"THE DEVELOPMENT OF AN ANCILLARY TEXTILE INDUSTRY WASTE TREATMENT PROCESS".

BY


A Thesis submitted in partial fulfilment of the requirements for the M.Sc.(Eng.) degree in the Faculty of Engineering, University of Cape Town.

Department of Civil Engineering,
University of Cape Town.
September, 1971.
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DECLARATION BY CANDIDATE.

I, Norman Greenblau, hereby declare that this thesis is my own work and that it has not been submitted for a degree of another University.

September 1971.
SYNOPSIS.

The objective of this thesis is twofold —

1. to design the unit processes for treating the waste from a specific industrial plant using the best practicable technology, and
2. to critically examine the theory of these unit processes for utilization in design.

The industrial waste was derived from a synthetic button and textile trimmings factory producing polyester and casein buttons. The effluent has a high pH and COD; contains metallic poisons; and is highly coloured. It exceeds the limitations for effluent quality promulgated by the Cape Town Municipality before discharge to the sewers. A certain measure of pretreatment before discharge to the sewers was therefore required.

The treatment process eventually selected consists of neutralization; flocculation; sedimentation; sand filtration of the sludge; and atmospheric drying of the sludge.

Each of the proposed unit operations was investigated experimentally to obtain data for optimum design:

Optimum flocculation occurred when the effluent was neutralized from a pH of 11 to a pH of 5; then flocculated with 500ppm of aluminium sulphate; and sufficient sodium aluminate added to bring the final pH value within the narrow range of pH=5.8 to pH=6.0. The sequence of chemical additions, and the pH limits, were found to be crucial.

No unusual pattern emerged from the sedimentation process experiments.

Experiments on the sand filtration of the sludge indicated that constant
head filtration was superior to falling head drainage in general, confirming the theoretical prediction in this respect. The sand filter loading rate was optimum when the sludge slurry was fed to a depth of 10-15 cm at constant head for approximately one day. Atmospheric drying could then proceed until the sludge is spadeable, at a moisture content of less than 8.5 g water/g dry solids. This required a total period of seven days on the sand filter. Experiments on the atmospheric drying of the drained sludge on the sand beds at constant temperature, humidity and air velocity, indicated that there was a significant deviation between theoretically predicted drying times and experimental observations.

A design chart is developed for simulating the operation of the settling tank with variable flow rates and influent solids concentrations. In addition, two modified filtration loading rate equations are proposed, the one taking into account the resistance of the filter media (which is significant at low sludge depths and concentrations), and the other one in which the filter media resistance is negligible (at higher sludge concentrations and at higher sludge depths).

Modifications of ideal drying theory equations are proposed in order to estimate the "constant" drying rate period, and the "critical" moisture content (at which value, the falling rate of drying period commences). The modifications account for the variation in these "constants", and appear to be exponential functions of the solids-loading to the bed.
ACKNOWLEDGEMENTS.

I gratefully acknowledge the assistance received from the following:

Professor G. v. R. Marais, Professor of Water Resources and Public Health Engineering, University of Cape Town, who supervised this thesis.

My wife, Maureen, who meticulously and patiently typed the manuscript.

The Board of Woolworths (Pty) Ltd., Cape Town, who gave permission to use their facilities in the printing of the manuscript, and their Technology Department for assisting in some of the chemical analyses.
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REFERENCES.

(Tables 1 - 36 inclusive, in the Appendix, Part 4.)
1. THE DEVELOPMENT OF AN ANCILLARY TEXTILE INDUSTRY WASTE TREATMENT PROCESS

1.1. Introduction.

In 1971, the Cape Town Municipality promulgated regulations controlling the discharge of waste water to the sewers of the city. The effect of these regulations is to protect the sewage works against the introduction of any waste which may injuriously affect the purification process, and thus prejudice the disposal of the treated sewage effluent, should it not meet with the requirements imposed in terms of the Water Act, Act No. 54 of 1956.

These regulations are of particular concern to industrial plants generating waste waters. These industries are now compelled to control the quality of their effluents by either modifying the process streams, or pre-treating the effluents. This thesis deals with the solution of this problem for an ancillary textile industry.

1.2. Description of the Manufacturing Process.

The industry described here is unusual in that its main activity is the production of synthetic buttons and trimmings for the clothing industry. At present, both polyester and casein-formaldehyde buttons and trimmings are manufactured. Essentially the process consists of the following operations:

Polyester raw material is admixed with a catalyst with or without added inert filler. The fluid mass is cast into moulds and cross-polymerisation allowed to proceed under precisely controlled conditions. The casein raw material, dissolved in an aqueous alkaline medium, is precipitated out
of solution; admixed with inert filler; and compressed and dried in moulds. Thereafter case-hardening is provided by means of a formaldehyde solution treatment, and further drying.

The cast buttons, etc., are further processed by machining in specially-tooled lathes to produce various designs and surface effects. The buttons, buckles and other trimmings are given a surface polish by tumbling in a slurry of 200-mesh pumice in hexagonally shaped tumbler drums. Finally, the polished products are given a variety of surface finishes by dyeing in a large number of vats in an assortment of colours, both with and without chemical or solvent surface etching. All the products are then thoroughly rinsed and dried. (See Photograph 1).

As a result of the appreciable number of processes and combinations of processes, a large variety of both organic and inorganic chemicals are used and these give rise to variation in the composition and quality of the waste effluent from the processing plant.

The effluent from the plant is highly alkaline (pH range normally 10.5 - 11.5) and toxic towards biological degradation processes. It also contains high concentrations of organic biodegradable material and is toxic towards aquatic life. It is thus a serious pollution hazard.

The local regulations (15) require inter alia that before a waste can be discharged to the sewers, the pH value must be within the range 5.5 - 12.0 and hexavalent chromium limited to a maximum of 50 mg/l (as CrO$_3$; i.e. 26 mg/l as Cr). In addition, the Council is empowered to impose certain restrictions as it may deem necessary, and in the case of this particular plant, the restrictions included limiting the effluent pH value to a maximum of 9.5; reduction of the intensity of the colour; and
limiting the Chemical Oxygen Demand (COD) to less than 1500 mg/l. Failure to comply with an acceptable quality of effluent can lead to closure of the processing plant.

The current installation has been treated as a large-scale pilot plant in aiding the formulation of a practical solution to the problem. It is of prime economic consideration that the evolved treatment process shall be relatively simple, and low-cost in installation and operation.

1.3. Survey of the Effluent Problem

A rational investigation of the waste water problem requires information about the following aspects:-

(1) chemicals used in the various processes;

(2) waste streams generated by the various processes and their polluting constituents; and

(3) magnitudes of the flow and concentration of pollution in the various streams, and their variability.

The nature of the processes involved suggests a combination of textile industry wastes; and chemical processing, plastics and dye-house wastes.

1.3.1. Chemicals used in the Processes.

A survey of the known chemical compounds found on the premises showed the presence of the following raw materials:-

A. Raw Base Materials

1. Polyester resin, being a maleic acid/propylene glycol/styrene mixture which is polymerised and cross-polymerised in situ in the presence of an organic peroxide catalyst. The styrene vehicle and diluent is also polymerised in this reaction.
2. Casein.
3. Sodium Hydroxide flake, and bulk 40% solution.

B. Solvents
1. Acetone.
2. Methyl Ethyl Ketone.
4. Thinners (hydrocarbon/ketone based).
5. Benzyl Alcohol.

C. Acids
1. Glacial Acetic Acid.
2. 90% Formic Acid.

D. Oxidation/Reduction Chemicals
1. Potassium Dichromate.
2. Sodium hypochlorite, 10% solution.
3. Hydrogen Peroxide (100 volume strength).

E. Textile Dyeing Chemicals
1. Calcolene Oil (employed in polyester dyeing).
2. Levelling Agents (essentially anionic, non-ionic and cationic detergents).
3. Multitude of organic dyestuffs (e.g. "Phthalocyanine Blue"; "Solochrome Black"; "Diamond Chrome Fast"; "Carbolan"; etc.).
4. Titanium Dioxide pigment and filler.
5. Calcium Carbonate pigment and filler.
F. Polishing and Cleaning Compounds

1. Pumice Powder (200 mesh).
2. Teepol (anionic detergent).

1.3.2. Waste Streams.

All processes continue batch-wise. The only interruptions are for vat cleaning between different dyeings among 16 different dye vats. (cf. Photograph 1 for position of vats). All chemical wastes, vat washings and floor sweepings are discharged down a sloping floor towards a common open channel along the wall and behind the dye vats; there is thus no question of isolation of selected pollution streams. A removable steel grid prevents large objects from blocking the channel outlet. The channel discharges directly into the sewer through an external drain at ground level.

1.3.3. Concentration and Variability of Waste Stream.

A 24-hour composite sample of effluent was collected at the external drain. Sampling was carried out at half-hourly intervals. An analysis gave the following constituents and concentration of pollution:

<table>
<thead>
<tr>
<th>TABLE 1.1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Effluent Analysis (24-hour Composite Sample)</strong></td>
</tr>
<tr>
<td>1. pH</td>
</tr>
<tr>
<td>2. Total Solids:</td>
</tr>
<tr>
<td>3. Settleable Solids:</td>
</tr>
<tr>
<td>4. COD</td>
</tr>
<tr>
<td>5. 4-hour O₂ absorbed (OA) from N/80 Permanganate, 27°C:</td>
</tr>
</tbody>
</table>
### Effluent Analysis (24-hour Composite Sample) (Continued)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6.</td>
<td>Ammoniacal Nitrogen (NH₃-N): 7,1 mg/l</td>
</tr>
<tr>
<td>7.</td>
<td>Total Nitrogen (N): 16,8 mg/l</td>
</tr>
<tr>
<td>8.</td>
<td>&quot;Grease&quot; (Hexane Solubles): 686 mg/l</td>
</tr>
<tr>
<td>9.</td>
<td>Total Chromium (Cr): 1,52 mg/l</td>
</tr>
<tr>
<td>10.</td>
<td>Hexavalent Chromium (Cr): 0,84 mg/l</td>
</tr>
<tr>
<td>11.</td>
<td>Copper (Cu): 0,27 mg/l</td>
</tr>
<tr>
<td>12.</td>
<td>Zinc (Zn): Negligible</td>
</tr>
<tr>
<td>13.</td>
<td>Manganese (Mn): Less than 1 mg/l</td>
</tr>
<tr>
<td>14.</td>
<td>Calcium (Ca): 24 mg/l</td>
</tr>
<tr>
<td>15.</td>
<td>Magnesium (Mg): 7 mg/l</td>
</tr>
<tr>
<td>16.</td>
<td>Chloride (Cl): 190 mg/l</td>
</tr>
<tr>
<td>17.</td>
<td>Sulphate (SO₄): 48 mg/l</td>
</tr>
<tr>
<td>18.</td>
<td>Hydroxide (OH): 416 mg/l</td>
</tr>
<tr>
<td>19.</td>
<td>Carbonate (CO₃): 85 mg/l</td>
</tr>
<tr>
<td>20.</td>
<td>Physical appearance: Murky dark red emulsion</td>
</tr>
</tbody>
</table>

To determine the variability of the polluting substances, samples were collected half-hourly throughout a 9 hour working day over a period of a five day week. Samples collected in the mornings, and those collected in the afternoons, were combined separately to provide 5 morning samples and 5 afternoon samples respectively. The associated flow rate was not recorded, but it was observed that it was fairly uniform throughout the day. Records of daily water meter readings for the processing section only were however available, and showed relatively small variation from day to day.
Photograph 1: General view of Processing Area. (Hexagonal Polishing Tumbler on right; Dye Vats against rear wall).

Photograph 2: The Half-Daily Variation in appearance of Effluent over one week (From left to right, Monday a.m. to Friday p.m.).
Photograph 1: General view of Processing Area. (Hexagonal Polishing Tumbler on right; Dye Vats against rear wall).

Photograph 2: The Half-Daily Variation in appearance of Effluent over one week (From left to right, Monday a.m. to Friday p.m.).
The following comparative analyses were carried out:

**TABLE 1.II**

Half-Daily Effluent Analyses

<table>
<thead>
<tr>
<th>Sample</th>
<th>Colour</th>
<th>pH</th>
<th>COD (mg/l)</th>
<th>Chloride (mg/l as Cl)</th>
<th>Daily Flow (m³)</th>
<th>Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Monday a.m.</td>
<td>Dark yellow</td>
<td>11.25</td>
<td>3074</td>
<td>896</td>
<td>22</td>
<td></td>
</tr>
<tr>
<td>2. Monday p.m.</td>
<td>Yellow</td>
<td>11.40</td>
<td>2971</td>
<td>358</td>
<td>7.96</td>
<td>22</td>
</tr>
<tr>
<td>3. Tuesday a.m.</td>
<td>Black</td>
<td>3.05</td>
<td>4145</td>
<td>1032</td>
<td>22</td>
<td></td>
</tr>
<tr>
<td>4. Tuesday p.m.</td>
<td>Grey</td>
<td>6.80</td>
<td>2139</td>
<td>717</td>
<td>7.32</td>
<td>22</td>
</tr>
<tr>
<td>5. Wed. a.m.</td>
<td>Red/Black</td>
<td>10.81</td>
<td>15607</td>
<td>673</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>6. Wed. p.m.</td>
<td>Blue/grey</td>
<td>11.25</td>
<td>2704</td>
<td>403</td>
<td>8.82</td>
<td>22</td>
</tr>
<tr>
<td>7. Thurs. a.m.</td>
<td>White</td>
<td>11.15</td>
<td>568</td>
<td>1287</td>
<td>23</td>
<td></td>
</tr>
<tr>
<td>8. Thurs. p.m.</td>
<td>Blue/grey</td>
<td>11.40</td>
<td>680</td>
<td>1755</td>
<td>10.23</td>
<td>23</td>
</tr>
<tr>
<td>9. Friday a.m.</td>
<td>Red</td>
<td>3.10</td>
<td>4287</td>
<td>1272</td>
<td>23</td>
<td></td>
</tr>
<tr>
<td>10. Friday p.m.</td>
<td>Very dark red.</td>
<td>3.48</td>
<td>2970</td>
<td>487</td>
<td>11.41</td>
<td>23</td>
</tr>
</tbody>
</table>

Arithmetical Means: 3915 888

Photograph 2 shows the appearance of each of the 10 effluent samples.

1.3.4. Evaluation of Results.

The half-daily fluctuations in flow, pH, COD and chloride are plotted on a time basis in Fig. 1. (COD values shown are uncorrected for chloride).

Graphical statistical analyses on cumulative probability paper (Fig. 2) show that the chloride and daily flow are approximately normally distributed, whereas in contrast the pH probability plot shows the presence
FIG. I: HALF-DAILY FLUCTUATION OF PARAMETERS.

(TABLE 1.11)
FIG. 2. DISTRIBUTION OF HALF-DAILY pH, CHLORIDE & FLOW.

(TABLE I.II)
of 2 distributions. On enquiry, the lack of statistical equilibrium in the pH values became apparent. On the Tuesday and Friday mornings, and on Friday afternoon, when the pH values were 3 to 3.5 respectively, sulphuric acid had been discharged to the drain in a crude attempt to neutralize the illegal effluent.

The COD values are log-normally distributed (Fig. 3), and it can be noted that the expected upper 95% confidence limit of 13200 mg/l is in good agreement with peak values actually found (13556 mg/l and 15607 mg/l respectively).

The statistical analyses yield the following results:-

<table>
<thead>
<tr>
<th>Flow (m³/d)</th>
<th>COD (mg/l)</th>
<th>Chloride (mg/l)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Arithmetic Mean, $\overline{x}$:</td>
<td>9.00</td>
<td>3795 ($\overline{x}=2600^\ast$)</td>
</tr>
<tr>
<td>2. Standard deviation, S:</td>
<td>440</td>
<td>2387 ($S$ geometric)</td>
</tr>
<tr>
<td>3. Upper 95% Confidence Limit:</td>
<td>12.96</td>
<td>15200</td>
</tr>
<tr>
<td>4. Lower 95% Confidence Limit:</td>
<td>5.09</td>
<td>400</td>
</tr>
</tbody>
</table>

Note: $\overline{x}$ = Geometric mean value.

The values for the COD indicate that considerable variation is to be expected and stresses the need for a balancing collection tank prior to any treatment process, to "smooth" the processing, rather than "on-stream" continuous treatment. The flow distribution provides an estimation of expected average and peak flow rates and their ratio for the present plant, and also for the intended expanded plant.

The normal pH value of the effluent is higher than that permitted for disposal to the sewer. The average COD value is in excess of the required maximum concentration, and the effluent is highly coloured. Treatment of the process waste water is therefore necessary.
FIG. 3. LOG-NORMAL DISTRIBUTION OF HALF-DAILY COD. (TABLE 1.11)
1.4. **Proposed Unit Operations.**

The variability of the composition of the effluent indicates the need for a balancing tank (section 1.3.4.). This tank should be of sufficient capacity to collect a reasonable and manageable quantity of effluent, such as one day's flow. It should have some means of mixing, such as a simple paddle mixer, to homogenize the contents.

1. **Neutralisation.**

A comparison of the analyses and the requirements of the regulations, indicates that after mixing the effluent must be neutralised. Most of the time, this will require sulphuric acid to reduce the pH value. However, if the pH value requires to be raised, an addition of caustic soda will suffice. Neutralization can conveniently be effected in the balancing tank. It is necessary to have a similar second tank to collect the effluent flowing during the neutralization operation in the first tank.

2. **Flocculation and Coagulation.**

To reduce the COD and colour of the effluent, a flocculation process may succeed. Simultaneously, it may be possible to reduce the chromate content. For this unit operation, it is necessary to investigate experimentally which chemical or combination of chemicals, will bring about a flocculation. Iron and aluminium salts are normally used in the flocculation of coloured and turbid waters, and it is appropriate to attempt flocculation with these common treatment reagents in the case of the waste water. Flocculation requires a particular minimum dosage of chemicals, and in addition, the growth of the floc and its subsequent coagulation is best effected in a particular pH range. The conditions
for optimum flocculation are determined by comparative experiments, in which the dosage and pH are separately varied. Flocculation of the neutralized effluent can be carried out batch-wise, or preferably on a semi-continuous basis.

In a semi-continuous procedure, the neutralized effluent from a batch is fed continuously through "on-stream" chemical dosers to the flocculation/sedimentation tank. The rate of dosing remains constant for each neutralized batch.

In a batchwise flocculation, dosing can be accomplished in the balancing tanks. However, each tank must have sufficient capacity to allow time for floc growth and coagulation in the one tank, while the other is receiving the flow.

(3) Sedimentation and Thickening.

After flocculation, the supernatant has to be separated from the coagulated floc. This operation is performed in a sedimentation tank, in which the floc is allowed to settle and thicken, or concentrate.

In the semi-continuous flow operation, the clarified supernatant overflows from the sedimentation tank to the sewer, while the thickened sludge may be drawn off continuously or intermittently from the bottom of the tank.

In batch operation, the flocculated effluent may be pumped to a separate sedimentation tank having a capacity equal to that of the balancing tank, and the floc allowed to settle and thicken. Alternatively, settling and thickening may proceed in the balancing tank, provided that the tank is designed as a sedimentation tank and the tank bottom is designed for removal of sludge.

In semi-continuous operation, a smaller sedimentation tank will be required than that necessary for batch operation, to obtain a desired
(4) **Dewatering and Drying of Flocculated Sludge.**

The thickened sludge from the sedimentation process still contains a high percentage of liquid. This liquid must be removed as much as possible (i.e. "dewatered") to reduce the bulk of the thickened sludge, and then the solids can be dried and conveniently handled for final disposal.

Dewatering can be accomplished by filtration (or drainage) on a sand filter. The wet sludge is fed onto a sand bed, where the solids are retained and the clear filtrate is underdrained to a sewer. This constitutes a filtration operation. As solids are deposited on the sand bed, the filtration resistance offered by the sludge cake itself increases.

There are three ways of feeding the thickened sludge onto the filter beds:

(i) The depth of slurry on the sand bed is maintained constant for a particular length of time ("constant head filtration"), and then allowed to drain without further addition of slurry ("falling head drainage").

(ii) The bed of sand is filled to a particular depth with slurry, and allowed to drain (falling head drainage only).

(iii) The bed may be topped-up intermittently with the feed. The depth of dewatered sludge increases progressively until the filtration resistance offered by the sludge cake reduces the filtration rate to a low level, when the cake is allowed to drain completely.

In the first method of operation, the mass of sludge solids increases progressively, but the depth of the slurry remains constant and the rate of filtration decreases. At some stage the slurry feed will have to be
diverted to a second sludge sand filter. Thus, to determine the minimum area necessary to dewater the sludge, the optimum depth of slurry feed is required.

(5) Sludge Drying.

Exposure of the draining sludge to the atmosphere causes evaporation of some of the water. Drying rates are controlled by climatic and atmospheric conditions, which are to a degree unpredictable. If the rate of drying is the controlling element, the area of sand beds for filtration will be dependent on the drying characteristics, rather than on the filtration characteristics. Thus, in this case, the unit operation of atmospheric drying is closely associated with that of dewatering by sand filtration.
2. INVESTIGATION OF UNIT OPERATIONS.

2.1. Flocculation pH

A plot of the neutralization curve of the composite effluent (Fig. 4a) shows two points of inflection; one at a pH value of 7.9; and the other, not as clearly defined, at a pH value of 5.95. The shape of the titration curve appears to be similar to that obtained by titrating a weak base with a strong acid, which raised the thought that it may indicate the "iso-electric pH" for the colloidal matter present in the effluent. However, many of the colloidal particles among the dyestuff present may exhibit various iso-electric pH values, and it therefore seemed necessary to establish if there was some composite "iso-electric" point, which may suggest a suitable flocculating pH. An initial attempt to find an iso-electric point for the colloidal matter where auto-coagulation might occur in the combined effluent, showed no clear-cut precipitation and separation in a reasonable time. However, it was noted that as the pH value was lowered in 0.5 unit decrements, when each sample was shaken an equal number of times by inverting, it was seen that there appeared to be a definite gradient in the head of foam produced above the liquid level. The foam layer broke readily at pH = 6.0 whereas there was a definite increase in the height of the foam and the time taken for the foam to breakdown as the pH value increased above 6.0 or decreased below 6.0. Shaking equal volumes of effluent at different pH values simultaneously, the foaming effect is clearly demonstrated in Photograph 3. This phenomenon was interpreted as indicating a pH value where the anionic and cationic surfactants in the effluent neutralised one another.
FIG. 4a. NEUTRALISATION CURVE OF ONE WEEK COMPOSITE EFFLUENT "A".

Photograph 4: Appearance of Effluent after Flocculation and settling one hour.

Photograph 5: Comparison of Supernatant (left) and Sludge (right) after Sedimentation.
Experiments on flocculating the colloidal matter (see below) confirmed that this pH was indeed an important one, and that the optimum pH value for flocculation with aluminium salts (aluminium sulphate alone, or in conjunction with sodium aluminate) was within the narrow range of pH = 5,8 to 6,0. In this pH range the floc grew rapidly and settled readily with minimum frothing difficulties, while the resultant supernatant was pale in colour and clear. Photographs 4 and 5 show the appearance of the supernatant and sludge after this treatment.

Based on previous experience for the treatment of an alkaline textile industry waste, the Niers Modified Flocculation Process (1) was applied to the waste. Ferrous sulphate was added as the flocculating agent, followed by aeration at various pH ranges from pH = 11 to pH = 3. A heavy black floc formed, but on settling, the supernatant remained colloidal and had an inky blue-black colour.

It thus appeared that aluminium sulphate, either alone or with the subsequent addition of sodium aluminate, was the most successful in producing a readily settleable floc, and was of more interest for further investigation.

To optimise the process, a program of comparative flocculation was put into action:–

(i) The concentration of aluminium sulphate was maintained constant while the pH was incrementally decreased from pH = 7,0 to pH = 3,0 until the conditions for best flocculation were obtained.

(ii) At the selected pH value, the concentration of aluminium sulphate was varied until the coagulation time was a minimum.

(iii) A dose of aluminium sulphate smaller than that found in (ii) was added to slightly acidic solutions, and both the pH and aluminium
sulphate dosage increased by the addition of sodium alinate.

Fig. 4 b shows these flocculation test results graphically (Data in Section 4.1).

The flocculation process eventually selected was as follows:-

1. Acidify effluent with 10% m/v sulphuric acid solution to pH = 6,5. Add one-half again of the volume of acid required to reach pH = 6,5. (pH drops to approximately 4 to 5).

2. Add 500 ppm aluminium sulphate hydrate in the form of a 10% m/v solution and rapid-mix for a few minutes.

3. Add 10% m/v sodium aluminate solution rapidly and immediately after mixing in the aluminium sulphate, until the pH value is in the range 5,8 - 6,0 (usually about 500 - 800 ppm).

Rapid-mix for 2 - 3 minutes, and then stir slowly for 20 minutes and allow to settle.

A comparative analysis of the original effluent and the clear supernatant after flocculation gives an indication of the efficacy of the process:-

<table>
<thead>
<tr>
<th>Raw Effluent</th>
<th>Flocculated Supernatant</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. COD (mg/l):</td>
<td>3193</td>
</tr>
<tr>
<td>2. pH value:</td>
<td>10,32</td>
</tr>
<tr>
<td>3. Hexavalent Chromium (As Cr) (mg/l):</td>
<td>0,52</td>
</tr>
<tr>
<td>4. Trivalent Chromium (As Cr) (mg/l):</td>
<td>0,84</td>
</tr>
<tr>
<td>5. Colour:</td>
<td>Murky dark red</td>
</tr>
</tbody>
</table>

Reduction in COD = 50%
FIG. 4b: FLOCCULATION AT (i) CONSTANT ALUMINIUM SULPHATE DOSAGE, (ii) CONSTANT pH, & (iii) CONSTANT ALUMINIUM SULPHATE DOSAGE AT pH=5, FOLLOWED BY SODIUM ALUMINATE DOSAGE. (Section 4.1., Appendix)
Of particular importance was the fact that the hexavalent chromium was reduced in the acidification stage, when all the chromium (1.36 mg/l) appeared to be present as trivalent chromium. The latter was completely removed with the sediment in the flocculation stage.

In addition, in the short time lag after acidification, free chlorine was liberated from the hypochlorite in the waste. Apparently, an oxidation/reduction reaction occurred with part of the oxidisable organic matter present, for no hypochlorite ion was detected in the settled supernatant after treatment, although it was invariably present in the raw effluent.

The estimated water consumption in excess of that employed in the processing plant is 25% (this water being used in ablution, urinals, toilets, dishwashing, etc.), which reduces the treated effluent COD to a value somewhat below 1000 mg/l (and less than 1500 mg/l) before discharge in the Municipal sewer. Thus there appears to be no necessity for further treatment before final discharge.
2.2. **Sedimentation and Thickening.**

Having established the conditions for optimum flocculation, it is necessary to determine the settling and thickening characteristics of the floc. The theoretical principles and experimental procedures were those adapted from the work of Eckenfelder (2a, 3), Wills (4), Talmage and Fitch (7), Kynch (16) and Fair and Geyer (5). Other procedures are possible (6a), but require the precise evaluation of physical constants such as size and density of the flocculating and settling particles.

During the sedimentation of a floc, there may be essentially two types of settling (2a) processes:

(a) **Flocculent settling**, which occurs when coalescence of the settling particles results in an increase in the settling velocity. The settling rate increases with time, and this results in a curvilinear settling path (Fig. 6).

With flocculent settling in a continuous process, the overflow rate and retention time of the settling tank are both significant. This type of settling is common in water treatment and primary settling in waste water treatment.

(b) **Zone settling**, which occurs when the floc particles are so concentrated that the settling is hindered by the appreciable upward displacement of fluid. The mass of flocculated sludge settles as a blanket, with a distinct interface between the top of the settling sludge and the supernatant.

Zone settling occurs when the solids concentration exceeds approximately 500 mg/l, and is common in the settlement of activated sludge particles and where heavy doses of flocculent chemicals are used. This type of settling
occurs in the treatment of textile wastes, such as that investigated in this thesis, and will therefore be considered in detail.

Kynch (16) presented a mathematical analysis of batch settling tests without analyzing the relationship between batch settling tests and continuous thickeners. To apply the Kynch method to continuous thickening, Talmage and Pitch (7) reviewed that part of the mechanism of continuous thickening which is pertinent to the problem.

In the initial stages of zone settling, the flocs fall through the liquid without pressing on layers of floc below. The quantity of solids which can settle through a unit horizontal cross-section in unit time (i.e. the "flux", kg/m²/hr) is equal to the product of the settling rate (m/hr) and the volumetric solids concentration (kg/m³).

In a continuous thickener, the solids must be able to subside through any concentration layers between the concentrations of feed and underflow at least as rapidly as they are fed to the unit. Otherwise, a layer of whatever concentration limits the solids-handling capacity will form and act as a barrier. If insufficient area is present to handle the solids, such a barrier layer would build up. All solids in excess of the amount which could fall through this zone would eventually have to overflow the thickener. Therefore, a thickener must have sufficient area to allow the solids to subside through whichever concentration layer has the least solids-handling capacity.

In a batch settling test, as the particles approach the bottom of the container, the settling rate approaches a small but finite value, and then decreases abruptly to zero because of the barrier presented by the bottom of the container. When the floc particles reach the bottom of the column, they must start accumulating and this causes a local increase in their concentration. Concentration layers with lower solids-handling
capacities (due to their higher local concentrations) than the initial concentration layers, propagate up from the bottom of the vessel and appear eventually at the upper boundary of the settling floc. These concentration layers with limited solids-handling capacity rest upon other such layers below them and compress them. A compression or consolidation of the thickened sludge commences, and this results in a concentration gradient from the bottom of the column of thickened sludge to its top surface. The propagation upwards of concentration layers with limited solids-handling capacity causes an ever-decreasing settling rate, so that there is a transition between the initial zone settling condition and the compression condition. Eckenfelder (2a) described this transition as a separate zone in the zone settling process. Kynch (16) described how in a settling dispersion initially at a uniform concentration, as the layers of higher concentration propagated from the bottom reach the surface, the concentration at the surface begins to increase and the initial constant settling rate of the surface begins to decrease. The change in concentration from one concentration layer to the next one below it may be very small (i.e. "continuous"), or it may be finite (i.e. "discontinuous"). This results in a series of concentration discontinuities. The whole adjustment of concentration which occurs when a dispersion settles can then be described as a series of small concentration discontinuities propagated through the fluid. In the compression zone, the thickened floc continues to settle at a reduced rate, and is consolidated until the minimum depth and maximum concentration is reached. The following diagrams show the floc height-concentration relationships at different times in a batch settling test:
During consolidation, compression causes liquid to be squeezed out of the floc and released upwards into the clarified supernatant, thereby concentrating or thickening the sludge.

Kynch (16) showed how the settling rate and concentration of any capacity-limiting concentration layer which may exist can be determined from the variation in settling rate observed in a batch settling test.

For the purposes of an analysis of the thickening process, the application of the Kynch theory used by Talmage and Fitch (7) will be considered:

Kynch postulated that the settling velocity, \( V \) (m/hr), of a particle is a function only of the local solids concentration, \( C(\text{kg/m}^3) \), around the particle, or mathematically,

\[
V = f(C).
\]

The function is not defined and may change in any manner as the concentration changes. The relationship between the two can be deduced from observations on the fall of the top boundary of the dispersion.

In a batch test starting at uniform concentration, all the solids start settling at a uniform velocity, since \( V = f(C) \). As the settling solids begin to collapse against the bottom of the vessel, they must pass through all concentrations between the initial concentration and that of the deposited solids. If, at any of these intermediate concentrations, the solids-handling capacity is less than that at the lower concentration
occuring immediately above it in the vessel, a zone of such intermediate concentration must start building up, since the solids cannot pass through it as fast as they are settling down into it.

Kynch showed that the rate of upward propagation of each such constant concentration zone is constant.

Consider the infinitely thin layer LM (diagram I, below) at the upper boundary of such a zone, having a concentration, C kg/m\(^3\), originating at the bottom at zero time and moving upward at a velocity of U m/hr.

The solids settling into this layer come from a layer having a concentration of (C - dC) kg/m\(^3\) and a settling velocity with respect to the vessel of (V + dV) m/hr, but with respect to the layer of (V + dV + U) m/hr.

The concentration of solids settling out of this layer will be C with a settling velocity of V with respect to the vessel and (V + U) with respect to the layer. Since the concentration of the layer is constant, the quantity of solids settling into the layer must equal the quantity of solids settling out of the layer, and a material balance can therefore be made.

If the cross-sectional area of the layer LM is A(m\(^2\)), then at time t (hr), the mass of solids settling into the layer is (C-dC)At(V + dV + U) kg, and the mass of solids settling out of the layer is CA(V + U) kg. Therefore,

\[(C - dC)At(V + dV + U) = CA(V + U) \quad (1)\]
Simplifying,
\[ C \cdot \frac{dV}{dC} - V \cdot \frac{dC}{dV} - U \cdot \frac{dC}{dV} = 0. \]

Solving for U, and ignoring second order differential terms,

\[ U = C \cdot \frac{dV}{dC} - V \quad (2). \]

Since, according to Kynch's assumption \( V = f(C) \), by differentiation,

\[ \frac{dV}{dC} = f'(C), \]

and substituting in (2),

\[ U = Cf'(C) - f(C) \quad (3). \]

Since C is constant for the layer in question, \( f(C) \) and \( f'(C) \) have fixed values and therefore U must also be constant by equation (3).

The constancy of U may now be used to determine the solids concentration of the layer at the upper boundary of the settling floc. Let \( C_0 \) (kg/m³) and \( H_0 \) (m) be the initial concentration and height respectively, of a column of floc in a batch settling test in a column such as that in Fig. 5. The total mass of solids in this column of floc is then \( C_0 H_0 A \) (kg).

When any capacity-limiting concentration layer reaches the floc/water interface, all solids in the column must have passed through it, since it was propagated up from the bottom of the column. If the concentration of this layer is \( C_2 \) (kg/m³), and it reaches the interface at time \( t \) (hr) with a settling velocity \( V_2 \) (m/hr), then the mass of solids having passed through this layer, \( C_2 A t_2 (V_2 + U_2) \) kg, must equal the total mass of solids in the column. Thus, equating the two expressions for the total mass of solids in the column,

\[ C_0 H_0 A = C_2 A t_2 (V_2 + U_2) \quad (4). \]
**FIG. 5.**

SEDIMENTATION COLUMN 0.055m DIAMETER.
FIG. 6. SETTLING PATHS OF EFFLUENT "A" FLOC: PERCENTAGE SUSPENDED SOLIDS REMOVAL AT INDICATED DEPTHS AND TIMES.

(TABLE 2A)

C_0 = 1796 mg/l

TIME (minutes)
If \( H_2(m) \) represents the height of the interface at time \( t_2 \) (hr), and since it has been shown by equation (3) that the upward velocity \( U \) m/hr of any specific layer is constant,

\[ U_2 = \frac{H_2}{t_2}. \]

Substituting for \( U_2 \) in equation (4), and solving for \( C_2 \),

\[ C_2 = \frac{C_0}{H_2 + V_2 t_2} \quad (5). \]

A plot of the height of the interface against time results in a settling curve as shown below in diagram II, and in Figs. 8 to 12 inclusive.

![Diagram II. Typical settling curve obtained in a batch settling test.](image)

\( V_2 \) is equal to \( dH/dt \) at the point on the plot at which the layer having a concentration of \( C_2 \) (kg/m\(^3\)) comes to the surface of the floc. \( V_2 \) is then the slope of the tangent to the curve at the point \( (H_2, t_2) \).

It follows geometrically that the intercept of this tangent on the \( H \) axis is

\[ H_1 = H_2 + (H_1 - H_2), \]

But

\[ V_2 = \frac{(H_1 - H_2)}{t_2}, \]

therefore

\[ (H_1 - H_2) = V_2 t_2. \]

Hence,

\[ H_1 = H_2 + V_2 t_2. \]

Substituting \( H_1 \) for \( H_2 + V_2 t_2 \) in equation (5),
From equation (6) it follows that \( H_1 \), the intercept height, is the height the floc would occupy if all the solids present were at the same concentration as the layer at the settling floc/water interface. Thus, for any arbitrarily selected value of \( C_2 \), the corresponding value of \( H_1 \) may be calculated from (6). \( V_2 \) can then be determined as the slope of the line drawn through point \( H_1 \) on the \( H \)-axis and tangential to the settling curve. A complete set of data showing \( V \) as \( f(C) \) can therefore be developed from one batch settling test.

2.2.1: Unit Area and Mass Loading.

