Using the Classification and Regression Tree (CART) model for stock selection on the S&P 700

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Declaration

I, Neil Deon Pienaar, declare that this dissertation is my own work. It is being submitted for the Degree of Master of Investment Management in the University of Cape Town. I declare that it has not been submitted before for a degree or examination of any kind at any other university.

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15 February 2016
Abstract

Traditionally, investment practitioners and academics alike have used stock fundamentals and a linear framework in order to predict future stock performance. This approach has been shown to have flaws as literature has shown that stock returns can exhibit non-linearity and involve complex relations beyond that of a linear nature (Hsieh, 1991; Sarantis, 2001; Shively, 2003). These findings present an opportunity to investment practitioners who are better able to model these returns.

This dissertation attempts to classify stocks on the S&P 700 index using a Classification and Regression Tree (CART) built during an in-sample period and then used for predicative purposes during an out-of-sample period deliberately comprising both a period of financial crisis and recovery. For these periods, various portfolios and performance measures are calculated in order to assess the models performance relative to the benchmark, the Standard and Poor (S&P) 700 index. The results of this paper indicate that the model built outperforms the S&P 700 index across all of the periods analysed but particularly over the recovery period which has previously not been incorporated under prior studies. The model also managed to outperform modestly over the crisis period but still realised negative absolute returns during this time of financial turmoil as the S&P 700 index fell.
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1. Introduction:

The predictability of stock returns has been of great concern to academics and asset managers alike for many years. Traditionally linear models have been proposed to predict these stock returns from some form of fundamental data surrounding a particular stock. These linear models include the Capital Asset Pricing Model (CAPM) (Sharpe, 1964; Lintner, 1965), the Arbitrage Pricing Theory (APT) (Ross, 1976), the Fama and French three-factor model (Fama & French, 1993) and more recently the Carhart four-factor model (Carhart, 1997). Each of these models have assumed a linear relationship between a stock’s fundamental characteristics and its mean return, however, there is a growing base of literature expressing the idea of non-linearity in stock prices (Hsieh, 1991; Sarantis, 2001; Shively, 2003).

This literature has spurred various non-linear approaches to modelling stock returns on the basis of two important contributions that these methods may be able to achieve according to Zhu, Philpotts and Stevenson (2012). First, if the true relationship between stock returns and the characteristics thought to influence them are non-linear, then it follows that profit opportunities may exist that have, as yet, not been exploited due to other managers use of linear modelling techniques. Second, since the vast majority of managers use linear models, non-linear modelling would provide a source of diversification from the majority of the industry. The practical implications of this wholesale use of linear models is clearly shown by the financial crisis of 2008 where the use of similar risk models and techniques led to massive losses amongst the majority of the industry and its managers. Khandania and Lo (2011) confirm the benefits of diversifying among models as they find evidence of profitable
strategies becoming unprofitable during the financial crisis due to similar moves made among the participants in the market.

This dissertation aims to apply a non-linear technique to a sub-sample of the stock universe on the Standard and Poor 700 (S&P 700) with a particular interest in assessing whether the CART model is able to outperform the S&P 700 index during a recessionary (crisis) period. The reason for the selection of the S&P 700 as the primary market to evaluate was based on the fact that prior papers have already assessed the North American stock markets (Zhu, et al., 2012) as well as the South African JSE (Giuricich, 2014) as recently 2012 and 2014 respectively. The Classification and Regression Tree (CART) model will be applied in order to contribute in the area of stock selection and the data will be taken monthly for the period January 2004 through April 2015. The CART model was chosen due to its properties which are well suited to stock selection. Firstly, since the CART model is non-parametric and as such it makes no assumption regarding the underlying distribution of returns and secondly, the CART model is invariant to monotone transformations of independent variables (Breiman, Freidman, Olshen & Stone, 1984). CART is particularly useful in its ability to compress very large samples of data into a simple graphical form displaying the fundamental characteristics which are then easily interpreted as well as being robust to outliers and having the ability to handle noisy data sets which are both common features of stock return data (Zhu, et al., 2012). Ultimately the CART algorithm helps produce a set of “if-then” rules through hierarchical prioritisation which allows complex interactions between variables and which can be used to guide decision making (Sorensen, Miller & Ooi, 2000). The purpose of this paper is to develop a CART model for stock selection within the global context and then verify the use of the model in an out-of-sample testing period which
will specifically include the 2008 financial crisis so as to test the impact of using such a model during this time period as well as during a recovery period thereafter.

This dissertation will proceed as follows: firstly, a background of the Classification and Regression Tree (CART) model will be presented in order to make the literature more useful to the reader and explain the steps and thinking behind the CART model. Then a review of the relevant literature within the CART space will be given with specific reference to application of the CART algorithm to stock selection in the past. The main focus of the literature review includes papers done by Sorensen et al. (2000), Zhu et al. (2012) and Giuricich (2014) which evaluated the use of CART in the technology sector of the United States, the United States more generally and South Africa respectively. Next, a description of the data and method behind this dissertation will be presented and finally a discussion of results will follow to determine the success of the model in its testing period.
2. **Background to Classification and Regression Trees (CART)**

The following is presented as a background to CART in order to help the reader understand the literature review to follow more clearly and to show how the CART model is initially constructed.

**2.1 Theoretical overview**

A Classification and Regression tree (CART) is a subset of a more general group of recursive partitioning algorithms (RPA’s) (Giuricich, 2014). The fundamental idea behind CART is to recursively partition a space until all sub-spaces are satisfactorily homogenous enough for a simple model to be applied to them (Zhu, et al., 2012). From a broader perspective, CART allows for two processes; it predicts to which group a dependent variable belongs and it shows this prediction process in the form of a tree-like diagram.

**2.1.1 Development of the CART methodology**

The CART methodology was developed by Breiman, Freidman, Olshen and Stone (1984) and it provides an alternative to the popular linear models currently used in the stock selection research in particular. These linear models impose a single predictive formula over the entire universe that they observe and so are termed global models while the CART model allows for complex, non-linear interactions between variables which is often the case particularly in financial data and therefore can better measure those relationships (Zhu, et al., 2012). In the past CART and its greater group of RPA’s have been used predominantly in medical research but there have also been a number of financial applications. Frydman, Altman and Kao (1985) used a recursive partitioning algorithm in a study where they

The power of using the CART model comes from four sources according to Sorensen et al. (2000):

1. Representing the results in a hierarchal and tree like structure is intuitive and easily understandable.

2. Non-linearities are taken into account.

3. Dependencies among variables can be observed from using the CART model.

4. Conditional probabilities are produced in the building of the model and can be very useful for analysis.

2.1.2 Building a CART model

In order to apply the CART model Zhu et al. (2012) specified two steps; firstly, the recursive splitting algorithm is used to split nodes and build a tree and secondly, use a pruning technique to prune the tree so as to avoid overfitting. These two steps will now be outlined in detail and will follow a similar process to that set out by Giuricich (2014).

2.1.2.1 Binary recursive partitioning (Breiman, et al., 1984, p.5)

The construction of a tree is always based on a learning sample which includes the observations of the explanatory variables and their respective responses $(x_1, j_1), \ldots, (x_n, j_n)$
on N cases where \( n = 1, ..., N \). The learning sample itself is denoted by \( L \), i.e.

Beginning with the learning sample the tree algorithm will begin splitting the subsets into two child subsets at a time. In order to do this a splitting rule or measure of diversity of the observations within a node is required. This measure is termed the diversity or Gini index which was outlined by Breiman et al. (1984) and will now be modified for this paper.

The Diversity or Gini index (GI) of any node is the probability that any two objects randomly chosen from that node will belong to different groups. In the cases when there are only two groups for the dependent variable the diversity index of a node can be calculated as:

\[
\text{GI} = \frac{1}{2} \left( \frac{N_1}{N} \cdot \frac{N_2}{N} \right)
\]

Where \( N_1 \) is the proportion of objects in a node belonging to Group 1 and \( N_2 \) is the proportion of objects in that same node belonging to Group 2.

More generally, when there are \( q \) groups within the dependent variable, the Gini index of a node can be computed as:

\[
\text{GI} = \sum_{i=1}^{q} \left( \frac{\rho_i}{N} \right)^2
\]

Where \( \rho_i \) is the proportion of observations within the node belonging to group \( i \).

From this point on the tree will be constructed using the following steps adapted from Wegner (2010).

