MAGIC FORMULA OPTIMISATION IN THE SOUTH AFRICAN MARKET

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I hereby declare that I have read and understood the regulations governing the submission of Master of Commerce dissertations, including those relating to length and plagiarism, as contained in the rules of the University, and that this dissertation conforms to those regulations.

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ABSTRACT

The purpose of this study is to investigate the performance of the value investing strategy commonly referred to as the “Magic Formula”, which was first introduced by Greenblatt (2006) and uses the return on capital and earning yield ratios as the basis for stock selection, in the South African market.

The study will build on the work previously performed by Howard (2015) by challenging the “Magic Formula” portfolio composition assumptions. In doing so, optimal combinations of holding period and portfolio size which: maximise the geometric mean return, minimise the volatility of returns and maximise the risk adjusted return, shall be determined.

The scope of this study includes all companies, excluding financial services entities, listed on the Johannesburg Stock Exchange, which exceed a market capitalisation of R 100 million, for the period 1 October 2005 to 30 September 2015.

The results showed that by adjusting certain portfolio parameters the overall performance of the “Magic Formula” on both a geometric mean and risk adjusted basis can be increased. However, the “Magic Formula” still provides an insufficient amount of evidence to conclude, on a statistically significant basis, an outperformance of the investment strategy relative to the Johannesburg Stock Exchange All Share Index.

Accordingly, the study makes several contributions to the literature. Firstly, it provides direct evidence of the relationship between value investing portfolio composition and the returns generated, indicating that excess returns can be achieved when the portfolio composition is adjusted. Secondly, albeit not on a statistical basis, the study provides further corroborating evidence of outperformance
of the “Magic Formula” in South African and global markets. Finally, the study provides the ‘optimal’ “Magic Formula” portfolio composition for the South African market as determined by an investors risk tolerance.
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CHAPTER 1 - INTRODUCTION

Background

The financial market is made up of different types of investors who follow differing investment strategies and have different investment styles. That being said, the common thread between most investors is attempting to achieve a similar goal of outperforming the market. This common thread results in these investors asking themselves the same fundamental question – how can I beat the market?

A solution to this fundamental question was provided when Joel Greenblatt published “The Little Book That Beats the Market” (Greenblatt, 2006). In this book it is explained how investors can outperform market averages (represented by broad based U.S. market indices) by simply following a formula that identifies businesses which are not only ‘good’ but are also currently ‘under-priced’ in the market. This formula, referred to as the “Magic Formula”, ranks shares based solely on two factors, namely: Return on Capital and Earnings Yield. Accordingly, the “Magic Formula” includes one component of value, represented by a high Earnings Yield and the other component which is investing in excellent companies as depicted by a high Return on Capital. The use of the “Magic Formula” is to capture possibilities of purchasing both cheap companies and also quality companies.

Empirically testing Greenblatt’s (2006) theory in the United States, Blij (2010) confirmed this theory and concluded that, by using this “Magic Formula”, investors could outperform the broad based U.S. Market Indices on a regular basis without incurring a higher level of risk as measured by the Sharpe Ratio. Applied in the South African market, the “Magic Formula” yielded an excess geometric mean return of 1% relative to the Johannesburg Stock Exchange (JSE) All Share Index Total Returns (ALSI), revealing no clear conclusions (Howard, 2015). However, a key
limitation of Howard (2015)’s testing was that certain variables, such as the “Magic Formula” portfolio holding period and the number of shares in the “Magic Formula” portfolio, were kept constant.

Any adjustment to these variables could result in a greater geometric mean return being generated in the South African market. This belief is evidenced by the contrasting results achieved by Olin (2011) and Howard (2015) for the Finnish and South African markets respectfully when the treatment of these variables differed.

The purpose of this study is therefore to observe the “Magic Formula” geometric mean return for the South African market when differing holding periods and portfolio sizes are applied in constructing the overall “Magic Formula” portfolio. This will enable us to conclude, on an optimal geometric mean return basis, whether it is possible to outperform the South African market, as represented by the JSE ALSI, on a consistent long term basis.

As a result, by addressing the purpose set out above, the study will make several contributions to both the South African and global markets. For the South African market specifically, the study shall outline whether it possible to outperform the JSE ALSI on a risk-adjusted basis when applying the “Magic Formula” investment strategy as well as outlining which combinations of portfolio holding periods and portfolio sizes results in the largest geometric mean returns. Further, the existence of any outperformance generated by the “Magic Formula” in the South African market would provide additional, substantiating, evidence against the ‘Efficient Market Hypothesis’. In application to the global market, the results of the study can be extrapolated in order to highlight whether an adjustment to the portfolio composition can influence the risk-adjusted returns generated by a value investing portfolio.
Structure of the dissertation

The study to be conducted will be divided into five chapters. The detail of these chapters, and the high level overview, is provided below:

The first chapter, ‘Introduction’, presents the contextualisation of the study to be conducted.

The relevant literature shall be reviewed in Chapter two, ‘Literature Review’, and shall cover the efficient market hypothesis, behavioural finance, a review of the “Magic Formula” investment strategy as well as a review of value investing strategies in the South African market.

The findings from the literature review shall form the basis of the research questions which is set out in Chapter three, ‘Methodology’. Chapter three will further include the research approach and strategic considerations of the study.

Chapter three will be followed by a discussion of the results, as well as key observations relevant to the study, which will be presented in Chapter four, ‘Results’.

Lastly, Chapter five, ‘Conclusion’, shall provide the resultant conclusions of the study conducted based on the results as set out in Chapter four. Additionally, areas of suggested further study will also be included in this chapter.
CHAPTER 2 - LITERATURE REVIEW

Introduction to Literature Review

The second chapter contains the overall literature review. As a result, findings from previous literature shall be presented, discussed and used to formulate the fundamental research questions of the study.

This chapter shall be sub-divided into five segments, an outline of this is provided below:

A basic overview of the ‘Efficient Market Hypothesis’ and ‘Behavioural Finance’ as well as the arguments for each shall first be introduced. This shall be used to establish an understanding of the two contradictory market theories.

This shall be followed by an introduction of the ‘value investing’ concept. Value investing forms part of the substantiation of behavioural finance as it can only be achieved through mispricing in the market. Accordingly, value investing is in sharp contrast to the principles of efficient market hypothesis.

A review of the principles underpinning the “Magic Formula” value investing strategy shall follow. The ‘Introducing the “Magic Formula”’ subsection will outline the basis of one of the many value investing strategies created, the “Magic Formula”. This is done as the “Magic Formula” investment strategy is to be the focal point of the study. Importantly, through review of literature, this section will further outline the existing research in the South African market and the limitations of such.

The penultimate subsection shall be a review of the value investing portfolio composition, namely an investigation into the portfolio holding period and number of shares making up the portfolio. As a result, the determination of whether an
adjustment to any of these factors could result in increasing returns shall be made. This shall indicate whether any adjustment to the assumptions applied in prior research of the "Magic Formula" in the South African market would result in an improved overall performance.

The last subsection shall be a review of the risk-adjusted returns, in particular a review of investor risk tolerance. This shall be performed in order to determine and quantify what constitutes ‘improved overall performance’.

Efficient Market Hypothesis and Behavioural Finance

Introduction

Decades ago, the ‘Efficient Market Hypothesis’ (‘EMH’) was widely accepted by most financial economists, with the exception of value investors such as Warren Buffett, Seth Klarman, Benjamin Graham, Walter Schloss, Joel Greenblatt, Howard Marks and the like, where the belief is that securities markets are extremely efficient in reflecting information about the share prices (Gupta, Preetibedi and Mlakra, 2014:56). In more recent times, since the introduction of ‘Behavioural Finance’, academic finance has evolved a long way from the days when efficient market theory was widely considered to be proved beyond doubt (Shiller, 2003:83).

At a high level, EMH is the notion that shares reflect all available information. This hypothesis is based on the theory that competition between profit-seeking investors drives prices to their correct value (Ritter, 2003:430). Contrastingly, behavioural finance encompasses research that drops the traditional assumptions of expected

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1 Please refer to Definitions of Key Terms, Concepts and Variables in the Glossary for more information on the Efficient Market Hypothesis concept.
2 Please refer to Definitions of Key Terms, Concepts and Variables in the Glossary for more information on the Behavioural Finance concept.
utility maximization with rational investors in efficient markets (Ritter, 2003:429). As such, the proponents of behavioural finance are those persons whose views are in sharp contradiction to the efficient markets theory (Shiller, 2003:83).

In this sub-section of the literature review, the basis for both EMH and Behavioural Finance shall be reviewed and discussed.

Efficient Market Hypothesis – An insight into the market theory

Should the EMH theory hold true, it would imply that the individual investor is therefore unable to consistently earn above-average returns without taking above-average risks (Malkiel, 2003:60).

According to Fama (1970), efficiency is distinguished in three different forms: Weak form, Semi-Strong form and Strong form of efficient market Hypothesis (Gupta, Preetibedi and Mlakra, 2014:57). An explanatory overview of these forms is provided below:

1. The weak form of the EMH holds that the share market prices follow a ‘random walk’\(^3\). As a result, share prices are independent from one another, making it impossible to predict a future price based on a series of past prices (Correia et al., 2011:4-25).

2. The semi-strong form of the EMH holds that all publicly available information is included immediately, and without bias, into the share price. Accordingly, it is not possible through fundamental analysis to extract new information which could result in superior returns being incurred consistently (Correia et al., 2011:4-25).

\(^3\) Please refer to Definitions of Key Terms, Concepts and Variables in the Glossary for more information on the Random Walk, Dividend Yield and Price-earnings ratio concepts.
3. The third, and last, level of efficiency is the *strong form*. In this form it is held that all information is impounded into the share price immediately, and without bias. As a result, it is impossible for any investor to outperform the market, even if ‘inside’ non-publicly available information is held (Correia et al., 2011:4-25).

Having distinguished the three forms of efficiency, Fama (1970) concluded that empirical evidence in support of both the *weak* and the *semi-strong forms* of the EMH is extensive, and that contradictory evidence is sparse (Howard, 2015:4).

**Behavioural Finance – An insight into the market theory**

Contrastingly, behavioural finance is a study of investor market behaviour that derives from psychological principles of decision making, to explain why people buy or sell shares (Gupta, Preetibedi and Mlakra, 2014:57). It encompasses two primary principles, namely ‘cognitive psychology’ and ‘limits to arbitrage’ whereby cognitive psychology refers to patterns regarding the behaviour of investors and limits to arbitrage refers to predicting in what circumstances arbitrage forces will be effective (Ritter, 2003:429-430).

Fundamentally, behavioural finance focuses upon how investors interpret and act on information to make informed investment decisions. Investors do not always behave in a rational, predictable and an unbiased manner. Behavioural finance places an emphasis upon investor behaviour leading to various market anomalies (Gupta, Preetibedi and Mlakra, 2014:58).

In recent times, behavioural finance has emerged as a model which, not only enhanced stagnating finance theories, such as EMH, but also refuted them (Gupta, Preetibedi and Mlakra, 2014:58).
Efficient Market Hypothesis vs. Behavioural Finance

According to the Efficient Market Hypothesis, investing markets are informationally efficient. All individuals can have access to available information, and as a result, investment news cannot be exploited (Gupta, Preetibedi and Mlakra, 2014:58). Stated simply, the current prices of securities are close to their fundamental values because of either the rational investors or the arbitragers’ buy and sell action of underpriced or overpriced shares (Yalçın, 2016:23).

Contrastingly, observed market anomalies have a challenge for EMH argument. They claim that irrational investment activities and the arbitrage opportunities’ being limited in markets cause some market anomalies that are inconsistent with efficient market hypothesis (Yalçın, 2016:23).

As a result of the differences between the two market theories noted above, the primary prevailing arguments of EMH and behavioural finance are discussed below:

The primary argument in support of behavioural finance is the proven, consistent, existence of market anomalies (DeBondt and Thaler, 1985; Black, 1986; De Long et al., 1990; Shleifer and Vishny, 1997; Thaler, 1999). Where an anomaly can be defined as: “a deviation from the presently accepted paradigms that is too widespread to be ignored, too systematic to be dismissed as random error, and too fundamental to be accommodated by relaxing the normative system” (Tversky and Kahneman, 1986:252).

Accordingly, behavioural finance proponents argue that the anomalies as observed are as a direct result of cognitive limitations (Kahneman and Tversky, 1979). These
cognitive limitations cause erroneous (irrational) investment decisions (Yalçın, 2016:35)

Contrastingly, in support of EMH, Malkiel (2005) acknowledges the arguments put forward by those opposed to the EMH theory by stating that “periods of large-scale irrationality, such as the technology-internet “bubble” of the late 1990s extending into early 2000, have convinced many analysts that the efficient market hypothesis should be rejected and, in addition, financial econometricians have suggested that stock prices are, to a significant extent, predictable on the basis either of past returns or of certain valuation metrics such as dividend yields and price-earnings ratios (Malkiel, 2005:2).

However, in spite of the arguments put forward by those opposed to the EMH theory, Malkiel (2003; 2005) remains steadfast in his support of this theory based on the following key observation: “Surely, if market prices often failed to reflect rational estimates of the prospects of companies, and if markets consistently overreacted (or under-reacted) to underlying conditions, then professional investors, who are richly incentivized to outperform passive investors, should be able to produce excess returns.

“The strongest evidence suggesting that markets are generally quite efficient is that professional investors do not beat the market” (Malkiel, 2005:2). This statement was supported by the finding that over 3, 5, 10 and 20 years 72%, 63%, 86% and 90% of equity funds were outperformed by the index, thus indicating that, on a consistent basis, the actively managed equity funds are outperformed by the S&P 500 (Malkiel, 2005:3).
Lastly, according to Fama and French (1998), value, as measured by low price to book, and small companies which have been found to outperform the Capital Asset Pricing Model (CAPM) is as a result of additional risk factors.

Conclusion

In summary, there are many occurrences of observable market anomalies. However, the fundamental question as posed by Yalçin (2016) is whether these anomalies occur because of inefficiency of the market or some other problems and by chance (Yalçin, 2016:34).

In addressing the question above, two contrasting views being presented:

1. The advocates for EMH maintain that share price movements approximate that of a random walk and that if new information develops randomly, then so will market prices, making the share market unpredictable apart from its long-run uptrend (Malkiel, 2005:1).

2. Contrastingly, behavioural finance treats investors as individuals and highlights that emotions, biases, and illusions cannot be rationalised; in addition, it emphasizes that information is inefficient resulting in anomalies occurring (Gupta, Preetibedi and Mlakra, 2014:60).

As evidenced, there is an ongoing debate about the possible reasons of observed market anomalies and whether they are the powerful sign for inefficiency of the market or not (Yalçin, 2016:35).

However, with the above being said, as existence of market anomalies continues to increase, the more difficult it becomes to maintain the belief of an efficient market and refute the claims of investor’s irrationality.
Value Investing Strategies

*Introduction*

The proponents of EMH believe that it is impossible to beat the market on a consistent basis over the long term. However, based on the anomalies and biases exhibited by investors, as addressed in the *Behavioural Finance* body of research, an increasing number of studies can be found surrounding ‘value investing’\(^4\) and how these investment strategies result in higher returns over an extended period of time without additional risk undertaken by the investor.

*Existence of Value Investing*

Since the seminal paper of Basu (1977), which documented that New York Stock Exchange (‘NYSE’) low price-earnings (P/E) ratio shares significantly outperformed high P/E shares on a risk adjusted basis, there has been substantial confirmation of the existence of a ‘value premium’ in global markets (Bird and Casavecchia, 2007; Larkin, 2009; Pätäri and Leivo, 2009; Sareewiwatthana, 2011; Fama and French, 2012). A value premium is the return achieved by buying (being long in an absolute sense or overweight relative to a benchmark) cheap assets and selling (shorting or underweighting) expensive ones (Asness et al., 2015:35).

*Value Investing in the South African Market*

In relation to the South African market, Rousseau and van Rensburg (2004) noted that similar results (‘to developed markets’) have been observed in the South African financial environment. Accordingly, the existence of a value premium is present on the JSE. This was confirmed to still be the case in more recent studies in which it

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\(^4\) Please refer to **Definitions of Key Terms, Concepts and Variables** in the **Glossary** for more information on the Value Investing concept.
was found that the top performing value investing portfolios, including earnings yield, dividend yield and market-to-book ratio all outperformed the market (Muller and Ward, 2013; Howard, 2015).

Further to the above, Hoffman (2012) found that the anomalous behaviour of shares on the JSE is, in many respects, similar to the behaviour observed by Fama and French (1992) on the NYSE, and that anomalous return behaviour is still present after compensating for risk. This indicates that the above-average returns generated by the value investing strategies were not as a result of taking above-average risks, accordingly leading to a deviation of the EMH principle as set out above.

Conclusion

The above research indicates that there is existence of ‘value’ in the South African market and that superior returns can be generated relative to the benchmark portfolio when using a singular value investing metric. Also, there is evidence that the behaviour of shares on the JSE carry the same anomalies as the NYSE which begs the question of whether a multi-factor model, namely the “Magic Formula”, which was found to hold a value premium on the NYSE would exhibit similar results in the South African market.

The “Magic Formula” investment strategy

Introduction

As indicated above, there are multiple value investing strategies which have been found to outperform the market. One such strategy is the “Magic Formula” investment strategy (Greenblatt, 2006; Greenblatt, 2010; Blij, 2011; Sareewiwathana, 2011; Wu, 2013; Howard, 2015).
This subsection reviews the basis for the “Magic Formula” investment strategy as well as the historical performance for both the global and South African markets.

Explanation of the “Magic Formula” investment strategy

The “Magic Formula” was first introduced by Greenblatt (2006) in his book titled, ‘The little book that beats the market’. In this publication, Greenblatt (2006) used two valuation metrics to construct his portfolio: Earnings Yield (EY) and Return on Capital (ROC) with the objective to combine an indicator of value with one of quality. ROC serves as the quality metric, while EY serves as the value metric. The formula is explicitly intended to ensure that investors are “buying good companies ... only at bargain prices” (Novy-Marx, 2014:6).

Greenblatt (2006) established that by ranking companies according to their combined ROC and EY values, excess returns could be earned relative to the market. Further, Greenblatt (2006) showed that the “Magic Formula” earned 30.8% in comparison to the market average of 12.3% for the period 1988 to 2004 (Howard, 2015:19).

The results shown by Greenblatt (2006; 2010) were subsequently back-tested in the US market. Regression results confirmed that the “Magic Formula” strategy is able to produce alpha\(^5\), accordingly it was concluded that the combination of earnings yield and return on capital might offer significant risk-adjusted abnormal returns (Blij, 2011:43).

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\(^5\) Please refer to Definitions of Key Terms, Concepts and Variables in the Glossary for more information on the alpha concept.
Empirical Testing of the “Magic Formula” in the South African market

With the “Magic Formula” investment strategy having outperformed the benchmark portfolio on the NYSE, as indicated above, along with previous studies showing that the behaviour of shares on the JSE carry the same anomalies as the NYSE, it leads to the question:

*Does the outperformance, as displayed by the “Magic Formula” investment strategy, hold true for the South African market?*

This question was addressed by Howard (2015) in which it was found that the “Magic Formula” investment strategy yielded an excess return of 1% per year in comparison to the JSE ALSI. While this excess return was noted, Howard (2015:65) concluded that based on overall geometric mean returns; there was not enough evidence to conclude that these returns were statistically significantly higher than the returns offered by the market benchmark. Furthermore, it was found that the “Magic Formula” investment strategy displayed a higher standard deviation in comparison with the JSE ALSI. This could imply, in support of the EMH theory, that the sole reason for the consistent above-average returns being earned is due to taking above-average risks (Howard, 2015).

The limitation of the aforementioned results was that the analysis on the “Magic Formula” was done on the basis of a fixed portfolio holding period of one year and a fixed portfolio size of 30 shares (Howard, 2015). As such, the returns shown by the “Magic Formula” could potentially be increased by determining the optimal holding period and number of shares within the portfolio.
Conclusion

An adjustment to the “Magic Formula”, in accordance with traditional value investing principles, will enable us to conclude whether the findings as set out by Rousseau and van Rensburg (2004) and Hoffman (2012) above, hold true for an alternative form of investment strategy in the South Africa market and whether it is possible to beat the Market in South Africa using the principles set out by Greenblatt (2006).

