

# Robo-Advising on South African Exchange Traded Funds utilizing Prospect Theory

University of Cape Town



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11 February 2019

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# Abstract

Robo-advising is an emerging trend in markets around the world. The term has come to refer almost exclusively to automated advisory services for financial investments or wealth management. Currently, in the South African market, financial services firms offer their own robo-advising platforms that only provide automated advice about their own products. This paper investigates the possibility of a robo-advising platform existing outside of these financial institutions. The paper reviews the preconditions that make robo-advising possible. Namely, risk profiling, portfolio allocation, availability of ETFs and accessible online trading platforms. The research shows that independent robo-advisers are possible in South Africa and a minimum viable implementation is presented.

# Declaration

I declare that this dissertation is my own, unaided work. It is being submitted for the Degree of Master of Philosophy in Financial Technology at the University of Cape Town. It has not before been submitted for any degree or examination.

Signed by candidate
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Ryan Jonathan Jacobson

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# Acknowledgements

I would like to express my gratitude to the following people:

To my family, without you I would not be in the fortunate position I am in today. I am eternally grateful for the sacrifices you make for me.

To my supervisor, Co-Pierre Georg, for his constant inspiration, guidance and support through the late hours of the evening.

To Allan Davids for the many hour-long calls. He was of great assistance in ideating numerous thesis topics. Once the topic was settled, he was a great sounding board and ensured my attention was focused on the most pressing issues.

To Andrew Soane for his guidance on the R coding. His statistics knowledge was of tremendous help.

To Kyle Roos for sacrificing many of his holiday hours to discuss this paper. His knowledge of stock analysis was essential.

To all the other people who helped along the way, I am grateful for your encouragement, time spent proof reading this paper and overall support of this process.

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# Chapter 1

## Introduction

### 1.1 Background

Digital disruption to financial services companies has become common place. Fin-Tech or financial technology is an industry dedicated to this endeavour. An area of increasing interest is the automation of financial advice or *Robo-Advising* - an industry that is estimated to reach nearly \$1 trillion by 2022 (Sorrell, 2018).

Robo-advising aims to automate the process of seeking investment advice and completing investment transactions. Modelling investors' preferences and recommending products is an established field that existed long before robo-advising. Robo-advising is made enticing by it's ability to remain unreliant on any financial institutions. Similar to independent financial advisers, robo-advising offers clients access to a wide universe of investment products from multiple fund managers. This process of automation drives down costs and makes financial advise accessible to a wider market.

In review of the South African market for robo-advisers, a number of surprising observations inspired this research. We observed that most robo-advisers in South Africa are owned by financial services institutions (Capital, 2019; Sygnia, 2019; Sanlam, 2019) and the few that aren't recommend funds of a single investment house (Bizank, 2019). Consider a customer who seeks investment advice but cannot assess the quality of that advice. In this scenario a conflict of interest exists (Mehran and Stulz, 2007). The advisor (both human or algorithmic) may recommend the product that earns the firm the most money. Alleviating this conflict is the primary concern of this paper. It is expected that robo-advisers can operate outside of financial services companies and, therefore, overcome such conflicts of interest.

Determining the value of investment advice will help determine whether or not it is worthwhile investigating a fund-agnostic robo-adviser. Allie (2015), conducted an extensive study wherein 4 147 advised and non-advised investors were reviewed. The findings indicated that the initial investment decision was the most significant and after that initial decision a financial advisor provided little value. Thus, making

the right investment decision at the outset is essential.

Conventional investment advisory services are often reserved for wealthy clients. ETFSA, an online investment platform, writes in their documentation that gaining access to financial advice is “only really worthwhile if you are a significant investor” (Brown, 2017). Furthermore, one source shows that financial advisory fees can range from R5 000 to R15 000 for an initial analysis (Ultima, 2015). This excludes anybody aiming to start with an investment that finds this fee is material. Therefore, greater accessibility to online fund-agnostic advice would be of value to these clients.

## **1.2 Objectives**

This research ascertains if robo-advising can be successfully implemented in a South African market. Currently no South African robo-advisers exist separately from financial institutions. This research will review all of the necessary preconditions to implement a robo-advisor and if these conditions are present, a minimum viable robo-advising implementation will be presented. Robo-advisers are made possible by risk profiling, portfolio optimization, online trading platforms and exchange traded funds (ETFs). Each of these areas will be researched and implemented.

## **1.3 Summary of Findings**

This paper finds that there is no insurmountable barrier that prevent the advent of independent robo-advisors in South Africa. While the industry as a whole has a number of hurdles to overcome, none of them are specifically unique to South Africa. The market provides access to a large variety of ETFs and several online trading platforms exist. Finally, Prospect Theory proved to be valuable in modelling investors’ preferences and determining an appropriate asset allocation.

# Chapter 2

## Robo-Advisors

### 2.1 Definition

Robo-advisors are digital platforms (Jung et al., 2018) that offer users automated investment solutions and/or advice. (Sironi, 2016) The platforms use interactive and intelligent components (Maedche et al., 2016) to guide users through a process of self-assessment and goal-setting. (Sironi, 2016). Robo-Advisors generally perform risk profiling and portfolio allocation (Jung et al., 2018). The term "Robo-advisor" has come to refer, almost exclusively, to offerings in financial services (Jung et al., 2018).

### 2.2 Background

The financial services industry is on the brink of digital disruption. (Baghai, Carson and Sohoni, 2016) The current wave of digitization is focused on creating intelligent services based on algorithms and machine learning. (Jung et al., 2018). The wide stated goal of these offerings is to enhance the customer experience and achieve cost reductions (Baghai, Carson and Sohoni, 2016). Robo-advisors aim to automate investment advisory and investment management for retail customers. It is therefore necessary to first understand how conventional investment advisors operate. The conventional investment advisory offering consists of the following four phases (Cocca et al., 2016): (1) Analysis of the investor's investment objectives, goals and risk profile, (2) Definition of investment strategy and appropriate asset allocation, (3) Implementation of that strategy with suitable products (4) Maintenance and adjustment of the investment strategy. Robo-advisors offer either static or dynamic management. A static robo-advisor will evaluate a customer's profile and risk preferences and provide a single investment solution. Re-balancing will occur only where the portfolio's constitution deviates from the defined parameters. A dynamic robo-advisor incorporates all the elements of a static robo-advisor but it will also periodically re-evaluate the customer and adjust her investment solution. Jung et al. (2018) For the purpose of this paper a static robo-advisor will be

considered.

The conventional structured advisory process is digitized inwardly but is seldom digitization outwardly (Cocca et al., 2016). Financial advisers have an in house technology stack that generates investment solutions based on the responses gathered from the client. On the other hand robo-advisors utilize models that are substantially similar and provide suggested portfolios to people who complete their survey. The difference between the two is that conventional financial advisers are licensed to offer advice while online robo-advisers continuously remind clients that the tool does not provide comprehensive financial advice. Limited (2018); Vanguard (2017*a*). Consequently, robo-advisers are limited in the extent to which they can fully automate the advisory process. Instead, robo-advisers take a hybrid approach where those who require professional advice can receive it through a licensed financial provider. This topic is covered in greater detail in Section 4.2.

## 2.3 Current state of the market

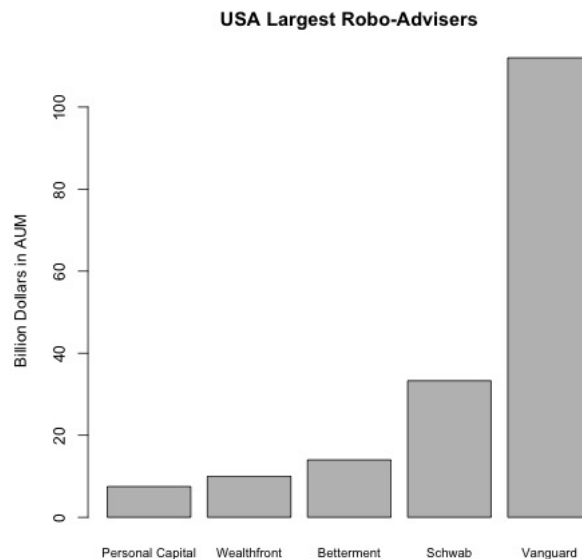


Figure 2.1: Largest Robo Advisers in the USA

In the USA, there are well established robo-adviser offerings. Those with the greatest assets under management are displayed in figure ???. In South Africa, the market is still in it's infancy with most robo-advisers being offered by the banks themselves. At the time of writing, the robo-advisers in South Africa were offered by Absa, Sanlam, Sygnia, Bizbank, Outvest (Kolver, 2018). As suspected, the international robo-advisers exist independently from the ETFs that they recommended (Betterment, 2019; Wealthfront, 2019) thus avoiding potential conflicts of interest.

The market for robo-advising has had slow uptake in assets versus the large actively managed funds. Internationally, most of these robo-advisers are reliant on

venture capital funding and a number of them have yet to become profitable. (Co, 2015) Robo-advising is a game of scale: additional volumes drives revenue without a commensurate increase in cost (Economist, 2015). Almost all robo-advisors offer portfolio allocation tools. There is then a risk that in the longer run, these services become ubiquitous becoming an offering to remain competitive instead of a profitable business model (Chishti and Barberis, 2016).

Where robo-advisers fall into the wealth-management value chain is determined by what decision complexity can be automated (Cocca et al., 2016). Primarily, the robo-advisors discussed above focus on investing clients' liquid assets. This process described in the beginning of this section is prone to automation. The decision logic can be easily implement. However, complexity balloons when one starts to focus on, *inter alia*, tax planning, legal advice, multiple jurisdiction investment management and efficient trust structure (Cocca et al., 2016). As such, many robo-advisors offer hybrid approaches where some investment processes are automated while others are addressed by financial practitioners.

# Chapter 3

## Decision Theory

### 3.1 Introduction

In Section 2.2, it was found that the first two stages of investment advisory require analysis of the investor's risk profile and provision of an investment strategy. In short, the objective is to link an investor to an asset allocation. This matching process begins with quantifying an investors risk profile and then feeding this information into a model that optimizes investments for this particular investor. Therefore, it must be understood how investors make decisions and how their preferences can be modelled.

Decision theory is a multidisciplinary field that focuses on all aspects of choice. This paper is concerned with decision theory as a means to quantify preferences. More specifically, how an investor's risk preferences translate into a portfolio allocation. In order to achieve this, a mathematical representation of preferences under risk and uncertainty is required. Decision theory concerning risky choices has been extensively studied. Historically, models assumed people are rational. However, Behavioural Finance adjusts for the emotional and cognitive errors in investor's decisions making. This Chapter will review Markowitz's seminal work on Modern Portfolio Theory, Expected Utility and Kahneman and Tversky's Prospect Theory.

### 3.2 Modern Portfolio Theory

*The Journal of Finance* published Markowitz's article "Portfolio Selection" in 1952. In this paper, Markowitz introduced Modern Portfolio Theory (MPT). The theory offers a framework to construct portfolios based on expected return of the underlying investments and the risk preference of the investor. (Fabozzi, Gupta and Markowitz, 2002)

MPT quantified diversification through the introduction of covariance and correlation (Fabozzi, Gupta and Markowitz, 2002). In brief, diversification has no value

if all of the assets move in the same direction. Holding assets that between them have negative or no correlations protects the investors from adverse movements in any one of the assets held.

