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Abstract

There is longstanding and widespread concern that students find the transition from school to university mathematics difficult. There have been various practical responses to supporting students in this transition. Research conducted on these responses tends to focus on student perceptions and the impact on academic performance. However, research which explores the pedagogy implemented in support courses is lacking. Yet such research is needed if we are to understand what knowledge is valued and how it is communicated in support courses, which is an important first step in establishing whether these courses are replicable and whether they might indeed provide access to the knowledge valued in mainstream mathematics courses.

My study investigates the implemented pedagogy of one particular mathematics support course for first-year engineering students. The pedagogy intended for the course is similar to the problem-centred approach (PCA), which is a competence pedagogy popular in selected white primary schools in South Africa in the 1990s. Critiques of school-level PCA – such as that it affords students insufficient “guidance” and that it is difficult to replicate – highlight the importance of understanding this support course’s pedagogy. I made video records of one activity of the course in order to explore what knowledge the course values and how that knowledge is communicated to students. My theoretical framework is founded on Bernstein’s (1996) theory of the pedagogic device, since it affords a language for speaking about the transformation of knowledge into pedagogic communication. I adopted theoretical tools from Davis’s (2001; 2005) investigation of PCA at the primary school level. My study demonstrates the generalisability of these theoretical tools.

Regarding what knowledge is valued in the course, I found that the central notion is problem solving. Problem solving serves as a vehicle for developing “sense-making”. However, the notion of problem solving remains implicit since it is not discussed with students and students do not have an opportunity to solve the given problem independently. Regarding how knowledge is communicated, I found the implemented pedagogy to be a hybrid of Bernstein’s competence and performance models. The former emerges in that much of the privileged knowledge remains implicit and the hierarchy between teacher and student is apparently flattened. The performance model is seen in teachers guiding students, both explicitly and implicitly. For example, they explicitly tell students to draw a diagram and how to check their answers. They implicitly guide students by modelling the problem-solving process and subtly positioning the students in complex ways.

My results raise questions about whether students acquire the notion of problem solving in the course. Furthermore, the pedagogy identified may mitigate against students acquiring the sense-making disposition that the course intends to develop. My results bring into question the replicability of the course and how it may support students in their transition to university mathematics.

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Chapter 1: Introduction

1.1 Overview

My study investigates a course intended to support engineering students in the transition from school to university mathematics (hereinafter “the course”). The course takes place in South Africa and is unusual in that it is run by a Non-Profit Organisation (NPO) in the mid-year¹ university vacation, independent from university structures. The course intends to employ a pedagogy reminiscent of the *problem-centred approach* (PCA) – a pedagogy popular in selected white² primary schools in South Africa in the 1980s and 90s – and to develop students’ skills for solving “realistic” (Human et al., 2010:2) mathematics problems. It also intends for students to develop, through solving such problems, an understanding of certain mathematical concepts and an attitude towards learning which privileges “sense-making”³ (ibid.:4).

My study aims to create a detailed description of the implemented pedagogy of the course, which makes visible what knowledge it privileges and how that knowledge is communicated to students. Bernstein’s (1996) notion of *evaluation* is central to my thesis, since privileged knowledge is that which is legitimated through evaluation. Davis’s (2005) theoretical construct of, and method for analysing, *pedagogic judgement* supplements Bernstein’s notion of evaluation, providing a lens to view what knowledge the course privileges and an aspect of how privileged knowledge is communicated. Davis’s (2005) notion of *transmission* and *acquisition functions* provides a second perspective on how privileged knowledge is communicated.

In this chapter, I elaborate on the general problem of students’ transition from school to university mathematics and discuss the background of the course I have investigated. I close the chapter by summarising the aim and purpose of my study, presenting my research question and outlining the structure of this dissertation.

¹ The academic and calendar years coincide in South Africa. Mid-year thus refers to June/July.

² Racial categories continue to be used to report educational participation and performance in South Africa, since access to educational opportunity continues to be impacted by race as a social construct. In addition to race being used as the identifier in the literature, it cannot be separated from socio-economic status, family history, language and geographic location.

³ The terms “sense-making” and “making sense” are used in various ways in mathematics education. Throughout this dissertation, I use the terms in the manner intended by the teachers of the course I have studied. See Human et al. (2010). I discuss the teachers’ intended meaning of sense-making in Section 1.3.2.4

1.2 The transition to university mathematics

1.2.1 The problem

There is longstanding concern in the mathematics education community that students exiting school are not adequately prepared for university mathematics. A recent study in South Africa suggests that there is a widening “gap” between school and university mathematics (Engelbrecht et al., 2010). However, this problem has been reported in various countries (Bergsten and Jablonka, 2015; Brandell et al., 2008; Heck & van Gastel, 2006; Hing, 2005; Hoyles et al., 2001; Kajander & Lovric, 2005)

Students’ difficulties in the transition to university mathematics have been attributed to numerous factors. Some point to a “mismatch” between the demands of university mathematics courses and students’ previous experiences (Hourigan & O’Donoghue, 2007). Others point to changes in school mathematics curricula (De La Paz, 2005; Engelbrecht et al., 2010). Still others (such as Anthony, 2000) attribute the problem to students’ and lecturers’ differing perceptions on issues such as workload, help-seeking and student effort.

In South Africa, students’ difficulties in moving from school to university mathematics need to be understood within the wider socio-economic and socio-political context and schooling more generally. The country has high levels of poverty and extreme socio-economic inequality – legacies of our apartheid history. This inequality is reflected in South Africa’s mathematics education (Graven, 2014:1039). In general, poorer learners receive “significantly inferior schooling [relative] to their more affluent peers” (Taylor et al., 2013, quoted by Walton et al., 2015:263). It is unsurprising, then, that poorer (usually black) students tend to experience more difficulty in the transition from school to university mathematics than their “more affluent peers”. This is reflected in the country’s notoriously low university-throughput rates, which are particularly low for black students. For example, only 41% of students who started a four-year engineering degree in 2006 completed said degree after five years (Council on Higher Education, 2013), with only 23% of black students in this cohort completing their degree in that time compared to 55% of white students. Such statistics highlight the need for supporting (all) South African students in their transition to university.

In post-apartheid South Africa, education has been treated as a vehicle for transforming society, there being a “political rhetoric of equity and quality education *for all*” (Graven, 2014:1039, emphasis in original). Nevertheless, there are indications (such throughput statistics) that inequality has worsened in mathematics education (Graven, 2014:1039). In addition, this post-apartheid political climate has seen numerous revisions to the school curriculum, which may also have contributed to the problem of transition to university (Engelbrecht et al., 2010).

1.2.2 Supporting students in the transition

Programmes for supporting students in the transition to university have been implemented in South Africa since the early 1980s at various levels of the education system (Rollnick, 2010). However, the abovementioned throughput rates indicate that the problem is serious and persists. Like the problem of transition, providing support for students in the transition is not unique to South Africa. At the higher-education level, practical responses to the problem have predominantly been positioned within university structures and have taken on forms such as:

- (a) inserting support into “mainstream”⁴ courses – see for example Craig (2007) and Hillock et al. (2013);
- (b) “bridging courses” prior to commencement of the university mathematics course – see for example Engelbrecht and Harding (2015) and Gordon and Nicholas (2013);
- (c) additional support alongside mainstream courses such as “supplemental instruction” and “mathematics support centres”. The former consists of regular workshops that (selected) students commit to for the duration of the mathematics course – see for example Congos and Schoeps (1993) and Engelbrecht and Harding (2015). Mathematics support centres, on the other hand, are designated physical spaces where students have access to additional resources on an ad hoc basis – see for example Croft (2000) and Engelbrecht and Harding (2015);
- (d) intensive revision programmes provided after students have failed the mainstream course but prior to their being re-assessed – see for example Campbell (2015);
- (e) credit-bearing “foundation” courses that replace mainstream first-year mathematics courses, but which proceed at a slower pace or have increased contact time with students – see for example le Roux (2011), Wood (2001) and Craig (2009).

The course which I have investigated is an unusual example of support for students in the transition, since it fits none of these models of support – for reasons mentioned in Section 1.1 and elaborated on in Section 1.3.

Wood (2001) argues that support courses provide space for innovation in pedagogy since they are often not subject to the constraints inherent in mainstream courses. Innovations used in mathematics support courses mentioned in the literature include the use of group work (Armien, 2007), “interactive white board tutorials” (Campbell, 2015) and

⁴ I use the term “mainstream” to refer to undergraduate programmes and courses that have traditionally been offered at universities.

“practical problems” (le Roux, 2011:iii). The pedagogy of the course in this study aims to employ other innovations (see Section 1.3.2). In general, I take “innovative” to mean pedagogies that are different to what is generally used in mainstream mathematics courses. The latter can be described broadly as follows (drawing on Bergsten et al., 2015, and my own experience):

The lecturer explains new content on the board while students watch and take notes. Students complete exercises related to this content on their own – at home and in weekly tutorials. The lecturer, tutor and textbook are authorities who hold answers to the exercises. The emphasis is on learning content and getting the right answers.

The pedagogy of mainstream mathematics courses fits Bernstein’s (1996) *performance model* of pedagogy, since students have little control over sequence, pace and selection of content and the teachers make the criteria for producing the right answers explicit by pointing out omissions and errors in students’ answers.

Le Roux and Adler (2016), however, argue that the freedom from constraints suggested by Wood is illusory, since any additional support is subject to wider power structures. Specifically, students who receive support need to succeed in mainstream university courses. Consequently, support courses should assist students in accessing the knowledge valued in mainstream courses. I therefore suggest that understanding what knowledge is valued and how it is communicated in support courses is a first step in establishing whether they might support students in the transition to university mathematics. In the next chapter I review research on support for students in the transition to university mathematics. I argue there that there is a dearth of research into what knowledge support courses make available to learn and how their pedagogies communicate this knowledge.

1.3 The course

The course which I investigated has been researched in terms of student perceptions thereof (Vosloo et al., 2012; Vosloo et al., 2013). The teachers themselves have reported on the design of the course and the pedagogy they intend for the course (Human et al., 2010). I discuss here the background of the course gleaned from these publications and other course materials.

1.3.1 Background

The course which I investigated is for undergraduate students supported by a single public benefit organisation (herein after “the foundation”) aimed at developing South Africa’s capacity in science, technology, engineering and mathematics (STEM). The foundation provides funding for post- and undergraduate STEM studies. I am a beneficiary

of the former, the students of the course are beneficiaries of the latter (this has ethical implications – see Section 4.7.2). The undergraduate students are selected on the basis of high academic performance at school, financial need, a preference for students from rural areas and the foundation’s stipulated ratio of three females to every two males (Vosloo et al., 2013:208).

The foundation aims to provide a “comprehensive support programme” for their undergraduate students (Vosloo et al., 2013) who are all studying mainstream courses, predominantly engineering. Financial support (bursaries) is complemented by other support such as access to psychologists, eye tests and the course I have investigated. The foundation conducts its own research on the programme “with a view to developing a replicable model for tertiary access support” (Vosloo et al., 2012:2). However, it has not researched the implemented pedagogy of the course. Yet clarity on what knowledge the course’s pedagogy makes available to students and how this knowledge is communicated is needed if the course is to be “replicable”. That others (Vithal & Volmink, 2005) have argued that PCA at school level is not easily replicated highlights this need (see Section 2.3.2).

The course is a week-long intensive residential programme in the mid-year university vacation, administered by an independent NPO. Following an introduction from the foundation, I was invited to observe the course, which I did twice prior to collecting data for this study. During that time, I collected course materials (see Appendix 1) and pilot data, which I used together with a publication by the teachers (Human et al., 2010) and personal correspondence with the teachers (see Appendix 2) to piece together a description of the course’s “intended curriculum” (Pelgrum et al., 1986).

1.3.2 The intended curriculum

The course’s intended curriculum informed my design of the project. In this section I present my description to highlight what the course sets out to accomplish.

1.3.2.1 Aim of the course

The course aims to

develop students’ skills and dispositions at solving problems using mathematical modelling, and at the same time to use the problem-solving experience as a means for students to make sense of mathematics that they have already learned (Human et al., 2010:2).

This statement points to three aspects of the intended curriculum. Students are to:

1. “make sense” of mathematical concepts they have previously encountered;
2. learn problem-solving skills, which the teachers also refer to as “modelling” and “the algebraic method” (personal communication, November 2011);
3. develop a particular disposition towards learning mathematics and solving problems, which the teachers refer to as sense-making.

1.3.2.2 Pedagogy to be used in the course

The teachers describe the pedagogy which they intend for the course (Human et al., 2010:3) as follows. Students are to “make sense” of concepts through solving problems, “investigate” and make “discoveries”, working mostly on their own and sometimes in groups. Teachers are to facilitate learning, through “critical listening and questioning”, introducing “counter-examples to show up illogical thinking”, suggesting “investigations that would lead to useful discoveries” and encouraging students to “explain their thinking”. As much as possible, teachers are to “refrain from ... telling students ‘what to do’”. The pedagogy of the course is thus innovative in the sense defined above.

The intended pedagogy constructs the student as already competent, since the student’s lack of knowledge is not to be explicitly addressed by the teacher, instead implied by other means (“counter-examples to show up illogical thinking” etc.). Consequently, I suggest that the intended pedagogy of the course fits Bernstein’s (1996:56) *competence model* of pedagogy which views:

- all students as intrinsically competent;
- the student as active in the creation of knowledge;
- the student as self-regulating, with “formal instruction” not effecting development since acquisition is an “invisible act” (ibid.:56);
- hierarchical relations sceptically, thus the conception of teaching as facilitation with teachers focusing on presences in students’ productions rather than absences, and students appearing to have control over sequence and pace.

I argue that the course’s intended pedagogy is similar to the problem-centred approach (PCA) (see Section 2.3.1) – a competence pedagogy popular at school level in the United States in the 1980s and ‘90s (Cobb et al., 1991) and in selected white primary schools in South Africa around the same time. PCA is a pedagogy informed by constructivist principles (Murray et al., 1998; Vithal & Volmink, 2005), in which students are to learn mathematics through solving problems (Murray et al., 1998). It has been researched extensively at school level; however, the literature is silent on research which explores the use of PCA-type pedagogy at the level of first-year university support. Yet critiques of

school-level PCA point to a need to understand how such pedagogy works at the level of university support (see Section 2.3.2).

1.3.2.3 Problems used in the course

The teachers claim that the learning activities used in the course employ problems which are “realistic”⁵ (ibid.:2). Consider for example *The Trains Activity* (Figure 1.1), which is used each time the course is run. I outline the intended curriculum with respect to *The Trains Activity*, since I focus on this activity in my study (for reasons discussed in Section 4.3). I offer my worked solution for *The Trains Activity* in Appendix 3.

There are two parallel railway tracks between the towns Lichtenstein and Schwarenstein. The length of the railway tracks from the one town to the other is 236 kilometres.

On a certain morning, one train needs to travel from Lichtenstein to Schwarenstein and arrive at Schwarenstein not later than 09h40. This train travels at an average speed of 96 km/h.

On the same morning, another train has to travel from Schwarenstein to Lichtenstein and arrive at Lichtenstein not later than 10h00. This train travels at an average speed of 139 km/h.

Figure 1.1 The Trains Activity problem statement (Human et al., 2010:4)

Students are to “investigate” the situation and suggest questions which they would like to answer about it (Course materials: Notes to teachers). However, the activity is only apparently open-ended, since teachers are to “Ask students to agree to focus on the question: ‘At what time and at what position on the railway will the trains cross one another?’” (ibid.). Students are to answer this question, first using the parameters specified in Figure 1.1 and then using unknown (but imagined fixed) parameters.

1.3.2.4 The three aspects of the intended curriculum in *The Trains Activity*

Mathematical concepts in *The Trains Activity*

In *The Trains Activity*, students are to make sense of the concept of co-variation. The teachers describe co-variation as “the structure of interdependencies” of variables (personal communication, 2012). Others describe co-variation as the concurrent variation of two (or more) quantities, understanding of which entails “persistent realization that, at every moment, the other quantity also has a value” (Saldanha & Thompson, 1998:1). It could be argued that students have already encountered co-variation, since it forms an implicit part of the simultaneous equations which students learn at school.

⁵ The value of using “realistic” problems in mathematics education is contested. Dowling (2008; 2008) and Cooper (Cooper & Dunne, 1998; Cooper & Harries, 2002) discuss the use of realistic problems in school level mathematics; le Roux (2011) attends to the use thereof at the level of university mathematics.

Co-variation is encoded in *The Trains Activity* in the simultaneous movement of the two trains, since “the position of one train varies together with the position of the other train, and the position of each train varies with time” (Course materials: Notes to teachers). Teachers anticipate that some students will “investigate the problem numerically, by calculating the position of each train at different times” (ibid.). Students who do this reveal the co-variation encoded in the problem, although possibly unaware that they are doing so.

Problem solving in *The Trains Activity*

The problem solving which students are to learn in *The Trains Activity* is not explicitly articulated in the source documents of the intended curriculum. In this section I argue that the actions which students are to be guided through in the activity are reminiscent of the problem-solving process articulated in Polya’s (1957) seminal work.

Polya suggests that the problem-solving process has four phases:

- (1) Understand the problem;
- (2) Devise a plan;
- (3) Carry out the plan;
- (4) Look back.

However, “in practice all the phases get mixed up and are carried out in parallel” (Stewart, 1957:xix in the foreword to Polya, 1957). Polya’s four phases and this mixing up are apparent in the intended *Trains Activity*, as discussed below.

In the activity, students are to first “investigate [the] situation – no question given” (Course materials: Notes to teachers), during which time the teachers are to encourage students to make a diagram to “help them think about the situation” (ibid.). At this point, the notes to teachers state:

We are hoping (and waiting patiently) that students come up with the idea to investigate questions like “When do the trains need to depart in order to arrive on time?”

Students are then to discuss with their peers “how they understand the situation” and propose “important/meaningful” questions about it (ibid.). These first parts of the activity (investigation, diagram drawing, group discussion and question posing) suggest that students are to first make sense of the problem – Polya’s phase 1.

The teachers are then to ask students to focus on the question(s): “At what time and at what position on the railway will the trains cross one another?” Teachers would like students to “first model (reveal the structure of

interdependencies), then calculate” when answering this question (personal communication, 2012). That is, students are to make a plan (phase 2) before they calculate to carry out that plan (phase 3). The final problem-solving phase suggested by Polya (look back) is apparent in the intended curriculum’s suggestion that students should “make sense of their mathematical representation of the problem” (Course materials: Notes to teachers).

Having completed Polya’s problem-solving process once, students are to do so again because, once students have solved the specific questions indicated above, the teachers are to “make the problem general” (ibid.):

ask them to imagine that they are the railway manager (transport planner). For any two trains for which the arrival times and the average speeds are specified, the railway manager needs to be able to say at what time and point they will cross. The students need to make a tool that will make it quick and easy for the railway manager to determine this for any two trains. (ibid.)

Implicit in this instruction is that students should create a formula, since all subsequent notes to the teachers are about assisting students with formulae. However, the notes to teachers state “no suggestion of formula-making!”

This new problem of making a “tool” for the “railway manager” needs to be understood (phase 1), and a plan devised (phase 2). Polya suggests that a useful heuristic in devising a plan is first to solve a related but simpler problem. In *The Trains Activity*, the teachers appear to set this heuristic up for students by providing them with a specific problem to solve prior to the more general problem:

Suggest to students that a good way to make formulas (that make sense) is to first do some calculations, and then to look back at their calculations. (ibid.)

Once students have carried out their plan (phase 3) and made their “tool”, they are to look back (phase 4) by testing their “tools” using “specifications for different trains journeys” (ibid.).

Sense-making in *The Trains Activity*

Efforts to develop a sense-making disposition in students are apparent throughout the *Trains Activity* documents. For example, teachers are “to point out their (the students’) lack of sense-making” (ibid.). Sense-making is set up by the teachers as being in opposition to “procedural thinking” (Human et al., 2010:12). The teachers define “procedural thinking” as being “when algebraic expressions are merely used to calculate answers” (ibid.:2) and

contrast it with “conceptual thinking”⁶, which they define as being “when algebraic expressions can be combined and re-arranged in a way that allows the problem to be solved” (ibid.). Consequently, I infer that these teachers equate “making sense” with “conceptual thinking”.

Interrelatedness of the three aspects of *The Trains Activity*

Although I have presented co-variation, problem solving and sense-making as three separate aspects of the intended curriculum, they are inextricably intertwined. Problem-solving in the course is to be “a means for students to make sense of mathematics that they have already learned” (Human et al., 2010:2), which suggests that problem solving is to be a vehicle for learning the other aspects of the intended curriculum, similarly to PCA. In addition, the intertwining is apparent in what I have likened to the second phase of the problem-solving process because, for these teachers, making a plan for solving the problem (Polya’s phase 2) means that students first “reveal the structure of interdependencies” before they calculate (personal communication, 2012).

1.4 This study

1.4.1 The aim

In this study, I aim to describe in detail the implemented pedagogy of one first-year engineering mathematics support course with particular attention to what knowledge is privileged and how that knowledge is communicated to students. The insight generated by such a description is important, firstly because understanding these aspects of the course is needed for developing a sense of whether the course might assist students in their transition to university mathematics – a problem which the course aims to address. While I do not evaluate the course’s success in this, my study does provide the ground work from which further research around this claim could be conducted. The need for establishing whether the course assists students in accessing the knowledge valued in mainstream courses is highlighted by critiques of PCA at school level: that it provides insufficient guidance to students (Kirschner et al., 2006) and that it inadvertently works against students’ understanding of the valued knowledge (Davis, 2005) (see Section 2.3.2).

Secondly, gaining a detailed, theoretically informed understanding of how the pedagogy of this particular course works is important because much of what it claims to achieve cannot be observed directly. Yet the funders intend for it to be “replicable” (Vosloo et al., 2012:2). My analysis of the course develops an understanding of what knowledge its implemented pedagogy privileges and how it communicates that knowledge. This understanding is needed if the

⁶ The conceptual-procedural distinction is widely deployed in mathematics education (see for example Hiebert and Lefevre, 1986). However, it has been argued that it is a false dichotomy (Kieran, 2013; Wu, 1999). I refer to the distinction here because it is referred to by the teachers of the course to clarify their use of the term “sense-making”.

course is to be administered by different teachers in different contexts. This need is highlighted by Vithal and Volmink's (2005) argument that PCA is a "weak pedagogy" and consequently not easily replicated (see Section 2.3.2).

1.4.2 The research question

In a mathematics support course for first-year engineering students, what knowledge is privileged and how does the pedagogy communicate this knowledge to students?

1.4.3 Outline of the thesis

The structure of this dissertation is as follows: In Chapter 2, I review literature which locates my study. Chapter 3 sets out my theoretical framework, while Chapter 4 attends to the methodology and methods which I have used. Chapters 5 and 6 are the analysis chapters; the former pertains to my analysis of pedagogic judgement, which attends to what knowledge is privileged in the course and an aspect of how it is communicated. Chapter 6 reports results on the distribution of transmission and acquisition functions, providing a second aspect of how knowledge is communicated to students. I conclude (Chapter 7) with a discussion of my results.

Chapter 2: Literature review

2.1 Introduction

In this chapter, I review research which investigates programmes aimed at providing support for students in the transition from school to university mathematics because my study is a response to this problem. I also explore literature which locates the pedagogy of the course examined in my study.

2.2 Supporting students in the transition from school to university mathematics

Research on the support of students studying first-year university mathematics can be divided into two types: that which explores students' perceptions and that which investigates the impact on academic performance.

2.2.1 Students' perceptions

Studies exploring students' perceptions of support generally demonstrate student satisfaction and suggest that, from this perspective, the support does indeed assist students in the transition to university mathematics. The specific focus of these studies, however, is varied. Some explore students' perceptions of the support's overall success in assisting them in the transition (Parnell & Statham, 2007; Fhloinn et al., 2014; Vosloo & Blignaut, 2010). Others focus on students' perceptions of how well the support has improved their learning of particular concepts and skills (Craig, 2007; Gordon & Nicholas, 2013). Some examine students' perceptions of the support's influence on their academic performance, often in conjunction with other foci (Fhloinn et al., 2014). Yet others explore students' perceptions of the social and physical space that the support affords. Solomon et al. (2010) conclude, for example, that students experience support centres as safe spaces in which they feel they establish a collaborative practice of doing mathematics. Only a minority of studies, however, explore students' perceptions of the pedagogy of support. One such study is Armien's (2007) investigation of a first-year foundational engineering mathematics course at a South African university of technology, where he found that students valued the support provided by group-work.

Most of this body of research looks at support located within university structures, although there are rare exceptions. Vosloo et al. (2012; 2013), for example, review student perceptions of the course examined in my study in their investigation of the funder's overall undergraduate support programme.

2.2.2 Impact on academic performance

An additional perspective on the problem is offered by studies which measure the impact of support on student academic performance. Such studies often show that support programmes improve the performance of students (Campbell, 2015; Congos & Schoeps, 1993; Hillock et al., 2013), or that student performance in bridging courses correlates well with student performance in subsequent mathematics courses (Yushau & Omar, 2007). This positive impact is evident, even in the performance of students who displayed lower predictors of academic potential prior to the support (Campbell, 2015; Congos & Schoeps, 1993).

Thus studies measuring the impact of support on academic performance tend to report success of the support. However, they say little about how the interventions work to support students in the transition to university mathematics.

2.3 The pedagogy of the course

As noted in Section 1.3.2, in the course examined in my study, students are to solve realistic problems and through doing so are to learn mathematical concepts, problem-solving skills, and a sense-making disposition. Learning through solving problems is reminiscent of PCA, a pedagogy which has been used and researched at the school level. I present here literature which supports my argument that PCA is similar to the pedagogy of the course. I also discuss literature which critiques PCA at the school level, since these critiques are potentially applicable to similar pedagogy used at the university first-year support level.

It could be argued that my study provides a detailed account of one way of teaching problem solving, since the intended curriculum suggests that the course aims to teach problem solving. Consequently, there is scope to locate the pedagogy of the course in the work of others who have explored the teaching of problem solving. However, I do not review that vast literature here, since the focus of my study is to analyse what knowledge is made available to students in the implemented pedagogy of the course, and how the pedagogy communicates that knowledge to students, rather than an investigation of problem-solving.

2.3.1 Similarity of PCA and the pedagogy of the course

PCA is a pedagogy informed by constructivist principles of learning (Murray et al., 1998; Vithal & Volmink, 2005), in which students are to learn mathematics through solving problems (Murray et al., 1998). Constructivist principles purport that learning is a process whereby the learner constructs knowledge. Constructivists vary in the degree to which they view learning (construction of knowledge) as being an individual activity. On one end of the spectrum are *radical constructivists* who follow Piaget, taking the stance that learning occurs completely individually (Philips,

1995). On the other end are *social constructivists* who consider social influences like language, classroom culture and interactions with the teacher as essential in learning (Cobb et al., 1992).

Certain forms of pedagogy are privileged by constructivist theorists (Confrey, 1990; Richardson, 2003), even though constructivist principles are a theory of learning and “not a description of teaching” (Fosnot & Perry, 2005:33). Richardson (2003:1626) characterizes this pedagogy by drawing on descriptions of the pedagogy of teachers who claim to adhere to constructivist principles. Her results indicate that constructivist pedagogy:

- is “student-centred” because it attends to and respects individual students’ backgrounds and beliefs;
- involves “group dialogue”;
- may involve introducing “formal domain knowledge ... through direct instruction”
- is “task” oriented;
- develops in students a “meta-awareness of their understandings and learning processes”.

Richardson acknowledges that not all constructivist pedagogy displays all of these features. Particularly, she clarifies that the use of “direct instruction” – which she describes as “telling” students (ibid.:1637) – is contentious. In the intended curriculum of the course in my study (see Section 1.3.2), all of Richardson’s features, except for “direct instruction”, can be identified.

PCA is one particular pedagogy informed by (social) constructivist principles (Murray et al., 1998; Vithal & Volmink, 2005). PCA regards “problem-solving as the *vehicle* for learning” (Murray et al., 1998:171, emphasis in original). According to Davis (2005:54), this stance is a consequence of their belief that that a learner’s ability to reproduce traditional school mathematics procedures is not a reliable indicator of whether they understand the mathematics. Davis (ibid.) goes on to say that these pedagogues contended that:

mathematics should be taught and learnt through problems meaningful to students, stated in terms that did not immediately reveal the required mathematical ideas and operations so that the student was obliged to ... “construct” the necessary mathematical contents for themselves ... then the teacher could be reasonably certain that they had learnt and understood the mathematics.

This idea of learning through solving problems resonates with the intended pedagogy of the course examined in my study, although it is not new mathematical knowledge which the first-year engineering students are to learn: students are to make sense of mathematical concepts previously encountered. In addition, Piet Human (founder of the NPO and a teacher of the course) has historically aligned himself with the social constructivists (Murray et al., 1998), particularly while he was promoting PCA in South African primary schools (Davis, 2005). Since the course is underpinned by social constructivist principles with problem solving being central to its pedagogy, we can safely conclude that the pedagogy of the course is similar to PCA.

2.3.2 Critiques of PCA

Numerous authors have problematised PCA. The pedagogy of the course I have investigated is similar to school-level PCA (argued above). Consequently, critiques of school-level PCA point to the need to understand how the pedagogy of this course for first-year engineering students works.

Kirschner et al. (2006), for example, are highly critical of any pedagogy in which students are to “discover or construct essential information for themselves” (ibid.:75). They cite constructivist pedagogy as an example of this and, since PCA is a specific type of constructivist pedagogy, their critique applies to PCA also. They claim that such pedagogies “fail” because they provide insufficient “direct instructional guidance” to students (ibid.:75). This points to a need to understand exactly what “guidance” is given to students in such pedagogies. This is something which my study addresses by analysing how privileged knowledge is communicated to students in a first-year mathematics support course.

Vithal and Volmink (2005:7) argue that the version of PCA implemented in South African primary schools had a well-defined theory of learning (constructivist principles) but an ill-defined pedagogy. So although successful in schools where it was piloted, PCA was not taken to scale due to it being unclear how the pedagogy should be used, particularly in the South African socio-economic and political context characterised by inequality. While the support course which I have investigated is not intended to be rolled out at such a large scale, the funders of the course do aim to develop a “replicable model for tertiary access support” (Vosloo et al., 2012:2). However, if the course is to be replicated, detailed description of its implemented pedagogy is needed in order to make clear what knowledge it privileges and how it communicates that knowledge.

Davis (2001) offers an analysis of PCA in South African primary school textbooks and classrooms. His focus is on how evaluation⁷ functions in this context. He contends (ibid.:11) that teachers of PCA affirm student productions (written text or symbols, oral statements etc.) indiscriminately, and therefore often inappropriately, and that this inadvertently works against students’ understanding of the mathematics which the pedagogy is trying to communicate. It is needful to understand whether the same can be said of similar pedagogies used at the level of first-year mathematics support. In addition, Davis (ibid.) contends that PCA’s conception of the student as an “autodidact” (ibid.:185) or one who “constructs” knowledge (ibid.:208), does not play out in practice (ibid.:207). My attention to how the pedagogy of the course communicates privileged knowledge explores this contention at the

⁷ Davis defines evaluation in the Bernsteinian sense as being more than just conventional assessment (tests, projects, examinations etc.); it also includes, amongst other things, judgments made during the course of teacher-student interactions (Davis, 2001:2). See Chapter 3 for elaboration on this theoretical construct.

level of first-year university support. I return to a discussion of Davis's work in the next chapter, as his theoretical perspective has informed my study.

2.4 Summary

In this chapter I have discussed research which investigates programmes aimed at providing support for students in the transition to university mathematics. I have highlighted that the primary focus in this literature is on student perceptions of support and the impact of support in terms of academic results. Such studies are encouraging because they suggest that support programmes are effective. However, my discussion has shown that the literature is silent on what knowledge is made available in the implementation of support courses and how the pedagogy of these courses works to communicate knowledge. My study therefore offers a perspective which seems to be absent in the literature and which is needed for reasons discussed in Chapter 1 (Section 1.4.1).

The discussion in this chapter has also shown that the pedagogy of the support course examined in my study is reminiscent of PCA, a pedagogy which has been used and researched at the school level. I have drawn attention to critiques of PCA at that level – such as that it affords students insufficient guidance, the difficulty in replicating such pedagogy and the way in which such pedagogy affirms student productions. These critiques further support the need for understanding the pedagogy of the course.

Chapter 3: Theoretical framework

3.1 Introduction

In this chapter, I present the theories which form the lens through which I have studied the course. The rationale for my use of each theory links to my aim of creating a detailed description of the implemented pedagogy of the course, focusing on what knowledge is privileged and how that knowledge is communicated. The transformation of knowledge into pedagogic communication is thus central to my study and Bernstein's (1996) *pedagogic device* gives me a general perspective on this. I follow Davis (2001; 2005) by supplementing Bernstein's theory, using Davis's application of Hegel's theory of judgement to pedagogic situations to view what and how knowledge is privileged and communicated.

3.2 The pedagogic device

The pedagogic device is a set of three rules (*distributive*, *recontextualising* and *evaluative*) which mediate between knowledge and what emerges as pedagogic communication, such as student-teacher talk, textbooks etc. (Bernstein, 1996). Bernstein's theory of the pedagogic device underpins my study, which seeks to investigate the workings of pedagogic communication in the course. Distributive rules determine who has access to what knowledge. Recontextualising rules govern the transformation of knowledge into what is intended to be taught and how it is intended to be taught. Evaluative rules effect the transformation of knowledge into pedagogic communication at the level of pedagogic practice, i.e. in the classroom.

The criteria for producing knowledge that is considered legitimate (or privileged) in a particular pedagogic context are regulated by the evaluative rules. In Bernstein's terms, evaluation is not just formal assessment; it includes all evaluative acts that occur in pedagogic situations, be they student-teacher interactions, questions, tests or the wording of textbooks. Furthermore, evaluation encompasses what knowledge is considered legitimate in a pedagogic situation, how that knowledge is communicated to students, and judgements about whether or not students have acquired that knowledge. Compare, for example, a high-school physical sciences classroom and the course investigated in my study. In the former context, kinematic equations are considered legitimate knowledge and the use thereof is encouraged, whilst in the latter, they are discouraged⁸.

⁸ See interaction 15 in Appendix 6 for an example of the teachers in my study discouraging the use of kinematics equations.

Evaluative rules are controlled by the teacher⁹ (Bernstein, 2000). Dooley (2001:61), drawing on Bernstein, says that “the relation between transmitter and acquirer is fundamentally asymmetrical: transmitter and acquirer are always unequal.” In other words, the teacher-student relationship is hierarchical. Hoadley (2005:238), who also draws on Bernstein, says that even when learners appear to have control over aspects of the pedagogy such as selection of content, sequence and pace, this control is only ever “apparent”; the hierarchy remains. Since the intended pedagogy of the course fits Bernstein’s competence model of pedagogy (as discussed in Section 1.3.2.2), I anticipate that, in the implementation of the course, the hierarchical relationship between teacher and student will appear flattened.

Evaluation is central to any pedagogy since it “condenses the meaning of the whole device” (Bernstein, 1996:50). My study focuses on the content and operation of the evaluative rule. Indeed, my research question can be reframed entirely in terms of Bernstein’s theory as: “How does evaluation operate in a mathematics support course for first-year engineering students and what knowledge is privileged in that evaluation?” However, according to Davis (2005), Bernstein’s theory does not provide adequate tools to operationalise the theoretical concept of evaluation. He argues¹⁰ that this stems from Bernstein’s silence on the theoretical structure of evaluation (other than that it is a function of the distributive and recontextualisation rules). Davis consequently recruits Hegel’s philosophical theory of judgement in order to address this apparent absence in Bernstein’s theory.

3.3 Pedagogic judgement

Davis (2005:82) introduces the term *pedagogic judgement* to distinguish “evaluative judgement” in pedagogic contexts from judgement in general. General judgement is the subject of Hegel’s theory, which Davis recruits and applies to judgement in pedagogic contexts in order to develop a theoretical structure of evaluation. Davis (ibid.:101) says that pedagogic judgement is the means by which students interpret what to do, how to do it and actually do it – or the teacher conveys what it means to do so.

The knowledge to be communicated is referred to as a *notion* in Hegel’s theory of judgement (Davis, 2001; Davis, 2005). In mathematics education, for example, a notion could be mathematical (such as the concept of a function) or dispositional (such as a strategy for approaching a certain class of mathematical problems). Any notion is constituted by a subject filled out by one or more predicates. In a nutshell, the theory of judgement says that for a notion to be

⁹ Bernstein (1996:103) uses the terms *transmitter* and *acquirer* to replace the terms teacher and student so as to allow for the device to extend to other knowledgeable-ignorant subject pairs (eg. priest-layperson, doctor-patient, social worker-client)

¹⁰ I acknowledge that Davis’s argument is contentious and refer the reader to his full explanation (2005:81-82)

judged, or “understood”¹¹, the subject and predicate must become associated. This association occurs by progression through four stages: *existence*, *reflection*, *necessity* and the *notion* (although it is conceivable that any one of these stages may be bypassed in practice, see Section 4.6.1).

3.3.1 The stages of pedagogic judgement

To elaborate the idea of judgement of a notion, I refer to the notion of a rectangle, adapted from Davis’s (2001) example involving a square. The subject of this notion is the term “rectangle” and the predicate is dependent on the level of the pedagogic interaction: a five-year old’s understanding of a rectangle (a four sided figure with square corners) is different to that of an older learner (a quadrilateral with interior angles of 90°). Regardless of the level, it is insufficient to say that “a rectangle is a rectangle” since this tautological statement does not predicate the subject “rectangle”. Rather, a student must offer a description of the geometrical shape (rectangle) which is distinct from its name.

The four stages of judgement of the notion of a rectangle could proceed as follows:

1. Existence: When the notion is first encountered it is merely there, lacking predication. This leads to a sense that the notion is impossible. For example, posing the question “what is a rectangle?” implies the existence of the notion of rectangle but at the same time, the notion seems impossible because no description of rectangle is offered.
2. Reflection: The apparent impossibility of the notion leads to consideration of various predicates, which creates a sense of possibility of the notion. For the notion of a rectangle, various properties of a rectangle – all interior angles being 90°, both pairs of opposite sides being equal and parallel, diagonals bisecting each other – could be listed.
3. Necessity: Attempts at predication cease in this stage and a necessary relation between the subject of the notion and a particular predicate(s) is established. In identifying a given figure as a rectangle, certain properties will necessarily be discarded and others retained, as the latter appropriately describe a rectangle whereas others do not.
4. Notion: Up until this point, the judgement of the notion in its necessity is contingent on the nature of the evaluative activity (the worksheet, group discussion etc.). This last stage of judgement is

¹¹ Davis (2005:149) links judgement with the common-sense use of the term “understanding”. He describes understanding in terms of predication. He says, when speaking of the first stage of judgement: “what we might call the ‘understanding’ of the notion is not yet apparent because of the absence of predication” (ibid.:83) and “to demonstrate that we ‘understand’ a notion we must display a series of predicates different from the signifier for the notion” (ibid.:83)

concerned with the sufficiency or adequacy of the object arrived at as the notion: “Is the object “good” or “bad”, “elegant” or “clumsy” ...?” (Davis, 2005:92). For example, the description of a rectangle deemed adequate depends on what properties of shapes have been suggested to or are known to a student, which is contingent on the student’s level of schooling. A Grade 7 teacher in South Africa would deem the judgment of a rectangle as a four-sided figure that has all four (internal angles) equal to 90° sufficient for his/her learners. However, a Grade 10 teacher who has taught his/her learners coordinate geometry would deem this predication of rectangle insufficient, instead wanting students to predicate rectangle as a four-sided figure whose diagonals are equal in length and have a common midpoint.

Moving from one stage of judgment to the next involves a breakdown – or negation – of the former stage (Davis, 2005). In pedagogic practice, Davis suggests that these movements predominantly take place in one-on-one discussion between teacher and student.

Davis (2005) contends that it is possible for a student to reproduce mathematical knowledge without establishing mathematical necessity. However, he argues (ibid.:208) that “the establishing of mathematical necessity [is] a necessary element of the “construction” of mathematics contents”. Since PCA – and therefore the pedagogy of the course – conceives of the student as an autodidact, I anticipate that the course will require students to judge necessity in the development of notions.

Davis (2005) originally used the construct of pedagogic judgement to analyse primary school mathematics textbooks and observations of classroom teaching which the textbook authors deemed exemplary instances of PCA. Davis’s work has since been taken up in other contexts, usually together with other tools for analysing the grounds that the teacher appeals to during judgement. For example, Adler and Davis (2011) and Parker and Adler (Parker & Adler, 2014) have used the notion of pedagogic judgement to analyse the constitution of mathematics for teaching in teacher education. Adler and Pillay (Adler & Pillay, 2007) have used it as a lens to view “how the concept of a function came to ‘live’” (ibid.:92) in the context of South African Grade 10 mathematics classrooms. Pillay (2013) has taken up the notion of pedagogic judgement further in his exploration of the potential of a learning study to enhance in-service teachers’ mediation of Grade 10 learners’ identification of mathematical functions.

My study employs Davis’s notion of pedagogic judgement as a lens to view how evaluation functions in a mathematics support course for first-year engineering students. It has provided me with analytic tools for determining what knowledge is privileged in the course: the notions that are being judged. It has also given me a way to examine how the privileged knowledge is communicated to students: following the judgement of the notion’s progress through the four stages of judgement.

3.3.2 The splitting produced by pedagogic judgement

In addition to proposing the structure of pedagogic judgement, Davis (2005:128) suggests that pedagogic judgement leads to “splitting” at two levels: firstly, at the level of content in that the notion to be acquired is represented differently at the outset and conclusion of judgement. Exploration of this first split gives me further insight into what knowledge is privileged in the course. The second split is in the “distribution of knowledge of the notion” (ibid.:115) to participants of the pedagogy, which gives an additional way to examine how knowledge is communicated in the course.

3.3.2.1 Splitting at the level of content

The split produced by pedagogic judgement at the level of content takes place in the stage of existence and is removed in the stage of necessity (Davis, 2005). That is, at the outset of judgement, the full notion is absent. Hence the need for the judgement. Consequently, in the stage of existence, the notion is identified with “something other than itself” (ibid.:96). Davis (ibid.:27) recruits Freud’s terminology to refer to these two representations of a notion in pedagogic contexts: that which indexes the notion in its necessity is the *missing representation* (MR), due to its initial absence; that which stands in place of the notion at the outset of judgement is the *representation of the missing representation* (RMR).

In the example of the notion of a rectangle, the question “what is a rectangle?” is the RMR. The verbal description of a rectangle (contingent on education level) is the MR.

3.3.2.2 The split in the distribution of knowledge

The second split produced by pedagogic judgement is in the distribution of knowledge of the notion and ignorance thereof to participants in the pedagogy (Davis, 2005). It takes on both inter- and intrasubjective forms.

The intersubjective form of this split is related to Davis’s (ibid.) observation that the operation of pedagogic judgement presupposes that one participant in the pedagogy is associated with knowledge and another with ignorance of that knowledge, and that the knowledge is to be transmitted by the former and acquired by the latter. Empirically, the former position is held by the teacher and the latter the student. Indeed, Bernstein’s (1996) pedagogic device describes a structure for the flow of knowledge between teacher and student at the level of pedagogic practice. Davis (2005) relates this intersubjective form of the second split back to the first split created by pedagogic judgement: the RMR stands for “the lack of the notion in the consciousness of the pedagogic subject” (ibid.:98).

The intrasubjective form of this split relates to the student (ibid.). At the outset of the judgement, the student is assumed to be ignorant of the notion. However, the student is required to (re)produce the necessary texts (the MR) by the end of the judgement of necessity. Until the student (re)produces the MR, or demonstrates that s/he cannot (re)produce the MR, it could go either way: the student may be associated with knowledge of the notion or not. This uncertainty during the first three stages of judgement generates the split at the intrasubjective level.

Thus the second split produced by pedagogic judgement can be recognised inter- or intrasubjectively in the first three stages of judgement (ibid.). However, in the final stage of judgement, the split at the level of knowledge of the notion is intersubjective, since the teacher holds “the symbolic mandate of mathematics education” (ibid.:110) and therefore judges the sufficiency of that which the student has (re)produced as representing acquisition of the notion.

In order to talk about the splitting produced by pedagogic judgement at the level of the relationship between knowledge and the participants in the pedagogy, Davis (ibid.:102) introduces the idea of the *transmission function* (TF) and the *acquisition function* (AF)¹². The participant in the pedagogy who is “positioned as doing the work of transmission” is considered to be distributed the TF (ibid.:102). Similarly for acquisition.

In the “paradigmatic form of evaluation” (ibid.:103), TF is distributed to the teacher and AF to the student. That is, the paradigmatic form of evaluation manifests the intersubjective form of the second split. He says that this distribution is paradigmatic because it is a “general condition of possibility for pedagogy” (ibid.:103) that the teacher is knowledgeable and the student not, and one expects the knowledgeable participant to do the work of transmission.

However, the teacher need not necessarily be the only one who is distributed TF. The intrasubjective form of the split suggests that the student could be distributed both TF and AF. Indeed, Davis (ibid.:102) points out that “any given empirical pedagogic subject can, of course, be positioned in both ways” i.e. be distributed either TF or AF. Nevertheless, Davis cautions that the distribution of TF and AF is always “embedded within the general form of the paradigmatic form of evaluation” (ibid.:103). So even if the student is manifestly distributed TF (perhaps s/he is explaining something to his/her peers) the student will also necessarily be distributed AF and the teacher TF.

¹² Davis (Davis, 2003; Davis et al., 2003) has applied Lacan’s theory of discourses to pedagogic situations in order to capture, theoretically, the social structures present in pedagogy. It appears that Davis’s (2005) constructs of TF and AF originate in his application of Lacan, although this link is not explicitly made. It is beyond the scope of this thesis to explain Davis’s use of Lacan and how this is related to the TF and AF.

Davis (ibid.:117) offers a network summarising the possible distributions of TF and AF which I have adapted, as discussed below. Consider the left-hand side of the network (Figure 3.1), which shows all possibilities for distribution of TF. As noted above, TF can be distributed to the teacher or to the student. It is also possible that TF be distributed to some other agent such as a textbook or a fictitious character in a written or oral text. When TF is distributed to the teacher, s/he may act as him/herself (teacher as teacher) or may speak (or be spoken about) as if s/he were someone or something other. For example, in the course, a teacher speaks as if he were an engineer when he says: "...this is something that engineers and scientists do – they say 'let us first look at this'" (line 4a₃). In such an instance, TF is distributed to the teacher as other. In a similar manner, the student may be distributed TF as him/herself or something other.

Analogously, AF can be distributed to the student, teacher or some other agent (see the right-hand side of Figure 3.1). The possibility that the teacher be distributed AF is alluded to by Davis (ibid.:130) but is not included in his original network. I have extended his network to include this possibility in order to account for what I saw in my data, because distribution of AF to the teacher emerged as a mechanism whereby the hierarchy between teacher and student was apparently flattened. For example the teacher speaks as if he were a student and suggests that he is aligned with ignorance while doing so: "if he tells me something that I don't understand, then I ask him 'show me on your drawing ... help me to understand...'" (line 3a₂). I considered such a statement to indicate distribution of AF to the teacher as other (a student).

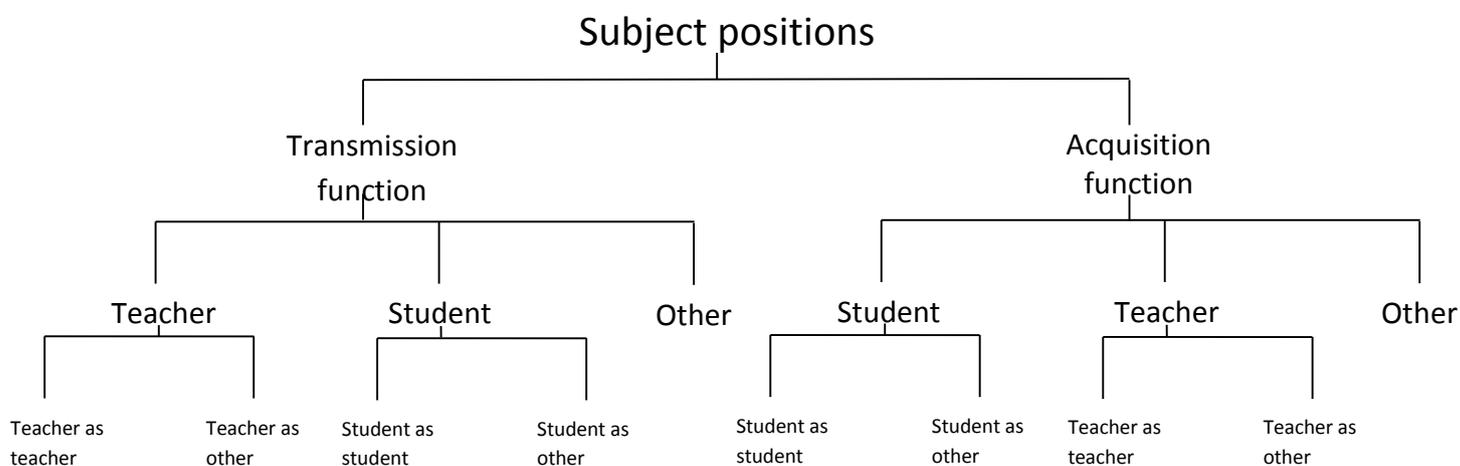


Figure 3.1 Possible subject positions created by the distribution of transmission and acquisition functions.

