Influence of Geometry on the Dynamic Behaviour of Steel Tubular Towers for Onshore Wind Turbines

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Abstract

South Africa has recently experienced challenges regarding electricity consumption and availability. As part of the country’s Integrated Resource Plan, these challenges are to be addressed. This involves a 20 year plan which aims to increase electricity supply capacity as well as reduce the reliance on coal power as part of the global trend to become more environmentally friendly. Wind power, specifically, is to account for a large portion of the renewable energy that is expected to become available by 2030. This results in the need for the understanding of wind turbine design by South African engineers. The dynamic analysis of wind turbine structures, is of particular interest to Civil Engineers.

Wind turbine towers are recently of the monopole or tubular type tower, predominantly constructed of either concrete or steel or a combination of both. Steel tubular towers above a height of 80m are generally not recommended for wind turbines owing to cost concerns as well as difficulties in meeting dynamic behaviour requirements. Concrete towers and steel-concrete hybrid towers are recommended for this height regime. The aim of this study was to assess the prospective use of steel tubular towers of varying geometric shape for wind turbines with tower heights of 80m or greater. The study focussed on the analysis of natural frequency and assessing the applicability of steel tubular towers of geometric shapes that have not been previously explored or reported. The turbine of choice for this study was the Vestas V112 3MW type as this is one of the most commonly used and more efficient turbines for towers of this height regime.

The results of this study showed that steel monopole towers of heights of 80m and more are still viable options for wind turbine towers. Various geometric tower cases of heights varying from 80m to 120m, produced acceptable fundamental natural frequencies within the allowable frequency range for a Vestas V112 3MW turbine.
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Notation

The notation used throughout this thesis is based on standard nomenclature adopted by Industry. Where they do not conform to this standard, distinctions are clearly indicated in the dissertation. Symbols are defined as they first appear within the text. Those which appear more than once are listed below. SI units are made use of for this study.

**Acronyms**

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Definition</th>
</tr>
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<tbody>
<tr>
<td>3D</td>
<td>Three Dimension/Dimension(s)</td>
</tr>
<tr>
<td>ASCE</td>
<td>American Society of Civil Engineers</td>
</tr>
<tr>
<td>AWEA</td>
<td>American Wind Energy Association</td>
</tr>
<tr>
<td>CFD</td>
<td>Computational Fluid Dynamics</td>
</tr>
<tr>
<td>DoE</td>
<td>Department of Energy</td>
</tr>
<tr>
<td>ECD</td>
<td>Extreme Coherent Gust with Direction Change</td>
</tr>
<tr>
<td>EDC</td>
<td>Extreme Direction Change</td>
</tr>
<tr>
<td>EOG</td>
<td>Extreme Operating Gust</td>
</tr>
<tr>
<td>ETM</td>
<td>Extreme Turbulence Model</td>
</tr>
<tr>
<td>EWS</td>
<td>Extreme Wind Shear</td>
</tr>
<tr>
<td>EWM</td>
<td>Extreme Wind Speed Model</td>
</tr>
<tr>
<td>FEA</td>
<td>Finite Element Analysis</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite Element Method(s)</td>
</tr>
<tr>
<td>GWEC</td>
<td>Global Wind Energy Council</td>
</tr>
<tr>
<td>HAWT</td>
<td>Horizontal Axis Wind Turbine</td>
</tr>
<tr>
<td>IEC</td>
<td>International Electrotechnical Commission</td>
</tr>
<tr>
<td>kW</td>
<td>Kilowatt</td>
</tr>
<tr>
<td>kWh</td>
<td>Kilowatt Hour</td>
</tr>
<tr>
<td>MW</td>
<td>Megawatt</td>
</tr>
<tr>
<td>NSW</td>
<td>New South Wales (Australia)</td>
</tr>
<tr>
<td>NTM</td>
<td>Normal Turbulence Model</td>
</tr>
<tr>
<td>NWP</td>
<td>Normal Wind Profile Model</td>
</tr>
<tr>
<td>IRP</td>
<td>Integrated Resource Plan</td>
</tr>
<tr>
<td>REIPPPP</td>
<td>Renewable Energy Independent Power Producer Procurement Programme</td>
</tr>
<tr>
<td>rpm</td>
<td>Rotations/revolutions per minute</td>
</tr>
</tbody>
</table>
TSR  Tip Speed Ratio
USA  United States of America
VAWT  Vertical Axis Wind Turbine
ZAR  Currency symbol for South African Rand

Symbols

1P  Rotor rotational frequency
3P  Blade passing frequency
A  Area
$C_p$  Power Coefficient
D  Outer boundary dimension
$D_n$  Outer base diameter of the $n^{th}$ section of tower
$d_n$  Inner base diameter of the $n^{th}$ section of tower
E  Young’s Modulus
$f_n$  Fundament natural frequency
$f_{n-\text{total}}$  Final fundamental frequency
$f_{ntower}$  Overall total fundamental natural frequency of tower
G  Dynamic shear modulus
H  Tower height
$h_n$  Height of $n^{th}$ section of tower
I  Moment of inertia
$I_n$  Moment of inertia of $n^{th}$ section of tower
$I_{\text{circle}}$  Moment of inertia for a circular section
$I_{\text{octagon}}$  Moment of inertia for an octagonal section
$k$  General structure stiffness
$k_n$  Stiffness of $n^{th}$ section of tower
$k_{\text{foundation}}$  Foundation stiffness
$k_{\text{horizontal}}$  Horizontal soil spring stiffness
$k_{\text{rotational}}$  Rotational soil spring stiffness
$k_{\text{torsional}}$  Torsional soil spring stiffness
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$k_{vertical}$</td>
<td>Vertical soil spring stiffness</td>
</tr>
<tr>
<td>$m$</td>
<td>General structure mass</td>
</tr>
<tr>
<td>$m_n$</td>
<td>Mass of $n^{th}$ section of tower</td>
</tr>
<tr>
<td>$m_{foundation}$</td>
<td>Mass of foundation</td>
</tr>
<tr>
<td>$m_{tower}$</td>
<td>Mass of tower</td>
</tr>
<tr>
<td>$m_{turbine}$</td>
<td>Mass of turbine</td>
</tr>
<tr>
<td>$P$</td>
<td>Power</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density</td>
</tr>
<tr>
<td>$R$</td>
<td>Radius</td>
</tr>
<tr>
<td>$v$</td>
<td>Velocity</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>$V_{ref}$</td>
<td>Reference wind speed average over 10 minutes</td>
</tr>
<tr>
<td>$\omega_n$</td>
<td>Natural frequency</td>
</tr>
<tr>
<td>$\omega_{n-fixed\ base}$</td>
<td>Fundamental frequency of tower with a fixed base</td>
</tr>
<tr>
<td>$\omega_{n-foundation}$</td>
<td>Fundamental frequency of foundation</td>
</tr>
<tr>
<td>$\omega_{n-total}$</td>
<td>Final fundamental frequency</td>
</tr>
<tr>
<td>$\omega_N$</td>
<td>Fundamental natural frequency of $n^{th}$ section of tower</td>
</tr>
<tr>
<td>$\omega_{tower}$</td>
<td>Overall total fundamental natural frequency of tower</td>
</tr>
<tr>
<td>$z$</td>
<td>Height above ground</td>
</tr>
<tr>
<td>$z_{hub}$</td>
<td>Height of hub</td>
</tr>
</tbody>
</table>
1. Introduction

1.1 Overview

1.1.1 Wind Energy in South Africa

South Africa’s long term electricity demands have been detailed in the Integrated Resource Plan (IRP, 2010) by the Department of Energy (DoE). This document provides information relating to the electricity capacity required, the type of capacity and basic timelines and cost. It is a 20 year plan to increase electricity capacity from 2010 to 2030. Additional new build of 17 800MW of renewable energy has been specified with wind power being 8400MW of the renewable allocation that is expected to become available by 2030 (IRP, 2010). Currently, South Africa’s main source of electricity comes from coal power plants. This is due to the fact that South Africa has an abundance of coal and in fact has the 7th largest coal reserves in the world (GWEC, 2014). South Africa’s energy mix also includes small percentages of natural gas, nuclear, hydroelectricity and more recently also wind and solar. However, it is South Africa’s substantial reliance on coal that has resulted in the country being the 12th largest CO₂ emitter in the world (GWEC, 2014). The implementation of the 8400MW of wind power will also aid in reducing the negative climate effects due to the prevention of a further increase in carbon emissions over the long term.

In line with the new build wind power specified in the IRP 2010, the South African government officially established the Renewable Energy Independent Power Producer Procurement Programme (REIPPPP) in 2011. According to Fourie et al. (2015), the REIPPPP has been set up in order to improve South Africa’s power generation capacity, to reduce the dependency of the country on fossil fuels and to create a local renewable energy industry that will aid the growth of socio-economic development and environmental sustainability.

1.1.2 Growth of Wind Energy in South Africa

The REIPPPP is focussed on the procurement of electricity by the power utility, Eskom, from the private sector (Fourie et al., 2015). This programme has been highly successful and resulted in a substantial increase in wind power in South Africa. Initially it took up to 10 years to achieve the first 10MW of wind power by 2013 and thereafter, rapid growth in the wind industry took place. 560MW of wind power was added to the national electricity grid during 2014 alone (GWEC, 2014) and a further 483MW during 2015 (GWEC, 2016). This rapid growth of the wind power industry in a period of approximately three years, has placed South Africa as one of the leading new wind markets globally (GWEC, 2014). This has resulted in an increased growth of local industry as well as a reduction of cost per kWh of electricity as depicted in Table 1. Local industry is evidenced in both steel and concrete towers now being manufactured locally and a local blade manufacturing facility which is expected to be established with an international partner in the near future (GWEC, 2014). One of these local industry examples includes the DCD Group with its head office based in Vereeniging, South Africa, who manufacture steel tubular towers for wind turbines. With the rapid growth of the wind power industry in South Africa, it is highly recommended that local engineers come to understand and grasp design concepts related to wind turbines. For civil engineers in particular, the dynamic load effects on turbine structures, are of particular interest.
Table 1: Reduction in bid price (Rand/kWh) for wind power throughout the REIPPPP bid rounds (Fourie et al., 2015)

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<tr>
<td></td>
<td>1.28</td>
<td>1.01</td>
<td>0.74</td>
</tr>
</tbody>
</table>

1.1.3 Dynamic Response of Wind Turbine Towers

Wind turbine structures are generally comprised of a turbine and blades supported on a tower and a foundation. There are various types of tower configurations in existence. Recently, the towers are largely of the monopole type – constructed of either concrete or steel material or a combination of both. Wind turbine towers are unique structures in that they are subjected to dynamic loads from more than one source. Dynamic loads exerted on the structure include those from rotation of the turbine itself as well as those imposed by the changing wind conditions. The dynamic analysis of any structure typically prioritises the avoidance of resonance. In order for resonance to be avoided, the wind turbine tower should be designed in such a way that its natural frequency does not coincide with the loading frequencies. The natural frequency of the wind turbine structure is largely dependent on its mass and stiffness. More specifically, the structure’s Eigen frequencies are governed by the stiffness of the tower, the foundation and the founding soil. These topics are explored within the literature review portion of this study.

1.2 Aim

The goal of this study is to assess the influence of altering the geometry and shape of the wind turbine tower on its natural frequency and to assess the applicability of steel tubular towers of geometric shapes that have not been previously explored or reported.

1.3 Objectives

The objective of this study is to assess the prospective use of steel tubular towers of varying geometric shape for wind turbines with tower heights of 80m or greater. It has been recommended in literature that concrete towers or steel-concrete hybrid towers be made use of for towers in excess of 80m in height instead of the uniformly tapering truncated cone shaped steel tubular towers. The objectives of this study include:

- Assessment of applicability of steel tubular towers of heights 80m, 100m and 120m,
- Assessment of applicability of steel tubular towers with different shape and geometric regimes from those typically used,
- Assessment of the influence of the shape and geometry on the dynamic behaviour of steel tubular towers.

1.4 Scope

Steel tubular towers above a height of 80m are generally not recommended for wind turbines, but rather concrete towers and steel-concrete hybrid towers are recommended for this height regime (or range). This is due, in part, to the limitation on base diameter sections subject to South African road transportation regulations. This limit is also based on cost of materials since it has been found that tower heights of more than 80m are more economical when constructed from concrete or a combination of steel and concrete. These studies have been carried out on tubular steel towers which are typically of a tapering conical shape only. It appears that there is no study in the current literature on effects of varying the geometry and shape of steel tower monopoles. This study focuses on the dynamic analysis of land based steel tubular towers of height 80m and greater with varying geometry.
and shape. The steel tubular towers are assessed for compatibility with the Vestas V112 3MW turbine as this is one of the most commonly used and more efficient turbines for towers of this height regime. For the purposes of this study the emphasis will be on large scale wind turbines for power utility application. Hence, the focus is placed on upwind type, three bladed, horizontal axis wind turbines (HAWT). The aerodynamics of the blade is a complex subject in its own right and is outside the scope of this study. A cost analysis will not be included in this study. Wind loading dynamic effects have not been taken into account in this study. Seismic loading and its dynamic effects too have not been considered.

1.5 Methodology
The dynamic analysis of this study, comprises the modal analysis in order to obtain natural frequencies. Initial tower geometries will be based on a static design of the tower. These will then be adjusted for suitability to ensure that dynamic requirements are met. This particular study will make use of both analytical approximations with the assistance of MATLAB as well as Finite Element Analysis (FEA) methods through the use of the commercial software, Abaqus, to obtain the fundamental natural frequencies. The results obtained from the FEA will then be compared to those obtained from simple analytical models assessed with MATLAB. The influence of the soil-structure interaction on the Eigen frequency will also be assessed by means of the spring stiffness method.
2. Literature Review

2.1 Wind Turbine Fundamentals

Wind is a natural resource which can be harnessed to produce electricity. This has been enabled through the use of wind turbines. A wind turbine comprises a foundation, a tower, a rotor consisting of blades (rotor blades) attached to hub (rotor hub) as well as a tower head. The tower head, more technically referred to as the nacelle, houses the gearbox and the generator as well as some other mechanical equipment and electronic controls. This is depicted below in Figure 1. It appears small in relation to the overall turbine and tower however, the nacelle alone can have a mass of up to 300 tonnes.

![Figure 1: Parts of a wind turbine](http://www.maglev.net/news/wind-turbine-technology/)

The moving air (wind) passes through the blades which causes them to rotate. In order to best utilise the wind energy, nacelles on upwind turbines (as considered in this study) are able to move so as to ensure they always face the approaching wind direction normally i.e. at 90°. An upwind turbine is one in which the rotor hub and blades always face the approaching wind direction. This is the most commonly selected turbine configuration (Burton et al., 2001). The turbines move to face the wind through the use of an anemometer, which measures the wind speed and a wind vane, measuring the wind direction. This data is then transmitted to the controller. The controller is responsible for signalling the starting up and shutting down of the turbine in minimum required and maximum allowable wind speeds respectively. A minimum wind speed is required in order to generate power and so the turbine will only start up once this speed is measured by the anemometer. The shutdown is precautionary so as to prevent any possible damage to the turbine due to excessive wind speeds.
Once wind is at adequate speed and start-up has occurred, the deviation of the wind direction measured by the anemometer, sends a signal to the controller, which in response then signals to the yawing mechanism to correct the orientation of the turbine. The yaw motor and drive located just below the nacelle then adjust the position of the rotor so that it faces the wind, no matter the wind direction, for optimal usage. Blade rotation by the wind then takes place due to their aerodynamic design and airfoil technology. This rotation is generally at a relatively low speed in terms of what is required for electricity generation; in most cases typically between about 30 and 60 rotations per minute (rpm). The rotational speed is internally enhanced through the use of a gearbox. A low speed shaft extends from the blade hub into the nacelle. These components are indicated in Figure 2. Typically, for most wind turbines, the rotational force is transferred to the gears which then increase the low rotational speed of the first shaft to the second high speed shaft. This high speed is then transferred to the generator which enables the conversion of kinetic energy to electrical energy, producing electricity. In the case of wind turbines used for large scale utility power production, the electricity produced by the generator is then transferred to a transformer and then a substation where voltages are increased for efficient long distance travel to the main grid lines.

Figure 2: Components of the turbine's nacelle (World Steel Association, 2012)

2.2 Energy Efficiency of Wind Turbines
Energy efficiency can be taken as the ratio of electrical energy extracted from that energy contained in the primary fuel i.e. the kinetic energy contained in the wind itself in the case of wind turbines. The fundamental wind power equation has been derived from the definition of kinetic energy. The fundamental power equation is given in equation 1.
**Equation 1: Fundamental wind power equation**

\[ P = \frac{1}{2} \cdot \rho \cdot A \cdot v^3 \]  

where:

- \( P \) = power (W)
- \( \rho \) = density of air (kg/m\(^3\))
- \( A \) = area swept by the rotor blades, \( A = \pi R^2 \) (m\(^2\)) where \( R \) = radius (m)
- \( v \) = velocity of the moving air/wind (m/s)

The equation is dependent on the air flowing through the turbine blade swept area. By this definition of efficiency, a turbine can only be considered to be 100% efficient if the total kinetic energy of the air stream is removed and converted to electrical energy. However this is not possible as it would mean that the air behind the blades would be stationary, therefore inhibiting any air flow through the blade swept area (Royal Academy of Engineering, 2014). The best possible efficiency available for the extraction of wind power by a turbine is 59.3%. This is the theoretical limit – sometimes also referred to as the Betz limit – and has been independently shown to be true by Albert Betz (1920), Frederick Lanchester (1915) and Nikolay Zhukovsky (1920). The highest achievable efficiency ratio (or power coefficient, \( C_p \)) in practice has been found to be about 0.47; that is around 80% of the theoretical Betz limit. However, in reality this power coefficient, \( C_p \), is normally within the range of 0.35 – 0.45 for even some of the world’s best designed wind turbines (RWE npower renewables, 2016). The \( C_p \) value differs from turbine to turbine depending on the location, the wind velocity that the specific turbine is operating in, the turbine type, strength and durability. Hence, to get a true indication of the turbine power output, the fundamental power equation needs to take the actual specific efficiency into account. Equation 1 has been adapted for this purpose as shown in equation 2.

