WAVE FORCES

ON

SUBMARINE PIPELINES.

A REVIEW

by

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in partial fulfilment of the
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DECLARATION

I hereby declare that this thesis is my own work and that it has not been submitted for a degree at another university.

ABSTRACT

In Part I theoretical aspects of hydrodynamic forces on immersed solids are considered.

Part II contains a chronological review of research projects, laboratory as well as ocean experiments, which have been carried out since 1950.

In Part III the recommended procedures, which emerge from Part II, are applied to calculate wave forces acting on the proposed Green Point outfall sewer which is to serve the central city area of Cape Town. The results are discussed and compared.

Part IV outlines the conclusions of this study and offers recommendations for future research in this field.
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**LIST OF SYMBOLS**

**Roman Letters**

- $a_1$, $a_2$, $a_3$: constants (Al-Kazily (1974))

- $A$: horizontal semi-axis of elliptical orbit described by particle area, projected at right angles to flow direction

- $A_r$: constant (Brater and Wallace (1972))

- $b$: vertical semi-axis of elliptical orbit described by particle

- $c$: wave celerity

- $c_0$: wave celerity in deep water

- $c_g$: group velocity

- $C$: force coefficient in Crooke's equation

- $C_D$: coefficient of drag

- $C_I$: coefficient of inertia

- $C_L$: coefficient of lift

- $C_m$: coefficient of added mass

- $C_W$: horizontal-vertical inertial coefficient (relating vertical force to horizontal acceleration), or conversely, a vertical-horizontal inertial coefficient (relating horizontal force to vertical acceleration)

- $C^*_D$, $C^*_I$, $C^*_L$, $C^*_W$: reference coefficients for the case $\alpha = 90^\circ$, $h/D = 0$ (Grace (1973))

- $C_M$, $C_{MM}$, $C_{MMM}$: types of inertial coefficients (used by Brater and Wallace (1972) and explained in the text)

- $d$: depth of water
D  
  diameter

e  
  base of Naperian logarithm = 2.718

\(f_o\)  
  frequency of oscillation

\(f_v\)  
  frequency of vortex shedding

\(f_L\)  
  frequency of lift force

F  
  horizontal force per unit length of body

\(F_l\)  
  horizontal force on length \(l\) of body

\(F'\)  
  horizontal force on entire length of body

\(F_D\)  
  horizontal drag force per unit length

\(F_I\)  
  horizontal inertial force per unit length

\(F_L\)  
  horizontal lift force per unit length

\(F_W\)  
  horizontal inertial force, per unit length (a horizontal force associated with the vertical acceleration)

\(F_D'\)  
  horizontal drag force, per unit length, due to a horizontal current

\(F_{friction}\)  
  horizontal frictional resistance force, per unit length, between sea bed and pipe

g  
  gravitational acceleration (\(\approx 9.81 \text{ m/s}^2\))

\(G\)  
  horizontal force on length \(l\) of pipe, acting perpendicularly to pipe, for the case \(\alpha = 90^\circ\) (Grace (1973))

h  
  distance between sea bed (or solid boundary) and underside of pipe (or cylinder)

H  
  wave height

\(H_o\)  
  wave height in deep water

j  
  constant (Brater and Wallace (1972))

k  
  effective spring constant

K  
  Keulegan-Carpenter period parameter

\(K_C\)  
  value of K at water surface (used by Isaacson and Maull (1976) and Sarpkaya (1976c))

\(K_U\)  
  a sort of horizontal drag coefficient \(\{\text{for the case } \alpha = 90^\circ\}\) (Grace (1973))

\(K_A\)  
  a sort of horizontal inertial coefficient
for the case $\alpha = 90^\circ$; $h/D = 0$ (Grace (1973))

$K_u$

$K_u$

$K_A$

$K_A$

length of pipe (or cylinder)

wave length

wave length in deep water

mass of body

mass of displaced fluid

so-called added mass

a sort of vertical drag coefficient

a sort of horizontal-vertical inertial coefficient

for the case $\alpha = 90^\circ$ (Grace (1973))

$M_U$

$M_U$

$M_A$

$M_A$

index

number of vortices shed in half a cycle

number of harmonics (in lift force, suggested by Chakrabarti, et al (1976))

pressure at a point

vertical force per unit length of body

vertical force on length $\ell$ of body

vertical force on entire length of body

vertical drag force per unit length

vertical inertial force per unit length

vertical lift force per unit length

vertical inertial force, per unit length

Buoyancy force, per unit length, in static water

Buoyancy force, per unit length, in moving water

vertical lift force, per unit length, due to a horizontal current
$q_r$ strength of the $r^{th}$ pulsating pressure point

$q$ vertical force on length $l$ of pipe, for the case $\alpha \neq 90^\circ$

(Grace (1973))

$r$ radius

$Re$ Reynolds number

$s$ travel distance of water particle, measured from centre of orbit

$S$ Strouhal number

$t$ time

$T$ wave period; period of oscillation

$T_n$ period of natural vibration

$u$ horizontal component of velocity

$\dot{u}$ horizontal component of acceleration

$\ddot{u}$ horizontal acceleration at the instant of zero horizontal velocity

$U_c$ horizontal current velocity

$v$ vertical component of velocity

$\dot{v}$ vertical component of acceleration

$V$ volume

$W$ weight

$W_o$ weight in air

$W_{eff}$ effective, or submerged, weight

$x$ horizontal axis direction

$y$ vertical axis direction

$y_s$ elevation of water surface above SWL

$Y$ vertical reactive force, per unit length, between sea bed and pipe

$z$ depth of submergence, positive when measured downwards from SWL
Greek Letters

\( \alpha \)
orientation angle, i.e. angle between pipe axis and wave direction

\( \beta \)
Grace's \( \beta \)-parameter

\( \gamma \)
phase angle for the \( n^{th} \) harmonic lift force (suggested by Chakrabarti, et al (1976))

\( \phi \)
phase angle, \( (\approx 2\pi t/T) \)

\( \phi_T \)
total velocity potential

\( \phi_i \)
velocity potential function due to the incident (undisturbed) wave

\( \phi_s \)
velocity potential function due to the scattered wave

\( \phi_r \)
potential originating from the \( r^{th} \) pulsating pressure point, with strength \( q_r \)

\( \lambda \)
the ratio between a (horizontal) force coefficient for a particular \( \alpha \) and \( h/D \), and the corresponding (horizontal) reference force coefficient

\( \nu \)
kinematic viscosity

\( \rho \)
mass density of water (or fluid)

\( \sigma \)
radian frequency, \( (\approx 2\pi/T) \)

\( \tau \)
shear stress at a point

\( \psi \)
the ratio \( [F_{\text{max}} \text{(measured)} - F_{\text{max}} \text{(calculated)}]/F_{\text{max}} \text{(calculated)} \)
(used by Sarpkaya (1975))

\( \Omega \)
the ratio between a (vertical) force coefficient for a particular \( \alpha \) and \( h/D \), and the corresponding (vertical) reference force coefficient

Subscripts

max maximum

Airy according to the linear, or Airy, wave theory

\( \tau \)
due to shear stresses

p due to direct pressures

vel due to velocity effects

accln due to acceleration effects
$u$ due to the horizontal component of velocity

$v$ due to the vertical component of velocity

$\dot{u}$ due to the horizontal component of acceleration

$\dot{v}$ due to the vertical component of acceleration
An assessment of anticipated wave forces is required in order to produce a rational design for a pipe which is to be laid in the sea. The primary object of a rational design is to provide an economic solution to a problem.

Should the pipeline be incapable of withstanding imposed hydrodynamic loads it will be displaced and may even be ruptured. Such failures are costly, and since pipelines often convey sewage, oil or harmful chemicals the resulting pollution may be anything between inconvenient and catastrophic.

On the other hand, if the pipe is overdesigned and constructed to conservative standards, the excess expenditure may be large and quite unjustified.

A popular method of protecting a pipe from wave effects in "shallow" water is to bury it. But at what water depth does one cease this practice? Grace (1971) has reported the displacement of pipelines in depths of as much as 70 m. Burying the pipe in a sandy sea bed is no foolproof protection either - according to Alterman (1962), for instance, 2.5 m changes in sea bed level have been observed at the site of a pipeline in the Mediterranean.

For almost three decades researchers have studied wave forces on rigid bodies, notably cylinders. The problem is by no means solved yet. A submarine pipeline is a special type of cylindrical structure, namely a cylinder near a solid boundary for which the results of general investigations are not necessarily valid. Consequently, even less positive information is available regarding wave forces on pipelines. This particular subject has, however, been receiving more and more attention in recent years.
PART 1

THEORETICAL BACKGROUND

TO THE

WAVE FORCE PROBLEM
CHAPTER 1

WAVE THEORY

1.1 SURVEY OF WAVE THEORIES

Numerous theories have been developed to describe wave phenomena; some are better than others in certain respects, but all the theories have shortcomings in some regards. The main reason is the extreme complexity of ocean waves - especially in the coastal zone - which is difficult to describe mathematically. Other reasons include the three dimensional characteristics and apparent random behaviour of water waves.

The two classical wave theories are those developed by Airy, in 1845, and Stokes, in 1880. Both theories predict wave behaviour better where water depth relative to wave length is not too small - that is, in deep water. Cnoidal wave theory, originally developed by Korteweg and De Vries in 1895 predicts fairly well the elongated trough - peaked crest wave form and associated motions encountered in shallow water regions. Certain features of wave behaviour in very shallow water near the breaker zone, are predicted satisfactorily by the so-called solitary wave theory. In 1802, Gerstner developed the trochoidal wave theory which predicts wave profiles quite accurately, but not water particle motions. A fairly new theory is the numerical stream function theory, developed by Dean in 1965. (Ref: Dean (1965), (1974)). Versions of Stokes' theory often encountered include Stokes' second order theory, as well as third, fourth and fifth order theories.

It is understood that D.H. Swart, et al, are currently developing a new wave theory in South Africa, and are due to present a paper titled "Vocoidal theory for all non-breaking waves" at the 16th International Conference on Coastal Engineering, to be held in Hamburg, West Germany, in September, 1978.

The Airy theory, which describes pure oscillatory waves, does not allow for mass transport and ignores the long trough, short crest configuration, has a few shortcomings, but has the advantage of being relatively uncomplicated. Mathematically, it can be considered a first approximation of a complete theoretical description of wave behaviour.
When concerned with submarine pipelines, one is usually interested in the movement of water particles close to the sea bed. Of all the theories, the Airy theory provides the best estimate of the true peak velocities. This was found by Le Méhauté, Divoky and Lin (1968), and also by Goda (1964), Grace (1971), Grace (1973) and Grace (1976). This theory, unfortunately, underestimates the accelerations of the water particles near the sea floor. It is, however, possible to compensate for this shortcoming when calculating the forces, as we will see later. Grace (1973) puts it as follows "The easiest wave theory to use is the linear, or Airy, wave theory. It turns out as well that this theory gives accurate predictions of the real bottom velocities beneath wave crests and is apparently as good or better than other theories in predicting maximum water particle accelerations near the sea floor. For design-type waves, it appears to predict such accelerations approximately one-third low". In another paper, Grace (1975) says that, regardless of how nonsinusoidal the surface feature of the wave appears, the Airy theory provides a "very creditable prediction of near-bottom water velocity under the wave crest". The velocity under the trough is predicted too high, but since the pipeline is designed for peak, or crest, conditions, this is irrelevant.

Vongvisessomjai and Silvester (1976) also make some interesting comments on the applicability of water wave theories: "Tests by Le Méhauté et al (1968) have indicated that no single theory predicts the (orbital) velocity distribution throughout depth as recorded in flume tests". Silvester (1974) found that, of eleven theories considered, the Airy theory as modified by Goda (1964), was the most accurate in predicting particle motions close to still water level as well as the bed.

Thus, to summarize, the Airy theory is accepted as the most suitable for obtaining particle velocities and accelerations required for calculating forces on pipelines close to the sea floor.

Unless indicated otherwise, we shall, in the remainder of this thesis, restrict ourselves to the use of the Airy theory.
1.2 SOME DEFINITIONS AND CONCEPTS RELEVANT TO WAVE KINEMATICS

Periodic wave: A wave is periodic if its motion and surface profile recur in equal intervals of time.

Gravity waves: The principal restoring force of these waves is gravity, i.e., the gravitational force attempts to bring the fluid back to its equilibrium position. The period is typically between about 0.1 second and 5 minutes, although, when concerned with critical forces on submarine pipelines, the 5 to 20 second period waves are usually of importance.

1.2.1 Notation

\[ T = \text{wave period (seconds)} \]
\[ d = \text{undisturbed or still-water depth (m)} \]
\[ L = \text{wave length (m)} \]
\[ H = \text{wave height (trough to crest) (m)} \]
\[ d/L = \text{relative depth} \]
\[ H/L = \text{wave steepness} \]
\[ f_0 = 1/T = \text{wave frequency (waves/second or cycles/second or hertz)} \]
\[ \sigma = 2\pi/T = \text{radian frequency (radians/second)} \]
\[ c = L/T = \text{wave celerity (m/s)} \]

Figure 1-1 Definition sketch.
1.2.2 Classification of waves

1.2.2.1 Progressive versus stationary waves

Progressive: A wave form which moves in some direction relative to the fluid is a progressive wave. Ocean waves, generated by wind, are examples.

Stationary: If the water surface merely moves up and down at a fixed position it is said to be a stationary wave or a clapotis. Waves, trapped between the vertical walls of a harbour, performing a see-saw motion known as seiche or range action, serve as examples.

1.2.2.2 Oscillatory versus translatory waves

Oscillatory: If the water particle has essentially a regular forwards-backwards motion about a fixed mean position, or alternatively, if the particle moves in a closed orbit, the wave motion is said to be oscillatory. Most sea waves outside the breaker zone are of this type.

Translatory: When the water particles advance with the wave, and do not return to their original positions, the wave is translatory. A solitary wave is an example of this type of wave.

1.2.2.3 Solitary wave versus wave train

The difference lies in the number of wave forms present.

Solitary wave: If only a single crest (or trough) is present, the wave is of the solitary type. Examples include the wave in a canal following the rapid opening of a sluice gate, and a wave at sea resulting from a landslide into coastal waters.

Wave train: If a series of repetitive wave forms is present, we have a wave train. Ocean swell is an example.
1.3 KINEMATICS OF WAVE ORBITS, ACCORDING TO THE AIRY THEORY

1.3.1 Wave form

Referring to Fig. (1-2), the equation describing the free surface of a simple sinusoidal, progressive, oscillatory wave as a function of time $t$ and horizontal distance $x$, is

$$y_s = \frac{H}{2} \cos 2\pi \left[ \frac{x}{L} - \frac{t}{T} \right]$$

(1-1)

where $y_s$ = elevation of the water surface relative to still-water level.

![Figure 1-2 Graphical description of terms, (Wiegel (1964), p.14)](image)

The wave form is propagated through the fluid by water particles performing orbital motions; the orbits generally being of elliptical shape, Fig. (1-3).
1.3.2 Hyperbolic functions

The functions sinh, cosh and tanh are widely used in linear wave theory, and are defined as

\[
\sinh x = \frac{e^x - e^{-x}}{2} \quad (1-2)
\]

\[
\cosh x = \frac{e^x + e^{-x}}{2} \quad (1-3)
\]

\[
\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} \quad (1-4)
\]

where \( e = \) Base of the Naperian logarithm = 2.718

1.3.3 Orbital Velocities

At any point \((x, y)\) within the fluid, and at time \(t\), the horizontal component of velocity, \(u\), is given by

\[
u = \frac{\pi H}{T} \cosh \left[ \frac{2\pi (y + d)/L}{2\pi d/L} \right] \cos 2\pi \left[ \frac{x - t}{T} \right] \quad (1-5)\]

and the vertical component of velocity, \(v\), is given by

\[
v = \frac{\pi H}{T} \sinh \left[ \frac{2\pi (y + d)/L}{2\pi d/L} \right] \sin 2\pi \left[ \frac{x - t}{T} \right] \quad (1-6)\]
The maximum values of $u$ and $v$ occur when the cosine and sine functions, respectively, are unity.

$$u_{\text{max}} = \frac{\pi H}{T} \frac{\cosh \left[ \frac{2\pi (y + d)}{L} \right]}{\sinh \left( \frac{2\pi d}{L} \right)}$$  \hspace{1cm} (1-7)$$

$$v_{\text{max}} = \frac{\pi H}{T} \frac{\sinh \left[ \frac{2\pi (y + d)}{L} \right]}{\sinh \left( \frac{2\pi d}{L} \right)}$$  \hspace{1cm} (1-8)$$

1.3.4 Orbital accelerations

The horizontal component of water particle local acceleration, $\dot{u}$, at any point $(x, y)$ within the fluid, at time $t$, is given by

$$\dot{u} = \frac{2\pi^2 H}{T^2} \frac{\cosh \left[ \frac{2\pi (y+d)}{L} \right]}{\sinh \left( \frac{2\pi d}{L} \right)} \sin \frac{2\pi}{L} \left( \frac{x}{L} - \frac{t}{T} \right)$$  \hspace{1cm} (1-9)$$

while the vertical component of local acceleration, $\dot{v}$, is

$$\dot{v} = -\frac{2\pi^2 H}{T^2} \frac{\sinh \left[ \frac{2\pi (y+d)}{L} \right]}{\sinh \left( \frac{2\pi d}{L} \right)} \cos \frac{2\pi}{L} \left( \frac{x}{L} - \frac{t}{T} \right)$$  \hspace{1cm} (1-10)$$

The maximum values of these accelerations, $\dot{u}_{\text{max}}$ and $\dot{v}_{\text{max}}$ respectively are

$$\dot{u}_{\text{max}} = \frac{2\pi^2 H}{T^2} \frac{\cosh \left[ \frac{2\pi (y+d)}{L} \right]}{\sinh \left( \frac{2\pi d}{L} \right)}$$  \hspace{1cm} (1-11)$$

$$\dot{v}_{\text{max}} = -\frac{2\pi^2 H}{T^2} \frac{\sinh \left[ \frac{2\pi (y+d)}{L} \right]}{\sinh \left( \frac{2\pi d}{L} \right)}$$  \hspace{1cm} (1-12)$$

1.3.5 Phase relationship

The phase relationship between $u$, $v$, $\dot{u}$, $\dot{v}$ and $y_s$ (the elevation of the free surface relative to the still-water level) for any fixed point in the fluid, through one complete wave period, can be found through inspection and is shown in Fig. (1-4).

It is important to note that, in any one direction, the velocity is always out of phase with the acceleration by a quarter period.

1.3.6 Displacement of water particles

The maximum horizontal displacement from the centre of the orbit, $A$, (see Fig. (1-3)), is given by

$$A = \frac{H}{2} \frac{\cosh \left[ \frac{2\pi (y+d)}{L} \right]}{\sinh \left( \frac{2\pi d}{L} \right)}$$  \hspace{1cm} (1-13)$$
Figure 1-4 Phase relationship between surface elevation, velocities and accelerations
The maximum vertical displacement from the centre of the orbit, $B$, is

$$B = \frac{H}{2} \frac{\sinh \left[ \frac{2\pi (y+d)/L}{\sinh (2\pi d/L)} \right]}{\sinh (2\pi d/L)}$$  \hspace{1cm} (1-14)

1.3.7 Wave celerity

The celerity, $c$, with which the wave form is propagated through a fluid is

$$c = \sqrt{\frac{gL}{2\pi}} \tanh (2\pi d/L)$$  \hspace{1cm} (1-15)

1.3.8 Classification of water waves according to relative depth

The ratio $d/L$, the relative depth, is an important parameter in linear wave theory.

The hyperbolic tangent function, defined as

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$  \hspace{1cm} (1-4)

tends to 1 when $x$ becomes large, and
tends to $x$ when $x$ tends to zero.

Therefore, the celerity $c$, as given by Eq. (1-15), can be expressed as

$$c \approx \sqrt{\frac{gL}{2\pi}}$$  \hspace{1cm} \text{for large values of } 2\pi \frac{d}{L}  \hspace{1cm} (1-16)

$$c \approx \sqrt{\frac{gL}{2\pi}} \times \frac{2\pi d}{L}$$

$$= \sqrt{gd}$$  \hspace{1cm} \text{for small values of } 2\pi \frac{d}{L}  \hspace{1cm} (1-17)

If $d/L > \frac{1}{4}$, the error when using Eq. (1-16) instead of Eq. (1-15) is smaller than 0.2%

If $d/L < 1/20$, the error when using Eq. (1-17) instead of Eq. (1-15) is smaller than 1.7%

It is very convenient to classify waves as follows:
Range of relative depth

<table>
<thead>
<tr>
<th>$d/L$</th>
<th>Type of wave</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 to 1/20</td>
<td>long, shallow waves</td>
</tr>
<tr>
<td>1/20 to 1/2</td>
<td>intermediate depth waves</td>
</tr>
<tr>
<td>1/2 to $\infty$</td>
<td>short, deep waves</td>
</tr>
</tbody>
</table>

(This is the classification recommended by Eagleson and Dean (1966) and by Kilner (1975). In the CERC (1975) classification, the limit "1/20" is replaced by "1/25").

Eq. (1-16) leads to a very useful relationship: the deep water wave celerity, $c_o$, as a function of wave period:-

From Eq. (1-16) : $c^2 = \frac{g}{2\pi} L$  

and since $L = cT$, it follows that $c = \frac{g}{2\pi} T$  

OR $c_o = 1.56 T$  

where $c_o =$ deep water wave celerity in m/s  

and $T =$ wave period in seconds.

1.3.9 Orbit characteristics of the three types of waves

Eagleson and Dean (1966), show, by considering Eqs. (1-13) and (1-14), that the shape of the particle orbits under short, deep waves is almost circular, and orbital motion virtually ceases at a distance of about half the wave length below the still-water level. This leads to the phenomenon that a wave in deep water does not "feel" the sea bed. Vice versa, objects on the sea bed are not affected by deep water waves passing overhead. It is usually accepted that the process of refraction of waves starts when the water depth becomes less than half the wave length. For the long, shallow and the intermediate depth type waves, the orbits are elliptical and occur right down to the sea bed.

For all three types of waves, the total vertical displacement of particles at the surface is equal to the wave height, $H$.

These characteristics are schematically illustrated in Fig. (1-5).
As waves move from deep water, through intermediate depths, to shallow water, various characteristics such as $L$, $H$, and $c$ change, while $T$ remains constant. The shoaling process will not be described herein, but a very useful alignment chart, developed by Prof. F.A. Kilner of Cape Town University, relating water depth, wave length, wave height, wave period, celerity and group velocity is included, see Fig. (1-6).

1.3.10 **Validity of the Airy theory**

According to Wiegel (1964), this theory is valid if the wave steepness, $H/L$, is small (a requirement stated by Stokes), and the ratio $L^2H/2d^3$ is small. Longuet-Higgins (1956) suggested that this ratio must be much smaller than $16\pi^2/3 = 52.6$.
Figure 1-6 ALIGNMENT CHART FOR DIMENSIONLESS BASIC WAVE RELATIONSHIPS (Copyright F.A. Kilner)
CHAPTER 2

MECHANISMS WHEREBY FORCES ARE GENERATED ON SUBMERGED BODIES

2.1 BODY FORCES

2.1.1 Weight

The weight, \( W \), of a body with mass, \( m \), is

\[
W = mg
\]  

(2-1)

where

- \( W \) = weight of the body (say in N)
- \( m \) = mass of body (say in kg)
- \( g \) = gravitational acceleration (usually taken as \( 9.81 \, \text{m/s}^2 \))

In the practical case of a pipeline, the total weight of a unit length of the line may consist of say the weight of the pipe tube, the weight of a protective coating around the tube, the weight of a concrete jacket, the weight of any saddles or anchors weighing the pipe down, and the weight of the contents of the pipe.

2.2 HYDROSTATIC FORCES

Under this category we shall consider those forces that are present when there is no relative motion between the body and the surrounding fluid.

2.2.1 Buoyancy force

The law of Archimedes states that if any body is wholly or partly immersed in a liquid, it will be buoyed by a force equal to the weight of the liquid displaced by the body.

This buoyancy force is obtained by integrating the pressures, exerted by the liquid, over the entire surface of the body. Consider the situation illustrated in Fig. (2-1).
Figure 2-1 Pressures on a body submerged in a liquid

The pressure \( p \), at a point at any depth \( z \) within the liquid is

\[
p = \rho gz
\]  

(2-2)

and acts normal to the surface of the body.

Since the liquid is static, the sum of the horizontal components of these pressures is zero; in other words, the net horizontal force on the body is zero.

Because the pressures increase with depth, the sum of the vertical components are not zero. A net vertical force results, its direction being upwards. This is the buoyancy force, \( P_B \).

This means that a system of pipeline (i.e. tube with or without coating, concrete jacket, saddles, contents etc) having a total weight \( W_0 \) in air and a total displacement volume \( V \), will, after immersion in a liquid have a smaller effective weight, \( W_{\text{eff}} \):

\[
W_{\text{eff}} = \text{weight in air} - \text{weight of displaced liquid}
\]

\[
= W_0 - \rho V g
\]  

(2-3)
2.3 HYDRODYNAMIC FORCES

Whenever a body obstructs the flow of a real fluid, such as water, it experiences some sort of force.

The dynamics of fluid flow around a cylinder (the shape usually applicable to pipelines) is quite complex, and therefore it is not easy to predict the induced forces accurately. Yamamoto, et al (1974) state that "... the subject has been difficult to understand thoroughly because of the complicated flow regimes which depend on the ambient velocity distribution, turbulence, nearby boundary conditions, cylinder shape, direction and time variation of the flow, presence of a free surface, and the motion response of the cylinder itself. The somewhat random phenomenon of vortex shedding adds further difficulty".

Bearing these complications in mind, we shall take a closer look at the types of forces and the manner in which they are generated.

In order to calculate a force on a body, it is necessary to consider the distribution of pressures and shear stresses over the surface of the body. Fig. (2-3) shows the direct pressure, $p$, and shear stress, $\tau$, on an elemental part of the surface, $dA_x$, of a body. The pressure and shear stress act in directions normally to
and tangentially to the elemental surface area, respectively.

![Elemental Surface Area](image)

Figure 2-3 Pressure and shear stress on an elemental surface area

By integrating these pressures and shear stresses, in other words, summing all the elemental forces on the total surface of the body, one arrives at the resultant water force on the body.

2.3.1 Hydrodynamic forces in steady flow

2.3.1.1 Drag forces

2.3.1.1.1 The concept of drag

If a body is placed in the steady current of a perfect fluid (i.e. a fluid without viscosity), the total force exerted by the fluid on the body, without circulation of velocity, is zero. This is known as the paradox of D'Alembert.

In the case of a real fluid (i.e. a viscous fluid), a boundary layer develops along the body and this leads to a drag force. This force is caused by shear stresses acting on the surface of the body. An additional effect of the boundary layer is a wake which is a phenomenon resulting from the separation of the boundary layer on the downstream side of the body.

Consider the flow around a blunt body such as a cylinder. The pressure along the surface of the body, which is impressed on the boundary from the external flow, is not constant, refer to Fig. (2-4).
The fluid particles are accelerated from A to B and decelerated from B to C. Consequently, according to the Bernoulli equation, the pressure decreases from A to B and increases from B to C. As a result of the viscosity of the fluid, which leads to friction within the boundary layer, a certain amount of kinetic energy is converted into heat and is thus not available to overcome the increasing pressure toward point C. The fluid particles, being influenced by the external pressure, may move in the reverse direction and cause flow separation behind the body at point S. The flow field behind the separation is very irregular, is characterized by turbulent eddies, and is called the turbulent wake.

Compared with frictionless flow, the flow field changes radically because of the existence of the wake. The main flow which separates from either side of the boundary does not meet immediately behind the body, but leaves a "dead zone" in which the pressure remains close to its value at the separation point, which is always less than the pressure at the forward stagnation point. Therefore, a large net force will act on the body resulting from the integration of the pressure differences across the body. This force is known as "form drag" or "pressure drag".

Form drag is critically dependent upon the existence and position of flow separation which, in turn, depends upon the shape of the body and the boundary layer characteristics. In general, the larger the
the width of the wake the larger the form drag and thus the total drag. The form drag can be reduced considerably by streamlining the body.

Drag usually consists of two components: that due to the skin friction giving rise to shear stresses which when integrated over the body yield surface drag or friction drag, and that due to the existence of the turbulent wake which leads to the form drag or pressure drag.

The drag force experienced by a stationary body immersed in the steady flow of a real fluid will depend on the Reynolds number characterizing the motion and upon the geometrical form and orientation of the body. Rouse (1938) gives the following equation for the drag force:

\[ F' = \text{function} \left[ \frac{u \ell}{v}, \text{form} \right] \ell^2 \rho u^2 \]  

(2-4)

where

- \( F' \) = resultant drag force acting on the body
- \( u \) = stream velocity
- \( \ell \) = the significant dimension of the body
- \( v \) = kinematic viscosity
- \( \rho \) = mass density of the fluid

Eq. (2-4) is derived from a dimensional analysis of the several variables involved, and is generally written in the more convenient form

\[ F' = \text{function} \left( R_e, \text{form} \right) \frac{A_r \rho u^2}{2} = C_D \frac{A_r \rho u^2}{2} \]  

(2-5)

where

- \( A_r \) = projected area of the body on a plane normal to the direction of fluid motion
- \( C_D \) = a variable coefficient of drag
- \( R_e = \text{Reynolds number} = \frac{u \ell}{v} \)

It follows from Eq. (2-5) that

\[ C_D = \text{function} \left( R_e, \text{form} \right) = \frac{F'}{\frac{A_r \rho u^2}{2}} \]  

(2-6)
The drag force per unit length, $F_D$, on a cylinder with a diameter $D$, placed with its axis at right angles to the flow, is expressed as:

$$F_D = \frac{1}{2} C_D D \rho u^2 \quad (2-7)$$

### 2.3.1.1.2 Drag characteristics of a cylinder in steady flow

The variation of the drag coefficient $C_D$ with Reynolds number for a smooth circular cylinder in steady flow is shown in Fig. (2-5).

![Figure 2-5](image)

**Figure 2-5** $C_D$ versus $Re$ for a smooth circular cylinder in steady flow

(Le Méhauté (1976), p.175)

Explanations for some of the characteristics of the curve in Fig. (2-5) are offered by Le Méhauté (1976) and Vennard and Street (1976).

As the Reynolds number, $Re$, increases the flow separates from the surface of the cylinder, beginning at the rear stagnation point, where the adverse pressure gradient is greatest.

For Reynolds numbers up to about 0.1 $C_D$ results from shear effects only, refer Fig. (2-6a). As the Reynolds number increases to about 10, separation and weak eddies begin to form, enlarging into a fully developed wake near a Reynolds number of 1000; in this range the drag coefficient results from a combination of friction drag and form drag.

When $Re = 1000$, friction drag constitutes only about 5% of the
total drag. For higher Reynolds numbers the contribution of friction drag becomes progressively smaller and form drag becomes the dominant component.

Figure 2-6  Flow about a smooth circular cylinder at various Reynolds numbers (Vennard and Street, p.668)

In the range of Reynolds numbers 50 to 5000, a regular pattern of vortices break loose from the leeside of the cylinder and get carried away by the main stream, see Fig. (2-7). These vortices are known as the von Karman vortex street. Alternate vortices rotate clockwise and counterclockwise. As the velocity is increased, and thus also the Reynolds number, the vortices are released more easily, the separation points move further downstream, the size of the wake is reduced and the form drag, now being the main component of the total drag decreases.
The value of $C_D$ is almost constant for Reynolds numbers of 5000 to $2.5 \times 10^5$. When the Reynolds number exceeds about $2.5 \times 10^5$ the drag coefficient suddenly reduces considerably. The reason apparently is that the boundary layer changes from the laminar state to turbulent. This transition brings a violent mixing in the boundary layer resulting in the fluid particles near the boundary gaining additional kinetic energy, which enables them to withstand the adverse pressure gradient better and to move the separation point still further downstream as shown in Fig. (2-6c). The size of the wake is thus further reduced, leading to a reduction in $C_D$.

Fig. (2-8) illustrates how the pressure distribution on the cylinder changes as result of the boundary layer transition. Since the transition from laminar to turbulent boundary layer depends on the roughness of the cylinder, and to a lesser extent on the turbulence level in the free stream, the drag coefficient near this critical region is not a unique function of the Reynolds number.
When the Reynolds number exceeds about $5 \times 10^5$ the drag coefficient starts increasing, probably due to excessive turbulence.

As mentioned earlier, the form drag of a body can be reduced by streamlining the body. Consider two objects: the one having a streamlined shape, Fig. (2-9a), the other being a flat plate held normal to the flow direction, Fig. (2-9b). Both have the same projected area on a plane normal to the flow direction.
It can be seen in Fig. (2-9a) that the boundary layer clings to the body over most of its surface and the width of the wake is very small; in other words, there is little separation. The total drag consists mainly of frictional drag, and little form drag.

In the case of the flat plate, Fig. (2-9b), the boundary layer is rapidly separated from the body, the wake is quite wide, resulting in substantial separation and a large form drag. The total drag of the flat plate is much greater than the total drag of the streamlined body.

A circular cylinder, which is also a blunt body, obviously falls in a category between the streamlined case and the flat plate.

2.3.1.2 The phenomenon of vortex shedding

The formation of the von Karman vortex street has been mentioned. Long blunt objects such as cylinders tend to shed large eddies regularly and alternately from opposite sides when placed with their axes perpendicular to the fluid flow. As a result of this unsymmetric periodic circulation of velocities, a varying pressure field is induced, in accordance with the Bernoulli principle, resulting in a periodic lateral force that reverses its direction continually. Therefore the cylinder may tend to oscillate from one side to the other, particularly if the frequency of vortex shedding approaches the frequency of natural oscillation of the cylinder, resulting in resonance.

The Strouhal number,
\[ S = \frac{f_D}{u} \tag{2-8} \]

is a useful parameter for determining the frequency of the vortex shedding, \( f_v \), in steady flow.

The Strouhal number depends uniquely on the Reynolds number (at least for steady flow) and is given in Fig. (2-10); \( S = 0.20 \) to \( 0.21 \) over a large part of the \( Re \) range.
2.3.1.3. **Lift forces**

When the buoyancy force, $P_B$, acting on a body immersed in a static fluid, was considered, it was shown that $P_B$ resulted from the integration of vertical components of pressures acting on the surface of the body.

When the body is placed in a moving fluid, the total vertical force may once again be obtained by such an integration. Let this vertical force be $P_{B\text{dyn}}$ at any instant.

In most cases $P_{B\text{dyn}}$ will not be equal to $P_B$. The reason is that the pressure field around the body is not hydrostatic.

It is however very convenient to subtract the assumed hydrostatic part from the overall force, so that one is left with $(P_{B\text{dyn}} - P_B)$ which represents that part of the vertical force due only to hydrodynamic effects.

In the remainder of this dissertation the hydrostatic component, $P_B$, of the vertical force will be subtracted. In other words, when referring to "lift" or "transverse" forces it is implied that these are merely the hydrodynamic force components.

If a circular cylinder is placed in the steady flow of an ideal fluid, away from any boundaries or free surfaces, the pattern of streamlines is perfectly symmetrical, as shown in Fig. (2-11), and

---

**Figure 2-10**  Strouhal number versus Reynolds number for circular cylinders in steady flow. (Le Méhauté (1976), p.175)
therefore the dynamic force perpendicular to the direction of flow is always zero.

Figure 2-11  Circular cylinder in an ideal fluid, no boundary effects, steady flow

If the ideal fluid is replaced by a real fluid, there may be separation on the downstream side of the cylinder accompanied by vortex shedding - which is asymmetrical - which, in turn, would lead to an oscillating transverse force.

Assuming for a moment that there were no vortex shedding but that the cylinder rotated about its axis, thereby creating an asymmetrical pattern of streamlines, a transverse force would also act upon the cylinder.

Furthermore, if the cylinder is placed in the steady flow of a real fluid, in the vicinity of a boundary, such as the channel floor, it will experience a transverse force. Refer Fig. (2-12).

Figure 2-12  Circular cylinder in a real fluid, in the vicinity of a boundary, steady flow
This transverse force is due to the streamlines around the cylinder being asymmetrical. This asymmetry of streamlines would generally have two causes: the first being the boundary alone, which would retard the particles close to the boundary more than those further away (viscosity effects), and the second being the combined cylinder-boundary system.

The side of the cylinder where the velocities are greater would, according to the Bernoulli principle, be subjected to smaller pressures, resulting in a net transverse force.

This transverse force is commonly known as the lift force since it is similar to the aerodynamic lift force exerted on an aerofoil. In fact, it is reported in Hydraulics Research (1962, p.74, that "the cylinder lying on the sea bed constitutes quite a good aerofoil, particularly when it is smooth...".