To specify the area requirement of a thickener, the concentration layer requiring the maximum area to pass a unit mass of solids must be determined. This may be done by calculating the unit area required for a series of concentrations, using the data showing \( V \) as \( f(C) \), developed in the previous section (2.2.). Whichever concentration layer gives the largest value of unit area is then used as a design basis. Talmage and Fitch\(^7\) used a simple geometrical construction on the settling curve (Diagram II) to obtain the unit areas directly.

At time \( t_2 \) (hr) the solids in the layer existing at the surface of the column of floc are settling at a linear rate of

\[
V_2 = \frac{H_1 - H_2}{t_2} \text{ (m/hr)}
\]

(i.e., the slope of the \( H \) vs. \( t \) curve).

If the solids of this layer are assumed as a datum, water is passing the solids at a bulk rate of

\[
A(H_1 - H_2)/t_2 \text{ m}^3/\text{hr}.
\]
In a continuous thickener, the solids in any zone do not have to settle past all of the water in the zone, since part of this water will accompany the solids to the underflow. The amount of water they must settle past is equal rather to the amount which would be released in bringing solids from layer concentration to underflow concentration. The corresponding quantity of water for the solids present in the batch test would be

\[ A(H_1 - H_u) \, m^3, \]

since \( H_1 \) is the height the floc would occupy if all solids present in the batch test were at layer concentration, and \( H_u \) is the height the floc would occupy if all solids present in the batch test were at the thickener underflow concentration. The time that would be required to release \( A(H_1 - H_u) \, m^3 \) of water through a layer concentration \( C_s \, kg/m^3 \) would then be

\[ t' = \frac{\text{amount of water to eliminate}}{\text{rate of eliminating water}} = \frac{A(H_1 - H_u)}{A(H_1 - H_u)/t_2} \, (hr). \]

Rearranging,

\[ t'/ (H_1 - H_u) = t_2/(H_1 - H_u). \]

Referring to the settling curve in Diag. II, by similar triangles,

\[ t_2/(H_1 - H_u) = t_u/(H_1 - H_u); \]

by equating the relationship in the latter equation to that in the previous one,

\[ t' = t_u. \]

Therefore,

\[ t_u = (H_1 - H_u)t_2/(H_1 - H_u) \quad (7). \]

The total quantity of solids in the batch test is \( C_o H_o A \, kg \) and it would take time \( t_u \) hr for this mass of solids to subside past a layer of
concentration $C_2 \text{ kg/m}^3$ in a continuous thickener. Therefore, the mass of solids that could be brought through layer concentration $C_2$ per unit time is

$$C_2 H A / t_u \text{ kg/hr},$$

or per unit area, 

$$ML = C_2 H / t_u \text{ kg/hr/m}^2. \quad (8)$$

"Unit Area" of a layer is, by definition, the area required to allow unit mass of solids to subside through the layer concentration in unit time. Thus, by (8),

$$\text{Unit Area, } UA = t_u / C_2 H \quad (9)$$

The "Mass Loading" is the reciprocal of unit area, and is defined as the mass of solids passed per unit area per unit time, as shown in equation (8).

The method for determining the unit area corresponding to any floc concentration $C_2$ (the value of $C_2$ is not known, but a series of values may be selected) in the free settling range is therefore as follows:

1. Determine $H_1$ and $H_u$ from the following material balances: point $H_1$ corresponding to an arbitrarily selected concentration, $C_2$; from equation (6),

$$H_1 = C_2 H / C_2$$

and

$$C_2 H = C_2 H_1 = C_2 H_u,$$

whence

$$H_u = C_2 H / C_2 \quad (10),$$

where $C_2 H_u$ is the underflow concentration (kg/m$^3$) at time $t_u$.

2. Draw an "underflow" line parallel to the time axis at $H = H_u$ on the settling curve (diagram II, and Figs. 8 - 12 inclusive).

3. Draw a tangent to the settling curve through point $H_1$ on the $H$ axis.

4. Read $t_u$ at the intersection of the tangent and the underflow line.
5. Calculate unit area from equation (9),

\[ UA = \frac{t}{C \cdot H_u} \]

When the underflow line, \( H_u \), intersects the settling curve above the point where the layer at the surface of the floc goes into compression, the time \( t_u \) corresponding to maximum unit area will be the co-ordinate of the intersection, since any other tangent will intersect the underflow line at a lesser value of \( t_u \). When the underflow line intersects the settling curve below the point where the layer at the surface of the floc goes into compression, the tangent giving maximum unit area will be drawn through this compression point, since this tangent gives the highest value of \( t_u \) in the free settling range and only free settling zones govern the unit area.

It should be noted that Talmage and Fitch assumed the surface layer of the compression zone was the limiting layer for water removal from the thickening sludge below it. The fallacy occurred in their further assumption that this limiting concentration had some particular value obtained by the geometric construction described above, and which was independent of the desired underflow concentration. However, for the purposes of calculation these assumptions have been adopted in the present investigation. Wills (1967) \(^4\) outlined the Talmage and Fitch method without comment on the possibility of there being a fallacy in the assumptions.

As the underflow concentration \( C_u \) increases, \( H_u \) decreases [from equation (10)] and \( t_u \) increases [by equation (7), and reference to the settling curve]. Therefore, from equation (9) the unit area \( UA \) increases. As \( C_u \) approaches its maximum concentration \( C_\infty \), the interfacial velocity approaches zero.
Equation (9) shows that the unit area $UA$ will vary with the initial floc solids concentration, $C_0$, so that the area required for the entire range of expected influent solids concentrations needs to be calculated. For each value of $C_u$ in each of the influent concentrations in this range, the interfacial settling velocity on the settling curve given by

$$ V_2 = \frac{(H_1 - H_2)}{t_2} = U_1, $$

is computed. The corresponding values of the unit areas $UA$ are plotted against the interfacial settling velocities $(U_i)$, as shown in Figs. 13 and 14, respectively. At each underflow concentration, $C_u$, the maximum value of $UA$ may be read from the plots. The maximum unit area $UA$ existing for each value of $C_u$ is observed to be related to a particular or "critical" interfacial settling velocity.

The time required for the settling sludge to initially go into compression, $t_c$ (hr) (i.e. the time for commencement of consolidation), is required so that the minimum average retention time may be calculated for a continuous thickener.

### 2.2.2. Estimation of the Time for Initial Sludge Compression

Eckenfelder and Melbinger (1957) suggests that the compression point can be estimated from the settling curve by projecting tangents from the hindered settling zone and the compression zone, and bisecting the angle formed by the two tangents. The point at which this bisection intersects the settling curve is the approximate compression point. Roberts (1949) has shown that the rate of sludge thickening with time can be expressed by the relationship,

$$ \log \frac{H - H_\infty}{H_c - H_\infty} = K (t - t_c) \quad (11), $$
where $H_{\infty}$ is the minimum height the sludge reaches at a maximum concentration $C_{\infty}$, and can be determined from a trial-and-error plot of $\log(H - H_{\infty})$ against time. The correct value of $H_{\infty}$ will yield a linear plot from the time $t_c$ that the sludge goes into compression until a later time $t$ when the sludge height is at an intermediate value $H$. The slope of this plot is $K$. When $t_c$ is greater than $t$, the sludge is not initially in compression.

To assist in the estimation of $H_{\infty}$, it has been found in the present investigation that $H_{\infty}$ is approximately 0.4 - 0.5 of the sludge height obtained by settling sludge samples for 18 hours. This approximate factor was obtained by centrifugation of the 18-hour settled sludge, the compacted sludge volume thus obtained being assumed to be an estimate of $H_{\infty}$ in relation to the 18-hour volume. These values are shown in the Appendix (contained in Tables 2a, 2b and 2c respectively). Values of $(H - H_{\infty})$ so obtained are given in Table 6 in the Appendix, and the plots of $(H - H_{\infty})$ against time are shown in Fig. 7. Time $t_c$ occurs at the beginning of the linear portion of the curves, and these values are indicated on the sedimentation curves (Figs. 8 - 12 inclusive). Figs. 8, 9 and 10, respectively, additionally show the initial compression point determined by the bisection of the tangents from the hindered settling zone and the compression zone respectively. Only in the case of Fig. 8 were these latter points (obtained by bisection of tangents) coincident with those obtained by the plot of equation (11).
FIG. 7. TIME FOR INITIAL SLUDGE COMPRESSION, $t_c$.

(TABLE 6)
FIG. 8: SEDIMENTATION CURVE FOR EFFLUENT "A" FLOC IN 2m COLUMN.

2m Column; pH=5.87; Temp.= 24°C.

\[ C_0 = 1796 \text{mg/l} \]

KEY:
- \( H_0, H_1 \) co-ordinates:
- \( H_u \) values:
- Tangents to curves, from \( H_1 \) values:
- Intersection of tangents with \( H_u \) horizontals = \( t_u \) values.

FIG. 9: SEDIMENTATION CURVE FOR EFFLUENT "A" FLOC IN 10 litre JAR.

10 Litre Rectangular Jar; pH=5.95; Temp. = 25°C.; \( C_0 = 1796 \text{mg/l} \)

KEY:
- \( H_0, H_1 \) co-ordinates:
- \( H_u \) values:
- Tangents to curves, from \( H_1 \) values:
- Intersection of tangents with \( H_u \) horizontals = \( t_u \) values.
FIG. 10: SEDIMENTATION CURVE FOR EFFLUENT "B" FLOC IN 2m COLUMN.

TABLE 1B

2m Column; pH = 5.9; Temp = 25°C.

$C_0 = 2054$ mg/l

KEY
- $H_2$, $t_2$ co-ordinates;
- $H_1$ values;
- Tangents to curves, from $H_1$ values;
- Intersection of tangents with $H_u$ horizontals = $t_u$ values.

Sludge Compression
- $t_c = 170$ min. (Intersection)
- $t_b = 225$ min. (Log $H-H_u$ plot)

FIG. 11: SEDIMENTATION CURVE FOR EFFLUENT "C" FLOC IN 2m COLUMN.

TABLE 1C

2m Column; pH = 5.9; Temp = 25°C.

$C_0 = 2018$ mg/l

KEY
- $H_2$, $t_2$ co-ordinates;
- $H_1$ values;
- Tangents to curves, from $H_1$ values;
- Intersection of tangents with $H_u$ horizontals = $t_u$ values.

Sludge Compression
- $t_c = 225$ min. (Log $H-H_u$ plot)

sludge compaction $t = 225$ min. (log $H-H_u$ plot)
FIG. 12: SEDIMENTATION CURVE FOR EFFLUENT "D" FLOC IN 1-litre MEASURING CYLINDER.

TABLE 2e)

1 Litre Cylinder; pH=5.90; Temp.=25°C.

\[ C_0 = 550 \text{ mg/l} \]

KEY.
- \( H_2 \), \( t_c \)-ordinates: ---
- \( H_u \) values: _____________
- Tangents to curves, from \( H_1 \) values: ————
- Intersection of tangents with \( H_u \) horizontals = \( t_u \) values.

Sludge Compression
\[ t_c = 170 \text{ min.} \text{(Log } H-H_{\infty} \text{ plot)} \]
2.2.3. **Correlation between Mass Loading and Underflow Concentration in Thickening.**

Eckenfelder (2a) has given a correlation between the mass loading, $ML = \frac{1}{UA}$, and underflow sludge concentration,

$$(\frac{C_u}{C_o} - 1) = K'_{b}/(ML)^n$$

(12)

(the mass loading value employed in this equation being the reciprocal of the $UA_{\text{max}}$ values obtained from the curves of the plots of $UA$ against interfacial settling velocity in Figs. 13 and 14).

A plot of $\log (\frac{C_u}{C_o} - 1)$ vs $\log (ML)$ yields a set of straight lines, one for each value of the initial concentration $C_o$, and all having the same slope, $n$. The limits of equation (12) are at $C_u = C_\infty$ and $C_u = C_c$, where $C_c$ is the concentration at the start of compression, $K_b$ is a constant for a given initial sludge concentration, $C_o$, and is the intercept on the plot at $(ML) = 1$. For the sludge investigated in this work, it was found that

$$(\frac{C_u}{C_o} - 1) = K_b / (ML)^{1.28}$$

The intercepts of each of these plots on a vertical axis where $(ML) = 1$ [i.e. $\log (ML) = 0$] gives the value of the constant, $K_b$ (Fig. 15). A further plot of $\log K_b$ vs $C_o$ yields another straight line which may be employed for design purposes for any selected or actual value of $C_o$ (Fig. 16). $K_b$ is seen to be an exponential function of the initial sludge solids concentration, $C_o$, and according to Eckenfelder (2a), also a function of the height of the sludge layer.

Thus,

$$K_b = k' e^{mC_o}$$

(12a),

where $k'$ and $m$ are experimentally determined constants, $k'$ being the intercept of the semi-log plot at $C_o = 0$, and $m$ being the slope of the line.
FIG. 13: DETERMINATION OF MAXIMUM UNIT AREA, \( U_{A_{\text{max}}} \), FOR FLOC 'A'.

\[ \text{(TABLE 3b)} \]

(i) EFFLUENT 'A' FLOC
10-litre JAR.
\[ C_0 = 1.796 \text{ g/l} \]
\[ H_0 = 0.29 \text{ m} \]

\[ \begin{align*}
  a: & \quad C_u = 12.5 \text{ g/l} \\
  b: & \quad C_u = 10.0 \text{ g/l} \\
  c: & \quad C_u = 7.5 \text{ g/l} \\
  d: & \quad C_u = 6.0 \text{ g/l} \\
  e: & \quad C_u = 5.0 \text{ g/l}
\end{align*} \]

\[ \text{(TABLE 3a)} \]

(ii) EFFLUENT 'A' FLOC
2 m COLUMN.
\[ C_0 = 1.796 \text{ g/l} \]

\[ \begin{align*}
  a: & \quad C_u = 17.96 \text{ g/l} \\
  b: & \quad C_u = 15.00 \text{ g/l} \\
  c: & \quad C_u = 14.00 \text{ g/l} \\
  d: & \quad C_u = 12.50 \text{ g/l} \\
  e: & \quad C_u = 10.00 \text{ g/l}
\end{align*} \]
FIG. 14: DETERMINATION OF MAXIMUM UNIT AREA, $U_{A_{\max}}$, FOR FLOCS 'B', 'C', & 'D'.

(i) EFFLUENT 'B' FLOC (TABLE 3a)

2 m COLUMN.

- $C_0 = 2,054 \text{ g/l}$
- $C_u = 6,000 \text{ g/l}$
- $C_u = 5,438 \text{ g/l}$
- $C_u = 4,513 \text{ g/l}$

(ii) EFFLUENT 'C' FLOC (TABLE 3b)

2 m COLUMN.

- $C_0 = 2,018 \text{ g/l}$
- $C_u = 4,000 \text{ g/l}$
- $C_u = 3,716 \text{ g/l}$
- $C_u = 3,500 \text{ g/l}$

(iii) EFFLUENT 'D' FLOC (TABLE 3c)

1-litre CYLINDER.

- $C_0 = 0,550 \text{ g/l}$
- $C_0 = 0,34 \text{ m}$
- $C_u = 2,50 \text{ g/l}$
- $C_u = 2,00 \text{ g/l}$
- $C_u = 2,25 \text{ g/l}$
- $C_u = 2,10 \text{ g/l}$
FIG. 15: RELATIONSHIP BETWEEN MASS LOADING & SOLIDS UNDERFLOW CONCENTRATION.

\[
\left( \frac{C_u}{C_0} - 1 \right) = \frac{3.2 \times 10^9 \cdot 8.11 C_0}{10^{1.28} \text{ ML}}
\]

(ML)
**Fig. 16**: Relationship between constant $K_b$ & initial sludge concentration.

$(TABLE\ 5)$
In the present investigation, it was found that

$$K_b = 3.2 \times 10^9 e^{-8.11C_0}.$$  

In the scale-up of a laboratory sedimentation test to actual continuous operation in a plant, the curves relating a tank constant $K_T$ to the laboratory apparatus constant $K_b$ must be known in order to apply a scaling-up correction factor. Usually a plot of $K_T$ vs sludge blanket depth yields a curve such that the correction factors $K_T/K_b$ are less than unity. This correction factor accounts for the lesser effects of tank walls on the sedimentation process in larger diameter vessels, and also any tendency for sludge scouring action in a continuous thickener.

In a continuous thickening operation, the sludge blanket height for a desired underflow concentration must be maintained.

The scaling-up factor $K_T$ may not be significant if the laboratory test apparatus is similar in height to the intended tank and of sufficiently large diameter. In many instances it is not possible to determine $K_T$ values unless a similar tank has operated with the particular floc in question. In these cases, the unit area is increased by an average design factor of 1.25 - 2.0.

2.2.4. Overflow Rate.

In thickening operations, the required unit area is obtained from equation (9),

$$UA = \frac{t_u}{H_0} C_0 H_0^{-2} \text{m}^2/\text{kg/day}. $$

Rearranging the terms,

$$\frac{H_0}{t_u} = \frac{1}{C_0 (UA)}.$$
where $H_o/t_u$ has the dimensions of a velocity, m/d.

When $C_o$ is expressed in kg/m$^3$, the equivalent overflow rate is thus

$$\frac{1}{C_o} (UA) \text{ m}^3/\text{m}^2/\text{d}.$$  

This overflow rate is based on the solids loading $(1/UA)$ and is not the same as the overflow rate for settling based on the settling velocities of the particles. In the latter case, the overflow rate in zone settling $(\text{m}^3/\text{m}^2/\text{d})$ is numerically equal to the settling velocity (m/d) of the smallest particle to be removed.

It is convenient to express the overflow rate based on the liquid throughput, given by

$$\text{(OR)} = \frac{1}{C_o (UA)} \times \frac{(C_u - C_o)}{C_u}$$  

(13).

Because $1 \text{ g} = \frac{1}{1000} \text{ kg}$, and $1 \text{ l} = \frac{1}{1000} \text{ m}^3$, expressing concentrations $C_o$ and $C_u$ in g/l, the units for (OR) in equation (13) remain as cubic metres per day/square metre.

For clarification, the initial zone settling velocity (m/day) is numerically equal to the (OR), $(\text{m}^3/\text{day/m}^2)$ (calculated from equation (13)), and whichever overflow rate is the smaller value, will determine whether clarification or thickening controls the surface area design of the settling tank. For clarification, the average rise velocity of the overflowing supernatant must be less than the zone settling velocity of the floc.

For thickening, the tank bottom surface area for a desired underflow concentration is related to the solids loading to the unit.

In the process proposed for the waste investigated in this thesis, it is essential to obtain thickening of the flocculated sludge. The overflowing supernatant is, however, to be clear.
2.2.5. Sedimentation/Thickening Tank Design.

For any selected initial sludge concentration, $C_o$ g/l, the mass of sludge obtained (kg/d) is given by

$$W_s = \text{[Flow rate]}(m^3/d) \times C_o (g/l)$$  \hspace{1cm} (14).

Dividing the mass of sludge obtained per day, $W_s$, [from equation (14)], by the calculated mass loading, (ML) (kg/m$^2$/d) [from equation (12)], the required surface area, $A$, m$^2$, is obtained. In the case of cylindrical tanks, the diameter $D$ (m) is given by

$$D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{1.275 W_s}{(ML)}}$$  \hspace{1cm} (15).

Equation (15) enables the calculation of corresponding tank diameters to provide designed unit areas necessary for different values of the underflow concentration $C_u$ [from equations (12) or (13) respectively].

At a selected constant flow rate, 45 m$^3$/day, a plot of $\log C_u$ against $\log D$ for different values of $C_o$ yields a family of parallel straight lines (Fig. 17) such that for each plot

$$C_u = K_D D^n,$$

where $K_D$ is the intercept at $D = 1$ and $n$ is the slope. Plotting the intercept $K_D$ of each parallel line of slope $n$ with the vertical axis at $D = 1$ (log $D = 0$) against the corresponding $C_o$ value on a semi-log plot, a further straight line resulted (Fig. 18). The equation of this line was of the form

$$K_D = c' e^{mC_o},$$

where $c'$ is the intercept at $C_o = 0$, and $m$ is the slope. The constant $K_D$ is thus an exponential function of $C_o$. 
$C_u = K_D \cdot D^{2,25} = 1,586 \times 10^{-5} \cdot D^{2,25} - 5,39C_0$ (AT 45 m$^3$/day)

**FIG. 17:** SLUDGE CONCENTRATION – TANK DIAMETER CRITERIA FOR FLOW RATE OF 45 m$^3$/day.

*(TABLE 7)*
\[ K_D = 1 \times 10^5 \cdot e^{-5.39C_0} \]

(At Flow Rate = 45 m³/day)

**FIG. 18:** RELATIONSHIP BETWEEN CONSTANT \( K_D \) & INITIAL SLUDGE CONCENTRATION.
FIG. 19a: SETTLING SLUDGE CONCENTRATION GRADIENTS WITH RESPECT TO THE INTERFACE AT DIFFERENT TIMES.

(TABLE 8a)

NOTE:
1. HORIZONTAL LINES CORRESPOND TO BOTTOM OF COLUMN.
2. TERMINAL CONCENTRATIONS AT BOTTOM OF COLUMN ARE ESTIMATES.

FLOC 'A', $C_0 = 1,796$ g/l
FIG. 19b: SETTLING SLUDGE CONCENTRATION GRADIENTS WITH RESPECT TO THE INTERFACE AT DIFFERENT TIMES.

(TABLE 85)

NOTE: I. HORIZONTAL LINES CORRESPOND TO BOTTOM OF COLUMN.
II. TERMINAL CONC. AT BOTTOM ARE ESTIMATES.

FLOC 'B', C₀ = 2,054 g/l
In the present investigation, it was found that at a design flow rate of 45 m$^3$/d,

$$C_u = K_D D^n = 1,586 \times 10^5 D^{2.25} e^{-5.39} C_0$$  \hspace{2cm} (16).

Hence,

$$D = \frac{C_u}{\sqrt{1,586 \times 10^5 e^{-5.39} C_0}}$$  \hspace{2cm} (17).

Equation (16) or (17) may be used in the design of thickening/clarification tanks with the flocculated sludge in question. They enable the calculation of a tank diameter, involving terms of the initial and underflow sludge concentrations at a fixed flow rate (45 m$^3$/d). However, it follows that when the flow is less than 45 m$^3$/d, the reduced overflow rate allows a longer retention time in the sedimentation tank, with a corresponding decrease in sludge interface height and an increased underflow concentration. Correspondingly, at peak flow rates greater than 45 m$^3$/d the converse will hold, and it would be useful to predict if the sludge height might be excessively high, and thus liable to cause scouring into the overflowing supernatant effluent.

2.2.6. **The Effect of Flow Rate on Sedimentation/Thickening Tank Design.**

Varying the flow rate $Q$ (m$^3$/day), and computing the data necessary for plotting families of curves of log $C_u$ against log $D$ at different values of $C_0$ (Figs. 20 and 21), the slopes of all the straight lines were found to be constant and independent of $Q$. (The slopes are normally distributed with a mean value of 2.165 and standard deviation 0.05 (Fig. 22)).

Plotting the respective intercepts, $K_D$, of the lines at $D = 1$ m against the initial concentration $C_0$ on a semi-log plot, a family of parallel straight lines resulted for each value of $Q$ ($Q$ ranging from 15 m$^3$/d to 75 m$^3$/d), as shown in Fig. 23.
FIG. 20: SLUDGE CONCENTRATION—TANK DIAMETER RELATIONSHIPS FOR $Q=15$ & $30$ m$^3$/d.

**TABLE 3**
FIG. 21: SLUDGE CONCENTRATION-TANK DIAMETER RELATIONSHIPS FOR $Q = 45, 60, \text{ & } 75 \text{ m}^3/\text{d}$.

(TABLE 9)
FIG. 22: MEAN EXPONENT OF TANK DIAMETER D, WITH VARIABLE FLOW-RATE, Q.

(ex FIGS. 20 & 21)
The equation of each line in Fig. 23 is of the form

\[ \log K_D = n C_0 + \text{constant} \]  

(18)

\( n \) = slope of each line, and is constant.

Equation (18) may be written as

\[ \log K_D = n C_0 + \log c' \]  

(19),

where \( \log c' = \log K_D \) at \( C_0 = 0 \).

Evaluating \( \log c' \) (and thus \( c' \)) from equation (19) at any selected convenient value of \( C_0 \) (e.g., \( C_0 = 2.00 \text{ g/l} \)), and for each value of \( Q \), yet a further plot of \( \log c' \) vs. \( \log Q \) yielded a straight line, as shown in Fig. 24. The equation of this line is given by

\[ c' = \text{constant}/Q^P \]

where the constant is the intercept at \( Q = 1 \), and the slope is given by \(-P\).

For the sludge under discussion in this work, it was found that

\[ c' = \frac{\text{constant}}{Q^{1.0914}} = \frac{33760}{Q^{1.0914}} \]  

(20).

Substituting the value of \( c' \) from equation (21) in (22), the latter equation becomes

\[ \log K_D = \log [33760/Q^{1.0914}] - 1.179 C_0 \]

from which

\[ K_D = 33760/Q^{1.0914} e^{-2.7142 C_0} \]  

(21).

The equations to the lines resulting from the plots in Figs. 20 and 21 (i.e., \( \log C_u \) versus \( \log D \)) have the general form,

\[ \log C_u = \text{constant} \times \log D + \log K_D \]  

(22),

where the constant is the value of the slope.

Substituting the value of \( K_D \) from equation (21) in (22), the latter equation becomes

\[ \log C_u = \text{constant} \times \log D + \log [33760/Q^{1.0914} e^{-2.7142 C_0}] \].
TABLE 10, & FIGS. 20 & 21

E-0

\[ \begin{array}{c|c|c|c|c}
\text{INITIAL SLUDGE CONCENTRATION, } C_0 \text{ (g/l)} & 0.1 & 0.15 & 0.2 & 0.25 \\
\text{SLOPE OF LINES} & -1.179 & & & \\
\end{array} \]

FIG. 23: RELATIONSHIP BETWEEN CONSTANT \( K_p \)
INTERCEPT & \( C_0 \) FOR VARIABLE FLOW RATE, \( Q \).
FIG. 24: RELATIONSHIP BETWEEN CONSTANT $C'$ ($K_D$ INTERCEPT AT $C_0 = 200\text{g/l}$) & FLOW RATE FOR DETERMINATION OF THE EXPONENT OF $Q$. (TABLE 11)
Solving for $C_u$,

$$C_u = \frac{33760}{Q^{1.0914}} e^{-2.7142 C_o} \text{ constant} \quad (23).$$

Evaluation of the exponent of $D$ (Figs. 20, 21 and 22) yielded a value of 2.165 for this constant. Substituting this value for the constant in equation (23), and rearranging,

$$D = 8.0975 \times 10^{-3} \times \frac{2.165}{\sqrt{C_u Q^{1.0914}} e^{2.7142 C_o}} \quad (24).$$

Equation (24) yields the optimum diameter of the thickening tank required, involving the parameters of flow rate; initial concentration of influent floe; and concentration of underflow sludge. Because the concentration and flow parameters may vary, the equation is proposed as a suitable one for simulation purposes.

When a thickening tank diameter has been selected, a log-log plot of $Q$ against $C_u$ for different values of $C_o$ produces a family of parallel straight lines. Each value of $C_u$ may be computed from equations similar to (23) or (24) respectively. The general form of equation (24) is seen to be

$$D = k \sqrt[n]{C_u Q^p e^{m C_o}} \quad (25),$$

where $k$, $m$, $n$, and $p$ are empirical constants or exponents determined from batch sedimentation tests on any given floe.

Inspection of equation (24) shows that the desired thickening tank diameter is closely a function of the square root of the flow rate,

$$i.e. \quad D = k \sqrt[n]{C_u e^{m C_o}} \times \sqrt{Q} \quad (25A),$$
and this latter equation may equally well be employed in calculations.

Fig. 25 is a plot of a simulation chart based on equation (24), and in which the thickening tank diameter is \( D = 3 \) m. The use of this chart in predicting sedimentation/thickening tank performance will be demonstrated at a later stage (section 3.2.5.).
Figure 25: The effect of flow rate on underflow sludge concentration at variable initial concentrations, for the simulation of a 3m dia. settling tank.

(Table 12)
2.3 Disposal of the Settled Flocculated Sludge by Sand Filtration.

The disposal of a chemical waste sludge requires its dewatering in the most economical manner as a prime objective. Continuous centrifugation will readily accomplish this objective, but mitigating against this procedure is the capital cost of plant and the additional power costs in operation. The cheapest procedure is drainage on a sand filter, where land availability permits. Further drying of dewatered sludge may be achieved at the expense of fuel or thermal energy costs. However, natural air-drying may readily dry the dewatered sludge to a sufficiently low moisture content, which will enable the residue to be dug out and carted away for final dumping.

Eckenfelder (2c) described the two mechanisms by means of which sludges are dewatered on sand beds, viz. (a) filtration through the sand bed, and (b) evaporation as a result of radiation and convection. In the case of filtration, there are two methods of operation to consider, as outlined in section 1.4.(4). Either the sludge may be continuously fed and spread onto the sand bed, until some predetermined thickness of sludge has built-up; or the bed may be filled intermittently, allowing sufficient drainage time between applications. The rate of filtration, which, among other parameters, depends on the specific resistance of the sludge cake, determines which operational procedure should be followed. Because the resistance of the cake increases with the depth of the cake formed on the bed, the intermittent application of feed to a given filtration bed will tend to be limited. Higher loading rates can be expected with smaller masses and thicknesses of the cake. For this reason, it was decided to investigate a continuous feed process to a sand filter. At some optimum manageable level, the feed is to be diverted to a
second filter while the first one drains to completion. After a bed has drained, air-drying would continue until the mass is of a spadeable consistency.

Evaporation occurs during the entire filtration cycle, and is strongly dependent on ambient conditions of temperature, humidity and wind velocity, as well as the physico-chemical properties of the sludge itself. Thus, in practice evaporation rates can be expected to vary widely. In the municipal area of Cape Town, it is a regulation that drainage beds leading directly to the sewer must be covered or roofed, in order to minimize rainwater infiltration. This factor reduces some of the expected variations such as when the bed is exposed to rainfall. It would appear sufficient to examine the effects of some of the variables under several constant ambient conditions.

In the filtration period, the effects of drying can essentially be neglected, and hence each process may be discussed separately.

2.4. Sludge Filtration.

Carman (10) and Coakley and Jones (11) formulated the rate of filtration of sludges based on Poiseuille's and D'Arcy's Laws. The rate equation is given by (10)

\[ \frac{dV}{dt} = \frac{PA^2}{\mu (r c V + R A)} \]  

(26),

where \( V \) = volume of filtrate (ml),  
\( t \) = form time for the cake (sec or min),  
\( P \) = vacuum (g/cm²),  
\( A \) = filtration area (cm²),  
\( \mu \) = filtrate viscosity (poise),
FIG. 26: DIAGRAM OF APPARATUS FOR THE CONSTANT HEAD FILTRATION OF SLUDGES ON A SAND BED.
\( r = \text{specific resistance of the cake} \ [\text{sec}^2/\text{g}, \text{or} \ (\text{sec})(\text{min})/\text{g}] \),
\[ c = \text{mass of solids per unit volume of filtrate} \ (\text{g/ml}), \]
\( R_m = \text{initial resistance of the filter media} \ [\text{sec}^2/\text{cm}^2 \ \text{or} \ (\text{sec})(\text{min})/\text{cm}^2]. \)

Integration and re-arrangement of equation (26) yields

\[
\frac{t}{V} = \frac{\mu rc}{2PA^2} V + \frac{\mu R_m}{PA} \tag{27}
\]

According to equation (27), a plot of the function \( t/V \) against \( V \) is linear (as shown in Figs. 28, 29 and 30 respectively). The slope \( b \) is equal to \( (\mu rc/2PA^2) \), and re-arranging,

\[
r = \frac{2bPA^2}{\mu c} \tag{28}
\]

The specific resistance, \( r \), is a measure of the filterability of the sludge. From equation (28) it is evident that it is numerically equal to the pressure difference required to produce a unit rate of filtrate flow of unit viscosity through unit mass of cake. However, \( r \) will only remain constant in the case of an incompressible sludge. For actual sludges \( r \) is dependent on pressure because of the reduced permeability as pressure increases, due to a reduction in the voids (Ruth et al. 1933(12)). The specific resistance is related to the pressure by an empirical relationship (Eckenfelder (2b)),

\[
r = r_o P^s \tag{29}
\]

where \( S = \text{coefficient of compressibility}, \)
and \( r_o = \text{specific resistance at unit pressure}. \)

Thus the filtration rate equation (27) is more precisely described by substituting equation (29) for \( r \) in equation (27).
FIG. 27: RATE OF EVAPORATION OF FILTRATE AT 20°C & 65% RELATIVE HUMIDITY.

MOISTURE LOSS PER UNIT AREA (mg/cm²)

EVAPORATION RATE = 0.142 mg/cm²/min

FIG. 28: CONSTANT HEAD SAND FILTRATION AT 20°C (TABLE 13a)
2. $C_0 = 18.798$ g/l

**FILTRATION HEADS**
- 2-1: 191.9 mm
- 2-2: 153 mm
- 2-3: 125 mm
- 2-4: 64 mm

**SLOPES, $b$ min/cm$^3$**
- 2-1: 0.02674
- 2-2: 0.03747
- 2-3: 0.03563
- 2-4: 0.04772

**FILTRATE VOLUME, $V_f$ (ml)**

**FIG. 29: CONSTANT HEAD SAND FILTRATION AT 20° C. (TABLE 13b)**

3. $C_0 = 20.930$ g/l

**FILTRATION HEADS**
- 3-1: 191 mm
- 3-2: 151 mm
- 3-3: 63.5 mm

**SLOPES, $b$ min/cm$^3$**
- 3-1: 0.03914
- 3-2: 0.04386
- 3-3: 0.05562

**FILTRATE VOLUME, $V_f$ (ml)**

**FIG. 30: CONSTANT HEAD SAND FILTRATION AT 20° C. (TABLE 13c)**
Thus, \( t = \frac{\mu r c}{2F(1-s)^2} \cdot v + \frac{\mu r m}{PA} \) \quad (30),

whence \( r_0 = \frac{2bP(1-s)^2}{\mu c} \) \quad (31).

In most filtration operations, it was proposed by Eckenfelder (2b) that \( R_m \), the initial filter media resistance, is small compared with the resistance developed by the filter cake, and suggested that it can be neglected. However, it will be shown that this simplification holds only approximately for low values of the pressure head, \( P \), across the filter concerned. Equation (26) was developed by Carman essentially for vacuum filtration, and in applying it to sand bed drainage, Eckenfelder (2b) defined the pressure term \( P \) as the average hydrostatic head of sludge in the bed during drainage, all other terms being defined as in the original Carman equation. In vacuum filtration, the time to form a cake from the beginning of the application of vacuum until the end when vacuum is released and the cake solids can be discharged, is the cycle time, \( t \). In sand filtration, it is interpreted as the time of filtration from the beginning to the time \( t \) to produce a filtrate volume \( V \).

It is essential for the proposed dewatering operation that the hydrostatic head be maintained at relatively low values, in order that the cake thickness for drying should be minimal, and consequently the value of \( P \) will be relatively low.

Equation (26) is primarily an equation for constant pressure filtration. In a similar form, Ruth et al. (12) evolved a simplified equation,

\[
\frac{dV}{dt} = \frac{AP}{\mu c},
\]
where $P_c$ is the pressure drop across the cake alone, and its resistance is $R$. The pressure head across the combined cake and filter medium is more readily measured. However, the resistance to the flow of the filtrate consists of the sum of the resistances offered by the cake and the filter medium. The latter resistance (filter medium) is accounted for in terms of an equivalent resistance offered by a mass of cake, which would have been deposited by a corresponding volume of filtrate.

In the present work it is confirmed that operating a filter at constant head, if the pressure head across the sludge and filter medium is the value of $P$ accepted for use in equations (30) and (31) respectively, inclusion of the filter media resistance $R_m$ results in equation (30) being closely satisfied. Consequently, the term $\mu R_m/PA$ in equation (30) may not always be insignificant. When the head of sludge varies (i.e. the liquid level falls during drainage), the rate equation could best be described by an empirical power function ($\log t/V$ plotted against $\log V$). In this instance, by simply assuming a mean value for the head or depth of sludge in the filter, it is an approximation to employ Carman's equation (Figs. 42 and 43 respectively).

Values of the specific resistance, $r$, may be determined from equation (28) for different experimental values of the pressure $P$. A plot of $\log r$ vs. $\log P$, which is approximately linear (Fig. 31), will resolve the constants $r$ and $s$ in equation (29). These are required in the calculation of loading rates for design purposes (section 2.4.1. and 2.4.2.).