Value Investing Portfolio Composition

Introduction

As a result of identifying that the returns could potentially be increased through an adjustment to the portfolio composition, the next subsection addresses each of these two variables, i.e. the portfolio holding period and portfolio size, in more detail.

Portfolio holding period implication

The holding period refers to the duration for which a certain position or share is held before being sold. Determining when, and how often, to alter a certain position in the market can have a large influence on the overall return of a portfolio. Levy (1972) emphasises this point by stating that in conducting empirical research or in evaluating the performance of the management of a portfolio, more attention should be devoted to the selection of the investment horizon, since the magnitude as well as the direction of the systematic bias is a function of this factor.

Traditional portfolio management theory suggests that the longer the assumed investment horizon, the higher the performance index of both aggressive and defensive shares (Levy, 1984:61). This is owing to a magnitude of factors, two of which are stated below:
1. Over a longer investment horizon the investor can choose to invest more aggressively in equities (Barberis, 2000:261).

2. A longer holding period mitigates the burden of illiquidity and increases the net expected return from illiquid assets (Amihud and Mendelson, 1986:46).

3. Share returns are also often sought to be less volatile over longer investment horizons (Pástor and Stambaugh, 2008).

Importantly, the liquidity factor impact on the portfolio investment horizon was found to still exist in modern day research where shares with higher (lower) liquidity risk are held by investors with shorter (longer) investment horizons (Vovchak, 2014:19).

The traditional portfolio management theory, establishing that a longer investment horizon should yield higher returns, was found to be true by Ahmed and Nanda (2000) who set out to determine whether value investment shares and growth shares should be classified as mutually exclusive. Ahmed and Nanda (2000) found that the extent of an investment strategy’s performance increases over longer investment horizons. This finding has subsequently been substantiated by Bird and Whitaker (2003), Rousseau and van Rensburg (2004) as well as Bird and Casavecchia (2007) who all suggested that longer holding periods will increase the returns of value portfolios.

For the South African market in particular, the research conducted by Rousseau and van Rensburg (2004) indicated that, on an annualised basis, the returns to the value portfolios become noticeably higher at time horizons extended beyond 12 months. This indicates that, in a South African market, a value investment strategy provides a more reliable source of outperformance as the investment horizon increases. Thus,
the results support the underlying basis for value investing, namely value investing is best approached as a long term strategy.

Portfolio size Implication

The Portfolio size refers to the number of shares to include within a portfolio. The implication is that portfolio size can have large influence on the overall return of a portfolio. This view is shared by Elton and Gruber (1977) who established that when an investor decides on the size of the portfolio that they will hold, they are making a trade-off between the decreased risk due to more effective diversification versus the increase in transaction costs from adding more securities to their portfolio.

The topic of diversification and how many shares constitute a diversified portfolio has seen conflicting differences in opinion as noted by Statman (1987). Evans and Archer (1968:767) found no economic justification of increasing portfolio sizes beyond 10 or so securities. Contrastingly, Statman (1987) concluded that a well-diversified portfolio of randomly chosen shares must include no-less than 30 shares.

The key concept for application to a private investor's portfolio rests in the trade-off between a decrease in diversification risk and the increase in transaction costs (Elton and Gruber, 1977). The point where the marginal increase in transaction costs is greater than the marginal saving as a result of a decrease in diversification should be the point where no economic justification is met. At this point no further shares should be included in the portfolio. The complication with this, giving rise to the differences in opinion by researchers, is that the marginal saving as a result of a decrease in diversification is difficult to quantify.
Evan and Archer (1968) showed, in the US market, that the marginal decrease in standard deviation of portfolio return only decreases by 3.298% between 10 shares and 30 shares. This number is reduced 1.033% when looking at the difference between portfolios of 20 and 30 shares. As a result, it would indicate that economic justification for the number of securities to include in the portfolio would rest somewhere between these two points (10 and 30 number of shares).

In relation to the South African market in particular, the research conducted by Bradfield and Munro (2017) indicated that, equally weighted portfolios require only between 15 and 29 stocks in order to achieve levels of risk reduction between 90 and 95%. This implies that, in a South African market, an investment strategies risk adjusted-return can be increased should the portfolio size be greater than 15 shares.

Application to the “Magic Formula”

As highlighted in the subsections above, there appears to be reasonable evidence that a change in the holding period and portfolio size can have an impact on a portfolio’s performance. Determining whether a change in the holding period and portfolio size could have an impact on the performance of the “Magic Formula” strategies in particular, Olin (2011) applied the “Magic Formula” to the Finnish market.

Olin (2011) established that altering the portfolio holding period as well as the number of shares within the portfolio both impacted on the “Magic Formula” strategies return. It was found that the optimal return for the “Magic Formula” strategy in the Finnish market was an annualised return of 20% comprising of a portfolio size of 5 shares which was held for a period of 6 months (Olin, 2011:50).
However, while the impact of the portfolio size on the “Magic Formula” investment strategy performance indicates that a portfolio size of five shares yields the greatest return for the Finnish Market (Olin, 2011:53), it could be argued that this additional return is due to additional risk incurred. The argument, that the excess return is only as a result of a greater risk exposure, would be based on the results indicating that the lowest volatility is observed when the portfolio composition comprises a portfolio size of 10 shares with an investment horizon of 18 months (Olin, 2011:51).

The implication of these results, for the Finnish market, is that the increase in the number of shares in the investment strategy may not be directly in line with the principles of diversification. This is as a result of, in relation to the 18 month portfolios, the volatility increases from 21.8% to 24.9% as the number of shares in the portfolio increased from 10 to 15 shares (Olin, 2011:51).

**Conclusion**

Value investing seeks to identify and invest in high quality companies which display low leverage, high profitability and low earnings volatility (Novy-Marx, 2013:12). Further, Novy-Marx (2013) suggests that shares that have these characteristics always win over longer holding periods.

Accordingly, as a result of:

1. The returns of value portfolios become noticeably higher at time horizons extending beyond 12 months (Rousseau and van Rensburg, 2004),
2. The economic justification for the number of securities to include in the portfolio would rest somewhere between 15 and 29 shares (Bradfield and Munro, 2017), and
3. The optimal “Magic Formula” portfolio in the Finnish market comprising a portfolio size of 5 shares and a portfolio holding period of 6 months, all being in contrast to the portfolio composition of ‘one year – 30 shares’ employed by Howard (2015) in the analysis of the “Magic Formula” in the South African market, it is likely that a change in the portfolio holding period and the number of shares could impact on the overall performance of the “Magic Formula” investment strategy.

Risk-Adjusted Returns

Introduction

Lastly, in accordance with the ‘risk premium’ concept, determining the overall performance for an investor will depend on the level of risk the investor is willing to accept (Bodie, Kane and Marcus, 2009). The subsection to follow investigates the various investor risk tolerance levels and the implication thereof relating to performance.

Investor Risk Tolerance

The risk tolerance of an investor can be defined as: An investor’s general predisposition toward financial risk (Hoffmann, Post and Pennings, 2013:62). Accordingly, one can expect individuals with low risk tolerance to act differently with regard to risk than individuals with a high risk tolerance (Grabe, 1997:13).

An investor with a high level of risk tolerance would be willing to accept a higher exposure to risk in the sense of taking sole responsibility, acting with less information, and requiring less control in comparison to an investor with a

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Please refer to Definitions of Key Terms, Concepts and Variables in the Glossary for more information on the risk premium concept.
low level of risk tolerance (MacCrimmon, Wehrung and Stanbury, 1988:34; Grabe, 1997:13).

In summary, high risk-tolerance individuals accept volatile events, while low risk-tolerance individuals require certainty (Grabe, 1997:13).

Conclusion

As a result of the varying risk tolerance levels, certain investors, in reducing the level of risk, would not purchase certain securities. This may result in a lower return being achieved, thus impacting on the overall performance of a particular investment strategy.

Accordingly, when addressing the overall performance of the “Magic Formula” investment strategy, in order to provide insightful and applicable recommendations to investors, the investor risk tolerance levels should form part of the discussion.

Conclusion to Literature Review

As supported by the literature review conducted above, there is uncertainty regarding whether or not the ‘Efficient Market Hypothesis’ holds true. There is increasing evidence surrounding market anomalies characterised through the identification of investor irrationality as documented under behavioural finance.

One such anomaly, which presents itself the form of long term consistent outperformance of the benchmark portfolio, is the “Magic Formula” value investing strategy.

The “Magic Formula” investment strategy was found to outperform the US (Greenblatt, 2006; Greenblatt, 2010; Blij, 2011), Finnish (Olin, 2011) and Thailand
Markets (Sareewiwatthana, 2011). In application to the South African Market, the “Magic Formula” was found to outperform the benchmark portfolio, albeit on a non-statistically significant basis (Howard, 2015).

It was noted that in the application to the South African market, however, Howard (2015) applied a fixed portfolio size and holding period in the “Magic Formula” analysis.

In review of literature surrounding the portfolio composition it was evident that an adjustment to these factors, the portfolio size and portfolio holding period, could increase the overall performance of the “Magic Formula” investment strategy. This was clearly demonstrated by the differences in portfolio composition between the top performing “Magic Formula” portfolio in the Finnish market of ‘six months – 5 shares’ and the portfolio constructed by Howard (2015) for the South African market of ‘one year – 30 shares’.
CHAPTER 3 - METHODOLOGY

Research Questions

The review of literature indicates that the “Magic Formula” may, in contrast to the findings of Howard (2015), outperform the South African market, as represented by the benchmark portfolio, should the “Magic Formula” portfolio holding period and portfolio size be altered.

Altering the “Magic Formula” portfolio holding period and portfolio size will enable us to determine which combinations of portfolio holding period and portfolio size would result in the ‘optimal’ “Magic Formula” portfolio from a return, risk and risk-adjusted return perspective. As a result, by constructing the ‘optimal’ “Magic Formula” portfolios for the South African market it would enable us to conclude, definitively, on each of the research questions set out below.

The research questions for this study are as follows:

1. Does the “Magic Formula” investment strategy yield superior returns relative to the benchmark portfolio in the South African market?
2. Does the “Magic Formula” investment strategy incur a higher level of risk in comparison to the benchmark portfolio in the South African market?
3. Does the “Magic Formula” investment strategy outperform the benchmark portfolio on a risk-adjusted basis in the South African market?

The null hypothesis for each of the 3 research questions outlined above is that the “Magic Formula” does not yield superior returns, incur a higher level of risk or outperform the benchmark on a risk-adjusted basis respectively. If the null hypothesis’ are rejected this would indicate the existence of a market anomaly and
highlight the opportunity to outperform the South African market on a consistent, long-term, basis.

Scope of the Study

The scope of this study is limited to shares which were listed on the main board of the JSE for the period between 1 October 2005 and 30 September 2015. Further, as outlined in greater detail in this chapter below, to limit the liquidity\(^7\) risk as presented by the Bid-Ask spread\(^8\), only shares which reflect a Market Capitalisation of greater than R100 million were included in the scope of this study.

As a result of these factors, along with the exclusion of the JSE financial sector as part of the “Magic Formula” investment philosophy, the number of companies included in the study ranges from a minimum of 137 companies in 2005 to a maximum of 235 companies being included during the period of analysis.

Details of these results, along with the number of shares included in the analysis at every ‘1 year’ portfolio rebalance date, are provided in Table 1 below:

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\(^7\) Please refer to Definitions of Key Terms, Concepts and Variables in the Glossary for more information on Liquidity.

\(^8\) Please refer to Definitions of Key Terms, Concepts and Variables in the Glossary for more information on the Bid Ask spread.
Table 1 - Number of shares included in the “Magic Formula” JSE analysis

<table>
<thead>
<tr>
<th>Financial Year</th>
<th>Shares included in Magic Formula Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005</td>
<td>137</td>
</tr>
<tr>
<td>2006</td>
<td>140</td>
</tr>
<tr>
<td>2007</td>
<td>155</td>
</tr>
<tr>
<td>2008</td>
<td>171</td>
</tr>
<tr>
<td>2009</td>
<td>176</td>
</tr>
<tr>
<td>2010</td>
<td>192</td>
</tr>
<tr>
<td>2011</td>
<td>208</td>
</tr>
<tr>
<td>2012</td>
<td>235</td>
</tr>
<tr>
<td>2013</td>
<td>225</td>
</tr>
<tr>
<td>2014</td>
<td>229</td>
</tr>
</tbody>
</table>

Comparably, the JSE ALSI, which encompasses 99% of the market by way of market capitalisation, consisted of 164 shares as at 1 October 2014. It is noted that the number of constituents of this benchmark has remained fairly stable since its inception date of 28 June 2002 in which 160 share were included.

As a result, the population data used in this study, as shown in Table 1 above, represent more shares than the benchmark for 7 of the 10 years included in the scope.

Research Design

The research design sub-section describes the overall strategy applied in addressing the three research questions. As a result, this section includes a detailed account of what quantitative information was obtained, details of the exclusions made to this information, the method for calculating the two “Magic Formula” ratios, how the “Magic Formula” portfolio is constructed as well as the way in which the overall “Magic Formula” portfolio return, portfolio risk and risk-adjusted return was calculated.
Importantly, as highlighted previously, altering the “Magic Formula” portfolio holding period and portfolio size will enable us to determine which combinations of portfolio holding period and portfolio size would result in the ‘improved’ “Magic Formula” portfolio from a return, risk and risk-adjusted return perspective. Accordingly, multiple “Magic Formula” portfolios which have differing holding periods and portfolio sizes were created.

The parameters for these portfolios are set as follows:

- Portfolio holding period: 6 months, 1 year, 2 years and 5 years
- Portfolio size: 5 shares, 10 shares, 15 shares and 20 shares.

The various combinations of portfolio holding period and portfolio size resulted in 16 “Magic Formula” portfolios being created, all of which were constituted in accordance with the design as outlined in the section below.

**Research design basis**

The research design, for the purposes of this study, was based on the methodology identified by Greenblatt (2006) and subsequently re-performed by Olin (2011) on the Finnish market.

**Sourcing information to perform the quantitative analysis**

A listing of all companies registered on the JSE was obtained for the period from 1 October 2005 to 30 September 2015. This listing included all relevant information required for the purposes of constructing the “Magic Formula” portfolio and determining its performance.

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9 It is noted that the primary basis for the portfolio holding period extending beyond the one year investment horizon prescribed by Greenblatt (2006), is based on one of the primary value investing principles being that value stocks which meet the criteria of low leverage, high profitability and low earnings have prevailed over longer holding periods (Novy-Marx, 2013:12).
Source data excluded from the “Magic Formula” analysis

As outlined by Greenblatt (2006), all shares which operate in the financial services sector were excluded from the “Magic Formula” analysis as these companies lack the underlying business fundamentals required to calculate Return on Capital (ROC) or Earnings Yield (EY) (Blij, 2011). Furthermore, the inclusion of these companies in the analysis could skew the results as a high leverage for an industrial firm could indicate financial distress whereas the same would not apply to financial services companies (Fama and French, 1992).

To ensure that liquidity constraints were negated, as far as practicably possible, all shares which have a Market Capitalisation of less than R100 million and all shares which are listed on the JSE Alternative Exchange (ALTx) were excluded from the analysis. The justification of these exclusions, as well as the minimum Market Capitalisation determination, is discussed in greater detail in the Liquidity Constraint section of the Strategic Considerations below.

Constructing the “Magic Formula” portfolio

Calculating the two “Magic Formula” ratios

Equations 1 and 2 were performed on both a 6 month and annual bases (1, 2 and 5 years) as part of the “Magic Formula” share selection process.

\[
\text{Return on Capital} = \frac{\text{Earnings before Interest and Tax}}{\text{Net Working Capital} + \text{Net Fixed Tangible Assets}}
\]

[Equation 1]

\[
\text{Earnings Yield} = \frac{\text{Earnings before Interest and Tax}}{\text{Market Capitalisation} + \text{Net Debt}}
\]

[Equation 2]
Equation 1 is computed to determine strong performing companies which exhibit long term growth. Equation 2 is computed to predict returns linked to the current share price (i.e. to identify discounted shares relative to their potential). Shares were then ranked from the best performing to worst performing for each of the computations above.

It is noted however that when performing the computation of ROC and EY, which form the foundation of the “Magic formula” share selection, should both the numerator and denominator contain negative values this would result in a positive indicator which may result in the incorrect share selection (Olin, 2011). In order to overcome this problem a function was included in the analysis to identify those instances where each of the variables contains negative figures. Accordingly, these instances were excluded from the “Magic Formula” share selection for that particular period.

“Magic Formula” share selection

In accordance with the “Magic Formula” investment strategy, the share ranking then became the starting point for the share selection process with the lowest combined rankings being used to select the shares to be included into the “Magic Formula” portfolios. The lowest combined rankings are selected as these are the companies which represent the ‘best’ combination of ROC and EY ratios relative to the alternative companies included in the data analysis.

The “Magic formula” share selection principle can be explained further using a hypothetical explanatory example.
Table 2 – Combined Ranking explanatory example

<table>
<thead>
<tr>
<th></th>
<th>ROC</th>
<th>Rank</th>
<th>EY</th>
<th>Rank</th>
<th>Combined Rank</th>
<th>Selected</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>10%</td>
<td>3</td>
<td>10%</td>
<td>3</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>BBB</td>
<td>20%</td>
<td>1</td>
<td>5%</td>
<td>4</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>CCC</td>
<td>15%</td>
<td>2</td>
<td>15%</td>
<td>2</td>
<td>3</td>
<td>Yes</td>
</tr>
<tr>
<td>DDD</td>
<td>5%</td>
<td>4</td>
<td>20%</td>
<td>1</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

The explanatory example, as shown in Table 2, is made up of 4 various shares (AAA – DDD) which all report differing ROC and EY ratios which are individually ranked amongst each other. Should 1 share be selected from this hypothetic population using the “Magic Formula” share selection principle it would result in share CCC being selected because, while it doesn’t report the highest ROC or EY ratios, it reports the lowest combined ranking amongst its peers.

“Magic Formula” portfolio construction

Using the lowest combined rankings, as set out above, equally-weighted portfolios of 5 shares, 10 shares, 15 shares and 20 shares were then selected for each of the six months, one year, two years and five years portfolio holding periods. These combinations resulted in the following synthetic “Magic Formula” portfolios:

Table 3 – “Magic Formula” constructed portfolios

<table>
<thead>
<tr>
<th></th>
<th>5 SHARES</th>
<th>10 SHARES</th>
<th>15 SHARES</th>
<th>20 SHARES</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIX MONTH</td>
<td>Portfolio 1</td>
<td>Portfolio 5</td>
<td>Portfolio 9</td>
<td>Portfolio 13</td>
</tr>
<tr>
<td>ONE YEAR</td>
<td>Portfolio 2</td>
<td>Portfolio 6</td>
<td>Portfolio 10</td>
<td>Portfolio 14</td>
</tr>
<tr>
<td>TWO YEAR</td>
<td>Portfolio 3</td>
<td>Portfolio 7</td>
<td>Portfolio 11</td>
<td>Portfolio 15</td>
</tr>
<tr>
<td>FIVE YEAR</td>
<td>Portfolio 4</td>
<td>Portfolio 8</td>
<td>Portfolio 12</td>
<td>Portfolio 16</td>
</tr>
</tbody>
</table>
An equally weighted portfolio method has been selected, in accordance with the core principles of the “Magic Formula” investment strategy (Blij, 2011), in order to determine whether the “Magic formula” yields superior returns relative to the market. As a result, to ensure calculation accuracy of the equally weighted portfolio and for ease of tracking the portfolio performance, a starting investment value of R1 000 was used and shares were purchased in their fractions.