The investment process of MPT performs portfolio selection by using mean-variance optimization. Given a set of assets' returns, variances and correlations, optimal portfolios can be created. These portfolios are optimal in that each one offers the greatest return for a given level of risk. The set of all of these portfolios creates the *Efficient Frontier*. (Fabozzi, Gupta and Markowitz, 2002). The explicit assumption in mean-variance optimization is that the first two moments of returns (that is mean and variation) are sufficient to determine asset selection (Maringer, 2008)

$$\min_w w^T \Sigma w \quad (3.1)$$

$$\text{subject to } w^T R = \mu \quad (3.2)$$

$$\sum_{w^i} = 1 \quad (3.3)$$

The optimization problem defined above computes the weighting of the assets that generate the lowest risk for a given return (Werner and Sjöberg, 2016). In this objective function  $\sigma$  is the covariance matrix of the asset.  $w$  is a vector of the assets weights in the portfolio.  $R$  is the expected return of the assets while  $\mu$  is the target portfolio return. Tobin (1958) extended this theory with the inclusion of the risk free asset. Thus introducing the Two-Fund Separation Theorem which dictates that an investor will split their investment between the market portfolio and the risk free asset. The Capital Market Line connects the risk free-rate to the market portfolio. The slope of this curve in the risk return space is given by:

$$\frac{\mu - r_f}{\sigma} \quad (3.4)$$

### 3.3 Expected Utility

Expected utility theory was formulated in 1944 by John von Neumann and Oscar Morgenstern but it has its roots dating back to Daniel Bernoulli in the 18<sup>th</sup> century (Levin, 2006). Expected Utility Theory asserts that there exists a preference plane over consequences. That is, future uncertain events can be ranked according to one's preferences and one can be seen to be maximizing the expected value of a function defined over this plane.

Von Neumann and Morgenstern explain that a utility function is defined for an individual who's preferences satisfy the following axioms: (Levin, 2006)

1. **Complete:** For any two possible events there exists either one that is clearly

preferred or the individual is indifferent between the two.

2. **Transitive:** If there exists an outcome A that is preferred to outcome B and B is preferred to outcome C it must be said that A is preferred to C.
3. **Continuous:** Preferences do not display erratic behaviour as a result of small changes in outcomes.
4. **Independent:** If outcome A is preferred to outcome B then outcome A plus outcome C should be preferred to outcome B plus outcome C.

Expected Utility Theory, therefore, can be used to model the utility of expected returns for different investors. The well known concept of diminishing marginal utility means that as expected return increases, additional utility will decrease at an increasing rate (returns are assumed to be proportional to risks here) (Correia, 2000). The rate of this decline will measure an investors risk preferences. Mathematically, a risk-averse investor will have a concave utility function while this curve will be linear for risk-neutral investors and convex for risk-seeking investors. Similarly, Expected utility theory introduces the concept of a Certainty Equivalent (CE). That is a certain amount preferred over an expected amount in a gamble. For a risk-averse investor the CE will be lower than the expected outcome, for a risk neutral investor they will be the same and for a risk seeking investor the CE amount will be above the expected amount. Risk preferences are modeled using the certainty equivalent (Levin, 2006)

Expected Utility Theory explains that a risk neutral investor will place all of her money in the asset that offers the highest return. Secondly, if the risky asset has a return above  $r_f$ , a risk-averse investor will always allocate some portion of her assets to the risky asset. Thirdly, the more risk-averse is an investor, the less she will allocate to the risky asset. A consequence of modelling diminishing marginal utility implies that the impact of a profit or loss is measured in relation to the individual's wealth. This tenet is challenged in Prospect Theory.

### 3.4 Prospect Theory

In 1979, *Econometrica* published Kahneman and Tversky's seminal work on Prospect Theory. Their theory is widely considered one of the most important frameworks for decisions under risk. PT combines behavioural phenomena and experimental evidence to explain observed deviations from expected utility while still incorporating most of the knowledge from this theory. The basic tenets of PT are that (1) values are placed on losses and gains relative to a reference point (2) individuals distort probabilities in their decision making. Consequently, an investor's risk attitude is a function of these two tenets. PT deviates from EU in that values are placed on losses and gains and not on the final value of assets. Let  $(x_1, p_1; \dots; x_n, p_n)$  denote a set of prospects that yield  $Rx_1$  with probability  $p_i$  where  $i = 1, 2, \dots, n$ . For convenience, assume that this set has been arranged such that  $x_1 < \dots < x_n$  PT assigns values to

both positive and negative gains. The theory was formulated for two prospects but it is straightforwardly extended to deal with  $n$  prospects. PT is therefore given as:

$$\sum_{i=1}^n w(p_i)v(x_i) \quad (3.5)$$

Kahneman (1979), define the piece-wise power value function,  $v(x)$  as:

$$v(x) = \begin{cases} (x - RP)^{a^+} & x \geq RP \\ -\beta(RP - x)^{a^-} & x < RP \end{cases} \quad (3.6)$$

The S-shaped curve places greater importance on losses than on gains. That is, an individual is not just risk averse but also loss averse. Here the reference point  $v(0) = 0$ . This value function violates the diminishing marginal utility assumption, where an additional unit is valued less by wealthier individuals. Moreover, the value function is concave for gains and convex for losses and steeper for losses than for gains. In this paper, the piecewise-power function will be used throughout. This paper will make use of the probability weighting function introduced by Tversky and Kahneman (1992). That is  $w(p)$  is given by:

$$w(p) = \frac{p^\gamma}{(p^\gamma + (1 - p)^\gamma)^{\frac{1}{\gamma}}} \quad (3.7)$$

The probability weighting function distorts probabilities. Specifically, it states that people overweight small probabilities and underweight larger probabilities. The tendency to underweight probable outcomes in comparison with certain outcomes, produces the Certainty Effect (Kahneman, 1979). This contributes to people being risk averse to choices with sure gains and risk seeking with choices involving sure losses.

Prospect Theory does not always uphold stochastic dominance. option A is first order stochastic dominance over B if for all values of  $x$ ,  $P[A > x] \geq P[B > x]$  and for some values of  $x$ ,  $P[A > x] > P[B > x]$ . As such Tversky and Kahneman (1992), introduced Cumulative Prospect Theory (CPT). CPT makes use of the same value function as PT but uses the rank-dependent function to transform probabilities. Probabilities associated with gains and losses are treated separately and then the sum of the two equations is taken to determine the cumulative prospect value. The decision weight function is defined separately for gains  $w^+$  and losses  $w^-$ . In CPT, greater weighting is placed on extreme events. Where  $x_1 < \dots < x_k < RP = 0 < x_{k+1} < \dots < x_n$ , CPT:

$$\sum_{i=1}^k \pi^-(p_i)v(x_i) + \sum_{i=k+1}^n \pi^+(p_i)v(x_i) \quad (3.8)$$

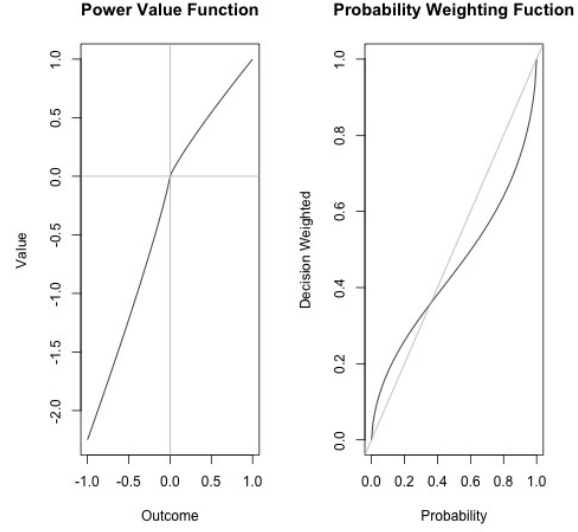


Figure 3.1: Prospect Theory Value Function and Decision Weighting Function

$$\pi_1^- = w^-(p_1), \quad \pi_i^- = w^-(p_1 + \dots + p_i) - w^-(p_1 + \dots + p_{i-1}) \quad 2 \leq i \leq k \quad (3.9)$$

$$\pi_n^+ = w^+(p_n), \quad \pi_j^- = w^+(p_j + \dots + p_n) - w^+(p_{j+1} + \dots + p_n) \quad k < j < n \quad (3.10)$$

# Chapter 4

## Risk Profiling

### 4.1 Introduction

Risk profiling is core to the wealth management industry. Suitable investments cannot be suggested to an investor without knowledge of, *inter alia*, her investment objectives, time horizon and risk aversion. Failure to properly assess the investor's needs is tantamount to providing inaccurate advice and is to the detriment of the investor. However, there is no consensus on how to best capture a client's risk preferences. Risk profilers primarily serve the purpose of matching investors to appropriate portfolios of assets. That is they address the asset allocation decision. The following section explores the respective focus areas of different stakeholders to the process of risk assessment.

Risk profiling has taken place through the use of questionnaires for over 30 years. Since the first risk questionnaire appeared in 1984 in *The Dow Jones-Irwin Mutual Fund Yearbook*, risk questionnaires have become ever-present in the industry (Droms and Strauss, 2003). For the purpose of this section, risk profiling will be assumed to refer exclusively to risk profiling through the use of a questionnaire. This decision is justified by the overwhelming use of the risk questionnaire in industry. Importantly, this section will highlight the design principle that will later be used to develop a risk questionnaire.

### 4.2 Legislation

Globally there are legislatively-instituted suitability rules that ensure products are appropriate for an investor's personal situation. In the United States of America, investment suitability guidance is provided by Rule 2111 of the Financial Industry Regulatory Authority. Article 25 of the Markets in Financial Instruments Directive II performs this function in the European Union (Klement, 2015). Similarly, the Financial Advisory and Intermediary Services Act (FAIS) governs suitability rules in South Africa (Board, 2014). Speaking specifically in a South African context,

providing advice places financial planners under onerous fiduciary duties and regulatory obligations. Equally severely, the FAIS Ombud (the enforcement function of the the FAIS Act) will assess every risk questionnaire given out by a financial planner when a client complains that she was provided with unsuitable investment advice. The Ombud, at times, found against the financial planners (Swanepoel, 2016). Advisers must be licensed as a financial service provider or must represent one in order to provide advice.

FAIS defines advice as “any recommendation, guidance or proposal of a financial nature furnished, by any means or medium, to any client or group of clients”. Advisers must be licensed by the Financial Services Board (FSB) and meet the requirements of the Act and be qualified to provide investment advice. Presently, the regulatory guidance on robo-advising is scarce. However, IOSCO’s Committee on the Regulation of Market Intermediaries have assessed how automated advice has affected investors (Preez, 2016). Their findings indicate that, *inter alia*, firms often give advice while labelling the output as “no advice” in order to comply with regulations. This is expected to come under increasing scrutiny as robo-advising becomes more invasive. To satiate the current regulatory requirements, one needs to make clear that a financial advisor should be contacted prior to investing.

### 4.3 Financial Risk Taking as a Personality Trait or Situation Specific

The literature is unclear on whether or not risk profiling is a general or specific trait. Weber, Blais and Betz (2002) found evidence that risk-taking was domain specific. They measured risk-taking across five content domains. While, Eysenck and Eysenck (1978), believed that risk preferences are a general personality trait and sought out evidence across risk-taking activities. Similarly, Wong and Carducci (1991) found that high “sensation seekers” showed a tendency to take greater risks in their everyday financial decisions. Where *sensation seeking* is defined by Zuckerman (1979) as “the need for varied, novel, and complex sensation and experiences and the willingness to take physical and social risks for the sake of such experiences”. Klement (2015), from the CFA Institute Research Foundation, published an article expressing that risk profiling can be determined by three categories: (1) An investor’s genetic predisposition towards financial risk-taking. (2) The people with whom an investor interacts. People were found to increase their equity holdings when they moved into communities who held a large amount of equity. (3) An investor’s life experiences (specifically during her formative years). Studies found that those who experienced the great depression were less likely to invest in stock. Despite the findings of the above, in practice, all risk profilers ask questions directly related to financial risk. Accordingly, this paper shifts its attention to decomposing financial risk.

