2} \sigma_R^2)T}{\sigma_R \sqrt{T}}, \text{ and} \]
\[ d_2 = d_1 - \sigma_R \sqrt{T}. \]

(We shall use this option price expression in the calculation of the model call option prices for the Displaced diffusion model.)

Note that the result is that the displaced diffusion model predicts volatility that will be positively correlated with stock price. Unfortunately, as was discussed in regard to the constant elasticity of variance diffusion model, there is evidence that these two variables tend to be negatively correlated. While the displaced diffusion model grants considerable versatility in terms of dividend payout schemes, stochastic firm value, and multiple firm parameters, it suffers from being difficult to use. There are more firm variables than in the compound option model model and their estimation is just as problematic. This difficulty is compounded by the fact that few, if any, firms can be so easily divided into risky and riskless components.

2.3.3 Non-zero Transaction Costs

Transaction costs are the costs incurred in buying and selling of the underlying, related to the bid-offer spread. The Black-Scholes analysis requires continuous hedging and assumes no transaction costs in the rebalancing. In liquid markets (government bonds in first-world countries, for example), costs are low and portfolios can be hedged often. Hedging continuously is in a sense taking the limit as the time between rebalances goes to zero \((\delta t \to 0)\). We therefore are hedging an infinite number of times, hence acquire infinite total transaction costs. There are economies of scale: the larger the amount a person trades, the less significant are his costs.

In contrast to the Black-Scholes model, we may expect there to be no unique option price, instead it depends on the investor. Not only do we expect different investors to have different values for contracts, but also expect an investor, if they are hedging, to have different values for long and short positions in the same contract. Why? It is because transaction costs are a sink of money for hedgers; they always lose out on the bid-offer spread of the underlying. Thus, we expect a hedger to value a long (short) position at less (more) than the Black-Scholes price: whether the position is long or short, hedging costs must be taken away from the value of the option.
2.3.3.1 The model of Leland (1985)

He adopted the hedging strategy of hedging at every time step. That is, every \( \delta t \) the portfolio is rebalanced, whether or not this is optimal. He assumes that the cost of trading \( \nu \) assets is proportional to the value traded, for both buying and selling (i.e. cost of trading = \( \kappa \nu S \), where \( \kappa \) is the proportionality constant).

The assumptions he makes, are:

- The portfolio is revised every \( \delta t \) where \( \delta t \) is a finite and fixed, small timestep.
- The random walk is given in discrete time by
  \[
  \delta S = \mu S \delta t + \sigma S \phi \delta t^{1/2}
  \]
  where \( \phi \) is drawn from a standardised normal distribution.
- Transaction costs are proportional to the value of the transaction in the underlying. Thus, if \( \nu \) shares are bought (\( \nu > 0 \)) or sold (\( \nu < 0 \)), then the cost incurred is \( \kappa |\nu| S \), where \( \kappa \) will depend on the individual investor.
- The hedged portfolio has an expected return equal to the risk-free rate.

2.3.3.2 The model of Hoggard, Whaley & Wilmot (1992)

Suppose we are going to hedge and value a portfolio of European options and allow for transaction costs. We can still follow the Black-Scholes analysis, but must allow for transaction costs. If \( \Pi \) denotes the value of the hedged portfolio (comprised of long a derivative \( V \) and short \( \Delta \) shares) and \( \delta \Pi \) the change in the portfolio over a time step \( \delta t \), then we must subtract any transaction costs from the equation for \( \delta \Pi \) at each time step. Thus, after each time step,

\[
\delta \Pi = \delta V - \Delta \delta S - \kappa S |\nu|.
\]

So, by Ito's lemma,

\[
\delta \Pi = \sigma S \left( \frac{\partial V}{\partial S} - \Delta \right) \phi \delta t^{1/2} + \left( \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} \phi^2 + \mu S \frac{\partial V}{\partial S} + \frac{\partial V}{\partial t} - \mu \Delta S \right) \delta t - \kappa S |\nu|,
\]

where we have used the form of \( \delta S \) given above, and we have the modulus sign because transaction costs are always positive. This is similar to the Black-Scholes expression, but contains a transaction cost term. (We choose \( \Delta = \frac{\partial V}{\partial S} \) to eliminate the riskiness in the portfolio.)

Now, the portfolio is rehedged at discrete intervals, so, at time \( t \)

\[
\Delta = \frac{\partial V}{\partial S}(S, t).
\]
After a time step $\delta t$ and rehedging (i.e. changing $S$ to $S + \delta S$), the number of assets we short, is

$$\frac{\partial V}{\partial S}(S + \delta S, t + \delta t).$$

Thus, the number of assets traded over $[t, t + \delta t]$ is

$$\nu = \frac{\partial V}{\partial S}(S + \delta S, t + \delta t) - \frac{\partial V}{\partial S}(S, t).$$

Since the time step and asset move are both small, we can apply Taylor's theorem:

$$\frac{\partial V}{\partial S}(S + \delta S, t + \delta t) = \frac{\partial V}{\partial S}(S, t) + \delta S \frac{\partial^2 V}{\partial S^2}(S, t) + \delta t \frac{\partial^3 V}{\partial S^3}(S, t) + ...$$

Since $\delta S = \sigma \phi \sqrt{\delta t} + O(\delta t)$, the dominant term is that which is proportional to $\delta S$; this term is $O(\delta t^{1/2})$ and the other terms are $O(\delta t)$. Thus, to leading order the number of assets traded, is

$$\nu \approx \frac{\partial^2 V}{\partial S^2}(S, t) \delta S \approx \frac{\partial^2 V}{\partial S^2} \sigma \phi \sqrt{\delta t},$$

so,

$$\kappa S |\nu| \approx \kappa S^2 \sigma \sqrt{\delta t} \left| \frac{\partial^2 V}{\partial S^2} \right|.$$

So, the expected transaction cost over a time step is

$$E[\kappa S |\nu|] = \sqrt{\frac{2}{\pi}} \kappa \sigma S^2 \left| \frac{\partial^2 V}{\partial S^2} \right| \sqrt{\delta t}, \quad (2.42)$$

where the factor $\sqrt{2/\pi}$ is the expected value of $|\phi|$.

We can now calculate the expected change in the value of our portfolio from equation (2.41). Using the fact that $E[\phi^2] = 1$,

$$E[\delta \Pi] = \left( \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2}(1) + \frac{\partial V}{\partial t} \right) \delta t - \sqrt{\frac{2}{\pi}} \kappa \sigma S^2 \left| \frac{\partial^2 V}{\partial S^2} \right| \sqrt{\delta t}. \quad (2.43)$$

Now, assuming that the holder expects to make as much from his portfolio as if he had put his money in the bank, we have that

$$E[\delta \Pi] = r \Pi \delta t = r (V - SV^2) \delta t.$$

So, equation (2.43) becomes

$$\frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} - \kappa \sigma S^2 \sqrt{\frac{2}{\pi \delta t}} \left| \frac{\partial^2 V}{\partial S^2} \right| + r S \frac{\partial V}{\partial S} - r V = 0. \quad (2.44)$$

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This is a non-linear, parabolic partial differential equation.

