Absolute Electrical Impedance Tomography and Spectroscopy

An Orthogonal Chirp Division Multiplexed (OCDM) Approach

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February 2021

A thesis submitted to the Department of Electrical Engineering, at the University of Cape Town, in fulfilment of the academic requirements for a Degree of Doctor of Philosophy in Electrical Engineering.
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Declaration

I declare that this thesis is my own work in substance. All assistance that I have received is gratefully acknowledged and detailed on page iv. It is being submitted for the degree of Doctor of Philosophy in the University of Cape Town. It has not been submitted before for any degree or examination in any other university.

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Signed by candidate

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February 2021
Abstract

Absolute Electrical Impedance Tomography and Spectroscopy (aEITS) is a non-intrusive imaging technique, that reconstructs images based on estimates of the absolute internal impedance distribution of an object. However, without the availability of a reference frame, it suffers from poor image quality when general assumptions are used to form the prior information about the object. This problem is intensified when selecting a multiplexing technique that introduces significant data inconsistencies.

Recent attempts to solve this problem are to use data from previous empirical studies that acquired scans from Magnetic Resonance Imaging (MRI). Another approach is to use statistical methods to estimate the boundaries of the expected internal domains of the object. These approaches have shown an improvement in the reconstructed images, but either rely on data from other imaging modalities or continue to use a reference frame taken at an earlier time. Therefore, this is a non-trivial problem.

In this thesis, the concept of Orthogonal Chirp Division Multiplexed aEITS (OCDM-aEITS) is introduced as an alternative multiplexing technique. OCDM-aEITS allows the simultaneous application of orthogonal wideband chirp current waveforms at all stimulation electrodes, while measuring the resultant boundary potentials. Given a single wideband measurement frame, a reference set, prior information, and several absolute images can be reconstructed.

Consequently, there no longer is a need to acquire reference data, from an earlier time, or prior information from other imaging modalities. Furthermore, OCDM-aEITS overcomes some of the data inconsistencies from other multiplexing techniques (such as the data inconsistencies caused by sequential stimulation or spikes from fast pseudorandom pulse stimulation), while reconstructing images with comparable quality to those in the related literature. The experimental results from this thesis (acquired from the reconstructed images of a phantom test tank containing biological specimen), achieved an average position and size error of 3.88 % and 2.49 %, respectively.
As I approach the end of this project, I am given the opportunity to ponder on the many forces that have come together to help me reach this point.

God deserves all the glory. He said yes when others said no. He sustained my inspiration when I thought that I had run empty. He guided me to a place that I never thought I could be.

To my family and friends: You have stood together as an undying force behind my success. You have provided unrivalled encouragement, support, critique, and love. And because of your involvement, I have the confidence to pursue my dreams.

Dr. M. S. Tšoeu was my academic supervisor. I have deep respect for his abilities as a supervisor and lecturer. Gratitude is paid to him. Furthermore, I recommend Dr. Tšoeu for anyone pursuing academic supervision. He provides the academic freedom, to follow the solution wherever it grips you, and sound advice for presenting exceptional research.

Finally, I would like to express gratitude to the Wilhelm Frank Trust Doctoral Scholarship, for their financial assistance. Their assistance has made my research journey more bearable. I hope and pray that they will continue to support talented students to achieve great milestones.
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Nomenclature

**EITS**: Electrical Impedance Tomography and Spectroscopy. A technique of imaging an object based on the internal impedance distribution by stimulating the boundary using low amplitude signals, and the measurement of the resultant boundary potentials.

**aEITS**: Absolute EITS. Like EITS, but the images are reconstructed from the absolute impedance distribution from a single measurement frame and a previously acquired reference frame.

**Orthogonal**: A term used to describe two or more objects that are statistically independent.

**OCDM**: Orthogonal Chirp Division Multiplexing. A technique of generating multiple orthogonal chirp signals that operate at the same period-bandwidth product. These signals are used to stimulate a multi-channel medium. At the receiving end of each channel, the signal is demodulated using matched filters, or by cross correlation.

**Prior information**: Information about an object under test to regularize and, therefore, stabilize the inverse solution of EITS.

**OCDM-prior**: A regularization matrix that incorporates prior information. This prior information is developed from a single wideband OCDM measurement frame.

**MATLAB**: A computer programme for developing mathematical functions that are applied to datasets.

**EIDORS**: Electrical Impedance and Diffuse Optics Reconstruction Software. A library commonly used with MATLAB to simulate and reconstruct images using EITS.

**FPGA**: Field Programmable Gate Array. A programmable device based on manipulating logic elements to perform an intended task.

**Stimulation**: Stimulation refers to the injection of current waveforms at the current electrodes, attached to the object being imaged.
This thesis involves the concept introduction, proof of concept, system design, implementation, and prototype phantom testing, of an Orthogonal Chirp Division Multiplexed Absolute Electrical Impedance Tomography and Spectroscopy (OCDM-aEITS) system.

Absolute Electrical Impedance Tomography and Spectroscopy (aEITS) is a non-intrusive imaging technique that generates images of cross-sections of an object. These images are reconstructed from estimates of its internal electrical properties [1], like the absolute internal impedance distribution. Excitation currents are applied to the object, through the surface mounted electrodes. The internal current distribution establishes resultant boundary potentials, commonly measured at the remaining surface mounted electrodes. The excitation currents and measured boundary potentials are used to estimate the internal impedance distribution [2]. To reconstruct images, of the internal impedance distribution, the following steps are routinely followed.

a) A forward model of the object and imaging system is derived to hold significant properties of the test object. These include the geometric dimensions of the object and electrodes, the current injection protocol, and the boundary voltage measurement protocol [1]. These properties are used when computing a sensitivity matrix, known as the Jacobian matrix. This matrix describes how the boundary potentials vary due to a change in the internal impedance distribution [3], [4] (when using the EITS sensitivity approach).

b) The next step is to solve the inverse problem of EITS. This involves inverting the Jacobian matrix before it is multiplied to the measured boundary potential vector to form a matrix of estimated internal impedances [5] (for a linearized EITS problem). The estimated internal impedance distribution is used to reconstruct an image of the internal electrical properties of the object.

The advantages of EITS include a more reasonable comparable cost to setup, radiation-free and fast imaging, portability, and a non-intrusive characteristic. A few fields that benefit from the advantages of EITS are the biomedical, industrial, and geophysical fields [1]. In the biomedical imaging field, EITS is employed to detect, monitor, and classify malignant tissue. In addition, EITS is used to measure the brain function, monitor cardiac systems, detect a haemorrhage, or image the thorax. In the industrial fields, EITS is utilized as a means of non-
destructive testing to detect cracks, image fluid flows and fluid distribution characterization in pipelines and mixing vessels. In the geophysical fields, EITS is used to measure the geophysical surfaces [6], cross boreholes [7], and prospecting [8].

Additionally, EITS present a non-linear and ill-posed problem, which involves computing many unknown internal variables of an object provided a significantly smaller dataset measured at distinct points on the physically limited boundary. Furthermore, one of the most widespread problems associated with EITS is the selection of the multiplexing technique. Frequently, Time Division Multiplexing (TDM), Frequency Division Multiplexing (FDM), or Code Division Multiplexing (CDM) is used. All of which introduce specific and significant data inconsistencies or signal contaminations.

TDM exhibits data inconsistencies when applied to specimens that have a fast changing or frequency sensitive impedance distribution, such as imaging the neuron firing in the brain. In [9], a novel time-difference approach is introduced to tackle the issue of inefficiently exploiting the multi-frequency information. This is achieved by imposing spectral constraints on the framework of a linear least squares problem. By increasing the step-iteration, the rank increases, and the image noise is reduced, which results in less position and size deformation errors. However, this method only works in cases where the spectral constraints are known. Furthermore, the results indicate that the system was asymmetrical when comparing the reconstructed images for anomalies placed in diametrically opposite positions. Further analysis of the results shows that the algorithm (Used in [9]) does not provide adequate detectability of multiple anomalies of different impedances within the tank. On the other hand, FDM simultaneously stimulates all current excitation electrodes with sinusoids. Each sinusoid operates at a unique frequency within the system band. Therefore, each electrode is stimulated at a specific frequency. This may cause inconsistencies in the measurement of boundary voltages when complex impedances are measured. Converse to FDM, CDM simultaneously applies wideband pseudorandom pulses to each current excitation electrode. This achieves a more uniform current distribution within the object. However, the pulses introduce capacitive signal spikes that are propagated to the measurement impedances, especially at high frequencies. These signal contaminations cause detrimental effects on the reconstructed images, such as a poor detectability or the introduction of image artefacts as revealed in [10], [11] for multi-source FDM systems. Although these approaches may introduce data inconsistencies or high-frequency signal contaminations, a flawless technique that does not suffer from any drawbacks does not exist. As the nature of engineering, some methods may be preferred over others due to accuracy, simplicity, or precision. As the nature of applied science, these techniques were not uniquely designed for EITS, instead they were adopted from other disciplines to benefit the field. Therefore, the aim of this research is not to
introduce a perfect technique, rather, it is to demonstrate a well-balanced and efficient technique to further benefit the field.

In addition, there are two conventional forms of EITs known as absolute or difference EITs. Absolute EITs assemble cross-sectional images of an object based on the absolute impedance distribution. And it can produce relatively high-quality images [12], [13], when provided with reliable prior information and a reference frame. However, this method does not perform well when these conditions are unsatisfied. On the other hand, difference imaging reconstructs images from the change of an objects’ impedance distribution over time or frequency. However, difference imaging requires several measurement frames to produce high-quality images and to improve the detectability. Therefore, to select which method of imaging to use, the current state of EITs must be established.

1.1 The Current State of EITs

A few intrinsic problems to EITs have been solved over the past decades. These include using difference imaging to alleviate image artefacts (that is caused by inaccuracies that result from insufficient skin contact of surface electrodes, impedance transfer or inter-individual anatomy) by simple image subtraction. Additionally, absolute EITs image reconstructions have been improved by incorporating, into the regularization matrix, basis constraint functions or data from previous empirical studies [14]. Furthermore, CDM was introduced to improve the frame rate of wideband EITs systems and ensure equal energy distribution, while overcoming the data inconsistencies of TDM and FDM.

Most recently, the field of EITs has largely shifted focus towards biological imaging and clinical studies. In [15], imaging of the neural activity over milliseconds throughout the brain was achieved. In [16], the investigation was set out to image the brain during a stroke. This was achieved by using capacitively coupled EITs. In [17], the aim was to quantify the frequency limits of an EITs carrier signal applied to a biological specimen. In [18], EITs was applied to advance hand gesture recognition, and in [19] rotational EIT was introduced to produce spatially accurate absolute reconstructions to image phantom breast-shaped tanks.

Subsequent research in EITs is presumed to continue with clinical testing. Therefore, some gaps in literature or disadvantages in EITs that need to be addressed (in academia) are the spatial resolution, effects of the electrode contact impedances, system efficiency and independency. When comparing EITs to more conventional imaging methods, such as Computed-tomography (CT) and X-rays, it still suffers from low spatial resolution.
Furthermore, for absolute EITS, prior information is compiled from previous empirical studies. These studies involve acquiring the average dataset from multiple images from more conventional forms of biological imaging. And makes absolute EITS dependent on data produced by other methods. This, therefore, leads to the questions that this thesis aims to answer.

1.2 Problem Statements and Research Hypotheses

Absolute EITS involves reconstructing images from the computation of the absolute internal impedance distribution (from a single measurement frame) [1]. However, without the availability of a reference frame, the image quality suffers when general assumptions are used to form the prior information about the object. This is exacerbated if the image reconstructions are limited to a few iterations. To improve the quality of the image reconstructions, statistical functions or data from previous empirical studies are incorporated into a regularization matrix, when using regularization. In one such study, the prior information was developed from 8000 chest CT scans across 800 patients and incorporated statistical shape-constraints [14]. In [20], the use of structure-aware sparse Bayesian Learning to construct the regularization matrix was proposed. In [21], a domain dependent deep D-bar learning method was introduced. This method involves the application of post-processing on the images that are reconstructed by a neural network based absolute EIT. These approaches have shown an improvement in the reconstructed images. Nevertheless, they continue relying on other imaging modalities, or a reference frame taken at an earlier time.

To the best of the author’s knowledge, no extant research has successfully explored a method to reconstruct absolute images and the prior information from a single wideband measurement frame, without the availability of a time different reference frame. Therefore, the approach considered in this thesis is to observe the frequency behaviour of the object using a heatmap of the computed channel impulse responses. Then identify frequencies at which the frame can be segmented to develop the regularization matrix. This removes the dependency of EITS on other imaging modalities and time different reference frames. Moreover, to observe the frequency characterization of an object, wideband excitation at all stimulation electrodes is preferred. This approach is presumed to allow the detection and observation of the possible internal structures of an object as images may be reconstructed over the entire frequency band. And by observing the characteristics of several domains within the object, one could identify the anomalies. Additionally, given that the regularization matrix is developed from a frequency-differenced set of information about the object (which includes information about
any internal structures or inhomogeneities of the object), fewer elements of a finite element model are reconstructed. Therefore, the rate of convergence of the solution improves, and the solution stabilizes.

One proposal to acquire the multi-frequency data is to use CDM. However, CDM commonly outputs sharp rectangular waves that may cause signal spikes at high frequencies. These signal spikes propagate to the channel measurement impedances, which creates image artefacts. Alternatively, FDM excites different electrodes at a different frequency. However, some signals possess more excessive energies that propagate further through the object, compared to lower frequency signals. Therefore, the impedance distribution is significantly asymmetrical which may cause large boundary measurement inconsistencies if complex impedances are to be measured. The next logical step is to use sinusoidal chirp signals. However, time multiplexing a single chirp signal will reduce the frame rate of the system, which makes it too slow for fast changing biological specimen. And the application of several chirp waveforms to the electrodes will cause inter-chirp interferences if they share the bandwidth and period and are not orthogonal. Therefore, an efficient method would be to generate orthogonal chirp waveforms that have the comparable period and bandwidth. This is the basis behind Orthogonal Chirp Division Multiplexing (OCDM) [22].

Therefore, below are the hypotheses of this thesis to tackle a few of the highlighted problems associated with absolute EITS:

1. Orthogonal Chirp Division Multiplexing (OCDM) can be used to simultaneously apply wideband orthogonal chirp current signals to the electrodes, attached to the boundary of a multi-channel electrically conductive object. Simultaneously, the resultant boundary potentials can be measured and cross-correlated with the stimulation currents to acquire the channel impulse responses. These responses are used to reconstruct images of the impedance distribution of the object from a single measurement frame.

2. The prior information can be developed from a single OCDM wideband measurement frame. Given this prior information, it is possible to improve the quality and rate of convergence of the absolute image reconstructions of an object. Compared to absolute image reconstructions that utilize generic prior information and do not have a reference frame. This is achieved by incorporating the OCDM information into the regularization matrix.
1.3 Objectives of this Study

In addition, to answer the research questions and test the hypotheses, a list of objectives needs to be established and fulfilled throughout this thesis. The first problem to be investigated in this research is to relate the concept of OCDM to aEITS. The OCDM method will allow one to observe the frequency characteristics of an object, at all measurement electrodes. This provides an opportunity to incorporate the impulse response data into the regularization matrix to enhance the performance of absolute image reconstructions.

Therefore, the following objectives need to be addressed to provide an adequate solution:

- Critically analyze existing literature, to acquire an understanding of the forward and inverse problems of EITS.
- Investigate recent works about absolute EITS and the development of prior information.
- Critically analyze the various configurations and designs of EITS systems.
- Present the concept of OCDM.
- Design a working EITS system, to answer the research questions, that is cost-effective and portable.
- Develop an understanding of various nonlinear image reconstruction algorithms and multiplexing techniques to control and regularize the inverse problem and to efficiently apply low amplitude orthogonal current waveforms.
- Provide a critical analysis of the results of the investigation.
- Provide recommendations for future research while outlining the practical and theoretical advantages and any significant drawbacks experienced during the investigation.
1.4 Scope of the Investigation

Given that the predefined objectives are broad in nature, a scope needs to be established to explicitly outline the focus of this thesis. Therefore, this thesis is limited to the following scope:

- Only 2D images will be reconstructed.
- This study does not present the development of an image reconstruction software. Existing software will be used. However, custom signal generation and data acquisition code will be developed when required.
- Although difference imaging may be used to highlight a comparison with the proposed method, it will not be the focus of this work. Absolute EITS is the focus of this work.
- Even though there are discussions about the application of this research to biological specimen, clinical trials are not explored.
- Continuing from the previous point, this work will not investigate how the electrodes move relative to one another, as would be the case for thoracic or abdomen measurements.
- The excitation waveforms will be in the form of low amplitude chirp signals only.
- The components of the prototype system will be made from easily accessible and cost-effective parts. Furthermore, the system is designed as an adequate system to answer the research questions, therefore, it is not intended to be the design with the best hardware capabilities.
1.5 Research Contributions

With the established scope of this thesis, the key contributions delivered by the author to the field are discussed in this section. While OCDM has been applied to telecommunications, it has not been applied to EITs to simultaneously excite the electrodes using several orthogonal chirp waveforms (that operate at the same period-bandwidth combination), and demultiplexing the resultant boundary potentials to acquire channel impulse responses. Additionally, to the best of the author’s knowledge, no work uses a single measurement frame from OCDM-aEITS to acquire prior information. This information is utilized to form a regularization matrix that enhances absolute image reconstructions, without the use of a time-different reference frame. This advances absolute EIT toward self-reliance, compared to other approaches that rely on data from alternative imaging modalities and statistical methods of constraining the solution. Therefore, this work extends the concept of OCDM to the field of EITs, as a competitive multiplexing technique that can be utilized to reconstruct multiple absolute images from a single measurement frame. Consequently, this thesis provides proof of the application of OCDM and exposes the challenges, advantages, and disadvantages thereof that will certainly facilitate future work.

Additionally, TDM suffers from low frame rates due to its sequential stimulation and measurement approach. This results in temporal inconsistencies when it is applied to rapidly changing or frequency dependent systems, such as imaging the neuron-firing in the brain. FDM simultaneously stimulates an object at all stimulation electrodes. However, each electrode is stimulated at a unique frequency to maintain orthogonality. This causes some electrodes to be excited at higher energies that propagate further through the object, which may cause an intolerable asymmetrical distribution. This result in significant data inconsistencies if complex impedances are measured. Alternatively, CDM excites all stimulation electrodes with wideband pseudorandom codes. However, at high frequencies, signal spikes that propagate to the impedance measurements are observed, when using pseudorandom rectangular pulse waveforms. In contrast, this work introduces a method that inherits the benefits of these widely used multiplexing techniques while negating the drawbacks, by efficiently applying smooth wideband chirp current signals at the electrodes. Therefore, the OCDM approach is targeted at applications that require imaging of the brain activity under certain psychological states.

Finally, discussions are provided on OCDM-aEITS, and generic-aEITS to highlight and compare the image quality of these methods.
1.6 Planning of Deliverables

To successfully complete the investigation, the following plan is proposed.

Figure 1.1 Proposed plan of execution of the investigation.

Figure 1.1 presents the process that will be followed to ensure successful completion of the research. The diagram starts with the formulation of the governing theoretical concepts, which need to be tested throughout the investigation. Subsequently, research constraints are established to identify the scope of work. The project then divides into concept simulations and system development. The concept simulations provide a proof of the theoretical concepts introduced in this work. Simultaneously, the hardware system is developed. The system development process involves simulating several viable circuit designs that consist of cost-
effective and easily accessible components which meet the system functional constraints. From the simulations, the best performing low-cost components and circuit designs are selected and developed. Successively, the assembled system is tested, and excitation waveforms are generated to stimulate the phantom test tank, while simultaneously acquiring boundary data before reconstructing images. The experimental results are then analysed and compared to those acquired in related works. Conclusions are drawn from the results and recommendations are made.

In addition, the following Gantt chart provides a project timeline to observe the expected duration of each component of the investigation.

![Chart of project timeline](image)

**Figure 1.2 Research Gantt chart showing the anticipated start date of various tasks and the corresponding duration.**

Figure 1.2 provides a Gantt chart of the tasks to complete the research. It starts on the 1st of August 2018 and is expected to end on the 17th of January 2021. The first six months are reserved for the development of a research proposal. The proposal includes a discussion around the background of EIT, the governing theoretical concepts, problem statements and hypotheses. Concurrently, a thorough review of relevant literature will be conducted. The review process will continue for most of the experimentation phase. The system design, component acquisition, assembly, and testing of the EIT system completes in successive order, starting on the 1st of January 2019. Successively, experimentation will take place and data acquisition. The experimental results will then be analysed and compared to most recent related works. The last task of the research is the compilation of the thesis.
1.7 Thesis Outline

The outline of this thesis follows:

1. **Chapter 1: Introduction** - Presents an introduction to this thesis to provide a setting, leading into the investigation. This includes a description of the current state, advantages, disadvantages, and challenges of EITS and the research hypotheses to tackle some of the challenges or gaps in literature.

2. **Chapter 2: Electrical Impedance Tomography and Spectroscopy (EITS)** - Discusses the most relevant literature on EITS. These include the methods used to acquire the reconstruction prior information and it is shown how these methods (based on assumptions, statistical analysis, and previous empirical studies) fail to make EITS a self-reliant imaging modality. The different multiplexing techniques and their advantages and disadvantages are explored to highlight the need for faster imaging systems that stimulate the electrodes with orthogonal signals. A typical EITS system layout is identified and discussed for design consideration. Finally, different image reconstruction methods are discussed to provide a mathematical framework which this work relies upon.

3. **Chapter 3: Orthogonal Chirp Division Multiplexed Absolute EITS** - This chapter introduces the concept of OCDM and its novel application to EITS. OCDM is proposed to provide a way to simultaneously apply orthogonal chirp waveforms that have the same period-bandwidth combination, while measuring the boundary potentials and estimating the impedance distribution of the object being imaged. Further discussion involves presenting the forward and inverse problems of EITS, the method of regularization and the implementation of prior information to constrain the solution to the inverse problem. The last section of this chapter outlines the performance figures of merit to be used to quantify the quality of the system performance and image reconstructions.

4. **Chapter 4: Proof of Concepts** - This chapter highlights the correlation and energy properties of the orthogonal chirp signals. Additionally, simulations are provided for image reconstructions using generic-prior and OCDM-prior absolute imaging methods.

5. **Chapter 5: System Design** - This chapter provides a motivation for the design of an OCDM-aEITs system. Followed by the system functional, non-functional and safety constraints. Different layout concepts are presented with a quantified approach of comparison and the most suitable layout for the purposes of this work is selected. The complete system design is presented together with a discussion on the different major components.
6. **Chapter 6: System Test** - This chapter explores the performance of the OCDM-aEITS system by measuring the signal integrity and image reconstruction capability. OCDM-prior absolute images are reconstructed for a phantom test tank containing 1% NaCl solution and several inhomogeneities. The NaCl solution is used to simulate the presence of the electrolyte in the human body while the inhomogeneity is a biological specimen (fruit or vegetable) to represent the tissue of the human body. These images are then reconstructed using generic prior information to analyze the difference in the image quality and reconstruction performance.

7. **Chapter 7: Results Analysis** - This chapter quantifies the OCDM-prior absolute image reconstruction performance and compares the results to those acquired from related works. Followed by discussions on the significant advantages, drawbacks, and challenges of using OCDM-aEITS compared to the alternative methods.

8. **Chapter 8: Conclusions, Recommendations, and Future Work** - This chapter draws conclusions from the results of chapters 6 and 7 and makes recommendations. Future work entails recommendations to combat the challenges of the proposed method.
The following chapter represents the relevant literature to the context of this thesis. Included in the discussion is a repertoire of distinct advantages, practical limitations, and applications reported on Electrical Impedance Tomography and Spectroscopy (EITS).

EITS represents a nonlinear and ill-posed problem, therefore, a discussion will be focused on algorithms that incorporate prior information to compute a unique solution. Some of the factors that affect the image reconstructions will be reviewed. Additionally, to reconstruct images involves resolving the forward and inverse problems. Therefore, the focus will be around developing the theoretical framework adopted to solve these problems.

Finally, to develop a functioning EITS system, an array of electronic instrumentation and their implementations are to be considered. Most of the reported research concludes the electronic instrumentation remains frequently the leading cause for substandard performance. The instrumentation performance is affected by the accuracy, effective bandwidth, tolerance, and other electronic hardware factors. Therefore, an overview of the critical hardware components and their notable performance is presented.