Adapted from Davis (2005:117)

Davis's constructs of TF and AF allow me to explore aspects of how the pedagogy of the course communicates knowledge to students. Firstly, I am able to relate the distribution of TF and AF to the development of judgement of the notion, thus adding a richness to that part of my analysis. In addition, they allow me a way of exploring how the idea that students construct their own knowledge plays out in practice. Davis (2005) expresses the PCA conception

of the student as autodidact in terms of TF and AF: the student will be distributed TF (since s/he constructs his/her own knowledge) and AF (since s/he needs to learn). Since the pedagogy of the course is similar to PCA (as argued in the previous chapter), I anticipate that the pedagogy of the course will distribute TF and AF to the student. Furthermore, I expect (following Davis) that when the student is distributed AF, the distribution will be to the student as other (as opposed to student as student), since the conception of student as autodidact implies that the student is already competent. Distribution of AF to the student positions the student as ignorant, which reveals the student as incompetent. Distribution of AF to the student as student highlights this revelation, whereas distribution of AF to the student as other projects the revealed incompetence onto something external to the student.

The constructs of TF and AF also allow me to frame my anticipation that the pedagogy of the course will appear to flatten the hierarchy inherent in pedagogy. That is, I expect that when the teacher is distributed TF, it will be to the teacher as other. Furthermore, I expect that that the teacher will be distributed AF in order to mask the hierarchical relationship between teacher and student.

3.4 Summary

In this chapter I have presented Bernstein's (1996) pedagogic device and Davis's concepts of pedagogic judgement (2001; 2005) and positioning (2003; 2005) as forming the theoretical perspective of my study. I have argued that Bernstein's theory of the pedagogic device (1996) forms a broad lens for my study, since it describes the structuring of the transformation of knowledge into pedagogic communication, and it is the working of the pedagogic communication in a mathematics support course for first-year engineering students that my study seeks to investigate. I have drawn on Davis's (2005) supplement to Bernstein's theory, pedagogic judgement, in order to aid my identification of what knowledge is privileged and how the judgement of that knowledge progresses. I have drawn on Davis's (2005) constructs of the TF and AF to view the positioning of the students and teachers with respect to the privileged knowledge.

The following propositions are summary of the theoretical tools outlined in this chapter, which inform my study:

1. Pedagogy is a process whereby knowledge is communicated between the transmitter (teacher) and acquirer (student). The pedagogic device with its three rules (distributive, recontextualising and evaluative) provides (amongst other things) a structure for this flow of knowledge between the transmitter teacher and acquirer student.
2. Evaluation, or pedagogic judgement, is central to the communication of knowledge in pedagogic contexts.

3. The relationship between teacher and student is hierarchical, since the teacher holds the symbolic mandate of knowledge and so is in possession of the evaluative rules.
4. Pedagogic judgement creates a split at the level of content, separating the notion to be judged into a subject and (one or more) predicates. This split is realised in pedagogic practice in the form of the MR and the RMR.
5. Pedagogic judgement also creates a split at the level of the relationship between knowledge and the participants in the pedagogy. The participant who is positioned as doing the work of transmission of knowledge is distributed a TF and the participant who is positioned as doing the work of acquisition is distributed an AF. Either participant can be distributed AF or TF.
6. The paradigmatic form of evaluation entails distribution of TF to the teacher and AF to the student.

In addition to these six propositions which summarize the theoretical framework of my study, I propose four hypotheses (predominantly drawn from Davis's (2005) discussion of PCA) which summarize my expectations for the pedagogy of the course in terms of the theoretical constructs laid out in this chapter:

- A. The pedagogy of the course will require students to judge necessity in the development of the notion(s).
- B. The pedagogy of the course will distribute both TF and AF to the student.
- C. When the student is distributed AF, the distribution will be to the student as other.
- D. When the teacher is distributed TF, the distribution will be to the teacher as other and/or the teacher will be simultaneously distributed AF.

The next chapter addresses the methodological implications for my study of adopting this theoretical framework.

Chapter 4: Methodology and methods

4.1 Introduction

In this chapter, I discuss the methodology and methods of my research process using Crotty's (2003) definitions thereof. Methods are procedures used to gather and analyse data. Methodology is the overall design of the research containing the rationale linking the study's choice of methods, purpose and theoretical framework.

I have used a case study methodology and my method of data collection was to video record part of an implementation of the course. I acknowledge, following Setati (2003), that the research process involves 're'-presenting data in ways which inevitably entail making choices influenced by the researcher's purposes, questions and theoretical framework. It is therefore important to be transparent about the methods of re-presentation used – which I aim to be in this chapter. I conclude with a discussion of quality in my study, paying specific attention to validity and ethics.

4.2 Methodology: case study

A case study is a “detailed examination of a single example of a class of phenomena” (Abercrombie et al. 1984 quoted by Flyvbjerg, 2006:220). My research project is a case study, since it focused on a single example of support for students in the transition to university mathematics: the course described in Chapter 1. I acknowledge that my selection of this case is opportunistic, since I was introduced to the course by a mutual funder. However, my research problem developed independently: although the funder suggested I study the course, it did not specify any aspect of my research focus or process.

In addition to their subject of study, case studies can be characterised by their purpose and approach (Thomas, 2011). The purpose of my study is to explore the implemented pedagogy of the course in order to understand two aspects of evaluation: what knowledge is privileged and how that knowledge is communicated to students. My approach is descriptive, since I aim to create a detailed description of these aspects of the course.

4.3 Empirical particulars of the case

I investigated an enactment of the course, since my aim was to explore aspects of its implemented pedagogy. The five-day course comprised tasks which the teachers refer to as “activities”. I focused on one complete activity (*The Trains Activity*) in order to make the amount of data collected manageable. Each activity was intended to develop the students' understanding of certain concepts, problem-solving skills and sense-making disposition. The activities

and their aims are summarised in Figure 4.1. Note that the first activity of the course was intended as a precursor to *The Trains Activity*, but this link was not mentioned to students.

Day	Time	Activity	Concept which the teachers aim to develop in the activity
1	Morning	Problem which is similar to but simpler than <i>The Trains Activity</i>	Making a formula
	Afternoon	Water flow activity	Rate of change and integration
2	Morning		
	Afternoon		
3	Morning		
	Afternoon		
4	Morning		
	Afternoon	The Trains Activity	Co-variation
5	Morning		
	Afternoon		

Figure 4.1 Outline of the course by activity

Throughout the course, the same activity was run in two classrooms, each with 22 students. Students were randomly (re)allocated to a classroom by a teacher at the outset of each activity. I observed and collected data in the classroom where the teacher who I had corresponded with previously was based, since he was eager to participate in my study. There were three other teachers present in this classroom, whose layout is shown in Figure 4.2.

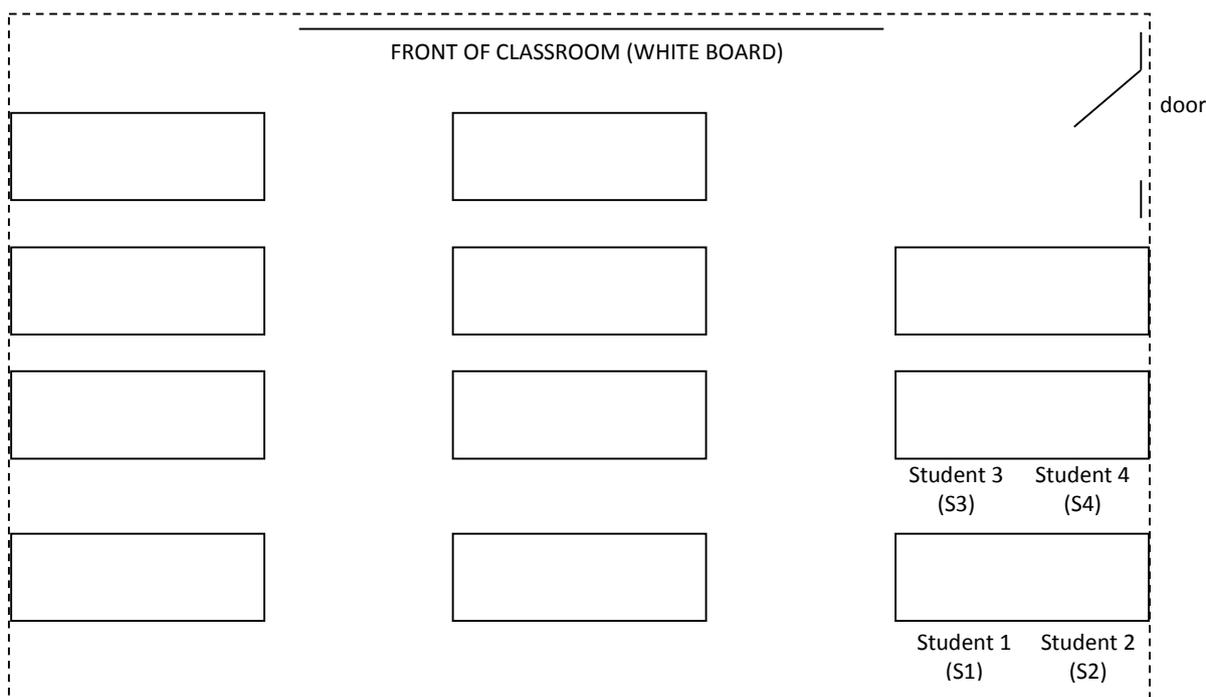


Figure 4.2. Layout of the classroom during implementation of *The Trains Activity* of the course

4.4 Data collection

I aimed to explore two aspects of evaluation in the implemented pedagogy of the course and evaluation is made visible in pedagogic communication. I worked on the premise that communication is contained in what students and teachers say, write and gesture at the level of pedagogic practice, particularly in the absence of inanimate pedagogic subjects. Consequently I used video records of interactions between teachers and students during the implemented *Trains Activity* as data, since videos provide an auditory record whilst allowing (repeated) observation of gestures and writing. I arranged for two third-year students (who had done the course previously) to video record the activity. Two video cameras were used:

Camera 1 followed one teacher continuously, recording all of his interactions with students, including whole class discussions. I followed this teacher – referred to as Teacher 1 – because he led all whole class discussions.

Camera 2 recorded a group of four students (hereinafter “the focus group”) continuously so as to capture all of their interactions with all teachers. I selected this group of students based on the teachers’ indication that they were neither the weakest students (so were likely to produce the privileged mathematics) nor the strongest students (so would likely need help, hopefully ensuring that they interacted with the teachers). The focus group were seated at the back of the classroom (Students 1 to 4 in Figure 4.2).

I collected pilot data in January 2012 at another of the NPO’s course’s in order to test my video recording equipment and plan. I used this data to develop my methods of data re-presentation and analysis before collecting the study data in June 2012.

4.5 Initial methods of re-presentation

In order to carry out my analysis (which itself generates a re-presentation), I re-presented the video records (themselves a re-presentation of the pedagogic communication) in written form. I discuss these methods here.

4.5.1 Written description

I first created a chronological written description of what I observed in my initial viewing of the video records to create a general sense of the lesson. Within this written description, I identified separate interactions in order to create a unit of analysis for the positioning of the teachers and students with respect to knowledge (see Section 4.6.2). An interaction was taken to be a single conversation between a teacher and one or more students, starting when the participants began speaking to each other and ending when they stopped speaking to each other. For

example, a student might ask a teacher for assistance (indicating the start of an interaction) and at the end, the teacher might walk away.

The written description captured oral, written, gestural and temporal aspects of each interaction, in addition to whether students were engaged in whole class instruction, individual work or group discussion. When students in the focus group were not interacting with a teacher, I recorded on whom the camera was focused and what s/he was doing. I also recorded information regarding the time on the video and duration of the interaction (see Table 4.1) to facilitate finding the interaction on video later in the research process. I also added notes about my initial interpretation to facilitate my subsequent detailed analysis. The descriptions from both camera recordings were placed alongside one another to establish an overall picture of the lesson at any given time.

Table 4.1. An example of the initial re-presentation of the video-record

Teacher camera				
File name	Time on video	Duration	Brief description	Notes (initial interpretation)
00089	00:00:38	00:00:46	A student asks Teacher 1 (privately) “what is the question here?” Teacher 1 addresses the whole class when ‘answering’ this question. Says: “You are the boss; there are no questions in front of you; you are the boss, you are the one that asks the questions. There is only what is. You have to decide what I want to investigate. What is interesting? Is there a problem?”	Note how Teacher 1 responds to a question with questions (“What is interesting? Is there a problem?”). These questions appear open-ended, but the teacher has specific answers in mind (so the questions are actually closed).

4.5.2 Sub-activities

In the written description, I observed a sequence of student actions similar to that in the intended curriculum (see Section 1.3.2). This sequence included drawing a diagram, posing questions and creating and checking formulae. I refer to these distinct periods of student action as “sub-activities”. Identifying sub-activities allowed me to place an initial structure on the data and gave me a first way to talk about what was happening in *The Trains Activity*.

I identified sub-activities by what students were doing, a shift in what they were doing indicating a new sub-activity. When one could not see on the video what students were doing, I inferred this from what was said. Details of my identification of sub-activities are provided in Appendix 4.

I then classified each interaction according to sub-activity and regrouped the written description of interactions accordingly. This second written re-presentation assisted with my analysis of the judgement of the notion.

4.5.3 Transcription

As mentioned earlier, the detail of what was said and referred to (such as diagrams or written symbols) during interactions was of particular interest in my study. Consequently, I transcribed in detail a selection of interactions between teachers and students using the rules and notation detailed in Appendix 5. The small amount of Sesotho used by students and one teacher was translated by a lecturer in Sesotho at the University of Cape Town.

Since there were just over 20 hours of video footage, the constraints of my study dictated that I transcribe (and subsequently analyse) a subset of my data. This subset consisted of all interactions of teachers with students in the focus group, including whole class discussions. I assumed that the teachers interacted in similar ways with other students in the classroom and, consequently, that this subset was illustrative of the pedagogy of the course.

4.6 Methods of analysis

I performed my analysis in two parts, each of which attends to a split generated by the operation of pedagogic judgement. Below I present the steps entailed in each part in a linear fashion. However, in practice, I moved between steps.

4.6.1 Method for my analysis of pedagogic judgement

There were two steps in my analysis of pedagogic judgement in *The Trains Activity*, both of which are based on Davis's (2005) adaptation of Hegel's theory of judgement for application to pedagogic situations (see Chapter 3).

Step 1: Identifying the privileged notions

I used the intended curriculum (see Section 1.3.2) as a guide to identify the notions privileged in the implemented curriculum, as the teachers do not explicitly name any notions in the implemented *Trains Activity*. Each aspect of the intended curriculum's aim was considered a notion and given a descriptive label: co-variation, problem solving and a sense-making disposition.

Co-variation was then deemed to be a notion privileged in the implemented curriculum because it was built into the problem (as it is in the intended curriculum). Problem solving was deemed to be a notion privileged in the implemented curriculum because it had a clear MR and RMR (discussed below). Sense-making was a notion privileged in the implemented curriculum because it was built into the development of problem solving. These three notions were closely interrelated, with problem solving a vehicle for developing the other two. I consequently focused my analysis on the central notion of problem solving, it being beyond the scope of this study to apply my

analytic tools to all the notions. I did, however, take note of actions and statements related to sense-making while analysing the judgement of problem solving in the implemented *Trains Activity*.

I then attended to the splitting generated by pedagogic judgement at the level of content by identifying the MR and RMR for the notion of problem solving. I identified these by examining the second written description of the video data. I recognised the MR by looking at what is to be arrived at in the stage of necessity. In *The Trains Activity*, the students produce a formula. Arrival at a formula indicates to the teachers that the student has solved the given problem, thus indexing the notion in its necessity. This formula is therefore the MR. I recognised the RMR as that which is presented to the students at the start of the judgement of the notion. In *The Trains Activity*, students are initially given a written description of a scenario (see Figure 4.3). This description is the RMR, since it marks the start of the problem solving.

There are two parallel railway tracks between the towns A and B. The length of the railway tracks from the one town to the other is 236 kilometres.

On a certain morning, one train needs to travel from A to B and arrive at B no later than 09h40. This train travels at an average speed of 96 km/h.

On the same morning, another train has to travel from B to A and arrive at A no later than 10h00. This train travels at an average speed of 139 km/h.

Figure 4.3 The problem description provided to the students, in writing, at the start of *The Trains Activity*

Once I had established the MR and RMR for the notion of problem solving, I considered whether judgement of the notion entailed judgement of any sub-notions along the way. I assigned a numbered, descriptive name to each sub-notion identified. For example, sub-notion 1 was found to be “asking appropriate questions about a given scenario”.

Each sub-notion is a notion in its own right. Consequently, I identified the MR and RMR for each sub-notion in the same way as for the notion. However, I used an additional indicator for recognising the MR: in the last stage of judgement, if students check their work to judge sufficiency of the MR, the work which students were checking was identified as the MR. The MR was sometimes subsequently modified in the final stage of judgement.

Step 2: Analysing the progression of judgement

I analysed the judgement of problem solving and each of its sub-notions by careful examination of the written descriptions of the video records and detailed transcript. I outline here how the features of the stages of judgement served as recognition criteria. Due to space constraints, I have not illustrated this analytic procedure with examples here. See Chapter 5 for my presentation of this analysis.

Existence

The start of this judgement is characterised by immediacy of the RMR, which I recognised as initial awareness of the RMR. I inferred this as occurring when students are first presented with the RMR. The stage of existence is also characterised by (seeming) impossibility of predicating the notion, which is negated. I recognised negation of impossibility when the teacher instructs students to do something which makes that which seemed impossible now seem possible.

Reflection

I recognised the second stage of judgement as taking place when possible predicate(s) are explored or students are instructed to do so.

Necessity

This stage of judgement is concerned with the shutting down of possibilities and establishing a necessary relationship between the subject of the notion and a particular predicate(s). I recognised this when conditions for identifying the privileged predicate(s) is (are) discussed and when some predicates are discarded.

Notion

The final stage of judgment is concerned with whether the representation of the notion arrived at in the previous stage is sufficient. I recognised this in what the teacher says and in what the students do. As mentioned earlier, this often takes the form of students checking (or being told to check) their work.

A caveat: not all stages of judgement always occur

The stages of judgement are theoretical concepts which are needed for the meaning of a notion to be established in a pedagogic context (Davis, 2005). However, in empirical pedagogic contexts, one or more of the stages may be absent (Davis, 2005). In my analysis, I identified omission of reflection when there is no consideration of possible predicates and omission of the judgement of necessity to be a consequence of the omission of reflection, since one predicate cannot be judged as necessary if there is not first a consideration of others. I also took the MR and RMR being mathematically tautological as a further indicator of the omission of the stages of reflection and necessity. Lastly, omission of judgements of reflection and necessity indicated to me that students are instead reproducing necessity established outside of the activity (again following Davis, 2005). This approach is borne out in my analysis of sub-notions 2 and 3 and the notion of problem solving.

4.6.2 Method for my analysis of positioning with respect to knowledge

The second part of my analysis attended to the split generated by pedagogic judgement at the level of the pedagogic subjects by analysing how they are positioned with respect to the privileged knowledge. I recognised this positioning by identifying, in the transcripts of interactions of the focus group, to whom transmission function (TF) and acquisition function (AF) are distributed. Recall that the pedagogic subject distributed TF is considered to be aligned with knowledge and the pedagogic subject distributed AF, ignorance. The available subject positions are summarised in Figure 4.4.

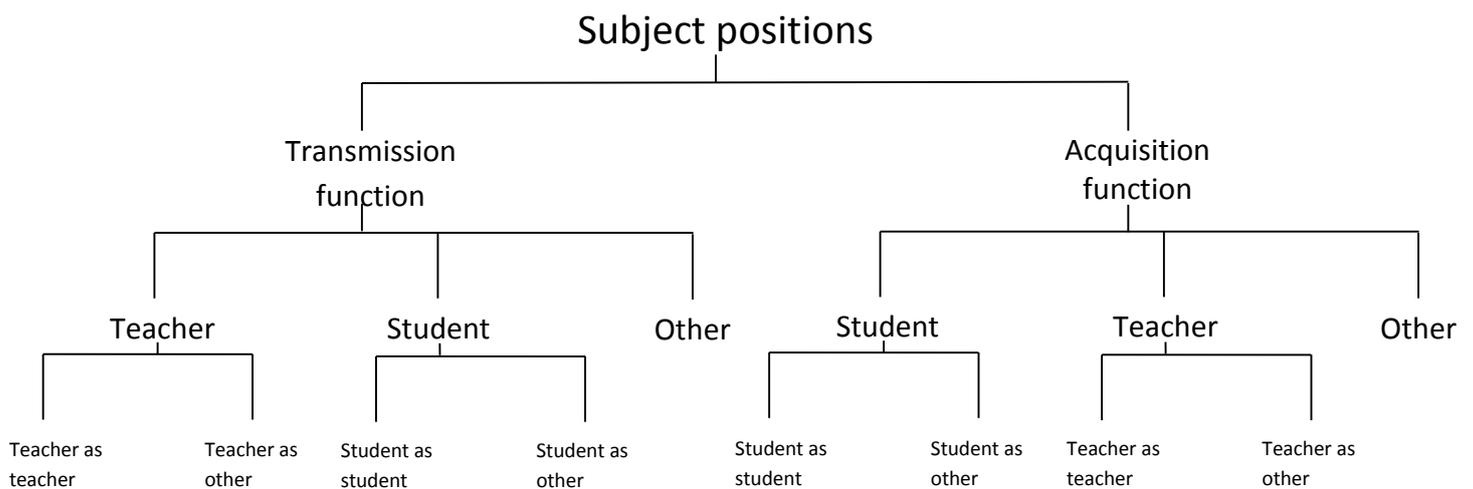


Figure 4.4 Possible subject positions created by the distribution of transmission and acquisition functions.

Adapted from Davis (2005:117)

Criteria for recognising distribution of TF and AF

The criteria for determining to whom TF and AF are distributed emerged in the interaction between the theory and data. Table 4.2 summarises these criteria. Notice that these allow for the possibility of AF being distributed to no-one

Most AF and TF indicators are paired, since the action of a speaker usually positions the receiver. I therefore considered the action of the speaker (e.g. explaining content), as primary and referred to the action of the receiver (e.g. listening to an explanation) as secondary. The secondary action is often inferred (not observed), it being a consequence of the primary action.

In two cases, one of the paired actions does not necessarily indicate positioning. However, whenever these actions occurred in my data, another indicator for positioning was also present. The first such action is “making a statement which aligns a pedagogic subject with knowledge/ignorance”. This does not necessarily position the speaker in a certain way, since s/he may be describing him/herself or someone else. For example, if the receiver is described as

ignorant, the speaker could be distributed AF (if describing him/herself) or TF (if making an unfavourable assessment of another person). The second action which does not necessarily position a pedagogic subject either way is “answering questions”. If the pedagogic subject answers correctly, s/he could be distributed TF; if incorrectly, AF. There may also be no correct response.

Table 4.2 Criteria used as indicators of distribution of TF and AF.

#	Actions		Distribution	
	Primary action Let the symbol α represent the pedagogic subject performing this action	Secondary action Let the symbol ρ represent the pedagogic subject performing this action	TF	AF
1a	Assessing a pedagogic subject's production in relation to the judgement of the notion.	Verbalising the assessment	$\alpha \rho$	none
1b		Having one's productions unfavourably assessed	α	ρ
1c		Withholding the assessment	$\alpha \rho$	none
2a	Making a statement which aligns a pedagogic subject with knowledge/ignorance	Being described (implicitly or explicitly) as aligned with the knowledge which is to be transmitted/acquired.	ρ	none
2b		Being described (implicitly or explicitly) as aligned with ignorance of the knowledge which is to be transmitted/acquired.	none	ρ
3	Explaining content in order to progress the judgment of the notion	Listening to an explanation which is intended to progress one's own judgment of the notion	α	ρ
4	Asking for assistance regarding the knowledge which is to be transmitted/acquired	Being asked for assistance regarding the knowledge which is to be transmitted/acquired	ρ	α
5	Posing questions regarding the knowledge which is to be transmitted/acquired which are not requests for assistance.	Answering questions posed regarding the knowledge which is to be transmitted/acquired.	α	none
6a	Giving instructions related to actions which are intended to lead to knowledge acquisition	Being instructed to perform an action which suggests alignment with the knowledge which is to be transmitted/acquired.	$\alpha \rho$	none
6b		Being instructed to perform an action which suggests ignorance of the knowledge which is to be transmitted/acquired.	α	ρ
7	Selecting what student productions to make public to the whole class with the intention of progressing judgment of the notion	None	α	none

The procedure for analysing positioning with respect to knowledge

This analysis involved three steps, with the basic unit of analysis being an interaction. Note that I signal reference to any criterion listed in Table 4.2 by a hashtag (#) and number.

Step 1. Identify distribution of AF and TF

To identify distribution of AF and TF, I examined each speech turn in an interaction's transcript, looking for the occurrence of any of the positioning criteria listed in Table 4.2. I then determined to whom each criterion indicated distribution of AF or TF. For example, in interaction 11 (see Appendix 6), Teacher 4 says to Student 4: "Talk to me. I'm just the railway conductor, but maybe I can talk about it. What's going on in your mind at the moment?" (line 11c). The teacher is asking a question regarding the privileged knowledge, since it is implied that he is asking about the student's thinking regarding the problem. The question is certainly not a request for assistance, but it does suggest that the teacher is offering his assistance to the student. It is thus an instance of criterion **#5** and indicates distribution of TF to the teacher.

I also determined whether distribution of TF and AF was to the pedagogic subject as him/herself or as something other. I took the former as the default and considered the latter to occur when the speaker takes on the voice of something other (as in line 11c above) or when the speaker asks the listener to imagine that s/he is someone or something other than him/herself (for example see interaction 25 where the teacher says to the students in line 25e₅, "imagine that you are the railway manager...").

In most of my analysis, I considered the student as a single analytic category. This is because there is usually either one student (only) involved in the interaction (as in interaction 11) or the teacher speaks to the whole class, indicating distribution of AF (or TF) to all students. When a single student speaks in a whole class discussion, I attributed the positioning of the vocal student to all students, because very few students participate in whole class discussions. In group discussions, however, I treated the individual students as separate analytic categories because, at times, one student is clearly distributed TF and the other students AF. For example, if one student explains something to the rest of the group, the student doing the explaining is distributed TF (**#3**) while the students listening to the explanation are distributed AF (**#3**). Interaction 17 (presented in Section 6.3) illustrates this.

Step 2. Subdivide the interaction based on distribution of TF and AF

In the second step of my analysis, I subdivided each interaction into sections (groupings of consecutive speech turns) within which the positioning of both student and teacher did not change. These sub-divisions facilitated my subsequent tracking of changes in positioning (see Step 3 below). At the speech turn in which the positioning of one or both pedagogic subjects changes, a new section was inserted and denoted by a sub-script. For example,

interaction 11 section 1 was notated as 11₁. In some interactions, a change in positioning (of teacher and/or student) takes place within a single speech turn. Consequently, I subdivided these interactions within a speech turn. For an example of this, see interaction 2 in Appendix 6.

Step 3. Summarise the positioning

The third step in my analysis of the positioning was to summarise the results in tabular form (see Appendix 7) and then group the analysis according to sub-activity and stage of judgment (see Appendix 6). While creating the tabular summary, I reviewed the positioning in the stages of judgment for each sub-notion, to ascertain what work the positioning does in terms of communicating the privileged knowledge. I explored trends within each sub-notion and within particular stages of judgement across sub-notions.

4.7 Quality in my study

In this section, I use Maxwell's (1992) critical realist approach to validity, and consideration of ethical issues, to reflect on the quality of my study.

4.7.1 Validity

The validity of a study is contained in the relationship between the account which it produces and that which it is an account of (Maxwell, 1992). However, since the realist perspective is that there is no absolute truth to which an account can be compared, one can only compare different accounts of the same thing (ibid.). Consequently, "validity pertains to the kinds of understanding that accounts can embody" (ibid.:284). These "kinds of understanding" are captured by Maxwell's categories of validity: *descriptive*, *interpretive*, *theoretical* and *evaluative validity*, and *generalisability*. These categories – and the need to reflect on a study in relation to them – relate to the importance which Setati (2003) places on the researcher's transparency regarding the process of re-presentation, since re-presentation "shapes the interpretations that we make and conclusions that can be drawn from the research" (ibid.:294).

Descriptive validity refers to the "factual accuracy" of an account (Maxwell, 1992:285). To achieve this I have presented all of my methods of re-presentation transparently in this chapter and have endeavoured to keep each re-presentation (which collectively form my account) as true as possible to what actually happens in the implemented *Trains Activity*. For example, the video cameras ran continuously throughout the activity, I did the transcriptions myself (endeavouring to record all relevant information whilst applying my transcription notation consistently) and I revisited the descriptions and transcriptions in relation to the videos throughout the research process.

Interpretive validity is achieved when an account “respects the perspectives” (ibid.:290) of the participants of the account. In describing the intended curriculum of the course, I linked my description closely to course documents and the teachers’ publication in order to respect their perspectives. In analysing the implemented curriculum, I make no claims about the participants’ perspectives.

Theoretical validity refers to the appropriateness of the choice of theoretical framework and its application to the empirical situation. I argued for this appropriateness when presenting my theoretical framework in Chapter 3. In the present and proceeding chapters, the appropriateness of the methods of my analysis (the application) is demonstrated. Theoretical validity is also dependent on agreement within a relevant research community regarding the “terms used to characterize the phenomena” (ibid.:292). To this end, I have made my study available to the research communities in which I participate.

Evaluative validity is not applicable to my study, as I describe the implemented pedagogy of the course, not evaluate it.

Generalisability, or “external validity” (Merriam, 1991:173), refers to the degree to which an account can be extended beyond its own empirical referents (Maxwell, 1992). Case studies are often criticised for apparent lack of generalisability (Yin, 2003). Yin says this is because case studies are “generalisable to theoretical propositions and not to populations” (ibid.:10). Such generalisability is apparent in my study, where I apply Davis’s (2005) theoretical concepts (pedagogic judgement, TF and AF) thus demonstrating the generalisability thereof. Furthermore, by giving a detailed description of what happens in the course, others are able to decide how my results might be applicable in their contexts.

4.7.2 Ethical considerations

I took various measures to prevent participants being harmed by my study. I discuss here these measures taken in relation to considerations of anonymity, confidentiality and voluntary participation.

Confidentiality and anonymity

I identified two factors as potentially affecting confidentiality and anonymity of participants: my use of video records and detailed writing. To protect participants in my use of video records, I stored the videos safely on an external hard drive and ensured that the videos were viewed only by myself, my supervisors and the teachers¹³ of the course. In my writing, I adopted the convention of assigning pseudonyms to all participants in my references to the video records. However, I did name some of the teachers in my writing about the background of the course and intended

¹³ The teachers specifically requested that they be allowed to view the videos for their own professional development.

curriculum. This naming was partly so that I could properly cite previous publications about the course and partly because the funder and NPO requested that they be named. All those concerned agreed to being named in this way. I have also attempted to write in a manner which respects each participant, particularly avoiding a deficit view of individuals. This is important because all participants of my study (even those not named) are potentially recognisable due to my detailed descriptions of the course, the cameras following the focus group and my naming of organisations and teachers.

Voluntary participation

I obtained informed consent from all participants. When negotiating participation, I explained (orally and in writing – see Appendix 8) the nature of my research, the issues above and that participation was voluntary. That participants felt assured that their involvement was voluntary is suggested by one student’s refusal to participate during pilot data collection.

The relationship of the funder to myself and the participants of the study could possibly have affected the voluntary nature of participation. Specifically, teachers and students may have felt obliged to participate for fear that their funding may otherwise be compromised since the funder was funding this present study, the students (see Chapter 1) and the NPO (who rely heavily on the income received for running the course). However, the funder indicated that refusal of any teacher or student to participate would not affect their funding and the funder has no record of which students/teachers participated. Furthermore, the funder made no specifications about what I should undertake as the subject of my research. I explained these issues to the students and teachers – orally and in writing – when obtaining their informed consent.

4.8 Summary

In this chapter, I have outlined my methodology and methods of data collection and analysis. The reader can see this at work in the next two chapters, where I present results of my analysis.

Chapter 5: Pedagogic Judgement

5.1 Introduction

In this chapter, I present the results of my analysis of pedagogic judgement in *The Trains Activity*. My analysis of what notions, and corresponding sub-notions, are judged in the activity shows what knowledge the pedagogy privileges. My detailed analysis of the development of problem solving (and its sub-notions) through the four stages of judgement gives further insight into what knowledge is privileged in addition to how that knowledge is communicated to the students. First, I give an overview of the implementation of *The Trains Activity* to orientate the reader.

5.2 The Implemented Trains Activity

Students are initially given a written description of a scenario involving two trains travelling in opposite directions on parallel railway tracks, with known speeds and arrival times (see Figure 5.1).

There are two parallel railway tracks between the towns A and B. The length of the railway tracks from the one town to the other is 236 kilometres.

On a certain morning, one train needs to travel from A to B and arrive at B no later than 09h40. This train travels at an average speed of 96 km/h.

On the same morning, another train has to travel from B to A and arrive at A no later than 10h00. This train travels at an average speed of 139 km/h.

Figure 5.1 The problem description provided to the students, in writing, at the start of *The Trains Activity*

Students first read the description in silence, without a teacher telling them to do so. However, a precedent is set on the first day of the course when Teacher 1 states to the whole class: “you will work individually on a problem – definitely at the start”. For most of *The Trains Activity*, students work individually and at their own pace, with brief periods of whole class instruction and group work.

Although students are not provided with any written instructions or questions, there is a specific sequence of sub-activities which the teachers orally direct the students through (the nature of this direction is discussed in Section 5.4). I identified nine sub-activities, which I refer to throughout my analysis. Table 5.1 summarizes the activity by listing all sub-activities.

Table 5.1. Summary of *The Trains Activity* according to sub-activity

Number	Description
1	Students read the problem/situation description.
2	Students create a graphical representation of the problem.
3	Students pose “interesting” questions, in group discussion.
4	Students solve the privileged questions.
5	Students check answers.
6	Students determine a formula for calculating the meeting time for any two trains travelling between any two towns.
7	Students simplify their formula (factorisation etc.)
8	Students check formula (by substitution of the values given originally).
9	Students refine the formula so that it uses the given quantities (arrival times, average speeds and total distance between the towns) <i>only</i> .

5.3 The privileged notions

In the implemented *Trains Activity*, students are not told explicitly what knowledge is privileged. Consequently, I used the intended curriculum as a guide to identify three closely related notions privileged in the implemented *Trains Activity*: co-variation, problem solving and a sense-making disposition (see Section 4.6.1). My analysis focused on the central notion of problem solving, and how sense-making was woven throughout.

The notion of problem solving

I identified the problem statement (Figure 5.1) as the RMR of problem solving and the formula for determining the meeting time of any two trains of fixed (but unknown) average speeds, arrival times and initial distance apart as the MR (as discussed in Section 4.6.1). In addition, I recognised three sub-notions of problem solving in *The Trains Activity*, evidence for which is provided in the remainder of this chapter.

Sub-notion 1: Asking appropriate questions about a given scenario

Sub-notion 2: Calculating specific values about a specific instance of given scenario

Sub-notion 3: Creating an algebraic formula to calculate values about the general case of a given scenario

Each sub-notion is a notion in its own right. They work together to assist the student to understand (judge) the notion of problem solving.

I liken the problem solving privileged in the activity to the problem-solving process articulated by Polya (1957), similarly to the intended curriculum (see Section 1.3.2.4). Recall that there are four phases in Polya's problem-solving process: understand the problem, devise a plan, carry out the plan and look back. I argue (below) that sub-notion 1 together with sub-notion 2 form one cycle of Polya's problem-solving process. In this cycle of the problem-solving process, students solve a specific problem – determining the meeting time of the two trains described in the problem description (Figure 5.1). Sub-notion 3 forms a second cycle of Polya's problem-solving process, in which students solve a related but more general problem. The relationship between problem solving and its sub-notions is summarised in Figure 5.2. I elaborate on this relationship in the remainder of this chapter.

Both the specific and general problems solved in *The Trains Activity* are subject to the same simplifying assumptions inserted by the teacher (see Section 5.4.1) and so both can be considered simplified versions of a more complex problem. Thus the activity models a heuristic identified by Polya: consider a simpler, related problem to devise a plan when solving a more complex problem. However, the students do not solve the more complex problem in the activity.

In addition, the specific problem seems to be an adaptation of this heuristic, since calculating the meeting time of two trains with known speeds etc. is a simpler but related problem to draw on when devising a plan for solving the more general problem (creating a formula for the meeting time of any two trains). This is indicated by the arrow in Figure 5.2. Evidence that the teachers insert the specific problem into *The Trains Activity* before the general problem to assist students with the more general problem is provided in Section 5.4.3.

In its original form, Polya's heuristic involves students setting up for themselves a simpler but related problem to aid their solution of a more complex (possibly general) problem with which they were originally presented. In contrast, in *The Trains Activity*, the teachers insert the simpler, related problem before presenting students with the general problem.

				Phases of Polya's Problem Solving Process
Notion (the problem solving process)	RMR Written description	Sub-notion 1 (asking appropriate questions about a given scenario)	RMR Written description	(1) Understand the problem
	MR The problem-solving process		MR The privileged questions	
		Sub-notion 2 (calculating specific values about a specific instance of a given scenario)	RMR The privileged questions	(2) Devise a plan (3) Carry out the plan (4) Look back
		MR Meeting time of the two trains		
		Sub-notion 3 (Creating an algebraic formula to calculate values about the general case of a given scenario)	RMR Teacher's verbal description of a "tool"	(1) Understand the problem (2) Devise a plan (3) Carry out the plan (4) Look back
	MR Algebraic formula			

Figure 5.2 A diagram showing the relationship between the four notions and the problem-solving process

Sub-notion 1: Asking appropriate questions about a given scenario

The first sub-notion entails students reading the problem description (sub-activity 1), drawing a diagram (sub-activity 2) and posing questions (sub-activity 3). The teacher implies that the lack of questions in the problem statement encourages sense making: students have to “think about what is going on here ... before anything is calculated” (line 2a₁). Sub-notion 1 therefore constitutes the first phase of Polya’s problem-solving process (understand the problem) for the specific problem in the activity.

The RMR for sub-notion 1 is the written description about the trains (Figure 5.1), since this is what students are initially presented with. The description is just there in its immediacy, suggested by the teacher when he says “There is only what is” (line 1b) in response to a student asking “What is the question here?” (line 1a). The MR is the list of questions deemed necessary by the teacher in a whole class discussion (see Section 5.4.1).

Sub-notion 2: Calculating specific values about a specific instance of a given scenario

The second sub-notion entails students solving the privileged questions (sub-activity 4) and checking their answers (sub-activity 5). Solving the questions requires students to make a plan (Polya’s phase 2) and carry out that plan (Polya’s phase 3) while checking their answers involves looking back (Polya’s phase 4).

The MR for sub-notion 2 is the time at which the two trains (described in Figure 5.1) meet, since the student’s statement of this time indicates that s/he has calculated a specific value about the given scenario. Furthermore, students judge the sufficiency of the meeting time (sub-activity 5). The RMR for sub-notion 2 takes the form of the list of privileged questions (which was the MR for sub-notion 1), since these stand in place of the meeting time of the trains at the start of the judgement of this sub-notion.

Sub-notion 3: Creating an algebraic formula to calculate values about the general case of a given scenario

The final sub-notion of problem solving in *The Trains Activity* entails students creating a formula for determining the meeting time of any two trains travelling on parallel tracks with specified (but unknown to the students) average speeds, arrival times and initial distance apart (sub-activity 6), simplifying that formula (sub-activity 7) and then checking and refining that formula (sub-activities 8 and 9).

Sub-notion 3 seems to be that which the teachers expect students to do for any problem: create an algebraic formula to calculate values about a given scenario. In *The Trains Activity*, students arrive at a formula by the end of the second cycle of the problem-solving process. Furthermore, students check the formula (sub-activity 8). Therefore this formula is the MR for sub-notion 3.

The RMR for sub-notion 3 is the idea of a “tool” (line 25e₈) for the work in practice of a railway manager, since this is what students are initially presented with (orally by the teacher) at the start of judgement of this sub-notion (see Section 5.4.3 for elaboration of this).

5.4 Development of the notion

My analysis of the pedagogic judgement of problem solving and its three sub-notions in the implemented *Trains Activity* shows that it is only in sub-notion 1 that all four stages (existence, reflection, necessity and the notion) are judged. For the other sub-notions and the notion of problem solving, there is no reflection in the activity and consequently no judgement of necessity in the activity; students merely reproduce necessity established elsewhere by the teachers. These results are summarised in Table 5.2 and the details thereof presented in what follows.

Table 5.2. Summary of judgement of notions in *The Trains Activity*

Description of notion / sub-notion		Existence	Reflection	Necessity	Notion
Sub-notion 1	Asking appropriate questions about a given scenario	✓	✓	✓	✓
Sub-notion 2	Calculating specific values about a specific instance of given scenario	✓	✗	(✓)	✓
Sub-notion 3	Creating an algebraic formula to calculate values about the general case of a given scenario	✓	✗	(✓)	✓
Notion: Problem solving		✓	✗	(✓)	✗

Key:

✓	Stage judged in the activity
✗	Stage of judgement absent from the activity
(✓)	The outcome of this stage of judgement is reproduced in the activity, but judged elsewhere

5.4.1 Sub-notion 1: Asking appropriate questions about a given scenario

In this section, I describe the development of sub-notion 1 in *The Trains Activity*, which provides evidence for my argument that it progresses through all four stages of judgement in the implemented *Trains Activity*. This argument supports my earlier claim that sub-notion 1 constitutes the first phase of Polya’s problem-solving process (understand the problem).

Existence

At the start of *The Trains Activity*, students read a written description of two trains (Figure 5.1) (sub-activity 1). Recall that this written description is the RMR for sub-notion 1.

The lack of questions in the activity is apparent to the students, as evidenced by a student asking Teacher 1: “What is the question here?” (line 1a). The lack of questions makes *The Trains Activity* seem impossible to the student (there is nothing to do) and impossibility is characteristic of the stage of existence. The lack of questions also appears to be a way that teachers encourage students to make sense of the problem, as discussed in Section 5.3.

Another characteristic of this stage is negation of impossibility. Here, impossibility is firstly negated by Teacher 1 telling the students that they “are the one that asks the questions” (line 1b), an action which appears to promote making sense of the problem and suggests students will “investigate” and “discover”. Secondly, teachers encourage students to draw a diagram of the situation (sub-activity 2): “make yourself a picture” says Teacher 1 (line 2a₃). The teacher suggests that making “some kind of drawing” (line 2a₂) will assist students to “visualise what is going on here” (line 2a₂), that is, to make sense of the problem.

Reflection

While the students are drawing diagrams individually, Teacher 1 addresses the whole class. He tells the students to discuss in groups “what is happening here ... in your own words” (line 3a₁) and then, in their groups, “to come up with ... important or interesting questions ... about this situation” (line 3a₁).

The second stage of judgement is characterised by consideration of possible predicates, which for sub-notion 1 are questions. Thus students consider possible predicates in their group discussions (sub-activity 3). The group discussions also encourage students to make sense of the problem by explaining to each other “what is happening here” and suggesting questions. However, the teacher stipulates how students are to make-sense: students are to refer only to their drawings in the discussions and not to their calculations, since the latter counters sense-making: “If you have done any calculations ... please turn them over. When you discuss it as a group, I want you just to point towards your drawing” (line 3a₁).

Necessity

In the third stage of judgement, one (or more) predicates are deemed essential and others are discarded. The discarding of possible questions (predicates) takes place primarily in a whole class discussion (Interaction 4, see Appendix 6). In this discussion, Teacher 1 inserts the assumption of constant speed which simplifies the problem for the students and serves as justification for discarding some of the questions which the students posed.

Prior to the whole class discussion, while students are engaged in group discussion, Teacher 1 writes some of the students' questions on the board (Figure 5.3). He claims he has "written most of the questions that (he) could remember you (the students) talking about" (line 4a₁). Teacher 1 has, however, only interacted with four of the seven groups of students, so the list is unlikely to be exhaustive. Furthermore, the teacher does not write two questions which students had suggested to him: "Which train leaves first?" and "What if there is only one driver?" Thus the teacher starts the process of discarding questions while writing the questions on the board, but does not make his selection explicit to the students.

The first two questions, written on the board in black, are judged by the teacher as necessary predicates for sub-notion 1, evidenced by their retention at the end of the class discussion. In comparison, the remaining questions are written in green and eventually erased. Teacher 1 tells students to assume that the trains are travelling at constant speed: "I'm simplifying matters by saying 'let's assume that the trains travel at a constant speed'" (line 4a₄). He uses this assumption as justification for discarding the green questions (see Extract 5.1). The colours and order in which the questions are written suggest that the teacher determined which questions were necessary prior to writing them on the board.

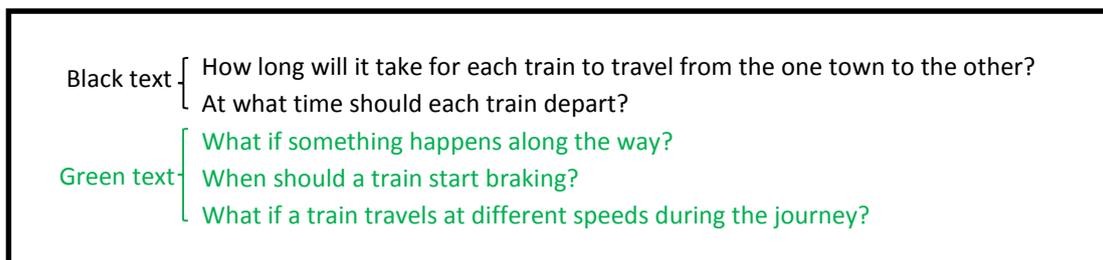


Figure 5.3 Questions which the teacher writes on the board about 20 minutes after the start of *The Trains Activity*

Teacher 1 claims that the simplifying assumption is necessary in order to deal with difficulty which he experiences: "Why am I making that assumption? Because it's easier for me to calculate things" (line 4a₄). However, he subsequently locates the justification external to himself: "this is something that engineers and scientists do" (line 4a₃). Yet the simplification is pedagogically necessary: the teacher has judged that it is the *students* who will experience difficulty. The students are not, however, given an opportunity to experience this difficulty. This is in opposition to the intended pedagogy, since the teacher is explicitly telling students what needs to be done to simplify the problem rather than students "discovering" this for themselves. In addition, I suggest that necessity is located in the notion of problem solving, since the simplifying assumption creates a simpler problem which Polya's (1957) problem-solving process suggests is useful to consider.

Lastly, I observe that the teacher appears to encourage sense-making while communicating the necessity of the simplifying assumption: he enacts the motion of the trains (walking across the room) whilst speaking about that motion in line 4a₄ (see Extract 5.1).

Extract 5.1

4a₄ Teacher 1 So I'm simplifying matters by saying "let's assume that the trains travel at a constant speed." ... So, from the time that train A departs until the time that it arrives at the other side [teacher walks across the front of the room as he says this], it's travelling at a constant speed [he is now standing still]. ... So that that deals actually with all three of these questions [points to the three questions on the board written in green – see Figure 5.3] ...

Notion

In the final stage of judgment, the sufficiency of the MR is judged. The MR for sub-notion 1 is the list of privileged questions, since it is what is arrived at in the stage of necessity. For sub-notion 1, the teacher judges the sufficiency and communicates it to students in the same whole class discussion in which he discards questions (interaction 4, line 4a₅ onwards).

Teacher 1 announces that he has another "question in mind" (line 4a₅), implying that he judges the two privileged questions posed by students as insufficient and indicating the start of this stage of judgement. Teacher 1 encourages the students to also judge their questions as insufficient by insisting that they pose this third question. He says: "I want you to think of another question" (line 4a₅).