Note that the direction of lift is essentially transverse for the stream direction and is not necessarily vertically upwards. A vertical pile, for example, in a stream, would experience a horizontal transverse or "lift" force due to vortex shedding.

Thus, in general, the lift force may be caused both by the modification to streamlines due to the presence of a solid boundary, and by vortex shedding.

The lift force per unit length of the cylinder, \( P_L \), is conventionally expressed as being proportional to the square of the velocity, \( u^2 \), and to the diameter, \( D \), and is given by

\[
P_L = \frac{1}{2} C_L D \rho u^2
\]

where \( C_L \) is a coefficient of lift.

2.3.2 A classification of hydrodynamic forces:- In-line forces and transverse forces:

Before proceeding to more complex flow situations, it is necessary to pause and consider the terminology we shall be using. The following discussion will be restricted to two-dimensional flow situations.
In the literature dealing with wave forces on submerged bodies reference is often made to the "in-line" force as opposed to the "transverse" force. The former is the total force in the dominant direction of movement of the water particles whereas the latter is the total force acting at right angles to the dominant direction of movement.

A horizontal submarine pipe with its axis parallel to the wave crests will, therefore, experience a horizontal in-line force, $F$, and a vertical transverse force, $P$. (A vertical pile, on the other hand, will have a horizontal in-line force and a horizontal transverse force acting on it).

For a horizontal cylinder $F$ is considered positive when it is in the same direction as that of wave propagation, whereas $P$ is considered positive when it is directed upwards - i.e. conforming with the directions of the positive $x$ and $y$ axes.

It has been explained that the force exerted by a fluid on a body results from integration of pressures and shear stresses that act upon the elemental areas of the body surface.

In the hydrostatic case the situation is fairly simple: the shear stresses are zero and the buoyancy force stems from the pressures only.

In the steady flow case, where there are no accelerations, the hydrodynamic forces are due to both pressures and shear stresses but these are linked to the main stream velocity only.

In the unsteady flow case, where temporal accelerations indeed occur, the hydrodynamic forces are due to pressures and shear stresses which are linked to accelerations as well as velocities.

The in-line force may be expressed in the following form

$$ F = F_{\text{vel}} + F_{\text{accln}} + F_{\text{vel}} + F_{\text{accln}} $$

(2-10)

whilst the transverse force may similarly be expressed as

$$ P = P_{\text{vel}} + P_{\text{accln}} + P_{\text{vel}} + P_{\text{accln}} $$

(2-11)
If temporal accelerations are zero the two above equations revert to the traditional separation of form and skin friction drag in steady flow.

It is noted that, whereas the two perpendicular forces, the in-line and transverse forces, are "real" forces and can be physically measured, it is not possible to separate the velocity and acceleration effects on the right-hand side of Eqs. (2-10) and (2-11).

It is however convenient to group the velocity linked components together and the acceleration linked components together.

Eq. (2-10) is therefore written as

\[ F = \left[ F_{vel} + F_{accln} \right] + \left[ F_{vel} + F_{accln} \right] \]  

(2-12)

Where the first two terms represent the velocity dependent part of the in-line force and the last two terms the acceleration dependent part.

The transverse force may be treated in an analogous way:

\[ P = \left[ P_{vel} + P_{accln} \right] + \left[ P_{vel} + P_{accln} \right] \]  

(2-13)

Where the first two terms represent the velocity dependent part of the transverse force and the last two terms the acceleration dependent part.

2.3.3 Hydrodynamic forces in oscillatory flow

2.3.3.1 Drag forces

Drag in unsteady flow differs from drag in steady flow mainly on account of the wake. Water particles flowing past the body have a temporal acceleration often reversing the direction of flow as in the case of oscillatory waves, and this results in a changing wake. The drag force will therefore not be constant: neither in magnitude nor in direction.

In the steady flow case the drag coefficient, \( C_D \), was found to be
dependent on the Reynolds number and form of the body. In unsteady, and especially in oscillatory flow, however, the past history of the flow plays a very important role in the behaviour of $C_D$. As explained by Wiegel (1964), p.250, we are dealing with a flow that reverses periodically, so that the eddies which are formed behind the body move against the body when the direction of water particle motion reverses - the wake is then at the "leading edge". As expressed by Beckmann and Thibodeaux (1962) "an object travelling in its own wake in an oscillatory motion experiences fully developed turbulent flow. The flow patterns and drag coefficients are comparable to those at supercritical Reynolds numbers".

Yamamoto, et al (1974) state that "no general exact theory has been developed to predict the velocity dependent drag forces, which result from wake formation, in unsteady flow".

Various researchers who have studied the behaviour of $C_D$ in oscillatory flow agree on one thing: $C_D$ is not constant but varies, to some extent, throughout the wave cycle. Nevertheless, it remains convenient to express the drag force in a way similar to that in steady flow:

$$F' = C_D A \frac{\rho u^2}{2} \quad (2-5)$$

This is an instantaneous value of the drag force because the approach velocity, $u$, keeps changing.

It is important to note that, at any particular instant, the direction of the drag force will always be the same as the direction of $u$. To prevent the minus sign getting lost when squaring the velocity, it is customary to write Eq. (2-5) in the form

$$F' = \frac{1}{2} C_D A \rho |u|^2 \quad (2-14)$$

The introduction of $\rho |u|$ (i.e. $|u|$) ensures that the correct sign is maintained.

The drag force per unit length, $F_D'$, on a cylinder with diameter $D$, thus becomes

$$F_D = \frac{1}{2} C_D D \rho |u|^2 \quad (2-15)$$
If the body is placed close to a solid boundary, like the sea bed, when subjected to unsteady flow, it will experience a streamline effect—a phenomenon similar to that in steady flow. The value of $C_D$ decreases and $C_L$ increases. In this respect, Beckmann and Thibodeaux (1962) report that from Wieghardt's (1943) experiments, it appears that the mere presence of the boundary causes a reduction of drag coefficient of about 33%. (The shapes tested by Wieghardt included flat plates, circular cylinders and diamond shaped columns).

The behaviour of $C_D$ will be dealt with in more detail later.

2.3.3.2 Lift forces

As in the case of drag, lift in unsteady flow situations differs from that in steady flow, mainly on account of different wake patterns. Yamamoto, et al (1974) say "...lift forces will exist from a nearby plane boundary and vortex shedding will occur and a rather confused flow state will exist as the wake is swept around the cylinder".

In unsteady flow, lift is generated from asymmetrical flow around the body, resulting from both vortex shedding and the proximity of a boundary, such as the sea floor.

The steady state equation for lift is still used, namely

$$ P_L = \frac{1}{2} C_L D \rho u^2 $$

(2-9)

but it must be borne in mind that $C_L$, for unsteady flow, is a variable coefficient and $P_L$ has an instantaneous value because $u$ is changing. The more the body is streamlined and the closer it is to a solid boundary, the larger $C_L$ will be. For a cylinder, the direction of lift is always perpendicular to the axis of the cylinder and to the direction of flow. If it is in contact with a boundary, the lift is always directed away from the boundary. If, however, even a very small gap exists between the cylinder and the boundary, then according to Yamamoto, et al (1974), the instantaneous force will be directed towards the boundary. This implies that, in certain cases, the lift coefficient $C_L$ may have negative values.

The importance of the lift force is often neglected in comparison
with other forces, viz. the drag and inertial forces. Laird, et al (1960), Laird (1961), and Laird (1962) investigated lift forces on rigid and flexible oscillating cylinders. They found that, in general, these transverse forces were highly dependent upon the dynamic response of the structure. Particularly large lift forces are generated if the frequency of natural vibration of the structure is about twice the wave frequency. For a flexible cylinder, Laird (1962) found that lift forces could occur which were, in magnitude, over four times the drag force, based on uniform steady flow at the same velocity.

At a later stage the behaviour of $C_L$ will be dealt with more closely.

2.3.3.3 Inertial forces

When considering unsteady flow, as opposed to steady flow, we are confronted with flows that are subjected to accelerations.

To cause the acceleration of a constant mass, $m$, or, more generally, to change the state of an existing motion, it is necessary to apply to this mass force, $F$, which causes an acceleration $\frac{du}{dt}$, such that

$$ F = m \frac{du}{dt} $$

This is known as Newton's second law.

The product $m \frac{du}{dt}$ is the so-called inertial force which characterizes the natural resistance of matter to any change in its state of motion.

Consider a body placed in the unsteady flow, of a frictionless incompressible fluid, Fig. (2-13).

---

Figure 2-13  Body in the unsteady flow of a frictionless, incompressible fluid
The force due to acceleration effects, acting on the body is given by

\[ F_I' = (M_O + M_a) \dot{u} \]  

(2-17)

where

- \( F_I' \) = inertial force acting on entire body
- \( M_O \) = mass of fluid displaced by body = \( \rho V \)
- \( M_a \) = so-called added mass
- \( (M_O + M_a) \) = so-called virtual mass
- \( \dot{u} = \frac{du}{dt} \) = acceleration of the fluid which would occur at the position of the centre of the body if the body were not there.

A physical interpretation of this force is the following:

The term \( M_O \dot{u} \) represents the force - sometimes referred to as the "Froude-Krylov force" - that would have acted on the mass of displaced fluid had the body not been there. But since the body is there, it is an obstruction and the fluid must be accelerated to flow around the body; this additional force is represented by the term \( M_a \dot{u} \).

Eq. (2-17) can be rewritten as

\[ F_I' = \left[ 1 + \frac{M_a}{M_O} \right] M_O \dot{u} \]  

(2-18)

or

\[ F_I' = C_I M_O \dot{u} \]  

(2-19)

where

\[ C_I = \left[ 1 + \frac{M_a}{M_O} \right] \]  

(2-20)

= the inertial coefficient

The inertial force per unit length, \( F_I' \), on a circular cylinder with diameter \( D \), can be expressed as

\[ F_I = C_I \frac{\pi}{4} D^2 \dot{u} \]  

(2-21)

The inertial coefficient, \( C_I \), is not constant, but like \( C_D \) and
$C_L$, dependent on many factors. These will still be discussed in detail. In classical non-viscous hydrodynamics, $C_I$ depends on the shape of the body only. Lamb (1945) has shown theoretically that, in this case, $C_I = 2.0$ for the right circular cylinder, and $C_I = 1.5$ for a sphere.
CHAPTER 3

THE KEULEGAN-CARPENTER PERIOD PARAMETER

3.1 INTRODUCTION OF THE PERIOD PARAMETER

Keulegan and Carpenter (1958) did laboratory experiments to determine drag and inertial coefficients of cylinders and plates in a sinusoidally oscillating fluid. They could not find any relationship between these coefficients and the Reynolds number, $R_e$, but found that the variation of the coefficients could be correlated with a parameter

$$K = \frac{u_{\text{max}} T}{D}$$  \hspace{1cm} (3-1)

where $u_{\text{max}}$ = the amplitude of the harmonically changing velocity $T$ = period of oscillation $D$ = diameter of the cylinder

Ever since the publication of Keulegan and Carpenter's paper, their work has been referred to and discussed by numerous researchers. The parameter $K$, known as the "period parameter" of the Keulegan-Carpenter number, is a very useful one. Yamamoto, et al (1976) aptly call it a "wake parameter", because it is closely related to the phenomenon of wake formation.

From linear wave theory we know that

$$u_{\text{max}} = \frac{\pi H}{T} \frac{\cosh \left[ \frac{2\pi (y+d)}{L} \right]}{\sinh \left( \frac{2\pi d}{L} \right)}$$  \hspace{1cm} (1-7)

whereas the maximum horizontal displacement from the centre of the orbit (i.e. semi-diameter), $A$, is

$$A = \frac{H}{2} \frac{\cosh \left[ \frac{2\pi (y+d)}{L} \right]}{\sinh \left( \frac{2\pi d}{L} \right)}$$  \hspace{1cm} (1-13)

Thus,

$$K = \frac{u_{\text{max}} T}{D} = \frac{1}{D} \frac{\pi H}{T} \frac{\cosh \left[ \frac{2\pi (y+d)}{L} \right]}{\sinh \left( \frac{2\pi d}{L} \right)}$$

$$= \pi \frac{2A}{D}$$  \hspace{1cm} (3-2)

$$= \pi \cdot \text{total displacement of water particles}$$

$$\text{diameter}$$
2A/D is called the relative amplitude and K is therefore also a measure of relative displacement of water particles.

3.2 SIGNIFICANCE OF THE PERIOD PARAMETER

In the course of their experiments, Keulegan and Carpenter (1958) also examined flow patterns around cylinders and plates for varying values of K. The flow pattern was visually examined by introducing a jet of coloured liquid on one side of the immersed object.

When K was small, of the order of 4, there was no separation of flow - the liquid near the cylinder clung to the cylinder, and the drag was "negligible". As K was increased to about 10, an eddy formed, but remained close to the body. For K = 17, it was found that there was complete separation of flow and eddy formation followed.

At K = 110 there were regular von Karman vortices forming alternately from above and below the horizontal cylinder, and a large drag force resulted.

A critical condition for cylinders occurred when K = 15. From Eq. (3-2).

\[
\frac{2A}{D} = \frac{1}{\pi} K = \frac{15}{\pi} = 4.8
\]

It was found that, at this value, the distance travelled by a water particle with respect to the cylinder diameter, was just sufficient to form a single eddy.

Thus K = 15 appeared to be a cut-off point: for smaller values of K the distance travelled was too small to achieve sufficient separation to form a complete eddy, and thus the drag remained small. For K > 15, separation took place, eddies formed and the drag was considerably greater.

Whereas the Strouhal number plays an important role in vortex shedding phenomena under steady flow conditions, it is the period parameter that is important in oscillatory flow.
Provided that the frequency of vortex shedding, \( f_v \), is exactly equal to the frequency of oscillation, \( f_o \), (which does not always happen), the Strouhal number is the reciprocal of the period parameter; i.e. provided that \( f_v = f_o \), then,

\[
S = \frac{f_v D}{u_{\text{max}}} = \frac{f_o D}{u_{\text{max}}} = \frac{1}{T} \frac{D}{u_{\text{max}}} = \frac{1}{K}
\]  

(3-3)

The exact numerical values of \( K \) at which the various eddy, or vortex, phenomena occur are not undisputed. Subsequent studies by, amongst others, Bidde (1971), Isaacson (1974), Isaacson and Maull (1976) and Chakrabarti, Wolbert and Tam (1976), have generally shown the transitions to occur at period parameter values lower than Keulegan and Carpenter (1958) suggested.

Table (3-1) outlines the \( K \) values at which the transitions are thought to take place.

In the Keulegan-Carpenter experiments the flow was essentially one-dimensional; in the other experiments it was two-dimensional. Chakrabarti, et al (1976) state that eddies may be expected to form earlier in two-dimensional flow than in one-dimensional flow.

Another possible reason for discrepancies is that, in the case of a vertical cylinder, the value of the period parameter is not constant along the cylinder length, but decreases with increasing depth, because the maximum horizontal component of orbital velocity, \( u_{\text{max}} \), decreases with depth. The period parameter for a vertical cylinder is therefore some weighted average value, whereas for a horizontal cylinder, which is deeply submerged and has a relatively small diameter, \( u_{\text{max}} \) and thus the period parameter may, rightly, be considered fixed for the cylinder as a whole.

The exact \( K \) values at which transitions occur are not important at this stage. What is important is the fact that when \( K \) becomes large, the flow approaches steady flow - with regular vortex shedding. It is thus not surprising if forces start resembling steady flow forces. The total in-line force at high period parameters in oscillatory flow is, therefore, mainly drag and to a lesser extent inertial.

Conversely, if the period parameter is small, the flow reverses
<table>
<thead>
<tr>
<th>REGIME</th>
<th>No Separation</th>
<th>Single eddy forms, remains close to cylinder</th>
<th>Eddy formation and separation of flow</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Keulegan and Carpenter (1958)</td>
<td>about 4</td>
<td>about 10</td>
<td>≥ about 15</td>
<td>Horizontal cylinder at node of stationary wave</td>
</tr>
<tr>
<td></td>
<td>≤ about 2</td>
<td>2 to 3</td>
<td>≥ about 3</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>≥ 3 to 4</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>One eddy shed per half cycle</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Two eddies shed per half cycle</td>
<td></td>
</tr>
<tr>
<td>Bidde (1971)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>≤ about 2</td>
<td>about 4</td>
<td>about 6 to 15</td>
<td>Vertical cylinder in wave flume</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>about 16</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Vertical cylinder in wave flume, and</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Vertical cylinder oscillated in still water</td>
</tr>
<tr>
<td></td>
<td>&lt; 6 to 7</td>
<td></td>
<td>about 7 to 15</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>about 16</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3-1 Values of $K = \frac{u_{max}}{T/D}$ associated with various regimes of eddy shedding
its direction before the wake structure can develop properly on the downstream side of the body. The drag force (consisting of a skin friction component and a pressure drag component) does not have time to be fully realised, with the result that the total in-line force is dominated by the inertial force.

Consequently, it can generally be said that "the shorter the wave period, the lower the wave height and the deeper the water and the greater the diameter, the more important is the effect of the inertial term with respect to the drag term". (Wiegel (1964), p.267). Note that all these factors lead to small period parameters.
CHAPTER 4

FORCES ON A HORIZONTAL PIPELINE NEAR THE SEA BED

A pipeline situated near the sea bed commonly experiences forces generated by waves as well as currents.

A current may be caused by tides, winds, the longshore energy component of waves, temperature gradients in the water etc., and may have a constant velocity and direction, or be unsteady.

When one is concerned with the forces acting on a pipeline, however, the velocity of a current may usually be treated as steady since it does not change at nearly the rate of wave-induced velocities.

In the general case, the direction of the current will differ from the direction of wave approach and both these directions will intersect the line of the pipe at some oblique angle.

Thus, the resultant forces acting on the pipeline will have components in the longitudinal direction (i.e. parallel to the pipe axis) as well as in the directions perpendicular to the axis of the pipeline. The longitudinal components are usually of lesser interest because of the strength of the pipeline in this direction.

The forces acting at right angles to the axis of the pipeline are:

1. Hydrostatic forces (vertical)
2. Hydrodynamic forces (horizontal and vertical)
   2.1 Due to currents - (steady flow)
   2.2 Due to waves - (unsteady, oscillatory flow)

One difficulty when having to calculate wave-induced hydrodynamic forces on a submerged body, is that the values of the water particle velocities and accelerations vary from point to point. Therefore, in estimating forces on submerged bodies weighted average values should be used in estimating these forces. In the case of by far most pipelines, the diameter is so small compared to the wave length of the stormwaves for which the pipeline is designed, that the weighted average values can conveniently be assumed to be the velocities and accelerations, respectively, of a water particle situated at the
4.1 HYDRODYNAMIC FORCES DUE TO STEADY CURRENTS

The resultant direction in which the current flows at the site of the pipeline may not necessarily be horizontal, due to thermal gradients, effects of wind, sea bed configuration or some other reason.

In most cases it will be very close to horizontal and can be treated as such within the accuracy of engineering computation.

Assuming a horizontal current velocity, \( U_c \), the forces created by such a flow will be a drag force, \( F_D \), in the direction of the current and a vertical lift force \( P_L \).

\[
P_L = C_L \cdot D \cdot \frac{1}{2} \cdot \rho \cdot U_c^2
\]

\[
F_D = C_D \cdot D \cdot \frac{1}{2} \cdot \rho \cdot U_c^2
\]

Figure 4-1 Hydrodynamic forces due to a steady current

4.2 HYDRODYNAMIC FORCES DUE TO WAVE ACTION

Wave-induced forces deserve special attention. Versowski and Herbich (1974) comment "Of primary concern to the designer working in an ocean environment (where water depths range from intermediate to shallow) are the forces due to gravity waves which are usually of considerably greater magnitudes than the forces due to currents".

Consider the basic case where the direction of wave attack is perpendicular to the pipe axis, that is, the wave crests are parallel to the pipe. Let the horizontal water particle velocity component,
at any instant, be \( u \), the vertical velocity \( v \), the horizontal acceleration \( \dot{u} \) and the vertical acceleration \( \dot{v} \).

It has been shown in Eqs. (2-12) and (2-13) that the in-line force, \( F \), on a horizontal pipeline, and the transverse force, \( P \), are generally due to velocity effects and acceleration effects.

In turn, the velocity effects may be generated both by \( u \) and by \( v \).

Thus, the first term in Eq. (2-12) may be written as

\[
[F_{vel} + F_{vel}^u] = [F_{vel} + F_{vel}^u] + [F_{vel} + F_{vel}^v]\]

\[
= F_{D_u} + F_{D_v}\quad (4-1)
\]

The acceleration effects may be attributed to both \( \dot{u} \) and \( \dot{v} \):

\[
[F_{accln} + F_{accln}^u] = [F_{accln} + F_{accln}^u] + [F_{accln} + F_{accln}^v]\]

\[
= F_{I\dot{u}} + F_{I\dot{v}}\quad (4-2)
\]

The in-line force, \( F \), may therefore, in very general terms, be expressed as

\[
F = F_{D_u} + F_{D_v} + F_{I\dot{u}} + F_{I\dot{v}}\quad (4-3)
\]

\[
= F_D + F_L + F_I + F_W\quad (4-4)
\]

Where \( F_D' \), \( F_L' \), \( F_I' \), and \( F_W' \) are merely different symbols but which conform more closely to the notation commonly used in the literature.

Following a similar reasoning, the transverse force, \( P \), may in quite general terms, be expressed as

\[
P = P_{D_u} + P_{D_v} + P_{I\dot{u}} + P_{I\dot{v}}\quad (4-5)
\]

\[
= P_L + P_D + P_W + P_I\quad (4-6)
\]
The eight possible hydrodynamic forces resulting from wave action are therefore:

1. Due to $u$: a horizontal drag and a vertical lift force.
2. Due to $v$: a horizontal lift and a vertical drag force.
3. Due to $\dot{u}$: a horizontal inertial and a vertical inertial force.
4. Due to $\dot{v}$: a horizontal inertial and a vertical inertial force.

This system of forces - which may presently be considered as conjectural - is summarized in Fig. (4-2).

![Figure 4-2 Instantaneous hydrodynamic forces due to waves](image)

From the outset, it must be stressed that, since the particle velocity in any one direction is always a quarter period out of phase with the particle acceleration, the maximum total force in any direction is not merely the linear sum of the maximum values of the various hydrodynamic forces.
Under certain conditions some of the forces listed in Fig. (4-2) are very small compared to others and can thus be omitted.

Close to the sea floor the vertical particle velocity, $v$, is very small, so that the vertical drag force $F_D$, and the horizontal lift force $F_L$, both of which are proportional to $v^2$ can be neglected.

This can be illustrated best by an example:-

![Diagram](image)

Figure 4-3 Hypothetical example of a pipeline subjected to a design wave

The maximum particle velocities and accelerations at a point 0,5 m above the sea floor, according to the Airy theory are:

$$
\begin{align*}
    u_{\text{max}} &= 2,812 \text{ m/s} \\
    v_{\text{max}} &= 0,048 \text{ m/s} \\
    \dot{u}_{\text{max}} &= 1,178 \text{ m/s}^2 \\
    \dot{v}_{\text{max}} &= 0,020 \text{ m/s}^2
\end{align*}
$$
The maximum values of the various forces would be:

<table>
<thead>
<tr>
<th>Maximum vertical force components</th>
<th>Forces due to horizontal components of motion</th>
<th>Forces due to vertical components of motion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( P_{L_{\text{max}}} = C_L \cdot 1.5 \cdot 1000 \cdot (2,812)^2 )</td>
<td>( P_{D_{\text{max}}} = C_D \cdot 1.5 \cdot 1000 \cdot (0.048)^2 )</td>
</tr>
<tr>
<td></td>
<td>( = C_L \times 3954 \text{ N/m} )</td>
<td>( = C_I \times 1 \text{ N/m} )</td>
</tr>
<tr>
<td></td>
<td>( P_{W_{\text{max}}} = C_W \cdot 1000 \cdot \frac{\pi}{4}(1)^2(1,178) )</td>
<td>( P_{I_{\text{max}}} = C_I \cdot 1000 \cdot \frac{\pi}{4}(1)^2(0.020) )</td>
</tr>
<tr>
<td></td>
<td>( = C_W \times 925 \text{ N/m} )</td>
<td>( = C_I \times 16 \text{ N/m} )</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Maximum horizontal force components</th>
<th>Forces due to horizontal components of motion</th>
<th>Forces due to vertical components of motion</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>( F_{D_{\text{max}}} = C_D \cdot 1.5 \cdot 1000 \cdot (2,812)^2 )</td>
<td>( F_{L_{\text{max}}} = C_L \cdot 1.5 \cdot 1000 \cdot (0.048)^2 )</td>
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<tr>
<td></td>
<td>( = C_D \times 3954 \text{ N/m} )</td>
<td>( = C_L \times 1 \text{ N/m} )</td>
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<td></td>
<td>( F_{I_{\text{max}}} = C_I \cdot 1000 \cdot \frac{\pi}{4}(1)^2(1,178) )</td>
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<td></td>
<td>( = C_I \times 925 \text{ N/m} )</td>
<td>( = C_W \times 16 \text{ N/m} )</td>
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</tbody>
</table>

Notes:

1. Assumption \( \rho = 1000 \text{ kg/m}^3 \)
2. The coefficients \( C_D, C_L, C_I \) and \( C_W \) have not been assigned any numerical values.
3. The maximum total (or combined) force in any one direction is not merely the linear sum of the maximum force components because the components are not in phase.

The coefficients \( C_D, C_L, C_I \) and \( C_W \) are of the same order of magnitude, the values being somewhere between 0.5 and 5.0 (this will be dealt with in more detail elsewhere).

For this (rather severe) case, it can be seen that the forces resulting from the vertical particle velocity, \( v \), are, indeed, much smaller than the forces generated by the horizontal velocity and horizontal acceleration.
A more general proof may be provided by considering the ratio

\[ \frac{v_{\text{max}}}{u_{\text{max}}} \]

From Eqs. (1-7) and (1-8) it follows that

\[ \frac{v_{\text{max}}}{u_{\text{max}}} = \frac{\sinh \left[ 2\pi \frac{(y+d)}{L} \right]}{\cosh \left[ 2\pi \frac{(y+d)}{L} \right]} = \tanh \left[ 2\pi \frac{(y+d)}{L} \right] \]

and, since \( \tanh x = x \) for a small \( x \)

and \( \frac{(y+d)}{L} = \) distance above sea bed \( \frac{\text{wave length}}{\text{is small}}, \)

it follows that

\[ \frac{v_{\text{max}}}{u_{\text{max}}} \approx 2\pi \frac{(y+d)}{L} \quad (4-16) \]

i.e. \( v_{\text{max}} \ll u_{\text{max}} \) near the sea floor.

Also the vertical particle acceleration, \( \ddot{v} \), and the corresponding inertial forces are relatively small compared to lift, drag and inertial forces resulting from the horizontal components of motion. The longer the wave period, and the deeper the water, the smaller \( \ddot{v} \) becomes.

The existence of \( F_D, F_I, P_L \) and \( P_I \) are commonly acknowledged in the literature dealing with wave forces on submarine pipelines. It has been shown that \( P_D \) and \( F_L \) are extremely small for a pipe close to the sea bed - in fact, they may be neglected. What about \( P_W \) and \( F_W \)?

Compared to the other horizontal forces \( F_W \) has almost no significance and can be omitted. \( P_W \) appears to be worth considering as a vertical force (unless the coefficient \( C_W \) is very small).

Grace and Nicinski (1976) state "very close to the bed under long waves \( \ddot{v} \) is virtually zero". After studying wave forces on pipelines in the ocean, they reported that the ratio \( \frac{v_{\text{max}} D}{u_{\text{max}}^2} \), according to stream function wave theory for the test waves, was never greater than 0.03.

Consequently, they preferred to disregard \( P_I \) and introduce what they called a "horizontal-vertical inertial" force. This force corresponds to \( P_W \); see Eq. (4-8), Table (4-2).

They commented that \( C_W \) "has no apparent theoretical justification". Nevertheless, Grace seems to have faith in this strategy because it is no
new idea of his; in earlier papers, Grace (1973) and Grace (1971), he already advocated this approach.

In a discussion on a paper by Yamamoto, Nath and Slotta (1974), Grace (1975) remarks that, although the vertical inertial force $P_I$, may be significant in some laboratory experiments, it has only a second order effect in real-life situations. He quotes two examples, the Sand Island Outfall No.2, off Honolulu, Hawaii, and a pipeline at Barber's Point near Honolulu. In the Sand Island case the ratio (maximum vertical inertial force)/(maximum vertical lift force), (i.e. $P_I/P_L$) is only 0.05; and in the second case the ratio is substantially smaller: only 0.0015. (In both these examples the ratio $(y+d)/d$ is approximately 0.02).

Grace then proceeds to say, that although it has been shown that, even under the best of design situations, the vertical inertial force, $P_I$, is only of secondary importance compared to the vertical lift force, $P_L$, "it does not appear complete generally to write the vertical force only as a lift term. A not entirely satisfactory attempt to make use of a horizontal acceleration in determining an inertial type of vertical force has been attempted and appropriate coefficients derived from experiments".

Yamamoto, Nath and Slotta (1974), however, consider both $P_L$ and $P_I$ to be important. (It has been indicated above that their practical work involved laboratory-scale forces and not prototype forces).

Also Garrison, Gehrman and Perkinson (1975) support the viewpoint that $P_I$ is significant. It must, however, be remembered that their research was directed more towards wave forces on large cylinders, in which case little or no separation takes place and inertial forces generally remain dominant.

Garrison, et al (1975) measured forces on cylinders mounted on the bed of a wave channel. They correlated the forces with two parameters, $2d/D$ and $gT^2/d$, where the symbols have their usual meanings.

They found that at small values of $gT^2/d$ (i.e. short period waves in deep water) the total vertical force was sinusoidal which indicated dominance of the inertial effect. At larger values of $gT^2/d$ the
effect of $P_L$ became relatively more important. The maximum total vertical force occurred when the wave trough passed over the cylinder since, at this instant, both $P_L$ and $P_I$ were directed upwards. At very large values of $gT^2/d$ and smaller values of $2d/D$ (i.e. long period waves in shallow water and large diameter cylinders) $P_L$ became the controlling factor and the maximum total vertical force now occurred at the passage of the wave crest. This is explained by the fact that in shallow water the wave crests are peaked and the orbital velocities are greater at the crests than at the troughs, so that $P_L$ at a crest is also larger. The total vertical force is thus also largest at the crest notwithstanding $P_I$ being directed downwards, in other words, opposing $P_L$.

From the preceding arguments the following conclusions are made:

1. The vertical drag force, $P_D$, as well as the horizontal lift force, $F_L$, on a pipeline near the sea bed can be ignored since the vertical velocities are very small.

2. The vertical lift force, $P_L$, resulting from the horizontal orbital velocity, is definitely significant.

3. The vertical inertial force, $P_I$, due to the vertical orbital acceleration may or may not be important. This very much depends on the parameters $2d/D$, $gT^2/d$ and probably $H/D$. In prototype situations, i.e. real pipes on the sea bed, Grace's arguments seem valid: the inertial force in the vertical direction is considerably smaller than the lift force. The conclusions of Garrison, et al (1975) also confirm this.

4. As an alternative to $P_I$, consideration may be given to $P_W$.

The significant wave-induced forces acting on a pipeline near the sea bed will thus be those shown in Fig. (4-4).
4.3 NOTES ON COMBINING CURRENT-INDUCED FORCES AND WAVE-INDUCED FORCES

In this dissertation it is assumed that the calculated force due to the current is added to the force calculated for the wave action.

Wiegel (1964), p. 255, is not sure about this - "it is difficult to know the proper way to determine the forces when both currents and waves are present". Reid (1956) suggested that the combined horizontal drag force be taken as:

$$F_D = C_D D \frac{1}{2} \rho \left| U_c + u \right| (U_c + u)$$  \hspace{1cm} (4-17)

Presumably, the combined horizontal drag when the wave motion opposes the current would be

$$F_D = C_D D \frac{1}{2} \rho \left| U_c - u \right| (U_c - u)$$  \hspace{1cm} (4-18)

The next uncertainty, according to Wiegel, is whether the drag coefficient obtained for oscillatory flow is satisfactory for the steady component, $U_c$, as well.

Hogben (1974) briefly mentions that there are at least three ways in which currents interact with waves to influence wave forces:

1. When a wave train enters a current field its properties are...
substantially modified. For example, if a current, of even low velocity, opposes the propagation direction of the waves, their steepness can be considerably increased. (This phenomenon is treated in papers by Taylor (1955), Longuet-Higgins and Stewart (1961) and Betts (1970)).

2. The modification of the flow velocity past the body, as indicated in Eqs. (4-17) and (4-18), may have a marked effect on wave forces. Since the drag force is proportional to the square of the velocity, such a modification in velocity can influence the drag force to a large extent.

3. The presence of a current can substantially modify the wake and eddy structure and lead to effects on the forces which do not even approximately correspond to those of the linear superposition of velocities (i.e. Eqs. (4-17) and (4-18)).

4.4 STABILITY OF A PIPELINE SUBJECTED TO WAVE- AND CURRENT-INDUCED FORCES

For a body to remain in equilibrium, the requirements are that the sum of forces in three perpendicular directions are zero, and the moments about three perpendicular axes are also zero.

Consider a pipeline resting on a horizontal rocky sea bed and subjected to current and wave action. Assume the direction of the current and the wave orthogonals are at right angles to the axis of the pipe. Refer Fig. (4-5).

![Balance of forces](image)

Figure 4-5 Balance of forces
In this ideal reference case the stability problem is reduced to a two-dimensional one. The pipeline will be stable if it is not lifted off the sea bed and does not slide along the sea bed or roll about its axis.

To prevent it being lifted off the sea bed, the submerged weight, \( W_{\text{eff}} \) must be larger than the total upward hydrodynamic force. (If it is not, lifting can still be prevented by, for instance, tying the pipe down to the sea floor. In other words, the reactive force, \( Y \), is kept positive).

To prevent the pipe from sliding horizontally, the frictional resistance force,

\[
F_{\text{friction}} = \text{friction coefficient} \times Y \quad (4-19)
\]

must always be capable of balancing the total horizontal hydrodynamic force.

It is important to note that \( F_{\text{friction}} \) is proportional to \( Y \), and \( Y \) is reduced by any upward acting hydrodynamic forces.

The most likely mode of failure is a horizontal displacement of the pipe, resulting from the friction force being reduced by the vertical hydrodynamic forces to such a level that it is exceeded by the horizontal hydrodynamic forces.

Rolling can normally be prevented by a combination of the torsional rigidity of the pipe and sufficiently wide base blocks spaced at intervals along the length of the pipe.

This ideal situation is, of course, seldom encountered in practice. The pipe is often unsupported over stretches along an uneven bottom and then bending moments are induced in the pipe. Hydrodynamic forces will rarely be uniform over the length of the pipe since orbital velocities and accelerations may be higher in some regions due to changes in water depth, profile of the sea floor, localized peaks in the wave crests, etc. Furthermore, apart from being laid on the sea bed, the pipe can, according to a classification by Grace (1973), be laid:
1. in a trench which is not backfilled;
2. on the sea bed and be stabilized laterally and on top by layers of stone;
3. in a trench and backfilled, in other words, completely buried.

These alternatives are only mentioned here. In all three cases the pipe is more protected and will be more stable than in the reference case dealt with above.
CHAPTER 5

METHODS OF CALCULATING IN-LINE FORCES

Wiegel, et al (1957) and Wiegel (1964) mention two methods: the method developed by Morison, O'Brien, Johnson and Schaaf (1950) which involves the formula, now commonly known as the Morison equation, and the method of Crooke (1955) which is based upon the study by Iversen and Balent (1951).

A third method, the so-called "diffraction theory method", has been developed by MacCamy and Fuchs (1954) for large-diameter vertical piles, and been modified by Garrison and Rao (1971) for a submerged hemisphere, and by Garrison and Chow (1972) for a submerged body of arbitrary shape.

There is also a fourth method, which this writer calls the "method of dimensionless wave parameters".

The first method is by far more popular and widely used than the second. The third method only becomes applicable if the object is large compared to the length of waves impinging on it, and only in exceptional cases would it be relevant to submarine pipelines. The fourth method has the disadvantage that it requires much experimental data to make it of general use; yet, it has its applications.