**Equation 2: Turbine power output equation**

\[ P_{\text{output}} = \frac{1}{2} \cdot \rho \cdot A \cdot v^3 \cdot C_p \]  

The number of blades on a wind turbine is also an aspect that would have an effect on the power coefficient and hence the efficiency of the turbine. As a blade rotates, it disturbs the air leaving a turbulent wake behind. As the number of blades increases, the probability of the next blade passing through air that is still turbulent from the blade before, is very high. This inhibits its ability to efficiently extract power from the wind, hence the motive to reduce the number of blades (Ragheb & Ragheb, 2011). As for the reduction in the number of blades, single bladed turbines have the advantage of less material and hence less costs. However this results in rotor balancing challenges and excessive tip speeds which can also introduce extreme vibrations. Two bladed turbines require what is known as a higher tip speed ratio (TSR) than that of three bladed turbines in order to operate at the same power efficiency. It is for this reason that most of world’s wind turbines have three blades. The blade efficiency factors are also dependent on the rotor speed, material solidity ratio, blade chord width and blade twist angle.

From equation 2, it is clear that the output power increases with increasing average steady wind velocity – which generally increases with height above ground, hence the trend in towers to become higher and higher. Typically, a turbine will only output power when experiencing wind speeds of 3m/s or more. Optimum power is achieved at steady wind speeds of about 12m/s and wind turbines are designed to cut out when wind speeds exceed around 25m/s. This is depicted in Figure 3. The output power also increases as the area through which the wind passes increases. This has resulted in an increase in turbine blade lengths, hence an increase in the swept area radius. Some of the first small
wind turbines in the early 1980s, had rotor diameters of about 15m and produced power outputs of around 50kW. By the year 2000, it was common to see turbines with rotor diameters of 80m that were outputting power of 2MW. As of April 2014, the largest turbine constructed had a rotor diameter of 164m and produced a power output of 8MW. To put this in perspective, the length of a single blade of this turbine is greater than the wingspan of an Airbus A380 (Royal Academy of Engineering, 2014). However this was an offshore turbine.

Onshore turbines are not expected to increase in size at the same fast rate that they have been in the past as turbine designers and manufacturers don’t expect onshore turbine diameters to be larger than about 100m with power outputs of around 3 to 5MW (Thresher, Robinson & Veers, 2007). However, as advances in technology have taken place over time, each group of turbine designers have anticipated that their machines have now reached the limits in terms of a size barrier and each time they were proven wrong by the next generation of turbines as sizes continued to increase and costs were reduced. Figure 4 shows the trend in the USA of increasing rotor diameters over time resulting in higher power output; with initial diameters of around 10m in 1980 and more recent diameters of 93m corresponding to power increases from about 50kW to 3MW respectively for onshore turbines.

One of the key arguments for the limit on the size of rotors has commonly been referred to as the “square/cube principle”. This principle goes to show that when the blade length increases, the power output increases by the ‘square’ of the length or diameter of the rotor blades since the power output is proportional to the area swept by the rotor. However, the volume of the blades – and hence mass and cost – increases by the ‘cube’ of the rotor blade length. This principle assumes that eventually the increased cost of the material for the turbine will far surpass the increased power output making it uneconomical to increase the rotor size. In reality however, engineers have come up with innovative designs in order to minimise the effects of this principle by making use of material more efficiently so as to minimise the increase in mass with increased size. Some of these innovations include the use of light weight carbon fibre material for example. Contrary to the “square/cube principle” a windPACT study has shown that recent turbine blade weights have actually been increasing by an exponent of approximately 2.3 and in fact not by 3. This can be seen in Figure 5 where the recent turbine designs, plotted with triangular plot points, have managed to minimise the increasing mass as the rotor diameters have increased due to the more recent design innovations. In this study limitations on sizes
are not viewed as causing a design restriction as research and development take place and innovation continues to occur.

Figure 4: Growth in wind turbine size in the USA from 1980 to 2015 (Thresher, Robinson & Veers, 2007)

Figure 5: WindPACT study results indicating reduction in blade mass growth due to design innovations (Thresher, Robinson & Veers, 2007)

2.3 Turbine Tower
2.3.1 Background
Wind turbine towers form the support structure of the turbine. The towers are available in various options constructed of either steel or concrete materials with varying tower heights. The most common tower configurations are steel lattice structures, steel monopoles, concrete monopoles and steel-concrete hybrid structures. Earlier wind turbine towers tended to be the lattice type, truss structure; this was most familiar as it was similar to the design of support structures for power lines already in use (Prowell, 2011).
2.3.2 Background to Monopoles

The tower configuration used almost exclusively worldwide is a steel monopole tower supported on a concrete foundation. These towers are generally in the height range of around 60-80m (Thresher et al., 2008). As the need for increased turbine sizes, rotor diameters and power output has emerged, so has the tower height requirement. Hence the trend of more recent tower heights of around 90 – 120m hub heights for turbines with power outputs of 3MW or more. This trend is shown clearly in Figure 6. This increase in height demanded larger diameters at the base of the steel tubular (monopole) as well as increased plate thicknesses. Due to road transport regulations of limitations on sections sizes of around 4.2m (Department of Transport, 2009), steel monopole structures' growth in tower height has been restricted. The increase in plate thickness also meant a rise in cost of steel tubular towers resulting in steel towers with heights of more than 80m being costly. This is confirmed in a study at BarcelonaTech stating that steel towers should only be used for hub heights up to 80m (Petcu & Mari-Bernat, 2007). In addition to the size limitation, steel tubular towers of heights exceeding 85m are reportedly no longer able to meet the vibration criteria (Harte & van Zijl, 2007). It is for these reasons that there has been an increased recent movement towards concrete monopole and steel-concrete hybrid structures for turbines with towers exceeding heights of 80m.

![Figure 6: Increase in tower height over time (IEA, 2013)](image)

2.3.3 Steel Lattice Tower

Steel lattice structures are comprised of four main legs braced with a truss like system of usually Angle sections bolted together. They are typically used for smaller size turbines. The main advantage of the steel lattice tower is the substantial reduction in mass of the actual tower compared to steel or concrete monopole structures resulting in cost savings. Due to the nature of this tower construction, there is also no limit on the size at the base unlike that of the steel tubular sections. Some of the drawbacks of lattice structures include the need for a high number of bolts subject to regular maintenance and replacement as well as problematic dynamic properties and torsional stiffness of the tower itself. Lastly, as far as visual aesthetics are considered, the lattice tower is considered to be unfavourable in modern times (Engström, et al., 2010).
2.3.4 Steel Monopole Tower

The steel monopole tower is one comprised of steel plate sections to form a large tubular shape. As mentioned above, this is the most common type of turbine tower on the market globally. Individual structural steel plates form half circle sections which are welded together (longitudinal weld) to complete cylinders of typically 2-3m high sections. These large circular hollow cylindrical sections are then joined together with transverse welds. Several cylinders join to form a section of the tower. The tower ends with a steel flange which allows for bolting to the end of the next section. This is shown in Figure 7. Vertical friction bolted connections and horizontal friction bolted connections of smaller 2-3m sections are not commonly used however, they have recently been used by Siemens for a 2.3MW prototype in Denmark (Wind Power Monthly, 2012). The overall tower structure shows weaknesses at positions of welds and flanges. Ordinary structural grade (S355JR) steel is sufficient for steel tubular towers since the structure’s strength and stiffness is mostly governed by its geometry, including diameter size and plate thickness, and its joints (Engström, et al., 2010). Typically steel monopole towers have a subtle taper from base to the top resulting in a cone like shape (Burton, et al., 2001). The advantage of steel tubular towers is the quick erection times on site and cost savings in construction time.

![Diagram of a typical steel tubular tower](image)

**Figure 7: Two sections of a typical steel tubular tower (Engström, et al., 2010)**

2.3.5 Concrete Monopole Tower

Concrete monopole towers are also generally tubular shell structures just like steel monopoles. They tend to have a similar cone like shape, although their shell thicknesses are greater. The concrete tower can be comprised of precast sections or the tower can be cast in situ. Concrete towers have the advantage that manufacturing of precast sections may be as small as necessary, or that the concrete can simply be placed on site, therefore having no limit on the size of the diameter of the section for transportation enabling taller towers (van Zyl & van Zijl, 2015). However, there are indirect limitations on the size of sections. This is based on the weight limitations which apply to the cranes used for erection. Concrete towers have the added advantage of longer life spans than that of steel towers,
provided the use of appropriate corresponding reinforcing cover thicknesses is observed. Therefore one concrete tower may be used for two generations of turbines with an obvious cost saving (Engström, et al., 2010). Precast concrete sections are manufactured in large factories, referred to as precast yards in South Africa, which allows for higher quality and quicker production times than that achieved with in situ casting on site. The precast concrete sections are then placed on top of one another with cranes on site and tied together with prestressing systems such as post tensioning tendons to form the tower (von der Haar & Marx, 2015).

In situ casting of concrete towers makes use of one of two methods. The first method is slip form construction, also known as sliding construction in Europe, which is continuous casting 24 hours a day. The disadvantage of this method is the added costs of night shift labour due to the continuous casting process. The prestressing tendons are positioned and tensioning takes place after the structure has cured (Engström, et al., 2010). The second method of in situ casting of towers is conventional climbing formwork. This method, with the use of cranes, is the most popular for concrete tower construction in Germany (von der Haar & Marx, 2015). The process involves the erection of the inner formwork along with the fixed reinforcing steel cage into position, followed by the hoisting of the outer formwork into position, after which the concrete can be poured. Pours are carried out in 4m lifts with allowance for the pole taper (von der Haar & Marx, 2015). At the end of tower construction, post tensioning takes place similar to the slip form method. The advantage of these two methods is the continuity of the structure that is monolithically cast without joints.

2.3.6 Steel-Concrete Hybrid Tower
A hybrid tower consists of a lower part that is a larger concrete shell structure with an upper part that is the conventional welded steel shell tower section. It is merely a combination of the two steel and concrete monopoles described previously. This resolves the transportation limit on the size of the steel sections and in reality makes it easier to design the concrete section and solve the Eigen frequency problem of the tower (Engström, et al., 2010). The steel-concrete hybrid tower has recently been shown to be very economical for larger turbines (von der Haar & Marx, 2015). The first hybrid tower, 133m high, from Advanced Tower Systems paired with a Siemens 2.3MW turbine, is shown in Figure 8.

Figure 8: Steel-concrete hybrid tower from Advanced Tower System (Engström, et al., 2010)
2.3.7 Other Tower Alternatives

Wooden towers are almost unheard of in current times. However, a wooden tower has recently been built for a large turbine in Germany. It was constructed by a company called Timber Tower. The advantage of this tower, is the known economic benefits due to the lower cost of timber material (Engström, et al., 2010). Other tower alternatives include lighter steel monopoles (thinner plate thicknesses) with supporting guy wires that provide added stiffness and stability however, these tower configurations are uncommon for large power output turbines (Prowell, 2011). Use of composite materials, such as glass/polyester sheets, for turbine towers could result in lighter weight structures having a positive impact on cost savings. There is however limited information available on these kinds of towers (Lim, Kong & Park, 2013).

![Figure 9: Wooden tower in Germany during and after construction (Engström, et al. 2010)](image)

2.3.8 Recent Studies and Research on Towers

Steel tubular towers have been studied by Way and van Zijl (2015). Three towers of tapering cone like shape were studied. The first had a tower of 80m in height with a base diameter of 4.5m and corresponding plate thickness of 34mm tapering to a diameter of 3m with a corresponding plate thickness of 15mm at the top. The second tower had a height of 100m with a base diameter of 4.5m and corresponding plate thickness of 55mm tapering to a diameter of 3m with a corresponding plate thickness of 15mm at the top. The third had a tower of 120m in height with a base diameter of 4.5m and corresponding plate thickness of 75mm tapering to a diameter of 3m with a corresponding plate thickness of 15mm at the top. The primary variants in this study of steel tubular structures were height and plate thickness (Way & van Zijl, 2015). It is clear from the study that the common base diameter of 4.5m for all towers is due to transportation limit on the size of sections in South African, as mentioned previously. The study compared these three steel tubular towers to three concrete tubular towers and finally three steel-concrete hybrid towers of same three height variations. The study focussed primarily on a cost comparison between the three different types of towers. The outcome revealed that the material costs (of towers and concrete foundations) were higher for the steel towers when compared to the concrete and hybrid towers, especially for tower heights in excess of 100m. The study also revealed that the dynamic stiffness requirement for the 120m high steel tower was not met. It should be noted that this study did not consider steel tubular towers of varying shape and...
geometry. Therefore the conclusion related to material cost and the dissatisfaction of the natural frequency requirement for the 120m high tower, is only valid for linearly tapering cone like steel tubular towers.

von der Haar and Marx (2015) have done dynamic analysis studies on both monolithically cast and precast concrete monopole towers. The towers were comprised of two predominant conical sections (between 0-90m and 90-140m) with a concrete strength class of C60/75. The towers modelled had a height of 140m consisting of a larger lower section which varied in base diameter between 8m and 14m tapering linearly to a diameter of 4.5m at a height of 90m (to ensure blade passing without collision). The top conical section varied from a lower diameter of 4.5m to 3m at the top. The primary variant in this study was the base diameter (von der Haar & Marx, 2015). The study indicated that the base diameter as well as the shell thickness of tower had an influence on the Eigen frequency of the tower however, the effect of the increase in base diameter was found to have a much larger influence. The study concluded that precast concrete towers, while being the fastest to construct, resulted in lower rotational stiffness at the position of joints when compared to monolithically cast reinforced concrete towers. For the particular two cone section shaped towers considered, it was revealed that the precast concrete tower resulted in larger rotations, deflections and bending moments. This study did not account for variance in concrete strength within the two tower cases. Nor did it consider alternative tower geometric shapes.

Information related to studies of hybrid tower structures are less readily available. Way and van Zijl (2015) have done research on steel-concrete hybrid towers of heights 80m, 100m and 120m. These towers consisted of a lower concrete shell structure of heights 40m, 60m, and 80m respectively with corresponding wall thicknesses of 200mm, 200mm and 300mm. All three concrete tower sections had base diameters of 7.5m tapering linearly to 4.6m at the top. The geometry of the upper steel tubular part of the hybrid towers was kept constant; steel sections were all 40m in height with a base diameter of 4.3m tapering to a diameter of 3m at the top with corresponding wall thicknesses varying from 40mm to 25mm respectively. The main design variants for this study on hybrid towers include the height and wall thicknesses of the lower concrete sections (Way & van Zijl, 2015). The study reveals a decrease in the first natural frequency as the lower concrete section increases in height. This is expected due to the increase in tower mass with increasing height. The study also points out an increase in pre-stress force losses with increasing height. Again, this is expected as the pre-stressing cables act over an increased distance. This study did not consider an increase in both concrete and steel sections equally, as the overall hybrid tower increased. Instead it maintained a constant steel upper section as the lower concrete portion of the tower increased in height and thickness. Nor did the study consider lower portion monolithically cast concrete hybrid towers; only precast concrete portions were considered.

A more unconventional complete steel hybrid tower has been studied by Malcolm (2004). The tower comprises a wide lattice/truss base supporting a steel tubular tower along with stabilising guy cables. Various configurations of this type of hybrid were studied. The findings of the study established that the overall cost of a hybrid design of this nature are likely to be greater than that of a single-tube design. This is due to the higher cost of fabrication assembly (Malcolm, 2004). The fact that lattice towers are aesthetically unpopular, occupy much more land space and that this unusual lattice tower-steel monopole hybrid resulted in much higher material costs, suggests that further research in this particular area of wind turbine towers would be futile as this type of design is unlikely to be commonly used. For this reason further research in this area is not recommended.
2.3.9 Deductions

There is an abundance of information available regarding studies that have been done on lattice structures and steel tubular structures. This is expected as it reaffirms the knowledge that majority of existing towers are steel and steel tubular towers in particular. Majority of studies carried out on steel tubular towers however, are only based on the typical tapering cone like tower. Therefore suggested limitations on tubular steel tower heights based on cost and base diameter transportation limitations, are only valid for that particular geometry and shape. There appears to be a lack of information available related to the behaviour of steel tubular towers of varying tower geometry and shape, in particular to towers above 80m in height. There have been some studies on tall concrete towers, both monolithic and precast, however there is room for further research on different tower geometries as this appears to be an unexplored area for wind turbine towers. There is limited information available on the dynamic behaviour of steel-concrete hybrid towers so this could potentially be an area where further studies are required. More specifically on varying heights of towers with respective equivalent increase in lower concrete and upper steel portion heights. Limited information and research is available for composite material towers, however these are unlikely to be used for large power output turbines. The particular area of most interest is that relating to steel tubular towers of varying shape and geometry with heights exceeding 80m. This forms the bases of the current study.

2.4 Wind Turbine Design

2.4.1 Wind Turbine Design Procedure

Wind turbine design is an intricate procedure. Material and construction costs of large wind turbines are substantial and in the order of millions of South African Rands (ZAR). If not designed correctly and appropriately, it can result in catastrophic failure and immense loss of costs invested. Proper due diligence should be carried out during the design of wind turbines. Several codes, standards and guidelines are available to assist with the design of wind turbines. Some of the most common and applicable include:

- Guidelines for Design of Wind Turbines (DNV/Risø – 2002)
- Guideline for the Certification of Wind Turbines, Edition 2010 (Germanischer Lloyd)
- Recommended practice for compliance of large onshore wind turbine support structures, Draft (ASCE/AWEA - 2011)

Several books are also often used as guidelines on design of turbines. These include:

- Wind Energy Handbook (Burton, Sharpe, Jenkins, Bossanyi – 2001)

Unfortunately these guidelines tend to only focus on wind turbines with steel towers. There is little information available regarding design guidelines related to concrete towers. This study focusses primarily on Guidelines for Design of Wind Turbines (DNV & Risø, 2002) and IEC-61400-1 Wind turbines – Design requirements (IEC, 2005).
There are numerous approaches in the design of wind turbines. One of these approaches has been outlined by Manwell, McGowan and Rogers (2002) in the book *Wind Energy Explained – Theory, Design and Application*. This approach makes use of the following 12 crucial design steps:

1. **Determine application**

This involves the determination of whether the wind turbine will be used for large scale distribution by power utilities or whether the turbine will be used on a much smaller scale, such as an individual household power supply. This will aid the selection of turbine size (power rating) and generator type.