The teacher then demonstrates the movement of the trains again, this time with the assistance of a student. The teacher hints at the third question by saying "hi" to the student (also acting as a train) as they pass each other. The demonstration appears to encourage sense-making by suggesting association of the physical movement of the trains with the mathematics at hand.

After demonstrating the movement of the trains, the teacher asks: "Is there something interesting that we could find out?" (line 4e). In response, students mumble proposals for the final privileged question (line 4f). The teacher reframes their responses: "At what time and at what position will the trains meet each other?" (line 4g). The declaration of this question completes the MR.

Judgement of the sufficiency of this question (which is really two questions) occurs when a student asks why this question is important: "But like of what importance is that though?" (line 4h). The teacher claims that the importance lies in that which students will learn by answering it: "It's of importance for the mathematics that you

will learn by finding it out” (line 4i). Thus the teacher locates sufficiency of this question in its pedagogic function which suggests that the pedagogy is aimed at the problem-solving process (the “mathematics”) rather than real-world solving of problems. Indeed, the question is needed so that this specific problem can become a related, simpler problem to draw on when devising a plan for solving the general problem. But this is not made explicit to students.

5.4.2 Sub-notion 2: Calculating specific values about a specific instance of given scenario

In this section, I present a detailed description of the development of sub-notion 2 in *The Trains Activity* to support my argument that students are not required to judge reflection or necessity in the judgement of this sub-notion. Instead, the pedagogy relies on students’ implicit acceptance of necessity established elsewhere. First, I orient the reader with a brief discussion of the privileged questions.

The privileged questions

There are four privileged questions left on the board at the conclusion of sub-notion 1 (Figure 5.4). I argue (below) that these questions are closely and hierarchically related. This suggests that the teacher writes these questions on the board in a preselected order.

How long will it take for each train to travel from the one town to the other?
 At what time should each train depart?
 When will the two trains meet one another?
 Where will the two trains meet one another?

Figure 5.4 Questions left on the board after whole class discussion

Answering the first question (“How long will it take for each train to travel from the one town to the other?”) involves a calculation using the speed-distance-time relationship (presumably familiar to students from their high-school maths and science experience) with the given speeds and distance between the towns, as demonstrated in Figure 5.5.

$$\text{time taken} = \frac{\text{distance travelled}}{\text{average speed}} = \frac{236 \text{ km}}{96 \text{ km/hr}} = 2.46 \text{ hours} = 2 \text{ hours} + 0.46 \times 60 \text{ min} = 2 \text{ hours } 28 \text{ min}$$

Figure 5.5 Calculation of the total time taken for train to travel from town A to town B

The second question, “At what time should each train depart?” builds on the first question, since answers to the first (the duration of each train’s journey) need to be combined with the given arrival times to obtain the starting time for each train. This is demonstrated in Figure 5.6.

Arrival time	09h40
Time travelled	2 hours 28 minutes
<hr/>	
Departure time	07h12
	$9 - 2$ $40 - 28$

Figure 5.6 Calculation of the departure time for train travelling from town A to town B

The first two questions appear to assist students to make sense of the problem, in addition to assisting students to answer the third question, “When will the two trains meet one another?” While there are various ways to solve the third question (see Appendix 3), a method used by a number of students (such as Students 2 and 3) is shown in Figure 5.7. This method requires using the answers to the first two questions, thus demonstrating the questions’ hierarchical relationship.

Let Train 1 be the train starting at town A and Train 2 the train starting at town B.

Train 1 departure time	07h12 (see Figure 5.6)	} (answer to second question)
Train 2 departure time	08h18	

Difference in starting times: 1.09 hours

So at 08h18, Train 1 has covered $98 \times 1.09 = 105$ km

\Rightarrow distance between the two trains at 08h18 is $236 - 105 = 131$ km

\Rightarrow sum of the distances travelled by trains from 08h18 until their meeting point = 131km

Let Δt represent the time taken for Train 2 to travel until the meeting point.

Distance travelled by Train 1 from 08h18 until the meeting point = $96\Delta t$

Distance travelled by Train 2 from 08h18 until the meeting point = $139\Delta t$

$$96\Delta t + 139\Delta t = 131$$

$$\Delta t = 0.56 \text{ hours}$$

$$= 33 \text{ minutes}$$

Therefore, the meeting time is $08h18 + 33 \text{ minutes} = 08h51$

Figure 5.7 A possible method for determining the meeting time of the two trains.

The teachers tell students to “work backwards” (line 22w), in a way which involves solving the final question (“Where will the two trains meet one another?”), to check their answers to the third question (the trains’ meeting time). Therefore, answering the fourth question constitutes sub-activity 5. I discuss the details of this when I attend to the final stage of judgement of sub-notion 2.

Now that the four questions have been unpacked, I turn to the development of judgement of sub-notion 2 in order to describe how evaluation functions in this part of the activity.

Existence

At the start of the judgement of sub-notion 2 students are faced with the list of privileged questions. Thus these questions, which were the MR for sub-notion 1, become the RMR for sub-notion 2 and existence for sub-notion 2 is judged at the same time as sufficiency for sub-notion 1.

For a student who knows how to solve the privileged questions, there is no impossibility or negation thereof for sub-notion 2. Impossibility is experienced only by students who do not know how to solve the privileged questions, in which case the teacher intervenes, negating the impossibility.

Reflection

There is no consideration of various possible predicates (that is, answers to the privileged questions) in the implemented *Trains Activity*, since there is only one correct answer for each question.

Necessity

Since there is no reflection, students also do not judge necessity for sub-notion 2. Instead, students answer the privileged questions (sub-activity 4) without first establishing a need for doing so – other than to follow an instruction from Teacher 1 just after the privileged questions are announced: “You can start” (line 4k). Thus students implicitly accept necessity established outside of and prior to the implementation of *The Trains Activity* and reproduce that necessity by answering the questions. I propose that this necessity is located in Polya’s heuristic of considering a related but simpler problem when devising a plan.

The students’ answer for the meeting time of the two trains (08h51) is the MR, since it is the culmination of sub-activity 4, indexing necessity of this sub-notion. That the MR and part of the RMR (the third question: “When will the two trains meet one another?”) are tautological supports my analysis that necessity for sub-notion 2 is not judged during the activity. If a student answers the questions correctly, s/he demonstrates that she does not disrupt the

tautology. In the event that the student does not answer the questions correctly, s/he disrupts the tautology and is taken back to the stage of existence where the impossibility experienced by the student can be negated, usually through assistance from a teacher or a peer.

Notion

In the final stage of judgement for sub-notion 2, students establish sufficiency of the MR by checking their answers for the trains' meeting time (sub-activity 5). However, the teacher explicitly tells the students both to check and how they are to check. He specifies that they should not "redo" their calculations but should instead "work backwards" (line 22w). He explains what he means by "work backwards": calculate the position of each train at their previously determined meeting time, the same position for each train confirming that the answer for the meeting time is correct (see lines 22y and 22aa). This checking process encourages students to make sense of the problem by making connections between their different calculations. Notice that a student's judgement of whether his/her answer (trains' meeting time) is sufficient is contingent on the checking procedure used, as indicated by the teacher in line 22w: "One way to check is to redo your calculations ... That will tell you whether you made a calculation error. But it won't tell you whether you made a logic or a method error."

5.4.3 Sub-notion 3: Creating an algebraic formula to calculate values about a general instance of a given scenario

My analysis of sub-notion 3 echoes that of sub-notion 2: reflection and necessity were not judged in the activity. In this section, I present a detailed description of the development of sub-notion 3 which provides evidence for this finding.

Existence

At the start of the judgement of sub-notion 3, Teacher 1 addresses the whole class (interaction 25), asking students to imagine that they are "the railway manager" who needs to know the meeting time of many pairs of trains (line 25e₅). The teacher suggests that it is impossible for "the railway manager" to calculate these meeting times for all pairs of trains in a normal work day: "you'll be happy if you leave work before ten o'clock in the evening doing these calculations for thirty trains" (line 25e₇). However, the teacher negates the impossibility by telling the students to "make yourself a tool that will make it quick and easy" to calculate these meeting times (line 25e₈) rather than the students negating impossibility for themselves.

That the teacher uses the word “tool” and not “formula” leaves the students to determine for themselves what constitutes a tool. This suggests an aspect of sense-making: students are to link the real-life scenario (“tool”) to an appropriate mathematical object (an algebraic formula). This description of an unspecified “tool” for the “railway manager” is the RMR, since it what students are presented with in place of the (sub-)notion at the outset of the judgement.

Reflection

For this sub-notion, there is no judgement of reflection in the implemented *Trains Activity*. Students go straight to creating an algebraic formula (sub-activity 6) in response to the teacher’s request that they make a “tool”. They do not consider any other predicates, even though alternatives are conceivable: look-up tables, a computer programme, graphical models etc.

Necessity

A consequence of the lack of reflection is that there is no judgement of necessity (in the activity) of a formula being the appropriate predicate for sub-notion 3. Yet students interpret (unchallenged by the teacher) the “tool” as being an algebraic formula and reproduce its necessity by creating one (sub-activity 6). I propose that this necessity can be traced back to students’ prior experiences. Firstly, students created formulae in an earlier activity of the course and the teachers intend for that activity “to introduce students to the idea that it can be useful to ... make a formula” (Course materials: Notes to teachers). Secondly, algebraic solutions have probably been privileged in students’ prior experiences of mathematics classrooms.

The teachers explicitly tell students what to do by specifying that they use their solution to the third question (“When will the two trains meet one another?”) to guide their creation of a “tool”. This occurs in numerous one-on-one interactions. For example, Teacher 3 says to Student 4: “you are supposed to follow the same procedure” (line 27cu). This relationship between the third question and the general problem is further evidence that the specific problem is to act as a simpler but related problem which students draw on when solving the general problem. However, this relationship is not mentioned to students.

After creating a formula, students “simplify” it (sub-activity 7). Many students do this of their own accord. Others only do so after a teacher tells them to: for example, Teacher 1 says to a student (not in the focus group): “Maybe you can take out some common factors or something and simplify that formula a bit”. Simplifying the formula (sub-activity 7) eases the process of substitution into the formula, thus enabling its subsequent checking (sub-activity 8). This checking constitutes the final stage of judgement of this sub-notion.

Notion

In the final stage of judgement, the sufficiency of the student's formula is judged on the basis of whether it correctly calculates any trains' meeting time using (only) the given quantities as inputs.

The students themselves judge whether their formula gives the correct meeting time by substituting the original values for speed, arrival times and distance and comparing the answer given by their formula with that calculated earlier (sub-activity 8). Some students check of their own accord, but many only do so after a teacher tells them to (and usually how to). See Extract 5.2, for example. This method of checking encourages sense-making by highlighting the connection between the formula and previous calculations. In Extract 5.2, notice that the teacher further tells the student what to do by insisting that she do the check in her presence: she says "I just want to see it" (line 27du).

Extract 5.2

- 27ds Teacher 3 ... can you use this formula [points to the student's formula for meeting time of the trains] ...
- 27dt Student 4 Yes.
- 27du Teacher 3 I just want to see it. If you get the same answer. It has to work, *akere* (right)? If it doesn't then it means something is wrong.

In addition to the student's checking, the teacher judges the student's formula as sufficient by inspecting whether it requires only the given information (speed, arrival time and distance values) as inputs. This criterion for sufficiency appears to stem from two sources: (1) the desired efficiency of the "railway manager" alluded to in the stage of existence and (2) algebraic facility often required in the problem-solving process. Teachers explicitly tell individual students (whose formulae are thus deemed insufficient) to refine their formula (sub-activity 9). For example, Teacher 1 says to a student (not in the focus group): "make another formula... that you don't first have to calculate the departure times?"

5.4.4 The notion: Problem solving

As discussed previously, the notion of problem solving privileged in *The Trains Activity* is like the problem-solving process described by Polya (1957). Judgement of problem solving is supported by the development of the three sub-notions, and takes place at the same time as their judgement. Consequently, I make reference to my analysis of the sub-notions in discussing the progression of judgement of problem solving.

Existence

The existence of the notion (problem solving) is judged during the first stage of judgement of sub-notion 1, when the students are presented with the written description of the trains (Figure 5.1) and they read it (sub-activity 1). Recall

(from Section 4.6.1) that this description is the RMR for problem solving. Reading the text makes students aware that a problem exists and by implication, a process for solving the problem exists. As for sub-notion 1, impossibility is contained in the students' experience that there is (initially) nothing to do and is negated by the teacher telling students to draw a diagram and that they will propose questions.

Reflection

There is no stage of reflection for the notion of problem solving in the implemented *Trains Activity*, since there is no consideration of various predicates. Reflection would require students to determine a general formula for the meeting time of the trains in a variety of different ways and compare these different methods. One of the teachers states: "it would be good if students solved it (the general trains' problem) with many different approaches, so that they can then compare the merits of the different approaches" (personal communication, 2012). However, there is no evidence of this taking place in the activity.

Necessity

For the notion of problem solving, there is no judgement of necessity in the activity since there is no reflection. Instead, students follow an adapted version of Polya's problem-solving process twice (see Section 5.3). In so doing, the students reproduce the necessity of this process – necessity established by the teachers prior to the activity. However, students are not told that they are following a particular problem-solving process.

The majority of judgement of sub-notion 1 together with the judgements of sub-notions 2 and 3 constitutes the reproduction of necessity of the privileged problem-solving process. This reproduction has therefore already been presented in my detailed description of the progression of judgement of each sub-notion. Recall (from Section 4.6.1) that the MR for this notion is the formula which the student (re)produces.

Notion

The final stage of judgement of the notion of problem solving is absent from *The Trains Activity*. Teachers do not judge whether the notion of problem solving which students have acquired is a sufficient reproduction of the privileged problem-solving process. Furthermore, students do not consider the adequacy of the problem-solving process they've used. To judge the sufficiency of the problem-solving process acquired by students, teachers would need to give students another problem to solve and then assess the process students use to solve it. Sufficiency would be judged against the problem-solving process privileged by the teachers. It is possible, however, that the final stage of judgement of the notion takes place in the course since students who complete *The Trains Activity* are given another (final) problem to solve.

5.5 Summary

In this chapter I have presented my analysis of the workings of pedagogic judgement in the implemented *Trains Activity*. The notions of co-variation, problem solving and sense-making constitute the privileged knowledge, although none of these notions are mentioned explicitly to students. Problem solving is central and is used as a vehicle to develop sense-making while co-variation is encoded in the problem, remaining implicit.

I focused on the central notion of problem solving, likening the notion of problem solving developed in the activity to an adaptation of the problem-solving process described by Polya (1957). I found that communication of this notion relied on students' implicit acceptance of necessity established elsewhere by the teachers. In addition, I found that there was no final stage of judgement of the notion of problem solving in the implemented *Trains Activity*.

I identified three sub-notions, which were judged in service of (and at the same time as) the notion of problem solving: (1) asking appropriate questions about a given scenario; (2) calculating specific values about a specific instance of a given scenario; and (3) creating an algebraic formula to calculate values about the general case of a given scenario. Only judgement of sub-notion 1 progressed through all four Hegelian stages. This supports my claim that the first sub-notion is primarily concerned with the student making sense of the problem and constituting Polya's first problem-solving phase (understand the problem). However, students do not judge necessity of sub-notion 1: teachers judge necessity and communicate it to students in a whole class discussion. For the notion of problem solving itself and sub-notions 2 and 3, there is no judgement of reflection or necessity. Instead, students accept necessity established elsewhere, and reproduce this necessity. While sense-making is promoted throughout the judgement of sub-notions 2 and 3, the focus of these sub-notions is to lead students to complete two cycles of the privileged problem-solving process.

Lastly, I found that the teachers' intention to "refrain from ... telling students 'what to do'" (Human et al., 2010:3) plays out in the implemented pedagogy in various respects: by not telling students the phases or the heuristics of the problem-solving process, the answers to the privileged questions, or the aims of the course. However, in many ways, they do tell students what to do. This "telling" varies in the degree to which it is implicit/explicit. During judgement of sub-notion 1, the teachers explicitly tell students to draw a diagram and pose questions. However, students appear to be given the opportunity to investigate and discover for themselves in that they pose questions about the trains. Yet the teachers implicitly "tell" students what to do in that they have predetermined what the privileged questions will be. The "telling" continues to be implicit at the start of the second sub-notion: students answer these predetermined questions in a predetermined order. However, at the end of sub-notion 2 and during sub-notion 3, the "telling" becomes more explicit in that the teacher instructs students to check their answers (and formulae) and

stipulates how to do so, and he tells students to create their formulae by following the same method used to solve the specific questions. In addition to this, the teachers implicitly “tell” students what to do by modelling the privileged problem-solving process in the set sequence of sub-activities, and in using sub-notions to communicate the notion of problem solving.

In the next chapter, I present the second part of my analysis in which I attend to the positioning of the teacher and student with respect to the privileged knowledge. This second part of my analysis gives additional insight into how the pedagogy communicates knowledge.

Chapter 6: Positioning

6.1 Introduction

In this chapter, I present a detailed description of the positioning of teachers and students with respect to knowledge in the implemented *Trains Activity*, based on my analysis of the distribution of transmission function (TF) and acquisition function (AF). Recall that distribution of TF indicates being positioned as knowledgeable and AF indicates being positioned as ignorant.

Given the constraints of this dissertation, I have not included the entire analysis here but have provided it in Appendix 6. Recall (from Chapter 5) that it is only in sub-notion 1 that all four stages of judgement are present in the activity. For sub-notions 2 and 3, there is no judgement of reflection or necessity in the activity but, rather, students reproduce necessity which has been established elsewhere. In my analysis, I found that the positioning in sub-notions 2 and 3 was quite similar. Consequently, I present here results of the positioning for sub-notions 1 and 2 only.

6.2 Sub-notion 1

In this section I describe the positioning of the student and teacher with respect to knowledge in the development of the first sub-notion in the implemented *Trains Activity* (asking appropriate questions about a given scenario). Recall that the privileged questions constitute the MR and the written description of the problem situation (see Figure 5.1) the RMR.

Existence

In the stage of existence for sub-notion 1, the teacher is distributed TF throughout and the student is alternately distributed TF and AF. This is summarised in Table 6.1 and elaborated below.

Table 6.1: Summary of the positioning of the pedagogic subjects during the judgement of existence of sub-notion 1 (asking appropriate questions about a given scenario)

	Interaction number	Speech turns	Transmission function					Acquisition function				
			T-T	T-O	S-S	S-O	O	S-S	S-O	T-T	T-O	O
Sub-Activity 1 (students read problem description)	1 ₁	1a	✓					✓				
	1 ₂	1b	✓		✓							
	2 ₁	2a ₁	✓					✓				
Sub-activity 2 (students create graphical representation)	2 ₂	2a ₂	✓		✓							
	2 ₃	2a ₃	✓		✓			✓			✓ (student)	

At the start of the activity, while students read the written description (sub-activity 1), one student appeals to Teacher 1 for assistance. Their conversation (Extract 6.1) demonstrates the positioning of both teacher and student at the beginning of the stage of existence. TF is distributed to the teacher because he is asked for assistance (#4) (line 1a) and he is instructing the students to ask questions (#6a) (line 1b). The student is initially distributed AF when she asks about the lack of questions (#4) (line 1a), this lack creating the impossibility characteristic of the stage of existence. However, the student is subsequently distributed TF when impossibility is negated by the teacher telling students that they will propose questions (#6a) and describing the students as knowledgeable about what questions to propose (#2a): “You are the boss” (line 1b).

Extract 6.1

- 1a Student What is the question here? [spoken quietly to the teacher only]
- 1b Teacher 1 You are the boss; there are no questions in front of you; you are the boss, you are the one that asks the questions. There is only what is. You have to decide what I want to investigate. What is interesting? Is there a problem? [spoken loudly to the whole class]

Further negation of impossibility occurs when students draw a diagram to represent the situation (sub-activity 2). The teacher tells them to do this (Extract 6.2, line 2a₃). Distribution of TF to the teacher is indicated in Extract 6.2 by his instructing students to draw a diagram (#6a), his selecting what student productions to make public (mentioning only students who had made a diagram and not those who had not in line 2a₂) (#7) and his explaining what the students should include in their diagram (line 2a₃) (#3). The student is also distributed TF throughout Extract 6.2, because s/he is described as having already drawn a diagram (#2a) (line 2a₂) and is instructed to draw such a diagram (#6a). However, at the end of Extract 6.2 both student and teacher are also simultaneously distributed AF (and TF). Distribution of AF to teacher as other (student) is suggested by the teacher speaking with the voice of a student while saying that the students “may get confused of ‘what am I doing now?’” (#2b). Distribution of AF to the student is suggested by the teacher’s justification for drawing a diagram which implicitly describes the students as ignorant (#2b): “it is not always easy to understand” and “you may get confused” (line 2a₃).

Extract 6.2

- 2a₂ Teacher 1 (addresses the whole class) And I see most of you have actually made some kind of drawing to visualise what is going on here 'cause this is a word problem.
- 2a₃ Teacher 1 It is not always so easy to understand the words. But once you've made yourself a picture of what is going on here, what is this distance? And what is this time? And where's this train and where is that train? So if you haven't made such a picture yet, make yourself a picture, it will help you. 'Cause later on, when we do have a question, things will get complicated and you may get confused of "what am I doing now?" And then you can always go back to your picture ...

Reflection

In the stage of reflection for sub-notion 1, the teacher is alternately distributed TF and AF, while the student is distributed TF throughout. This is summarised in Table 6.2.

Table 6.2: Summary of the positioning of the pedagogic subjects during the judgement of reflection of sub-notion 1 (asking appropriate questions about a given scenario)

	Interaction number	Speech turns	Transmission function					Acquisition function				
			T-T	T-O	S-S	S-O	O	S-S	S-O	T-T	T-O	O
Sub-activity 3 (students pose questions)	3 ₁	3a ₁	✓		✓							
	3 ₂	3a ₂			✓			✓			✓ (student)	
	3 ₃	3a ₃	✓		✓							

Possibility is opened up through group discussions where the students pose questions (predicates) about the given scenario, as instructed by the teacher (**#6a**) (line 3a₁ and 3a₃ of Extract 6.3). Thus the student and teacher are both initially distributed TF. Continued distribution of TF to the student is indicated by the student being hypothetically asked for assistance (**#4**). However, at the same time, the student and teacher (as student) are also distributed AF since the teacher takes on the voice of a student whilst asking for assistance (**#4**) and describing himself as aligned with ignorance ("if he tells me something that I don't understand...") (**#2b**).

Extract 6.3

- 3a₁ Teacher 1 ... as a group I want you to come up with questions. Are there some important or interesting questions that you can make about this situation? ...
- 3a₂ Teacher 1 ... if he tells me something that I don't understand, then I ask him "show me on your drawing ... help me to understand what you are meaning".
- 3a₃ Teacher 1 But please, turn over any calculations or formulas that you have made. That's not the purpose of the group discussion...

Necessity

In the stage of necessity for sub-notion 1, questions (possible predicates) posed by students in the preceding group discussion are discussed in a whole class setting (interaction 4), where the teacher inserts a simplifying assumption and subsequently discards some questions and fixes others. The teacher is distributed TF throughout and often simultaneously distributed AF. The student, although initially distributed TF, is mostly distributed AF. This positioning is summarised in Table 6.3.

Table 6.3: Summary of the positioning of the pedagogic subjects during the judgement of necessity of sub-notion 1 (asking appropriate questions about a given scenario)

	Interaction number	Speech turns	Transmission function					Acquisition function				
			T-T	T-O	S-S	S-O	O	S-S	S-O	T-T	T-O	O
Sub-activity 3 continued (students pose questions)	4 ₁	4a ₁	✓		✓							
	4 ₂	4a ₂	✓					✓		✓		
	4 ₃	4a ₃		✓ (as engineer)				✓			✓ (as engineer)	
	4 ₄	4a ₄	✓					✓		✓		

The teacher judges necessity of the questions (predicates) posed by students and is distributed TF when doing so. Specifically, he selects what questions to make public (#7): “Ok, I have written most of the questions that I could remember you talking about... on the board...” (line 4a₁). The student is also distributed TF at the start of the whole class discussion, indicated by the teacher describing the students as knowledgeable about this problem (#1a): “I think you all understand what’s going on” (line 4a₁).

For the remainder of the whole class discussion, the student is distributed AF due to listening to explanations from the teacher (#3). First, the teacher explains the simplifying assumption (constant speed) (#3):

4a₂ Teacher 1 Firstly, I I want to make a simplifying assumption. I want to say: let us assume that the trains travel at a constant speed [writes this on the board]. Why am I making that assumption? Because it’s easier for me to calculate things.

This indicates distribution of TF to the teacher (#3) and simultaneous distribution of AF the teacher, since he implicitly describes himself as ignorant: “it’s easier for me to calculate things” (#2b).

Partway through his explanation, the teacher speaks as if he were an engineer when justifying the simplifying assumption’s by the way engineers work:

4a₃ Teacher 1 ... this is something that engineers and scientists do – they say “let us first look at this problem in a simple way. And see if we can solve that. And once we’ve solved that we can make it more complicated ... But if I cannot solve a problem for the simple case, I will also not be able to solve it for the difficult case.” ...

Thus the positioning of the teacher changes to being distributed TF and AF (simultaneously) as something other (engineer). Distribution of TF is because the teacher explains the need for the simplifying assumption (#3) and AF because needing the simplifying assumption implies that the teacher (as engineer) is ignorant of how to solve the more complicated problem (#2b).

At the end of this stage of judgement, TF and AF are once again distributed to the teacher as teacher when he discards some questions (predicates). He says, “So I’m simplifying matters ... now it’s easier to do calculations... just treat it as a constant speed. So that that deals actually with all three of these questions” (line 4a4). Distribution of TF is because he is still explaining (#3), and AF, because he is still implicitly describing himself as ignorant: “now it’s easier to do calculations” (#2b).

Recall that two privileged questions are left on the board at the end of the whole class discussion: “How long will it take for each train to travel from the one town to the other?” and “At what time should each train depart?”

Notion

In the final stage of judgement for sub-notion 1, the teacher is distributed TF throughout, although this alternates between distribution as teacher and as other. At one point, the teacher is simultaneously distributed AF and TF. The student is both alternately and simultaneously distributed TF and AF. This is summarised in Table 6.4.

Table 6.4: Summary of the positioning of the pedagogic subjects during the final stage of judgement of sub-notion 1 (asking appropriate questions about a given scenario)

	Interaction number	Speech turns	Transmission function					Acquisition function				
			T-T	T-O	S-S	S-O	O	S-S	S-O	T-T	T-O	O
Sub-activity 3 continued (students pose questions)	4 ₅	4a ₅	✓		✓			✓				
	4 ₆	4a ₆ – 4b		✓ (as train)				✓				
	4 ₇	4c ₁	✓					✓				
	4 ₈	4c ₂	✓					✓		✓		
	4 ₉	4c ₃ – 4d		✓ (as train)				✓				
	4 ₁₀	4e – 4f	✓		✓							
	4 ₁₁	4g ₁		✓ (as train)	✓							
	4 ₁₂	4g ₂ – 4m	✓		✓			✓				

The final stage of judgement for sub-notion 1 takes place at the end of the same whole class discussion as the stage of necessity (interaction 4). Judgement shifts from necessity to notion when the teacher judges the students' questions (predicates) as insufficient, suggested by his statement: "I have a certain question in mind and I also thought of the first two questions but I want you to think of another question" (line 4a₅). This statement indicates distribution of TF to the teacher and AF to the student, since it suggests insufficiency of the students' questions and so is an unfavourable assessment thereof (#1b). At the same time, the students are also distributed TF, since favourable assessment is implied by the teacher saying he thought of the first two questions (#1a).

The teacher then explains and demonstrates (with a student) the movement of the trains (Extract 6.4) (#3), speaking as if he were the train (line 4a₆). He is therefore distributed TF as other (a train). The student, however, is distributed AF (only) during this explanation, indicated by listening to the explanation (#3). Furthermore, the teacher makes an explicit unfavourable assessment of a student's statement: "You don't know that" (line 4c₁) (#1b).

Extract 6.4

- 4a₆ Teacher 1 ... We need to do a demonstration. We're going to be the two different trains ...
But before we depart ... do you know that we will depart at the same time?
- 4b Students (Student responses are inaudible.)
- 4c₁ Teacher 1 You don't know that. Maybe it will be like that.
- 4c₂ Teacher 1 We can check by calculating but we don't know it.

The student's positioning shifts again: after listening to the explanation, the student is distributed TF when the teacher asks the students "Is there something interesting that we could find out?" (line 4e). This question serves as an instruction to the students to pose the final question (#6a). The teacher continues to be distributed TF, since he gives the instruction (#6a).

At the end of this final stage of judgement, a final question (which is actually two questions) is added which renders the list of questions sufficient:

- 4g₂ Teacher 1 Now those first two questions are also important. But I want to add the third one... At what time and at what position will the trains meet each other?

The students are again distributed both AF and TF, while the teacher continues to be distributed TF (only). This is indicated similarly to the positioning at beginning of this stage of judgement: the teacher is simultaneously favourably and unfavourably assessing the two questions which students have already posed (#1a & b).

6.3 Sub-notion 2

In this section I describe the positioning of pedagogic subjects with respect to knowledge in the development of sub-notion 2 (calculating specific values about a specific instance of a given scenario). Recall that, for this sub-notion, the MR is the time at which the two trains (in Figure 5.1) meet, and the RMR the list of questions remaining on the board at the end of judgement of sub-notion 1 (see Figure 5.4).

Existence

Existence for sub-notion 2 is judged at the same time as the final stage of judgement for sub-notion 1 (see Section 5.4.2). Thus my analysis of the positioning of pedagogic subjects during this stage of judgement has already been presented (Section 6.2).

Necessity

Recall (from Section 5.4.2) that there is no judgement of reflection or necessity of sub-notion 2 during the implemented *Trains Activity*. Instead, students reproduce necessity established elsewhere by answering the privileged questions (sub-activity 4). This occupies most of the time of the implemented *Trains Activity*. The constraints of this dissertation prevent me from presenting my detailed analysis of all focus-group interactions pertaining to this stage of judgement. However, teachers interacted similarly with all students in the focus group. Consequently, I provide a brief summary of the positioning in this stage of judgement, followed by the detail of selected interactions. This selection follows a single student, Student 4 (S4) for continuity and is intended to cover the range of ways in which pedagogic subjects were positioned.

The positioning in the stage of necessity for sub-notion 2 can be summarized as follows:

- a) The teacher is distributed TF throughout;
- b) The student is also almost always distributed TF;
- c) The student is often simultaneously distributed AF (and TF);
- d) There are only a small number of instances of the student being distributed AF only.

The detail of my analysis, which follows, supports these findings.

Individual interactions

The positioning of the student and teacher in two one-on-one interactions pertaining to the reproduction of necessity is summarised in Table 6.5 and the details provided below.

Table 6.5: Summary of the positioning of the pedagogic subjects during one-on-one interactions pertaining to the reproduction of necessity for sub-notion 2 (calculating specific values about a specific instance of a given scenario)

	Interaction number	Speech turns	Transmission function					Acquisition function				
			T-T	T-O	S-S	S-O	O	S-S	S-O	T-T	T-O	O
Sub-Activity 4 (Students solve the privileged questions)	5	5	✓					✓				
	15	15	✓		✓			✓				

Interaction 5 (see Appendix 6) takes place about ten minutes after the students start solving the privileged questions, and is preceded by Teacher 3 approaching Student 4 and silently reading her written work. The teacher is distributed TF and the student AF in this interaction. This is indicated by the teacher explaining how to do the calculation (lines 5c and 5e) (#3) which also suggests unfavourable assessment of the student’s solution to the first privileged question (#1b). Also, the teacher instructs the student to revise her answer: “Why don’t you go back to question 1 and try to do it again?” (line 5g) (#6b). The distribution of TF to the teacher is further indicated by the teacher posing questions to the student, such as “what are you calculating?” (5c) (#5).

Interaction 15 (see Appendix 6) takes place at the start of the following day when Teacher 1 approaches Student 4 and discusses her written attempts at determining the meeting time of the trains. Much of the conversation is about the acceleration of the trains. The student acknowledges (line 15h) that the trains have zero acceleration, yet she also says: “I know the initial speed of the train but I don’t know the final” (line 15l). The teacher is distributed TF and the student AF because the teacher unfavourably assesses the student’s work, suggested by his saying that the student should not use standard kinematics formulae (#1b): “The formula is not going to tell you that (Δt)” (line 15as). Also, the teacher explains to the student (#3), for example: “In this situation, speed at the start and speed at the end are the same because speed is constant so ... you do not need a formula to calculate speed at the end” (line 15au).

Throughout interaction 15, the student is also simultaneously distributed TF, indicated by the teacher implicitly describing the student as knowledgeable by acknowledging the validity of the kinematics formula the student used: “it’s not that the formula is not true. It’s just that formula is not helping you” (line 15au) (#2a). Furthermore, the teacher gives instructions which align the student with knowledge such as: “You have to ask yourself” (line 15as) (#6a) and the student wants to assess her own answers: “I want to check that” (line 15al) (#1c).

Whole class discussion

Almost immediately after interaction 15, Teacher 1 addresses the whole class (interaction 16, see Appendix 6), instructing them to discuss, in groups, their plans for answering the questions. However, he tells students who have

already determined the trains' meeting time not to participate in a group discussion. The positioning in interaction 16 is summarised in Table 6.6.

Table 6.6: Summary of the positioning of the pedagogic subjects during a whole class address pertaining to the reproduction of necessity for sub-notion 2 (calculating specific values about a specific instance of a given scenario)

	Interaction number	Speech turns	Transmission function					Acquisition function				
			T-T	T-O	S-S	S-O	O	S-S	S-O	T-T	T-O	O
Sub-Activity 4 (Students solve the privileged questions)	16 ₁	16a ₁	✓		✓							
	16 ₂	16a ₂	✓					✓				
	16 ₃	16a ₃	✓		✓							
	16 ₄	16a ₄		✓ (as student)	✓			✓			✓ (as student)	
	16 ₅	16a ₅	✓		✓			✓				
	16 ₆	16a ₆		✓ (as student)	✓			✓			✓ (as student)	
	16 ₇	16a ₇	✓		✓			✓				

The student is distributed both TF and AF in this whole class address. Distribution of TF is evidenced by the teacher instructing the students to make plans and to explain these plans to their peers (**#6a**) and describing the students as knowledgeable: “You can calculate...” (line 16a₅) (**#2a**). Distribution of AF to the student is suggested by the teacher instructing the students not to say any numbers, implying that they would otherwise have said numbers and so are ignorant with regards to how to make a plan for solving the question (**#6b**). The students are also explicitly described as ignorant when the teacher says “you forgot what it (a number previously calculated) meant” (line 16a₅) (**#2b**).

The teacher is distributed TF throughout this whole class address, since he explains what he means by “making a plan” (line 16a₄) (**#3**) and he gives the students instructions about what to do (tell their peers their “plans”) and what not to do (say any numbers) in the group discussions (**#6**). During parts of the whole class address, the teacher is distributed AF and TF as other (student), since he speaks as if he were a student. For example, the teacher asks himself a question as if he were a student: “How will I look at this problem?” (line 16a₄). This question indicates simultaneous distribution of TF and AF to the teacher (as student), since he is simultaneously asking for and being asked for assistance (**#4**).

TF is distributed to each student at some point in the group discussion (see Table 6.7). Distribution of TF to S2 is shown in Extract 6.5: the teacher instructs (in line 17ab₂) S2 to explain something which she said previously about two points on the group's diagram (#6a). Furthermore, S2 follows the instruction and explains (line 17ac) to her peers the significance of the two points (#3). In contrast, the rest of the students are distributed AF since they listen to her explanation (#3).

Extract 6.5

- 17ab₂ Teacher 1 Will you [points to S2] now explain again what you said about that and that [points to A₁ and A₂B₂, and then B₁ and A₂B₂] and then I just want to check that everybody agrees with that [pause]. Or maybe there is a problem with it.
- 17ac Student 2 Ok. So. The change in time that it takes for A to get to that point [points to A₁ and then A₂B₂] will be equal to the change in time that it takes for B to get to that point [points to B₁ and then A₂B₂]. So then you can say that the distance over the speed of train 1

Indication of distribution of TF to the other students in the group discussion is by favourable assessment of something which s/he says (#1a). Consider Extract 6.6, for example. S3 is distributed TF because the teacher favourably assesses (in 17af) S3's explanation given in line 17ae. Distribution of TF to S3 is also suggested by his explaining to his peers the reason for the two changes in time being equal (line 17ae) (#3). The other students are again distributed AF because they are listening to the explanation (#3).

Extract 6.6

- 17ad Teacher 1 ... why is that change in time for train A from there to there [points to A₁ and A₂B₂, see Figure 6.1] the same as the change in time for train B from there to there [points to B₁ and A₂B₂]?
- 17ae Student 3 'Cause the question was: ... when do they meet? So therefore the time you take to get there it might not be, oh well, from this point [points to A₁ and B₁]. That's why we had to move this guy here first [points from A₀ to A₁]. Because the time he [points to A₀] took to get there [points to A₂B₂] was not the same time as he [points to B₁] took to get there [points to A₂B₂].
- 17af Teacher 1 Oh, 'cause they started at different times. Oh I see.

The teacher continues to be distributed TF, indicated by his posing questions about the equality of two travelling times in 17ad (#5) and by his indicating favourable assessment in line 17af (#1a). However, AF is also distributed to the teacher when he feigns newfound understanding (following the student's explanation) and implies that he previously was ignorant (#2b) "Oh ... Oh I see" (line 17af).

Notion

Sufficiency of the MR for sub-notion 2 is established through students checking their answer for the meeting time of the two trains (sub-activity 5). I present here my analysis of the whole class address (interaction 25) in which the teacher tells students to check their answers and how to do so. The positioning in this interaction is summarised in Table 6.8 and then elaborated in what follows.

Table 6.8: Summary of the positioning during the final stage of judgement of sub-notion 2 (calculating specific values about a specific instance of a given scenario)

	Interaction number	Speech turns	Transmission function					Acquisition function				
			T-T	T-O	S-S	S-O	O	S-S	S-O	T-T	T-O	O
Sub-activity 5 (Students check answers)	25 ₁	25a – 25e ₁	✓		✓							
	25 ₂	25e ₂	✓		✓			✓				
	25 ₃	25e ₃	✓			✓ (engineer)			✓ (engineer)			
	25 ₄	25e ₄		✓ (student)	✓			✓			✓ (student)	

Consider Extract 6.7, part of the aforementioned whole class address. Distribution of TF to the teacher throughout is evidenced in a number of ways: the teacher poses questions (line 25e₁) (**#5**); he hypothetically withholds assessment in his indicating that the students are to check their own answers (line 25e₁, 25e₂ and 25e₃) (**#1c**); he also hypothetically assesses the student’s method of checking when he says “it’s good to check your calculations but it’s not enough” (line 25e₂) (**#1a** and **#1b**); he explains content (line 25e₄) (**#3**); and lastly, he gives instructions (**#6a**). Furthermore, in line 25e₄, TF is distributed to the teacher as other, since the teacher speaks as if he were a student.

Extract 6.7

- 25e₁ Teacher 1 ... typically once you do get an answer, we’ll come to you and we’ll ask you: have you checked your answer? How have you checked it? Have you redone your calculations? Yes. And did you get the same answer? Yes. So what does that tell you? It tells you you didn’t make a calculation error. But that’s all it tells you.
- 25e₂ Teacher 1 If there was a error in your method or your logic your reasoning and you just redid the same calculations, you will get the same answer. So it’s good to check your calculations, but it’s not enough.
- 25e₃ Teacher 1 You design that aeroplane wing, you checked your calculations, but there was something wrong – some assumption that you made that was wrong – and now the aeroplane ... goes down in smoke and flames. Ok. You also need to check it in a different way. ...
- 25e₄ Teacher 1 Often it’s very useful to work backwards. To say: “no, no, let me work back from my answer and see if I can work back to the original information. Let me work from my answer and then pretend that some of the original information I don’t know. Maybe the total distance or whatever. Now let me work from my answer backwards and see: do I get the ... two hundred and thirty six (236) kilometres? If I get two hundred and fifty (250) kilometres, then my answer couldn’t have been right.” ...

The student is also distributed TF throughout interaction 25, indicated by the students being expected to assess their own productions (line 25e₁) (**#1c**); described as aligned with knowledge (the teacher implies that they would “get an answer” in line 25e₁) (**#2a**); hypothetically having their productions favourably assessed (**#1a**) (“it’s good to check your calculations,” line 25e₂); and being instructed to check their work (line 25e₃ and 25e₄) (**#6a**). For part of the whole class address (line 25e₃), the student is distributed TF as other (engineer), since the teacher speaks about the students as if they were engineers who have designed an aeroplane wing.

For most of the interaction the student is simultaneously distributed AF (and TF). This is indicated by the students hypothetically having their method of checking unfavourably assessed (line 25e₂) (**#1b**); being described as aligned with ignorance (“there was something wrong...”) (**#2b**); and listening to explanation from the teacher (line 25e₄) (**#3**).

6.4 Summary

I summarise here the results of my analysis of the positioning of students and teachers with respect to knowledge of the notion, but leave my detailed discussion thereof for the next chapter. Firstly, my results show that the pedagogy of the course distributes both TF and AF to the student. This distribution varies depending on the stage of judgement and works in service of the judgement of the respective (sub-)notion. Secondly, I found that when AF is distributed to the student, it is predominantly as student (and not as other). Thus when the pedagogy reveals the student as ignorant, this is rarely masked. Finally, my analysis suggests that TF is distributed to the teacher throughout. However, this distribution of TF is often to the teacher as other and/or at the same time as the teacher being distributed AF.

Chapter 7: Discussion

7.1 Introduction

This study set out to describe in detail the implemented pedagogy of a mathematics support course for first-year engineering students with particular attention to what knowledge is privileged and how that knowledge is communicated to students. The course is a response to the problem of supporting students in the transition from school to university mathematics, a transition which many students find difficult. While support for students in this transition abounds, throughput rates for South African universities suggest that this support is not sufficiently effective. Furthermore, most research on support focuses on student perceptions and the impact on students' academic performance. Very little closely examines what knowledge the pedagogies implemented in support courses privilege and how that knowledge is communicated to students. Yet understanding support from this perspective is needed because it is a first step in (a) establishing whether support courses might assist students in accessing the knowledge valued in mainstream courses and (b) rendering support courses replicable. Furthermore, the need for understanding these aspects of the course in my study is highlighted by its use of a pedagogy which has been critiqued in other contexts.

I began this dissertation by describing the intended curriculum of the course. In summary, the course intends to develop students' understanding of previously encountered mathematical concepts (co-variation, in the case of *The Trains Activity*) and problem-solving skills while encouraging a sense-making disposition towards learning mathematics and solving problems. The intended pedagogy is set up as being in contrast to explicitly "telling students 'what to do'" (Human et al., 2010:3). Instead, students are to learn through solving realistic problems, investigating and discovering. I argued that the course's intended pedagogy is a competence pedagogy, similar to the problem-centred approach (PCA) promulgated in selected white South African primary schools in the 1990s.

The theoretical framework for this study is founded on Bernstein's (1996) theory of the pedagogic device, since it affords a language for speaking about the transformation of knowledge into pedagogic communication. In this theory, privileged knowledge is that which is legitimated in the pedagogic context by the operation of evaluation. The detail of my theoretical framework is adopted from Davis's (2001; 2005) investigation of PCA at the primary school level. Davis draws on Hegel's theory of judgement to flesh out the working of the evaluation. I used Davis's tools for describing pedagogic judgement and the positioning of students and teachers with respect to knowledge to analyse what knowledge is privileged in one activity of the course (*The Trains Activity*) and how the privileged knowledge is communicated to students in that activity.

This study is limited in that it examines only a single activity of one support course with a focus on the development of a single – albeit central – notion. Consequently, my study could be criticised for an apparent lack of generalisability. I contend, however, that my study demonstrates generalisability of the theories developed by Davis (2005). Furthermore, I counter that the strength of my study comes from the detail of the description I have generated with such a limited focus. In this concluding chapter, I discuss that detail in relation to the research question I posed in Chapter 1. I flag opportunities for further research as they arise throughout the chapter, and conclude with a discussion of the implications of my findings.

7.2 What knowledge is privileged in the course?

My analysis of pedagogic judgement in the implemented *Trains Activity* showed that the knowledge privileged in the course consisted of three closely intertwined notions – co-variation, problem solving and sense-making – none of which were explicitly mentioned to students. Co-variation was encoded in the problem in that the position of each train varies simultaneously with respect to time. This was not discussed with students. Problem solving was both a notion in its own right and a vehicle for developing sense-making. Sense-making was woven throughout the activity. My analysis focused on the central notion of problem solving, although application of my analytic tools to the other notions is conceivable, which suggests opportunity for further research. In my analysis of problem solving, I highlighted points at which sense-making was encouraged. This gave me an impression of how these teachers work to counter what they perceive as “procedural” thinking (as discussed in Section 1.3.2.4).

The notion of problem solving was communicated through the judgement of three sub-notions, namely:

1. asking appropriate questions about a given scenario;
2. calculating specific values about a specific instance of a given scenario;
3. creating an algebraic formula to calculate values about a general case of a given scenario.

I liken the problem solving privileged in the implemented *Trains Activity* to the problem-solving process articulated by Polya (1957). For Polya, problem solving is a process made up of four phases: understand the problem, make a plan, carry out the plan and look back. In *The Trains Activity*, sub-notions 1 and 2 form one cycle of the problem-solving process in which students solve a specific problem. Sub-notion 3 forms a second cycle of the problem-solving process in which students solve a generalised version of the specific problem. Both problems are simplified versions of a more complex problem. My analysis showed that the teachers insert the specific problem into *The Trains Activity* before the general problem to assist students with the latter.

7.3 How is knowledge communicated in the course?

Communication of the privileged knowledge occurred in complex and subtle ways, primarily in one-on-one interactions, where teachers guide students through a set sequence of sub-activities. This “instructional guidance” (Kirschner et al., 2006:75) took on various forms (discussed below), which I investigated from two perspectives: the development of pedagogic judgement and the positioning of pedagogic subjects with respect to knowledge.

7.3.1 Pedagogic judgement

In addition to revealing what knowledge was privileged in the implemented *Trains Activity* (see Section 7.2), my analysis of pedagogic judgement through the four stages (existence, reflection, necessity and notion) made visible some of the complexity of how the privileged knowledge was communicated. I focus here on three aspects of that complex communication.

Firstly, recall that the teachers intend to “refrain from ... telling students ‘what to do’” (Human et al., 2010:3). I found that they achieve this goal by not discussing with students the phases or heuristics of the problem-solving process, the answers to the privileged questions, or the aims of the course. However, in many ways, they do tell students what to do, but this telling varies in the degree to which it is implied. They explicitly tell students to draw a diagram, to pose questions, to check their answers/formulae, how to check etc. They implicitly “tell” students what to do by predetermining what questions the students will answer and in what order, by modelling the privileged problem-solving process in a set sequence of sub-activities and in subtly positioning students in various ways (see Section 7.3.2). However, this telling obscures the problem-solving process for students since they are not given the opportunity to grapple with the problem independently of the sub-activities set up by the teacher. Indeed, the teachers’ modelling of the problem-solving process during this final activity of the course suggests that they set up the process for students precisely because students had not yet acquired the notion, despite having completed numerous other course activities also intended to develop problem solving. Nevertheless, my study shows that the implemented pedagogy of the course, in contrast to Kirschner et al.’s criticism of constructivist pedagogy, provides students with much “instructional guidance” (Kirschner et al., 2006:75), although it varies in the degree to which it is “direct” (ibid.). This suggests that the implemented pedagogy contains aspects of the performance model of pedagogy, since some criteria for producing legitimate texts are made explicit.

A second aspect of pedagogic communication made visible by my analysis of pedagogic judgement was that the teachers judge necessity of the problem-solving process and its sub-notions – contrary to the expectation that the student, as autodidact, would do so. Davis argues (2001; 2005) that for a student to understand a notion, the student him/herself must demonstrate necessity. My results show that students did not have opportunity to do that in the implemented *Trains Activity*. In his analysis of PCA in South African primary schools, Davis (2005:208) found a

similar lack of student judgement of necessity. In my study, this lack of student judgement of necessity suggests that students may leave the course without understanding the notion of problem solving. Further research is required to establish if this is indeed the case.

Thirdly, there was no judgement of the sufficiency of the notion of problem solving in the implemented *Trains Activity* – by student or teacher. Consequently, the pedagogy does not reveal (to the student, teacher or researcher) whether the student actually acquired the notion of problem solving. Indeed, since it is not discussed with students, it is questionable whether the students even know that they should have acquired this notion. However, the absence of this final stage of judgement suggests that teachers do not provide “uncritical affirmation” of students’ productions – as Davis’s (2005:96) study of PCA at primary school level suggests might be the case. Nevertheless, the absence does suggest opportunity for further research: students could be given another problem to solve, similar to the general problem in *The Trains Activity*, and how they go about solving the problem recorded. However, it is conceivable that students’ acquisition of problem solving may not be visible in this next activity and that more extensive research is needed.