5.1 MORISON EQUATION METHOD

It is assumed that the total in-line force on a submerged body in unsteady flow may be obtained by linear addition of the instantaneous values of inertial force and drag force, i.e.

\[ F' = F'_D + F'_I = C_D \rho \frac{D^2}{4} \ddot{u} + \frac{1}{2} C_I \rho |u| u \]  

(5-1)

where

\[ F' = \text{the total in-line force acting on the entire body.} \]

More specifically, for a circular cylinder with diameter D and placed at right angles to the flow, the Morison equation becomes

\[ F = C_I \frac{D}{4} \ddot{u} + \frac{1}{2} C_D \rho |u| u \]  

(5-2)
where

\[ F = \text{the total in-line force per unit length of cylinder}. \]

This method, originally developed for vertical piles, is well entrenched in ocean engineering practice. Since its formulation in 1950 ideas may have changed as to how the equation should be interpreted, but the basic concept that the in-line force generally consists of a velocity dependent drag component and an acceleration dependent inertial component remains unchanged.

It is worthwhile to note some of the assumptions on which the equation is based, as well as some of its limitations.

Wiegel (1964) points out that the equations for the inertial force and drag force are based on two assumptions:

1. The object is at a sufficient depth below the free surface so that it may be considered to be in a fluid of infinite extent.

2. The flow is treated as though it were uni-directional.

The Morison equation usually violates both these assumptions. The drag force in particular, is open to question as the wave-induced flow periodically reverses direction causing the wake formed on the leeside to become the fluid at the leading edge of the body at the beginning of the next phase in the flow cycle.

Provided that the calculated results are useful and within the range of engineering accuracy, there is no objection to the assumptions being violated.

It is important to realise from the start that the user of the Morison equation generally needs to know, apart from the cylinder and fluid properties, four quantities: \( C_B \), \( C_I \), \( u \) and \( \dot{u} \). The latter two are calculated with the aid of some wave theory whilst the former two are obtained empirically. It is, however, essential that the coefficients are compatible with the wave theory used to predict the kinematics; the reason being that different wave theories may yield different values for the particle kinematics.
The linear, or Airy, wave theory is commonly used for calculating the kinematics, and in most cases the coefficients have been derived for use in conjunction with this theory.

5.1.1 Some basic shortcomings of the Morison equation approach

Almost all problems related to wave force computations can be blamed on the complex behaviour of the coefficients, because if it were possible to know the appropriate coefficient values at any instant, under any condition, the force could be calculated without difficulty. Unfortunately, evaluations of the coefficients, over the years, have displayed a great deal of scatter.

1. An apparent reason for the scatter in coefficient values is the degree of dimensionality of the flow. In most laboratory studies the flow has been two-dimensional or virtually one-dimensional. Coefficient values obtained under such circumstances have proved to be somewhat inaccurate for three-dimensional flow as encountered in the sea.

2. It has been established beyond doubt that $C_D$ and $C_I$ vary during the wave cycle, but their instantaneous values cannot be determined without making some assumptions (such as that they have "symmetrical" values about certain phases of the cycle). Thus it is customary to assign average-over-the-wave-cycle values to the coefficients. But since the designer is primarily interested in the maximum force and not the average force, this is not entirely satisfactory.

3. Hogben (1974) draws attention to the fact that unless the body is quite secluded, there may be interference between neighbouring bodies. (A number of pipelines bunched together and a group of piles serve as examples). The result is that the coefficients as well as the kinematics change and the Morison equation will yield an inaccurate estimate of the in-line force.

4. He also states that the equation is not adequate for predicting forces in the "splash zone" of, say, lattice
structures where, for instance, certain horizontal members may be subjected to severe impact loads.

5. He issues a warning on a further point: whereas the Morison equation may be used to design structural members in areas having relatively moderate sea conditions with the occasional severe storm, (e.g. the Gulf of Mexico, where the Morison equation was developed), it may prove inadequate to design similar members situated in areas with a heavy incidence of rough weather (such as the North Sea), because fatigue failure may be the more important criterion.

6. A very important objection to the Morison equation, made by Hogben (1974), is the difficulty associated with the non-linearity of the drag term containing the factor $|u|u$. In a spectrum the velocity, $u$, at any instant, is the sum of a number of contributions from component waves of different frequencies, and the corresponding drag is proportional to the square of this sum which cannot be determined by linear addition of components.

7. In steady flow $C_D$ for a cylinder is dependent on the Reynolds number, $R_e$. Indications are that $C_D$ in unsteady flow is dependent on at least $R_e$ and the period parameter, $K$. As prototype experiments in the ocean are expensive and difficult to control, most investigations to determine the coefficients have been performed in laboratories. But in laboratories it is difficult to achieve the high Reynolds numbers that are often experienced in the ocean. As a result, the designer usually has to employ coefficient values derived at rather low $R_e$.

8. Nath and Yamamoto (1974) argued that drag forces were actually due to the convective acceleration of the flow; in fact, all hydrodynamic forces on submerged objects were considered to be due to acceleration effects. They felt that the Morison equation could give "a misconception that drag forces are not due to acceleration effects".

9. It must be ensured that the object is small enough not to influence the incident wave (by, for instance, diffracting
or reflecting it). Garrison and Chow (1972) agree that the Morison equation gives a valid approximation of the in-line force if the lineal dimension of the object is small compared to the wave length. Hogben (1974) and Vongvisessomjai and Silvester (1976) quantify this limit:

If \( \frac{D}{L} > 0.2 \) the Morison equation is no longer applicable.

5.1.2 The maximum value of the total in-line force predicted by the Morison equation:

According to the Airy wave theory the water particle velocity in any direction is always a quarter cycle out of phase with the acceleration. Consequently, the drag force is also a quarter cycle out of phase with the inertial force, and the maximum value of their sum generally occurs at some intermediate stage - between the stages of maximum drag and maximum inertial force, see Fig. (5-1).

![Diagram showing wave profile and forces](image)

According to the Airy wave theory the water particle velocity in any direction is always a quarter cycle out of phase with the acceleration. Consequently, the drag force is also a quarter cycle out of phase with the inertial force, and the maximum value of their sum generally occurs at some intermediate stage - between the stages of maximum drag and maximum inertial force, see Fig. (5-1).

Grace (1973) gives the following equations for the maximum value of the combined in-line force, \( F_{\text{max}} \) in the horizontal direction:

\[
F_{\text{max}} = \begin{cases} 
F_{I_{\text{max}}} & , \text{if } \beta > 1 \\
F_{D_{\text{max}}} & , \text{if } \beta \leq 1 \\
(1 + \beta^2) F_{D_{\text{max}}} & 
\end{cases}
\] (5-3)
where

\[ F_{\text{max}} = \text{the maximum inertial force during the wave cycle} \]
\[ = C_D \frac{\pi}{4} D^2 \max \]
\[ F_D = \text{the maximum drag force during the wave cycle} \]
\[ = \frac{1}{2} C_D D \max \max \]
\[ \beta = \frac{\pi}{4} \frac{C_I}{C_D} \frac{1}{A/D} \]
\[ = \frac{\pi^2}{2} \frac{C_I}{C_D} \frac{1}{K} \]
\[ A = \text{the horizontal semi-diameter of the particle orbit.} \]

These equations are based on the Morison equation and the Airy theory.

As can be seen in Fig. (5-1), \( F_{\text{max}} \) (positive) (i.e. in the direction of wave propagation) occurs between the rising "still-water" stage and the wave crest stage, whilst \( F_{\text{max}} \) (negative) occurs between the falling "still-water" stage and the wave trough stage.

An alternative equation for \( F_{\text{max}} \), in dimensionless form, is provided by Rance (1969): 
\[ \frac{F_{\text{max}}}{\rho D^2 \sigma^2} = \frac{C_D}{2} \left( \frac{A}{D} \right)^2 + \frac{\pi}{32} \frac{C_I^2}{C_D} \]
where \( \sigma = 2\pi/T \)

Also this equation is derived from the Morison equation and the Airy theory.

5.2 CROOKE'S METHOD

The method, described by Crooke (1955), is based on a study by Iversen and Balent (1951) who found that, when the velocity was linearly dependent upon the acceleration, the total force exerted on a flat disk, being accelerated in one direction through still water,
could be expressed as

\[ F = \frac{1}{2} C_A \rho |u| u \]  

(5-7)

where \( A_x \) is the projected area normal to the flow and \( C \) is a coefficient which depends not only upon the shape of the body, Froude modulus, Reynolds number, friction, and Mach number, but also upon a modulus, \( \dot{u} D / u^2 \), where \( D \) is the diameter of the disk. (Wiegel (1964), Keulegan and Carpenter (1958)).

This modulus, now known as Iversen's modulus, is related to the period parameter, \( K \), in the case of sinusoidal oscillatory flow. Using linear wave theory, it is easy to show that

\[ \frac{\dot{u}_{\text{max}} D}{u_{\text{max}}^2} = \frac{2\pi}{K} = \frac{1}{A/D} \]  

(5-8)

This method has not gained nearly as much support as the Morison equation and will not be considered further in this dissertation.

5.3 DIFFRACTION THEORY METHOD

The present writer suspects that the reason for calling this method the "diffraction theory method" may be that it was originally developed, by MacCamy and Fuchs (1954), for large diameter cylinders standing vertically and piercing the water surface - a situation where the phenomenon of diffraction would be noticeable.

This method may be used to calculate the in-line force in cases where the object size is so large that the waves are diffracted and/or reflected. This is deemed to occur when the lineal dimension of the object exceeds one fifth of the wave length, or, for a cylinder, when \( D/L > 0.2 \).

Vongvisessomjai and Silvester (1976) also specify a second prerequisite: the period parameter must be smaller than \( \pi \); in other words the relative amplitude, \( A/D = K/2\pi \), must be less than \( \frac{1}{4} \), i.e. the overall horizontal displacement of the water particles, \( 2A \), must be smaller than the cylinder diameter. If \( K < \pi \), flow separation
takes place to such an extent that viscous effects - which are not allowed for in the diffraction theory analysis - cannot be neglected.

Typical diffraction theory calculations are performed by computer as vast volumes of numerical operations are involved.

The three basic steps in the process are:

1. A total velocity potential is defined in the vicinity of the object.

2. From this, the dynamic pressure distribution around the object is calculated.

3. The forces are found by integrating the pressures over the surface of the object.

The procedure will be dealt with more closely in the review of the work by Garrison and Chow (1972).

5.4 METHOD OF DIMENSIONLESS WAVE PARAMETERS

Some investigators, such as Garrison, Gehrman and Perkinson (1975), have determined relationships between maximum forces and certain dimensionless parameters describing the wave.

Garrison, et al (1975), plotted dimensionless values of the maximum force, measured in wave flume experiments, versus the parameters \(2d/D, gT^2/d\) and \(H/D\) and found that the plotted points followed consistent curves.

Vongvisessomjai and Silvester (1976), recommend this method for calculating forces on vertical members (where the maximum particle velocities change considerably along the length of the member) in fairly shallow water, i.e. when \(d/L < 0.3\). Silvester (1974) adopts this approach in his suggested procedure for computing forces on vertical piles. (See his Tables 8 - IV, 8 - V, and 8 - VI). Paape and Breusers (1966) used a similar approach in their study of forces on square piles.

One important disadvantage of this method is the overwhelming amount of data that is required. (Garrison, Gehrman and Perkinson (1975)).
PART II

A CHRONOLOGICAL REVIEW OF RESEARCH

INTO HYDRODYNAMIC FORCES

ON SUBMERGED CYLINDERS
CHAPTER 6

PUBLICATIONS PRIOR TO 1960

The more significant studies of hydrodynamic forces on cylinders and the behaviour of the associated force coefficients performed before about 1960 are dealt with by Wilson and Reid (1963), Wiegel (1964) and Dean and Harleman (1966) and are mentioned briefly below.

6.1 MORISON, et al (University of California, at Berkeley)

The popular method for calculating wave-induced forces on rigid bodies, and more specifically, cylindrical piles, now known as the Morison equation, was described in a paper by Morison, et al (1950). Further laboratory investigations followed and the results published in papers by Morison (1951) and Morison, Johnson and O'Brien (1954).

6.2 MacCamy AND FUCHS, 1954 (Beach Erosion Board, U.S. Army Corps of Engineers)

MacCamy and Fuchs (1954) did not use the same approach as Morison, et al (1950) because they were interested in wave forces acting on large diameter vertical piles, where the wave did not pass the object without being affected.

They therefore formulated a diffraction theory, to describe the interaction between waves and large piles. In the limiting case of small diameter pile to wave length ratios, the results of this theory become equivalent to those produced by the Morison equation. (Garrison and Rao (1971)).

6.3 HARLEMAN AND SHAPIRO (Massachusetts Institute of Technology)

Their approach was to calculate wave forces on piles, assuming $C_D = 2.0$, as given by the so-called diffraction theory (MacCamy and Fuchs (1954)) or potential flow considerations, and assuming a $C_D$ value as given by the steady-state results versus Reynolds number relationship. Stokes' third-order wave theory was used to predict
water particle velocities and accelerations. To calculate the Reynolds number, the root-mean-square velocity along the piling was used.

The next step was to compare these calculated forces with forces measured in a laboratory. According to Dean and Harleman (1966), the agreement was, "in general, good".

The relevant publications are Harleman and Shapiro (1955) and Harleman, Shapiro and Marlow (1957).

6.4 WILSON AND REID (Texas A. and M. University)

Reid (1957) and Wilson (1957) reported on field measurements they had done in the Gulf of Mexico. They measured total forces and fluctuations of the water surface, calculated the kinematics of the water particles, and then the values of $C_D$ and $C_I$ using a best least-squares fit technique. (Dean and Harleman (1966)).

6.5 WIEGEL, et al (University of California, at Berkeley)

Wiegel, Beebe and Moon (1957) studied ocean wave forces on vertical piles. Typical wave periods were in the 10 to 20 second range, water depth approximately 15 m and the wave height as high as 6 m. Pile test sections were 0.3 m high, mounted on supporting piles, and varied from 0.17 m to 1.5 m in diameter. With the aid of the linear wave theory, water particle kinematics were predicted from measured waves, the relevant characteristics being wave height and period and water depth.

$C_D$ was computed from the total in-line force measured on the pile at the instant of maximum horizontal velocity and zero horizontal acceleration, in other words, the crest and trough phases. Similarly, $C_I$ was obtained at the "still-water" phases - when the horizontal acceleration was maximum and the horizontal velocity zero.

The Reynolds number, calculated from the peak velocity value, was between $3 \times 10^6$ and $9 \times 10^5$. They could find no well-defined relationship between $C_D$ and $R_e$. Fig. (6-1) shows a plot of $C_D$ values obtained by Wiegel, et al (1957) as well as Reid (1956),
Figure 6-1  \( C_D \) versus \( R_e \) for circular cylindrical piles (Wiegel (1964), p.258)
Wilson and Reid (1963) and Morison, et al (1950), versus $R_e$.

They also found no relationship between $C_I$ and $R_e$, or the water particle acceleration. $C_I$ showed a very slight tendency to increase with increasing wave period.

They found $C_I$ to be a normal Gaussian distribution with mean value $2.5$ and standard deviation $1.2$. The mean of $2.5$ is somewhat greater than the theoretical value of $2.0$ for ideal flow as derived by Lamb (1945) and MacCamy and Fuchs (1954).

Although they were not interested in transverse forces, they noticed large lateral vibrations due to vortex shedding under certain conditions of testing. In fact, one of the test piles actually broke after several days; this turned out to be a fatigue failure.

6.6 KEULEGAN AND CARPENTER, 1958

The paper by Keulegan and Carpenter (1958) has become a milestone in its field.

They investigated the drag and inertial coefficients of horizontal cylinders and plates, fixed normal to the flow direction, at the node of stationary waves oscillating in a rectangular tank. The motion was horizontal (not elliptical orbits) and varied sinusoidally. The flow was thus essentially one-dimensional. Transverse forces were not analyzed, but the mechanism of vortex formation was studied.

Keulegan and Carpenter found that the average values of $C_D$ and $C_I$ over a wave cycle changed when the maximum (calculated) velocity of water flowing past the cylinder, $u_{max}$, was altered and when the diameter, $D$, of the cylinder was varied. Neither $C_D$ nor $C_I$ appeared to be dependent on the Reynolds number, $u_{max} D/\nu$, but both were found to be a function of the period parameter, $K = u_{max} T/D$.

Fig. (6-2) shows the variation of $C_D$ and $C_I$ with respect to the period parameter.
Figure 6-2 $C_D$ and $C_I$ versus $K$ for a circular cylinder

Bearing in mind that the period parameter, $K = \pi \cdot 2A/D$, is also a measure of the relative displacement of water particles, the explanation given by Keulegan and Carpenter for the behaviour of $C_D$ and $C_I$ is worth noting:

For very small values of $K$, when there is no separation, $C_D$ is fairly small (about 0.9) while $C_I$ is approximately equal to the theoretical value of 2.0. As $K$ increases towards 15, where, according to the authors, a complete single eddy forms, $C_D$ attains its maximum value (about 2.5) and $C_I$ its minimum (about 1.0). For still larger values of $K$, when numerous eddies form, $C_D$ gradually decreases to the value that would be applicable to steady flow, and $C_I$ gradually increases to a value of 2.5 at $K = 120$.

The values of $C_D$ and $C_I$, shown in Fig. (6-2) are mean values for the entire wave cycle. Keulegan and Carpenter also studied the instantaneous values of the coefficients.

They achieved this by assuming that the coefficient values at an instant $t_1$ seconds before a particular still-water phase was equal to the values at an instant $t_1$ seconds after that particular phase, on the grounds that the accelerations at these two instants were equal and the velocities, although of opposite sign, were equal in absolute magnitude. Similar assumptions were made regarding the crest phase and trough phase. (Incidentally, Sarpkaya (1975) observed that this
assumption was not quite correct because the distribution of skin friction at the times considered was not identical, and that there was no simple way to accurately determine the instantaneous values of \( C_I \) and \( C_D \).

Keulegan and Carpenter concluded that the coefficients could vary during a wave cycle, (K being kept constant).

\( C_D \) was found to remain at its mean value except for a small interval about the zero velocity phases, i.e. maximum acceleration phases, when \( C_D \) would become several times greater than the mean value. On the other hand, \( C_I \) was found to remain virtually constant throughout the wave cycle provided that \( K \) was either very small or large. If, however, \( K \) was in the vicinity of 15, \( C_I \) displayed considerable variation: between about -2 and 2. (Neither Keulegan and Carpenter (1958) nor Wiegel (1964) could explain the existence of negative values for \( C_I \). McNown and Learned (1978) recently suggested an explanation for negative added mass coefficients, \( C_m \), in other words, for \( C_I < 1.0 \). This is dealt with in the review of Garrison, et al (1977)).

The variation of \( C_D \) and \( C_I \) during a wave cycle, for three different \( K \) values, is shown in Fig. (6-3).

![Figure 6-3 Variation of \( C_D \) and \( C_I \) during a wave cycle, at various values of \( K \)](image)
Wilson and Reid (1963) summarized the results of significant research programmes carried out before 1960, see Table (6-1).

### Table 6-1 \( C_D \) and \( C_I \) values for Circular Cylinders in Accelerating Flows

<table>
<thead>
<tr>
<th>Authority and date</th>
<th>Nature of Experiments</th>
<th>Cylinder diameter in mm</th>
<th>Coefficient Value</th>
<th>Type of flow; Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crooke, 1955</td>
<td>Model</td>
<td>50, 25, 13</td>
<td>( C_D ) 1.60, ( C_I ) 2.30</td>
<td>Oscillatory</td>
</tr>
<tr>
<td>Keulegan and Carpenter, 1958</td>
<td>Model</td>
<td>75, 63, 50</td>
<td>( C_D ) 1.34, ( C_I ) 1.46</td>
<td>Oscillatory (av. of 29 tabulated values)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>38, 30</td>
<td>( C_D ) 1.52, ( C_I ) 1.51</td>
<td>(av. of 57 tabulated values)</td>
</tr>
<tr>
<td>Keim, 1956</td>
<td>Model</td>
<td>25, 13</td>
<td>( C_D ) 1.00, ( C_I ) 0.93</td>
<td>Accelerated, non-oscillatory</td>
</tr>
<tr>
<td>Dean, 1956</td>
<td>Model</td>
<td>75</td>
<td>( C_D ) 1.10, ( C_I ) 1.46</td>
<td>Accelerated, non-oscillatory</td>
</tr>
<tr>
<td>Wiegel et al, 1957</td>
<td>Prototype</td>
<td>610</td>
<td>( C_D ) 1.00, ( C_I ) 0.95</td>
<td>Ocean Waves, Californian Coast (Based on their Fig.15).</td>
</tr>
<tr>
<td>Reid, 1957</td>
<td>Prototype</td>
<td>170</td>
<td>( C_D ) 0.53, ( C_I ) 1.47</td>
<td>Ocean Waves, Gulf of Mexico</td>
</tr>
<tr>
<td>Bretschneider, 1957</td>
<td>Prototype</td>
<td>400</td>
<td>( C_D ) 0.40, ( C_I ) 1.10</td>
<td>Ocean Waves, Gulf of Mexico</td>
</tr>
<tr>
<td>Wilson, 1957</td>
<td>Prototype</td>
<td>760</td>
<td>( C_D ) 1.00, ( C_I ) 1.45</td>
<td>Ocean Waves, Gulf of Mexico</td>
</tr>
</tbody>
</table>

Note that Wilson and Reid disregarded the effect of period parameter and merely computed average values when they drafted Table (6-1). Dean and Harleman (1966), in reviewing this table, again computed averages and found:

\[
\frac{C_D}{C_I} = 1.05 \\
\frac{C_D}{C_I} = 1.40
\]

The roughness of the cylinder would also influence the drag coefficient and is another reason for the wide scatter of values in Table (6-1).
CHAPTER 7

PUBLICATIONS BETWEEN 1960 AND 1969

7.1 BRATER, McNOWN AND STAIR, 1961 (University of Michigan, Ann Arbor)

With the aid of models, they studied the magnitude and characteristics of wave forces on submerged barge-like structures. They concluded that since there was no flow separation from the large objects, the inertial force was all-important compared to the drag force — in fact, only the coefficient of inertia was required to predict the maximum force.

7.2 RESEARCH AT WALLINGFORD, 1961

The Hydraulics Research Station at Wallingford, England, has been involved for apparently almost two decades in investigations on wave forces. The reports, usually produced annually, are rather brief on account of results being kept confidential.

Nevertheless, some useful information can be extracted from these reports and will be summarized in this dissertation.

Hydraulics Research (1961) states that wave (and current-) induced forces on a 1/16 model of a submarine pipe lying on the sea floor were studied during that year. Orbital velocities were measured throughout the wave cycle with a miniature current meter and orbital diameters by direct observation of a float.

Values were obtained for $C_D$ and $C_I$ for horizontal forces as well as vertical forces. It was found that the coefficients varied both with wave characteristics and throughout the wave cycle. Another approach was then adopted:

<table>
<thead>
<tr>
<th>Maximum horizontal force</th>
<th>and</th>
<th>force at instant of maximum velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D L \cdot \frac{1}{4} \rho u^2$</td>
<td></td>
<td>$D L \cdot \frac{1}{4} \rho u^2$</td>
</tr>
</tbody>
</table>
these coefficients were plotted as a family of curves of equal $D^2/TV$ against $2A/D$, where all the symbols have their usual meanings.

It was thought that these dimensionless parameters would be sufficient to determine the nature of the motion. $2A/D$ (which is the period parameter divided by $T$) describes the geometry of motion, while $D^2/TV$ actually is a Reynolds number, $D.2A/TV$, divided by $2A/D$.

Some results were given, refer Fig. (7-1).

![Graph](image)

**Figure 7-1** Horizontal force coefficient versus relative displacement amplitude for cylinder lying on floor

The investigators came to the conclusion that $D^2/TV$ was an important parameter in determining the coefficient, but since its value in the prototype was large it could not be reproduced in the model.

Subsequently, Grace (1971) re-analyzed the raw data obtained in this study, and calculated drag, inertial and lift coefficients. It appeared that $C_D$ roughly decreased with increasing Reynolds number.
Also, $C_D$ values for the unsteady flow case were considerably larger than in the steady flow case.

The point to be made is that the steady-state drag and lift coefficients should, in general, not be applied to the unsteady flow case. The same conclusion was made by Font (1967).

Fig. (7-2), by Grace (1971) shows $C_D$ values for unsteady flow (from the Wallingford data) for a pipe lying on a floor plotted versus Reynolds number, together with the steady-state $C_D$ curve and some other experimental results.

![Figure 7-2](image-url)

Figure 7-2 $C_D$ versus $R_e$ for a cylinder near and remote from a boundary, (data from various sources), Grace (1971)
Up to this stage most research had been done on coefficients for cylinders away from a solid boundary, such as the sea floor. Beckmann and Thibodeaux (1962) investigated the case where the cylinder was in contact with a solid boundary. They were among the first to point out that hydrodynamic lift forces could be of critical importance to a submarine pipeline, and since its publication, their paper has been quoted and discussed by numerous authors.

They considered drag and lift forces on two shapes: a horizontal circular cylinder resting on a floor and a trapezoidally profiled block resting on a floor. Their conclusions were based upon basic principles and aerodynamic studies, performed mainly on steady flow situations, by various researchers. Alterman (1962), in discussion, stated that Beckmann and Thibodeaux "had to make simplifications, the validity of which have yet to be justified".

They estimated that $C_D$ for a smooth circular cylinder would be in the range 0,35 to 0,40 regardless of whether it was in contact with the solid boundary or not. Their recommended $C_D$ value for a rough pipe was about 0,5.

Regarding the lift coefficient, $C_L$, their estimate for a smooth pipe in contact with the boundary was in the range 0,35 to 0,65; their recommended value for a rough pipe was about 0,5. It is important to note that they took into account only one mechanism whereby lift can be generated: non-symmetrical flow around the body caused by the presence of the boundary; in other words, the role played by vortex shedding was ignored. Therefore, $C_L$ was taken as zero for the case where the cylinder was unaffected by the proximity of the floor. They suggested that a pipe could be considered to be so positioned if the clearance between ground and pipe was at least 0,1 pipe diameter.

Beckmann and Thibodeaux also considered the inertial coefficient, $C_I$. They stated that the $C_I$ values obtained by Wiegel, et al (1957) for free-standing piles showed a wide spread, and no experimental data
for $C_I$ for the case of a cylinder attached to a boundary were available by 1962, and therefore an estimate of the $C_I$ value would be crude. They expected $C_I$ to fall between 1.0 and the theoretical potential flow value of 2.0, and suggested a value of 1.5.

In a discussion of this paper, Wilson and Reid (1963) express the opinion that the recommended values for $C_D$, $C_L$, and $C_I$ are all on the low side, and produce their table (Table 6-1). They admit that factual experimental information is very scarce, but suggest that $C_D$ should be taken as at least 1.0 for pipelines in contact with the sea bed.

For a cylinder, in contact with a solid boundary, they derive an upper limit of $C_L$ for special case of potential flow, namely 4.48. They also derive a lower limit for the real flow case, namely 0.74. They conclude that, for design purposes, $C_L > 1.0$ for a pipe in contact with the sea bed.

Similarly, they show that for potential flow a theoretical upper-boundary for $C_I$, for a cylinder in contact with the bed, is 3.30. They reason that, since the average value of $C_I$ in their table for the case of a circular cylinder away from the influence of a boundary is approximately 1.5, and since the theoretical value of $C_I$ for such a case is 2.0, $C_I$ for a cylinder in contact with the bed could be $3.30 \times 1.5 = 2.5$.

7.4 RESEARCH AT WALLINGFORD, 1962

The oscillatory flow tests done during 1961 were confined to Reynolds numbers not exceeding $1.7 \times 10^3$, which were lower than those applying in the prototype. In order to achieve higher Reynolds numbers ranging between $3 \times 10^3$ and $9.5 \times 10^5$, 1/3-scale models of pipe sections together with a plate representing the sea bed were towed through still water.

The experiments performed during 1962 were, therefore, carried out under uni-directional flow conditions.
Lift and drag forces were measured separately. Six different shapes of pipe sections were investigated including a rough and a smooth circular cylinder.

The $C_D$ and $C_L$ versus $Re$ plots, as determined from these tests, are shown in Fig. (7-3).

![Figure 7-3](image)

Figure 7-3 $C_D$ and $C_L$ versus $Re$ for a circular cylinder in contact with a boundary, uni-directional unsteady flow, Wallingford (1962)

It appears that typical $C_D$ values for a smooth pipe in contact with the bed, are about 0.6 to 0.9 and for a rough pipe 0.8 to 1.1, while $C_L$ is mostly around 0.5 to 0.6 for a smooth pipe, and about 0.2 for a rough pipe.

Roughening of the surface increased the drag only marginally but reduced the lift substantially (about 67%). As stated in Hydraulics Research (1962), "the cylinder lying on the sea bed constitutes quite a good aerofoil, particularly when it is smooth...".

The fact that the tests were performed in uni-directional flow must make one hesitant to apply these results to oscillatory wave motion. Further errors, although small, were introduced by the presence of a free surface close to the pipe model in the channel, and the existence of a wave crest in front of the obstacle and a trough shortly behind.

7.5 RESEARCH AT WALLINGFORD, 1965, 1966

According to Hydraulics Research (1965), the research done on submarine pipelines in that year was confined to aspects of sediment movement and scour around the pipe.
During approximately 1966 a pulsating water tunnel was constructed at Wallingford whereby forces acting under oscillatory flow at high Reynolds numbers, could be measured. Full size circular pipe sections up to 760 mm diameter could be installed in this water tunnel.

Tests showed that lift forces were higher than previous work (e.g. 1961 - 1962 results) had indicated, and were in fact of the same order as the drag forces. "However, the variation of forces during the wave-cycle showed a pattern other than that given by a consideration of the summation of drag and inertia terms", that is, as given by the Morison equation. "It was found that the lift force was at no time zero and that secondary peaks or maxima occurred".

It was also reported that "an analytical study demonstrated that all the characteristics could be accounted for by including wake effects. The new theory evolved closely predicts the forces measured".

The "new theory", involving wake effects, is presumably that covered by P.J. Rance in Hydraulics Research (1969) and will be discussed in the review of the Wallingford, 1969, research programme.

7.6  **BROWN, 1967**  (Bechtel Corporation, San Francisco)

Brown (1967) produced a paper describing experiments performed during the early 1960's in which the effects of transverse horizontal currents on a horizontal pipeline, in contact with the sea bed, were studied. In actual pipe environmental conditions, velocities to 3.6 m/s had been observed in rivers and areas of large tidal variations.

He thought that the hydrodynamic forces caused by such currents could be of the utmost importance to the lifespan and stability of a pipeline, but were usually the least recognized. Numerous failures of pipelines indicated underdesign of the stability aspect, whereas many pipelines were excessively over-designed with resultant excess expenditure.

The work of Brown (1967), although not specifically on wave-induced forces, but on current-induced forces, is described herein because it does improve one's understanding of hydrodynamic forces.
acting on a pipe in contact with a solid boundary.

The experiments were carried out in this manner:

A length of pipe was fixed to the floor of a test flume, perpendicular to the flume axis. While a steady current flowed past the pipe, the pressure distribution around the periphery of the pipe was measured. A typical pressure distribution is shown in Fig. (7-4).

![Diagram showing positive and negative pressures on a pipe](image)

Figure 7-4  Typical pressure distribution on a pipe in contact with the floor, steady flow

From these pressures, forces were obtained which were then resolved into horizontal and vertical components. By summation of all the horizontal components the total horizontal drag force, $F_D$, was obtained. The total vertical lift force, $P_L$, was found by summation of all the vertical components.

Coefficients of drag, $C_D$, and lift, $C_L$, and the Reynolds number, $R_e$, were calculated:

$$C_D = \frac{F_D}{\frac{1}{2} \rho u^2 D}$$

$$C_L = \frac{P_L}{\frac{1}{2} \rho u^2 D}$$

$$R_e = \frac{u D}{v}$$
Pipe diameters used in the experiments were 150 mm and 250 mm, water depth 460 mm and the maximum current velocity 1.8 m/s. The Reynolds number varied between $0.6 \times 10^5$ and $3.0 \times 10^5$. The pipes had smooth surfaces, although an attempt was made to simulate the effect of discontinuities of smoothness by attaching a "spoiler" to the pipe.

Brown, although giving no graphs of $C_D$ and $C_L$ versus $Re$, draws the following conclusions from this study:

1. A pipe resting on the sea bed experiences lift and drag forces like the wing of an airplane.
2. In the range of Reynolds numbers $0.6 \times 10^5$ to $3.0 \times 10^5$, $C_D$ varied from 0.90 to 0.55 and $C_L$ from 1.3 to 0.8.
3. $C_L$, therefore, exceeded $C_D$ by about 50% in the range of $Re$ tested.
4. The drag coefficient, $C_D$, was smaller when the pipe was in contact with the bed than when there was a clearance between pipe and bed.
   (Beckmann and Thibodeaux (1962) had come to the same conclusion).
5. Spoilers on the pipe altered the hydrodynamic forces and their coefficients to a considerable extent.

Larock (1967) criticized Brown by saying that, although the experiment involved the direct application of basic principles, the scope of the project was too limited and the results thus obtained correspondingly limited. He, as well as Font (1967) and Sutco (1967), felt that the proximity of the free surface could have influenced the hydrodynamic forces considerably because the water depth was only about two or three pipe diameters. He pointed out that the Froude number was an appropriate index to use in describing shallow submergence flow situations whereas the Reynolds number became important with deep submergence.

Font (1967) reported that his organization (Universidad Central de Venezuela) had performed similar experiments (apparently also involving the measurement of pressure distributions), but obtained
different values for $C_D$ and $C_L$, refer Table (7-1). (It appears that the submergence was 5.5 diameters as opposed to 2 or 3, in Brown's case).

<table>
<thead>
<tr>
<th>Reynolds Numbers $R_e = uD/\nu$</th>
<th>Froude Numbers $F_r = u/\sqrt{gd}$</th>
<th>Drag Coefficient $C_D$</th>
<th>Lift Coefficient $C_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.97 \times 10^5$</td>
<td>0.29</td>
<td>0.85</td>
<td>1.48</td>
</tr>
<tr>
<td>$0.79 \times 10^5$</td>
<td>0.24</td>
<td>0.83</td>
<td>1.35</td>
</tr>
<tr>
<td>$0.68 \times 10^5$</td>
<td>0.20</td>
<td>0.90</td>
<td>1.13</td>
</tr>
<tr>
<td>$0.53 \times 10^5$</td>
<td>0.16</td>
<td>0.93</td>
<td>1.11</td>
</tr>
</tbody>
</table>

Table (7-1) $C_D$ and $C_L$ in steady flow, Font (1967)

Possible sources for these discrepancies are:

1. Brown could have overestimated the reference velocity, and therefore, the Reynolds number.
2. The influence of the free surface, mentioned above.
3. Differences in separation patterns on the leeside of the pipe would have a marked effect on the hydrodynamic forces generated.

Font then warned against using drag and lift coefficients obtained from steady flow experiments, for calculating forces caused by oscillatory waves. He quoted the Wallingford Report No. Ex. 158 (1961), on wave forces, wherein $C_L$ was found to be as large as 4.5, if the Morison equation were assumed to be valid. This value was very close to the theoretical value of 4.48 derived from potential flow considerations and quoted by Wilson and Reid (1963). Font thought that this agreement was not too strange because since it took time for a wholly separated zone to develop on the downstream side of a body, the peak lift forces (in the case of the Wallingford oscillatory flow experiments, anyway) probably occurred when potential-like flow was still present.
7.7 **JOHANSSON, 1968** (Royal Institute of Technology, Stockholm)

As reported by Grace (1971), Johansson (1968) performed wave force experiments in a flume. Regular waves were generated and the particle kinematics predicted from the Airy theory. Reynolds numbers ranged from approximately $3 \times 10^3$ to $2 \times 10^4$.

$C_I$ was found to lie between 2.8 and 4.0 and $C_L$ between 1.8 and 6.0 for a constant clearance between the pipe and the bottom.

The effect of clearance on the coefficients was studied to a limited extent and the results are reflected in Table (7-2).

<table>
<thead>
<tr>
<th>Relative clearance h/D</th>
<th>Horizontal drag coefficient $C_D$</th>
<th>Horizontal inertial coefficient $C_I$</th>
<th>Lift coefficient $C_L$</th>
<th>Vertical inertial coefficient $C_I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>2.0</td>
<td>3.3</td>
<td>4.0</td>
<td>3.0</td>
</tr>
<tr>
<td>0.25</td>
<td>1.1</td>
<td>2.4</td>
<td>0.0</td>
<td>1.9</td>
</tr>
<tr>
<td>1.0</td>
<td>1.1</td>
<td>2.0</td>
<td>0.0</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Table 7-2 Effect of bottom clearance on coefficients, Johansson (1968)

7.8 **LEBRETON AND CORMAULT, 1969**

According to Garrison and Chow (1972), Lebreton and Cormault (1969) derived a diffraction theory and applied it to large diameter vertical cylinders. Their results agreed with those of MacCamy and Fuchs (1954) and with their own experimental results for small amplitude waves. Viscous effects appeared to become significant at larger wave heights but they could not establish a clear definition of non-linear effects.
Rance (1969) reported on research into wave forces on cylinders at Wallingford with the aid of a pulsating water tunnel. The tests were done on cylinders placed fairly remote from boundaries.