2. **Review previous experience**

Once the application is known, research on lessons learnt from previously designed and constructed wind turbines of a similar nature should be carried out. This will provide guidance.

3. **Select topology**

Selection of turbine topology mostly involves the specifications related to the rotor. Some of these include the orientation of the axis of rotation i.e. HAWT or VAWT, the position of the rotor (upwind of downwind tower) and the number of blades. As previously mentioned, for the purposes of this study the emphasis will be on large scale wind turbines for power utility application. Hence, the focus is placed on upwind type, three bladed, horizontal axis wind turbines (HAWT).

4. **Estimate preliminary loads**

An assessment of the approximate preliminary loads that the wind turbine will be subjected to is then made. This loading is then used to design the various components of the turbine. Initial load estimation may make use of simplified methods such as scaling of loads from turbines of similar design. The loading will be refined as the details and specifics of the turbine are developed throughout the design phase.

5. **Develop tentative design**

Once the topology has been decided and the initial loads estimated, a preliminary design can be carried out.

6. **Predict performance**

It is necessary to predict the expected performance of the turbine. This involves the efficiency of rotor and many mechanical and electrical components within the nacelle. This will not be of significant focus for civil engineers.

7. **Evaluate design**

The preliminary design will have accounted for loads that the wind turbine is most likely to be exposed to and which it should be able to withstand. Other loads that will be considered during this phase, would be those the turbine is exposed to during normal operation such as static wind loads, steady loads associated with rotor rotation, cyclic loads due to yaw motion and transient loads due to start up and shut down. Also to be considered are extreme loads that are less likely to occur such as impulsive loads due to wind gusts, stochastic loads due to wind turbulence and resonance induce loads (although this will be avoided in the design of the tower). Some of these loads are assessed, in industry, through wind tunnel testing and computational fluid dynamics (CFD) modelling.
8. Estimate costs and cost of energy

A good design should be the most suitably economical, as is the case with any type of engineering design. In order to ensure the design is economical, material and construction costs need to be estimated during the design phase as well as the anticipated costs of energy production i.e. the associated price per kWh. This gives an indication on the rate of return on the cost of production.

9. Refine design

It is necessary to refine the design based on the outcomes from steps 6 – 8. This then allows for an improvement on the initial preliminary design. It is during this phase of design that the critical load combinations will be assessed. Design, being an iterative process, means that multiple design refinements may take place until the optimal design is achieved.

10. Build prototype

Once the design has been refined, a prototype of the turbine is then constructed. This allows for assessment of ease of installation, monitoring of operation and the execution of testing (if required).

11. Test prototype

Field testing can proceed once the prototype has been constructed. Power output can be measured and performance assessed. Actual loads can also be measured and compared to those used as inputs in the design.

12. Design production machine

Based on the prototype testing, improvements may be made on the design to lower costs or improve performance for example. However, when all improvements have been made and satisfactory test results are obtained from step 11, the final design of the turbine should take place. This design should be very similar to the final prototype.

The design steps are shown schematically in Figure 10.
Design Procedure

1 – Determine application
2 - Review previous experience
3 – Select topology
4 – Estimate preliminary loads
5 – Develop tentative design
6 – Predict performance
7 – Evaluate design
8 – Estimate cost and costs of energy
9 – Refine design
10 – Build prototype
11– Test prototype
12 – Design production machine

Design not optimised
Design optimised
Unsatisfactory test results/Further improvement required
Satisfactory test results/No further improvements required

Figure 10: Summary of design process (adapted from Manwell, McGowan & Rogers, 2002)

2.4.2 Turbine Tower Design
The design of the turbine tower would follow a similar design process as set out above. The tower would be one of the main components of the turbine to be designed by civil engineers. The primary aspect of interest in this study would be the dynamic analysis, comprising two parts namely the modal analysis and the vibration response of the structure. van Zyl and van Zijl (2015) have stated that there are two options available to ensure that a wind turbine is dynamically sound. The first option is to
avoid the occurrence of resonance altogether so as to ensure that the fundamental natural frequency of the structure differs as far as practicably possible from the applied excitation frequency that the structure is likely to be exposed to over its lifespan. The second option is to make use of damping systems to decrease the amplification factor as a result of applied excitation load. The first option is most favourable in industry (van Zyl & van Zijl, 2015). The two dominant loading/excitation frequencies are those from the rotor rotational frequency and the blade passing frequency referred to as “1P” and “3P” respectively (1P = rotational frequency, 3P = blade passing frequency). The tower should be designed so that the natural frequency is kept out of range of both of these frequencies by 10%. If this is the case, then there will normally not be any problems as the occurrence of load amplification or resonance is unlikely (DNV & Risø, 2002). Three classifications of towers exist according to their stiffness. The first is a stiff tower (also referred to as stiff-stiff) where the natural frequency is greater than the blade passing frequency ($\omega_n > 3P$), the second is a soft tower (also referred to as soft-stiff) where the natural frequency lies between the rotor rotational frequency and the blade passing frequency ($1P < \omega_n < 3P$) and the third is a soft-soft tower where the natural frequency is less than that of the rotor rotational frequency ($\omega_n < 1P$) (Manwell, McGowan & Rogers, 2002). This is well illustrated in the Campbell Diagram in Figure 11. While the stiff tower is structurally favoured since it is insensitive to motions of the actual turbine during any kind of operational situation i.e. start up and shut down (Manwell, McGowan & Rogers, 2002), it has been shown that the most economical of these three tower types is the soft tower (Harte & Van Zijl 2007). This results in a safe working frequency for the soft tower such that:

**Equation 3: Safe working frequency for a soft tower**

$$1.1P < \omega_n < 2.7P$$

The fundamental natural frequency may be approximated with analytical methods or obtained from finite element software packages. For a simplified model, where the tower and turbine are approximated as a uniform cantilever beam with a mass on the top, the following equation (Manwell, McGowan & Rogers, 2002) provides an estimate of the fundamental natural frequency:

**Equation 4: Approximation of natural frequency (Manwell, McGowan & Rogers, 2002)**

$$f_n = \frac{1}{2\pi} \sqrt{\frac{SEI}{(0.23m_{tower} + m_{turbine})L^3}}$$

where:

- $f_n$ = the fundamental natural frequency (Hz)
- $E$ = Young's Modulus
- $I$ = Moment of Inertia
- $m_{tower}$ = mass of the tower
- $m_{turbine}$ = mass of the turbine
- $L$ = height of the tower

Equation 3 would ensure that the natural frequency of the structure does not coincide with the excitation frequencies related to the turbine rotor. These properties can be seen in Table 2. This correlates well with the established knowledge that a typical 3MW wind turbine is expected to have normal rotational speeds of between 6rpm and 13rpm (Way & van Zijl, 2015.)
Figure 11: Typical Campbell diagram for wind turbines (von der Haar & Marx, 2015)

Table 2: Typical rotational speeds and corresponding frequencies for various turbine models
(http://www.aweo.org/windmodels.html - cited 29/04/2016)

<table>
<thead>
<tr>
<th>Turbine Model Name</th>
<th>Rotational speed range (rpm)</th>
<th>Corresponding rotational frequency calculated (Hz)</th>
<th>Nominal Power (MW)</th>
<th>Hub height (m)</th>
<th>Rotor diameter (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Vestas V112</td>
<td>6.9 – 17</td>
<td>0.115-0.283</td>
<td>3</td>
<td>84 – 119</td>
<td>112</td>
</tr>
<tr>
<td>2 Vestas V100</td>
<td>7.2 – 15.3</td>
<td>0.12-0.255</td>
<td>2.75</td>
<td>80</td>
<td>100</td>
</tr>
<tr>
<td>3 Vestas V90</td>
<td>8.8 – 14.9</td>
<td>0.147-0.248</td>
<td>1.8</td>
<td>80</td>
<td>90</td>
</tr>
<tr>
<td>4 Siemens</td>
<td>6 – 16</td>
<td>0.10-0.267</td>
<td>2.3</td>
<td>80</td>
<td>93</td>
</tr>
<tr>
<td>5 GE 1.5s</td>
<td>11.1 – 22.2</td>
<td>0.185-0.37</td>
<td>1.5</td>
<td>64.7</td>
<td>70.5</td>
</tr>
</tbody>
</table>
2.4.3 Turbine Tower Loads
Wind turbine loading is a complex topic. IEC 61400-1 standard provides eight design situations to be taken into account for the design of turbines. These eight situations involve the various operation phases of the turbine. These load situations are then combined with some of the various external conditions including normal and extreme wind conditions. The wind loading is described in more detail in Section 2.5 Wind-Structure interaction. These combinations are used to perform the structural design of the turbine and involve the following load cases:

- normal design situations and appropriate normal or extreme external conditions
- fault design situations and appropriate external conditions
- transportation, installation and maintenance design situations and appropriate external conditions (IEC, 2005)

Turbine loads have been simplified to include primarily the dead load (self-weight) and wind load by van Zyl and van Zijl (2015). These loads acting on rotor blades, hub and nacelle are then transferred to the tower and are known as turbine loads. The wind load results in a direct pressure along the tower (van Zyl & van Zijl, 2015). This is illustrated in Figure 12.

![Wind pressure profile diagram](image)

**Figure 12: Typical wind turbine tower loads (Way & van Zijl, 2015)**

For this study, the main analyses that took place for the turbine tower design includes:

- A static analysis to determine the initial sizing of the tower including minimum plate thickness required with a corresponding tower base dimension of 4.2m,
- Eigen value analysis to ensure that the fundamental natural frequency is maintained within the allowable working frequency.
2.5 Wind-Structure Interaction

Wind flowing past a wind turbine results in aerodynamic loads. Wind fields acting on the structure are of a stochastic nature meaning that they vary in speed and direction with time. This is a result of turbulent activity in the atmosphere (Hansen, 2008). This turbulent wind field can be seen in Figure 13. The wind passes through the blades creating lift and drag forces which can be broken up into component vectors that are parallel and perpendicular to the wind direction. While the perpendicular component vector causes the blades to rotate, the parallel component vector acts as a load on the structure itself (Van der Woude & Narasimhan, 2010). Wind loading is just one of the external loads mentioned in design guidelines and standards that act on a wind turbine.

![Illustration of turbulent wind field applied to the turbine](Hansen, 2008)

Wind turbines are subjected to external loading. According to *IEC 61400-1* (2005), this loading is comprised of environmental and electrical loading. Environmental loading being those such as wind, temperature, lightening, ice and earthquakes for example. The primary environmental loading to be considered when designing a turbine is that loading due to wind conditions. Wind conditions include both normal and extreme conditions. Normal wind conditions are those which occur often during normal operation of the turbine where as extreme wind conditions are defined as those which could occur in a 1 year in 50 year return period (IEC, 2005). Both IEC 61400-1 and DNV/Risø propose various wind models for both normal and extreme conditions.

The two normal wind condition models presented in *IEC 61400-1* are:

- Normal wind profile model (NWP)
- Normal turbulence model (NTM)

There are six extreme models presented in *IEC 61400-1*, these include:

- Extreme wind speed model (EWM)
- Extreme operating gust (EOG)
- Extreme turbulence model (ETM)
- Extreme direction change (EDC)
- Extreme coherent gust with direction change (ECD)
- Extreme wind shear (EWS)
These wind models may be applied to different types of wind turbines according to their IEC 61400-1 classification. Wind turbine classes are defined in terms of parameters including wind speeds and turbulence. The parameters are not site specific and are intended to cover a wide range of applications.

Table 3: Basic parameters according to wind turbine classification (IEC, 2005)

<table>
<thead>
<tr>
<th>Wind Turbine Class</th>
<th>Class I</th>
<th>Class II</th>
<th>Class III</th>
<th>Class S</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{ref}$ (m/s)</td>
<td>50</td>
<td>42.5</td>
<td>37.5</td>
<td>Special conditions</td>
</tr>
<tr>
<td>$A$</td>
<td></td>
<td></td>
<td></td>
<td>- values to be specified by the designer</td>
</tr>
<tr>
<td>$B$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

where:

$V_{ref}$ = reference wind speed average over 10 minutes

Categories A, B and C represent cases of high, medium and low turbulence respectively.

$I_{ref}$ = expected value of the turbulence intensity at 15 m/s

Classes I, II and III cover the different site conditions ranging from high wind speeds to lower wind speeds. Class S relates to special conditions which are not covered by the other three turbine classifications. Under these conditions, the designer is expected to carry out simulations based on design parameters provided in Annex A of the IEC 61400-1. The Kaimal Spectrum Model and Exponential Coherence Model are turbulence models provided for the purpose of design load calculations for a Class S turbine.

Authors Van der Woude and Narasimhan (2010) have conducted research based on turbulent models. This included the simulation of turbulent wind with the use of Power Spectral Density (PSD) functions. The PSD function outputs the magnitude of a single directional component of the changing wind speed for a specific frequency. This research was carried out based on the IEC 61400-1 specified Kaimal Spectrum Model and Exponential Coherence Model.

Equation 5: PSD function for the Kaimal spectrum model (Van der Woude & Narasimhan, 2010)

$$PSD(f) = \frac{4\sigma^2 L}{V_{hub}^5} \left(1+6\frac{fL}{V_{hub}}\right)^{-5}$$

where:

$\sigma$ = standard deviation of wind speed
$L$ = a length scale based on the wind velocity component integral scale parameter
$V_{hub}$ = 10 minute mean wind speed at hub height
$f$ = frequency of the turbulent wind load (Hz)

Van der Woude and Narasimhan (2010) made use of the coherence function – in conjunction with the Kaimal spectrum model – to represent the coherence between wind time histories at two points.
**Equation 6: Exponential coherence model (Van der Woude & Narasimhan, 2010)**

\[
Coh(r, f) = \exp \left[ -12 \left( \left( \frac{fr}{V_{hub}} \right)^2 + \left( \frac{0.12r}{L} \right)^2 \right)^{0.5} \right]
\]  

where:

\( r \) = separation distance between the two points on a plane normal to the average wind direction

Coherence tends towards a value of 1 as the value of \( r \) approaches 0 and coherence decreases with increasing of the separation distance (Van der Woude & Narasimhan, 2010). Together the PSD and coherence functions can be used to create a set of time histories of wind speeds at various positions. These time histories can then ultimately be used to obtain a resultant effective wind speed for a Class S turbine.

Author de Kock (2015) performed research based on the steady extreme wind speed model (EWM). These models were approximated as follows:

- \( V_{e50} \), with a 50 year return period
- \( V_{e1} \), with a 1 year return period

**Equation 7: Steady EWM for 50 year return period (de Kock, 2015)**

\[
V_{e50} = 1.4V_{ref} \left( \frac{z}{z_{hub}} \right)^{0.11}
\]  

**Equation 8: Steady EWM for 1 year return period (de Kock, 2015)**

\[
V_{e1} = 0.8V_{e50}
\]

where:

\( V_{ref} \) = Reference wind speed average over 10 minutes  
\( z \) = Height above the ground  
\( z_{hub} \) = Height of hub

The equations were used in the research to create an approximate wind profile over the height of the turbine (de Kock, 2015).

IEC 61400-1 provides a total of 8 different wind models (including normal and extreme conditions) as well as 8 different turbine operational situations. This results in the suggested 22 design load cases to be used and checked for when designing wind turbines. In this study however, dynamic effects of wind loading will not be considered.

### 2.6 Foundation Design Aspects

Simplified models are available for the dynamic analysis of the turbine tower. Analytical equations such as that presented in equation 4 provide a simplified approach to determining natural frequency. However, these simplified approaches do not take into account the foundation of the turbine structure. The foundation itself has a mass and stiffness and therefore it influences the natural frequency of the structure.
Foundations form part of the support structure of the wind turbine and are typically constructed with reinforced concrete. According to the South African design code for reinforced concrete, SANS 10100-1, foundations are typically designed to be able to resist bearing pressure from vertical loads, sliding from lateral loads and overturning from bending moments experienced at the base (South African National Standards, 2000: Clause 4.10). Wind turbines are generally founded on either slab foundations – sometimes also referred to as gravity foundations, spread/pad footing or raft foundations – or pile foundations. The type of foundation required would depend on the specific soil conditions present at the chosen site. In order to establish the soil conditions, it is recommended in design guidelines, that geological studies be carried out to establish historical geological information pertaining to the area of interest. Geophysical studies are recommended to extrapolate information obtained from single bore holes regarding soil layers within the area. Lastly, and perhaps most importantly, geotechnical investigations – including soil sampling for laboratory testing and in-situ soil testing – should be carried out (DNV & Risø, 2002). This will aid in the decision of most suitable foundation type. A slab foundation is generally made use of when favourable strong stiff soils are present however, where poorer quality soft soils are present, pile foundations are made use of to transfer the loads to greater depths where soil strengths are superior (Olariu, 2013).

2.6.1 Types of Foundations
2.6.1.1 Slab or Gravity Foundation
There are many variations of slab foundations. According to Burton et al. (2001) foundation slabs of uniform thickness are generally selected when rock is near the ground surface. The slab is reinforced with predominantly top and bottom steel in two perpendicular directions and the slab thickness is governed by its shear capacity so as to eliminate the need for shear reinforcing steel. Where rock is located further away from the ground surface, other slab options exist. These include slabs mounted with pedestal type stubs, tapering slab thickness towards outer edges of foundation and rock anchorage of slab foundations so as to resist overturning while reducing self-weight of foundation normally required. Different types of slab foundations are illustrated in Figure 14. Ideally these gravity foundations should be circular in shape when observed from above however, circular formwork is often more costly and less readily available. Hence, the common use of either octagonal or square shaped gravity foundations (Burton et al., 2001).

