The simulation testing framework used during the development of OMP-13

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This document details the framework used to simulation test candidate MPs during the development of OMP-13. A summary of assumptions made in this simulation testing framework are listed below, with yellow highlights indicating updates since de Moor and Butterworth (2012f). Appendix A provides the full details, with data used listed in the tables at the end of the Appendix.

Summary list of assumptions made in the framework to be used to simulation test OMP-13

1) Half the sardine is caught between 1 November and 30 April and half from 1 May to 31 October.
2) Half the juvenile anchovy is caught between 1 November and 15 July and half from mid-July to 31 October.
3) Half the adult anchovy is caught between 1 November and 31 March and half from 1 April to 31 October.
4) The assumptions made during the development of the underlying operating models (de Moor and Butterworth 2012d,e), such as age at maturity and stock-recruitment relationships and differences in these assumptions between alternative operating models (robustness tests), are carried forward during projections.
5) The recruit survey is simulated to commence mid-May each year.
6) During implementation, all of the directed (>14cm) sardine TACy and TABbycatch ≥ 14cm = 7000t (bycatch ≥14cm) are 1+ year old sardine.
7) All 1+ sardine catch is split into age groups according to the selectivity-at-age estimated by the underlying operating model.
8) During implementation, all TABs for <14cm sardine translate into 0-year-old sardine bycatch.
9) During implementation, all <14cm sardine bycatch with round herring, TABy,small, rh =1000t, is caught between the time of the recruit survey (mid-May) and the end of the normal season.
10) During implementation, half of the juvenile (<14cm) sardine bycatch with directed sardine is caught by the time of the recruit survey (mid-May), and there is an implicit assumption that all of this bycatch is caught by the end of the normal season (the latter extends to different cut-off dates depending on the assumption made by each MP variant with regards to an additional sub-season).

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11) During implementation, the maximum amount of \( \leq 14 \text{cm} \) sardine bycatch in the directed (>14cm) sardine catch used to set the sardine TAB, \( \sigma \), is not always assumed taken; a proportion is drawn from a distribution based on the historic proportions with a maximum of \( \sigma \).

12) During implementation, half of \( TAB^A = 500 \text{t} \) is taken by the end of June, with the remaining half taken by the end of the normal season.

13) During implementation, the initial normal season anchovy TAC, \( TAC^{IA}_y \), is caught by the end of June, and 65% of this is caught by the end of May with the remaining 35% caught during June.

14) During implementation, 26\% of the anchovy catch landed by the end of June \( (TAC^{IA}_y + \frac{1}{2}TAB^A) \) are juveniles caught by mid-May.

15) During implementation, 33\% of the anchovy catch landed by the end of June \( (TAC^{IA}_y + \frac{1}{2}TAB^A) \) are adult anchovy; 69\% of the adult anchovy catch is landed by the time of the recruit survey (mid-May).

16) During implementation, the juvenile (<14cm) sardine bycatch with anchovy from January to 31 May is 1.436 times that from January to mid-May.

17) During implementation, juvenile (<14cm) sardine bycatch with anchovy over the months of June to December is taken to be a proportion of the anchovy catch during these months, with the monthly proportions and variances being estimated from the monthly juvenile sardine to anchovy ratios, based upon historic catch monthly observations and draws from model predicted recruitment.

18) In the implementation of sardine bycatch with anchovy, correlations in the juvenile sardine to anchovy ratios apply between successive months only.

19) In the implementation of sardine bycatch with anchovy, if the additional sub-season begins 1 September, 60\% of the July and August anchovy catch is taken in July.

20) In the implementation of sardine bycatch with anchovy, if the additional sub-season begins 1 October, 45\% of the July to September anchovy catch is taken in July and 30\% in August.

21) In the implementation of sardine bycatch with anchovy, if there is no additional sub-season, 42\% of the July to December anchovy catch is taken in July, 26\% in August and 22\% in September.

22) For all catches simulated, an upper limit is placed on the industry’s efficiency by assuming that no more than 95\% of the selectivity-weighted stock abundance may be caught.

23) The ratio of juvenile sardine to anchovy in May (and used in the Harvest Control Rule), \( r_y \), is restricted to a maximum of 1.

24) The ratios of juvenile sardine to anchovy in the months of June, July, August, September and October to December, used in simulating how much juvenile sardine is actually caught, are restricted to a maximum of 2.
25) Sardine bycatch with anchovy in the additional sub-season is at most \( r_y \), the ratio of juvenile sardine to anchovy “in the sea” during May, of that portion of the anchovy final TAC taken in the additional sub-season.

26) Implementation simulation accounts for the closure of the anchovy fishery if the sardine bycatch with anchovy allowance is reached, by proportionally decreasing the amount of juvenile anchovy catch simulated to be taken within a year.

27) Future survey observations are generated taking the historic correlation between the species into account, and the variance is based on a regression between historic survey CV and model predicted abundance.

28) Survey and catch-related observations already known for 2012 have been used instead of model simulated observations. The undercatch of the revised anchovy normal season TAC has been taken into account. The recruitment in November 2011, and the corresponding recruitment residual are obtained by combining information from both the stock recruitment relationship and the now known June 2012 survey results.

References


de Moor, C.L., and Butterworth, D.S. 2012f. A summary list of the assumptions made in the simulation testing framework used during the development of OMP-13. DAFF: Branch Fisheries Document FISHERIES/2012/OCT/SWG-PEL/55. 3pp

Appendix A: The framework used to simulation test a joint MP for South African sardine and anchovy: OMP-13

In this appendix, the framework used to simulation test OMP-13 is detailed. The framework consists of a population dynamics model for future simulation of the effects of alternative MPs on the sardine and anchovy populations, an implementation model which generates future catches-at-age given annual TAC/Bs [calculated using the harvest control rules detailed in de Moor and Butterworth 2012a], and an observation model which generates the necessary data (in this case, catch and survey data) to be input into the MP. Catches-at-age are given in numbers of fish (billions), whereas the TACs and TABs are given in biomass (in thousands of tons). All parameters which are drawn from the Bayesian posterior distributions of de Moor and Butterworth (2012b,c) are listed in Table A1.

Population dynamics model
Given the numbers-at-age at the beginning of the projection period [i.e., November 2011, drawn from the posterior distributions output from the operating models (de Moor and Butterworth, 2012b,c)], values for future catches output from the implementation model, \( C_{j,y,a}^i, \ i = S, A \) (see below), the population dynamics model projects numbers-at-age and spawning biomass at the beginning of November for \( y = 2012, \ldots, 2032 \) as follows. The sardine adult catch is assumed to be taken half way between 1\(^{st}\) November and 31\(^{st}\) October each year. (The sardine stock assessment was fit to quarterly commercial proportion at length data and thus catch was modelled to be taken quarterly (de Moor and Butterworth 2012d). The catch tonnage between 1984 and 2011, however, is almost equally split from 1 November to 30 April and 1 May to 31 October.) The anchovy juvenile catch is assumed to be taken as a pulse at 15\(^{th}\) July and the adult catch is assumed to be taken as a pulse at 1\(^{st}\) April (de Moor and Butterworth 2012e). Sardine are assumed to mature at age 2 (de Moor and Butterworth 2012e) and anchovy at age 1 (de Moor and Butterworth 2012d). All notation allows for multiple stocks of both species, though only a single stock for anchovy is considered in all operating models.