- This means that should we be performing a computation of a 5 share portfolio, R 200 (\( \frac{R\ 1000}{5} \)) will be used to purchase each of the top ranked shares.
- Should a share be trading at a price of R500 at the time of selection, then 0.4 (\( \frac{R\ 200}{R\ 500} \)) of that share will be added to the portfolio.

It is important to note that while the above process may result in returns that may be unrealisable in practice due to a fully equally weighted portfolio having practical constraints, it will result in a more meaningful analysis between investment alternatives. This view is shared by Olin (2011) whereby it was identified that, in order to ensure the calculation is as real as possible, the weight of each share included in the portfolio must be the same. Accordingly, by using an initial investment of R 1000 as a proxy for the starting portfolio value, along with all investments being equally weighted, it could result in a fraction of a share being purchased which is not possible in practice.

**Calculation of Portfolio Return, Portfolio Risk and Risk Adjusted Return**

*“Magic Formula” Portfolio Return*

The “Magic Formula” portfolio return was calculated by determining the net increase (or decrease) in the share price between the share selection date and the
rebalancing date for each of the shares selected in accordance with the “Magic Formula” share selection methodology.

Further, for each of the shares making up the “Magic Formula” portfolio, the dividends, which theoretically would have been received when holding the share, were added to the return generated from an increase (or decrease) in share price as described above. The sum of the net increase (or decrease) in share price and the dividends received presents the real return which would have been received through following the “Magic Formula” investment strategy.

The aforementioned portfolio return, as discussed above, is presented in the equation below:

\[
R_p = \left( \frac{\frac{1}{N} \sum_{i=1}^{N} (SP_{X+1} - SP_X) + d}{p} \right)
\]

[Equation 3]

Where:
\( R_p = \) Portfolio Return
\( N = \) Number of shares included in the Portfolio
\( SP = \) Share Price
\( d = \) Dividend received during the period
\( p = \) Value of Portfolio

Returns on the various synthetic portfolios constructed were then compared to the returns for the benchmark portfolio in order to determine whether the “Magic Formula” investing strategy yields superior returns in the South African market. This led us to the conclusion of first research question, namely whether the “Magic Formula” investment strategy yields superior returns relative to the benchmark portfolio in the South African market.
As a further subset of addressing the first research question, the following additional substantive research questions were consequentially addressed:

i. Are the returns, which are generated by the “Magic Formula” investment strategy, generated randomly?

ii. Are the returns generated in this study, when calculated using the same investment parameters, consistent with the results achieved by the comparable “Magic Formula” study conducted by Howard (2015)?

The research design of these two additional substantive research questions, 1(i) and 1(ii), is provided below.

“Magic Formula” investment strategy impact on returns

In order to address research question 1(i), as set out above, the following methodology has been carried out:

- ‘Inverse “Magic Formula” portfolios’\(^{10}\) were constructed in accordance with the ‘Construction of the “Magic Formula” portfolio’ section set out above.

- The results of synthetically created ‘inverse “Magic Formula” portfolios were then compared to the results achieved from the traditional “Magic Formula” portfolios.

- Lastly, a paired t-Test\(^{11}\) was performed over representative portfolios in order to determine whether the differences noted, if any, between the traditional

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\(^{10}\) The ‘inverse’ “Magic Formula” portfolio is whereby shares are selected based on the same underlying characteristics, that being EY and ROC, as the traditional “Magic Formula” portfolio with the sole exception being that the highest combined rankings (i.e. worst performing ratios) are selected as opposed to the lowest combined rankings of ROC and EY.

\(^{11}\) Please refer to Definitions of Key Terms, Concepts and Variables in the Glossary for more information on t-Test.
“Magic Formula” portfolio and the inverse “Magic Formula” portfolio, were statistically significant.\textsuperscript{12}

The principle argument for the methodology applied above was based on the following reasoning:

- Should the returns generated from the “Magic Formula” investment strategy be generated randomly, then the results achieved in the primary “Magic Formula” portfolio analysis should be able to be mimicked by constructing alternative portfolios using any combinations of ROC and EY.

- Accordingly, should the return generated from the inverse portfolios differ from the return achieved from the primary portfolio analysis then there is a causal relationship between the “Magic Formula” investment strategy and the returns generated.

The methodology, as outlined above, enabled us to conclude on research question 1(i), namely whether the returns generated by the “Magic Formula” were generated randomly.

Comparison to alternative study

In order to address research question 1(ii), as set out above, the following methodology has been followed:

- An additional synthetic “Magic Formula” portfolio, over and above the 16 portfolios initially created as shown in Table 1, was created. This additional “Magic Formula” portfolio (‘Portfolio 17’) was constructed based on the same

\textsuperscript{12} Please refer to Definitions of Key Terms, Concepts and Variables in the Glossary for more information on Statistical Significance.
investment criterion as the comparable study, that being a holding period of 1 year with a portfolio size of 30 shares.

- The return generated by Portfolio 17 was then compared to the return which was reported in the comparable study for the periods over which the scope overlapped, namely 2005 to 2013.

- Lastly, a paired t-Test was performed in order to determine whether the differences noted, if any, between the “Magic Formula” portfolio constructed in this study (Portfolio 17) and the results shown in the comparable study, were statistically significant.

The comparison of the returns generated by Portfolio 17 to the comparable study enabled us to reach a conclusion on research question 1(ii), that being a determination of whether the results shown under this study, when using the same investment parameters, are consistent with the comparable study conducted by Howard (2015).

“Magic Formula” Portfolio Risk

In order to address the second research question, the risk was determined for each of the 16 synthetic portfolios, as shown in Table 1. This was done by examining the volatility of the returns as measured by the standard deviation at each of the portfolios rebalancing dates.
The aforementioned portfolio risk is presented in the equation below:

\[
\sigma_p = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^2}
\]

[Equation 4]

Where:
- \( \sigma_p \) = Portfolio Standard Deviation
- \( N \) = Number of shares included in the Portfolio
- \( x_i \) = Total return for the share for the period \( i \)
- \( \mu \) = Average return of all Shares included in the Portfolio

The risk for each of the synthetic portfolios constructed was compared to the benchmark portfolio, the JSE ALSI, in order to determine whether the “Magic Formula” investing strategy incurs a higher level of risk in comparison to the benchmark.

This led us to the conclusion of second research question, namely whether the “Magic Formula” investment strategy yield incurred a higher risk relative to the benchmark portfolio in the South African market.

“Magic Formula” portfolio risk-adjusted return

In the final phase of the analysis, the Sharpe Ratio was calculated for each of the synthetic portfolios as well as the benchmark portfolio.
The Sharpe ratio (Sharpe, 1994) was calculated as follows:

\[ \text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p} \]  

[Equation 5]

Where:

- \( R_p \) = Portfolio Return
- \( R_f \) = Risk – Free Rate
- \( \sigma_p \) = Portfolio Standard deviation

The Sharpe ratio calculation is used as a measure of risk adjusted return and enables us to conclude whether any of the synthetic “Magic Formula” portfolios outperform the benchmark portfolio, the JSE ALSI, on a risk adjusted basis. This led us to the conclusion of a third and final research question, namely whether the “Magic Formula” investment strategy yielded a higher risk-adjusted return relative to the benchmark portfolio in the South African market.

Strategic Considerations

In order to adequately address the 3 research questions, by performing the “Magic Formula” analysis in accordance with the research design, additional factors needed to be considered. These additional factors are highlighted in this section of the study and include:

1. A consideration of the liquidity constraint, whereby the determination and justification for companies to be included in the study from a liquidity perspective is provided.

2. A consideration of the availability of information, whereby the determination of what publicly available financial information would have
been available at the time of performing the calculation of the EY and ROC ratios.

3. A consideration of data accuracy, whereby the determination, documentation and adjustments of potential data outliers has been addressed.

4. A consideration of the risk free rate, whereby the determination and justification for the most appropriate risk free rate for the use in Equation 5 is provided.

1. Liquidity Constraint

Liquidity is generally viewed as the ability to trade large quantities quickly at low cost with little price impact (Liu, 2006). Accordingly, liquidity is an important consideration in the construction of any portfolio as the ability to realise a certain return, when required, is imperative (Eltringham, 2014). This statement holds true to the application of the “Magic Formula” and accordingly a liquidity element was incorporated into this study, the details of which are provided below:

Minimum Market Capitalisation

Olin (2011), in the analysis of the “Magic Formula” on the Finnish Stock Exchange, accounted for the liquidity risk by excluding all shares listed on the exchange which reported a market capitalisation of less than £10 million from the portfolio determination.

The analysis of various investing strategies on the JSE however has shown differing approaches with regards to the population size.

Howard (2015) considered only the top 160 companies on the JSE. The same holds true for Muller and Ward (2013) in their analysis of style-based effects on the JSE.
Van Rensburg and Robertson (2003) differed when performing their analysis of cross section returns, as they only included the highest 100 shares by market capitalisation. Contrastingly to the aforementioned studies, Hoffman (2012) included the entire JSE in his analysis of share return anomalies.

As a result of these differences, there appeared to be no concrete solution to the most appropriate population size to use in the analysis of the “Magic Formula” for the South African market. Accordingly, it was undertaken to construct the synthetic “Magic Formula” portfolios based on a population size which was greater than a certain threshold, being a market capitalisation of R100 million, as opposed to a certain fixed number of shares. This treatment coincides with that of Olin (2011) in the application of the “Magic Formula” to the Finnish Market.

**JSE ALTx**

An additional consideration related to liquidity was whether or not to include the JSE Alternative exchange (‘AltX’) into the “Magic Formula” analysis.

In order to address this question, a comparison, with particular emphasis on the liquidity, between the JSE and JSE AltX was performed. Through this comparison it was established that merely due to the differences in listing requirements (between the JSE and the JSE AltX) it would imply that the risk of purchasing shares in the AltX companies are slightly higher than on the JSE, simply because they (the shares) may be harder to liquidate (Van Heerden, 2015).

Further to the above, the listing requirements of the JSE AltX also do not require a profit history to be provided (Business Blue-Book of South African, 2008:6). The implication of this is that by including companies which do not display a ‘proven track
record’ it could distort the underlying “Magic Formula” analysis as these companies would ordinarily be weaned out by traditional market listing requirements.

As a result of the aforementioned factors, the JSE AltX was excluded from the scope for this study.

2. Data Availability and Selection

In order to be sure of the “Magic Formula” investment strategy one must be certain that the information was available at the time that the investment decision was made (Olin, 2011).

In light of the above statement, in order to ensure that the information for the calculation of the ROC and EY ratios was available at the time that the investment decision was made, a fixed rebalancing date was set out in this analysis.

- Six month “Magic Formula” portfolios were rebalanced semi-annually on 1 April and 1 October each year.
- One year, two year and five year “Magic Formula” portfolios were rebalanced on 1 October as required.

For each of the companies included in the dataset, using their respective financial year end dates, it was determined whether interim financial results or year-end financial results would be publicly available as at the applicable rebalancing dates (1 April or 1 October).

The determination of which company financial information would be available (i.e. interim-year or final-year) was made based on the assumption that the release of financial results trails the financial year-end and financial interim-end dates by three months (KPMG, 2013). This assumption is supported by the JSE listing
requirements, whereby the provisional report or interim report must be made available to the public at a minimum of three months after the respective interim-year or final-year close (KPMG, 2013). Hence, the required information to calculate ROC and EY would have been available for a particular share a minimum of three months after the reporting date.

The details of the financial year-end dates and the applicable information assumed to be available for the purposes of this study, in which to compute the ROC and EY ratios, is provided in Table 4:

**Table 4 - Publicly available information to compute ROC and EY ratios**

<table>
<thead>
<tr>
<th>Year-end Date</th>
<th>Information available at 1 April</th>
<th>Information available at 1 October</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>Interim</td>
<td>Final</td>
</tr>
<tr>
<td>February</td>
<td>Interim</td>
<td>Final</td>
</tr>
<tr>
<td>March</td>
<td>Interim</td>
<td>Final</td>
</tr>
<tr>
<td>April</td>
<td>Interim</td>
<td>Final</td>
</tr>
<tr>
<td>May</td>
<td>Interim</td>
<td>Final</td>
</tr>
<tr>
<td>June</td>
<td>Interim</td>
<td>Final</td>
</tr>
<tr>
<td>July</td>
<td>Final</td>
<td>Interim</td>
</tr>
<tr>
<td>August</td>
<td>Final</td>
<td>Interim</td>
</tr>
<tr>
<td>September</td>
<td>Final</td>
<td>Interim</td>
</tr>
<tr>
<td>October</td>
<td>Final</td>
<td>Interim</td>
</tr>
<tr>
<td>November</td>
<td>Final</td>
<td>Interim</td>
</tr>
<tr>
<td>December</td>
<td>Final</td>
<td>Interim</td>
</tr>
</tbody>
</table>

**Data Availability – Illustrative Examples**

In order to explain Table 4 above, two illustrative examples have been included below:

1. For the purposes of this illustrative example, assume the theoretical company has a 30 June financial year end:
- At the rebalancing date of 1 October, three months would have passed since financial year end (July, August and September) and, as a result of the JSE listing requirements, the latest final year financial results would be publicly available.
- Accordingly, the calculation of the EY and ROC ratios would be based on this information as stated in the table above.

2. Assuming the theoretical company has a 31 July financial year end:
   - At the rebalancing date of 1 October, only two months would have passed since financial year end (August and September) and as a result the final year financial results would not yet be available for the purposes of calculating the EY and ROC ratios.
   - Accordingly, the latest publicly available financial information, for use as inputs in the EY and ROC calculations, would have been the interim-year end results which would have been published three months after the interim-year end of 31 January (31 January being six months after financial year end).

It is noted that the interim financial information was used in the “Magic Formula” analysis as it would represent the latest available financial information at the time of the rebalancing date. Further, in order to ensure comparability to an annualised figure, all income statement metrics at interim reporting were annualised prior to the calculation of the ROC ratio.
3. Data accuracy – a reasonability check

In order to ensure the validity and accurateness of the results in this study, when calculating the returns generated from the respective shares selected under the synthetic “Magic Formula” portfolios, reasonability checks were performed on all statistical outliers\(^\text{13}\) as determined using the Tukey (1997) methodology as described below.

Statistical Outlier Determination

In determining what constituted a statistical outlier, the John Tukey outlier filter (Tukey, 1977:43-44) was used as the quantitative basis. Importantly, for the computation thereof, the JSE ALSI was used as a proxy as it represented the benchmark portfolio (this is discussed in greater detail in the ‘Selecting the Benchmark Portfolio’ section below).

Accordingly, by following the Tukey outlier filter methodology the annual JSE ALSI returns were used to determine the 1\(^{\text{st}}\) and 3\(^{\text{rd}}\) quartiles as well as to calculate the interquartile range as shown in Table 5 below:

\(^{13}\) Please refer to Definitions of Key Terms, Concepts and Variables in the Glossary for more information on statistical outliers.
Table 5 – Determination of what constitutes a statistical outlier

<table>
<thead>
<tr>
<th>OBSERVATION</th>
<th>JSE ALSI RETURN</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>37%</td>
</tr>
<tr>
<td>9</td>
<td>36%</td>
</tr>
<tr>
<td>8</td>
<td>27%</td>
</tr>
<tr>
<td>7</td>
<td>24%</td>
</tr>
<tr>
<td>6</td>
<td>21%</td>
</tr>
<tr>
<td>5</td>
<td>15%</td>
</tr>
<tr>
<td>4</td>
<td>8%</td>
</tr>
<tr>
<td>3</td>
<td>5%</td>
</tr>
<tr>
<td>2</td>
<td>4%</td>
</tr>
<tr>
<td>1</td>
<td>-18%</td>
</tr>
</tbody>
</table>

3rd Quartile (Q3) 26%
1st Quartile (Q1) 4%

Interquartile Range (IQR) 22%
(Q3 - Q1)

1.5 x IQR 32%

Upper Outlier Determination 58%
(Q3 + 1.5 x IQR)

Lower Outlier Determination -28%
(Q1 - 1.5 x IQR)

As shown in table 5 above, in accordance with the Tukey outlier filter methodology (Tukey, 1977:43-44), upper outliers were determined to be where the return generated was greater 58%. This upper outlier determination was calculated as the sum of the 3rd quartile return and 1.5 times the ‘interquartile range’\(^\text{14}\). Similarly, the lower outliers were determined to be where the return was less than -28% which was calculated as the 1st quartile less 1.5 times the interquartile range.

\(^{14}\) The Interquartile range, as shown in Table 5, refers to the difference between the 3rd and 1st quartiles (Tukey, 1977:43-44).
**Reasonability Check Performed**

The reasonability check performed on all statistical outliers, included, but was not limited to:

- Verifying the inputs used to calculate the share return by agreeing the respective share purchase and sale price information, as obtained from Bloomberg, to a secondary source, namely McGregor BFA.
- Reviewing the SENS announcements of the company for the particular period in question to identify any significant events, for which adjustments must be made, such as share splits.

**Adjustments made as a result of Reasonability Check performed on Identified Outliers**

Of the various outliers investigated as part of the reasonability check in this study, **only** two differences requiring adjustment were encountered.

These two outliers were adjusted for accordingly, details of which are provided below:

1. Combined Motor Holdings was selected in the 2006 rebalance date. The initial analysis, as summarised in **Figure 1** below, indicated a return on this share of -82% thus triggering a statistical outlier.

   Investigation into this outlier identified that, as shown in **Appendix 2**, the reason for the decline was due to a share split. This was adjusted for by multiplying the number of shares purchased, under the terms of the “**Magic Formula**” by the share split ratio of 1:5.
**Figure 1** – Combined Motor Holdings statistical outlier key information

<table>
<thead>
<tr>
<th>Period</th>
<th>2006</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>TICKER</th>
<th>Share Price Bought</th>
<th>Share Price Sold</th>
<th>% Change</th>
<th>Investigate</th>
</tr>
</thead>
<tbody>
<tr>
<td>CMH</td>
<td>94</td>
<td>16.5</td>
<td>-82%</td>
<td></td>
</tr>
</tbody>
</table>

**Market Cap** 1,990,995,200  
**McGregor BFA**

**Adjustment** Number of Shares purchased x5

2. Assore Limited was selected in the 2009 rebalance date. The initial analysis, as summarised in **Figure 2** below, indicated a return on this share of -75% thus triggering a statistical outlier.

Investigation into this outlier identified that, as shown in **Appendix 3**, the reason for the decline was due to a share split. This was adjusted for by multiplying the number of shares purchased, under the terms of the “Magic Formula” by the share split ratio of 1:5.

**Figure 2** – ASR statistical outlier key information

<table>
<thead>
<tr>
<th>Period</th>
<th>2009</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>TICKER</th>
<th>Share Price Bought</th>
<th>Share Price Sold</th>
<th>% Change</th>
<th>Investigate</th>
</tr>
</thead>
<tbody>
<tr>
<td>CMH</td>
<td>629.99</td>
<td>157</td>
<td>-75%</td>
<td></td>
</tr>
</tbody>
</table>

**Market Cap** 17,370,141,390  
**McGregor BFA**

**Adjustment** Number of Shares purchased x5
4. Risk-Free Rate

As established in the Sharpe Ratio formula set out in Equation 5, in order to calculate the Sharpe Ratio, a risk-free rate was required to be determined. For the purposes of this determination, bond yields are frequently used as the proxy for risk free rates (EY, 2014). Accordingly, in determining an appropriate proxy in which to measure the South African risk-free rate, details of the South African government bonds were reviewed.

Based on the review performed, the long term government R186 bond was chosen as the proxy for the risk free rate. The reason for choosing this long term government bond was due to the following reasons:

i. The R186 becomes redeemable between 2025 and 2027 representing a long term investment horizon. As the coupon rate of the R186 government bond reflecting 10.5% does not change between the inception, being 1998, and redemption dates, the yield thereon would represent an accurate reflection of the long term interest free-rate.

ii. The bond covered the scope of the analysis. An alternative to the R186 government bond was the R157 government bond as used by Howard (2015) in the comparable study. However, with the R157 maturing on 15 September 2014, this did not cover the entire investment period analysed in this analysis and accordingly could not be used as a suitable proxy.