2.1 Electrical Impedance Tomography and Spectroscopy

Electrical impedance tomography and Spectroscopy (EITS) refer to the practise of imaging an object at distinct sections, using low amplitude current waves that penetrate through the object [23]. Electrodes are mounted in a single cross-section plane along the boundary of the object to arrange an EITS system that reconstructs two-dimensional images. Considering a system that does not reuse the electrodes for stimulation and measurement. The current waves can be time division multiplexed (TDM) to each subsequent current stimulation electrode pair on the boundary of the test object as shown in Figure 2.1 (left). The resulting boundary voltages are measured using a second multiplexer that transmits the separate voltages to an analog-to-digital convertor [24]. However, this approach severely reduces the frame rate due to the
delays that are introduced by the multiplexers. And may induce pulses on the stimulation signals caused by the switching process [25]. Consequently, temporal data inconsistencies occur when TDM is applied to rapid changing objects like a biological specimen. Alternatively, using parallel EITS, several orthogonal current sources can simultaneously be applied to the test object as shown in Figure 2.1 (right), which improves the temporal resolution of EITs. However, a parallel EITS frequency division multiplexing (FDM) approach introduces data inconsistencies due to the substantial asymmetrical changes in the impedance distribution of a frequency sensitive specimen. This occurs when applying multiple sinusoidal current waveforms at different frequencies. Therefore, it is imperative an adequate range of frequencies are selected to prevent these substantial changes in the specimen and observe a smooth change in the impedance distribution.

![Figure 2.1 Shows the current injection, voltage measurement scheme used in (left) serial stimulation and (right) parallel stimulation. Serial stimulation involves a single source that is time multiplexed to each stimulation electrode. Parallel stimulation involves applying a source to each stimulation electrode.](image)

In [26], Code Division Multiplexing (CDM) is used to simultaneously excite all current electrodes with pseudorandom binary codes. To optimize the separation between different measurement channels, Walsh-Hadamard codes were implemented. The system consisted of nine parallel excitation sources and 18 parallel measurement channels. In addition, the task of demodulating the measurements was shown to be more laborious, compared to the demodulation of the measurement channels that were separated using nearly orthogonal Gold Codes, as achieved in [25]. However, the results from [25] included instances of ghosting and signal spikes at higher frequencies, which formed image artefacts.
Furthermore, some studies have demonstrated the use of chirp signals to characterize the imaged object. In [27], a feasibility study is presented, which proposes a method to employ a FPGA-generated single wideband chirp signal to a three-element impedance load. The measured results illustrated the driving capability of the current source and that chirp signals can be used to obtain the sensitive spectral response of the load. However, the approach was limited to measuring the impedance of a particular homogeneous tissue and not an inhomogeneous medium. In [28], a magneto-acoustic electrical tomography system is introduced. The system used a single 2-3 MHz chirp signal applied to a phantom via a probe. The researchers concluded that the reconstructed B-scan image of the impedance distribution was consistent with that of the ultrasound B-scan image. This was observed when a 1,000 µs pulse duration chirp signal stimulated a homogeneous phantom with 0.5 % concentrated NaCl. However, the results were limited to measuring the phantom thickness caused by the variation of the NaCl concentration. And did not illustrate the use of chirp signals to identify the frequency characteristics of a medium containing sub-domains.

2.2 Development of the Reconstruction Prior Information

EIT is a non-linear problem and typically requires iterative image reconstruction methods to acquire a unique solution. The most daunting challenges with iterative methods are precisely the convergence of the solution and residual errors for several iterations [29]. Convergence and residual errors are directly related to the initial guess made for the first operation. For instance, if the distinctive shape and size of a phantom object, and electrode positions are known precisely, then making an adequate initial guess is possible to improve the forward model of the EIT problem. However, if the internal distribution of a medium is unknown or poorly estimated, most likely the solution will contain significant errors or require more iterations to resolve. A good initial guess, for convergence purposes, is to collect an initial measurement and use this measurement as a reference [30]. However, a direct subtraction between the reference and subsequent measurements could poorly detect an internal inhomogeneity if both frames were acquired at the same frequency and the impedance does not change over time. Additionally, a reference frame may be unavailable. Therefore, frequency difference imaging is preferred for frequency dependent specimen. Alternatively, to reconstruct absolute images from a single measurement frame, absolute EIT is preferred.

Absolute EIT has a strong dependence on prior information [31]. Customarily, this prior information is formed from the results of previous empirical studies like those obtained from Magnetic Resonance Imaging (MRI) [32]. However, in the author’s opinion, the recent literature has not yet sufficiently moved EIT to the point of self-reliance. One proposal, as
investigated in this thesis, is to use a parallel EITS system to obtain the frequency differenced prior information about the impedance distribution. This prior information is developed, specific to the medium being imaged, from a single wideband measurement frame.

A few EITS reconstruction approaches that incorporate prior information based on the object’s expected properties have been carefully studied in the literature [32], [33], [34], [35], [36] and [37]. In [34], the prior information was developed from the assumption that the unknown conductivity was sharp and “blocky” by nature. This resulted in prior information that was based merely on well-defined conductivity and relatively simple level sets. With these assumptions, the inverse problem was formulated as a problem of finding the minimal total variation conductivity.

In [35], the prior information was developed from the knowledge of the geometry of the human chest and the structures of the internal organs, from previous empirical studies. A finite element mesh model was constructed from the knowledge of this geometry. The mesh was implemented to identify five distinctive areas of elements. All elements belonging to a specific area was constrained to a single conductivity value. This form of constraint indicated a change in the conductivity of the lungs during the breathing cycle. A similar approach was followed in [33], with supplementary information on the anisotropy of the myocardium and skeletal muscle included to constrain the solution more significantly.

In [35], the prior information was developed in isolation from a basis constraint method. The basis constraint method approximates the expected geometry and internal structures of an object with a linear combination of pre-selected basis functions. Followed by the iterative minimization of the difference between the measured and computed boundary potentials. The result represents a vector of constants used to constrain the solution. This vector is consumed to construct an impedance distribution random variable. The random variable is manipulated to develop the regularization matrix. Furthermore, it was demonstrated that satisfactory images are reconstructed only when there is a direct correlation between the subspace of the random variable and that of the actual impedance distribution. Otherwise, the images are misleading and present false anomaly detection.

### 2.3 EITS Image Reconstruction

The problem of EITS is separated into a forward problem and an inverse problem. The forward problem develops a model of the electrostatic behaviour of a medium. This is done by constructing a relationship between the current density and the resultant potentials at distinct
elements within the medium, after discretizing the domain. The inverse problem aims to exploit this relationship to estimate the impedance distribution within the medium, given the measured boundary potentials. The task subsequently is to regularize the problem by accounting for modelling and data errors. The following sections demonstrate these concepts.

2.3.1 The Forward Problem of EITS

The forward problem of EITS refers to the computation of the boundary potentials from the injected currents, for a known impedance distribution [38]. This forward model is adopted to predict subsequent observations. In this case, the spatial electric field, and hence the boundary potentials are computed by injecting known currents to a model of known impedance distribution [39]. This allows one to compute a Jacobian matrix which can be consumed to resolve the inverse problem.

Let the internal impedance distribution of an object be represented as $z(r)$, at points $r = (x, y, z)$ (where $x$, $y$ and $z$ are cartesian coordinates) within the domain of the object. This internal impedance is assumed to be linear and isotropic. The current that stimulates the test object produces current patterns within the object, setting up the current density. The current density causes spatial electric potentials inside and on the boundary of the object as shown in Figure 2.2.

![Figure 2.2 Current, applied to an electrode pair, distributes within the object. Resultant equipotential lines are formed and measured at the boundary.](image-url)
Chapter 2: Electrical Impedance Tomography and Spectroscopy (EITS)

The corresponding relationships, in Figure 2.2, are defined by Maxwell’s equations [40], which follows.

\[ \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \]  
(2.3-1a)

\[ \nabla \cdot \mathbf{B} = 0 \]  
(2.3-1b)

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]  
(2.3-1c)

\[ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \]  
(2.3-1d)

Where:

- \( \rho \) is the charge density.
- \( \mathbf{E} \) is the electric field.
- \( \mathbf{B} \) is the magnetic flux intensity.
- \( \varepsilon_0 \) is the permittivity.
- \( \mu_0 \) is the permeability.
- \( \mathbf{J} = \sigma \mathbf{E} \) is the current density and \( \sigma \) is the conductivity.

The presence of an electric field, \( \mathbf{E} \), sets up a vector represented electric potential distribution, \( \mathbf{v}(r) \). The electric field may be computed from the negative gradient of these vectors.

\[ \mathbf{E} = -\nabla \mathbf{v}(r) \]  
(2.3-2)

The Fourier transform of equation (2.3-1d) shows the direct relationship between the curl of the magnetic field and the complex admittivity.
\[ \nabla \times \mathbf{B}(r) = \mu_0(\sigma(r) + \varepsilon_0 j \omega) \mathbf{E}(r) = \mu_0 \gamma(r) \mathbf{E}(r) \]  

(2.3-3)

The complex admittivity is represented by \( \gamma(r) = \sigma(r) + j \omega \varepsilon_0 \), where \( \omega \) is the angular frequency and \( j = \sqrt{-1} \). The inverse of the admittivity is the complex impedance, \( z(r) = R(r) + j \omega X(r) \).

Furthermore, no sources originate from within the object, causing zero charge build-up. Therefore, the divergence of the curl is zero.

\[ \nabla \cdot (\nabla \times \mathbf{B}(r)) = \nabla \cdot (\gamma(r) \mathbf{E}(r)) = 0 \]  

(2.3-4)

Substituting equation (2.3-2) into (2.3-4) yields the following Laplacian equation for the internal electric potential of a medium:

\[ \nabla \cdot (\nabla \times \mathbf{B}(r)) = \nabla \cdot (\gamma(r) \nabla \varphi(r)) = 0 \]  

(2.3-5)

This Laplacian suggests that the flow of electrons and the gradient of the electric potential distribution are in the direction of the negative electric field.

After observing the behavior of the current in a medium. The following step involves observing how these currents and internal potentials affect the formation of the boundary potentials at the electrodes. Therefore, boundary conditions must be developed for stimulating electrodes, grounded electrodes, boundary potential measuring electrodes and the spaces on the boundary between the electrodes. The formation of the boundary conditions is based on the Complete Electrode Model (CEM) [41] which provides an accurate representation of the electrode system.

Consider neglecting the effects at the edge of the stimulating electrodes. The instantaneous total stimulating current \( I_{\text{stim}} \) perpendicular \( \mathbf{n} \) to a given stimulating electrode (electrode \( i \)) is the surface integral of the current density \( j(r, t) \), over the surface \( S \) of the electrode [25].
\[ I_{\text{stim}} = \int_{\text{electrode}_i} J(r, t) \cdot \hat{n} dS \] (2.3-6)

The grounded electrode is responsible for sinking all current that is injected through the remaining stimulating electrodes, to ensure charge conservation [25]. For systems that multiplex the ground or opposite polarity electrode for each injection, the instantaneous total current at that electrode is the negative value of the corresponding stimulating current. This ensures that a balanced alternating current signal is maintained for each injection. And avoids polarization of the contents of the object and that there is a conservation of charge. For systems that use separate stimulation and measurement electrodes, the measurement electrodes ideally have zero current due to the high input impedance amplifiers at the output of these electrodes. In addition, one boundary condition of a medium would be formed by fixing the normal current \( J\hat{n} \) at every point on the boundary [1].

\[ J\hat{n} = -z \frac{\partial v}{\partial \hat{n}}, \text{at a boundary} \] (2.3-7)

Equation (2.3-7) illustrates that the current density normal to the boundary of the object is determined by the product of the impedance and the normal electric potential vector at the boundary. This condition can be modified to account for the electrodes at the boundary of the medium, such as the electrode contact impedances.

Equations (2.3-5) and (2.3-7) form the forward problem of EITS, which are used to estimate the boundary potentials based on known injected currents and an estimated internal impedance distribution. Models that have arbitrary boundary geometries require numerical methods like the Finite Element Method (FEM) of modelling, which converts a continuous problem to a discrete algebraic problem [42]. This model is produced of discrete triangular elements, with each element containing a value of the electric potential variable. The potential is approximated by a shaping function which is defined at the nodes of the elements. Collectively, these elements define the behaviour of the electric potential over the medium. Furthermore, the FEM is used to solve the weak form of the EITS boundary value problem. It linearizes the problem by constraining the dynamic variables to minor changes. Additionally, the FEM provides substantial accuracy but is more complicated to implement compared to other problem discretization methods. One such alternative is the Finite Difference Method (FDM). The FDM uses square elements to discretize a domain. It is therefore more
straightforward to implement. But introduce significant truncation errors of finite divided difference approximations, inaccuracies in the discretization of irregular boundaries and a reduced numerical computation precision [25]. Apart from these disadvantages, FDM can be improved by increasing the refinement of the strong form partial differential equations (PDE) of the boundary value problem. That is, the elements of the model are reduced. However, this increases the computational burden. Consequently, compared to FDM, the FEM relaxes the stringent requirements. Such that the trial solution of the PDE be smooth and improves the accuracy at modelling the boundary of arbitrary shapes.

2.3.2 The Inverse Problem of EITS

In EITS, the forward problem solves a Laplace equation to compute the boundary potentials, given an initial estimate of the internal impedance distribution and the stimulation signals. This is required to deduce the properties and conditioning of the image reconstruction process. The solution to the inverse problem follows the process of first computing the internal impedance distribution. Followed by developing an image of a medium based on its internal impedance distribution. The impedance of a medium can be measured absolutely or as a variation with time, frequency, or a combination of the two. The most conventional methods proposed to resolve the inverse problem of EITS are the Back-projection, Newton-Raphson, and Sensitivity methods.

*The Back-Projection Method:*

The back-projection method is derived from the methods utilized in Computed Tomography [23]. Signals are propagated through the medium in a straight line to observe the attenuation profile. In addition, injecting signals at different angles results in different attenuation profiles as shown in Figure 2.3, which shows the attenuation voltage profiles for two sources. Furthermore, combining all profiles results in the localization of the attenuating object or anomaly within the medium. However, in EITS, current does not travel in a straight line due to the low current amplitude. This results in an inequitable current distribution through the medium, and assuming that current is evenly distributed or that the impedance changes are small, results in poor system performance and image quality.
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Figure 2.3 Back-projection method of reconstructing images by solving the EITS inverse problem. It shows two stimulating sources that inject a waveform that penetrates through the medium in a straight line. The resulting boundary potential profile is observed to extract information about the size and location of the anomaly (solid black circle).

**The Modified Newton-Raphson Method:**

The Newton-Raphson method employs a function, \( f(\gamma(r)) \), which contains the geometry, electrode positions and the discrete forward model of the test object. It bases the relationship between the image pixels and the computed boundary potentials \( v_c = f(\gamma(r)) \) [25].

This method is based on the following minimization equation, which minimizes the sum of the squared errors between the measured and computed boundary potentials.

\[
\min_{\gamma(r)} \{ \delta = [v_c - v_m]^T[v_c - v_m]\} 
\]  

(2.3-8)

Where:

- \( \delta \) is the pre-defined tolerance.
- \( v_m \) is the measurement boundary potentials.
The solution to equation (2.3-8) yields a change in the local admittivity:

\[ \Delta \gamma (r) = (K^T K)^{-1} K (v_m - v_c) \]  \hspace{1cm} (2.3-9)

Where:

- \( K \) is the Jacobian of \( v_c \).
- \( T \) is the matrix transpose operator.

The solution is computed iteratively, \( \gamma_{i+1}(r) = \gamma_i(r) + \Delta \gamma(r) \), until the pre-defined tolerance or maximum specified number of iterations is achieved. And produces exceptional results but suffers from a high sensitivity to the measurement noise [43].

In [44], the image resolution using Electrical Resistance Tomography (ERT) was improved. The approach was to develop a particle swarm optimization (PSO) image reconstruction algorithm that employs the modified Newton-Raphson algorithm to resolve the inverse problem of ERT before applying prior information and clustering to constrain the solution. The results show an improvement compared to those acquired from the standard modified Newton-Raphson algorithm. However, their comparisons were limited to the Newton-Raphson algorithm and were unexpanded to alternative image reconstructions methods. Furthermore, the method employed does not focus on the contents of the prior information, and no clear process could be deduced to identify the assumptions made when forming the prior information.

The Sensitivity Method:

Compared to the methods discussed, a more reliable approach is to use a regularization method to resolve the inverse problem of EITS; known as the Sensitivity method. The EITS inverse problem can be modelled as \( z = B v \). A linear equation that relates the measurement boundary electric potentials, \( v \), to the internal impedance distribution, \( z \). Using the inverse of a Jacobian matrix, \( B = K^{-1} \), where \( K \) is a Jacobian matrix. Hence, to compute the impedance distribution within a medium requires the measurement of the boundary potentials and a Jacobian matrix [38].
For a linearized forward problem, the Jacobian is derived from:

\[ \mathbf{v} = \mathbf{Kz} + \mathbf{h} \]  

*(where \( \mathbf{K} \) is the Jacobian and \( \mathbf{h} \) is the measurement white noise vector per channel.)*

Each element in the Jacobian can be computed as:

\[ K_{i,j} = \frac{\partial v_i}{\partial z_j} \]  

Which shows that each element in the Jacobian relates a change in the electric potential to the change in impedance at each node inside a forward discrete model. To compute the complete Jacobian matrix requires the computation of the power dissipation through the medium using Green’s identity. Furthermore, given that the potential and the impedance variables represent unequal vectors, the Jacobian will be a non-square matrix.

Additionally, by convention, regularization is used to invert the Jacobian by placing penalties to acquire a unique solution. Regularization involves using a hyperparameter to filter the high-frequency singular values and uniquely resolve the inverse problem. One method to maintain continuity of the solution is the generalized Tikhonov regularization method. This method relies on placing prior information, about the system, into the least squares minimization solution shown below.

\[ \mathbf{z} = \arg \min_{\mathbf{z}} \left\{ ||\mathbf{Kz} - \mathbf{v}||^2 + \lambda^2 ||\mathbf{Rz}||^2 \right\} \]  

Here:

- \( \mathbf{R} \) is a regularization matrix.
- \( \lambda \) is the hyperparameter.
- \( ||\mathbf{Kz} - \mathbf{v}||^2 \) is used to constrain any data errors.
- \( \lambda^2 ||\mathbf{Rz}||^2 \) uses prior information to constrain any modelling errors.
The solution to the minimization problem is:

\[ z = (K^T K + \lambda^2 R^T R)^{-1} K^T v = Bv \]  \hspace{1cm} (2.3-13)

Furthermore, using a quadratic iso-surface, the regularization method returns a radial vector. This vector belongs to a circle that has the solution subspace as a tangent [45], and uses a hyperparameter to filter the high-frequency singular values of the Jacobian. Consequently, a regularization matrix is used to constrain the solution implementing a prior or assumed solution [32]. This matrix helps to speed up the convergence of the solution while penalizing or regulating any modelling errors. Therefore, of all the possible solutions to the ill-posed problem, a unique solution is selected, and the problem presents a well-posed problem.

Additionally, in [46], a proposal is presented based on a self-weighted noser-prior EIT using internal electrodes in cardiac radiofrequency (RF) ablation. The aim was to use EIT to track the electrical variation due to temperature changes within the myocardium. The approach included the development and application of a time-efficient algorithm with a self-weighted noser-prior and optimizing the number of measurements using filtration to monitor the size of a lesion during the cardiac RF ablation. A tank model was used with a circular myocardium of 12 mm thickness, 16 boundary electrodes and three internal catheter-based electrodes. The acquired results indicated successful measurement of the lesion size and adequate accuracy and tolerance of noise. However, given that a noser-prior was used, suggests the experiment was constrained to a single generic prior matrix. In addition, the method used to select the position of the weights was not provided. And little information is known about the frequency characteristic of the specimen other than the reaction to distinct frequency-controlled ablations.

In [47], the focus is on prior information to improve the sparsity reconstruction using partial data. This partial data was formed from Cauchy data measured on the boundary subsets. The approach was to enforce the sparsity using a L1-norm of the basis coefficients as a penalty term in the Tikhonov function, and incorporate a spatially distributed regularization parameter within the prior information. The simulated results illustrated an improvement when applying this prior information even when using partial data to penalize the Tikhonov function. The method was shown to be comparable to the Total Variation (TV) approach. However, the reconstructions became more unreliable the greater the mismatch between the partial data and the measurement subsets.
In [48], the problem of selecting the prior information for a specimen that varies with time and frequency, like the heart, is explored. It identifies the generic implementation using standard prior information based on the boundary numerical estimates of the internal domains of a thorax. However, for internal domains like the heart during a cardiac cycle, it becomes challenging to accurately specify the structural domains. Consequently, it is proposed that the prior information only contains the structural information of the internal domains, of a thorax, that can be estimated with adequate accuracy. In this case, the regularization matrix is modified anisotropically to include sub-domains as prior information, and the regularization parameter is assigned with different weights to each sub-domain. The results show an improvement compared to the conventional employment of generic prior information.

### 2.4 EITS System Design

In EITS, applied voltage signals are generated using analogue hardware or digital waveform synthesizes. The voltage signals are converted to electrical currents, by using voltage-to-current convertor modules, which stimulate the test object. Regarding the analogue electronics, the hardware is based on simple oscillators. Regarding the digital waveform synthesizes, the first approach is to store the waveform in read-only memory (ROM) and output the samples using a digital-to-analog convertor (DAC). The DAC introduces signal discrepancies caused by the bit resolution, quantization, and signal-to-noise ratio (SNR). Alternatively, the signals can be generated using direct digital synthesis (DDS). DDS stores samples in a look-up table (LUT), and a phase increment function is used to index these samples based on the clock frequency. Therefore, the stored waveform can be varied in frequency. DDS remains the preferred method as it provides flexibility and synchronization between the source and demodulation.

Regarding the stimulation method, a signal can be time multiplexed to each current stimulation electrode using single-ended or floating current sources. In [49], it is demonstrated that mirrored modified Howland voltage-controlled current sources provide a uniform current spatial distribution compared to single-ended sources with a fixed ground electrode. In multiple source EITS systems, cumulative instrumentation errors are observed and analysed stochastically [50].

In addition, several considerations about the separate components of the EITS system follow. Foremost, all electronics that make up the EITS system should be placed close to the source. And a common ground should be used to limit the effects of leakage currents. Although,
leakage currents are also introduced through parasitic circuit components. Moreover, regarding the type and configuration of electrodes, copper or stainless-steel electrodes are frequently used due to their high conductivity and reasonable cost [51]. Stainless steel electrodes are preferred because of its strong anti-corrosion properties. Ag/AgCl electrodes have been used in biomedical applications [52], however, the cost of these electrodes is excessively high for the prototyping stage. Additionally, to avoid injecting harmful metal ions into a patient, electrodes are made of carbon, at a high price, in [1] and [53]. Furthermore, some papers explore the use of shared electrodes for stimulation and measurement [54] as shown in Figure 2.4 a. While other papers explore the use of separate electrodes [1], [2], [55], [56]. Commonly using compound electrodes as shown in Figure 2.4 b. Compound electrodes are used to reduce the effects of the electrode contact impedances. Compound electrodes have been shown to provide good current distribution through the medium and reliable boundary potential measurements [57]. Furthermore, the size of the electrodes remains a critical factor to improve current distribution and limit electric field distortion.

Separation of voltage and current electrodes is done to limit the effects of the electrode contact impedances in the measurements. This is achieved by attaching high input impedance voltage measuring devices to the measuring electrodes to attenuate the output current. However, this method increases the number of electrodes without improving the spatial resolution, because the spatial resolution depends on the number of potential-measuring electrodes [24]. Possible solutions are to decrease the size of the electrodes to increase the number of boundary electrodes [58], apply voltages and measure currents [1] or re-use the electrodes [59]. The first solution is limited by the length of the boundary, and the size of the electrodes is limited.
by the effects of the contact impedance. The smaller the electrode, the larger the contact impedance as most of the applied current passes through a smaller electrode patch. Therefore, a sufficiently large electrode surface area is required to reduce the contact impedance. The second solution of applying a voltage to the boundary eliminates the contact impedance. However, supplying a constant voltage results in a variable current through the medium. Minor changes at the surface of the medium causes considerable current variation in the domain due to the ill-posedness of the inverse problem. Furthermore, this method does not produce an optimal system. The third solution represents the most desired method of reusing electrodes; dual-purpose electrodes [59]. Reusing electrodes allow one to employ the identical number and size of electrodes as is used in the first and second solution. Yet, simultaneously measures and stimulates at all electrodes. However, this method has not been tested enough to be benchmarked. It is also more difficult to make these electrodes and place them on the surface.