1. Each explanatory variable must be carefully considered.
2. Calculate every possible binary split of values for each independent variable by creating two categories or intervals (i.e. "x" or "not x").

3. Calculate a weighted average of the diversity (WAGI) or Gini indices (GI) for each possible split using the following formula:

\[ WAGI = \frac{n_{child_1} GI_{child_1} + n_{child_2} GI_{child_2}}{n_{child_1} + n_{child_2}} \]

Where \( n_{child_1} \) and \( n_{child_2} \) are the number of observations in the two child nodes respectively.

4. The aim of CART is to create groups with the greatest homogeneity possible. In order for this to occur the partition with the greatest reduction in diversity is selected as the one to use in the tree. The reduction in diversity is calculate as follows:

\[ RiD = GI_{parent} - WAGI \]

5. This process is then repeated for each of the new child nodes.

2.1.2.2 Tree Pruning
The process specified above creates the tree like structure as defined by CART with the parent node being split first into two child nodes and then those child nodes being split further into two subsequent child nodes with the eventual outcome being a set of terminal nodes. These terminal nodes help determine a set of rules by which new data can be classified. Theoretically, the algorithm will continue to split nodes until each terminal node contains only observations belonging to the same class. This is not always optimal as although each split was able to decrease heterogeneity within a learning sample, correspondingly the tree is being split so many times that eventually even small disturbances such as noise in the data set could be splitting nodes and becoming part of the
final decision rules (Andriyashin, et al., 2008). The goal of the CART model is to create a reliable set of rules which represent the inner fundamental data patterns in order for the tree to best classify in an out-of-sample scenario (Zhu, et al., 2012). Correspondingly, a tree which is too small is also not advisable since it would place fewer parameters on the data leading to rules which do not account for large portions of the data (Andriyashin, et al., 2008).

The solution to this problem is to have a tree pruning rule such as the misclassification error approach first proposed by Breiman et al. (1984). This method is based on a measure of misclassification error called the cross-validation cost (CV cost). Each tree constructed by CART has a complexity measure which is the number of terminal nodes in that tree as well as a CV cost which is calculated using the following approach as used by Giuricich (2014):

1. Divide the entire learning sample into K sub-samples, as equal in size as possible.
2. Specify the desired size of the classification tree, starting at the root node and following through to the terminal nodes of the un-pruned classification tree.
3. The tree is then computed K times while each time leaving out one of the K sub-samples from the computations in order to use these sub-samples as a hold-out sample. From this hold out sample, a misclassification rate is calculated as the complement of the overall hit rate.
4. The average misclassification rate (CV cost) can then be calculated for the specified tree as well as the standard error.

From then the tree can be pruned using the CV cost rule. This requires choosing the tree with the fewest terminal nodes with a CV cost equal to:
Where $\alpha$ is a complexity parameter that is pre-specified by the modeller and is considered to be the marginal cost of each additional node added to the decision tree. Therefore, the larger $\alpha$, the smaller the tree and often a cross-validation sample is used in determining the optimal value of $\alpha$. 
3. Literature review:

This chapter presents prior research done on CART with emphasis placed on CART within the stock selection setting. First, traditional methods of predicting stock returns are discussed with a view to exploring the opportunity for CART within this setting, following which a review of prior literature regarding CART within stock selection is presented.

3.1 Traditional models and space for CART

The overwhelming majority of academic literature surrounding the predictability of stock returns focuses on a linear relationship between a stock and its fundamental characteristics. Internationally, examples of such models are the CAPM (Sharpe, 1964; Litner, 1965), APT (Ross, 1976), the Fama and French three-factor model (Fama & French, 1993) and the Carhart four-factor model (Carhart, 1997) mentioned in the introduction to this dissertation. This assumption of a linear relationship has no absolute theoretical grounding and has been proven to be violated on a number of occasions (Hsieh, 1991; Qi, 1999; Sarantis, 2001; Shively, 2003; Kim, Mollick & Nam, 2008). Additionally, these models commonly assume a normal underlying distribution for stock returns which has also been shown to be violated in practice in both developed and developing stock markets particularly over shorter time periods (Aparicio & Estrada, 2001; Cont, 2001; Simonsen, Jensen & Johansen, 2002; Hueng & McDonald, 2005; Zhu et al., 2012).

Sorensen et al. (2000) explains that investment managers commonly attempt to reduce an investment universe to a group of stocks which possess a common set of desired characteristics. This process involves “quantitatively screening” rather than mathematically optimising for the desired stock characteristics and leaves significant room for expansion since certain profitable stocks may be excluded on the basis of one fundamental
characteristic but still possess all the remaining criteria necessary to fall into a certain group. Additionally, these linear models are plagued with problems surrounding multicollinearity, outliers and missing data which are common features of datasets (Zhu, et al., 2012).

A series of surveys conducted by the Chartered Financial Analyst (CFA) Institute asked asset managers in Europe and the United States about the techniques they used to model (Fabozzi, Focardi & Jonas, 2008). The surveys found that linear modelling remained the cornerstone for financial modelling and was used by the vast majority while non-linear methods such as neural networks and decisions trees were used sparingly, if at all, and were never relied upon solely (Zhu, et al., 2012). Managers justified this approach by citing the lack of theory behind these non-linear methods, although managers did believe the technique had explanatory power, as well as the lack of appropriate skills with regard to these models within the assets manager’s investment house (Zhu, et al., 2012).

The implications of this study point towards further research needed in this field of non-linear models which could provide unexplored profit opportunities and diversification from traditional linear models. Non-linear models have been used to model stock returns before, however, in the South African context Bong-Bonga and Makakabule (2010) used a Smooth Transition Regression (STR) model to assess the relationship between stock returns and macroeconomic variables specifically on the Johannesburg Stock Exchange (JSE) ALL Share Index. The authors compared the predictions of the non-linear model, STG, with that of an Ordinary Least Squares Model (OLS) and random walk models and found that the STG outperformed the linear models out-of-sample.
3.2 Prior research on CART within a stock selection framework

Sorensen et al. (2000) pioneered using decision trees, CART in particular, in stock selection. They used cross-sectional data comprising the United State’s Russell 1000 index and focused specifically on technology stocks. The aim was to forecast whether a stock would outperform or underperform during the period January 1996 through October 1999 by using the CART framework. The dependent variable used by the researchers was whether a stock belonged to the outperform group or underperform group. The explanatory variables were selected due to their prominence among money managers and comprised variables representing consensus expectations, valuation, profitability and a price-momentum criterion.

Sorensen et al. (2000) first constructed a static CART model assuming a high level of stability between the explanatory variables and the dependent variable and termed it a “static tree”. The data were divided into two periods with February 1993 to December 1995 used as the period in which the model was formed, often called the in-sample period. Thereafter the model was tested through January 1996- October 1999 in what is called the out-of-sample period in order to evaluate the ability of the tree to classify observations that it has not yet seen.

The tree built during the in-sample period had five terminal nodes with the primary split due to “12 week change in forecasted earnings”. The next split was based on return on assets, a profitability indicator, from this node on the other variables were not able to effect the final classification of a stock although splitting continued. It was determined that the change in forecasted earnings was thus the most powerful factor in deciding whether a stock would
belong to the outperform group or the underperform group and Sorensen et al. (2000) proceeded to test the model of their out-of-sample period (i.e. 1996-1999).

In order to evaluate the performance of the model out-of-sample, Sorensen et al. (2000) created two separate portfolios each month using the rules derived from the in-sample model. One portfolio comprised an equal weighting of the “outperform” technology stocks which built a portfolio of stocks an investor would like to be long in. Conversely, the second portfolio comprised an equal weighting of the “underperform” stocks and thus an investor would prefer to be short these. The results over the out-of-sample analysis shows that on a monthly basis the stocks classified as outperform had a 1.4% greater return on average than those classified as underperform by the model, a result which was statistically significant at the 5% level using both a t-test and Wilcoxon ranked test. Additionally, the long portfolio outperforms the overall universe of stock used in this study not taking into account transaction costs or taxes.

Sorensen et al. (2000) then adjusted their first approach to be more dynamic and called it the “evolving tree approach”. In this approach the tree formed by CART was re-estimated on a monthly basis using all available data from the beginning (February 1993) to the current month. At the start of a new month, new samples from the latest month are added to the model estimation procedure and new forecasts are then produced taking this data into account. This approach is justified by the dynamic nature of markets which are ever-changing as well as allowing for more observations as new samples are added to the data set, thereby helping with the statistical significance of the analysis.