Please refer to Definitions of Key Terms, Concepts and Variables in the Glossary for more information on the Risk-Free rate concept.
Selecting the ‘Benchmark Portfolio’

As highlighted by Kruger and Van Rensburg (2008), benchmarks form an integral component of fund management, both for active management who seek an appropriate index against which to evaluate their performance as well as passive management who seek an index to track.

The objective of the benchmark in this study is to serve as a proxy for the market. As a result, the starting point was the JSE ALSI. This was due to the JSE ALSI incorporating the top 99% of eligible listed companies ranked by full market capitalisation (Grayswan, 2013) thus representing the majority of the market.

In assessing the appropriateness of this index as a viable benchmark, the traditional inherent limitations of the ALSI, being the high levels of market concentration in terms of market capitalisation and liquidity as well as a volatile resources sector (Kruger and Van Rensburg, 2008), were considered. It was assumed that these limitations all presented underlying market risks which would have been faced by an investor when investing in the broad based South Africa market. As such, it was concluded that these factors did not result in the JSE ALSI being excluded as a viable proxy for the market.

Over and above the widespread representation of the market which the ALSI provides, a further factor weighted in favour of selecting this index as the benchmark portfolio is that it ensured comparability with the benchmark used by Howard (2015). As a result of this it was concluded that the JSE ALSI was most appropriate benchmark for the purposes of this study as it enabled comparability between the two studies.
However, with the above being noted, it is understood that the JSE ALSI would present specific limitations of the study. These limitations, impacting the comparability between the “Magic Formula” return and the ‘market’ return, include:

1. The JSE ALSI is a market weighted index, whereas the “Magic Formula” analysis constructs an equally weighted index.
2. The JSE ALSI includes financial services firms within the index, contrastingly all financial services companies are excluded from the “Magic Formula”.

Validated Data Sources

Sources of Quantitative Data

In performing the quantitative aspects of the “Magic Formula” analysis, the data was obtained from a variety of sources and exported into excel for further analysis. Details of the sources used in the analysis, as well as the reasoning thereof, are as follows:

The company financial statements data\(^{16}\) was sourced from S&P Capital IQ. S&P Capital IQ was chosen for this component of the data gathering phase as it returned specified data for certain shares, i.e. those shares included in the scope of analysis, for an extended period of time in an easily usable format.

The market capitalisation and share price data, as at the 1 October and 1 April dates, was sourced from a Bloomberg terminal. Bloomberg was chosen for this aspect of the data gathering phase as it provided accurate, historical, market information for the entire JSE.

\(^{16}\) The Company financial statements data refers to the information which appears on the annual financial statements. Of the inputs required for the “Magic Formula” analysis it includes: EBIT, Total Assets, Total Liabilities, Current Assets, Current Liabilities, Cash & Cash Equivalents, Goodwill and DPS.
The benchmark, the JSE ALSI (J203), as well as the share price data for the validation checks was sourced from McGregor BFA. This platform was used for this section of the data gathering phase as it provided specific detail required from the benchmark, namely the price data and annualised volatility, as well as providing an appropriate secondary source in which to verify certain share price information obtained from the Bloomberg terminal above.

**Validating Quantitative Data Gathered**

The effectiveness of any study is only as accurate as the information used in performing the analysis. Accordingly, all the data was sourced from reputable companies which specialise in providing financial information. It must be noted however, that while using reputable sources such as S&P Capital IQ, Bloomberg and McGregor BFA minimises the risk of data inaccuracy, there still exists the possibility of certain data imperfections.

As a result, in order to limit the impact of potential data imperfections in the study, certain data validation checks were put into place when constructing the financial model. These validation checks are described briefly below:

- Should information, relevant for the purposes of calculating the ROC or EY ratios be missing then a 0% ratio was returned resulting in the specific company being excluded from the lowest combined rankings selection criteria. Accordingly, this resulted in the specific company being excluded from the analysis at the specific rebalance date for which information was missing.
- As described under the *Strategic Considerations* section, certain validation checks were also performed on results where statistical outliers were noted. Where adjustments were required to accurately reflect the substance of the
company return, they were made accordingly using the results obtained from alternative data sources.

As an additional data reliability test, the market data used in performing the statistical analysis was used to re-calculate the return of the JSE ALSI. The result of which, as shown in Figure 3, was plotted against the actual return of the JSE ALSI for comparison.

In completing this reconstruction of the JSE ALSI and determining the total returns generated from the index the below formulas, as obtained from the FTSE/JSE (2004), were used:

\[
\text{Index Calculation} = \sum_{1}^{n} \frac{(p \cdot e) \cdot s \cdot f}{d}
\]

[Equation 6]

Where:

- \( n \) = Number of Shares in the Index
- \( p \) = Latest traded price of the share
- \( e \) = Exchange rate required to convert the shares currency to that of the index
- \( s \) = Number of shares in issue on the JSE
- \( f \) = Free Float factor to be applied on each share to allow weighting amendments
- \( d \) = Divisor representing the total issued share capital of the index at base level

In determining the Index value above, for the purposes of the recalculation, the top 165 shares were included into the reconstructed index with the following treatment being applied:

- The exchange rate \((e)\) was not applicable as all figures, in the data which was collected, were denominated in South Africa Rands (ZAR). Accordingly, a factor of 1 was used for this input in the calculation.
- The resultant share price \( (p) \) multiplied by the number of shares \( (s) \) would equate to the market capitalisation. As a result, the market capitalisation figure was used to represent this portion of the computation.

- A free float factor \( (f) \) of 1 was used in the calculation as it represented an assumption that all shares in the index were tradable (FTSE/JSE, 2014:11). This is consistent with the treatment applied when conducting the study.

- The divisor \( (d) \) is included in the index calculation, as shown in equation 6 above, to account for rights issues initiated during the period. By including the divisor into the calculation it ensures that the index falls in line with the reduction in share price on the right ex-date (FTSE/JSE, 2014:19). With this being the case, in order to simplify the reconstruction, no divisor was used in the reconstruction performed in this study. This may result in certain differences between the reconstructed and the actual JSE ALSI however, as the purpose of this is merely to provide a reasonability test, the potential differences are accepted.

Having calculated the index value, the returns of the reconstructed index were required to be compared to the returns generated from the actual JSE ALSI. Accordingly, the total returns were calculated in accordance with the following formula as outlined in FTSE/JSE (2004):

\[
R_t = R_y \cdot \frac{I_t}{I_y - XD}
\]

[Equation 7]

Where:
\( R_t = \text{Total Return Index (todays value)} \)
\( R_y = \text{Total Return Index (yesterdays value)} \)
\[ I_t = \text{Underlying Capital Index (today's value)} \]
\[ I_y = \text{Underlying Capital Index (yesterday's value)} \]
\[ XD = \text{Adjustment to underlying capital} \]

It must be noted, that for the purposes of calculating the Total Returns Index for comparison, the only adjustment which was applied to the capital index was that of the dividends earned by all the companies included in the listing.

**Figure 3** – Reconstructed ALSI using annual market data gathered for statistical analysis

As can be seen from **Figure 3** above, the theoretic return generated from the reconstructed market data approximates that of the JSE ALSI and as a result it would indicate that the information used for the purposes of performing the analysis is accurate.
Ethical Considerations

No ethical clearances were required for any component of this study as no interest in gender nor racial differences and no participation human participants was necessary for the completion of this research.

In relation to the confidentiality of information used, all of the information which was obtained is publicly and readily available from the sources as highlighted above.

Limitations of the Study

In determining the optimal “Magic Formula” portfolio, transaction costs and taxes were excluded from the analysis. This treatment was consistent with the work performed in alternative studies by Howard (2015), Hoffman (2012) and Muller and Ward (2013) on the basis that these costs would not differ significantly amongst portfolios and as a result they are immaterial to the investment decision.

In respect of the portfolio construction, the ‘five year’ portfolio only results in two observable periods. Accordingly, the limited number of observations may impact on the ability to draw accurate and meaningful conclusions in relation to this portfolio holding period.

Lastly, in relation to the measurement of risk, the volatility of portfolio returns has been used as a proxy. As a result, the study, relies on a normal distribution of returns to evaluate risk for a value investing metric which assumes that market efficiency does not apply.
CHAPTER 4 - RESULTS

This chapter presents the findings of using the “Magic Formula” investment strategy in the South African market for various portfolio sizes and holding periods. The returns generated, risk incurred and risk adjusted returns manufactured from the synthetic portfolios which were created will be compared to the JSE ALSI as well as to each other in order to address the three primary research questions.

The results for each of the three primary research questions, as set out on page 29, will be addressed independently and sequentially throughout this section and in addressing the results for each of the three research questions, the results will be presented followed by a discussion thereof.

Performance of “Magic Formula” investment strategy

The first section of the results chapter will cover the first two research questions by determining and discussing the returns generated as well as the risk incurred from the “Magic Formula” investment strategy.

In addressing the “Magic Formula” portfolio return, the ‘portfolio size’ and ‘portfolio holding period’ shall first be discussed independently followed by a discussion on which combination of these two portfolio structure constituents yields the highest return for the “Magic Formula” in the South African market. This will lead us to the conclusion of the first research question.

Subsequently to addressing the first research question, the two additional research questions (research questions 1(i) and 1(ii)) which will provide substantiation to the results shown for research question one shall be discussed.
The final component to be addressed in this section of the chapter is the “Magic Formula” portfolio risk. Similarly to the manner in which the portfolio return was presented, the “Magic Formula” portfolio risk shall address first the ‘portfolio size’ and ‘portfolio holding periods’ followed by which combinations of these two portfolio structure constituents results in the highest “Magic Formula” portfolio risk being incurred. This will lead us to the conclusion of the second research question.

“Magic Formula” Portfolio Return

The synthetic “Magic Formula” portfolios, which vary the holding period and number of shares, created by selecting the best shares based on a combined ranking of ROC and EY prove to be highly successful in the South African market. As shown in Figure 4, the majority of these constructed portfolios yielded a greater geometric mean return than the benchmark with only the five year – 5, 10, 15 & 20 share and the two year – 5 share combinations underperforming the JSE ALSI. A summary of these results, presented numerically, is provided in Appendix 1.

Figure 4 – Portfolio Geometric Mean Returns
**Portfolio Size**

When looking at the performance of the different portfolios from a *number of shares to be included in the portfolio* perspective, it can be seen in Figure 4 above that each of the respective rebalance frequencies (with the exception of the 5 year portfolio) has a trend of an increasing mean return as the number of shares included in the portfolio increases. This trend exists up to a “breaking point” at which time the mean return of the portfolio starts to diminish.

**Table 6** – Breaking point of number of shares to be included in the “Magic Formula” portfolio

<table>
<thead>
<tr>
<th>BREAKING POINT</th>
<th>[15 Shares]</th>
<th>[10 Shares]</th>
<th>[15 Shares]</th>
<th>N/A</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIX MONTH</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ONE YEAR</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TWO YEAR</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FIVE YEAR</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The implication of a breaking point being observed in these portfolios (i.e. 10 and 15 shares) has a twofold application:

Firstly, the breaking point of a one year portfolio, being 10 shares, indicates that the use of 30 shares by Howard (2015) in prior research performed on the “Magic Formula” did not optimise the mean return from the investment strategy. This was corroborated through the reconstruction of a one year – 30 shares portfolio in which it was found that a mean return of 19% was generated (*Appendix 1*).

The mean return of 17% generated from the one year – 30 share portfolio falls below the mean return of 18% generated from the one year – 20 share portfolio. This would
indicate that once the breaking point is surpassed, each share added to the portfolio would bring about a marginal decrease in the total mean return.

Secondly, the breaking point of six month, one and two year portfolios suggest that the optimal number of shares, from a mean return generating perspective falls between these two portfolio sizes. This is corroborated by Table 7 which, once removing the five year portfolio, indicates that portfolio sizes of between 10 and 20 shares offer similar returns and similar rates of outperformance on a geometric mean return basis.

**Table 7 – Average Geometric returns generated from the various portfolio sizes**

<table>
<thead>
<tr>
<th>SHARE SIZE</th>
<th>5 SHARES</th>
<th>10 SHARES</th>
<th>15 SHARES</th>
<th>20 SHARES</th>
</tr>
</thead>
<tbody>
<tr>
<td>AVERAGE GEOMETRIC RETURN</td>
<td>15.19%</td>
<td>18.82%</td>
<td>19.40%</td>
<td>18.23%</td>
</tr>
</tbody>
</table>

In relation to the five year portfolio, it is important to note that the potential reason for this portfolio not incurring a breaking point, as shown by the other portfolios, may be as a direct result of only being exposed to two rebalancing periods (1 October 2005 and 1 October 2010) thus potentially skewing the results contained therein.

*Holding Period*

When looking at the holding period impact on the overall return, as shown in Figure 4 above and Table 8 below, it indicates that the optimal holding period lies between ‘six months’ and ‘one year’ as these holding periods generate the highest mean returns and are the least sensitive to change in the number of shares making up the portfolio.
The five year portfolios are found to have the lowest geometric mean returns. This would indicate that fewer ‘winners’ are picked as a result of the less frequent rebalancing resulting in diminishing returns for “Magic Formula” portfolios which have a holding period of greater than two years. Further implication of this result is that the five year portfolio also contrasts to the fundamental principle of value investing which prescribes that value investing stocks should always ‘win’ over longer holding periods (Novy-Marx, 2013).

Table 8 – Average returns generated from the various holding periods

<table>
<thead>
<tr>
<th></th>
<th>SIX MONTHS</th>
<th>ONE YEAR</th>
<th>TWO YEAR</th>
<th>FIVE YEAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>AVERAGE GEOMETRIC RETURN</td>
<td>18.26%</td>
<td>18.28%</td>
<td>17.18%</td>
<td>8.96%</td>
</tr>
<tr>
<td>SENSITIVITY OF RETURNS TO A CHANGE IN NUMBER OF SHARES WITHIN PORTFOLIO</td>
<td>7.95%</td>
<td>8.56%</td>
<td>14.23%</td>
<td>34.64%</td>
</tr>
</tbody>
</table>

When looking at the geometric mean return in isolation, it would suggest, contrastingly to the portfolio size, that the one year holding period recommendation by Greenblatt (2006) and subsequently applied by Howard (2015) in his application of the “Magic Formula” to the South African market would result in the highest geometric return.

However, in relation to the holding period, the one year portfolio yields the highest mean return yet suffers from a higher standard deviation of returns in comparison to the six month and two year portfolios. This implies that on a risk adjusted basis we cannot determine, without additional analysis, whether the outperformance generated by the one year portfolio is as a result of superior performance of the underlying investment strategy or as a result of additional risk exposure to which the one year portfolios are exposed. More detail and analysis, relating to the risk
adjusted return, is provided in the ‘Risk-adjusted returns of the “Magic Formula” portfolio’ section to follow.

In relation to the sensitivity shown in Table 8, the portfolio least sensitive to the number of shares included within it relates to the six month portfolio, in other words the period with the most frequent rebalancing. This may indicate that by perusing a six month “Magic Formula” holding period you are taking advantage of a short term mispricing in the market as identified by the ROC and EY ratios resulting in “smaller” less volatile gains.

*Optimal mean-return generating combination of Portfolio size and Holding Period*

Based on the isolated discussions regarding the portfolio size and holding period above, the top three mean generating “Magic Formula” portfolios all consist of various derivations of the optimal constituents. A summary of Appendix 1, displaying the values of the top three mean return synthetic portfolios, is provided in Table 9 below.
Table 9 – Highest mean return generating “Magic Formula” portfolios

<table>
<thead>
<tr>
<th>TOTAL RETURN</th>
<th>SIX MONTH 15 SHARES</th>
<th>ONE YEAR 10 SHARES</th>
<th>TWO YEAR 15 SHARES</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005 to 2006</td>
<td>35%</td>
<td>36%</td>
<td>161%</td>
</tr>
<tr>
<td>2006 to 2007</td>
<td>78%</td>
<td>52%</td>
<td></td>
</tr>
<tr>
<td>2007 to 2008</td>
<td>-21%</td>
<td>-26%</td>
<td>-13%</td>
</tr>
<tr>
<td>2008 to 2009</td>
<td>13%</td>
<td>9%</td>
<td></td>
</tr>
<tr>
<td>2009 to 2010</td>
<td>10%</td>
<td>10%</td>
<td>28%</td>
</tr>
<tr>
<td>2010 to 2011</td>
<td>14%</td>
<td>4%</td>
<td></td>
</tr>
<tr>
<td>2011 to 2012</td>
<td>38%</td>
<td>48%</td>
<td>62%</td>
</tr>
<tr>
<td>2012 to 2013</td>
<td>32%</td>
<td>45%</td>
<td></td>
</tr>
<tr>
<td>2013 to 2014</td>
<td>17%</td>
<td>30%</td>
<td>30%</td>
</tr>
<tr>
<td>2014 to 2015</td>
<td>6%</td>
<td>19%</td>
<td></td>
</tr>
<tr>
<td>ANNUALISED GEOMETRIC MEAN</td>
<td>19.8%</td>
<td>20.3%</td>
<td>19.8%</td>
</tr>
</tbody>
</table>

| Initial Portfolio Value | 1 000 | 1 000 | 1 000 |
| Ending Portfolio Value  | 6 106 | 6 374 | 6 099 |
| Compound Annual Growth Rate | 19.8% | 20.3% | 19.8% |

As shown in Table 9, the “Magic Formula” investment strategy does beat the market on an annualised mean return basis with the top three portfolio size and holding period combinations of one year – 10 shares, six month – 15 shares, and two years – 15 shares yielding a Compound annual growth rate (‘CAGR’) of 20.3%, 19.8% and 19.8% respectively. Comparatively, the JSE ALSI\textsuperscript{17} showed a CAGR of 14.8% for the same period, thus resulting in an outperformance of 5.5%, 5.0% and 5.0% for the same aforementioned portfolios over the 10 year period.

\textsuperscript{17} For a graphical representation of the cumulative performance of the “Magic Formula” against the JSE ALSI please refer to Appendix 12.
Importantly, the investment strategy does not only beat the benchmark portfolio, on a pure mean return basis, for the constructed portfolios shown in Table 9 above but also for 8 of the remaining 13 synthetic “Magic Formula” portfolios created.

Conclusion on Research Question 1

As a result of the above discussion, along with the returns shown in appendix 1, it is concluded, consistently with the findings from Howard (2015), that the “Magic Formula” investment yields superior returns relative to the benchmark portfolio in the South African market.

“Magic Formula” investment strategy impact on returns

Introduction

Having determined that the returns generated by the “Magic Formula” investment strategy outperforms the benchmark portfolio, it would lead us to the consideration and execution of research question 1(i), as outlined under Chapter 3 of this study.

Accordingly, in this section it shall be determined whether the returns generated by the “Magic Formula” investment strategy were random through the construction of the inverse “Magic Formula” investment portfolios.

Results and Discussion

The results of the inverse portfolios, as shown in Table 10, all differ from the traditional “Magic Formula” portfolios. This would indicate that the returns generated by the “Magic Formula” investment strategy are not generated randomly and that there is a causal relationship between the “Magic Formula” investment basis and the returns generated.
Table 10 – Inverse “Magic Formula” portfolios

<table>
<thead>
<tr>
<th>TOTAL RETURN</th>
<th>1 YEAR</th>
<th>2 YEAR</th>
<th>3 YEAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005 to 2006</td>
<td>39%</td>
<td>75%</td>
<td>86%</td>
</tr>
<tr>
<td>2006 to 2007</td>
<td>47%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2007 to 2008</td>
<td>-34%</td>
<td>-41%</td>
<td>-17%</td>
</tr>
<tr>
<td>2008 to 2009</td>
<td>-28%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2009 to 2010</td>
<td>-14%</td>
<td>-14%</td>
<td>14%</td>
</tr>
<tr>
<td>2010 to 2011</td>
<td>-32%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2011 to 2012</td>
<td>-5%</td>
<td>25%</td>
<td>47%</td>
</tr>
<tr>
<td>2012 to 2013</td>
<td>20%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2013 to 2014</td>
<td>-6%</td>
<td>-33%</td>
<td>-14%</td>
</tr>
<tr>
<td>2014 to 2015</td>
<td>-16%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Annualised Average: -3% 1% 12%

Initial Portfolio Value: 1 000 1 000 1 000

Ending Portfolio Value: 514 752 2 217

CAGR: -6% -3% 8%

In order to support the conclusion made above, which indicates that the returns generated by the “Magic Formula” portfolio are not random and that the “Magic Formula” selection basis influences the returns generated, a paired t-Test was performed over representative portfolios. Performing this t-Test allows us to determine whether the differences in the mean return, noted between Table 9 and 10, are statistically significant.