Regarding the potential measurement component of the system, the difference in voltage between two electrodes is measured and scaled. This is performed to reduce the large voltage ranges associated with single boundary voltage measurements [25]. To achieve this, instrumentation amplifiers are used. After accurately measuring the boundary electric potentials, they should be converted to a digital format, using an analog-to-digital convertor (ADC), to allow further digital computations and image reconstruction. Consequently, meticulous attention should be placed upon recording enough samples to accurately extract the amplitude and phase shift of the measured signal.

In [60], a multi-frequency EITS system was developed for biomedical imaging. Research included practical tests on a phantom filled with a saline solution and vegetables. The Electrical Impedance Tomography and Diffuse Optical Tomography Reconstruction Software (EIDORS) was used to efficiently reconstruct the images from the measured data. In [61], an EIT system that comprised of electrical circuitry to obtain data signals from the surface of the object (the arm) was designed. After acquiring the data, the image was then reconstructed using direct and iterative methods. Research concluded that the human body impedance varies in the range of 0.7 kΩ to 1 kΩ. The accuracy of the system depended upon the constant applied current and a reference resistance source. The images were successfully reconstructed by employing the MATLAB EIDORS toolbox and the method of imaging was non-invasive and avoided any effects of radiation. The reconstructed images were validated from the distinctive shape of the arm.
2.5 Summary

This chapter outlined the literature relevant to the context of this work. It started with a qualitative description of EITS, which included a discussion on the distinct advantages, practical limitations and applications that were reported in related works. Furthermore, a multiple source EITS system was described as a method that improves the temporal resolution compared to time-multiplexed systems. Regarding the type of stimulation signals to use, it was indicated that orthogonal sinusoidal, pseudorandom binary codes and single chirp waveforms have been used in related works. The sinusoidal waveforms are made to be orthogonal by operating at different frequencies. However, the selection of these frequencies may cause significant changes in the impedance distribution of the object. Which increases the number of iterations and rate of convergence of the solution. The pseudorandom binary codes are made orthogonally by using a random code generator. Most commonly, these codes are based on Gold or Walsh-Hadamard sequences. However, this method may introduce ghosting, which will include false anomaly detection. In addition, signal spikes are observed in the stimulation signals, particularly at high frequencies when rectangular pulses are employed. Alternatively, the chirp waveforms are time-multiplexed and provides a way of observing the entire frequency spectrum of the medium from a particular frame. However, because of the use of time-multiplexing, this method suffers from low temporal resolution.

Subsequently, the importance of utilizing prior information was presented jointly with the ways in which it is developed to improve the convergence of EITS. Comparable works were discussed with a focus being on the data used to develop the prior information and if an improvement was observed. Most commonly, the prior information is developed from previous empirical studies that provide a dataset from the average of multiple measurements made by MRI. This provides some insight into the possible locations and boundaries of the internal domains of the object and therefore improves the convergence rate of the solution. Ultimately, of the image reconstruction algorithms presented, the Sensitivity method is preferred. As it explicitly allows the incorporation of prior information as a penalty on the errors in the system.

Finally, several factors that affect EITS hardware performance are described. For a multiple source EITS system, a typical approach is to generate the applied signals as voltages, using analogue hardware or digital waveform synthesizes. These voltages are converted to current waveforms using either single-ended or floating current sources. For the single-ended sources, a common ground electrode is fixed. However, this produces an asymmetrical current spatial distribution. Alternatively, mirrored modified Howland voltage-controlled current sources may be used to overcome this problem. To stimulate the object, the choice of the
electrode configuration is significant. The preferred approach represents the use of compound electrodes to separate the voltage from the current electrodes, to alleviate the effects of electrode contact impedances. On the other hand, one could use dual-purpose electrodes to improve the effective number of boundary measurements. However, meticulous attention needs to be paid to the method of eliminating the electrode contact impedances. Woefully, this remains a crux issue for EITS.
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Absolute EITS is used to reconstruct an image of an object, using the computed absolute impedance distribution of that object from a single measurement data frame and a reference frame [1]. If a particular source is time multiplexed to each current electrode of a dynamic system, there will be temporal data inconsistencies. Alternatively, signals can (simultaneously) be applied to each non-redundant current electrode pair to eliminate this problem. These signals must be orthogonal to prevent interferences within the object. One such method is to apply sinusoidal waveforms at each electrode, with each sinusoid operating at a unique frequency [62]. Nevertheless, this may cause considerable asymmetrical changes in the impedance distribution or may get swamped with repetitive data due to the selected set of stimulation frequencies. These issues could adversely affect the rate at which the solution converges [25]. In contrast, pseudorandom binary codes can be applied (using code division multiplexing (CDM)) to the electrodes. These codes are orthogonal (or nearly so) and can, therefore, reuse the stimulation frequency, which eliminates the data inconsistencies due to considerable asymmetrical changes in the impedance distribution. However, the generated pulses may cause signal spikes, which can cause detrimental effects on the measurement data [25].

Therefore, a multi-frequency sinusoid (or chirp) is the preferred signal waveform because these smooth signals operate over a wideband. Additionally, these signals incorporate the advantages of CDM, without any signal spikes. Lamentably, one cannot simultaneously stimulate the test object at each electrode with perfectly overlapping chirp waveforms, without incorporating a method to orthogonalize the stimulation set. A method must be introduced to produce orthogonal chirp waveforms that have the same bandwidth and period. This is recognized as Orthogonal Chirp Division Multiplexing (OCDM) [22].

This chapter introduces OCDM and refers to its application to acquire wideband measurements, used to develop prior information that reduces the rate of convergence of the absolute image reconstructions. Followed by the discussion of a multi-channel system to test the ability of OCDM to identify the channel impulse responses. The chapter ends with a review of the employed performance figures of merit.
3.1 Data Acquisition

In this thesis, to extract frequency-dependent data involves stimulating the object surface electrodes with multiple wideband chirp currents and measuring the resultant boundary potentials. The cross-correlations of these signals are used to determine the internal impedance distribution of the object.

Providing a classical EIT scenario, suppose a single electrical current waveform is applied to a test object at any instant. Furthermore, suppose this current is injected into the object during the $k^{th}$ cycle, $i_k(t)$, and that the voltage at electrode $m$ is measured, $v_{m,k}(t)$, as shown in Figure 3.1.

\[ v_{m,k}(t) = z_{m,k}(t) i_k(t) \]  

Here $z_{m,k}(t)$ is the resultant impedance along path $m, k$.

Subsequently, to compute the total measured voltage during cycle $k$, at electrode $m$, requires a convolution between the applied current during cycle $k$ and the impedance along path $m, k$. 

![Figure 3.1 Current used to stimulate an object during the $k^{th}$ stimulation cycle and the voltage measured at electrode $m$ due to the current passing path $m, k$.](image-url)
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\[ v_{m,k}(t) = \int_{-\infty}^{\infty} z_{m,k}(\alpha) i_k(t - \alpha) d\alpha \]  \hspace{1cm} (3.1-2)

Similarly, for parallel EITS, multiple current waveforms are simultaneously applied to the surface of a test object. The total voltage at electrode \( m \) due to all currents, as indicated in Figure 3.2, is determined from a discrete summation of the convolutions of the different input signals, \( i_h \), with the impedances along the paths \( m, h \). In the multi-stimulant case, path \( m, h \) refers to the net equipotential path, to electrode \( m \), due to the \( h^{th} \) stimulant.

\[ v_m(t) = \sum_{h=1}^{n} v_{m,h}(t) = \sum_{h=1}^{n} \int_{-\infty}^{\infty} z_{m,h}(\alpha) i_h(t - \alpha) d\alpha \]  \hspace{1cm} (3.1-3)

For \( n \) total stimulant current waveforms.

Figure 3.2 Simultaneous orthogonal current stimulation resulting in the total voltage measured at electrode \( m \) due to all currents. The measurement voltage is computed as the convolutions between the stimulation currents and the impedance along the corresponding paths to the measurement electrode.

Furthermore, to uncover information about the paths or channels between the points of transmission and detection, the cross-correlations between the transmitted and received
signals must be computed. In EITS, this channel information contains the impedance along those paths. This is demonstrated below when the current injected at electrode $k$ is cross correlated with the measured voltage at electrode $m$ to extract the impedance between these electrodes [25], [63].

$$R_{v_m,i_k}(\tau) = \int_{-\infty}^{\infty} v_m(t + \tau)i_k(t)dt$$  \hspace{1cm} (3.1-4a)

$$= \int_{-\infty}^{\infty} \left[ \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} z_{m,n}(\alpha)i_n(t + \tau - \alpha) d\alpha \right] i_k(t)dt$$  \hspace{1cm} (3.1-4b)

$$= \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} z_{m,n}(\alpha) \left[ \int_{-\infty}^{\infty} i_n(t + \tau - \alpha)i_k(t)dt \right] d\alpha$$  \hspace{1cm} (3.1-4c)

$$= \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} z_{m,n}(\alpha)R_{i_n,i_k}(\alpha) d\alpha$$  \hspace{1cm} (3.1-4d)

In this case $R_{i_n,i_k}(\alpha)$ is the cross-correlation between the injected current at electrode $k$ and all injected currents. Therefore, by cross correlating each boundary potential with the applied currents, returns several impulse responses if the applied currents are (at least nearly) orthogonal. For a 2-dimensional test object, these impulse responses are implied as being the impedance along the path from the applied currents to measurement electrode $m$. 

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3.2 Orthogonal Chirp Division Multiplexing

Orthogonal Chirp Division Multiplexing (OCDM) involves generating orthogonal chirp signals that simultaneously stimulate a multi-channel medium and the recovery of the channels’ impulse responses by cross correlating the received with the transmitted signals. Chirp signals are used for their spread-spectrum properties which guarantee robust transmission [64], [65], [66], while being resistant to the detrimental effects of an environment like channel noise.

By convention, chirp signals are generated by analog devices employing one of two approaches. The filter approach uses surface acoustic wave (SAW) devices [67]. The frequency modulation approach employs voltage-controlled oscillators based on CMOS technology [68]. Additionally, in chirp spread-spectrum systems, a wide bandwidth is used to modulate information. And, based on an intrinsic trade-off, the spectral efficiency is sacrificed for higher processing gain and enhanced multi-path resolution [22].

Furthermore, to generate orthogonal chirp signals, starting from a fundamental understanding, each signal operates over a unique bandwidth and period as shown in Figure 3.3.

![Figure 3.3](image)

*Figure 3.3 The bandwidth B and period T of six orthogonal chirp signals [22]. It shows an inefficient method of generating orthogonal chirp waveforms that have unique period-bandwidth products.*

In Figure 3.3, an illustration of several orthogonal chirp signal blocks is presented. Each chirp signal has a unique bandwidth B and period T combination. This approach is desirable for low-rate applications in which reliable transmission remain a priority. However, for high-rate applications that require reliable and efficient signal transmissions, like EITS, this approach is inadequate. Furthermore, generating chirp signals that occupy the same period and bandwidth
causes inter-chirp interference within the multi-channel medium. To overcome this issue and to improve the rate of the system, each chirp can be modulated implementing a pseudorandom code like the Walsh-Hadamard code [22]. In this way, several orthogonal chirp signals can be generated in the same bandwidth and over the same period, as shown in Figure 3.4. This improves the rate of transmission. At the receiving end, the transmitted information is recovered by retrieving the codes from the modulated chirp signals over the entire bandwidth.

![Figure 3.4 Orthogonal chirp signal modulation using orthogonal codes [22]. The chirp signals are modulated using pseudorandom codes and can operate with the same period-bandwidth product while remaining orthogonal.](image)

Figure 3.4 illustrates the dimensions of the chirp signals when using orthogonal codes. Modulation of this kind allows one to generate several chirp signals that overlap temporally and spectrally. However, the use of orthogonal codes to modulate the signal adds an extra level of complexity to implement. This may include more computational burden when storing the codes or more electronic circuits. Therefore, a more efficient approach is required.

Typically, a chirp signal is defined as a frequency modulated signal whose frequency evolves linearly or phase quadratically over time. The equation of a spread-spectrum chirp signal is therefore defined as:

\[ i(t) = e^{j(\pi \alpha t^2 + \theta_0)} \]  

(3.2-1)

Where:

- \( \alpha \) is the chirp rate.
- \( \theta_0 \) is the initial phase.
Additionally, the instantaneous frequency of this chirp signal is:

\[ f(t) = \left( \frac{1}{2\pi} \right) \left( \frac{d}{dt} [\pi \alpha t^2 + \theta_0] \right) = \alpha t \]  \hspace{1cm} (3.2-2)

The time bandwidth product \((\alpha = BT)\) describes the processing gain of the signal. The larger the product \(BT\), the less spectral efficient it becomes. Subsequently, consultation of the Fresnel transformation is required to generate enhanced spectrally efficient orthogonal chirp signals.

### 3.2.1 The Fresnel Transformation

The Fresnel transformation [22] is an integral transformation used to describe the near-field optical diffraction as shown in Figure 3.5.

Figure 3.5 An illustration of monochromatic plain light waves originating behind a surface. The light passes an opening in this surface and diffracts, forming circular ring patterns on a second (parallel) surface a distance \(d\) away. The Fresnel transformation describes these patterns.

Figure 3.5 illustrates a monochromatic light wave, of wavelength \(\lambda\), is incident to a wavelength proportional opening on a surface. The light then diffracts and forms circular patterns on a parallel surface placed a distance \(d\) from the first.
In optics, the Fresnel transformation describes this pattern by the following equation.

\[
\tilde{s}(\tau) = \mathcal{F}_a\{s(t)\}(\tau) = \frac{e^{-j\left(\frac{\pi}{4}\right)}}{\sqrt{a}} \int_{-\infty}^{\infty} s(t)e^{j\left(\frac{\pi}{2a}\right)(\tau-t)^2} dt
\]  

(3.2-3)

Where:

- \( \mathcal{F}_a\{\} \) is the Fresnel transformation.
- \( a = \lambda d \) is the normalized Talbot distance.
- \( s(t) \) is the complex transmittance of the grating.
- \( \tilde{s}(\tau) \) is the Fresnel transformation of \( s(t) \).

This may be represented in the following convolution form:

\[
\tilde{s}(\tau) = s(\tau) \ast \varphi_a(\tau)
\]  

(3.2-4)

here:

\[
\varphi_a(t) = e^{j\left(\frac{\pi}{a^2} - \frac{\pi}{4}\right)}
\]  

(3.2-5)

To extend the Fresnel transformation to applications involving multiple equidistant openings through the first surface, one should consider the discrete Fresnel transformation (DFnT). The DFnT incorporates the Talbot effect, which describes the periodic image of the openings repeated at uniform distances from the incident surface. Essentially, the DFnT develops a matrix that describes the optical coefficients of the Talbot image. And, therefore, the pattern of diffraction that is repeated at periodic distances \( d = \frac{d_T}{N}, d_T = d_g^2/\lambda \) from the first surface, shown in Figure 3.6.
Consequently, the \((m, n)^{th}\) entry into the \(DFnT\) matrix is:

\[
\phi(m, n) = \frac{e^{-j\frac{\pi m}{4}}}{\sqrt{N}} \times \begin{cases} 
    e^{\frac{j\pi (m-n)^2}{N}}, & N \equiv 0 \ (\text{mod} \ 2) \\
    e^{j\pi (m+0.5-n)^2}, & N \equiv 1 \ (\text{mod} \ 2) 
\end{cases}
\]  

(3.2-6)

Furthermore, the \(DFnT\) matrix is unitary, and benefits from the properties associated with the discrete Fourier transformation [22]. As a result, just as the discrete Fourier transformation is the basis for frequency division multiplexing, the \(DFnT\) is the basis for orthogonal chirp division multiplexing.
3.2.2 The Translation from Optics to OCDM

As with CDM, the code lengths are restricted to maintain orthogonality, so too (for OCDM) should the chirp signals be time limited. Furthermore, to translate the Fresnel transformation from classical optics to OCDM, require the adaptation of the Talbot effect. This adaptation involves replacing the discrete distance $d_g$ used in optics with the chirp period, $T$.

$$d_T = \frac{T^2}{\lambda}$$  \hspace{1cm} (3.2-7)

And the Talbot distance is:

$$d = \frac{d_T}{N}$$  \hspace{1cm} (3.2-8)

This suggests that, as with classical optics, there are $N$ distinct incident surface openings that cause $N$ diffraction patterns. So too are there $N$ stimulation sources that generate $N$ orthogonal chirp signals.

To formulate the root chirp, $i_0(t)$, signal, the modified Talbot distance is substituted into equation (3.2-5) and the result is shown below:

$$i_0(t) = \Pi_T(t)\varphi_a^*(t)\left|_{a=\frac{T^2}{N}}\right. \hspace{1cm} (3.2-9)$$

$$= e^{j\pi\left(\frac{N}{2}t^2 - \frac{1}{4}\right)}, 0 \leq t < T$$

Where the superscript * refers to the complex conjugate and the rectangular function is defined as:

$$\Pi_T(t) = \begin{cases} 1 & 0 \leq t < T \\ 0 & elsewhere \end{cases}$$  \hspace{1cm} (3.2-10)
Therefore, the substitution of the modified Talbot distance into the Fresnel transformation gives the equation for a root chirp signal with a signal rate of \( \alpha = \frac{N}{T^2} \).

To formulate \( N \) orthogonal chirp waveforms based on the root chirp signal, consider the following extension of the \( DF\pi T \):

\[
i_k(t) = \Pi_T(t)\varphi_k^*(t - k\frac{T}{N})|_{a=\frac{T^2}{\pi}} = e^{j\pi\left(\frac{N}{T^2}(t-k\frac{T}{N})^2 - \frac{1}{4}\right)}, 0 \leq t < T
\]

Where \( k = 0, 1, 2, ..., N - 1 \).

A plot of eight chirp signals based on equation (3.2-11) is provided below.

![Figure 3.7 Plot of eight chirp signals, illustrating orthogonality. The chirp signals operate at the same period-bandwidth product.](image_url)

In Figure 3.7, eight arbitrary chirp waveforms were developed from the root chirp equation. Immediately it can be seen that these waves are not identical, over the entire period. In addition, the waveforms operated over the same period-bandwidth product. But, to draw a conclusion on the level of orthogonality, an additional test is required.
To further prove that the chirp waveforms, in equation (3.2-11), are mutually orthogonal, the inner product for function spaces is computed between two arbitrary waveforms for $x \neq y$:

\[
\int i_x(t)i_y^*(t) \, dt = \int_0^T e^{j\pi \frac{N}{T} (t-x\frac{T}{N})^2} e^{-j\pi \frac{N}{T} (t-y\frac{T}{N})^2} \, dt \cong \delta(y - x) \tag{3.2-12}
\]

Which concludes that the computed integral is a Dirac-delta function, and therefore the signals are mutually orthogonal.

Furthermore, the orthogonal chirp signals are used to stimulate an EITS test medium and the resultant boundary potentials are measured over the entire chirp period. Following the measurement of the potentials, the channel impedance must be estimated by demodulating the output waveforms. The measurement potential $v_e(t)$ at any arbitrary electrode $e$ is modelled as the summation of convolutions between the stimulation signals and the corresponding impedances. This is modelled along the paths between the stimulant and measurement electrodes of the test medium, shown in Figure 3.8.

![Figure 3.8 The principle of orthogonal chirp division multiplexing. Each channel of a test object is simultaneously stimulated with orthogonal chirp waveforms. The sum of the convolutions is correlated with the different stimulation signals using the inner product. The result produces impulse responses that are directly related to the impedance over the specific channel.](image)
Figure 3.8 shows the process of simultaneously stimulating an EITS test object with orthogonal chirp waveforms and the demodulation of the measured signal at an arbitrary measurement electrode. Demodulation is achieved by using the inner product to extract the corresponding channel impedances.

Following from section 3.1, the equation for the total measurement potential at any given electrode $e$ as illustrated above, is indicated below:

$$v_e(t) = \sum_{h=0}^{N-1} \int_{-\infty}^{\infty} z_{e,h}(\beta) i_h(t - \beta) \, d\beta, \quad 0 \leq t < T$$

(3.2-13)

Hence, the inner products are computed as:

$$z_{e,h}^* = \int_{0}^{T} v_e(t) i_k(t) \, dt = \sum_{k=0}^{N-1} \int_{0}^{T} z_{e,h}(\beta) \delta(h - k) \, d\beta = z_{e,h}$$

(3.2-14)

Therefore, the result of a specific inner product is equal to the impedance of the channel measured at electrode $e$ along path $e, h$. And given the measurement channel impedances, first a forward problem must be resolved before images can be reconstructed.

3.3 The Forward Problem

The forward problem of EITS refers to the computation of the boundary voltages and Jacobian, given a known internal impedance distribution and the boundary conditions. Further information included in the problem is the geometry of the physical system, electrode shape and positions, and the applied stimulation signals.

To obtain the weak form of the forward problem involves applying a weighting function, $\omega(r)$, to the Laplacian equation and integrating over the interior, $\emptyset$, of the object.
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\[
\int_{\emptyset} \omega(r) \nabla \cdot (\gamma(r) \nabla v(r)) d\emptyset
\]

(3.3-1)

It must be ensured that the weighting function is at least the linear Lagrange interpolants, to be used for EITS. Furthermore, this equation must be accompanied by the boundary conditions, over the boundary \( \mathcal{B} \), using the divergence and Green’s theorem [25], as summarized in equation (3.3-2).

\[
\int_{\mathcal{B}} \omega(r)(\gamma(r) \nabla v(r)).\hat{n} dB
\]

(3.3-2)

\[
= \begin{cases} 
0, & \text{between electrodes on the boundary} \\
\int_{B_e} \omega(r) \left( \frac{I}{A_e} \right) dB_e, & \text{at each electrode} \\
\int_{B_g} \omega(r) \left( \frac{I}{A_g} \right) dB_g, & \text{at each ground electrode}
\end{cases}
\]

Where:

- \( A_e \) represents the area of the electrode.
- \( B_e \) and \( B_g \) are the boundaries of the electrodes.

The conditions are assumed to be zero for all spaces between the electrodes on the boundary. And are maintained by the following equation.

\[
\int_{\emptyset} \omega(r) \nabla \cdot (\gamma(r) \nabla v(r)). d\emptyset
\]

(3.3-3)

\[
= \int_{\mathcal{B}} \omega(r)(\gamma(r) \nabla v(r)).\hat{n} dB - \int_{\emptyset} \nabla \omega(r)(\gamma(r) \nabla v(r)) d\emptyset = 0
\]

And substituting the boundary conditions into equation (3.3-3) yields the following weak form of the forward problem of EITS in equation (3.3-4).
\[
\int_{\Phi} \nabla \omega(r)(y(r)\nabla v(r)) d\Phi = \int_{B} \omega(r)(y(r)\nabla v(r)).\hat{n} dB
\]

\[
= \int_{B_e} \omega(r) \left( \frac{l}{A_e} \right) dB_e + \int_{B_g} \omega(r) \left( \frac{l}{A_e} \right) dB_g
\]

(3.3-4)

Subsequently, the continuous EITS problem must be discretized before applying these conditions.

### 3.4 Problem Discretization

The finite element method (FEM) is a technique that models a system based on the approximate solutions to a set of partial differential equations [69]. The FEM is currently the most utilized method employed to numerically resolve the EITS problem [1]. It has the advantage of being able to model arbitrary geometries with various boundary conditions [1]. Other methods, like the finite difference and volume methods, possess more efficient solvers. However, these methods use regular mesh grids, which makes it difficult to accurately model irregular systems.

Moreover, the FEM reduces the EITS continuum problem to one which is discrete. It achieves this by discretizing the medium into non-overlapping triangular elements, forming a finite element mesh as shown in Figure 3.9.

Figure 3.9 (left) A test phantom and (right) the corresponding 256-element two-dimensional circular FEM model.
Each element in this mesh, in Figure 3.9 (right), contains a value of the internal electric potential variable, which is approximated by a shaping or interpolation function defined only by the nodes of the discrete element [1]. These elements collectively define the behaviour of the electric potential over the entire medium.

Currently, there are three different FEM techniques [69]. These techniques are listed below:

1. Direct approach: Provide an intuitive way of understanding the FEM. However, it is limited to elements with constant conductivity.

2. Variation approach: Uses calculus to compute the extremes of a potential function. It is employed to work with arbitrary element shapes and to solve high order interpolation functions.

3. Method of Weighted Residuals (MWR): The MWR does not depend on a variation statement. It works with a set of governing equations and is predominantly utilized to derive element properties for non-structural applications like fluid flow.

Each of these approaches involves the following sequence, considering a 2D EITS problem [69]:

1. Mesh generation: A finite element mesh made of finite, non-uniform, non-overlapping elements, connected at nodes, to discretize the spatial domain of a medium.