This shift in approach means that the tree could essentially change significantly every month but Sorensen et al. (2000) state that the tree structures tend to be relatively stable over the
short term with small changes occurring month to month. The new tree model’s primary split was again based on the 12 week change in forecasted earnings followed by return on assets which is now significantly affected by the value indicator (cashflow-to-price). This finding reiterated the findings of the “static tree” as well as further informing the researchers of the importance of a value criterion in stock selection.

In order to assess the reliability of the model, the tree was again used to predict performance during the out-of-sample period. The same procedure was followed in terms of creating portfolios and it was found that the monthly return differential between the stocks classified as outperform and those classified as underperform was 1.47% on average, a marginal improvement on the static model (Sorensen, et al., 2000). This result was again statistically significant at the 5% level and the long portfolio outperformed the stock universe over the sample period.

In a German context, Andriyashin, Härdle & Timofeev (2008) used CART to analyse weekly observations of XETRA DAX companies. The data was taken over the period 27 April 2000 to 30 October 2003 and consisted of historical stock prices as well as a set of fundamental and technical indicators. In contrast to other studies done on stock selection using CART, Andriyashin et al. (2008) built individual trees for each stock on the XETRA DAX and divided their data set into a learning sample, a test set and a validation set. The dependent variable was categorical and consisted of long, short or neutral representing undervalued, overvalued and fairly priced respectively.

Stocks were first categorised into one of the respective classes and a tree was built from the learning set. Second, the model was optimised using the test set data and lastly, the model was tested using the validation section of data. For the validation period, an equally
weighted portfolio was built and updated weekly using the CART recommended active positions, long or short. This portfolio was then compared to various benchmark indices and Andriyashin et al. (2008) found that an active CART strategy outperformed indices such as the DAX30, the DJIA and the FTSE 100.

Zhu et al. (2012) followed on the work done by Sorensen et al. (2000) and attempted to apply the CART model to a larger universe of stocks over a longer time period. The primary aim was to compare the outcome produced by this CART model against two linear factor models based on the same underlying inputs. One linear model was based on linear regressions on forward excess returns while the second was built in accordance with mean-variance optimisation. Zhu et al. (2012) used monthly stock data from December 1986 to August 2010 comprising stocks from the North American stock markets, including the United States and Canada but excluding financial stocks as specified by the Global Industry Classification Standards. Additionally, in order to be included a company had to have a market capitalisation of $1 billion in 2010 and the adjusted equivalent historically which prevented the potential for over-representation by less liquid companies.

Explanatory variables were selected according to investment intuition and prominence in prior studies attempting to explain stock returns. In total, 25 factors were deemed important variables and in order to avoid correlation among these variables they were compiled into 9 equally weighted composite factors which were then used as explanatory variables (Zhu, et al., 2012). Stocks were first sorted into two groups; outperformers and underperformers which produced the categorical variable used as the dependent variable for the model. Similar to the work of Sorensen et al. (2000) the data analysed was divided
into in-sample and out-of-sample periods comprising of data from December 1986 to April 2007 and May 2007 to August 2010 respectively.

The results of the in-sample data show that the primary split is on “value” which comprised an equally weighted average of the five stock metrics including dividends-to-price, cash flow-to-price, book-to-price, earnings-to-price and sales-to-price. This value composite factor represented whether a stock was relatively expensive or inexpensive and showed that relatively cheap stocks had the highest probability of outperforming the universe of stocks (Zhu, et al., 2012). Not surprisingly if a stock was categorised as being expensive and exhibiting low profitability then it formed part of the worst performing group of stocks. The two linear models also found “value” to be the most important determinant of outperformance but Zhu et al. (2012) noted that a key advantage of the CART method was the ability of the tree to identify conditional relevance. For example, although the decision tree’s primarily split was on value, a stock which was not cheap still had a chance to outperform the market if it had desirable characteristics such as high profitability and stability under CART. This again emphasised the non-linear relationships present which were not captured by the linear weighting methodology (Zhu, et al., 2012).

In terms of the results from the out-of-sample period, there was a clear strong correlation between the linear weighting models used of 83% while the CART model exhibited a significantly lower correlation of 56% and 57% with the two linear weighted models respectively. It was argued that this result shows that there are clear model diversification benefits made available by using CART relative to the linear models (Zhu, et al., 2012).

Additionally, stocks were split equally on the predicted outperformance probabilities each month and a long portfolio created for those expected to outperform while a short portfolio
was created for those expected to underperform. The two long linear weighting schemes both marginally outperformed the universe in the out-of-sample period before transaction costs while the CART long portfolio outperformed the universe by 2.6% with similar relative risk to the linear weighted models. A similar scenario was observed for the short portfolio comparisons. In order to see whether the better performance came predominantly from greater exposure to common risk factors, the performance of each portfolio was adjusted for the Fama-French-Carhart risk factors, namely factors relating to; market, value, size and momentum. The result was that the linear models’ performance can be completely explained by these risk factors while the CART model still managed to outperform significantly even after adjusting for the risk factors (Zhu, et al., 2012).

Zhu et al. (2012) reached the conclusion that the improved performance of CART relative to the linear weighted approaches was primarily down to its ability to capture non-linear relationships between debt sustainability and leverage over the period analysed which was characterised by financial distress.

A study done by Giuricich (2014) attempted to build on the work done by Zhu et al. (2012) from a South African perspective. Giuricich (2014) used monthly cross-sectional stock data taken from the top 100 shares on the Johannesburg Stock Exchange (JSE) for the period January 2000 to December 2012. All rand denominated data was adjusted to December 2012 prices using the Consumer Price Index (CPI) in order to remove any inflationary effects. To account for liquidity issues, each stock included in the data had to have a market capitalisation of at least R1 billion in 2012 or its equivalent historically. Stock metrics used as explanatory variables were based on their prominence in prior South African research in explaining stock returns on the JSE. Ultimately, explanatory variables were grouped into 9
categories of stock metrics and a categorical variable was created as the dependent variable with a stock being grouped as either outperforming or underperforming. Two linear models were built to test against the CART model using the same explanatory variables with the dependent variable attempting to model excess returns. The models built were a linear regression model and a mean-variance model respectively.

Data from January 2000 to April 2007 was used to construct the CART and linear models while the models would be tested in the period May 2007 to December 2012 and it is important to note that Giuricich (2014) did not use the evolving model tree as Sorensen et al. (2000) had before but rather the static tree approach. Giuricich (2014) created four portfolios in which to compare two linear models against the CART model; a long portfolio, a short portfolio, a 130-30 strategy portfolio and a hedge fund style portfolio combining both long and short positions in the top and bottom 10 shares predicted to outperform and underperform respectively.

The in-sample CART model showed that the primary split was on the value criterion including earnings-to-price, dividends-to-price, cashflow-to-price, sales-to-price and book-to-price. It was also evident that both momentum and stability, encompassing volatility in corporate earnings, sales and cash flows, played key roles in a stocks categorisation. As was the case under Zhu et al. (2012), conditional relevance with regards to the CART approach was highlighted as stocks could be included even if they performed poorly on one of the criterion. The non-linear relationships were also better captured by CART as a number of decisions based on the same criterion factor were often needed in order to determine whether a stock was an outperformer or underperformer (Giuricich, 2014).
The model was then tested in an out-of-sample period in which the majority of traditional managers performed poorly. It was found that the linear models exhibited weak positive linear correlations (rank correlation= 0.11) while there was almost no correlation between the CART model and the linear models (rank correlation= -.01, -.03 respectively) showing the diversification benefits of CART with respect to models. A hit rate was established for the CART model in order to determine how successful the model was at classifying a stock as either an underperformer or an underperformer. Giuricich (2014) found that the CART model successfully forecasted 72% of the stocks that outperformed while only 42% of the stocks that underperformed were predicted to underperform by the model. Overall the CART model was marginally better at classifying stocks when compared to the two linear models.