Should the null hypothesis, presented in Equation 8 below, not be rejected in the t-Test analysis then it is concluded that the differences observed between the “Magic
"Formula" and the inverse thereof are not statistically significant. Accordingly, this would indicate that returns generated are random from a statistical perspective.

The t-Test was constructed using the annual mean returns generated from the respective portfolios with the significance level being set at 95%.

\[ H_0: \mu_{\text{"Magic Formula" Portfolio}} = \mu_{\text{Inverse "Magic Formula" Portfolio}} \]

[Equation 8]

Where:

\[ \mu = \text{Mean Return} \]

The portfolios used to perform this t-Test were the one year – 10 shares "Magic Formula" portfolio and its inverse counterpart. It is noted that the one year portfolios were selected for ease of computation as all the information contained therein was already annualised allowing for accurate comparison.

**Figure 5** – t-Test results of “Magic Formula” influencing portfolio returns

<table>
<thead>
<tr>
<th></th>
<th>&quot;Magic Formula&quot;</th>
<th>Inverse &quot;Magic Formula&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arithmetic mean</td>
<td>0.227</td>
<td>-0.028</td>
</tr>
<tr>
<td>Variance</td>
<td>0.060</td>
<td>0.084</td>
</tr>
<tr>
<td>Observations</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Pearson Correlation</td>
<td>0.800</td>
<td></td>
</tr>
<tr>
<td>Hypothesized Mean Difference</td>
<td>0.000</td>
<td>9.000</td>
</tr>
<tr>
<td>Df</td>
<td>4.635</td>
<td></td>
</tr>
<tr>
<td>P(T&lt;=t) one-tail</td>
<td>0.001</td>
<td></td>
</tr>
<tr>
<td>t Critical one-tail</td>
<td>1.833</td>
<td></td>
</tr>
<tr>
<td>P(T&lt;=t) two-tail</td>
<td>0.001</td>
<td></td>
</tr>
<tr>
<td>t Critical two-tail</td>
<td>2.262</td>
<td></td>
</tr>
</tbody>
</table>
As the paired t-Test result indicates a $P(T\leq t)$ two-tail, which represents the *p value*, value of 0.001 which is lower than the significance level of 0.05 it would lead us to reject the null hypothesis.

**Conclusion on Research Question 1(i)**

As the null hypothesis, as shown by **Equation 8** above, is rejected it would lead us to conclude, on a statistically significant basis, that the “Magic Formula” selection basis influences the returns. Accordingly, this leads to the conclusion of research question 1(i) whereby it is determined that the returns generated by “Magic Formula” investment strategy are not random.

Comparison to alternative study

**Introduction**

The 20.3% return generated from the top performing “Magic Formula” portfolio, on a geometric mean basis as shown in **Table 9**, differing from the 18.75% return shown by Howard (2015) in the alternative study conducted over the “Magic Formula” in the South African market begs the question of whether the studies are comparable.

Answering the above question, as represented by additional research question 1(ii), is of high importance because should the studies *not be comparable* and it is found that “Magic Formula” outperforms the benchmark then we will be unable to conclude that this outperformance is due to altering the portfolio size and holding period as the outperformance may relate to the differences in periods over which the “Magic Formula” is applied.
Results and Discussion

In accordance with Chapter 3, an additional synthetic portfolio (Portfolio 17), using the dataset of this study, was created under the terms set out by Howard (2015). This portfolio reflected a one year holding period with a portfolio size of 30 shares.

Table 11 – Comparison to “Magic Formula” to alternative study

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2005 to 2006</td>
<td>48%</td>
<td>33%</td>
</tr>
<tr>
<td>2006 to 2007</td>
<td>25%</td>
<td>46%</td>
</tr>
<tr>
<td>2007 to 2008</td>
<td>-20%</td>
<td>-24%</td>
</tr>
<tr>
<td>2008 to 2009</td>
<td>34%</td>
<td>3%</td>
</tr>
<tr>
<td>2009 to 2010</td>
<td>30%</td>
<td>19%</td>
</tr>
<tr>
<td>2010 to 2011</td>
<td>4%</td>
<td>19%</td>
</tr>
<tr>
<td>2011 to 2012</td>
<td>23%</td>
<td>41%</td>
</tr>
<tr>
<td>2012 to 2013</td>
<td>10%</td>
<td>28%</td>
</tr>
<tr>
<td>Geometric Mean</td>
<td>17.52%</td>
<td>18.38%</td>
</tr>
</tbody>
</table>

Refer to Appendix 4 for a full listing of the results recorded by Howard (2015:38)

The comparison, as shown in Table 11, was generated for all years of overlap between the two studies, namely 2005 to 2013, and resulted in a geometric mean return difference of 1% being found.

The reason for the 1% geometric mean return difference being observed is believed to be derived primarily from two key factors. These factors are discussed below:

The first factor which is believed to give rise to the differences observed, particularly at a yearly breakdown, is the differing portfolio rebalancing dates. In the study conducted by Howard (2015), the portfolio is stated to be rebalanced in December annually. Comparatively, this study rebalances the portfolio annually in October. The
implication of this difference, as outlined in *Chapter 3 – Strategic Considerations*, would be that different information would be publicly available in which to calculate the ROC and EY ratios. This could result in there being a difference in the respective portfolio compositions which would lead to the geometric mean return differences as observed. Due to the nature of this factor, quantitatively measuring the difference was impracticable without re-performing the entire study. As a result, the differences caused by this factor were deemed to be un-adjustable and were accepted as being reasonable for the purposes of the study.

The second factor is that certain adjustments for statistical outliers, as previously discussed, were made in carrying out the execution of this study. As there was no indication of adjustments being made in the comparable study, processing adjustments for share splits, as performed above for JSE:CMH in 2006 and JSE:ASR in 2010, would give rise to a measurable difference between this study and that of the comparable study.

A comparison of the “*Magic Formula*” portfolio constructed in this study, whereby the adjustments which were made in respect of share splits of the statistical outliers have been removed, and that presented by Howard (2015) is shown in *Table 12* below:
**Table 12** – Comparison of unadjusted “Magic Formula” portfolio to alternative study

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2005 to 2006</td>
<td>48%</td>
<td>33%</td>
</tr>
<tr>
<td>2006 to 2007</td>
<td>25%</td>
<td><strong>40%</strong></td>
</tr>
<tr>
<td>2007 to 2008</td>
<td>-20%</td>
<td>-24%</td>
</tr>
<tr>
<td>2008 to 2009</td>
<td>34%</td>
<td>3%</td>
</tr>
<tr>
<td>2009 to 2010</td>
<td>30%</td>
<td><strong>16%</strong></td>
</tr>
<tr>
<td>2010 to 2011</td>
<td>4%</td>
<td>19%</td>
</tr>
<tr>
<td>2011 to 2012</td>
<td>23%</td>
<td>41%</td>
</tr>
<tr>
<td>2012 to 2013</td>
<td>10%</td>
<td>28%</td>
</tr>
<tr>
<td>Geometric Mean</td>
<td><strong>17.52%</strong></td>
<td><strong>17.30%</strong></td>
</tr>
</tbody>
</table>

Refer to **Appendix 4** for a full listing of the results recorded by Howard (2015:38)

**Table 12** indicates that the two studies yielded similar results with respect to the *one year – 30 share* portfolios. However, in order to conclude that the studies do not differ significantly from one another, a paired t-Test was performed over the two comparable portfolios.

Should the null hypothesis, presented in **Equation 9** below, *not be rejected* in the t-Test analysis then it is concluded that the differences in geometric mean returns observed between this study and the comparable study are not statistically significant. The t-Test was constructed using the annual mean returns generated from the respective portfolios with a 95% significance level.
\[ H_0: \mu_{\text{Ker-Fox "Magic Formula" Portfolio}} = \mu_{\text{Howard "Magic Formula" Portfolio}} \]

[Equation 9]

Where:

\[ \mu = \text{Mean Return} \]

The paired t-Test result, shown in Figure 6, indicates a \( P(T<=t) \) two-tail p-value of 0.997 which is higher than the significance level of 0.05. This would lead us to not reject the null hypothesis and conclude, on a statistically significant basis, that the "Magic Formula" portfolio constructed in this study and the "Magic Formula" portfolio constructed by Howard (2015) do not differ significantly.

**Figure 6** – t-Test results of South African "Magic Formula" portfolio comparisons

t-Test: Paired Two Sample for Means

<table>
<thead>
<tr>
<th></th>
<th>Howard &quot;Magic Formula&quot; Portfolio</th>
<th>Ker-Fox &quot;Magic Formula&quot; Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arithmetic mean</td>
<td>0.1929</td>
<td>0.1932</td>
</tr>
<tr>
<td>Variance</td>
<td>0.0438</td>
<td>0.0474</td>
</tr>
<tr>
<td>Observations</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Pearson Correlation</td>
<td>0.603</td>
<td></td>
</tr>
<tr>
<td>Hypothesized Mean Difference</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>df</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>t Stat</td>
<td>-0.004</td>
<td></td>
</tr>
<tr>
<td>( P(T&lt;=t) ) one-tail</td>
<td>0.498</td>
<td></td>
</tr>
<tr>
<td>t Critical one-tail</td>
<td>1.895</td>
<td></td>
</tr>
<tr>
<td>( P(T&lt;=t) ) two-tail</td>
<td>0.997</td>
<td></td>
</tr>
<tr>
<td>t Critical two-tail</td>
<td>2.365</td>
<td></td>
</tr>
</tbody>
</table>

**Conclusion on Research Question 1(ii)**

As a result of the t-Test shown in Figure 6 above, we have established that this study is comparable to that presented by Howard (2015). The implication of this is that any
additional return generated by altering either the holding period or the portfolio size is
as a direct result of changing those variables and not due to differences in underlying
data.

Accordingly, finding the combination of portfolio size and holding period which yields
the greatest geometric mean return per given level of risk would present the optimal
“Magic Formula” in the South African market. Comparison of this portfolio to the
benchmark would enable us to conclude on whether the same findings of
outperformance as observed by Greenblatt (2006), for the US market, and Olin
(2011), for the Finnish market, hold for the South Africa.

“Magic Formula” Portfolio Risk

The volatility of returns, as measured by standard deviation, was carried out across
all synthetic portfolios constructed. The results, as shown in Appendix 5, are
summarised for analysis in Figure 7 below:

Figure 7 – Portfolio Standard Deviation of Returns
Similar to the comparison performed between the geometric mean returns and JSE ALSI, the synthetic portfolios yielded a higher variability of return in comparison to the benchmark portfolio for ten of the sixteen portfolios constructed. This may indicate that the additional return generated by the “Magic Formula” investment strategy was only as a result of the increased risk to which the investor was exposed.

At a high level, it is evident that the five year “Magic Formula” portfolios yield the lowest variability in returns. This indicates that the “Magic Formula” may produce lower, but more consistent returns over a longer investment horizon. This is an important consideration when bearing in mind the investor risk profile as well as the resultant overall risk adjusted return. These factors are discussed in further detail in the ‘risk-adjusted returns of the “Magic Formula” portfolio’ section below.

*Portfolio Size*

As can be seen in **Figure 7**, all of the varying holding periods follow the same trend when it comes to the portfolio size. This trend being that as the number of shares increases to a peak portfolio size of 15 shares so too does the variability of returns. Thereafter, as the portfolios size increases, the standard deviation (measuring the variability of returns) starts to diminish. This trend is expected to continue until all constituents of the JSE ALSI are represented in the portfolio at which point the JSE ALSI annualised volatility of 19% should be achieved.
The aforementioned trend is displayed numerically in Table 13 below:

**Table 13 – Average Standard deviation generated from the various portfolio sizes**

<table>
<thead>
<tr>
<th>SHARE SIZE</th>
<th>AVERAGE GEOMETRIC STD. DEVIATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 SHARES</td>
<td>17.45%</td>
</tr>
<tr>
<td>10 SHARES</td>
<td>20.86%</td>
</tr>
<tr>
<td>15 SHARES</td>
<td>23.56%</td>
</tr>
<tr>
<td>20 SHARES</td>
<td>22.26%</td>
</tr>
</tbody>
</table>

The impact of a change in portfolio size on the geometric standard deviation is closely aligned to that observed with the geometric mean return. This observation, as displayed in Figure 8 below, is supported by the high correlation between these two trends. The implication of this observation is that any additional return generated, as a result of an adjustment to the portfolio size, is a direct result of the increased risk to which the investor is exposed. Accordingly, any adjustment made to the “Magic Formula” portfolio size, in isolation, will not generate significant excess risk-adjusted returns.
A further observation which can be made from Figure 8 is that the margin between the geometric mean return and the geometric standard deviation is at its minimum at the 10 share portfolio size. This indicates that, on average, the 10 share portfolio size would generate the lowest risk adjusted returns in comparison to the alternative portfolio sizes in the study. This observation, and confirmation on whether or not it is found to be true, shall be reviewed in greater detail in the section to follow.

In review of Table 13, while the 5 share portfolios displayed the lowest volatility, this was also the only grouping where more than one holding period yielded a standard
deviation lower than that of the benchmark. The implication of this, while having a lower volatility is most often regarded as a positive in the construction of any portfolio, may be that the smaller portfolio size is limiting the inclusion of potential winners which potentially fall outside the top five ranked ROC and EY ratios. This occurrence would not only result in the lower volatility, as shown by the 17.45% in Table 13, but also a lower geometric mean return as is the case, as shown in Table 7, whereby the 5 share portfolios reflect the lowest geometric mean return of all four of the portfolio size groupings.

*Portfolio Holding Period*

**Figure 9** – Portfolio holding period varying risk profiles

As seen in **Figure 7** and **Figure 9** above, the portfolio holding period results in different levels of risk being incurred for each of the respective portfolio holding period groupings. The five year grouping, for all of the various portfolio sizes contained therein, has less volatile earnings in comparison to all the other holding
period groupings. On the other end of the spectrum, the entire one year portfolio, for all of the various portfolio sizes contained therein, has more volatile earnings in comparison to the other holding period groupings.

The only holding period groupings which result in a change in the level of risk as the number of shares in the grouping increased is the six month and two year portfolios. The overlap of these two groupings, indicating a change in the level of risk, occurs approximately at a portfolio size of 12 shares. At this point of overlap, the risk of the six month and two year portfolios are approximately the same. However, a portfolio size smaller that 12 would indicate that the six month portfolio is exposed to a higher volatility of returns whereas a portfolio size of more than 12 shares would indicate that the two year portfolio is exposed to a higher volatility.

The aforementioned observation ties into the numerical representation provided in Table 14 which shows the six month portfolios as being the least sensitive to a change in the number of shares within the portfolio. Contrastingly, the two year portfolios are the most sensitive to a change in the number of shares. The differences in sensitivity, coupled with the similar overall geometric standard deviation observed, result in the overlap between these two portfolio groupings being observed as the number of shares included in the portfolio increases.
Table 14 – Average Standard Deviation generated from the various portfolio holding periods

<table>
<thead>
<tr>
<th></th>
<th>SIX MONTH</th>
<th>ONE YEAR</th>
<th>TWO YEAR</th>
<th>FIVE YEAR</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>AVERAGE GEOMETRIC STD DEVIATION</strong></td>
<td>21.01%</td>
<td>30.63%</td>
<td>21.65%</td>
<td>10.85%</td>
</tr>
<tr>
<td><strong>SENSITIVITY OF RETURNS TO A CHANGE IN NUMBER OF SHARES WITHIN PORTFOLIO</strong></td>
<td>7.09%</td>
<td>11.45%</td>
<td>15.46%</td>
<td>11.74%</td>
</tr>
</tbody>
</table>

The advantage of being able to identify the level of risk incurred is valuable as this can be used to tailor the best performing “Magic Formula” portfolios to specific individual investor requirements. In other words, by being able to identify which holding period results in the overall highest or lowest level of risk, a risk-averse investor wishing to follow the “Magic Formula” would be able to construct the portfolio which is found to outperform the market on a risk adjusted basis while incurring the lowest level of risk, namely the five year portfolio.

*Combination of Portfolio Size and Holding Period*

In summary of the above, it was found that the portfolio holding period is a large determinant in the level of risk faced by the “Magic Formula” portfolio with one year portfolios yielding the highest variability of returns. Further, as presented by the small differences in standard deviation observed between portfolio sizes of 10 to 20 years, as shown in Table 13, along with the low sensitivity of portfolio returns to portfolio size, as shown in Table 14, it can be confirmed that the portfolio size factor has little impact on this risk.
As a result, due to one year portfolios yielding the highest variability of returns, the portfolios which have the highest geometric standard deviation all consist of a one year holding period.

A summary of Appendix 5, displaying the values of the top 3 portfolios generating the highest variability of returns, is provided in Table 15.

**Table 15** – Highest geometric average standard deviations “Magic Formula” portfolios generated

<table>
<thead>
<tr>
<th>YEARS</th>
<th>10 SHARES</th>
<th>15 SHARES</th>
<th>20 SHARES</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005 to 2006</td>
<td>18%</td>
<td>40%</td>
<td>38%</td>
</tr>
<tr>
<td>2006 to 2007</td>
<td>36%</td>
<td>35%</td>
<td>32%</td>
</tr>
<tr>
<td>2007 to 2008</td>
<td>30%</td>
<td>25%</td>
<td>25%</td>
</tr>
<tr>
<td>2008 to 2009</td>
<td>26%</td>
<td>26%</td>
<td>25%</td>
</tr>
<tr>
<td>2009 to 2010</td>
<td>25%</td>
<td>29%</td>
<td>27%</td>
</tr>
<tr>
<td>2010 to 2011</td>
<td>25%</td>
<td>29%</td>
<td>27%</td>
</tr>
<tr>
<td>2011 to 2012</td>
<td>56%</td>
<td>51%</td>
<td>46%</td>
</tr>
<tr>
<td>2012 to 2013</td>
<td>29%</td>
<td>26%</td>
<td>24%</td>
</tr>
<tr>
<td>2013 to 2014</td>
<td>37%</td>
<td>44%</td>
<td>40%</td>
</tr>
<tr>
<td>2014 to 2015</td>
<td>44%</td>
<td>38%</td>
<td>35%</td>
</tr>
<tr>
<td><strong>GEOMETRIC AVERAGE</strong></td>
<td><strong>32.2%</strong></td>
<td><strong>34.0%</strong></td>
<td><strong>31.6%</strong></td>
</tr>
</tbody>
</table>

After a review of all the synthetic “Magic Formula” portfolios variability of returns, it is important to note, as eluded to in the discussion of the geometric mean returns, that the one year holding period assumption applied by Howard (2015) in the application of the “Magic Formula” to the South African market does not present the optimal portfolio from a risk adjusted return perspective.

This statement is made as a result of, as shown and supported by the figures in Table 16, the one year holding period marginally earning the highest geometric
mean return, however the risk incurred therein is significantly higher in comparison to the next best geometric mean return generating portfolio grouping, the six month portfolio.

**Table 16** – One year vs six month holding periods impact on risk adjusted returns

<table>
<thead>
<tr>
<th></th>
<th>SIX MONTHS</th>
<th>ONE YEAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>GEOMETRIC MEAN RETURN</td>
<td>18.26%</td>
<td>18.28%</td>
</tr>
<tr>
<td>GEOMETRIC STD DEVIATION</td>
<td>21.01%</td>
<td>30.63%</td>
</tr>
</tbody>
</table>

The assessment made above is further supported by the finding that, of the three portfolios which generate the highest variability of returns as shown in Table 15, only one, being the one year – 10 shares portfolio, is on the list of the top mean generating portfolios as shown in Table 9. This would imply that the additional risk, to which the one year portfolios are exposed, does not result in excess mean generating returns being earned.