Sardine:
\[
N_{j,y,a}^{S,\text{pred}} = \left( N_{j,y-1,a}^{S,\text{pred}} e^{-M_{a,0}/2} - C_{j,y,0}^{S,\text{pred}} \right) e^{-M_{a,0}/2}
\]
\[
N_{j,y,a}^{S,\text{pred}} = \left( N_{j,y-1,a-1}^{S,\text{pred}} e^{-M_{a,0}/2} - C_{j,y,a-1}^{S,\text{pred}} \right) e^{-M_{a,0}/2}, \quad a = 2, \ldots, 4
\]
\[
N_{j,y,5+}^{S,\text{pred}} = \left( N_{j,y-1,4}^{S,\text{pred}} e^{-M_{a,0}/2} - C_{j,y,4}^{S,\text{pred}} \right) e^{-M_{a,0}/2} + \left( N_{j,y,5+}^{S,\text{pred}} e^{-M_{a,0}/2} - C_{j,y,5+}^{S,\text{pred}} \right) e^{-M_{a,0}/2}
\]
\[
B_{j,y,N}^{S,\text{pred}} = \sum_{a=1}^{5+} N_{j,y,a}^{S,\text{pred}} S_j^a
\]
\[
SSB_{j,y,N}^{S,\text{pred}} = \sum_{a=2}^{5+} N_{j,y,a}^{S,\text{pred}} S_j^a
\] (A.1)
Anchovy: 

\[
N_{i,j,y}^{A,\text{pred}} = \left( N_{i,j,y-1}^{A,\text{pred}} e^{-8.5M_{j-1}^{0} / 12} - C_{i,j,0}^{A,\text{pred}} \right) e^{-3.5M_{j}^{0} / 12}
\]

\[
N_{i,j,y}^{A,\text{pred}} = \left( N_{i,j,y-1}^{A,\text{pred}} e^{-5M_{j-1}^{0} / 12} - C_{i,j,1}^{A,\text{pred}} \right) e^{-7M_{j}^{0} / 12}
\]

\[
N_{i,j,y}^{A,\text{pred}} = N_{i,j,y-1}^{A,\text{pred}} e^{-M_{j}^{0}}
\]

\[
N_{i,j,y}^{A,\text{pred}} = N_{i,j,y-1}^{A,\text{pred}} e^{-M_{j}^{0}} + N_{i,j,y-4}^{A,\text{pred}} e^{-M_{j}^{0}}
\]

\[
B_{i,j,y}^{A,\text{pred}} = \sum_{a=4}^{4} N_{i,j,y}^{A,\text{pred}} \bar{w}_{j,a}^{A}
\]

(A.2)

where 

\[
N_{i,j,y}^{i,\text{pred}}
\]

is the operating model predicted numbers at age \( a \) (in billions) of species \( i \) \((i=S, A)\), stock \( j \), at the beginning of November in year \( y \);

\[
M_{j}^{i}
\]

is the natural mortality rate (in year\(^{-1}\)) of juvenile (age 0) fish of species \( i \) \((i=S, A)\) (de Moor and Butterworth 2012d,e);

\[
M_{j}^{i}
\]

is the natural mortality rate (in year\(^{-1}\)) of age 1+ fish of species \( i \) \((i=S, A)\) (de Moor and Butterworth 2012d,e);

\[
C_{i,j,y}^{i,\text{pred}}
\]

is the model predicted future catches at age \( a \) in year \( y \) of species \( i \) \((i=S, A)\), stock \( j \) output from the implementation model (given below);

\[
B_{i,j,y}^{i,\text{pred}}
\]

is the operating model predicted November 1+ biomass (in thousands of tons) of species \( i \) \((i=S, A)\), stock \( j \);

\[
SSB_{i,j,y}^{i,\text{pred}}
\]

is the operating model predicted spawning stock biomass (in thousands of tons) of species \( i \) \((i=S, A)\), stock \( j \);

\[
SSB_{i,j,y}^{A,\text{pred}}
\]

is the operating model predicted anchovy spawning stock biomass (in thousands of tons);

and

\[
\bar{w}_{j,a}^{i}
\]

is the average weights-at-age \( a \) of species \( i \) \((i=S, A)\), stock \( j \) from the historic November spawner biomass surveys (Table A2);

Letting \( f(SSB_{i,j,y}^{i,\text{pred}}) \) denote the stock recruitment curve of the chosen model, with parameters \( a_{j}^{i} \) and \( b_{j}^{i} \) (Table A1), then future recruitment \( N_{i,j,y}^{i,\text{pred}} \) \((i=S, A)\) is assumed to be log-normally distributed about a stock recruitment relationship as follows:

\[
N_{i,j,y}^{i,\text{pred}} = f(SSB_{i,j,y}^{i,\text{pred}}) e^{e_{j,y}^{i}}
\]

(A.3)

where

\[
e_{j,y}^{i} = s_{j,cor} e_{j,y-1}^{i} + \sqrt{1-(s_{j,cor})^{2}} \omega_{j,y}^{i}
\]

where \( \omega_{j,y}^{i} \sim N(0;1) \)

(A.4)

and
\( \varepsilon_{j,y} \) is the standardised recruitment residual for stock \( j \) of species \( i \) \( (i = S, A) \) in year \( y \), see below for \( \varepsilon_{2011} \);

\( \sigma_{j,r}^{i} \) is the standard deviation of the recruitment residuals for stock \( j \) of species \( i \) \( (i = S, A) \), drawn from posterior distributions output from the operating models (de Moor and Butterworth, 2012b,c);

\( s'_{j,cor}^{i} \) is the recruitment serial correlation for stock \( j \) of species \( i \) \( (i = S, A) \), drawn from posterior distributions output from the operating models (de Moor and Butterworth, 2012b,c).

**Implementation model**

The MP variants outputs the following TAC/Bs (see de Moor and Butterworth 2012a):

1. An annual directed >14cm sardine TAC, \( TAC_{S}^{y} \).
2. An initial and revised normal season anchovy TAC (\( TAC_{y}^{1,A} \) and \( TAC_{y}^{2,A} \)), either applying to i) January to 31 August, ii) January to 30 September, or iii) January to December depending on the assumptions regarding the additional sub-season in the MP variant being tested. In cases i) and ii), where an additional sub-season is included from September/October to December, a total anchovy TAC, which includes the additional sub-season with the normal season, is given by \( TAC_{y}^{3,A} \).
3. An annual constant anchovy TAB, \( TAB^{A} \).
4. An annual constant >14cm sardine TAB, \( TAB_{big}^{S} \).
5. An annual constant \( \leq 14 \)cm sardine bycatch with round herring, and to a lesser extent with anchovy, \( TAB_{small, rh}^{S} \).
6. An annual \( \leq 14 \)cm sardine bycatch with directed (>14cm) sardine, \( TAB_{y,small}^{S} \).
7. For each anchovy TAC in 2) there is a corresponding \( \leq 14 \)cm sardine TAB with anchovy, \( TAB_{y,anch}^{A} \); \( TAB_{y,anch}^{2,S} \) and, when applicable, \( TAB_{y,anch}^{3,S} \).

Given these TAC / TABs output from the MP (in thousands of tons), the implementation model simulates the implementation of these catch limits by the industry to yield future catches-at-age (in billions).