The concentration term, $c$, is defined by the mass of solids per unit volume of filtrate. It is therefore a measure of the mass of
FIG. 31:
RELATIONSHIP BETWEEN SPECIFIC RESISTANCE OF SLUDGE FILTER-CAKE & PRESSURE DURING CONSTANT HEAD SAND FILTRATION AT 20º C. (TABLE 14)

INITIAL SPECIFIC RESISTANCE, \( r = 0.512 \times 10^6 \cdot P^{0.325} \)

COMPRESSIBILITY OF SLUDGE FILTER CAKE, \( s = 0.325 \)

\( C_0 = 14,573 \text{ g/l} \)
\( C_0 = 18,798 \text{ g/l} \)
\( C_0 = 20,930 \text{ g/l} \)
solids removed, regardless of their initial concentration. In terms of the initial and final moisture contents of the sludge, $c_i$ and $c_f$ (g water/100g of sludge) respectively, $c$ is derived as follows:-

Initially, 100 g of sludge contains $c_i$ grams of water and $(100-c_i)$ grams of solids.

After filtration, 100 g of sludge contains $c_f$ grams of water and $(100 - c_f)$ g of solids.

Thus, the final sludge contains $c_f/(100-c_f)$ g water per 1 g of dry solids.

But the mass of dry solids removed in the filtration = $(100 - c_i)$ g.

Therefore the mass of water associated with $(100-c_i)$ g of solids in the final sludge =

$$\frac{c_f}{(100-c_f)} \times (100 - c_i) \text{ g}$$

Mass of water removed in the filtration as filtrate

$$= \left[ c_i - \frac{c_f (100 - c_i)}{(100 - c_f)} \right] \text{ g}$$

Assume the density of water is unity (g/ml); then the mass of solids removed per 1 ml of filtrate = $c$

$$= \frac{(100 - c_i)}{c_i (100 - c_f) - c_f (100 - c_i)} \text{ g/ml}$$

whence

$$c = \frac{(100 - c_i) (100 - c_f)}{c_i (100 - c_f) - c_f (100 - c_i)} \text{ g/ml}$$

or

$$c = \frac{1}{c_i/(100 - c_i) - c_f/(100-c_f)} \quad (32)$$

This is the relationship given by Eckenfelder (2b).
However, if $C_0 =$ initial average solids content of the feed (units, g/l) and $C_f =$ final average solids content of the drained sludge (g/l); $\rho_o$ and $\rho_f$ being the initial and final densities of the sludge respectively (g/ml); then per 100 g mass of initial sludge,

$$c_i = 100 - \frac{C_0}{10.\rho_o}$$

and per 100 g mass of final sludge,

$$c_f = 100 - \frac{C_f}{10.\rho_f}$$

Substituting in equation (32) and simplifying,

$$c = \frac{C_0}{1000} \times \frac{C_f}{(C_f \rho_o - C_o \rho_f)}$$

(33)

With most sludges during drainage, due to their high degree of hydration, $\rho_o$ is not very much less than $\rho_f$.

$$\rho_o \approx \rho_f = \rho,$$

where $\rho =$ mean density of initial and final sludges on the bed. Hence, equation (33) may be approximated to.

$$c = \frac{C_0}{1000} \frac{C_f}{\rho (C_f - C_o)}$$

(34)

Equation (34) enables the solids removal term $c$ (g/ml) to be calculated from the more conveniently available values of initial sludge slurry concentration $C_o$ (g/l); the final sludge concentration after completion of filtration, $C_f$ (g/l); and the mean sludge density during the filtration period, $\rho$ (g/ml).
2.4.1. Loading Rate Equations Neglecting Filter Media Resistance.

If the filter medium resistance \( R_m \) in equation (30) is neglected, solving for the filtrate volume,

\[
V = \sqrt{\frac{2P(1-s)}{\mu r_c}} \frac{2}{A} \frac{t}{c}
\]  

Multiplying both sides of the latter equation by the term \( c/tA \), the filtration rate, or loading rate \( L \) \((2b)\) is obtained. Thus,

\[
L = \frac{Vc}{tA} = \sqrt{\frac{2P(1-s)c}{\mu r_o} \frac{t}{c}}
\]  

where \( L \) = mass of solids removed per unit area per unit time (kg/m\(^2\)/d).

To account for deviations resulting from variations in the permeability of the cake as more solids are deposited, and also as a result of variations in the feed concentration, equation (36) needs to be modified. Eckenfelder \((2b)\) suggested that the terms \( c \) and \( t \) each be raised to some exponent \( m \) and \( n \) respectively.

Hence equation (36) becomes,

\[
L = \sqrt{\frac{2P(1-s)}{\mu r_o} \frac{t}{c^m}}
\]  

The values of \( m \) and \( n \) can be determined experimentally by selecting a series of constant filtration times \( t \) and calculating values of \( L \) and \( c \). The slope of the log-log plot of \( L \) versus \( c \) evaluates \( m \). Similarly, when a series of constant solids removal values \( c \) is selected, values of \( L \) and \( t \) may be calculated. The slope of the log-log plots of \( L \) against \( t \) evaluates \( n \). These plots are shown in Fig. 37 which give values for \( m \) and \( n \) in \( c^{m/n}t^n \) as \( 0.52/0.78 \).

The data for calculating \( L \) at constant time or constant solids removal, may be obtained from the original experimental observations of the
filtrate volume \( v_t \) at each time \( t \), these being used in equations (27) and (30). By plotting \( v_t \) against \( t \) for the different filtrations (as shown in Fig. 36), the curves are read at different selected times, and the calculations are described in the Appendix (section 4.3.5.).

When equation (37) has been used to evaluate the loading rate \( L \), the results have been found to be very low compared with the actual loading rates possible. The relationship is therefore still unsatisfactory for design. However, in this investigation it was found that comparing experimentally observed values of \( L \) with those obtained from equation (37), an empirical correlation may be derived as follows:-

The ratio between observed loading rates and those calculated from (37) was found to be approximately constant.

i.e. \( \frac{L_{\text{observed}}}{L_{\text{calculated}}} \) is constant.

Substituting for \( L_{\text{calculated}} \) from equation (37),

\[
L_{\text{observed}} = k \sqrt{\frac{2P(1-s)}{\mu r_0}} \frac{c^m}{t^n}
\]

(37A).

Rearranging,

\[
L_{\text{observed}} \cdot t^n \sqrt{\frac{\mu r_0}{2P(1-s)}} = kc^m
\]

(37B).

Plotting the function on the left-hand side of this equation against the term \( c \) on a log-log plot, a straight line is now obtained as shown in Fig. 39, \( k \) thus being an experimentally derived constant. A least-squares fit of the data in Fig. 39 (correlation coefficient = 0.83) yields, after re-arranging, the following empirical equation:-

\[
L = 8.55 \sqrt{\frac{2P(1-s)}{\mu r_0}} \cdot \frac{c^{0.56}}{t^{0.78}}
\]

(37C).
Equation (37c) now replaces equation (37), and was found to be suitable at higher sludge concentrations (Table 2.III (c); $C_0 > 18 \text{g/l}$) when the resistance of the filter media is relatively small in comparison with that of the sludge, and $R_m$ may be ignored.

2.4.2. Theoretical Loading Rate Equations Including Filter Media Resistance.

When the sludge concentration is at lower values ($C_0 < 18 \text{g/l}$) and the pressure heads are also low, the resistance of the filtering sludge is reduced, and the filter medium resistance $R_m$ is significant. In this investigation the following sequence of equations was derived:

For a given constant pressure head $P$ and a constant filtration time $t$, re-arrangement of the terms of the theoretical equation (30) yields the equation of a parabola;

$$\text{thus, } \left( \frac{\mu r c}{2P(1-s)A^2} \right) \times V^2 + \left( \frac{\mu R_m}{PA} \right) \times V - t = 0 \quad (38).$$

Selecting a given fixed value of the filtration time $t$, the quadratic equation (38) may be solved for $V$. Thus,

$$V = \frac{\sqrt{\left( \frac{\mu R_m}{PA} \right)^2 - \frac{2\mu r c t}{P(1-s)A^2} - \frac{\mu R_m}{PA}}}{\mu r c \sqrt{\frac{P(1-s)A^2}{}}$$

(39).

The loading rate, $L$, defined by $Vc/tA$ [equation (36)] may be calculated in a similar manner to equation (36) by multiplying both sides of equation (39) by the term $c/tA$, and simplifying. Thus,

$$L' = \frac{\left( \frac{\mu R_m}{\mu r c P^s t} \right)^2 + \frac{2\mu r c P(1+s)}{}}{\mu r c P^s t} \cdot t - \frac{R_m}{r_o P^s t}$$

(40).
Equation (40) is a theoretical loading rate equation, which indicates that the filter media resistance $R_m$ can be of significance in the calculation of the loading rate, depending on its value relative to the sludge cake resistance. In addition, this equation indicates that the physical properties of the sludge (initial specific resistance $r_0$, and compressibility $s$) and their variation with respect to the pressure head $P$ and time $t$, also affect the significance of $R_m$. As the pressure head and/or time increase, the term $R_m / r_0 P^m t$ decreases in value, and $R_m$ then becomes of lesser importance. It is therefore necessary to determine the value of $R_m$.

The value of $R_m$ may be obtained as follows:

Reference to the linear equation (30) shows that the intercept, $I$ (min/cm³), of the plot of $t/V$ versus $V$ on the $t/V$ axis, yields

$$I = \frac{\mu R_m}{PA}$$

which enables the calculation of an estimate of the value of $R_m$.

Theoretically, for a given sand filter and a sludge having a constant filtrate viscosity, it is expected that the filter media resistance should remain constant, and thus the intercept $I$ should also be a constant. However, reference to Figs. 28, 29 and 30 respectively, showed that this was not always true. Only at a high sludge concentration, was this approximately so (Fig. 30). In the other experiments at lower sludge concentrations, there appeared to be a variation in the permeability of the sand layer which may account for variations in $I$, and thus in $R_m$. Nevertheless, values of $R_m$ calculated for eleven constant head filtrations showed that the set of individual values was normally distributed (Fig. 32), and it is
**FIG. 32.**

**DISTRIBUTION OF LAB. FILTER MEDIA RESISTANCE, \( R_m \).**

\( R_m = 4.31 \times 10^4 \text{ (min/sec)} \)

\( \sigma = 2.0 \times 10^4 \text{ /cm}^2 \)

**FIG. 33: RELATIONSHIP BETWEEN OBSERVED & CALCULATED FILTER LOADING RATES.**

(Constant Head Sand Filtration)

\[ \frac{R_m}{R_{o, t, P}^a} \times 10^5 \]

\[ L_{obs} = L_{calc} - \frac{1.026 R_m}{R_{o, t, P}^a} + 0.0236 \times 10^{-4} \]
FIG. 34: SAND FILTER LOADING RATE FOR CONSTANT HEAD FILTRATION AT 20°C & C₀ = 14 g/l. (TABLE 16)
appropriate to use the mean value of $R_m$ in evaluating equation (40).

Comparing the calculated theoretical loading rates from equation (40) with those actually observed (Table 2.III (d)), agreement was not obtained in all the filtration experiments and even this equation requires empirical modification, as follows:-

Equation (36) is based on the theoretical equation (30) in which the term $\mu R_m/PA$ is ignored because $R_m$ is assumed to be negligible. It seems that the calculated loading rates obtained from equation (36) need to be corrected by a function involving $R_m$ and the properties of the filtering sludge. These properties concern the specific resistance of the sludge; its variation with time and pressure head; and the sludge compressibility (as described above). From the theoretical loading rate equation (40), a term involving these parameters is $R_m/r_P^S t$. Attempting the use of the latter as a correction function to the simplified equation (36), the correction function was plotted against the difference between observed loading rates and those calculated from equation (36). This plot is shown in Fig. 33, and results in a straight line. The best fitting straight line to this plot obtained by least squares, gives an empirical equation of the form,

$$L_{\text{observed}} - L_{\text{calculated}} = k' - k'' \frac{R_m}{r_P^S t},$$

where $k'$ and $k''$ are experimentally determined constants.

In this investigation it was found that:

$$L_{\text{observed}} - L_{\text{calculated}} = 0.0236 \times 10^{-4} - \frac{1.028 R_m}{r_P^S t}.$$

Rearranging, $L_{\text{observed}} = L_{\text{calculated}} - \frac{1.028 R_m}{r_P^S t} + 0.0236 \times 10^{-4}$ (41),
<table>
<thead>
<tr>
<th>Note:</th>
<th>(a) $I_{CALC} = \sqrt{\frac{2 \mu (1-S)}{\mu_r r_0^2}}$</th>
<th>(equation (36), neglecting $R_m$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(b) $L_{EMP} = \frac{L_{CALC}}{r_0 R_p}$</td>
<td>$1.029 \frac{R_m}{R_p} + 0.0236 \times 10^{-4}$ (equation (41); $R_m = 4.31 \times 10^4$) (FIG. 39).</td>
</tr>
<tr>
<td></td>
<td>(c) $L_{EMP} = 8.55 \sqrt{\frac{2 \mu (1-S)}{\mu_r}} \cdot \frac{0.56}{0.76}$</td>
<td>(equation (37c)) (FIG. 39).</td>
</tr>
<tr>
<td></td>
<td>(d) $L_1 = \sqrt{\frac{1\mu R_m}{\mu_x r_0^2 t}} + \frac{2 \mu R_p (1 + S)}{r_0 R_p t}$</td>
<td>$\frac{R_m}{r_0 R_p t}$ (equation (40); $R_m = 4.31 \times 10^4$).</td>
</tr>
</tbody>
</table>
Loading rates calculated from the modified empirical equation (41) yielded very good agreement with those observed [Table 2.III (b)] over the entire range of sludge concentrations and pressure heads to be expected. Thus for equation (41) in which $R_m$ is taken into consideration, $L_{\text{Empirical}}$ is approximately equal to $L_{\text{Observed}}$.

2.4.3. Selection of a Design Loading Rate Equation.

When the filter bed loading rates are calculated from both modified empirical equations (37c) and (41) respectively, it is now possible to ascertain whether the filter medium resistance $R_m$ may be neglected. It has been found that at higher initial sludge concentrations, and at higher pressure heads or depths of sludge, the resistance $R_m$ is relatively small compared to that of the sludge, and may be neglected (section 2.4.1.). Equation 37(c) may then be used as a design equation. For lower initial sludge concentrations and low pressure heads or depths of sludge, the resistance $R_m$ becomes increasingly significant and needs to be taken into consideration to ensure that the required drainage area is not under-designed. In this case, equation (41) is then suitable for the design of sand filters (section 2.4.2.).

Thus a method is now available which enables one to select the appropriate design equation. By comparing the experimentally observed loading rates obtained over the range of initial sludge concentrations and pressure heads expected, with those calculated from each of the equations in turn, the equation giving the best agreement is the appropriate one. In Table 2.III, the ratio $L_{\text{Observed}}/L_{\text{Calculated}}$ is shown for each of the equations in columns (a), (b), (c) and (d) respectively.
The closeness of approach to unity is a measure of the suitability of the equation concerned. For the sludge and filtration experiments in this work, equation (41) was found to be the most suitable to use.

2.5. *The Parameters in Constant Head Drainage.*

Inspection of equation (36)

\[ L = \sqrt{\frac{2P(1-e)}{\mu r_0 t}} \]

shows that the filter loading rate varies as the square root of a power of the head or depth of sludge, and also directly as the square root of the concentration term. For a given sludge \((r_o \text{ constant})\) the filter loading rate varies inversely as the square root of the viscosity of the filtrate and the cycle time, (or time of application of sludge).

Thus, higher loading rates will be expected by increasing the depth of sludge and operating at high values of the concentration term, \(c\).

However, excessively high values of the pressure head \(P\) tend to create problems with the subsequent drying of the filter cake (longer drying times will be expected, and thus larger areas required). The operating head would then be dependent essentially on the drying characteristics of the cake, and on the availability of land.

The concentration term, \(c\), calculated from equation (34), (which is plotted in Fig. 35) shows that for any given initial sludge concentration \(C_0\), larger values of \(c\) were obtained when the final drained sludge concentration \(C_f\) is relatively low. However, the object of the drainage was to obtain as high a value of \(C_f\) as possible. The family of curves for \(c\) tends to have diminishing slopes as \(C_f\) increases, and thus during prolonged
FIG. 34a: EFFECT OF TIME OF FILTRATION ON LOADING RATE IN CONSTANT HEAD FILTRATION (20°C).

FIG. 35: CALCULATION OF MASS OF SOLIDS PER UNIT VOLUME OF FILTRATE, \( c \).
**FIG. 36:**
CONSTANT HEAD FILTRATION. DETERMINATION OF FILTRATE VOLUME AT CONSTANT TIMES.
(TABLES 13a, 13b & 13c)

**FIG. 37:** DETERMINATION OF EXPONENTS OF $t$ & $c$ IN CONSTANT HEAD FILTRATION.
(TABLES 18 & 19)
FIG. 38: DISTRIBUTION OF THE EXponent OF TIME, n.

Mean, \( \bar{x} = -0.78 \)
S.D. = -0.04
\[ L = 8.55 \frac{2P(l-S)}{\mu \cdot r_0} \cdot c^{0.56} \cdot t^{0.78} \text{ g/cm}^2/\text{min} \]

\[ f(c) = L \cdot t^{0.78/2P(l-S)} \]

**CONCENTRATION TERM, c g/ml**

**FIG. 39**: DETERMINATION OF RELATIONSHIP BETWEEN c & f(c) TO ESTABLISH A MODIFIED EMPIRICAL LOADING RATE EQUATION FOR CONSTANT HEAD SAND FILTRATION OF SLUDGE. (TABLES 20 & 2.111c)
drainage, relatively small changes in c result in greater increases in $C_f$. In this work, it was found that the sludge concentration term ranges from $c = 0.03 \text{ g/ml}$ to $c = 0.06 \text{ g/ml}$, and the initial sludge concentration $C_o$ from 10 - 20 g/l. Selecting a fixed value of $c = 0.03 \text{ g/ml}$, the following table shows the negligible effect of the initial concentration $C_o$ on the loading rate:

TABLE 2.IV
Calculation of Constant Head Filtration Loading Rates at Different Initial Sludge Concentrations and Depths.

<table>
<thead>
<tr>
<th>$C_o$ (g/l)</th>
<th>$C_f$ (g/l)</th>
<th>$\bar{\rho}$ (g/ml)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>15.0</td>
<td>1.0095</td>
</tr>
<tr>
<td>20</td>
<td>59.3</td>
<td>1.0165</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$C_o$ (g/l)</th>
<th>Bed Depth h(cm)</th>
<th>Pressure Head, $P$ (g/cm$^2$)</th>
<th>$\dot{L}_{\text{CALC}}$ (kg/m$^2$/d)</th>
<th>$\dot{L}_{\text{EMP}}$ (kg/m$^2$/d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>30</td>
<td>31.90</td>
<td>4.126</td>
<td>3.879</td>
</tr>
<tr>
<td>20</td>
<td>30</td>
<td>32.12</td>
<td>4.136</td>
<td>3.889</td>
</tr>
<tr>
<td>10</td>
<td>25</td>
<td>26.85</td>
<td>3.893</td>
<td>3.630</td>
</tr>
<tr>
<td>20</td>
<td>25</td>
<td>27.04</td>
<td>3.902</td>
<td>3.640</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
<td>21.81</td>
<td>3.629</td>
<td>3.345</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
<td>21.96</td>
<td>3.638</td>
<td>3.355</td>
</tr>
<tr>
<td>10</td>
<td>15</td>
<td>16.76</td>
<td>3.321</td>
<td>3.008</td>
</tr>
<tr>
<td>20</td>
<td>15</td>
<td>16.87</td>
<td>3.328</td>
<td>3.017</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>11.71</td>
<td>2.942</td>
<td>2.587</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
<td>11.79</td>
<td>2.949</td>
<td>2.595</td>
</tr>
</tbody>
</table>
### TABLE 2.IVa (Fig. 34a)

The Effect of Filtration Time on Loading Rate in Constant Head Filtration.

<table>
<thead>
<tr>
<th>Pressure Head, ( P \text{(g/cm}^2 \text{)} )</th>
<th>10 ( \times 10^{-4} )</th>
<th>15 ( \times 10^{-4} )</th>
<th>25 ( \times 10^{-4} )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Filtration Time, ( t \text{(days)} )</strong></td>
<td>( L_{\text{EMP}} )</td>
<td>( L_{\text{EMP}} )</td>
<td>( L_{\text{EMP}} )</td>
</tr>
<tr>
<td>0.25</td>
<td>2.76</td>
<td>3.47</td>
<td>4.46</td>
</tr>
<tr>
<td>0.50</td>
<td>2.19</td>
<td>2.67</td>
<td>3.33</td>
</tr>
<tr>
<td>0.75</td>
<td>1.88</td>
<td>2.26</td>
<td>2.79</td>
</tr>
<tr>
<td>1.00</td>
<td>1.68</td>
<td>2.00</td>
<td>2.45</td>
</tr>
<tr>
<td>1.25</td>
<td>1.53</td>
<td>1.81</td>
<td>2.22</td>
</tr>
<tr>
<td>1.50</td>
<td>1.42</td>
<td>1.67</td>
<td>2.04</td>
</tr>
<tr>
<td>1.75</td>
<td>1.33</td>
<td>1.56</td>
<td>1.90</td>
</tr>
<tr>
<td>2.00</td>
<td>1.25</td>
<td>1.47</td>
<td>1.78</td>
</tr>
<tr>
<td>2.50</td>
<td>1.14</td>
<td>1.33</td>
<td>1.61</td>
</tr>
<tr>
<td>3.00</td>
<td>1.05</td>
<td>1.22</td>
<td>1.48</td>
</tr>
<tr>
<td>4.00</td>
<td>0.92</td>
<td>1.07</td>
<td>1.29</td>
</tr>
<tr>
<td>5.00</td>
<td>0.83</td>
<td>0.97</td>
<td>1.16</td>
</tr>
</tbody>
</table>

* **NOTE:**

\[
L_{\text{EMP}} = \sqrt{\frac{2P(1-s)c}{\mu rt}} - 1.028 \frac{R_m}{r_o P s^2} + 0.0236 \times 10^{-4},
\]

where \( r_o = 0.512 \times 10^6 \text{ (sec)(min)/g} \); \( \mu = 0.01026 \text{ poise} \);

\( s = 0.325 \); \( c = 0.03 \text{ g/ml} \); \( R_m = 4.31 \times 10^4 \text{ (min)(sec)/cm}^2 \).
Figure 34 is a plot of the filter loading rate against sludge depth at constant head, for a range of values of \( c \) calculated at the average expected value of \( C_0 = 14 \text{ g/l} \). The curve for \( c = 0.03 \text{ g/ml} \) is plotted for both \( C_0 = 10 \text{ g/l} \) and \( C_0 = 20 \text{ g/l} \). This shows the negligible influence of \( C_0 \). The cycle time has been kept constant at \( t = 1440 \text{ minutes (1 day)} \). This set of curves may be employed at a later stage for design purposes, once the depth of sludge has been selected.

Calculating the loading rate from equation (41),

\[
L_{\text{EMP}} = \sqrt{\frac{2 \rho (1-s) c}{\mu r t}} - 1.028 \frac{R_m}{r_c} + 0.0236 \times 10^{-4},
\]

in which the solids removal term \( c = 0.03 \text{ g/ml} \) and the mean value of \( R_m = 4.31 \times 10^4 \text{ (min)(sec)/cm}^2 \), the effect of constant head filtration for different filtration times and pressure heads is apparent in Table 2.IVa. The data are plotted in Fig. 34a. It is seen that as the filtration time increases, the rate of drainage decreases, and the rate at which slurry may be fed to the filter to maintain a constant head, will thus also decrease.

After a period of approximately one day's constant head filtration, the loading rate is considerably reduced. This was confirmed in the experimental filtrations of this particular sludge, when regardless of the initial concentration, \( C_0 \), or operating depth, \( h \), a filtration time in excess of one day produced an insignificant amount of filtrate. Although a reduction of the filtration time increases the loading rate for a given sludge dewatering, too short a filtration time achieves insignificant sludge dewatering. It was thus apparent that for this sludge, the constant head filtration time should not exceed approximately one day.
The loading rate is also temperature dependent, by virtue of 1 varying inversely as the square root of the viscosity [Equations (36), (37c) and (41)]. Thus it would be appropriate to operate at as high a temperature as possible, in order to reduce the liquor viscosity, and thereby increase the loading rate. Hence, for a given filtration, \( \log T/\mu \). If \( L_1 \) and \( L_2 \) are respective loading rates at temperatures \( T_1 \) and \( T_2 \) at which the viscosities are \( \mu_1 \) and \( \mu_2 \), respectively, then
\[
\frac{L_1}{L_2} = \sqrt{\frac{\mu_2}{\mu_1}}.
\]
The effect of temperature is indicated in the following examples:

\[
L_{10^\circ C} = 0.87 \times L_{20^\circ C},
\]
\[
L_{30^\circ C} = 1.12 \times L_{20^\circ C}.
\]

2.6. Falling Head Drainage.

As an alternative procedure to constant head filtration, a sand filter may be filled and then allowed to drain [section 1.4 (4)]. In this case, filtration is under a falling hydrostatic head of sludge. Although the head of sludge reduces with time, the resistance of the filter progressively increases as a result of compaction of the mass. Application of the Carman equation (27) produces a plot which is approximately linear, as shown in Fig. 40. The linearity is not as close as that observed in the case of constant head operation (Figs. 28 - 30 inclusive).

In calculating the specific resistance according to equation (28), the pressure term suggested by Eckenfelder is defined by the average hydrostatic head, \( H \), of sludge during drainage. However, a typical plot on semi-log paper of the head of sludge against time, shows approximate
FIG. 40: FALLING HEAD DRAINAGE AT DIFFERENT INITIAL CONCENTRATIONS OF SLUDGE, AT 20°C. (TABLE 22)

<table>
<thead>
<tr>
<th>EXP. No.</th>
<th>C₀ g/l</th>
<th>SLOPE, b (min/cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>11.191</td>
<td>0.02893</td>
</tr>
<tr>
<td>5</td>
<td>12.650</td>
<td>0.03096</td>
</tr>
<tr>
<td>6</td>
<td>14.286</td>
<td>0.03654</td>
</tr>
<tr>
<td>7</td>
<td>22.580</td>
<td>0.04646</td>
</tr>
<tr>
<td>8</td>
<td>23.110</td>
<td>0.08789</td>
</tr>
</tbody>
</table>

FIG. 41: RELATIONSHIP BETWEEN SPECIFIC RESISTANCE OF SLUDGE & PRESSURE. (FALLING HEAD)

(TABLE 23)
linearity (Fig. 44), which rather suggests the use of a log-mean head, defined by

\[
H_{(L.M.)} = \frac{H_0 - H_f}{2,303 \log \frac{H_0}{H_f}}
\]  
(42)

The log-log plot of specific resistance of sludge against pressure, where the latter is calculated both at the arithmetic-mean and logarithmic-mean heads respectively, shows considerable scattering of the data, as shown in Fig. 41. This indicates the variable permeability of the draining sludge. Fitting straight-lines by least-squares, there is virtually no correlation for the data plotted against the arithmetic-mean head (correlation coefficient = 0.03). The log-mean head plot, however, shows a slightly better correlation (correlation coefficient = 0.33). The initial specific resistance, \(r_0\), is lower than that obtained during constant head filtration, and the compressibility, \(s\), is higher.

A log-log plot of \(t/v_t\) against \(v_t\) (Figs. 42 and 43), shows that for each initial sludge concentration, two straight lines may be obtained. The first line approximately describes the filtration during the first day's drainage, and thereafter the characteristics of the sludge change, and a second line with increased slope (reduced rate of filtration) applies during the following 2 to 5 days. This change may result from the compaction of the sludge. Each of the log-log plots has the form

\[
t/v_t = k x v_t^n
\]  
(43)

where \(k\) and \(n\) are empirical constants. (In the present investigation, it was found that during the first day of drainage, \(n = 0.7 - 0.8\); during the following 5 days, \(n = 1.1 - 1.2\)).

Table 2.V shows the extent of divergence between observed loading rates, and those calculated from the Carman equation. Table 2.VI compares the loading rates when operating under falling head drainage, with those
FIG. 42: FALLING HEAD DRAINAGE AT 20°C.

(TABLES 22 & 26)

![Diagram showing falling head drainage at 20°C.](image)

<table>
<thead>
<tr>
<th>EXP. No.</th>
<th>C₀ g/₁</th>
<th>SLOPE</th>
<th>INTERCEPT</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>11.191</td>
<td>0.66</td>
<td>0.012 (a)</td>
</tr>
<tr>
<td>5</td>
<td>12.630</td>
<td>0.72</td>
<td>0.099 (a)</td>
</tr>
<tr>
<td>6</td>
<td>14.286</td>
<td>0.78</td>
<td>0.096 (a)</td>
</tr>
<tr>
<td>7</td>
<td>22.380</td>
<td>0.78</td>
<td>0.182 (a)</td>
</tr>
</tbody>
</table>

FILTRATE VOLUME, Vₜ (ml)

FIG. 43: FALLING HEAD DRAINAGE TO COMPLETION AT 20°C.

(TABLES 22 & 26)

![Diagram showing falling head drainage completion at 20°C.](image)

EXP. No. 8

\[
\frac{C₀}{23.11 \, g/l}
\]

SLOPE (a) = 0.77
(b) = 1.10
INTERCEPT (a) = 0.276
(b) = 0.052

FILTRATE VOLUME, Vₜ (ml)
FIG. 44: FALLING HEAD DRAINAGE AT 20°C., SHOWING EXPONENTIAL RELATIONSHIP BETWEEN SLUDGE HEAD & DRAINAGE TIME.

(TABLE 22)

<table>
<thead>
<tr>
<th>EXP. No.</th>
<th>$C_0$ g/l</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>11,191</td>
</tr>
<tr>
<td>5</td>
<td>12,630</td>
</tr>
<tr>
<td>6</td>
<td>14,286</td>
</tr>
<tr>
<td>7</td>
<td>22,380</td>
</tr>
</tbody>
</table>
### TABLE 2, V

**Falling Head Drainage on Sand Filter**

(Temp. = 20°C; \( r_0 = 0.466 \times 10^{-6} \) \((\text{sec})^{-1}/\text{g}; \ c = 0.623\))

<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>( C ) (g/l)</th>
<th>( H_{L,H} ) (cm)</th>
<th>( P_{L,H}) (g/cm²)</th>
<th>( t ) (min)</th>
<th>( \varepsilon ) (g/ml)</th>
<th>( L_{OBS} \times 10^4 ) (g/(min)(cm²))</th>
<th>( L_{CALC} \times 10^4 ) (g/(min)(cm²))</th>
<th>( L_{OBS} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>11.191</td>
<td>10.60</td>
<td>10.76</td>
<td>2730</td>
<td>0.01372</td>
<td>0.67</td>
<td>0.72</td>
<td>0.94</td>
</tr>
<tr>
<td>5</td>
<td>12.300</td>
<td>13.46</td>
<td>13.62</td>
<td>1510</td>
<td>0.02032</td>
<td>1.31</td>
<td>1.19</td>
<td>1.10</td>
</tr>
<tr>
<td>6</td>
<td>14.286</td>
<td>11.95</td>
<td>12.14</td>
<td>2985</td>
<td>0.01690</td>
<td>0.80</td>
<td>0.82</td>
<td>0.87</td>
</tr>
<tr>
<td>7</td>
<td>22.380</td>
<td>12.70</td>
<td>12.93</td>
<td>2902</td>
<td>0.03562</td>
<td>1.42</td>
<td>1.20</td>
<td>1.18</td>
</tr>
<tr>
<td>8</td>
<td>23.110</td>
<td>14.00</td>
<td>14.32</td>
<td>4740</td>
<td>0.03055</td>
<td>0.85</td>
<td>0.97</td>
<td>0.87</td>
</tr>
</tbody>
</table>

**NOTE:**

\[ L_{OBS} = \frac{V_0}{tA} ; \quad L_{CALC} = \frac{2p(1-0.623)c}{\mu \tau} \]

---

### TABLE 2, VI

**Comparison of Falling Head & Constant Head Drainage at Equivalent Hydrostatic Heads**

\( (c = 0.035 \text{ g/ml}; \ t = 1440 \text{ min}) \)

<table>
<thead>
<tr>
<th>Pressure Head ( P ) (g/cm²)</th>
<th>( L_{CALC} ) (Falling Head) ( \times 10^{-4} ) (g/(min)(cm²))</th>
<th>( L_{CALC} ) (Constant Head) ( \times 10^{-4} ) (g/(min)(cm²))</th>
<th>( L_{CONSTANT HEAD} )</th>
<th>( L_{FALLING HEAD} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10,76</td>
<td>1.58</td>
<td>2.09</td>
<td>1.32</td>
<td>1.32</td>
</tr>
<tr>
<td>12,14</td>
<td>1.61</td>
<td>2.23</td>
<td>1.36</td>
<td>1.36</td>
</tr>
<tr>
<td>12,93</td>
<td>1.64</td>
<td>2.29</td>
<td>1.39</td>
<td>1.39</td>
</tr>
<tr>
<td>13,62</td>
<td>1.65</td>
<td>2.32</td>
<td>1.41</td>
<td>1.41</td>
</tr>
<tr>
<td>14,32</td>
<td>1.67</td>
<td>2.36</td>
<td>1.42</td>
<td>1.42</td>
</tr>
</tbody>
</table>

**NOTE:**

\[ L_{FALLING HEAD} = \frac{2p(1-0.623)c}{\mu \tau \times 0.466 \times 10^6} \]

\( L_{CONSTANT HEAD} = \sqrt{\frac{2p(1-0.325)c}{\mu \tau \times 0.512 \times 10^6}} \)
obtained at constant heads equal to the log-mean hydrostatic head.

during falling head drainage. In both cases, the simplified Carman equation (36) has been used, ignoring filter media resistance. It is seen that for a given sludge draining on a given area of sand bed, the loading rate in the pressure head range 10 - 14 g/cm² is 30 - 40% higher in the case of constant head filtration. Table 2.VII compares the two methods of operation at different depths of sludge. These calculations are plotted in Fig. 34. It is thus apparent that a process which is initially operated as a constant head filtration and then followed by drainage, requires less bed area to handle a given sludge than one which is operated only as a falling head drainage process.

It is therefore proposed to operate the sand filter at constant head for one day, followed by drainage and drying. Hence, only the loading rate derived from the constant head equation (41) need be considered. During the subsequent falling head drainage, the filter bed has no further solids handling capacity. Equation (43) has no useful significance under these conditions.
<table>
<thead>
<tr>
<th>Sludge Depth (cm)</th>
<th>Hydrostatic Head H (cm)</th>
<th>Pressure Head $P$ (g/cm²)</th>
<th>$L^FALLING$ HEAD ($\text{kg/m}^2/\text{d}$)</th>
<th>$L^CONSTANT$ HEAD ($\text{kg/m}^2/\text{d}$)</th>
<th>$L^CONSTANT$ HEAD $L^FALLING$ HEAD</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>11.6</td>
<td>11.76</td>
<td>2.05</td>
<td>2.82</td>
<td>1.38</td>
</tr>
<tr>
<td>15</td>
<td>16.6</td>
<td>16.83</td>
<td>2.26</td>
<td>3.29</td>
<td>1.45</td>
</tr>
<tr>
<td>20</td>
<td>21.6</td>
<td>21.90</td>
<td>2.41</td>
<td>3.64</td>
<td>1.51</td>
</tr>
<tr>
<td>25</td>
<td>26.6</td>
<td>26.97</td>
<td>2.53</td>
<td>3.95</td>
<td>1.56</td>
</tr>
<tr>
<td>30</td>
<td>31.6</td>
<td>32.04</td>
<td>2.64</td>
<td>4.22</td>
<td>1.60</td>
</tr>
</tbody>
</table>

Note: (i) In Falling Head Drainage, $h$, $H$ and $P$ are log-mean values.

(ii) In Constant Head Filtration, values of $P$ are the same as in Falling Head Drainage at corresponding sludge depths.
2.7. **Atmospheric Sludge Drying.**

Evaporation of moisture from the sludge in the sand filter will proceed throughout the filtration period. At some stage, the rate at which filtrate drains from the filter becomes negligible, the rate of drying becoming significant. Drying is a diffusion process, the rate being dependent on the temperature, humidity of the air, and air velocity.