We know that \( \Gamma > 0 \) for a single call or put in the absence of transaction costs, so we postulate that this is true for a single call or put option when transaction costs are included. If this is the case, then we can drop the modulus sign in equation (2.44) to obtain

\[
\frac{\partial V}{\partial t} + \frac{1}{2} S^2 \frac{\partial^2 V}{\partial S^2} (\tilde{\sigma}^2) + rS \frac{\partial V}{\partial S} - rV = 0, \tag{2.45}
\]

where \( \tilde{\sigma}^2 = \sigma^2 - 2\kappa \sigma^2 \sqrt{\frac{2}{rM}}. \) (We shall use this equation to obtain model call option values when transaction costs are considered.)
Chapter 3

Data and Methodology

The data sample is drawn from the set of closing prices for all call options on the S&P500 index traded on the Chicago Board Options Exchange on 15 August 2001. This amounted to 251 options with expiration lengths varying from 2 days to 857 days - 20 December 2003. The interest rate used in my calculations was the United States deposit rate (continuously compounded) applicable over the life of each option. I have ignored the dividend yield of the index. (See Appendix A for the specifications of all the call options.) I have used only the programming languages Matlab and Excel in my work. All code is given as Appendix C.

All models require the historical volatility \( \sigma \) (however, for the GARCH(1,1) model we calculate this estimate of \( \sigma \) differently to formula (2.16).) Using \( m = 10 \) (i.e. we’re using the continuously compounded returns for the last 10 days), we find from formula (2.16) that

\[
\sigma = 0.052.
\]

For almost all of the parameter estimations, we use the following options:

<table>
<thead>
<tr>
<th>Option status</th>
<th>Expiry</th>
<th>Option price</th>
<th>Strike</th>
<th>Interest rate (nacc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short maturity, in-the-money</td>
<td>18 Aug 2001</td>
<td>278.9</td>
<td>900</td>
<td>1.45%</td>
</tr>
<tr>
<td>Long maturity, out-of-the-money</td>
<td>21 Dec 2002</td>
<td>0.25</td>
<td>1900</td>
<td>1.44%</td>
</tr>
</tbody>
</table>

(where nacc means continuously compounded).

Pure Jump model

The parameters that are needed here are \( u \) and \( \lambda_pJ \). We find that the average
value of $u$ and $\lambda_{PF}$ that minimise the difference between model and market prices for the options tabled above, is

$$u = 1.52 \quad \text{and} \quad \lambda_{PF} = 0.02.$$ 

Mixed Jump-Diffusion model
The parameter that is needed here is $\lambda_{MJD}$. We find that the average value of $\lambda_{MJD}$ that minimises the difference between model and market prices for the options tabled above, is

$$\lambda_{MJD} = 0.27.$$ 

Compound Option model
This model is actually not applicable to index options because, to begin with, what is the firm? I circumvent this problem by defining a hypothetical firm with $M = \$1$ million and $T = 1$ year, and then calculate the values of $V$ and $\sigma_V$.

Stochastic volatility model
The parameter that is needed here is $\xi$. We find that the average value of $\xi$ that minimises the difference between model and market prices for the options tabled above, is

$$\xi = 0.05.$$ 

Displaced Diffusion model
The parameters that are needed here are $\alpha$ and $\beta$. We find that the average value of $\alpha$ and $\beta$ that minimise (in the least squares sense) the difference between model and market prices for the options tabled above, is

$$\alpha = 0.2 \quad \text{and} \quad \beta = 0.8.$$ 

3.1 Non-zero Transaction Costs

We solve equation (2.45) using the Crank-Nicolson finite difference scheme. Now, we need to initialise the parameter $\kappa$. I decided to speak to traders at Investec and the general consensus is that transaction costs comprise about 0.1% of the value of most traded financial instruments, hence $\kappa = 0.001$. (It turns out that this value of $\kappa$ gives pretty accurate values for the options tabled above.)

3.1.1 Derivation of the Crank-Nicolson scheme
If we make the substitution $x = \ln S$, then (2.45) becomes

$$V_t + \frac{1}{2} \sigma^2 V_{xx} + \nu V_x = rV_t,$$ 

(3.1)
where \( v := r - \frac{1}{2} \sigma^2 \). Crank and Nicolson suggested an alternative way to utilise centred differences. The forward difference in time

\[
V_t \approx \frac{V(t + \Delta t) - V(t)}{\Delta t}
\]

may be interpreted as the central difference around \( t + \Delta t/2 \). The error in approximating \( \frac{\partial V}{\partial t} (t + \Delta t/2) \) is \( O(\Delta t)^2 \). Thus, we discretise the second derivative at \( t + \Delta t/2 \) with a centred difference scheme. Since this involves functions evaluated at this in-between time, we take the average at \( t \) and \( t + \Delta t \). So, letting time and space be indexed by \( n \) and \( i \), respectively, we obtain that

\[
\begin{align*}
V &\approx V_{i+\Delta t/2} \\
V_x &\approx \frac{V_{x,i+\Delta t} + V_{x,i}}{2} \\
V_{xx} &\approx \frac{V_{xx,i+\Delta t} + V_{xx,i}}{2}
\end{align*}
\]

we obtain that

\[
\begin{align*}
V &\approx V_{i+\Delta t/2} \\
V_x &\approx \frac{V_{x,i+\Delta t} + V_{x,i}}{2} \\
V_{xx} &\approx \frac{V_{xx,i+\Delta t} + V_{xx,i}}{2}
\end{align*}
\]

Substituting these approximations into equation (2.45), yields

\[
\begin{align*}
r_d V_{i-1}^n + r_m V_i^n + r_u V_{i+1}^n &= -r_d V_{i-1}^{n+1} - (r_m - 2) V_i^{n+1} - r_u V_{i+1}^{n+1}, \quad (3.2)
\end{align*}
\]

where \( i = 1, ..., N \) and \( n \) is the time index. If we restate this in matrix form, then we have that

\[
QV^n = RV^{n+1}, \quad (3.3)
\]

where \( V^n \) is the price of the derivative at time \( n \), \( Q = \text{tridiagonal}(r_u, r_m, r_d) \), \( R = \text{tridiagonal}(-r_d, 2 - r_m, -r_u) \), and

\[
\begin{align*}
r_u &= \frac{1}{4} \Delta t \left( \frac{\sigma^2}{(\Delta x)^2} + \frac{v}{\Delta x} \right), \\
r_m &= 1 + \frac{\Delta t}{2} \left( \frac{\sigma^2}{(\Delta x)^2} + r \right), \text{ and} \\
r_d &= r_u + \frac{v \Delta t}{2 \Delta x}.
\end{align*}
\]

(Remember that we want to calculate \( V^0 \).)