**Sardine adult catch**

The adult sardine catch is simulated using selectivity-at-age estimated by the operating model:

\[
C_{j,y,a}^{S, pred} = N_{j,y-1,a}^{S, pred} S_{j,a}^{S} F_{j,y} e^{-M_{w}/2}, \quad a = 1,...,5 + (A.5)
\]

where \( F_{j,y} = \frac{\sum_{a=1}^{5} N_{j,y-1,a}^{S, pred} S_{j,a}^{S} W_{j,ac}^{S} e^{-M_{w}/2}}{\sum_{a=1}^{5} N_{j,y-1,a}^{S, pred} S_{j,a}^{S} W_{j,ac}^{S} e^{-M_{w}/2}} \), \( (A.6) \)
$S_{j,a}^s$ are the sardine stock $j$ fishing selectivities-at-age $a^1$ drawn from posterior distributions estimated from the operating model (de Moor and Butterworth 2012c);

$\overline{w}_{j,ac}^i$ are the historic average weights-at-age $a$ in the catches for stock $j$ of species $i$, $i = A, S$ (Table A2);

$\tau_j$ is the proportion of the directed sardine TAC assumed caught west/south of Cape Agulhas; and

$\tau_j'$ is the proportion of the big sardine TAB assumed caught west/south of Cape Agulhas.

**Anchovy 1-year-old catch**

Between 1984 and 2011, the total (annual) 1-year-old catch in tons constituted, on average, 37% of the anchovy catch biomass between January and June (the period to which $TAC_{y}^{1,A}$ and half of $TAB^A$ is taken to apply). This percentage drops to 33% if only the past 10 years are considered. As the most recent history is likely a better reflection of future catch patterns, the anchovy 1 year old catch is thus taken to be 33% of the initial normal season anchovy TAC:

$$C_{1,y,1}^{A,\text{pred}} = 0.33 \times \frac{\left( TAC_{y}^{1,A} + \frac{1}{2} TAB^A \right)}{\overline{w}_{1c}^A}.$$ (A.7)

**Anchovy 0-year-old catch**

Between 1984 and 2011 the anchovy juvenile catch in tons from 1$^{\text{st}}$ January to 30$^{\text{th}}$ April, together with half the May juvenile catch in tons was 26% of the total anchovy catch biomass from January to June. This percentage remains unchanged if only the past 10 years are considered. Using the above assumption that $TAC_{y}^{1,A}$ and half of $TAB^A$ is caught by the end of June, the anchovy 0-year-old catch taken prior to the recruit survey is:

$$C_{1,y,\text{blue}}^{A,\text{pred}} = 0.26 \times \frac{\left( TAC_{y}^{1,A} + \frac{1}{2} TAB^A \right)}{\overline{w}_{0c}^A}.$$ (A.8)

and for the normal season as a whole:

$$C_{1,y,0}^{A,\text{pred}} = \frac{1}{\overline{w}_{0c}^A} \left( TAC_{y}^{2,A} + TAB^A - C_{1,y,1}^{A,\text{pred}} \times \overline{w}_{1c}^A \right)$$ (A.9)

**Sardine 0-year-old catch prior to the recruit survey**

The 0-year-old sardine catch prior to the recruit survey is based on the January to mid-May bycatch occurring with i) round herring, ii) adult sardine in the directed fishery, and iii) adult and juvenile anchovy. It is assumed that all juvenile sardine bycatch with round herring occurs after the recruit survey. It is further

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1 Taken to be the average of model predicted commercial selectivity-at-age in quarters 2 and 3 of de Moor and Butterworth (2012e). The selectivities-at-ages 1 to 5+ are re-normalised such that the largest selectivity is 1.
assumed that half the juvenile sardine in the directed sardine catch is caught by the time of the survey. As an average of 69%\(^2\) of adult anchovy catch has been landed by mid-May between 2002 and 2011, 69% of the anchovy adult catch together with the juvenile anchovy catch prior to the survey is used to calculate the 0-year-old sardine catch prior to the survey:

\[
C_{S, pred}^{y, 0, obs} = \frac{1}{2} \sigma_y^{draw} \tau_1 TAC_y^S + k_{JanMay} \frac{N_{S, pred}^{y, 1, 0}}{N_{A, pred}^{y, 1, 0}} \sigma_{anom, traw} \frac{(C_{A, pred}^{y, 0, obs} w_{0, A}^A + 0.69 C_{A, pred}^{y, 1} w_{1, A}^A)}{w_{1, 0c}^S},
\]

\[
C_{S, pred}^{y, 2, obs} = \frac{1}{2} \sigma_y^{draw} \tau_2 TAC_y^S,
\]

where \(\eta_{y, JanMay} \sim N(0; 1)\)

and \(k_{JanMay}\) and \(\sigma_{JanMay}\) are given in equations (A.41) and (A.43) respectively. \(\sigma\) is the estimate of the maximum amount of \(\leq 14\) cm sardine bycatch in the directed (>14 cm) sardine catch used to set the sardine TAB. During simulation, this maximum amount is not always assumed taken. Instead, the proportion, \(\sigma_y^{draw}\), of the directed catch assumed taken is drawn from a distribution based on the historic proportions (Figure A1).

**Sardine 0-year-old catch (in billions)**

In modelling the total sardine juvenile bycatch, the following approach is used. If the full TAB with anchovy were caught, the total juvenile sardine catch by mass would be

\[
\bar{W}_{1, 0c}^S C_{S, A}^{y, pred} = (\lambda_y TAC_y^{1, A} + r_y (TAC_y^{2, A} - TAC_y^{1, A})) + \tau_y TAB_{small, rh}^S + \omega_y^{draw} \tau_1 TAC_y^S,
\]

\[
\bar{W}_{2, 0c}^S C_{S, 2, obs}^{y} = \tau_2 TAB_{small, rh}^S + \omega_y^{draw} \tau_y TAC_y^S,
\]

where \(\lambda_y = \max \{\gamma_y, r_y\}\)

\(\gamma_y\) is the percentage of the initial anchovy TAC used to set the initial \(\leq 14\) cm sardine TAB with anchovy. We now need to model how the sardine bycatch with anchovy fishery develops during the season; and

\(\tau_y\) is the proportion of the small sardine TAB with redeye assumed caught west/south of Cape Agulhas.

The ratio of juvenile sardine to anchovy “in the sea” during May, \(r_y\), is calculated from two sources as follows:

\[
r_y = \frac{1}{2} (r_{y, sur} + r_{y, com})).
\]

When implementing OMP-13, both \(r_{y, sur}\) and \(r_{y, com}\) will be observations that will be available to input into the Harvest Control Rules, with

\(^2\)78% average from 1984 to 2011.
During simulation, the sardine bycatch to anchovy ratio in commercial catches in May, is given by:

\[
r_{y,\text{com}} = k_{\text{may}} \frac{N_{y,\text{may}}^{S,\text{pred}}}{N_{y,\text{may}}^{A,\text{pred}}} e^{\sigma_{\text{may}} \varepsilon_{y,\text{may}}}.
\] (A.14)

where \( \varepsilon_{y,\text{may}} = \rho_{\text{may}} \eta_{y,\text{jan,may}} + \sqrt{1 - (\rho_{\text{may}})^2} \eta_{y,\text{may}} \). (A.15)

with \( \eta_{y,\text{may}} \sim N(0;1) \) and \( \eta_{y,\text{jan,may}} \) is given by equation (A.10). As \( r_{y,\text{com}} \) is based on simulated commercial catches, the model predicted numbers-at-age, \( N_{y,r}^{i,\text{pred}} \), are used rather than those simulated to be survey observations. Here we have

\( N_{j,y,r}^{i,\text{obs}} \) - the acoustic survey estimate of recruitment (in billions) for fish of stock \( j \) of species \( i \) \((i = S, A)\) for year \( y \), which will be an observation available for input into the Harvest Control Rules; during simulation these observations are derived using equation (A.31).