Using linear wave theory, the Morison equation can be written as

\[ F = C_D \frac{1}{2} \rho \pi D^2 \sigma^2 \sin^2 \varpi t + C_I \rho \frac{1}{2} \pi D^2 \sigma^2 \cos \varpi t \]  

\( (7-1) \)

(Note:

1. In Eq. (7-1) it is implied that \( \varpi t = 0, 2\pi, \) etc. coincides with the "rising still-water" phase. The convention generally followed in this thesis is that \( \varpi t = 0, 2\pi, \) etc. coincides with the crest phase.

2. Strictly speaking, "mod" signs should have been included in the first term of the right hand side of Eq. (7-1) (instead of writing "\( \sin^2 \)") in order to allow for the reversibility of the drag force.)

The instant at which the maximum total force occurs is given by:

\[ \cos \varpi t = \frac{\pi}{4} \frac{C_I}{C_D} \frac{D}{A} = \cos \phi \]  

\( (7-2) \)

where \( \phi \) is the phase angle of the maximum force, with respect to the phase of "rising still-water".

The maximum in-line force, \( F_{\text{max}} \), is given by:

\[ F_{\text{max}} = \frac{C_D}{2} \left[ \frac{A}{D} \right]^2 + \frac{\pi}{32} \frac{C_I^2}{C_D} \]  

\( (5-6) \)

Since the phase angle, \( \phi \), could be determined from the records it was possible to calculate the values of \( C_D \) and \( C_I \) as follows:

\[ C_D = \frac{2F_{\text{max}}}{\rho D^3 \sigma^2 (A/D)^2 (1+\cos^2 \phi)} \]  

\( (7-3) \)

and

\[ C_I = \frac{4}{\pi} \frac{A}{D} \cos \phi. C_D \]  

\( (7-4) \)
Rance and his co-investigators found that if the A/D ratio was less than 10 and the total force measured was of reasonable magnitude, $C_I$ was reasonably constant with a value of 2.0.

For A/D values greater than 10, the second term in Eq. (5-6), contributes very little to the total force. Above this limit, it was also not possible to determine $\phi$ with sufficient accuracy to yield a reasonable estimate of $C_I$.

For a very small value of A/D, the forces measured were very small, giving rise to a large scatter in $C_D$ values, and consequently also in $C_I$.

In view of the apparent constancy of $C_I$, namely 2.0 in the range mentioned, Rance, et al re-analyzed their data using $C_I = 2.0$ rather than relying on the estimate of $\phi$.

In this way a reduction in scatter was achieved, and they proceeded to seek a relationship between $C_D$ and the Reynolds number.

The Reynolds number at the instant of maximum force was found to be:

$$R_e = \frac{D}{V} \sigma a \left[ 1 - \left( \frac{\pi C_I D}{4 C_D A} \right)^2 \right]^2 \quad (7-5)$$

They produced Fig. (7-5), which shows $C_D$ as a function of $R_e$ for various A/D ratios.
Rance points out that the A/D ratio has a marked effect, particularly for values less than $10$ at $R_e < 5 \times 10^4$. The critical point which occurs at about $R_e = 5 \times 10^5$ in uni-directional flow becomes less pronounced and occurs earlier in oscillatory flow.

Transverse forces generated by vortex shedding, were studied at the same time as in-line forces.

The lift coefficient, $C_{L_{\text{max}}}$, was calculated from

$$C_{L_{\text{max}}} = \frac{2P_{I_{\text{max}}}}{\rho \sigma^2 \lambda^2}$$

(7-6)
Fig. (7-6) illustrates the dependence of $C_L$ upon Reynolds number and $A/D$ ratio.

![Graph showing $C_{L_{max}}$ as a function of $Re$ and $A/D$ for cylinders (fairly remote from a boundary).](image)

The variable nature of the lift force is reflected by the considerable scatter in results. Rance draws the attention to the fact that the transverse forces can be as large or larger than the in-line forces, particularly at low Reynolds numbers.

The frequency of the lift force, $f_L$, appeared to be much higher than the frequency imposed by wave action. A Strouhal number, defined by Rance, as

$$S = \frac{f_L D}{0.5 A} \quad (7-7)$$

was found to be remarkably consistent, being $0.2 \pm 6\%$, and did not vary with either the $A/D$ ratio or the Reynolds number.

This seems to imply that $f_L$ can be calculated from the equation

$$f_L = 0.2 \times \frac{2\pi}{T} \cdot \frac{A}{D} \quad (7-8)$$

or

$$\frac{f_L}{f_o} = 0.2 \quad (7-9)$$

where $f_o$ is the wave frequency.
The high frequency of the lift force may be even more significant than its magnitude. Fatigue failure of a member could result if the exciting frequency of the force should coincide with the natural frequency of the member.

Because of the comparable size of the transverse force, Rance concludes that it is not enough to look at the in-line forces alone. One should consider the resultant force which is the vectorial sum of the transverse and in-line forces.

Hogben (1974) pointed out that although the Wallingford experiments yielded important results at relatively high Reynolds numbers the need for prototype tests at high Reynolds numbers remained urgent. The reason was that the pulsating water tunnel had some artificial characteristics: the rectilinear motion, for instance, led to trapping of the eddy structure behind the cylinder. It was also possible that wall ratio and aspect ratio effects existed.

7.10 GRACE AND CASCIANO, 1969 (University of Hawaii, Honolulu)

This research project, which dealt with spheres, but not cylinders, is mentioned here as it was carried out in the sea and led to further useful field experiments specifically aimed at analyzing wave forces on pipelines near the sea floor.

Grace and Casciano (1969) appraised the behaviour of $C_D$ and $C_I$ under oscillatory wave action, qualitatively:

1. Neither $C_D$ nor $C_I$ was constant over a wave cycle, and they were not independent.

(Note: In the remaining items, reference is presumably made to mean values of $C_D$ and $C_I$, obtained by averaging over the entire wave cycle).

2. Consider the situation of a body positioned away from any boundaries, in oscillatory flow at time $t = 0$, where $u = 0$ and $du/dt$ is a maximum. A short time later, the flow begins to move past the body, but remains, essentially, potential flow, and one would expect $C_I$ to have its theoretical, potential flow, value - in the case of a cylinder it would be 2.0. The drag force remains quite insignificant even at a small velocity, implying that $C_D$ is extremely small.
If the flow velocity returns very quickly to zero before moving in the opposite direction, then a true wake structure never really develops and $C_I$ stays at its theoretical, potential flow, value. The inertial force is large and the drag force small.

If, however, the particle travel distance becomes large in relation to the body size ($D$, in the case of a cylinder), in other words, if the ratio $A/D$, or the period parameter, $k = \frac{u_{\text{max}}}{T/D} = \frac{\pi A}{D}$, becomes large, separation of flow occurs, and a wake develops. The drag force becomes significant and the inertial force reduces.

Thus with larger wake formation and larger period parameter, $C_D$ increases and $C_I$ decreases.

This phenomenon has been examined by, amongst others, McNown and Keulegan (1959), and demonstrated experimentally by Sarpkaya and Garrison (1963) for circular cylinders.

3. $C_D$ could possibly be dependent upon the Reynolds number unless, perhaps, the eddy size in the flow was relatively large.

4. $C_D$ could be influenced by the relative turbulence intensity of the approach flow.

5. $C_D$ was dependent on the relative roughness of the body.

6. If the body was not completely rigid, vibrations could exert a very large influence on the coefficients. (Note that vibrations may be caused by transverse forces).

7. It was unreasonable to expect to obtain single values of $C_D$ and $C_I$ from experiments. A distribution of values was more likely.

In fact, they found that different waves with approximately identical wave heights and periods in the same water depth produced a spread in maximum forces and in coefficients. (Wiegel, et al (1957) also experienced this).

Possible reasons for this may be inevitable experimental errors and also subtle differences in the ambient turbulence of the flow.
CHAPTER 8

PUBLICATIONS BETWEEN 1970 AND 1978

8.1 JOHNSON, 1970 (Oregon State University, Corvallis)

Johnson (1970), using a dimensional analysis, derived an expression for the dimensionless total force on a bottom-mounted pipe:

\[
\frac{F}{\rho g D^3} = f \left( \frac{d}{D}, \frac{H}{D}, \alpha \right)
\]  

(8-1)

Where \( \alpha \) is the angle, in radians, between the axis of the pipe and the approaching wave orthogonal.

After obtaining data from flume tests for which the angle, \( \alpha \), was always \( \pi/2 \), and using only one test cylinder, he established equations for predicting \( F/\rho g D^3 \) in terms of \( d/D \) and \( H/D \).

He accomplished this with the aid of regression analysis. The emphasis of his work, as he himself admitted in the final discussion of his paper, was on the application of this statistical technique.

As a result, and also because he confined his study to the shallow water wave case (\( d/L < 1/20 \)), and neglected the effect of wave length and consequently wave period, on the total force, as pointed out by Petrauskas (1971), his contribution is considered of limited value.

A further serious shortcoming in Johnson's work, which has, to this writer's knowledge, not yet been pointed out by another critic, is that the effect of pipe length has been neglected in the dimensional analysis. The force, \( F \), used by Johnson, is not a force per unit length of pipeline, but the force for the entire length. This is borne out by the fact that he writes the dimensionless force as \( F/\rho g D^3 \) and not \( F/\rho g D^2 \). In the final discussion of his paper, he said that the test cylinder used was precisely one foot long. Thus, in his experiments the force per unit length and the force over the entire length happened to be numerically equal.
Bidde (1971) investigated the magnitude and frequencies of transverse forces acting on vertical circular cylinders placed in a wave flume. The cylinders were cantilevered into the water from above and reached only halfway down towards the floor of the flume. End effects were thought to be insignificant on the grounds that measured forces compared favourably with those of Jen (1967), in whose experiments the gap between cylinder and flume floor had been very small.

Bidde pointed out that the lift force was not necessarily zero at the instant of zero velocity. This fact was incompatible with the popular lift force equation

$$F_L = C_L \cdot D \cdot \frac{1}{2} \rho u^2$$  \hspace{1cm} (2.9)

This problem could be sidestepped by defining a relationship for the maximum value of the force only,

$$F_{L_{\text{max}}} = C_{L_{\text{max}}} \cdot D \cdot \frac{1}{2} \rho u_{\text{max}}^2$$  \hspace{1cm} (8.2)

However, he chose to study the ratio of lift force to in-line force, rather than $C_L$.

Lift forces of appreciable magnitude were measured—as high as 60% of the in-line force. There were indications that the maximum ratio of forces might occur at a period parameter value of 15. Dattatri (1972) suggested an explanation to this: When $K > 15$, the flow became so highly turbulent, with so many eddies, that the lift force, which depended on alternate eddy formation, actually decreased.

By spreading a very fine powder on the water surface, detailed visual observations were made of eddy shedding patterns at the surface. The study confirmed that lift forces were associated with eddy shedding.

The period parameter, $K$, was found to be the all-important parameter in studying vortex phenomena and lift forces in waves, while the Reynolds number could be ignored.
Unlike a horizontal pipe with a small diameter relative to water depth and wave length, for which \( K \) may safely be assumed to have one value only, a vertical pile extends through a whole spectrum of maximum water particle velocities, with the result that there is also a whole spectrum of \( K \) values. It is not clear from Bidde (1971) exactly at which depth the period parameter was calculated in the experiments, but it apparently represents some average value of \( K \) over the pile length (Dattatri (1972), Isaacson and Mault (1976)).

Bidde observed the following eddy shedding regimes associated with the period parameter:

- \( K < 2 \) : No flow separation.
- \( K = 2 \) to \( 3 \) : Small separation, no eddies.
- \( K = 3 \) : First eddies formed and shed; lift force begins to be non-zero.
- \( K = 3 \) to \( 4 \) : More than 2 eddies shed within a half cycle; beginning of a von Karman vortex street.
- \( K = 5 \) to \( 7 \) : Turbulent wake.
- \( K > 7 \) : Extremely turbulent wake.

Keulegan and Carpenter (1958) found that vortex shedding started when \( K \approx 15 \), while Bidde found \( K \approx 3 \). He commented that the difference could be due to a variety of factors: they tested horizontal cylinders in virtually one-dimensional flow at the node of a stationary wave, whereas he tested vertical cylinders in the two-dimensional flow of oscillatory waves.

Bidde found the lift force frequency to be random at high values of \( K \). He could not agree with Chang (1964) who had established the frequency of lift force to be twice the wave frequency. Wiegel (1953) had noted that the period of lateral vibrations in a prototype installation was 2.5 seconds whilst the wave period was 13 seconds. This showed that Chang's conclusion was not necessarily correct.
8.3 HERBICH AND SHANK, 1971 (Texas A. and M. University)

Like Brater, et al (1961), Herbich and Shank (1971) found that the forces on models of large submerged structures were almost entirely inertial. They calculated inertial coefficients for the models; the values varied between 1.4 and 2.2 for all the tests. For the various models, they plotted dimensionless forces as families of curves in terms of the wave height to wave length ratio, $H/L$, versus the wave length to water depth ratio, $L/D$. These curves were intended to help designers of large rectangular and cylindrical tanks.

8.4 GRACE, 1971. (University of Hawaii, Honolulu)

Grace (1971) pointed out that, up to 1971, all coefficients available to the pipeline designer had been derived from low Reynolds numbers and were questionable for prototype applications that often involved Reynolds numbers in excess of $10^6$. It was also uncertain whether these coefficients would be valid for the real sea bed as they had been obtained from pipes resting on flat, hard flume floors.

The fact that the roughness of a pipe increased in the sea due to the inevitable growth of marine fauna and flora also clouded the issue.

He came to the conclusion that the design coefficients had to be obtained from experiments in the sea.

In the meantime, he investigated, in a laboratory, the effects upon the horizontal and vertical wave-induced forces, of,

1. clearance between the bottom of the pipe and the flume floor,

and 2. orientation of the pipe with respect to the incident wave fronts.

Because of his further investigations and since some of his earlier findings have been updated, only an outline of his 1971 conclusions is now given:-

He found that the horizontal force decreased substantially as the pipe orientation approached the direction of the wave orthogonals (i.e.
small \( \alpha \). The horizontal force appeared to be rather insensitive to the effect of bottom-clearance.

Regarding the vertical force, it was found to decrease at small \( \alpha \) values and it also decreased with increasing bottom-clearance.

The most severe direction of wave approach to a submarine pipe is perpendicular to the axis of the pipeline, i.e. \( \alpha = 90^\circ \). This has been established by tests performed at Wallingford (Crisp, Stewart and Fletcher (1970)), and according to Grace (1973), by Krieg (1966). Such an angle is, however, rather unlikely to occur in nearshore waters because waves tend to be refracted so that the wave crests turn parallel to the coastline. A more likely approach angle, \( \alpha \) would be at 45\(^\circ\) or less.

8.5 **GARRISON AND RAO, 1971** (Naval Postgraduate School, Monterey, California)

Garrison and Rao (1971) pointed out that the Morison equation was only valid as long as the presence of the body did not affect the incident wave. If the body was large enough to have a scattering effect on the wave, another approach was necessary.

Such an approach would have to account for the effect of relative size as well as the free water surface. "In such an analysis, generally referred to as diffraction theory, viscous effects are neglected and the problem is set up in terms of a velocity potential. Once the velocity potential is found, the pressure distribution on the surface of the object can be determined and, therefore, the forces on the object can be computed".

Using this approach, they developed an analysis for wave interaction with submerged objects, and applied the results to calculate wave forces on a submerged hemisphere. They showed that for small values of the wave height to sphere diameter ratio, \( H/D \), the wave force coefficients could be well represented by the diffraction theory and correlated as functions of two parameters, namely \( \pi D/L \) and \( 2d/D \), where \( L \) is the wave length and \( d \) the water depth. (Garrison and Chow (1972)).
They concluded that diffraction effects were of primary importance in shallow water (small 2d/D ratios), especially for large \( \pi D/L \). However, if the H/D ratio became too large, the linear relationship between wave force and wave height would eventually break down.

8.6 **GARRISON AND CHOW, 1972** (Naval Postgraduate School, Monterey, California)

The authors derived a diffraction theory for submerged objects of arbitrary shape. Theoretically predicted forces were then compared with experimental results obtained from wave channel testing of oil storage tank models.

Over the range of the experimental results, the wave force was found to vary linearly with the wave height so that the dimensionless force coefficients were independent of the parameter H/D. The results could therefore be represented as functions of \( \pi D/L \) and 2d/D (i.e. similar to Garrison and Rao (1971)).

The basic principles involved in performing diffraction theory computations have been summarised by Hogben (1974):

First, the total velocity potential, \( \Phi_T \), which satisfies the boundary conditions is sought. It is assumed to be the linear sum of two components,

\[
\Phi_T = \Phi_I + \Phi_S
\]  

(8-3)

where \( \Phi_I \) is the known velocity potential function of the incident wave and \( \Phi_S \) is an unknown velocity potential function due to the scattered wave.

\( \Phi_S \) is in fact generated by the disturbance pressures of the body surface and may thus be thought of as compounded from a set of component potentials \( \Phi_r \), originating from an array of pulsating pressure points, or sources, of strength \( q_r \) distributed over the surface so that

\[
\Phi_S = \sum_{r=1}^{R} q_r \Phi_r
\]  

(8-4)
Garrison and Chow (1977) express the functions $J_r$ as Green's functions which are suitable for analysis involving arbitrary three-dimensional bodies in shallow water. The boundary condition, with respect to the body, is zero relative velocity normal to the surface. The input information required for formulating this condition includes coordinates defining body geometry by a mesh of surface elements, and the area and direction cosines of the normal for each element. If, in addition, the body is moving, information to define the motion uniquely is also required.

Secondly, having found the total velocity potential, the dynamic pressure distribution around the body is determined from the linearized Bernoulli equation.

Thirdly, the forces are obtained by integrating the pressure distribution over the surface of the body.

The complexity of the body shape which can be handled by diffraction analysis is limited only by the size of computer storage available and the run time that can be afforded.

8.7 BRATER AND WALLACE, 1972 (University of Michigan, Ann Arbor)

Brater and Wallace (1972) produced a report on a laboratory investigation of horizontal in-line forces produced by oscillatory waves impinging at right angles on submerged pipelines.

In this study, a continuous record was obtained of wave height and horizontal force. Four pipe diameters, three wave heights and three wave lengths were used. Forces were measured with the pipe at four locations below the water surface, that is, at various clearances above the bed. Further tests were conducted with the pipes in various positions within trenches.

A large set of $C_1$ values, and to a lesser extent, $C_D$ values were derived.

Brater and Wallace, unlike Grace, believe that the inertial force is usually the predominant force and that the drag force only makes a significant contribution under special circumstances, such as when the
pipe diameter is very small, and the wave length and height are very large. Consequently, more attention was devoted to the inertial coefficient.

It is noted that Brater and Wallace confined their experiments to situations where the wave period was relatively short (6 to 12 seconds in the prototype) and the pipe diameters relatively large (2.4 to 4.5 m in the prototype). Short periods and large diameters tend to make the inertial force large compared to the drag force since flow separation from the body is limited.

The phase angle, $\phi$, at which the total force measured was a maximum, was plotted against the dimensionless parameter $D^2/HL$, and they decided that notwithstanding the scatter, the contribution of the drag force only became important when $D^2/HL$ was very small, say smaller than 0.02 or 0.025. They did not mention the Keulegan-Carpenter period parameter, but this will be considered in due course.

$C_I$ was obtained from the equation

$$ F_I = C_I \rho \frac{\pi}{4} D^2 \cdot \ddot{u} $$  \hspace{1cm} (8-5) $$

The force $F_I$, which was assumed to be the same as the total horizontal force, was assessed in three ways. As a result, three "types" of inertial coefficients were obtained; called CM, CMM and CMMM by the authors, and defined below.

Forces acting in the same direction as the wave propagation were regarded as positive. This can be seen in Fig. (8-1), the inertial force would theoretically reach its peak positive value when the horizontal forward acceleration was maximum, or according to the Airy theory, at the stage when the rising water surface passed through the "still-water" position, i.e. phase angle $= 3\pi/2$. 
Figure 8-1 Phase relationship between water surface elevation, $u, \dot{u}, F_D$ and $F_I$

**CM:** The forces taken were the average forces at phase angles $\frac{\pi}{2}$ and $\frac{3\pi}{2}$, and CM calculated from Eq. (8-5).

Brater and Wallace suggest that CM might be considered the true value of $C_I$.

**CMM:** The forces taken were the average of the maximum positive and negative forces, and CMM calculated from Eq. (8-5).

The authors suggest that "CMM could be used to estimate the average total force, assuming the drag force to be negligible".

**CMMM:** The forces taken were the maximum forces (regardless of sign) obtained from each test.

The authors suggest that CMMM "might be used for conservative estimates of the maximum forces, assuming the drag force to be negligible".
It was found that CM, CMM and CMMM were relatively independent of the diameter, D, and the wave height, H, but varied in a very orderly linear manner with the ratio \( z/L \), where \( z \) is the depth of submergence, measured downwards (positive) from the still-water level to the centre of the pipe.

Statistical techniques were employed and equations of the form

\[
C = b + j \left( \frac{z}{L} \right)
\]  

(8-6)

were derived for CM, CMM and CMMM.

Values of \( b \) and \( j \) are given in Table (8-1).

<table>
<thead>
<tr>
<th>D (mm)</th>
<th>( z/d )</th>
<th>CM</th>
<th></th>
<th>CMM</th>
<th></th>
<th>CMMM</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. 60</td>
<td>1.37</td>
<td>6.56</td>
<td>1.59</td>
<td>5.82</td>
<td>1.79</td>
<td>5.33</td>
<td></td>
</tr>
<tr>
<td>2. 48</td>
<td>1.41</td>
<td>6.36</td>
<td>1.69</td>
<td>5.41</td>
<td>1.91</td>
<td>4.96</td>
<td></td>
</tr>
<tr>
<td>3. 43</td>
<td>1.26</td>
<td>5.87</td>
<td>1.56</td>
<td>5.50</td>
<td>1.75</td>
<td>5.22</td>
<td></td>
</tr>
<tr>
<td>4. 32</td>
<td>1.49</td>
<td>6.65</td>
<td>2.04</td>
<td>5.32</td>
<td>2.38</td>
<td>4.84</td>
<td></td>
</tr>
<tr>
<td>5. ALL</td>
<td>1.38</td>
<td>6.36</td>
<td>1.73</td>
<td>5.37</td>
<td>1.97</td>
<td>4.87</td>
<td></td>
</tr>
<tr>
<td>6. ALL except 32</td>
<td>1.34</td>
<td>6.41</td>
<td>1.61</td>
<td>5.64</td>
<td>1.81</td>
<td>5.22</td>
<td></td>
</tr>
<tr>
<td>7. ALL</td>
<td>2.59</td>
<td>4.83</td>
<td>3.07</td>
<td>4.13</td>
<td>3.20</td>
<td>4.16</td>
<td></td>
</tr>
<tr>
<td>8. ALL</td>
<td>1.0</td>
<td>0.83</td>
<td>3.11</td>
<td>1.01</td>
<td>2.85</td>
<td>1.14</td>
<td>2.78</td>
</tr>
</tbody>
</table>

Table 8-1 Values for the constants \( b \) and \( j \) in Eq. (8-6)

Lines 1 to 4 in Table (8-1) provide the \( b \) and \( j \) values for the four pipe diameters tested at various clearances above the bottom,
line 5 the average values for all four diameters and line 6 the average values for the first three diameters. (It was felt that in the case of the smallest diameter, \( D = 32 \text{ mm} \), the drag force could not be neglected).

Line 7 contains the coefficients for the case where the pipe is approximately in contact with the bed, and line 3 represents a half buried pipe.

Brater and Wallace suggest that, whenever \( D^2/HL < 0.02 \), the equations indicated in line 6, namely:

\[
\begin{align*}
CM &= 1.34 + 6.41 \frac{z}{L} \\
CMM &= 1.61 + 5.64 \frac{z}{L} \\
CMMM &= 1.81 + 5.22 \frac{z}{L}
\end{align*}
\]

be used for coefficients of inertia (for a pipe away from the bottom).

For \( D^2/HL < 0.02 \), they suggested the use of the equations of line 4. They pointed out that such refinements in the coefficient of inertia were perhaps not justified in the light of other uncertainties. The equations of line 5 could, therefore, be accepted regardless of what the pipe diameter was.

It was suggested that design values for the inertial coefficient for a pipe on the bottom could be obtained from the equations of line 7.

From the tests done on pipes located within a trench the following equation emerged:

\[
CMM = 1.0 + 1.4 \frac{z}{L}
\]

It also appeared that when the width of the trench was about 7 times the pipe diameter, the conditions were approaching those without a trench.

To summarise, Brater and Wallace plotted various equations for \( CMM \) versus \( \frac{z}{L} \). See Fig. (8-2).
Figure 8-2  Equations of CMM versus z/L for various clearances

In cases where \( D^2/H \text{L} < 0.02 \), it could be "desirable or necessary to include the drag force". They investigated the behaviour of the drag coefficient and produced two graphs of \( C_D \) versus Reynolds number. The scatter of data was, however, very wide and the Reynolds number quite small (less than \( 2 \times 10^4 \)), and therefore their results are not reproduced herein.

They concluded that the inertial force was the principal term in the in-line horizontal force and, unless \( D^2/H \text{L} \) was very small, this total force could, for design purposes, be calculated with the aid of Eq. (8-5) and either CMM or CMMM.

In an attempt to explain the conflicting statements made by Brater and Wallace on the one hand, and Grace on the other, regarding the dominance or non-dominance of the inertial force, this writer referred to the period parameter of Keulegan and Carpenter.

From the available data in Brater and Wallace's paper it appears that the largest period parameter, \( K \), was about 8. (This is the \( K \)
obtained when combining the largest wave height, the largest wave length and the smallest pipe diameter used in the test series). The smallest $K$ in the series was about 0.3.

It will be recalled that inertial force effects are much greater than drag force effects at low period parameters because proper flow separation from the body does not occur.

Thus it seems that Brater and Wallace's work may be criticised on two points: drag effects were neglected on account of the small period parameters at which the experiments were executed, and the usefulness of the derived coefficients is limited by the fact that the Reynolds numbers were much smaller than would be encountered in typical prototype situations.

8.8 GRACE, 1973 (University of Hawaii, Honolulu)

Grace (1973) reviewed the data available for the assessment of wave forces on pipelines, and suggested numerous values for the drag, inertial and lift coefficients. Also the effects of pipe orientation relative to wave direction and those of pipe clearance from the sea bed were taken into account.

Grace points out that when using the Morison equation, it is necessary to select values for $C_D$ and $C_I$ that are compatible with the values of the particle velocities and accelerations. In other words, a drag coefficient derived from a true measured force and a true measured velocity may be considered the true drag coefficient, but if it is used to calculate a drag force by combining it with a velocity which is obtained through some wave theory, it will give an incorrect drag force unless the estimated velocity happens to be precisely correct.

The analyst must therefore ensure that the coefficients he intends to use, match the wave theory he employs to calculate the wave kinematics. Theoretical coefficients are, almost always, to be used in conjunction with the Airy theory since they are, in most cases suited to this theory alone.
It has been shown earlier that Grace expressed the maximum total force occurring during a wave cycle as follows:

\[
F_{\text{max}} = \begin{cases} 
F_{I_{\text{max}}} & \text{for } \beta > 1 \\
F_{D_{\text{max}}} (1 + \beta^2), & \beta \leq 1
\end{cases}
\]  

(5-3)

where \( \beta = \frac{\pi}{4} \frac{C_I}{C_D} \cdot \frac{1}{A/D} \)  

(5-4)

In the first case, the drag force is very small due to the small \( A/D \) ratio, whereas in the second case, where the relative displacement of the water particles becomes large, the drag force is quite significant.

For the sake of convenience, Grace labelled these two wave force situations Class 1 and Class 2 respectively, and further subdivided the latter into Classes 2A, 2B and 2C, as explained in Table (8-2).
<table>
<thead>
<tr>
<th>Class</th>
<th>Range of $\beta$</th>
<th>Remarks</th>
<th>Appropriate values for coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$C_D$</td>
</tr>
<tr>
<td>1</td>
<td>$\beta &gt; 1.0$</td>
<td>Maximum force all inertial, small relative displacement of water particles; no separation</td>
<td>Not required</td>
</tr>
<tr>
<td>2A</td>
<td>$0.3 \leq \beta \leq 1.0$</td>
<td>Maximum force both inertial and drag, usual wave force situation</td>
<td>Twice or more steady state value</td>
</tr>
<tr>
<td>2B</td>
<td>$0.07 &lt; \beta &lt; 0.3$</td>
<td>Maximum force both inertial and drag, very long waves, and/or tiny objects</td>
<td>Between values for classes 2A and 2C</td>
</tr>
<tr>
<td>2C</td>
<td>$0.0 \leq \beta \leq 0.07$</td>
<td>Maximum force all drag, infinite wave period - i.e. steady flow</td>
<td>Steady state value</td>
</tr>
</tbody>
</table>

Table 8-2 Classification of wave force situations
Grace introduced several equations of similar form to that of the Morison equation in order to cover the "non-standard" cases:

1. **Horizontal force on a pipe at orientation \( \alpha \) to wave direction (i.e. \( \alpha \neq 90^\circ \)).**

   The horizontal force on length \( l \) of the pipe, \( \mathcal{F}_G \), not in-line with the water motion, but normal to the pipe of which the axis is oriented at an angle \( \alpha \) to the wave direction, is

   \[
   \mathcal{F}_G = \mathcal{F}_{GU} + \mathcal{F}_{GA} \tag{8-11}
   \]

   where

   \[
   \mathcal{F}_{GU} = K_u \frac{D}{2} (Dl) u_{\text{max, Airy}}^2 \tag{8-12}
   \]

   \[
   \mathcal{F}_{GA} = K_A \rho \left[ \frac{\pi D^2}{4} \right] \hat{u}_{\text{max, Airy}} \tag{8-13}
   \]

   Note that the length, \( l \), and hence the area taken are not those normal to the direction of water motion.

   When \( \alpha = 90^\circ \), we get the "standard" Morison equation

   and

   \[
   \mathcal{F}_{GU} = F_D \times \hat{\ell} ; K_u = C_D
   \]

   \[
   \mathcal{F}_{GA} = F_I \times \hat{\ell} ; K_A = C_I
   \]

2. **Vertical force on a pipe at right angles to wave direction (i.e. \( \alpha = 90^\circ \)).**

   It has been mentioned before that Grace prefers to include \( P_W \) (rather than \( P_I \)) in the vertical force equation; refer to Fig. (4-2) for the notation. \( P_L \) is, however, the principal term in the equation.

   \[
   \mathcal{F}_{P} = C_L \frac{D}{2} (Dl) u_{\text{max, Airy}}^2 \sin \alpha t \sin \alpha t
   \]

   \[
   + C_W \rho \left[ \frac{\pi D^2}{4} \right] \hat{u}_{\text{max, Airy}} \cos \alpha t \tag{8-14}
   \]

   (Note that Eqs. (8-14) and (8-15) imply that \( \alpha t = 0 ; 2\pi \), etc. coincides with the "rising still-water" phase).
3. Vertical force on a pipe at orientation $\alpha$ to wave direction (i.e. $\alpha \neq 90^\circ$).

\[
\mathcal{Q} = M_U \frac{D}{2} (\mathrm{fl}) \max_{\text{Airy}} u^2 \sin \omega t \sin \omega t \\
+ M_A \rho \left[ \frac{\pi D^2}{4} \right] \max_{\text{Airy}} u^2 \cos \omega t
\]  

Again the length and area are not perpendicular to the wave direction, and when $\alpha = 90^\circ$,

\[
\mathcal{Q} = \mathcal{P}; M_U = C_L \text{ and } M_A = C_N.
\]

The effects of orientation, $\alpha$, and relative clearance, $h/D$, where $h$ is the vertical distance between the sea bed and the bottom of the pipe, upon the various coefficients were dealt with as follows:

a) Reference force coefficients, written as $C_D$, $C_I$, $K_U$, $K_A$, $C_L$, $C_W$, $M_U$, and $M_A$ were defined for the case $\alpha = 90^\circ$, $h/D = 0$, i.e. for the pipe resting on the sea bed with its axis perpendicular to the wave direction.

b) Coefficient values were given in terms of these reference force coefficients.

In order to make allowance for orientation and/or clearance effects such a reference force coefficient would be multiplied by a factor $\Omega$ or $\lambda$. Where horizontal forces are involved, $\Omega$ is defined as the ratio of a particular force coefficient for a specific $\alpha$ and $h/D$ to the corresponding reference force coefficient.

Similarly, for vertical forces, $\lambda$ is the ratio (force coefficient at certain $\alpha$, $h/D$) / (reference force coefficient).

Examples:

\[
\Omega_D = \frac{C_D}{C_D} ; \quad \lambda_A = \frac{M_A}{M_A}
\]
Having defined four classes of flow situations, eight reference force coefficients and the $\Omega$, $\lambda$ ratios, Grace considered and compared the results of numerous researchers—much in the same way as Wilson and Reid (1963) had done—and suggested the reference coefficient values given in Table (8-3), and the $\Omega$ and $\lambda$ curves, this can be seen in Figs. (8-3), (8-4) and (8-5).

<table>
<thead>
<tr>
<th>Class</th>
<th>Range of $\beta$</th>
<th>Reference force Coefficients for Kinematics Predicted by Airy Wave Theory</th>
<th>Applicable figures for $\Omega$, $\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\beta &gt; 1.0$</td>
<td>$C_D$, $C_I$, $C_L$, $C_W$, $K_U$, $K_A$, $M_U$, $M_A$</td>
<td>Fig. (8-3)</td>
</tr>
<tr>
<td>2 A</td>
<td>$0.3 \leq \beta \leq 1.0$</td>
<td>2.0, 3.5, 2.0, 5.0</td>
<td>Figs. (8-3); (8-4)</td>
</tr>
<tr>
<td>2 B</td>
<td>$0.07 \leq \beta &lt; 0.3$</td>
<td>4.37$\beta + 0.69$, 3.5, 4.37$\beta + 0.69$, 5.0</td>
<td>Figs. (8-3); (8-4)</td>
</tr>
<tr>
<td>2 C</td>
<td>$0.0 \leq \beta &lt; 0.07$</td>
<td>1.0, Not applicable</td>
<td>Fig. (8-5)</td>
</tr>
</tbody>
</table>

Table 8-3: Suggested reference force coefficients

$\Omega$ and $\lambda$ curves could not be provided for the behaviour of $K_A$ and $M_A$ as no related studies were to be found.

Regarding the practical application of these coefficients, Grace explains:

Given a value of $\beta$, the analyst obtains the reference design coefficients, refer to Table (8-3). He then modifies the coefficients for any orientation or relative clearance effects with the aid of $\Omega$, $\lambda$ curves. But since $\beta$ is in itself a function of the same coefficients which it is used to obtain, an iterative procedure must be employed in the assignment of design force coefficients. A value of $\beta$ is assumed and the coefficients then selected; $\beta$ is then computed using the coefficients obtained and the design wave characteristics required; if the value of $\beta$ so-obtained results in the same coefficient values found earlier, then these are the proper design values.
Figure 8-3 Variation of acceleration-dependent force coefficients with relative clearance

CLASSES 1, 2A, 2B

Figure 8-4 Variation of velocity-dependent force coefficients with relative clearance and orientation

CLASSES 2A and 2B

Figure 8-5 Variation of velocity-dependent force coefficients with relative clearance and orientation

CLASS 2C
Yamamoto, et al (1974) did a theoretical analysis of the influence of a nearby plane boundary on the vertical and horizontal wave-induced forces on a horizontal cylinder. In addition, they performed laboratory experiments and compared these, as well as data obtained by Schiller (1971), with their theoretical results.

Only horizontal cylinders oriented perpendicularly to the wave direction, and being sufficiently far from the free water surface to avoid surface effects, were considered in this study.

Regarding the coefficient of inertia, \( C_I \), and the added mass coefficient, \( C_m \), where

\[
C_I = 1.0 + C_m
\]

they found from the theoretical analysis that \( C_m \) depended on the distance of the cylinder from the boundary. On the grounds of both theoretical and experimental evidence, they concluded that the horizontal and vertical added mass coefficients were equal for a cylinder close to the boundary and increased to 2.3 when in contact with the boundary. In other words, for zero clearance, \( C_I \) was 3.3. (The same value had been quoted by Wilson and Reid (1963)). The behaviour of \( C_I \) versus the relative clearance \( h/D \) can be seen in Fig.(8-6).