![Variations of slab or gravity foundations](image)

*Figure 14: Variations of slab or gravity foundations (Burton et al., 2001)*
2.6.1.2 Pile Foundations

Pile foundations are available in various options depending on the particular application. Burton et al. (2001) describes three principle types of pile foundations. The first being multi-pile foundations which are primarily “pile caps” or a slab foundation supported on a number of cylindrical piles positioned in a circle. The second is referred to as a mono-pile foundation which is a single large cylindrical pile. This is a favourable option in cases where the water table is low and soils around the hole perimeter do not collapse when deep excavations take place. It can be costly so a third option available is the hollow mono-pile foundation. This is a single large walled cylindrical pile with its centred filled with a material fill of sorts that is less costly than concrete. This is illustrated in Figure 15. Pile foundations are typically used for lattice towers since the wide spread of the legs would alternatively result in an uneconomically large slab foundation (Ibid.).

![Various pile foundations](image)

**Figure 15: Various pile foundations (Burton et al., 2001)**

2.6.2 Recent Studies and Research on Foundations

Various studies have been carried out on the dynamic analysis and design of turbine towers where foundation influence has been taken into consideration. Some of these studies focussed on the various different types of foundations and geometries. Way and van Zijl (2015) carried out research involving rectangular shallow gravity foundations for varying heights of steel, concrete and steel-concrete hybrid towers with a specific focus on cost. As expected, larger size foundations and greater volume of concrete were required for towers with increased height. When comparing towers of the same height, it was found that larger foundations were required for steel towers. This was due to the fact that steel towers being generally lighter than the concrete and hybrid towers, requiring additional weight in the foundation to resist overturning (Way & van Zijl, 2015).

Authors von der Haar and Marx (2015) carried out comparative research on circular raft foundations and circular annular foundations used to support concrete towers. The foundations studied, had an outer diameter of 10m and the inner diameter varied from 0m (circular raft) to 6m (circular annular) as seen in Figure 16. They found that the annular foundation had improved stability against overturning when compared to the raft foundation but that as the annular foundation became more slender (as the inner diameter became closer to the outer diameter dimension) the bearing stresses increased (von der Haar & Marx, 2015).
2.7 Soil-Structure Interaction

2.7.1 Background

It has been previously stated that the foundation of the wind turbine, which has a mass and stiffness, influences the fundamental natural frequency of the structure. Similarly, it has been shown in research, that the interaction between the soil and the foundation also has an effect on the fundamental natural frequency of the wind turbine structure. Wind turbines have often been modelled as a beam element (the tower) with a lumped mass (the turbine) on a fixed support. In structural modelling, the fixed support is indicative of no vertical or horizontal motion as well as no rotation at the support position i.e. a rigid support that is infinitely stiff. In reality it is known that settlements and rotations of foundations do in fact take place to some degree. This is due to the fact that the soil below the foundation does not behave in a rigid manner. In regular structures, such as small buildings, the effects of these support conditions can be neglected however, this is not advisable when designing wind turbines. Olariu (2013) has stated that the assumption of a rigid base for the frequency analysis of turbine towers, has been known to provide misleading results. When soil stiffness is not taken into account, the resulting natural frequency is normally larger than the frequency obtained when soil stiffness is taken into account. In order to obtain more accurate frequency results, the soil-structure interaction should be considered (Olariu, 2013).

There are two primary methods commonly used for the consideration of the effects of soil-structure interaction on the dynamic analysis of wind turbines. The first method is to model the soil with the use of a system of springs. The springs will have a certain stiffness – corresponding to the soil properties – and can be applied in various directions. Typically, the spring system will provide a vertical, horizontal and rotational stiffness to the structure support. This is depicted in Figure 17. Typical equations commonly used for the calculation of the soil spring stiffness for circular foundations include those presented by Gazetas (1983). These equations are shown below in Table 4. The spring stiffness method is a simple, cost effective and time efficient manner of considering the effects of soil-structure interaction on the dynamic behaviour of a turbine tower (van Zyl & van Zijl, 2015). The other method of modelling soil-structure interaction is to make use of the Finite Element Method (FEM). The soil below the foundation as well as the structure itself can be modelled in commercially available software such as Abaqus, Diana, Adina, etc. FEM is dependent on soil properties such as Young’s Modulus and Poisson’s ratio. This method has been said to be more computationally expensive than simpler spring models however, results obtained are considered to be a better representation of the actual realistic behaviour (Olariu, 2013). Warren-Codrington (2013) has stated that FEM results are
highly dependent on input parameters which are not always investigated during geotechnical testing, such as Poisson’s ratio. Results are only as reliable as the accuracy of the input requirements and one’s correct usage of the software. Warren-Codrington further states that the accuracy of results are dependent on the interpretation by the individual who may or may not have an in depth understanding of FEM (Warren-Codrington, 2013). 

![Figure 17: Spring stiffness model of soil-structure interaction (adapted from Olariu, 2013)](image)

<table>
<thead>
<tr>
<th>Stiffness:</th>
<th>Vertical</th>
<th>Horizontal</th>
<th>Rotational (Rocking)</th>
<th>Torsional</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$k_{vertical} = \frac{4GR}{1-v}$</td>
<td>$k_{horizontal} = \frac{8GR}{2-v}$</td>
<td>$k_{rotational} = \frac{8G^3R}{3(1-v)}$</td>
<td>$k_{torsional} = \frac{16G^3R}{3}$</td>
</tr>
<tr>
<td>Where:</td>
<td>$R =$ radius of the foundation, $G =$ dynamic shear modulus of the soil, $v =$ Poisson’s ratio of the soil</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Typically manufacturers of turbines provide a limit on the stiffness of the soil. Soil stiffness can be calculated based on calculations taken from Gazetas (1983).

2.7.2 Recent Studies and Research on Soil-Structure Interaction

There has been much research conducted on the effect of soil-structure interaction on the dynamic behaviour of wind turbines in the recent past. Authors van Zyl and van Zijl (2015) carried out research on circular foundations and looked closely at the effect that different soils had on the overall fundamental natural frequency of the tower. They considered the foundation to be rigid and supported on various soil types, modelled by springs with a certain stiffness. A comparison was then made on the effect of the different soil types on the fundamental frequency of the tower for a rigid fixed foundation model (neglecting the soil-structure interaction) and a spring stiffness model. Difference in frequencies of the two models of between 15% (for stiffer soils such as gravel) and 39% (for softer soils such as clay) were obtained for the various soil types (van Zyl & van Zijl, 2015). These large discrepancies, obtained from this two-step analysis process, are indicative of the importance of the need to consider the soil-structure interaction. The results from this study also concurred with the information stated in design guidelines that the effect of soil stiffness is much greater than that of the foundation stiffness on the tower’s fundamental frequency (DNV & Risø, 2002).

Prowell, Elgamal and Lu (2010) carried out a study on a 5MW wind turbine comprising 90m tower and 126m rotor diameter. A finite element model (FEM) of the structure was developed and included a full 3D soil mesh. Three different soil layers – soft, medium and stiff soil – of 15m thickness were assessed to obtain an understanding of the influence of soil-structure interaction on the tower
moment and shear demand. It was found that softer soils had a more significant influence, resulting in a reduction in frequencies of the structure as well as an increase in rotation of the foundation when compared to the influence of soil-structure interaction of stiff soils (Prowell, Elgamal & Lu, 2010). This correlates with the results obtained from the study by van Zyl and van Zijl (2015).

Olariu (2013) performed research which involved the comparison of the two main methods of modelling soil-structure interaction for wind turbines, namely the spring stiffness method and FEM. In the study a 67.6m high steel monopole tower – conical in shape – with a base diameter of 4.2m gradually decreasing to a diameter of 1.85m at the top, was modelled. The tower was modelled with shell elements of varying thickness and diameter in SAP 2000 with a lumped mass at the top to account for the turbine weight. The tower had a mass of 85,150kg and the turbine, including the rotor hub and blades, a mass of 47,000kg. The foundation was a circular raft with 16m diameter and a depth of 3m. For Case A of the study, the soil support was modelled with four situations; three with spring stiffness for different soil conditions and one with a rigid support. For Case B, the entire soil-structure was modelled with elements including the tower, foundation and soil. In Case A, when comparing the three different soil types with the spring stiffness method to the rigid base, Olariu found that a smaller frequency of the tower was obtained for the dynamic analysis when making use of the spring stiffness method. This reiterates the importance of the consideration of the soil-structure interaction. When comparing the results of the three different soils with the spring stiffness method, in Case A, to those with FEM, in Case B, Olariu found the obtained natural frequencies to be very similar. The dynamic analysis for the two different methods resulted in differences of frequencies for the various soil types of between 0.19% and 1.2%. Olariu used these results to emphasise the importance of obtaining the correct soil properties suggesting that either method is sufficient since results were so similar. The choice of method used to consider the soil-structure interaction is up to the discretion of the engineer (Olariu, 2013).

Mawer and Kalumba (2016) conducted research on the effect of soil stiffness on the dynamic behaviour of wind turbines. This study focussed specifically on the typical soil conditions found on the three main wind development regions in South Africa, the Eastern Cape, the Western Cape and the Karoo. The turbine type, a Vestas V112, 3MW output, was analysed using a simplified model with a lumped mass supported on a massless cantilever column. Results were then compared for an infinite soil stiffness model and a finite soil stiffness model. The finite soil stiffness model made use of spring supports, the stiffness of which was calculated based on the equations presented by Gazetas (1983), while the infinite stiffness model neglected the effects of soil-structure interaction. In the case of the Western Cape soils, where stiffness governed the design of the foundation as opposed to structural strength governing the design for the other cases, natural frequency reductions of between 8% and 10% were obtained. According to design guidelines and standards for wind turbines, the allowable frequency range only has a safety factor of ±10%. With frequency discrepancies as large as those obtained in the study by Mawer and Kalumba, this practically annihilates any kind of safety factor. Once again this reiterates the significance of the need to account for the soil stiffness effects on the dynamic behaviour of the tower (Mawer & Kalumba, 2016).

In this study, soil-structure interaction was considered by means of the spring stiffness method.

2.8 Vibration/Dynamic Response

2.8.1 Dynamic Analysis Overview

The dynamic analysis and response of the wind turbine is the focus of this research. This is the aspect of the design of wind turbine structures that is of most interest to civil engineers and requires the most in depth understanding. The dynamic analysis of a structure has been described by Zingoni
as the response of the structure to a load that varies with time. The response of the structure refers to the displacement, velocity and acceleration of the structure’s components, which in turn results in stresses and strains. In civil engineering applications, this response needs to be either eliminated or controlled. A structure may not have an excitation load applied in the case of free vibration or such a load may be applied in the case of forced vibration (Zingoni, 2015).

Dynamic analysis and structural response is commonly explained with a single degree of freedom system. This system is comprised of a mass, a spring providing stiffness and a damper to dissipate energy as depicted in Figure 18. The displacement response, $x$, of the mass is dependent on the excitation frequency, $\Omega$. This has been well explained by van der Tempel and Molenaar (2002). When the excitation frequency is far less than natural frequency of the system the response is said to be quasi-static. In the case of the excitation frequency being close in value to natural frequency and in the case of the excitation frequency much larger than that of the natural frequency, the responses are referred to as resonance and inertia dominated, respectively. This is illustrated in Figure 19. In the case of resonance it is clear that the response force is far greater than the excitation force as the response has been magnified. This is solely due the close relationship of the frequency of the excitation force and that of the structure’s natural system (van der Tempel & Molenaar, 2002).

\[ F(t) = \cos(\Omega t) \]

Figure 18: Single degree of freedom system (adapted from van der Tempel & Molenaar, 2002)

The solid black line represents the excitation force and the dashed blue line represents the response

Figure 19: Three different response regions (adapted from van der Tempel & Molenaar, 2002)

In the case of wind turbines, load or excitation frequencies are primarily those resulting from the wind causing rotation of the turbine rotor. Amplification of static loads as well as the amplification of the response of the structure, has been known to occur at certain loading frequencies. This occurrence is the well known effect of resonance, as described previously, when the structure is loaded with an excitation frequency similar or numerically close to its natural frequency. This should be avoided at all
costs. It is the primary reason for the limitations on the allowable fundamental natural frequency range of the wind turbine as described previously in Section 2.4.2.

2.8.2 Recent Studies and Research on Dynamic Response

One of the vital tasks for an engineer designing a wind turbine, is the determination of the natural frequency of the structure. This has been carried out in predominantly two ways. The first method to perform a dynamic analysis is to make use of approximation methods such as simplified discrete models or distributed parameter models. These models can be further improved by increasing accuracy and considering more masses (discrete model) or more elements (distributed parameter model). With the assistance of numerical method approximations, natural frequency and response of the structure can be easily computed in software such as MATLAB. The second method of dynamic analysis is to make use of FEA methods and through software such as Abaqus, Diana, Adina and ANSYS for example, the natural frequency can be obtained. This makes use of FEM where various options of discretisation are available such as consideration of quadrilateral or triangular elements with various number of nodes or the consideration of plate elements of varying thickness.

Van der Woude (2011) carried out studies at the University of Waterloo involving the effects of vibration isolators (dampers) on the response of a wind turbine structure subject to wind and seismic loading. The FEA software, COMSOL Multiphysics, was used to carry out the investigation as well as MATLAB to simulate the wind load over time (Van der Woude, 2011). Ahlström (2002) made use of the FEA package SOLVIA to perform a dynamic analysis on a wind turbine model under various wind loading cases. The goal was to produce a model accurate enough that it could be used for research and investigation purposes to assess response under different dynamic loads (Ahlström, 2002).

Prowell, Elgamal and Lu (2010) carried out a study on a 5MW wind turbine. A dynamic analysis was executed using FEM with the software OpenSees to assess the structure’s response to seismic and wind loading. The FEA model consisted of a beam-column section with a simple fixed base. The tower beam-column section consisted of 100 beam elements with each element’s stiffness dependent on the cross section of the tower at the centre of that element (Prowell, Elgamal & Lu, 2010). Nuta conducted research on the dynamic response of a Vestas, 1.65MW output, wind turbine to seismic loading. The research involved the modelling of the tower with shell elements and a lumped mass on top to account for the rotor mass. The model was conducted through the use of FEM software package, ANSYS. Different mesh sizes were examined (Nuta, 2010). The study carried out by van Zyl and van Zijl (2015) comprised a non-linear finite element analysis of a high strength concrete tubular tower. Curved shell elements, each with 8 nodes, were used to model the tower. This study assessed the dynamic behaviour of the tower with the FEA software Diana. The research was extended to include an analysis of the tower under crack development and assess this influence on the tower stiffness and natural frequency (van Zyl & van Zijl, 2015).

Depending on the purpose of the dynamic analysis study to be carried out, it is up to the discretion of the engineer to decide which method of evaluation is more suitable in each particular case. However, it is worth noting the following statement by Asareh (2015); “although multi-body dynamics models that are used in recent wind turbine computational tools may be adequate for preliminary design, a more detailed structural finite element model is necessary to enhance the verification of the final design” (Asareh, 2015). This particular study will make use of both numerical method approximations with the assistance of MATLAB as well as FEA methods through the use of Abaqus to obtain natural frequencies.
2.9 References


3. Detailed Study and Analysis

3.1 Introduction
It has been recommended against making use of steel monopole towers for wind turbines with heights in excess of 80m. Steel turbine towers with heights of 80m and greater are considered to be limited in terms of their stiffness. This stiffness limitation is based on the limitation of the size of the base diameter due to road transportation restrictions. However, the above is based on typically linearly tapering conical shaped steel towers. This study assesses the influence of altering tower geometry and shape on the tower’s natural frequency. The aim is to determine the suitability of other geometric steel towers with heights of 80m or more which may have not yet been studied.

3.2 Overview
The study involved the assessment of three main types of tower geometries. Each of these three tower cases were assessed for three typical height scenarios. The different tower cases also comprised various plate thicknesses. A modal analysis of the various tower cases was carried out to assess the fundamental natural frequency of each case. The towers were analysed for compatibility with a Vestas V112 3MW wind turbine. The various tower cases were all analysed in commercial software using Finite Element Analysis (FEA). The results were adjusted to account for the soil-structure interaction based on the spring stiffness method. The FEA results were then compared to those results obtained from analytical methods as well as simplified calculations.

3.3 Tower Case Geometry
The three main geometries and shapes considered for the analysis are relatively unique. The first main tower case is a linearly tapering octagonal shaped tower. The second tower case is a linearly tapering conical circular lower section with a uniform circular upper section. The third type of tower case is a quadratically tapering circular lower section with a uniform circular upper section forming a trumpet like shape. Each of the tower geometry cases were analysed for heights of 80m, 100m and 120m. Their shapes are depicted in the figure below.

![Figure 20: Three tower cases for analysis shown in plan and section](image-url)
For each of the three geometry cases, 1, 2 and 3, sub-cases a, b and c exist for each of the three height cases. Sections of 20m in height with constant plate thicknesses – across the section – were utilised to form four, five and six sections of sub-cases a (80m tower), b (100m tower) and c (120m tower) respectively. In total, 60 cases were assessed and analysed in order to obtain cases suitable for the Vestas V112 3MW turbine, from a trial and error modelling approach. Some of these geometric case details have been provided in Table 5.