In an out-of-sample test of the CART model, the four portfolios built were evaluated on a variety of measures such as the annualised excess stock return, the tracking error, the portfolio holding period information and the Sharpe ratio among others (Giuricich, 2014). It was found that the CART model performed better than the linear models on both a non-risk adjusted basis as well as a risk adjusted basis with regard to the long only portfolio. Additionally, returns generated from CART appeared to be more stable than both of the linear model’s returns which were described as “erratic”. With regards to the short only portfolio, the CART model produced the lowest excess return, albeit still positive, which confirms previous findings in the paper that the CART model seemed to be a poor predictor of underperforming stocks. This finding was also confirmed for the linear models so that it can be concluded that none of the models were excellent predictors of underperforming stocks. Giuricich (2014) made a final point concerning these two portfolios by stating that
that the stocks comprising the long and short portfolios were vastly different between CART and the two linear models, thus providing evidence of the diversification potential of using CART.

In evaluating a 130/30 portfolio Giuricich (2014) found both a regression based linear model as well as the CART model performed similarly and advised the use of both within this strategy. The hedge fund style portfolio saw the CART model outperform the linear models quite strongly across multiple performance evaluation criteria, however, Giuricich (2014) stated that the strategies involved in this portfolio were simplistic. It also appeared that the holding period over the linear and CART models was relatively shorter than under the other strategies which could suggest that the extra returns were due to higher stock turnover.

In reviewing the literature it is clear that the research done on CART and stock selection is still very much in its infancy and in the cases where it has been studied, it has often resulted in superior performance particularly over a period of financial distress. Considering the evidence of non-linear dynamics shown in stock returns and the inherent flaws of traditional linear models the case can be made that methods such as CART should be at the forefront of stock return research as a way of exploiting opportunities missed by the more traditional linear models.
4. **Data and methodology:**

This chapter sets out the data used for this research as well as building on the theoretical background of the CART model discussed in chapter 2 of this dissertation. Further, performance metrics and portfolios constructed in order to compare various strategies are detailed.

**4.1 Data:**

The data consists of monthly cross-sectional stock data for the period January 2004 through April 2015 on the S&P 700 index. This index measures the non-U.S. component of the global equity market and is built to be highly liquid and efficient to replicate. The index represents regions from across the globe as included in the S&P Global 1200 with the exception of the U.S. stock market which is represented by the S&P 500. There does appear to be some overlap, however, with a very small percentage of the S&P 700 index being made up of US stocks with a potential reason being dual listings. The data set used comprised of 95900 observations in total over the 700 stocks and is denominated in United States Dollars. Due to the index being constructed with liquidity in mind, it was not necessary to remove shares for these purposes as has been done in prior studies. It is important to note that transaction costs were not taken into account and newly listed or delisted shares from the S&P 700 index were not explicitly accounted for. Winsorisation was performed on the stock fundamentals in order to mitigate the effect of outliers using a method established by Tukey (1977) and further discussed by Hoaglin and Iglewicz (1987). Under this method observations are classified as outliers if they are either above:
Where $q_1$ is the lower first quartile, $k$ is chosen as 2.2 and $q_3$ is the upper third quartile. If an observation is above $E.1$, it is adjusted to equal the upper third quartile. Correspondingly, if an observation is below $E.2$ it is adjusted to equal the lower first quartile.

Data work was performed using Eviews, R studio and Microsoft Office Excel. The data was taken from Bloomberg and I-Net. Figures 4.1 and 4.2 below show respective breakdowns of the shares that constitute S&P 700 index by country as well as by Global Industry Classification Standards (CICG’s).

**Figure 4.1: S&P 700 constituents by country**
A number of stock fundamentals thought to explain returns were required to be chosen and were done so on the basis of prior research within the CART in stock selection field as previously discussed in the literature review chapter of this paper. The main groups of fundamentals used include value, profitability, momentum and financial strength ratios which were incorporated under Zhu et al. (2012) and Giuricich (2014).

Value: Anomalies within the value sphere have been shown to be present in the Price-to-earnings (P/E) ratio as well as the Book-to-market-ratio by Fama and French (1992) from a more international perspective and Muller and Ward (2013) showed Earnings yield (EY) and Cash-flow-to-price as important anomalies in their study conducted from the South African perspective when building on the work of other authors.

Debt service and Leverage: Bhandari (1988) found that the expected returns to common equity were positively related to the ratio of debt to equity when controlling for stock beta and firm size.
Momentum: Muller and Ward (2013) validated the findings of prior authors within a South African space with regards to momentum as a style characteristic stating that a momentum style with a 12 month formation period and a 3 month holding period outperformed the All Share Index (ALSI) by 14% per annum. More generally, Jegadeesh and Titman (1993) found that momentum could be used to forecast future stock returns from past returns and also stated that the best results were achieved with a 12 month formation period and a 3 month holding period.

Profitability: Fama and French (2008) found that profitability anomalies appear to be less robust but that firms with high profitability tend to be associated with higher abnormal returns while firms with low profitability are not necessarily associated with low returns.

In order to avoid correlations, several stock fundamentals may be combined into one and an equally weighted average taken of the metrics comprising that group. One such group could be, for example, a value factor which includes stock fundamentals such as earnings-to-price, sales-to-price and book-to-price. Each of the grouped metrics is assessed at the end of each month in the sample period over the 700 stocks.

Table 4.1 below displays the groups created which will be referred to as factors for remainder of this dissertation. The categories are similar to those used by Giuricich (2014) and Zhu et al. (2012) before him but within the limitations of the data set. A total of 5 factors were created from 12 stock fundamentals observed in the data set. In the below table the profitability factor was calculated as, for example, an equally weighted average of return-on-equity, 12 month-trailing-profit-margin and asset turnover.
Table 4.1: Factors created in order to build the CART model

<table>
<thead>
<tr>
<th>Factor</th>
<th>Stock metrics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>Earnings-to-price, book-to-price, sales to price</td>
</tr>
<tr>
<td>Profitability</td>
<td>Return-on-equity, 12 month trailing profit margin, asset turnover</td>
</tr>
<tr>
<td>Debt service</td>
<td>Interest coverage ratio, free-cash-flow -to-debt</td>
</tr>
<tr>
<td>Leverage</td>
<td>Long-term-debt-to-equity, long-term-debt-to-market-capitalisation</td>
</tr>
<tr>
<td>Momentum</td>
<td>6 month momentum factor, 12 month momentum factor</td>
</tr>
</tbody>
</table>

As stated, the above combining of stock metrics was done in an attempt to reduce the potential for correlations between these metrics and Table 4.2 shows the Spearman rank correlation matrix for the 5 factors with the majority of correlations lying between -0.2 and 0.2.

Table 4.2: Spearman rank correlation matrix for factors created

<table>
<thead>
<tr>
<th></th>
<th>Momentum</th>
<th>Value</th>
<th>Profitability</th>
<th>Debt service</th>
<th>Leverage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Momentum</td>
<td>1.00</td>
<td>-0.21</td>
<td>0.07</td>
<td>0.04</td>
<td>-0.10</td>
</tr>
<tr>
<td>Value</td>
<td>-0.21</td>
<td>1.00</td>
<td>0.01</td>
<td>-0.10</td>
<td>0.17</td>
</tr>
<tr>
<td>Profitability</td>
<td>0.07</td>
<td>0.01</td>
<td>1.00</td>
<td>0.12</td>
<td>-0.05</td>
</tr>
<tr>
<td>Debt service</td>
<td>0.04</td>
<td>-0.10</td>
<td>0.12</td>
<td>1.00</td>
<td>-0.46</td>
</tr>
<tr>
<td>Leverage</td>
<td>-0.10</td>
<td>0.17</td>
<td>-0.05</td>
<td>-0.46</td>
<td>1.00</td>
</tr>
</tbody>
</table>

4.2 Sample periods and categorical variable construction

The data was divided into two primary subsets, an in-sample subset comprising January 2004 to July 2007 and a subset reserved for out-of-sample testing comprising August 2007 to April 2015. Following this, the out-of-sample subset will include both a crisis period of August 2007 to June 2012 and a recovery period of July 2012 to April 2015 in order to assess whether the CART model performs better over a crisis period in this particular study.

In order to formulate the categorical dependent variable required by CART, forward returns including dividends over the period are calculated at month end as:
Where $Y_{t+1}$ is the Dollar stock price at time $1$, $Y_t$ is the Dollar stock price at time $t$ and is the value of dividends received between times $t$ and $1$.

A median return for all of the companies is then calculated as a proxy for the market return and subtracted from this forward return in order to get an excess return. These excess returns allow the monthly categorisation of the returns into either “outperformers” or “underperformers” which forms the basis of the dependent variable. One benefit of using a categorical variable to model instead of raw returns is that it alleviates the effect of outliers (Zhu, et al., 2012).