## 4.4 Decomposing Financial Risk

Cordell (2001) argues that an investors acceptance of investment risk can be broken up into four components: propensity (risk acceptance in every day situation), attitude (willingness to accept financial risk), capacity (as explained above) and knowledge (understanding financial products and their risk/return trade offs). However, the foreword of the Profile's Unit Trusts & Collective Investments Handbook highlighted the necessity for risk profiling tools to separate their questions into risk capacity and risk appetite. (Swanepoel, 2016). Similar views are echoed widely in the industry (See for example Klement (2015)). For this reason we place majority of the focus into risk capacity and risk attitude.

Yook and Everett (2003) and Klement (2015) explain that risk capacity speaks to an investor's objective *ability* to take on risk. This considers the investors financial situation and stage of life. For example: a person approaching retirement with minimal retirement savings will have a low risk capacity as any loss of capital will have a harsh impact on her financial well being. Risk attitude, on the other hand, investigates an individuals *willingness* to take on risk. Exploring the investor's view on the trade off between risk and return - taking on greater The more risk she takes the greater the potential returns.

## 4.5 Drawbacks

Swanepoel (2016), explains that the industry (both globally and locally) are struggling to determine an risk profiling industry standard that can be used with confidence. In a survey conducted by the Financial Intermediaries Association of South Africa (FIA), 85% of 554 financial advisers surveyed expressed that risk questionnaires in South Africa are insufficient to provide appropriate advice. Rice (2005), analyzed 131 risk questionnaires and found discouraging results. 11% of the questionnaires directly asked investor to pick their fund, 35% failed to ask about the time horizon and questionnaires answered in the most conservative way ranged in equity allocation from 0% to 70%. Undoubtedly, the risk profiling questionnaire is a contentious issue and one that won't be resolved simply.

# Chapter 5

## Exchange Traded Funds

### 5.1 Background

An exchange-traded fund (ETF) is an investment vehicle that ordinarily aims to track the performance of a specific index (Lettau and Madhavan, 2018). “An ETF combines the diversified portfolio of a unit trust investment with the tradability features of a listed security” Brown (2018). The fund pools together money from investors and divides the fund up into individual portions that can be traded on a stock exchange. Since their inception in Canada in 1990 (Vanguard, 2017b), ETFs have become ubiquitous in capital markets around the world. Globally, ETFs have \$5.2 trillion in assets under management as at September 2018 (BlackRock, 2018). ETFs have gained popularity due to their low cost structures and the ability to be purchased like any listed security (Brown, 2018). While ETFs have been acclaimed as one of the most important financial innovations, they still account for only a small fraction of the world’s \$160 trillion equity and fixed income securities global total market value (Lettau and Madhavan, 2018). South Africa has 72 ETFs listed on the JSE, with a total market capitalization of over R72 billion as at March 2018 (BusinessTech, 2018). In South Africa, ETFs are regulated by the Financial Services Board under the Collective Investment Schemes Act (ETFSA.co.za, 2018).

ETFs are similar to mutual funds in that they hold the basket of underlying assets. For example, an ETF tracking the FTSE/JSE Top 40 will hold shares in those top 40 companies in the same proportion as they are represented in the index. The ETF manager will buy and sell shares in these companies to insure these proportions remain constant. These transactions are known as re-balancing. (Hesse, 2017)

An ETF - as with any share of a publicly listed company- is purchased through a stock broker. (ETFSA.co.za, 2018) As such there must be a willing buyer and a willing seller for there to be liquidity for the ETF. The *Market Makers* also known as *Authorized Participants* (AP) provide this market liquidity by taking on the risk of holding a number of the ETF shares in order to facilitate trading (Hesse, 2017).

While ETFs will have a value close to the underlying NAV, the buy and sell price

will differ by the bid-offer spread Hesse (2017). The bid price represents the amount at which someone is willing to buy the stock while the offer price is the amount at which someone is willing to sell. The difference between these two amounts is the bid-offer spread. The spread will vary for a number of reasons. Most importantly, ETFs with high trading volumes will have narrow spreads, while thinly-traded ETFs or ETFs holding highly illiquid underlying assets will lead to wider spreads (ETF.com, 2017).

Unlike mutual funds, ETFs do not interact directly with capital markets. Instead, ETF managers interact with APs who then interact with the capital markets. The ETF manager will issue shares (known as Creation Units) to the AP to increase the supply of an ETF (Lettau and Madhavan, 2018). The AP will then issue the ETF with a basket of stock and/or cash. The reverse is true for decreasing the supply of an ETF. Authorized participants can sell and redeem shares in the secondary market and directly with the ETF. ETFs generally track the benchmark better than equivalent unit trusts and display lower volatility. Brown (2018), hypothesizes that this is owing to the JSE requirements to be a listed ETF. A requirement that does not apply to unit trusts.

## 5.2 The case for Passive Investing

There is a growing body of research that advocates for passive investment strategies. Such research illustrates that low-cost index investing outperforms most actively manage funds (Indices, 2015). Jr., Walker and Ning (2018) illustrates three key drivers of the efficacy of index investing: (1) Zero-Sum Game. Since the market is made up of the cumulative holdings of all of the investors, the market return is the asset-weighted return of all market participants. Since this return is the average market return, for every position that outperforms the market there must be an equal number of positions that underperform such that the excess returns of all assets is zero. Thus there are even odds at beating the market. This may be attractive until the following section is discussed. (2) Fees. To participate in this active market, there are management fees, commissions, bid-ask spreads and taxes. As a result, the distribution is shifted to the left, making it that much more difficult to beat the market. Finally, (3) persistent out-performance is rare. Research dating back to 1960s shows that past performance is not indicative of future performance. Most recently Fama and French (1993) report on a 22-year study that active managers do not regularly outperform their benchmarks. The above offers a compelling case to consider low-cost passive investment strategies. To that end, determining which ETF to invest in is explored.

## 5.3 Evaluating ETFs

Data on ETFs is more readily available as they are publicly listed. As such, one can access a fund's daily prices, bid-offer spread, and trading volumes. Thus ana-

lyzing ETFs is easier than analysing unit trusts or any other unlisted product. This increased transparency makes ETFs suitable candidates for robo-advising applications.

1. **Bid-Offer Spread:** A narrow bid-offer spread leads to a lower cost of trading (ETF.com, 2017). For example, consider a bid price of R120 and offer price of R100, an investor who buys this ETF and then sells it immediately will incur a cost of R20. Assuming the spread remains constant, the price of the ETF needs to increase by R20 before the investor can realize any returns.
2. **Liquidity:** Liquidity can be determined by the volume of shares that are traded daily. This may have several implications. Firstly, purchasing an ETF with low liquidity may be difficult if an investor demands more than the daily traded volume. She may have to offer higher prices to inspire holders to sell their holdings. An investor's ability to drive up the market price is known as *Market Impact* (ETF.com, 2017). Secondly, selling a relatively illiquid ETF may prompt an investor to offer the security at a lower price if there is insufficient demand for her stock. An investor holding this security will sacrifice potential profits by selling at a lower price. Speculating on low-liquidity ETFs is not considered here. Finally, low liquidity can make it difficult for an investor to realize her investment. Consider an investor that holds 1000 ETF units but the average trading volume is only 100 per day.
3. **Costs:** Total Expense Ratio (TER) is the percentage an investor must pay on a yearly basis. The expense directly reduces the potential returns for the investor. Johnson (2018) explains that it is essential to keep costs low in an index tracking fund. TER's differ widely across the industry as such investor should interrogate this cost prior to making an investment.
4. **Tracking Error:** The tracking errors measures an ETF manager's efficiency. A small tracking error proves that the managers closely replicate the performance of the index (Cummans, 2015). Consequently, it is a reliable measure of the quality of management and the index tracking technique that has been employed.
5. **Assets under Management (AUM):** An ETF's AUM indicate where the market is concentrating their funds. High AUM may signal that the ETF is attractive. High AUM also increases the managers revenue allowing them to sufficiently support the operations of the fund. Since ETFs are low cost structures, high volumes are important.

## 5.4 Accessibility in South Africa

Robo-advising is greatly simplified where a market offers accessibility to listed instruments through online platforms. Automating the purchase of a listed instrument in their absence would increase complexity. In South Africa, several such platforms are already in existence. Easy Equity is an online platform that allows

anyone to purchase listed instruments. Similarly, ETFSA.co.za is another platform through which ETFs can be purchased. ETFSA offers a lower degree of automation than does Easy Equities. With these platforms up and running the minimum viable robo-advisor offered in this paper becomes attractive. Using this implementation an investor is able to constitute her own portfolio through these platforms.

# Chapter 6

## Methodology

### 6.1 Introduction

The literature reviewed highlighted that there are no insurmountable hurdles preventing the creation of an independent South African robo-advisor. Thus, a minimum viable robo-advisor will be presented. The objective of this advisor is to replicate the phases of robo-advising that have been discussed previously. Specifically, this implementation must assess an investor's risk profile, compute an optimal portfolio allocation and then present products that can be used to constitute her portfolio. This implementation will be achieved through the use of C(PT). The optimization problem needs risk parameters as inputs and the model will provide an asset allocation as the output.

### 6.2 Modelling

In Section 3.4 Prospect Theory & Cumulative Prospect Theory were introduced. They will now be used to compute an asset allocation for given risk preferences. The optimization problem is defined below where  $\xi$  is a vector of asset returns,  $\lambda_i$  is the weight of the  $i^{th}$  asset and  $\mathbb{1}$  is the identity matrix. (Hens and Mayer, 2014)

$$\max_{\lambda} \left. \begin{array}{l} W(\lambda) := V(\xi^T \lambda) \\ \mathbb{1}^T \lambda = 1 \\ \lambda \geq 0 \end{array} \right\} \quad (6.1)$$

The PT objective function is defined in equation 6.1. Similarly, CPT objective function is defined in equation 6.2 where  $(\eta^1)^T \lambda, \dots, (\eta^S)^T \lambda$  is a sorted vector of portfolio returns.

$$W_{PT}(\lambda) := V_{PT}(\xi^T \lambda) = \sum_{i=1}^S w(p_i) v((\xi^i)^T \lambda) \quad (6.2)$$

$$W_{CPT}(\lambda) := V_{CPT}(\xi^T \lambda) = \sum_{i=1}^S \pi_i v((\eta^i)^T \lambda) \quad (6.3)$$

## 6.3 Numerical Solutions

(C)PT value functions are non-differentiable, non-concave and includes probability distortions. Thus computing an asset allocation using (C)PT is non-trivial. Levy and Levy (2003), presented an simple solution where one's attention is focused only along the optimal mean-variance portfolio set. This assumptions dramatically reduces the set of possible solutions, thus, making numerical solutions possible. A fine mesh along the efficient frontier is generated. For each resulting portfolio the C(P)T value is calculated and the portfolio with the highest value is selected. The major drawback of this method is that higher order moments are not taken into consideration. However, Levy and Levy (2003), stressed that while skewness may be priced into the stock and may be important for C(P)T investors, the optimal mean variance efficient portfolios are very similar to the prospect efficient portfolios.