The Vestas V112 3MW wind turbine has a rotational speed range of 6.9 – 17rpm which is equivalent to a loading frequency range of 0.115 – 0.283Hz. In order to ensure optimal economic design of a tower, classified as soft, which falls within the safe working frequency of 1.1P and 2.7P as previously seen in Figure 11, the allowable frequency range of the tower must fall within 0.283Hz and 0.345Hz.
<table>
<thead>
<tr>
<th>Case 1</th>
<th>Case 1a1 - 80m linearly tapering octagonal tower</th>
<th>Case 1b1 - 100m linearly tapering octagonal tower</th>
<th>Case 1c2 - 120m linearly tapering octagonal tower</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sketch</td>
<td>Section</td>
<td>Outer Base Diameter (m)</td>
<td>Outer Base Side Length (m)</td>
</tr>
<tr>
<td>Case 1</td>
<td></td>
<td>1</td>
<td>4.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>4.03</td>
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<td>6</td>
<td>3.35</td>
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<td></td>
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<td>7</td>
<td>3.18</td>
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<td>8</td>
<td>3.01</td>
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<td></td>
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<td></td>
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<td>2.5</td>
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</table>

<table>
<thead>
<tr>
<th>Case 2</th>
<th>Case 2a1 - 80m linearly tapering conical lower and uniform upper</th>
<th>Case 2b1 - 100m linearly tapering conical lower and uniform upper</th>
<th>Case 2c11 - 120m linearly tapering conical lower and uniform upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sketch</td>
<td>Section</td>
<td>Radius - for sketch in Abaqus (m)</td>
<td>Shell thickness (mm)</td>
</tr>
<tr>
<td>Case 2</td>
<td></td>
<td>1</td>
<td>4.2</td>
</tr>
<tr>
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<td></td>
<td>2</td>
<td>3.86</td>
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<td>3.18</td>
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<td>2.5</td>
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<td>2.5</td>
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<td></td>
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<td>8</td>
<td>2.5</td>
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<tr>
<td></td>
<td></td>
<td>9</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10</td>
<td>2.5</td>
</tr>
<tr>
<td>Sketch</td>
<td>Case 3 - 80m quadratically tapering lower and uniform upper</td>
<td>Case 3b - 100m quadratically tapering lower and uniform upper</td>
<td>Case 3c - 120m quadratically tapering lower and uniform upper</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------</td>
<td>-----------------------------------------------------------</td>
<td>----------------------------------------------------------</td>
</tr>
<tr>
<td>Section</td>
<td>Outer Base Diameter (m)</td>
<td>Radius - for sketch in Abaqus (m)</td>
<td>Section Base Height (m)</td>
</tr>
<tr>
<td>1</td>
<td>4.2</td>
<td>2.1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>3.19</td>
<td>1.595</td>
<td>8</td>
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<td>3</td>
<td>3.04</td>
<td>1.52</td>
<td>16</td>
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<td>4</td>
<td>2.92</td>
<td>1.46</td>
<td>24</td>
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<tr>
<td>5</td>
<td>2.81</td>
<td>1.405</td>
<td>32</td>
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<td>6</td>
<td>2.71</td>
<td>1.355</td>
<td>40</td>
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<td>7</td>
<td>2.62</td>
<td>1.31</td>
<td>48</td>
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<td>8</td>
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<td>1.27</td>
<td>56</td>
</tr>
<tr>
<td>9</td>
<td>2.5</td>
<td>1.25</td>
<td>64</td>
</tr>
<tr>
<td>10</td>
<td>2.5</td>
<td>1.25</td>
<td>72</td>
</tr>
</tbody>
</table>
3.4 Initial Sizing Process

Sizing of the tower cases was carried out initially by means of a static analysis. Initial tower base diameters were fixed at 4.2m. This correlates well with existing research and existing constructed steel towers based on previous tower optimisation. A static analysis was then carried out accounting for the self-weight of the tower, the weight of the turbine and the wind loading as per the extreme wind model (EWM) discussed previously. For simplification of wind loading, a constant wind line load was assumed. When fixing the lower base diameter at 4.2m, this resulted in a top diameter of 2.5m for each of the tower heights – in order to obtain a constant wind line load. It is noted that the top diameter is in practice also governed by the nacelle mechanism. This enabled the calculation of the minimum plate thickness required for the initial geometric parameters which provided a starting point for the dynamic analysis of the three tower cases. Minimum plate thicknesses of 6mm, 10mm and 13mm respectively were obtained for the 80m, 100m and 120m tower heights. Plate thicknesses were then increased until the dynamic requirements were met. While the static analysis provided a starting point for the geometric parameters, generally these parameters were governed by the dynamic analysis resulting in much thicker plate sizes required.

In Case 1 particularly, it was found that the initial tower diameters provided more stiffness than required and so the tower base and top diameters were decreased in order to obtain economical and light tower structures. In Cases 2 and 3, top diameters were increased slightly to provide additional stiffness such that plate thicknesses could be reduced in order to obtain more economical structures. In Cases 2c and 3c it was found that very thick plates were required to provide the stiffness such that dynamic requirements were met. For this reason greater top diameters were used corresponding to base diameters of 4.2m and then when maintaining a top diameter of 2.5m, alternative geometric cases were explored for these two cases which allowed for base diameters beyond the transportation limit of 4.2m. This is achieved by means of a vertical joint connection between four sections within the lower section of the tower such that individually transported sections will not exceed the 4.2m limitation. This is depicted in Figure 21.

![Figure 21: Alternative geometries with base diameters greater than 4.2m for Cases 2c and 3c](image-url)
3.5 Finite Element Analysis

3.5.1 Overview

The tower cases were assessed by means of finite element analysis (FEA). The towers were modelled as shell structures in the commercial software, Abaqus. They were modelled using the structural steel of grade S355JR. The material properties include:

Density, $\rho = 7850\text{kg/m}^3$
Young’s Modulus, $E = 200\times10^3\text{MPa}$
Poisson’s Ratio, $\nu = 0.3$

Each of the tower cases were modelled with quadrilateral shaped elements, specifically 4-noded doubly curved shell elements. Shells are thin structural elements that have a relatively high strength and stiffness as a result of their geometry, more specifically the shell’s surface and curvature give it these favourable properties. Shell elements distribute loading predominantly through extensional action which is the in-plane direct force action (Zingoni, Mwakali & Salahuddin, 2000). Shells with aspect ratios less than 10 were used in the analysis with a corresponding mesh size of approximately 0.5m. Shell boundary conditions were automatically applied due to the selection of the associated free meshing within Abaqus. The shell’s mid-surface has been selected as the reference surface of the shell. Boundary conditions were made use of in Abaqus to ensure that the towers were modelled with a fixed base, fixed against both translation and rotation. The towers were also modelled with a lumped mass applied to the top of each tower. This lumped mass represents the self-weight of the nacelle as well as the turbine rotor and blades for a Vestas V112 3MW turbine. The lumped mass was considered in Abaqus through the addition of an inertia point mass, as what is referred to as an engineering feature, equivalent to a total of 137 000kg.

![Figure 22: Typical 4-noded doubly curved quadrilateral shell element mesh type for Case1b](image)

![Figure 23: Fixed base and lumped mass for Case2a](image)
3.5.2 Convergence Study

A convergence study was carried out on the model to establish a suitable shell element size. An initial element size of 3m was used and then this size was reduced until an insignificant difference in the overall frequency was obtained while ensuring the analysis did not become computationally expensive and time consuming. As the mesh element size was decreased, the resulting frequency converged. This is shown in Graph 1. The difference between the 0.5m element size and 0.4m element size, resulted in a frequency difference of 0.028% which is considered to be negligible. For this reason, the 0.5m element size was selected as the converged size to be used for this study.

![Graph 1: Convergence of mesh element size](image)

3.6 Model Calibration

Prior to commencing of the analysis of the geometric cases chosen for this study, an Abaqus FEA calibration procedure was carried out. The calibration process involved the dynamic analysis of a specific 80m steel tower which had previously been analysed by Way and van Zijl (2015). Their analysis involved the modelling of both the actual tower and the foundation with 8-noded linear hexagonal brick solid 3D elements. A lumped mass, of value 158 000kg, was also added to the top of the tower to account for the self-weight of the nacelle and the turbine rotor and blades. The results obtained by Way and van Zijl were compared to those obtained from the calibration model of only the tower – with the exact same geometry, material properties and lumped mass – comprising 4-noded quadrilateral doubly curved shell elements. The aim of the calibration procedure was to assess whether a significant difference in the fundamental frequency would be obtained due to the use of different element types. The calibration process output acceptable results with a percentage difference between the two models of 1.37%. This small difference is also due to the difference in modelling a fixed base tower alone and a tower along with foundation supported on a spring elastic support. The results of the calibration process were deemed acceptable and the geometric case towers for this study were then modelled in Abaqus with the same techniques.
### Table 6: Summary of calibration results

<table>
<thead>
<tr>
<th>Description</th>
<th>Base Diameter (m)</th>
<th>Top diameter (m)</th>
<th>Section Thickness (mm)</th>
<th>Abaqus 1st Natural Frequency with soil stiffness interaction (Hz)</th>
<th>Frequency obtained by Way and van Zijl (Hz)</th>
<th>Percentage difference between Way and van Zijl value and Abaqus Calibration (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>80m steel tower with equivalent mass of 158t on top of tower</td>
<td>4,5</td>
<td>3</td>
<td>34</td>
<td>27,67</td>
<td>21,33</td>
<td>15</td>
</tr>
</tbody>
</table>

#### 3.7 Soil-Structure Interaction Consideration

The soil-structure interaction was taken into account after the results of the fixed base model had been obtained from Abaqus. The fundamental natural frequency obtained for each individual case was then adjusted so that the soil stiffness as well as a typically suitable foundation was taken into account in the final result. The soil stiffness was considered by means of the spring stiffness method as previously discussed in Section 2.7. The specific soil chosen for this study was a typical Eastern Cape soil located in one of the main corridors, found between the Western Cape and Port Elizabeth. This region was one of those investigated for the South African Wind Farm projects and is one of the three soil types considered in the assessment by Mawer and Kalumba (2016). This study also makes use of this specific Eastern Cape soil. The soil properties assumed for the analysis include:

- Dynamic shear modulus, \( G = 285 \text{MPa} \)
- Poisson's ratio, \( \nu = 0.3 \)

The foundations considered in this study were based on typical dimensions which have been used prior in both research and reality. For the purpose of this study, foundations were not designed according to limit state design, instead realistic typical foundation sizes have been assumed for the various tower cases in order to credibly account for the soil-structure interaction. All foundations assumed are of a circular gravity type with diameters of 20m. The thicknesses of the foundation vary according to the height of the particular tower case. The foundations assumed are 2.2m thick, 2.5m thick and 2.75m thick for their corresponding heights of 80m, 100m and 120m for subcases a, b and c respectively. The mass of the foundation could then be calculated as well as the foundation stiffness based on the soil rotational rocking spring stiffness equation as previously seen in Table 4. This enables the calculation of fundamental frequency of the foundation itself as shown below.

**Equation 9: Frequency of foundation calculation (Mawer & Kalumba, 2016)**

\[
\omega_{n=\text{foundation}} = \frac{k_{\text{foundation}}}{m_{\text{foundation}}} = \frac{k_{\text{rotational}}}{m_{\text{foundation}}H^2} \quad [9]
\]

where:

- \( k_{\text{foundation}} \) = stiffness of foundation
- \( m_{\text{foundation}} \) = mass of foundation
- \( k_{\text{rotational}} \) = rotational spring soil stiffness for the Eastern Cape
- \( H \) = tower height
The tower and the foundation can be seen to behave in series and therefore Dunkerley’s Method of addition can apply to obtain the overall final frequency as follows:

**Equation 10: Dunkerley’s method to obtain overall frequency with soil stiffness effects**

\[
\frac{1}{\omega_{n-\text{total}}^2} = \frac{1}{\omega_{n-\text{fixed base}}^2} + \frac{1}{\omega_{n-\text{foundation}}^2}
\]  

[10]

**Equation 11: Final frequency in Hz**

\[
f_{n-\text{total}} = \left(\frac{1}{2\pi}\right) \omega_{n-\text{total}}
\]  

[11]

where:

- \(\omega_{n-\text{total}}\) = final fundamental frequency including the soil stiffness effects (rad/s)
- \(\omega_{n-\text{fixed base}}\) = fundamental frequency of tower with a fixed base obtained from FEA model (rad/s)
- \(\omega_{n-\text{foundation}}\) = fundamental frequency of foundation on Eastern Cape soil (rad/s)
- \(f_{n-\text{total}}\) = final fundamental frequency including the soil stiffness effects (Hz)

Using the above equations, the soil-structure interaction could be accounted for in the final fundamental frequency of each of the tower geometry cases.

### 3.8 Comparison with Analytical Approximation Method and Simplified Calculation Method

An analytical approximation method was used to further perform a dynamic analysis of the steel towers for the geometric Case 3 – the quadratically tapering lower circular section with a uniform upper circular section, trumpet like shaped tower. This was carried out as a cross check on the FEA results in order to assess the percentage difference obtained between the results of the two methods. The analytical approximation method made use of the lumped mass principle from fundamental dynamic analysis theory. The tower was treated as a cantilever beam and was cut into 10 sections which allowed for the lumping of 10 masses as well as an additional mass at the top equivalent to that of the nacelle and turbine rotor and blades for a Vestas V112 3MW turbine (137 000kg). This lumping method was used for all three tower height subcases a, b and c.

![Figure 24: Sketch of the principles used in the analytical method](image)
The particular mass, height and section properties were calculated for each individual 10 sections of the tower as indicated in Figure 24. This then enabled the calculation of the fundamental natural frequency for each section of the tower. For example when considering mass 4, this section of the tower:

- is located at height, $h_4$,
- has a mass, $m_4$ (which is a function of the average cross sectional area)
- has a moment of area, $I_4$,
- has a stiffness, $k_4$ (which is a function of the average cross sectional moment of area) and
- a frequency, $\omega_4$.

**Equation 12: Height of 4th section**

$$h_4 = 4h = 4\left(\frac{H}{10}\right)$$

**Equation 13: Mass of 4th section**

$$m_4 = \left[\frac{\pi}{4}\left(D_4^2 + D_5^2\right)^2 - \left(D_4 + D_5\right)^2\right] \rho$$

**Equation 14: Moment of area of 4th section**

$$I_4 = \left[\frac{\pi}{64}\left(D_4^4 - d_4^4\right)\right]$$

**Equation 15: Stiffness relating to 4th mass**

$$k_4 = \frac{3E}{\rho \left(I_1 + I_2 + I_3 + I_4\right)}$$

**Equation 16: Natural frequency of 4th section**

$$\omega_4 = \frac{k_4}{m_4}$$

where:

- $h$ = height of tower cut sections
- $H$ = tower height
- $\rho$ = material density
- $D_4$ = outer base diameter of the 4th section of tower
- $D_5$ = outer top diameter of the 4th section of tower (i.e. outer base diameter of section 5)
- $D_n$ = outer base diameter of the $n$th section of tower
- $d_4$ = inner base diameter of the 4th section of tower (shell plate thickness dependent)
- $d_5$ = inner top diameter of the 4th section of tower (i.e. inner base diameter of section 5)
- $d_n$ = inner base diameter of the $n$th section of tower
- $I_n$ = moment of area of the $n$th section of tower
- $\omega_4$ = fundamental natural frequency of the 4th section of tower (rad/s)

Once the natural frequencies have been obtained for each of the 10 tower sections/lumped masses, the overall tower frequency can be obtained by means of Dunkerley’s Method as follows:

**Equation 17: Dunkerley’s Method for lumped masses**

$$\frac{1}{\omega_{\text{tower}}^2} = \frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} + \ldots + \frac{1}{\omega_9^2} + \frac{1}{\omega_{10}^2}$$
Equation 18: Overall tower frequency in Hz

\[ f_{ntower} = \left( \frac{1}{2\pi} \right) \omega_{total} \]  

where:

\[ \omega_N \] = fundamental natural frequency of the \( n \)th section of tower (rad/s)
\[ \omega_{tower} \] = overall total fundamental natural frequency of the tower (rad/s)
\[ f_{ntower} \] = overall total fundamental natural frequency of the tower (Hz)

The soil-structure interaction was then again taken into account as described previously in Section 3.7. This analytical calculation process, while simple in method, is laborious when carried out numerous times for each tower subcase in order to assess the dynamic frequency. The calculations depend on inputs relating to the tower geometry, the shell plate thickness, the material properties, foundation dimension, soil properties, etc. It is for this reason that the approximation method based on the lumped masses was coded using MATLAB with general variables. This allowed for multiple input variations while still able to analyse every individual case using the same method. The use of the MATLAB code based on the analytical approximation method described above, allowed for comparison of results with those obtained from FEA.

The FEA results obtained from Abaqus were also compared to results obtained from a simplified approximation method. The chosen simplified method was that based on equation 4 by Manwell, McGowan and Rogers (2002). This equation approximates a cantilever beam to represent the fixed base tower and accounts for both the mass of the tower as well as the mass of the actual turbine. The overall tower mass was assessed as for the above analytical method in MATLAB. The moment of inertia was approximated in a similar manner as above by assessing the average cross section over the tower height for each of the subcases a, b and c. For ease of dynamic assessment of various tower cases, Manwell’s simplified method was also coded using MATLAB. The soil-structure interaction was again considered as mentioned previously. The complete MATLAB code for both the analytical approximation, based on Dunkerley’s method, as well the simplified approximation calculation by Manwell, McGowan and Rogers, for the geometric tower Case 3 has been provided in the appendix.
4. Results

4.1 Successful Frequency Results

60 tower models were assessed and analysed. The results of each case vary depending on geometry and tower height. The individual model cases were adjusted accordingly so that acceptable results could be obtained for each of the three main geometric cases; the outcome being a fundamental natural frequency within the allowable frequency range for the Vestas V112 3MW turbine. Adequate results were obtained for 25 tower cases which are all compatible with this turbine type. The results of which are tabulated below. Details for the entire set of all 60 cases analysed – some of which did not produce acceptable results within the allowable frequency range – can be found in the appendix.