2. Shaping function selection: Involve selecting a shaping function, which is defined at the connecting nodes of the elements, to approximate the electric potential over an element. These functions are piecewise, linear, or quadratic. Some papers report using higher order functions [70].

3. System Modelling: For each element, a local stiffness matrix is computed from a set of partial differential equations. This matrix is used collectively with the shaping function to compute the electric potential solution at the element nodes. A global stiffness matrix is formed from all local stiffness matrices.

4. System Solver: This step imposes the boundary conditions, categorized into three stages (fixed field variable, derivative of the fixed field variable and a combination of the first two stages). After imposing these conditions onto the global matrix, it is then inverted to compute the electric potential value at each node.

5. System Solution: Using the shaping functions, and the global matrix, a solution to the entire mesh network is approximated.
In accordance with the steps listed above, after generating the triangular-element mesh, come the development of the shaping functions that approximate the potential at each element.

Considering a linear approximation function of the following form [25].

\[
\varphi^e(r) = \alpha_0^e + \alpha_1^e x^e + \alpha_2^e y^e = \begin{bmatrix} a_0^e \\ a_1^e \\ a_2^e \end{bmatrix} = k^e T \alpha^e \tag{3.4-1}
\]

In equation (3.4-1), the following definitions are offered for the elements:

- The superscript \( e \) refers to the element number in the mesh.
- The numbers in the subscript refer to the local node number within the element.
- The superscript \( T \) represents a transpose operator.

This step is repeated for each of the three nodes of the triangular element, and the approximation functions are given by:

\[
C^e(r) = \begin{bmatrix} \varphi_1^e(r) \\ \varphi_2^e(r) \\ \varphi_3^e(r) \end{bmatrix} = \begin{bmatrix} 1 & x_1^e & y_1^e \\ 1 & x_2^e & y_2^e \\ 1 & x_3^e & y_3^e \end{bmatrix} \begin{bmatrix} a_0^e \\ a_1^e \\ a_2^e \end{bmatrix} = K^e \alpha^e \tag{3.4-2}
\]

Solving for \( \alpha^e \) in equation (3.4-2) and substituting the result into equation (3.4-1) yields the following:

\[
\varphi^e(r) = k^e T [K^e]^{-1} C^e(r) = H^e C^e(r) \tag{3.4-3}
\]

The matrix \( H^e \) contains the shaping functions. To ensure compatibility, the Kronecker delta property is used to constrain the polynomial functions of this matrix to 1 at the respective node and 0 elsewhere.
Moreover, consider an arbitrary shape that is discretized using two triangular elements, shown below.

![Figure 3.10 Arbitrary shape that is discretized using two triangular elements. The corner nodes of the shape are the global nodes, while the nodes at the corners of the discrete triangular elements are the local nodes. The aim is to identify a relationship between the global and local nodes.](image)

In Figure 3.10, an arbitrary shape is discretized using two triangular elements. The nodes at the corners of this shape are the global nodes of the shape while the nodes at the corners of the discrete triangular elements are the designated local nodes. From Figure 3.10, element 1 and 2 contain nodes at the same location as the “1” node of the arbitrary shape, similarly if we repeat this observation, the following pattern is discovered.

**Table 3.1** Defines the relationship between the local and global nodes.

<table>
<thead>
<tr>
<th>Arbitrary shape node</th>
<th>Element 1</th>
<th>Element 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 (local node 1)</td>
<td>1 (local node 1)</td>
</tr>
<tr>
<td>2</td>
<td>0 (no local node)</td>
<td>1 (local node 2)</td>
</tr>
<tr>
<td>3</td>
<td>1 (local node 2)</td>
<td>1 (local node 3)</td>
</tr>
<tr>
<td>4</td>
<td>1 (local node 3)</td>
<td>0 (no local node)</td>
</tr>
</tbody>
</table>

Table 3.1 defines the observed relationship between the local and global nodes. A “1” is placed in the element column whenever a respective element has a node in the same location as a global node.
Therefore, the approximations at the global nodes can be formed from the superposition of the two local elements as described below.

\[
\mathbf{C} = \begin{bmatrix}
\frac{\partial_1(r)}{\partial_2(r)} & \frac{\partial_3(r)}{\partial_4(r)}
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\frac{\partial_1}{\partial_2} & \frac{\partial_2}{\partial_2} & \frac{\partial_3}{\partial_2} & \frac{\partial_4}{\partial_2}
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\frac{\partial_1(r)}{\partial_2(r)} + \frac{\partial_2(r)}{\partial_2(r)} + \frac{\partial_3(r)}{\partial_2(r)} + \frac{\partial_4(r)}{\partial_2(r)}
\]

\[= e^{1T}C^1(r) + e^{2T}C^2(r) = \sum_{e=1}^{2} e^{eT}C^e(r)\]

And for an \(n^{th}\) element of the model:

\[
\mathbf{C} = \sum_{e=1}^{n} e^{eT}C^e(r) \quad (3.4-5)
\]

Following the process above in reverse order allows one to observe the effect of the local nodes on the global nodes (i.e., to compute \(C^e(r)\)) for each triangular element. Furthermore, given that the weak form of the forward problem aims to compute \(\nabla v(r)\), it is intuitive to compute the gradient of the approximation function. This function is substituted into the weak forward problem equation. This is demonstrated in the equation below.

\[
\nabla \partial^e(r) = \nabla (H^eC^e(r)) = \begin{bmatrix}
\frac{\partial H^e}{\partial x} \\
\frac{\partial H^e}{\partial y}
\end{bmatrix} C^e(r) = E^eC^e(r) \quad (3.4-6)
\]
Substituting equation (3.4-6) into the weak form forward equation (3.3-4) and taking the sum over all elements yields the following set of equations that define the global stiffness matrix and boundary values:

\[
F^e = \int_{\Omega} E^T \chi(r) E^e d\Omega 
\]  \hspace{1cm} (3.4-7a)

\[
F = \sum_{e=1}^{n} c^e F^e c^e 
\]  \hspace{1cm} (3.4-7b)

\[
b_{Be} = \sum_{e=1}^{n} c^e \int_{B_e} \left( \frac{l}{A_e} \right) dB_e 
\]  \hspace{1cm} (3.4-7c)

\[
b_{Bg} = \sum_{e=1}^{n} c^e \int_{B_g} \left( \frac{l}{A_e} \right) dB_g 
\]  \hspace{1cm} (3.4-7d)

Equation (3.4-7a) is the local stiffness matrix. Equation (3.4-7b) is the global stiffness matrix. Equations (3.4-7c) and (3.4-7d) are the boundary values.

Therefore, the weak form forward equation after evaluating the integrals, yields:

\[
\omega^T(r)Fv(r) = \omega^T(r)[b_{Be} + b_{Bg}] 
\]  \hspace{1cm} (3.4-8)

For systems that require adaptive currents and mesh refinements, an iterative approach should be employed. Furthermore, as the order of the shaping function increases or as the number of elements increases, the solution converges to a tolerable level of uncertainty.

After resolving the forward EITS problem, one needs resolving the inverse problem. An inverse problem is formulated to reconstruct an image of the medium, based on the mapping of its internal impedance distribution.
3.5 The Inverse Problem

To reconstruct an image, first a FEM model is produced with a homogeneous impedance distribution. Exploiting the generated signals to stimulate this model allows one to compute the boundary potentials, \( v_c(r, t) \). In the same way, the generated signals are exploited to stimulate the actual test object, and the boundary potentials are measured, \( v_m(r, t) \). The measured and calculated potentials are individually cross correlated with the generated signals. Followed by the Fast Fourier transform (FFT) of the cross-correlations and the subsequent sampling of the frequency spectrum to produce impulse responses, \( h_{ij}^m(jw, t) \) and \( h_{ij}^c(jw, t) \) respectively. The image reconstruction algorithm is then used to minimize the error between the measured and calculated potentials. The error is implemented to update the FEM model and the final FEM model is expected to represent a reliable estimate of the internal impedance distribution of the actual test object. This complete operation is shown in the control loop diagram in Figure 3.11.

Figure 3.11 Image reconstruction control loop diagram showing the process of computing and measuring the boundary potentials, followed by computing the error using the respective impulse responses. The error is used to update the FEM model until the model’s admittivity matches that of the actual test object, with some tolerable uncertainty.

Figure 3.11 shows the process of computing the error \( e \) between the measured and calculated boundary potentials and implementing this error to update the FEM model. The reconstruction algorithm is commonly selected from those described in section 2.3.2. In this thesis, the Sensitivity approach (or method of regularization) is used because it possesses many robust properties and allows the incorporation of prior information.
3.5.1 The Method of Regularization

The continuous EITS problem is often discretized using the FEM [69], which generates a discrete model of finite elements that represent the physical object. In EIT, the aim is to compute the matrix of internal impedances $z$, given the measured boundary voltages $v$, such that $z = K^{-1}v$. However, the dimensions of $z$ and $v$ are unequal, therefore, the Jacobian, $K$, is not a square matrix and cannot be inverted by ordinary linear algebra. To compute the Jacobian involves solving the forward equation with an initial assumption of the impedance distribution and the known input. This results in each element of the Jacobian matrix representing a change in potential to a change in impedance at distinct elements in the discrete model. The inverse of the Jacobian can be determined by the Moore-Penrose inverse or pseudoinverse method. This method returns the following equation to invert a non-square matrix, if the weighting matrix, $W \neq I = identity$ matrix:

$$K^{-1} = (K^T W K)^{-1} K^T W$$

(3.5-1)

Therefore, the solution to the problem is given by:

$$z = (K^T W K)^{-1} K^T W v$$

(3.5-2)

Conversely, if the system is considerably nonlinear, then $z$ cannot (accurately) be determined from a linear combination of $K^{-1}$ and $v$. If this nonlinearity is assumed to be weak, in a sense that the governing partial differential equations and Neumann boundary conditions are weighted and integrated over the object domain and boundary, then the solution may be linearized about the initial estimation of the internal impedance distribution, $z_0$, of the object, as indicated below.

$$z = z_0 + (K^T W K)^{-1} K^T W (v - v_0)$$

(3.5-3)
Furthermore, the nonlinear problem can be solved using the Gauss-Newton iterative algorithm. Here a first approximation $\psi(z)$ of the solution $z$ is determined by using an initial condition, $z_0$, with a second-order Taylor expansion.

\[
||Wv - WKz||^2 \approx \psi(z) = \psi(z_0) + \left( \frac{\partial \psi}{\partial z}(z_0) \right)^T (z - z_0) + \frac{1}{2} (z - z_0)^T \left( \frac{\partial^2 \psi}{\partial z^2}(z_0) \right) (z - z_0)
\]

Here $W = \text{weighted matrix}$.

Differentiating this approximation function and equating the result to zero, gives a solution of the estimated impedance that closely matches the actual impedance.

Therefore, the approximation becomes:

\[
z_{\text{estimated}} = z_{0_{\text{estimated}}} - \left( \frac{\partial^2 \psi}{\partial z^2}(z_{0_{\text{estimated}}}) \right)^{-1} \left( \frac{\partial \psi}{\partial z}(z_{0_{\text{estimated}}}) \right)
\]

And the Hessian can initially be approximated by:

\[
\frac{\partial^2 \psi}{\partial z^2}(z_{0_{\text{estimated}}}) = 2K_i^TWK_i
\]

As a result, the Gauss-Newton recursion has the form:

\[
z_{i+1_{\text{estimated}}} = z_{i_{\text{estimated}}} + u_i(K_i^TWK_i)^{-1} \left( K_i^TW(v - K_iz_{i_{\text{estimated}}}) \right)
\]

Where $u_i$ is a unit step function.
However, equation (3.5-7) assumes that the problem is well-posed, which is not the case with EITs. Alternatively, the problem needs to be modified or regularized. This ensures that the ill-posed problem is replaced by a somewhat well-posed equivalent problem.

Therefore, to regularize the problem requires an augmented least squares function:

\[ a(z) = ||W_1(v - Kz)||^2 + \lambda ||W_2(z - z_0)||^2 \]  (3.5-8)

Here \( a(z) \) is the augmented least squares function, \( 0 < \lambda < 1 \) is the regularization hyperparameter, \( W_1 \) is the square-root of the weighting matrix and \( W_2 \) is a regularization matrix.

And one method, among others, to regularize the problem is the Tikhonov regularization. The standard method defines \( W_1 = I, W_2 = R \) (An alternative representation of the regularization matrix) and \( z_0 = 0 \), and the solution becomes:

\[ \min_z \{ ||(v - Kz)||^2 + \lambda ||(Rz)||^2 \} \]  (3.5-9)

Consequently, below are the results of this computation, for the linear and nonlinear cases [25]:

\[ z = (K^T K - \lambda R^T R)^{-1} K^T v \]  (3.5-10)

And,

\[ z_{i+1} = z_i + \left( K_i^T K_i - \lambda R^T R \right)^{-1} (K^T (v - Kz_i) - \lambda R^T Rz_i) \]  (3.5-11)

Furthermore, the regularized version of the EITs problem provides a unique solution and is well-posed because the matrix \( K_i^T K_i - \lambda I \) has positive eigenvalues if \( \lambda > 0 \). To improve the rate of convergence of the solution will involve reducing the number of elements in the discrete forward model that need to be reconstructed. To reduce the number of elements to be reconstructed, while accounting for a possible change in the geometry of the internal domains
of an object, prior information is needed to regularize the inverse problem. This idea will be expanded upon in section 3.5.2.

3.5.2 Using OCDM Prior Information

To reconstruct an accurate image from the computed absolute internal conductivity (or impedance) distribution of an object, accurate prior information is required. Incorporating this prior information into the regularization matrix, fewer elements of a finite element model need to be reconstructed. This improves the rate of convergence and stabilizes the solution to the inverse problem. Frequently, the regularization matrices are represented by the identity matrix or the matrices that correspond to the first or second difference operators [71], [72], [73]. The prior information is frequently based on general assumptions when these matrices are used. These include the assumption that the resistivity values are small or that the solution is constant or smooth, which may not be the case. Alternatively, the prior information is formed from previous empirical studies on the expected internal boundaries. One such method (Proposed in [32]) uses a basis constraint method. In this method, basis functions are developed from the data of previous empirical studies to introduce weights that constrain the solution. It was assumed that the conductivity distribution $\sigma$ could be well approximated as a linear combination of these basis functions $w_m$. Such that, $\sigma = \sum_{m=1}^{M} \alpha_m w_m(r)$, $\alpha_m \in \mathbb{R}$ where $M$ is small. However, the prior information must be in accordance with the actual impedance distribution to produce good results. Otherwise, the results are misleading and may result in false positive anomaly detection, especially when there are significant geometrical changes in the internal structures of the object. This idea was extended in [74]. Which is known as the subspace regularization method (SSRM). The SSRM uses the generalized Tikhonov regularization method to converge the solution towards the null subspace of the regularization matrix. This extension provided good results even when the prior information was incorrect.

The approach followed in this thesis employs a similar idea to that in [74]. But instead of using prior information from previous empirical studies, it is proposed that the prior information be developed from a single OCDM wideband frame. This frame contains the unique conductivities to the object under test. Therefore, the solution to the inverse problem converges towards the subspace $S_R$ of the corresponding prior information about the instantaneous geometry of the object’s internal structures. This is achieved by using the generalized Tikhonov regularization method with an adequately constructed regularization matrix, $R$. In this way, EITS (and more specifically absolute EITS) shifts closer towards the point of self-reliance.
Moreover, consider a procedure to construct the subspace $S_R$, within which the solution is assumed to exist [57]. To begin with, the task is to develop a set of the expected conductivity distribution vectors from a single OCDM measurement frame. The frame is segmented into a heatmap that defines the absolute channel conductivities for $N$ channels, across the frequency spectra as shown in Figure 3.12.

![Figure 3.12](image)

*Figure 3.12 A heatmap illustrating the channel conductivity vectors for a $N$ channel FEM model. Each vector is defined at a single frequency within the frequency spectra. The vectors collectively form a training set, which is used to develop the regularization matrix.*

Figure 3.12 demonstrates the segmentation of a single wideband OCDM measurement frame. To construct the heatmap, the boundary potentials are cross correlated with the stimulation chirp signals, to attain the channel conductivities. The fast Fourier transform is used to observe the absolute conductivities in the frequency domain. The result is a heatmap of the channel number versus frequency. The magnitude of the heatmap is defined by the absolute channel conductivities at the corresponding frequencies and across the various channels. These conductivities are used to develop a collection of absolute conductivity distributions for a $P$ element FEM model. The set of conductivity distributions define the training set, from which the following covariance matrix may be computed:

$$
C = n^{-1} \tilde{\sigma} \tilde{\sigma}^T
$$

(3.5-12)

Where:

- $n$ is the number of discrete frequencies used in the heatmap (i.e., Number of channel conductivity vectors).
- $\tilde{\sigma} = [\sigma_1, ..., \sigma_n] \in \mathbb{R}^{P \times n}$ is a matrix of conductivity vectors. $\sigma_i, i = 1, 2, ..., n$ is the $i^{th}$ conductivity distribution vector of the model formed from the channel conductivities measured at the $i^{th}$ frequency. And $P$ is the number of elements in the FEM model.
In addition, to compute the basis functions, the task is to find a $M \ll n$ dimensional subspace such that the most adequate approximation of the training set is established from the smallest mean square error. This is performed by computing the $M$ largest eigenvalues and corresponding orthonormal eigenvectors $w_m (m = 1, ..., M)$ of the covariance matrix, using the orthogonal iteration method [75]. From the principal component analysis [76], it is deduced that these eigenvectors span the subspace $S_R$.

To converge the inverse solution to this subspace requires resolving the minimization equation defined in equation (3.5-9). This equation is solved with respect to the conductivity and using a regularization matrix $R$ with a null space of $S_R$. A regularization matrix that conforms to these conditions is defined below [74].

$$R = I - QQ^T$$  \hspace{1cm} (3.5-13)

Where:
- $I$ is the identity matrix.
- $Q$ is a matrix having the basis vectors $w_m$ as columns.

Consequently, the solution to the minimization equation (3.5-9) is defined by equation (3.5-11), which is shown below with respect to the conductivity.

$$\sigma_{i+1} = \sigma_i + \Delta \sigma_i$$  \hspace{1cm} (3.5-14)

Where:
- $\Delta \sigma_i = (K^T K - \lambda R^T R)^{-1}[K^T (v_m - v(\sigma_i)) - \lambda R^T R \sigma_i]$ 
- $K$ is a Jacobian matrix.
- $R$ is the regularization matrix.
- $T$ is the transpose operator.
- $\lambda$ is the regularization hyperparameter.
- $v_m$ is the measured boundary potential vector.
- $v(\sigma_i)$ is the computed boundary potential vector.
Chapter 3: Orthogonal Chirp Division Multiplexed Absolute EITS

To understand how the generalized Tikhonov regularization method converges the solution (of the inverse problem) to the null space of the regularization matrix, consider the generalized singular value decomposition of the pair of matrices $K \in \mathbb{R}^{m \times n}$, $R \in \mathbb{R}^{p \times n}$ and $r = \text{rank}(R)$ [74].

\[
(K) = (U \ 0) \begin{pmatrix} \text{diag}(a_i) & 0 \\ 0 & I_{n-r} \end{pmatrix} X^{-1}
\]

(3.5-15)

Where:

- $U = [u_1, ..., u_n] \in \mathbb{R}^{m \times n}$, $u_i$ are orthonormal to $v_i$
- $V = [v_1, ..., v_p] \in \mathbb{R}^{p \times p}$, $v_i$ are orthonormal to $u_i$
- $X = [x_1, ..., x_n] \in \mathbb{R}^{n \times n}$, $x_i$ are linearly independent.
- The positive diagonal matrices ($\text{diag}(a_i)$ and $\text{diag}(\tau_i)$) refer to matrices of dimension $r \times r$.

Subsequently, this generalized singular value decomposition of the pair of matrices can then be defined as the ratios $\mu_i = \frac{a_i}{\tau_i}$ for $i = 1, 2, ..., n$, in descending order of value.

Moreover, the change in conductivity can now be represented by the following form using the singular value decomposition.

\[
\Delta \sigma = \sum_{i=1}^{r} \mathcal{F}(\mu_i^2) \left( \frac{u_i^T \Delta v}{a_i} \right) x_i + \sum_{i=r+1}^{n} u_i^T \Delta v x_i
\]

(3.5-16)

Where:

\[
\mathcal{F}(\mu_i^2) = \frac{\beta}{\beta + \lambda}
\]

is a Tikhonov filter function [77].

Equation (3.5-16) demonstrates the generalized singular value decomposition representation of a change in conductivity. The first sigma term depends on the regularization hyperparameter, while the second sigma term is the component of the solution in the null space of the regularization matrix. Therefore, it becomes apparent that increasing the hyperparameter reduces the effect of the first sigma term and conversely, increases the effect of the second sigma term. Essentially, the solution converges towards the null space of the
regularization matrix, which effectively converges toward the true subspace. Furthermore, considering the edge cases as \( \lambda \to 0 \) and \( \lambda \to \infty \), returns the linearized ordinary least squares problem for which the solutions remain within the null space [74].

Subsequently, given that the conductivity of a biological specimen may vary with the applied frequencies, it serves well to implement a method of developing the regularization matrix from a change in conductivity due to a change in the stimulation frequencies. This involves a unique case where the \( ||R\sigma||^2 \) in the conductivity representation of equation (3.5-9) is replaced by \( ||R(\sigma - \bar{\sigma})||^2 \). In this unique case, \( \bar{\sigma} \) refers to the conductivity distribution which is the least affected by the conductivity of the inhomogeneities. And contains the frequency independent homogeneous distribution of the system. In this thesis, a heatmap is consulted to identify the distribution with the least absolute maximum conductivity as \( \bar{\sigma} \). Therefore, the difference in conductivity distributions will alleviate the similarities between these distributions. It is therefore assumed that the constants between the distributions are the equivalent system resistance per channel and homogeneous distribution of the electrolyte. The variation between these distributions will be the frequency dependent change in conductivity of the inhomogeneities. Subsequently, the solution to the inverse problem may be represented by the following equation.

\[
\bar{\sigma} = (K^TK - \lambda R^TR)^{-1}K^Tv_m = \sigma - \bar{\sigma} \tag{3.5-17}
\]

And the following method demonstrates the computation of the square regularization matrix that maintains this relationship or at least tends towards this solution.

\[
(K^TK - \lambda R^TR)^{-1}K^Tv_m = \sigma - \bar{\sigma} \tag{3.5-18}
\]

\[
K^Tv_m = (K^TK - \lambda R^TR)(\sigma - \bar{\sigma}) \tag{3.5-19}
\]

\[
K^Tv_m(\sigma - \bar{\sigma})^T = (K^TK - \lambda R^TR)(\sigma - \bar{\sigma})(\sigma - \bar{\sigma})^T \tag{3.5-20}
\]

\[
K^Tv_m(\sigma - \bar{\sigma})^T[(\sigma - \bar{\sigma})(\sigma - \bar{\sigma})^T]^{-1} = K^TK - \lambda R^TR \tag{3.5-21}
\]

\[
R^TR = \left(\frac{1}{\lambda}\right)[K^TK - K^Tv_m(\sigma - \bar{\sigma})^T((\sigma - \bar{\sigma})(\sigma - \bar{\sigma})^T)^{-1}] \tag{3.5-22}
\]
Equation (3.5-22) presents a square regularization matrix that converges the solution to the true conductivity distribution while alleviating the effects of the equivalent system resistance and homogeneous conductivity distribution. The method does not force the solution to the expected distribution, but merely draws the solution towards it. Therefore, good quality images can be reconstructed even if the prior information is not accurate [74].

3.6 The Performance Figures of Merit

After developing a complete parallel OCDM-aEITS system, the performance must be tested. An adequate measure of the performance of an EITS system is to compute various errors in the system. To observe the spatial qualities of the reconstructed images, requires computing the difference between the reconstructed image and the actual inhomogeneous environment. These differences can be grouped into the size error, position error, noise error ratio, and the magnitude error for a given inhomogeneity inside the phantom [25], [78]. Furthermore, the errors in the reconstructed images are caused by the type of reconstruction algorithm and the errors in the hardware.