Accordingly, a set of portfolios is generated along the efficient frontier. In this case, 1000 portfolios have been generated. The weights are generated by optimizing the objective function in Equation 3.1. The process is repeated for 1000 equidistant subdivisions of the range of expected returns. The resulting efficient frontier can be seen in figure ?? With a set of weights constituted the objective function can be solved numerically for a set of given risk parameters.

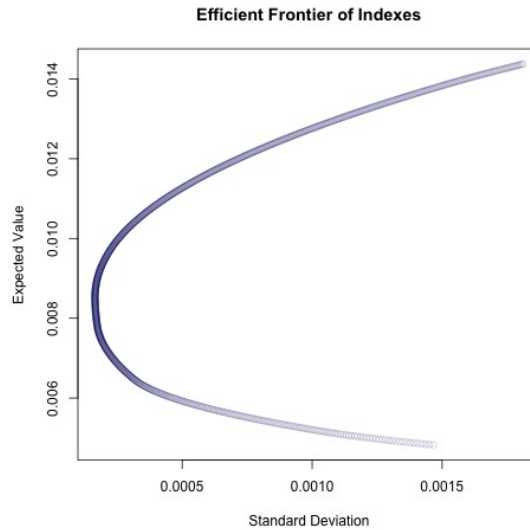


Figure 6.1: Efficient Frontier

## 6.4 Derivation of Risk Parameters

The question now arises as to how one can derive risk preferences for a specific investor and then determine her optimal asset allocation. First, the authors eliciting the exact risk parameters for an investor's value function. However, this process is non-trivial and time consuming. Abdellaoui, Bleichrodt and Paraschiv (2007), provides a comprehensive procedure for eliciting these the parameter to compute  $C(PT)$ . The process requires a value function and decision weighting function be derived for each participants. Participants are required to choose between two prospects until a number of indifference values can be ascertained. Abdellaoui, Bleichrodt and Paraschiv, is then able to extract all the parameters of the participant's value function and decision weighting function.

Initially, this seems like an attractive approach to replicate. However, there are several drawbacks that disqualify this option. Firstly, this procedure takes an average 1 hour to complete and consists of over 100 questions. The proposition of the robo-advisor is useful only to the extent that consumers are likely to use it and questionnaire response rate have been shown to be inversely related to the questionnaire length (Roszkowski and Bean, 1990). It is unlikely that users will dedicate 1 hour to selecting between prospects. A questionnaire of this nature will impose a significant barrier to people interested in this robo-advisor. Secondly, selecting between prospects is prone to being misunderstood. Abdellaoui, Bleichrodt and Paraschiv (2007), included only economic students in the experiment where an interviewer explained the experiment to them and gave them multiple practice questions. This level of individual attention is difficult to replicate in an automated environment and failing to provide it may lead to spurious results. Finally, convincing the average layman that selecting prospects is the best way to select an appropriate asset allocation may prove difficult especially where one has little financial literacy.

An alternative approach is to directly ask investors for their indifference between a prospect and a sure gain/loss. This approach was used by Werner and Sjöberg (2016). Unfortunately, research has shown asking for indifference values directly is unreliable (Bostic, Herrnstein and Luce, 1990). Moreover, Werner and Sjöberg (2016) use one question for each of the three parameters required in the value function they use. The question to elicit the  $\alpha^+$  parameter is as follows. "Suppose you have a 50% chance of gaining  $x\%$  ( $X$  [Rands]) on your investment in a year, while otherwise you gain nothing. What sure gain would you prefer over this opportunity?" The problem with this approach is that small changes in the answer to this question produces large changes in the derived  $\alpha^+$ . Moreover, it is difficult to very easily understand the exact amount that makes one indifferent with a hypothetical answer. Cognizant of this, Abdellaoui, Bleichrodt and Paraschiv (2007) uses the bisection method to find a more reliable indifference point. Accordingly, this is not a viable method.

Consequently, inferring these parameters indirectly is now of interest. There is much debate in the wealth management industry about how to accurately assess an investors risk preferences. The risk questionnaire literature reviewed in Section 4 offered little by way of inferring these parameters. (Swanepoel, 2016; Klement,

2015; Resnik, 2015). Risk profiling in academia and industry do not always share the same focus. While academia is focused on sophisticated decision theory frameworks, industry is focused customer-centric and legally compliant tools. Moreover, the algorithms used by industry are their own intellectual property so it is difficult to gain insights on precisely how their algorithms function.

## 6.5 Designing the Risk Profiler

The literature uncovered the trade-offs and compromises that must be considered when creating a risk profiler as well as their inherent limitations. It was found that there is no widely accepted risk profiling technique and legislators themselves offer little by way of practical guidance. Surmounting this hurdle is in itself an interesting research question. As such, this paper takes inspiration from best practices in the market and incorporates several insights from the literature into designing a risk questionnaire. A brief table comparing the risk profilers of local and international risk profilers can be found in Appendix C. The resulting risk profiler is a subjective measure of risk that assigns investors to one of five risk profiles. Parameters are then assigned to each risk profile. It must be clearly noted at the outset that it is not the aim of this paper nor in its scope to formulate the ideal risk profiler and cross-validate its results.

Table 6.1 presents the final risk profiler. Each question and the motivation for its inclusion will now be discussed. Question 1 (Q1) will be used to determine the rand value of the investment Q2 determines the investors time horizon. Q3 to Q6 are dedicated to capturing risk capacity. Q3 asks for an investor's age. Dempster et al. (2015), explains that as a person nears closer to retirement so her ability to take on risk decreases. Intuitively this makes sense as she will require those funds to support herself. The age intervals were taken from Betterment (2019). Q4 speaks to income. A question, asked by many US robo-advisors, illustrates that the ability to withstand losses is a function of how that loss effects one's livelihood. Income intervals were taken from the SARS Tax Tables (SARS, 2019). Q5 directly asks about an investors day-to-day financial reality. If the investor has financial constraints such as paying debt or bills they are obviously unable to withstand high risk. (An argument can be made that they should dedicate all "liquid funds" to satisfying their obligations. (Note, a future tooltip can be added to the questionnaire that confronts the investor if she selects "Paying overdue debts". The tip can tell the investor to focus on those obligations first and avoid the interest penalty before investing in stock). Q6 speaks to income stability an important factor in determining risk capacity. Q7 to Q10 captures an investors risk appetite. Q7 attempts to capture the community effect discussed earlier and presented by Klement (2015). One's community's equity holding can predict one's own desired equity position. Q8 asks about a user's investment experience. As part of the regulated requirements by the FAIS, the question seeks to determine if the individual experientially understands the risks she to which she is exposing herself. Q9 assess an investors loss aversion by testing her behaviour in adverse market conditions. Finally, Q10 asks an investor to define her investment focus (that is on gains, losses or both).

	Risk Question	Column A	Column B	Column C	Column D	Column E
1	What amount do you intend to invest?	Rand amount				
2	How long do you plan to hold your investment? this investment?	< 1 year	1-3 years	3-5 years	6-10 years	> 10 years
3	How old are you?	20-40	41-55	56-70	71-85	>85
4	What is your annual gross income?	0-196k	196k-423k	423k-708k	708k-1.5m	> 1.5m
5	At the end of each month which of the following best describes your financial concerns?	Paying overdue debts	Paying bills	Saving enough money	Optimising investments	I have no financial concerns
6	Current future income is	Very unstable	Unstable	Moderately stable	Stable	Very stable
7	Your friends/family/co-workers talk about their investments in the stock market	Never	Rarely	Sometimes	Often	Always
8	My level of investment experience	Not existent	Very little	Some experience	Experienced investor	Highly experienced
9	During market downturns a portfolio can decreased by 20% (<insert amount>). In such an event, you would	Sell all of your investment	Sell some of your investment	Hold	Buy more of this investment	
10	In making this investment are you more concerned about losses or gain	Only the losses	Mostly the losses	Both equally	Mostly the gains	Only the gains

Table 6.1: The final risk profiler questionnaire

For the minimum viable implementation of this risk profiler questions in column A, B, C, D and E take on a value of 1, 2, 3, 4 and 5 respectively. The answers in the risk capacity section and the risk appetite section are treated separately. In both cases the the sum of the scores are taken and then divided by the total possible score for that section. The composite score is computed by taking the weighted sum of the risk capacity score and risk appetite score where each carries a weight 70% and 30% respectively. Risk capacity is seen as an investors objective ability to take on financial risk (Klement, 2015) as such it must be the driving force of her risk profile regardless of her risk appetite. Finally, the investment time horizon adjusts the score by -50% if the horizon is less than 1 year and by -30%, -20%, -10% and 0% if a time horizon of 1-3 years, 3-5 years, 6-10 years and greater than 10 years respectively. Droms and Strauss (2003), propose that a risk profiler should profile risk as a function of investment time horizon. That is, for any risk preferences of an investor, a short-term investment horizon should result in a more conservative allocation than a long-term horizon. Ultimately, scores can take on a value from 0.1 to 1.

These risk scores are spread equally across 5 risk categories (Next to each name the score range is given.) Risk Averse (0.1-0.28), Conservative (0.29-0.46), Moderate (0.47-0.64), Moderately Aggressive (0.65-0.82) and Aggressive (0.83-1). Once an investor has been allocated to one of these profiles an appropriate asset allocation needs to be determined. For this exercise, risk parameters for the C(PT) objective functions need to be assigned to each category. As discussed earlier, this exercise will be done indirectly. To this end, a number of simplifying assumptions will be made. Firstly, it is assumed that an investor's loss aversion, risk aversion for gains and risk seeking for losses all move together. That is, as an investor becomes more loss averse so too does she become more risk averse for gains and more risk seeking for losses.

# Chapter 7

## Results

### 7.1 Introduction

This section will review the outputs of the portfolio optimization problem under different risk parameters. Two approaches are taken here. Firstly, asset allocations is observed when changing one risk parameter at a time (holding all others constant). In this section, the difference between PT and CPT will be reviewed. Secondly, the asset allocations are observed for the 48 subjects obtained via parameter-free measurement by Abdellaoui, Bleichrodt and Paraschiv (2007). **Finally, this section concludes with the application of the end-to-end robo-advisor - applying the risk profiler discussed in Section ?? to the optimization problem and potential ETFs.**

### 7.2 Comparison of C(PT)

Using the risk parameters found by Tversky and Kahneman (1992) for the median investor, PT with decision weighting (DW), PT without DW and CPT are compared. In all models,  $\alpha^+ = \alpha^- = 0.88$ ,  $\beta = 2.25$  and  $RP = 0\%$  are used.  $\gamma = 0.65$  For PT,  $\gamma = 0.65$  while in CPT  $\gamma = 0.61$  and  $\delta = 0.69$

Table 7.1 shows that asset allocation under PT is unaffected by changes in decision weightings while under CPT the allocation depends on this weighting. It is also noted that by CPT placing greater weight on extreme events the optimal portfolio

Theory	$w_{SWIX}$	$w_{SP500}$	$w_{MSCIworld}$	$w_{SAPY}$	$w_{GOVI}$	$w_{GOLD}$
PT with DW	0%	34%	0%	0%	64%	2%
PT without DW	0%	34%	0%	0%	64%	2%
CPT	0%	39%	0%	0%	61%	0%
Min Var	0%	23%	0%	0%	71%	6%

Table 7.1: Comparison of portfolio allocation under variations of Prospect Theory

for the median investor becomes slightly more risk seeking. Since there is no difference between PT with DW and PT without DW, there is no need to continue the comparison. When comparing these allocations with the minimum variance portfolio (Min Var), found by MV-optimisation, we see that median prospect theory investors only take on slightly more risk than the minimum variance portfolio.