*Table 7: Successful geometric tower case parameters and results*

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Description</th>
<th>*Base Diameter (m)</th>
<th>*Top Diameter (m)</th>
<th>Part 1: 0-20m</th>
<th>Part 2: 20-40m</th>
<th>Part 3: 40-60m</th>
<th>Part 4: 60-80m</th>
<th>Part 5: 80-100m</th>
<th>Part 6: 100-120m</th>
<th>Abaqus 1st Natural Frequency - fixed base (Hz)</th>
<th>Abaqus 1st Natural Frequency accounting for soil-structure interaction (Hz)</th>
<th>Tower Mass from Abaqus (kg)</th>
<th>Tower Height (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a1</td>
<td>Linearly tapering with octagonal cross section.</td>
<td>4.2</td>
<td>2.5</td>
<td>15</td>
<td>12</td>
<td>8</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>0.31021</td>
<td>0.3045</td>
<td>219331.3</td>
<td>80</td>
</tr>
<tr>
<td>1a3</td>
<td>Linearly tapering with octagonal cross section.</td>
<td>2.6</td>
<td>1</td>
<td>35</td>
<td>28</td>
<td>22</td>
<td>15</td>
<td>NA</td>
<td>NA</td>
<td>0.29617</td>
<td>0.2912</td>
<td>173470.5</td>
<td>80</td>
</tr>
<tr>
<td>1a5</td>
<td>Linearly tapering with octagonal cross section</td>
<td>2.1648</td>
<td>1</td>
<td>70</td>
<td>60</td>
<td>50</td>
<td>40</td>
<td>NA</td>
<td>NA</td>
<td>0.31728</td>
<td>0.3112</td>
<td>303126.4</td>
<td>80</td>
</tr>
</tbody>
</table>

Allowable Frequency Range for Vestas V112 3MW (Rotational Speed = 6.9 - 17 rpm)

<table>
<thead>
<tr>
<th>Allowable Frequency Range</th>
<th>1.1P</th>
<th>2.7P</th>
<th>Mass of Vestas V112 3MW including nacelle, turbine rotor and blades (kg)</th>
<th>E (kN/m²)</th>
<th>Soil G (kN/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0,283</td>
<td>0,345</td>
<td>137000</td>
<td></td>
<td>2.00E+08</td>
<td>285000</td>
</tr>
<tr>
<td>Case No.</td>
<td>Description</td>
<td>*Base Diameter (m)</td>
<td>*Top Diameter (m)</td>
<td>Section Thickness (mm)</td>
<td>Abaqus 1st Natural Frequency - fixed base (Hz)</td>
</tr>
<tr>
<td>---------</td>
<td>------------------------------------------------------------------------------</td>
<td>--------------------</td>
<td>-------------------</td>
<td>------------------------</td>
<td>------------------------------------------------</td>
</tr>
<tr>
<td>1b2</td>
<td>Linearly tapering with octagonal cross section.</td>
<td>4,2</td>
<td>2,5</td>
<td>50 40 30 22 15 NA</td>
<td>0.33289</td>
</tr>
<tr>
<td>1b3</td>
<td>Linearly tapering with octagonal cross section.</td>
<td>3,2</td>
<td>1,2</td>
<td>45 40 35 30 20 NA</td>
<td>0.32562</td>
</tr>
<tr>
<td>1b5</td>
<td>Linearly tapering with octagonal cross section.</td>
<td>2,6</td>
<td>1</td>
<td>70 60 50 40 30 NA</td>
<td>0.29778</td>
</tr>
<tr>
<td>1c2</td>
<td>Linearly tapering with octagonal cross section.</td>
<td>4,2</td>
<td>2,5</td>
<td>60 50 40 32 25 18</td>
<td>0.30676</td>
</tr>
<tr>
<td>1c3</td>
<td>Linearly tapering with octagonal cross section.</td>
<td>3,8</td>
<td>1,4</td>
<td>60 50 40 32 25 18</td>
<td>0.31897</td>
</tr>
<tr>
<td>1c5</td>
<td>Linearly tapering with octagonal cross section.</td>
<td>3,2</td>
<td>1,2</td>
<td>80 70 60 50 40 30</td>
<td>0.30636</td>
</tr>
<tr>
<td>2a1</td>
<td>Linearly tapering conical circular lower section with a uniform circular upper section.</td>
<td>4,2</td>
<td>2,5</td>
<td>50 40 30 20 NA NA</td>
<td>0.30311</td>
</tr>
<tr>
<td>Case No.</td>
<td>Description</td>
<td>*Base Diameter (m)</td>
<td>*Top diameter (m)</td>
<td>Section Thickness (mm)</td>
<td>Abaqus 1st Natural Frequency - fixed base (Hz)</td>
</tr>
<tr>
<td>---------</td>
<td>------------------------------------------------------------------------------</td>
<td>--------------------</td>
<td>------------------</td>
<td>------------------------</td>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td>2a3</td>
<td>Linearly tapering conical circular lower section with a uniform circular upper section.</td>
<td>4,2</td>
<td>3</td>
<td>35 28 22 15 NA NA</td>
<td>0,30392</td>
</tr>
<tr>
<td>2b1</td>
<td>Linearly tapering conical circular lower section with a uniform circular upper section.</td>
<td>4,2</td>
<td>2,5</td>
<td>100 90 80 70 60 NA</td>
<td>0,29513</td>
</tr>
<tr>
<td>2b5</td>
<td>Linearly tapering conical circular lower section with a uniform circular upper section.</td>
<td>4,2</td>
<td>3</td>
<td>70 60 50 40 30 NA</td>
<td>0,29163</td>
</tr>
<tr>
<td>2c10</td>
<td>Linearly tapering conical circular lower 80m section with a uniform circular upper 40m section.</td>
<td>4,2</td>
<td>3,6</td>
<td>150 110 95 85 55 45</td>
<td>0,29731</td>
</tr>
<tr>
<td>2c11</td>
<td>Linearly tapering conical circular lower 80m section with a uniform circular upper 40m section.</td>
<td>8,4</td>
<td>2,5</td>
<td>70 60 50 40 30 20</td>
<td>0,31577</td>
</tr>
<tr>
<td>Case No.</td>
<td>Description</td>
<td>*Base Diameter (m)</td>
<td>*Top diameter (m)</td>
<td>Section Thickness (mm)</td>
<td>Abaqus 1st Natural Frequency - fixed base (Hz)</td>
</tr>
<tr>
<td>---------</td>
<td>------------------------------------------------------------------------------</td>
<td>--------------------</td>
<td>-------------------</td>
<td>------------------------</td>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td>3a1</td>
<td>Quadratically tapering circular lower 60m section with a uniform circular upper 20m section.</td>
<td>4,2</td>
<td>2,5</td>
<td>60 45 38 30 NA NA</td>
<td>0,29646</td>
</tr>
<tr>
<td>3a3</td>
<td>Quadratically tapering circular lower 60m section with a uniform circular upper 20m section.</td>
<td>4,2</td>
<td>3</td>
<td>40 32 25 18 NA NA</td>
<td>0,31564</td>
</tr>
<tr>
<td>3a5</td>
<td>Quadratically tapering circular lower 60m section with a uniform circular upper 20m section.</td>
<td>4,2</td>
<td>3</td>
<td>35 28 22 15 NA NA</td>
<td>0,29843</td>
</tr>
<tr>
<td>3b1</td>
<td>Quadratically tapering circular lower 60m section with a uniform circular upper 40m section.</td>
<td>4,2</td>
<td>2,5</td>
<td>200 190 180 100 80 NA</td>
<td>0,3215</td>
</tr>
<tr>
<td>3b3</td>
<td>Quadratically tapering circular lower 60m section with a uniform circular upper 40m section.</td>
<td>4,2</td>
<td>3</td>
<td>90 80 65 50 40 NA</td>
<td>0,29558</td>
</tr>
<tr>
<td>Case No.</td>
<td>Description</td>
<td>*Base Diameter (m)</td>
<td>*Top diameter (m)</td>
<td>Section Thickness (mm)</td>
<td>Abaqus 1st Natural Frequency - fixed base (Hz)</td>
</tr>
<tr>
<td>----------</td>
<td>------------------------------------------------------------------------------</td>
<td>--------------------</td>
<td>-------------------</td>
<td>------------------------</td>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td>3b4</td>
<td>Quadratically tapering circular lower 60m section with a uniform circular upper 40m section.</td>
<td>4,2</td>
<td>3</td>
<td>100 90 80 70 60 NA</td>
<td>0.29755</td>
</tr>
<tr>
<td>3b6</td>
<td>Quadratically tapering circular lower 80m section with a uniform circular upper 20m section.</td>
<td>4,2</td>
<td>3</td>
<td>85 75 60 35 NA</td>
<td>0.29277</td>
</tr>
<tr>
<td>3c11</td>
<td>Quadratically tapering circular lower 60m section with a uniform circular upper 60m section.</td>
<td>4,2</td>
<td>3</td>
<td>200 190 140 70 50 35</td>
<td>0.29812</td>
</tr>
<tr>
<td>3c14</td>
<td>Quadratically tapering circular lower 60m section with a uniform circular upper 60m section.</td>
<td>8,4</td>
<td>2.5</td>
<td>200 180 160 100 80 60</td>
<td>0.3053</td>
</tr>
<tr>
<td>3c15</td>
<td>Quadratically tapering circular lower 90m section with a uniform circular upper 30m section.</td>
<td>8,4</td>
<td>3</td>
<td>90 80 70 60 50 40</td>
<td>0.31954</td>
</tr>
</tbody>
</table>

* Diameters provided in the table above for Case 1 are those circumscribing the octagonal section.
4.2 Results Comparison between FEA and Analytical Methods

A comparison between the Abaqus FEA results and the MATLAB analytical approximation and simplified calculation results was carried out for some of the cases of tower geometric Case 3. The results obtained for the analytical approximation, based on the principle of lumped masses and Dunkerley’s method, were relatively similar to those results obtained from Abaqus FEA with percentage differences of between 6.05% and 10.53% being noted. Percentage differences of between 3.12% and 20.87% were observed for the difference in results between Manwell’s simplified method and Abaqus FEA. The results for each subcase of the quadratically tapering tower as well as the percentage difference in results, can be seen in Table 8 below.

Table 8: Percentage difference between Abaqus FEA and analytical MATLAB results for Case 3

<table>
<thead>
<tr>
<th>Allowable Frequency Range for Vestas V112 3MW (Rotational Speed = 6.9 - 17 rpm)</th>
<th>1.1P</th>
<th>2.7P</th>
<th>Mass of Vestas V112 3MW including nacelle, turbine rotor and blades (kg)</th>
<th>E (kN/m²)</th>
<th>Soil G (kN/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0,283</td>
<td>0,345</td>
<td>137000</td>
<td>2.0E+08</td>
<td>285000</td>
<td></td>
</tr>
</tbody>
</table>

| Case No. | Description | Base Diameter (m) | Top diameter (m) | Part 1: 0-20m | Part 2: 20-40m | Part 3: 40-60m | Part 4: 60-80m | Part 5: 80-100m | Part 6: 100-120m | Abaqus 1st Natural Frequency - fixed base (Hz) | Abaqus 1st Natural Frequency accounting for soil-structure interaction (Hz) | Approximation Method – 10 lumped masses - fixed base (Hz) | Approximation Method – 10 lumped masses with soil stiffness (Hz) | Percentage difference accounting for soil-structure interaction (%) | Simplified Natural Frequency - fixed base (Manwell) (Hz) | Simplified Natural Frequency with soil stiffness (Manwell) (Hz) | Percentage difference accounting for soil-structure interaction (%) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3a3 | Quadratically tapering circular lower section with a uniform circular upper section – 80m high tower. | 4,2 | 3 | 40 | 32 | 25 | 18 | NA | NA | 0,31564 | 0,3096 | 0,2952 | 0,2903 | 6,25% | 0,3259 | 0,3193 | 3,12% |
| 3a5 | Quadratically tapering circular lower section with a uniform | 4,2 | 3 | 35 | 28 | 22 | 15 | NA | NA | 0,29843 | 0,2933 | 0,2798 | 0,2756 | 6,05% | 0,3095 | 0,3038 | 3,57% |
circular upper section – 80m high tower.

| Case No. | Description                                                                 | Base Diameter (m) | Top diameter (m) | Part 1: 0-20m | Part 2: 20-40m | Part 3: 40-60m | Part 4: 60-80m | Part 5: 80-100m | Part 6: 100-120m | Abaqus FEA 1st Natural Frequency - fixed base (Hz) | Abaqus 1st Natural Frequency accounting for soil-structure interaction (Hz) | Approximation Method – 10 lumped masses - fixed base (Hz) | Approximation Method – 10 lumped masses with soil stiffness (Hz) | Percentage difference accounting for soil-structure interaction (%) | Simplified Natural Frequency - fixed base (Manwell) (Hz) | Simplified Natural Frequency with soil stiffness (Manwell) (Hz) | Percentage difference accounting for soil-structure interaction (%) |
|---------|------------------------------------------------------------------------------|-------------------|------------------|---------------|---------------|---------------|---------------|---------------|---------------|------------------------------------------------|------------------------------------------------|------------------------------------------------|------------------------------------------------|------------------------------------------------|------------------------------------------------|
| 3b3     | Quadratically tapering circular lower section with a uniform circular upper section – 100m high tower. | 4,2               | 3                | 90            | 80            | 65            | 50            | 40            | NA            | 0,29558                                               | 0,2870                                               | 0,2685                                               | 0,262                                               | 8.70%                                                  | 0,2666                                               | 0,2602                                               | 9.32%                                                  |
| 3b4     | Quadratically tapering circular lower section with a uniform circular upper section – 100m high tower. | 4,2               | 3                | 100           | 90            | 80            | 70            | 60            | NA            | 0,29755                                               | 0,2887                                               | 0,2689                                               | 0,2623                                              | 9.16%                                                  | 0,2758                                               | 0,2687                                               | 6.94%                                                  |
| 3c11    | Quadratically tapering circular lower section with a uniform circular upper section – 120 high tower. | 4,2               | 3                | 200           | 190           | 140           | 70            | 50            | 35            | 0,29812                                               | 0,2845                                               | 0,2642                                               | 0,2545                                              | 10.53%                                                 | 0,2316                                               | 0,2251                                               | 20.87%                                                 |
4.3 Modal Analysis Graphical Results
Mode shapes obtained from Abaqus for some of the successful tower cases have been included in the results below. As expected, the tower dynamic behaviour becomes more extreme with increasing mode shape. These mode shapes have been included in the results for the sake of interest. However, in this study, the first mode shape and its related first natural frequency are of significance.

Table 9: Mode shapes for some acceptable tower cases of height 80m

<table>
<thead>
<tr>
<th>Case No.</th>
<th>A: 80m Tower</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: Linearly tapering with octagonal cross section</td>
<td><img src="image1" alt="Mode Shape 1 for Case 1a" /></td>
</tr>
<tr>
<td>2: Linearly tapering conical circular lower section with a uniform circular upper section</td>
<td><img src="image4" alt="Mode Shape 1 for Case 2a1" /></td>
</tr>
<tr>
<td>3: Quadratically tapering circular lower section with a uniform circular upper section</td>
<td><img src="image7" alt="Mode Shape 1 for Case 3a1" /></td>
</tr>
<tr>
<td>Case No.</td>
<td>B: 100m Tower</td>
</tr>
<tr>
<td>---------</td>
<td>--------------</td>
</tr>
<tr>
<td>1: Linearly tapering with octagonal cross section</td>
<td><img src="image" alt="Mode Shape 1 for Case 1b2" /> <img src="image" alt="Mode Shape 2 for Case 1b2" /> <img src="image" alt="Mode Shape 3 for Case 1b2" /></td>
</tr>
<tr>
<td>2: Linearly tapering conical circular lower section with a uniform circular upper section</td>
<td><img src="image" alt="Mode Shape 1 for Case 2b1" /> <img src="image" alt="Mode Shape 2 for Case 2b1" /> <img src="image" alt="Mode Shape 3 for Case 2b1" /></td>
</tr>
<tr>
<td>3: Quadratically tapering circular lower section with a uniform circular upper section</td>
<td><img src="image" alt="Mode Shape 1 for Case 3b1" /> <img src="image" alt="Mode Shape 2 for Case 3b1" /> <img src="image" alt="Mode Shape 3 for Case 3b1" /></td>
</tr>
</tbody>
</table>
Table 11: Mode shapes for some acceptable tower cases of height 120m

<table>
<thead>
<tr>
<th>Case No.</th>
<th>C: 120m Tower</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: Linearly tapering with octagonal cross section</td>
<td></td>
</tr>
<tr>
<td><img src="image1" alt="Mode Shape 1 for Case 1c1" /></td>
<td><img src="image2" alt="Mode Shape 2 for Case 1c1" /></td>
</tr>
<tr>
<td>2: Linearly tapering conical circular lower section with a uniform circular upper section</td>
<td></td>
</tr>
<tr>
<td><img src="image4" alt="Mode Shape 1 for Case 2c10" /></td>
<td><img src="image5" alt="Mode Shape 2 for Case 2c10" /></td>
</tr>
<tr>
<td>3: Quadratically tapering circular lower section with a uniform circular upper section</td>
<td></td>
</tr>
<tr>
<td><img src="image7" alt="Mode Shape 1 for Case 3c11" /></td>
<td><img src="image8" alt="Mode Shape 2 for Case 3c11" /></td>
</tr>
</tbody>
</table>
5. Discussion

5.1 Overview
Steel monopole towers for wind turbine structures were assessed for suitability to support an operating Vestas V112 3MW turbine. Various tower geometries were analysed for dynamic behaviour. For each of the three main geometric cases, suitably acceptable towers were identified. The parameters of the towers were retained within the road transportation limits; in most cases the base diameters were limited to 4.2m and in Cases 2c and 3c specifically, the base diameters exceeded 4.2m however individual sections remained within these limits. The frequency results obtained for each case, show that steel towers of heights in excess of 80m are viable options for wind turbine towers. Deviating from the typical linearly tapering conical circular section steel tower, allows for possible solutions for a steel monopole wind turbine tower with heights of 80m and more. Other geometries are able to provide solutions to this problem. Although factors such as cost, buckling, fatigue and deflection have not been considered in this study.

5.2 Influence of Tower Height
From examination of the results, it is clear that a greater mass is obtained with increasing tower height. This is to be expected with the increase in material required for the additional height. This increase in tower mass with tower height can be seen in Graph 2 for all tower geometry cases. Given the fundamental equation for natural frequency, one can see that an increasing mass – for a constant stiffness – results in a decreasing natural frequency.

\[ \omega_n = \sqrt{\frac{k}{m}} \]  \hspace{1cm} [19]

where:

\( \omega_n \) = natural frequency of structure
\( k \) = stiffness of structure
\( m \) = mass of structure

For this reason, it can become more challenging to ensure an acceptable frequency of towers as the height increases. This is based on the requirement to increase the stiffness of the tower without the base diameter surpassing the limit of 4.2m. A stiffer section is required to achieve an acceptable frequency for towers at increased heights. This additional stiffness of section is achieved through the use of a section shape with a greater moment of inertia, an increased plate thickness or an increased effective diameter.