3.6.1 The System Hardware Performance

*The Signal to Noise Ratio (SNR) and Noise Error Ratio (NER)*:

The SNR is computed as:

\[
SNR_i = \frac{\text{mean}(v)_i}{sd(n)_i}
\]  

The SNR \((SNR_i)\) for a given measurement channel \((i)\), represent the ratio between the mean of the measured signal \((\text{mean}(v)_i)\) and the mean of the noise \((sd(n)_i)\). An adequate estimate of the mean of the noise in the measured data frame, is the standard deviation of that frame [79]. An SNR > 0 \(dB\) means that there is more signal than noise. Furthermore, for an EITS problem, minimum SNR constraints are placed upon the system design. This is based on the minimum impedance that the system is required to measure. After establishing this constraint,
one can then compare the SNR for different measurement frames to this constraint to show the system adheres to the design.

Additionally, the noise error ratio (NER) is defined as the inverse of the SNR, [25]. The NER must be a negligible value to indicate the noise level is significantly lower than the measured signal. In this paper, a negligible NER refers to any value less than 20 %, which corresponds to a SNR > 14 dB.

The System Measurement Accuracy:

The measurement accuracy of a system is the smallest change in the internal impedance distribution of the object, which the system can detect [25]. The system measurement accuracy is computed by observing the minimum voltage that can reliably be measured for a selected current amplitude. A threshold is applied to the measured data to extract only the measurements that lie outside of the expected noise level of the system. This threshold produces the minimum voltage that the system can reliably measure. Furthermore, using Ohms law, the minimum resistance that the system can detect is computed. This is replicated using the amplitude and frequency of the applied current and measured potential signals to compute the minimum impedance.

Other methods for computing this quantity are to directly measure the impedance of an inhomogeneity. Place it into the phantom tank and observe the impedance of the inhomogeneity in the reconstructed image data [80]. However, this method requires repetitive labour whenever an inhomogeneity, having a different impedance distribution, is inserted into the phantom. This method itself produces errors as the system used to measure the exact impedance of the inhomogeneity, may contain uncertainties. Alternatively, the reciprocity error can be computed by applying the system to a resistor network or saline solution. The error is then defined as the ratio between the measured resistance and the actual resistance [81].

The Symmetry of the System:

To observe the symmetry of a system, require reconstructing images of inhomogeneities that are placed, opposite to each other, at equal distances from an axis of symmetry [25]. The reconstructed image of an inhomogeneity placed at a certain position should be the mirror image of that which is produced when an inhomogeneity is placed diametrically opposite the first position, for an ideal system. However, practical systems have electrode position errors,
and the manual positioning of an inhomogeneity could all cause a system to appear to be asymmetrical. In this thesis, to test the symmetry of a system is to compare the position and size errors of the reconstructed images for inhomogeneities that are placed diametrically opposite one another. Another method for observing the systems symmetry, is to compute the amplitude response (AR) of the system. The AR is computed by subtracting the mean pixel value of a given image from each pixel and dividing the result by the standard deviation of all pixels in the image [79]. The AR for one image is then compared to that of another image, when an inhomogeneity is placed in the mirror position.

_The Low Frequency Drift (LFD):_

The low frequency drift (LFD) of the system is the observed drift of a measurement frame of the measured signal's amplitude and offset when a DC signal is injected [25]. This is typically caused by ionization and polarization in the medium, temperature, and hardware properties [82]. One way to eliminate LFD, is to reduce prolonged measurement times or only acquire measurements after the system has entered a steady state.

_The Measurement Frame Rate (MFR):_

The MFR of a system is the combined time taken to measure the voltages at all measurement electrodes. This is done until current has been injected through all possible injection electrode pairs, for a given current injection pattern. Reducing the MFR also reduces the LFD, at the increased cost of fabricating a fully parallel system.

3.6.2 The Image Reconstruction Performance

The image reconstruction (IR) performance of a system is a measure of how close the reconstructed images resemble the actual inhomogeneous case. This includes how well the size, position and shape of the inhomogeneity is preserved in the image. The following figures of merit are employed to estimate the IR performance of a system.
Chapter 3: Orthogonal Chirp Division Multiplexed Absolute EITS

The Position Error (PE):

The PE refers to the error between the actual and reconstructed image positions of an inhomogeneity, at a selected position in the phantom [78]. It is computed as:

$$PE = r_{recon} - r_{actual}$$

(3.6-2)

Here, \(r_{recon} - r_{actual}\) is the difference between the radial distances to the centre of the actual inhomogeneity and the inhomogeneity in the reconstructed image. However, a more useful measure of the position error, is to compare this difference to the radius of the test tank, \(r_{tank}\), shown below.

$$PE[\%] = \frac{|r_{recon} - r_{actual}|}{r_{tank}} \times 100$$

(3.6-3)

The Size Error (SE):

The SE refers to the difference in diameter of the reconstructed image inhomogeneity to the actual case [78]. In a similar way to computing the PE, the difference between the diameters of the inhomogeneities is divided by the diameter of the test tank, as shown below.

$$SE[\%] = \frac{|d_{recon} - d_{actual}|}{d_{tank}} \times 100$$

(3.6-4)

This method is, however, limited to circular inhomogeneities.

The Residual Error:

The residual error refers to the absolute difference between the computed impedance distribution and the estimated impedance distribution as the solution converges toward the estimated or prior distribution. This is a measurement ratio given by an iterative computation
such as the Gauss-Newton computation. Essentially, it measures how accurate the solution is, compared to the prior information.

*The Rate of Convergence*

The number of iterations performed to attain a solution to the EITS reconstruction problem, is a measure of the rate of convergence of the solution.

*System Distinguishability*

Distinguishability refers to the ability of the system to distinguish between several inhomogeneities, placed inside the phantom, which could have different impedance distributions, shapes, or sizes [25]. To quantify this performance merit requires computing the level of shape deformation, size error, position error and measured impedances. Alternatively, knowing the size and position error and *observing* that the shape of the inhomogeneities in the reconstructed image is close to the shape of the actual object is adequate information, in this work, to conclude the system can distinguish between various inhomogeneities.

*Image Ringing*

Ringing (or more generally noise) is the evidence of artefacts in the reconstructed image that have an opposite polarity to the inhomogeneities [78]. It is caused by the reconstruction algorithm and the hardware. In this paper, to remove ringing that is caused by the hardware of the system, requires the use of thresholding and applying balanced currents.

*System Detectability*

Detectability is a measure of the most minor change of impedance that can be detected by the system. Furthermore, knowing the systems measurement accuracy already presents this information. Other reports suggest that detectability is a measure of how well the reconstructed images detect an inhomogeneity [25]. However, this definition of detectability is like that of distinguishability, which means that once the system can distinguish between several inhomogeneities, then it is reasonable to conclude that the system is able to detect a single
inhomogeneity. Intuitively, if several inhomogeneities can be detected, then a single inhomogeneity can be detected.

System Repeatability:

Repeatability refers to the ability of a system to produce repeatable measurements, under the identical environmental conditions, over time. To observe this, involve taking several measurement frames for an arbitrary inhomogeneity position, and reconstructing images thereof. How well these images resemble each other, gives an indication on the repeatability of the system.

3.7 Summary

In this chapter, a description was issued to demonstrate the computation of the impulse responses from the cross-correlations between the stimulation and measurement signals of an EITS system. These impulse responses represented the impedances along different paths or channels within an object. In addition, the concept of OCDM was presented to illustrate an efficient method of developing orthogonal chirp waveforms that maintain identical period-bandwidth products. At that time, the weak form of the forward problem was resolved using the FEM to discretize the problem. After resolving the forward problem, the method of Tikhonov regularization was represented to resolve the inverse problem. Subsequently, a demonstration was provided on the incorporation of OCDM prior information to constrain the solution to the inverse problem. In addition, the OCDM prior information was assumed to contain information about the homogeneous conductivity distribution and the system equivalent resistance per channel. This provided a way of modelling and therefore the subsequent convergence of the solution to the inhomogeneous conductivity distribution. At the last moment, a set of figures were introduced, which will be used in this study to examine the quality and performance of the solution.
Chapter 4: Proof of Concepts

Simulations performed with MATLAB and EIDORS will be presented to provide a proof of concept that aids the hypotheses of this thesis. In MATLAB, several orthogonal chirp sequences are developed and used to stimulate a test FEM model. The aim is to use OCDM to attain a wideband measurement frame of channel conductivities. This frame is then segmented to develop vital frequency-dependent prior information about the conductivity (or impedance) distribution of the test model. This prior information is utilized, in a regularization matrix, to constrain the absolute EITS solution by accounting for any inherent modelling irregularities. It therefore allows the absolute EITS system to detect true anomalies. Accordingly, the prior information is unique to the test model and will improve the quality and rate of convergence of absolute EITS image reconstructions.

To begin, the correlation properties of the developed orthogonal chirp signals are discussed. Subsequently, a FEM model will be presented as the test model that contains a pre-existing inhomogeneity. Several orthogonal chirp sequences will be used to simultaneously stimulate the model. The output voltages at the different boundary electrodes will be measured and cross-correlated with the stimulation signals (before using the fft) to acquire the channel impulse responses. The impulse responses will be used to develop the regularization matrix. Furthermore, images will be reconstructed using absolute EITS with commonly used generic prior matrices. In comparison, the OCDM-prior regularization matrix will be used to regularize the subsequent absolute image reconstructions. The performance of all reconstructed images will then be compared, utilizing a few performance figures that were outlined in section 3.6.2.

4.1 The Properties of Orthogonal Chirp Signals

For direct sequence spreading of orthogonal alternating signals, the following constraints must be obeyed [83]:

- The sequence must have sharp autocorrelation peaks to allow synchronization.
- A low cross-correlation profile is needed, to be able to increase the number of orthogonal sequences that are used to stimulate the test object.
By conforming to these constraints, the generated signals will have good spectral density properties. And this allows equal spreading of the energy, across the different stimulation electrodes, over the frequency-band.

From section 3.2.2, the orthogonal chirp signals are formulated by substituting a modification of the Talbot effect into the Fresnel transformation as shown below, for \( N \) orthogonal chirp signals:

\[
i_k(t) = e^{-\frac{j\pi}{4}} e^{j\pi\left(\frac{N}{T}\right)^2(t-k\frac{T}{N})^2}, 0 \leq t < T
\]  

\[ (4.1-1) \]

Where \( k = 0, 1, 2, ..., N - 1 \).

Mutual orthogonality (of these signals) was demonstrated by the inner product:

\[
\int_{-\infty}^{\infty} i_x(t) i_y^*(t) dt = \delta(y - x)
\]  

\[ (4.1-2) \]

In addition, the cross correlations between the root chirp and three arbitrary chirp signals are shown in Figure 4.1.
Figure 4.1 The correlation plots of four orthogonal chirp signals. The plots show that the autocorrelation contains a large identifiable peak, while the cross-correlations are significantly lower. This conforms to the requirements set out to identify orthogonality and to achieve equal energy distribution across the stimulation electrodes.

Figure 4.1 provides the correlation plots between four orthogonal chirp signals. The autocorrelation of the first signal produces a sharp distinguishable peak. Furthermore, the chirp signals do possess a lower amplitude cross-correlation which further confirms orthogonality between the chirp signals. However, given that the cross-correlations are not completely flat at zero magnitudes, it is anticipated that the measurements will contain some spectral contamination. Subsequently, it is observed that the cross-correlations become flatter as the difference between the k and T values, in equation (4.1-1), for two arbitrary signals are increased. Therefore, more minuscule k and T value differences may lead to more significant spectral contaminations. One way to reduce the effects of spectral contamination is to (at least) compute $N^2$ signals for an N stimulation system. In this way, the signals are adequately separated. Figure 4.2 illustrates the impact of increasing the separation between signals and increasing the period.
Correlation between orthogonal chirp signals with increased separation and period

Figure 4.2 The correlation plots of four orthogonal chirp signals, after increasing the separation and period, \( T \). Separation was increased by computing every fourth signal in a set of 16 orthogonal signals, for a 4-stimulation system. The plots show that the autocorrelation contains a large identifiable peak, as in Figure 4.1, while the cross-correlations are significantly lower. Therefore, the cross-correlations are improved, and the spectral contaminations are reduced.

Figure 4.2 demonstrates a reduction in the cross-correlation magnitudes due to an increased separation between orthogonal signals and their period. For a 4-stimulation system, every fourth signal was computed from a set of 16 orthogonal chirp signals. Computation speed is identical to the adjacent separation approach shown in Figure 4.1, because four signals are still being computed. But the value of \( N \) is set to 16 (instead of 4), and \( k=0,3,6,9 \) (instead of \( k=0,1,2,3 \)). To select the signal length depends on the capability of the system and the desired frame rate. Furthermore, a signal length of 1024 samples is adequate to acquire enough information about the object being imaged [25].

Additionally, to observe the spreading of energy over the frequency-band, given that the energy of a chirp signal represents a function of the stimulation signal magnitudes, the combined channel magnitudes are presented in Figure 4.3.
Figure 4.3 presents the spectrogram of 16 orthogonal chirp signals. It is shown that the collective magnitude across the sample frame remains consistent, indicating an equal energy distribution across the electrodes. However, comparing the magnitude profiles between different channels indicates that some electrodes are stimulated at lower comparable energies at different instances. It may appear that OCDM suffers from the same data inconsistencies as FDM. However, FDM does not allow one to observe the entire spectra at all stimulation electrodes, whereas OCDM stimulates all stimulation electrodes with wideband chirp signals that operate over the same period-bandwidth product. Therefore, it is anticipated that the injection of orthogonal chirp signals will maintain uniform current distribution. And will not suffer from the data inconsistencies or signal contaminations due to TDM, FDM, and CDM. And given that the properties of the orthogonal chirp signals are favourable (The orthogonal signals provide a flat energy distribution and wideband stimulation at all electrodes.), simulations are needed to provide proof of the capability to reconstruct images using the OCDM measurement data.
4.2 Absolute EITS Image Reconstructions

To investigate the concept of using OCDM prior information to constrain, and therefore improve the rate of convergence and quality of the absolute EITS image reconstructions, consider the following test FEM model, having background conductivity set to a value of one. The model in Figure 4.4 contains a pre-existing inhomogeneity. The aim is to detect this inhomogeneity and observe its frequency behaviour from a single measurement frame (acquired using the adjacent measurement protocol) using absolute EITS.

![Figure 4.4 A FEM model containing a single pre-existing internal inhomogeneity, at 100 kHz. The aim is to detect and characterize this inhomogeneity using a single measurement frame.](image)

To evaluate the hypotheses of this thesis, 16 orthogonal chirp signals were computed and used to stimulate the model in Figure 4.4. The conductivity of the inhomogeneity was defined by a frequency dependent rising exponential function, such as that of a banana. The instantaneous values of the chirp signals (each signal contained 1000 samples) were mapped to the different electrodes, and the corresponding boundary potentials were measured. After the instantaneous values of the chirp signals were applied to each corresponding electrode, the resultant matrix of boundary potentials was cross correlated with the chirp current signals. Which was performed to compute the channel impulse responses. This was reiterated for ten equidistant frequencies within the 1 kHz to 1 MHz frequency spectra. The computed channel conductivity magnitudes are shown in Figure 4.5.
In Figure 4.5, ten different channel magnitude profiles are presented. It shows there is an increase in the magnitude of the channel conductivities as the stimulation frequency rises (for single source systems, the same information can be acquired at a much slower rate, because the single chirp must be time multiplexed to each stimulation electrode). This is caused by the conductivity of the inhomogeneity that rises exponentially. Since the conductivity rises exponentially (for the proposed method) the channel impulse responses from the frequency-differenced measurements taken between the 1 kHz ($\bar{\sigma}$) and 100 kHz ($\sigma$) distributions are used in the prior information ($R^T R$) about the object. In contrast, to compare the proposed method to the frequently used generic prior matrices, absolute images are reconstructed using a Laplace, Tikhonov, or Noser prior. The generic prior absolute image reconstructions are given below. In all image reconstructions, the only variation is the type of prior matrix used.
Figure 4.6 Generic absolute EITS image reconstructions of the test model (at 100 kHz), using (from left to right) a Laplace, Noser, and Tikhonov prior. The measurement frame included -12 dB of white noise. The sidebars display the range of conductivity magnitudes in S/m.

Figure 4.6 (using several conventional priors for absolute EITS) shows the detection of the pre-existing internal structure, at 100 kHz with -12 dB of white noise. The hyperparameter (of value 0.8) was selected using the well-known Heuristic approach. A 256-element mesh is implemented for all image reconstructions. However, there are some significant impedance distribution errors present in the reconstructed images. Note that the sidebars of all reconstructions represent the conductivity magnitudes in S/m. Furthermore, on average the reconstructions completed after six iterations with a residual error of 45.58%. Subsequently, Figure 4.7 illustrates the improvements in the image quality with fewer image artefacts, when utilizing the OCDM prior information to constrain the solution.

Figure 4.7 OCDM Absolute EITS image reconstruction of the test model, at 100 kHz, with OCDM frequency-differenced prior information (using the difference between the 1 kHz and 100 kHz distributions). Comparatively, the image contains fewer image artefacts, lower residual error, and a stronger resistance to image noise. The sidebar displays the range of conductivity magnitudes in S/m.
Figure 4.7 was reconstructed using OCDM-aEITS. The inverse solution was constrained by a frequency-differenced OCDM-prior regularization matrix. This was the same measurement frame used for the generic prior case, with -12 dB of added white noise. The OCDM data were incorporated into the regularization matrix, utilizing the frequency difference method proposed in section 3.5.2. The difference between the proposed method and FDM (such as the weighted frequency method proposed in [84]) is that the current FDM methods still rely on information from previous empirical studies. And the electrodes are stimulated at unique frequencies, to maintain orthogonality, which is insufficient when measuring complex impedances. In contrast, OCDM applies wideband signals at all stimulation electrodes and develops a wideband measurement frame. Given the measurement frame, the prior information can be computed from a difference between the distribution at the imaging frequency and the distribution obtaining the lowest variance to its mean. Essentially, the prior information in the case of OCDM-aEITS includes the expected perturbations and converges the inverse solution to this information. This ensures the reconstruction of fewer discrete elements. Furthermore, the OCDM prior adapts to changes in the internal domains of the object, because the images are reconstructed using prior information that is unique to the object at the time of imaging.

Furthermore, the reconstructions completed (on average) after three iterations and produced a residual error of 25 %. Additionally, the images show that the shape and location of the inhomogeneity closely resembles that of the original test model. Furthermore, practically all artefacts are eliminated from the image. This indicates OCDM can be applied to EITS to reconstruct images of an object.

Subsequently, to compute and compare the quality of the reconstructed images, the image pixel intensities need to be plotted for the generic-prior and the OCDM-prior absolute EIT. This was performed by first converting the figures to grayscale before plotting the pixel complement values. The relevant pixel intensity plots are shown in Appendix A: Simulated Image Pixel Intensity Plots. The following table presents the computed image quality as a measure of the size and position error, as outlined in section 3.6.2. These errors were computed in relation to the actual test object that contained an internal inhomogeneity diameter equal to 19.16 % of the object diameter and a radial distance, from the centre of the object, of 9.30 % of the diameter of the test object.
Table 4.1 Performance figures of the simulated image reconstructions for absolute EITs using different regularization matrices.

<table>
<thead>
<tr>
<th>Performance figure</th>
<th>Absolute EIT with Laplace prior</th>
<th>Absolute EIT with Tikhonov prior</th>
<th>Absolute EIT with Noser prior</th>
<th>Absolute EIT with OCDM prior</th>
</tr>
</thead>
<tbody>
<tr>
<td>SE [%]</td>
<td>7.00</td>
<td>11.54</td>
<td>3.46</td>
<td>2.69</td>
</tr>
<tr>
<td>PE [%]</td>
<td>3.19</td>
<td>3.72</td>
<td>1.33</td>
<td>1.06</td>
</tr>
<tr>
<td>Iterations</td>
<td>7</td>
<td>6</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>RE [%]</td>
<td>46.25</td>
<td>48.00</td>
<td>42.50</td>
<td>25.00</td>
</tr>
<tr>
<td>Performance rank</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4.1 indicates the size and position errors that were computed from the corresponding pixel intensity plots. In addition, the table includes the average residual error and number of iterations for the image reconstructions for various prior information matrices. Comparing the generic prior information methods, the Tikhonov method performed the worst, while the Noser method provided the most outstanding overall performance. Subsequently, the generic prior information methods performed worse compared to the OCDM prior information method. Focusing on the rate of convergence and residual errors. The use of an OCDM-prior reduced the rate of convergence from a maximum of seven down to three iterations. And reduced the residual error by 54.84%. Furthermore, these figures suggest that indeed the performance of absolute EITs image reconstructions can be improved by incorporating OCDM prior information.

Subsequently, Figure 4.8 illustrates the change in impedance distribution, at ten equidistant frequencies, across the measurement frame.
Figure 4.8 Absolute EITS image reconstructions, using the OCDM prior information. The prior information was developed from the frequency-differenced impulse responses between the 1 kHz and 100 kHz sections of the measurement frame. Images were then reconstructed with ten equidistant frequency sections to observe the change in impedance as the frequency increases. The sidebars display the range of conductivity magnitudes in S/m.

Figure 4.8 demonstrates the change in the absolute impedance of the object at ten equidistant frequency sections of the measurement frame. The prior information was developed from the frequency-differenced impulse responses between the 1 kHz and 100 kHz sections of the measurement frame. The reconstructed images affirm that the conductivity of the inhomogeneity increases with the applied frequencies. Therefore, using a single OCDM wideband measurement frame, one could observe the frequency behaviour of the object at all stimulation electrodes.

### 4.3 Summary

In this chapter, the method of generating orthogonal chirp signals were discussed. This was discussed to identify the correlation properties and energy distribution of the orthogonal signals. The orthogonal chirp signals were shown to have a high autocorrelation peak, while maintaining lower cross-correlation amplitude profiles. Consequently, it was concluded that indeed the chirp signals were orthogonal and ensured that these signals maintained adequate energy across the entire frequency spectrum. However, given that the cross-correlations were not perfectly zero, some spectral contamination was expected.
Furthermore, OCDM was utilized to attain information about a test FEM model, which contained a pre-existing inhomogeneity. It was shown that the measurement frame contained sufficient information to reconstruct images and detect the inhomogeneity. In addition, absolute EITS image reconstructions were performed for several generic prior matrices. The results showed an identification of the inhomogeneity. But the images contained artefacts that may be misrepresented as additional anomalies, especially when -12 dB of white noise were added to the measurements. Consequently, on average the reconstructions took 6 iterations to reach a residual error of 45.58 %. Subsequently, the frequency-differenced impulse responses between the 1 kHz and 100 kHz sections of the OCDM measurement frame were used to form the OCDM prior information for absolute EITS. In this case, the reconstructed images showed clear identification of the inhomogeneity, reduced errors, faster convergence, and fewer image artefacts. Subsequently, it was concluded that the OCDM prior information can improve the image quality and rate of convergence of absolute EITS.
Chapter 5: System Design

In this chapter, a motivation for the development of an EITS system will be presented. Thereafter, several system specifications and requirements will be discussed. Furthermore, the procedure followed to design an EITS system for this research (which includes creating a phantom test tank within which inhomogeneities will be placed) are outlined. The final step is to construct a data acquisition system that measures the resultant boundary potentials and transmits this data to a controller module. The controller will process the data before conveying the result to a computer, or to resume onboard computations.

5.1 Motivation for an EITS System

To the best of the author’s knowledge, an absolute EITS system that develops the prior information using OCDM was unavailable. Hence, based on the following system requirements, a new system is required to answer the research questions.

1. *Current injection and voltage measurement:*

To significantly reduce the effects of source-on-source interference in a multi-source system, devices will be introduced to provide uninterrupted orthogonal current stimulation and voltage measurement. These devices will provide efficient and reliable excitation signals to improve the system accuracy [57].

2. *Absolute imaging:*

As the research question places a constraint on the type of imaging, an absolute EITS system will be designed. Although the system will be developed to produce absolute images, it will remain flexible to be used for other methods as well.
5.2 EITS System Specifications

The aim of designing an EITS system for this research, is to design an anomaly detection system by mapping the internal impedance distribution of the system. Furthermore, most geological and process monitoring applications require cumbersome and expensive tomographic systems compared to the biomedical requirements [85]. The reason is that the biological specimen, like the lungs, brain and thorax all require low amplitude and frequency stimulating signals; signals operating under 1 mA at a maximum frequency of 1 MHz [60]. Hence, the most cost-effective application of EITS for this research is one which is sought to the biomedical industries. The following sections provide carefully documented engineering standards to ensure system reliability and biomedical safety.