### 7.3 Impact of Risk Parameters on Portfolio Selection

It is now of interest how the portfolio allocation changes as the risk parameters change. As a starting point, the parameters shown by Coelho et al. (2014) in his sensibility assessment will be used to assess how the portfolio changes for different risk parameters.  $\beta$  of 1 shows an investor who exhibits no loss aversion while an investor with a  $\beta$  of 3 has high loss aversion. Figure 7.1 show the percentage of the portfolio concentrated in bonds as beta changes. As the results illustrate, as the investor becomes more loss averse, the portfolio becomes more conservative. That is a greater percentage of the portfolio is invested into bonds. In both C(PT), the bond percentage rises dramatically as beta changes from 1 to 2. The change is more gradual under CPT than PT but investors. Furthermore, the maximum that an investor allocates to the bond peaks and then plateaus between 60% and 70%.

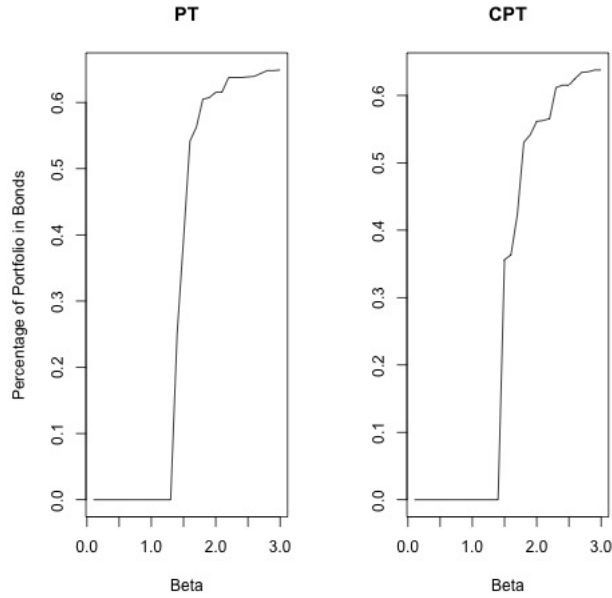


Figure 7.1: % allocation to bonds as  $\beta$  changes

Again fixing all other parameters,  $\alpha^+$  and  $\alpha^-$  will be altered simultaneously. Values between 0.1 and 1 are assessed.  $\alpha^+$  measures the curvature of the value function in the positive region while  $\alpha^-$  measures the curvature of the value function in the negative region. For  $\alpha^+ = \alpha^- = 1$  an investor is risk neutral while for  $\alpha^+ = \alpha^- = 0.5$  an investor is risk averse in gains and risk seeking in losses. Finally

for  $\alpha^+ = \alpha^- = 0.1$  the investor is a highly risk averse for gains and highly risk seeking for losses. Figure 7.2 shows the resulting portfolio's bond allocation. As an investor becomes more risk averse in gains and more risk seeking in losses, her portfolio allocation becomes more concentrated in bonds.

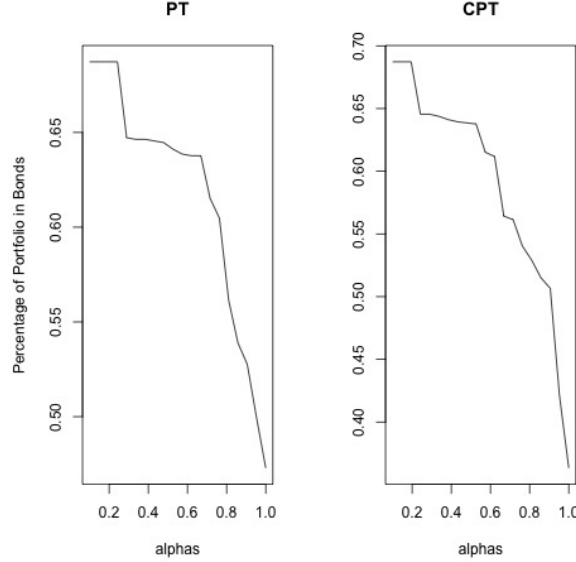


Figure 7.2: % allocation to bonds as  $\alpha^+$  and  $\alpha^-$  changes

For the next two sections only CPT will be reviewed. First, changes in decision weights are examined.  $\gamma$  &  $\delta$  will be altered simultaneously while holding all other variables constant.  $\gamma = \delta = 1$  shows linear probability weighting. That is no distortion of the actual probabilities is observed. Conversely,  $\gamma = \delta = 0.44$  shows non-linear probability weightings. As such, under CPT, more extreme events will be given greater importance. In Figure 7.3, it is observed that investors take on more risk as their distortion of probabilities becomes larger. As such, the investor gives little importance to returns of bonds as the returns are temperate while the more extreme returns of equity are brought into focus.

## 7.4 Application to the Risk Profiler

### 7.4.1 Allocating Risk Parameters to the Risk Categories

In Section 6.5, five risk categories were extracted from the risk profiler. Now risk parameters must be assigned to these categories. Here it is achieved by looking at the empirical data derived by Abdellaoui, Bleichrodt and Paraschiv (2007). Table 7.4.1 shows the  $\alpha^+$ ,  $\alpha^-$ ,  $\beta$  from the empirical data. Parameters at the for the 10<sup>th</sup>, 25<sup>th</sup>, 75<sup>th</sup>, 90<sup>th</sup> percentile are found together with the mean of the data. Excluding the topic 10% and bottom 10% prevents outliers from skewing the results. The reference point, measured in % p.a., is artificially grown by 6% at each interval.

Assigning these parameters to the risk categories and setting  $\gamma = 0.61$  &  $\delta = 0.69$ ,

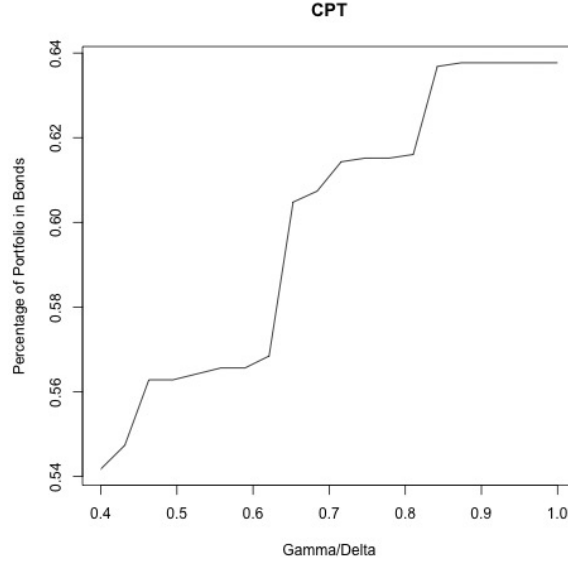


Figure 7.3: % allocation to bonds as  $\gamma$  and  $\delta$  changes

	$\alpha^+$	$\alpha^-$	$\beta$	RP
10%	0.51	0.51	3.87	0.00
25%	0.61	0.60	2.35	0.06
Mean	0.86	0.80	2.04	0.12
75%	1.00	0.83	1.18	0.18
90%	1.29	1.04	0.83	0.24

Table 7.2: Empirical risk parameters found by Abdellaoui, Bleichrodt and Paraschiv (2007)

Table 7.4.1 shows the resulting asset allocation. The bond allocation systematically increases as the riskiness of the investor increases. There is, however, an steep drop from 45% held in bonds to 0% when the category changes from moderately aggressive to aggressive. Table shows the expected returns and standard deviation for these optimal portfolios.

## 7.4.2 Reasonability checks

ABSA's robo-advisor offers proposed portfolio allocations for different levels of risk preferences (Capital, 2019). They divide a portfolio up into 3 assets types - low risk, medium risk and high risk. From least risk averse to most risk averse they propose the following allocation. Denoted (High/Medium/Low) (0/0/100), (0/30/70), (10/40/50), (30/40/30), (50/40/10), (70/30/10), (100/0/0). Firstly, it is clear that the robo-advisor proposed in this paper does reallocate assets in the same manner as the ABSA advisor. Secondly, the least risky portfolio presented here still allocates 25% of the portfolio to equities while Absa recommends a 0% allocation to the risky asset.

	SWIX	SP500	MSCIworld	SAPY	GOVI	GOLD
Risk Averse	0.00	0.25	0.00	0.00	0.69	0.05
Conservative	0.00	0.34	0.00	0.00	0.64	0.02
Moderate	0.02	0.44	0.00	0.00	0.54	0.00
Mod. Aggressive	0.03	0.52	0.00	0.00	0.45	0.00
Aggressive	0.01	0.99	0.00	0.00	0.00	0.00

Table 7.3: Asset allocation by applying the parameters in Table 7.4.1

	Risk Averse	Conservative	Moderate	Mod Aggressive	Aggressive
Exp. Return	10.47%	11.22%	12.21%	13.00%	17.21%
Std. Deviation	4.47%	4.81%	5.80%	6.92%	14.67%

Table 7.4: Performance of Optimal Portfolios 7.4.1

Betterment’s robo-adviser proposes that the least risky portfolio be constituted by 33% Equity and 67% Bonds while there most risky portfolio suggests 95% Equity and 4% Bonds. (Betterment, 2019) The robo-advisor presented in this paper comes close to that recommended by Betterment. Despite Absa’s superficial guidance, this is reassuring that the robo-advisor presented here holds up to reasonability checks.

## 7.5 Showcasing specific ETFs

The portfolios have been constituted on the benchmark or index data. Now it is necessary to recommend a specific ETF in which the individual can invest. Comparing and contrasting ETFs is an another area of complexity. The complexity is sufficient to warrant entire business based on providing in-depth analysis of ETFs. Johnson (2018), among others, dive deep into the complexities of ETF evaluation. However, using a few easily accessible metrics it is possible to give investors the means to compare the performance of an ETFs relative to its peers. Table 7.5 shows all of the ETFs that track the indexes modelled in Section ?? . Therefore, with a table similar to the one below investors can quickly ascertain which funds are optimal investment vehicles. The data in Table 7.5 was collected from ETFSA (2019) and Equities (2019). The following table can be made understandable to investors by including a tool tip above each column that briefly explains how to interpret each metric. T

Index Tracked	Provider	Ticker	TER	Tracking Error	Spread	AUM (R'm)
SWIX	Absa	NFSWIX	0.37%	0.09%	0.25	16.15
SWIX	Satrix	STXSWX	0.45%	NA	0.07	410
SWIX	Stanlib	STANSX	0.29%	0.09%	0.08	1850
SWIX	Sygnia	SYGSW4	0.19%	0.07%	0.01	193.08
SP500	CoreShares	CSP500	0.60%	0.13%	0.01	619.82
SP500	Satrix	STX500	0.25%	NA	0.03	394
SP500	Stanlib	ETF500	0.27%	NA	0.16	9.70
SP500	Sygnia	SYG500	0.16%	-0.03%	0.03	1137.47
MSCIworld	Satrix	STXWDM	0.35%		0.31	669
MSCIworld	Stanlib	ETFWLD	0.40%	NA	0.09	48.19
MSCIworld	Sygnia	SYGWD	0.68%	0.09%	0.02	7293
SAPY	CoreShare	PTXSPY	0.57%	0.55%	0.003	132.47
S&P SA Prop	Satrix	STXPRO	0.32%	0.16%	NA	109
GOVI	Absa	NFGOVI	0.29%	0.09%	0.01	690.15
GOLD	Absa	GLD	0.40%	NA	0.003	10607.04
GOLD	Standard Bank	ETFGLD	0.25%	NA	0.32	108.95

Table 7.5: Key Metrics for the evaluation of ETFs

## 7.6 Limitations & Future Work

Financial Technology, as an academic discipline, lends itself to a multi-disciplinary approach. This paper touches on all of the key components of independent robo-advising with the focus on implementing a solution that matches industry standards. This focus comes at the cost of not interrogating any one of these components in depth. As such, the paper should be interpreted in this light. An implementation for an independent robo-advisor is offered that can enable investment into a diversified portfolio. Importantly, an investor can constitute the portfolio herself through online trading platforms.