**Conclusion on Research Question 2**

As shown in Appendix 5 and discussed in detail in the section above, the majority of the “Magic Formula” portfolios, with the majority being 11 of the 16 synthetically constructed portfolios, incurred a higher level of risk in comparison to the benchmark portfolio. Accordingly it is concluded that, on a pure volatility of returns basis, the “Magic Formula” investment strategy is exposed to a higher level of risk relative to the benchmark portfolio in the South African market.
Risk-adjusted returns of the “Magic Formula” portfolio

The results of the initial research questions, which found that both the “Magic Formula” geometric mean return as well as the “Magic Formula” standard deviation of returns exceeded the benchmark portfolio, leads us directly to research question 3. Accordingly, the determination and discussion of whether the “Magic Formula” investment strategy outperforms the benchmark, as represented by the JSE ALSI, on a risk adjusted basis shall be included in this section of the study.

This segment of the results chapter will be outlined as follows:

In the first subsection, the portfolio returns, for each of the synthetic portfolios created (i.e. Portfolio 1 to Portfolio 17), shall be reviewed with reference to the risk incurred for each of those respective portfolios.

Subsequently, in accordance with the research design set out in chapter 3 above, the Sharpe ratio for each of the synthetic portfolios shall be calculated. This shall be performed in order to quantify the risk-adjusted returns and determine whether any of the synthetic “Magic Formula” portfolios outperform the benchmark portfolio, as represented by the JSE ALSI.

Based on the results of the Sharpe ratio, a discussion surrounding the ‘optimal’ “Magic Formula” portfolio in the South African market shall be provided. In addressing the question of: ‘What is the optimal “Magic Formula” portfolio for the South African market?’ the risk tolerance of investors shall be taken into account and shall form the basis of the discussion.

Lastly, it shall be determined whether the outperformance of the “Magic Formula” risk-adjusted returns relative to the JSE ALSI, if any, is statistically significant. This
will lead us to the conclusion of research question 3, namely: ‘Does the “Magic Formula” investment strategy outperform the benchmark portfolio on a risk-adjusted basis in the South African market?

“Magic Formula” Portfolio Return relative to Portfolio Risk

In addressing research question 3 and determining whether the “Magic Formula” outperforms the JSE ALSI on a risk adjusted basis, the returns must be viewed relative to the risk incurred. A graphical representation to this effect, namely a representation of the synthetic “Magic Formula” portfolios geometric mean returns relative to the geometric standard deviations, is provided in Figures 10 to 13.

Figure 10 – Return relative to Risk represented for the Six month portfolios

![Return vs. Risk - Six month "Magic Formula" portfolios](image)

Figure 11 – Return relative to Risk represented for the One year portfolios

![Return vs. Risk - One year "Magic Formula" portfolios](image)
Figures 12 – Return relative to Risk represented for the Two year portfolios

Figures 13 – Return relative to Risk represented for the Five year portfolios

Figures 10 to 13 would indicate outperformance of the benchmark where the geometric return generated is higher than that of the benchmark and the geometric standard deviation is lower than that of the benchmark. In other words, all portfolios plotted to the top left of the JSE ALSI.

Only one such portfolio meeting the aforementioned criteria exists, namely the Six month – 5 shares “Magic Formula” portfolio. Accordingly, this would lead us to conclude that the portfolio size and holding period combination of 6 months and 5 shares results in an outperformance of the benchmark. This would further indicate,
without additional analysis and computations, that the “Magic Formula” investment strategy outperforms the benchmark portfolio representing the South African market.

**Figures 10 to 13** also provide confirmation on some of the observations and conclusions drawn in the ‘Performance of the “Magic Formula” investment strategy’ section of this study. Each of these is briefly discussed below:

*Six month portfolios*

In relation to the six month portfolios, as shown in Figure 10 above, it can be seen that the all the portfolios have generated a return in excess of the benchmark portfolio. Further, it can be seen that the dispersion of the returns generated for each of the different portfolio sizes is minimal indicating a low sensitivity as initially highlighted in Table 14.

*One year portfolios*

Regarding the one year portfolios as presented in Figure 11, it can again, similarly to Figure 7, be seen that this holding period grouping results in a large risk being incurred relative to the benchmark portfolio. However, with this being said, the additional risk to which the investor is exposed has also resulted in excess an return being earned and as a result we are unable to conclude, at this stage of the study, whether the return per given level of risk is higher than that of the benchmark.

*Two year portfolios*

In the two year portfolios, as shown in Figure 12, it can be seen that the only synthetic portfolio falling within the six month to two year holding period, which results in an underperformance of the benchmark on a pure geometric mean basis, was the ‘two year – 5 shares’ portfolio.
Five year portfolios

Lastly, in relation to the five year portfolios as shown in Figure 13, it can be seen that all these portfolios incurred both a lower geometric mean return as well as a lower variability in returns in comparison with the benchmark.

“Magic Formula” Sharpe Ratio

As a direct result of all synthetic “Magic Formula” portfolios, with the exception of the ‘six month – 5 shares portfolio’, incurring either:

1. A higher geometric mean return and a higher volatility of returns, or
2. A lower geometric mean return and a lower volatility of return

It cannot be succinctly seen whether the “Magic Formula” outperforms the benchmark portfolio on a risk adjusted basis. Accordingly, in order determine whether the “Magic Formula” outperformed the benchmark on a risk-adjusted basis, as established in Chapter 3, the Sharpe Ratios were calculated for all the respective synthetic portfolios.

It must be noted that the Sharpe Ratio was chosen because, based on the results achieved and set out in Figures 10 to 13, it is difficult to determine which synthetic portfolio contains the best risk-reward ratio and this ratio provides a measure which is easily comparable with that calculated from the JSE ALSI.

Using the yields of the R186 long term government bond as a proxy for the risk free rate, along with the geometric mean returns and geometric standard deviation as set out in Appendix 1 and 5 respectively, the synthetic “Magic Formula” portfolios Sharpe Ratios were calculated. Quantitative details of the results can be found in Appendix 6 with the graphical representation provided in Figure 14.
As can be seen in Figure 14, the “Magic Formula” investment strategy outperforms the benchmark, the JSE ASLI, on a risk-adjusted basis for eight of the sixteen synthetic portfolios constructed. This indicates that, under the correct portfolio construction conditions, the “Magic Formula” yields a higher return for each unit of volatility.

At a high level, the six month holding period was the only grouping to outperform the JSE ALSI on a consistent basis for all portfolio sizes. As a result, this may indicate that this grouping represents the most stable “Magic Formula”.

“Magic Formula” portfolio risk-adjusted return

As a result of eight of the “Magic Formula” synthetic portfolios’ yielding a Sharpe Ratio in excess of the benchmark it creates some uncertainty as to which of the portfolios represents the optimal portfolio from an investment perspective.
To illustrate the uncertainty, as referred to above, the highest Sharpe Ratios for each of the respective holding periods are presented in Table 17 below. In this table, it is clearly shown that each holding period contains at least one portfolio size which has been found to generate excess risk adjusted returns relative to the benchmark portfolio in the South African market.

Multiple portfolios, with significantly different compositions, which all outperform the benchmark portfolio poses the question: ‘Which of the portfolio compositions should actually be chosen to represent the optimal “Magic Formula” portfolio?’

Table 17 – Optimal “Magic Formula” portfolios

<table>
<thead>
<tr>
<th></th>
<th>SIX MONTH</th>
<th>ONE YEAR</th>
<th>TWO YEAR</th>
<th>FIVE YEAR</th>
<th>JSE ALSI</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20 SHARES</td>
<td>10 SHARES</td>
<td>10 SHARES</td>
<td>20 SHARES</td>
<td></td>
</tr>
<tr>
<td>2005 to 2006</td>
<td>1.29</td>
<td>1.63</td>
<td>3.63</td>
<td>1.37</td>
<td>1.24</td>
</tr>
<tr>
<td>2006 to 2007</td>
<td>3.20</td>
<td>1.23</td>
<td></td>
<td></td>
<td>1.68</td>
</tr>
<tr>
<td>2007 to 2008</td>
<td>-1.30</td>
<td>-1.16</td>
<td>-0.91</td>
<td></td>
<td>-1.03</td>
</tr>
<tr>
<td>2008 to 2009</td>
<td>0.22</td>
<td>0.00</td>
<td></td>
<td></td>
<td>-0.02</td>
</tr>
<tr>
<td>2009 to 2010</td>
<td>0.23</td>
<td>0.04</td>
<td>0.47</td>
<td>0.71</td>
<td></td>
</tr>
<tr>
<td>2010 to 2011</td>
<td>0.66</td>
<td>-0.19</td>
<td></td>
<td>-0.28</td>
<td></td>
</tr>
<tr>
<td>2011 to 2012</td>
<td>1.47</td>
<td>0.72</td>
<td>1.12</td>
<td>1.24</td>
<td></td>
</tr>
<tr>
<td>2012 to 2013</td>
<td>1.28</td>
<td>1.25</td>
<td>1.01</td>
<td>1.40</td>
<td></td>
</tr>
<tr>
<td>2013 to 2014</td>
<td>0.11</td>
<td>0.59</td>
<td>0.88</td>
<td>0.67</td>
<td></td>
</tr>
<tr>
<td>2014 to 2015</td>
<td>-0.06</td>
<td>0.26</td>
<td></td>
<td>-0.18</td>
<td></td>
</tr>
<tr>
<td><strong>Geometric Average</strong></td>
<td><strong>0.53</strong></td>
<td><strong>0.37</strong></td>
<td><strong>0.54</strong></td>
<td><strong>0.43</strong></td>
<td><strong>0.34</strong></td>
</tr>
</tbody>
</table>

In answering the question posed above, it is noted that the combination of many portfolios beating the benchmark on a risk adjusted basis, and differing levels of risk being incurred for each of the respective holding periods as identified in the Volatility of Returns section, presents a unique opportunity in the sense that the
optimal “Magic Formula” portfolio can be tailored to the specific risk profile of an investor.

Accordingly, the discussion of the optimal “Magic Formula” to follow shall be set out based on the risk tolerance of investors:

**Risk-averse investor**

For a risk-averse investor, the optimal “Magic Formula” investment strategy would be the ‘five year – 20 share’ portfolio. This portfolio yields a Sharpe Ratio of 0.43 which, being positive and greater than the JSE ALSI Sharpe Ratio of 0.34, is a return greater than that of the risk free rate and the benchmark on a risk-adjusted basis. The primary reason why this portfolio would suit a risk-averse investor would be due to the low levels of volatility observed as the geometric standard deviation was only recorded at 11%.

**Risk-seeking investor**

A risk-seeking investor or a risk-loving investor would most likely be suited to the ‘one year – 10 share’ portfolio. This portfolio was found to match the market on a risk-adjusted basis as it recorded a Sharpe Ratio of 0.37 in comparison to a Sharpe Ratio of 0.34 for the JSE ASLI. The reason for the appeal of this portfolio to a risk seeking investor would be due to the potential higher returns which could be generated as this portfolio showed the highest geometric mean of all the “Magic Formula” portfolios of 20.3%.

**Risk-neutral investor**

For a risk-neutral investor the optimal “Magic Formula” portfolio would be that portfolio yielding the highest risk-adjusted return. As the difference in Sharpe Ratio
between the ‘two year – 10 share’, ‘six months – 20 share’ and ‘six months – 15 share’ is marginal, all of these portfolios would be considered for investment from a risk neutral investor.

In determining which of these three portfolios is the most appropriate for investment the underlying factors making up the portfolio should be considered.

From a mean generating return perspective, it was noted in previous discussions that the breaking point, for both the six month and two year portfolios, was 15 shares. Accordingly, the portfolio generating the highest return was that of the ‘six month – 15 share’ as it reported a geometric mean return of 19.8%.

From a variability of returns (risk) perspective, it was noted in previous discussions that the 15 share portfolio groupings not only reported the highest geometric mean returns but also reported the highest standard deviation. With this being said, it was further established that of all the portfolio holding periods created (i.e. the six month, one year, two year and five year), the six month portfolios presented the lowest sensitivity to changes in portfolio size. This implies that the difference in standard deviation between the ‘six month – 15 share’ and ‘six month – 20 share’ portfolios is marginal. Accordingly, the portfolio which yields the lowest variability in its returns of 19.6% is the ‘two year – 10 share’ portfolio.

As a result of these factors a risk-neutral investor may be indifferent between the ‘six month – 15 share’ and ‘two year – 10 share’ portfolios. It must be noted however that the only additional factor which is not taken into account in this analysis is the transaction costs. At a qualitative level, should the rebalancing transaction costs be taken into account then the returns of the ‘six month – 15 share’ portfolio would be reduced relative to the ‘two year – 10 share’ portfolio as it will require four times
more rebalancing. As a consequence, should this impact result in an incremental decrease in geometric return of more than 1% between the six month and two year portfolios it would lead to the conclusion that the ‘two year – 10 share’ portfolio is the optimal portfolio for a risk-neutral investor.

Statistical significance of the benchmark outperformance

Having found that the certain “Magic Formula” portfolios outperform the JSE ALSI on a risk adjusted basis, it must be determined whether this outperformance, on a geometric mean basis, is statistically significant. In order to make this determination a paired t-Test was performed over representative portfolios. Performing this t-Test allows us to determine whether the differences in the geometric mean return of the “Magic Formula” portfolio and the JSE are fundamentally different.

Should the null hypothesis, presented in Equation 10 below, not be rejected in the t-Test analysis then it is concluded that the differences observed between the “Magic Formula” and the JSE ALSI are not statistically significant. Accordingly, this would indicate that the excess geometric mean return of the “Magic Formula” portfolios, as seen in Appendix 1, are random from a statistical perspective.

\[ H_0: \mu_{\text{"Magic Formula" Portfolio}} = \mu_{\text{JSE ALSI}} \]  

[Equation 10]

Where:

\( \mu = \text{Mean Return} \)
The portfolio used to perform this t-Test was the ‘6 month – 20 share’ “Magic Formula” portfolio. This portfolio was chosen for the t-Test analysis as it would represent, in the absence of transaction costs, the returns generated from an ordinary, risk-neutral, investor. It is to be noted that this portfolio was favoured over the ‘two year – 10 share’ portfolio due to the number of observations available for the statistical analysis.

Figure 15 – t-Test results of South African “Magic Formula” portfolio outperformance

<table>
<thead>
<tr>
<th></th>
<th>SIX MONTHS - 20 SHARE</th>
<th>JSE ALSI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arithmetic mean</td>
<td>0.22</td>
<td>0.16</td>
</tr>
<tr>
<td>Variance</td>
<td>0.06</td>
<td>0.03</td>
</tr>
<tr>
<td>Observations</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Pearson Correlation</td>
<td>0.88</td>
<td></td>
</tr>
<tr>
<td>Hypothesized Mean</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Difference</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>df</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>t Stat</td>
<td>1.429</td>
<td></td>
</tr>
<tr>
<td>P(T&lt;=t) one-tail</td>
<td>0.093</td>
<td></td>
</tr>
<tr>
<td>t Critical one-tail</td>
<td>1.833</td>
<td></td>
</tr>
<tr>
<td>P(T&lt;=t) two-tail</td>
<td>0.187</td>
<td></td>
</tr>
<tr>
<td>t Critical two-tail</td>
<td>2.262</td>
<td></td>
</tr>
</tbody>
</table>

The paired t-Test result, shown in Figure 15 above, indicates a $P(T<=t)$ two-tail value, which represents a $p$ value, of 0.185 which is higher than the significant level of 0.05 (i.e. 95% confidence interval\(^\text{18}\)). As a result, and similarly to the findings presented by Howard (2015), using this figure would lead us to not reject the null hypothesis and conclude, on a statistically significant basis, that the “Magic Formula” portfolio constructed in this study does not result in outperformance of the JSE ALSI.

\(^\text{18}\) Please refer to Definitions of Key Terms, Concepts and Variables in the Glossary for more information on confidence interval.
Conclusion on Research Question 3

The comparable study conducted by Howard (2015) resulted in a t-Test value of 0.514 being observed. Through the adjustment to the portfolio size and portfolio holding period the “Magic Formula”, as calculated in this study, resulted in an increased t-Test value of 1.435 shown in Figure 15.

The higher t-Test value indicates an increased likelihood that the two means are different, the increase in the t stat between Howard (2015) and this study indicates, similarly to the results of Olin (2011), that the performance of the “Magic Formula” can be enhanced by adjusting certain portfolio parameters.

In reaching a conclusion to Research Question 3 however, while adjusting certain “Magic Formula” portfolio parameters has increased the overall performance on both a geometric mean and risk adjusted basis, there is still, similarly to Howard (2015), insufficient evidence to conclude that the “Magic Formula’s” outperformance of the JSE ASLI, on a geometric mean basis, is statistically significant.
Other Observations in the “Magic Formula” Portfolio Composition

In determining the optimal “Magic Formula” portfolio and reviewing the performance of the various synthetic portfolios some additional interesting insights, relating to the portfolio composition, were established.

The most prominent insights, these being the portfolio share turnover as well as the identification of a share selection bias, are discussed below.

Portfolio Share Turnover

Introduction

The Portfolio Share Turnover reviews the percentage of the portfolio which is changed, on average, on the respective portfolio rebalance date.

In the section to follow, the portfolio share turnover percentage will be reviewed with reference to the “Magic Formula” portfolio holding period, portfolio size as well as the geometric mean return.

The portfolio holding period will be discussed in order to identify whether shares which have been identified as being mispriced by the “Magic Formula” investment strategy have subsequently been corrected by the market, resulting in them being replaced by new shares in the “Magic Formula” portfolio.

Subsequently, the portfolio size shall be examined in order to establish whether the portfolio share turnover decreases, as expected, as the portfolio size increases.

Lastly, the portfolio share turnover shall be linked to the geometric mean return in order to establish whether there is any relationship between the realised “Magic Formula” geometric mean return and the portfolio share turnover.
Results and Discussion

Portfolio Holding Period

The results of this analysis, as contained in Appendix 8 and summarised graphically in Figure 16, are somewhat expected as the increase in holding period results in a higher turnover percentage.

The higher portfolio share turnover which is observed as the holding period increases implies that the combinations of the ROC and EY ratios result in the correct identification of a mispricing. This mispricing, which is identified as being a ‘cheap’ share as a result of the EY ratio as well as a ‘good’ share as a result the ROC ratio (Blij, 2011:7), is subsequently adjusted for in the short to medium term by the market as indicated by the high portfolio share turnover.

Figure 16 – Average “Magic Formula” portfolio share turnover

<table>
<thead>
<tr>
<th>Portfolio Stock Turnover (%)</th>
<th>Five year</th>
<th>Two year</th>
<th>One year</th>
<th>Six month</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 Shares</td>
<td>100%</td>
<td>85%</td>
<td>87%</td>
<td>100%</td>
</tr>
<tr>
<td>10 Shares</td>
<td>100%</td>
<td>85%</td>
<td>87%</td>
<td>100%</td>
</tr>
<tr>
<td>15 Shares</td>
<td>87%</td>
<td>85%</td>
<td>87%</td>
<td>100%</td>
</tr>
<tr>
<td>20 Shares</td>
<td>85%</td>
<td>85%</td>
<td>87%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Portfolio Size

It can be seen in Figure 16 above, with the exception of the six month portfolios, that an increase in the portfolio size reduces the turnover percentage. This is expected
because as the portfolio size increases, the likelihood of having similar shares in the portfolio increases as you move from one period to the next.

*Portfolio Turnover impact on Geometric Mean Return*

When the turnover percentage is viewed in relation to the geometric mean return, as shown in Figure 4, it would indicate that a lower portfolio turnover percentage may result in certain shares being held for too long, reducing return. This can be seen as the optimal portfolio size, from a geometric mean perspective, is 10 to 15 shares, thus translating to an optimal portfolio turnover percentage of between 59% and 65%.