\( N_{j,y,r}^{i,\text{pred}} \) - the model-predicted recruitment (in billions) for fish of stock \( j \) of species \( i \) \((i = S, A)\) in November of year \( y-1 \), projected forward to the time of the recruit survey in year \( y \)
(equation A.34).

\( k_{\text{may}} \) - the constant of proportionality from equation (A.44),

\( \sigma_{\text{may}} \) - the residual standard deviation from equation (A.46); and

\( \rho_{\text{may}} \) - the correlation coefficient from equation (A.47)

Equation (A.11) assumes that the ratio of juvenile sardine to anchovy “in the sea” during May, \( r_s \), will remain a constant for the remainder of the normal season. However, there is usually a drop-off in this ratio as the year progresses (Figure A2). This effect is simulated by adjusting equation (A.11) to reflect the actual level of 0-year-old sardine to be expected in the catches, given the historical pattern of sardine bycatch to anchovy ratio changes (usually a drop-off) from May to August/September/October-December\(^3\).

Over the past 10 years (2002-2011), the sardine bycatch with anchovy from January to 31\(^{\text{st}}\) May has been 1.436 times that from January to mid-May\(^4\). Adjusting the sardine bycatch prior to the survey to take account of this additional bycatch by the end of May, equation (A.11) is modified as follows.

i) If the MP variant assumes the additional sub-season runs from 1 September to 31 December:

\(^3\) The choice of month depends on the end of the normal season in the MP variant.

\(^4\) Bycatch from 1\(^{\text{st}}\) to 15\(^{\text{th}}\) May approximated by half the bycatch from the full month of May.
\[ C_{1,y,0}^{S_{\text{pred}}} = 1.436 \left( C_{1,y,0\text{obs}} - \frac{1}{2} \frac{\sigma_y \tau_1 TAC_y^S}{W_{1,0c}^S} \right) + \frac{\tau_1 TAC_y^S}{W_{1,0c}^S} + \frac{1}{W_{1,0c}^S} \left( r_{y,\text{jun}} C_{y,\text{jun}}^A + r_{y,\text{aug}} C_{y,\text{aug}}^A + r_{y,\text{sep}} C_{y,\text{sep}}^A \right) \]  
(A.16a)

ii) If the MP variant assumes the additional sub-season runs from 1 October to 31 December:

\[ C_{1,y,0}^{S_{\text{pred}}} = 1.436 \left( C_{1,y,0\text{obs}} - \frac{1}{2} \frac{\sigma_y \tau_1 TAC_y^S}{W_{1,0c}^S} \right) + \frac{\tau_1 TAC_y^S}{W_{1,0c}^S} + \frac{1}{W_{1,0c}^S} \left( r_{y,\text{jun}} C_{y,\text{jun}}^A + r_{y,\text{aug}} C_{y,\text{aug}}^A + r_{y,\text{sep}} C_{y,\text{sep}}^A \right) \]  
(A.16b)

iii) If the MP variant assumes no additional sub-season:

\[ C_{1,y,0}^{S_{\text{pred}}} = 1.436 \left( C_{1,y,0\text{obs}} - \frac{1}{2} \frac{\sigma_y \tau_1 TAC_y^S}{W_{1,0c}^S} \right) + \frac{\tau_1 TAC_y^S}{W_{1,0c}^S} + \frac{1}{W_{1,0c}^S} \left( r_{y,\text{jun}} C_{y,\text{jun}}^A + r_{y,\text{aug}} C_{y,\text{aug}}^A + r_{y,\text{sep}} C_{y,\text{sep}}^A \right) \]  
(A.16c)

The sardine bycatch to anchovy ratios, \( r_{y,m} \), are simulated in a similar way to \( r_{y,\text{com}} \) (equation A.14) as follows:

\[ r_{y,m} = k_m \frac{N_{S_{\text{pred}},y,m}}{N_{A_{\text{pred}},y,m}} e^{\sigma_m \epsilon_{y,m}}, \text{where } m = \text{jun, jul, aug, sep, octdec} \]  
(A.17)

where \( k_m \) and \( \sigma_m \) are from equations (A.44) and (A.46), summing over years for which anchovy directed catch is non-zero, and:

\[ \epsilon_{y,jun} = \rho_{y,jun} \epsilon_{y,yay} + \sqrt{1 - (\rho_{y,jun})^2} \eta_{y,jun} \]

\[ \epsilon_{y,jul} = \rho_{y,jul} \epsilon_{y,jun} + \sqrt{1 - (\rho_{y,jul})^2} \eta_{y,jul} \]

\[ \epsilon_{y,aug} = \rho_{y,aug} \epsilon_{y,jul} + \sqrt{1 - (\rho_{y,aug})^2} \eta_{y,aug} \]

\[ \epsilon_{y,sep} = \rho_{y,sep} \epsilon_{y,aug} + \sqrt{1 - (\rho_{y,sep})^2} \eta_{y,sep} \]

\[ \epsilon_{y,octdec} = \rho_{y,octdec} \epsilon_{y,sep} + \sqrt{1 - (\rho_{y,octdec})^2} \eta_{y,octdec} \]  
(A.18)
The equations above reflect the correlative relationships between adjacent months, where \( \varepsilon_{y,m} \) is from equation (A.15), \( \rho_m \) is from equation (A.47)\(^5\) and \( \eta_{y,m} \sim N(0;1) \), \( m = \text{jun, jul, aug, sep, octdec} \). (A.19)

Between 2002 and 2011 the average total anchovy catch from January to May was 65\(^6\)% of that from January to June. Assuming 65\(^6\)% of \( \text{TAC}_{y}^{1,A} \) is caught by the end of May, and given the assumption that \( \text{TAC}_{y}^{1,A} \) is caught by the end of June, the anchovy catches in equation (A.16), \( \text{C}_{y,m}^{A,\text{pred}} \ (m = \text{jun, jul, aug, sep}) \), are derived as follows (in thousands of tons):

\[
\text{C}_{y,\text{jun}}^{A,\text{pred}} = 0.35 \times \text{TAC}_{y}^{1,A} \tag{A.20}
\]
\[
\text{C}_{y,\text{jul}}^{A,\text{pred}} = \rho_{\text{jul}} (\text{TAC}_{y}^{2,A} - \text{TAC}_{y}^{1,A}) \tag{A.21}
\]

i) For the case where a MP variant assumes the additional sub-season begins 1 September:

\[
\text{C}_{y,\text{aug}}^{A,\text{pred}} = (1 - \rho_{\text{jul}}) (\text{TAC}_{y}^{2,A} - \text{TAC}_{y}^{1,A}) \tag{A.22a}
\]

where \( \rho_{\text{jul}} = 0.60 \)\(^7\) is taken to be the average 2002 to 2011 proportion of total anchovy catch during July and August that is taken in July.

ii) For the case where a MP variant assumes the additional sub-season begins on 1 October:

\[
\text{C}_{y,\text{aug}}^{A,\text{pred}} = \rho_{\text{aug}} (\text{TAC}_{y}^{2,A} - \text{TAC}_{y}^{1,A}) \tag{A.22b}
\]
\[
\text{C}_{y,\text{sep}}^{A,\text{pred}} = (1 - \rho_{\text{jul}} - \rho_{\text{aug}}) (\text{TAC}_{y}^{2,A} - \text{TAC}_{y}^{1,A}) \tag{A.23b}
\]

where \( \rho_{\text{jul}} = 0.45 \)\(^8\) and \( \rho_{\text{aug}} = 0.30 \)\(^9\) are taken to be the average 2002 to 2011 proportion of total anchovy catch during July to September that is taken in July and August, respectively.