### 3.1.2 Boundary conditions

Consider a typical plot of a call option price \( c \) as a function of the underlying \( S \) given in Figure 1 below. We see that when \( S \) is very small or very big, \( c \) is
approximately linear, hence the boundary conditions used, are

\[
\frac{\partial^2 c}{\partial S^2} = 0 \text{ when } S \text{ is small, and} \quad (3.4)
\]

\[
\frac{\partial^2 c}{\partial S^2} = 0 \text{ when } S \text{ is large.} \quad (3.5)
\]

Using equation (3.2) and the Crank-Nicolson approximation for second derivatives, we find that

\[
Q(1,1) = r_m + 2r_d \quad Q(1,2) = r_u - r_d \\
R(1,1) = -2r_d - r_m + 2 \quad R(1,2) = r_d - r_u \\
Q(N,N) = r_m + 2r_u \quad Q(N,N-1) = r_d - r_u \\
R(N,N) = -2r_u - r_m + 2 \quad R(N,N-1) = r_u - r_d.
\]

(So, in my program I shall simply just loop over the statement

\[ V^n = Q^{-1} RV^{n+1} \]

to obtain the solution.)

### 3.2 GARCH(1,1)

The parameters \( \kappa, \alpha_G \) and \( \beta_G \) are estimated from historical data via maximum likelihood estimation. It involves choosing values for the parameters that max-
imise the likelihood of the data occurring.

3.2.1 Estimating a Constant Variance

We try to estimate a constant variance \( \sigma^2 \) from \( m \) observations \( u_1, u_2, \ldots, u_m \) when the underlying distribution is normal (with mean zero) and the variance is assumed to be constant. The probability density of the \( m \) observations occurring in the order that they are observed, is

\[
\prod_{i=1}^{m} \left[ \frac{1}{\sqrt{2\pi \sigma^2}} \exp \left( -\frac{u_i^2}{2\sigma^2} \right) \right].
\]

Now, maximising an expression is equivalent to maximising its logarithm. So, taking the logarithm of this expression and ignoring constants, it can be seen that we wish to maximise

\[
-\frac{1}{2\sqrt{2\pi}} \ln \sigma^2 - \sum_{i=1}^{m} \frac{u_i^2}{2\sigma^2}.
\]

Differentiating with respect to \( \sigma^2 \) and setting the derivative to zero, the maximum likelihood estimate is

\[
\sigma^2 = \frac{2}{m} \sum_{i=1}^{m} u_i^2.
\]

3.2.2 Extension to estimate parameters in a volatility updating scheme

We now suppose that the variance is subject to a volatility updating scheme such as GARCH(1,1). Define \( \sigma_i^2 = \sigma_i^2 \) as the variance estimated for day \( i \). A similar analysis to the one just given shows that we wish to maximise

\[
\prod_{i=1}^{m} \left[ \frac{1}{\sqrt{2\pi \sigma_i^2}} \exp \left( -\frac{u_i^2}{2\sigma_i^2} \right) \right].
\]

Taking logarithms, we see that this is equivalent to maximising

\[
\sum_{i=1}^{m} \left[ -\frac{1}{2} \ln \sigma_i^2 - \frac{u_i^2}{2\sigma_i^2} \right].
\]

We search iteratively for the parameters in the model which maximise this expression. From Matlab, we find that

\[
\kappa = 0.000000067889, \quad \alpha_G = 0.0868, \quad \beta_G = 0.8917;
\]

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3.3 A note on implied volatilities

The volatility input I use in my models is not an implied volatility, but a historical volatility estimate of the underlying, since I am assuming that the Black-Scholes model is not correct (i.e. it does not produce theoretical prices that are exactly the same as the market values). Recall that implied volatilities are "backed out" of the option prices via the Black-Scholes formula, which implies that it is assumed that the Black-Scholes model is correct. Added to this, if implied volatilities were used as an input for the Black-Scholes formula, then it (Black-Scholes) would produce theoretical prices that are exactly the same as the market prices everytime, and thus it would be the perfect pricing model - it definitely is not the perfect model.

Another reason for using the historical volatility is that in all of the nine models that I consider it assumed that the underlying $S$ follows

$$\frac{dS}{S} = \sigma dB_t + \text{other terms},$$

where the important point is that $\sigma$ is the volatility of the underlying (i.e. the variance of its sample path, and not the implied volatility which is almost never even close to $\sigma$).
Chapter 4

Results and Statistics

The principal finding is that no one model prices the options invariably better than any other i.e. no one model produces a relative error which is less than that of any other model in each of the five (see below) categories of options. (Remember that the market prices need not be the correct arbitrage-free prices, so, even though our models don’t match the market prices very well, it does not mean that they are poor models. One of the fundamental reasons why market prices are not necessarily the arbitrage-free prices is due to supply and demand.)

We also find that for the lower boundary condition $S \exp(-\gamma t) - Ke^{-\gamma t} \leq c$. (All the model call option prices is given in Appendix B.)

If we have a look below at the relative error (expressed as a percentage) for each of the models for the different types of options,

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<th>Im,lm</th>
<th>Ct,at</th>
<th>Om,sm</th>
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(where Im, Om, sm, Im, and Ct,at, denote in-the-money, out-of-the-money, short-maturity, long-maturity, and close to at-the-money, respectively) then we notice that for all the options, the models produce values which are very different from the market. We find that all the models (except for the Nonzero
Transaction costs model) price the relatively deep in-the-money options about the same, but then differ as the options get closer to being at-the-money. The Nonzero transaction costs model prices for the relatively deep in-the-money options are similar to any other model, but the reason why its relative error is so high is because its accuracy decreases more rapidly than any other model as the options get closer to being at-the-money. The different model prices for the close to at-the-money options are very similar. The Pure jump model prices for the short-maturity, out-of-the-money options are very different from the other eight. The Mixed jump-diffusion model prices for the long-maturity, out-of-the-money options are very different from any of the other eight.

The Stochastic volatility model has the least error in each category except for the out-of-the-money, short-maturity options.

If we have a look at plots of the relative errors (given in Appendix C), then we observe for each of the different types of options, the error in the model predictions decreases rapidly as the options that we consider get closer to being at-the-money.