This paper was limited by the industry's advancements and its own scope. The single greatest risk to any advisory offering is providing poor advice. Presently, there is no consensus in the literature nor in the industry on how to best profile risk preferences and then provide investment advice. This sets a ceiling on best possible results that can be achieved. Moreover, the model suggested in this paper is focused on matching industry standards but it has not been thoroughly tested. Therefore, before an implementation is offered to the public, an in-depth analysis of its performance is required.

# Chapter 8

## Conclusion

This thesis has demonstrated the end-to-end process of creating an independent robo-advisor. Through the process, the nuances and complexities involved herein were made clear. Firstly, robo-advising provides an extensive and interesting research field. Robo-advising must not only overcome the hurdles in automating advice but also address systemic issues in conventional robo-advising. Secondly, the paper highlighted how risk profiling is a contentious issue and one that will likely continue to spark debate among all stakeholders. Furthermore, decision theory frameworks offered valuable models to understand investor behaviour. Unfortunately, the extent to which these models are used in risk profiling algorithms could only be inferred and not directly observed. Thirdly, the rise of ETFs and simultaneous support for passive investment strategies have sparked great interest in the robo-advising space. Finally, an independent robo-advising offering was found to be possible for the South African market with of the preconditions necessary for its success present.

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# Appendix A

## Data Exploration

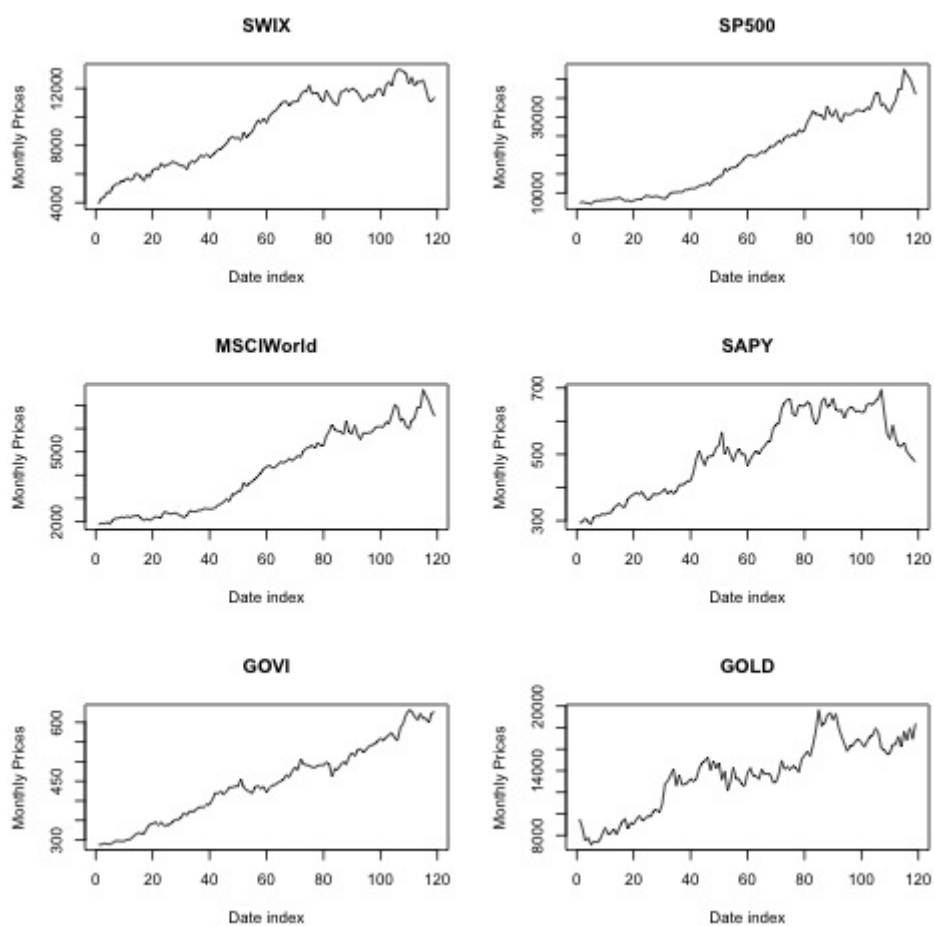


Figure A.1: Asset Monthly Price

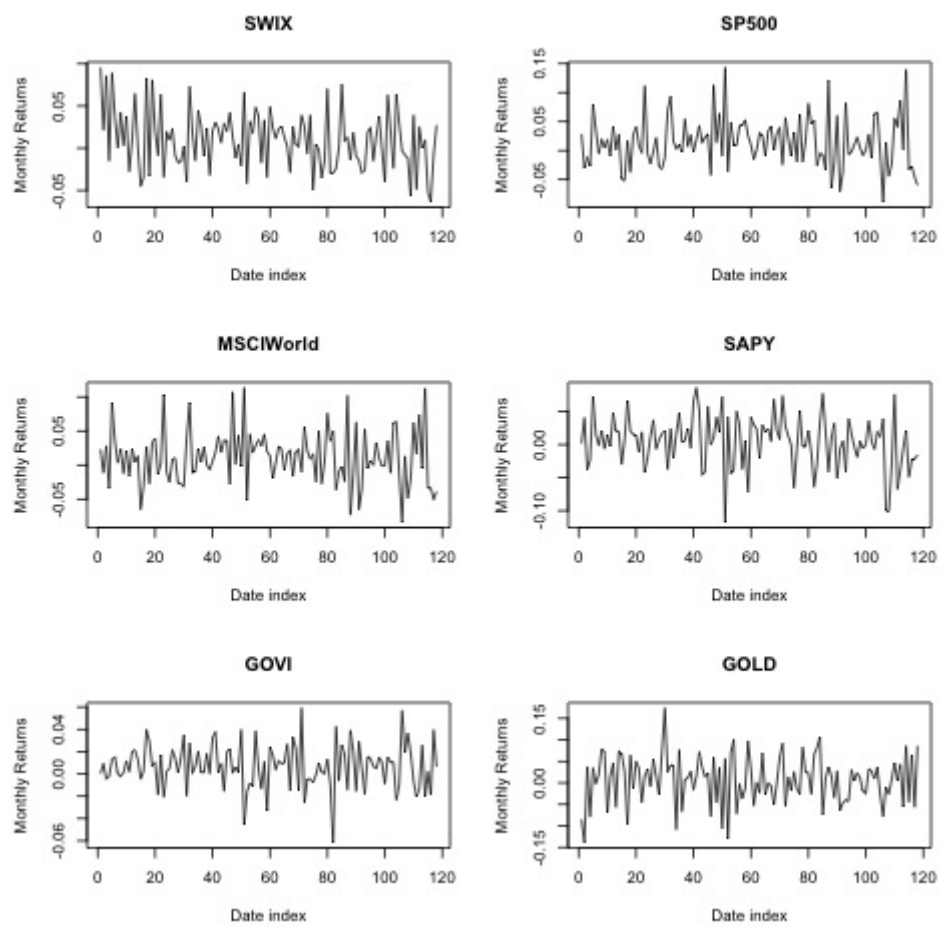


Figure A.2: Asset Monthly Return

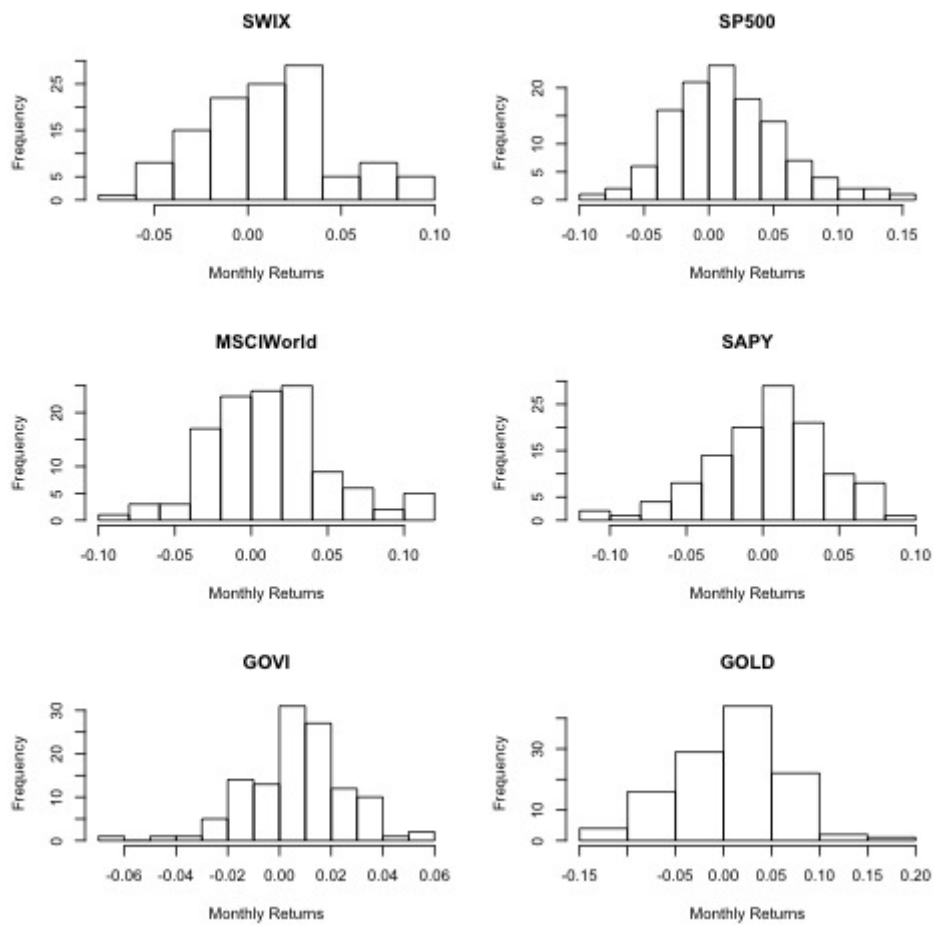


Figure A.3: Asset Returns Histogram

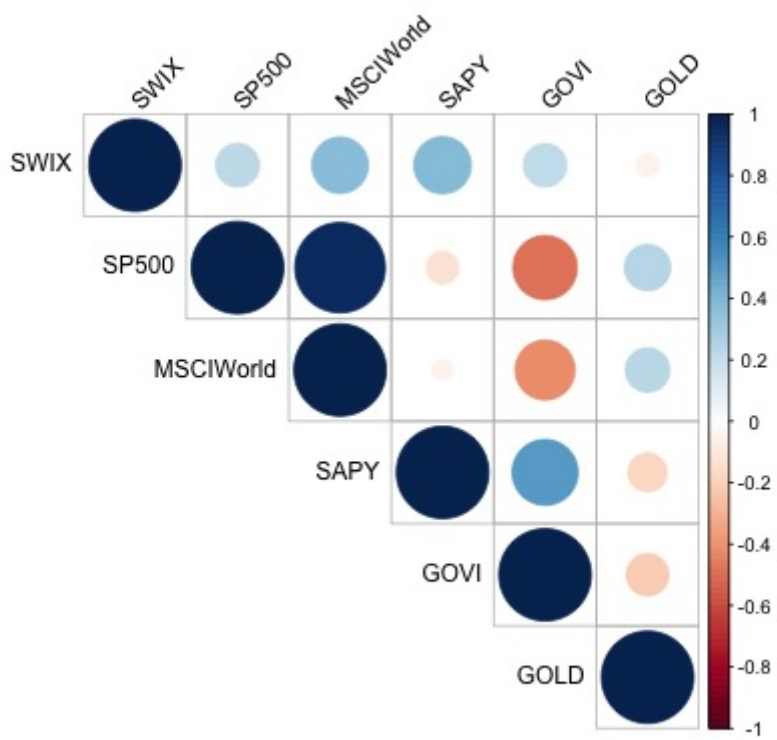


Figure A.4: Correlation between Assets

# Appendix B

## R Code

```
1 ---
2 title: "R Notebook"
3 output: html_notebook
4 ---
5
6 ```{r}
7 library(readxl)
8
9 #Daily prices for each ETF are extracted from Thomson Reuters Eikon & Sharenet.
10 #These files are then amalgamated into a single xls file & read into R
11
12 prices <- read_excel("IndexesFinal.xlsx", sheet = "Sheet1",
13                      col_types = c("date", "numeric", "numeric", "numeric", "
14                                   numeric", "numeric", "numeric"))
15
16 #Name each row with the date of the observation
17 prices<-as.data.frame(prices)
18 rownames(prices)<-as.Date(prices[,1], origin = "1904-01-01")
19 prices<-prices[,-1]
20
21 ```
22
23 ```{r}
24 #Follow the same procedure with the returns
25 returns <- read_excel("IndexesFinal.xlsx", sheet = "Sheet2",
26                       col_types = c("date", "numeric", "numeric", "numeric", "
27                                   numeric", "numeric", "numeric"))
28
29 #Name each row with the date of the observation
30 returns<-as.data.frame(returns)
31 rownames(returns)<-as.Date(returns[,1], origin = "1904-01-01")
32 returns<-returns[,-1]
33
34 ```
35
36 ```{r}
37 jpeg("cor.jpeg")
38 library(corrplot, quietly = T)
39 corrplot(cor(returns), type= "upper", method = "circle", tl.col = "black", diag=T,
40          tl.srt = 45)
41 dev.off()
42
43 ```
44
45 ```{r}
46 jpeg("price.jpeg")
47 par(mfrow=c(3,2))
48 for (i in 1:6){
49   plot(prices[,i], type="l", ylab="Monthly Prices", xlab="Date index", main=
50        colnames(returns)[i])
51 }
52 dev.off()
```

```

48 ""
49
50
51 ""{r}
52 jpeg("ret.jpeg")
53 par(mfrow=c(3,2))
54 for (i in 1:6){
55   plot(returns[,i], type="l", ylab="Monthly Returns", xlab="Date index", main=
56     colnames(returns)[i])
57 }
58 dev.off()
59 ""
60
61
62 ""{r}
63 jpeg("rethist.jpeg")
64 par(mfrow=c(3,2))
65 for (i in 1:6){
66   hist(returns[,i], xlab="Monthly Returns", main=colnames(returns)[i], breaks = 10)
67 }
68 dev.off()
69 ""
70
71 ""{r}
72 allmeans<-apply(returns, 2, mean)
73 allsds<-apply(returns, 2, sd)
74 ""
75
76 ""{r}
77 jpeg("retVSrisk.jpeg")
78 plot(allsds, allmeans, type="n", ylim=c(0, 0.02), xaxt='n', yaxt='n', xlab="
79   Standard Deviation", ylab="Expected Return")
80 text(allsds, allmeans, labels=colnames(returns), cex= 0.7, pos=3)
81 dev.off()
82 ""
83
84 ""{r}
85 library(fPortfolio)
86
87 ts_returns<-as.timeSeries(returns)
88
89 spec <- portfolioSpec()
90 setNFrontierPoints(spec)<-1000
91
92 eff_ports<-portfolioFrontier(ts_returns, spec = spec, constraints = "LongOnly")
93 weights<-eff_ports@portfolio@portfolio[["weights"]]
94
95 colnames(weights) = colnames(returns)