7.3.2 Positioning

During the development of pedagogic judgement, students and teachers were positioned in complex ways in relation to the privileged knowledge. My study has made visible how this positioning works in service of the judgement of the sub-notions, and in tandem with both the explicit and implicit guidance which teachers gave to students. The hypotheses presented in Section 3.4 guide my discussion of this positioning (below).

a) Positioning of the student

The student was positioned as both knowledgeable and ignorant

I found that the student was routinely positioned as both knowledgeable and ignorant of the privileged knowledge, sometimes simultaneously, sometimes alternately. Although this is a simplification of my results (the detail of which follows here), it was expected due to the pedagogy’s conception of the student as autodidact. Yet it contrasts with Davis’s (2005) finding that the student was primarily positioned as ignorant in his study of PCA at primary school level. He consequently concludes that the PCA conception of the student as autodidact is a “fiction” (2005:207). While my abovementioned result on the student positioning seems counter to this conclusion, my other findings do support it: for example, the teachers do not demand that the students judge necessity and they have particular ways of guiding students through a set sequence of sub-activities which constitute the problem-solving process, often explicitly telling students what to do (Section 7.3.1).

In the stage of existence in the judgement of all three sub-notions, the student was initially positioned as ignorant at the outset. Since the paradigmatic form of evaluation probably dominated students' previous pedagogic interactions (at school and university), the initial positioning of the student as ignorant is unsurprising. Furthermore, it is the student who experiences the impossibility characteristic of the stage of existence, and impossibility is due to ignorance of the notion. However, impossibility is also negated in the stage of existence, and this seems to work together with later positioning the student as knowledgeable.

Recall that reflection was judged only in the first sub-notion (asking appropriate questions about a given scenario). The student was positioned as knowledgeable throughout this stage, which worked together with this aspect of judgement – consideration of possible predicates. Students posed questions (possible predicate) which required them to draw on their knowledge, thus positioning the student as knowledgeable.

In the judgement of necessity for the first sub-notion, the student was briefly positioned as knowledgeable at the outset after which the student was positioned as ignorant. This worked together with the teacher (rather than the student) judging the necessity of the questions and the teacher trying to convince the students to agree with this judgement.

During the reproduction of necessity for the other two sub-notions (calculating specific values about a specific instance of a given scenario, and creating an algebraic formula to calculate values about a general case of a given scenario), the student was positioned as both knowledgeable and ignorant throughout. I suggest that this positioning stems from the teachers treating the students as if they had judged necessity for themselves (hence knowledgeable), even though it is the teachers who have judged the necessity prior to implementing the activity (hence ignorant).

In the final stage of judgement, the student was positioned as both knowledgeable and ignorant for all three sub-notions. This positioning worked together with the manner in which judgement of sufficiency usually occurred: the students checked their own work (hence knowledgeable), but the way in which they check is explicitly stipulated by the teacher (hence ignorant).

Positioning of the student as other when positioned as ignorant

I found that the student rarely took on the role of something other when s/he was positioned as ignorant. In the rare (specifically two) instances when this did occur, this positioning appeared to do the work of masking the ignorance of the student, as anticipated from the course's conception of the student as autodidact. However, that this result does not occur more often in *The Trains Activity* supports my previous argument that the intended curriculum's conception of the student as autodidact is not evident in practice.

b) Positioning of the teacher

My analysis showed that the teacher was almost always positioned as knowledgeable. This was expected, since the teacher holds the symbolic mandate of knowledge. As for the student, the positioning of the teacher worked in service of the judgement of the sub-notions. For example, in the stage of necessity, the teacher judged that a particular predicate was more appropriate than others (either in the activity, or external to the activity). In the final stage of judgement, the teacher either judged the sufficiency of the predicates or stipulated how the students were to check their solutions.

The hierarchy was apparently flattened

Often when the teacher was positioned as knowledgeable it was while he was taking on the role of something other and/or being positioned as ignorant at the same time. This positioning worked to apparently flatten the inherent hierarchy between teacher and student. Consequently, this result was expected following the intended pedagogy's similarity to Bernstein's (1996) competence model of pedagogy.

The positioning of the teacher as ignorant and as other when positioned as knowledgeable also worked in service of the judgement of the sub-notions. For example, in the stage of existence, impossibility is key. However, the teacher experiences no impossibility, since he already "understands" the notion. It is thus unsurprising that the teacher is positioned as knowledgeable. Yet the pedagogy retains the appearance of impossibility (in sub-notions 1 and 2) by positioning the teacher as simultaneously ignorant and/or masking his alignment with knowledge by positioning him as something other.

7.4 Conclusion

My study has made visible that problem solving is the central notion privileged in the implementation of one activity of a mathematics support course. The teaching of problem solving is "very difficult" (Craig, 2007:14) and my study has demonstrated how one course attempts to do this: teachers guide students through a set sequence of sub-activities, which constitutes the judgement of three sub-notions and replicates two cycles of a problem-solving process similar to that articulated by Polya (1957). Communication of the notion of problem solving (and its sub-notions) entails intricate positioning of pedagogic subjects in relation to the privileged knowledge, which works in service of the judgement of the notion and in tandem with the guidance which teachers give students.

While the pedagogy intended for the course fits Bernstein's (1996) competence model of pedagogy in some respects, the implemented pedagogy emerges as a hybrid of the competence and performance models. The former

is evident because students are given opportunities to “investigate” and “discover”, the hierarchy inherent in pedagogy is apparently flattened and the teachers “refrain... from telling students ‘what to do’” (Human et al., 2010:3) in many respects. For example, teachers do not explicitly discuss with students the privileged problem-solving process, answers to the privileged questions or aims of the course. However, the performance model of pedagogy emerges in that the teachers do tell students what to do in various respects. They explicitly tell students to draw a diagram, to check their answers/formulae etc. They also implicitly “tell” students what to do by predetermining what questions the students will answer and in what order, modelling the problem-solving process and subtly positioning the students in complex ways.

In what follows, I focus on three implications of my results which raise concerns. Firstly, the central notion which the course aims to teach (problem solving) and the intricate positioning of the student suggest that the course requires extremely complex work from the teacher. This has potential implications for the funder’s aim of developing a “replicable model for tertiary access support” (Vosloo et al., 2012:2), since this complex work may be difficult to communicate to other teachers. Further research is needed to investigate this.

Secondly, the complex positioning of the students in the course may not be familiar to students and so may not work in the ways which I have suggested to develop the notion, instead possibly hindering communication of the privileged knowledge. I ask whether the switching between being positioned as knowledgeable and ignorant has the (unintended) consequence of the students leaving the course feeling uncertain as to whether they are indeed knowledgeable. I tentatively suggest that this complexity of the course’s pedagogy may detract from the “main role” which the funder claims their overall support programme plays: “to hold students through their periods of self-doubt, to support their sense of self-efficacy so that they can continue with their studies” (Vosloo et al., 2013:217). Again, further research is needed to investigate this.

Finally, regarding the privileged knowledge, the notion of problem solving remains implicit since neither it, nor its sub-notions, are discussed with students and students do not have an opportunity to solve the general problem independently. Indeed there is no evidence that the students “understand” the notion at the end of the course. This raises questions about how the course may support students in their university mathematics courses. In addition, the pedagogy identified may mitigate against the acquisition of the sense-making disposition that the course intends to develop.

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Appendices

1. Course materials for *The Trains Activity*
2. Email correspondence with the NPO teachers
3. Solution to *The Trains Activity*
 - 3.1 The problem statement
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Appendix 1: Course materials for *The Trains Activity*

Two trains

There are two parallel railway tracks between the towns ° and ~. The length of the railway tracks from the one town to the other is 236 kilometres.

On a certain morning, one train needs to travel from ° to ~ and arrive at ~ no later than 09h40. This train travels at an average speed of 96 km/h.

On the same morning, another train has to travel from B to A and arrive at A no later than 10h00. This train travels at an average speed of 139 km/h.

Notes to the presenter(s):

a) Only ask to investigate situation – no questions given. *And no suggestion of formula-making!*

Explain what is meant by ‘investigate’:

“You were provided with *some* information about the problem. Use this information to produce *more* information about the situation, where such additional information is useful and/or interesting.”

Ask students to make some kind of picture to that will help them to think about the situation, and to visualise all the important information.

Ask students to imagine themselves being in the shoes of the railway manager (transport planner) when they investigate this situation.

We are hoping (and waiting patiently) that students come up with the idea to investigate questions like “When do the trains need to depart in order to arrive on time?” and “At what time and on what point will the two trains cross each other?”

b) Discussion in small groups:

Students tell to one another how they understand the situation. They have to check themselves that they understand correctly.

Students share ideas about what are important/meaningful questions to ask in this situation.

c) Now answer the questions individually. *Once again, no suggestion of formula-making!*

Ask students to agree to focus on the question: “At what time and at what position on the railway will the trains cross one another?”

Some students will investigate the problem numerically, by calculating the position of each train at different times. It is a very good learning process if they first investigate the problem numerically.

At some time during the learning trajectory, after some students have spontaneously made drawings and/or number lines and or tables to represent the problem, we may show that representations on the board. Some students will not do this by themselves unless they are shown the example of other students.

Some students may make formulas straight away, or even try to set up and solve an equation. It is great if they do so, but only if they make sense of their mathematical representation of the problem. If not, we need to point out their lack of sense-making (e.g. by using a counter-example), and then suggest that they first do some numerical investigations before trying to make formulas/equations.

The problem 'invites' students to form an idea of *co-variation*: the position of one train varies together with the position of the other train, and the position of each train varies with time. Students may explore this by means of number lines, or by adding tick marks to their drawings of the railway to show the positions of the two trains at different times. Thereby they would in effect have made a double number line. It would be great if they did that by their own initiative.

Some students may also by their own initiative make graphs to show the positions of the two trains at different times. This is also great, as long as they make sense of their graphs (we can test this by asking them to relate their graphs to the real problem).

d) Complicate the problem: many different train journeys at different time of the day.

Point out to students that there are actually many trains travelling at different times of the day in both directions on the railway. Once again, ask them to imagine that they are the railway manager (transport planner). For any two trains for which the arrival times and the average speeds are specified, the railway manager needs to be able to say at what time and point they will cross. The students need to make a tool that will make it quick and easy for the railway manager to determine this for any two trains.

Notice that both the arrival times at the average speeds of trains are now variable.

Students will probably try to make a formula when asked to make such a tool. But many of them may not make sense of what they are doing, and make senseless formulas. We need to guide them by using the following techniques (preferably by engaging with students one-on-one):

(i) Ask questions that will create a conflict in students' minds about the formulas that they made, so that they will discover themselves that their formulas do not make sense.

(ii) Suggest to students that a good way to make formulas (that make sense) is to first do some calculations, and then to look back at their calculations. If they *write all their calculations* (and no intermediate answers), and if they write all of the calculations in a *one-line-calculation*, then they will discover a 'pattern' in the way that something can be calculated. It is a good idea to do different calculations, for different values of the input variables, and then to ask oneself which quantities change and which remain the same in the different calculations.

You may want to repeat to the students: "What is a formula? A formula is a calculation instruction that tells you *how to calculate* something that depends on another thing or things. So if you have to make a formula, it helps to first calculate the thing, and then look back on how you calculated it."

e) Give some specifications for different trains journeys, each time giving the arrival times and average speeds of the two trains, and let students test their tools on these specifications.

f) Are the answers reasonable?

Ask students to read the description of the situation again, and then think about whether their answers make sense: does it sound reasonable that the trains will cross at the time and point predicted by their calculations?

e) Group discussion: Reflection on process and new learning

Students have to tell one another about the *process* that they followed to solve the problem, and about *what they have learned* about *doing* mathematics (not what they learned about *mathematics*).

f) Reality-check: how good a description of reality does the mathematical representation give?

Ask students to read the description of the situation again, and to forget about the mathematics for a while.

Then ask them to think about whether things are really so simple as it was assumed to be when they made the mathematical representation of the problem. Are there some complexities that have been ignored? We are hoping here that some students will point out that a train does not travel at a constant speed.

Two trains activity

Learning trajectory

Dear colleagues

In the past we have started the support course to 1st year engineering and science students by doing the two trains problem. The great majority of students responded by first solving the problem in a step-wise arithmetic way, by using the result of one calculations in the next calculation, and so forth. (The final step, however, required solving a simple equation.)

When we then asked them the general problem: make a tool that will enable you to quickly calculate the time when any two trains, with any arrivals times, any speeds, etc. will meet, the students struggled, and many never succeeded in making such a formula.

Could we do something different this year so that more students will eventually be able to solve this problem in an algebraic manner, but so that they still have responsibility for and ownership of what they are doing (i.e. that they are not merely following our instructions)?

We've had various conversations about this in the past week. The suggestion has been made from Teacher 4 and my side, to use two prior activities. The first prior activity would be to introduce students to the idea that it can be useful to write how something (y) that depends on something else (x) *could* be calculated (i.e. to make a formula), even though the something else (x) is not known, so that nothing *can* be calculated. One cannot expect students to find out the value of such a seemingly futile pursuit by their own – they certainly did not learn that at school, and not likely in their first year calculus course.

The second activity would be for students to overcome the obstacle of converting times from decimal form (e.g. 2.75h) to hours-and-minutes form (e.g. 2h45min) and back. Some students have in the past greatly struggled with this during the trains activity, and this have often lead to them never engaging with the main objective of the activity, namely formula-making.

Yet there are some arguments against using prior activities:

- a) The prior activities may waste time for the students who do not need them, and may bore them.
- b) Students who could have figured things out with less structured support from us, will now not need to do so, and thereby lose out on a sense of responsibility and ownership.
- c) Students may get the unintended message early in the course that the week's activities will be easy, and that we will often demonstrate to them what to do. This may lead to them being less engaged, and less thinking-for-themselves, than what is good for their learning.
- d) Students might not transfer knowledge from the prior activities to the trains activity, in which case the prior activities would be a waste of time.
- e) Students might unthinkingly re-apply what they did in the prior activities in the trains activity, if they perceived that the prior activities were intended to 'train' them for the more difficult trains activity.

In the light of these arguments for and against the prior activities, we have tried to come up with a learning trajectory that includes the benefits of the prior activities, whilst minimising the drawbacks.

Monday morning 8h00 to 10h00:

Give students a simpler problem, that can be solved numerically and algebraically, to work on individually for 45 minutes. Tell the students that this is an easy problem, and that they should not expect easy problems for the rest of the week.

R380 was divided between 10 students for transport money. The students come from two towns, A and B. The students from town A did not have to travel very far, and received R10 transport money each. The students from town B had to travel far, and received R50 transport money each. How many students come from town A and how many from town B?

Students who solve this quickly with a numerical search, will then be asked if they can solve it by making and solving an equation. If they do that quickly as well, they will be asked an additional, more challenging problem.

After 45 minutes, one of the presenters will lead a class discussion on different ways of solving this problem. The purpose of this class discussion will be for students to distinguish between a numerical and an algebraic way of solving a problem. The presenter should try to use examples of different ways in which actual students have solved the problem. But the examples should not be discussed in full. For example, for the algebraic method, one needs only to say:

“We have identified the two things that are unknown, namely the number of students who received R10 each, and the number of students who received R50 each. Let us call these to amounts A and B (calling them x and y will make things more difficult as we will have to remember which is for town A and which is for town B). There is something we know about A and B, and that is that together they give 10 students. So we can write this as an equation.

Can we solve for A and B in this equation? No, we need another equation. How can we get that, is there something else we know that we can write as an equation? Have we written all the important information in a mathematical way?”

One would then give the students another 30 minutes to work on this individually.

One would end with solving a similar problem (but with different numbers) on the board. This time the presenter will show the full solution. The presenter should focus students’ attention on the fact that the formula: total amount of money = $10A + 50B$ could not be used to calculate anything, because A and B were (still) unknown. Yet without making this formula, the problem would never have been solved, except by trial-and-error.

The presenter should make no suggestion that the students will later the week be given a problem for which they have to use a similar solution strategy.

Note that this problem does not contain the idea of interval thinking (duration from starting time to end time, etc.), that is an important and challenging part of the trains problem. Also, the problem contains only two unknowns, whereas in the trains problem, there are more unknowns, and the clear and unique labelling (with symbols) of unknowns becomes a more important issue.

The rest of Monday: water flow activities

Tuesday morning: 08h00 to 9h00

Students will write a numeracy test, which will include questions on time conversions.

The rest of Tuesday: water flow activities

Wednesday morning: 08h00 to 9h00

The test questions on time conversion will again be handed out, without answers. Students will be given 15 minutes to work on the questions again. The presenter will then draw a triple number line on the board, for reading time on three different scales: decimal notation, fraction notation, and hours-and-minutes notation. The number line will be for a time span of 1 hour, and the presenter will only show numbers for the end-points of the 1-hour time span, namely for whole numbers of hours. The students will then be asked work in small groups and explain to each other how to convert between the different notations. They have to refer to the triple number line in their explanations to one another, and to fill in the values on the triple number line as a group.

The presenters should aim to choose the groups in such a manner that there and a student in each group how understands time conversion at least more-or-less. Yet overconfident or overbearing students who can already do time conversions should be grouped separately.

The rest of Wednesday: water flow activities: integration**Thursday morning: spatial reasoning activities****Thursday 13h30 to Friday 17h00: the trains activity**

Note that the trains activity is well separated in time from the prior activities, which would hopefully avoid that students unthinkingly re-apply what they did in the prior activities in the trains activity.

Appendix 2: Email correspondence with the NPO teachers

Clarification

To: Teacher 1

Sent: Tue, Oct 2, 2012 at 10:18 AM,

From: Renee Rix

Subject: Clarification

Dear Teacher 1

I hope you are well. It feels like ages since I have seen or spoken to you!

In working on my research project, I have a couple of queries which I hope that you can help me to clarify. I have emailed Teacher 4 as well, but I thought it would be useful to hear both of your perspectives...

Firstly, please recall the following:

A statement Teacher 4 made in an email last year (which I think you forwarded to me and we chatted about over dinner in Pretoria): "What we try to achieve includes better understanding of concepts like variable, function and rate of change, algebraic thinking (method), modelling as a mindset, modelling as a strategy, and learning attitudes."

(November 2011)

And then a comment in your mind the gap paper: "The intention of the learning activities reported here was to develop students' skills and dispositions at solving problems using mathematical modelling, and at the same time to use the problem-solving experience as a means for students to make sense of mathematics that they have already learned." (Human et al., 2010:2)

And lastly, in your write up / course materials for the Trains Activity, you state that: "The problem 'invites' students to form an idea of co-variation: the position of one train varies together with the position of the other train, and the position of each train varies with time."

With this in mind, can you please tell me:

1. a) What is it that you mean by "modelling" (as a mindset and as a strategy)? and
b) What, during the trains problem, would you consider an indication that a student has mastered "modelling"?
2. a) What is it that you mean by "algebraic thinking (method)"? and
b) What, during the trains problem, would you consider an indication that a student has mastered the "algebraic method"?
3. How is modelling related to the algebraic method?
4. a) What exactly do you mean by "covariation"? and again,
b) what would count as a student demonstrating an understanding of "covariation" in the trains problem?

I have a sense of what you might say in answer to some of these questions and but for some of them, I feel quite unsure. In either case, I think it is important to understand what you and Teacher 4 (who, like I said, I have emailed separately with these questions) would answer.

Warm regards

Renee

From: Teacher 1

Sent: 02 October 2012 01:10 PM

To: Renee Rix

Cc: all NPO teachers

Subject: Re: clarification

Hi Renee

It's good to hear from you. Yes, it's been too long.

Your timing is good, since just yesterday I started reflecting on our objectives with [the foundation's] courses. Your questions are helping me to do this.

I do not have a final articulation of this, but here goes:

The purpose of mathematical modelling is to reveal the structure of interdependencies between different quantities in a certain situation.

The practice of mathematical modelling is to use algebraic expressions to represent the way(s) in which different quantities are interdependent (in other words: how they vary together). The first step is to identify what are the different quantities that can vary.

The awareness of quantities that vary together (covariation), the making of individual algebraic expressions, and the overall inventory of quantities and expressions that constitute the model, are not, in my mind, separate concepts or skills.

I believe we have used algebraic thinking as a synonym for mathematical modelling.

Teacher 4 added in brackets: "algebraic thinking (method)". I believe he called it a method in the following sense: The approach/method/discipline of first trying to represent algebraically how quantities are interdependent, before doing calculations. So the 'method' says: first model (reveal the structure of interdependencies), then calculate. But other than this, mathematical modelling are anything but methodical/procedural.

What is not mathematical modelling/algebraic thinking?

Algebraic manipulations are not mathematical modelling, they are indeed procedural, and concern only immediate detail and no 'big-picture' thinking. With this in mind, we should reconsider the use of the term 'algebraic thinking'. Maybe rather talk about 'meta-algebraic thinking' as a synonym for mathematical modelling. What the 'meta' would indicate is that this kind of thinking are about the purpose and plan with the algebraic actions, and are not the algebraic actions themselves.

What we observed through the years, is that many students can 'do algebra', but they cannot solve a fairly simple real-life problem like the trains problem. And we hypothesise that this is because they have not developed meta-

algebraic (purpose and plan) thinking, and that such thinking is more a mindset/disposition/attitude than a skills/concept.

How do we know that a student has developed the mindset of mathematical modelling/ meta-algebraic thinking? When it is evident from his/her written work that he/she first aims to mathematically represent the structure of interdependencies between quantities in the given situation, before they calculate things.

Hence when a student, in the trains problem, first calculates where train A was by the time that train B was departing, that does not indicate the mindset of mathematical modelling. Whereas when a student did not bother with the aforementioned 'intermediate answer', but directly made formulas for the positions of both trains with respect to time, that does indicate the mindset of mathematical modelling.

The great majority of students first went for the intermediate answer (where train A was by the time that train B was departing). Many of them tried later to make equations for what they have done (and that was a complicated algebraic task). But some of them later discarded their earlier approach, and then made formulas for positions of the two trains with respect to time (and that was a much simpler algebraic task). It is the latter students who I would venture to say 'converted' to the mindset of mathematical modelling.

However, the trains activity is meant as a 'demonstration-by-experience' of the mindset of mathematical modelling, and of its usefulness. And it may well be the first demonstration of this to some of the students. So students who did not 'convert' the first time round, may well convert later because of some experience in their university studies, or during the 'three masses' problem in the second year [course for the foundation].

I hope this help. Please ask more -- I am thinking somewhat new things as I am trying to answer you.

Keep
Teacher 1

well

From: Teacher 4
Sent: 02 October 2012 03:27 PM
To: Teacher 1; Renee Rix
Cc: all NPO teachers
Subject: RE: clarification

Thanks Teacher 1. I fully agree and can not add anything of value to this at the moment, But please Renee and everybody, let us keep challenging each other on these things!

From: Renee Rix

Sent: 02 October 2012 04:12 PM

To: Teacher 1

Cc: all NPO teachers

Subject: RE: clarification

Dear Teacher 1

Thank you for these comments, they are very helpful. I had thought that perhaps the modelling and algebraic thinking were synonymous, but my sense was that, although covariation is intertwined with this, that it could be extracted and perhaps there are certain things (student productions) which could perhaps be evidence that they understand covariation (possibly independent of whether or not they have mastered the algebraic thinking / method / modelling). It is interesting to me that you don't consider it possible to consider covariation in any way separate to the modelling.

At this stage I don't have any more questions, but I will let you know if/when I do.

Warm regards

Renee

Clarification: part 2

From: Teacher 4

Sent: Tue, Oct 2, 2012 at 6:03 PM

The way I understand Stacey et al.** to use "the algebraic method" is to indicate a specific way of tackling a real problem, namely to set up an algebraic model (eg an equation) rather than to work numerically. This may or may not involve thinking of covariation, depending on whether the problem situation involves related variables or not.

In this particular case, (the two trains) there definitely are related variables: the distance of both trains from their respective stations, the distance from each other, and time all covary and this covariation may be expressed algebraically. BUT, my sense is that the problem can be solved algebraically without thinking specifically in terms of covariation.

From: Teacher 1

Sent: 02 October 2012 07:37 PM

To: Renee Rix

Cc: all NPO teachers

Subject: Re: clarification

Hi Renee

Your comment made me think twice about whether it is possible for a student to have used covariational thinking without having done mathematical modelling.

a) My short answer is yes: I remember one or two students this year who solved the problem by trial-and-error (numerically), by calculating at different times the positions of both trains and comparing.

b) In the 2010 [course for the foundation], we tried to encourage students to first use such a trial-and-error (numerical) approach, by making a table with different times, and the corresponding positions of the two trains. However, this did not seem to work well. Most of the students who made such a table made something like the following:

time	train A	time	train B
7h12	0km	8h18	0km
7h42	48km	8h48	70km
8h12	96km	9h18	140km
8h42		9h48	
9h12		10h58	

They missed the point that the positions of both trains were varying with the same variable time. So although they were now thinking covariationally, and correctly so for each train on its own, they missed the overall structure of the situation.

We have since (during the 2011 and 2012 courses) encouraged only some students to first use such a numerical approach, and it was not a clear objective in our lesson plan.

** Stacey, K. and MacGregor, M. (2000) Learning the Algebraic Method of Solving Problems. *Journal of Mathematical Behavior*, 18(2), 149-167

c) If a student can use the numerical approach with sense of the overall structure of the situation, I suspect that it may then be a fairly easy step to represent this process in mathematical expression, as the meta-thinking (purpose and plan) has already been done.

d) The last two points beg the question, should we not encourage (more) students to first engage numerically with the covariation in the situation? And could we do this in a way that will be more successful than in 2010?

e) Yesterday I made a note to myself of an activity we could try before students start solving the trains problem: Describe qualitatively how the 'meeting time' will change if some of the parameters (train speeds, arrival time, distance between town) are changed.

f) About Y's comment: Yes, some students solved the problem algebraically without covariational thinking. They are the ones who made a formula for calculating where train A was by the time that train B was departing, and then made a formula involving the remaining distance between the trains, and the remaining time (duration) until they meet: $(v_A * t + v_B * t) = D$. (Note in this equation that "t" is a constant to be solved, not a variable.)

However, as mentioned earlier, it was a lot of algebraic work for them to then make a formula for the time (on the clock) when the trains will meet. So the step of first making a formula for where train A was by the time that train B was departing, complicates matters algebraically. They did it this way because before tackling the general problem, their approach was to first calculate where train A was by the time that train B was departing.

Such students placed higher value on intermediary answers than on revealing the structure of interdependencies in the situation. They avoided (deliberately or not) to make mathematical expressions while they could still do some seemingly useful calculations.

So I do not regard this as mathematical modelling proper, even though it made use of some algebraic expressions. In this problem, a lack of covariational thinking signifies to me an avoidance of modelling/'structural' thinking in favour of calculations.

There may be problems where there are no covariation and then a student could do modelling without thinking covariationally. However, I suspect that these will only be very simple problems.

Regards

Teacher 1

Clarification (part 3)

From: Teacher 4

Sent: 04 October 2012 10:28 PM

To: 'Renee Rix; Teacher 1

Cc: all NPO teachers

Subject: covariation extended

I now think the term "covariation" may be used in a wider sense than I have used it before.

Before, I used it only in connection with different aspects of the same function covarying, eg the argument of the function itself and the rate of change of the function.

So the three covarying quantities are of different kinds, especially in models of real situations.

But in a situation like the two trains, the distance of the two trains from their respective destinations (or departure points) also covary, and here we have two quantities of the same kind covarying.

I am pretty sure the wider view is also taken in some of the attached papers, but I do not have the energy to read them now.

In concert with the above, I now realize (the simple and obvious thing) that there are at least two quite different kinds of situations that may be modelled by an equation of the form $ax + b + cx + d$, namely

A. static situations in which the unknown x is really constant, and does not signify a varying quantity

B. Dynamic situations where

Clarification: Part 4

To: Teacher 1

Sent: Thu, Oct 11, 2012 at 10:20 PM

From: Renee Rix

Cc: Teacher 4

Hi Teacher 1 and Teacher 4

Thanks again for your detailed replies to my questions last week. Sorry it has taken me a little while to come back to you on the last one.

I follow all of what you are saying but do query two things, both contained in point (f) of your email.

1) I hear what you say about t being a constant to be solved for in the equation $(v_A * t + v_B * t) = D$. But in what equation is this not the case (ie that the variable to be solved for is truly a variable, and not just an unknown to be solved for)?

and

2) I do not understand how someone who calculates the position of the first train at the time the second train departs, and works from there is thinking any less 'covariationally' than someone who does not do this. Indeed, when Teacher 4 did the problem (for himself whilst assisting at the course this year, but which he graciously shared with me - it is attached here), he did exactly this - solved for the position of the first train

at the departure time of the second and then worked from there, even creating the general formula from there. I am unsure if Teacher 4 did this because it was what the students were doing (and he wanted to clarify intermediate answers for himself) or if it is because this is how he felt it should be done. Teacher 4, perhaps you can tell us?

Teacher 1, perhaps you can show me what a solution which does not involve this intermediate step and so to your mind demonstrates covariational thinking would look like?

Teacher 4, I have also not yet read the papers you sent about covariation - perhaps tomorrow I will have the energy. But I felt I wanted to try clarify the above in the mean time. I hope that is ok.

Sorry to keep pestering you!

Many thanks for helping me to understand these things.

Warm regards

Renee

From: Teacher 1

Sent: 12 October 2012 02:09 PM

To: Renee Rix

Cc: all NPO teachers

Subject: More about mathematical modelling and covariation

Hi Renee

I've responded to your questions in a Word document, since I needed to do formula editing.

Also I made it a bit formal, which might help you referring to it, and might help me to build up the body of text that I want to use in a subsequent paper, that will concern both the 'trains' and the 'three masses' activities.

Next week will be very busy for me, so I may not be able to respond to emailed questions then.

If you would like to talk about this further, you could meet me at the airport on Monday from 12h30-13h45, since I will be flying to Johannesburg 14h30.

Keep well, and hope to see you soon.

Warm regards

Teacher 1

Friday 12 October 2012

To: Renée Rix
 School of Education
 University of Cape Town

Hi Renée

You asked: *"I hear what you say about t being a constant to be solved for in the equation $(v_A t + v_B t) = D$. But in what equation is this not the case (i.e. that the variable to be solved for is truly a variable, and not just an unknown to be solved for)?"*

If any problem/situation is described by only *one* equation (and one that can be solved algebraically), then one can indeed always regard the unknown as a constant to be solved, and not a variable.

But if the problem/situation is described by more than one equation (and more than one variable), where each equation on its own cannot be solved, it is different. Then any equation on its own cannot be used to calculate any variable values; it merely represents the relationship between two or more variables, in other words the covariation of the variables. Only later, when all the different equations that together represent the problem/situation are combined in some way, can one eventually make equations with one unknown in each equation. (There are of course some problems where the set of equations can only be solved numerically.) So even though each unknown can only have one value (i.e. could be considered a constant), the original equations represent the unknowns as inter-related variables, and therefore the making of those equations require a sense of covariation. Note also that at least some of original equations will never be used to calculate anything with, yet it was essential to make them.

Appreciating the usefulness (if not necessity) of making equations that can't be used (one their own) to calculate anything with is, in my mind, at the core of the mindset of mathematical modelling. Our main aim with the trains activity is for students to have an experience of the usefulness of making such equations. If a student made only one equation with one unknown, i.e. $v_A t + v_B t = D$ (1), it is not certain that he/she had such an experience. Of, course, the student could have, in his/her mind or on paper, first made the equations $D_A = v_A t$ (2), $D_B = v_B t$ (3), and $D = D_A + D_B$ (4), and then combined them to get equation (1). However, such a student *would not have needed to suspend her/his urge to calculate something for very long*. (Note that t above means 'the duration after the departure time of train B that the two trains will keep on travelling until they meet'.)

Compare the above with the following approach:

Position of train A at after it has travelled for a duration Δt_A (from its departure time):

$$x_A = v_A \Delta t_A \text{ (a)}$$

Position of train B at after it has travelled for a duration Δt_B (from its departure time):

$$x_B = D - v_B \Delta t_B \text{ (b)}$$

Expressing the durations Δt_A and Δt_B in terms of 'any point in time', t , and the departure times of the two trains:

$$\Delta t_A = t - t_{\text{departure}_A} \text{ (c)}, \text{ and } \Delta t_B = t - t_{\text{departure}_B} \text{ (d)}$$

Substituting equation (c) and (d) into equations (a) and (b):

$$x_A = v_A(t - t_{\text{departure}_A}) \quad (a'), \text{ and } x_B = D - v_B(t - t_{\text{departure}_B}) \quad (b')$$

At the meeting time t_{meet} , the trains will be at the same position:

$$x_A(t) = x_B(t)$$

$$\therefore v_A(t_{\text{meet}} - t_{\text{departure}_A}) = D - v_B(t_{\text{meet}} - t_{\text{departure}_B})$$

Then solve for t_{meet} .

A student who followed the latter approach had to wait longer, after starting to make equations, until she/he could calculate anything. It looks above like the students following the latter approach had a more arduous algebraic task, but the opposite is actually true. The former approach looks simple only because the unknown there (the duration after the departure time of train B that the two trains will keep on travelling until they meet) was not actually the unknown that answers the question directly; additional and complicated algebraic manipulations are then necessary to make a formula for the *point in time* when the trains meet (t_{meet}).

The approach of gathering enough equations to 'fix' the system (no degrees of freedom left) whilst temporarily suspending attempts to calculate answers, is essential for solving more difficult problems like the three masses problem. The trains problem is meant as a first experience with this approach, so that it can later be applied in more difficult problems.

Before we first decided on using the trains problem in 2010, I looked at 'mass balance' problems, which chemical engineering students have to learn to solve at the end of their first year, and with which they struggle tremendously (also in the time that I studied it). I hypothesise that the lack of familiarity with the approach discussed above is a major contributing factor to students' struggles with mass balance problems: when they can't 'see' how they can calculate an unknown, they become jittery, and start working in a haphazard manner. (I include a mass balance problem from an examination at the chemical engineering department in Stellenbosch. One needs to read some parameters off 'humidity charts', so these are also provided.)

The two graphics I made of different approaches to the trains problem in the 'Mind the Gap' paper of 2010 (pp. 10-11) illustrate what I mean by suspension of doing calculations. Also, the discussion of student responses to the trains problem, as well as to the wood factory problem (pp. 5-8) correlates with the issues discussed here, although we may not have articulated it as clearly then. (I attach an extended version of the paper, since I'm not sure whether you have the this version or the shorter original version.)

I believe the above also answered your second question, but feel free to ask more – you are helping me to be clearer about these issues myself.

Regards

Teacher 1

From: Renee Rix
Sent: 15 October 2012 09:04 AM
To: Teacher 1
Cc: Teacher 4
Subject: RE: More about mathematical modelling and covariation

Dear Teacher 1

This is all very interesting and extremely helpful! Thank you SO much - for all your careful thought and time in preparing and sending such a detailed response. I really really appreciate it.

I think I understand what you are saying and don't have new questions at this stage. But I will mull this all over and come back to you when I do.

I have attached here something which I typed up last week (it is dated for today because it is intended as a discussion point for a meeting with my supervisors this afternoon, but you will see the time stamp is last week Wednesday or so). It is a proposed solution path, based on your write up to teachers of how the Trains Activity should proceed (the same document continues on to report of how it went at (a certain tertiary institution) - I have attached this here, just so you know which 'course material' I was referring to). I have a feeling that your asking students to solve the specific problem first (before the general) may help them to 'make sense' of the situation, and although the numerical approximation approach (which your course materials say you will encourage - which sometimes does and sometimes does not happen, but that is immaterial) perhaps encourages an understanding of the covariation, solving the specific problem algebraically - which is surely needed if one is to write one's work in a single line calculation - perhaps actually discourages the type of modelling that you speak of in your response? I don't know. It is just a thought I had as I read your formal reply.

[note that the last two pages of my 'proposed solution' document - i.e. the addendum - is a solution which I wrote up a while ago, before trying to write a solution which seems to follow the proposed learning trajectory indicated in the course document].

Thanks also for the offer to meet at the airport this afternoon. Unfortunately I won't be able to make it at that time. I do hope you have a good week up north.

Warm regards

Renee

From: Teacher 1
Sent: 15 October 2012 10:00 AM
To: Renee Rix
Cc: Teacher 4
Subject: Re: More about mathematical modelling and covariation

Hi Renee

And thanks for your detailed reply.

I'm a bit late in packing my bags, so I just glanced through the word documents with different possible solution methods. It seems to be a very thorough 'inventory' of solution methods. I will print it and read it in more detail later. I have not always taken the time to do that, and I should! So thanks.

I believe there is a printing error in the equation in the middle of page 11 (route 2, 3rd equation):
It should read: $d^*/v_2 = (k-d^*)/v_1 - \dots$. The first two fractions have been inverted. This mistake was followed through to the final formula for d^* , which you can see by checking the units of that formula.

About the mass balance problem: Two years ago when I did it I drew some analogies. But I'll have to do it again to refresh my memory, and be critical about whether there are such analogies. Will get back to you about that in two weeks.

About the doing the specific solution first possibly discouraging the kind of modelling that I have in mind:
Yes, I think that may be the case for some students. I remember that last year, when we first gave students the 'cars overtaking' problem, and immediately asked them to make a formula to determine whether, in any case, it would be safe to overtake or not, we were surprised by how many students made those formulas. But thereafter some of the same students struggled to make formulas for the trains problem (after first solving the specific train problem). I have not yet quite got my head around that. It may be that students will fairly easily produce a formula if the question asks for a formula, but tend not to use the modelling approach when the question asks for a numerical answer. That would be a dispositional issue then, once again. If students used the numerical method (trial-and-error, tabular search), I believe that is indeed a good precursor to the modelling approach. Although, they would probably still have broken calculations into small bits, never writing a longer one-line calculation such as: $t_{1dep} = t_{1arr} - 236/96$.
But if students used the calculate step-by-step far as can go approach (where train A was when train B departed, then make and solve one equation), I believe that may make it harder for them to eventually use the modelling approach. But at the end of the day, it would be good if students solved it with many different approaches, so that they can then compare the merits of the different approaches. That may require that they spend even more time on this problem.

Ok, more later.
Keep well.
Teacher 1

PS: But the 'cars overtaking' problem did not have the complicating factor of different departure times, so that may also be a reason why some students more easily did modelling for that problem than for the trains problem.

Appendix 3: Solution to *The Trains Activity*

What follows is my proposed solution for *The Trains Activity*, constructed in a manner implied by the intended curriculum. However, note that none of the specifics of this solution are given in the intended curriculum source documents. Instead, they describe what the teachers should do to move students through a specific sequence of actions (see Appendix 1).

A3.1 The problem statement

The students are given a written description of the problem situation, as follows:

There are two parallel railway tracks between the town A and B. The length of the railway tracks from the one town to the other is 236km.

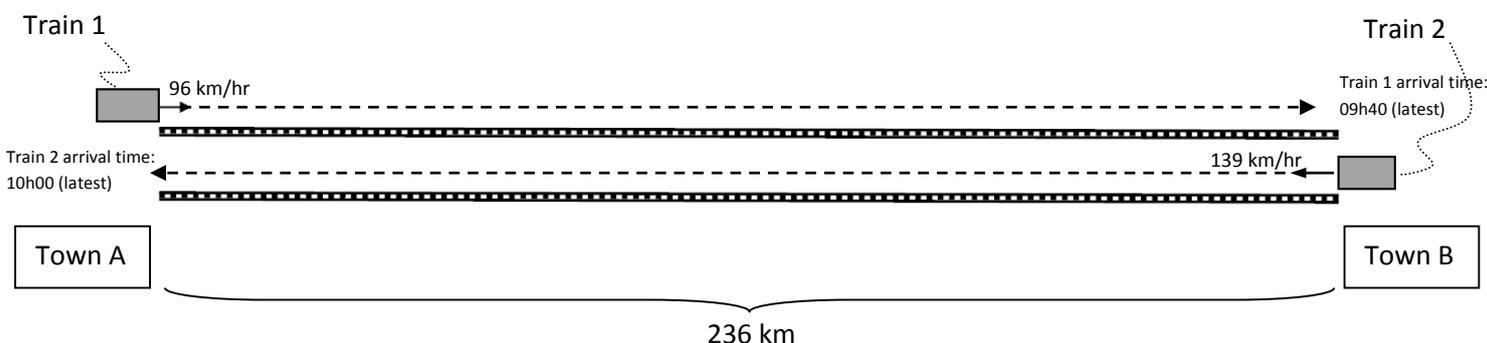
On a certain morning, one train needs to travel from A to B and arrive at B no later than 09h40. This train travels at an average speed of 96km/h.

On the same morning, another train has to travel from B to A and arrive at A no later than 10h00. This train travels at an average speed of 139km/h.

A3.2 Graphical representation

Notes to the teachers in the Course materials state that teachers should “Ask students to make some kind of picture to that will help them to think about the situation, and to visualise all the important information.”

I propose the following picture:



A3.3 Students pose questions

Notes to the teachers in the Course materials state that students are to have “discussion in small groups” and that in these discussions, “Students share ideas about what are important/meaningful questions to ask in this situation.”

The following is a list of questions which I (in conversation with colleagues) imagine that students could pose:

- How long does it take each train to travel between the towns?
- What is the latest time you could leave town A so that you still had time to catch the other train back to town B?
- What time should each train depart by?
- At what point (in time and in terms of distance before the end) should the trains start braking?
- What is the maximum speed that each train reaches?
- If the trains don't leave at the same time, how far has the first train travelled at the time when the second train departs?
- At what time would the trains pass each other?
- What distance has each train covered at the point when they pass each other?

A3.4 Privileged questions

Notes to teacher in the Course Materials state, “We are hoping (and waiting patiently) that students come up with the idea to investigate questions like ‘When do the trains need to depart in order to arrive on time?’ and ‘At what time and on what point will the two trains cross each other?’” Thus the questions which the teachers are to ask the students focus on are:

1. At what time should each train depart?
2. At what time do the trains meet / pass each other?
3. At what position do the trains meet / pass each other?

A3.5 Simplifying assumptions

In order to answer the privileged questions, one needs to assume that:

- a) The trains arrive exactly at the (latest) time indicated in the problem statement.
- b) The trains are travelling at constant speed, given by the average speed in the problem statement.

Note that the introduction of simplifying assumptions is not mentioned in the course documents (intended curriculum). However, these assumptions are necessary if one is to answer the (privileged) questions. Further, these assumptions are made by the course designer in the solution which he provided the researcher.

A3.6 Students answer the first question:

At what time should each train depart?

Let Train 1 be that travelling from town A to town B at 96km/hr and arriving at town B at 09h40 and

Let Train 2 be that travelling from town B to town A at 139km/hr, arriving at town A at 10h00.

Let t_{1_depart} be the departure time of Train 1.

Let t_{2_depart} be the departure time of Train 2.

$$\begin{aligned} \text{speed} &= \frac{\Delta\text{distance}}{\Delta\text{time}} \\ 96 &= \frac{236}{9\frac{40}{60} - t_{1_depart}} \\ 96\left(9\frac{40}{60} - t_{1_depart}\right) &= 236 \\ 9\frac{40}{60} - t_{1_depart} &= \frac{236}{96} \\ t_{1_depart} &= 9\frac{40}{60} - \frac{236}{96} \\ &= 7\frac{5}{24}\text{ hours} \\ &= 07\text{h}12 \text{ and } 30 \text{ seconds} \end{aligned}$$

$$\begin{aligned} \text{speed} &= \frac{\Delta\text{distance}}{\Delta\text{time}} \\ 139 &= \frac{236}{10 - t_{2_depart}} \\ 139\left(10 - t_{2_depart}\right) &= 236 \\ 10 - t_{2_depart} &= \frac{236}{139} \\ t_{2_depart} &= 10 - \frac{236}{139} \\ &= 8\frac{42}{139}\text{ hours} \\ &= 08\text{h}18 \text{ and } 8 \text{ seconds} \end{aligned}$$

Alternatively, one can first calculate the total travel time of each train before determining the departure time:

Let t_1 be the total time taken for Train 1 to travel from town A to town B.

Let t_2 be the total time taken for Train 2 to travel from town B to town A.

$$\text{speed} = \frac{\Delta\text{distance}}{\Delta\text{time}}$$

$$96 = \frac{236}{t_1}$$

$$96t_1 = 236$$

$$t_1 = \frac{236}{96}$$

$$= 2\frac{11}{24}$$

$$t_{1_{\text{depart}}} = 9\frac{40}{60} - 2\frac{11}{24}$$

$$= 7\frac{5}{24} \text{ hours}$$

$$= 7.21 \text{ hours}$$

$$= 07\text{h}12 \text{ and } 30 \text{ seconds}$$

$$\text{speed} = \frac{\Delta\text{distance}}{\Delta\text{time}}$$

$$139 = \frac{236}{t_2}$$

$$139t_2 = 236$$

$$t_2 = \frac{236}{139}$$

$$= 1\frac{97}{139}$$

$$t_{1_{\text{depart}}} = 10 - 1\frac{97}{139}$$

$$= 8\frac{42}{139} \text{ hours}$$

$$= 8.30 \text{ hours}$$

$$= 08\text{h}18 \text{ and } 8 \text{ seconds}$$

A3.7 Students answer the second question:

At what time do the trains meet each other?

Note that there are numerous ways in which one can answer this question. However, most solutions can be classified as either numerical or algebraic. Both solution methods will be discussed here. Indeed, the course documents indicate that students will use both methods, starting with the numerical and then moving on to the algebraic.

The course materials state that:

Most/all students will investigate the problem numerically by calculating the position of each train at different times. It is good for the learning process if they do so.

Further, the document states that:

Some students may make formulas straight away, or even try to set up and solve an equation. It is great if they do so, but only if they make sense of their mathematical representation of the problem. If not, we need to point out their lack of sense-making (e.g. by using a counter-example), and then *suggest that they first do some numerical investigations before trying to make formulas/equations.*

(emphasis added)

Thus the course designers expect that students will start by solving the problem numerically, and that students who do not (i.e. those who initially use algebraic methods) will (probably) not be “making sense” and so should be directed towards first using numerical methods.

Later in the course materials (notes to teachers document), it is stated that

Suggest to students that a good way to make formulas (that make sense) is to first do some calculations, and then to look back at their calculations. If they *write all their calculations* (and no intermediate answers), and if they write all of the calculations in a *one-line calculation*, then they will discover a 'pattern' in the way that something can be calculated.

(emphasis in the original)

It is only possible to write all of one's "calculations in a *one-line calculation*" if one has used algebraic methods to solve the problem.

What follows is a brief description of my attempt at each solution method (numerical and algebraic), followed by a description of my creation of a one-line calculation.

1. Numerical (approximation)

The position of each train is determined at various times. The various times would be strategically chosen, perhaps initially at regular intervals which are carefully reduced, so as to hone in on the meeting time.

One possibility is to determine the position of train 1 at the time when train 2 departs (08h18 and 8 sec) and then to work in 15 minute (quarter of an hour) intervals after that:

Time	Total distance travelled (km)		Distance between trains (km)
	Train 1	Train 2	
08h18 and 8 sec	105.01	0	130.99
08h33 and 8 sec	129.01	34.75	72.24
08h48 and 8 sec	153.01	69.5	13.49

Calculation required to complete the first line of the table:

$$\begin{aligned}\Delta\text{distance} &= \text{speed} \times \Delta\text{time} \\ &= 96 \times (8.3 - 7.21) \\ &= 105.01 \text{ km}\end{aligned}$$

$$\begin{aligned}\text{distance between trains} &= 236 - 105.01 \\ &= 130.99 \text{ km}\end{aligned}$$

Calculation required to complete the second line of the table:

Train 1:

$$\begin{aligned}\Delta\text{distance} &= \text{speed} \times \Delta\text{time} \\ &= 96 \times (0.25) \\ &= 24 \text{ km}\end{aligned}$$

$$\begin{aligned}\text{Total distance} &= 105.01 + 24 \\ &= 129.01 \text{ km}\end{aligned}$$

Train 2:

$$\begin{aligned}\Delta\text{distance} &= \text{speed} \times \Delta\text{time} \\ &= 139 \times (0.25) \\ &= 34.75 \text{ km}\end{aligned}$$

$$\begin{aligned}\text{Total distance} &= 0 + 34.75 \\ &= 34.75 \text{ km}\end{aligned}$$

$$\text{Distance between the trains: } 130.99 - 24 - 34.75 = 72.24 \text{ km}$$

Calculation required to complete the third line of the table:

Train 1:

$$\begin{aligned} \text{Total distance} &= 129.01 + 24 \\ &= 153.01 \text{ km} \end{aligned}$$

Train 2:

$$\begin{aligned} \text{Total distance} &= 34.75 + 34.75 \\ &= 69.5 \text{ km} \end{aligned}$$

$$\text{Distance between the trains: } 72.24 - 24 - 34.75 = 13.49 \text{ km}$$

Following the calculations at these three times, a student who is “making sense” will realise that travelling for a further 15 minutes will result in the trains having passed each other (since the distance between the trains reduces by $24 + 34.75 = 58.75$ km every 15 minutes, and the trains only have 13.49 km left between them).