<table>
<thead>
<tr>
<th>PORTFOLIO SIZE</th>
<th>TURNOVER PERCENTAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIX MONTH</td>
<td>15 SHARES</td>
</tr>
<tr>
<td>ONE YEAR</td>
<td>10 SHARES</td>
</tr>
<tr>
<td>TWO YEAR</td>
<td>15 SHARES</td>
</tr>
<tr>
<td>FIVE YEAR</td>
<td>N/A</td>
</tr>
</tbody>
</table>

*Table 18* – Portfolio turnover percentage generating highest mean generating returns

In Table 18 the five year portfolio has been excluded, as was the case under the geometric mean return discussion, due to the limited number of observations included within the scope of this study.

As can be seen in Figure 16 above, the six month portfolios all yielded a similar turnover percentage with the difference between the highest and lowest share turnovers amounting to a nominal 3% (*Appendix 8*).

This consistent turnover percentage, as highlighted in in the paragraph above, could be a further reason for the low variability in geometric mean returns generated by the six month portfolio grouping. Furthermore, since the six month portfolio turnover
percentages, as shown in Appendix 9, all approximate to the optimal portfolio rebalance percentages, as set out in Table 18, it could explain the reason for all the six month synthetic portfolios consistently outperforming the benchmark portfolio as shown in Figure 14.

Conclusion on additional observation 1 – Portfolio Share Turnover

The investigation of the portfolio share turnover resulted in two key observations being identified. These key observations provided further insight into the “Magic Formula” investment strategy and the ‘optimisation’ thereof.

The first key observation was the identification that the portfolio share turnover provides further evidence, over and above the outperformance shown in Figure 14, in support of the “Magic Formula” investment strategy identifying share mispricing in the market. This was found to be the case as the share turnover percentage increases with the increase in the portfolio holding period, thus indicating an identification and subsequent market correction of a share mispricing in the short to medium term.

The second key observation was the identification that, similarly to the optimal portfolio being between 10 and 20 as shown in Table 6, the returns are maximised when the portfolio share turnover percentage falls between the ranges of 59% and 63%.

The statement, “Magic Formula” portfolio returns are maximised when the portfolio share turnover falls between the ranges of 59% and 63%, was supported by six months “Magic Formula” portfolios which all outperformed the benchmark portfolio and reported a share portfolio turnover percentage approximating the portfolio
grouping average of 59%. Accordingly, this may be the reason for the consistent ‘excess’ risk-adjusted returns achieved by this particular portfolio grouping as shown in Figure 14.

Selection Bias

Introduction

The second insightful observation is the possibility of a potential bias in the selection of shares stemming from the financial information available.

As a result, in the section to follow, the potential share selection bias is reviewed in detail with the share selection percentage being compared to the availability of financial information. The existence of the bias will subsequently be confirmed with reference to its statistical significance.

Should the existence be found to be statistically significant, the impact of this observation will be reviewed with reference to the impact on this study.

Results and Discussion

Identification of share selection bias

As stated in the introduction of this section, the second insightful observation is the possibility of a potential bias in the selection of shares. This was observed, as shown in Figure 17, where, on average, 74% of the synthetic portfolios were made up of shares chosen based on final results.

Comparatively, as shown in Figure 18, the breakdown of financial information, using an overall weighted average of one year, two year and five year portfolios rebalanced on 1 October and six month portfolios rebalanced semi-annually on 1
October and 1 April, indicated that only 60% of shares had final results available at the time the selection was made.

Figure 17 – “Magic Formula” share selection percentage

![Magic Formula share selection](chart)

Figure 18 – Percentage of publicly available information

![Availability of financial information](chart)

Confirmation of share selection bias

In order to confirm this bias, as per Appendix 10, a statistical single factor ANOVA\textsuperscript{19} table was created with the purpose to test whether the means of the “Magic Formula” share selection differed significantly from the weighted average availability of financial results.

\textsuperscript{19} Please refer to Definitions of Key Terms, Concepts and Variables in the Glossary for more information on ANOVA table.
The results of the statistical test showed a p-value of 0.015 which, being lower than the 5% significant level of 0.05, indicates a rejection of the null hypothesis and accordingly it is concluded that the “Magic Formula” share selection differs significantly from that of the available information.

**Impact on the Study**

A potential reason for the occurrence of this bias may be as a result of, but is not limited to, more accurate financial information being provided at the financial year-end due to the compulsory statutory audit which is not required for interim reporting purposes (KPMG, 2013:16).

As a result of the “Magic Formula” using ROC and EY ratios as a basis for share selection, which require company financial reported information, it would indicate that the underlying bias stems from differences in market and company financial information between interim-year results and final-year results as opposed to the “Magic Formula” investment strategy. Consequently, for the purposes of this study, the share selection bias falls outside the scope of the intended objectives. As such, no further research is performed and a more thorough analysis is recommended under **Suggestions for further study** below.

**Conclusion on additional observation 2 – Selection Bias**

The investigation into the share selection percentage (i.e. shares selected based on final information vs. shares selected based on interim available information) identified a bias in the shares selected skewing in favour of shares based in final year-end information.
When the share selection percentage was compared to the availability of information at the portfolio rebalance date, it resulted in a statistically significant difference which confirmed the existence of this bias.
CHAPTER 5 - CONCLUSION

Research Summary

The purpose of this study was to conduct an analysis covering the effectiveness of the “Magic Formula” investment strategy, as set out by Greenblatt (2006), in the South African market. The scope of this analysis included all companies listed on the JSE main board for the period between 1 October 2005 and 30 September 2015 with the exclusion of financial services companies and companies with a market capitalisation, at rebalance date, of less than R100 million.

It was found that the geometric mean returns generated from the “Magic Formula” investment strategy are maximised when the portfolio size falls between 10 and 15 shares. However, with this being said, when comparing the geometric mean returns of the portfolio to the portfolio standard deviation, a correlation of 0.93 was found to exist. This indicated that an adjustment to the “Magic Formula” portfolio size would not result in excess risk-adjusted returns being earned in the South African market.

From a portfolio holding period perspective, the shorter holding periods of six months and one year yielded the highest geometric returns. However, contrastingly to the portfolio size, each of the differing holding periods exposed the investor to differing levels of risk with the one year and five year portfolios yielding the highest and lowest volatilities respectively.

As a result of these findings, after calculating the respective Sharpe Ratios, the following optimal “Magic Formula” portfolios were determined for the various investor risk profiles:
- **Risk-averse:** A five year – 20 shares portfolio as this yields the lowest level of volatility while still maintaining a Sharpe Ratio, i.e. greater risk-adjusted return, in excess of the benchmark portfolio.

- **Risk-seeking:** A one year – 10 shares portfolio as it yields the highest geometric mean return. It is further noted that this portfolio has a Sharpe Ratio consistent with the benchmark portfolio.

- **Risk-neutral:** A two year – 10 shares portfolio or a six month – 15 shares portfolio as these portfolios exhibit the highest risk-adjusted returns, both of being in excess of benchmark portfolio.

When investigating the causality between the geometric mean returns generated from the “Magic Formula” and the principles used in the share selection process it was found, on a statistically significant basis, that the “Magic Formula” share selection process influences the returns. As a result, this confirmed that the returns generated by the investment were not done so randomly, and that by using the ROC and EY ratios as a basis for share selection influences the returns generated.

Lastly, in comparing the highest geometric mean generating “Magic Formula” portfolio with the JSE ALSI (through the use of a paired t-Test), it was concluded that while adjusting certain portfolio parameters has increased the overall performance on both a geometric mean and risk adjusted basis, there is still, similarly to Howard (2015), insufficient evidence to conclude that the “Magic Formula’s” outperformance of the JSE ASLI, on a geometric mean basis, is statistically significant at a 95% confidence interval. This was found to be the case even though, as shown in **Appendix 12**, the “Magic Formula” accumulated value over 10 years was 6.33 in comparison to the 3.69 generated by the benchmark portfolio.
Conclusions and Recommendations

The analysis in this study demonstrates that the performance of the “Magic Formula” investment strategy, and by extension value investing strategies, can be increased when the portfolio composition is adjusted.

However, while an adjustment to the portfolio holding period and portfolio size increased both the geometric mean return as well as the risk-adjusted return, the increase was not sufficient to conclude on a statistical significant basis that the “Magic Formula” investment strategy outperforms the JSE ALSI.

In applying the “Magic Formula” investment strategy to the South African market it is suggested that, based on historical performance, a ‘five year – 20 share’ and ‘one year – 10 share’ portfolio be constructed by a risk-averse and risk-seeking investor respectively with a risk-neutral investor to create either a ‘two year – 10 share’ or ‘six month – 15 share’ portfolio.

Suggestions for future research

While completing the various stages of this study, three primary areas of future research presented themselves, namely: ‘Determining portfolio compositions which maximise the returns of traditional value investing strategies’, ‘Combination of alternative investment metrics’ and ‘Investigation of the share selection bias’. More detail of these suggestions is provided below:

*Determining portfolio compositions which maximise the returns of traditional value investing strategies*

The results of this study found that by altering the portfolio composition, i.e. the holding period and the portfolio size, the risk-adjusted return could be increased.
Accordingly, it is suggested that for each of the traditional value investing metrics, such as: Price-to-earnings (P/E) ratio, Dividend yield and Market-to-Book ratio etc., the portfolio size and portfolio holding period which results in the largest risk-adjusted returns be determined for both the South African and global markets.

Combination of alternative investment metrics

The majority of the information currently available on value investing in South Africa compares the benchmark portfolio to a singular metric such as the P/E ratio, dividend yield or Market-to-Book ratio.

Further research is recommended in addressing, much like the “Magic Formula”, whether combining various variables can add a ‘value premium’. This could be expanded into determining which combinations of investment metrics result in consistent long-term outperformance of the benchmark portfolio.

Investigation of the share selection bias

As identified in the results chapter of the “Magic Formula” analysis, an investigation of the observed selection bias could be performed. More specifically, investigating the impact, both qualitatively and quantitatively, of year-end financial information on key investment metrics, such as ROC and EY, and performing a comparison of these results with the results observed for interim-year financial information.

This investigation could be further extended into determining the reliability of unaudited interim financial information through the comparison of grossed up interim results to audited year-end financial information.
REFERENCES


https://www.jse.co.za/content/JSEIndexClassificationandCodesItems/Calculation%20Guide.pdf [2016, June 28]


Available at:


GLOSSARY
Definitions of Key Terms, Concepts and Variables

The following definitions of key terms, concepts and variables are provided:

- **Alpha**: The abnormal rate of return on a security in excess of what would be predicted by an equilibrium model like ‘CAPM’ or ‘APT’.
- **ANOVA**: Analysis of variance (ANOVA) is an analysis tool used in statistics. It summarises the sources of variation (Underhill and Bradfield, 2014:299)
- **Behavioural finance**: Models of financial markets that emphasise implications of psychological factors impacting on investor behaviour.
- **Bid-Ask spread**: The difference between a dealers bid and asked price.
- **Confidence Interval**: A confidence interval measures the probability that a population parameter will fall between two set values (Underhill and Bradfield, 2014:178)
- **Dividend Yield**: The percentage rate of return provided by a shares dividend payments.
- **Earnings Yield (EY)**: The ratio of earnings to price (i.e. The earnings yield is the inverse of the price earnings ratio as mentioned above).
- **Efficient market hypothesis (EMH)**: The prices of securities fully reflect available information. Investors buying securities in an efficient market should expect to obtain an equilibrium rate of return.
- **Growth Shares**: A growth share is a share in a company whose earnings are expected to grow at an above-average rate relative to the market (Thrivent Financial, 2016:1)
- **Liquidity**: The speed and ease with which an asset can be converted into cash.
- **Net Debt**: Net Debt is determined as *Short term debt* plus *Long term debt* less *Cash and cash equivalents* (Blij, 2011).

- **Net fixed assets**: Net fixed assets is determined as *Total Assets* less *Total current assets* less *total goodwill* (Blij, 2011).

- **Net working capital (NWC)**: Net working capital is all current assets less current liabilities. It is used to measure the liquidity of a business (Blij, 2011).

- **Price Earnings Ratio**: The ratio of a shares price to its earnings per a share.

- **Random Walk**: Describes the notion that share price changes are random and unpredictable.

- **Return on Capital (ROC)**: Return on Capital is a financial ratio that measures the efficiency with which capital is employed (Blij, 2011).

- **Risk-Free Rate**: The interest rate which can be earned with certainty.

- **Risk Premium**: An expected return in excess of that on risk-free securities. The premium provides compensation for the risk of an investment.

- **Sharpe’s Ratio**: Reward to volatility ratio, ratio of portfolio excess return to standard deviation.

- **Statistical Outliers**: An outlier is an observation strikingly far from some central value. It is an unusual value relative to the bulk of the data (Tukey, 1977)

- **Statistical Significance**: This relates to how we infer or draw conclusions from data (Underhill and Bradfield, 2014:182). It relates to the likelihood that a relationship between 2 or more variables is caused by something other than a random chance.
- **t-Test:** This refers to the analysis of two population’s means through the use of statistical examination. The t-tests is used for comparing means of two populations (Underhill and Bradfield, 2014:211)

- **Value Investing:** This is a strategy of selecting shares that trade for less than their intrinsic values. Value investors actively seek shares that they believe the market has undervalued. They believe the market overreacts to good and bad news, resulting in share price movements that do not correspond with the company's actual long-term fundamentals. The result is an opportunity for value investors to profit by buying when the price is deflated (Sanlam, 2016:1)

The definitions of key terms, concepts and variables as set out above is intended to provide the reader with additional, explanatory, information on technical concepts contained herein.

Unless otherwise indicated, the information contained in the glossary was sourced in accordance with the reference below:

APPENDIXES

Appendix 1 – Return generated by the “Magic Formula” investment strategy on the South African market

<table>
<thead>
<tr>
<th>SIX MONTHS²⁰</th>
<th>ONE YEAR</th>
<th>TWO YEARS</th>
<th>FIVE YEARS</th>
<th>JSE ALSI</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 SHARES</td>
<td>10 SHARES</td>
<td>15 SHARES</td>
<td>20 SHARES</td>
<td>5 SHARES</td>
</tr>
<tr>
<td>2005 to 2006</td>
<td>21%</td>
<td>33%</td>
<td>35%</td>
<td>36%</td>
</tr>
<tr>
<td>2006 to 2007</td>
<td>68%</td>
<td>70%</td>
<td>78%</td>
<td>73%</td>
</tr>
<tr>
<td>2007 to 2008</td>
<td>-8%</td>
<td>-24%</td>
<td>-21%</td>
<td>-22%</td>
</tr>
<tr>
<td>2008 to 2009</td>
<td>25%</td>
<td>11%</td>
<td>13%</td>
<td>14%</td>
</tr>
<tr>
<td>2009 to 2010</td>
<td>-2%</td>
<td>4%</td>
<td>10%</td>
<td>13%</td>
</tr>
<tr>
<td>2010 to 2011</td>
<td>24%</td>
<td>9%</td>
<td>14%</td>
<td>20%</td>
</tr>
<tr>
<td>2011 to 2012</td>
<td>23%</td>
<td>31%</td>
<td>38%</td>
<td>35%</td>
</tr>
<tr>
<td>2012 to 2013</td>
<td>39%</td>
<td>39%</td>
<td>32%</td>
<td>31%</td>
</tr>
<tr>
<td>2013 to 2014</td>
<td>-5%</td>
<td>21%</td>
<td>17%</td>
<td>11%</td>
</tr>
<tr>
<td>2014 to 2015</td>
<td>-2%</td>
<td>5%</td>
<td>6%</td>
<td>6%</td>
</tr>
<tr>
<td>Geometric Mean</td>
<td>16%</td>
<td>17%</td>
<td>19.8%</td>
<td>19%</td>
</tr>
</tbody>
</table>

The six month portfolios, represented in the table above, all present the annual equivalent return which is achieved through semi-annual rebalancing on 1 April and 1 October. The annual equivalent is shown in order to allow for easy comparison with the respective one year portfolios.

²⁰ The six month portfolios are highlighted above as the purpose of this synthetic “Magic Formula” portfolio was, contrastingly to the other portfolios shown above, to allow for a direct comparison with a comparable study performed in the South African market.
Appendix 2 – SENS announcement for the Share Split of Combined Motor Holdings (JSE:CMH)

CMH
Combined Motor Holdings Limited - Results of general meeting
Combined Motor Holdings Limited
(Incorporated in the Republic of South Africa)
(JSE share code: CMH & ISIN number: ZAE000001541
("CMH" or "the company")
RESULTS OF GENERAL MEETING
Shareholders are advised that, at a general meeting of CMH held on Thursday 7 December 2006, that the ordinary resolutions relating to the group restructuring, introduction of a BEE shareholder, and subdivision of shares were passed unanimously.

The special resolutions pertaining to the approval of the 5-for-1 subdivision and the amendment to the Articles of Association were also passed unanimously and will be lodged at the Companies and Intellectual Property Registration Office.

All the conditions precedent, as set out in the circular dated 15 November 2006, have accordingly been fulfilled.

Shareholders are reminded of the following dates pertaining to the subdivision of shares:

Last day to trade in CMH shares: Friday, 26 January 2007
Trading commences in the subdivided CMH ordinary shares of no par value under the new ISIN ZAE000088050. The JSE code will remain the same as that currently applicable to CMH

Recall date for the subdivision: Monday, 29 January 2007
Replacement certificates reflecting the subdivision will be posted to certificated shareholders whose share certificates have been received by Friday, 2 February 2007 on or about

If share certificates have not been received by Friday, 2 February 2007, share certificates will be posted within five business days of receipt of the certificates
Dematerialised shareholders will have their accounts at their CSDP or broker updated on Monday 5 February 2007

SK JACKSON
Company Secretary
Durban
7 December 2006
Sponsor
PricewaterhouseCoopers
Corporate Finance (Pty) Ltd
(Registration number 1970/003711/07)
Date: 07/12/2006 12:06:08 PM Produced by the JSE SENS Department

Source: McGregor BFA
Appendix 3 – SENS announcement for the Share Split of Assore Limited (JSE:ASR)

ASR
ASR – Assore Limited - Sub-Division of ordinary shares and Notice of General Meeting
Assore Limited
(Incoporated in the Republic of South Africa)
(Registration number 1996/037594/06)
Share code: ASR
ISIN: ZAE000017117
(“Assore” or the “Company”)

SUB-DIVISION OF ORDINARY SHARES AND NOTICE OF GENERAL MEETING

1. Introduction
Shareholders of ordinary shares of Assore (“Assore Shareholders”) are advised that the board of directors of Assore has proposed a sub-division of the ordinary share capital of Assore (“Assore Shares”) on a 5 for 1 basis (“the Sub-division”).

2. The Sub-division

2.1 Rationale
The Assore Share price has increased from R11.00 per share on 7 December 1998, the date of the last sub-division, to R785.00 per share on 6 August 2010. The purpose of the Sub-division is to encourage more participation by private investors in the Assore Shares and increase the liquidity of the Assore Shares on the securities exchange operated by the JSE Limited (the “JSE”).

2.2 Effects of the Sub-division
The Sub-division should have the effect of decreasing the market price, earnings per share and net asset value per Assore Share on the JSE by a factor of 5. On 6 August 2010, Assore’s closing share price on the JSE was R785.00 per share. Consequently, had the Sub-division been implemented on such date, Assore’s closing share price on the JSE subsequent to the Sub-division would have been approximately R157.00 per share. The effect of the Sub-division on the ordinary share capital of Assore is as follows:

<table>
<thead>
<tr>
<th>Before the Sub-division</th>
<th>After the Sub-division</th>
</tr>
</thead>
<tbody>
<tr>
<td>Par value per Assore Share</td>
<td>2.5 cents</td>
</tr>
<tr>
<td>Number of authorized Assore Shares</td>
<td>40 000 000</td>
</tr>
<tr>
<td>Number of issued Assore Shares</td>
<td>27 921 400</td>
</tr>
</tbody>
</table>

3. Conditions precedent
The implementation of the Sub-division is subject to the approval of the following resolutions by the requisite majority of votes required at the general meeting of Assore Shareholders, as detailed in 4 below, and the registration (where applicable) of such resolutions by the Registrar of Companies:
- the special resolution for the amendment of the Memorandum of Association of Assore,
- the special resolution for the Sub-division of the authorised and issued Assore Shares and
- the ordinary resolution for directors to execute the Sub-division.