Drying theory \(^{(2c, 6c)}\) shows that evaporation proceeds at a constant rate, until a critical moisture content is reached. Thereafter, evaporation proceeds at a falling rate, until the equilibrium moisture content is reached. During the constant rate period, the plane of vaporisation is near to the surface, and the rate will not be greatly affected by the nature of the sludge. At, and beyond the critical moisture content, the rate of diffusion of water to the surface decreases, and this results in a falling rate of drying period. During the latter period, the physical and chemical properties; thickness; and atmospheric conditions; will be dependent variables. Eventually, a sub-surface drying phase occurs, until the equilibrium moisture content is reached. The latter is dependent on the humidity and temperature of the air, as well as on the characteristics of the dried sludge itself. However, although the equilibrium moisture content under specific conditions is the ultimate dried state resulting from air drying, it is not necessary to dry to this degree, if the ultimate requirement is only the ability to be lifted from a sand-bed.

During the constant drying rate period, physical changes occur on the surface of the sludge. These can affect the permeability of the surface layer. If the surface evaporation rate is higher than the mass transfer rate
from within the mass to the surface, "case-hardening" of the latter may occur, and a surface skin is formed which reduces permeability. This results in shrinkage of the surface and an overall fall in the evaporation rate. If the shrinkage is sufficiently large, surface cracking and fissure formation occurs. The latter increases the exposed surface area, tending to produce a temporary increase in the evaporation rate. Thus, the physical characteristics of the hydrated sludge are responsible for complex variations in the rate of drying, during the normally so-called constant drying rate period.

2.7.1. Ideal Drying Theory.

Under ideal conditions, when the drying sludge is "inert", the rate of drying may be described by the following equation (Badger and McCabe):

\[
\frac{M}{t} = k' G A (H_i - H_a)
\]

where \( M \) = mass of moisture (g) evaporated in time \( t \) (hr),

\( k' \) = film coefficient for the diffusion process \([g/(hr)(m^2)]\),

\( G \) = area of surface/air interface \((m^2)\),

\( H_i \) = humidity of the air at the interface \((g \text{ water/g dry air})\)

(equivalent to saturation humidity at the interface temperature, \( g \text{ water/g dry air} \)),

\( H_a \) = humidity of the drying air \((g \text{ water/g dry air})\).

The diffusion coefficient \( k' G \) is a function of the mass velocity of the air \((6c)\), i.e.

\[
k' G = \alpha (V p)^n
\]

where \( V \) = velocity of the air \((m/hr)\),

\( \rho \) = density of the air \((g/ml)\),

\( \alpha \) = a constant, and

\( n \) = experimentally determined exponent \( \approx 0.8 \).
Equations (44) and (44A) respectively, indicate that the drying rate is proportional to the humidity, as well as being a function of the velocity of the drying air. It is assumed that the temperature of the diffusion interface is equal to that of the surrounding air. In practice, surface evaporation causes the temperature of the interface to be slightly lower than the air temperature, and is equivalent to the interface wet-bulb temperature.

When the critical moisture content is reached, a falling rate period follows, as shown by CE in the diagram below:

\[
\begin{align*}
\text{Evaporation rate,} & \quad f(W), \\
(g/m^2/hr) & \\
\text{moisture content, } W (g H_2O/g dry solids) &
\end{align*}
\]

In the falling rate period, the evaporation rate is a function of the moisture content. Commencing at the critical moisture content \( W_c \) (g water/g dry solids), the mass transfer rate of moisture from within the mass begins to be less than the surface evaporation rate. The rate reduces progressively, until the equilibrium moisture content \( E \) (g water/g dry solids) is reached, and no further evaporation occurs.

At any instant during the drying phases, the moisture is present as both bound and unbound (or free) water. The free moisture content is equal to the difference between the total moisture content and the equilibrium moisture content.

It is assumed that the rate of drying during the falling rate period CE is a function of the free moisture content (6c). The mass of water evaporated in a short time interval is equal to the product of the mass of dry solids, and the change in free moisture content per unit mass of dry solids.
If \( m \) = mass of free moisture (g) at time \( t \) (hr),
\[ D = \text{mass of dry solids (g)}, \]
\[ A = \text{drying surface area (m}^2\text{)}, \]
\[ W = \text{total moisture content of the mass (g water/g dry solids)}, \]
\[ E = \text{equilibrium moisture content (g water/g dry solids)}, \]
\[ f(W-E) = \text{ordinate of the rate of drying curve (g/m}^2\text{/hr)}, \]

where the free moisture content is \((W-E)\) g/g, then
\[ m = D(W-E) \] (45).

Differentiating (45) with respect to the free moisture content, \((W-E)\),
\[ \frac{dm}{d(W-E)} = D \times \frac{d(W-E)}{dt} \] (46).

But by the assumption, that the rate of drying during the falling rate period is a function of the free moisture content \((6c)\),

\[ \frac{-dm}{dt} = f(W-E) \] (47).

Substituting (46) in (47)

\[ - \frac{D}{A} \times \frac{d(W-E)}{dt} = f(W-E), \]

or

\[ \frac{dt}{f(W-E)} = \frac{d(W-E)}{f(W-E)} \] (48).

Integrating (48) over the limits from the beginning of the falling rate period, to a time \( t_f \) in this period; and from the critical moisture content \( W_c \), to the value \( W_f \) \((W_c > W_f > E)\),

\[ \int^{t_f}_{0} dt = t_f = \frac{D}{A} \int^{W_c}_{W_f} \frac{d(W-E)}{f(W-E)} \] (49) \((6c)\).

Equation (49) may be solved graphically by plotting \( 1/f(W-E) \) against \((W-E)\), the latter being obtained from an experimental drying-rate curve, and the former from the area under this curve between the limits \( W_f \) and \( W_c \).

An approximation \((13)\) may be made by assuming that the drying rate in
the falling-rate period is directly proportional to the free moisture content. The approximation was found to be valid at lower loadings, as shown in plot 1 of Fig. 46. Equation (47) then becomes

\[ - \frac{dm}{A \, dt} = b(W-E) = \frac{R_c}{(W-E)} \]  

(50).

\( (R_c \) is the ordinate of the rate at the critical moisture content, and \( b \) is the slope of the falling rate curve, CE in above diagram). Substituting (46) in (50), transposing and integrating,

\[ \int_0^t D \cdot \frac{(W_c-E)}{R_c} \, dt = \int_{W_f}^{W} \frac{d(W-E)}{(W-E)} \]  

(51).

Equation (51) evaluates to give

\[ t_f = \frac{D}{A} \cdot \frac{(W_c-E)}{R_c} \ln \frac{(W_c-E)}{(W_f-E)} \]  

(52).

This is the equation given in Badger and McCabe (6c).

Under given constant conditions, \( W_c \) and \( E \) are constant. A plot of the log of the free moisture content \((W_f-E)\) against time, results in a straight line. This is shown in Fig. 49 during the earlier stages of the falling rate period. At later stages the plot deviates from linearity. If the slope of the semi-log plot is \( K_f \), equation (52) may be written as

\[ \ln \frac{(W_c-E)}{(W_f-E)} = K_f \, t_f \]  

(52A).

\((W_c-E)\) is the free moisture content at the "critical" moisture content, and is the intercept of the plot at \( t_f = 0 \).
2.7.2. Modification of Drying Theory.

The problem with the use of the theoretical equations (section 2.7.1.) for sludge drying in the present work, is that the values of $R_c$ and $W_c$ are not known unless they are determined experimentally.

The varying characteristics of the sludge and its interface during drying, results in the constant drying rate and critical moisture contents being ill-defined, as shown in Fig. 46. Maintaining constant air temperature, humidity and air mass velocity, the diffusion constant varies in a complex manner. Varying the loading, different values of $W_c$ and $R_c$ were obtained. Experimental variation of the exposed surface area and the depth of the drying sludge, varies the mass loading term, $D/A$ (defined as "the mass of dry solids per unit area"). The deeper the sludge bed, the greater the mass of free moisture beneath the interface. Because the change in free moisture content is measured on the entire mass (the moisture content at any time thus being an average value), the physical dimensions of the surface, and the depth of the sludge, will be expected to contribute towards deviations from the theoretical analysis.

The writer proposes the following sequence of plots to obtain values of $R_c$ and $W_c$, which may then be used in the theoretical drying equations for design purposes:-

With aliquots of the same sludge drying under variable loading conditions, the relationships between the deviations of the theoretical constants may be estimated as follows:-

Calculating the average rate of evaporation per unit area at each time $t$ from $t=0$ to $t=t$, and plotting these values against the moisture content at $t$, Fig. 46 results. It was observed that an artificial falling-rate line is obtained, which is linear. The intersection of this line with the horizontal constant - rate values, yields co-ordinates for $R_c$ and $W_c$. 
FIG. 45: DRYING CURVES FOR SLUDGE INITIALLY AT MOISTURE CONTENT 12.5 g water/g dry solids. (21°C; 65% R.H.)

(TABLES 28 - 32 INCLUSIVE)

<table>
<thead>
<tr>
<th>EXP. No.</th>
<th>DRY SOLIDS LOADING, D/A, g/m²</th>
<th>INITIAL MASS OF WET SLUDGE, g</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>533</td>
<td>9,0380</td>
</tr>
<tr>
<td>2</td>
<td>1533</td>
<td>21,0515</td>
</tr>
<tr>
<td>3</td>
<td>1799</td>
<td>45,2600</td>
</tr>
<tr>
<td>4</td>
<td>4552</td>
<td>27,5934</td>
</tr>
<tr>
<td>5</td>
<td>5776</td>
<td>43,3539</td>
</tr>
</tbody>
</table>

**NOTE**
- \( t_c = 107.5 \text{ hr} \)
- \( m_c = \text{mass of moisture evaporated at } t_c \)
- \( m_c = \text{mass of moisture evaporated at } t_c \)
- \( t_c = \text{time to reach "critical" moisture content} \)

**EQUILIBRIUM MOISTURE CONTENT**

- \( m_c = 12.47 \text{ g} \)
- \( m_c = 6.46 \text{ g} \)
- \( m_c = 1.80 \text{ g} \)
- \( m_c = 1.94 \text{ g} \)
- \( t_c = 44.5 \text{ hr} \)
- \( t_c = 94.5 \text{ hr} \)
- \( t_c = 156.5 \text{ hr} \)
Interchange the symbols $R_c$ and $W_c$ within the plot.

**Fig. 46:** Rate of Drying Curves at Different Bed Loadings (21°C., 65% R.H., 0.8m/min Air Velocity).

*Note: Solid data points correspond to average values of $R$ & $W$. Open points are plots of $R=m/tA$ vs. $W$. (Tables 28 - 32 inclusive)*
This appears to be reasonable. Joining the intersection point with the value of the equilibrium moisture content $E$ at zero rate of evaporation, the course of the falling rate period may be approximated. This method seems to give good agreement at low loadings (Plot 1), but progressively deviates at higher loadings (Plots 2 - 5 inclusive).

From the plots at different loadings $D/A$ in Fig. 46, it seemed that the parameters $R_c$ and $W_c$ in the theoretical equation (52) are functions of the mass loading $D/A$. Inspection of equation (52) and (52A) shows that because the falling rate constant, $K_f$, is given by

$$K_f = \frac{R_c}{D(W_c - E)} \quad \text{(units of } K_f = \text{hr}),$$

$K_f$ is already a function of the mass loading. However, since $K_f$ includes the parameters $R_c$ and $W_c$, the theoretical value of $K_f$ should be modified to a value given by

$$K_f = \frac{R_c}{(W_c - E)} \times f(D/A),$$

This modification is shown to hold experimentally, in the following manner:

Plotting the "critical" moisture content $W_c$ against the bed loading $D/A$ on log-log paper, a straight line is obtained, as shown in Fig. 47. The equation of this line is of the form

$$W_c = c' (D/A)^n \quad \text{(53)},$$

where $c'$ is the intercept on the $W_c$ axis at $D/A = 1 \text{ g/m}^2$, and $n$ is the slope of the line. In Fig. 47, it was found that

$$W_c = 0,045 \left( \frac{D}{A} \right)^{0,655},$$

$$W_c = 0,045 \left( \frac{D}{A} \right)^{0,655}.$$
**FIG. 47:** RELATIONSHIP BETWEEN "CRITICAL" MOISTURE CONTENT & DRY SOLIDS LOADING, D/A.

(\( W_c = 0.045(D/A)^{0.655} \)) AT 21°C & 65% R.H.; 0.8 m/min air velocity.

**FIG. 48:** RELATIONSHIP BETWEEN "CONSTANT" RATE OF DRYING, \( R_c \), & DRY SOLIDS LOADING, D/A.

(\( R_c = 158 - 0.0012D/A \)) AT 21°C; 65% R.H.; 0.8 m/min air velocity.
Plotting the "constant" drying rate $R_c$ against the bed loading on semi-log paper, a further straight line is obtained (Fig. 48). The equation of this line is given by

$$R_c = R_0 e^{c'' D/A}$$  \hspace{1cm} (54),

where $R_0$ is the intercept at zero loading, and $c''$ is the slope of the plot. In this work (Fig. 48), it was found that

$$R_c = 158 e^{-0.00012 D/A}.$$

Plotting the free moisture contents ($W_f - E$) against time $t_f$ in the falling rate period on a semi-log plot (Fig. 49), the slopes of the straight lines enable the evaluation of the values of constant $K_f$ [equation (52A)]. Plotting the values of $K_f$ against the bed loading $D/A$ on log-log paper a straight line is obtained, as shown in Fig. 50. From the equation to this line, the modification to $K_f$ is given by

$$K_f = c'''' (D/A)^m$$  \hspace{1cm} (55),

where $c''''$ is the intercept at $D/A = 1$, and $m$ is the slope of the line. In this work (Fig. 50), it was found that the following equation was obtained:

$$K_f = 2.78 \times 10^6 (D/A)^{-2.67}.$$

Experimental equations (53) – (55) inclusive show the parameters $W_c$, $R_c$ and $K_f$ respectively, as functions of the mass loading, $D/A$. The terms may now be used in the theoretical equations (52) or (52A), thus enabling an estimation of drying times in the falling rate period. These equations are also useful in scaling-up constant temperature and humidity drying experiments to actual bed loadings.
FIG. 49: RELATIONSHIP BETWEEN FREE MOISTURE CONTENT & TIME IN THE FALLING RATE PERIOD OF DRYING AT 21°C & 65% R.H. (TABLE 33)
FIG. 50:
VARIATION OF FALLING RATE "CONSTANT," $K_f$, WITH BED LOAD.
(21° C.; 65% R.H.; 0,8 m/min air velocity)

$K_f = 2,78 \times 10^6(D/A)^{-2,87}$

FIG. 51:
DRYING TIME CURVES, $W_0=13,5 g/g$. (TABLE 36)
2.7.3. **Drying Time Equations**

Having determined $W_c$, $R_c$, and $K_f$, the drying times under identical constant ambient air conditions may be predicted. If the maximum final moisture content enabling the sludge to be spadeable is $W$, and the moisture content at the commencement of drying is $W_0$, then if $W$ is greater than or equal to the "critical" moisture content $W_c$, the drying time in the "constant" rate period, $t_c$, is given by

$$t_c = \frac{(W_0 - W_c)D}{R_c} \cdot \frac{D}{A}$$ \hspace{1cm} (56).

In this case, all the drying will occur in the "constant" drying rate period.

However, if the desired moisture content is less than the "critical" moisture content, the drying time will be the sum of the time $t_c$ in the "constant" rate period, up to the "critical" moisture content, and the time $t_f$ in the falling rate period.

$$t = t_c + t_f$$

$$\ln\left(\frac{W_c - E}{W_c - 0}\right) = \frac{2,303\log\left(\frac{W_c - E}{W - E}\right)}{K_f}$$ \hspace{1cm} (from equation 52A).

Thus,

$$t = \frac{(W_0 - W_c)D}{R_c} \cdot \frac{D}{A} + \frac{2,303\log\left(\frac{W_c - E}{W - E}\right)}{K_f}$$ \hspace{1cm} (57).

Employing these equations, Table 36 (Section 4.4.5.) has been calculated for the range of bed loadings resulting from the sand filtration (Table 2. IV section 2.5).

These results are shown graphically on the semi-log plot in Fig. 51. The total drying time (in days) to reach a desired final moisture content, $W_f$ (g/g), is plotted against the solids loading to the drying bed, $D/A$ (kg/m$^2$).
A separate curve is plotted for each final moisture content in the range expected. When the maximum moisture content for spadeability has been determined by experiment, the effect of the solids loading on the total drying time can be seen from this set of plots.

In the present investigation, a maximum moisture content of 8.5 g/g dry solids was found to be satisfactory for spadeability, and at a bed loading of 2.6 kg/m², drying was achieved in 7 days (see Appendix, section 4.4.6.).
3. DESIGN CALCULATIONS.

The calculations in connection with the proposed treatment process are limited essentially to the criteria necessary for the design of the sedimentation/thickening tank, and the sand filtration and drying beds.

It is intended that dosing of flocculation chemicals occurs en route to the sedimentation tank, immediately after the balancing tank. Flocculation, coagulation, and sedimentation take place in the sedimentation tank.

The plant is to be designed to process an expected effluent flow of 45 m³/d. From Fig. 2, the ratio of the upper 95% confidence limit of the distribution of the flow rate, to the mean value, is given by 12.96 m³/d to 9.00 m³/d = 1.44.

Therefore, the plant is to be designed to handle $1.5 \times 45$ m³/d = 67.5 m³/day, so that peak flows may be accounted for.

3.1. Neutralisation and Flocculation.

Two balancing and neutralisation tanks are required, each having a minimum capacity of 67.5 m³ (1 day's storage with peak flow).

A suitable size for each tank is 5 m x 5 m square, and 2.75 m deep (volumetric capacity = 68.75 m³). This size allows a freeboard of

$$\frac{(68.75 - 45)}{5 \times 5} = 0.95 \text{ m},$$

when filled with the normal 45 m³ of raw effluent.

The tanks may be constructed in concrete, the internal surfaces being treated with a chemically resistant coating. The reason for this is the relatively long periods of contact with solutions at pH values varying from pH = 11 to pH = 3. Each tank requires a simple low-speed
mechanical stirrer, equipped with chemically resistant paddle blades.

With 2 balancing tanks, the effluent flow feeds into each tank on alternate days. While the one tank collects the flowing waste, the other tank (which has been filled with the previous day's flow) is ready for treatment.

The treatment consists of mixing the collected effluent, and then neutralising it with sulphuric acid to a pH = 6.5. A further dose of acid is added, equal to half the quantity required in the first neutralisation to pH = 6.5.

Slow stirring of the slightly acidic contents of the treated tank is maintained with advantage throughout the continuous dosing for flocculation, which follows. This procedure will ensure uniformity of the composition of the feed to the continuous flocculation, in the event of a tendency for partial separation of components in the balancing/neutralisation tank.

Aluminium sulphate and sodium aluminate solutions are injected into the feed line connecting the balancing tanks with the sedimentation/thickening tank. The two chemical feeders are flow-operated, and the dosing rate for each is preferably predetermined experimentally. However, a constant dose of 500 ppm of aluminium sulphate has been found suitable (section 4.1, Appendix), and if the pH in the balancing tank is controlled, the sodium aluminate dosage will also remain fairly constant from batch to batch. For the relatively small volume of effluent to be treated, rapid mixing after each chemical addition is simply effected by over-and-under-baffled pipe sections. The neutralised effluent is normally pumped through the chemical dosers to the sedimentation tank at a rate of 45 m³/d. This is equivalent to 45/24 x 3600 m³/sec. The
desired velocity for baffled pipe/rapid mixing should be in the range 0.3 - 1 m/sec, and head losses 0.3 - 1 m (14).

Using 40 mm i.d. piping, the velocity in the baffled pipe is approximately

\[
\frac{45 \times 4 \times 10^6}{24 \times 3600 \times \pi \times 40^2} = 0.42 \text{ m/sec},
\]

which should be satisfactory.

3.2.1. Sedimentation Tank Design.

The typical mixed effluent after neutralization and flocculation, has been found to have a suspended solids concentration of \( C_0 = 2.0 \text{ g/l} \). This is similar to effluent "B" (Table 2B, Appendix; and Fig. 10).

From Fig. 16, when \( C_0 = 2.00 \text{ g/l} \), constant \( K_b = 310 \).

However, because data are not available for the tank constant, \( K_T \), the value of \( K_b \) found experimentally for floe "B" (\( C_0 = 2.054 \text{ g/l} \)), \( K_b = 210 \), is acceptable. Thus, the value of \( K_T \) assumed is \( 210/310 = 0.68 \).

The lower value for \( K_b \) will compensate inter alia for wall effects, and lead to a conservative estimate for the mass loading [equation (12)].

3.2.2. Thickening.

Selecting extreme values for the underflow concentration, \( C_u = 10 \text{ g/l} \) and \( C_u = 30 \text{ g/l} \) respectively, Fig. 17 shows that at \( C_o = 2.00 \text{ g/l} \) and flow rate \( Q = 45 \text{ m}^3/\text{d} \), the tank diameters required to provide sufficient unit area for the thickener, are respectively \( D = 1.98 \text{ m} \) and \( D = 3.2 \text{ m} \) for the minimum and maximum expected values of \( C_u \).

To reduce the dewatering load on the sand drainage beds, it is advantageous to obtain an underflow concentration as high as possible.
Therefore selecting a nominal tank diameter $D = 3,0 \text{ m}$, Fig. 17 shows that at $C_0 = 2,00 \text{ g/l}$, $C_u = 25,8 \text{ g/l}$. (This latter underflow concentration is close to the maximum experienced in the experimental tests for floc "A", where $C_u = 27,38 \text{ g/l}$; Table 2A, p A.13).

When $C_u = 25,8 \text{ g/l}$, $(C_u/C_0 - 1) = 11,9$ and the mass loading calculated from equation (12),

$$(ML)^{1,28} = K_b (C_u/C_0 - 1) = 210/11,9 ,$$

yields,

$$(ML) = 9,42 \text{ kg/m}^2/\text{d} .$$

At lower values of $C_u$, say $C_u = 10 \text{ g/l}$, $(C_u/C_0 - 1) = 4,0$ and thus

$$(ML) = 22,08 \text{ kg/m}^2/\text{d} .$$

The overflow rate, based on liquid throughput and calculated from equation (13)(Section 2.2.4.),

$$(OR) = \frac{1}{C_0 (UA)} \times \frac{(C_u - C_0)}{C_u} = \frac{(C_u - C_0)(ML)}{C_u C_0} ,$$

yields for maximum and minimum values of the underflow concentration as follows:-

Overflow rate: (Based on liquid throughput)

At $C_u = 25,8 \text{ g/l}$, $(OR)_\text{c} = 4,35 \text{ m}^3/\text{d/m}^2$;

and at $C_u = 10,0 \text{ g/l}$, $(OR)_\text{c} = 8,83 \text{ m}^3/\text{d/m}^2$.

The overflow rate based on the solids loading to the thickener.
is given by

\[(\text{OR})_T = \frac{1}{C_0(\text{UA})} = \frac{(\text{ML})}{C_0},\]

(Section 2.2.4.). Thus,

Overflow rate: (Based on solids loading)

\[(\text{OR})_T = \frac{9.42}{2.00} = 4.7 \text{ m}^3/\text{d/m}^2, \quad (C_u = 25.8 \text{ g/l});\]

\[(\text{OR})_T = \frac{22.08}{2.00} = 11.04 \text{ m}^3/\text{d/m}^2, \quad (C_u = 10.0 \text{ g/l}).\]

From section 2.2.4., because the overflow rates based on liquid throughput, \((\text{OR})_C\), are lower than those based on the solids loading to the thickener \((\text{OR})_T\), the former (i.e. clarification) controls the surface area required.

Hence the average rise velocities of the overflowing supernatant will vary from 4.35 to 8.83 m/d as the sludge underflow concentration is varied from \(C_u = 25.8 \text{ g/l}\) to \(C_u = 10.0 \text{ g/l}\).

Using the sedimentation curve for floc "B" (Fig.10), the slope of the initial linear portion of the curve gives the initial constant velocity of the settling floc during zone settling, \(V = 14.68 \text{ m/d}\).

Because 14.68 m/d > 8.83 m/d or 4.35 m/d, the overflow rise velocity is less than the zone settling velocity, and clarification of the supernatant will be ensured.
The tangent to the settling curve (Fig. 10) at the initial sludge compression point \( (t_c = 225 \text{ minutes}; H = 0.58 \text{ m}) \), intersects the vertical axis at \( H_1 = 0.94 \text{ m} \). This is the height the interface would have reached if all the sludge were at the same uniform concentration, \( C_2 \text{ g/l} \), as that at the interface.

For this curve, \( C_{\infty} = 2.054 \times 2.065 = 4.242 \text{ g/10}^{-2} \text{ m}^2 \).

But by equation (6), \( C_{\infty} = C_2 H_1 \).

\[ C_2 = \frac{C_{\infty} H_1}{H} = \frac{4.242}{0.94} \text{ g/l} = 4.50 \text{ g/l}. \]

This is the interfacial layer sludge concentration at the estimated commencement of sludge compression.

However, it is desirable to have sufficient retention time, to ensure that the floe is well into the compression zone. Operating at a sludge interface height equal to 15% of the tank depth, i.e. \( H_u = 0.30 \text{ m} \), the underflow concentration is given by

\[ C_u = \frac{C_{\infty} H_u}{H} = \frac{4.242}{0.30} = 14.14 \text{ g/l}. \]

At the latter expected underflow sludge concentration,

\[ (ML)^{1.28} = \frac{210}{(14.14/2.00 - 1)}, \]

whence \( (ML) = 14.09 \text{ kg/m}^2/d \) [equation (12)].

The overflow rate [by equation (13)]

\[ = (OR)_c = (14.14 - 2.00) 14.09/14.14 \times 2 = 6.05 \text{ m}^3/d/m^2. \]

The average overflow rise velocity = 6.05 m/d < 14.68 m/d. Thus, clarification of the supernatant will occur in the continuous thickener when the sludge depth is 0.30 m. This height of sludge at an underflow concentration \( C_u = 14.14 \text{ g/l} \) occurs at a time \( t \) greater than the value of \( t_c \) (section 3.2.3).
For a 3m diameter tank, cross-sectional area =

\[ \frac{\pi}{4} (3)^2 = 7.07 \text{ m}^2. \]

\[ \therefore \text{overflow rate (OR)} = 6.05 \times 7.07 \text{ m}^3/\text{d} = 42.77 \text{ m}^3/\text{d}. \]

This is the expected overflow rate from the continuous thickener when the influent flow rate is 45 m³/d, and the underflow concentration is 14.14 g/l.

3.2.3. Retention Time.

The settling curve for floc "B" (Fig. 10) gives the time for the sludge to initially go into compression as

\[ t_c = 225 \text{ min} = 3.75 \text{ hours}. \]

This is the minimum retention time.

Volume of cylindrical portion of 3m diameter settling tank at a depth of 2m = \( \frac{\pi}{4} \times 3^2 \times 2 = 14.14 \text{ m}^3. \)

\[ \therefore \text{average retention time at flow rate } Q = 45 \text{ m}^3/\text{d}, \]

\[ = 14.14/45 \times 24 = 7.54 \text{ hours}. \]

When \( Q = 67.5 \text{ m}^3/\text{d} \) (i.e. peak flow rate), retention time

\[ = 14.14/67.5 \times 24 = 5.03 \text{ hours}. \]

For both normal and peak flow rates, the tank retention times are greater than that estimated for the sludge to initially go into the consolidated condition (\( t_c = 3.75 \text{ hours} \)).

Therefore the tank size is acceptable, because the underflow sludge will always be in the compression zone, and thus consolidated.
3.2.4. Mass of Sludge Solids Produced.

The average dry mass of sludge produced is given by equation (14).

\[ W_s = \text{Flow Rate} \times C_o = 45 \times 2.00 = 90 \text{ kg/d}. \]

At peak flow rate, \( W_s = 67.5 \times 2.00 = 135 \text{ kg/d}. \)

This is the dry solids loading of wet sludge to be dewatered on the filtration beds when the initial influent solids concentration to the thickener is 2.00 g/l.

3.2.5. Simulation of Sedimentation Tank Performance in Overload and Underload Conditions.

Using Fig. 25 for rapid computation, the proposed 3 m diameter tank may be tested under conditions of variable flow rates, and variable initial floc concentrations.

It has been calculated (Section 3.2.2) that the sludge concentration must not be less than 4.50 g/l, for the sludge to be in the compression zone at an interface height of 0.58 m; and that the underflow concentration is 14.14 g/l for an underflow height of 0.30 m. The underflow concentrations computed from equation (23) may be read from the chart for a 3 m diameter tank (Fig. 25). Using the sedimentation data for floc "B" Fig. 10, the interface height is estimated from

\[ H_u = \frac{C_o H_o}{C_u} = C_o \times 2.065/C_u. \]
The above calculations show that the tank will function at all loadings up to peak flow rate (Q = 67.5 m³/d) at initial floc solids concentrations less than $C_o = 2.25$ g/l.

At $C_o = 2.50$ g/l, the sludge is not in initial compression at peak flow rate (67.5 m³/d). The flow rate is limited to a maximum of...
50 m\(^3\)/d at this influent concentration.

At higher values of \(C_o > 2.25 \text{ g/l}\), there is a tendency for the underflow sludge interface height to be too high (i.e. > the proposed 0.30 m). However, at \(C_o = 2.00 \text{ g/l}\), the underflow sludge interface height will always be less than the design value of 0.30 m.

The final tank is now designed to have a diameter of 3 m, and a cylindrical height of 3 m (2 m, as designed for thickening and clarification; plus 1 m free-board and safety factor, for flow-rates in excess of peak flow-rate = 67.5 m\(^3\)/d). The tank is fitted with a conical bottom having a vertical height = 0.25 m. This provides a slope of 0.25:1.5 = 1:6, so that sludge can slide down the bottom and concentrate in the conical bottom for withdrawal.

Volume of vertical cylindrical portion, including freeboard = \(\pi \times 3^2 \times 3/4 = 21.21 \text{ m}^3\).

Volume of conical bottom = \(\pi \times r^2h/3 = \pi \times 1.5^2 \times 0.25/3 = 0.59 \text{ m}^3\).

Therefore retention time ratio (section 3.2.3) = 21.80/14.14 = 1.54.

The increased volume of the tank thus increases the retention times by approximately 50\%. This will be an additional safety factor to ensure that the underflow sludge is at least in initial compression.

It is proposed that the feed of floc will be introduced into the middle of the tank at a height of 1 m above the bottom of the tank. This will then allow an upward flow through 2.25 m of liquid to a peripheral V-notched weir at the top of the tank. The weir overflow would feed into a peripheral collection channel which drains to the sewer. The inlet velocity should not exceed 0.15 m/sec (14) (a 75 mm i.d. inlet, without allowing for head losses; will yield an inlet velocity of 0.12 m/sec @ Q = 45 m\(^3\)/d; and 0.18 m/sec @ Q = 67.5 m\(^3\)/d. This would appear to be a
satisfactory size for the inlet nozzle).

A slow speed paddle stirrer (4-6 revs/min) supported such that the bottom of the vertical paddle is about 0.25 m above the inlet nozzle, will promote floc growth, sedimentation, and thickening.

Sludge is drawn off from the bottom of the tank and pumped to the sand filtration beds.

The maximum volume of sludge occurs when the underflow concentration is at a minimum. At a peak estimated sludge production of 135 kg/d, when the underflow sludge concentration is \( C_u = 10 \) g/l, the maximum rate of sludge withdrawal = \( 135 \times \frac{1000}{10} = 13500 \) l/d,

\[ \text{i.e.} \quad \frac{13500}{24} \times 60 = 9375 \text{ l/min (13.5 m}^3\text{/d)}. \]

A sludge pump selected to handle 10 l/min, may be operated intermittently when the sludge withdrawal rate is less than 13.5 m³/d (which will occur at underflow sludge production values of less than 135 kg/d).

The velocity of the underflow sludge at the outlet should be minimal in order to avoid disturbing the sludge blanket and its underflow level. Thus it is proposed that the outlet will be covered by a baffle hood supported above the outlet, and the orifice should therefore be as large as is convenient, and preferably 150 mm diameter.

3.3. Sand Filtration.

The underflow effluent from the sedimentation tank (which is now the feed to the sand filters), will normally have a solids concentration in the range of 10 to 20 g/l. Table 2.IV (Section 2.5) shows the negligible effect of the usual range of variation in initial concentration on the filter loading rate in obtaining equal solids removals by constant head filtration. This applies at all sludge depths, from

Fig. 34 shows how the filter loading rate may be increased by increasing the depth of the sludge on the sand bed. However, although a higher filtration rate indicates that a smaller area of filter is required, Fig. 51 shows how the drying time of the sludge to spadeability increases exponentially with the sludge solids loading. During drying, the sand filter cannot be used to drain further quantities of wet sludge. Additional drainage area is required for the additional flocculated sludge until the first bed is dried. Thus, the drying time to spadeability is the limiting parameter.

As a guide to the estimation of a suitable filter loading, referring to Fig. 51 (at 65% R.H. and 21°C), the drying time to spadeability at moisture content 8.5 g H₂O/g dry solids is 7 days at a bed loading of 2.6 kg/m². The moisture content at the beginning of drying was assumed to be 13.5 g/g for the purposes of calculating the data for the curves in Fig. 51.

The feed to the filter from sedimentation has a concentration range $C_o = 10$ to 20 g/l (typically, 14 g/l).

The experimental work showed that operating the filter at constant head for 1 day, the average sludge concentration would be $C_f = 25$ to 30 g/l, (typically, 14 g/l).

From Figs. 35 and 34 respectively, for constant head filtration only, the following data were obtained:-

$h = 10\,\text{cm}, \text{ to } h = 30\,\text{cm}.$ (Fig. 34, where $C_o = 14\,\text{g/l}$).
Thus to obtain a filter sludge with a concentration of about 25 g/l after 1 day's operation, a head of up to 25 cm of sludge is required. This depth of sludge will account for all normal variations in the sand filter feed solids concentration. However, for the typical sludge ($C_o = 14 \text{ g/l}; C_f = 25 - 30 \text{ g/l}$) the range of head required is seen to be $9,3 - 12,8 \text{ cm}$ from the above data.

The experimental work showed that after stopping constant head feed to a sand filter, drainage is essentially complete after an additional 3 days of falling head drainage (Fig. 43). The average sludge concentration at the end of drainage is approximately in the range 60 - 70 g/l.

Neglecting the evaporation during the first day's constant head feed, dewatering is achieved by both drainage and evaporation during the falling head drainage. Thereafter, drying by evaporation alone dewateres the residual sludge.

Considering the average sludge concentration at the beginning of falling head drainage to be 25 to 30 g/l (approximately equivalent to moisture contents of 39,0 to 32,3 g\text{H}_2\text{O/g dry solids}, respectively); and that at the end of drainage, the moisture content $W_0 = 13,5 \text{ g/g (sludge}$
concentration = 69 g/l; 93.1% moisture content by mass);
water removed by drainage = (39.0 - 13.5) or (32.3 - 13.5) =
25.5 g H₂O/g dry or 18.8 g H₂O/g dry.

At a bed loading of 2.6 kg/m², water removed by drainage
= 25.5 x 2.6 or 18.8 x 2.6 kg/m²
= 66.30 kg/m² or 48.88 kg/m².

Additionally, during the 3 day falling head drainage, the evaporation
rate of moisture will be entirely in the constant rate period.

By equation (54), the constant rate of evaporation will be at least
\[ R_c = 158 e ^{-0.00012 \times 2600} = 115.7 \text{ g/m}^2/\text{hr}. \]
The corresponding "critical" moisture content,
\[ W_c = 0.045 \times (2600)^{0.655} = 7.76 \text{ g/g} \text{ [equation (53)].} \]

water lost by evaporation during a 3-day falling head drainage
= \[ R_c \times 3 \times 24 = 115.7 \times 72 = 8,33 \text{ kg/m}^2 \].

Hence, total dewatering during falling head drainage over the initial
concentration range 25 to 30 g/l = (66.30 + 8.33) to (48.88 + 8.33) kg/m²
= 74.63 kg/m² to 57.21 kg/m².

At a bed loading = 2.6 kg/m², the corresponding dewatering is
74.63/2.6 to 57.21/2.6 = 28.7 g H₂O/g dry solids to 22.0 g H₂O/g dry solids.

The average sludge moisture content at the end of falling head
drainage, allowing for simultaneous draining and evaporation, is
\[ W_o = (39.0 - 28.7) \text{ or } (32.3 - 22.0) \text{ g H}_2\text{O/g dry solids} \]
= 10.3 g/g or 10.3 g/g (i.e. identical, regardless of the
sludge concentration at the beginning of falling head drainage being
25 or 30 g/l).