Thus, the student will reduce the time interval of calculation, perhaps to 1 minute:

Train 1:

$$\begin{aligned} \Delta \text{distance} &= \text{speed} \times \Delta \text{time} \\ &= 96 \times \frac{1}{60} \\ &= 1.6 \text{ km} \end{aligned}$$

Train 2:

$$\begin{aligned} \Delta \text{distance} &= \text{speed} \times \Delta \text{time} \\ &= 139 \times \frac{1}{60} \\ &= 2.32 \text{ km} \end{aligned}$$

Time	Distance (km)		
	Train 1 total	Train 2 total	Between trains
08h18 and 8 sec	105.01	0	130.99
08h33 and 8 sec	129.01	34.75	72.24
08h48 and 8 sec	153.01	69.5	13.49
08h49 and 8 sec	154.61	71.82	9.57
08h50 and 8 sec	156.21	74.13	5.66
08h51 and 8 sec	157.81	76.45	1.74

At this point, the student should realise that the distance between the trains is decreasing by 3.92km each minute. Consequently, after another full minute, the trains will have passed each other and be about 2km apart.

Thus the time interval needs to be reduced, perhaps to 30 seconds:

Train 1:

$$\begin{aligned} \Delta \text{distance} &= \text{speed} \times \Delta \text{time} \\ &= 96 \times \frac{0.5}{60} \\ &= 0.8 \text{ km} \end{aligned}$$

Train 2:

$$\begin{aligned} \Delta \text{distance} &= \text{speed} \times \Delta \text{time} \\ &= 139 \times \frac{0.5}{60} \\ &= 1.66 \text{ km} \end{aligned}$$

Time	Distance (km)		
	Train 1 total	Train 2 total	Between trains
08h18 and 8 sec	105.01	0	130.99
08h33 and 8 sec	129.01	34.75	72.24
08h48 and 8 sec	153.01	69.5	13.49
08h49 and 8 sec	154.61	71.82	9.57
08h50 and 8 sec	156.21	74.13	5.66
08h51 and 8 sec	157.81	76.45	1.74
08h51 and 38 sec	158.61	78.11	-0.72

The trains have thus passed and are already 0.72km apart after 30 seconds.

The student may then try intervals of 5 seconds:

Train 1:

$$\begin{aligned}\Delta\text{distance} &= \text{speed} \times \Delta\text{time} \\ &= 96 \times \frac{5}{3600} \\ &= 0.13 \text{ km}\end{aligned}$$

Train 2:

$$\begin{aligned}\Delta\text{distance} &= \text{speed} \times \Delta\text{time} \\ &= 139 \times \frac{5}{3600} \\ &= 0.19 \text{ km}\end{aligned}$$

Time	Distance (km)		
	Train 1 total	Train 2 total	Between trains
08h18 and 8 sec	105.01	0	130.99
08h33 and 8 sec	129.01	34.75	72.24
08h48 and 8 sec	153.01	69.5	13.49
08h49 and 8 sec	154.61	71.82	9.57
08h50 and 8 sec	156.21	74.13	5.66
08h51 and 8 sec	157.81	76.45	1.74
08h51 and 38 sec	158.61	78.11	-0.72
08h51 and 13 sec	157.94	76.64	1.42
08h51 and 18 sec	158.07	76.83	1.1
08h51 and 23 sec	158.20	77.02	0.78
08h51 and 28 sec	158.33	77.21	0.46
08h51 and 33 sec	158.46	77.40	0.14

At this point, the time interval needs to be further reduced (since we know from the previous attempt that the trains will have passed and be 0.72 km apart at 08h51 and 38 seconds). Perhaps use 1 second intervals:

Train 1:

$$\begin{aligned}\Delta\text{distance} &= \text{speed} \times \Delta\text{time} \\ &= 96 \times \frac{1}{3600} \\ &= 0.03 \text{ km}\end{aligned}$$

Train 2:

$$\begin{aligned}\Delta\text{distance} &= \text{speed} \times \Delta\text{time} \\ &= 139 \times \frac{1}{3600} \\ &= 0.04 \text{ km}\end{aligned}$$

Time	Distance (km)		
	Train 1 total	Train 2 total	Between trains
08h18 and 8 sec	105.01	0	130.99
08h33 and 8 sec	129.01	34.75	72.24
08h48 and 8 sec	153.01	69.5	13.49
08h49 and 8 sec	154.61	71.82	9.57
08h50 and 8 sec	156.21	74.13	5.66
08h51 and 8 sec	157.81	76.45	1.74
08h51 and 13 sec	157.94	76.64	1.42
08h51 and 18 sec	158.07	76.83	1.1
08h51 and 23 sec	158.20	77.02	0.78
08h51 and 28 sec	158.33	77.21	0.46
08h51 and 33 sec	158.46	77.40	0.14
08h51 and 34 sec	158.49	77.44	0.07
08h51 and 35 sec	158.52	77.48	0

Thus the trains meet / pass each other at 08h51 and 35 seconds.

Of course, one could obtain this numerically approximated solution using different time intervals, slightly different calculations and even working ‘backwards’ from the arrival times (as opposed to ‘forwards’ from the departure times as has been done above). Furthermore, the same methods can be applied to answer the third question, that is, to determine the meeting position (or the total distance covered by each train at the time when they meet) – by systematically and strategically varying the positions of the trains and calculating the time at which each train is at that position. None of the above will be demonstrated here, as it is very similar (analogous, even) to what has been shown above. It is left to the reader to do so, should s/he desire.

2. Algebraic solution

An example of this approach to solving for the time at which the trains meet is given below and is based on a solution given to the researcher by Teacher 4, who is also the course designer. This solution follows on from the solution to question 1 (see above) where it was determined that Train 1 departed at 07h12 and Train 2 departed at 08h18:

At 08h18, Train 1 has covered $96 \times 1.09 = 105$ km (note that 1.09 is the time difference between 07h12 and 08h18)

Therefore, distance between the two trains at 08h18 is $236 - 105 = 131$ km

From 08h18,

$$96\Delta t + 139\Delta t = 131 \quad (\text{Note that } \Delta t \text{ is the change in time (i.e. physical travelling time) from 08h18 to the meeting time})$$

$$235\Delta t = 131$$

$$\Delta t = \frac{131}{235}$$

$$= 0.56 \text{ hours}$$

$$= 33 \text{ minutes } 27 \text{ seconds}$$

Therefore, meeting time is $08\text{h}18 + 33 \text{ min} + 27 \text{ sec} = 08\text{h}51 \text{ and } 27 \text{ sec}$

Alternatively, one can avoid calculating the position of Train 1 when Train 2 departs. This alternative solution deviates from that provided by Teacher 4, but it is heralded by Teacher 1 (who is an assistant course designer) as a solution method which signals that the student has “the mind set of mathematical modelling,” while he states that the former (calculating the position of Train 1 when Train 2 departs) does not (personal communication, October 2012). Note that the Teacher 1 indicated agreement with this (personal communication, October 2012).

A solution which avoids calculation of Train 1’s position at the time Train 2 departs, could look as follows:

Let t^* be the meeting time of the two trains.

At the meeting time:

$$\begin{array}{r} \text{Total distance} \\ \text{covered by} \\ \text{Train 1} \end{array} + \begin{array}{r} \text{Total distance} \\ \text{covered by} \\ \text{Train 2} \end{array} = 236$$

Hence,

$$\begin{aligned}
96(t^* - 7\frac{5}{24}) + 139(t^* - 8\frac{42}{139}) &= 236 \\
96t^* - 96 \times 7\frac{5}{24} - 139t^* - 139 \times 8\frac{42}{139} &= 236 \\
235t^* - 96 \times 7\frac{5}{24} - 139 \times 8\frac{42}{139} &= 236 \\
235t^* &= 236 + 96 \times 7\frac{5}{24} + 139 \times 8\frac{42}{139} \\
t^* &= \frac{236 + 96 \times 7\frac{5}{24} + 139 \times 8\frac{42}{139}}{235} \\
&= 8\frac{202}{235} \text{ hours} \\
&= 08\text{h}51 \text{ and } 34 \text{ seconds}
\end{aligned}$$

3. Students create a single line calculation

to answer question 2 (with no intermediary answers)

A single line calculation for determining the meeting time of the two trains, if it is to have no intermediary answers, must necessarily follow from one of the two algebraic solutions given above.

Firstly, one must note that the departure times of the two trains (answer the question 1 above) are given by

$9.6 - \frac{236}{96}$ and $10 - \frac{236}{139}$ for Trains 1 and 2 respectively.

Following from the first algebraic solution given in the previous section, one obtains the following:

At the departure time of Train 2, Train 1 has covered $96 \times \left[\left(10 - \frac{236}{139} \right) - \left(9.6 - \frac{236}{96} \right) \right]$

Therefore, distance between the two trains at 08h18 is $236 - 96 \times \left[\left(10 - \frac{236}{139} \right) - \left(9.6 - \frac{236}{96} \right) \right]$

From the departure time of Train 2,

$$\begin{aligned}
96t + 139t &= 236 - 96 \left[\left(10 - \frac{236}{139} \right) - \left(9.6 - \frac{236}{96} \right) \right] \\
(96 + 139)t &= 236 - 96 \left[\left(10 - \frac{236}{139} \right) - \left(9.6 - \frac{236}{96} \right) \right] \\
t &= \frac{236 - 96 \left[\left(10 - \frac{236}{139} \right) - \left(9.6 - \frac{236}{96} \right) \right]}{96 + 139}
\end{aligned}$$

Therefore, meeting time is

$$\left(10 - \frac{236}{139} \right) + \frac{236 - 96 \left[\left(10 - \frac{236}{139} \right) - \left(9.6 - \frac{236}{96} \right) \right]}{139 + 96}$$

The one-line calculation

Which (when plugged into a calculator) gives 8.86 hours i.e. 08h51 and 34.5 seconds

Following from the second algebraic solution offered in the previous section, one obtains the following one-line calculations:

Let t^* be the meeting time of the two trains.

At the meeting time:

$$\begin{array}{ccccc} \text{Total distance} & & \text{Total distance} & & \\ \text{covered by} & + & \text{covered by} & = & \\ \text{Train 1} & & \text{Train 2} & & 236 \end{array}$$

Hence,

$$\begin{aligned} 96\left(t^* - \left(9.6 - \frac{236}{96}\right)\right) + 139\left(t^* - \left(10 - \frac{236}{139}\right)\right) &= 236 \\ (96 + 139)t^* - 96\left(9.6 - \frac{236}{96}\right) - 139\left(10 - \frac{236}{139}\right) &= 236 \\ (96 + 139)t^* &= 236 + 96\left(9.6 - \frac{236}{96}\right) + 139\left(10 - \frac{236}{139}\right) \end{aligned}$$

The one-line calculation

$$t^* = \frac{236 + 96\left(9.6 - \frac{236}{96}\right) + 139\left(10 - \frac{236}{139}\right)}{96 + 139}$$

Which (when plugged into a calculator) gives 8.86 hours i.e. 08h51 and 34.5 seconds

A3.8 Students create a formula

for calculating the meeting time of any two trains between any two towns.

The intended curriculum says that students should create a formula for the meeting time of any two trains, travelling on parallel railway tracks, at any given average speeds, between any two towns where the distance between the towns is given.

Students are to use their previous work (particularly the one-line calculation) to arrive at a formula.

It seems that, if students have managed to write down a one-line calculation, then the most important (new and/or difficult) part of this sub-activity of creating a formula is simply the introduction of appropriate symbols for the variables. What follows is a suggestion for this:

Let the arrival times of Train 1 and Train 2 be $t_{1\text{arrival}}$ and $t_{2\text{arrival}}$ respectively.

Let the average speeds of Train 1 and Train 2 be v_1 and v_2 respectively.

Let the distance between the two towns be d .

Then, following the first one-line calculation given above, the formula for t^* (the meeting time of the two trains) would be:

$$t^* = \left(t_{2\text{arrival}} - \frac{d}{v_2}\right) + \frac{d - v_1\left[\left(t_{2\text{arrival}} - \frac{d}{v_2}\right) - \left(t_{1\text{arrival}} - \frac{d}{v_1}\right)\right]}{v_2 + v_1}$$

Whereas following the second one-line calculation above, the formula would look like this:

$$t^* = \frac{d + v_1 \left(t_{1_{arrival}} - \frac{d}{v_1} \right) + v_2 \left(t_{2_{arrival}} - \frac{d}{v_2} \right)}{v_2 + v_1}$$

Clearly, it is possible to perform algebraic manipulation on either one or both of these formula so that they 'look' the same.

A3.9 Alternative methods for determining a formula

Lastly, I offer two alternative solutions for the general trains problem, that is:

If two trains travelling at different (average) speeds leaving and arriving at different times from two different stations are such that the destination of each train is the starting point of the other, when will they meet?

These solutions do not follow the “trajectory” which the course designers and teachers intend for their students.

Let the train travelling from town A to B have

average speed v_1 (known)

arrival time $t_{1_{arrival}}$ (known)

departure time $t_{1_{depart}}$ (unknown)

Let the train travelling from town B to A have average speed and arrival time

average speed v_2 (known)

arrival time $t_{2_{arrival}}$ (known)

departure time $t_{2_{depart}}$ (unknown)

Let the distance between town A and town B be d kilometres (known).

$$t_{1_{arrival}} - t_{1_{depart}} = \frac{d}{v_1} \quad \text{and} \quad t_{2_{arrival}} - t_{2_{depart}} = \frac{d}{v_2}$$

$$\text{i.e.} \quad t_{1_{depart}} = t_{1_{arrival}} - \frac{d}{v_1} \quad \text{and} \quad t_{2_{depart}} = t_{2_{arrival}} - \frac{d}{v_2}$$

Let t^* be the time at which the trains meet and d^* the distance of the meeting point from town B

Route 1

Express the sum of the distances travelled in two ways and solve for the time at which the trains meet (t^*):

$$\begin{aligned}v_1(t^* - t_{1_{depart}}) + v_2(t^* - t_{2_{depart}}) &= d \\ \text{i.e.} \quad t^*(v_1 + v_2) &= d + v_1 t_{1_{depart}} + v_2 t_{2_{depart}} \\ \text{i.e.} \quad t^* &= \frac{d + v_1 t_{1_{depart}} + v_2 t_{2_{depart}}}{v_1 + v_2} \\ &= \frac{d + v_1 \left(t_{1_{arrival}} - \frac{d}{v_1} \right) + v_2 \left(t_{2_{arrival}} - \frac{d}{v_2} \right)}{v_1 + v_2} \\ &= \frac{d + v_1 t_{1_{arrival}} - d + v_2 t_{2_{arrival}} - d}{v_1 + v_2} \\ &= \frac{v_1 t_{1_{arrival}} + v_2 t_{2_{arrival}} - d}{v_1 + v_2}\end{aligned}$$

Route 2

Equate time intervals and solve for the position at which the trains meet (d^*), i.e. the distance from town B:

Let us assume (arbitrarily) that $t_{1_{depart}} < t_{2_{depart}}$

$$t^* - t_{2_{depart}} = t^* - t_{1_{depart}} - (t_{2_{depart}} - t_{1_{depart}})$$

$$\text{i.e.} \quad \frac{d^*}{v_2} = \frac{d - d^*}{v_1} - \left(\left(t_{2_{arrival}} - \frac{d}{v_2} \right) - \left(t_{1_{arrival}} - \frac{d}{v_1} \right) \right)$$

This can be solved for d^* , which is technically the only unknown in the equation:

$$\begin{aligned}v_1 d^* &= v_2 \left(d - d^* \right) - \left(t_{2_{arrival}} - t_{1_{arrival}} - \frac{d}{v_2} + \frac{d}{v_1} \right) v_1 v_2 \\ d^* &= \frac{v_2 d - \left(t_{2_{arrival}} - t_{1_{arrival}} - \frac{d}{v_2} + \frac{d}{v_1} \right) v_1 v_2}{v_1 + v_2} \\ &= \frac{v_2 d - \left(t_{2_{arrival}} - t_{1_{arrival}} \right) v_1 v_2 + v_1 d - v_2 d}{v_1 + v_2} \\ &= \frac{v_1 d - \left(t_{2_{arrival}} - t_{1_{arrival}} \right) v_1 v_2}{v_1 + v_2}\end{aligned}$$

The time of arrival, t^* , can then easily be solved for, since (for example):

$$t^* - t_{1_{depart}} = \frac{d^*}{v_1}$$

i.e. $t^* = t_{1_{depart}} + \frac{d^*}{v_1}$

For *The Trains Activity*, with one train arriving a 09:40, speed 96km/hr and other train arriving 10:00, speed 139km/hr, the solution is then:

$$t_{1_{depart}} = 7.21 \text{ hours} = 07\text{h}12$$

$$t_{2_{depart}} = 8.3 \text{ hours} = 08\text{h}18$$

$$t^* = 8.86 \text{ hours} = 08:51 (+32\text{sec})$$

$$d^* = 77.5 \text{ km} \quad \text{i.e. they meet at a distance of 77.5 km from town B.}$$

Appendix 4: Sub-activities

Identification of Sub-activities of *The Trains Activity* in the Implemented curriculum

#	Sub-activity	Justification for identifying this as a sub-activity	Markers & recognition rules for identifying this sub-activity in the video data
1	Students read the problem/situation description (individually)	Reading the given information was what students did at the start of <i>The Trains Activity</i> . This sub-activity took place for all students in the first 3 minutes of the activity.	The researcher asked the question of the data: are the students reading the given information in this segment of the video record?
2	Students create a graphical representation of the problem.	<p>The Teacher 1 instructs student to create a graphical representation of the situation:</p> <p>“... I see most of you have actually made some kind of drawing to visualise what is going on here ‘cause this is a word problem. It is not always so easy to understand the words. But once you’ve made yourself a picture of what is going on here, what is this distance? And what is this time? And where’s this train and where is that train? So if you haven’t made such a picture yet, make yourself a picture, it will help you. ‘Cause later on, when we do have a question, things will get complicated and you may get confused of what am I doing now. And then you can always go back to your picture and see ok but this is the situation. So let me now think in terms of my picture. How will I approach it? So our picture will help us in the sense of a road map. Later, when things get tricky, you can go back to your picture. So, if you haven’t done so, make yourself a picture. What is going on here.”</p> <p style="text-align: right;">(At time 00:04:10 on video file 00089)</p>	This sub-activity is evidenced and so identified by the student physically drawing a diagram on their page, depicting the initial information given.
3	Students pose “interesting” questions, in group discussion	<p>At about 7 minutes into <i>The Trains Activity</i>, the teacher stops the students’ individual work and instructs them to move into group discussions where each student is to explain to the others in his/her own words “what is happening here” and then, as a group, “to come up with questions”. He also instructs students to hide any calculations and to only make reference to their diagrams when explaining themselves to each other:</p> <p>Teacher 1 Ok, let’s let’s stop there for a minute. And in two minutes I’ll ask you to get into groups of three. But before you do so, let me first tell you what the group discussion will be about. I want you, each person in the group, to have a chance of explaining to the others what is happening here. Maybe that’s quite easy but I want you to say it in your own words, each person say it in your own words just what is happening with these trains here today. Then, once you’ve all done that, then as a group I want you to come up with questions. Are there some important or interesting questions that you can make about this situation? Are there some important or interesting questions that you can make as a group. But before you do so, first quickly, quickly tell each other in your words what is going on here. If you have done any calculations – aw, I see formulas here and there, I don’t know what questions you answered – please turn them over. When you discuss it as a group, I want you just to point towards your drawing. So that if he tells me something that I don’t understand, then I ask him “show me on your drawing; show me on your drawing; help me to understand what you are meaning”. But please, turn over any calculations or formulas that you have made. That’s not the purpose of the group discussion. The purpose is just to explain what is happening with the trains and in the second place, to try to make a question – not to answer a question yet, that we will do later individually.</p>	This sub-activity was identified by what students were doing – were they or were they not participating in a group discussion about what questions to pose. Due to the nature of the group discussions, which started and ended following the teacher’s instruction, this sub-activity spanned a specific time in the activity (from 00:10:00 to 00:20:24) for all students.

4	Students solve the privileged questions	<p>After the group discussion (sub-activity 3), the teacher leads a whole class discussion which essentially clarifies what questions students are to answer. This moves the students from the sub-activity of posing questions, to answering those questions. The indication by the teacher “You can start” (see transcription below) signals the beginning of this sub-activity.</p> <p>Teacher 1 Ok. (pause). Alright. I’ll ... You can start. But I’ll write the question on the board as well. Any any uncer... anything that I need to clear up about that question? <i>(four second pause)</i> Austen?</p> <p>Austen Is the question “at what time will they pass each other?” or “at what distance will they pass each other”?</p> <p>Teacher 1 Both. Both. At what time and at what position will the two trains cross each other. <i>(pause)</i> You can make two questions out of it. Where and when. Where and when will the two trains meet each other. <i>(Students begin to work in silence and teacher one writes the third question on the board, as well as erase the three questions which the teacher has referred to as “taken care of”).</i></p>	<p>When observing an interaction on the video records, the researcher identified the sub-activity as “students solve the privileged questions” by inspection of the student work done in a video-record (if it was after 20 minutes into the activity). The researcher asked the question: has the student obtained an answer for the departure times and the meeting time of the two trains? If yes, then the student was considered to have completed this sub-activity. If no, then the student was considered to be working on sub-activity 4. Note that, in some instances, the student work could not be seen on the video record, but this above determination was made by inferences from the teacher’s and/or student’s speech.</p>
5	Students check answers	<p>Teachers encouraged students to check their answers to the privileged questions – particularly that concerning the meeting time of the two trains – themselves.</p> <p>For example: Teacher 1 says to a student, after having a discussion with him about his work thus far:</p> <p>“I like to suggest to something to you, ‘cause as I said, you’ll check your answers yourself. If you can check your answer by maybe working backwards. Like saying ‘what if, what if that is the correct time when they meet?’ can you now calculate for that time where will train A be at that time? And then calculate where will train B be at the time? And then see, is that the same place? ‘cause if it is the same place, then you’ve then you’ve checked.”</p> <p style="text-align: right;">See 00:01:22 on file 0091</p>	<p>When observing a video-recorded interaction, the researcher determined whether a student was working on this sub-activity by inspecting the work which the student had already done. If s/he had already obtained an answer for the meeting time of the original two trains but was still working on this original question in some way, s/he was considered to be checking his/her work.</p>

6	<p>Students determine a formula for calculating the meeting time for any two trains travelling between any two towns</p>	<p>When students have solved the privileged question <i>and</i> checked their answer (i.e. completed sub-activities 4 and 5), they are to move on to “a new problem” (teacher 1, at 00:20:25 on file 00092), which is either given to the student personally/individually by a teacher who has seen that the student has completed sub-activities 4 and 5 or, in some instances, the students move on of their own accord once they have completed sub-activities 4 and 5, since sub-activity 6 was described to the whole class at the start of the last day of the course.</p> <p>Teacher 1 (at 00:20:40 on file 00092) says to a student:</p> <p>“Imagine you are the railway manager. On every day there are actually many trains going in both directions. And actually there is not only this one railway track there are also other railway tracks between your town and another t.. and other towns. So there are different railway tracks with different distances. There are different trains, some slower some faster depending on how heavy it is. So these are just two trains on one specific railway track. Now every day, if you wanna an... imagine every day you wanna answer this question for every two trains. Now you have to do the whole calculation again. That will take a lot of time. Can you make a tool that will make it quick and easy for you, so that for any two trains with any speeds with any distance between the towns with any arrival times, you can quickly calculate when they will meet?”</p> <p>At the start of the last day of the course, Teacher 1 – when speaking to the whole class – Indicated the sub-activities which students should be working through. At this time (00:09:51 on file 00094) he said:</p> <p>“Once you’ve checked your answer we will ask you another question about the trains. Ok. And, in case you are finished, the other question is: you’ve now answered it for these two trains. But imagine you are the railway manager at a big station in the city. And every day there are tens maybe twenty or different thirty different trains coming in and out. They don’t only come from the one town. They come from different towns. Some of them are long and heavy and slow. Some of them are short and fast. They’ve got different speeds. They even have different arrival times. Can you now make yourself a tool so that every day, if you are the railway manager, and let’s say – and I don’t I know it’s not really that realistic – but let’s say we want to know, for any two trains that go between the same two towns, when will they meet? Let’s say that’s your job for every day. Now for every day for the thirty trains, you can do all of these calculations. And um you’ll be happy if you leave work before ten o’clock in the evening doing these calculations for thirty trains. Can you not make yourself a tool that will make it quick and easy to calculate at what time will the two trains meet, or, pass each other? Can you not quickly just use the arrival times of those two trains? use the total distance between the towns? use the speeds of the two trains and just quickly calculate at what time will they meet? Can you make yourself a tool to do that quickly and easily for many trains? So that’s once you have finished it for this particular two trains. You don’t want to do the same work over and over again for thirty trains. So, if it takes us the whole day, it takes us a whole day. I think you can get there yourself. You you can make that tool yourself. We have enough time. We have more than enough brain power. So let’s go on.”</p>	<p>An interaction on the video records was identified as pertaining to this sub-activity if the student was determining a formula for the meeting time of any two trains. This may involve alternating between working with self-introduced symbols and the numerical values originally provided, or working only with self-introduced symbols. Students will not necessarily have completed sub-activities 4 and 5 before they start working on determining a formula.</p>
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7	Students simplify their formula (factorisation etc)	<p>Teachers explicitly told many students to do this, hence it was considered a sub-activity.</p> <p>For example, at 00:06:16 on video file 00100 (emphasis added):</p> <p>Student My formula is here, Teacher 1 'K Student Using all the information that we're already given. Teacher 1 Ok. Alright. Can you write a bit... could you find a simpler formula? Is it possible that that could be simplified? Student Mm Teacher 1 So that is departure time of train 2 minus departure time of train 1 everything multiplied by speed of train 1. That's how far train 1 has travelled by the time train 2 has just depart. Plus speed of train1 everything multiplied by total distance minus... ok. I see the same thing in two places. This departure time of train 2 minus departure time of train 1 times speed 1. Is there? Ok, maybe you can take out some common factors and something and simplify that formula a bit. And then once you've done so, just test it to the original problem again. Then just test does it really work.</p> <p>Note that not all students performed this action of simplifying their formula as a separate sub-activity. Many students created a "simplified" formula when originally creating a formula i.e. whilst working on sub-activity 6, thus for some students sub-activities 6 and 7 were merged.</p>	An interaction on the video records was considered to pertain to this sub-activity if the student had created a formula for calculating the meeting time of any two trains travelling between any two towns at any average speeds but was instructed by a teacher to first simplify their formula before checking it by substitution of the original values (sub-activity 8).
8	Students check formula (by substitution of the values given originally)	Teachers tell students to do this. For example, consider the piece of transcript provided above in sub-activity 7. At the end, the teacher says: "And then once you've done so, just test it to the original problem again. Then just test does it really work."	A student was considered to be working on this sub-activity if s/he had created a formula and was substituting the values of arrival times, speeds and distance give at the outset of <i>The Trains Activity</i> .
9	Students refine the formula so that it uses the given quantities (arrival times, average speeds and total distance between the towns) <i>only</i>	<p>Teachers explicitly told students to do this when the student had created formulas that required a preliminary calculation of departure time.</p> <p>For example, at the start of video file 00093, a student explains his formula to Teacher 1, to which Teacher 1 responds:</p> <p>Teacher 1 Now I want you to make it even easier for us, for the railway manager... In your tool, the railway manager first needs to calculate the arrival times because, like let's say in the first problem, it's only the arrival times that is specified. So you'll first have to use the arrival time to calculate the departure time. And then you have to compare the departure times to see which one departs first.</p> <p>Student Yes</p> <p>Teacher 1 Plus you have to calculate the difference between the departure times. And then only you can use this formula. Can you make it easier so that it's one formula for the time when they meet. I just say time when they meet and then I put in the arrival times – not the departure times – I put in the arrival times and I put in the speeds and I put in the total distance. I don't have to calculate departure times, I don't have to know which one is starting first. It is one formula. It immediately calculates when they will meet. So I am not saying this is wrong. I am just saying as a next step.</p>	Students were considered to be working on this sub-activity if they had already created a formula which required calculation of departure times before substitution (and possibly checked it) but they were currently adjusting their formula so that such preliminary calculation was not needed.

Appendix 5: Transcription notation

Rules for transcription:

1. Non-italicized text is used for words spoken in English.
2. Italicized text is used for words spoken in any language other than English.
3. Square brackets [text] are used for descriptions of gestures
4. Round brackets (text) are used for any researcher's notes that are added for clarification. Such clarification includes indicating mathematical symbols which speakers refer to, translation of words spoken in another language (usually Sesotho), indication of the length of pauses in speech and indicating (inaudible) when it was clear on the video record that someone was speaking but what was said was inaudible to the transcriber.
5. Each new speech turn is numbered, numerically and alphabetically. The numeral indicates the interaction number while the letter indicates the speech turn within that interaction. For example, the second speech turn in the third interaction was coded as 3b

Rules for quoting transcribed text:

1. Each speech turn's number is indicated with the quote.
2. In the case where text is omitted from a speech turn, this is indicated by an ellipsis i.e. ...

Appendix 6: Positioning analysis (detailed)

I present here my analysis of the positioning of the pedagogic subjects with respect to the privileged knowledge in the implemented *Trains Activity*. As discussed in Chapters 4 and 6, the privileged knowledge I focused on in this analysis was the sub-notions of problem solving. My presentation of these results is therefore organised around these sub-notions, and within that, the sub-activities which comprise each sub-notion (as discussed in Chapter 5).

While I have analysed all 30 interactions of students in the focus group with teachers (as discussed in Chapter 4), I present here only the interactions with teachers in which Student 4 participated. This includes all whole class discussions and a group discussion. As discussed in Chapter 6, this selection was deemed necessary due to the constraints of this document, and it was deemed a sufficient representation of my data since the teachers interacted (individually) similarly with the four students in the focus group.

Sub-notion 1: Asking appropriate questions about a given scenario

Table 6.1 Positioning in sub-activity 1 (students read the problem description)

Interaction number	Transcript	Transmission function	Acquisition function
1 ₁	<p>While the students are reading the information on <i>The Trains Activity</i> page, a student asks Teacher 1 a question privately to which Teacher 1 addresses the whole class in response:</p> <p>1a Student What is the question here? [spoken quietly to the teacher only]</p>	<p>Distributed to: Teacher as teacher</p> <p>Notes: The teacher is being asked for assistance. (#4)</p>	<p>Distributed to: Student as student</p> <p>Notes: The student is asking for assistance. (#4)</p>
1 ₂	<p>1b Teacher 1 You are the boss; there are no questions in front of you; you are the boss, you are the one that asks the questions. There is only what is. You have to decide what I want to investigate. What is interesting? Is there a problem? [spoken loudly to the whole class]</p>	<p>Distributed to: Teacher as teacher AND Student as other</p> <p>Notes: The teacher is giving instructions related to actions which are intended to lead to knowledge acquisition (#6a)</p> <p>The student is</p> <ul style="list-style-type: none"> - Being described implicitly as aligned with knowledge (#2a). That is, the student is given the role of someone other than himself who is knowledgeable (“the boss”; “the one that asks the questions”). - Being instructed to do something which aligns with knowledge: “you have to decide” (#6a) 	<p>Distributed to: No-one</p> <p>Notes:</p>

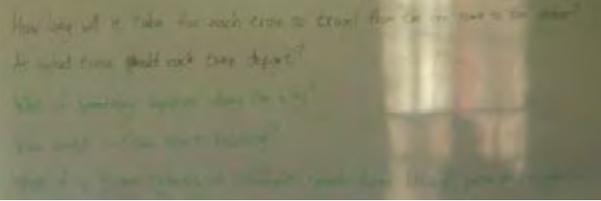
<p>2₁</p>	<p>Teacher 1 addresses the whole class. This interaction occurs about 3 minutes after interaction 1 and nothing has been said to the class as a whole since interaction 1; the students have just continued to read the written description in silence.</p> <p>2a₁ Teacher 1 Ok, there's another reason why we didn't ask you a question. It's that, often when a student gets a question, they immediately grab a calculator for help. In calculators we trust. I'm just making a little bit of a joke about it. Often, when you ask the question, you immediately want to start calculating. We spoke about this in the previous days as well. Often it's good not to start calculating too quickly. First to plan. First to make a plan. And then once you have a good plan, the calculations are easy. But if you don't have a plan, you can do all the calculations in the world and it won't help you. So we didn't ask you a question also so that you cannot calculate anything 'cause there's nothing to answer. But you have to think about what is going on here. And we just wanted to give you a chance before anything is calculated just to think about what is going on here.</p>	<p>Distributed to: Teacher as teacher</p> <p>Notes: The teacher is explaining content in order to progress the judgement of the notion (#3).</p>	<p>Distributed to: Student as student</p> <p>Notes: The students are</p> <ul style="list-style-type: none"> - listening to an explanation which is intended to progress their own judgement of the notion (#3) - being described as ignorant (#2b): "often when a student gets a question, they immediately grab a calculator for help..." <p>Requiring "help" suggests ignorance.</p>
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Table 6.2 Positioning in sub-activity 2 (students create a graphical representation)

Interaction number	Transcript	Transmission function	Acquisition function
2 ₂	<p>2a₂ Teacher 1 And I see most of you have actually made some kind of drawing to visualise what is going on here 'cause this is a word problem.</p>	<p>Distributed to: Teacher as teacher AND Student as student</p> <p>Notes: The teacher is selecting what student productions to make public to the whole class (#7).</p> <p>The students are being described as being aligned with knowledge (#2a) "most of you have actually made some kind of drawing to visualise what is going on here."</p>	<p>Distributed to: No-one</p> <p>Notes:</p>
2 ₃	<p>2a₃ Teacher 1 It is not always so easy to understand the words. But once you've made yourself a picture of what is going on here, what is this distance? And what is this time? And where's this train and where is that train? So if you haven't made such a picture yet, make yourself a picture, it will help you. 'Cause later on, when we do have a question, things will get complicated and you may get confused of "what am I doing now?" And then you can always go back to your picture and see "ok but this is the situation. So let me now think in terms of my picture. How will I approach it?" So our picture will help us in the sense of a road map. Later, when things get tricky, you can go back to your picture. So, if you haven't done so, make yourself a picture. What is going on here?</p>	<p>Distributed to: Teacher as teacher AND Student as student</p> <p>Notes: The teacher is - Explaining (#3) - Giving instructions (#6a).</p> <p>The student is being instructed to do something which aligns with knowledge ("So, if you haven't done so, make yourself a picture.") (#6a).</p>	<p>Distributed to: Student as student AND Teacher as other (student)</p> <p>Notes: The students are: - Listening to an explanation (#3) - Being described as aligned with ignorance: "it is not always easy to understand", "you may get confused" and "when things get tricky". (#2b)</p> <p>The teacher speaks with the voice of a student, whilst implying that the students are ignorant: "you may get confused of 'what am I doing now?'" (#2b)</p>

Table 6.3 Positioning in sub-activity 3 (Students pose “interesting” questions)

Interaction number	Transcript	Transmission function	Acquisition function
3 ₁	<p>At about 7 minutes into <i>The Trains Activity</i>, Teacher 1 brings the students’ individual diagram drawing to a halt and addresses the whole class as follows:</p> <p>3a₁ Teacher 1 Ok, let’s let’s stop there for a minute. And in two minutes I’ll ask you to get into groups of three. But before you do so, let me first tell you what the group discussion will be about. I want you, each person in the group, to have a chance of explaining to the others what is happening here. Maybe that’s quite easy but I want you to say it in your own words, each person say it in your own words just what is happening with these trains here today. Then, once you’ve all done that, then as a group I want you to come up with questions. Are there some important or interesting questions that you can make about this situation? Are there some important or interesting questions that you can make as a group? But before you do so, first quickly, quickly tell each other in your words what is going on here. If you have done any calculations – aw, I see formulas here and there, I don’t know what questions you answered – please turn them over. When you discuss it as a group, I want you just to point towards your drawing.</p>	<p>Distributed to: Teacher as teacher AND Student as student</p> <p>Notes: The teacher is - Giving instructions (#6a).</p> <p>The students are being instructed to perform actions which presuppose their alignment with knowledge (“explaining to others” and “say it in your own words” and “come up with questions”). (#6a)</p>	<p>Distributed to: No one</p> <p>Notes:</p>
3 ₂	<p>3a₂ Teacher 1 So that if he tells me something that I don’t understand, then I ask him “show me on your drawing; show me on your drawing; help me to understand what you are meaning”.</p>	<p>Distributed to: Student as student</p> <p>Notes: The student is - Being asked for assistance (hypothetically) regarding the knowledge which is to be transmitted/acquired by the teacher as student. (#4)</p>	<p>Distributed to: Teacher as other (student) AND Student as student</p> <p>Notes: The teacher takes on the voice of a student whilst he - Describes himself – and therefore also the students – as being aligned with ignorance (“if he tells me something that I don’t understand”) (#2b) - Asks for assistance (#4).</p>
3 ₃	<p>3a₃ Teacher 1 But please, turn over any calculations or formulas that you have made. That’s not the purpose of the group discussion. The purpose is just to explain what is happening with the trains and in the second place, to try to make a question – not to answer a question yet, that we will do later individually.</p>	<p>Distributed to: Teacher as teacher AND Student as student</p> <p>Notes: The teacher is giving instructions (#6a).</p> <p>The students are being instructed to perform actions (explain to their peers) which presuppose their alignment with knowledge (#6a).</p>	<p>Distributed to: No one</p> <p>Notes:</p>

<p>4₁</p>	<p>This interaction occurs on Day 4 of course, 20 minutes into the beginning of <i>The Trains Activity</i>. Prior to this segment of whole class discussion, the students have been working in groups, discussing “what is going on here”. The teacher, towards the end of the group discussions writes the questions on the board which students have posed. The first two questions are written in black, while the 3rd, 4th and 5th (which are eventually erased) are written in green:</p>  <p>The questions written on the board (shown in the screen shot above):</p> <ul style="list-style-type: none"> How long will it take for each train to travel from [illegible]? At what time should each train depart? What if something happens along the way? When should a train start braking? What if a train travels at different speeds during the journey? <p>Once the teacher has finished writing the questions on the board, he addresses the whole class:</p> <p>4a₁ Teacher 1 Ok [spoken slowly]. May I stop the discussion there? I think you’ve generated some questions and I think you all understand what’s going on with these two trains travelling. Can you move back to your desks? and then we’ll have a short whole class discussion. (20 second pause while students move out of their groups and back to their own, original desks)</p> <p>‘k, I have written most of the questions that I could remember you talking about, most of those questions on the board. [Reading the questions on the board to the class, whilst still mostly facing the class, making eye contact with students:]</p> <p>How long will a train take for a train to travel from the one town to the other? So the journey time. At what time should each train depart so that it will arrive on time? Somebody said what if a what if something happens to a train along the way? Maybe it breaks down or something. When should a train start braking before it reaches the next town? It should start braking early enough, that’s a important question. What if the train travels at different speeds at different parts of the journey? That person pointed out that we are only given the average speed. So that doesn’t tell us that it travelled at the same speed always. Maybe some parts it travelled at 120 kilometres per hour, some part at 90 kilometres per hour (pause). ‘K. One or two of those questions I want to answer for you. First. And then I want you to. And then I want us to think of making yet another question.</p>	<p>Distributed to: Student as student AND Teacher as teacher</p> <p>Notes: The teacher is: - Selecting what student productions (questions) to make public to the whole class (#7) - Assessing the students’ productions when he says “I think you all understand what’s going on with these two trains travelling” (#1a)</p> <p>The students are having their productions favourably assessed (#1a)</p>	<p>Distributed to: No-one</p> <p>Notes: Initially, while the teacher writes the questions on the board, the students are still engaged in group discussions where they are listening to their peers’ explanations, so the positioning – student distributed AF – is maintained. However, as soon as the group discussions end, this positioning is lost.</p>
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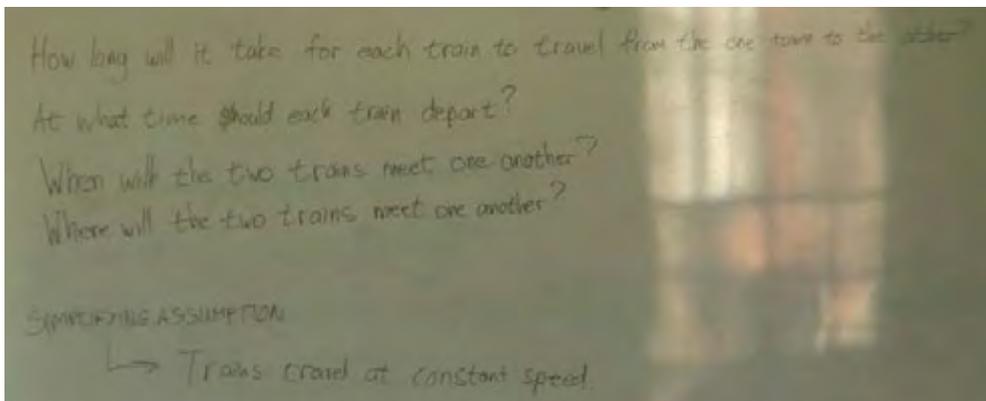
4 ₂	<p>4a₂ Teacher 1 Firstly, I I want to make a simplifying assumption. I want to say: let us assume that the trains travel at a constant speed [writes this on the board]. Why am I making that assumption? Because it's easier for me to calculate things. If the speed changes all of the time, it's very difficult to tell you where will the train be at a certain time. Because what was the speed in this interval and it gets complicated.</p>	<p>Distributed to: Teacher as teacher</p> <p>Notes: The teacher is explaining content (#3)</p>	<p>Distributed to: Student as student AND Teacher as teacher</p> <p>Notes: Students are: - Listening to an explanation (#3) - Being described as aligned with ignorance implicitly in the teacher's explanation, since he says he makes the assumption because it's "easier" and "it gets complicated" (#2b).</p> <p>Teacher is implicitly describing himself as ignorant by highlighting that "it's easier for me" and "it's very difficult to tell you" (#2b).</p>
4 ₃	<p>4a₃ Teacher 1 So (pause) just as a first as a first way of approaching this problem – and this is something that engineers and scientists do – they say "let us first look at this problem in a simple way. And see if we can solve that. And once we've solved that we can make it more complicated and add acceleration and braking and all of that. But if I cannot solve a problem for the simple case, I will also not be able to solve it for the difficult case." So they say: "let me first simplify matters."</p>	<p>Distributed to: Teacher as other (engineer)</p> <p>Notes: The teacher continues to explain content. (#3)</p> <p>The teacher takes on the persona of an engineer: "...this is something that engineers and scientists do – they say "let us... but if I..."</p>	<p>Distributed to: Student as student AND Teacher as other (engineer)</p> <p>Notes: Students continue to listen to explanation (#3)</p> <p>The teacher is being described as aligned with ignorance (#2b) in his own speech (whilst speaking with the voice of an engineer): "if I cannot solve a problem..."</p>
4 ₄	<p>4a₄ Teacher 1 So I'm simplifying matters by saying "let's assume that the trains travel at a constant speed." Ok, of course the the speed of a train is not going to change a lot, hopefully, but it still it's a approximation, it's not quite the truth, but now it's easier to do calculations. So for today, let's stick with that assumption. Just to make matters simpler.</p> <p>So, from the time that train A departs until the time that it arrives at the other side [teacher walks across the front of the room as he says this], it's travelling at a constant speed [he is now standing still]. So we're not thinking about how long it takes to accelerate up to speed and then to decelerate down to stand still again. We just treat it as a constant speed. So that that deals actually with all three of these questions [points to the three questions on the board written in green – see screen shot – "what if something happens along the way? When should a train start braking? What if a train travels at different</p>	<p>Distributed to: Teacher as teacher</p> <p>Notes: Same as for 4a₂:</p> <p>The teacher is explaining (#3)</p>	<p>Distributed to: Student as student AND Teacher as teacher</p> <p>Notes: Same as for 4a₂:</p> <p>Students are: - Listening to an explanation (#3) - Being described as aligned with ignorance (#2b)</p> <p>Teacher is implicitly describing himself as ignorant by highlighting that "now it's easier to do calculations" (#2b).</p>

		speeds during different parts of the journey?] We're assuming or we're approx... Ja. We're simplifying the problem by saying "no, it's not going to break down in the middle of the track. It'll just keep on travelling at the same speed." We don't have to think about braking. And we don't think have to think about speed changing 'cause we're approximating that it'll travel at a constant speed.		
4 ₅	4a ₅ Teacher 1	The two top questions still remain now (pause). Now I want (pause) of course I have a certain question in mind and I also thought of the first two questions but I want you to think of another question.	<p>Distributed to: Teacher as teacher AND Student as student</p> <p>Notes: The teacher is - Assessing the student's production: "I also thought of the first two questions." (#1a) and "of course I have a certain question in mind" (#1b) - Being described implicitly as aligned with knowledge: "I have a certain question in mind" (#2a) - Giving instructions to students (#6a)</p> <p>The students are - Having their productions favourably assessed: "I also thought of the first two questions." (#1a) - Being given instructions which align with knowledge (expected to pose questions): "I want you to think of another question." (#6a)</p>	<p>Distributed to: Student as student</p> <p>Notes: The student's productions are being unfavourably assessed by the teacher (#1b) when he says "of course I have a certain question in mind" since this statement suggests that the teacher has judged the questions posed by the students as inadequate.</p>
4 ₆	4a ₆ Teacher 1 4b Students	Will you help me Austen? We need to do a demonstration. We're going to be the two different trains. [Austen stands up and stands at front of class on by the door and teacher 1 walks to the opposite wall, still in front of the class]. But while. So he's (Austen is) at town A, I'm at town B. But before we depart, do you think we will depart at the same time? (2 second pause) Or do you know that we will depart at the same time? (Students respond but their responses are inaudible.)	<p>Distributed to: Teacher as other (train)</p> <p>Notes: The teacher is - Explaining (#3) - Posing questions: "do you think we will depart at the same time?" (#5)</p>	<p>Distributed to: Student as student</p> <p>Notes: The students listen to (and watch) an explanation (#3)</p>
4 ₇	4c ₁ Teacher 1	You don't know that. Maybe it will be like that. [Teacher 1 looks at the students who offered the response. His arms are wide open, elbows at his side, eyebrows raised].	<p>Distributed to: Teacher as teacher</p> <p>Notes: The teacher is assessing the students' production (response to his question) (#1b)</p>	<p>Distributed to: Student as student</p> <p>Notes: The students are having their production (response to the teacher's question) unfavourably assessed (#1b)</p>

4 ₈	<p>4c₂ Teacher 1 We can check by calculating but we don't know it.</p>	<p>Distributed to: Teacher as teacher</p> <p>Notes: The teacher is</p> <ul style="list-style-type: none"> - Assessing the students' production (answer to question) (#1b) - Explaining content (#3) 	<p>Distributed to: Student as student AND Teacher as teacher</p> <p>Notes: Students are</p> <ul style="list-style-type: none"> - Having their production unfavourably assessed. (#1b) - Listening to explanation (#3) <p>The teacher is</p> <ul style="list-style-type: none"> - Being described as aligned with ignorance "we don't know it". (#2b)
4 ₉	<p>4c₃ Teacher 1 So let's say that we won't depart at the same time. So Austen, you come first [Teacher 1 gestures with his hand for Austen to come to him] but walk very slowly [Austen starts to walk towards Teacher 1]. Time goes on and now I also depart [Teacher 1 starts walking towards Austen]. Hello Austen [said at the time when Teacher 1 and Austen pass each other].</p> <p>4d Austen Hi Steven [other students laugh]</p>	<p>Distributed to: Teacher as other (train)</p> <p>Notes: The teacher explains content (#3)</p>	<p>Distributed to: Student as student</p> <p>Notes: The students listen to (and watch) an explanation (#3)</p>
4 ₁₀	<p>4e Teacher 1 Is there something interesting that we could find out?</p> <p>4f Students Yeah. (Many students respond, most responses are inaudible.)</p>	<p>Distributed to: Teacher as teacher AND Student as student</p> <p>Notes: The teacher is</p> <ul style="list-style-type: none"> - Posing a question (#5) and thereby tacitly instructing the students to pose another question, that is, he is - Giving instructions (#6a) <p>The students are being instructed to do something which aligns with knowledge (pose another question) (#6a)</p>	<p>Distributed to: No-one</p> <p>Notes:</p>
4 ₁₁	<p>4g₁ Teacher 1 When will we pass each other [voice inflects down, seems to be repeating and affirming a student response given in 4f].</p>	<p>Distributed to: Teacher as other (train) AND Student as student</p> <p>Notes: The teacher is assessing the students' production (answer to question) (#1a)</p> <p>The students are having their answer favourably assessed. (#1a)</p>	<p>Distributed to: none</p> <p>Notes:</p>

412	4g ₂	Teacher 1	Thank you Austen. [Austen returns to his seat]. Ok. Now those first two questions [points to the board] are also important. But I want to add the third one. At what time – and maybe I can even ask – at what position along the railway track will the two trains meet each other. At what time and at what position will the trains meet each other?	<p>Distributed to: Teacher as teacher AND Student as student</p> <p>Notes: The teacher is: - Assessing the students' productions favourably (in 4g₂: "Now those first two questions are also important.") (#1a) and simultaneously unfavourably (in 4g₂: "But I want to add the third one." (#1b). - Posing questions in 4g₂ and 4m (#5) - Explaining in 4i and 4m (#3) - Teacher is selecting what student production to make public when he writes the question on the board in 4k (#7) - Teacher is being asked for assistance in 4h and 4l (#4)</p> <p>The students are: - Having their production favourably assessed in 4g₂. (#1a) - Described as being aligned with knowledge (#2a) in 4i when the teacher says the problem must be "difficult enough also for you"</p>	<p>Distributed to: Student as student</p> <p>Notes:</p> <ul style="list-style-type: none"> - The students are having their productions unfavourably assessed in 4g₂ since the teacher is suggesting that the two questions they have posed are insufficient by his wanting to add a third question. (#1b) - The students are asking for assistance regarding the knowledge which is to be transmitted/acquired (4h and 4l) (#4) - The students are being described as aligned with ignorance in the teacher's statement "a realistic problem which is simple enough" (#2b) - The students listen to explanation (#3)
	4h	Student A	But like of what importance is that though?		
	4i	Teacher 1	It's of importance for the mathematics that you will learn by finding it out, to be honest [teacher smiles, some students laugh]. Ok. So we're trying to give you problems that are realistic, yes, but it's sometimes difficult to find a realistic problem which is simple enough but difficult enough also for you. So sometimes we bend the reality a little bit. And we just say: this is a interesting question. Ok of course if there was only one railway track, this would be the time at which the two trains would crash and everybody would be dead. But we didn't want to choose that example because it's a bit you know depressive you know [students laugh]. And we want you to be positive about life. So we said no it's not crashing. They are just greeting each other. So maybe it's not so important, but it's interesting. And the mathematics that you will learn when you try to answer that question will be very interesting. Have I answered you?		
	4j	Student A	Ja		
	4k	Teacher 1	Ok (pause). Alright. I'll. You can start. But I'll write the question on the board as well. Any any uncer' anything that I need to clear up about that question? (four second pause) Austen?		
	4l	Austen	Is the question "at what time will they pass each other?" or "at what distance will they pass each other"?		
4m	Teacher 1	Both. Both. At what time and at what position will the two trains cross each other (pause). You can make two questions out of it. Where and when. Where and when will the two trains meet each other. [Students begin to work in silence and teacher 1 writes the third question on the board, as well as erases the three questions which the teacher has referred to as "taken care of"].			