4. General meeting
Notice is hereby given of a general meeting of Assore Shareholders to be held at 10:00 on Friday, 10 September 2010, at the registered office of Assore, being Assore House, 15 Fricker Road, Tlovo Boulevard, Johannesburg.

5. Salient dates and times
Circular and notice of general meeting posted to Assore Shareholders on
Last day for receipt of forms of proxy for the general meeting by 10:00 on Wednesday, 8 September
General meeting to be held at 10:00 on Friday, 10 September
Special resolutions lodged with the Registrar of Companies Friday, 10 September
Results of the general meeting released on Securities Exchange News Service (“SENS”) on September
Results of the general meeting published in the press on Monday, 13 September
Finalisation announcement Thursday, 16 September
Last day to trade in Assore Shares with a par value of 2.5 cents each on Thursday, 23 September
Shares with a par value of 0.5 cent each listed on Monday, 27 September
Shares with a par value of 2.5 cents each suspended on Monday, 27 September
Trading in shares with a par value of 0.5 cent each commences at 9:00 under the new ISIN: ZAE0000146932 and the same JSE code ASR on September
Record date for the Sub-division Friday, 1 October
Replacement certificates reflecting the Sub-division will be posted to certificated Assore Shareholders whose share certificates have been received by 12:00 on Friday, 1 October 2010 on or about Monday, 4 October
If share certificates have not been received by 12:00 on Friday 1 October 2010 replacement share certificates will be posted within five business days of receipt of the share certificates. Dematerialised Assore Shareholders will have their accounts at their CSDP or broker updated on

Shares with a par value of 2.5 cents each terminated at the commencement of trade on

Notes:
1. The abovementioned dates and times are South African local dates and times, and are subject to change. Any such material change will be released on SENS and published in the South African press.
2. If the date of the general meeting is adjourned or postponed, forms of proxy must be received by no later than 48 hours prior to the time of the adjourned or postponed general meeting, provided that, for the purposes of calculating the latest time by which forms of proxy must be received, Saturdays, Sundays and South Africa's public holidays will be excluded.
3. Share certificates with a par value of 2.5 cents each per share may not be dematerialised or rematerialised after Friday, 23 September 2010.

Illovo Boulevard
Johannesburg
20 August 2010
Sponsor to Assore
Standard Bank
Legal adviser to Assore
Webber Wentzel

Date: 20/08/2010 12:19:01 Produced by the JSE SENS Department. The SENS service is an information dissemination service administered by the JSE Limited ('JSE'). The JSE does not, whether expressly, tacitly or implicitly, represent, warrant or in any way guarantee the truth, accuracy or completeness of the information published on SENS. The JSE, their officers, employees and agents accept no liability for (or in respect of) any direct, indirect, incidental or consequential loss or damage of any kind or nature, however arising, from the use of SENS or the use of, or reliance on, information disseminated through SENS.

Source: McGregor BFA
Appendix 4 – “Magic Formula” portfolio annual return generated under an alternative study

<table>
<thead>
<tr>
<th>Year</th>
<th>Portfolio annual return</th>
<th>ALSI TRI annual return</th>
<th>Portfolio monthly mean return</th>
<th>ALSI TRI monthly mean return</th>
<th>Portfolio standard deviation</th>
<th>ALSI TRI standard deviation</th>
<th>Risk-adjusted annual return</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998</td>
<td>-22.42%</td>
<td>-6.33%</td>
<td>-1.32%</td>
<td>0.29%</td>
<td>0.1272</td>
<td>0.1293</td>
<td>-22.68%</td>
</tr>
<tr>
<td>1999</td>
<td>73.10%</td>
<td>71.31%</td>
<td>4.78%</td>
<td>4.71%</td>
<td>0.0474</td>
<td>0.0528</td>
<td>73.30%</td>
</tr>
<tr>
<td>2000</td>
<td>-7.30%</td>
<td>0.15%</td>
<td>-0.53%</td>
<td>0.15%</td>
<td>0.0466</td>
<td>0.0549</td>
<td>-8.62%</td>
</tr>
<tr>
<td>2001</td>
<td>30.47%</td>
<td>35.89%</td>
<td>2.43%</td>
<td>2.88%</td>
<td>0.0629</td>
<td>0.0799</td>
<td>29.01%</td>
</tr>
<tr>
<td>2002</td>
<td>13.11%</td>
<td>-10.23%</td>
<td>1.09%</td>
<td>-0.76%</td>
<td>0.0352</td>
<td>0.0535</td>
<td>25.28%</td>
</tr>
<tr>
<td>2003</td>
<td>24.03%</td>
<td>17.60%</td>
<td>1.95%</td>
<td>1.57%</td>
<td>0.0562</td>
<td>0.0682</td>
<td>25.40%</td>
</tr>
<tr>
<td>2004</td>
<td>37.99%</td>
<td>25.36%</td>
<td>2.76%</td>
<td>1.96%</td>
<td>0.0288</td>
<td>0.0372</td>
<td>41.69%</td>
</tr>
<tr>
<td>2005</td>
<td>36.60%</td>
<td>47.70%</td>
<td>2.70%</td>
<td>3.39%</td>
<td>0.0384</td>
<td>0.0430</td>
<td>35.25%</td>
</tr>
<tr>
<td>2006</td>
<td>48.10%</td>
<td>40.58%</td>
<td>3.45%</td>
<td>2.98%</td>
<td>0.0529</td>
<td>0.0468</td>
<td>47.23%</td>
</tr>
<tr>
<td>2007</td>
<td>24.64%</td>
<td>19.40%</td>
<td>1.91%</td>
<td>1.55%</td>
<td>0.0368</td>
<td>0.0365</td>
<td>24.59%</td>
</tr>
<tr>
<td>2008</td>
<td>-19.89%</td>
<td>-23.20%</td>
<td>-1.65%</td>
<td>-1.95%</td>
<td>0.0622</td>
<td>0.0693</td>
<td>-19.51%</td>
</tr>
<tr>
<td>2009</td>
<td>34.00%</td>
<td>31.67%</td>
<td>2.57%</td>
<td>2.52%</td>
<td>0.0480</td>
<td>0.0669</td>
<td>34.92%</td>
</tr>
<tr>
<td>2010</td>
<td>30.48%</td>
<td>18.70%</td>
<td>2.32%</td>
<td>1.57%</td>
<td>0.0410</td>
<td>0.0537</td>
<td>34.13%</td>
</tr>
<tr>
<td>2011</td>
<td>3.94%</td>
<td>4.45%</td>
<td>0.34%</td>
<td>0.39%</td>
<td>0.0216</td>
<td>0.0257</td>
<td>3.84%</td>
</tr>
<tr>
<td>2012</td>
<td>22.71%</td>
<td>26.21%</td>
<td>1.75%</td>
<td>1.99%</td>
<td>0.0243</td>
<td>0.0244</td>
<td>22.70%</td>
</tr>
<tr>
<td>2013</td>
<td>10.33%</td>
<td>19.83%</td>
<td>0.85%</td>
<td>1.56%</td>
<td>0.0236</td>
<td>0.0284</td>
<td>8.40%</td>
</tr>
</tbody>
</table>

Source: Howard, 2015:38
### Appendix 5 – Standard Deviation of the “Magic Formula” investment strategy on the South African market

<table>
<thead>
<tr>
<th></th>
<th>SIX MONTHS*</th>
<th>ONE YEAR</th>
<th>TWO YEARS</th>
<th>FIVE YEARS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5 SHARES</td>
<td>10 SHARES</td>
<td>15 SHARES</td>
<td>20 SHARES</td>
</tr>
<tr>
<td>2005 to 2006</td>
<td>14%</td>
<td>14%</td>
<td>24%</td>
<td>22%</td>
</tr>
<tr>
<td>2006 to 2007</td>
<td>23%</td>
<td>21%</td>
<td>23%</td>
<td>20%</td>
</tr>
<tr>
<td>2007 to 2008</td>
<td>21%</td>
<td>24%</td>
<td>24%</td>
<td>24%</td>
</tr>
<tr>
<td>2008 to 2009</td>
<td>24%</td>
<td>28%</td>
<td>27%</td>
<td>26%</td>
</tr>
<tr>
<td>2009 to 2010</td>
<td>21%</td>
<td>17%</td>
<td>21%</td>
<td>19%</td>
</tr>
<tr>
<td>2010 to 2011</td>
<td>17%</td>
<td>16%</td>
<td>19%</td>
<td>18%</td>
</tr>
<tr>
<td>2011 to 2012</td>
<td>10%</td>
<td>20%</td>
<td>20%</td>
<td>19%</td>
</tr>
<tr>
<td>2012 to 2013</td>
<td>17%</td>
<td>21%</td>
<td>19%</td>
<td>18%</td>
</tr>
<tr>
<td>2013 to 2014</td>
<td>12%</td>
<td>26%</td>
<td>23%</td>
<td>23%</td>
</tr>
<tr>
<td>2014 to 2015</td>
<td>29%</td>
<td>26%</td>
<td>26%</td>
<td>26%</td>
</tr>
<tr>
<td>Geometric Average</td>
<td>19%</td>
<td>21%</td>
<td>23%</td>
<td>21%</td>
</tr>
</tbody>
</table>

### Appendix 6 – Risk Adjusted Return of the “Magic Formula” investment strategy on the South African market

<table>
<thead>
<tr>
<th></th>
<th>SIX MONTHS*</th>
<th>ONE YEAR</th>
<th>TWO YEARS</th>
<th>FIVE YEARS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5 SHARES</td>
<td>10 SHARES</td>
<td>15 SHARES</td>
<td>20 SHARES</td>
</tr>
<tr>
<td>2005 to 2006</td>
<td>1.01</td>
<td>1.78</td>
<td>1.15</td>
<td>1.29</td>
</tr>
<tr>
<td>2006 to 2007</td>
<td>2.60</td>
<td>2.92</td>
<td>3.05</td>
<td>3.20</td>
</tr>
<tr>
<td>2007 to 2008</td>
<td>-0.83</td>
<td>-1.35</td>
<td>-1.24</td>
<td>-1.30</td>
</tr>
<tr>
<td>2008 to 2009</td>
<td>0.67</td>
<td>0.08</td>
<td>0.17</td>
<td>0.22</td>
</tr>
<tr>
<td>2009 to 2010</td>
<td>-0.50</td>
<td>-0.28</td>
<td>0.06</td>
<td>0.23</td>
</tr>
<tr>
<td>2010 to 2011</td>
<td>0.94</td>
<td>0.04</td>
<td>0.29</td>
<td>0.66</td>
</tr>
<tr>
<td>2011 to 2012</td>
<td>1.48</td>
<td>1.15</td>
<td>1.46</td>
<td>1.47</td>
</tr>
<tr>
<td>2012 to 2013</td>
<td>1.69</td>
<td>1.51</td>
<td>1.26</td>
<td>1.28</td>
</tr>
<tr>
<td>2013 to 2014</td>
<td>-1.13</td>
<td>0.46</td>
<td>0.36</td>
<td>0.11</td>
</tr>
<tr>
<td>2014 to 2015</td>
<td>-0.33</td>
<td>-0.09</td>
<td>-0.07</td>
<td>-0.06</td>
</tr>
<tr>
<td>Geometric Average</td>
<td>0.44</td>
<td>0.43</td>
<td>0.65</td>
<td>0.53</td>
</tr>
</tbody>
</table>
Appendix 7 – Single Factor ANOVA table and related Equation showing consistent returns for the 6 month holding period grouping

\[ H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 \]

Where:

\[ \mu = \text{Mean Return} \]

Anova: Single Factor

<table>
<thead>
<tr>
<th>Groups</th>
<th>Count</th>
<th>Sum</th>
<th>Average</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIX MONTH - 5 SHARES</td>
<td>10</td>
<td>1.831</td>
<td>0.183</td>
<td>0.056</td>
</tr>
<tr>
<td>SIX MONTH - 10 SHARES</td>
<td>10</td>
<td>1.990</td>
<td>0.199</td>
<td>0.065</td>
</tr>
<tr>
<td>SIX MONTH - 15 SHARES</td>
<td>10</td>
<td>2.222</td>
<td>0.222</td>
<td>0.067</td>
</tr>
<tr>
<td>SIX MONTH - 20 SHARES</td>
<td>10</td>
<td>2.172</td>
<td>0.217</td>
<td>0.061</td>
</tr>
</tbody>
</table>

**ANOVA**

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>P-value</th>
<th>F crit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Groups</td>
<td>0.00961561</td>
<td>3</td>
<td>0.003</td>
<td>0.052</td>
<td>0.984</td>
<td>2.866</td>
</tr>
<tr>
<td>Within Groups</td>
<td>2.233934992</td>
<td>36</td>
<td>0.062</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>2.243550601</td>
<td>39</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As \( p > 0.05 \), cannot reject \( H_0 \)
### Appendix 8 – “Magic Formula” average share turnover on rebalancing date

<table>
<thead>
<tr>
<th></th>
<th>SIX MONTHS</th>
<th>ONE YEAR</th>
<th>TWO YEARS</th>
<th>FIVE YEARS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5 SHARES</td>
<td>10 SHARES</td>
<td>15 SHARES</td>
<td>20 SHARES</td>
</tr>
<tr>
<td></td>
<td>5 SHARES</td>
<td>10 SHARES</td>
<td>15 SHARES</td>
<td>20 SHARES</td>
</tr>
<tr>
<td></td>
<td>30 SHARES¹⁷</td>
<td>5 SHARES</td>
<td>10 SHARES</td>
<td>15 SHARES</td>
</tr>
<tr>
<td>Portfolio Average Stock T/O %</td>
<td>60%</td>
<td>59%</td>
<td>59%</td>
<td>57%</td>
</tr>
<tr>
<td>Yearly Average Stock T/O %</td>
<td></td>
<td></td>
<td></td>
<td>59%</td>
</tr>
</tbody>
</table>

### Appendix 9 – “Magic Formula” basis of selection percentage

<table>
<thead>
<tr>
<th></th>
<th>SIX MONTHS</th>
<th>ONE YEAR</th>
<th>TWO YEARS</th>
<th>FIVE YEARS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5 SHARES</td>
<td>10 SHARES</td>
<td>15 SHARES</td>
<td>20 SHARES</td>
</tr>
<tr>
<td></td>
<td>5 SHARES</td>
<td>10 SHARES</td>
<td>15 SHARES</td>
<td>20 SHARES</td>
</tr>
<tr>
<td></td>
<td>30 SHARES¹⁷</td>
<td>5 SHARES</td>
<td>10 SHARES</td>
<td>15 SHARES</td>
</tr>
<tr>
<td>Percentage Final Data</td>
<td>69%</td>
<td>77%</td>
<td>76%</td>
<td>74%</td>
</tr>
</tbody>
</table>
Appendix 10 – *Single Factor ANOVA table and related Equation showing the potential selection bias of year-end financial results*

\[ H_0: \mu_1 = \mu_2 \]

Where:

\[ \mu = \text{Mean Return} \]

Anova: Single Factor

<table>
<thead>
<tr>
<th>SUMMARY</th>
<th>Groups</th>
<th>Count</th>
<th>Sum</th>
<th>Average</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Portfolio based on Final year-end information</td>
<td>17</td>
<td>12.48602</td>
<td>0.73447</td>
<td>0.00223</td>
</tr>
<tr>
<td></td>
<td>Publicly available information at rebalance date</td>
<td>1</td>
<td>0.60251</td>
<td>0.60251</td>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ANOVA</th>
<th>Source of Variation</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>P-value</th>
<th>F crit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Between Groups</td>
<td>0.0164</td>
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<td>0.0164</td>
<td>7.3891</td>
<td>0.0152</td>
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<tr>
<td></td>
<td>Within Groups</td>
<td>0.0356</td>
<td>16</td>
<td>0.0022</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>Total</td>
<td>0.0521</td>
<td>17</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

As \( p < 0.05 \), reject \( H_0 \) and conclude that there is a difference between the information selected in the “Magic Formula” analysis and available financial information.
Appendix 11 – JSE ALSI TRI, benchmark portfolio, relative performance over the explicit period of the study

![JSE ALSI TRI Index Close Graph]

Appendix 12 – Cumulative performance of top performing “Magic Formula” portfolios

![Cumulative Performance of Top Performing "Magic Formula" Portfolios Graph]
### 2005 to 2010 - RETURN CALCULATED

<table>
<thead>
<tr>
<th>RANKED STOCK</th>
<th>COMBINED RANK</th>
<th>TICKER</th>
<th>Share Price Bought</th>
<th>Shares bought</th>
<th>Share Price Sold</th>
<th>Share Price Return</th>
<th>Cash Return (Div)</th>
<th>Total Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>AME</td>
<td>11.60</td>
<td>4.31</td>
<td>33.00</td>
<td>142</td>
<td>-</td>
<td>142</td>
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<tr>
<td>2</td>
<td>27</td>
<td>SAB</td>
<td>126.00</td>
<td>0.40</td>
<td>221.20</td>
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<td>9</td>
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<tr>
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<td>20.83</td>
<td>1.35</td>
<td>28</td>
<td>9</td>
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<tr>
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<td>5</td>
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<tr>
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<td>0.88</td>
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<td>73</td>
<td>16</td>
<td>89</td>
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<td>39</td>
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<td>71.80</td>
<td>0.70</td>
<td>113.50</td>
<td>79</td>
<td>14</td>
<td>93</td>
</tr>
<tr>
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<td>40</td>
<td>IVT</td>
<td>16.20</td>
<td>3.09</td>
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<td>100</td>
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<td>20.83</td>
<td>1.35</td>
<td>28</td>
<td>9</td>
<td>38</td>
</tr>
<tr>
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<td>43</td>
<td>PNC</td>
<td>1.40</td>
<td>35.71</td>
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<td>195</td>
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<td>71.00</td>
<td>0.70</td>
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<td>79</td>
<td>22</td>
<td>100</td>
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<td>11</td>
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<tr>
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<td>19</td>
<td>69</td>
<td>EOH</td>
<td>4.90</td>
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</table>

#### Initial Investment (ZAR) 1000

<table>
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<tr>
<th>Total Return (ZAR)</th>
<th>1,887</th>
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</thead>
<tbody>
<tr>
<td>Investment (ZAR)</td>
<td>1000</td>
</tr>
<tr>
<td>Return (ZAR)</td>
<td>887</td>
</tr>
<tr>
<td>5 year Total Return (%)</td>
<td>89%</td>
</tr>
<tr>
<td>Annualised Geometric Return</td>
<td>14%</td>
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<tr>
<td>Standard Deviation</td>
<td>59.90</td>
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<tr>
<td>Standard Deviation (%)</td>
<td>63%</td>
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<tr>
<td>Annualised Geometric Std. Dev (%)</td>
<td>10%</td>
</tr>
<tr>
<td>RANKED STOCK</td>
<td>COMBINED RANK</td>
</tr>
<tr>
<td>--------------</td>
<td>---------------</td>
</tr>
<tr>
<td>1</td>
<td>13</td>
</tr>
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<td>2</td>
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<td>19</td>
<td>74</td>
</tr>
<tr>
<td>20</td>
<td>83</td>
</tr>
</tbody>
</table>

**Initial Investment (ZAR)** 1,887

**Total Return (ZAR)** 3,428

**Investment (ZAR)** 1,887

**Return (ZAR)** 1,540

**5 year Total Return (%)** 82%

**Annualised Geometric Return** 13%

**Standard Deviation** 135.17

**Standard Deviation (%)** 79%

**Annualised Geometric Std. Dev (%)** 12%