![Figure 8-6](image)

**Figure 8-6** \( C_I \) versus \( h/D \) for a cylinder near a plane boundary.
Nath and Yamamoto (1974) argued that all hydrodynamic forces on submerged objects were actually due to acceleration effects of the fluid flow. Thus, the drag force was a convective acceleration force.

Yamamoto, Nath and Slotta (1974) showed that the frictional drag force (excluding form drag) on an oscillating cylinder, under laminar conditions without flow separation was 45° out of phase with the cylinder velocity. Although flow separation and the consequent form drag would alter this phase angle, they suspected that the maximum drag force acting on a cylinder would be out of phase with the maximum velocity. They therefore suggested that some of the past evaluations of $C_d$, where the drag force was assumed to be zero when the water particle velocities were zero, could be in error. (Should the drag force, in fact, be significantly out of phase with the velocity, the credibility of the Morison equation would be further reduced).

Although they admitted that the drag force could, under certain conditions, contribute significantly to the total horizontal force, they found it was negligible in their test program, and drag coefficients were omitted from the study. Grace (1975) pointed out that the Keulegan-Carpenter period parameter, in the course of these experiments, was confined to the range 0.1 to 2. If the parameter were larger, as it usually is in real-life situations, the picture would be completely different: drag forces would be of primary importance.

Lift forces, which were also considered to be due to effects of convective acceleration, and the behaviour of $C_L$ with varying relative clearance were closely investigated. Although it was recognized that the total transverse lift force on a cylinder near a boundary consisted of two parts, one due to the Bernoulli lift resulting from asymmetrical flow and the other due to vortex shedding, attention was devoted only to the former.

Calculations based on potential flow theory yielded that $C_L = 4.49$ if the cylinder was in contact with the boundary (Nath and Yamamoto (1974)). (Wilson and Reid (1963) stated 4.48). For the cylinder far away from a boundary (i.e. symmetrical flow pattern), $C_L = 0.0$.  

104.
Although the lift force would be directed away from the boundary when the cylinder was in contact, it would be directed towards the boundary if even a very small gap existed between cylinder and boundary. This meant that $C_L$ was negative and increased in size as the relative clearance approached zero; this can be seen in Fig. (8-7). Some experimentally obtained points are also plotted therein.

![Figure 8-7](image)

**Figure 8-7**: $C_L$ versus $h/D$ for a cylinder near a plane boundary

It is interesting to note that Knoblock and Troller conducted experiments to find the transverse forces acting on a cylinder situated near a solid boundary and subject to an air stream. Their results are to be seen in Dean and Harleman (1966). The general tendency was that $C_L$ varied from a maximum at no clearance to zero at large clearance. At a particular Reynolds number and in a certain range of relative clearance, $C_L$ was negative. An explanation put forward by Dean and Harleman is that the boundary layer remains laminar on the surface of the cylinder facing the boundary for higher Reynolds numbers than on the surface away from the boundary. This unsymmetrical wake pattern at the downstream side of the cylinder could give rise to a force directed towards the boundary.

Also Garrison, et al (1975) noticed that the slightest gap between boundary and cylinder had a sizable effect on the lift force, and thus on $C_L$. 
The reader is reminded that the lift coefficient derived by Yamamoto, et al, does not allow for vortex shedding effects which are quite significant, especially at large values of the period parameter.

Grace (1975) commented that negative lift coefficients were hardly of interest to a designer because downward forces implied greater stability of the pipeline. The authors' reply to this was that the real significance lay in the reversibility of the lift force. Subsequent experiments performed at Oregon State University at larger period parameters where vortex shedding also became important, indicated that the lift force could be directed downwards when wake formation was small and then be directed upwards at large wake formation. All this happened within half a wave cycle. The cylinder could therefore experience an oscillating vertical lift force with twice the wave frequency. The actual load on the pipe as well as the vibration frequency could be of importance in the design of pipelines.

Yamamoto, et al (1974) expressed the total vertical force on the cylinder as

\[ P = C_p \frac{\pi}{4} D^2 \frac{V_{\max}}{u} \cos \omega t + C_L \frac{D}{2} \frac{u_{\max}^2}{u} \cos^2 \omega t \]  

(8-16)

Grace (1975) felt that the contribution of the first term would be substantially smaller than that of the second term, in any design pipe situation. He illustrated his argument with two examples: the Sand Island Outfall no.2 near Honolulu, and an oil pipelines at Barber's Point also near Honolulu. In the first example, where the Keulegan-Carpenter period parameter generally was small and inertial forces outweighed drag forces, the ratio of the first term to the second term was only 0.05. In the case of Barber Point where the period parameter is much larger and drag forces are completely dominant, this ratio is only 0.0015.

Notwithstanding the secondary importance of the vertical inertial force compared to the vertical lift force, it did not appear complete to Grace to write the total vertical force as a lift term only. That had been the reason for his introducing the less common "horizontal-vertical inertial force", \( P_W \), and writing

\[ P = P_W + P_L \]

\[ = C_W \frac{D}{4} \frac{\rho u^2}{\omega} + C_L D \frac{\rho u^2}{\omega} \]  

(8-17)

He stated that this was a "not entirely satisfactory attempt".
Fadhil Al-Kazily (1974) studied the behaviour of the coefficients $C_D$ and $C_I$ and in-line wave forces on cylinders located away from the floor in a laboratory flume.

The average-over-the-wave-cycle coefficient values were obtained by two methods:

1. $C_I$ was calculated from the force measured at the instant that the particle velocity was zero, and $C_D$ from the force when the particle acceleration was zero.

2. $C_I$ and $C_D$ were calculated from the maximum measured force together with its phase lag with respect to the maximum velocity phase. (In other words, in much the same way as Rance (1969) had done).

Observations regarding $C_I$ were:

1. $C_I$ decreases as the wave height increases, for constant wave period. (The same observation was apparently made by Evans (1970)).

2. $C_I$ increases as the depth of the cylinder beneath the still-water level increases for fixed wave characteristics. (Evans (1970) apparently observed the opposite).

3. $C_I$ increases as the wave period increases, for constant wave height.

4. $C_I$ varies with the diameter of the cylinder.

5. $C_I$ computed from vertical forces appeared slightly smaller than $C_I$ computed from horizontal forces.

6. A relationship seemed to exist between $C_I$ and the ratios $2A/D$ and $2B/D$ (both of which are relative displacements). This can be seen in Fig. (8-8).
It is noted that the above-mentioned ratio for the horizontal diameter, $2A/D$, is in fact equal to $K/\pi$, where $K$ is the period parameter.

Keulegan and Carpenter also found $C_1$ to vary with the period parameter - they found $C_1$ to have a minimum value of about 0.75 at $K \approx 15$, and $C_1$ over 2.0 in the range of $K$ values at which Al-Kazily performed his experiments.

7. $C_1$ varies within the wave cycle, for fixed wave characteristics. The variation is of sinusoidal shape with frequency twice that of the wave. $C_1$ was found to be positive at all times. A sample plot of $C_1$ versus $t/T$ can be seen in Fig. (8-9).

![Figure 8-8 $C_1$ versus $2A/D$](image)

**Figure 8-8** $C_1$ versus $2A/D$

![Figure 8-9 Variation of $C_1$ within a wave cycle](image)

**Figure 8-9** Variation of $C_1$ within a wave cycle
The instantaneous values of $C_I$ and $C_D$ were determined by Al-Kazily in this manner:

The time history of the total in-line force was measured. The Morison equation was assumed to be valid and was written in the form

$$a_1 = a_2 \cdot C_I + a_3 \cdot C_D$$  \hspace{1cm} (8-18)

where $a_1$ represented the total in-line force, and $a_2$, $a_3$ and $a_4$ were constants and could be computed from the time history of the wave characteristics and cylinder diameter.

Eq. (8-18) was evaluated at time instants $t_1$, $t_2$, $t_3$, .... and any two consecutive equations were solved simultaneously to yield values of $C_I$ and $C_D$ at time $t = \frac{t_1 + t_2}{2}$ etc. It is recalled that Keulegan and Carpenter (1958) also investigated the instantaneous values and concluded that when the period parameter was either small or large (i.e. not close to 15), $C_I$ was reasonably constant. If the period parameter was about 15, however, $C_I$ showed much variation and even had large negative values at $t/T = 0.0, 0.5$ and 1.0. They could not explain this.

A closer look at the range of wave characteristics and cylinder diameters used in Al-Kazily's experiments revealed that the period parameter must have been smaller than about 4.5 throughout the test programme.

8. Values of $C_I$ obtained by method 1. above, were larger than those obtained by method 2.

Since the maximum total horizontal force (consisting of inertial and drag forces) occurs shortly before the crest and the trough pass over the cylinder, and since method 2. involves calculating $C_I$, for horizontal movement, at still-water level phases, one would not expect the two methods to yield the same results because $C_I$ does vary within the wave cycle. The scatter of data can be seen in Fig. (8-9), however, makes it difficult to predict just which method would give the higher $C_I$ values.
Regarding $C_D$, Al-Kazily made the following observations:

1. $C_D$ varies with wave height.
2. $C_D$ varies with cylinder depth below the still-water level.
3. $C_D$ varies with the wave period.
4. $C_D$ varies with the diameter of the cylinder.
5. $C_D$ varies within the wave cycle as can be seen in Fig. (8-10).

Figure 8-10  Variation of $C_D$ within a wave cycle.

(It is suspected that this is derived from vertical motion)

Comparing this with Keulegan and Carpenter's results, a serious discrepancy is found here: the maximum $C_D$ values occur at phases $t/T = 0; 0.5$ and $1.0$, that is at the instant when a crest or trough passes over the cylinder. Keulegan and Carpenter (1958) found $C_D$ to remain fairly constant over most of the cycle and increase when the water surface passes through the still-water level, in other words at $t/T = 0.25$ and $0.75$. Their explanation was that it could be expected as the velocities became zero at these stages.

It is possible that Fig. (8-10) shows the $C_D$ (apparently ten times too large) that would be applicable to vertical drag forces. Many of Al-Kazily's tests were done on cylinders quite close to the water surface where vertical drag forces, would, indeed, be significant.

Careful reading of Al-Kazily's paper does, unfortunately, not clear up the doubt as to whether Fig. (8-10) is applicable to vertical or horizontal drag.

Another possibility is that Fig. (8-10) deviates from Al-Kazily's
usual convention that $t/T = 0$ coincides with a wave crest; i.e. a phase shift of a quarter period is implied.

Al-Kazily did not plot $C_D$ versus either of the two (orbit diameter) : (cylinder diameter) ratios. Keulegan and Carpenter did, however, plot $C_D$ against the period parameter and found that $C_D$ achieved a peak value at $K \approx 15$, this can be seen in Fig. (6-2).

8.11 SARPKAYA, 1974, 1975 (Naval Postgraduate School, Monterey, California)

Sarpkaya (1975) described a well-controlled laboratory research project that was aimed at gaining a better understanding of the hydrodynamic forces on a cylinder fixed in the oscillatory flow of a viscous fluid, and assessing the applicability of the Morison equation in the range of subcritical Reynolds numbers.

Test cylinders were placed horizontally in a U-shaped, streamlined, vertical tunnel with a cross-section of approximately 460 mm by 510 mm. A perfectly harmonic motion of the water was produced and sustained by a simple pneumatic system. Displacement of the free water surface, the instantaneous velocity and acceleration as well as the in-line and transverse forces were simultaneously recorded.

Sarpkaya pointed out that experience had shown that of all things that may be relevant in wave-structure interaction problems, including diffraction and free surface effects, separation and viscous effects were the most important and the least understood. There were many possible reasons why the coefficients $C_D$ and $C_I$, as determined by numerous researchers, displayed such a wide scatter. The investigations by Keulegan and Carpenter (1958) appeared to Sarpkaya to be the most systematic and therefore he prepared his results in a similar way in order to make meaningful comparisons.

A. In-line forces

From the time-history of the measured force, values for $C_D$ and $C_I$ were determined as average values over the entire wave cycle, and were then plotted against the period parameter, $K = u_{\text{max}} T/D$. 
The results, together with those of Keulegan and Carpenter can be seen in Figs. (8-11) and (8-12).

Figure 8-11 $C_D$ versus $K$ for a cylinder (remote from a boundary)

Figure 8-12 $C_I$ versus $K$ for a cylinder (remote from a boundary)

The agreement between the $C_D$ values of the two studies is very good, particularly in view of the fact that the two experiments were carried out in completely different type test rigs.

The agreement between $C_I$ values for $K > 10$ is not so good. Sarpkaya suggests that the relatively few data points obtained by Keulegan and Carpenter at large $K$ values could have been in error, or the stationary waves could have deviated from the required characteristics.

The experiments were carried out in the Reynolds number range of $2.5 \times 10^3$ to $5 \times 10^6$. Sarpkaya concluded that there was "absolutely no correlation" between the Reynolds number and $C_D$ and $C_I$. 
Sarpkaya and Garrison (1963) had already found that a unique relationship existed between $C_D$ and $C_I$. Fig. (8-13), taken from Sarpkaya (1974), shows the variation of $C_D$ and $C_I$ for various values of $K$.

![Diagram showing variation of $C_D$ and $C_I$ for various values of $K$.]

Figure 8-13 $C_D$ and $C_I$ for various values of $K$

With regard to the instantaneous values of $C_D$ and $C_I$, Sarpkaya commented that these could differ considerably from their average-over-the-cycle values. He did not determine the instantaneous values because he disagreed with the assumption of Keulegan and Carpenter that $C_D$ and $C_I$, respectively, had equal values at $t_1$ seconds before a particular still-water phase and $t_1$ seconds after that phase. He thought that different distributions of skin friction at the two instants invalidated the assumption, and there was no simple way to determine the instantaneous values of the coefficients accurately, "save an exact analytical solution of the problem".

Chakrabarti and Wolbert (1975) commented that the accuracy of the data was the result of the controlled environment in which the experiments had taken place. Both Keulegan and Carpenter's and Sarpkaya's studies had been conducted in essentially one-dimensional flow with no changing free surface. Chakrabarti and Wolbert, who had performed tests on wave forces acting on vertical tubes and also found the coefficients to be dependent on the period parameter, thought that $C_D$ and $C_I$ changed drastically in two-dimensional flow under a
changing free surface caused by a progressive wave. The applicability of Sarpkaya's results to prototype situations was, therefore, considered to be limited.

Miller (1975) also discussed Sarpkaya's work. He strongly suspected that the Reynolds number had an influence on $C_D$ and $C_I$. Using the data of Keulegan and Carpenter and Sarpkaya, he plotted contours of these coefficients on a Reynolds number - period parameter plane, this can be seen in Figs. (8-14) and (8-15).

![Diagram showing contours for $C_D$ and $C_I$ on a Reynolds number - period parameter plane.]

He commented that the disagreement of $C_I$ values as determined by Keulegan-Carpenter and Sarpkaya, (see Fig. (8-12)), was due to different Reynolds number ranges in the two studies. Miller recommended that research on Reynolds number influence be continued, also in the range of higher numbers.

Apart from investigating the relationship between the coefficients and the period parameter, Sarpkaya (1975) also analysed phase angles and the errors involved when using the Morison equation:

The phase angle between the maximum in-line force and the maximum velocity was plotted against period parameter. The result is shown in Fig. (8-16).
The important observation to make here is that at small $K$ values, where inertial forces predominate, the maximum in-line force occurs close to the still-water phases, while at large values of the period parameter, where drag becomes all-important, the maximum force occurs very shortly before the passage of a crest or a trough.

The relative magnitude of error, $\psi$, where

$$\psi = \frac{F_{\text{max}}(\text{measured}) - F_{\text{max}}(\text{calculated})}{F_{\text{max}}(\text{calculated})}$$

was plotted versus period parameter. It was found that the measured in-line force was almost always larger than the force calculated by the Morison equation (and, presumably, using the $C_D$ and $C_I$ values given in Figs. (8-11) and (8-12)). The relative error, $\psi$, could be as large as 15%, as may be seen from Fig. (8-17).
Sarpkaya concluded that the in-line force on a cylinder could be calculated with the aid of the Morison equation and the appropriate averaged coefficient values, and then be corrected by the factor $\psi$.

B. **Transverse forces**

The maximum lift coefficient, $C_{L\text{max}}$, within a wave cycle, defined as

$$C_{L\text{max}} = \frac{\text{maximum amplitude of the transverse force in a cycle}}{\frac{1}{2} \rho u_{\text{max}}^2 D L}$$

was determined as a function of the period parameter; refer Fig. (8-18).

![Figure 8-17](image.png)

**Figure 8-17** Relative error, versus K for a cylinder

![Figure 8-18](image.png)

**Figure 8-18** $C_{L\text{max}}$ versus K for a cylinder (remote from a boundary)
It is noted that this lift coefficient is probably due entirely to the shedding of vortices and not to the presence of a solid boundary as the cylinder models were installed relatively far away from the tunnel walls.

Fig. (8-18) displays several interesting features:

1. $C_{L_{\text{max}}}$ may reach a value as high as 3.0 and exhibits several maxima.

2. In the period parameter range 4 to 10, the lift force can be substantial. In this range the in-line force is essentially of inertial nature, and one may be inclined to ignore the drag and transverse forces. Such a step would lead to a severe underestimate of the resultant force acting on the cylinder. Such a procedure would only be acceptable for $K < 4$ where there is no lift force.

In order to compare the transverse force with the in-line force, the ratio $(\text{maximum measured transverse force}) : (\text{maximum measured in-line force})$ was plotted against the period parameter, this can be seen in Fig. (8-19).

![Figure 8-19](image)

**Figure 8-19** Ratio $\frac{\text{max transverse force}}{\text{max in-line force}}$ versus $K$ for a cylinder

Since the transverse force can be significantly larger than the in-line force, it is essential, for design purposes, to consider the vectorial sum of these two forces.
As in the case of the other coefficients, Sarpkaya found no correlation between \( C_{L_{max}} \) and the Reynolds number.

He also made interesting observations regarding the frequency of the lift force. Fig. (8-20) shows a plot of the ratio of vortex shedding frequency, \( f_v \), to lift force frequency, \( f_L \), versus the period parameter.

![Plot of frequency ratio vs. period parameter](image)

**Figure 8-20**  Frequency ratio, \( f_v/f_L \), versus \( K \) for a cylinder

The predominant tendency in Fig. (8-20) is for the lift force frequency to be one half of the vortex shedding frequency.

Sarpkaya (1974) stated that the frequency of the lift force depended on the period parameter, \( K \), and that it varied from the wave frequency, \( f_o \), to \( 5f_o \). In certain ranges of \( K \), several lift force frequencies, such as \( f_o, 2f_o, 3f_o, 4f_o \) and \( 5f_o \) could exist simultaneously.

**8.12  U.S. ARMY, COASTAL ENGINEERING RESEARCH CENTRE, 1975**

The 1975 edition of the CERC Shore Protection Manual discusses the use of the Morison equation for in-line forces on piles, the transverse forces on piles, and the associated force coefficients.

Being a design manual, the theory is not refined and rather rough-and-ready rules are given as guidelines.
A. In-line forces

A laboratory study of Thirriot, et al (1971) is mentioned, wherein it has apparently been found that for

\[
\frac{A}{D} > 10, \quad C_D = C_D \text{ (steady flow)} \\
1 < \frac{A}{D} < 10, \quad C_D > C_D \text{ (steady flow)} \quad (8-20)
\]

where \( \frac{A}{D} \) is the ratio \( \frac{\text{horizontal semi-axis of particle orbit}}{\text{pile diameter}} \).

(Seeing that \( K = \frac{2\pi A}{D} \), the value of the period parameter corresponding to \( \frac{A}{D} = 10 \) is \( K = 62.8 \).

(The comments of Rance (1969) come to mind. Referring to Fig. (7-5), he noted that the \( \frac{A}{D} \) ratio had a marked effect on \( C_D \), particularly if the ratio was smaller than 10 and the Reynolds number less than \( 5 \times 10^4 \). Under these conditions the unsteady state \( C_D \) was considerably larger than the steady state \( C_D \).

Thirriot, et al (1971) also found that at Reynolds numbers greater than \( 4 \times 10^5 \), \( C_D \) for unsteady flow was equal to \( C_D \) for steady flow, regardless of the value of \( \frac{A}{D} \).

Consequently, the CERC Manual recommends that \( C_D \) values be obtained from Fig. (8-21), which has been drawn up by Thirriot, et al. Fig. (8-21) primarily shows the variation of \( C_D \) versus Reynolds number for steady flow conditions, although some experimental points from unsteady flow experiments are superimposed thereon. The solid line is regarded by the Manual as "generally conservative".

The CERC Manual recommends the following values for \( C_I \):

\[
C_I = \begin{cases} 
2.0 & \text{when } R_e < 2.5 \times 10^5 \\
2.5 - \frac{R_e}{5 \times 10^5} & \text{when } 2.5 \times 10^5 < R_e < 5 \times 10^5 \\
1.5 & \text{when } R_e > 5 \times 10^5 
\end{cases} \quad (8-21)
\]
Figure 8-21  $C_D$ versus $Re$ for a circular pile, steady and unsteady flow
These values are based on the results of Keulegan and Carpenter (1958) and six subsequent investigators.

B. Transverse forces

Regarding transverse forces, the laboratory study of Chang (1964) is mentioned. He was to have found that eddies were shed at a frequency twice the wave frequency (i.e. \( f_v/f_o = 2 \)); two eddies were shed after the passage of the wave crest and two on the return flow after the passage of the trough. (It must be realised that this is a special case. If the period parameter were greater, more eddies could form at a time - Keulegan and Carpenter found that von Karman vortex streets developed at large \( K \) values, of the order of say \( K = 100 \)).

In Chang's study \( C_L \) was found to be dependent on \( K \), averaged over the length of the pile. For rigid piles, \( C_L \) increased with increasing \( K \) until it approximately equalled \( C_D \). For \( K < 3 \), \( C_L \) was virtually zero.

(Unfortunately, this report on Chang's work conflicts with the reports by Bidde (1971) and Isaacson and Maull (1976): -

Bidde (1971) states that Chang concluded that \( f_L/f_o = 2 \). Isaacson and Maull (1976) state that Chang found \( f_L/f_o \) to be 2 or 3 depending on the value of the period parameter at the water surface. Although one would have to read Chang's thesis to get the full story, it is clear that the CERC Manual refers to the ratio \( f_v/f_o \) whereas Bidde and Isaacson and Maull refer to \( f_L/f_o \).

The CERC Manual also quotes the work of Bidde (1970, 1971). It is stated that he "investigated the ratio of maximum lift force to the maximum drag force", \( F_{L_{max}}/F_{D_{max}} \). This appears to be incorrect, since Bidde (1971) explains that he plotted ratios of "average maximum lift forces to average maximum longitudinal forces versus Keulegan-Carpenter number and Reynolds number", in other words \( F_{L_{max}}/F_{max} \), where \( F_{max} \) is the (average) maximum total in-line force, including both drag and inertial forces. As the Keulegan-Carpenter period parameter was fairly small in many cases, the inertial force was not insignificant.
Be that as it may, the Manual reasons that \( \frac{F_{L_{\text{max}}}}{F_{D_{\text{max}}}} = \frac{C_L}{C_D} \), if there is no phase difference between the lift and drag forces. The dependence of \( \frac{C_L}{C_D} \) upon \( K \) is then given, Fig. (8-22).

To obtain a value for \( C_L \), the recommendation of the Manual is to:

1. select a \( C_D \) value,
2. calculate the parameters \( K \) and \( \frac{H}{gT^2} \),
3. obtain the \( \frac{C_L}{C_D} \) ratio from Fig. (8-22), and
4. calculate \( C_L = C_D \times \frac{C_L}{C_D} \).

In the event of \( K \) being beyond the range of Fig. (8-22), the lift coefficient is taken as \( C_L = C_D \).

The reader is reminded that all these recommendations are intended, more specifically, for piles unaffected by nearby boundaries. Furthermore, these coefficient values are not to be used with Airy wave theory, but in conjunction with design charts, in the CERC Manual, which have been based on Dean's stream function theory.


Garrison, et al (1975) made two contributions whereby the horizontal and vertical force components, acting on a large-diameter horizontal cylinder in contact with a solid boundary, may be predicted:

1. Using experimental data, they plotted maximum horizontal forces in terms of dimensionless parameters containing only the basic wave variables, namely wave height and period, in a specified water depth. (The plotted points appeared to follow consistent curves). In other words, they avoided the Morison approach which requires a knowledge of coefficients on the one hand, and water kinematics on the other hand, and resorted to what this writer calls, the "method of dimensionless wave parameters".

2. By assuming that the displacement of water particles is so small compared to the diameter of the cylinder, that separation does not occur, they formulated approximate equations for the maximum horizontal and vertical forces. This is referred to as their "unseparated flow model".
Figure 8-22 Variation of $C_L / C_D$ with Keulegan-Carpenter Number and $H/gT^2$
From dimensional analysis they found the maximum force, per unit length, on a bottom-mounted cylinder to be dependent on four dimensionless groups: $2d/D$, $gT^2/d$, $H/D$ and $D^2/\nu T$.

The fourth group is a kind of Reynolds number. On the assumption that when this number is large it hardly affects the force, the authors disregarded it. Data acquired from wave flume tests were plotted as points of maximum horizontal and vertical forces (expressed in dimensionless form) versus the remaining three parameters, $2d/D$, $gT^2/d$ and $H/D$ - see Fig. (8-23).

The ranges covered in the experiments were approximately:

- $2d/D$ - 4.0 to 9.0
- $gT^2/d$ - 10 to 300
- $H/D$ - 0.0 to 2.0

The unseparated flow model

Since this theory is intended for rather large amplitude waves in the shallow water range, Garrison, et al (1975) recommend the use of Stokes' fifth-order wave theory (as presented by Skjelbreia and Hendrickson (1961)) to predict particle velocities and accelerations.

A. Horizontal force

Assuming that separation does not occur - i.e. no wake formation and negligible viscous effects - the maximum horizontal force, per unit length, on a bottom-mounted cylinder is considered to consist of an inertial term only:

$$F_{\text{max}} = \rho \frac{\pi}{4} D^2 (1 + C_m) \dot{u}_{\text{max}}$$  \hspace{1cm} (8-22)

where $C_m$ is the added mass coefficient (discussed extensively in the review of Garrison, Field and May (1977)).

$C_m$ is assigned to value of 2.29. (Wilson and Reid (1963) calculated $C_{\text{f}} (= 1 + C_m)$ for a bottom-mounted cylinder from potential flow theory and found it to be 3.30; Yamamoto, et al (1974) found $C_m = 2.30$).
Figure 8-23 Maximum wave forces, in dimensionless form, on a large-diameter bottom-mounted cylinder

<table>
<thead>
<tr>
<th>2d/D = 4.0</th>
<th>2d/D = 5.5</th>
</tr>
</thead>
</table>

LEGEND: --- THEORY (i.e. linear flow model) ○: HOR. FORCE, \( \frac{F_{\text{max}}}{\rho g D^4} \) Δ: (UP) VERT. FORCE, \( \frac{P_{\text{max}}}{\rho g D^4} \)
Figure 8-23  Maximum wave forces, in dimensionless form, on a large-diameter bottom-mounted cylinder

C. $2d/D = 7.0$

D. $2d/D = 9.0$

LEGEND:

--- THEORY (i.e., 1USER FLOW MODEL)

O: HOR. FORCE $F_{max}/\rho g D^2/4$

$A$: (up)

$O$: (down)

VERT. FORCE $P_{max}/\rho g D^4$
Eq. (8-22) is valid only under the following circumstances:

1. Large $2d/D$, because $C_m$ is 2.29 if the water depth, $d$, becomes so small that the free surface affects the flow.
2. Large $L/D$, to ensure that diffraction effects remain negligible.
3. Small $H/D$, because if the wave height is too large, flow separation will occur.

In order to compare their unseparated flow theory with the experimental results, the authors used Stokes' fifth-order wave theory and Eq. (8-22) to compute values of the maximum horizontal force (in dimensionless form, i.e. $F_{max}/\rho g D^2/4$) and plotted these as solid lines in Fig. (8-23).

As can be seen, the agreement between theoretical and experimental results, within the test range is good.

The maximum horizontal force varies rather linearly with wave height, i.e. the parameter $H/D$, - the only significant non-linear variation occurs at large $gT^2/d$ values and small $2d/D$ values.

B. Vertical force

The authors assumed that the vertical force consisted of two components: the hydrodynamic lift force due to the asymmetrical streamline pattern caused by the presence of the boundary, and the vertical inertial force.

As an approximation, the second component which is much smaller than the first at large $gT^2/d$ values, was omitted. Thus, the simplified equation for the maximum vertical force, per unit length, on a bottom-mounted cylinder became:

$$P_{max} = \frac{1}{2} \rho C_L D u_{max}^2$$  \hspace{1cm} (8-23)

where the coefficient of lift, $C_L$, was assigned the value of 4.48, as determined by Dalton and Helfinstein (1971). (Wilson and Reid (1963) derived the same value, whereas 4.49 is quoted by Nath and Yamamoto (1974) and Yamamoto, et al (1974)).
Since 4.48 is the result of an unseparated flow analysis, Eq. (8-23) with \( C_L = 4.48 \) cannot be expected to be valid under conditions where the relative particle displacement is so large, that gross flow separation occurs.

Fig. (8-23) shows that the vertical force varies rather non-linearly with the wave height, or the parameter H/D.

As with the horizontal force, the authors compared Eq. (8-23) with the experimental results, by using Stokes' fifth-order wave theory and calculating values of the dimensionless force \( \frac{P_{\text{max}}}{\rho g D^2/4} \) and plotting them as solid lines in Fig. (8-23).

The agreement between theoretical and experimental results is fairly good for small H/D values - i.e. for H/D < about 1.0, depending on the values of 2d/D and gT^2/d.

A possible reason for the disagreement of theoretical values with experimental values at larger H/D, is that gross separation of flow sets in when H/D (and consequently the relative amplitude of oscillation of water particles) exceeds certain limits. When such separation takes place, the actual \( C_L \) decreases. Therefore the unseparated flow model, involving Eq. (8-23) with \( C_L = 4.48 \), "will always tend to give conservative results".

Vongvisessomjai (1976), in discussing Garrison, et al.'s work, thought that the Morison equation was still most suitable if the relative size of the cylinder, D/L < 0.2. If the relative size exceeded 0.2 and the period parameter, K < \( \pi \) (or say 3), the diffraction theory method should be favoured.

8.14 GRACE AND NICINSKI, 1976 (University of Hawaii, Honolulu)

As far as can be ascertained, the research programme described by Grace and Nicinski (1976), was the first prototype study of wave forces acting on a circular pipe near the sea floor. Prior to this, several wave force experiments had been carried out in the ocean, but these were mainly concerned with vertical piles, and, to a lesser extent, with spheres.
8.14.1 General description of the apparatus and experiments

The test rig consisted of a 400 mm outside diameter steel pipe having a wall thickness of about 6 mm, which was mounted on, and 75 mm above, a flat base. The base was made of steel angles and T-beams which could be stabilized by sliding concrete blocks into them after the rig had been placed on the sea bed. The base together with the concrete inserts had a mass of almost five tons.

The overall length of the pipe rig was about 5,3 m, but the test section was only 1 m long and located in the middle between two flanking wing sections, which were fixed to the base. To increase the stability after placement, lengths of chain were loaded inside the wing sections. The test section, designed to be neutrally-buoyant in sea water, was supported from the two flanking wing sections, which also helped to eliminate "end effects". Horizontal and vertical forces on the test section could be measured by a strain gauge arrangement which could be linked to electronic recording devices housed on board a boat on the surface.

Other instruments which were also linked to the boat, included a wave staff for measuring wave heights, and a propeller-type current meter for measuring velocities. This meter was placed about one meter off the one end of the pipe rig, on its centreline, and 375 mm above the sea bed. This location was chosen, as a compromise, to represent the water motion incident upon the test pipe, and yet to be far enough away not to be influenced by flow around the end of the pipe.

Unfortunately, the wave staff did not perform satisfactorily, with the result that most of the processed data are of the form of force coefficients associated with measured water motion, rather than being related to water kinematics predicted by some wave theory.

The experiments were carried out in a water depth of 11 to 11.5 m, some 430 m offshore of the Hawaiian island of Oahu. The sea floor was relatively flat and consisted of coral rock.

The test waves were generated in the distant southern hemisphere, and arrived at the site as a very consistent swell. After refraction the wave direction was reduced to quite a narrow band, and since it had
been decided to devote one and a half years of the project to the situation where the wave attack was perpendicular to the pipe axis, (i.e. $\alpha = 90^\circ$), the test rig could be installed without an adjustable orientation feature. During this first stage the relative clearance, $h/D$, was also kept constant at $75/400 \approx 0.2$.

The wave period hardly varied over the short term so that the swell could be considered as virtually uniperiod. In the long term, the periods involved ranged from 12 to 20 seconds. The wave heights during the tests were up to 2.1 m. The Reynolds number varied between $10^5$ and $4 \times 10^5$, and the relative roughness of the pipe was estimated to be $2.5 \times 10^{-3}$.

Notwithstanding the poor performance of the wave measuring staff, the values of drag, lift, vertical and horizontal inertial coefficients were investigated for two types of situations:

1. where the coefficients are to be used in conjunction with measured kinematics, and
2. where they are to be used with kinematics predicted by the Airy wave theory.

The authors refer to the first type as "true" coefficients and the second type as "theoretical" coefficients. Whereas the first type may be of interest to researchers, it is the second type that is of real interest to designers.

8.14.2 Results of the experiments

8.14.2.1 True coefficient $C_I$ in predominantly inertial force situations

The case where the flow did not separate from the pipe received special attention. Grace and Nicinski (1976) associated this with the design situation where the relative displacement of water particles, $A/D$, was less than $\frac{1}{7}$, or equivalently, if $K < \frac{\pi}{2}$.

This condition could arise, even for high, long period design waves, if the pipe diameter, $D$, was large.
Dalton and Helfinstein (1971) used potential flow theory to derive \( C_I \) for a cylinder at varying relative clearance, \( h/D \). Yamamoto, Nath and Slotta (1974), incidentally derived a similar curve. Grace, et al felt that the Dalton-Helfinstein curve could be of practical use in cases where the flow remained virtually unseparated, and attempted to verify this curve by an experiment that was independent of wave action.

Their procedure, which appears rather unique, was the following:-

A steel plate, mounted on plywood shims around its edges, was placed under the test section of pipe. The test section was struck several times in the horizontal direction and then in the vertical direction to obtain its period of natural vibration, in either direction, at that particular clearance.

The natural (horizontal) period, \( T_n \), of a spring-mounted body immersed in a liquid is given by the equation

\[
T_n = 2\pi \sqrt{\frac{C_I m}{k}}
\]  

(8-24)

where \( m \) is the mass of the neutrally-buoyant pipe, and

\( k \) is the effective spring constant of the mounting arrangement.

Since both \( m \) and \( k \) were known, and \( T_n \) was measured, \( C_I \) could be calculated.

Their calculated \( C_I \) values, superimposed upon the Dalton-Helfinstein curve can be seen in Fig. (8-24).

8.14.2.2 Various coefficients in situations where inertial effects are not predominant

Whereas the investigation described above did not involve wave forces in any way, the experiments that are about to be described included the measurement of forces, wave properties and flow velocities at the pipe. From these data both true and theoretical coefficients could be calculated.
Figure 8-24  $C_I$ versus $h/D$, relative clearance, for a cylinder in predominantly inertial force situations
8.14.2.2.1 **Horizontal forces**

The Morison equation was accepted as the formula whereby the total horizontal force per unit length of a cylinder could be calculated:

\[ F = \frac{1}{2} C_D \rho |u| u + C_I \rho \frac{u}{2} D^2 \dot{u} \quad (5-2) \]

8.14.2.2.1.1 **True instantaneous value of \( C_I \)**

From a sample of 48 readings, inertial coefficients were obtained at the instant at which the velocity was (apparently) zero. (The acceleration was not necessarily at its maximum).

The mean value was \( C_I = 1.93 \), with a coefficient of variation of 22.8%.

A plot of individual \( C_I \) values versus Reynolds number, \( R_e = u_{\text{max}} \frac{D}{\nu} \), displayed no discernible trends.

Since the peak acceleration did not necessarily coincide with the zero velocity phase, the ratio \( \dot{u}_* / \dot{u}_{\text{max}} \) was considered, where

- \( \dot{u}_* = \) the acceleration at the instant of zero velocity
- \( \dot{u}_{\text{max}} = \) the peak acceleration.

This ratio had a mean value of about 0.92, and provided a crude estimate of what proportion of peak inertial force occurred at the time of zero velocity.

The inertial force at the point of zero velocity was compared to the maximum total in-line force measured on the pipe. On the average, this inertial force was about 45% of the maximum total force.