Final, privileged questions written on the board:



Sub-notion 2: Calculating specific values about a specific instance of given scenario

Table 6.4 Positioning in sub-activity 4 (students solve the privileged questions)

Interaction number	Transcript	Transmission function	Acquisition function
5	<p>Day 4 of course, about 16 minutes into <i>The Trains Activity</i>. While the students are all working on their own to solve the privileged questions, Teacher 3 approaches Student 4, reads the student's written work and then starts speaking:</p> <p>5a Teacher 3 How do you calculate the average speed?</p> <p>5b Student 4 Average speed? it is when you do finally position minus initial position (pauses for 2 seconds) over final time minus (2 second pause) initial (speaking very slowly, hesitantly).</p> <p>5c Teacher 3 So then this thing, this thing, on the left hand side [points on student's page, unclear] is the same as this one on the right hand [points on student's page, unclear] so then what are you calculating when you add it like this? You can add it like this [points, unclear] or like this [points, unclear]. So what are you calculating? You can add it like this [points on student's page, unclear] or like this [points on student's page, unclear].</p> <p>5d Student 4 Average speed</p> <p>5e Teacher 3 So then which means you have the average speed and you have the average distance then it means you can calculate the average time, isn't it?</p> <p>5f Student 4 (2 second pause) oh ja</p> <p>5g Teacher 3 Why don't you back to question 1 and try to do it again?</p>	<p>Distributed to: Teacher as teacher</p> <p>Notes: The teacher is</p> <ul style="list-style-type: none"> - Explaining content in 5c and 5e (#3) - Posing a question in 5c (#5) - Instructing the student to do something which aligns with ignorance in 5g (#6b) - Assessing the student's production unfavourably (#1b). That is, by asking the student to re-do question 1 (in 5g) the teacher implies that her first attempt at answering question 1 was incorrect. 	<p>Distributed to: Student as student</p> <p>Notes: The student is</p> <ul style="list-style-type: none"> - Listening to an explanation (#3) - Being instructed to do something which aligns with ignorance in 5g (#6b) - Having her production unfavourably assessed (#1b)

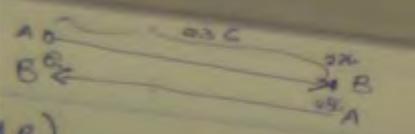
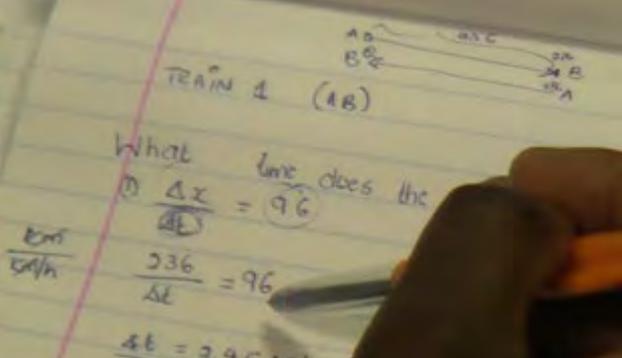
<p>11₁</p>	<p>Teacher 4 approaches student 3 immediately after interaction 10. He does not read her work first. She is trying to determine the meeting time of the two trains.</p> <p>11a Teacher 4 Tawela, you seem like you want to talk to somebody</p> <p>11b Student 4 [laughs, says something inaudibly]</p> <p>11c Teacher 4 [Kneeling down to her table level]. Talk to me. I'm just the railway conductor, but maybe I can talk about it. What's going on in your mind at the moment?</p> <p>11d Student 4 Like I'm finished with the calculations. I think that. I was trying to discuss with this guy [indicates Student 3 next to her] is that how can we di' distinguish between this two templates where actually. After we've calculated initially they've departed on their durations he came up with the thought that we should like we should restart what we did and make them start at the same time.</p>	<p>Distributed to: Teacher as other (railway conductor)</p> <p>Notes: The teacher is posing a question in 11c (#5)</p>	<p>Distributed to: Student as student</p> <p>Notes: The student is being described implicitly as ignorant in 11d when she points out that "he came up with the thought..." as opposed to it being her own thought. (#2b)</p>
<p>11₂</p>	<p>11e Teacher 4 So you are at that point now, you are starting to analyse what happens at the restart. Well, you surely have to do that, there's no doubt about that. [Walks away]</p>	<p>Distributed to: Teacher as teacher AND Student as student</p> <p>Notes: The teacher assesses the student's production (#1a)</p> <p>The student has her production favourably assessed (#1a)</p>	<p>Distributed to: None</p>

<p>14₁</p>	<p>Teacher 1 and Teacher 4 address the whole class and call an end for the day, but indicate that they will continue working on this activity tomorrow</p> <p>14a Teacher 1 Right everyone. Let's stop there for the day.</p> <p>14b Student No</p> <p>14c Teacher 1 No? ok, we will go on 'til six o'clock. Yes</p> <p>14d Student Yes</p> <p>14e₁ Teacher 1 Ok. I we going to stop there for the day. We you carrying on with this problem tomorrow. And I want to ask you a big favour and it's for yourself. Don't go to the cleverest person in the class tonight and ask him or her</p>	<p>Distributed to: Student as student AND Teacher as teacher</p> <p>Notes: The teacher is giving instructions (#6a)</p> <p>The students are</p> <ul style="list-style-type: none"> - Being instructed to do something which aligns with knowledge (not ask for assistance) (#6a) - Being described as aligned with knowledge when the teacher refers to "the cleverest person in the class" (#2a) 	<p>Distributed to: No-one</p> <p>Notes:</p>
<p>14₂</p>	<p>14e₂ Teacher 1 'just show me how to do it so that I can get the right answer'. Because then it will you will always remember that that person told you what to do. But if you just let it lie and you come back tomorrow and you try on your own again after tomorrow you will always remember</p>	<p>Distributed to: Student as student</p> <p>Notes: The students are</p> <ul style="list-style-type: none"> - hypothetically being asked for assistance: "just show me how to do it" (#4) - Being described as aligned with knowledge: "that person told you what to do" (#2a) 	<p>Distributed to: Teacher as other (student) AND Student as student</p> <p>Notes: The teacher is</p> <ul style="list-style-type: none"> - Asking for assistance with the voice of a student (#4) <p>The student is</p> <ul style="list-style-type: none"> - hypothetically asking for assistance (#4) - being described (implicitly) as aligned with ignorance: "that person told you what to do" (#2b)
<p>14₃</p>	<p>14e₃ Teacher 1 'I solved this problem myself. Nobody told me what to do'</p>	<p>Distributed to: Teacher as other (student) AND Student as student</p> <p>Notes: The teacher takes on the voice of a student and describes himself (the hypothetical student) as knowledgeable. (#2a)</p>	<p>Distributed to: No-one</p> <p>Notes:</p>
<p>14₄</p>	<p>14e₄ Teacher 1 because we're not telling you what to do. We're trying to help you without telling you what to do.</p>	<p>Distributed to: Teacher as teacher AND Student as student</p> <p>Notes: The teachers are</p> <ul style="list-style-type: none"> - withholding assessment (#1c) and - helping students, which suggests that they are (hypothetically) being 	<p>Distributed to: Student as student</p> <p>Notes: The students are</p> <ul style="list-style-type: none"> - (hypothetically) asking for assistance (#4) - Implicitly being described as ignorant, since the teacher

			asked for assistance (#4) The students are - expected to assess for themselves “what to do” (#1c)	suggests that they need “help” (#2b)
14 ₅	14e ₅ Teacher 1	So I would like – and I believe all of you will get there – by the end of tomorrow you will feel	Distributed to: Student as student AND Teacher as teacher Notes: The students are being described as aligned with knowledge in “I believe all of you will get there” (#2a) This comment doesn’t position the teacher in any way. He retains the positioning indicated in 14e ₄ .	Distributed to: No-one Notes:
14 ₆	14e ₆ Teacher 1	‘uh-ah the clever boy didn’t tell me what to do. I figured out what to do. I solved it myself.’	Distributed to: Teacher as other (student) AND Student as student Notes: As per 14e ₃	Distributed to: No-one Notes:
14 ₇	14e ₇ Teacher 1	It’s very nice to have that memory. So guard that memory. And don’t let somebody just tell you what to do. Tell them,	Distributed to: Student as student AND Teacher as teacher Notes: (#6a) and (#2a) As per 14e ₁	Distributed to: No-one Notes:
14 ₈	14e ₈ Teacher 1	‘listen this is my thinking. Don’t intrude on my thinking.’	Distributed to: Teacher as other (student) AND Student as student Notes: (#2a) As per 14e ₃	Distributed to: Notes:
14 ₉	14e ₉ Teacher 1 14f ₁ Teacher 4	Your thinking is precious and very is valuable. And it’s not just about that memory and the pride of doing it yourself. It’s about something else too and I think many of you know it. [Points to different students in the classroom each time he says: “you know it”].	Distributed to: Student as student AND Teacher as teacher Notes: The student is being described as aligned with knowledge in both teachers’ comments (#2a)	Distributed to: Notes:
14 ₁₀	14f ₂ Teacher 4	That by wrestling with this problem, you are learning a lot. The moment this problem is solved, it has lost its value for you. (Inaudible) The moment you know the solution – <i>especially</i> if someone else has shown you how to solve it – then this problem has no value for you anymore. This problem is valuable for you as long as	Distributed to: Teacher as teacher AND Student as student Notes: The teacher is	Distributed to: Student as student Notes: The students are - listening to explanation (#3)

	<p>you wrestle with it. Work at it. So it's a matter of protecting your own opportunities. That's why you should not consult somebody who's already solved it. You can talk to other people. But talk to other people who are also struggling. To share ideas. But don't talk to the person who has done it. Because if that person gives you the solution it will just take it away. (Inaudible). Protect your own opportunities.</p>	<ul style="list-style-type: none"> - Explaining (#3) - Giving instructions (#6a) <p>The students are</p> <ul style="list-style-type: none"> - Being instructed to do something which aligns with knowledge (not ask for assistance) (#6a) - Being described as aligned with knowledge "the person who has done it." (#2a) 	<ul style="list-style-type: none"> - Being described as aligned with ignorance ("who are also struggling") (#2b)
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15	<p>This interaction takes place at the start of the last day of the course (ie the day after interaction 14)</p> <p>15a Teacher 1 Hi Tawela 15b Student 4 Hey 15c Teacher 1 I see you considering that formula [points to student's written work, unclear]. Um, it's got the initial speed, time duration and veloc' 15d Student 4 Acceleration 15e Teacher 1 Acceleration. Do you know the acceleration of these trains? 15f Student 4 Yes, they mention the the the speed is constant. 15g Teacher 1 So what is the acceleration? 15h Student 4 Zero 15i Teacher 1 Ok [nods head. Pause for 15 seconds while teacher and student look at her written work]. Here it is, speed speed final and speed initial? 15j Student 4 Uh 15k Teacher 1 What do you know about them? 15l Student 4 (4 second pause) I know the initial speed of the train but I don't know the final. 15m Teacher 1 You don't know the final speed. So you know the initial speed of the train but not the final speed? 15n Student 4 Yes 15o Teacher 1 Is the train accelerating or decelerating? 15p Student 4 Decelerating? 15q Teacher 1 Decelerating means the, is the opposite of de dedece' decelerate is the opposite of accelerate. So at the start when you start a car you accelerate. And then when you brake you decelerate. Is the train train 1 accelerating or decelerating at any time? 15r Student 4 Decelerating 15s Teacher 1 It's decelerating. (5 second pause) And what will happen to the speed of a train when it decelerates? 15t Student 4 (6 second pause) I don't know 15u Teacher 1 Will the speed stay the same when you decelerate will it will the speed increase or will the speed decrease? 15v Student 4 (inaudible) 15w Teacher 1 Say again? 15x Student 4 What happens to the time? 15y Teacher 1 Time is going on, ok? So let's forget about the formulas for a moment. You you're you're in a car and you said the trains are traveling at a constant speed. So what can you say about the speed at the start and the speed at the end if you are travelling at a constant speed? 15z Student 4 It's decelerating at the end. 15aa Teacher 1 It's? how do you know that it's decelerating at the end? 15ab Student 4 Because it has to stop. 15ac Teacher 1 Ok. So what will be the speed right at the end when it has stopped?</p>	<p>Distributed to: Teacher as teacher AND Student as student</p> <p>Notes: The teacher is</p> <ul style="list-style-type: none"> - Posing questions in 15e, 15g, 15o, 15q, 15s, 15u, 15y, 15aa, 15ac, 15ae, 15ag, 15ai and 15as (#5) - Explaining content in 15c, 15q, 15y, 15ae, 15ai, 15as and 15au (#3) - Being asked for assistance in 15x (#4) - Giving an instruction in 15y ("let's forget about the formulas") (#6b) and in 15as ("you have to ask yourself...") (#6a) - Making assessments of the student's productions (answers to his questions and her written work) (#1) <p>The student is</p> <ul style="list-style-type: none"> - Being expected (and herself expecting) to assess her own production (#1c). This is seen in the discussion between 15a and 15al, where the teacher is withholding assessment. The student is making incorrect statements in answer to the teacher's questions (15z, 15ab, 15ad, 15af) but the teacher does not directly correct her. He acknowledges her responses ("ok") and 	<p>Distributed to: Student as student</p> <p>Notes: The student is</p> <ul style="list-style-type: none"> - Listening to explanation (#3) - Being described as aligned with ignorance in 15l and 15t (#2b) - Asking for assistance in 15x (#4) - Being instructed to do something which aligns with ignorance in 15y (#6b) - Having her production unfavourably assessed in 15as ("your formula is not going to tell you") and 15au ("that formula is not helping you about this situation") (#1b) <p>When the teacher says "Ok" (in 15i, 15ac, 15ae and 15ai) note that it is merely acknowledgement of the student's response to his questions. It is not indication of favourable assessment (that is, #1a). Sometimes the student's response which he acknowledges is correct, sometimes it is incorrect.</p>
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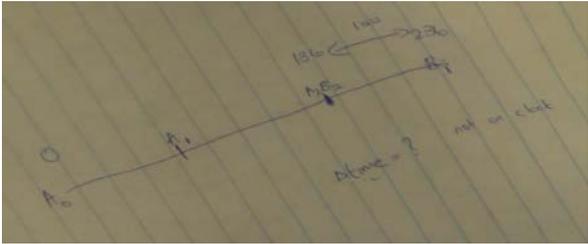
15ad	Student 4	Zero.	<p>then goes on to explain or pose another question (eg. 15ac, 15ae and 15ai). The withholding of assessment culminates in the student indicating wanting to assess herself (15al).</p> <ul style="list-style-type: none"> - Being described as aligned with knowledge in 15au (#2a) when the teacher says “you told me” and “it’s not that the formula is not true” - Being instructed to do something which aligns with knowledge in 15as: “You have to ask yourself” (#6a)
15ae	Teacher 1	<p>Ok. In the way that we look at these two, um, so can you show me a drawing of [he trails off and she shows him her diagram]. Ok, so let’s say train one goes from A to B [points to left and right ends of her diagram] and train one travels at ninety-six (96) kilometres per hour. Will there be any point on this railway track between A and B where the speed is less than (96) kilometres per hour?</p>	
			
15af	Student 4	(12 second pause) Because time is increasing, the speed is also increasing.	
15ag	Teacher 1	You say if time increases, speed increases?	
15ah	Student 4	Hm (yes)	
15ai	Teacher 1	<p>Ok, so let’s we’ve made the assumption of a constant speed. Is it possible that the time that the train travels at a constant speed and then as time increases, speed increases but it’s still travelling at a constant speed? Can I increase my speed and still travel at a constant speed?</p>	
15aj	Student 4	[shakes head] No	
15ak	Teacher 1	(11 second pause) what are you thinking now? What are you feeling?	
15al	Student 4	(11 second pause) I want to check that	
15am	Teacher 1	What would you want to check?	
15an	Student 4	I need the increase in the time.	
15ao	Teacher 1	Sorry?	
15ap	Student 4	I need the time [circles Δt on her page]	
			
15aq	Teacher 1	Hm	
15ar	Student 4	(Inaudible)	
15as	Teacher 1	<p>Your formula is not going to tell you. This is going to tell you that. The formula is not going to tell you that. You have to ask yourself: what, is this a constant speed? Or does this speed change? (22 second pause) You have to think about this situation first. And then if the formula doesn’t apply to the situation, then can you use that formula?</p>	
15at	Student 4	No	
15au	Teacher 1	<p>So, if this was a constant acceleration situation, if this was a constant acceleration situation, car started from zero and constantly accelerated, then that would be a useful formula [points to formula on page, unclear]. But here, you told me the train is not accelerating at all. The acceleration is zero. So it’s not that the formula is not true. It’s just that formula is not helping you about this situation. In this situation, speed at the start and speed at the end are the same because speed is constant so you do not</p>	

	even you do not need a formula to calculate speed at the end because you know it's still ninety-six (96). (Pause for a few seconds) [teacher stands up and walks away].		
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16 ₁	<p>Teacher 1 addresses the whole class about 45 minutes into the start of the last day of the course. He gives instructions for group discussions.</p> <p>16a₁ Teacher 1 Hi everyone. I want you to stop for a minute and um I want you to get into groups and have a discussion in groups. But before you do so, I want you to put all of your work away. Just put it in your book and close it. But before ah sorry it's one more thing. Not everybody will have a group discussion now. Ok. If you have found the time at which the two trains meet, if you've checked your calculations then if you've also checked your answer in a different way working backwards to check whether maybe there was a problem with your method. If you have done that already, so now you are busy making that tool for any two trains, I don't want you to be in on the discussion. I want you to get into groups of 3. Only if you are still busy with finding when the two trains meet, and now before you go in to the groups, I want to tell you what the purpose is. I don't want you to take any of your work with you. It will be about how we talk about this. But more specifically, about what your plan is. All of you have by now calculated the departure times.</p>	<p>Distributed to: Student as student AND Teacher as teacher</p> <p>Notes: The students are being described as aligned with knowledge ("now you are busy making that tool for any two trains" and "All of you by now calculated the departure time") (#2a)</p> <p>The teacher is technically not positioned in this part of the speech turn but his distribution of TF – clearly seen in 16a₃ – is taken to be in effect from the start.</p>	<p>Distributed to: No-one</p> <p>Notes:</p>
16 ₂	<p>16a₂ Teacher 1 But you want to find the time when the two trains will meet. So what now? And (pause) at the start you don't know what now.</p>	<p>Distributed to: Teacher as teacher</p> <p>Notes: See 16a₁ notes.</p>	<p>Distributed to: Student as student</p> <p>Notes: The students are described as aligned with ignorance (#2b)</p>
16 ₃	<p>16a₃ Teacher 1 You have to make a plan what to do. There's not a formula in your physics textbook that will tell you what to do now. You have to make a plan. Now that you have the departure times</p>	<p>Distributed to: Teacher as teacher AND Student as student</p> <p>Notes: The teacher is giving instructions (#6a)</p> <p>The students are being instructed to do something which aligns with knowledge (making "a plan what to do" requires knowledge) (#6a)</p>	<p>Distributed to: No-one</p> <p>Notes:</p>

16 ₄	<p>16a₄ Teacher 1</p> <p>How will I look at this problem? How will I approach it? What (pause) how will I approach it to find out when the two trains meet? That is making a plan. And you can only make a plan by thinking about the situation. You have to think about the situation. You have to look at your drawing of the two trains on the railway track and think how can I how can I approach it.</p>	<p>Distributed to: Teacher as other (student) AND Student as student</p> <p>Notes: The student and the teacher (who is speaking as if he were a student) are hypothetically being asked for assistance, since the questions “how will I look at this...” are being asked by himself to himself (#4)</p> <p>The teacher is also</p> <ul style="list-style-type: none"> - giving instructions (#6a) - explaining what he means by “making a plan (#3) <p>The students are being instructed to do something which aligns with knowledge (“make a plan”) (#6a)</p>	<p>Distributed to: Teacher as other (student) AND Student as student</p> <p>Notes: The teacher is speaking as if he were a student asking for assistance (#4)</p> <p>The students are also listening to explanation (#3).</p>
16 ₅	<p>16a₅ Teacher 1</p> <p>So when you get in to groups, I want you to tell each other what your plans are. But rules for the group discussion: you may not say a single number. You may not say a hundred and thirty nine (139) kilometres per hour. You have to give it a name. You may not say thirty (30) minutes after eight (8). You have to give it a name in words. There’s a reason why I ask this to you. Is in this problem, one often gets confused with exactly what a number means. You can calculate something but then later you forgot what it meant. The number the answer doesn’t tell you what it means. It’s only words can tell you what that number means. So that’s wha’ and this will really help you not to get confused. So that so in your group discussions don’t say a single number. If somebody in the group says a single number, somebody else should just say ‘red flag. Don’t say a number, give it a name’. Instead of saying numbers, give it names. And just by talking about these names of different times and different distances and whatever, I want each one to explain to the other, what is your plan for finding out when the two trains meet. And you can make a drawing. Once again, when you make the drawing, don’t write numbers.</p>	<p>Distributed to: Teacher as teacher AND Student as student</p> <p>Notes: The teacher is giving instructions (#6)</p> <p>The students are</p> <ul style="list-style-type: none"> - Being instructed to do something (explain to peers) which aligns with knowledge (#6a) - Being described as aligned with knowledge (“You can calculate”) (#2a) 	<p>Distributed to: Student as student</p> <p>Notes: The students are</p> <ul style="list-style-type: none"> - Being instructed to do something (listen to peers’ explanation and not saying/writing numbers) which aligns with ignorance (#6b) - Being described as aligned with ignorance (#2b) “one often gets confused” and “you forgot what it meant” and “help you not to get confused”

16 ₆	<p>16a₆ Teacher 1</p> <p>And if somebody says a time if he [points to student at the front of the class] says a time and I'm not sure what he means, once again, I ask him, 'Show me on the drawing. Is that time or is it (pause) is it a when question like this point in time at eight o'clock [looks at his watch] or is it a how long question like it's from eight o'clock until ten o'clock so it's two hours long. That eight hours is the time on the clock. The two hours is not a time on the clock, it's a how long. So what kind of time are you talking about? And if it's a how long time, if it's a duration, then show me on your picture. From when until when is that how long?'</p>	<p>Distributed to: Student as student AND Teacher as other (student)</p> <p>Notes: The student is hypothetically being asked for assistance (#4)</p> <p>The teacher is explaining whilst speaking with the voice of a student (#3)</p>	<p>Distributed to: Teacher as other (student) AND Student as student</p> <p>Notes: The teacher is speaking as if he were a student and is:</p> <ul style="list-style-type: none"> - Described as being aligned with ignorance (#2b) "I'm not sure what he means" - Hypothetically asking for assistance (#4) <p>The student is also listening to explanation (#3)</p>
16 ₇	<p>16a₇ Teacher 1</p> <p>So if you not sure what time or what distance he's talking about, let him explain more in words and using the picture. But not by saying a single number. Also, not by saying a single x or y or t. Names in words. 'Cause even if you wrote a t, what exactly does that is that now the arrival time or is that now the departure time? Or is that the time when they meet? Or it the total time for covering the whole distance between the two towns? The total journey duration. I've already mentioned four possible meanings of t. The arrival time of A, arrival time of B, departure time of A, departure time of B, time when they meet on the clock, total journey time for A, total journey time for B, time for A from the start until they meet. Sho. There's already seven or eight different meanings of t. Which one which one are you talking about now? So I want you to clarify to each other exactly what you are meaning by words. But I want you to share your plans. And maybe by sharing your plans you will improve your own plan so that after the group discussion you can go back and then you can implement the plan. But the plan isn't the calculations. The plan isn't the numbers. The plan is something higher. It's above the formulas and the calculations. So we don't need to talk about the formulas and calculations now. First have a plan. And then it will be easy to know what calculations to do. Once you have a plan, it's easy to know what calculations to do. Ok. So groups of three at a table unless you've already... ok. Those three. Three or four. Four of them, sorry Renee. So those four have to be together (indicating the four members of the research focus group). The others, in groups of three. Share your plans. And those who are already making formulas or making a tool for when for any two trains between any two towns, don't go in to groups. Just go on where you are. Let's give about ten or fifteen minutes for the group discussions. But put away your work. In the group discussion, don't show your work. Get some I will hand out some fresh pages and you can make drawings on the new pages.</p>	<p>Distributed to: Teacher as teacher AND Student as student</p> <p>Notes: The teacher is</p> <ul style="list-style-type: none"> - Explaining (#3) - Giving instructions (#6) <p>The student is being instructed to do something which aligns with knowledge (explain) (#6a)</p>	<p>Distributed to: Student as student</p> <p>Notes: The student is</p> <ul style="list-style-type: none"> - Described as being aligned with ignorance (#2b) "if you not sure" - Being instructed to do something which aligns with ignorance (listen) (#6b) - Listening to explanation (#3)

Interaction number	Transcript	Transmission function	Acquisition function
17 ₀	<p>The group is discussing their “plans” as they were instructed to do in interaction 16. Teacher 1 joins the group’s discussion about half way in. Whilst speaking, they are referring to the following diagram, which is drawn on a piece of paper on the table in the middle of the group:</p>  <p>17(i)¹ Student 2 What I said was (pause) if we look at the I said that they both start moving from here right [points to A₁ and B₁]?</p> <p>17(ii) Student 3 So you moved that one first [points to A₀] ?</p> <p>17(iii) Student 2 Ja.</p> <p>17(iv) Student 3 Ok.</p> <p>17(v) Student 2 So the time it takes for this one to get to here [points to A₁ and then A₂B₂] will be equal to the time it takes for this one to get to here [points to B₁ and then A₂B₂]. Alright. So then you get distance over speed.</p> <p>17(vi) Student 3 For time?</p> <p>17(vii) Student 2 Mm-hm. Then you know the speeds right?)</p> <p>17(vii) Student 3 Hm</p> <p>17(vii) i)</p> <p>17(ix) Student 2 So then you can say distance over speed of train 1 is equal to distance over speed is speed of train 2. And then</p>	<p>Distributed to: Not coded</p> <p>Notes: This dialogue occurs before the start of interaction 17 (when teacher 1 starts talking to the group). It is included here in order to allow the reader to locate Teacher 1’s comment in 17a.</p>	
17 ₁	<p>17a Teacher 1 I stop you (student 2) there. Just because you are talking about a calculation. And I. Let’s not talk about the calculation too much here. Ok. That you can do on your own. But when you (student 2) explained something about A moving first [points from A₀ to A₁] and then the time from there to there [points to B₁ and then A₂B₂] is the same as the time from there to there [points to A₁ and then A₂ B₂]. Is that really so? What do the others think? So she said the time from there to there [points to A₁ and then A₂B₂] is the same as the time from there [points to B₁ and then A₂B₂]. Is it really so?</p> <p>17b Student 3 I think what she meant was the change in time there they take to (inaudible)</p> <p>17c Student 2 Oh</p> <p>17d Student 3 Not the time on the clock. Well the time on the clock is the same there [points to A₂B₂]. But she’s not talking about time that A travelled to get there [points to A₀ and A₂B₂] is equal to the time that B [points to B₁ and A₂B₂] I that’s my understanding of what she said.</p> <p>17e Teacher 1 Ok. Somebody else. What is special about</p>	<p>Distributed to: Student as student (all) AND Teacher as teacher</p> <p>Notes: The teacher is - Posing questions regarding the knowledge which is to be transmitted / acquired (#5). Note that the questions posed seem to serve the purpose of giving the teacher access to their thinking and direct the students to think in a particular way (eg. 17e, the special point) - Withholding assessment. This is particularly apparent in 17g: “I know. Ja</p>	<p>Distributed to: No-one</p> <p>Notes:</p>

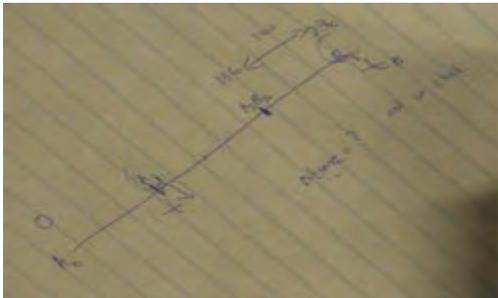
¹ This interaction is taken to start at the time when the teacher starts speaking to the group. Prior to that, the group members are talking to each other. The transcript here starts when the teacher steps up and starts listening to the group’s discussion. All conversation recorded before the teacher starts speaking is numbered with Roman numerals to indicate clearly where the interaction starts (17a)

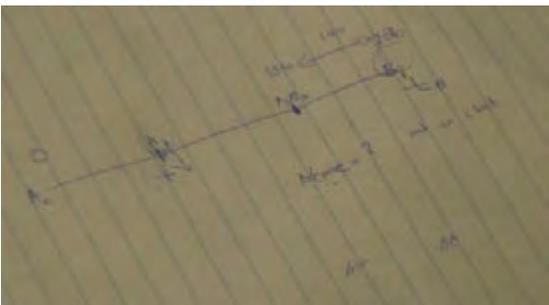
	<p>17f Student 1</p> <p>17g Teacher 1</p> <p>17h Student 1</p> <p>17i Teacher 1</p>	<p>here [points to A₁]? 'cause she talked about this time it takes to travel the time it takes for train A to travel from here to here [points to A₁ and then A₂B₂]. The time it takes for train A to travel from here to here [again points to A₁ and then A₂B₂]. What is here [points to A₁]? What is here [points to A₂B₂]? What is special about those two points? That you want to know the time it takes to travel between them. So what is this point [points to A₁]? What is special about it and what is special about that point [points to A₂B₂]? Somebody else. What is special about this point and that point [points to A₁ and then A₂B₂] concerning the time or what?</p> <p>No. Why do you want to know the time it takes to travel from there to there [points to A₁ and then A₂B₂]? Why what is special about that point [points to A₁]? Can that [points to A₁] just be any point? Can I also have drawn that point there or there [points to two different, unmarked points to the left and right of A₀ respectively]? Why why that point [points to A₁] and not this one [points to an unmarked position to the left of what is marked as A₁]? I know. Ja I'm not going to. Why that point and not this one [points to A₁ and then to an unmarked position to the left of what is marked as A₁]?</p> <p>Why not that point but this one [points to same positions on diagram as Teacher did in 17g]?</p> <p>Yes</p>	<p>I'm not going to" (#1c)</p> <p>The students are</p> <ul style="list-style-type: none"> - Being described as aligned with knowledge in 17a: "That you can do on your own" suggests that the students have the knowledge to do the calculations (#2a) - Being expected to assess their own productions (their peer, student 2's, claim). This is apparent in 17a: "is that really so? What do the others think?" and 17g (#1c) 	
<p>17₂</p>	<p>17j Student 1</p> <p>17k Teacher</p> <p>17l Student 2</p> <p>17m Student 1</p> <p>17n Teacher 1</p> <p>17o Student 1</p>	<p>I don't know what you are talking about honestly.</p> <p>You you don't know what she (student 2) was talking about.</p> <p>He's asking why, what's important about this point [points to A₁]. Why can't we draw it here or there [points to two positions on diagram, one on the left and one on the right of A₁]? What's special about this one [points to A₁]?</p> <p>Oh, because you can say like train A is moving with train B [points to A₁ and B₁].</p> <p>Oh. So. Ok.</p> <p>Yeah. Ok. 'Cause of that they they moving together now that one with that one over there [points to A₁ and then B₁].</p>	<p>Distributed to: Teacher as teacher</p> <p>Notes: The teacher is posing questions (#5)</p>	<p>Distributed to: Student as student (S1)</p> <p>Notes: The student is described as being aligned with ignorance (#2b) in 17j and 17k</p>

<p>17₃</p>	<p>17p Teacher 1 Ok. So at what time will train A be at this point [points to A₁]?</p> <p>17q Student 1 At what time?</p> <p>17r Teacher 1 At what time will train A be at that point [points to A₁]?</p> <p>17s Student 1 Ok. Let me just say this that, um, at what time, neh? Because you consider t zero [points to A₀] and then you gonna have t final [points to A₁], right? Let's just forget about those [covers A₂B₂ and B₁ with his hand, palm face down].</p> <p>17t Teacher 1 Ok.</p> <p>17u Student 1 Let's talk about this point [points to A₁]. These two [points to A₁ and A₀]. Then we gonna have t zero there [points to A₀] but we gonna have t final there [points to A₁]. And then here to get from there to there [points to A₁ and A₂B₂] we gonna consider t final as what? t initial [pointing at A₀].</p> <p>17v Teacher 1 I I hear you. But um but you said but so what is the time there [points to A₁]?</p> <p>17w Student 1 How will I know what the t final there is [points to A₁]?</p> <p>17x Teacher 1 You you told me when I asked the question what is special about that point [points to A₁], you said, that is the point at which train A and B now travels together [points to A₁ and B₁ simultaneously and then slides his two pointing fingers towards each other]. Before that, only train A travelled [slides finger from A₀ to A₁]. But from that point onwards, both travel together [points to A₁ and B₁ simultaneously and then slides his two pointing fingers towards each other]. So what is the time when train A is at that point [points to A₁]?</p> <p>17y Student 4 I think it's the same time as B will start moving.</p>	<p>Distributed to: Teacher as teacher AND Student as student (S1)</p> <p>Notes: The teacher is - Posing questions (#5) - Assessing the student's production (verbal response given in 17o) in 17p (#1a)</p> <p>The student (S1) is being favourably assessed in 17p (#1a)</p>	<p>Distributed to: No-one</p> <p>Notes:</p>
<p>17₄</p>	<p>17z Teacher 1 Ok. She said train A will be there [points to A₁] at the time when train B starts moving. It's similar to what you (student 1) said. Because then they move together. So train A will be there [still pointing to A₁] at the departure time of train B. And what's special about that point [points to A₂B₂]?</p> <p>17aa Student 4 It's where they going to pass each other or meet.</p> <p>17ab₁ Teacher 1 Ok. So that's the position where they pass each other and the time when they pass each other [points to A₂B₂] (6 second pause).</p>	<p>Distributed to: Teacher as teacher AND Student as student (S1&S4)</p> <p>Notes: The teacher is - Posing questions (#5) - Assessing student 4 and student 1's verbal responses. His re-stating their explanations (in 17z and 17ab₁) suggests favourable assessment. (#1a)</p> <p>The students (specifically students 1 and 4) and having their productions favourably assessed (#1a)</p>	<p>Distributed to: No-one</p> <p>Notes:</p>

<p>17₅</p>	<p>17ab₂ Teacher 1 Will you [points to Student 2] now explain again what you said about that and that [points to A₁ and A₂B₂, and then B₁ and A₂B₂] and then I just want to check that everybody agrees with that (pause). Or maybe there is a problem with it.</p> <p>17ac Student 2 Ok. So. The change in time that it takes for A to get to that point [points to A₁ and then A₂B₂] will be equal to the change in time that it takes for B to get to that point [points to B₁ and then A₂B₂]. So then you can say that the distance over the speed of train 1</p> <p>17ad Teacher 1 Ok. Let's let's stop. So why why is that change in time for train A from there to there [points to A₁ and A₂B₂] the same as the change in time for train B from there to there [points to B₁ and A₂B₂]?</p> <p>17ae Student 3 'Cause the question was: where do they meet? And if two people meet, if I say let's meet at a mall and I get to the mall first then I leave and then you get there and you also took the same time that I took to get there, but we not meeting, so then we not answering the question. But the question was: when do they meet? So therefore the time you take to get there it might not be, oh well, from this point [points to A₁ and B₁]. That's why we had to move this guy here first [points from A₀ to A₁]. Because the time he [points to A₀] took to get there [points to A₂B₂] was not the same time as he [points to B₁] took to get there [points to A₂B₂].</p>	<p>Distributed to: Teacher as teacher AND Student as student (S2)</p> <p>Notes: Teacher is</p> <ul style="list-style-type: none"> - Posing questions (#5) - Giving instructions related to actions which are to lead to knowledge acquisition (#6a). That is, he is asking a student to explain something which she explained earlier. That he is asking her to repeat it suggests that her explanation was aligned with the knowledge which is to be transmitted and it useful in progressing the judgement of the notion. His statement "or maybe there's a problem with it" (17ab₂) seems to be an afterthought, said only to remind of his (feigned) withholding of judgement. <p>The student (specifically student 2) is:</p> <ul style="list-style-type: none"> - Being instructed to do something which aligns her with knowledge (explain to peers) (#6a) - Explaining (#3) 	<p>Distributed to: Student as student (all)</p> <p>Notes: The students are</p> <ul style="list-style-type: none"> - listening to explanation from their peer in order to progress their judgement of the notion in 17ac (#3)
<p>17₆</p>	<p>17af Teacher 1 Oh, 'cause they started at different times. Oh I see.</p> <p>17ag Student 3 At different times. But from here [points to A₁ and B₁] they we can make them start at the same time and get the same change in time and then therefore they meet.</p> <p>17ah Teacher 1 Ok. So that is the same time for both of them [points to A₁ and B₁]. And then when they meet [points to A₂B₂] it's also the same time for both of them.</p> <p>17ai Student 3 On the clock, yes.</p> <p>17aj₁ Teacher 1 So that change in time [points from A₁ to A₂B₂] must be the same as that [points from B₁ to A₂B₂]. Ok. (Pause for 10 seconds)</p>	<p>Distributed to: Teacher as teacher AND Student as student (S3)</p> <p>Notes: The teacher is</p> <ul style="list-style-type: none"> - Feigning newfound understanding, which implies alignment of himself with knowledge (#2a) - Assessing the student's production in 17af, 17ag and 17aj₁ (#1a) seen in his saying "ok" and also in his re-stating what the student has said. In 17af, the teacher's (feigned) newfound understanding implies that the student has said something which 	<p>Distributed to: Teacher as teacher</p> <p>Notes: The teacher is feigning newfound understanding in 17af, which implies a description of himself as previously aligned with ignorance (#2b)</p>

			is considered correct.	
			The student (specifically student 3) is having his productions (verbal statements) favourably assessed. (#1a)	
17 ₇	<p>17aj₂ Teacher 1 I want to ask you [looks at student 2] to say something more but without saying how you calculated. What can you say about the distances here? You've got that distance [points from A₁ to A₂B₂] and that distance [points from B₁ to A₂B₂]. You've got the total distance [points from B₁ to A₀]. You also have that distance [points from A₁ to B₁]. Is there something special about these different distances?</p> <p>17ak Student 2 The distance from A [points to A₀] to the point where they meet [points to A₂B₂] plus the distance from B [points to B₁] to the point where they meet [points to A₂B₂] should give us the total distance. Oh oh oh. And we've g' – somebody else – if we just look at that [covers up the part of the diagram to the left of A₁]. Can we say there are three distances. There's that distance [points between A₁ and A₂B₂], there's that distance [points between B₁ and A₂B₂] and there's that distance [points from A₁ to B₁]. Is there some relationship between those three distances?</p> <p>17al Teacher 1 Oh oh oh. And we've g' – somebody else – if we just look at that [covers up the part of the diagram to the left of A₁]. Can we say there are three distances. There's that distance [points between A₁ and A₂B₂], there's that distance [points between B₁ and A₂B₂] and there's that distance [points from A₁ to B₁]. Is there some relationship between those three distances?</p> <p>17am Student 1 Ok there's this one, that one, that one and that one [points to the same distances teacher 1 indicated in 17al]. Is there some relationship between those three distances?</p> <p>17an Teacher 1 [Nods]</p> <p>17ao Student 1 Ok. Mm. Ok. Obviously the sum of this one [points between A₁ and A₂B₂] and this one [points between B₁ and A₂B₂] will give you that one [points from A₁ to B₁].</p>	<p>Distributed to: Teacher as teacher AND Student as student (S2)</p> <p>Notes: The teacher is - Posing questions (17aj₂) (#5) - Giving instructions (#6a)</p> <p>The student (specifically student 2) is - Being instructed to do something which aligns with knowledge, in the same way as in 17ab₂ (#6a) - Explaining to her peers (#3) in 17ak</p>	<p>Distributed to: Student as student (all)</p> <p>Notes: The students are - Listening to explanation from their peer in order to progress their judgement of the notion in 17ak (#3)</p>	
17 ₈	<p>17ap Teacher 1 Ok</p> <p>17aq Student 1 Yeah?</p> <p>17ar Teacher 1 Ok. So if you know that distance [points from A₁ to B₁], then you know that one [points between A₁ and A₂B₂] plus that one [points between B₁ and A₂B₂] should give you that [points from A₁ to B₁].</p> <p>17as Student 1 Yeah. Ok. And um what else? (pause) um</p> <p>17at Teacher 1 If you knew the time there [points to A₂B₂], could you calculate some of these distances?</p> <p>17au Student 1 If you knew the time [points to A₂B₂]?</p> <p>17av Teacher 1 If you... Let's say you knew the time [points to A₂B₂]. Could you calculate some of these distances? What do you think? 'cause you've already told me you know the time there [points to A₁]. Ok? Which also means you can find the distance there [points to A₁]. Um. So if you know the time there [points to A₂B₂] can you maybe calculate this distance [points between A₁ and A₂B₂]? What do you think? (directs the latter question to Student 4 by looking at her)</p> <p>17aw Student 4 Yeah, I think it will calculate.</p> <p>17ax Teacher 1 And that distance [indicates between B₂ and B₁]? If you knew the time when they meet [points to A₂B₂] could you calculate</p>	<p>Distributed to: Teacher as teacher AND Student as student (all)</p> <p>Notes: The teacher is - Assessing the student's production in 17ap, 17ar and 17bb (#1a) The favourable assessment is particularly apparent in the teacher's repeating back to the students (although he rephrases) what the students have already said (in 17ar and 17bb) - Posing question (#5)</p> <p>The student (student 1) is having his production</p>	<p>Distributed to: No-one</p> <p>Notes:</p>	

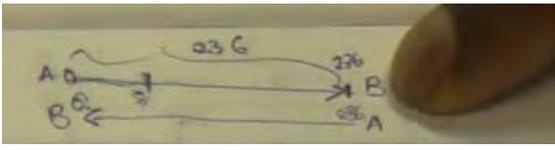
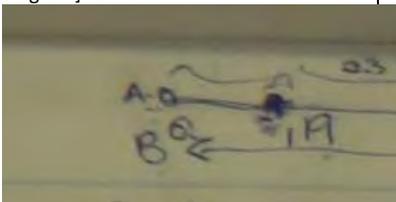
	<p>17ay Student 4</p> <p>17az Teacher 1</p> <p>17ba Student 4</p> <p>17bb Teacher 1</p>	<p>that distance [indicates between B_2 and B_1]?</p> <p>Yes</p> <p>But you don't know the time there. But do you know that distance [indicates A_1 to B_1]? The remaining distance after train A travelled that [indicates from A_0 to A_1]. Can you find out that distance [indicates from A_1 to B_1]?</p> <p>After you've got the time you can find the distance.</p> <p>Find it. For that. As you (student 1) said, that [points to A_1 and B_1] is the time when train B starts moving. So you have a time you can find, if you have that distance [points to A_0 to A_1], you know the remaining distance [points to A_1 to B_1]. If you knew the time [points to A_2B_2], you could calculate that [points between A_1 and A_2B_2] using the time and you could calculate that [points between B_1 and A_2B_2] using the time. As he said, you know the one [points between A_1 and A_2B_2] plus the other [points between B_1 and A_2B_2] gives you that [points between A_1 and B_1].</p> <p>(pause for 10 seconds)</p> <p>Does that make? Dunno. Is that does that make sense? Is there something that's that's bothering you?</p>	<p>favourably assessed (#1a)</p> <p>All students are being described as aligned with knowledge (#2a) "you could calculate..." in 17bb</p>	
<p>17₉</p>	<p>17bc Student 1</p> <p>17bd Teacher 1</p> <p>17be Student 1</p> <p>17bf Teacher 1</p> <p>17bg Student 1</p> <p>17bh Teacher 1</p> <p>17bi Student 1</p> <p>17bj Teacher 1</p> <p>17bk Student 1</p>	<p>Ok. Um. There's a question that I wanna ask there. Um. Like I. You have some time here [writes t on the diagram below A_1] like a t there [writes t_b below B_1]. So I'm talking about the departure time. 'Cause we're gonna consider like A is departing here [circles A_1] and B is departing there [circles B_1]</p>  <p>Right</p> <p>So 'cause the departure time here [points to A_1], from from my point of view, is not equal to the departure time there [points to B_1].</p> <p>Not?</p> <p>Is not.</p> <p>Let me ask again, what is special about that time [points to A_1]?</p> <p>Ah ok. Um. Um. Ok. That is the question that I was thinking but it came out wrong.</p> <p>Give yourself some time. It's ok.</p> <p>Um. That's a stupid thing</p>	<p>Distributed to: Teacher as teacher</p> <p>Notes: The teacher is</p> <ul style="list-style-type: none"> - Being asked for assistance (#4) - Posing questions (#5) - Assessing the student's production (in 17bf and 17bh) (#1b) 	<p>Distributed to: Student as student (S1)</p> <p>Notes: The student is</p> <ul style="list-style-type: none"> - Asking for assistance (#4) in 17bc - Is being unfavourably assessed (#1b) in 17bf and 17bh. Although it should be noted that although the assessment that the student is not on the right track is explicit, the teacher is withholding information, redirecting the student's attention to the specialness of time instead of answering his question.