From this stage onwards, dewatering is accomplished solely by
evaporation.
Additional time required to dry to "critical" moisture content

\[ W_c = 7.76 \, \text{g/g}, \]
\[ t_c = \frac{(W_c - W) - W}{A} \cdot \frac{D}{R} \quad \text{[equation (56)].} \]

i.e.
\[ t_c = \frac{(10.30 - 7.76) \times 2600}{115.7} = 57 \text{ hours (2.4 days)}. \]

\[ W_c = 7.76 < 8.5; \] therefore the sludge is spadeable in a
total time from commencement of sand filtration = (time for constant
head filtration) + (time in falling head drainage) + (time to dry to
critical moisture content) = 1 + 3 + 2.4 = 6.4 days.

After a nominal convenient period of 7 days altogether on the sand
bed, drying will continue in the falling drying rate period for a further
0.6 day (= 15 hours).

The final expected moisture content \( W_f \) is calculated from equation (52b):

\[ 2.303 \log \left( \frac{(W_c - E)}{(W_f - E)} \right) = K_f \times t_f. \]

The falling rate constant, \( K_f = 2.78 \times 10^5 (2600)^{-2.67} \quad \text{[Equation (55)]} \)
i.e. \( K_f = 0.0027 \).

Equilibrium moisture content, \( E = 0.10 \, \text{g/g}. \)

\[ \therefore \text{by equation 52 (a),} \]
\[ 2.303 \log \left( \frac{(7.76 - 0.10)}{(W_f - 0.10)} \right) = 0.0027 \times 15, \]
whence \[ W_f = 5.21 \, \text{gH}_2\text{O/g dry solids, which is less than} \ W_f = 8.5 \, \text{g/g}. \]

Thus, at a bed loading of 2.6 kg solids/m\(^2\), the sludge will be
liftable after a total residence of 7 days on the sand filter.

During constant head filtration feed, the head of sludge can be
maintained in the range 9 - 13 cm. However, this is not very critical,
from Fig. 34, and operation at a nominal 10-15 cm will be satisfactory.
3.3.2. Calculation of Filtration/Drying Bed Areas.

The average mass of sludge solids produced daily is 90 kg; and at peak loading, 135 kg. (Section 3.2.4.).

At a bed solids loading = 2,6 kg/m², area of drying beds = 90/2,6 = 34,62 m²/d (average flow rate),

which is approximately = 35 m²/d;

or at peak flow rate, 135/2,6 = 51,92 m²/d,

which is approximately = 52 m²/d.

Each bed is designed to be in operation for 7 days to accomplish drying (section 3.3.1.), therefore, total bed area required for average flow of effluent,

Q = 45 m³/d,

= 7 x 35 = 245 m²;

or at peak effluent flow

Q = 67,5 m³/d,

bed area = 7 x 52 = 364 m².

Allowing for a lower mean temperature of operation such as 10°C, the filter loading being reduced by the increased viscosity, from section 2.5,

\[ L_{10°C} = 0,87 \times L_{20°C} \]

and the total bed areas required are

90/2,6 x 7/0,87 m² and 135/2,6 x 7/0,87 m²,

for average and peak flow rates respectively,

i.e. 280 m² and 420 m², respectively.

At lower mean temperatures, and under wintry conditions, sludge was actually found to yield a moisture content \( W = 5,43 \) g/g in 6 days. (Appendix, section 4.4.6.). This is a similar moisture content to that which was calculated for drying in 7 days at 65% R.H. and 21°C. (section 3.3.1.). Therefore the reduced rate of evaporation to be expected at the lower temperature of 10°C., need not be considered in
these calculations.

If each filtration/drainage bed is 6 m x 10 m, 7 such beds provide a surface area = $7 \times 60 = 420$ m$^2$. This is the area required to cope with peak flow (as above). For average flow conditions, 5 beds of this size will have an area of $5 \times 6 \times 10 = 300$ m$^2$, which is more than the 280 m$^2$ required above, and will thus be sufficient.

If each bed is subdivided into 3 by means of wooden baffles, such that the partitioned beds are each 6 m x 3.3 m, constant head loading feeding is more readily obtained. After the first subdivision is filled to a depth of 10 - 15 cm, the flow is diverted to the second, and then the third, subdivision. Thereafter, sludge feed is diverted among the 3 subdivisions to "top up" and thus maintain an approximately constant head filtration throughout the day. The sludge may be distributed in wooden channels traversing the length of the sand beds, and distributed into each subdivision in turn by means of sliding baffles operated manually.

Each bed is proposed to consist of approximately a 5 cm layer of 20 - 30 mesh graded sand supported on approximately 15 - 20 cm of graded stone. The stone rests on open glazed drain pipes, draining to the sewer. A simple supported roof, with open sides, covers the drainage beds. A transparent plastic material is suitable for the roofing. This would promote increased evaporation by radiation. The open sides would allow wind access, and from equation (44a) (Section 2.7.1.), the increased wind velocity would promote the drying rate.

The waste treatment process plant layout, is shown in the flow diagram of Fig. 52.
FIG. 52: WASTE TREATMENT PROCESS FLOWSHEET; TO PROCESS 45 m³ per day.

UNIT OPERATIONS:

Neutralization

Flocculation

Sedimentation

Thickening

Filtration & Drying

SLUDGE SOLIDS TO CARTING AWAY

SLUDGE UNDERFLOW

DISH FILTERS & DRYING BEDS

SLUDGE UNDER DRAIN

TO SEWER

OVERFLOW

TO SEWER

RAW EFFLUENT

BALANCING TANK No.1

H₂SO₄

68.75 m³ capacity.

NaOH

BALANCING TANK No.2

AI (BO₃)₄

FEEDER

BAFFLE PIPE MIXERS

NaAlO₂

FEEDER

SEDIMENTATION TANK

3 m dia. 3 m depth
4. EXPERIMENTAL PROCEDURES, RESULTS AND CALCULATIONS.

4.1 Flocculation

20 litres of alkaline composite effluent were neutralised with 10% sulphuric acid solution to a pH value = 7.0. 400 ml aliquots of the neutralised solution were placed in a set of six identical 600 ml beakers. Individual pH adjustments were made by the dropwise addition of 10% sulphuric acid solution (to reduce the pH), or the dropwise addition of 10% sodium hydroxide (to increase the pH).

10% m/v solutions of aluminium sulphate hydrate and sodium aluminate were used to dose the effluent aliquots. The aluminium sulphate hydrate was analysed and found to have the composition Al₂(SO₄)₃.14H₂O. The solid sodium aluminate contained 80% NaAlO₂. In each 400 ml aliquot of effluent, the addition of 0.1 ml of the flocculating reagent solution is equivalent to 25 ppm.

In the initial experiments, 2 ml of 10% aluminium sulphate (i.e. 500 ppm) was added to each aliquot, and the test solution rapid-mixed for 1 minute. The pH was adjusted to the value indicated with 10% sodium hydroxide, and slow-mixed (60 rpm) for exactly 20 minutes. Stirring was stopped, and the time noted for the flocculating mass to first commence separating and show a clear supernatant layer below the liquid surface.

The following results were obtained:-

(a) Constant Alum Dose = 500 ppm (20°C.)

<table>
<thead>
<tr>
<th>pH:</th>
<th>7</th>
<th>6</th>
<th>5</th>
<th>4</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time to separate (min):</td>
<td>&gt;120</td>
<td>20</td>
<td>35</td>
<td>90</td>
<td>&gt;120</td>
</tr>
</tbody>
</table>
(b) Constant Alum Dose = 500 ppm (20°C.)

<table>
<thead>
<tr>
<th>pH:</th>
<th>6,1</th>
<th>6,0</th>
<th>5,9</th>
<th>5,8</th>
<th>5,7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time to separate (min):</td>
<td>25</td>
<td>20</td>
<td>15</td>
<td>18</td>
<td>27</td>
</tr>
</tbody>
</table>

The tests in (b) are at fractional pH values. The best time for separation of the floc occurred at pH = 5,9. In the next series of tests, incremental dosages of alum were given to aliquots of the neutralised effluent. The pH was adjusted to 5,9 in each case with sodium hydroxide solution, and the time for the floc to separate again noted.

(c) Constant pH = 5,9 (20°C.)

<table>
<thead>
<tr>
<th>Alum Dose (ppm):</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>350</th>
<th>400</th>
<th>450</th>
<th>500</th>
<th>550</th>
<th>600</th>
<th>700</th>
<th>800</th>
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</thead>
<tbody>
<tr>
<td>Time to separate (min):</td>
<td>&gt;120</td>
<td>96</td>
<td>75</td>
<td>72</td>
<td>66</td>
<td>45</td>
<td>32</td>
<td>23</td>
<td>20</td>
<td>15</td>
<td>14</td>
</tr>
</tbody>
</table>

The best floc appeared to form with a minimum of 6 - 700 ppm of alum. However, the addition of a smaller dose of alum to effluent initially acidified to approximately pH = 5,0 was found by a trial and error procedure, to give a better floc when the pH was finally increased with increments of sodium aluminate solution, as follows:-

(d) Constant Alum Dosage = 500 ppm (Effluent at pH = 5,0 at 20°C.)

<table>
<thead>
<tr>
<th>Sodium Aluminate Dosage (ppm):</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
<th>700</th>
<th>800</th>
</tr>
</thead>
<tbody>
<tr>
<td>Final pH value:</td>
<td>5,52</td>
<td>5,70</td>
<td>5,85</td>
<td>5,95</td>
<td>6,10</td>
<td>6,21</td>
</tr>
<tr>
<td>Time to separate (min):</td>
<td>25</td>
<td>20</td>
<td>10</td>
<td>10</td>
<td>14</td>
<td>17</td>
</tr>
</tbody>
</table>

The optimum sodium aluminate dosage for this effluent appeared to be 5 - 600 ppm; the final pH value being 5,85 - 5,95.
Following this procedure, aliquots of 4 different composite bulk effluents were collected and flocculated in a similar manner. In each preparation, the pH was first adjusted to 6.5 with sulphuric acid; an additional one-half of the volume of acid used added to pH ± 5.0; 500 ppm of aluminium sulphate added, and the solution rapid-mixed; sodium aluminate added to pH 5.85 - 5.95, and again rapid mixed. Finally, the solution was slow-mixed for 20 minutes, and allowed to settle.

The comparative analyses of influent and effluent was as follows:

<table>
<thead>
<tr>
<th>Flocculation</th>
<th>Effluent &quot;A&quot; (red)</th>
<th>Effluent &quot;B&quot; (white)</th>
<th>Effluent &quot;C&quot; (Brown)</th>
<th>Effluent &quot;D&quot; (Dark Red)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Total Sulphuric Acid added:</td>
<td>607</td>
<td>1083</td>
<td>587*</td>
<td>1868</td>
</tr>
<tr>
<td>(equiv. 98%) (ppm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Aluminium Sulphate:</td>
<td>475</td>
<td>477</td>
<td>500</td>
<td>483</td>
</tr>
<tr>
<td>(Commercial Al₂(SO₄)₃.14H₂O)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(ppm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Sodium Aluminate:</td>
<td>300</td>
<td>631</td>
<td>786</td>
<td>888</td>
</tr>
<tr>
<td>(Commercial NaAlO₂, 80%) (ppm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Analyses</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Initial COD (mg/l):</td>
<td>3193</td>
<td>1129</td>
<td>8013</td>
<td>2930</td>
</tr>
<tr>
<td>5. Supernatant COD (mg/l):</td>
<td>1600</td>
<td>641</td>
<td>5054</td>
<td>1400</td>
</tr>
<tr>
<td>6. % Reduction in COD:</td>
<td>50%</td>
<td>43%</td>
<td>37%</td>
<td>52%</td>
</tr>
<tr>
<td>7. Initial pH value:</td>
<td>10,32</td>
<td>10,90</td>
<td>4,20</td>
<td>11,30</td>
</tr>
<tr>
<td>8. Supernatant pH value:</td>
<td>5,87</td>
<td>5,90</td>
<td>5,90</td>
<td>5,90</td>
</tr>
</tbody>
</table>
Laboratory Batch Flocculation Data (continued)

<table>
<thead>
<tr>
<th>Effluent</th>
<th>Effluent</th>
<th>Effluent</th>
<th>Effluent</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;A&quot; (red)</td>
<td>&quot;B&quot; (white)</td>
<td>&quot;C&quot; (Brown)</td>
<td>&quot;D&quot; (Dark Red)</td>
</tr>
<tr>
<td>9. Initial S.S.</td>
<td>1,796</td>
<td>2,054</td>
<td>2,018</td>
</tr>
<tr>
<td>C₀ (g/l):</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10. Final Sludge S.S.:</td>
<td>27,380</td>
<td>11,130</td>
<td>11,038</td>
</tr>
<tr>
<td>(18 hr.) Cᵤ (g/l)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11. Concentration</td>
<td>15,24</td>
<td>5,43</td>
<td>5,47</td>
</tr>
<tr>
<td>ratio: (Cᵤ / C₀)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12. Initial Chloride</td>
<td>688</td>
<td>1298</td>
<td>659</td>
</tr>
<tr>
<td>(as Cl): (mg/l)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>13. 4 hour OA on</td>
<td>298</td>
<td>336</td>
<td>1185</td>
</tr>
<tr>
<td>Supernatant: (mg/l)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>14. Ammoniacal-N on</td>
<td>18,2</td>
<td>11,2</td>
<td>15,4</td>
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<tr>
<td>Supernatant:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(mg/l)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15. Total-N on</td>
<td>48,3</td>
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<td>31,3</td>
</tr>
<tr>
<td>Supernatant:</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>(mg/l)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note:
This effluent had been acidified at the factory. For consistency, 688 ppm of NaOH was added to bring it to pH = 7.0, and then 587 ppm of sulphuric acid added. This was equivalent to treating the original effluent with 209 ppm of sodium hydroxide, from its original pH of 4.20 to a pH value of 5.0.

4.2.1 Procedure for Determination of the Sedimentation Characteristics and Tank Design Parameters.

Batches of floc suspension were prepared from effluents A to D inclusive (section 4.1). Immediately after ceasing the slow stirring (after completion of flocculation), the contents of the 5 l (or 10 l) beakers, in which flocculation was effected, were introduced into the
sedimentation column (Fig. 5). The column was clamped in a vertical position, adjusting the alignment with spirit-levels. An air sparge was introduced at the bottom for a few minutes, in order to obtain an initially uniform suspended solids concentration; and then it was closed. Known volumes of samples of the original suspension were retained for analysis.

Space in the diameter of the column was insufficient to employ a slow-speed stirrer operating at about 4 - 6 revolutions per hour, as recommended by Eckenfelder (2a). The floc was therefore allowed to settle quiescently.

For convenience, all measurements of liquid surface levels and sludge levels were measured from the top of the column itself as a reference point. The sludge level was observed by shining a low-wattage torch behind the column, in order not to heat the liquid, and thus avoid production of convection currents. This procedure was successful, and provided that the column was not sited in a draughty position, the temperature remained essentially constant throughout the test. (Flocculation and sedimentation were effected in the same laboratory, and at the same temperature).

At fixed time intervals, after recording both the height of the sludge interface and liquid surface below the reference point (i.e. from the top of the column), each of the 4 taps was bled to rinse out residual suspension, and 10 ml samples collected in weighed; numbered; and graduated centrifuge tubes. This process was repeated over a period of 120 minutes and longer where necessary, and also after an
overnight settling (18 hours nominally).

Samples of the bottom sludge after 18 hours settling, were collected in 40 ml tared graduated centrifuge tubes. The initial slurry was also collected in a 40 ml tared graduated centrifuge tube. Each of the centrifuge tubes was allowed to stand 18 hours, and the sludge volume recorded. After centrifugation, the sludge volume was again recorded, and in the case of the completely settled 18-hour sludge, these results were used to assist in the estimation of $H_{\infty}$ and $C_{\infty}$ (Section 2.2.2.).

The supernatant was drained out of each tube, and the contents dried for several hours at $120^\circ$C. The concentration of sludge at each depth and at each time interval, was thus estimated.

These results are tabulated in Tables 1A, 1B and 1C respectively, and in the case of the 10l rectangular jar and 1l measuring cylinders, Tables 2 (d) and 2 (e) respectively. (Only initial and final samples were taken in the latter two cases, and no intermediate samples).

For computation of Tables 1A, 1B and 1C, the following procedure was employed:

Each time samples were taken, the liquid surface height and the sludge interface height in the column decreased. In order to correct the sludge height and obtain the correct depths of sampling taps 1, 2, 3, and 4 respectively (Fig. 5) below the liquid surface, the samples at time $t$ minutes are assumed to have been drawn from a depth below the surface, which was the mean value immediately before sampling and immediately after sampling at time $t$. 
<table>
<thead>
<tr>
<th>Time (Min.)</th>
<th>Distance of liquid surface from top (cm)</th>
<th>Sludge height from top of column (cm)</th>
<th>Liquid depth in column (cm)</th>
<th>Sludge interface face from liquid surface (cm)</th>
<th>Incremental fall in liquid depth (cm)</th>
<th>Top tap to liquid surface (cm)</th>
<th>No. 2 tap to liquid surface (cm)</th>
<th>No. 3 tap to liquid surface (cm)</th>
<th>No. 4 tap to liquid surface (cm)</th>
<th>Sludge height from bottom (cm)</th>
<th>Sludge volume (L)</th>
<th>Corrected sludge height (M cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>17.00</td>
<td>17.00</td>
<td>166.50</td>
<td>0</td>
<td>0</td>
<td>28.50</td>
<td>74.50</td>
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<td>100.00</td>
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<td>191.80</td>
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<td>4.70</td>
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<td>72.15</td>
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<td>163.15</td>
<td>181.50</td>
<td>92.37</td>
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<td>12.40</td>
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<td>5.78</td>
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<td>89.33</td>
<td>134.83</td>
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<td>50.43</td>
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<td>86.75</td>
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<td>16.17</td>
<td>31.77</td>
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<td>84</td>
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<td>127.90</td>
<td>14.90</td>
<td>9.38</td>
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</tbody>
</table>
### Table 1B: Effluent Floc Settling Rate
(Data for Fig. 10)

<table>
<thead>
<tr>
<th>Min</th>
<th>A (cm)</th>
<th>B (cm)</th>
<th>C (cm)</th>
<th>D (cm)</th>
<th>E (cm)</th>
<th>F (cm)</th>
<th>G (cm)</th>
<th>H (cm)</th>
<th>J (cm)</th>
<th>K (cm)</th>
<th>L (%)</th>
<th>M (cm)</th>
</tr>
</thead>
<tbody>
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<td>7.20</td>
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<td>90.32</td>
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<td>201.50</td>
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</tbody>
</table>
**TABLE I C**

**EFFLUENT "C" FLOC SETTLING RATE**

(Data For FIG. 11)

<table>
<thead>
<tr>
<th>Time (Min)</th>
<th>Distance of liquid surface from top (cm)</th>
<th>Sludge height from top of column (cm)</th>
<th>Liquid depth in column (cm)</th>
<th>Sludge interface from liquid surface (cm)</th>
<th>Incremental fall in liquid depth (cm)</th>
<th>Top tap to liquid surface (cm)</th>
<th>No. 2 tap to liquid surface (cm)</th>
<th>No. 3 tap to liquid surface (cm)</th>
<th>No. 4 tap to liquid surface (cm)</th>
<th>Sludge height from bottom (cm)</th>
<th>Sludge volume (L %)</th>
<th>Corrected sludge height (cm)</th>
</tr>
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<tbody>
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<td>94.40</td>
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<td>178.20</td>
<td>36.80</td>
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<td>8.65</td>
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<td>100.15</td>
<td>145.65</td>
<td>141.40</td>
<td>79.35</td>
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</tr>
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<td>90</td>
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<td>82.50</td>
<td>175.10</td>
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<td>5.85</td>
<td>51.85</td>
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</table>
Let $A =$ distance measured from liquid surface to the top of the column reference point. ($A_i =$ initial distance)

$B =$ distance measured from the top of the sludge interface to the top of the column reference point.

The distance from the reference point at the column top to the top tap is 45.5 cm; the inter-tap distances down the column are 46.0; 45.5; and 45.5 cm respectively; the total length of the apparatus is 213.5 cm.

Let $C =$ liquid depth in the column ($C_i =$ initial depth)

$D =$ sludge interface height in the column

$E =$ incremental fall in level of the surface as a result of sampling at each interval.

$F =$ mean depth of top tap below liquid level

$G =$ mean depth of No. 2 tap below liquid level

$H =$ mean depth of No. 3 tap below liquid level

$J =$ mean depth of bottom tap below liquid level

$K =$ sludge height calculated from the bottom of the column

$L =$ sludge height at each interval as a fraction of total liquid height.

$M =$ corrected sludge height at each interval.

(Units for all the above are centimetres, excepting $L$, which is dimensionless).
Then at period $t$, (from $t = 0$ ........ $t = t_n$),

$$C_t = 213.5 - A_t$$

$$D_t = B_t - A_t$$

$$E_t = A_t + 1 - A_t$$

$$F_t = 45.5 - \left( A_t + \frac{1}{2} E_t \right) = 45.5 - \frac{1}{2} (A_t + A_t + 1)$$

$$G_t = F_t + 46.0 = 91.5 - \frac{1}{2} (A_t + A_t + 1)$$

$$H_t = G_t + 45.5 = 137.0 - \frac{1}{2} (A_t + A_t + 1)$$

$$J_t = H_t + 45.5 = 182.5 - \frac{1}{2} (A_t + A_t + 1)$$

$$K_t = C_t - D_t = 213.5 - B_t$$

$$L_t = \left( \frac{K_t}{C_t} \right) \times 100 = \frac{213.5 - B_t}{213.5 - A_t}$$

$$M_t = \frac{L_t \times C_t}{100} = \frac{(213.5 - B_t)(213.5 - A_t)}{(213.5 - A_t)}$$

In the form above, calculations were readily effected in a computer.

The letters $A$, $B$, etc., refer to column headings in Tables 1A, 1B, and 1C respectively.

For each corrected sludge height at time $t$ (column $M$), the settling curves in Figures 8, 10 and 11 have been plotted. Tables 2 (D) and 2 (E) (batch sedimentation in a 10 ℓ jar and 1 ℓ measuring cylinder respectively) required no correction to sludge height, and the settling curves are plotted directly to give Figures 9 and 12 respectively.

Tables 2A, 2B and 2C were also computed on a computer, as follows:
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<tr>
<th>Top Position</th>
<th>Time</th>
<th>Sample No.</th>
<th>Mean tap depth from surface (cm)</th>
<th>Total volume of sample (ml)</th>
<th>18 hr. sludge volume (ml)</th>
<th>Centrifuged sludge volume (ml)</th>
<th>Mass of sludge (g)</th>
<th>Sludge concentration (g/l)</th>
<th>18 hr. % sludge volume</th>
<th>Compacted sludge volume %</th>
<th>Compacted sludge vol. g/ml</th>
<th>Concentration of compacted sludge solids g/ml</th>
<th>Concentration of 18 hr. sludge solids g/ml</th>
<th>Sludge solids removal on original sludge %</th>
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<th>Sample No.</th>
<th>Mean tap depth from surface (cm)</th>
<th>Total volume of sample (ml)</th>
<th>18 hr. sludge volume (ml)</th>
<th>Centrifuged sludge volume (ml)</th>
<th>Mass of sludge (g)</th>
<th>Sludge concentration g/l</th>
<th>18 hr. %</th>
<th>Concentrated sludge volume g/ml</th>
<th>Concentration of concentrated sludge solids g/ml</th>
<th>Sludge solids removal on original solids %</th>
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Initial suspended solids concentration = 1,796 g/l

Final sludge suspended solids concentration = 27,380 g/l
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<th>Mass of sludge (g)</th>
<th>Sludge concentration g/l</th>
<th>18 hr. &amp; sludge volume %</th>
<th>Compacted sludge volume</th>
<th>Concentration of 18 hr. sludge solids</th>
<th>Concentration of compacted sludge solids</th>
<th>Sludge solids removal on original sludge solids</th>
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Initial suspended solids concentration = 2.054 g/l
Final sludge suspended solids concentration = 11.150 g/l
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Initial suspended solids concentration = 2.018 g/l
Final sludge suspended solids concentration = 11.038 g/l
### TABLE 2 (g) (Data For Fig. 9)

**Effluent "A" Floc Sedimentation Curve in 10 L Rectangular Jar**

**Liquid Depth = 0.29 metres**

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<td>120</td>
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<td>220</td>
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<td>11.2</td>
</tr>
<tr>
<td>1050</td>
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<td>10.4</td>
</tr>
</tbody>
</table>

### TABLE 2 (E) (Data For Fig. 12)

**Effluent "D" Floc Sedimentation Curve in 1 L Cylindrical Jar**

**Liquid Depth = 1 litre = 0.34 metres**

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>Sludge Interface Height (cm)</th>
<th>Sludge Volume % V/V</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1000</td>
<td>100.0</td>
</tr>
<tr>
<td>7</td>
<td>902</td>
<td>90.2</td>
</tr>
<tr>
<td>15</td>
<td>730</td>
<td>73.0</td>
</tr>
<tr>
<td>28</td>
<td>542</td>
<td>54.2</td>
</tr>
<tr>
<td>38</td>
<td>452</td>
<td>45.2</td>
</tr>
<tr>
<td>53</td>
<td>360</td>
<td>36.0</td>
</tr>
<tr>
<td>64</td>
<td>321</td>
<td>32.1</td>
</tr>
<tr>
<td>80</td>
<td>279</td>
<td>27.9</td>
</tr>
<tr>
<td>100</td>
<td>245</td>
<td>24.5</td>
</tr>
<tr>
<td>120</td>
<td>226</td>
<td>22.6</td>
</tr>
<tr>
<td>150</td>
<td>207</td>
<td>20.7</td>
</tr>
<tr>
<td>180</td>
<td>199</td>
<td>19.9</td>
</tr>
<tr>
<td>210</td>
<td>193</td>
<td>19.3</td>
</tr>
<tr>
<td>244</td>
<td>192</td>
<td>19.2</td>
</tr>
<tr>
<td>1056</td>
<td>142</td>
<td>14.2</td>
</tr>
</tbody>
</table>
Let $A_1 =$ Initial distance from liquid surface to the column-top reference point (cm)

$A_t =$ Liquid surface distance to the reference point at period $t$ (cm)

$A_{t+1} =$ Liquid surface distance to the reference point at $(t+1)$ (cm)

$B_t =$ sludge interface distance to the reference point at period $t$ (cm)

$V_t =$ volume of sample tapped at period $t$ (ml)

$V_s =$ volume of sludge (ml) in the sample after 18 hours settling; tube in a vertical position.

$V_c =$ volume of sludge in the sample after centrifuging (ml)

$W_s =$ mass of the sludge in the sample of volume $V_t$ after drying (g)

$C_0 =$ initial s.s. concentration (g/l)

(i) Then sludge concentration at period $t$,

$$C_t = \frac{W_s}{V_t} \times 1000 \text{ g/l}$$

(ii) 18 hour % sludge volume = \frac{V_s}{V_t} \times 100\%

(iii) Compacted % sludge volume = \frac{V_c}{V_t} \times 100\%

(iv) Concentration of compacted sludge = \frac{W_s}{V_c} \text{ g/l}

(v) Concentration of 18 hour sludge = \frac{W_s}{V_s}
(vi) Percentage sludge solids removal on original settled solids conc\(^n\),
\[ \frac{(C_o - C_t)}{C_o} \times 100\% \]

Only in the case of effluent "A" (Table 2A) was it possible to obtain some resemblance of flocculent settling, and a corresponding set of suspended solids removal curves at different times and depths. However, the resemblance was extremely poor, and was not supported in any of the other experiments, and was thus abandoned. Fig. 6 shows the approximate curves, but in the other experiments no similar curves could be obtained. (The results of (vi) above were essentially negative in the cases of effluents "B" and "C" respectively (Tables 2B and 2C), and clearly indicated thickening). Results (ii), (iii), (iv) and (v) above were employed in the estimation of \( H_\infty \). (Table 6; Fig 7; and section 2.2.2.).

4.2.2. Calculation of Unit Area And Mass Loading.

Table 3 summarises the initial and final conditions for calculation of the product \( C \cdot H_\infty \) in each of the different settling apparatus, and for different flocs. The expected variation in the range of concentrations in the effluent is thus covered.

Table 6 lists the plotting points for the plot of \( \log (H - H_\infty) \) vs. time, required for the estimation of the initial sludge compression point at time \( t_c \) (equation (11), Fig. 7). The intersection of \( t_c \) with the settling curves served as a guide in selecting values of \( C_u \) and
### TABLE 3

<table>
<thead>
<tr>
<th>Settling Apparatus:</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column</td>
<td>Column</td>
<td>Column</td>
<td>Column</td>
<td>1-l Jar</td>
<td>Rect. Jar</td>
</tr>
<tr>
<td><strong>EFFLUENT:</strong></td>
<td>&quot;A&quot;</td>
<td>&quot;B&quot;</td>
<td>&quot;C&quot;</td>
<td>&quot;D&quot;</td>
<td>&quot;A&quot; 10 L</td>
</tr>
<tr>
<td><strong>CALCULATIONS h</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. 18-hour settling, S.S. concentration, ( C_0 ) (mg/l)</td>
<td>27380</td>
<td>11150</td>
<td>11038</td>
<td>8550</td>
<td>27380</td>
</tr>
<tr>
<td>2. Initial S.S. Concentration, ( C_0 ) (mg/l)</td>
<td>1796</td>
<td>2054</td>
<td>2018</td>
<td>550</td>
<td>1796</td>
</tr>
<tr>
<td>3. Initial Height of Interface, ( H_0 ) (metres)</td>
<td>1965</td>
<td>2065</td>
<td>2000</td>
<td>1945 L+</td>
<td>0.29</td>
</tr>
<tr>
<td>4. ( C_0 ), ( H_0 ) (mg/10 cm²)</td>
<td>3527</td>
<td>4242</td>
<td>4036</td>
<td>575 mg.</td>
<td>521</td>
</tr>
</tbody>
</table>

* Sludge height in Litres (1 - Litre = 0.34 metres)
<table>
<thead>
<tr>
<th>C2</th>
<th>H1 CoHo</th>
<th>H2</th>
<th>t2</th>
<th>H1-H2</th>
<th>V2</th>
<th>t2 (m/day)</th>
<th>UA+</th>
<th>t2 (min)</th>
<th>UA+</th>
<th>t2 (min)</th>
<th>UA+</th>
<th>t2 (min)</th>
<th>UA+</th>
<th>t2 (min)</th>
<th>UA+</th>
<th>t2 (min)</th>
<th>UA+</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
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<td>48,0</td>
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<td>0,0262</td>
<td>37,64</td>
<td>54,0</td>
<td>0,01531</td>
<td>56,7</td>
<td>0,01608</td>
<td>57,5</td>
<td>0,01630</td>
<td>58,5</td>
<td>0,01650</td>
<td>60,0</td>
<td>0,01701</td>
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<td>2500</td>
<td>1,412</td>
<td>0,460</td>
<td>50,5</td>
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<td>0,0159</td>
<td>27,18</td>
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<td>59,8</td>
<td>0,01694</td>
<td>61,0</td>
<td>0,01729</td>
<td>62,0</td>
<td>0,01764</td>
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<td>0,390</td>
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<td>0,765</td>
<td>0,0165</td>
<td>20,55</td>
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<td>0,01617</td>
<td>61,9</td>
<td>0,01754</td>
<td>64,0</td>
<td>0,01813</td>
<td>65,0</td>
<td>0,0185</td>
<td>68,0</td>
<td>0,01928</td>
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<tr>
<td>4000</td>
<td>0,882</td>
<td>0,310</td>
<td>60,5</td>
<td>0,572</td>
<td>0,00946</td>
<td>13,62</td>
<td>56,0</td>
<td>0,01588</td>
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<td>0,01791</td>
<td>67,0</td>
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<td>0,705</td>
<td>0,290</td>
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<td>0,415</td>
<td>0,00659</td>
<td>9,48</td>
<td>54,0</td>
<td>0,01531</td>
<td>64,0</td>
<td>0,01837</td>
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<td>0,01955</td>
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<td>0,589</td>
<td>0,260</td>
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<td>49,5</td>
<td>0,01403</td>
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<td>0,01837</td>
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<td>0,02010</td>
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<td>83,0</td>
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<td>74,0</td>
<td>0,230</td>
<td>0,00311</td>
<td>4,48</td>
<td>58,5</td>
<td>0,01092</td>
<td>60,5</td>
<td>0,01714</td>
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<td>89,0</td>
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<tr>
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<td>0,353</td>
<td>0,200</td>
<td>90,0</td>
<td>0,155</td>
<td>0,00170</td>
<td>2,45</td>
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<td>0,01176</td>
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<td>0,02692</td>
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<td>15000</td>
<td>0,235</td>
<td>0,192</td>
<td>104,0</td>
<td>0,043</td>
<td>0,00044</td>
<td>0,59</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

+ Consistent units for UA = \( \text{m}^2\text{mg/min}) \times 10^{-3} \n
**TABLE 3 (a) (Data For FIG. 13(ii))**

Effluent "A" - Column, CoHo - 3527 mg/10 cm²

<table>
<thead>
<tr>
<th>Cu (mg/l)</th>
<th>Hw = ( \frac{C_2 H_1}{C_2} ) (metres)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10000</td>
<td>0,353</td>
</tr>
<tr>
<td>12500</td>
<td>0,262</td>
</tr>
<tr>
<td>14000</td>
<td>0,252</td>
</tr>
<tr>
<td>15000</td>
<td>0,235</td>
</tr>
<tr>
<td>17960</td>
<td>0,197</td>
</tr>
<tr>
<td>C₂</td>
<td>( H₁ = \frac{C_{\text{Collo}}}{C₂} ) (m)</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>2000</td>
<td>0,2604</td>
</tr>
<tr>
<td>2500</td>
<td>0,2084</td>
</tr>
<tr>
<td>3000</td>
<td>0,1736</td>
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<tr>
<td>4000</td>
<td>0,1302</td>
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<tr>
<td>5000</td>
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<td>6000</td>
<td>0,0868</td>
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<td>7000</td>
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<tr>
<td>7500</td>
<td>0,0695</td>
</tr>
<tr>
<td>8000</td>
<td>0,0560</td>
</tr>
<tr>
<td>9000</td>
<td>0,0521</td>
</tr>
</tbody>
</table>

+ CONSISTENT UNITS FOR UA = \( \frac{m^2(mg/min)}{10^{-3}} \)
### Table 3 (c) (Data for FIG. 14(i))

**Effluent "B" - Column, CoHo = 4242 mg/10 cm²**

<table>
<thead>
<tr>
<th>Cu (mg/l)</th>
<th>H₂ = C₂H₁ (metres)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4513</td>
<td>0,940</td>
</tr>
<tr>
<td>5438</td>
<td>0,780</td>
</tr>
<tr>
<td>6000</td>
<td>0,707</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>C₂</th>
<th>C₂H₁</th>
<th>H₁H₂</th>
<th>H₁-H₂</th>
<th>V₂²</th>
<th>V₂</th>
<th>H₂ = 0,940 m</th>
<th>H₂ = 0,780 m</th>
<th>H₂ = 0,707 m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C₂H₂</td>
<td>t₂</td>
<td>V₂²</td>
<td>V₂</td>
<td>t₂</td>
<td>(m/min)</td>
<td>(m/min)</td>
<td>(m/min)</td>
</tr>
<tr>
<td>2250</td>
<td>1880</td>
<td>1061</td>
<td>105,5</td>
<td>0,824</td>
<td>0,00781</td>
<td>11,24</td>
<td>110,0</td>
<td>0,0281</td>
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<tr>
<td>2500</td>
<td>1697</td>
<td>1061</td>
<td>124,0</td>
<td>0,757</td>
<td>0,00610</td>
<td>123,5</td>
<td>125,0</td>
<td>0,0290</td>
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<tr>
<td>3000</td>
<td>1414</td>
<td>1061</td>
<td>154,0</td>
<td>0,634</td>
<td>0,00412</td>
<td>115,0</td>
<td>115,0</td>
<td>0,0271</td>
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<tr>
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<td>1212</td>
<td>1061</td>
<td>186,5</td>
<td>0,545</td>
<td>0,00289</td>
<td>92,0</td>
<td>92,0</td>
<td>0,0217</td>
</tr>
<tr>
<td>4000</td>
<td>1131</td>
<td>1061</td>
<td>198,0</td>
<td>0,491</td>
<td>0,00248</td>
<td>3,57</td>
<td>76,5</td>
<td>0,0180</td>
</tr>
<tr>
<td>4500</td>
<td>1061</td>
<td>1061</td>
<td>204,3</td>
<td>0,440</td>
<td>0,00215</td>
<td>3,10</td>
<td>66,0</td>
<td>0,0155</td>
</tr>
</tbody>
</table>

*CONSISTENT UNITS FOR UA = M² (mg/min) x 10⁻³*
<table>
<thead>
<tr>
<th>( C_2 ) (mg/l)</th>
<th>( H_1 ) (m)</th>
<th>( H_2 ) (m)</th>
<th>( t_2 ) (min)</th>
<th>( V_{1c}^{+} ) (m/day)</th>
<th>( V_2 ) (m/min)</th>
<th>( H_U = 1,153 ) m</th>
<th>( H_U = 1,086 ) m</th>
<th>( H_U = 1,009 ) m</th>
</tr>
</thead>
<tbody>
<tr>
<td>2100</td>
<td>1,922</td>
<td>1,280</td>
<td>119,0</td>
<td>0,642</td>
<td>7,76</td>
<td>137</td>
<td>0,0339</td>
<td>149,0</td>
</tr>
<tr>
<td>2200</td>
<td>1,835</td>
<td>1,220</td>
<td>128,0</td>
<td>0,615</td>
<td>6,91</td>
<td>140</td>
<td>0,0347</td>
<td>153,0</td>
</tr>
<tr>
<td>2400</td>
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<td>1,120</td>
<td>154,0</td>
<td>0,562</td>
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<td>143</td>
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<td>0,514</td>
<td>4,69</td>
<td>140</td>
<td>0,0347</td>
<td>160,0</td>
</tr>
<tr>
<td>2600</td>
<td>1,552</td>
<td>1,060</td>
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<td>0,472</td>
<td>4,19</td>
<td>137</td>
<td>0,0339</td>
<td>158,0</td>
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<tr>
<td>2800</td>
<td>1,441</td>
<td>0,970</td>
<td>204,3</td>
<td>0,471</td>
<td>3,31</td>
<td>123</td>
<td>0,0305</td>
<td>153,0</td>
</tr>
</tbody>
</table>

+ CONSISTENT UNITS FOR \( UA = m^2/(mg/min) \times 10^{-3} \)
TABLE 3 *(e) (Data For FIG. 14(iii))

Effluent "D" = 1 Litre Cylindrical Jar. CoHo = 575 mg ( = 195.5 mg/10 cm)

<table>
<thead>
<tr>
<th>$C_u$ (mg/l)</th>
<th>$H = \frac{C_1 H_1}{C_u}$ (Litres)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>0.288</td>
</tr>
<tr>
<td>2250</td>
<td>0.256</td>
</tr>
<tr>
<td>2500</td>
<td>0.230</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$C_u$ (mg/l)</th>
<th>$H = \frac{C_1 H_1}{C_u}$ (Litres)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>0.288</td>
</tr>
<tr>
<td>2250</td>
<td>0.256</td>
</tr>
<tr>
<td>2500</td>
<td>0.230</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>N.B.</th>
<th>1 Litre = 0.34 metres</th>
<th>$H = 0.298 L$</th>
<th>$H = 0.274 L$</th>
<th>$H = 0.256 L$</th>
<th>$H = 0.230 L$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C_2$</td>
<td>$H_1$</td>
<td>$H_2$</td>
<td>$t_2$</td>
<td>$H_1 - H_2$</td>
</tr>
<tr>
<td></td>
<td>(mg/l)</td>
<td>(l)</td>
<td>(l)</td>
<td>(min)</td>
<td>(l)</td>
</tr>
<tr>
<td>750</td>
<td>0.765</td>
<td>0.495</td>
<td>32.5</td>
<td>0.270</td>
<td>0.00831</td>
</tr>
<tr>
<td>1000</td>
<td>0.575</td>
<td>0.360</td>
<td>53.0</td>
<td>0.215</td>
<td>0.00406</td>
</tr>
<tr>
<td>1250</td>
<td>0.460</td>
<td>0.278</td>
<td>79.0</td>
<td>0.182</td>
<td>0.00230</td>
</tr>
<tr>
<td>1500</td>
<td>0.383</td>
<td>0.256</td>
<td>93.0</td>
<td>0.127</td>
<td>0.00137</td>
</tr>
<tr>
<td>1750</td>
<td>0.329</td>
<td>0.221</td>
<td>127.0</td>
<td>0.108</td>
<td>0.000850</td>
</tr>
<tr>
<td>2000</td>
<td>0.266</td>
<td>0.211</td>
<td>142.0</td>
<td>0.077</td>
<td>0.000542</td>
</tr>
</tbody>
</table>

* CONSISTENT UNITS FOR UA ABOVE: $\frac{m^2}{(mg/min)} \times 10^{-3}$
calculating $H_u$, as listed in the unit area calculations of Table 3 (a) - 3 (e) inclusive, respectively. The point of intersection of the tangents to the curve drawn from the $H_1$ values of Table 3, yielded the co-ordinates $H_2$, $t_2$, from which the interfacial settling velocities, given by $V = \frac{H_1 - H_2}{t_2}$ were calculated.