iii) For the case where a MP variant assumes no additional sub-season:

\[
\text{C}_{y,\text{aug}}^{A,\text{pred}} = \rho_{\text{aug}} (\text{TAC}_{y}^{2,A} - \text{TAC}_{y}^{1,A}) \tag{A.22c}
\]
\[
\text{C}_{y,\text{sep}}^{A,\text{pred}} = \rho_{\text{sep}} (\text{TAC}_{y}^{2,A} - \text{TAC}_{y}^{1,A}) \tag{A.23c}
\]
\[
\text{C}_{y,\text{octdec}}^{A,\text{pred}} = (1 - \rho_{\text{jul}} - \rho_{\text{aug}} - \rho_{\text{sep}}) (\text{TAC}_{y}^{2,A} - \text{TAC}_{y}^{1,A}) \tag{A.24c}
\]

where \( \rho_{\text{jul}} = 0.42 \)\(^10\), \( \rho_{\text{aug}} = 0.26 \)\(^11\) and \( \rho_{\text{sep}} = 0.22 \)\(^12\) are taken to be the average 2002 to 2011 proportion of total anchovy catch during July to December that is taken in July, August and September, respectively.

---

\(^5\) Note that \( \varepsilon_{y,m-1} \) is replaced by \( \varepsilon_{y,m}^{*} \) in the numerator of equation (A.47) for \( m = \text{jun, jul, aug, sep} \).

\(^6\) Average from 1984 to 2011 is 70%.

\(^7\) Average from 1984 to 2011 is 0.59

\(^8\) Average from 1984 to 2011 is 0.49.

\(^9\) Average from 1984 to 2011 is 0.30.

\(^10\) Average from 1984 to 2011 is 0.44.

\(^11\) Average from 1984 to 2011 is 0.28.

\(^12\) Average from 1984 to 2011 is 0.18.
Closure of the anchovy fishery

The anchovy catch, \( C_{1,y,0}^A \), is adjusted if the adjusted \( C_{1,y,0}^S \) exceeds \( \frac{\text{TAB}_{2,y,\text{anch}}}{w_{1,0c}} \) (equation (OMP.12) of de Moor and Butterworth 2012a), in order to reflect the closure of the anchovy fishery once the sardine bycatch allowance linked to anchovy is reached. If \( C_{1,y,0}^{S,y,\text{pred}} \frac{S_{1,y,0}}{w_{1,0c}} > \text{TAB}_{2,y,\text{anch}} \), then the anchovy fishery would be closed once the full bycatch allowance was taken. This is simulated by assuming that the anchovy TAC is taken at the same rate as the sardine bycatch:

\[
C_{1,y,0}^{S,y,\text{pred}} = \min \left\{ C_{1,y,0}^{S,y,\text{pred}}, \frac{\text{TAB}_{2,y,\text{anch}}}{w_{1,0c}} \right\}
\]  

(A.24)

\[
C_{1,y,0}^{A,y,\text{pred}} = \begin{cases} 
1 \frac{TAC_{2,y}^{A,y,\text{pred}}}{W_{0c}} \left[ \frac{\text{TAB}_{2,y,\text{anch}}}{C_{1,y,0}^{S,y,\text{pred}} w_{1,0c}} + \text{TAB}_{A}^{A} - C_{1,y,0}^{A,y,\text{pred}} \frac{W_{0c}}{W_{1,0c}} \right] & \text{if } C_{1,y,0}^{S,y,\text{pred}} \frac{S_{1,y,0}}{w_{1,0c}} \leq \text{TAB}_{2,y,\text{anch}} \\
\frac{1}{W_{0c}} \left( TAC_{2,y}^{A,y,\text{pred}} + TAC_{A}^{A} - C_{1,y,0}^{A,y,\text{pred}} \frac{W_{0c}}{W_{1,0c}} \right) & \text{if } C_{1,y,0}^{S,y,\text{pred}} \frac{S_{1,y,0}}{w_{1,0c}} > \text{TAB}_{2,y,\text{anch}} 
\end{cases}
\]

(A.25)

Additional sub-season

A final adjustment is made to \( C_{1,y,0}^{S,y,\text{pred}} \) and \( C_{1,y,0}^{A,y,\text{pred}} \) as given by equations (A.24) and (A.25), only for the MP variants that assume an additional sub-season, to reflect the catches taken in the additional sub-season, as follows:

\[
C_{1,y,0}^{S,y,\text{pred}} = C_{1,y,0}^{S,y,\text{pred}} + \frac{1}{W_{1,0c}} \min \left\{ \text{TAB}_{\text{sub}}, r_{y} TAC_{1,y,0}^{A,y,\text{pred}} - TAC_{A}^{A} \right\}
\]  

(A.26)

and

\[
C_{1,y,0}^{A,y,\text{pred}} = C_{1,y,0}^{A,y,\text{pred}} + \frac{1}{W_{0c}} \left( TAC_{1,y,0}^{A,y,\text{pred}} - TAC_{A}^{A} \right)
\]

(A.27)

which ensures that the bycatch in the additional sub-season is at most \( r_{y} \) of that portion of the anchovy final TAC taken in the sub-season.

General

For all catches simulated in the operating model, an upper limit is placed on the industry’s efficiency by assuming that no more than 95% of the selectivity-weighted stock abundance may be caught. Furthermore, appropriate adjustments are made to ensure non-negative values for catches.

Observation Model

The survey estimates for spawner biomass and recruitment are generated by the observation model as follows (\( i = A, S \)):

\[
B_{j,y,N}^{i,\text{obs}} = k_{j,y}^{i} B_{j,y,N}^{i,\text{pred}} \epsilon_{j,y,\text{Nov}}^{i}
\]

(A.28)

where \( \epsilon_{j,y,\text{Nov}}^{i} = \eta_{j,y,\text{Nov}}^{i} \tilde{\sigma}_{j,y,\text{Nov}}^{i} \), where \( \eta_{j,y,\text{Nov}}^{i} \sim N(0,1) \)
and \( \varepsilon_{k,y,Nov}^A = \left( \rho_{1,y,Nov}^S \eta_{1,y,Nov}^A + \sqrt{1 - (\rho_{1,y,Nov}^S)^2} \eta_{1,y,Nov}^A \right) \bar{\sigma}_{1,y,Nov}^A, \) where \( \eta_{1,y,Nov}^A \sim N(0;1) \)

Here \( \bar{\sigma}_{1,y,Nov}^S = \sqrt{\min \left( 1.1181^2, 0.000 + \frac{136.6338}{B_{1,y,N}^{S,pred}} \right) + (\phi_{ac}^S)^2 + (\lambda_N^S)^2 } \)  

(A.29)

and \( \bar{\sigma}_{1,y,Nov}^A = \sqrt{\min \left( 0.4096^2, 0.0242 + \frac{11.1157}{B_{1,y,N}^{A,pred}} \right) + (\lambda_N^A)^2 } \)  

(A.30)

obtained from a regression of the observed CV against the base case assessment model predicted biomass between 1984 and 2011 (Figure A3). Here