From this argument, Grace, et al concluded that the overall maximum forces on the pipe - in the course of these experiments - were drag force dominated.

An interesting observation was made: The peak horizontal force always preceded the development of the peak velocity by 1 to 2 seconds.
Since they had reasoned that inertial forces constituted only a small proportion of the total force, they searched for other explanations.

A further observation was that while the velocity peaked at a value that often persisted for a second or more, the total horizontal force dropped rapidly.

Two possible explanations were offered:

1. The true drag coefficient dropped rapidly as time progressed, or equivalently, as the water particles travelled a longer distance, \( s \).

2. The velocity that was measured at a point one meter off the end of the pipe was not representative of the real flow incident on the pipe. This could occur because as the direction of flow reversed, the pipe would be bathed in its own highly turbulent wake.

Unfortunately, Grace and his team could not prove or disprove the second alternative with their measuring techniques and they largely ignored it and preferred the first explanation.

8.14.2.2.1.2 True, instantaneous value of \( C_D \)

The trough and crest data of 66 separate waves were utilized to obtain values for \( C_D \). Each wave yielded between 1 and 4 values depending on the length of time interval over which the velocity peaked. \( C_D \) was plotted against \( s/D \), the relative travel distance of the water particles. The results can be seen in Fig. (8-25). The mean value in Fig. (8-25) is \( \overline{C_D} = 1.125 \).
In Fig. (8-25), the points have been coded in terms of the Reynolds number and an acceleration parameter, \( \hat{u}_{\text{max}}/g \). Grace, et al (1976) comment that both these parameters play minor roles compared to \( s/D \), in the behaviour of \( C_D \). Some correlation was found between the Reynolds number and \( s/D \).

Much of the scatter in Fig. (8-25) could be attributed to incorrect estimates of the real \( s \), due to wake effects.

8.14.2.2.1.3 True, average values of \( C_D \) and \( C_I \) on wave

Using force and velocity histories and a best least-squares fit statistical technique, two samples of wave data were analyzed to obtain average-over-the-wave-cycle values of \( C_D \) and \( C_I \). The approximate
results were \( C_D = 1.12 \) and \( C_I = 2.20 \). Both are close to the instantaneous mean values, 1.125 and 1.93, respectively, quoted above.

8.14.2.2.1.4 Theoretical, instantaneous values of \( C_D \) and \( C_I \)
derived from maximum total horizontal force

When measured forces, theoretical velocities calculated from Airy theory equations, and a regression line technique were used together, the data from 65 waves yielded \( C_D = 1.50 \) and \( C_I = 2.57 \).

As mentioned earlier in this thesis, the Airy theory gives very good predictions of the true peak velocities near the sea bed under ocean waves, but underestimates the true accelerations near the sea bed (by as much as 33%). This has been noted by e.g. Wiegel (1964), Le Méhaute, et al (1968) and recently confirmed by Grace (1976) who studied water particle kinematics that occurred in the ocean.

This explains why the theoretical \( C_I (= 2.57) \) is about one third more than the true \( C_I (= 1.93) \).

The discrepancy between the theoretical \( C_D (= 1.50) \) and the true \( C_D (= 1.125) \) is rather surprising. Grace and Nicinski (1976) suggest this could be due to subtle errors in the regression line technique.

8.14.2.2.2 Vertical forces

The equation for the total vertical force on a pipeline near the sea bed, as preferred by Grace, is

\[
P = P_L + P_W = C_L \rho u^2 + C_W \rho g D^2 |\dot{u}| \quad (8.25)
\]

It is noted that Grace and Nicinski (1976) imply that both \( P_L \) and \( P_W \) are positive at all times (i.e. acting upwards). (Earlier papers, such as Grace (1973) and Grace (1971) did not include a "mod" sign for \( \dot{u} \), while \( u^2 \) was written as \(|u| \cdot u\).
8.14.2.2.2.1 True instantaneous value of $C_L$

Fig. (8.26), showing the variation of $C_L$ versus relative travel distance, $s/D$, was derived from a sample of 33 waves. (It is suspected that Fig. (8-26) was obtained in a similar way as Fig. (8-25), so that some of the scatter can be attributed to poor estimates of the actual $s$).

Figure 8-26 True, instantaneous $C_L$ versus $s/D$, relative travel distance
There appears to be a tendency for $C_L$ to decrease with increasing $s/D$, then increase somewhat and decrease again.

The authors compare Fig. (8-26) to Fig. (8-18) of Sarpkaya (1975). Attention is drawn to the fact that Sarpkaya's experiments, apart from being performed on cylinders away from a boundary, determined $C_L$ as the maximum lift coefficient within the wave cycle, at different period parameter values, where $K = 2\pi A/D$. Fig. (8-18) is thus, effectively, a plot of $C_L$ versus $A/D$, whereas Fig. (8-26) is a plot of $C_L$ versus $s/D$, where $s$ can vary from zero to a particular value of $A$. Nevertheless, the comparison is interesting.

8.14.2.2.2 True, average values of $C_L$ and $C_W$

More than one peak value of the vertical force occurred during a wave cycle. There were, generally, peak (upward) forces shortly before the trough or the crest passed over the pipe. The force associated with crest passage was by far the larger - perhaps twice the force associated with the trough. Refer Fig. (8-27).

![Diagram](image-url)
This "double bounce" behaviour of the vertical force led to very poor results when the coefficients were determined with a best least-squares fit technique.

8.14.2.2.2.3 True, instantaneous value of \( C_{w} \)

From the data of 28 waves, at the instants of zero horizontal velocity, instantaneous values of \( C_{w} \) were calculated. The mean value was \( C_{w} = 0.32 \), with a coefficient of variation of 100%.

No correlation was found between \( C_{w} \) and the Reynolds number.

8.14.2.2.2.4 Theoretical, instantaneous values of \( C_{L} \) and \( C_{w} \), derived from maximum total vertical force

When measured forces, theoretical kinematics calculated from Airy theory equations, and a regression line technique were used together, the data of 65 waves yielded \( C_{L} = 0.56 \) and \( C_{w} = 2.18 \).

8.14.3 Conclusions and recommendations by Grace and Nicinski

8.14.3.1 Situations where inertial forces are predominant

After finding a favourable fit between the experimental results and the Dalton-Helfinstein curve, the authors concluded that \( C_{I} \), for both the horizontal and vertical directions, could be obtained from Fig. (8-24), for those design situations where pipe sizes and wave conditions were such that inertial effects were predominant.

8.14.3.2 Situations where inertial forces are not predominant

8.14.3.2.1 Horizontal forces

8.14.3.2.1.1 Theoretical drag coefficient, \( C_{D} \)

Since the Airy theory predicts peak near-bottom velocities under ocean waves very well, it is accepted that the theoretical \( C_{D} \) would be equal to the true \( C_{D} \). In other words, the true \( C_{D} \) values (which were determined more accurately in the experiments) are also taken to be the theoretical \( C_{D} \) values.
Guven, et al (1975), found that even if the relative roughness of a circular cylinder in steady flow was increased considerably above $2.5 \times 10^{-3}$, it caused an increase in supercritical drag coefficient of only 4%. (It is also recalled that it was found at Wallingford, in 1962, that roughening of the pipe increased drag only marginally).

Beattie, et al (1971), found that the drag coefficient of both rough and smooth pipes in steady flow, close to the floor, hardly changed in the Reynolds number range $5 \times 10^5$ to $2 \times 10^6$. For a rough pipe, $C_D$ was about 0.7.

From these findings, the authors concluded that their $C_D$ results could be applied also to cases where the relative roughness of the pipe and the Reynolds number were larger than in the experiments.

The drag coefficient in Fig. (8-25) is asymptotic to 0.7 - the same value as found by Beattie, et al. (Incidentally, according to the CERC Shore Protection Manual (1975), Thirriot, et al (1971) also found $C_D$ for unsteady flow to be approximately equal to $C_D$ for steady flow if the Reynolds number was greater than $4 \times 10^4$).

In consideration of this, and seeing that Jones (1971) found that $C_D$ for a cylinder close to a boundary in steady flow, with the Reynolds number $< 5 \times 10^5$, was rather insensitive to changes in relative clearance, $h/D$ (within the range zero to 0.16), and in boundary roughness, Grace and Nicinski (1976) concluded that Fig. (5-25) could be used as a design curve for $C_D$ for pipes at any relative clearance.

**8.14.3.2.1.2 Theoretical inertial coefficient, $C_I$**

The theoretical curve in Fig. (8-24) is used as a base reference.

Because the theoretical coefficient of inertia was found to be 2.57, for a relative clearance of about 0.18 to 0.2, the authors suggest a $C_I = 4.0$ for zero pipe clearance. (This is obtained through proportioning from Fig. (8-24): $2.57 \times \frac{3.29}{2.17} = 3.90$, or say 4.0).

They also suggest that, for any other relative clearance, the theoretical $C_I$ be derived, in a similar manner, from Fig. (8-24).
8.14.3.2.2 Vertical forces

8.14.3.2.2.1 Theoretical lift coefficient, $C_L$

The wide range covered by the true $C_L$ for varying $s/D$, made it difficult for the authors to produce a design curve. They suggested that a design curve could be plotted through the upper bound of the data in Fig. (8-26) - this has been done by the present writer.

The values of the true $C_L$ may be assumed valid also for the theoretical $C_L$, for the same reason as mentioned in the case of the drag coefficient.

Since the lift coefficient is strongly influenced by the relative clearance, no suggestions have been made regarding $C_L$ for other relative clearances.

8.14.3.2.2.2 Theoretical horizontal-vertical inertial coefficient, $C_W$

Grace and Nicinski admit that they "do not have great confidence" in the $C_W$ values, but suggest a design value of 3,0 for the theoretical $C_W'$ for $h/D = 0.2$. To the present writer, the reason for this value seems to be the following: Because the Airy theory underpredicts the actual near-bottom acceleration under ocean waves by as much as one-third, the theoretical $C_W$ is assumed to be:

$$\frac{4}{3} \times \text{the true } C_W = \frac{4}{3} \times 2.18 = 2.91 \text{, or say, 3.0}.$$

8.14.3.2.3 Adjustment of coefficients for orientation angle, $\alpha \neq 90^\circ$

To make allowance for the case where the wave attack is not at right angles to the pipe axis, the authors suggest the use of the curves produced by Grace (1971) and Grace (1973); refer Figs. (8-3), (8-4) and (8-5).

8.14.4 Final remarks by Grace and Nicinski

They point out that considerable research still remains to be done on the interaction between waves and pipes despite the apparent saturation of published material on wave-cylinder (whether horizontal or vertical) interaction.
Their research programme for 1976 included experiments with the pipe clearance reduced to 13 mm, that is, a relative clearance, \( h/D = 0.03 \), while the orientation angle \( \alpha \) remained at 90°. From correspondence it is understood that Prof. Grace is currently (1978) writing a report on the last two years’ research.

8.15 ISAACSON AND MAULL, 1976 (apparently at University of Cambridge)

According to the authors, their work "may be considered an extension of Bidde's investigation, with emphasis being placed on the generation of shed vortices during the wave period and the relation between these vortices and the lift force". Tests were done on vertical circular cylinders placed in a wave flume, as well as by oscillating a vertical cylinder in still water.

Apart from their observations regarding the various vortex shedding regimes at different values of the period parameter (which has been referred to earlier), they made interesting comments on the lift force frequency.

They thought that the question of whether lift would or would not be developed, and at what frequency compared to the wave frequency, could be answered best by considering the shedding of vortices from the cylinder during half a period of the oscillatory motion. They formulated the following model:-

As a vortex develops and detaches from the cylinder, the lift force attains a maximum, then reduces and changes sign as the next vortex develops. When the relative velocity between fluid and cylinder changes direction the next vortex formed has the same sign as the previous one. If \( N \) vortices are shed in each half cycle, the ratio of lift force frequency, \( f_L \), to oscillation frequency, \( f_o \), is \((N + 1)\).
This may be illustrated by an example (based partly on Isaacson and Maull's (1976) Figs. (8-28) and (8-29), and partly on the present writer's own interpretation, refer Fig. (8-30): -

Consider the case where two vortices are shed per half cycle, i.e. \( N = 2 \). Since \( N = 2 \), the lift force frequency is 3 times the oscillatory frequency. Mercier (1973) apparently found \( f_L / f_0 = 2 \) if, \( K < 18 \), and 3 if \( K > 18 \). This implies that, according to the Isaacson-Maull model, two vortices were shed per half cycle once the period parameter exceeded 18.

Sarpkaya (1975) found that \( C_{L_{\text{max}}} \) peaked at period parameter values of about 10 and 18 (refer Fig. (8-18)). If, in fact, one vortex were just shed per half cycle when \( K = 10 \), and two vortices were just shed when \( K = 18 \), the results of Sarpkaya (1975) would reinforce the credibility of Isaacson and Maull's theory.

Sarpkaya subsequently extended his research on cylinder forces in oscillatory flow to a Reynolds number range up to \( 5 \times 10^5 \), that is,
Figure 8-30 A schematic representation of how the lift force may be generated where two vortices are shed per half cycle (i.e. $N = 2$)
higher than in the research programme described in Sarpkaya (1975). (The later publication is Sarpkaya (1976a), and an extract of his results appears in Sarpkaya (1976c).

Sarpkaya (1976c) contains a graph of the lift frequency to oscillation frequency ratio, $f_L/f_0$, as a function of the Reynolds number and the surface Keulegan-Carpenter number, $KC$, - see Fig. (8-31). (KC is not the local period parameter value, $K$, at a specific depth, as one would normally use in the case of say a pipeline, but the value at the water surface, i.e. it characterizes the wave motion).

Fig. (8-31) shows that $f_L/f_0$ increases with increasing $KC$.

![Figure 8-31 Variation of the $f_L/f_0$ ratio with the Reynolds number, $R_e$, and the surface period parameter, $KC$](image)

8.16 CHAKRABARTI, WOLBERT AND TAM, 1976 (Chicago Bridge and Iron Company, Plainfield, Illinois)

The study of Chakrabarti, et al (1976) included the calculation of $C_D$, $C_I$ and $C_L$ from wave force data obtained from tests on a circular
cylinder placed vertically in a wave tank. They also investigated the relationship between lift force frequency and wave frequency.

Because the vertical cylinders were remote from a solid boundary, the results are not to be applied blindly to submarine pipelines near the sea floor.

A. In-line forces

The coefficients $C_D$ and $C_I$, for use in the Morison equation, were determined from the record of in-line force, by a best least-squares fit method. The water particle kinematics, substituted into the Morison equation were obtained from the Airy wave theory, whilst the coefficients were assumed to remain constant over the whole wave cycle. $C_D$ and $C_I$, plotted against the period parameter, $K$, are shown in Fig. (8-32).

![Figure 8-32](image)

Figure 8-32 $C_I$ and $C_D$ versus $K$ for vertical cylinders remote from a boundary

The coefficient values were actually derived as weighted average values, corresponding to the total horizontal force, over the entire length of the cylinder. (This restricts the general usefulness of the curves). Using these curves and the Morison equation, the authors
calculated in-line forces which they then correlated with measured forces. They found a very good correlation.

They also attempted to plot $C_D$ and $C_I$ versus the Reynolds number, but this proved unsuccessful.

Chakrabarti, et al (1976) attribute the scatter in drag coefficient, in Fig. (8-32), to the relatively small drag force compared to the inertial force, at small $K$ values. The fact that the drag force becomes important at larger period parameters, has been pointed out on numerous occasions in this dissertation.

The reason given by the authors for the difference between their results and those of Keulegan and Carpenter (1958) and Sarpkaya (1974, 1975), was that their own experiments had been conducted in two-dimensional flow under a changing free surface, whereas the other experiments had taken place in virtually one-dimensional oscillatory flow.

B. Transverse forces

The measured profile of the lift force was generally found to be irregular: typically of the form shown in Fig. (8-33).

![Figure 8-33 Typical profiles of the lift force](image)
The frequency of the peak lift forces was found (almost) always to be a multiple of the wave frequency, in other words, 1, 2, 3, ... times the wave frequency. (In a few instances peaks occurred at frequencies 1.5 and 2.5 times the wave frequency but these peaks were generally very weak).

Chakrabarti, et al, like Isaacson and Maull (1976) and Isaacson (1974), associated lift forces with vortex shedding and agreed that if the ratio of lift force frequency to wave frequency $f_L/f_o$, was $(N + 1)$, the number of vortices formed and shed per half cycle would be $N$.

Considering only the principal (i.e. most predominant) peaks of the lift force, they plotted the frequency ratio $f_L/f_o$ versus the period parameter; as can be seen in Fig. (8-34).

![Figure 8-34](image)

**Figure 8-34** Ratio of predominant lift frequency to wave frequency versus period parameter. (Possible transition zones are shown by dotted vertical lines)

Although Fig. (8-34) seems, at a glance, similar to Fig. (8-20) of Sarpkaya (1975), there is a difference: Sarpkaya (1975) plotted the ratio $f_v/f_L$ versus the period parameter. (Chakrabarti, et al (1976), incidentally, incorrectly state that Sarpkaya (1975) found the dominant vortex shedding frequency to be twice the "wave frequency". What he did, in fact, find was that the dominant vortex shedding frequency was twice the lift force frequency).
Chakrabarti, et al (1976) concluded that the number of eddies shed per half cycle and the lift force frequency were directly related to the period parameter. "The higher the parameter the higher is the number of eddies shed by the cylinder". They suggested, on the strength of Fig. (8-34), that no eddies were shed for $K < 6$ to 7, one eddy was shed for $K$ between about 7 and 15, and two eddies were shed for $K > 15$. (Unfortunately they used the word "formed" instead of "shed"; this led to confusion as well as criticism by Maull and Isaacson (1977)). They stated that similar observations had been made by others such as Isaacson (1974), Sarpkaya (1975) and Tuter (1974). The transition points were, however, different in the latter two cases, mainly for the following two reasons: Firstly, Sarpkaya and Tuter had measured the particle velocities whereas the authors had calculated the velocities from linear wave theory. Secondly, Sarpkaya and Tuter performed their tests in one-dimensional flow without a changing free surface. (It is not clear how Chakrabarti, et al (1976) can state that Sarpkaya (1975) made such observations because nowhere does the latter make any such comments. Perhaps Chakrabarti, et al (1976) inferred this, incorrectly, from Fig. (8-20). The present writer suspects that this is the case because Chakrabarti and Wolbert (1975) casually compare Fig. (8-20) with their own results as though there were no difference between $f_v/f_L$ and $f_L/f_o$).

Maull and Isaacson (1977), in discussion of the paper, said that if a lift force is generated, its frequency will normally be at least twice the wave frequency. Thus the number of vortices shed per half cycle, $N$, would be at least one. They questioned the existence of a lift force at the wave frequency at values of $K$ less than about 5, where $N = 0$.

The first reason they suggested for this phenomenon was that sufficient flow asymmetry to create lift may occur during only one half of the wave cycle, thus resulting in a lift force frequency equal to the wave frequency. The second and third possible reasons, which they considered more likely than the first, were misalignment of the balance system with respect to the experimental waves or an undue cross-coupling of the balance whereby a lift reading would contain a significant amount of drag force (which has the same frequency as that of the wave).
Chakrabarti, et al (1977), replied that neither the second nor the third were valid reasons. The first was more likely and could have been caused by small nonuniformities or out-of-roundness of the cylinder. They also stated that the ratio \( f_/f_0 = 1 \) was uncommon, but not unprecedented because also Tuter (1974), who had experimented in a more controlled environment, found the same at small \( K \) values.

With regard to the coefficient of lift, \( C_L \), Chakrabarti, et al (1976) pointed out that the popular equation for the lift force

\[
F_L = \frac{1}{2} C_L D \rho u^2
\]

(2-9)

was, in fact, unsuitable because the lift force was quite irregular and had multiple frequencies. (It is recalled that Biddé (1971) made a similar observation).

They proposed to express the lift force per unit length in a series form:

\[
F_L(t) = \frac{1}{2} D \rho u^2 \max \sum_{n=1}^{N_H} C_L(n) \cos [2\pi n f_0 t + \gamma(n)] \quad (8-26)
\]

where

- \( F_L(t) = \) the time-dependent lift force.
- \( u \max = \) the maximum horizontal velocity component, obtained by linear wave theory.
- \( C_L(n) = \) the lift coefficient of the \( n \)th harmonic.
  
  \( (C_L(n) \) assumed to be a function of \( K \).
- \( N_H = \) number of harmonics.
- \( \gamma(n) = \) the phase angle of the \( n \)th harmonic force

and the other symbols have their usual meanings.

The values of \( C_L(n) \) and \( \gamma(n) \) are calculated by numerical integrations:

\[
C_L(n) \cos \gamma(n) = \frac{2}{T} \int_0^T \frac{2 f_L(t)}{\rho u^2 \max} \cos (2\pi n f_0 t) \ dt \quad (8-27)
\]

\[
C_L(n) \sin \gamma(n) = \frac{2}{T} \int_0^T \frac{2 f_L(t)}{\rho u^2 \max} \sin (2\pi n f_0 t) \ dt \quad (8-28)
\]
\( \gamma(n) \) was not calculated in the analysis, but the lift coefficient for the first five harmonics was evaluated from the experimental data, and is shown in Fig. (8-35).

![Figure 8-35](image)

Figure 8-35 First five coefficients of lift obtained by harmonic analysis

Chakrabarti, et al (1976) pointed out that these curves should be considered as provisional and should be verified with further tests as they were based on a number of widely scattered points. A further limitation appears to be that no values were obtained for \( K > 16 \).

Over most of the investigated range of \( K \) the second component of lift predominates. This is associated with a lift force having a frequency twice that of the incident wave, and the shedding of one vortex per half cycle.

For \( K \) smaller than about 6, the first component of lift associated with a lift force with a frequency equal to the wave frequency, is the largest.

Chakrabarti, Wolbert and Tam (1976) also considered the resultant force stemming from both the in-line and transverse forces. The contribution of the transverse (or lift) force was negligible at \( K \) values below about 5, but if \( K \) approached about 15, the resultant force could be as much as 60% higher than the in-line force alone.
The paper by Vongvisessomjai and Silvester (1976) contains useful information regarding the calculation of water particle velocities and accelerations on the one hand, and in-line forces on a variety of object shapes, on the other hand. The writer will briefly mention some of the points that are of particular interest to this thesis; the reader is referred to the original paper for more detail.

Because water particle kinematics are of basic importance for calculating wave forces on submerged objects, the authors produced valuable graphs and tables whereby the most likely maximum velocity (and acceleration) values at any depth, and for any wave steepness, may readily be found.

These graphs and tables are based on linear wave theory (deep water), cnoidal theory and hyperbolic theory (shallow water), and have, where necessary, been modified to suit experimental data.

Having determined the maximum horizontal velocity, the period parameter, K, which is recognized as most important, can be calculated.

The other parameter that may be important for some objects - but rarely for submarine pipelines, since their diameters are usually small compared to wave length and water depth - is the relative size, \( D/L \).

From dimensional analysis the authors derived the maximum in-line force and expressed it in dimensionless form as:

\[
\frac{F_{max}}{\rho^A u_{max}^2} = \text{function} \left( \frac{D}{L}, \frac{u_{max}}{D}, \frac{u_{max}}{v}, \frac{u_{max}}{\sqrt{g (d-h)}} \right) \quad (8-29)
\]

or:

\[
\frac{F_{max}}{\rho V_{max}} = \text{function} \left( \frac{D}{L}, \frac{u_{max}}{D}, \frac{u_{max}}{v}, \frac{u_{max}}{\sqrt{g (d-h)}} \right) \quad (8-30)
\]
where $F_{max}$ is the maximum value of the total force per unit length, and all the other symbols have their usual meanings.

Eq. (8-29) represents a drag formulation whereas Eq. (8-30) represents an inertial formulation.

The four dimensionless groups are relative size, period parameter, Reynolds number and Froude number respectively.

As stated in Vongvisessomjai (1976) and Vongvisessomjai and Silvester (1976), the Reynolds number was thought to be of minor importance and could be disregarded (with some reservation, however). Also the Froude number could be disregarded provided the body was not too close to a solid boundary or to the water surface.

The behaviour of the total force within the ranges of the two remaining parameters, viz. relative size, $D/L$, and period parameter, $K$, was then considered as well as the methods to calculate it.

Vongvisessomjai (1976) found the Morison equation to be the most suitable method for calculating in-line forces on small objects, i.e. for $D/L < 0.2$.

Fig. (8-36) shows the ratios $C_I/C_I$ (potential flow) and $C_D/C_D$ (steady flow, at similar $Re$) versus $K$.

It appears that

1. For $K < 3$, $C_I \approx C_I$ (potential flow) ($= 2.0$ for a cylinder away from a boundary).

2. For $K > 25$, $C_I \approx 0.75 C_I$ (potential flow) ($= 1.5$ for a cylinder away from a boundary).

3. For $K \approx 12$ (i.e. $A/D = 12/2\pi \approx 2$), the coefficients display the greatest deviations from the values they are compared with.
If $D/L > 0.2$ the presence of the object has a "scattering" effect on the incident wave, i.e. the flow field in the vicinity of the object is affected and the Morison equation can give unreliable results. If, in addition, $K < 3$, the force can be calculated via the so-called diffraction theory because for such small $K$ values the force is predominantly inertial. If $K > 3$, viscous and drag effects may lead to inaccurate results by the diffraction theory.

This reasoning is borne out by Fig. (8-36a), from which it can be seen that:

1. For $K < 3$, the inertial force completely dominates the drag force.
2. For $5 < K < 12$, both inertial and drag components make up the total in-line force.
3. For $12 < K < 25$, the drag force becomes more and more important.

4. For $K > 25$, the drag force is completely predominating and the dimensionless force remains essentially constant.

Using experimental data of previous investigators, Vongvisessomjai and Silvester (1976) plotted the dimensionless force with $K$ and found good correlations.

For a cylinder they found the following relationship to be valid if $K < \pi$ (or about 3), and $D/L$ generally $< 0.32$ :-

$$\frac{2 F_{\text{max}}}{\rho D u_{\text{max}}^2} = \frac{\pi^2 C_I(\text{potential flow})}{K}$$

(8-31)

where

$$C_I(\text{potential flow}) = \begin{cases} 2,0 & \text{for a cylinder remote from a boundary} \\ 3,3 & \text{for a cylinder in contact with a boundary} \end{cases}$$

They were of the opinion that if the relative clearance, $h/D$, exceeded 0.5, $C_I$ did not differ much from its potential flow value away from a boundary, i.e. $C_I \approx 2,0$.

Incidentally, Eq. (8-31) is merely a different way of writing Grace's Eq. (5-3) for the inertial case, and then assuming $C_I = C_I(\text{potential flow})$.

From Eq. (5-3),

$$F_{\text{max}} = \frac{F_I_{\text{max}}}{u_{\text{max}}}$$

if $\beta > 1$

$$= C_I \rho \frac{D^2}{4} \frac{2\pi \rho}{u_{\text{max}}^2}$$

$$= \frac{1}{2} \rho D u_{\text{max}}^2 \frac{\pi^2 C_I}{K}$$

thus

$$\frac{2 F_{\text{max}}}{\rho D u_{\text{max}}^2} = \frac{\pi^2 C_I}{K}$$

There is slight disagreement as to exactly when this equation becomes valid:
The derivation of Eq. (5-3) indicates validity when \( \beta > 1 \), or \( K < \frac{\pi^2 \, C_l}{2 \, C_D} \),

whilst it is stated that Eq. (8-31) holds for \( K < \pi \), in other words, for \( \Lambda/D < 1 \).

This is considered to be of academic interest only.

8.18  GARRISON, FIELD AND MAY, 1977  (Naval Postgraduate School, Monterey, California)

Garrison, et al (1977) studied the behaviour of the in-line force coefficients applicable to cylinders, remote from a solid boundary, in oscillating flow.

Since dimensional analysis suggests that wave forces on a body in unsteady flow are dependent on more than one dimensionless group, they expressed the coefficients as functions of two parameters: the Reynolds number and the period parameter.

Although their approach was not completely new (similar strategies had been adopted in the Wallingford (1961) study, and by Rance (1969) and Miller (1975)), their paper as well as the discussions it has aroused, contains stimulating ideas and will be reviewed closely. It is unfortunate that the reply by the authors was not yet available at the time of publication of this dissertation.

Garrison, Field and May (1977) recognized the difficulties involved in determining drag and inertial coefficients for use in calculating wave loads, be it with the aid of laboratory tests or full scale tests in the ocean.

One problem in the laboratory is to achieve large Reynolds numbers that are comparable to those often encountered in prototype situations, namely \( 10^6 \) and higher. (This difficulty has also been mentioned in numerous Wallingford reports, e.g. in 1961, 1962, 1966 and appears to be a prime reason for acquiring the pulsating water tunnel from which, inter alia, Rance (1969) obtained his test data). A second problem with laboratory experiments is that Froude scaling is necessary and that one generally does not have independent control over the Reynolds
number with the result that it is practically impossible to achieve both Reynolds and Froude similarity. The authors comment "it is doubtful that much can be learned about prototype wave forces from small-scale wave channel testing" of "piling or structures composed of small diameter tubular members although there seems to be no scarcity of such tests reported in the literature".

Experiments performed in the ocean suffer from the disadvantage that no control can be exercised over the incident waves. The irregular and complex wave motion "does not lend itself to description through a simple dimensionless parameter or two".

The present result is that neither laboratory nor ocean tests have yielded very much systematic information regarding drag and inertial coefficients.

Keulegan and Carpenter (1958) were the first to investigate a special, more basic, aspect of the wave force problem, namely the in-line force acting on a circular cylinder in essentially one-dimensional flow oscillating with a simple sinusoidal motion. Although they had recognized, from dimensional analysis, the possibility of Reynolds number dependence of the drag and inertial coefficients, they concluded that no such correlation with Reynolds number, in fact, existed.

Garrison, et al (1977) replotted the Keulegan-Carpenter data which spanned a range of Reynolds numbers between $7 \times 10^3$ and $3 \times 10^6$ and found that both coefficients were dependent on the Reynolds number. They suspected that the apparatus used by Keulegan and Carpenter was not well-suited to the study in so far as the Reynolds number changed whenever the period parameter was changed. In other words, a series of tests could not be performed at constant period parameter and varying Reynolds number unless the test cylinder diameter was changed every time. A further handicap was that the Reynolds number range covered in the experiments was rather limited, and it so happened that the coefficients underwent some rather unexpected variations in this range; Keulegan and Carpenter then interpreted this as 'scatter'.

Garrison, Field and May also noted that Sarpkaya (1975) failed to find a correlation between the coefficients and Reynolds number.
They devised a test apparatus capable of holding the period parameter constant throughout a series of experiments wherein only the Reynolds number was varied.

The apparatus consisted of a carriage supported on rails set over a water channel. A pair of thin vertical struts, connected to the carriage, held a horizontal cylinder at about mid-depth in the channel. The carriage was driven backwards-forwards in a virtually sinusoidal motion by a linkage connected eccentrically to a flywheel. By varying the rotation speed of the flywheel without varying the eccentricity, the Reynolds number could be altered without changing the amplitude of motion of the carriage and thus the period parameter. (It will be recalled that the period parameter \( K = \frac{u_{\text{max}} T}{D} = \frac{\pi}{2} \frac{2A}{D} \)). The authors actually preferred to use the relative amplitude parameter, \( \frac{2A}{D} \), rather than \( K \), although they are, of course, closely related.

The flywheel was capable of attaining a high rotational speed, so that fairly large Reynolds numbers, apparently up to about \( 6 \times 10^5 \), could be achieved.

The period parameter, or relative amplitude, could be changed to a new fixed value by adjusting the eccentricity.

Sarpkaya and Collins (1978) severely criticized the authors on two points. Firstly, they objected because the authors had neglected to refer to certain references (i.e. Collins (1976), Onur (1975), Sarpkaya (1976a, and 1976b)) which dealt with experiments performed at Reynolds numbers higher than those in Sarpkaya (1975). Subsequent publications that have become available are Sarpkaya (1977a) and Sarpkaya (1977b). Secondly, the test apparatus was criticized. Severe vibrations were to have occurred which reduced the reliability of the test data.

In the experiments of Garrison, et al (1977) the relative flow was caused by the oscillation of the cylinder in still water, and not by the oscillation of water with respect to a fixed cylinder. They explained that the only difference between these two situations lies in the appearance of a buoyancy-like force (often called the Froude-Krylov force, e.g. by Hogben (1974), Garrison, Gehrman and Perkinson
in the second case. This force is exactly equal to the product of the mass of water displaced by the cylinder and the acceleration. In other words, it is the force that would have accelerated the mass of water if the cylinder had not been there.

An additional force is required to accelerate the water around the cylinder. It has been shown earlier that the combined force due to acceleration effects is:

\[ F' = M_0 \ddot{v} + M_a \dot{v} \]  

(2-17)

Froude-Krylov force to accelerate water around cylinder

\[ F'_I = \left[ 1 + \frac{M_a}{M_0} \right] M_0 \dot{v} \]  

(2-18)

i.e.

and the coefficient of inertia, \( C_I \), was defined as

\[ C_I = \left[ 1 + \frac{M_a}{M_0} \right] \]  

(2-20)

we can write Eq. (2-20)

\[ C_I = 1 + C_m \]  

(8-32)

where \( C_m \) is the so-called added mass coefficient.

Since the test cylinder was made to oscillate in still water, the Froude-Krylov force was absent and Garrison, et al (1977) actually derived and plotted \( C_m \), and not \( C_I \), versus the relevant parameters.

The simplifying assumption was made that \( C_m \) and \( C_D \) did not vary within the oscillatory cycle. This was done for two reasons. Firstly, constant coefficients are more convenient for application and, secondly, "experience has indicated that the constant coefficient represents the force variation fairly well in most cases".

McNown and Learned (1978) criticized the authors on the latter point. The practice of expressing the coefficients as average and constant values for the entire cycle was convenient, but could be misleading. The designer was more interested in maximum forces than
in average trends during a cycle. They realised that instantaneous values of the coefficients could not be determined directly, but predicted forces based on the average values could be compared with measured forces.

Having made this simplifying assumption, Garrison, et al applied a best least-squares fit technique of the Morison equation to the measured forces to find the constant values of the two coefficients.

Values of $C_m$ and $C_D$ were determined at ten different integer values of the amplitude parameter, $2A/D$, and at various Reynolds numbers, $Re$, where

$$Re = \frac{u_{\text{max}} D}{\nu}$$

Figs. (8-37a) and (8-37b) show the consistency of the data obtained for one cylinder. Data were obtained from tests on three cylinders. (Fig. (8-38) indicates the mean lines drawn through all the data points.

It is interesting to note that $C_m$ becomes negative for large $2A/D$ values at small $Re$ (i.e. $C_I$ becomes less than 1.0). Although the authors do not comment on this phenomenon at all, McNown and Learned (1978) do.

They point out that forces on cylinders in oscillating flow vary considerably with the degree of unsteadiness, which is closely linked to the time required for the shedding of the first vortex. For shorter periods (and thus small period parameters and small $2A/D$ values) the motion is quasi-potential and inertial forces are dominant. For longer periods (and thus large period parameters and large $2A/D$ values) the motion is quasi-steady, drag forces are larger, and inertial forces can only be determined empirically. For intermediate periods "the motion is strongly affected by the growth of the initial vortex and the dramatic consequences of its shedding".

Their explanation for the existence of negative values for the added mass coefficient, $C_m$, (which implies negative masses and thus does not represent physical reality) is the following:-
Figure 8-37 Coefficients of drag and added mass versus $R_e$ for various 2A/D values (data from one cylinder)

Figure 8-38 Coefficients of drag and added mass versus $R_e$ for various 2A/D values (data from three cylinders)
The mass of water accelerated together with the cylinder varies with time - especially if the period of motion is approximately equal to that of the formation of the first vortex. As this mass increases, \( C_m \) also increases (and becomes considerably larger than the value that would correspond to the net mass of water actually displaced by the cylinder). When the vortex sheds from the cylinder, the mass is reduced abruptly, and so is the value of \( C_m \) - even to negative values.

An algebraic explanation, based upon McNown and Keulegan (1959), is also offered. The inertial force component per unit length, \( F_I \), is written in the form

\[
F_I = \frac{d}{dt} \left( \frac{\pi}{4} D^2 \rho \ C_m u \right) = \frac{\pi}{4} D^2 \rho \left[ C_m \frac{du}{dt} + u \frac{dC_m}{dt} \right]
\]  

(8-33)

A rapid reduction in accelerated mass is reflected by a large negative value for \( \frac{dC_m}{dt} \). The term \( u \frac{dC_m}{dt} \) dominates the inertial component values for \( 2A/D \) between 3.5 and 6.0 in Fig. (8-38b), causing \( C_m \) to become negative.

Therefore, "negative masses" and negative values of \( C_m \) are "a consequence of the definitions and procedures used by the authors and do not actually contradict reality".

