The challenge: We need to sample according to a distribution \( p(y) \) with \( y \in [a, b] \) but have access just to a uniform random generator for \( x \) over the interval \([0, 1]\).

One option is to consider the cumulative distribution function of \( p(y) \):

\[
F(y) = \int_a^y p(y') \, dy'.
\]

We identify \( F(y) \) with \( x \), perform the integration and invert the result to obtain a transformation law \( y(x) \). The so-called ‘transformation method’ then proceeds as follows:

- Use the uniform random generator to return a random \( x_i \) in the interval \([0, 1]\)
- Use the generated \( x_i \) to find \( y_i \), using the derived transformation law \( y(x) \)
- Repeat until enough samples are collected
- The set \( \{y_i\} \) then follows the distribution \( p(y) \)

The greatest advantage of this method is that no work is wasted; every sampled \( x_i \) leads to an accepted \( y_i \). Infinite domains are also no problem, provided the distribution can be analytically integrated and the result inverted. One disadvantage, however, is that the sample distribution \( p(y) \) has to be normalised.

Questions:

1. Use the transformation method to sample according to the following probability distribution functions (check your results using the associated EJS simulation):
   
i) \( p(y) = \frac{1}{2} \) with \( y \in [2, 4] \);
   
ii) \( p(y) = \cos(2y) \) defined on the region \(-\pi/4 \leq y \leq \pi/4 \);
   
iii) \( p(y) = \frac{1}{\sqrt{8-\sqrt{3}} \sqrt{y^2-1}} \) defined on the region \( 2 \leq y \leq 3 \);
   
iv) \( p(y) = \frac{1}{\sqrt{8} \sqrt{y^2-1}} \) defined on the region \( 1 < y \leq 3 \);
   
v) \( p(y) = e^{-y} \) with \( y \in [0, \infty) \).