\( B_{1,y,N}^{\text{obs}} \) is the November acoustic survey estimate of stock \( j \) of species \( i \) (\( i = S, A \)) 1+ biomass (in thousands of tons) in year \( y \);

\( k_{j,N} \) is the constant of proportionality (multiplicative bias) between survey estimated and model predicted 1+ biomass of stock \( j \) of species \( i \) (\( i = S, A \)), drawn from posterior distributions output from the operating models (de Moor and Butterworth, 2012b,c);

\( \rho_{1,y,Nov} \) is the correlation in the residuals between the sardine and anchovy November survey estimates, given by equations (A.37);

\( \phi_{ac}^S \) is the CV associated with the factors which cause bias in the sardine acoustic survey estimates and which vary inter-annually rather than remain fixed over time (de Moor and Butterworth 2012c);

\( (\lambda_{j,N1,rec}^S)^2 \) is the additional variance (over and above the squares of the survey sampling CV and of the CV \( \phi_{ac}^S \)) associated with the November/recruit surveys of stock \( j \) of species \( i \) (\( i = S, A \)); and

\( N_{1,y,rec}^{i,\text{obs}} = k_{j,r}^i N_{1,y,rec}^{i,\text{pred}} \varepsilon_{j,y,rec}^S, \)  

(A.31)

where \( \varepsilon_{1,y,rec}^S = \eta_{1,y,rec}^S \bar{\sigma}_{1,y,rec}^S, \) where \( \eta_{1,y,rec}^S \sim N(0;1) \)

and \( \varepsilon_{1,y,rec}^A = \left( \rho_{1,y,rec}^S \eta_{1,y,rec}^S + \sqrt{1 - (\rho_{1,y,rec}^S)^2} \eta_{1,y,rec}^A \right) \bar{\sigma}_{1,y,rec}^A, \) where \( \eta_{1,y,rec}^A \sim N(0;1) \).

Here \( \bar{\sigma}_{1,y,rec}^S = \sqrt{\min \left( 1.0785^2, 0.0987 + \frac{1.5010}{N_{1,y,rec}^{S,pred}} \right) + (\phi_{ac}^S)^2 + (\lambda_r^S)^2 } \)  

(A.32)

and \( \bar{\sigma}_{1,y,rec}^A = \sqrt{\min \left( 0.2830^2, 0.0372 + \frac{0.3226}{N_{1,y,rec}^{A,pred}} \right) + (\lambda_r^A)^2 } \)  

(A.33)

obtained from a regression of the observed CV against the base case assessment model predicted recruitment between 1985 and 2011 (Figure A3).

Here

\( \bar{\sigma}_{1,y,rec}^S = \sqrt{\min \left( 1.0785^2, 0.0987 + \frac{1.5010}{N_{1,y,rec}^{S,pred}} \right) + (\phi_{ac}^S)^2 + (\lambda_r^S)^2 } \)  

(A.32)

and \( \bar{\sigma}_{1,y,rec}^A = \sqrt{\min \left( 0.2830^2, 0.0372 + \frac{0.3226}{N_{1,y,rec}^{A,pred}} \right) + (\lambda_r^A)^2 } \)  

(A.33)
\( k_{j,r} \) is the constant of proportionality (multiplicative bias) between survey estimated and model predicted 1+ biomass, drawn from posterior distributions estimated by the operating models (de Moor and Butterworth, 2012b,c); and

\( \rho_{j,rec} \) is the correlation in the residuals between the sardine and anchovy recruit survey estimates, given by equation (A.40).

Assuming that the recruit survey begins mid-May each year, and that juvenile sardine are caught half-way between 1 November and the start of the survey, while juvenile anchovy caught prior to the survey are taken in a pulse at 1 May (in line with the assumptions made in de Moor and Butterworth 2012d,e), we simulate:

\[
N_{j,y}^{S,\text{pred}} = (N_{j,y-1,0}^{S,\text{pred}} - C_{j,y,0\text{obs}}^{S,\text{pred}}) e^{-3.25M_{j,y}^{S,1/12}}
\]

\[
N_{j,y}^{A,\text{pred}} = (N_{j,y-1,0}^{A,\text{pred}} - C_{j,y,0\text{obs}}^{A,\text{pred}}) e^{-0.5M_{j,y}^{A,1/12}}.
\]

(A.34)

Assumptions made for 2011 and 2012

As the stock assessments (de Moor and Butterworth 2012b,c) covered the period to November 2011, the MP testing framework begins from November 2011 and projects to November 2032. A number of parameters that would be simulated in the testing framework for 2012, have however already been observed. Thus the following changes are made to the simulation framework above for 2011/2012:

i) The TAC/TABs (in thousands of tons) for 2012 have already been set using OMP-08, thus

\[
TAC_{2012}^{S} = 100.595, \quad TAC_{2012}^{1A} = 202.718, \quad TAB_{2012,\text{anch}}^{1S} = 25.4466 - 3.5 = 21.9466,
\]

\[
TAC_{2012}^{2A} = 352.718, \quad TAB_{2012,\text{anch}}^{2S} = 35.8791 - 3.5 = 32.3791,
\]

\[
TAC_{2012}^{3A} = TAC_{2012}^{2A} + 120, \quad TAB_{2012,\text{anch}}^{3S} = TAB_{2012,\text{anch}}^{2S} + 2
\]

ii) The anchovy catch from 1\textsuperscript{st} January to 30\textsuperscript{th} June 2012 closely resembles \( TAC_{2012}^{1A} \). However, the anchovy catch from 1\textsuperscript{st} July to 31\textsuperscript{st} August 2012 of about 66 thousand tons (van der Westhuizen pers. comm.) is much less than \( TAC_{2012}^{2A} - TAC_{2012}^{1A} = 150 \). Thus during implementation (equations (A.9), (A.21)-(A.27)), \( TAC_{2012}^{2A} \) is replaced with 268.5 thousand tons and \( TAC_{2012}^{3A} \) is replaced with 388.5 thousand tons.

iii) As the May 2012 survey observations are available, no error is required, thus equation (A.31) is replaced by \( N_{1,2012,r}^{c,\text{obs},S} = 8.103 \) billion and \( N_{1,2012,r}^{c,\text{obs},A} = 210.563 \) billion. The survey CVs were 0.321 for sardine and 0.138 for anchovy (Mhlongo et al. 2012).

iv) The ratio of juvenile sardine to anchovy “in the sea” used in equation (A.26) is

\[
r_{2012} = 0.5 \times (0.0933 + 0.0458).
\]

v) The model predicted recruitment in November 2011 is an inverse variance weighted average of the logarithms of two estimates (logarithms are taken as the distributions of the estimates themselves are assumed to be log-normal). The first estimate comes from the recruitment observed in the 2012 recruit survey:
\[ N_{j,2012,r}^{S,\text{pred}} = \frac{1}{k_j^S} N_{j,2012,r}^{\text{obs},S} \] (being the best estimate from equation (A.31))

\[ N_{j,2012,r}^{A,\text{pred}} = \frac{1}{k_j^A} N_{j,2012,r}^{\text{obs},A} \] (being the best estimate from equation (A.31))

\[ N_{j,2011,0}^{S,\text{pred}} = (N_{j,2012,r}^{S,\text{pred}} e^{0.5(6+1.5)M_j^r/12} + C_{j,2012,0bs}^S) e^{0.5(6+1.5)M_j^r/12} \] (from equation (A.34))

\[ N_{j,2011,0}^{A,\text{pred}} = (N_{j,2012,r}^{A,\text{pred}} e^{1.5M_j^r/12} + C_{j,2012,0bs}^A) e^{0.5M_j^r} \] (from equation (A.34))

where \( C_{j,2012,0bs}^S = 32.050 \) billion, and \( C_{j,2012,0bs}^S = 0.013 \) billion being the juvenile anchovy and sardine catch, respectively from 1 November 2011 to the day before the recruit survey in June 2012.