Reasons for errors:

1. Inputs have been incorrectly measured.
2. The mathematical structure of the models is incorrect.

The first reason is a definite possibility since, for some of the models, I used only two options to estimate parameters, and then priced the other options. (I could have used more options in the estimation of the parameters, but I did not want to bias my results by trying to match too many of the market option prices.) Also, let us not forget machine error. This is very important when the model option price is close to zero and we have to do many calculations as in the case of the Compound option model and Nonzero Transaction costs model. If, further, we suppose that the options market, although possibly sporadically different, exhibits no systematic inefficiencies, then we shall be forced to conclude that the trouble lies with the form of the models.

*Correlation between index value and volatility*

If we look at Figures 2 and 3 below, then we notice that there is little correlation between the volatility and the index value between 15 August 1996 and 15 August 2001. The correlation coefficient for this period is

$$ r = -0.1655. $$

This implies that those models which model the index value and its volatility to be negatively correlated, should price the options better than the others. The only model which incorporates this negative correlation is the CEV model and it produces prices which are very different from the market.
Chapter 5

Conclusion

The point is that not one of these nine, fairly distinct models produces prices even close to the market prices (despite ignoring the dividend yield), but their results are very similar. If they are all incorrect, then there must be some underlying assumption (which does not hold in the real world) common to all of them, or their inputs were incorrectly measured.

Is the deviation of the model option prices from market prices economically significant? This will depend on what you plan to do with options. If you are a market maker who trades frequently with high turnover, then the answer may be yes; if you are a positioner holding for the longer term, then the answer may be no.

A number of problems arise in carrying out empirical research to test the Black-Scholes and other option pricing models. The first is that any statistical hypothesis about how options are priced has to be a joint hypothesis to the effect that (1) the option pricing formula is correct and (2) markets are efficient. If the hypothesis is rejected, then it might be the case that (1) or (2) is not true, or both aren't true. The second problem is that the asset price volatility is an unobservable variable. One can use historical data or implied volatilities to estimate it. The third is that the researcher needs to ensure that the asset price and option price data are synchronous. In most cases, the mispricing by Black-Scholes is not sufficient to present profitable opportunities to investors when transaction costs and bid-ask spreads are taken into account.

One of the biggest problems I encountered, involved obtaining historical data for option prices. I only obtained data for one day's sale of options, and this is insufficient to draw accurate conclusions about the performance of the models over a long period of time.

Since none of the models price the options much more accurately than Black-Scholes, I'll choose Black-Scholes because it takes the least computing time. If I am trader, then I want to be able to quote a price quickly to a potential buyer and not want to have to run a model, which should give me a fair price, but
that takes about a few minutes to run. So, for example, it is definitely not advisable for traders to employ the Nonzero Transaction costs model because it takes about 3 minutes to price the simplest type of option, viz European vanilla options.

The ideal model would be one which would be able to incorporate all of the models, but then we'd sacrifice computing time. However, even if this model is derived, there might still be so many other factors responsible for the price of a derivative which have not been discovered yet. So, as long as we cannot accurately predict what the price of certain instruments will be in the future, portfolio management comes down to theory, risk management, and experience.


Bibliography


Appendix A

Specifications of S&P500 index options sold on 15 August 2001 on the CBOE

S&P500 close on 15 August 2001 = 1178.02.

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## Appendix B

Call option prices generated by the different models.

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### Notes
- The table contains values that appear to be financial or currency-related, with columns indicating amounts and possibly dates or other identifiers.
- The values range from 1000.00 to 14670.00, with increments that suggest a structured dataset, possibly for financial reporting or accounting.
Appendix C

In-the-money, short maturity

Relative error of Pure Jump model.

Total relative error = 4.68

Relative error of CEV model.

Total relative error = 4.47

Relative error of Mixed Jump-Diffusion model.

Total relative error = 4.39

Relative error of Stochastic volatility model.

Total relative error = 4.10
Relative error of Black-Scholes model.

Total relative error = 4.15
In-the-money, long maturity

Relative error of Pure Jump model.

Total relative error = 4.82

Relative error of Mixed Jump-Diffusion model.

Total relative error = 4.52

Relative error of CEV model.

Total relative error = 4.52

Relative error of Stochastic volatility model.

Total relative error = 4.15
Relative error of Compound option model.
Total relative error = 4.36

Relative error of Displaced Diffusion model.
Total relative error = 4.51

Relative error of Nonzero Costs model.
Total relative error = 18

Relative error of GARCH(1,1) model.
Total relative error = 4.2
Relative error of Black-Scholes model.

Total relative error = 4.2
At-the-money

Relative error of Pure Jump model.

Total relative error = 4.20

Relative error of Mixed Jump-Diffusion model.

Total relative error = 3.81

Relative error of CEV model.

Total relative error = 4.10

Relative error of Stochastic volatility model.

Total relative error = 3.0
Relative error of Compound option model.

Total relative error = 0.35

Relative error of Nonzero Costs model.

Total relative error = 4.70

Relative error of Displaced Diffusion model.

Total relative error = 3.87

Relative error of GARCH(1,1) model.

Total relative error = 3.07
Relative error of Black-Scholes model.
Total relative error = 3.07
Out-of-the-money, short maturity

Relative error of Pure Jump model.
Total relative error = 155.05

Relative error of Mixed Jump-Diffusion model.
Total relative error = 86.48

Relative error of CEV model.
Total relative error = 92

Relative error of Stochastic volatility model.
Total relative error = 90.45
Relative error of Compound option model.

Total relative error = 91.30

Relative error of Displaced Diffusion model.

Total relative error = 91.92

Relative error of Nonzero Costs model.

Total relative error = 92

Relative error of GARCH(1, 1) model.

Total relative error = 90.68
Relative error of Black-Scholes model.

Total relative error = 90.68
Out-of-the-money, long maturity

Relative error of Pure Jump model.

Total relative error = 43.03

Relative error of Mixed Jump-Diffusion model.

Total relative error = 258.14

Relative error of CEV model.

Total relative error = 38

Relative error of Stochastic volatility model.

Total relative error = 36.17
Relative error of Compound option model.

Total relative error = 37.12

Relative error of Nonzero Costs model.

Total relative error = 38

Relative error of Displaced Diffusion model.

Total relative error = 37.81

Relative error of GARCH(1,1) model.

Total relative error = 36.42
Relative error of Black-Scholes model.

Total relative error = 36.42
Appendix D

The main program

% This function calculates the model option prices for the models I consider in my
% thesis, viz the Pure Jump, Mixed Jump-Diffusion, CEV, Stochastic volatility,
% Displaced Diffusion, Non-zero Transaction costs, and GARCH(1,1) models.