5.2.1 Functional Characteristics

*Functional Characteristic Overview*

The EITS system must be designed to reconstruct cross-sectional images using the computed internal impedance distribution of objects. This is performed by surrounding the object with a ring of electrodes and using additional circuitry to acquire and analyse the measured quantities. A few constraints regarding the hardware of the complete system are that the system needs to be portable, cost-effective, lightweight, and produced from an assembly of conveniently accessible products. The data is collected using an analogue-to-digital converter (ADC), which sends the measured and formatted quantities to a controller module. Images are reconstructed from these measurements using the MATLAB EIDORS image reconstruction libraries or customized Python image reconstruction libraries.

*Signal Levels*

In biomedical applications, the stimulation current that can be sensed by biological tissue and penetrate through the resistance of the skin is 1 mA at 1 kHz [86]. Applications that require lower amplitudes must adhere to higher frequency signals. While lower frequency requirements involve larger proportional amplitudes. This amplitude-frequency trade-off is based on the heart’s vulnerability at frequencies higher than 60 Hz [25].
Frame Rate

For this research, an adequate maximum time required reading all measurements (1,000,000 samples) is one second for a 1 kHz applied signal. This limitation is based on the desired acquisition speed after considering other systems in industry. Consequently, a frame rate of greater than 10 frames/s is desirable. Nonetheless, this thesis focuses on reconstructing multiple images from single wideband measurement frames. Therefore, one frame per second remains the minimum requirement.

System Accuracy

A system measurement accuracy of 0.9 % would be deemed satisfactory for a 16-electrode single plane (phantom based) system, which uses a 10 – bit resolution ADC. And can detect a voltage in a range of 5 V, [25]. Furthermore, the measurement accuracy per electrode is:

\[
\frac{\text{total measurement accuracy}}{\text{total electrodes}} = \frac{0.9\%}{16} = 0.05625\%
\]

Any higher resolution data acquisition system is adequate to achieve this level of accuracy.

5.2.2 Safety Characteristics

A few biomedical safety precautions (as stipulated in the IEC 601-1, IEC 601-1-2, IEC 601-1-4 and ISO 13485, ISO13488) are listed below.

- To protect a patient from an electric shock during testing, the system will be isolated from the mains supply using pulse-transformers or opto-isolators.
- Overcurrent fuses will be used to prevent possible fires and electrical shorts caused by an electrical fault.
- Common ground will be used throughout the system.
- Multiple points of failure will be integrated into the system.
- System diagnostic tests will be performed regularly using risk preventative methods embedded into the computer algorithm.
- Implement fail-safe computer algorithms and hardware.
5.2.3 Non-Functional Characteristics

A few non-functional characteristics of the system are described below. These characteristics do not directly describe the operational requirements of the system nor do they place any test subject safety constraints. These characteristics define the user-end side of the design for the final system after the prototyping stage.

**System Quality**

The system is to be assembled from conveniently accessible, high quality and reliable components to ensure repeatable performance over long-term usage.

**Fabrication of the Data Acquisition System**

The data acquisition system should be fabricated using university resources. This includes free sourced PCB design software and board printing and component assembly resources.

**Economic Factors**

The project budget is dictated by the university resources quota assigned to the author. Furthermore, the total cost of the project should be reduced to a minimum compared to standard available EITS systems which cost roughly 500 000 $ZAR$ [25]. For the prototype, the budget will be capped at 50 000 $ZAR$.

**Ergonomic Factors**

To provide an ergonomic system, the following requirements should be achieved.

- It should be portable, lightweight, and user-friendly.
- It should be capable to withstand reasonable user-device abuse.
- It should not create user discomfort.
- The design layout should be of a logical manner.
- It should be powered by a 12 $V_{DC}$ power supply, as these are common voltage source outlets in vehicles and buildings.
5.3 EITS System Layout Concepts

A few common EITS hardware concepts need to be reviewed to procure measurement data. This involves connecting the phantom tank to a central processor via a data acquisition system. The central processor is required to reconstruct the images and control the acquisition board and the multiplexers. Several EITS system layout concepts are discussed below.

5.3.1 Concept 1: Computer and Data Acquisition Cards

Utilizing a computer to receive data from a data acquisition card, like the \( \mu \text{DAQ} \) cards by Eagle Electronics, provide the simplest system layout, shown in Figure 5.1. These cards come as an ADC, DAC, or a combination of the two. If two cards are handled, then the signal from the DAC card is passed to a voltage controlled current source (VCCS). And the resulting signal is passed to the surface mounted electrodes on a test tank. At the voltage measurement end, the voltages at the boundary of the tank are measured using difference amplifiers. The result is passed to the ADC card, which converts the analog voltage signals to a digital format before conveying the result to a computer for image reconstruction. These cards provide high sampling rates (as high as \( 1 \, \text{MSps} \)) shared among 16 channels. Consequently, the sampling rate is \( 62.5 \, \text{kHz} \) per channel. Furthermore, the slower of the two cards dictate the sampling speed of the system, according to the Nyquist sampling criterion.

![Figure 5.1 Personal computer and data acquisition board system setup. It shows the complete process in which this layout collects data.](image)

Figure 5.1 shows the complete data acquisition cycle used to acquire data from the test tank, when using the data acquisition cards. This method is the simplest to implement.
5.3.2 Concept 2: Computer and Microcontroller

Most applications that require signal transmission and data acquisition, require devices that can collect samples of the measured data, in memory. This is performed before transmitting the acquired data to a primary device that does further formatting of this data before producing useful information. Microcontrollers have severely limited storing capabilities. One of several other solutions to this problem, if microcontroller compatibility is essential, is to store all code on the primary device. Subsequently use the microcontroller as a secondary control device. The supply signal could be processed on the computer, and sent through the microcontroller's onboard DAC, to produce a supply signal. However, most onboard DACs have limiting output frequencies and bit resolutions that are not adequate for EITS applications. Regarding the storage problem, a secure digital (SD) card may be used to increase the memory of the controller. This design layout is shown in Figure 5.2 below.

Figure 5.2 shows the data acquisition process when using a computer and microcontroller as the controlling mechanism. The flow chart shows that a voltage signal is generated by a signal generator and converted to a constant current. The current is then multiplexed to the electrodes around the test object, if TDM is employed. Otherwise, 16 current sources are required to simultaneously stimulate all current electrodes. Furthermore, output voltage differences are measured and converted to a digital format before the microcontroller transmits the results to the computer.
5.3.3 Concept 3: Computer and Field Programmable Gate Array

The concept of using a Field Programmable Gate Array (FPGA), coupled with a computer, is like the previous concept. The advantages of using an FPGA, over a microcontroller, is the processing speed and system flexibility. However, the FPGA shares some inherent disadvantages with the microcontroller such as the low onboard memory (typically, 5 GB storage space is needed for EITS applications.) while being less cost-effective. A stand-alone FPGA system was considered, using the Altera SocKit Cyclone V development board. It provided 1 GB random access memory (RAM) with 128 MB quad serial peripheral interface (QSPI) Flash and 925 MHz processing speed.

5.3.4 Concept 4: Microcomputer

Microcomputers provide some promise, as they remove the cost and need for a personal computer. Two microcomputers were considered for the task: the PCDuino 3B and the BeagleBone Black. Of the two choices, the PCDuino provided the most onboard storage of 1 GB with 1 GHz onboard processing capabilities. Furthermore, it has storage upgrade capabilities via a SD card slot. However, the PCDuino required other software to provide a supply signal, read and format the measured data. Furthermore, for the research questions that need to be answered, the microcomputer provides an unnecessary level of complexity during the implementation phase. As compared to the other concepts, which will dramatically affect implementation times. Additionally, the standard EITS image reconstruction libraries are not commercially available for these devices. As a result, new image reconstruction libraries are to be written precisely for the microcomputer, which is outside of the scope of this work.

5.3.5 Concept Evaluation

The above-mentioned concepts are evaluated towards achieving the functional, safety and non-functional characteristics that were previously outlined. A weighting system is employed to assign scores to each concept based on portability, cost, design simplicity, implementation simplicity, supply and control signal output rate, and the data acquisition rate. All system concepts need to be portable, which is weighted at 5%. The net cost represents a crucial factor and is weighted at 15%. Design simplicity establishes how simple the design is, which affects the time required for implementation and is given a weight of 20%. This implies a design possessing a high level of complexity will drastically affect the design and implementation time. The implementation simplicity refers to the accessibility of the system
components. And how long it will take from ordering components to setting up the complete system and is given a weight of 10%. This was selected to record the time required to develop the system and how it will directly affect the time it takes to complete the research objectives. The signal output rate is given a weight of 20%, because the output rate affects the frame rate of the system. The data acquisition rate is given the remaining weight of 30%, because it affects the frame rates while restricting the range of frequencies that can be used to reconstruct an image. The table underneath shows the scoring for each concept.

Table 5.1 Scoring table of different EITS system layout concepts.

<table>
<thead>
<tr>
<th>Concept</th>
<th>Portability [5%]</th>
<th>Cost [15%]</th>
<th>Design Simplicity [20%]</th>
<th>Implementation Simplicity [10%]</th>
<th>Signal Output Rate [20%]</th>
<th>Data Acquisition Rate [30%]</th>
<th>Total [100%]</th>
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As seen in Table 5.1 above, concept four provides the second-best overall score, however, the complexity of implementing the system layout will cause too many delays. Concept three provides the best overall score and functional characteristics, however, the cost of implementing this concept is higher than the alternatives. Concept two provides reduced complexity and cost. These are two significant factors for the time to completion and the cost of the investigation. However, this concept exhibits the lowest number of adequate functional characteristics. Concept one provides the least complexity, but it has a low portability, signal output rate and data acquisition rate score. Therefore, Concept three is best suited among the concepts presented, for EITS in this research.

The EITS hardware system design carries out a crucial role in acquiring data about the object being imaged. Due to the nonlinear and ill-posed nature of the inverse problem, errors in the hardware significantly affect the quality of the image reconstructions. A typical 16-measurement electrode EITS system uses the four-electrode measurement approach [87]. And the current electrodes are stimulated using a single or multi-source architecture [88].
A block diagram of a complete signal injection and voltage measurement procedure is given below, for an equivalent eight-electrode system. This system follows the standard EITS system layout presented in [89], and is extended to a multi-source architecture. The final prototype contains 32 electrodes. Sixteen (equidistant) of which are used for current stimulation, and the remaining sixteen are used for sampling the boundary potentials. The boundary potentials are sampled and used to reconstruct images of the cross-section impedance distribution of a test object [90].

![Block diagram of a complete eight-electrode EITS system](image)

*Figure 5.3 Block diagram of a complete eight-electrode EITS system. An eight-electrode diagram is presented to illustrate the layout.*

Figure 5.3 shows the flowchart for an eight-electrode layout equivalent of an EITS system. Voltages are generated using Numerically Controlled Oscillators (NCO) [91], implemented in Verilog Hardware Description Language (VHDL), onboard the FPGA. This design uses a high-precision single-channel DAC driven by the FPGA and is based on a single channel DAC multiple output channel architecture [92]. Followed by a mirrored modified Howland voltage-controlled current source [93] that drives the inputs of two multiplexers, via optocouplers for source isolation. The two optocouplers output a mirrored current pair that is multiplexed to several sample-and-hold (S&H) devices [94]. The waveforms from the S&H devices are
sampled before and after the respective output resistors, to measure the injected current waveforms that will stimulate a test tank. The chirp current waveforms at the output of the resistors are applied to the electrodes (of dimensions 1 cm x 2 cm) of the test tank. The boundary voltages are simultaneously measured (via the four-electrode adjacent-measurement approach [87]) using voltage instrumentation amplifier circuits to difference the voltages and reduce the effective voltage ranges [95]. These voltages are sampled by a second ADC. The FPGA reads and transfers the samples to a computer that computes the inner product of the samples, followed by a \textit{fft}, to extract the impulse responses.

5.4 Supply Signal Transmission and Data Acquisition

Stable chirp current waveforms are to be used to stimulate non-redundant electrode pairs. To achieve this, a voltage-controlled current source with a FPGA, DAC, multiplexer, and S&H circuits are used to supply the current at an amplitude of 1 mA with a frequency in the range up to 1 MHz. The boundary electrode potentials are sampled and used to reconstruct an image of the cross-section impedance distribution of a test object. The following sections outline the essential characteristics of each component in the data acquisition system.

5.4.1 Signal Generation

The FPGA board consumed in this work is the Altera Cyclone V SOCKit, with a 925 MHz clock. This board contains Altera’s Numerically Controlled Oscillator intellectual property (NCO IP) core which generates signals using an NCO module. The NCO, shown in Figure 5.4, is used to synthesize a discrete-time and discrete-valued representation of a sinusoidal signal [96]. The NCO core features include:

- 32-bit angle and phase precision.
- An IP core that allows multiple NCO architectures, which can generate several parallel signals.
- A ROM-based implementation.
- Individual NCO’s that can simultaneously output a sine/cosine pair.
- The allowance for phase and amplitude modulation.
- Phase dithering.
- Frequency-hopping and multi-channel capability.
The generated sine wave at the output of the NCO module in Figure 5.4, is defined below:

\[ s(nT) = A \sin(2\pi(f_O + f_{FM})nT + f_{PM} + f_{Dither})) \] (5.4-1)

Here:

- \( T \) is the clock period.
- \( f_O \) is the unmodulated output frequency based on the phase increment.
- \( f_{FM} \) is the frequency modulation parameter.
- \( f_{PM} \) is derived from the input phase modulation and the number of bits used to describe the modulation.
- \( f_{Dither} \) is the internal dither parameter.
- \( A \) is the amplitude of the sine wave based on the magnitude precision.

To convert a sinusoid to a chirp waveform, consider the following sinusoidal waveform:

\[ s(t) = A \sin(\theta(t)) \] (5.4-2)
Chapter 5: System Design

The instantaneous angular chirpiness, for this sine wave, is defined as the second derivative of the instantaneous phase:

\[ \beta(t) = \frac{d^2 \theta(t)}{dt^2} = \frac{df(t)}{dt} \]  

(5.4-3)

Here, \( f(t) \) is the instantaneous frequency.

For a linear chirp signal, the frequency varies linearly with time:

\[ f(t) = ct + f_0 \]  

(5.4-4)

Here, \( c \) is a frequency modulation constant and \( f_0 \) is the initial unmodulated frequency.

Furthermore, we define the frequency scalar multiple, with a final frequency of \( f_1 \) over period \( T \), as:

\[ c = \frac{f_1 - f_0}{T} \]  

(5.4-5)

The time-domain function for the phase of an alternating signal is defined as:

\[ \theta(t) = \theta_0 + 2\pi \int_0^t f(\tau) d\tau \]  

(5.4-6)

Therefore, the linear chirp signal is defined as:

\[ v(t) = A \sin \left( 2\pi \left( \frac{c}{2} t + f_0 \right) t + \theta_0 \right) \]  

(5.4-7)

Comparing equation (5.4-7) to (5.4-1) concludes that \( f_{FM} = \frac{c}{2} t \). Consequently, a linear chirp signal is a sinusoid with the frequency modulated as a linear function of time.
Furthermore, to generate orthogonal chirp signals, unique steps are used for individual signals according to the Talbot distance parameter as shown below.

\[
v_k(t) = A \sin \left( \pi \left( \frac{N}{T^2} \left( t - \frac{2kT}{N} \right) t + \left( \frac{k^2}{N} - \frac{1}{4} \right) \right) \right)
\]  

(5.4-8)

To output the orthogonal chirp analog current waveforms, a single DAC architecture is employed as shown in Figure 5.5. An alternative, multi-channel DAC will produce a parallel design, at a significantly higher cost trade-off.

Figure 5.5 Multiple channel output, single DAC architecture.

In Figure 5.5, a single DAC, multi-channel output architecture is employed to generate multiple analog signals using a single DAC, VCCS, multiplexers and sample and hold devices. The DAC used in this work is the Altera DA/AD conversion card, which houses dual 14-bit DACs with a sample rate of 250 MSps. Considering the Onboard 14-bit ADC that has a sample rate of 150 MSPS, the time to read a particular frame of 1,000,000 samples is 6.7 ms. Therefore, the system is designed to have a frame rate of 150 frames/s, which is significantly higher than the functional constraints defined in section 5.2.1. In addition, the frame rate can be improved by reducing the number of samples per frame, at the cost of a loss of information.
Furthermore, to output 16 orthogonal chirp signals to the DAC, a sequential output approach is used as shown in Figure 5.6.

---

| S15_1 | S14_1 | S13_1 | S12_1 | S11_1 | S10_1 | S8_1 | S7_1 | S6_1 | S5_1 | S4_1 | S3_1 | S2_1 | S1_1 | S0_1 |

Figure 5.6 Packet transmission from FPGA to DAC. Each snippet in a packet consists of the individual 14-bit codes that represent the instantaneous values of the chirp signals. The subscript i indicates the sample number of the corresponding signal.

---

Figure 5.6 illustrates the body of a data packet that is transmitted from FPGA to DAC. Each packet consists of the corresponding bytes of each chirp signal. Therefore, the DAC receives the corresponding instantaneous values of the chirp signals in a sequential manner.

The analog output of the DAC is sent to a mirrored modified Howland voltage-controlled current source (MMHVCCS) that outputs a balanced current pair corresponding to the input analog voltage. The MMHVCCS includes optocouplers, at each output, for electrical isolation. Each output analog current is sent to a multiplexer that is controlled to switch to the correct output channel S&H circuit. This multiplexer inherits low input and output capacitance, low power-up impedance to allow short settling times and low leakage currents to reduce the voltage droop. Additionally, the precision S&H circuit limits the allowable sample output rate to 4.38 ms and frame rate to 14 frames/s if each frame consists of 1,000,000 samples. Indeed, the frame rate can be improved by reducing the frame size. Therefore, utilizing a settling time of 70 ns, and to output 1024 samples per signal, results in a frame rate of 871 frames/s, which is significantly higher than the functional constraints of this work. Additionally, an output rate of 1024 samples per signal was shown to contain adequate information for EITS applications [25]. And Figure 5.7 displays the concept of operation, for a single DAC architecture.
Figure 5.7 The concept of operating a single DAC, 2-channel output architecture. This idea is extended to 16 channels in this thesis.

Figure 5.7 illustrates the concept of operating a single DAC, 2-channel output architecture. In this thesis this idea is extended to a multi-channel output architecture to simultaneous output 16 orthogonal chirp signals. The figure depicts the process of first generating the orthogonal signals. The instantaneous values of the signals are ordered in packets transmitted to a single high-precision DAC via SPI, for its rate of transmission benefits. The DAC outputs the corresponding analog voltage value of every subsequent 14-bit number received. The voltage
waveform is converted to the corresponding 1 mA current waveform pairs. Each current waveform is transmitted to a multiplexer that conveys the instantaneous current value to the precise S&H circuit. The S&H outputs the instantaneous value and sustains this value until a new value is applied at the input. The output waveform from the S&H circuits could be made ‘smoother’, or more closely matched to the generated waveform, by reducing the time it takes to update the circuit and by using polynomial fit estimates.

5.4.2 Design of a Current Source

Orthogonal chirp voltage signals are supplied by the FPGA. Current sources, shown in Figure 5.8, based on the mirrored modified Howland voltage-controlled current sources (MMHVCCS) were designed using wideband OPA657 operational amplifiers (op-amps). These sources are needed to convert the orthogonal chirp supply voltages to orthogonal chirp current waveforms, which will stimulate a phantom test tank. Furthermore, the MMHVCCS configuration was chosen to avoid grounding at a fixed electrode that introduces asymmetry in the current distribution within a test object [25].

![Diagram](image)

*Figure 5.8 mirrored modified Howland voltage-controlled current source attached to two electrodes on the boundary of a test subject, via optocouplers to isolate the sources. In this thesis, the output currents are attached to electrodes via multiplexers and sample-and-hold circuits.*
The MMHVCCS, shown in Figure 5.8, was designed to produce a stable load current of 1 mA from 100 Hz to 1.2 MHz. And have an output impedance of 100 kΩ. Furthermore, research has shown that the output impedance of the VCCS should be larger than the expected impedance of the test load [1]. Therefore, the circuit above was tested (using a droop method), on a variable resistor load, and observed to manage the expected loads (in the range 1 Ω to 10 kΩ) and was eminently capable of supplying the designed current as shown in Figure 5.9.

![Figure 5.9 Plot of the load current over the load resistor range of 1-10kohm. It shows that the current is held at 1mA.](image)

### 5.4.3 Design of a Difference Amplifier

The voltage between each adjacent pair of electrodes should be measured and amplified. The difference between adjacent electrode voltages is measured to avoid large voltage ranges, common-mode rejection ratios and gain bandwidth when measuring individual electrode voltages [25]. A few signal amplifier circuits were considered such as the differential, operation, and instrumentation amplifiers. Of the three amplifier circuits, the instrumentation amplifier had the advantages of using one external resistor to set the gain. And no current flows in or out of the device, irrespective of a variation in the input resistance or load. Therefore, the instrumentation amplifier described below was selected. As it provides adequate bandwidth, reduced implementation complexity, high signal transmission speed and a high input impedance to reduce current flowing out of the tank and into the amplifier circuit.
The instrumentation amplifiers were configured from OPA657 operational amplifiers. These amplifiers provide:

- A bandwidth of 1.6 GHz.
- Dual voltage supply range of ±5 V.
- Low complexity for implementation.
- Fast signal transmission.
- A typical slew rate of 700 V/μs.
- A typical input voltage noise density of 4.8 nV/√Hz.

Typically, these circuits are configured in the following way:

![Diagram of instrumentation amplifier circuit](image)

*Figure 5.10 Employed application of a OPA657. The gain of this instrument amplifier can be controlled with $R_{gain}$ [97].*

Figure 5.10, above, shows the typical application of a OPA657 incorporated instrumentation amplifier circuit. The circuit receives two boundary voltages. The difference between these voltages is measured before the result is amplified. The output is defined by the following equation.

$$V_{out} = 3(V_2 - V_1)$$

(5.4-9)

In equation (5.4-9), the resistors (all 100 kΩ) are selected to be significantly greater than the internal resistance of the phantom test tank (to minimize the resistive loading of the source) and well-matched to obtain an acceptable Common Mode Rejection Ratio (CMRR). This ratio provides an adequate gain of three if all resistors are equal.
In addition, multiple variations in the input voltage were implemented to verify the functionality of the circuit, while it was applied to a 10 kΩ variable resistor. The maximum output voltage was observed to be $V_{\text{supply}} - 0.1 \ V$; where $V_{\text{supply}} = 5 \ V$. Sinusoidal signals were applied to the inputs and the frequency was swept to observe the maximum input frequency of 0.98 MHz. After which the circuit reached a saturation and a roll-off occurred at the output. Subsequently, the voltage differences are transmitted to a multiplexer before an ADC converts the voltages to digital sequences that are used by the FPGA to produce impulse responses.

5.5 Summary

In this chapter, a motivation was provided for the need to design and develop an EITS system for this research. Part of the motivation included an outline of the functional, non-functional and safety constraints. To meet these constraints, a system was designed to work as follows.

Orthogonal chirp signals are generated in a program preserved on an FPGA development board. These signals are converted to equivalent voltage analog signals using a single DAC. The output of the DAC is converted to balanced current pairs, using MMHVCCS circuits, that are correctly multiplexed to corresponding S&H circuits, via opto-couplers to electrically isolate the current source. The outputs from the S&H circuits are sent to the corresponding electrodes. Simultaneously, adjacent electrode voltages are differenced using instrumentation amplifiers to reduce the effective voltage range. These differences are transmitted to an ADC, which converts the analog voltages to a digital format and stored on a FPGA. The FPGA then transmits the sampled data to a computer. The computer performs a cross-correlation between the supply currents and the measured voltages. A $\text{fft}$ is performed on the results to produce impulse responses in the frequency domain. These responses are used to reconstruct cross-section images.

Considering the propagation speed of the components of the design, the limiting factor was found to be the acquisition speed of the S&H circuits of 70 ns. Therefore, after considering the Nyquist sampling criterion and period of the signals. The time taken to transmit the chirp signals and measure the boundary potentials is 1.15 ms per 16,384-sample frame. Resulting in a frame rate of 870 frames/s, which meets the design constraints of the system.
Chapter 6: System Test

In this chapter, several tests for validating the full functionality of the EITS system are presented. These tests include measuring the system measurement accuracy, distinguishability, detectability, and confirming the research hypotheses. A discussion is drawn from the image reconstructions of a phantom test tank.

6.1 System Measurement Accuracy

The accuracy of the system is assessed by its inherent signal to noise ratio (SNR). The SNR is computed to identify the weighting of the measured signals to the zero-input noise level. In Figure 6.1, the noise level is deduced to be $0.1 \, mV_{pp}$. Furthermore, the system was designed to measure a load impedance in the range of $100 \, \Omega$ to $25 \, k\Omega$ with an applied current of $1 \, mA$. This results in a system SNR of $57 \, dB$. A reasonable result when considering the effect of noise on the lowest expected potential measurement of $0.1 \, V$, and the lowest expected difference potential of $0.01 \, V$. These expectations are based on the design of the system to measure a load resistance in the range defined above.