The standard errors associated with the logarithms of these estimates are:

\[ \hat{\sigma}_{1,2012,\text{rec}}^S = \sqrt{0.321^2 + (\sigma_{ac}^S)^2 + (\lambda_{j}^S)^2} \]

\[ \hat{\sigma}_{1,2012,\text{rec}}^A = \sqrt{0.138^2 + (\lambda_{j}^A)^2} \]

vi) The second estimate comes from the stock recruitment curve, but needs to take account of the serial correlation in residuals about this curve, and so depends on the residual estimated about this curve for November 2010. Thus:

\[ N_{j,2011,0}^{*,\text{pred}} = f(\hat{SSB}_{j,2011N}) e^{s_{j,rec}^j + \mu_{i,j}} \]

with a standard error of the logarithm of this estimate being given by

\[ \tilde{\sigma}_{j,2011} = \sqrt{1 - (s_{j,rec}^j)^2} \sigma_{j,r} \]

vii) The inverse variance weighted average of the logarithms of these two estimates is then given by:

\[ \ln(N_{j,2011,0}^{*,\text{pred}}) = \frac{\ln(N_{j,2011,0}^{*,\text{pred}})}{\left(\hat{\sigma}_{j,2012,\text{rec}}^S\right)^2} + \frac{\ln(N_{j,2011,0}^{*,\text{pred}})}{\left(\tilde{\sigma}_{j,2011}\right)^2} \]

\[ \ln(N_{j,2011,0}^{*,\text{pred}}) = \frac{1}{\left(\hat{\sigma}_{j,2012,\text{rec}}^S\right)^2} + \frac{1}{\left(\tilde{\sigma}_{j,2011}\right)^2} \]

Note that this process is essentially shrinking the estimate provided by the survey towards the mean provided by the stock recruitment relationship (adjusted for serial correlation)

viii) The recruitment residual in November 2011, required in the calculation of the recruitment residual in November 2012 (equation A.4), is obtained from equation (A.3) as follows:

\[ e_{2011}^j = \ln\left(\frac{N_{j,2011,0}^{*,\text{pred}}}{f(\hat{SSB}_{j,2011N})}\right) / \sigma_{j,r} \]

External inputs into the MP testing framework

Some of the parameters required in the observation model were sampled from the posterior distributions of the underlying operating models (de Moor and Butterworth, 2012d,e). In addition, historic catches were
used in the calculation of sardine bycatch to anchovy ratios used in the implementation model. These parameters are detailed in this section.

**Correlation in survey residuals**

The sardine and anchovy November survey residuals are given by \((i = S, A)\):

\[
e_{j,y,Nov}^i = \ln B_{j,y,Nov}^{i,obs} - \ln(k_{j,y,Nov}^i \hat{B}_{j,y,Nov}^i), \quad y = 1984, \ldots, 2011
\]  

(A.35)

where \(\hat{B}_{j,y,Nov}^i\) is the operating model estimate of historic November 1+ biomass (in thousands of tons) of stock \(j\) of species \(i (i = S, A)\) in year \(y\), drawn from posterior distributions estimated by the operating models (de Moor and Butterworth, 2012b,c).

The standard deviations of the residuals are given by \((i = S, A)\):

\[
\sigma_{j,Nov}^i = \sqrt{\frac{\sum_{y=1984}^{2011} (e_{j,y,Nov}^i)^2}{\sum_{y=1984}^{2011} 1}}.
\]  

(A.36)

The correlation in the residuals between the sardine and anchovy November survey estimates is:

\[
\rho_{j,Nov}^i = \frac{\sum_{y=1984}^{2011} e_{j,y,Nov}^i e_{A,y,Nov}^A}{\sqrt{\sum_{y=1984}^{2011} (\sigma_{j,Nov}^i)^2 \sigma_{1,Nov}^A}}.
\]  

(A.37)

Similarly, the sardine and anchovy May recruit survey residuals are given by \((i = S, A)\):

\[
e_{j,y,rec}^i = \ln N_{j,y,rec}^{i,obs} - \ln(k_{j,y,rec}^i \hat{N}_{j,y,rec}^i), \quad y = 1985, \ldots, 2011
\]  

(A.38)

where \(\hat{N}_{j,y,rec}^i\) is the operating model estimate of historic May recruitment (in billions) of stock \(j\) of species \(i (i = S, A)\), at the time of the recruit survey in year \(y\), drawn from posterior distributions estimated by the operating models (de Moor and Butterworth, 2012b,c).

The standard deviations of the residuals are given by:

\[
\sigma_{j,rec}^i = \sqrt{\frac{\sum_{y=1985}^{2011} (e_{j,y,rec}^i)^2}{\sum_{y=1985}^{2011} 1}}.
\]  

(A.39)

The correlation in the residuals between the sardine and anchovy recruit survey estimates is:

\[\text{The sum is taken over all years for which a survey estimate of recruitment exists.}\]

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Ratio of sardine bycatch to anchovy between January and May

The ratio of sardine bycatch to anchovy in the commercial catches from January to May is needed to simulate the 0-year-old sardine caught prior to the recruit survey (equation (A.10)). The relationship between the historical sardine bycatch to anchovy ratio in the catches from January to May, together with the stock assessment model prediction for the ratio of sardine to anchovy November recruitment, is used to provide this ratio (the predicted recruitment ratio is used because the catch of 0-year-old anchovy dominates that of older anchovy, so applying the ratio also to the early season adult anchovy catch will not introduce substantial error). Only the most recent 10 years data is used in the below equations as future catches are assumed to more closely simulate those over the past decade, rather than earlier periods when fishing patterns may have differed, particularly since the additional anchovy sub-season was only introduced in 1999.

The constant of proportionality estimated and the associated time series of residuals are as follows:

\[
\begin{align*}
\hat{k}_{\text{jan-may}} &= \exp \left\{ \sum_{y=2002}^{2011} \left[ \ln \left( \frac{C_{S,\text{byc}}^{y,\text{jan-may}}}{C_{A,\text{y,}\text{jan-may}}} \right) - \ln \left( \frac{\hat{N}^{S}_{1,y-1,0}}{\hat{N}^{A}_{1,y-1,0}} \right) \right] \right\} \\
\epsilon_{y,\text{jan-may}} &= \ln \left( \frac{C_{S,\text{byc}}^{y,\text{jan-may}}}{C_{A,\text{y,}\text{jan-may}}} \right) - \ln \left( \frac{k_{\text{jan-may}} \hat{N}^{S}_{1,y-1,0}}{\hat{N}^{A}_{1,y-1,0}} \right) \quad y = 2002, ..., 2011
\end{align*}
\]  

(A.41)

where

- \( \hat{N}^{i}_{j,y,0} \) is the model estimated number of recruits of stock \( j \) of species \( i (i = S, A) \) in November of year \( y \) (from which catches of 0-year-old sardine and anchovy are made in year \( y+1 \)), drawn from posterior distributions estimated by the operating models (de Moor and Butterworth, 2012b,c).