96 ""
97
98
99 ""{r}
100 #This function calculates the portfolio variance
101
102 portvarfunc <- function(weights, returns){
103   sdevs <- apply(returns, MARGIN = 2, sd)
104   sdwt<-matrix(sdevs*weights, ncol = 1)
105   portvar<-(t(sdwt)%*%cor(returns)%*%sdwt)^0.5
106   return(portvar)
107 }
108
109 ""
110
111
112 ""{r}
113 jpeg('eff_frontier.jpg')
114 eff_mean<-apply(weights, 1, function(x) mean(as.matrix(returns) %*% x))
115 eff_sd <- apply(weights, 1, function(x) portvarfunc(x, as.matrix(returns))^2)
116 plot(x=eff_sd, y=eff_mean, ylab="Expected Value", xlab="Standard Deviation",
117   col=rgb(0, 0, 100, 50, maxColorValue=255),
118   main="Efficient Frontier of Indexes")

```

```

119 dev.off()
120 ""
121
122 ""{r}
123 min_var_port<-efficientPortfolio(ts_returns, spec = portfolioSpec(), constraints =
    "LongOnly")
124 min_var_port@portfolio@portfolio[["weights"]]
125 ""

```

images/Code/dataExploration.Rmd

```

1 ---
2 title: "R Notebook"
3 output: html_notebook
4 ---
5
6 This workbook contains the code to perform the prospect theory optimization.
7
8 ""{r}
9 #Prospect Theory's power value function
10 power_val<-function(par, x){
11   ap<-par[1]
12   an<-par[2]
13   b<-par[3]
14   rp<-par[4]
15
16   val<-rep(0, length(x))
17
18   for (i in 1:length(x)){
19     if (x[i] >= rp){
20       val[i]<-(x[i]-rp)^ap
21     }
22     else {
23       val[i]<-b*((rp-x[i])^an)
24     }
25   }
26
27   return(val)
28 }
29
30 ""
31
32
33 ""{r}
34 #Probability weighting function
35
36 prob_weight<-function(y, p){
37   (p^y)/((p^y+(1-p)^y)^(1/y))
38 }
39
40 ""
41
42
43 ""{r}
44 #Plot the value function and probability weighting function
45 jpeg('prospgraphs.jpg')
46 par(mfrow=c(1,2))
47
48 x1<-seq(-1, 1, length.out = 100)
49 plot(x1, power_val(c(0.88, 0.88, 2.25, 0), x1), type = "l",
50      main="Power Value Function",
51      ylab="Value",
52      xlab="Outcome")
53 abline(h=0, v=0, col="grey")
54
55
56 x2<-seq(0,1, length.out = 100)
57 plot(x2, prob_weight(0.65, x2), xlim=c(0,1), ylim=c(0,1), type="l",
58      main="Probability Weighting Function",
59      ylab="Decision Weighted",
60      xlab="Probability")
61 abline(a=0, b=1, col="grey")
62 dev.off()

```

```

63 ""
64
65
66 ""{r}
67 #Objective function for prospect theory, historical data (all outcomes are equally
    likely)
68
69 pt_obj_func<-function(risk_par, p_weights, p_returns, y){
70
71     #Matrix where each column is a portfolio and each row is that portfolio's return
        at a simulated date
72     w_returns<-as.matrix(p_returns) %*% t(as.matrix(p_weights))
73
74     #Find the derived value for every observed portfolio return
75     ret_vals<-apply(w_returns, 1:2, function(x) power_val(risk_par, x))
76
77     #Find the derived value for each portfolio over the simulated period
78     port_quad_vals<-apply(ret_vals, 2, function(x) x*prob_weight(y, 1/length(x)))
79
80     #Calculate the value of the portfolio over all historical returns
81     pros_val<-apply(port_quad_vals, 2, sum)
82
83     #Return the weights for the portfolio that maximises the pros_val
84     return(p_weights[which(pros_val == max(pros_val)),])
85 }
86
87 #Values for par are taken from the litterature
88 pt_obj_func(c(0.88, 0.88, 2.25, 0), weights, returns, 0.65)
89
90 ""
91
92 ""{r}
93 pt_obj_func_no_prob_weighting<-function(risk_par, p_weights, p_returns){
94
95     #Matrix where each column is a portfolio and each row is that portfolio's return
        at a simulated date
96     w_returns<-as.matrix(p_returns) %*% t(as.matrix(p_weights))
97
98     #Find the derived value for every observed portfolio return
99     ret_vals<-apply(w_returns, 1:2, function(x) power_val(risk_par, x))
100
101     #Find the derived value for each portfolio over the period
102     pros_val<-apply(ret_vals, 2, mean)
103
104     #Return the weights for the portfolio that maximises the pros_val
105     return(p_weights[which(pros_val == max(pros_val)),])
106 }
107
108 #Values for par are taken from the litterature
109 pt_obj_func_no_prob_weighting(c(0.88, 0.88, 2.25, 0), weights, returns)
110
111 ""
112
113 ""{r}
114 #Coding of the probability weighting function under cumulative prospect theory
115
116 cum_prob<-function(ret_vals, sample_prob, rp, yp, yn){
117     probs<-seq(1:length(ret_vals))
118     t<-which(ret_vals >= 0)[1]
119
120     for (i in 1:length(ret_vals)){
121         if (ret_vals[i] <= rp){
122             probs[i]<-prob_weight(yn, i*sample_prob)-prob_weight(yn, (i-1)*sample_prob)
123         }
124         else {
125             probs[i]<-prob_weight(yp, (length(ret_vals)-(i-1))*sample_prob)-
126                 prob_weight(yp, (length(ret_vals)-i)*sample_prob)
127         }
128     }
129
130     return(probs)
131 }
132