![Figure 6.1 Measured zero-input noise. It indicates some channel offsets and DC bias caused by the ADC and multiplexers. These measurements were acquired from a phantom tank filled with homogeneous saline solution.](image-url)
Figure 6.1 depicts channel offsets or DC bias introduced by the ADC and multiplexers. The software compensates for these fixed-value offsets before processing the results, to avoid image reconstruction errors.

Additionally, the multiplexing method was applied across the test network shown in Figure 6.2.

![Image](image1.png)

*Figure 6.2 The circuit used for system identification tests, using the OCDM multiplexing approach. The circuit is made of two channels. Channel 1 is a pure resistor channel, and channel 2 is a low-pass RC channel, with a cut-off frequency of 159 Hz.*

Figure 6.2 presents the resistor-capacitor network used for testing the ability of the multiplexing method, to adequately identify the system. The acquired system channel magnitude Bode plots are provided in Figure 6.3.

![Image](image2.png)

*Figure 6.3 Corresponding channel magnitude plots for the circuit in Figure 6.2. The Bode plots show adequate identification and distinguishability between the channels, and correctly characterizes the channels. The first channel is correctly identified as a pure resistor channel of magnitude 100 Ω, while the second channel is correctly characterized as a low-pass RC network with cut-off frequency at 159 Hz. However, the spectral contamination affects the measurements significantly at high frequencies.*

Figure 6.3 correctly characterizes the RC-network. However, the spectral contaminations significantly affect the measurements at higher frequencies.
For completeness, the corresponding 8th-order voltage output polynomial fits of 16 FPGA generated orthogonal chirp current signals are shown in Figure 6.4. These signals were applied to a phantom test tank and the resulting boundary potentials are shown in Figure 6.5. By repeating these measurements across a known channel input resistor, the channel input current can be computed, and cross correlated with the boundary potentials.

**Figure 6.4** The 8th-order polynomial fits of 16 FPGA generated orthogonal chirp signals. Polynomial fits were used to smoothen the output of the sample-and-hold circuits. These measurements were acquired from a phantom tank filled with inhomogeneous saline solution.

**Figure 6.5** Plot of the boundary potentials. There are observable variations in the measurement potentials due to the orthogonal stimulation frequencies and the impedance distribution of the object. These measurements were acquired from a phantom tank filled with inhomogeneous saline solution.
In Figure 6.5, there are occurrences of signal amplitude variations, due to the stimulation frequencies and the impedance distribution. Therefore, the measurements indicate the magnitude of the impedance along a particular channel within the test tank reduces as the frequency reduces. This is indicative of the frequency behaviour of the object. However, further irregularities occur due to some spectral contamination of the stimulation signals, which returns an observable resultant absolute impedance with a reduced magnitude. Furthermore, the effects of the measurement irregularities on the absolute impedances are demonstrated below.

![Absolute impedance per channel at the 100 kHz section of measurement frame](image)

*Figure 6.6 Absolute channel impedances at the 100 kHz section of a measurement frame. The measurements show some irregularities due to the spectral contamination. These measurements were acquired from a phantom tank filled with homogeneous saline solution.*

Figure 6.6 illustrates the effects that the measurement irregularities exert on the channel impedance measurements. The irregularities are defined as the unexpected dips and shifts in the U-curve, compared to the simulated results. To quantify the irregularities the U-curve of the experimental system is compared to a simulation, as shown below.
Figure 6.7 Quantifying the effects of the signal irregularities by comparing the experimental results to simulations. It presents the U-curves for a homogeneous environment. The difference plot indicates that the irregularities create a shift upward and to the left, which will most likely affect the position error of the reconstructed images.

Figure 6.7 is used to quantify the effects of the irregularities on the experimental results. Subsequently, the experimental results are deduced to have shifted upward by 7% and shifted to the left by 10%. Furthermore, the irregularities are unidentical at all channels, with some channels experiencing lower effects of the irregularities compared to the average. This may be due to the use of the adjacent separation of orthogonal signals, which varies the channel irregularities, as shown in Figure 4.1.

Consequently, the practical application of OCDM to aEITS suggests that there are non-ideal effects. These include an unequal energy distribution and spectral contamination across the frequency band, that causes irregularities in the measurement data. However, to reduce these effects may require all signals to inject at the same frequency, which will increase the spectral contamination. Alternatively, CDM may allow the injection of all stimulation signals at the same frequency. This may be at the cost of high frequency signal spikes which introduces other signal contaminations and measurement irregularities. But the cross-correlations between the measured potentials and orthogonal chirp stimulation signals allows the determination of the impulse responses along the respective channel. Therefore, for OCDM-aEITS, irregularities due to an inequitable distribution of energy across the frequency band is not detrimental to the process of reconstructing images. Additionally, it provides further insight into the frequency behaviour of the object. Furthermore, the problem of spectral contamination is ordinary to parallel multiplexing methods in EITS. Therefore, OCDM remains the preferred method. And given that the system can adequately stimulate and measure at the electrodes of the phantom test tank, the following section demonstrates the capability of the system to reconstruct absolute EITS images.
6.2 OCDM-Prior Absolute EITS Phantom Testing

This section involves reconstructing images based on the absolute impedance distribution of a phantom test tank, using absolute imaging and a regularization matrix. The regularization matrix will be developed from the frequency differenced prior information. To replicate the NaCl concentration of the electrolyte in the human body, the phantom test tank was filled with a 1% concentrated NaCl solution. To reconstruct images involves injecting mirrored orthogonal chirp signals into all non-redundant stimulation electrode pairs with a bandwidth of 1MHz. The boundary potentials are sampled (using the adjacent measurement protocol), and the channel impulse responses are recorded. The hyperparameter (of value 0.8) was selected using the well-known Heuristic approach. A 1024-element mesh is used for all image reconstructions. As described in section 3.5.2, the regularization matrix was developed from the difference between the impulse responses at the 1kHz section and another frequency section of the measurement frame. The choice was based on a controlled environment where the frequency behaviour of the inhomogeneity was known beforehand. However, in cases where the internal distribution of an object is unknown, more work is needed to select an adequate section of the measurement frame. This is performed to develop the regularization matrix. In this thesis, a heatmap is used to observe the frequency behaviour of the object, across the entire measurement frame as shown below.

![Heatmap of impedance magnitude versus frequency](image)

Figure 6.8 A heatmap of a single measurement frame taken from an object with an unknown internal distribution. The heatmap illustrates how the magnitude of the impedance per channel varies with the applied frequency. These measurements were acquired from a phantom tank filled with inhomogeneous saline solution.
In Figure 6.8, a heatmap is presented for an arbitrary object, with an initially unknown conductivity distribution. To construct a heatmap, first the stimulation signals and boundary potential measurements are cross correlated to compute the wideband channel conductivities. For a 16-measurement electrode system, 256 channels exist (16 measurements x 16 stimulation). Therefore, 256 cross-correlations are computed across the stimulation frequency band (up to 1 MHz in this work). The heatmap is then plotted as the channel number (1 to 256) versus frequency (1 kHz to 1 MHz).

The heatmap is developed to observe how the magnitude of the impedance per channel varies as the frequency changes. In this case, at around 1 kHz the channel impedances are low. The magnitudes then rise until the stimulation frequency reaches 300 kHz, after which the impedance drops, and the distribution appears to become uniform. Therefore, the regularization matrix may be developed from the difference between the 1 kHz (or any section above 600 kHz) and the 300 kHz sections. Furthermore, it appears that (across the entire frame) the 14th measurement electrode (every 14th channel) is most sensitive to a change in impedance and on average shows a more considerable magnitude. This indicates that there most likely is an inhomogeneity that is closest to the 14th measurement electrode with a peak magnitude of approximately 0.03 kΩ. Therefore, to observe the greatest change in the absolute impedance distribution, the image may be reconstructed from the 300 kHz section of the measurement frame when using the developed frequency-differenced regularization matrix. In cases where it may not be simple to select which distributions to use, when computing the prior information matrix, the first distribution is defined as the one having the lowest variance from its mean. The second distribution is taken at the imaging frequency. Therefore, the prior information is formed from the difference between the imaging distribution and the distribution having the least variation, which would be most affected by the homogenous distribution. An extreme case would be to randomly select the distributions to develop the prior information matrix. In this case, the quality of the image is proportional to the selected distributions. And may introduce additional image artefacts. This would also be true for noisy measurements, which is not unique to the methods introduced in this thesis.

Furthermore, the images in this chapter are reconstructed for several inhomogeneities and their positions. The inhomogeneities include cross-sections of a banana or cucumber. These items were selected to simulate the presence of a biological specimen. Absolute images are reconstructed after a single or several inhomogeneities are placed within the tank, and by the consultation of the corresponding heatmap to identify which sections to use for the image reconstructions. Further tests involve comparing OCDM-prior to generic-prior absolute image reconstructions, to quantify any improvements in the proposed approach.
To begin with, a cross-section of a banana was placed inside the test tank near the 13th measurement electrode. Orthogonal chirp current signals stimulated the electrodes and the corresponding channel magnitudes were recorded in the following heatmap.

Figure 6.9 A heatmap of the channel magnitudes measured across a measurement frame, when a banana cross-section was placed near the 13th measurement electrode.

Figure 6.9 presents a heatmap made from the corresponding channel magnitudes at 50 equidistant frequencies, for better legibility and to reduce repetitive data. From the heatmap it is observed that the magnitude across the frame at the 13th measurement electrode (every 13th channel) remains considerably higher than the other electrodes. This confirms that at least the measurements detected an inhomogeneity near the 13th measurement electrode. However, to observe the frequency behaviour of this inhomogeneity, the following figure illustrates the magnitude profile at channel 189 (the channel corresponding to the 12th stimulation source, and the measurement at the 13th measurement electrode) across the measurement frame.
Figure 6.10 The conductivity measured at channel 189 (the channel corresponding to the 12th stimulation source, and the 13th measurement electrode), due to the insertion of a banana cross-section beside electrode 13. The profile follows an exponential curve and indicates the low frequency magnitude is mainly made up of the saline solution.

Figure 6.10 displays the frequency behaviour of the conductivity measured at channel 189, after a banana cross-section was placed near the 13th measurement electrode. The low frequency magnitude is substantially affected by the saline solution and the energy of the stimulation signals. This agrees with the observation in [98], about the increase in conductivity of the banana as the stimulation frequency increases. Subsequently, considering the complete heatmap and the frequency behaviour at the closest electrode to the inhomogeneity (this is identified by the heatmap in cases where the contents of the object are initially unknown.). The prior information was developed from the difference between the 1 kHz and 1 MHz sections of the frame (given that the 1 kHz section is chiefly affected by the homogeneous impedance distribution.). And an image was reconstructed, as shown in Figure 6.11.
Similarly, when a cross-section of a cucumber was placed near the 11th measurement electrode inside the tank, the stimulation electrodes stimulated the tank with the orthogonal chirp signals. Subsequently, the boundary potentials were measured, and the channel impulse responses were computed. The magnitude impulse responses are shown in Figure 6.12.

Figure 6.11 A picture of a banana cross-section, placed inside the phantom test tank, and a corresponding OCDM-prior absolute reconstructed image. The image was reconstructed at 1 MHz, and the prior information was developed from the difference between the 1 kHz - 1 MHz sections of the measurement frame.

Figure 6.12 A heatmap of the channel magnitudes measured across a measurement frame, when a cucumber cross-section was placed near the 11th measurement electrode. Every 11th channel measured the most considerable magnitude within the respective set of 16 measurement electrodes. And the 139th channel (the channel corresponding to the 11th measurement electrode and the 9th stimulation source) showed the highest magnitude.
Figure 6.12 demonstrates the measurement frame heatmap acquired when a cross-section of a cucumber was placed inside the test tank. From the heatmap it is deduced that the 11\textsuperscript{th} measurement electrode (every 11\textsuperscript{th} channel) detected a significantly higher conductivity compared to the other electrodes. Furthermore, the conductivity increases with an increase in the stimulation frequencies. The peak magnitude occurred at around 0.96 MHz, while the low frequency measurements are chiefly affected by the saline solution. The following figure illustrates the change of conductivity at the 11\textsuperscript{th} measurement electrode.

![Conductivity measured at electrode 11 (closest electrode to the inhomogeneity)](image)

Figure 6.13 The conductivity measured at channel 139 (corresponding to the 11\textsuperscript{th} measurement electrode and the 9\textsuperscript{th} stimulation source), due to the insertion of a cucumber cross-section near electrode 11. The profile follows an exponential curve and that the low frequency magnitude is mainly made up of the saline solution, while the peak occurs at 0.96 MHz.

Figure 6.13 shows the profile measured at channel 139. The low frequency measurements are attributed to the saline solution, while the profile follows the change of impedance of the inhomogeneity. Additionally, given that the maximum magnitude was observed at channel 139 (the 9\textsuperscript{th} stimulation source) and not 171 (the 11\textsuperscript{th} stimulation source) indicates the inhomogeneity may not be at electrode 11. But, most likely, it is between the 10\textsuperscript{th} and 11\textsuperscript{th} measurement electrodes. And by considering the entire measurement frame and the conductivity profile for the cucumber cross-section, the regularization matrix can be developed. The regularization matrix was developed from the difference between the 1 kHz and 0.96 MHz sections of the frame, and the following image was reconstructed.
Figure 6.14 A picture of a cucumber cross-section, placed inside the phantom test tank, and a corresponding OCDM-prior absolute reconstructed image. The image was reconstructed at 0.96 MHz, and the prior information was developed from the difference between the 1 kHz and 0.96 MHz sections of the measurement frame.

The reconstructed images, for the OCDM-prior absolute EITS system, presents an explicit detection of diverse types of inhomogeneities. The reconstructions show minimal presence of false anomalies or image artefacts. Furthermore, based on the mirrored adjacent current stimulation protocol, the current stimulation is symmetrical, and the spatial distribution is most sensitive near the boundary. Additionally, there is a minimal presence of the effects of the non-ideal correlations on the reconstructions. Hence, the quality of the reconstructions is comparable to those reported in the current literature. Specifically, those reported for CDM, TDM and FDM in [25], [24] and [52], respectively.

Figure 6.15 and Figure 6.16 show the distinguishability and detectability of the system, when two cross-sections of a banana are placed within the tank. However, significant warping of the shapes and sizes of the inhomogeneities are observed. This phenomenon gets worse as the distance between inhomogeneities is reduced or when the differences between impedances are increased. Ultimately, the shape warping is caused by the linearization of the FEM.
Figure 6.15 Two banana cross-sections placed inside the phantom test tank, and the corresponding OCDM-prior absolute reconstructed image. The image was reconstructed at 1 MHz, and the regularization matrix was developed from the difference between the 1 MHz and 1 kHz sections of the measurement frame.

Figure 6.16 A picture of two banana cross-sections, placed inside the phantom test tank, and the corresponding OCDM-prior absolute reconstructed image. The image was reconstructed at 1 MHz, and the prior information was developed from the difference between the 1 MHz and 1 kHz sections of the measurement frame. The distance between the banana cross-sections have been reduced to observe the effect on the shapes and sizes of the detected inhomogeneities.

From the figures presented in this section. It is concluded that OCDM can be used to acquire channel impulse responses to reconstruct images about the impedance distribution of an object. Furthermore, OCDM impulse responses contain sufficient information to clearly detect and distinguish between inhomogeneities.
6.3 Comparing OCDM-Prior to Generic-Prior Absolute EITs

To compare the approach of using an OCDM-prior to generic-prior absolute EITs, the only changes are to be made to the prior matrix when reconstructing the images. Therefore, in both cases the physical system, stimulation protocol and measurement protocol are unchanged, as described in section 6.2. In the generic-prior case, the reconstruction algorithm utilizes a prior matrix based on the Noser, Laplace, or Tikhonov algorithms. In contrast, the OCDM-prior case constructs prior information based on the change in the spatial distribution between two frequency-different sections of the measurement frame. Therefore, the regularization matrix is developed from a model that incorporates this change in the impedance distribution, as described in section 3.5.2. And should therefore reduce the number of elements that need to be reconstructed. This stabilizes the solution to the inverse problem and improves the rate of convergence of the solution. Figure 6.17 and Figure 6.18 present the image reconstructions for diverse types of inhomogeneities, when using either method of absolute EITs.

![Image 1](image1.png)

*Figure 6.17 The corresponding absolute image reconstructions of a single banana cross-section inhomogeneity, using (left) a Noser-prior and (right) an OCDM-prior. The Noser-prior image reconstruction required a reference frame to detect the inhomogeneity. The sidebars display the range of conductivity magnitudes in S/m.*
Chapter 6: System Test

Figure 6.18 The corresponding absolute image reconstructions of a single, cucumber cross-section, inhomogeneity, using (left) a Noser-prior and (right) an OCDM-prior. The Noser-prior image reconstruction required a reference frame to detect the inhomogeneity. The sidebars display the range of conductivity magnitudes in S/m.

Figure 6.17 and Figure 6.18 presents the absolute EITS image reconstructions when using a Noser-prior or OCDM-prior. These were performed when a banana or cucumber cross-section was placed inside the test tank. Comparing the quality of the image reconstructions to those presented in chapter 4, shows that the experimental case produced images of a lower quality. This is a direct result of the combination of a non-ideal hyperparameter, noise, reference frame, and some modelling uncertainties. Figure 6.17 shows that the conductivity of the banana cross-section is 2 S/m and 0.55 S/m, for the Noser-prior and OCDM-prior image reconstructions, respectively. Of the two image reconstructions, the OCDM-prior method provides the closest magnitude correlation. As compared to the difference between the measurements conducted at 1 MHz and 1 kHz, as shown in Figure 6.10. Additionally, the Noser-prior was unable to adequately detect an inhomogeneity unless a reference frame (using a conductivity profile as jacobian_bkg) was used. Whereas the OCDM-prior method could clearly detect an inhomogeneity without using a reference frame. Furthermore, comparing a frequency differenced magnitude of approximately 0.39 S/m, as reported in [99], between 1 MHz and 1 kHz. The measurement accuracy between the OCDM- and Noser-prior absolute image reconstructions are 41.02 % and 412.82 %, respectively. Similarly, Figure 6.18 shows that the conductivity of the cucumber cross-section is 0.38 S/m for the OCDM-prior, and 2.8 S/m for the Noser-prior. This indicates the measurement errors are 26.67 % for the OCDM-prior, and 900 % for the Noser-prior. As compared to the 1 MHz - 1 kHz frequency-differenced magnitude of 0.3 S/m, deduced from [100]. Therefore, the incorporation of an OCDM-prior provides the lowest measurement errors compared to the Noser-prior method.
And since the Noser-prior image reconstructions in general produce better image reconstructions compared to the other common generic-priors, this observation can be extended to the other ordinary generic priors.

Furthermore, to observe the ability of the system to detect and distinguish between multiple inhomogeneities of different impedances, a cross-section of the banana and cucumber was placed diametrically opposite each other inside the test tank. Once more, OCDM was used to acquire the channel impulse responses for the banana-cucumber case. In this case, the corresponding absolute images, revealed in Figure 6.19, were reconstructed using a Tikhonov, Noser, or Laplace prior.

![Figure 6.19 Absolute image reconstructions using a (from left to right) Tikhonov, Noser, and Laplace prior, when cross-sections of a banana and cucumber were placed diametrically opposite each other, in the test tank. These images could only detect the inhomogeneities when a reference frame was used.](image)

Figure 6.19 illustrates the absolute image reconstructions when the test tank included two different inhomogeneities (Cross-sections of a banana and cucumber were used as the two inhomogeneities.). To distinguish between the inhomogeneities, an image colour reference was set to 0.5. This was performed to reconstruct an image where the cucumber is represented by the blue pixels, and the banana is represented by the red pixels. Additionally, the image reconstructions were unable to detect the inhomogeneities unless a reference frame was used. From the reconstructions, the Tikhonov-prior algorithm poorly detected the cucumber. The Laplace-prior algorithm reconstructed images with considerable size and position errors. And the Noser-prior algorithm performed the best in preserving the location of the inhomogeneities. However, it also showed large shape deformation.

In contrast, an OCDM-prior was formed from the difference between the 1 MHz – 1 kHz sections of the measurement frame. Furthermore, this prior information and the impulse
responses of the 1 MHz section of the frame were incorporated in the inverse solution before reconstructing the following image without a reference frame.

![Image](image.png)

*Figure 6.20 Absolute image reconstruction using an OCDM prior when cross-sections of a banana and cucumber were placed inside the test tank. This image was reconstructed without a reference frame.*

From the results between generic-prior and OCDM-prior absolute EITs, the generic-prior approach has poor comparable localization, distinguishability, and detectability. This is substantially valid when a reference frame is unused. Furthermore, image reconstructions from the generic-prior approach contain significant image artefacts. These observations highlight the problems discussed in section 3.5. These include the inaccurate reconstructions due to an unreliable or non-existent time-different reference dataset when using generic prior information. The image reconstructions for the generic-prior approach are comparative to those reported in [12]. Furthermore, for both inhomogeneities, the generic-prior absolute EITs image reconstructions completed, on average, after nine iterations. Alternatively, the OCDM-prior approach sufficiently identifies and characterizes the inhomogeneities. This is achieved by the insertion of a regularization matrix (that was arranged from the OCDM frequency-differenced impulse responses) to the reconstruction algorithm. On average, for both inhomogeneities, the OCDM-prior absolute EITs image reconstructions completed in three iterations. Therefore, using OCDM prior information significantly lessens the image artefacts, precisely identifies inhomogeneities, improves the rate of convergence and reduced the average residual or reconstruction error.
6.4 Summary

In this chapter, several tests for validating the full functionality of the OCDM-aEITS system were presented. These tests included measuring the system measurement accuracy, distinguishability, detectability, and confirming the research hypotheses. To measure the system accuracy, the SNR was computed to be $-57 \, dB$. This was considered adequate for fulfilling the objectives of this thesis. Subsequently, several boundary potential measurements were presented. The measurements contained some irregularities caused by the unequal energy distribution within the test object and spectral contamination. The spectral contamination was declared to remain a familiar recurring issue for parallel EITS systems and is therefore not unique to OCDM. The unequal energy distribution was attributed to the frequency behaviour of the test object and is not considered to be an issue that will significantly affect the image reconstructions.

Furthermore, the chapter describes the testing environment and how the OCDM prior information was developed, as elaborated in chapter 3. Then, inhomogeneities representing biological specimens were placed inside the test tank. And OCDM was used to acquire subsequent impulse responses. These impulse responses were used to develop heatmaps that offered additional insight into the frequency behaviour of the test object. Some valuable insights acquired from this method included the detection, identification or estimation of the location and observing the conductivity profile of the inhomogeneity. Using these insights, the images were reconstructed. The image reconstructions showed minimal observable presence of the non-ideal correlations.

To observe the distinguishability and detectability of the system, two banana cross-sections were placed at various locations inside the tank. The results showed adequate distinguishability and detectability. However, shape warping was more imminent as the distance between the inhomogeneities were reduced. The warping was identified as being caused by the linearization in the FEM.

Subsequently, the last test involved the incorporation of an OCDM-prior to reconstruct images about the absolute conductivity distribution. These images were compared to those acquired with a generic-prior for absolute EITS. The generic-prior matrices were formed from the widely employed Tikhonov, Noser, and Laplace algorithms. The comparisons identified that the incorporation of the OCDM-prior information significantly reduced image artefacts, precisely identified inhomogeneities, improved the image reconstruction rate of convergence, and reduced the average residual error.
Chapter 7: Results Analysis

This chapter presents the quantitative and qualitative analysis, described in section 3.6, of the acquired image reconstructions. This analysis aids in affirming the research hypotheses by adopting quantitative measures. The chapter pursues the approach of first analysing the performance of the physical system, followed by the performance of the image reconstructions. Ultimately, the image reconstruction performance is compared to those inferred from current literature that deal with the relevant technologies.