The subset of years used is that for which the catch data and assessed recruitment estimates for both species are available. The standard deviation of the residuals is given by:

\[
\sigma_{\text{jan-may}} = \sqrt{\frac{\sum_{y=2002}^{2011} \left( \epsilon'_{y,\text{jan-may}} \right)^2}{\sum_{y=2002}^{2011} 1}}. 
\]

(A.43)
Ratio of sardine bycatch to anchovy in the commercial fishery during May

For equation (A.17), the estimated constant of proportionality and the associated time series of residuals for the juvenile sardine to anchovy ratio from the commercial catches during May are as follows:

\[
\hat{k}_m = \exp \left\{ \sum_{y=2002}^{2011} \left[ \ln \left( C_{y,m}^{S,byc} / C_{y,m}^A \right) - \ln \left( \hat{N}_{1,y,r}^S / \hat{N}_{1,y,r}^A \right) \right] / \sum_{y=2002}^{2011} 1 \right\} \tag{A.44}
\]

and

\[
\hat{\epsilon}_{y,m}^* = \ln \left( C_{y,m}^{S,byc} / C_{y,m}^A \right) - \ln \left( \hat{k}_m \hat{N}_{1,y,r}^S / \hat{N}_{1,y,r}^A \right), \quad y = 2002, \ldots, 2011 \text{ and } m = \text{may}. \tag{A.45}
\]

The associated residual standard deviation is:

\[
\sigma_m = \sqrt{\frac{\sum_{y=2002}^{2011} (\hat{\epsilon}_{y,m}^*)^2}{\sum_{y=2002}^{2011} 1}}, \quad m = \text{may}. \tag{A.46}
\]

A correlation coefficient between the January to May and May residuals, for use in equation (A.18) above, is then calculated by:

\[
\rho_m = \frac{\sum_{y=2002}^{2011} \hat{\epsilon}_{y,m-1} \hat{\epsilon}_{y,m}}{\left( \sum_{y=2002}^{2011} 1 \right) \sigma_{m-1} \sigma_m}, \quad \text{for } m = \text{may} \text{ and } m-1 = \text{janmay}. \tag{A.47}
\]
Table A1. Parameters sampled from the Bayesian posterior distributions of de Moor and Butterworth (2012b,c)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{S/A,\text{pred}}^{a,2011}$</td>
<td>Operating model predicted numbers at age of sardine/anchovy in November 2011 (in billions)</td>
</tr>
<tr>
<td>$\varepsilon_{S/A}^{\text{e},2010}$</td>
<td>Operating model estimated sardine/anchovy recruitment residual in November 2010</td>
</tr>
<tr>
<td>$a_j^{S/A}$</td>
<td>Maximum deterministic sardine/anchovy recruitment (in billions)</td>
</tr>
<tr>
<td>$b_j^{S/A}$</td>
<td>Sardine/anchovy spawner biomass below which median recruitment declines linearly with this biomass (in thousands of tons)</td>
</tr>
<tr>
<td>$K_j^{S/A}$</td>
<td>Sardine/anchovy average pristine level (“carrying capacity”)</td>
</tr>
<tr>
<td>$\sigma_j^{S/A}$</td>
<td>Standard deviation in the sardine/anchovy recruitment residuals</td>
</tr>
<tr>
<td>$s_j^{S/A}$</td>
<td>Sardine/anchovy recruitment serial correlation</td>
</tr>
<tr>
<td>$S_{j,a}^{S,A}$</td>
<td>Sardine commercial selectivity at age</td>
</tr>
<tr>
<td>$k_j^{S/A}$</td>
<td>Multiplicative bias associated with the hydroacoustic survey estimate of sardine/anchovy recruitment</td>
</tr>
<tr>
<td>$\hat{B}_{j,y,N}^{S/A}$</td>
<td>Operating model estimated sardine/anchovy November 1+ biomass (in thousands of tons)</td>
</tr>
<tr>
<td>$\hat{N}_{j,y,N}^{S/A}$</td>
<td>Operating model estimated sardine/anchovy May recruitment (in billions)</td>
</tr>
<tr>
<td>$\hat{N}_{j,y,0}^{S/A}$</td>
<td>Operating model estimated sardine/anchovy recruitment in November (in billions)</td>
</tr>
<tr>
<td>$\left(\lambda_{N/\gamma}^A\right)^2$</td>
<td>Additional variance (over and above the survey sampling CV) associated with the November/recruit anchovy surveys</td>
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</table>
Table A2. Average 1984 to 2011 weights-at-age (in grams) from the historic catches ($\bar{W}^i_{j,a}$, $i=S,A$) and from the historic November spawner biomass surveys ($\bar{W}^i_{j,a}$, $i=S,A$).

<table>
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<tr>
<th>Weights-at-age in the catch</th>
<th>Weights-at-age in the survey</th>
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</thead>
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<tr>
<td>Sardine</td>
<td>Anchovy</td>
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<tr>
<td>$\bar{W}^S_{1,0c}$</td>
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<td>$\bar{W}^S_{1,2c}$</td>
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<td>$\bar{W}^S_{1,5+c}$</td>
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Table A3. Anchovy catch (in thousands of tons) from landings that have targeted anchovy \( (C_{y,m}^A) \), for five-month ("janmay"), five single month ("may", "jun", "jul", "aug", "sep"), and a three-month ("octdec") periods, with the associated recorded landings of sardine bycatch \( (C_{y,m}^{S,by}) \), also in thousands of tons).

<table>
<thead>
<tr>
<th>Year</th>
<th>( C_{y,janmay}^A )</th>
<th>( C_{y,may}^A )</th>
<th>( C_{y,jul}^A )</th>
<th>( C_{y,aug}^A )</th>
<th>( C_{y,sep}^A )</th>
<th>( C_{y,octdec}^A )</th>
<th>( C_{y,janmay}^{S,by} )</th>
<th>( C_{y,may}^{S,by} )</th>
<th>( C_{y,jun}^{S,by} )</th>
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<td>2.8</td>
<td>#</td>
</tr>
</tbody>
</table>

* A landing is assumed to have targeted anchovy when the ratio anchovy : (anchovy + directed sardine + horse mackerel + round herring) exceeds 0.5 (in terms of mass).

# As no anchovy were landed during these months, sardine bycatch with anchovy is not applicable.
Figure A1. The historic ratio of ≤14cm sardine to >14cm sardine in the directed sardine fishery. The two ratios above 7%, shown as open diamonds, are fixed at 7% in the distribution from which future samples are made (equations (A.11) and (A.16)).
Figure A2. The regressions of the ratio of small sardine bycatch : anchovy\(^{16}\) in the monthly commercial catch against that observed in the recruit survey, i.e. minimising

\[
\sum_{y=2002}^{2011} \left[ \frac{C_{y,m}^{S,byc}}{C_{y,m}^A} - k_m \frac{N_{1,y,r}^{S,obs}}{N_{1,y,r}^A} \right]^2
\]

w.r.t. \(k_m\). The outliers of commercial ratio of 0.69 in October to December 2010 (shown as an open diamond) is removed, as this could have been biased by the mid-water trawl experiments which occurred during this time. The regression including this outlier is given by the dotted line.

\(^{16}\) For cases where anchovy is the most common species by mass in the landing
Figure A3. The regressions between observed survey CV and model predicted abundance for a) sardine single stock November, b) anchovy November, c) sardine single stock May and d) anchovy May surveys, for use in equations (A.29), (A.30), (A.32) and (A.33). In b) the outlier (333,0.17) was excluded from the regression.