When examining Case 1 specifically, it can be seen that when reducing the base diameter to less than the limiting dimension of 4.2m, the required base diameter (diameter circumscribing the octagonal base) increased as the tower height increased in order to attain acceptable frequencies. Cases 1a3, 1b3 and 1c3 comprised towers with outer base diameters of 2.6m, 3.2m and 3.8m respectively. This is plotted on Graph 3. The plate thickness of the base diameter also played a role in providing stiffness to the section. When maintaining a constant base diameter of 4.2m for all towers, a similar trend is noticed as this plate thickness too, increased along with the increase in tower height. Cases 1a1, 1b2 and 1c2 consisted of base diameter plate thicknesses of 15mm, 50mm and 60mm respectively. This increase in plate thickness with increasing tower height – to provide acceptable frequency results – can be seen in Graph 4. A similar trend of increasing plate thickness is observed for both Cases 2 and 3 as well.
Graph 2: Increasing tower mass with increasing tower height for acceptable geometries for all three cases

Graph 3: Increasing base diameter with increasing tower height for Case 1 (octagonal cross section) acceptable geometries

Graph 4: Increasing plate thickness with increasing tower height for Case 1 (octagonal cross section) acceptable geometries
5.3 Influence of Cross Section

Case 1 is of an octagonal cross section where as Cases 2 and 3 are of circular cross sections. The geometric shape of the tower cross section too, has a significant influence on its natural frequency. The octagonal cross section of Case 1 allows for a smaller base diameter (diameter circumscribing the octagonal base) than that of Cases 2 and 3 while ensuring an acceptable natural frequency. Examining Graph 5, it can be seen that a maximum base diameter of only 4.2m for Case 1 is required, while a maximum base diameter of 8.4m (four sections with vertical joint connections to ensure individual section does not exceed road transportation limit) is required for Case 2 and Case 3. This octagonal cross section also allows for a decreased plate thickness of the 20m sections comprising the tower, when compared to the plate thicknesses required for Cases 2 and 3 such that an acceptable tower frequency is obtained. The results of this can be seen in Graph 6. The base diameter section required a plate thickness of 70mm for Case 2 and a plate thickness of 90mm for Case 3. In contrast, a plate thickness of only 60mm was required for the base section of Case 1.

Graph 5: Base diameters of 120m high tower for Cases 1c, 2c and 3c acceptable geometries

Graph 6: Plate thicknesses of 120m high tower for Cases 1c, 2c and 3c acceptable geometries
Both the reduction in base diameter as well as plate thickness for Case 1 results in a tower which has a lower mass. This can be seen in Graph 7 below. The mass of the tower for Case 1 is only 575 tonnes, while the masses for Cases 2 and 3 are 690 tonnes and 857 tonnes respectively for a tower height of 120m. This comparison is based on the lowest masses for the cases reported.

![Graph 7: Mass of 120m high tower for Cases 1c, 2c and 3c acceptable geometries](image)

It is due to this substantial reduction in mass that Case 1, the linearly tapering octagonal cross sectional tower, is considered to be a more economical tower in terms of both structural loading and material usage. This is based on the influence of the tower’s geometry and shape on the natural frequency. Examination of the formulae for the moment of inertia of a solid octagon and a solid circle can be carried out as follows:

**Equation 20: Moment of inertia for a solid octagonal section**

\[ I_{octagon} \approx 0.0547D^4 \]  

**Equation 21: Moment of inertia for a solid circular section**

\[ I_{circle} = \frac{\pi D^4}{64} \approx 0.0491D^4 \]

where:

\[ D \] = outer boundary dimension for the octagonal and circular shape (see Figure 25)

![Figure 25: Outer boundary dimensions for octagon and circle](image)

Assuming the same boundary dimension D, for both the above equations, yields a larger moment of inertia for the octagon than the circle. This increased moment of inertia influences the stiffness of the tower section resulting in an 11.4% increase in stiffness when comparing the octagonal to the circular shaped cross section for towers of the same height and material. This is the reason for the increased
stiffness of geometric Case 1, the linearly tapering octagonal tower. This added geometric cross sectional stiffness is what allows for the thinner shell plate thickness, the decreased base diameter and the overall lighter tower resulting in a more economical design.

5.4 Influence of Tower Geometry

When focussing on the circular cross sectional tower Cases 2 and 3, it needs to be noted that the actual tower geometry will influence the frequencies. These cases require base diameters of 4.2m for subcase b (100m tower height) and base diameters of 8.4m (extending of base beyond limit to reduce plate thicknesses) for subcase c (120m tower height) in order to ensure natural frequencies within the allowable frequency range for a Vestas V112 3MW turbine. Since they are both of the same cross sectional shape, the only differing parameters which influence their frequencies are the plate thickness and tower geometric shape. When comparing the plate thicknesses of these two cases for the 100m tower height, subcase b, it can be seen that Case 2b5 requires a base section plate thickness of 70mm while Case 3b6 requires a base section plate thickness of 80mm. This is illustrated in Graph 8. The quadratically tapering portion of the tower for Case 3 has a narrower tower diameter than that of Case 2 and this results in the need for an increased section stiffness. This is achieved through the 14% increased plate thickness for Case 3. However, this also results in a 7% increase of tower mass for Case 3 as seen in Graph 9 below. Case 2 is generally considered to be more economical – again in terms of structural load and material quantity – than Case 3. For this reason the lower conical tapering section of Case 2 is more favourable than the lower quadratically tapering section of Case 3. This favourable geometric shape provides an increased overall tower stiffness allowing for a lighter and more economical structure that is still dynamically suitable. Similarly, the linearly tapering geometric shape of tower Case 1 also results in an increased tower stiffness. It achieves this through the provision of wider intermediate cross sections between the base and tower top, as compared to those present in the geometric shapes of Cases 2 and 3. This reiterates the fact that Case 1 is the most favourable and economical tower type.

![Graph 8: Plate thickness of 100m high tower for Cases 2b and 3b acceptable geometries](image-url)
5.5 Comparison of Results between FEA and MATLAB

When looking at the differences between results obtained from FEA methods and those obtained from analytical methods, while accounting for soil-structure interaction, it should be noted that these results compare favourably. The difference between the Abaqus FEA results and the MATLAB analytical results for the lumped mass and Dunkerley method, range between 6.05% for Case 3a5 and 10.53% for Case 3c11. It is observed that this difference in results tends to increase with increasing tower height. This is clearly illustrated for Cases 3a5 (80m), 3b4 (100m) and 3c11 (120m) in Graph 10 below.

This increase in percentage difference with increasing tower height is expected. This is due to the fact that each tower assessed with the analytical method has been divided into 10 sections, forming a cantilever beam with 10 lumped masses. For subcase a, the 80m tower, each tower section considered was 8m in height. Whereas tower sections considered for subcases b and c, the 100m and 120m towers, were of heights 10m and 12m respectively. This relates to finite element theory where it is
known that smaller elements, comprising a structure under consideration, generally produce more accurate results. Since larger tower sections were made use of for subcases b and c, a reduced accuracy and hence larger percentage difference is the outcome. This difference in result could be reduced for subcases b and c by simply dividing the tower into more sections and analysing as a cantilever beam with an increased number of lumped masses. This will enable an increase in accuracy for higher towers when making use of the analytical method in MATLAB. None the less, percentage differences between results for all subcases are considered to be acceptable. These small percentage differences are indicative of robust results since these results, obtained from two different analyses methods, yield similar outputs. For this reason, both methods – the FEA and analytical – are considered to be acceptable for the dynamic analysis of steel towers for wind turbines of heights varying from 80m to 120m.

The differences between results obtained from Abaqus FEA and those obtained from simplified methods based on Manwell’s equation, while accounting for soil-structure interaction, also generally follow a similar trend of increasing percentage difference with increasing tower height. This trend can be seen in Graph 11 with the differences having been plotted for Cases 3a5 (80m), 3b4 (100m) and 3c11 (120m). The simplified method appears to be a very accurate approximation for towers of 80m height, obtaining differences in results of 3.12% and 3.57% for Cases 3a3 and 3a5 respectively. The simplified method yields better results than those from the Dunkerley method when compared to the results obtained from Abaqus FEA for lower tower heights. Conversely, the simplified method yields the least accurate results for the 120m tower Case 3c11, with a percentage difference of 20.87% being obtained when compared to the FEA results. For this reason, the simplified method is considered to be accurate for approximations of natural frequencies for towers with heights of up to 100m for this particular geometry. The simplified method is not considered accurate enough for prediction of the frequencies of towers of heights of 120m and more of this geometry and is therefore not recommended for these particular cases. But it could be considered acceptable for linear tapering geometries.

![Graph 11: Difference between FEA and simplified results for Case 3 acceptable geometries](image-url)
6. Conclusion

6.1 Overview
At the commencement of this study a literature review of existing studies was carried out. The review indicates that the use of steel monopole towers for wind turbines with heights exceeding 80m, are discouraged from being constructed. Instead, literature shows that concrete or steel-concrete hybrid monopole towers are recommended for towers of heights greater than 80m. This is based in part, on the limitation of the base diameter section to 4.2m for road transportation purposes. Taller steel towers typically require larger base diameters for added stiffness. It has also been shown that towers of these heights are more economical when constructed with concrete or both concrete and steel in combination. These steel towers considered in literature however, include the typical linearly tapering, circular cross sectional, conical shaped towers. This study assessed other types of steel monopole towers with different geometric shapes. The study involved the dynamic analysis of various geometric tower cases, with heights of 80m and more, to assess their compatibility with a Vestas V112 3MW turbine. Three main tower geometries were considered. The first case, Case 1, is a linearly tapering octagonal shaped tower. The second case, Case 2, is a linearly tapering conical circular lower section with a uniform circular upper section and the third case, Case 3, is a quadratically tapering circular lower section with a uniform circular upper section forming a trumpet like shape. These geometric cases were considered for three tower heights of 80m, 100m and 120m. These tower heights formed the subcases a, b and c respectively. Initial geometries were determined based on static analysis requirements. The initial base diameters were fixed at 4.2m with a corresponding top diameter of 2.5m to ensure a constant wind line load. Minimum plate thicknesses were then calculated which enabled initial geometric parameters in order to commence with the dynamic analysis of the three tower cases. Plate thicknesses were then increased – as well as tower diameters adjusted in some cases – until the dynamic requirements were met as this was generally the governing factor. The tower dynamics were analysed by means of Finite Element Analysis, with the commercially available software Abaqus. These results were compared and verified by carrying out a second dynamic analysis analytically in MATLAB based on the discrete, lumped mass method, for tower Case 3.

6.2 Findings
The results of this study show that steel monopole towers of heights of 80m and more are still viable options for wind turbine towers. 25 geometric tower cases of heights varying from 80m to 120m, produced acceptable fundamental natural frequencies within the allowable frequency range of 0.283Hz to 0.345Hz for a Vestas V112 3MW turbine. Acceptable frequencies were obtained for all three tower geometric cases. On closer inspection of the results, one can observe the parameters that are influential over the natural frequency. This study has shown the influence of the geometry and shape of the tower on its natural frequency. The cross sectional shape as well as the overall geometric shape of the tower have significant influence on the stiffness of the overall structure. By making use of tower cross sectional shapes with greater moment of areas, such as that of the octagonal cross section, less material is required in terms of both plate thickness and base dimensions in order to provide an adequate tower stiffness. It has been shown that the most favourable tower shape and geometry is that of Case 1, the linearly tapering octagonal cross sectional tower. This specific case resulted in the lightest tower, with the smallest circumscribing base diameter and the thinnest shell plate thickness. This enables one to draw upon the conclusion that this case is the most economical both in terms of structural loading as well as costs based on material usage. Based on the same deductions, the second most favourable tower shape and geometry is that of Case 2, while Case 3 is considered to be the least favourable as this tower had the greatest mass and required the largest
shell plate thickness in order to achieve an acceptable frequency. While certain shapes are more favourable than others, all three tower cases could be configured with geometric parameters such that acceptable frequency results were obtained. Compatibility with a Vestas V112 3MW turbine has been confirmed for 25 tower cases. This study shows the significant influence that geometry and shape have on the natural frequency of a tower. By merely deviating from the typically used, linearly tapering circular cross section, conical shaped tower and investigating other geometric options, it has been shown that valid acceptable solutions are available for steel tubular towers with heights in excess of 80m.

6.3 Recommendation of Further Research
Based on the acceptable results obtained from the dynamic analysis of the various steel tower cases, it is recommended that an assessment of the cost implications is carried out for the three main tower geometric cases. This cost assessment can include material cost, plate section manufacturing cost and cost of onsite assembly and construction. This information can further influence the favourability of a particular tower case. It can also influence the current perceived perception, as seen in literature, that steel towers above a height of 80m tend be less economical than concrete or steel-concrete hybrid towers. Further research can also include fatigue assessment of vertical connections used for larger base sections where overall base diameters exceed 4.2m. Research can also be conducted in order to assess the vulnerability of these towers when located in earthquake prone regions. Another option recommended for further research is the evaluation of further variations in geometry and shape of steel towers with heights exceeding 80m. The results have shown that towers with sections that have greater moment of areas prove to be structurally stiffer and economically favourable. For this reason it is recommended that towers with pentagonal and hexagonal cross sections are investigated. Further areas of study may include the evaluation of steel tower geometries for tower heights of more than 120m. There is limited information available on the dynamic behaviour of steel-concrete hybrid towers. Especially the less conventional hybrid towers such as those comprising steel tubular shells encasing or encased by concrete shells. This too, could potentially be an area where further studies are required. The use of optimisation functions could be researched in order to obtain a better understanding of tower geometry optimisation.
Appendices
## Detail Tower Parameters and Frequency Results for All Analysed Cases

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Description</th>
<th>*Base Diameter (m)</th>
<th>*Top diameter (m)</th>
<th>Part 1: 0-20m</th>
<th>Part 2: 20-40m</th>
<th>Part 3: 40-60m</th>
<th>Part 4: 60-80m</th>
<th>Part 5: 80-100m</th>
<th>Part 6: 100-120m</th>
<th>Abaqus 1st Natural Frequency - fixed base (Rad/s)</th>
<th>Abaqus 1st Natural Frequency with soil stiffness (Hz)</th>
<th>Abaqus 1st Natural Frequency (Hz)</th>
<th>Tower Mass from Abaqus (kg)</th>
<th>Tower Height, H (m)</th>
<th>Tower Case Acceptable</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a1</td>
<td>Linearly tapering with octagonal cross section.</td>
<td>4.2</td>
<td>2.5</td>
<td>15</td>
<td>12</td>
<td>8</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>1,9491</td>
<td>0.31021</td>
<td>0.3045</td>
<td>219331.3</td>
<td>80</td>
<td>Yes</td>
</tr>
<tr>
<td>1a2</td>
<td>Linearly tapering with octagonal cross section.</td>
<td>2.6</td>
<td>1</td>
<td>45</td>
<td>40</td>
<td>35</td>
<td>30</td>
<td>NA</td>
<td>NA</td>
<td>2,2631</td>
<td>0.36018</td>
<td>0.3513</td>
<td>255106.4</td>
<td>80</td>
<td>No</td>
</tr>
<tr>
<td>1a3</td>
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<td>2.6</td>
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<td>Section Thickness (mm)</td>
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<td>Section Thickness (mm)</td>
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<td>Abaqus 1st Natural Frequency - fixed base (Hz)</td>
<td>Abaqus 1st Natural Frequency with soil stiffness (Hz)</td>
<td>Tower Mass from Abaqus (kg)</td>
<td>Tower Height, H (m)</td>
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<td>60 45 38 30 NA NA</td>
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<td>40 32 25 18 NA NA</td>
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<td>3</td>
<td>35 28 22 15 NA NA</td>
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<td>Case No.</td>
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<td>Top Diameter (m)</td>
<td>Section Thickness (mm)</td>
<td>Abaqus 1st Natural Frequency - fixed base (Rad/s)</td>
<td>Abaqus 1st Natural Frequency - fixed base (Hz)</td>
<td>Abaqus 1st Natural Frequency with soil stiffness (Hz)</td>
<td>Tower Mass from Abaqus (kg)</td>
<td>Tower Height, H (m)</td>
<td>Tower Case Acceptable</td>
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<td>3b1</td>
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<td>75 68 60 50 40 NA 1,7383</td>
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<td>Section Thickness (mm)</td>
<td>Abaqus 1st Natural Frequency - fixed base (Rad/s)</td>
<td>Abaqus 1st Natural Frequency - fixed base (Hz)</td>
<td>Abaqus 1st Natural Frequency with soil stiffness (Hz)</td>
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<td>3</td>
<td>85 75 60 35 25 NA</td>
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<td>80 70 60 50 40 30</td>
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<td>70 60 50 40 30 20</td>
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<td>150 130 110 70 60 50</td>
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<td>Top diameter (m)</td>
<td>Part 1: 0-20m</td>
<td>Part 2: 20-40m</td>
<td>Part 3: 40-60m</td>
<td>Part 4: 60-80m</td>
<td>Part 5: 80-100m</td>
<td>Part 6: 100-120m</td>
<td>Abaqus 1st Natural Frequency - fixed base (Rad/s)</td>
<td>Abaqus 1st Natural Frequency with soil stiffness (Hz)</td>
<td>Tower Mass from Abaqus (kg)</td>
<td>Tower Height, H (m)</td>
<td>Tower Case Acceptable</td>
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<td>140</td>
<td>110</td>
<td>70</td>
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<td>110</td>
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<td>3c7</td>
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<td>3</td>
<td>180</td>
<td>160</td>
<td>115</td>
<td>70</td>
<td>50</td>
<td>40</td>
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<td>0,2677</td>
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<td>3</td>
<td>200</td>
<td>180</td>
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<td>Case No.</td>
<td>Description</td>
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<td>Section Thickness (mm)</td>
<td>Abaqus 1st Natural Frequency - fixed base (Rad/s)</td>
<td>Abaqus 1st Natural Frequency - fixed base (Hz)</td>
<td>Abaqus 1st Natural Frequency with soil stiffness (Hz)</td>
<td>Tower Mass from Abaqus (kg)</td>
<td>Tower Height, H (m)</td>
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<td>3c10</td>
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<td>4.2</td>
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<td>200 190 140 70 50 35</td>
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<td>0.2845</td>
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<td>3c12</td>
<td>Quadratically tapering circular lower 60m section with a uniform circular upper 60m section.</td>
<td>8.4</td>
<td>2.5</td>
<td>90 80 70 60 50 40</td>
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<td>0.22799</td>
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<td>3c13</td>
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<td>8.4</td>
<td>2.5</td>
<td>200 180 160 100 80 60</td>
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<td>Case No.</td>
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<td>Top Diameter (m)</td>
<td>Section Thickness (mm)</td>
<td>Abaqus 1st Natural Frequency - fixed base (Rad/s)</td>
<td>Abaqus 1st Natural Frequency - fixed base (Hz)</td>
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<td>3c15</td>
<td>Quadratically tapering circular lower 90m section with a uniform circular upper 30m section.</td>
<td>8.4</td>
<td>3</td>
<td>Part 1: 0-20m</td>
<td>Part 2: 20-40m</td>
<td>Part 3: 40-60m</td>
<td>Part 4: 60-80m</td>
<td>Part 5: 80-100m</td>
<td>Part 6: 100-120m</td>
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</table>

* Diameters provided in the table above for Case 1 are those circumscribing the octagonal section.
Appendix B
This specific code below contains the inputs for tower Case 3a5. The following code can be copied into MATLAB and run to obtain outputs.