% The price of the S&P500 index at close on 15 August 2001 was
% So = 1178.02;
%
% Firstly, there are 194 market option prices that we consider. So, most vectors will
% be this length. So,
no_of_options = length(K);

% The most important calculation is that of the constant volatility estimate for
% 15 August 2001 using the most recent 11 index values.
returns = returns(1:10);

% We now calculate call option prices using the PURE JUMP MODEL.
% But first we need to a_star, u and lambda_PJ.
% By minimising the difference between the market call prices and Pure Jump model
% prices of the following three call options, we found that the optimal value of u and
% lambda_PJ are
u = 1.52;
lambda_PJ = 0.02;

% Now we calc. the Black-Scholes model option prices.

d1 = ( log(So /K) + ( r +0.5 .*sigma^2) .*tau) ./ (sigma .*sqrt(tau));
d2 = ( log(So /K) + ( r -0.5 .*sigma^2) .*tau)/ (sigma .*sqrt(tau));

% Now we calculate the Pure Jump option prices.
a_star = ceil(log(K./So).* log(u));
for i = 1:no_of_options
    first_sum = 0;
    second_sum = 0;
    % We only calculate up to the 100th term in each sum because we're dividing by j!,
    % and 100! is huge!!!!!
    for j = a_star(i):100
        first_sum = first_sum + (u.*lambda_PJ.*tau(i)).^j.*exp(-u.*lambda_PJ.*tau(i)) ./ prod(1:j);
        second_sum = second_sum + (lambda_PJ.*tau(i)).^j.*exp(-lambda_PJ.*tau(i)) ./ prod(1:j);
    end
    % Here we calc. all the errors between model and market price and add them.
    purejump_prices(i) = So.*first_sum - K(i).*exp(-lambda_PJ.*tau(i).* (u - 1)).*second_sum;
end

lambda_MJ = 0.27;
% Now, for the MIXED JUMP-DIFFUSION call option pricing formula.
d1 = ( log(So / K) + ( r +lambda_MJ +0.5.*sigma^2 ).*tau ) / (sigma .*sqrt(tau));
d2 = ( log(So / K) + ( r +lambda_MJ -0.5.*sigma^2 ).*tau ) / (sigma .*sqrt(tau));
mixedjumpyrices = So .*0.5 .*erfc(-d1./sqrt(2)) - K .*exp(-r.*tau).*0.5 .*erfc(-d2./sqrt(2));

% Now, we calc. the CEV call option prices.
 omega = [0 4];
 y = 4 .*r .*So ./ ( sigma^2 .*( 1 -exp(r .*tau) ));
 z = 4 .*r .*K ./ ( sigma^2 .*( -1 +exp(-r .*tau) ));
 for j = 1:2
     h = 1-2./3.*(omega(j) +y).* (omega(j) +3 .*y).* (omega(j) +2 .*y).^(-2);
     q1 = (omega(j) +2 .*y)./(omega(j) +y).^2;
     q2 = (h -1).*(1-3 .*h);
     top_q = 1 + h.*q1 -0.5 .*h.* (2 - h).*q2 - (q1.^2 -(q1.*2 -(2 .*h).*q2 - (-z ./ (omega(j) +y) ).^h;
     bottom_q = sqrt ( 2 .*h.^2 .*q1 .* (1 +q2).*q1);
     q(j,:) = (top_q ./bottom_q);
 end
% q(0) = q(1) and q(2) = q(4).
ccv_prices = So .*0.5 .*erfc(-q(2,:)./sqrt(2)) - K .*exp(-r .*tau).*0.5 .*erfc(-q(1,:)./sqrt(2));
% Now we calc. the STOCHASTIC VOLATILITY call option prices.

% First get the BS price. Already done above.

% Now calc. second term.

\[ xsi = 0.05; \]

\[ k = xsi^2 \cdot \sqrt{t}; \]

\[ \text{product1} = 0.5 \cdot \text{sqrt}(\text{tau}) \cdot \exp(-0.5 \cdot d_1^2) / \sqrt{2 \cdot \pi} \cdot \exp(-0.5 \cdot d_2^2) \cdot (d_1 \cdot d_2 - 1) / (4 \cdot \sigma^3); \]

\[ \text{product2} = 2 \cdot \sigma^4 \cdot (\exp(k) - k - 1) / k^2 \cdot \sigma^4; \]

\[ \text{Second_term} = \text{product1} \cdot \text{product2}; \]

% Now calc. third term.

\[ \text{product1} = \text{So} \cdot \text{sqrt}(\text{tau}) \cdot \exp(-0.5 \cdot d_1^2) / \sqrt{2 \cdot \pi} \cdot (d_1 \cdot d_2 - 1) \cdot (d_1 \cdot d_2 - 1) / (4 \cdot \sigma^5); \]

\[ \text{product2} = \sigma^6 \cdot (9 + 18 \cdot k) \cdot \exp(k) + (8 + 24 \cdot k + 18 \cdot k^2 + 6 \cdot k^3) / (3 \cdot k^3); \]

\[ \text{Third_term} = \text{product1} \cdot \text{product2}; \]

% Display Stochastic voly call option price.

\[ \text{stoch_prices} = \text{bs_price} + \text{Second_term} + \text{Third_term}; \]

% Now for the Compound option model prices.

% We define M = 1e6.
% Firstly, we use equations 2.35 and 2.39 to calc. V and sigmaV.

\[ [V \ sigmaV] = \text{Get_all}(M); \]

if \( V / M > 0 \)

\[ k = (\log(V / M) + (r - 0.5 \cdot \text{sigmaV}^2) \cdot \text{tau2}) / (\text{sigmaV} \cdot \sqrt{\text{tau2}}); \]

else

error('U are taking the log of a negative number')

end

\[ t_{\text{star}} = \text{funct_t_star}(V, k, \text{sigmaV}); \]

\[ \text{tau1} = t_{\text{star}} - t; \]

\[ V_{\text{bar}} = V_{\text{bar}}(V, \text{sigmaV}, M, t_{\text{star}}); \]

if \( V / V_{\text{bar}} > 0 \)

\[ h = (\log(V / V_{\text{bar}}) + (r - 0.5 \cdot \text{sigmaV}^2) \cdot \text{tau1}) / (\text{sigmaV} \cdot \sqrt{\text{tau1}}); \]

\[ \text{co_prices1} = V \cdot \text{BivNorm}(h + \text{sigmaV} \cdot \sqrt{\text{tau1}}, k + \text{sigmaV} \cdot \sqrt{\text{tau2}}, \sqrt{\text{tau1} / \text{tau2}}); \]
\[
\text{co\_prices2} = M \cdot \exp(-r \cdot \tau) \cdot \text{BivNorm}(h, k, \sqrt{\tau_1 \cdot \tau_2});
\]
\[
\text{co\_prices} = \text{co\_prices1} - \text{co\_prices2} - K \cdot \exp(-r \cdot \tau) \cdot 0.5 \cdot \text{erfc}(-h / \sqrt{2});
\]

```matlab
else
    error('You are taking the log of a negative number')
end
```