4.3 Portfolio construction
The out-of-sample test will proceed in accordance with Sorensen et al. (2000). For each month, two portfolios are formed, one with an equal weighting of the stocks predicted to outperform and one with an equal weighting of stocks predicted to underperform. These portfolios then represent the stocks an investor would want to be long and short respectively. These portfolios will be broken up into the crisis period and recovery period for the out-of-sample subset. Excess returns above the benchmark will be calculated and assessed or the portfolios over a number of performance measures for statistical significance and economic significance. Additionally, the portfolios will be tracked throughout the out-of-sample period by means of wealth curves showing the amount of wealth achieved if $1$ was invested at the beginning of the period. These wealth curves will then be assessed versus the wealth curve of the S&P 700 index graphically. With regard to portfolio construction it is important to note that this paper follows the static tree methodology set out by Sorensen et al. (2000) rather than the evolving tree approach as the
model was not updated with data as time progressed, however, Sorensen et al. (2000) found that the tree models were relatively stable across both methods.

The following 3 portfolios were formed for this paper:

**Long portfolio:** An equally weighted portfolio of stocks forecasted to outperform by the model updated monthly.

**Short portfolio:** An equally weighted portfolio of stocks forecasted to underperform by the model updated monthly.

**Net portfolio:** A simplistic long minus short portfolio comprising the net position if an investor were to follow the equally weighted long and short portfolios listed above. In each month the return of the short portfolio is subtracted from that of the long portfolio. Following this a geometric average is taken and an annual return calculated which includes compounding effects. It must be emphasised that this portfolio is unlikely to be practically investible due to the number of stocks required to arrive at this net position, however, it provides a useful tool for assess the relative performances of the long and short portfolios respectively.

All three portfolios mentioned above were formed from the stock universe comprising 700 shares and will be measured against the S&P 700 index itself over the various periods.

Further, in order to assess the model’s performance over various periods a number of performance measures were calculated for the crisis period (August 2007 to June 2012), the recovery period (July 2012 to April 2015) and the entire period (August 2007 to April 2015). These measures included:
Average excess portfolio return: This measure encompasses the excess annual return above the benchmark which was selected as the S&P 700 index.

$$\left[ \prod \left( \frac{\text{Return}_t - \text{Benchmark return}_t}{\text{Risk free rate}_t} \right)^{\frac{1}{n}} \right]$$

Where $n$ is the number of months in the period (crisis, recovery or entire, $n=\{1, 2, ..., 93\}$), $r_t$ is the return of the portfolio for a given month (for $t=\{1, 2, ..., 93\}$) and $\eta_{bt}$ is the return of the benchmark, S&P 700 index, for a given month (for $t=\{1, 2, ..., 93\}$)

Sharpe ratio: This measure calculates the risk adjusted excess return for the particular period under observation following the methodology of Sharpe (1975).

$$\left[ \prod \left( \frac{\text{Return}_t - \text{Risk free rate}_t}{\sigma_t} \right)^{\frac{1}{n}} \right]$$

Where $\eta_{ft}$ is the risk free rate for a given month (for $t=\{1, 2, ..., 93\}$). The US 30 day T-bill was used as a proxy for the risk free rate which is stated by DeFrusco, McLeavey, Pinto, Runkle and Anson (2015) as a widely used short term risk free rate proxy. The reason for this proxy is also evident from the fact that the index used is denominated in United States Dollars. $\sigma$ is the standard deviation of the portfolio’s returns at time $t$ (i.e. crisis period, recovery period or entire period)

Jensen’s Alpha (Jensen, 1967): Jensen’s alpha is calculated as the average return over the Capital Asset Pricing Model (CAPM). It is a risk adjusted excess return with a positive number indicating a portfolio has “beaten the market”. The formula for Jensen’s alpha is as follows:
\[ \beta(r, \ ) \]

Where \( \alpha_{pt} \) is the risk adjusted excess return, \( \mu \) is the expected return to the portfolio, \( r_f \) is the return of the risk free asset, \( \beta \) is the beta of the portfolio and \( \mu_f \) is the return of the market.

During the out-of-sample period the long and short CART portfolios were regressed monthly on the returns of the index, the S&P 700 index, in order to calculate a monthly alpha and beta. All portfolios were in excess of the risk free rate. These results will later be presented graphically relative to the corresponding returns of the benchmark, the S&P 700 index.

**T-statistic:** This measure is used in order to test the significance of the excess returns calculated over a given period at the 5% level.

\[
\frac{\mu - \mu_f}{\sqrt{n}}
\]

Where \( \mu \) is the return being tested against, being zero, \( S \) is the annualised sample standard deviation of the portfolio at time \( t \) (i.e. crisis period, recovery period or entire period) and \( n \) is the number of observations in the given period (\( n = \{1,2,\ldots, 93\} \)).
5. Discussion and results

This chapter displays the descriptive statistics of the factors used as well as the CART model built during the period January 2004 to July 2007. This CART model is then used during an out-of-sample period and the findings presented by means of performance measures and wealth curves in relation to the chosen benchmark, the S&P 700 index.

5.1 Descriptive statistics

There were 5 factors created as referenced in chapter 3 of this dissertation, namely; a value factor, profitability factor, leverage factor, momentum factor and a debt service factor. The distributions of these factors as well as a summary of their respective descriptive statistics are shown below in figures 5.1 through 5.5:

Figure 5.1: Value factor

Figure 5.2: Profitability factor
Figures 5.1 through 5.5 above allow for a more thorough analysis of the decisions within the CART model by providing a reference point for each of the decisions as discussed in the next section of this dissertation. It can be noted that factors had varying numbers of observations due to missing data at times.

### 5.2. CART model

The CART model was built for the universe of 700 stocks over the period January 2004 to July 2007 with the resulting hierarchical tree structure shown below in Figure 4.6. The methodology of Zhu et al. (2012) was used to construct the tree with a stock being classified as an “Outperformer” at a particular node if it has a 50% or greater chance of outperforming at that node. Conversely, a stock was classified as an “Underperformer” at a particular node.
if it had a 50% or greater chance of underperforming at that node. The probability of belonging to the chosen group at any given node is given below at each node in the CART tree. The CART tree is presented below in figure 5.6.

Figure 5.6: CART model built January 2004 - July 2007

As can be seen in figure 5.6 above the primary split is on a profitability showing the importance of this factor which was made up of an equally weighted average on return-on-equity, 12 month-trailing-profit-margin and asset turnover. Note in the tree that if a stock
meets the criteria stated it moves to the left branch indicating a “yes” while if it fails to meet the criteria it moves down to the right indicating a “no”. A high value here indicates high profitability and vice versa and it is clear from figure 5.6 that if a stock falls into the high profitability criterion category it moves further down the tree and has a chance of outperforming if it meets various other criteria while if profitability is below that level, as specified by the profitability factor, the stock is immediately classified as an underperformer. Interestingly, the level of profitability is not high in relation to the mean for this variable which can be seen in figure 5.2. In general it would be assumed that stocks with higher profitability would outperform.

It can also be seen that the second split is on momentum which was built as an equally weighed average of two momentum factors, one representing momentum over the past 6 months and one representing momentum over the past 12 months. A momentum factor greater than 0.42 appears to be relatively high momentum according to the mean for this factor seen in figure 5.4 and it is apparent that stocks with profitability greater than or equal to 5.4 and momentum greater than or equal to 0.42 fall into node 2 and outperform with 66% of the observations in this node falling into the outperform group.

The third split is on value which comprised earnings-to-price, book-to-price and sales to price. It appears that stocks classified as cheap (having a high value factor in this case) underperform while stocks classified as relatively more expensive proceed further down the tree. This is contrary to conventional finance research which suggests that cheaper stocks, stocks with a higher value factor, outperform more often than not. In relation to the mean for this factor seen in figure 5.1, a value factor of 4 is relatively high. This unexpected split can possibly be explained by the so called “value trap” as particular stocks are cheap for a
reason rather than being undervalued. Interestingly, the other splits on value throughout the tree follow more conventional finance wisdom and stocks with a high value factor (i.e. cheap) outperform. These value factor splits are below the mean of 2.55 shown in figure 5.1 indicating that at some level a higher value factor is desirable.