Figures 13 and 14 are plots of the unit areas vs. interfacial settling velocities for different values of $C_u$, the underflow concentration.

Table 4, records the maximum values of UA for each value of $C_u$ from these plots, and the mass loading values at these UA maxima.

Fig. 15 is a log-log plot of the correlation,

$$\left(\frac{C_u}{C_o} - 1\right) = \frac{K_b}{(ML)^n} \quad \text{(equation (12))}.$$  

Figure 16 is a plot of log $K_b$ vs. initial concentration $C_o$ obtained from Fig. 15 and tabulated in Table 5. The slope and intercept gave the relationship:

$$K_b = 3.2 \times 10^9 \cdot e^{-8.11C_o}$$

Thus, from the data of Figs. 15 and 16, Eckenfelder's correlation equation was found to be:

$$\left(\frac{C_u}{C_o} - 1\right) = \frac{3.2 \times 10^9 \cdot e^{-8.11C_o}}{(ML)^{1.28}}$$

Table 7 summarises calculations of tank diameters for different values of $C_o$ and $C_u$ and ML, calculated from Eckenfelder's equation, and at a fixed flow rate of $45m^3/d$. 
### TABLE 4

**Maximum Unit Areas & Minimum Mass Loading**

*(Data For Fig. 15, ex Flgs. 13 & 14)*

<table>
<thead>
<tr>
<th>Cu (g/l)</th>
<th>(a) FLOC &quot;A&quot; - COLUMN, Co = 1,796 g/l</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>UA Max. (m²/mg/min) x 10⁻⁵</td>
<td>UA Max. 10⁰ UA² (m²/Kg/day)</td>
<td>ML = 1/UA (Kg/m²/day)</td>
<td>Co / Co⁻¹</td>
</tr>
<tr>
<td>17.96</td>
<td>0.0272</td>
<td>0.0189</td>
<td>53.0</td>
<td>9.00</td>
</tr>
<tr>
<td>15.00</td>
<td>0.0212</td>
<td>0.0147</td>
<td>68.0</td>
<td>7.35</td>
</tr>
<tr>
<td>14.00</td>
<td>0.0205</td>
<td>0.0142</td>
<td>70.5</td>
<td>6.80</td>
</tr>
<tr>
<td>12.50</td>
<td>0.0186</td>
<td>0.0129</td>
<td>77.5</td>
<td>5.97</td>
</tr>
<tr>
<td>10.00</td>
<td>0.0160</td>
<td>0.0111</td>
<td>90.2</td>
<td>4.57</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cu (g/l)</th>
<th>(b) FLOC &quot;A&quot; - 10’6 Rectangular Jar, Co = 1,796 g/l</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12.50</td>
<td>0.2120</td>
<td>0.1472</td>
<td>6.8</td>
<td>5.97</td>
</tr>
<tr>
<td>10.00</td>
<td>0.0997</td>
<td>0.0692</td>
<td>14.6</td>
<td>4.57</td>
</tr>
<tr>
<td>7.50</td>
<td>0.0560</td>
<td>0.0389</td>
<td>25.7</td>
<td>3.17</td>
</tr>
<tr>
<td>6.00</td>
<td>0.0440</td>
<td>0.0306</td>
<td>32.7</td>
<td>2.34</td>
</tr>
<tr>
<td>5.00</td>
<td>0.0372</td>
<td>0.0256</td>
<td>35.8</td>
<td>1.79</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cu (g/l)</th>
<th>(c) FLOC &quot;B&quot; - COLUMN, Co = 2,054 g/l</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>6.00</td>
<td>0.0407</td>
<td>0.0283</td>
<td>35.4</td>
<td>1.92</td>
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<tr>
<td>5.44</td>
<td>0.0363</td>
<td>0.0252</td>
<td>39.7</td>
<td>1.65</td>
</tr>
<tr>
<td>4.51</td>
<td>0.0290</td>
<td>0.0201</td>
<td>49.7</td>
<td>1.20</td>
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<table>
<thead>
<tr>
<th>Cu (g/l)</th>
<th>(d) FLOC &quot;C&quot; - COLUMN, Co = 2,018 g/l</th>
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<th></th>
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<td></td>
<td></td>
</tr>
<tr>
<td>3.50</td>
<td>0.0354</td>
<td>0.0246</td>
<td>40.7</td>
<td>0.74</td>
</tr>
<tr>
<td>3.72</td>
<td>0.0396</td>
<td>0.0275</td>
<td>36.4</td>
<td>0.84</td>
</tr>
<tr>
<td>4.00</td>
<td>0.0467</td>
<td>0.0324</td>
<td>30.9</td>
<td>0.98</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Cu (g/l)</th>
<th>(e) FLOC &quot;D&quot; - 1 Litre Cylindrical Jar, Co = 0.550 g/l</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
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</tr>
<tr>
<td>2.00</td>
<td>0.582</td>
<td>0.404</td>
<td>2.48</td>
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<tr>
<td>2.25</td>
<td>0.455</td>
<td>0.316</td>
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<td>3.09</td>
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<td>2.50</td>
<td>0.585</td>
<td>0.406</td>
<td>2.47</td>
<td>3.54</td>
</tr>
<tr>
<td>2.70</td>
<td>0.419</td>
<td>0.291</td>
<td>3.44</td>
<td>2.82</td>
</tr>
</tbody>
</table>
TABLE 5 (Data For FIG. 16)

INTERCEPT ($K_b$) AND SLOPE ($n$) FROM PLOT $\log \left( \frac{C_u}{C_0} \right)$ vs. $\log$ (ML)
(EX FIG. 15)

<table>
<thead>
<tr>
<th></th>
<th>Co. (g/l)</th>
<th>Slope &quot;n&quot;</th>
<th>Intercept $K_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Floe &quot;A&quot; (Column)</td>
<td>1.796</td>
<td>1.32</td>
<td>1900</td>
</tr>
<tr>
<td>2. Floe &quot;B&quot; (Column)</td>
<td>2.054</td>
<td>1.33</td>
<td>210</td>
</tr>
<tr>
<td>3. Floe &quot;C&quot; (Column)</td>
<td>2.135</td>
<td>1.16</td>
<td>55</td>
</tr>
<tr>
<td>4. Floe &quot;A&quot; (Rect. 10 l. jar)</td>
<td>1.796</td>
<td>1.32</td>
<td>225</td>
</tr>
<tr>
<td>5. Floe &quot;D&quot; (Mes. 1 l. Cyl.)</td>
<td>0.550</td>
<td>1.05</td>
<td>10</td>
</tr>
</tbody>
</table>

Mean Slope "n", (except "D") = 1.28
<table>
<thead>
<tr>
<th>Time (min.)</th>
<th>Sludge Height (cm)</th>
<th>H-H₀ (cm)</th>
<th>Sludge Height (cm)</th>
<th>H-H₀ (cm)</th>
<th>Sludge Height (cm)</th>
<th>H-H₀ (ml)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>196,5</td>
<td>187,5</td>
<td>29,0</td>
<td>27,5</td>
<td>1000</td>
<td>930</td>
</tr>
<tr>
<td>10</td>
<td>153,0</td>
<td>154,0</td>
<td>20,0</td>
<td>18,5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>139,0</td>
<td>130,0</td>
<td>10,0</td>
<td>8,5</td>
<td>625</td>
<td>555</td>
</tr>
<tr>
<td>30</td>
<td>106,0</td>
<td>99,0</td>
<td>6,7</td>
<td>5,2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>77,0</td>
<td>66,0</td>
<td>5,8</td>
<td>4,3</td>
<td>440</td>
<td>370</td>
</tr>
<tr>
<td>50</td>
<td>47,0</td>
<td>36,0</td>
<td>5,6</td>
<td>4,1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>32,0</td>
<td>23,0</td>
<td>5,0</td>
<td>3,5</td>
<td>335</td>
<td>265</td>
</tr>
<tr>
<td>70</td>
<td>26,0</td>
<td>17,0</td>
<td>4,7</td>
<td>3,2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>23,0</td>
<td>14,0</td>
<td>4,6</td>
<td>3,1</td>
<td>275</td>
<td>205</td>
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<tr>
<td>90</td>
<td>20,5</td>
<td>11,5</td>
<td>4,5</td>
<td>3,0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>19,0</td>
<td>10,0</td>
<td>4,3</td>
<td>2,8</td>
<td>245</td>
<td>175</td>
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<tr>
<td>120</td>
<td>18,5</td>
<td>9,5</td>
<td>4,1</td>
<td>2,6</td>
<td>220</td>
<td>150</td>
</tr>
<tr>
<td>140</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>210</td>
</tr>
<tr>
<td>160</td>
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<td></td>
<td></td>
<td>204</td>
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<tr>
<td>180</td>
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<td>192</td>
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<tr>
<td>240</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>191</td>
</tr>
</tbody>
</table>

(Continued)
TABLE 6 (CONTINUED)

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>Sludge Height H (cm)</th>
<th>( H - H_\infty ) (cm)</th>
<th>Sludge Height H (cm)</th>
<th>( H - H_\infty ) (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>206,3</td>
<td>193,3</td>
<td>192,5</td>
<td>174,5</td>
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<tr>
<td>50</td>
<td>156,0</td>
<td>143,0</td>
<td>166,5</td>
<td>150,5</td>
</tr>
<tr>
<td>100</td>
<td>110,0</td>
<td>97,0</td>
<td>137,0</td>
<td>119,0</td>
</tr>
<tr>
<td>150</td>
<td>80,0</td>
<td>67,0</td>
<td>113,5</td>
<td>95,5</td>
</tr>
<tr>
<td>200</td>
<td>53,0</td>
<td>50,0</td>
<td>98,0</td>
<td>80,0</td>
</tr>
<tr>
<td>250</td>
<td>55,5</td>
<td>42,5</td>
<td>87,0</td>
<td>69,0</td>
</tr>
<tr>
<td>300</td>
<td>52,0</td>
<td>39,0</td>
<td>81,5</td>
<td>63,5</td>
</tr>
<tr>
<td>350</td>
<td>50,0</td>
<td>37,0</td>
<td>77,5</td>
<td>59,5</td>
</tr>
<tr>
<td>400</td>
<td>48,0</td>
<td>35,0</td>
<td>73,5</td>
<td>55,5</td>
</tr>
</tbody>
</table>
### TABLE 7 (Fig. 17)

*Design Criteria for Thickener/Clarifier For Flocculated Sludge*  
(Maximum Flow: 45 m³/day)

<table>
<thead>
<tr>
<th>$C_0$ (g/I)</th>
<th>$K_b$</th>
<th>$C_0$ = 1,5 g/I</th>
<th>$K_b$ = 17800</th>
<th>$C_0$ = 1,75 g/I</th>
<th>$K_b$ = 2330</th>
<th>$C_0$ = 2,00 g/I</th>
<th>$K_b$ = 310</th>
<th>$C_0$ = 2,25 g/I</th>
<th>$K_b$ = 38,2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cu</strong></td>
<td><strong>Mass Loading</strong></td>
<td><strong>Tank Diameter</strong></td>
<td><strong>Mass Loading</strong></td>
<td><strong>Tank Diameter</strong></td>
<td><strong>Mass Loading</strong></td>
<td><strong>Tank Diameter</strong></td>
<td><strong>Mass Loading</strong></td>
<td><strong>Tank Diameter</strong></td>
<td><strong>Mass Loading</strong></td>
</tr>
<tr>
<td>g/l</td>
<td>ML (Kg/m²/day)</td>
<td>(Metres)</td>
<td>ML (Kg/m²/day)</td>
<td>(Metres)</td>
<td>ML (Kg/m²/day)</td>
<td>(Metres)</td>
<td>ML (Kg/m²/day)</td>
<td>(Metres)</td>
<td>ML (Kg/m²/day)</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>7,5</td>
<td>708</td>
<td>0,348</td>
<td>169,8</td>
<td>0,768</td>
<td>40,1</td>
<td>1,688</td>
<td>8,91</td>
<td>3,810</td>
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<tr>
<td>10,0</td>
<td>637</td>
<td>0,400</td>
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<td>1,956</td>
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<td>4,425</td>
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<tr>
<td>12,5</td>
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<td>0,439</td>
<td>103,5</td>
<td>0,983</td>
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<td>2,175</td>
<td>5,27</td>
<td>4,950</td>
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<tr>
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<td>88,1</td>
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<td>20,6</td>
<td>2,367</td>
<td>4,44</td>
<td>5,380</td>
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<tr>
<td>20,0</td>
<td>295</td>
<td>0,538</td>
<td>68,3</td>
<td>1,210</td>
<td>15,9</td>
<td>2,690</td>
<td>3,43</td>
<td>6,125</td>
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<tr>
<td>25,0</td>
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<td>0,593</td>
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<td>1,330</td>
<td>13,1</td>
<td>2,960</td>
<td>2,89</td>
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<tr>
<td>30,0</td>
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<td>0,643</td>
<td>48,7</td>
<td>1,434</td>
<td>11,3</td>
<td>3,190</td>
<td>2,42</td>
<td>7,290</td>
<td></td>
</tr>
</tbody>
</table>

**Sludge/ day**  
67,5 Kg. | 78,8 Kg | 90,0 Kg | 101,3 Kg
Fig. 17 is a set of plots at a flow rate of 45 m³/d (design flow-rate) of log $C_u$ vs. log D, where D = cylindrical tank diameter.

The slope of the linear plots for different $C_o$ values is found to be $m = 2.25$. The intercepts provided a "tank diameter constant", $K_D$. Figure 18 is a plot of log $K_D$ vs. $C_o$.

From the slope and intercept of this plot,

$$K_D = 1.586 \times 10^5 e^{-5.39 C_o}.$$

The final equation developed, relating all thickener calculations for this particular floc sedimentation, and a flow of 45 m³/d, is

$$C_u = K_D D^{2.25} = 1.586 \times 10^5 D^{2.25} e^{-5.39 C_o} \quad \text{[equation (16)]}$$

### 4.2.3. Concentration Profiles in the Settling Sludge

Plots of the height of the thickening column vs. concentration (as determined by analysis) at separate time intervals, did not provide distinct profile curves. (The concentrations were more or less constant above the compression zone. Data was extracted from Tables 1 and 2). This may be due to the lowest sampling point not being situated at the column bottom and hence the bottom layer concentration could not be obtained analytically. However, calculation of the suspended solids concentrations at particular intervals, and at known depths below the sludge interface level, provided a profile of the concentration gradient in the sludge at different intervals. Tables 8 (a) – (c) inclusive, list these sludge concentration gradients. Figures 19a and 19b are plots of the concentration gradients at various times for flocs 'A' and 'B' respectively.
<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>Corrected Sludge height (cm)</th>
<th>Sampling Depth below corrected interface (cm)</th>
<th>Sludge concentration (g/l)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>196.50</td>
<td>28.50</td>
<td>1.796</td>
</tr>
<tr>
<td></td>
<td></td>
<td>74.50</td>
<td>1.796</td>
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<tr>
<td></td>
<td></td>
<td>120.00</td>
<td>1.796</td>
</tr>
<tr>
<td></td>
<td></td>
<td>165.50</td>
<td>1.796</td>
</tr>
<tr>
<td>6</td>
<td>181.50</td>
<td>13.50</td>
<td>1.292</td>
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<td></td>
<td>59.50</td>
<td>1.442</td>
</tr>
<tr>
<td></td>
<td></td>
<td>105.00</td>
<td>1.500</td>
</tr>
<tr>
<td></td>
<td></td>
<td>150.50</td>
<td>1.194</td>
</tr>
<tr>
<td>12</td>
<td>157.57</td>
<td>32.58</td>
<td>1.240</td>
</tr>
<tr>
<td></td>
<td></td>
<td>79.19</td>
<td>1.302</td>
</tr>
<tr>
<td></td>
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<td>125.81</td>
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<tr>
<td>120</td>
<td>18.43</td>
<td>18.45</td>
<td>27.380</td>
</tr>
</tbody>
</table>
### CONCENTRATION GRADIENTS DURING SLUDGE THICKENING

**TABLE 8 (B) (FIG. 19(b))**

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>Corrected Sludge height (cm)</th>
<th>Sampling Depth below corrected interface (cm)</th>
<th>Sludge concentration (g/l)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>80.60</td>
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<td>171.80</td>
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<td>186.34</td>
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<td>1.810</td>
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<td></td>
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<td>108.02</td>
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<td>113.42</td>
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<td>69.01</td>
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<tr>
<td>1020</td>
<td>27.24</td>
<td>27.24</td>
<td>11.130</td>
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</table>
# Concentration Gradients during Sludge Thickening

**Table 8 (C)**

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>Corrected Sludge Height (cm)</th>
<th>Sampling Depth below corrected interface (cm)</th>
<th>Sludge concentration (g/l)</th>
</tr>
</thead>
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<td></td>
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<tr>
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<td></td>
<td>67.94</td>
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<td>113.46</td>
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<td>158.98</td>
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<td>93.48</td>
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<td>81.93</td>
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<td>914</td>
<td>35.96</td>
<td>35.96</td>
<td>11.038</td>
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<tr>
<td>$C_o$ g/l</td>
<td>Mass Loading ML (Kg/d$^2$/day)</td>
<td>Flow Rate, m$^3$/Day</td>
<td>Mass Loading ML (Kg/d$^2$/day)</td>
</tr>
<tr>
<td>----------</td>
<td>-------------------------------</td>
<td>----------------------</td>
<td>-------------------------------</td>
</tr>
<tr>
<td></td>
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<td>45</td>
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<td>7,5</td>
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<td>447</td>
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<td>0,388</td>
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<td>Sludge per day, Kg:</td>
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<td>45,0</td>
<td>67,5</td>
</tr>
<tr>
<td>Sludge per day, Kg/d</td>
<td>Mass loading, ml/(kg.f/d)</td>
<td>Flow rate, m³/Day</td>
<td>Tank diameter, D metres</td>
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<td>------------------------</td>
</tr>
<tr>
<td>30,0</td>
<td>30,0</td>
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<td>1,381</td>
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<td>24,2</td>
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<td>1,777</td>
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<td>15.0</td>
<td>20,6</td>
<td>1,362</td>
<td>1,926</td>
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<td>15,9</td>
<td>1,550</td>
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<td>13,1</td>
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</table>
### TABLE 10

INTERCEPTS, \( k_0 \), AT DIA. = 1 metre (LOG \( l = 0 \))

(EX FIGURES 20 and 21)

<table>
<thead>
<tr>
<th>Flow Rate ( Q_3 ) (cubic metre/day)</th>
<th>Mean Slope, ( n )</th>
<th>( C_0 = 1.50 ) g/l</th>
<th>( C_0 = 1.75 ) g/l</th>
<th>( C_0 = 2.00 ) g/l</th>
<th>( C_0 = 2.25 ) g/l</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>2,136</td>
<td>268</td>
<td>44.2</td>
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<td>1.505</td>
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<td>30</td>
<td>2,103</td>
<td>125</td>
<td>20.5</td>
<td>4.20</td>
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<td>2.31</td>
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<td>2,222</td>
<td>57.3</td>
<td>9.46</td>
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<td>0.335</td>
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<tr>
<td>75</td>
<td>2,158</td>
<td>43.1</td>
<td>7.55</td>
<td>1.425</td>
<td>0.252</td>
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</table>
4.2.4. **Effect of Flow Rate on Tank Design Parameters**

For maximum flow rates, $Q = 15, 30, 45, 60, 75$ m$^3$/d respectively, as in Figures 20 and 21 respectively. The slopes are constant and independent of $Q$. Graphical statistical analysis shows these to be normally distributed with mean slope $\bar{x} = 2.165$ and S.D. = 0.05 (Fig. 22). The slopes of these lines, and their intercepts, are tabulated in Table 10. The plots of the log of the intercepts $K_D$ versus initial sludge concentration, $C_0$, are shown in Figure 23.

At each value of $Q$, the slopes were constant and equal to $-1.179$. From the intercept co-ordinates (read at $C_0 = 2.00$ g/l) the intercepts at $C_0 = 0$ were calculated as follows:

$$\log K_D = -1.179 C_0 + \log c' \quad \text{[equation (19)]}$$

When $C_0 = 2.00$, $\log c' = \log K_D + 2.3571$.

Thus the constant $c'$ is found, and these calculations are summarised in Table 11:

<table>
<thead>
<tr>
<th>$Q$ (m$^3$/d)</th>
<th>$K_D$ intercept ($C_0 = 2.00$ g/l)</th>
<th>Constant $c'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>7.90</td>
<td>1797.6</td>
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<tr>
<td>30</td>
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<td>905.7</td>
</tr>
<tr>
<td>45</td>
<td>2.31</td>
<td>525.7</td>
</tr>
<tr>
<td>60</td>
<td>1.70</td>
<td>387.0</td>
</tr>
<tr>
<td>75</td>
<td>1.345</td>
<td>306.1</td>
</tr>
</tbody>
</table>
Plotting log $c'$ vs. log $Q$ (Fig. 24), gave a linear plot with a slope of $-1.0914$; and intercept of $c' = 2735$, at $Q = 10 \text{ m}^3$/day.

The equation to this line is

$$
\log 2735 = -1.0914 \log 10 + \log \ c''.
$$

When $\log \ c'' = -4.5284$, $c'' = 33760$.

Thus, $\log \ c' = \log 33760 - 1.0914 \log Q$

$. \ c' = 33760/Q^{1.0914}$ \hspace{1cm} [equation (20)]

Substituting the value of $c'$ in equation (20) into equation (19), the latter now evaluates as

$$
\log K_D = \log \frac{33760}{Q^{1.0914}} - 1.179 \ C_o
$$

i.e. $2.303 \log K_D = 2.303 \log \frac{33760}{Q^{1.0914}} - 2.303 \times 1.179 \ C_o$

or $\ln K_D = \ln \frac{33760}{Q^{1.0914}} - 2.7142 \ C_o$

$. \ K_D = \frac{33760}{Q^{1.0914}} e^{-2.7142 \ C_o} \hspace{1cm} [equation (21)].$

From Figures 20 and 21,

$$
\log C_u = 2.165 \log D + \log K_D
$$

[equation (22)].

Substituting equation (21) in equation (22) gives

$$
\log C_u = 2.165 \log D + \log \left[ \frac{33760}{Q^{1.0914}} e^{-2.7142 \ C_o} \right]
$$

$. \ C_u = \frac{33760}{Q^{1.0914}} e^{-2.7142 \ C_o} \ D^{2.165} \hspace{1cm} [equation (25)]$

Rearranging, $D = \sqrt{\frac{C_u Q^{1.0914} e^{2.7142 \ C_o}}{33760}}$
or, \[ D = 8,0975 \times 10^{-3} \sqrt{\frac{2,165}{Q_{1,0914}}} \sqrt{\frac{1,0914}{Q_{1,0914}}} e^{2,7142 C_o} \]  

4.2.5. Construction of a Design Chart for Simulation.

With the selected tank diameter of \( D = 3,0 \) m, Table 12 shows the concentration of the underflow sludge for various flow rates, \( Q \text{ m}^3/\text{d} \), and for the expected range of initial flow sludge concentrations, \( C_o = 2,00 \) g/l and \( C_o = 2,50 \) g/l respectively:

<table>
<thead>
<tr>
<th>Flow Rate</th>
<th>( Q_{1,0914} )</th>
<th>( C_o = 2,00 ) g/l</th>
<th>( C_o = 2,50 ) g/l</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>19,213</td>
<td>83,30</td>
<td>21,45</td>
</tr>
<tr>
<td>30</td>
<td>40,935</td>
<td>39,10</td>
<td>10,07</td>
</tr>
<tr>
<td>45</td>
<td>63,724</td>
<td>25,12</td>
<td>6,47</td>
</tr>
<tr>
<td>60</td>
<td>87,138</td>
<td>18,37</td>
<td>4,73</td>
</tr>
<tr>
<td>75</td>
<td>111,30</td>
<td>14,38</td>
<td>3,70</td>
</tr>
</tbody>
</table>

A plot of log \( Q \) vs. log \( C_u \) is shown in Fig. 25. The set of parallel lines led to a family of lines, ranging in initial concentration \( C_o = 1,50 \) g/l to \( C_o = 3,50 \) g/l.

The effect of varying the flow rate and initial concentration of the floc sludge on the underflow sludge concentration, may now be rapidly evaluated.
4.3. **Sand Filtration of Sludge.**

Sludges of different concentrations were prepared from the same flocculated composite sample. By settling in the sedimentation column for different times, different average sludge concentrations were obtained in the settled sludge.

To minimise the effect of temperature, experiments were carried out in a constant temperature and constant humidity laboratory (20°C; 65% R.H.). Because of the extended duration of the filtration experiments, all filtrate volumes have been corrected for evaporation loss. The rate of evaporation of the filtrate at 20°C and 65% Relative Humidity was found to be 0.142 mg/cm²/min. (Fig. 27). This is equivalent to an evaporation loss of 0.14 ml/cm²/10³ min.

The apparatus employed is shown diagrammatically in Fig. 26. It consists of a 5.35 cm i.d. glass filtration tube with 21 cm wall height. A 5 mm No. 2 sintered glass disc is fused in at the bottom. The lower end is tapered to fit into the neck of a tall 250 ml graduated cylinder. The sintered glass disc acts as a support for a shallow sand bed (and also provides a source of resistance to filtration). An 11 mm bed of graded 20/30 mesh washed sand rests on top of the sintered disc, representing the top layer of a sand filter. The filtration tube is clamped in a vertical position above the measuring cylinder.

For constant head filtration, a 300 ml polythene bottle was clamped inverted over the filtration tube. A 5 mm glass tube, in which a 2 mm vent was blown approximately 5 - 10 mm from the tapered outlet end, was fitted through a bung into a length of 15 mm Tygon tubing. The latter fits over the neck of the bottle. In operation, when the bottle was one-half to two-thirds filled with feed sludge, and clamped in an
inclined position as shown in the diagram, sludge fed out of the bottle until the surface of the liquid in the filtration tube just covered the small vent. (The latter was positioned facing upwards). When the liquid level in the filter fell to the lower edge of the vent, sludge fed into the filter once again, until the upper edge of the vent was again covered. During feeding, air bubbles rose into the neck of the bottle and into the reservoir above. In this latter operation, the sludge feed in the reservoir was gently mixed, thus promoting the feeding of a constant concentration of slurry. This apparatus provided a simple automatic feeder which maintained a fairly constant head in the filter. The depth of sludge in the bed was varied by adjusting the height of clamping of the bottle, and the length of the feeder tube.

At the commencement of each filtration, the sand bed and supporting sintered disc were wetted by filtering a portion of filtrate and allowing to drain. At the end of each filtration, after removal of the sludge, the sand bed was back-washed with mains water. It was then rinsed with filtrate before commencing further filtrations.

4.3.1. **Constant Head Filtration. Determination of Specific Resistance, r.**

The volume $v_t$ of filtrate collected at each time interval $t$ was recorded for different constant heads of sludge. Three initial sludge concentrations were used, viz. $C_0 = 14,573$ g/l; $C_0 = 18,798$ g/l; and $C_0 = 20,930$ g/l. The results are shown in Tables 13 (a) to 13 (c) inclusive. Figures 28, 29 and 30, are the respective plots of the function $t/v_t$ against $v_t$ for each initial concentration.
# Table 13 (A) (Fig. 28)

<table>
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<tr>
<th></th>
<th>Exp. 1</th>
<th>Exp. 1</th>
<th>Exp. 1</th>
<th>Exp. 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
</tr>
<tr>
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<td>14,573</td>
<td>14,573</td>
<td>14,573</td>
</tr>
<tr>
<td><strong>Final Solids Concentration, Cᵢ (g/l)</strong></td>
<td>23,365</td>
<td>21,763</td>
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<td>23,395</td>
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<tr>
<td><strong>Mean Sludge Density, µ (g/ml)</strong></td>
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<td>1,010</td>
<td>1,010</td>
<td>1,010</td>
</tr>
<tr>
<td><strong>Concentration Term, c (g/ml)</strong></td>
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</tr>
<tr>
<td><strong>Initial Compacted Volume of Sludge, %</strong></td>
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<td>19,00</td>
<td>19,00</td>
<td>19,00</td>
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**TIME**

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<th>t/Vₜ</th>
<th>VOL.Vₜ (ml)</th>
<th>t/Vₜ</th>
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<td>FILTRATE VOLUME</td>
<td>t/V t (min/ml)</td>
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**TABLE 13 (B) (FIG. 29)**

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<th>CONSTANT HEAD FILTRATION</th>
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</table>

<table>
<thead>
<tr>
<th>Initial Solids Concentration, C o (g/l)</th>
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</thead>
<tbody>
<tr>
<td>Final Solids Concentration, C f (g/l)</td>
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<tr>
<td>Mean Sludge Density, (\rho) (g/ml)</td>
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</tr>
<tr>
<td>Concentration Term, c (g/ml)</td>
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<td>Initial Compacted Volume of Sludge, %</td>
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<tr>
<td>Final Compacted Volume of Sludge, %</td>
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**Exp. 2 - 1:** Head = 19,19 cm

**Exp. 2 - 2:** Head = 15,25 cm

**Exp. 2 - 3:** Head = 12,45 cm

**Exp. 2 - 4:** Head = 8,40 cm
## Table 13 (c) (Fig. 30)

### Constant Head Filtration

<table>
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<th>EXP. 3 - 2</th>
<th>EXP. 3 - 3</th>
</tr>
</thead>
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<tr>
<td><strong>Initial Solids Concentration, ( C_0 ) (g/l)</strong></td>
<td>20,930</td>
<td>20,930</td>
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<td><strong>Final Solids Concentration, ( C_f ) (g/l)</strong></td>
<td>32,995</td>
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<td><strong>Mean Sludge Density, ( \bar{\rho} ) (g/ml)</strong></td>
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<td>1,016</td>
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<td><strong>Concentration Term, ( c ) (g/ml)</strong></td>
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<td><strong>Final Compacted Volume of Sludge, %</strong></td>
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<td>46,90</td>
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</table>

### EXP. 3 - 1: Head = 19, 10 cm

<table>
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<th>TIME ( t ) (min)</th>
<th>FILTRATE VOLUME ( V_t ) (ml)</th>
<th>( t/V_t ) (min/ml)</th>
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<tr>
<td>0</td>
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<tr>
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<td>90</td>
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</tr>
<tr>
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<tr>
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<td>48,5</td>
<td>3,096</td>
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<tr>
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<td>3,398</td>
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<td>3,917</td>
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<tr>
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<td>78,2</td>
<td>4,220</td>
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<tr>
<td>420</td>
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<td>4,674</td>
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<tr>
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<td>5,185</td>
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<tr>
<td>640</td>
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### EXP. 3 - 2: Head = 15, 10 cm

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<th>FILTRATE VOLUME ( V_t ) (ml)</th>
<th>( t/V_t ) (min/ml)</th>
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<tr>
<td>0</td>
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<tr>
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<td>8,381</td>
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### EXP. 3 - 3: Head = 8, 35 cm

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<th>( t/V_t ) (min/ml)</th>
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<td>0</td>
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<td>0</td>
</tr>
<tr>
<td>5</td>
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<tr>
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<td>184,4</td>
<td>8,381</td>
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</table>
From the slope of the lines, \( b \text{ min/cm}^6 \), the specific resistance was calculated, \( r = \frac{2bPA^2}{wc} \), [equation (28)]. The filter medium resistance, \( R_m = \frac{IFA}{\mu} (\text{min}) (\text{sec})/\text{cm}^2 \) (section 2.4.2.), was calculated from the intercept, \( I \text{ min/cm}^3 \). These results are given in Table 14.