% Next, we calc. the DISPLACED DIFFUSION model call option prices.
% By minimising the difference between the market call prices and Displaced diffusion model prices of the following three call options, we found that the optimal value of alpha and beta are

\[
\alpha = 0.2;
\]
\[
\beta = 0.8;
\]

\[
a = \alpha \cdot (1 + \beta);
b = (1 - a) \cdot \exp(r \cdot \tau);
\]
\[
d1 = (\log(a \cdot S_0 - b \cdot S_0) + (r + 0.5 \cdot \sigma^2 \cdot \tau) / (\sigma \sqrt{\tau})) / \sigma \sqrt{\tau};
d2 = (\log(a \cdot S_0 - b \cdot S_0) + (r - 0.5 \cdot \sigma^2 \cdot \tau) / (\sigma \sqrt{\tau})) / \sigma \sqrt{\tau};
\]
\[
d1(195:251) = \text{imag}(d1(195:251));
d2(195:251) = \text{imag}(d2(195:251));
\]

% Display Displaced diffusion model call option prices.
\[
\text{disdiff\_prices} = a \cdot S_0 \cdot 0.5 \cdot \text{erfc}(-d1 / \sqrt{2}) - (K - b \cdot S_0) \cdot \exp(-r \cdot \tau) \cdot 0.5 \cdot \text{erfc}(-d2 / \sqrt{2});
\]

% Now we calc. model call option prices taking into account transaction costs.
% There is one problem and that is the initialisation of kappa. From speaking to traders in the market, it is generally agreed that kappa is almost never greater than 0.1% of the value of any traded financial instrument. So,

\[
kappa = 0.001;
\]

% Initialise
\[
dt = 0.01;
dx = 0.01;
\]
\[
\text{sigma\_bar\_squared} = \sigma^2 - 2 \cdot \kappa \cdot \sigma \cdot \sqrt{2 / (\pi \cdot dt)};
\]
\[
S_{\text{max}} = 2000;
x_0 = \log(S_0);
\]
\[
\text{spatial\_steps} = \text{ceil}(\log(S_{\text{max}} / S_0) / dx);
\]
\[
x = x_0 - \text{spatial\_steps} \cdot dx : dx : x_0 + \text{spatial\_steps} \cdot dx;
\]
\[
N = \text{length}(x);
\]

global nu rm rd N
for i = 1:no_of_options
    v = r(i) -0.5.*sigma_bar_squared;
    ru = -0.25.*dt.*( sigma_bar_squared /dx^2 + v /dx);
    rm = 1 +0.5.*dt.*( sigma_bar_squared /dx^2 + r(i));
    rd = ru + 0.5.*v.*dt/dx;

    Q = Get_Q,
    R =Get_R;

    % ICs: The final condition (which has become an initial condition) is V(0,S)=+(S-K).
    % So, V(0,x) = max(exp(x)-K, 0). The values I obtain in Excel are correct because
    % I have checked it in Matlab.
    solution = max(exp(x)-K(i).*ones(1,N), 0);

    % We loop over time T/dt many times, from the start of the option to expiry.
    for t = 1:tau(i)/dt
        solution = inv(Q) *R *solution;
    end

    % Now display the prices in column ...
    nonzeroTC_prices(i) = solution(spatial_steps+1);
end

% Now we calc. the GARCH(1,1) model call option prices. From a different function, we have
% that
kappa = 0.0000067889;
alpha = 0.0868;
beta = 0.8917;

% Thus, by GARCH, we have a better estimate of the volatility for the next 10-day period
% starting on 15 August 2001.

sigma = sqrt( kappa +alpha*returns(1)^2 +beta*sigma^2);

% So, basically, we calculate the BS price with THIS value of sigma...
for i = 1:no_of_options
    d1 = ( log(So /K) + ( r +0.5.*sigma^2) .*tau) /
          (sigma.*sqrt(tau));
    d2 = ( log(So /K) + ( r -0.5.*sigma^2) .*tau) /
          (sigma.*sqrt(tau));

    GARCH_prices = So.*0.5.*erfc(-d1./sqrt(2)) -K.*exp(-r.*tau).*0.5.*erfc(-d2./sqrt(2));
end

% Display the GARCH(1,1) model call option prices...
The pure jump model parameters.

% The aim of this function is to calculate the value of u in the Pure Jump model that % makes the model prices of the two calls equal to their market value.

%Initialise

So = 1178.02;  
sigma = 0.0520;  
no_of_options = 2;  
best_abs_error = 1000000;  
K=strikes;  
r=rates;  
candidates = 0;  
for u = 1:0:0.01:2  
    for lambda_PJ = 0:0.01:1  
        disp(u)  
        disp(lambda_PJ)  
    
        %But first we need to get a_star  
        a_star = ceil(log(K/So)* log(u));  
    
        %Now we calculate the Pure Jump option prices  
        for i = 1:no_of_options  
            first_sum = 0;  
            second_sum = 0;  
            abs_error = 0;  

            %We only calculate up to the 100th term in each sum because we're dividing by j!,  
            %and 100! is huge!!!!!  

            for j = a_star(i):100  

first_sum = first_sum + (u * lambda_PJ * tau(i)).^j* exp(-u * lambda_PJ * tau(i)) / prod(1:j);
second_sum = second_sum + (lambda_PJ * tau(i)).^j* exp(-lambda_PJ * tau(i)) / prod(1:j);

end

% Here we calc. all the errors between model and market and add them.
modelprice(i) = So * first_sum - K(i)* exp(-lambda_PJ * tau(i) * (u - 1)) * second_sum;
abs_error = sum(abs(marketprices(i) - modelprice(i)));
end

% Keep a record of the best u and lambda_PJ.
if abs_error < best_abs_error
    candidates = candidates + 1;
    best_abs_error = abs_error;
    best_u = u;
    best_lambda_PJ = lambda_PJ;
end
end

% Display the best values...
disp(candidates)
disp(best_abs_error)
disp(best_u)
disp(best_lambda_PJ)

The Mixed jump and Stochastic volatility model parameters.