Figs. (8-39) and (8-40) show the Keulegan-Carpenter data, obtained at fairly low Reynolds numbers, plotted together with Garrison, et al.'s results (shown as broken lines). Contours of equal \( 2A/D \) values were drawn (solid lines) through the Keulegan-Carpenter data points. The authors comment that although the overlap is small, "the results appear to agree well in the overlap region".

![Figure 8-39](image-url)  
Figure 8-39 \( C_D \) versus \( Re \) for various values of \( 2A/D \)
From Figs. (8-38), (8-39) and (8-40), Garrison, et al (1977) made several observations, then drew four formal conclusions which, in turn, were discussed and modified by McNown and Learned (1978).

Observations concerning $C_D$

1. From Fig. (8-38a) and within the $R_e$ range of the data, it appears that $C_D$ decreases with increasing $R_e$.

2. The change in $C_D$ is fairly gradual except in the range $10^4 < R_e < 2.5 \times 10^5$ where some very drastic variations take place, this can be seen in Fig. (8-39).

3. $C_D$ seems to approach a constant value at high $R_e$.

4. The variation of $C_D$ with $R_e$, as shown in Fig. (8-39), appears to represent a transition from subcritical to supercritical flow in a somewhat similar manner as in the case of steady flow. (Refer Fig. (2-5)). The transition process, however, begins at smaller values of $R_e$ than in steady flow and extends over a greater $R_e$ range. (This was also noted by Rance (1969), see Fig. (7-5)). Garrison, et al suspected that the earlier start of the transition is due to the cylinder moving through its own highly turbulent wake, whilst the extended $R_e$ range over which the transition takes place may be attributed to the fact that the instantaneous $R_e$ during a cycle of oscillation ranges from zero to $u_{max}D/V$. 

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**Figure 8-40** $C$ versus $R_e$ for various values of $2A/D_m$.
5. Although data are not available for high $Re$ at small $2A/D$ values, it appears from Fig. (8-38a) that $C_D$ increases with increasing $2A/D$. The authors comment that this is consistent with the notion that if $2A/D$ is quite small, separation does not occur, and the drag coefficient is reduced.

This writer draws attention to the fact that Fig. (8-38a) is in conflict with Fig. (7-5) of Rance (1969) who found that $C_D$ decreases with increasing $2A/D$. In deriving Fig. (7-5), Rance, et al assumed that $C_I = 2$ and remained constant at all values of $Re$. Since the $Re$ in Fig. (7-5) is not the maximum Reynolds number in the cycle, but the Reynolds number occurring at the time of maximum in-line force, one cannot compare the two figures directly.

6. At large $2A/D$ values and high $Re$, $C_D$ seems to approach a constant value of about 0.6. Research at higher $Re$ is, however, necessary to confirm this trend.

7. The authors quote the following investigations to substantiate their own findings regarding $C_D$:

7.1 Kim and Hibbard (1975) and, apparently, Blumberg and Rigg (1961) report that towing-tank tests indicated that $C_D$ increased from 0.57 to 0.60 for smooth cylinders over the $Re$ range $3 \times 10^5$ to $6 \times 10^6$. (It seems that these tests were carried out in steady-state flow).

Garrison, et al (1977) observed in their oscillatory flow tests that $C_D$ reached its minimum value of about 0.57, for large $2A/D$, near $Re = 2 \times 10^5$ and increased again to about 0.61 at higher values of $Re$; refer Fig. (8-38a).

7.2 Dean and Aagaard (1970) determined $C_D$ for piles and found that $C_D$ decreased from about 1.3 to 0.6 over the $Re$ range $2 \times 10^5$ to $6 \times 10^6$.

7.3 Wheeler (1963) found $C_D \approx 0.6$ to correspond with the peak force.

7.4 Kim and Hibbard (1975) also measured both wave kinematics and the force on a test pile in the ocean and found $C_D = 0.61$ within the $Re$ range $2 \times 10^5$ to $7 \times 10^6$. This is in agreement with their steady flow results as well as Fig. (8-38a).
Observations concerning $C_m$

1. From Fig. (8-38b) and within the $R_e$ range of the data, it appears that $C_m$ increases with increasing $R_e$. The explanation offered for this phenomenon is that the width of the wake reduces and the separation point moves downstream as the $R_e$ increases and changes from subcritical to supercritical. Supercritical flow past the cylinder resembles inviscid flow more closely than subcritical flow, so that it is not unexpected that $C_m$ approaches the value of 1.0 as predicted by potential flow theory, (i.e. $C_f$ approaches 2.0).

2. In the range $10^4 < R_e < 3 \times 10^4$ considerable changes in $C_m$ occur, see Fig. (8-40).

3. $C_m$ seems to approach a constant value at high $R_e$. The explanation offered for observation 1, above, may also be valid here: The more stable wake configuration characteristic of supercritical $R_e$ would reduce the variability of $C_m$.

4. It appears from Fig. (8-40) that $C_m$ tends to become independent of $R_e$, at low $R_e$ values, but highly dependent on $2A/D$.

5. As $2A/D$ tends to zero, $C_m$ approaches the value of 1.0 as would be expected since this is the theoretical value for unseparated inviscid flow.

6. At large $2A/D$ values and high $R_e$, $C_m$ seems to approach a constant value of about 0.7. Research at higher $R_e$ would be necessary to confirm whether $C_m$ becomes completely $R_e$ independent at high $R_e$.

Since both $C_D$ and $C_m$ were found to be highly dependent on $R_e$, unless the $R_e$ was very large, the authors stated that results based on tests performed at low $R_e$ (as usually is the case in laboratory work) could be misleading. This implies that the validity of small-scale model tests to predict prototype forces must be questioned.

Conclusions by Garrison, Field and May (1977)

1. Both $C_D$ and $C_m$ (and thus $C_f$) are strongly $R_e$ dependent, unless $R_e > 2 \times 10^5$. 
2. $C_D$ and $C_m$ generally depend on both parameters, $R_e$ and $2A/D$. When $2A/D$ gets large, both coefficients become independent of this parameter, and when the $R_e$ becomes large, the coefficients become independent of this, too.

3. When both parameters are large, $C_D$ approaches about 0.61, and $C_m$ about 0.70 (i.e. $C_I = 1.70$).

4. As $2A/D$ decreases, "$C_m$ and $C_D$ tend to their potential flow values of 1.0 and zero, respectively".

In discussion, McNown and Learned (1978) commented on these conclusions, as follows:

**Conclusion 1.**

This conclusion needs modification because the Keulegan-Carpenter values of $C_m$ are, in fact, independent of $R_e$ over the greater part of the range. (McNown and Learned (1978), who apparently replotted the data, found that the curve for $2A/D = 7.0$ in Fig. (8-40) had been drawn incorrectly: it had been "placed much too high"; the most probable limiting value of $C_m$ was about 0.2 and not 0.65). The curves are actually all horizontal lines for $R_e < 1.5 \times 10^4$, and $C_m$ shows no variation for $2A/D < 3$. (The data are, however, too sparse for $R_e < 10^4$ to make a definite conclusion). $C_m$ does vary somewhat for $2A/D > 3$ and for $R_e > 10^4$.

**Conclusion 2.**

This conclusion is "equivalent to saying that a quasi-steady flow approaches steady flow as $2A/D$ becomes large". If several vortices are shed during one cycle, the pattern of flow much resembles that for steady flow. That is why the curves for $C_m$ and $C_D$, respectively, in Fig. (8-38), have almost the same trends for $2A/D \geq 4$, and only the curves for $2A/D = 2$ and 3 are significantly different.

**Conclusion 3.**

The values given as convergence values for $C_D$ and $C_m$ are considered unreasonable on two grounds: both sets of curves have a
definite slope at the highest $R_e$ tested, and the spread of points as indicated in Figs. (8-37a) and (8-37b), is more than would justify two-digit accuracy.

**Conclusion 4.**

It is agreed that $C_m$ tends to its theoretical value of 1.0 (as $2A/D$ decreases), although the Keulegan-Carpenter data clearly exceed that value.

The conclusion that $C_D$ tends to zero is not supported by the data; it appears to the present writer that this conclusion must have originated in Sarpkaya and Garrison (1963). (McNown and Learned (1978) then discuss the behaviour of $C_D$ with varying $R_e$, and not varying $2A/D$. The present writer suspects that they confused the two parameters). Be that as it may, they state that most of the $C_D$ curves rise as $R_e$ decreases and though the magnitude of drag force may become small, it is more likely that the drag coefficient will be large at small $R_e$ values. This growth of $C_D$ is a consequence of the conventional use of a coefficient based on the square of the velocity in a range where the resistance varies with a lesser power, tending toward unity for truly laminar flow.

Finally, McNown and Learned (1978) expressed the hope that Garrison, et al would contribute towards the understanding of phenomena such as why negative values of $C_m$ can exist, the associated formation and shedding of vortices, changes that occur as the boundary layer goes from laminar to turbulent, how well average-over-the-cycle coefficient values describe the conditions of maximum and zero force. (It is anticipated that they will describe these forces better if $2A/D > 4$ than when $2A/D < 4$. In prototype situations in the ocean both $R_e$ and $2A/D$ are likely to be large—except in cases of large cylinder diameters with small values for the latter parameter, where inertial forces become more and more important.
8.19 WALLINGFORD INVESTIGATIONS ON PROTOTYPE PIPE INSTALLATIONS,  
1970 - 1978

8.19.1 Scour

The Hydraulics Research Institute, Wallingford, undertook an experimental study to provide information on scour around pipes buried in sand and subjected to tidal flow. Part of this programme was devoted to the study of the movement of a flexible pipeline.

The experiments took place in the Taw-Torridge estuary on the north coast of Devon. Concrete coated steel pipes with outside diameters of 270 mm, 500 mm and 740 mm as well as a 160 mm outside diameter high density polyethylene flexible pipeline, encased in a steel wire wound armoured sheath, were used.

The flexible pipeline proved to have certain advantages over the rigid pipelines. It could be laid with less preparation on an undulating sand bed. It then conformed very well to the bed form. Being close to the bed, drag forces were reduced (and although not stated, lift forces were probably increased), and the local high scour-producing currents that occur where gaps are present between pipe and bed, were greatly eliminated. The flexible pipe remained stable throughout the test period and progressively buried itself under the sand.

It was concluded that the flexible pipe was a most useful type of pipe where severe bed movements were anticipated, and could lead to considerable economies where it was acceptable to lay a surface pipe, which would gradually bury itself, in areas where obstructions to navigation were no problem.

As this study was primarily concerned with scour and not with wave forces; it will be discussed no further; for more information, the reader is referred to the well-illustrated report by Littlejohns (1977).

8.19.2 Wave-induced forces

Experiments are presently (1978) being conducted in the Atlantic Ocean at Perranporth, Cornwall, to study wave forces on pipelines near the sea bed and the associated coefficients.
P.S.G. Littlejohns informed the writer that effects of orientation of the pipe with respect to wave direction, were also being examined, and that research had also been done on large diameter cylinders resting on the bed in the Wallingford pulsating water tunnel. Unfortunately, the results of the latter as well as the Perranporth experiments still remain confidential at this stage.

The installation used at Perranporth reminds one of that described by Grace and Nicinski (1976), but the data logging system seems to be more sophisticated.

The 760 mm diameter steel pipe, 15 m in length, consists of two flanking wing sections and two 1 m long test sections. The test sections are cantilevered from the wing sections, which in turn, are mounted on a steel platform measuring 15 m x 12 m x 0.3 m. See Fig. (8-41).

This rig rests on the sea bed in 20 or 25 m of water.

Figure 8-41 Diagrammatic sketch of the Perranporth installation

When subjected to forces, the test sections, cantilevered by sets of stiff springs, can move slightly, both horizontally and vertically; such movements are then measured by very sensitive displacement transducers, and can be interpreted as force measurements.

Electro-magnetic velocity meters, set at three different heights above the platform measure the horizontal velocity components normal
and parallel to the pipe. Also pressures at sea bed level are measured.

Electronic signals from all these instruments are fed into a magnetic tape data logging system contained in a pressure capsule. All data are recorded on a common time base and can therefore be correlated. The capsule, which also contains the power supply for the instrumentation, is recovered periodically for data retrieval and servicing.

The recording system has been modified so that conditions are monitored continuously only when moderate to high surface waves impinge on the pipe.

In addition, a wave rider buoy, on the water surface, records the wave heights and periods for statistical analysis.

Each record is analyzed in the following manner:

1. The pressure and velocity component are correlated to determine the mean direction, and angular spread about the mean, of the wave energy at the site for each frequency band.

2. A wave by wave analysis of the vertical and horizontal forces, $F$ and $F$ respectively, (actually called "lift" and "drag" forces in the Wallingford information pamphlet, but "vertical" and "horizontal" appear to be the more correct terms), and the velocity component normal to the pipe, $u$, is carried out to determine their maximum values for each wave. Coefficients

\[
\frac{F_{\text{max}}}{\frac{1}{2}} \rho D \frac{u^2}{\text{max}} \quad \text{and} \quad \frac{P_{\text{max}}}{\frac{1}{2}} \rho D \frac{u^2}{\text{max}}
\]

are calculated and correlated with the relative displacement parameter, $A/D$, and the Reynolds number, $Re = \frac{u_{\text{max}} D}{V}$.

(Thus it seems to be an extension of the work by Rance (1969), and is similar to that of Garrison, Field and May (1977)).
3. Instantaneous values of \( u \) and \( \dot{u} \) are obtained from the velocity record.

The coefficients of drag and inertia, \( C_D \) and \( C_I \), are then determined by finding the best fit of the theoretical force, predicted by the Morison equation, to the measured horizontal force.

It is anticipated that when the results of this Wallingford-HRS research project are released, a wealth of useful design information will become available.

8.19.3 Current-induced forces

A 13 m length of pipe, mounted on two suspension bars in 7 m of water in a tidal estuary, was used to study the response to currents of a pipe, not supported along its length, but spanning freely between points on an uneven sea bed.

It was found that the pipe could vibrate under certain conditions. (Although not stated in the information pamphlet, these vibrations would depend highly on the degree of resonance achieved. The natural frequency of the pipe depends on, among other things, the spanning distance, its diameter and wall thickness, its fixity at the support points and its modulus of elasticity. The frequency of the variable force is related to the frequency of vortex shedding, which in steady flow, may be calculated with the aid of the Strouhal number).

Maximum allowable spans were determined for a range of heights above the sea bed.

8.20 SUMMARY OF THE INVESTIGATIONS REVIEWED IN PART II

Table (8-4) outlines and compares the various research programmes that have been considered.

Although all are somehow concerned with hydrodynamic forces only about half deal with hydrodynamic forces due to oscillatory flow around horizontal cylinders in contact with or in the vicinity of a solid plane boundary.
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**TABLE 8-4**

A BRIEF COMPARISON OF THE VARIOUS INVESTIGATIONS CONSIDERED IN PART II
Of these, few have produced (or published) enough information to enable the designer of a submarine pipeline to actually calculate wave forces.

The coefficient values suggested by Beckmann and Thibodeaux (1962) have been found to be too small. The results of Brater and Wallace (1972), Yamamoto, Nath and Slotta (1974) and Garrison, Gehrman and Perkinson (1975) are useful, but only when the period parameter is sufficiently small to ensure unseparated flow and thus negligible drag and lift forces. This situation is seldom encountered in practice unless the pipe diameter is very large, the wave period fairly short, the water depth quite large and the wave height rather small.

The apparent result is that two papers are of use to the pipeline designer: Grace (1973) and Grace and Nicinski (1976). Whereas the first is primarily based upon numerous laboratory experiments the second stems from force measurements done on a real pipe in the sea. Vertical forces can be calculated with the aid of the latter paper for relative clearances of $h/D = 0.2$ only.
PART III

A PRACTICAL DESIGN PROBLEM
The proposed Green Point outfall sewer is of local (Cape Town) interest and is here used to illustrate how wave forces on submarine pipelines may be calculated.

Present indications are that this pipeline will have an outside diameter of about 1 m and will extend to a point where the water depth is approximately 28 m. The sea bed is essentially hard and solid with occasional small outcrops of rock.

The pipeline is to be laid into the sea off Green Point which is not far from Granger Bay - the site of a proposed small boat harbour, for which wave climate data has been collected and analysed. Storm waves can impinge on Granger Bay as well as on Green Point from both the north-west and the south-west. The NW waves experience little refraction and arrive at both sites with wave heights of up to about 7 or 8 m and wave periods of about 12 to 14 seconds. The SW waves, with typical periods of 12 to 18 seconds, are refracted more and may have wave heights of approximately 6 m when they arrive. These waves are however more critical to the stability of the Green Point outfall than the NW waves because they act almost at right angles to the pipe axis - i.e. $\alpha = 90^\circ$; see Fig. (9-1).

For about the first 250 or 300 m of its length the sewer pipe is to be laid on the north-eastern side of an existing groin which accommodates a stormwater outfall. The groin will effectively act as a breakwater and prevent the nearshore length of the pipeline from experiencing the full fury of the SW waves. Consequently, it is estimated that the shallowest depth in which the pipeline will be subjected to the full effects of a design wave is about 12 m; see Fig. (9-2).
TO DIFFUSER (WHICH WILL BE SITUATED ABOUT 1600 OR 1700 M FROM THE SEA WALL).

APPROX. LINE OF PROPOSED GREEN POINT SEWER OUTFALL

APPROX. POSITION OF EXISTING SEWER OUTFALL

EXISTING GROIN ACCOMMODATING OVERFLOW SEWER AND STORMWATER OUTFALL

GREEN POINT SEWERAGE PUMP STATION

GREEN POINT LIGHT HOUSE

SEAWALL

TABLE BAY

TO GRANGER BAY

BEACH ROAD

FIGURE 9-1 SKETCH PLAN OF THE PROPOSED GREEN POINT SEWER OUTFALL SCALE 1:5000
Processes of shoaling and refraction alter the wave height as the wave moves shorewards. These minor variations have however been disregarded; for the purpose of demonstrating how wave forces may be calculated it has been assumed that 6 m high waves with periods between 12 and 18 seconds and with crests parallel to the pipe axis, produce critical loadings.

Three water depths have been chosen for analysis: 12 m, 20 m and 28 m, and allowance has been made for a nominal clearance of 200 mm between the ocean bed and the underside of the pipe.

Wave forces have been calculated for the case $\alpha = 90^\circ$ and relative pipe clearance, $h/D \approx 0.2$, according to the suggestions in Grace (1973) and Grace and Nicinski (1976). Examples of the calculations are to be seen in Appendices A and B; vector envelopes of the resultant forces—consisting of total in-line forces and total transverse forces—are shown in Figs. (9-3) and (9-4). A line joining the origin with any point on the envelope represents the instantaneous magnitude and direction of the resultant force vector.

Whereas the force coefficients in Grace (1973) are assumed to be
Figures 9-3(a) and 9-3(b) Resultant force envelopes, according to Grace (1973)

\[ T = 12 \text{ s} \]
\[ H = 6 \text{ m} \]
\[ d = 12 \text{ m} \]
\[ D = 1 \text{ m} \]

\[ T = 18 \text{ s} \]
\[ H = 6 \text{ m} \]
\[ d = 12 \text{ m} \]
\[ D = 1 \text{ m} \]
Figures 9-3(c), 9-3(d), 9-3(e) and 9-3(f)

Resultant force envelopes, according to Grace (1973)
Figure 9-4(a) Resultant force envelope, according to Grace and Nicinski (1976)
Figure 9-4(b) Resultant force envelope, according to Grace and Nicinski (1976)
Figures 9-4(c), 9-4(d), 9-4(e) and 9-4(f) Resultant force envelopes, according to Grace and Nicinski (1976)
constant throughout the wave cycle, $C_D$ and $C_L$ vary over a wide range in Grace and Nicinski (1976); see Figs. (8-25) and (8-26). Both these coefficients are thought to achieve their maximum values when the relative travel distance of water particles, $s/D = 0$. Figs. (8-25) and (8-26), however, do not show the values of the coefficients for $s/D$ values below about 1.0 and 1.5, respectively.

Since the designer is primarily interested in the maximum horizontal and vertical forces, both of which occur at the instant that the wave crest passes over the pipe (i.e. when $s/D = 0$), this writer has assumed that the tentative design curves in the said Figs. can be extended linearly, yielding $C_D \approx 3.25$ and $C_L \approx 3.05$ at $s/D = 0$.

To indicate the range of $s/D$ (taken as $|s/D|$ between 0 and 1.5) where the curves have been extended, the force envelopes in Figs. (9-4 (a) to (f)) have been drawn in broken lines.

A second, but more conservative, assumption is that Figs. (8-25) and (8-26) are actually plots of $C_D$ and $C_L$, respectively, versus $|s/D|$ and not just $s/D$; in other words, that the coefficients display symmetrical behaviour before and after crest (and trough) phases.

The dissimilarity between the results obtained by Grace (1973) and Grace and Nicinski (1976) is rather startling. The first method produces oval-shaped envelopes, half above and half below the horizontal axis and the second method produces upturned mustache-shaped envelopes which are entirely above the horizontal axis.

The two most important reasons for the different envelope shapes are:

1. Grace (1973) assumes that the vertical force acts upwards during one half of the wave cycle and downwards the other half. Grace and Nicinski (1976), on the other hand, assume that the vertical force always acts upwards.

2. The 1973-paper implies constant-over-the-wave-cycle coefficient values whereas the 1976-paper allows $C_D$ and $C_L$ to achieve large peak values as $s/D$ approaches zero, which in turn lead to steeply peaked horizontal and vertical forces.
The question is: which set of envelopes is likely to be more correct?

The writer believes that Grace and Nicinski (1976) probably provides a more reliable prediction of the real forces. The grounds for this conclusion are the following:

1. The direction in which the instantaneous vertical force will presumably act, was considered.

Of all the papers considered in Part II, only two show curves of measured vertical force versus time for a horizontal cylinder in the vicinity of a solid plane boundary.

1.1 Yamamoto, Nath and Slotta (1974) contains a sample record of the instantaneous vertical (and horizontal) force; refer Fig. (9-5).

Figure 9-5 Sample record of instantaneous vertical and horizontal forces on a horizontal cylinder close to a plane boundary, Yamamoto, et al (1974)

By scaling the wave height and period from Fig. (9-5) and using the Airy theory it appears that the period parameter, in this case, is only about 0.73. Also Grace (1975) pointed out that K had been in the range 0.1 to 2 throughout these experiments.
It is also noted that \( D \) is rather large compared to the water depth and even to the wave length so that it does not nearly resemble a typical pipe-in-the-sea situation. In fact, the diameter is so large that one can hardly refer to one \( K \) value for the cylinder - \( K \) varies from 0.88 at the top of the cylinder to 0.68 at the bottom.

Clearly, there is virtually no separation of flow from the cylinder. It has generally been agreed that vortex shedding only starts when \( K > 3 \) to 5, or more; also refer to Table (3-1).

Thus it appears that the vertical forces measured by Yamamoto, et al (1974) did not include vortex shedding effects. As they have rightly explained, the vertical force consisted of a Bernoulli lift component and an inertial force component.

1.2 Grace and Nicinski (1976) produced Fig. (8-27) showing peak upward forces shortly before the crest and before the trough. In this case \( K \) was somewhere between 25 and 30 and vortex shedding effects were definitely significant, together with Bernoulli lift effects and inertial forces.

Although reference could be made to only these two vertical force traces, it seems that if the period parameter is so large that the flow separates from the cylinder, i.e. vortex shedding takes place, then peak, upward, vertical forces may be expected to occur shortly before (or perhaps simultaneously with) the passage of a crest as well as the passage of a trough. This helps to partially justify the fact that Grace and Nicinski (1976) predict upward vertical forces throughout the wave cycle.

2. The fact that the recommendations by Grace and Nicinski (1976) are based on prototype experiments performed in the ocean whilst Grace (1973) is based on the results of various
laboratory investigations, acts in favour of the 1976 paper. It is important to note that the prototype tests were done in K and $R_e$ ranges that are comparable to real-life design situations. Also the relative roughness of the pipe, namely about $2.5 \times 10^{-3}$, was fairly realistic.

3. It can be inferred that the 1976-paper contains more up to date ideas of Grace and his co-investigators than the earlier paper.

Two worthwhile observations can be made from Figs. (9-3) and (9-4):

The first is that the shallower the water the larger are the forces on the pipe. This is to be expected - almost intuitively. The reason is that (provided the force coefficients remain constant) the drag and lift forces are proportional to the square of the horizontal water particle velocity and the two inertial forces are proportional to the horizontal acceleration. Both the velocity and acceleration decay quite rapidly with increasing depth.

The second, less obvious, observation is that, according to the Grace (1973) model, the greatest forces do not necessarily occur under the long period waves that have the large $u_{\text{max}}$ values. There seem to be two reasons for this:

1. $C_D$ and $C_L$ are functions of $\beta$ and thus of $K$, refer Table (8-3). Long period waves have large $K$ values which tend to reduce $\beta$ and thus reduce $C_D$ and $C_L$.

2. Since $\dot{u} = 2\pi u/T$, it follows that the larger the period the smaller the acceleration, because $u$ increases at a slower rate than $T$ increases. The result is that the inertial forces become smaller as the wave period gets longer.
PART IV

CONCLUSIONS
10.1 FORCE COEFFICIENTS

10.1.1 Notes on the various force coefficients

It is very important to be aware of the following:

1. Coefficient values derived from steady flow, or even unsteady but uni-directional flow, should not be used in wave force calculations.

2. Coefficients that have been derived for vertical cylinders (such as piles remote from a wall), and also for horizontal cylinders away from a solid boundary, are not to be applied to a horizontal cylinder close to a boundary because the situations are quite different. Potential flow calculations (e.g. by Wilson and Reid (1963)) indicate that for a cylinder in contact with a boundary, $C_L = 4.48$, $C_I = 3.30$ and $C_D = 0$, and for a cylinder remote from a boundary $C_L = 0$, $C_I = 2.0$ and $C_D = 0$. (This observation has also been made by Grace (1971) and Grace (1971a)).

3. Another difference between the vertical cylinder and the horizontal cylinder is that the maximum particle velocities and accelerations vary along the length of a vertical cylinder whilst for a horizontal cylinder they may be assumed to be constant. The result is that coefficients derived for vertical cylinders often represent some weighted average values.

4. It must be ensured that the correct wave theory is used together with a certain set of coefficient values because such values have been derived for a specific wave theory only.
10.1.2 Some reasons for the scatter in coefficient values

The considerable scatter in values derived for the various coefficients is due to more than just experimental error. Other reasons include:

1. The extent to which the flow relative to the cylinder was one-, two- or three-dimensional would affect the formation and shedding of vortices, which, in turn, would affect the various hydrodynamic forces and thus their coefficients.

2. Not all researchers used the Airy wave theory to calculate particle kinematics for deriving the coefficients; some used other theories whilst some measured the actual particle velocities and accelerations.

3. Different researchers assumed the coefficients to be dependent on different parameters. On the whole, the period parameter, introduced by Keulegan and Carpenter (1958), has become recognized as of basic importance when concerned with oscillatory flow.

4. Yamamoto, et al (1974) pointed out that the drag force could perhaps be out of phase with the particle velocity, leading to incorrect estimates of $C_I$ if the assumption is made that the drag force is zero when the velocity is zero.

10.2 IMPORTANT PARAMETERS REGARDING WAVE FORCES ON CYLINDERS

Two parameters appear to be of importance: the Reynolds number, $R_e$, and especially the Keulegan-Carpenter period parameter, $K$.

$K$ has also been called a "wake parameter" because it is a measure of the extent to which the wave-induced flow resembles either steady flow or potential flow.

When $K$ is large, the wake pattern approaches that of steady flow; i.e. vortices are regularly shed on the downstream side of the cylinder, alternately from above and below. Consequently one may expect the coefficients of drag and lift to somehow approach those values that
would be applicable to steady flow conditions. In fact, Vongvisessomjai and Silvester (1976) found $C_D$ to become about 1.5 times its steady state value when $K > 25$. (They did not consider $C_L$, but found $C_I$ to become about 0.75 times its value as derived from potential flow theory).

Conversely, when $K$ is small, the water particles do not travel far enough to detach from the cylinder - the flow remains unseparated and resembles potential flow, so that one would expect the coefficients of inertia and lift to approach the values derived from potential flow considerations. The finding of Vongvisessomjai and Silvester (1976) was that $C_I$ became equal to the potential flow value when $K < 3$, or $\pi$, that means, when the total travel distance of the particles, $2A$, did not exceed the cylinder diameter.

For steady flow, $C_D$ is dependent on $R_e$, and numerous investigators have suspected that the coefficients in unsteady flow are also $R_e$ dependent but most have failed to find meaningful correlations between the coefficients and $R_e$.

Recent work by, amongst others, Garrison, Field and May (1977) has shown that the coefficients are, in fact, functions of both $K$ and $R_e$.

With regard to wave forces on submarine pipelines - as opposed to cylinders in general - the present writer suggests that the behaviour of force coefficients be examined in terms of $K$ and $R_e$ as well as the relative clearance parameter $h/D$. There are two reasons for this recommendation :-

Firstly, since the coefficient values for a cylinder close to a boundary undoubtedly differ from the values for a cylinder remote from a boundary, the results of studies belonging to the latter category (e.g. the numerous investigations into wave forces on piles, etc.) are not directly applicable to pipelines near the sea bed.

Secondly, it may be possible to reduce the apparent scatter in coefficient values presently available, by expressing the coefficients as functions of not just one parameter (such as either $K$ or $R_e$) but of both $K$ and $R_e$ for a specific $h/D$. 
It would be particularly useful if prototype test data, such as those of Grace and Nicinski (1976) and the recent Wallingford experiments, could be analysed in this way because these tests allow for high Reynolds numbers, real-life pipe roughnesses (with marine flora and fauna), and the proximity of a fairly uneven, natural sea bed.

10.3 RANGES OF APPLICABILITY OF VARIOUS METHODS TO CALCULATE IN-LINE FORCES

Fig. (10-1) suggests the ranges in which various methods appear to be valid.

\[ F_{\text{max}} = F_{\text{max}} \]
\[ F_{\text{max}} = F_{\text{max}} \left( 1 + \beta^2 \right) \]
where \( \beta = \frac{\pi^2 c_s}{2 c_p} \frac{1}{K} \)

Figure 10-1 Applicability ranges of various methods to calculate in-line forces

Vongvisessomjai and Silvester (1976) have also drawn attention to the fact that, if \( d/L < 0.3 \), better results are obtained for vertical cylinders (where the maximum particle velocities vary considerably along the length of the cylinder) by employing a method of dimensionless wave parameters, e.g. Tables 8-IV, 8-V and 8-VI in Silvester (1974).
10.4 COMMENTS ON TRANSVERSE FORCES

In comparison, less attention was initially paid to transverse forces due to wave action than to in-line forces. It has, however, been realised that the transverse force may contribute significantly to the resultant force on the body, and since it may attain frequencies high enough to match the natural frequency of the body, it may induce undesirable or even catastrophic oscillations. Consequently, more and more efforts have been made, particularly in the last decade, to understand and be able to predict the magnitudes and frequencies of transverse forces.

The phenomenon of vortex shedding and thus the behaviour of transverse forces are very complex and it is clear that a great deal still needs to be learnt before the designer of a submarine pipeline can predict the resultant hydrodynamic forces with confidence. This is borne out by the fact that much confusion and disagreement still exist amongst researchers regarding:

1. the magnitude of the peak lift force,
2. the direction of the lift force at any instant, and
3. the frequency of the lift force.

The rather unsatisfactory result is that, in order to assess the resultant force, the designer of today has to make conservative assumptions about these three characteristics of the lift force and then combine them with a conservative estimate of the in-line force.

In assessing the stability of a submarine pipeline it is essential that the vectorial combination of total horizontal force and lift force - or, better still - total horizontal force and total vertical force, be considered. The reason is twofold: not only may the resultant be much larger than the in-line force alone, but the ability of the pipeline to resist being displaced by horizontal forces may be reduced considerably when it experiences simultaneous vertical upward forces.

The lift force, \( P_L \), constitutes the greater part of the total vertical force on a pipeline near the sea bed. The remaining part is
either the vertical inertial force, $P_I$, as assumed by Yamamoto, et al (1974) and Garrison, et al (1975) – or it may be considered to be a vertical inertial force due to a horizontal acceleration, $P_W'$ as assumed by, for instance, Grace and Nicinski (1976).

$P_L$ generally has two causes: it is generated by the asymmetrical shedding of vortices and by the modification of streamlines due to the proximity of a solid boundary, i.e. the Bernoulli lift effect. The first causes the force to reverse its direction periodically whereas the second causes the force to be directed away from the boundary if the pipe is in contact with the boundary, and towards the boundary if even a small gap exists between the pipe and the boundary.

When $K$ is smaller than about 4, the part of the lift force due to vortex shedding can be regarded as negligible, but the part due to the proximity of the boundary still has to be taken into account.

Many researchers experimented with vertical cylinders (e.g. Bidde (1971), Isaacson and Maull (1976), Chakrabarti, et al (1976)) or horizontal cylinders remote from a boundary (e.g. Sarpkaya (1975), Garrison, et al (1977)) and thus studied only the vortex shedding effect.

Other researchers, such as Dalton and Helfstein (1971) and Yamamoto, et al (1974) considered only the Bernoulli effect resulting from a nearby boundary. The lift force, and thus $C_L$, is highly dependent upon the distance between the cylinder and the boundary, or equivalently, on the relative clearance $h/D$.

Grace and Nicinski (1976) produced a plot of $C_L$ values for $h/D \approx 0.2$, Fig. (8-26). This included both vortex shedding and Bernoulli effects. The dependence upon $h/D$ was, however, so great that they could not make any statement about the behaviour of the $C_L$ at other $h/D$ values.

Thus, for the time being, the best general information regarding $C_L$ for a pipeline near the sea bed can be obtained from Grace (1973).

It has been shown in the example, considered in Part III, that the vector envelopes of the resultant force as determined by Grace (1973) on the one hand, and Grace and Nicinski (1976) on the other hand, are however, strikingly different – mainly on account of the
fact that the former paper assumes that the transverse force acts upwards during half a wave cycle and downwards the other half, whereas the latter paper assumes that the transverse force always acts upwards.

To summarise, aspects of the transverse force that need further research are:

1. Values of the lift coefficient for pipes at various clearances above the sea bed.
2. The direction in which the lift force will act at any instant under certain conditions (i.e. at a specific relative clearance, h/D, and subject to certain K values).
3. The frequency of the lift force under various conditions, and its relationship with the frequency of vortex shedding and with the wave frequency.
4. The contribution of a secondary term, be it $P_I$ or $P_W$, to the total vertical force needs further investigation.

Regarding point 4: R.A. Grace supported the existence of $P_W$, which may appear more strange than $P_I$. But since not only the water particle velocities but also the accelerations are much greater in the horizontal direction than in the vertical direction, this writer recommends that the matter be given further thought—perhaps along the lines of reasoning set out in Section 4.2.

10.5 USEFUL INFORMATION FOR CALCULATING WAVE FORCES ON SUBMARINE PIPELINES

The applicability of the results of the various investigations into wave forces on cylinders to the special case of a pipe near the sea bed needs to be considered before such results are used to calculate wave forces for reasons outlined earlier in this chapter.

From the material covered in Part II it appears to the writer that the papers most relevant to submarine pipelines, in order of preference, are:

1. Grace and Nicinski (1976)
2. Grace (1973)
3. Garrison, Gehrmann and Perkinson (1975) - unseparated flow model, for cases where the inertial force predominates.

4. Brater and Wallace (1972) - for cases where the inertial force predominates; \( \frac{D^2}{HL} \geq 0.02 \); in-line forces only.

The first paper, based on prototype tests, bears the most resemblance to real-life situations. Vertical forces can be calculated for the case where the relative clearance, \( h/D \approx 0.2 \) only whereas horizontal forces can be calculated for any \( h/D \).

The second paper, although partly superseded by the first, has the advantage that it covers a wider spectrum of design cases; it allows for any value of \( h/D \) and any orientation angle \( \alpha \).

Peak forces - both in-line and transverse - are larger when calculated with the aid of the first paper because it allows for variable values of \( C_D \) and \( C_L \) within the wave cycle whereas the second paper assumes the coefficients to have constant values throughout the cycle.

As mentioned, a point of direct conflict is that the vertical force, according to Grace and Nicinski (1976) is always directed upwards, whilst according to Grace (1973), it is directed upwards during half a cycle and downwards during the other half cycle. From the viewpoint of stability, the pipeline designer is however only interested in the upward force. The first paper, therefore, gives more conservative estimates of the peak forces.