Splits of lesser importance include the leverage factor and debt service factor. With regards to the leverage factor which comprised long-term-debt-to-equity and long-term-debt-to-market-capitalisation, it appears that stocks should be leveraged above 5.1 but below 124 which leaves a wide range of possible leverage factor values. This is not altogether surprising as certain firms with high leverage will generate good returns while others might find it hard to repay that debt and suffer in the process. The debt service factor, comprising interest coverage ratio and free-cash-flow-to-debt, is more specific but appears to require a low level of ability to cover their interest payments and generate cash flows relative to debt if the distribution of this factor is taken into account. It must be noted that the debt service factor’s observations are significantly lower than the other respective factors which could be a limitation of the factor to create splits within the tree as well as for these splits to be meaningful.

A key finding that follows from prior studies is the impact of conditional relevance allowed for by the CART model. For example, a stock can have a momentum factor below 0.42 and still be classified as an outperformer if it possesses a high value factor among other attributes. Additionally, it seems that complex, possibly non-linear, relationship are captured within the tree as arriving at outperforming nodes requires multiple decisions on profitability, momentum and value in particular.
5.3. **Out-of-sample performance**

This section evaluates the portfolios built during the out-of-sample period through the use of a number of performance measures as well as graphically investigates the portfolios in relation to the market with regard to their betas, alphas and wealth curves over the entire sample period.

5.3.1 **Performance in excess of the benchmark, the S&P index**

The out-of-sample people comprised both a crisis period (August 2007-June 2012) and a recovery period (July 2012-April 2015) in an attempt to isolate performance over these two varying periods and assess further the findings of previous research that CART performs particularly well over crisis periods. Table 5.1 below illustrates the portfolios’ performance over the various periods.

*Table 5.1: Performance of portfolios in out-of-sample period*

<table>
<thead>
<tr>
<th></th>
<th>Crisis period</th>
<th>Recovery period</th>
<th>Entire period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Long</td>
<td>Short</td>
<td>Net</td>
</tr>
<tr>
<td><strong>Excess return</strong></td>
<td>2.92%</td>
<td>0.17%</td>
<td>3.53%</td>
</tr>
<tr>
<td><strong>Standard deviation</strong></td>
<td>26.87%</td>
<td>23.62%</td>
<td>6.99%</td>
</tr>
<tr>
<td><strong>T-stat</strong></td>
<td>1.05</td>
<td>0.07</td>
<td>4.87</td>
</tr>
<tr>
<td><strong>Sharpe ratio</strong></td>
<td>-0.05</td>
<td>-0.14</td>
<td>0.31</td>
</tr>
</tbody>
</table>

In Table 5.1 above it can be seen that in each period the long portfolio as well as the short portfolio outperform the benchmark as excess returns for all of these portfolios are positive. The greatest excess performance for both the long and short portfolios occurred during the recovery period with the long portfolio earning 5.82% above the benchmark which was
significant at the 5% level. The short portfolio also significantly outperformed the benchmark during the recovery period but still fell short of the long portfolios performance. In absolute terms the recovery period portfolios again performed the best with Sharpe ratios of 1.69 and 1.59 respectively which is not surprising given that this period is characterised by a recovery from the slump of the financial crisis and a return to positive growth. It can be noted that the model’s portfolios performed better on both a risk adjusted and non-risk adjusted basis as shown by the findings of the excess return portfolios and the Sharpe ratios of those portfolios.

The model earned positive excess returns above the benchmark during the crisis period with the long portfolio doing particularly well although the results were insignificant due in part to the high volatility during this period. The short portfolio marginally outperformed the benchmark but was highly insignificant. These findings appear to indicate that the model more correctly classified underperformers during the crisis period than over the recovery period as the returns to the short portfolio performed well above the benchmark during the recovery period but had a highly insignificant excess return over the crisis period. It can be seen from the crisis period’s Sharpe ratios that the two portfolios earned negative returns which is not surprising given the period under observation.

The finding that each of the short portfolios generate excess positive returns is consistent with Giuricich (2004) where it was found that the short portfolio earned excess returns above the benchmark, albeit lower than the long portfolio. Guirich (2014) went on to state that the tree appeared to perform poorly when predicting underperforming stocks but much better when identifying outperformers.
The net position encompasses a simplistic long minus short portfolio which performed well over each period analysed with the return being statistically significant in each case. The net portfolio performed best over the crisis period with a return per annum of 3.53% which was significant at the 1% level. This was due to the long portfolio performing well during this period from an excess return perspective while the short portfolio earned excess returns close to zero with the difference between the long and short portfolios and the corresponding net portfolio being due to compounding. This finding again supports the idea that the model more accurately predicted underperforming stocks during the crisis period but caution must be exercised when attaching weight to this result as this strategy is not strictly investible.

Over the entire period it can be seen that both portfolios make excess returns above the benchmark with the long portfolio outperforming the short. The returns are statistically significant for the recovery period but insignificant for the crisis period and over the entire period, however, the results may still be economically significant for an investor. The variation in performance between the crisis and recovery period suggests that one must be exceedingly careful when deciding on what period to build the model over as well as over what period it will used to forecast.

5.3.2 Analysing portfolio alpha and beta in relation to market return
In order to further assess the portfolio in relation to the market it is useful to estimate the portfolios’ beta and alpha for a given month. This was done through monthly regressions described in chapter 3 of this dissertation. Figures 5.7 and 5.8 below show these relationships in any given month within the out-of-sample period along with the corresponding return on the benchmark, the S&P 700 index.
In this figure betas derived from regressions of the CART long and short portfolios on the S&P 700 index are displayed against the return of the S&P 700 index for a given month. Beta is viewed as the portfolio’s respective exposure to the market for a given month. Returns were in excess of the risk free rate, stated previously as the US 30 day T-bill. No transaction costs were considered.

In figure 5.7 above it can be seen that both portfolio betas vary significantly over the period. It can also be seen that betas close to or in excess of 1 are not uncommon for either portfolio. Figure 5.8 above is useful for examining the effect beta at a particular point in
time may have had on the corresponding risk adjusted excess return, here on out referred
to as alpha. Using both figures above it can be seen that initially having a higher beta
allowed the long portfolio to reap significantly more alpha than the short portfolio, but as
returns of the market turned negative that higher exposure to the market led to a sharp
decline in alpha over the October 2009 to November 2009 period while the short portfolio’s
lower exposure meant a small period of alpha above the long. It was seen in Table 5.1 that
over the crisis period in particular, the short portfolio’s return above the benchmark was
highly insignificant and figure 5.8 gives further evidence of this as the short portfolio
appears to earn high alpha for only the small period mentioned before and alpha close to
zero otherwise between August 2007 and June 2012.

Over the entire period it appears that the long portfolio outperforms the short in terms of
alpha and, for the most part, has a higher market exposure as measured by beta. This higher
alpha over the entire period for the long portfolio is again consistent with the findings in
table 4.1 and gives credence to the idea that the CART model was successful in predicting
outperforming and underperforming shares to a degree. One spike of note occurs in
November 2012 where the short portfolio’s beta is exceedingly high and results in a
significantly negative alpha as seen in figures 5.7 and 5.8 which seems to suggest that in that
month in particular the criteria that the CART model set out seemed to predict
underperformers well. Table 5.2 below shows the respective alpha of the long, short and
net portfolio as examines them for significance.
### Table 5.2: Portfolios’ respective alphas together with test for significance
*Note: Average and standard deviation are taken over the entire period (i.e. August 2007-April 2015).*

<table>
<thead>
<tr>
<th></th>
<th>Long alpha</th>
<th>Short alpha</th>
<th>Net alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>0.73%</td>
<td>0.33%</td>
<td>0.40%</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.49%</td>
<td>0.70%</td>
<td>0.80%</td>
</tr>
<tr>
<td>T-stat</td>
<td>14.40</td>
<td>4.61</td>
<td>4.78</td>
</tr>
</tbody>
</table>

In table 5.2 above it can be seen that for each portfolio alpha is significant over the entire period at the 5% level, however the long portfolio far outperforms the short and correspondingly the net portfolio also manages a higher alpha than the short portfolio. This is evidence that the CART model was able to predict outperforming shares in particular but also underperforming shares to some degree.