% This function finds the optimal values of xsi and lambda_MJ which minimise the % difference between market option price and model price.
best_abs_error_stoch = 1e6;
best_abs_error_lambda = 1e6;

for variable = 0:0.01:2
    xsi = variable;
    lambda_MJ = variable;
    % Now we calc. the STOCHASTIC VOLATILITY call option prices
    % First get the BS price.
\[
\begin{align*}
\text{d1} &= \frac{\log(S_0/K) + (r + \lambda_{MJ} + 0.5 \times \sigma^2) \times \tau}{\sigma \sqrt{\tau}}; \\
\text{d2} &= \frac{\log(S_0/K) + (r + \lambda_{MJ} - 0.5 \times \sigma^2) \times \tau}{\sigma \sqrt{\tau}}; \\
\text{bs\_price} &= S_0 \times \text{erfc}\left(-\frac{\text{d1}}{\sqrt{2}}\right) - K \times \exp\left(-r \times \tau\right) \times \text{erfc}\left(-\frac{\text{d2}}{\sqrt{2}}\right); \\
\% \text{ Now calc. second term.} \\
\text{xsi} &= 0.05; \quad \% \text{See how well model prices calls to see whether need change xsi} \\
\text{k} &= \text{xsi}^2 \times \sigma \sqrt{\tau}; \\
\text{product1} &= 0.5 \times S_0 \times \text{sqrt}(\text{tau}) \times \exp(-0.5 \times \text{d1}^2) \times \sqrt{\text{2}\pi} \times (\text{d1} \times \text{d2} - 1) \times (4 \times \sigma^3); \\
\text{product2} &= 2 \times \sigma^4 \times (\exp(\text{k}) - \text{k}) \times \text{k}^2 \times \text{sqrt}(\text{tau}); \\
\text{Second\_term} &= \text{product1} \times \text{product2}; \\
\% \text{ Now calc. third term.} \\
\text{product1} &= S_0 \times \text{sqrt}(\text{tau}) \times \exp(-0.5 \times \text{d1}^2) \times \sqrt{\text{2}\pi} \times ((\text{d1} \times \text{d2} - 3) \times (\text{d1} \times \text{d2} - 1) \times (\text{d1} + \text{d2} + 2)) \times (48 \times \sigma^5); \\
\text{product2} &= \sigma^6 \times (\exp(3 \times \text{k}) - (9 + 18 \times \text{k}) \times \exp(\text{k}) + (8 + 24 \times \text{k} + 18 \times \text{k}^2 + 6 \times \text{k}^3)) \times (3 \times \text{k}^3); \\
\text{Third\_term} &= \text{product1} \times \text{product2}; \\
\% \text{ Display Stochastic voly call option price.} \\
\text{stoch\_prices} &= \text{bs\_price} + \text{Second\_term} + \text{Third\_term}; \\
\text{abs\_error\_stoch} &= \text{sum}((\text{marketprices} - \text{stoch\_prices}) / \text{marketprices}); \\
\% \text{Keep a record of the best xsi.} \\
\text{if abs\_error\_stoch < best\_abs\_error\_stoch} \\
&\hspace{1cm} \text{best\_abs\_error\_stoch} = \text{abs\_error\_stoch}; \\
&\hspace{1cm} \text{best\_xsi} = \text{xsi}; \\
\text{end} \\
\% \text{ Now, for the MIXED JUMP-DIFFUSION call option pricing formula.} \\
\text{d1} &= \frac{\log(S_0/K) + (r + \lambda_{MJ} + 0.5 \times \sigma^2) \times \tau}{\sigma \sqrt{\tau}}; \\
\text{d2} &= \frac{\log(S_0/K) + (r + \lambda_{MJ} - 0.5 \times \sigma^2) \times \tau}{\sigma \sqrt{\tau}}; \\
\text{mixedjump\_prices} &= S_0 \times \text{erfc}\left(-\frac{\text{d1}}{\sqrt{2}}\right) - K \times \exp\left(-r \times \tau\right) \times \text{erfc}\left(-\frac{\text{d2}}{\sqrt{2}}\right); \\
\text{abs\_error\_lambda} &= \text{sum}((\text{marketprices} - \text{mixedjump\_prices}) / \text{marketprices}); \\
\% \text{Keep a record of the best lambda\_MJ.} \\
\text{if abs\_error\_lambda < best\_abs\_error\_lambda} \\
&\hspace{1cm} \text{best\_abs\_error\_lambda} = \text{abs\_error\_lambda}; \\
&\hspace{1cm} \text{best\_lambda} = \text{lambda\_MJ}; \\
\text{end}
\end{align*}
\]
 Compound option model parameters

function y_new = Get_all(M)
% This function calculates the value of V that satisfies eqn 2.35 for the particular value of M.
% First get the value of V. Use that to get the value of \( \sigma V \), and so on...
global T t tau2 r K tau sigma

So = 1178.02;
V_old = M/2 +1000;
sigmaV_old = 0.5;
V_new = M/2;
sigmaV_new = 0.05 +0.1;
count = 0;
necessary_V = 1;
necessary_sigmaV =1;
necessary = 1;

while necessary %& count < 50

necessary_V = 0;
necessary_sigmaV = 0;

% Second get the value of V if necessary.

countV =0;

while abs (V_new -V_old) > 0.01 | V_new < 0%& countV <50

necessary_V = 1;
countV = countV +1
V_old = V_new;

if V_old/M > 0
A program for calculating the value of \( \sigma \) if necessary. Details of the program are described in the text that follows.

1. Initialize:
   - \( \sigma_{old} \)
   - \( \sigma_{new} = 0 \)
   - \( V_{new} = \log(V_{old}/M) + \left( r - 0.5 \cdot \sigma_{old} \cdot \tau^2 \right) / \tau \)
   - \( k = \log(V_{new}/M) + \left( r - 0.5 \cdot \sigma_{new} \cdot \tau^2 \right) / \tau \)

2. Calculate:
   - \( \text{dist2} = 0.5 \cdot \text{erfc}(k + \sigma_{old} \cdot \sqrt{\tau^2}) / \tau \)
   - \( \text{dist}_{2,prime} = 0.5 \cdot \text{erfc}(k + \sigma_{old} \cdot \sqrt{\tau^2}) / \tau \)
   - \( \text{top} = \sigma_{new} \cdot \text{dist2} \cdot \sqrt{\tau^2} \)
   - \( \text{bottom} = \text{dist2}_{prime} \cdot \sqrt{\tau^2} \)

3. Check:
   - If \( \text{bottom} = 0 \):
     - \( \sigma_{new} = \sigma_{old} \cdot \text{dist2} \cdot \sqrt{\tau^2} \)
   - Else:
     - \( \sigma_{new} = \sigma_{old} \cdot \text{dist2} \cdot \sqrt{\tau^2} \)
end

else
    \( \sigma_V' = \sigma_V' \) + 0.02;
end

end

\text{necessary} = \text{necessary} + \text{necessary} \cdot \sigma_V;
\text{count} = \text{count} + 1

end

\text{y_new} = [V_{new} \sigma_V_{new}];

-----------------------------------

function \text{y_new} = \text{V_bar}(V, \sigma_V, M, t_{star})

\% This function calculates the value of \text{V_bar} which is the solution to the integral equation
\% S - K = 0.
\% Use Newton's method with any starting point because we know there is a unique solution.