The third and fourth papers become useful when the period parameter is sufficiently small so that the flow remains virtually unseparated, i.e. large diameter cylinders in deep water and subjected to short period waves of low wave height.
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GREEN POINT SEWER OUTFALL: CALCULATION OF WAVE FORCES
ACCORDING TO GRACE (1973).

CASE CONSIDERED:

\[
\begin{align*}
T &= 12 \text{ seconds} \\
H &= 6 \text{ m} \\
d &= 12 \text{ m} \\
D &= 1 \text{ m} \\
h/D &= 0.2 \\
\alpha &= 90^\circ
\end{align*}
\]

\[
\frac{d}{gT^2} = 0.0085
\]

FROM FIG. (1-6): \( \frac{d}{L} = 0.097 \)

\[
L = 124 \text{ m}
\]

\[
U_{\text{max}} = 2.43 \text{ m/s}; \quad U_{\text{max}} = 1.27 \text{ m/s}^2
\]

\[
K = 2.43 \times 12 = 29.11
\]

FIND THE APPROPRIATE FORCE COEFFICIENTS: (Iteration Procedure)

ROUND 1: Assume class 2B; Assume \( \beta = 0.25 \) : from Table (8.3) \( C_D = 4.37(0.23) + 0.03 = 1.78 \)

\( C_I = 3.5 \)

From FIG. (8-4) \( \lambda_I = 1.0 \) (for \( h/D = 0.2 \)) : \( C_D = 1.0 \times 1.78 = 1.78 \)

From FIG. (8-3) \( \lambda_I = 0.66 \) (for \( h/D = 0.2 \)) : \( C_I = 0.66 \times 3.5 = 2.31 \)

From Eq. (5-5) \( \beta = \frac{\pi^2 2.31 - 1}{2 \times 1.78} = 0.22 \) (\( \neq 0.25 \))

ROUND 2: Assume \( \beta = 0.23 \) : from Table (8.3) \( C_D = 4.37(0.23) + 0.03 = 1.70 \)

\( C_I = 3.5 \)

For \( h/D = 0.2 \) : \( C_D = 1.0 \times 1.70 = 1.70 \)

\( C_I = 0.66 \times 3.5 = 2.31 \)

Thus, Transverse force coefficients: from Table (8.3): \( C_l = 1.70 \); \( C_w = 5.0 \)

From FIG. (8-4) \( \lambda_I = 0.5 \) (for \( h/D = 0.2 \)) : \( C_L = 0.5 \times 1.70 = 0.85 \)

From FIG. (8-3) \( \lambda_W = 0.65 \) (for \( h/D = 0.2 \)) : \( C_W = 0.65 \times 5.0 = 3.25 \)

COMPUTE THE FORCES:

\[
F_D = F_{\text{max}} \cos \left( \beta \frac{\pi}{4} \right) \cos \left( \frac{\pi}{4} \frac{t}{T} \right)
\]

\[
F_L = F_{\text{max}} \left( -\sin \left( \beta \frac{\pi}{4} \right) \right)
\]

\[
P_W = P_{\text{max}} \left( -\sin \left( \frac{\pi}{4} \right) \right)
\]

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Symmetrical values for \( 0.5 < \frac{t}{T} < 1.0 \)
APPENDIX B

GREEN POINT SEWER OUTFALL: CALCULATION OF WAVE FORCES
ACCORDING TO GRACE AND NICINSKI (1976).

CASE CONSIDERED:

\[ T = 12 \text{ Seconds} \]
\[ H = 6 \text{ m} \]
\[ d = 12 \text{ m} \]
\[ D = 1 \text{ m} \]
\[ h/D = 0.2 \]
\[ \alpha = 90^\circ \]

\[ \frac{d}{\sqrt{T}} = 0.0085 \]

\. FROM FIG. (1-6) \: \frac{d}{L} = 0.097

\. \quad L = 124 \text{ m} \]
\[ u_{\text{max}} = 2.43 \text{ m/s} \]
\[ u_{\text{max}} = 1.27 \text{ m/s}^2 \]
\[ \frac{A}{D} = \frac{29.11}{2\pi} = 4.63 \]

IN-LINE FORCES:

\[ F_D = F_{D_{\text{max}}} \left| \cos 2\pi \frac{t}{T} \right| \cos 2\pi \frac{t}{T} \]
\[ = C_D \cdot 1.2 \cdot 1000 \cdot (2.43)^2 \left| \cos 2\pi \frac{t}{T} \right| \cos 2\pi \frac{t}{T} \left[ \frac{N}{m} \right] \]
\[ = C_D \cdot (2.952) \left| \cos 2\pi \frac{t}{T} \right| \cos 2\pi \frac{t}{T} \left[ \frac{KN}{m} \right] \]

\[ F_I = F_{I_{\text{max}}} (-\sin 2\pi \frac{t}{T}) \]
\[ = C_I \cdot 1000 \cdot \frac{2}{\pi} \left| (1.27) (-\sin 2\pi \frac{t}{T}) \right| \]
\[ = (2.563) (-\sin 2\pi \frac{t}{T}) \left[ \frac{KN}{m} \right] \]

\[ \text{where} \quad C_D \text{ depends on } s/D; \quad \text{refer to Fig. (8-2S)} \]
\[ \text{where} \quad C_I = 2.57 \text{ for } h/D = 0.2 \]

TRANSVERSE FORCES:

\[ P_L = P_{L_{\text{max}}} \left| \cos 2\pi \frac{t}{T} \right|^2 \]
\[ = C_L \cdot 1.2 \cdot 1000 \cdot (2.43)^2 \left( \cos 2\pi \frac{t}{T} \right)^2 \left[ \frac{N}{m} \right] \]
\[ = C_L \cdot (2.952) \left( \cos 2\pi \frac{t}{T} \right)^2 \left[ \frac{KN}{m} \right] \]

\[ P_W = P_{W_{\text{max}}} \left| \sin 2\pi \frac{t}{T} \right| \]
\[ = C_W \cdot 1000 \cdot \frac{2}{\pi} \left| (1.27) \sin 2\pi \frac{t}{T} \right| \]
\[ = (2.992) \sin 2\pi \frac{t}{T} \left[ \frac{KN}{m} \right] \]

\[ \text{where} \quad C_L = 3.0 \text{ for } h/D = 0.2 \]

\[ \text{and} \quad C_W = 3.0 \text{ for } h/D = 0.2 \]

\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline
\multicolumn{2}{|c|}{t/T} & \multicolumn{2}{|c|}{s/D} & \multicolumn{1}{|c|}{C_D} & \multicolumn{1}{|c|}{F_D} & \multicolumn{1}{|c|}{F_I} & \multicolumn{1}{|c|}{F} & \multicolumn{1}{|c|}{C_L} & \multicolumn{1}{|c|}{P_L} & \multicolumn{1}{|c|}{P_W} & \multicolumn{1}{|c|}{P} & \multicolumn{1}{|c|}{RESULTANT} \\
\hline
\hline
CREST & & \multicolumn{5}{|c|}{0} & 0 & 3.25 & 9.6 & 0 & 9.6 & 3.05 & 9.0 & 0 & 9.0 & 13.2 \ 
0,01 & 0,0 & 3.1 & 9.1 & -0.2 & 9.0 & 2.85 & 8.4 & 0.2 & 8.6 & 12.4 \ 
0,03 & 0,9 & 2.85 & 8.1 & -0.5 & 7.6 & 2.5 & 7.1 & 0.6 & 7.7 & 10,3 \ 
0,05 & 1.5 & 2.6 & 7.9 & -0.8 & 5.9 & 2.1 & 5.5 & 1.0 & 6.6 & 8.8 \ 
0,07 & 2.1 & 2.3 & 5.4 & -1.2 & 4.2 & 1.75 & 4.1 & 1.4 & 6.5 & 6.9 \ 
0,1 & 2.7 & 2.0 & 3.9 & -1.5 & 2.4 & 1.35 & 2.9 & 1.8 & 4.4 & 5.0 \ 
0,15 & 3.7 & 1.55 & 1.6 & -2.1 & -0.5 & 0.8 & 0.8 & 2.4 & 3.2 & 3.3 \ 
0,2 & 4.4 & 1.25 & 0.4 & -2.4 & -2.1 & 0.61 & 0.2 & 2.8 & 3.0 & 3.7 \ 
0,25 & 4,6 & 1.2 & 0 & -2.6 & -2.6 & 0.61 & 0 & 3.0 & 3.0 & 3.9 \ 
\hline
SWL & & \multicolumn{5}{|c|}{0} & 0 & 3.25 & 9.6 & 0 & 9.6 & 3.05 & 9.0 & 0 & 9.0 & 13.2 \ 
0,3 & 4.4 & 1.25 & 0 & -2.4 & -2.8 & 0.61 & 0.2 & 2.8 & 3.0 & 4.1 \ 
0,35 & 3.7 & 1.55 & 1.6 & -2.1 & -3.7 & 0.8 & 0.8 & 2.4 & 4.4 & 4.9 \ 
0,4 & 2.7 & 2.0 & -3.9 & -1.5 & -5.4 & 1.35 & 2.9 & 1.8 & 4.4 & 6.9 \ 
0,448 & 1.5 & 2.55 & -6.7 & -0.8 & -7.6 & 2.1 & 5.5 & 1.0 & 6.5 & 10,0 \ 
0,5 & 0 & 3.25 & -9.6 & 0 & -9.6 & 3.05 & 9.0 & 0 & 9.0 & 13.2 \ 
\hline
\end{tabular}

\[ \text{S\Y\M\E\T\R\I\A\L \VALUES \FOR \ 0.5 \leq t/T \leq 1.0 \]
APPENDIX C

COURSES COMPLETED IN PARTIAL FULFILMENT OF

THE REQUIREMENTS FOR THE DEGREE M.Sc. (ENG.)

AT THE UNIVERSITY OF CAPE TOWN

<table>
<thead>
<tr>
<th>COURSE</th>
<th>YEAR</th>
<th>CREDIT VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>CE 506 Properties of Concrete</td>
<td>1974</td>
<td>4</td>
</tr>
<tr>
<td>CE 504 Probability and Engineering Statistics</td>
<td>1974</td>
<td>4</td>
</tr>
<tr>
<td>CE 525 Coastal Engineering</td>
<td>1975</td>
<td>5</td>
</tr>
<tr>
<td>CE 511 Sediment Transportation</td>
<td>1976</td>
<td>5</td>
</tr>
<tr>
<td>CE 535 Engineering Economy</td>
<td>1977</td>
<td>3</td>
</tr>
<tr>
<td>CE 533 Bridge Engineering</td>
<td>1977</td>
<td>4</td>
</tr>
<tr>
<td>CE 526 Coastal Engineering Practice</td>
<td>1977</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Number of credits required = 40
1. Give formulae for:
   (a) the coefficient of variation;
   (b) the mean deviation about the mode.

2. Find the standard deviation of the data: 2; 6; 10.

3. In a particular experiment the result of 10 weighings showed 4 values between 20 and 25 g, 4 values between 25 and 30 g and 2 values between 30 and 35 g. What was the median weight?

4. An engineering firm has 100 electrical components in stock, 25 manufactured by process A and 75 manufactured by process B. Unknown to the firm, 13 of those manufactured by A are defective and 16 of those manufactured by B are defective.

   A component is chosen at random from the 100 components. What is the probability that this component is:
   (i) manufactured by B and defective?
   (ii) either manufactured by A or is a defective component?

5. An item of radar equipment has three critical components A, B, C. The frequency of defect for component A was found to be 5 per 100, for B to be 6 per 100, and for C to be 8 per 100. Estimate the probability that a given item of equipment is defective.

6. A biased coin which has twice the probability of falling heads as falling tails is tossed with two unbiased coins. What is the probability:
   (i) of at least two heads occurring?
   (ii) of no heads occurring?

7. At a telephone exchange the average number of calls passed per hour in the morning is 96 and the rate can be regarded as constant. Calculate the probability of:
   (i) exactly 3 calls in a period of 5 minutes;
   (ii) more than 3 calls in a period of 5 minutes.
Packets are filled automatically and on the average, 5 per cent are underweight. An inspector takes a batch of twelve collected randomly. What is the probability that he will find 25 per cent or more underweight?

The mean diameter of steel rods produced by a process is 2 cm and the standard deviation is 0.05 cm. Assuming the diameters are normally distributed, find the value such that only 5 per cent of the rods will have a diameter exceeding this value.

A sample of 11 lengths of plastic were tested and found to have a standard deviation of 35. A second sample of 9 lengths of plastic, treated by a different process, was tested and found to have a standard deviation of 20. Test whether the standard deviations differ significantly.

State the assumptions required for the use of the t-test for the difference between the means of two independent samples.

If one denotes by $y'$ the values of $y$ which are calculated by means of the equation of the regression line, what is the least squares criterion?
SECTION B

Answer these questions in the answer books provided.

1. A laboratory balance is used to weigh the same object 100 times. The values are given in the table below.

<table>
<thead>
<tr>
<th>Weight in g</th>
<th>Number of observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.55 - 4.65</td>
<td>10</td>
</tr>
<tr>
<td>4.65 - 4.75</td>
<td>20</td>
</tr>
<tr>
<td>4.75 - 4.85</td>
<td>40</td>
</tr>
<tr>
<td>4.85 - 4.95</td>
<td>15</td>
</tr>
<tr>
<td>4.95 - 5.05</td>
<td>10</td>
</tr>
</tbody>
</table>

(a) Using this data calculate:

(i) the mean,
(ii) the mode,
(iii) the median,
(iv) the variance and standard deviation,
(v) the coefficient of variation.

(b) By fitting a normal distribution to the data, find the expected frequencies in the first two class intervals.

2. (a) Derive the binomial distribution from first principles and hence derive the Poisson distribution from the binomial distribution.

(b) The probability of a light bulb failing during the first twelve hours of service is 0.0049. If 1000 light bulbs are installed, use the Poisson distribution to find the probability of exactly ten bulbs failing within the first twelve hours.

(c) A machine is known to produce piston rings of which 10 per cent are defective. Find the probability that in a random sample of 400 rings:

(i) at most 35 rings will be defective;
(ii) between 55 and 50 will be defective.

3. (a) Define with diagram a type I error, type II error and the power of a test.

(b) The outputs from two production plants A and B were measured on each of 5 days. The data was given as follows:

<table>
<thead>
<tr>
<th>Plant A</th>
<th>Plant B</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>2.2</td>
</tr>
<tr>
<td>1.7</td>
<td>2.0</td>
</tr>
<tr>
<td>2.6</td>
<td>2.7</td>
</tr>
<tr>
<td>1.7</td>
<td>1.7</td>
</tr>
<tr>
<td>2.0</td>
<td>2.9</td>
</tr>
</tbody>
</table>

Test whether ......

SECTION C

Answer this question in a separate answer book.

1. (a) In a set of 10 compressive tests on concrete cubes, two of the tests exceeded the capacity of the testing machine (12 MPa). The 8 definite test results were (in MPa):

11.9, 8.0, 8.8, 11.3, 10.7, 10.7, 9.9, 9.7.

For the set of 10 cubes, determine graphically the mean compressive stress and its standard deviation.

(b) On two succeeding days a set of data was obtained on the concentration of bacteria in the effluent from a sewage works. The two sets of ranked data are:

Set 1: 400 700 850 1300 1900
Set 2: 750 1200 1700 1300 2400 3100 3600 5000 6600 10000

The data is expected to be log-normally distributed.

3. (b) (Continued)

Test whether the output of Plant B is significantly higher than that of Plant A at the 5 per cent level of significance if:

(i) sample A was considered to be independent of sample B;
(ii) it was believed that the day on which the observation was made was a relevant factor, and the observations were considered to be paired.

4. An experiment was carried out to measure the resistance of wire from three sources by taking five samples from each source.

(a) Use analysis of variance to determine whether or not there is a significant difference between the resistance of the wire from the three sources.

<table>
<thead>
<tr>
<th>Source</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.2</td>
<td>8.5</td>
<td>8.7</td>
</tr>
<tr>
<td>2</td>
<td>7.3</td>
<td>8.6</td>
<td>8.6</td>
</tr>
<tr>
<td>3</td>
<td>7.4</td>
<td>8.0</td>
<td>8.6</td>
</tr>
<tr>
<td>4</td>
<td>7.9</td>
<td>8.7</td>
<td>8.7</td>
</tr>
<tr>
<td>5</td>
<td>7.7</td>
<td>8.7</td>
<td>8.8</td>
</tr>
</tbody>
</table>

(b) It is believed that the 3 samples for each source were taken on consecutive days and that the resistance increased each day due to an external factor. Explain how you would test this hypothesis for source A only.

3. (b) (Continued)
1. (b) (Continued)

(i) Determine (using graphical procedures) the log-mean, geometric-mean of each set of data.

(ii) Test if the log-means are significantly different at 95 per cent level of significance.

(iii) Briefly explain why you performed the test for significance (in (ii) above), on the differences of the log-means and not on the differences of the geometric and arithmetic-means.

(c) List the conditions which must prevail for (i) a normal, (ii) a log-normal, distribution to arise.
1. Explain and illustrate the following:

(a) the hydration of the mineral compounds constituting Portland cement with particular reference to their respective contributions to strength and heat of hydration.

(b) the structure of hardened cement paste, with particular reference to the different categories of water contained in the paste.

2. (a) Show why a cement paste having a W/C ratio < 0.36 (by mass) and continuously cured under water will never achieve 100% hydration.

(b) Three cement pastes made with 314 g cement and having W/C ratios of 0.4, 0.4 and 0.6 respectively are placed in stoppered test tubes.

(i) What is the maximum hydration that is possible for each of the respective pastes?

At maximum hydration of the 0.6 W/C ratio paste calculate:

(iii) the volume of gel formed,

(iv) the water in the capillary pores.

3. The results from a trial mix of 33 kg of water; 50 kg of cement; 140 kg dry sand and 370 kg dry stone are slump of 130 mm and a real mortar excess of 6%. It is assumed that the densities of water, cement, sand and stone are 1000, 3100, 2600 and 2800 kg/m³ respectively and that the dry stone contains 50% voids.

(a) What mix would you suggest if the slump and real mortar excess required are 150 mm and 5% respectively?

4. Discuss:

(a) factors which influence concrete strength, and

(b) the stress/strain relation of concrete in terms of crack initiation and crack propagation.

5. Discuss drying shrinkage and carbonation shrinkage of concrete.

/No. 14. Elive.......

/No. 18. Elive...
All Questions may be attempted  

**Time: 3 hours**

**Constants**

Sea water density = 1025 kg/m³

Sea water weight = 10 kN/m³

1. A swell of 10 second period with a deep water wave height of 3 m approaches a beach with the wave crests parallel to the shore. Trace the progress of this wave in shoaling water through to the breaker point including the following calculations:

(a) the wave length and wave celerity in deep water
(b) the water depth at which the wave begins to be affected by the presence of the sea bed,
(c) the wave length and wave celerity for water depths at 10 m intervals between d=30 m and d=10 m, and at 1 m intervals between d=10 m and d=1 m.
(d) the depth of water in which the wave breaks, the type of breaker, and the wave height at breaking. Ignore the effect of wave setup or down.
(e) sketch the effect of wave set up and down including an estimate of depths.
(f) estimate the wave heights in the surf zone.
(g) calculate the energy flow in W/m² in water depths of 10 m, 5 m, and 2 m.

2. A cylindrical pipe is laid on the sea bed across a harbour entrance in 10 m of water, the pipe diameter being 1 m and the axis of the pipe is parallel to the local wave crests. If the local wave length is 50 m, estimate the wave period, and find the peak magnitude of the celerity and acceleration force components per metre length of pipe. Estimate the peak resultant force in the inshore direction, and the timing of this in relation to the passage of a wave crest.

\[ H = 2H, C_D = 1.2, C_n = 2.5 \]

**3.**

(a) A storm at sea generates waves with a period range of 6 to 12 seconds. The resulting swell travels towards a harbour 400 km away. Estimate the time required for the longest waves to cover the intervening distance, assuming deep water throughout. Also estimate how much later the shortest waves will begin to arrive.

(b) A refraction diagram is constructed for a bay and the spacing between a particular pair of adjacent orthogonal doubles in travelling from deep water to the 10 m depth, the wave period being 7 seconds. Estimate the percentage change in wave height occurring between these zones on the assumption that no breaking waves are present between the zones.

(c) Suggest some of the requirements you would incorporate into a specification for armour blocks.

4. The overleaf page shows the plan view of three separate coastal structures on which oblique waves impinge. In each case indicate areas where you consider deposition or erosion will occur, and also estimate the shape of the breaker line once stable conditions are established.

5. There is a continuous dissipation of energy due to tidal movements of water over the earth’s surface, and in some instances useful power is abstracted from the sea in tidal power schemes. Suggest what effect this may have on the dynamics of the earth-moon system over very long periods of time.
This is an 'open book' examination. Scripts are to be collected at 5.30 p.m. on Thursday, 24th February 1977 and returned by 9.00 a.m. on Monday, 28th February, 1977. The attached affidavit is to be signed by each student on receipt of the examination script.

1. One criterion for determining whether suspensions will be settling or non-settling is the particle Reynolds number of 2.0. For sand of relative density $S_d = 2.65$, indicate into which category "average" particles of sand, of mean particle diameter $d = 100, 250, 1000$ or $2000$ $\mu$m at concentrations of 0, 10, 20 and 30% will fall. Tabulate your results and show how they can be presented graphically. Hence determine for each of the four concentrations, the particle size which designates the boundary between settling and non-settling suspensions.

(a) Compare the above results with those obtained by two other methods.

(b) Repeat the above procedure for coal of sphericity 0.7 and relative density as given by Fig. 3.11.

(c) Repeat the above procedure for iron ore assuming spherical particles and a relative density of 4.0. In this case include particles of size $d = 2.5$ mm. Assume $v = 1.14$ $\text{m}^2/\text{s}$

2. Carry out a feasibility study for transporting 10 million metric tonnes of iron ore of relative density 4.0, a distance of 600 km in a horizontal pipeline with a load factor of 95%. Assume the pipe roughness is constant at $k = 0.06$ $\text{mm}$ and the kinematic viscosity of water is $v = 1.14$ $\text{m}^2/\text{s}$.

In order to carry out the feasibility study, consider five alternative proposals.

(i) Assume that the ore is crushed to an average particle size of $d = 2.5$ mm and the mean drag coefficient can be taken as $C_D = 0.44$.

(*) Which flow regime would you consider as being feasible for transportation of this material and why?

(b) Assume that the delivered volumetric concentration is $C_d = 20\%$. Determine the limit deposit velocity according to Denard and show that the pipe diameter required to operate at the minimum energy loss is approximately 900 mm.

(c) What is the total power required per km? Compute this value as the average obtained by four methods.

(d) Compare the pipe diameter obtained above with that obtained by another method for determining the value of the limit deposit velocity.
2. (Continued)

(iii) The finer material can also be transported at a lower mean mixture velocity as a heterogeneous suspension in the 300 mm diameter pipe.

(a) What is the power requirement in this case? Assume that the heterogeneous mixture is transported at the minimum deposit velocity as determined from the Durand equation with the coefficient $P_{D} = 0.95$. Note that the delivered volumetric concentration will be greater than 20% in this case.

$C_{vD} = \frac{r_{p}}{r_{s}}^{2} \frac{1}{\mu_{p}}$

(iv) The material is ground further to give a non-Newtonian suspension at 20% volumetric concentration.

(a) What would the average size of particle have to be in this case?

(b) The rheological properties as determined by means of a capillary tube viscometer, 3 mm in diameter and 3 m long, are as follows:

<table>
<thead>
<tr>
<th>Mass Flow (g/s)</th>
<th>0.848</th>
<th>1.69</th>
<th>2.54</th>
<th>4.24</th>
<th>8.48</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure Drop (kPa)</td>
<td>4.0</td>
<td>7.2</td>
<td>10.8</td>
<td>16.3</td>
<td>36.0</td>
</tr>
</tbody>
</table>

Determine the power requirement for the same flow rate as in schemes (i) and (ii).

(v) Consider a pipe diameter of 500 mm with the average roughness size of 0.06 mm and for the same mixture flow rate and material concentration as in scheme (iv), determine the power requirement.

3. Coal of size $d_{50} = 225 \mu m$ was tested in pipelines test loops of 100 mm and 200 mm diameters at a concentration by weight of $C_{w} = 57\%$. The relative density of the coal is 1.5 and the kinematic viscosity of water $\nu = 1.14 \text{ mm}^2/\text{s}$. The following test results were obtained at $\rho = 1.75 \text{ g/ml} -$

<table>
<thead>
<tr>
<th>$D(\text{mm})$</th>
<th>100</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_{w}$</td>
<td>0.0457</td>
<td>0.0224</td>
</tr>
</tbody>
</table>

The pipes .......

3. (Continued)

The pipes were found to be hydraulically smooth.

$C_{vD} = \frac{r_{p}}{r_{s}}^{2} \frac{1}{\mu_{p}}$

Determine the head loss in a 300 mm diameter pipe at a mean mixture velocity of 1.75 m/s. Use three different methods of scaling and compare them.

4. Calculate the mixture head loss in units of clear fluid (water) for the coal described above, assuming a heterogeneous flow regime in a pseudo homogeneous mixture (i.e. the method of Wasp et al) in a 300 mm diameter pipe. Compare with the Durand equation.

Assume that the Wastman Green equation $\mu = (\mu_{0} + A) e^{\gamma_{v} r}$ applies with $\gamma_{v} = 1.14 \times 10^{-3} \text{ kg/m} \cdot \text{s}$

$A = 0.000064 \text{ kg/m} \cdot \text{s}$

$B = 4.29$. Ignore the effect of hindered settling of the particles.
1. Outline and discuss the rationale behind four basic methods of making economy studies. 

Indicate the relative advantages (if any) of each method. [15 marks]

2. Interest and annuity relationships are used to establish the equivalence between sums of money when time and interest rates are taken into consideration.

Solve the following with interest at 6% compounded annually.

(a) If R3 500 is deposited now, what uniform amount could be withdrawn at the end of each year for 15 years and have nothing left at the end of the 15th year?

(b) What present investment is necessary to secure a perpetual income of R2 000 a year?

(c) How much will be accumulated in a fund at the end of 21 years if R2 500 is invested now?

(d) What annual saving for 20 years must be expected to justify a present expenditure of R7 000? [12 marks]

3. Suppose a company can save R4 000 a year by replacing a manual process with a machine costing R12 000, but rejects this opportunity. Describe the resulting loss in money if the economic life of the machine is 10 years with R3 000 salvage and if the minimum required rate of return is 15%. On the other hand, describe the loss, if any, if the company approves the installation and the economic life proves to be 4 years with R3 000 salvage value. Neglect tax. [12 marks]

4. You have the opportunity of making one of two investments. The first will cost R1 000 now and R100 per year at the end of this and the next four years. The second will cost R700 now and R200 per year at the end of this and the next four years. All other factors are equal.
- At what rate of interest are the two equal?
- Over what range of interest rates will you favour the first?
- Discuss your result.  

[7 marks]

5. You have purchased a machine for R40 000 and negotiated a life of four years, with the Receiver. What is the present value of tax allowances if:

(a) The machine was a truck used for transporting personnel and equipment.

(b) The machine was a truck used for transporting material for road construction.

Motivate your answers. Use \( i = 15\% \)

6. The Plant Director, in working up the hourly charge out rate for a model D digger (which cost Rx to buy) includes the amount of \( \text{Ry} \) as a provision for replacement where

\[
L = \text{expected life of the present digger in years},
\]

\[
n = \text{number of hours worked in a year}.
\]

\[
\text{Ry} = \text{expected cost of the new generation Model D digger in } L \text{ years time}.
\]

Set out an argument to convince him that his formula for calculating the provision is not entirely correct. Propose an alternative formula and argue for its adoption.  

[7 marks]

7. A R39 000 investment in machinery directly used in the process of manufacture is proposed. It is anticipated that this investment will cause a reduction in net annual operating disbursements of R10 500 a year for 12 years. The investment will be depreciated for income tax purposes by the straight-line method assuming a 12 year life and zero salvage value. The forecast of zero salvage value is also to be used in the economy study. The effective tax rate is 55%. What is the prospective rates of return after income taxes?  

[12 marks]

8. Depreciation has been defined as that amount which must be invested annually for \( N \) years at \( i \% \) p.a. in order to create a fund in \( N \) years, which equals the first cost of the asset. A minimum required profit of \( i \times \text{first cost} \) is also defined in the annual worth method.

Show that the sum of depreciation and minimal required profit as defined above equals the capital recovery factor \((A/P, i\% N.)\)  

[10 marks]
9. A new machine can be acquired for R25 800. Its operating disbursements for the first year of operation are expected to be R16 500; thereafter they are expected to increase R692 a year as a result of deterioration. The analyst predicts that the proposed machine will be replaced in the future by "like" machines having the same first cost and operating disbursements as the proposed new machine. Replacements will be necessary because of increasing operating disbursements. The salvage values of all the new machines in any year \( t \) are expected to conform to the formula,

\[
L = \left[ \frac{(15 - t)(16 - t)}{15 \times 16} \right] (25 800)
\]

The present machine has a net salvage value today of R3 000, and this is expected to decrease R500 a year for every year that the machine is kept by the company. Its operating disbursements will be R22 728 for the coming year, and these are expected to increase R650 a year thereafter. The minimum required rate of return is 20%. Should the present machine be replaced? Neglect tax. 

[20 marks]
1. Write a brief critical review of the main ideas contained in the road traffic bridge loading specifications covered in this course and discuss in particular the effects of using simplified or equivalent loading systems. [20 marks]

2. Give a critical evaluation of the methods of analysis for hollow concrete slab bridges, considering different deck plan shapes and different void configurations. [20 marks]

3. For each of the four bridge sites on the attached sheets, select the most suitable type of bridge structure and construction method.

   Draw adequate sketches of the superstructure, substructure and foundations directly on these sheets and hand them in. State all assumptions clearly and describe the construction method adequately. List brief reasons for all the major decisions. [15 marks each]

3a/ . . . . .
3(a) A straight and level single railway line crossing a gorge of great scenic beauty 100 km inland from the coast.

rail level

1 km overall

2:1

1:6

Sound rock on surface

Max. flood level

ELEVATION

SCALE - 1:5000
3(b) A straight and level 2 m diameter steel water pipeline crossing an estuary on the coast. The water depth in the estuary is between 2 m and 6 m, with a 1 m tidal fluctuation. A minimum headroom of 10 m is required for yachting.
3(c) A straight and level single carriageway two-lane road with sidewalks, 12.0 m overall width, crossing an industrial estate situated in a 20 m deep valley.
A dual carriageway rural freeway crossing an existing RC irrigation canal on level ground in the Karoo. No piers are allowed in the canal and the canal may not be subjected to additional loads.
1. Answer all questions on the attached sheets, in the space provided. If additional space is required the answer is to be completed in an Examination Answer Book where the answer must be clearly numbered.

2. The attached plan shows the bathymetry of False Bay to M.S.L. Using this plan and annotating it if necessary, answer the following questions, stating all assumptions and sources of information.

   a) The time taken to develop a fully arisen sea
   b) The significant wave height: $H_s$
   c) The significant wave period: $T_s$
   d) The depth at which this wave would break

3. If a wave recorder of the 'Wave Rider' type were to generate a record with the following characteristics:
   - Record length = 340 seconds
   - Number of 'zero-up-crossings' = 43
   - Number of crests = 10k

   a) Calculate the zero crossing period
   b) Calculate the mean crest period
   c) Calculate the spectral width parameter
   d) What type of waves are these (i.e. swell, sea, mixed etc)
   e) Give your reason

4. A lake of area $4 \times 10^6$ m$^2$ is to be joined to the sea by a navigation channel with sides formed by vertical sheet piles driven into the sand bed. The tidal range is 1.8 m.

   a) What dimension would you recommend for the width of the channel assuming a bed depth of 2 m below M.S.L.? (4)
   b) Estimate the average outflow velocity (2)
   c) If the size and grading of the sand is typical of the Cape Flats at what velocity would you anticipate scour would commence? (1)

5. Two vertical aerial photographs of the coast taken at 12 seconds apart are mounted in a viewer. Two adjacent wave crests (A and B) approaching a shallow shoreline are examined. In the first photograph the distance between A and B is 127 m apart. In the second photograph the distances between A and B is 117 m apart. The second position of A is 84 m ahead of its position in the first photograph.

   a) Estimate the average wave celerity of crest A and of crest B during the twelve second interval (2)
   b) Assuming the water is effectively shallow, estimate the average water depth under each crest, and check that the assumption is valid. (3)
   c) Calculate the wave period for each crest (2)
   d) To what do you attribute the difference in period (2)
   e) Have these waves been generated locally, or at a considerable distance (1)

6. a) Explain the term 'spectral window' as applied to electromagnetic radiation in the region 0.2 to 20 micrometres (3)
   b) Explain why colour-false infrared film is particularly suitable for demarcating the tide line in an estuary (3)
   c) Explain what is meant by the term 'spectral signature' of a ground material such as sand or grass; and hence explain how a 'classification' of a set of multi-spectral images of a ground scene can be achieved (7)
The attached plan shows contours of the sea bed at the Strand, near Gordons Bay. It will be seen that rocks outcrop in many places and provide a relatively calm area which is considered to have some potential for a small craft harbour and in particular a boat ramp.

a) Outline briefly the investigations and work you would recommend to establish the feasibility of constructing a small craft harbour in this location (7)

b) Identify the personnel and equipment required to undertake each of the investigations outlined above in (a). Estimate the time, rates and hence the cost of undertaking this work. (7)

c) Draw on the plan provided the main features of a proposal to provide a small craft harbour at this location (10)

d) Identify the number of boats at moorings and in dryboat storage that can be accommodated (3)

e) Give a rough estimate of quantities for any harbour protection works (e.g. breakwaters) proposed. State any assumptions. (5)
1. The optimum orientation for the mooring of sailing craft is _________________________________ (1)

2. The economic advantage in providing locks in a tidal harbour is in respect of _________________________________ (1)

3. The optimum location for waterside fuelling facilities is _________________________________ because _________________________________ (2)

4. Minimum dredged depths in a small craft harbour are the sum of individual depths allowed for the following:

   - Typically __________________ m
   - __________________ m
   - __________________ m
   - __________________ m
   - __________________ m
   - __________________ m
   - __________________ m
   - __________________ m

   Use additional lines if required (3)

5. Detail in plan and dimension typical floating berthing to provide double occupancy for boats of length 8 m. Show the system of mooring proposed. (3)

6. Assuming an average overall cost of R100/m² of floating berth deck area, estimate the cost of providing this berthing per boat: __________________ m² @ R100/m² = __________________ (2)

6. Give a local example of a leeshore anchorage _________________________________ (1)

7. Why is a leeshore disadvantageous to a harbour _________________________________ (1)

8. Explain the significance of providing a turning basin in the harbour on the Buffalo River at East London _________________________________ (2)

9. Explain briefly how the position of a dredger may be ascertained by using a sextant _________________________________ (3)

10. Explain what is meant by the term 'controlling depth' of a harbour _________________________________ (2)
1.11 Sketch the main elements of a float type tide recorder such as is installed at Hout Bay.

1.12 Explain how a 'clinometer' is used for wave recording. Identify the main elements of the system.

1.13 Tabulate the advantages and disadvantages of a 'clinometer' system as compared with a 'wave rider' system of wave measurement.

<table>
<thead>
<tr>
<th>Clinometer</th>
<th>Wave Rider</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advantages</td>
<td>Disadvantages</td>
</tr>
</tbody>
</table>

1.14 Draw the approximate form of the wave orthogonals to reach A, B, C and D as they approach the coastline drawn in plan below. (Assume refraction without diffraction).

For the harbour entrance detailed below sketch:

a) the approximate form of 4 wave orthogonals as they enter the harbour
b) 3 wave crests
c) If the gap width is equal to 1 wave length draw in a dotted line the location of diffracted wave heights of one half the incident wave height. (Refer to Fig. 2-44 in CERC Shore Protection Manual)
5.

1.16
a) What is the significance of waves entering a harbour with a period equal to the fundamental period of oscillation of one of the basins?

b) Name two design features that may be incorporated in a harbour design to reduce reflection? (1) (2)

c) Sketch where you would site the two features described above in 16b) in the basin shown below

[Diagram of Basin and Entrance]

d) Sketch sectional elevations of the two features described above

1.17 Give an example of a situation in which it would be appropriate to commission:

a) A 3-dimensional hydraulic model

b) A 2-dimensional hydraulic model

c) A mathematical model

6.

1.18 Explain briefly the function of coastal sanddunes in maintaining the stability of a sandy coastline

1.19 Identify by means of annotated sketches the procedures involved in implementing the following stages which might occur in the construction of a jetty on a rock bed covered in a thin layer of sand.

a) Temporary staging

b) Airlift

c) Placement of precast bases

d) Placement of bearer piles

e) Placement of concrete underwater