### 5.3.3 Comparing portfolios from an absolute, Dollar perspective through wealth curves

In order to compare the results of the long and short portfolio from an absolute perspective wealth curves were constructed where the dollar value of a portfolio if $1 was invested at t=0 is graphed over the entire period against the value of a portfolio in the S&P 700 index if $1 was invested at t=0. Figure 5.9 below shows the respective wealth curves over the entire period August 2007 to April 2015.
In Figure 5.9 above it can be seen that the long portfolio, stocks predicted to outperform by the model in each month, performs the best over the entire period using absolute cumulative returns. In contrast, the short portfolio appears to mainly vary around the S&P 700 index but on the whole return similar performance which is consistent with figure 4.7 where it was seen that the short portfolios beta varies roughly around 1 with the exception of certain spikes and table 5.1 where it was seen that the short portfolio had positive but insignificant excess returns over the entire period. The performance of the long portfolio appears to strengthen over time relative to the others starting from roughly July 2009 where it begins to open a substantial gap on the other 2 main portfolios which it continues into the recovery period. This same relationship was seen in figure 5.8 where July 2009 marked a spike in alpha for the long portfolio and a corresponding dip in alpha for the short portfolio allowing the long portfolio to open a gap on the other two portfolios.
In order to further examine the relative performances of the long and short portfolios it is helpful to look at the cumulative wealth graph from a net perspective, that is the long portfolio less the short portfolio in every month added to an initial $1 investment at t=0. This relationship is shown below in figure 5.10.

Figure 5.10: Cumulative wealth graphs:
This figure plots the cumulative wealth graphs of the net portfolio if $1 is invested at t=0. Note: Returns are absolute and taken monthly over the period August 2007 to April 2015 with portfolios being rebalanced monthly. Cumulative return is the Dollar amount at time=t. Transaction costs are not included.

In figure 5.10 above the difference between the long and the short portfolio can more easily be seen across the various periods through the net portfolio created. It can be see that initially the long portfolio greatly outperforms the short portfolio in the early months of 2008, however, this gap is quickly closed as the short portfolio begins to gain on the long portfolio throughout the remained of 2008 and the early stages of 2009. In figure 5.8 this relationship was also observed as the lower exposure to the market, beta, of the short portfolio relative to the long portfolio led to a dramatic increase in alpha generated over this period with the long portfolio actually generating negative alpha for November 2009.
As the graph approaches midway through 2009 the long portfolio again starts to outperform the short and this gain continues throughout the remained of the out-of-sample period. During the out-of-sample period it can be seen that this simplistic long minus short portfolio earned a cumulative return of approximately 25% at an average annual return of roughly 2.9% taking compounding into account.

Overall it appears as though the CART model offers superior return to the benchmark through both the long, short and net portfolios, however, these results were not always statistically significant. The long portfolio is each case was especially good at outperforming the benchmark and was statistically significant during the recovery period while also outperforming the short portfolio to a degree during the crisis period and over the entire period. The short portfolio appeared to select stocks best during the recessionary period where it displayed negligible excess return above the benchmark but overall the long portfolio was able to dominate the short and a net return in each case was found to be statistically significant. In summary, if $1 had been invested in the long portfolio at the beginning of the period it would have outperformed the benchmark, the risk free rate and the short portfolio.
6. Conclusion

This dissertation shows evidence of the stock picking ability of the Classification and Regression Tree (CART) model over the out-of-sample period (i.e. August 2007-April 2015) as well as the individual sub-periods within this sample. The results provide motivation for the inclusion of the CART model into stock selection decisions.

The idea behind the CART model is to come up with a set of decision rules which can then be able to classify stocks as either underperformers or outperformers based on various fundamentals attributable to those stocks. These decision trees allow for the inclusion of possibly non-linear returns and complex relationships between variables that may not be picked up by traditional methods of asset pricing. Through the CART model 3 portfolios were built; a long portfolio of stocks an investor would wish to be long in, a short portfolio comprising stocks an investor would like to be short in and a net portfolio incorporating a simplistic long minus short portfolio.

The results show that the CART model successfully picked a long portfolio that beat the benchmark, the S&P 700 index, in terms of excess returns over the entire period but these returns were only statistically significant over the recovery period. Crucially, during the crisis period, a period CART is said to outperform traditional models most, the tree was shown to more correctly predict underperforming stocks as shown by a negligible short portfolio return while the long portfolio return could be considered to be economically significant.

To further analyse the where returns were earned in relation to the benchmark, alpha and beta estimations were derived from regressions of the CART portfolios’ returns on the S&P 700 index returns. These results were then then graphically presented and helped explain
where alpha was earned and the beta, exposure to the market, which helped generate these alphas. It was found that for the most part the long and short portfolios generated positive alpha which was shown to be statistically significant at the 5%, however, the long portfolio’s alpha was substantially larger than that generated by the short portfolio.

Finally, in an attempt to show the long term effect of investing in the portfolios built, wealth curves were displayed showing the effect of investing $1 at \( t=0 \) for the entire period. It was again seen that the long portfolio ended with substantially larger wealth while the short portfolio marginally outperformed the S&P 700 index. A net position wealth curve showed that average returns of roughly 2.9% per annum were possible if an investor were to go long the outperform stocks and short the underperform stocks as classified by the CART model.

Despite these results it was still observed that excess returns over the crisis and entire period were not statistically significant although they may prove economically significant to an investor. This leaves space for further research to use a larger sample than the 43 months used to build the CART model in this case and then test it over a crisis and recovery period. Furthermore, factors that were not considered in this paper could be introduced with the expectation of further improving the classification of outperforming and underperforming stocks. Another point to note is that the long and short portfolios created were equally weighted returns of the stocks forecasted to outperform and underperform respectively which provides an area for further research with a different weighting approach. A final point is that this paper only considered the returns relative to the S&P 700 index as a benchmark in terms of its excess returns and cumulative wealth graphs. This leaves scope to test whether the CART model performs better relative to linear models in the future as was done by the authors discussed in the literature review of this paper.
7. References:


Appendix A: Eviews code

Appendix A.1: Loop for winsorisation

```
smpl @all
for %factor
smpl @all

genr Q1Q3= (@quantile(%factor,0.75))-(@quantile(%factor,0.25))

smpl if %factor>@quantile(%factor,0.75) + (2.2*(Q1Q3))
(%factor)= @quantile(%factor,0.75))

smpl @all

smpl if %factor>@quantile(%factor,0.25) - 2.2*(Q1Q3)
(%factor)= @quantile(%factor,0.25)

smpl @all

hist %factor

next
```
Appendix A.2: Coding to build equally weighted portfolios from CART model

!smplstart=480   '1999.12
!minmonth=500
!maxmonth=665
!Totalshares=700
!Totalmonths=(!maxmonth-!minmonth)+1

delete results
delete mrLong
delete mrShort
delete mrLong
delete mrShort
delete obsLong
delete obsShort

TABLE RESULTS
'label the results table

results(2,1)="Average summed return, long"
results(3,1)="Average summed return, short"
results(4,1)="Average summed return, long-short"
results(5,1)="T-statistic: Average summed long-short return"
results(6,1)="Standard deviation of long-short return"

results(8,1)="Average observations, long"
results(9,1)="Average observations, short"
results(10,1)="Average net long, long-short"

results(12,1)="Average return, long"
results(13,1)="Average return, short"
results(14,1)="Average return, long-short"
results(15,1)="T-statistic: Average long-short return"
results(16,1)="Standard deviation of long-short return"

!num = 0

series mrLong     'mean return to long stocks
series mrShort     'mean return to short stocks
series rLong      'return to long stocks
series rShort       'return to short stocks
series obsLong     'no. of observations long
series obsShort     'no. of observations short

***********
!row=1
!column=1
!col=1

smpl @all

for !month = !minmonth to !maxmonth-1
'NODE 1 - Short
smpl if monthnum=!month and profitability_criterion < 5.4
if @obs(returnsfwd)<>0 then
  rShort(!month -(!smplstart-1)) = @sum(returnsfwd)
  obsShort(!month -(!smplstart-1)) = @obs(returnsfwd)
endif

'NODE 2 - Long
smpl if monthnum=!month and profitability_criterion >= 5.4 and mom_criterion>=0.42
if @obs(returnsfwd)<>0 then
  rLong(!month -(!smplstart-1)) = @sum(returnsfwd)
  obsLong(!month -(!smplstart-1)) = @obs(returnsfwd)
endif

'NODE 3 - short
smpl if monthnum=!month and profitability_criterion >= 5.4 and mom_criterion<0.42 and
value_criterion>=4
if @obs(returnsfwd)<>0 then
  rShort(!month -(!smplstart-1)) = rShort(!month -(!smplstart-1)) + @sum(returnsfwd)
  obsShort(!month -(!smplstart-1)) = obsShort(!month -(!smplstart-1)) + @obs(returnsfwd)
endif