%Structural inputs - dimensions, properties, etc. (all parameters in %kN, meters, seconds, radians, etc.)

%Case 3a - Quadratically Tapering Lower (till 60m) with Upper Uniform (20m) Section - 80m
%Quadratic function used to find diameters at various heights \( y = 176.5728 \times x^2 - 5.9437 \times x \)

\( H = 80; \) %in m
\( n = 10; \) %number of masses and tower section
\( h = H/n; \) %individual tower section height
\( E = 200e6; \) %Young's Modulus in kN/m^2

%Second Moment of area for 20m long sections of circular shaped tubular %tower (i.e. 4 different sections for 80m tower with a total of 10 parts)

\( t1 = 0.035; \) %Thickness of Section 1 (0-20m)
\( t2 = 0.028; \) %Thickness of Section 2 (20-40m)
\( t3 = 0.022; \) %Thickness of Section 3 (40-60m)
\( t4 = 0.015; \) %Thickness of Section 4 (60-80m)

\( Db = 4.2; \) %Outer base diameter at bottom of tower in m
\( db = Db - 2*t1; \) %Inner base diameter at bottom of tower
\( Dt = 3.00; \) %Outer diameter at top of tower

\( D1 = Db + t1/2; \) %Outer diameter of part 1 at base for mass 1
\( d1 = db; \) %Inner diameter of part 1 at base for mass 1

\( D2 = D1 - 2*0.23; \) %Outer diameter of part 2 at base for mass 2
\( d2 = D2 - 2*t1; \) %Inner diameter of part 2 at base for mass 2

\( D3 = D1 - 2*0.32; \) %Outer diameter of part 3 at base for mass 3
\( d3 = D3 - 2*t2; \) %Inner diameter of part 3 at base for mass 3

\( D4 = D1 - 2*0.386; \) %Outer diameter of part 4 at base for mass 4
\( d4 = D4 - 2*t2; \) %Inner diameter of part 4 at base for mass 4

\( D5 = D1 - 2*0.445; \) %Outer diameter of part 5 at base for mass 5
\( d5 = D5 - 2*t2; \) %Inner diameter of part 5 at base for mass 5

\( D6 = D1 - 2*0.495; \) %Outer diameter of part 6 at base for mass 6
\( d6 = D6 - 2*t3; \) %Inner diameter of part 6 at base for mass 6

\( D7 = D1 - 2*0.54; \) %Outer diameter of part 7 at base for mass 7
\( d7 = D7 - 2*t3; \) %Inner diameter of part 7 at base for mass 7

\( D8 = D1 - 2*0.58; \) %Outer diameter of part 8 at base for mass 8
\( d8 = D8 - 2*t4; \) %Inner diameter of part 8 at base for mass 8
%Upper part with constant cross section

D9 = Dt + t4/2;  %Outer diameter of part 9 at base for mass 9
d9 = D9 - 2*t4;  %Inner diameter of part 9 at base for mass 9

D10 = Dt;  %Outer diameter of part 10 at base for mass 10
d10 = D10 - 2*t4;  %Inner diameter of part 10 at base for mass 10

D11 = Dt;  %Outer diameter of top of tower (i.e. top of part 10)
d11 = D11 - 2*t4;  %Inner diameter of top of tower

I1 = (pi/64)*(D1^4 - d1^4);  %Second moment of area in m^4 for Part 1
I2 = (pi/64)*(D2^4 - d2^4);  %Second moment of area in m^4 for Part 2
I3 = (pi/64)*(D3^4 - d3^4);  %Second moment of area in m^4 for Part 3
I4 = (pi/64)*(D4^4 - d4^4);  %Second moment of area in m^4 for Part 4
I5 = (pi/64)*(D5^4 - d5^4);  %Second moment of area in m^4 for Part 5
I6 = (pi/64)*(D6^4 - d6^4);  %Second moment of area in m^4 for Part 6
I7 = (pi/64)*(D7^4 - d7^4);  %Second moment of area in m^4 for Part 7
I8 = (pi/64)*(D8^4 - d8^4);  %Second moment of area in m^4 for Part 8
I9 = (pi/64)*(D9^4 - d9^4);  %Second moment of area in m^4 for Part 9
I10 = (pi/64)*(D10^4 - d10^4);  %Second moment of area in m^4 for Part 10
I11 = (pi/64)*(D11^4 - d11^4);  %Second moment of area in m^4 at top of tower

%Stiffness of each individual 8m part of tower

k1 = (3*E*I1)/((1*h)^3);  %Stiffness of part 1 for mass 1
k2 = (3*E*(I1+I2)/2)/((2*h)^3);  %Stiffness of part 2 for mass 2
k3 = (3*E*(I1+I2+I3)/3)/((3*h)^3);  %Stiffness of part 3 for mass 3
k4 = (3*E*(I1+I2+I3+I4)/4)/((4*h)^3);  %Stiffness of part 4 for mass 4
k5 = (3*E*(I1+I2+I3+I4+I5)/5)/((5*h)^3);  %Stiffness of part 5 for mass 5
k6 = (3*E*(I1+I2+I3+I4+I5+I6)/6)/((6*h)^3);  %Stiffness of part 6 for mass 6
k7 = (3*E*(I1+I2+I3+I4+I5+I6+I7)/7)/((7*h)^3);  %Stiffness of part 7 for mass 7
k8 = (3*E*(I1+I2+I3+I4+I5+I6+I7+I8)/8)/((8*h)^3);  %Stiffness of part 8 for mass 8
k9 = (3*E*(I1+I2+I3+I4+I5+I6+I7+I8+I9)/9)/((9*h)^3);  %Stiffness of part 9 for mass 9
k10 = (3*E*(I1+I2+I3+I4+I5+I6+I7+I8+I9+I10)/10)/((10*h)^3);  %Stiffness of part 10 for mass 10

%Stiffness due to soil-structure interaction to be taken into account

G = 285000;  %Shear Modulus for Eastern Cape Soil in kN/m2
v = 0.3;  %Poisson's ration for Easter Cape Soil
R = 10;  %Radius of circular foundation in m
t = 2.2;  %Thickness of foundation in m
m0 = (24*pi*R^2*t)/9.81;  %Mass of foundation in tonnes (kN/m/s2)
k0 = ((8*G*R^3)/(3*(1-v)))/H^2;  %Rotational stiffness of soil
x0 = k0/m0;  %Square of natural frequency due to soil-structure interaction (w0^2)
w0 = sqrt(x0);  %Natural frequency due to soil-structure interaction in rad/s

%Masses of individual 8m parts of tower

mturbine = 137;  %Mass of nacelle and rotor of Vestas V112 3MW (tonnes = kN/m/s^2)
P = 7850;  %Density of structural steel in kg/m^3 as per SAISC Red Book
\[ Ar_1 = \left(\frac{\pi}{4}\right) \left(D_1^2 - d_1^2\right); \quad \text{%area at base of part 1 in m}^2 \]
\[ Ar_2 = \left(\frac{\pi}{4}\right) \left(D_2^2 - d_2^2\right); \quad \text{%area at base of part 2} \]
\[ Ar_3 = \left(\frac{\pi}{4}\right) \left(D_3^2 - d_3^2\right); \quad \text{%area at base of part 3} \]
\[ Ar_4 = \left(\frac{\pi}{4}\right) \left(D_4^2 - d_4^2\right); \quad \text{%area at base of part 4} \]
\[ Ar_5 = \left(\frac{\pi}{4}\right) \left(D_5^2 - d_5^2\right); \quad \text{%area at base of part 5} \]
\[ Ar_6 = \left(\frac{\pi}{4}\right) \left(D_6^2 - d_6^2\right); \quad \text{%area at base of part 6} \]
\[ Ar_7 = \left(\frac{\pi}{4}\right) \left(D_7^2 - d_7^2\right); \quad \text{%area at base of part 7} \]
\[ Ar_8 = \left(\frac{\pi}{4}\right) \left(D_8^2 - d_8^2\right); \quad \text{%area at base of part 8} \]
\[ Ar_9 = \left(\frac{\pi}{4}\right) \left(D_9^2 - d_9^2\right); \quad \text{%area at base of part 9} \]
\[ Ar_{10} = \left(\frac{\pi}{4}\right) \left(D_{10}^2 - d_{10}^2\right); \quad \text{%area at base of part 10} \]
\[ Ar_{11} = \left(\frac{\pi}{4}\right) \left(D_{11}^2 - d_{11}^2\right); \quad \text{%area at very top of tower (top of part 10)} \]

\[ \begin{align*}
  m_1 &= P \times \left(\frac{Ar_1 + Ar_2}{2}\right) \times \frac{h}{1000}; \quad \text{%mass of part 1 in kN/m/s^2 based on average volume/area} \\
  m_2 &= P \times \left(\frac{Ar_2 + Ar_3}{2}\right) \times \frac{h}{1000}; \quad \text{%mass of part 2 in kN/m/s^2} \\
  m_3 &= P \times \left(\frac{Ar_3 + Ar_4}{2}\right) \times \frac{h}{1000}; \quad \text{%mass of part 3 in kN/m/s^2} \\
  m_4 &= P \times \left(\frac{Ar_4 + Ar_5}{2}\right) \times \frac{h}{1000}; \quad \text{%mass of part 4 in kN/m/s^2} \\
  m_5 &= P \times \left(\frac{Ar_5 + Ar_6}{2}\right) \times \frac{h}{1000}; \quad \text{%mass of part 5 in kN/m/s^2} \\
  m_6 &= P \times \left(\frac{Ar_6 + Ar_7}{2}\right) \times \frac{h}{1000}; \quad \text{%mass of part 6 in kN/m/s^2} \\
  m_7 &= P \times \left(\frac{Ar_7 + Ar_8}{2}\right) \times \frac{h}{1000}; \quad \text{%mass of part 7 in kN/m/s^2} \\
  m_8 &= P \times \left(\frac{Ar_8 + Ar_9}{2}\right) \times \frac{h}{1000}; \quad \text{%mass of part 8 in kN/m/s^2} \\
  m_9 &= P \times \left(\frac{Ar_9 + Ar_{10}}{2}\right) \times \frac{h}{1000}; \quad \text{%mass of part 9 in kN/m/s^2} \\
  m_{10} &= \left(P \times \left(\left(\frac{Ar_{10} + Ar_{11}}{2}\right) \times h\right)/1000\right) + m_{turbine}; \quad \text{%mass of part 10 in kN/m/s^2} \\
\end{align*} \]

%Approximation of natural frequency according to Manwell, McGowan & Rogers(2002)
\[ \begin{align*}
  l_{total} &= \left(\left(1 + i4 + i5 + i11\right)/4\right); \quad \text{%Average stiffness of tower} \\
  m_{tower} &= m_1 + m_2 + m_3 + m_4 + m_5 + m_6 + m_7 + m_8 + m_9 + m_{10} - m_{turbine}; \quad \text{%Total mass of tower} \\
  f_n &= \left(1/(2\pi)\right) \times \sqrt{\left(3 \times E \times l_{total}\right)/\left((0.23 \times m_{tower} + m_{turbine}) \times H^3\right)}; \quad \text{%This approximation in Hz is said to be accurate within 15%} \\
\end{align*} \]

%Approximation of natural frequency according to Manwell, McGowan & Rogers(2002) with soil-structure interaction taken into account
\[ \begin{align*}
  w_n &= \sqrt{\left(3 \times E \times l_{total}\right)/\left((0.23 \times m_{tower} + m_{turbine}) \times H^3\right)}; \\
  x_n &= 1/\left(wo^2 + 1/\left(w_n^2\right)\right); \quad \text{%Addition of frequencies in series} \\
  f_{n0} &= \left(1/(2\pi)\right) \times \sqrt{1/x_n}; \quad \text{%This is an approximation including soil stiffness effects in Hz} \\
\end{align*} \]

%Natural frequencies of individual parts according to Dunkerley's Method
\[ \begin{align*}
  x_1 &= k_1/m_1; \\
  x_2 &= k_2/m_2; \\
  x_3 &= k_3/m_3; \\
  x_4 &= k_4/m_4; \\
  x_5 &= k_5/m_5; \\
  x_6 &= k_6/m_6; \\
  x_7 &= k_7/m_7; \\
  x_8 &= k_8/m_8; \\
  x_9 &= k_9/m_9; \\
  x_{10} &= k_{10}/m_{10}; \\
  x_{total} &= 1/x_1 + 1/x_2 + 1/x_3 + 1/x_4 + 1/x_5 + 1/x_6 + 1/x_7 + 1/x_8 + 1/x_9 + 1/x_{10}; \\
\end{align*} \]
\[ w_{ntotal} = \sqrt{\frac{1}{x_{total}}}; \]
\[ f_{ntotal} = \frac{1}{2\pi} \frac{1}{\sqrt{x_{ntotal}}}; \]
\%Overall 1st Natural Frequency according to Dunkerley's Method (excluding soil stiffness effects) in Hz
\[ x_{n2} = \frac{1}{w_0^2} + \frac{1}{(w_{ntotal})^2}; \]
\%Taking into account soil-structure stiffness
\[ f_{ntotal_0} = \frac{1}{2\pi} \frac{1}{\sqrt{x_{n2}}}; \]
\%Overall 1st Natural Frequency according to Dunkerley's Method (including soil stiffness effects) in Hz

\%Outputs
\%Simplified Method according to Manwell's equation
\[ f_n \quad \%\text{Without soil stiffness} \]
\[ f_{n0} \quad \%\text{With soil stiffness} \]
\%Analytical Method based on lumped mass/discrete method and Dunkerley's method
\[ f_{ntotal} \quad \%\text{Without soil stiffness} \]
\[ f_{ntotal_0} \quad \%\text{With soil stiffness} \]
### Application for Approval of Ethics in Research (EiR) Projects
Faculty of Engineering and the Built Environment, University of Cape Town

**APPLICATION FORM**

Please Note:
Any person planning to undertake research in the Faculty of Engineering and the Built Environment (EBE) at the University of Cape Town is required to complete this form before collecting or analysing data. The objective of submitting this application prior to embarking on research is to ensure that the highest ethical standards in research, conducted under the auspices of the EBE Faculty, are met. Please ensure that you have read, and understood the EBE Ethics in Research Handbook (available from the UCT EBE, Research Ethics website) prior to completing this application form: [http://www.ebe.uct.ac.za/usr/ebe/research/ethics.pdf](http://www.ebe.uct.ac.za/usr/ebe/research/ethics.pdf)

#### APPLICANT’S DETAILS

<table>
<thead>
<tr>
<th>Name of principal researcher, student or external applicant</th>
<th>Kaylee Folster (Kaylee Hewitt)</th>
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<tbody>
<tr>
<td>Department</td>
<td>Civil Engineering</td>
</tr>
<tr>
<td>Preferred email address of applicant</td>
<td><a href="mailto:hewitt.kaylee@gmail.com">hewitt.kaylee@gmail.com</a></td>
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**If a Student**

<table>
<thead>
<tr>
<th>Your Degree: e.g., MSc, PhD, etc.,</th>
<th>MEng Structural Engineering</th>
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<tbody>
<tr>
<td>Name of Supervisor (if supervised)</td>
<td>Kenny Mudenda</td>
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**If this is a research contract, indicate the source of funding/sponsorship**

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**Project Title**

Influence of Geometry on the Dynamic Behaviour of Steel Tubular Towers for Onshore Wind Turbines

I hereby undertake to carry out my research in such a way that:

- there is no apparent legal objection to the nature or the method of research; and
- the research will not compromise staff or students or the other responsibilities of the University;
- the stated objective will be achieved, and the findings will have a high degree of validity;
- limitations and alternative interpretations will be considered;
- the findings could be subject to peer review and publicly available; and
- I will comply with the conventions of copyright and avoid any practice that would constitute plagiarism.

### SIGNED BY

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<thead>
<tr>
<th>Full name</th>
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<th>Date</th>
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<tbody>
<tr>
<td>Principal Researcher/Student/External applicant</td>
<td>Kaylee Folster (Kaylee Hewitt)</td>
<td>08 Dec 2016</td>
</tr>
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### APPLICATION APPROVED BY

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<td>Kenny Mudenda</td>
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</table>

**HOD (or delegated nominee)**

Final authority for all applicants who have answered NO to all questions in Section 1, and for all Undergraduate research (including Honours).

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**Chair: Faculty EIR Committee**

For applicants other than undergraduate students who have answered YES to any of the above questions.

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