7.1 System Performance

The designed OCDM-aEITS system was intended to provide a benchmark for testing the research hypotheses. For this thesis, it was required that a multifrequency system be developed to enable the application of OCDM, to estimate the cross-section impedance distribution of an object. The wideband measurement data from this system is then used to develop the prior information, before reconstructing absolute images. By using frequency-difference impulse responses from a single measurement frame to develop the prior information, fewer elements of the FEM model need to be reconstructed. This stabilizes and improves the convergence of the inverse solution. Therefore, the imaging process begins by filling a phantom test tank with the NaCl saline solution to simulate the electrolyte in the human body. Subsequently, inhomogeneities are placed in the solution to simulate the presence of the internal biological tissue. At that time, multiple orthogonal chirp current waveforms are generated and simultaneously applied to all non-redundant stimulation electrode pairs of the tank. Cross-correlations between the measured boundary potentials and stimulation signals are computed to acquire channel impulse responses. Therefore, to quantify the performance of the system, this section examines the accuracy of the physical EITS system. This is performed to ensure the system provides reliable data which can be employed to reconstruct images of the impedance distribution of an object.
Chapter 7: Results Analysis

7.1.1 Impedance Measurement Accuracy

In [101], Code Division Multiplexing (CDM) and frequency difference EITS were utilized on the Sheffield Mk3.5 EITS system to measure the conductance change of a cross-section of a banana. The banana was placed in a background conductance comprised of a 0.02 % concentrated NaCl saline solution. The results showed the system could detect the banana up to a frequency of 128 kHz, with a minimum conductance change measured to be 0.035 S/m. Albeit a deteriorating detectability as the frequency increases over this range. Additionally, the results were compared to those from direct impedance measurements of the conductance of the banana. From this comparison, it is concluded that the system could only provide reliable measurements up to 128 kHz, thereafter, the difference is exponential.

In [38], a similar approach, as implemented in [101], was implemented on the UCLH Mk2.5 EIT system. The results from the system were compared to those obtained from direct measurements. This involves placing the specimen inside a Perspex tube, between two silver-chloride discs and measuring the conductance, using the HP4284A impedance analyser, from 20 Hz to 1 MHz. The direct measurements showed a minimum conductance change of 0.04 S/m. And the image reconstructions reliably detected the banana up to 80 kHz and a conductance change of 0.1 S/m. This infers a measurement error of 250 %. Compared to the results reported in [69] that recorded a measurement error of 50 % with a minimum conductance change of 0.01 S/m on the KHU Mk1 EIT system.

From the relevant reported literature, it is inferred that the ability of EITS systems in directly quantifying the conductance of a biological tissue is severely inaccurate. Possible approaches to rectify the measurement accuracy is to improve or calibrate the modelling and frequency errors. However, this requires simultaneous estimation of the conductance and a reference, and therefore increasing the problem complexity. Nevertheless, the localization, size, and conductivity profile characterizations of the inhomogeneities (deduced from the heatmaps and image reconstructions in this thesis), provide valuable information about the imaged object. In addition, section 6.3 compared the acquired results using OCDM-aEITS to the direct measurements of the conductivity of a banana and cucumber from empirical studies. From this comparison, it was concluded that the conductivity magnitude profile of the inhomogeneities closely resembles that of the direct measurements. However, the image reconstructions for the generic-prior and OCDM-prior methods obtained the average measurement errors of 656.41 % and 33.85 %, respectively. A few factors that may have caused these large measurement errors include the ripeness of the inhomogeneities, environment conditions, and hardware inaccuracies. Therefore, the measurement accuracy of
the system cannot be (conclusively) determined as there is evidence for errors in directly measuring the conductance, which make it difficult to benchmark the results. Nonetheless, the measurement signals must be analysed statistically, to measure the level of signal integrity.

7.1.2 Signal Integrity

In Figure 6.1, the channel noise and DC offsets were presented, and it was concluded that the peak-to-peak noise level is significantly lower than the expected minimum voltage measurement. Furthermore, the DC offsets introduced by the ADC and multiplexers can be compensated programatically. Figure 7.1 provides a statistical analysis of the channel characteristics.

![Channel statistics](image)

*Figure 7.1 Plot of the channel noise measurement statistics and channel DC offsets. This information is used to regularize the measurements. These measurements were acquired from a phantom tank filled with homogeneous saline solution.*

In Figure 7.1, the maximum DC offset is estimated (from the channel mean values) to be -0.7 mV and a maximum channel variation of 0.1 mV. That is a maximum variation of 0.1 %, compared to the minimum expected measurement of 0.1 V, when stimulating a 100 Ω load with 1 mA. This information was utilized to compute the channel compensation parameters. Furthermore, the compensation of the channel low-frequency drift is achieved by leaving the inhomogeneities in the saline solution for a prolonged period before taking any measurements.
7.2 Image Reconstruction Performance

In this section, the performance figures of merit, presented in section 3.6.2, are used to analyse the image reconstructions for several positions of an inhomogeneity. Images are reconstructed for an inhomogeneity of size and position equal to 26 % and 21 %, respectively, relative to the diameter and radius of the test tank. These images are imported to MATLAB to apply the performance tests. In MATLAB, the images are converted to grayscale and inverted to set the pixel intensities (which do not detect an inhomogeneity) to zero. This approach does not incur any loss of information because the grayscale image includes the RGB data from the original image. Furthermore, the image pixel values are plotted along a centre horizontal axis, vertical axis, or within a particular quadrant of the image matrix. From the pixel intensity plot, the diameter and centroids of the inhomogeneity and tank can be estimated. This data is then used to compute the size and position errors for the reconstructed image.
7.2.1 Image Analysis

Below are the reconstructed images, their grayscale equivalents, and the corresponding pixel intensity plots, used to analyse the size and position errors.

Figure 7.2 A reconstructed image, the grayscale equivalent, and pixel intensity plot for an inhomogeneity placed at 11 o’clock. The datapoints in the pixel intensity plot reveals the size and position errors of 2.07 % and 9.78 %, respectively.
Figure 7.3 A reconstructed image, the grayscale equivalent, and pixel intensity plot for an inhomogeneity placed at 8 o’clock. The datapoints in the pixel intensity plot reveals the size and position errors of 2.07 % and 0.10 %, respectively.

Size Error
\[
\frac{680 - 497}{1070 - 418} \times 100 = -26\% = 2.07\%
\]

Position Error
\[
\frac{311}{1070 + 418} \times 100 = -21\% = 0.10\%
\]
Figure 7.4 A reconstructed image, the grayscale equivalent, and pixel intensity plot for an inhomogeneity placed at 5 o’clock. The datapoints in the pixel intensity plot reveals the size and position errors of 3.29 % and 4.67 %, respectively.

Size Error
\[
\frac{961 - 770}{1070 - 418} \times 100 = 26\% = 3.29\%
\]

Position Error
\[
\frac{243}{1070 + 418} \times 100 = 21\% = 4.67\%
\]
Figure 7.5 A reconstructed image, the grayscale equivalent, and pixel intensity plot for an inhomogeneity placed at 2 o’clock. The datapoints in the pixel intensity plot reveals the size and position errors of 2.53 % and 0.97 %, respectively.
Chapter 7: Results Analysis

The following table presents the computed position and size errors, given the data points in Figure 7.2, Figure 7.3, Figure 7.4 and Figure 7.5, compared to the total number of pixels for each image.

Table 7.1 The computed position and size errors of the reconstructed images for four different positions of an inhomogeneity.

<table>
<thead>
<tr>
<th>Position</th>
<th>11 o’clock</th>
<th>8 o’clock</th>
<th>5 o’clock</th>
<th>2 o’clock</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position error [%]</td>
<td>9.78</td>
<td>0.1</td>
<td>4.67</td>
<td>0.97</td>
<td>3.88</td>
</tr>
<tr>
<td>Size error [%]</td>
<td>2.07</td>
<td>2.07</td>
<td>3.29</td>
<td>2.53</td>
<td>2.49</td>
</tr>
</tbody>
</table>

7.2.2 Discussion: The Image Reconstruction Size Error

In this chapter, a few images were reconstructed for several positions of the cross-section of a banana, producing an average size error of 2.49 %. The size error was computed by analysing the pixel intensity plots, of the reconstructed images, to identify the number of pixels which have detected an inhomogeneity. The range of pixels which have detected an inhomogeneity is divided by the combined number of pixels to estimate the ratio of the diameter of the inhomogeneity to the diameter of the test tank. The absolute difference between this percentage and the ratio between the effective diameter of the inhomogeneity and the test tank is computed to give the size error. The average size error of the prototype system can be compared to those deduced from the following related works.

- 28 % obtained using absolute imaging, in [12].
- 6.2% obtained using Code Division Multiplexing, in [25].
- 14 % obtained using radially symmetric multi-frequency EIT, in [102].
- 40 % obtained using Time Division Multiplexing, in [85].
7.2.3 Discussion: The Image Reconstruction Position Error

In this chapter, a few images were reconstructed for several positions of the cross-section of a banana, producing an average position error of 3.88%. The position error was computed using the pixel intensity plots of the reconstructed images. By plotting the pixel intensity plot along a vertical or horizontal line within the vicinity, where the inhomogeneity was placed, one can compute the radial position of the centroid of an inhomogeneity in the reconstructed image. And compare the result to the exact position of the inhomogeneity. The average position error of the prototype system can be compared to those deduced from the following related works.

- 25% obtained using static imaging, in [12].
- 3.5% obtained using Code Division Multiplexing, in [25].
- 12% obtained using multi-frequency EIT, in [80].
- 25% obtained using EITS, in [103].

7.2.4 Discussion: Detectability, Symmetry, and Image Artefacts

From the reconstructed images in this chapter, the cross-section of a banana was adequately detected. Although using absolute EITS, the figures showed that the OCDM-aEITS reconstructed images have a lower size error and a competitive position error to the widely used methods. This agrees with the simulations performed in section 4.2.

Additionally, the system is deduced as having a slight position asymmetry when considering the reconstructed images of an inhomogeneity placed in an arbitrary position, then placed diametrically opposite. The asymmetry was caused by the imperfect positions of the electrodes. This may be resolved by machine fitting the electrodes to the tank or accounting for these imperfections in the forward model.

Subsequently, the image reconstructions presented evidence of blurring when imaging multiple objects. These effects of blurring are mainly contributed by current volume effects rather than FEM errors. Therefore, increasing the number of elements in the FEM mesh should reduce the effects of blurring.

Furthermore, sections 6.2 and 6.3 illustrated the ability of the system to detect and distinguish between inhomogeneities that vary in impedance and size. This was demonstrated by placing two banana cross-sections into the tank, to observe the distinguishability of two similar
inhomogeneities. Followed by inserting cross-sections of a banana and a cucumber into the tank. Although the system could identify and distinguish between inhomogeneities that vary in impedance and size, the images still contained some errors. It was revealed that the individual shapes would become warped when the distance between the inhomogeneities are reduced. This is caused by the linearization in the FEM.

Additionally, after reviewing all OCDM absolute reconstructed images, no significant presence of the image artefacts was observed. In this work, significant presence refers to the size, and number (i.e., severity) of artefacts compared to the known number, size, and location of the inhomogeneities. This is attributed to the use of an OCDM-prior that incorporates a frequency-differenced impedance distribution of the test tank and converging the inverse solution to this distribution. Therefore, given that the OCDM-prior obtain a more adequate representation of the actual impedance distribution. The rate of convergence and residual errors improved, when compared to the frequently used generic-prior algorithms such as the Laplace, Noser, and Tikhonov prior. In addition, the generic-prior methods were unable to adequately detect the inhomogeneities without the use of a reference frame, which demonstrates the problem presented in chapter 1 of this thesis. Furthermore, it does become apparent that incorporating the OCDM-prior impulse responses as a reference will improve the detectability and performance of the Noser-prior image reconstructions. This has the advantage of developing the prior information at a more rapid processing speed of $O(n)$. As compared to the OCDM-prior method that develops the prior information at a processing speed of $O(n^2)$. Although the Noser-prior reconstructed images can be improved by using a reference from a different section of the OCDM measurement frame, the solution still converges toward the Noser prior information. Therefore, there is a trade-off between the processing speed of developing the prior information and the rate of convergence and stability of the solution to the inverse problem.

7.2.5 Discussion: The System Frame Rate

To compare the frame rate of various systems appears to be an unjust comparison, provided that some systems achieve higher frame rates by utilizing better performing hardware. Therefore, incurring a higher cost. Consequently, a more appropriate comparison would be between alternative multiplexing methods if the hardware remains the same. For example, to obtain frequency dependent information about an object, a wideband chirp signal can be time multiplexed about the system. If this signal operates, say, over a 1 MHz bandwidth with a period of 5 ms, then the time required to record a particular frame would be 0.08 s, or 12
frames/s, when the current is sequentially injected into 16 non-redundant electrode pairs and the boundary potentials are simultaneously measured. For this system, an adequate sampling requirement would be 2 MHz per channel. Therefore, to measure 13 boundary potentials and the single stimulation signal at any instant would require a multiplexer with a combined channel switching and settling speed of 35.7 ns. In a similar fashion, if this multiplexer is used for a 32-electrode OCDM-aEITS system, the effective imaging bandwidth will reduce to 437.5 kHz, when 16 boundary potentials and 16 stimulation signals need to be measured. As a result, the frame rate of the system improves to 200 frames/s. Therefore, it appears that a multi-stimulation system such as the OCDM-aEITS system retains a particular advantage over TDM. This advantage represents its application to fast changing objects. In other cases where the object being imaged does not vary rapidly with time and has low temporal resolution requirements, a TDM system using a single wideband chirp signal becomes the preferred option. An application of this TDM would be the detection of malignant tissue in the breast.

7.2.6 Discussion: The System Imaging Technique

This thesis focused on absolute EITS to reconstruct images. It was preferred over the alternative difference EITS imaging techniques because images can be reconstructed from a single measurement frame. However, conventional absolute imaging prior information was generic and therefore, required a reference frame to improve the residual error of the reconstructed images. Additionally, a reference frame may be difficult to acquire or may not be readily available. To remove the dependence on a reference frame, the wideband OCDM measurement frame was examined to identify sections of the frame at which the prior information can be developed. This was based on the aim of incorporating the frequency behaviour of the object, into the regularization matrix. Therefore, fewer elements in a FEM model needed to be reconstructed, while using a single measurement frame and without a reference frame. However, the regularization matrix was developed from the difference between two sections of the measurement frame to observe the greatest change in conductivity. As a result, most of the measurement frame is unused. In this case, it may be swifter to perform subsequent data acquisition only at the frequencies selected from the heatmap.

Alternatively, the prior information can be developed from the M largest eigenvectors of the conductivity distributions developed from the heatmap, as described in section 3.5.2. In this way, the prior information contains as much information about the frequency behaviour of the
object, without performing a difference between sections of the frame. This is anticipated to further improve the residual error of the image reconstructions because the solution converges toward the subspace of the absolute prior information, and not toward the frequency-differenced prior information. In addition, multiple images can be reconstructed from a single measurement frame and the prior information, without a reference frame.

To image non-circular objects, the forward model mesh must be refined at the boundaries and along the perimeter of the anomalies to improve the accuracy. Although this was not investigated in this work, the limitation of the tests does not reflect a limitation in the concepts because the mesh refinement is a matter of adjusting the mesh to be finer at the boundaries of the anomalies and the object being imaged. Furthermore, forthcoming work should consider the improvement of the inverse and forward problems to improve the quality of the image reconstructions. Additionally, a more realistic experiment would be to replace the saline solution with a frequency response background conductivity. In this case, the heatmaps will be affected by the frequency responses of the anomalies and the background conductivity. It is therefore recommended that the prior information be constructed using the generalized method described in section 3.5.2. That is, the regularization matrix will contain the orthonormal eigenvectors having the largest eigenvalues. The eigenvalues are computed from the conductivity distributions at different frequencies in the measurement frame. In this way, the prior information contains the expected conductivity distribution. This is anticipated to average out the background conductivity and the reconstruction will converge to the largest perturbations, assumed to be due to the anomalies. However, this method is limited and will produce a blank or uniform prior information matrix if the background conductivity and anomalies have the same frequency responses. This may also be seen when applying OCDM-aEITS to image a biological specimen having a slow varying frequency response. As the variations are reduced, the prior information will deteriorate. However, it is noted that most biological specimen have a similar impedance profile to those acquired with a banana or cucumber (as shown in this work) [104]. Therefore, OCDM-aEITS is equipped to adequately image the majority of biological specimen. Subsequently, this limitation (of extremely slow variation) is not unique to OCDM-aEITS and affects all forms of frequency difference imaging.

Considering the movement of electrodes, contact impedance mismatch and the movement of the boundary. If the electrode contact impedances vary slowly then the convergence of the solution will average out or alleviate the effects of these impedances. Movement of the electrodes and the boundary can be modelled using an augmented Jacobian, that is sensitive to impedance changes and electrode movement. Therefore, the FEM model imposes a smoothness constraint on these movements. Consequently, the technique of OCDM-aEITS...
can be improved, to account for a changing boundary and movement of electrodes, by using an augmented Jacobian [105].

Furthermore, to extend OCDM-aEITS to a 3-dimensional case would require more electrodes and stimulation channels. This may increase the spectral contamination, which is not unique to OCDM. For OCDM, the spectral contaminations can be alleviated by increasing the period and the separation between orthogonal signals.
Chapter 8: Conclusions, Recommendations, and Future Works

This chapter draws conclusions based on the research hypotheses, questions, simulations, and experimental results. Subsequently, recommendations are issued to improve the OCDM-aEITS system and future works are discussed.

8.1 Confirmation of the Research Hypotheses

This work was set out to investigate the use of Orthogonal Chirp Division Multiplexing (OCDM) as an alternative multiplexing method for absolute Electrical Impedance Tomography and Spectroscopy (aEITS). In addition, the aim was to incorporate frequency-differenced OCDM-prior information about the channel impulse responses of an object, into the inverse solution. Therefore, the following hypotheses were presented.

1. Orthogonal Chirp Division Multiplexing (OCDM) can be used to simultaneously apply wideband orthogonal chirp current signals to the electrodes, attached to the boundary of a multi-channel electrically conductive object. Simultaneously, the resultant boundary potentials can be measured and cross-correlated with the stimulation currents to acquire the channel impulse responses. These responses can be managed to reconstruct images of the impedance distribution of the object from a single measurement frame.

2. Given the prior information created from a single OCDM measurement frame, it is possible to improve the quality and rate of convergence of the absolute image reconstructions of an object. This is achieved by incorporating this information into a regularization matrix. These image reconstructions will maintain an improvement of the general performance of an absolute EITS system, compared to absolute image reconstructions that utilize generic prior information and do not have a time-different reference frame.
Therefore, utilizing the designed OCDM-aEITS system, these hypotheses were confirmed. The obtained results from the experiments showed an average image reconstruction size and position error of 2.49% and 3.88%, respectively. This was compared to the effective size and position of the actual inhomogeneity. These errors are lower than the reported results in literature that uses alternative multiplexing techniques such as TDM, FDM, and CDM. In addition, the incorporation of an OCDM-prior reduced the average measurement error and number of iterations by 94.84% and 66.67%, respectively. As compared to the incorporation of a Noser-prior. Another benefit of using the OCDM-prior (over the generic alternatives) is the independence of a reference frame or prior information from other imaging modalities. However, it was noted that the chirp waveforms had non-ideal correlation properties that introduced a problem of spectral contamination. This limited the attainable quality of the image reconstructions. Further errors were introduced in the measurements due to the use of polynomial fit estimates to refactor the impulse responses. Ultimately, the use of OCDM-aEITS makes spectroscopy easier, self-reliant, and more reliable by using a single measurement frame. As compared to TDM and other methods that require prior information from previous empirical studies and a time different reference frame, which may be unavailable.

8.2 Prototype OCDM-aEITS System

A developed prototype OCDM-aEITS system was successfully designed and adequately tested. The system uses 32 copper electrodes, 16 of which are stimulated by orthogonal chirp current waveforms while the electrode potentials are accurately measured at the remaining electrodes. The system was assessed, and the direct results showed adequate signal integrity, measurement accuracy, and repeatability. Additionally, the system was capable to correctly apply orthogonal chirp signals with an amplitude of 1 mA and a frequency range of [1 kHz, 1 MHz]. And achieved a frame rate of 870 frames/s when acquiring a 16,384-sample frame. The imaging frequency of up to 1 MHz is adequate for this research and an industry standard when imaging biological specimen. This provided a good range over which to observe the frequency behaviour of the object being imaged. The cost of the system was below 30,000 ZAR as compared to the commercial platforms that cost more than 500,000 ZAR. However, the designed system is a prototype, and it is estimated that an additional 200,000 ZAR is required to convert the prototype to a commercially viable standard. The system portrayed some DC-offsets and low frequency drift. The developed software incorporated DC-offset compensation penalties and inhomogeneities remained inside the test tank for a prolonged
period before imaging. This prevented the effects of the low-frequency drift in the measurements, which are most likely caused using copper electrodes in a saline solution.

8.3 Recommendations

Based on the limitations of the designed prototype system, the following recommendations are issued:

- Compare images reconstructed with an OCDM-prior, to those reconstructed utilizing prior information from previous empirical studies. This should be performed to quantify the mismatch between the OCDM-prior information and those acquired from other mainstream imaging technologies.
- A multi-channel DAC and ADC should be used to reduce certain signal irregularities. Additionally, the incorporation of multi-channel components will increase the frame rate, and the effective imaging bandwidth.
- Investigate the benefits of using OCDM-prior frequency-differenced impulse responses as a reference set rather than a prior matrix. This should be done to efficiently reconstruct images of comparable performance. As compared to the OCDM-prior application, as the time to generate the OCDM-prior increases with the number of finite elements.
- The prior information was developed from a difference between sections of the measurement frame. To incorporate as much information about the frequency behavior of the object into the regularization matrix, the matrix should be developed from the M largest eigenvectors from the subspace of the wideband conductivity distributions.
- Develop an efficient solver to compute the OCDM prior information. Currently, the rate at which the OCDM prior information is developed, increases as more elements are used in the FEM at a rate of $O(number \ of \ elements \times n^2)$. Therefore, the task should initially be to quantify the effect that the number of finite elements in a forward model affects the rate at which images are reconstructed. Subsequently design a more efficient algorithm to compute the OCDM prior information.
- Incorporate system power isolation using fast-switching circuit breakers. This is required to safeguard the system and test subjects from a mains fault.
- Practically test the system on live biomedical subjects. This moves OCDM-aEITS toward live biomedical testing, after ethical clearance.
Chapter 8: Conclusions, Recommendations, and Future Works

• The average of several impulse responses acquired from longer period stimulation waveforms should be performed to improve the spectral purity. Therefore, an optimal signal length should be investigated.

8.4 Future Works

From the success of this investigation, future work aimed at improving the system capabilities and extend the system’s applications are outlined below.

• Since the system already provides parallel stimulation, it should be adapted to support other widely employed modes as well. Precisely, the following modes should be supported: TDM, FDM and CDM.

• Regarding CDM, the most conventional codes utilized are maximum length, Gold and Hadamard codes. With Hadamard codes achieving the best correlation properties of the three. Alternative to these methods, forthcoming work will explore the use of quantum random walks to relate quantum theory toward achieving faster ways of computing orthogonal sets.

• The current system experiences measurement and stimulation irregularities due to the switching mechanism employed. Therefore, a multi-channel DAC and ADC will be used to improve the system frame rate to 1,000 frames/s. And operate over a wider effective operating bandwidth of 10 MHz.

• Explore the biomedical testing capability of the system, starting from non-critical organs to imaging brain activity. This will require that the system meets all biomedical safety requirements outlined in section 5.2.2 and passes all ethical clearances.

• Implement custom software that uses neural network methods to identify and characterize malignant tissue and activity from the wideband measurement data. This includes analyzing ex vivo OCDM-aEITS and serial digital histopathology of specimens, to better understand the anatomical variability and to quantify biomarkers.

• Improve system instrumentation with a focus on stability. This includes developing more stable mirrored current sources (by increasing the output impedance) and incorporating higher resolution data convertors.

• Extend the current method to more practical three-dimensional applications.

• Investigate the stability of OCDM-aEITS to the movement of electrodes and the boundary of the object, using an augmented Jacobian.


Bibliography


Appendix A: Simulated Image Pixel Intensity Plots

Figure A.1 Grayscale reconstructed image and pixel intensity plot for the true object under test.
Figure A.2 Grayscale reconstructed image and pixel intensity plots for absolute EITS using a Laplace prior.
Figure A.3 Grayscale reconstructed image and pixel intensity plots for absolute EITS using a Tikhonov prior.
Figure A.4 Grayscale reconstructed image and pixel intensity plots for absolute EIT using a Noser prior.
Figure A. 5 Grayscale reconstructed image and pixel intensity plots for absolute EITS using an OCDM frequency-differenced prior.

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\begin{align*}
\text{size error} &= |100x \left( \frac{c - b}{d - a} \right)| - 19.16 | = 2.69 \\ 
\text{position error} &= |100x \left( \frac{e}{d + a} \right) + 9.30 | = 1.06 \\
\end{align*}
\]