global T t tau2 r K tau

y_old = V;
y_new = V + 100;
count_{V_bar} = 0;

while abs(y_old - y_new) > 0.01 & count_{V_bar} < 1e3
    y_old = y_new;
    if y_old / M > 0
        k = ( log(y_old / M) + (r - 0.5 \cdot \sigma_V^2) \cdot \tau^2 ) / (\sigma_V \cdot \sqrt{\tau^2});
        top = y_old \cdot 0.5 \cdot \text{erfc}(-k) \cdot \text{erfc}(T-a_2) / (\sqrt{2} \cdot \pi) - M \cdot \exp(-r \cdot \sqrt{T - a_2}) / (\sqrt{2} \cdot \pi);
        bottom1 = 0.5 \cdot \text{erfc}(-k) \cdot \text{erfc}(T-a_2) / (\sqrt{2} \cdot \pi);
        bottom2 = y_old \cdot \exp(-0.5 \cdot (k + \sigma_V \cdot \sqrt{T - a_2})^2) \cdot (\sqrt{2} \cdot \pi) / (\sqrt{T - a_2})^2;
        bottom = bottom1 + bottom2 - M \cdot \text{erfc}(-0.5 \cdot (k^2) \cdot \sqrt{2} \cdot \pi);
        y_new = y_old - top / bottom;
        count_{V_bar} = count_{V_bar} + 1;
    else
        y_new = y_old + 2;
    end
end
function y = BivNorm(a,b,rho)

% This approximates the Bivariate normal distribution.

A = [0.3253030 0.4211071 0.1334425 0.006374323];
B = [0.1337764 0.6243247 1.3425378 2.2626645];

a_prime = a ./ sqrt(2.*(1 - rho.^2));
b_prime = b ./ sqrt(2.*(1 - rho.^2));

y = 0;
for i = 1:4
    for j = 1:4
        sum = 0;
        first = a_prime.*(2.*B(i) - a_prime) + b_prime.*(2.*B(j) - b_prime);
        second = 2.*rho.*(B(i) -a_prime).* (B(j) -b_prime);
        f = exp( first +second);
        sum = sum + A(i).*f;
    end
    y = y +A(i).*sum;
end

y = sqrt((1 -rho.^2)/pi).*y;

--------------------------------------------------------

The Displaced diffusion parameters

% The aim of this function is to calculate the value of alpha and beta in the Displaced
% Diffusion model that makes the model value equal to the market value.

% Initialise

So = 1178.02;
K=900;
tau=0.849315068;
r = 0.014436528;
sigma = 0.0520;
count = 0;
for alpha = 0:0.01:2
  for beta = 0:0.01:2
    a = alpha * (1 + beta);
    b = (1 - a) * exp(r * tau);
    if (a * So / (K - b * So)) > 0
      count = count + 1;
      d1 = (log(a * So / (K - b * So)) + (r + 0.5 * sigma ^ 2) * tau) / (sigma * sqrt(tau));
      d2 = (log(a * So / (K - b * So)) + (r - 0.5 * sigma ^ 2) * tau) / (sigma * sqrt(tau));
      term1 = a * So * 0.5 * erfc(-d1/sqrt(2));
      DD_price = term1 - (K - b * So) * exp(-r * tau) * 0.5 * erfc(-d2/sqrt(2));
      end
  end
end

The Nonzero Transaction costs model

function y = Get_Q
% This function calculates the Q matrix in the Crank-Nicolson matrix equation.

global rd rm ru N
a = rd*ones(1,N-1);
b = rm*ones(1,N);
c = ru*ones(1,N-1);
Q = diag(a,-1) + diag(b) + diag(c,1);
Q(1,:) = Qrow1;
Q(N,:) = QrowN;
y = Q;

function y = Qrow1
% This function sets up the first row (which is a 1 x N vector) of the Q matrix as determined from
% the Crank-Nicolson equation as well as the left boundary condition,
% viz c_{SS}(t,0) = 0.

global ru rm rd N
firstrow(1) = rd;
firstrow(2) = ru +rm;
y = [firstrow(1), firstrow(2), zeros(1, N-2)];

function y = QrowN(N, ru, rm, rd)
%
% This function sets up the last row (which is a 1 x N vector) of the Q matrix as determined from
% the Cranck-Nicolson equation as well as the left boundary condition,
% viz c_{S}(t,X) = 1.
%
global ru rm rd N

lastrow(N) = ru - rd;
lastrow(N-1) = rm + 2*rd;
y = [zeros(1, N-2), lastrow(N-1), lastrow(N)];

function y = Get_R
%
% This function calculates the R matrix in the Crank-Nicolson matrix equation.
%
global rd rm ru N

u = -rd * ones(1, N-1);
v = (2 - rm) * ones(1, N);
w = -ru * ones(1, N-1);

R = diag(u, -1) + diag(v) + diag(w, 1);
R(1, :) = Rrow1;
R(N, :) = RrowN;
y = R;

function y = Rrow1(N, ru, rm, rd)
%
% This function sets up the first row (which is a 1 x N vector) of the R matrix as determined from
% the Cranck-Nicolson equation as well as the left boundary condition,
% viz c_{SS}(t, 0) = 0.
%
global ru rm rd N

firstrow(1) = -rd;
firstrow(2) = -ru + 2 - rm;
y = [firstrow(1), firstrow(2), zeros(1, N-2)];

function y = RrowN(N, ru, rm, rd)
%
% This function sets up the last row (which is a 1 x N vector) of the R matrix as determined from
% the Cranck-Nicolson equation as well as the left boundary condition,
% viz c_{(S)}(t, X) = 1.
%
global ru rm rd N

lastrow(N) = ru + rd;
lastrow(N-1) = (2 - rm) - 2*rd;
\( y = \text{zeros}(1,N-2), \text{lastrow}(N-1), \text{lastrow}(N) \); \\

\\

\textbf{The term structure of volatility}\\

\% This function calculates the term structure of volatility from the S&P500 index history.\\
\% First, we have to invert it.\\
\l = \text{length}(\text{SP500});\\
for i = 1:l \\
\quad \text{index}(i) = \text{SP500}(i+1);\\
end \\
\text{plot}(\text{index}), \text{pause} \\
\% Let's calculate the 10-day vols.\\
for i = 1:l-10 \\
\quad u\_\text{bar} = \text{sum}((\text{returns}(i:i+9))./\text{length}(\text{returns}(i:i+9)));\\
\quad \text{SSD} = \text{sum}((\text{return}(i:i+9) - u\_\text{bar})^2); \\
\quad \text{sigma\_10day}(i) = \text{sqrt}((\text{SSD} \times 262/(\text{length}(\text{return}(i:i+9))-1));\\
end