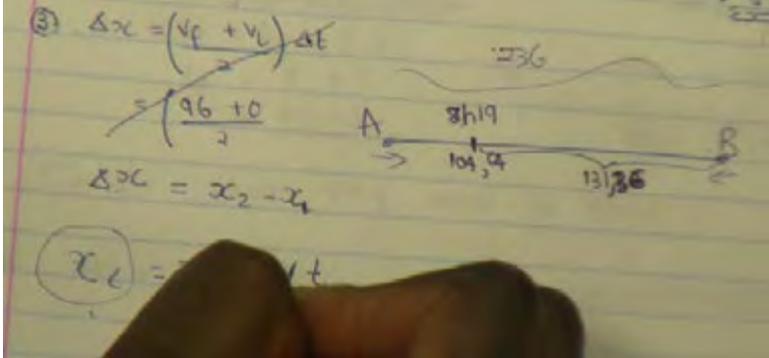
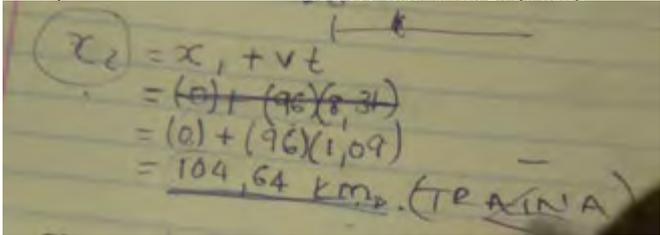
<p>17₁₀</p>	<p>17bl Student 3 Can I ask something? 17bm Student 1 Ok 17bn Student 3 There's some girl over there, that one, she was asking me, I don't know if I explained to her. But I would like to hear what you [indicates Teacher 1] said about this one. So the time there [points to A₀] and the time [points to B₁]. If I just use one equation and calculate where they meet, I'm calculating the change in time right [writes Δt below the diagram]? 17bo Teacher 1 The change in time from when until when? 17bp Student 3 For ah let's say let's say they meet there [points to A₂B₂]. 17bq Teacher 1 Ok. 17br Student 3 And I'm saying and I'm using an equation and I say, let's say her equation [points to Student 2] of equating the two times. And I say and I get a time [points to Δt which he just wrote] which says maybe one hour after travelling. 17bs Teacher 1 Ok. So that's a duration. 17bt Student 3 Yes 17bu Teacher 1 What. I just want to be sure we all understand what duration you are talking about. From when until when are you talking about? What is the starting point and what is the ending point for that duration you are talking about? 17bv Student 3 So it's the duration for both trains to meet [points to A₀ and B₁]. 17bw Teacher 1 The duration from what starting time? 17bx Student 3 From. Um. Let's say from there [points to A₀]. From there (A₀). From from when this one leaves [still pointing to A₀]. 17by Teacher 1 From the departure time of train A. 17bz Student 3 Train A 17ca Teacher 1 Ok. Until when? 17cb Student 3 Until until they meet. 17cc Teacher 1 Ok. So the duration from when A departs [points to A₀] until when A meets B [points to A₂B₂]. 17cd Student 3 So my question is can I use one formula to get where they meet [points to A₂B₂]?</p>	<p>Distributed to: Teacher as teacher AND Student as student (S3)</p> <p>Notes: The teacher is - Being asked for assistance (#4) - Posing questions (#5)</p> <p>The student is recounting his being asked for assistance by a peer (#4)</p>	<p>Distributed to: Student as student (S3)</p> <p>Notes: The student is asking for assistance (#4) in 17bℓ</p>
<p>17₁₁</p>	<p>17ce Teacher 1 That I don't know. 17cf Student 3 'Cause she was asking me ah what's the difference between us let's say what's the difference between these two trains meeting at exactly let's say ten o'clock [points to A₂B₂] and um ok, this one left at nine [points to B₁] and this one left at eight [points to A₀] but they both meet at ten o'clock so she was asking me if I was using the equation of delta t (Δt) [writes Δt below diagram again], which means</p>  <p>17cg Teacher 1 Ah. That's going too fast. Sorry. I don't want us to talk about equations sorry to interrupt you [covers the places he's</p>	<p>Distributed to: Teacher as teacher AND Student as student (all)</p> <p>Notes: The teacher is - Withholding assessment (#1c) – made particularly apparent in 17ce, where he feigns ignorance which effects avoiding answering the student's question. - Giving instructions (#6a) ("make your own equation" in 17cg) - Posing questions (#5)</p> <p>The student is - Recounting being asked for assistance by a peer (#4)</p>	<p>Distributed to: Teacher as teacher AND Student as student (S3)</p> <p>Notes: The teacher is described as being aligned with ignorance (#2b) in 17ce and 17cm. 17cg ("That's going too fast")</p> <p>The student (specifically student 3) is - Asking for assistance (#4) - Describing himself as aligned with ignorance ("I wasn't sure" in 17cn) (#2b)</p>

		<p>written Δt with his hand]. The reason being is I don't want you to think that if you just get the right equation you'll solve it. Because in this problem, you may need to make to make (emphasizes the second make) your own equation instead of taking some equation from somewhere that you remember. So that's why I'm asking let's not talk about equations 'cause so long as you understand your plan, you will be able to make your own formula or your own equation. But it's not an equation that you just borrow from somewhere. You have to think about this [circles the diagram with his finger] to make an equation.</p>	<ul style="list-style-type: none"> - Being instructed to do something which aligns with knowledge ("make your own equation" in 17cg) (#6a) - Being described as aligned with knowledge ("you will be able to" in 17cg) (#2a)
17ch	Student 3	It's just that what she was asking was since the change in time is t final minus t initial	
17ci	Teacher 1	Right.	
17cj	Student 3	Right	
17ck	Teacher 1	For train A?	
17cl	Student 1	Yes	
17cm	Teacher	um I'm not sure	
17cn	Student 3	T final for both trains she was saying t final for both trains must be the same. So I wasn't sure how to explain to her. It's just I knew she had to move this one first [points to A_0] but she was using one equation. And I just couldn't explain to her why the t-two is not the same.	
17co	Teacher 1	Do do you think that she thought that the time it takes train A from there until there [points to A_0 and then A_2B_2] is the same as the time it takes for train B from there to there [points to B_1 and then A_2B_2]?	
17cp	Student 3	Yes, according to her statement and her equation that's what she meant.	
17cq	Teacher 1	But is it true that the time it takes for train A from its departure until when they meet [points to A_0 and then A_2B_2] is the same as the time it takes train B from its departure until they meet [points to B_1 and then A_2B_2]? (3 second pause) [Student 1 and 3 shake their heads as if to say "no"] Why? Why are they not the same?	
17cr	Student 1	Um. I've remembered I I think um because you said the time from that A takes from there to there [points to A_0 and then A_2B_2] it's not you you. That's what we think. Ah. From there to there [points to A_0 and then A_2B_2] it's not equal it's not equal the times [points to B_1 and then A_2B_2] of both it's not equal. But the way I think of it is because because train A has already moved [points from A_0 to A_1] while train B stayed there [points to B_1].	
17cs	Teacher 1	Ok.	
17ct	Student 1	So it's impossible for them to just be equal.	
17cu	Teacher 1	Oh. So train A has moved longer than train B.	
17cv	Student 1	Ja.	
17cw	Teacher 1	Ok. Ok.	
17cx	Student 3	That was what I needed to explain to her.	

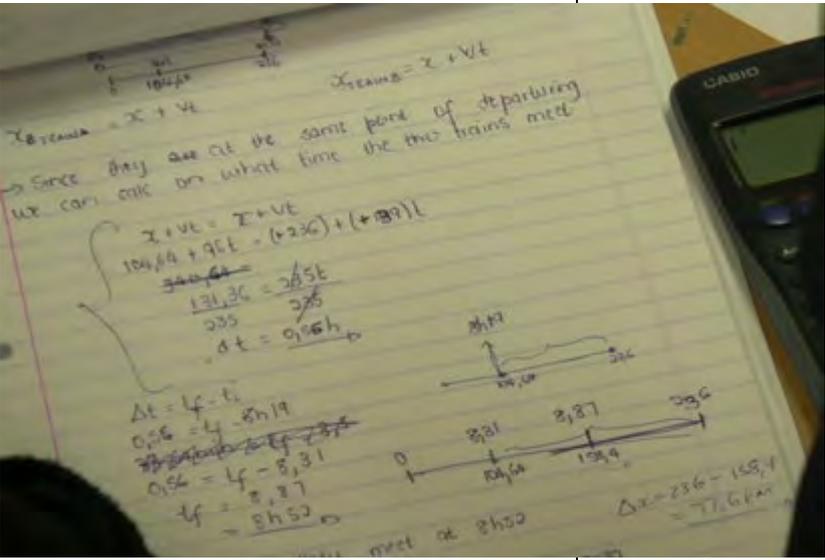
<p>17₁₂</p>	<p>17cy Teacher 1 Ok. Can you (Student 1) remember what you wanted to ask?</p> <p>17cz Student 1 Ah. No. No, it's fine.</p> <p>17da Teacher 1 I think you're done. Remember thinking about these distances [points to A₁ and B₁]. You told me that you can find that distance [points between A₀ and A₁] until train A for train A from the departure of train A [points to A₀] until the departure of train B [points to A₁ and B₁]. You said that's [points to A₁] the same as the departure time of train B. Because now they move together [pointing to A₁ and B₁]. So you can calculate that distance [points between A₀ and A₁]. Then you'd know the remaining distance [points between A₁ and B₁]. And you told me the remaining distance is the same as that distance [points to A₁ and A₂B₂] plus that distance [points to B₁ and A₂B₂]. Maybe that's useful. ([ause for 7 seconds] Let's end there. I think you have more than enough ideas. Sorry to cut you short. [Teacher 1 stands up as if to walk away]</p>	<p>Distributed to: Teacher as teacher AND Student as student (all)</p> <p>Notes: The teacher is assessing the students' productions (#1a) seen in his summarizing what the students have said to him, following his statement in 17cy "I think you are done."</p> <p>The students are having their productions favourably assessed (#1a)</p>	<p>Distributed to: No-one</p> <p>Notes:</p>
<p>17₁₃</p>	<p>17db Student 3 What if they gave us different speeds, like ah if one of the trains accelerates</p> <p>17dc Student 1 Like it accelerates?</p> <p>17dd Teacher 1 Worry about that later. First first solve a simple problem. This is something engineers and scientists do. They make a simplifying assumption like constant speed. Then they first make sure that they can solve that problem then you can always add more of the real life complexity later. But then you know at least I've solved the simple problem [walks away from group.]</p>	<p>Distributed to: Teacher as other (engineer)</p> <p>Notes: The teacher is being asked for assistance (#4)</p> <p>The teacher is positioned as other when he talks about engineers and scientists solving a problem and then switches to "I've solved the simple problem"</p>	<p>Distributed to: Student as other (engineers & scientists) (all)</p> <p>Notes: In 17db, the student is requesting assistance (#4).</p> <p>The students are positioned as engineers and scientists in 17dd.</p> <p>The students, even though positioned as engineers and scientists, are described as ignorant in that they must "first solve a simple problem" (#2b)</p>

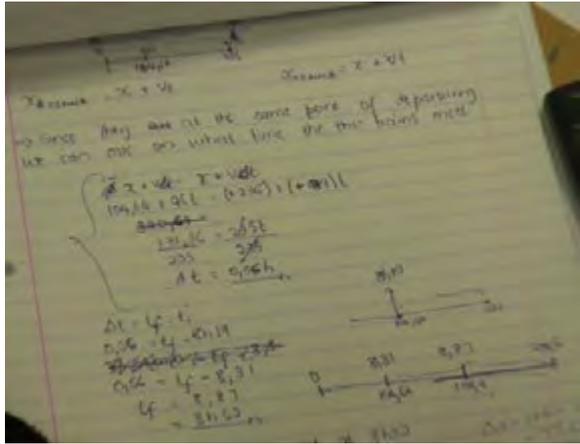
<p>18</p>	<p>Teacher 1 re-joins the focus group, after having left them for about 10 minutes during which they spoke about topics other than <i>The Trains Activity</i>.</p> <p>18a Teacher 1 Stay away from formulas. That you can do on your own. You can make your own formulas and equations. I see parabolic motion and all kinds of interesting things. And gravitational acceleration. It seems that the train is falling now. Off the bridge and into the big big river valley. [Students laugh]. I think you can work individually again.</p>	<p>Distributed to: Teacher as teacher</p> <p>Notes: The teacher is assessing the students' productions (written work) (#1b)</p>	<p>Distributed to: Student as student</p> <p>Notes: The students are having their productions (written work) unfavourably assessed (#1b) since the teacher is pointing out the nonsensical scenario "that the train is falling now" which he reads on their page shows that he assesses what they are doing as incorrect.</p>
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Interaction number	Transcript		Transmission function	Acquisition function
20 ₁	20a	Teacher 3 [Teacher 3 reads Student 4's work for 20 seconds before speaking] What are you calculating here? [points to student's work]	Distributed to: Teacher as teacher Notes: The teacher is - Posing questions in 20a, 20c, 20d, 20e, 20g, 20k etc. (#5) - Assessing the student's productions (verbal responses & written work) seen particularly in 20e: the question "Come again?" implying that what the student has just said is not correct. An unfavourable assessment is also apparent in 20y. (#1b) - Giving instructions in 20y (#6b)	Distributed to: Student as student
	20b	Student 4 [turns back to her previous page] I started I started here [points to work on her page] to calculate this. I said here by calculating the distance that A would be when it's eight the initially of B		Notes: The student is - Having her productions unfavourably assessed (#1b)
	20c	Teacher 3 Where does the 8 come from?		- Being instructed to do something which aligns her with ignorance in 20y (#6b)
	20d	Student 4 The initially time of B?		
	20e	Teacher 3 Come again? So you have A here and you have B here [points to A and B on the student's diagram]		
				
	20f	Student 4 Yes. They are departing at different times. So I make I just make A to be at some position at this time of B when the B when the B is going to leave. So that I will I will be able to calculate the time that's where they were where they are going to meet.		
	20g	Teacher 3 What is the departure time of A?		
	20h	Student 4 It's seven		
	20i	Teacher 3 Seven thirteen		
	20j	Student 4 Yes		
	20k	Teacher 3 And then the departure time of B?		
	20l	Student 4 It's eight nineteen.		
	20m	Teacher 3 And then this eight is for which train? [points to the 8 in the middle of the student's diagram]		
	20n	Student 4 Ok. Um. Train A is train A is one hour ahead of B		
	20o	Teacher 3 How do you know that?		
	20p	Student 4 Train A train A moved first then B. At seven thirteen and B at eight nineteen		
	20q	Teacher 3 Which is one hour ahead		
	20r	Student 4 Of B. So I calculated the distance that A would be when it's after an hour.		
	20s	Teacher 3 Ja		
	20t	Student 4 train A will be here [points to the 8 on the student's diagram] and it's 8 o'clock?		
20v	Student 4 Eight nineteen. I calculated the distance on [points between the A and 8 on her diagram]			
20w	Teacher 3 Mm			
20x	Student 4 And the distance will be eight on this time of B. And B will also be starting to move.			
20y	Teacher 3 Oh, this [points to the eight on student's diagram] is not eight it's eight nineteen. Then write eight nineteen (8:19). [Student writes this ":19" onto her diagram]. Ok?			
				
20z	Student 4 Ok. Then I calculated that position and it was			
20aa	Teacher 3 The position of?			
20ab	Student 4 The position of this eight nineteen [points to 8:19 on her diagram].			

	<p>20ac Teacher 3 So what does the x two (x_2) stand for? [points to x_2 in the student's written calculation]</p>  <p>20ad Student 4 The position where A would be from</p> <p>20ae Teacher 3 The position as in the distance or the position as in ?</p> <p>20af Student 4 The position as in where A would be when it's eight nineteen (8:19).</p> <p>20ag Teacher 3 I hear what you saying. My question is, the position is it are are you talking about this distance [waves finger over the diagram, not pointing at any one thing clearly]? I mean, are you talking about the the position as in distance? Could this train will be so many kilometres away. Ok. The x two (x_2). The kilometres. Ok.</p> <p>20ah Student 4 For A</p> <p>20ai Teacher 3 For train A.</p> <p>20aj Student 4 Yes.</p> <p>20ak Teacher 3 Let me get this straight. So then it means that this point [points to the 8:19 point on the student's diagram] these are the kilometres [points to the 104.,64km on the student's diagram.] (pause). Ok. Carry on.</p> <p>20al Student 4 Then I calculated the time when they are the two trains are going to move. And that's the equation [points to the Δt and the 3 preceding lines of working on her page of calculations]</p>		
<p>20₁ cont.</p>	<p>20am Teacher 3 What is this formula for? [points to $x_2 = x_1 + vt$ on student's page]</p> <p>20an Student 4 Ok. To calculate the position.</p> <p>20ao Teacher 3 Ok. The distance for which train?</p> <p>20ap Student 4 Sorry?</p> <p>20aq Teacher 3 For which train?</p> <p>20ar Student 4 For train A.</p> <p>20as Teacher 3 Ok</p> <p>20at Student 4 I calculated the distance for train A.</p> <p>20au Teacher 3 So write A by it so that I can see you are talking about train A [student writes "TRAIN A" next to the 104.64km].</p>  <p>20ay Teacher 3 where the train (inaudible) meets. So I see two formulas here [points to the $x + vt = x + vt$ on the student's page] and which are the same. You have equated one formula to</p> <p>20az Student 4 another formula but with with different</p>	<p>Distributed to: Teacher as teacher</p> <p>Notes: The teacher</p> <ul style="list-style-type: none"> - Poses questions in 20am, 20ao, 20bc etc. (#5) - Giving instructions in 20au (#6b) - Assessing the student's productions (written work) in 20ba, 20cf and 20ch₁ (#1b) 	<p>Distributed to: Student as student</p> <p>Notes: The student is</p> <ul style="list-style-type: none"> - Having her work unfavourably assessed in 20ba, 20cf and 20ch₁ (#1b) - Being instructed to do something which aligns her with ignorance in 20au, that is, she is told to write an indication of "train A" next to her calculation with an implication that she should have done so of her own accord (#6b). - Being described as aligned with ignorance in 20ca, since the teacher's question "do

<p>20ba Teacher 3 20bb Student 4 20bc Teacher 3 20bd Student 4 20be Teacher 3 20bf Student 4</p>	<p>position. But they they they look the same. Ok, for train A and for train B [student writes A and B as subscripts on the x's written in her work so that this line teacher pointed to becomes $x_A + vt = x_B + vt$] And these two [points to the v on either side of the equation referred to above], are they the same? What? The v-t and the v-t? V for train A and V for train B [student writes subscripts of A and B on to the v symbols in her work]</p>		<p>you understand this" carries an implication that the teacher thinks that the student does not understand. (#2b)</p>
<p>20bg Teacher 3 20bh Student 4 20bi Teacher 3 20bj Student 4 20bk Teacher 3 20bl Student 4 20bm Teacher 3 20bn Student 4 20bo Teacher 3 20bp Student 4 20bq Teacher 3 20br Student 4 20bs Teacher 3 20bt Student 4 20bu Teacher 3 20bv Student 4 20bw Teacher 3 20bx Student 4 20by Teacher 3 20bz Student 4 20ca Teacher 3 20cb Student 4 20cc Teacher 3 20cd Student 4</p>	<p>And what does the V-A-T (v_AT) stand for? The velocity (inaudible) The? The velocities to... they are moving um the speed The speed of what? Of the train A and where train B Ok. Go on (inaudible and then pause of silence for 10 seconds) This is the distance for train A [points to student's work; in video, teacher's arm obscures exactly what she is pointing to] Mm This is the distance for train B [continues pointing to student written work with video record unclear what she points to] so the distance for train B. Get the total distance. Train B this two three six (236) is the total distance. Train B's starting to move from here and that's this side (points on her diagram) Train B started at the two-hundred and thirty six. Yes So why you subtracted the kilos? It's minus (pause) Ok. And train A is starting from the distance of one-oh-four (104) Do you understand this? [points to students work, broadly indicating towards her whole page of writing]. Yes I do understand it Ok, let's carry on. And then I calculated the time when both trains will meet. (10 second pause)</p>		

	<p>20ce Teacher 3 Let me see. [Teacher moves students work upward on the desk, takes the students' calculator, seems to get ready to use it, but then does not use it. The teacher moves the calculator out of the way and then the student turns the page of her work over. Teacher and student appear to read what is written on this next page.] (It is a 30 second pause in total between 20ce and 2cf).</p>  <p>20cf Teacher 3 But this is no longer negative, it's positive. (pause) And you know what my problem is, you have x v-t, x plus v-t [points to first line of working, $x + vt = x + vt$] but here [points to 4 lines later on student's page where she has written $\Delta t = 0,56h$] you have delta t and I don't have it in the formula [points to first line of working, $x + vt = x + vt$ again]. Where does it come from?</p> <p>20cg Student 4 I'm calculating the change in t not the (inaudible)</p> <p>20ch₁ Teacher 3 The change in time, I agree with you. But I don't see it here. [Teacher points to the student's first line on the second page, after which the student writes in Δ symbols in from of the t symbols]. Where does it come from? Because I only see it at the end. Now you are changing the meaning of the whole thing. You didn't understand. (pause) Because you are changing this but this still the velocity. It's not the time. And we don't have the velocity changing time.</p>		
20 ₂	<p>20ch₂ Teacher 3 Don't change it because I'm asking you. If you can just try to explain it to me so that I understand. <i>ke hore</i> (I mean) when I ask you changing don't don't do that. Don't change it. A <i>kere wena</i> (since you) you understand what to do. Just explain to me. Make me understand. Because what I'm talking about is I have x plus v-t ($x + vt$) <i>akere</i> (right)? equals to x plus v-t ($x + vt$). And next time please just make sure you identify that <i>akere o a bona</i> (like you see) which is which.</p>	<p>Distributed to: Teacher as teacher AND Student as student</p> <p>Notes: The teacher is - Giving instructions in 20ch₂ and 20cr (#6a and #6b) - Is assessing the student's work. In 20ch₂, 20cj and 20cl she is repeating the</p>	<p>Distributed to: Teacher as teacher AND Student as student</p> <p>Notes: The teacher is - Describing herself as ignorant in 20ch₂ "...so that I understand" and "I don't know" and in 20cn, where she feigns ignorance (#2b)</p>



20ci	Student 4	Ok.
20cj	Teacher 3	But at the end, I see change in time equals to this. But in the formula, I don't have change in time. Do you get my point?
20ck	Student 4	Yeah, I do.
20cl	Teacher 3	Because <i>Ntho eo ke tlamehileng ho e bona mona ke ntho eo e tshwanetseng hoba ka mona</i> (What I have to see here it is what is supposed to be here) And it's not there in the formula. But your final answer, you are calculating something that is not in here [points to the first line of student's written work $x + vt = x + vt$ again]. That is why I'm asking where does this (Δx) come from because I don't see it here [points to first line again]. Are you with me?
20cm	Student 4	Yeah
20cn	Teacher 3	(20 second pause) Something is wrong with your formula. Your calculations I see they make sense but your formulas they don't make sense. I don't understand them. I don't understand them.
20co	Student 4	Ok. This formula is the right one. [Student proceeds to write on to her page: $\Delta x = v_i \Delta t + \frac{1}{2} a \Delta t^2$, mumbling the symbols as she writes]. So there's the (pause) these trains are moving at a constant speed so the acceleration is zero seconds so that part is zero. Then what is (inaudible) delta x (Δx) equals (inaudible). So we are looking for the (pause) First we look for where the two trains are going to move.
20cp	Teacher 3	Mm
20cq	Student 4	So we transpose this x to the other side.
20cr	Teacher 3	Making what the subject of the formula? Making x the [turns back the student's page to read the first page of her work] (pause for 30 seconds). Just can you can you try to fix these formulas for me so that I can come and just put it more in such a way that I understand it
20cs	Student 4	I can name that (not clear what student is referring to)
20ct	Teacher 3	Yes. [Student 4 turns over to a clean page and begins to rewrite her work; teacher 3 walks away.]

statement of an assessment made earlier. In 20cn, she verbalises a new assessment. (#1b)

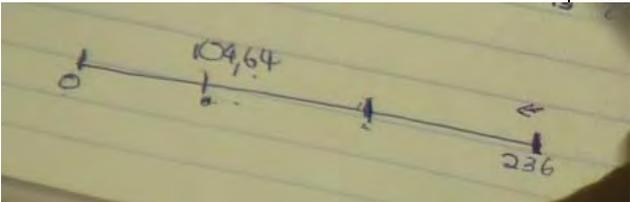
The student is

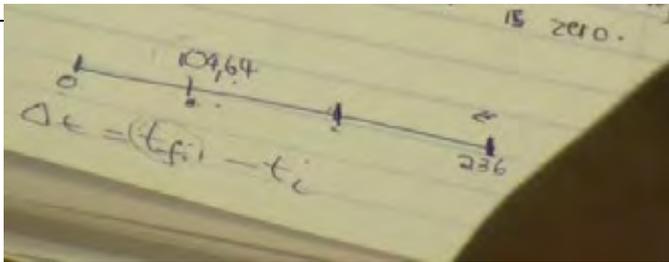
- Being instructed to do something which aligns her with knowledge in 20ch₂ (explain to the teacher so that she can understand) and 20cr (there is an implication that the student can "put it in such a way" that the teacher understands it) (#6a)

The student is

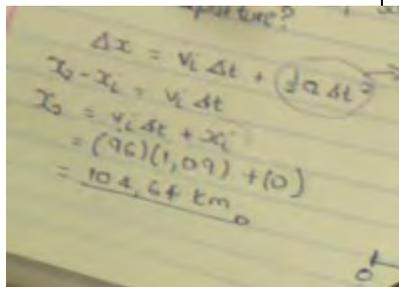
- Being instructed to do something which aligns her with ignorance in 20ch₂ (don't just change something because the teacher says so) (#6b)
- Being unfavourably assessed (#1b)

Interaction number			Transcript	Transmission function	Acquisition function
21 ₁	21a	Teacher 1	Hi Tawela. How are you going? Oops. [Teacher 1 knocks her work. Teacher 1 bends down and reads student's work.] (25 second pause in speaking)	Distributed to: Teacher as teacher AND Student as student Notes: The teacher is - Posing questions in 21b, 21d, 21l, 21n, 21af and so on (#5) - Assessing the student's productions (verbal explanation of written work) in 21f, 21h, 21l, 21r, 21z, 21ah, 21aj and so on (#1a) The student is - Having her productions favourably assessed (#1a) - Being described as aligned with knowledge, for example, in 21a "I can see you can now calculate..." (#2a)	Distributed to: No-one Notes:
21b	Teacher 1	Ok. Can you talk to me? What what what are you thinking at the moment?			
21c	Student 4	I'm thinking I must calculate the time that it takes for each train to reach its destination.			
21d	Teacher 1	Ok. Sorry. Maybe I want to go to the end. I I can see how you thought there. Can you tell me what you are thinking now?			
21e	Student 4	Ok. Since the two the two trains are their departing times are (inaudible) I firstly calculated (inaudible) time. In this instance train A will be in in one hour, since it is one hour ahead of train B.			
21f	Teacher 1	Right right. How far t (pause) Right, I'm with you.			
21g	Student 4	Ok, then I I (pause) now I was calculating the distance			
21h	Teacher 1	Right			
21i	Student 4	that's where train A will be			
21j	Teacher 1	when train B is just starting			
21k	Student 4	Yes. For now I calculated that.			
21l	Teacher 1	Ok. And without doing more calculations what, do you have a plan of what to do now?			
21m	Student 4	Yeah I do.			
21n	Teacher 1	What what's your plan?			
21o	Student 4	Ok. [She draws a line on her page.] Um. This [points to the far left of her line] is the position where train A is			
21p	Teacher 1	when train B is just departing.			
21q	Student 4	Ja.			
21r	Teacher 1	Right			
21s	Student 4	No. Initially			
21t	Teacher 1	Initially. Oh at the start.			
21u	Student 4	Yes. I calculated the distance that's where			
21v	Teacher 1	Ok			
21w	Student 4	train A will be when ah train B is departing.			
21x	Teacher 1	Right			
21y	Student 4	That's its final four comma six three (4,63) [writes 104,63 just above a point about half way along on her line.]			
21z	Teacher 1	Ok			
21aa	Student 4	Then now I want to calculate that's when on what time the two trains are going to meet.			
21ab	Teacher 1	Right			
21ac	Student 4	So I think the (inaudible) it's [points to far right hand side of her line] when the position it's where train B will be			
21ad	Teacher 1	Ok			
21ae	Student 4	And this position is where train A is [points to the 104,64 point on her line].			
21af	Teacher 1	Say that again please. What about the position of ? (pause) You said something about the position where train B will be			
21ag	Student 4	Train B is two			
21ah	Teacher 1	Right right			
21ai	Student 4	thirty-six (236). So now it is no longer two thirty-six away from train A since train A has			
21aj	Teacher 1	Yes yes it is now closer.			
21ak	Student 4	Yeah. I will get this distance and calculate the time for train B			
21al	Teacher 1	I can see you can now calculate the remaining distance between them [indicates the space between the point			

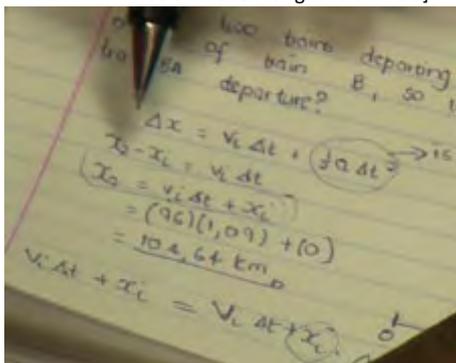
	<p>marked 104,64 and the far right hand side]. But then what will you do, what will you do then? (4 second pause) So let's say initially the distance between the two trains was two hundred and thirty-six (236). But now you can see what's the remaining distance between them. And now they both moving at the same time. How how do you think you can go on then?</p> <p>21am Student 4 Ah. I think they are they are their starting time is the same. Ah I can see that's where they are going to meet their change in time will be the same.</p> <p>21an Teacher 1 Oh. So let's make a mark make a mark somewhere where, more or less, somewhere [student makes a mark on her diagram about half way between the 104,64 and the far right].</p>  <p>21ao Student 4 They say they are going to meet here.</p> <p>21ap Teacher 1 So what change in time is going to be the same?</p> <p>21aq Student 4 From for train A from here [points to the 104,64] at this distance to</p> <p>21ar Teacher 1 There [student points to the new point the diagram]</p> <p>21as Student 4 And for train B from here to here. [Points to far right of line and the new point]</p>		
21 ₂	<p>21at Teacher 1 Oh. That that change in time is the same. Because it starts at the same time [points to 104,64] and it ends at the same time [points to the new point in the middle of the line]. I'm with you. (3 second pause) And how can you find that change in time?</p> <p>21au Student 4 I will simply calculate oh ok</p>	<p>Distributed to: Teacher as teacher AND Student as student</p> <p>Notes: The teacher is assessing the student's production in 21at (#1a)</p> <p>The student is having her production favourably assessed in 21at, indicated both by the teacher's repetition of the student's statement and his saying "I'm with you". (#1a)</p>	<p>Distributed to: Teacher as teacher</p> <p>Notes: The teacher is feigning previous ignorance and newfound understanding in 21at implying previous alignment of himself with ignorance (#2b)</p>
21 ₃	<p>21av Teacher 1 You know that distance, so how can you find that change in time?</p> <p>21aw Student 4 Without using any formulas?</p> <p>21ax Teacher 1 No you can use a formula. No. In whatever way you can.</p> <p>21ay Student 4 Ok. Since I know their position now, I can equate the two equations and solve for change in time.</p> <p>21az Teacher 1 So ok. Go on a bit</p> <p>21ba Student 4 Ok. After I got the change in time I know the time that (2 second pause) [student writes $\Delta t = t_f - t_i$ below her diagram]</p>	<p>Distributed to: Teacher as teacher AND Student as student</p> <p>Notes: The teacher is</p> <ul style="list-style-type: none"> - Posing questions in 21av, 21bp and 21bz (#5) - Explaining how the 	<p>Distributed to: Student as student</p> <p>Notes: The student is</p> <ul style="list-style-type: none"> - Listening to explanation (#3) - Being unfavourably assessed in 21bp (see



21bb Teacher 1 Right
 21bc Student 4 They are going to meet on and I know the departure time is the same so
 21bd Teacher 1 Ok
 21be Student 4 So I can calculate the time that they
 21bf Teacher 1 Ok. But *how* will you get the change in time? [points to the Δt which student has just written]. So the change in time from there to there [points to 104,64 and then middle point] which is the same as the change in time from there to there [points to 236 and then to middle point]. How can you get that change in time?
 21bg Student 4 By equating the two equations?
 21bh Teacher 1 That equation? [student is pointing to the line of her written work which says:
 $x_2 = v_i \Delta t + x_i$]



21bi Student 4 Yes
 21bj Teacher 1 Ok. And what speed will you put there? [Points to the v_i on the line of the student's work indicated in 21bh above]
 21bk Student 4 I know the speed for train A
 21bl Teacher 1 Ok.
 21bm Student 4 as well as for train B
 21bn Teacher 1 Ok. So what speed
 21bo Student 4 And then we have position (3 second pause)
 21bp Teacher 1 But but but this equation [points to $x_2 = v_i \Delta t + x_i$] is this equation for one object that is moving or is it for two objects that is moving?
 21bq Student 4 It's for one
 21br Teacher 1 But now you have two objects that are moving
 21bs Student 4 So equate the two
 21bt Teacher 1 (3 second pause) Just write that down. [student writes $v_i \Delta t + x_i = v_i \Delta t + x_i$ below her existing written work]



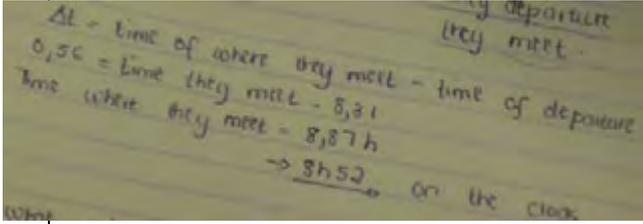
student is to move on to the next stage of the calculation through posing questions (#3)
 - Assessing the student's production favourably in 21av, 21az and 21cd
 In 21bp he withholds verbalising this assessment and instead asks her a question prefaced by "but" which indicates to the student both that she is expected to assess her own productions and that there is an unfavourable assessment from the teacher (since up until this point, the teacher has verbalised when he makes a favourable assessment) (#1a 1b & #1c)
 - Giving the student instructions in 21bt and 21cd (#6a)

The student is
 - Having her production favourably assessed 21az, 21bb, 21bd, 21bf, 21bj, 21bl, 21bn (#1a)
 - Being expected to assess her own production (written work) (#1c)
 - Being instructed to do things (writing down what she has said, equating two equations in 21bt and remember what she has just said in 21cd) which aligns her with knowledge (#6a).
 -

explanation adjacent) (#1b)

<p>21bu Student 4 21bv Teacher 1 21bw Student 4 21bx Teacher 1 21by Student 4</p>	<p>I know that So so let's see what you said now. X-two (x_2). You made x-two (x_2) the subject of the formula. Now you saying It's for train Oh oh oh Train A this train A it's moving like this [writes "TRAIN A" below the x_1 on the left hand side of her equation] and this position for train B [writes "TRAN" above x_1 on the right hand side]. And then</p>		
<p>21bz Teacher 1 21ca Student 4 21cb Teacher 1 21cc Student 4 21cd Teacher 1 21ce Student 4 21cf Teacher 1</p>	<p>And then let's just think about for train A will x-two (x_2) there [points to the middle point on the line] be bigger than x-one (x_1) [points to 104,64 on diagram]? No And for train two will x-two (x_2) [points to the middle point on diagram] be bigger than x-one (x_1)? [points to 236 on the diagram] Yes, it will be less for. It will be less 'cause it's moving on this side. Ah, you have to remember that. 'Cause if you take x-one (x_1) [points to x_1 in student's calculation] plus the speed it will be bigger than x_1. Mm. I will consider also the direction. Oh. Ok. You will consider the direction. Ok. (Teacher walks away, without either pedagogic subject saying anything more.)</p>		

Table 6.5 Positioning in sub-activity 5 (students check answers).

Interaction number	Transcript		Transmission function	Acquisition function
24 ₁	 <p>24a Teacher 1 Hi Tawela, how are you? 24b Student 4 (inaudible) 24c Teacher 1 You've found when they met? 24d Student 4 Yeah 24e Teacher 1 Wow 24f Student 4 I was given the time. 24g Teacher 1 Ok 24h Student 4 Then I from this duration 24i Teacher 1 Alright 24j Student 4 I calculated the distance that's where they meet. The position is where they meet. 24k Teacher 1 Ok. Did you check your answer?</p>		<p>Distributed to: Teacher as teacher AND Student as student</p> <p>Notes: The teacher is - Withholding verbalising assessment, expecting the student to assess her own work in 24k "Did you check your answer?" (#1c) - Posing questions in 24c and 24k (#5).</p> <p>The student is - Being expected to assess her own productions in 24k (#1c)</p>	<p>Distributed to: No-one</p> <p>Notes:</p>
24 ₂	24l	Student 4 Do I have to work backwards?	<p>Distributed to: Teacher as teacher</p> <p>Notes: The teacher is being asked for assistance (#4)</p>	<p>Distributed to: Student as student</p> <p>Notes: The student is asking for assistance (#4)</p>
24 ₃	24m 24n 24o 24p 24q 24r 24s 24t	<p>Teacher 1 I think so, ja. What I want to su... That's that's a good way always to check your thinking, to work back (pause) So let's take this time [points to 08:52]. And then you ask yourself, or you calculate the position of train A at that time and then you also calculate the position of train B at the same time. And then you check are the positions the same. 'Cause if the positions are the same, it means yes, they are meeting at that time. But if train A and train B have different positions at this time, they are not meeting. Does it make sense?</p> <p>Student 4 Yeah. It does make sense. But I think it's when. Ok. If I calculate it for B.</p> <p>Teacher 1 Right</p> <p>Student 4 It won't be won't be exact. The distances I get will be the distance that train B have travelled.</p> <p>Teacher 1 Ok</p> <p>Student 4 So I have to minus it from 236</p> <p>Teacher 1 Ok. And then you can check whether that distances are the same (said in unison). [teacher walks away]</p> <p>Student 4 and Teacher</p>	<p>Distributed to: Teacher as teacher AND Student as student</p> <p>Notes: The teacher is - Assessing the student's (verbal) productions in 24m, 24o, 24q and 24s (#1a) - Giving instructions in 24m (#6a) - Explaining in 24m and 24s (#3) - Posing a question in 24m (#5)</p> <p>The student is - Having her response (in 24n) favourably assessed (#1a) - Being instructed to do something which aligns her with knowledge in 24m (#6a) - Describing herself as aligned with knowledge in 24n (#2a)</p>	<p>Distributed to: Student as student</p> <p>Notes: The student is - Listening to explanation in 24m and 24s (#3)</p>

<p>25₁</p>	<p>This occurs at the start of the last day of the course and is a whole class address from Teacher 1.</p> <p>25a Teacher 1 Good morning. We can just go on where we left yesterday.</p> <p>25b Student Our test?</p> <p>25c Teacher 1 Ok. No no. No no. Let me just get the test</p> <p>25d Student No no no no (shouting, high pitched)</p> <p>25e₁ Teacher 1 Ok. Let's just go on. Let me just say, our plan is. Sorry, I have not put on the speaker but this is not important. Ah, if you work on this problem until the end of today, that was the plan. But typically, once you've finished it once you've got an answer. Sorry [puts on the microphone]. Let me say that for the record. So in a court of law, I did tell you this. I did warn you. Um, typically once you do get an answer, we'll come to you and we'll ask you: have you checked your answer? How have you checked it? Have you redone your calculations? Yes. And did you get the same answer? Yes. So what does that tell you? It tells you you didn't make a calculation error. But that's all it tells you.</p>	<p>Distributed to: Teacher as teacher AND Student as student</p> <p>Notes: The teacher is - Posing questions in 25e₁ (#5) - (Hypothetically) withholding assessment in his indicating that the students are to check their own answers (#1c)</p> <p>The students are - Being described as aligned with knowledge in that the teacher implies that the students can and will "finish": "once you've finished it once you've got an answer" in 25e₁ (#2a) - Being expected to assess their own productions (#1c)</p>	<p>Distributed to: No-one</p> <p>Notes:</p>
<p>25₂</p>	<p>25e₂ Teacher 1 If there was a error in your method or your logic your reasoning and you just redid the same calculations, you will get the same answer. So it's good to check your calculations, but it's not enough.</p>	<p>Distributed to: Teacher as teacher AND Student as student</p> <p>Notes: The teacher is (hypothetically) assessing the student hypothetical productions (#1a and #1c)</p> <p>The students are (hypothetically) having their productions favourably assessed "it's good to check your calculations" (#1a)</p>	<p>Distributed to: Student as student</p> <p>Notes: The students are (hypothetically) having their productions unfavourably assessed: "but it's not enough" (#1b)</p>
<p>25₃</p>	<p>25e₃ Teacher 1 You design that aeroplane wing, you checked your calculations, but there was something wrong – some assumption that you made that was wrong – and now the aeroplane goes off... goes up in the air and then it goes down in smoke and flames. Ok. You also need to check it in a different way. So, you try to do another calculation and as I told earlier this week – some of you were not then in that class.</p>	<p>Distributed to: Teacher as teacher AND Student as other (engineer)</p> <p>Notes: The students are being given the persona of engineer by implication in the teacher's speaking.</p> <p>The teacher - Withholds assessment of the student's hypothetical design of an</p>	<p>Distributed to: Student as other (engineer)</p> <p>Notes: The student is being described as aligned with ignorance "there was something wrong..." (#2b)</p>

		<p>aeroplane wing (#1c) - Giving instructions (#6a)</p> <p>The student is</p> <ul style="list-style-type: none"> - Being expected to assess his own productions “you checked...” (#1c) - Being instructed to do something which aligns him with knowledge “you also need to check it in a different way” (#6a) 	
25 ₄	<p>25e₄ Teacher 1</p> <p>Often it’s very useful to work backwards. To say: “no, no, let me work back from my answer and see if I can work back to the original information. Let me work from my answer and then pretend that some of the original information I don’t know. Maybe the total distance or whatever. Now let me work from my answer backwards and see: do I get the two hundred and thirt sixty two hundred and thirty six kilometres? If I get two hundred and fifty kilometres, then my answer couldn’t have been right.” So I want you to think about how I can check my answer by working backwards. And then once you’ve checked your answer we will ask you another question about the trains. Ok.</p>	<p>Distributed to: Teacher as other (student) AND Student as student</p> <p>Notes: The teacher speaks with the voice of a student while he is</p> <ul style="list-style-type: none"> - Explaining (#3) and - Giving instructions (#6a) - Posing questions: “we will ask you another question” (#5) <p>The student is being given instructions (to check their work) which align them with knowledge (#6a)</p>	<p>Distributed to: Student as student AND Teacher as teacher</p> <p>Notes: The student is</p> <ul style="list-style-type: none"> - Listening to explanation (#3) - Being described as aligned with ignorance when the teacher says: “...then my answer couldn’t have been right.” (#2b) <p>The teacher, speaking of himself as if he were a student, is being described as aligned with ignorance when the teacher says: “...then my answer couldn’t have been right.” (#2b)</p>

Sub-notion 3: Creating an algebraic formula to calculate values about the general case of a given scenario

Table 6.6 Positioning in sub-activity 6 (students determine a formula)

Interaction number	Transcript		Transmission function	Acquisition function
25 ₅	25e ₅	Teacher 1 And, in case you are finished, the other question is: you've now answered it for these two trains. But imagine you are the railway manager at a big station in the city. And every day there are tens maybe twenty or different thirty different trains coming in and out. They don't only come from the one town. They come from different towns. Some of them are long and heavy and slow. Some of them are short and fast. They've got different speeds. They even have different arrival times.	Distributed to: Teacher as teacher Notes: The teacher is - Explaining the scenario (regarding being a "railway manager at a big station in the city") in order to justify the need for a formula i.e. to motivate the necessity of a formula. (#3)	Distributed to: Student as other (railway manger) Notes: The student is explicitly given the persona of "railway manager". The student is listening to explanation (#3)
25 ₆	25e ₆	Teacher 1 Can you now make yourself a tool so that every day, if you are the railway manager, and let's say – and I don't I know it's not really that realistic – but let's say we want to know, for any two trains that go between the same two towns, when will they meet? Let's say that's your job for every day.	Distributed to: Teacher as teacher AND Student as other (railway manager) Notes: The teacher is - Posing questions: "can you now make yourself a tool...?" and "when will they meet?" (#5) The student retains the persona of "railway manager" from 25e ₆ and at the same time is being described as aligned with knowledge when the teacher asks him to make "yourself a tool", since the question contains the implication that the student can do this (#2a)	Distributed to: No-one
25 ₇	25e ₇	Teacher 1 Now for every day for the thirty trains, you can do all of these calculations. And um you'll be happy if you leave work before ten o'clock in the evening doing these calculations for thirty trains.	Distributed to: Teacher as teacher Notes: The positioning for the teacher remains as it was in 25e ₅ .	Distributed to: Student as other (railway manager) Notes: The student is still given the persona of railway manager ("if you leave work...") and is described as being aligned with ignorance, implied in the railway manager having to work very late to complete all calculations (#2b)

25 ₈	<p>25e₈ Teacher 1 Can you not make yourself a tool that will make it quick and easy to calculate at what time will the two trains meet, or, pass each other? Can you not quickly just use the arrival times of those two trains? use the total distance between the towns? use the speeds of the two trains and just quickly calculate at what time will they meet? Can you make yourself a tool to do that quickly and easily for many trains? So that's once you have finished it for this particular two trains. You don't want to do the same work over and over again for thirty trains. So, if it takes us the whole day, it takes us a whole day. I think you can get there yourself. You you can make that tool yourself. We have enough time. We have more than enough brain power. So let's go on</p>	<p>Distributed to: Teacher as teacher AND Student as student</p> <p>Notes: The teacher is posing questions (#5)</p> <p>The student is being described as aligned with knowledge (#2a)</p>	<p>Distributed to: No-one</p> <p>Notes: Although the teacher refers to taking a long time to complete work here in 25e₆, this is not considered an indication of ignorance and so AF to student (as was done in 25e₅). The teacher is indicating (in 25e₆) that they are expecting the students to take the whole day to create a formula and that this length of time is acceptable.</p>
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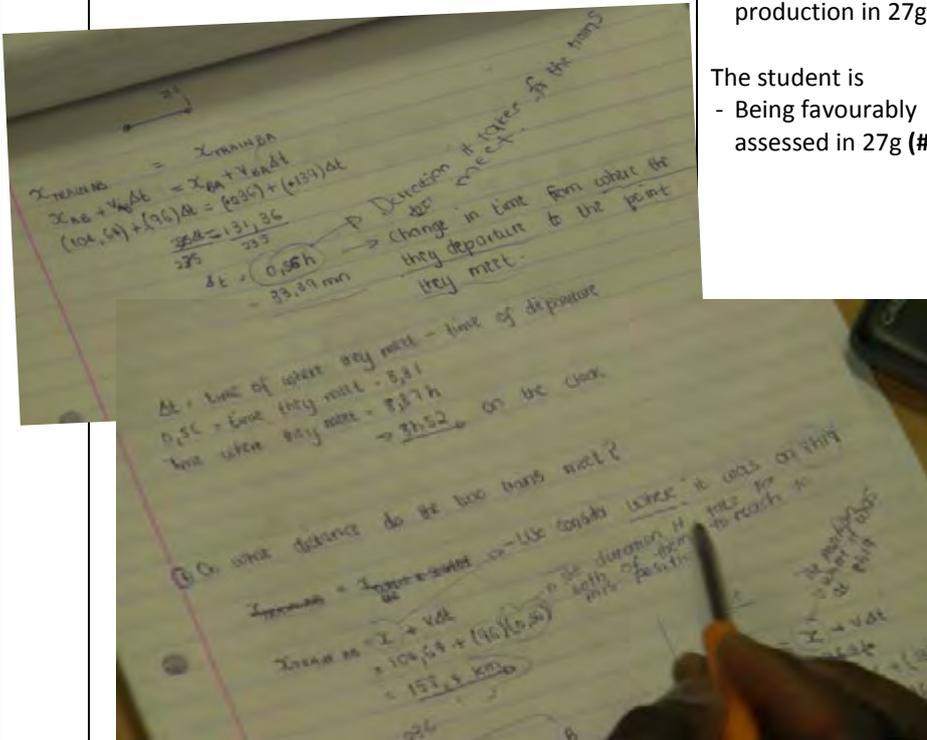
26 ₁	<p>26a Teacher 1 Hi Tawela. How's it going? [sits down next to student]</p> <p>26b Student 4 It is the distance they are going.</p> <p>26c Another student (Another student, not in focus group, approaches teacher 1. Says something inaudible.)</p> <p>26d Teacher 1 You are not done yet. You haven't made a formula for the time when they meet yet (spoken to other student). Just give me a second (spoken to student 4). [Teacher gets up and walks with the other student to his desk and separate interaction ensues.]</p> <p>26e Teacher 1 Sorry Tawela. [Teacher 1 sits back down next to Tawela.] I'm listening to you.</p> <p>26f Student 4 Ok. I've calculated the distance to meet.</p> <p>26g Teacher 1 Ok</p> <p>26h Student 4 For A for train A</p> <p>26i Teacher 1 Ok</p> <p>26j Student 4 and you said I must also check</p> <p>26k Teacher 1 Check for train B</p> <p>26l Student 4 for train B. So I did that also. Minus 158, which is the opposite of this one.</p> <p>26m Teacher 1 Sorry? So are they at the same position at eight fifty-two?</p> <p>26n Student 4 Yes.</p> <p>26o Teacher 1 Ok.</p> <p>26p Student 4 And this minus I think it's the (inaudible) since (inaudible).</p> <p>26q Teacher 1 Cool. Now.</p> <p>26r Student 4 What?</p> <p>26s₁ Teacher 1 [laughs] Congratulations. Now, we have a second problem.</p>	<p>Distributed to: Teacher as teacher AND Student as student</p> <p>Notes: The teacher is assessing the student's productions in 26q and 26s₁ (#1a)</p> <p>The student is</p> <ul style="list-style-type: none"> - Being favourably assessed (#1a) - Assessing his own production in 26j (#1c) 	<p>Distributed to: No-one</p> <p>Notes:</p>
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26 ₂	<p>26s₂ Teacher 1 Imagine once again that you are the railway manager, you working at the station at a big city. Every day there are many many trains coming in and coming out of the station from different towns. And some trains are faster, some trains are slower. Now let's say that every day you want to know for any two trains, at what time will they pass each other, what time will they meet each other. Now it will be a lot of work if you have thirty trains in a day to do this calculation over and over again.</p>	<p>Distributed to: Teacher as teacher</p> <p>Notes: - Explaining the scenario (regarding being a "railway manager working in a big city") in order to justify the need for a formula i.e. to motivate the necessity of a formula. (#3)</p>	<p>Distributed to: Student as other (railway manager)</p> <p>Notes: The student is explicitly given the persona of "railway manager" and is also listening to explanation (#3)</p>
26 ₃	<p>26s₃ Teacher 1 Can you make yourself a tool that will make it quick and easy to calculate for any two trains, if those two trains are going between two towns,</p> <p>26t Student 4 Mm</p> <p>26u Teacher 1 For any two trains with any speeds, any total distance between the towns, and any departure times, it will quickly and easily it will calculate at what time on the clock they will meet?</p> <p>26v Student 4 Hm.</p> <p>26w Teacher 1 Ok. (Teacher 1 walks away)</p>	<p>Distributed to: Teacher as teacher AND Student as other (railway manager)</p> <p>Notes: The teacher is - Posing questions in 26s₃ and 26u: "Can you make yourself a tool that...will calculate at what time on the clock they will meet?" (#5)</p> <p>The student retains the persona of "railway manager" (from 26s₂) and at the same time is being described as aligned with knowledge when the teacher asks him to make "yourself a tool", since the questions contains the implication that the student can do this (#2a)</p>	<p>Distributed to: No-one</p>

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On the afternoon of the last day of the course, Teacher 3 sits down next to Student 4 at the student's desk. Student 4 does not appear to have called her over. Teacher 3 reads the student's work without saying anything for about 30 seconds.