```

```

133 ""
134
135 ""{r}
136 #Cumulative Prospect Theory objective function
137
138 cpt_obj_func<-function(risk_par, p_weights, p_returns, rp, yp, yn){
139
140   #Matrix where each column is a portfolio and each row is that portfolio's return
141   #at a simulated date
142   w_returns<-as.matrix(p_returns) %*% t(as.matrix(p_weights))
143
144   #Sort Weighted returns
145   sorted_w_ret<-apply(w_returns, 2, sort)
146
147   #Find the derived value for every observed portfolio return
148   ret_vals<-apply(sorted_w_ret, 1:2, function(x) power_val(risk_par, x))
149
150   #Find the derived value for each portfolio over the simulated period
151   port_quad_vals<-apply(ret_vals, 2, function(x) x*cum_prob(x, 1/length(x), rp, yp,
152     yn))
153
154   pros_val<-apply(port_quad_vals, 2, sum)
155
156   #Return the weights for the portfolio that maximises the pros_val
157   return(p_weights[which(pros_val == max(pros_val)),])
158 }
159
160 cpt_obj_func(c(0.88, 0.88, 2.25, 0), weights, returns, 0, 0.61, 0.69)
161 ""
162
163 PT: Optimal portfolio as Beta changes
164 ""{r}
165 betavar<-seq(0.1, 5, length.out = 50)
166
167 PT_portfolio_beta<-matrix(nrow=50, ncol=6)
168
169 for (i in 1:50){
170   PT_portfolio_beta[i,]<-pt_obj_func(c(0.88, 0.88, betavar[i], 0), weights, returns
171     , 0.65)
172 }
173
174 CPT_portfolio_beta<-matrix(nrow=50, ncol=6)
175
176 for (i in 1:50){
177   CPT_portfolio_beta[i,]<-cpt_obj_func(c(0.88, 0.88, betavar[i], 0), weights,
178     returns, 0, 0.61, 0.69)
179 }
180
181 ""
182
183 ""{r}
184 jpeg('betachange.jpg')
185 par(mfrow=c(1,2))
186 plot(betavar[1:30], PT_portfolio_beta[1:30,5], type="l", xlab="Beta", ylab="
187   Percentage of Portfolio in Bonds", main = "PT")
188 plot(betavar[1:30], CPT_portfolio_beta[1:30,5], type="l", xlab="Beta", main = "CPT"
189   , ylab="")
190 dev.off()
191 ""
192
193 ""{r}
194 #Compute the different portfolios as alpha negative changes
195 alpha_neg_var1<-seq(0.1, 1, length.out = 20)
196 PT_portfolio_alpha_neg1<-matrix(nrow = 20, ncol = 6)
197 CPT_portfolio_alpha_neg1<-matrix(nrow = 20, ncol = 6)
198
199 for (i in 1:20){
200   PT_portfolio_alpha_neg1[i,]<-pt_obj_func(c(alpha_pos_var1[i], alpha_pos_var1[i],
201     2.25, 0), weights, returns, 0.65)
202 }
203
204 for (i in 1:20){

```

```

199   CPT_portfolio_alpha_neg1[i,]<-cpt_obj_func(c(alpha_pos_var1[i], alpha_pos_var1[i
      ], 2.25, 0), weights, returns, 0, 0.61, 0.69)
200 }
201
202 ```
203
204 ```{r}
205 jpeg('alphachange.jpg')
206 par(mfrow=c(1,2))
207 plot(alpha_neg_var1, PT_portfolio_alpha_neg1[,5], type="l", ylab="Percentage of
      Portfolio in Bonds", main="PT", xlab="alphas")
208 plot(alpha_neg_var1, CPT_portfolio_alpha_neg1[,5], type="l", ylab="", main="CPT",
      xlab="alphas")
209 dev.off()
210 ```
211
212 ```{r}
213 gammadelta<-seq(0.40, 1, length.out = 20)
214 CPT_portfolio_gd<-matrix(nrow = 20, ncol = 6)
215
216 for (i in 1:20){
217   CPT_portfolio_gd[i,]<-cpt_obj_func(c(0.88, 0.88, 2.25, 0), weights, returns, 0,
      gammadelta[i], gammadelta[i])
218 }
219
220 ```
221
222 ```{r}
223 jpeg('gammadeltachange.jpg')
224 plot(gammadelta, CPT_portfolio_gd[,5], type="l", xlab="Gamma/Delta", ylab="
      Percentage of Portfolio in Bonds", main = "CPT")
225 dev.off()
226 ```
227
228 ```{r}
229 #Take outputs from the litterature
230
231 utpars<-matrix(c(
232 0.70, 0.40, 1.03, 0.70, 0.50, 1.03, 4.99, 4.11,
233 0.65, 0.61, 1.20, 1.24, 0.73, 0.88, 1.08, 1.08,
234 0.56, 0.56, 1.02, 0.60, 0.84, 0.49, 2.49, 2.25,
235 0.60, 0.58, 0.85, 0.80, 0.60, 0.60, 2.69, 2.75,
236 0.50, 0.68, 0.57, 0.85, 0.57, 0.46, 0.89, 0.85,
237 0.42, 0.68, 0.56, 1.01, 0.72, 0.55, 2.16, 2.34,
238 0.60, 0.69, 0.96, 0.79, 0.69, 0.43, 1.86, 1.85,
239 0.61, 0.61, 2.17, 1.09, 1.39, 0.78, 6.67, 6.80,
240 0.76, 0.42, 1.28, 0.66, 0.44, 0.85, 2.10, 2.08,
241 0.74, 0.56, 1.02, 0.70, 0.42, 0.58, 1.52, 1.28,
242 0.44, 0.60, 0.72, 0.68, 0.90, 0.48, 1.65, 1.72,
243 0.64, 0.70, 0.74, 0.91, 0.46, 0.47, 1.19, 1.20,
244 0.58, 0.58, 0.72, 0.67, 0.56, 0.52, 1.54, 1.39,
245 0.80, 0.75, 1.93, 3.07, 0.54, 1.19, 0.30, 0.30,
246 0.56, 0.56, 0.59, 0.82, 0.49, 0.67, 1.45, 1.53,
247 0.52, 0.66, 0.61, 0.82, 0.57, 0.47, 2.00, 2.01,
248 0.71, 0.77, 1.00, 0.85, 0.47, 0.31, 3.19, 3.06,
249 0.40, 0.52, 0.68, 0.82, 0.93, 0.77, 1.54, 1.63,
250 0.63, 0.66, 0.96, 0.80, 0.61, 0.49, 1.45, 1.54,
251 0.52, 0.54, 0.90, 0.71, 0.84, 0.64, 7.23, 7.01,
252 0.56, 0.62, 0.87, 1.02, 0.72, 0.69, 1.79, 1.93,
253 0.54, 0.58, 0.69, 0.66, 0.61, 0.52, 1.13, 1.13,
254 0.54, 0.58, 0.84, 0.55, 0.73, 0.42, 1.99, 1.70,
255 0.65, 0.62, 0.50, 0.54, 0.30, 0.36, 1.66, 1.57,
256 0.59, 0.63, 0.61, 0.33, 0.47, 0.23, 3.14, 2.53,
257 0.34, 0.38, 1.11, 0.75, 1.98, 1.11, 3.50, 2.94,
258 0.84, 0.48, 0.38, 0.48, 0.10, 0.50, 0.86, 0.87,
259 0.22, 0.39, 0.51, 0.51, 1.25, 0.69, 4.75, 4.35,
260 0.62, 0.63, 0.87, 0.75, 0.58, 0.51, 1.89, 2.06,
261 0.62, 0.40, 1.24, 0.66, 0.69, 0.85, 0.58, 0.63,
262 0.58, 0.56, 2.05, 0.71, 1.45, 0.59, 2.30, 1.92,
263 0.44, 0.64, 0.49, 1.01, 0.58, 0.64, 0.52, 0.41,
264 0.58, 0.68, 0.72, 1.08, 0.55, 0.50, 1.48, 1.46,
265 0.64, 0.60, 0.46, 0.80, 0.30, 0.56, 1.04, 1.06,
266 0.47, 0.61, 0.71, 0.60, 0.78, 0.43, 4.12, 3.77,

```

```

267 0.40, 0.30, 0.56, 0.58, 0.78, 1.16, 2.05, 2.05,
268 0.67, 0.71, 0.86, 0.81, 0.48, 0.38, 4.68, 4.32,
269 0.61, 0.44, 1.32, 0.74, 0.87, 0.88, 1.62, 1.57,
270 0.72, 0.66, 0.66, 0.60, 0.29, 0.36, 1.73, 1.76,
271 0.40, 0.38, 0.71, 0.59, 0.95, 0.82, 1.72, 1.68,
272 0.68, 0.22, 0.50, 0.51, 0.29, 1.09, 0.86, 0.87,
273 0.68, 0.67, 0.69, 0.66, 0.39, 0.37, 1.21, 1.14,
274 0.60, 0.58, 0.67, 0.81, 0.48, 0.62, 1.66, 1.66,
275 0.60, 0.63, 0.60, 0.62, 0.43, 0.40, 2.51, 2.41,
276 0.74, 0.44, 0.51, 1.57, 0.23, 2.01, 0.58, 0.40,
277 0.77, 0.14, 0.63, 0.50, 0.25, 2.04, 3.22, 2.37,
278 0.52, 0.50, 0.66, 0.87, 0.61, 0.87, 0.80, 0.80,
279 0.58, 0.62, 1.35, 0.43, 1.03, 0.30, 2.30, 1.64), ncol=8, byrow = T)
280
281 colnames(utpars)<-c("probGain", "probLoss", "alphaGain", "alphaLoss","skip", "skip"
    , "betaMean", "betaMed")
282
283 ```
284
285
286 ```{r}
287 #Risk Parameters used in the optimal asset allocation for different risk levels
288 riskpars_finalport<-apply(utpars, 2, function(x) quantile(x, c(0.1, 0.25, 0.5,
    0.75, 0.9)))
289 riskpars_finalport[,5]<-sort(riskpars_finalport[,5], decreasing = T)
290 riskpars_finalport[,1]<-sort(riskpars_finalport[,1], decreasing = T)
291 riskpars_finalport[,2]<-sort(riskpars_finalport[,2], decreasing = T)
292 riskpars_finalport[,3]<-apply(utpars, 2, mean)
293
294 refpoint<-c(0, 0.06, 0.12, 0.18, 0.24)/12 #convert to monthly returns
295
296 riskpars_finalport
297 ```
298
299 ```{r}
300 library(xtable)
301 print(xtable(round(CPT_portfolio_final, 2), type = "latex"), file = "CPTportFinal.
    tex")
302 print(xtable(riskpars_finalport[, -c(1,2)], type = "latex"), file = "CPTriskparFinal
    .tex")
303 ```
304
305 ```{r}
306 #Compute performance for optimal portfolios
307 port_performance<-matrix(ncol=5, nrow=2)
308
309 port_performance[1,]<-apply(CPT_portfolio_final, 1, function(x) mean(as.matrix(
    returns) %*% x))*12*100
310 port_performance[2,]<-apply(CPT_portfolio_final, 1, function(x) portvarfunc(x, as.
    matrix(returns)))*(12^0.5)*100
311
312 print(xtable(port_performance, type = "latex"), file = "portPerformance.tex")
313 ```

```

images/Code/Final\_bot\_maybe.Rmd

# Appendix C

## Risk Profilers in Industry

Vanguard	Wealthfront	Betterment	Sanlam	Absa	Sygnia
Time Horizon	Reason for saving	Retired	Goal-based	Time horizon	Age
Divestment period	Outcome Desired	Age	Target outcome	Risk versus return preference	Retirement age
Define long-term period	Annual gross income	Annual Income	Installments/lump sum	Investment experience	Children
Behaviour in adverse market (Equities)	Income (single/dual) dependents			Investment knowledge	Property
Risk versus return preference	Current value of savings			Willingness to take on risk	Discretionary investments
Financial decisions made off conversations	Gains/losses/botl			Adaptability in financial hard times	Investor income statement
Behaviour in adverse market (Bonds)	Behaviour in adverse market (Portfolio)			Association with word "risk"	Retirement savings contribution
Greatest loss versus greatest gain				Large investments made for the thrill	Accumulated retirement savings
Income stability				Focus on gains or losses	Current investment strategy risk profile
Investment experience				Borrowed money for investment	Growth or capital protection
Current allocation				Maximum drawdown	
				Purchasing power or value	
				Expected returns	
				Ability to withstand loss at end of investment	

Table C.1: Industry Risk profilers reviewed (USA & RSA)