The viscosity of the filtrate, \( \mu \), was determined by timing the flow between two fixed marks in a U-tube capillary viscometer \( t_1 \), and comparing with the time required for distilled water, \( t_2 \), at the same constant temperature. The densities of the filtrate, \( d_1 \), and distilled water, \( d_2 \), were determined at the same temperature in an S.G. bottle. The following results were obtained:-

Mean time for flow of filtrate, \( t_1 = 86.3 \text{ sec} \).
Mean time for flow of water, \( t_2 = 85.1 \text{ sec} \).
Density of filtrate at \( 20^\circ\text{C} \), \( d_1 = 1.0044 \text{ g/ml} \).
Density of water at \( 20^\circ\text{C} \), \( d_2 = 0.9982 \text{ g/ml} \).
Viscosity of water at \( 20^\circ\text{C} \), \( \mu' = 0.01005 \text{ poise} \).


dot\cdot \text{ Viscosity of filtrate at } 20^\circ\text{C}, \mu = \mu' \times \frac{d_1}{d_2} \times \frac{t_1}{t_2}

\mu = 0.01005 \times \frac{1.0044 \times 86.3}{0.9982 \times 85.1} = 0.01026 \text{ poise}

The mean pressure head \( P \) was evaluated from the sum of the depth of sludge \( h(\text{cm}) \) plus the height of the filter media (16 mm), and the average of the sludge density at the beginning and at the end of each filtration respectively. The densities were determined in an S.G. bottle.

Then \( P = (h + 1.6) \rho \text{ g/cm}^2 \).
TABLE 14 (FIGS. 31 & 52)
CALCULATION OF SPECIFIC RESISTANCE OF SLUDGE, \( r \), & FILTER MEDIUM RESISTANCE, \( R_m \) FROM PLOTS OF \( t/V_t \) vs. \( V_t \)

<table>
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<th>Filter Area A, ( A = 22.5 \text{ cm}^2 ); Temperature = 20° C; Filtrate Viscosity, ( \mu = 0.01026 \text{ cp} )</th>
<th>Initial Sludge Concentration ( C_0 ) (g/l)</th>
<th>Exp. No.</th>
<th>Fig. No.</th>
<th>Head Pressure, ( H ) (cm)</th>
<th>Concentration Term ( C ) (g/ml of filtrate)</th>
<th>Specific Resistance ( 1 = \frac{(\text{min/cm})^3}{\mu\text{cm}} )</th>
<th>Intercept ( \frac{r}{\mu} ) (min)(sec)/g</th>
<th>Filter Medium Resistance ( R_m = \frac{1}{\mu} \frac{L}{P} ) (min)(sec)/cm ( ^2 )</th>
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<td>19.19</td>
<td>0.03834</td>
<td>0.02610</td>
<td>1.29 ( \times 10^6 )</td>
<td>1.303</td>
<td>5.48 ( \times 10^4 )</td>
</tr>
<tr>
<td></td>
<td>1 - 2</td>
<td>15.10</td>
<td>15.25</td>
<td>0.04367</td>
<td>0.03766</td>
<td>1.30 ( \times 10^6 )</td>
<td>2.212</td>
<td>5.74 ( \times 10^4 )</td>
</tr>
<tr>
<td></td>
<td>1 - 3</td>
<td>12.20</td>
<td>12.32</td>
<td>0.03429</td>
<td>0.03257</td>
<td>1.16 ( \times 10^6 )</td>
<td>1.998</td>
<td>5.40 ( \times 10^4 )</td>
</tr>
<tr>
<td></td>
<td>1 - 4</td>
<td>8.75</td>
<td>8.84</td>
<td>0.03826</td>
<td>0.04584</td>
<td>1.05 ( \times 10^6 )</td>
<td>2.441</td>
<td>4.73 ( \times 10^4 )</td>
</tr>
<tr>
<td>18,798</td>
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<td>19.19</td>
<td>19.52</td>
<td>0.03844</td>
<td>0.02674</td>
<td>1.34 ( \times 10^6 )</td>
<td>1.676</td>
<td>7.18 ( \times 10^4 )</td>
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<tr>
<td></td>
<td>2 - 2</td>
<td>15.25</td>
<td>15.46</td>
<td>0.04574</td>
<td>0.03747</td>
<td>1.25 ( \times 10^6 )</td>
<td>0.535</td>
<td>1.81 ( \times 10^4 )</td>
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<tr>
<td></td>
<td>2 - 3</td>
<td>12.45</td>
<td>12.62</td>
<td>0.04400</td>
<td>0.03963</td>
<td>1.12 ( \times 10^6 )</td>
<td>0.831</td>
<td>2.30 ( \times 10^4 )</td>
</tr>
<tr>
<td></td>
<td>2 - 4</td>
<td>8.40</td>
<td>8.52</td>
<td>0.03737</td>
<td>0.04772</td>
<td>1.08 ( \times 10^6 )</td>
<td>0.721</td>
<td>1.35 ( \times 10^4 )</td>
</tr>
<tr>
<td>20,930</td>
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<td>19.41</td>
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<td>0.03915</td>
<td>1.33 ( \times 10^6 )</td>
<td>1.175</td>
<td>5.00 ( \times 10^4 )</td>
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<tr>
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<td>3 - 2</td>
<td>15.10</td>
<td>15.34</td>
<td>0.05326</td>
<td>0.04386</td>
<td>1.25 ( \times 10^6 )</td>
<td>1.306</td>
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<td>3 - 3</td>
<td>8.35</td>
<td>8.48</td>
<td>0.04529</td>
<td>0.05362</td>
<td>1.04 ( \times 10^6 )</td>
<td>1.250</td>
<td>2.33 ( \times 10^4 )</td>
</tr>
</tbody>
</table>
4.3.2. Determination of Initial Specific Resistance, $r_0$, and Sludge Compressibility, $s$.

A log-log plot of the specific resistance, $r$, against the pressure head $P$, calculated in Table 14, is shown in Fig. 31. The equation of this line is given by

$$r = 0.512 \times 10^6 P^{0.325}$$

The intercept at unit pressure yielded the initial specific resistance,

$$r_0 = 0.512 \times 10^6 \text{ (sec)(min)/g}$$

The slope of the line gave the sludge compressibility,

$$s = 0.325$$

(The units of $r_0$ are consistent with those of the measured parameters, and are convenient in the subsequent calculations. However, converting minutes to seconds,

$$r_0 = 3.072 \times 10^7 \text{ sec}^2 / \text{g}$$

4.3.3. Estimation of Filter Media Resistance, $R_m$.

Table 14 shows that there was considerable variation in the filter media resistance calculated from the experimental results, although the order of the resistance was similar in each case. With the same apparatus and filtrate, the filter media resistance is expected to remain constant. However, variations in permeability and compaction of the sludge in the filter will influence the calculated values of $R_m$.

Fig. 32 is a cumulative probability plot of the experimental results, and showed a normal distribution, with mean resistance

$$\bar{R}_m = 4.31 \times 10^4 \text{ (min)(sec)/cm}^2$$

and standard deviation $= 2.0 \times 10^4 \text{ (min)(sec)/cm}^2$. 
4.3.4. Calculation of Rate of Loading in Constant Head Filtration.

Using the experimental data obtained at the end of each filtration from Tables 13(a) to 13(c) inclusive and the calculated values of the filter media resistance $R$ in Table 14, Table 15 shows the observed loading rates ($L_{OBS} = \frac{V_L c_m}{t_A}$), compared with those calculated on the assumption that $R$ is negligible

$$L_{calc} = \sqrt{\frac{2P(1-S)}{\mu r_0}} c, \quad \text{equation (36)}$$

and those calculated to include $R$

$$L_{TEMP} = L_{calc} - \frac{1,028 R_m}{r_o t p s} + 0,0236 \times 10^{-4}, \quad \text{equation (41)}$$

Fig. 33 is a plot of the function $R_m$ against the difference $r_0 t p s$ between the loading rates ($L_{calc} - L_{OBS}$).

This plot showed the relationship between the two values, leading to the equation of the straight line:

$$L_{OBS} = L_{CALC} - \frac{1,028 R_m}{r_o t p s} + 0,0236 \times 10^{-4} \quad \text{[equation (41)]}$$

Table 16 shows the calculation of the loading rate based on the latter equation, and at a time $t = 1$ day (1440 minutes), and initial concentration $C_0 = 14$ g/l. The sludge depth has been varied from 10 to 30 cm. These results were plotted in Fig. 34, which also shows the curves for different
<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>Time (min)</th>
<th>Filtrate Volume $V_t$(ml)</th>
<th>Pressure $P$ (g/cm²)</th>
<th>Concentration $c$ (g/ml)</th>
<th>Median Resistance $R \times 10^4$ (min/10 ml/cm²)</th>
<th>Observed Loading Rate $L_{OBS}$ $g/(min)(cm^2)$</th>
<th>Calculated Loading Rate $L_{CALC} = 1,029 R$ $g/(min)(cm^2)$</th>
<th>Empirical Loading Rate $L_{EMP} = L_{CALC} - 0,0236 \times 10^{-4}$ $g/(min)(cm^2)$</th>
<th>$\frac{R}{r_0 \times 10^{-5}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - 1</td>
<td>1600</td>
<td>238,9</td>
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<td>0,03834</td>
<td>5,46</td>
<td>2,26</td>
<td>2,41</td>
<td>2,41</td>
<td>1,52</td>
</tr>
<tr>
<td>1 - 2</td>
<td>1285</td>
<td>156,5</td>
<td>15,25</td>
<td>0,04367</td>
<td>7,40</td>
<td>2,39</td>
<td>2,85</td>
<td>2,85</td>
<td>2,40</td>
</tr>
<tr>
<td>1 - 3</td>
<td>1475</td>
<td>184,3</td>
<td>12,32</td>
<td>0,05429</td>
<td>5,40</td>
<td>1,90</td>
<td>2,20</td>
<td>2,20</td>
<td>1,89</td>
</tr>
<tr>
<td>1 - 4</td>
<td>1505</td>
<td>157,0</td>
<td>8,84</td>
<td>0,03626</td>
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<td>1,77</td>
<td>2,05</td>
<td>2,05</td>
<td>1,77</td>
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<tr>
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<td>19,52</td>
<td>0,03844</td>
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<td>2,39</td>
<td>2,54</td>
<td>2,54</td>
<td>2,24</td>
</tr>
<tr>
<td>2 - 2</td>
<td>1432</td>
<td>187,5</td>
<td>15,46</td>
<td>0,04574</td>
<td>1,81</td>
<td>2,66</td>
<td>2,79</td>
<td>2,79</td>
<td>2,70</td>
</tr>
<tr>
<td>2 - 3</td>
<td>1395</td>
<td>178,3</td>
<td>12,62</td>
<td>0,04400</td>
<td>2,30</td>
<td>2,50</td>
<td>2,58</td>
<td>2,58</td>
<td>2,46</td>
</tr>
<tr>
<td>2 - 4</td>
<td>1375</td>
<td>163,2</td>
<td>8,52</td>
<td>0,05737</td>
<td>1,30</td>
<td>1,97</td>
<td>2,10</td>
<td>2,10</td>
<td>2,02</td>
</tr>
<tr>
<td>3 - 1</td>
<td>1545</td>
<td>184,4</td>
<td>19,41</td>
<td>0,05634</td>
<td>5,00</td>
<td>2,99</td>
<td>3,20</td>
<td>3,20</td>
<td>2,98</td>
</tr>
<tr>
<td>3 - 2</td>
<td>1380</td>
<td>163,1</td>
<td>15,34</td>
<td>0,05326</td>
<td>4,39</td>
<td>2,90</td>
<td>3,05</td>
<td>3,05</td>
<td>2,81</td>
</tr>
<tr>
<td>3 - 3</td>
<td>1470</td>
<td>154,5</td>
<td>8,48</td>
<td>0,04329</td>
<td>2,33</td>
<td>2,02</td>
<td>2,18</td>
<td>2,18</td>
<td>2,04</td>
</tr>
</tbody>
</table>

Note: (1) $+ L_{CALC} = \sqrt{ \frac{2P(t - 3) \mu \rho \tau}{\mu \rho \tau}}$ (equation (36))

(11) $+ L_{EMP} = L_{CALC} - 1,029 R \frac{c}{r_0 \times 10^{-5}} + 0,0236 \times 10^{-4}$ (equation (41))
<table>
<thead>
<tr>
<th>Constant Head Drainage of Sludge at Initial Solids Concentration, $C_0 = 14$ g/l (20°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Loading Rate</strong></td>
</tr>
<tr>
<td>-------------------</td>
</tr>
<tr>
<td>Loading Rate at $c = 0.020$ g/ml, L Kg/m$^2$ (day)</td>
</tr>
<tr>
<td>Loading Rate at $c = 0.025$ g/ml, L Kg/m$^2$ (day)</td>
</tr>
<tr>
<td>Loading Rate at $c = 0.030$ g/ml, L Kg/m$^2$ (day)</td>
</tr>
<tr>
<td>Loading Rate at $c = 0.035$ g/ml, L Kg/m$^2$ (day)</td>
</tr>
<tr>
<td>Loading Rate at $c = 0.040$ g/ml, L Kg/m$^2$ (day)</td>
</tr>
<tr>
<td>Loading Rate at $c = 0.045$ g/ml, L Kg/m$^2$ (day)</td>
</tr>
<tr>
<td>Loading Rate at $c = 0.050$ g/ml, L Kg/m$^2$ (day)</td>
</tr>
</tbody>
</table>
### TABLE 17

**Calculation of Final Sludge Solids Concentration**

After Constant Head Filtration

(Data for Fig. 35)

<table>
<thead>
<tr>
<th>( c ) (g/l)</th>
<th>( C_{o} ) (g/l)</th>
<th>( C_{f} ) (g/l)</th>
<th>( C_{f} ) (g/l)</th>
<th>( C_{f} ) (g/l)</th>
<th>( C_{f} ) (g/l)</th>
<th>( C_{f} ) (g/l)</th>
<th>( C_{f} ) (g/l)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>11,81</td>
<td>11,99</td>
<td>12,21</td>
<td>12,48</td>
<td>12,84</td>
<td>13,31</td>
<td>13,97</td>
</tr>
<tr>
<td>12</td>
<td>14,70</td>
<td>14,98</td>
<td>15,52</td>
<td>15,76</td>
<td>16,33</td>
<td>17,10</td>
<td>18,20</td>
</tr>
<tr>
<td>14</td>
<td>17,81</td>
<td>18,27</td>
<td>18,74</td>
<td>19,40</td>
<td>20,27</td>
<td>21,47</td>
<td>23,24</td>
</tr>
<tr>
<td>16</td>
<td>21,18</td>
<td>21,77</td>
<td>22,51</td>
<td>23,46</td>
<td>24,75</td>
<td>26,56</td>
<td>29,33</td>
</tr>
<tr>
<td>18</td>
<td>24,84</td>
<td>25,65</td>
<td>26,68</td>
<td>28,03</td>
<td>29,88</td>
<td>32,57</td>
<td>36,83</td>
</tr>
<tr>
<td>20</td>
<td>28,81</td>
<td>29,91</td>
<td>31,32</td>
<td>33,20</td>
<td>35,83</td>
<td>39,76</td>
<td>46,30</td>
</tr>
<tr>
<td>22</td>
<td>33,15</td>
<td>34,62</td>
<td>36,52</td>
<td>39,10</td>
<td>42,80</td>
<td>48,54</td>
<td>58,64</td>
</tr>
<tr>
<td>24</td>
<td>37,92</td>
<td>39,84</td>
<td>42,36</td>
<td>45,90</td>
<td>51,08</td>
<td>59,47</td>
<td>75,38</td>
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<tr>
<td>26</td>
<td>43,16</td>
<td>45,67</td>
<td>49,05</td>
<td>53,82</td>
<td>61,08</td>
<td>73,47</td>
<td></td>
</tr>
</tbody>
</table>

\[
C_{f} = \frac{1}{C_{o}} - \frac{1}{\bar{C}_{c}}
\]

where \( \bar{C} = 1,006 \text{g/ml} \)

\[ A.54 \]
solids removals, c g/ml. The negligible effect of \( C_o \) on the loading rate curves, was shown by the plots for \( C_o = 10 \) g/l and \( C_o = 20 \) g/l, at \( c = 0.03 \) g/ml.

At incremental values of the concentration term c, the final filtered sludge solids concentration \( C_f \) has been calculated from equation (34),

\[
c = \frac{C_o \cdot C_f}{1000 \cdot \rho} (C_f - C_o)
\]

\( \rho \) has been taken as the mean sludge density = 1.006 g/ml; and different values of the initial sludge solids concentration \( C_o \) taken from \( C_o = 10 \) g/l to \( C_o = 26 \) g/l.

The calculations for \( C_f \) are contained in Table 17, and plotted in Fig. 35.

### 4.3.5. Calculation of Rate of Loading Based on Modified equation

\[
L = \sqrt{\frac{2P(1-S)}{\mu r_o}} \cdot \frac{c}{t^n}\quad [equation \ (37)].
\]

A plot of the filtrate volume \( v_t \) against time \( t \), for values of \( t \) between 1000 and 1800 minutes, is shown in Fig. 36. Experiment 1-2 was omitted because it terminated earlier at \( t = 1285 \) minutes.

The data were extracted from the experimental results in Tables 13 (a) to 13 (c) inclusive. (In this range of times, the parabolic curves are essentially linear). At constant time intervals, \( t = 1000 \) min; 1200 min; 1400 min; and 1600 min, respectively, the filtrate volume \( v_t \) was read from each curve in Fig. 36. The initial solids concentration, \( C_o \), was known by analysis. However, the final average solids concentration
C_f at each constant time t, is calculated as follows:-

If V ml = constant volume of sludge in the filter during constant
head filtration,

Initial mass of dry solids on filter = C_o V/1000 grams.

Since v_t ml of feed at concentration C_o g/l have been fed to the filter
at time t to maintain a constant head, mass of dry solids on filter at t
= C_o V/1000 + C_o v_t/1000 grams.

Average concentrations of solids on filter at t
= C_f = C_o (V/1000 + v_t/1000) x \frac{1000}{V}

i.e. C_f = C_o \left(1 + \frac{v_t}{V}\right) g/l \quad \text{-----}(a).

But c = \frac{C_f C_o}{1000 P_\beta (C_f - C_o)} \quad \text{-----}(b).

Substituting (a) in (b),

\[ c = \frac{C_o (V + v_t)}{1000 P_\beta v_t} \quad \text{-----}(c). \]

Equation (c) is used to evaluate the solids removal or concentration

The loading rate is given by \[ L = \frac{v_t c}{t \Delta} \]

Substituting the value of c obtained in equation (c), in this loading
rate equation

\[ L = \frac{C_o (V + v_t)}{1000 P_\beta t \Delta} \quad \text{-----}(d). \]

If h = depth of sludge in the filter above the sand surface in cm,
then V = hA.

Substituting in (d), the loading rate is calculated,

\[ L = \frac{C_o (h + v_t/A)}{1000 P_\beta t} \quad \text{-----}(e). \]

Table 18 shows the calculation of the parameters c and L at various
TABLE I8 (FIG. 37 or FIG. 36)

CALCULATION OF CONCENTRATION c & LOADING RATE L AT CONSTANT TIMES

<table>
<thead>
<tr>
<th>Initial Condition</th>
<th>Exp. No.</th>
<th>Sludge depth (cm)</th>
<th>Mean density (\rho) (g/ml)</th>
<th>(V_t) (ml)</th>
<th>(C_f) (g/l)</th>
<th>(c) (g/ml)</th>
<th>(L \times 10^{-4}) (g/cm²/min)</th>
<th>(V_t) (ml)</th>
<th>(C_f) (g/l)</th>
<th>(c) (g/ml)</th>
<th>(L \times 10^{-4}) (g/cm²/min)</th>
<th>(V_t) (ml)</th>
<th>(C_f) (g/l)</th>
<th>(c) (g/ml)</th>
<th>(L \times 10^{-4}) (g/cm²/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C - 0</td>
<td>1-1</td>
<td>17.50 1,010</td>
<td>175.0 21,087</td>
<td>0.04671</td>
<td>5.6</td>
<td>191.5 21,701</td>
<td>0.0439</td>
<td>3.11</td>
<td>207.2 22,286</td>
<td>0.04169</td>
<td>2.74</td>
<td>225.5 22,892</td>
<td>0.03970</td>
<td>2.46</td>
<td></td>
</tr>
<tr>
<td>14,573 g/l</td>
<td>1-3</td>
<td>10.60 1,010</td>
<td>146.5 23,525</td>
<td>0.03792</td>
<td>2.46</td>
<td>160.5 24,380</td>
<td>0.03567</td>
<td>2.13</td>
<td>174.8 25,254</td>
<td>0.03412</td>
<td>1.89</td>
<td>188.0 26,060</td>
<td>0.03273</td>
<td>1.70</td>
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</tr>
<tr>
<td>1-4</td>
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<td>125.0 25,896</td>
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<td>150.5 28,206</td>
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<td>162.0 29,246</td>
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<tr>
<td>C - 0</td>
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<td>4.64</td>
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<td>3.50</td>
<td>214.0 28,962</td>
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<td>3.14</td>
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<tr>
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<td>1.65</td>
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<td></td>
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<tr>
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<td>146.5 28,717</td>
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<td>4.94</td>
<td>160.3 29,451</td>
<td>0.07120</td>
<td>4.22</td>
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<td>0.06708</td>
<td>3.71</td>
<td>188.0 30,923</td>
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<td></td>
</tr>
<tr>
<td>20.230 g/l</td>
<td>3-2</td>
<td>15.50 1,016</td>
<td>136.5 30,336</td>
<td>0.06644</td>
<td>4.03</td>
<td>150.7 31,314</td>
<td>0.06212</td>
<td>3.46</td>
<td>163.7 32,210</td>
<td>0.05883</td>
<td>3.05</td>
<td>178.0 33,195</td>
<td>0.05575</td>
<td>2.75</td>
<td></td>
</tr>
<tr>
<td>3-3</td>
<td>6.75 1,016</td>
<td>125.0 38,156</td>
<td>0.04563</td>
<td>2.53</td>
<td>130.0 39,948</td>
<td>0.04327</td>
<td>2.21</td>
<td>151.2 41,767</td>
<td>0.04129</td>
<td>1.98</td>
<td>163.0 43,939</td>
<td>0.03979</td>
<td>1.60</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
constant times. Fig. 37 is a log-log plot of the loading rate $L$ against the concentration term $c$.

The best fitting line has a slope $m = 0.56$. This is the exponent of the parameter $c$.

Transposing equation (c) above,

$$v_t = \frac{hA}{1000 \cdot 5 \cdot cC_0} - 1$$

Thus, at constant values of solids removal, $c$ g/ml, the experimental filtrate volume $v_t$ was calculated. Referring to the curves in Fig. 36, the times $t$ were read corresponding to each value of $v_t$. The loading rate was again calculated from

$$L = \frac{v_t \cdot c}{tA}$$

The calculated values of $t$ and $L$ at constant $c$ are given in Table 19. Only results which are within the expected loading range, have been tabulated. A log-log plot of the loading rate $L$ against time $t$ is shown in Fig. 37. A series of nearly parallel straight lines resulted. The slopes of these lines were plotted on cumulative probability paper, as shown in Fig. 38. The mean slope, $n = 0.78$ is the exponent of the parameter $t$.

To obtain an empirical fit of the modified equation ignoring filter media resistance,

$$\sqrt{\frac{2 \cdot P \cdot (1-S)}{m}} \cdot \frac{c}{t^n}$$

[equation (37)]

the expression

$$L \cdot \sqrt{\frac{2 \cdot P \cdot (1-S)}{2 \cdot P \cdot (1-S)}} \cdot t^{0.78}$$

has been calculated [equation (378)]

from the observed values of $L$, $P$, and $t$ given in Table 15, and Table 20 was constructed.
The equation of each line in Fig. 23 is of the form
\[ \log K_D = n C_0 + \text{constant} \quad (18) \]
\((n = \text{slope of each line, and is constant}). \)

Equation (18) may be written as
\[ \log K_D = n C_0 + \log c' \quad (19), \]
where \(\log c' = \log K_D' \text{ at } C_0 = 0\).

Evaluating \(\log c' \) (and thus \(c'\)) from equation (19) at any selected
convenient value of \(C_0\) (e.g. \(C_0 = 2.00 \text{ g/l}\)), and for each value of \(Q\),
yet a further plot of \(\log c' \) vs. \(\log Q\) yielded a straight line, as shown
in Fig. 24. The equation of this line is given by
\[ c' = \text{constant}/Q^P, \]
where the constant is the intercept at \(Q = 1\), and the slope is given by \(-P\).

For the sludge under discussion in this work, it was found that
\[ C_0 = \frac{\text{constant}}{Q^{1.0914}} = 33760/Q^{1.0914} \quad (20). \]

Substitution of equation (20) in equation (19) yields
\[ \log K_D = \log [33760/Q^{1.0914}] - 1.179 C_0, \]
from which
\[ K_D = 33760/Q^{1.0914} e^{-2.7142 C_0} \quad (21). \]

The equations to the lines resulting from the plots in Figs. 20 and 21
(i.e. \(\log C_u \) versus \(\log D\)) have the general form,
\[ \log C_u = \text{constant } \log D + \log K_D \quad (22), \]
where the constant is the value of the slope.

Substituting the value of \(K_D\) from equation (21) in (22), the latter
equation becomes
\[ \log C_u = \text{constant } \log D + \log [33760/Q^{1.0914} e^{-2.7142 C_0}]. \]
Table 20: Empirical Loading Rate Equation ($R_m$ negligible)

(Data for Fig. 39)

<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>Time t (min)</th>
<th>Concentration c (g/ml)</th>
<th>Pressure P (g/cm²)</th>
<th>Observed Loading Rate $L_\text{OBS}$ (g/cm²/min) $\times 10^{-4}$</th>
<th>$\frac{\sqrt{w}}{\sqrt{2P(1-S)}} t^{0.78}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-1</td>
<td>1800</td>
<td>0.03834</td>
<td>19.19</td>
<td>2.26</td>
<td>1.496</td>
</tr>
<tr>
<td>1-2</td>
<td>1285</td>
<td>0.04367</td>
<td>15.25</td>
<td>2.39</td>
<td>1.301</td>
</tr>
<tr>
<td>1-3</td>
<td>1475</td>
<td>0.03429</td>
<td>12.32</td>
<td>1.90</td>
<td>1.239</td>
</tr>
<tr>
<td>1-4</td>
<td>1505</td>
<td>0.03826</td>
<td>8.84</td>
<td>1.77</td>
<td>1.311</td>
</tr>
<tr>
<td>2-1</td>
<td>1688</td>
<td>0.03844</td>
<td>19.52</td>
<td>2.39</td>
<td>1.386</td>
</tr>
<tr>
<td>2-2</td>
<td>1432</td>
<td>0.04574</td>
<td>15.46</td>
<td>2.66</td>
<td>1.567</td>
</tr>
<tr>
<td>2-3</td>
<td>1395</td>
<td>0.04400</td>
<td>12.62</td>
<td>2.50</td>
<td>1.544</td>
</tr>
<tr>
<td>2-4</td>
<td>1375</td>
<td>0.03737</td>
<td>8.52</td>
<td>1.97</td>
<td>1.375</td>
</tr>
<tr>
<td>3-1</td>
<td>1545</td>
<td>0.05634</td>
<td>19.41</td>
<td>2.99</td>
<td>1.727</td>
</tr>
<tr>
<td>3-2</td>
<td>1380</td>
<td>0.05326</td>
<td>15.34</td>
<td>2.80</td>
<td>1.605</td>
</tr>
<tr>
<td>3-3</td>
<td>1476</td>
<td>0.04329</td>
<td>8.48</td>
<td>2.02</td>
<td>1.488</td>
</tr>
</tbody>
</table>

Fig. 39 is a log-log plot of the function

$$f(c) = \sqrt{\frac{w}{2P(1-S)}} t^{0.78}$$

against the concentration term $c$. Fitting the best line to the scattered results by least squares, the equation of the line was evaluated:

$$f(c) = 8.55 c^{0.56}$$  (coefficient of correlation $= 0.83$).

From these calculations, the empirical fit to the data is given by
\[ L = 8.55 \sqrt{\frac{2p(1-S)}{\mu r_o}} \times \frac{c^{0.56}}{t^{0.78}} \]  

[equation (37c)]

Using this equation, Table 21 shows the loading rates calculated empirically, and compared with the observed loading rates:

**TABLE 21**

Comparison of Loading Rates Calculated Empirically, \((R_m \text{ negligible})\) and Observed Values.

<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>(L_{OBS} \times 10^{-4}) (g/cm(^2)/min)</th>
<th>(L_{EMP} \times 10^{-4}) (g/cm(^2)/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-1</td>
<td>2.26</td>
<td>2.08</td>
</tr>
<tr>
<td>1-2</td>
<td>2.39</td>
<td>2.73</td>
</tr>
<tr>
<td>1-3</td>
<td>1.90</td>
<td>1.99</td>
</tr>
<tr>
<td>1-4</td>
<td>1.77</td>
<td>1.86</td>
</tr>
<tr>
<td>2-1</td>
<td>2.24</td>
<td>2.23</td>
</tr>
<tr>
<td>2-2</td>
<td>2.66</td>
<td>2.55</td>
</tr>
<tr>
<td>2-3</td>
<td>2.50</td>
<td>2.41</td>
</tr>
<tr>
<td>2-4</td>
<td>1.97</td>
<td>1.97</td>
</tr>
<tr>
<td>3-1</td>
<td>2.99</td>
<td>2.95</td>
</tr>
<tr>
<td>3-2</td>
<td>2.80</td>
<td>2.89</td>
</tr>
<tr>
<td>3-3</td>
<td>2.02</td>
<td>2.00</td>
</tr>
</tbody>
</table>

The agreement was not as close as that obtained by using empirical equation (41), and reported in Table 15.
4.3.6. **Falling Head Drainage**

Measured volumes of sludges at different initial solids concentrations, $C_0$, were applied to the sand filter. At different time intervals $t$, the volume of filtrate collected, $v_t$, and the height of sludge above the sand layer were measured. These results are shown in Table 22. Values of $t/v_t$ plotted against $v_t$ are shown in Fig. 40. It is seen that the data are not as linear as in the case of constant head filtration (Figs. 28, 29, and 30). Drainage continued for approximately 2 days before slowing down to an extremely low rate. However, one experiment (No. 8) was allowed to proceed to its ultimate over a period of 7 days.

Because of the continuous compaction of the sludge during falling head drainage, homogenising of the final sludge for analytical sampling was unsatisfactory. Thus, the final average solids content, $C_f$, was calculated as follows, rather than determined analytically:

If $V$ ml = volume of sludge originally applied to the filter at a solids concentration $C_0$ g/l,

$$\text{Mass of dry solids on filter} = \frac{C_0 \times V}{1000} \text{ g}$$

Since no solids have been added to or taken from the filter, concentration of solids at time $t = C_f = \frac{C_0 V}{(V - v_t)}$ g/l \hspace{1cm} \text{(a)}

**Solids "removal" concentration term**

$$c = \frac{C_f - C_0}{1000 \frac{V}{v_t} (C_f - C_0)} \text{ g/ml}$$

Substituting for $C_f$ from equation (a),

$$c = \frac{C_0 V}{1000 \frac{V}{v_t}}$$ \hspace{1cm} \text{(b)}
# Table 22

**Falling Head Drainage at 20°C**

*(Figures 40, 42, 43 & 44)*

<table>
<thead>
<tr>
<th>EXPERIMENT</th>
<th>EXPERIMENT 5</th>
<th>EXPERIMENT 6</th>
<th>EXPERIMENT 7</th>
<th>EXPERIMENT 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>TIME (min)</td>
<td>SLUDGE HEAD</td>
<td>SLUDGE V (ml)</td>
<td>SLUDGE HEAD</td>
<td>SLUDGE V (ml)</td>
</tr>
<tr>
<td>t(min)</td>
<td>t(min)</td>
<td>t(min)</td>
<td>t(min)</td>
<td>t(min)</td>
</tr>
<tr>
<td>5</td>
<td>16.8</td>
<td>19.99</td>
<td>0.500</td>
<td>19.99</td>
</tr>
<tr>
<td>10</td>
<td>20.0</td>
<td>19.99</td>
<td>0.500</td>
<td>19.99</td>
</tr>
<tr>
<td>15</td>
<td>24.6</td>
<td>19.99</td>
<td>0.500</td>
<td>19.99</td>
</tr>
<tr>
<td>20</td>
<td>28.1</td>
<td>19.99</td>
<td>0.500</td>
<td>19.99</td>
</tr>
<tr>
<td>25</td>
<td>31.8</td>
<td>19.99</td>
<td>0.500</td>
<td>19.99</td>
</tr>
<tr>
<td>30</td>
<td>35.9</td>
<td>19.99</td>
<td>0.500</td>
<td>19.99</td>
</tr>
<tr>
<td>35</td>
<td>39.9</td>
<td>19.99</td>
<td>0.500</td>
<td>19.99</td>
</tr>
<tr>
<td>40</td>
<td>43.9</td>
<td>19.99</td>
<td>0.500</td>
<td>19.99</td>
</tr>
<tr>
<td>45</td>
<td>47.9</td>
<td>19.99</td>
<td>0.500</td>
<td>19.99</td>
</tr>
<tr>
<td>50</td>
<td>51.9</td>
<td>19.99</td>
<td>0.500</td>
<td>19.99</td>
</tr>
<tr>
<td>55</td>
<td>55.9</td>
<td>19.99</td>
<td>0.500</td>
<td>19.99</td>
</tr>
<tr>
<td>60</td>
<td>59.9</td>
<td>19.99</td>
<td>0.500</td>
<td>19.99</td>
</tr>
<tr>
<td>65</td>
<td>63.9</td>
<td>19.99</td>
<td>0.500</td>
<td>19.99</td>
</tr>
<tr>
<td>70</td>
<td>67.9</td>
<td>19.99</td>
<td>0.500</td>
<td>19.99</td>
</tr>
<tr>
<td>75</td>
<td>71.9</td>
<td>19.99</td>
<td>0.500</td>
<td>19.99</td>
</tr>
<tr>
<td>80</td>
<td>75.9</td>
<td>19.99</td>
<td>0.500</td>
<td>19.99</td>
</tr>
<tr>
<td>85</td>
<td>79.9</td>
<td>19.99</td>
<td>0.500</td>
<td>19.99</td>
</tr>
<tr>
<td>90</td>
<td>83.9</td>
<td>19.99</td>
<td>0.500</td>
<td>19.99</td>
</tr>
<tr>
<td>95</td>
<td>87.9</td>
<td>19.99</td>
<td>0.500</td>
<td>19.99</td>
</tr>
<tr>
<td>100</td>
<td>91.9</td>
<td>19.99</td>
<td>0.500</td>
<td>19.99</td>
</tr>
</tbody>
</table>

**Initial Solids Concentration, \( C_0 \) (g/l)**

**Final Solids Concentration, \( C_f \) (g/l)**

**Mean Sludge Density, \( \rho \) (g/ml)**

**Concentration Term, \( c \) (g/ml)**

**Initial Compacted Volume of Sludge, %**

**Final Compacted Volume of Sludge, % (after drainage ceases)**

\[ \frac{C_f}{C_0} = \frac{\rho V}{V_{\text{m}} - V} \]
Equations (a) and (b) have been used in Table 22 for the calculation of the variables $C_f$ and $c$ respectively.

From the slopes of the plots in Fig. 40, the apparent specific resistance of each sludge was calculated. Pressure head terms were calculated for both the arithmetic-mean depth of the sludge, and the log-mean depth of sludge.

The latter is given by

$$H_{LM} = \frac{(H_0 - H_f)}{2.303 \log \frac{H_0}{H_f}}$$

[equation (42)]

The specific resistance $r$ was calculated at both the arithmetic-mean pressure, and the log-mean pressure (Exp. 8 was calculated at $t = 4740$ minutes, where the slope of the plot $t/v_t$ against $v_t$ in Fig. 40, changed).

The results are given in Table 23.

Fig. 41 is a log-log plot of the specific resistance against pressure. The data were seen to be scattered. Least-squares best-fit lines drawn for the mean-head calculations and the log-mean head, showed hardly any correlation in the former case (correlation coefficient = 0.03), and only 0.32 coefficient of correlation in the latter case. The results for the arithmetic-mean head were thus abandoned. The intercept and slope of the log-mean plot yielded the initial specific resistance, $r_o = 0.47 \times 10^6$, and compressibility $s = 0.623$, respectively.

These results are shown in Table 23.