- 27a Teacher 3 Can you explain what you did?
- 27b Student 4 Ok, I've done with this [points to work, not clear where]. And checked it right?
- 27c Teacher 3 Mm. Where did you start?
- 27d Student 4 Ok. Where I started. I've calculated the time that the two trains meet. [Student points at 0,56h on her page]



- 27e Teacher 3 Come again?
- 27f Student 4 I've calculated the the duration time when the two trains meet.
- 27g Teacher 3 Mm. So then it means that they are going to meet at zero comma five six hours [points to 0,56h on student's page]. Let me see here. [Teacher takes the student's calculator] where are the numbers? One two three four five six. [Teacher types in 0.56x60]. Good. The trains will meet at thirty-three comma six minutes.



- 27h Student 4 Yeah. I think that's it.
- 27i Teacher 3 Starting from where?
- 27j Student 4 Ok.

Distributed to:
Teacher as teacher
AND
Student as student

Notes:
The teacher is
- Posing questions in 27a and 27i (#5)
- Assessing the student's production in 27g (#1a)

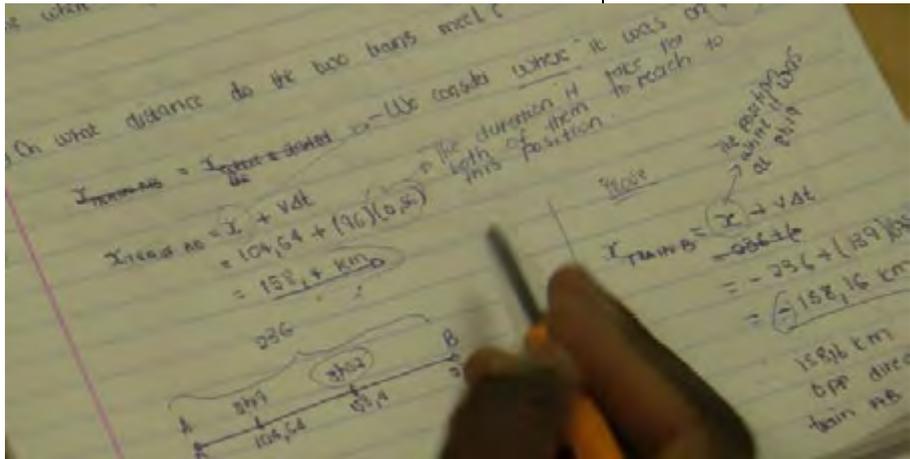
The student is
- Being favourably assessed in 27g (#1a)

Distributed to:
No-one

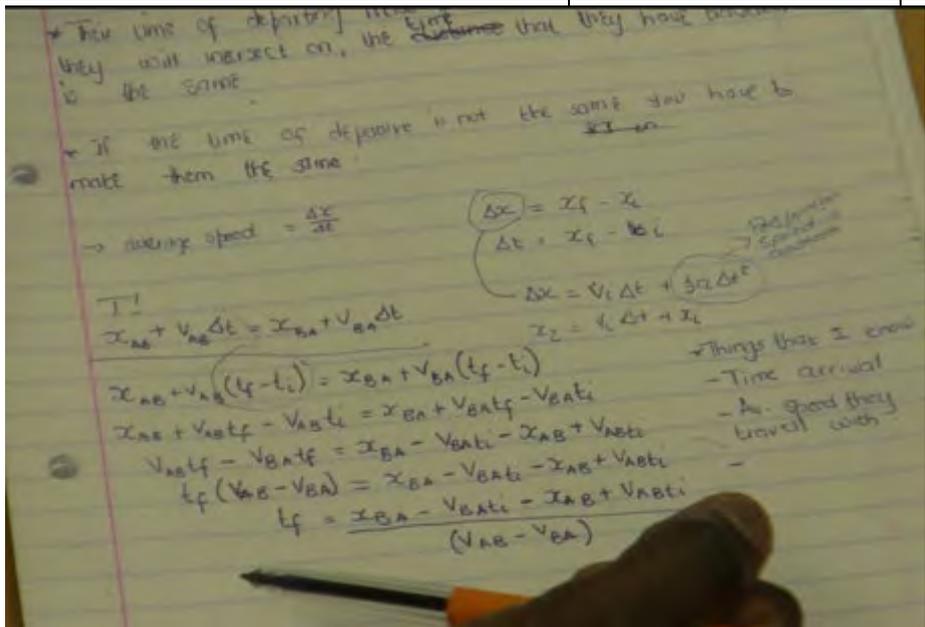
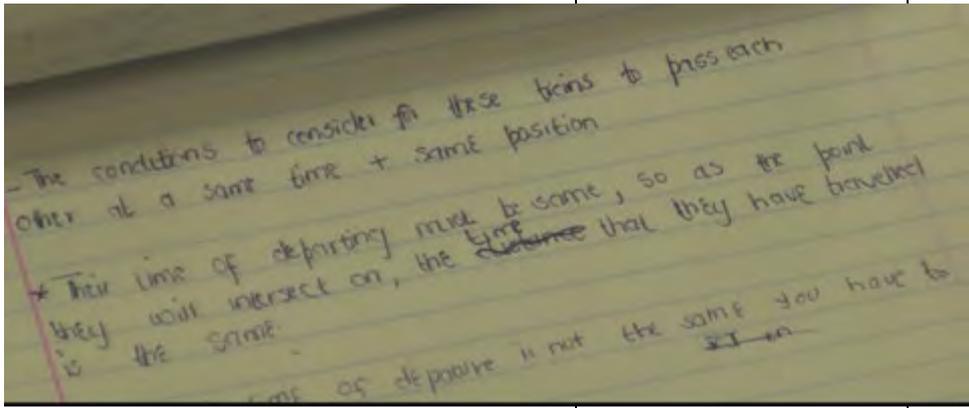
Notes:

27 ₂	<p>27k Teacher 3 Akere (in fact) You know why I'm asking this? Because the other one is going to start one hour after the other one has started. So what I want to know is, thirty-three point six minutes from</p> <p>27l Student 4 From where.</p> <p>27m Teacher 3 From when.</p> <p>27n Student 4 After <i>u</i>position <i>u</i>Train (position Train) A has travelled a certain distance until it's the time it is the same as B.</p> <p>27o Teacher 3 Mm</p> <p>27p Student 4 Ja. Then the duration is take for them to meet.</p> <p>27q Teacher 3 <i>Ke hore</i> (Meaning) after starting from the same at the same time.</p> <p>27r Student 4 At the same time.</p> <p>27s Teacher 3 Ok.</p> <p>27t Student 4 They are</p> <p>27u Teacher 3 So, <i>ke hore</i> (meaning) after starting at eighteen past eight, both of them, so they are going to meet after thirty-three comma three nine (33,39) minutes.</p> <p>27v Student 4 Yes</p> <p>27w Teacher 3 So uh you have equated what? [points to student's work where she has written</p> $\begin{aligned} x_{\text{TRAIN AB}} &= x_{\text{TRAIN BA}} \\ x_{\text{AB}} + v_{\text{AB}}\Delta t &= x_{\text{BA}} + v_{\text{BA}}\Delta t \end{aligned}]$ <p>X train A ($x_{\text{TRAIN AB}}$) what does this mean?</p> <p>27x Student 4 It's the train does move from A to B and other one from B to A.</p> <p>27y Teacher 3 The distance? Come again</p> <p>27z Student 4 This formula [points to $x_{\text{AB}} + v_{\text{AB}}\Delta t$] is for this train moving from A to B and the other one [points to $x_{\text{BA}} + v_{\text{BA}}\Delta t$] from</p> <p>27aa Teacher 3 So is the formula for what? What were you calculating? The distance, the time or the speed?</p> <p>27ab Student 4 The time.</p> <p>27ac Teacher 3 Ok. Hm.</p> <p>27ad Student 4 After I got the time [points to 0,56h], um, I went back and calculated the position where um that the both of them were going to meet. So I firstly calculated for the train that is moving from A to B.</p> <p>27ae Teacher 3 Mm</p> <p>27af Student 4 This [points to 158,4km] is the distance I got.</p> <p>27ag Teacher 3 So this [points to 158,4km] is the?</p> <p>27ah Student 4 From here to here [points to 104,64 and then 158,4 on her diagram]. They met at this time [circles 8h52 on diagram], which means, so I used this the duration from here to here [points to 104,64 and then 158,4 on her diagram] to calculate on one position.</p> <p>27ai Teacher 3 You used this? [points to the 8h52 on the student's diagram]</p> <p>27aj Student 4 No, I used the duration [points to 0,56].</p> <p>27ak Teacher 3 This [points to 0,56h] uh to calculate this [points to 158,4km]</p> <p>27al Student 4 Yes. I used...</p> <p>27am Teacher 3 <i>Ke hore</i> (Meaning) after calculating this you got the distance [points to 158,4km]? And then you went back, used the speed formula to calculate the the time [points to 8h52 on diagram]?</p> <p>27an Student 4 Ja.</p> <p>27ao Teacher 3 Ok.</p> <p>27ap Student 4 So I got this distance [points to 158,4km]</p> <p>27aq Teacher 3 Mm</p> <p>27ar Student 4 I, for A, I did this for A (referring to what</p>	<p>Distributed to: Teacher as teacher</p> <p>Notes: The teacher is</p> <ul style="list-style-type: none"> - Explaining in 27k (#3) - Assessing the student's production in 27m (#1b) - Posing questions in 27w, 27aa, 27ai, 27am and 27ba (#5) 	<p>Distributed to: Student as student</p> <p>Notes: The student is</p> <ul style="list-style-type: none"> - Listening to explanation (#3) - Having her production (verbal response) unfavourably assessed in 27m and 27be. The unfavourable nature of the assessment is apparent in that the teacher repeats what the student has said but adds in a correction. (#1b) <p>In 27bk, the teacher claims that she doesn't understand, so one could perhaps conclude distribution of AF to the teacher here. However, since the teacher is trying to gain access to student's thinking, it is the student's thinking – and not the notion – which she is claiming ignorance of. Thus there is no distribution of AF to teacher here.</p>
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27as Teacher 3 has just been discussed). Then I have to prove whether it's right.
 27at Student 4 Mm
 If I do it for B. Then I do it for B also [student points to writing on the right hand side of her page.]



27au Teacher 3 Mm
 27av Student 4 And I got it and a negative, which means, obviously, train A is moving on this (inaudible) [points to -158,16km]
 27aw Teacher 3 Mm
 27ax Student 4 Then after that I had to make the formula [turns to next page of her written work].



27ay Teacher 3 Mm?
 27az Student 4 I had to make the formula.
 27ba Teacher 3 Ok. Can you explain the formula for me?
 27bb Student 4 Ok. Ok. It's like there's a condition I have to consider.

	<p>27bc Teacher 3 Mm</p> <p>27bd Student 4 Ok. Firstly, we've got these trains that meet at the same, they must, their time that they start to moving must be the same.</p> <p>27be Teacher 3 Mm. Mm. They are the time at which they are going to meet must be the same.</p> <p>27bf Student 4 Yes.</p> <p>27bg Teacher 3 Mm (Yes). I agree.</p> <p>27bh Student 4 As well as the position that they. Ok. Ja. As well as the time that they departing from, the position they are intersect must be the same.</p> <p>27bi Teacher 3 Mm</p> <p>27bj Student 4 Um. Ok. I've worked on what I did (inaudible)</p> <p>27bk Teacher 3 [laughs] But I don't understand what's going on .</p> <p>27bl Student 4 Ok. Ok. To use this formula, our aim was to get the duration, right?</p> <p>27bm Teacher 3 Mm</p> <p>27bn Student 4 Was to get the duration time. What I did is that I took the formula as it is and our aim was to get the time exactly [turns back to previous page of work, points to $\Delta t = 0,56 \text{ h}$].</p>		
27 ₃	<p>27bo Teacher 3 At what time are they going to meet. Mm-hm.</p> <p>27bp Student 4 So I [circles $(t_f - t_i)$ on her page] ok, the formula [underlines $x_{AB} + v_{AB}\Delta t = x_{BA} + v_{BA}\Delta t$], on what I did the formula was delta t (Δt) on what I did before.</p> <p>27bq Teacher 3 What does this one [points to x_{AB}] stand for? The first one.</p> <p>27br Student 4 It's for</p> <p>27bs Teacher 3 The x-A-B (x_{AB})?</p> <p>27bt Student 4 We've got a train that's moving from A to B.</p>	<p>Distributed to: Teacher as teacher AND Student as student</p> <p>Notes: The teacher is - Assessing the student's (verbal) production in 27bo, where she adds clarification and then agrees with what the student has said in 27bn (#1a) - Posing questions in 27bq and 27bs (#5)</p> <p>The student is - Having her production favourably assessed in 27bo. (#1a)</p>	<p>Distributed to: No-one</p> <p>Notes:</p>
27 ₄	<p>27bu Teacher 3 x-A-B (x_{AB}) is the train?</p> <p>27bv Student 4 Oh, this is the position where it starts move. It's the position where train A is.</p> <p>27bw Teacher 3 Oh, <i>ke hore</i> (meaning) it's at the at zero point. Ok. And then this one [points to v_{AB}] is the</p> <p>27bx Student 4 The (inaudible)</p> <p>27by Teacher 3 So then it means is zero plus the the distance. Is this [points to $v_{AB}\Delta t$] the distance? <i>Akere</i> (in fact) the velocity multiplied by change in time.</p> <p>27bz Student 4 This is distance [points to x_{AB}]</p> <p>27ca Teacher 3 This [points to x_{AB}] is the distance, and then what is this [points to $v_{AB}\Delta t$] ?</p> <p>27cb Student 4 Velocity times time.</p> <p>27cc Teacher 3 Hm?</p> <p>27cd Student 4 Velocity times change in time.</p> <p>27ce Teacher 3 How do you calculate the speed? o</p>	<p>Distributed to: Teacher as teacher</p> <p>Notes: The teacher is - Posing questions in 27bu, 27ca, 27ce, 27ci, 27cq and 27cs (#5) - Assessing the student's production in 27bu, 27cc, 27ck, 27cm and 27cu. (#1b) In 27bu, that the question also represents an unfavourable assessment is clear in that the teacher is questioning the student's</p>	<p>Distributed to: Student as student</p> <p>Notes: The student is - Having her production unfavourably assessed. (#1b) - Listening to explanation (#3) - Asking for assistance in 27cx and 27cz (#4)</p>

		<i>nngolle</i> (write for me) the formula of calculating the speed? [gives student a new, clean page]	response which was given in 27bt and then the student adjusts her response in 27bv. That the student corrects herself in 27cd suggests that 27cc was an unfavourable assessment. (Note also that the teacher's statements in 27co indicate that the student has misunderstood the teacher's unfavourable assessment.)
27cf	Student 4	Speed?	
27cg	Teacher 3	Mm	
27ch	Student 4	Distance over time. [Student writes $s = \frac{d}{t}$]	
27ci	Teacher 3	Mm. And then make d the subject of the formula. [Student writes $d = st$]. And then and then what is the difference between this [points to st] and this [points to $v_{AB}\Delta t$] ?	
27cj	Student 4	This [points to $v_{AB}\Delta t$] is change in time and this [points to st] is time at a certain point.	
27ck	Teacher 3	And did you hear yesterday, or the day before tomorrow, I mean the day before yesterday. They said this [points to $s = \frac{d}{t}$] is not the correct formula because the speed it is the rate of change. It's good we don't calculate the speed at a certain point.	- Explaining in 27ck, 27cm, 27co, 27cq, 27cu and 27cw (#3)
27cl	Student 4	Yeah	
27cm	Teacher 3	We calculate the speed <i>ke hore</i> (meaning) at an interval. <i>ke hore</i> (meaning) change in distance, because akere (in fact) you are moving from one point to the other, divided by the change in time. Akere (right)? [student writes $s = \frac{\Delta d}{\Delta t}$]. So then it means this thing [circles $\frac{d}{t}$] it's wrong because <i>ka mokgwa o e leng ka teng ke</i> (it is the way it is) that is why we ended up talking about the instantaneous, talking about the average flow rate, <i>ke hore</i> (meaning) what is the difference between the two. If you write it like this [points to $\frac{d}{t}$], then it means that you are calculating a speed at one point. And if you are at one point, then it means you are not moving.	- Being asked for assistance in 27cx and 27cz (#4)
27cn	Student 4	Ok	
27co	Teacher 3	Ok. Akere (Right)? Akere (in fact) If <i>ke bua ka ke na le le</i> (I talk about I have) At this point, at this point, at this point, <i>ke ha hona</i> (it there is no) movement, you cannot calculate the speed while you are standing. Akere (Right)? So then that is why I am asking <i>hore mona ke bona</i> (that here I should see) change in time, then I see the velocity. And to me, I see a distance.	
27cp	Student 4	Ah. Ok.	
27cq	Teacher 3	Because distance <i>ke</i> (is) velocity multiplied by change in time. Akere (Right)? And if this [points to $v_{AB}\Delta t$] is the distance, so <i>mona</i> (here) what is this [points to x_{AB}]? Are you, is it distance plus distance equals to distance plus distance?	
27cr	Student 4	I [laughs]	
27cs	Teacher 3	Mm? (12 second pause) <i>o wa nunderstanda</i> (do you understand)?	
27ct	Student 4	Yeah. <i>Ke ya understanda</i> (Yes. I understand) but how to, how to derive it.	
27cu	Teacher 3	You know what, you were supposed to <i>ke hore</i> (meaning) write something like keywords. <i>Ke hore</i> (Meaning) and tell us <i>ke hore</i> (in that) you were going to let	

	<p>27cv Student 4 27cw Teacher 3</p> <p>27cx Student 4</p> <p>27cy Teacher 3 27cz Student 4</p>	<p>what equals to what, what equals to what, what equals to what. So that when you substitute them here [points to $x_{AB} + v_{AB}\Delta t = x_{BA} + v_{BA}\Delta t$]. Akere (in fact) whatever. The way you were explaining this to me [turns back to student's previous page where $\Delta t = 0,56h$ is calculated], <i>ke hore</i> (meaning) you are supposed to follow the same procedure like you did here [points to student's algebraic formula for Δt].</p> <p>Yeah, I'm trying to</p> <p>Mm. If you you <i>ke hore</i> (meaning) you first calculated whatever, you must first do the same here. Then in the end you'll put everything together. (8 second pause)</p> <p>So am I going to write this one [points to $x_{AB} + v_{AB}\Delta t$]?</p> <p>Hm? (yes?)</p> <p>Am I supposed to write this one [points to $x_{AB} + v_{AB}\Delta t$]?</p>		
27 ₅	<p>27da Teacher 3</p> <p>27db Student 4</p> <p>27dc Teacher 3</p> <p>27dd Student 4</p> <p>27de Teacher 3</p>	<p>(5 second pause) I'm not saying it's wrong or it's right.</p> <p>[Laughs]</p> <p>Akere (In fact) we are only talking [laughs]. We are talking.</p> <p>Ok.</p> <p>Mm. Then carry on and maybe it will make sense on the way.</p>	<p>Distributed to: Teacher as teacher AND Student as student</p> <p>Notes: The teacher is - Withholding assessment in 27da and 27de (#1c). - Giving instructions (#6a)</p> <p>The student is - Being expected to assess her own productions in 27da and 27de (#1c) - Being instructed to do something which aligns with knowledge in 27de (#6a).</p>	<p>Distributed to: No-one</p> <p>Notes:</p>
27 ₆	<p>27df Student 4</p> <p>27dg Teacher 3 27dh Student 4</p> <p>27di Teacher 3</p> <p>27dj Student 4 27dk Teacher 3</p>	<p>Ok. Then ah. Ok. Our aim on on this side [turns back to previous page] was to calculate the duration time, isn't it? So after we've got the duration time, we we split up this total (not clear what this total is) and we got the finally exactly the time when they meet.</p> <p>Mm (yes)</p> <p>So I I break down this [points to $(t_f - t_i)$] and on the other side, then I do it (inaudible) to get [Reading the student's written calculations] So change in time [points to Δt on left hand side of equation], you took it back to t-f (t_f) minus t-l (t_i) [points to $(t_f - t_i)$ on the next line of student's work] and then the same thing applies to this one [points to Δt and $(t_f - t_i)$ on the right hand side of student's work] .</p> <p>Yes.</p> <p>And then when you came here [points to $v_{AB}t_f - v_{AB}t_i$ on next line of student's written work], you multiplied everything</p>	<p>Distributed to: Teacher as teacher</p> <p>Notes: The teacher is - Being asked for assistance in 27df (#4) - Explaining in 27do (#3) - Assessing the student's production in 27do (#1b)</p>	<p>Distributed to: Student as student</p> <p>Notes: The student is - Asking for assistance in 27df (#4) - Listening to explanation 27do (#3) - Having her production (written work) unfavourably assessed in 27do (#1b)</p>

		by this [points to v_{AB} and then t_f , again to v_{AB} and then t_i .]		
27dl	Student 4	Yes.		
27dm	Teacher 3	And then the same here [points to $v_{BA}t_f - v_{BA}t_i$ on right hand side of student's work]. And then and then this one [points to $v_{BA}t_f$] to the other side. And this one [points to x_{AB}] this side. And then you have this [points to $v_{BA}t_f$]. And then this one [points to $v_{AB}t_i$] to the side. And then		
27dn	Student 4	I took out t-f (t_i) [points to t_f where she has written $t_f(v_{AB} - v_{BA})$], the one I'm looking for. Then I... it becomes v-a-b (v_{AB}).		
27do	Teacher 3	By the way, your final time is not the final time. Is the time where the two trains meet.		
27dp	Student 4	Mm (yes). Because there it's equal		
27dq	Teacher 3	Because for t-f (t_i) I'm thinking about final time as the final time. So if it's the final time of. Why don't we just write m [writes on student's page, changing t_f to say t_m in her final line]? so that it's the midpoint.		
27dr	Student 4	Ok.		

Table 6.7 Positioning in sub-activity 7 (students simplify their formula).

Interaction number	Transcript	Transmission function	Acquisition function
	No data (No student in the focus group participates in any interactions related to this sub-activity)	Distributed to: Notes:	Distributed to: Notes:

Table 6.8 Positioning in sub-activity 8 (students check formula)

Interaction number	Transcript		Transmission function	Acquisition function	
27 ₇	27ds	Teacher 3	Because f is like (pause). And then, can you try can you use this formula [points to $t_m = \frac{x_{BA} - v_{BA}t_i - x_{AB} + v_{AB}t_i}{v_{AB} - v_{BA}}$] using the <i>ntho</i> (thing)	Distributed to: Teacher as teacher AND Student as student	Distributed to: Student as student
	27dt	Student 4	Yes.	Notes: The teacher is	Notes: The student is
	27du	Teacher 3	I just want to see it. If you get the same answer. It has to work, <i>akere</i> (right)? If it doesn't then it means something is wrong.	- Giving instructions to the student in 27ds and 27eg (#6a)	- listening to explanation (#3)
	27dv	Student 4	(Pause for 3 minutes 15 seconds) [student rewrites her formula using t_m in place of t_i and substitutes the values given originally in place of all other symbols. Then puts in on her calculator. Teacher 1 watches].	- Explaining to the student how to check her formula in 27ds and 27du (#3)	- being unfavourably assessed in that the teacher points out that "it is not working" in 27eg (#1b)
	27dw	Teacher 3	Which university are you from?	- Assessing the student's productions in 27eg (#1b)	
	27dx	Student 4	Hm?		
	27dy	Teacher 3	From which university are you from?	The student is being instructed to do something (check her work, although this is not explicitly stated) which requires alignment with knowledge (#6a)	
	27dz	Student 4	The university of (states name of institution)		
	27ea	Teacher 3	Are you from (states name of place)		
	27eb	Student 4	From (states name of another place).		
	27ec	Teacher 3	Ok		
	27ed	Student 4	(inaudible)		
	27ee	Teacher 3	It doesn't give you the same answer?		
	27ef	Student 4	Maybe the direction.		
	27eg	Teacher 3	Then try it with everything and check. Because it means if it is not working, then it means that the formula is not right. It has to work. [Teacher sits next to the student, watching her work for a few more minutes but does not interact further, then walks away.]		

Table 6.9 Positioning in sub-activity 9 (students refine their formula).

Interaction number	Transcript	Transmission function	Acquisition function
	No data (Student 4 does not participate in any interactions related to this sub-activity)	Distributed to: - Notes:	Distributed to: Notes:

Appendix 7: Positioning analysis (summary tables)

The following tables summarize the positioning of pedagogic subjects with respect to knowledge in *The Trains Activity*. The colour coding matches that in figure 5.2 – an attempt to indicate the stages of judgment of the first three notions.

Table 1: summary of the positioning of the pedagogic subjects during the judgement of sub-notion 1 (asking appropriate questions about a given scenario)

	Interaction number	Speech turns	Transmission function					Acquisition function						
			T-T	T-O	S-S	S-O	O	S-S	S-O	T-T	T-O	O		
Sub-Activity 1 (students read problem description)	Existence	1 ₁	1a	✓					✓					
		1 ₂	1b	✓		✓								
		2 ₁	2a ₁	✓					✓					
Sub-activity 2 (students create graphical representation)	Existence	2 ₂	2a ₂	✓		✓								
		2 ₃	2a ₃	✓		✓			✓			✓ (student)		
Sub-activity 3 (students pose interesting questions & explain “what’s going on here”)	Reflection	3 ₁	3a ₁	✓		✓								
		3 ₂	3a ₂			✓			✓			✓ (student)		
		3 ₃	3a ₃	✓		✓								
	Necessity	4 ₁	4a ₁	✓		✓								
		4 ₂	4a ₂	✓					✓		✓			
		4 ₃	4a ₃		✓ (as engineer)				✓			✓ (as engineer)		
		4 ₄	4a ₄	✓					✓		✓			
	Notion	4 ₅	4a ₅	✓		✓			✓					
		4 ₆	4a ₆ – 4b		✓ (as train)				✓					
		4 ₇	4c ₁	✓					✓					
		4 ₈	4c ₂	✓					✓		✓			
		4 ₉	4c ₃ – 4d		✓ (as train)				✓					
4 ₁₀		4e – 4f	✓		✓									
4 ₁₁		4g ₁		✓ (as train)	✓									
4 ₁₂		4g ₂ – 4m	✓		✓			✓						

Table 2: summary of the positioning of the pedagogic subjects during the judgement of sub-notion 2 (calculating specific values about a specific instance of a given scenario)

	Interaction number	Speech turns	Transmission function					Acquisition function					
			T-T	T-O	S-S	S-O	O	S-S	S-O	T-T	T-O	O	
Sub-Activity 4 (Students solve the privileged questions)	Reproduction of Necessity	5	5	✓					✓				
		6	6	✓		✓			✓				
		7 ₁	7a – 7j	✓					✓				
		7 ₂	7k – 7o	✓		✓							
		8	8	✓					✓				
		9	9	✓					✓				
		10 ₁	10a – 10k	✓					✓				
		10 ₂	10l – 10m	✓		✓							
		11 ₁	11a – 11d		✓ (as railway conductor)				✓				
		11 ₂	11e	✓		✓							
		12 ₁	12a – 12q	✓					✓				
		12 ₂	12r – 12s ₁	✓		✓							
		12 ₃	12s ₂ – 12t	✓					✓				
		12 ₄	12u ₁	✓		✓							
		12 ₅	12u ₂ – 12v	✓					✓				
		12 ₆	12w ₁	✓		✓							
		12 ₇	12w ₂ – 12x	✓					✓				
		12 ₈	12y – 12ad	✓		✓							
		12 ₉	12ae – 12af	✓					✓				
		12 ₁₀	12ag ₁	✓		✓							
		12 ₁₁	12ag ₂ – 12q	✓		✓			✓				
		13 ₁	13a – 13ab	✓					✓				
		13 ₂	13ac ₁						✓		✓		
		13 ₃	13ac ₂ – 13an	✓					✓				
		13 ₄	13ao – 13ap	✓		✓							
		14 ₁	14e ₁	✓		✓							
		14 ₂	14e ₂			✓			✓			✓ (as student)	
		14 ₃	14e ₃		✓ (as student)	✓							
		14 ₄	14e ₄	✓		✓			✓				
		14 ₅	14e ₅	✓		✓							
		14 ₆	14e ₆		✓ (as student)	✓							
		14 ₇	14e ₇	✓		✓							
		14 ₈	14e ₈		✓ (as student)	✓							
		14 ₉	14e ₉ – 14f ₁	✓		✓							
14 ₁₀	14f ₂	✓		✓			✓						

	Interaction number	Speech turns	Transmission function					Acquisition function				
			T-T	T-O	S-S	S-O	O	S-S	S-O	T-T	T-O	O
Sub-Activity 4 cont... (Students solve the privileged questions)	Reproduction of Necessity	15	✓		✓			✓				
		16 ₁	16a ₁	✓		✓						
		16 ₂	16a ₂	✓				✓				
		16 ₃	16a ₃	✓		✓						
		16 ₄	16a ₄		✓ (as student)	✓		✓			✓ (as student)	
		16 ₅	16a ₅	✓		✓		✓				
		16 ₆	16a ₆		✓ (as student)	✓		✓			✓ (as student)	
		16 ₇	16a ₇	✓		✓		✓				
		17 ₁	17a – 17i	✓		✓ (all)						
		17 ₂	17j – 17o	✓				✓ (S1)				
		17 ₃	17p – 17y	✓		✓ (S1)						
		17 ₄	17z – 17ab ₁	✓		✓ (S1 & S4)						
		17 ₅	17ab ₂ – 17ae	✓		✓ (S2)		✓ (all)				
		17 ₆	17af – 17aj ₁	✓		✓ (S3)				✓		
		17 ₇	17aj ₂ – 17ao	✓		✓ (S2)		✓ (all)				
		17 ₈	17ap – 17bb	✓		✓ (all)						
		17 ₉	17bc – 17bk	✓				✓ (S1)				
		17 ₁₀	17b ^l – 17cd	✓		✓ (S3)		✓ (S3)				
		17 ₁₁	17ce – 17cx	✓		✓ (all)		✓ (S3)		✓		
		17 ₁₂	17cy – 17da	✓		✓ (all)						
		17 ₁₃	17db – 17dd		✓ (engineer & scientist)				✓ (all) (engineer & scientist)			
		18	18	✓				✓				
		19 ₁	19a – 19aj	✓		✓						
		19 ₂	19ak – 19al	✓				✓				
		19 ₃	19am – 19an	✓		✓						
		19 ₄	19ao – 19ar	✓				✓				
		19 ₅	19as – 19av	✓		✓						
		19 ₆	19aw – 19ba ₁	✓				✓		✓		
19 ₇	19ba ₂ – 19bh	✓				✓						
19 ₈	19bi – 19bk	✓		✓		✓						

	Interaction number	Speech turns	Transmission function					Acquisition function				
			T-T	T-O	S-S	S-O	O	S-S	S-O	T-T	T-O	O
Reproduction of Necessity	19 ₉	19bl – 19cf	✓					✓				
	19 ₁₀	19cg – 19cm	✓		✓			✓				
	20 ₁	20a – 20ch ₁	✓					✓				
	20 ₂	20ch ₂ – 20ct	✓		✓			✓		✓		
	21 ₁	21a – 21as	✓		✓							
	21 ₂	21at – 21au	✓		✓					✓		
	21 ₃	21av – 21cf	✓		✓			✓				

Sub-activity 5 (Students check answers)	Notion	22 ₁	22a – 22e	✓		✓			✓				
		22 ₂	22f – 22l	✓					✓				
		22 ₃	22m – 22u ₁	✓		✓							
		22 ₄	22u ₂ – 22ab	✓					✓				
		23 ₁	23a – 23b	✓					✓				
		23 ₂	23c – 23aa	✓		✓			✓				
		23 ₃	23ab – 23ae		✓ (student)	✓			✓				
		23 ₄	23af – 23am	✓		✓			✓				
		24 ₁	24a – 24k	✓		✓							
		24 ₂	24l	✓					✓				
		24 ₃	24m – 24t	✓		✓			✓				
		25 ₁	25a – 25e ₁	✓		✓							
		25 ₂	25e ₂	✓		✓			✓				
		25 ₃	25e ₃	✓			✓ (engineer)			✓ (engineer)			
		25 ₄	25e ₄		✓ (student)	✓			✓			✓ (student)	

Table 3: summary of the positioning of the pedagogic subjects during the judgement of sub-notion 3 (Creating an algebraic formula to calculate values about the general case of a given scenario)

	Interaction number	Speech turns	Transmission function					Acquisition function					
			T-T	T-O	S-S	S-O	O	S-S	S-O	T-T	T-O	O	
Sub-activity 6 (Students determine a formula)	Existence	25 ₅	25e ₅	✓						✓ (railway manger)			
		25 ₆	25e ₆	✓			✓ (railway manger)						
		25 ₇	25e ₇	✓						✓ (railway manger)			
		25 ₈	25e ₈	✓		✓							
		26 ₁	26a – 26s ₁	✓		✓							
		26 ₂	26s ₂	✓						✓ (railway manger)			
		26 ₃	26s ₃ – 26w	✓			✓ (railway manger)						
	Reproduction of Necessity	27 ₁	27a – 27j	✓		✓							
		27 ₂	27k -27bn	✓					✓				
		27 ₃	27bo – 27bt	✓		✓							
		27 ₄	27bu – 27cz	✓					✓				
		27 ₅	27da – 27de	✓		✓							
		27 ₆	27df – 27dr	✓					✓				
Sub-activity 8 (Students check their formula)	Notion	27 ₇	27ds – 27eg	✓		✓			✓				
Sub-activity 9 (Students refine their formula)		28 ₁	28a –	✓		✓			✓				
		29 ₁	29a – 29ao	✓		✓			✓				
		29 ₂	29ap – 29aq		✓ (student)	✓			✓				
		29 ₃	29ar – 29ay	✓		✓			✓				
	30 ₁	30a – 30bn	✓		✓								

Appendix 8: Ethical considerations

Information Sheet for Facilitators

24 June 2012

Dear

Invitation to participate in Master's Research project

I am a student at the University of Cape Town studying towards a Master's degree in Education. One of the requirements of the degree is that I complete a research project. Through my research project I hope to better understand *how* mathematics is taught and *what* mathematics is taught in a support course when the focus is on problem-solving. It is believed that such an understanding could prove useful to future courses of this nature (which you may offer) as well as to the wider community grappling with how to support the transition from school to university.

In order to achieve an understanding of how teaching through problem solving works in the classroom, I would firstly like to construct a description of your intended curriculum, so as to guide my investigation of how the problem-solving pedagogy works in the classroom. I do *not* wish to compare the intended and the implemented curricula. To this end I would like to make reference to our email and other conversations as well as your previous publications about the support course.

Secondly, in order to analyse your teaching, I would like to video record a sequence of problem solving sessions in which your students tackle *The Trains Problem*. In recording the sessions, I would like to focus on the following:

1. Any whole group instruction which you engage in;
2. Any other interactions which you have with students;
3. The (possibly private) conversations between facilitators as the session progresses;
4. A group of four students, when they are working as a group as well as when they are working on their own and especially when they – collectively or individually – interact with you. This is to capture what is said and done (by them and you) during their engagement with the problem.

It is important to note that participation is voluntary. The NPO, its facilitators and its students are under no obligation to participate and there will be no consequences for you should you choose not to. All participants have the right to withdraw from the study at any future point. If necessary, please talk to me about any concerns which you have.

I undertake to ensure the anonymity of all participants and the organisation, as well as the confidentiality of the data. To this end, the following measures will be put in place:

- a) All data will be stored securely during the research process, and will then be destroyed when it is no longer required.
- b) All persons involved in the data collection (eg. the camera persons) will be required to commit to ensuring confidentiality of the data.
- c) Only a select few people will view the video-recordings. I will obviously view the recordings and my Master's supervisors may view them in order to obtain a better appreciation for the object of my research. In addition, someone else may help me transcribe what is said in the lesson and this person would also need to watch the videos. Lastly, the workshop facilitators will also have access to the videos. All people who view the footage will be required to commit to ensuring confidentiality of this data. No-one other than the four parties listed (me, my supervisors, transcriber and facilitators) will be allowed to view the videos without first obtaining written consent from all persons who have been video recorded.
- d) I will present the results of the study to the wider community. While busy with the study I will get feedback on my progress by presenting my preliminary work to my supervisors, my bursar, The Centre for Research in Engineering Education (CREE) and fellow Master's students. Once I have completed the study, the results will be presented in my final Master's dissertation, and subsequently in academic research papers and conferences. In all reports and presentations of the study, the names of all participants and that of the organisation will be changed to ensure anonymity. Please note, however, that my use and proper citation of your previous publications about the course and its pedagogy will mean that you will not remain completely anonymous.

I would be very grateful if you are agreeable to this research process and ask that you please read and complete the attached consent forms.

Should you have any concerns or questions about the study, please chat to me at any time while I am here with you. Alternatively, you can contact me by email at rixrenee@gmail.com or telephonically at 082 219 7295. If you want to talk to someone else about your involvement, you may also contact my supervisors (Dr Kate Le Roux, kate.leroux@uct.ac.za and Ms Shaheeda Jaffer, shaheeda.jaffer@uct.ac.za).

Yours sincerely

Renee Rix
School of Education, UCT

Consent form for Facilitators

Facilitator consent form: Participation in Master's research project

I,, consent to participate in this research project.

I am aware that participation will involve

- a) facilitating the Workshops when data is collected for the research project;
- b) the use of my email and other conversations with the researcher.

Facilitator initial:

I am satisfied that the aims of the study and my role in the study have been explained at the beginning of the project and that I am free to discuss my involvement with the researcher at any time.

Facilitator initial:

I am aware that I can withdraw my participation in the study at any time during the process.

Facilitator initial:

I hereby waive my right to anonymity so that the researcher can properly cite my previous publications.

Facilitator initial:

Signature of Facilitator:

Date:

Signature of Witness:

Date:

Facilitator consent form: Data collection and data usage

I consent to the video-recording of my interactions with the students and with other facilitators during the Workshop sessions.

Facilitator initial:

I consent to the use of my email and other conversations with the researcher.

Facilitator initial:

I am aware that the results of the study will be presented as part of the researcher's Master's dissertation, amongst the CREE community, at academic conferences and in journal articles.

I consent to the results being used in this way.

Facilitator initial:

I undertake to obtain written consent from the students should I want to show the video-recordings to anyone other than the researcher, her supervisors or my colleagues who were present during the Workshop sessions.

Facilitator initial:

Signature of Facilitator:

Date:

Signature of Witness:

Date:

26 June 2012

Dear

Invitation to participate in Master's Research project

I am a student at the University of Cape Town studying towards a Master's degree in Education. One of the requirements of the degree is that I complete a research project. In my study, I plan to investigate how mathematics is taught, through problem solving, in the (the foundation's) support course. The hope is that this will help to improve this and other similar courses in the future.

Your participation in the study will involve being video recorded while you work on one of the problems during this week (25th – 29th June). The aim the study is to understand how mathematics is *taught* in the course and so the main focus will be on your facilitators. The video records of you and your group will be used to help me better understand the teaching. You will be able to work as you usually do, but we will record your group interaction, your individual work and any contributions that you make to whole class discussions using a video-recorder which uses a microphone placed on the desk. At the end of the Workshop I will collect your written work to make copies of your solutions and then return your work to you.

It is important to note that participation is voluntary. You are under no obligation to participate and there will be no consequences for you (related to your funding or anything else) should you choose not to. All participants have the right to withdraw from the study at any future point. If necessary, please talk to me about any concerns which you have, especially if you are hesitant about participating.

Should you agree to participate, I can assure that that this will not affect you involvement with (the foundation). My findings are for research purposes

only and will not affect your facilitators' or funder's interactions with you in any way.

I undertake to ensure the anonymity and of all participants and the organisation, as well as the confidentiality of the video data. To this end, the following measures will be put in place:

- a) All data will be stored securely during the research process, and will then be destroyed when it is no longer required.
- b) All persons involved in the data collection (e.g. the camera persons) will be required to commit to ensuring confidentiality of the data.
- c) Only a select few people (me, my supervisors, a transcriber and your facilitators) will view the video-recordings. I will, however, present the written results of my study to other people. In all written work, I will refer to you only by a pseudonym.

I would be very grateful if you are agreeable to this research process and ask that you please read and complete the attached consent forms.

Should you have any concerns or questions about the study, please chat to me at any time during the Workshops. Alternatively, you can contact me by email at rixrenee@gmail.com. If you want to talk to someone else about your involvement, you may also contact my supervisors (Dr Kate Le Roux, kate.leroux@uct.ac.za and Ms Shaheeda Jaffer, shaheeda.jaffer@uct.ac.za).

Yours sincerely

Renee Rix
School of Education, UCT

Consent form for Students

Group Members

Student consent form: Participation in Master's research project

I,, consent to participate in this research project.

I am aware that participation will involve:

- a) The observation of how my group solves problems.
- b) The observation of how I personally solve problems.
- c) The observation of my interactions with members of my group and the facilitators.
- d) Collection and photo-copying of my written work at the end of the Workshop.

Student initial:

I am satisfied that the aims of the study and my role in the study have been explained at the beginning of the project and that I can ask questions about and discuss my participation with the researcher at any time.

Student initial:

I am aware that I can withdraw from participation in the study at any time during the process.

Student initial:

Signature of Student: Date:

Signature of Witness: Date:

Student consent form: Data collection and data usage

I consent to the video-recording of my work when I solve problems during the Workshop sessions.

Student initial:

I consent to the video-recording of my interactions with the facilitators during the Workshop sessions.

Student initial:

I consent to the video-recording of my interactions with other members of my group during the Workshop sessions.

Student initial:

I am aware that the results of the study will be presented (anonymously) as part of the researcher's studies, at academic conferences, in journal articles and in the final Master's dissertation. I consent to the results being used in this way.

Student initial:

Signature of Student: Date:

Signature of Witness: Date:

26 June 2012

Dear

Invitation to participate in Master's Research project

I am a student at the University of Cape Town studying towards a Master's degree in Education. One of the requirements of the degree is that I complete a research project. In my study, I plan to investigate how mathematics is taught, through problem solving, in (the foundation's) support course. The hope is that this will help to improve this and other similar courses in the future.

This is an invitation for you to participate in this research project. I have selected a group of students whose work I will follow closely as the problem solving activities unfold. Although you are not a member of this group, I ask that you read the following and information and sign the attached consent form, as I will require your participation in the study in two small ways. I would like to video record all whole class discussions and many of the individual interactions between you and the facilitator(s) which take place during the Workshops this week (25th – 29th June). Mostly, the video will focus on the facilitator (since I am interested in how mathematics is *taught*). However, should you contribute to the discussion (eg. by asking a question or perhaps by answering a question asked by the teacher), I would like to also record what you say.

It is important to note that participation in the study is voluntary. You are under no obligation to participate and there will be no consequences for you should you choose not to. All participants have the right to withdraw from the study at any future point. If necessary, please talk to me about any concerns which you have, especially if you are hesitant about participating.

Should you agree to participate, I can assure that that this will not affect your involvement with (the foundation). My findings are for research purposes only and will not affect your facilitators' or funder's interactions with you in any way.

I undertake to ensure the anonymity of all participants and the organisation, as well as the confidentiality of the data. To this end, the following measures will be put in place:

- a) All data will be stored securely during the research process, and will then be destroyed when it is no longer required.
- b) All persons involved in the data collection (eg. the camera persons) will be required to commit to ensuring confidentiality of the data.
- c) Only a select few people (me, my supervisors, a transcriber and your facilitators) will view the video-recordings. I will, however, present the written results of my study to other people. In all written work, I will refer to you only by a pseudonym.

I would be very grateful if you are agreeable to this research process and ask that you please read and complete the attached consent forms.

Should you have any concerns or questions about the study, please chat to me at any time during the Workshops. Alternatively, you can contact me by email at rixrenee@gmail.com. If you want to talk to someone else about your involvement, you may also contact my supervisors (Dr Kate Le Roux, kate.leroux@uct.ac.za and Ms Shaheeda Jaffer, shaheeda.jaffer@uct.ac.za).

Yours sincerely

Renee Rix
School of Education, UCT

Consent Form for Student

General

Student consent form: Participation in Master's research project

I....., consent to participate in this research project.

I am aware that participation will involve the observation of

- a) my contribution to any whole class discussions;
- b) my interactions with the facilitator(s).

Student initial:

I am satisfied that the aims of the study and my role in the study have been explained at the beginning of the project and that I can discuss my participation with the researcher at any time.

Student initial:

I am aware that I can withdraw from participation in the study at any time during the process.

Student initial:

Signature of Student: Date:

Signature of Witness: Date:

Student consent form: Data collection and data usage

I consent to the video-taping of my contribution to any whole class discussions during the Workshop. **Student initial:**

I consent to the video-taping of my interactions with the facilitator(s) during the Workshop. **Student initial:**

I am aware that the results of the study will be presented (anonymously) as part of the researcher's studies, at academic conferences, in journal articles and in the final Master's dissertation. I consent to the results being used in this way. **Student initial:**

Signature of Student: Date:

Signature of Witness: Date: