CURRENT TRANSFORMERS
TRANSIENT RESPONSE MODELLING
USING
ELECTROMAGNETIC TRANSIENT PROGRAM
(EMTP)

By

NOEL CHUTHI

Submitted to the University Of Cape Town in fulfilment of the requirements for the Degree of Masters in Electrical Engineering.

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Firstly, I would like to thank Professor S. Darie who has been a wonderful supervisor. His inspiration and help whenever I was stuck really put me through. Without him this work would not have been successful.

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Victor Shikoana and many other friends who made my stay in Cape Town pleasant need a special tribute.

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Lastly thanks to all the people who helped me in various other ways for the success of this work.

May the Lord bless you all.
The subject of this thesis is Current Transformer Transient response study using an electromagnetic Transient program (EMTP).

Current transformers are considered eyes for power system protection. Behaviours of protection systems depend largely on information fed to them by instrument transformers. Ferromagnetic current transformers have for many years provided practical method of current measurement, however there are limitations associated with current transformer operation: notably, difficult in maintaining accuracy over the full range of operating conditions, and most particularly current transformers tendency to suffer saturation of iron core during severe faults, with accompanying severe ratio or loss of output. These limitations might lead to maloperation of protective relays due to distorted inputs from current transformers particularly in transient periods.

This thesis involved studying the behaviour of current transformers in both steady state and transient periods. An emphasis being put on transient periods which are very crucial in behaviour of current transformers because transformation errors are greatest in these periods. Errors in current transformer transformation might affect operation of entire protection schemes. Maloperation of current transformers in transient periods have very bad effect on relay co-ordination and worst condition might be failure of protection scheme operation altogether.

Over the past decades engineers have been trying to develop a current transformer model that would represent a current transformer well in transient periods. It has proved to be rather difficult to come up with a single detailed model that would satisfy all possible conditions. This is due to non-linearity of magnetising curve and saturation effects of current transformer iron cores.

The author has considered different current transformer models with their merits and demerits being highlighted. It has been shown that different current transformer models have to be used when considering different operating conditions of a current transformer in a power system.

ATP-EMTP an Electromagnetic Transient Program was developed in the sixties for the study of electromagnetic transients in power systems. It has proved to be a very useful tool in this regard. The program development is still going on today to accommodate a wide application in power systems. Several components have been developed to represent different components in a power system. It is only recently that there has been a growing interest to include modelling of protective equipment. This has been accelerated by the inclusion of MODELS in the EMTP program.
This thesis explores the effect of transients taking into account different conditions like transient fault currents, effects of high frequency waves and surges. Effects of different types of burdens on current transformers were explored as well.

Due to limitations of EMTP, simulation results are only applicable to current transformers with ARMCO M4 oriented steel with ungapped cores.

The author arrived at several conclusions. The most important conclusion is that maloperation of protective relays due to current transformer saturation can be avoided easily if proper current transformer selection is given priority in power protection design.

EMTP package proved to be very useful and handy when studying transients though one has to be careful with numerical oscillations which might be present during simulations. Problems of numerical oscillations have been discussed under current transformer simulation tools.
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CHAPTER ONE

1 Introduction

1.1 Current Transformer Application

It is necessary to accurately measure current flowing in a power system network for metering purposes and protection of the power system in times of faults or equipment maloperation. To obtain the best performance possible from such equipment it is desirable to connect the equipment, where practically possible, directly into the circuits which are being monitored. These types of equipment are however not available for operation at high current levels. The insulation also cannot withstand the high operating voltages associated with typical electrical reticulation systems. Therefore direct measurement becomes dangerous and expensive as the magnitude of current increase. Instrument transformers become useful in these circumstances for scaling down the required quantities to values that can be handled safely.

Current transformers are instrument transformers which are used to scale down large quantities of current to smaller values for metering and protection purposes. Standard values for transformed currents (current transformer secondary current output) are 0.5 A (metering purposes), 1 A and 5 A at 50 Hz or 60 Hz. Current transformers also act as high voltage isolators for auxiliary equipment like meters and relays which operate at low voltages.

1.2 Problems Associated With Current Transformers

As pointed out in the last section, the best current measurement is obtained with direct measurement. Introduction of current transformers in a power system brings along some errors in measurement. These errors are due to losses incurred in current transformer windings due to resistance, leakage inductance and magnetization losses in current transformer iron cores. These losses are minimal when a current transformer is operated within its specified limits. BS standards 3938:1973, BS 7626:1993 and ANSI Standards C57.13 give recommended limitations for current transformer accuracy [1,2].
Current transformer errors become more and more when they are operated in situations where fault levels become more than the current transformer can handle or when dc biasing is present. Recently, the increase in harmonic content and non-sinusoidal waves in power networks have raised many questions on the operation of current transformers. The increase of harmonic and non-sinusoidal waves can be attributed to increased use of electronic equipment which have non-linear characteristics.

Complexity of power networks has also increased tremendously in recent years which has led to:

- Increased number of relays connected to each current transformer to drive primary and back up relays for two or more adjacent zones which overlap at the current transformer, therefore increasing current transformer secondary burden.
- Fault current levels to increase.
- Increase in system voltages and have imposed the fundamental requirement for more elaborate insulating systems. This increased insulation requirement has brought with it problems of capacitance between current transformer windings. This has been described in detail under high frequency models in this thesis.

The discussion above underlines the extent of problems to which relay engineers are faced with in recent years due to increased complexity in power networks.

1.3. Importance Of Current Transformer Modelling

Evidently current transformers play a very important role in power system protection and measurement. Because of costs involved in maintaining a sound protection system as well as for safety purposes, it is vital to have a thorough knowledge of current transformers and their response under certain conditions. Equivalent models are useful to predict the response of any system under specified conditions, provided that the model parameters can be determined accurately enough. The important requirement in this regard is that these models faithfully represent the real systems.

Models of varying accuracy and complexity can be used to simulate typical current transformers. The goal however is to obtain the most accurate and the most simplistic model. In the case of current transformers, equivalent circuit diagram representations are used as models. The next step is to determine the values of the circuit parameters. With the parameters known, the response of the current transformer can be determined for a specific input condition.
Computer simulations will be used in this thesis to simulate different conditions in current transformer operation. ATP-EMTP an electromagnetic transient program provides a very useful tool in modelling transformer core characteristics and electromagnetic transients though the nonlinear inductor (Type 98) and nonlinear hysteresis model (Type 96). Another merit of this program is that it uses trapezoidal rule in solving problems which gives it a capability of solving virtually all non linear problems. EMTP can also be used to model entire protection systems from system networks to relay responses which most computer packages can not achieve.
1.4 Layout Of The Thesis

The purpose of this thesis was to investigate and simulate various current transformer models, study their responses to various current inputs and to determine impact of various simulation conditions on the current transformers during transient periods. A thorough understanding of current transformer operation theory was therefore required. Current transformer theory has been presented in chapters two and three. These chapters provide current transformer theoretical background information for steady state and transient state operation respectively. A summarised transient circuit theory has been presented at the beginning of chapter three to build up understanding of current transformer behaviour in transient periods.

In chapter four current transformer modelling is covered. Factors affecting current transformer modelling which include leakage flux, magnetisation core losses, resistance and capacitance are discussed. Different models for low frequency modelling and high frequency modelling are presented and discussed. A section of different source current affecting current transformer modelling has also been included.

Current transformer saturation is the main problem associated with operation of current transformers. Saturation causes undesired relay operation (in differential protection scheme), delay in relay operation (instantaneous and delayed time schemes) and some times total failure of protection schemes. Chapter five looks at logistics of current transformer saturation under different conditions. It will be seen that different types of secondary burden on the current transformer influence current transformer saturation differently.

Chapter six presents simulation tools used in the thesis. These include ATP-EMTP an electromagnetic transient program, MODELS which is a high level programming language for transient studies which works hand in hand with EMTP and PCPLOT for plotting ATP-EMTP outputs. Problems associated with these simulation tools have been highlighted.

Chapter seven presents simulation results, analysis and discussions. Conclusions are drawn at the end. The appendices provide selected data file programs used in EMTP simulations and results of selected simulations.
CHAPTER TWO

2. Current Transformer Steady State Theory

2.1 Introduction

Current transformer theory in steady state is basically a very simple one, but its behaviour, particularly under saturation and transient conditions, is rather complex. This chapter presents the theory behind operation of current transformer in steady state. There are several assumptions which will be made in steady state to simplify description and in some cases the mathematics involved. These assumptions mainly include the behaviour of magnetisation inductance and iron losses. Analytical methods have been used to try and explain current transformer behaviour in this state.

2.2 General Steady State Current Transformer Theory

A current transformer is essentially an iron core with two windings. The winding connected in series to circuit to be measured being called the primary and that connected to the burden, the secondary. The flow of current in the primary winding produces an alternating flux in the core and this flux induces an emf in the windings connected to an external closed circuit. Magnetic effects of the secondary current, in accordance with the value of the secondary current automatically adjusts itself to such a value, that the resultant magnetic effect of the primary and secondary currents, produce the flux required to induce emf necessary to drive the secondary current against the impedance of the circuit in which it flows.

In an ideal transformer, the primary ampere-turns are always exactly equal to the secondary ampere-turns and the secondary current is therefore, always proportional to the primary current. In an actual current transformer, however this is never the case. Every core material requires a certain number of ampere-turns to induce in it magnetic flux which in turn causes the second voltage.
The above description can be illustrated as shown in the Figure 2.1 below. $E_p$, and $E_s$ are voltages in the primary and secondary windings, respectively, which are in anti phase. To maintain the flux, the primary current must supply a current $I_e$ in phase with the voltage to overcome the iron loss, and a current $I_m$ at right angles to the voltage and in phase with the flux, to magnetise the core. These two currents combine to form, the exciting current, $I_e$. If a burden, is connected to the secondary and draws a current $I_s$ lagging behind voltage ($V_s$) by an angle $\theta$, a corresponding $I_{s2}$ must flow in the primary. The total primary current, therefore, is the sum of $I_e$ and $I_{s2}$ or $I_p$.

![Figure 2.1
Vector Diagram Of A Current Transformer Operation](image)

Where:

- $E_p$ = Primary induced rms voltage
- $E_s$ = Secondary induced rms voltage
- $I_s$ = Secondary rms current
- $I_p$ = Primary rms current
- $I_{s2}$ = Secondary Current refereed to primary side
- $I_e$ = Iron core loss current
- $I_x$ = Excitation current
- $I_m$ = Magnetisation current
- $\theta$ = Angle between secondary induced voltage and secondary current
- $\phi$ = Iron core flux
- $r$ = Current (ratio) error
- $\beta$ = Phase angle error
If the primary is reduced, the secondary current will also be reduced, and since the secondary impedance is fixed, the secondary voltage and flux in the core will be reduced in the same proportion. However due to the shape of magnetisation curve of the iron core (See Figure 2.2), exciting current $I_e$ decreases in a different ratio. The result is that the ratio and phase angle curves of current transformer generally are not straight lines but tend to tip upwards at low values of primary current. That is, with a given impedance in the secondary circuit, the exciting ampere-turns form a larger proportion of the total small primary currents than at larger primary current. It is this exciting current required to magnetise the core that brings the errors. Detailed theory of transformer and magnetic circuit can be found in references [1-3].

In a current transformer which is series connected in a power system, primary current is determined by the power system variables. Unlike in a power transformer where primary current is determined by the burden of the secondary whose value reflects through the ampere turns ratio to the primary.

In a current transformer the secondary circuit is virtually shorted such that the secondary ampere turns and the core flux density are low. The primary and secondary currents are approximately in inverse proportion to the turns on the windings.

A distinguishing feature is that for a shunt connected transformer, the core flux density is basically constant under normal operating conditions, while in the case of a series connected transformer, it is dependent on the size of the primary current as well as the impedance of the secondary circuit.

### 2.3 Induced Emf

Emf induced in the secondary winding of a current transformer by an alternating current can be calculated from the usual transformer formula.

$$E_{RMS} = 4.44 f B A T$$  \hspace{1cm} (2.1)

where $f$ = frequency  
$B$ = Peak flux density  
$A$ = cross sectional Area  
$T$ = number of secondary turns

This induced voltage in the secondary winding causes current to flow through the external burden in the secondary winding.
2.4 Current Transformer Magnetisation Curve

The primary current contains two components:

- The secondary current which is transformed in the inverse ratio of the turns' ratio.
- An exciting current, which supplies the eddy and hysteresis losses and magnetises the core.

The latter current is not transformed and therefore, is the cause of transformer errors. It is therefore not sufficient to assume a value of secondary current and to work backwards to determine the value of primary current by invoking the constant ampere-turn rule, since this does not take into account the excitation current.

The amount of exciting current drawn by a current transformer depends upon the core material and the amount of flux which must be developed in the core to satisfy the burden requirements of the current transformer. This may be obtained directly from the excitation characteristics of the transformer since the secondary emf and therefore the flux developed is proportional to the product of secondary current and burden impedance.

A general shape of an excitation characteristic for a typical current transformer core is as shown in figure 2.2 below. The characteristic is divided into three regions, defined by the ankle point and the knee point. The working range of a protective transformer extends over the full range between the ankle point, the knee point and sometimes beyond, while a measuring current transformer usually only operates in the region of the ankle point.

![Magnetisation Curve](image)

Figure 2.2 Magnetisation Curve
2.5 **Knee-Point**

The knee point of an excitation characteristic is defined as the point at which a 10% increase in secondary voltage produces a 50% increase in exciting current. It may, therefore, be regarded as a practical limit beyond which a specified ratio may not be maintained. Beyond the knee point the CT is said to enter saturation region. In this saturation region almost all the primary current is used to maintain the core flux and since the shunt admittance is not linear, both the exciting and secondary currents depart from sine wave.

2.6 **Losses In Magnetic Cores Containing Time Varying Fluxes**

Losses arise from two causes:

a) The tendency of the material to retain magnetism or to oppose a change in magnetism, often referred as magnetic hysteresis

b) $I^2R$ heating which appears in the material. The eddy current loss produced by the currents in the magnetic material, and these currents are caused by the electromotive forces set up by the varying fluxes. The sum of hysteresis and eddy current losses is called total core loss.

2.7 **Current Transformer Performance**

Current Transformer accuracy performance is ultimately determined by the magnetising and watt loss ampere turns required to maintain excitation of the core. These are in turn determined by the core material used, the type of core construction as well as the operating flux density. The operating flux density again is determined by the burden and secondary winding impedances, the power factor of the secondary circuit, the frequency, the number of secondary turns and the core dimensions. In effect it is the combination of core design and core material effects that will produce a magnetising curve that would display the $L_z$ relationship.
2.8 Current Transformer Errors

Errors associated with current transformers are the current ratio error and the phase angle error which have been described below. The error limits are defined by BS 6726. A measuring current transformer requires fewer errors than a protective current transformer since a measuring current transformer is only supposed to work in the linear region and rarely below the ankle point.

2.8.1 Composite Error

Composite error is the rms value of the difference between the ideal and actual secondary current including the effect of phase displacement and harmonics.

It has been seen already that current transformer errors are due to a component of the primary current being utilised in magnetising the core. Therefore only the remainder of the primary current is available for passing on to the secondary circuit.

2.8.2 Ratio Error (Current error)

If the scalar quantities in Figure 2.1 of primary current $I_p$ and secondary current referred to primary side $I_{s2}$ are compared, a difference in magnitude will be observed. This error is due to magnetising and capacitive currents and is called ratio error. The ratio error is defined as the error in the secondary current due to the incorrect ratio and is expressed as a percentage, by the expression:

$$\frac{nI_s - I_t}{I_p} \times 100\%$$  \hspace{1cm} 2.2

Where:

- $n$ = the nominal ratio (rated primary current/rated secondary current)
- $I_s$ = actual secondary current
- $I_p$ = actual primary current

The ratio is considered positive when the actual ratio of the transformer is less than the nominal ratio, that is when the actual secondary current of the transformer is less than the rated current.
2.8.3 Phase Angle Error

The phase angle error is the angle by which the secondary current vector, when reversed, differs in phase from the primary current. In Figure 2.1 this phase angle error is $\beta$ which is the angle difference between $I_{2}$ and $I_{p}$.

The phase angle error originates because of the fact that a small current is needed to magnetize the transformer core. There is also a small loss component present in this magnetization of the transformer core. The total magnetizing or no-load current consists of the phasor sum of this magnetizing and loss current components. This total magnetizing current $I_m$ has to be supplied by the primary current. Since it is out of phase with the primary current it is obvious that the phase angle error $\beta$ would occur if the magnetizing current is subtracted from the primary current to obtain the secondary current of the current transformer.

This angle is reckoned as positive, if the reversed secondary current vector leads the primary current vector. On very low power factors the phase angle error may be negative. Rarely, if ever, is necessary to determine the phase angle error of a current transformer used for relaying. One reason for this, is that the load on the secondary of a current transformer is generally of such highly lagging power factor, that the secondary current is practically in phase with the exciting current and hence the effect of the exciting current on the phase angle error is negligible. Most relaying applications can tolerate phase angle errors which for metering would be intolerable.

2.9 Rated Overcurrent Factor

Is the ratio of the maximum current that can pass the transformer without exceeding the designed electromagnetic forces to the rated primary current of the circuit. (the ratio of the rated short time current to the rated current).

2.10 Rated Saturation Factor

A current transformer is designed to maintain its ratio within specified limits up to a certain value of primary current, expressed as a multiple of its rated primary current. This multiple is termed its rated saturation factor.
2.11 **Current Transformer Vector Representation**

A current transformer vector presentation has been given in figure 2.1. From the vector diagram and performance discussion that followed, several equations can be derived. The most important derived equations being those for current and phase angle errors as follows [4]:

Relationships between \( I_p, I_{s2}, I_m, I_c, \) and \( \theta \)

\[
I_p \cos \beta = I_{s1} + I_m \sin \theta + I_c \cos \theta
\]

and

\[
I_p \sin \beta = I_m \cos \theta - I_c \sin \theta
\]

Note:

\[
\cos(90 - \theta) = \sin \theta
\]

\[
\sin(90 - \theta) = \cos \theta
\]

Squaring both equations, adding on both sides and simplifying we get:

\[
\frac{I_p}{I_s} = \frac{1 + I_m \sin \theta + I_c \cos \theta}{I_s}
\]

where the top part of the second term in the brackets represents 'r'

1. **Percentage error (with no compensation)**

\[
I_p = -100 \left( \frac{I_m \sin \theta + I_c \cos \theta}{I_p} \right) \%
\]

Where the negative sign shows that the actual secondary current will be less than the nominal current.

Similarly:

2. **Phase error**

\[
\text{Phase error } \beta = \arctan \frac{I_m \cos \theta - I_c \sin \theta}{I_{s1}}
\]

3. **Percentage error (unity power factor)**

\[
100 \left( \frac{I_c}{I_p} \right) \%
\]
2.12 Remanent Flux In Current Transformer Cores

Any iron core device will retain a flux level even after the excitation current falls to zero. This is determined by transformer secondary current and secondary burden. The remanence flux may either aid or detract from transient flux performance, depending on the relative directions of the residual flux and the required flux variation. In most cases, the alternating component of fault current or load current will generate small minor B-H loops such that the remanence is not destroyed [5].

The presence of remanent flux in a CT would cause core flux in normal steady state conditions to operate round a minor hysteresis loop displaced along the B axis of the B-H curve. This would reduce the accuracy of CT transformation. Remanence will also reduce the available flux swing in one direction this would make avoidance of saturation during fault condition more difficult. Remanence can be effectively eliminated by the introduction of small air gaps in a current transformer core.

2.13 Transformer Accuracy Under Normal Conditions

In steady state condition with linear magnetising inductance, it can be assumed that the degree of distortion of the secondary current waveform is always very much less than that of the excitation current [6]. For example, if a transformer has an exciting current which contains a 10% - 3rd harmonic, and the ampere turns provided by its fundamental component are 1% of those provided by the secondary current fundamental component, then the secondary current will contain a third harmonic component providing ampere turns of 10% of the fundamental component of the excitation current. The harmonic current necessary to do this will be only 10% of the secondary current fundamental component. The distortion produced by such small harmonics is virtually undetectable and thus for all practical purposes the secondary current waveform can be regarded as sinusoidal.
2.14 Choice of Secondary Current Rating

In BS 3938:1973 and BS 7626:1993 preferred values of rated secondary current of 1 A and 5 A are recommended. A secondary rating of 0.5 A might be used for metering purposes. In ANSI C57.13 standards, only 5 A is recommended as rated secondary output current.

The choice of whether a 1 A or 5 A secondary rating current transformer should be used depends largely on the value of secondary burden. A 1 A secondary current rated current transformer might be recommended for secondary burdens which are high. For example consider a 15 VA rated current transformer, in this case a burden of 15 Ohms might be accommodated sufficiently with a 1 A secondary rated current transformer. Whereas if a 5 A current transformer was used a high rated current transformer capacity would be required to accommodate the same secondary burden.

All current transformer accuracy considerations require knowledge of the current transformer burden, which is the load applied to the secondary of the current transformer. This should preferably be expressed with impedance of the load and its resistance and reactance components. In most cases the burden is expressed in terms of VA and a power factor. The VA being what would be consumed in the burden impedance at rated secondary current.
3 Current Transformer Transient State Theory

3.1 Introduction

A transient component by definition is that part of a current that diminishes to zero as time increases without limit. On the other hand a steady state component will continue to flow unchanged in value as long as voltage is applied to a circuit as presented in the previous chapter.

In current transformers transient period occurs immediately a fault has occurred in a power system. This is the most important stage in system protection because it will determine behaviour and response of protective relays. Mostly, transient periods are accompanied by dc transient currents which present problems in current transformer operation. This chapter gives the required theory for understanding the behaviour of current transformers in these transient states.

This chapter has been divided into three main sections. The first section outlines a general transient circuit theory, then some different current sources that might be subjected to current transformers are presented and lastly the current transformer transient theory is presented.

The summerised electric circuit transient theory has been included to understand more about the current transformer transient operation.

It will be seen that it is difficult to find the actual current transformer transient behaviour from mathematics because of the non linearity of iron core inductance. Therefore illustrations have been used in most cases to explain and clarify some concepts.
3.2 Electric Circuits Transient Theory

Transients can also be defined as an interval when energy is transferred from one form to another. It is a well-known phenomenon that energy cannot be instantaneously converted from one form to another. Theory states that for every action, there is a reaction in nature. When an action is performed in a short period, for example, stopping a bullet from a gun by a wall, a lot of heat and sound will be produced. Similarly, when an electric circuit is broken instantly a spark might be produced.

Extensive study and mathematical analysis of electric circuit transient theory have been presented in references [1, 2, 3, 4].

There are two main forms of current transients as follows:

- DC components of exponential form such as those produced at the start of fault conditions. Similar currents can be produced under load conditions by the switching of reactive circuits.

- High frequency oscillatory currents caused by switching operations and restriking conditions in a circuit breaker.

Of great concern are the DC transients as they may take a considerable longer time than the high frequency oscillatory currents.

Several mathematical presentations are available for understanding of electric transients and they include, differential equations, Laplace method, Z-transforms, and many other methods. In this thesis, a differential equation's method has largely been used.

The most important thing in transient studies is to express a circuit in a mathematical form depending on the method of analysis. In general, the solution of any circuit will have transient and steady-state parts.

Consider an electrical circuit represented by a first order linear differential equation as follows:

\[ A \frac{di}{dt} + Bi = f(t) \]

where: \( i \) and \( t \) are instantaneous current and time respectively
\( A \) and \( B \) are any constants according to a circuit.
\( f(t) \) indicates some function of variable \( t \)
Solving the above linear differential equation gives the following solution:

\[ i = Ke^{-\lambda t} + \frac{1}{A} e^{\lambda t} \int e^{-\lambda t} f(t) dt \tag{3.2} \]

where \( \lambda = -\frac{A}{B} \) and \( K \) is a constant.

It should be observed that the expression in equation 3.2 above is a sum of two terms as described below.

### 3.2.1 Complimentary Function

A complimentary function is the part in the differential equation solution which gives the transient component.

From Equation 3.2, a complimentary function can be defined as:

\[ Ke^{-\lambda t} \tag{3.3} \]

If \( \lambda \) is negative, the function will diminish to zero as time becomes infinite.

If \( \lambda \) is positive, the function will increase without limit. This condition is not practical as it implies infinite current and power which are impossible to obtain in practice because each and every circuit has some resistance. Therefore a transient complimentary function will always have a negative \( \lambda \) for it to diminish with time.

### 3.2.2 Particular Integral

As discussed earlier, a transient period will have two parts; the transient and steady state. A particular integral is the steady state function.

From equation 3.2, the general solution of first order differential equation. The particular integral is:

\[ \frac{1}{A} e^{\lambda t} \int e^{-\lambda t} f(t) dt \tag{3.4} \]

The type and order of a differential equation for a particular circuit is largely dependent on elements comprising the circuit. The number of transient types in a particular circuit depends on complexity of the circuit and its parameter connection which determine the differential equation's order.
References [2,3] showed that the amount of transient surges or train of oscillations that gradually die away in a circuit are determined by the amount of resistance in the circuit, relative to the inductance and capacitance that determine which form the transient current will take.

If the roots are real in equation 3.8, then the outputs are simple surges but if complex, the current transients are oscillatory. Oscillations are usually found in circuits where capacitance and inductance are present.

3.2.3 Circuit Response To Alternating Voltages

When alternating voltage is suddenly applied to a circuit, the steady flow of alternating current is preceded by a period of adjustment during which transient current flows. The form and amplitude of the steady state current will depend on the form and amplitude of applied voltage.

The amplitude of the transient component depends on circuit parameters, amplitude of applied voltage and instant of alternating voltage cycle at which voltage is applied to the circuit. However, the form of the transient component will depend only on the nature of the circuit as pointed out in section 3.2.2.

Generally, the transient response of a circuit does not depend on type of voltage driving it. Rather the composition of the circuit itself plays a big role in the transient response.

3.2.4 Effects Of Various Initial Times On Circuits.

When a circuit and its applied voltage are known, the steady state component of current is defined. However, the transient component cannot be determined until the instant of closing a switch is known.

The transient current at the instant of closing the switch has very nearly its maximum value in a circuit whose reactance is high compared to its resistance. There is an instant, however, at which the switch may be closed without producing any transient component of current at all.

If a circuit to which alternating voltage is applied consists purely of inductance, with resistance so small as to be neglected, the greatest transient component of current appears when the switch is closed at the instant of zero voltage.
A transient component magnitude may be of any magnitude, low resistance in the circuit compared to reactance allows a large transient component of current.

Closing a switch in capacitive circuit when voltage is near its maximum will produce an extremely high initial rush of current in the circuit.

Low resistance gives high current compared to steady state current whose magnitude is limited by impedance of the circuit. The steady state current leads the voltage by nearly \(90^\circ\). Because of low resistance in the circuit, the transient component dies out very rapidly.

If the resistance is reduced to zero, the amplitude of the current will be infinite with zero duration. However it is practically impossible, to have zero resistance and zero inductance.

A combination of inductance and capacitance in parallel acts as a short circuit at the first instant of impact of an alternating wave due to the presence of the capacitor. Finally it also acts as a short circuit due to presence of the inductor as time progresses. During the intermediate period a certain voltage will grow and then disappear across the inductor and capacitor.

A combination of inductance and capacitors in series acts as an open circuit at the first instant because current can not flow instantly through the inductor. Finally current flows continuously through the capacitor. At the first instant, the total voltage acts across the inductance, later it grows across the capacitor as time proceeds.

### 3.2.5 Non Sinusoidal Alternating Voltage

When the voltage applied to a circuit is alternating in a periodical manner but in a non sinusoidal form, the total current may be found readily by analysing the voltage into several sinusoidal components using Fourier series.

If the voltage is expressed as fourier series, and the current response to each term of the series is found separately, the total steady state current is merely the sum of the individual components. However, it is not necessary to determine transient component for each of the terms of the fourier series voltage. As pointed out earlier, the transient response is independent of applied voltages or currents.
3.2.6 Non Periodic Applied Voltage

When the applied voltage is neither a constant unidirectional voltage nor a periodically varying voltage, there is no steady state of either voltage or current. In fact, most of these non-periodic voltages or currents are transients. The task would be to find circuit response to transient input. This can be found by integrating the input function if its time function can be described.

3.2.7 Coupled Circuit With Widely Different Natural Frequencies

When two electromagnetically circuits have different natural frequencies, for example, a transformer (secondary and primary sides), the output waves for transients will be different.

This is an important point as it explains the different waveforms obtained between secondary output burden current and excitation current in current transformers. With a non-linear inductor, it would be presumed to have a wide range of natural frequencies as the inductor values keep changing.

Note that for two coupled circuits, the current in each circuit induces a voltage in the other circuit so that there is a relatively small component of the natural frequency of each circuit in the other. From this point of view, the current is unlike the response of a single circuit to which alternating voltage is applied because of an additional mutual inductance.

3.2.8 Circuits With Variable Parameters

Consider a situation where inductance is not constant but instead variable. When inductance is variable, its relationship to current is best expressed in terms of flux. It may be considered that current produces flux and rate of change of flux induce a voltage.

Symbolically:

\[ e = \frac{N\phi}{dt} \quad 3.9 \]

If iron is present in a magnetic field, and particularly if the iron becomes magnetically saturated, flux is not proportional to current anymore, but may be described as a function of current as follows:

\[ N\phi = f(t) \quad 3.10 \]
Inductance of a circuit in which flux linkages \( N \phi \) are produced by current \( i \) may be expressed as

\[
L = \frac{N \phi}{di}
\]

Therefore by substitution:

\[
e = L \frac{di}{dt} = \frac{N \phi}{di} \frac{di}{dt} = \frac{N \phi}{dt}
\]

**Voltage equation with Variable Inductance**

When inductance varies as a function of current as in equation 3.11, the analytical method of the foregoing discussion can not be applied. The voltage equation of a circuit with variable inductance is obviously not a differential equation with constant coefficients and no other kind of differential equation has yet been considered. Analytical solutions for current in such circuits are sometimes possible.

Consider an analytical expression proposed by Fröhlich's equation [3].

\[
N \phi = \frac{di}{b + i}
\]

The fundamental form of Fröhlich's equation is:

\[
B = \frac{aH}{b + H}
\]

but \( N \phi \) is proportional to \( B \) and \( i \) to \( H \)

Consider a simple RL circuit:

\[
Ri + L \frac{di}{dt} = E
\]

\[
\therefore Ri + N \frac{d\phi}{dt} \frac{di}{dt} = E
\]

By simple transposition

\[
\frac{di}{dt} = \frac{N d\phi}{E - Ri}
\]
The above equation can be solved by substituting Fröhlich’s equation. However, the mathematics involved would be rather complex. Graphical solutions that make direct use of magnetisation curves are preferred instead.

### 3.3 Other Sources Of Currents Subjected to Current transformers

The discussion in section 3.2 described transient currents due to different circuits with an assumption of direct and alternating currents. There are other sources of transient currents or some transient conditions which might be subjected to current transformers as well. This section explores some of these different transient sources and harmonic sources in a power system.

#### 3.3.1 Capacitor Inrush Currents

Capacitors might be installed in high transmission lines for power factor correction. These big capacitors are constantly switched on and off from power networks depending on system requirements. As discussed earlier, pure capacitances have an effect of producing extremely high inrush current and high output frequencies [1]. These currents might be subjected to current transformers in the network which might operate high speed protection unnecessarily. Therefore, it is highly recommended that proper care and calculations be taken into account before installing capacitors in a network.

#### 3.3.2 Transformer Inrush Magnetising Current

Another transient source which might affect current transformers is power transformer inrush current. This current might be experienced when switching a transformer into circuit. Below is an illustration of how this happens.

Under normal excitation, a transformer draws a magnetising current of between 0.5-2% of its rated current. Because of saturation effects in transformer iron, the excitation current is not sinusoidal. The amount of distortion depends on the flux density to which the core is worked.
When voltage is increased probably due to switching, more flux is demanded from the core, the peak current increases sharply and the core becomes saturated. Consider a transient condition that occurs when voltage is first applied to the transformer windings. Using figure 3.1, suppose at time of switching off a transformer, the remanent flux is $\pm \phi_R$ in the core. This may usually be less than $\phi_R$ because a transient current will flow in the winding after current ceases in the disconnecting device, as a consequence of the transformer discharging its capacitance.

Suppose remanent flux is $\phi_1$ and suppose again that next time transformer is energised, the polarity of the voltage is such that it calls for the flux to increase positively. If the applied voltage is just passing through zero, the resulting flux will have to be an increment equal to $\phi_m$ before the voltage returns to zero again. Since the flux started from an initial value of $\pm \phi_1$, it will have to reach $\phi_m$, before reversing. It is clear from the figure 3.1 that an enormous current ($i_1$) will be required. During the next half cycle, the flux will return to $\phi_1$, when the current, though negative will be less than normal peak. In a power transformer this current might be several times normal full load current. A current transformer located immediately before the power transformer might cause relay operation if the relay is not properly graded.
3.3.3 Lightning

Lightning is another concern for power system engineers as it affects power systems very much and most system outages in United States and Europe are attributed to it [5]. It would be voluminous to go into details about lightning phenomenon because it covers a wide area. For purposes of this thesis, only the wave and impact generated by it will be of interest.

From Greenwood’s point of view [1], apparently when lightning strikes a power line, a current is injected into the power system. The voltage of this current depends upon its waveshape and the impedance through which it flows.

Current strokes suggest that current can go up to or over 100 kA during a thunderstorm. It is noted that almost 50% of lightning strokes exceed 50 kA [6].

A lightning wave is a double edged surge as shown in a standard lightning waveform below [1]:

![A standard Lightning Wave.](image)

Figure 3.2
A standard Lightning Wave.

\[ T_1 = \text{Front time, typically } 1.2 \, \mu s \]
\[ T_2 = \text{Virtual time for half-value, typically } 50 \, \mu s \]

A lightning wave is characterised by a high amplitude, with very short rising time of about 2 micro-seconds or less and a falling time of more than 50 micro-seconds at half its peak value. Because of its fast rise, it is mostly difficult to prevent its impact on current transformers and other power system equipment. In current transformers, lightning has an effect of weakening winding insulation therefore shortening life span of current transformers [7,8].
3.3.4 Harmonics

Studies by various engineers have shown that there is growth in harmonics presence in power system networks recently. These harmonics, if not checked, can cause maloperation of protective relays and instrumentation and other effects. Therefore, inclusion of harmonic studies in power system analysis and design has become a necessity nowadays.

1 Harmonic Sources

Harmonics can be caused by several factors in a power system.

1. They can be generated by non-linear loads in a distribution system such as adjustable speed drives, electric furnaces and electronic loads whose effects might seem small but cumulatively become very significant. Such loads include television sets, computers, etc.

2. Power factor correction capacitors may resonate leading to significant voltage waveform distortion. This is not a real problem in distribution networks because the capacitor banks are connected in delta configuration (no earth connection).

3. Magnetisation inrush current; when a transformer is energised there is magnetisation inrush current into the transformer. This inrush current might be different for different phases. Inrush currents tend to be greatest when a phase is closed at voltage amplitude zero. As phases have different amplitudes at any given time, their inrush currents will be different as well even if a circuit breaker might be a three pole one. This will cause a resultant zero sequence current to flow into earth. This problem might be amplified by capacitance of line or cable to ground and might cause a time delay in decay of this current or, even worse, resonate the current.

4. Distribution line configuration, voltage regulators, transformer configurations and capacitance.

5. Single pole switching.

6. Ferroresonance; this might be caused by system subharmonics which are below fundamental frequency. Subharmonics might be generated by iron core saturation in transformers. Ferroresonance can also occur on other frequencies depending on system conditions.
2 Harmonics During Transient Periods

When a transformer is required to develop a high secondary emf under steady state conditions, the non-linearity of the excitation impedance causes some distortion of the output waveform. Such a wave form will contain along with the fundamental current odd harmonics. A power transformer acts as a harmonic generator even in normal operation as explained in reference [9].

3.4 DC Transient Phenomenon In Current Transformers

Consider an ideal current transformer connected in a power network as shown in figure 3.3. \( R_1 \) and \( L_1 \) are resistance and inductance respectively of the power network. Assume an ideal current transformer with ratio \( N_1/N_2 \). \( L_m \) is an infinite magnetising inductance and \( Z_2 \) is a secondary impedance connected to the current transformer. Assume the current transformer has no winding impedance.

Let driving voltage for the power system be:

\[
v_1 = V_1 \sin(\omega t + \theta) \tag{3.19}
\]

where: \( V_1 \) is rms value and \( \theta \) is the angle of sine wave at \( t = 0 \).

The transient current for the network will therefore be:

\[
i_t = \frac{V_1}{Z} \sin(\omega t + \theta - \varphi) - \frac{V_1}{Z} \sin(\theta - \varphi)e^{-\frac{R}{L_1}t} \tag{3.20}
\]

Figure 3.3
Ideal Current Transformer Model
Where: 
\[ Z = \sqrt{R_1^2 + \omega^2 L_1} \]

and 
\[ \varphi = \tan^{-1} \frac{\alpha L_1}{R_1} \]

The first term in equation 3.20 represents steady state alternating current and the second term a dc transient component. Below is a diagram to illustrate the primary current.

![Diagram of primary DC exponential current transient and total flux](image)

**Figure 3.4**
Illustration Of A Simple Transient Current And Total Flux

Current output from ideal current transformer is:

\[ i_2 = i_1 \frac{N_1}{N_2} \]  
\[ \therefore i_2 = \frac{V_1}{Z} \frac{N_1}{N_2} \left( \sin(\omega t + \theta - \varphi) - \sin(\theta - \varphi) e^{-R_1 t/L_1} \right) \]  
\[ \therefore i_2 = i_1 \frac{N_1}{N_2} \left( \sin(\omega t + \theta - \varphi) - \sin(\theta - \varphi) e^{-R_1 t/L_1} \right) \]  
\[ \therefore i_2 = i_2 \left( \sin(\omega t + \theta - \varphi) - \sin(\theta - \varphi) e^{-R_1 t/L_1} \right) \]
Magnetising flux requirement for the ideal current transformer under transient condition will be calculated as follows.

We know that
\[
e = N \frac{d\phi}{dt}
\]
\[\therefore \phi = \int \frac{e}{N} dt \tag{3.25}\]

Where \(\phi\) is flux and \(e\) is instantaneous voltage.

If \(Z_2\) figure 3.3 is assumed to be pure resistive and denote it \(R_2\). Also assume the current transformer to be of bar type such that \(N_1 = 1\). The instantaneous emf in the secondary side of current transformer will be:

\[
e = i_2 R_2 = i_{2a} R_2 + i_{2b} R_2 \tag{3.26}\]

\[\therefore \phi = \int \frac{i_{2a} R_2}{N_2} dt + \int \frac{i_{2b} R_2}{N_2} dt \tag{3.27}\]

\[
\phi = I_2 \frac{R_2}{N_2} \left\{ \sin(\omega t + \theta - \varphi) - \sin(\theta - \varphi) e^{-R_2/\omega} \right\} \tag{3.28}\]

Solving equation 3.28 gives the following result

\[
\phi = -I_2 \frac{R_2}{N_2 \omega} \left( \cos(\omega t + \theta - \varphi + \gamma) \right) + I_2 \frac{P_2 L_2}{N_2 R_1} \sin(\theta - \varphi) e^{-R_2/\omega} + k \tag{3.29}\]

The flux equation can be shown diagramatically as below:

![Figure 3.5](current_transformer_core_flux_during_transient_period.png)

**Figure 3.5**
Current Transformer Core Flux During Transient Period
In practice the dc component of \( I_2 \) becomes less than that of \( I_1 \) by the amount of the dc component exciting current (\( I_m \)). Thus the dc component of \( E_2 \) and the unidirectional flux required to produce it are reduced.

For a current transformer with insufficient core section, the flux reaches saturation region during one of the positive excursions of primary currents. The saturated exciting inductance shunts most of the primary current \( I_1 \), thus distorting the secondary current \( I_2 \) to the wave shape shown in figure 3.3. This will be explored further in the next chapter.

![Figure 3.6 Current Transformer Transient Flux Response To Saturation](image)

During the negative excursions of current, the flux is required to reduce. The core becomes unsaturated during this negative current and for part of the following positive current excursion before becoming saturated again. As the dc component of the primary current decays, the negative excursions of the current and flux become greater. The core eventually runs out of saturation during the complete cycle, when the secondary current becomes normal again. The core material of current transformers is non-linear, therefore with a sinusoidal applied voltage, the magnetising current will have a distorted waveform having third and fifth harmonic components. These will become significant around the knee point and increase extremely rapidly for voltages above this point [10].
3.4.1 Basic Forms Of Transient Magnetising Current

The transient magnetising current is in the form of the difference of two exponential terms, having the same initial conditions but different time constants. One has a time constant equal to that of the secondary circuit resistance and shunt capacitance while the other is that for primary circuit. In most cases the secondary time constant is bigger than that of the primary circuit. From this point, the transient condition in the current transformer core may persist after the dc primary transient has disappeared. The rise of transient exciting current is largely dependent on the primary time constant whereas it decays largely according to the secondary time constant.

3.4.2 Effect Of The Non Linearity Of The Core Excitation Characteristics And Losses

Practically the core has a non-linear characteristic as pointed out earlier, therefore small harmonics will be introduced in steady state secondary current. However the magnitude of the sinusoidal term in the secondary current is not appreciably affected unless saturation occurs.

The exponential component of the secondary current which is dependent on the primary circuit parameters is not appreciably affected. The component affected is $e^{-R_t/2L_m}$ influenced by secondary winding parameters.

The effect of core losses is that it tends to reduce the flux swing.

The current transformer is saturated uni-directionally while being simultaneously subjected to alternating current quantity. As discussed above, the output will contain harmonics. Even harmonics are usually neglected because they have very small values.

3.4.3 Core Flux Variation In A Current Transformer

In figure 3.3, several assumptions were made one of which was that $L_m$ was an infinite linear inductance. In the following sections equations developed by Wright will be given. In these equations $L_m$ will be taken into account though the other assumptions made in figure 3.3 remain unchanged.

1 Purely Resistive Secondary Circuit

The leakage reactance of a toroidally wound transformer is negligible. Therefore, the secondary circuit of such a transformer can be regarded as purely resistive if a resistive burden is connected to it.
Using figure 3.3, assume $Z_2$ is a pure resistive and denoted $R_2$.

Wright [6] derived, the flux ($\phi$) to be:

$$\frac{I_1P_2}{\omega N_2} \left[ \frac{\sin(\theta - \varphi)}{R_1 - R_2} e^{-R_1/I_1} + \frac{1}{\omega L_m} \cos(\omega t + \theta - \varphi + \gamma) \right] \left[ 1 + \left( \frac{R_2}{\omega L_m} \right)^2 \right]^{\frac{1}{2}} + \frac{\omega L_m}{R_2} k_2 e^{-\alpha t/\rho} + k_3$$

The above equation assumes $I_1 = 0$, $i_m = 0$ and $I_2 = 0$ at instant of switching.

$$k_3 = 0$$

$k_3 = 0$ because there are no constant components in either primary and secondary windings current.

2 Inductive Secondary Circuit

Using figure 3.3, assume $Z_2$ to be pure inductance and denoted $L_2$. Assume $R_1 >> \omega L_s$ and $L_m >> L_2$. The secondary current and core flux will be:

$$i_2 = I_2 \left( \sin(\omega t + \theta - \varphi) - \sin(\theta - \varphi) e^{-R_1/I_1} \right)$$

$$\phi = \left\{ \frac{I_2 L_1}{N_1} \right\} \left( \sin(\omega t + \theta - \varphi) - e^{-R_1/I_1} \sin(\theta - \varphi) \right)$$

The flux expression shows that it has almost the same waveform as the primary current.

Generally, the peak fluxes reached after switching a circuit always tend to be lower with an inductive secondary circuit than would be with a resistive burden having the same ohmic impedance at the power supply frequency [11].
Wright showed that core saturation would not occur in a transformer with a purely inductive circuit if its saturation level was greater than the sum of the possible residual flux, and twice the peak flux value during steady state fault conditions. That is \( \phi_{sat} \geq \phi_R + 2\phi_{psa} \). It is much more difficult therefore to go into saturation with an inductive burden.

The effect of reducing power factor on inductive circuit is the reduced maximum flux which may be reached for a given primary current and secondary circuit impedance level.

3 Capacitive Secondary Circuit

Assuming magnetising inductance \((L_m)\) value to be infinite and again assume primary current to be a standard fault current (with dc component), the flux can be simplified to be:

\[
\phi = I_p \left[ \frac{1}{\omega C_2} \left( \cos(\omega t + \theta - \phi) - \left( \frac{\omega^2 L_1^2}{R_1^2} \right) e^{-\left( \frac{R_1}{l_1} \right) t} \sin(\theta - \phi) \right) \right] + k_4 + k_6 \] 3.34

The flux variation is made up of four components:

- Steady state sinusoidal.

- The second being flux required to drive the exponentially varying current component. This component, because it results from double integration of the secondary winding current, is proportional to \( \omega^2 L_1^2 / R_1^2 \). It might be of a very large magnitude if the primary circuit into which the transformer is connected is highly inductive.

- The third component varies linearly with time and is proportional to a constant \( k_{16} \). This constant is dependent on the voltage across capacitor \( C_2 \) at the instant of switch closure. If switching occurs when \( \alpha = \gamma_1 \), which is the condition that gives maximum asymmetry and if at this time there is zero voltage across the capacitor then:

\[
k_4 = -\frac{\omega L_1}{R_1 C_2} \] 3.35

This third term is very significant in determination of the output current.

- The fourth term is constant flux which is dependent of residual flux at the instant of switching.
It was shown in reference [12] that flux level can reach a peak of sixteen times in a period corresponding to two to three cycles of the power system frequency.

It is possible in capacitive burden to have large exponential components of long time constants present, during transient conditions, in the core flux wave of a transformer. These make the on set of saturation almost inevitable.

When saturation is reached in capacitive burden circuit, the capacitor discharges through the transformer secondary winding, the discharge current being of infinite magnitude and zero duration in the case of ideal transformer. If the capacitor voltage before saturation had the same polarity as the secondary current, then the discharge peak would be of opposite polarity, and after its passage the secondary current would remain zero until the primary current zero, when desaturation would occur.

In practice, the discharge current peaks have finite magnitude, and they have short but significant duration, the actual values being very dependent on the excitation characteristics of the core material.

Generally it should be noted that as frequency becomes higher, the flux required to induce a given voltage is inversely proportional to the frequency.

For most cases, the maximum value of transient flux density is dependent only on the resistive component of the secondary burden.

3.5 Conclusions

It has been established in this chapter that the transient presence in a circuit will depend on its descriptive equation order. More important is the fact that it is rather difficult to solve equations with variable parameters as discussed in Section 3.2.8.

Transient currents, such as inrush currents into capacitor banks or transformers, are frequently high in magnitude and often contain a significant dc component. These might have an effect on current transformer next to them.

When a current transformer has some dc bias current, it will go into saturation when the current (sinusoidal) is increasing in the same direction as the bias current. This will cause the excitation current to be non uniform.

The most important conclusion in this chapter is that transient currents in current transformers make flux in the core increase very much. This has an effect of saturating the core and produces distorted output current as will be seen in chapter five.
CHAPTER FOUR

4 Current Transformer Modelling

4.1 Introduction

The concept of a model representation of any electrical, electronic, mechanical, chemical or biological system is of significant importance since it simplifies to a certain extent the analysis of such a system. The behaviour of the system under various conditions can then be predicted with the aid of the model. The complexity of the model is largely determined by the linearity of the system. In most cases it is required to make certain assumptions as to avoid major complications due to the non-linear characteristics of a system. This is not an exception with current transformers as will be shown later.

Over the years engineers and scientists have been trying to develop a universal model of a current transformer that would represent its performance satisfactorily. However, it is rather difficult to come up with a model that will cover every detail because of presence of iron core and the broadband of frequencies a current transformer might be subjected to.

This chapter presents different current transformer models studied and some of them used in EMTP simulations of this thesis. Current transformer models can be categories into two main groups: low frequency and high frequency models. The low frequency models can be used to simulate current transformers with linear magnetising inductance characteristics, while high frequency models can be used for broadband simulations which take into account winding capacitances as well.
4.2 Principles Of Current Transformer Modelling

A model is a circuit representation in the form of inductance, resistance and capacitance that responds very much like the component(s) it represents when provoked by the stimulus of interest.

When modelling a current transformer, the most important thing to consider is the non-linearity of magnetising inductance due to iron core. The non-linearity of current transformer magnetising curve has the following consequences:

- Under steady-state conditions, the non-linearity of transformer iron core introduces harmonics into the current and voltage waveforms.

- Under transient conditions it can lead to very high transient currents in the core resulting in high current output error.

The first step to modelling is to find values of parameters affecting the system to be modelled. In current transformers, values of leakage inductance, magnetising inductance, winding resistance, inter turn and stray capacitance have to be found. Several articles have discussed these at length and will not be repeated here [1-5]. In transients modelling, inductance and capacitance of natural frequencies at which the model oscillates when the system is disturbed have to be calculated as well.

The detail of modelling depends on the intended use of a model. For example, a current transformer can be modelled as a simple current divider when operating in steady state with no losses. On the other hand, a study of transient stresses on current transformer windings requires a more detailed model.

In high voltage electrical networks current transformers with a bar-primary winding are normally used to ensure that the voltage drop across the primary winding will always be negligible compared to the network voltage it is connected to. The inclusion of the current transformer in the circuit will thus not affect the flow of current in the circuit. Consequently the primary impedance is omitted for the purposes of this thesis.
The performance of a protection scheme very much depends on the dynamic behaviour of the current transformer it is connected to. To evaluate the transient behaviour of a protective scheme it is necessary to present an accurate model of the current transformer. The simulation of the magnetizing current is the preferred method in developing a model of the current transformer since it is the non-linear characteristics of the magnetizing current that presents the difficulties in modelling the current transformer. When a fully offset fault current flows in the primary of a current transformer, the dc offset generally causes a rise in flux in the core several times greater than that required to transform the 50 Hz component of the current. This may lead to core saturation and the current transformer may not accurately transform the fault-current to the secondary side.

Current transformers are operated in power system frequency and therefore in low frequency range of 50 Hz or 60 Hz. However because of other factors like surges and short circuits, a high frequency model is also very important.

Current transformer models can be divided into two broad categories:

- Low frequency models
- High frequency models

The voltage drop across a primary winding of a current transformer should always be negligible relative to the driving voltage of the circuit into which it is connected, such that its inclusion does not affect the current flowing in the circuit. In consequence, the voltage drop across the series impedance of the primary winding is seldom, if ever of interest and for this reason the basic equivalent circuit was usually simplified.

4.3 Modelling Current Transformer Losses

To model a current transformer under steady or transient states, factors that contribute to current transformer transformation errors have to be modelled and represented correctly. Below are parameters which contribute to current transformer errors.

4.3.1 Winding Resistance

Winding resistance is one source of current output error in current transformers. In current transformers, the resistance which is more pronounced is the secondary winding resistance because the winding contains big length of conductor. Resistance values are independent of low frequencies. However skin effect might become influential in high frequencies.
4.3.2 Flux Leakage

The concept is to represent flux by so-called secondary reactance, which is considered to add an effect to the burden. In general, leakage flux should be avoided in protective circuit. However, it can be beneficial in current transformers with certain forms of operation for the following reasons [6]:

- In a current transformer supplying a resistive burden, additional core exciting current due to leakage flux leads the exciting current due to mutual flux by $90^\circ$. Thus the total exciting current is brought more nearly into phase with primary current. This reduces the phase error. Reduction in secondary current magnitude, due to leakage flux can be compensated by an increase in turns correction (reducing number of secondary turns).

- Secondly an increase in leakage flux is accompanied by a decrease in mutual air flux. This leads to transformation errors. Thus in current transformers with high leakage flux the secondary current is more effectively limited at high system fault currents.

4.3.3 Core Loss Branch

The core loss is represented by a non-linear resistor as it has eddy current loss and hysteresis loss that is non-linear. However the losses of the core in a current transformer are minimal as their sizes are made small and the current transformers are designed to work with very small powers. It is therefore convenient to represent the core loss branch by a very high linear resistor [7].

As the frequency becomes higher, the flux required to induce a given voltage is inversely proportional to the frequency.

In this thesis direct magnetising curves were used in EMTP by converting current transformer V-I characteristics supplied by manufacturers.
4.3.4 Transformer Capacitance

Capacitance will be formed between secondary and primary windings and between the secondary layers themselves. Like any other capacitors, their values depend upon the plates, their separation and permittivity of material separating them. Therefore one would expect a current transformer with higher power rating to have higher capacitance than those with lower ratings.

It might also be supposed that capacitance of a high voltage transformer would be more than a low voltage transformer of comparable power rating since high voltage calls for more insulation.

The bottom line is that capacitance in current transformer will depend on geometric construction of the current transformer and type of materials used in core or insulation.

It can be concluded from reference [8] that transformer natural frequencies increased with reduction in transformer voltage level and size. This agrees with earlier statement that higher power rating current transformers will have higher capacitance. In this reference, power transformers were considered and their natural frequencies ranged from 5 kHz to 100 kHz. In a current transformer a wider range of frequency would be expected than in power transformer because of the non-linearity of magnetising inductance which is more severe in current transformers.

4.4 Measuring Characteristics Of Transformers

A very important task in current transformer modelling is to find exact values of parameters in a current transformer model. Conventional open circuit and short circuit methods are used to measure current transformer characteristics. In current transformer open circuit measurement is done with primary winding open. Voltage is varied on the secondary winding and current output monitored. Impedance measurement in open circuit measurement would be secondary winding impedance in series with core impedance which consists of $Z_m$ in parallel with capacitive reactance. Results by Douglas [9] revealed that the core impedance obtained from open circuit measurement are much larger than secondary impedance. Thus the assumption that the open circuit input impedance results are a parallel combination of magnetising impedance and capacitive reactance.

Short circuit measurements can be done on both windings of the current transformer. In this test it is assumed that magnetising impedance is much bigger than either winding primary and secondary impedance.
Winding resistance can easily be measured by using direct current measurement method. Once resistance is known, it would be easy to calculate leakage inductance from the short circuit results.

To measure natural frequency of a transformer, one way would be to pass a small direct current through the primary winding of the transformer and then interrupt the current suddenly, the current should be measured on the secondary winding. The sudden suppression of the current releases magnetic energy stored in its inductance. The energy oscillates back and forth at natural frequency of the transformer. This is manifested in the transient voltage that appear at the terminals [10].

Neglecting damping, the voltage is given by

\[ V = I_cZ_o = I_c \left( \frac{L}{C} \right)^{\frac{1}{2}} \]  

where:  
- \( I_c \) is the current chopped.  
- \( Z_o \) is combined impedance of current transformer

The natural frequency is:

\[ f_o = \frac{1}{2\pi(LC)^{\frac{1}{2}}} \]  

Therefore:

\[ L = \frac{1}{2\pi f_o I_c}, \quad C = \frac{I_c}{2\pi f_o V} \]

\( I_c \) is measured before opening the switch. \( V \) is obtained by projecting back the envelope of the oscillation to the zero time axis.
4.5 Current Transformer Models

4.5.1 Low Frequency Models

Low frequency models were used to simulate current transformer without taking into account capacitance. These models are very useful because current transformers are operated in power system frequency which is either 50 Hz or 60 Hz.

MODEL I

A current transformer primary winding is usually a single bar therefore it has no significant impedance. It is convenient therefore to transfer all parameters into the secondary side. Similarly, all parameters would be referred to the primary side easily using turns ratio. Figure 4.1 shows a current transformer model with parameters referred to either primary or secondary side of current transformer.

![Current Transformer Model](image)

Figure 4.1
Current Transformer Model With Parameters Referred To One Side [6]

<table>
<thead>
<tr>
<th>Parameters referred to the primary</th>
<th>Parameters referred to the secondary</th>
</tr>
</thead>
<tbody>
<tr>
<td>a = R_2 n^2</td>
<td>R_2</td>
</tr>
<tr>
<td>b = L_2 n^2</td>
<td>L_2</td>
</tr>
<tr>
<td>c = v_1 n</td>
<td>v_2</td>
</tr>
<tr>
<td>d = i_m</td>
<td>-i_m n</td>
</tr>
<tr>
<td>e = i_e</td>
<td>-i_p n</td>
</tr>
<tr>
<td>f = i_p</td>
<td>-i_l n</td>
</tr>
<tr>
<td>g = i_l</td>
<td>v_1/n</td>
</tr>
<tr>
<td>h = v_1</td>
<td></td>
</tr>
</tbody>
</table>

Note: $n = \frac{N_1}{N_2}$
MODEL II

Model I was modified into figure 4.2. In this model winding inductance and resistance were ignored. Only magnetising impedance was included. This model was used to model conditions where different types of burden parameters $Z_b$ were considered. That is pure resistive, inductive or capacitive. Combinations like resistance and capacitor, resistor and inductor, capacitor and inductor, etc.

![Figure 4.2](current_transformer_model.png)

**Figure 4.2**
Current transformer Model
With No Secondary Winding impedance [6]

Where:

- $R_p$ = iron core resistance losses
- $L_m$ = non-linear magnetising inductance
- $i_{1n}$ = primary current referred to secondary winding
- $i_e$ = excitation current
- $e_2$ = induced emf in secondary winding
- $Z_b$ = current transformer burden
- $I_2$ = secondary current
Chaudhary [11] developed a current transformer model for EMTP simulations which includes a non-linear hysteretic inductance (type 96) representing iron core magnetising inductance. In this representation, the secondary winding resistance and the secondary winding leakage inductance are included with the lead impedance and the burden impedance. The non-linear characteristics of the current transformer are represented by type 96 pseudo-non-linear hysteretic inductor.

Referring to figure 4.3, note that $Z_m$ is placed after $Z_{emtp}$. This representation is required because finite values must be given for $Z_{pr}$ and $Z_{emtp}$. More precisely, $L_{emtp}$ must be non-zero, and $R_{emtp}$ can be zero, and either $R_{pr}$ or $L_{pr}$ can be zero, but not both. More detailed limitation of impedance values in EMTP can be found in reference [9]. This data requirement of EMTP transformer model was satisfied by separating $Z_{sec}$ into two parts $Z_{emtp}$ and $Z_{sec}$. $Z_{emtp}$ should be small, however, very small values of $Z_{emtp}$ give rise to very large values of the admittance $(1/Z_{emtp})$ in the matrices $[Y]$ and $[G]$, which are steady state complex matrix and the real coefficient matrix within the time step loop respectively. If $Z_{emtp}$ is very small an error message in the steady state solution in EMTP is given.

**Figure 4.3**

Current Transformer Model
With Type 96 Nonlinear Element [11]

In the figure:
- $Z_{pr}$ is transformer primary resistance and leakage inductance
- $Z_{sec}$ is transformer secondary resistance and leakage inductance
- $Z_m$ is transformer magnetising impedance
- sw is switch which closes at time = 0
MODEL IV

Another equivalent model to Model III was by use of a type 98 non-linear inductor model available in EMTP. In this case, the magnetising branch was purposely made dormant (that is no saturation is modelled) by choosing artificially small (e.g. 1.0e-6) values for the secondary resistance and leakage inductance. The type 98 non-linear inductor model was then connected at the secondary as shown in figure 4.4.

![Figure 4.4](image)

**Figure 4.4**
Current Transformer Model
With Type 98 Nonlinear Element [12]

The saturation characteristics are referred to secondary side of a current transformer since the V-I characteristics are available for the secondary side of a current transformer.
4.5.2 Dual Models

In Model II figure 4.2, \( L_m \) was assumed non-linear. In the following discussion using the same Model II assume \( L_m \) linear. The following equations developed by Wright [13] are valid for this type of model.

\[
-\frac{N_2}{N_1} i_1 = i_2 - i_p - i_m \quad (4.4)
\]

\[
L_m s_i_m(s) = R_j p(s) \quad (4.5)
\]

\[
Z_j(s) j_2(s) = R_j p(s) \quad (4.6)
\]

\[
e_j(s) = Z_j(s) j_2(s) \quad (4.7)
\]

Note: \( s \) in the above equations is a Laplace operator.

A dual circuit for the equations above can be represented as follows:

![Diagram of Dual Circuit]

Figure 4.5
Current Transformer Dual Model Circuit [13].

From figure 4.5 which is a dual circuit, the following relationships apply:

\[
-\frac{N_2}{N_1} V_1 = V_2 - V_p - V_m \quad (4.8)
\]

\[
C_m s V_m(s) = G_p V_p(s) \quad (4.9)
\]

\[
Y_j(s) v_j(s) = G_p V_p(s) \quad (4.10)
\]
The correspondence of these two sets of equations show that the circuits in Figure 4.2 and 4.5 are dual of one another, an exact equivalence would be obtained at any instant if:

\[ C_m = k_1 L_m \]
\[ G_p = k_1 R_p \]
\[ Y_2 = k_1 Z_2 \]

where \( k_1 \) is a constant

For these conditions, if the voltage applied to the dual circuit in figure 4.5 is given by:

\[ \frac{N_2}{N_1} V' = k_2 \frac{N_1}{N'_{e}} i_1 \]

then:

\[ V_2 = k_3 I_2 \]
\[ V_p = k_3 I_p \]
\[ V_m = k_3 I_m \]
\[ i = k_1 k_2 e_2 \]

The dual circuit components \( C_m \) and \( Y_2 \) must be controlled to match automatically any changes which occur in transformer exciting impedance and load.
Figure 4.6 is divided into two stages. Stage 0 to C represent unsaturated region with inductance $L_{mu}$ and stage C to D representing saturated region with inductance $L_{ms}$.

To advance more, consider a two stage B-H curve in figure 4.6 above. This curve required $C_m$ to change between two stages accordingly. This was achieved by redrawing figure 4.5 to that shown in figure 4.7.

Figure 4.7 has two capacitors in parallel. When point C is reached on the B-H curve, contactor 'r' opens. This opening increases voltage across $V_m$ and effectively reducing the circuit current 'i'. This point is assumed to be the saturation point of the current transformer.
Another way to achieve perfect chopping which occurs when saturation occurs, was by use of a circuit with diodes as shown in figure 4.8.

Contacts 1 and 2 open when the core flux reaches the positive and negative knee point levels, respectively. When contact 1 opens, there is a voltage across the capacitor corresponding to the knee point magnetising current. At this instant there is zero voltage across the diode and contact, but, as saturation deepens, the extra magnetising current is represented by the build-up of reverse voltage across the diode. When the saturation decreases, the voltage across the diode falls and when it is zero, the voltage reverses and the diode conducts making re-closing of the contact at exactly the correct instant unnecessary. Contact 2 and diode 2 are needed to cater for conditions of opposite core flux polarity.

A close look at output waves from a resistive burden circuit show that the secondary current is a differentiated wave of flux. Refer to Figure 5.2.

Note that values used in a dual circuit are those of conductance which are inverse of resistance. Therefore the dual circuit discussed are applicable to current transformer with pure resistive burdens only.

A simpler dual circuit model developed for this thesis is shown in figure 4.9. This model was used to write a MODELS program. Details of the program will be given in chapter seven. In figure 4.9 diodes 1 and 2 operate the same way as described for figure 4.8. The operation of the diodes is assumed to be similar to flux chopping in an iron core. The output of a pure resistive burden is a differentiated wave of the flux. A differentiator is placed in series with the diodes for this reason. Resistor R is used for scaling purposes only. The dual models were successfully used by describing them using MODELS language. A MODEL program describing Figure 4.9 has been given in Appendix A, File 5.
4.5.3 High Frequency Models

A power system may be subjected to sudden impact of high frequency waves (which may consist, in effect, of a single half cycle unidirectional impulse) arising from switching in operations, atmospheric lightening discharges, arcing grounds, short circuits and from almost any change in the electromagnetic and electrostatic conditions of the circuit involved. These disturbances might have an effect on current transformers as well. While transformer windings are generally considered merely as large inductances, they also contain capacitance distributed throughout the winding in different ways, depending upon the type of coils and the arrangement of the windings.

At normal operating frequencies, the effect of electrostatic capacitance between turns and layers of individual coils is negligible and as a result the windings act as simple concentrated inductances giving uniform distributed voltages (parameters). When the windings are subjected to the sudden impact of high voltage and high frequency or steep fronted waves, the effect of electrostatic capacitance in determining the initial voltage distribution becomes important due to the fact that capacitances which are unimportant at low frequencies may have very low impedances or even become virtually short circuits when subjected to high frequency waves or to steep fronted impulses.

Moreover at high frequencies, conditions of resonance may be reached for the various combinations of inductances and capacitances.
Stray capacitances are also eminent in high frequency responses and their inclusion is therefore imperative for a high frequency model. Stray capacitances in a transformer are physically distributed and very complicated to model in detail. However Chimklai [8] developed a model, figure 4.10, that includes:

1. Capacitances from winding to ground.
2. Capacitances between windings.
3. Turn to turn capacitance.

\[ \begin{align*}
C_1 &= \text{capacitor between winding to ground in primary winding.} \\
C_{12} &= \text{capacitor between secondary and primary windings.} \\
C_2 &= \text{capacitor between turns and winding to ground in secondary winding of current transformer.} \\
Z_1 &= \text{Primary winding impedance.} \\
Z_2 &= \text{secondary winding impedance.}
\end{align*} \]

In a current transformer \( Z_1 \) is negligible, and stray capacitance \( C_1 \) can also be neglected. The current transformer high frequency circuit diagram therefore becomes the one below:
CHAPTER FIVE

5 Current Transformer Saturation

5.1 Introduction

One of the problems facing protection engineers is current transformer saturation. Current transformer saturation causes maloperation of relays. This chapter looks at problems associated with saturation. Effects of different burdens on current transformer output secondary currents. Both steady state and transient state transient performances have been studied. In transient periods, the dc component plays a big role as it determines how quickly a current transformer goes into saturation.

When the iron core saturates, its inductance becomes zero and the total primary current becomes expended on exciting the core, the secondary output current disappearing. This condition will last until the primary current has reduced to the value corresponding to the saturation point. From this point onwards, the core comes out of saturation and the core flux decays in a transient manner largely decided by the secondary time constant.

5.2 Effect Of Current Transformer Saturation

If a protective relay is given a high over current setting, the current transformer may saturate below the setting in which case the relay will not operate. The limit of the useful range is dependent upon the secondary knee point voltage and may be reached at high current when the burden is low or alternatively at low currents if the burden is high.

During saturation there is no flux change and therefore no voltage induced. The peak value of the resulting open circuit secondary voltage spikes are given by \(-\frac{N\phi}{dt}\) where \(N\) is turns ratio, \(\phi\) is flux change, \(dt\) is proportional to inverse of current \(i\). Hence induced emf is proportional to \(Ni,\phi\). Thus the higher the turns number \(N\) and the greater the primary current \(i\) for a given current transformer, the higher the peak value of the spike voltage and the shorter its duration will be.
Where:

\[ C_{12}^{'} = C_u \frac{N_1}{N_2} \quad 4.13 \]

\[ C_2^{'} = C_2 + C_{12}(1 - \frac{N_1}{N_2}) \quad 4.14 \]

\( C_2 \) includes winding to ground and turn to turn capacitances.

Figure 4.11
Current transformer High Frequency Model [14].
During saturation, the shunt impedance $Z_m$ rapidly reduces leading to an increase in current error. The non linearity of iron core results in magnetising current having harmonic content even when the current transformer is operating in the unsaturated region.

The quality of current transformer is most critical for different schemes, where the performance of all the transformers must match. In such cases accuracy of reproduction is required not only at load currents, but at all fault currents as well. Dissimilar saturation in any differential scheme will produce operating current.

5.3 Consequences Of Current Transformer Saturation

Current transformer saturation can cause the current transformer fail to deliver true reproduction of the primary current and this might cause undesirable operations as follows [1]:

False Trip:

This is common for current differential relays when one current transformer goes into saturation while others do not as this causes current imbalance. Partial or full saturation of one current transformer will allow the other current transformers to deliver the necessary operating current to the differential relay for a through fault condition and thus causing a false trip.

Delayed Trip:

When there is partial or a distorted current reproduction of the primary current to a protective device which has inverse time current characteristics, additional delay in tripping of the power circuit will result. This might cause relay co-ordination problem.

An overcurrent relay may be unduly delayed in seeing a fault for the same reason. The time delay is directly proportional to system ratio $(X/R)$.

Failure To Trip:

The most undesirable condition that may result due to current transformer saturation is failure of a protective device to trip when the secondary current is either very low or extremely distorted. This is possible when extremely high fault current is expected to trip an instantaneous relay. Consequently this might lead to back up relays to operate after extended time delay resulting in isolating entire bus or greater part of the system network.

A distance relay may under reach or not see a fault in its zone of protection due to current transformer saturation and loss of secondary current.
Failure To Block Tripping Of Overdutied Devices:

Overdutied fault interrupting equipment (starter, load switch or circuit switcher) is sometimes applied where the tripping is blocked by high speed fault sensing relays if the fault current exceeds the equipment interrupting capability. Saturation of a current transformer could prevent proper blocking by the fault sensing relay and permit interruption attempts of fault current in excess of the equipment rating.

Desirable:

Prevent Damage To Current Circuit Devices:

It is a common practice to use the same current transformers for both protective and metering functions. When the protective device is used purely for overloads, a saturated current transformer will have no detrimental effect. On the contrary, it may serve to limit damaging high current to these devices.

Current transformer saturation for high current will cause protection relay time current characteristic to be slower than published values or to fail to operate as discussed in this section In addition the saturated current transformer output is rich in odd harmonics which can be detrimental to the mechanical and electrical operation of connected relays.

5.4 DC Saturation

Dc transients result in current transformers when:

- The current in an inductance can not change instantaneously.
- The steady state current, before and after a change must lag (or lead) the voltage by the proper power factor angle.
- There is remanence in the current transformer iron core.

If \[ V_x \geq 2 I_2 R_2 T_1 \]  \hspace{1cm} 5.1

the dc component of a fault current will not produce current transformer saturation [2]. In this expression:

- \( V_x \) = voltage at the knee of the saturation curve.
- \( I_2 \) = symmetrical secondary current (amp rms.)
- \( R_2 \) = total secondary resistance
- \( T_1 \) = dc time constant of the primary circuit in cycles \( T_1 = (L_1 / R_1) \)
L₁ and R₁ are primary circuit inductance and resistance respectively.

When saturation has occurred, magnetising inductance Lₘ decreases to almost zero. When this occurs, the ability of the mutual inductance to retard the flow of direct current through it collapses so that practically all the dc component goes through Lₘ. This condition will continue until the dc component has subsided, after which the transformer can again perform with its normal alternating current characteristics. During the saturation time, the inductance also offers much less impedance to the flow of alternating current, consequently, it is to be expected that the percentage of the alternating current component which is transmitted to the secondary will be materially less.

5.5 **Time To Saturate**

Time to saturate in a current transformer depends on several factors like primary circuit time constant, secondary circuit time constant, secondary burden, current transformer ratio, etc. To calculate the time required for saturation, the flux required for current transformer operation in that particular situation should be known. Derivations for time to saturate can be done using the flux equation.

A relationship between the flux required and the flux available for current transformers with resistive burdens and fully offset primary current is given by equation below [4]:

\[
\phi_{\text{required}} = \frac{I₁R₂}{N^2\omega} \left( \frac{\omega T₂}{T₂ - T₁}(e^{-\omega T₁} - e^{-\omega T₂}) - \sin \omega t \right)
\]

where:
- \(\phi\) = Core flux
- \(R₂\) = Secondary resistance
- \(N\) = turns ratio
- \(T₁\) = Primary circuit time constant
- \(T₂\) = Secondary circuit time constant
- \(\omega\) = angular velocity at power frequency (50 Hz)
- \(I₁\) = primary rms fault current

The flux available can be calculated from the saturation voltage obtained from the current transformer magnetising curve. This curve is based on test results using fundamental sine wave voltage.

55
If $V_x$ is the rms. saturation voltage from these tests then:

$$\sqrt{2} V_x = \omega N \Phi_{\text{available}}$$

$$\therefore \Phi_{\text{available}} = \frac{\sqrt{2} V_x}{\omega N}$$

For correct relay operation:

$$\Phi_{\text{available}} = \Phi_{\text{required}}$$

$$\frac{\sqrt{2} V_x}{N \omega} = \frac{I_1 R_1}{N^2 \omega} \left( \frac{\omega T_1 T_2}{T_2 - T_1} \left( e^{\omega T_1} - e^{-\omega T_1} \right) - \sin \omega t \right)$$

$$\therefore V_x = \frac{I_1 R_2}{N} \left( \frac{\omega T_1 T_2}{T_2 - T_1} \left( e^{\omega T_1} - e^{-\omega T_1} \right) - \sin \omega t \right)$$

A reasonable approximation is to let $-\sin \omega t = 1$ then:

$$V_x = \frac{I_1 R_2}{N} \left( \frac{\omega T_1 T_2}{T_2 - T_1} \left( e^{\omega T_1} - e^{-\omega T_1} \right) + 1 \right)$$

This gives the saturation voltage $V_x$ necessary for the primary current to be correctly transformed up to time $t'$. It can be seen that for any value of $V_x$, the current transformer will saturate after time $t'$. The saturation factor $K_s$ is defined as the ratio of $V_x/I_1 R_2$. That is the ratio of saturation voltage to transform the offset current to the voltage required to transform the steady state current.

$$K_s = \frac{V_x}{I_1 \omega T_1} = \left( \frac{\omega T_1 T_2}{T_2 - T_1} \left( e^{\omega T_1} - e^{-\omega T_1} \right) + 1 \right)$$

It can be seen that as $V_x$ and $K_s$ are increased, the time $t$ taken for the current transformer to saturate increases as well.

Other interesting factors to note are:

a) The value of $K_s_{\text{max}}$ required, increases as the primary system time constant $T_1$ increases.

b) The value of $K_s_{\text{max}}$ required, reduces as the current transformer secondary time constant $T_2$ reduces.
The effect of burden inductance is to increase the required current transformer size to achieve the same "time to saturate" as with a resistive burden or to reduce the "time to saturate" if the same size is used.

From the discussion above a general equation for time to saturate developed by Wu is as follows [4]. In this general equation, the effect of short circuit current offset and the effect of remanence in current transformer cores have been included:

\[ t_s = -T_2 \ln\left\{ 1 - \frac{T_2 - T_1}{\omega T_2 T_1} \left[ (1 - \%\text{remanence}) V_s N - \frac{1}{\cos \phi} \right] \right\} \]

where:
- \( V_s \) = saturation voltage.
- \( t_s \) = time to saturate
- \( \cos \phi \) = secondary burden power factor

5.5.1 Generally Time To Saturate Is Dependent On The Following:

1. Short circuit rms. current magnetising \( I_1 \).
2. Current transformer turns ratio.
3. System time constant \( T_1 \).
4. Current transformer time constant \( T_2 \).
5. Saturation voltage
7. Burden power factor (\( \cos \phi \))
8. Remanence in the transformer core
9. Percentage of short circuit offset

The above discussion has been verified by results in Appendix B

5.6 Current Transformer Error Due To Magnetic Saturation

A fault current may generally be considered as consisting of three components; the fundamental frequency component, the dc offset component and the high frequency component (both harmonic and non-harmonic)

The high frequency components of fault current will have little effect on the flux level in the current transformer core. This can be illustrate in the equations below.

Faraday's law states that:

\[ \nu = N \frac{d\phi}{dt} \] (Faraday’s Law)  \[ 5.11 \]
For a sinusoidal voltage:

\[ v = \sqrt{2}V \sin 2\pi ft = N \frac{d\phi}{dt} \]  \hspace{1cm} 5.12

\[ \phi(t) = \phi_e + \frac{\sqrt{2}V_{ms}}{N\pi f} \cos 2\pi ft \]  \hspace{1cm} 5.13

\[ \Phi_{\text{max}} = \phi_e + \frac{\sqrt{2}V_{ms}}{N\pi f} \]  \hspace{1cm} 5.14

where:

- \( \phi_e \) = remnant flux
- \( f \) = power network frequency
- \( N \) = turns ratio
- \( V_{\text{ms}} \) = primary rms voltage

From equation 5.14, the maximum flux will be affected by a dc component and a fundamental frequency component. In the case above a pure voltage source has been used. However if a source with harmonics was used, the harmonic components would not contribute much to the flux because the flux is inversely proportional to harmonic order. It is also unlikely that any high frequency component measured during a fault will exceed ten per cent of the fundamental term, therefore harmonics contribution to core saturation will be negligible.

### 5.7 Open Circuit Secondary Voltage

When a current transformer is operated with secondary winding open circuited, a high voltage is developed on the secondary winding of the current transformer. This section presents the reason for this effect.

The emf induced in the secondary windings necessary to drive the secondary current through the total impedance of the secondary circuit and that the core flux inducing this emf is provided by a small difference between the primary and secondary ampere turn. With the secondary circuit open, there are no secondary ampere turns to oppose those due to the primary current and the entire primary ampere turns act on the core as an excessive exciting force, which might drive the core into saturation on each half wave of current as shown in figure 5.1 below.
The high rate of flux change in the region where the primary current is zero, induces an emf $E_2$ of high peak value in the secondary winding. With rated current in the primary winding this peak value may be a few hundred volts in a small measuring current transformer, but it might reach many kilovolts in large current transformers.

When a current transformer is working in saturation region, most of the current (alternating current) wave drawn is virtually used as magnetising current. This results in greatly distorted secondary wave shape and high peak voltage “spikes” across the burden each time the primary current passes through zero because of the rapid rate of flux change.

The magnetising impedance $Z_e$ is given by the slope of the magnetisation curve

$$Z_e = \frac{V}{I}.$$ \hspace{1cm} 5.15

Again, almost all input current is used to magnetise the saturated core that has the characteristic of an air path (very high reluctance). The current transformer output current collapses to almost zero in saturation.

The distorted emf in a current transformer saturated state causes secondary current distortion in a linear burden.
5.8 **Current Transformer Output Currents With Different Types Of Burden**

This section presents different current outputs from current transformers with different types of secondary burden. That is; resistive, inductive, capacitive, resistive + inductive, etc. Theory on flux variation in chapter three is vital to understanding the current outputs presented here. Both steady state and transient saturation conditions are considered for each particular burden. As usual, several assumptions are have been made for particular cases.

5.8.1 **Output With Resistive Secondary Circuit**

For simplicity, in the following analysis assumptions are made:

- Magnetic component of the exciting current remains zero until saturation is reached.
- No increase of flux above saturation level is possible.

a) **Steady State Saturation**

In a resistive burden, the secondary current leads the magnetising current by $90^\circ$. When a saturation level is reached, the core flux density will become constant and no further emf will be produced. Therefore the secondary current will collapse instantly to zero, the whole of the primary current then being available to hold the core in saturation. The core will remain in this state until the exciting current falls to its saturation value.

The above description can be represented diagramatically as follows:

![Resistive Burden Waveforms In Steady State](image.png)
The illustration above shows that the general effect for a resistive saturated circuit is for a portion to be chopped out of the trailing edge of each half cycle of the secondary current waveform.

However the above description is idealistic. In actual transformer there would be finite exciting current, therefore the actual secondary output current would be slightly different from the one above.

During the portion of the cycle when the core is saturated, the magnetising inductance \( L_m \) reduces from the high value obtained before saturation, to a very small value. The core loss resistance \( R_p \) rises as both the hysteresis and eddy current losses become low. A very small emf and secondary current are produced as the core goes more deeply into saturation, because of the slight flux increase which is possible.

The energy loss in a given core due to the hysteresis effect, during the period when the flux changes from saturation level of one polarity to that of the other is significant. The power loss tends to be inversely proportional to the period for which the core is unsaturated. The emf induced in the core as the flux changes from the saturation level of one polarity to that of the other, is inversely proportional to the time taken for the flux change, and the square of this time. For this reason, the eddy current loss becomes large and very important when a transformer is operated with a high degree of saturation. The increased losses cause the loss component of exciting current to be above its normal value and thus their effect, when the secondary circuit is resistive is to directly reduce the secondary winding current.

b) Behaviour During Transient Conditions

During transient conditions, the primary current has two components as already discussed. The alternating and direct components. The dc component dependent on primary inductance \( L_1 \) and primary resistance \( R_1 \) values. The greater the primary current magnitude and the higher the primary circuit impedance ratio \( \omega L_1 / R_1 \), the earlier saturation would occur and therefore the greater would be the amount chopped out of the first oscillation of the secondary winding current. For a primary circuit with low \( \omega L_1 / R_1 \) ratio, the flux swing which would be required during this period might exceed that available. In this event some saturation would occur, and a slight chopping of the tail in the secondary current wave would accompany it.
An illustration of a transient condition behaviour in a core with a residual flux is given in figure 5.3 below.

![Diagram of primary current, core flux, and secondary current and voltage waveforms.]

Figure 5.3
Resistive Burden Waveforms In Transient State [5]

5.8.2 Output From Transformer With Inductive Secondary Circuit

a) Steady State Saturation

Considering ideal conditions during periods of saturation, no emf would be produced. No more secondary current change would occur and therefore its value would remain constant and equal to the value at the instant of saturation. For a period after saturation has occurred, the primary current must continue to increase, a surplus of ampere turns so being provided to hold the core in its saturated state. This condition would continue until the primary current had decreased to the magnitude it had when saturation occurred, the resultant ampere turns then being zero again. The core would come out of saturation at this time as the flux change required to provide the secondary emf would be in a direction away from the saturation level. This process would be repeated each half cycle, the general effect being to chop off the tops of the half cycle of secondary current as shown in figure 5.4.
The effect of saturation in an inductive burden is the loss of peaks of the secondary current waveform. Phase comparison systems would be less affected by saturation of this type.

![Diagram of Inductive Burden Waveforms Steady State Condition]

Practically, the $L_m$ is finite and its presence in parallel with the secondary circuit inductance causes a reduction in the secondary circuit. The core power losses during these periods increase in a given transformer. The effect of these losses can be determined by considering presence of a loss resistance $R_p$ in parallel with the secondary circuit inductance in the equivalent circuit. This resistor causes the secondary circuit current to lag behind its ideal position, the angle of lag depending on the magnitude of the losses.

b) Transient State Behaviour

In a current transformer with pure inductive burden, the core flux varies in the same manner as the primary current until saturation occurs. If a primary current contains a dc transient component as it happens in transient state, the core flux would vary in the same manner until saturation is reached when the top part would be chopped. No emf would be induced when saturation is reached and the secondary current would remain at the value which it had at the instant of saturation. This condition would continue until the primary current once again had the same magnitude as it possessed when saturation occurred. Therefore, the flux would decrease and the emf needed to cause the secondary current to vary in the correct manner, would be produced.
As in steady state, the general effect of saturation is to chop off the tops of the secondary current wave, the greater the degree of saturation the greater the amount chopped off as illustrated in figure 5.5.

![Inductive Burden Waveforms - Transient State](image)

5.8.3 Effect Of R-L Secondary Burden

Wright [5] concluded that effects of R-L burden always tend to be intermediate between those obtained with purely resistive and purely inductive burdens. Exponential decay of secondary circuit, during saturation periods, starts before and finishes after a time at which peak flux would have been reached if saturation had not occurred. The rate of decay increase with an increase in the secondary circuit power factor.

The presence of residual flux at the beginning of transient condition not only affects the time taken for saturation to be reached initially, but it continues thereafter to have an effect which reduces more slowly, the lower the secondary circuit power factor.
For sinusoidal steady state primary current condition, the secondary current would be made up of the exponentially decaying wave and part of the sine wave. This can be clearly seen in the diagrams above.

\[
e_2 = i_1 R_s + L_i \left( \frac{di_2}{dt} \right) = 0
\]  

5.16
5.8.4 Current Transformer With Capacitive Burden

The combination of capacitor and non-linear inductance is known to produce complex waveforms through the action of ferro-resonance. Saturation in capacitive burden takes place even with small primary current values. This is because the capacitor pauses as an open circuit and this will push the non-linear inductance into saturation quickly. Ferro-resonance causes the cyclic peak of current distortion, the current peak being due to the flux level in the inductance being driven beyond its normal level, giving saturation and consequent discharge of the capacitor through this saturated inductance.

Because of the risk of resonance and distortion, the use of a capacitive burden is not common. Magnetising current lags secondary current by $180^\circ$.

Capacitors may be included in the secondary circuit of CT to effect filtering or tuning or to compensate for the capacitance of equipment being protected.

Below are equations assuming no saturation occurs with a capacitive burden when carrying a sinusoidal primary current.

\[ t_2 = -\frac{I_1 N_1}{N_2} \sin \omega t \]  
5.17

\[ e_2 = \frac{1}{C_2} \int t_2 \, dt = \frac{I_1 N_1}{\omega C_2 N_2} \cos \omega t \]  
5.18

\[ \phi_e = -\frac{1}{N_2} \int e_2 \, dt = -\frac{I_1 N_1}{\omega^2 C_2 N_2^2} \sin \omega t \]  
5.19

Therefore unsaturated performance should allow for maximum flux $\phi_{\text{max}} \geq \frac{I_1 N_1}{\omega^2 C_2 N_2^2}$.

In current transformer operation with capacitive burden, at the end of the unsaturated period of operation that is, when the flux reaches saturation level in the opposite direction, a voltage would be built up across the capacitor. The transformer would then be unable to produce any further emf and the capacitor would then discharge through the secondary winding of the transformer. With these assumptions, the magnetising impedance of the transformer would become zero when its core enters saturation and therefore an infinite current of zero duration would flow to reduce the capacitor voltage to zero.

Because of this, the mmf produced by the transformer secondary winding would exceed that produced by the primary winding, during the moment of saturation.
5.8.5 Current Transformer With Resistor And Capacitor Secondary Circuit

a) Steady State Behaviour

During saturation, assuming ideal saturation

\[ t_z R_z + \left(1/C_z\right) \int t_z dt = 0 \quad \text{(5.20)} \]

that is;

\[ t_z = k_z e^{-t/C_z R_z} \quad \text{(5.21)} \]

At instant of saturation, the capacitor would begin to discharge, the current being driven by voltage across the capacitor \(V_{cs}\) and controlled by the secondary circuit resistance.

An equation for \(\phi_{sat}\) can be represented by [6]

\[ \phi_{sat} = \left( I_{pk} N/2\alpha N \right) \left( \cos(y_z + \alpha t_z) + \cos(y_z + \alpha t_z) - \left( \pi + \alpha t_z - \alpha t_z \right) \sin(y_z + \alpha t_z) \right) \]

Analysing the above equation shows that there are two possible secondary current waveforms, both of them shown in figure 1.12 below.
b) Transient Behaviour

When core flux saturation level is reached, the secondary current changes from its value to the one in equation below:

$$i_2 = -(V_a / R_s) e^{-	au / R_s C}$$  \(5.23\)

This condition provides the imbalance of primary and secondary current decays to the value which it would have had, had saturation not occurred. At this time the remaining voltage across the capacitor must be equal and opposite to that across the secondary circuit resistor and the core flux must be of the saturation level.

The complete behaviour may be established as
The secondary current waves will evidently consist of sinusoidal and exponentially decaying sections. Sudden changes in magnitude which occur on saturation and the subsequent rates of current changes being greater, the lower the power factor of the secondary circuit. With capacitive burden, under certain circumstances, three different steady state secondary current waveforms may be obtained for one primary current. This shows how unpredictable the output from capacitive burden can be.
CHAPTER SIX

6 Simulation Tools Used On Current Transformer Performance Modelling

6.1 Introduction

It is very difficult to solve transient problems by hand except in the simplest circuit containing a small number of elements. The algebra for more complicated circuits is just too burdensome. It is for this reason that computer becomes a handy tool.

This chapter looks at simulation tools used in this thesis. An electromagnetic transient program EMTP-ATP was chosen as a simulation tool based on its ability to model non-linear elements in time domain.

The Alternative Transients Program (ATP) is the most widely used version of the Electromagnetic Transients program (EMTP) in the world today - by far! In no small part, the acceptance of ATP is due to its availability to nearly everyone in the world free of royalty, and its compatibility with many computers.

Being a royalty free software, it is constantly being upgraded and developed by several experts. This makes it a more relevant software as it is always up to date. Several other packages related to EMTP were used as well.

These include:

- ATPDRAW which is a graphical pre-processor interface.
- PCPLOT a post processor used as a tool for plotting ATP outputs and
- MODELS which is a high level programming language for transient studies. It can also work independently of EMTP.

Individual elements used for current transformer modelling have been presented and their merits and demerits discussed.

EMTP is capable of solving any power network which consists of interconnections of resistance, inductance, capacitance, single and multiphase π circuits and certain other elements.
6.2 Solution Method Used In EMTP

Node voltages are used as state variables in EMTP [1]. It is therefore necessary to express branch current as functions of the node voltages. For clarification, consider the network below:

From Kirchoff's Current Law:

\[ i_1(t) = i_{12}(t) + i_{13}(t) + i_{14}(t) \]  

Expressing current in terms of node voltages for the above example gives:

For the resistance:

\[ i_{12}(t) = \frac{1}{R}(v_1(t) - v_2(t)) \]  

For the inductor, a simple relationship is obtained by replacing the differential equation:

\[ v = L \frac{di}{dt} \]
With a central difference equation below:

$$\frac{v(t) + v(t - \Delta t)}{2} = \frac{i(t) - i(t - \Delta t)}{\Delta t} \quad 6.4$$

This can be rewritten to make current subject for the case of figure 6.1 as follows:

$$i_{13}(t) = \frac{\Delta t}{2L} \{v_1(t) - v_3(t)\} + hist_{13}(t - \Delta t) \quad 6.5$$

with $hist_{13}$ known from the values of the preceding time step.

$$hist_{13}(t - \Delta t) = i_{13}(t - \Delta t) + \frac{\Delta t}{2L} \{v_1(t - \Delta t) - v_3(t - \Delta t)\} \quad 6.6$$

Similarly current through the capacitor is:

$$i_{14}(t) = \frac{2C}{\Delta t} \{v_1(t) - v_4(t)\} + hist_{14}(t - \Delta t) \quad 6.7$$

with $hist_{14}$ again known from values of the preceding time step.

$$hist_{14}(t - \Delta t) = -i_{14}(t - \Delta t) + \frac{2C}{\Delta t} \{v_1(t - \Delta t) - v_4(t - \Delta t)\} \quad 6.8$$

If equations 6.2, 6.5 and 6.7 are inserted into equation 6.1, then the node equation for node 1 becomes:

$$\left(\frac{1}{R} + \frac{\Delta t}{2L} \right) v_1(t) - \frac{1}{R} v_3(t) - \frac{\Delta t}{2L} v_3(t) - \frac{2C}{\Delta t} v_4(t)$$

$$= i_1(t) - hist_{13}(t - \Delta t) - hist_{14}(t - \Delta t) \quad 6.9$$

which is simply a linear algebraic equation in unknown voltages, the right hand side known from values of preceding time steps.
For any type of network with \( n \) nodes, a system of \( n \) such equations can be formed as follows:

\[
\begin{bmatrix} G \end{bmatrix} \begin{bmatrix} v(t) \end{bmatrix} = \begin{bmatrix} i(t) \end{bmatrix} - \begin{bmatrix} hist \end{bmatrix} \tag{6.10}
\]

with:

\[
\begin{align*}
[G] & = \quad n \times n \text{ Symmetric nodal conductance matrix.} \\
[v(t)] & = \quad \text{vector of } n \text{ node voltages} \\
[i(t)] & = \quad \text{vector of } n \text{ current sources and} \\
[hist] & = \quad \text{vector of } n \text{ known history terms.}
\end{align*}
\]

Normally some nodes have known voltages either because voltage sources are connected to them, or because the node is grounded. In this case equation 6.10 is partitioned into a set \( A \) of nodes with unknown voltages, and a set \( B \) of nodes with known voltages. The unknown voltages are then found by solving for \([v_A(t)]\) in the following matrix equation.

\[
\begin{bmatrix} G_{AA} \end{bmatrix} \begin{bmatrix} v_A(t) \end{bmatrix} = \begin{bmatrix} i_A(t) \end{bmatrix} - \begin{bmatrix} hist_A \end{bmatrix} - \begin{bmatrix} G_{AB} \end{bmatrix} \begin{bmatrix} v_B(t) \end{bmatrix} \tag{6.11}
\]

The actual computation in the EMTP proceeds as follows:

Matrices \([G_{AA}]\) and \([G_{AB}]\) are built, and \([G_{AA}]\) is triangularized with ordered elimination of sparsely. In each time step, the vector on the right hand side of equation 6.11 is assembled from known current and voltage history terms, and known current and voltage sources. Then the system of linear equations is solved for \([v_A(t)]\), using the information contained in the triangularized conductance matrix. In this repeat process, the symmetry of the matrix is exploited in the sense that the same triangularized matrix used for downward operations is also used in the back-substitution.

Before proceeding to the next time step, the history terms of equation 6.6 and 6.8 are then updated for use in the future time steps. This process continues for the whole solution.
6.3 **Structure Of EMTP Input Data**

EMTP problems are described by a collection of data cards that are called data cases. Each data case is presented in a chronologically ordered manner of different cards as explained below:

1. Cards to begin a new data case.
2. Cards that define any TACS or MODEL modelling
3. Cards for linear and non-linear branches, transformers and transmission lines.
4. Cards for electric network switches, diodes and thyristors.
5. Source cards for the electrical networks: voltage and current sources.
6. Output variable specification cards.
7. Data cards to end data case.

Detailed structures of EMTP data cases are given in references [2-5]. An outline of a data case has been presented in Appendix B, File 1.

The final outputs in EMTP simulations consist of component variables (e.g. branch currents, or voltages, etc) as functions of time, for those variables that were requested by the user. The outputs are in ASCII format. Graphs can be plotted from the output data using PCPLOT which is a graphical plotter for EMTP.

6.4 **Transformer Models In EMTP**

A transformer can be represented in several ways. Three fundamentally different types of transformer models exist in EMTP

1. Ideal transformer model
2. Saturable transformer model
3. Model based on mutually coupled coils.

A Current transformer is basically a single phase transformer which can effectively be modelled using models 1 and 2 only. Model 3 is not applicable. This can be used with three phase transformers and in cases where there are several windings on the primary winding. Unlike in current transformer where the primary winding might be a solid bar.
6.4.1 Saturable Transformer Model

This model considers leakage as well as the reluctance of the magnetic material. The model assumes that a finite reluctance magnetic path exists, and that around each individual coil a separate possible magnetic leakage path exists.

This model is the most ideal for single phase transformers. A piece-wise linear flux-current magnetisation curve must be supplied. This curve is input point by point, starting from the point nearest to the origin and increase monotonically moving away from the origin.

Both saturable and pseudo-non-linear reactance (Type 98) branches use a flux-current magnetisation curve. An auxiliary program CONVERT is designed to perform conversions from rms values of voltage and current or current versus incremental inductance data to flux -current data.

Transformer cores in saturable transformer model can be represented as:

1. Pseudo non-linear hysteretic reactance (Type 96) or
2. Pseudo-nonlinear reactance (type 98)

6.4.2 Ideal Transformer (Type 18)

In this model leakage flux is ignored with the assumption that all flux is confined to the magnetic core. In addition, magnetisation current is ignored assuming no reluctance in the magnetic material. It has no impedances and simply changes voltages and current from side one to side two using turns ratio.

This model was successfully used for current transformer modelling by building other elements around it to fully represent current transformer models.

6.4.3 Frequency Dependent Transformer Models

Meanwhile, no frequency dependent effects have been included in transformer models. Therefore to account for; frequency dependence in exciting current and influence of stray capacitances at frequency above 1-10 kHz, external components should be added to the transformer models. Suggested modelling of frequency dependent models are inclusion of capacitances as follows [6]:

1. Between the winding closest to the core and the core.
2. Between any two windings.
3. Across each winding from one end to the other.
6.5 **Support Routines**

Support routines are used to translate some input data, which is known to the user, to some other output data which can directly be used in an EMTP application. Two groups of these supporting routines used in this thesis are presented in this chapter.

6.5.1. CONVERT

Saturation curves supplied by manufacturers often give rms voltages as a function of rms currents which cannot be immediately used in EMTP modelling. The support routine CONVERT changes the \( V_{\text{rms}}/I_{\text{rms}} \) curves into flux/current curves which can be used in EMTP simulations. The following simplifying assumptions are made in the conversion.

1. Hysteresis and eddy current losses in the iron core are ignored.
2. Resistance in winding is ignored and
3. The flux/current curve is generated point by point at such distances that linear interpolation is acceptable in between points.

It is also assumed that flux varies sinusoidally at fundamental frequencies as a function of time.

For illustration of CONVERT process consider the diagrams below:

![Diagram of CONVERT process](image)

**Figure 6.2**
Recursive Conversion Of A \( V_{\text{rms}}/I_{\text{rms}} \) Curve Into A Flux-Current Curve.
With the assumption pointed out earlier, conversion of $V_{rms}$ value to flux values becomes a simple re-scaling exercise as follows.

$$\phi = \frac{V_{rms} \sqrt{2}}{\omega} \quad 6.12$$

The re-scaling of current is more complicated, except for point $i_B$ at the end of the linear region A-B (figure 6.2) which is:

$$i_B = \sqrt{2} i_{RMS-B} \quad 6.13$$

The following points $i_C, i_D, \ldots$ are found recursively. Assume that $i_E$ is the next value to be found. Assume further that the sinusoidal flux just reaches the value $\Phi$ at its maximum.

$$\phi = \Phi \sin \omega t \quad 6.14$$

Within each segment of the curve already defined by its end points, in this case A-B, B-C and C-D, current is known as a function of flux (namely piecewise linear), and equation 6.14 shows that flux is also a function of time. Only the last segment is undefined in as much as $i_E$ is still unknown. Therefore $i = f(t, i_E)$ in the last segment.

If the integral needed for RMS values,

$$F = \frac{2}{\pi} \int_{t_1}^{t_2} i^2 d(\omega t) \quad 6.15$$

is evaluated segment by segment, the result will contain $i_E$ as an unknown variable. With the trapezoidal rule of integration (reasonable step size = $1^\circ$), $F$ has the form:

$$F = a + bt_E + ci_E^2 \quad 6.16$$

with $a, b$ and $c$ known, since $F$ must be equal to $i_{RMS-E}^2$ by definition, equation 6.16 can be solved for the unknown value $i_E$; this process is repeated recursively until the last point $i_N$ has been found.
HYSDAT is another support routine designed to provide data needed to represent hysteresis in transformer cores. HYSDAT is an attempt to catalogue the hysteresis characteristic for some common transformer core materials. Only 1 ARMCO M4 oriented silicon steel is supported in EMTP. The HYSDAT program stores the shape of the hysteresis loop for the material specified. Even though, saturation effects have been modelled by adding extra non-linear inductance or inverse inductance matrix representations, an accurate representation of hysteresis and eddy current effects of skin effects in the coil and of stray capacitance is still difficult at this time.

6.6 Simple Voltage And Current Sources

Frequently used source functions $f(t)$ are built into the EMTP data base. Besides the built in functions, a user can define functions through FORTRAN subroutines, TACS and MODELS programs.

If a voltage or current source is specified at a node, it is assumed to be connected between that node and local ground. If more than one voltage sources is connected to the same node, then the EMTP simply adds their functions, $f_1(t) - f_0(t)$ to form one voltage source, implying a series connection. Similarly, if current sources are connected to the same node, then EMTP adds their functions to form one current source.

Commonly encountered source functions are inbuilt in EMTP are documented in reference [2], these sources include:

- Step function (Type 11)
- Ramp function (Type 12)
- Sinusoidal function (Type 14)
- Impulse function (Type 15)

A special impulse function has been provided for representation of lightning or switching impulses. A standard lightning source has been defined in EMTP as given in function below [2].

$$f(t) = k \left( e^{-\alpha_1 t} - e^{-\alpha_2 t} \right)$$

where:

- $k$ is an amplitude constant
- $\alpha_1$ and $\alpha_2$ are functions calculated according to time constants $T_1$ and $T_2$. 6.17
The table below shows values of standard lightning.

### Table 6.1 Standard Lightning Data Input In EMTP

<table>
<thead>
<tr>
<th>$T_1/T_2$ ($\mu$s)</th>
<th>$1/\alpha_1$ ($\mu$s)</th>
<th>$1/\alpha_2$ ($\mu$s)</th>
<th>$k$ to produce $f_{\text{max}} = 1.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2/50</td>
<td>68.2</td>
<td>0.405</td>
<td>1.037</td>
</tr>
</tbody>
</table>

6.7 **Switches**

The strength of EMTP lies in its ability to model transient events in a power system usually initiated by the closing or opening of a switch. Any switching operation in a power system can potentially produce transients. So far all switching devices in EMTP are represented as ideal, with zero current ($R = \infty$) in the open position and zero voltage ($R = 0$) in the closed position. Below are two types of switches which were used in current transformer model simulations.

6.7.1 **Time Controlled Switch**

This type of switch is intended for modelling circuit breakers, disconnectors and similar switching devices, as well as short circuits. The switch is originally open and closes at a specified time $T_{\text{close}}$. Its flexibility in being able to open and close anytime makes it possible to simulate fault conditions perfectly.

6.7.2 **Measuring Switch**

This is always closed in the transient simulation as well as in the alternating steady state solution. It is used to obtain current, or power and energy, in places where these quantities are not otherwise available. This feature is necessary for calculation of currents for certain types of branches.
6.8 TACS

EMTP is a very flexible program because a user can define his/her functions using TACS (Transient Analysis of Control Systems)[5] which use FORTRAN language. TACS was designed to incorporate control system modelling into EMTP. A TACS program can simulate a control system separately and input the results into an EMTP program or results from EMTP used as input to TACS.

Tacs allows the user to:

- express parameter relationships using FORTRAN statements
- express parameter relationships using controller blocks and
- express parameters relationships using miscellaneous special devices.

6.9 MODELS

MODELS is a general purpose description supported by a set of simulation tools for the representation and study of time variant systems. The MODELS language provides a format which focuses on the description of the structure of a model and on the function of its elements.

Models is high level programming language for modelling electric power components. Models can work independently or it can be incorporated with EMTP. Advantages of MODELS are:

1. It allows detailed descriptions of pre-simulation initial state of modelled components.
2. It provides a simple 'measure and control' interface for connecting other programs to EMTP.

A system can be described in MODELS as an arrangement of inter-related submodels, independent from one another in their internal description and in their simulations. Unlike an EMTP data case, the description of each model uses a free format and does not require fixed formatting in its representation.

Another merit feature is that syntax of MODELS allows the representation of a system to closely follow the system's final structure, supporting the explicit description of composition, sequence, concurrence, selection, repetition and replication. A more detailed description of MODELS language can be found in references [7,8].
6.10 ATPDRAW

ATPDRAW is a graphical pre-processor interface program used to compile data into file form (ASCII editor) which EMTP can process. The user can build up a circuit in ATPDRAW by picking components from menus and connect them using a mouse. ATPDRAW creates ATP input file according to the user's assembly and parameters and automatically add node names to unspecified nodes [9].

6.11 Obtaining Flux Using EMTP

There is no direct way to request "flux" (actually flux-linked) output with a column 80 request. However there are four choices which could be used:

- TACS integration of related voltage
- MODELS integration of related voltage.
- Placing RC integrator across related voltage branch and outputting the capacitance voltage (flux linked = 1/(1+RCs)). In this case, it was made sure that RCs >> 1 so that results reasonably close to an ideal integration could be obtained. Also, provide the correct scaling to Vc to obtain the correct value of flux linked. Vc being voltage across the capacitor. R=100K and C=10uF values have been successfully used, if the frequencies involved are not too low (If RC=1, Vc equals the flux linked and no scaling was required). In any case, the initial flux linked to the integrator must be supplied, if the initial flux is not zero. Initial conditions' cards were used to initialise Vc in the case of the RC integrator, or appropriate initialisation with TACS or MODELS could be used.
- The branch in the network whose flux is desired could be monitored by means of a large parallel inductance. The inductive current is proportional to flux. Correct value of flux was obtained by scaling inductive current using TACS as shown in Appendix A File 4.
6.12 Important Points To Note In EMTP Simulations

6.12.1 Problems Of Using Very Small Values Of Resistors And Inductors

Very small values of resistors create very large conductance values in matrix \([Y]\) for steady state solutions and in the matrix \([G]\) for transient solutions, which can "swamp out" effects of other elements connected to those resistors. Consequently, wrong results can be obtained or an error message can be displayed. Very small values of inductors create similarly problems as in way small resistors do. Therefore care should be taken when using small resistance and inductance values in EMTP simulations.

6.12.2 Physical Reasons For Parallel Resistance

Magnetising impedance of transformer and iron core reactors are needed for a crude approximation of the hysteresis and eddy current losses.

Saturation effects have been modelled by adding extra non-linear inductance and resistance branches to the inductance or inverse inductance matrix representations. Accurate representation of hysteresis, eddy current, skin and stray capacitance effects are still difficult at this time. Therefore external elements have to be added to simulate these effects.

The EMTP is structured to generate odd harmonics only up to the 15th order when considering non-linear elements. This could bring some errors to practical results as higher orders of even harmonics are neglected.

6.13 Numerical Oscillation Problem

There have been several concerns by EMTP user group about the presence of numerical oscillations in EMTP simulations. In most cases, numerical oscillations is an indication of improper or inadequate modelling. This feature can be regarded a useful bug indicating that the model needs more work either to accurately represent the oscillation or to represent its realistic damping.

The numerical oscillations are always associated with switching that causes abrupt flux changes, essentially current chopping in an inductive circuit. That can include switching the "hanging inductor", slope transitions in Type 98 non-linear inductors, improper initialisation of Type 96 Hysteresis or related cases. The current chopping can be deliberate or inadvertent, as when chopped near zero due to the time step not allowing switch opening at exactly zero current.
Getting rid of the noise is usually easy. Just represent the capacitances and their series damping resistances that are actually present on the inductances and use a small enough time step. In general any switch noise problem can be eliminated by snubber circuitry. That is putting damped capacitances across the switch to limit \( \frac{dv}{dt} \) and series inductances to limit \( \frac{di}{dt} \). Paralleling inductors with resistances was also a useful technique.

Problems with (Type 98) non-linear reactors were easy to solve by avoiding large changes in slope in the model user. The rule would be to double the current of successive data points. The use of parallel resistance is recommended, since only iron core reactors are non-linear and all iron core devices have core losses. Again, the more realistic the model the more damping of noise.

Type 98 Hysteresis models must be adequate for the highest voltages (fluxes) that will ever occur in the model and transition from the steady state simulation to the non-linear must not involve a flux transition. As a general rule the Type 96 model is not realistic for minor loop representation anyhow. True hysteresis requires current injections, not a B-H path model. It is after realising this fact that Type 98 was preferred to Type 96.

A fictitious capacitance could be added across the switch or on both sides of the switch to ground whose capacitive reactance at the highest frequency of concern is at least ten times that of the system reactance at the highest frequency of concern, with no more than ten percent error at the highest frequency of concern.

The series resistances of fictitious capacitances can sometimes be entered as two to three times the value needed for critical damping. This will help to over-damp at time setting frequency, without affecting the highest frequency of concern.

From the discussion on numerical in EMTP simulations, it can be concluded that the time step used should always be very small. This has an added advantage of smoothening output waveforms.

It was foundout that the type 96 element was not able to correctly model, predict ferroresonance over a wide range of core voltages. However, if operation does not wander too far from rated voltage, type 96 works fine. It was found that for a single phase transformer, type 98 paralleled with a linear core loss resistance whose value was determined for rated voltage performed much better. More recent work with single phase transformers has borne this out. However there is still some concern with the accuracy of CT models that use the type 96 element, as core voltage covers an extremely broad range depending on the short circuit current and type of burden attached.
Conclusion is that at the moment, until some better core representations are developed, verified and implemented, fairly good results can be obtained with a type 98 inductance for the core magnetisation. ATP is a very good simulation package to use for this type of simulation. MODELS gives nearly unlimited possibilities should modeling work require special user-defined modeling elements.

The trapezoidal rule used in EMTP has a problem of amplifying high frequency voltages across inductance in situations where currents are forced into them. In this situation, the trapezoidal rule works as a differentiator and the problem shows up as numerical oscillations in cases where the derivative of the current changes abruptly, for example opening a switch.

6.14 Conclusions

1. EMTP based Current transformer models are a convenient way of simulating fault transient.

2. Existing models in EMTP for the single phase power transformer and the non-linear inductor Type 98 can be used to build Current transformer models if these phenomena are relevant in the studies.

3. EMTP based current transformer models using Type 98 elements are sensitive to the change in the V-I curve slope. If the slope in the saturation region is high, the simulation results will not represent current transformer transient behaviour well enough.

4. Current transformer models which use Type 96 elements for the hysteresis representation are sensitive to the selection of saturated point needed for hysteresis generation. The point is not precisely determined for the V-I curve. However, if a point is selected deeper into saturation region, results that are closer to the actual values are obtained.

5. Some differences between practical waveforms might arise from differences and problems related to the use of element Type 98 and Type 96. This requires further investigation.
CHAPTER SEVEN

7 Analysis Of EMTP Simulated Results And Conclusions

7.1 Introduction

This chapter presents some EMTP simulation results obtained in this thesis. Detailed procedures used in each case have been included. Observations on the results obtained have been highlighted and discussed.

Current transformer data used for simulations in this thesis were obtained from Haefely Trech (pty) Ltd and ESKOM Brackenfell, Cape Town.

Simulation results presented in this report include:

- Effects of different levels of input current dc biasing on current transformer performance.
- Effect of different amplitudes of third harmonic on current transformer performance.
- Effect of increasing fault level current on current transformer performance.
- Effects of different primary current fault angles on current transformer performance.
- Different waveforms generated from different types of burden on current transformer secondary winding.
- Effects of high frequency input currents on current transformer performance.
7.2 CASE I: Effects Of Different Levels Of Input Current Dc Biasing On Current Transformer Performance

There are various methods for inducing dc currents on a power system. Electric power systems, particularly at higher latitudes, often experience effects of geomagnetic disturbances. These geomagnetic or auroral activity generates an earth-surface potential which, if present between ground points of a transmission line, produces geomagnetically-induced currents (GIC) limited only by the direct current resistance between the ground points [1].

Another probable main source of dc currents on power systems is silicon controlled rectifiers (SCR's). The lack of symmetry in the triggering angles of SCR's causes a dc current to be injected into the power network.

The aim of this simulation was to investigate effects of primary current dc biasing on current transformer secondary outputs. In chapter three, it was shown that dc offset transient currents cause excursions in the flux level of current transformers. In the case of dc bias due to geomagnetic or remnant in iron core, the flux excursion can be thought of as a steady state dc offset.

Dc bias simulations could also be used to study effect of remnant flux in current transformers. Remanence does not gradually disappear but remain constant once the appropriate equilibrium condition has been attained.

A circuit diagram shown below was used for these simulations.

Figure 7.1
Current Transformer High Frequency Model Used for EMTP Simulations
Where:

\[ a = \text{current source at power frequency (50 Hz)} \]
\[ b = \text{variable direct current source} \]
\[ L_1 = \text{power network inductance} \]
\[ R_1 = \text{power network resistance} \]
\[ \text{sw1} = \text{switch on primary side to simulate a fault condition} \]
\[ R_{\text{Cntp}} = \text{resistance required to avoid matrix singularity. (Refer to section 6.12.1)} \]
\[ Z_m = \text{non-linear magnetising impedance} \]
\[ \text{sw2} = \text{measuring switch (refer to section 6.7.2)} \]
\[ Z_b = \text{current transformer secondary burden.} \]
\[ C_{12} = \text{Inter-winding capacitance.} \]
\[ C_2 = \text{Winding to ground stray capacitance.} \]

**CT Data**

<table>
<thead>
<tr>
<th>Ratio:</th>
<th>200/1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class:</td>
<td>5P20</td>
</tr>
<tr>
<td>Burden:</td>
<td>15 VA</td>
</tr>
<tr>
<td>Insulation:</td>
<td>138 kV</td>
</tr>
</tbody>
</table>

**Procedure:**

Resistor \( R_1 \) and inductor \( L_1 \) between node SRC and ground were used as source impedance. In this model big values of \( 1.0 \times 10^6 \Omega \) and \( 3.0 \) H were used for resistance and inductance respectively to inhibit effects of dc transient. This was done to investigate the effect of dc bias only without dc transient current.

Capacitance values were \( 1.0 \) nF and \( 30 \) nF for \( C_{12} \) and \( C_2 \) respectively. These capacitance values were according to experiments done by Douglas in reference [2]. \( R_{\text{Cntp}} \) was put to avoid matrix singularity as required by EMTP. This value was kept very small at \( 1.0 \times 10^{-5} \) \( \Omega \).

Values of \( Z_m \) were calculated internally in the EMTP program from flux - current data calculated from manufacturers specified V-I curves. A pure resistive burden \( Z_2 \) was used at a rated value of \( 15 \) \( \Omega \).

A primary current amplitude of twenty times rated input current value was used (4000 A). This value was chosen to operate the current transformer within its specified accuracy range. BS 3938, states that a composite error of 5% should not be exceeded at 20 times rated primary current for a class 5P20 current transformer. The fault current was maintained constant for all simulations by reducing the amplitude of power frequency source current as the direct current amplitude value was increased.
The value of direct current bias component added to the primary current was increased from 0% to 25% in steps of 5%. A maximum value of 25% was used to cover the widest range of dc bias possible. In a study by Kappenman [1], it was presented that dc bias in a power network can reach as high as 200 A in regions experiencing geomagnetic disturbances. 100% dc offset current are possible during transient periods as well.

**Observations:**

Results for dc biasing have been presented in Appendix B, Case I. The results shown are for simulations with dc bias of 0%, 5%, and 25%.

Figures B1(a - c) are results for excitation current versus primary current. Figure B1(a) has a symmetrical oval shaped figure. This indicates that the ratio of excitation current versus the primary current are equal for any particular angle on all cycles considered irrespective of time. This figure indicates that the excitation current is constant and symmetrical on both positive and negative cycles when no biasing is considered.

There were no current or phase errors when the current transformer was simulated with no dc biasing. The input and output currents are in phase as seen in figure B2(a).

When a dc bias was introduced, the symmetry obtained for figure B1(a) disappeared. Shapes of figures B1(b) and B1(c) indicate that excitation current was mainly unidirectional. A lot of errors were introduced when the sine wave of primary current was in a positive going direction.

Figures B2(a - c) are results for primary and secondary currents. The secondary currents were referred to primary side of current transformer. When no biasing was introduced into the primary current, the secondary and primary current waves fit on to each other perfectly. This can be observed in figure B2(a) where no errors are present. Figure B2(b) shows some transformation errors. Note that errors start showing after the second cycle. Figure B2(c) which is a curve for 25% dc bias, current error is very prominent. The current transformer goes into saturation after one cycle. These results agree with those by Kappenman [1].

Figures B3(a - c) are plots of secondary currents versus primary currents. Extent of transformation error is vividly seen for a 25% dc bias where an output secondary current of less than 18 A is obtained as opposed to 20 A for 0% dc bias. A current transformation error of more than 10% was obtained for the 25% dc bias simulation. This is outside the specified range in BS 3938 standards.
Transformation errors are introduced as soon as a dc bias is present in current transformers. A dc bias as low as 5% introduced a significant error in transformation. Errors increased as the dc component was increased further. Emanuel [3] showed that dc levels as low as 0.4 percent will cause unacceptable errors in current transformers.

From the results, the value of excitation current is about 200 mA for primary current with no dc bias. Increasing the dc bias to 5%, accelerates the excitation current increase to 7 A, an increase of 35 times the excitation current with no dc bias.

Discussion

Transformer ratio is a function of the primary dc excitation. With dc biasing of very small magnitudes causes an increase in error levels that would exceed the limits established for metering and protection accuracy. Results obtained by Rowling [4] also indicated that current transformers with high permeability cores are affected the most by dc biasing. It was also found that even harmonics are affected the most by primary dc excitation. Phase difference between primary and secondary currents of current transformers is also affected by primary direct current excitation according to results by Rowling.

Experimental results by Aspnes in reference [5], showed a phase angle error of 4 degrees when a dc current was varied between 0% and 5%. Current transformer ratio changed by 2.93%. A primary current of 0.143 pu was used in experiments by Aspnes. This meant it was operated in the linear region of the magnetising curve. His results showed that harmonics were generated to 17th order. Increasing the dc bias produced high amplitudes of harmonics, however, the type of harmonics remained the same. Another interesting point to note is that errors were present in current transformers with dc bias currents, even when operating at normal rated primary current. Therefore more distortions could be expected with higher levels of primary current having dc bias.

Results by Rawling [4] showed that even a dc bias as little as 2% has a very pronounced effect on the transformation ratios of all the frequency components. Since direct current is not transformed by means of transformer action, all the direct current is used as magnetising current. Thus it would be expected that larger values of dc biasing current would affect the transformation ratios even more. Although two percent is quite small compared to the load current, and since direct current is not transformed by means of transformer action, all the direct current is magnetising current. Thus it would be expected that larger values of dc biasing current would affect the transformation ratios even more.
Results published by Aspenes [5] showed that besides the transformation errors, even harmonics, are generated due to the presence of the dc biasing current. It would seem that the even harmonics are generated on the primary winding and then reflected back to the secondary winding because they are of larger magnitude on the primary winding side. Therefore an increase in dc biasing current caused an increase in the even harmonic magnitude.

7.3 CASE II  **Effect Of 3 rd Harmonic In Primary Current On Secondary Output Current.**

References [6, 7] indicate that third harmonic values in a power system can be as high as eighty per cent in distributed single phase power electronic loads. It was for this reason that effects of third harmonics had to be simulated. The third harmonic is the most prominent harmonic in power systems. In three phase systems a third harmonic appears as a zero sequence component. The zero sequence currents might operate sensitive earth fault relays unintentionally in a power system. It was therefore, necessary to study this commonest source of zero sequence in current transformers.

Below is a low frequency model which was used in simulating this case.

![Diagram of a current transformer with a low frequency model](image_url)

**Figure 7.2**
Current Transformer
Low Frequency Model Used For EMTP Simulations
Where:

\[ a = \text{current source at power frequency (50 Hz).} \]
\[ b = \text{third harmonic current source.} \]
\[ L_1 = \text{power network inductance.} \]
\[ R_1 = \text{power network resistance.} \]
\[ \text{sw1} = \text{switch on primary side to simulate a fault condition.} \]
\[ R_{\text{emtp}} = \text{resistance required to avoid matrix singularity. (Refer to section 6.12.1.)} \]
\[ Z_m = \text{non-linear magnetising impedance.} \]
\[ R_f = \text{resistor for measuring core flux (Refer to section 6.11).} \]
\[ C_f = \text{capacitor required for measuring core flux.} \]
\[ \text{sw2} = \text{measuring switch (Refer to section 6.11).} \]
\[ Z_b = \text{current transformer secondary burden.} \]

**CT Data:**

- **Ratio:** 2000/5
- **Class:** C800
- **Burden:** 200 VA
- **Insulation:** 138 kV

**Procedure**

Figure 7.2, which is a low frequency model, was used to simulate the effect of varying a third harmonic content. An EMTP data case for this model has been presented in Appendix A Case 10. In the figure \( R_1 = 0.1 \Omega \) and \( X_1 = 1 \Omega \). \( X_1 = \omega L_1 \). A primary impedance ratio \( \frac{X_1}{R_1} \) ratio of ten was used to obtain a full offset asymmetry when switch (sw1) is closed to simulate a fault condition.

A resistor and capacitor combination in branch IR-IRG-ground is used for measuring core flux. A discussion on operation of this branch is given in chapter six. \( R_f = 1 \text{k} \Omega \) and \( C_f = 10 \mu \text{F.} \)

\( R_{\text{emtp}} \) was put to avoid matrix singularity as required by EMTP. This value was kept very small at 1.0E-5 \( \Omega \).

Values of \( Z_m \) were calculated internally in the EMTP program from flux - current data calculated from manufacturers specified V-I curves.

A rated resistive burden of 8 Ohm was used as \( Z_b \). The primary current amplitude was kept constant at 80 kA for all conditions simulated. A value of 80 kA was chosen to simulate a very high possible fault level in a system. (A fault level forty times rated primary current is assumed in this case.)
Values of third harmonic component were increased from 0 - 50% in steps of 5%. The effective primary current value was kept constant at 80 kA to ensure equal operating point on the saturation curve for all simulations.

Results with 0, 10, 20, 40, and 50 per cent third harmonics in their primary current have been presented in this report.

Observation

Transformation improved with increase in third harmonic content. This implies that high frequency components transformed faithfully in the secondary winding.

Generally, increasing third harmonic content on input current reduced transformation errors. Compare figures B4(a) and B4(e). In figure B4(a), an error between primary and secondary currents can be observed up to 120 msec while in figure B4(e) errors can be observed up to about 90 msec only.

A high dc transient offset can be observed in figure B4(a) and a very low offset in figure B4(e). This means increasing third harmonic content in primary current has the influence of reducing dc transient offset component.

Figures B5(a - e) present excitation current results obtained for different third harmonic content levels. Figure B5(a) has a maximum amplitude of about 225 A and this amplitude reduces when the third harmonic component is increased. At 50%, the excitation current amplitude has reduced to a maximum of 215 A. Another interesting observation is that exciting current reduced quickly when the third harmonic content was increased. In figure B5(a), an exciting current magnitude was still pronounced after 200 msec while in figure B5(e) it died away as quickly as after 100 msec.

Increasing third harmonic content did not have much influence on the first few cycles (one to three) of output current. However this increase had an effect of reducing excitation current as the current transformer went into steady state operation. See Figures B5(a - e).
7.4 CASE III Effect Of Increasing Fault Current

Current transformers are mostly subjected to high input currents in power systems due to high fault levels. Performance of current transformers in these situations need to be well understood to avoid maloperation of protection schemes. Current transformer mostly exposed to high fault levels are the low ratio type found in generation station [8]. Simulations in this case investigates the high input current effect on current transformers.

Procedure

A current transformer model and data in Case II with figure 7.2 were used. Source current ‘b’ was made dormant in this case.

Primary current source ‘a’ was varied from rated current of 2000A to 200000 A, hundred times the rated current. A value of hundred times rated primary current was chosen to simulate worst conditions that might occur for current transformer nearest to generating stations where fault levels are very high.

A rated secondary burden of 8 Ω resistive was used for all simulations. Core flux was measured for all conditions by measuring voltage across capacitor Cr. A detailed description of core flux measurement is presented in section 6.11.

Results obtained in these simulations were secondary current, core flux and excitation current. Results presented in this report include those with fault levels of 1, 10, 50 and 100 times rated primary current.

Observation

Output current became more distorted as primary current was increased. This means more harmonic content was present in the output current with a high fault level. The effect of increasing primary current was the same as keeping current constant and increase secondary burden.

Saturation occurred quicker when primary current was increased. Figure B6(a) which is an output with a primary current at rated value, has no saturation effects. However, when the primary current was increased to ten times, figure B6(b), saturation effects could be observed on the secondary output current. In this case one half cycle had to elapse before the secondary output current was distorted. This meant a delay of 10 msec before saturation occurred. Saturation effects only show in the first 100 msec when dc transient was still present. As the primary current was increased, the current transformer saturated much more quickly as evidenced in figures B6(c) and B6(d) where saturation happened in the first half cycle in a period less than 10 msec. Saturation continued into the steady state condition for these two cases.
Flux curves obtained in figures B7(a - d) can be described as follows. For an input primary current of rated value, the flux was sinusoidal though it had a transient dc component that gradually died down as the primary transient component diminished. In figure B7(b), the flux is seen chopped in the first few cycles when the current transformer went into saturation due to dc transient current. The dc component of the flux died out quickly as well due to the magnitude of the alternating flux which quickly reduced total offset flux value when it went into the opposite direction to that of the offset flux. The flux became sinusoidal when the offset flux died out as time increased.

For high primary current, the flux was chopped more which means the iron core saturated quickly. These saturation offsets are prominent on both positive and negative cycles of the sinusoidal waves. No dc transient can be seen with the high input currents. This was because the magnitude of alternating flux was very big and over shadowed effects of the unidirectional flux. Desaturation of this unidirectional flux was also possible with one or two opposite going cycles.

Excitation current for unity primary current input was very small at about 60 mA. Mainly this excitation current was composed of a dc component due to the dc transient primary current. As expected in unsaturated operation of current transformers, the dc component was not transferable to secondary side. Therefore it was used for excitation. Marshall and Langguth [9] explained that the primary current has but a small unidirectional magnetising current and hence cannot supply the unidirectional magnetising current required by the transformer. The magnetising current must therefore fall gradually to zero. This decreasing flux generates a unidirectional voltage which causes a current through the burden circuit. The current enables the flux to gradually decrease to steady state form and also causes an apparent shift in the zero line of the secondary current which is noticeable in figure B6(a).

At ten times input primary current, saturation occurred during the transient period only. This can be observed from the quickly dying excitation current with time in figure B7(b). The current transformer went into saturation on both positive and negative cycles with high primary current input. The excitation current had therefore asymmetrical output value when in steady state operation as seen in figures B7(c) and B7(d).
7.5 CASE IV Effect Of Different Fault Current Closing Angle

Current transformer simulations with different primary current closing angles were done to study their effect on current transformer performance. This study is significant for differential protection schemes. Different types of distorted current transformer output waves might be produced on different phases. These waves might have different resultants from expected ones having a consequence of protection failure, delayed operation or unwanted operation of protection schemes in differential protection schemes.

Procedure

Figure 7.1 with the same current transformer data was used for simulating effect of different primary current closing angles on current transformer performance. A rated burden of 15 Ω pure resistive was used as $Z_2$.

Primary resistive and inductive impedances, $R_1 = 0.1\Omega$ and $X_t = 1\Omega$, were used respectively. These values were used to simulate a full offset primary current. Only a power frequency source current 'a' was used. Source 'b' was neglected.

A fault current of fifty times rated current transformer primary current was used for all simulation cases. Fault angles were increased from $0^\circ$ to $330^\circ$ in steps of $30^\circ$. In this report, results for $0^\circ$, $30^\circ$, $60^\circ$, $90^\circ$, $120^\circ$, $180^\circ$, $210^\circ$, $270^\circ$ and $330^\circ$ are shown.

Observation

It can be observed from exciting current curves, figures B10(a - i), that different switching angles produced different type of curves. It can be concluded therefore that an angle on which a fault occurs on the primary current will have an influence on current transformer saturation behaviour. Figures B9(a - i) are curves of secondary output current versus primary input current. The aim of these curves was to investigate effects of different closing angles on output currents and transformation.

An important point to note in the curves is that current transformers with closing angles at $0^\circ$, $270^\circ$ and $330^\circ$ have maximum amplitudes on negative half cycles while simulation results with closing angles at $60^\circ$, $90^\circ$, $120^\circ$ and $180^\circ$ have maximum amplitudes on positive half cycles. Figures B9(b) and B9(c) with closing angles of $30^\circ$ and $210^\circ$ have got almost symmetrical curves but figure B9(b) have one cycle offset on the negative, while figure B9(g) has an offset on the positive side. These offsets in figures B9(b) and B9(g) are due to the first half cycle in output current before saturation occurred.
Maximum transformation errors occurred at $120^\circ$ on the positive half cycle and $330^\circ$ on the negative half cycle according to figures B10(e) and B10(i) respectively. It can be concluded that different closing angles on a current transformer will influence the amplitude of input current and consequently transformer errors.

7.6 CASE V  **Current Transformer Simulations Having Different Types Of Secondary Burden.**

The aim of simulations in this case was to compare EMTP simulated results with transient and steady state saturation theory results in chapter five.

**Procedure**

Current transformer model of Figure 7.2 was used for simulations in this case. Current source (b) was neglected. The burden impedance $Z_b$ was changed to different burden types for different simulations.

Types of burden used were:

1. Resistive
2. Resistive and inductive in series
3. Inductive
4. Capacitive
5. Capacitive and resistive in series

A rated burden resistance value of 8 $\Omega$ was used for pure resistive burden condition. A fault level of fifty times rated input current was used.

In R -L burden, $R = 4$ $\Omega$ and $L = 6.3$ mH. For this combination, saturation could not be obtained quickly so the primary current had to be increased to 200000 A.

A value of 1$\Omega$ was used for a pure inductive burden. This value was within the range of typical inductive values for current transformers. In this simulation winding resistance was ignored.

For a pure capacitive burden a capacitor value of 1$\mu$F was used. In the last simulation for resistive and capacitive burden, a resistor value of 4 $\Omega$ and capacitor value of 1$\mu$F in series were used.
Observation

Output with a resistive burden was more distorted in the first few cycles during a period when dc transient was still present. The simulated result compared very well with the theoretical ones in figure 5.3. The only difference was that some output current was still being produced even after saturation had been reached in the simulated result. This is practically correct as slight flux increase is possible even after saturation level is reached.

In an inductive burden, the secondary output is supposed to have a flat top when saturation occurs as shown theoretically in figure 5.4. A similar result is obtained in the EMTP simulation as presented in figure B11(c) though the tops are not exactly flat. A high primary current of 200000 A was used before the current transformer could not go into saturation with an inductive burden. A pure resistive burden with the same value saturates the current transformer faster.

The result for resistive and inductive burden in series is an intermediate between pure resistive and pure inductive burdens. The result in figure B11(b) shows that the wave had exponential decay on both positive and negative half cycles. In the theoretical result in figure 5.6, saturation is only unidirectional because unidirectional offset saturation was assumed.

Saturation occurred the quickest for pure capacitive burden. A fault current of as little as 1000 A, which is half times the rated current transformer input current, saturates the current transformer severely as observed in figure B11(d). This result compares well with those in Figure 5.7. Note that spikes on the secondary output current are very high making secondary output current very impropotional to current transformer ratio. Secondary current amplitudes of more than 100 A are obtained with a primary current of 1000 A only. The high spikes in current can be attributed to effects of ferroresonance.

Results for RC burden are comparative as well as observed in figures 5.8 and B11(e). However there is a slight difference in that the simulation result has two spikes on the positive side followed by one on the negative. This shows how unpredictable a capacitive burden output can be. More capacitive burden output can be found in reference [10].

The behaviour of series capacitive and resistive burden is better than that of pure capacitive burden. However capacitive and resistive burden proved to produce the most unpredictable secondary current waveforms. There are three possible outputs with each burden values. However results obtained agree with those in theory. A rated primary input current of 2000A was used as fault current. With this current, the current transformer went into saturation. The output current had spikes as high as 150 A.
Figures B12(a to e) are results of excitation current for different types of burdens simulated and figures B13(a to e) are core flux results. Rather different results between theoretical and simulated results are obtained for RC burden. It has to be borne in mind that three possible output waves are possible for this type of burden [10].

In general, there are several factors affecting the shape of secondary output current. Factors ranging from remnance, fault angle, type of burden, fault level, etc. It is very important therefore for protection engineers to be very conversant with these influences on current transformers when designing protection schemes.

7.7 CASE VI Effect Of High Frequency On Current Transformers

Study of high frequency effects on current transformers is very important due to increased presence of harmonics in power systems. Meanwhile, there are no standards for current transformer operation and performance with polluted primary current conditions. Study of high frequency currents include; transient frequencies which could be as high as 30 kHz. High voltages might be set up in the secondary circuit when current transformers are operated with high transient currents. Such voltages are of short duration due to the rapid decay of the transient. Other high frequency sources are lightning and switchings.

Procedure

Current transformer model and data of figure 7.1 were used. A pure resistive burden of rated value (15Ω) was used in all simulations. Current source ‘b’ should be neglected. Primary resistive and inductive impedances, $R_1 = 0.1\Omega$ and $X_1 = 1\Omega$, were used respectively. A fault current of twenty times rated primary input current was used in all simulations in order to operate the current transformer around knee point.

Frequency input to a current transformer was varied from 50 Hz to 1 MHz. In this report presentations are made for 50 Hz, 100 Hz, 150 Hz, 1 kHz, 1.5 kHz, 10 kHz and 15 kHz.
Observation

Figures B14(a to g) are results of secondary current versus primary current. Figure B14(a) shows some errors in about one half cycles during transient period, thereafter the transformation was linear. No vivid transformer errors were present for second order harmonic source as seen in figure B14(b). Note that figures B14a, d, f and g which are curves for 50 Hz, 1000 Hz, 10 kHz and 15 kHz respectively have maximum amplitudes on primary current on negative going cycles. While 150 Hz and 1500 Hz which are figures 14(c) and 14(e) respectively have maximum amplitudes on positive cycles. It can be concluded therefore that a frequency component will influence offset primary current and consequently affect current transformer transformation. Results by Douglas [2] showed that high frequency fault current errors due to stray capacitance become prominent after 10 kHz. It can be concluded that source of error at frequencies above 50 Hz does not depend primarily on the core exciting current, but rather on the secondary leakage inductance and secondary capacitance to ground.

Figures B15(a- g) are results for exciting current versus primary current curves. These results show excitation errors due to each frequency studied. The worst error occurred for a 50 Hz simulation where excitation current of more than 20A was obtained, the general trend was that excitation current decreased gradually when frequency was increased. This trend reverses when frequencies become very high due to effect of stray capacitance which become prominent and errors increase. This was verified in this thesis but results could not be printed as the output data were very big to be transferred from PCPLOT to MS Word graphic document.

Increasing the harmonic order, reduced excitation current magnitudes. An exception was the second order harmonic, 100 Hz input current, which had the least error, about 140 mA. Increasing the even harmonic order, increased the exciting current output. This was only true with low order harmonics up to tenth order. However, as frequency is increased to several kHz, there is no real difference between excitation current outputs of odd and even harmonics.

In general even harmonics are almost perfectly transformed in lower harmonic orders. On the other hand odd harmonics introduce a lot of errors with lower harmonic orders. As pointed out earlier, as frequencies become very high errors are mainly due to capacitances and there are no vivid difference between even and odd harmonic losses.

In another study by F. Leon and A. Semlyen [11], the value of secondary impedance increased up to a certain extent before it started to decrease. Note that in this study the value of secondary impedance considered included resistance, inductance and capacitance. The decrease in impedance when increasing frequency can be attributed to presence of capacitance.
At very high frequencies, the effect of inductance will tend to cancel with the effect of capacitance, in the end the effective impedance is not that much as it is reduced naturally. The effective impedance might be that of resistance with skin effect.

**Discussion**

Eddy current losses in a core are proportional to $\omega^2 \phi_0^2$ [12] and the hysteresis losses resistance increases with frequency. This increase in resistance will reduce excitation current and thus a more accurate transformation would be obtained. Any high frequency components present in the current of a transformer will be satisfactorily transformed. In fact, the accuracy of reproduction will be higher than that obtained at the normal power system frequency, unless the burden is predominantly inductive, in which event a slightly poorer, but nevertheless acceptable performance should be obtained.

The above generalisation holds true for frequencies up to several kHz only. When the frequencies become extremely high, currents are filtered by the stray capacitances. However, for power system conditions current transformers will perform satisfactorily in presence of high frequencies.

In the higher frequency range there are two factors that affect the ratio characteristic namely the increase in the core losses and the effect of the stray capacitances and inductances of the windings. The stray inductances and capacitances might cause resonant peaks to occur at high frequencies.
7.8 Conclusions

This section concludes results obtained in this thesis and summarises conclusions of current transformer behaviour under different conditions studied have been presented. Some recommendations for current transformer selection have been given.

Below are some conclusions drawn from simulations carried out in this thesis.

The presence of any dc current component severely affects the transformation ratio as well as the phase angle error. Not only is the ratio error increased but because of the offset, the phase angle between the voltage and the magnetizing current changes dramatically, causing an increase in the phase angle between the primary and the referred secondary current. The phase angle error increases because the primary current is obtained by adding the referred secondary current and the magnetizing current phasors.

Dc biasing affects even harmonics quite dramatically. This was observed from increased even harmonics output from current transformer output current. However, the odd harmonics are not too adversely affected by the presence of a dc component.

Secondary output currents are affected by dc bias very much. The current ratio is a function of primary dc excitation. A primary dc excitation has a small effect on the secondary current frequency components. Even harmonics are most greatly affected when a current transformer is operating with a dc bias.

The effect of dc bias is to lower effective knee point of the current transformer. This effectively permits the current transformer to saturate at lower levels of primary current.

Increasing a third harmonic content in current transformers input current had the effect of reducing dc transient offset component. This reduction in dc transient component has a consequence of improving transformation.

Primary offset transient current is affected by phase angle on which a fault occurs in the power network. The offset transient current in turn, determines current transformer saturation behaviour. A large offset will saturate a current transformer quickly. Closing angles also determined whether the offset current would be on positive or negative amplitudes.
High primary fault currents led to more distortion in secondary output current. This is similar to effect of high secondary burden which have the effect of quickly saturating current transformers.

When different types of burden, that is resistive inductive and capacitive, were considered. It was observed that current transformer with capacitive burden saturated with primary current less than its rated value. A current transformer with inductive burden did not saturate very fast.

7.9 Recommendations

A solution for dc bias current would be to have air gapped cores. However careful consideration should be done as air gaps in current transformers increase the magnitude of excitation current required and more errors might be introduced.

Care should be taken when testing for polarity of a current transformer as a common procedure is to use a battery which might leave some remnance in the current transformer. Therefore, it should be recommended that the test facility must have a means to demagnetise the current transformer before and after the test.

Apply high ratio current transformers near high fault level locations. A conservative limiting condition would be for maximum fault current not to exceed 20 times the selected current transformer current rating with the total secondary external burden not greater that the current transformers standard burden rating.

In locations with high fault current but high ratio current transformer can not be installed, restrict the ground fault current level with a system neutral resistor sufficient to limit the fault current to an acceptable value no more than 20 times the current transformer load rating.

Relays with characteristics to withstand heavy faults with saturation should be used in places where fault levels are very high. Alternatively, optical fibre measurement systems could be used in high fault level current locations.

All in all the current transformer performance was successfully simulated using EMTP. However, improved modelling of non-linear elements to include eddy current losses are still required in EMTP. A wide choice of iron core material data base for magnetising inductance is also required in EMTP.
REFERENCES

Chapter One


Chapter Two


Chapter Three


**Chapter Four**


Chapter Five


Chapter Six


Chapter Seven


APPENDIX A

DATA FILES

Some data files used during simulations have been presented in this appendix.

FILE 1: A general outline of an ATP-EMTP data file.

FILE 2: Data file CONVERT for changing V-I (rms.) values of saturation curve to flux - current curve.

FILE 3: Data file HYSDAT for interpreting the knee point V-I (rms.) values of saturation curve into a hysteretic non-linear reactor.

FILE 4: A TACS incorporated file for measuring flux of current transformers.

FILE 5: A MODEL program for simulating a transient high frequency current source.

FILE 6: A current transformer model with all parameters referred to secondary side. A TACS Hybrid to generate magnetising flux wave is incorporated.

FILE 7: A MODEL program for simulating Dual Circuit in Figure 4.9

FILE 8: A MODELS program for simulating current transformer with high frequency input transient current source. This program takes into account saturation.

FILE 9: A data case used to simulate current transformer model of Figure 7.1

FILE 10: A data file used to simulate current transformer model of Figure 7.2
A General Outline Of An ATP-EMTP Data File.

FILE 1
C A GENERAL DATA CARD LAYOUT
BEGIN NEW DATA CASE
C This file is a data case layout generated by ATPDRAW
$PREFIX,F:\ATPDRAW\LIB$
$SUFFIX, .LIB$
$DUMMY, XYZ000$
C Miscellaneous Data Card ....
POWER FREQUENCY 5.0E+01
1.0E-06 1.0E-03 0.0E+00 0.0E+00
500 1 1 1 0 0 1 0
C 1 2 3 4 5 6 7 8
C 345678901234567890123456789012345678901234567890123456789012345678901234567890
C BLANK BRANCH
BLANK SWITCH
BLANK SOURCE
BLANK OUTPUT
BLANK PLOT
BEGIN NEW DATA CASE
BLANK

FILE 2
Data File CONVERT For Changing V-I (Rms.) Values Of Saturation Curve To Flux - Current Curve

C CONVERT DATA CARD
BEGIN NEW DATA CASE
$ERASE$
$SATURATION$
C --Voltage Current
50.0 1.0E-3 1.0E-6 1 0
C Irms Vrms
C This is the section where V-I values from manufacture are entered.
9999
$PUNCH$
BEGIN NEW DATA CASE
BLANK CARD

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FILE 3  Data File HYSDAT For Interpreting The Knee Point V-I (Rms.)
Values Of Saturation Curve Into A Hysteretic Non-Linear Reactor.

C file ctv.atp
C HYSDAT DATA FILE
BEGIN NEW DATA CASE
HYSTERESIS
C ITYPE  LEVEL
  1  3  0
C CURSAT  FLUX
 28.145960.675237
SPUNCH
BLANK CARD
BEGIN NEW DATA CASE

FILE 4  A TACS Incorporated File For Measuring Flux Of Current
Transformers.

C This TACS subroutine can be used in conjunction with a main routine in EMTP
/BRANCH
C 10000-HENRY FLUX MONITORING INDUCTANCES TO INITIALIZE TACS.
C
51SPRE1ASPX1DA               3769911.185
/SWITCH
C MEASURING SWITCHES TO MONITOR CURRENT IN 10000-HENRY FLUX-
MONITORING INDUCTOR.
C <NAME><NAME><T CLOSE ><T. OPEN ><AMP MARG> 67890123 MEASURING
56789012345678 P
SPX1DA                     MEASURING
/TACS
C NFLUX IS PROPORTIONAL TO THE INDUCTIVE CURRENT IN THESE SWITCHES:
C TYPE 91 TACS SOURCES DERIVED FROM NETWORK SWITCH CURRENTS:
C <NAME>  <----A----> <----A----> <----A----> <----A---->
C <START>  <STOP>
91SPX1DA                   -1.0 999
C FORTRAN STATEMENTS, 99= INPUT, 98= OUTPUT, 88= INSIDE
C <NAME>  <- FREE FORMAT FORTRAN TO COL 80
C MULTIPLY BY 10E4 TO GET FLUX-TURNS
99SPX2DA = SPX1DA • 1.0E4
C TACS OUTPUT REQUESTS - TYPE 33
C
33SPX2DA

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A MODEL Program For Simulating A Transient High Frequency Current Source.

BEGIN NEW DATA CASE
SOPEN, UNIT=4 FILE=F:ISO.PL4 FORM=FORMATTED
C miscellaneous data cards
  1.0E-6  1.0E-3
  100  1  1
MODELS
OUTPUT scr
MODEL so
  DATA tstart {dflt:0}
   tstop {dflt:1000}
    io {dflt:1.0}
    freq {dflt:550}
    angle {dflt:0}
  VAR cosine, omega, cfin, ampl
  OUTPUT cosine
  INIT omega:=2*pi*freq
ENDINIT
EXEC
  ampl := io * exp(-2.0E3 * t)
  cosine := ampl*cos(omega*(t-tstart) + angle*pi/180) * AND((t-tstart),(tstop-t))
ENDEXEC
ENDMODEL
USE so AS so
OUTPUT scr := cosine
TIMESTEP MIN 0
ENDUSE
RECORD
  so cosine AS cosine
  so cfin AS cfin
MODEL sol
INPUT dcos
VAR dcos, dcosa
EXEC
  dcosa := DERIV(dcos)
ENDEXEC
ENDMODEL
USE sol AS sol
TIMESTEP MIN 0
INPUT dcos := scr
ENDUSE
RECORD
  sol dcosa AS dcosa
ENDMODELS
C
C BRANCH CARDS
SCR 10.0 3
BLANK CARD ENDING BRANCHES
BLANK CARD ENDING SWITCHES
14SCR 100.0 50.0
60SCR 0.0 1.0
BLANK CARD ENDING SOURCES
SCR
BLANK CARD
BLANK CARD ENDING PLOTS
BEGIN NEW DATA CASE
BLANK
FILE 6  A Current Transformer Model With All Parameters Referred To Secondary Side. A TACS Hybrid To Generate Magnetising Flux Wave Is Incorporated

C FILE MAVUTO.ATP
BEGIN NEW DATA CASE
C
POWER FREQUENCY  5.0E+01
C Miscellaneous data cards
5.0E-05  1.0E-1  5.0E+01  5.0E+01
1  1  1  0  0  0  0  1
TACS HYBRID
90V2
99VSUM  58+V2  1.0  0.0  1.0
33V2  VSUM
BLANK CARD ENDING TACS DATA
C
$OPEN, UNIT=4 FILE=F:\CTF.pl4 FORM=FORMATTED
C
1  2  3  4  5  6  7  8
C 34567890123456789012345678901234567890123456789012345678901234567890
C Non linear Inductor
C    curr flux resid
C
96V2     8888 .47042
C HYSTERESIS
C C ITYPE  LEVEL
C 1  3  0
C C CURSAT FLUX
C 28.145960  675237
1.05547350E+01 -6.59349071E-01
-27736750E+00 -6.51405106E-01
-1.75912250E+00 -6.31545194E-01
-3.51824500E-01 -6.16852828E-01
6.15692875E-01 -5.64021494E-01
1.16102085E+00 -4.76637882E-01
2.11094700E+00  3.41590482E-01
3.34233275E+00  4.88553829E-01
4.74963075E+00  5.48133565E-01
7.03649000E+00  5.95797353E-01
1.03788227E+01  6.27573212E-01
1.61839270E+01  6.51405106E-01
2.81459600E+00  6.75237000E-01
3.87006995E+00  6.79208982E-01
9999

V2  VR   0.253
VL   1.300
E    1.0E6
DUMMY  1.0
FILE 6  continued....

BLANK CARD ENDING BRANCH DEFINITIONS
VR  VL MEASURING 1
E  V2 .0090 10. 1
BLANK CARD ENDING SWITCH CARDS
14DUMMY 1.41 50 -1. 1.000
14E -1 200.00 50 -1 1.000
BLANK CARD ENDING SOURCE DEFINITIONS
BLANK CARD ENDING PLOT CARDS
BLANK END OF SIMULATION
BEGIN NEW DATA CASE
BLANK CARD
A MODEL Program For Simulating Dual Circuit.

BEGIN NEW DATA CASE
SOPEN, UNIT=4 FILE=F:\SO PL4 FORM=FORMATTED
C miscellaneous data cards
1.0E-4 1.0E-1
1000 1 1

MODELS
OUTPUT scr
MODEL so
DATA tstart (dflt:0)
tstop (dflt:1000)
ampl (dflt:10.0)
freq (dflt:50)
angle (dflt:0)
VAR cosine, omega, cfin
OUTPUT cosine
INIT omega = 2*pi*freq
ENDINIT
EXEC
cosine := ampl*(cos(omega*(t-tstart) + angle*pi/180))
*AND((t-tstart),(tstop-t)) { MAX : 9 MIN : -9}
IF t < 1.0E-8 THEN
cfin := 0
ELSE
cfin := DERIV(cosine)*2.880E-3
ENDIF
ENDEXEC
ENDMODEL
USE so AS so
OUTPUT scr := cosine
TIMESTEP MIN:0
ENDUSE
RECORD
so.cosine AS cosine
so.cfin AS cfin
ENDMODELS
C BRANCH CARDS
SCR 10.0
BLANK CARD ENDING BRANCHES
BLANK CARD ENDING SWITCHES
14SCR 100.0 50.0
60SCR
BLANK CARD ENDING SOURCES
SCR
BLANK CARD
BLANK CARD ENDING PLOTS
BEGIN NEW DATA CASE
BLANK
A MODELS Program For Simulating Current Transformer With High Frequency Input Transient Current Source. This Program Takes Into Account Saturation.

BEGIN NEW DATA CASE
C Current transformer simulation having a decaying transient input source
C This is a MODELS program.
$OPEN, UNIT=4 FILE=F:\SO PL4 FORM=FORMATTED
C miscellaneous data cards
1.0E-4 1.0E-1
100 1 1
MODELS
OUTPUT scr
MODEL so
DATA tstart {dflt:0}
tstop {dflt:1000}
ampl {dflt:1.0}
 freq {dflt:550}
angle {dflt:0}
VAR cosine, omega, cfin
OUTPUT cosine
INIT omega:=2*pi*freq
ENDINIT
EXEC
  cosine:=ampl*exp(-4.5E1*(t-tstart))*(cos(omega*(t-tstart)+angle*pi/180))  
  *AND((t-tstart),(tstop-t)) {MAX : 9 MIN :-9}
IF t < 1.0E-8 THEN
  cfin =0
ELSE
  cfin =DERIV(cosine)*2.880E-3
ENDIF
ENDEXEC
ENDMODEL
USE so AS so
OUTPUT scr := cosine
TIMESTEP MIN:0
ENDUSE
RECORD
  so cosine AS cosine
  so cfin AS cfin
ENDMODELS
C
C BRANCH CARDS
SCR 10.0 3
BLANK CARD ENDING BRANCHES
BLANK CARD ENDING SWITCHES
14SCR  100.0  50.0
60SCR  0.0  1.0
BLANK CARD ENDING SOURCES
SCR
BLANK CARD
BLANK CARD ENDING PLOTS
BEGIN NEW DATA CASE
BLANK
CASE 9  A Data Case Used To Simulate Current Transformer Model Of Figure 7.1

BEGIN NEW DATA CASE
SOPEN, UNIT=4 FILE=F:\THES1.PL4 FORM=FORMATTED
2.0E-5 3.0E-1 00. 00.
2000 30 1 1
C TRANSFORMER
TRANSFORMER 1.0E-41.E-5 BUSA
9999
01CTH01 6.0E-51.E-4 0.001
02CTX01 0.41 4.E-4 0.200
C SATURATION
C 50.0 1.00E-3 1.00E-6 1 0
C Irms Vrms
C 0.9900E-01 0.215E+03
C 0.4500E+00 0.300E+03
C 0.8100E+00 0.308E+03
C 0.1000E+01 0.312E+03
C 0.1300E+01 0.316E+03
C 9999
98IR
3.95979797E-02 1.89066426E-01
5.59812411E-02 3.60126526E-01
9.16669484E-02 6.75237237E-01
1.42113995E-01 9.67840040E-01
2.36011828E+00 1.38648713E+00
9999
CTX01 IR 1.E-5
CTX01 LDX 1.E-3
LDX 0.03
LDX 15. 1
SCR 1.E6 3.E3
BLANK CARD TO END BRANCH DATA
C SWITCH DATA CARDS
SCR CTH01 -1 0.27
IR LDX -0.10 10.0
BLANK CARD TO END SWITCH DATA
C SOURCE CARD
14SCR -1 4000 50.0
11SCR -1 1000
BLANK CARD TO END DATA
BLANK CARD END OF NODAL VOLT OUTPUT REQUEST
BLANK CARD
BEGIN NEW DATA CASE
BLANK

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FILE 10: A Data File Used To Simulate Current Transformer Model Of Figure 7.2

BEGIN NEW DATA CASE
2.0E-5 3.0E-1 50 50
2000 30 1
$OPEN, UNIT=4 FILE=F:THES2.PL4 FORM=FORMATTED
C TRANSFORMER
TRANSFORMER 1.0E-4 1.E-5 BUSA
9999
01CTH01 6.0E-5 E-4 0.001
02CTX01 1.E-4 0.500
C <++++++> Cards punched by support routine on 08-Nov-95 11:35:20 <++++++>
C SATURATION
C C CT ratio => 2000:5
C C 1 2 3 4 5 6 7
C C 3457890123456789012345678901234567890123456789012345678901234567890123456789
C C freq Vbase Phase punch quads
C C Hz  kV  MVA
C C 50.0 1.0E-3 1.0E-6 1 0
C C I rms Vrms
C 0.0400 200.0
C 0.0500 300.0
C 0.0600 380.0
C 0.0900 610.0
C 0.1600 755.0
C 0.2200 890.0
C 0.3980 850.0
C 0.7000 873.0
C 0.7800 877.0
C 1.4000 893.3
C 2.0000 900.0
C 3.5080 903.0
C 9999
98IR
5.65685425E-02 9.00316316E-01
6.26494650E-02 1.35047447E+00
8.17343081E-02 1.71060100E+00
9.90654813E-02 2.25079079E+00
1.27259218E-01 2.74596476E+00
2.80825713E-01 3.39869409E+00
4.52510250E-01 3.60126526E+00
9.31346073E-01 3.82634434E+00
2.27066698E+00 3.84788705E+00
4.35937667E+00 4.02126283E+00
6.98956683E+00 4.05142342E+00
1.67692882E+01 4.06492817E+00
9999
IR  IRG  1.0E5  
IRG  318.3  
CTX01 IR  1.E-5  
LDX  8.0  
SCR  0.10 1.00  
BLANK CARD TO END BRANCH DATA  
C SWITCH DATA CARDS  
SCR CTH01 0.00318  0.27  
IR LDX -0.10  10.0  
BLANK CARD TO END SWITCH DATA  
C SOURCE CARD  
14SCR -1  80000  50.0  
BLANK CARD TO END DATA  
BLANK CARD END OF NODAL VOLT OUTPUT REQUEST  
BLANK CARD  
BEGIN NEW DATA CASE  
BLANK
APPENDIX B

SIMULATION RESULTS


CASE II: Effect Of Different Amplitudes Of Third Harmonic Content On Current Transformer Performance.


CASE V: Different Waveforms Generated By Different Types Of Burden On Current Transformer Secondary Winding.

CASE I

Effect Of Input Current De Bias On Current Transformer Performance

- **(a)**
  - 0% DC Bias
  - Exciting Current vs Primary Current.

- **(b)**
  - 5% DC Bias
  - Exciting Current vs Primary Current.
Figure B1
Case I: Excitation Current vs Primary Current Curves
   a) Primary Current 0% De Bias
   b) Primary Current 5% De Bias
   c) Primary Current 25% De Bias
CASE I Continued.

(a) 0% DC Bias
Primary Current And Secondary Current Waves.

(b) 5% DC Bias
Primary Current And Secondary Current Waves.
25% DC Bias
Primary Current And Secondary Current Waves

Figure B2, CASE I
Primary Current And Secondary Current Waves Of DC Bias Simulations
Solid line - Primary Current Referred To Secondary Side
Broken line - Secondary Current
CASE I Continued..

(a) 0% DC Bias
Secondary Current Vs Primary Current.

(b) 5% DC Bias
Secondary Current Vs Primary Current.
Figure B3, Case I
Secondary Current vs Primary Current Curves Of
De Bias Simulations.
CASE II

Effect Of Different Amplitudes Of Third Harmonic Content On Current Transformer Performance

(a) 0% - 3rd Harmonic Content. Secondary And Primary Current Outputs.

(b) 10% - 3rd Harmonic Content. Secondary And Primary Current Outputs.
(c)
20% - 3rd Harmonic Content
Secondary And Primary Current Outputs.

(d)
40% - 3rd Harmonic Content
Secondary And Primary Current Outputs.
Figure B4, Case II
Primary and Secondary Current Waveforms for Effect of 3rd Harmonic Simulations.

- Solid Line - Primary Current
- Broken Line - Secondary Current Referred To Primary Side

50% - 3rd Harmonic content
CASE II Continued

(a) 0% - 3rd Harmonic Content
Exciting Current Waveform.

(b) 10% - 3rd Harmonic Content
Exciting Current Waveform

(c) 20% - 3rd Harmonic Content
Exciting Current Waveform
Figure B5, Case II
Excitation Current For Various 3rd Harmonic Contents
CASE III

Effect Of Increasing Fault Current On Current Transformer Performance

(a) Secondary Current Output With Input Current 1 Times Rated Current.

(b) Secondary Current Output With Input Current 10 Times Rated Current
Secondary Output Currents For Simulations With Different Fault Levels.
CASE III Continued...

(a) Magnetising Flux For 1 Times Rated Input Current

(b) Magnetising Flux For 10 Times Rated Input Current
(c) Magnetising Flux For 50 Times Rated Input Current.

(d) Magnetising Flux For 100 Times Rated Input Current

Figure B7, Case III
Magnetising Flux Output For Different Multiples Of Input Current
CASE III Continued...

(a) Exciting Current At 1 Times Rated Input Current

(b) Exciting Current At 10 Times Rated Input Current

(c) Exciting Current At 50 Times Rated Input Current
Exciting Current At 100 Times Rated Input Current

Figure B8, Case III

Excitation Currents For Simulations With Different Multiple Input Currents.
CASE IV

Effects Of Different Fault Angles Of Primary Currents On Current Transformer Performance

(a)
Secondary Current Vs Primary Current
Fault Angle At 0 Degrees

(b)
Secondary Current Vs Primary Current
Fault Angle At 30 Degrees
(c) Secondary Current Vs Primary Current
Fault Angle At 60 Degrees

(d) Secondary Current Vs Primary Current
Fault Angle At 90 Degrees
(e) Secondary Current Vs Primary Current
Fault Angle At 120 Degrees

(f) Secondary Current Vs Primary Current
Fault Angle At 180 Degrees
(g) Secondary Current Vs Primary Current
Fault Angle At 210 Degrees

(h) Secondary Current Vs Primary Current
Fault Angle At 270 Degrees
Figure B9

Secondary Current Vs Primary Current Curves For Different Primary Current Fault Angles.
CASE IV: Continued..

(a) Exciting Current At Fault Angle Of 0 Degrees

(b) Exciting Current At Fault Angle Of 30 Degrees
Exciting Current At
Fault Angle Of 60 Degrees

Exciting Current At
Fault Angle Of 90 Degrees
(e) Exciting Current At
Fault Angle Of 120 Degrees

(f) Exciting Current At
Fault Angle Of 180 Degrees
(g) Exciting Current At Fault Angle Of 210 Degrees

(h) Exciting Current At Fault Angle Of 270 Degrees
Exciting Current At Fault Angle Of 330 Degrees

Figure B10
Excitation Current For Primary Currents With Different Fault Angles
CASE V

*Waveforms Generated From Different Types Of Burden On Current Transformer Secondary Winding.*

(a) *Secondary Current With Resistive Burden*

(b) *Secondary Current With Resistive And Inductive Burden*
(c) Secondary Current With Inductive Burden

(d) Secondary Current With Capacitive Burden
Secondary Output Currents For Different Types Of Current Transformer Burdens

Figure B11

Secondary Current With Capacitive and Resistive Burden
CASE V: Continued

(a) Exciting Current With Resistive Burden

(b) Exciting Current With Resistive And Inductive Burden

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(c) Exciting Current With Inductive Burden

(d) Exciting Current With Capacitive Burden
Exciting Current With Capacitive And Resistive Burden

Figure B12

Excitation Current For Current Transformer With Different Types Of Secondary Burden.
CASE V: Continued ..

(a) Magnetising Flux With Resistive Burden

(b) Magnetising Flux With Resistive And Inductive Burden
(c) Magnetising Flux With Inductive Burden

(d) Magnetising Flux With Capacitive Burden
Figure 13

Magnetising Core Flux Due To Different Types Of Current Transformer Secondary Burdens
CASE VI

Effect Of High Frequency Input Current On Current Transformer Performance

(a) Secondary Current Vs Primary Current At 50Hz

(b) Secondary Current Vs Primary Current At 100Hz

164
(c) Secondary Current Vs Primary Current
At 150 Hz

(d) Secondary Current Vs Primary Current
At 1000 Hz
(e) Secondary Current Vs Primary Current
At 1500 Hz

(f) Secondary Current Vs Primary Current
At 16 kHz
(g)
Secondary Current Vs Primary Current
At 15 kHz

Figure B14
Secondary Current Vs Primary Current Of Frequency
Scan On A Current Transformer
CASE VI Continued..

(a)  
50 Hz  
Excitation Current vs Primary Current

(b)  
100 Hz  
Excitation Current vs Primary Current
(c) 150 Hz
Excitation Current vs Primary Current

(d) 1000 Hz
Excitation Current vs Primary Current
(e) 1500 Hz
Excitation Current vs Primary Current

(f) 10 kHz
Excitation Current vs Primary Current
Figure B15
Excitation Current vs Primary Current
For Input Frequencies Between 50 Hz and 15 kHz