Table 24 compares the loading rate observed ($L = \frac{V_t C}{tA}$), with that calculated at the log-mean pressure (ignoring filter media resistance). The experimentally determined initial sludge resistance and compressibility have been used ($r_o = 0.47 \times 10^6; s = 0.623$).
<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>Slope of $\frac{t}{V}$ vs $\ln \frac{R}{p}$ (min/cm$^2$)</th>
<th>Concentration Term, $c$ (g/ml)</th>
<th>Mean Sludge Density, $\rho$ (g/ml)</th>
<th>Initial Sludge Depth, $H_i$ (cm)</th>
<th>Final Sludge Depth, $H_f$ (cm)</th>
<th>Mean Sludge Density, $\rho$ (g/ml)</th>
<th>Log Mean Depth, $H_L$ (cm)</th>
<th>Mean Pressure, $P$ (g/cm$^2$)</th>
<th>Log Mean Pressure, $P_L$ (g/cm$^2$)</th>
<th>Specific Resistance, $\frac{R}{\rho}$ (sec)(min)/g</th>
<th>Specific Resistance, $\frac{R}{\rho}$ (sec)(min)/g</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.0289</td>
<td>0.01372</td>
<td>1.015</td>
<td>19.66</td>
<td>4.87</td>
<td>12.77</td>
<td>10.60</td>
<td>12.45</td>
<td>10.76</td>
<td>2.59</td>
<td>2.24</td>
</tr>
<tr>
<td>5</td>
<td>0.0310</td>
<td>0.02052</td>
<td>1.012</td>
<td>19.68</td>
<td>8.72</td>
<td>14.23</td>
<td>13.46</td>
<td>14.40</td>
<td>13.62</td>
<td>2.17</td>
<td>2.05</td>
</tr>
<tr>
<td>6</td>
<td>0.0265</td>
<td>0.03680</td>
<td>1.016</td>
<td>19.89</td>
<td>6.47</td>
<td>14.18</td>
<td>11.95</td>
<td>14.41</td>
<td>12.14</td>
<td>2.76</td>
<td>2.33</td>
</tr>
<tr>
<td>7</td>
<td>0.0465</td>
<td>0.03362</td>
<td>1.018</td>
<td>19.36</td>
<td>7.79</td>
<td>13.58</td>
<td>12.70</td>
<td>13.82</td>
<td>12.93</td>
<td>1.89</td>
<td>1.76</td>
</tr>
<tr>
<td>8</td>
<td>0.0870</td>
<td>0.03885</td>
<td>1.017</td>
<td>20.20</td>
<td>9.35</td>
<td>14.78</td>
<td>14.08</td>
<td>15.03</td>
<td>14.32</td>
<td>3.36</td>
<td>3.20</td>
</tr>
</tbody>
</table>
### TABLE 24

Comparison of Observed and Calculated Loading Rates During Falling Head Drainage

<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>Time t (min)</th>
<th>Concentration c (g/ml)</th>
<th>Log-Mean Pressure (v_t c) (g/cm²)</th>
<th>(L_{OBS} = \frac{v_t c}{tA}) (g/min/cm² x 10⁻⁵)</th>
<th>(L_{CALC} = \frac{2P (1-0.623)}{cu 0.47 x 10^6 t})</th>
<th>(L_{OBS}^{CALC} = )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>2730</td>
<td>0.01372</td>
<td>10.76</td>
<td>6.75</td>
<td>7.18</td>
<td>0.94</td>
</tr>
<tr>
<td>5</td>
<td>1610</td>
<td>0.02032</td>
<td>13.62</td>
<td>13.06</td>
<td>11.91</td>
<td>1.10</td>
</tr>
<tr>
<td>6</td>
<td>2985</td>
<td>0.01880</td>
<td>12.14</td>
<td>7.96</td>
<td>8.22</td>
<td>0.97</td>
</tr>
<tr>
<td>7</td>
<td>2582</td>
<td>0.03362</td>
<td>12.93</td>
<td>14.19</td>
<td>11.98</td>
<td>1.18</td>
</tr>
<tr>
<td>8</td>
<td>4740</td>
<td>0.03885</td>
<td>14.32</td>
<td>8.46</td>
<td>9.67</td>
<td>0.87</td>
</tr>
</tbody>
</table>

The agreement between calculated loadings and observed values is not as close as in the case of constant head filtration (Table 15).

#### 4.3.7 Falling Head Drainage Versus Constant Head Filtration

The effect of feeding the same sludge to constant head or falling head filters, is shown in Table 25. At the same solids removal \(c = 0.035\) g/ml and at the same time; mean pressure; and depth of sludge (log-mean values in the case of the falling head filter), the loading rates have been calculated using equation (41) (Section 4.3.4) in the case of constant head filtration, and from

\[ L = \sqrt{\frac{2P (1-S)}{u t_o t}} \]

in the case of falling head drainage:
TABLE 25
Comparison of Falling Head Drainage and Constant Head Drainage Loading Rates. (Part data for Fig. 34)

<table>
<thead>
<tr>
<th>Sludge Depth h(cm)</th>
<th>Head Pressure $P$ (g/cm$^2$)</th>
<th>Loading Rate, $L_{\text{const.head}}$ (kg/m$^2$/d)</th>
<th>Loading Rate, $L_{\text{fall.head}}$ (kg/m$^2$/d)</th>
<th>$\frac{L_{\text{constant head}}}{L_{\text{falling head}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>31.6</td>
<td>32.04</td>
<td>4.22</td>
<td>2.64</td>
</tr>
<tr>
<td>25</td>
<td>26.6</td>
<td>26.97</td>
<td>3.95</td>
<td>2.53</td>
</tr>
<tr>
<td>20</td>
<td>21.6</td>
<td>21.90</td>
<td>3.64</td>
<td>2.41</td>
</tr>
<tr>
<td>15</td>
<td>16.6</td>
<td>16.83</td>
<td>3.28</td>
<td>2.26</td>
</tr>
<tr>
<td>10</td>
<td>11.6</td>
<td>11.76</td>
<td>2.83</td>
<td>2.05</td>
</tr>
</tbody>
</table>

(c = 0.035 g/ml; t = 1440 min.)

The plot of loading rate against sludge depth for falling head drainage at $c = 0.035$ g/ml, is included in Fig. 34. Under similar conditions to produce similar dewatering of sludge, constant-head filtration has a greater loading rate on a given filter bed than falling head drainage.

4.3.8 Empirical Relationship between $t/v_t$ and $v_t$ in Falling Head Drainage

Figs. 42 and 43 are log-log plots of the function $t/v_t$ against $v_t$ for falling head drainage. The data were taken from Table 22. Two distinct lines were obtained in each case, the slope changing within approximately 1 day of the commencement of drainage. In the case of prolonged drainage (Exp. 8, Fig. 43), there was a further distinct change in the slope after 5 days.
Reading the co-ordinates of the intersection \((t/v, v)\) of the plots, the time at which the change occurred was calculated from \(t/v \times v\). The slopes and intercepts of the two lines are tabulated in Table 26:

<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>Initial Solids Conc., (C_0) (g/l)</th>
<th>(t/v) at Intersection (min/cm³)</th>
<th>(v) at Intersection (ml)</th>
<th>Drainage Time at Intersection (t_i) (min)</th>
<th>SLOPE</th>
<th>INTERCEPT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(&lt; t_i)</td>
<td>(&gt; t_i)</td>
</tr>
<tr>
<td>4</td>
<td>11,19</td>
<td>2,5</td>
<td>97</td>
<td>243</td>
<td>0,66</td>
<td>1,12</td>
</tr>
<tr>
<td>5</td>
<td>12,63</td>
<td>2,9</td>
<td>104</td>
<td>302</td>
<td>0,72</td>
<td>1,12</td>
</tr>
<tr>
<td>6</td>
<td>14,29</td>
<td>4,4</td>
<td>135</td>
<td>594</td>
<td>0,78</td>
<td>1,19</td>
</tr>
<tr>
<td>7</td>
<td>22,38</td>
<td>4,9</td>
<td>106</td>
<td>519</td>
<td>0,75</td>
<td>1,05</td>
</tr>
<tr>
<td>8</td>
<td>23,11</td>
<td>13,0</td>
<td>152</td>
<td>1976</td>
<td>0,77</td>
<td>1,10</td>
</tr>
</tbody>
</table>

Table 26 shows that the slopes of the plots are similar in both phases of the drainage. However, the time at which the change occurs is a complex function of both the initial solids concentration, and the change in the permeability characteristics of the compacting sludge.

The intercept is similarly a complex function of the initial solids concentration, and permeability of the sludge.

Each equation is in the form of functions given respectively by

\[
t/v = k_1 v^{0.75}, \quad \text{whence } t = k_1 v^{1.75}
\]

and

\[
t/v = k_2 v^{1.12}, \quad \text{whence } t = k_2 v^{1.12}
\]
Both of these equations are of little use in calculating the loading rate, because the time for the change in permeability and slope to occur, is not predicted.

4.4. **Sludge Drying.**

The drying which occurs throughout the constant head and falling head filtrations, has not been considered. A large mass of drained sludge was prepared in several batches, by applying settled sludge to the filtration apparatus (Fig. 26). Drainage was essentially complete in 3 days. The composite sample of drained sludge was analysed and found to contain 12.50 g moisture/g bone dry solids (i.e. 92.59% water; 74.07 g solids/kg).

To establish drying curves and investigate the effect of drying bed loadings, aliquots of the drained sludge were poured into a set of tared flat-bottomed stainless steel tubes of varying diameters (4.00 cm; 3.60 cm; 4.87 cm; 2.39 cm; and 2.66 cm respectively); and to varying depths (1.00 cm; 3.20 cm; 4.10 cm; 6.35 cm; and 7.90 cm respectively). This gave a range of loadings of 0.533 to 5.776 kg/m² (Table 27).

Prior to commencing the drying experiments, the bulk mass of drained sludge was thermally equilibrated in a closed bottle placed overnight in a laboratory with controlled atmosphere (65% Relative Humidity ± 2%; 21°C. ± 1°C.)

The 5 drying beds were mounted together in a plastic stand.

The latter consisted of a panel drilled with holes slightly larger in diameter than that of the outside diameter of the tubes, and supported about 1 cm above a solid panel bottom.

Throughout the 15 day experimental work, the drying beds were weighed
at intervals in the controlled atmosphere laboratory.

The air velocity was constant at 0.8 m/min.

4.4.1. **Determination of Drying Curves and "Critical" Moisture Content at 65% R.H. and 21°C.**

Results are recorded in Tables 28-32 inclusive.

Instantaneous moisture contents were calculated in the following manner:

If \( M \) = initial mass of wet sludge (g),

\[ M_d = \text{bone-dry mass of solids (g)} \]

\[ m = \text{mass of moisture (g) evaporated up to time } t \ (\text{hours}) \]

\[ A = \text{surface area of drying bed (cm}^2\text{)} \]

Initial moisture content, \( W_0 = \frac{(M-M_d)}{M_d} \cdot \frac{gH_2O}{g \text{ dry solids}} \)

Instantaneous moisture content, \( W = \frac{(M-M_d-m)}{M_d} \cdot \frac{gH_2O}{g \text{ dry solids}} \)

Instantaneous average rate of evaporation,

\[ R = \frac{m}{At} \times 10^4 \text{ g/m}^2/\text{hr.} \]

If \( m_1 \) = mass of moisture (g) evaporated up to time \( t_1 \) (hours), and

\( m_2 \) = mass of moisture (g) evaporated up to time \( t_2 \) (hours),

average drying rate during period \( (t_2-t_1) \) =

\[ \bar{R} = \frac{(m_2-m_1) \times 10^4}{(t_2-t_1) A} \text{ g/m}^2/\text{hr.} \]

If \( W_1 \) = moisture content (gH_2O/g dry solids) at time \( t_1 \) (hr.), and

\( W_2 \) = moisture content (gH_2O/g dry solids) at time \( t_2 \) (hr.),

Average moisture content in period \( (t_2-t_1) \) =

\[ \bar{W} = \frac{W_1 + W_2}{2} \text{ (g/g)} \]

Fig. 45 is a plot of the drying curves obtained during the first 200 hours (mass of moisture evaporated, \( m \), against time, \( t \)). Fig. 46 is a plot
### TABLE 28

<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Sludge Depth (cm)</td>
<td>1.0</td>
<td>3.2</td>
<td>4.1</td>
<td>6.25</td>
<td>7.9</td>
</tr>
<tr>
<td>Initial Surface Area, A (cm²)</td>
<td>12.56</td>
<td>10.17</td>
<td>18.63</td>
<td>4.49</td>
<td>5.56</td>
</tr>
<tr>
<td>Initial Mass of wet Sludge, M (g)</td>
<td>9.0360</td>
<td>21.0615</td>
<td>45.2460</td>
<td>27.5934</td>
<td>43.3539</td>
</tr>
<tr>
<td>Mass of Bone Dry Solids, M_d (g)</td>
<td>0.6694</td>
<td>1.5593</td>
<td>3.3524</td>
<td>2.0439</td>
<td>3.2112</td>
</tr>
<tr>
<td>Loading, D/A = [-(\cdot M_d/A) \times 10^6 \text{g dry solids/cm}^2]</td>
<td>533</td>
<td>1535</td>
<td>1799</td>
<td>4565</td>
<td>5776</td>
</tr>
</tbody>
</table>

Initial Moisture Content = 92.59 g/100 g sludge
Initial Moisture W_o = 12.50 g H₂O/g dry solids
Initial Solids Content = 74.07 g/kg sludge.

### TABLE 29 (FIGS. 45(1) & 46(1))

<table>
<thead>
<tr>
<th>Time (hours)</th>
<th>Mass H₂O Evaporated (g)</th>
<th>Drying Rate, R m x 10⁴ (g/m²/hr)</th>
<th>Moisture Content, W (g H₂O/g dry)</th>
<th>Average Drying Rate, R (g/m²/hr)</th>
<th>Average Moisture Content, W (g H₂O/g dry)</th>
</tr>
</thead>
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<td>0</td>
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<td>12.06</td>
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<td>11.36</td>
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<td>114.4</td>
<td>7.27</td>
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<td>124.8</td>
<td>6.43</td>
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<td>127.8</td>
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<td>4.23</td>
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</tr>
<tr>
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<td>72.1</td>
<td>1.61</td>
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<td>2.5</td>
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<td>0</td>
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</table>

\[
\frac{t_2 - t_1}{R} \times 10^4 = \frac{(m_2 - m_1)}{t_2 - t_1} \cdot A
\]

\[
W = \frac{(W_1 + W_2)}{2}
\]
### TABLE 29
SLUDGE DRYING AT CONSTANT AMBIENT CONDITIONS (FIGS. 45(2) & 46(2)).

Exp. 2: Loading, \(D/A = 1533 \text{ g/m}^2\); Area, \(A = 10,17 \text{ cm}^2\)

<table>
<thead>
<tr>
<th>Time (hours)</th>
<th>Mass (\text{H}_2\text{O} ) Evaporated (m ) (g)</th>
<th>Drying Rate ( R_m \times 10^4 ) (g/m²/hr.)</th>
<th>Moisture Content, ( W ) (g/H₂O/g dry)</th>
<th>Average Drying Rate, ( R + ) (g/m²/hr.)</th>
<th>Average Moisture Content ( W + ) (g/H₂O/g dry)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>12.50</td>
<td>133.9</td>
<td>12.32</td>
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<td>52.25</td>
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</tbody>
</table>

\[
R = \frac{(m_2 - m_1) \times 10^4}{(t_2 - t_1)A}
\]

\[
W = \frac{(W_f + W_s)}{2}
\]
<table>
<thead>
<tr>
<th>Time (hours)</th>
<th>Mass H₂O Evaporated (g)</th>
<th>Drying Rate, $R \times 10^4$ (g/m²/hr)</th>
<th>Average Drying Rate, $R_+ (g/m²/hr)$</th>
<th>Average Moisture Content, $\bar{W} + \bar{W}$ (g H₂O/g dry)</th>
</tr>
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<tbody>
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<td>0</td>
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</table>

$$\bar{R} = \frac{(m_2 - m_1)}{(t_2 - t_1)} \times 10^4$$

$$\bar{W} = \frac{(\bar{W}_1 + \bar{W}_2)}{2}$$

**Note:**

- Table 31: Sludge Drying at Constant Ambient Conditions (Figs. 45(4) & 46(4)).
- Exp. 4: Loading, $O/A = 4552$ g/m²; Area, $h = 4.49$ cm².
### SLUDGE DRYING AT CONSTANT AMBIENT CONDITIONS

(FIGS. 45(5) & 46(5))

**Table 3:**

<table>
<thead>
<tr>
<th>Time (hours)</th>
<th>Mass H₂O Evaporated (mg)</th>
<th>Drying Rate R = ( \frac{m}{tA} \times 10^4 ) (g/m²/hr)</th>
<th>Moisture Content, W (gH₂O/g dry)</th>
<th>Average Drying Rate R (g/m²/hr)</th>
<th>Average Moisture Content, W (gH₂O/g dry)</th>
</tr>
</thead>
<tbody>
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<td>0</td>
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<tr>
<td>49.00</td>
<td>0.6994</td>
<td>28.0</td>
<td>12.29</td>
<td>30.0</td>
<td>12.28</td>
</tr>
<tr>
<td>52.25</td>
<td>0.7456</td>
<td>28.4</td>
<td>12.27</td>
<td>18.0</td>
<td>12.25</td>
</tr>
<tr>
<td>69.75</td>
<td>0.9192</td>
<td>26.2</td>
<td>12.22</td>
<td>23.7</td>
<td>12.17</td>
</tr>
<tr>
<td>95.50</td>
<td>1.2583</td>
<td>26.2</td>
<td>12.11</td>
<td>25.2</td>
<td>12.06</td>
</tr>
<tr>
<td>117.25</td>
<td>1.5629</td>
<td>26.6</td>
<td>12.01</td>
<td>26.8</td>
<td>12.00</td>
</tr>
<tr>
<td>124.25</td>
<td>1.6749</td>
<td>26.8</td>
<td>11.98</td>
<td>25.2</td>
<td>11.80</td>
</tr>
<tr>
<td>165.50</td>
<td>2.2075</td>
<td>26.6</td>
<td>11.81</td>
<td>25.1</td>
<td>11.80</td>
</tr>
<tr>
<td>172.50</td>
<td>2.3051</td>
<td>26.6</td>
<td>11.78</td>
<td>17.4</td>
<td>11.76</td>
</tr>
<tr>
<td>189.25</td>
<td>2.4670</td>
<td>26.0</td>
<td>11.75</td>
<td>28.2</td>
<td>11.72</td>
</tr>
<tr>
<td>196.25</td>
<td>2.5769</td>
<td>26.2</td>
<td>11.70</td>
<td>28.5</td>
<td>11.72</td>
</tr>
<tr>
<td>239.25</td>
<td>3.0199</td>
<td>25.2</td>
<td>11.56</td>
<td>17.0</td>
<td>11.63</td>
</tr>
<tr>
<td>294.00</td>
<td>3.3563</td>
<td>24.0</td>
<td>11.40</td>
<td>16.4</td>
<td>11.48</td>
</tr>
<tr>
<td>316.00</td>
<td>3.7374</td>
<td>23.6</td>
<td>11.34</td>
<td>15.0</td>
<td>11.37</td>
</tr>
<tr>
<td>340.00</td>
<td>3.9569</td>
<td>23.0</td>
<td>11.28</td>
<td>11.8</td>
<td>11.31</td>
</tr>
<tr>
<td>358.00</td>
<td>4.0552</td>
<td>22.6</td>
<td>11.24</td>
<td>11.26</td>
<td></td>
</tr>
</tbody>
</table>

\[
+ \frac{R}{R} = \frac{(w_2 - w_1)}{(t_2 - t_1) \cdot A} \quad \frac{w}{W} = \frac{(w_1 + w_2)}{2}
\]
of the rate of evaporation $R$ against moisture content $W$. Data for both of these sets of plots have been taken from Tables 28-32 inclusive. In Fig. 46, the intersection of the sloping lines and the horizontal "constant"-rate lines, was accepted to be the points at which the "critical" moisture content $W_c$ had been reached. Straight lines joining the latter points with the equilibrium moisture content $E$ approximate the falling rate-of-drying period. Average drying rates $\bar{R}$ at average moisture contents $\bar{W}$, were also plotted, and show good co-linearity with the falling rate period at low loadings [Plot 1 in Fig. 46]. At higher loadings, there was an increasing divergence [Plots 2-5 in Fig. 46].

4.4.2. Determination of The Falling Rate Drying Curves At 65% R.H. and 21°C.

The "critical" moisture contents $W_c$ read from Fig. 46 enabled the estimation of the drying times required to reach these values, as follows:

- Mass of Residual Moisture at "critical" moisture content $= W_c M_d$.
- Mass of Moisture evaporated in the "constant" rate period to the time $t_c$ of the "critical" moisture content $= M - M_d - W M_c = m$.

(symbols defined as in section 4.4.1.)

Referring to the drying curves in Fig. 45, the time to evaporate the $m$ grams of moisture was read. These values yielded the $t_c$ times. The $t_c$ times occurred at the end of the "constant" rate period, and were also the zero times for the commencement of the falling rate periods. Subtracting corresponding $t_c$ values from the total times ($t \geq t_c$) in Tables 28-32 inclusive, the nett times $t_f$ in the falling rate period were obtained. Simultaneously, subtracting the equilibrium moisture content, $E = 0.10 \text{ g/g}$, from the total moisture contents $W$ at these corresponding times, the free moisture contents $W_f$ were obtained. These calculations are tabulated in Table 33, and plotted on semi-log paper in Fig. 49.
TABLE 33
SLUDGE DRYING IN FALLING RATE PERIOD AT CONSTANT AMBIENT CONDITIONS (FIG. 49)

<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>'Critical' Moisture Content, $W_c$ (gH₂O/g dry)</th>
<th>Mass of Dry Solids, $M_d$ (g)</th>
<th>Mass of Residual Moisture at 'Critical' Moisture Content, $W_{d-c}$ (g)</th>
<th>Initial Mass of Moisture Evaporated at 'Critical' Moisture Content, $W = M - W_c$ (g)</th>
<th>Time at 'Critical' Moisture Content, $t_c$ (Fig. 45) (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.85</td>
<td>0.6984</td>
<td>1.91</td>
<td>8.37</td>
<td>44.5</td>
</tr>
<tr>
<td>2</td>
<td>4.50</td>
<td>1.5593</td>
<td>7.02</td>
<td>12.49</td>
<td>107.5</td>
</tr>
<tr>
<td>3</td>
<td>7.40</td>
<td>3.3524</td>
<td>24.80</td>
<td>41.91</td>
<td>108.0</td>
</tr>
<tr>
<td>4</td>
<td>11.62</td>
<td>2.0458</td>
<td>23.75</td>
<td>25.55</td>
<td>94.5</td>
</tr>
<tr>
<td>5</td>
<td>11.90</td>
<td>3.2112</td>
<td>38.20</td>
<td>40.14</td>
<td>156.5</td>
</tr>
</tbody>
</table>

EQUILIBRIUM MOISTURE CONTENT AT 65% R.H. & 21°C, $E = 0.10$ g H₂O/g dry solids.

<table>
<thead>
<tr>
<th>$D/A = 533$ g/m²</th>
<th>$D/A = 1533$ g/m²</th>
<th>$D/A = 1799$ g/m²</th>
<th>$D/A = 4552$ g/m²</th>
<th>$D/A = 5776$ g/m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time in Falling Rate Period</td>
<td>Free Moisture Content $W_f = W_E$ (g)</td>
<td>Time in Falling Rate Period</td>
<td>Free Moisture Content $W_f = W_E$ (g)</td>
<td>Time in Falling Rate Period</td>
</tr>
<tr>
<td>$t_f - t_c$ (hr)</td>
<td>$t_f - t_c$ (hr)</td>
<td>$t_f - t_c$ (hr)</td>
<td>$t_f - t_c$ (hr)</td>
<td>$t_f - t_c$ (hr)</td>
</tr>
<tr>
<td>0</td>
<td>2.75</td>
<td>0</td>
<td>4.40</td>
<td>7.30</td>
</tr>
<tr>
<td>0.75</td>
<td>2.33</td>
<td>9.75</td>
<td>3.74</td>
<td>12.25</td>
</tr>
<tr>
<td>4.50</td>
<td>1.73</td>
<td>13.50</td>
<td>3.56</td>
<td>16.00</td>
</tr>
<tr>
<td>7.75</td>
<td>1.29</td>
<td>16.75</td>
<td>3.41</td>
<td>19.25</td>
</tr>
<tr>
<td>25.25</td>
<td>0.12</td>
<td>58.00</td>
<td>1.68</td>
<td>60.50</td>
</tr>
<tr>
<td>40.50</td>
<td>0</td>
<td>65.00</td>
<td>1.45</td>
<td>67.50</td>
</tr>
<tr>
<td>81.75</td>
<td>0.78</td>
<td>91.25</td>
<td>1.29</td>
<td>101.75</td>
</tr>
<tr>
<td>88.75</td>
<td>0.13</td>
<td>154.25</td>
<td>0.78</td>
<td>101.75</td>
</tr>
<tr>
<td>131.75</td>
<td>0.04</td>
<td>199.00</td>
<td>0.01</td>
<td>144.75</td>
</tr>
<tr>
<td>186.50</td>
<td>0.01</td>
<td>211.00</td>
<td>0.21</td>
<td>199.50</td>
</tr>
<tr>
<td>208.50</td>
<td>0.01</td>
<td>235.00</td>
<td>0.36</td>
<td>221.50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>253.00</td>
<td>0.93</td>
<td>245.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>263.00</td>
<td>9.68</td>
<td>263.50</td>
</tr>
</tbody>
</table>
The slopes of these lines gave the falling rate constants, \( k_f \); the intercepts are the "critical" moisture contents, \( W_c \).

4.4.3. Effect of Bed Loading on Critical Moisture Content, \( W_c \), and Constant Drying Rate, \( R_c \).

Table 34 tabulates the co-ordinates at the "critical" moisture contents for the different loadings. These values were obtained from Fig. 46.

**TABLE 34**

Co-ordinates of the Intersection Points of "Constant" and Falling Rate Periods. (Data for Figs. 47 and 48, ex Fig. 46)

<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>Loading, ((D/A \times 10^{-3}) ) (kg/m²)</th>
<th>&quot;Critical&quot; Moisture Content, ( W_c ) (g/g)</th>
<th>&quot;Constant&quot; Drying Rate, ( R_c ) (g/m²/hr.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0,533</td>
<td>2,85</td>
<td>126,8</td>
</tr>
<tr>
<td>2</td>
<td>1,533</td>
<td>4,50</td>
<td>120,0</td>
</tr>
<tr>
<td>3</td>
<td>1,799</td>
<td>7,40</td>
<td>93,7</td>
</tr>
<tr>
<td>4</td>
<td>4,552</td>
<td>11,62</td>
<td>50,0</td>
</tr>
<tr>
<td>5</td>
<td>5,776</td>
<td>11,90</td>
<td>26,8</td>
</tr>
</tbody>
</table>

A log-log plot of the "critical" moisture content \( W_c \) against the bed loading \( D/A \) is shown in Fig. 47.

The equation of the resulting line was calculated to be

\[
W_c = 0,045 \cdot (D/A)^{0,655} \quad \text{[equation (53)]}
\]

A semi-log plot of the "constant" drying rate \( R_c \) against the bed loading \( D/A \) is shown in Fig. 48.

The equation of this line gave the relationship
\[ R_c = 158 e^{-0.00012 \frac{D}{A}} \]  
[equation (54)].

The intercept of the latter plot at zero loading 
\( (R_c = 158 \text{ g/m}^2/\text{hr}) \) corresponded to the constant rate evaporation of 
moisture from the filtrate at 65\% R.H; 21^\circ\text{C}; and 0.8 m/min air flow 
velocity.

4.4.4. Effect of Bed Loading on the Falling Rate Period Constant, \( K_f \)

The values of the falling rate constant \( K_f \) [equation (52A)] were 
calculated from the slopes of the lines in Fig. 49, as follows:

**TABLE 34A:**

Variation of Falling Rate Constant \( K_f \) with loading

(Data ex Fig. 49 for Fig. 50)

<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>Slope</th>
<th>Falling Rate Constant, ( K_f ) (=(-2.303 \times \text{slope})</th>
<th>Loading ( \frac{D}{A} ) kg/m(^2) x10(^{-3})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.0432</td>
<td>0.0995</td>
<td>0.533</td>
</tr>
<tr>
<td>2</td>
<td>-0.00743</td>
<td>0.0171</td>
<td>1.533</td>
</tr>
<tr>
<td>3</td>
<td>-0.00266</td>
<td>0.00612</td>
<td>1.799</td>
</tr>
<tr>
<td>4</td>
<td>-0.000223</td>
<td>0.000515</td>
<td>4.552</td>
</tr>
<tr>
<td>5</td>
<td>-0.000117</td>
<td>0.00027</td>
<td>5.776</td>
</tr>
</tbody>
</table>

Fig. 50 shows a log-log plot of \( K_f \) against the loading \( \frac{D}{A} \), 
from which the following relationship was established:

\[ K_f = 2.78 \times 10^6 \left( \frac{D}{A} \right)^{-2.67} \]  
[equation (55)].
4.4.5. Calculation of Drying Bed Loadings At 65% R.H. and 21°C.

Selecting a range of bed loadings \( D/A = 2000 - 4000 \, g/m^2 \), [corresponding to the loading rates expected (Section 4.3.4)], values of \( W_c, R_c \) and \( K_f \) were calculated from the experimental relationships obtained in sections 4.4.3 and 4.4.4 respectively. Table 35 shows the results of these calculations:

**TABLE 35**

Drying Bed Loading Calculations at 65% R.H. and 21°C.

<table>
<thead>
<tr>
<th>Bed Loading D/A (g/m²)</th>
<th>&quot;Critical&quot; Moisture Content ( W_c ) (g/g)</th>
<th>&quot;Constant&quot; Rate of Drying ( R_c ) (g/m²/hr)</th>
<th>Falling Rate Period Constant ( K_f )</th>
<th>Drying Time * To Critical Moisture Content, ( t_c ) (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>6.54</td>
<td>91</td>
<td>0.0043</td>
<td>153</td>
</tr>
<tr>
<td>2500</td>
<td>7.56</td>
<td>79</td>
<td>0.0024</td>
<td>185</td>
</tr>
<tr>
<td>3000</td>
<td>8.53</td>
<td>69</td>
<td>0.0015</td>
<td>216</td>
</tr>
<tr>
<td>3500</td>
<td>9.43</td>
<td>60</td>
<td>0.00096</td>
<td>237</td>
</tr>
<tr>
<td>4000</td>
<td>10.29</td>
<td>52</td>
<td>0.00067</td>
<td>247</td>
</tr>
</tbody>
</table>

* Assuming a wet sludge initially at \( W_0 = 13.5 \, g \, H_2O/g \, dry \, solids \) (93.1% mass/mass moisture content).

The time to arrive at the critical moisture content at the end of the constant rate period, was calculated as follows:

\[
t_c = \frac{(W_0 - W_c)}{R_c \, D/A}
\]  
[equation (56)]

The drying time required to reach a desired moisture content \( W \), when \( W < W_c \), was obtained from this equation, and is included in Table 35.

Drying times to reach desired moisture contents which are less than the "critical" moisture contents, were calculated from the sum of the...
TABLE 36 (FIG. 51)

Drying Times for Sludge Initially at Moisture Content \( W = 13.5 \text{ g H}_2\text{O/ g dry solids; 65\% R.H. & 27^\circ C.} \)

<table>
<thead>
<tr>
<th>Moisture Content ( W \text{ g/g} )</th>
<th>Drying in 'Constant Rate Period', ( t_c )</th>
<th>Drying in 'Falling Rate Period', ( t_f )</th>
<th>Time to 'Critical' Moisture Content ( W \text{ (hrs)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W \text{ g/g} )</td>
<td>Time (hrs)</td>
<td>Time (hrs)</td>
<td>Time (hrs)</td>
</tr>
<tr>
<td>2.0</td>
<td>77</td>
<td>99</td>
<td>121</td>
</tr>
<tr>
<td>2.5</td>
<td>111</td>
<td>142</td>
<td>174</td>
</tr>
<tr>
<td>3.0</td>
<td>152</td>
<td>195</td>
<td>-</td>
</tr>
<tr>
<td>3.5</td>
<td>204</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

**NOTE:** (-) = Drying to moisture content \( W \) cannot terminate in this period.

---

**TOTAL DRYING TIME, \( t_c + t_f \) (FIG. 51)**

<table>
<thead>
<tr>
<th>Moisture Content ( W \text{ g/g} )</th>
<th>Bed Loading ( 2 \text{ kg/m}^2 )</th>
<th>Time (Days)</th>
<th>Time (Days)</th>
<th>Time (Days)</th>
<th>Time (Days)</th>
<th>Time (Days)</th>
<th>Time (Days)</th>
<th>Time (Days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W \text{ g/g} )</td>
<td>( t_c )</td>
<td>( t_f )</td>
<td>( t_c )</td>
<td>( t_f )</td>
<td>( t_c )</td>
<td>( t_f )</td>
<td>( t_c )</td>
<td>( t_f )</td>
</tr>
<tr>
<td>2.0</td>
<td>5.2</td>
<td>4.1</td>
<td>4.7</td>
<td>5.0</td>
<td>6.0</td>
<td>7.2</td>
<td>9.0</td>
<td>11.3</td>
</tr>
<tr>
<td>2.5</td>
<td>4.6</td>
<td>5.9</td>
<td>6.5</td>
<td>7.3</td>
<td>9.0</td>
<td>11.8</td>
<td>15.0</td>
<td>19.0</td>
</tr>
<tr>
<td>3.0</td>
<td>6.3</td>
<td>8.2</td>
<td>9.1</td>
<td>10.5</td>
<td>14.6</td>
<td>18.9</td>
<td>24.1</td>
<td>31.0</td>
</tr>
<tr>
<td>3.5</td>
<td>8.5</td>
<td>12.0</td>
<td>14.5</td>
<td>17.1</td>
<td>23.0</td>
<td>29.8</td>
<td>37.8</td>
<td>47.8</td>
</tr>
<tr>
<td>4.0</td>
<td>12.9</td>
<td>18.7</td>
<td>22.3</td>
<td>26.1</td>
<td>34.5</td>
<td>44.3</td>
<td>55.8</td>
<td>70.0</td>
</tr>
</tbody>
</table>
drying times $t_c$ to "critical" moisture content (Table 35), and drying time in the falling rate period $t_f$, given by

$$t_f = \frac{2.303}{K_f} \log \frac{(W_c - E)}{(W_f - E)}$$  \[ \text{from equation (52A)} \]

Negative values of $t_f$ indicated that the desired final moisture contents were less than the "critical" moisture contents. These values have been omitted from Table 36.

The total drying times required to obtain a range of final moisture contents, $3<W_f<10$, were calculated in Table 36 and plotted in Fig. 51.

4.4.6. Sludge Moisture Content for Spadeability.

A mass of drained sludge (approximately 6.5 kg) at moisture content $W_o = 12.50 \, \text{g/g}$, was exposed to the weather under adverse conditions.

The drying bed consisted of an open box, 25 cm x 25 cm x 10 cm deep.

A transparent perspex sheet, 60 cm x 60 cm, was supported at a height of 30 cm above the box. The covering perspex sheet was inclined to the horizontal to allow rainwater drainage to flow to the one edge. A maximum- and minimum thermometer and recording hygrometer were placed beneath the cover. Wind velocities were obtained from the local weather station.

The mass of sludge was examined daily for spadeability. Small core samples were taken through the depth of the bed, for the determination of average moisture content. The following observations were recorded:
From the third day of drying onwards, the average mass was sufficiently coherent to be spadeable ($W_0$ approx. = 8.5 g/H₂O/g dry solids).

From the sixth day onwards, the mass had a "dry" consistency, and was readily spadeable. After the seventh day, a strong wind disrupted the apparatus. On the following day, the dried sludge was found in fairly hard lumps, which could be hand-lifted. An average moisture content of the lumps gave $W = 1.90$ g H₂O/g dry solids.

Accepting the maximum moisture content for spadeability as 8.5 g/g, this value is shown in Fig. 51. Drying bed loadings may now be estimated such that a moisture content of less than 8.5 g/g is obtained at 65% R.H. and 21°C. The actual exposure of sludge to wind and radiation produced an increased drying rate, despite the reduced temperatures and higher humidities during this experiment. Thus, a bed-loading of 2.6 kg/m² (Fig. 51) will produce sufficient drying within 7 days of exposure, which will enable the lifting of the sludge from the sand bed.
REFERENCES.


(a) ibid, "Sedimentation", 28-51; "Thickening", 228-235;
(b) ibid,"Filtration", 236-247;
(c) ibid, "Sludge Drying", 247-249.


(a) ibid, "Settling", 571-579; "Sedimentation", 586-589;
(b) ibid, "Filtration", 494-501;
(c) ibid, "Drying", 307-310.


(11) Coakley, P. and Jones, P.R.S., Sewage and Industrial Wastes, 28, 8, 963, 1956.