'NODE 4 - short
smpl if monthnum=!month and profitability_criterion >= 5.4 and mom_criterion<0.42 and
value_criterion<4 and mom_criterion>=0.2 and debt_service_criterion<2.3
if @obs(returnsfwd)<>0 then
  rShort(!month -(!smplstart-1)) = rShort(!month -(!smplstart-1)) + @sum(returnsfwd)
  obsShort(!month -(!smplstart-1)) = obsShort(!month -(!smplstart-1)) + @obs(returnsfwd)
endif

'NODE 5 - Long
smpl if monthnum=!month and profitability_criterion >= 5.4 and mom_criterion<0.42 and
value_criterion<4 and mom_criterion<0.2 and profitability_criterion<6.1
if @obs(returnsfwd)<>0 then
  rLong(!month -(!smplstart-1)) = rLong(!month -(!smplstart-1)) + @sum(returnsfwd)
endif
obsLong(!month -(!smplstart-1)) = obsLong(!month -(!smplstart-1)) + @obs(returnsfwd)
endif

'NODE 8 - Long
smpl if monthnum=!month and profitability_criterion >=5.4 and mom_criterion<0.42 and value_criterion>=4 and mom_criterion<0.2 and profitability_criterion>=6.1 and mom_criterion>=0.18
if @obs(returnsfwd)<>0 then
    rLong(!month -(!smplstart-1)) = rLong(!month -(!smplstart-1)) + @sum(returnsfwd)
    obsLong(!month -(!smplstart-1)) = obsLong(!month -(!smplstart-1)) + @obs(returnsfwd)
endif

'NODE 9 - Long
smpl if monthnum=!month and profitability_criterion >=5.4 and mom_criterion<0.42 and value_criterion>=4 and mom_criterion<0.2 and profitability_criterion>=6.1 and mom_criterion<0.18 and value_criterion>=3.1
if @obs(returnsfwd)<>0 then
    rLong(!month -(!smplstart-1)) = rLong(!month -(!smplstart-1)) + @sum(returnsfwd)
    obsLong(!month -(!smplstart-1)) = obsLong(!month -(!smplstart-1)) + @obs(returnsfwd)
endif

'NODE 10 - Short
smpl if monthnum=!month and profitability_criterion >=5.4 and mom_criterion<0.42 and value_criterion>=4 and mom_criterion<0.2 and profitability_criterion>=6.1 and mom_criterion<0.18 and value_criterion<3.1 and mom_criterion>=0.16
if @obs(returnsfwd)<>0 then
    rShort(!month -(!smplstart-1)) = rShort(!month -(!smplstart-1)) + @sum(returnsfwd)
    obsShort(!month -(!smplstart-1)) = obsShort(!month -(!smplstart-1)) + @obs(returnsfwd)
endif

'NODE 11 - Short
smpl if monthnum=!month and profitability_criterion >=5.4 and mom_criterion<0.42 and value_criterion>=4 and mom_criterion<0.2 and profitability_criterion>=6.1 and mom_criterion<0.18 and value_criterion<3.1 and mom_criterion<0.16 and leverage_criterion<5.1
if @obs(returnsfwd)<>0 then
    rShort(!month -(!smplstart-1)) = rShort(!month -(!smplstart-1)) + @sum(returnsfwd)
    obsShort(!month -(!smplstart-1)) = obsShort(!month -(!smplstart-1)) + @obs(returnsfwd)
endif

'NODE 12 - Short
smpl if monthnum=!month and profitability_criterion >=5.4 and mom_criterion<0.42 and value_criterion>=4 and mom_criterion<0.2 and profitability_criterion>=6.1 and mom_criterion<0.18 and value_criterion<3.1 and mom_criterion<0.16 and leverage_criterion>=5.1 and leverage_criterion >=124
if @obs(returnsfwd)<>0 then
    rShort(!month -(!smplstart-1)) = rShort(!month -(!smplstart-1)) + @sum(returnsfwd)
    obsShort(!month -(!smplstart-1)) = obsShort(!month -(!smplstart-1)) + @obs(returnsfwd)
endif

'NODE 13 - Long
if monthnum=!month and profitability_criterion >=5.4 and mom_criterion<0.42 and value_criterion>=4 and mom_criterion<0.2 and profitability_criterion>=6.1 and mom_criterion<0.18 and value_criterion<3.1 and mom_criterion<0.16 and leverage_criterion>=5.1 and leverage_criterion <124

if @obs(returnsfwd)<>0 then
  rLong(!month -(!smplstart-1)) = rLong(!month -(!smplstart-1)) + @sum(returnsfwd)
  obsLong(!month -(!smplstart-1)) = obsLong(!month -(!smplstart-1)) + @obs(returnsfwd)
endif

mrLong(!month -(!smplstart-1)) = rLong(!month -(!smplstart-1))/obsLong(!month -(!smplstart-1))
mrShort(!month -(!smplstart-1)) = rShort(!month -(!smplstart-1))/obsShort(!month -(!smplstart-1))

smpl @all
next

genr netLong=obsLong-obsShort

!col=!col+1

results(1,!col)=%0
results(2,!col)=@mean(rLong)
results(3,!col)=@mean(rShort)
results(4,!col)=@mean(rLong)-@mean(rShort)
results(5,!col)=((@mean(rLong))^2+(@mean(rShort))^2 - 2*@mean(rLong)*@mean(rShort)*@cor(rLong,rShort))^0.5
results(6,!col)=((@stdev(rLong))^2+(@stdev(rShort))^2 - 2*@stdev(rLong)*@stdev(rShort)*@cor(rLong,rShort))^0.5
results(7,!col)=(!Totalmonths^0.5)/((@stdev(rLong))^2+(@stdev(rShort))^2 - 2*@stdev(rLong)*@stdev(rShort)*@cor(rLong,rShort))^0.5
results(8,!col)=@mean(obsLong)
results(9,!col)=@mean(obsShort)
results(10,!col)=@mean(netLong)
results(11,!col)=results(4)!col
results(12,!col)=@mean(mrLong)
results(13,!col)=@mean(mrShort)
results(14,!col)=@mean(mrLong)-@mean(mrShort)
results(15,!col)=((@stdev(mrLong))^2+(@stdev(mrShort))^2 - 2*@stdev(mrLong)*@stdev(mrShort)*@cor(mrLong,mrShort))^0.5
results(16,!col)=(!Totalmonths^0.5)/((@stdev(mrLong))^2+(@stdev(mrShort))^2 - 2*@stdev(mrLong)*@stdev(mrShort)*@cor(mrLong,mrShort))^0.5
Appendix A.3: Coding to calculate alpha and beta

!smplstart=480   '1999.12
!minmonth=500
!maxmonth=665
!Totalshares=700
!Totalmonths=(!maxmonth-!minmonth)+1

series longAlpha
series shortAlpha
series longBeta
series shortBeta

for !month = 0 to 100 '2007.08 to 2015.05
smpl 2007.08 + (!month-5) 2007.08 + !month

ls mrLong c spint_returnfwd
longAlpha((!month+571)-(!smplstart-1)) = c(1)
longBeta((!month+571)-(!smplstart-1)) = c(2)

ls mrShort c spint_returnfwd
shortAlpha((!month+571)-(!smplstart-1)) = c(1)
shortBeta((!month+571)-(!smplstart-1)) = c(2)

next
Appendix B: Wealth graphs

Appendix B.1: Crisis period wealth graph

This figure plots the cumulative wealth graphs of each of the portfolios if $1 is invested at t=0 for the long portfolio, short portfolio, the S&P 700 index and the risk free portfolio over the crisis period (i.e. August 2007-June 2012). Note: Returns are absolute and taken monthly over the period August 2007 to April 2015 with portfolios being rebalanced monthly. Cumulative return is the Dollar amount at time=t. Transaction costs are not included.
Appendix B.2: Recovery period wealth graph

This figure plots the cumulative wealth graphs of each of the portfolios if $1 is invested at $t=0$ for the long portfolio, short portfolio, the S&P 700 index and the risk free portfolio over the recovery period (i.e. July 2012-April 2015). Note: Returns are absolute and taken monthly over the period August 2007 to April 2015 with portfolios being rebalanced monthly. Cumulative return is the Dollar amount at time=$t$. Transaction costs are not included.